Effect of Boundary Conditions on Uplift Pressure and Exit Gradients Due to Sub-Surface Flow Under Hydraulic Structures

A Dissertation
submitted in partial fulfilment
of the requirements for the Degree
of
MASTER OF ENGINEERING
in
WATER RESOURCES DEVELOPMENT

by
A. S. CHAWLA

1967

WATER RESOURCES DEVELOPMENT TRAINING CENTRE
UNIVERSITY OF ROORKEE,
ROORKEE.
(INDIA)

Certified that this dissertation entitled "Effect of loading conditions on uplift pressure and exit gradients due to cutoffs on flow under hydraulic structures" is a record of the bonafide work done by Sri [illegible] a post-graduate student in the course of Master of Engineering (Water Resources Development) in the sessions 1965-66 and 1966-67. He worked under our guidance on this dissertation from October 10, 1966 to October 10, 1967.

To the best of our knowledge this dissertation has not been published anywhere nor submitted for the award of any other degree.

(Dr. [illegible])
[illegible]
U.P. Irrigation Research Institute
Roorkee

(Virendra Singh)
Superintending Engineer (Design) III
Central Design Directorate U.P.
Lucknow.

ACKNOWLEDGEMENTS

The author expresses his sincerest thanks to Dr. S.P.Garg, Director, U.P. Irrigation Research Institute, Roorkee and Prof. Vijendra Singh, Superintending Engineer III (Designs), Central Design Directorate, I.D., U.P., Lucknow, external and internal guides respectively, for their able guidance in compilation of this dissertation.

The help and guidance received from Dr. N.C.Misra of Civil Engineering Department, University of Roorkee is sincerely appreciated.

The author is also grateful to the authorities of Computer Centre, Structural Engineering Research Centre, Roorkee for providing facilities for the use of IBM 1620 Computer.

INDEX

CHAPTER ONE

CHAPTER I

I N T R O D U C T I O N

1.1 HISTORICAL BACKGROUND :

A rational basis to the study of subsoil flow was, for the first time, given in 1856 when Darcy (8) gave his law governing ground water flow in alluvial and sedimentary formations. On the basis of Darcy's work, significant contribution to the subsoil hydraulics were made by Boussinesq(3), Dupit (9), Forchheimer (10) and Thein (20) in the later half of the nineteenth century and more recently by Dachler (7), Kozeny (17), Hamel (13), King and Slichter(20)

In case of hydraulic structure founded on permeable soil, a major force of uplift comes in. One of the first basis for the design of hydraulic structure built on pervious foundations was developed in India, as a result of investigations of Clibborn and Beresford (1). As a result of experiments by Clibborn the hydraulic gradient theory of design of weirs was developed by Cotley and Hughes.

A considerable amount of research on the flow of water through soils, with special regard to the Transmission constant (K) was carried out in 1899 by Slichter and King(20). The main object of this investigation, was to determine the amount of water percolating through various types of soils with a view to ascertain the extent and behaviour of ground waters of United States. It did not deal with the distribution of pressures across structures built on pervious soils.

In 1910 Bligh in his book on "Practical Design of Irrigation Works" enunciated the "Hydraulic Gradient Theory" which later came to be known as the "Bligh Creep Theory". Because of its simplicity, this theory found general acceptance. By a detailed analysis of failure of structures designed by Bligh Theory, Lane (10) introduced his emperical "weighted creep theory".

Pavlovsky, in 1922, introduced the technique of electrical analogy in the investigations of problems of flow of sub-soil water below hydraulic structures.

In 1929 Terzaghi(27) stated that failures occured by undermining if the hydraulic gradient at exit was in excess of what he called the "Floatation gradient". The concept of flow net was enunciated by Forchheimer(10) and Terzaghi(27). A mathematical treatment of the flow of water through permeable subsoil under dams was evolved by Weaver (20). In 1936 Khosla Bose and Taylor(16) published the method of "Independent variables" which gave a general solution to problem of uplift over various base profiles. These findings were supported by an exhaustive series of laboratory and prototype observations.

1.2 LAPLACE EQUATION AND ITS SOLUTION

Flow can be laminar or turbulent depending upon the Reynolds number of flow. Normally for consolidated sand, the maximum size of the pore spaces is approximately 0.08 cm (21), and for the velocity 0.1 cm/sec. and 0.15 cm/sec., the Reynolds number ranges from 0.8 to 0.78. For Reynolds number less than one it can be assumed that flow is laminar and most of the flows under structures founded on sand fall in this range.

It is well known (11, 19, 21) that flow according to Darcy's Law satisfies the Laplace Equation.

$$V_x = -K \frac{\partial \phi}{\partial x} \qquad \text{...... (1.1)}$$

$$V_y = -K \frac{\partial \phi}{\partial y} \qquad \text{...... (1.2)}$$

and

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad \text{...... (1.3)}$$

where

V_x = Velocity in x direction

V_y = velocity in y-direction

ϕ = potential function

ψ = stream function

K = coefficient of permeability

Laplace Equation is satisfied by the conjugate harmonic function ϕ and ψ and that $\phi(x, y)$ = constant and $\psi(x, y)$ = constant are orthogonal curves.

It is seen that a variety of real solution to the Laplace equation can be obtained by taking various functions of z, by separating out real and imaginary parts ϕ and ψ. Which solution to apply in a particular problem depends upon the boundary condition which that particular solution satisfies.

1.3 DIFFERENT METHODS FOR SOLUTION OF LAPLACE'S EQUATION

Following methods are commonly employed for the solution of Laplace's equation.

(i) Graphical method

(ii) Hydraulic models

(iii) Electrical analogy models

(iv) Analytical methods.

1.3.1 The method of graphical field plotting is a widely used method, more popularly known as "Forchheimer solution". This method is confined to two dimensional cases and is capable of considerable accuracy. The process is begun on a scale drawing of boundaries with assigned potentials by plotting any suitable number of intermediate equipotential lines. Flux lines or stream lines are then drawn to cut the equipotential lines orthogonally and to form curvilinear squares. The errors are corrected by systematic improvements as suggested by Taylor(31). Much experience and foresight are needed for correct representation of flow net by the trial and error method. The greatest advantage of this method is that it requires little equipment and gives quick results. It gives clear idea about the subsoil flow of water.

1.3.2 HYDRAULIC Models may be

(a) Viscous fluid models

(b) Sand models

Viscous fluid models are based on the principle that flow through two parallel plates placed at a very small distance apart follows the Laplace Equation. Such a model consists of a small tank constructed with two parallel sides of glass plates-placed a small distance apart. The model is fitted in and the fluid (usually water or glycerine) is made to flow under the model from the upstream to downstream side. When the flow becomes steady the stream lines can be traced by adding a coloured solution of the same fluid at points and marking or photographing the path of such lines. Then equipotential lines are drawn orthogonal to the stream lines.

In hydraulic scale model sand is filled between two parallel plates. The pressures are observed by means of piezometric pipes introduced at proper points. An ingenious method of tracing the stream lines was followed in Punjab. The flow lines were made visible through sand by injecting potassium dischromate and silver nitrate separately. As a result of chemical reaction red precipitate of silver chromate was produced along the stream lines which became clearly visible.

1.3.3 In the electrical analogy method, the analogy developed between the flow of fluid through porous media which follows Darcy's Law and flow of electricity through an electrolyte which follows Ohm's Law is used. Two dimensional as well as three dimensional problems can be solved by this technique.

The chief advantage of this method are that the laboratory equipment is cheap and handy and flow probelems can be analysed speedily and with reasonable degree of accuracy. Many problems which might have been difficult to solve, find an easy solution in electrical analogy method. Models of much smaller scales can be used with electrical analogy set up while capillary forces will vitiate the results if the scale is reduced very much in hydraulic models.

Thus amongst the various experimental method electrical analogy is the most accurate and quick.

1.3.4 Analytical Method.

Analytical method aims at the solution of Laplace equation mathematically which can be done by help of conformal transformations and conjugate functions. In this method boundary conditions of the problem are expressed by equations and solution obtained mathematically. The method gives exact solution of the problem, although the method becomes involved for complicated boundary conditions. Classical work in this field has been done by A.N.Khosla(10). N.Dover (20), N.N.Pavlovsky (22,23), N.E. Zhukovsky (30). The present work is an extension of Khosla's approach and so a brief review of Khosla's method will be pertinent.

1.4 KHOSLA'S METHOD

If the Laplacian equation could be integrated for a given set of boundary conditions, the mathematical solution of the flow net could be obtained for those conditions. This equation is amenable to mathematical solution for simple boundary condition but becomes too involved for complex boundary conditions. With the help of the method of "Independent variables", evolved by Khosla, an approximate but sufficiently precise results can be arrived at by splitting a complicated foundation profile into several elementary forms. Mathematical solutions for a number of such elementary forms were obtained by Khosla. The basic idea underlying this solution is that the effect of every additional sheet piling is concentrated mainly in its immediate neighbourhood and dies out gradually as the distance from the pile increases. At a certain given distance from the pile its effect may therefore be estimated with enough accuracy by an emperical formula.

1.8 Limitations of Khosla's pressure and Exit Gradient Diagrams.

The formulae and graphs given by Khosla are applicable for boundary condition when length of pervious reaches on the upstream and downstream are infinite and depth of pervious strata below the foundation of structure is also infinite. In lined canals the boundary conditions are different in that the canal bed is impervious except for downstream inverted filter length. If the lining is effective on the upstream of the structure no water can seep below its foundation and the structure will not be subjected to any uplift. The development of pressures depends upon the extent of assumed pervious bed or width of crack in the lining. The release of pressures downstream of hydraulic structures on lined canals can only be effected through a predetermined filter length. In addition to find cases where depth of pervious strata below the foundations of structure is limited. The formulae and curves given by Khosla are not applicable for such boundaries. Exact solutions for such boundaries are given in the subsequent chapters.

CHAPTER TWO

CHAPTER II

MATHEMATICAL ANALYSIS FOR DIFFERENT BOUNDARY CONDITIONS

2.1 STATEMENT OF THE PROBLEM

If a structure founded on permeable foundation is considered, the flow through pervious medium satisfies the two dimensional Laplace equation

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = 0 \qquad \ldots.(2.01)$$

where

H = the difference in upstream and downstream water levels.

In the present study two cases A and B with boundary conditions as described below have been considered for analysis.

A. FINITE PERVIOUS REACHES ON THE UPSTREAM AND DOWNSTREAM OF THE STRUCTURE FOUNDED ON INFINITE PERVIOUS FLOW ZONE (FIGURE 2.1a).

(i) Impervious floor length, b1 + b2.

(ii) Cut off depth below canal bed, d.

(iii) Length of upstream pervious bed, f1.

(iv) Length of downstream pervious bed or filter length, f2.

(v) Impervious bed HF and CH extending upto infinity on either side.

(vi) Depth of permeable subsoil extends upto infinity.

Referring to figure 3.1a, the foundation profile ABCD forms the inner boundary and represents stream line $\psi = 0$. The extreme impervious boundary FH and HG (both extending upto infinity) represents stream line $\psi = q$, where q is the total discharge per unit width infiltering through the foundation. The other two boundaries are AF at the upstream bed level along which potential $\phi = H$, representing the total head and at the downstream end DG, along which potential $\phi = 0$.

B. FINITE PERVIOUS REACHES ON THE UPSTREAM AND DOWNSTREAM OF THE STRUCTURE FOUNDED ON FINITE PERVIOUS FLOW ZONE (FIGURE 3.2a).

(i) Impervious floor length, $b_1 + b_2$

(ii) Cut off depth below canal bed, d.

(iii) Length of upstream pervious reach, f_1.

(iv) Length of downstream pervious reach, f_2.

(v) Impervious bed FG and HJ extending upto infinity on either side.

(vi) Depth of permeable subsoil, T.

(vii) Impermeable layer at a depth T, extending upto infinity on either side.

Referring to figure 3.2a, the foundation profile ABCDE forms the inner boundary and represents stream line $\psi = 0$. The extreme impermeable boundary FGHIJ represents stream line $\psi = q$, where q is the total discharge per unit width infiltering through the foundation. The other two boundaries are AF at the upstream bed level along which potential $\phi = H$, representing the total head and at the downstream end EJ, along which potential $\phi = 0$.

Further for the purpose of making the procedure simple, it is assumed that the coefficient of percolation $k = 1$ and permeable strata is homogeneous.

2.2 METHOD OF SOLUTION

Referring to figures 2.1 and 2.2 both the true profile of the hydraulic structure and the rectangular flow field represented in w-plane are transformed on to the semi-circle of infinite radius i.e. on the semi infinite ζ - plane. The ζ - plane serves as the link joining the two phases of analysis Z-plane is transformed into ζ-plane through an auxiliary t-plane.

Using Schwarz-Christoffel transformation, the area in Z-plane (see figure 2.1a) bounded by the boundaries mentioned above is transformed to the lower half of the t-plane which in its turn is transformed into the lower half of the ζ-plane. Thus

$$Z = f(t)$$

$$= f'(\zeta) \qquad \ldots\ldots(2.02)$$

where

$$Z = x + iy, \; t = r + is \; \text{ and } \; \zeta = \xi + i\eta$$

gives the solution of Laplace equation for the Z-plane.

The second operation in this process consists in transforming the rectangular flow field in the W-plane on to the same semi-infinite plane, giving the relation

$$W = F(\zeta) \qquad \ldots\ldots\ldots(2.03)$$

where $W = \phi + i\psi$

and $\zeta = F^{-1}(W)$

on combining the two operations (2.02) and (2.03) into one, we obtain

$$Z = f'(\zeta) = f' F^{-1}(t) \qquad \text{.....(2.04)}$$

and $\quad t = F(\zeta) = F f'^{-1}(z)$

The relation (2.04) hold good for curves defined by their equations, e.g. equipotentials and stream lines. Also for the arrangement of the rectangular flow field shown in Figures 2.1 d and 2.2 d, the potential function is linearly correlated with the path of filtration.

2.3 ANALYTICAL SOLUTION FOR THE BOUNDARY CONDITION 'A'

2.3.1 THEORETICAL SOLUTION

(a) First Operation

Referring to Figure 2.1 the profile of the hydraulic structure in Z-plane, is transformed on to the real axis of the t-plane with

$$\begin{aligned}
Z &= Z_A = -b_1 & t &= t_A = -L_1 \\
Z &= Z_B = +b_2 & t &= t_B = +L_2 \\
Z &= Z_C = +0 & t &= t_C = +\sigma \\
Z &= Z_D = id & t &= t_D = 0 \qquad \text{...(2.05)} \\
Z &= Z_E = -0 & t &= t_E = -\sigma \\
Z &= Z_F = -(b_1+\ell_1) & t &= t_F = -\gamma_1 \\
Z &= Z_G = +(b_2+\ell_2) & t &= t_G = +\gamma_2
\end{aligned}$$

Schwarz-Christoffel transformation that maps Z-plane on to the lower half of t-plane is given by

$$\frac{dZ}{dt} = A(t-t_1)^{\frac{\alpha_1}{\pi}-1} \, (t-t_2)^{\frac{\alpha_2}{\pi}-1} \, (t-t_3)^{\frac{\alpha_3}{\pi}} \, (t-t_4)^{\frac{\alpha_4}{\pi}-1}$$

In this case

$$\frac{dZ}{dt} = At\,(t-\sigma)^{-\frac{1}{2}}\,(t+\sigma)^{-\frac{1}{2}} \qquad \text{.......(2.06)}$$

On integration

$$Z = A\int_0^t \frac{t \cdot dt}{\sqrt{t^2-\sigma^2}} + B.$$

$$Z = A\sqrt{t^2-\sigma^2} + B$$

Constants A and B can be calculated with the help of corresponding known points in Z and t-planes.

At C and E

$$Z = 0 \quad \text{and} \quad t = \pm\sigma$$

$$B = 0$$

and at D,

$$Z = id \text{ and } t = 0$$

$$id = A\sqrt{-\sigma^2} = Ai\sigma$$

$$A = \frac{d}{\sigma}$$

Therefore the transformation equation becomes

$$Z = \frac{d}{\sigma}\sqrt{t^2-\sigma^2}$$

$$\text{or} \quad t = \pm\sigma\sqrt{1+\left(\frac{Z}{d}\right)^2} \qquad \text{.......(2.07)}$$

This gives

$$L_1 = \sigma\sqrt{1+\left(\frac{b_1}{d}\right)^2}$$

or $$t_B = \sigma\sqrt{1+\left(\frac{b_1}{d}\right)^2}$$

$$\gamma_1 = \sigma\sqrt{1+\left(\frac{b_1+2l}{d}\right)^2} \qquad ......(2.08)$$

$$\gamma_2 = \sigma\sqrt{1+\left(\frac{b_2+2l}{d}\right)^2}$$

Second operation can be made easy by transforming t-plane on to ζ-plane such that F and G are placed symmetrically along real axis at -1 and +1 respectively. Therefore t-plane is transformed on to ζ-plane such that

$$\zeta_A = -\beta_1$$

$$\zeta_B = +\beta_2 \qquad ......(2.09)$$

$$\zeta_F = -1$$

$$\zeta_G = +1$$

Transformation of t-plane to ζ-plane is given by

$$t = A'\zeta + B'$$

To find the constants consider the values of corresponding coordinates at F and G.

at F, $t = -\gamma_1$, $\zeta = -1$

at G, $t = +\gamma_2$, $\zeta = +1$

$$-\gamma_1 = B' - A'$$

$$+\gamma_2 = B' + A'$$

Solving these equations and substituting the values of A' and B', the transformation equation becomes

$$t = \frac{\gamma_1+\gamma_2}{2}\zeta + \frac{\gamma_2-\gamma_1}{2}$$

or $$\zeta = \left[t - \frac{\gamma_2-\gamma_1}{2}\right]\frac{2}{\gamma_1+\gamma_2} \qquad ...... (2.10)$$

Substituting value of t from equation (2.07), equation (2.10) becomes

$$\zeta = \left[\sigma\sqrt{1+\left(\frac{Z}{d}\right)^2} - \frac{\gamma_2-\gamma_1}{2}\right]\frac{2}{\gamma_1+\gamma_2}$$

Let

$$\frac{\gamma_1}{\sigma} = \Delta_1 = \sqrt{1+\left(\frac{b_1+f_1}{d}\right)^2}$$

$$\frac{\gamma_2}{\sigma} = \Delta_2 = \sqrt{1+\left(\frac{b_2+f_2}{d}\right)^2}$$

$$\frac{L_1}{\sigma} = \lambda_1 = \sqrt{1+\left(\frac{b_1}{d}\right)^2} \qquad (2.11)$$

$$\frac{L_2}{\sigma} = \lambda_2 = \sqrt{1+\left(\frac{b_2}{d}\right)^2}$$

$$\frac{t}{\sigma} = \lambda = \sqrt{1+\left(\frac{Z}{d}\right)^2}$$

Substituting equation (2.11) in equation (2.10),

$$\zeta = \left[\lambda - \frac{\Delta_2-\Delta_1}{2}\right]\cdot\frac{2}{\Delta_1+\Delta_2}$$

$$= 1 - \frac{2(\Delta_2-\lambda)}{\Delta_1+\Delta_2}$$

$$\beta_1 = 1 - \frac{2(\Delta_1-\lambda_1)}{\Delta_1+\Delta_2} \qquad (2.12)$$

and

$$\beta_2 = 1 - \frac{2(\Delta_2-\lambda_2)}{\Delta_1+\Delta_2}$$

are obtained

(II) Second Operation :

Referring to figure 2.1, the rectangular flow field can be transformed on to the same real axis of the ζ-plane noting that

$$\zeta_G > \zeta_B > \zeta_A > \zeta_F$$

Transformation formula is given by

$$\frac{dw}{d\zeta} = \frac{C}{\sqrt{(\zeta-\zeta_G)(\zeta-\zeta_B)(\zeta-\zeta_A)(\zeta-\zeta_F)}}$$

on integration

$$U = C\int_0^{\zeta} \frac{d\zeta}{\sqrt{(\zeta-\zeta_G)(\zeta-\zeta_B)(\zeta-\zeta_A)(\zeta-\zeta_F)}} + D \qquad ....(2.13)$$

Since the uplift pressures on the base of the structure and exit gradient at the downstream end of structure are to be evaluated the integration process is therefore confined to this Zone only.

The Zone for which the uplift pressures are to be determined corresponds to AB in ζ-plane. Pertaining to this zone

$$\zeta_G > \zeta_B > \zeta \geqslant \zeta_A > \zeta_F$$

and the integral equation becomes

$$U = C\int_0^{\zeta} \frac{d\zeta}{\sqrt{(-\zeta+\zeta_G)(-\zeta+\zeta_B)(\zeta-\zeta_A)(\zeta-\zeta_F)}} + D \qquad ....(2.14)$$

Now, substituting the characteristic coordinates of ζ-plane given by equation (2.17) in equation (2.10b) the corresponding values of U are determined. Thus for

$$\zeta = \zeta_B \quad (\text{at } B)$$

equation (2.10b) reduces to (Appendix I equations 8a, 8b, 8c, and 8d)

$$sn^2 U = 0$$

or $\quad U = 0$

for $\quad \zeta = \zeta_G \quad$ (at G)

$$sn^2 U = \infty$$

or $\quad U = iK$

for $\quad \zeta = \zeta_A \quad$ (at A)

$$sn^2 U = 1$$

$$U = K$$

and for $\quad \zeta = \zeta_F \quad$ (at F)

$$sn\, U = \sqrt{\frac{(\zeta_G - \zeta_A)(\zeta_F - \zeta_B)}{(\zeta_B - \zeta_A)(\zeta_F - \zeta_G)}} = 1/k$$

or $\quad U = K + iK'$

Thus the four coordinate of the U-plane are

$$\begin{array}{ll} \zeta = \zeta_G & U = iK' \\ \zeta = \zeta_B & U = 0 \\ \zeta = \zeta_A & U = K \\ \zeta = \zeta_F & U = K + iK' \end{array} \qquad \ldots\ldots(2.18)$$

in which K and K' are complete elliptic integrals of the first kind with modulus k and k' respectively. The which on integration, using a bilinear transformation, yields (refer to appendix II)

$$sn^2\frac{\mu}{c}W = \frac{(\zeta_G-\zeta_A)(\zeta-\zeta_B)}{(\zeta_B-\zeta_A)(\zeta-\zeta_G)} \qquad (2.15)$$

where

$$\mu = \frac{1}{2}\sqrt{\frac{(\zeta_B-\zeta_F)(\zeta_G-\zeta_A)}{1}}$$

and the modulus, k is given by

$$k = \sqrt{\frac{(\zeta_B-\zeta_A)(\zeta_G-\zeta_F)}{(\zeta_G-\zeta_A)(\zeta_B-\zeta_F)}}$$

from which the co-modulus defined by

$$k' = \sqrt{1-k^2} = \sqrt{\frac{(\zeta_A-\zeta_F)(\zeta_G-\zeta_B)}{(\zeta_G-\zeta_A)(\zeta_B-\zeta_F)}}$$

and from equation (2.10) it follows that

$$\zeta = \frac{\zeta_B(\zeta_G-\zeta_A)-\zeta_G(\zeta_B-\zeta_A)\,sn^2\frac{\mu}{c}W}{(\zeta_G-\zeta_A)-(\zeta_B-\zeta_A)\,sn^2\frac{\mu}{c}W} \qquad (2.15b)$$

Further to simplify the procedure, the constant C is assumed equal to μ. This will yield

$$\zeta = \frac{\zeta_B(\zeta_G-\zeta_A)-\zeta_G(\zeta_B-\zeta_A)\,sn^2W}{(\zeta_G-\zeta_A)-(\zeta_B-\zeta_A)\,sn^2W} \qquad (2.16)$$

$$\text{and } sn^2W = \frac{(\zeta_G-\zeta_A)(\zeta-\zeta_B)}{(\zeta_B-\zeta_A)(\zeta-\zeta_G)} \qquad (2.16b)$$

Referring to Figure 2.14 the characteristic coordinates of the transformation are

$$\begin{aligned} \zeta &= \zeta_G = +1 \\ \zeta &= \zeta_B = +\beta_2 \\ \zeta &= \zeta_A = -\beta_1 \\ \zeta &= \zeta_F = -1 \end{aligned} \qquad (2.17)$$

four values of U thus calculated determine the layout of the basic rectangular flow field for the second operation.

The values of characteristic coordinate given by the equation (2.17) when substituted in k and k' yield

$$k = \sqrt{\frac{2(\beta_1+\beta_2)}{(1+\beta_1)(1+\beta_2)}} \qquad \ldots\ldots(2.19)$$

$$k' = \sqrt{\frac{(1-\beta_1)(1-\beta_2)}{(1+\beta_1)(1+\beta_2)}}$$

Similarly the substitution of characteristic coordinate values given in equation (2.17) in equation (2.10b) yield

$$Sn^2 W = \frac{(1+\beta_1)(\xi-\beta_2)}{(\beta_1+\beta_2)(\xi-1)}$$

$$= \frac{(1+\beta_1)(\beta_2-\xi)}{(\beta_1+\beta_2)(1-\xi)}$$

$$\text{or} \quad Sn W = \sqrt{\frac{(1+\beta_1)(\beta_2-\xi)}{(\beta_1+\beta_2)(1-\xi)}} \qquad (2.20)$$

2.3.2 EVALUATION OF UPLIFT PRESSURES

Along the bottom line of the foundation

$$\psi' = 0$$

hence $U = \phi'$

therefore equation (2.20) is reduced to

$$Sn\,\phi' = \sqrt{\frac{(1+\beta_1)(\beta_2-\zeta)}{(\beta_1+\beta_2)(1-\zeta)}} \qquad ....(2.21)$$

The value of β_1 and β_2 are given by equation (2.12). The pressures decrease from maximum head, H at point A in the begining to 0 at point B in the end. The length of line ABCD in W-plane equals to complete integral of first kind(K) having its modulus equal to k (figure 4d).

Potential ϕ for H =100, is found from the relation

$$\phi = \frac{\phi}{K} \times 100$$

To evaluate the value of potential or pressure at any point along the floor the value of ζ is found from equation (2.12) corresponding to its location in Z-plane. Substituting the value of β_1, β_2 and ζ in equation (2.10) and (2.21), the values of modulus and $Sn\,\phi'$ are found. Using tables for Jacobian functions (20,14) the required potential corresponding to any point in Z-plane can be evaluated.

2.3.3 EXIT GRADIENT

In addition to the characteristic pressure intensities investigated in the last paragraph, it is also important to know the hydraulic gradient at the down stream end of the first percolation trajectory. According to the principle enunciated by Terzaghi in 1930 and discovered independently by Khosla a few years later, the stability of the granular soil depended on the limiting value of the hydraulic gradient at the upper surface of the granular material. Khosla describes this limiting gradient as the exit gradient. According to Terzaghi's theory of 1920 failure by piping could occur in two different manners

(a) Starting with the lifting and removal of the top particles of soil from the surface of the earthen bed of the downstream channel.

(b) Originating in the deeper layers of the granular material in the case of very steep pressure gradients of the water filtering through those lower layers were to be responsible for the local concentration of the uplift force, capable of lifting the superposed buoyant, weight of the upper strata.

In so far as failure of type (a) is concerned Khosla's exit gradient furnishes indeed the significant design criterion; but in order to ensure safety in regard to failure of type (b) gradients in the deeper soil levels should also be investigated. Since however, type(b) failure is supposed to be a less frequent occurance (10) there is enough reason to believe that Khosla's analysis of exit gradient covers the main part of the problem

Gradient at any point is given by

$$I = \frac{dh}{ds}$$

where h = head at any point along floor or cut off.

Therefore $\quad T = \frac{\Pi}{K} \cdot \frac{d\Omega'}{ds}$ ......(2.33)

where K = complete Elliptic Integral of first kind with modulus k

It is not possible to find out $\frac{d\Omega'}{ds}$ directly

$$\frac{d\Omega'}{ds} = \frac{d\Omega'}{d\zeta} \cdot \frac{d\zeta}{dZ} \cdot \frac{dZ}{ds} \qquad \text{......(2.29a)}$$

Now $\quad Z = x + iy$

$$dZ = dx + i\,dy$$

$$\frac{dZ}{ds} = \frac{dx}{ds} + i\frac{dy}{ds}$$

$$= \cos\theta + i\sin\theta$$

where θ = angle of stream line with horizontal

At the exit

$$\theta = 0^{\circ}$$

and therefore

$$\frac{dZ}{ds} = 1 \qquad \text{......(2.30)}$$

Again $\quad \frac{d\zeta}{dZ} = \frac{d\zeta}{dt} \cdot \frac{dt}{dZ}$

From equation (2.90)

$$\frac{dt}{dZ} = \frac{\sqrt{t^2 - \sigma^2}}{A_0 t}$$

Substituting the value of constant A_0 we get

$$\frac{dt}{dZ} = \frac{\sigma}{dt}\sqrt{t^2 - \sigma^2} \qquad \text{......(2.28)}$$

Differentiating Equation (2.10)

$$\frac{d\zeta}{dt} = \frac{2}{\gamma_1 + \gamma_2} \qquad \text{......(2.30)}$$

Combining equations (2.25) and (2.26)

$$\frac{d\zeta}{dz} = \frac{2\sigma\sqrt{t^2-\sigma^2}}{(\gamma_1+\gamma_2)\,d\,.\,t.}$$

$$= \frac{2}{\Delta_1+\Delta_2}\cdot\frac{1}{d}\cdot\frac{1}{t/\sigma}\cdot\sqrt{(t/\sigma)^2-1}$$

where Δ_1 and Δ_2 are defined in Equation (2.11)

$$\frac{t}{\sigma} = \sqrt{1+\left(\frac{z}{d}\right)^2} = \lambda$$

$$\frac{d\zeta}{dz} = \frac{2}{\Delta_1+\Delta_2}\cdot\frac{1}{d}\cdot\frac{\sqrt{\lambda^2-1}}{\lambda} \qquad \text{......(2.27)}$$

$$W = \phi' + i\psi'$$

along the foundation profile $\psi' = 0$

$$W = \phi'$$

$$\frac{dW}{d\zeta} = \frac{d\phi'}{d\zeta}$$

$$= \frac{C}{\sqrt{(\zeta-\zeta_G)(\zeta-\zeta_B)(\zeta-\zeta_A)(\zeta-\zeta_F)}}$$

where

$$C = \tfrac{1}{2}\sqrt{(\zeta_B-\zeta_F)(\zeta_G-\zeta_A)}$$

$$\frac{d\phi'}{d\zeta} = \frac{1}{2}\sqrt{\frac{(\zeta_B-\zeta_F)(\zeta_G-\zeta_A)}{(\zeta-\zeta_G)(\zeta-\zeta_B)(\zeta-\zeta_A)(\zeta-\zeta_F)}}$$

Substituting characteristic coordinates of transformation.

$$\frac{d\phi'}{d\zeta} = \frac{1}{2}\sqrt{\frac{(1+\beta_1)(1+\beta_2)}{(\zeta^2-1)(\zeta+\beta_1)(\zeta-\beta_2)}} \qquad \text{.....(2.28)}$$

From equation (2.12)

$$
\begin{aligned}
1+\beta_1 &= \frac{2(\Delta_2+\lambda_1)}{\Delta_1+\Delta_2} \\
1-\beta_2 &= \frac{2(\Delta_1+\lambda_2)}{\Delta_1+\Delta_2} \\
\zeta-1 &= \frac{2(\lambda-\Delta_2)}{\Delta_1+\Delta_2} \\
\zeta+1 &= \frac{2(\Delta_1+\lambda)}{\Delta_1+\Delta_2} \\
\zeta+\beta_1 &= \frac{2(\lambda+\lambda_1)}{\Delta_1+\Delta_2} \\
\zeta-\beta_2 &= \frac{2(\lambda-\lambda_2)}{\Delta_1+\Delta_2}
\end{aligned}
\qquad (2.29)
$$

Substituting equation (2.29) in equation (2.28) and simplifying

$$\frac{d\phi'}{d\zeta} = \frac{\Delta_1+\Delta_2}{4}\sqrt{\frac{(\Delta_2+\lambda_1)(\Delta_1+\lambda_2)}{(\lambda-\Delta_2)(\lambda+\Delta_1)(\lambda+\lambda_1)(\lambda-\lambda_2)}} \qquad (2.29a)$$

Combining equations (2.24), (2.27) and (2.29a) and substituting in (2.23), gives after simplification

Exit gradient $= G_E$

$$= \frac{Hi}{2Kd\,\lambda}\sqrt{\frac{(\Delta_2+\lambda_1)(\Delta_2+\lambda_2)(\lambda-1)(\lambda+1)}{(\lambda-\Delta_2)(\lambda+\Delta_1)(\lambda+\lambda_1)(\lambda-\lambda_2)}}$$

With sheet pile at the end of floor, at exit

$\lambda = 1.0$

and $\lambda_2 = 1.0$

$$G_E = \frac{Hi}{2kd\,\lambda}\sqrt{\frac{(\Delta_2+\lambda_1)(\Delta_1+1)(\lambda-1)(\lambda+1)}{(\lambda-\Delta_2)(\Delta_1+1)(\lambda+\lambda_1)(\lambda-1)}}$$

On simplification this equation reduces to

$$C_1 = \frac{H}{2Kd\lambda}\sqrt{\frac{2(\Delta_2+\lambda_1)}{(\Delta_2-\lambda)(1+\lambda_1)}} = \frac{H}{dK}\sqrt{\frac{\Delta_2+\lambda_1}{2(\Delta_2-1)(1+\lambda_1)}} \qquad (2.30)$$

where

$$\lambda_1 = \sqrt{1 + \left(\frac{b_1}{d}\right)^2}$$

$$\Delta_2 = \sqrt{1 + \left(\frac{b_2 + f_2}{d}\right)^2}$$

d = depth of cut off

K = complete elliptic integral of first kind for modulus $k = \sqrt{\frac{2(\beta_1+\beta_2)}{(1+\beta_1)(1+\beta_2)}}$.

H = Total head in feet of water.

2.4 ANALYTICAL SOLUTION FOR THE BOUNDARY CONDITION 'B'

2.4.1 THEORETICAL SOLUTION

(I) First Operation

Referring to figure 2.2 the profile of hydraulic structure in Z-plane is transformed on to the real x axis of the t-plane with

$$\begin{aligned}
Z &= Z_A = -b_1 & t &= t_A = -l_1 \\
Z &= Z_B = +b_2 & t &= t_B = +l_2 \\
Z &= Z_C = +0 & t &= t_C = +\sigma \\
Z &= Z_D = id & t &= t_D = 0 \\
Z &= Z_E = -0 & t &= t_E = -\sigma \\
Z &= Z_F = -(b_1 + f_1) & t &= t_F = -\gamma_1 \\
Z &= Z_H = +(b_2 + f_2) & t &= t_H = +\gamma_2 \\
Z &= Z_G = -\infty & t &= t_G = -1 \\
Z &= Z_J = +\infty & t &= t_J = +1 \\
Z &= Z_K = i\infty & t &= t_K = \pm\infty
\end{aligned} \qquad \ldots (2.31)$$

Chwarz.christoffel transformation that maps Z-plane on the lower half of t-plane is given by

$$\frac{dz}{dt} = A(t - t_1)^{\frac{\alpha_1}{\pi}-1} \; (t - t_2)^{\frac{\alpha_2}{\pi}-1} \; (t - t_3)^{\frac{\alpha_3}{\pi}-1} \; (t - t_4)^{\frac{\alpha_4}{\pi}-1}$$

In this case

$$\frac{dz}{dt} = A(\sigma - t)^{-\frac{1}{2}} \, (\sigma + t)^{-\frac{1}{2}} \, (1 - t)^{-1} \, (1 + t)^{-1} (t) \quad \ldots (2.32)$$

On integration

$$Z = A \int_0^t \frac{t \cdot dt}{(1-t^2)\sqrt{\sigma^2 - t^2}} + B.$$

Let

$$\sigma^2 - t^2 = \lambda^2$$

which gives $- t. \; dt = 2\lambda . d\lambda$

and $1 - t^2 = \lambda^2 - \sigma^2 + 1$

$$= \lambda^2 + \sigma'^2$$

where

$$\sigma'^2 = 1 - \sigma^2$$

$$Z = -A \int \frac{d\lambda}{\lambda^2 + \sigma'^2} + B.$$

$$= -\frac{A}{\sigma'} \cdot \tan^{-1}\frac{\lambda}{\sigma'} + B.$$

$$= -\frac{A}{\sqrt{1-\sigma^2}} \cdot \tan^{-1}\sqrt{\frac{\sigma^2 - t^2}{1-\sigma^2}} + B.$$

Constants A and B can be evaluated with the help of corresponding known points in Z and t-planes.

At C and E

$Z = 0 \; , \; t = \pm \sigma$

Substituting we get

$$B = 0$$

At H,

$$Z = iT \text{ and } t = \alpha$$

Substituting we get

$$iT = -\frac{A}{\sigma'} \tan^{-1}\alpha = -\frac{A\pi}{2\sigma'}$$

or $$A = -\frac{2iT\sigma'}{\pi}$$

The transformation equation therefore becomes

$$Z = \frac{2iT}{\pi} \tan^{-1}\frac{\sqrt{\sigma^2 - t^2}}{\sigma'} \qquad \text{.....(2.33)}$$

$$\text{or} \quad \frac{\sqrt{\sigma^2 - t^2}}{\sigma'} = \tan\left(\frac{\pi Z i}{2T}\right)$$

$$= -\tanh\left(\frac{\pi Z}{2T}\right)$$

$$\text{or} \quad t = \sqrt{\sigma^2 + \sigma'^2 \tanh^2\left(\frac{\pi Z}{2T}\right)} \qquad (2.33a)$$

To evaluate σ, consider D,

where $t = 0, \quad Z = id$

$$0 = \sqrt{\sigma^2 + \sigma'^2 \tanh\left(\frac{\pi i d}{2T}\right)}$$

$$\text{or} \quad \frac{\sigma}{\sigma'} = \tan\frac{\pi d}{2T}$$

$$\sigma = \sin\frac{\pi d}{2T}$$

and $$\sigma' = \cos\frac{\pi d}{2T} \qquad \text{.....(2.34)}$$

Equation (2.33a) can be written as

$$t = \cos\frac{\pi d}{2T}\sqrt{\tan^2\frac{\pi d}{2T} + \tanh^2\left(\frac{\pi Z}{2T}\right)} \qquad \text{.... (2.35)}$$

This gives

$$L_1 = \cos\frac{\pi d}{2T}\sqrt{\tan^2\frac{\pi d}{2T} + \tanh^2\frac{\pi b_1}{2T}}$$

$$L_2 = \cos\frac{\pi d}{2T}\sqrt{\tan^2\frac{\pi d}{2T} + \tanh^2\frac{\pi b_2}{2T}}$$

$$\gamma_1 = \cos\frac{\pi d}{2T}\sqrt{\tan^2\frac{\pi d}{2T} + \tanh^2\frac{\pi(b_1+f_1)}{2T}}$$

$$\gamma_2 = \cos\frac{\pi d}{2T}\sqrt{\tan^2\frac{\pi d}{2T} + \tanh^2\frac{\pi(b_2+f_2)}{2T}}$$

Second operation can be made easy by transforming t-plane on the ζ-plane such that F and M are placed symmetrically at -1 and +1 respectively. Therefore t-plane is transformed onto ζ-plane such that

$$t_A = -I_1 \qquad \zeta_A = -\beta_1$$
$$t_D = +I_2 \qquad \zeta_D = +\beta_2$$
$$t_F = -\gamma_1 \qquad \zeta_F = -1$$
$$t_M = +\gamma_2 \qquad \zeta_M = +1$$

Transformation of t-plane to ζ-plane is given by

$$t = A'\zeta + B'$$

To evaluate the constants consider the values of corresponding coordinates F and M

$$-\gamma_1 = B' - A'$$
$$+\gamma_2 = B' + A'$$

Solving these equations and substituting the values of A' and B', the transformation equation becomes.

$$t = \frac{\gamma_1+\gamma_2}{2}\zeta + \frac{\gamma_2-\gamma_1}{2}$$

or $$\zeta = \left[t - \frac{\gamma_2 - \gamma_1}{2}\right]\frac{2}{\gamma_1+\gamma_2} \tag{2.38}$$

let
$$\frac{L_1}{\sigma'} = D_1 = \sqrt{\tan^2\frac{\pi d}{2T} + \tanh^2\frac{\pi b_1}{2T}}$$

$$\frac{L_2}{\sigma'} = D_2 = \sqrt{\tan^2\frac{\pi d}{2T} + \tanh^2\frac{\pi b_2}{2T}}$$

$$\frac{\gamma_1}{\sigma'} = D_3 = \sqrt{\tan^2\frac{\pi d}{2T} + \tanh^2\frac{\pi(b_1+f_1)}{2T}} \tag{2.39}$$

$$\frac{\gamma_2}{\sigma'} = D_4 = \sqrt{\tan^2\frac{\pi d}{2T} + \tanh^2\frac{\pi(b_2+f_2)}{2T}}$$

and
$$\frac{t}{\sigma'} = \mu = \sqrt{\tan^2\frac{\pi d}{2T} + \tanh^2\frac{\pi Z}{2T}}$$

Combining equations (2.35) and (2.38) and substituting equations (2.39),

$$\zeta = 1 - \frac{2\,(D_4 - M)}{D_3 + D_4}$$

$$\beta_1 = 1 - \frac{2(D_3 - D_1)}{D_3 + D_4} \qquad \text{.....(2.40)}$$

$$\beta_2 = 1 - \frac{2\,(D_4 - D_2)}{D_3 + D_4}$$

(II) Second Operation

Referring to figure 2.2, the rectangular flow field can be transformed onto the same real axis of ζ-plane noting that

$$\zeta_M > \zeta_B > \zeta_A > \zeta_F$$

Transformation formula is given by

$$\frac{dw}{d\zeta} = \frac{c}{\sqrt{(\zeta-\zeta_M)(\zeta-\zeta_B)(\zeta-\zeta_A)(\zeta-\zeta_F)}} + D \tag{2.41}$$

Since the uplift pressures on the base of the structure and exit gradient at the downstream end of structure are to be evaluated the integration process is confined to this zone only.

The zone for which the uplift pressures, are to be determined corresponds to AB in ζ-plane. Pertaining to this zone

$$\zeta_M > \zeta_B \geq \zeta \geq \zeta_A > \zeta_F$$

and the integral equation becomes

$$W = C\int \frac{d\zeta}{\sqrt{(\zeta_M-\zeta)(\zeta_B-\zeta)(\zeta-\zeta_A)(\zeta-\zeta_F)}} \qquad ....(2.42)$$

which on integration using Bilinear transformation yields (refer to Appendix II)

$$sn^2 \frac{\mu}{C} W = \frac{(\zeta_M-\zeta_A)(\zeta-\zeta_B)}{(\zeta_B-\zeta_A)(\zeta-\zeta_M)} \qquad ....(2.43)$$

where

$$\mu = \frac{1}{2}\sqrt{(\zeta_B-\zeta_F)(\zeta_M-\zeta_A)}$$

and the modulus k is given by

$$k = \sqrt{\frac{(\zeta_B-\zeta_A)(\zeta_M-\zeta_F)}{(\zeta_M-\zeta_A)(\zeta_B-\zeta_F)}}$$

from which the co-modulus defined by

$$k' = \sqrt{1-k^2} = \sqrt{\frac{(\zeta_A-\zeta_F)(\zeta_M-\zeta_B)}{(\zeta_M-\zeta_A)(\zeta_B-\zeta_F)}}$$

and from equation (2.43) it follows that

$$\zeta = \frac{\zeta_B(\zeta_M-\zeta_A) - \zeta_M(\zeta_B-\zeta_A)\,Sn^2 \tfrac{\mu}{c} W}{(\zeta_M-\zeta_A) - (\zeta_B-\zeta_A)\,Sn^2 \tfrac{\mu}{c} W} \qquad \ldots\ldots(2.43b)$$

Further to simplify the procedure, we assume the constant C equal to μ. This will yield

$$\zeta = \frac{\zeta_B(\zeta_M-\zeta_A) - \zeta_M(\zeta_B-\zeta_A)\,Sn^2 W}{(\zeta_M-\zeta_A) - (\zeta_B-\zeta_A)\,Sn^2 W} \qquad \ldots\ldots(2.44a)$$

$$Sn^2 W = \frac{(\zeta_M-\zeta_A)(\zeta-\zeta_B)}{(\zeta_B-\zeta_A)(\zeta-\zeta_M)} \qquad \ldots\ldots(2.44b)$$

Referring to figure 2.2d, the characteristic coordinates of the transformation are

$$\begin{aligned} \zeta_M &= +1 \\ \zeta_B &= +\beta_2 \\ \zeta_A &= -\beta_1 \\ \zeta_F &= -1 \end{aligned} \qquad \ldots\ldots(2.45)$$

Now substituting the characteristic coordinates of ζ- plane given by equation (2.45) in equation (2.44b) the corresponding values of w are determined. Thus for

$\zeta = \zeta_B$ (at B)

equation (3.10b) reduces to (refer appendix I equations 8a, 8b, 8c and 8d)

$Sn^2 W = 0$

or $W = 0$

for $\zeta = \zeta_M$ (at M)

$Sn^2 W = \infty$

or $W = iK'$

for $\zeta = \zeta_A$ (at A)

$$\operatorname{sn}^2 U = 1$$

$$U = K$$

and for $\zeta = \zeta_F$ (at F)

$$\operatorname{sn} U = \sqrt{\frac{(\zeta_M - \zeta_A)(\zeta_F - \zeta_B)}{(\zeta_B - \zeta_A)(\zeta_F - \zeta_M)}} = 1/k$$

or $U = K + iK'$

Thus the four coordinates of the w-plane are

$$\begin{aligned} \zeta &= \zeta_M & U &= iK' \\ &= \zeta_B & U &= 0 \\ &= \zeta_A & U &= K \\ &= \zeta_F & U &= K + iK' \end{aligned}$$

in which K and K' are complete elliptic integrals of the first kind with modulus k and k' respectively. The four values of U thus calculated, determine the layout of the basic rectangular flow field for the second operation.

The value of characteristic coordinates given by the equation (2.45) when substituted in k and k' yield

$$k = \sqrt{\frac{2(\beta_1 + \beta_2)}{(1+\beta_1)(1+\beta_2)}}$$

$$k' = \sqrt{\frac{(1-\beta_1)(1-\beta_2)}{(1+\beta_1)(1+\beta_2)}} \qquad \ldots\ldots(2.47)$$

Similarly the substitution of characteristic coordinate values given in equation (2.45) in equation (2.44b) yields

$$sn^2\, U = \frac{(1+\beta_1)(\beta_2-\xi)}{(1+\beta_1)(1+\beta_2)}$$

or

$$sn\, U = \sqrt{\frac{(1+\beta_1)(\beta_2-\xi)}{(1+\beta_1)(1+\beta_2)}} \qquad \ldots\ldots(2.48)$$

2.4.3 EVALUATION OF UPLIFT PRESSURES

Along the bottom line of the foundation

$$\psi' = 0$$

and $U = \phi'$

therefore equation (2.48) is reduced to

$$sn\, \phi' = \sqrt{\frac{(1+\beta_1)(\beta_2-\xi)}{(1+\beta_1)(1+\beta_2)}} \qquad \ldots\ldots(2.49)$$

The value of β_1 and β_2 are given by equation (2.40). The pressures decrease from maximum head H at point A in the begining to 0 at point B in the end. The length of line ABDCB in w-plane equals to complete elliptic integral of first kind (K) having its modulus equal to k (figure 2.2d)

Potential for H = 1, is found from the relation

$$\phi = \frac{\phi'}{K} \qquad \ldots\ldots(2.50)$$

and in percentage

$$\phi^r = \frac{\phi'}{K} \times 100$$

To evaluate the value of potential or pressure at any point along the floor the value of ζ is found from equation (2.60) corresponding to its location in Z-plane. Substituting the value of β_1, β_2 and ζ in equations (2.47) and (2.49) the values of modulus and Sn are found. Using tables for Jacobian functions [14, 20] the required potential corresponding to any point in Z-plane can be evaluated.

2.4.3 EXIT GRADIENT

It has been indicated in para 2.3.3 that it is also important to know the hydraulic gradient at the down stream end of the first percolation trajectory.

Gradient at any point is given by

$$I = \frac{dh}{ds}$$

where h = head at any point

$$= \frac{H}{K} \times \phi'$$

Therefore $I = \frac{H}{K} \frac{d\phi'}{ds}$ ....(2.81)

where K = complete Elliptic integral of First kind with modulus k.

It is not possible to find out $\frac{d\phi'}{ds}$ directly therefore differentiation by stages is adopted as shown below

$$\frac{d\phi'}{ds} = \frac{d\phi'}{d\zeta} \cdot \frac{d\zeta}{dz} \cdot \frac{dz}{ds} \quad ......(2.81a)$$

$$Z = x + iy$$

or $$\frac{dz}{ds} = \frac{dx}{ds} + i \frac{dy}{ds}$$

$$= \cos\theta + i \sin\theta$$

where θ = angle of stream line with horizontal.
At the end

$$\theta = 90^{\circ}$$

and therefore

$$\frac{dn}{ds} = 1 \qquad \text{.....(2.52)}$$

Again

$$\frac{d\zeta}{dz} = \frac{d\zeta}{dt} \cdot \frac{dt}{dz}$$

From equation (2.38)

$$\frac{dt}{dz} = \frac{(1-t^2)(\sigma^2-t^2)^{\frac{1}{2}}}{At}$$

$$= -\frac{\pi(1-t^2)(\sigma^2-t^2)^{\frac{1}{2}}}{2\, l\, \gamma\sigma t} \qquad \text{.....(2.53)}$$

Differentiating equation (2.36)

$$\frac{d\zeta}{dt} = \frac{2}{\gamma_1+\gamma_2} \qquad \text{....(2.54)}$$

Combining equations (2.53) and (2.54), we obtain

$$\frac{d\zeta}{dz} = -\frac{2\pi}{\gamma_1+\gamma_2} \cdot \frac{(1-t^2)(\sigma^2-t^2)^{\frac{1}{2}}}{2\, l\, \gamma\sigma t}$$

$$= -\frac{\pi}{(D_3+D_4)} \cdot \frac{(1-\sigma'^2\mu^2)(\sigma^2-\sigma'^2\mu^2)^{\frac{1}{2}}}{i \cdot T \cdot \sigma' \cdot \sigma \cdot \mu} \qquad \text{.....(2.55)}$$

where D3, D4 and μ are defined in equation (2.30)

$$U = \phi' + i\psi'$$

along the foundation profile, $\psi' = 0$

$$U = \phi'$$

$$\frac{dU}{d\zeta} = \frac{d\phi'}{d\zeta} = \frac{C}{\sqrt{(\zeta-\zeta_M)(\zeta-\zeta_B)(\zeta-\zeta_A)(\zeta-\zeta_F)}}$$

where

$$C = \frac{1}{2}\sqrt{(\zeta_B - \zeta_F)(\zeta_M - \zeta_A)}$$

$$\frac{d\phi'}{d\zeta} = \frac{1}{2}\sqrt{\frac{(\zeta_B - \zeta_F)(\zeta_M - \zeta_A)}{(\zeta - \zeta_M)(\zeta - \zeta_B)(\zeta - \zeta_A)(\zeta - \zeta_F)}}$$

substituting characteristic coordinates of transformation

$$\frac{d\phi'}{d\zeta} = \frac{1}{2}\sqrt{\frac{(1+\beta_1)(1+\beta_2)}{(\zeta^2-1)(\zeta+\beta_1)(\zeta-\beta_2)}} \qquad \ldots.(2.56)$$

From equation (2.40)

$$1 + \beta_1 = \frac{2(D_4 + D_1)}{D_3 + D_4}$$

$$1 - \beta_2 = \frac{2(D_3 + D_2)}{D_3 + D_4}$$

$$\zeta - 1 = \frac{2(\mu - D_4)}{D_3 + D_4}$$

$$\zeta + 1 = \frac{2(D_3 + \mu)}{D_3 + D_4} \qquad \ldots..(2.57)$$

$$\zeta + \beta_1 = \frac{2(\mu + D_1)}{D_3 + D_4}$$

$$\zeta - \beta_2 = \frac{2(\mu - D_2)}{D_3 + D_4}$$

Substituting values from equals (2.57) in equation (2.56) and simplifying to obtain

$$\frac{d\phi}{d} = \frac{1}{2}(D_3 + D_4)\sqrt{\frac{(D_3 + D_1)(D_3 + D_2)}{(\mu - D_4)(\mu + D_3)(\mu + D_1)(\mu - D_2)}} \qquad \ldots(2.57a)$$

Combing equation (2.52), (2.53) and (2.57a) and substituting in equation (2.51) to get after simplification

Unit gradient $= G_3 = \dfrac{H\pi(1-\sigma'^2\mu^2)}{4KT\sigma'^3\mu}\sqrt{\dfrac{(D_4+D_1)(D_3+D_2)(\sigma^2-\sigma'^2\mu^2)}{(D_4+\mu)(D_3+\mu)(D_2-\mu)(D_1+\mu)}}$

with sheet pile at the end of floor, at exit

$$\mu = \frac{\sigma}{\sigma'}$$

$$D\,S = \frac{\sigma}{\sigma'}$$

$$G_E = \frac{H\,\pi}{4\,K}\sqrt{\frac{(D_4+D_1)\;2\,\sigma\cdot\sigma'}{(D_4+\mu)(D_1+\mu)}}$$

$$= \frac{H\,\pi}{4\,K}\sqrt{\frac{(D_4+D_1)\;2\,\cot(\pi d/2T)}{(D_4-\tan\frac{\pi d}{2T})(D_1+\tan\frac{\pi d}{2T})}}$$

where

H = Total head in feet of water

K = Complete elliptic integral of first kind for modulus $k = \sqrt{\frac{2(\beta_1+\beta_2)}{(1+\beta_1)(1+\beta_2)}}$

T = Depth of permeable soil strata

$$D_1 = \sqrt{\tan^2\frac{\pi d}{2T} + \tan h^2\left(\frac{\pi b_1}{2T}\right)}$$

$$D_4 = \sqrt{\tan^2\left(\frac{\pi d}{2T}\right) + \tan h^2\,\frac{\pi(b_2 + f_2)}{2T}}$$

2.8 Application of above Equations for computations

The equations as derived above can be used for computations of uplift pressures and exit gradients. These computations involve the use of elliptic functions which have to be read from tables (14,20). The computations thus becomes rather cumbersome. In order to facilitate the use of above equations for finding the uplift pressures and exit gradients at key points, values have been computed for different combinations of the variables involved and plotted in the form of curves. These curves can be directly used for practical design purposes. As will be seen interpolation of values is very easy and accurate and can be safely adopted for design of most of hydraulic structures governed by already mentioned boundary conditions.

These curves have been worked out and plotted for simple elementary profile consisting of a flat floor with a cut off at one end. In any actual problem of hydraulic structure, the foundation profile is more complicated. But it can be reduced to simple elementary profile and pressures at key points worked out. The values have to be corrected by the application of method of "independent variables".

The computation of various values and reading of elliptic function tables by linear interpolation for plotting the curves were carried out on an IBM 1620 digital Computer. Computations were carried out for boundary conditions as mentioned below and the curves obtained from each have been shown in figures against corresponding case.

Boundary condition	Figure no.
A. Structure founded on infinite depth of pervious flow zone	
i. uplift pressure at D	
(a) upstream pervious length = 1.0d	2.3
(b) upstream pervious length = 2.0d	2.4
(c) upstream pervious length = 3.0d	2.5
ii. uplift pressure at E	
(a) upstream pervious length = 1.0d	2.6
(b) upstream pervious length = 2.0d	2.7
(c) upstream pervious length = 3.0d	2.8

iii. Exit gradient

(a) upstream pervious length = 1.0d 2.9

(b) upstream pervious length = 2.0d 2.10

(c) upstream pervious length = 3.0d 2.11

D. Structure founded on finite pervious flow zone

i. uplift pressures at B

(a) upstream pervious length = 0.5T 2.12

(b) upstream pervious length = 1.0T 2.13

(c) upstream pervious length = 1.5T 2.14

ii. uplift pressures at D

(a) upstream pervious length = 0.5T 2.15

(b) upstream pervious length = 1.0T 2.16

(c) upstream pervious length = 1.5T 2.17

iii. Exit gradient

(a) upstream pervious length = 0.5T 2.18

(b) upstream pervious length = 1.0T 2.19

(c) upstream pervious length = 1.5T 2.20

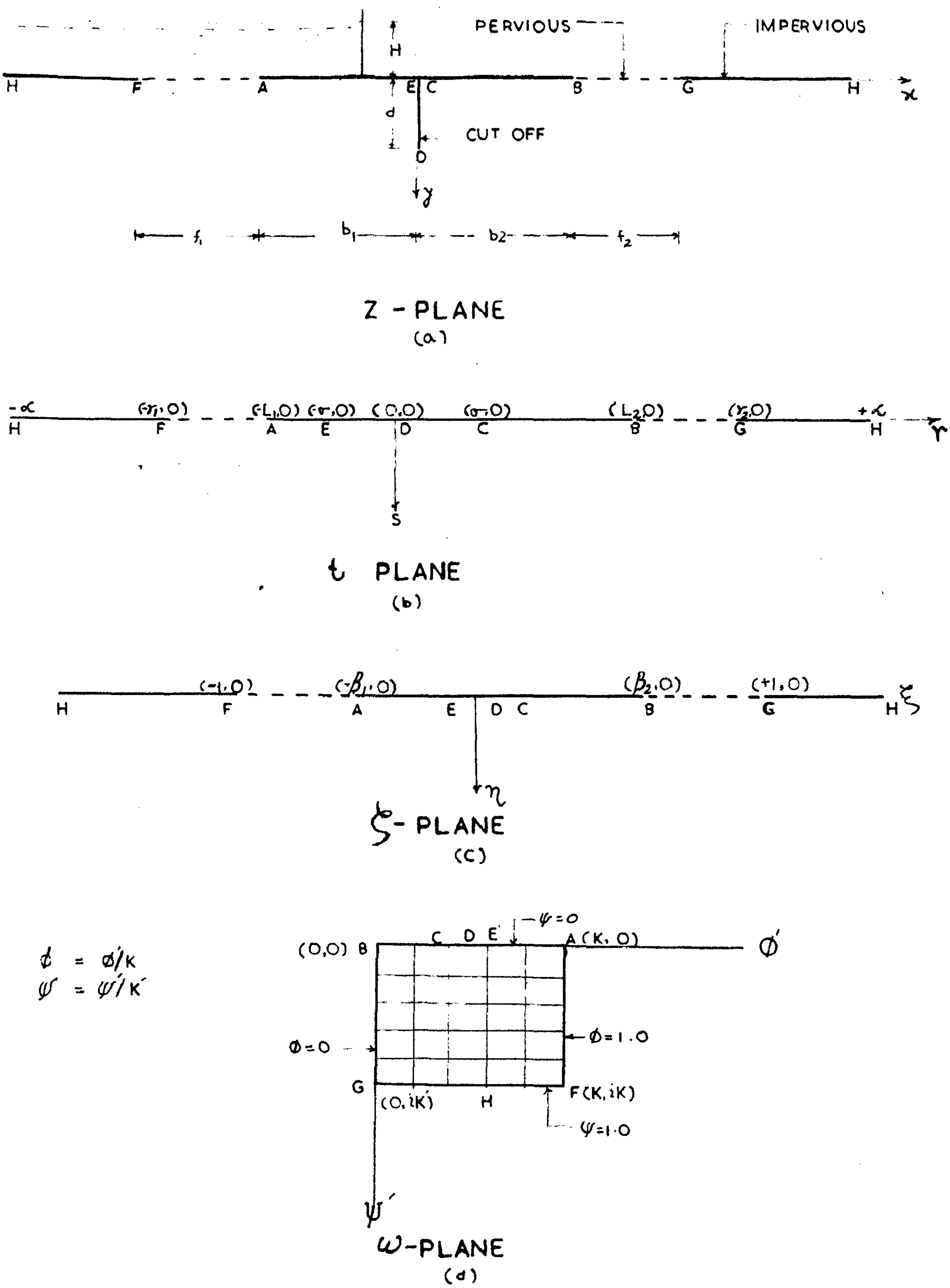

FIG. 2·1 SCHWARTZ-CHRISTOFFEL TRANSFORMATION

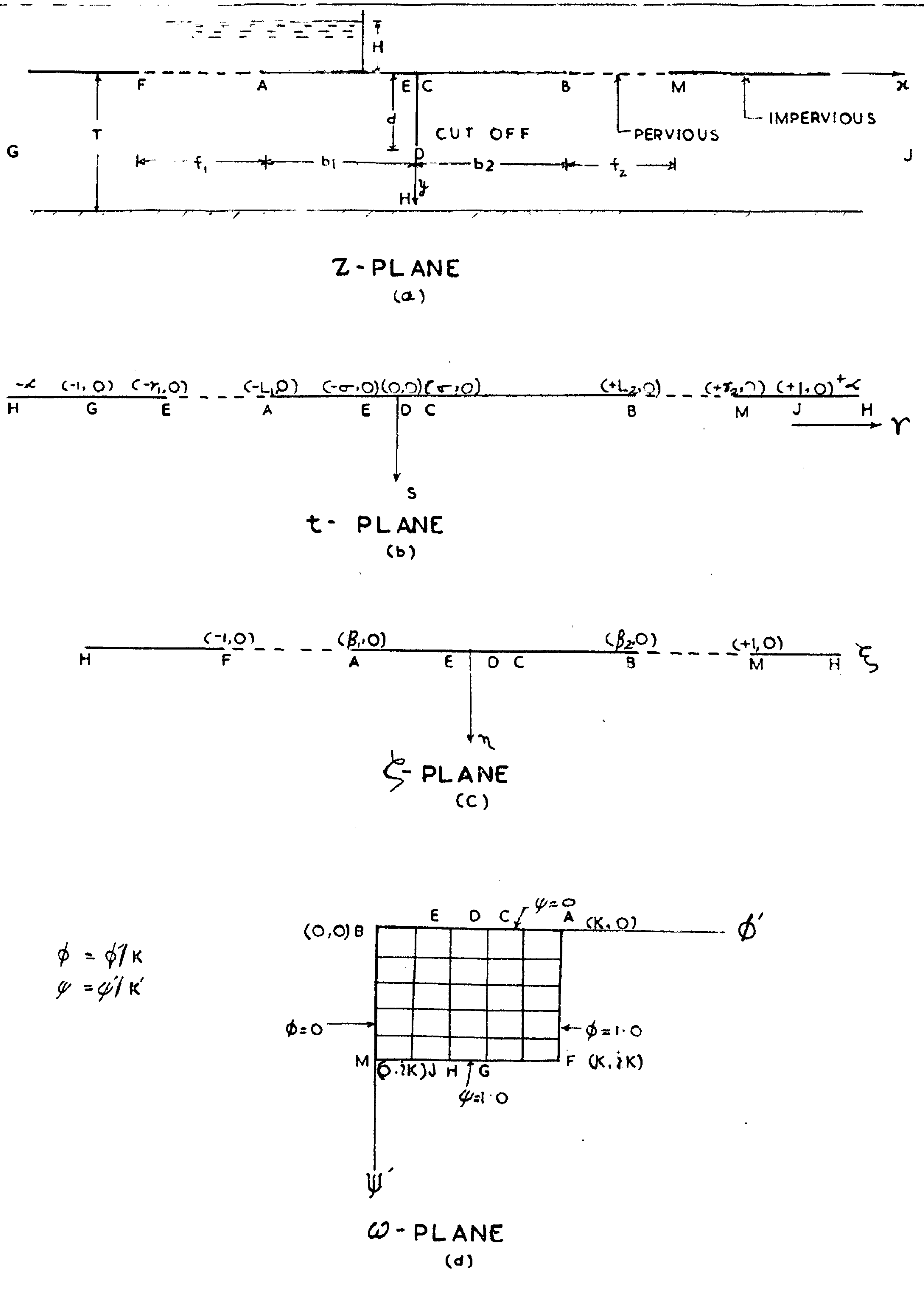

FIG. 2·2. SCHWARTZ–CHRISTOFFEL TRANSFORMATIONS

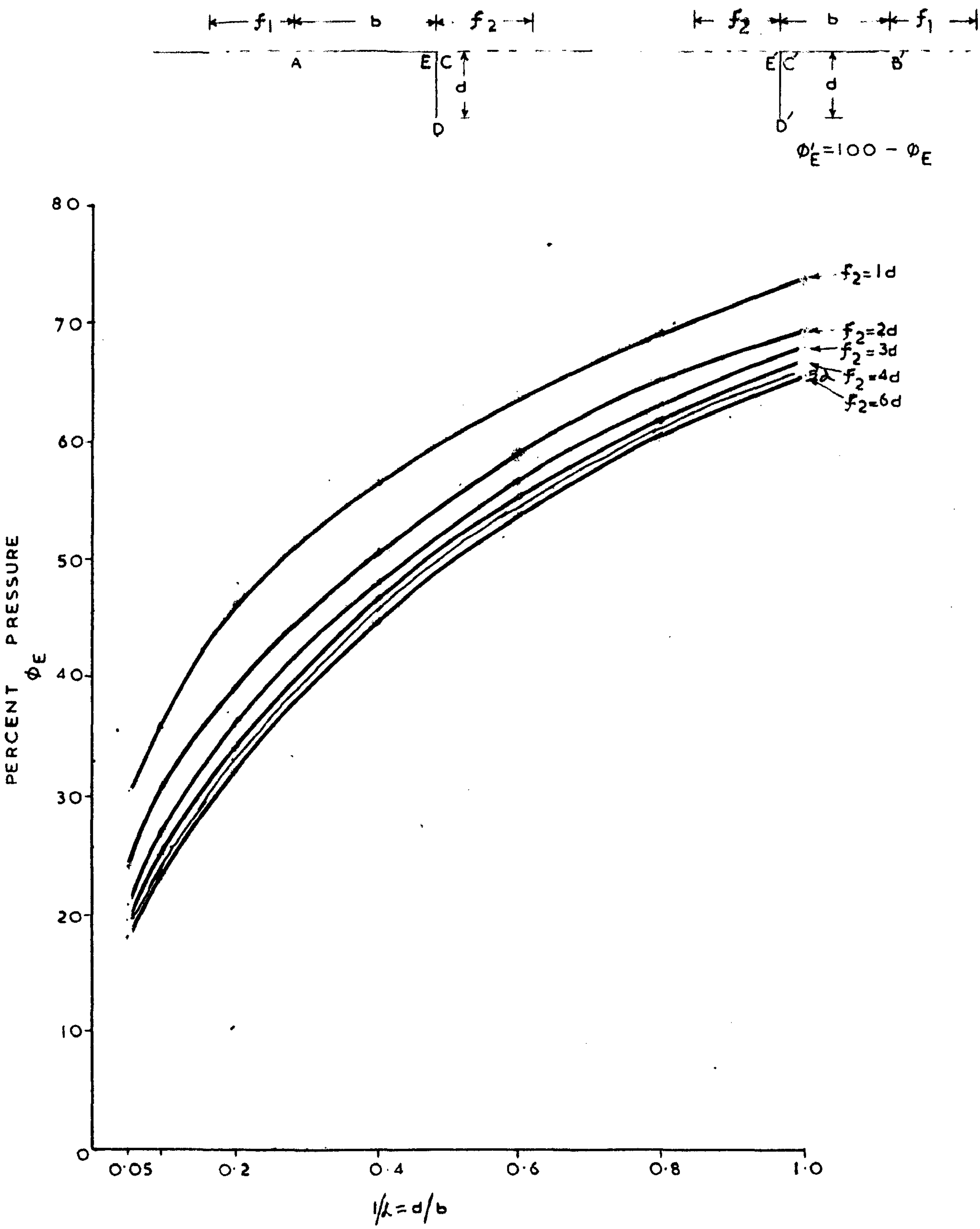

FIG. 2.3 PERCENT PRESSURE AT E FOR $f_1 = 1.0d$.

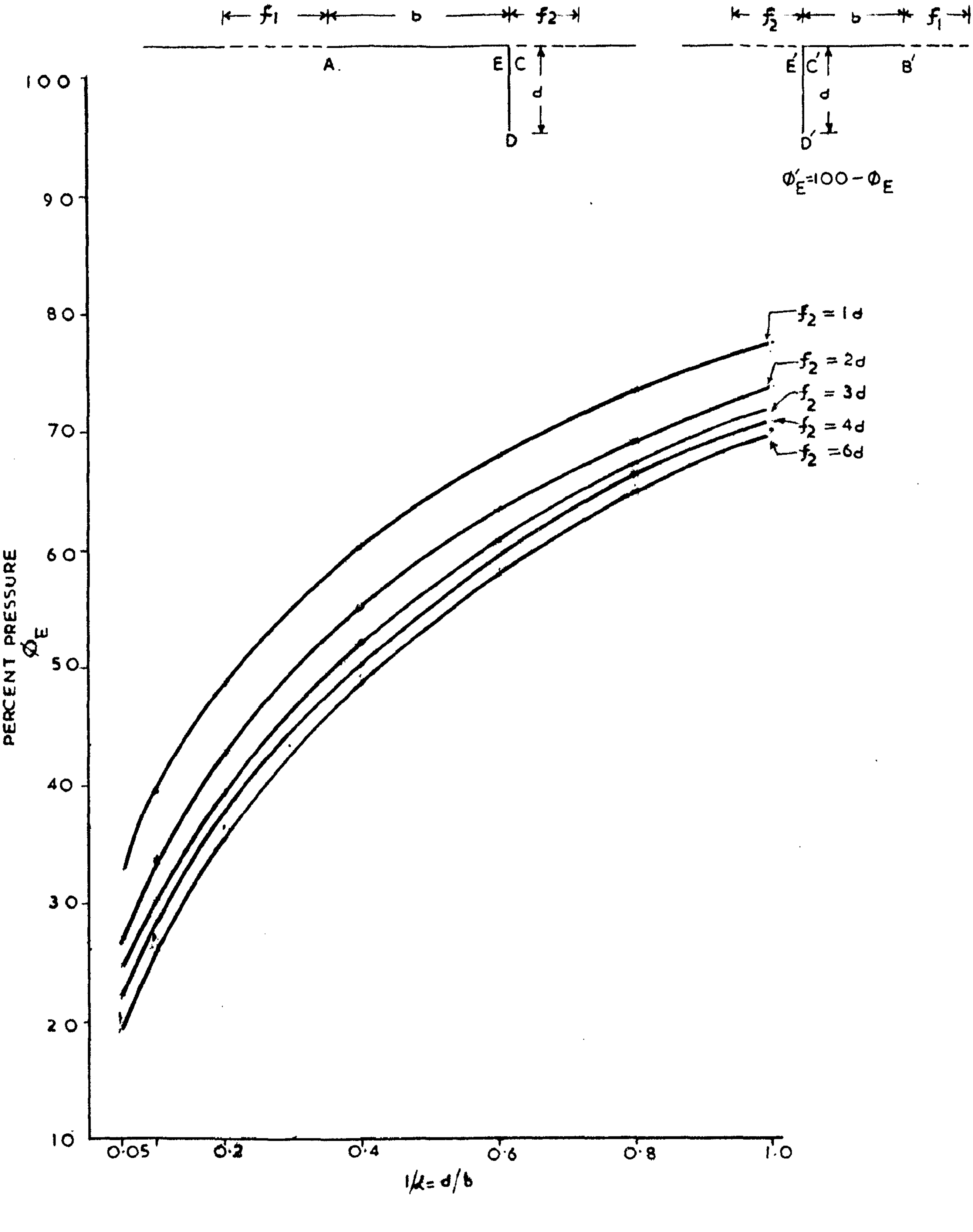

FIG. 2·4. PERCENT PRESSURE AT E FOR f_1 = 2·0 d.

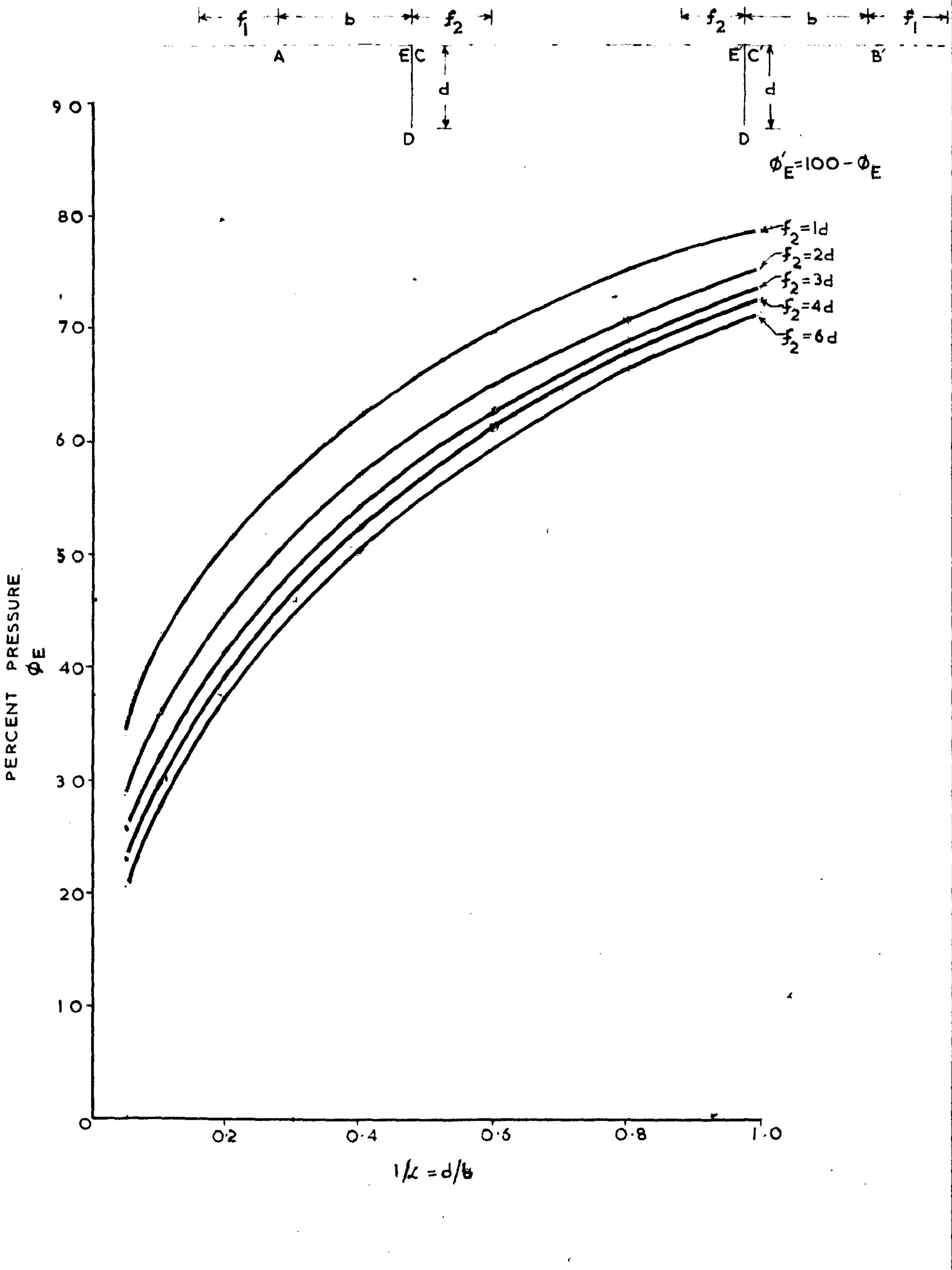

FIG. 2·5. PERCENT PRESSURE AT E FOR f_1 = 3·0 d.

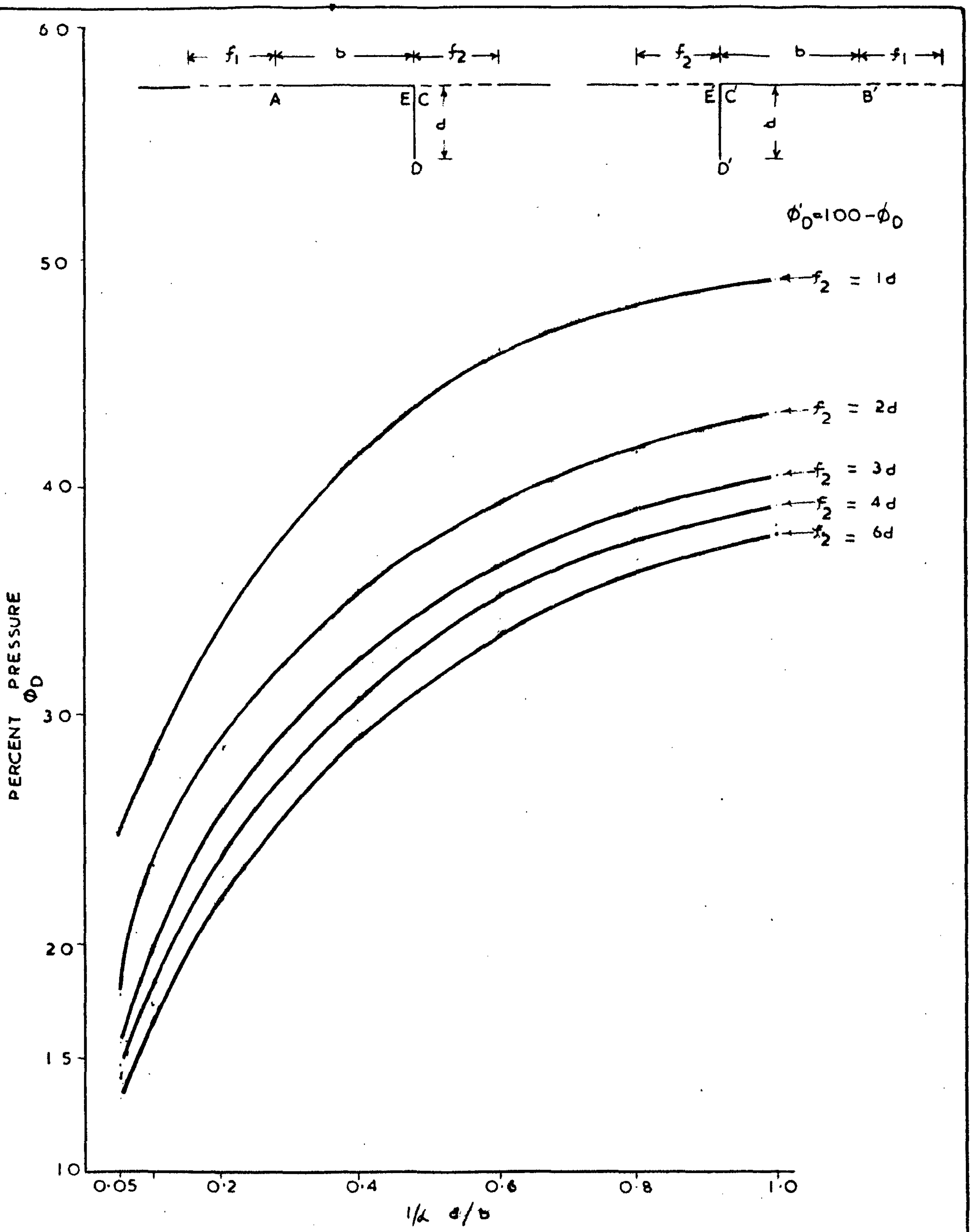

FIG. 2·6. PERCENT PRESSURE AT D FOR f_1 = 1·0d.

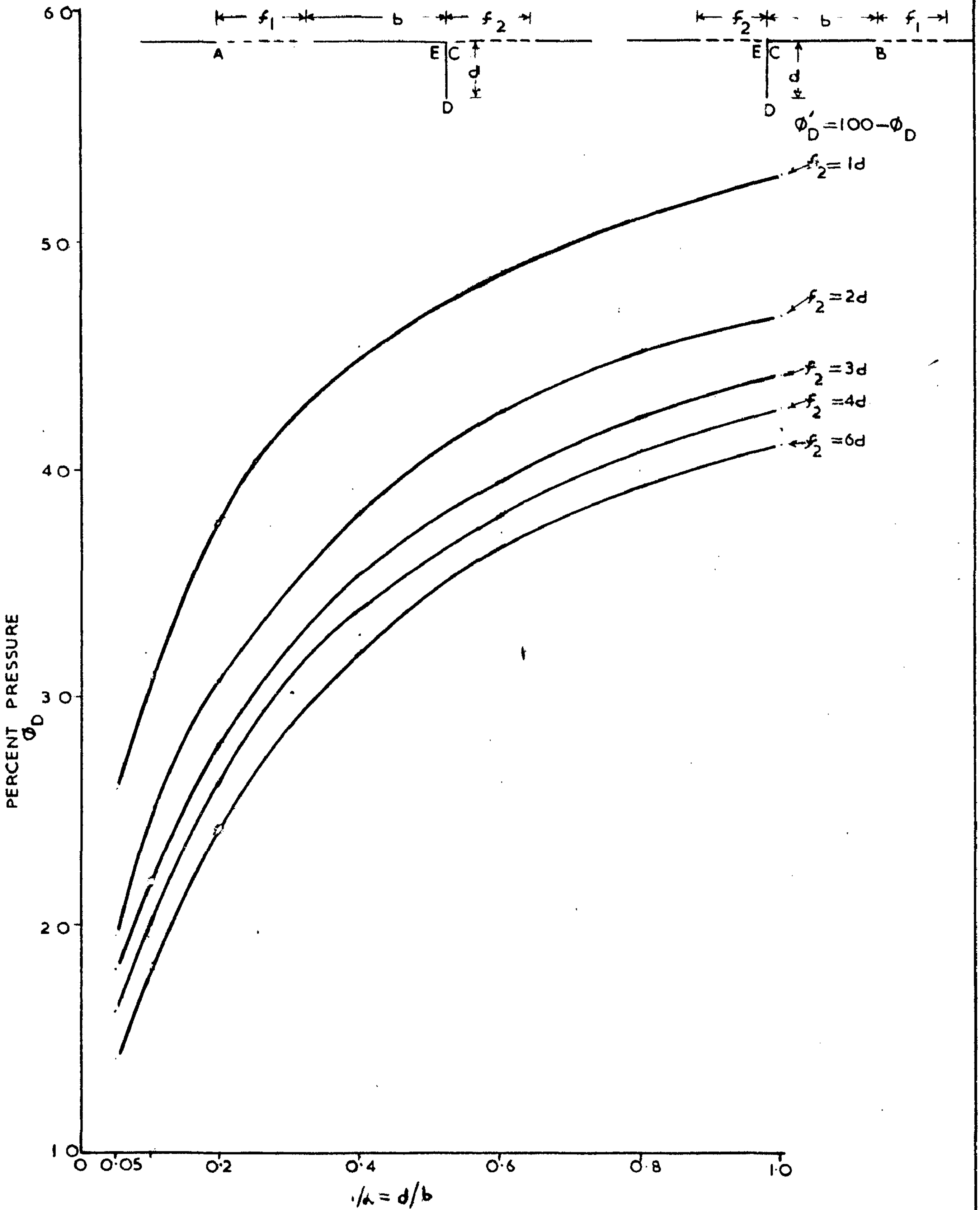

FIG. 2·7· PERCENT PRESSURE AT D FOR $f_1 = 2·0$ d.

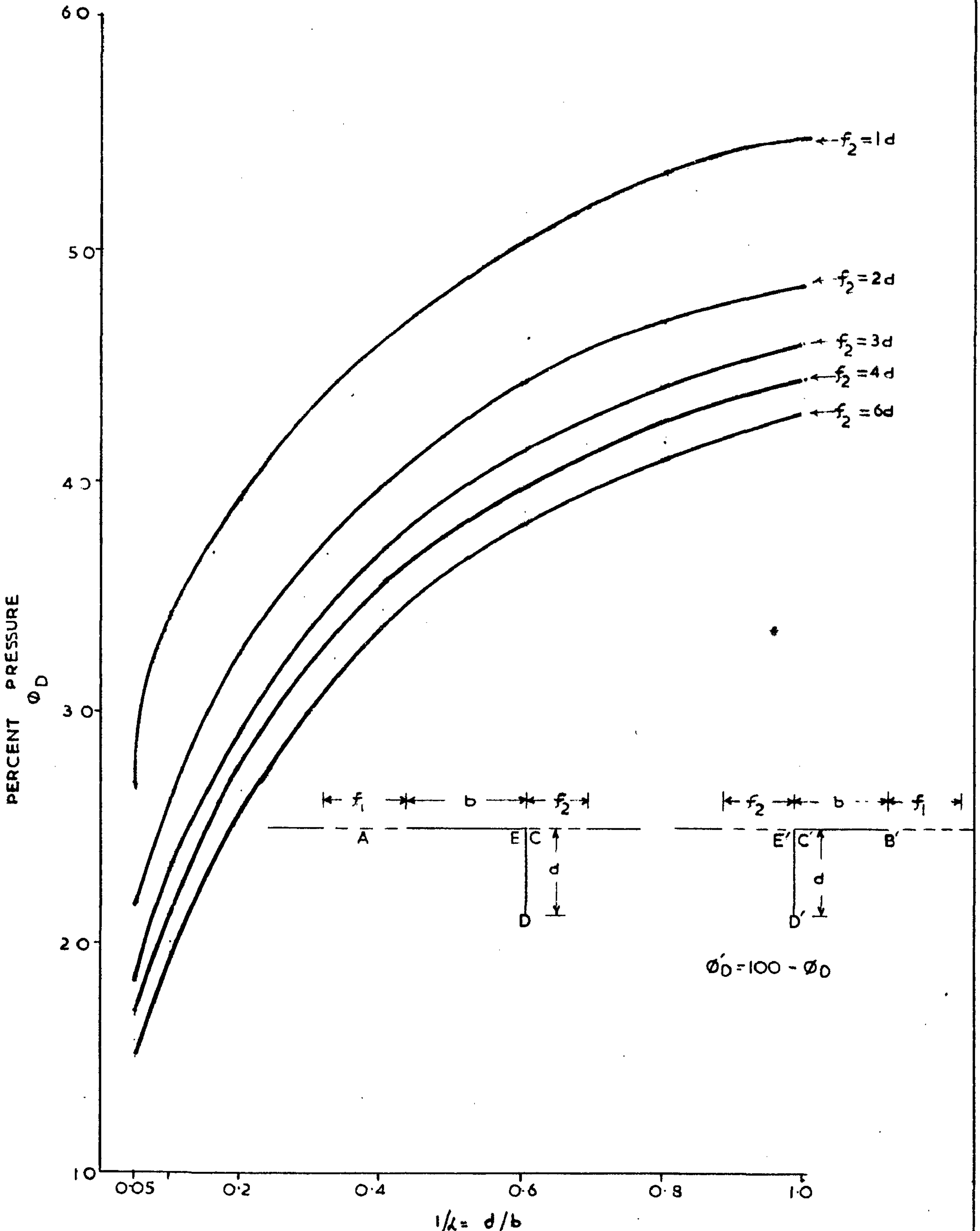

FIG. 2·8· PERCENT PRESSURE AT D FOR f_1 = 3·0d.

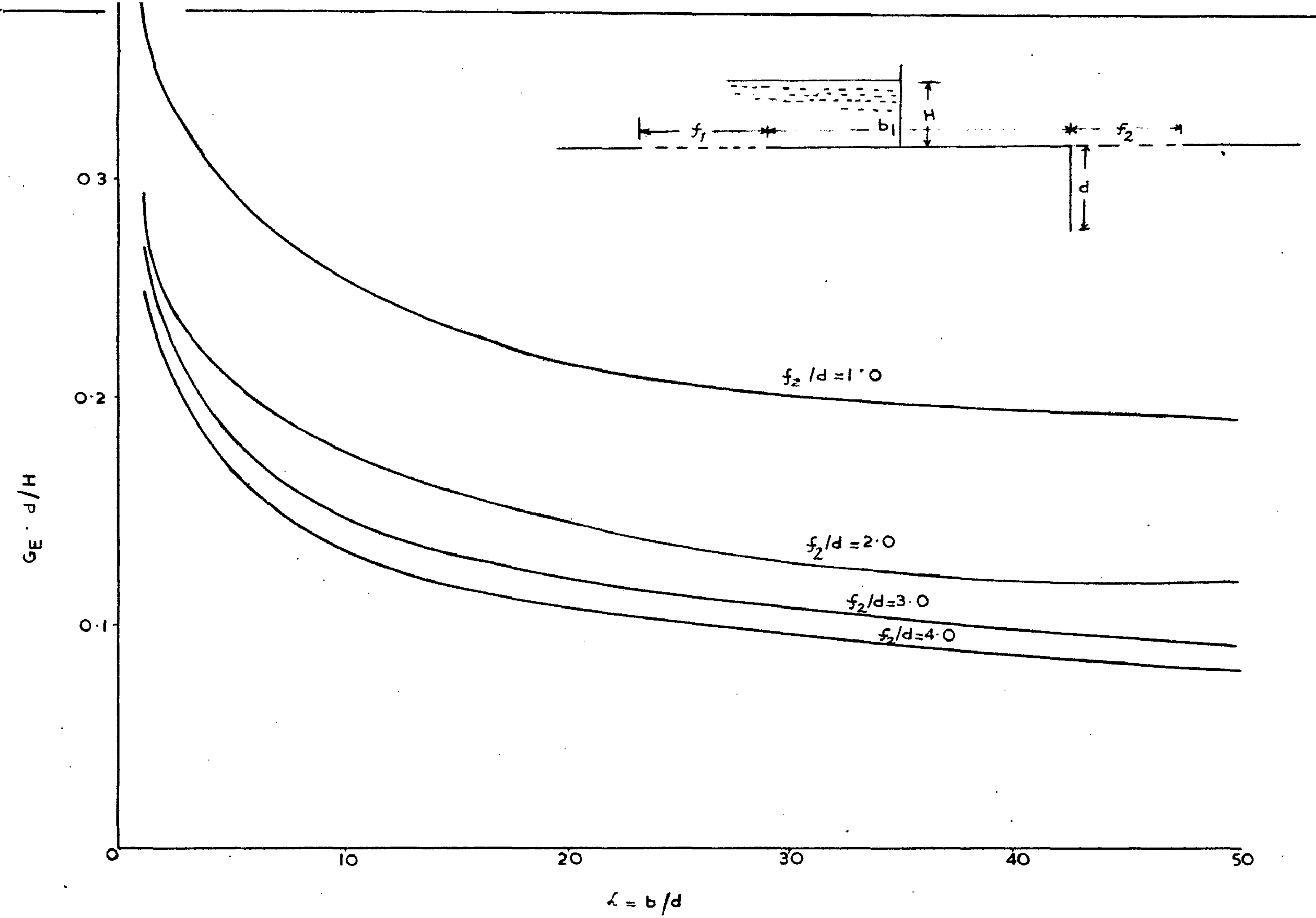

FIG. 2·9. EXIT GRADIENT FOR f_1 = 1·0 d.

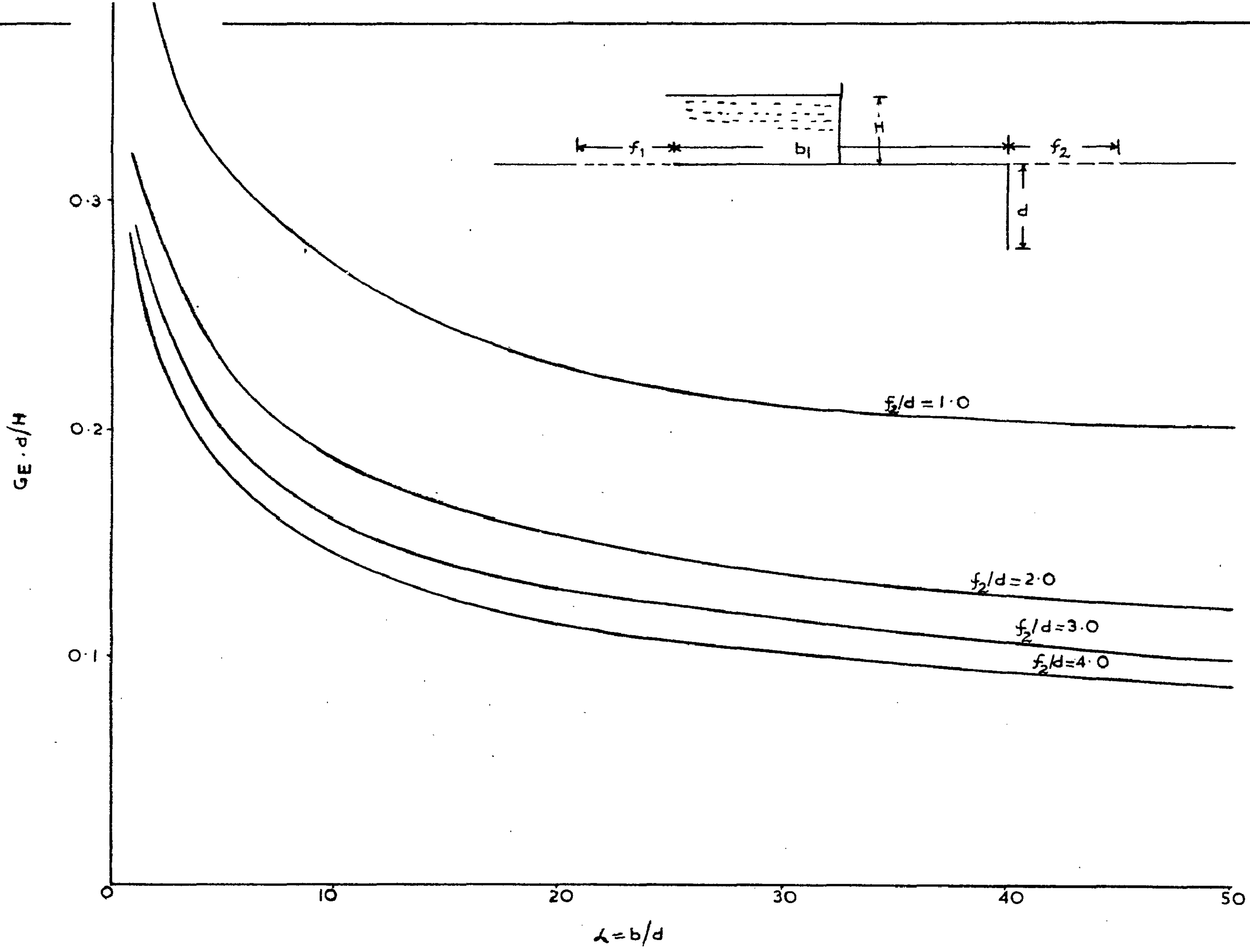

FIG. 2·10. EXIT GRADIENT FOR f_1 = 2·0d.

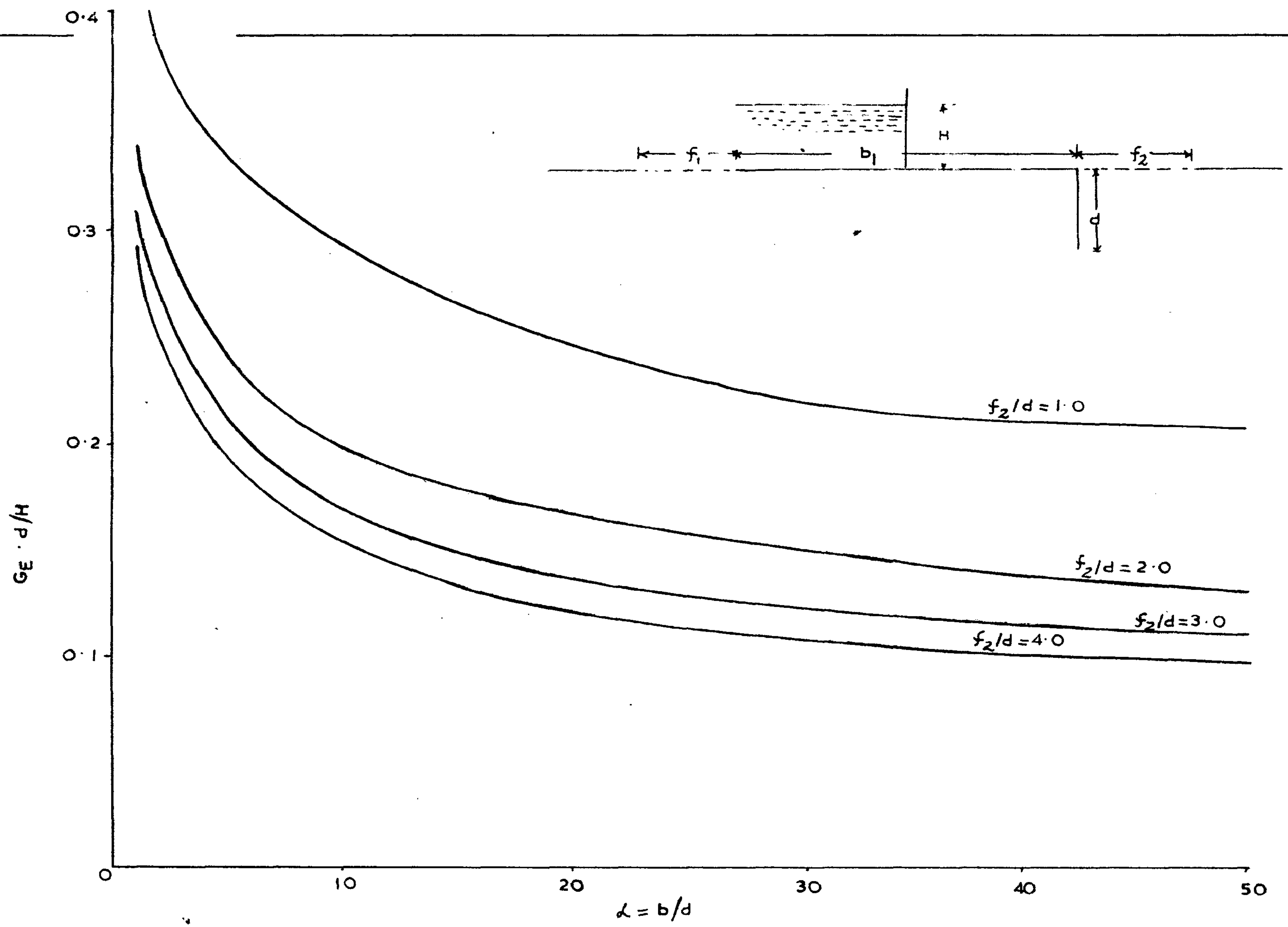

FIG. 2·11. EXIT GRADIENT FOR $f_1 = 3.0\,d$.

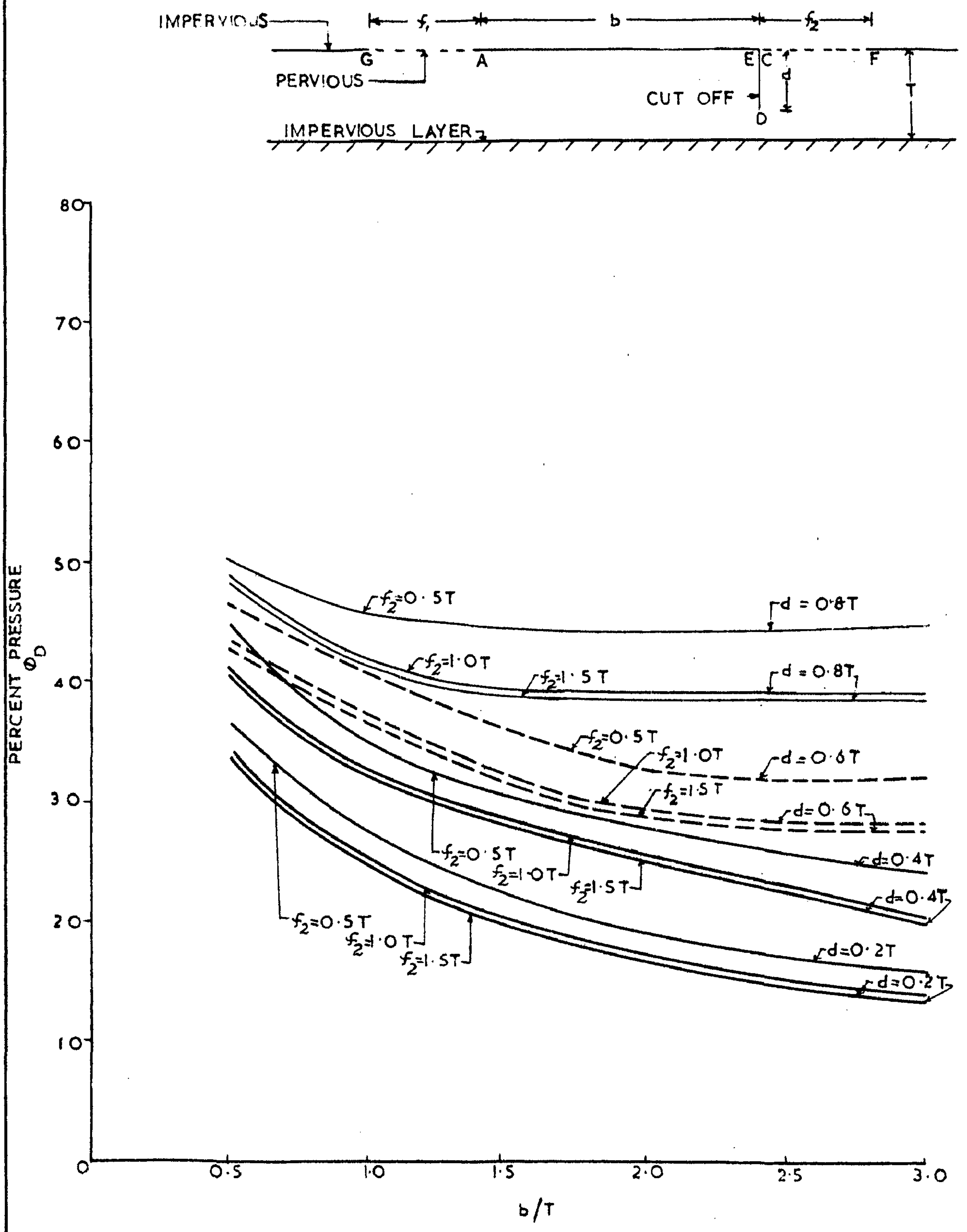

FIG.2.12. PERCENT PRESSURE AT D FOR f_1 = 0·5 T.

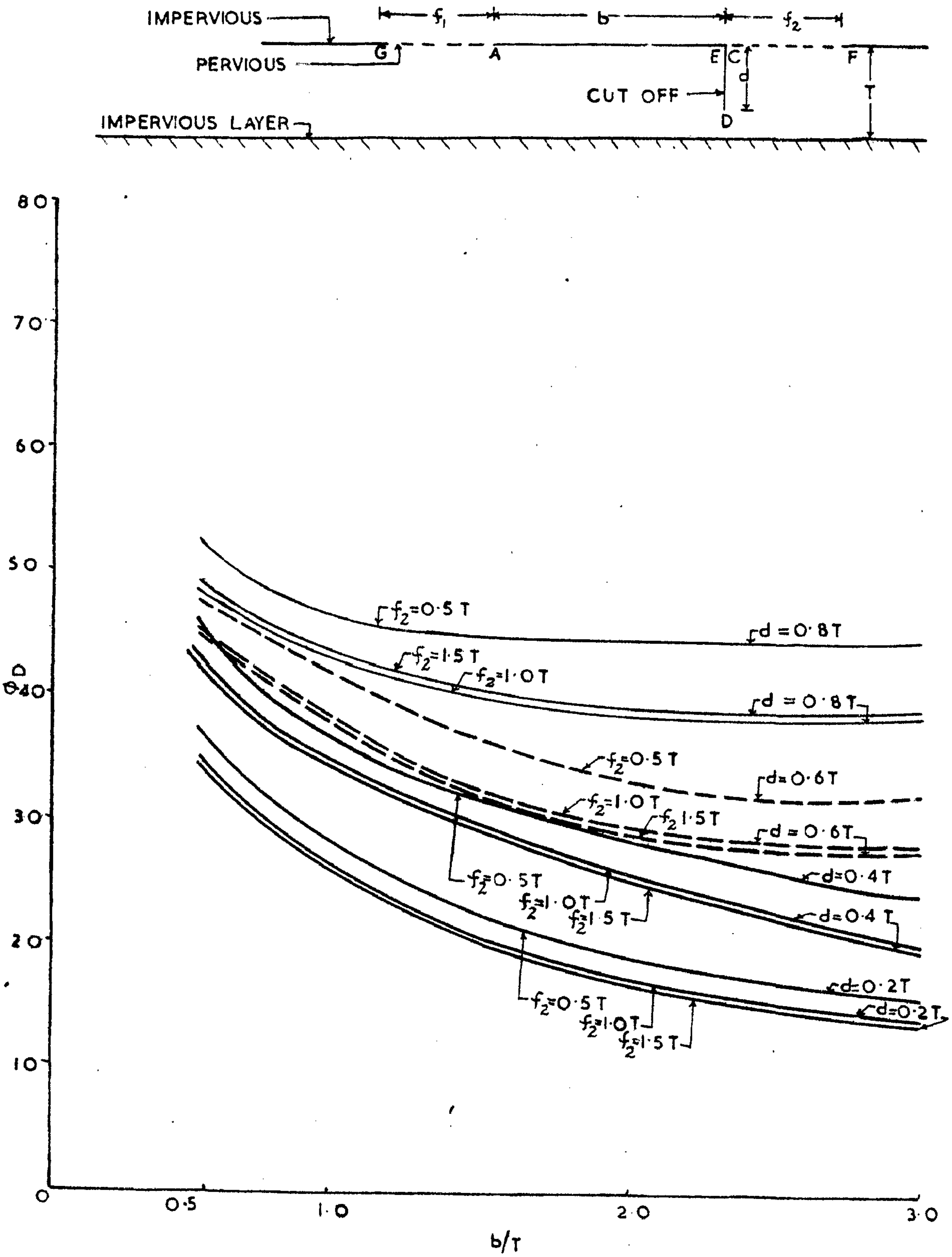

FIG. 2·13. PERCENT PRESSURE AT D FOR f_1=1·0T.

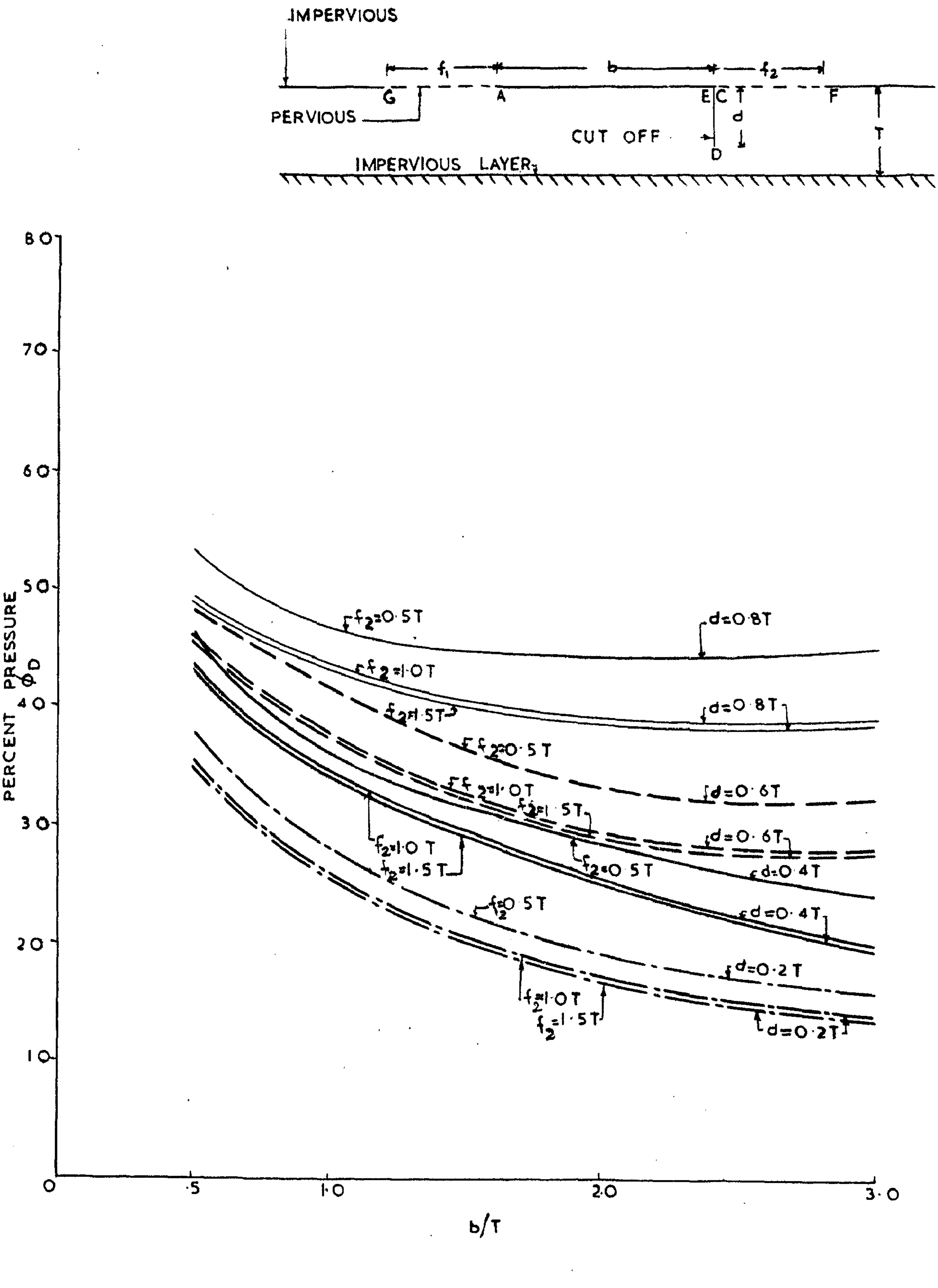

FIG. 2·14 PERCENT PRESSURE AT D FOR $f_1 = 1 \cdot 5\ T$

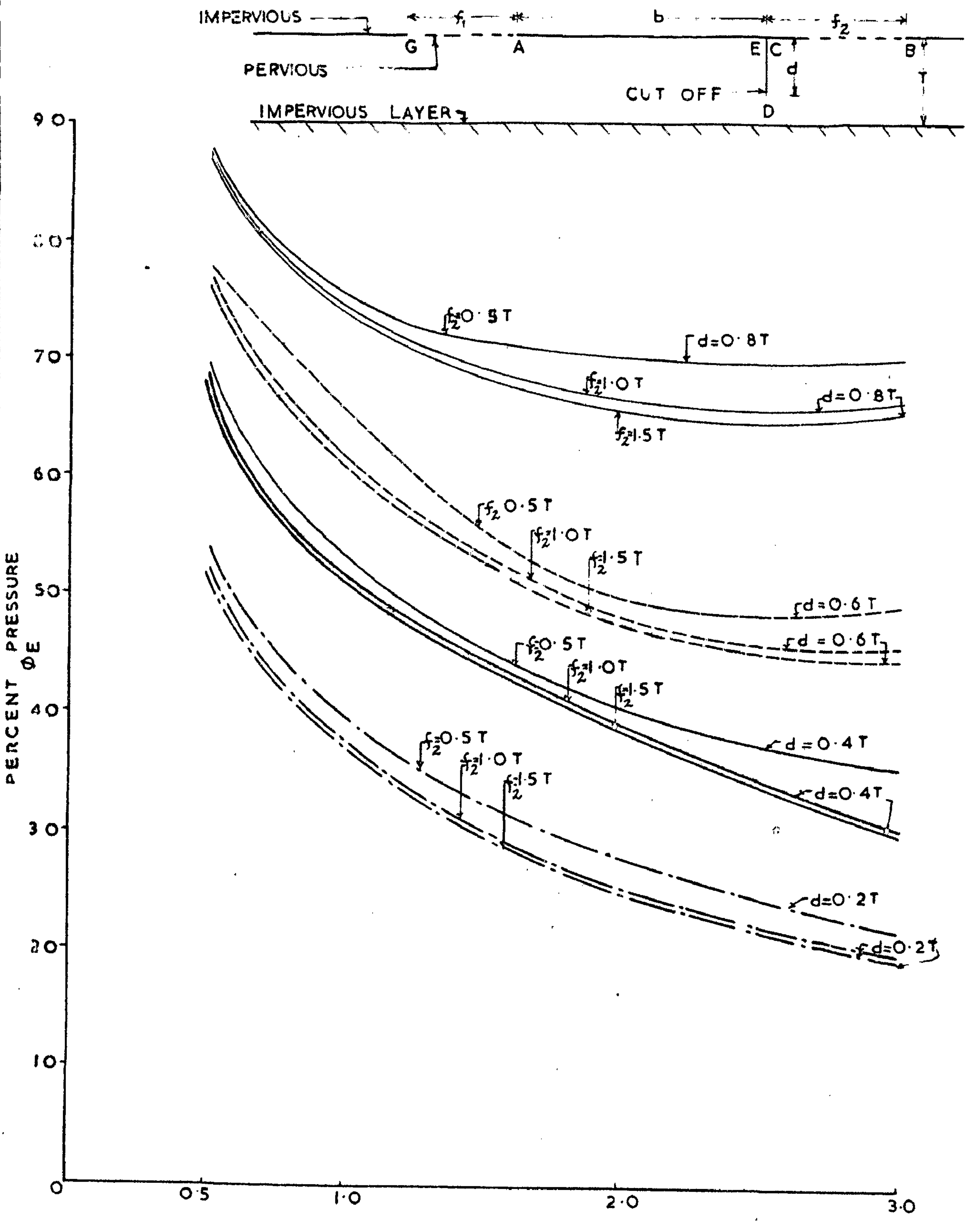

FIG.2.15 PERCENT PRESSURE AT E FOR f_1=0.5T.

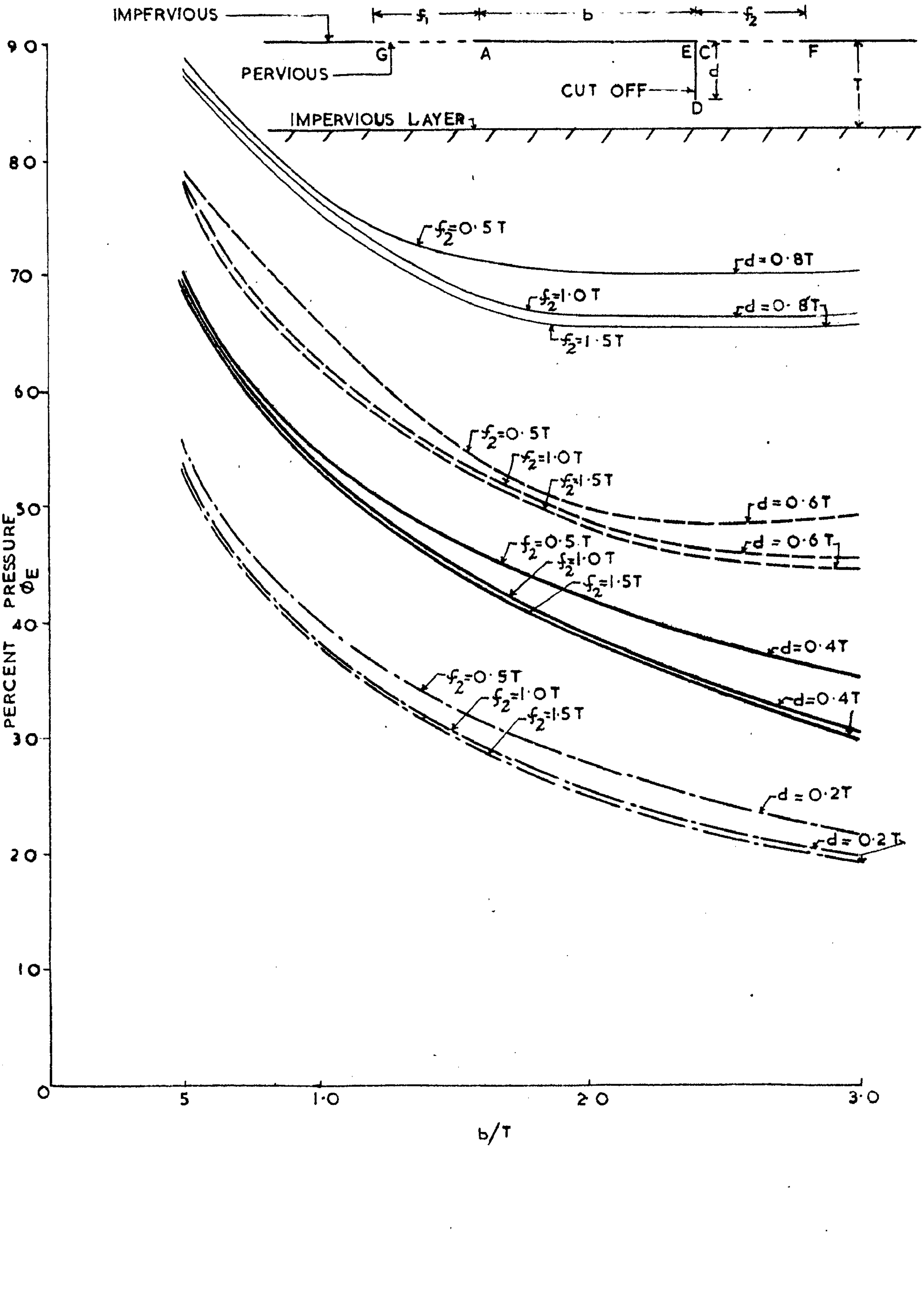

FIG. 2·16. PERCENT PRESSURE AT E FOR f_1 = 1·0T.

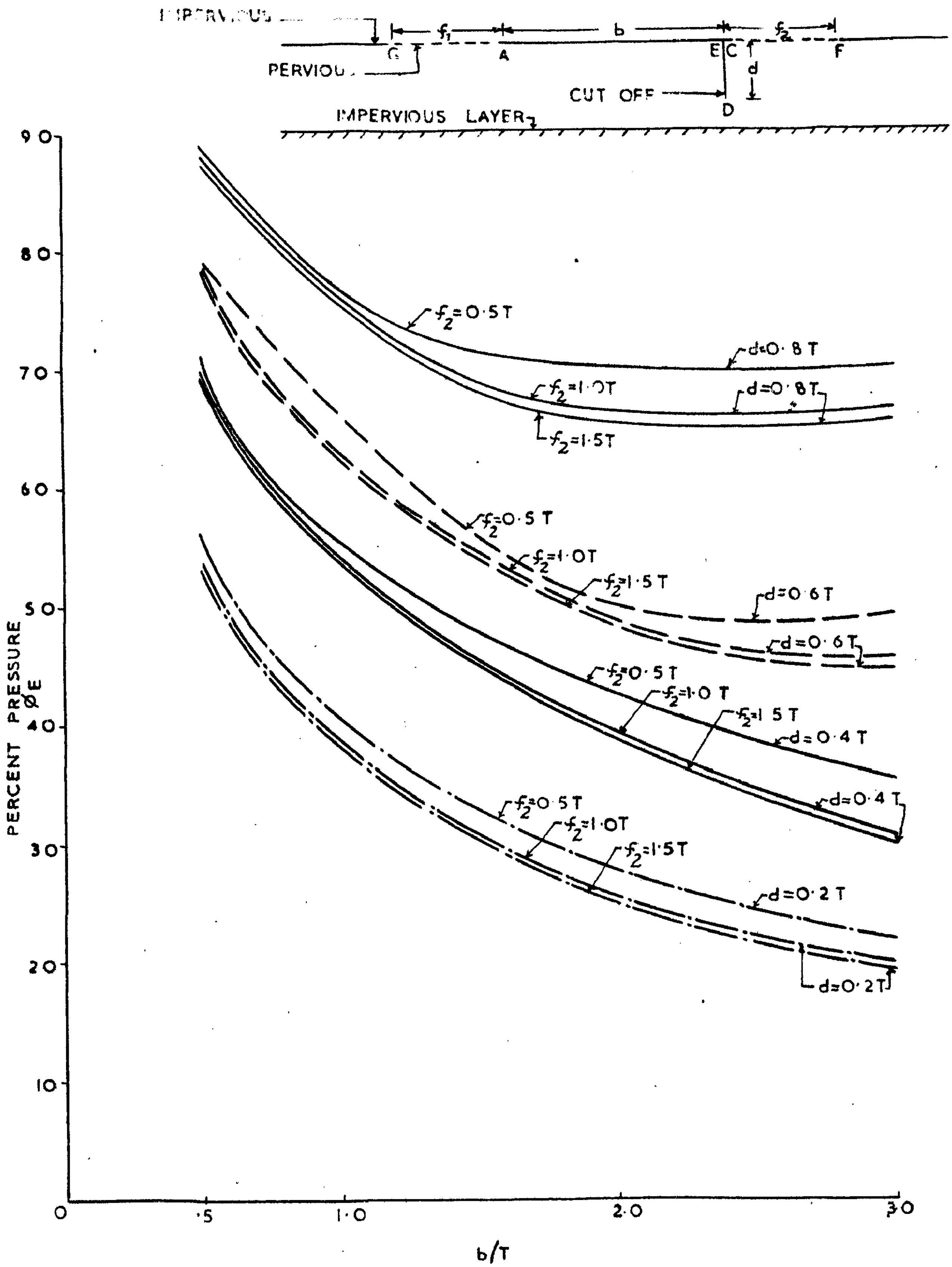

FIG. 2·17. PERCENT PRESSURE AT E FOR f_1 = 1·5T.

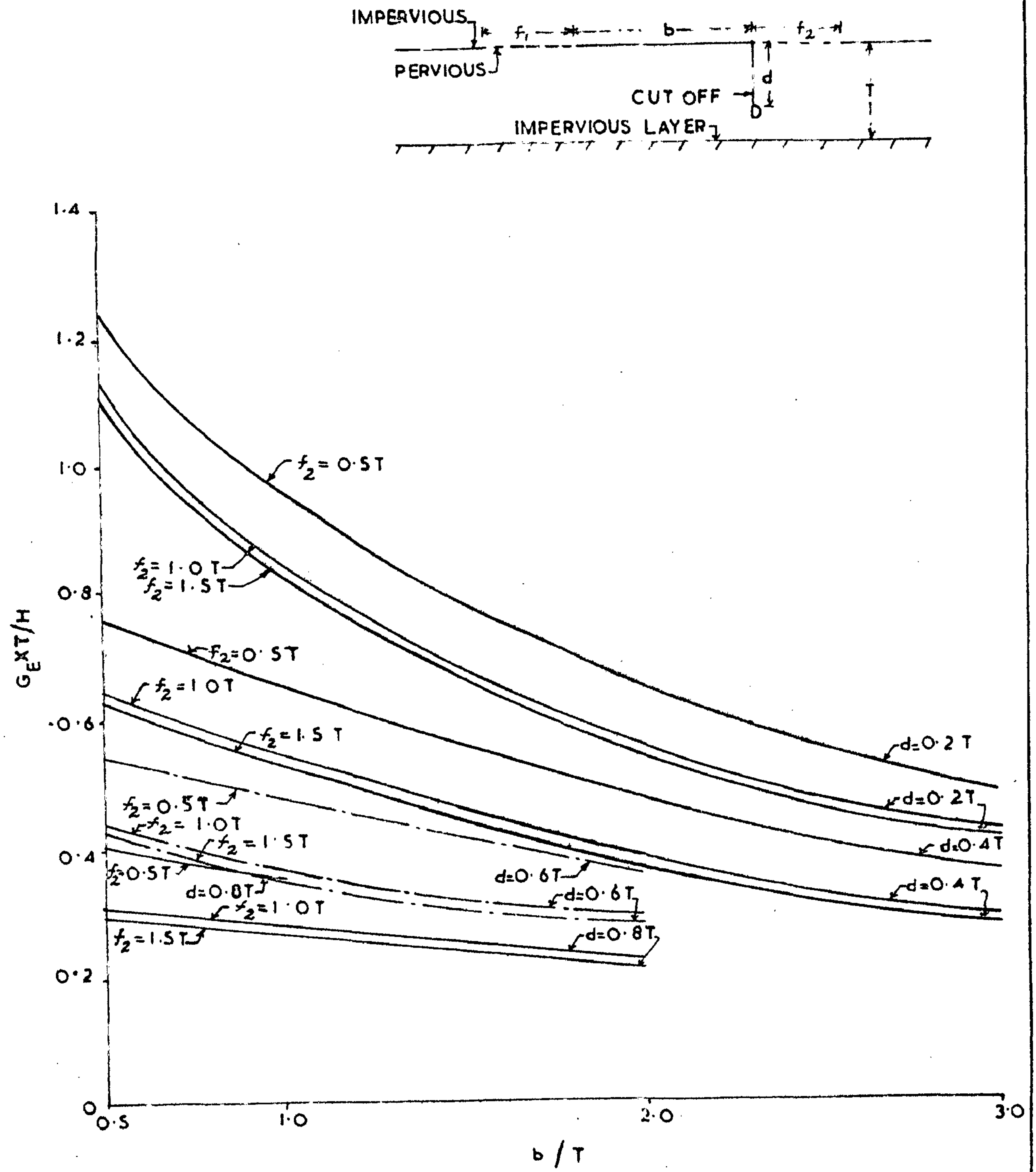

FIG. 2·18. EXIT GRADIENT FOR f_1 = 0·5 T.

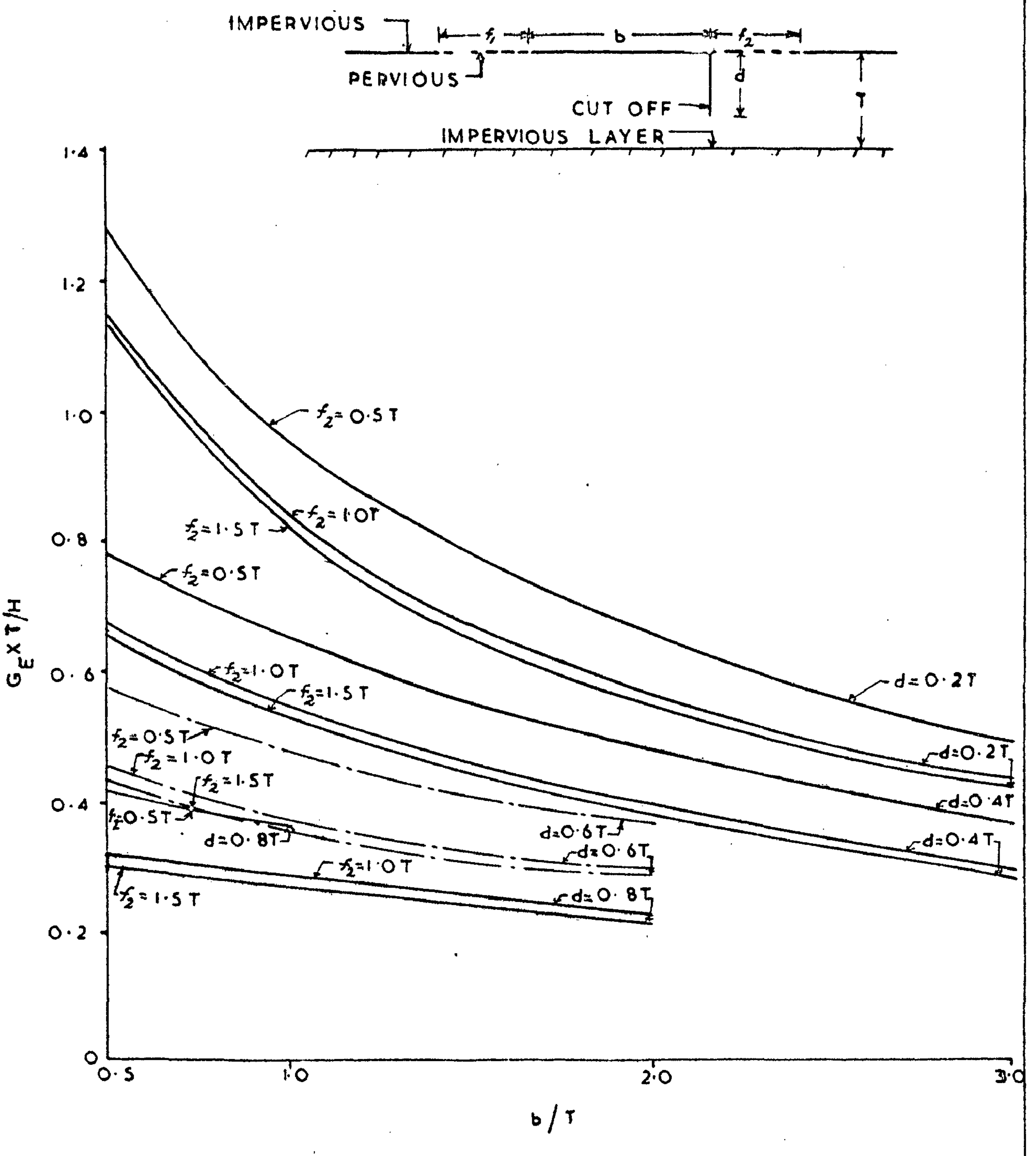

FIG 2·19. EXIT GRADIENT FOR f_1 = 1·0 T.

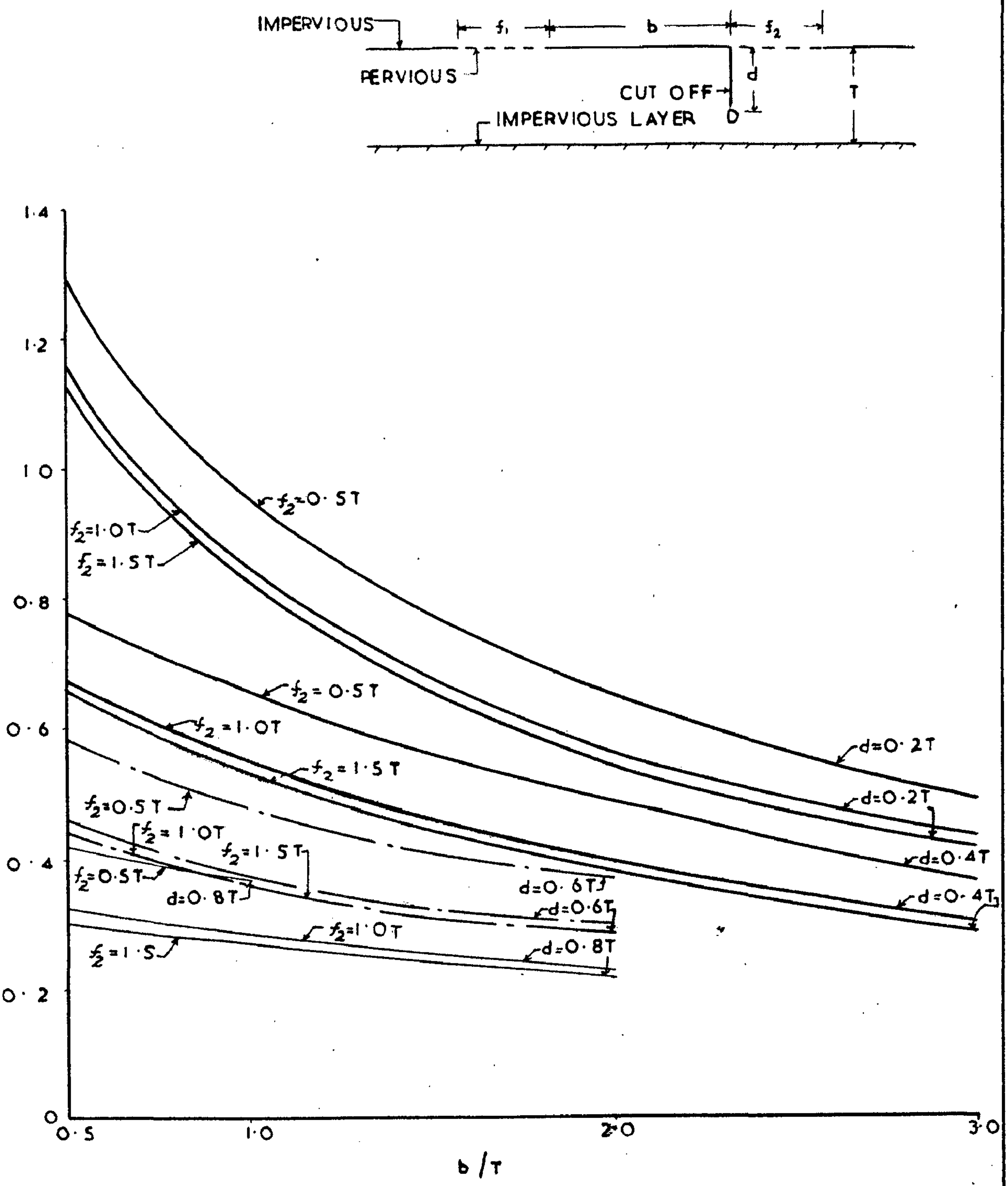

FIG. 2·20. EXIT GRADIENT FOR $f_1 = 1\cdot 5T$.

C H A P T E R   THREE

CHAPTER III

EXPERIMENTAL VERIFICATION

3.1 TEST CONDITIONS

Results obtained from exact solutions for the following boundary conditions were verified by two dimensional electrical analogy model technique and analog field plotter.

A. Finite pervious reaches on the upstream and downstream of the structure founded on Infinite pervious flow zone.

(i)	Depth of cut off	= d
(ii)	Impervious floor length upstream of the cut off, b1	= 5.0 d
(iii)	Impervious floor length downstream of cut off = b2	= 5.0 d
(iv)	Depth of permeable subsoil extends upto infinity	
(v)	Length of upstream pervious bed, f1	= 3.0 d
(vi)	Length of downstream pervious bed, f2	= 3 d

B. Finite pervious Reaches on the upstream and downstream of the structure founded on finite pervious flow zone.

(i)	Depth of pervious flow zone	= T
(ii)	Depth of cut off d	= 0.4T and 0.6T
(iii)	Length of the impervious floor on the upstream of cut off be b_1	= 1.0T
(iv)	Length of the impervious floor on the downstream of cut off b2	= 1.0T
(v)	Length of pervious bed on the upstream, f1	= 0.5T
(vi)	Length of pervious bed on the downstream, f2	= 0.5T

3.2 EXPERIMENTAL SET UP

3.2.1 Electrical Analogy Technique

The experimental set up consisted of a tank filled with electrolytic solution. Ordinary tape water was used as electrolytic solution. The electrodes used to simulate the upstream and downstream pervious reaches in the model were 1/8" to 1/32" thick copper plates. An electrical analogy model showing the foundation profile of the structure to scale was constructed and fitted in the tray. The foundation profile of the structure along with the electrical connections done on the principle of Wheat stone Bridge and the equipment needed to get the accurate null point are shown vide figure no. 3.1

3.2.2 ANALOG FIELD PLOTTER

An analog field plotter consists of a control unit which houses the power supply, a voltage dividing potentiometer and a sensitive null detector. Its other accessories are a conducting paper, silver paint, resistance wire, plotting board and test leads. In the sheet of conducting paper, which is the "Current Sheet" of the plotter, the electric current analog field pattern is established by means of suitability attached and energised electrodes. The resulting potential drop pattern is marked directly on the conducting paper by means of a moving probe.

3.2.3 The 0.004 inch thick conducting paper is made by uniform spraying carbon or graphite over a uniform sheet of paper. It has an approximate resistance of 4000 ohms per square. For experimental studies, the section of the foundations of structure was drawn to scale on the conducting paper. The pervious reaches were represented by silver painted electrodes having a resistance of 1 to 4 ohms/square. The sub-soil flow zone was represented by the conducting paper itself and the impervious foundation was simulated by removing the conducting paper from those portions thus rendering it nonconducting to the flow of current. The electrical connections and the model profile are shown vide Drawing no. 3.2

3.3 TEST RESULTS

3.3.1 For test condition 'A' when the structure is founded on the infinite depth of pervious flow zone, the observed uplift pressures along bottom of the floor both by two dimensional electrical analogy model and analog field are given in Table I. The uplift pressure along bottom of the floor calculated from the formulae for above boundary condition are also given in the above table. The results are plotted in Figure 3.3 for comparison.

A perusal of Table I and Figure 3.3 indicates that observed uplift pressures are very close to the calculated values.

3.3.2 For the test condition 'D' when the structure is founded on the finite depth of pervious flow zone, the observed uplift pressures along bottom of the floor both by two dimensional electrical analogy model and analog field plotter are given in Table II and III. The uplift pressures, along bottom of the floor, calculated from the formulae for the above boundary conditions are also given in the above tables. The results are plotted in Figure 3.4 and 3.5 for comparison. A perusal of table II and III and Figure 3.4 and 3.5 indicates that observed uplift pressures are sufficiently close to the calculated values.

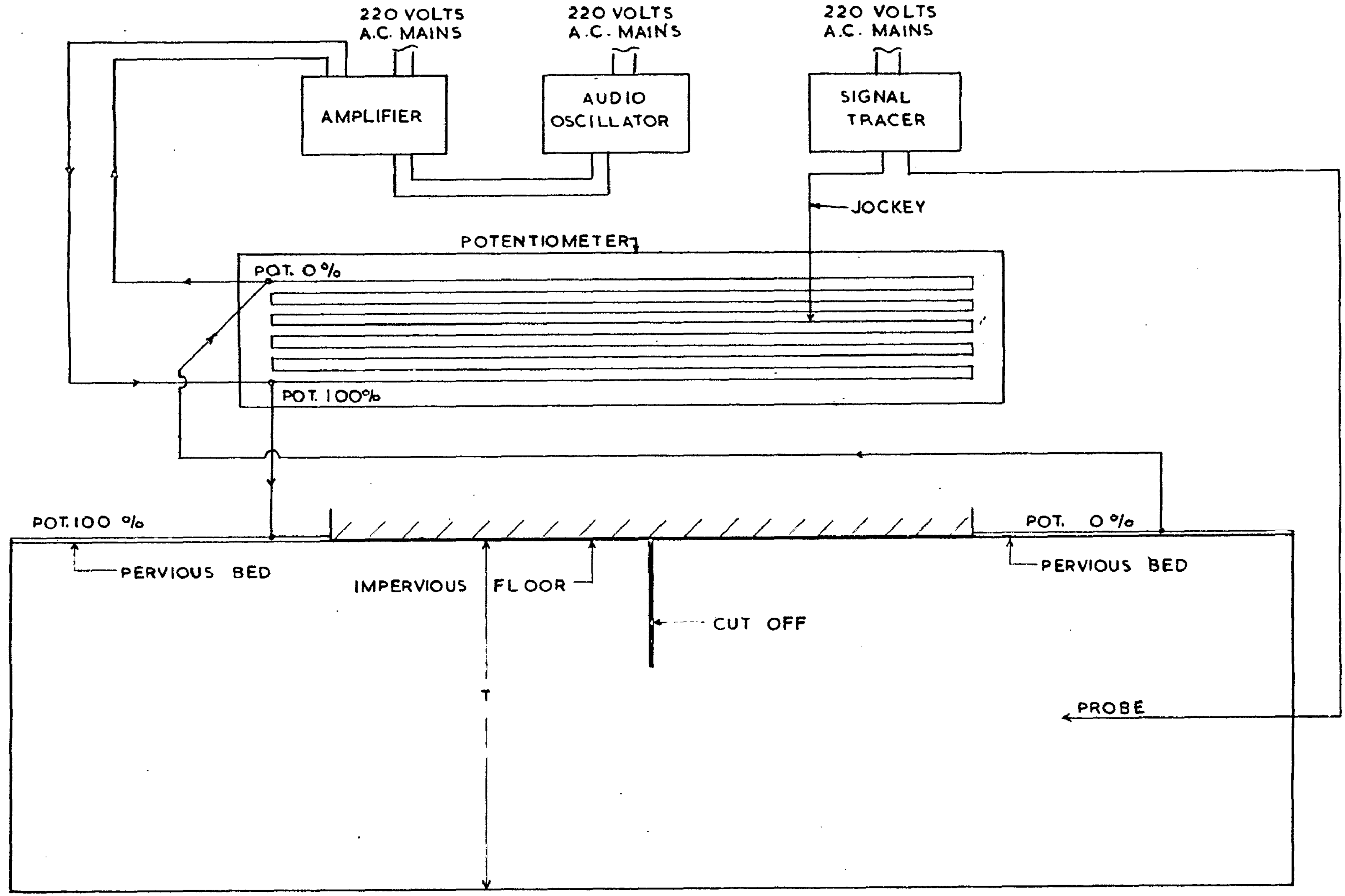

FIG. 3.1. ELECTRICAL CONNECTIONS –2–D ELECTRICAL ANALOGY MODEL.

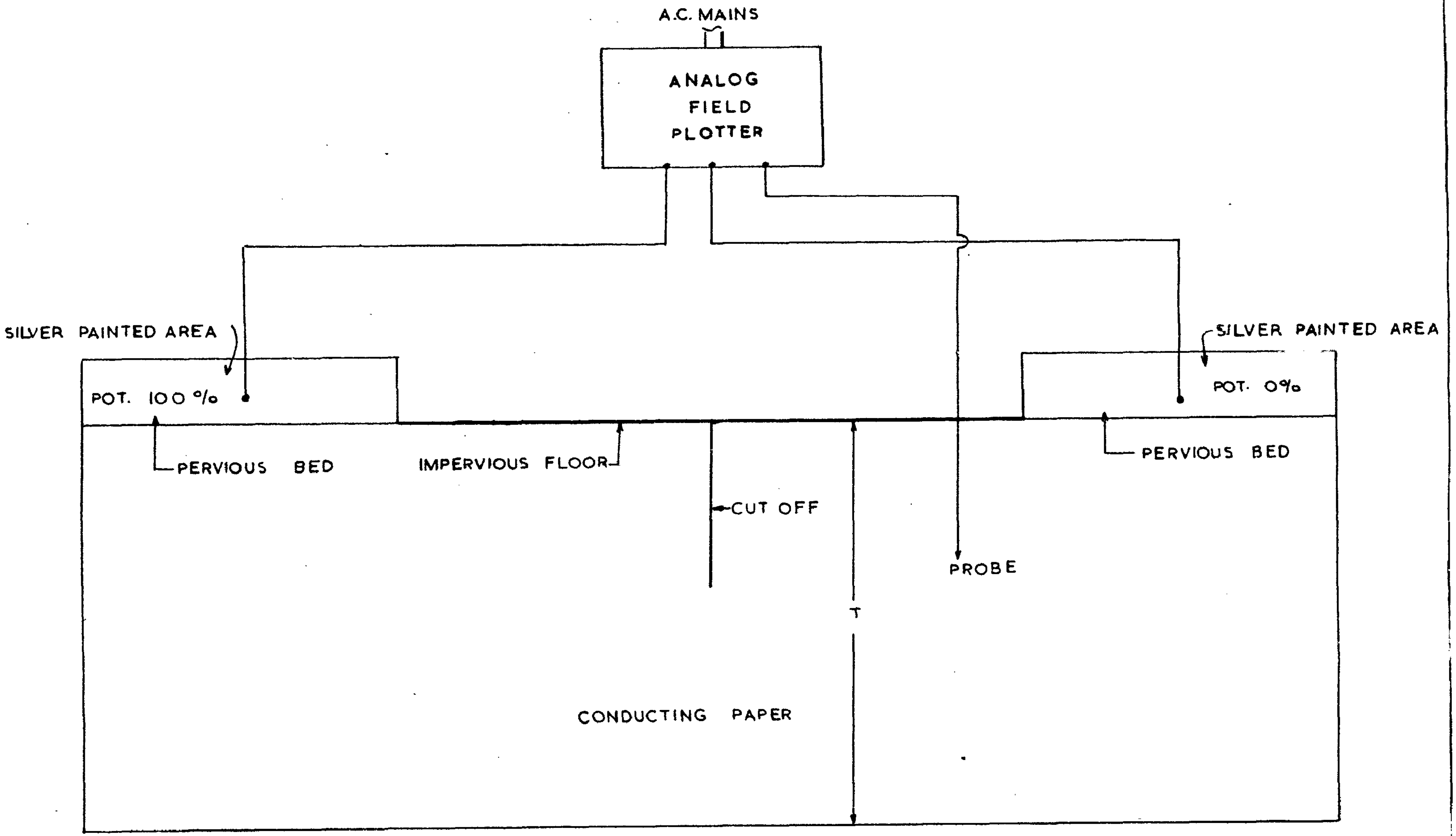

FIG. 3.2. ELECTRICAL CONNECTIONS—ANALOG FIELD PLOTTER.

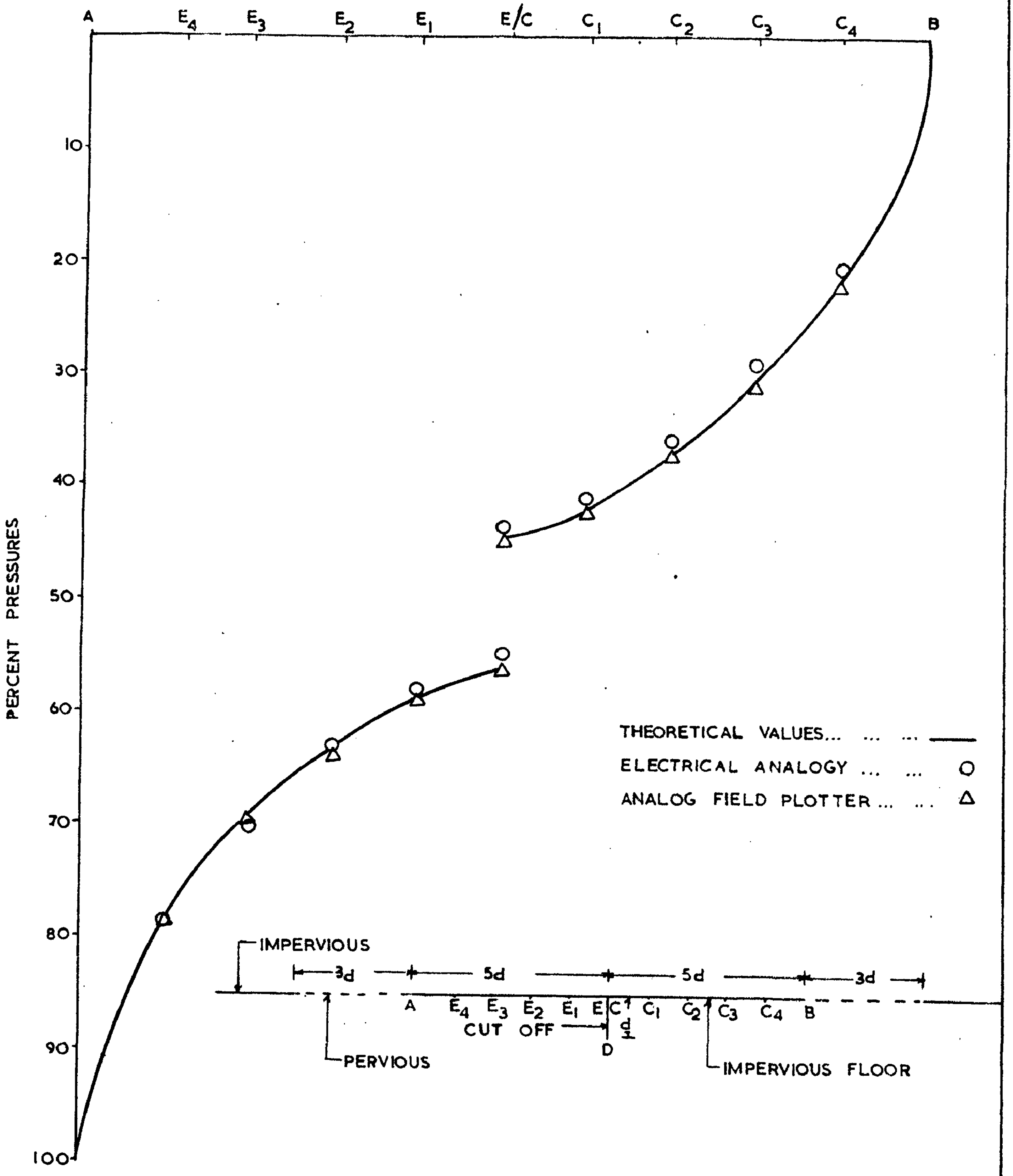

FIG. 3.3 UPLIFT PRESSURE ON THE BOTTOM OF FOUNDATION

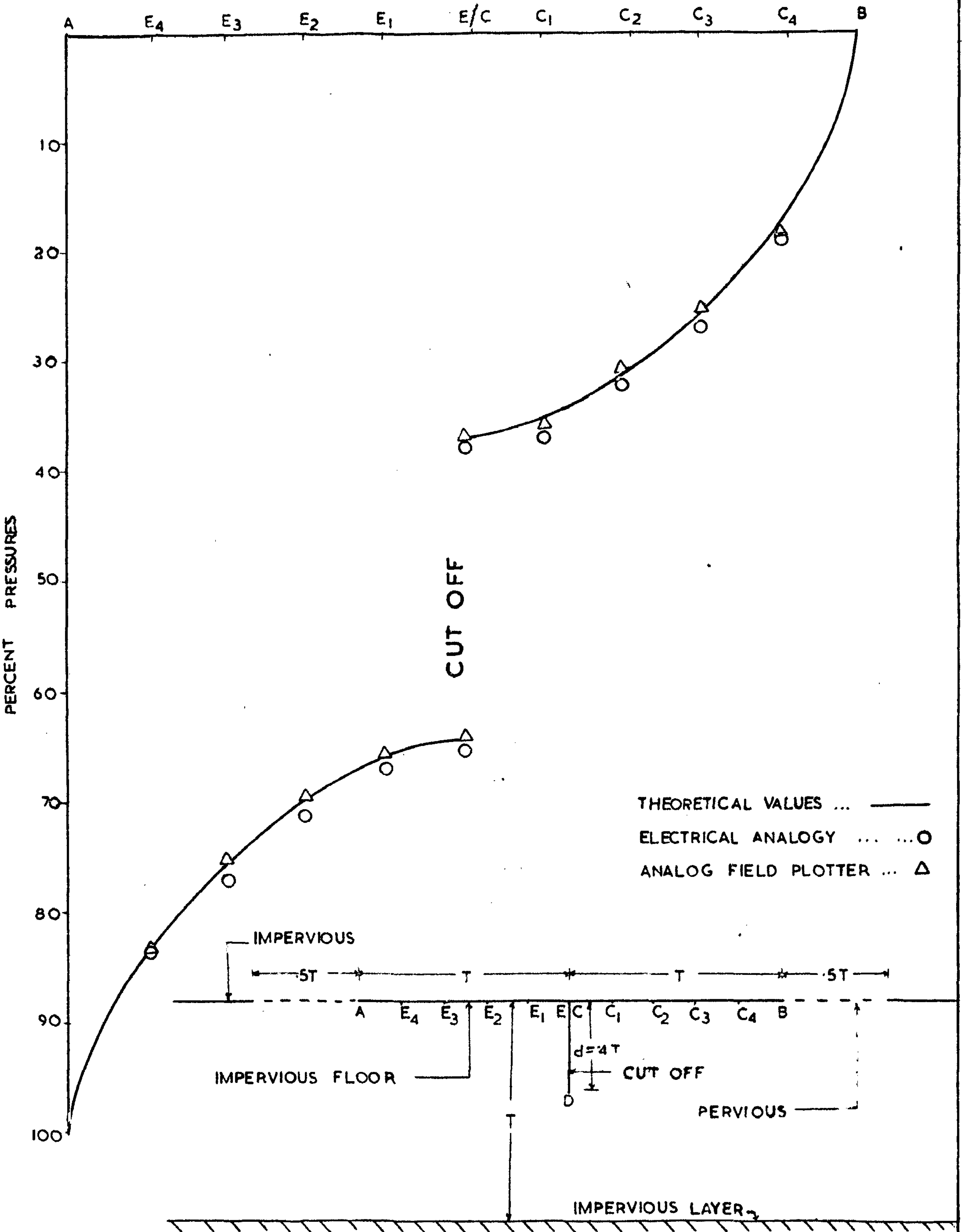

FIG. 3.4. UPLIFT PRESSURE ON THE BOTTOM OF FOUNDATION

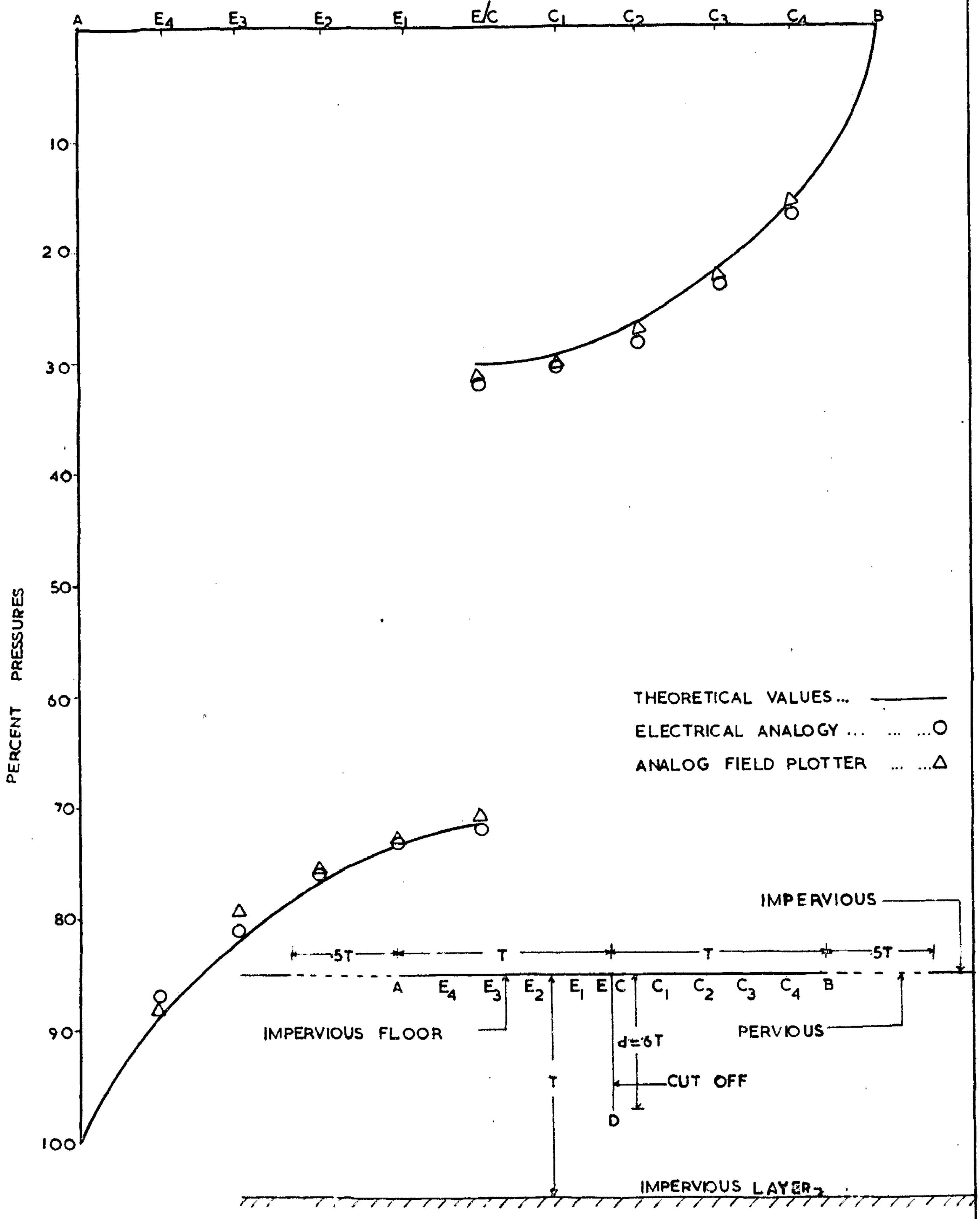

FIG.3·5· UPLIFT PRESSURE ON THE BOTTOM OF FOUNDATION.

CHAPTER FOUR

CHAPTER IV

DISCUSSION OF RESULTS

4.1 A perusal of Figure 4.1 and 4.2 indicate that uplift pressures increase throughout the floor length of the structure founded on infinite as well as finite pervious flow zone with the increase in the upstream pervious reach. The increase in the uplift pressure due to increase in the upstream pervious reach is slightly more below the floor upstream of the cut off.

4.2 A perusal of Figure 4.3 and 4.4 indicate that uplift pressures decrease throughout the floor length of the structure founded on infinite as well as finite pervious flow zone with the increase in the downstream pervious reach. The decrease in uplift pressures is more on the downstream side of the floor.

4.3 When the pervious reaches on the upstream and downstream are equal, the uplift pressures increase on the floor upstream of the cut off and decrease on the floor downstream of the cut off with the increase in the pervious reaches. This is true for the structure founded on the infinite as well as finite pervious flow zone. This is evident from Figure 4.5 and 4.6. The effect is more pronounced on the floor downstream of the cut off.

4.4 A perusal of figure no. 4.7 indicates that for the same length of pervious reaches both on the upstream and downstream of the structure founded on the finite depth of pervious flow zone the uplift pressures below the floor upstream of the cut off increase where as those on the floor downstream of cut off decrease with the increase in the depth of cut off.

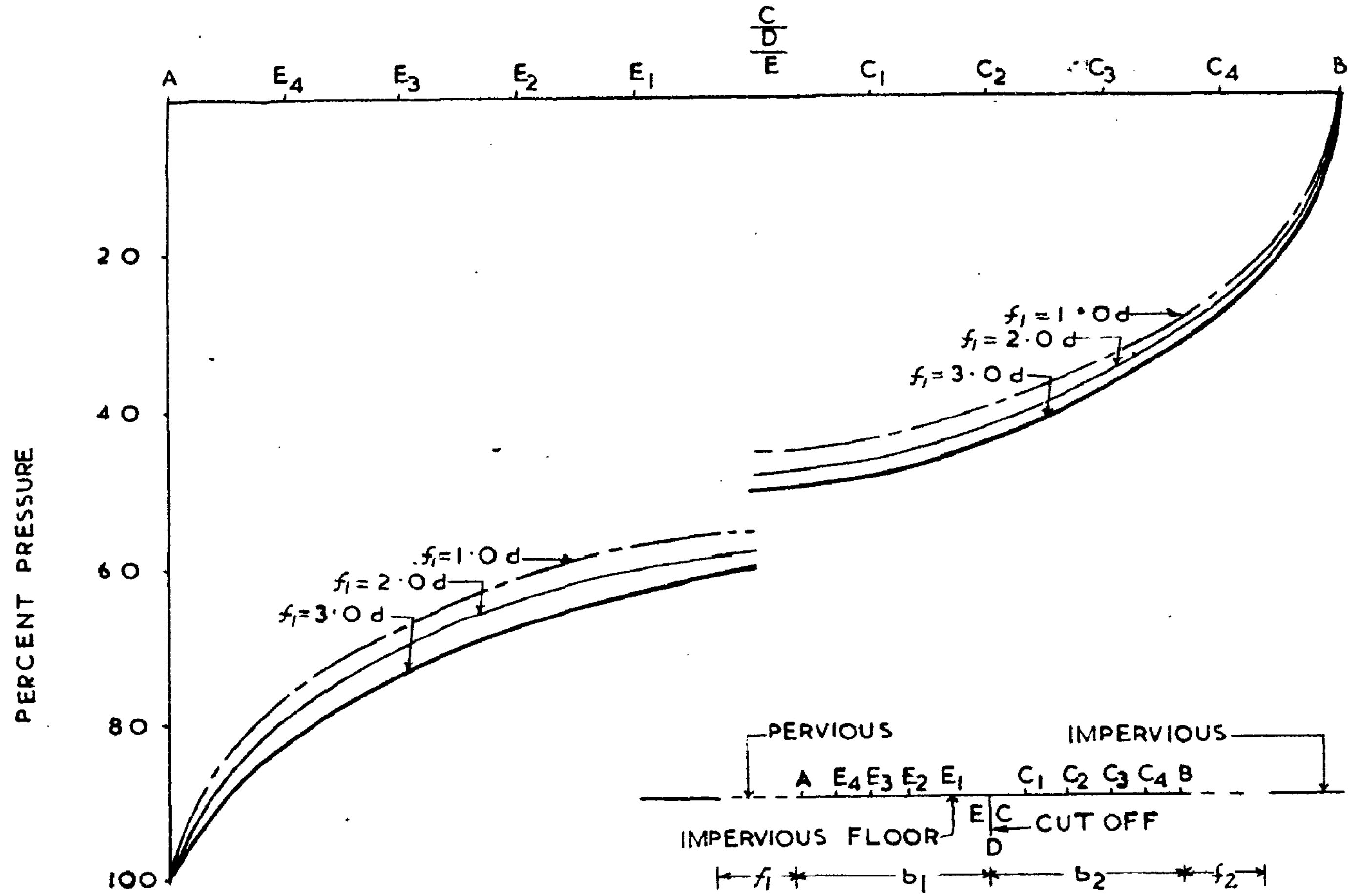

FIG.4·1. UPLIFT PRESSURES ON THE BOTTOM OF FOUNDATIONS FOR $f_2 = 1.0\,d$.

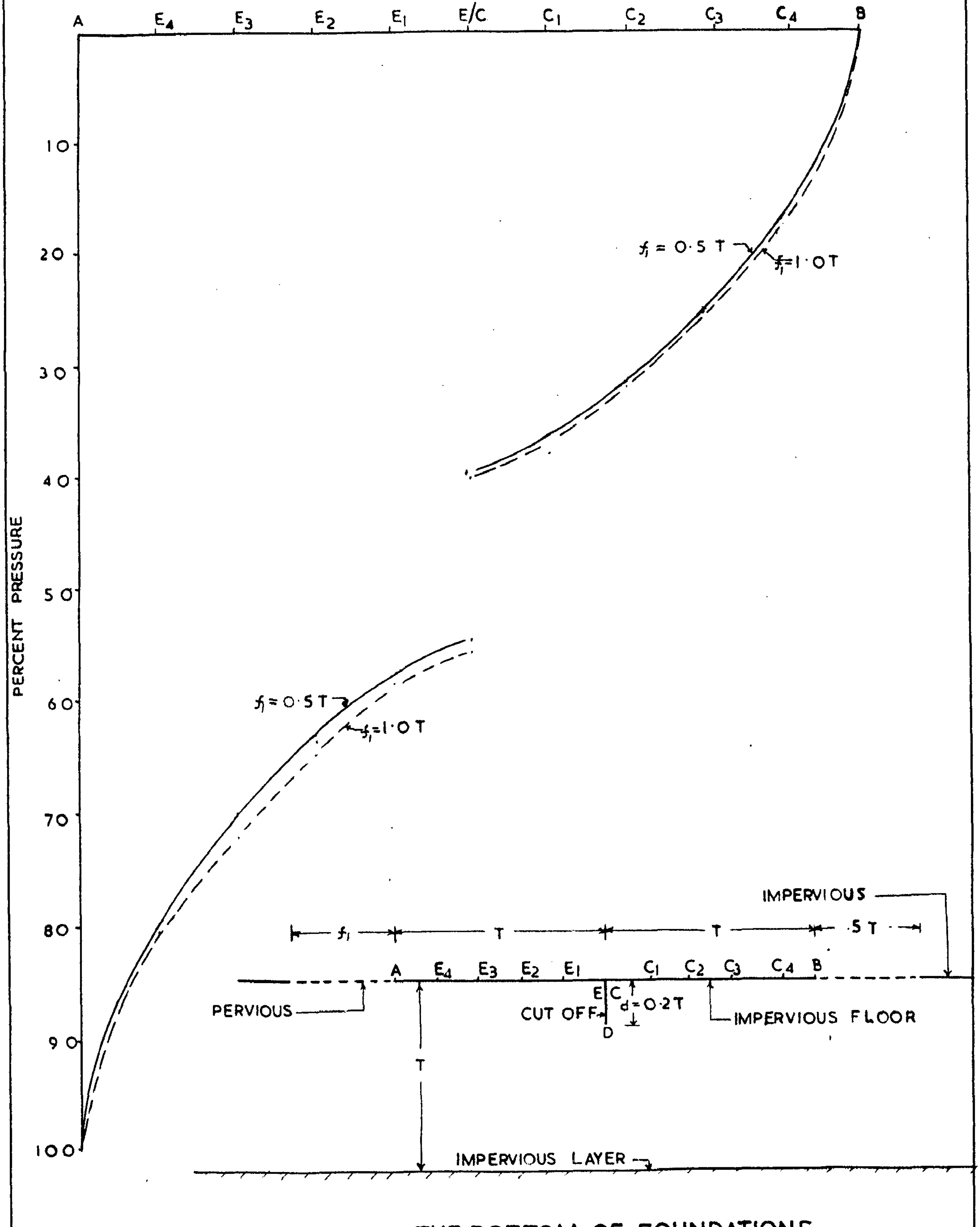

FIG.42.UPLIFT PRESSURE ON THE BOTTOM OF FOUNDATIONS

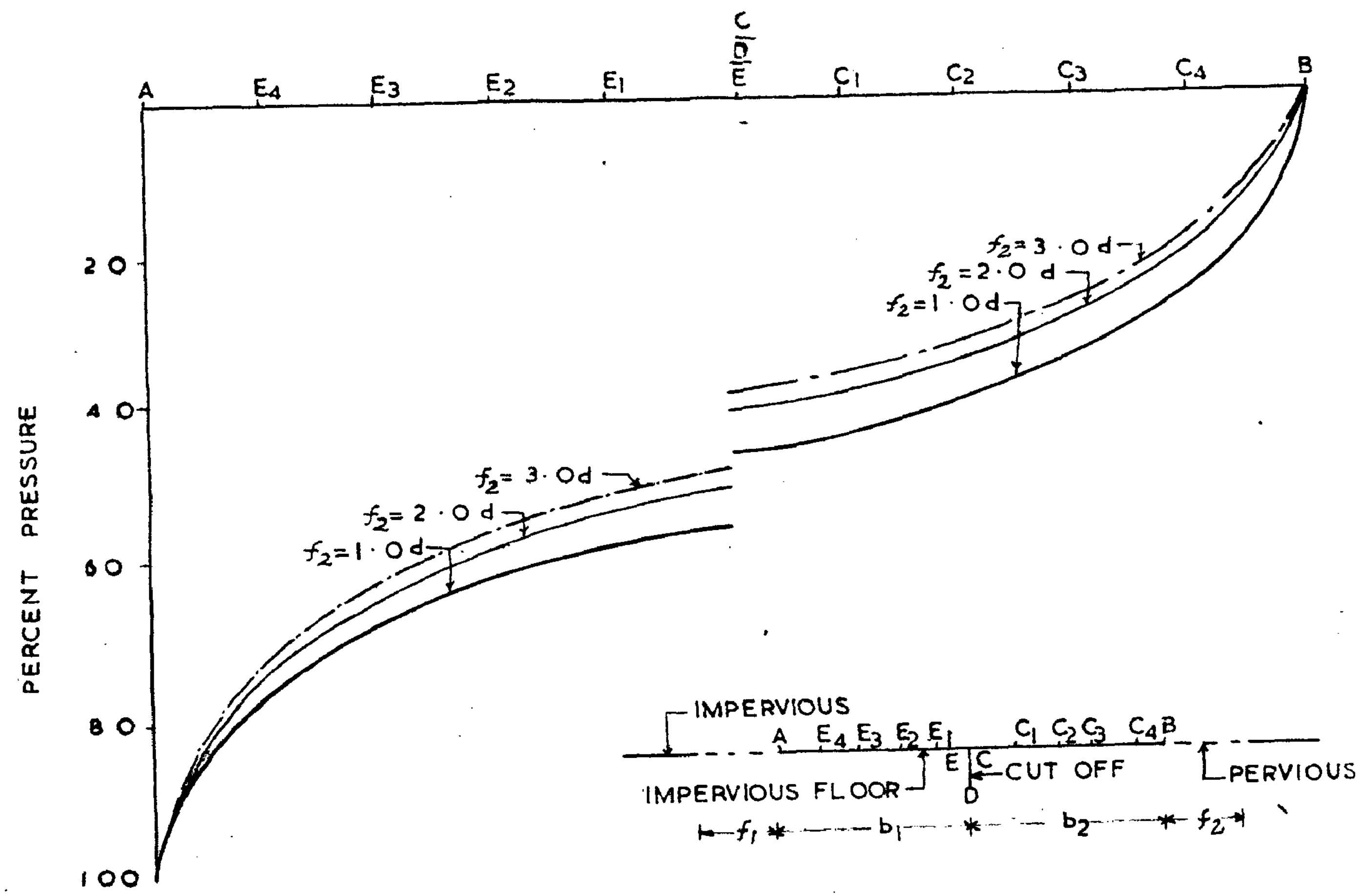

FIG. 4·3. UPLIFT PRESSURES ON THE BOTTOM OF FOUNDATIONS FOR $f_1 = 1{\cdot}0\,d$.

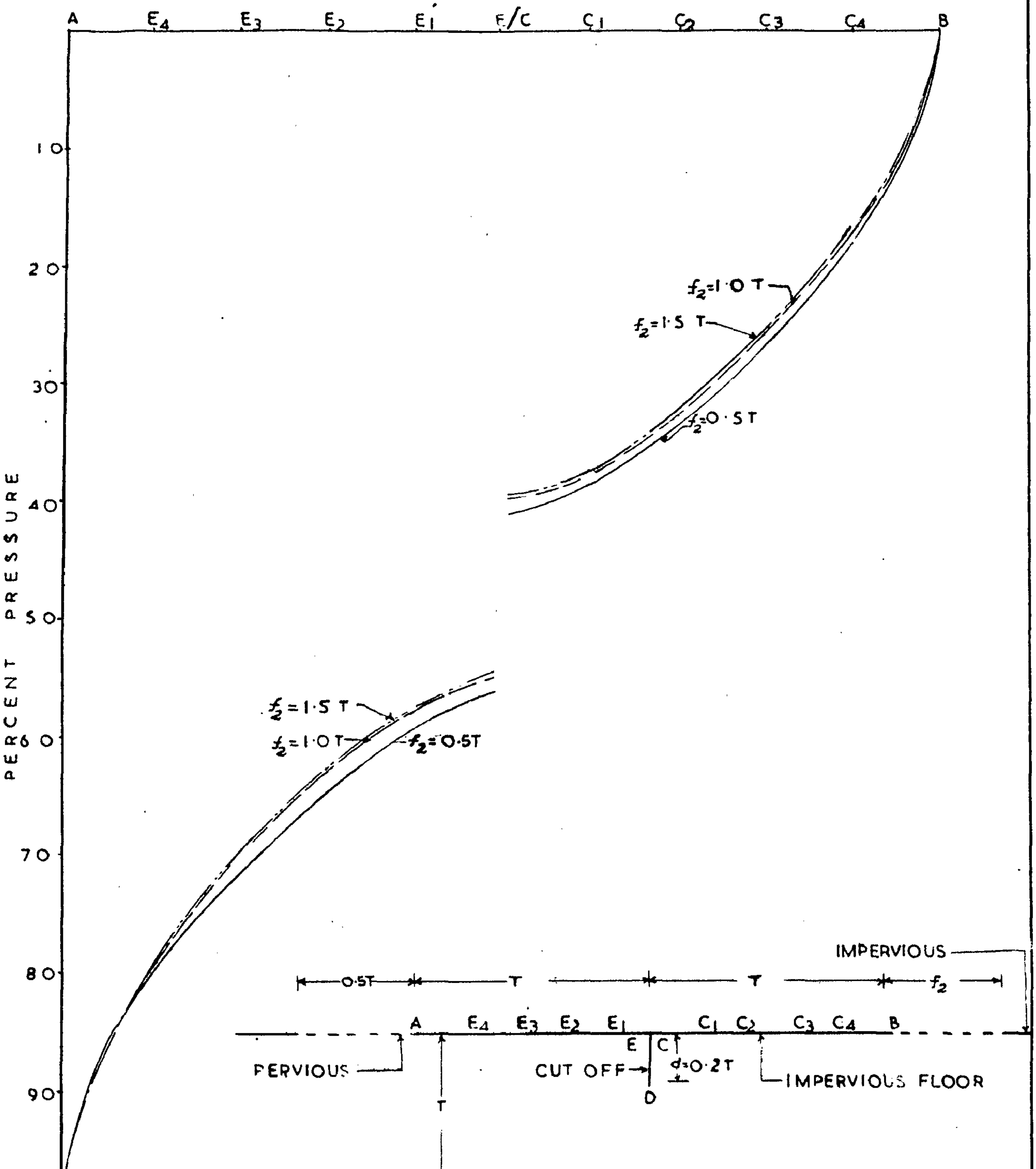

FIG. 4·4. UPLIFT PRESSURE ON THE BOTTOM OF FOUNDATION

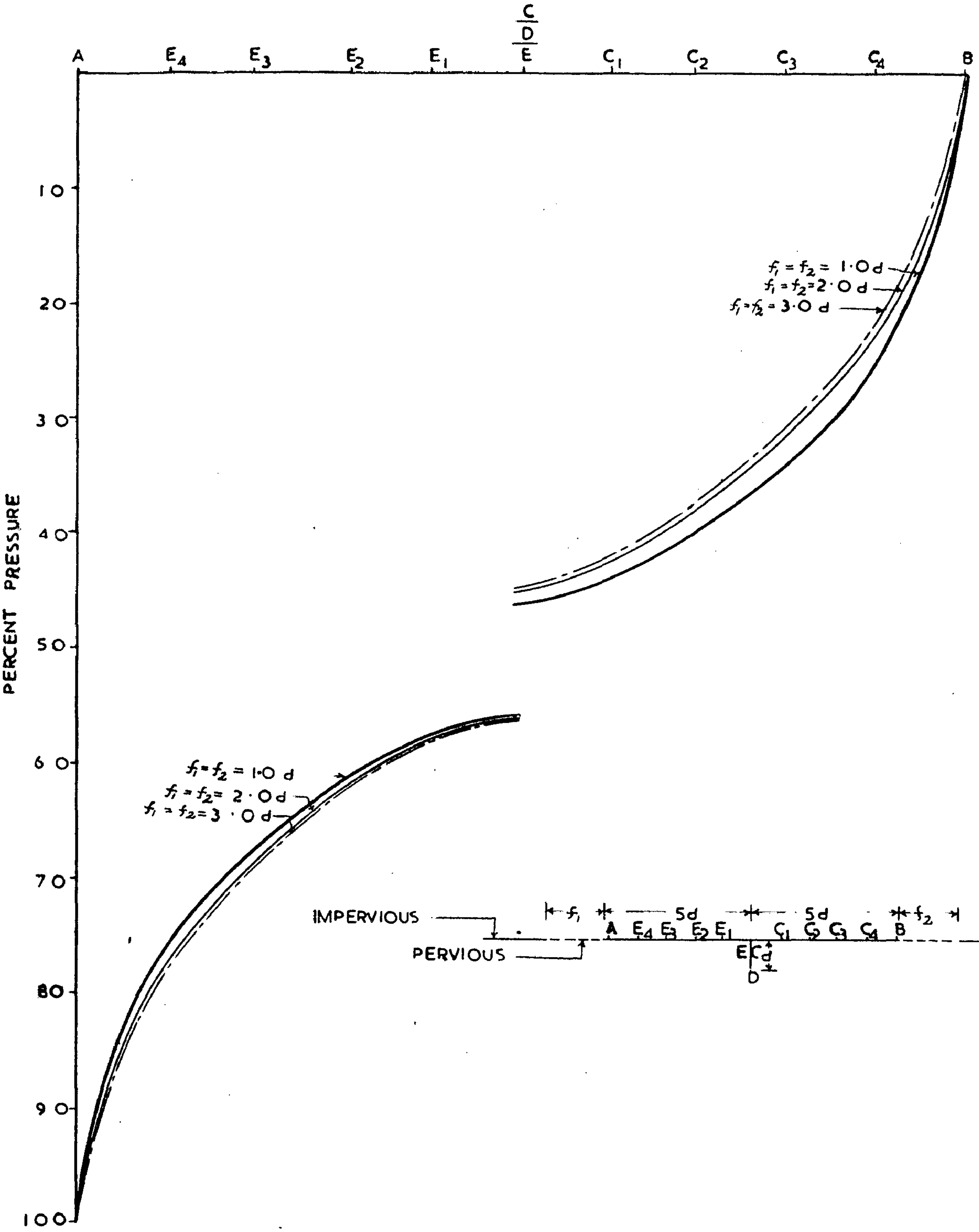

FIG.4.5. UPLIFT PRESSURE ON THE BOTTOM OF FOUNDATIONS

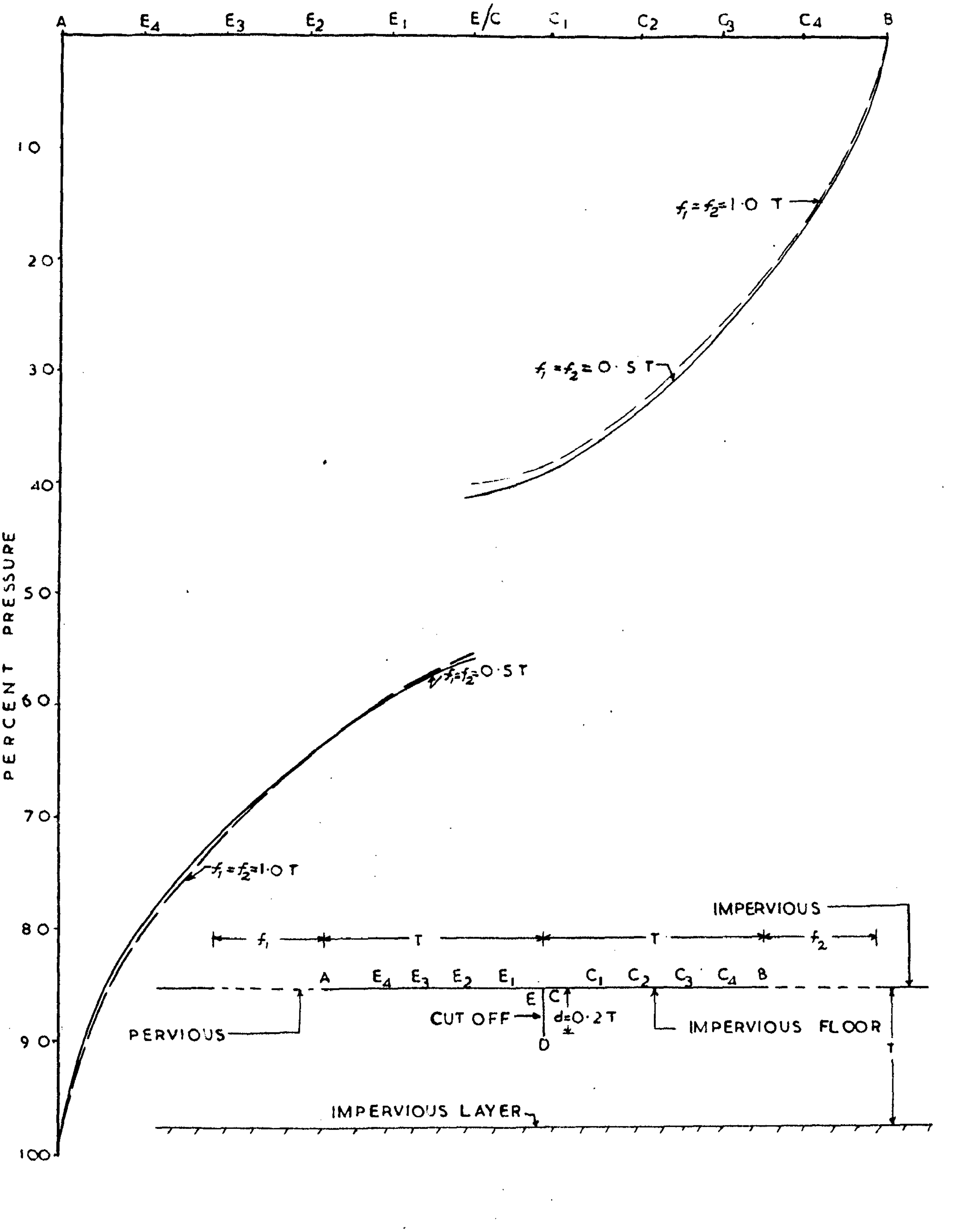

FIG. 4·6 UPLIFT PRESSURE ON THE BOTTOM OF FOUNDATION

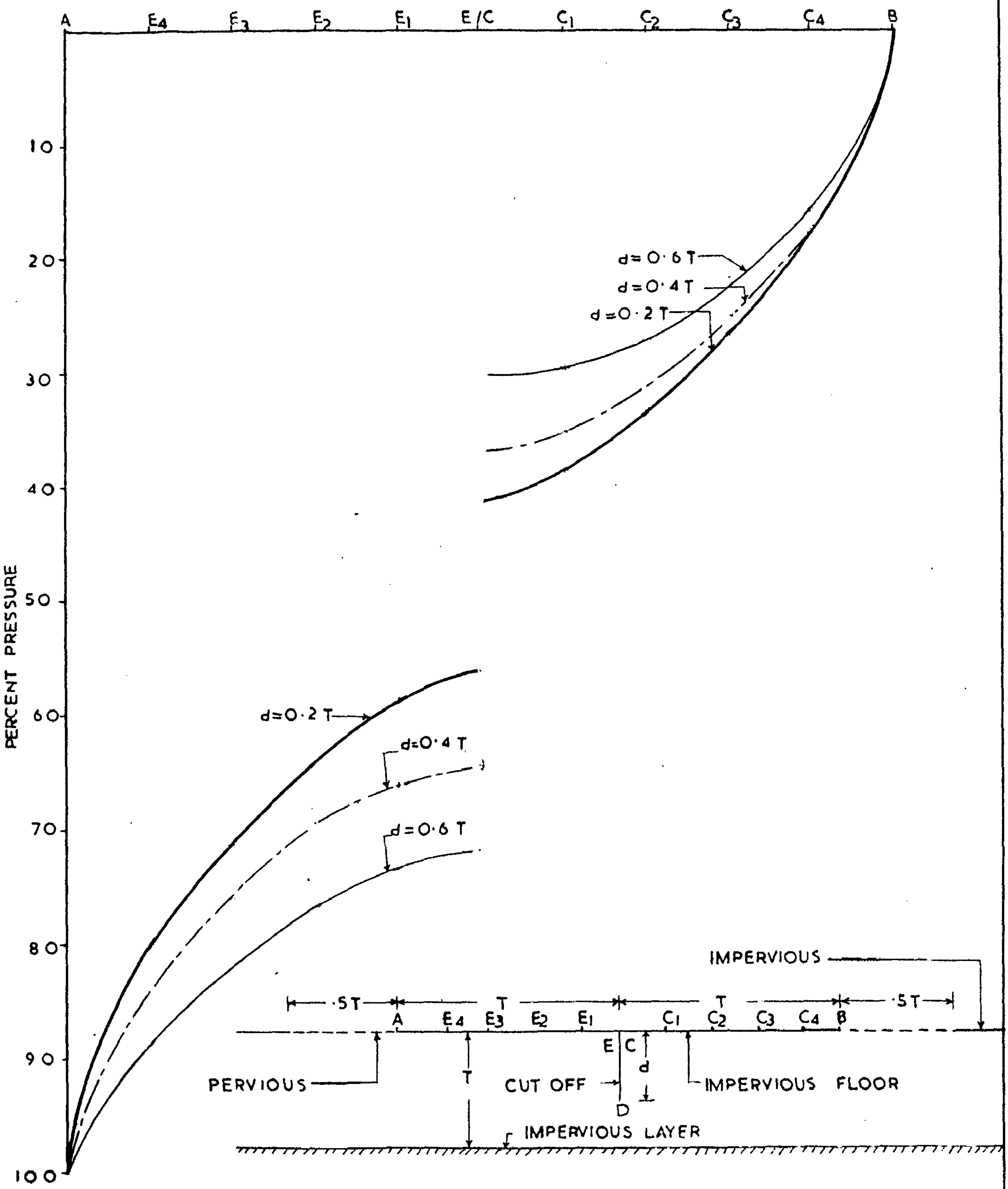

FIG. 4·7. UPLIFT PRESSURE ON THE BOTTOM OF FOUNDATIONS

CHAPTER FIVE

CHAPTER V

C O N C L U S I O N S

1. Mathematical solution is given to find out exit gradient and uplift pressure on the bottom of foundation of hydraulic structures founded on infinite and finite pervious flow zone.

2. Computations have been made to find out values of exit gradient and uplift pressures at the key points of an elementary profile of a hydraulic structures with various combinations of the variables involved. The computed results have been plotted in the form of curves. These curves can be used for practical design purposes.

3. The computed uplift pressures along the foundation of three simple and elementary profiles of hydraulic structures have been verified by Analog Field Plotter and on two dimensional electrical analogy model. The observed pressures are close to the calculated values.

4. The uplift pressures below the hydraulic structure increase with the increase of upstream pervious reach and decrease with the increase of downstream pervious reach.

5. For the structure founded on finite depth of pervious flow zone the uplift pressures below the floor upstream of the cut off increase whereas those on the floor downstream of cut off decrease with the increase in the depth of cut off.

TABLE I

EXPERIMENTAL VERIFICATION
UPLIFT PRESSURES ON THE BOTTOM OF FOUNDATION

Test Conditions :-

(i) Depth of permeable sub-soil extends upto infinity.

(ii) Depth of cut off = d.

(iii) Impervious floor length upstream of the cut off b1 = 5.0d

(iv) Impervious floor length downstream of the cut off b2 = 5.0d

(v) Length of upstream pervious bed f1 = 3.0d

(vi) Length of downstream pervious bed f2 = 3.0d

Sl. No.	Name of the points	Distance from D.S. edge of the structure	Uplift pressures in %		
			Calculated values	Observed on Electrical analogy model	Analog field plotter
1.	C_4	d	22.0	20.5	22.0
2.	C_3	2d	30.4	29.0	30.8
3.	C_2	3d	37.2	30.0	37.2
4.	C_1	4d	42.1	41.0	42.1
5.	C	5d	44.7	43.5	44.6
6.	D	5d	49.7	48.6	49.3
7.	E	5d	55.9	54.0	56.1
8.	E_1	6d	58.1	59.0	58.6
9.	E_2	7d	63.2	63.0	63.7
10.	E_3	8d	69.3	70.0	69.8
11.	E_4	9d	70.3	70.0	70.4

TABLE II

EXPERIMENTAL VERIFICATION
UPLIFT PRESSURES ON THE BOTTOM OF FOUNDATION

Test Conditions :-

(i)	Depth of pervious flow zone	T
(ii)	Depth of cut off	d = 0.4T.
(iii)	Impervious floor length upstream of the cut off b1	= 1.0T.
(iv)	Impervious floor length downstream of the cut off b2	= 1.0T.
(v)	Length of upstream pervious bed	f1 = 0.5T
(vi)	Length of downstream pervious bed	f2 = 0.5T

Sl. No.	Name of the points	Distance from D.S. edge of the structure	Uplift pressures %		
			Theoretically calculated values	By Electrical analogy experiments	By Analog field plotter
1.	C_4	.8T	17.8	19.0	18.2
2.	C_3	.6T	25.5	27.0	28.2
3.	C_2	.4T	30.0	32.0	30.6
4.	C_1	.2T	35.5	37.0	35.8
5.	C	T	36.9	37.0	36.0
6.	D	T	49.5	54.0	49.8
7.	E	T	64.1	65.2	63.9
8.	E_1	1.2T	65.8	67.0	65.4
9.	E_2	1.4T	69.2	71.0	69.2
10.	E_3	1.6T	75.9	77.0	75.3
11.	E_4	1.8T	83.8	83.6	83.3

TABLE XII

EXPERIMENTAL VERIFICATION
UPLIFT PRESSURES ON THE BOTTOM FOUNDATION

Test Conditions :-

(i)	Depth of pervious flow zone	= T
(ii)	Depth of cut off	d = 0.6T
(iii)	Impervious floor length upstream of the cut off b1	= 1.0T
(iv)	Impervious floor length downstream of the cut off b2	= 1.0T
(v)	Length of upstream pervious bed	f1 = 0.5T
(vi)	Length of downstream pervious bed	f2 = 0.5T

Sl. No.	Name of the points	Distance from D.S. edge of the structure	Uplift pressures %		
			Theoretically calculated value	By Electrical analogy experiments	By Analog field plotter
1.	C_4	.2T	15.8	17.0	16.0
2.	C_3	.4T	21.7	23.0	22.3
3.	C_2	.6T	27.1	28.1	27.0
4.	C_1	.8T	29.6	30.8	30.0
5.	C	T	30.2	32.0	31.3
6.	D	T	50.9	54.4	51.4
7.	E	T	71.6	71.0	70.8
8.	E_1	1.2T	73.8	73.0	72.8
9.	E_2	1.4T	76.6	76.1	75.8
10.	E_3	1.6T	79.0	81.0	79.2
11.	E_4	1.8T	88.5	87.0	88.2

A P P E N D I X I

APPENDIX I

NOTES ON ELLIPTIC FUNCTIONS

For finding a mathematical solution of problem of seepage below hydraulic structure with complex boundary conditions, use of elliptic integrals is made for solving the differential equation.

The existence of the integral of a continuous function is assured; however, it does not necessarily follow that the integral can be expressed by elementary functions alone. In ground water problems, as a consequence of the Schwarz-Christoffel transformations, such functions are more commonly encountered.

Liouville was the first to demonstrate the existence of genuine elliptic integrals viz. elliptic integrals that can not be expressed in terms of elementary functions. The main classical work on the subject is Legendre's "Traite des fonctions elliptiques" in which the analysis of all elliptic integrals is reduced to the following three normal forms

$$F(x, k) = \int_0^x \frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}} \qquad \ldots.(1a)$$

$$E(x, k) = \int_0^x \frac{x^2 \cdot dx}{\sqrt{(1-x^2)(1-k^2x^2)}} \qquad \ldots.(1b)$$

$$\Pi(x, k, n) = \int_0^x \frac{dx}{(x^2+n)\sqrt{(1-x^2)(1-k^2x^2)}} \qquad \ldots.(1c)$$

These basic types are termed as first, second and third kinds. Ready made solutions are available for their calculations, as a function of x, k and n. For the modulus, k the limits of the tabulated values are $0 \leq k^2 \leq 1.0$

As regards x, it may have any value (real, imaginary, or complex), but in usual tables real values only are listed, for $0 \leq x^2 \leq 1.0$

If the upper limit of integration is made $x = 1.0$, the elliptic integral is described as complete i.e. considering equation (1a) it becomes

$$K = F(x, k) = \int_0^x \frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}} \qquad \text{....(2a)}$$

and

$$K' = F(x, k') = \int_0^x \frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}} \qquad \text{....(2b)}$$

where k' is the co-modulus or complimentary modulus defined by

$$k' = \sqrt{1 - k^2}$$

The "Jacobi's functions" are defined as follows considering the elliptic integrals of first kind

let

$$u = F(x, k)$$

$$= \int_0^x \frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}} \qquad \text{....(3a)}$$

substituting

$$\sin \phi = x \qquad \text{....(3b)}$$

$$u = \int_0^\phi \frac{d\phi}{\sqrt{1-k^2\sin^2\phi}} \qquad \text{....(3c)}$$

Jacobi uses the term "amplitude" to denote the function inverse to this integral and he uses the symbol $\phi = \text{am}\, u$, to represent it.

Then, according to equation (3.b)

$$x = \operatorname{Sin} am\ U$$

In referring to elliptic functions, a more common notation is used, viz.,

$$Sn\ U = \operatorname{Sin} am\ U \text{ (sinus amplitudinis U)} \qquad ....(4)$$

In addition to this Jacobi introduced two other functions, which are correlated with $x = \sin am\ u$ in the following manner

$$\sqrt{1 - x^2} = \cos am\ u = Cn\ U$$

$$\sqrt{1 - \quad x^2} = \Delta\ am\ u = dn\ u$$

These functions are termed "Jacobi's elliptic functions".

PROPERTIES OF JACOBI'S ELLIPTIC FUNCTIONS

Referring to equation (3.a), since U = 0, when x=0

$$Sn = 0$$

$$\text{or} \quad Sn^{-1}0 = 0 \qquad ....(5a)$$

Considering equation (3.a), since U = K, when x=1

$$Sn\ K = 1$$

$$\text{or} \quad Sn^{-1}1 = K \qquad ....(5.b)$$

In the same way, $u = iK'$, when $x = \infty$

$$Sn\ iK' = \infty$$

$$\text{or} \quad Sn^{-1}\infty = i\,K' \qquad ....(5.c)$$

Similarly $W = K + iK'$, when $x = 1/k^2$

$$Sn\,(K + iK') = 1/k^2$$

$$Sn^{-1}(1/k^2) = K + iK' \qquad \ldots\ldots(5.d.)$$

These four points given by 5.a, 5.b, 5.c, and 5.d are the four corners in order of the rectangle in W plane (see main text)

APPENDIX II

APPENDIX II

REPRESENTATION OF A RECTANGLE UPON A HALF PLANE

The w-plane represented in figure 4.4 is transformed into the ζ-plane as indicated in fig. 4c. Corresponding points in the w and ζ-planes are indicated by the same names.

The integral equation (2.10) corresponding to zone AB in the ζ-plane as given in the text, is

$$W = A \int \frac{d\zeta}{\sqrt{(\zeta_G-\zeta)(\zeta_B-\zeta)(\zeta-\zeta_A)(\zeta-\zeta_F)}} \qquad ....(1)$$

where $\zeta_G > \zeta_B \geqslant \zeta \geqslant \zeta_A > \zeta_F$

In order to integrate this we first make a bilinear transformation of the ζ-plane into t-plane. There is just one bilinear transformation that maps three given distinct points t1, t2 and t3 in w-plane respectively. The transformation is given by

$$\frac{(t - t1)\,(t2 - t3)}{(t - t3)\,(t2 - t1)} = \frac{(\zeta-\zeta_1)\,(\zeta_2-\zeta_3)}{(\zeta-\zeta_3)\,(\zeta_2-\zeta_1)} \qquad ....(2)$$

The corresponding points in t and ζ-plane in our case are as below

$$\begin{aligned} t1 &= 0 & \zeta_1 &= \zeta_B \\ t2 &= 1 & \zeta_2 &= \zeta_A \\ t3 &= \alpha & \zeta_3 &= \zeta_G \\ t4 &= 1/k^2 & \zeta_4 &= \zeta_F \end{aligned} \qquad ....(3)$$

The transformation then, can be written

$$t = \frac{(\zeta-\zeta_B)(\zeta_A-\zeta_G)}{(\zeta-\zeta_G)(\zeta_A-\zeta_B)}$$

and $t_A = 1/k^2 = \dfrac{(\zeta_F-\zeta_B)(\zeta_A-\zeta_G)}{(\zeta_F-\zeta_G)(\zeta_A-\zeta_B)}$

from which it follows

$$1-t = \frac{(\zeta_A-\zeta)(\zeta_B-\zeta_G)}{(\zeta-\zeta_G)(\zeta_A-\zeta_B)}$$

and $1-k^2t = \dfrac{(\zeta_G-\zeta_B)(\zeta-\zeta_F)}{(\zeta_F-\zeta_B)(\zeta-\zeta_G)}$

and $dt = \dfrac{(\zeta_G-\zeta_B)(\zeta_F-\zeta_G)}{(\zeta_F-\zeta_B)(\zeta-\zeta_G)}\, d\zeta$

and hence after little reduction equation (1) becomes

$$W = \frac{2A}{\sqrt{(\zeta_B-\zeta_F)(\zeta_G-\zeta_A)}} \int_0^t \frac{dt}{\sqrt{4t(1-t)(1-k^2t)}} + B \qquad \ldots\ldots(4)$$

where integration constant B is zero in this case

If we put $\mu = \frac{1}{2}\sqrt{(\zeta_B-\zeta_F)(\zeta_G-\zeta_A)}$

$$\frac{\mu}{A} W = \int_0^t \frac{dt}{\sqrt{4t(1-t)(1-k^2t)}} \qquad \ldots\ldots(5)$$

If we introduce intermediate variable x,

such that $x = \sqrt{t}$, the above equation (5) becomes

$$\mu/A\, W = \int_0^x \frac{dx}{\sqrt{(1-x^2)(1-x^2k^2)}}$$

Since the above equation (6) is of the form of elliptic integral of the first kind, using the results of equation(4) of Appendix I we can write

$$Sn\ {}^{M}\!/_{A}\ W = x$$

or $$Sn^2\ {}^{M}\!/_{A}\ W = x^2$$

On substituting the value of $x^2 = t$, we arrive at

$$Sn^2\ {}^{M}\!/_{A}\ W = \frac{(\zeta - \zeta_B)(\zeta_A - \zeta_G)}{(\zeta - \zeta_G)(\zeta_A - \zeta_B)} \qquad \ldots\ldots(7)$$

The above equation (7) is the same as equation (2.15) and equation (2.43) of the text.

A P P E N D I X   III

APPENDIX III

NOTATIONS

The following symbols are used in this dissertation:

b_1, b_2 = upstream and downstream impervious floor lengths respectively.

d = depth of cut offs

f_1, f_2 = upstream and downstream pervious lengths respectively.

k = modulus of the elliptic integral

k' = co-modulus defined as $\sqrt{1-k^2}$

q = seepage discharge per unit width

t = $r + is$, complex variable representing auxiliary semi-infinite plane;

U_x, U_y = velocity components in x and y directions respectively

U = complex potential, i.e. $\phi' + i\psi'$, complex plane representing rectangular flow field.

Z = complex potential, i.e. $x + iy$, complex plane representing physical plane;

A, C = Magnification factors in Schwartz-christoffel Transformation

B, D = constants of integration

H = Head, difference between upstream and downstream water levels.

L_1, L_2 = transformation parameters.

K, K' = complete elliptic integrals of the first kind, with modulus k and k', respectively.

T = Depth of permeable flow zone

σ, β, γ = transformation parameters.

ζ = $\xi + i\eta$, complex variable representing auxiliary semi-infinite plane

ξ, η = real and imaginary axes in ζ-plane

K = permeability coefficient,

ϕ' = potential function

ϕ' = reduced value of ϕ'

ψ' = stream function

ψ = reduced value of ψ', and

Sn = elliptic sine

A N N E X U R E   I

```
C  C  A S CHAHLA Z IRI  UPLIFT PRESSURES FOR BOUNDRY CONDITION A
      DIMENSION AM(20),SN(11,20),U(11),SNFX(11)
      READ 10,(AM(J),J=1,20)
10    FORMAT (10F7.5)
      READ 20, ((SN(I,J),J=1,20),I=1,11)
20    FORMAT  (10F7.5)
      READ 30, (U(I),I=1,11)
30    FORMAT  (11F6.4)
C     UFOR=B1/D
C     DFOR=B2/D
C     EL IS THE LOCATION OF KEY POINT IN  T-PLANE
200   READ 700,EL,UFOR
700   FORMAT(2F10.4)
C     UFIR=F1/D
C     DFIR=F2/D
40    UFIR=1.0
50    DFIR=1.0
      ELON=SQRTF(1.0+(UFOR)**2)
      ELT=1.0
      DON=SQRTF(1.0+(UFOR+UFIR)**2)
60    DT=SQRTF(1.0+(DFIR)**2)
      BON=1.0-2.0*(DON-ELON)/(DON+DT)
      BT=1.0-2.0*(DT-ELT)/(DON+DT)
      T=1.0-2.0*(DT-EL)/(DON+DT)
      AMX =SQRTF(2.0*(BON+BT)/(1.0+BON+BT+BON*BT))
      SNFU=SQRTF((BT+BON*BT-T-T*BON)/(BON+BT-BON*T-BT*T))
      IF (AMX-.9999)  11,11,12
12    J=20
      GO TO 2
11    DO1    J=1,20
      IF (AMX-AM(J))  3,2,1
 1    CONTINUE
 2    K=J
      DO 4  I=1,11
      IF (SNFU-SN(I,K)) 6,5,4
 4    CONTINUE
 5    M=I
      PHI=U(M)
      GO TO 100
 6    M=I
      PHI=U(M-1)+(U(M)-U(M-1))/(SN(M,K)-SN(M-1,K))*(SNFU-SN(M-1,K))
      GO TO 100
 3    K=J
      DO 7  I=1,11
      A=AMX-AM(K-1)
      SNFX(I)=SN(I,K-1)+(SN(I,K)-SN(I,K-1))/(AM(K)-AM(K-1))*A
 7    CONTINUE
 8    DO 9  I=1,11
      IF (SNFU-SNFX(I)) 14,5,9
 9    CONTINUE
14    M=I
      PHI=U(M-1)+(U(M)-U(M-1))/(SNFX(M)-SNFX(M-1))*(SNFU-SNFX(M-1))
100   PUNCH  300,EL,UFOR,UFIR,DFIR,AMX,SNFU,PHI
 300  FORMAT (7F10.7)
```

```
      IF(DFIR-6.0)70,80,80
70    DFIR=DFIR+1.0
      GO TO 60
  80  IF(UFIR-3.0)90,200,200
  90  UFIR=UFIR+1.0
      GO TO 50
      END
 00175  00872  01737  02588  03420  04226  05000  05736  06428  07071
 07660  08192  08660  09063  09397  09659  09848  09962  09999  099999
0000000
0000000
 01564  01567  01576  01591  01612  01640  01676  01720  01773  01838
 01917  02012  02128  02272  02456  02702  03054  03655  04956  05947
 03090  03096  03112  03139  03178  03229  03293  03373  03469  03585
 03724  03891  04093  04340  04649  05047  05595  06452  07958  08787
 04540  04547  04568  04602  04652  04717  04800  04900  05021  05166
 05336  05539  05779  06066  06415  06846  07403  08184  09262  09674
 05878  05887  05907  05944  05997  06066  06152  06257  06382  06529
 06701  06901  07134  07405  07723  08099  08552  09121  09745  09917
 07071  07078  07098  07132  07180  07243  07321  07414  07525  07654
 07802  07972  08165  08384  08632  08913  09231  09591  09914  09978
 08093  08095  08112  08138  08176  08225  08285  08357  08442  08539
 08649  08778  08912  09065  09234  09417  09613  09817  09971  09995
 08910  08914  08924  08942  08966  08998  09037  09083  09137  09198
 09267  09343  09427  09517  09615  09717  09822  09924  09991  099986
 09511  09512  09517  09526  09538  09554  09573  09595  09621  09650
 09683  09719  09758  09799  09843  09887  09932  09973  099975 099997
 09877  09877  09879  09881  09884  09888  09893  09899  09906  09914
 09922  09931  09941  09952  09963  09974  09985  09994  099993 099999
 10     10     10     10     10     10     10     10     10     10
 10     10     10     10     10     10     10     10     10     10
  0      1      2      3      4      5      6      7      8      9     10
0000000           1000
  00000           1250
   0000           16667
   0000000        2500
   00000          5000
   000           10000
00000            20000
    -10           10000
    -100          125
    -1000         16667
    -100          2500
    -100          500
    -10          10000
    -100         200000
```

```
C  C  A S CHAWLA IRI Z EXIT GRADIENT- BOUNDRY CONDITION A
      DIMENSION  AM(10),CK(10)
      READ 10, (AM(J),J=1,10)
10    FORMAT (10F7.5)
      READ 10, (CK(J),J=1,10)
C     UFOR=B1/D
C     DFOR=B2/D
 200  READ 700,UFOR
 700  FORMAT (F10.4)
C     UFIR=F1/D
C     DFIR=F2/D
 30   UFIR=1.0
 40   DFIR=1.0
      ELON=SQRTF(1.0+(UFOR)**2)
      ELT=1.0
      DON=SQRTF(1.0+(UFOR+UFIR)**2)
60    DT=SQRTF(1.0+(DFIR)**2)
      BON=1.0-2.0*(DON-ELON)/(DON+DT)
      BT=1.0-2.0*(DT-ELT)/(DON+DT)
      AMX =SQRTF(2.0*(BON+BT)/(1.0+BON+BT+BON*BT))
      DO 1  J=1,10
      IF (AMX-AM(J))  3,2,1
1     CONTINUE
2     K=J
      CKX=CK(K)
3     K=J
      CKX=CK(K-1)+(CK(K)-CK(K-1))/(AM(K)-AM(K-1))*(AMX-AM(K-1))
      S=SQRTF((DT+ELON)/(2.0*(DT-1.0)*(ELON+1.0)))
      SKR=S/CKX
300   FORMAT (7F10.7)
      PUNCH 300,UFOR,UFIR,DFIR,AMX,CKX,S,SKR
      IF(DFIR-4.0) 70,80,80
 70   DFIR=DFIR+1.0
      GO TO 60
  80  IF(UFIR-3.0)90,100,100
  90  UFIR=UFIR+1.0
      GO TO 40
 100  GO TO 200
      END
 08192  08660  09063  09397  09659  09848  09962  09986  099985 099993


 20347  21565  23088  25046  27601  31534  38317  433P7  56349  57914
    20000
    3000
    4000
    50000
     10000
     5000
    1000
```

```
C  C  A S CHAWLA Z  PRESSURE VARIATION   FOR BOUNDRY CONDITION A
      DIMENSION AM(20),SN(11,20),U(11),SNFX(11)
      READ 10,(AM(J),J=1,20)
10    FORMAT (10F7.5)
      READ 20, ((SN(I,J),J=1,20),I=1,11)
20    FORMAT  (10F7.5)
      READ 30, (U(I),I=1,11)
30    FORMAT  (11F6.4)
C     EL IS THE LOCATION OF KEY POINT IN  T-PLANE
200   READ 700,EL
700   FORMAT (F10.6)
C     UFOR=B1/D
C     DFOR=B2/D
C     UFIR=F1/D
C     DFIR=F2/D
      UFOR=5.0
      DFOR=5.0
      UFIR=1.0
 60   DFIR=1.0
 50   ELON=SQRTF(1.0+(UFOR)**2)
      ELT=SQRTF(1.0+DFOR**2)
      DON=SQRTF(1.0+(UFOR+UFIR)**2)
      DT=SQRTF(1.0+(DFOR+DFIR)**2)
      BON=1.0-2.0*(DON-ELON)/(DON+DT)
      BT=1.0-2.0*(DT-ELT)/(DON+DT)
      T=1.0-2.0*(DT-EL)/(DON+DT)
      AMX =SQRTF(2.0*(BON+BT)/(1.0+BON+BT+BON*BT))
      ENU=BT+BON*BT-T-T*BON
      DENU=BON+BT-BON*T-BT*T
      SNFU=SQRTF(ENU/DENU)
      IF (AMX-.9999)  11,11,12
12    J=20
      GO TO 2
11    DO1   J=1,20
      IF (AMX-AM(J))  3,2,1
 1    CONTINUE
 2    K=J
      DO 4  I=1,11
      IF (SNFU-SN(I,K)) 6,5,4
 4    CONTINUE
 5    M=I
      PHI=U(M)
      GO TO 100
 6    M=I
      PHI=U(M-1)+(U(M)-U(M-1))/(SN(M,K)-SN(M-1,K))*(SNFU-SN(M-1,K))
      GO TO 100
 3    K=J
      DO 7  I=1,11
      A=AMX-AM(K-1)
      SNFX(I)=SN(I,K-1)+(SN(I,K)-SN(I,K-1))/(AM(K)-AM(K-1))*A
 7    CONTINUE
```

```
 8      DO 9  I=1,11
        IF (SNFU-SNFX(I)) 14,5,9
 9      CONTINUE
14      M=I
        PHI=U(M-1)+(U(M)-U(M-1))/(SNFX(M)-SNFX(M-1))*(SNFU-SNFX(M-1))
100     PUNCH 300,EL,UFIR,DFIR,AMX,SNFU,PHI
 300    FORMAT (6F10.7)
        IF(DFIR-3.0) 15,16,16
 15     DFIR=DFIR+1.0
        GO TO 50
 16     IF(UFIR-3.0) 17,18,18
17      UFIR=UFIR+1.0
        GO TO 60
18      GO TO 200
        END
 00175   00872   01737   02588   03420   04226   05000   05736   06428   07071
 07660   08192   08660   09063   09397   09659   09848   09962   09999   099999
00000000
0000000
 01564   01567   01576   01591   01612   01640   01676   01720   01773   01838
 01917   02012   02128   02272   02456   02702   03054   03655   04956   05947
 03090   03096   03112   03139   03178   03229   03293   03373   03469   03585
 03724   03891   04093   04340   04649   05047   05595   06452   07958   08787
 04540   04547   04568   04602   04652   04717   04800   04900   05021   05166
 05336   05539   05779   06066   06415   06846   07403   08184   09262   09674
 05878   05887   05907   05944   05997   06066   06152   06257   06382   06529
 06701   06901   07134   07405   07723   08099   08552   09121   09745   09917
 07071   07078   07098   07132   07180   07243   07321   07414   07525   07654
 07802   07972   08165   08384   08632   08913   09231   09591   09914   09978
 08090   08095   08112   08138   08176   08225   08285   08357   08442   08539
 08649   08773   08912   09065   09234   09417   09613   09817   09971   09995
 08910   08914   08924   08942   08966   08998   09037   09083   09137   09198
 09267   09343   09427   09517   09615   09717   09822   09924   09991   099986
 09511   09512   09517   09526   09538   09554   09573   09595   09621   09650
 09683   09719   09758   09799   09843   09887   09932   09973   099975  099997
 09877   09877   09879   09881   09884   09888   09893   09899   09906   09914
 09922   09931   09941   09952   09963   09974   09985   09994   099993  099999
 10      10      10      10      10      10      10      10      10      10
 10      10      10      10      10      10      10      10      10      10
  0      1       2       3       4       5       6       7       8       9      10
 0000000000
  -1000000
  -1414214
  -2236068
  -3162278
  -4123106
  +1000000
  +1414214
  +2236068
  +3162278
  +4123106
```

```
C  C  A S CHAWLA Z IRI  UPLIFT PRESSURES FOR BOUNDRY CONDITION B
      DIMENSION AM(20),SN(11,20),U(11),F(5),X(5),D(5),SNFX(11)
      READ 10, (AM(J),J=1,20)
10    FORMAT (10F7.5)
      READ 20, ((SN(I,J),J=1,20),I=1,11)
20    FORMAT  (10F7.5)
      READ 30, (U(I),I=1,11)
30    FORMAT  (11F6.4)
C     F(1)=B1/T
C     F(2)=B2/T
C     UFIR=F1/T
C     DFIR=F2/T
C     CUT=D/T
C     EMU IS THE LOCATION OF KEY POINT IN  T-PLANE
      F(1)=0.5
114   F(2)=0.0
      CUT=0.2
54    UFIR=0.5
53    DFIR=0.5
52    F(3)=F(1)+UFIR
      F(4)=F(2)+DFIR
      S=SINF(1.5708*CUT)
      C=COSF(1.5708*CUT)
      DO 51  I=1,4
      X(I)=3.1416*F(I)
      D(I)=SQRTF(S*S/(C*C)+((EXPF(X(I))-1.0)/(EXPF(X(I))+1.0))**2)
51    CONTINUE
      BN=1.-2.*(D(3)-D(1))/(D(3)+D(4))
      BT=1.-2.*(D(4)-D(2))/(D(3)+D(4))
115   EMU=-S/C
5*    T=1.-2.*(D(4)-EMU)/(D(3)+D(4))
      AMX=SQRTF(2.*(BN+BT)/((1.+BN)*(1.+BT)))
      SNFU=SQRTF(((1.+BN)*(BT-T))/((BN+BT)*(1.-T)))
      IF(SNFU)  62,62,63
62    PHI=0.000
      GO TO 100
63    IF(AMX-.99999)  11,12,12
12    J=20
      GO TO 2
11    DO1   J=1,20
      IF(AMX-AM(J))  3,2,1
 1    CONTINUE
 2    K=J
      DO 4  I=1,11
      IF (SNFU-SN(I,K)) 6,5,4
 4    CONTINUE
 5    M=I
      PHI=U(M)
      GO TO 100
 6    M=I
      PHI=U(M-1)+(U(M)-U(M-1))/(SN(M,K)-SN(M-1,K))*(SNFU-SN(M-1,K))
      GO TO 100
 3    K=J
      DO 7  I=1,11
```

```
      A=AMX-AM(K-1)
      SNFX(I)=SN(I,K-1)+(SN(I,K)-SN(I,K-1))/(AM(K)-AM(K-1))*A
 7    CONTINUE
 8    DO 9  I=1,11
      IF (SNFU-SNFX(I)) 14,5,9
 9    CONTINUE
14    M=I
      PHI=U(M-1)+(U(M)-U(M-1))/(SNFX(M)-SNFX(M-1))*(SNFU-SNFX(M-1))
100   PUNCH 60,FMU,F(1),F(2),UFIR,DFIR,CUT,BN,AMX,SNFU,PHI
60    FORMAT  (F10.3,4F6.3,4F8.5,F6.3)
      IF(DFIR-1.5)    31,32,32
31    DFIR=DFIR+0.5
      GO TO 52
32    IF(UFIR-1.5)    33,34,34
33    UFIR=UFIR+0.5
      GO TO 53
34    IF(CUT-0.8)    35,36,36
35    CUT=CUT+0.2
      GO TO 54
36    IF(F(1)-1.0)  111,112,112
111   F(1)=F(1)+0.5
      GO TO 114
112   IF(F(1)-3.0)  113,49,49
113   F(1)=F(1)+1.0
      GO TO 114
49    STOP
      END
 00175  00872  01737  02588  03420  04226  05000  05736  06428  07071
 07660  08192  08660  09063  09397  09659  09848  09962  09999  099999
00000000
0000000
 01564  01567  01576  01591  01612  01640  01676  01720  01773  01838
 01917  02012  02128  02272  02456  02702  03054  03655  04956  05947
 03090  03096  03112  03139  03178  03229  03293  03373  03469  03585
 03724  03891  04093  04340  04649  05047  05595  06452  07958  08787
 04540  04547  04568  04602  04652  04717  04800  04900  05021  05166
 05336  05539  05779  06066  06415  06846  07403  08184  09262  09674
 05878  05887  05907  05944  05997  06066  06152  06257  06382  06529
 06701  06901  07134  07405  07723  08099  08552  09121  09745  09917
 07071  07078  07098  07132  07180  07243  07321  07414  07525  07654
 07802  07972  08165  08384  08632  08913  09231  09591  09914  09978
 08090  08095  08112  08138  08176  08225  08285  08357  08442  08539
 08649  08773  08912  09065  09234  09417  09613  09817  09971  09995
 08910  08914  08924  08942  08966  08998  09037  09083  09137  09198
 09267  09343  09427  09517  09615  09717  09822  09924  09991  099986
 09511  09512  09517  09526  09538  09554  09573  09595  09621  09650
 09683  09719  09758  09799  09843  09887  09932  09973  099975 099997
 09877  09877  09879  09881  09884  09888  09893  09899  09906  09914
 09922  09931  09941  09952  09963  09974  09985  09994  099993 099999
 10     10     10     10     10     10     10     10     10     10
 10     10     10     10     10     10     10     10     10     10
  0      1      2      3      4      5      6      7      8      9     10
```

```
C  C  A S CHAWLA Z IRI EXIT GRADIENT      FOR BOUNDRY CONDITION B
      DIMENSION  X(5),F(5),AM(14),CK(14),D(5)
      READ 10, (AM(J),J=1,14)
10    FORMAT  (7F10.7)
      READ 10, (CK(J),J=1,14)   /
C     F(1)=B1/T
C     F(2)=B2/T
C     UFIR=F1/T
C     DFIR=F2/T
C     CUT=D/T
      F(1)=0.5
114   F(2)=0.0
      CUT=0.2
54    UFIR=0.5
53    DFIR=0.5
52    F(3)=F(1)+UFIR
      F(4)=F(2)+DFIR
      S=SINF(1.5708*CUT)
      C=COSF(1.5708*CUT)
      DO 51  I=1,4
      X(I)=3.1416*F(I)
      D(I)=SQRTF(S*S/(C*C)+((EXPF(X(I))-1.0)/(EXPF(X(I))+1.0))**2)
51    CONTINUE
      BN=1.-2.*(D(3)-D(1))/(D(3)+D(4))
      BT=1.-2.*(D(4)-D(2))/(D(3)+D(4))
55    AMX=SQRTF(2.*(BN+BT)/((1.+BN)*(1.+BT)))
      IF (AMX-1.000)  5,6,6
6     K=14
      GO TO 2
5     K=1
11    IF(AMX-AM(K))  3,2,1
1     K=K+1
      IF(K-14)  11,11,2
2     CKX=CK(K)
      GO TO 4
3     CKX=CK(K-1)+(CK(K)-CK(K-1))*(AMX-AM(K-1))/(AM(K)-AM(K-1))
4     ENU=2.0*C*(D(4)+D(1))/S
      DENU=(D(4)-S/C)*(D(1)+S/C)
      S=0.7854*SQRTF(ENU/DENU)
      G=S/CKX
      PUNCH 60,F(1),F(2),UFIR,DFIR,CUT,BN,AMX,CKX,S,G
60    FORMAT  (5F5.2,2F8.5,F12.5,2F8.5)
      IF(DFIR-1.5) 31,32,32
 31   DFIR=DFIR+0.5
      GO TO 52
32    IF(UFIR-1.5) 33,34,34
33    UFIR=UFIR+0.5
      GO TO 53
34    IF(CUT-0.8)35,36,36
35    CUT=CUT+0.2
      GO TO 54
36    IF(F(1)-1.0) 111,112,112
111   F(1)=F(1)+0.5
      GO TO 114
112   IF(F(1)-5.0)  113,49,49
113   F(1)=F(1)+1.0
      GO TO 114
49    STOP
      END
  .7660     .8192     .8660     .9063     .9397     .9659     .9848
  .996195   .99863    .99985    .99993    .9999756  .9999985 1.0000000
 1.9356    2.0347    2.1565    2.3088    2.5046    2.7681    3.1534000
```

```
C  C  A S CHAWLA Z PRESSURE VARIATION     FOR BOUNDRY CONDITION B
      DIMENSION AM(20),SN(11,20),U(11),F(5),X(5),D(5),SNFX(11)
      READ 10, (AM(J),J=1,20)
10    FORMAT (10F7.5)
      READ 20, ((SN(I,J),J=1,20),I=1,11)
20    FORMAT  (10F7.5)
      READ 30, (U(I),I=1,11)
30    FORMAT  (11F6.4)
C     F(1)=B1/T
C     F(2)=B2/T
C     UFIR=F1/T
C     DFIR=F2/T
C     CUT=D/T
C     EMU IS THE LOCATION OF KEY POINT IN  T-PLANE
      F(1)=1.0
      F(2)=1.0
      CUT=0.2
54    UFIR=0.5
53    DFIR=0.5
52    F(3)=F(1)+UFIR
      F(4)=F(2)+DFIR
      S=SINF(1.5708*CUT)
      C=COSF(1.5708*CUT)
      TAN=S/C
      DO 51  I=1,4
      X(I)=3.1416*F(I)
      D(I)=SQRTF(S*S/(C*C)+((EXPF(X(I))-1.0)/(EXPF(X(I))+1.0))**2)
51    CONTINUE
      BN=1.-2.*(D(3)-D(1))/(D(3)+D(4))
      BT=1.-2.*(D(4)-D(2))/(D(3)+D(4))
      L=1
      Z=0.8
56    Y=3.1415927*Z
      EMU=SQRTF(TAN*TAN+((EXPF(Y)-1.0)/(EXPF(Y)+1.0))**2)
55    T=1.-2.*(D(4)-EMU)/(D(3)+D(4))
      AMX=SQRTF(2.*(BN+BT)/((1.+BN)*(1.+BT)))
      SNFU=SQRTF(((1.+BN)*(BT-T))/((BN+BT)*(1.-T)))
      IF(AMX-.99999)  11,12,12
12    J=20
      GO TO 2
11    DO1    J=1,20
      IF(AMX-AM(J))  3,2,1
 1    CONTINUE
 2    K=J
      DO 4  I=1,11
      IF (SNFU-SN(I,K)) 6,5,4
 4    CONTINUE
 5    M=I
      PHI=U(M)
      GO TO 100
 6    M=I
      PHI=U(M-1)+(U(M)-U(M-1))/(SN(M,K)-SN(M-1,K))*(SNFU-SN(M-1,K))
      GO TO 100
 3    K=J
      DO 7  I=1,11
      A=AMX-AM(K-1)
      SNFX(I)=SN(I,K-1)+(SN(I,K)-SN(I,K-1))/(AM(K)-AM(K-1))*A
 7    CONTINUE
 8    DO 9  I=1,11
      IF (SNFU-SNFX(I)) 14,5,9
 9    CONTINUE
14    M=I
```

```
      PHI=U(M-1)+(U(M)-U(M-1))/(SNFX(M)-SNFX(M-1))*(SNFU-SNFX(M-1))
100   PUNCH 60, Z ,F(1),F(2),UFIR,DFIR,CUT,BN,AMX,SNFU,PHI
60    FORMAT  (5F6.3,4F8.5,F8.3)
      IF(L-4)  120,121,122
120   L=L+1
      Z=Z-0.2
      GO TO 56
121   L=L+1
      Z=0.0
      EMU=+S/C
      GO TO 55
122   IF(L-6)  123,124,125
123   L=L+1
      Z=CUT
      FMU=0.0
      GO TO 55
124   L=L+1
      Z=-0.0
      EMU=-S/C
      GO TO 55
125   IF(L-10)  126,126,128
126   L=L+1
      Z=Z-0.2
      Y=3.1415927*(-Z)
      EMU=-SQRTF(TAN*TAN+((EXPF(Y)-1.0)/(EXPF(Y)+1.0))**2)
      GO TO 55
128   IF(DFIR-1.5)   31,32,32
31    DFIR=DFIR+0.5
      GO TO 52
32    IF(UFIR-1.5)  33,34,34
33    UFIR=UFIR+0.5
      GO TO 53
34    IF(CUT-0.8)  35,36,36
35    CUT=CUT+0.2
      GO TO 54
36    STOP
      END
 00175  00872  01737  02588  03420  04226  05000  05736  06428  07071
 07660  08192  08660  09063  09397  09659  09848  09962  09999  099999
00000000
00000000
 01564  01567  01576  01591  01612  01640  01676  01720  01773  01838
 01917  02012  02128  02272  02456  02702  03054  03655  04956  05947
 03090  03096  03112  03139  03178  03229  03293  03373  03469  03585
 03724  03891  04093  04340  04649  05047  05595  06452  07958  08787
 04540  04547  04568  04602  04652  04717  04800  04900  05021  05166
 05336  05539  05779  06066  06415  06846  07403  08184  09262  09674
 05878  05887  05907  05944  05997  06066  06152  06257  06382  06529
 06701  06901  07134  07405  07723  08099  08552  09121  09745  09917
 07071  07078  07098  07132  07180  07243  07321  07414  07525  07654
 07802  07972  08165  08384  08632  08913  09231  09591  09914  09978
 08090  08095  08112  08138  08176  08225  08285  08357  08442  08539
 08649  08773  08912  09065  09234  09417  09613  09817  09971  09995
 08910  08914  08924  08942  08966  08998  09037  09083  09137  09198
 09267  09343  09427  09517  09615  09717  09822  09924  09991  099986
 09511  09512  09517  09526  09538  09554  09573  09595  09621  09650
 09683  09719  09758  09799  09843  09887  09932  09973  099975 099997
 09877  09877  09879  09881  09884  09888  09893  09899  09906  09914
 09922  09931  09941  09952  09963  09974  09985  09994  099993 099999
 10     10     10     10     10     10     10     10     10     10
 10     10     10     10     10     10     10     10     10     10
  0      1      2      3      4      5      6      7      8      9    10
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