ULTIMATE STRENGTH

OF

SINGLE-BAY PRESTRESSED CONCRETE PORTAL FRAMES

BY

T. VENKATACHARYULU

A THESIS

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FOR THE DEGREE OF

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CONCRETE TECHNOLOGY.

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CERTIFICATION

This is to certify that the thesis entitled "ULTIMATE STRENGTH OF SINGLE-BAY PRESTRESSED PORTAL FRAMES" which is being submitted by Mr. T. Venkatacharyulu, as a part fulfilment for the degree of Master of Engineering of the University of Roorkee, Roorkee, is a record of bonafide work carried out by him under my supervision and guidance. The results embodied in this thesis have not been submitted for the award of any degree or diploma.

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CONTENTS

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The author extends his thanks to the Director, Central Building Research Institute, Roorkee for permitting to submit this thesis to the University of Roorkee, Roorkee.

SYNOPSIS.

The object of this investigation was to stu ultimate strength of single-bay prestressed portal fra the moment redistribution beyond the elastic limit. The controversial opinions about the extent of redistribution moments in statically indeterminate structures at their ultimate failure. For example, Guyon is of the opinion that the distribution of moments in statically indeterminate structures, say portal frames, under service loads will not be the one deduced from the elastic theories. Some redistribution, Guyon says, is bound to take place and <u>cracking</u>[2] will be delayed beyond the point indicated by the elastic theory. In contrast to this, Professor R. H. Evans and J. S. Raftery found from their test on a three dimensional prestressed concrete frame that the failure occurred without the frame ~~that the failure occurred~~ becoming a mechanism by forming a sufficient number of plastic hinges i.e., without the full redistribution of moments. P. B. Morice and H.E. Lewis, by performing a number of tests on prestressed concrete continuous beams and portal frames, came to the conclusion that there was full redistribution of bending moment at failure. But the degree of moment redistribution, or adaptation, obtained by them demands special examination on account of the contrary evidence produced by Macchi.

An attempt has been made to find the discrepancy between the actual and the calculated ultimate loads and the degree of redistribution of bending moments, or adaptation, that takes place. The relation between the curvature and the moment of resistance is also studied.

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NOTATION

b = breadth of beam

d_1 = effective depth

nd = depth to neutral axis

rnd = depth to centre of compression

R = cube strength in lbs.

A_s = Area of high tensile steel

f_u = ultimate stress in steel

$\overline{W}$ = Weighted percentage

$= \frac{A_s f_u}{b d_1 R}$

Fsu = Total ultimate Force in the steel

K = a coefficient depending on the shape of the stress diagram

$\alpha_1 d_1$ = lever arm

ε = Strain in the concrete

ε' = Strain in the steel

ε_0 = initial strain in the concrete after prestressing.

ε'_0 = initial strain in the concrete after prestressing

r = radius of curvature

fs = actual stress in the steel

λ = tension factor $\frac{f_s}{f_u}$

Mr = ultimate moment of resistance

l = length of the portal between the centre lines of legs

g = height of the portal frame

ρ = ratio of stiffness of leg to stiffness of transom.

CHAPTER I

INTRODUCTION.

1.1 Advantages of continuity:

Continuity in prestressed concrete offers the same advantages as in non-prestressed structures, namely that the moments may be more evenly distributed between the centre portion and the ends of the members than is possible in simply supported beams or in structures which contain a number of hinges to reduce the degree of indeterminacy. The cross-section of the members may therefore be reduced.

A further advantage lies in the use that can be made of pre-casting for the production of high-quality concrete under factory conditions. Individual members may be formed from an assembly of pre-cast blocks, connected only by the prestressing cables and the logical development of this technique is the assembly, by means of prestress, of complete members into continuous beams or frames.

1.2 Problems in the prestressing of indeterminate structures.

(a) Prestressing an indeterminate structure introduces redundant reactions and therefore secondary moments.

(b) Cables correctly placed to give the ~~plasters~~ prestress, having regard to the secondary moments, are not necessarily in a position which gives a high ultimate moment, as in simply supported beams. Moreover the calculation of the failure load is complicated by the redistribution of moments in the structure.

(c) The loss of prestress in tensioning due to friction is appreciable in long cables with considerable curvature. This largely offsets the possibility of a saving in the cost of end anchorages, by the use of long cables.

1.3 <u>Possible practical solutions</u>.

(a) Stressing in the determinate condition, and then rendering the structure indeterminate.

(b) Stressing, and adjusting the redundant reactions by use of jacks or other means to elinunate the secondary forces, or to control them to any desired value.

(c) Stressing in a way that does not affect the reactions.

(d) Stressing in the most convenient way and calculating the secondary moments.

1.4 <u>Plastic Theory and ultimate load</u>.

(a) Existing ultimate strength theories only apply for statically determinate structures where the moments etc., are known statically, and cannot be used as such for the analysis of redundant structures where a moment deformation relation is needed for all stages of moment at the sections upto their ultimate strength. Some work has been published previously by Professor A. L. Baker' in order to estimate the strength of reinforced and prestressed concrete redundant structures. This work is based on lines similar to the plastic theory involving the use of plastic hinges as applied to mild steel redundant structures. Mild steel, being a highly ductile material, the necessary rotation required at the plastic hinges is always available without the strains exceeding the ultimate limit. Concrete is a material having much smaller ultimate strain and it crumbles if over strained. Therefore, the necessity arises in the case of reinforced and prestressed concrete redundant structures, when applying the concept of plastic hinges to such structures, that the hinge rotation required for the assumed moment redistribution is available without putting strain in the concrete which is more than the ultimate that it can carry. This means that some variation from the methods applicable to steel structures becomes necessary. So a method of analysis and calculation of ultimate strength of prestressed concrete redundant structures were given. It is based on the modification of the elastic theory results due to the formation of plastic hinges. Actual test results are given in verification of the theoretical deductions and it will be seen that the proposed theory estimates the ultimate strength of the structure fairly closely to the actual strength. The ultimate strength calculated on the basis of the elastic theory as is the practice nowadays, is as low as 46 per cent of the actual strength.

(b) It is well known that most building materials like steel and concrete are not fully elastic tight upto their ultimate strength and plastic deformations accompany the elastic strains.

Steel is elastic upto a stress of about 60 per cent of its ultimate strength, after which if shows excessive plastic deformations known as yield of steel. Concrete shows some plastic deformations right from the beginning, but after a certain stage of loading, these plastic deformations increase abnormally. The methods of analysing structures and predicting their behaviour under load, after taking into account of these plastic deformations, is called Plastic Theory. The prevalent methods of structural analysis are based on Elastic Theory which assumes that the material is perfectly elastic at all stresses. It is thus evident that the analysis based on elastic theory is only an idealised one, and the results may be different from those obtained in practice. The importance of taking account of the plastic strains was realised as early as 1892 when A.E.H. Love, the famous elastician, wrote in his classic work "A Treatise on the Mathematical Theory of Elasticity", "there exists no adequate mathematical theory of set or of after strain, or in fact of any of the phenomena exhibited by materials strained beyond their elastic limits, yet it is imperatively necessary that effects which can not be calculated exactly should be taken into account in construction, and it is in this sense that the elastic theory is at this time behind the Engineering Practice." It is only in recent years that some attention is being paid to this aspect of the problem. Glanville[2] had shown in 1936 that the plastic deformations of concrete and steel have considerable influence on the behaviour of continuous beams and frames of reinforced concrete.

If the stresses created in a structure are within the elastic range of the material, then the relationship between the load on the structure and the stresses is linear in most cases. However, as stated earlier, steel and concrete both show plastic deformations after a certain stage of loading and then the load-stress relationship can not be linear, i.e., the increase of stress is not in the same proportion as the increase of load. Hence if we fix certain factors of safety on the stresses and calculate the working load on the structure, based on these reduced stresses which are within the elastic range of the material, then by multiplying the working load with the factor of safety, we do not get the collapse load. The actual collapse load may be less(as in columns) or more (asxcollapsexloadxmayxbexless (as in continuous beams than the one obtained above. In order to determine the real factor of safety of a structure, its collapse load should be found and then the working load may be kept a fraction of this collapse load. The ratio of the collapse load to the working load is called "Load Factor" and is different from the factor of safety applied to stresses. The plastic theory aims to find the collapse load of a structure rather than the working load as is the practice now.

C. METHOD OF CALCULATING COLLABSE LOAD

A structure, in general, is subjected to (i) direct forces (ii) bending moments (iii) shear forces and (iv) torsion at any section under the action of a given system of loads.

These actions may be present individually or in combination with each other. Before the collapse load of a structure can be found, it is essential to know (i) the relationship between the external loads on the structure and the bending moment, etc. caused at any of its sections, (ii) the ultimate strength of a section under the action of the particular type of force e.g., direct thrust, bending moment, etc. suppose the ultimate moment of resistance of the section is Mr, and the relation between the collapse load W and the maximum bending moment M caused in the structure is, $M = KW$, then the collapse load $W = \frac{Mr}{K}$

(d) Analysis of moments, thrusts, etc.

In the case of statically determinate structures, the relationship between moments and load etc., is easily found by statics and does not depend upon the properties of the material of which the structure is made. This relationship remains the same at all loads till the collapse of the structure. Hence, in such structures, the collapse load can be calculated as described above, once the ultimate strength of the individual sections is known. The ultimate strength of a section can be calculated by any of the existing theories. 2, 3, 4, 5, 6, 7, 8.

For statically indeterminate structures, the relationship between load and moments depends upon the properties of the material and the relative dimensions of the structure. The analysis of moments, thrusts, etc. in redundant structures is done with the help of elastic theory and needs calculation of angular and direct deformations of the various members of the structure. In this calculation, it is assumed that the material

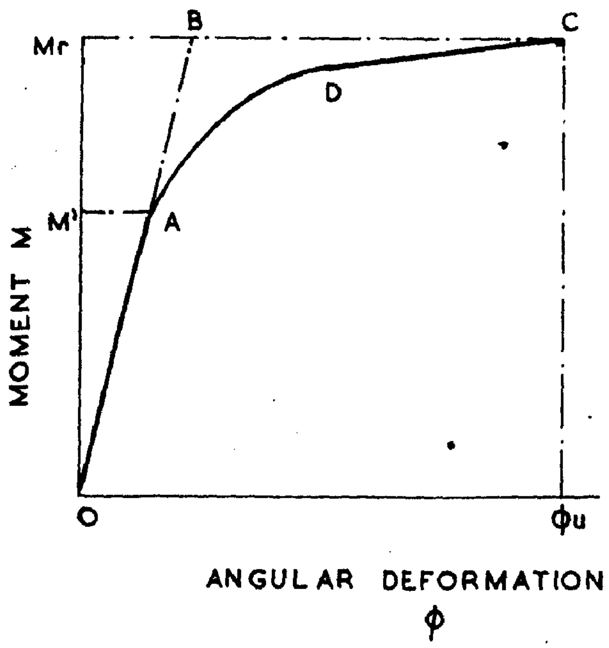

FIGURE :– I

is perfectly elastic at all sections of the structure and that the angular deformation of any section of the structure is proportional to the bending moment acting there. With the help of these calculations, a certain distribution of bending moments, etc. is obtained in the structure under a given system of loading.

This distribution of moments will not be affected by the magnitude of the external load so long as the load is of the same pattern and the material is elastic. It means that the relation between the moment at any section and the load on the structure is linear.

However, in the case of reinforced or prestressed concrete section, the relation between bending moment and angular deformation is not linear right upto the ultimate moment of resistance of the section. Fig. 1 shows a typical curve giving a relation between moment and angular deformation of a unit length of a prestressed concrete member. Upto a moment M', the curve OA is almost a straight line showing that the behaviour is elastic. Beyond M', the angular rotation increases very rapidly till the section collapses on reaching its ultimate moment of resistance Mr. the value of ϕ_u i.e. the maximum angular rotation, which a unit length of a prestressed concrete member is capable of undergoing before crushing of concrete, depends upon the depth of the section, the percentage reinforcement and the ultimate compressive strain of concrete.

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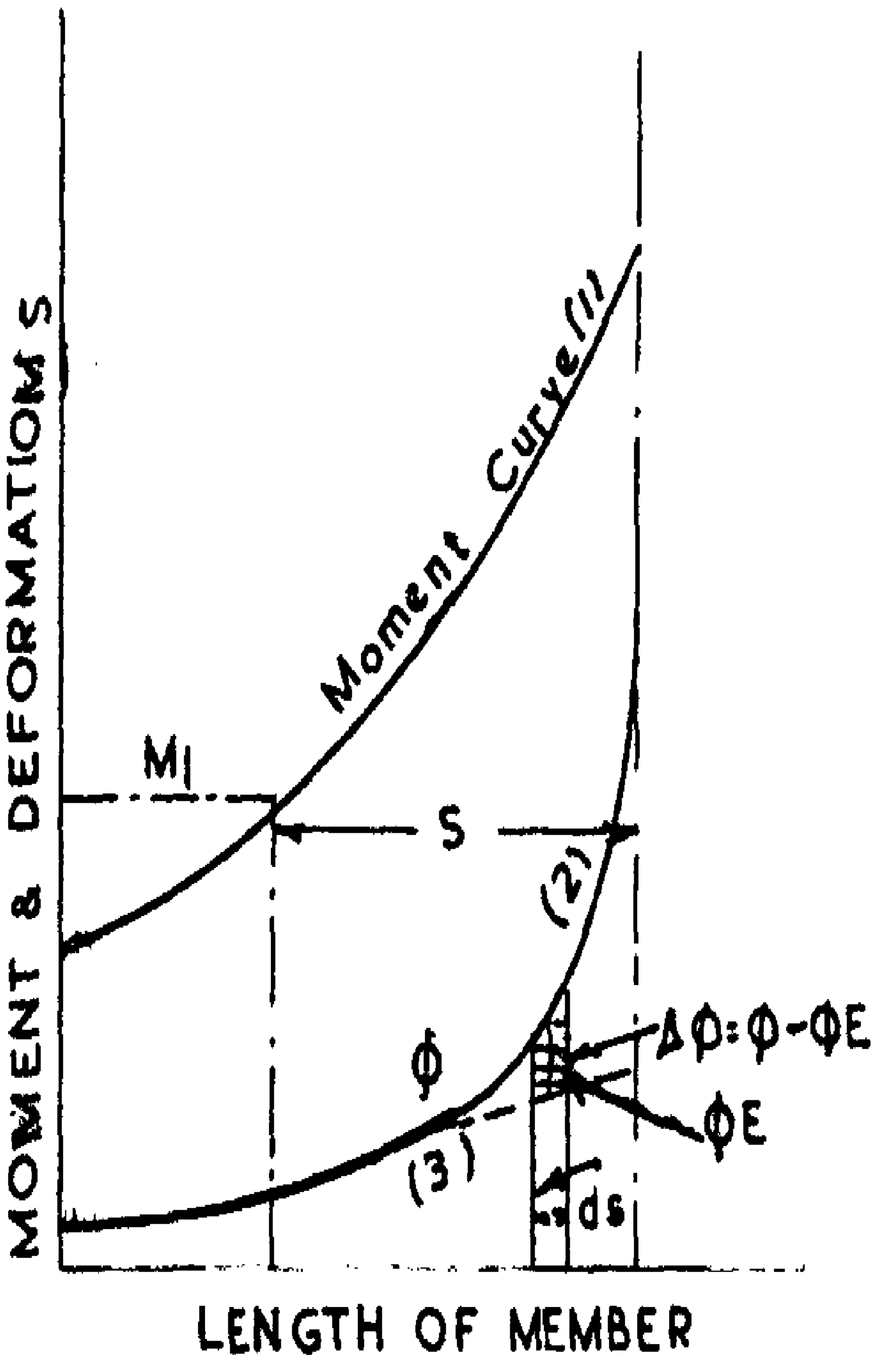


MOMENT & DEFORMATIONS
Moment Curve (1)
M1
S
(2)
(3)
ϕ
Δϕ = ϕ - ϕE
ϕE
ds
LENGTH OF MEMBER

FIGURE :— 2

It is assumed here, as in all the usual formulae, that the failure of a prestressed concrete section is always accompanied by the steel reaching its ultimate tensile strength, Fsu. The ratio of the moment M' upto which the curve is a straight line, to the ultimate moment of resistance Mr depends upon the quality of steel and its percentage in the section. Another critical point on the curve is D beyond which a slight increase in bending moment results in a large angular deformation of the section. The section at this stage of bending, is said to have formed into a plastic hinge.

The distribution of moments, etc., in prestressed concrete redundant structures will, therefore, be given by the elastic theory fairly correctly till the maximum moment in the structure is less than M'. There may be minor deviations due to creep of concrete or its cracking. But even when the load on the structure is increased beyond this stage, there will only be small zones of the structure where the moments will exceed M' and the rest of the structure will still be in the elastic stage, for which the elastic theory is applicable. Let one of these zones be of length 'S'

Fig. 2 shows the length of the member on which the moment exceeds M' in a length S. Curve (1) shows the distribution of bending moment on this length and curve (2) gives the corresponding angular deformations ϕ at various points as obtained from Fig. 1. The dotted curve (3) gives the angular deformation for the same moments on the assumption that the material was perfectly elastic. This curve can be drawn with the help of the straight line OB in Fig. 1. The vertical ordinates between curves (2) and (3) give the extra angular deformation per unit length of the member due to plasticity of the materials. The total plastic deformation in a small length ds will be the ordinate multiplied by this length i.e., equal to the shaded area in Fig. 3. Hence, total plastic deformation in the lengths will be the area between the curves (2) and (3) in figure 3. This is called the rotation of the plastic hinge and the length S of the member in which these plastic deformations occur is called the "length of the plastic hinge".

In the analysis based on elastic theory, only elastic deformations were considered. The existence of these plastic deformations is bound to alter the distribution of moments in the structure. This is called the "redistribution of moments" beyond the elastic limit in the structure. Thus, when a structure is about to collapse, the distribution of moments is different from that given by the elastic theory. The calculation for the actual moment distribution at failure, will involve the knowledge of rotation of the plastic hinge. This rotation cannot be calculated accurately unless the distribution of moments is known. Hence the problem becomes one of trial e.g. a probable distribution of moments is assumed and then the rotation of the plastic hinge calculated.

It can now be checked whether the assumed moment distribution was correct.

Since the plastic hinge* is not located at a section ~~alo~~ alone, i.e., on an infinitesimally small length, but is distributed over a certain length which has reached the plastic state, the rotation of hinge is the sum of the elements of curvature, in other words if θ is the rotation which is required at a ~~hinge~~ hinge to permit full adaptation, θ is the sum of the curvatures $\frac{d\theta}{ds}$ of the elements ds of the plastic zone on both sides of the theoretical hinge (or $\theta = \int \frac{d\theta}{ds} ds$) the integral being extended to the plastic zone). Since along this plastic zone the moment varies from the elastic limit to the ultimate moment, it is seen that the basis for the justification of redistribution (or the study of the rate of redistribution) is the relationship between the moment and the curvature, which may be written $\frac{d\theta}{ds} = M$.

If this relationship is known, the maximum possible rotation of the hinge may be determined for a given assumed distribution and the assumption will be justified if this maximum is not exceeded.

*It has already been stated (at the F.I.P. Congresses) that this designation is merely an abbreviation, and that the plastic hinge is in fact a certain length of the structure of which the section is the centre.

In theory a knowledge of this relationship allows the behaviour of the structure from the end of the elastic phase right up to failure to be followed. Professor Levi and his assistants have obtained some interesting results in this direction.

1.5 Calculation of the ultimate moment of resistance of a section.

Simple formulae have been given for the ultimate moment, and these are certainly sufficient for the simple cases of a statically determinate beam with a small percentage of steel, this percentage being based on the area of surrounding concrete (i.e., to bd_1). The question is if they are still sufficient for the higher percentages which often occur in a statically indeterminate structure, due to the reversal of bending moments when the cable is very close to a flange.

Further, the real parameter is not the percentage $\frac{As}{bd_1}$ but the ratio of the strengths $\frac{As\,fu}{bd_1R}$, which is called the weighted percentage denoted by $\overline{\omega}$.

The usual formulae assume that the steel reaches its ultimate tensile strength when the section fails, and thus the direct force is Fsu. A certain shape of the concrete stress diagram is also assumed. Hence the resultant compression is kbndr, where k is a coefficient depending on the shape of the stress diagram; the centre of compression is at a distance r and d from the extreme flange, r being a constant also dependent on the shape of the stress diagram. The lever arm is equal to d_1 - rnd, and the resisting moment Mr is equal to Fsu (d_1-rnd).

From considerations of equilibrium, we have

$$F_{su} = kbndr \text{ or } nd = \frac{F_{su}}{kbr}$$

Hence $$Mr = F_{su}\, d_1 \left(1 - \frac{r}{k} \quad \frac{F_{su}}{bd_1R}\right)$$

If the constant coefficient $\frac{r}{k}$ is denoted by α and the weighted percentage $\frac{Fsu}{bd_1R}$ by $\bar{W}$, the formula becomes*

$$Mr = Fsu\ d_1(1- \alpha\bar{\omega})$$

various values have been given for the coefficient α . When R is the cube strength, most authors give $\alpha = \frac{1}{2}$.

The value of α was discussed at the first congress of the Federation Internationale de la Precontrainte held in London in 1953. Walley gives $\alpha = 0.74$ ($\alpha = 0.59$ when R is the prism strength, assumed to be 0.8 R cube).

Morice suggests $\alpha = 0.8$, and that for Post-tensioned members a reduction coefficient should be applied to F_{su}, to allow for the imperfact bond.

*The formulae only apply where the bond is perfect; They also assume that b is constant over the depth nd.

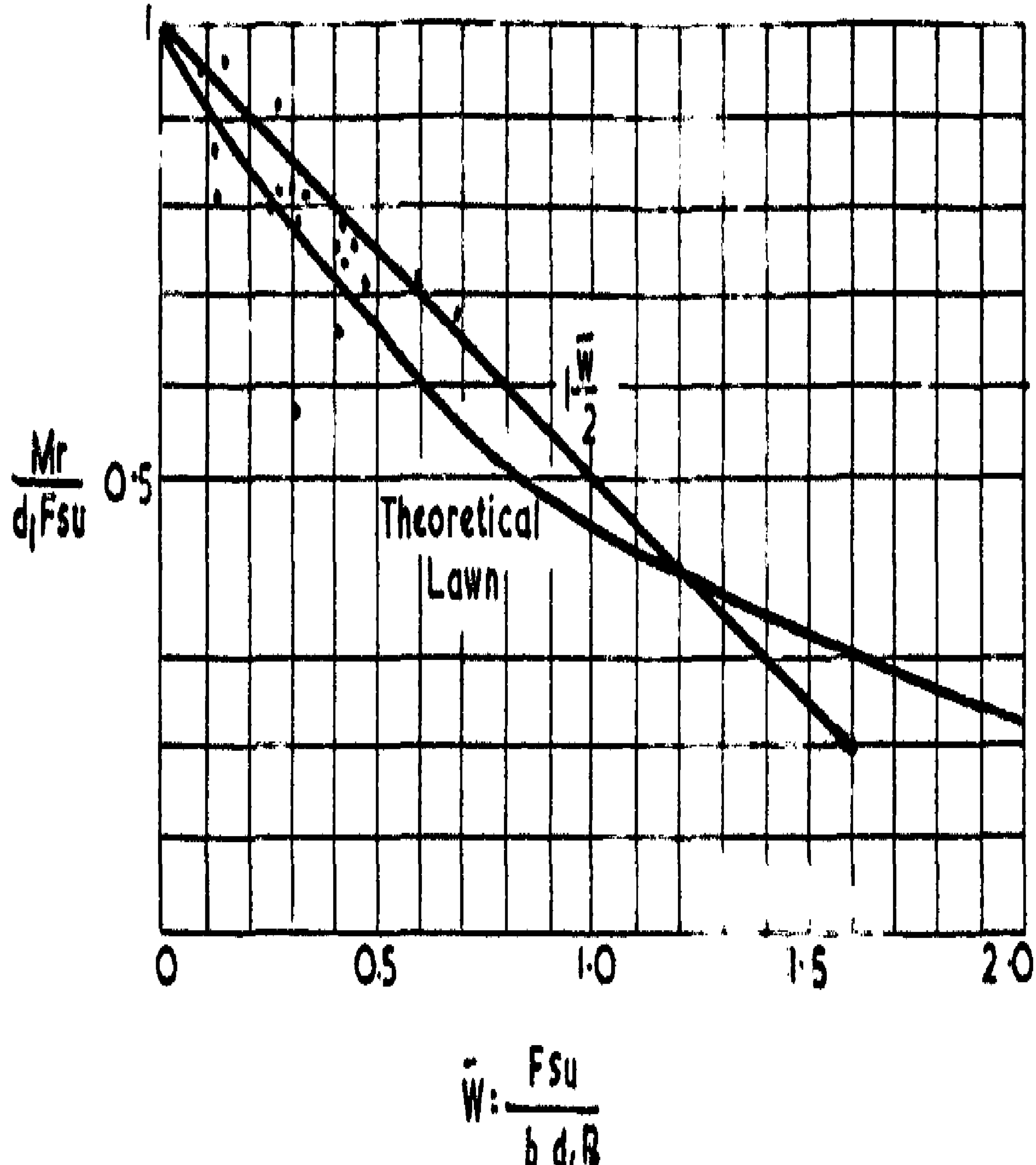


FIGURE:—4

As can be seen, opinions vary appreciately.

To attempt a comparision between these formulae and the results of tests on statically determinate beams (which ensure a knowledge of Mr, is very difficult, due in particular to the lack of uniformity of the standards concerning the strength of concrete in different countries. The value of the concrete strength varies in meaning according to the test procedure; cube, prism, cylinder, size of sample, the interposition of material between the concrete and the platens of the testing machine, the method of curing the samples before testing, etc.

If the above mentioned formulae hold good, then a plot of the results on a diagram relating $\frac{M_r}{d_1 F_{su}}$ and $\overline{\omega}$ should give a straight line ($1 - \frac{M_r}{d_1 F_{su}} = \alpha\overline{\omega}$), from which an average value of α could be deduced. The results are plotted in Fig. 4. It may be said that, in spite of considerable scatter, the relationship $M_r = F_{su} d_1 (1 - \frac{1}{2}\overline{\omega})$ seems to give reasonable agreement for low percentages.

By "reasonable agreement" is meant that, although there is an appreciable variation in α (i.e in the slope of the straight line joining the point $\overline{\omega} = 0$ and $\frac{M_r}{d_1 F_{su}}$ to the point under consideration), the approximation in the value of the moment is not so bad, due to the low value of the corrective term $\alpha\overline{\omega}$ when $\overline{\omega}$ is small. The error is generally no greater than 10%, and could not be expected to be smaller.

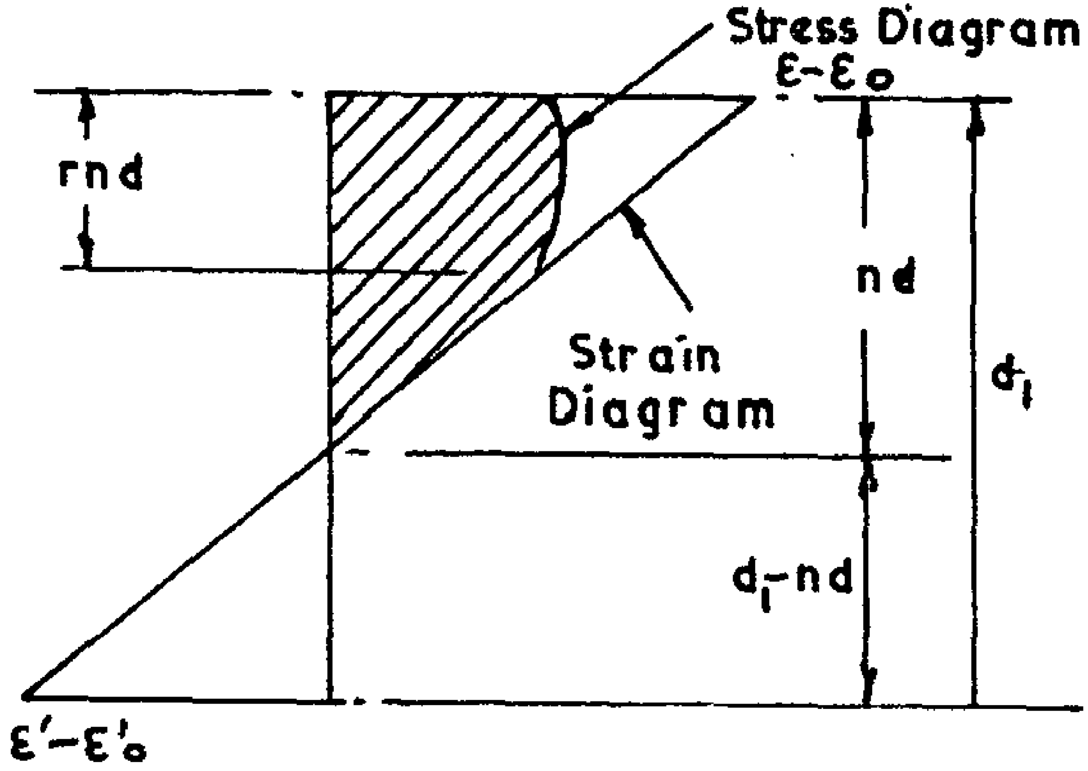


FIGURE:—5

The situation is different at higher percentage of steel, For high percentages, it is certainly impossible for the steel to reach its ultimate tensile strength. In certain cases, when the cable is in the compressive zone (which may happen in statically indeterminate structures, due to a reversal of moments), not only will the steel not reach its ultimate tensile strength, but it will lose part of its initial tension. Therefore formulae covering the whole range of percentages should be established.

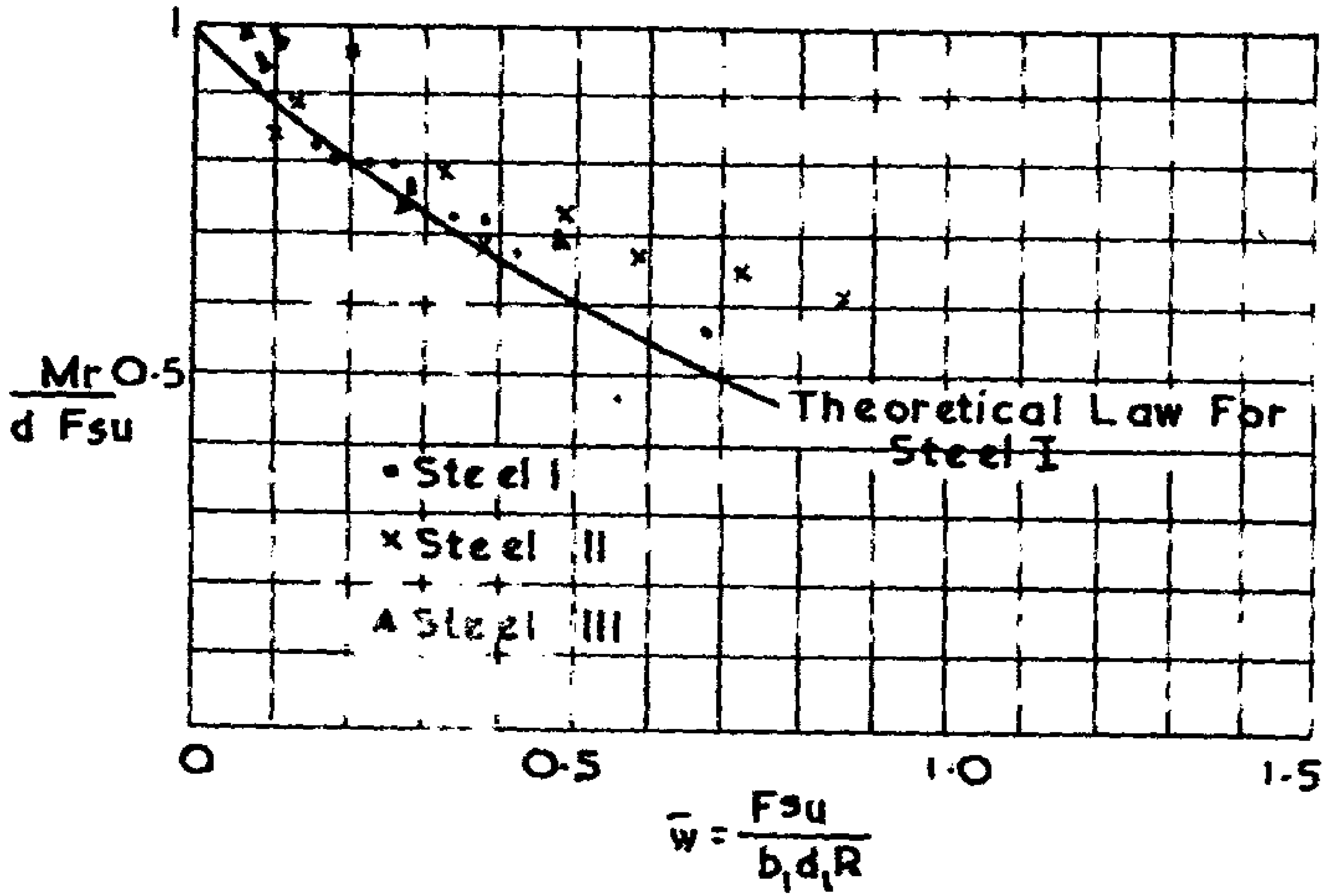
It seems reasonable to adopt the assumption that plane sections remain plane. The validity of this assumption appears to be justified by tests made by Billet and Appleton[9] on twenty-four statically determinate beams.

Figure 5 explains the assumption; let $\mathcal{E}$ be the strain in the concrete, $\mathcal{E}'$ the strain in the steel, let $\mathcal{E}_0$ and $\mathcal{E}'_0$ be the initial strains after prestressing. ($\mathcal{E}'_0$ is the strain corresponding to the initial tension in the cable after relaxation.) Under the effect of the ultimate moment Mr, the strain in the section will vary from $\mathcal{E} - \mathcal{E}_0$ to $\mathcal{E}' - \mathcal{E}'_0$. Let fs be the actual stress in the steel. Then using the same coefficients K and α as defined above

$$K\, b\, n\, d\, R = A_s f_s \qquad (1)$$

$$\frac{n d}{d_1} = \frac{\mathcal{E} - \mathcal{E}_0}{\mathcal{E} - \mathcal{E}_0 + \mathcal{E}' - \mathcal{E}'_0} \qquad (2)$$

Since there is a definite relationship between $\mathcal{E}'$ and fs



Test Results Of Billet 2nd Appletion

FIGURE. — 6

(the stress-strain diagram) these two equations allow the two unknowns fs and nd to be determined, assuming ε and ε_0 to be known.

It is assumed that ε_o is small in comparison with ε (which is generally the case) and have taken . obviously other values could be taken if thought to be better.

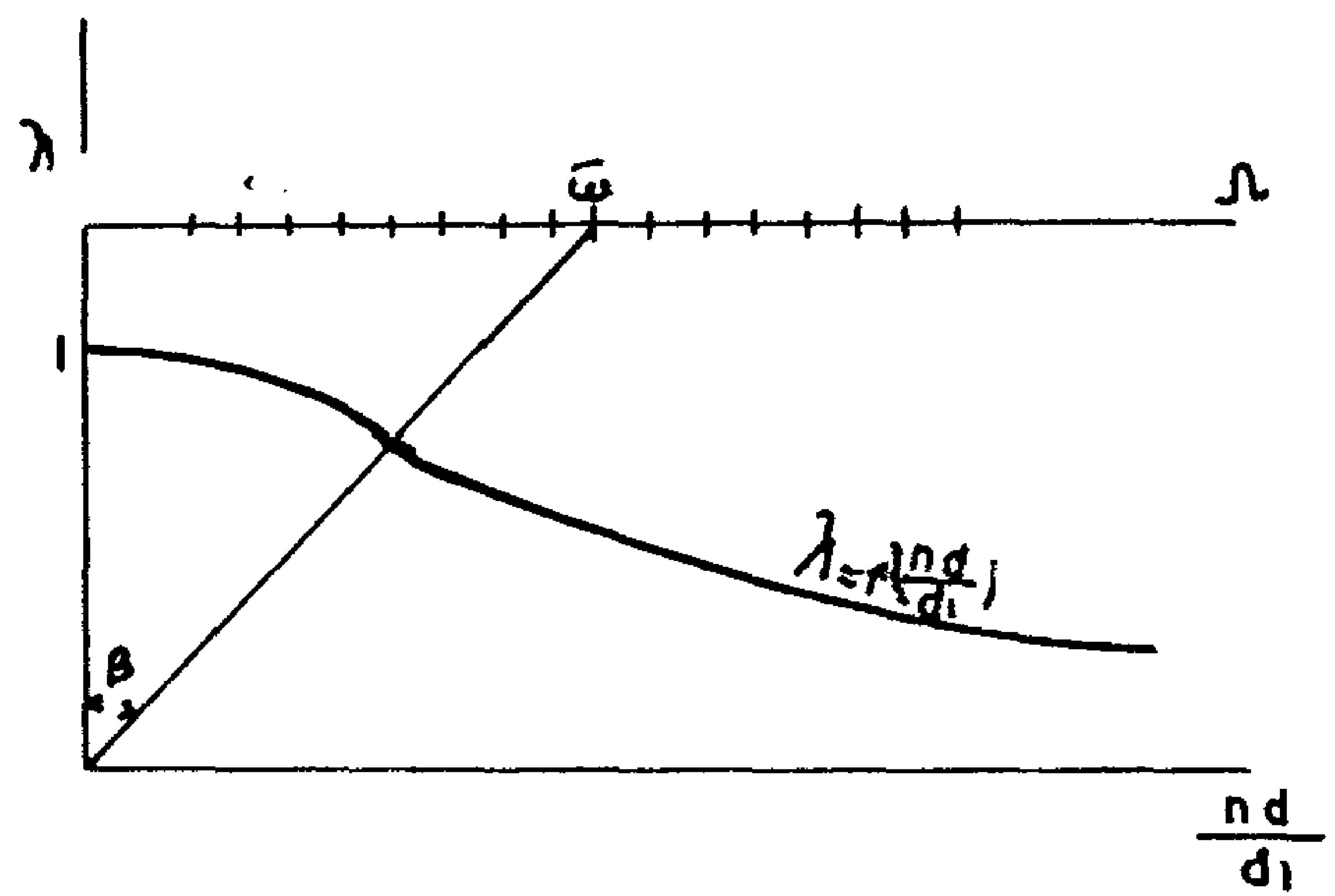
When fs and nd are known, Mr is found from the equation

$$M_r = A_s f_s (d_1 - \gamma n d) \qquad (3)$$

Knowing also the radius of curvature r,

$$\frac{1}{r} = \frac{\varepsilon}{nd} \qquad (4)$$

Tests with three kinds of steel have been made for a large range of values of $\overline{\omega}$ upto 0.85, the characteristics of both concrete and steel being known. Figure b shows the results plotted in terms of $\frac{M_r}{d_1 F_{su}}$ and $\overline{\omega}$ as above, compared with the theoretical formula given by equation (3). The theoretical law corresponding to steel I is shown on this figure, and it is seen that the experimental points corresponding to this steel are in good agreement with the theoretical curve. The agreement is equally good for the other steels.


λ
Ω
ω̄
1
β
λ = f(nd/d1)
nd/d1

Below is given a practical way of solving equations (1) and (2), and hence finding Mr and $\frac{1}{r}$,

Let the ratio $\frac{fs}{fu}$ (the "tension factor", referred to the ultimate tension taken as unity) be denoted by λ. Then the stress-strain diagram for any given steel may be represented by the equation $\mathcal{E}' = F(\lambda)$. Let λ_0 be the value of λ corresponding to the initial tension. Since $\frac{As\, fs}{b d_1 R} = \lambda \bar{\omega}$, equations (1) and (2) may be written as follows:

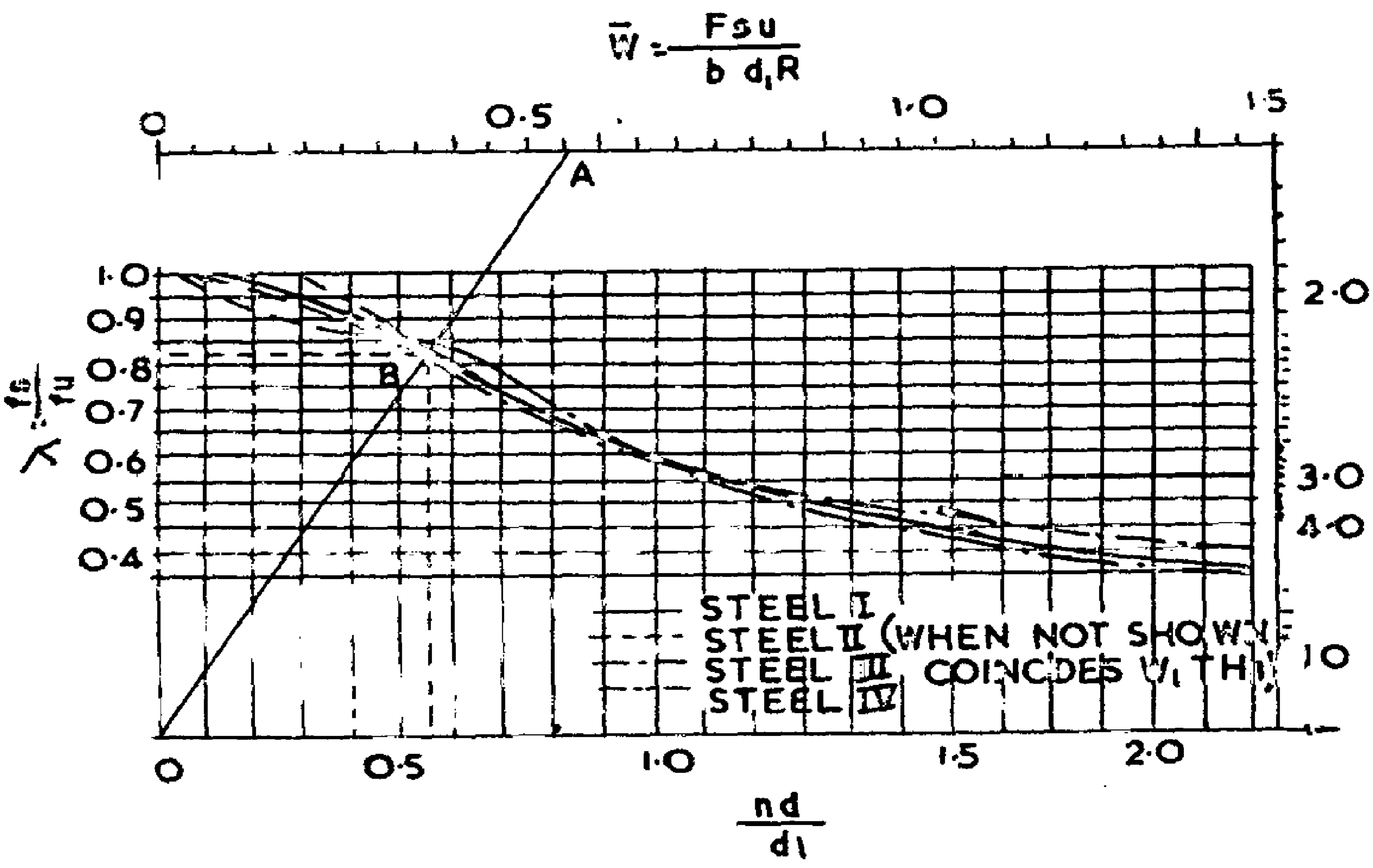
$$\frac{nd}{d_1} = \lambda \frac{\bar{\omega}}{k} \qquad (1a)$$

$$\frac{nd}{d_1}\left[F(\lambda) - F(\lambda_0) + \mathcal{E} - \mathcal{E}_0\right] = \mathcal{E} - \mathcal{E}_0 \qquad (2a)$$

Taking $\frac{nd}{d1}$ and λ as coordinates, equation (2a) expresses a relationship between λ and $\frac{nd}{d_1}$ which we may call $\lambda = f\,\frac{nd}{d_1}$. This curve may be plotted as shown in Figure 7 . This is easily done if the stress - strain diagram $\mathcal{E}' = F(\lambda)$ is known (if is assumed that $\mathcal{E}_0'$ is given). We may call this curve the characteristic curve for the steel.

The point ($\frac{nd}{d_1}$, λ) lies on this curve, from equation (1a) the point also lies on the straight line through the origin given by $\frac{nd}{d_1}/\lambda = \frac{\bar{\omega}}{k}$ or $\tan\beta = \frac{\bar{\omega}}{k}$.

Thus, if, to a suitable scale, we mark the scale $\bar{\omega}$ (a uniform scale) on the horizontal Ω , the line β is obtained by joining o to the point representing $\bar{\omega}$ on this scale.



W̄ = Fsu / b d₁R
0
0·5
1·0
1·5
A
B
1·0
0·9
0·8
0·7
0·6
0·5
0·4
λ = fs/fu
2·0
3·0
4·0
10
STEEL I
STEEL II (WHEN NOT SHOWN
STEEL III COINCIDES WITH I)
STEEL IV
0
0·5
1·0
1·5
2·0
nd / d₁

FIGURE:- 8

Thus, if we construct this graph, the solution of equations (1) and (2) is obtained for a given value of $\overline{\omega}$ by marking this value on the scale Ω and joining it to the origin. The intersection of this straight line and the characteristic curve has the coordinates λ and $\frac{nd}{d_1}$ and hence Mr and $\frac{1}{r}$. Special scales for the lever arm ($\frac{a_1 d_1}{d_1} = 1 - \gamma \frac{nd}{d_1}$) and for $\frac{1}{\gamma}$ ($\frac{1}{\gamma} / \frac{1}{d_1} = \varepsilon / \frac{nd}{d_1}$) facilitate the calculations.

A graph of this kind is shown in figure 8 for the initial tension $\lambda_0 = 0.6$ and for four different steels (I, II, III, IV) corresponding to the types of steel usually employed in Great Britain, France, U.S.A., and Italy respectively. It will be seen that the results (and hence the values of the ultimate moments) depend on the shape of the stress - strain diagram (corresponding to the different characteristic curves) and that a knowledge of this diagram is indispensable when interpreting test results.

The graph shown in Figure 8 corresponds to the assumptions : $k = 0.84$, $\gamma = 0.44$, $\varepsilon - \varepsilon_0 = 3.6 \times 10^{-3}$. It is assumed that the concrete strength is measured on cubes cast wx without smoothing.

Guyon has constructed a similar graph for $\lambda_0 = 0.5$ The differences in the values of Mr are very small (same 3%)

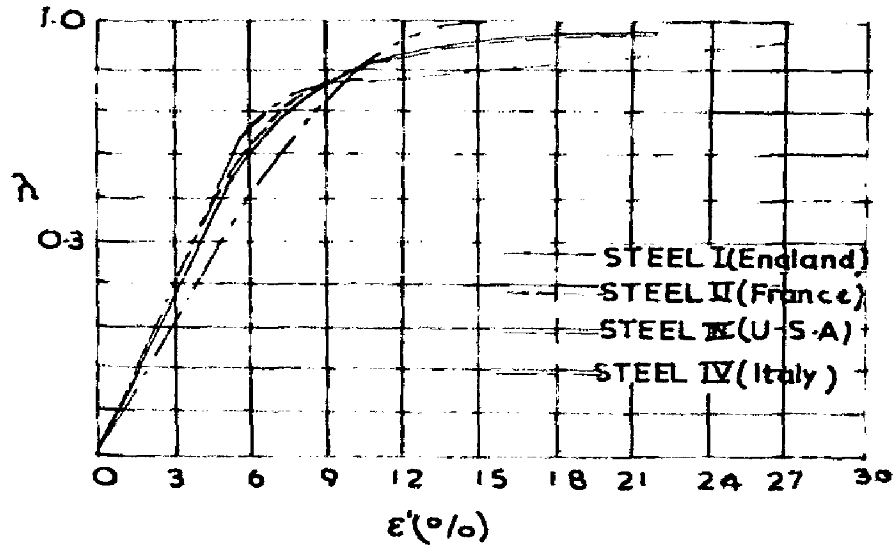


FIGURE:- 9

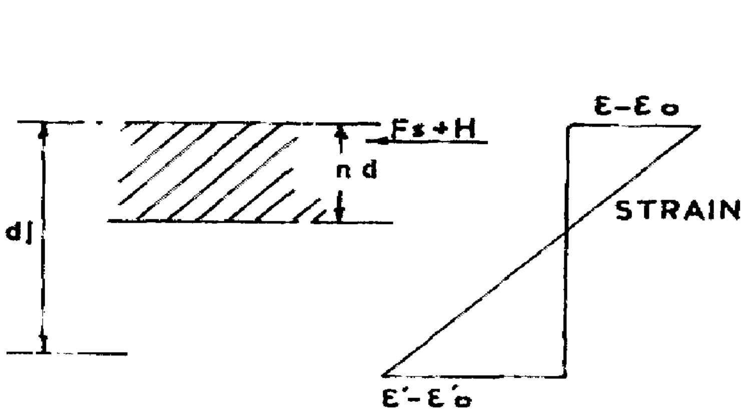


FIGURE:-10

Figure 9 shows the stress - strain diagrams in terms of λ for the four steels I to IV.

Special case where the external loads introduce a direct force.

In cases where external loads introduce a direct force into a member, for example a thrust H, the problem is to determine the depth nd to the neutral axis and the value of λ.

From figure 10, equation (1) becomes

$$k b n d R = H + A_s f_s \qquad (1b)$$

and equation (2) remains, $$\frac{nd}{d_1} = \frac{\mathcal{E} - \mathcal{E}_0}{\mathcal{E} - \mathcal{E}_0 + \mathcal{E}' - \mathcal{E}'_0} \qquad (2)$$

If we denote by $\overline{\omega}$ the weighted percentage $\frac{A_s f_{su}}{b d_1 R}$ and by $\overline{\omega'}$ the quantity $\frac{H}{b d_1 R}$ equation (1b) may be written.

$$\frac{nd}{d_1} = \lambda \frac{\overline{\omega}}{k} + \frac{\overline{\omega'}}{k} \qquad (1c)$$

The representative point ($\frac{nd}{d_1}$, λ) is to be found now. This point is on the characteristic curve, since equation (2) is the same as before. The point is also on the straight line represented by equation (IC). It is therefore at the intersection of the characteristic curve and the line given by equation (IC). The scale of the graph shown in Figure 11 has been chosen so that this line can be

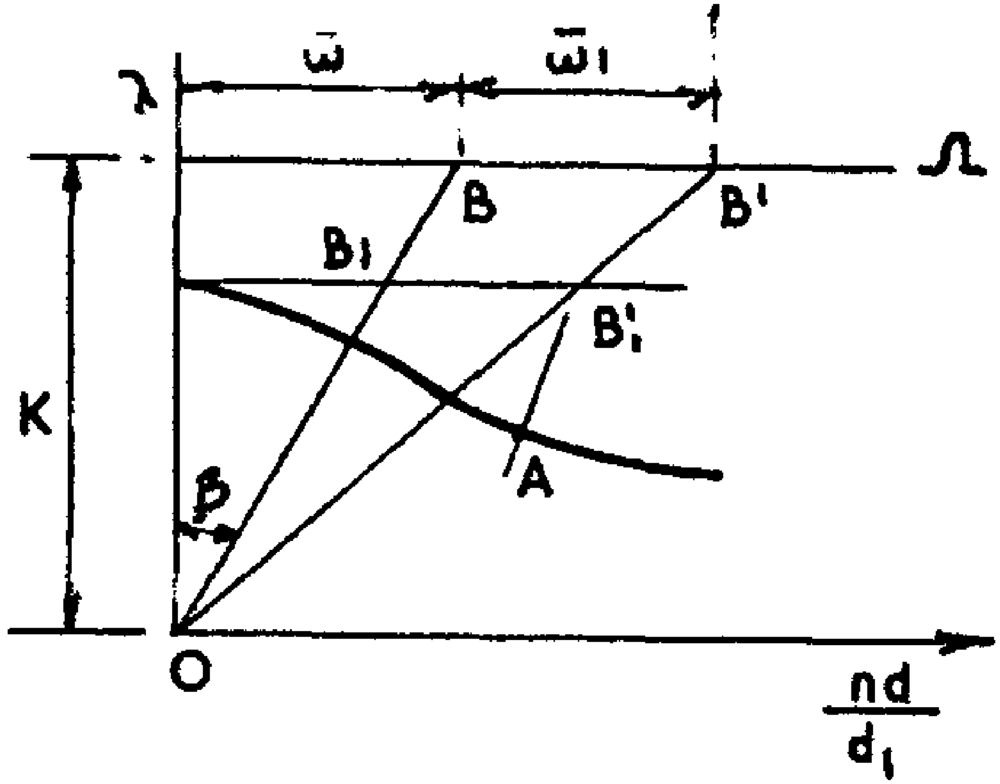


FIGURE: — II

traced easily. The scale for $\overline{\omega}$ has been marked on the line Ω ; to the same scale, the distance between the line Ω and the horizontal axis is k (since $\tan\beta = \frac{\overline{\omega}}{k}$). Further the scale for λ and $\frac{nd}{d_1}$ have been made the same.

On the scale Ω, the point B at $\overline{\omega}$, and B' at $\overline{W}+\overline{w'}$ are marked (see figure 11). B and B' are joined to the origin. These two lines OB and OB' intersect the horizontal $\lambda = 1$ at B_1 and $B_1{}^1$ - Through $B_1 1$., a line is drawn parallel to OB. It intersects the characteristic curve at the point A, which is the point required since $\frac{B_1B_1'}{1} = \frac{\overline{W'}}{k}$; the line $B_1'A$ represents the equation (IC).

The coordinates of the point A are $\frac{nd}{d_1}$ and λ, and the problem is easily solved.

CHAPTER 2.

REVIEW OF LITERATURE

2-1 GENERAL

Considerable amount of work has been done to study the moment distribution in statically indeterminate prestressed structures beyond the elastic phase. Below is given a review of the work done by various investigators.

2.2 GUYON'S WORK

(a) Tests on beams:

Monsieur Guyon conducted some tests on beams in October, 1952. The beams were 5 x 10 in. in cross-section, and each consisted of two spans of 13 ft. 1 in. Each beam was prestressed by one cable consisting of 12 wires of 0.196 in. diameter. The concrete strength (tested on 5½ in. cubes smoothed before test) was 7,000 lb/in^2. This was considered as equivalent to 0.8 x 7000 = 5,600 lb/in^2 due to the smoothing. The ultimate strength of the cable was F_{su} = 80,5000 lb. Mild steel was added in beams A_2, B and C (two bars of 0.196 in. diameter, Fy = 2000 lb, 1 in. from the top and bottom flanges respectively). Two equal loads W were applied, one at the centre of each span. Let M_r be the ultimate moment in the span, M^1r the ultimate moment over the intermediate support, and W the dead load.

If redistribution is complete, we should have at failure

$$Mr + \frac{M^1r}{2} = Wr\frac{l}{4} + W\frac{l^2}{8}$$

where wr is the ultimate load.

The experimental data and observstions are given below. The measured effective depth at mid-span and over the supports are denoted by d_1 and d_1^1 respectively. The values of Mr and M^1r have been calculated from figure 8.

$$W\frac{l^2}{8} = 1,090 \text{ lb.ft.}$$

If Mr and M^1r are expressed in 16 Ft. x 10^3, we should have

$$Wr = \frac{4}{13.1}\left(Mr + \frac{M^1r}{2} - 1.09\right)$$

In the table below are given the calculated ultimate load, the observed ultimate load, and the ultimate load assuming no redistribution.

TABLE 'I'

Beams	d_1 (in)	$d_1 1$ (in)	Mr (lb.ft) x 10^3	M^1r (lb.ft x 10^3)	W Calculated (lb x 10^3)	W Observed (lb x 10^3)	Error (%)	W (assuming no redistribution) (lb x 10^3)
A_1*	1.38	8.8	2.28	46.6	7.46	7.70	-3	4.1
A_2	1.5	8.85	3.59	47.8	8.00	8.36	-4	4.1
B	6.3	3.27	31.20	12.5	11.05	11.90	-8	11.0
C	2.96	8.09	10.80	42.7	9.45	11.00	-14	9.4

* No added mild steel.

Note: All the differences are negative (i.e. the ~~xx~~ calculated loads are smaller than the observed loads). The concrete strength was probably overestimated (i.e. coefficient of reduction smaller than 0.8).

Monsieur Guyon witnessed a test on a three span continuous beam carried out in London in November, 1952. The following data are reproduced from the notes made at the time.

The beam consisted of three spans of 10 ft. each, plus two cantilevers of 3 ft. 10 in. The depth at the mid span was 1 ft. 7½ in. and above the supports 2 ft. The breadth b was 6½ in. The beam was prestressed by two cables each consisting of 12 wires of 0.196 in. diameter. The total ultimate force for the two cables was 168,000 lbs. Three mild steel bars of 3¾ in. diameter were provided over a length of 4 ft. 9 in. over the two intermediate supports, 1 in. from the top flange, stirrups of ¼ in. diameter bar were arranged at a pitch of 9 in.

The concrete cube strength was 5,600 lb/in^2.

Loads were applied at the third points in each span, plus one load on each cantilever 1 ft 3½ in. from the extreme supports.

It is seen from the test that the ultimate load would be 20% less than the full redistribution load.

b. TEST ON FRAME:

Monsieur Guyon has tested a frame identical to that tested by Lebelle, with the same prestressing force in the horizontal truss but arranged differently, the tendon had a double curvature in one truss, and was centred in the other.

The main difference between this test and Lebelle's test is that in this case the load was applied asymmetrically, at mid-span of one truss only. Due to the arrangement of the wires in t the members, the theoretical ultimated load is greater than that for Lebelle's tests.

The ultimate moments were 13,300 lb.ft. for the posts, 1,300 lb.ft. for section A (see figure 12), 24,600 lb.ft for M (positive) and B (negative), and ± 15,900 lb.ft. for D, N and E.

Assuming complete redistribution, the theoretical ultimate loads are 18,600 x 10^3 lb. for DNE and 23,000 x 10^3 for AMB.

The observed ultimate loads were respectively 18,100 x 10^3 and 22,500 x 10^3 lb. It is difficult to compare these results with the assumption of no redistribution since failure would have occurred immediately at the point A.

2.3 LABELLE's TESTS ON FRAMES:

Lebelle tested a double frame of the dimensions shown in Fig. 13. All the members were 8 x 5 in. in cross-section. The horizontal members were prestressed by four wires of 0.276 in. diameter, the wires being central in one of the members and eccentric in the other. The posts were of reinforced concrete. As the full details have already been published(10), only the data concerning failure are given below.

The ultimate moments were 13,200 lb. ft for the outside posts and 15,050 lb. ft. for the upper truss (with the central prestressing). If we denote the net span of this truss by l, the theoretical ultimate load Wr is given by $Wr\frac{1}{4} = 15{,}050 + \frac{15050 + 13200}{2}$ which gives Wr = 18,000 lb. The observed ultimate loads were 17,700 lb (with the central prestressing) and 17,000 lb (with the ecentric prestressing).

According to the elastic theory, the ultimate load would have been 15,100 lb. The observed loads are therefore 2% and 6% smaller than the load assuming complete redistribution, and 17% and 1 12% greater than the elastic ultimate load.

2.4 LIN'S TESTS

Tests have been carried out by Lin at the Magnel Laboratory on four continuous beams, each having two spans of 24.6 ft, the beams were 8 x 16 in. in cross-section, prestressed by a curved concordant cable consisting of 32 wires of 0.196 in. diameter. The concrete strength measured on 8 in. cubes (cured) was 8,280 lb/in^2. The ultimate strength of the cable was 25,000 lbs.

Two of the beams had additional untensioned steel (two bars of 0.55 in. diameter at a distance of 1 in. from the extreme ~~fib~~ fibres at mid-span and above the support, in the tensile zones).

Loads were applied symmetrically 8.6 ft. from the central support (see figure 14). The reactions were measured.

The results have already been published." The following ultimate moments are calculated.

<u>Beam A</u> (without additional mild steel)

Ultimate moment in the span (loaded section): 145,000 lb.ft.

ultimate moment above the support : 176,000 lb.ft.

The theoretical ultimate load assuming complete redistribution (Wr) is given by

$$Wr \times \frac{16 \times 8.6}{24.6} + W\frac{l^2}{8} = 145,000 + 176,000 \times \frac{16}{24.6}$$

$$\text{or } 5.57\ Wr + W\frac{l^2}{8} = 259,000$$

since $W\frac{l^2}{8} = 8,700$ lb.ft., we have Wr = 45,300 lb.

The observed value of W was 39,200 lb. (a difference of -13%)

The ultimate moments may be calculated from the measurements of the reactions. These were 162,000 lb.ft. over the support instead of 176,000 lb.ft. (difference 8%) and 115,000 lb.ft. under the load instead of 145,000 lb.ft. (difference 21%).

<u>Beam B</u> (with additional mild steel).

Ultimate moment in the span (loaded section) : 164,000 lb.ft.

Ultimate moment above the support: 197,000 lb.ft.

The theoretical ultimate load Wr is therefore 50,500 lb.

The observed value of W was 46,000 lb [illegible] (a difference of -9%)

The measured moments were as follows:

under the load : 136,000 lb.ft. instead of 164,000 lb. ft. (difference 17%)

above the support: 193,000 lb. ft. instead of 197,000 lb.ft. (difference 2%)

If there had been no redistribution, the loads would have been 35,900 lb. for beam A and 40,300 lb. for beam B. The ratio $\frac{\text{observed load}}{\text{No redistribution load}}$ is therefore 1.09 in the one case and 1.14 in the other.

2.5 TESTS BY LEVI AND MACCHI.

Tests carried out by Levi and Macchi are described in a paper by Macchi presented at the second congress of the Federation Internationale de la precontrainte[12] held in Amoterdam.

Tests were made on three continuous three span beams, 4 x 10 in. in section, in two of the beams (C_1 and C_2). The spans were 6.55 ft. 13.1 ft. and 6.55 ft., one beam (C_3) had spans of 9.85 ft. 13.1 ft. and 9.85 ft.

A prestress was applied by one cable consisting of 8 wires of 0.196 in. diameter, the ultimate force F_{su} = 64,500 lb.

Untensioned steel wires of 0.196 in. diameter were provided at each corner(with1in. cover)

The cable was at the upper edge of the middle third in the side spans. In the central span the cable was curved, the arrangement of the cable over the intermediate supports was symmetrical with the arrangement at mid-span, hence the ultimate moments are the same for both sections. If Mr is their common value, then the theory of full redistribution would lead to an ultimate load Wr such that

$$Wr\,\frac{1}{4} + W\,\frac{\ell^2}{8} = 2\,Mr$$

Macchi measured Mr for two of these beams, by testing them to distruction in the statically determinate condition (as a cantilever) after the test. For the third beam, the ultimate moment was evaluated by comparison with an identical beam CR, which was tested upto failure under statically determinate conditions. The strength of the concrete was not the same for beams C_3 and CR.

The concrete cube strengths were as follows:

C_1 and C_2 : 5,600 lb/in^2

C_3 : 7,400 lb/in^2

In the above equation, $W \frac{l^2}{8} = 1,000$ lb.ft.

The theoretical values of Wr are given by this equation, using the respective values of Mr. A comparison between theory and experiment is given below:

TABLE 2.

Beam	Mr (lb. ft. x 10^3)	Wr Calculated (lb. x 10^3)	Wr observed (lb x 10^3)	$\frac{\text{observed}}{\text{Calculated}}$	$\frac{\text{Observed load}}{\text{No redistribution load}}$
C_1	20.6	12.76	11.1	0.87	1.13
C_2	27	16.8	14.7	0.87	1.14
C_3	30.4	18.9	14.9	0.78	1.08

As has been said above, M. Guyon believes that the value of Mr has been over-estimated for beam C_3, as was stated in his general report at Amsterdam; the coefficient 0.78 should in his opinion be raised to something like 0.84. In any case, there is a lack of redistribution.

2.6 TESTS OF MORICE AND LEWIS

Tests by Morice and Lewis are described in an unpublished report[13]

Tests were made on 28 continuous beams, each of two spans of 7 ft 6 in., the beam section being 4 x 6 in. Two concentrated loads W were applied symmetrically at the mid-spans. The tests were made in order to check that linear transformations of the cables (i.e., transformations of the tendon profile without altering the end anchorages or the intrinsic shape of the tendon in each span) would not affect the ultimate load. This is an intrinsic property of full redistribution, as will appear from what has been said earlier in this paper.

The tests did in fact show that this assumption was approximately correct. The series of beams with different profiles, one of them concordant and the others transformed up or down, failed under loads differing from the average by not more than 5%. The full details are given in the report.

With reference to these tests, it seems appropriate to draw attention to the necessity of standardising the test procedure followed in determining the strength of concrete. The meaning of the strength of concrete varies from one country to another, and sometimes within the same country. In the graph shown in figure 8, the concrete strength is that measured on the cubes <u>as cast</u> . When the cubes are smoothed, the strength is multiplied by 1.25.

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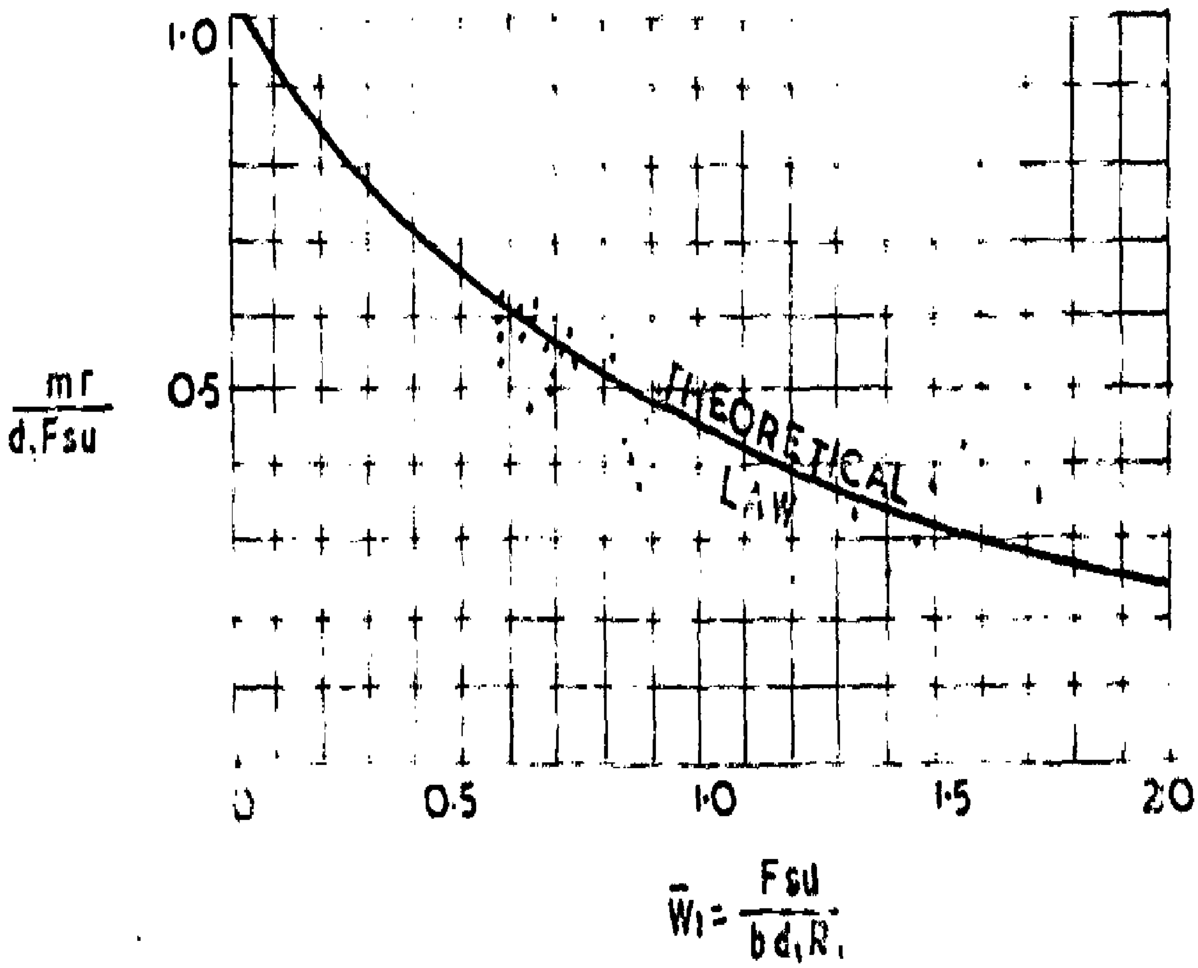


FIGURE 15 TEST RESULTS OF MORICE A

In the tests reported by Morice and Lewis, the strengtn of concrete was measured on 4 in. cubes, cast in accurately machined moulds, the effect of this perfect finish of the moulds may be considered as equivalent to smoothing. Further, the cubes were cured in water and tested wet, which, compared with our tests, is another difference which will increase the strength.

In order to compare the results with our graph, therefore, the cube strengths reported by Morice and Lewis must be multiplied by some factor C, and the weighted percentage which should be marked on the scale $\overline{\omega}$ of figure 8 should be $\overline{\omega} = \frac{F_{su}}{b d_1 R'}$ where R' = CR.

When the strength is measured on smoothed cubes, the value of C should be taken as 0.8. It seems that in this case the value to be taken is $\frac{1}{1.6}$ or 0.625. In other words if the simple formula $Mr = d_1 F_{su} (1 - \alpha \overline{\omega})$ is used (with $\overline{\omega} = \frac{F_{su}}{b d_1 R}$), then α becomes $\frac{0.5}{0.625}$ = 0.8 instead of 0.5. This factor of 0.8 is the one suggested by Morice and Lewis and appears to be justified. If, instead of this formula, the graph of figure 8 is used, we must mark on the scale $\overline{\omega}$ a value equal to 1.6 times the value $\frac{F_{su}}{b d_1 R}$. With this corrected value of $\overline{\omega}$, the agreement is perfactly satisfactory, and as the tests give the statistical result (from 28 beams) the assumptions made by Morice and Lewis are justified.

In their tests, the reactions were measured, the moments actually reached in the critical sections may, therefore, be calculated. These are plotted in terms of $\overline{w'}$ (where $\overline{w'} = 1.6\,\overline{\omega}$) in figure 15. The test results agree satisfactorily with the "theoretical" law relevant to the steel used.

In addition, statically determinate control beams, possessing the same characteristics, were tested. The results are marked as points C on the figure 15. It will be seen that their ultimate moments are of the same order of magnitude as those of the statically indeterminate beams. It should also be pointed out that these orders of magnitude agree with results obtained by Baker(6) and by Prentis(8).

It may be concluded from the tests carried out by Morice and Lewis that in the case of these 28 beams, redistribution was practically complete.

2.7 TEST OF THE FESTIVAL OF BRITAIN FOOTBRIDGE.

Results of the test of the Festival Footbridge(14-16) do not indicate full redistribution but, having studied them very carefully, we think that they do not contradict the theories of redistribution. The differences are due to the absence of bond between the cables and the concrete, the bad quality of the grouting was noted after the structure had been tested to destruction, in fact, the grouting was quite ineffective. This should not be taken as criticism of the construction, the hurry which was necessary is a sufficient explanation of the difficulties, the aim here is to examine objectively the reasons why failure occurred sooner than had been expected.

It can be said that, under the conditions of the test, i.e., with the actual ultimate moments (reduced due to the lack of bond), the redistribution was as near complete as it could have been.

2.8 SUMMARY OF EXPERIMENTAL DATA:

TABLE 3

Tests	Beam	Observed Load / Full Redistribution Load	Observed Load / No Redistribution Load
4 Two-span beams (Guyon)	A_1	1.03	1.88
	A_2	1.04	2.04
	B	1.08	1.07
	C	1.16	1.16
Three-span beam		1.04	1.25
Frames — Lebelle		0.98	1.17
Frames		0.94	1.12
Frames — Guyon		0.97	
		0.97	
2 Continuous beams (LIN)	A	0.87	1.09
	B	0.91	1.14
3 Three-span beams (Macchi)	C_1	0.87	1.13
	C_2	0.87	1.14
	C_3	0.78 (0.84)	1.08 (1.16
28 Two-span beams (Morice and Lewis)		0.95 - 1.05	1.03 - 1.51 (3.16 for test 11)
Festival Footbridge		0.97*	

*Taking into account lack of bond (0.82 referred to the maximum load which could have been supported with good bond.

Under normal conditions (absence of bond being considered as an abnormal condition) the following conclusions may be drawn from the above experimental data.

1. The increase of strength due to redistribution of the moments depends upon the discordance. In Guyon's tests B & C, there was no increase due to lack of discordance. On the otherhand when the ultimate moment of one of the plastic hinges is very small in comparison with the ultimate moments at other hinges (which often occurs due to the reversal of moments, e.g. positive moments over supports, or negative moments at mid span over certain structures, the increase may be considerable (e.g. Guyon's tests A_1 amd A_2, and Morice's test 11).

2. The assumption of full redistribution allows the ultimate load to be estimated to within at worst 15%, and generally more accurately. It might be said that this accuracy is sufficient, and of the same order as that of the ultimate moments themselves, further, that the designer does not require any greater accuracy, since he never knows the true strength of his materials, particularly the concrete.

The assumption of full redistribution cannot be other than an approximation, the accuracy of which depends on definite conditions of compatibility of the strains, or more precisely of the rotations of the ends of the spans on both sides of a given support, or, which is still equivalent, of the rotations at the successive plastic hinges.

The assumption merely means that small variations in the reactions give rise to large variations in these rotations, which allows the solution to be adjusted with very small modifications to the conditions of compatibility to be fulfilled.

CHAPTER 3.

THEORY AND DESIGN OF PORTAL FRAMES

3.1 SUMMARY

Continuous prestressed concrete beams are, in general more difficult to design than simply supported beams. The fundamental difference between the two is that in the latter the line of pressure (i.e. the line along which the prestressing force acts) coincides with the mean line (the centroid axis) of the cables, while in the former this is not necessarily the case. This is due to the fact that, if a cable is placed arbitrarily in a statically indeterminate beam, the tensioning of the cable causes the beam to deflect, and this deflection creates statically indeterminate reactions which distort the line of pressure so that, in general, it no longer concides with the cable.

It will be shown that the most important factor is not the cable profile, but the shape of the line of pressure. The problem therefore consists in determining a cable profile corresponding to a given line of pressure.

However, an arbitrary line of pressure is not necessarily a possible one. For a given line of pressure to be possible, it must fulfil the condition that the tensioning of a cable placed along it causes no statically indeterminate reactions. Such a line of pressure is called "stable."

One possible position of the cable for obtaining a stable line of pressure is along the line itself and such a cable is called "Concordant". It can be shown that this is not the only possible cable profile which will give this particular line of pressure; by translating the cable i.e., by adding linear functions to its coordinates with respect to the neutral axis of the beam, we can, it is possible in theory to deduce from a concordant cable an infinite number of cable profiles, each of which will give the stable line of pressure from which they originate. A particular cable profile determined in this way may often be an improvement on the concordant cable, reducing the friction between the curved cable and the concrete or making it easier to accommodate the jacks and anchorages.

It is shown that the stable line of pressure must be entirely within a limiting zone (there being no such zone for each span or member of the structure) which is determined by the shape of the numbers and by the external loads which they have to support.

The general problem therefore consists in determining a stable line of pressure lying entirely within the limiting zone. In certain cases, it may be necessary to allow the line of pressure to be outside the limiting zone in some parts of the beam, thus causing tensile stresses in the concrete at those parts greater than the permissible stresses. In this case, complementary mild steel reinforcement is introduced in the regions of high tensile stresses.

There are two methods of ~~showing~~ solving the general problem. In both methods, if there are n statically indeterminate reactions, there will be n unknowns. In the first method, the n unknowns which are determined are the points through which the stable line of pressure must pass in order that the statically indeterminate reactions due to the prestressing shall be zero. In the second method, the unknowns are conditions concerning the area included between the stable line of pressure and the neutral axis.

It can be shown that although the problem is usually rather complicated, it can always be replaced by a corresponding problem involving an imaginary load q(x) acting on each span. The line of pressure is then considered as being the fenicular polygon of this load and of the compressive force due to the prestressing force F (to which may be added a compressive force N due to external loads). This imaginary load is the second derivative of the prestressing moment Fy and is therefore equivalent to the transverse load created by the tension in the curved cable. Conversely the ordinate y in each span or element is the second integral of the imaginary load q(x). This second integral includes two constants representing the ordinates y at the ends of each span; these depend on the continuity conditions at those ends i.e., on the partial or complete restraints assumed at those ends.

In the case of the hinged portal, the only statically indeterminate reaction is the horizontal thrust. The problem therefore consists in determining a line of pressure lying entirely within the limiting zone and of such a shape that no horizontal thrust is created when the cable is tensioned. The problem is complicated by the fact that in portal frames, the magnitude of the compressive force depends not only on the prestress but also upon the reactions due to vertical loads.

Thus, in the legs, the prestressing force is increased (algebrically) by the vertical reactions and in the transom increased by the horizontal thrust.

3.2 Concordant cable. Stable line of Pressure.

The cable is said to be 'concordant' when the line of pressure caused by the tensioning of this cable coincides with it.

Such a line of pressure is said to be 'stable' in order to emphasize the fact that tensioning the cable does not displace the assumed line of pressure.

A line can only be chosen as the line of pressure if its eccentricities at the supports correspond to the moments created at the supports by the prestress exercised by a cable coinciding with this line, in other words, only if it is stable.

This can also be expressed in another form:

1. A line may only be chosen as the line of pressure, if for a cable coinciding with this line, the statically indeterminate support reactions caused by the tensioning of the cable are nil.

and also in this form:

2. A stable line of pressure is the feunicular curve of a certain imaginary load function q(x) and of the reactions exercised by the supports of the continuous beam under this system of loading, i.e., the bending moment diagram created in the beam by the loading q(x), taking into account the constrants and drawn to a scale depending on the magnitude F of the prestressing force. This line is a solution of the problem if, when drawn

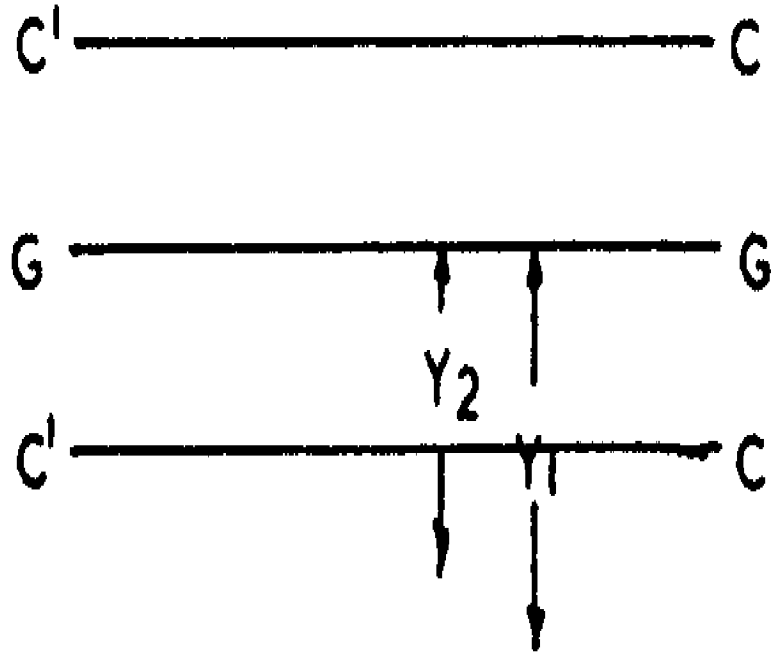


FIGURE :- 16

to a proper scale, it remains entirely within the predetermined limiting zone.

If necessary, end moments, caused by eccentric anghorages, may be added to the system of loads q(x), this is equivalent to adding to the real constrants partial or complete constrants at the two ends of the beam.

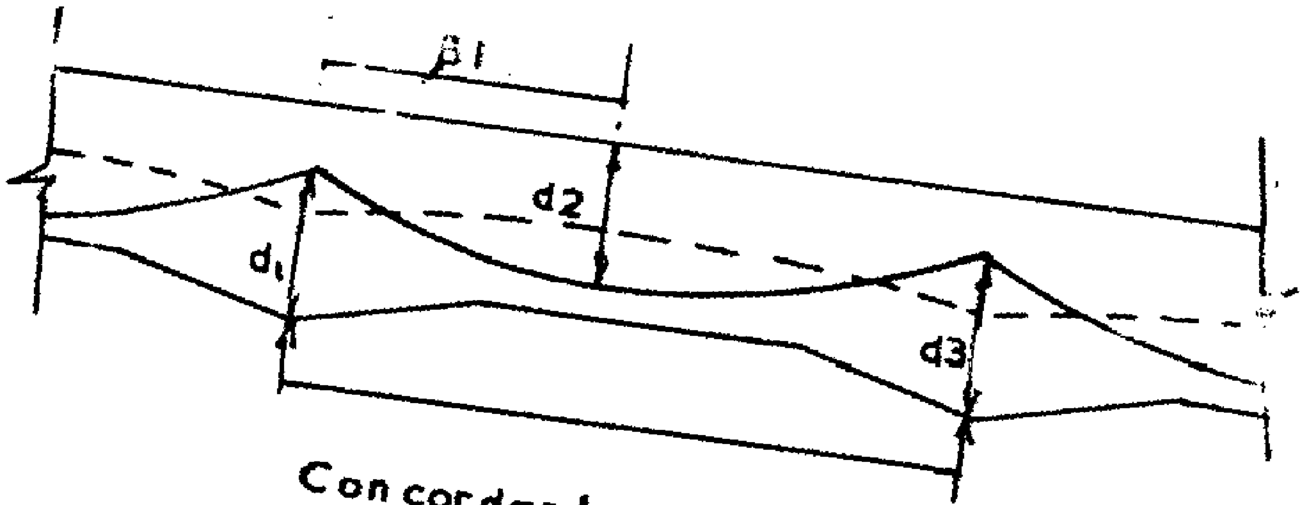
Once the line of pressure has been found, all the possible cables can be obtained by linear transformations.

The above two principles are obviously equivalent, the load q is equal to $F \frac{d^2y}{dx^2}$, where y is the ordinate of the line of pressure, and conversely, y is the second integral of the expression $\frac{q}{F}$. The arbitrary integration constants are only two in number because the ordinate of the line of pressure at a support has a single value which is the same for the span to the left and for the span to the right of that support. End moments can be introduced by choosing adequate values for these two arbitrary constants.

The methods of solution which (are given lower down), are based on these two principles. They consist in determining either the line of pressure or the imaginary loading of which the line of pressure is a funicular curve (18).

3.4 <u>The invariance of elastic design stresses under linear tendon transformations</u>.

It was first shown by Guyon[17] that the working load conditions are invariant under a linear transformation.



Concordant Tendon Profile

FIGURE:- 17

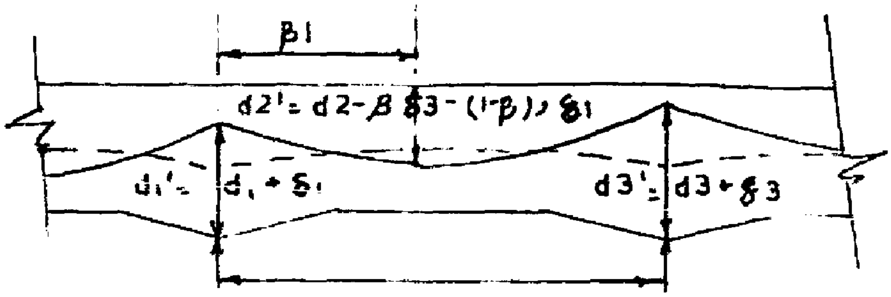
A study of the use of continuity in prestressed concrete frames leads to the conclusion that it is not necessarily advantageous and often may be uneconomical as far as the elastic condition is concerned, since prestressed design is principally based upon moment variations and not upon absolute moment values.

However, considerable advantageous will almost always follow from continuiting when attention is turned to the ultimate load carrying capacity of the whole frame, particularly if the moment curvature relationship contains a platean at the maximum moment, since this will enable the critical sections of the structure to hold their maximum moments simultaneously, irrespective of the order in which they were first developed. Except in the case in which all the critical sections have a constant ratio of maximum moment of resistance to working moment, the failure condition for the whole frame will have a load factor greater than that of the weakest critical section.

Consider the ultimate moment conditions of a span of a beam or frame having two alternative tendon profiles, one of which is a linear transformation of the other (figures 17 and 18). The free bending moment on the span is $\bar{M}$ and M_1 M_2, M_3 are the ultimate moments at the three critical sections (figure 19). It can easily be seen that M_1, M_2 and M_3 are linear functions of the corresponding effective depths d_1, d_2 and d_3.

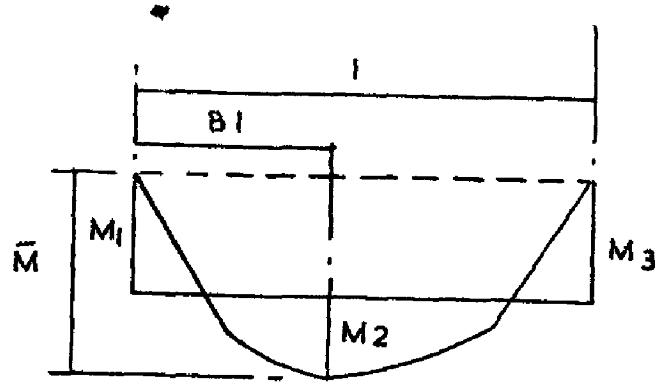
The ultimate moment equation for the span is

$$\bar{M} = (1-\beta)M_1 + M_2 + \beta M_3 \qquad (1)$$



Transformed Tendon Profile

FIGURE:- 18



Moment Diagram At Failure With Concordant Tendon Profile

FIGURE:- 19

which can be written

$$\overline{M} = (1-\beta)(Ad_1+B)+Ad_2+B+\beta(Ad_3+B)$$

i.e.

$$\overline{M} = A\left\{(1-\beta)d_1+d_2+\beta d_3\right\}+2B \qquad (2)$$

where $A = A_s f_u$ and $B = -\frac{A_s^2 f_u^2 Y}{\alpha u b}$

under a linear transformation of the tendon profile, the critical effective depths will be changed to the following(figure 18).

$$d_1' = d_1 + \delta_1$$
$$d_3' = d_3 + \delta_3$$
$$d_2' = d_2 - (1-\beta)\delta_1 - \beta\delta_3$$

It is seen that expression (2) is invariant under such a transformation and therefore the ultimate load remains the same for both tendon profiles.

3.5 Hinged Portal Frame;

The line of pressure, for the prestress only, consists in this case of three separate lines: One for each leg and one for the transom.

The prestressing force is usually different for the legs and for the transom; if the loading and the frame itself are unsymmetrical, the prestressing force may even have different values for the two legs.

The problem is again pased in the same general manner; Given the limiting zone within which the line of pressure must be (more exactly, given the three limiting zones, one per member, within which the three lines of pressure must be), the line of pressure (i.e., the three branches of this line) cannot be drawn arbitrarily. For a line of pressure to be possible, the statically indeterminate reactions caused by the tensioning of cable placed along this line (i.e. of the three cables placed along the three lines of pressure) must be nil.

Having found such a line, we may apply to the cable profiles linear transformations of a certain type without altering the line of pressure.

We shall assume that the portal frame is symmetrical. We shall call F the magnitude of the prestressing force in the transom and F' and F'' the magnitude of the prestressing force in the left and the right leg respectively.

We shall call 'g' the height of the legs, measured from the centre line of the hinge to the neutral axis of the transom, and l the span of the transom. We shall also assume that the legs and the transom have constant rectangular cross-sections of equal width 'b' and of a depth 'h' for the transom and 'k' for the legs.

The results which we shall obtain below can easily be extended to other cases (unsymmetrical portal frames, non-rectangular cross-sections, different widths b and b' for the transom and the legs).

For the sake of simplicity, we shall call the three separate lines of pressure (i.e. for the left leg, for the transom and for the right leg) the line of pressure of the frame, and similarly we shall call the three separate cables, the cable of the frame.

The only statically indeterminate reaction in this case is the horizontal thrust. The problem therefore consists in determining a cable profile lying entirely within the limiting zone and of such a shape that the tensioning of the cable causes no horizontal thrust.

Let 'I' be the moment of inertia of the transom and 'J' that of the legs. Let 'y' be the distance of a point on the centroid axis of the cable to the neutral axis of the transom or of the legs, as the case may be; y will be positive or negative according to whether the point in question lies outside or inside the rectangle ABCD formed by the neutral axes of the members and which we shall call for short the <u>neutral line</u> of the frame. Finally, let Z be the ordinate of a point on the neutral line of the frame.

The limiting zone is determined by the same general considerations as previously, i.e., by the condition that the line of pressure must remain within the limit core for all possible loading cases. The case of the portal frame differs however in one particular from that of the continuous beam. in the latter case the compressive force had a constant magnitude throughout the beam, equal to the prestressing force, whereas in the case of portal frames, the magnitude of the compressive force depends not only on the prestress but also on the reactions due to the external loads. Thus, in the legs, the prestressing force F' is increased by the vertical reactions, added algebrically in the transom the prestressing force F is increased by the horizontal thrust.

If M is the moment and N the normal force in a point of a member due to any given loading, F the prestressing force and e its eccentricity, the eccentricity of the line of pressure of this point will be given by

$$\frac{M + Fe}{F + N}$$

The line of pressure must remain within the limit core. Calling M_1, N_1 and M_2, N_2 the values of M and N corresponding to the smallest and to the greatest value of the expression $\frac{M + Fe}{F + N}$ respectively, this condition is translated by

$$\frac{M_1 + Fe}{F + N_1} > -\frac{h}{6}$$

and $$\frac{M_2 + Fe}{F + N_2} < +\frac{h}{6}$$

Let point E be the centre of pressure for prestress only and let y_1 and y_2 be its ordinates with respect to the line $c'c'$ and $c\,c$ respectively; $c'c'$ and $c\,c$ are as previously the lower (inner) and the upper (outer) limit core boundary lines.

we have

$$e = y_1 - \frac{h}{6}$$

and also

$$e = y_2 + \frac{h}{6}$$

and the above inequalities become, after transformation:

$$y_1 > -\frac{M_1 + N_1 \frac{h}{6}}{F}$$

and $$y_2 < -\frac{M_2 - N_2 \frac{h}{6}}{F}$$

It follows that if we measure the ordinates $\frac{M_1 + N_1 \frac{h}{6}}{F}$ and $\frac{M_2 - N_2 \frac{h}{6}}{F}$ from the lines $c'c'$ and cc respectively, positive values being measured downwards and the negative values being measured upwards (Note the negative sign in the previous expressions), the centre of pressure must remain within the zone unshaded in figure .

This rule is similar to that given previously for simple bending, the moments M_1 and M_2 now being replaced by $M_1 + N_1 \frac{h}{6}$ (moment with respect to the lower edge of the limit core) and $M_2 - N_2 \frac{h}{6}$ (moment with respect to the upper edge of the limit core) respectively.

The magnitude of the horizontal thrust Q is usually small compared to that of the prestressing force F in the transom. Its effect may therefore be neglected in a first approximation and the limiting zone determined in the same manner as before.

In the frame legs, on the contrary, the magnitude of the compression, due to the external loads and reactions is always large compared to that of the prestressing force F' and F'' and must therefore always be taken into consideration for the determination of the limiting zone.

Calling M_o the bending moment at each point due to prestressing only, and not taking into consideration the member shortening due to axial thrust, the condition that the thrust caused by the prestress must be nil can be written:

$$\int \frac{M_o z\,ds}{I} = 0$$

the integration extending over the whole contour ACBD.

The moment M_o is equal to Fy in the transom and to $F'y$ and $F''y$ in the left and the right legs respectively. The transom is horizontal and therefore $Z = g = \text{Const.}$

The integration may be written

$$(1) \quad \frac{F'}{J}\int_0^g yz\,dz + \frac{Fg}{I}\int_0^\ell y\,dx + \frac{F''}{J}\int_0^g yz\,dz = 0$$

Calling stiffness of a member the ratio of its moment of inertia to its length, the stiffness of the transom is $\frac{I}{\ell}$ and that of each leg is $\frac{J}{g}$. The ratio of leg stiffness to transom stiffness is:

$$\rho = \frac{J\ell}{Ig}$$

The one extreme value of this ratio, $\rho = 0$, corresponds to a simply supported transom, the other, ρ infinite, corresponds to a transom fully fixed at both ends. Introducing this ratio into equation (1) we may write it in the form:

$$(2) \quad F'\int_0^g yz\,dz + F''\int_0^g yz\,dz + F\rho g\frac{g}{\ell}\int_0^\ell y\,dx = 0$$

The two integrals for the legs, although they have the same form, may not have the same value if the function defining y is not the same for both.

For the sake of simplicity we shall assume that the prestressing conditions are identical for the two legs i.e., that $F' = F''$ and that the function y is the same. The case of unsymmetrical prestress presents no special difficulties (and we shall see one below).

If the prestressing conditions are identical in both legs, equation (2) may be written in the form:

$$(3) \qquad 2F'\int_0^g y z\,dz + F\rho\frac{g^2}{\ell}\int_0^\ell y\,dx = 0$$

If $\rho = 0$, equation (3) becomes $F'\int_0^g y\,dz = 0$

This can be satisfied either by $\int_0^g y\,dz = 0$ which means that the prestressing cable is placed in the frame leg in such a manner it does not tend to displace point C out of plumb with respect to A; a particular case is that of axial prestressing or by $F' = 0$, i.e. noprestressing. This solution is usually inadequate because the legs can not withstand without prestressing the effects of the subsequently applied external loads, unless it is made in ordinary reinforced concrete.

If ρ is infinite, equation (3) becomes $\int_0^\ell y\,dx = 0$, an expression which we have already met for a beam fixed at both ends.

The cables in the legs are often straight and pass through the centre lines of the hinges. Assuming that both legs are prestressed identically and calling u the eccentricity of the leg cables at the level of the horizontal straight line cb (the neutral axis of the transom), we have in the legs:

$$y = u\frac{z}{g}$$

$$\text{and} \int_0^g yz\,dz = \int_0^g \frac{uz^2}{g}\,dz = \frac{ug^2}{3}$$

Equation (3) may then be written:

$$(4) \quad \frac{2}{3} F'.ug^2 + F\rho\frac{g^2}{l}\int_0^l y\,dx = 0$$

$$\text{or } (4) \quad \frac{2}{3}\frac{F'u}{\rho} + \frac{F}{l}\int_0^l y\,dx = 0$$

In this case the general problem is posed as follows:

The cross sections of the legs and the transoms and magnitudes F' and F of the required prestressing forces are determined by the analysis of stresses due to the external loads. This determines the three limiting zones.

Equation (4) furnishes a relationship between u and $\int y\,dx$.

If the limiting zones of the legs are reduced to a point at the top (i.e. at the neutral axis of the transom), as is the case when the legs are dimensioned strictly, the

the eccentricity u is known " a priori".

The problem is then reduced to finding a curve lying entirely within the limiting zone of the transom and with a shape such that the expression $\int_0^l y\,dx$

$$\int_0^l y\,dx = -\frac{2}{3} \times \frac{F'}{F} \times \frac{ul}{p}$$

If the legs are over dimensioned, we have a certain latitude in the choice of u provided that equation (4) remains satisfied.

However, the condition that the cables in the legs should be straight and should pass through the centre lines of the hinges, is by no means compulsory; the general equation(3) is always applicable. Thus in some cases where it is difficult or impossible to satisfy equation (4) corresponding to straight leg cables passing through the centre lines of the hinges, it may be advantageous to choose some other shape for these cables,

the eccentricity u is known " a priori".

The problem is then reduced to finding a curve lying entirely within the limiting zone of the transom and with a shape such that the expression $\int_0^l y\,dx$

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provided that they remain within the limiting zones of the legs.

In effect, the meaning of the general equation (3) and of its particular form (4) is that the prestressing of the legs deforms the legs in such a manner that the leg ends A and B, which, were they free to slide on their bearings would have moved under the influence of the transom deformation to A' and B' respectively, are brought back to their original positions A & B.

Therefore, if the lines of pressure which it is possible to place within the limiting zone of the transom result in deformations which of is impossible to compensate by the action of straight leg cables passing through the centre lines of the hinges, we must endeavour to give the leg cable a more effective shape.

A straight cable parallel to the neutral axis of the leg, i.e. with a constant ecentricity V, provokes a deformation:

$$\int_0^{8} \gamma z \, dz = \frac{V 8^2}{2}$$

A parabolic cable passing through the centre line of the hinge and having a vertical tangent at the top with an ecentricity W (say of the parabola), provokes a deformation:

$$\int_0^{8} \gamma z \, dz = \frac{5 W 8^2}{12}$$

FIGURE :- 25

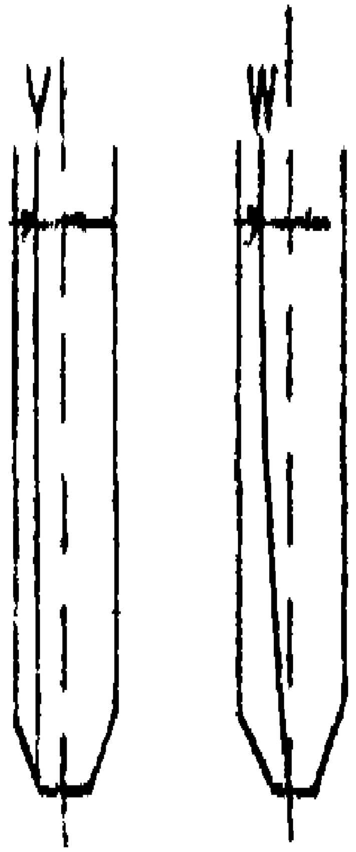

FIGURE :- 26

Comparing these values with that for the straight cable passing through the centre line of the hinge and having the same ecentricity at the top so that u = V = W, we see that the parallel straight cable and the parabolic cable are $1\frac{1}{2}$ and $1\frac{1}{4}$ times more respectively.

Possible profile of cables obtained from a stable line of pressure, on the assumption that the two frame legs are prestressed identically by straight cables passing through the centre lines of the hinges.

On this assumption the stable line of pressure, which must be entirely within the limiting zone must satisfy the condition expressed by equation (4).

$$\int_0^l y\,dx = -\frac{2}{3} \times \frac{F'}{F} \times \frac{ul}{P}$$

Let us assume that some other cable profile gives the same line of pressure as that defined by this equation. This cable profile will still obviously consist of two straight lines, one in each leg and passing through the centre lines of the respective hinges, and of a curve, in the transom, which must be parallel to the stable curve defined by the equation.*

In effect, the second profile is not stable, since it results in a line of pressure different from itself, it causes therefore a thrust Q which causes in the transom a constant

*The general rule requires that the two curves must have the same curvature. The "parallelism" is due to symmetry.

bending moment $Q z$ which displaces the line of pressure by the amount $-\frac{Q z}{F}$.

Let us assume, therefore, that the transom cable has been obtained by translating the original stable line of pressure by an amount a and let us now try to determine the amount by which we have to rotate the leg cables (about the centre lines of the hinges) in order to obtain a thrust Q equal to $\frac{Fa}{z}$ which will make the line of pressure in the transom coincide with the original stable line of pressure. Let us call V the supplementary eccentricity of the leg cables at the top (i.e. at the neutral axis of the transom); the total eccentricity of the leg cable at the top is thus increased from U to U + V.

This new state differs from the original by the presence of bending moments $\frac{F' V z}{z}$ in the legs and F a in the transom due to the supplementary eccentricities. Since the thrust in the original state was not, the thrust Q in the new state is due to these supplementary bending moments only and therefore equal to:

$$Q = \frac{2\frac{F'V}{8J}\int_0^8 z^2 dz + \frac{Fa8}{I}\int_0^l dx}{\frac{2}{J}\int_0^8 z^2 dz + \frac{8^2}{I}\int_0^l dx}$$

$$= \frac{\frac{2}{3}\frac{F'V}{J}8^2 + \frac{Fa8l}{I}}{\frac{2}{3}\frac{8^3}{J} + \frac{8^2 l}{I}}$$

$$= \frac{\frac{2}{3}\frac{F'V}{J}8 + \frac{Fal}{I}}{\frac{2}{3}\frac{8^2}{J} + \frac{8l}{I}}$$

$$= \frac{\frac{2}{3}F'V \cdot \frac{I8}{Jl} + Fa}{\frac{2}{3}8 \cdot \frac{I8}{lJ} + 8}$$

$$= \frac{1}{8}\,\frac{\frac{2}{3}F'V\frac{I8}{Jl} + Fa}{\frac{2}{3}\frac{I8}{Jl} + 1}$$

$$= \frac{1}{8}\,\frac{\frac{2}{3}F'\frac{V}{\rho} + Fa}{\frac{2}{3\rho} + 1}$$

But $Q = \dfrac{Fa}{8}$

$$\therefore\ Fa = \frac{\frac{2}{3}F'\frac{V}{\rho} + Fa}{\frac{2}{3\rho} + 1}$$

Finally $F'V = Fa$ or $V = \dfrac{Fa}{F'}$.

The value V is the measure of the rotation we set out to determine. With this value of V, the thrust $Q = \frac{Fa}{8}$ will bring the line of pressure in the legs back to its original position because this thrust will cause at the top of the leg a bending moment - F a equal to - F'V and the resulting total moment at the leg will be $F'(U+V) - F'V$, i.e. the same as originally.*

In the transom the line of pressure remains the same because the curvature of the cable has not altered.

The bending moment increments at the corners of the frame being Fa for the transom and F'V for the legs, we can obtain from a given concordant cable and on the above assumptions (identical straight leg cables passing through the centre lines of the hinges) all the possible corresponding cable profiles by giving the transom cable any arbitrary translation and at the same time giving the leg cables a corresponding rotation, this rotation being calculated in such a manner that the increments of the bending moments due to the tensioning of these three cables should be equal for the transom and for the legs respectively because this common moment increment is then cancelled by the resulting statically indeterminate thrust.

This was in fact evident "a priori" because it is the statically indeterminate thrust which brings the unstable line of pressure back to its original stable shape; this thrust causes at the frame angle the same bending moments in the transom and in the legs.

This rule is equivalent to that established for continuous beams. In a continuous beam, we are free to alter the ordinate of the cable over a support with respect to the ordinate of the concordant cable because in a continuous beam with a uniform cable, the bending moment over a support can only have one single value which is the same immediately to the left and immediately to the right of the support. A translation of the cable over a support alters therefore the moments immediately to the left and immediately to the right of the support by the same amount and as we have seen, the statically indeterminate support reactions caused by such a translation bring the bending moment back to its original value corresponding to the concordant cable. In a portal frame, the bending moment at the angle has two values (equal), one for the leg, the other for the transom. The simultaneous translations of the two lines of pressure at that point must be so correlated to each other that they cause the same apparent moment increment in the leg and in the transom, these being equivalent to two spans of a continuous beam lying on either side of a support.

In the case of portal frames, the possinilities offered by such profile tranformations present somewhat less practical interest than they do in the case of continuous beams because they cannot be used to reduce the duration of the cables, the transom and leg cables being distinct, these are no angular points or breaks of profile which could be eliminated.

However, cable profile transformations may some times be used to advantage to displace the leg cable into a position where it can be more easily tensioned and anchored. In particular, it is often advantageous to have straight vertical leg cables placed along the neutral axis of the leg; this can be achieved by adequately translating the transom cable.

A safety check must of course always be carried out according to the principles set out on page 9.[18]

Correction for transom shortening

Under the influence of the prestressing force F, the transom CD shortens.

If point D could slide freely, the leg BD would take the position B'D' and in order to bring point B' back to B, we would have to exercise a certain thrust a acting in the sense shown by the arrows. Points A and B being held in position, the effect of transom shortening is to reduce the prestressing force from F to F-a and to increase the bending moments developed by prestressing by the amount ag in the transom and az in the legs. These total or effective forces and moments are the ones which must satisfy the previously established equations (2), (3) and (4).

Let us make the assumption for which we established equation (4), i.e., straight leg cables passing through the centre lines of the hinges. The effects of transom shortening can be analysed as follows.

1. In order to obtain an effective compression F in the transom, we must apply a pre-stressing force $F_1 > F$. To the transom shortening due to F_1, corresponds a thrust.

(5) $$a_1 = a \times \frac{F_1}{F}$$

and we must have $F_1 - a_1 = F$

This gives $F_1 - a \times \frac{F_1}{F} = F$

and $F_1 \left(1 - \frac{a}{F}\right) = F$

$$\therefore F_1 = \frac{F}{1 - \frac{Q}{F}}$$

Let us calculate Q

For a prestressing force F in the transom, the effective compression in the transom is $F - Q$ causing a shortening of the transom.

$$\Delta l = l \times \frac{F-Q}{ES}$$

On the other hand, the thrust Q caused by this shortening is given by

$$\Delta l = \frac{Q}{E} \int \frac{z^2 \, dz}{I}$$

The integration being taken over the whole contour ACDB. We thus have the equation.

$$Q \left(\frac{2}{3} \frac{\delta^3}{I} + \frac{\delta^2 l}{I} \right) = \frac{F-Q}{S} l$$

which can also be written

$$Q \frac{\delta^2}{I} \left(1 + \frac{2}{3\rho} \right) = \frac{F-Q}{S}$$

Introducing the radius of gyration r of the transom (in order to reduce sand I to a single parameter), we obtain:

$$Q \left\{ \frac{\delta^2}{S r^2} \left(1 + \frac{2}{3\rho} \right) \right\} + \frac{Q}{S} = \frac{F}{S}$$

$$\propto Q \left\{ \frac{\delta^2}{r^2} \left(1 + \frac{2}{3\rho} \right) + 1 \right\} = F$$

$$\propto Q = \frac{r^2}{\delta^2 \left(1 + \frac{2}{3\rho} \right) + r^2} F$$

Introducing this value into equation (5), we see that the required prestressing force is not F, but

$$(6) \quad F_1 = \frac{F}{1 - \frac{Q}{F}}$$

$$= \frac{F}{1 - \dfrac{r^2}{g^2\left(1 + \frac{2}{3\rho}\right) + r^2}\dfrac{F}{F}}$$

$$= F\left\{1 - \frac{r^2}{g^2\left(1 + \frac{2}{3\rho}\right) + r^2}\right\}^{-1}$$

$$\text{or} \quad F_1 = F\left[1 + \frac{r^2}{g^2\left(1 + \frac{2}{3\rho}\right) + r^2}\right]$$

2. The thrust Q_1 corresponding to F_1 will bring B' back to B, but it will also modify the lines of pressure: it rotates the lines of pressure in the legs outwards and raises the line of pressure in the transom. The amount of rotation, at the top of leg, is $d_1' = \frac{Q_1 g}{F'}$ and the amount of translation in the transom is $d = \frac{Q_1 g}{F}$

We wish the final or effective line of pressure to be entirely within the limiting zone.

To achieve this we shall rotate this limiting zone for (F') (for the legs) inwards into the position (F_1') by an amount displacing the top of this zone by the distance d_1' . We shall also have to transform the limiting zone for (F) (for the transom as follows).

The ordinates of the zone (F) with respect to the neutral axis of the transom will be reduced in the ratio $\frac{F_1}{F}$, which gives the limiting zone (f), and this limiting zone (f) will be lowered by the distance $d_1 = \frac{a_1 g}{F_1}$.

The line of pressure corresponding to the prestressing forces F_1 and F and making no correction for the transom shortening should be entirely within these transformed limiting zones (F_1') and (F_1). Then the real line of pressure, taking account of the thrust a caused by the shortening of the transom, will lie within the original limiting zones determined from the external loads.

In effect, let us consider a cable lying within the transformed limiting zone (F_1') and (F_1) and which would be concordant if no allowance had to be made for transom contraction. Let us tension this cable, the tension being equal to F_1 in the transom and F' in the legs. The transom contraction creates a thrust a . This thrust does not alter the value F' of the compression in the legs but causes the line of pressure in the legs to rotate (about the centre line of the hinge) outwards by an amount $d_1' = \frac{a g}{F'}$ at the top. In the transom, on the contrary the compression is reduced from F_1 to F and at the same time the bending moments increase by $a_1 g$. Had the compression remained equal to F_1, the line of pressure, the line of pressure would have risen by $\frac{a_1 g}{F_1}$ and consequently into the limiting zone (f). At the same time, however, the compression is reduced from F_1 to F. The ordinates of the line of pressure corresponding to the effective pressure F can be obtained from those corresponding to F_1 by multiplying them by the ratio $\frac{F}{F_1}$. Thus the line of pressure finally comes into the original limiting zone (F).

These considerations can be summarized in the following rule: For the legs we take a cable tensioned to (F') obtained by rotating the zone (F') outwards by a distance equal to $\frac{a g}{F'}$ at the top of the leg. For the transom we take a beam tensioned to F_1, which is given as a function of F by equation (6), and lying within a limiting zone obtained by reducing the zone (F) in the ratio $\frac{F}{F_1}$ and then lowering it by a distance $d_1 = \frac{a_1 g}{F_1}$.

The ordinates y of the lines of pressure lying within the transformed limiting zone (F_1') and (F_1) will be determined in such a manner that the tensioning of the cables causes no thrust other than the thrust a due to transom shortening, these ordinates m must then satisfy the following equation equivalent to equation (4)

$$(7) \qquad \int_0^l y\,dx = -\frac{2}{3} \times \frac{F'}{F_1} \times \frac{ul}{\rho}$$

It should be mentioned that the increase of prestress required by formula (6) is usually slight. The limit values of F_1 are $F_1 = F$ for $\rho = 0$ ("Soft" Portal) and $F_1 = F(1 + \frac{r^2}{g^2})$ for ρ infinite ("Hard" portal); the radius of gyration (r) of the transom is usually small in comparison to the height 'g' of the legs.

Assuming the transom to be of rectangular cross-section with a depth 'h', the height of leg 'g' of the portal frame must necessarily be greater than $\frac{h}{2}$ and therefore g^2 will necessarily be greater than $\frac{h^2}{4}$. The radius of gyration 'r' of a rectangular section being given by $r^2 = \frac{h^2}{12}$, the amplifying factor $1 + \frac{r^2}{g^2}$ will always be very much smaller. than $1 + \frac{1}{3}$.

As can be seen from equation (7), an increase of the transom prestress from F to F_1 requires a corresponding increase of the leg prestress F'.* The total increase of the prestress in the whole of the portal frame, made necessary by the shortening of the transom, is therefore only slight. This is no longer the case, if th the frame legs are fixed.

As has already been mentioned, all the above arguments which have been limited to the case of both portal legs being prestressed identically by straight cables passing through the centre lines of the hinges, can easily be generalised.

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*Only if it is required to keep to the same value of U and to a given shape of the line of pressure.

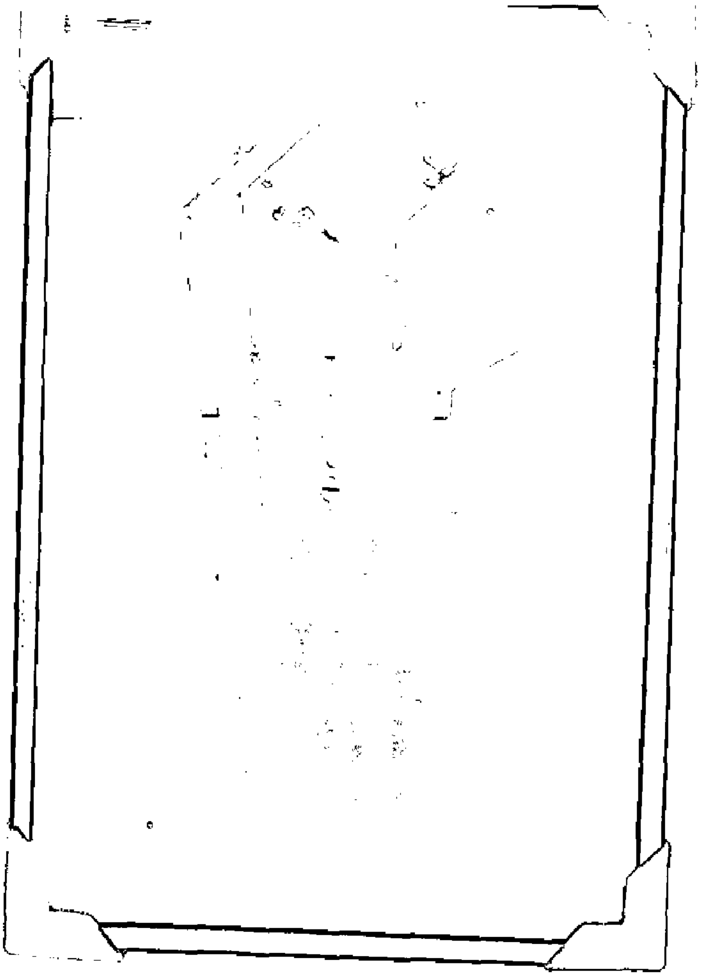


Plate 1. Details of reinforcement Cables and the end block spirals at the left frame angle.

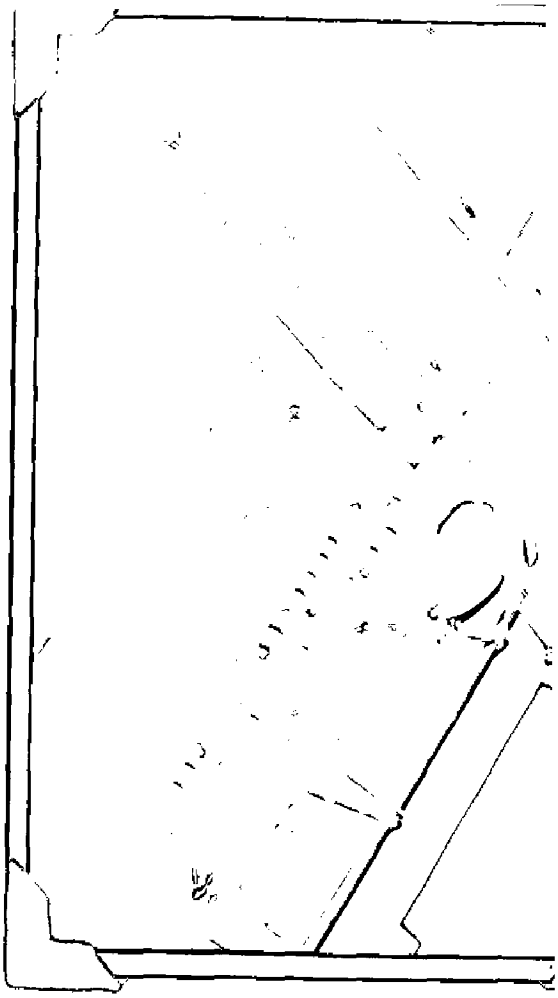


Plate 2. Details of re Cables and th spirals at th frame angle.

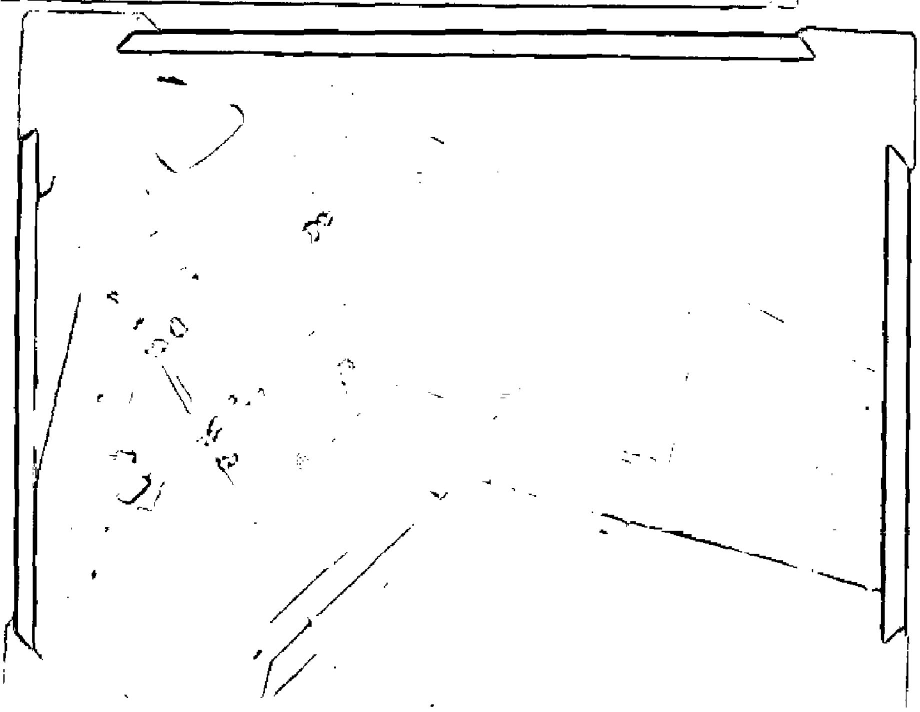


Plate 3. Top view frame angl reinforcem pipe fork Concrete the necess handling

CHAPTER 4.

MATERIALS AND TEST SPECIMENS

4.1 MATERIALS

All the portals were cast with concrete consisting of round gravel 3/4" maximum size, Bhādripur sand which were made available at the Institute and A.C.C. brand normal Portland Cement conforming to I.S. 269 of 1951. The properties of materials and fineness modulii of the aggregates are given in Appendix III.

4.2 THE MIX

The concrete mix was 1:2:4 by weight, with a water cement ratio of 0.55. The mix proportion and the water-cement ratio were designed to give a 28 - day cube strength of 4,000 p.s.i.

4.3 REINFORCEMENT

A nominal reinforcement of 4 Nos. 3/8" dia. M.S. longitudinal bars were used. The ends of the rods were hooked 1/4" dia. M.S. rods ties were used at 9" c/c in the legs, 1/4" dia two-legged stirrups at 6" c/c were used in the transom. The stirrups and the ties were welded to the main longitudinal reinforcement bars. The welding of the stirrup and the ties to the main rods ensured that the spacing was not disturbed at the time of concreting.

(Contd.)

4.4 TEST SPECIMENS

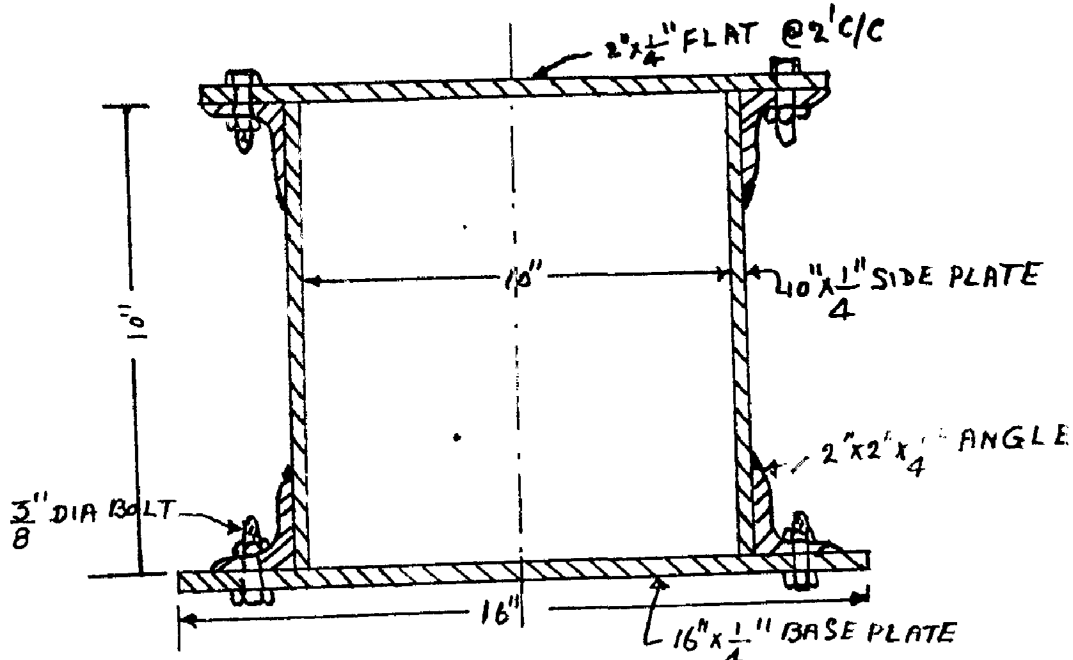
Three two-hinged Portal frames: Two for a central load of 3.8T and one for a third point loading of 2.8T have been designed on elastic theory considerations and have been tested to failure. The frames tested have a span of 9 FT. between leg centre-lines and a height of 4.5 FT. from the centre of hinge to the centre line of the transom. The portals have been cast monolithically thereby establishing continuity between the legs and the transom, each having a cross-section of 10" x 10". The Gifford-udall system of prestressing was adopted for the portals designed for a central load of 3.8T, there being ten 0.2 in. high tensile steel wires in the transom and six of 0.2 in. in each of the legs. The Magnet-Blaton system of prestressing was adopted for the portal designed for a third point loading of 2.8T there being eight 0.2 in. high tensile steel wires in the transom and eight of 0.2 in high tensile steel wires in each of the legs. The transom wires were anchored on the end face and leg wires anchored on the underside of the feet and on the top surface of the transom.

The tests were conducted on the prototypes. The frame supports were formed from 1½ in. dia round bars of hardened steel passing from front to back of the feet. A knife-edge was machined on these bars for a length of 2" from the ends. They were positioned in the moulds so that the centre-lines of the knife-edges were in the same direction as the resultant thrus due to the vertical one horizontal reactions. Each end of each knife-edge was supported in a V-groove machined in the head of a 1" dia bolt. The bolts were fixed in structural steel channel (6" x 3") assemblies which were themselves welded to the base plate . Thus each knife-edge was adjustable for both vertical and horizontal movement to suit small variations between frames. A close-up of one support assembly is shown in Plate 9 .

In order to prevent the feet moving together while the transom was prestressed, a steel strut was wedged between the feet.

Once the frame was raised into its test position in the seatings and a small loading had been applied, the horizontal ~~thr~~ thrust is developed and measured by means of the tie connecting the two legs. This tie consists of two brackets connected by means of a 5/8 in. dia. rod, two U-bolts and a spring balance. This is shown in Plate 8 .

The interference between the leg wires and the transom wires at the frame angle was avoided by passing the transom wires through a pipe-fork as shown in Plate 2 . Similarly the interference between the leg wires and the round bar was avoided by using a similar pipe-fork as shown in Plate 1 . The ducts, both in the transom and the legs, were formed by embedding corrugated metal sheath tubes to the required profile in the concrete. The corrugated metal sheath tubes were held in the required position by suspending them in the steel moulds by means of wires. Necessary hoop steel had been



CROSS-SECTION OF STEEL MOULD

FIGURE 30

provided in the end-blocks to take care of cracking stresses. The required splay for prestressing the wires was provided by means of funnels provided at the ends of the pipe-forks. The slope for the arms of the pipe-forks was so provided that there was no friction caused while the wires were prestressed. Care was taken to see that no concrete or cement mortar entered the duct by putting plaster of paris around all the joints and putting cotton wastes in the funnels.

4.5 MOULDS

The section of the steel mould is as shown in Figure 30 . It consisted of a base plate 1/4" thick and the side plates (also 1/4" thick) strengthened by welding 2"x2"x1/4" angles along their edges. The side plates were fixed to the base plate by means of 3/8" dia. bolts. The top edges of the side-plates were tied together by a flat to withstand the effect of vibration. in holes were were drilled in the end-plates and sideplates of the mould for passing the high tensile wires. 2 in. dia. holes were drilled in the base-plate to carry the supporting round bars. The whole assembled mould was placed on 4" x 4" x 16" wooden pieces so that the bolts can be inserted or removed at ease. They also facilitate the lifting of the portal.

4.6 FIXING OF CAGES

The reinforcement cages were actually centred in the formwork by putting 1" cubes in between the cage and the base plate. Incidentally, they ensured the exact cover on all sides.

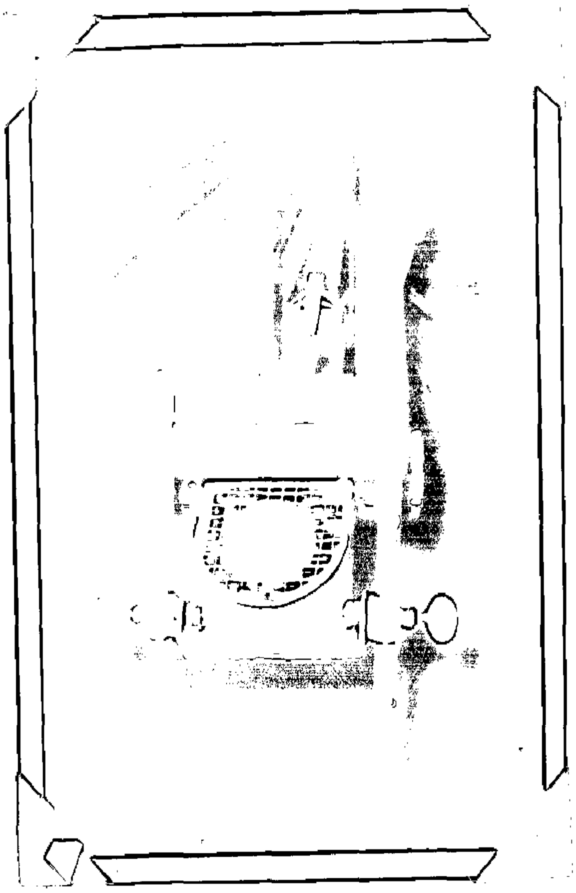


Plate 4. Prestressing the transom wires with Gifford Udall Jack.

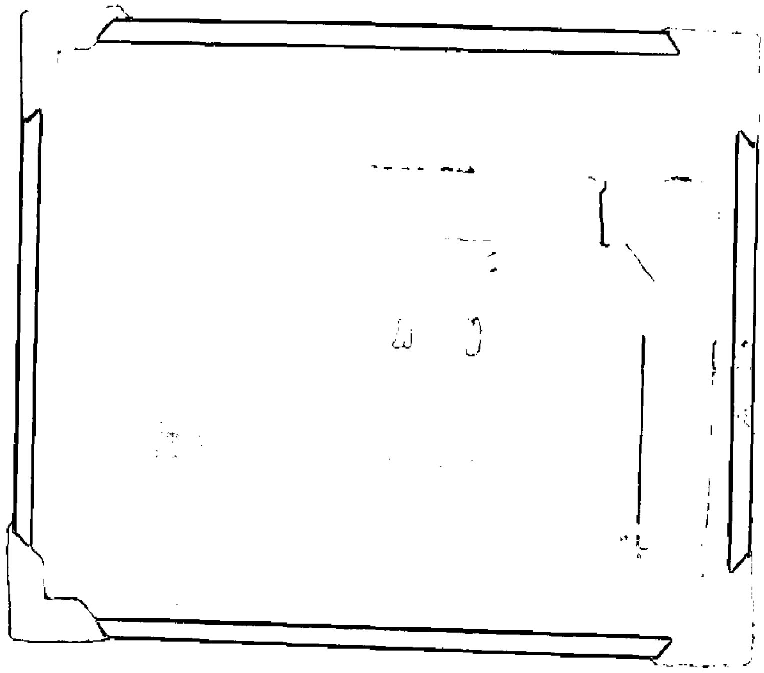


Plate 6. Portal in the raised position showing the mobile yale crane.

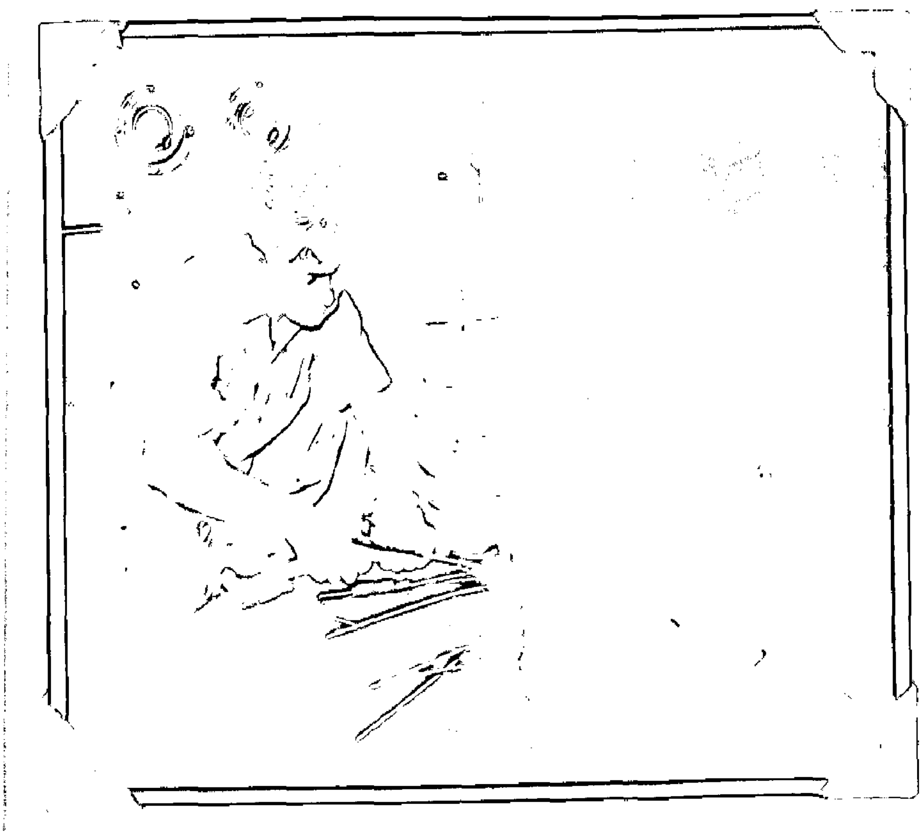


Plate 5. Grouting of the transom after prestressing through grout hole.

4.7 CASTING

The materials were mixed in batches of about 3.5 Cft. for five minutes in the petrol driven knicker broker mixer. The concrete was placed in the mould in small quantities and thoroughly vibrated with the electrically driven vibrators. Three 4" Control cubes, three numbers 6" x 12" cylinders and three numbers 4"x4"x16" beams were cast for each portal with vibrator identical to those of the portal. While casting, care was taken to see that the sheath was not disturbed from the required profile. The bearing plates were cemented to the portal frame at one at each end of the transom and are at each end of the leg.

4.8 CURING AND STORAGE

The portals, cubes, cylinders and control beams were stripped from the moulds 24 hours after casting and were stored on ground with damp gunny bags, kept damp all the time. After 28 days of water curing prestressed and grouted and then ~~were~~ they were stored away till the day of testing.

4.9 PRESTRESSING AND GROUTING

The first two portals were prestressed with the Gifford-udall system and the third portal was prestressed with the Magnet-Blaton system. Reliance was placed on the extension of the w wires than on the pressure dial gauge readings as the pressure gauges were not giving consistent readings. Any slip occuring from the anchoring of wires was allowed for.

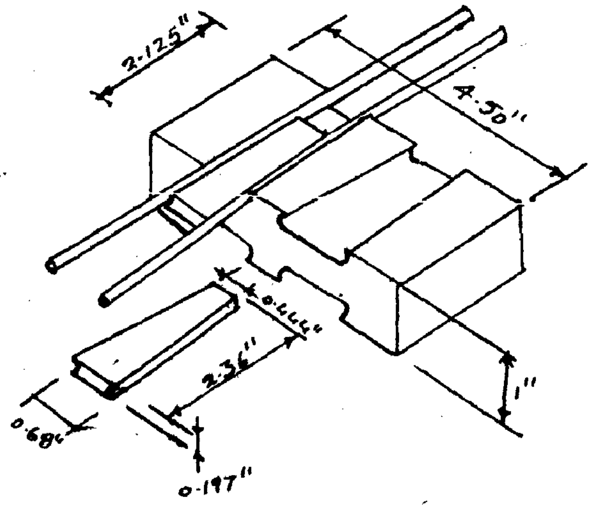


Fig: 30(A) A Magnel sandwich plate with wedges (for 0.2 · in. wires)

After prestressing was over, the ducts were cleaned with water pumped under pressure. Incidentally this also showed th that the ducts were not clogged. Then cement-sand grout in the ratio 2:1 with a water-cement ratio 0.67 was injected under sufficient pressure into the duct through the grout hole till it freely came out through the other end. Then the hole was closed. The grout was allowed to set and then the portal was ready for testing.

CHAPTER 5.

5.1 TESTING MACHINE

The portals were tested on a 500 ton Losenhausenwerk Universal Testing Machine. It permits besides of the simple compression test on cubes etc., the execution of compression and collapsing tests on longer columns and the execution of bending tes tests on beams, portals etc.

The transverse head with the upper pressure plate can be adjusted to a required height between zero and the maximum distance of pressure plates i.e. about 15'

The machine columns are equipped as threaded spindles f for this purpose, on which, the transverse head can be shifted by means of nuts moved by means of electric motor by warm gear. The adjustment is done by a switch on the pump case.

The bending device consists of the bending table of length 12 ft., and the bending supports which are slidably arranged on the bending table and the bending stamp. The bending stamp is fastened on the upper pressure plate by means of fish plates attached to it and the supplied screws. The bending table moves on the 5 meter ratio. Initially the bending table is moved out of the machine columns and the test specimen is put on it. Then the bending table along with the specimen is pushed into the machine columns.

The load is applied by hydraulic pressure of oil with the aid of an electric motor by operating a valve. The load can be applied at any desired rate by adjusting the valve, and decompressing and releasing valves are also provided to take off the load when necesssry. The load applied is measured from the dial gauge.

For the Stress - strain observations on concrete cylinders and for testing the cubes, the 200 ton amsler compression testing machine was used. The strain observations were made by means of the Lamb's Roller Extensometer.

The control beams were tested in the 50 ton Avery Universal Testing Machine. They all work on the same principle of hydraulic pressure.

5.2 REACTION MEASURING EQUIPMENTS:

The vertical reaction under each leg is measured by means of a 10 ton hydraulic capsule.

The horizontal reaction is measured by means of a spring balance inserted in the horizontal tie. A turn-buckle is also inserted in the tie to keep the centre lines of the legs ~~aba~~ always 9 ft. apart. The whole arrangement is shown clearly in the plate.

The hydraulic capsule is placed on a 16" x 16" x 1/4" plate resting on 8 Nos. rollers (each 1" dia and 16" long and the surface is finished smooth) free to roll on the bending table.

The base plate of the hinge rests on the top plate of the hydraulic capsule.

5.3 10 TON PROVING RING

In order to measure the applied load accurately, a 10 ton proving ring was inserted between the upper pressure plate of the testing machine and the centre of the transom. The calibration chart supplied by the manufacturer enables the load to be determined from the reading in the dial gauge attached to the proving ring along the vertical diameter.

5.4 STRAIN MEASURING EQUIPMENTS

As the electric resistance strain gauges of suitable gauge length were not available, Demec's demountable mechanical strain gauges having 8" gauge length was used to measure the strains in the concrete. The brass studs were fixed to the concrete surface with a special type of glue at the required gauge length. This was done by making use of the gauge bar possessing two centre punches - at a distance apart equal to the length of measurement - with different conical points. The correctness of the Demec's strain gauge is assessed by means of comparing the gauge-length with that of the invar test bar.

5.5 DEFLECTION MEASURING EQUIPMENTS

The deflection measurements were made by using Baty's deflectometers with stands having a magnetic base.

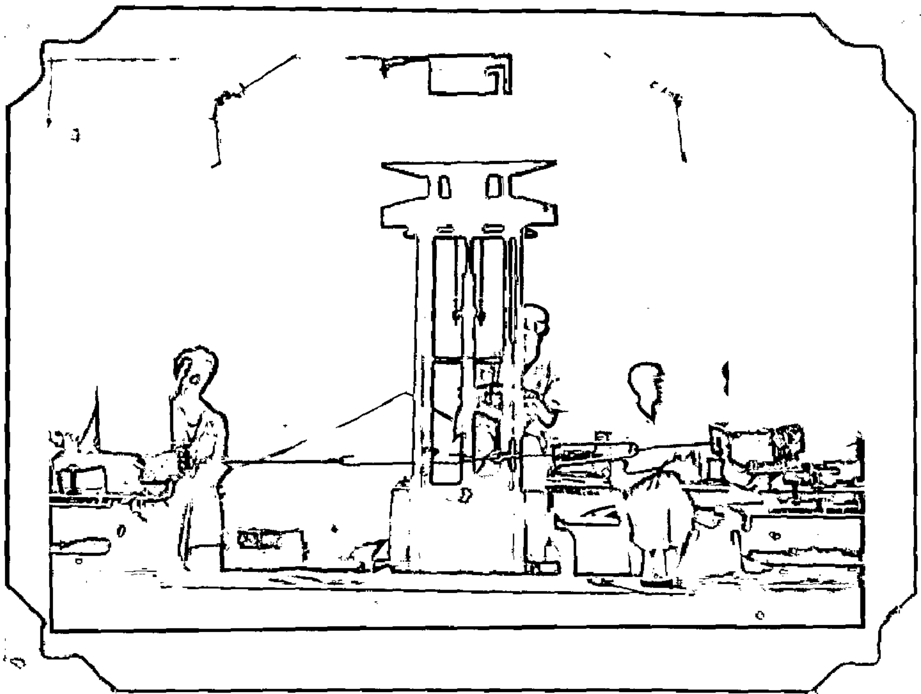


Plate 7. Portal being lifted and mounted on the bending table of the 500 Ton testing machine.

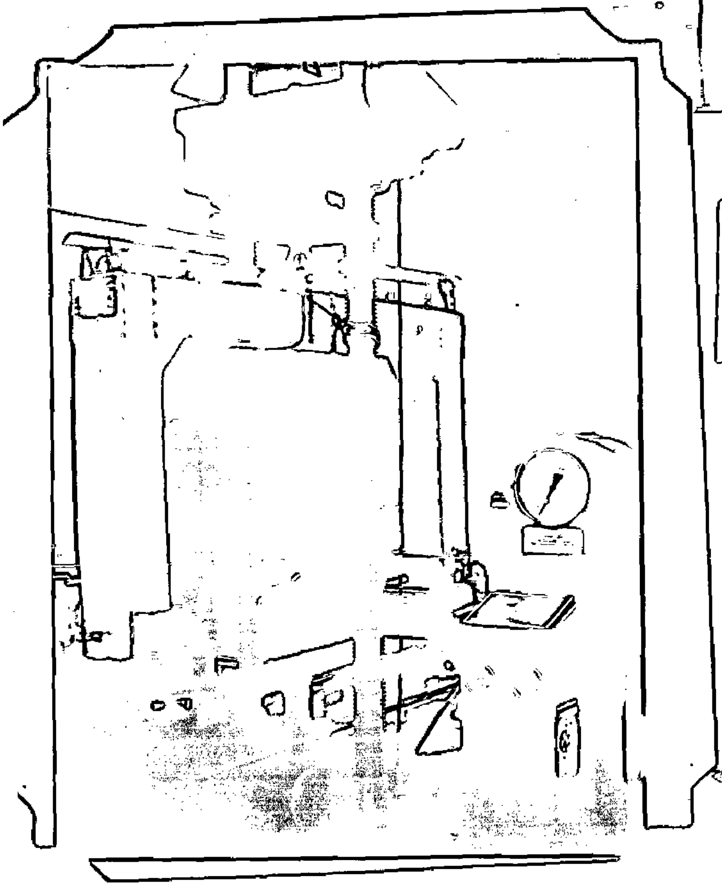
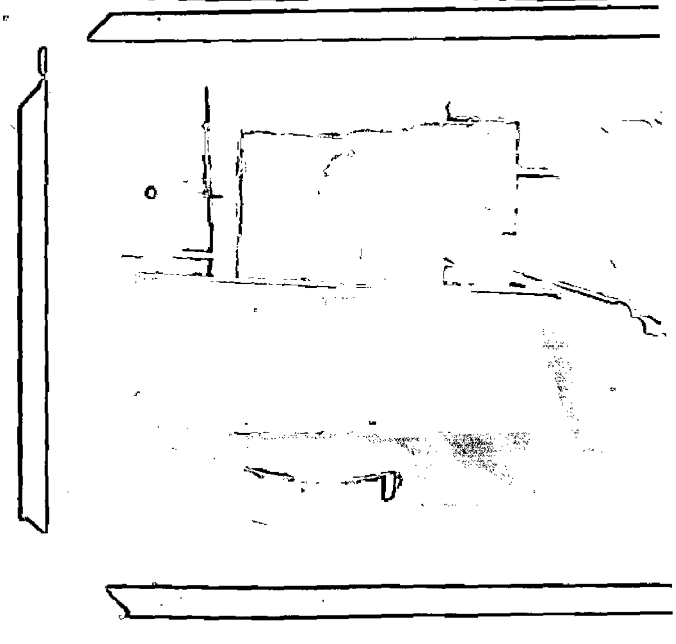


Plate 8. Portal ready for test u the 500 Ton testing m c showing dial gauges, di ... stands, provin with the spring balanc

Plate 9. Close up of the hinge s the hydraulic capsule a rollers.



The deflectometers were attached to a stand as shown in the plate 8. All the dial gauges were fixed outside the portal so that they were not damaged when the portal was tested to failure.

5.6 PREPARATION OF TEST SPECIMENS

The portals were cleaned with a brush and were given two coats of white-washing. The centre lines of the transom and the legs were clearly drawn. The strain measuring studs were fixed at 8" apart to take in the strain gauge points. Strains were measured both at the centre of the transom and at the frame-angles.

5.7 TESTING PROCEDURE:

The first step in testing was to ensure that the load was applied centrally and perpendicularly to the transom axis for the portals I and II. For the portal III, the load was applied perpendicularly at the third points. This was also ensured by measuring the vertical reactions in the hydraulic pressure capsules. Since the portal is symmetrical and the load is also symmetrical about the central axis of the portal, the reactions are to be equal. The diagonal length of the portal was also measured now and then to ensure that there was no sidesway. An initial load of 1 ton was applied and the strains, deflections and the reactions were noted. Before measuring the horizontal thrust, care was always taken to see that the distance between the centre lines of the legs was always 9 ft. For this purpose, two dial gauges were fixed at the hinge points and their readings were constantly noted.

The load was applied at a constant rate in increments of 1 ton. The readings were taken at 1 ton interval. The load was kept constant for 2 minutes while the readings were taken.

The load at the first crack was noted and the portal was loaded to failure. All the cracks were clearly marked with japan black.

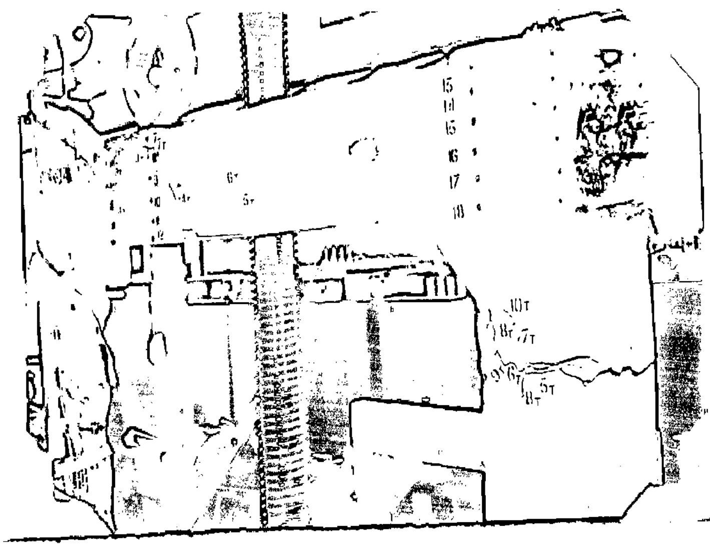


Plate 10. Close-up of the cracks on the right-knee and at the centre of the transom.

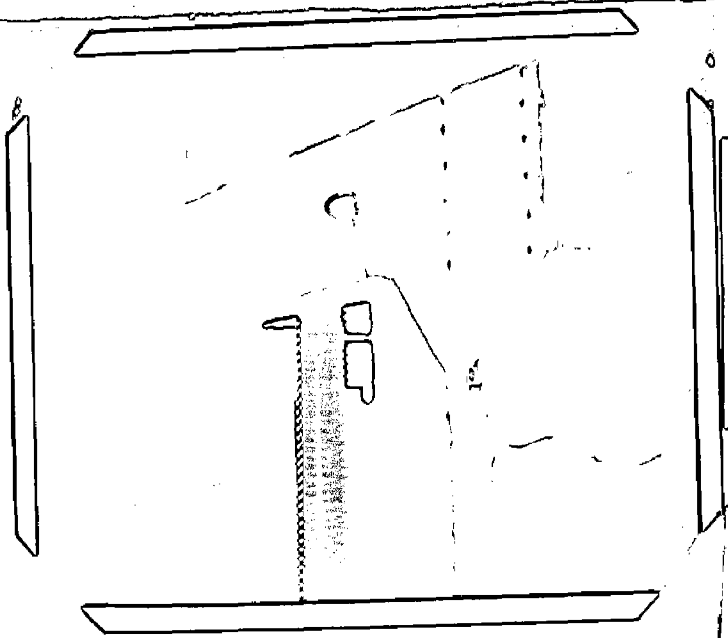
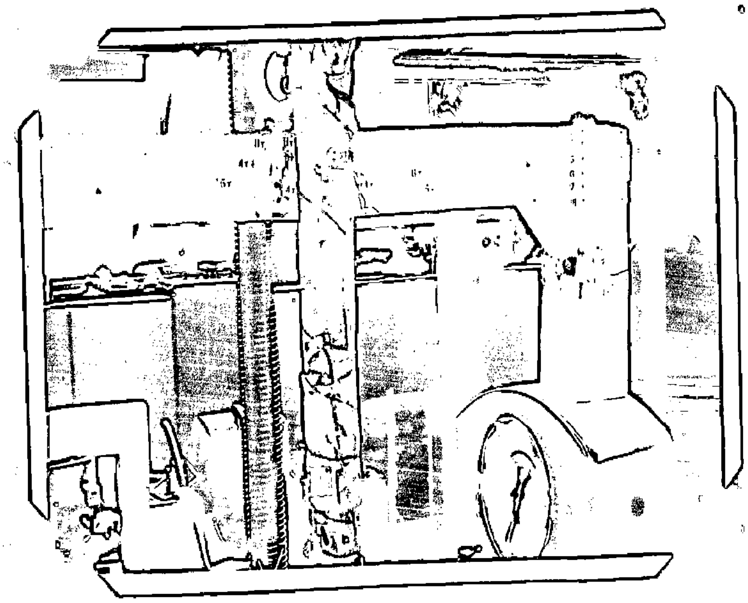


Plate 11. Crack on the side of the right knee.

Plate 12. Just crushing of the concrete on the right knee.



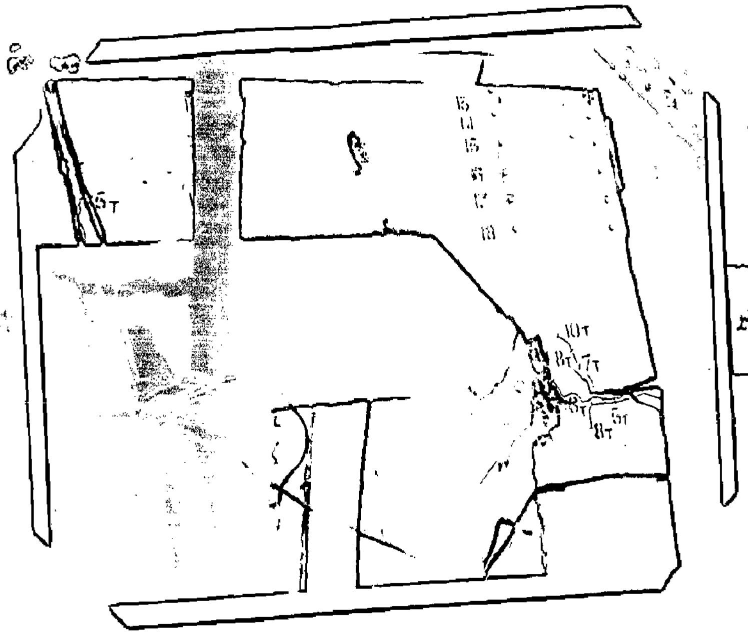


Plate 13. Complete crushing of the concrete on the right knee

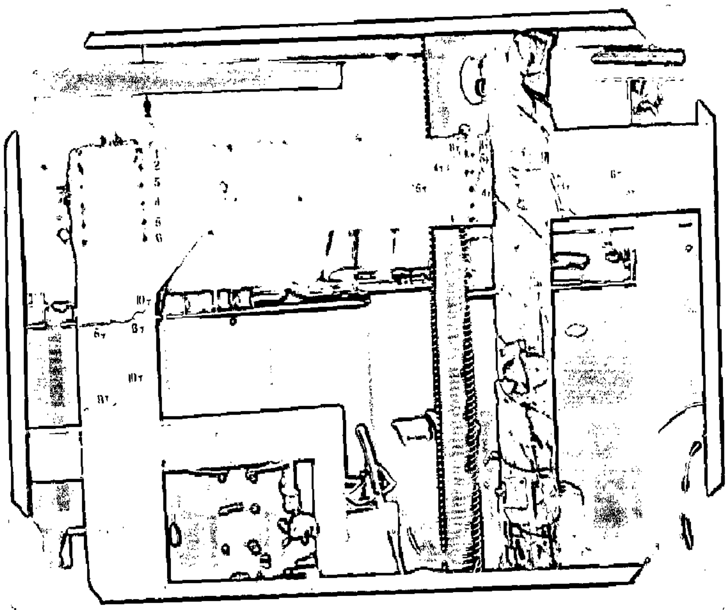


Plate 14. Cracks on the left knee and at the centre of transom.

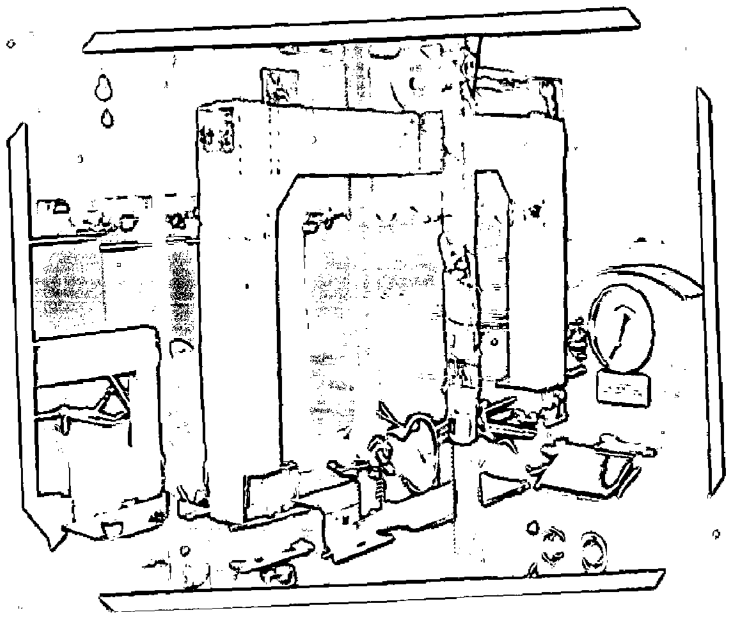


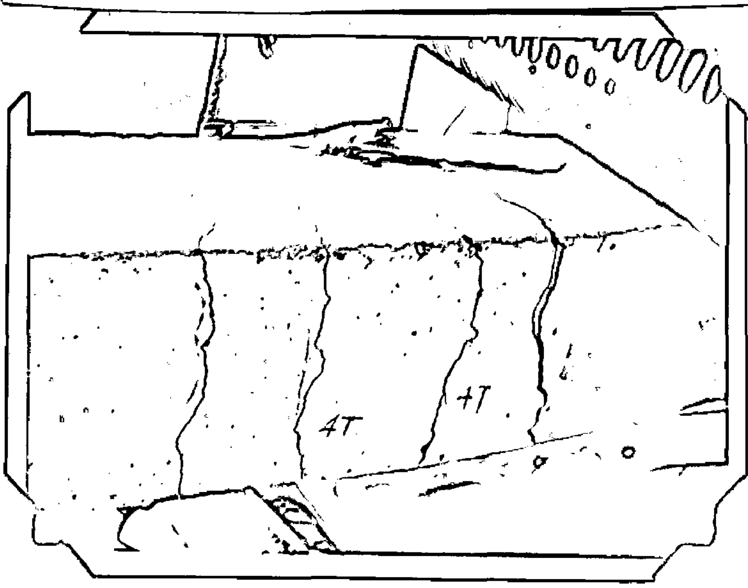
Plate 15. View of the portal just on the point of failure from left side.

CHAPTER 6.
OBSERVATIONS & CALCULATIONS
PORTAL I.

Loading Tons	Proving Ring 1401 10 Tons. 100 Div. L.C 0.0001"	Horizontal Thrust. lbs.	Hydraulic capsule V_1 Tons.	Hydraulic capsule V_2 Tons	Deflection at the centre (INS)	Remarks
0	0	112.50	0.4	0.4	0	
1.0	28.0	485.02	0.9	0.9	0.005	
2.0	58.0	292.08	1.4	1.4	0.011	
3.0	88.2	1322.77	1.9	1.9	0.015	
4.0	117.0	1763.70	2.4	2.4	0.019	Crack at centre of Transom formed.
5.0	145.8	2425.08	2.9	2.9	0.022	Cracks at both the Knees formed.
6.0	176.0	2976.24	3.4	3.4	0.025	
7.0	213	3527.39	3.9	3.9	0.029	Cracks appeared on legs. Dial Gauges removed.
8.0	243	4078.55	4.4	4.4		Cracks advanced both at centre and knees.
9.0	270	4739.93	4.9	4.9		Cracks widening
10.0	291	5070.63	5.4	5.4		Central crack remained the same and knee crack advanced.
11.8	51.5	6064.91	6.3	6.3		Inserted 50T proving ring.
12.5	57.5	6172.94	6.65	6.65		Portal failed

Plate 17. Crushing of the concrete on the left knee.

Plate 18. Crack pattern at the centre of transom.



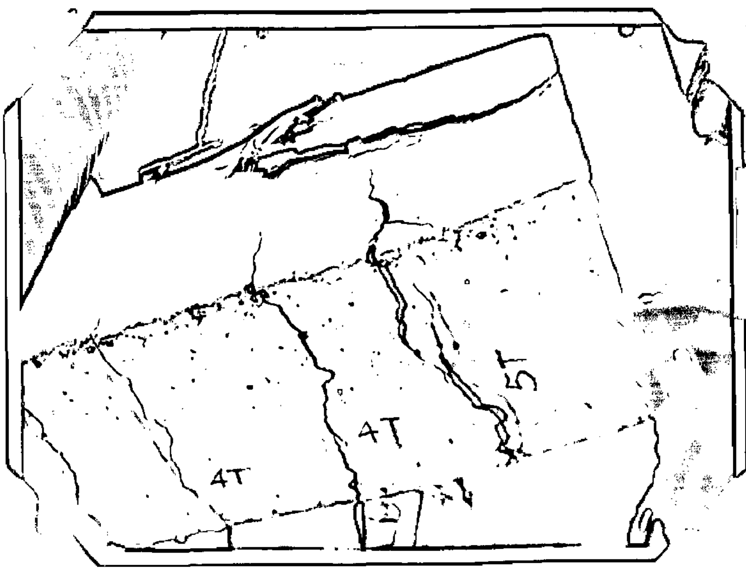


Plate 19. Crack pattern at the centre of transom from different angle.

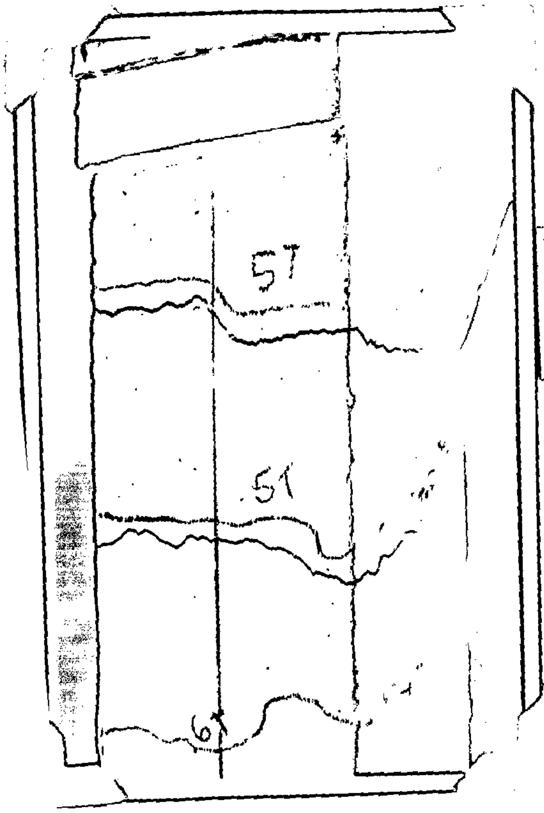


Plate 20. Cracks on the sides of the left knee before crushing of concrete.

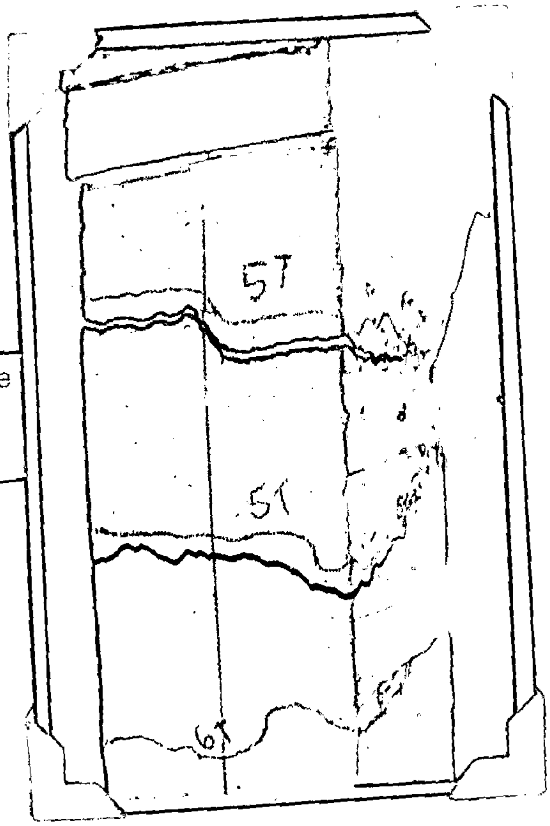


Plate 21. Cracks on the sides of the left knee of the crushing of the concrete.

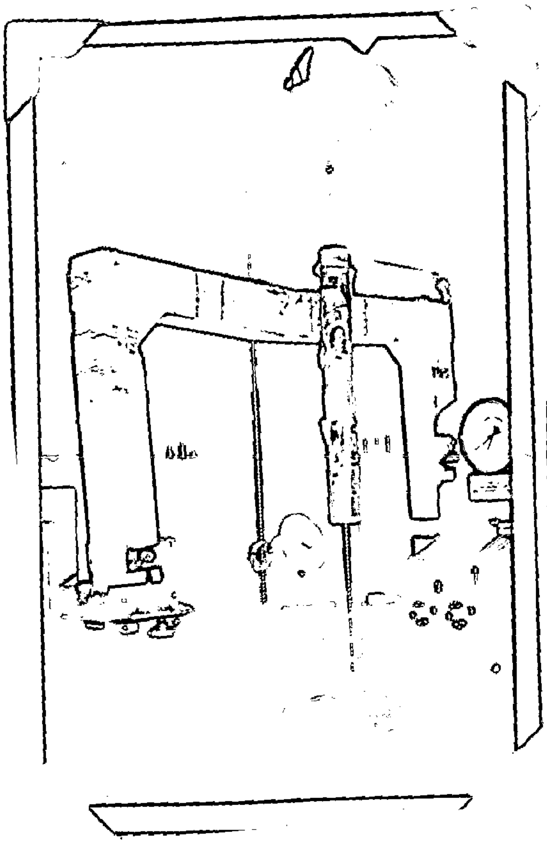
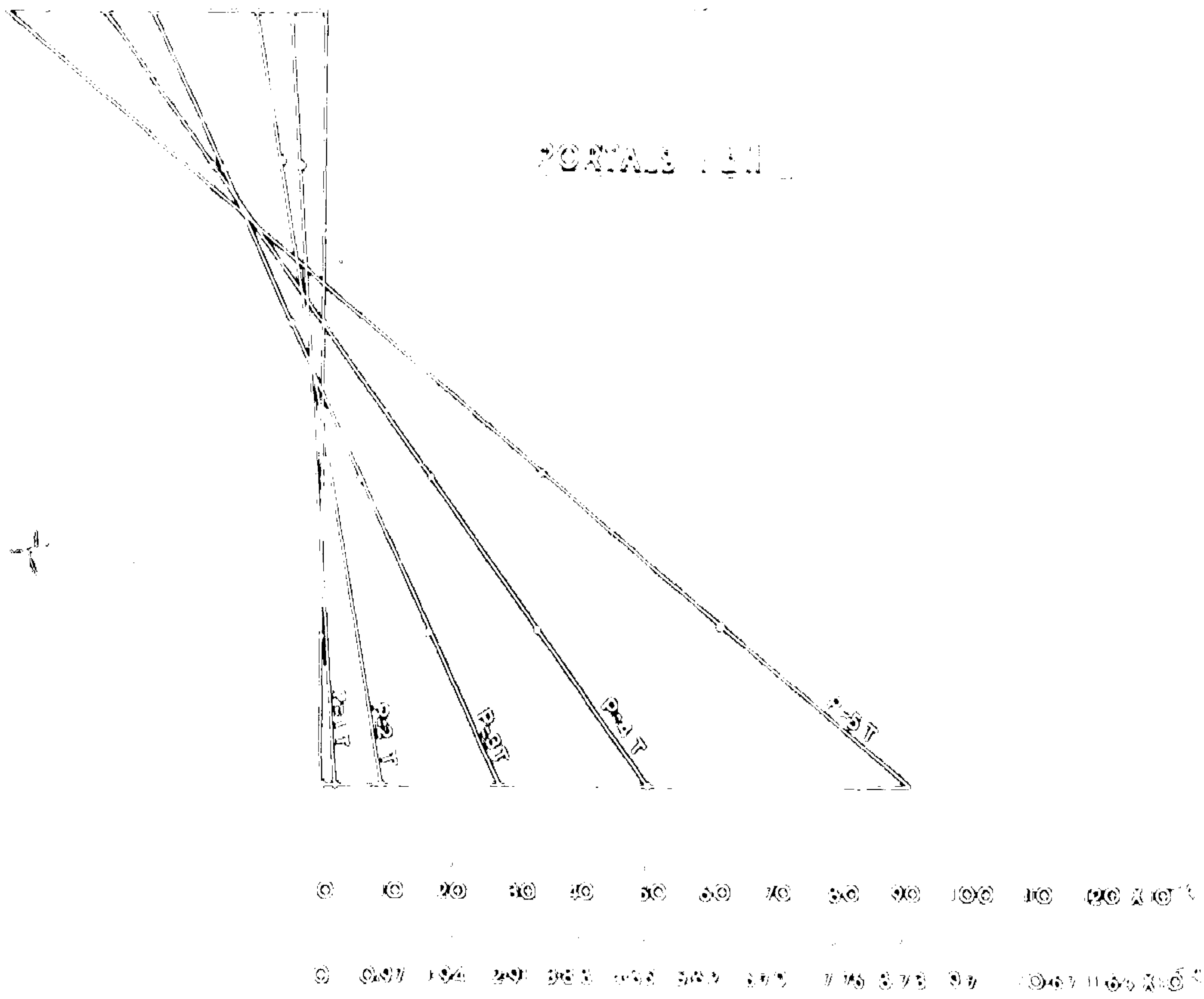


Plate 22. View of the portal after failure.

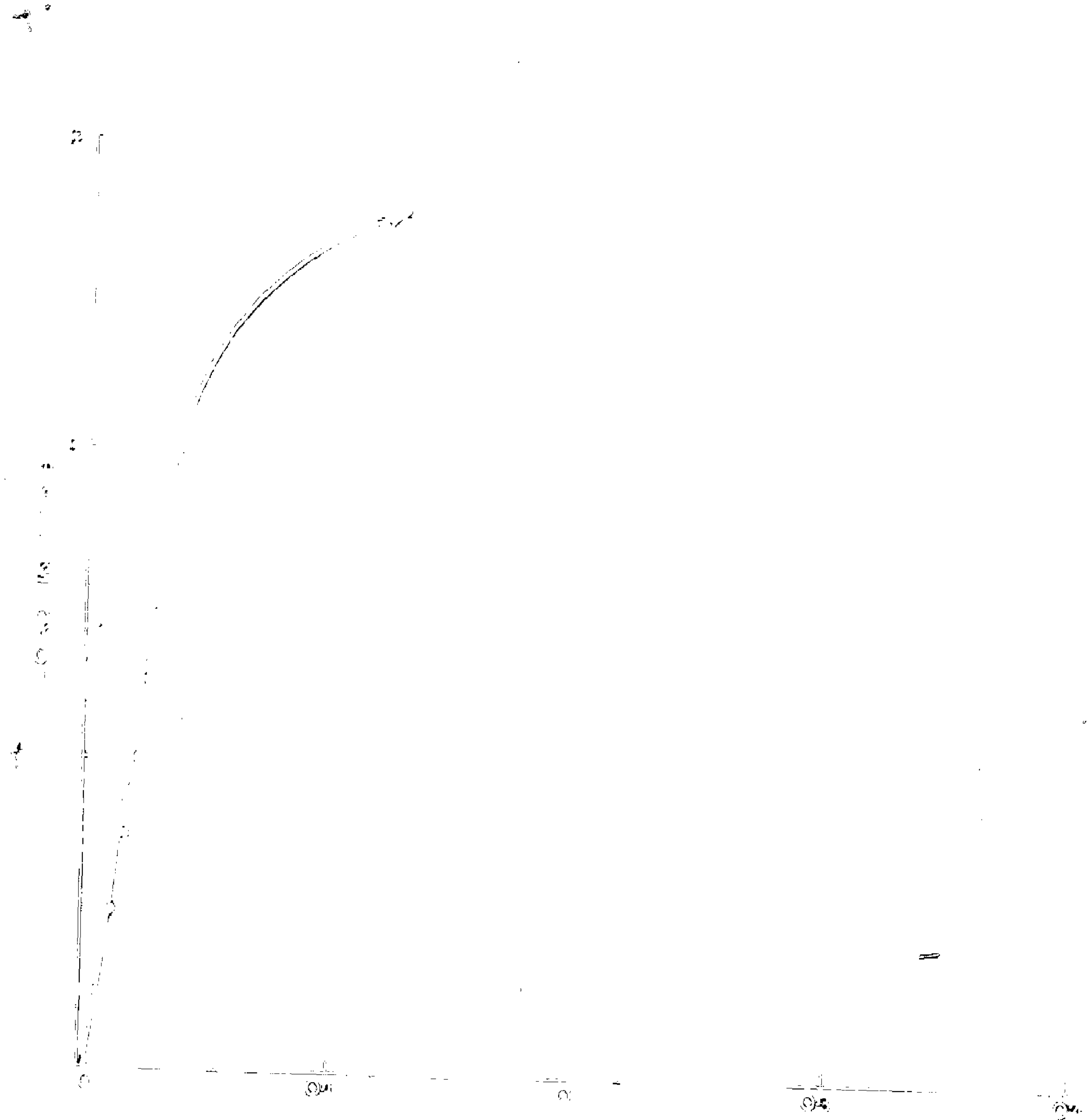
PORTAL II

Loading Tons	Proving Ring 1399 (50 Tons) 100 Div. L.C 0.0001"	Horizontal Thrust Lbs.	Hydraulic capsule V_1 Tons	Hydraulic capsule V_2 Tons.	Deflection at the centre (INS)	Remarks.
0	0	112.50	0.4	0.4	0	
1.0	4.9	485.02	0.9	0.9	0.005	
2.0	9.8	222.08	1.4	1.4	0.011	
3.0	14.7	1322.77	1.9	1.9	0.015	
4.0	19.6	1763.70	2.4	2.4	0.019	First crack formed at centre.
5.0	24.6	2425.08	2.9	2.9	0.022	Cracked started at left knee
6.0	29.5	2976.24	3.4	3.4	0.025	
7.0	34.4	3527.39	3.9	3.9	0.029	
8.0	39.3	4078.55	4.4	4.4	0.033	
9.0	44.2	4739.93	4.9	4.9	0.042	Left leg cracked fully
9.75	47.5	5070.63	5.28	5.27		



STRAIN AT THE CENTRE SECTION

FIGURE 3-3



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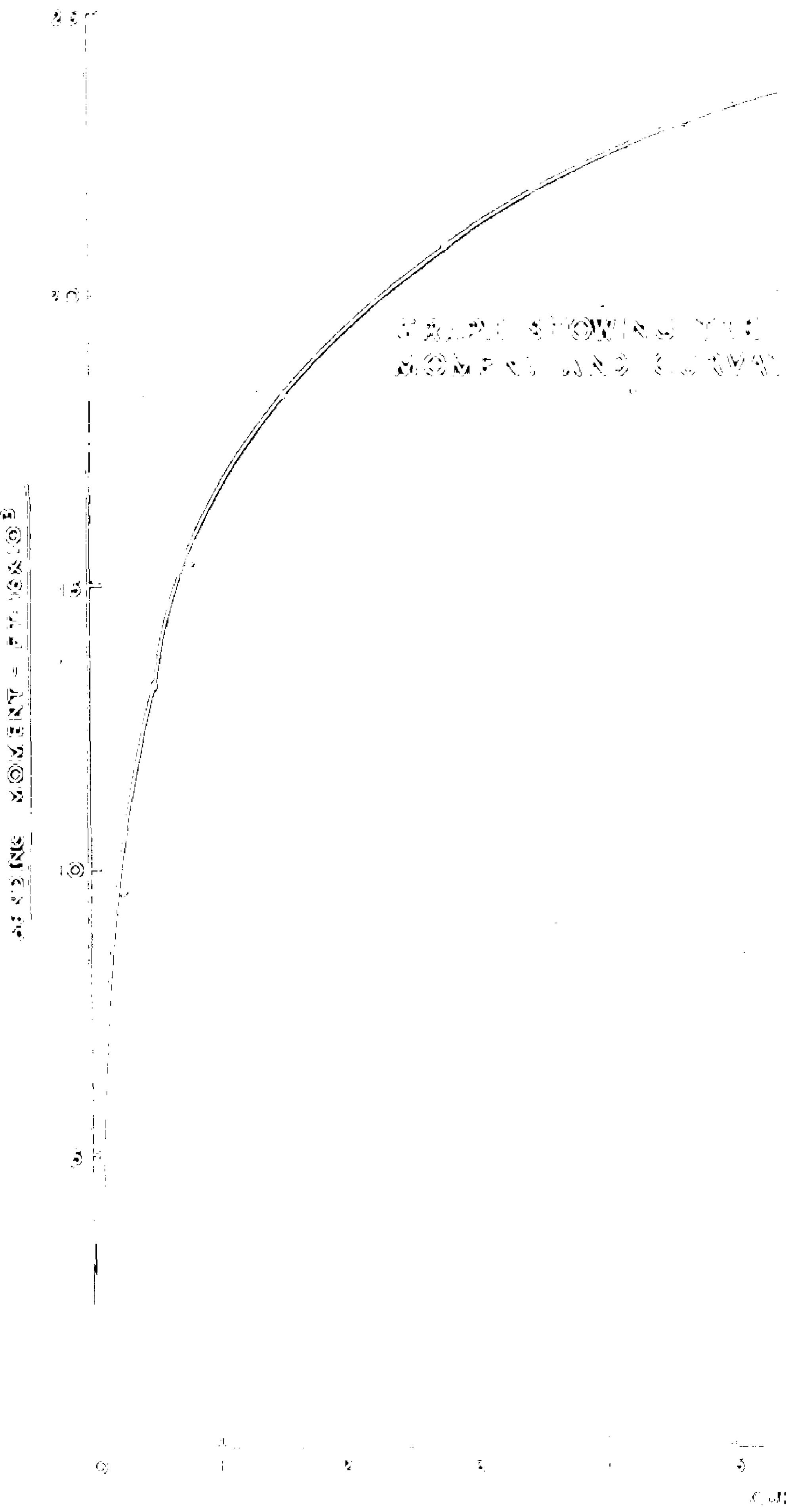
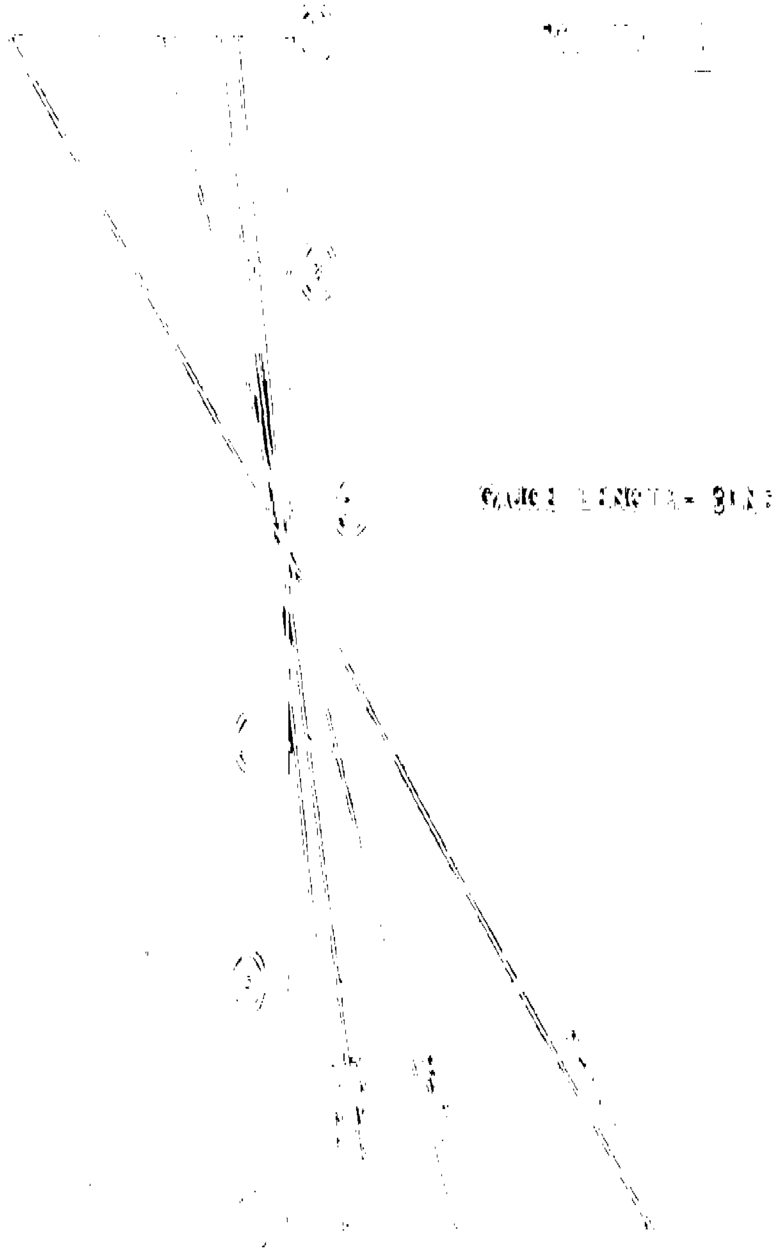
PORTALS I & II

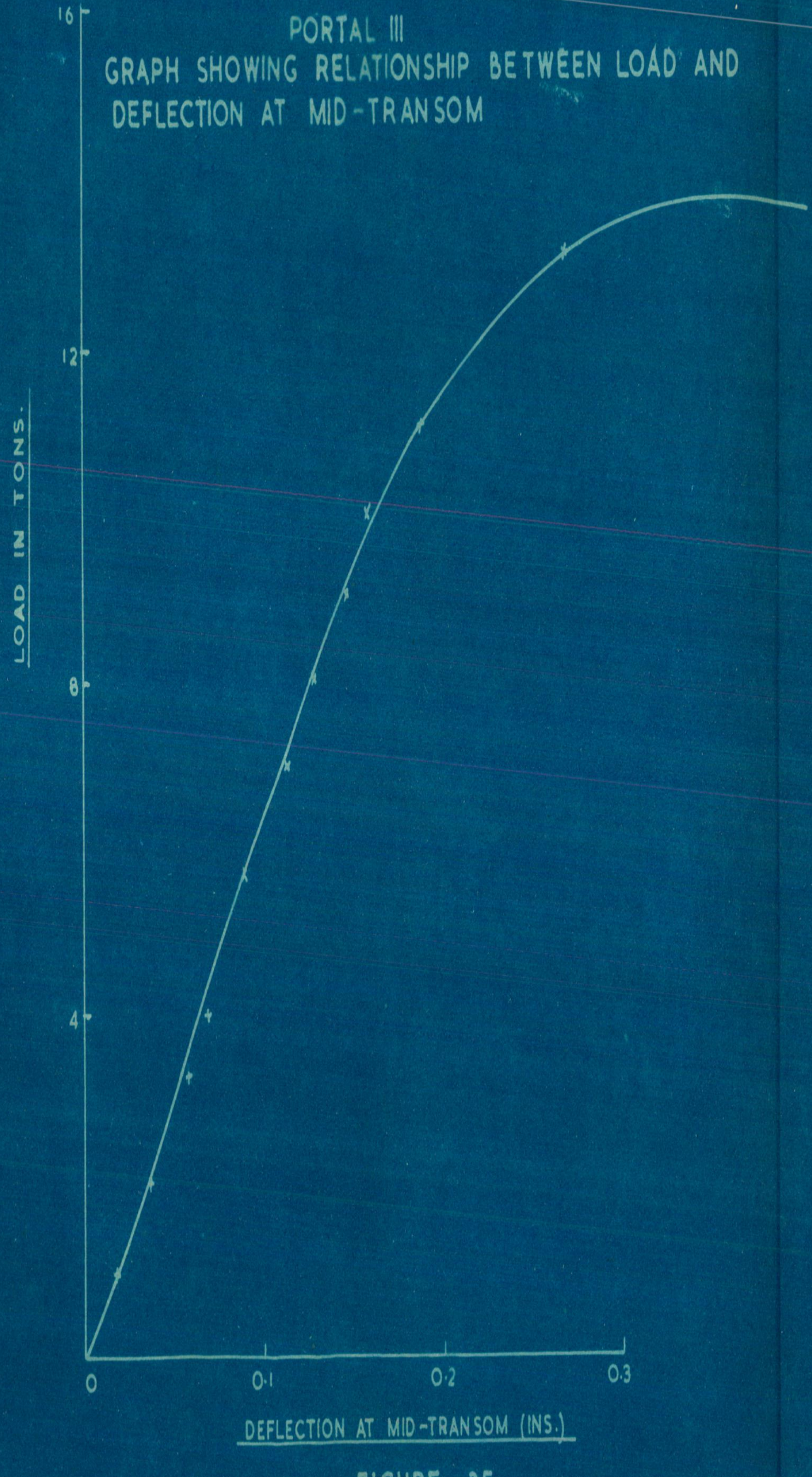
Loading Tons	Bending moment at Mid-Transom M (ft.lbs).	Depth of neutral Axis from the top fibre, nd (INS)	Compressive Strain in the Concrete.	Radius of Curvature.
0				
1.0	6.89×10^3	7.0	0.582	0.083×10^{-4}
2.0	9.65×10^3	5.2	1.067	0.206×10^{-4}
3.0	13.2×10^3	4.9	2.619	0.534×10^{-4}
4.0	16.25×10^3	4.0	3.395	0.849×10^{-4}
5.0	18.32×10^3	3.5	5.335	1.52×10^{-4}
6.0	20.88×10^3	2.9	7.954	2.74×10^{-4}
9.0	24.01×10^3	2.136	16.248	7.5 x 10-4



PORTAL III.

Loading Tons	Proving ring 1399 (50 tons) 100 Div. L.C 0.0001"	Horizontal Thrust (lbs)	Hydraulic capsule V_1 (TONS)	Hydraulic Capsule V_2 Tons)	Deflection at the centre (INS)	Remarks.
0	0	112.5	0.4	0.4	~~0.0245~~	~~0.0245~~
1.0	4.9	418.87	0.9	0.9	0.0245	
2.0	9.8	837.76	1.4	1.4	0.046	
3.0	14.7	1256.63	1.9	1.9	0.062	
4.0	19.6	1675.51	2.4	2.4	0.075	
5.0	24.5	2094.39	2.9	2.9	0.086	
6.0	27.6	2314.85	3.2	3.2	0.091	
7.0	34.4	2976.24	3.9	3.9	0.112	
~~8.0~~	~~39.3~~	~~3395.12~~	~~4.4~~	~~4.4~~	~~0.128~~	
8.0	39.3	3395.12	4.4	4.4	0.128	Crack started at both the knees.
9.0	44.2	3747.86	4.9	4.9	0.146	Crack at the bottom fibre of the centrel section
10.0	48.8	4078.55	5.4	5.4	0.160	
11.0	53.7	4585.61	5.9	5.9	0.187	
13.0	63.5	5379.27	6.9	6.9	0.265	
16.0	76.5	6007.59	8.4	8.4		Crushing of concrete started on right knee at 15T


GAUGE LENGTH = 3 IN.



PORTAL III
GRAPH SHOWING RELATIONSHIP BETWEEN LOAD AND DEFLECTION AT MID-TRANSOM
LOAD IN TONS.
16
12
8
4
0
0·1
0·2
0·3
DEFLECTION AT MID-TRANSOM (INS.)

FIGURE:- 35

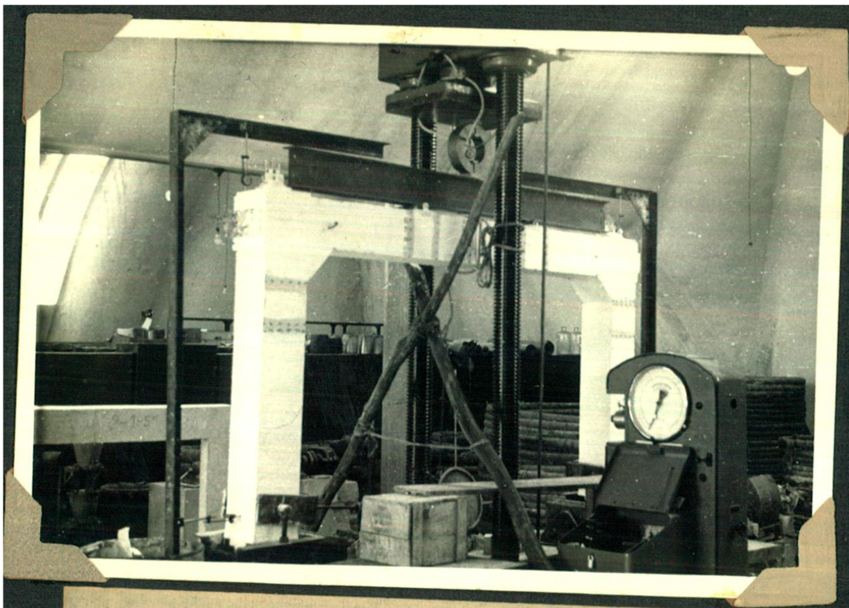


Plate 23. View of the portal before testing showing the arrangement for third-point loading.

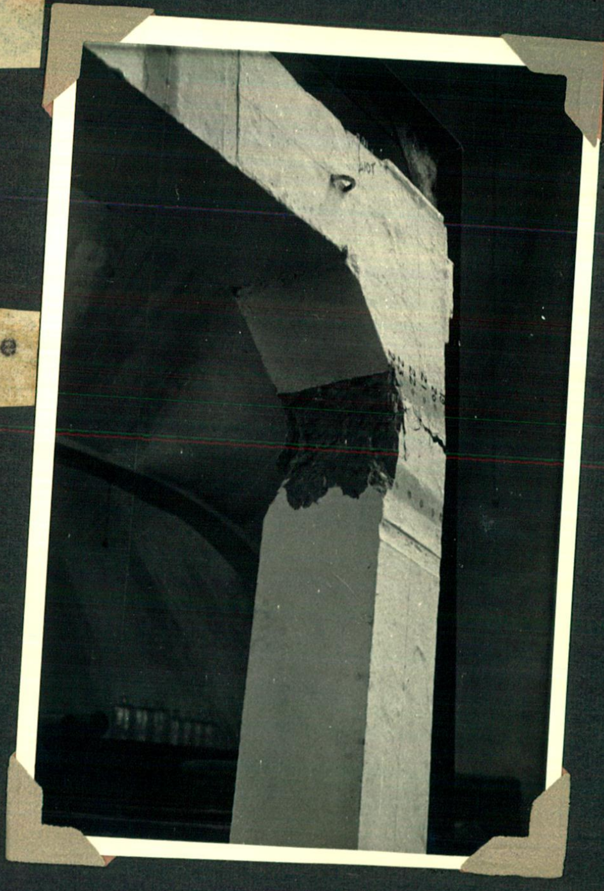


Plate 24. Crushing of the concrete on the right knee.

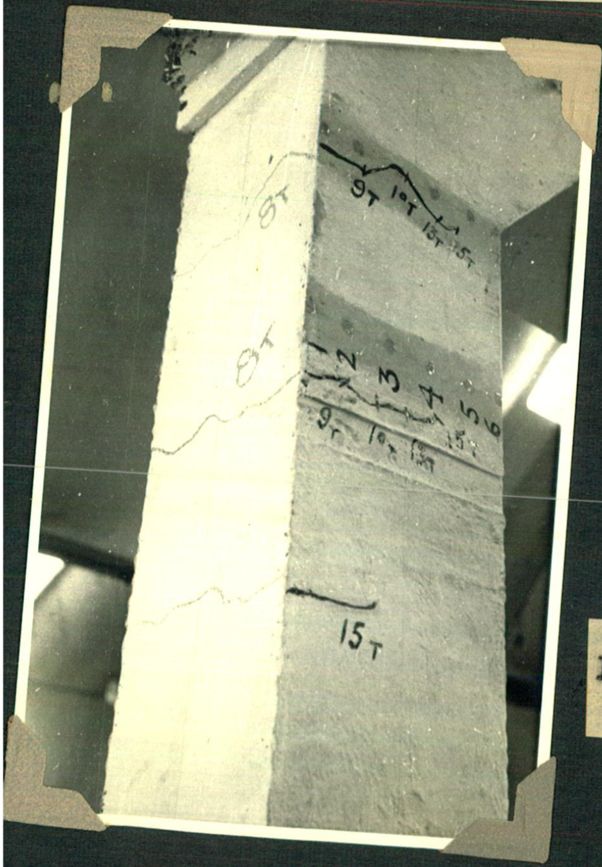


Plate 25. Crack-pattern on the left knee.

Plate 26. Close-up of the Magnel-Blaton anchorage along with the grout plugs.

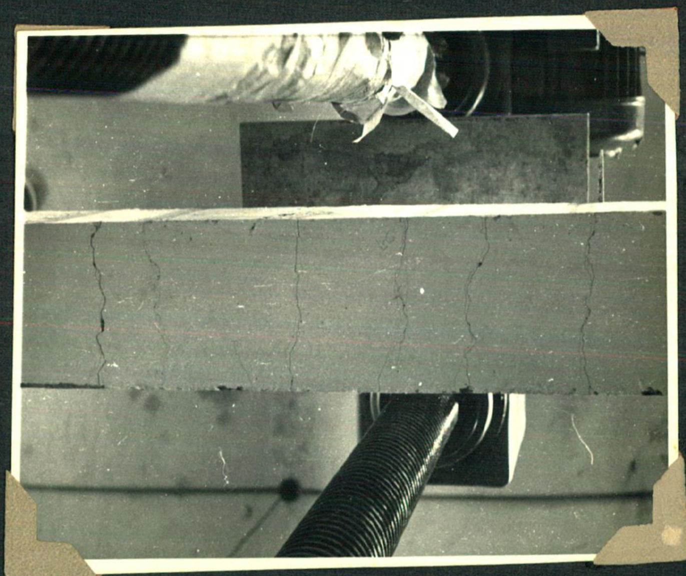
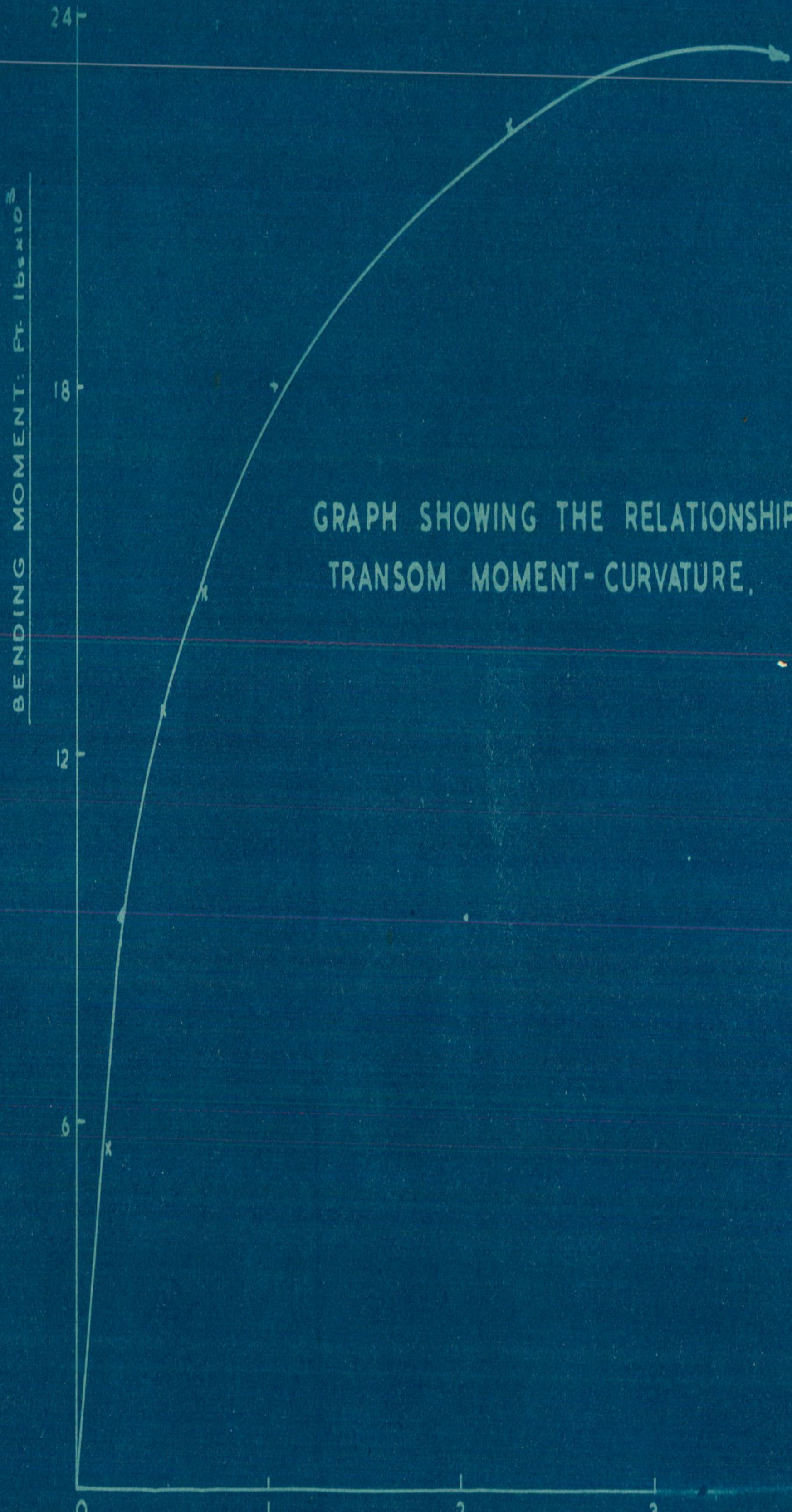


Plate 27. Crack pattern at the bottom of the middle third of the transom.

Plate 28. View of the portal after failure.

TABLE III.

Loading $P_1=P_2=\frac{P}{2}$ (tons)	Bending moment at third-point of the section M (in. lbs).	Depth of neutral axis from the top fibre, nd (in.)	Compressive strain in the concrete	Radius of curvature.
1.0	5.64×10^{3}	4.6	0.776	$0.169 \times 10^{-}$
2.0	8.59×10^{3}	4.5	1.067	$0.237 \times 10^{-}$
3.5	12.81×10^{3}	4.4	1.843	$0.42 \times 10^{-}$
4.0	14.51×10^{3}	4.3	3.298	$0.765 \times 10^{-}$
5.0	17.93×10^{3}	4.0	4.074	$1.02 \times 10^{-}$
6.5	22.16×10^{3}	3.3	7.469	$2.26 \times 10^{-}$



GRAPH SHOWING THE RELATIONSHIP BETWEEN THIRD - POINT TRANSOM MOMENT - CURVATURE.

FIGURE:- 36

I(a) Calculated load = 3.8T

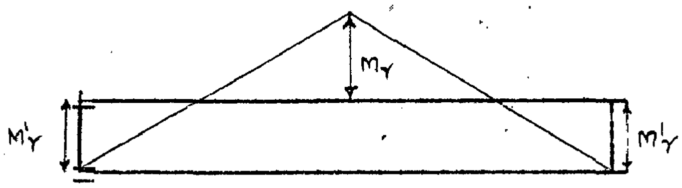
A Cable consisting of 10 wires of 0.2" dia. is used.

Ultimate strength of the cable F_{su} = 10 x 0.03 x 249,984

= 0.3 x 249,984

= 74,995.2 lbs.

Let a load W be applied at the centre of the span.



Let Mr be the ultimate moment in the mid transom

And M'_r the ultimate moment over the knee. W_u, the ultimate load and ω, self-weight for foot run of the transom.

d_1, the measured effective depth at mid-transom = 5.209"

d'_1, the measured effective depth at knee = 5.059"

(a) $\overline{W} = \frac{F_{su}}{bd1R}$

$= \frac{74,995.2}{10 \times 5.209 \times 4,550}$

$= \frac{74,995.2}{52.09 \times 4550}$

$= \frac{74,995.2}{237009.5}$

b = 10"

d_1 = 5.209"

R = The concrete strength tested on 4" cubes.

$\frac{nd}{d_1} = 0.41$

$\lambda = \frac{fs}{fu} = 0.9$

$\therefore fs = 0.9\ fu$

$= 0.9 \times 249,984$

$= 224,985.6$ lbs/sq.in.

$nd = 0.41 \times 5.209 = 2.136$ in.

$\therefore Mr = Asfs(d_1 - \gamma nd)$

~~xx10xxx10~~

$= 10 \times 0.03 \times 224,985.6\ (5.209 - 0.44 \times 2.136)$

$= 67495.68 \times 4.269$

$= 288,139$ in.lbs $= 24011.58$ ft.lbs. $= 24.011 \times 10^3$ ft.lbs.

Theoretical Mr = 24011.58 ft. lbs.

observed Mr = 30404.57 ft.lbs.

(b) <u>M'r</u>

$= \frac{Fsu}{bd1R}$

$= \frac{74995.2}{10 \times 5.059 \times 4550}$

$= \frac{74995.2}{50.59 \times 4550}$

$= \frac{74995.2}{23,0184.5} = 0.324"$

$\frac{nd}{d_1} = 0.37$

$\lambda = \frac{fs}{fu} = 0.92$

$\therefore fs = 0.92\ fu$

$= 0.92 \times 249,984$

$= 229,985.28$ lbs.

$nd = 0.37 \times 5.059$

$= 1.872$ in.

$\therefore M'r = Asfs\ (d_1 - \gamma nd)$

$= 10 \times 0.03 \times 229,985.28(5.059 - 0.44 \times 1.872)$

$= 68995.584 \times 4.235$

$= 292,196.3$ in lbs. $= 24,349.7$ ft.lbs.

$= 24.3497 \times 10^3$ ft.lbs.

$\therefore$ Theoretical M'r = 24,349.7 ft.lbs.

observed M'r = 22,817.84 ft.lbs.

(c) Calculation of the ultimate load. Assuming redistribution is complete.

$$\frac{W_u l}{4} + \frac{\omega l^2}{8} = Mr + M'r$$

$$\frac{W_u \times 9}{4} + \frac{100 \times 9 \times 9}{8} = 24011.58 + 24349.7 = 48361.28 \text{ ft.lbs.}$$

$$\text{or } W_u \times \frac{9}{4} = 48361.28 - 1012.5 = 47,348.78$$

$$\therefore W_u = \frac{4}{9} \times 47348.78$$

$$= 21,043.88 \text{ lbs.}$$

$$\simeq 9.5 \text{ tons.}$$

II. THIRD POINT LOADING

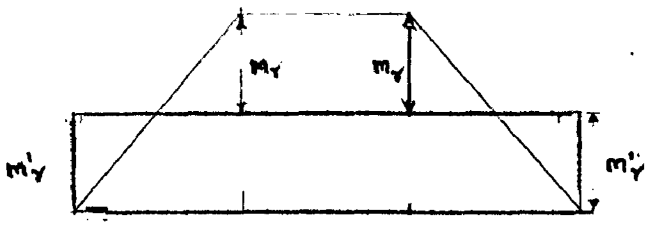
A cable consisting of 8 wires of 0.2" dia. is used.

Ultimate strength of the cable, ~~xxxxxx8xxxx10~~

$$F_{su} = 8 \times 0.03 \times 249{,}984$$

$$= 8 \times 7499.52$$

$$= 59{,}996.16 \text{ lbs.}$$



Let Mr. be the ultimate moment at the third point of the transom and M'r the ultimate moment over the knee.

Wu, the ultimate load, and ω, self-weight per foot run of the transom.

d1, the measured effective at third-point of the transom = 5.289 in.

d'1 , the measured effective depth at knee = 5.001 in.

(a) $\bar{\omega} \doteq \dfrac{F_{su}}{b d_1 R}$

$$= \frac{59{,}996.16}{10 \times 5.289 \times 4{,}242}$$

$$= \frac{59{,}996.16}{52.89 \times 4{,}242}$$

$$= \frac{59{,}996.16}{224359.38} = 0.266$$

b = 10"

d1 = 5.289

R = The concrete strength tested on 4" cubes

$$\frac{nd}{d_1} = 0.31$$

$$\lambda = \frac{f_s}{f_u} = 0.94$$

$$\therefore f_s = 0.94 \times f_u$$

$\therefore$ nd = 0.31 x 5.289 = 1.639 in.

= 0.94 x 249,984

= 234,984.96 lbs/sq.in.

$\therefore$ Mr = As fs (d1 - γnd)

= 8 x 0.03 x 234,984.96 (5.289 - 0.44 x 1.639)

= 56,396.39 x 4.568 = 257,618.71 in.lbs. = 21,468.22 ft.lbs.

THEORETICAL ULTIMATE MOMENT OF RESISTANCE = 21,468.22 ft.lbs.

OBSERVED ULTIMATE MOMENT OF RESISTANCE = 19,012.76 ft.lbs.

(b) $M'r = \frac{Fsu}{bd1R}$

$= \frac{59,996.16}{10 \times 5.001 \times 4,242}$

$= \frac{59,996.16}{212142.42} = 0.282$

$\frac{nd}{d'_1} = 0.33$

$\lambda = \frac{f_s}{f_u} = 0.93$

$\therefore f_s = 0.93 \times f_u$

= 0.93 x 249,984

= 232,485.12 lbs/sq.in.

$\therefore$ nd = 0.33 d1

= 0.33 x 5.001

= 1.65

$\therefore$ M'r = Asfs (d1 - γnd)

= 8 x 0.03 x 232,485.12 (5.001 - 0.44 x 1,65)

= 55,796.43 x 4.275

= 238,529.74 in.lbs = 19,877.31 ft.lbs.

Theoretical ultimate moment of resist nce = 19,877.31 ft.lb.s

Observed ultimate moment of resistance = 20,635.24 ft.lbs.

B.M. $= \frac{W_u l}{3}$

$\frac{W_u l}{3} + \frac{w l^2}{9}$ = Mr + MIr

$3W_u + \frac{100 \times 9 \times 9}{9} = 21,468.22 + 19,877.31 = 41,345.53$

$\therefore 3W_u + 900 = 41345.53$

$\therefore 3W_u = 40,445.53$

$\therefore W_u = 13,481.84$ lbs = 6T

ne	Tendon profile.	Load	Measured Mid-transom (In)	eel r- n- ge -transom.	Knee	Depth to neutral axis at failure Mid-transom (In)	Depth to neutral axis at failure Knee (In.)	$\frac{M_1 e}{M_2 e}$ X^a	$\frac{M_1 r}{M_2 r}$ Y^a	Disproportion X/Y or Y/X	REMARKS
	2	3	4	5	17	18	19	20	21	22	23
	Transformed.	Central loading	5.20	57	0.59	2.136	1.872	1.62	0.99	1.64	*
	"	"	5.20	57	0.59	2.136	1.872	1.62	0.99	1.64	**
	"	Third point loading.	5.28	45	0.48	1.639	1.65	1	1.08	1.08	***

CHAPTER - 7

ANALYSIS OF RESULTS

TABLE

...me	Tendon profile.	Load	Measured effective depth of tendon: Mid-transom (In).	Measured effective depth of tendon: Knee (In)	Age at Test (days)	Average cube strength lb/in^2	Cracking load (Tons)	Calculated ultimate moment of resistance: mid-transom (lb/ft)	Calculated ultimate moment of resistance: Knee (Lb/ft)	Ultimate load (Tons)	Mean ultimate load Tons	Ultimate load / Mean ultimate load.	Theoretical ultimate load. Tons	Ultimate load / Theoretical U.L.	Steel percentage: Mid-transom.	Steel percentage: Knee	Depth to neutral axis at failure: Mid-transom (In)	Depth to neutral axis at failure: Knee (In.)	$\frac{M_1 e}{M_2 e}$ X^a	$\frac{M_1 r}{M_2 r}$ Y^a	Disproportion X/Y or Y/X^a	REMARKS
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
	Transformed.	Central loading	5.209	5.059	120	4,550	P=4	24011.58	24349.7	11.8		1.1	9.5	1.24	0.57	0.59	2.136	1.872	1.62	0.99	1.64	*
	"	"	5.209	5.059	45	4,452	P = 4	24011.58	24349.7	9.75	10.77	0.905	9.5	1.03	0.57	0.59	2.136	1.872	1.62	0.99	1.64	**
	"	Third point loading.	5.289	5.001	37	4,242	P_1 = 4 P_2=4	21468.22	19877.31	P_1=8 P_2=8	P_1=8 P_2=8	1	P_1=6 P_2=6	1.33	0.45	0.48	1.639	1.65	1	1.08	1.08	***

Notes: a) Suffix 1 applies to the load section in transom and 2 to the knee section.

b) Inverse ratio taken where necessary to give disproportion greater than unity.

* Failure in Mid-transom and right knee

** Failure in Mid-transom and left leg

*** Failure in middle third of transom and right knee.

CHAPTER 8.

CONCLUSIONS AND DISCUSSION.

The conclusions arrived at from the foregoing analysis are:-

1). The moment-curvature relationship as found in all the three portal tests followed the curve 'b' labelled in the adjoining figure. The initial shape of the moment-curvature relation is not particularly significant in determining the degree of moment redistribution in statically indeterminate prestressed structures. More important is the form of the relation after the peak moment has been reached. The adjoining figure shows two idealised forms of the moment-curvature relation. In that labelled "a", the moment falls off rapidly with increasing curvature after reaching its peak value, it indeed represents a brittle behaviour. In such a case, there could be no moment redistribution as it no longer retains its ultimate moment of resistance to enable the other critical sections to attain their ultimate moments. In the case of the relation labelled "b" that is what is exactly obtained in the present tests, curvature ~~xxxxxx~~ increases at a constant ultimate moment representing aplastic material in which moment redistribution will occur. The length of the moment plateau will determine the amount of redistribution which is possible. In all the cases, the observed ultimate load was greater than the theoretical ultimate loads.

2). All the portals have failed by forming two plastic hinges one under the load, the other at one of the two knees.

Since there is only one redundancy, two hinges are required to form a mechanism. The mode of failure was first by cracking and then the crushing of the concrete at the critical sections. Incidentally this confirms the hypothesis that a system of N redundancies will fail, by an increase in load, when the $(N + 1)^{th}$ hinge appears. So all the portals are completely adapted. Without cracking, complete redistribution is not possible.

3). The ultimate load will be invariant under a linear transformation which provides an artificial redistribution of strength. Such transformations can be arranged so that each of the critical sections has an ultimate moment in a constant ratio to its elastic moment so that there will be no moment redistribution necessary to achieve the same ultimate strength.

4). In machi's tests on continuous beams, it appears that at failure, the support moments had reached only the following proportions of their ultimate values: 75% for C_1 and C_2, 57% for C_3. Macchi attributes this lack of redistribution to what he terms the "disproportion" between the ultimate moments of the critical sections when they are related to the elastic moments to which the l load gives rise at these sections. This disproportion can be represented by a parameter which is the ratio between two other ratios, namely the ratio between elastic moments at critical sections, and the ratio between ultimate moments of resistance at the same sections. Macchi calls this parameter d, t us

$$d = \frac{M_1e}{M_2e} \qquad \frac{M_2r}{M_1r}$$

where the suffixes 1 and 2 refer to the critical sections and e and r respectively stand for elastic and rupture.

Thus a d value of unity implies that the ratio of elastic moments is equal to the ratio of rupture moments and therefore no redistribution of bending moments is necessary to produce failure of both critical sections. On the other hand, the greater the value of d, the greater is the redistribution of bending moment required to produce failure of both critical sections. (It is obvious that d is never less than unity, since it is quite legitimate to invert each ratio before multiplication).

The d values for Macchi's tests are 1.67 for both C_1 and C_2 and 2.0 for C_3. The larger value for C_3 is due to the larger end spans, and this results in a larger elastic moment ratio between mid-span and supports. The M_{2r}/M_{1r} value is of course the same (unity) for all three specimens. The ratio, r, of maximum load sustained to the calculated load, assuming that the full strength of all critical sections is developed, is 0.870 for C_1, 0.876 for C_2 and 0.783 for C_3. This ratio, r is termed by Macchi the "efficiency" of the system. On the basis of these results, Macchi tentatively proposes that there may exist a unique relationship between d and r for this type of beam and loading arrangement such that the efficiency of the system is inversely proportion to d, its disproportion. Certainly for beams C_1 and C_2, which have the same d value, the r values are practically identical, while for C_3, with its larger d value, r is substantially less.

That some relationship should exist appears at first sight to be highly probable since, if elastic "ultimate" analysis demonstrates that one critical section is subjected to a bending moment approaching its ultimate moment of resistance while the other critical section is subjected to a moment which is only a small fraction of its ultimate resistance, one naturally doubts the capacity of the highly stressed section to rotate sufficiently, without less of strength, to permit the development of full strength at the other section. of the 23 members (12 beams and 11 frames) tested by Morice, only in four instances, one beam and three frames, was the failing load less than the load calculated on the basis of full redistribution of bending moment. of greater significance, of course, was the visual evidence in each case of crushing of the concrete both in the span and the knee while the maximum load was held. Morice and Lewis found that there was no indication of a reduction of efficiency with increased disproportion. The acid test for complete redistribution of moments is not whether the maximum sustained load is consistent with the calculated ultimate moments of resistance of the critical sections, but rather the visual evidence of complete simultaneous rupture of these sections under that load. From a consideration of normal structural proportioning, it is evident that high disproportion ratios are unlikely to occur, except when they are primarily due to high ultimate moment of resistance ratios. In Morice experiments, it was found that weaker sections, e.g., the centre support section of beam 4, have very high percentages of steel in relation to the width of the concrete section and the depth of the steel. Sections such as these, being the weaker of the two critical sections, must be capable of suffering quite large rotations to enable full redistribution of moment to take place. This at first sight seems to be inconsistent with the behaviour of normally heavily reinforced sections which we know will not suffer such large rotations before failure as sections of the same dimensions with a much smaller percentage of steel. The reason for this apparent inconsistency is that a section may be given a high percentage of ste l in two quite dissimilar ways. The effective depth of the steel may be kept constant and the quantity of steel increased or the quantity of steel may be kept constant and the effective depth to the steel may be diminished. In the former case, the rotation capacity of the section will be decreased, but in the latter case it may well be increased. The smaller the depth of the neutral axis, the greater is the ultimate rotation. So larger rotation is always associated with the higher steel percentage. This result is consistent with failure occurring at a lower steel stress for higher steel percentages.

Of the frame tests, it can be said that they exhibited full moment redistribution for elastic moment ratios of the same order of magnitude as is found in Macchi's beams C_1 and C_2. In the present state of knowled e, it is not admisible, however, to attempt a comparison between the frames and the beams.

5. No failure of end block, either at the transom end or at the leg end took place even at the ultimate collapse.

6. In the third point loading of the third portal frame, it was found that the strains under the loads were more than the strain at the centre section, ~~through~~ though the moment was constant in the middle third of the transom. This was due to the "Karman effect". In finding the curvatures, the strain at the centre was taken into calculation.

7. Guyon found plastic phenomena occurring before ~~crack-~~ cracking in tests on continuous beams. Any section with a normal scatter - has a certain strength against cracking, or more exactly two strengths, one positive and the other negative. Adaptation amounts to a tendency of the structure to use its strength pos ibilities to the full due to a redistribution of the moments under the effect of unit plastic rotations* which are produced in those zones where the bending moment reaches its ultimate value.

For other materials the term "plastic hinge" is often used. Professor Magnel says, quite rightly, that this expression is incorrect since in prestressed concrete it is a ~~questi~~ question of cracking; one cannot speak of hinges unless the moment remains constant while the deformation increases. In the case of cracking, the moment is not constent, but it only increases slightly as a function of the unit rotation (or inversely, the curvature increases at a greater rate as a function of the moment).

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*By unit rotation is meant the rotation per unit length, i.e., the ratio of the rotation d to the length ds. The unit rotation is thus equal to the curvature .

Nevertheless, it is not a major inconvenience to use the word "hinge", if one knows what is meant by it.

In fact, a hinge is not a section but a zone of a certain length, at the centre of which a section reaches its limiting moment; the magnitude of the total possible rotation of the hinge is equal to the sum of the plastic rotations (i.e. the difference between the actual rotation and that which it would have had elastically) over the length of this zone. This magnitude is limited, the limit being a function of the possible curvature and of the length of the zone.

8. Discordance is a second condition of adaptation. The same strength can/ be reached with a discordant system and with a concordant system, where we call discordant a system where the stresses in the hinge sections are different and concordant a s; system where they are the same. Returning to an example quoted by Professor, Magnel, for the case of failure it is true, but it applies equally to cracking - if one considers a beam with two equal spans which is ~~xxpparxxx~~ supposed to be subjected to a central ~~pres-~~ pr.stress and in which are either two equal loads one at the middle ~~xpxx~~ of each span, or a single load at the middle of one span, the value of the load causing cracking will be the same, in the case of complete adaptation, whether there are the two loads or one. But on testing it was found that the cracking load P is bigger in the case of two loads than in the case of a single load. The reason for this is the first system is adaptable and that the second is not, because of too great a discordance.

9. Bennett, before testing continuous beams, made strain and deflexion measurements on simply supported beams, deduced a moment - curvature equation and worked out load-moment curves. The **amount** of moment redistribution thus predicted was much less than that found experimentally.

He believes this to be due to two main causes. The first is that the strain or deflexion measurements upon which the moment - curvature relation is based are made over an appreciable length, ranging from 0.20 to 1.0 m, and it is assumed that over this length the curvature is uniform. But this will not be true if, as seems probable, the deformation of a pre-stressed beam after cracking is brought about chiefly by very severe curvature in the vicinity of the cracks.

Another error of the same type is the calculation of steel stress in a cracked section of pre-stressed or reinforced concrete, using strain measurements made over a relatively large gauge length.

The second difficulty, which Mr. Guyon has mentioned, is that that at failure the curvature is increasing extremely rapidly with respect to the moment. According to him, the moment - curvature relation should be carefully used because of the above two facts.

As far as the present tests on frames are concerned, the writer finds complete redistribution of moments.

It is not out of place here to mention the use of edistribution factor suggested by Bennett as a guide to designers where complete redistribution of moments does not invariably occur. The redistribution factor is a slight modification of Sgr. Macchi's "r". It would be defined by the equation:

$$r = \frac{W_{ult} - W_{el}}{W_{pl} - W_{el}}$$

where W_{ult} = Actual ultimate load

W_{el} = Ultimate load assuming that the structure behaves elastically.

W_{pl} = Ultimate load assuming full plastic redistribution.

A redistribution factor 0 would then denote elastic behaviour, and 1.0, complete redistribution. The designer could work to a suitable intermediate value, which could be xxxk established from tests. According to him, 0.5 would be a safe figure.

****

CHAPTER 9.

SCOPE FOR FURTHER WORK.

The assumption of full redistribution cannot be other than an approximation, the accuracy of which depends on definite conditions, compatibility of the strains at the successive plastic hinges. It is necessary to check that, in the final phase, compatibility is effectively ensured or can be easily adjusted. Some more work has to be done in this direction.

The moment-curvature relationships have to be studied in still greater detail to get precise knowledge of the plastic hinges. Accurate instruments are needed for experimentally finding the plastic hinge rotations.

This study of ultimate strength should be extended to multi-storey and multi-bay frames along the lines suggested by Guyon. The ultimate strength of precast (legs and transom separately) prestressed beam portals should be studied in contrast to monolithically cast ~~[illegible]~~ prestressed portals.

To the best of our knowledge, no systematic experimental work has been carried out on shear failure. It appears that the elastic theory leads to a slightly excessive factor of safety as shown by the tests made by Lebelle on ordinary reinforced concrete beams. This important question should still be pursued. It is necessary to find if there is any interaction between moment and shear at the sections of failure.

The behaviour of similar portals under different forms of loading is another topic of research requiring early attention.

It is likely that the photo-elastic studies on the scale models of the frames may throw some more light on this actual behaviour under different types of loading.

As suggested by Levi, at the F.I.P. Congress in Amsterdam, the complete behaviour from cracking right upto failure should be investigated. It is also to be seen if plastic phenomena occur before cracking.

The following require still more attention:

(a) It has been suggested that before cracking that there is a plastic redistribution of the tensile zone similar to that of the compressive zone at failure, but some of the experimental evidence contradicts this* and indicates that the tensile stress is almost linear upto the point at which cracking occurs.

(b) The effect of crack spacing on the radius of curvature.

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*Evans, R. H. Extensibility of Concrete and Modulus of rupture of Concrete, Stract Engr, 24, 636-658(1946).

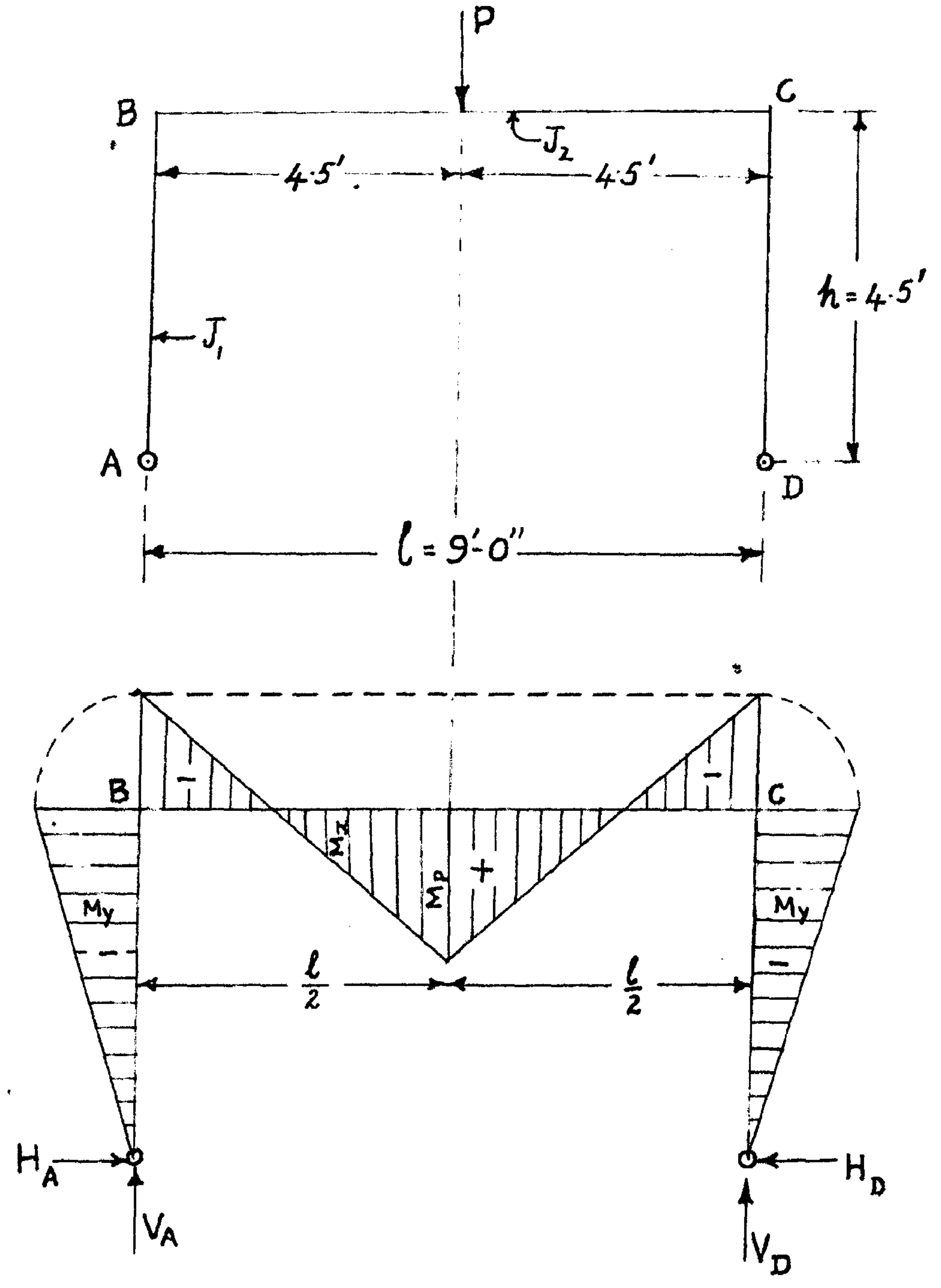


FIG. 37

$$M_B = M_C = -\frac{3Pl}{8N}. \qquad V_A = V_D = \frac{P}{2}. \qquad H_A = H_D = -\frac{M_B}{h}.$$

$$M_P = \frac{Pl}{4} + M_B. \qquad M_x = \frac{Px}{2} + M_B. \qquad M_y = \frac{y}{h} \cdot M_B.$$

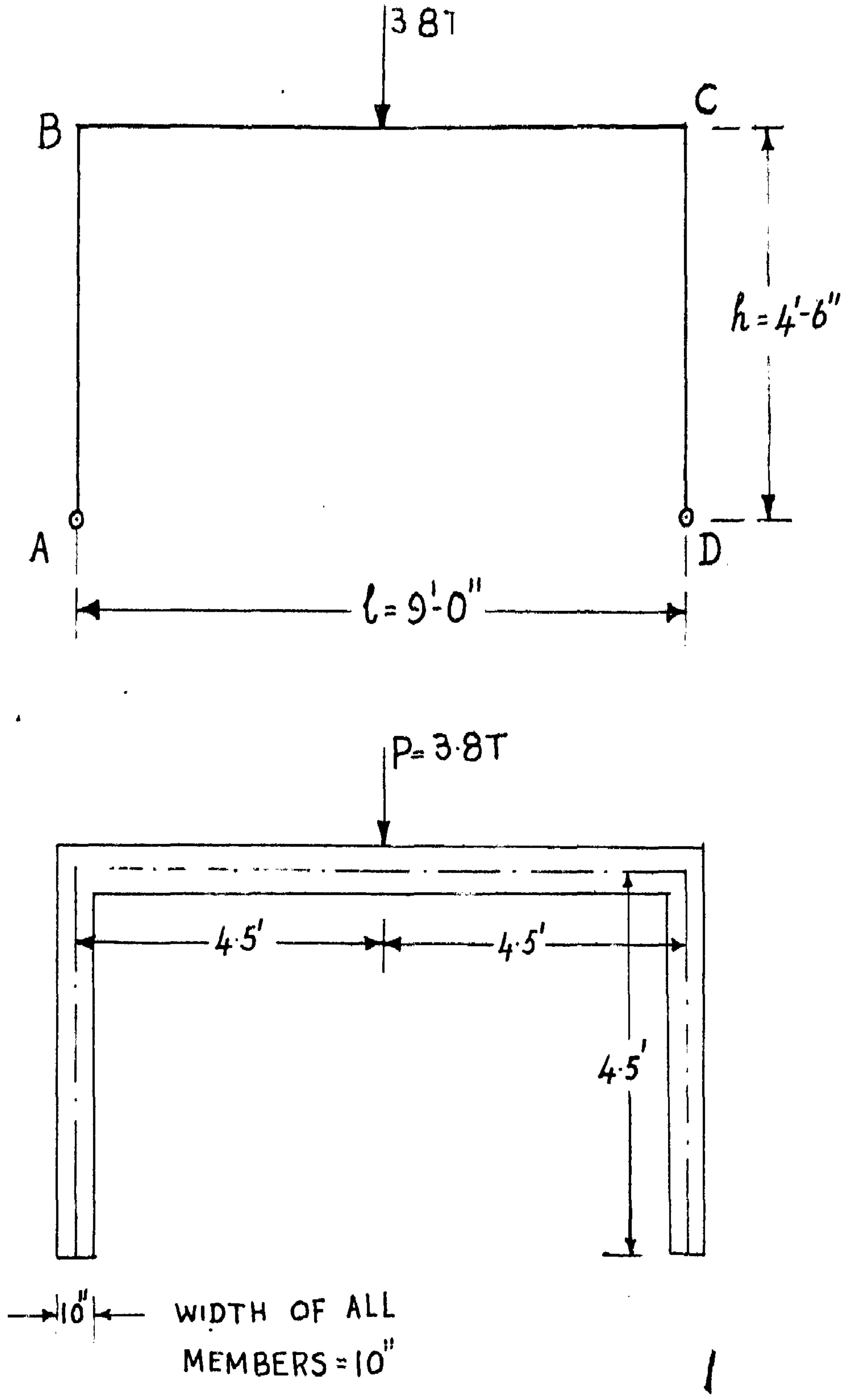


FIG. 38

Let the c/c distance of the Portal frames = 12'-0"

Width of the Portal frame = 9'-0"

Total load = 150 lb/ft^2

Total load per foot run of the transom = 150 x 12

= 1800 lbs./ft run.

$$\text{Maxm. B.M.} = \frac{1800 \times 9^2}{8} \text{ ft.lb}$$

$$\text{Maximum Bending moment} = \frac{PL}{4} = \frac{P}{4} \times 9 = 2.5 \text{ P ft. lb.}$$

$$\text{or } P = \frac{4}{9} \times \frac{1800 \times 9 \times 9}{2} = 8100 \text{ lbs.}$$

= 3.62 tons.

= 3.8 tons.

UNIFORMITY DISTRIBUTED LOAD

Section of transom = 10" x 10"

Section of legs - 10" x 10"

$$K = \left(\frac{J2}{J1}\right)\frac{h}{l} = \frac{4.5}{9.0} = \frac{1}{2}$$

q = 100 l = 9 h = 4.5

$$MB = -\frac{ql^2}{4N} = -100 \times \frac{9 \times 9}{4 \times 4}$$

$$HA = HD = -\frac{MB}{h} = -100 \times \frac{9 \times 9}{4 \times 4} \times \frac{1}{4.5}$$

= 112.5 lbs

CONCENTRATED LOAD

$P = 3.8\ T \qquad l = 9'\text{-}0" \qquad h = 4.5$

$N = 4$

$$MB = MC = -\frac{3PL}{8N}$$

$$MP = \frac{Pl}{4} + MB$$

$$MB = -\frac{3 \times 3.8 \times 9}{8 \times 4} = 3.21 \text{ ft. tons.}$$

$$\text{Horizontal Thrust} = -\frac{MB}{h} = \frac{3.21}{4.5} = 0.715 \text{ tons.}$$

$$MB = -\frac{3}{8 \times 4} PL = -\frac{3}{32} PL$$

$$MP = \frac{PL}{4} + MB = \frac{PL}{4} - \frac{3}{32} PL = \frac{5}{32} PL$$

Span $l = 9'-0"$

Height $g = 4'-6"$

Depth of members h, Transom = 10"

Legs = 10"

Width of members Transom b = 10"

Legs b = 10"

Loads: Self load of 100 lbs/ft plus a concentrated load $p = 3.8T$ placed at the centre of the transom.

The permissible limit stresses are

Compression $R_b = 1000$ p.s.i.

Tension $R^1{}_b = 0$

The stiffness ratio legs/transom is:

$$\frac{Jl}{Ig} = \left(\frac{10}{10}\right)^3 \times \frac{9}{4.5} = 2$$

1. Reactions due to external load.

The numerical computations are given on page 3.

The results are as follows:

(a) Self-weight of the transom : 100 lbs/ft.

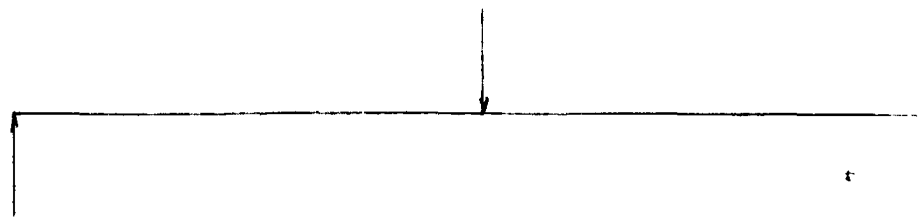
Vertical reaction $= \frac{100 \times 9}{2} = 450$ lbs.

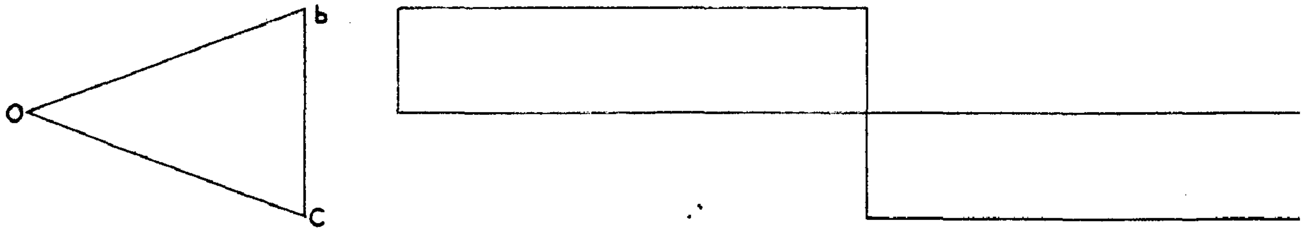
Horizontal Thrust = 112.5 lbs.

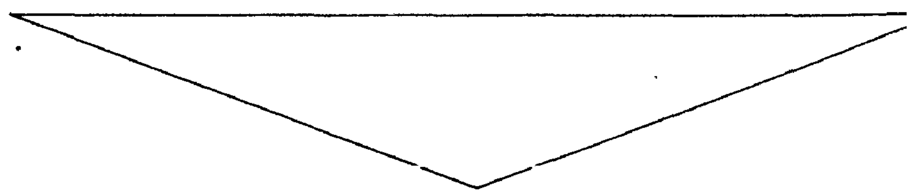
(b) Concentrated load : $P = 3.8T$

Vertical reaction = 1.9T

Horizontal Thrust = 0.715 T = 1600 lbs.







BENDING MOMENT DIAGRAM

FIG. 39

SCALE :- 1″ = 1·8 ft = α_1

1 = 3·8 T = α_2

H = 1·5″

SCALE OF BENDING MOMENT DIAGRAM :-

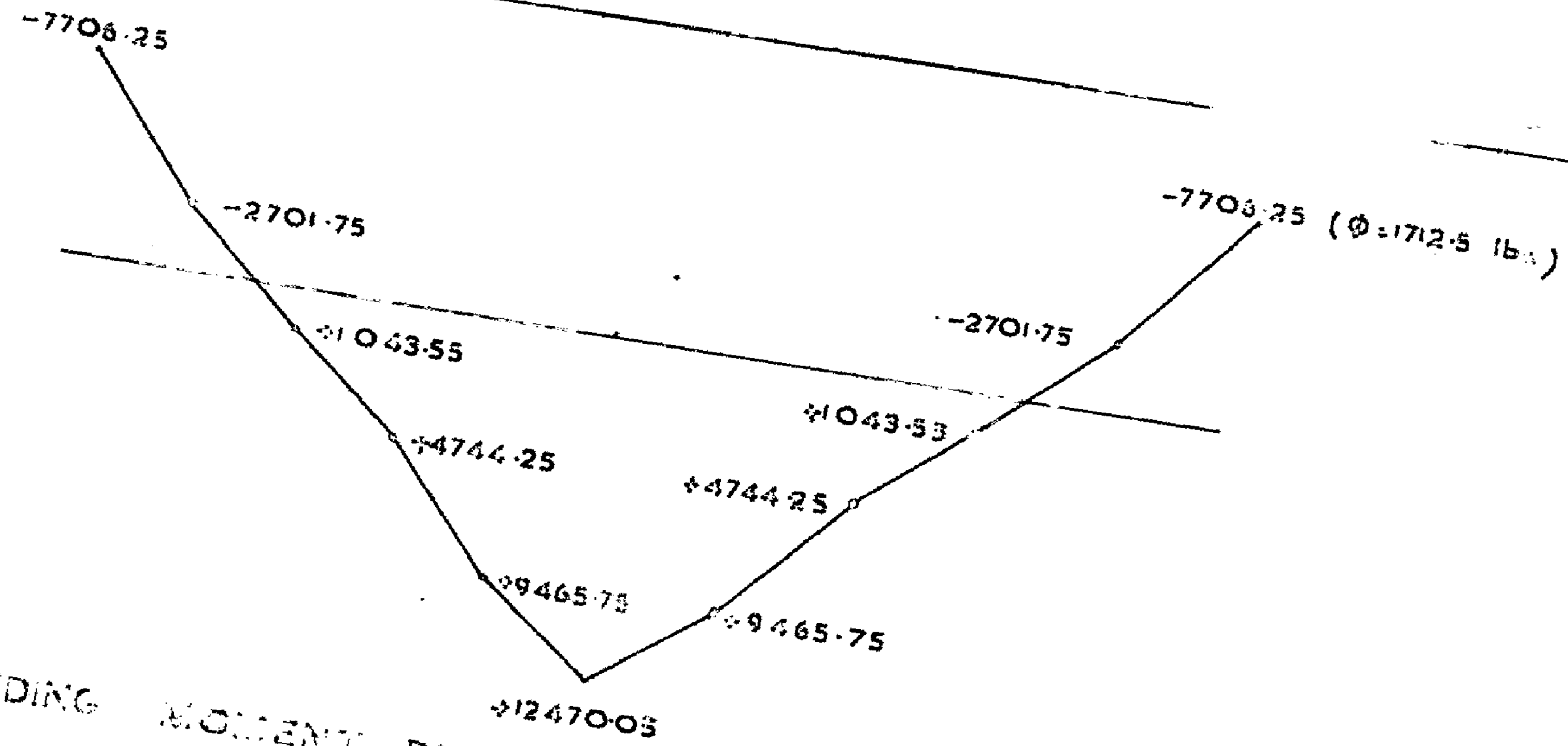
1 = H α_1 α_2

= 1·5 × 1·8 × 3·8

= 10·34 ft T

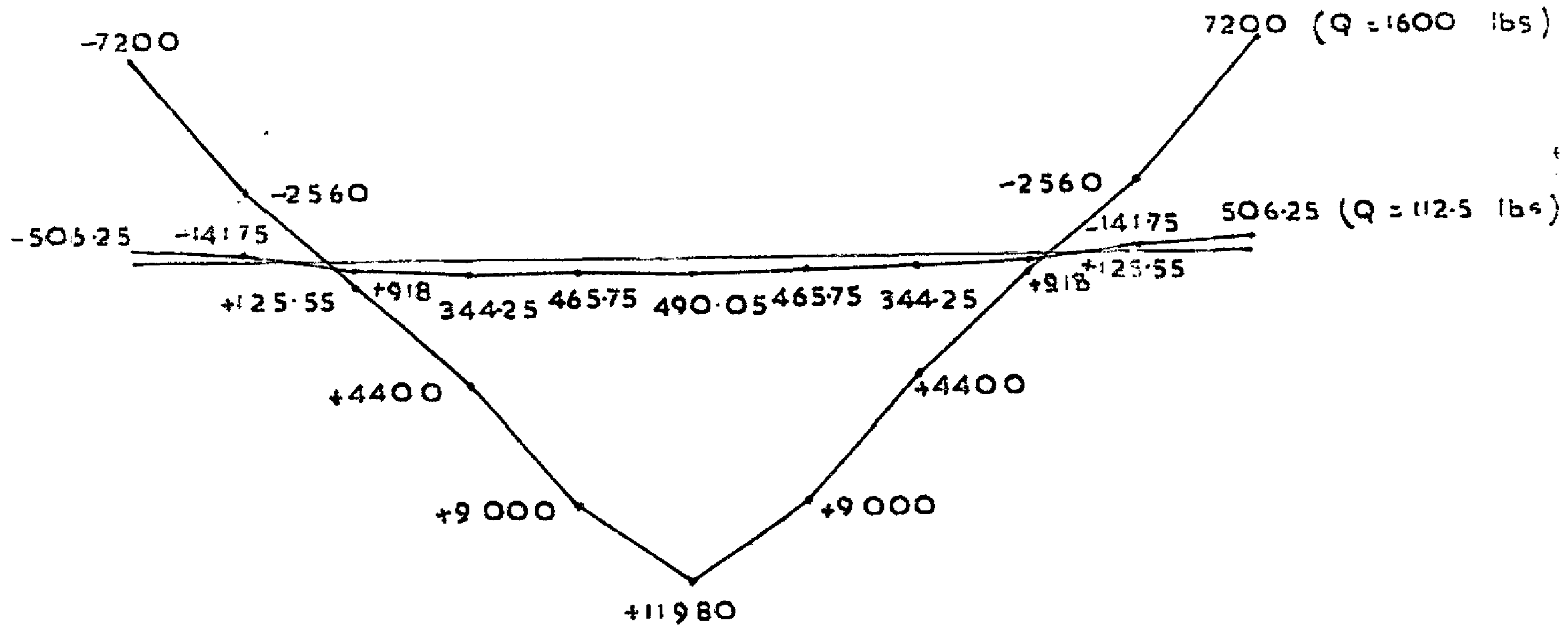
TABLE

	0	0.11	0.21	0.31	0.41	0.51	0.61	0.71	0.81	0.91	1
Self-weight(ft.lbs.)	- 506.25	- 141.75	+ 125.55	+ 344.25	+ 465.75	+ 490.05	+ 465.75	+ 344.25	+ 125.55	- 141.75	- 506.25
Concentrated load (ft.lbs.)	- 7200	- 2560	+ 918	+ 4400	+ 9000	+11980	+ 9000	+ 4400	+ 918	- 2560	- 7200
Total Moment due to self-weight and concentrated load (ft. lbs.)	- 7706.25	- 2701.75	+ 1043.55	+ 4744.25	+ 9465.75	+12470.05	+ 9465.75	+ 4744.25	+ 1043.55	- 2701.75	- 7706.25



BENDING MOMENT DIAGRAM DUE TO SELF WEIGHT PLUS CONCENTRATED LOAD IN THE TRANSOM.

FIG. 41



SCALE :- 1" = 8000 lbs

BENDING MOMENT DIAGRAMS FOR SELF WEIGHT AND CONCENTRATED LOAD RESPECTIVELY IN TRANSOM.

FIG. 40

2. T R A N S O M.

The Bending moments in the transom are given in the table overleaf.

Fig. (21) shows the bending moment diagrams corresponding to the two possible loading cases.

The greatest variation of stress occurs at the point x = 0.5 l. The smallest moment in this section occurs for the case of self-weight only and is equal to + 490.05 ft.lbs., the corresponding horizontal thrust being 112.5 lbs., the corresponding horizontal thrust being 1712.5 lbs.

The cross-section characteristics are:

b = 10", h = 10", S = 100 Sq.in.

$$Z, \text{ Section modulus} = \frac{10 \times 10 \times 10}{6} = 166.7 \text{ in}^3$$

The variation of stresses is:

$$\frac{1712.5 - 112.5}{100} + \frac{(12470.05 - 490.05) \times 12}{166.7} = 16 + 862.6$$

$$= 878.6 \text{ lbs/sq.in.}$$

$$< 1000$$

.·. The section is sufficient.

Let F be the magnitude of the prestressing force throughout the transom and e its eccentricity at the ~~plant~~ point x = 05 l. In order to have the least amount of steel (the minimum value of F), the centre of thrust should travel from edge to edge of the limit core when the bending moment varies from (f) are extreme value to the other, the edges of the limit

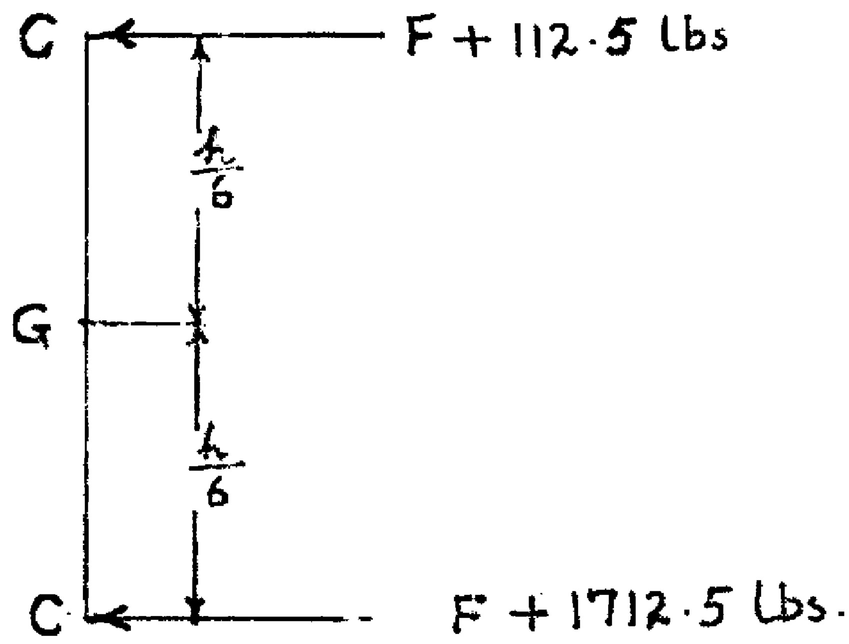


FIG. 43

core at $\frac{h}{6} = \frac{10}{6}$ = 1.67 in. above and below the neutral axis respectively.

The forces and moments in the section are:

Prestress plus self-weight only: compression = F + 112.5 lbs.

Moment = Fe + 490.05 ft.lbs.

Prestress plus concentrated loads: compression = F + 1712.5 lbs + Self-weight

Moment = Fe + 12,470.05 ft.lbs.

The above conditions can therefore be written:

$$+\frac{Fe + 490.05}{F + 112.5} = -\frac{1.67}{12}$$

$$+\frac{Fe + 12470.05}{F + 1712.5 \text{ lbs.}} = +\frac{1.67}{12}$$

The geometrical measuring of these relationships is shown in Fig. 43.

Solving the above equations,

F = 42,170 lbs. and e = - 0.151 ft = 1.812"

As has been explained in the general principles, the boundary lines of the limiting zones are obtained by drawing the diagrams

$$\frac{M_1 + e_1 \frac{h}{6}}{F} \quad \text{and} \quad \frac{M_2 - e_2 \frac{h}{6}}{F}$$

from the upper and lower core edge lines respectively.

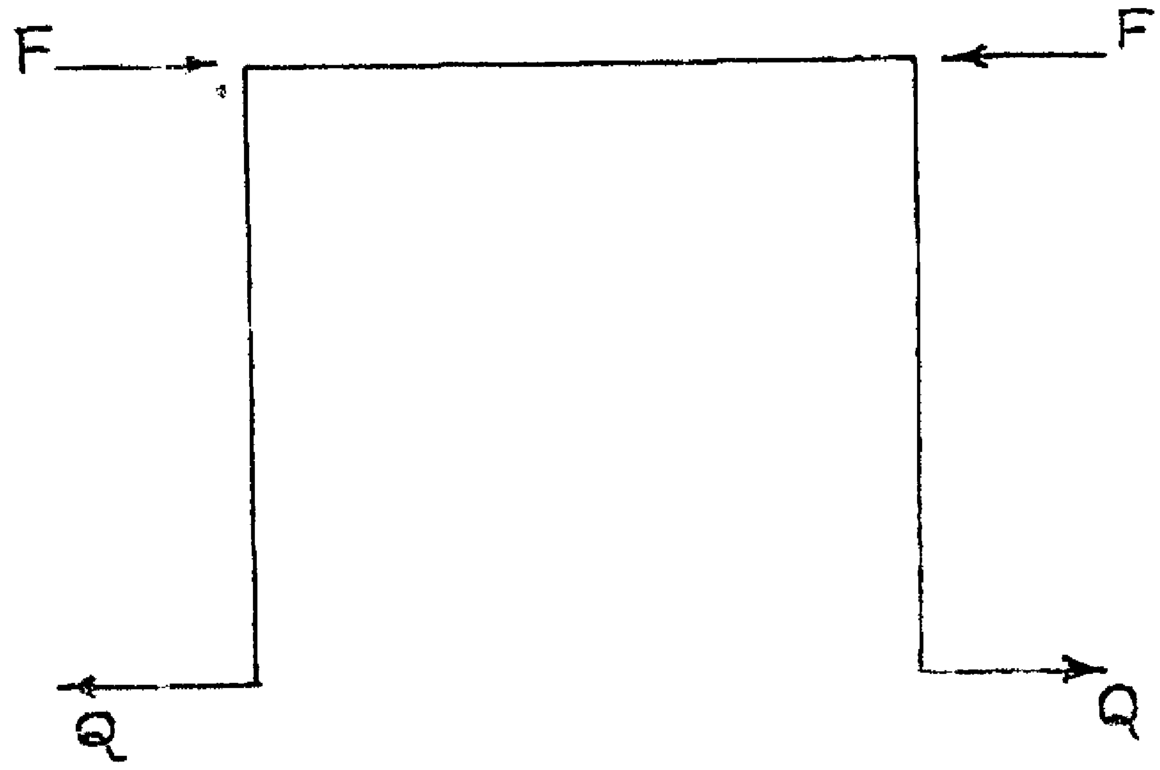


FIG. 44

The values of M_1 and M_2 and Q_1 and Q_2 are the smallest and the greatest bending moments and the corresponding horizontal thrusts respectively.

CORRECTION FOR THE TRANSOM SHORTENING:- The thrust caused by the shortening of the transom is approximately equal to

$$Q = F \frac{r^2}{g^2(1 + \frac{2}{3f})}$$

We have $r^2 = \frac{10 \times 10}{144} \times \frac{1}{12} = 0.058 \text{ ft}^2$

$$f = 2$$

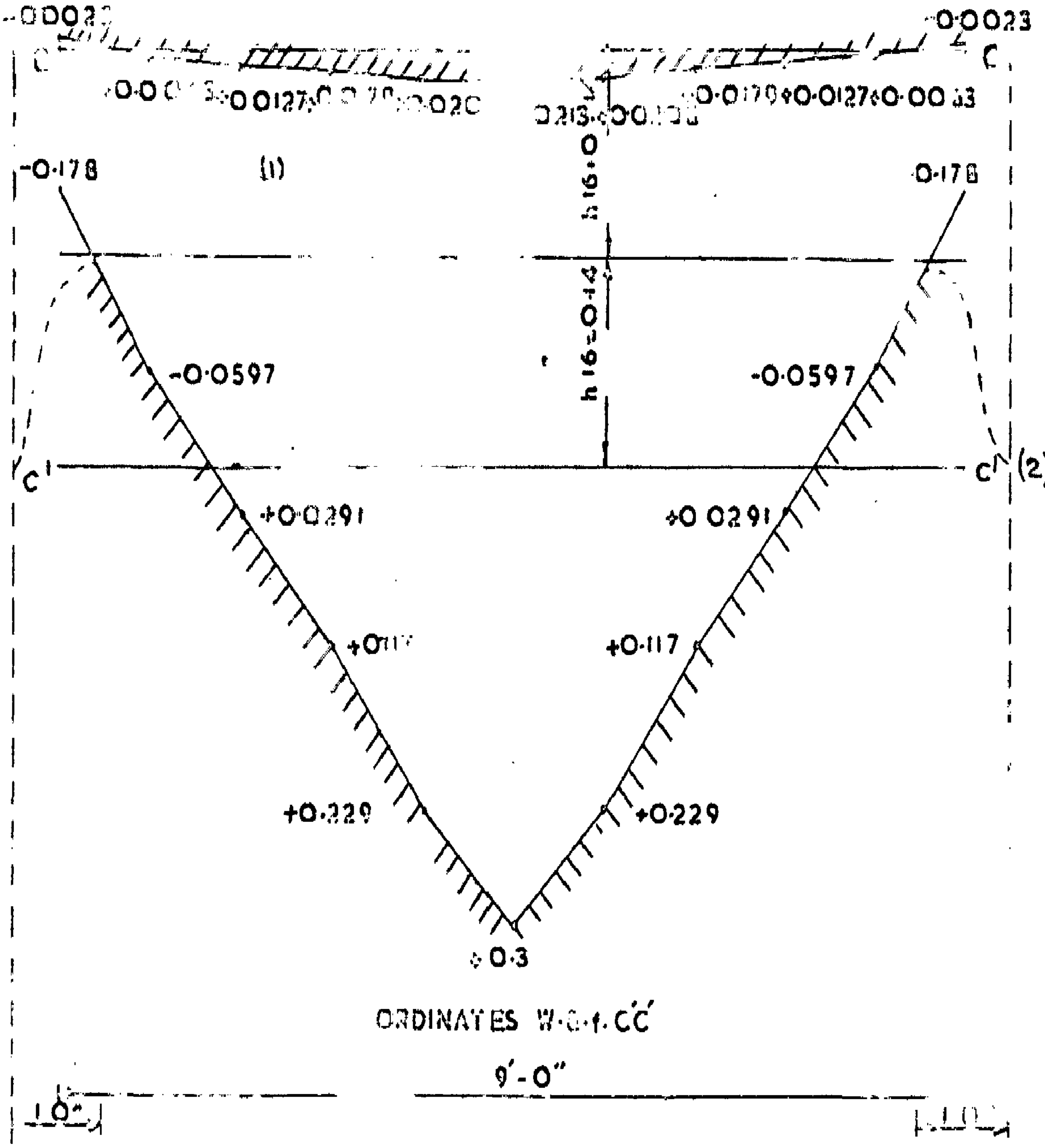
$$Q = 42{,}170 \times \frac{0.058}{20.25(1 + \frac{1}{3})}$$

$$= 90.5 \text{ lbs.}$$

Note:

The above formula is approximate. The value of the thrust correction being very small compared with F, the approximatio is sufficiently exact. Had the correction been considerable, it would have been necessary to use the exact formula given previously.

The action of the thrust correction is shown on figure 44.



LIMITING ZONE ... THE TRANSOM.

1. THE SCALE O. ... SCISSAE ... RENT FROM THAT OF ORDINATES

2. TRANSITION C... OVER TH... ..DTH OF THE LEG

FIG. 42

Throughout the transom, the bending moments are increased by 90.5 x 4.5 = 407.25 ft. lbs. whilst at the same time the compression is decreased by 90.5 lbs.

The values M_1, M_2, Q_1, Q_2 used in the expressions

$$\frac{M_1 + (Q_1 \frac{h}{6})}{F} \quad \text{and} \quad \frac{M_2 - (Q_2 \frac{h}{6})}{F}$$

for determining the boundary lines of the limiting zone, are values taking the correction into account. The limiting zone for the transom is shown in figure 41.

$$\frac{h}{6} = \frac{1.67}{12} = 0.14 \text{ ft.}$$

3 LEGS

The most dangerous section is not the theoretical point of intersection of the neutral axes of the transom and of the leg, but the section of the leg which is at the level of the transom soffit. For the sake of simplicity, the ~~transom~~ calculations are carried out for the theoretical intersection point only.

(a) Right leg, taking account of transom shortening

~~Self~~ Corrections : M = 407.25 ft.lbs. Q =-90.5 lbs.

Self-weight only : M = - 506.25 + 407.25 = - 99 ft.lbs.

N = + 450 lbs.

Self-weight plus concentrated loads:

M = -7706.25 + 407.25 = - 7299 ft.lbs.

N = + 4256 + 450 = 4706 lbs.

TABLE

	0	0.11	0.21	0.31	0.41	0.51	0.61	0.71	0.81	0.91	1
Self-weight Moment (ft.lbs). Horizontal Thrust = 112.5 lbs.	- 506.25	- 141.75	+ 125.55	+ 344.25	+ 465.75	+ 490.05	+ 465.75	+ 344.25	+ 125.55	- 141.75	- 506.25
Total Moment due to Self-weight and concentrated load(ft.lbs) Horizontal Thrust = 1712.5 lbs.	- 7706.25	- 2701.75	+ 1043.55	+ 4744.25	+ 9465.75	+12470.05	+ 9465.75	+ 4744.25	+ 1043.55	- 2701.75	- 7706.25

II. Correction for transom shortening a_1 = 90.5 lbs; Increase of Moment in transom = + 407.25 ft.lbs.

	0	0.11	0.21	0.31	0.41	0.51	0.61	0.71	0.81	0.91	1
Moment due to Self-weight(ft.lbs) Horizontal Thrust = 22 lbs.	- 99.00	+ 265.50	+ 532.80	+ 751.50	+ 873.00	+ 897.30	+ 873.00	+ 751.50	+ 532.80	+ 265.50	- 99.00
Total moment due to Self-weight and concentrated load(ft.lbs) ~~Horizontal Thrust = 1622.0 lbs.~~	- 7299.00	- 2294.50	+ 1450.80	+ 5151.50	+ 9873.00	+12877.30	+ 9873.00	+ 5151.50	+ 1450.80	- 2294.50	- 7299.00

III. M_1 Smallest moment. a_1 = 22 lbs.; $\frac{h}{6} = \frac{1.67}{12}$ ft; $a_1 \frac{h}{6} = 22 \times \frac{1.67}{12}$ = + 3.06 ft.lbs. F = 42,170

	0	0.11	0.21	0.31	0.41	0.51	0.61	0.71	0.81	0.91	1
$\dfrac{M_1 + a_1 \frac{h}{6}}{F}$ ft.	- 0.0023	+ 0.0063	+ 0.0127	+ 0.0179	+ 0.0208	+ 0.0213	+ 0.0208	+ 0.0179	+ 0.0127	+ 0.0063	- 0.0023

IV. M_2 greatest moment. a_2 = 1622 lb; $\frac{h}{6} = \frac{1.67}{12}$ ft; $a_2 \frac{h}{6} = 1622 \times \frac{1.67}{12}$ = - 225.7 F = 42,170 lbs.

	0	0.11	0.21	0.31	0.41	0.51	0.61	0.71	0.81	0.91	1
$\dfrac{M_2 - a_2 \frac{h}{6}}{F}$ (ft.)	- 0.178	- 0.0597	+ .0291	+ .117	+ 0.229	+ 0.3	+ 0.229	+ .117	+ 0.0291	- 0.0597	- 0.178

SECTION PROPERTIES:

b = 10" = 0.83 ft , h = 0.83 ft, S = 100 sq.in = 0.69 sq.ft.

Section modulus = 166.7 in^3

Stresses due to external load only:

Self-weight only:

Outer edge : $n = \frac{450}{100} - \frac{99 \times 12}{166.7} = 4.5 - 7.13 = -2.63$ lbs/sq.in.

Inner edge : $n^1 = \frac{450}{100} + \frac{99 \times 12}{166.7} = 4.5 + 7.13 = 11.63$

Full load:

at the outer edge $n_o = \frac{4706}{100} + \frac{7299 \times 12}{166.7} = 47.06 - 525.4$

$= -478.34$

at the inner edge $n^1{}_o = \frac{4706}{100} + \frac{7299 \times 12}{166.7} = 47.06 + 525.4$

$= +572.46$

By realising a prestress such that it gives:

at the outer edge $n_o = +478.34$

at the inner edge $n^1{}_o = -11.63$

We obtain the following total stresses:

outer edge: Self weight only $n = +475.71$, full load $n_o = o$

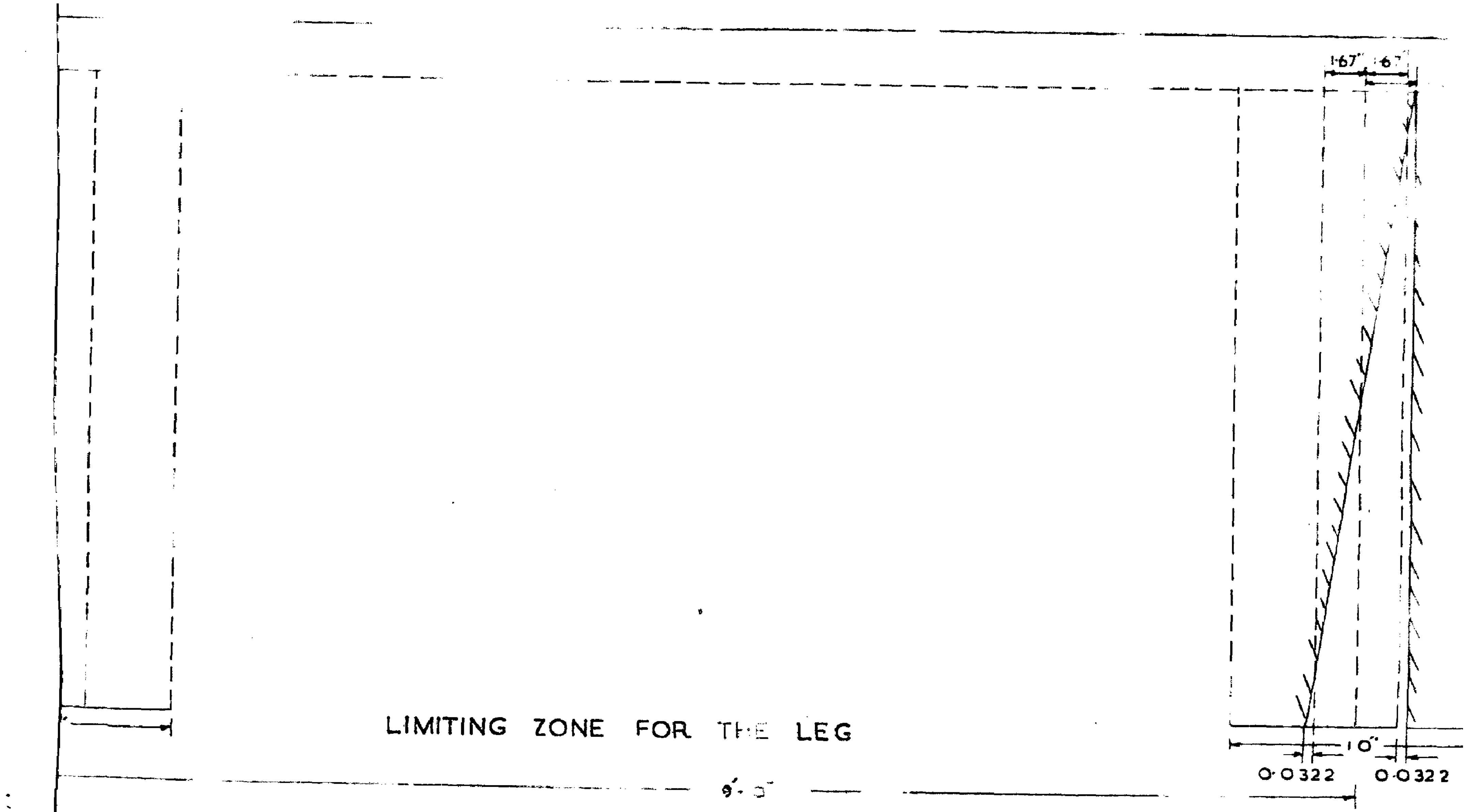
Inner edge: Self weight only $n^1 = o$, full load $n^1{}_o = +560.83$

These stresses are permissible.

Magnitude of prestress $F^{11} = 100 \quad \frac{(478.34 - 11.63)}{2}$

$= 100 \times 233.355$

$= 23,335.5$ lbs.



LIMITING ZONE FOR THE LEG

SCALE OF WIDTHS IS DOUBLE THE SCALE OF LENGTHS

FIG 45

Eccentricity of prestress $e^{11} = \frac{h}{6} \times \frac{n_o - n^1{}_o}{n_o + n^1{}_o}$

$$= \frac{10}{6} \times \frac{478.34 + 11.63}{478.34 - 11.63}$$

$$= + 1.75 \text{ in.}$$

Limiting Zone for the leg: We draw from the two vertical core edge lines, the boundary edge lines determined by the ordinates

$$\vartheta_1 = \frac{M_1 + (N_1 \frac{h}{6})}{F} \quad \text{and} \quad \vartheta_2 = \frac{M_2 - (N_2 \frac{h}{6})}{F}$$

respectively.

The moments M_1 and M_2 are proportional to the absassal Z(Vertical). The boundary lines are therefore straight lines intersecting at the top of the leg in the point determined by the eccentricity $e^{11} = + 1.75$ in. and passing through the points

$$\vartheta_1 = +\frac{N_1 h}{6F} \quad \text{and} \quad \vartheta_2 = -\frac{N_2 h}{6F}$$

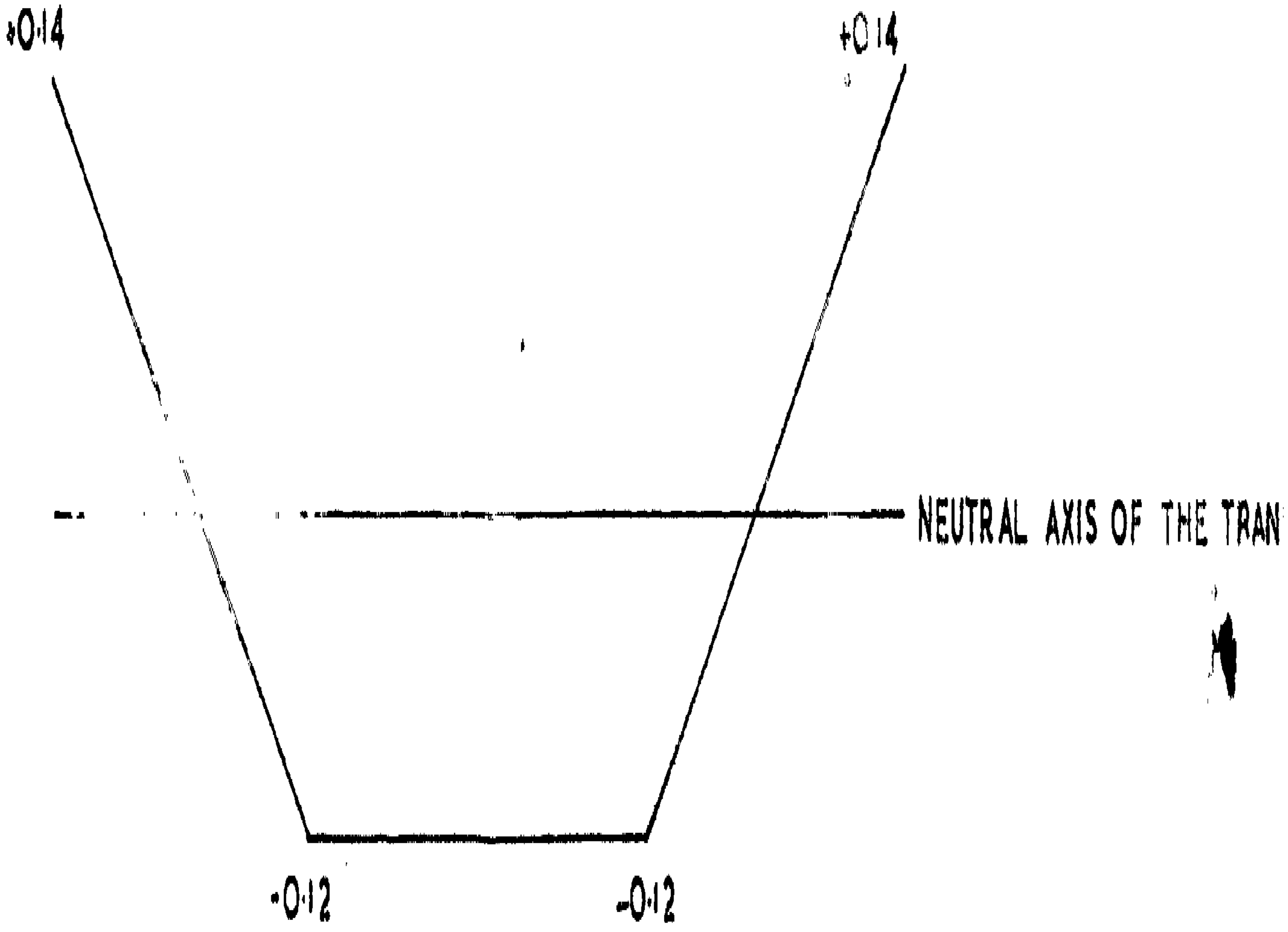
for $Z = M_1 = M_2 = 0$

$$\vartheta_1 = + \frac{1.67 \times 450}{23335.5} = + 0.032 \text{ in.}$$

$$\therefore \vartheta_1 = - \frac{1.67 \times 4706}{23,335.5} = - 0.337 \text{ in.}$$

(b) Left leg, taking into account the transom shortening

Same as for right leg due to symmetry.



CONCORDANT CABLE PROFILE IN THE TRANSOM

FIG. 46

4. <u>Determination of the concordant cable profile for the transom.</u>

Straight cables passing through the centre lines of the hinges are taken. Then

F^1 = 23,335.5 lbs. u^1 = 1.75 in. F'u' = 40,837.2 in lbs

F^{11} = 23,335.5 lbs, u'' = 1.75 in. F''u'' = 40,837.2 in. lbs.

In the transom F = 42,170 lbs.

Equation (4) becomes $= -\frac{2}{3} \times 108 \times \frac{40837.2}{42,170} \times 2$

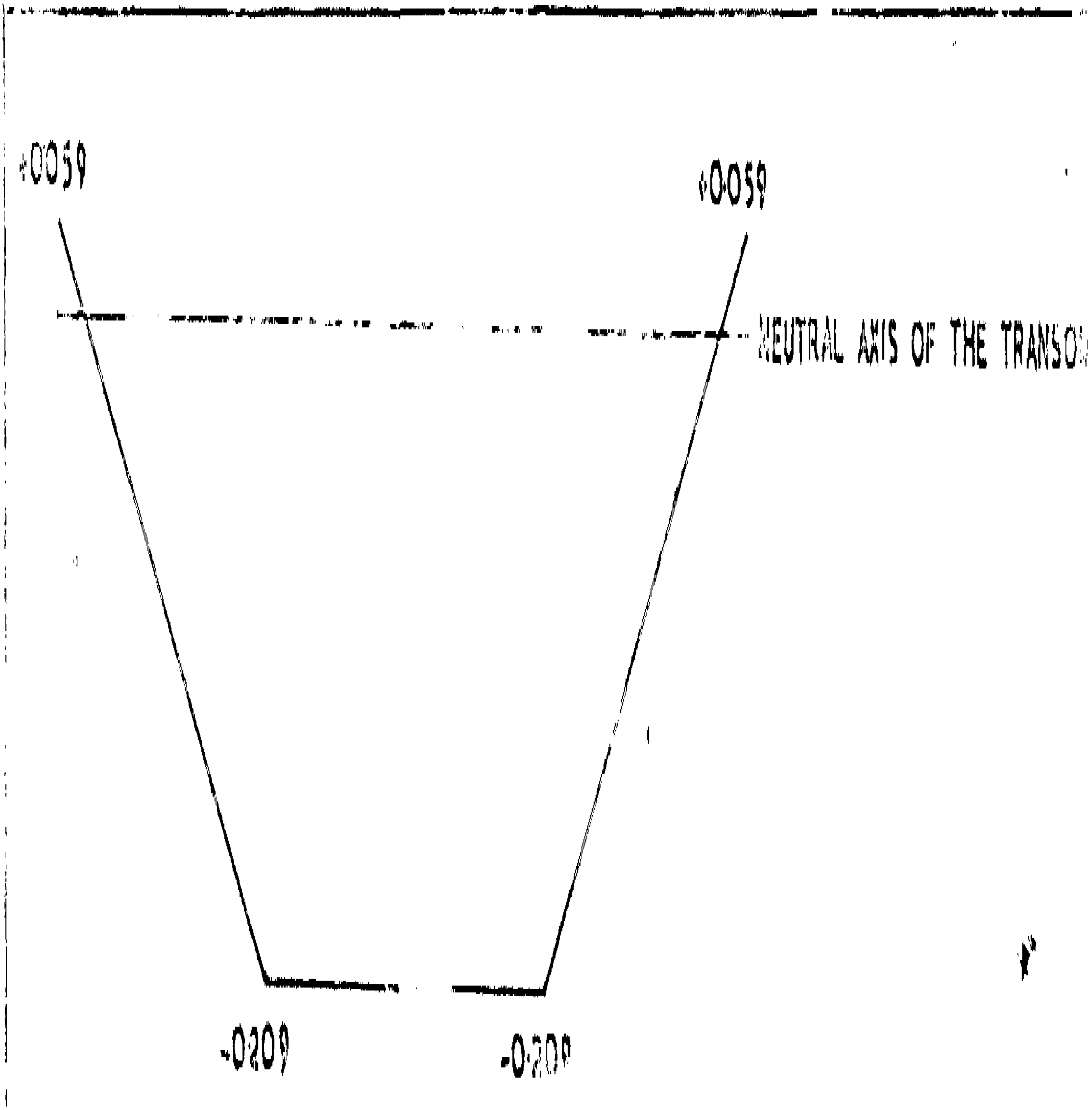
$= -34.86$ sq.in.

$= -\ 0.242$ sq.ft.

A curve lying entirely within the limiting zone and enclosing between itself and the neutral axis of the transom an area -34.86 sq.in must be found out.

This curve can be determined by trial and error. However, the gene ral method can be applied which consists in the determination of an imaginary load function q(x) which would give for a compression of 42,170 lb and assuming partial restraints at the two ends of the transom, a funicular curve lying entirely within the neutral zone and making with the neutral axis an area equal to - 34.86 sq.in.*

* The negative sign means that the area is below the neutral axis. The imaginary loads therefore act upwards.



+0059
+0059
NEUTRAL AXIS OF THE TRANSO
-0209
-0209

In the present case, the curve is determined by trial and error method imposing the condition that it should have at the ends of the transom an ordinate equal to $+ \frac{h}{6} = 0.14$ ft and the area be tween itself and the neutral axis of the transom an area of -34.86 sq.in.

The adequacy of this line of pressure is checked by sketching it on the drawing of the limiting zone and by verifying that it determines with the neutral axis an area equal to 34.86 sq.in.

5. <u>Transformation of the cable profile</u>.

As has already been mentioned, the transom cable may be moved up or down provided at the same time the leg cables are rotated about the centre lines of the hinges by such an amount that the apparent variations of the bending moment at the frame angle caused by these cable movements should be the same for the transoms and for the legs.

This positively can be made use of to give leg cables an approximately vertical position.

This means that the prestressing moment in the transom has to be reduced by the amount.

$F'u' = 40837.2$ in.lbs = 3403.1 ft.lbs.

In other words the transoms cable has to be lowered by $a = \frac{3403.1}{F} = \frac{3403.1}{42,170} = -0.081$ ft.

$= -0.972$ in.

The final prestressing is shown in fig. 48

Various losses due to prestressing are taken as 15%

Ultimate strength of the high tensile wire = 249,984 lbs/sq.in.

Working stress of the high tensile wire = 0.65 x 2,49,984

= 161,952 lbs/sq.in.

Stress in steel after 15% loss would be 0.85 x 161,952 = 137,500 p.s.i

∴ Steel area required for transom $= \frac{42,170}{137,500}$

$= 0.306$ sq.in.

Adopt 10 wires of dia. 5 mm (0.2") each giving an area of 0.314 sq.in

Steel required per leg $= \frac{23,335.5}{137,500}$

$= 0.17$ sq.in.

Adopt 6 wires of 0.2" dia each.

<u>Design of shear reinforcement.</u>

Maximum shear force at the support = 450 + 4256 = 4706 lbs.

Shear Force taken by the cables in the transom = 42,170 Sin α

$= 42,170 \times \sin 44^{\circ}$

$= 42,170 \times 0.69$

$= 29,090$ lbs.

DETAILS OF THE PRESTRESSED CONCRETE PORTAL FRAMES

FIG. 48

The maximum shear force at the support is taken care of by the vertical component of the force in the prestressing cables. Yet a nominal reinforcement of 1/4" dia. stirrups at 6" C/C is used in the transom. Ties of 1/4" dia. at 9" C/C are used in the legs. Dia of the longitudinal bars is 3/8". The diagram of the portal frame along with the prestressing wires and the reinforcement cage is shown clearly in the drawing.

Weight of the portal = 0.805 Tons.

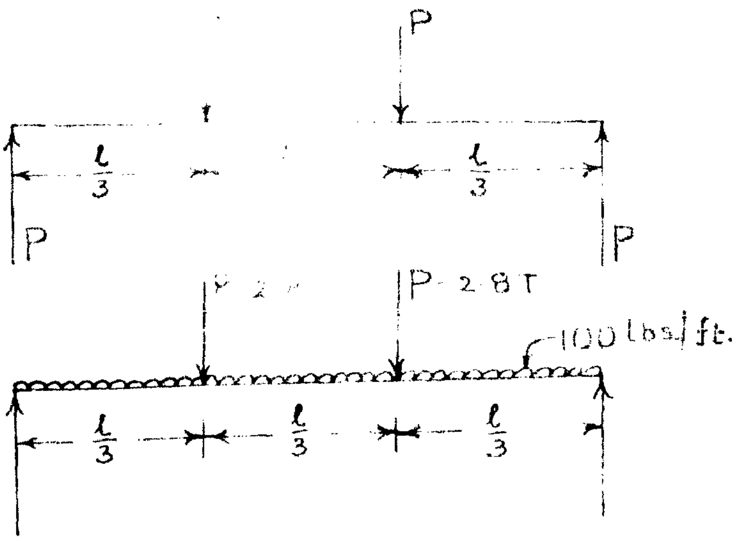
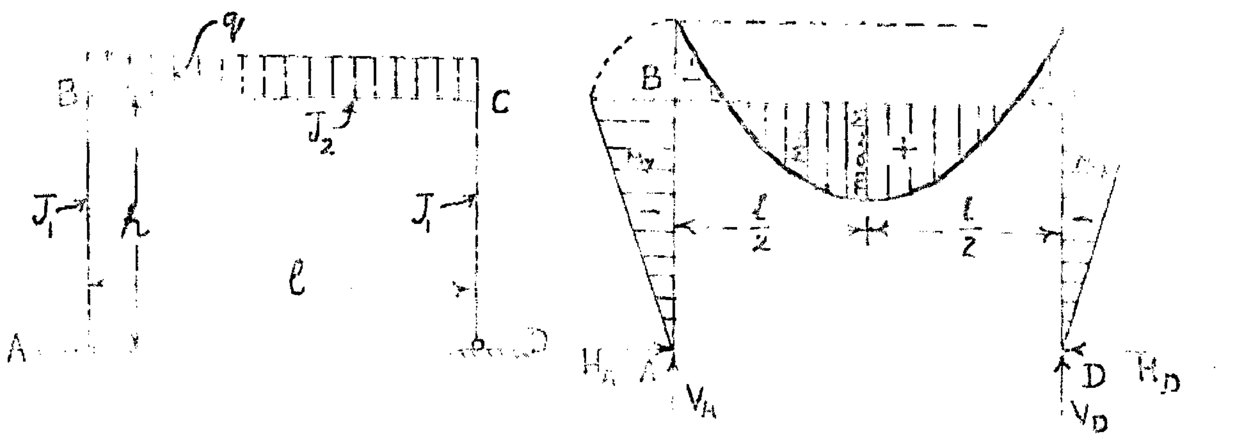


FIG. 49

COEFFICIENTS. $K = \frac{J_2}{J_1} \cdot \frac{h}{l}$, $N = 2K +$ [illegible]



$M_B = M_C = -\frac{q l}{4N}$ $\max M = \frac{q l^2}{8} + M_B$ $M_x = \frac{q x x'}{2} + M_B$

$V_A = V_D = \frac{q l}{2}$ $H_A = H_D = -\frac{M_B}{h}$ $M_y = \frac{y}{h} M_B$

FIG. 50

Let the c/c distance of the portal frames = 12' - 0"

Width of the portal frame = 9' - 0"

Total load = 150 lbs/ft^2

Total load per foot run of the transom = 150 x 12

= 1800 lbs/ft.run.

$$\text{Maxm. B.M.} = \frac{1800 \times 9^2}{8} \text{ ft.lb.}$$

$$\text{Maximum Bending moment} = \frac{pl}{3} = \frac{P \times 9}{3} = 3\,P$$

$$\therefore 3\,P = \frac{1800 \times 9 \times 9}{8}$$

$$\text{or } P = \frac{1800 \times 9 \times \not{9}^{3}}{8 \times \not{3}} = 225 \times 27 \text{ lbs.}$$

$$\approx 2.8 \text{ tons}$$

P = 2.8T P = 2.8T

Let the section be 10" x 10"

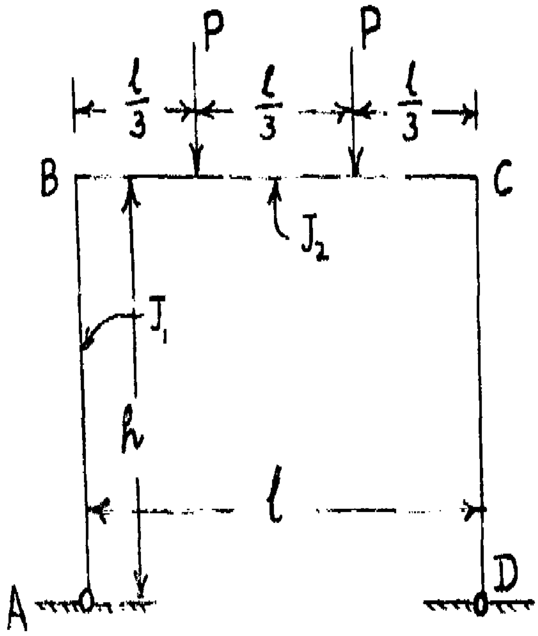
$$\text{Coefficients : } K = \frac{J2}{J1} \times \frac{h}{l}$$

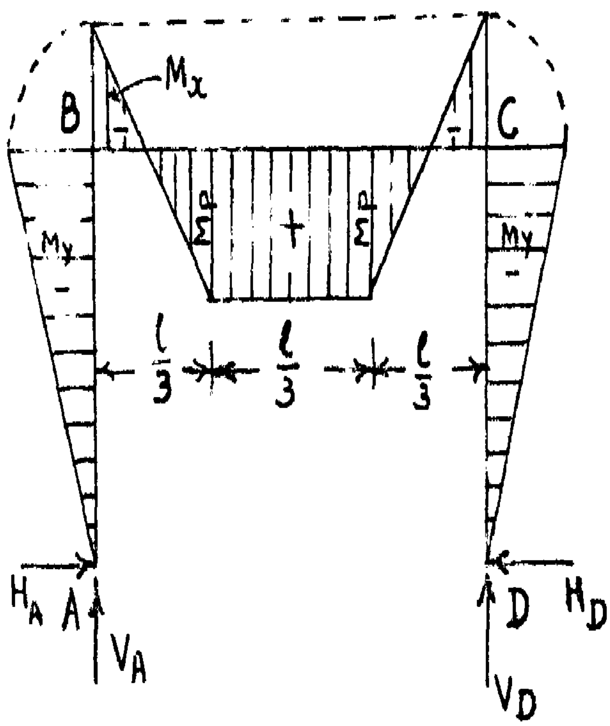
$$MB = Mc = -\frac{ql^2}{4N} \quad \text{Max } M = \frac{ql^2}{8} + MB \quad Mx = \frac{qxx'}{Z} + MB$$

$$VA = VD = \frac{ql}{2} \quad HA = HD = -\frac{MB}{h} \quad My = \frac{y}{h} M_B$$

$$MB = Mc = \frac{2PL}{3N} \quad VA = VD \quad HA = HD = -\frac{MB}{h}$$

$$MP = \frac{pl}{3} + MB \quad Mx = Px + MB \quad My = \frac{y}{h} MB$$





$M_B = M_C = -\frac{2Pl}{3N}$ $\quad V_A = V_D = P$ $\quad H_A = H_D = \frac{M_B}{h}$

$M_P = \frac{Pl}{3} + M_B$ $\quad M_x = Px + M_B$ $\quad M_y = \frac{y}{h} M_B$

FIG. 51

UNIFORMITY DISTRIBUTED SELF LOAD

Section of transom = 10" x 10"

Section of legs = 10" x 10"

$$K = \left(\frac{J_2}{J_1}\right) = \frac{h}{l} = \frac{4.5}{9.0} = 1/2$$

$$N = 2K + 3 = 2 \times \tfrac{1}{2} + 3 = 4$$

$$q = 100 \; ; \quad l = 9; \quad h = 4.5$$

$$MB = \frac{ql^2}{4N} = -\frac{100 \times 9 \times 9}{4 \times 4}$$

$$HA = HD = -\frac{MB}{h} = -\frac{100 \times 9 \times 9}{4 \times 4} \times \frac{1}{4.5}$$

$$= 112.5 \text{ lbs.}$$

CONCENTRATED LOADS

$$P = 2.8T; \; l = 9'\text{-}0"; \; h = 4.5'$$

$$N = 4$$

$$MB = MC = -\frac{2PL}{3N} = -\frac{2 \times 2.8 \times 9}{3 \times 4} = -4.2 \text{ ft.tons.}$$

$$\text{Horizontal Thrust} = \frac{MB}{h} = \frac{4.2}{4.5} = 0.93 \text{ tons.}$$

$$MB = MC = -\frac{2PL}{3x4} = -\frac{1}{6} PL$$

$$MP = \frac{PL}{3} + MB = \frac{PL}{3} - \frac{1}{6} PL = \frac{PL}{6}$$

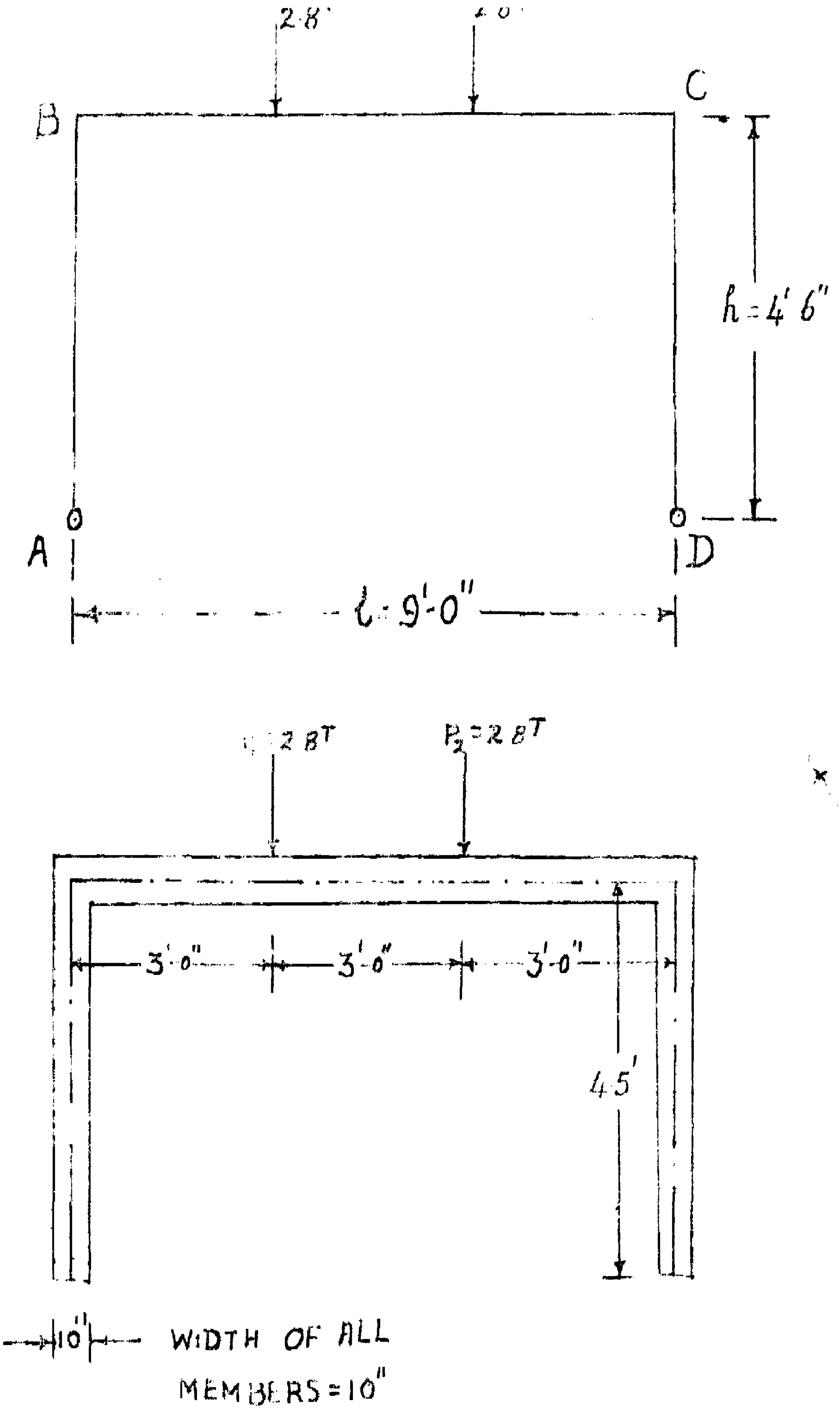


FIG 52

Span = 9'-0"

Height = 4'-6"

Depth of Members : h, Transom = 10"

Legs = 10"

Width of members : Transom b = 10"

Legs b = 10"

LOADS: Self load of 100 lbs/ft. plus two concentrated loads P_1 and P_2 placed at 3 ft. from the centre lines of the support points

The loads P_1 and P_2 may assume the following values.

Loading case : $P_1 = 2.8^T$ $P_2 = 2.8^T$

The permissible limit stresses are

Compression R_b = 1200 p.s.i.

Tension R_b^1 = 0

The stiffness ratio legs/transom is:

$$= \frac{Il}{Ih} = \left(\frac{10}{10}\right)^3 \times \frac{9}{4.5} = 2.$$

1. Reactions due to external loads:

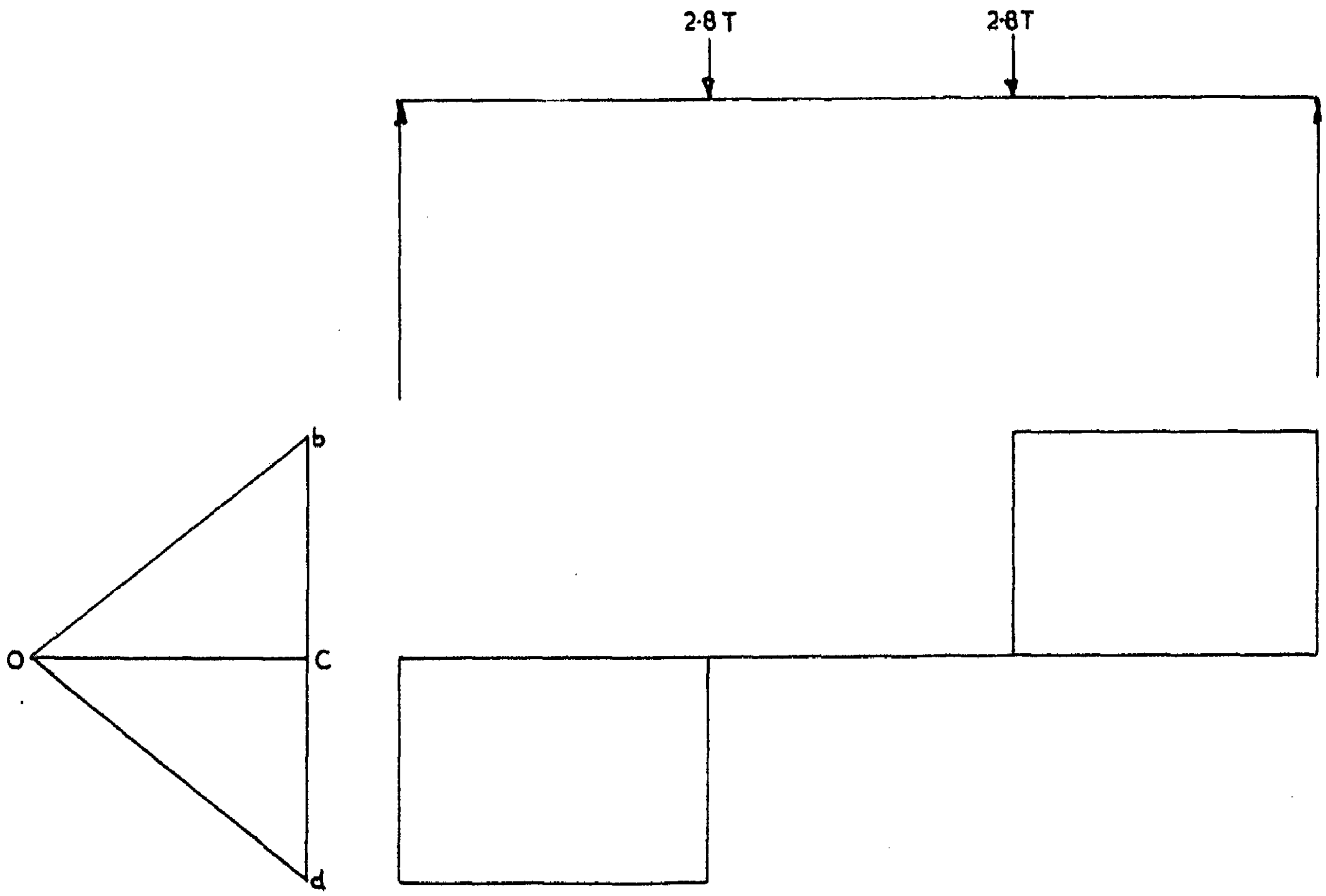
The numerical computations are given on page 112.

The results are as follows:

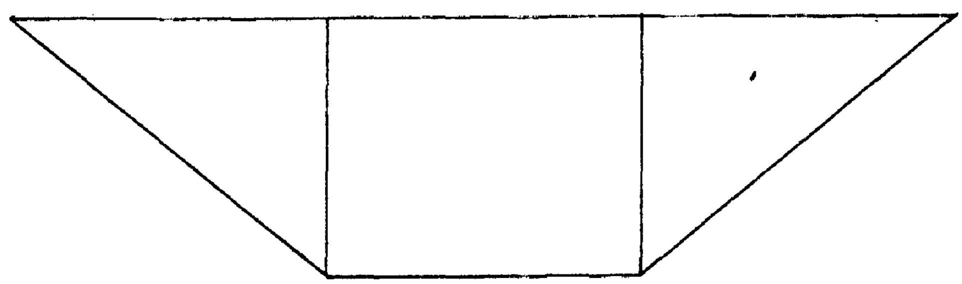
(a) Self-weight of the transoms : 100 lbs/ft.

Vertical reaction : $\frac{100 \times 9}{2}$ = 450 lbs.

Horizontal thrust : 112.5 lbs.



SHEAR FORCE DIAGRAM



BENDING MOMENT DIAGRAM

FIG. 54

SCALE :- $1'' = 1{\cdot}8' = \alpha_1$

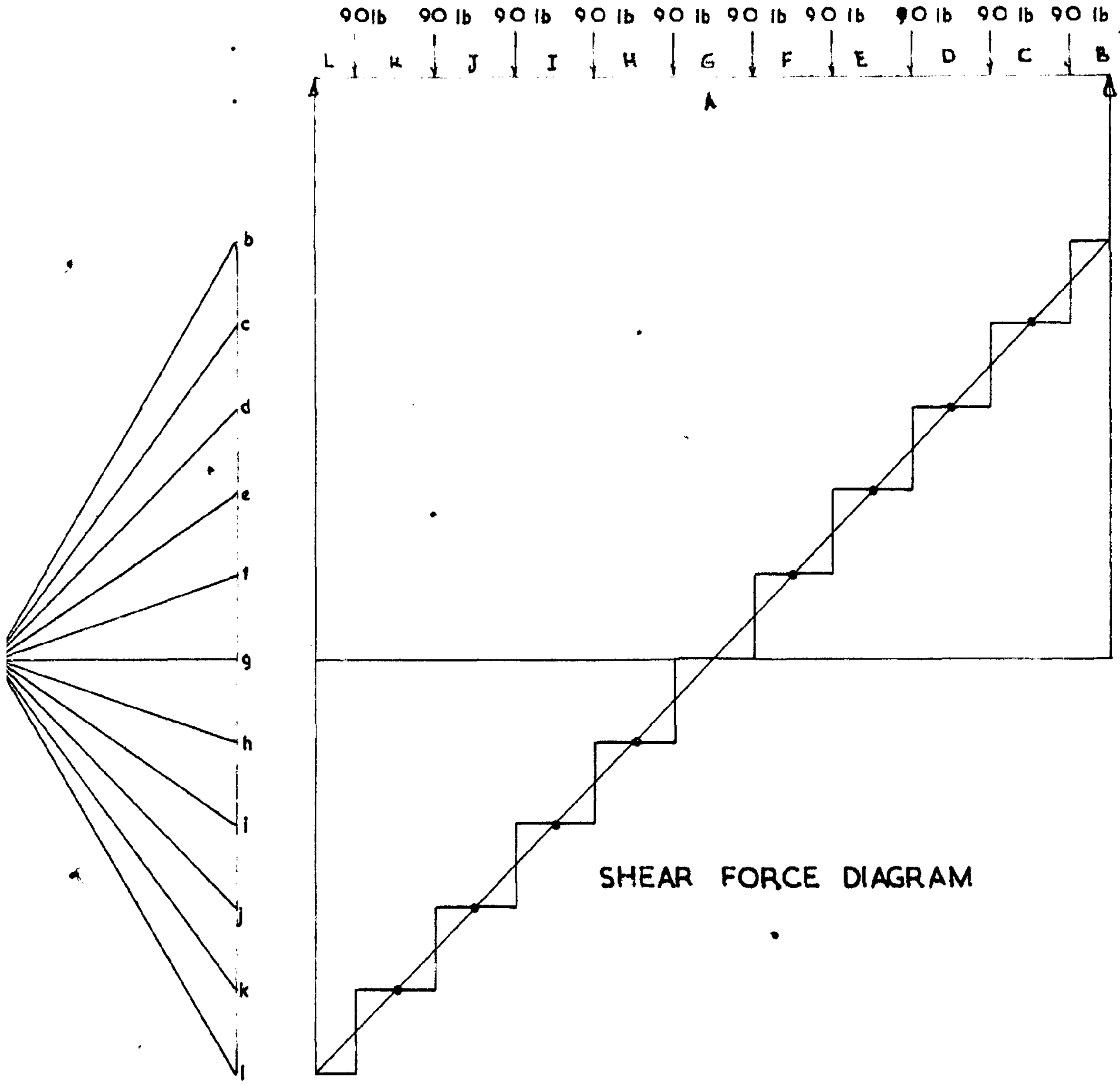
$1'' = 2{\cdot}8' = \alpha_2$

$H = 1{\cdot}5''$

SCALE :- $1'' = H \alpha_1 \alpha_2$

$= 1{\cdot}5 \times 1{\cdot}8 \times 2{\cdot}8$

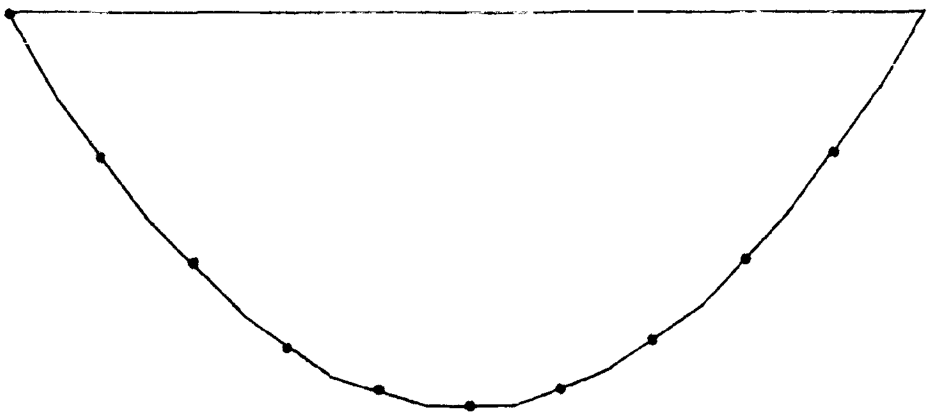
$7{\cdot}56'$ T



SHEAR FORCE DIAGRAM

AL E:- 1'' = 1·8 ft = α
1'' = 180 lb = α2
H = 1.5''

ALE OF B.M. DIAGRAM:-
1'' = Hα1 α2
= 1·5 X 1·8 X 180
= 486 ft. lb.

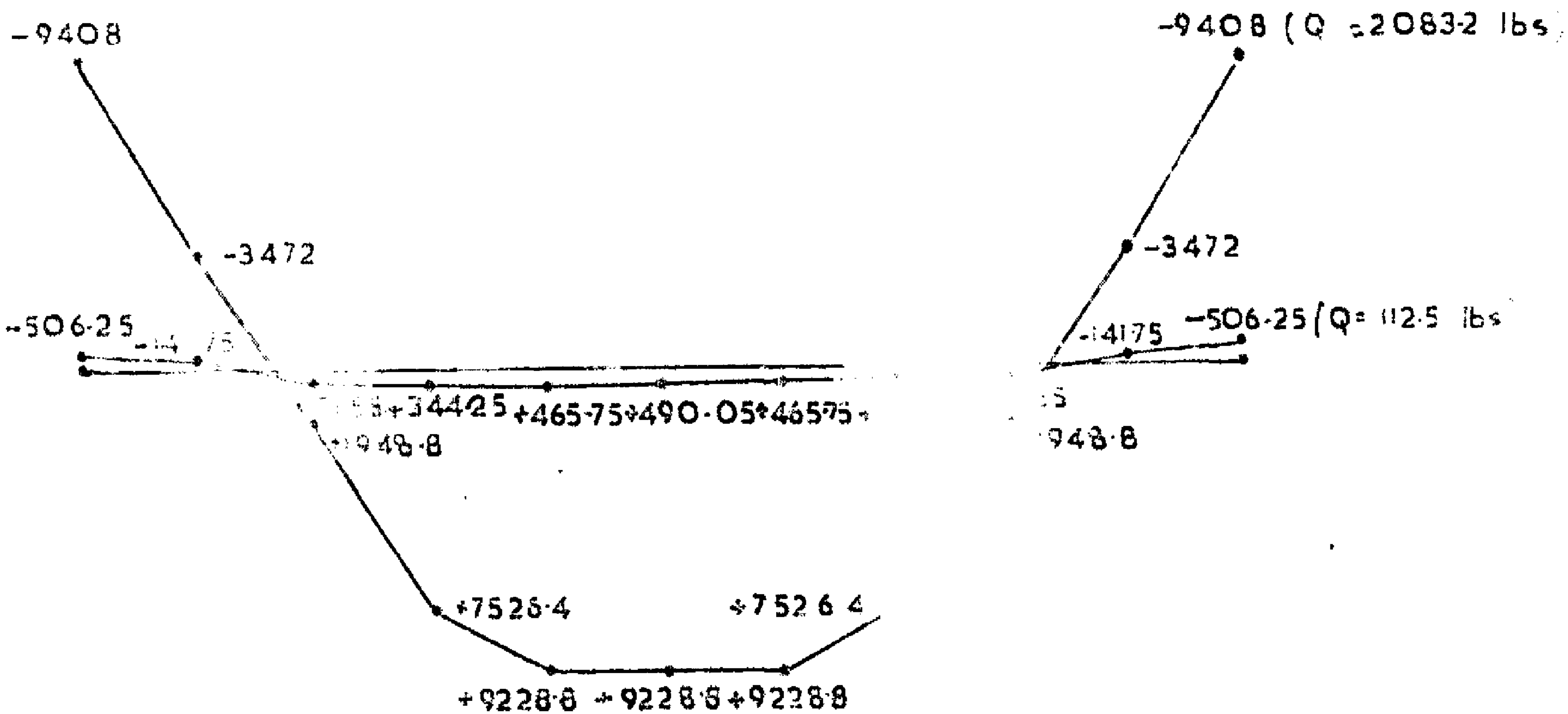


BENDING MOMENT DIAGRAM

FIG. 53

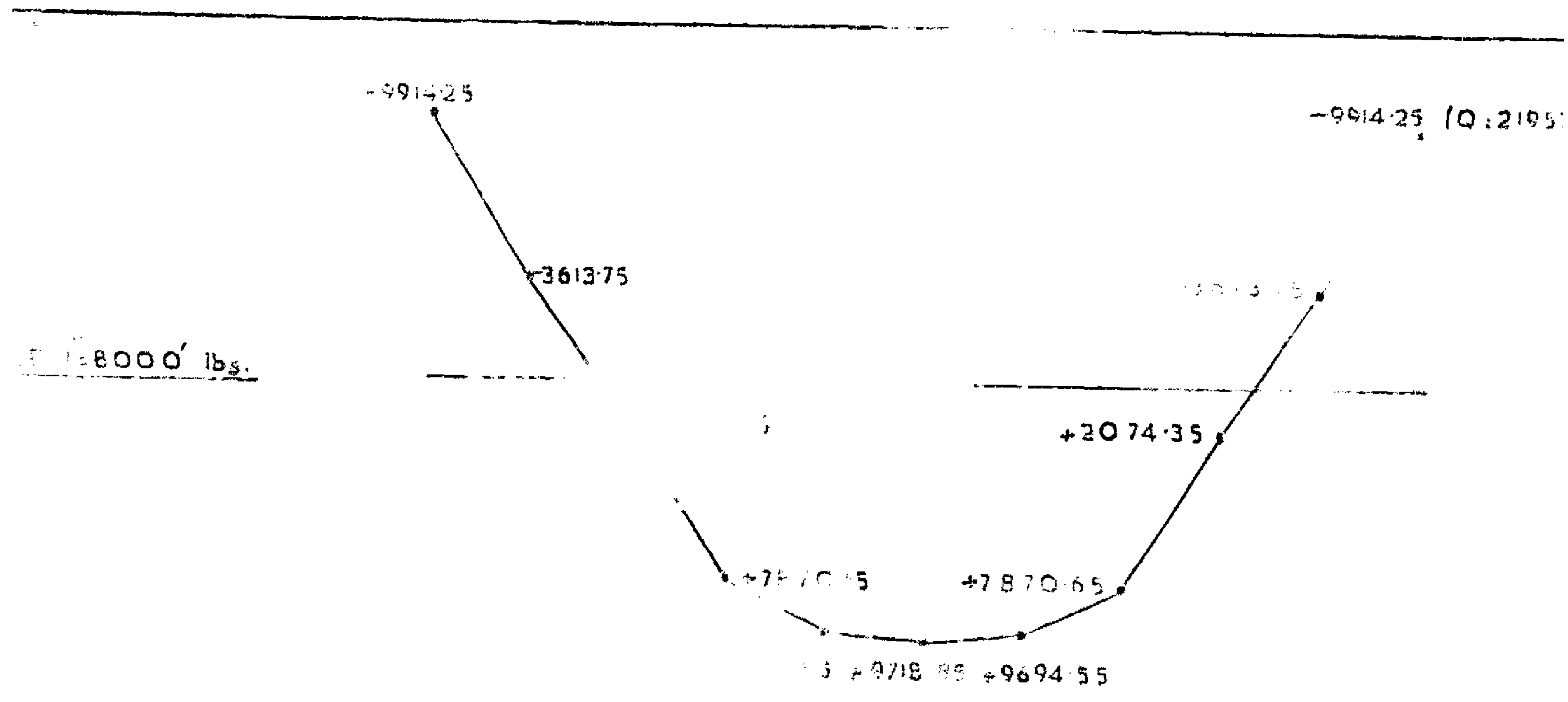
TABLE

	0	0.11	0.21	0.31	0.41	0.51	0.61	0.71	0.81	0.91	1
1. Selft-weight (ft.lbs.)	- 506.25	- 141.75	+ 125.55	+ 344.25	+ 465.75	+ 490.05	+ 465.75	+ 344.25	+ 125.55	- 141.75	- 506.25
2. Concentrated loads (ft. lbs).	- 9408	- 3472	+ 1948.8	+ 7526.4	+ 9228.8	+ 9228.8	+ 9228.8	+ 7526.4	+ 1948.8	- 34.72	- 9408
3. Total Moment due to Self-weight and concentrated load (ft. lbs).	- 9914.25	- 3613.75	+ 2074.35	+ 7870.65	+ 9694.55	+ 9718.85	+ 9694.55	+ 7870.65	+ 2074.35	- 3613.75	- 9914.25



BENDING MOMENT DIAGRAMS CORR... DING TO SE... WEIGHT AND COCENTRATED LOADS RE... ELY.

FIG. 55



BENDING MOMENT DIAGRAM OF SELF WEIGHT PLUS CONCENTRATED LOADS

(b) Concentrated Loads : $(P_1 = P_2 = 2.8^T)$

Vertical reaction $= 2.8^T = 6272$ lbs.

Horizontal Thrust $= 0.93^T = 2083.2$ lbs.

2. TRANSOM:

The beinding moments in the transom are given in the table overleaf.

Fig. 56 shows the bending moment diagrams corresponding to the two possible loading cases. These have been calculated from the table as follows:

The greatest variation of stresses occurs at the point X = 0.5 l. The smallest moment in this section occurs for the case of self-weight only and is equal to + 490.05 ft.lbs., the corresponding horizontal thrust is equal to 112.5 lbs. The greatest moment is equal to + 9718.85 ft.lbs., the corresponding horizontal thrust being 2195.7 lbs.

The cross-section characteristics are:

b=10", h=10", S = 100 sq.in.

$$Z, \text{ Section modulus} = \frac{10 \times 10 \times 10}{6} = 166.7 \text{ in}^3$$

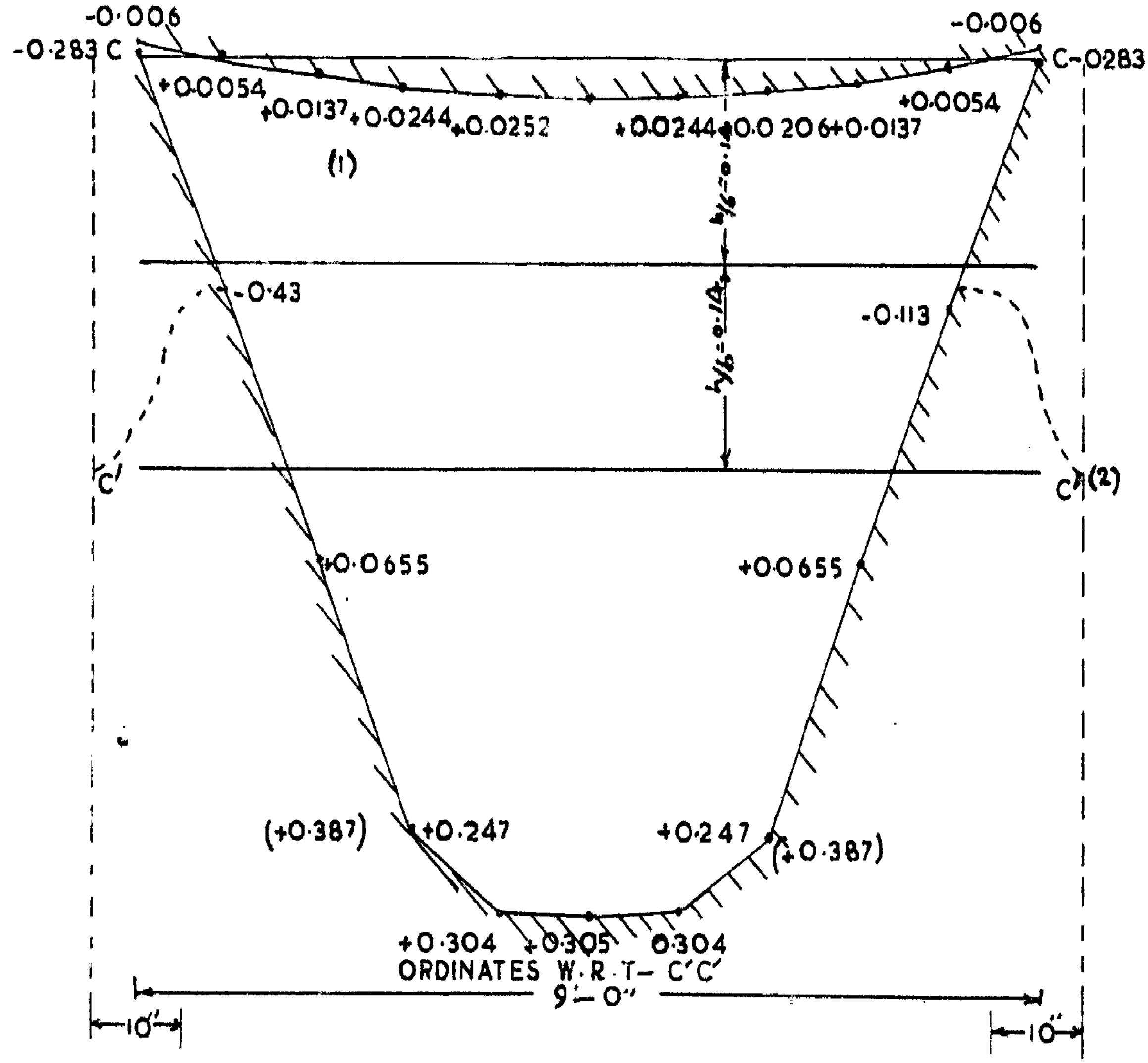
The variation of stresses is:

$$\frac{2195.7 - 112.5}{100} \quad \frac{(9718.85 - 490.05) \times 12}{166.7} = 2083 + 666$$

$$= 687 \text{ lbs/sq.in.}$$

$$< \quad 1000$$

∴ The section is sufficient.



1. THE SCALE OF ABSCISSAE IS DIFFERENT FROM THAT OF ORDINATES

2. TRANSITION CURVE OVER THE WIDTH OF THE LEG LIMITING ZONE FOR THE TRANSOM

FIG. 57

As has been explained in the general principles, the boundary lines of the limiting zones are obtained by drawing the diagrams

$$\frac{M_1 + a_1 \frac{h}{6}}{F} \quad \text{and} \quad \frac{M_2 - a_2 \frac{h}{6}}{F}$$

from the upper and lower core edge lines respectively.

The values M_1 and M_2 and a_1 and a_2 are the smallest and the greatest bending moments and the corresponding horizontal thrusts respectively.

CORRECTION FOR THE TRANSOM SHORTENING: - The thrust caused by the shortening of the transom is approximately equal to

$$a = F \frac{r^2}{g^2 (1 + \frac{2}{3\rho})}$$

we have

$$r^2 = \frac{10 \times 10}{144} \times \frac{1}{12}$$

$$= \frac{8.33}{144} = 0.058 \text{ ft.}^2$$

$$\rho = 2$$

We obtain

$$a = 31900 \frac{0.058}{20.25 (1 + 1/3)}$$

$$= \frac{319 \times 5.8}{20.25 \times 4/3}$$

$$= \frac{319 \times 5.8}{27} = 68.5 \text{ lbs.}$$

Note:-

The above formula is approximate. The value of the thrust correction being very small compared with F, the approximation is sufficiently exact. Had the correction been considerable, we would have had to use the exact formula given previously.

The action of the thrust correction is shown in figure Throughout the transom the bending moments are increased by 68.5 x 4.5 = 308.25 ft. lbs. whilest at the same time the compression is decreased by 68.5 lbs.

The values M_1, M_2, Q_1, Q_2 used in the expressions $\frac{M_1 + Q_1(\frac{h}{6})}{F}$ and $\frac{M_2 - Q_2(\frac{h}{6})}{F}$ for determining the boundary lines of the limiting zone, are values taking this correction into account.

The limiting zone for the transom is shown in fig. 57 .

$$\frac{h}{6} = \frac{1.67}{12} = 0.14 \text{ ft.}$$

3. LEGS

The most dangerous section is not the theoretical point of intersection of the neutral axes of the transom and of the leg, but the section of the leg which is at the level of the transom soffit. For the sake of simplicity, the calculations are carried out for the theoretical intersection point only.

TABLE

	0	0.11	0.21	0.31	0.41	0.51	0.61	0.71	0.81	0.91	1
Self-weight(ft.lbs). Horizontal Thrust = 112.5 lbs.	- 506.25	- 141.75	+ 125.55	+ 344.25	+ 465.75	+ 490.05	+ 465.75	+ 344.25	+ 125.55	+ 141.75	+ 506.25
Horizontal Thrust = 2195.7 lbs. 3. Total Moment due to Self-weight & concentrated loads (ft.lbs).	- 9914.25	- 3613.75	+ 2074.35	+ 7870.65	+ 9694.55	+ 9718.85	+ 9694.55	+ 7870.65	+ 2074.35	- 3613.75	- 9914.25

II. Correction for Transom shortening: a_1 = 68.5 lbs; Increase of Moment in transom = + 308.25 ft.lbs.

	0	0.11	0.21	0.31	0.41	0.51	0.61	0.71	0.81	0.91	1
1. Self-weight(ft.lbs) Horizontal Thrust = 44 lbs.	- 198.00	+ 166.50	+ 433.80	+ 652.50	+ 774.00	+ 798.30	+ 774.00	+ 652.50	+ 433.80	+ 166.50	- 198.00
3. Total Moment due to Self-weight & concentrated Loads. (ft.lbs). Horizontal Thrust = 2127.2 lbs.	- 9606.00	- 3305.50	+ 2382.60	+ 8178.90	+10002.80	+10027.10	+10002.81	+ 8178.90	+ 2382.60	- 3305.50	- 9606.00

III M_1 Smallest moment a_1 = 44 lbs. $\frac{h}{6} = \frac{1.67}{12}$ ft; $a_1 \frac{h}{6} = 44 \times \frac{1.67}{12}$ = + 6.12 ft.lb. F = 31,900 lbs.

	0	0.11	0.21	0.31	0.41	0.51	0.61	0.71	0.81	0.91	1
$\frac{M_1 + a_1 \frac{h}{6}}{F}$ ft.	- 0.006	+ 0.0054	+ 0.0137	+ 0.0206	+ 0.0244	+ 0.0252	+ 0.0244	+ 0.0206	+ 0.0137	+ 0.0054	- 0.006

IV M_2 greatest Moment a_2 = 2127.2 lb; $\frac{h}{6} = \frac{1.67}{12}$ ft; $a_2 \frac{h}{6} = 2127.2 \times \frac{1.67}{12}$ = - 296.02 ft.lbs. F = 31,900 lbs.

	0	0.11	0.21	0.31	0.41	0.51	0.61	0.71	0.81	0.91	1
$\frac{M_2 - a_2 \frac{h}{6}}{F}$ (ft.)	- 0.283	- 0.113	+ 0.0655	+ 0.247	+ 0.304	+ 0.305	+ 0.304	+ 0.[illegible]47	+ 0.0655	- 0.113	- 0.283

(a) <u>RIGHT LEG, TAKING ACCOUNT OF TRANSOM SHORTENING</u>.

Corrections: M = 308.25 ft. lbs, a = - 68.5 lbs.

Self-weight only : M = -506.25 + 308.25 = -198.00 ft.lbs.

N = + 450 lbs.

Self-weight plus concentrated loads:

M =-9914.25 + 308.25 = -9606.00 ft.lbs.

N = 6272 lbs. + 450 lbs. = 6722 lbs.

<u>SECTION PROPERTIES</u>:

~~Exx*x*)X~~

b = 10" = 0.83 ft. h = 0.83 ft; S = 100 sq.in = 0.69 sq.ft.

Section modulus = 166.7 in^3

Stresses due to external load only:

Self-weight only:

Outer-edge: $n = \frac{450}{100} \quad \frac{198 \times 12}{166.7} \quad 4.5 - 14.3 = -9.8$ lbs/sq.in.

~~Innerxedge: xn'z=z 6722/100 z +z 9606xxx12~~

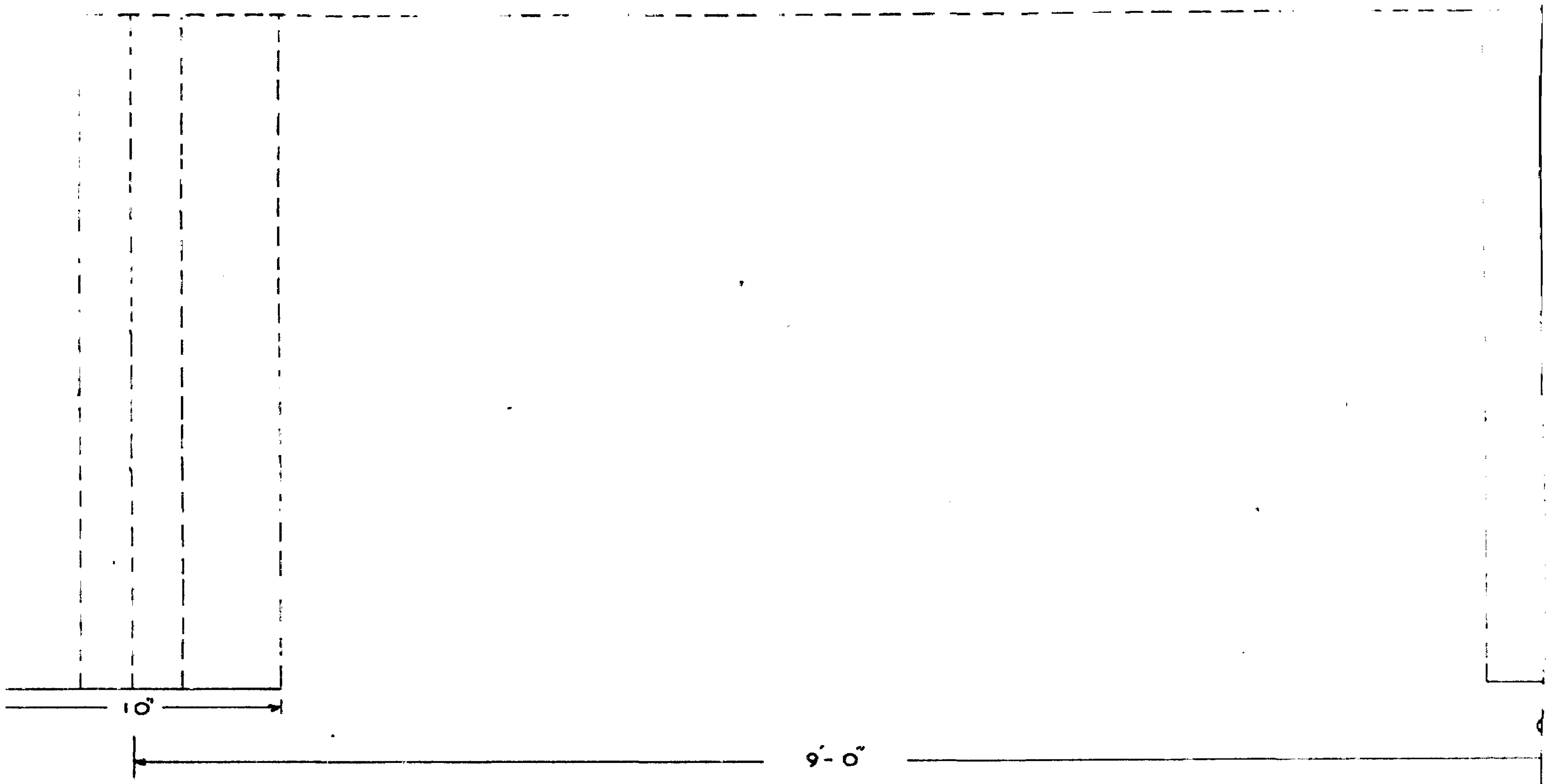
Inner edge: $n' = \frac{450}{100} + \frac{198 \times 12}{166.7} = 4.5 + 14.3 = 18.8$

Full Load:

at the outer edge $n_o = \frac{6722}{100} - \frac{9606 \times 12}{166.7} = 67.22 - 693$

$= - 625.78$ lbs./sq.in.

at the inner edge $n_o' = \frac{6722}{100} + \frac{9606 \times 12}{166.7} = 67.22 + 693 = + 760.22$



THE SCALE OF WIDTHS IS DOUBLE THE SCALE OF LENGTHS
LIMITING ZONE FOR THE LEGS
FIG. 60

By realising a prestress such that it gives:

at the ~~inner~~ outer edge n_o = <u>625.78</u> lbs/sq.in.

at the inner edge n'_o = -<u>18.8</u> lbs/sq.in.

we obtain the following total stresses:

Outer-edge self-weight only n = 615.98 lbs/sq.in.

full load $n_o = 0$

Inner edge self-weight only n' = 0

full load n'_o = 741.42 lbs/sq.in.

These stresses are permissible.

Magnitude of prestress $F'' = 100 \times \left(\frac{625.78 - 18.8}{2} \right)$

$= 100 \times 303.49$

$= 30{,}349$ lbs.

Eccentricity of prestress $e'' = \frac{h}{6} \times \frac{n_o - n'_o}{n_o + n'_o}$

$= \frac{10}{6} \times \frac{625.78 + 18.8}{625.78 - 18.8}$

$= 1.77$ in.

<u>Limiting zone for the leg</u>: we draw from the two vertical core edge lines, the boundary lines determined by the ordinates

$y_1 = \frac{M_1 + (N_1 \frac{h}{6})}{F}$ and $y_2 = \frac{M_2 - (N_2 \frac{h}{6})}{F}$ respectively.

The moments M_1 and M_2 are proportional to the abscissae Z(vertical). The boundary lines are therefore straight lines intersecting at the top of the leg in the point determined by the

~~eccentricity e'' = $\frac{M_1 h}{6F'}$ and $T_{2x} = -\frac{N_2 h}{6F'}$ for Z = M_1 = M_2 = 0~~ .

eccentricity e'' = 1.77 in. and passing through the points.

$$\mathfrak{T}_1 = + \frac{N_1 h}{6F'} \quad \text{and} \quad \mathfrak{T}_2 = - \frac{N_2 h}{6F'} \quad \text{for } Z = M_1 = M_2 = 0$$

$$\mathfrak{T}_1 = + \frac{1.67 \times 450}{30,349} = + 0.0248 \text{ in.}$$

$$\text{and } \mathfrak{T}_2 = - \frac{1.67 \times 6722}{30,349} = - 0.37 \text{ in.}$$

(b) <u>Left leg, taking into account transom shortening</u>
same as for the Right leg.

4. <u>Determination of the concordant cable profile for the transom.</u>

Straight cables passing through the centre lines of the hinges are taken. Then, F' = 30,349 lbs; u' = 1.77 in. F'u'=53500 in.lbs.

F'' = 30,349 lbs. u'' = 1.77 in. F''u'' = 53,500 in. lbs.

In the transom [illegible] = 31,900 lbs.

Equation (4) becomes $\int_0^l y\,dx = -\frac{2}{3} \times 108 \times \frac{53,500}{31,900 \times \rho}$

As $\rho = 2$

$$\int_0^l y\,dx = -\frac{2}{3} \times 108 \times \frac{53500}{31900 \times 2} = -60.5 \text{ sq. in} = -0.42 \text{ sq. ft.}$$

A curve lying entirely within the limiting zone and enclosing between itself and the neutral axis of the transom an area -60.5 sq.in. must be found out.

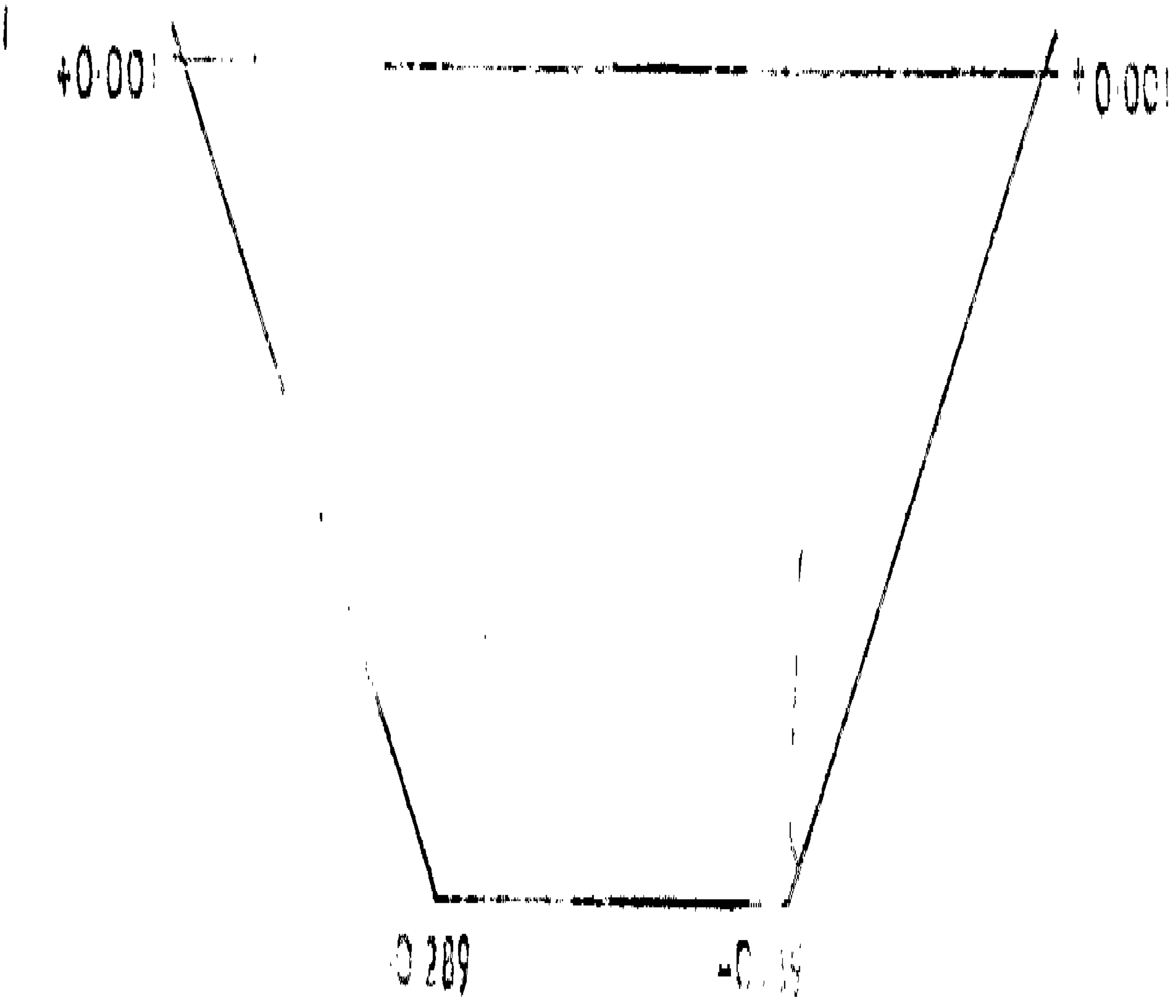
This curve can be determined by trial and error. However, the general method can be applied which consists in the determination of an imaginary load function q(x) which would give for a compression of 31,900 lbs and assuming partial restraints at the two ends of the transom, a furnicular curve lying within the neutral axis an area equal to - 60.5 sq.in*

In the present case, the curve is determined by trial and error method imposing the condition that it should have at the ends of the transom an ordinate equal to $+ \frac{h}{6} = 0.14$ ft. and the area between itself and the neutral axis of the transom an area of - 60.5 sq.in.

The adequacy of this line of pressure is checked by sketching it on the drawing of the limiting zone and by verifying that it determines with the neutral axis an area equal to 60.5 sq.in.

---

*The negative sign means that the area is below the neutral axis. The imaginary loads therefore act upwards.



TRANSF[illegible] CABLE PROFILE IN THE TRANSOM

FIG.62

5. TRANSFORMATION OF THE CABLE PROFILE:

As has already been mentioned, the transom cable may be moved up or down provided at the same time the leg cables are rotated about the centre lines of the hinges by such an amount that the apparent variations of the bending moment at the frame angle caused by these cable movements should be the same for the transom and for the legs.

This possibility is made use of to give the leg cables an approximately vertical position.

This means that the prestressing moment in the transom has to be reduced by the amount.

$$F'u' = 53{,}500 \text{ in.lbs.} = 4{,}458.3 \text{ ft.lbs.}$$

In other words, the transom cable has to be lowered by

$$a = \frac{4{,}458.3}{31{,}900} = 0.139 \text{ ft.} = 1.668 \text{ in.}$$

The final prestressing is shown in Fig. 63

Ultimate strength of the high tensile wire = 0.65 x 249,984

= 161,952 lbs/sq.in.

Stress in steel after 15% loss would be=0.85 x 161,952

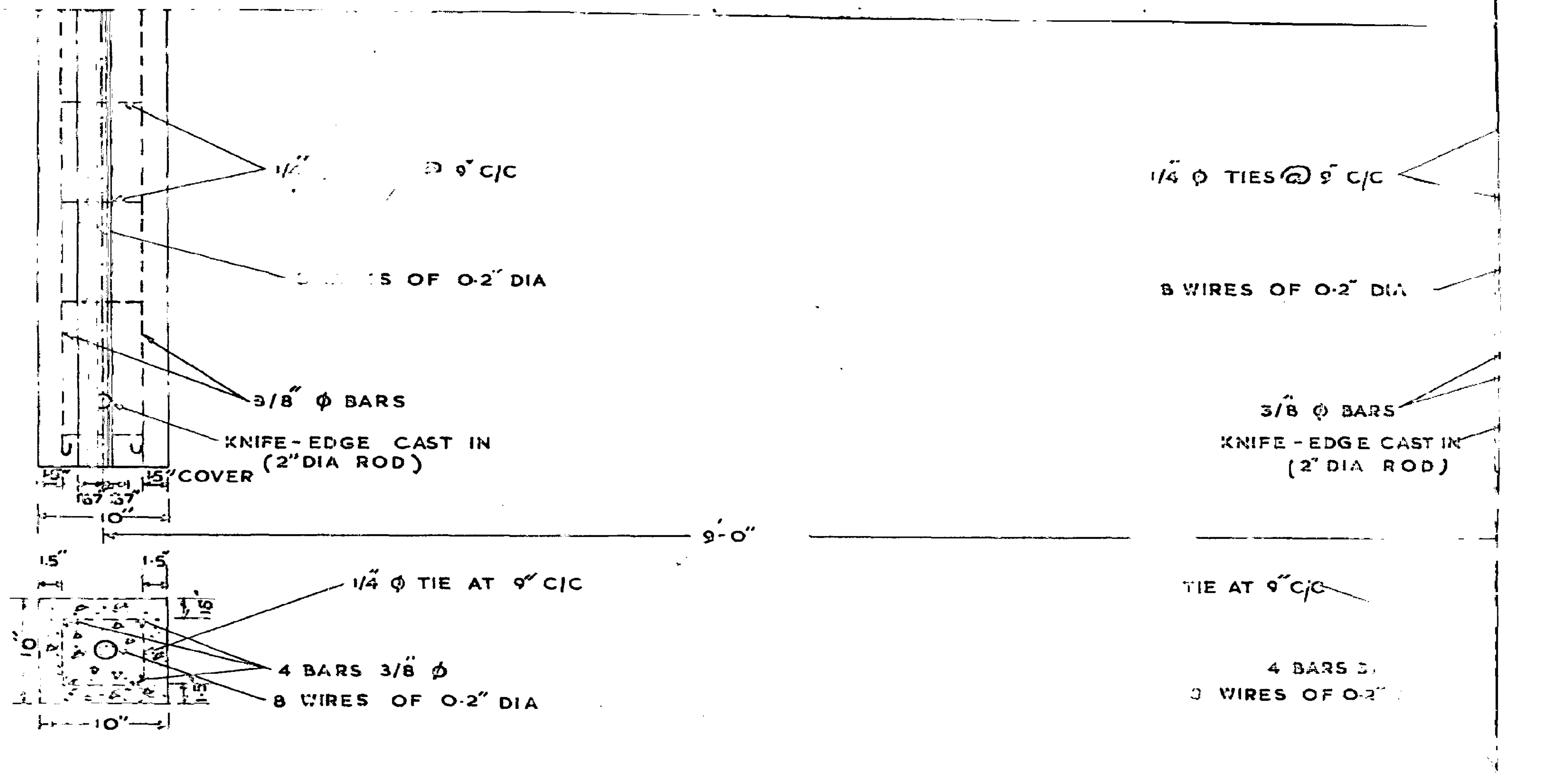
= 137,500 lbs/sq.in.

∴ Steel area required per transom = $\frac{31900}{137{,}500}$ = 0.232 Sq. in.

Adopt 8 wires of 0.2" dia. giving 0.24 sq.in.

Steel area required per leg = $\frac{30{,}349}{137{,}500}$ = 0.22 sq.in.

Adopt 8 wires of 0.2" dia. giving 0.24 sq.in.



DETAILS OF THE PRESTRESSED CONCRETE PORTAL FRAME.

FIG 63

DESIGN OF SHEAR REINFORCEMENT

Maximum shear Force at the support = 450 lbs + 2.8^{T}

= 6,722 lbs.

Shear Force taken by the cables = 31900 Sin

= 31900 x ~~0x62~~ Sin. 38^{o}

= 31900 x 0.62

= 19,778 lbs.

The maximum shear force at the support is taken core of by the vertical component of the force in the prestressing cables. Yet a nominal reinforcement of 1/4" stirrups at 6" c/c/ is used in the transom. This of 1/4" bars at 9" c/c are used in the legs. The diameter of the longitudinal bars is 3/8".

The diagram of the portal frame along with the prestressing wires and the reinforcement cage is shown clearly in the drawing.

Weight of the portal = 0.805 Tons.

APPENDIX III.

DESIGN OF THE CONCRETE MIX.

Minimum Compressive Strength = 1000 p.s.i.

Workability : Medium

Control : Very good

$$\text{Average crushing strength} = \frac{1000 \times 3}{0.75}$$

40

$$= \frac{3000 \times 100}{75} = 4{,}000 \text{ p.s.i.}$$

Note: Cube Strength should be 3 times working strength.

Water - Cement ratio : 0.55

Slump required : 3 in.

Aggregates available:

Coarse aggregate: 3/4" round gravel.

Weight of coarse aggregate : 110 lbs. per Cft.

Bulking percentage : 2.56

Fine Aggregates.	% Passing	100	sieve	1.69
	"	52	"	5.49
	"	25	"	37.46
	"	14	"	81.29
	"	7	"	97.85
	"	3/8	"	100.00

Weight of Fine Aggregate 100 lbs. per Cft.

Bulking Percentage 14.3

Determination of the weight of cement per 100 Cu.ft. of Concrete

$$\text{Weight of cement per 100 Cu.ft. of Concrete} = \frac{\text{Total quantity of water per 100 Cft. of concrete}}{\text{Water-cement ratio}}$$

$$= \frac{1149}{0.55}$$

$$= 2080 \text{ lbs.}$$

Absolute volumes of water, cement and mixed aggregates.

$$\text{Absolute Volume of water} = \frac{1149}{62.4} = 18.4$$

$$\text{Absolute Volume of cement} = \frac{2080}{3.15 \times 62.4} = 10.6$$

$$\text{Absolute volume of water and cement} = 18.4 + 10.6 = 29 \text{ Cft.}$$

$$\text{Therefore absolute volume of mixed aggregates} = 100 - 29 = 71 \text{ Cft.}$$

F.M. of fine and coarse aggregates and proportions of fine and coarse aggregates.

F.M. of coarse Aggregates = FC

SIEVE	PASSING	RETAINED
3/8"	40%	60%
3/4"	100%	0%
3/16"	0%	100%
7	0%	100%
14	0%	100%
25	0%	100%
52	0%	100%
100	0%	100%
		660%

$$FC = \frac{660}{100} = 6.6$$

F.M. of fine aggregates =

SIEVE	PASSING	RETAINED
100	1.69	98.31
52	5.49	94.51
25	37.46	62.54
14	81.29	18.71
7	97.85	2.15
3/8	100.00	0.00
		276.22

$$F = \frac{276}{100} = 2.76$$

Average F.M. of mixed aggregates = 5.05

$$\% \text{ of fine aggregate} = \frac{F_c - F_m}{F_c - F_f} \times 100$$

$$= \frac{6.6 - 5.05}{6.60 - 2.76} \times 100$$

$$= \frac{1.55}{3.84} \times 100$$

$$= 40\%$$

% of Coarse aggregate = 60.0%

Absolute volumes and weights of fine and coarse agretages:

Absolute volume of fine aggregate = 71.0 x 0.4 = 28.4

Absolute volume of coarse aggregate = 71.0 x 0.6 = 42.6

∴ Weight of fine aggregate per 100 Cu.ft. of concrete = 28.4 x 2.65 x 62.4

= 4696.2 lbs.

Weight of coarse aggregate per 100 Cu.ft. of concrete. = 42.6 x 2.55 x 62.4

= 6778.5 lbs.

∴ Nominal Mix.

$$\text{Nominal Mix} = \frac{2080}{2080} : \frac{4696.2}{2080} : \frac{6778.5}{2080}$$

$$= 1 : 2.25 : 3.26$$

APPENDIX 'IV'

CUBE CRUSHING STRENGTHS OF CONCRETE

DETAILS OF CUBE TESTS
(SIZE 4" x 4" x 4")

***

No.	Date of casting	Date of testing	Age (days).	Weight lb.	Weight Oz.	Load at Failure (Tons)	Average Tons.	Fcu p.s.i.	Portal No.
1.	28.5.59	28.9.59	120	5	8	34			
2.	"	"	"	"		32.5	32.5	4,550	I
3.	"	"	"	"		31			
4.	19.8.59	4.10.59	45	"		31			
5.	"	"	"	"		32.5	31.8	4,452	II
6.	"	"	"	"		32			
7.	1.9.59	8.10.59	37	"		31			
8.	"	"	"	"		30	30.3	4,242	III
9.	"	"	"	"		30			

APPENDIX 'V'

CYLINDER CRUSHING STRENGTHS OF CONCRETE
DETAILS OF CYLINDER TESTS (SIZE 6" X 12")

No. &	Date of casting	Date of testing	Age (days).	Weight lb.	oz.	Load at Failure (Tons)	Average (Tons)	Fcu p.s.i.	Portal No.
1.	28.5.59	28.9.59	120	29	--	46.5			
2.	"	"	"	"		47	47	3720	I
3.	"	"	"	"		47.5			
4.	19.8.59	4.10.59	45	"		47.5			
5.	"	"	"	"		46	46.8	3640	II
6.	"	"	"	"		47			
7.	1.9.59	8.10.59	37	"		47.5			
8.	"	"	"	"		47.5	47.7	3780	III
9.	"	"	"	"		48			

APPENDIX 'VI'

RESULT OF TENSILE TEST ON HIGH TENSILE STEEL WIRE 0.2" DIA..

Diameter of Rod = 0.2" (Average of 8 readings)

Area of cross section = 0.03 sq.in.

Machine used: Avery 50 Ton Universal Testing Machine.

Extensometer : Lindley's No. 1

Gauge length : 2 In.

Value of each division on dial : $\frac{1}{20,000}$" = 0.5×10^{-4} in.

Load(Tons)	Dial Reading Div.	Stress(Tons/in2)	Strain x 10-4
1	2	3	4
0.0	0	0.0	0.00
0.1	11	3.3	2.75
0;2	21	6.6	5.25
0.3	32	9.9	8.00
0.4	43	13.2	10.75
0.5	53	16.5	13.25
0.6	62	23.1	18.50
0.7	74	26.4	20.75
0.8	83	29.7	23.50
0.9	94	33.0	26.00
1.0	104	36.3	28.75
1.1	115	39.6	31.00
1.2	124	42.9	33.00
1.3	132	46.2	36.25
1.4	145	49.5	38.25
1.5	153	52.8	41.50
1.6	166	56.1	44.00

1	2	3	4
1.7	176	59.4	46.75
1.8	187	62.7	48.50
1.9	194	66.0	51.00
2.0	204	69.3	53.00
2.1	212	72.6	55.50
2.2	222	75.9	58.25
2.3	233	79.2	60.50
2.4	242	82.5	63.00
2.5	252	85.8	65.00
2.6	260	89.1	67.25
2.7	269	92.4	69.00
2.8	276	95.7	71.75
2.9	287	99.0	74.25
3.0	297	102.3	
3.1		106.6	
3.2		112.0	

Ultimate strength of the wire = 112T.S.I.

The extensometer was removed at a load of 3.0 tons

Working stress = 161,952 lb.sq.in.

$E = 30 \times 10^6$ p.s.i.

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NOTATION

b = breadth of beam

d_1 = effective depth

nd = depth to neutral axis

rnd = depth to centre of compression

R = cube strength in lbs.

A_s = Area of high tensile steel.

f_u = ultimate stress in steel

$\overline{W}$ = Weighted percentage

$$= \frac{A_s f_u}{b d_1 R}$$

F_{su} = Total ultimate Force in the steel

K = a coefficient depending on the hsape of the stress diagram

$\alpha_1 d_1$ = lever arm

ε = Strain in the concrete

ε' = Strain in the steel

ε_o = initial strain in the concrete after prestressing

ε'_o = initial strain in the concrete after prestressin_

r = radius of curvature

fs = actual stress in the steel

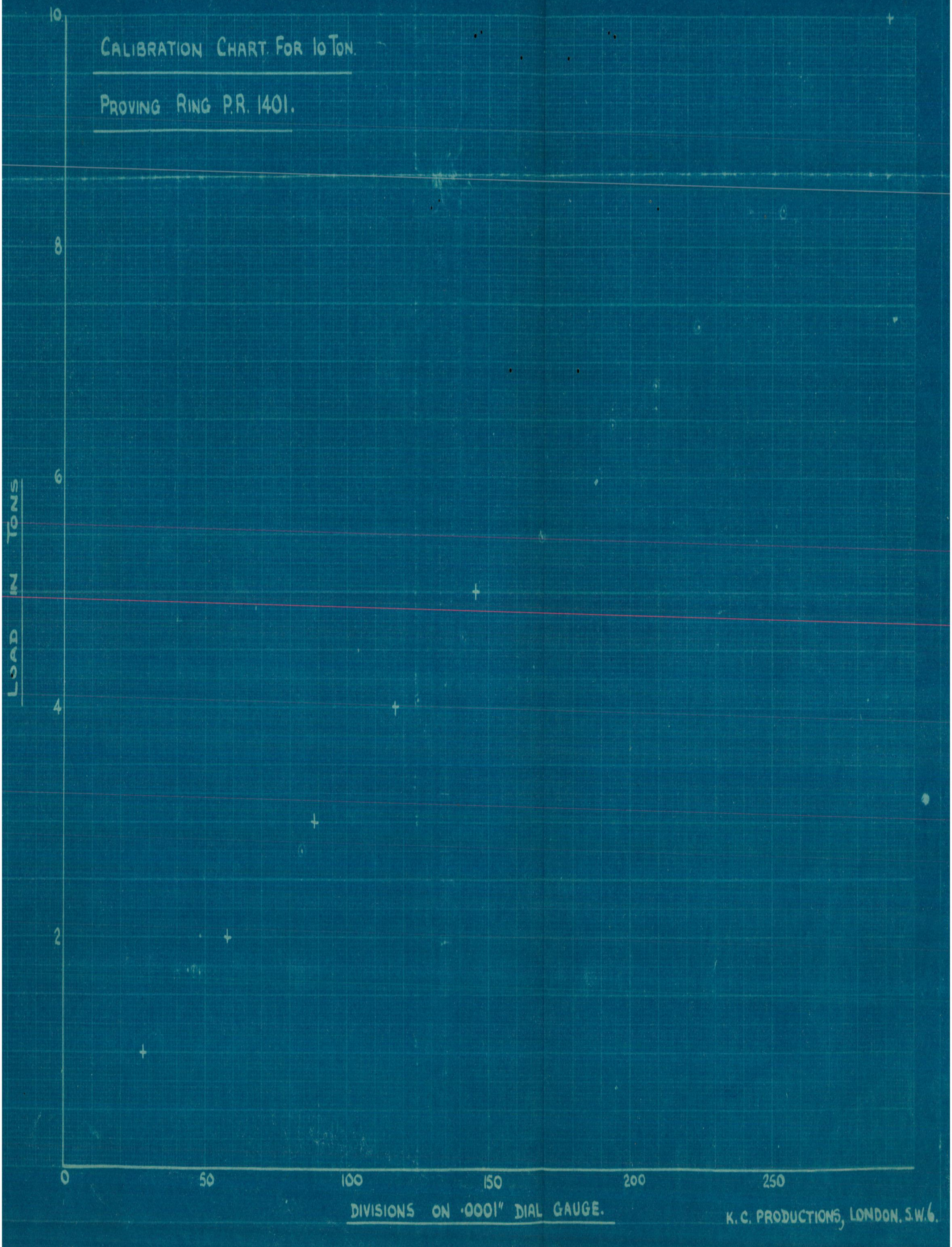
λ = tension factor $\frac{f_s}{f_u}$

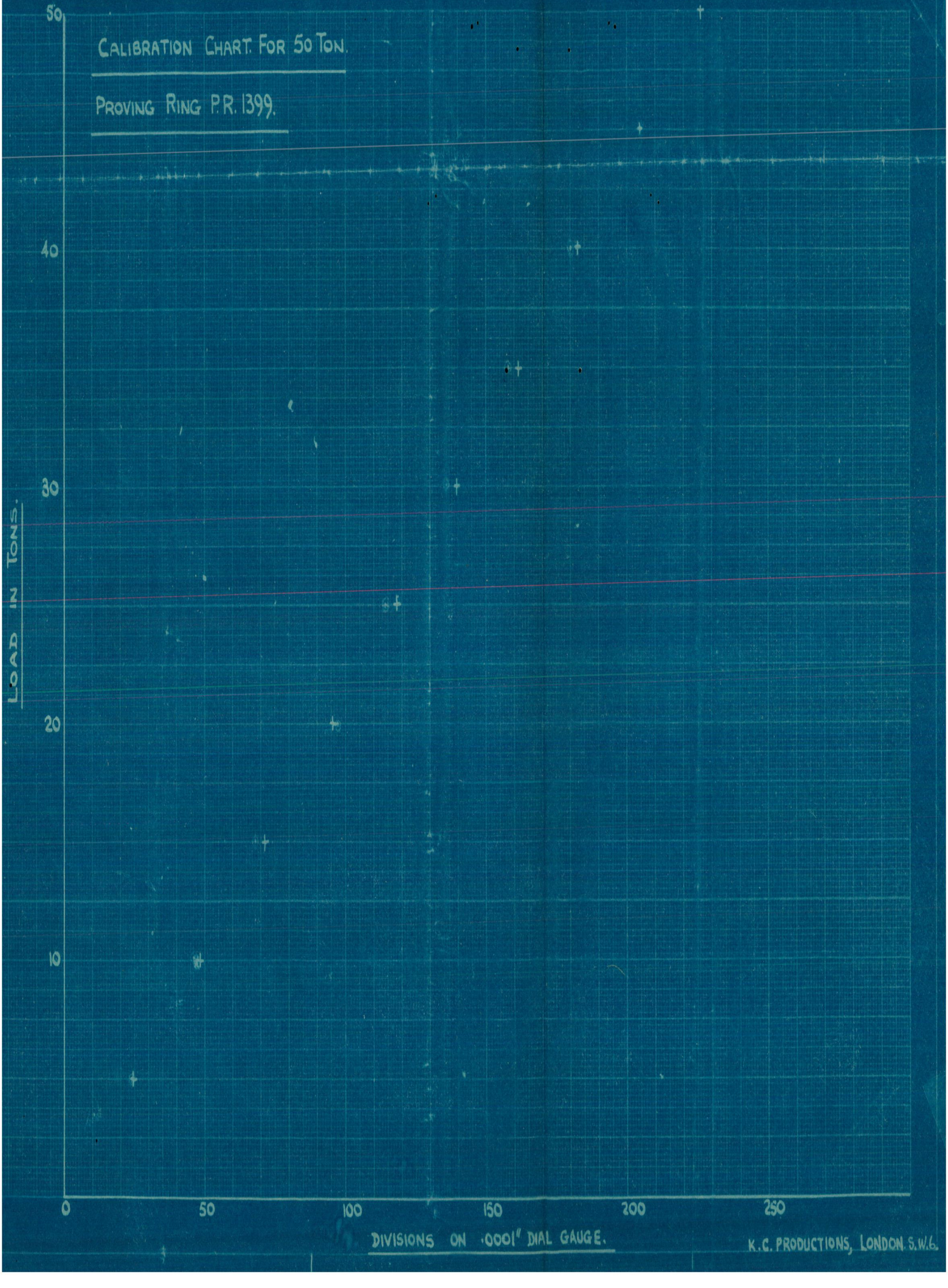
Mr = ultimate moment of resistance

l = length of the portal between the centre lines of legs

g = height of the portal frame

ρ = ratio of stiffness of leg to stiffness of transom


CALIBRATION CHART FOR 10 TON.
PROVING RING P.R. 1401.
LOAD IN TONS
0
2
4
6
8
10
50
100
150
200
250
DIVISIONS ON ·0001" DIAL GAUGE.
K.C. PRODUCTIONS, LONDON. S.W.6.


CALIBRATION CHART. FOR 50 TON.
PROVING RING P.R. 1399.
LOAD IN TONS.
50
40
30
20
10
0
50
100
150
200
250
DIVISIONS ON ·0001" DIAL GAUGE.
K.C. PRODUCTIONS, LONDON. S.W.6.