OPTIMAL SCHEDULING
IN
THERMAL GENERATING SYSTEMS

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by
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CERTIFICATE

Certified that the thesis entitled "OPTIMAL SCHEDULING IN THERMAL GENERATING SYSTEMS" which is being submitted by Mr. Jagdish Chander Kohli in fulfilment of the requirements for the degree of Doctor of Philosophy (Electrical Engineering) of the University of Roorkee, is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this thesis has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for a period of two years and ten months, from November 1972 to August, 1975 for preparing this thesis for the Doctor of Philosophy degree, at this University.

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ABSTRACT

As power system generating facilities increase in size, number and complexity, the power utility is faced with a range of decision scheduling problems. The simple rules of thumb, based on human judgement alone are no longer applicable in the solution of intricate cases. The application of mathematical programming techniques as a supplement to human judgement has aroused considerable interest among power system engineers. The present work is primarily concerned with the development of mathematical models and scheduling algorithms for a range of decision making situations arising in the daily and/or periodic functioning of a power plant or group of power plants under centralized administration. Based on the structure of the mathematical models used the work is classified into two parts. In the first part, a number of maintenance scheduling and allied problems are formulated as integer linear and nonlinear programs. In the second part, generation scheduling problems are formulated as mixed integer nonlinear and continuous variable nonlinear programs.

First of all, the problem of preventive maintenance scheduling is discussed. A 0-1 integer programming model is presented for obtaining minimum maintenance cost schedules. A set of comprehensive and interacting constraints, such as sequencing of generating units, security

considerations , resources limitation etc. are transformed into the integer programming format. The problem becomes an involved one, when a large number of units are to be maintained during the multiperiod scheduling horizon. A new, simple and efficient optimization technique is developed for the solution of the problem. The method is superior to the other known integer programming procedures as, it exploits the special properties of the model. In the overhauling of power plants, the maintenance staff is interchanged between stations at times of overhauls. A mathematical description of the problem of staff interchange scheduling is presented and solved through the 0-1 programming approach. Thus, the program makes available the required number of craftsmen of each category at the minimum cost.

Next, the problem of corrective maintenance scheduling is presented. To have built-in maintenance at the design or planning stage is referred to as the problem of corrective maintenance. A system analyst / designer is faced with the problem of designing systems having failure free operation. Such an objective is fulfilled by designing critical subsystems having a high degree of reliability. A nonlinear programming formulation of the corrective maintenance scheduling problem is presented. The analysis results in the optimal number of standby components and repair facilities to achieve a specific level of system reliability. A new scheduling algorithm is devised and the results of computation are presented for 'generator excitation system' and 'turbine cooling system'.

In the next section, the problem of maintenance budgetary control is discussed. Choosing a sound and effective maintenance policy reduces the system down-time and thus increases the revenue to the utility. The objective is aimed at selecting that set of proposals which will maximize the net present value of its total expected return. The problem is discussed under conditions of certainty and uncertainty. A mathematical version of the problem is presented and scheduling algorithms are developed for deterministic and probabilistic cases.

After the units have been scheduled for preventive maintenance on annual basis, the next problem is the selection of units out of the available set for real time operation. This is referred to as the problem of unit commitment scheduling. The total production cost to be minimized is the sum of running cost, shut down cost and time dependent start up cost. The security model incorporated provides a means for assessing system security in hour-to-hour operation on a probabilistic basis. An efficient computation procedure is developed based on the premise of feasibility and economic dispatch. Results of computation are presented to obtain a 24-hour schedule

for a medium size system drawn from the literature. In the end a continuous variable nonlinear model is presented for the real power scheduling. A linearized representation of the network is used to include the effect of transmission losses. An efficient multivariable constrained search iterative procedure is developed for the solution of coordinating equations. A scheduling algorithm is developed and results are presented for a sample system. The computation time and storage are encouragingly small. Avenues of future research in the area are discussed.

A C K N O W L E D G E M E N T S

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Jagdish Chander Kohli

LIST OF ILLUSTRATIONS

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LIST OF SYMBOLS

The following is the list of principal symbols, which is common for the text. The other symbols used at specific locations are defined separately in the text .

a_{ij}	-	constraint coefficients for the binary variables x
A_{1i}, A_{2i}, A_{3i}, A_{4i}	-	cost function constants associated with ith generating unit or bus
b_i	-	resources available for ith type constraint
C_i	-	cost coefficients for the binary variables x
De	-	number of descendants at a parent branch
ITL	-	incremental transmission loss
k	-	number of stages or subsystems in a system
ℓ_i	-	ith level of the tree diagram
L	-	number of levels in a tree
NPV	-	net present value of a set of proposals
n	-	total number of binary variables in a solution vector
n_i	-	number of components in the i th subset
P_{G_i}	-	active power generation at ith node
$Pmax_i$, $Pmin_i$	-	upper and lower limits of generation for ith unit or generating bus
P_D	-	load demand
P_L	-	transmission loss
R_i	-	reliability of ith subsystem
S_i	-	ith subset corresponding to ith generator
S	-	n component solution vector

S^T	-	a non-redundant solution set
U_i	-	unreliability of ith subsystem
x y	-	binary variables
z	-	cost function value
λ	-	failure rate
μ	-	repair rate
γ	-	incremental production cost
(δ_i+j)	-	subscript of the binary variables x for ith subset

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CHAPTER I

INTRODUCTION

STATEMENT OF THE PROBLEM

As power generating facilities increase in size, number and complexity, the power utility is faced with a range of decision scheduling problems . In today's economy technological, environmental and competitive factors interact in a complicated fashion and it becomes difficult to make up a schedule that is both realistic and economical. A broader meaning is attached to the word scheduling and it implies, the preparation of a time table, a plan, a programme or a scheme. It is a rational approach to execute decisions according to a prepared schedule, as the preparation involves, the diagnosis and detailed analysis of the problem. In the past , human judgement has been used in the preparation of schedules and this works well for tackling simple situations. In the present times, the simple rules of thumb based on judgement alone are no longer applicable in the solution of intricate cases. The application of mathematical programming techniques, as a supplement to human judgement has aroused considerable interest among power system engineers. The present work is primarily concerned with the development of mathematical models and scheduling algorithms for a range of decision making situations arising in the daily and/or periodic functioning of a power plant or a group of power plants under centralized administration.

APPROACH

A generating system is considered to be composed of a management system and the equipment. Both of these could be further classified into their subsystems , which are interconnected and interdependent in operation, concepts and objectives. The problems of importance faced by the utility are identified and posed to the system analysts . The problems are approached from a systems view point as, such an approach is inter-disciplinary and takes benefit of the experience of the designers of equipment, financial experts, administrators ; system operators and dispatchers. A prelude to a quantitative analysis of a decision scheduling problem is a thorough qualitative analysis. Thus, a realistic appraisal of the specifics of the problem is obtained by the system analyst. After obtaining a feel of the problem, a suitable mathematical model, which keeps a balance between detail and tractability is formulated. The model building involves the choice of an objective or a measure of effectiveness. In pursuit of its objective all systems operate within a set of constraints, some self imposed and some stemming from technical considerations. The time horizon for which the decisions are to remain valid is specified. The input data requirements for the model are also specified. Some of the data may be readily available and the other may be gathered from past historical records.

Now depending upon the nature of the objective function, constraints and the type of decision variables, the problems are usefully classified by a number of attributes. The problems may be , linear or nonlinear, discrete or continuous, deterministic or probabilistic. Examples of these various models are available in the work presented.

After the problem is casted into a suitable mathematical model, the next question is the selection of a particular technique of analysis. At this stage, the structure of the model could be usefully exploited in simplifying the solution procedure. The choice of a particular technique is primarily governed by the type of the model and the experience of the system analyst. Developing new, simplified and efficient algorithms is an active area of research where the system scientists can exhibit their ingenuity.

REVIEW AND SCOPE

The reliability of operation, production cost and capital expenditures on a power system are all affected by the maintenance outages of generating facilities. The statistics available for the year 1970 indicate that the

U.S. industry alone has spent 1.5 billion dollars on the preventive maintenance of thermal generting units during this year [40] . In addition to rising costs, present trends in the business are to large generating units, stronger interconnections, and greater difficulties in maintaining adequate reserves. The objective of maintenance scheduling is to evolve an overall maintenance policy considering the various facets of the problem.

First of all the problem of preventive maintenance scheduling is discussed. The task of scheduling preventive maintenance involves specifying dates at which man power is to be allocated to an overhaul of a major functional element or group of elements. The problem has been attacked by many authors [11,12,14,22,24,26,28,32,46,52,59] and useful contributions have been reported. In the early attempts [11,12,22,46,52] rigorous approaches have been rejected as impractical and adhoc computer algorithms have been developed in an attempt to do this scheduling automatically. With all their limitations, the contributions are valuable, in the sense that these have served the need of the industry in the absence of algorithms resulting in global optimal solutions. The problem of preventive maintenance scheduling has been also discussed in the literature with reference to the generation expansion schedules [28] and the unit commitment schedules [24,26]. The problem becomes an involved one, when a large number of generating units are to be scheduled for maintenance in the multi-period scheduling horizon.

The application of mathematical programming techniques to the problem of preventive maintenance is a recent trend. Gruhl [24] has advanced a mixed-integer model for the general scheduling problem, in which, maintenance scheduling is solved as a sub-problem. Zurn and Quintana [59] have presented a valuable contribution proposing "group sequential scheduling" to find a compromising or good feasible solution to the problem. A large class of scheduling problems are also formulated as 0-1 integer programs [44,45] . Dopazo and Merrill [14] have used an integer programming formulation for the preventive maintenance problem. The solution procedure uses Bala's algorithm for finding the optimal solution. In this procedure some of the variables are initially assigned values and these are called partial solutions. The completion of these partial solutions are tested and in this process some of the infeasible solutions are ignored. All solutions are generated explicitly or implicitly . Lot of book - keeping has to be done in order to keep a record of the initial position of the variables. A back-tracking procedure is used in the method to avoid redundancy.

The present work offers the 0-1 integer programming model that includes the new constraining equations.

The elements of uncertainty associated with the availability of resources are considered [33]. The model has the advantage that a diversity of constraints are easily transformed into the problem format. The mathematical model is critically analyzed and arranged in a systematic fashion. A new and efficient tree search optimization technique is developed exploiting the special structure of the model. Programming considerations for reducing the storage of binary vectors are presented. New skipping rules are evolved for eliminating many infeasible solution vectors.

Next, the problem of corrective maintenance scheduling is presented. To have built-in maintenance in the system at the design phase or planning stage is referred to as the problem of corrective maintenance scheduling. The system analyst/ designer is posed with the problem of preparing design schedules, which result in the failure free operation of the systems, as far as possible. Such an approach reduces the expenditure on maintenance during the operating life of the system and also enhances the system reliability [9, 51]. Billinton and Krasmodekski [5] have also emphasized the inclusion of reliability and maintainability analysis at the design phase. In the present work a nonlinear integer programming formulation of the corrective maintenance problem is presented. The analysis is useful in deciding power plant sub-system configuration and size [32]. The objective is aimed at maximizing the system reliability or minimizing the cost subject to the attainment of a specific level of system reliability. The analysis results in the optimal number of standby components and repair facilities. A scheduling algorithm is developed and results are presented for a 'generator excitation system' and 'turbine cooling system'.

In the next section, the problem of maintenance budget scheduling is discussed. The selection of a portfolio of proposals out of a set of alternatives involving capital expenditure is referred to as the problem of budgetary control or budget scheduling [9]. The future success of a policy, consisting of a set of proposals, depends upon the investment decisions made today. Choosing a sound and effective maintenance policy enhances the life of the equipment, reduces system down-time and thus increases the revenue to the utility. Bierman and Smidt [6] have emphasised the role of net present value, as a measure of total expected return to the utility. Mao and Wallingford [41] have solved investment decision scheduling problems using Lawler and Bell [37] method of integer programming. The method does not take advantage of the special structure of the model. In this method, the problem is arranged in a special form and this increases the value of the constraints to double. The generation of the solution is started by keeping one variable as unity and remaining variables as zero. Then tests are applied for generating the next vector. In this process some of the solutions are skipped. In going from one solution vector to the next many intermediate steps are involved and these are also time consuming. In the present work integer linear and nonlinear models are presented for the control of maintenance expenditure on thermal generating units. The objective is aimed at selecting that set of proposals for maintenance, which will maximize the net present value of its total expected return . New and efficient maximization algorithms are developed for the deterministic and probabilistic cases.

After the units have been scheduled for preventive maintenance on annual basis, the next problem is the selection of units out of the available set for real time operation. This is referred to as the problem of unit commitment scheduling. The objective function to be minimized is the total production cost, which is a summation of the running cost, shut down cost and time-dependent start up cost. The earlier practice was to start up and shut down units in accordance with a priority list based on unit heat rates [4,29] . Such an approach could impair reliability and economics of operation. The work in the area of application of mathematical techniques to the problem of unit commitment started about a decade ago and many useful contributions [1,15,23,25,26,38,39,43] have been presented. Garver [23] has advanced an integer programming formulation of the problem. Muckstadt and Wilson [39] use a mixed-integer model and employ Benders Decomposition to find a solution. Lowery [38] has recommended the use of dynamic programming and subsequently Guy [25] and Ayoub and Patton [1] have used this approach incorporating the evaluation of security into the model. A useful comparative study of some methods for the hydro-thermal generating unit commitment has been reported by Nagrath and Kothari [43] in a recent contribution. The present work takes a different approach and develops a new direct iterative procedure based on the premise of feasibility and economic dispatch. The heuristics developed limit the search in the region of interest and help to speed up the enumeration of binary vectors. A scheduling algorithm is designed and its applicacy tested on a system drawn from the literature.

In the end the problem of real power scheduling is discussed. The theory of this subject is well developed and many useful contributions [7,16,31,42,48,49,50,53,54] have appeared in the literature. In the work reported a linearized representation of the network is used to include the effect of transmission losses. An efficient multi-variable, constrained search iterative procedure is developed for the solution of coordinating equations. The limit on the line flows is included in the model. A scheduling algorithm is developed and its applicacy is tested on a sample system. The data is assumed to be available for the models presented.

ORGANIZATION OF THE THESIS

The work is classified into four chapters in addition to the first and the last chapters, which contain the introduction and conclusion of the subject. A chart showing the type of the problem tackled and the nature of the mathematical model used is displayed in Fig. 1. In the second chapter, the preventive maintenance scheduling of power plants is discussed. An efficient scheduling algorithm is developed for the solution of the problem. In the third chapter of the thesis, the problem of corrective maintenance scheduling is presented. The applicacy of the analysis is demonstrated for a 'generator excitation system' and 'turbine cooling system' [10] . In the fourth chapter, the problem of maintenance budget scheduling is presented. Two

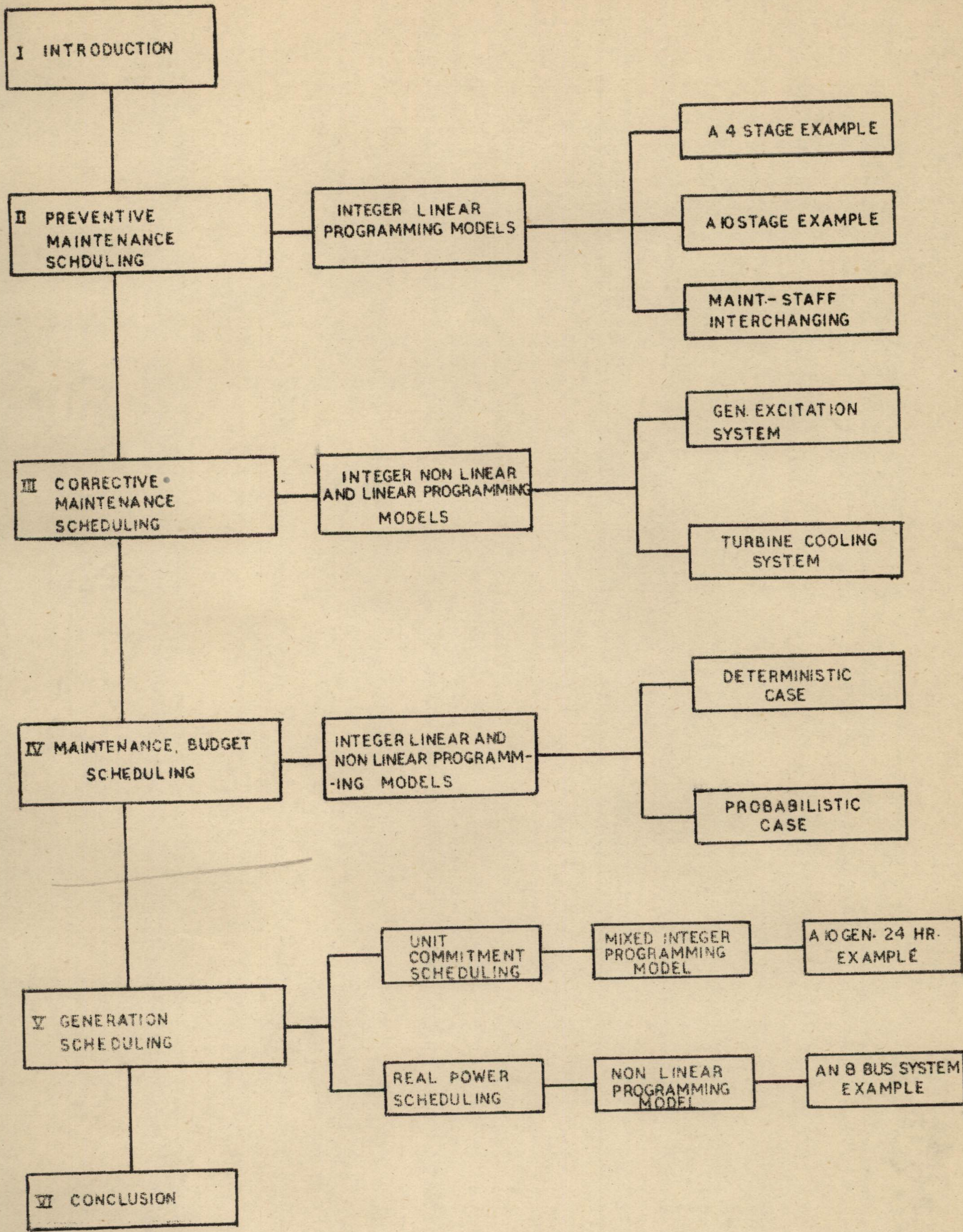

FIG.1 ORGANIZATION OF THE THESIS

maximization algorithms are developed for the deterministic and probabilistic cases. In the fifth chapter the generation scheduling problems are discussed. Simplified and efficient algorithms are devised for the solution of the problems. In the last chapter, the conclusions regarding the contributions made by the author are drawn. Some suggestions, for further investigations in this field which might lead to some interesting results, have been included in the concluding discussion.

CHAPTER II

PREVENTIVE MAINTENANCE SCHEDULING

The task of scheduling preventive (that is routine or planned) maintenance involves specifying dates at which manpower is to be allocated to an overhaul of a major functional element or a group of elements. The scheduling interval between two successive maintenance events is decided based on the type and the state of the unit to be maintained. As the number and complexity of the units increase, manual scheduling becomes both difficult and tedious. A detailed description of the present day practice for the scheduling of maintenance for the fossil fuel generating facilities is available [9].

The problem of preventive maintenance has been attacked by many authors . In the early attempts [11,12, 22,46,52] rigorous approaches have been rejected as impractical and adhoc computer algorithms have been developed in an attempt to do this scheduling automatically. The three serious drawbacks of these methods are :

(i) they may fail to find a schedule satisfying the problem constraints, even when one does exist,

(ii) while they implicitly incorporate a criterion of goodness, they do not always find the best schedule in terms of this criterion; and

(iii) the criterion of goodness is limited to either equalizing net reserve or equalizing an approximation to "Loss of Load Probability".

The application of mathematical programming techniques to the maintenance scheduling is a recent trend. Gruhl [24] has advanced a mixed integer model for the general scheduling problem in which , maintenance scheduling problem of thermal generating units is solved as a sub-problem. However, this algorithm is suboptimal and may fail to find a schedule satisfying the constraints , even when one exists [14] . Recently Zurn and Quintana [59] have proposed "Group sequential scheduling" to find a compromising or good feasible solution to the problem. The grouping criterion is the same as used by Hara et al [26] . A large class of scheduling problems are also formulated as 0-1 integer programs [44,45] . Dopazo and Merrill [14] have used an integer programming model in formulating the preventive maintenance scheduling problem . The solution procedure uses Bala's additive algorithm [2]. The method is not an efficient one and the programming is involved. The present work offers the 0-1 integer programming model that includes new constraining equations. There is always an element of uncertainty associated with the availability of resources specially, during the later intervals of the scheduling horizon. Such a requirement is modelled with the help of chance constraints. Some of the old units requiring maintenance more than once during the scheduling horizon are also modelled. These constraints have not been considered in the earlier formulation [14]. The model has the advantage that a diversity of constraints associated with the problem are easily transformed into the 0-1 format.

2.1 MODELLING

The problem of preventive maintenance of fossil fuel generating units involves the determination of the periods or intervals during which the overhauling is to be done on each of its units in a multiperiod scheduling horizon. The cost of preventive maintenance is to be minimized subject to the satisfaction of a set of interacting and comprehensive constraints.

The maintenance scheduling problem is set up as a 0-1 integer program, whose general form is

$$\text{Minimize : } z = \sum_{j=1}^{n} c_j x_j \qquad (2.1)$$

Subject to:

$$\sum_{j=1}^{n} a_{ij}\, x_j \leqslant b_i \qquad (2.2)$$

$$(i = 1,2,\ldots, m)$$

$$x_j = 0 \text{ or } 1 \qquad (2.3)$$

where, c_j are the cost coefficients and a_{ij} are the constraint coefficients. b_i are the limits on the m constraints. A variable x_j is unity when the maintenance starts on a generating unit in a particular interval and is zero if the maintenance does not start in that interval. In accordance with the generally accepted terminology, a verctor $S = (x_1, x_2, \ldots, x_n)$ is a solution to the problem. Moreover, if the constraints (2.2) and (2.3) hold, then it is a feasible solution. A feasible n-vector S^* is optimal if and only if, the corresponding objective function value $z^* \leqslant \text{Min } z_f$; where z_f corresponds to all feasible z values.

VARIABLES

Each x_j is associated with beginning maintenance on some unit G_i during some interval k . Maintenance on unit G_i is scheduled to begin during week k if and only if the corresponding $x_j = 1$. For each problem, tables relating j,i,k are developed. For further explanation and ease of exposition, Table 2.1 is presented for a 4- unit example. The variables x_j are assigned to each unit for the start of maintenance in the allowed intervals.

TABLE 2.1

Unknowns Associated with a 4-unit Example

Unit (i)	Capacity MW	Allowed period weeks	Outage duration weeks	Associated variables (j)	Maintenance start in week (k)
G_1	80	1-4	2	x_1 x_2 x_3	1 2 3
G_2	70	2-4	1	x_4 x_5 x_6	2 3 4
G_3	50	2-4	2	x_7 x_8	2 3
G_4	110	1-3	1	x_9 x_{10} x_{11}	1 2 3

COST FUNCTION

Depending upon the choice of the system analyst and subject to the availability of appropriate data, the objective of preventive maintenance could be to optimize any of an important class of useful criteria . Some of the important cost functions are : Minimum Lateness Penalty Schedule, Minimum change from existing schedule and the Minimum Cost Schedule.

TABLE 2.2

'Minimum Lateness Penalty Schedule' Cost Function

$$c_j = [0\ 1\ 2 \mid 0\ 1\ 2 \mid 0\ 1 \mid 0\ 1\ 2]$$

Table 2.2 shows a possible cost function for the 4 unit system of Table 2.1. For each generating unit there is a penalty of 0 associated with beginning maintenance as early as possible, during the first allowed interval. There is a cost of 1 imposed for beginning maintenance in the second allowed week and cost of 2 for third allowed week. The schedule that minimizes, this cost function is the "Minimum Lateness Penalty" maintenance schedule for the system. Such penalty factors are similar to the one used in the general problem of machine scheduling [58]. An example of "Minimum changes" cost function is demonstrated for the following case. Suppose, an optimal maintenance schedule has been obtained for a particular period and plans have been made with plant personnel and parts suppliers to implement it. It then turns out that a unit that was not originally on the list to be maintained, needs to be scheduled for outage during this period. This has to be done with the minimum possible disruption of the existing schedule. Minimizing the cost function shown in Table 2.3, provides the least disruptive changes from the existing schedule [14]. The minimum disruptive changes are obtained by keeping the cost coefficients for the new unit as zero.

TABLE 2.3

'Minimum Changes' Cost Function

	Present Schedule ↓	
c_j = - - - 2 3	0 1 2 - - -	0 0 0 0
	old unit	New Unit

Particular attention is directed to a new criterion [14] incorporating Rupees costs / benefits incurred by delaying or advanging maintenance on a unit. Performing maintenance is viewed as a capital investment that is expended over a 12 months period : for the maintenance expenditure one purchases 12 months of operation of the unit. Fig. 2.1a shows that there is a cost associated with maintaining a unit too early.

If the maintenance is delayed too long, the expected maintenance cost will rise dramatically. Such a behaviour is displayed in Fig. 2.1b. The increased cost is due to the accelerated deterioration of neglected machinery, that makes maintenance, when it is performed, more expensive. There is also an expected cost associated with rapid increase in the likelihood of forced outage.

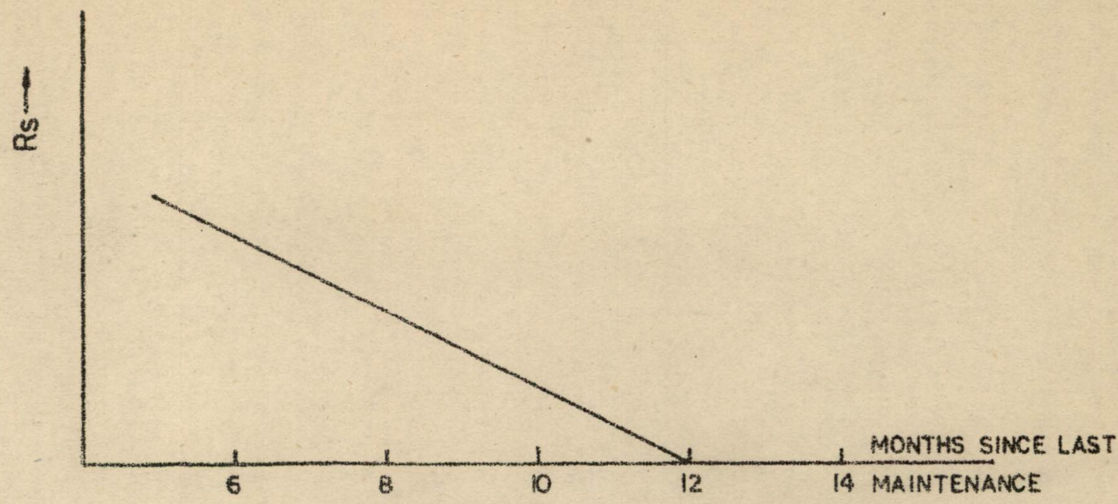

FIG. 2.1(a) MAINTENANCE INVESTMENT LOST BY MAINTAINING A UNIT TOO EARLY

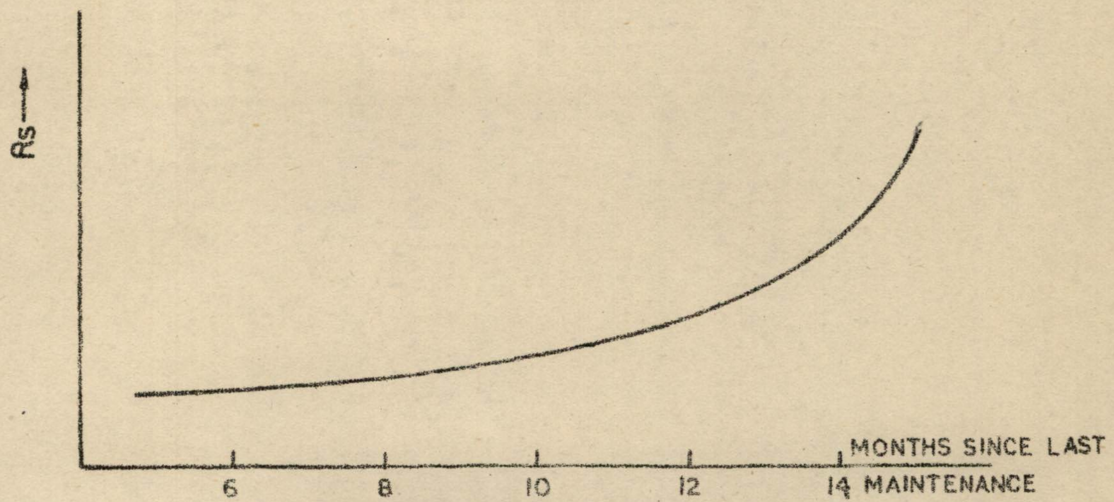

FIG. 2.1 (b) EXPECTED OUT-OF-POCKET COST IN TERMS OF TIME SINCE PREVIOUS MAINTENANCE

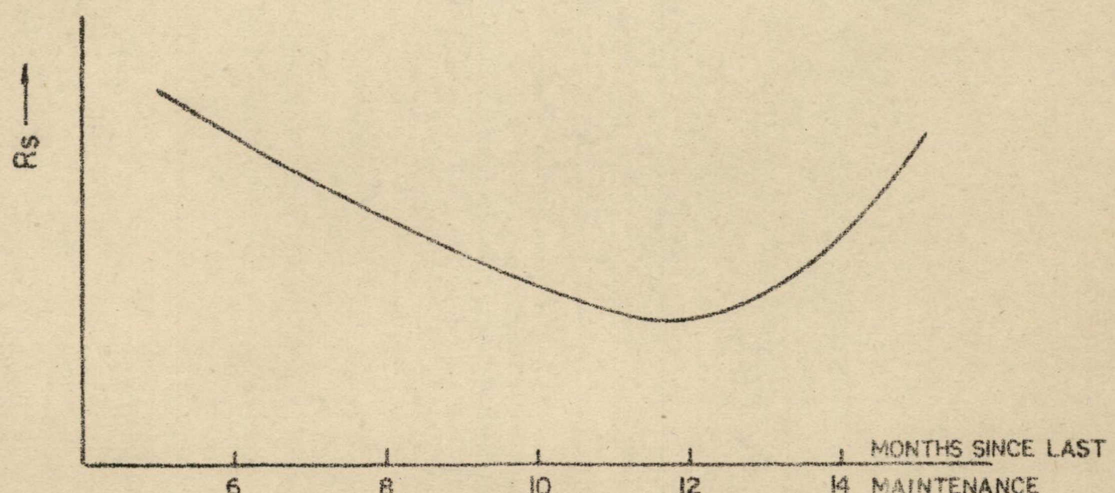

FIG. 2.1(c) TOTAL MAINTENANCE COST IN TERMS OF TIME SINCE PREVIOUS MAINTENANCE

Fig. 2.1c is the sum of the costs of Fig.2.1a and Fig. 2.1b. The optimum time to begin maintenance on this unit is available from the figure. Cost functions similar to Fig. 2.1c have been developed [14] . In the absence of constraints each unit is maintained at its individual optimum time. The presence of a set of constraints complicates the issue and the need for employing a mathematical programming technique arises.

CONSTRAINTS

Some of the important constraints are very neatly embedded implicitly in the model and need not be expressed explicitly as (2.2). These constraints are : that each unit must be maintained exactly once, that the maintenance for each unit must occupy the required time duration, without interruption, in a specified allowed time period.

A required precedence constraint is expressed in words as "unit m must be taken down exactly k weeks after unit j comes back on line".

A resource constraint is a limit on the resources (megawatt, manpower) available for maintenance at any given time.

Because of security considerations, it may not be possible to execute simultaneous maintenance on some of the units. This is expressed by an exclusion constraint in words as "No more than one of the units i,j,k,..

be maintained simultaneously.

Some of the old units, if, required to be maintained twice during the scheduling horizon can be easily included in the problem format. Such a requirement is modelled by replacing the single unit by two equivalent units of the same capacity and the maintenance on these two units is separated by a fixed time horizon.

TABLE 2.4

Constraints Description for the 4 Unit Example

Constraint No.	a_{ij}											
	Unit G_1			Unit G_2			Unit G_3		Unit G_4			b_i
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	
											c	
1	0	0	0	0	0	0	-1	0	1	0	0	0
2	0	0	0	0	0	0	0	-1	0	1	0	0
3c	0	0	0	0	0	0	0	0	0	0	1	0
4	80	0	0	0	0	0	0	0	110	0	0	150
5	80	80	0	70	0	0	50	0	0	110	0	170
6	0	80	80	0	70	0	50	50	0	0	110	180
7	0	0	80	0	0	70	0	50	0	0	0	120
8	1	0	0	0	0	0	0	0	1	0	0	1
9	1	1	0	0	0	0	0	0	0	1	0	1
10c	0	1	1	0	0	0	0	0	0	0	1	1
Starting week	1	2	3	2	3	4	2	3	1	2	3	-
Cost coeff.	1	2	3	1	2	4	1	2	2	3	6	-

A description of precedence constraint, resource constraint and exclusion constraint is obtainable from Table 2.4 for the 4 unit example presented in Table 2.1. Constraints 1,2,3 correspond to the sequence of maintenance on units G_4 and G_3. These are referred to as the precedence constraints. Constraints 4,5,6,7 represent the limit on the resources. Constraints 8,9 and 10 are for preventing simultaneous maintenance on the units G_1 and G_4. These are also called the exclusion constraints.

CHANCE CONSTRAINTS

The uncertainty associated with the availability of resources is modelled with the help of chance constrained programming [58]. The constraints (2.2) are divided into a set of deterministic and a set of probabilistic constraints as follows :

$$\sum_{j=1}^{n} a_{ij} x_j \leqslant b_i \quad (i=1,2,\ldots,g) \tag{2.4}$$

and

$$P\left[\sum_{j=1}^{n} a_{ij} x_j \leqslant b_i\right] \geqslant \beta_i \quad (i = g+1,\ldots,m) \tag{2.5}$$

Eq. (2.5) is interpreted as constraining the unconditional probability to be no smaller than β_i, where $0 \leqslant \beta_i \leqslant 1$, that the actual value for b_i is at least as large as $\sum_{j=1}^{n} a_{ij} x_j$.

It is possible [13] to transform probabilistic constraints (2.5) into deterministic equivalent constraints (2.6) as follows :

$$\sum_{j=1}^{n} a_{ij} x_j \leqslant B_i \quad \text{for } i = g+1, \ldots, m \qquad (2.6)$$

where B_i is the largest number satisfying

$$P\left[b_i < B_i\right] \leqslant 1 - \beta_i \qquad (2.7)$$

A numerical example is given to explain as to how to determine B_i from a marginal distribution. Suppose the marginal distribution for b_1 is $P\left[b_1 = 10\right] = 0.2$; $P\left[b_1 = 30\right] = 0.4$; $P\left[b_1 = 80\right] = 0.3$, $P\left[b_1 = 100\right] = 0.1$ giving the graph of $P\left[b_1 \geqslant B_1\right]$ shown in Fig. 2.2.

Thus if $0.8 \leqslant \beta_1 \leqslant 1.0$ then $B_1 = 10$

and $0.4 \leqslant \beta_1 \leqslant 0.8$ then $B_1 = 30$ etc.

The chance constrained model has two desirable properties. First, it leads to an equivalent linear program that has the same size and structure as a deterministic version of the model. Consequently, the computational burden of the stochastic version is no greater after the proper right hand side values have been determined. Second, the only information required about each uncertain element b_i is the $(1-\beta_i)$ fractile for the unconditional distribution of the right-hand-side coefficient.

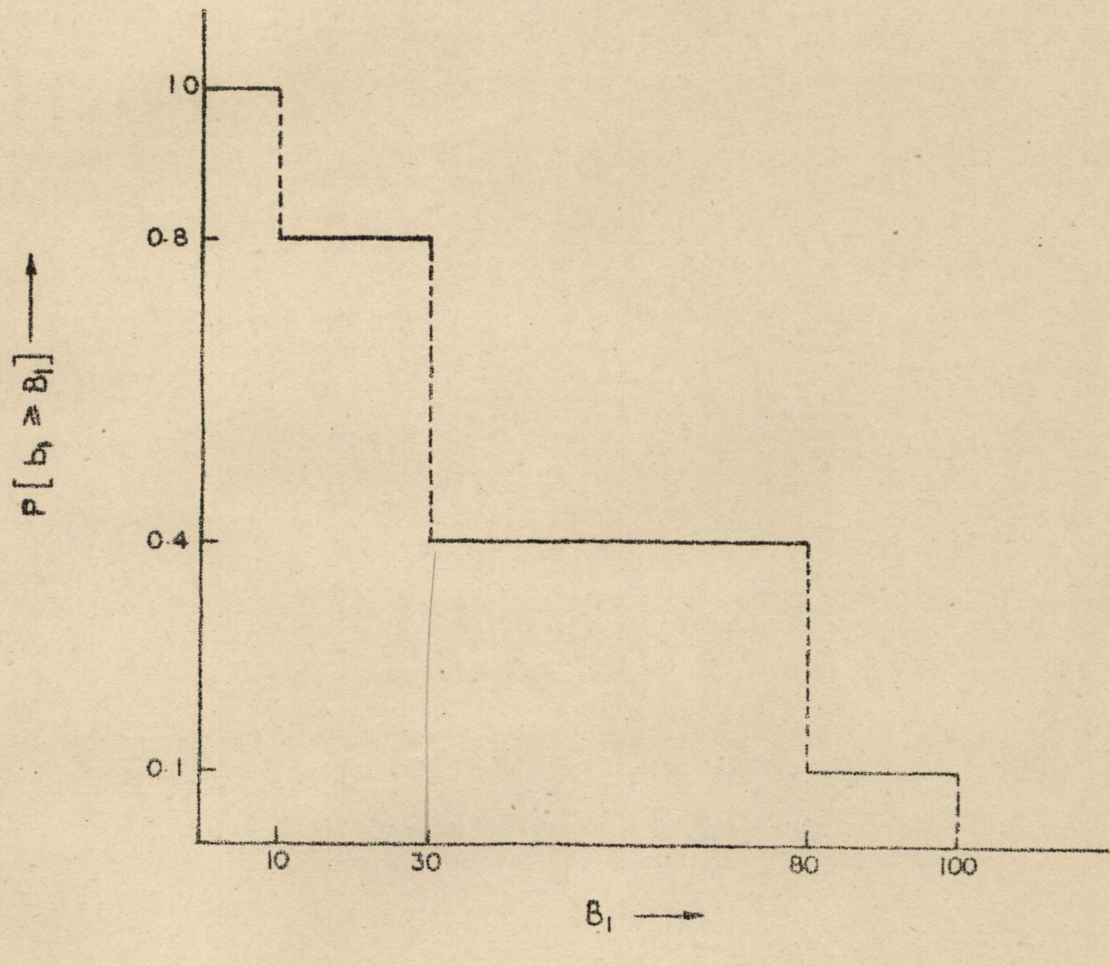

FIG.2.2 MARGINAL PROBABILITY DISTRIBUTION FOR b_1

2.1.1 PREFILTERING ANALYSIS

Prefiltering analysis is important mainly from a dimensionality standpoint. By studying the properties of the constraining equations, simple test rules are evolved. The test rules are applied at the pre-computational stage and obviously infeasible variables and irrelevant constraints are filtered out. It is also advantageous to aggregate the sets of constraints with integer coefficients into equivalent single constraints [21] . This helps in reducing the dimensionality of a large size problem with many constraints.

CANCELLING VARIABLES

A variable x_j is cancelled when $x_j = 1$ cannot be part of an optimal feasible completion of any solution S. When x_j is cancelled it is considered to be fixed at $x_j = 0$ and is removed from the set of variables. If for some variable x_m corresponding to some unit G_c , the following inequality (2.8) holds, then that variable is cancelled.

$$\sum_{\substack{v = 1 \text{ to } k \\ v \neq c}} \inf (a_j(S^v)) + a_m(S^c) \not\leq b_i \qquad (2.8)$$

where c is the Generator, whose corresponding variable x_m is under test. S^v corresponds to the vth generator.

$\inf (a_j(S^v))$ is the smallest element of ith constraint corresponding to vth unit

k are the total number of units.

Applying the above test to the constraint set of Table 2.4 , the variable x_{11} stands cancelled. Since x_{11} can never be unity in the enumerated solutions, therefore, it is removed from the set of variables.

CANCELLING CONSTRAINTS

Sometimes, some of the constraints, always remain satisfied, whatever, the values of the variables may be . Such constraints should be eliminated or ignored in order to enhance the efficiency of the solution procedure. The following test (2.9) helps in cancelling such constraints.

$$\sum_{v=1 \text{ to } k} \sup (a_j(S^v)) \leqslant b_i \qquad (2.9)$$

where

$\sup (a_j(S^v))$ is the largest element of constraint i corresponding to the vth unit.

Applying the above test to the constraint set of Table 2.4, the constraints 3 and 10 stand ignored. Since, these constraints always remain satisfied, therefore, these are not considered while solving the problem.

AGGREGATING CONSTRAINTS

In many practical systems, the number of constraints may be very large. Thus, the checking for feasibility is very time consuming. There is a synergetic effect associated with the set of constraints taken as a whole or in certain groups. Geffrion [19,20] has recommended the development of additional 'surrogate constraints', which are linear combinations of original constraints. The new constraints are obtained from the linear programming subroutine embedded in the algorithm. In the maintenance scheduling problem most of the constraints are with integer coefficients. Even if this is not so for some of the constraints, it can always be achieved by simple manipulations. Glover and Woolsey [21] have developed a procedure for aggregating the constraints with integer coefficients. Thus the multi-constraint problem can be converted into a single constraint problem by the procedure detailed below :

By adding the slack variables x_{n+i} $(i=1,2,\ldots,m)$ the inequalities (2.2) are transformed into the equalities, as

$$\sum_{j=1}^{n} a_{ij} x_j + x_{n+i} = b_i \qquad (2.10)$$

$$i = 1,2,\ldots,m$$

Consider first two constraints, i.e., $i = 1,2$. Therefore,

$$\sum_{j=1}^{n} a_{1j}x_j + x_{n+1} = b_1 \qquad (2.11)$$

and

$$\sum_{j=1}^{n} a_{2j}\, x_j + x_{n+2} = b_2 \qquad (2.12)$$

Equalities (2.11) and (2.12) can be combined to form a new constraint by using multipliers t_1 and t_2 respectively satisfying the following conditions [21].

1. t_1 and t_2 are relatively prime
2. t_1 does not divide b_2 and t_2 does not divide b_1
3. $t_1 > b_2 - a_2$ and $t_2 > b_1 - a_1$ where a_i represents the smallest of the positive a_{ij}.

The equivalent constraint which has the same solution as constraints (2.11) and (2.12) is written as :

$$t_1\left(\sum_{j=1}^{n} a_{1j}\, x_j + x_{n+1}\right) + t_2\left(\sum_{j=1}^{n} a_{2j}\, x_j + x_{n+2}\right)$$

$$= t_1b_1 + t_2b_2 \qquad (2.13)$$

Recursively using the construction (2.13) for the constraint set (2.2), the single equivalent constraint obtained is :

$$\sum_{j=1}^{n} d_jx_j \leqslant D \qquad (2.14)$$

where d_j are the new constraint coefficients and D is the right-hand-side value.

By employing the above procedure, the constraints 8 and 9 of Table 2.4 are combined to form the single equivalent constraint as given by (2.15a).

$$4x_1 + 3x_2 + x_9 + 3x_{10} \leqslant 4 \qquad (2.15a)$$

Similarly, the single equivalent constraint for the sequence of units G_4 and G_3 is given by 2.15(b)

$$-x_7 - 3x_8 + x_9 + 3x_{10} = 0 \qquad (2.15b)$$

To the knowledge of the author, this procedure of aggregating the constraints has not been discussed elsewhere in the power system engineering literature. Thus, the prefiltering analysis helps in reducing the storage and computational burden of the scheduling algorithm.

2.2 OPTIMIZATION TECHNIQUE

The problem, whose solution is to be obtained is stated in (2.1), (2.2) and (2.3). Dopazo and Merrill [14] have used Balas implicit enumeration algorithm [2]. The method is not computationally efficient, the storage is large for practical size systems and the programming is involved. In the present work, a new strategic tree search method is developed for the solution of the problem. The other important existing tree for the 0-1 programming problems is the Balas tree [2], where the number of branches are very large.

Balinski [3] rightly pointed out "that various clever methods of enumeration and other specialized approaches are the most efficacious means existent by which to obtain solutions to practical problems". His belief is strengthened by the development reported below:

The problem to be tackled is a multi-variable, multistage one. As a first step, systematization is introduced by rearranging the variables of the cost function to be minimized. The cost function coefficients are arranged in a monotonic increasing sequence for each stage of the problem. To the knowledge of the author, such systematic modelling has not been done earlier. The intutive reasoning for such a systematic arrangement is that in a minimization problem, all the low cost solutions are generated first thus avoiding the search for an optimal solution over the whole solution space. Next, it can be observed that the model of the problem has a specialized structure. It is signified by the constraints (2.16) which form a part of the constraint set (2.2). That is,

$$\sum_{i \in k} x\,(S_i) = 1 \qquad (2.16)$$

Thus, k of the m constraints are absorbed in the solution procedure and only (m-k) constraints remain associated with the problem. Stated in words (2.16) says that the summation of all the variables corresponding to each stage is unity. Thus in any enumerated solution vector only one variable for each stage will be unity and all the remaining zero. Therefore, taking advantage of this property of the model, a large number of imfeasible solutions are never generated. This results in an efficient search process. Using the above mentioned simple, but powerful ideas, the new optimization technique comes out to be superior to the existing methods [2,19].

2.2.1 THEORY

Any solution vector S is composed of k independent subsets (or subvectors), where k are the number of stages in the problem. Each subset S_i further consists of n_i components or variables, where the index i varies from 1 to k. The following notation is used in developing the theory of the method.

$$S_1 = (x_{\delta_1+1}, x_{\delta_1+2}, \cdots, x_{\delta_1+n_1})$$

$$S_2 = (x_{\delta_2+1}, x_{\delta_2+2}, \cdots, x_{\delta_2+n_2})$$

$$\vdots$$

$$S_i = (x_{\delta_i+1}, x_{\delta_i+2}, \cdots, x_{\delta_i+n_i})$$

$$\vdots$$

$$S_k = (x_{\delta_k+1}, x_{\delta_k+2}, \cdots, x_{\delta_k+n_k}) \qquad (2.17)$$

where

$$\delta_i = \delta_{i-1} + n_{i-1}$$

$$i = 2,3,\ldots, k \qquad (2.18)$$

and $\delta_1 = 0$. , initial.

The subscript of any variable x gives the number of the variable in any solution vector S. n_i are the number of variables in the ith subset. Therefore, the equality (2.19) holds

$$\sum_{i=1}^{k} n_i = n \qquad (2.19)$$

where , n are the total number of variables in any solution vector S. The subscript of δ , gives the subset to which a particular variable belongs.

The following properties of the subsets and sets are useful in depicting their characteristic behaviour. The properties hold true for the model presented.

1. The subsets of any solution vector S are mutually independent and the union of these subsets results in the solution vector S.

$$\therefore \; S = S_1 \cup S_2 \cup S_3 \ldots\ldots \cup S_k$$

or $$S = \bigcup_{i \in k} S_i \qquad (2.20)$$

2. All the subsets $S_i (i=1,2,\ldots,k)$ belong to the set S.

$$\therefore \; S_i \subset S \quad (i= 1,2,\ldots,k). \qquad (2.21)$$

3. The subsets S_i and the set S are never empty.

$$\therefore \; S_i \neq \emptyset$$

$$S \neq \emptyset \qquad (2.22)$$

GENERATION OF A NON-REDUNDANT SOLUTION SET

In an n variable problem, there are 2^n possible solutions, which are to be searched implicitly or explicitly for finding an optimal solution to the problem(2.1)- (2.3). Many of the enumerated solutions are infeasible. If the constraint set (2.16) is kept satisfied, then many of the infeasible solutions need not be generated at all. Thus, the search procedure should be in a position to generate the remaining set of non-redundant solutions.

Now, if there are n_i components in the ith subset, then, these correspond to n_i locations which are to be occupied by σ_i objects. Then the number of arrangements of σ_i objects at n_i locations, without repetition are termed as permutations and symbolized by $A_{n_i}^{\sigma_i}$.

Where,

$$A_{n_i}^{\sigma_i} = \frac{\lfloor n_i}{\lfloor n_i - \sigma_i} \qquad (2.23)$$

Thus for k subsets, the number of possible solutions S^T is given by expression (2.24).

$$S^T = A_{n_1}^{\sigma_1} \times A_{n_2}^{\sigma_2} \times A_{n_3}^{\sigma_3} \ldots A_{n_k}^{\sigma_k}$$

$$= \frac{\lfloor n_1}{\lfloor n_1 - \sigma_1} \times \frac{\lfloor n_2}{\lfloor n_2 - \sigma_2} \times \frac{\lfloor n_3}{\lfloor n_3 - \sigma_3} \ldots \times \frac{\lfloor n_k}{\lfloor n_k - \sigma_k} \tag{2.24}$$

Now, for the specific case at hand, there are n_i locations in the ith subset, over which the unity element has to move.

Therefore,

$$\sigma_1 = \sigma_2 = \ldots = \sigma_i = \ldots = \sigma_k = 1 \tag{2.25}$$

and

$$S^T = \frac{\lfloor n_1}{\lfloor n_1 - 1} \times \frac{\lfloor n_2}{\lfloor n_2 - 1} \times \ldots \times \frac{\lfloor n_k}{\lfloor n_k - 1}$$

$$= n_1 \times n_2 \times \ldots \times n_k$$

or

$$S^T = \prod_{i=1}^{k} n_i \tag{2.26}$$

Thus, the non-redundant (no arrangement is repeated) solution set given by (2.26), is the product of the number of variables belonging to all the subsets. For this specialized problem, this is much smaller than 2^n solutions in a general case.

DEVELOPMENT OF THE TREE

It is desirable that the S^T solutions for any n variable problem be generated systematically and also efficiently. After lot of experimentation and intutive thinking, a new tree is developed. The enumerated solutions are represented at the branches of the tree. In order to initiate the generation of solutions, the initial solution S^0 has its left-hand-side elements x_{δ_i+1} $(i = 1,2,\ldots, k)$ as unity and the remaining elements as zero. The first element i.e. x_{δ_1+1} is underlined or put under a bar. This serves as a reference point for generating further descendants. For any n-variable problem having n_i variables $(i = 1,2,\ldots, k)$ for its various subsets, the tree developed is unique. There are a definite number of branches and a fixed number of levels associated with the tree. The number of branches at each level of the tree is also fixed. The generation of solutions from the first level is initiated and a fixed number of descendents appear at level 2 of the three. The generalized procedure for generating descendents from any parent branch is - starting from the underlined element, shift the unity entry one position towards the right-hand-side with respect to the subsets of solution vector at the parent branch. Thus, by the shifting of unity elements towards the right-hand-side and proceeding in a systematic fashion, the complete tree is enumerated. The last solution at the extreme level (tail) of the tree has all the $x_{\delta_i + n_i}$ ($i = 1,2,\ldots,k$), extreme right-hand-side elements as unity and the remaining elements as zero. The generation procedure takes care of the fact that none of the solution vector duplicates, while enumerating the tree. For further discussion and ease of exposition, a 4 stage, 10 variable problem tree is enumerated and is shown in Fig. 2.3 . The variables x_1 , x_2 , x_3 correspond to the first stage and hence the first subset, the variables x_4, x_5, x_6 correspond to the second stage, x_7, x_8 to the third and x_9, x_{10} to the 4th stage. The corresponding cost coefficients are arranged in a monotonic increasing order which means $c_1 \leqslant c_2 \leqslant c_3$; $c_4 \leqslant c_5 \leqslant c_6$; $c_7 \leqslant c_8$ and $c_9 \leqslant c_{10}$.

Looking at the tree diagram Fig. 2.3, one finds that one of the unity entry is underlined at every branch. The underlining of an element of a solution vector is a very important concept, as, the underlined element is the reference point for generating further descendants and also tells us as to how many descendants are associated with any parent branch. The next section explains the generation of descendants.

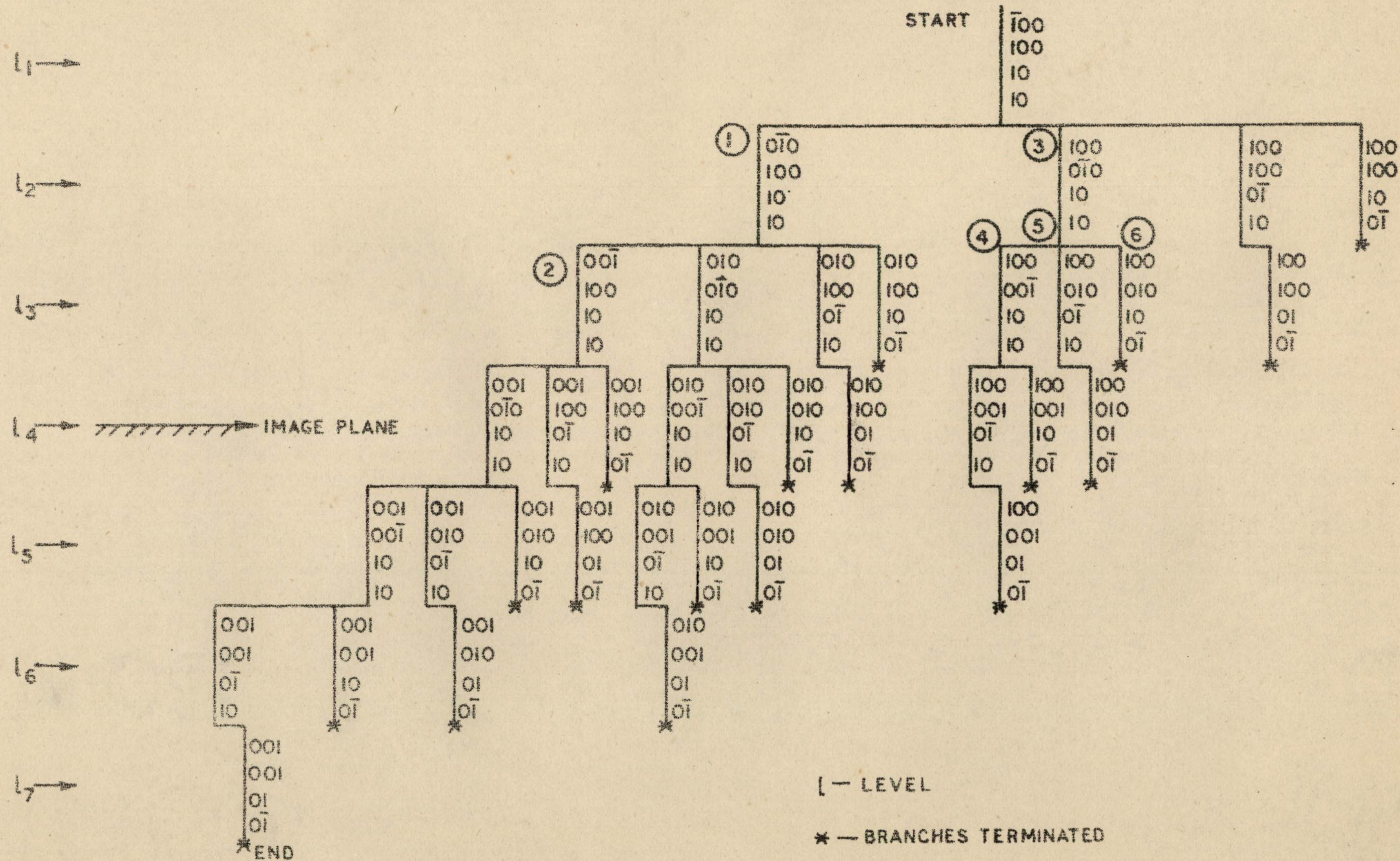

FIG. 2.3 TREE DIAGRAM FOR A 4 STAGE 10 VARIABLE EXAMPLE

GENERATION OF DESCENDANTS

For the systematic generation of the branches of the tree, it is important to know, as to how many descendants are associated with any parent branch. It is also essential to know, which unity element in a solution is to be underlined.

Let us consider a solution vector (1), indicated on Fig. 2.3. There are 4 subsets i.e. S_1, S_2, S_3 and S_4. The unity element x_2, which belongs to S_1 is underlined. The number of descendants associated with this branch is 4. In general, let the underlined element ($\bar{x}$) belong to the ith subset and $\bar{x}$ is not the last element of ith subset (i.e. $\neq x_{\delta_i+n_i}$), then the number of descendants D_e is given by (2.27)

$$D_e = k - i + 1 \qquad (2.27)$$

The second possibility is that the $\bar{x}$ is the last element of ith subset (i.e. $= x_{\delta_i+n_i}$). Then D_e is given by (2.28)

$$D_e = k - i \qquad (2.28)$$

This is explained by solution vector (2), indicated in Fig. 2.3. Here, the last element x_3 of S_1 is underlined. Therefore, in this case D_e is 3.

As regards, the underlining of unity elements of descendants, consider solution vector (3), whose descendants are ④ , ⑤ and ⑥ shown in Fig. 2.3. If $\bar{x}$ belongs to the ith subset at the parent branch and is not the last element (i.e., $\neq x_{\delta_i+n_i}$) , then ith subset $\bar{x}$ is shifted one position towards the right-hand-side in the 1st descendant and the subsequent solutions have $x_{\delta_{i+1}+2}$, $x_{\delta_{i+2}+2}$, $x_{\delta_{i+3}+2}$, ..., x_{δ_k+2} elements as underlined. If $\bar{x}$ is the last element of the ith subset (i.e. $= x_{\delta_i+n_i}$) , then it remains fixed and the rest of the procedure is same as explained above. To the knowledge of the author the newly developed tree and the procedure of generating descendants has not been discussed elsewhere in the literature.

PROPERTIES OF THE TREE

1. The total number of levels present in any tree is given by (2.29)

$$L = \sum_{i=1}^{k} n_i - k + 1 \qquad (2.29)$$

(Proof :- The number of levels associated with any tree is given by the expression (2.29). For this the basic requirement is to find the number of stages from which the first solution vector having all the left-hand-side elements passes in going to the last solution vector having all the right-hand-side elements in the various subsets as unity. Therefore, the number of stages to be traversed is $(\sum_{i=1}^{k} n_i - k)$, because k positions are initially occupied by the unity entries. Including the first level or stage from where the enumeration is started, the total number of levels $L = \sum_{i=1}^{k} n_i - k + 1)$.

For the 4 unit, 10 variable example L = 7.

2. The number of branches in a tree is given by S^T, where

$$S^T = \prod_{i=1}^{k} n_i \qquad (2.30)$$

(Refer to the proof given in Eqs (2.23) - (2.26))
For the example discussed, $S^T = 36$.

3. At the centre of the tree, if we consider an image plane, then the solution vectors below the image plane are the images of the solution vector above it. For example, the last vector (0010010101) of Fig. 2.3 is the image of the first vector (1001001010).

4. A spread of the solutions at the various levels of the tree follows a symmetric distribution. For the 4 unit example, the distribution is shown in Fig. 2.4.

5. The values of z at the ith level of the tree are always greater than the minimum of z at the

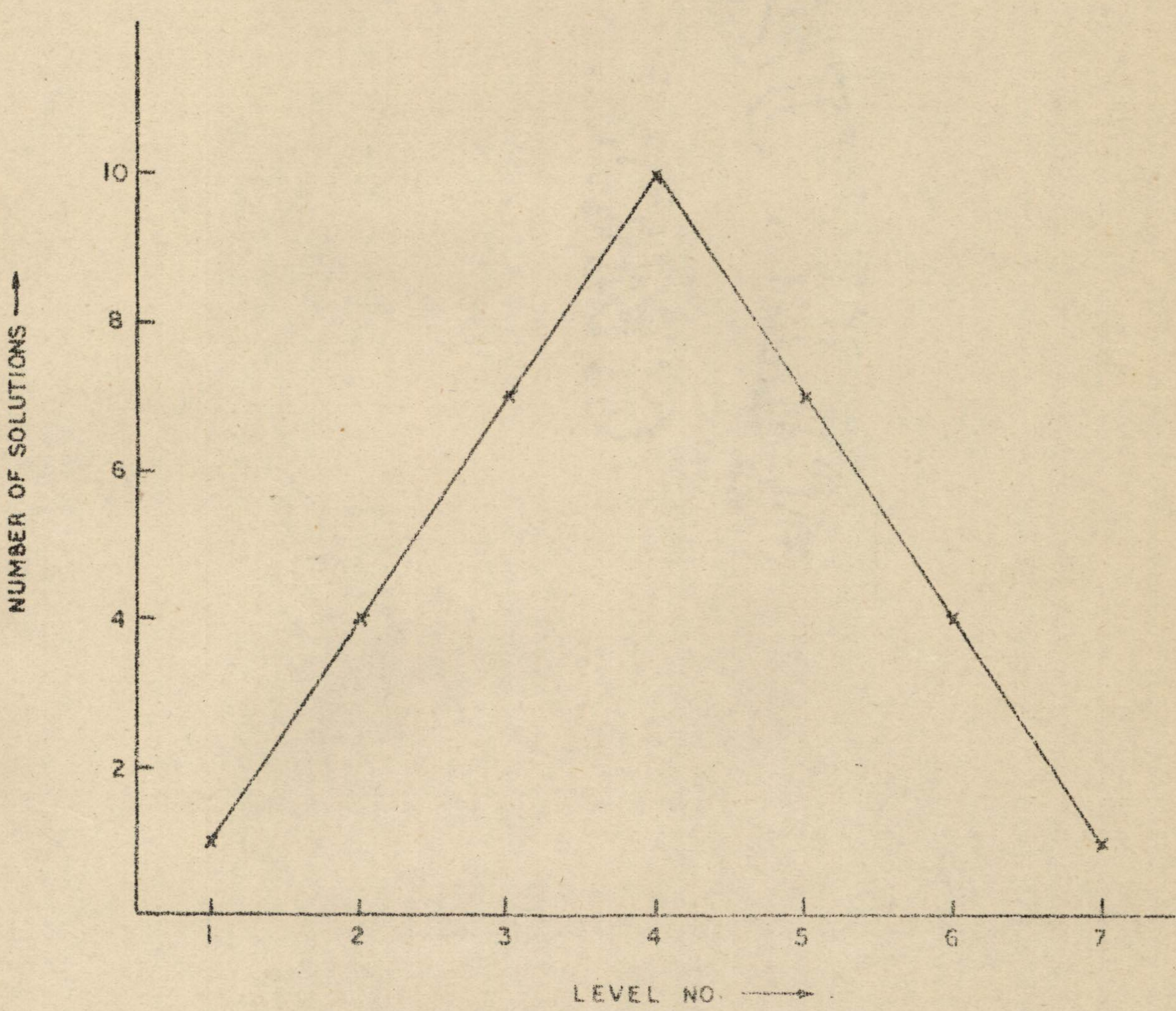

FIG. 2.4 DISTRIBUTION OF SOLUTIONS AT THE LEVELS OF THE TREE DIAGRAM

(i-1)th level. Therefore, the inequality (2.31) holds.

$$z^{i} \geqslant \text{Min } z^{i-1} \qquad (2.31)$$

The above statement is factual because of the monotonic behaviour of the cost function for each stage.

2.2.2 ALGORITHM

Based upon the concepts explained above a computer algorithm is developed for the solution of Eqs.(2.1), (2.2) and (2.3). The effective size of the tree to be searched is further reduced by enumerating many solutions at the subsequent levels with a very little computational effort. The reduced tree diagram for a 4 stage 10 variable example is shown in Fig.2.5. Such a move enhances the efficiency of the search process and also gives a relief to the computer core. The efficiency of the scheduling algorithm can be further enhanced by evolving simple rules for skipping a large number of solution vectors at the levels of the tree. A simple SKIP RULE I is evolved by taking advantage of the structure of constraining equations with positive coefficients. The rule is based on the premise that if the summation of the constraint coefficients upto (w-1)th subset violates the particular constraint, then all subsequents solution vedtors associated with this parent branch are infeasible and therefore are skipped. Where, w

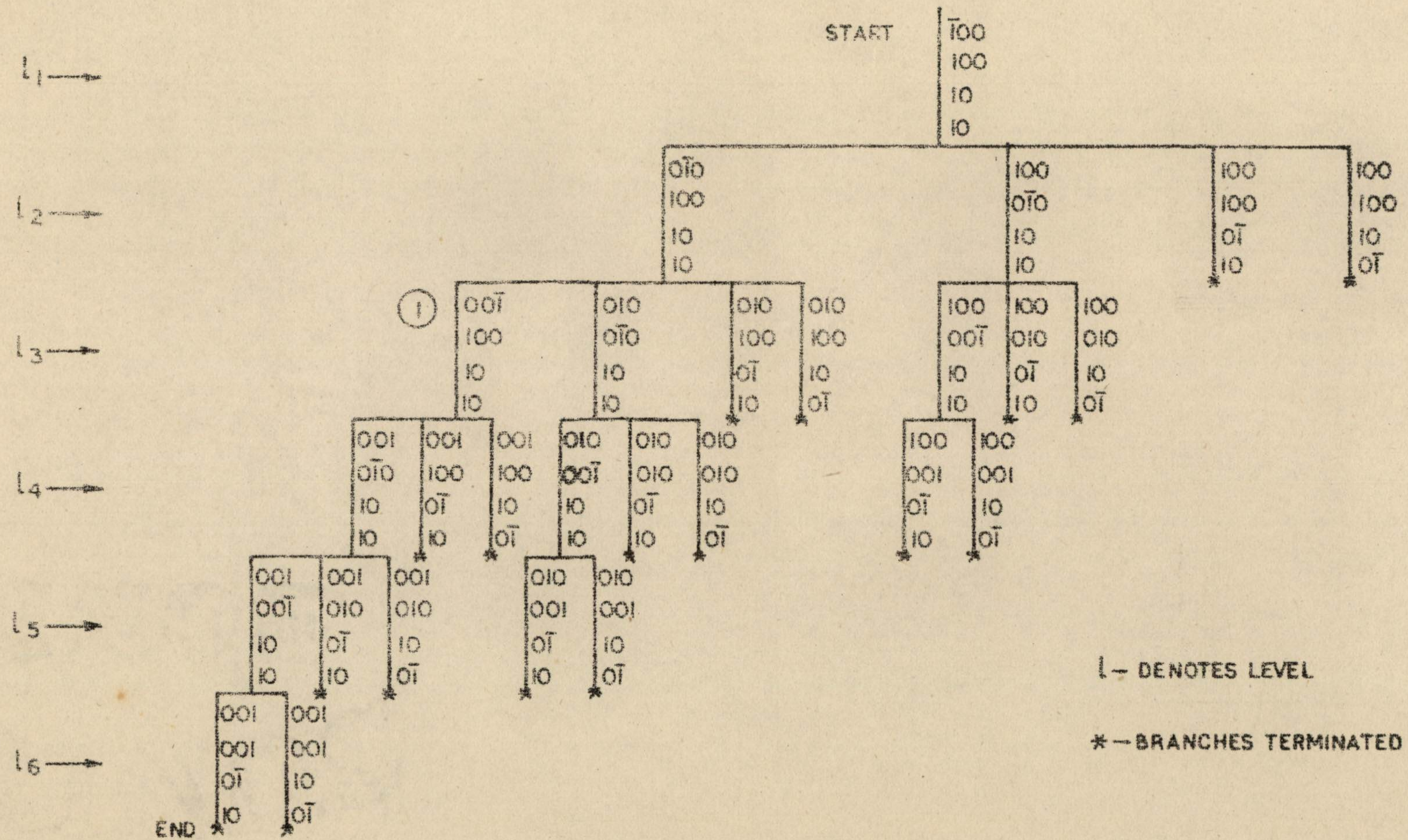

FIG. 2.5 REDUCED TREE DIAGRAM FOR A FOUR STAGE 10 VARIABLE EXAMPLE

refers to the subset with an underlined element. Stated mathematically, if

$$\sum_{j \in (w-1)} a_{ij} x_j \nless b_i \quad \text{for constraints}$$

with positive coefficients a_{ij}, then skip all descendants associated with this parent branch. This is called SKIP RULE I.

The steps of the algorithm are detailed below :

1. Read System Parameters - Cost coefficients, constraint coefficients and constraint limits.
2. Initialize solution vector at $\ell = 1$. Enter x_{δ_i+1}, $(i = 1,2,\ldots k)$ as unity entries and the remaining $x = 0$. Underline element x_{δ_1+1}.
3. Check constraints. If satisfied, stop, otherwise go to step 4.
4. Advance the level counter by one i.e. $\ell = \ell + 1$. Generate descendants by using Eqs. (2.27) and (2.28). Go to 5.
5. Search for a feasible schedule at the ℓth level. Terminate the branches having $x_{\delta_{k-1}} + n_{k-1}$ element as underlined and the branches having any of the elements x_{δ_k+j} $(j = 1,2,\ldots, n_k)$ as underlined. Apply SKIP RULE I. Go to 6.

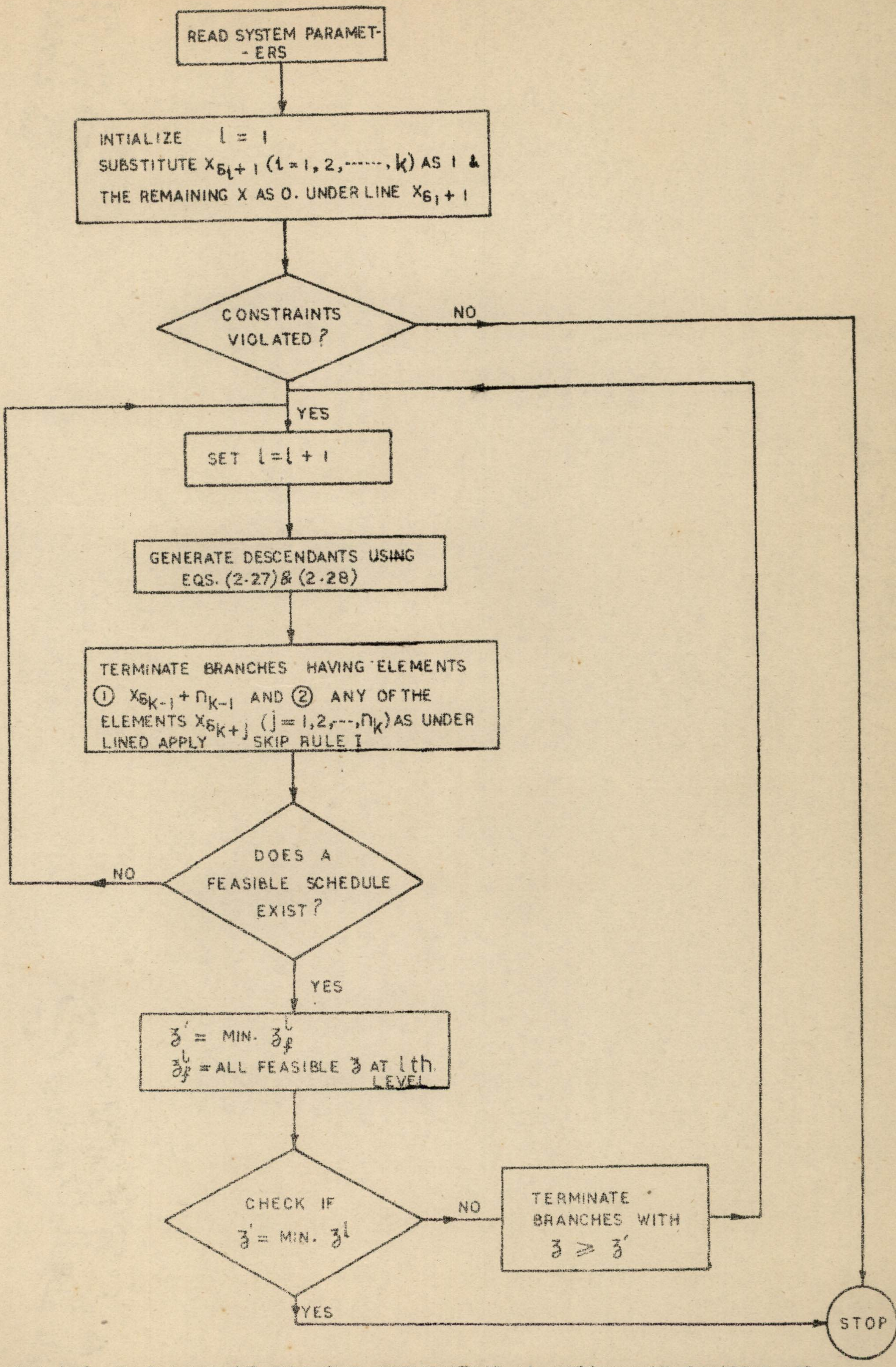

FIG. 2.6 FLOW CHART FOR THE PREVENTIVE MAINTENANCE SCHEDULING ALGORITHM

6. Does a feasible schedule exist ? If yes, go to 7, else go to 4.

7. Store z' , which is the minimum feasible z at the ℓth level, as z interesting. Check if z' is the minimum of all z at this level. If not go to 8, else stop.

8. Terminate branches having $z \geqslant z'$. Go to 4.

A flow chart of the algorithm is given in Fig. 2.6.

2.2.3 PROOF OF OPTIMALITY

Theorem :

The procedure of flow chart Fig. 2.6 results in an optimal solution to the problem given by Eqs. (2.1), (2.2), (2.3) and (2.16).

Proof :

The monotonic non-decreasing characteristic of the multistage cost function is displayed in Fig. 2.7. For any stage i, the following inequality (2.32) holds

$$c_{\delta_i+1} \leqslant c_{\delta_i+2} \leqslant c_{\delta_i+3} \cdots \leqslant c_{\delta_i+n_i} \qquad (2.32)$$

The initial solution z^0 corresponds to the state, when c_{δ_i+1} $(i = 1,2,\ldots,k)$ are active . Therefore,

$$z^0 = \sum_{i=1}^{k} c_{\delta_i+1} \qquad (2.33)$$

If the constraints (2.2) are satisfied corresponding to this solution, then, the optimal is reached, because z^0

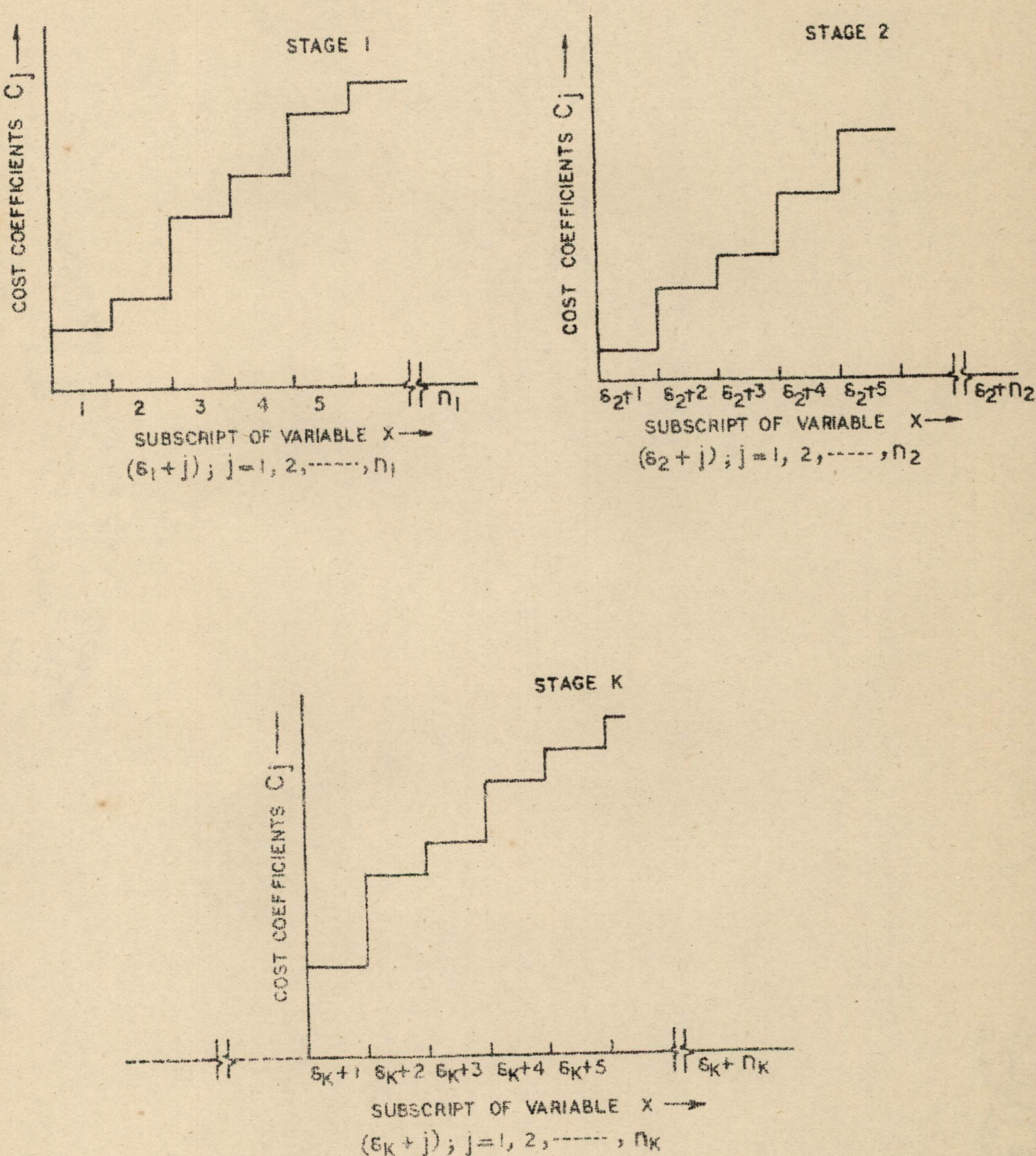

FIG. 2.7 MONOTONIC CHARACTERISTIC OF COST FUNCTION

is the summation of all the least values of $c's$ for the k stages.

If the above condition does not hold, then the descendants are generated by the shift of unity entry by just one position, towards the right-hand-side. This helps in generating various low cost combinations of z.

Let us suppose, there are some feasible z values at any level ℓ of the tree. Thus $z' = \text{Min } z_f^{\ell}$. Now, if z' is the minimum of all z at this level. Then, the optimal is reached. This is so, because of the non-decreasing characteristic (2.32) of the cost function.

The only alternative left is that $z' \nleq \text{Min} z^{\ell}$. Under this condition, moving down the tree, $z' \leq \text{Min } z^{\ell}$ gets satisfied and the optimal solution is achieved.

This completes the proof that the flow chart of Fig. 2.6 results in an optimal solution.

2.2.4 ADVANTAGES

The optimization technique developed above has the following advantages.

1. Every move is in the forward direction. No back-tracking [2] is required and hence the book-keeping is minimum.

2. The computational time is small as the z values and the constraints at the ith level are obtained from the z values and the constraints at the (i-1)th level, only by a minor change.

3. The storage is small, as it needs storing solutions, corresponding to only two levels at a time.

4. A large number of infeasible solution, which do not satisfy Eq.(2.16) are never generated.

5. As the constraint set (2.16) remains satisfied, the total number of constraints are reduced.

6. For practical problems, only a part of the tree has to be searched for finding the optimal solutions. Many branches are also terminated during the search process.

2.2.5 PROGRAMMING CONSIDERATIONS

From the programming stand point, it is desirable that the computer storage be economized and the speed of computations should be fast. This enhances the value of the algorithm in solving problems of large dimensionality. A storage scheme for the enumerated solution vectors is shown in Fig. 2.8. Such an arrangement take advantage of the systematization present in the search procedure. Because of the known fact that there is one and only one element of a subset which is unity, one needs storing only

STAGE NO. \ NO. OF SOLS.	1
1	$\bar{1}$
2	4
3	7
4	9

← l_1

STAGE NO. \ NO. OF SOLS.	1	2	3	4
1	$\bar{2}$	1	1	1
2	4	$\bar{5}$	4	4
3	7	7	$\bar{8}$	7
4	9	9	9	$\overline{10}$

← l_2

STAGE NO. \ NO. OF SOLS.	1	2	3	4	5	6	7	8
1	$\bar{3}$	2	2	2	1	1	1	1
2	4	$\bar{5}$	4	4	$\bar{6}$	5	5	4
3	7	7	$\bar{8}$	7	7	$\bar{8}$	7	8
4	9	9	9	$\overline{10}$	9	9	$\overline{10}$	$\overline{10}$

← l_3

FIG. 2.8 STORAGE SCHEME FOR THE ENUMERATED VECTORS the index of this element. Fig. 2.8 displays the storage of solution vectors corresponding to the first three levels of the tree diagram shown in Fig. 2.3. For example, the numbers 1,4,7,9 refer to the variables x_1, x_4 , x_7 and x_9 . Thus, for the enumerated solution, these variables are currently active and have unity values.

ACCELERATING THE CONVERGENCE

The efficiency of the scheduling algorithm can be further increased by taking advantage of the systematic behaviour of the constraining equations. This requires the evolvement of skipping rules, with the aid of which , many branches of the tree are terminated during the search process. Also, sets of constraints need not be checked for many enumerated solutions. The applicacy of these ideas is demonstrated by evolving SKIP RULE I. Another skipping rule is evolved in section (3.2) on the basis of the systematic behaviour of the constraining equations. Thus, the inclusion of skipping rules enhances the efficiency of the algorithms.

2.3 SAMPLE APPLICATIONS

The problem of preventive maintenance is discussed in detail in section (2.1). A scheduling algorithm is developed for the solution of the problem and is given in Fig. 2.6 . A computer program of the algorithm is prepared in Fortran II for an IBM 1620 computer. The applicacy of the algorithm is demonstrated by solving two examples. Another important problem concerning overhauling of generating units is the interchanging of staff among power plants, during the maintenance periods. A mathematical description of the problem is presented and an illustrative example is given. The results of computation are displayed.

2.3.1 A 4-STAGE EXAMPLE

EXAMPLE 2.1

Generating units G_1,G_2, G_3 and G_4 are to be maintained during a time horizon of four weeks. The resources available during these intervals are 150, 170, 180, and 120 MW respectively. A sequence constraint specifies that the maintenance on unit G_3 , must begin immediately after maintenance on unit G_4 is completed. From security considerations, simultaneous maintenance on units G_1 and G_4 is to be avoided. The data of the problem is given in Table 2.1. The details of constraint coefficients and cost coefficients are displayed in Table 2.4. Constraints 1,2,3 correspond to the sequence of maintenance on units G_3 and G_4 . Constraints 4,5,6,7 represent the resources. Constraints 8,9, and 10 are for preventing simultaneous maintenance on the units G_1 and G_4 . A minimum cost maintenance schedule is to be obtained.

With the help of prefiltering rules (2.8) and (2.9) , the variable x_{11} stands cancelled and the constraints 3 and 10 get eliminated.

The problem is solved as a 10 variable one. The tree diagram of Fig. 2.5 is applicable for this case. The optimal solution appears at branch 1 of level 3 of the tree diagram.

Thus the optimal solution is :

$$x_3 = x_4 = x_7 = x_9 = 1$$

$$x_1 = x_2 = x_5 = x_6 = x_8 = x_{10} = x_{11} = 0.$$

$$z^* = 7$$

The solution obtained is the first feasible solution and is also the optimal. Further search at this level is terminated. The computer memory requirement and the execution time for the IBM 1620 computer is 1970 words length and 2.0 mins. respectively. A very useful representation of the results of computation is in the form of bar charts. Such charts may be obtained directly from the computer, thus saving lot of manual labour. The bar chart for the case discussed is shown in Fig. 2.9. The bar charts are of value to plant managers for executing maintenance decisions.

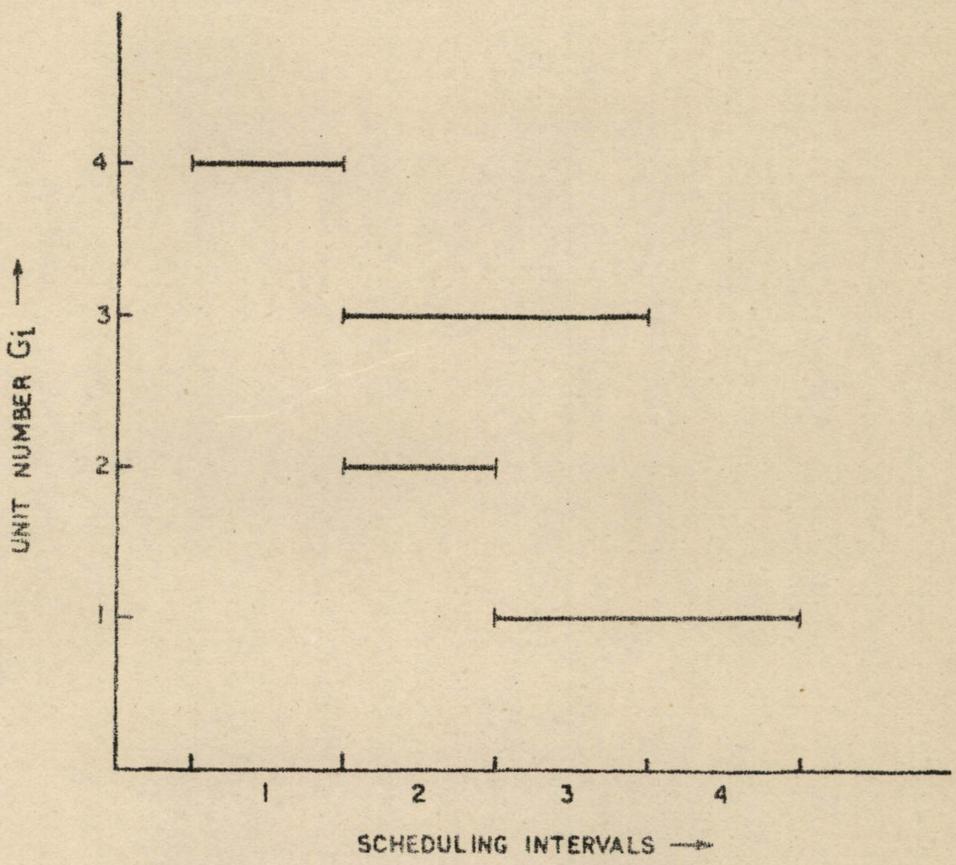

FIG. 2.9 BAR CHART FOR A FOUR STAGE EXAMPLE

2.3.2 A 10-STAGE EXAMPLE

EXAMPLE 2.2

A power utility has to maintain 10 generating units (G_i , i = 1,2,...,10) during a scheduling horizon of one year having 12 equal monthly intervals. The data of the problem is given in Tables 2.5 , 2.6, and 2.7 . In Table 2.5 , the units G_4 and G_5 are equivalent to a single unit which requires maintenance twice during the scheduling horizon. The maintenance on these two units is to be separated by a six months period. In the Table 2.6 , the constraints (1-12) are for the resources limitation. The constraints (13-15) are for the old unit which is replaced by G_4 and G_5 . The constraints (16-18) are for sequencing of maintenance on the units G_2 and G_3 . The constraints (19-21) are for the exclusion of simultaneous maintenance on the units G_1 and G_2 . Table 2.7 portrays the constraint limits for the constraints of Table 2.6 . The data used is hypothetical.

The optimal solution obtained to the 10 generator, 35 variable, 12 interval problem is :

$$x_3 = x_4 = x_7 = x_9 = x_{12} = x_{15} = x_{19} = x_{23} = x_{27} = x_{32} = 1$$

$$x_1 = x_2 = x_5 = x_6 = x_8 = x_{10} = x_{11} = x_{13} = x_{14} = x_{16} = x_{17} = x_{18}$$
$$= x_{20} = x_{21} = x_{22} = x_{24} = x_{25} = x_{26} = x_{28} = x_{29} = x_{30} = x_{31}$$
$$= x_{33} = x_{34} = x_{35} = 0.$$

$$z^* = 160.00$$

TABLE 2.5

Data and the Associated unknowns for Example 2.2

Unit	Capacity MW	Allowed period	Outage duration months	Associated variables	Maintenance starts in month
G_1	80	1-4	2	x_1	1
				x_2	2
				x_3	3
G_2	40	1-3	1	x_4	1
				x_5	2
				x_6	3
G_3	80	2-4	2	x_7	2
				x_8	3
G_4	40	2-4	1	x_9	2
				x_{10}	3
				x_{11}	4
G_5	40	8-10	1	x_{12}	8
				x_{13}	9
				x_{14}	10
G_6	120	4-9	3	x_{15}	4
				x_{16}	5
				x_{17}	6
				x_{18}	7
G_7	80	5-9	2	x_{19}	5
				x_{20}	6
				x_{21}	7
				x_{22}	8
G_8	120	2-7	3	x_{23}	2
				x_{24}	3
				x_{25}	4
				x_{26}	5
G_9	40	8-12	1	x_{27}	8
				x_{28}	9
				x_{29}	10
				x_{30}	11
				x_{31}	12
G_{10}	120	7-12	3	x_{32}	7
				x_{33}	8
				x_{34}	9
				x_{35}	10

TABLE 2.7

Constraint limits b_i for Example 2.2

Constraint Number	b_i	Constraint Number	b_i
1	100.0	12	100.0
2	250.0	13	00.0
3	300.0	14	00.0
4	350.0	15	0.0
5	200.0	16	0.0
6	200.0	17	0.0
7	150.0	18	0.0
8	200.0	19	1.0
9	150.0	20	1.0
10	250.0	21	1.0
11	100.0		

The computer memory requirement and the execution time for the IBM 1620 computer is 3760 words length and 12.0 mins. respectively. A graphical display of the results is presented in Fig. 2.10.

2.3.3 MAINTENANCE STAFF INTERCHANGE SCHEDULING

The reliability and economics of operation dictates the need for pool coordination of resources (men and material) within a utility. The pooling is also beneficial to the mutual interest of the neighbouring utilities.

employed from an alternative source D. The data pertaining to the problem is given in Table 2.8. The data used is hypothetical. A schedule which minimizes the total cost of interchanging the labour is to be obtained.

A diagrammatic view of the staff interchange scheduling problem is presented in Fig. 2.11. The arrows on the lines connecting the stations indicate the direction in which the labour is transferred. The dotted line indicates that there are constraints between the stations.

TABLE 2.8

Data of Example 2.3

Source	No. of Repair teams	Associated variables	Cost coefficient
Station A (Labour of category I)	1	x_1	5
	2	x_2	10
	3	x_3	25
Station B (Labour of category II)	1	x_4	4
	2	x_5	8
	3	x_6	20
Alternative source D (Labour of Category I)	0	x_7	0
	1	x_8	10
	2	x_9	20

TABLE 2.7

Constraint limits b_i for Example 2.2

Constraint Number	b_i	Constraint Number	b_i
1	100.0	12	100.0
2	250.0	13	00.0
3	300.0	14	00.0
4	350.0	15	0.0
5	200.0	16	0.0
6	200.0	17	0.0
7	150.0	18	0.0
8	200.0	19	1.0
9	150.0	20	1.0
10	250.0	21	1.0
11	100.0		

The computer memory requirement and the execution time for the IBM 1620 computer is 3760 words length and 12.0 mins. respectively. A graphical display of the results is presented in Fig. 2.10.

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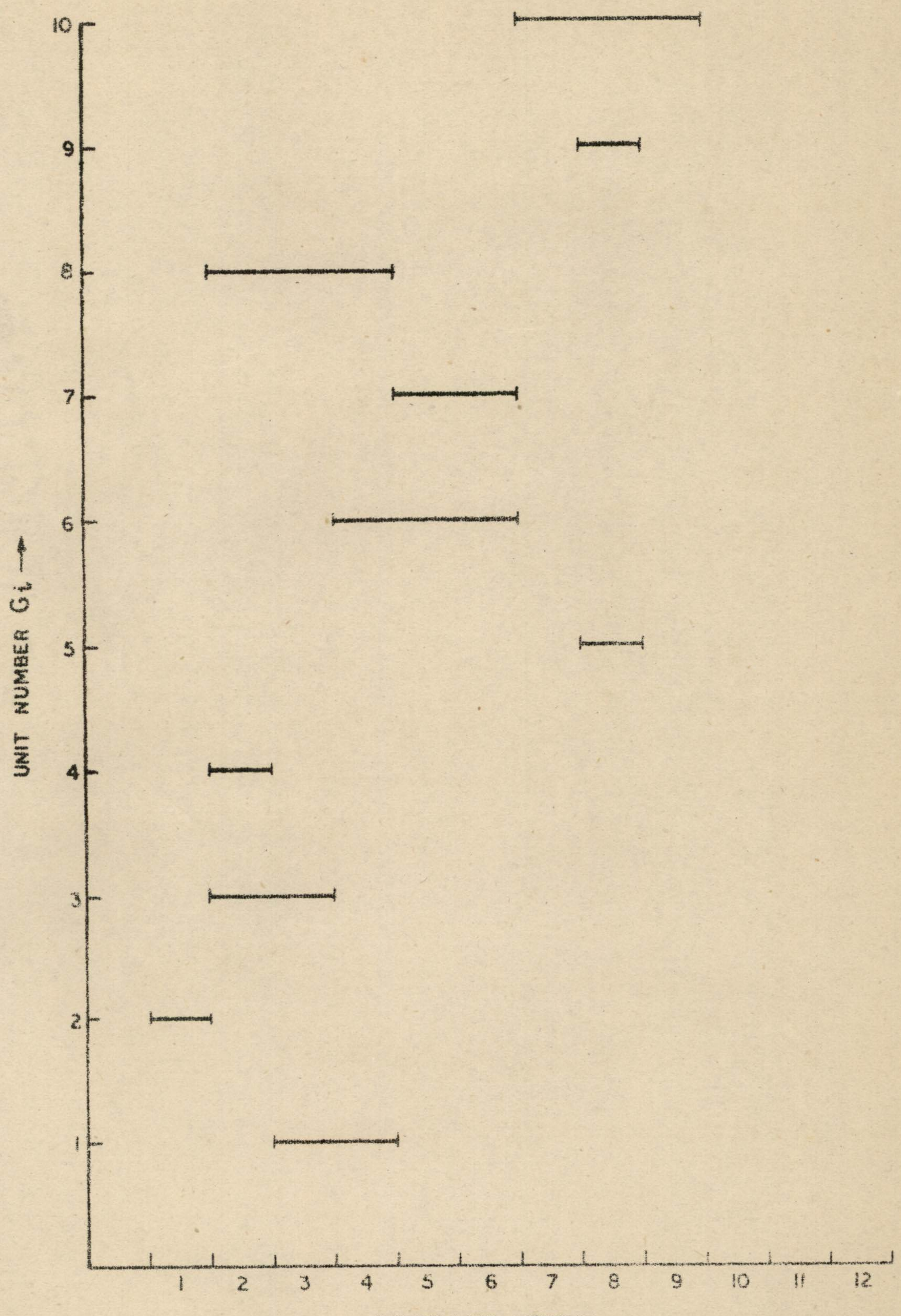


FIG. 2.10 BAR CHART FOR A 10 STAGE EXAMPLE

In the overhauling of power plants, maintenance staff is to be interchanged between stations at times of overhauls. In this way a larger labour force is concentrated in two or more nearby stations. Then, instead of two or more units being out of service simultaneously with less than optimum manpower working on them, they may be dealt with one after the other, each overhaul employing a combined labour force and taking a considerable shorter time. A logical extension of this concept of interchanging labour is to establish Divisional or Regional pools of labour, which can supplement the station maintenance teams.

The problem of staff interchanging [9] is formulated as an integer program. The model falls within the format of Eqs. (2.1), (2.2), (2.3) and (2.16). The scheduling algorithm of Fig. 2.6 is applicable. The following example 2.3 illustrates the above concepts. A schedule, which minimizes the total cost of interchanging the labour is obtained.

<u>EXAMPLE 2.3</u>

An electric utility is managing three power plants named as A,B, and C. The maintenance is to be concentrated on plant C and its maintenance teams are to be supplemented by pooling of repair men from stations A and B. A minimum labour force is required at any of the plants A and B for emergency purposes. The additional labour, if required, is employed from an alternative source D. The data pertaining to the problem is given in Table 2.8. The data used is hypothetical . A schedule which minimizes the total cost of interchanging the labour is to be obtained.

A diagrammatic view of the staff interchange scheduling problem is presented in Fig. 2.11. The arrows on the lines connecting the stations indicate the direction in which the labour is transferred. The dotted line indicates that there are constraints between the stations.

TABLE 2.8

Data of Example 2.3

Source	No. of Repair teams	Associated variables	Cost coefficient
Station A (Labour of category I)	1	x_1	5
	2	x_2	10
	3	x_3	25
Station B (Labour of category II)	1	x_4	4
	2	x_5	8
	3	x_6	20
Alternative source D (Labour of Category I)	0	x_7	0
	1	x_8	10
	2	x_9	20

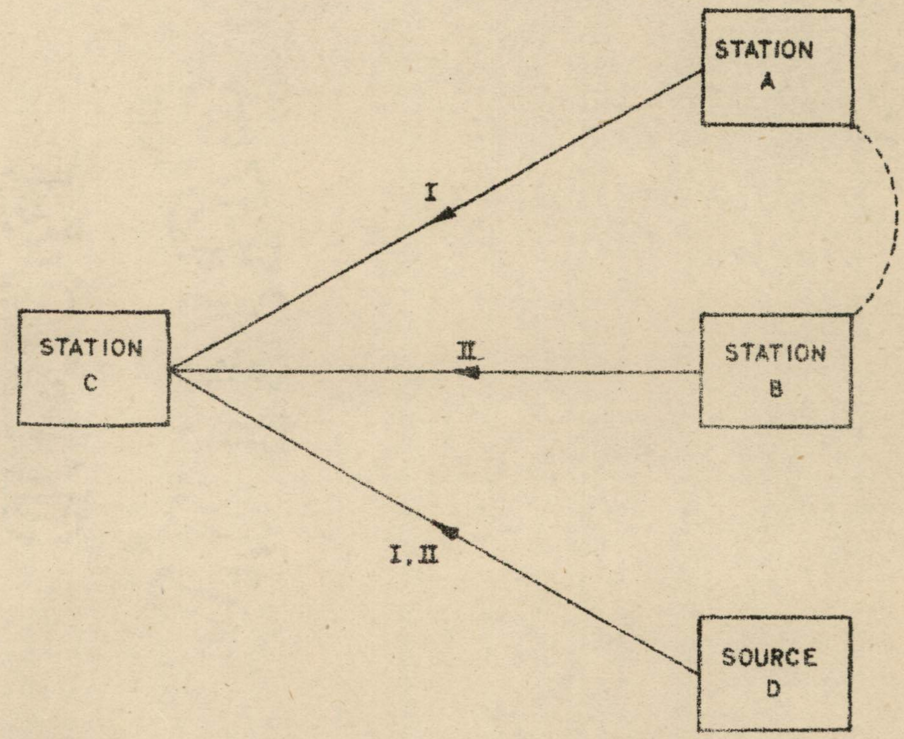


FIG. 2.11 A DIAGRAMMATIC VIEW OF STAFF-INTERCHANGE SCHEDULING PROBLEM

Table 2.8 contd..

Source	No. of Repair teams	Associated variables	Cost coefficient
Alternative	0	x_{10}	0
Source D	1	x_{11}	12
(Labour of	2	x_{12}	24
Category II)			

Thus, the objective is to

$$\text{Minimize} \quad z = 5x_1 + 10x_2 + 25x_3 + 4x_4 + 8x_5 + 20x_6 + 0x_7 + 10x_8 + 20x_9 + 0\,x_{10} + 12x_{11} + 24x_{12} \qquad (2.34)$$

Subject to the constraints

(i) To assure that the required labour teams of category I are available at plant C, the equality (2.35) is written. That is,

$$1\,x_1 + 2x_2 + 3x_3 + 0x_7 + 1x_8 + 2x_9 = 4 \qquad (2.35)$$

(ii) To assure that the required labour teams of category II are available at plant C, the equality (2.36) is written. That is

$$1x_4 + 2x_5 + 3x_6 + 0x_{10} + 1x_{11} + 2x_{12} = 3 \qquad (2.36)$$

(iii) To account for the minimum labour force at any of the plants A and B for emergency purposes, the inequality (2.37) is written. That is,

$$1x_1 + 2x_2 + 3x_3 + 1x_4 + 2x_5 + 3x_6 \leqslant 5 \qquad (2.37)$$

The optimal solution obtained is :

$$x_2 = x_6 = x_9 = x_{10} = 1$$

$$x_1 = x_3 = x_4 = x_5 = x_7 = x_8 = x_{11} = x_{12} = 0$$

$$z^* = 50$$

The computer memory requirement and the execution time for the IBM 1620 computer is 1926words length and 4.0 mins. respectively. The optimal solution is obtained at level 6 of the tree. The solution indicates that 2 repair teams of category I are transferred from station A, 3 repair teams of category II are transferred from station B, 2 repair teams of category I are transferred from source D and no repair team of category II is transferred from source D. The cost of transferring labour is 50 units.

CHAPTER III

CORRECTIVE MAINTENANCE SCHEDULING

It is conceivably true that a system should be designed to have built-in maintenance as far as possible. A system analyst is posed with the problem of preparing the best schedule out of a host of available alternatives. Such an approach reduces the expenditure on maintenance during the operating life of the system and also enhances the system reliability . Billinton and Krasnodekski [5] have also emphasized the inclusion of maintainability analysis at the design phase. In designing systems with regard to reliability and repairability typical considerations involve trade offs between system mean time to failure (MTTF) , and system mean time to repair (MTTR). Criteria for the tradeoffs are based on cost and availability. These techniques are particularly applicable to the determination of the number and capacity of boiler feed-pumps, feed heating train arrangements, pulverizer configuration, auxiliary electric power systems and cooling water etc. The tradeoff techniques are also applicable to such situations where :

i) A choice is to be made between easily replaceable modular components against piece parts.

ii) A decision is to be made on the layout of the system, subsystem or equipment, whether to economize space or facilitate easy accessibility.

Thus, it is possible to increase the auxiliary system reliability by varying the configuration and using mixed redundancy. The tradeoff analysis results in the optimal number of standby components and repair facilities required in order to achieve a specific level of system reliability.

3.1 MODELLING

A huge amount of literature is available in the area of reliability modelling for Defence and Aerospace systems [5,51] . The basic models are also applicable to many situations in Power System Engineering. It is required that power system engineers take full advantage of the available literature and develop a unified approach for corrective maintenance scheduling. Some of the important reliability models, under which the power plant equipment may be classified are for :. standby systems, standby systems with repair facilities, systems subject to two types of failures, (m/n) systems ,(m/n) systems with repair facilities. The following treatment is presented in order to explain the behaviour of these models in an expository form. Graphical plots of the reliability expression of various types of subsystems are useful to the system analyst in understanding the characteristic behaviour of these subsystems and in the analysis of corrective maintenance scheduling problem.

1. STANDBY SYSTEM

A system consisting of n independent components is said to be a standby system provided the system operates in the following manner. Component 1 operates until failure then component 2 is switched on, ..., component i operates until failure ~~failure~~, then component i+1 is switched on, ..., component n operates until failure, then the system is declared as failed. The reliability expression, which is of interest to system analyst, for such a system is given by Eq. (3.1) [51].

$$R_1' = e^{-\lambda t} \sum_{i=0}^{n-1} \frac{(\lambda t)^i}{i!} \qquad (3.1)$$

where,

λ is the failure rate, t is the mission time and n are the total number of components. The network configuration of such a system is provided in Fig. 3.1 , where S and SD denote the switch and the failure sensing device. Typical assumption in analysing such systems are :

i) If switching is necessary due to failure in a parallel component, the time required is insignificant and does not affect the desired operation.

ii) No warm up time is necessary for components being switched in.

iii) Failure is detected with probability one and the subsequent component is then switched in automatically.

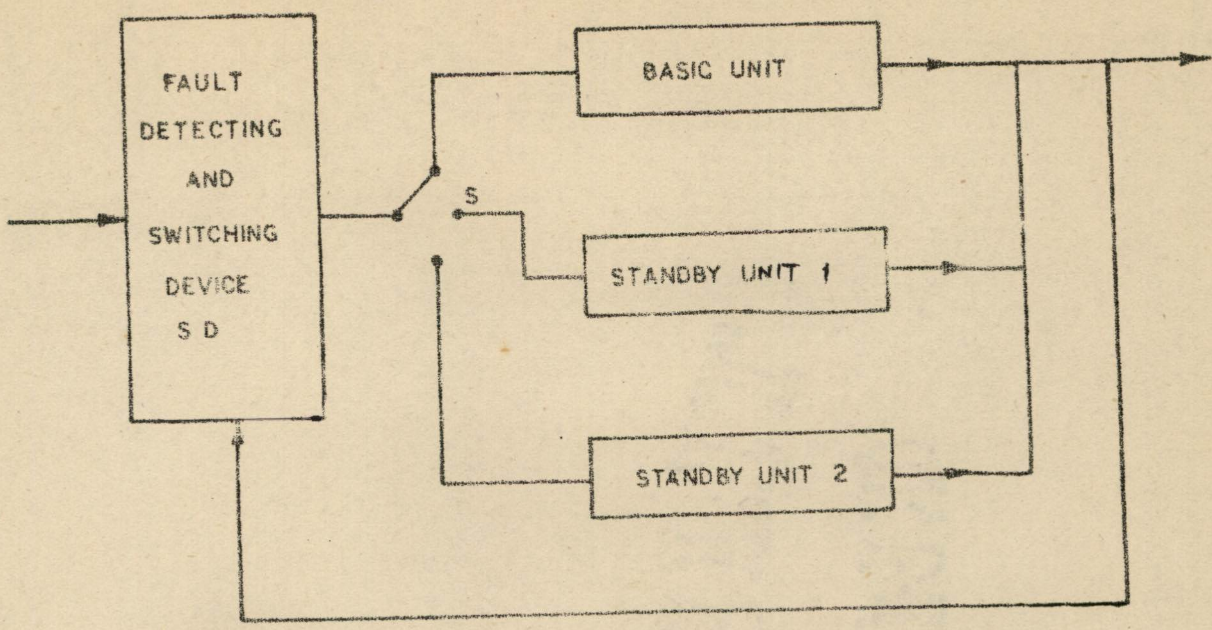


FIG.3.1 A SUB SYSTEM WITH STANDBY UNITS

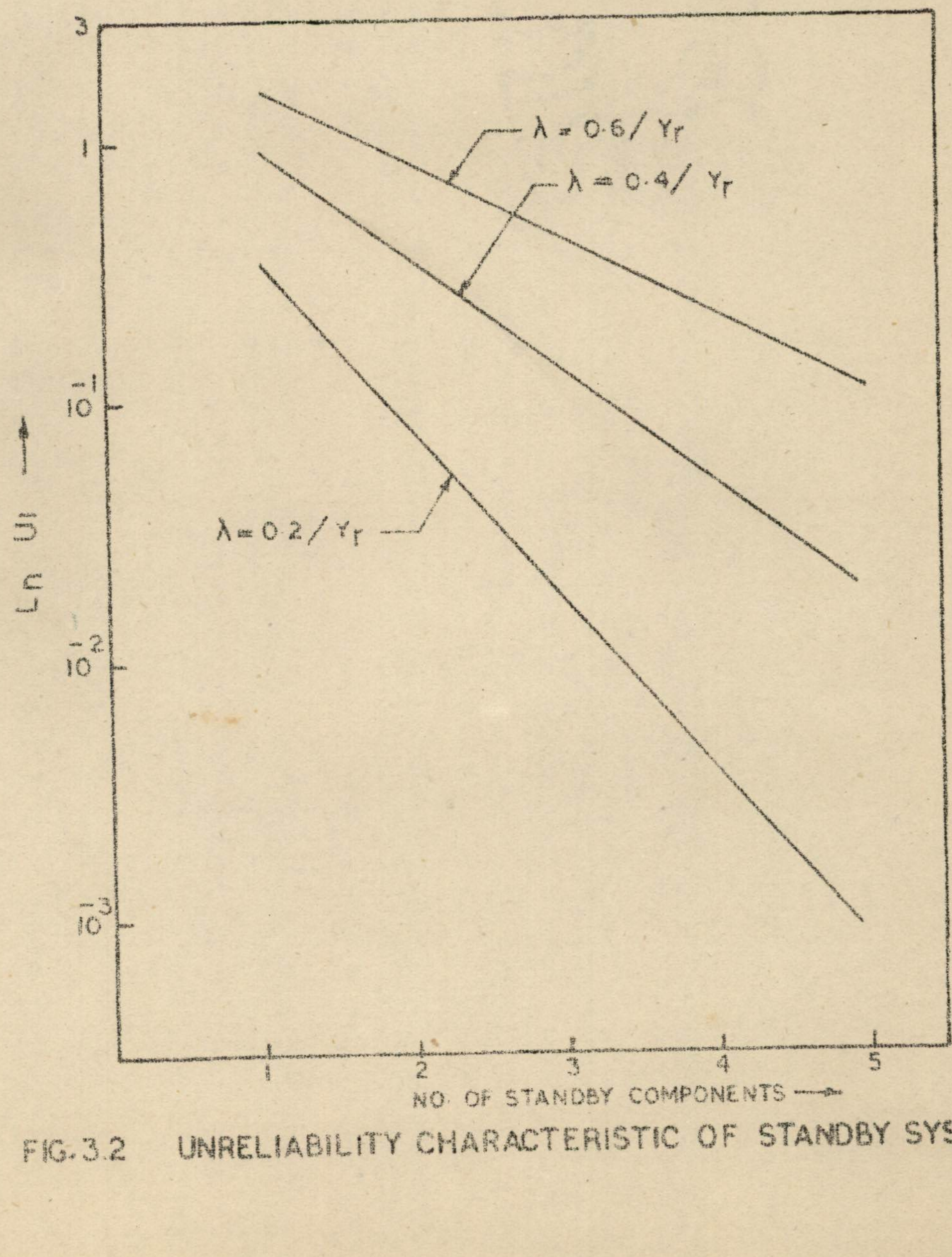


FIG.3.2 UNRELIABILITY CHARACTERISTIC OF STANDBY SYSTEM

A useful graphical plot of expression (3.1) is obtainable from Fig. 3.2 , where $U_1 = 1 - R'_1$ is the standby system unreliability.

2. STANDBY SYSTEM WITH REPAIR

When a fault in a system is non-recoverable the failed component is disconnected and repair is performed. It is possible that at a time more than one component may fail simultaneously . This requires more than one repair crew in order to increase the operating life of the system. In the case of irredundant systems, repair alone does not help in increasing the system reliability. It is enhanced by providing the spare components. The behaviour of such type of system is explained as follows: Initially one component is kept in operation and the others are kept as spare. When a component fails, it is replaced by a spare component and the failed component is sent for repair. When repaired, it is kept as a standby component.

If a standby system has n components each with a constant failure rate λ and there are r repair facilities, each with constant repair rate μ , where $1 \leqslant r \leqslant n$, then, the reliability expression of such a system [51] , which is of interest to system analyst is given by Eq.(3.2).

$$R'_2 = 1 - p_n \qquad (3.2)$$

where,

$$p_i = \frac{1}{r!\ \ r^{i-r}} \left(\frac{\lambda}{\mu}\right)^i p_o \qquad (3.3)$$

for $r+1 \leqslant i \leqslant n$

and

$$p_o = \left[1 + \sum_{i=1}^{r} \frac{1}{i!} \left(\frac{\lambda}{\mu}\right)^i + \sum_{i=r+1}^{n} \frac{1}{r!\ r^{i-r}} \left(\frac{\lambda}{\mu}\right)^i \right]^{-1}$$

R_2' is also known as system uptime ratio (3.4)

A useful graphical plot of expression (3.2) is provided in Fig. 3.3 , where $U_2 = 1 - R_2'$ is the corresponding system unreliability.

3. SYSTEM SUBJECT TO TWO TYPES OF FAILURES

Many types of systems [56] consists of components which fail in the mutually exclusive ways, and the result is that the system fails in either of the two mutually exclusive ways. For example, a network consisting of n relays in parallel has the property that a short-circuit failure on any one relay would cause a system failure, and an open circuit failure of all the n relays would again cause a failure of the system. Diodes also exhibit the behaviour of two failure modes given by an open circuit failure or a short circuit failure. A series parallel arrangement of diodes is used, if the probability of open circuit failure is high. The reliability expression for

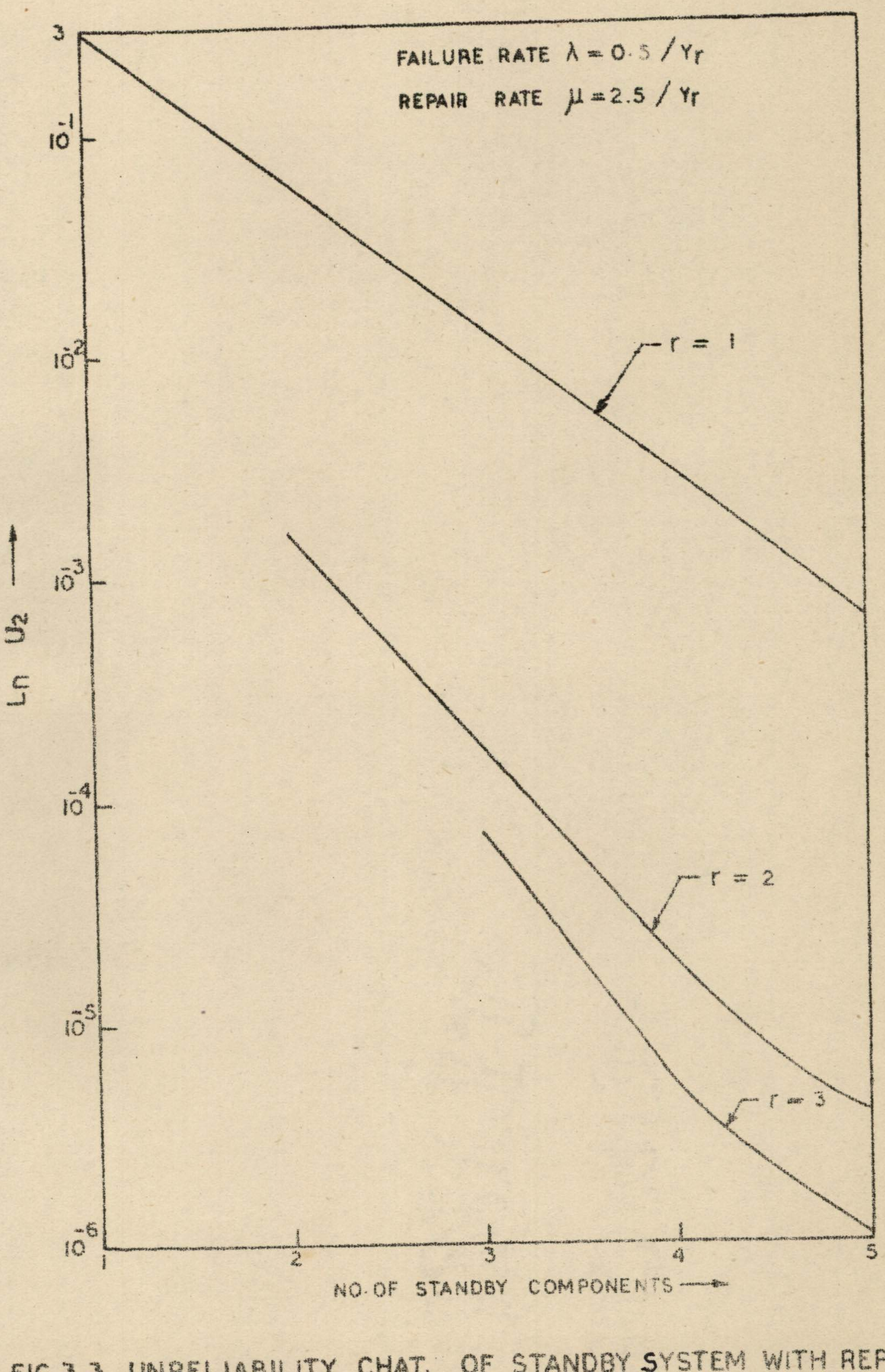


FIG. 3.3 UNRELIABILITY CHAT. OF STANDBY SYSTEM WITH REPAIR

such a system, which is of interest is given in Eq.(3.5)

$$R_3' = \left[1-(Q_1)^n\right]^m - \left[1 - (1-Q_2)^n\right]^m \qquad (3.5)$$

where

Q_1 is the probability of open circuit failure

Q_2 is the probability of short circuit failure

m are the number of sections in series.

n are the number of diodes in each section connected in parallel.

A graphical plot of expression (3.5) is given in Fig. 3.4 , where $U_3 = 1-R_3'$ is the corresponding unreliability.

4. (m/n) SYSTEM

A system with n independent components out of which m components must operate is called (m/n) or 'm our of n ' system. In a data processing system with five video displays, a minimum of three displays operable may be sufficient for full data display, in which case the display subsystem behaves as a (3,5) system. In such a system, when (m-n+1) component fail, the system is said to have failed. The reliability expression, which is of interest for such a system is given by Eq.(3.6)

$$R_4' = \sum_{i=m}^{n} \binom{n}{i} (e^{-\lambda t})^i (1-e^{-\lambda t})^{n-i} \qquad (3.6)$$

where,

λ is the failure rate, t is the mission time, n are the total number of components and m are the number of

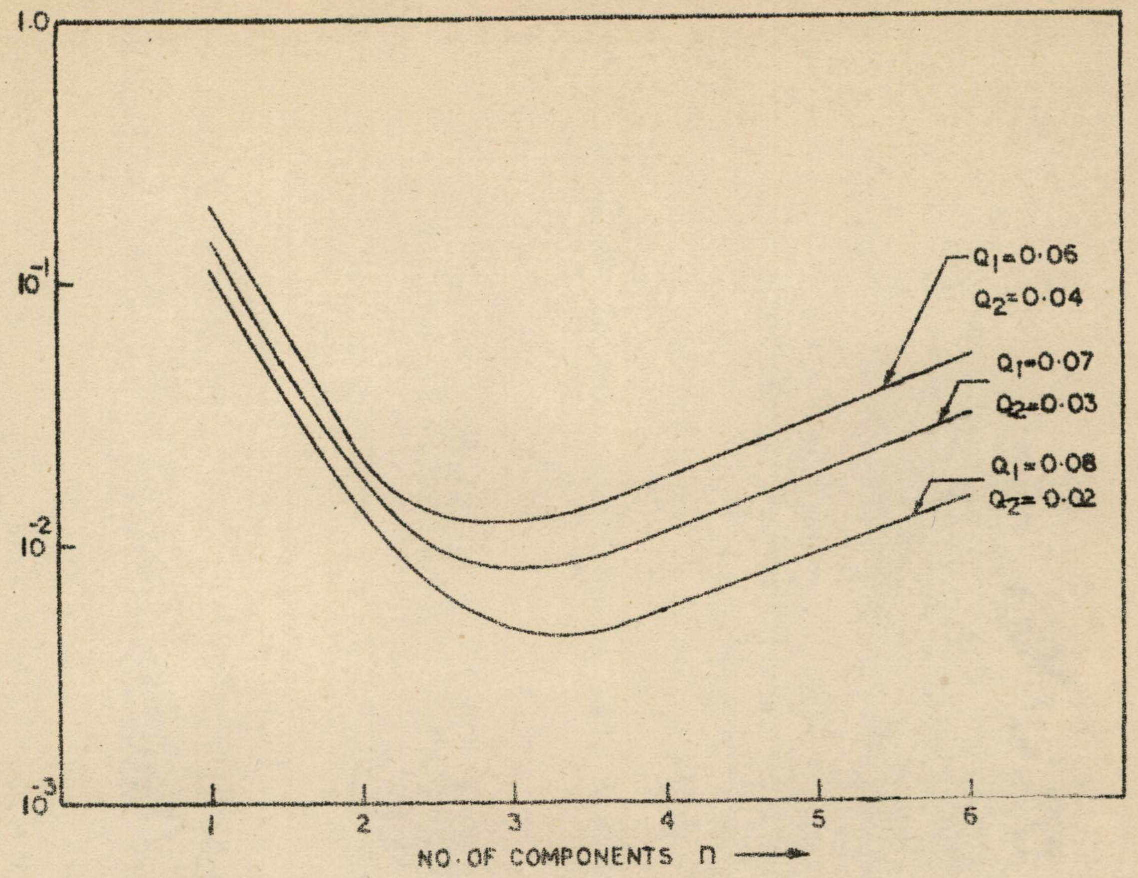


FIG. 3 .4 UNRELIABILITY CHAT. FOR A SERIES-PARALLEL STRUCTURE

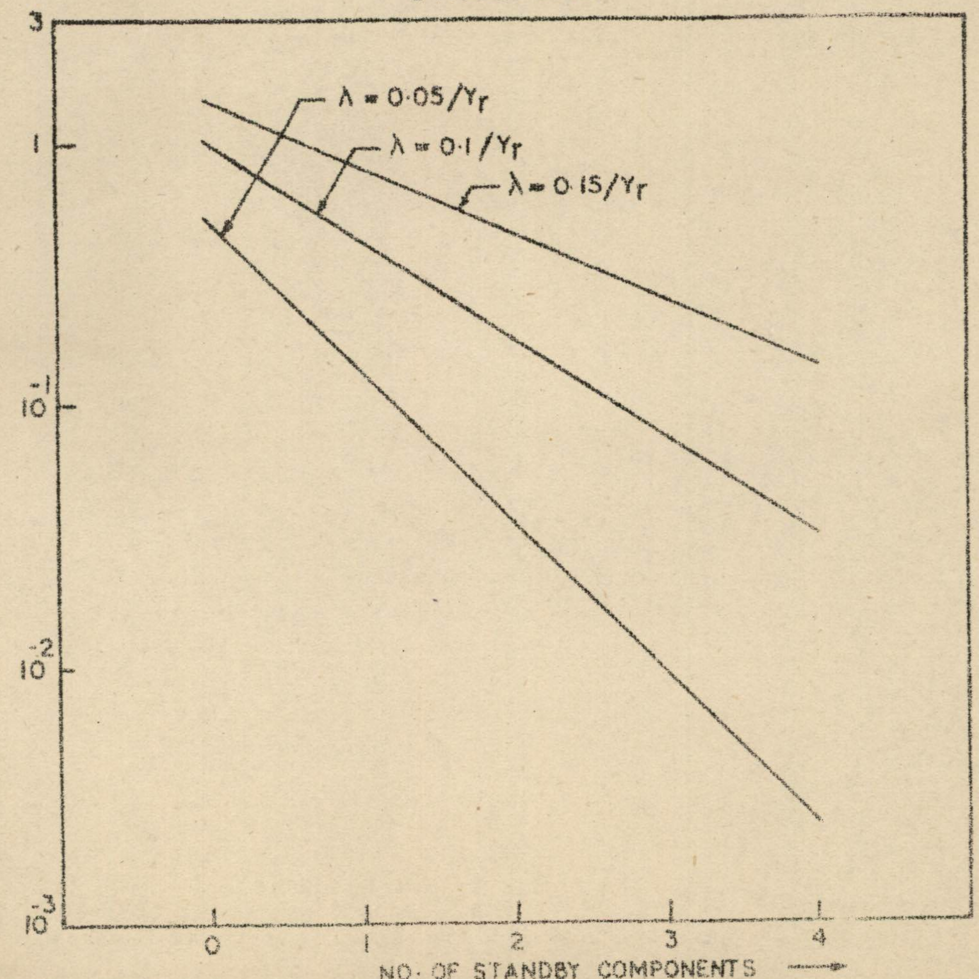


FIG.3.5 UNRELIABILITY CHARACTERISTIC OF M/N SYSTEM

components, which must be in an operable state.

A graphical plot of Eq. (3.6) is available from **Fig.**3.5 , where $U_4 = 1-R_4'$ is the corresponding unreliability.

OBJECTIVE

Consider a system composed of k independent subsystems, which are functionally in series. The reliability of a such a system in terms of subsystem reliabilities is given by the expression (3.7) . That is,

$$R_s = \prod_{i=1}^{k} R_i(m_i, r_i) \qquad (3.7)$$

where, R_i is the reliability of ith subsystem and is a function of the number of repair facilities r_i and the number of spare components m_i.

The product reliability expression (3.7) is converted into the summation (3.8), by taking the logarithm of the both sides of expression (3.7). Thus,

$$\text{Ln } R_s = \sum_{i=1}^{k} \text{Ln } R_i(m_i, r_i) \qquad (3.8)$$

The advantage of the above transformation is that the reliability expressions for each stage get separated and a separable objective function obtained.

One of the important objectives on which the resulting schedule is based, is the maximization of the system reliability, R_s, subject to the constraints on cost, weight, volume (space) and power consumption etc. Stated mathematically, the problem is :

$$\text{Maximize : } Ln\ R_s = \sum_{i=1}^{k} Ln\ R_i(m_i, r_i) \qquad (3.9)$$

subject to:

$$\sum_{i=1}^{k} g_{ij}\ (m_i, r_i) \leqslant b_j \qquad (3.10)$$

$$j = 1, 2, \ldots, w$$

where,

g_{ij} is the jth type resource requirement associated with the ith subsystem and b_j is the amount allocated for the jth resources and w are the total number of constraints.

The objective of maximizing the system reliability can also be realized by minimizing the system unreliability as given by expression (3.11)

$$U_s = 1 - R_s \qquad (3.11)$$

where, U_s stands for the system unreliability. Therefore, the objective function (3.9) is transformed to the objective function (3.12) . That is,

$$\text{Minimize : } Ln\ U_s = - \sum_{i=1}^{k} Ln\ R_i(m_i, r_i) \qquad (3.12)$$

subject to the constraints (3.10).

Another important objective could be to minimize the cost of additions or changes in the system, subject to the satisfaction of achieving a specific level of reliability besides satisfying ~~other~~ associated constraints.

The problem of minimizing the unreliability (3.12) or minimizing the cost of additions or changes in the system is transformed into the integer linear program as given below.

$$\text{Minimize :} \quad z = \sum_{i=1}^{n} c_i \, x_i \tag{3.13}$$

subject to :

$$\sum_{i=1}^{n} a_{ij} \, x_i \leq b_j \tag{3.14}$$

$$j = 1,2,\ldots,w$$

$$x_i = 0 \text{ or } 1 \tag{3.15}$$

where, z is the objective function to be minimized and and c_i are the unreliability values, when the objective is : system unreliability minimization. For the case, when the cost is to be minimized, c_i are the cost coefficients. a_{ij} are the constraint coefficients and b_j are the limits on the available resources for the w constraints.

In the formulation (3.13) - (3.15) , k of the w constraints have a special structure as given by (3.16) . That is,

$$\sum_{i \in k} x(S_i) = 1 \tag{3.16}$$

The equality (3.16) signifies that for each subsystem or stage, the summation of the variables is unity. To the knowledge of the author such systematic modelling for the corrective maintenance scheduling problem has not been done earlier. In this case also the cost coefficients for each stage are arranged in a monotonic increasing sequence.

3.2 ALGORITHM

The problem whose solution is to be obtained is given by equations (3.13) - (3.16) . The mathematical structure of the problem is identical to the problem of preventive maintenance scheduling discussed in Chapter II. Thus, the scheduling algorithm of Fig. 2.6 is directly applicable.

In many of the corrective maintenance scheduling problems a part of the constraints (3.14) have a systematic structure. Thus, the constraint set (3.14) is represented by the equations (3.17) and (3.18) as given below :

$$\sum_{i=1}^{n} a_{ij} \; x_i \leqslant b_j \qquad j = 1,2,\ldots,h \tag{3.17}$$

and

$$\sum_{i=1}^{n} a_{ij} \; x_i \leqslant b_j \qquad j = h + 1 , \ldots, w \tag{3.18}$$

Thus , h of the w constraints given by Eq. (3.17) have a special structure such that the constraint coefficients a_{ij} for each stage appear in a monotonic non-decreasing sequence. Advantage is taken of this property of the constraint set (3.17), in order to enahance the efficiency of the scheduling algorithm. A simple skipping rule is evolved by exploiting the monotonic characteristic of the constraint set (3.17). The rule says that if any of the h constraints (3.17) gets violated at any parent branch then ignore or skip all subsequent descendants at this branch as these will be all infeasible solutions. This is called SKIP RULE II. Stated, mathematically, the rule is :

$$\text{if} \quad \sum_{i=1}^{n} a_{ij} \, x_i \nleq b_j \qquad (3.19)$$

For any of the constraints (j = 1,2,....,h), then, terminate the branch, where the rule is applied.

The modified algorithm incorporating skip rule II is given in Fig. 3.6. A computer program of the flow chart of Fig. 3.6 has been prepared for an IBM 1620 computer. The next section demonstrates the applicacy of the algorithm.

3.3 SAMPLE APPLICATIONS

EXAMPLE 3.1

The excitation system of a generator consisting of a sub-pilot excitor, a pilot excitor, a main excitor and a rectifier unit is shown in Fig. 3.7. The reliability of the system consisting of four stages is to be

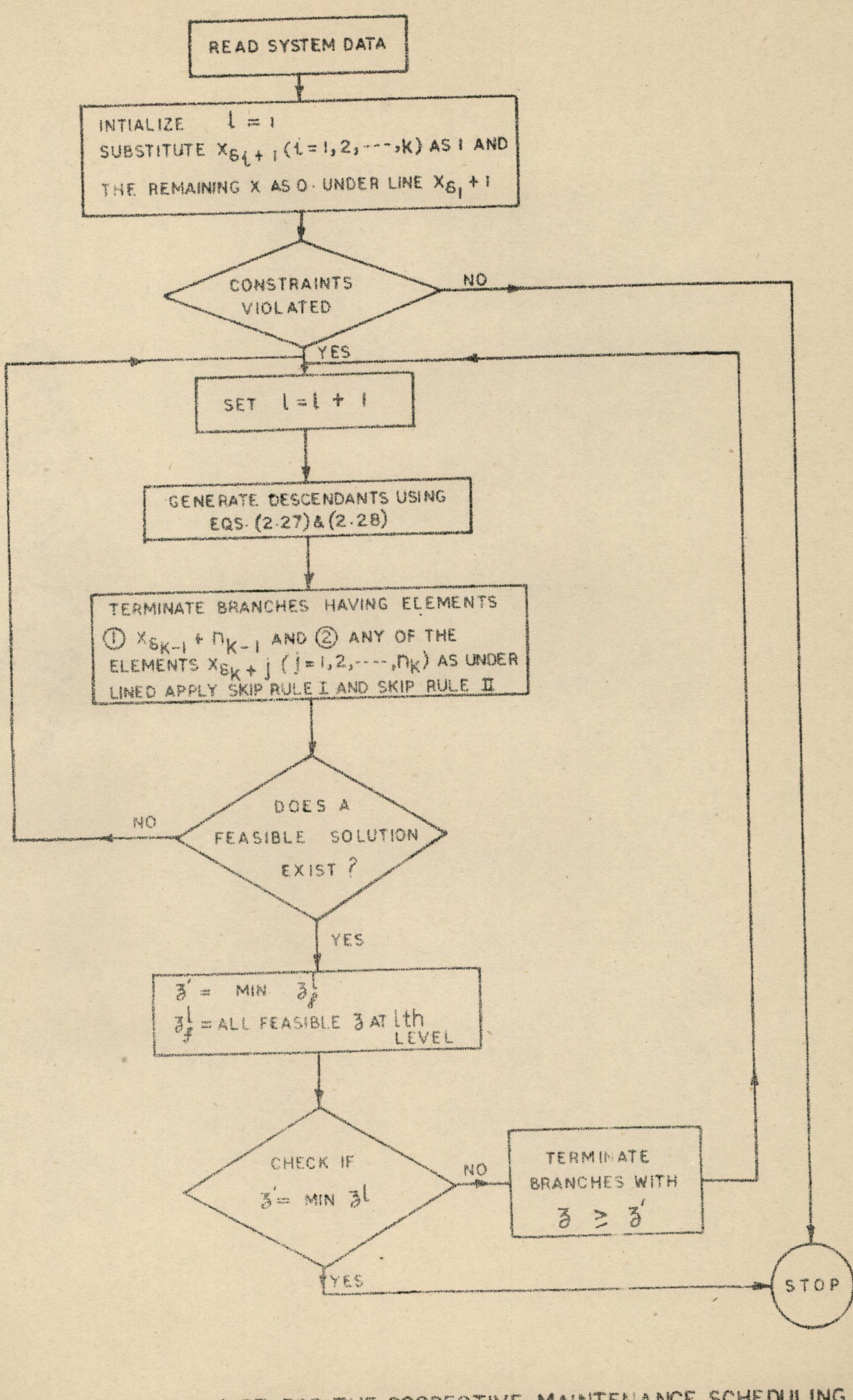


FIG. 3.6 FLOW CHART FOR THE CORRECTIVE MAINTENANCE SCHEDULING ALGORITHM

maximized through the use of mixed redundancies. The stages 1 and 2 are to be supported by standby components and the stage 3 with standby components along with repair facilities. The open circuit failure probability of the rectifiers, because of the voltage spikes arriving, is higher as compared to the short circuit probability of failure. Under these conditions series-parallel arrangement of the rectifiers is to be used. The entire cost and space of the system should not exceed 25.2 and 17 units respectively, the available resources, From the design consideration the maximum number of redundant components and repair facilities at each stage is known.

TABLE 3.1

Failure and Repair Rate Data For Example 3.1

Sub-System	Failure rate (1/yr) ; λ	Repair rate (1/yr) ; μ
Sub Pilot Excitor	0.2	-
Pilot Excitor	0.3	-
Main Excitor	0.5	1.25
Rectifier Diode	O.C. Prob. of failure	0.18
	S.C. Prob. of failure	0.02

The failure and repair rate data for the various subsystems is given in Table 3.1. The complete description of the problem is obtained from Table 3.2.

TABLE 3.2

Detailed Description of Example 3.1

Sub-system	Number of stand by components	Number of Repair facilities	Associated variables	Objective function coeff.	Cost constraint coeff.	space constraint coeff.
Rectifier unit	4	-	x_1	0.00084	0.8	0.0
	3	-	x_2	0.00423	0.6	0.0
	2	-	x_3	0.02340	0.4	0.0
	1	-	x_4	0.13174	0.2	0.0
Main exciter	2	2	x_5	0.01075	20.0	12.0
	2	1	x_6	0.04020	16.0	12.0
	1	1	x_7	0.10821	10.0	6.0
Pilot Excitor	3	-	x_8	0.00027	6.0	6.0
	2	-	x_9	0.00361	4.0	4.0
	1	-	x_{10}	0.03764	2.0	2.0
Sub Pilot Excitor	2	-	x_{11}	0.00115	1.0	1.0
	1	--	x_{12}	0.01768	0.5	0.5

The optimal solution obtained is :

$$x_2 = x_5 = x_9 = x_{12} = 1.$$

$$x_1 = x_3 = x_4 = x_6 = x_7 = x_8 = x_{10} = x_{11} = 0$$

The computer memory requirement and the execution time for the IBM 1620 computer is 1918 words length and 3.0 mins. respectively. The optimal solution is obtained at level 4 of the tree.

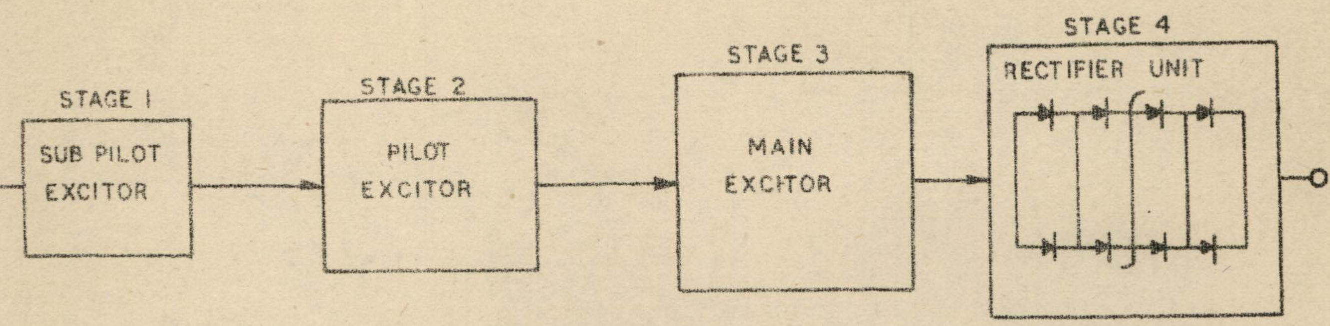


FIG. 3.7 BLOCK REPRESENTATION OF GENERATOR EXCITATION SYSTEM

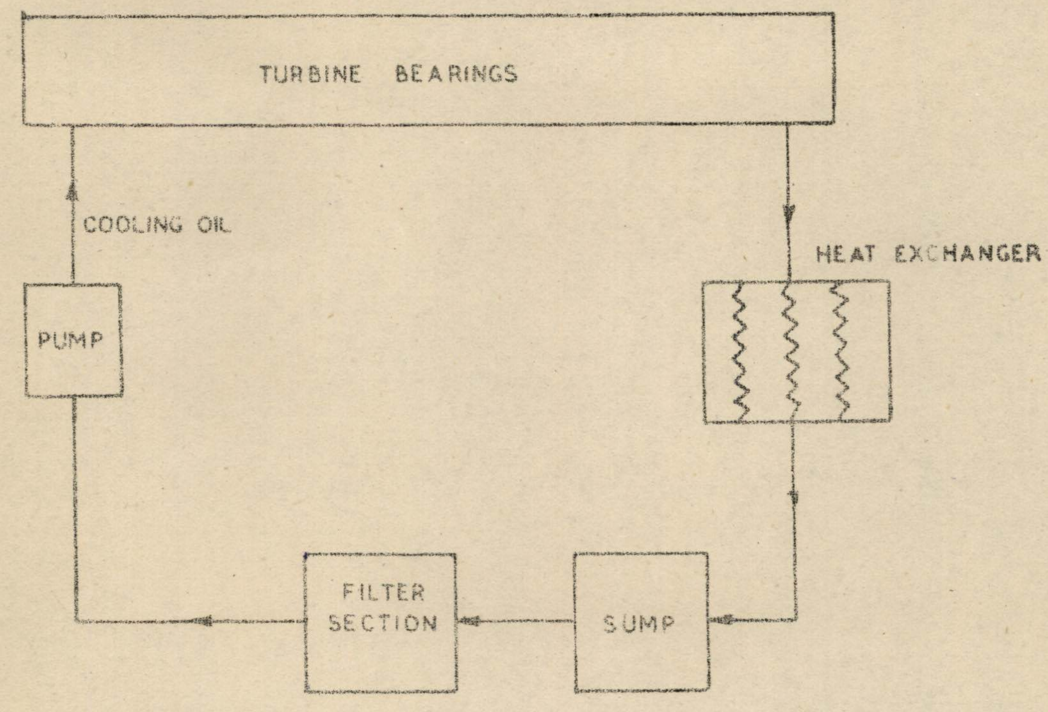


FIG. 3.8 SCHEMATIC REPRESENTATION OF TURBINE COOLING SYSTEM

The results show that the redundant component for stage one and two are 1 and 2 respectively. . Stage three has 2 standby components with 2 repair facilities. For stage four, the number of diodes in each section connected in parallel is 3.

The system reliability achieved is 0.9623.

EXAMPLE 3.2

The cooling arrangement of a turbine consisting of heat exchangers, sump, filter section and an oil circulating pump is shown in Fig. 3.8. The temperature of the turbine bearings is maintained within very precise limits [10]. A schedule of repair and spare components is to be prepared to achieve a minimum system reliability of the value of 0.981 The heat exchanger, which form a m/n system is to be supported with standby units. The filter section is to be supported with standby sections and the oil circulating pump with standby units along with repair facilities.

TABLE 3.3

Failure and Repair Rate Data for Example 3.2

Subsystem	Failure Rate (1/yr) λ	Repair Rate (1/yr) μ
Pump	0.5	2.5
Heat Exchanger section	0.05	-
Filter section	0.2	-

TABLE 3.4

Detailed Description of Example 3.2

Sub-system	Number of standby componen-ts	Number of Repair facili-ties	Associa-ted variables	Object-ive function cost coeff.	Reliabi-lity const. coeff.	Space const. coeff.
Heat exchanger unit	1	-	x_1	2.00	0.13369	2.00
	2	-	x_2	4.00	0.03677	4.00
	3	-	x_3	6.00	0.00990	6.00
	4	-	x_4	8.00	0.00259	8.00
Filter Section	1	-	x_5	0.50	0.30685	1.00
	2	-	x_6	1.00	0.08371	2.00
	3	-	x_7	1.50	0.01917	3.00
	4	-	x_8	2.00	0.00367	4.00
Pump	1	1	x_9	1.50	0.03278	0.5
	2	1	x_{10}	2.25	0.00643	1.0
	2	2	x_{11}	3.00	0.00163	1.0

From design considerations, the maximum number of standby components and repair facilities for each stage is known. The sump is taken to be perfect. The decisions are to remain valid for a period of five years. The entire space should not exceed 12 units, the available resources. A minimum of 2 units are required in the cooling section which means $m = 2$, The cost of obtaining the required schedule is to be minimized.

The failure and repair rate of the various subsystem is given in Table 3.3. The complete description of the problem showing the variables, cost and constraint coefficients is displayed in Table 3.4.

The optimal solution obtained is :

$$x_3 = x_8 = x_{11} = 1$$

$$x_1 = x_2 = x_4 = x_5 = x_6 = x_7 = x_9 = x_{10} = 0$$

$$z^* = 11$$

The computer memory required and the execution time for the IBM 1620 computer is 1898 words length and 5.0 mins. respectively. The optimal solution is obtained at level 7 of the tree.

The results show that the standby component for stage one and two are 3 and 4 respectively. The stage three is to be supported with 2 standby components along with 2 repair facilities. The system reliability achieved is 0.98480.

CHAPTER IV

MAINTENANCE BUDGET SCHEDULING

The financial management of a power utility frequently chooses among competing investments either because the firm's capital is rationed or because some of the projects are interdependent. The future success of a policy, consisting of a set of proposals, depends upon the investment decisions made today. Choosing a sound and effective maintenance policy reduces the system down-time. enhances the life of the equipment and thus increases the revenue to the utility. Bierman and Smidt [6] have duly emphasized , the role of net present value, as a measure of total expected return to the utility. In the present work, integer programming linear and non-linear models are presented for the control of maintenance expenditure on thermal generating units. The objective is aimed at selecting that set of proposals which maximizes the net present value of its total expected return. New and efficient maximization algorithms are developed for the solution of the problems.

4.1 MODELLING

The maintenance budget scheduling problem belongs to an important class of capital budgeting decisions. In the evolvement of an optimal maintenance policy, the selection of a portfolio of proposals is of the utmost importance to the financial management of the utility. Advantage is taken of the existing models [8,27,41], in the formulation of the problem. The problem of maintenance budgeting is discussed both under conditions of certainty and uncertainty. Some of the new terms used in the analysis are discussed.

DETERMINISTIC CASE

First of all, the concept of net present value is explained. This is also referred to as the present discounted value of a return. By investing on the preventive maintenance of a generating unit, the power company purchases the successful operation of the unit for a specific period. As a consequence of this, there is a return associated with this investment. Thus, the net present value of the return to be obtained at a later time is a very useful measure of the alternative proposals. In terms of the formula framework, the promised future reward F of the present sum M is [27] .

$$F = M(1 + r)^n \tag{4.1}$$

$$\text{or} \quad M = \frac{F}{(1+r)^n} \tag{4.2}$$

Eq. (4.2) says that the offer of a promised future reward F, in n years, is worth the present sum M if one's time value of money is r.

Next, the concept of cash flows is of importance in the analysis of capital budgeting problems. The company's directors insist that the total cash outflow, during the time horizon for which the prepared schedule is to remain operative, should be restricted. The cash out flow associated with each alternative proposal during the intervals of the scheduling horizon is known for the analysis.

Thus, under conditions of certainty, the maintanance budgetary control problem is set as a 0-1 integer linear program, whose general form is :

Maximize : $z = NPV$ (Net Present Value)

$$= \sum_{i=1}^{n} c_i x_i \qquad (4.3)$$

subject to :

$$\sum_{i=1}^{n} a_{ip} \; x_i \leqslant b_p \qquad (4.4)$$

$$(p = 1,2,\ldots,m)$$

and $x_i = 0$ or 1 (4.5)

where,

c_i's are the net present value of the various proposals.

a_{ip}'s are constraint coefficients

b_p's are the right-hand-side values of m constraints

n are the total number of proposals.

PROBABILISTIC CASE

When the various available proposals are interacting, there is a risk involved in the achievement of the objective of the firm. In such a case, it is not enough to maximize the function (4.3) alone. The interaction among various proposals are measured by the variances and covariances associated between them . The portfolio manager derives utility or satisfaction from the return obtained. Most of the managers set their objectives, so as, the firm's utility function is quadratic or cubic [27] . Thus a combined objective function is formulated, which results in a maximum return to a firm at the minimum of risk involved. In the present work, a quadratic cost function is considered.

Under conditions of uncertainty, the cash outflows and hence net present values are random variables with forecast means (expected values) and variances. If it is assumed that the company's utility function is quadratic, then the power generating firm should choose that 'portfolio' of proposals, which maximize the following function:

$$\text{Maximize}: \quad z = E(NPV) - A\left\{ \left[E(NPV)\right]^2 + V(NPV) \right\} \quad (4.6)$$

where

E(NPV) Expected Net present value for the set of proposals.

V(NPV) Variance of the Net present value for the set of proposals.

A Power generating firm's coefficient of risk aversion.

From the computations stand point, the expression (4.6) is put in the following convenient form :

$$\text{Maximize} \quad z = \sum_{i=1}^{n} E(NPV)_i \, x_i - A \left[\sum_{i=1}^{n} E(NPV)_i \, x_i \right]^2$$

$$- A \left[\sum_{i=1}^{n} x_i \, x_j \, C\,(NPV)_{i,j} \right] \qquad (4.7)$$

Subject to the constraints (4.4) and (4.5)

where,

$$C(NPV)_{i,j} = \begin{cases} V(NPV) \text{ if } i = j \\ \text{Covariance of the ith and jth} \\ \text{proposals' NPV , otherwise.} \end{cases}$$

The expression (4.7) has the characteristic that if proposal i is rejected, then proposal i makes no contribution to the objective function. If i = j, then no covariance is associated with proposal i. Finally, if proposal j is rejected, no covariance is associated with proposal i.

In the linear (4.3) and nonlinear (4.7) formulations a verctor $S = (x_1, x_2, \ldots, x_n)$ is composed of a number of subsets S_i $(i = 1, \ldots, k)$, where k are the number of stages in the problem. These k of the m constraints (4.4) have a structure as given by Eq.(4.8). That is,

$$\sum_{i \varepsilon k} x(S_i) = 1 \qquad (4.8)$$

Eq. (4.8) signifies that for each stage of the problem, only one proposal is to be selected from a set of proposals. It is shown in the section to follow that k of the m constraints are eliminated because of the desirable attributes of the solution procedure.

4.2 ALGORITHMS

DETERMINISTIC CASE

First, the development of the algorithm is discussed for the deterministic case given by Eqns. (4.3), (4.4) and (4.5). The Eq. (4.8) also holds true for this case. The problem whose solution is to be obtained is the maximization of the objective function. As a first step, systematization is introduced in the model. The objective function coefficients are so arranged that these appear in a monotonic decreasing sequence for each stage or subset. The intutive reasoning for such a move is to obtain the optimal solution with a minimum of the computations, without searching the solution over the whole solution space. Thus, for any stage i , the following inequality (4.9) holds. That is,

$$c_{\delta_i+1} \geqslant C\, c_{\delta_i+2} \geqslant c_{\delta_i+3} \quad \cdots\cdots \quad \geqslant c_{\delta_i+n_i} \tag{4.9}$$

where, n_i are the number of variables in the ith subset, Also,

$$\delta_i = \delta_{i-1} + n_{i-1} \qquad i=2,3,\ldots, k \tag{4.10}$$

$$\delta_1 = 0 \quad \text{initial}$$

The subscript of the cost coefficient c gives the number of the variable to which this coefficient precedes. A graphical display of the characteristic behaviour of the cost function is given in Fig. 4.1 . The tree search method discussed in section 2.2 is applicable for the case at hand. In this case, the problem is that of maximization one, therefore, the property no.5 of the tree given in section 2.2.1 is modified as follows :

The values of z at the ith level of the tree are always less than the maximum of z at the (i-1)th level. Therefore, the inequality (4.11) holds. That is

$$z^i \leqslant \text{Max} \quad z^{i-1} \tag{4.11}$$

The rules for generating the descendants are :

i) $$D_e = k - i + 1 \tag{4.12}$$

$$(\text{if } \bar{x} \neq x_{\delta_i+n_i})$$

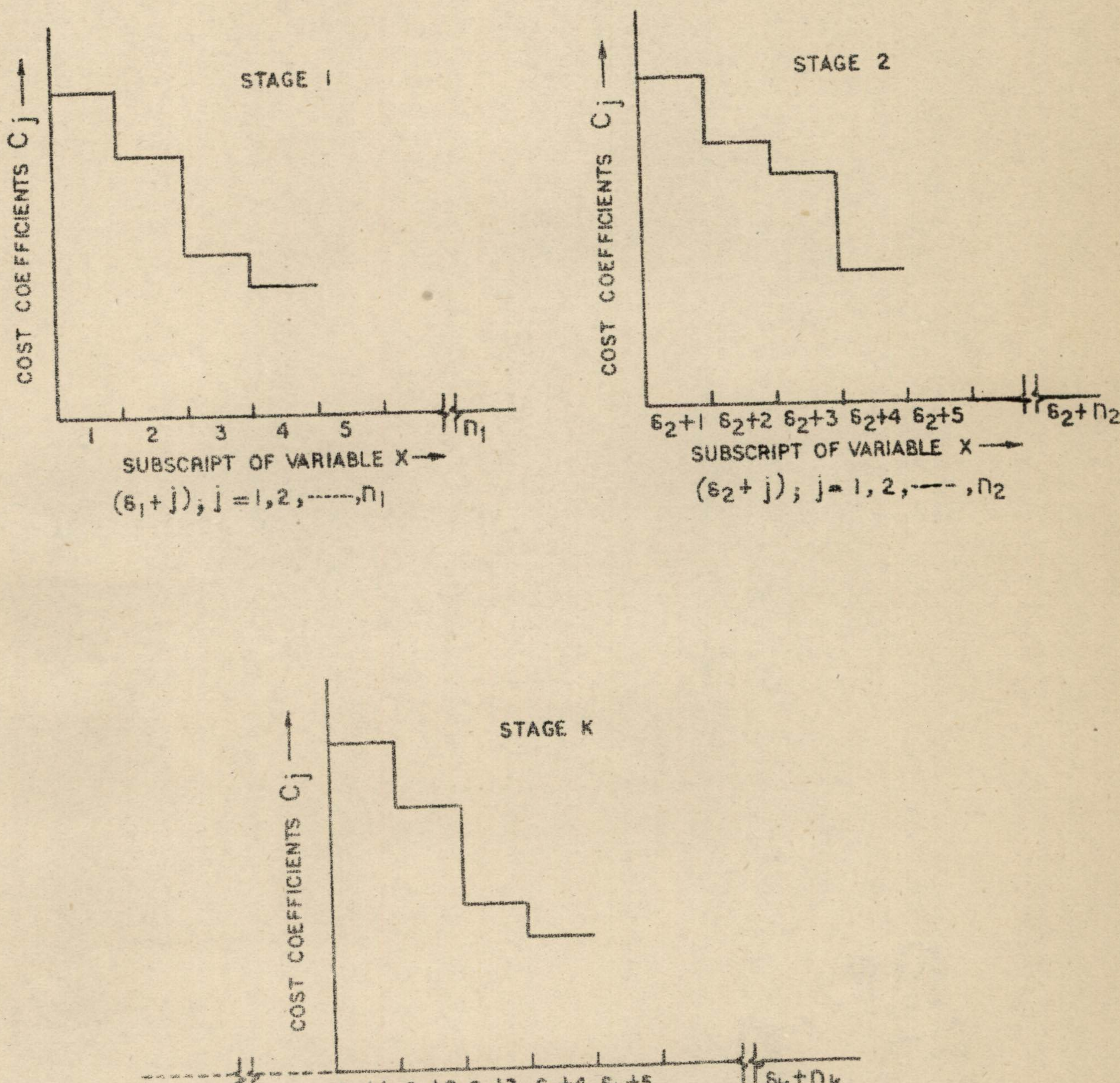


FIG. 4.1 MONOTONIC DECREASING CHARACTERISTIC OF COST FUNCTION

and

ii) $D_e = k - i$ (4.13)

(if $\bar{x} = x_{\delta_i + n_i}$)

where k are the number of stages or subsets and the underlined element lies in the ith subset. Thus, using the concept given above, a computer algorithm is developed. The steps of the algorithm are detailed below :

1. Read System Parameters
2. Initialize solution vector at $\ell = 1$. Enter x_{δ_i+1} $(i = 1,2,\ldots k)$ as unity entries and the remaining $x = 0$. Underline element x_{δ_1+1} . (In the computer program the sign of negation (-) is used for this reference element and other underlined elements).
3. Check constraints, if satisfied, stop, else go to 4.
4. Advance the level counter by one i.e. $\ell = \ell + 1$. Generate Descendants by using Eqs (4.12) and (4.13). Go to 5.
5. Search for a feasible solution at the ℓth level. Terminate the branches having $x_{\delta_{k-1}+n_{k-1}}$ element as underlined and the branches having any of the elements x_{δ_k+j} $(j = 1,2,\ldots, n_k)$ as underlined. Apply SKIP RULE 'I (Refer section 2.2.2). Go to 6.
6. Does a feasible solution exist ? If yes, go to 7, else go to 4.

7. Store z' , which is the maximum feasible z at the ℓth level as z interesting. Check if z' is the maximum of all z at this level. If No, go to 8, else stop.

8. Terminate branches having $z \leqslant z'$. Go to 4.

A flow chart of the algorithm for the deterministic case is given in Fig. 4.2. The algorithm has been programmed in Fortran IIon an IBM 1620 computer.

PROBABILISTIC CASE

Now, the development of the algorithm is presented for the case given by Eqs. (4.7) , (4.4) , (4.5). In this case also the equality (4.8) holds true. Here the objective function is nonlinear and has a random behaviour. If the objective function has a monotonic characteristic, one could exploit this property and proceed as in the linear case. This advantage is not present here. Thus, for the nonlinear case the solutions are searched along the branches of the tree. When a feasible solution is obtained, it is stored as an interesting case. Now, if a better feasible solution results, it replaces the earlier interesting solution. A computer algorithm is devised for the case discussed. The steps of the algorithm are :

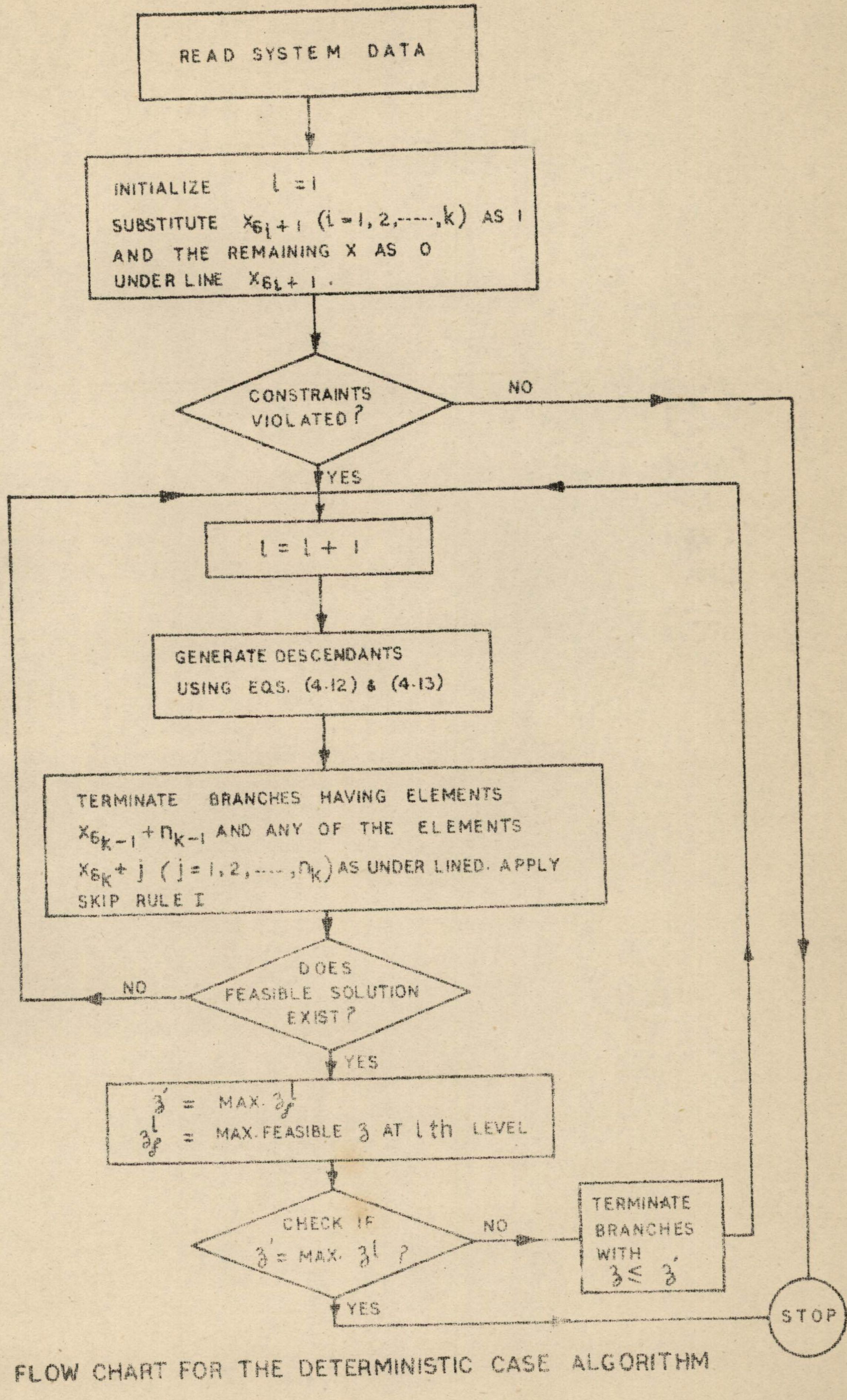


FIG. 4.2 FLOW CHART FOR THE DETERMINISTIC CASE ALGORITHM

1. Read system Parameters

2. Initialize solution vector at $\ell = 1$. Enter x_{δ_i+1} $(i = 1,2,\ldots,k)$ as unity entries and the remainning $x = 0$, Underline element x_{δ_1+1}.

3. Check constraints. If satisfied, store $z' = z$ feasible . Go to 4.

4. Advance the level counter by one i.e. $\ell = \ell+1$. Generate descendants using Eqs.(4.12) and (4.13). Go to 5.

5. Search for a feasible solution at the ℓth level. Terminate the branches having $x_{\delta_{k-1} + n_{k-1}}$ element as underlined and the branches having any of the elements x_{δ_k+j} $(j = 1,2,\ldots,n_k)$ as underlined. Apply SKIP RULE I (Refer section 2.2.2) Go to 6.

6. Does a feasible solution exist ? If yes, go to 7 else go to 4.

7. Store z', which is the maximum feasible z upto the ℓth level. Check if $x_{\delta_i+n_i}$ $(i = 1,2,\ldots,k)$ vector with unity entries has reached. If yes, stop, else go to 4.

A flow chart of the algorithm for a probabilistic case is given in Fig. 4.3 . A computer program for the algorithm has been prepared in Fortran II for IBM 1620 computer.

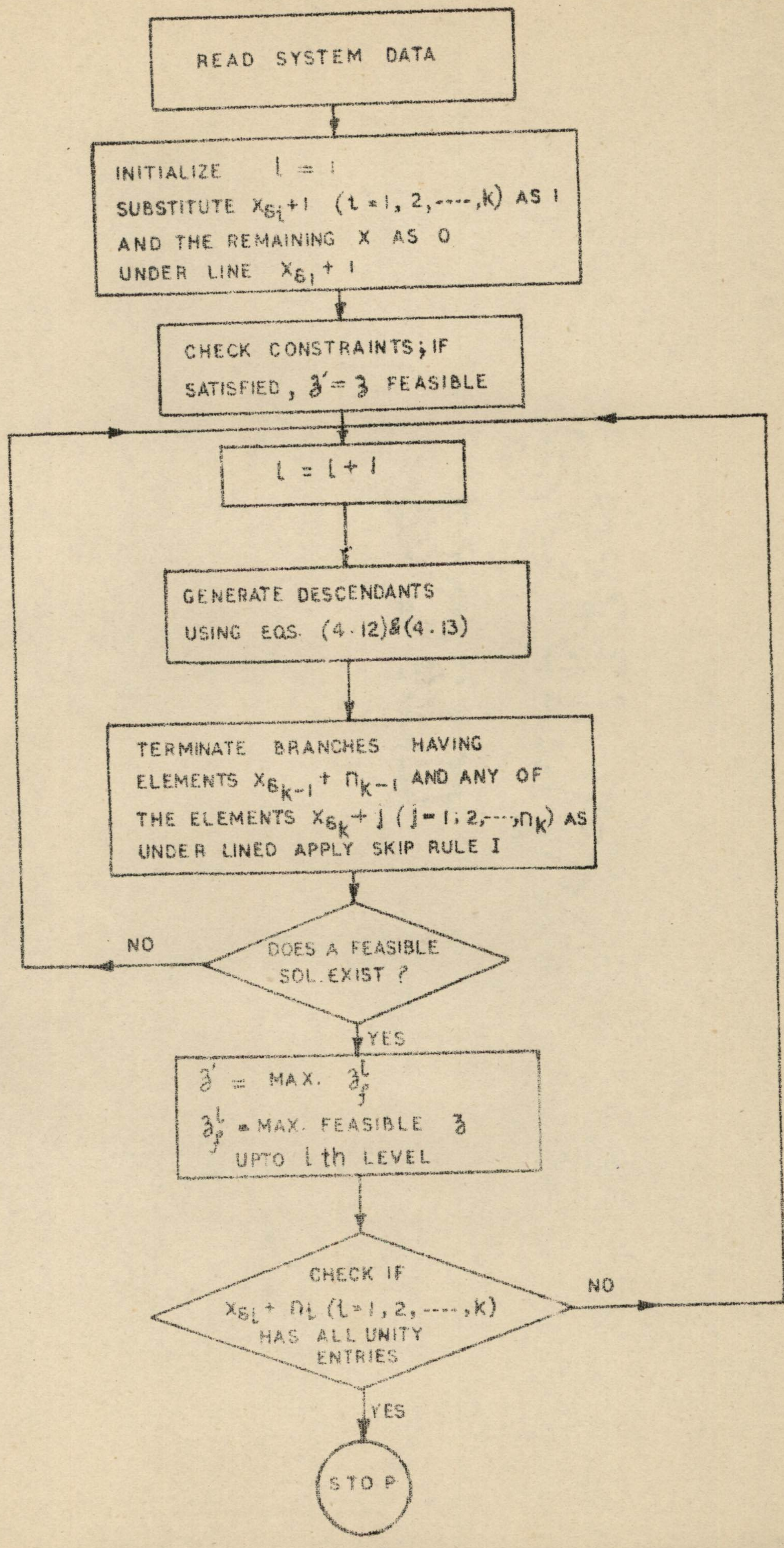


FIG. 4.3 FLOW CHART FOR THE PROBABILISTIC CASE ALGORITHM

4.3 SAMPLE APPLICATIONS

EXAMPLE 4.1 DETERMINISTIC CASE

A power generating utility is obliged to choose one alternative from each of the available sets of alternative proposals for its three newly installed units. Each generating unit is made up of a boiler, a turbine and a generator. From practical considerations it is

TABLE 4.1

Data of Alternative Proposals

Stage No.	Generator Cap. MW	Proposal No.	Nature of Investment	NPV	Repair men Reqd.	Cash aoutflows				
						Yr 1	Yr 2	Yr 3	Yr 4	Yr 5
1	20	1	Contract Maintenance to the manufacturer	1425	45	14	14	14	14	14
		2	Expand the existing maintenance facilities.	1420	40	100	20	10	0	0
2	10	3	Contract maintenance to the manufacturer	650	30	30	4	4	4	4
		4	Sub contract maintenance to another firm	596	20	14	8	8	8	8
		5	Expand the existing maintenance facilities	350	15	20	20	20	12	6
3	15	6	Contract the maintenance to the manufacturer	987	30	60	4	0	0	16
		7	Subcontract maintenance to another firm	825	25	30	24	8	8	8
		8	Expand the existing maintenance facilities	757	20	10	10	15	10	4

decided that the maximum number of repairmen should not be more than 100. The power utility is not in a position to expand its existing maintenance facilities for both of the 10 MW and 15 MW generating units simultaneously. The complete data for the problem is given in Tables 4.1 and 4.2. The data used are hypothetical. Select that set of proposals, which will maximize the net present value of the return to the utility over the next five years.

TABLE 4.2

Maximum cash outflows

Year	Max. cash outflow
1	140
2	60
3	30
4	30
5	30

Thus, our objective is to

Maximize z =

$$1425\,x_1 + 1420\,x_2 + 650\,x_3 + 596\,x_4 + 350\,x_5 + 987\,x_6 + 825\,x_7 + 757\,x_8 \quad (4.14)$$

Subject to the constraints

$$45x_1 + 40x_2 + 30x_3 + 20x_4 + 15x_5 + 30x_6 + 25x_7 + 20x_8 \leqslant 100 \quad (4.15)$$

$$14x_1 + 100x_2 + 30x_3 + 14x_4 + 20x_5 + 60x_6 + 30x_7 + 10x_8 \leqslant 140$$

$$14x_1 + 20x_2 + 4x_3 + 8x_4 + 20x_5 + 4x_6 + 24x_7 + 10x_8 \leqslant 60$$

$$14x_1 + 10x_2 + 4x_3 + 8x_4 + 20x_5 + 0x_6 + 8x_7 + 15x_8 \leqslant 30$$

$$14x_1 + 0x_2 + 4x_3 + 8x_4 + 12x_5 + 0x_6 + 8x_7 + 10x_8 \leqslant 30$$

$$14x_1 + 0x_2 + 4x_3 + 8x_4 + 6x_5 + 16x_6 + 8x_7 + 4x_8 \leqslant 30$$

(4.16)

$$x_5 + x_8 \leqslant 1$$

(4.17)

$$x_1 + x_2 = 1$$

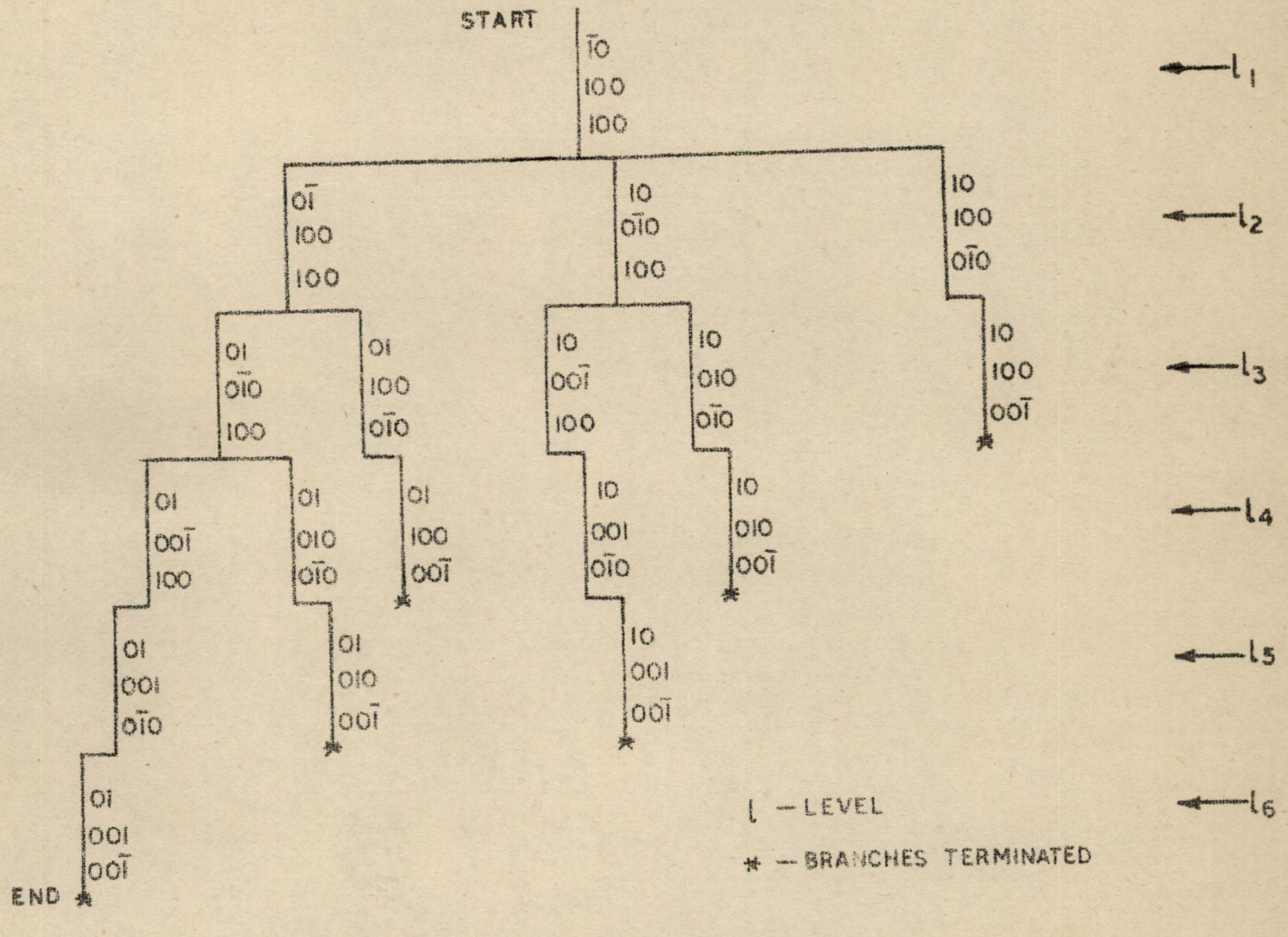
$$x_3 + x_4 + x_5 = 1$$

$$x_6 + x_7 + x_8 = 1$$

(4.18)

In the above formulation, constraint (4.15) represents the limit on maximum number of repairmen. Constraint set (4.16) restricts the cash flows. Constraint (4.17) is included because of power generating firm's limitation, of not being able to expand the existing maintenance facilities for both the 10MW and 15 MW generating units simultaneously. The constraint set (4.18) signifies that only one alternative is to be chosen from each set. This set is eliminated because of the desirable attributes of the solution procedure.

The algorithm of Fig. 4.2 is used for finding the solution to the above problem. The tree diagram applicable for this case is shown in Fig. 4.4.



FIG·4·4 COMPLETE DECISION TREE FOR THREE STAGE EXAMPLE

Here $c_1 \geqslant c_2$; $c_3 \geqslant c_4 \geqslant c_5$; $c_6 \geqslant c_7 \geqslant c_8$.

The optimal solution obtained is

$$x_1 = x_3 = x_7 = 1$$

$$x_2 = x_4 = x_5 = x_6 = x_8 = 0$$

$$z^* = 2900$$

The computer memory and the execution time for the IBM 1620 computer is 1948 words length and 2.0 mins. respectively. The solution is obtained at level 2 of the tree. Thus, the power generating firm accepts the proposals 1,3, and 7 with a return of 2900.

EXAMPLE 4.2 PROBABILISTIC CASE

The variance and co-variance data associated with a set of proposals of Table 4.1 is portrayed in Table 4.3. The constraints for the problem are the same as given by (4.15), (4.16) , (4.17) and (4.18). The utility's coefficient of risk aversion is of the value $(2.0) \times 10^{-4}$. A schedule which results in a maximum expected return is to be obtained.

TABLE 4.3

Variance and Co-variance Data of Various Proposals

→ j Proposal No. ↓ i	1	2	3	4	5	6	7	8
1	1000	0	3000	1500	-1000	3000	4500	- 990
2	0	400	1200	1000	- 800	1200	960	- 600
3	3000	1200	14000	0	0	10000	4800	-3300
4	1500	1000	0	4900	0	6500	3400	-2100
5	-1000	- 800	0	0	3600	-3960	-2800	1800
6	3000	1200	10000	6500	-3960	12000	0	0
7	2000	960	4800	3400	-2900	0	6400	0
8	- 990	- 600	-3300	-2100	1800	0	0	2500

Thus, the objective is to

Maximize $z =$

$$1425x_1 + 1420x_2 + 650x_3 + 596x_4 + 350x_5 + 987x_6 + 825x_7 + 757x_8$$

$$-A\left[1425x_1 + 1420x_2 + 650x_3 + 596x_4 + 350x_5 + 987x_6 + 825x_7 + 757x_8\right]^2$$

$$-A\Big[1000x_1^2 + 3000x_1x_3 + 1500x_1x_4 - 1000x_1x_5 + 3000x_1x_6 + 4500x_1x_7 - 990x_1x_8\Big) \; i = 1$$

$$\vdots$$

$$-990x_8x_1 - 600x_8x_2 - 3300x_8x_3 - 2100x_8x_4 + 1800x_8x_5 + 2500x_8^2\Big) \; i = 8\Big]$$

(4.19)

The algorithm of Fig. 4.3 is used for obtaining the solution to (4.19) subject to the constraints (4.15), (4.16), (4.17) and (4.18). The tree diagram of Fig. 4.4 is also applicable for this case. The optimal solution obtained is

$$x_2 = x_3 = x_8 = 1$$

$$x_1 = x_4 = x_5 = x_6 = x_7 = 0$$

$$z^* = 1226$$

The computer memory and the execution time for the IBM 1620 computer is 2012 words length and 4.0 mins. respectively. The optimal solution is obtained at level 4 of the tree. Thus, under conditions of uncertainty the power generating firm accepts the proposals 2,3 and 8 with an expected return of 1226.

4.4 COMPARISON OF RESULTS

A study was carried out to compare the efficiency of the newly developed tree search optimization technique with the earlier used Lawler and Bell [37] method for the investment decision scheduling problems [41]. Example 4.1 was solved using both these methods and results of comparison are portrayed in Table 4.4.

TABLE 4.4

Results of comparison

Basis for comparison	Lawler and Bell Method	Tree search method developed by the Author
Number of constraints	13	7
Solution space size	256	18
Number of searches required to arrive at the optimal solution	81	10

In the Lawler and Bell method, the number of constraints are large, because each equality constraint has to be transformed into two inequality constraints, one of the 'less than type' and second of the 'greater than type'. Thus, the constraint set (4.18) having 3 constraints is replaced by 6 constraints. In the tree search method these equality constraints are absorbed in the search procedure and thus get eliminated from the problem. The example 4.1 has 7 constraints when solved by the tree search method and 13 constraints when solved via Lawler and Bell method. Further the size of each constraint is doubled in the Lawler and Bell procedure for applying the skip rules.

The size of the solution space over which the optimal solution has to be searched is much smaller

$\left(\prod_{i=1}^{k} n_i = 18 \right)$ in the case of tree search method as compared to the total solution space $(2^8 = 256)$. Such a vast reduction in the solution space results as the tree search method exploits the special structure of the model which has not been done earlier. A large number of infeasible solutions are never generated.

In the Lawler and Bell method the search is initiated by assigning one variable as unity and the remaining variables as zero. Two side vectors are generated for the implementation of the skip rules. A number of intermediate steps are involved in going from one vector to the next and these are time consuming. Some of the infeasible solution vectors are skipped in the search process. In the tree search method, the search process is direct and systematic. By employing the Lawler and Bell method to example 4.1, 81 searches have to be made in order to find the optimal solution. In the tree search method only 10 searches are made for arriving at the optimal solution. Thus, the newly developed technique of analysis is superior to the earlier used method for the specialized problem discussed.

CHAPTER V

GENERATION SCHEDULING

An important class of scheduling problems are discussed in literature under the title of "Generation Scheduling". These relate to the scheduling of real and reactive power in power systems. Many useful contributions have appeared in this area and many more are appearing. The problems are diverse in nature and are discussed under the mathematical models of mixed integer programs, continuous variable programs and integer programs. In the work reported, a unit commitment scheduling problem and a real power scheduling problem are discussed. New and efficient scheduling algorithms are developed and sample applications are presented. The work is primarily useful for real time operation of power systems and also for study purposes.

5.1 UNIT COMMITMENT SCHEDULING

For the reliable operation of a power system, it is first necessary to evolve an optimal maintenance policy [32] for the generating units and its auxiliary systems. After the units have been scheduled for preventive maintenance on annual basis, the next problem is the selection of units out of the available set for real time operation. This is referred to as the problem of unit commitment scheduling. The scheduling horizon over which such a decision has to be valid may be a couple of hours, a day or even a week. The objective function to be minimized is the total production cost, which is a summation of the running cost, shut down cost and time dependent start-up costs during the intervals of the scheduling horizon.

The earlier practice was to start up and shut down units in accordance with a priority list based on unit heat rates [4, 29]. Many a times this would also involve the discretion of the system operator or dispatcher based on his own experience. As a consequence of this such rules as - units are shut down if not required for a preselected interval are very common. Such an approach could impair reliability and economics of operation. The work in the area of application of mathematical programming techniques to the problem of unit commitment started about a decade ago and many useful contributions have been presented. Garver [23] has advanced an integer programming formulation of the problem. Muckstadt and Wilson [39] use a mixed integer linear model and employ Benders Decomposition to find a solution. The start up cost model included in the mixed integer formulation is approximate as it is not time dependent. The cost function considered is linear and thus the solution procedure cannot accommodate nonlinear (quadratic or cubic) cost functions. The computation burden of the algorithm presented is large and is only useful for very small size systems. A 2 generator 2 interval problem is solved. Lowery [38] has recommended the use of dynamic programming. Subsequently Guy [25] and Ayoub and Patton [1] have used dynamic programming approach incorporating the evaluation of security into the model. The computer storage and the computation burden are large.

The present work takes a different approach to the problem of unit commitment and presents a new direct iterative optimization procedure based on the premise of feasibility and economic dispatch. The heuristics developed limit the search in the region of interest and help to speed up the enumeration of binary vectors. The security function incorporated provides a means for assessing system security in hour-to-hour operation on a probabilistic basis. Based on the concepts detailed above, a scheduling algorithm is designed and its applicacy tested on a medium size system. The results of computation are presented.

5.1.1. MODELLING

The problem of unit commitment is set up as a mixed-integer program, whose general form is

$$\text{Minimize } f(P_G, y) = f_1(P_G) + f_2(y) \qquad (5.1)$$

Subject to

$$g_1(P_G) + g_2(y) \geq 0 \qquad (5.2)$$

$$h_1(P_G) \geq 0 \qquad (5.3)$$

$$h_2(y) \geq 0 \qquad (5.4)$$

$$P_G \geq 0 \quad , \quad y = 0 \text{ or } 1 \qquad (5.5)$$

Where P_G is the real variable vector, corresponding to the generations of various units, y is the 0-1 variable vector corresponding to the status of the units; $g_1 + g_2$ in (5.2) represent m constraint functions and h_1, h_2 are the constraints involving only P_G and only y respectively. For example security constraint belongs to the g_1+g_2 set, the generation equals load demand belongs to h_1 and if because of limited labour force available say only one unit can be started at a time, this belongs to constraint set h_2. The function $f_1(P_G)$ varies nonlinearly with the power output P_G and $f_2(y)$ is a function of y and represents the start up and shut down costs.

FUEL COST CALCULATION MODEL

The cost curves (or input-output curves) of fossil fuel generating units are assumed to be nonlinear. The normal practice [1,25] is to approximate these curves by quadratic functions. Since, in a composite system units are of different ages and types, therefore, for some or all units cubic [42] or higher order functions may give a more faithful representation of the input-output curves. In view of this, it is desirable that the method used should be able to handle a set of polynomials of the form given in (5.6).

$$f_i(P_G) = A_{1i} + A_{2i}P_G + A_{3i}P_G^2 + A_{4i}P_G^3 + \dots \quad (5.6)$$

where,

$f_i(P_G)$ = fuel cost of ith generator supplying P_G MW in Rs./hr.

$A_{1i}, A_{2i}, A_{3i}, A_{4i}$ = cost function constants associated with ith generator.

START-UP AND SHUT-DOWN COSTS

The act of starting or removing a unit from line involves labour and money. The decision to shut down a unit depends upon a number of factors which include:

1. The number of hours the unit can be shut down before it is required again, i.e. the shape of the hourly integrated load time curve for the period considered.
2. The cost of start up and the shut down costs.
3. The relative efficiency of the unit to be shut down compared to the efficiencies of the units left running.

Now, if a decision is to be taken to shut down a unit, the boiler will either be shut down and allowed to cool on it will be banked and continue to be supplied with fuel to maintain boiler pressure and temperature. Clearly, the latter alternative will be chosen when the unit is to be required again in a short time.

It is assumed that when a unit is shut down the boiler is allowed to cool and hence its temperature will fall exponentially. The cost of restart [4] the unit is given by Eq. (5.7).

$$S(y) = B\left[1 - e^{-\alpha(T-1)}\right] + K_T \qquad (5.7)$$

where, T is the number of hours the unit has been shut down. B is the cost of starting the boiler cold. K_T is the cost of starting the turbine alone. α is the cooling time constant of the boiler. For simplicity, the above expression is written as [1],

$$S(y) = S_{\gamma}\left[\frac{\alpha T}{1 - \alpha T}\right] \qquad (5.8)$$

where S_{γ} is the cold start up cost of the complete unit.

From the above discussion it transpires that a case arises for making trade off analysis between total production cost, including start up and shut down costs by closing a unit or keeping it on line.

UPPER AND LOWER BOUNDS ON THE VARIABLES

The power P_{Git} for the generating unit i at any time t may be zero or between the two limits P_{min_i}

and P_{max_i} . A bivalent variable represents this two state condition :

$$y_{it} = 0 \qquad \text{if } P_{Git} = 0$$

$$y_{it} = 1 \qquad \text{if } P_{min_i} \leqslant P_{Git} \leqslant P_{max_i}$$

The above may also be represented by the constraint Eq. (5.9) , i.e.,

$$P_{min_i} \; y_{it} \leqslant P_{Git} \leqslant P_{max_i} \; y_{it} \qquad (5.9)$$

SPINNING RESERVE

The availability of spinning reserve is a basic requirement for the reliable operation of any system. Its location in the system will be a result of the computation. It is a common practice to maintain spinning reserve equal to or larger than the operating set. If R_t MW is the amount of spinning reserve during any hour t , then the constraint eq. (5.10) must hold. That is,

$$\sum_{i=1}^{n} P_{max_i} \; y_{it} - \sum_{i=1}^{n} P_{Git} - R_t \geqslant 0 \qquad (5.10)$$

Also,

$$\sum_{i=1}^{n} P_{Git} = P_{D_t} \qquad (5.11)$$

Eq.(5.11) signifies that the load demand is always met. P_{D_t} is the load demand for the interval t and n are the total number of units on-line.

The missing ingradient in the above representation is the failure and repair rate consideration of different units. Therefore, a more rational way of meeting the spinning reserve need is through the evaluation of short term reliability [47] or security of the system. For incorporating the security model into the scheduling algorithm a maximum tolerable insecurity level (MTIL) is defined through simulation studies. A quantitative measure of system security is obtained by calculating a dimensionless security function S(t) as follows :

$$S(t) = \sum_{i} P_i(t)\, Q_i(t) \qquad (5.12)$$

where,

$P_i(t)$ = Probability that the system is in state i at time t

$Q_i(t)$ = Probability that state i is a condition for which the load exceeds the generation at time t.

The summation (5.12) is in theory carried out over all possible system states, but in practice only needs to be carried out over states reflecting a relatively small number of forced outages. $Q_i(t)$ functions as a true probability only when uncertainty exists as to wheather or not a certain system state constitutes a breach of security at time t . In the case at hand, uncertainty may arise due to the fact that the load at the future time t is not known precisely at the time the security function is calculated, and the values of $Q_i(t)$ will then depend on the load forecasting method. However, it is assumed that the load forecasts are exact and consequently $Q_i(t)$ takes on values of either zero or unity. That is, either a system state constitutes a breach of security or it does not . $Q_i(t)$ assumes that value of unity if the system load exceeds the available capacity, and the value zero, otherwise. The remaining problem is the determination of the state existence probabilities, $P_i(t)$. Assuming that the operations of generating units is modelled as a discrete-state continuous transition Markov process, it is shown [47] that $P_{dn_m}(t)$, the probability that any unit m is down at time t , given that it was up at time zero , is equal to :

$$P_{dn_m}(t) = \frac{\lambda_m}{\lambda_m + \mu_m}\left[1 - e^{-(\lambda_m + \mu_m)t}\right] \tag{5.13}$$

where λ_m and μ_m are the failure and repair rates of unit m, respectively. The probability of finding the mth unit in the upstate is given by (5.14), i.e.,

$$P_{up_m}(t) = 1 - P_{dn_m}(t) \tag{5.14}$$

Now, assuming no standby generators or t less than the time required to start a standby generator, the probability of system state is found as follows:

$$P_i(t) = \prod_{j \in X_i^*} P_{dn_j}(t) \prod_{k \in Y_i^*} P_{up_k}(t) \qquad (5.15)$$

where,

X_i^* = set of generators which are down in state i

Y_i^* = set of generators which are up in state i

Therefore, to calculate the probability of the existance at any state at any time, it is necessary only to combine the appropriate unit state probabilities in a multiplicative process.

5.1.2 ALGORITHM

For evolving practical algorithms, it is conceivably true that the problem model be critically analysed and its properties used in order to reduce the computational burden of the algorithm. The solution procedure should be based on simple logic and the computations should also be arranged in a systematic fashion, as far as possible, in order to obtain the solution with a minimum of the computations. Such desirable attributes are present in the method discussed below.

DIRECT ITERATIVE PROCEDURE

In the formulation (5.1) y is called a vector of complicating variables in the sense that (5.1) is a much easier optimization problem in P_G when y is temporarily held fixed. This is called as the continuous nonlinear subproblem.

The solution procedure for the continuous nonlinear subproblem is based on the premise of feasibility and economic dispatch. A feasible solution vector P_G corresponds to the conditions when generation meets the load demand and the variables are within their upper and lower bounds. Thus, during any interval t , the equality (5.11) must hold. In the procedure, starting from any feasible point and reaching the final solution point in a finite number of iterations, the total change in the variables is zero. Thus, the equality (5.16) holds, That is,

$$\sum_{i=1}^{n} P_{Git} + \sum_{i=1}^{n} \Delta P_{Git} = P_D \qquad (5.16)$$

Also

$$\sum_{i=1}^{n} \Delta P_{Git} = 0 \qquad (5.17)$$

Provided changes in losses are neglected.

For example, if P_{G_1} and P_{G_2} are the two pivot variables corresponding to the lowest and the highest incremental cost values, then for the next solution point, these variables are incremented by $+\Delta P_G$ and $-\Delta P_G$ respectively . The change ΔP_G is so chosen that there is no overcrossing of new P_{G_1} and P_{G_2} variable values. During this process if a variable reaches at its boundary, it is held fixed and no further change is made in this variable. Thus, the method is called a multivariable constrained search procedure. In this method, every move is a move of success and this enhances the efficiency of the procedure. Such a procedure terminates to a convergent solution in a finite number of step moves.

The next important point is about the enumeration of binary vectors. Supposing we have started our computation by fixing all binary variables to unity and the convergent solution to the nonlinear sub-problem is obtained. The question to be answered is that which of the binary variables be made zero next. In power system parlance, it means that which of the generators be switched off in order to obtain a better feasible solution. This decision is guided by the indices developed from the continuous nonlinear sub-problem in the preceding step. These indices are obtained by the ratio of the fixed charge cost and the variable cost of each generator corresponding to expression (5.6). Thus, the generator having the highest index is eliminated first and so on. Such heuristics limit the search in the region of interest by limiting the number of binary vectors to be enumerated.

The security calculation sub-program is very neatly embedded in this procedure. This further helps in reducing the number of binary vectors to be enumerated. Thus, by following the above procedure, the minimum cost solution for a specific interval is obtained by enumerating a few binary vectors.

Based on the concepts detailed above, a scheduling algorithm is developed. A tradeoff analysis with start up cost is included within the scheduling horizon This is based on the premise that there is a possibility of achieving a less costlier operating cost schedule by not shutting down some of the units for various periods of time. Thus, if the operating cost by keeping a unit on line is smaller than the operating cost (running cost plus later start up cost) by not keeping the unit on line, then the first alternative is accepted. The procedure is similar to the one given in the reference [1]. The main steps of the algorithm are :

1. Read system Parameters. Initialize time $t = 0$, Go to 2.
2. Advance the time counter by 1 i.e. $t = t+1$, Go to 3

3. If time t is greater than t_{max}, the maximum time of the scheduling horizon, stop, else go to4.

4. Generate a feasible schedule and minimize by direct iterative procedure. Evaluate secuity function and compare with MTIL Go to 5.

5. Check convergence ? If No, go to 6 , else go to 7.

6. Enumerate next binary vector and go to 4.

7. Compare $(y_{it} - y_{it-1})$? If zero or minus one go to 2 , else go to 8.

8. Calculate start up costs. Is trade off for start up cost needed ? If no , go to 2 . If yes, reset $t = t^*$, go to 2.

A flow chart for the unit commitment algorithm is presented in Fig. 5.1. In this t_{max} corresponds to the number of intervals in the scheduling horizon. $(y_{it} - y_{it-1})$ establishes as to which of the units have been started or shut down for the time interval t. Also t^* corresponds to an earlier time interval when a unit was shut down . This helps in making trade off analysis with the start up cost between different intervals within the scheduling horizon. A computer program for the algorithm has been prepared in fortran II for an IBM 1620 computer.

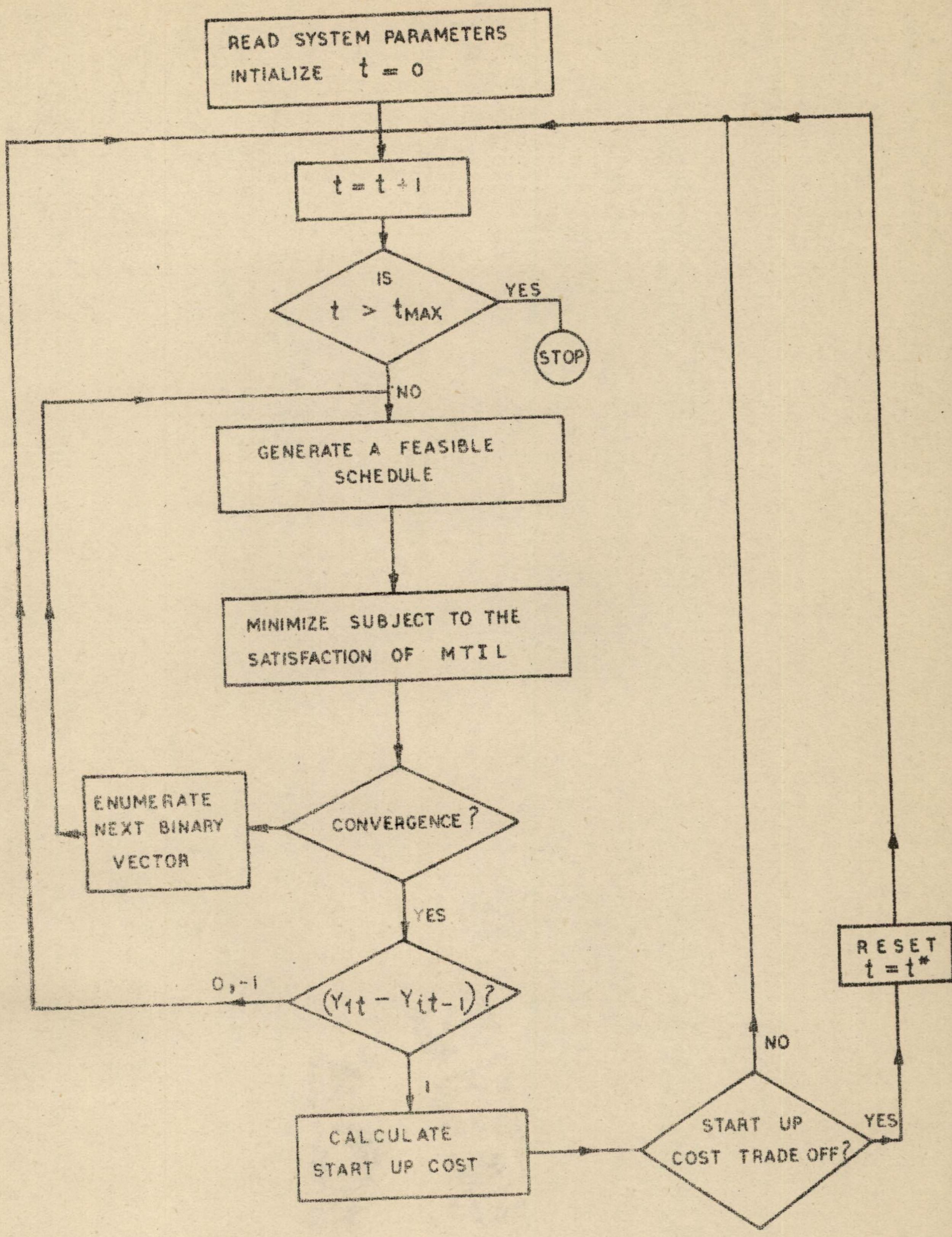


FIG. 5.1 FLOW CHART FOR THE UNIT COMMITMENT ALGORITHM

5.1.3 SAMPLE APPLICATION

The method of the paper has been applied to a sample system [1] having ten generating units the parameters of which are given in Table 5.1 . The cooling rate of all units, α is taken to be 0.25 . All units

TABLE 5.1

Generating Unit Parameters* for Sample Application

Unit No.	Capacity (MW)	Minimum output (MW)	cost curve parameters			Cold start up time (hr)	Cold start up cost Rs.	Failure rate 1/yr	Repair rate 1/yr
			A_1 Rs.	A_2 Rs./MW	A_3 Rs/MW^2				
1	60	15	105	15.4238	0.03570	3	595	1.2	151
2	80	20	175	13.4127	0.02772	3	707	1.2	151
3	100	30	280	12.9626	0.02751	3	798	1.2	151
4	120	25	224	11.8752	0.02674	4	658	2.5	585
5	150	50	203	12.6105	0.01484	4	791	2.5	585
6	280	75	504	10.7470	0.01827	6	1232	2.6	638
7	320	120	343	8.8501	0.02023	8	1309	2.6	638
8	445	125	574	8.4952	0.01136	10	1589	2.6	638
9	520	250	735	8.3678	0.00889	12	1869	4.0	638
10	550	250	700	7.8995	0.00945	12	1974	4.0	219

* The data is taken from reference [1] . A conversion factor of one dollar = seven rupees is used.

TABLE 5.2

Minimum Cost Schedule

Hr	Unit status										Running cost Rs	Start up cost Rs	Load MW
	1	2	3	4	5	6	7	8	9	10			
1	1	1	1	1	1	1	1	1	1	1	29642.697	-	2000
2	1	1	1	1	1	1	1	1	1	1	29312.472	-	1980
3	1	1	0	1	1	1	1	1	1	1	28485.618	-	1940
4	0	1	0	1	1	1	1	1	1	1	27729.226	-	1900
5	0	1	0	1	1	1	1	1	1	1	26750.080	-	1840
6	0	1	0	1	1	1	1	1	1	1	27237.973	-	1870
7	0	0	0	1	1	1	1	1	1	1	26323.171	-	1820
8	0	0	0	1	1	1	1	1	1	1	24390.268	-	1700
9	0	0	0	1	1	0	1	1	1	1	21234.570	-	1510
10	0	0	0	0	1	0	1	1	1	1	19580.183	-	1410
11	0	0	0	0	1	0	1	1	1	1	18201.057	-	1320
12	0	0	0	0	0	0	1	1	1	1	17198.461	-	1260
13	0	0	0	0	0	0	1	1	1	1	16293.403	-	1200
14	0	0	0	0	0	0	1	1	1	1	15701.007	-	1160
15	0	0	0	0	0	0	1	1	1	1	15408.106	-	1140
16	0	0	0	0	0	0	1	1	1	1	15701.007	-	1160
17	0	0	0	0	0	0	1	1	1	1	17198.461	-	1260
18	0	0	0	1	0	1	1	1	1	1	19116.307	474.60	1380
19	0	0	0	1	1	1	1	1	1	1	22204.595	1335.60	1560
20	0	0	0	1	1	1	1	1	1	1	24390.268	-	1700
21	0	0	0	1	1	1	1	1	1	1	26323.171	-	1820
22	0	1	0	1	1	1	1	1	1	1	27729.226	-	1900
23	0	1	1	1	1	1	1	1	1	1	28722.183	665.00	1950
24	0	1	1	1	1	1	1	1	1	1	29380.414	-	1990

are assumed to be operating at time zero, the time at which the 24-hour schedule is computed. In practice of course no such assumption would be required since the past operating history of all the units would be known. Knowledge of operating history prior to scheduling period is required for calculation of start up time and costs.

A scheduling period of 24 hours is assumed for the sample application. The predicted load cycle, assumed to be error free, is given in Table 5.2. The value of MTIL used in this study is 0.000450. Operation according to this value of MTIL should result in a loss of load about once every ten years for the sample system according to the results of a Monte Carlo simulation study.

Table 5.2 shows the minimum cost schedule subject to the satisfaction of the security constraint and also including trade off for start up cost.

The total Running cost is Rs. 555,253.92 and the start up cost is Rs. 3033.10.

5.2 REAL POWER SCHEDULING

The optimal scheduling of real power is important from the system economic and other operating considerations. The problem is also referred to as "Economic Power Dispatch".

The theory of the subject is well developed and many useful contributions have appeared in the literature. A well known method which is representative of the state of the art in the developments is the optimal power flow developed by Dommel and Tinney [16]. It schedules real and reactive power, represents losses exactly and satisfies constraints on load voltages, reactive sources and tie line power angles. The method is based upon Newton power flow solution and is much more elaborate than the simplified approach presented. However, the method is based upon explicit and exact solution of the network and is computationally much more difficult and involved. The computer memory and time required are beyond the range of many dispatch-office computer systems. The coupling between the real and reactive power flow is small and the phase angle difference between the two nodes mainly controls the active power flow through the connecting branch, and the difference between voltage magnitudes is mainly related to the reactive power flow [54]. Thus, the static real power scheduling problem is considered decoupled from the reactive power of the system.

In the past, Kirchmayer's method [30] has been well established as an effective method for economic scheduling. Such an approach evaluates losses by loss formula based on Kron's B coefficients [7, 31].

However, the calculation of the loss formula itself involves a considerable computation effort. In the present work, a simplified method is used forthe calculation of transmission loss coefficients. The method is based upon the linearized or d.c. load flow approximation. The accuracy of the method has been extensively tested and is well within the practical requirements of economic dispatch [48]. The method requires a very small computation time and introduces the possibility of updating loss coefficients on line according to actual network status and load conditions. Another limitation of the Kirchmayer's method is that there is no guarantee that the schedules will not violate secure line limits. In the actual operation of power system, there are peak load periods, when one or more lines may get over loaded, if not checked [15]. Therefore, it is necessary to reallocate the generation so that the line constraints are satisfied. In practice, this operation is most commonly performed by manual adjustment of generator output limits. With the demands upon the power system reliability and the increasing size and complexity, it is desirable that the scheduling algorithm accounts for the network constraints.

The problem of real power scheduling has also been solved by gradient projection method [49]. The gradient projection method is basically an extension of the method of steepest descent and has the characteristic limitations of slow convergence near the optimum and oscillations along a steep sided valley . Thus, the method become unsuitable when the cost function is represented by a cubic or higher order polynomial. In this procedure, the evaluation of optimal step size at each iteration is also time consuming. In the work reported, a simple and efficient multi-variable constrained scheduling method is presented. The computer storage and computation time are encouragingly small. The applicacy of the algorithm is tested on a sample system and the results of computation are presented.

5.2.1 MODELLING

In the scheduling of real power, the consideration of losses is important. In the present work a simplified method [48] is used for the calculation of transmission loss coefficients. The DC load flow loss formula provides a fast means for calculating incremental losses without the need for direct network solutions. Thus, transmission losses are coordinated into economic schedules without large overheads in computer time and storage.

SYSTEM LOSSES

The basic problem is to express the system losses as a function of the generator power outputs. As a first step, the losses are separated into voltage magnitude and angle dependent components.

The real power loss in the branch k m in Fig.5.2 is ,

$$P_L = E_{km}^2 \; g_{km} \tag{5.18}$$

Using the cosine rule to expand E_{km},

$$P_L = (E_k^2 + E_m^2 - 2E_kE_m \cos (\sigma_k - \sigma_m)) \, g_{km} \tag{5.19}$$

In power system operation $(\sigma_k - \sigma_m)$ is small and therefore the approximation (5.20) is valid.

$$\cos (\sigma_k - \sigma_m) = 1 - (\sigma_k - \sigma_m)^2 / 2 \tag{5.20}$$

Thus, from above,

$$P_L = (E_k - E_m)^2 \; g_{km} + E_k \, E_m (\sigma_k - \sigma_m)^2 \, g_{km} \tag{5.21}$$

In expression (5.21) the branch loss is separated into two parts. The first component is dependent only on voltage magnitudes and the second component is predominantly dependent on voltage phase angles. Thus, for the complete system, the losses are summed over all the branches. Therefore, the system loss is expressed as,

$$P_L = P_\sigma + P_E \tag{5.22}$$

where,

$$P_E = \sum_k \sum_m (E_k - E_m)^2 g_{km} \tag{5.23}$$

and

$$P_\sigma = \sum_k \sum_m E_k E_m (\sigma_k - \sigma_m)^2 g_{km} \tag{5.24}$$

The expression (5.22) is put in a more convenient form as,

$$P_L = \sigma^t G_{NN} \sigma + P_E \tag{5.25}$$

Where, the elements of G_{NN} are defined by,

$$G_{NN}(k,k) = \sum_m E_k E_m g_{km} \tag{5.26}$$

and

$$G_{NN}(k,m) = -E_k E_m g_{km} \tag{5.27}$$

Thus, the losses are separated into voltage magnitude and angle dependent components.

LINEARIZED LOAD FLOW

The linearized dc load flow is applied to express the bus angles linearly in terms of the bus powers. The power flow in line km of Fig. 5.2 is given by [48] Eq. (5.28).

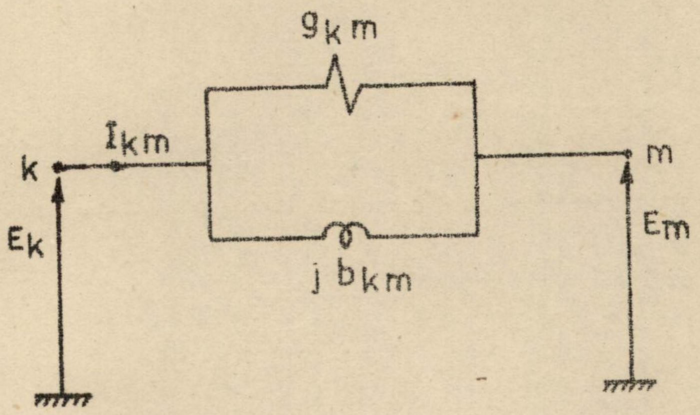


FIG.5.2(a) BRANCH km OF A SYSTEM

FIG.5.2 (b) BRANCH VOLTAGE AND CURRENT RELATIONSHIPS

$$P_{km} = E_k \left[E_k - E_m \cos(\sigma_k - \sigma_m) \right] g_{km}$$

$$- E_k E_m \sin(\sigma_k - \sigma_m)\, b_{km} \qquad (5.28)$$

The terms of (5.28) are the components of power flow through the branch conductance and the branch susceptance respectively. For the dc load flow, the following approximations are used,

$$\text{Sin}\,(\sigma_k - \sigma_m) = \sigma_k - \sigma_m \qquad (5.29)$$

and $$\cos(\sigma_k - \sigma_m) = 1 \qquad (5.30)$$

The effect of the error inthe latter approximation is reduced by the high X/R ratios of the modern transmission systems. Thus, from above,

$$P_{km} = E_k(E_k - E_m)\, g_{km} - E_k E_m\, b_{km}(\sigma_k - \sigma_m) \qquad (5.31)$$

Considering the power balance at each bus, one obtains,

$$P_N = M_N + B_{NN}\, \sigma \qquad (5.32)$$

Where, the $M_N(k)$, the net conductive flow from bus k, is given by (5.33)

$$M_N(k) = \sum_m E_k(E_k - E_m)\, g_{km} \qquad (5.33)$$

and B_{NN} by

$$B_{NN}(k,k) = - \sum_{m} E_k E_m b_{km} \qquad (5.34)$$

and $$B_{NN}(k,m) = E_k E_m b_{km} \qquad (5.35)$$

Eqn (5.32) expresses the bus powers P_N linearly in terms of the bus angles σ and it is commonly known as the dc load flow equation. For convenience, M_N is lumped with the load powers, thus Eq. (5.32) becomes

$$P_N = B_{NN} \sigma \qquad (5.36)$$

THE DC LOAD-FLOW LOSS FORMULA

The dc load-flow Eq.(5.36) is solved for the bus angles,

$$\sigma = Z_{NN} P_N \qquad (5.37)$$

Substituting (5.37) into (5.25), the losses are obtained as quadratic function of the bus powers,

$$P_L = P_N^t Z_{NN} G_{NN} P_N + P_E \qquad (5.38)$$

Separating, the bus powers into the generator power P_G and the load powers P_{NL},

$$\begin{bmatrix} P_N \end{bmatrix} = \begin{bmatrix} P_G \end{bmatrix} + \begin{bmatrix} P_{NL} \end{bmatrix} \qquad (5.39)$$

Also, Z_{NG} is defined as a submatrix of the generator bus columns of Z_{NN}. Substitution of (5.39) into (5.38) thus gives,

$$P_L = P_G^t \; Z_{NG}^t \; G_{NN} \; Z_{NG} \; P_G + 2 \; P_G^t \; Z_{NG}^t \; G_{NN} \; Z_{NN} \; P_{NL} + P_{NL}^t \; Z_{NN} \; G_{NN} \; Z_{NN} \; P_{NL} + P_E \qquad (5.40)$$

The dc load-flow loss coefficients are defined as,

$$E_{GG} = Z_{NG}^t \; G_{NN} \; Z_{NG}$$

$$E_{GO} = 2 \; Z_{NG}^t \; G_{NN} \; Z_{NN} \; P_{NL}$$

$$E_{OO} = P_{NL}^t \; Z_{NN} \; G_{NN} \; Z_{NN} \; P_{NL} \qquad (5.41)$$

Thus, the system losses are expressed in terms of the generator powers as,

$$P_L = P_G^t \; E_{GG} \; P_G + P_G^t \; E_{GO} + E_{OO} + P_E \qquad (5.42)$$

Eq. (5.42) is referred to as the dc load-flow loss formula. The incremental losses are therefore,

$$\frac{\partial P_L}{\partial P_G} = 2 \; E_{GG} \; P_G + E_{GO} \qquad (5.43)$$

MATHEMATICAL STATEMENT OF THE PROBLEM

Stated mathematically, the objective is :

$$\text{Minimize } f(P_G) = \sum_{i \in NG} A_{1i} + A_{2i} \; P_{Gi} + A_{3i} P_{Gi}^2 + A_{4i} P_{Gi}^3 \qquad (5.44)$$

Subject to

$$\sum_{i \in NG} P_{Gi} = P_D + P_L \tag{5.45}$$

$$P_{min_i} \leqslant P_{Gi} \leqslant P_{max_i} \tag{5.46}$$

$$i = 1,2,\ldots,NG$$

and

$$P_{SL_j} \leqslant P_{aj} \leqslant P_{SU_j} \quad j=1,2,\ldots,g \tag{5.47}$$

Where,

A_{1i}, A_{2i}, A_{3i}, A_{4i}	Cost function constants associated with ith generating bus
NG	Number of generating buses
P_{min_i}	Minimum active power generation at node i
P_{max_i}	Maximum active power generation at node i
P_{Gi}	Actual generation at ith bus
P_D	Total system load demand
P_L	Total system losses
P_{aj}	Active power flow in line j
P_{SLj}	Specified lower power limit in the jth line
P_{SUj}	Specified upper power limit in the jth line
$f(P_G)$	Total system running cost.

The operating cost of a generating unit is generally approximated by a polynomial of the generated active power with a degree three or higher. Thus, the objective function, $f(P_G)$ is non-linearly related to the variables P_G' s . The system losses and the incremental transmission losses are evaluated by the dc load-flow loss formula (5.42), (5.43). The line flow limits (5.47) are represented [49] as linear inequality constraints on the control variables P_G . Thus, the problem to be solved is : minimize the nonlinear objective function subject to the linear constraints. The next section presents an algorithm for the solution of the problem.

5.2.2 ALGORITHM

The economic dispatch of power systems is based upon scheduling generation so that the cost of supplying the system load is minimized. This involves the coordination of generator production costs with system transmission losses. The coordinating equations, whose solution is to be obtained are,

$$\gamma = \frac{IC_i}{1 - ITL_i} \tag{5.48}$$

where, $$IC_i = \frac{df(P_G)}{dP_{Gi}} \tag{5.49}$$

and $$ITL_i = \frac{\partial P_L}{\partial P_{Gi}} \tag{5.50}$$

where λ is the incremental generation cost.

The problem becomes difficult to solve, under the constraints (5.45),(5.46) and (5.47). In developing the algorithm advantage is taken of the constraints (5.45). If the constraint (5.45) remains satisfied, we always have a feasible solution , provided the constraints (5.46) and (5.47) are also satisfied. Thus, using the concept of feasibility, perturbations are given in the generation levels at the generating nodes. Stated mathematically,

$$\sum_{i \in NG} P_{Gi} + \sum_{i \in NG} \Delta P_{Gi} = P_D + P_L \qquad (5.51)$$

where ΔP_{Gi} is the change in generation at the ith node.

Also

$$\sum_{i \in NG} \Delta P_{Gi} = 0 \qquad (5.52)$$

The above signifies that the perturbations are so given that the total generation remains unchanged. The size of the increment depends upon the scope available for the improvement of the feasible solution. The small change in transmission loss, which is likely to result is absorbed by the slack bus. Thus, at every iteration a decision is to be made regarding the magnitudes of changes ΔP_G .

For this, the ψ_i values are evaluated for all the generating nodes. An average value of ψ_{av} is obtained. The differences between ψ_i and ψ_{av} are measured and also the closeness of generations with their upper or lower limits is recorded. Based on these two factors the changes ΔP_G are affected. Care is taken during the search process that if any P_G violates the constraints (5.46) or (5.47), that P_G is also constrained. The input to the dispatch program consists of the following data,

(i) loss coefficients E_{GG}, E_{GO} and E_{OO}

(ii) branch-generator power matrix S_{BG} (required for calculating line flows)

(iii) generator cost data

(iv) generator power limits, P_{min} and P_{max}.

(v) Total load P_D

(vi) Branch power limits P_{SL} and P_{SU}

(vii) initial generation P_G^o and initial line flows P_a^o.

LOSS COEFFICIENTS EVALUATION

The following steps are used for evaluating the loss coefficients, which are needed as input to the scheduling algorithm.

1. Read branch admittances and load powers. Also, read the bus voltage magnitudes and generator power.
2. Assemble the matrices B_{NN} and G_{NN}.
3. Form $P_N = B_{NN}\ \sigma$
4. Invert B_{NN} to get Z_{NN}. Seperate Z_{NG}.
5. Calculate E_{GG}, E_{GO} and E_{OO}.

LINE FLOWS

The power flowing in a branch is represented [49] as linear function of generation P_G. Thus,

$$P_a = S_{BG}\ P_G + R_B \tag{5.53}$$

where

$$R_B = P_a^o - S_{BG}\ P_G^o \tag{5.54}$$

P_a^o is the initial line flow and P_G^o is the initial generation level .. Also, the branch-generator power matrix is,

$$S_{BG} = B_{BB}\ A_{BN}\ Z_{NG} \tag{5.55}$$

where,

B_{NN} diagonal matrix of branch susceptances

A_{BN} branch-node transformation matrix

Z_{NG} generator node columns of Z_{NN}

Based on the concepts detailed above, a scheduling algorithm is developed. The main steps of the algorithm are :

1. Read System Parameters. Initialize $i = 0$, $NG_R = [0]$ Go to 2.

2. Advance iteration counter by 1, i.e. $i = i+1$, Go to 3.

3. Calculate $\psi_i = \dfrac{IC_i}{1 - I\,TL_i}$, $i = 1,\ldots,NG$ and $i \neq NG_R$. Go to 4.

4. Calculate ψ_{av}. From $(\psi_i - \psi_{av})$ and $(P_{max} - P_G)$ and $(P_{min} - P_G)$. Calculate ΔP_{Gi}, $i = 1,\ldots,NG$; $i \neq NG_R$ Go to 5.

5. Check constraints (5.46) and (5.47). If satisfied, go to 6, else go to 7.

6. Calculate $f(P_G) = \sum_{i \in NG} A_{1i} + A_{2i}P_{Gi} + A_{3i}P_{Gi}^2 + A_{4i}P_{Gi}^3$
   Check, if $(\psi_{max} - \psi_{min}) \leqslant TOL$, stop, else go to 2.

7. Form set NG_R. Go to 2.

Where NG_R is the restricted generator set, in which increments are not possible, TOL is the tolerance for convergence of the solution. A flow chart of the algorithm is given in Fig. 5.3. The next section demonstrates the application of the algorithm.

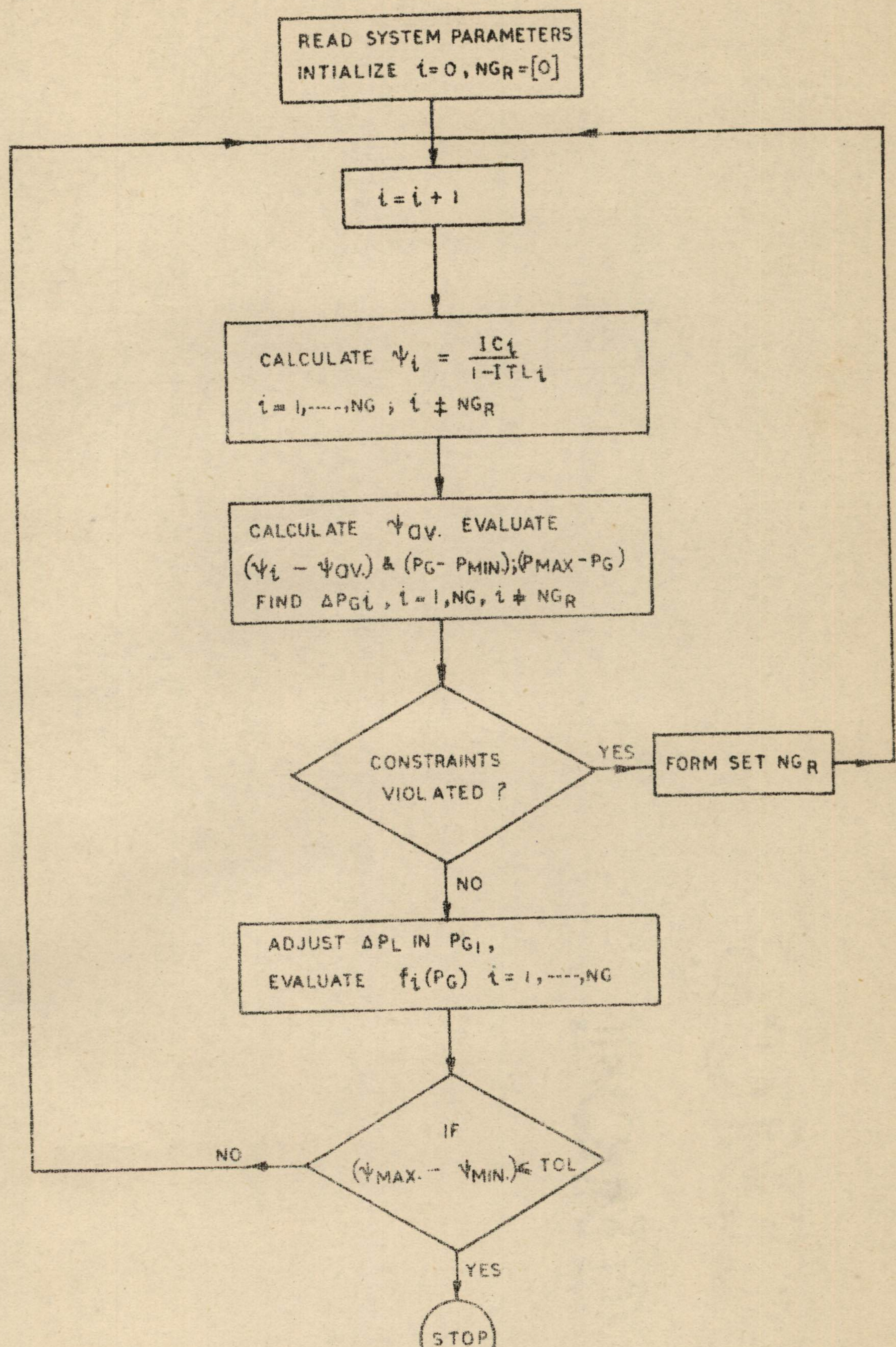


FIG. 5.3 FLOW CHART FOR THE REAL POWER SCHEDULING ALGORITHM

5.2.3 SAMPLE APPLICATION

The line diagram of an eight bus system [17] is given in Fig. 5.4. The percentage admittance data is portrayed in Table 5.3. The cost data for the generating nodes is given in Table 5.4. The Table 5.4 also gives the upper and lower generation limits at the generating nodes . Table 5.5 records the initial load flow bus voltage magnitudes and real bus powers. The data is on a base MVA = 10 .The line L_{10} is constrained to carry a maximum line flow of 15.00 pu . A minimum production cost schedule is to be obtained.

TABLE 5.3

Line Data

Line	g %	b %
1	0.0200	0.1400
2	0.1920	0.9620
3	0.0330	0.3300
4	0.0187	0.1520
5	0.0850	0.4850
6	0.0187	0.1570
7	0.0443	0.6640
8	0.0467	0.4300
9	0.0165	0.1210
10	0.0351	0.3290
11	0.2010	0.9580
12	0.1160	0.7500
13	0.0361	0.4750
14	0.0261	0.3200

TABLE 5.4

Cost Data and Generator Limits

Bus i	A_{1i} cost	A_{2i} cost/MW	A_{3i} cost/MW^2	A_{4i} cost/MW^3	P_{min_i} MW	P_{max_i} MW
1	25.00	1.91610	0.00396	0.00000	20.00	100.00
2	75.00	1.53540	0.00261	0.00000	120.00	300.00
3	50.00	1.85180	0.00393	0.00000	80.00	250.00
4	90.00	1.12850	0.00135	0.00000	150.00	400.00

TABLE 5.5

Initial Load Flow Results for the Sample System

Bus i	Voltage Magnitude pu	Bus angles δ Degrees	Generation pu	Load pu
1	1.0000	0.000000	3.4386	-
2	1.0572	6.25464	25.0000	-
3	1.00000	10.67595	15.0000	-
4	0.9540	8.52400	25.000	-
5	1.0000	-17.92220	-	23.300
6	0.9276	0.35247	-	22.000
7	1.0000	- 8.12036	-	20.000
8	0.9350	9.01675	-	-

The algorithm of Fig. 5.3 is used for attaining the most economic schedule of generation. A computer program of the algorithm has been prepared in Fortran II for the IBM 1620 computer. The optimal schedule of real

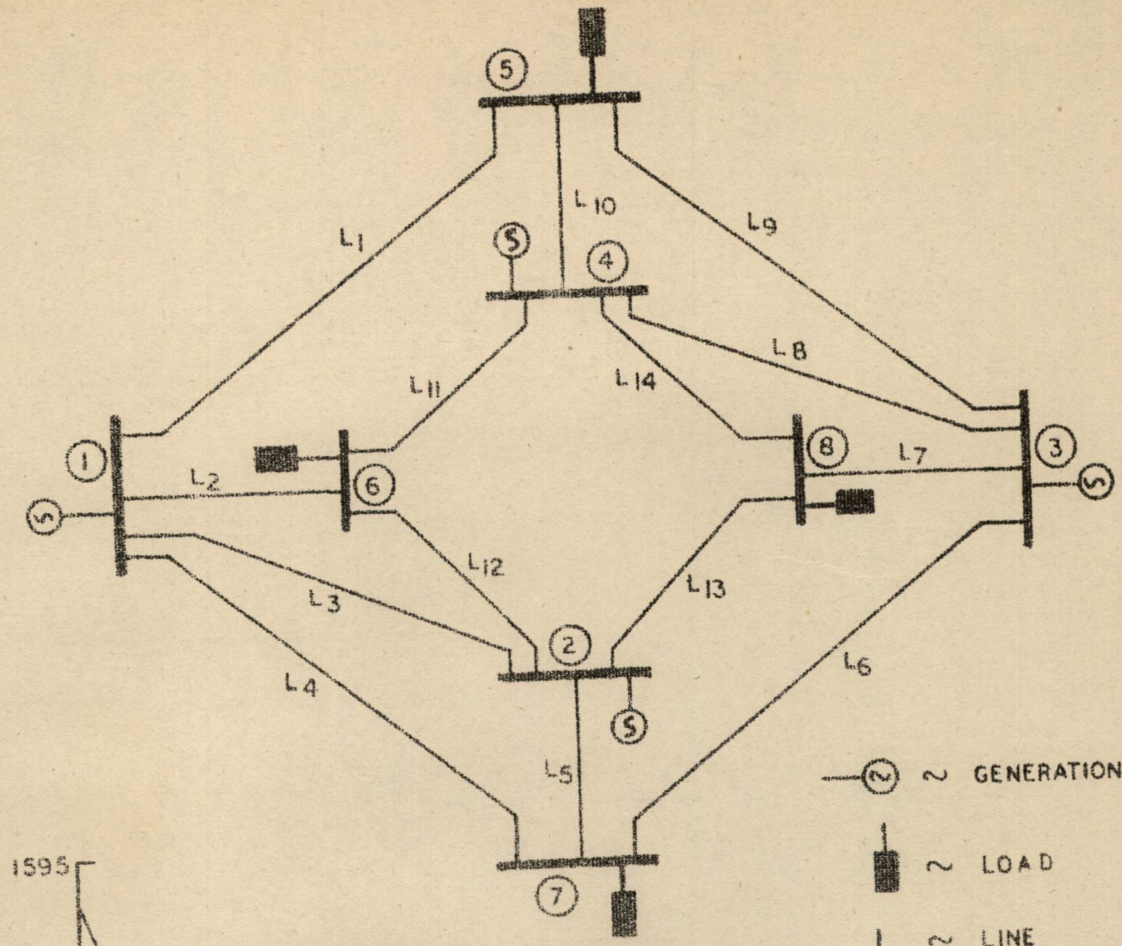


FIG. 5.4 EIGHT BUS SAMPLE SYSTEM

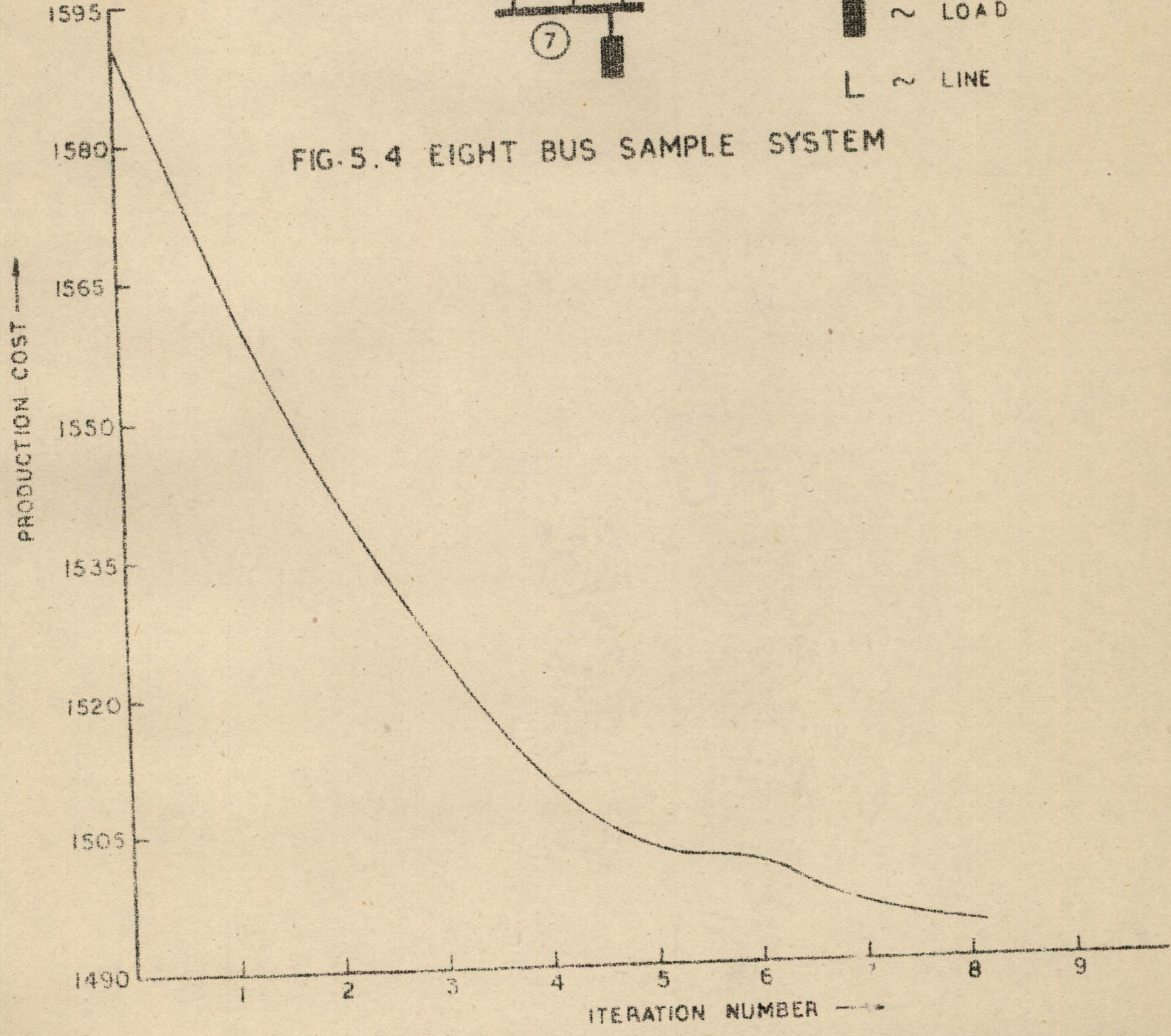


FIG. 5.5 CONVERGENCE CHARACTERISTIC FOR THE SAMPLE SYSTEM

power generation for the generator buses 1,2,3 and 4 is ; 8.012, 15.00, 8.00 and 37.500 units of power (pu) respectively with a total production cost of 1494.30 cost units. A tolerance of 0.075 is used for the convergence of the solution. The convergence characteristic of the solution procedure is displayed in Fig.5.5. The computer memory and the execution time requiremert for the IBM 1620 computer is 4160 words length and 6.0 mins. respectively. A total of 8 iterations are needed for the convergence of the solution. Most of the cost reduction occurs in the first four iterations.

CHAPTER VI

CONCLUSIONS

The application of mathematical programming techniques in the solution of complex scheduling problems is of a great interest and importance to the utilities. The present work is an attempt in presenting mathematical models and algorithms for a range of important decision scheduling problems. Avenues of future work in the area are also detailed in the concluding discussion.

First of all the problem of preventive maintenance scheduling is discussed. A set of different objectives of preventive maintenance, such as to obtain minimum lateness penalty schedule, minimum change from the existing schedule or minimum maintenance cost schedule, are explained. Now depending upon the choice of the system analyst and subject to the availability of appropriate data, a suitable objective is aimed at. An important objective is to minimize the cost of maintenance during the intervals of the scheduling horizon. A set of comprehensive and interacting constraints such as, precedence constraints, security considerations and resources limitations are included in the mathematical formulation of the problem. The uncertainty associated with the availability of resources is modelled with the help of chance constrained programming. Some of the old generating units requiring maintenance more than once are considered in the formulation. After the formulation, a prefiltering analysis is applied in order to reduce the dimensionality of the program. Simple rules are developed in order to cancel some of the variables and constraints at this stage. A procedure for aggregating groups of constraints into single equivalent constraints is presented. To the knowledge of the author this procedure has not been used earlier in the power system engineering literature. This helps in reducing the computation burden of the algorithm as the constraints checking for feasible solutions is time consuming.

The present work offers an integer programming model for the preventive maintenance scheduling problem. The formulation has the advantages of having only bivalent variables and the diversity of constraints are easily transformed into the problem format. Dopazo and Merrill [14] have made an arduous effort in modifying Bala's algorithm [2] and making it suitable for the solution of the problem. The solution procedure is not an efficient one, as the optimal solution has to be searched over the whole solution space. In the present work, the problem has been approached from a different view point. First of all, the problem model is critically analyzed and systematically arranged. To the knowledge of the author such systematic modelling for the multistage problem has not been presented elsewhere in the literature. The properties of the mathematical model are exploited in evolving a new and efficient Tree search optimization technique. The development of the tree, its properties and the method of generating non- redundant descendants are the new additions in the existing literature of Operations Research. In the tree search method no back-tracking procedure is required as compared to the Bala's method [2]. This results in a lot of saving in the computation effort and core as regards the book-keeping for avoiding redundancy is concerned. New skip rules are developed by taking advantage of the properties of the constraining equations. This helps in reducing the size of the tree to be searched as many branches are terminated at a very early stage in the search process. Thus, by employing this procedure the region of search for finding the optimal solution is drastically reduced. The applicacy of the algorithm developed is demonstrated for two sample studies. First a 4 unit, 10 variable, 4 interval problem is solved. The second case study is made for a 10 unit, 35 variable 12 interval preventive maintenance scheduling problem. The results of computation are presented in the form of bar charts, which are useful for plant managers in executing maintenance decisions. These charts can be directly obtained from the computer resulting in lot of saving of manual effort. Next, a mathematical version of the maintenance staff interchange scheduling problem is presented. An illustrative example is given. The algorithm makes available the required number of craftsmen of each category, at the plant where the maintenance is currently active, at the minimum cost. The same concepts can be used forthe pool coordination of other facilities in the system.

The present work might prove to be of value in the solution of integrated scheduling problems, where the preventive maintenance is coordinated either with the unit commitment scheduling or generation expansion scheduling. The sparsity of the preventive maintenance matrices can be exploited in further enhancing the efficiency of the solution procedure.

Next, a mathematical description of the corrective maintenance scheduling problem is presented. A graphical display of the reliability characteristics of some types of subsystems is presented. There is a huge amount of literature available in the area of reliability and maintainability analysis of aerospace and Defence systems. The power system engineers can take advantage of the existing models and develop a unified approach for the corrective maintenance scheduling of power plant equipment. This analysis is of value in the preparation of design schedules or for affecting system configuration changes using reliability as a figure of merit. The mathematical model of the corrective maintenance is nonlinear. A transformation is used for converting the nonlinear problem to a linear-integer program. The algorithm developed for the preventive maintenance scheduling problem is suitably modified for the solution of this problem. A new skip rule is developed taking advantage of the systematic characteristic of the constraining equations . The applicacy of the analysis is demonstrated for a'generator excitation system' and 'turbine cooling system'. Further work would include classifying the power plant critical subsystems into the known types of models and extending the analysis to these systems.

Next , the problem of selecting a set of proposals or schemes in the formulation of a maintenance policy is viewed as a maintenance budgeting problem. The objective is aimed at maximizing the net present value of the total expected return to the utility. Integer programming linear and nonlinear models are presented for the deterministic and probabilistic versions of the problem. Mao and Wallingford [41] have solved a similar investment decision scheduling problem employing Lawler and Bell [37] method of integer programming. The method is not efficient as it does not exploit the special structure of the model. In the present work, the problem model is critically analyzed and arranged in a systematic fashion. Thus, taking advantage of the properties of the model, new maximization algorithms are developed for the deterministic and probabilistic cases. The algorithms are used for 3 stage 8 variable sample system studies and results of computation are given. A study was carried out to compare the efficiency of the newly developed tree search optimization technique with the earlier used Lawler and Bell method [37] for the investment decision scheduling problems [41]. The results of comparison have been portrayed in Table 4.4. The results show that the new technique of analysis is much superior to the earlier used solution procedure. The future work would include the utilization of the maximization algorithms developed, for other investment decision scheduling problems.

Next the problem of unit commitment or predispatch scheduling is discussed. The consideration of the security of the system is incopporated into the model. The heuristics developed limit the search in the region of interest. A simplified and efficient algorithm is presented for the solution of the problem. As, the number of binary vectors to be enumerated is relatively small, it is a superior approach. The computation time and the storage requirement of the algorithm are encouragingly small.

In the end, a mathematical model of the problem of real power scheduling is presented. A linearized load flow representation of the transmission network is used. The accuracy of such a representation is very high for the modern transmission systems having high X/R ratio. A simplified and efficient algorithm is developed for the solution of the problem.

The future work would include the development of a single algorithm for the solution of the total problem of generation scheduling i.e. the combined problem of unit commitment and the real power scheduling including transmission losses. It is in this context that the present work might prove to be of value. Thus new heuristic rules need to be developed for the enumeration of binary vectors considering system losses . During the scheduling process load changes occur in the system which may be frequent and also violent. Therefore, another cost term is associated with the act of changing the load conditions. This excess cost is due to the transient degradation in unit efficiency produced by fluctuations in boiler and turbine parameters when generator output is changed. Stadlin [55] has done some work on a similar problem, which relates to the economic allocation of the regulating margin. Perhaps, what is required is that the scheduling algorithm should also advise the system operator as to how best to accept the load changes. This may amount to reallocating the power on some or all of the generators for the transient period in order to accept the additional load with the minimum of change related cost. Further research would clarify the position about this aspect of the scheduling problem.

APPLICATION OF THE TREE SEARCH METHOD

The tree search technique of analysis developed in this thesis is of a fundamental nature. Unlike the situation with basic research in other sciences, however, relatively little time elapses between an important discovery in operations research and its implementation by experienced practitioners in industrial groups.

A potential application of this new technique is demonstrated [36] for the scheduling of reactive power, when banks of fixed and switchable capacitors are to be allocated in the system for the control of system voltage profile under different contingencies. A mathematical statement of the problem solved is given in the appendix. The problem has been earlier solved by Lawler and Bell method [37], which is not an efficient procedure. It has been identified that the integer programming models presented for the maintenance scheduling problems have a natural matching with many real life situations. The author hopes that the tree search method will find application in the solution of other fascinating practical problems, in power system engineering and in the field of business management. The problems, which can be tackled are : optimal allocation of shunt reactors in EHV systems, optimal allocation of lightning arrestors, optimal design of a series compensated line, optimal design of systems using mixed redundencies and a class of planning problems. The method is also useful for the solution of optimal routing problems. The solution of some class of contractual scheduling and machine scheduling problems also falls within the perview of the present technique of analysis.

It is hoped that the work reported would help to pave the way to a better understanding and control of the inherent complex aspects of modern thermal generating systems.

APPENDIX

OPTIMAL CAPACITOR ALLOCATION

In electric power system, low voltages may occur at various points of the system due to large loads or line/generator outages. Adding capacitor banks to the system for voltage control, is a very economical way of correcting low-voltage levels. With the increase in size and complexity of the system, the determination of size and location of the capacitor banks has become quite an involved problem as many alternate solutions are possible. A new 0-1 programming formulation of the capacitor allocation problem is presented [36]. The objective is to minimise the total number of capacitors and buses at which these are placed, out of the chosen application set, under all anticipated contingencies.

The tree search optimization technique developed in this thesis for the preventive maintenance scheduling problem is very successfully applied to the capacitor allocation problem. The mathematical statement of the problem is detailed below.

MATHEMATICAL MODEL

$$\text{Minimize : } f = \sum_{i=1}^{n} c_i x_i \qquad \text{(i)}$$

subjectt to :

$$V_\ell(CF, B, s_o) \leq V_{max} \qquad \ell = 1,2,\ldots,L \qquad \text{(ii)}$$

$$V_\ell(C,B,s_j) \geq V_{min}^{s_j} \qquad \ell = 1,2,\ldots,L, \quad j = 1,2,\ldots,K \qquad \text{(iii)}$$

$$x_i = 0 \text{ or } 1 \qquad \text{(iv)}$$

where,

n = total number of binary variables

K = total number of low voltage or outage conditions to be studied.

L = total number of load buses

s_o = state corresponding to the highest expected voltage condition

s_j = state corresponding to jth low voltage on outage case to be studied

$V_{min}^{s_j}$ = L vector representing lower limits on bus voltage magnitudes corresponding to jth case to be studied.

V_{max} = L vector representing upper limits on bus voltages

f = cost function accounting for cost of capacitor units, cost of any associated breakers and controls and the cost of installation

In accordance with the generally accepted terminology, a vector x $(x_1, \ldots, x_n)$ with 0-1 components is a solution to the problem. If the constraints (ii),(iii) and (iv) hold, then it is a feasible solution. A feasible n-vector x^* is optimal if and only if, $\sum_{i=1}^{n} c_i x_i^* \leq \sum_{i=1}^{n} c_i x_i$ for all feasible x.

The algorithm for the problem (i), (ii), (iii) and (iv) is developed using the tree search method [36]. The results are presented for a sample system drawn from the literature [57]. The other details are available from the reference [36].

LIST OF PUBLICATIONS

The following is the list of contributions by the author related with this study.

1. "Optimal preventive and corrective maintenance scheduling in power systems - models and technique of analysis" Paper C75 146-6 , IEEE , Winter Power Meeting , New York, Jan, 1975.
2. "Optimal generator maintenance scheduling using integer programming," Paper T74 162-4, Discussion, to appear in , IEEE Trans. on Power Apparatus and Systems.
3. "Optimal maintenance budgetary control in generating systems", Paper to appear in Int. J. of Systems Science (U.K.).
4. "Optimal thermal generating unit commitment considering system security", Proc. IEEE - India, ACE-75, Banglore, Feb. 1975.
5. "Optimal capacitor allocation by 0-1 programming" Paper A75 476-2,IEEE,Summer Power Meeting, San Francisco, July , 1975.
6. "Optimum capital budgeting decisions in industrial systems",Paper, submitted for presentation and publication in the Proc. National Systems Conference, Roorkee India, Feb. 1976.

7. "A new 0-1 programming method for a class of problems" Paper, submitted for publication in Management science (U.S.A).

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