BEHAVIOUR OF SCHRAGE MOTOR
UNDER
ABNORMAL CONDITIONS OF OPERATION

BY

SANTOSH KUMAR JAIN

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Abstract

In this thesis, a systematic and detailed analysis of the behaviour of a Schrage motor under the following abnormal conditions of operation is described:

(i) Supply voltage sinusoidal but unbalanced,

(ii) Single phase supply,

(iii) Supply balanced but injected voltage to secondary unbalanced, and

(iv) Non-sinusoidal but periodic supply.

As a prelude to subsequent analysis, the steady state operation of the motor under balanced conditions has been investigated using quaderature component approach (Two axis method). Through the use of appropriate connection matrices and a few derived parameters of time constant nature, comprehensive expressions for performance equations have been obtained. The resultant expressions are explicit functions of slip, brush separation and brush axis shift, and brings out easily, the dependence of a desired quantity on any operational adjustment. Methods are given to determine experimentally all the machine parameters.

The method of symmetrical components is used to analyse the performance with unbalanced supply voltages. The nature and variation of positive, negative and zero sequence impedances for various mode of operations have been discussed. The technique of power flow concept has been used to provide physical interpretation of the

<u>Certificate</u>

Certified that the thesis entitled 'Behaviour of Schrage Motor under Abnormal Conditions of Operation' which is being submitted by Mr. Santosh Kumar Jain in fulfilment of the requirements for the degree of Doctor of Philosophy(Electrical Engineering) of the University of Roorkee, is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this thesis has not been submitted for the award of any other degree or diploma.

This is to further certify that he has worked for a period of four and a half years from June, 1965 to November, 1969 for preparing this thesis for Doctor of Philosophy Degree at the University.

L.M. Ray

Dated December 30, 1969

(L.M. Ray)
Professor of Electrical Engg.,
University of Roorkee,
Roorkee, U.P.

behaviour of the motor under different operating conditions. It is found that the negative sequence operation is characterized with many abnormalities compared to an induction motor and depends upon the brush adjustment. In the light of the aforesaid investigations, a criterion has been suggested to predetermine the allowable output of a Schrage motor under unbalanced supply voltage conditions. The approach is based on the concept of conduction coefficient for thermal considerations. The commutation aspect is also explored to find its possible contribution to limit the output of the motor. It has been shown that the commutation of a Schrage motor deteriorates under voltage unbalance but it is the heating of the secondary winding which restricts the output. It is found that this motor is capable of sustaining more severe unbalances with normal brushes than with crossed brushes.

The generalized performance equations of a Schrage motor reveal an interesting quality of the motor, i.e. it may develop starting torque with single phase supply under favourable operational adjustment of brush separation and axis shift. A detailed mathematical analysis is carried out for single phase operation. The speed and direction of motor rotation with single phase supply or on occurrence of single phasing while running with 3-phase supply, depends upon the combination of brush separation, axis shift and load torque. The starting behaviour is similar to that of a single phase repulsion motor but

running characteristics conform to shunt nature. The experimental results corroborate the theoretical deductions.

If the voltages injected to three phases of secondary through commutator are unequal due to faulty brush gear settings, the secondary circuit becomes a source of negative and zero-sequence voltages. A method to analyse secondary unbalance has been outlined. The equations obtained are, however, not amenable to usual method of solution and physical interpretation.

It is shown that the behaviour of a Schrage motor supplied with non-sinusoidal voltages may be predicted with the help of performance equations deduced earlier. The machine with normal brush settings behave similar to an induction motor, but the effect of harmonics on torque in this operating condition, expressed as percentage of fundamental voltage torque, increases with increase of brush separation. For crossed brush setting, the effect is more pronounced but once more it is a function of brush separation. The copper losses with non-sinusoidal supply are less than with purely sinusoidal supply of the same r.m.s. value but it is shown that there is little difference in overall heating.

The entire analysis is general, and the induction motor performance under the above stipulated conditions may be deduced as a boundary value case.

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CONTENTS

List of Principal Symbols

C	- conduction coefficient,
C_L, C_M	- self and mutual coefficients of temperature rise of a phase winding,
E_2	- secondary induced voltage,
E_j	- injected voltage,
h	- order of time harmonic,
I_{1h}, I_{1p}, I_{1n}, I_{2h}, I_{2p}, I_{2n}	- currents; suffix 1 denotes primary and 2 secondary current, suffix h stands for hth harmonic, p for positive sequence and n for negative sequence,
I_N	- normal current of a winding,
I_{2m}	- maximum permissible current of secondary winding,
k, k_1, k_2	- constants,
k_e	- ratio of effective resistance of secondary circuit without, to with finite, brush separation; $R_o/(R_o+R_{3\beta})$,
k_n	- R_{ep}/R_{en}
n_s, N_s	- synchronous speed,
n_r, N	- speed of the motor,
p	- order of space harmonic, operator d/dt,
P	- number of poles, A constant,
Q	- constant,
r_3	- total resistance of tertiary winding per phase,
R_1, R_2, $R_{3\beta}$	- resistance per phase of primary, secondary and tertiary windings respectively,
R_b	- brush contact resistance (effective per phase),
R_e, R_{ep}, R_{en}	- effective resistance of secondary circuit, p-positive sequence, n-negative sequence,

R_o - R_2+R_b,

s - slip,

s_o - no load slip,

s_m - slip at which torque is maximum

T, T_p, T_n - torque; p-positive and n-negative sequence,

T_s - starting torque with single phase supply,

T_{1s} - starting torque with sinusoidal fundamental balanced voltages,

T'_{2o} - X_{22}/R_o

T'_{3o} - X_{33}/R_o

$T'_{\rho o}$ - X_{23}/R_o

T'_{2s} - $(X_{22}-X_{12}^2/X_{11})/R_o$

T'_{3s} - $(X_{33}-X_{13}^2/X_{11})/R_o$

$T'_{\rho s}$ - $(X_{23}-X_{12}X_{13}/X_{11})/R_o$

T'_{op1} - $k_e(T'_{3o}\sin\beta+T'_{\rho o}\cos\rho_1)\sin\beta$

T'_{sp1} - $k_e(T'_{3s}\sin\beta+T'_{\rho s}\cos\rho_1)\sin\beta$

T'_{on1} - $k_e k_n(T'_{3o}\sin\beta+T'_{\rho o}\cos\rho_2)\sin\beta$

T'_{sn1} - $k_e k_n(T'_{3s}\sin\beta+T'_{\rho s}\cos\rho_2)\sin\beta$

T'_{op2} - $k_e(T'_{2o}+T'_{\rho o}\sin\beta\cos\rho_1)$

T'_{sp2} - $k_e(T'_{2s}+T_{\rho s}\sin\beta\cos\rho_1)$

T'_{on2} - $k_e k_n(T'_{2o}+T'_{\rho o}\sin\beta\cos\rho_2)$

T'_{sn2} - $k_e k_n(T'_{2s}+T'_{\rho s}\sin\beta\cos\rho_2)$

V - primary applied voltage per phase,

V_1 - fundamental component of V,

X_{11}, X_{22}, X_{33} - self reactance per phase of primary, secondary and tertiary windings respectively,

X_{12}, X_{23}, X_{13} - mutual reactances; suffix 1 stands for primary, 2 for secondary and 3 for tertiary winding,

Z - impedance, suffix s is for starting value,

λ - unbalance factor, suffix v is for voltage, i for current unbalance, e.g. $\lambda_v = V_n/V_p$,

2β - angle of brush separation in electrical degrees,

ρ - angle by which the axis of brushes is shifted from the axis of secondary winding in electrical degrees,

ρ_1 - effective brush axis shift for positive sequence or forward field operation = $(\pi + \rho)$ or ρ,

ρ_2 - effective brush axis shift for negative sequence or backward field operation = $(\pi - \rho)$ or $-\rho$,

CHAPTER I

Introduction

Soonafter the development of polyphase induction motor, efforts were made to improve its powerfactor and to vary the speed. One of the most effective methods in that direction for a slip ring induction motor comprises of an injection of a voltage into the secondary circuit. This injected e.m.f. must be of slip frequency. The necessary frequency conversion may either be accomplished by a separate auxiliary machine or by incorporating suitable commutator arrangement as a part of induction motor itself. The latter approach led to the development of various three phase a.c. commutator motors. These motors are, in fact, modifications of an ordinary induction motor so that either both speed and powerfactor control or only one of them is available by suitable adjustments.

The Schrage motor was proposed in 1914 as a modification of the Osmos motor and goes with the name of its inventor. It is basically an inverted induction motor with a frequency changer and phase modifier built into it. It is a versatile motor and is one of the most popular amongst commutator motors in industrial application. The Schrage motor, although limited to about 500 h.p. rating only, is remarkably suited to any drive requiring variable speed.

It is strange that inspite of so many desirable qualities, so ingeniously incorporated in this machine, studies on this motor are limited to only its steady state balanced operation. Arnold[2] carried out a comprehensive analysis from circle diagram point of view. Same approach was followed by Rudra[3,4], Conard, Clark and Zewig[5], Franklin[8] and De and Ray[12], Gibbs[7] adopted tensorial approach and obtained the equations to circle diagram in terms of self and mutual inductances of the machine windings. There is a large deviation in theoretical and experimental circles and this deviation is attributed to the varying nature of brush contact resistance with current. The equivalent circuit approach was pursued by Coultherd[6], Shang[14], Kron[73], Taylor[75], Mainer[23] and Ogino[24]. The purpose of Coultherd's paper is to lay a common foundation to the theory of all polyphase commutator motors. He developed an equivalent circuit for only speed control of a Schrage motor. He did not consider the variation of the angle between the axes of secondary and commutator windings and its effect on speed and power factor. Shang's derivation is conventional and is an extension of an induction motor equivalent circuit. The secondary and adjustable windings are combined into a single equivalent impedance. The effect of mutual leakage reactance of primary and tertiary windings has not been clearly brought out by most of these authors.Taylor has obtained an equivalent circuit from the flux linkage considerations but individual fluxes associated with primary and tertiary have been assumed to be negligible. Bauman[9] and Charlton[13] did incorporate the effect of various leakages in their theories but failed to obtain satisfactory results. It is surprising that none of these authors have given any experimental method of determining the equivalent circuit constants or method of predetermining the performance of the machine otherwise.

The analysis of a Schrage motor is more complex compared to an induction motor, as this machine has three electrical circuits in addition to a commutator interposed between secondary and tertiary windings. At the outset, balanced operation of the motor has been presented in this thesis on a more sound footing than hitherto available in technical literature. The resulting performance equations are considerably simpler and amenable to further manipulation and interpretation. The parameters involved can all be determined experimentally. Theoretical predictions on the basis of these experimentally determined constants are found to be in close agreement with test records.

The unbalance operation of a Schrage motor is almost completely unexplored. In spite of wide application of the Schrage motor, there is no record of systematic attempt to analyse its behaviour under unbalance conditions.Though, Arnold, Thomas[18] and Taylor have indicated that due to the asymmetry in secondary circuit of a Schrage motor, a time varying torque with double the slip frequency is responsible for a phenomenon similar to hunting in synchronous motor, but none of them have analysed the reasons for the presence of such a pulsating torque at synchronous speed. Recently Panda[35] made an attempt to study some of the aspects of unbalance operation. His analysis is based on approximate equivalent circuit and suffers from lack of experimental corroboration.

For analysis of unbalanced conditions in which the impedances of rotating machine play an important part, the method of symmetrical components is almost indispensible.Therefore, for successful application of this method, a knowledge of the sequence impedances of the machine is essential.

The study of performance of even an induction motor fed with non-sinusoidal supply voltages is rather limited in scope[71,72], because non-sinusoidal voltage waveform is uncommon if supply source is conventional. But due to growing use of static frequency convertor as source of power supply systems which are rich in time harmonics, analysis of induction class a.c. machines subjected to non-sinusoidal voltages has gained added importance.

In this thesis, the general theory of a Schrage motor operation under unbalanced supply voltages is

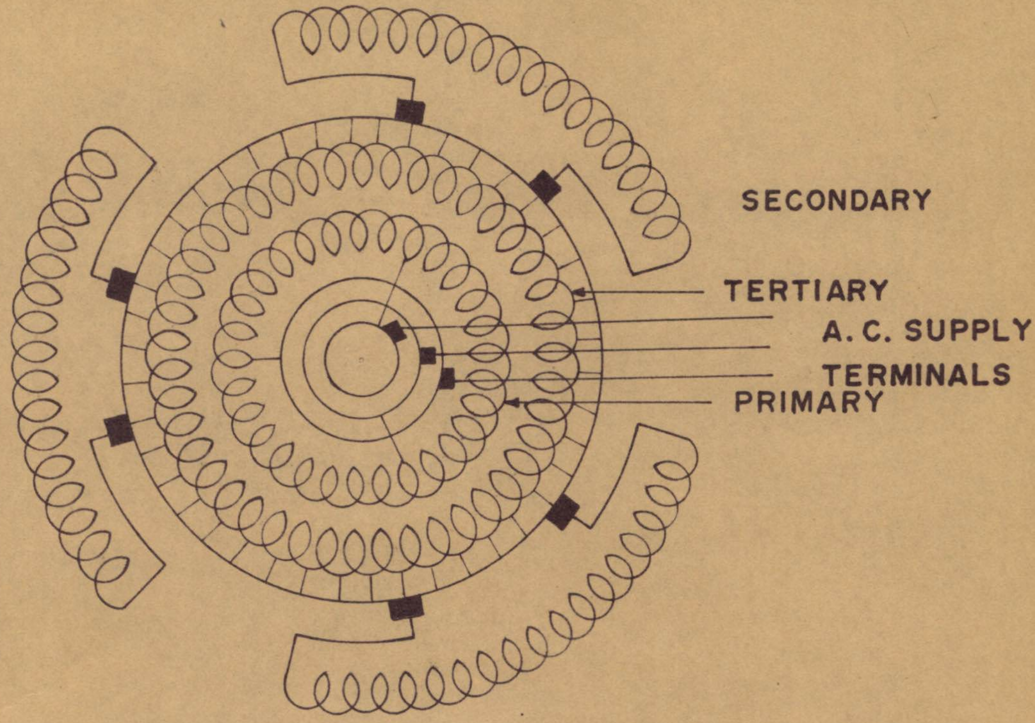
CHAPTER II

Analysis under Balanced Conditions

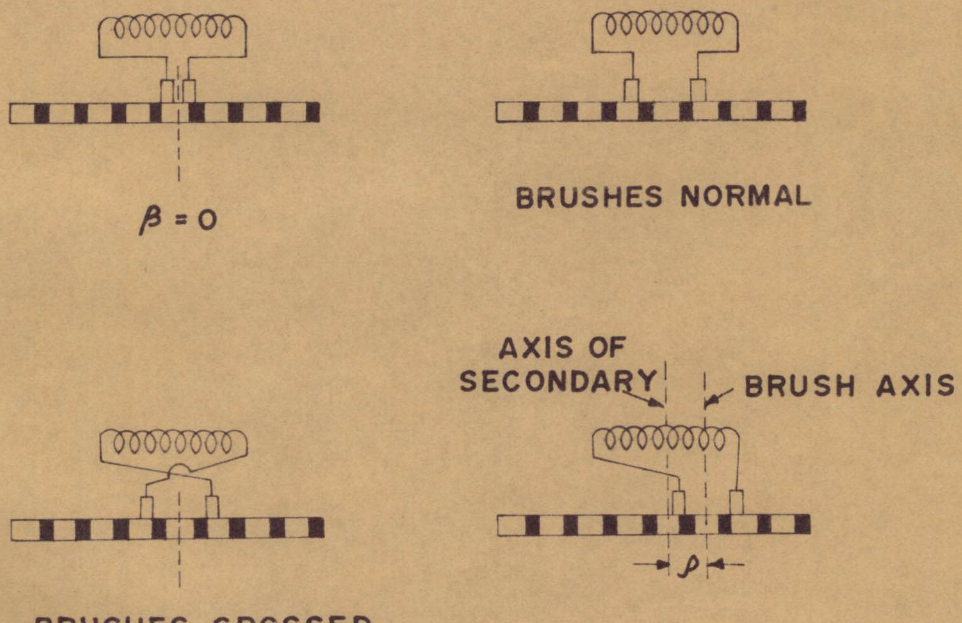
2.1 General:

A Schrage motor is a rotor fed three phase commutator machine. It has three sets of windings: primary and tertiary windings housed in rotor slots, secondary winding in the stator slots. The primary is fed from supply through slip rings at one end of the shaft. The tertiary, which is a commutated winding, is located above the primary conductors. Line frequency voltage is induced in the tertiary winding due to current in primary but slip frequency voltage appears across brushes mounted on commutator and is injected into the secondary circuit. The magnitude of the voltage picked up from commutator depends upon the separation of the brush halves of a phase, and the relative time phase angle of this voltage with respect to secondary induced voltage depends upon the position of the axis of the brush pairs relative to corresponding secondary winding axis. The schematic arrangement is shown in Fig.2.1.

The analysis of a machine using two axes method depends only on the relative motion of different sets of windings, and makes no distinction between the stationary and the rotating member so long as the effect discussed. The method of analysis has been extended to the cases of single phase operation, secondary circuit unbalance and the calculation of performance in the presence of time harmonics. The analysis is based on symmetrical component theory and method has been developed for predicting motor torque, current etc. A criterion has been suggested to predetermine the allowable output of the motor fed with unbalanced voltages. Recommendations are also made regarding the feasibility of single phase starting of the motor as well as the limitations of its operation on non-sinusoidal voltages.



(a)



(b)

FIG. 2·1 SCHEMATIC ARRANGEMENT OF WINDINGS AND BRUSH ADJUSTMENTS OF A SCHRAGE MOTOR.

of rotation and flux linkages is correctly reproduced. Therefore, for this analysis, it is assumed that the reference frame is attached to rotor and is stationary. The direct axis is arbitrarily choosen coincident with the axis of one primary phase winding. Consequently, the tertiary winding is equivalent to a set of quasi-stationary coils.

Thus, the roles of stator and rotor are interchanged. If the direction of rotation of the field produced by primary excitation is presumed to be forward, then in accordance with the aforesaid simulation, secondary would run steadily along the flux direction, although in actual Schrage motor, the direction of primary flux rotation and that of rotor are opposite to each other.

2.2 The ideal machine:

The circuit representation of a rotating machine is accomplished conveniently only by idealizing it. The following assumptions are inherent with such an idealized model.

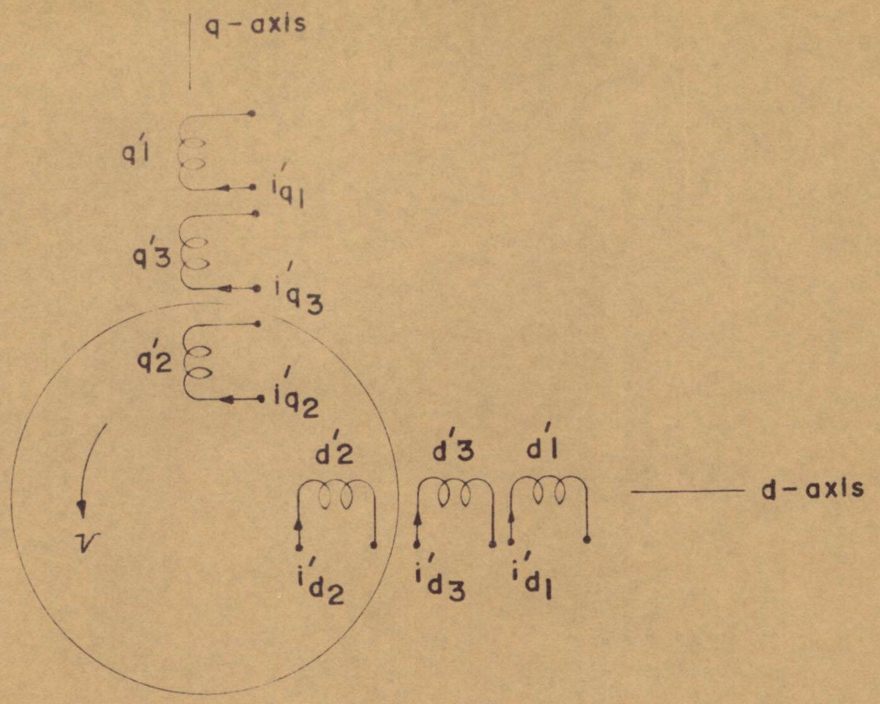
(i) Both the stator and rotor are having symmetrical three phase sinusoidally distributed windings.

(ii) The effects of magnetic saturation, hysteresis and eddy currents are negligible. Therefore, the superposition of magnetic fields is permissible and inductances of various windings may be considered to be independent of the currents. Further, the air gap flux produced by either the rotor or stator currents is sinusoidally distributed in space.

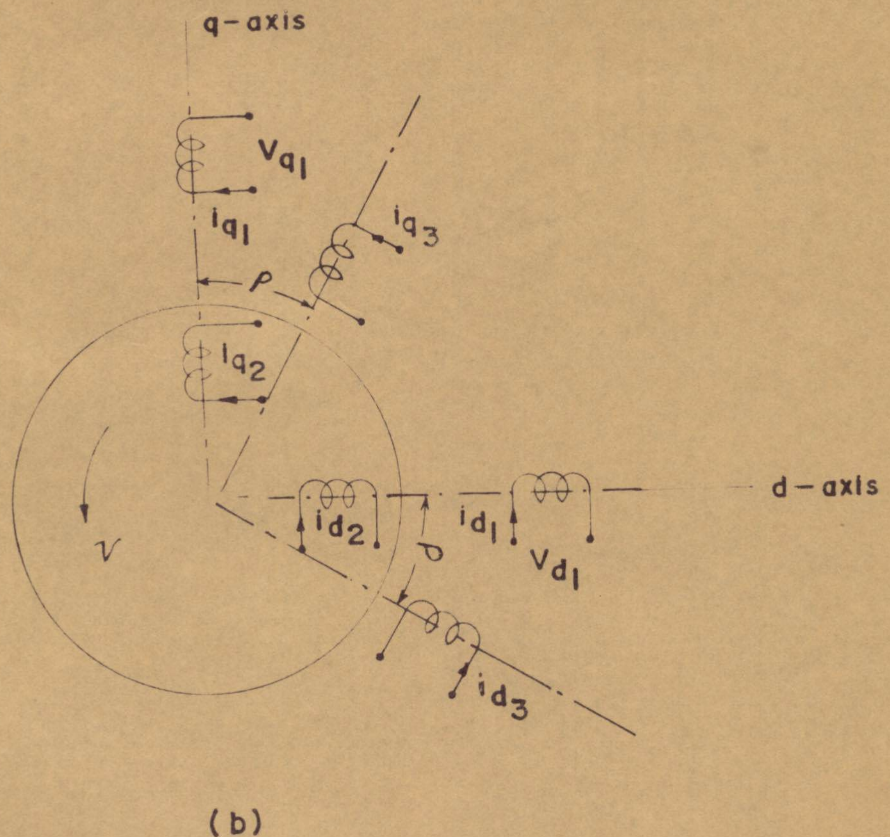
(iii) Since both the stator and rotor of a Schrage motor are cylindrical, air gap is uniform all around the periphery. The effect of slot openings is negligible. Also, the self inductances of the windings and mutual inductances of the phase windings on the same member are independent of rotor position.

(iv) The effects of interturn capacitances are of little consequence.

The primitive arrangement of an idealized model of a Schrage motor is shown in Fig. 2.2(a). Since, a symmetrical three phase winding can be represented by two identical coils in space quadrature, the primitive machine has six coils corresponding to three windings of the actual machine. The positive or forward direction of rotation is assumed to be from d-axis to q-axis, the quadrature axis being ahead of direct axis in the direction of rotation by 90°. If ν is the angular speed of rotation, the following relations of the variables for the arrangement of Fig. 2.2(a) are obtained.



(a)



(b)

FIG. 2·2 PRIMTIVE ARRANGEMENT.

$$
\begin{bmatrix} e'_{d1} \\ e'_{d3} \\ e'_{d2} \\ e'_{q1} \\ e'_{q3} \\ e'_{q2} \end{bmatrix}
=
\begin{bmatrix}
R_1+L_1p & M_{13}p & M_{12}p & & & \\
M_{13}p & R_{3\beta}+L_3\sin^2\beta p & M_{23}\sin\beta p & & & \\
M_{12}p & M_{23}\sin\beta p & R_o+L_2p & \nu M_{21} & M_{23}\sin\beta\,\nu & L_2\nu \\
 & & & R_1+L_1p & M_{13}p & M_{12}p \\
 & & & M_{13}p & R_{3\beta}+L_3\sin^2\beta p & M_{23}\sin\beta p \\
-\nu M_{12} & -\nu M_{23}\sin\beta & -\nu L_2 & M_{12}p & M_{23}\sin\beta p & R_o+L_2p
\end{bmatrix}
\begin{bmatrix} i'_{d1} \\ i'_{d3} \\ i'_{d2} \\ i'_{q1} \\ i'_{q3} \\ i'_{q2} \end{bmatrix}
\quad .. \ (2.1)
$$

In these equations, 2β is the angle of brush separation in electrical degrees. The self and mutual inductances between primary and secondary windings are independent of brush separation. But the self as well as mutual inductance of tertiary with respect to primary and secondary is function of brush separation. The effective turns of the tertiary winding for a brush separation of 2β is $T_3 \sin\beta$ where, T_3 is the number of turns per phase[75]. Therefore, for a given brush separation, the self inductance of tertiary is $L_3 \sin^2\beta$, L_3 being the total self inductance per phase. Similarly, the mutual inductances are $M_{13}\sin\beta$ and $M_{23} \sin\beta$. The effective resistance of tertiary is also effected by β. If r_3 is the total resistance of the tertiary winding per phase, then $R_{3\beta} = r_3(\beta/180)$.

The effective resistance of secondary circuit comprises of the resistance of brushes along with that of secondary winding. Thus, $R_o=(R_2+R_b)$.

2.2.1 Effect of Brush Axis Shift:

The m.m.f's of coils 2 and 3 are in addition in Fig. 2.2(a) and correspond to crossed brushes without any brush axis shift in actual machine. To represent the operation with normal brushes, the m.m.f. of coil 3 should be in opposition to that of coil 2. In general, let ρ be the angle of brush axis shift against the direction of flux rotation, as shown in Fig. 2.2(b)(corresponding to crossed brushes). In order to obtain power factor improvement, a component of the voltage across coil 3 to be injected to coil 2 should lead the voltage of coil 2 by 90^o. With normal brushes, this is achieved by shifting the brush axis suitably along the direction of flux rotation in an actual machine. Thus, if ρ_1 is the effective angle to account for the relative phase of the e.m.f. obtained from commutator for injection into secondary circuit, with respect to secondary induced e.m.f., ρ_1 is either zero or π corresponding to crossed and normal brush separations in a neutral set machine. But with axis shift of ρ, ρ_1 is equal to $(\pi + \rho)$ for normal brush settings and ρ for crossed brushes. The brush axis shift may be accounted for by modifying the impedance matrix Z' of eqn. (2.1) to $[\,Z''\,]$ in accordance with

$$[\,Z''\,] = [\,C_t'^{*}\,][\,Z'\,][\,C'\,] \qquad .. \; (2.2)$$

such that the C' is a connection matrix given by

$$[\,C'\,] = \begin{array}{c} \\ i'_{d1} \\ i'_{d3} \\ i'_{d2} \\ i'_{q1} \\ i'_{q3} \\ i'_{q2} \end{array} \begin{array}{cccccc} i_{d1} & i_{d3} & i_{d2} & i_{q1} & i_{q3} & i_{q2} \\ \hline 1 & & & & & \\ & \varepsilon^{-j\rho_1} & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & \varepsilon^{-j\rho_1} & \\ & & & & & 1 \end{array}$$

$$.. \; (2.3)$$

$[\,C_t'^{*}\,]$ is the transpose of the conjugate of $[\,C'\,]$.

In eqn. (2.3), the effective angle of axis shift ρ_1 is taken to be negative. Since in actual machine the brush axis is shifted against the direction of rotation but along that of air gap flux while the primitive arrangement of Fig. 2.2(b), shows the axes of coils d_3 and q_3 shifted through an angle ρ against the direction of flux.

2.2.2 Steady State Analysis:

During steady polyphase operation of the machine, each pair of axes currents form a two phase system, the only difference between the currents in phase d and q being that of time. The axes d and q are in space quadrature, the d and q phase quantities would differ in time phase by 90°. Therefore, the operator j may be used to represent phase relation between d and q axes quantities.

In the model simulated the resultant air gap flux rotates in the same direction as the rotor. Therefore, quadrature axis current lags behind the direct axis current by 90°. Accordingly,

$$\left.\begin{aligned} i_{d1} &= I_1 \quad ; \quad i_{q1} = -j\ I_1 \\ i_{d2} &= I_2 \quad ; \quad i_{q2} = -j\ I_2 \\ i_{d3} &= I_3 \quad ; \quad i_{q3} = -j\ I_3 \end{aligned}\right] \quad .. \ (2.4)$$

and the connection matrix is

$$[C''] = \begin{array}{c} \\ i_{d1} \\ i_{d3} \\ i_{d2} \\ i_{q1} \\ i_{q3} \\ i_{q2} \end{array} \begin{array}{c} \begin{array}{ccc} I_1 & I_2 & I_3 \end{array} \\ \left[\begin{array}{ccc} 1 & & \\ & 1 & \\ & & 1 \\ -j & & \\ & -j & \\ & & -j \end{array}\right] \end{array} \qquad .. (2.5)$$

The new impedance matrix Z''' is obtained as

$$\left[Z''' \right] = \frac{1}{2} \left[C''^{*}_{t} \right] \left[Z'' \right] \left[C'' \right] \qquad .. (2.6)$$

Under steady conditions, the operator p may be replaced by $j\omega$ and ν by $(1-s)\omega$ in impedance matrix Z'''. Further, the tertiary and secondary windings are connected together to obtain speed and powerfactor control of the machine. Thus, the final impedance matrix Z is given by

$$\left[Z \right] = \left[C'''^{*}_{t} \right] \left[Z''' \right] \left[C''' \right] \qquad .. (2.7)$$

where,

$$[C'''] = \begin{array}{c} \\ 1 \\ 3 \\ 2 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \left[\begin{array}{cc} 1 & \\ & 1 \\ & 1 \end{array}\right] \end{array} \qquad .. (2.8)$$

With appropriate substitutions in eqn. (2.7), the impedance $[Z]$ reduces to the following form

$$[Z] = \begin{array}{c|c|c|} & 1 & 2 \\ \hline 1 & R_1 + jX_1 & j(X_{12} + X_{13}\sin\beta\, \epsilon^{-j\rho_1}) \\ \hline 2 & j(sX_{12} + X_{13}\sin\beta\, \epsilon^{j\rho_1}) & R_o + R_{3\beta} + j[sX_{22} + X_{33}\sin^2\beta + X_{23}\sin\beta(\epsilon^{j\rho_1} + s\,\epsilon^{-j\rho_1})] \\ \hline \end{array} \quad .. (2.9)$$

2.2.3 Torque-slip Relationship:

The instantaneous electric power in a three phase machine in terms of two axis voltages and currents is given by

$$P_e = \frac{3}{2}\,(e_d i_d + e_q i_q + e_o i_o) \quad .. (2.10)$$

The quantities e_o, i_o are non-existant when phase voltages are balanced.

Therefore,

$$P_e = \text{Real part of } \frac{3}{2}\omega\{[i^*][G][i]\} \quad .. (2.11)$$

where $[G]$ is torque matrix and is obtained from motional impedance matrix (Appendix 2.1). The resulting expression for power is, therefore,

$$P_e = 3\nu R_e[I_2^*(-jM_{12})I_1 + I_2^*(-jM_{23}\sin\beta\,\epsilon^{-j\rho_1})I_1] \quad .. (2.12)$$

The electromagnetic torque developed by machine corresponds to power due to speed voltages. Thus, the steady torque per phase in synchronous watts is given by $\dot{P}_e.(\omega/3\upsilon)$, or

$$T = R_e \left[I_2^*(-jX_{12})I_1 + I_2^*(-jX_{23}\sin\beta\epsilon^{-j\beta_1})I_1 \right] \qquad .. (2.13)$$

The torque T is considered positive if it is in the direction of rotation of the machine. For normal motoring under balanced operating conditions, T is positive.

2.3 Power Flow in Schrage Motor

The following equations for steady operation of the motor are obtained with the help of impedance matrix in eqn.(2.9).

$$\left.\begin{aligned} V_1 &= I_1(R_1+jX_{11}) + I_2(jX_{12}+jX_{13}\sin\beta\epsilon^{-j\beta_1}) \\ 0 &= jI_1(sX_{12} + X_{13}\sin\beta\epsilon^{j\beta_1}) + \\ &\qquad \left[R_o+R_{3\beta}+j(sX_{22}+X_{33}\sin^2\beta + \right. \\ &\qquad \left. sX_{23}\sin\beta\epsilon^{-j\beta_1} + jX_{23}\sin\beta\epsilon^{j\beta_1}) \right] \end{aligned}\right] \qquad .. (2.14)$$

It may be recognised that two voltages are injected by tertiary into the secondary circuit: $I_1X_{13}\sin\beta\epsilon^{j\beta_1}$ and $I_2sX_{23}\sin\beta\epsilon^{-j\beta_1}$. The former is dependent on primary current and therefore is equivalent to a voltage approximately proportional to secondary induced voltage E_2 (as the leakage between tertiary and secondary is small), while the latter is proportional to secondary current I_2. Thus, the injected voltages proportional to secondary induced e.m.f and current are simultaneously present. Both of them are affected by brush separation and brush axis shift. Substantial power factor variation is possible even with small values of ρ since the resulting quadrature component of injected voltage is appreciable. On the other hand there is no significant change in inphase component of injected voltage for these values of ρ thereby unaltering the speed adjustment. Therefore, as far as speed control is concerned, only brush separation is important with limiting values of ρ_1 i.e. π and zero. For an ideal machine, I_2 is zero at no load. Thus, the no load speed is governed by $\pm I_1 X_{13} \sin\beta$ $(= \pm\, k_1 E_2 \sin\beta)$ only. On load, additional injected voltage $sI_2X_{23}\sin\beta$ $(=k_2 I_2 \sin\beta)$ is available to influence the speed as well as power factor.

It has been shown by Teogo[1] that the e.m.f's across the brushes and stator terminals are always of identical polarity and hence, oppose each other except in a narrow zone when brushes just cross over but speed of the machine still remains subsynchronous or when the machine with crossed brush setting is so loaded as to run at subsynchronous speed. In this region, two e.m.f's add each other instantaneously. At subsynchronous speeds, i.e. with normal brushes, secondary e.m.f. exceeds $k_1E_2\sin\beta$ while at supersynchronous speeds with crossed brushes (leaving aside the zone in which two voltages are of same polarity) brush e.m.f. is more than sE_2 (Appendix 2.2). For both speed ranges the difference increases with an increase in β. However, for the same speed variation with two settings of brushes, β required with normal brush setting is less than with crossed brushes. This follows from the fact that for a certain slip s, secondary voltage is same whether the machine is running at subsynchronous or super-synchronous speed. But as the injected voltage is to exceed sE_2 at supersynchronous speed while it is to be less at subsynchronous speed, β should be comparatively more with crossed brushes. Conversely, for a given brush separation, speed variation with normal brush setting is more as compared with that on crossed brush setting.

As there is an electric connection between tertiary and secondary windings, energy transfer can occur from and to the secondary through the brushes because the voltages at stator terminals as well as at brushes are of same slip frequency. When the speed is subsynchronous, this power transfer is from secondary to tertiary as $SE_2 > k_1E_2\sin\beta$. The tertiary winding, in turn, dissipates a part as copper loss and returns the rest to primary through magnetic coupling. Larger the brush separation more is the difference between $S\ E_2$ and $k_1\ E_2\ \sin\beta$ and consequently increased backward power flow and reduced speed.

In case of speed control of an ordinary induction motor by secondary circuit rheostat, some of the energy transferred to rotor is consumed by this additional resistance. This decreases the energy available to shaft and thereby causes a reduction in speed. In case of Schrage motor with normal brushes, the energy which is otherwise wasted in the control rheostat is returned to the source through tertiary and primary windings, rendering speed reduction without lowering efficiency. As far as the secondary circuit of Schrage motor is concerned, this concept brings in an effect of increased resistance for torque production. Larger the brush separation more pronounced is this effect of increased secondary resistance. This explains as to why Schrage motor develops high starting torque without excessive primary current on low speed brush settings.

Rheostatic method of speed control of an induction motor is incapable of increasing its speed above synchronism, for, that would call for a negative resistance, which is equivalent to a source. Comparing a Schrage motor with crossed brushes, same situation exists as additional power to secondary is available through tertiary to raise the speed. An increase in β represents large flow of energy from supply circuit which results in increased speeds. Therefore, the secondary circuit of a Schrage motor is similar to that of an induction motor with reduced resistance. It is shown later that this concept renders the necessary explanation to the shifting of the maxima of the torque-slip curves to higher speeds as β is increased with crossed brush settings.

To verify experimentally the foregoing concept of conductive power flow in a Schrage motor, a wattmeter was connected between secondary and tertiary winding of one phase with a dead load on the shaft. The record of wattmeter reading confirmed that the power is transferred to and from tertiary as brush separation is changed from normal to cross and that power transfer increases as β is increased. (Details in Appendix 2.2).

The injected voltage $k_2 I_2 \sin\beta$ on load, transfers energy to the secondary through induction but does not contribute to the energy transfer through conduction between tertiary and secondary. This leads to modification of characteristics as discussed in Chapter 4.

2.4 Experimental determination of the parameters

To advance experimental evidence in support of any theoretical findings on a machine, a prior knowledge of its parameters involved, is essential. Therefore, in this section tests are described to determine experimentally all the parameters used in the impedance matrix of the machine.

Usually the brush gear of commercially available Schrage motors is calibrated in terms of full load speed instead of brush separation in degrees. This calibration may be done by recording voltage E_3 across the brushes 120^o electrical apart on the same rocker with secondary open and primary fed with balanced rated voltages. If E_b is the voltage across the brushes of a phase for a separation 2β, the angle is given by

$$\beta = \sin^{-1}\left(\frac{E_b}{E_3} \sin 60^o \right) \qquad .. \ (2.15)$$

2.4.1 Primary and secondary resistances

The resistance measurement may be carried out using voltmeter-ammeter method or Kelvin's double bridge. For primary resistance, the voltage may be applied to brush leads but the voltage drop is measured directly at the slip rings to eliminate brush contact resistance. Measurements are made on all the three phases separately and average taken.

2.4.2 Tertiary winding and Brush contact resistance

The tertiary winding resistance cannot be measured directly, being a commutator winding. The carbon brush contact resistance forms a significant part of the total resistance of tertiary circuit. Further, the non-linearity of contact resistance makes its determination necessary at different current densities. Fig. 2.3(a) shows the test arrangement for measuring these resistances. The

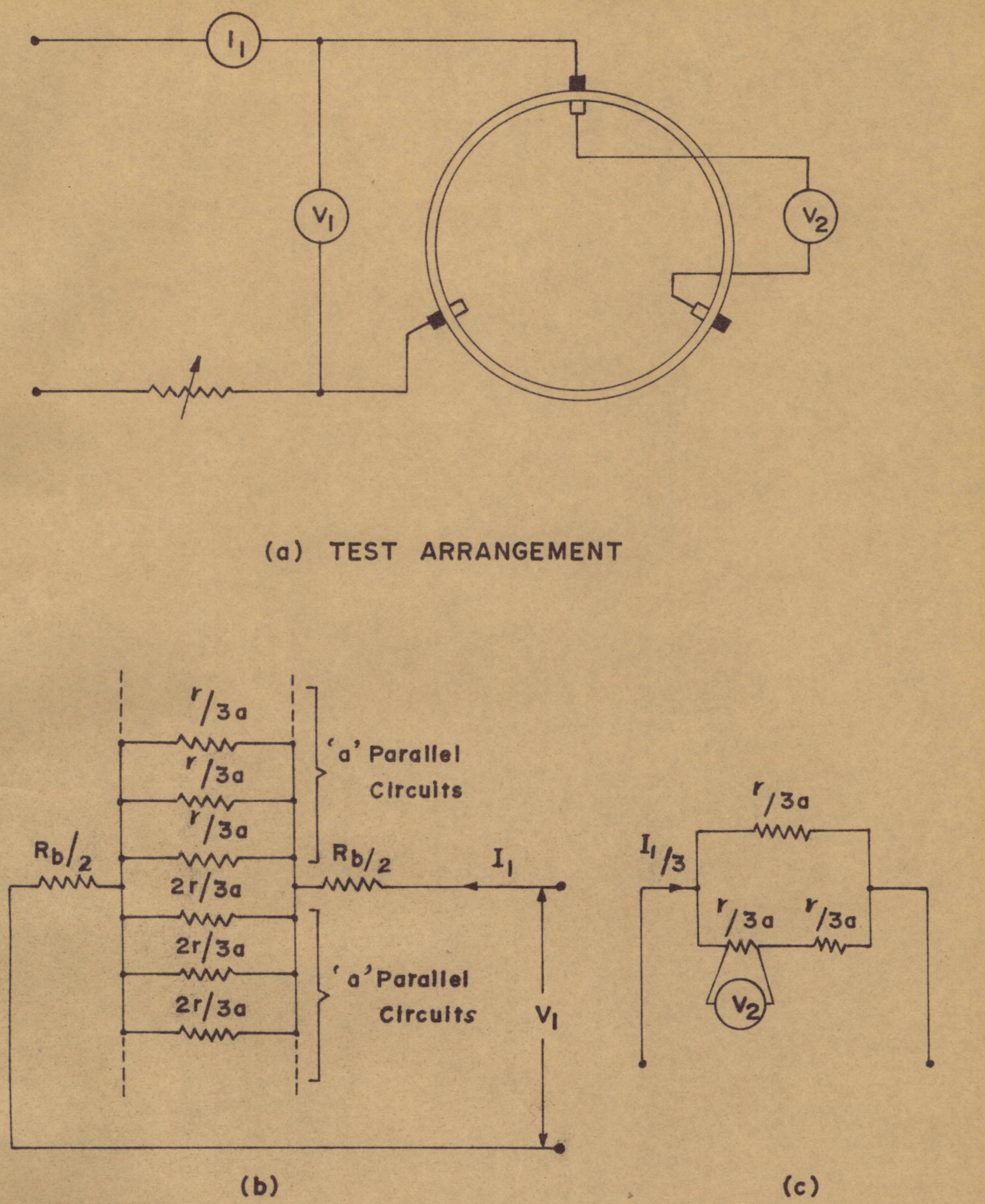


FIG. 2·3 MEASUREMENT OF TERTIARY WINDING AND BRUSH CONTACT RESISTANC brush separation is adjusted to be zero and a suitable d.c. voltage applied to the tertiary through two consecutive brushes on the same brush rocker. A record of V_1, V_2 and I_1 yields the brush resistance and the resistance of the complete closed circuited tertiary winding.

For the test circuit of Fig. 2.3(a), the corresponding equivalent circuit is as shown in Fig. 2.3(b) and 2.3(c). It may be easily deduced that if R_b is the brush contact resistance per phase and r is the total resistance of the complete tertiary winding in series, then

$$\frac{V_1}{I_1} = R_b + \frac{2}{9} \cdot \frac{r}{a^2} \qquad .. (2.16)$$

and

$$\frac{V_2}{I_1} = \frac{1}{9} \frac{r}{a^2} \qquad .. (2.17)$$

where, 2a is the number of parallel paths of commutator winding. Therefore,

$$R_b = \frac{V_1 - 2V_2}{I_1} \qquad .. (2.18)$$

$$r = 9a^2 \frac{V_2}{I_1} \qquad .. (2.19)$$

$$r_3 = \frac{r}{a^2} \qquad .. (2.20)$$

The test is conducted for various values of the secondary current to obtain the variation of R_b.

2.4.3 Measurement of X_{11}, X_{12} and X_{13}.

The primary winding is fed with rated voltage with secondary and tertiary circuits open. The primary applied voltage V_1, primary phase current I_1, secondary induced voltage per phase E_2 and the tertiary induced voltage E_t between two consecutive brushes on the same rocker are recorded.

Assuming, as it is true for most induction type motors, that the coreloss component of primary current is negligible, the total self reactance is

$$X_{11} = \left[\left(\frac{V_1}{I_1} \right)^2 - R_1^2 \right]^{1/2} \qquad .. (2.21)$$

also $$X_{12} = \frac{E_2}{I_1} \qquad .. (2.22)$$

If E_t is divided by I_1, the value of mutual reactance between primary and tertiary for a brush separation of 120^o will be obtained. Since X_{13} is proportional to the effective number of tertiary winding turns,

$$X_{13} = \frac{E_t}{I_1 \sin 60^o} \qquad .. (2.23)$$

2.4.4 Measurement of X_{22} and X_{23}

The secondary winding terminals are connected either in star or delta and a reduced three phase voltage applied to it. The primary and tertiary circuits are left open. The voltage induced across two consecutive brushes on the same rocker is measured. The secondary current is adjusted to give approximately rated voltage across primary. Now,

$$X_{22} = \left[Z_{22}^2 - R_2^2 \right]^{1/2} \qquad .. (2.24)$$

$$Z_{22} = \frac{V_2}{I_2} \qquad .. (2.25)$$

$$X_{23} = \frac{E_t}{I_2 \sin 60^\circ} \qquad .. (2.26)$$

The primary induced voltage may be recorded to check the value of X_{12}.

2.4.5 Measurement of X_{33}

A three phase low voltage is applied to the tertiary winding at the three brushes which are 120° electrical apart on the same rocker, keeping primary and secondary windings open. Now,

$$Z_3 = R + j\,X'_{33} = \frac{V_3}{I_3/\sqrt{3}} \qquad .. (2.27)$$

and $$X'_{33} = (Z_3^2 - R^2)^{1/2} \qquad .. (2.28)$$

In eqn. (2.28), X'_{33} is the total self reactance of the tertiary winding for a brush separation of 120° electrical, and R is the equivalent resistance per phase corresponding to the test circuit of Fig. 2.4(a). In

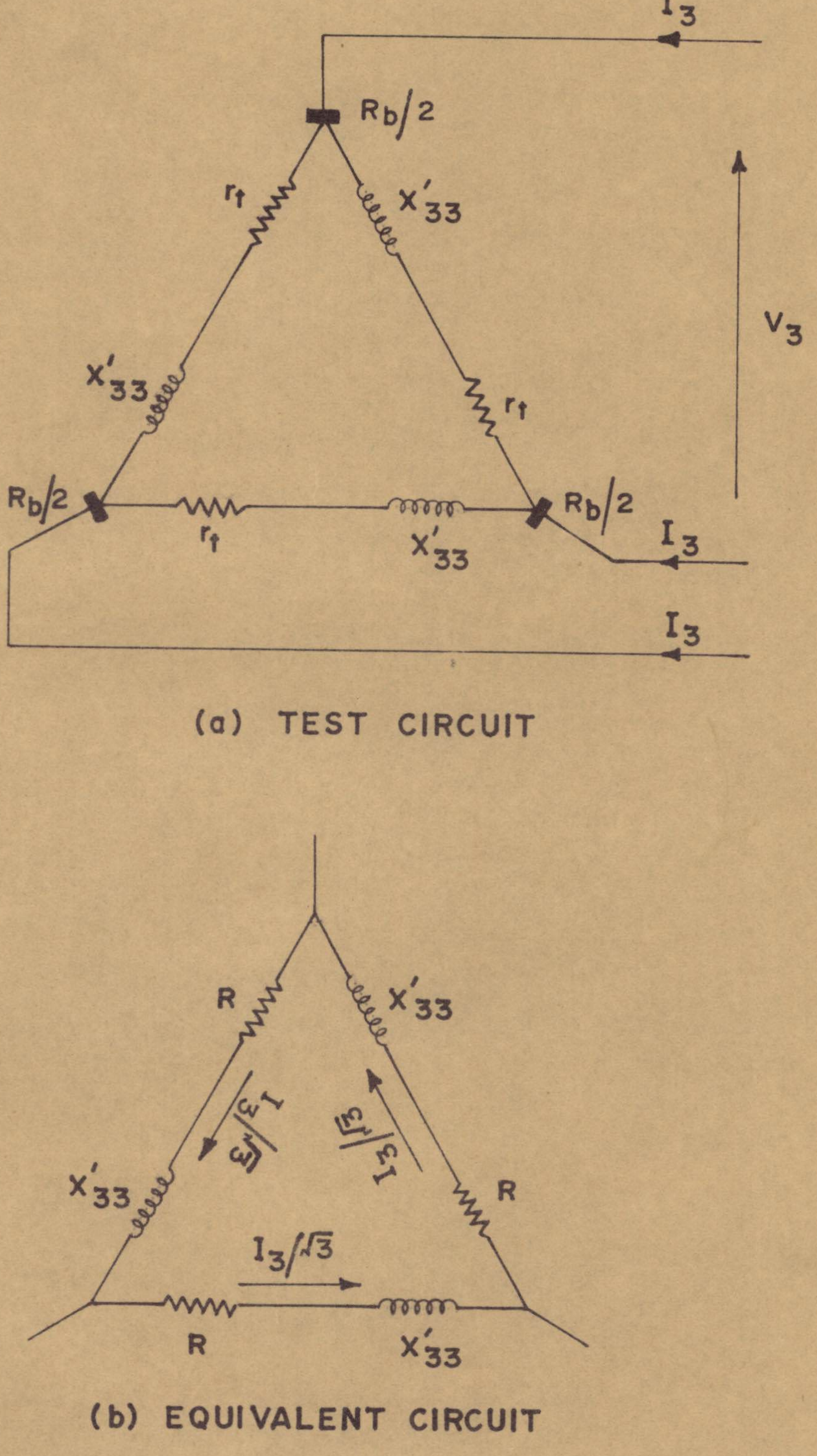


FIG. 2·4 MEASUREMENT OF TERTIARY WINDING REACTANCE.

each phase, the effective resistance of tertiary winding is given by

$$R_{3\beta} = r_3 \frac{60^\circ}{180^\circ} = \frac{r_3}{3} \qquad .. (2.29)$$

To determine effective resistance of equivalent circuit of Fig. 2.4(b), power loss is being equated with that of test circuit.

$$3I_3^2 \frac{R_b}{2} + 3 \frac{I_3^2}{3} R_{3\beta} = 3 \frac{I_3^2}{3} R$$

Therefore,

$$R = \frac{3}{2} R_b + \frac{r_3}{3} \qquad .. (2.31)$$

The self inductance of tertiary winding per phase is proportional to the square of the effective turns of tertiary. Therefore,

$$X_{33} = \frac{X'_{33}}{\text{Sin}^2 60} \qquad .. (2.32)$$

If the primary and secondary voltages are also measured in this test, then X_{13} and X_{23} can be determined and used to check the values obtained in previous tests. In fact, it is desirable to determine mutual reactances as geometric mean of the two values obtained by open circuit test on two windings separately, in order to minimize the effect of leakage flux.

CHAPTER III

Sequence Impedances of Schrage Motor

3.1 General

The method of symmetrical components is excellently suited to the analysis of an unbalanced system. Accordingly unbalanced voltages and currents are resolved into their balanced symmetrical components. The system behaviour is then investigated for these symmetrical components. But the impedances offerred to different components by a rotating machine are not equal in general. Therefore, a thorough study of magnitude and nature of the positive-, negative- and zero-sequence impedances of a Schrage motor, under varying adjustments of brush separation and brush axis shift with slip, is essential before the method of symmetrical components can be successfully applied for analysis of performance.

It is to be noted that the application of symmetrical component theory is based on the assumption that there is no interaction between the different sequence systems and that the impedance parameters which are determined under balanced conditions preserve their identity unchanged when the conditions are unbalanced. There is no direct evidence in support of these assumptions but an observed overall agreement between theory so deduced and practice provides necessary proof. Another important aspect of the application of symmetrical component theory is the preassumption of magnetic linearity and validity of theorem of superposition. But saturation does effect the machine parameters. It has been found, however, that error introduced in symmetrical component analysis due to non-linear nature of machine parameters is small and can in most cases be neglected.

3.2 Positive Sequence Impedance

It has been shown that the eqn.(2.14) relates the applied balanced voltage with the current drawn by a Schrage motor. Introducing the derived parameters of the nature of the time constants in radians, the positive sequence impedance is given by

$$Z_p = R_1 + jX_{11}\,\frac{1+jk_e\left[(T'_{3s}\sin\beta + T'_{\rho s}\,\epsilon^{j\rho_1})\sin\beta + s(T'_{2s} + T'_{\rho s}\sin\beta\,\epsilon^{-j\rho_1})\right]}{1+jk_e\left[(T'_{3o}\sin\beta + T'_{\rho o}\,\epsilon^{j\rho_1})\sin\beta + s(T'_{2o} + T'_{\rho o}\sin\beta\,\epsilon^{-j\rho_1})\right]} \qquad .. (3.1)$$

These time constants have been defined under the heading of symbols and are not only independent of brush setting and slip, but they are also constant for a motor. The factor k_e accounts for the change in the effective resistance of secondary circuit with brush separation β. The variation of $k_e = R_o/(R_o+R_{3\beta})$ for the machine under test is shown in Fig.3.1. The curve has been drawn with the value of R_b pertaining to full load secondary current

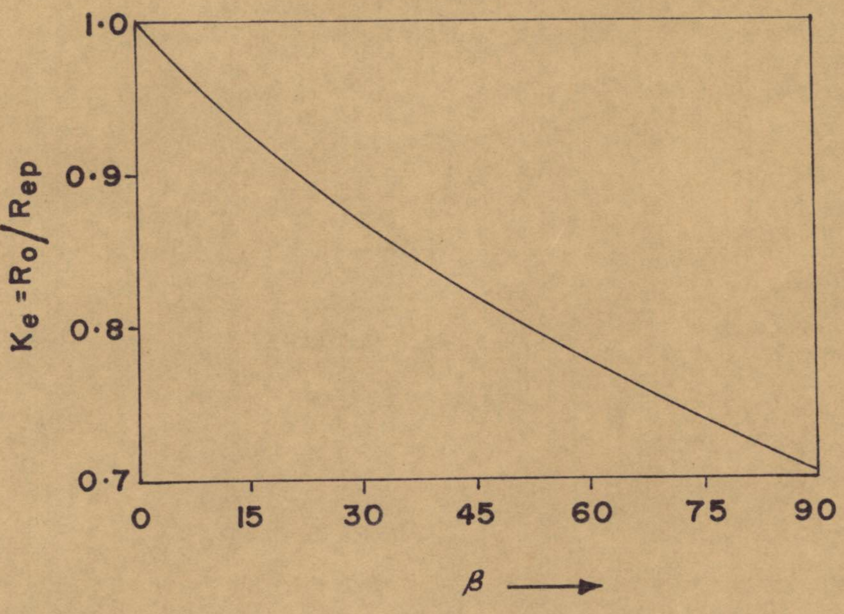


FIG. 3·1 EFFECTIVE POSITIVE SEQUENCE RESISTANCE OF SECONDARY CIRCUIT AT DIFFERENT VALUES OF β

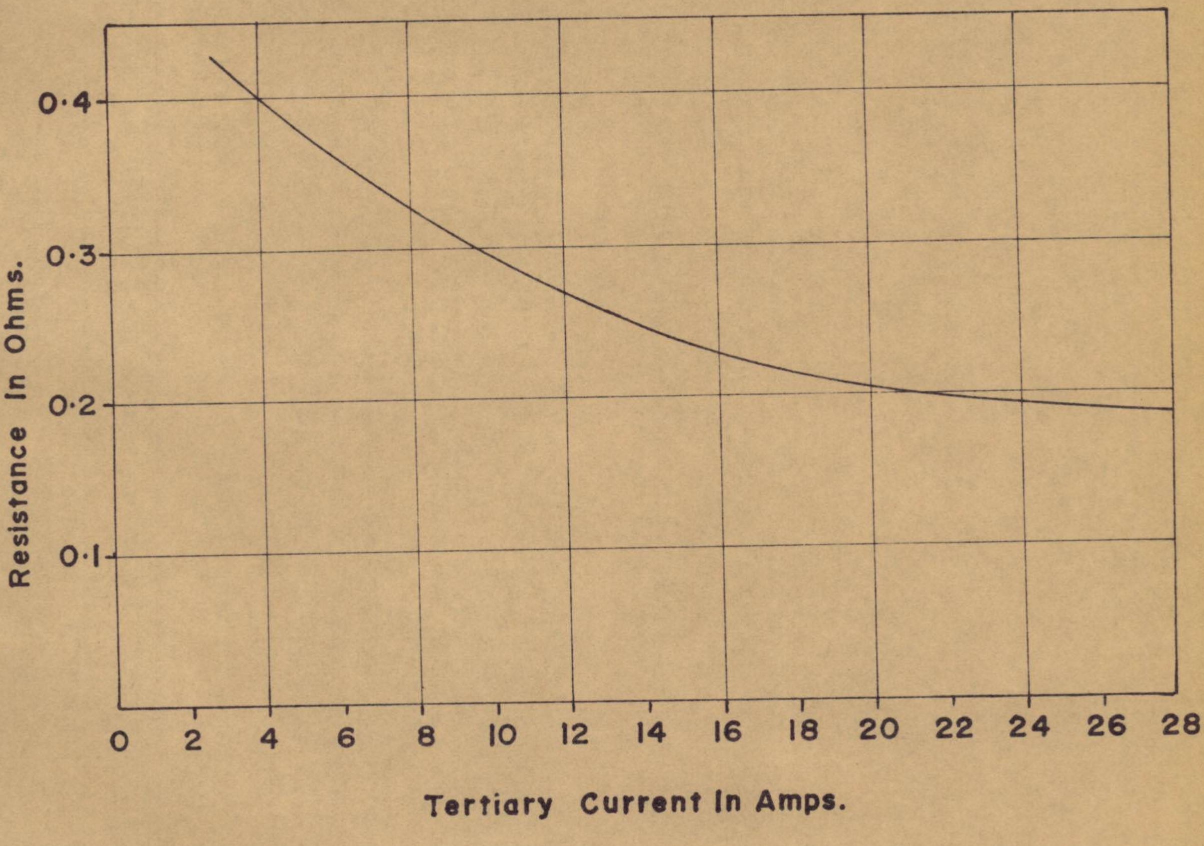


FIG. 3·2 VARIATION OF BRUSH CONTACT RESISTANCE.

as shown in Fig. 3.2. Any change in R_b can easily be incorporated by modifying R_o.

The expression (3.1) brings out the dependence of Z_p on slip, β and ρ explicitly as it has been reduced to a usable form. Z_p can, therefore, be easily determined for any operational adjustment once the machine parameters are known.

To examine the validity of expression (3.1), a neutral set motor was tested (Appendix 3.1). The calculated curves and experimental points for various values of β are shown in Fig. 3.3. To calculate Z_p, brush contact resistance R_b was taken corresponding to actual secondary circuit current. In experimental determination, resistive component of the input impedance was suitably corrected for iron loss corresponding to the applied voltage and speed during test. It may be seen that there is a very close agreement between calculated and experimental values of Z_p.

Fig. 3.3 reveals some interesting features of the dependence of positive sequence impedance on slip and brush separation. The following points are worth noting:-

(i) The impedance offerred by machine is same at theoretical no load speed corresponding to a given value of β and ρ_1, irrespective of brush separation. This impedance is equal to that of an equivalent induction motor i.e. β=0. (This is because there is no current in secondary circuit at theoretical no load speed of the motor)

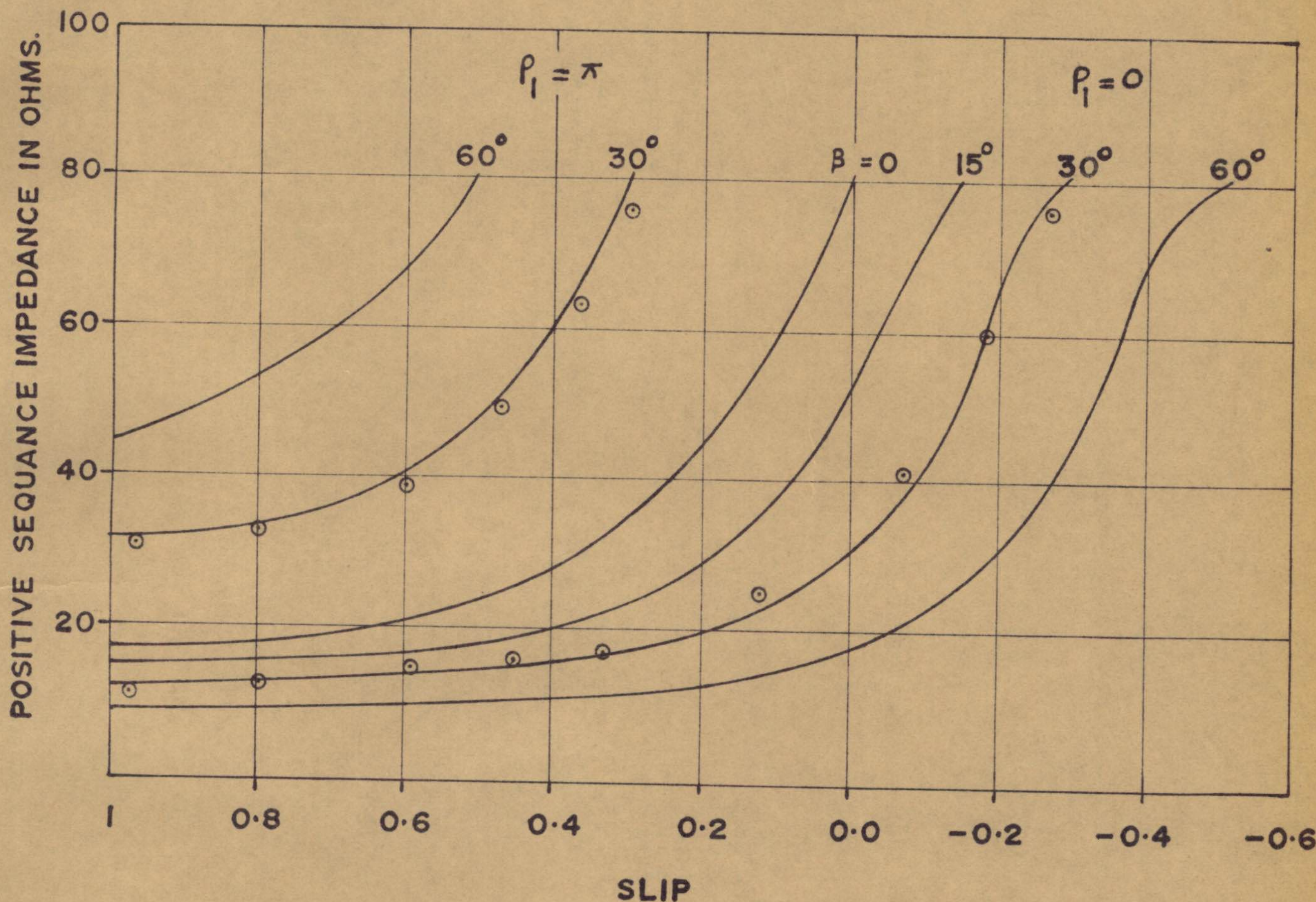


FIG. 3·3 POSITIVE SEQUENCE IMPEDANCE FOR DIFFERENT VALUES OF β

(ii) The variation of Z_p with slip for different values of β is similar to the variation of input impedance of an induction motor.

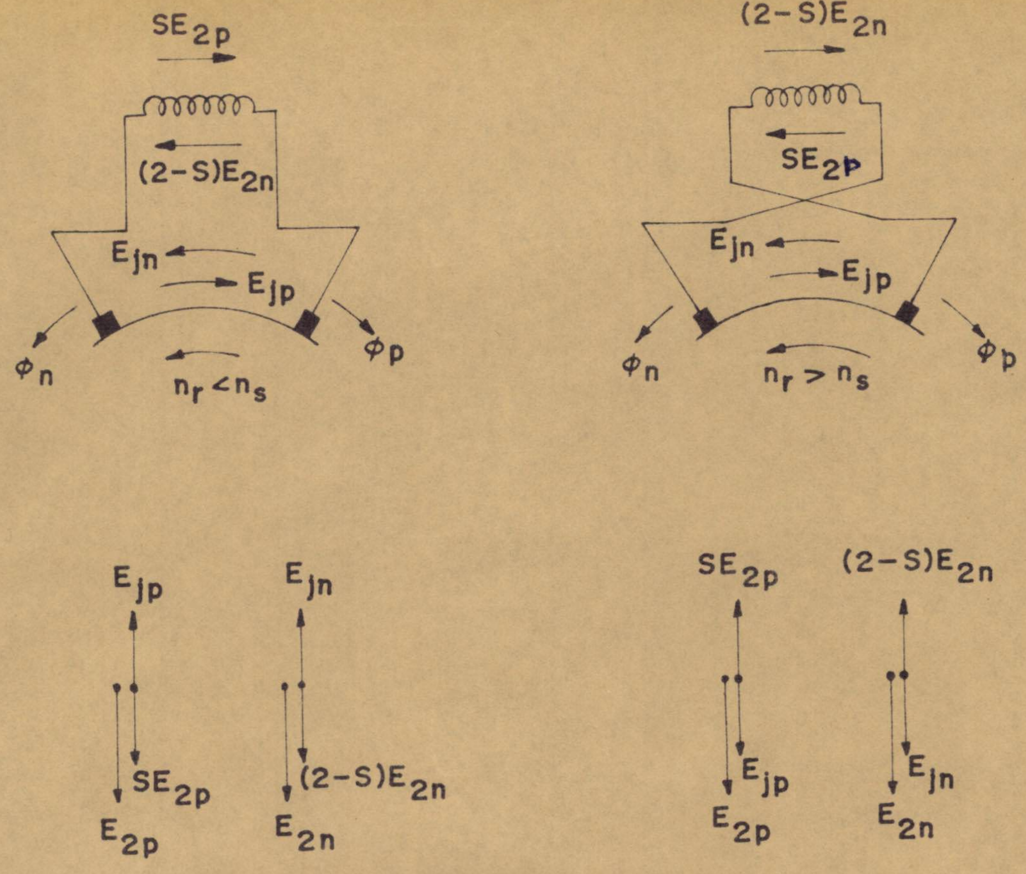
(iii) Larger the brush separation, more is the value of Z_p. for normal brush separations. Therefore, the motor will draw less and less current from source as β is increased with $\rho_1 = \pi$.

(iv) With crossed brushes, Z_p decreases as β is increased. Thus, with this setting, motor current would exceed the limiting value of that of an induction motor action, with $\beta=0$.

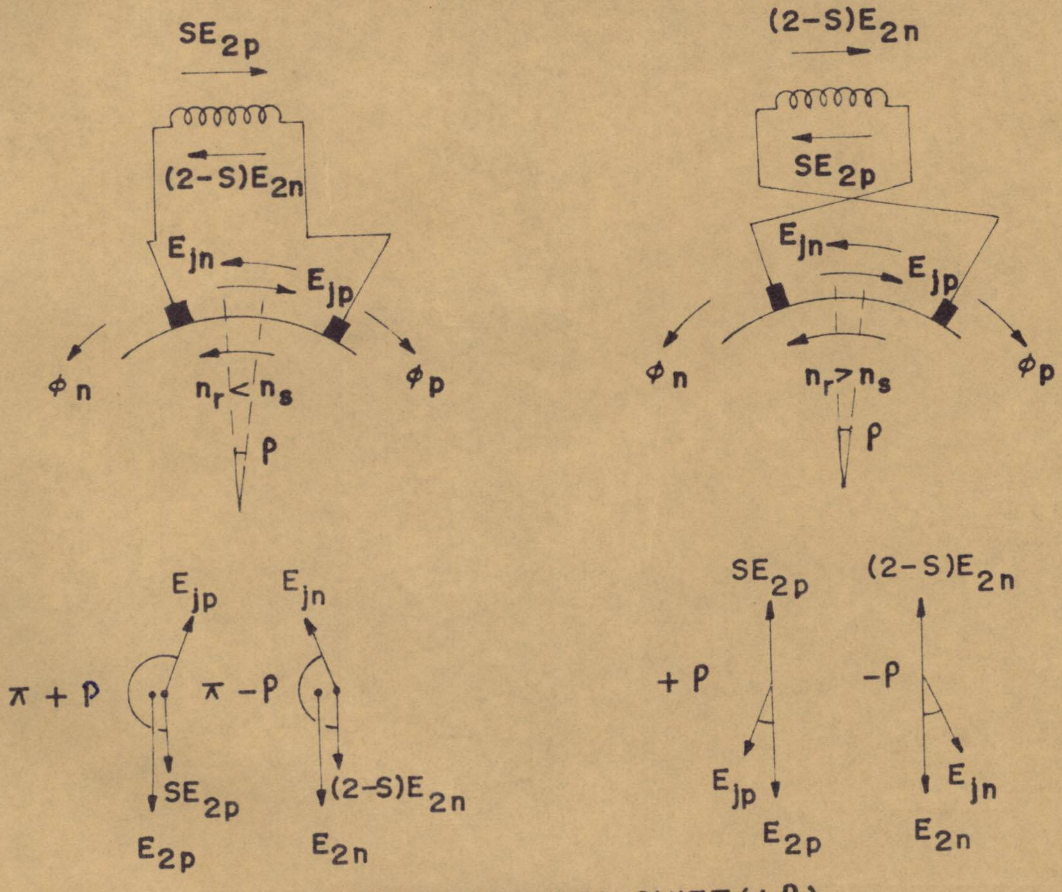
3.3 Negative Sequence Impedance

The negative-sequence impedance of a Schrage motor may be obtained from eqn. (3.1), replacing s by (2-s) and ρ_1 by ρ_2.

The direction of rotation of the negative-sequence flux is opposite to that of positive-sequence flux. But the frequency of the negative sequence voltage injected through commutator and brushes into secondary circuit remains equal to that of secondary winding negative sequence induced e.m.f., because the speed of the air gap field is same with respect to both brushes and secondary. Further, since the direction of flux rotation is reversed in relation to both tertiary and secondary windings Fig.3.4, the phase angle relation between the induced e.m.f's in secondary and tertiary windings of a neutral set motor due to negative-sequence field is same as with positive-sequence field. On the other hand, if ρ is the brush axis



(a) NEUTRAL SET MOTOR



(b) WITH BRUSH SHIFT (+ρ)

FIG.3·4 PHASE RELATIONSHIP BETWEEN SECONDARY AND INJECTED VOLTAGE shift from its neutral position in the direction of positive sequence field, it results in a quadrature component of injected voltage with respect to secondary induced e.m.f. For normal brush separations, this component leads the secondary voltage consequent to positive sequence flux, while it lags the secondary e.m.f induced by negative sequence flux. Therefore, to account for phase angle of injected e.m.f. relative to secondary e.m.f induced by negative sequence field, ρ_1 is changed to ρ_2 such that $\rho_2 = (\pi - \rho)$ for normal brush separations and $(-\rho)$ for crossed brushes. For a neutral set motor, however, ρ_2 is still equal to π (normal brushes) or zero (crossed brushes).

The secondary winding resistance is fairly constant at low frequencies but it does change at higher frequencies in its d.c. value. For usual operating range of the motor, effective value of secondary resistance may be taken to be equal to its d.c. value for positive sequence currents. For negative sequence currents, the frequency is $(2-s)f$ and in consequence, the secondary resistance is no more equal to its d.c. value. To account for this increased resistance, a factor $k_n = R_{ep}/R_{en}$ is introduced. As R_{2n} is a function of speed of the motor , its variation was determined experimentally by variable frequency test. The experimental details are described in Appendix 3.1. The variation of k_n for different values of β with respect to slip is then determined and is shown in Fig. 3.5.

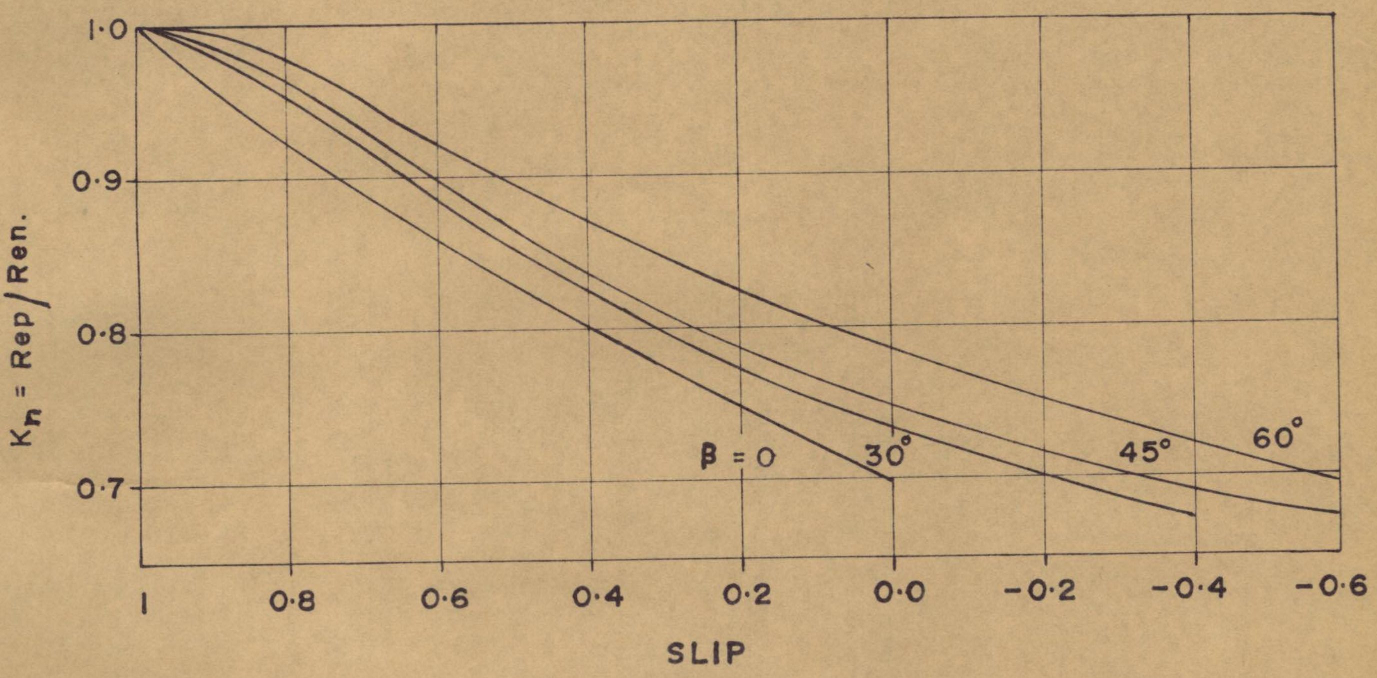


FIG. 3·5 RATIO OF EFFECTIVE POSITIVE TO NEGATIVE SEQUENCE RESISTANCES OF SECONDARY CIRCUIT.

Therefore, in the light of the above discussion, the negative sequence impedance is given by,

$$Z_n = R_1 + jX_{11} \frac{1 + jk_e k_n \left[(T'_{3s} \sin\beta + T'_{\rho s} \epsilon^{j\rho}) \sin\beta + (2-s)(T'_{2s} + T'_{\rho s} \sin\beta \, \epsilon^{-j\rho_2}) \right]}{1 + jk_e k_n \left[(T'_{3o} \sin\beta + T'_{\rho o} \epsilon^{j\rho_2}) \sin\beta + (2-s)(T'_{2o} + T'_{\rho o} \sin\beta \, \epsilon^{-j\rho_2}) \right]} \quad .. (3.2)$$

The calculated and experimentally determined values of Z_n for various brush separation β are shown in Fig.3.6. As for Z_p, the variation in brush contact resistance was properly accounted for. Also k_e and k_n were suitably substituted. It is evident from Fig.3.6 that the theoretical values are in complete confirmation with experimental ones. Therefore, eqn.(3.2) gives Z_n with sufficient accuracy.

Fig. 3.7 shows the variation of Z_n/Z_s with speed as obtained from Fig.3.6. The following inferences may be drawn from these two sets of curves regarding negative sequence impedance.

(i) The negative sequence impedance decreases with speed for all values of brush separation β as for an induction motor ($\beta=o$), both for normal as well as crossed brush settings.

(ii) For normal brush setting, Z_n increases with β while for crossed brushes Z_n decreases with brush separation.

(iii) For $\rho_2 = \pi$ i.e. normal brush setting, as β is increased, the value of Z_n/Z_s decreases

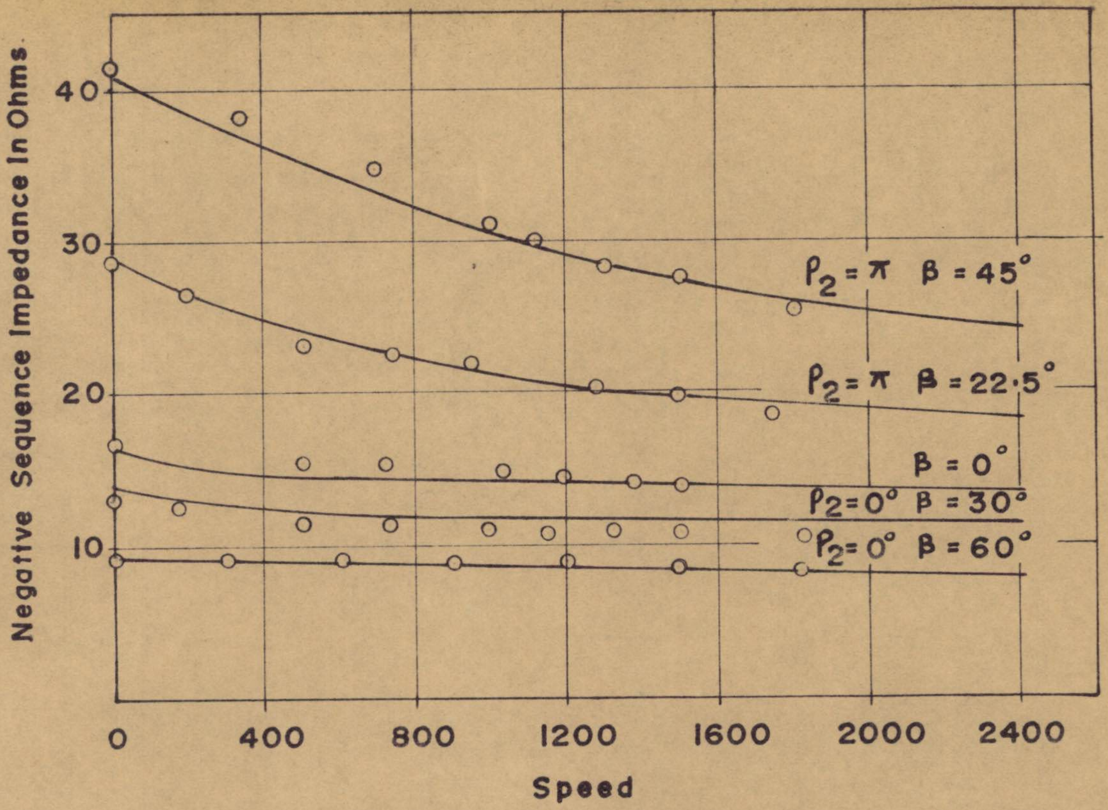


FIG. 3·6 NEGATIVE SEQUENCE IMPEDANCE FOR DIFFERENT VALUES OF β

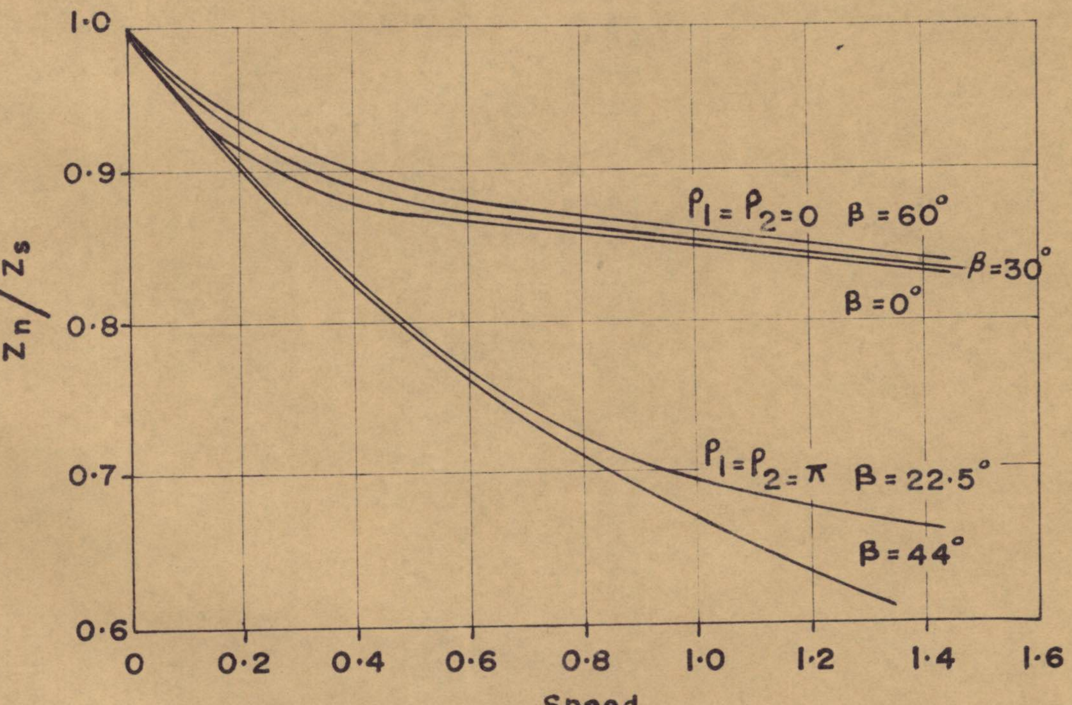


FIG. 3·7 VARIATION OF THE RATIO OF NEGATIVE TO STAND STILL IMPEDANCE WITH SPEED.

while for $\rho_2 = 0$, Z_n/Z_s increases above the limiting case of an induction motor ($\beta = o$).

(iv) The change in the value of Z_n at a particular speed from its standstill value Z_s is a function of brush separation. But with $\rho_2 = 0$, the decrease in Z_n is not appreciable and reduces as β is increased. Therefore, for a particular brush separation with crossed brushes, Z_n may be taken equal to corresponding Z_s for that value of β for all speeds without any substantial error. The percentage error with this assumption goes on decreasing as β increases. When the motor is run with normal brush separation, it is not possible to assume a fixed value of negative sequence impedance at all speeds. Although for an induction motor, very often Z_n is assumed to be constant and equal to Z_s irrespective of speed. With $\rho_2 = \pi$, as β is increased, Z_n deviates from Z_s inordinately.

Hence, for an accurate analysis of a Schrage motor under unbalance operation, it is not desirable to assume negative sequence impedance constant irrespective of slip for different brush separations, especially with normal brush settings. The calculation of Z_n for any value of slip, β and ρ_2 is a involved process by conventional methods[35], but becomes much simpler by the use of the expression in eqn. (3.2). If simplifying approximation prevails and not so accurate calculations are necessary, Z_n may be presumed constant for crossed brushes.

3.4 Zero-sequence Impedance.

The zero-sequence impedance of a machine is its impedance when all of its phases carry equal co-phasal currents in the same sense. The zero-sequence excitation of a machine may be simulated by connecting all the three phases in series to a single phase source. If the current flowing is $I_m \cos\omega t$, the resultant space distribution of m.m.f. is given by $\Sigma F_\nu \sin\nu \frac{\pi x}{\tau_p} (1-2\cos\frac{\nu\pi}{3})\cos\omega t$, ν being any integer.

It shows that m.m.f is zero for $\nu = 1,5,7$, etc., but is equal to $3F_\nu \sin(\nu\pi x/\tau_p)\cos\omega t$ for $\nu = 3,9,15$, etc. Therefore, the resultant m.m.f consists of only triplen space harmonics. Although the space distribution of m.m.f with single phase excitation is likely to have high triplen harmonic content, harmonics of the order higher than third may be neglected being of very small amplitude in a properly designed machine. Therefore, the zero-sequence current may be looked upon as origin of pulsating field of fundamental frequency but having three times the number of poles in space, for which the machine has actually been wound. It is to be stressed that the nature of the field established by zero-sequence currents is radically different in waveform and amplitude from that produced by either positive or negative sequence currents of the same magnitude and flowing in the same winding[41].

The pulsating field $3F_3 \sin(3\pi x/\tau_p)\cos\omega t$ may be resolved into two fields, each of half the total amplitude but rotating in opposite directions w.r.t exciting winding at a speed of $n_s/3$. Correspondingly, the speed of these revolving fields with respect to a winding rotating relative to exciting winding at a speed of $n_r = n_s(1-s)$ are $(-2+3s)\frac{n_s}{3}$ and $(4-3s)\frac{n_s}{3}$.

In case of a Schrage motor, the primary winding is on the rotor along with tertiary winding and secondary winding is housed on stator. The two rotating fields in air gap due to zero-sequence excitation of primary winding induce corresponding voltages in both tertiary and secondary windings. As tertiary winding is always stationary with respect to primary, the induced voltage and hence current in tertiary is of supply frequency irrespective of rotor speed. Further, the tertiary winding is a closed delta winding and therefore, would behave as short circuited to these zero-sequence voltages. Thus, most of the pulsating flux due to primary excitation will be balanced by flux arising out of tertiary current of the same frequency as of supply, and only a small flux is expected to link with secondary.

The secondary winding of a Schrage motor is similar to the rotor winding of a slip ring induction motor but with individual phases short circuited through a portion of tertiary winding. The zero-sequence current in primary is looked upon as origin of an alternating field of fundamental frequency but having three times the number of poles, for which the machine has actually been wound. Therefore, voltage induced due to this field in three phases of secondary will be in same time phase. As single phase current can flow in three phase circuit of a winding only if it is either delta connected or having each phase short circuited, the zero-sequence current will exist in each phase of secondary. The resultant m.m.f. due to this secondary current will also be having only 3rd and multiple of 3rd space harmonics unless the chording factor of secondary is $2\pi/3$. With the same consideration as with primary, harmonics of the order higher than 3rd may be neglected. Consequently secondary winding also behaves as a single phase winding carrying currents of frequency $(-2+3s)f$ and $(4-3s)f$. The m.m.f. set-up by these currents once more correspond to three times the number of poles. This state of affairs is basically different to a cage induction motor where secondary can adapt itself to any number of poles corresponding to stator excitation, although the result is same.

Thus, to sum up, the zero sequence behaviour of a Schrage motor may be represented by three single phase windings; primary and tertiary having more or less identical magnetic circuit while secondary a different one. Tertiary behaves as a short circuited winding of a transformer to any single phase excitation of primary. Secondary is also short circuited through a portion of tertiary depending upon brush separation. Tertiary winding is called upon to carry two currents, one of its own and another of secondary circuit. Corresponding to a brush separation 2β under balanced operation, the effective angle for zero sequence simulation would be 6β. The secondary induced e.m.f. is a function of the space angle θ between equivalent secondary winding axis and the position of the resultant primary m.m.f[42]. The portion of the tertiary winding forming a part of secondary circuit is hardly in a position to inject any appreciable e.m.f. into secondary and would simply be equivalent to an impedance in circuit. Moreover, the mutual flux between secondary and tertiary due to secondary current is expected to be negligibly small. Therefore, zero sequence performance equations may be written as

$$\begin{bmatrix} v_o \\ \\ \\ \end{bmatrix} = \begin{array}{c} \\ p \\ s \\ t \end{array} \!\!\begin{array}{c} \begin{array}{ccc} \quad p \quad & \quad s \quad & \quad t \quad \end{array} \\ \begin{bmatrix} r_1+pL_1 & pM_{12}\cos\theta & pM_{13} \\ pM_{12}\cos\theta & r_2+r_3+p(L_2+L_3\sin^2 3\beta) & \\ pM_{13} & & R_3+pL_3 \end{bmatrix} \end{array} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} \qquad \text{.. (3.3)}$$

In this equation,

$v_o = \sqrt{2}\, V_o \cos\omega t$; zero sequence voltage per phase

$\theta = \frac{P}{2}\omega_m t +$; ω_m being rotor speed

i_3 = current through short circuited tertiary.

The steady state solution of currents leads to an infinite sequence of even harmonics $(\omega \pm 2n \frac{P'}{2}\omega_m)$ in primary and odd harmonics $\left[\omega \pm (2n+1)\frac{P'}{2}\omega_m\right]$ in secondary. $P' = 3P$ is effective number of poles and n is any integer including zero.

It has been shown by Elco[42] that the input impedance of a single phase machine with single phase rotor may be approximated to be time varying at all speeds except when $(p'/2)\omega_m$ equalsto ω , provided secondary reactance is assumed to be small compared to r_2. Although this approximation is not valid in general but it gives a fair estimate of the variation of zero-sequence impedance with rotor speed. Qualitatively, the variation of input inductance is very nearly linear with respect to ω_m. Further, it may be concluded reasonably that near synchronous speed of rotating field in forward direction, the impedance would have a maxima in magnitude similar to a cage induction motor[43]. However, this increase is not likely to be appreciable as only a small part of the total zero-sequence flux is able to cross the airgap.

To verify the foregoing conclusions, tests were conducted on a Schrage motor with all the three phases of primary connected in series to a single phase supply. The observations were recorded with secondary winding open as well as short circuited through tertiary for various values of β including zero, both with crossed and normal brush settings.

The standstill value of Z_o with secondary in circuit was found to be a function of angular position of rotor and confirmed to three times the actual number of poles, as expected. The Fig. 3.8 shows the variation of zero sequence impedance with speed. The standstill value has been obtained by extrapolating the curves.

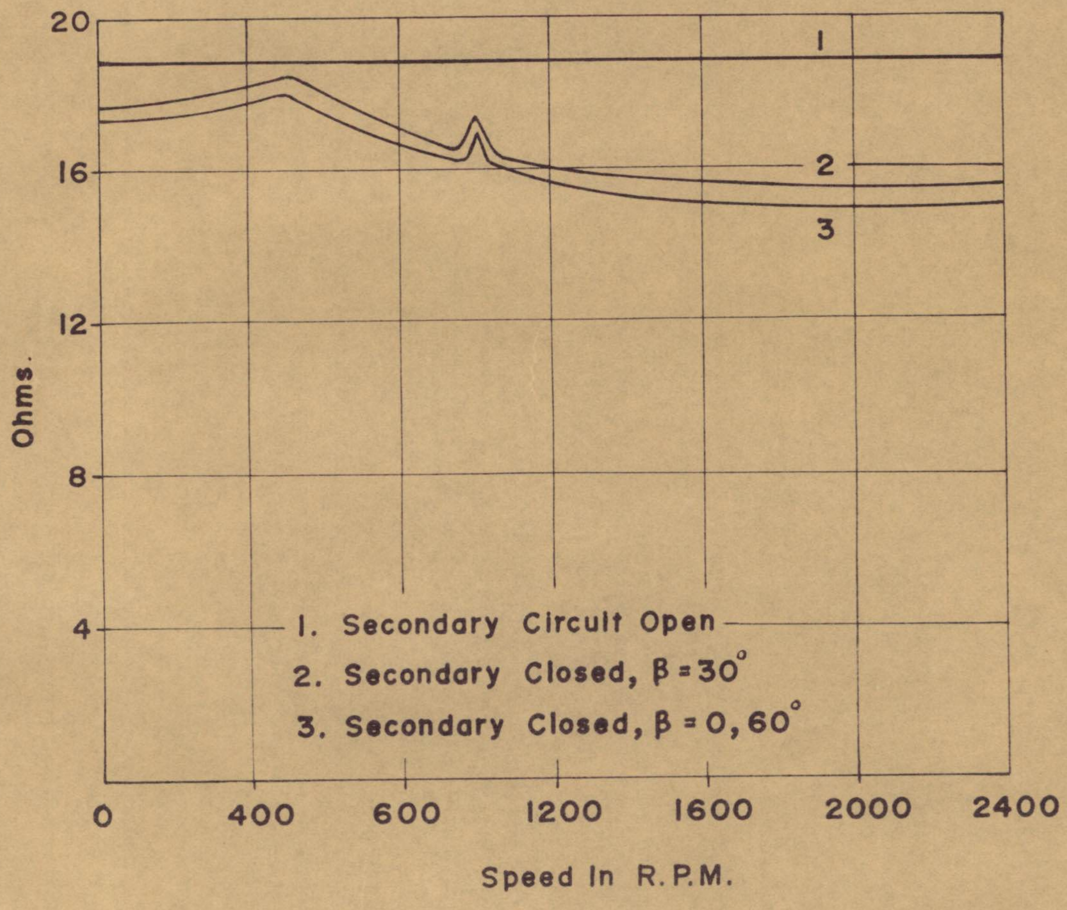


FIG. 3·8 ZERO SEQUENCE IMPEDANCE.

The magnitude of the impedance is highest and independent of speed, if secondary winding is open circuited. But if secondary circuit is closed, an impedance parallel to tertiary is formed and net impedance is reduced. This decrease as discussed earlier is not substantial because of shielding effect of tertiary winding. Curves 2 and 3 (Fig. 3.8) depict the extreme limit of Z_o with secondary winding in circuit. The zero sequence impedance for all values of β both for $\rho = \pi$ or 0, lies more or less within these limits, justifying the arguments regarding the tertiary-secondary coupling. The value of Z_o for $\beta = 0$ as well as $\beta=60^o$ is almost the same confirming the fact that there is a very little voltage injection from tertiary to secondary and portion of tertiary winding in secondary circuit is just equivalent to a reactance proportional to $\sin^2 3\beta$, being zero both for $\beta=0$ and 60^o. A slight deviation was observed due to the resistance of the winding which obviously is unaffected by the change in number of poles and simply increases with β. For the sake of clarity only limiting values of Z_o has been plotted in Fig. 3.8, although the zero-sequence impedance was measured for a large number of values of β, both for $\rho= 0$ and π, to verify the above inference.

The peculiar features of the variation of Z_o can be conveniently explained from torque considerations. A rotating machine develops torque when two of its members carry currents of such frequencies that a space harmonic of one winding m.m.f. due to a component of the current through that winding, is relatively at rest with the same space harmonic of another winding (on the other member) m.m.f. due to some time harmonic of this winding exciting current. The torque occurs at all speeds upto corresponding synchronous speed if the relevant component in one winding is induced by the corresponding harmonic of the other winding current. This asynchronous torque changes sign as the rotor passes through the relevant synchronous speed. In the present case, asynchronous torque exists at all speeds but is positive in the region $0 < \frac{p'}{2} \omega m < \omega$. At $\frac{p'}{2} \omega m = \omega$ or $\frac{p}{2} \omega m = \frac{\omega}{3}$ = 500 rpm, the forward direction torque is zero and input impedance is maximum.

On the other hand synchronous locking occurs only when the current giving rise to relevant space harmonic of one winding is not induced by the corresponding component of the other winding and the speed of the rotor is correct one to make the two rotating fields relatively stationary. If h_1 is the time harmonic order of primary winding current, h_2 that of the current in secondary and ν is space harmonic order, then synchronous locking is possible at a speed of

$$\frac{p}{2} \omega_m = \left(\frac{h_1 - h_2}{\nu - h_2} \right) \omega \qquad .. (3.4)$$

h_1, h_2 and ν are positive if harmonic order is 3,7,9,13 etc., or positive sequence fundamental component; negative for harmonic order 5, 11, 17 etc., or negative sequence current. For single phase zero-sequence excitation,

$$\nu = 3$$

$h_1 = \pm 1$ as m.m.f. is pulsating (n=0)

$h_2 = \pm (\omega - \frac{3P}{2}\omega m)/\omega$ corresponding to $h_1 = +1 (n=o)$

$= \pm (\omega + \frac{3P}{2}\omega m)/\omega$ for $h_1 = -1$ (n = o)

A little manoeuvring reveals that a strong tendency for synchronous locking exists due to the interaction of primary forward rotating component with oppositely rotating component of secondary due to backward field of primary. The crawling speed is given by,

$$\frac{P}{2}\omega m = \left[\frac{1+(\omega + \frac{3P}{2}\omega m)/\omega}{3+(\omega + \frac{3P}{2}\omega m)/\omega}\right]\omega$$

$$= \frac{2}{3}\omega \; ; \; -\omega \qquad .. \; (3.5)$$

At the speed of $\frac{2}{3}\omega$, current drawn from the source decreases due to synchronization and consequently input impedance is characterised with a very sharply defined peak. Thereafter, once more the variation of $\dot{Z}_o$ is almost linear.

CHAPTER IV

Operation with Unbalanced Supply

4.1 General

The unbalance phase voltages at the terminals of a motor may occur owing to the presence of unbalanced loads on the system or line disturbances.Sometimes deliberate unbalance is created to control and brake a.c. motors.

The magnitude of sequence components of current consequent to unbalanced operation, depends upon the parameters of the motor and the nature of unbalance. The zero-sequence may be present if warranted under that operating condition[38,40]. Usually motors are fed with three wire supply i.e. neutral of the motor if star connected is left isolated. For this condition, zero-sequence current cannot flow through motor windings. A simple method to resolve unbalance voltages irrespective of their relative phase displacement has been given by Williams[36]. The methods to determine the nature and magnitude of sequence components and their effect have been covered for induction class of machine, by Brown and Butler[38] for asymmetrical primary conditions, and by Butler and Wallace[39] for any number of asymmetrical primary windings fed with asymmetrical power supply with asymmetrical impedances connected to motor terminals.

Each of the sequence voltages produces its own current and torque, and these currents and torques are combined to obtain the behaviour of the machine under specified operating condition. Once the basic nature of these currents, torque etc. is known, the machine performance may be easily visualized for any unbalance operation. Therefore, in this chapter, attempt has been made to deduce theoretical expressions to describe the Schrage motor performance with unbalance supply voltages. These theoretical formulations have been checked by experimental results and physical explanations have been advanced wherever possible.

4.2 Primary and Secondary Currents

If V_p and V_n are respectively the positive and negative sequence components of applied voltage, the corresponding currents in primary winding are given by

$$\left.\begin{aligned} I_{1p} &= V_p/Z_p \\ I_{1n} &= V_n/Z_n \end{aligned}\right| \qquad ..(4.1)$$

Z_p and Z_n are defined and obtained from eqns.(3.1) and (3.2) respectively. From eqn.(2.14),

$$I_{2p} = \frac{V_p}{X_{11}} \frac{k_e}{R_o} \frac{-(sX_{12}+X_{13}\sin\beta\cos\rho_1)+jX_{13}\sin\rho_1\sin\beta}{\left[\left\{1-(1-s)k_e T'_{\rho s}\sin\beta\sin\rho_1\right\}+\frac{R_1}{X_{11}}(T'_{op1}+sT'_{op2})\right]^2 - j\left[\frac{R_1}{X_{11}}\left\{1-(1-s)k_e T'_{\rho o}\sin\beta\sin\rho_1\right\} - (T'_{sp1}+sT'_{sp2})\right]^2} \qquad ..(4.2)$$

Similarly,

$$I_{2n} = \frac{V_p k_e}{X_{11} R_o} \lambda_v k_n \frac{-\left[(2-s)X_{12}+X_{13}\sin\beta\cos\rho_2\right]+jX_{13}\sin\beta\sin\rho_2}{\left[\left\{1+(1-s)k_e k_n T'_{\rho s}\sin\beta\sin\rho_2\right\}+\frac{R_1}{X_{11}}\left\{T'_{on1}+(2-s)T'_{on2}\right\}\right]^2 - j\left[\frac{R_1}{X_{11}}\left\{1+(1-s)k_e k_n T'_{\rho o}\sin\beta\sin\rho_2 - \left\{T'_{sn1}+(2-s)T'_{sn2}\right\}\right]^2} \quad .. (4.3)$$

From eqns.(4.1) to (4.3), primary and secondary circuit currents of any phase may be easily computed.

In order to study the influence of degree of voltage unbalance, brush separation and speed range setting on the relative magnitudes and nature of primary and secondary currents unbalances, variations of λ_{i1}, λ_{i2}, $\lambda_{i2}/\lambda_{i1}$, I_{2p} and I_{2n} with respect to slip are shown in Figs. 4.1 to 4.5 respectively. To check the validity of expressions (4.2) and (4.3), test points are also recorded in Figs. 4.4 and 4.5. For experimentation, applied voltage was reduced so that the current does not exceed the rated value at any operating condition.

It is evident from these figures that the nature of the current flowing in two circuits of a Schrage motor is similar in general to that of an induction motor (β=o). But considerable modifications result as brush separation and brush setting are changed. For example in Fig.4.1, the primary current unbalance increases with speed as for an induction motor but rate of change of λ_{i1}, is different with different values of brush separation. For crossed brushes, the value of λ_{i1} is less at lower speeds compared to a corresponding induction motor operation but rises sharply as theoretical no load speed approaches. This is further influenced by brush separation. More is the brush separation, lesser is the value of λ_{i1} for lower speeds. For normal brush separation, λ_{i1} is always more compared to the case of $\beta=0$, and increases with increase in β. Therefore, even a small voltage unbalance may result in the presence of large negative sequence currents. The primary unbalance is more pronounced with crossed brushes than with normal brush setting in the usual operating region of the motor for a given degree of voltage unbalance.

The current unbalance prediction was further checked experimentally and is described in Appendix 4.1.

If a Schrage motor is operated at its theoretical no load speed corresponding to a particular value of β and ρ, λ_{i2} is always infinite as in case of an induction motor, for at s_o, theoretically no positive sequence secondary circuit current exists. In general the secondary current unbalance follows the same pattern as primary current unbalance.

Figure 4.3 presents a ready comparison of degree of secondary to primary current unbalance. The following

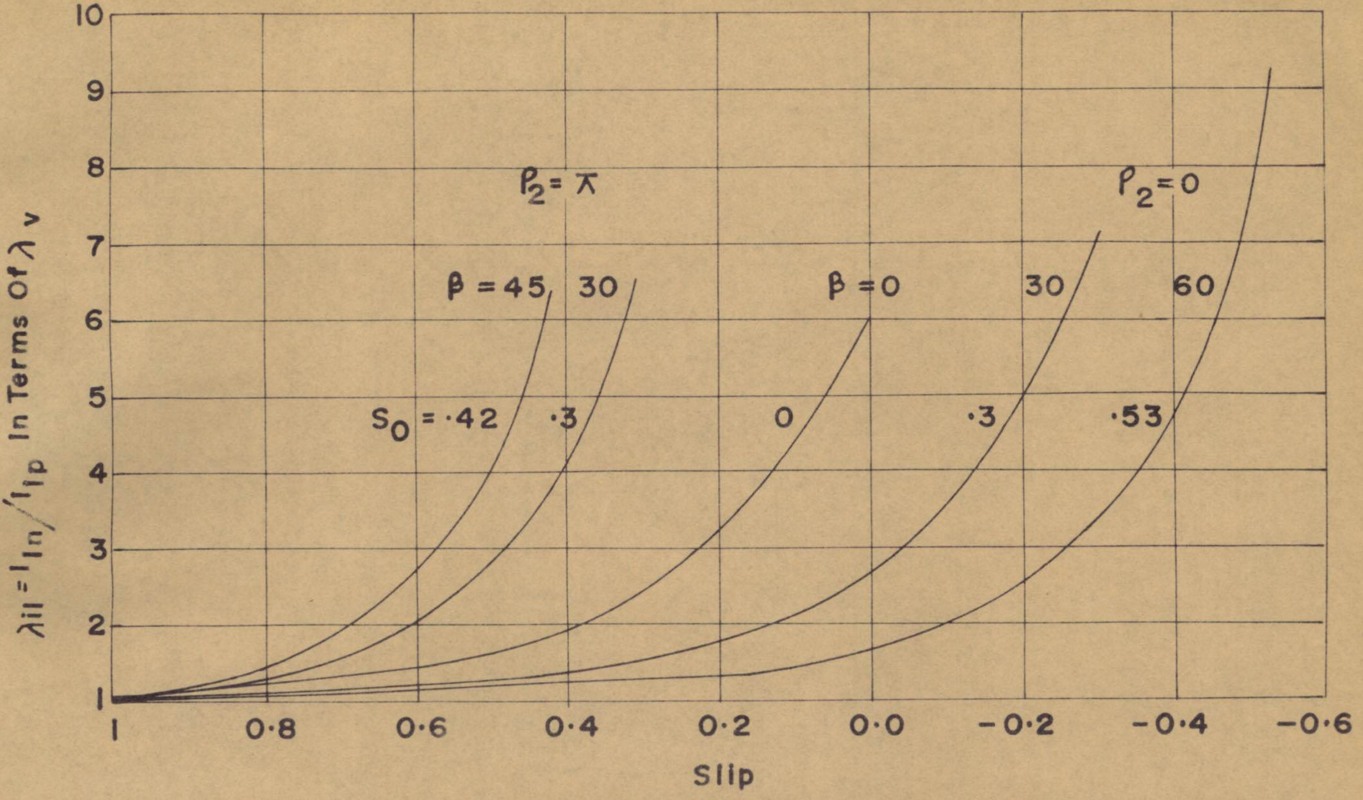


FIG. 4·1 VARIATION OF λ_{i_1} IN TERMS OF DEGREE OF VOLTAGE UNBALANCE FOR DIFFERENT VALUES OF β WITH SLIP (λ_v=1)

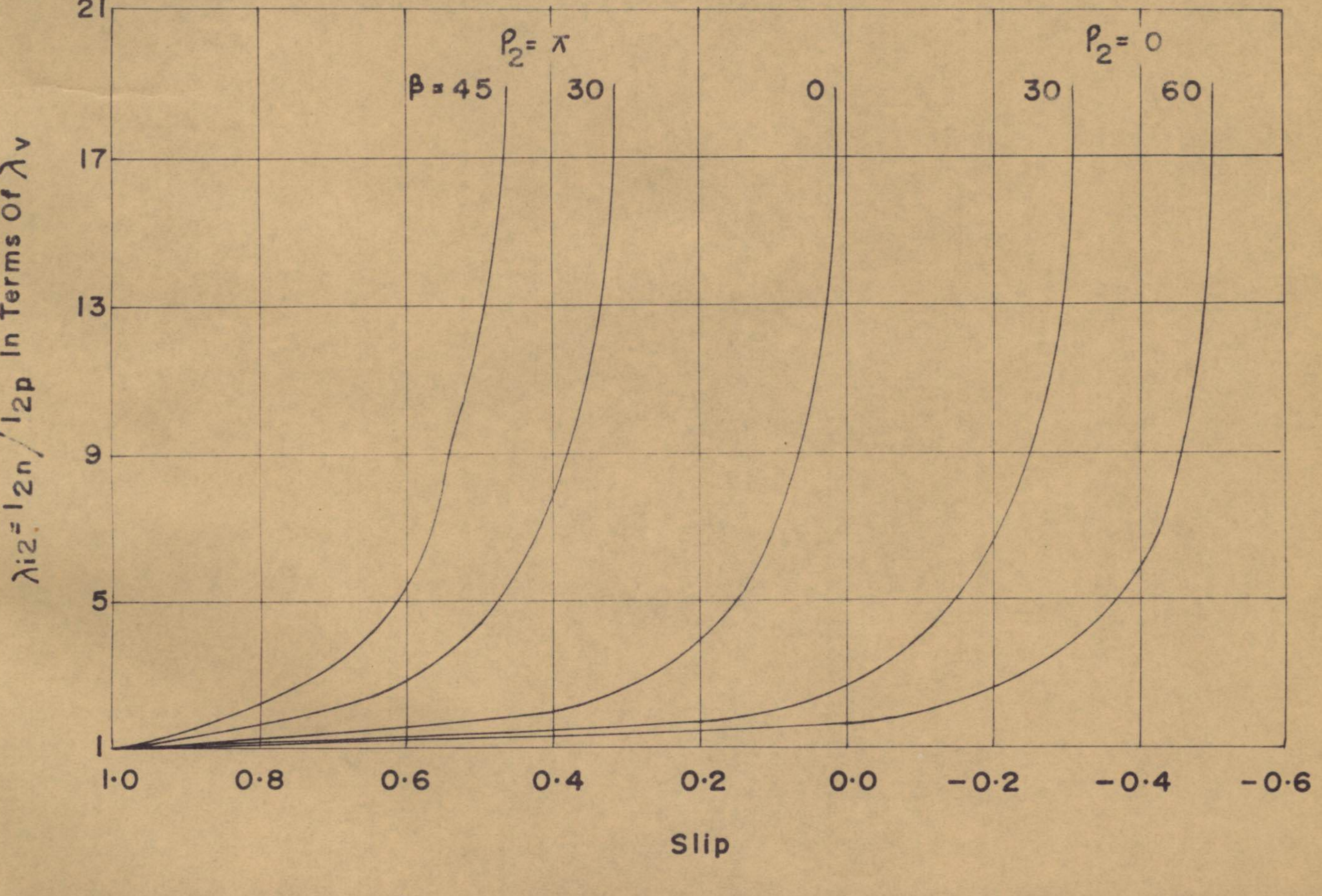


FIG. 4·2 VARIATION OF λ_{i_2} IN TERMS OF DEGREE OF VOLTAGE UNBALANCE

FOR DIFFERENT VALUES OF β WITH SLIP (λ_v=1)

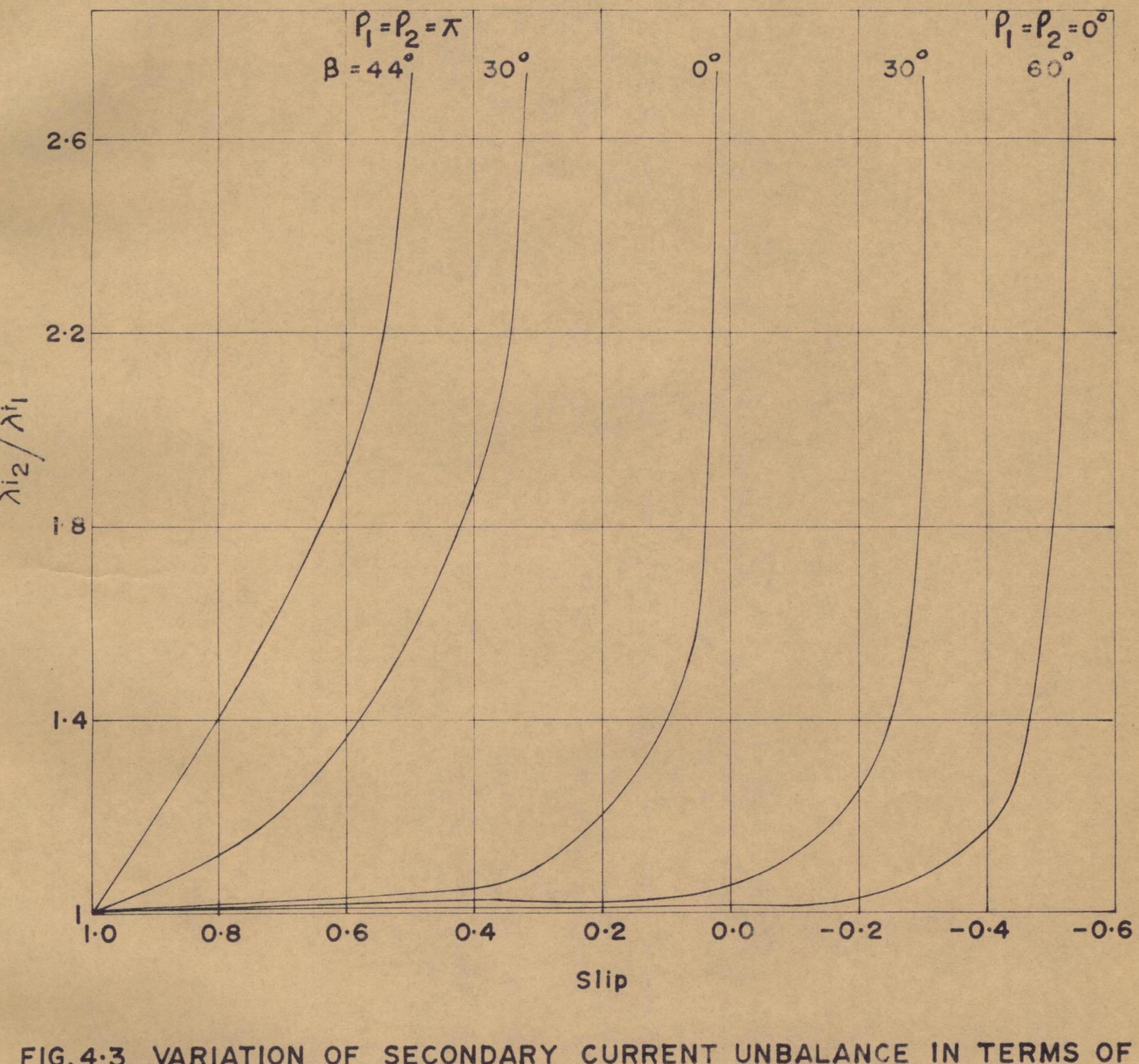


FIG. 4·3 VARIATION OF SECONDARY CURRENT UNBALANCE IN TERMS OF PRIMARY CURRENT UNBALANCE w.r.t. SLIP ($\lambda_v = 1$)

features of interest may be noted.

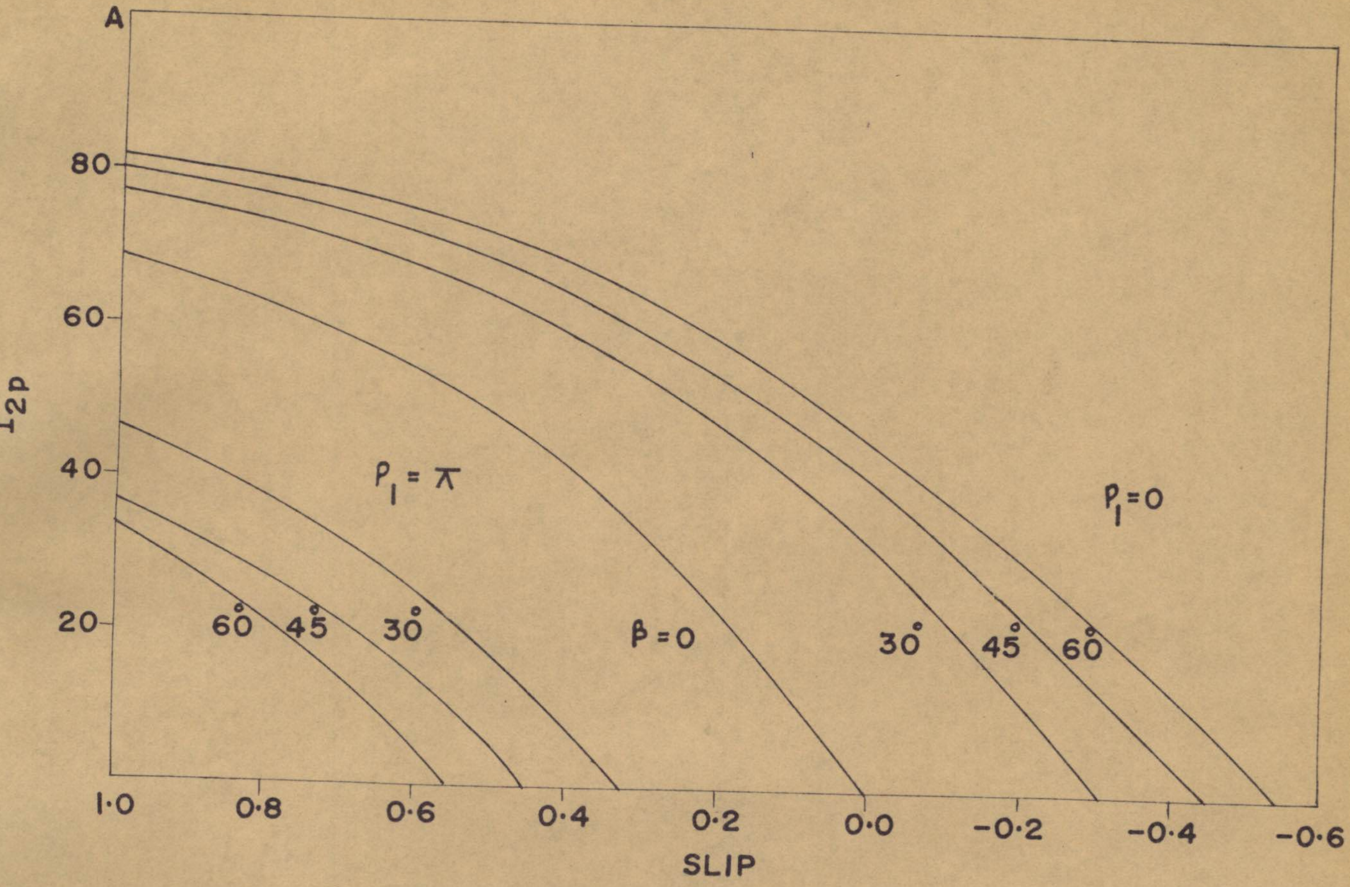
(i) For a given voltage unbalance factor λ_v, current unbalance in secondary circuit is always more than primary current unbalance at a particular speed, both for normal as well as crossed brush separations.

(ii) For crossed brushes, λ_{i1} is only slightly less than λ_{i2} at lower speeds for a given value of β, but as the speed of the motor reaches no load speed for that value of brush separation, λ_{i2} increases more rapidly than λ_{i1}.

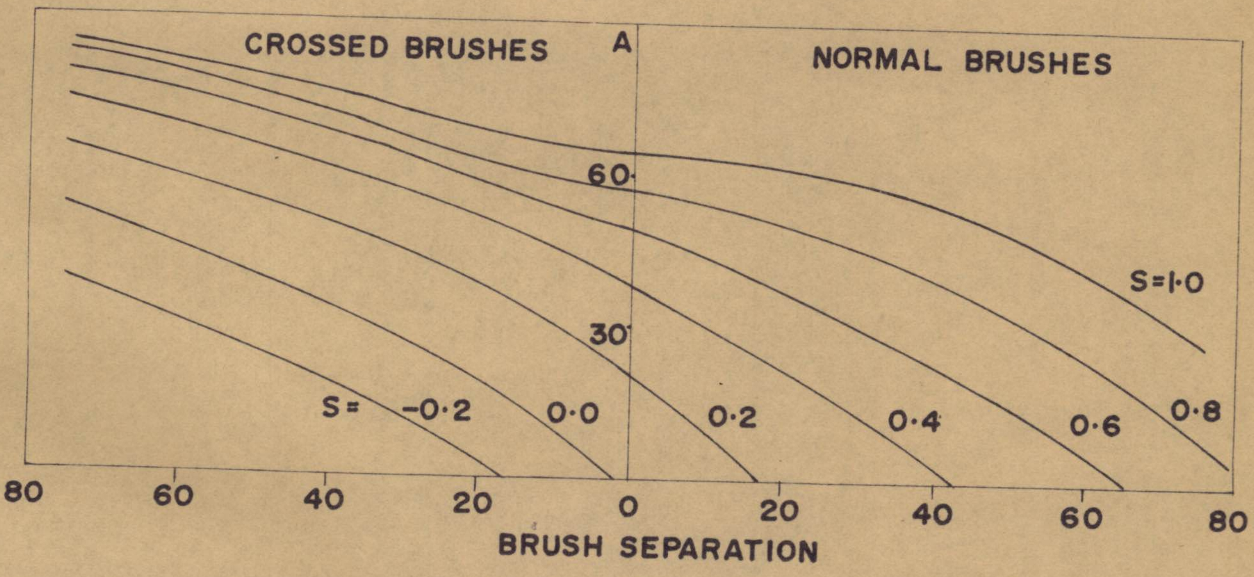
(iii) For normal brush separations, secondary circuit unbalance increases at a faster rate than primary unbalance even at lower speeds. Further, for large brush separations, degree of secondary unbalance for same primary unbalance is more pronounced.

The nature of variation of secondary circuit positive sequence current I_{2p} and negative sequence current I_{2n} with different brush settings is shown in Figs. 4.4 and 4.5 respectively. Figs. 4.4(b) and 4.5(b) relate these current variations with brush separation at different speeds. Some interesting inferences follow from the two figures.

The secondary current depends upon the brush adjustments and may vary widely in the operational range of Schrage motor. When the brushes are crossed and the motor is running at supersynchronous speed, the injected voltage and secondary induced voltage are in opposition. The

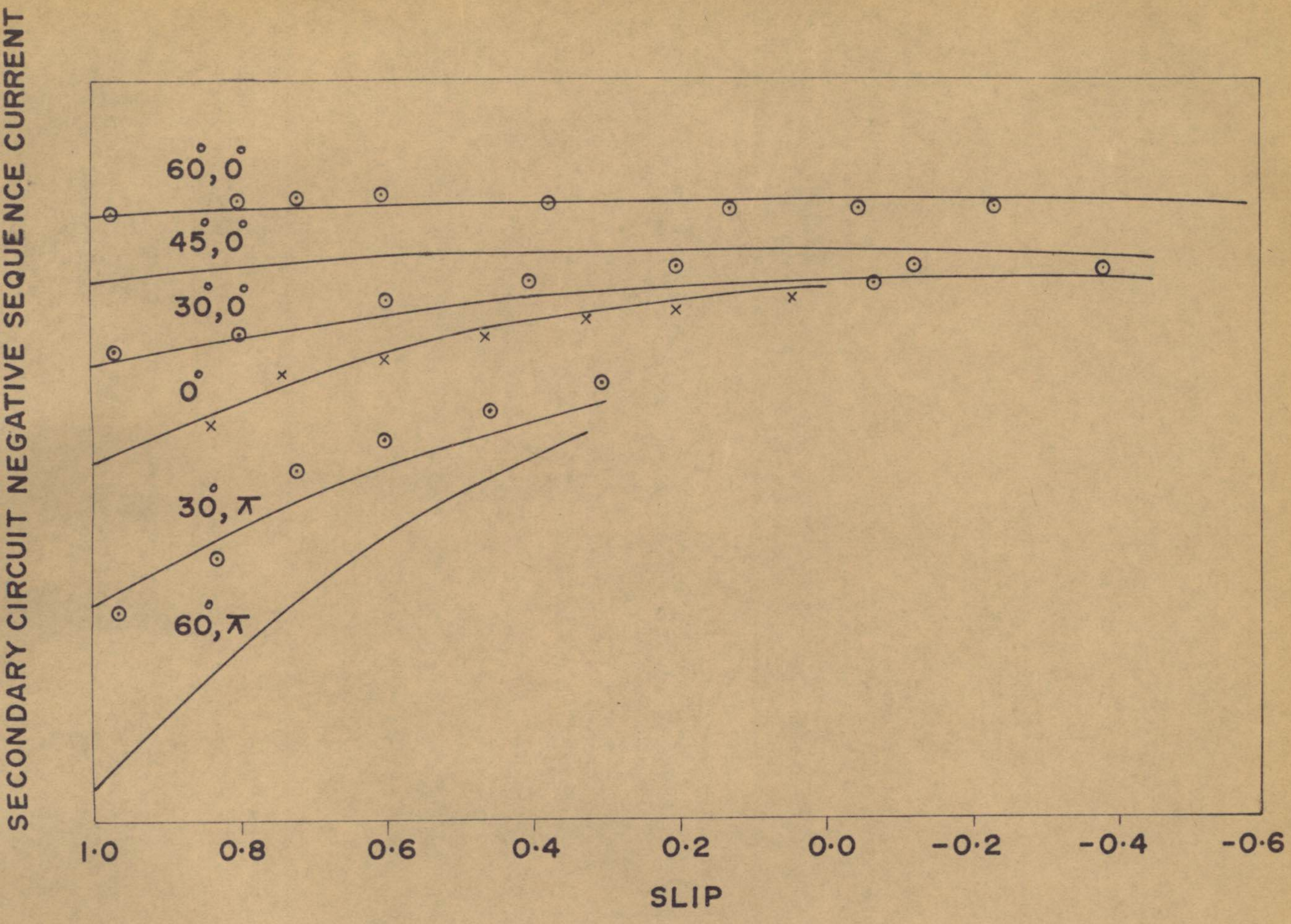


(a) POSITIVE-SEQUENCE CURRENT OF SECONDARY CIRCUIT FOR DIFFERENT β .

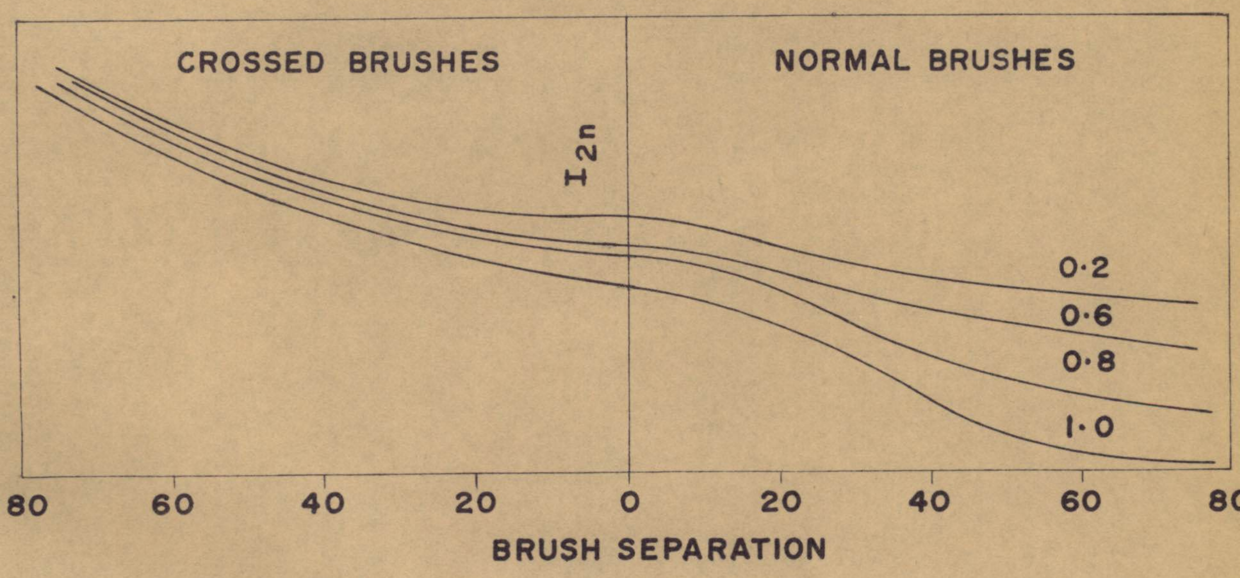


(b) VARIATION OF I_{2p} WITH BRUSH SEPARATION β .

FIG. 4·4 POSITIVE SEQUENCE CURRENT OF SECONDARY CIRCUIT .



(a) NEGATIVE SEQUENCE CURRENT OF SECONDARY CIRCUIT FOR DIFFERENT β.



(b) VARIATION OF I_{2n} WITH BRUSH SEPARATION β.

FIG. 4·5 NEGATIVE SEQUENCE CURRENT OF SECONDARY CIRCUIT.

difference between the two voltages increases with decrease in speed. Consequently I_{2p} rises as slip reduces. If the speed falls below synchronism, the two voltages add round the secondary circuit. Besides the induced voltage $I_1(X_{13}\sin\beta+sX_{12})$, another voltage $sI_2X_{23}\sin\beta$ is induced in secondary winding. Therefore, as the speed decreases, net voltage across secondary circuit due to positive sequence voltage increases thereby increasing the current further. Larger the brush separation more is the magnitude of secondary circuit current I_{2p}. If the brush separation is normal, $I_1X_{13}\sin\beta$ and $sX_{12}I_1$ are in opposition. The difference decreases with fall of speed, but the injected voltage proportional to secondary current i.e. $sX_{23}\sin\beta I_2$ increases. It may be easily visualized that increase in sk_2I_2 is more than decrease in $(k_1I_1-sX_{12}I_1)$. Therefore, with normal brushes also, I_{2p} increases with slip. However, obviously the magnitude of current is less than in the case of corresponding induction motor operation.

Under the usual operation with unbalance supply voltages, negative sequence current is present simultaneously with predominating positive sequence current. The motor is supposed to run in forward direction. In experimental study, the phase sequence of applied voltage was reversed to simulate negative sequence operation. The motor was made to run in forward direction with another prime mover.

Figure 4.5 shows that the negative sequence current I_{2n} rises as the motor picks up speed whatever be the value of brush separation. This nature is same as for an induction motor although the rate of increase is different with different brush settings. The rise is more rapid for normal brushes in comparison with that for crossed brush setting. Physically this behaviour may be explained once more by analysing secondary circuit voltages. The net voltage forcing current through secondary circuit is $I_{1n}\left[\pm X_{13}\sin\beta + (2-s)X_{12}\right]$. The minus sign holds for $\beta_2 = \pi$ and plus sign for $\beta_2 = 0$. As a result of mutual coupling between tertiary and secondary windings, secondary current causes two voltage drops; $X_{23}\sin\beta I_{2n}$ and $(2-s)X_{23}\sin\beta I_{2n}$. The drop $(2-s)X_{23}\sin\beta I_{2n}$ varies with slip and is responsible for inductive power transfer from tertiary to secondary. The magnitude of both $(2-s)X_{12}I_{1n}$ and $(2-s)X_{23}\sin\beta I_{2n}$ increases with speed, but while former is additive, latter is additive for crossed brushes and subtractive for normal brushes. Thus, with normal brushes, the rate of decrease of effective reactance of secondary circuit is faster than rise in voltage, thereby contributing to the increase of current I_{2n} rapidly with speed. In contrast, the increase in voltage is comparatively slower than reactance with crossed brushes. Therefore, with crossed brush setting, increase in I_{2n} is marginal.

It has already been shown that the current unbalance of primary circuit is comparatively more with crossed brushes than with normal brush setting for a given value of λ_v. In addition, secondary unbalance always exceeds primary current unbalance and I_{2n} increases with speed. Therefore, total secondary current $(I_{2p}^2 + I_{2n}^2)^{1/2}$ may reach prohibitive proportions when Schrage motor is operated on unbalanced voltages with crossed brushes.

4.3 Torque-speed Characteristic

The torque developed by a Schrage motor, as shown earlier, is given by

$$\text{Torque} = \text{Real part of } \left[I_2^*(-jX_{12})I_1 + I_2^*(-jX_{23}\sin\beta\, \epsilon^{j\rho_1})I_2 \right]$$

By substituting appropriate values of currents, the following expression for positive sequence torque is obtained.

$$T_p = \left(\frac{V_p}{X_{11}}\right)^2 \frac{X_{12}k_e}{R_o} \cdot \left[\frac{(sX_{12}+X_{13}\sin\beta\cos\rho_1)+X_{13}\sin\beta\sin\rho_1\, k_e \left\{ s\left(T'_{2s}-X_{12}\frac{T'_{\rho s}}{X_{13}}\right)+ \sin^2\beta\left(T'_{3s} - X_{13}\frac{T'_{\rho s}}{X_{12}}\right) \right\}}{\left\{ \frac{R_1}{X_{11}}(T'_{op1}+sT'_{op2})+1-(1-s)k_eT'_{\rho s}\sin\beta\sin\rho_1 \right\}^2 + \left\{ \frac{R_1}{X_{11}}\left[1-(1-s)T'_{\rho o}k_e\sin\beta\sin\rho_1\right]-(T'_{3p1}+sT'_{sp2}) \right\}^2} \right] \quad .. \quad (4.4)$$

The positive sequence torque is zero at a slip given by

$$s_o = \frac{-X_{13}\sin\beta\left[\cos\rho_1 + k_e\sin^2\beta\sin\rho_1\left(T'_{3s} - X_{13}T'_{\rho s}/X_{12}\right)\right]}{X_{12} + X_{13}\sin\beta\sin\rho_1\left(T'_{2s} - X_{12}T'_{\rho s}/X_{13}\right)k_e} \qquad .. (4.5)$$

For a neutral set motor, the well-known expression is obtained from (4.5)

$$s_o = -\frac{X_{13}\sin\beta\cos\rho_1}{X_{12}} \qquad .. (4.6)$$

The negative sequence torque is similarly given by

$$T_n = \lambda_v^2\left(\frac{V_p}{X_{11}}\right)^2\frac{X_{12}k_e k_n}{R_o}\left[\frac{(2-s)X_{12} + X_{13}\sin\beta\cos\rho_2 + X_{13}\sin\beta\sin\rho_2\left\{(2-s)\left(T'_{2s} - T'_{\rho s}X_{12}/X_{13}\right) + \sin^2\beta\left(T'_{3s} - T'_{\rho s}X_{13}/X_{12}\right)\right\}k_e k_n}{\left\{1 + (1-s)k_e k_n T'_{\rho s}\sin\beta\sin\rho_2 + \frac{R_1}{X_{11}}\left[T'_{on1} + (2-s)T'_{on2}\right]\right\}^2 + \left\{\frac{R_1}{X_{11}}\left[1 + (1-s)T'_{\rho o}k_e\sin\beta\sin\rho_2\right] - \left[T'_{sn1} + (2-s)T'_{sn2}\right]\right\}^2}\right] \qquad .. (4.7)$$

The net torque in forward direction is given by

$$T = T_p - T_n - T_r \qquad .. (4.8)$$

T_r being rotational losses.

The magnitude and variation of torque with slip is shown in Fig.4.6. In practice, the effects of stray load losses and harmonics modify the nature of calculated torque and thus, it is different from that actually developed by the machine[44]. But the determination of

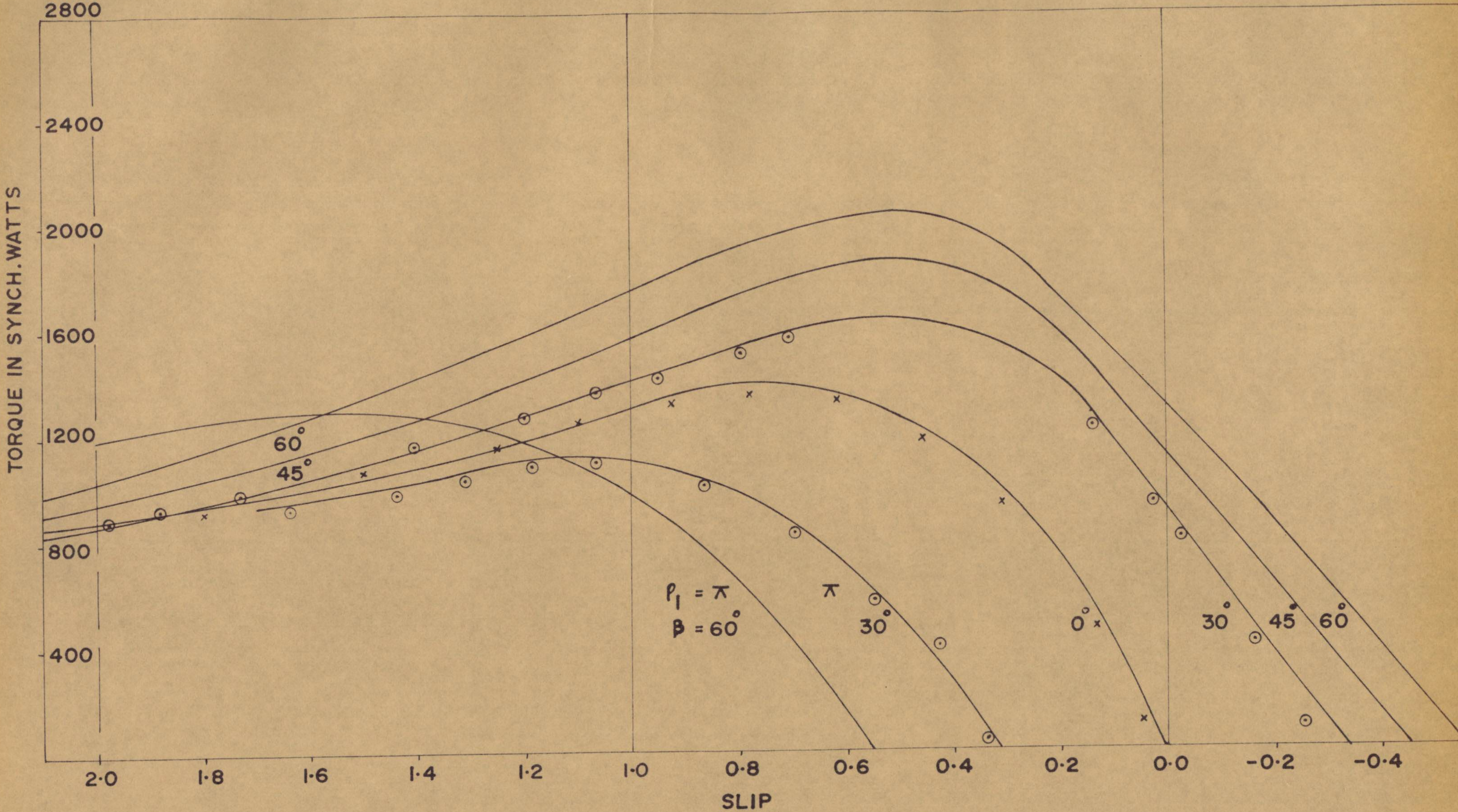


FIG. 4·6 TORQUE–SLIP CHARACTARISTIC OF A SCHRAGE MOTOR FOR DIFFERENT BRUSH SEPARATIONS AND SETTINGS.

impedance/speed curves for positive and negative sequence operation for a fixed voltage (limiting value being rated current) is usually sufficient for subsequent deductions[38]. As the validity of impedance expressions has already been established, it is possible to deduce some general conclusions from the torque expressions derived on the basis of the variation of machine parameters with speed. However, in order to verify the theoretical results, torque was determined experimentally. The test points are also recorded in Fig.4.6. It may be seen that the observed results are in close agreement with calculated ones providing complete confirmation of the theoretical deductions.

4.4 Discussion on torque-speed variation

The variation of the torque developed by a Schrage motor with speed is inherently similar to that of an induction motor (or with β=0). Modifications of the characteristic are brought about by the voltage injection into secondary circuit with the help of tertiary winding. For ease of analysis, the total voltage across the secondary winding in addition to the one induced directly due to primary excitation, may be split up into two components: $k_1E_2\sin\beta\epsilon^{j\rho_1}$ and $k_2sI_2\sin\beta\epsilon^{-j\rho_1}$,

The component $E_{j1} = k_1E_2\sin\beta\epsilon^{j\rho_1}$ is proportional to the airgap flux and is present under all operating conditions except when β=o. The other component $E_{j2}=k_2sI_2\sin\beta\epsilon^{-j\rho_1}$ owes its existance to secondary current, therefore is negligible at no load and when $\beta=0$. The running light behaviour is, thus, governed by only former component. As already discussed, the no load speed may be controlled to any desired value within designed limits by adjusting brush setting. The speed control is possible because of power flow through conduction between secondary and tertiary windings.

The magnitude of torque at a given speed depends upon the total power transfer to secondary through conduction as well as induction. The current injection component E_{j2}, transfers power by induction only. Therefore, its contribution is in addition to normal airgap power of an induction motor. Consequently the torque characteristic should retain exactly the same nature as for an ordinary induction motor when voltage injected to its secondary is proportional to I_2 except that an additional torque component adds to the otherwise developed torque. Larger the brush separation, more is the injected voltage E_{j2} at a given speed. However, the change due to this component is not very appreciable. It may be easily seen that due to larger brush separation, the increase in the effective impedance of tertiary winding is comparatively less than the increase in the voltage acting round the secondary circuit. Therefore, as the brush separation is increased, motor develops more torque at that speed. The slip at which torque is maximum, is given by

$$s_m \simeq -\frac{X_{13}}{X_{12}} \sin\beta \cos\rho_1 + \left[\left(\frac{X_{13}}{X_{12}} \sin\beta \cos\rho_1 - \frac{T'_{s1}}{T'_{s2}} \right)^2 + \frac{1}{T'^{2}_{s2}} \right]^{1/2} \quad .. \ (4.9a)$$

or $$s_m \simeq s_o + \left[\left(s_o - \frac{T'_{s1}}{T'_{s2}} \right)^2 + \frac{1}{T'^{2}_{s2}} \right] \quad .. \ (4.9b)$$

The eqn.(4.9b) shows that the maxima of torque-speed curve for a given value of β and. is dependent upon no load slip s_o. If s_o is changed by adjusting brushes, torque maxima shifts its position. For crossed brushes, shifting is towards higher speeds while for normal brushes, to lower speeds. However, the rate of change of position with crossed brushes is lesser than that with normal brush setting since s_o is negative with crossed brushes. The shifting of the position of maxima may further be explained physically by considering the slip power flow. As discussed in Chapter II from the view point of power flow concept, it may be said that operation of a Schrage motor with normal brush setting is equivalent to an addition of resistance in secondary circuit of an induction motor. Whereas, with crossed brush setting, it is equivalent to operation of an induction motor with rotor of lower resistance.Since the resistance of secondary circuit gets changed in Schrage motor, the slip at which torque is maximum for a fixed value of β shifts to either side of the case when β=o. It may be noted that with β=0, (induction motor operation), the slip for maximum torque as given by eqn.(4.9) is

$$s_m = \frac{1}{T'_{2s}} \approx \frac{r_2}{x_{L2}} \text{ (neglecting primary leakage)} \quad .. (4.10)$$

This is a well known result for an induction motor.

In a Schrage motor, therefore, the limiting speeds for which motoring torque is available as well as the magnitude of torque is adjustable by brush settings. While the component E_{j1} controls the no load speed, magnitude of torque and the region of stable operation, E_{j2} mainly contributes to the magnitude of torque though not to an appreciable extent.

With normal brush setting, it seems that a high starting torque may be obtained by suitably adjusting the brush separation. Although higher torque is developed with crossed brushes but at the expense of increased starting current. Therefore, it is safer to start a Schrage motor with normal brushes so that a reasonably high starting torque is available with moderate current which flows when brushes are normal.

The variation of starting torque with β is shown in Fig. 4.7. As such, the starting torque with normal brushes is lesser than corresponding induction motor case (β=0). But by suitably adjusting the design parameters of the machine, it seems possible to obtain equal or higher than equivalent induction motor torque at starting with normal brushes. From eqn. (4.9), neglecting

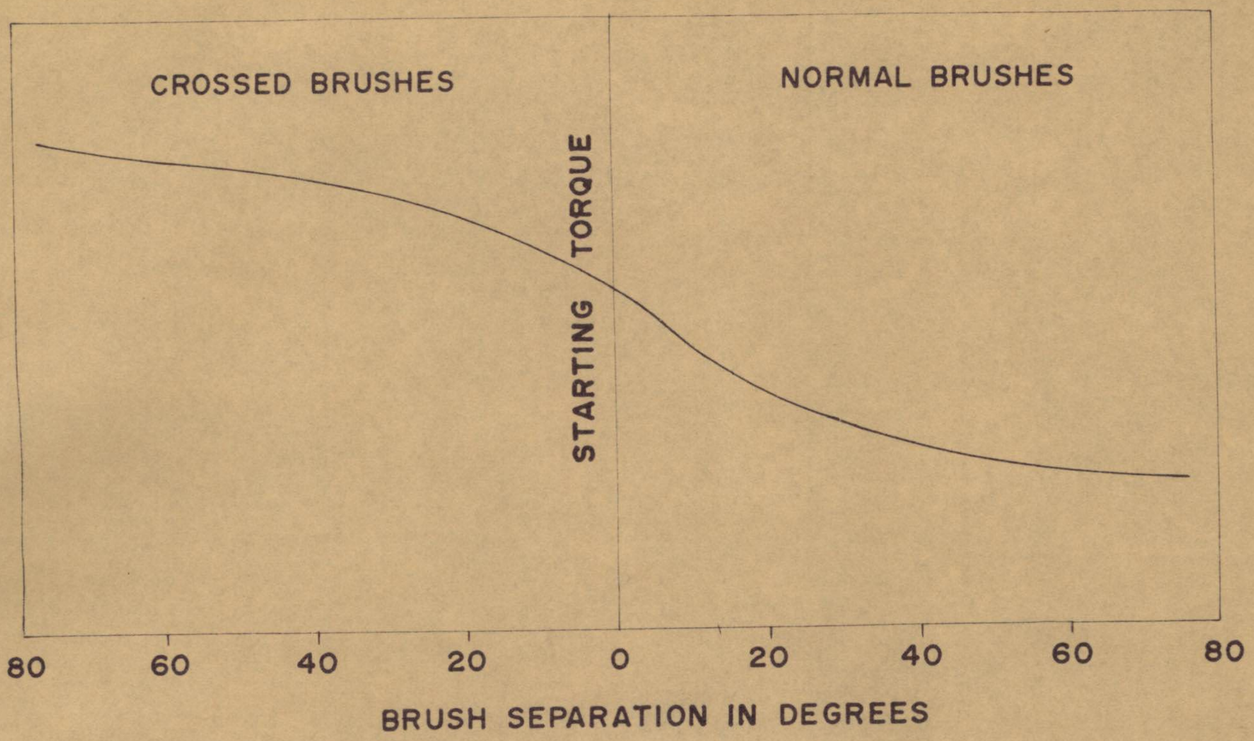


FIG. 4·7 VARIATION OF STARTING TORQUE WITH BRUSH SEPARATION β.

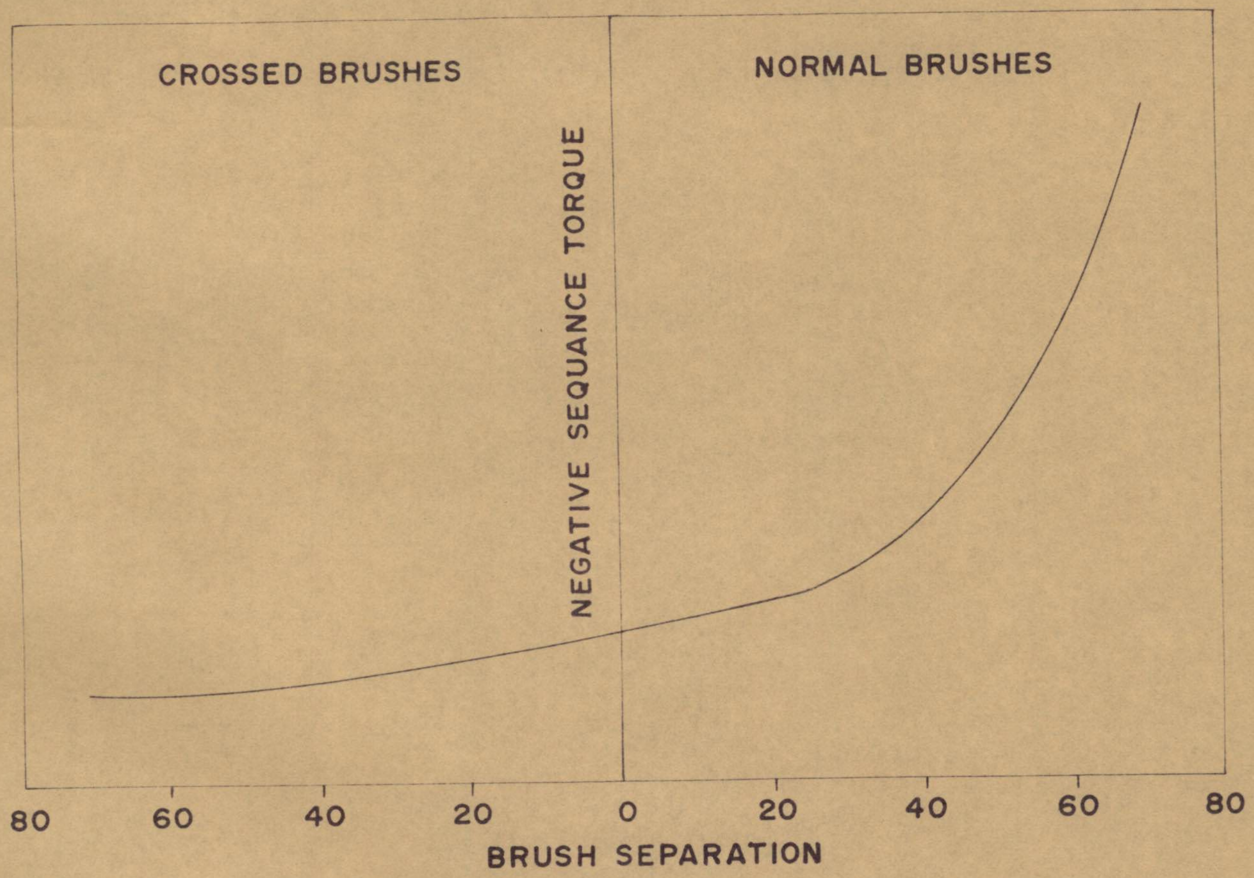


FIG. 4·8 APPROXIMATE MAGNITUDE OF T_n AT NO LOAD FOR DIFFERENT β..

primary leakage,

$$s_m \approx s_o + \left[\left(s_o - \frac{x_{L3} \sin^2\beta}{x_{L2}} \right)^2 + \frac{R_{ep}}{x_{L2}} \right]^{1/2} \quad .. \ (4.11)$$

Normally $s_m > 1$ for any brush separation with $\rho_1 = \pi$. But, if for a fixed value of β and no load speed s_o, the maxima of the corresponding torque/slip curve is to be obtained at $s_m = 1$, the right hand side of eqn. 4.11 is to be reduced. This reduction may be accomplished either by increasing x_{L3} or reducing R_{ep}. The former proposition is not desirable in view of its adverse effect on other operating features of the motor. The latter may be easily realized by reducing brush contact resistance through the use of better quality material.

The negative sequence torque is also dependent upon the value of β and ρ_2. The nature of torque/speed characteristic is appreciably different in two types of brush settings. The behaviour of the motor may once more be attributed to the handling of slip power by tertiary winding. Under normal operation of the motor with subsynchronous setting, airgap power is transmitted from primary to secondary and slip power from secondary to primary through tertiary. If the direction of airgap field rotation is reversed retaining that of rotor, the negative sequence operation is simulated and mechanical power is to be fed to rotor to continue its rotation. A part of slip power is still transmitted to tertiary since for a given primary excitation, the voltage induced across secondary continues to be higher than that across the brushes, (as this induced voltage is proportional to (2-s) instead of s which is in case of positive sequence field and (2-s)>s). Therefore, secondary circuit has to draw more power from shaft and is equivalent to induction motor secondary with increased resistance. The negative sequence torque, therefore, increases as brush separation is increased with normal setting. For crossed brushes, slip power which is now conducted to secondary is in addition to the one available from shaft. Therefore, comparatively lesser shaft power is needed to maintain rotation or lesser negative sequence torque is developed.

The Fig. 4.8 shows the variation of negative sequence torque at no load speed of forward direction with different brush separations. It is evident that the magnitude of T_n increases with an increase in β for normal brushes while it decreases slightly with crossed brushes. It follows, therefrom, that the torque reduction due to unbalance supply voltages is more pronounced with normal setting than with crossed brush setting. The difference in reduction of torque with a certain degree of voltage unbalance increases with increase of brush separation. However, for moderate voltage unbalance the torque reduction is small and the full load speed is not likely to be effected appreciably.

If the degree of unbalance is high, like an induction motor, the output of a Schrage motor is effected substantially. The worst case of voltage unbalance occurs with single phase supply when the positive and negative sequence voltages are equal at standstill. It is well known that a polyphase induction motor does not develop any starting torque with single phase supply, for, the magnitude of both T_p and T_n are equal. Same is the case with a neutral set Schrage motor, irrespective of brush separation. If the brush axes are shifted by an angle ρ, it may be seen from eqns. (4.4), (4.7) and (4.8) that a torque exists at standstill. This interesting aspect of Schrage motor operation forms the subject matter of study in Chapter VI.

CHAPTER V

Allowable Output Under Unbalance Supply Voltages

5.1 General

Most electric motors are designed to operate satisfactorily with balanced voltage supply. One of the important factors which is taken into consideration in the design of an electrical machine is the permissible temperature rise in the windings. This temperature rise is above the ambient temperature, when the machine is run with rated balanced voltage to deliver rated load. Usually, allowance is made in design to meet contingencies such as marginal increase in voltage, rise in ambient temperature, overload etc. It is commonly accepted that a voltage unbalance upto 2 percent is easily tolerated by a machine. Therefore, it becomes important from operational as well as design point of view to predict motor capability under more severe conditions of voltage unbalance.

An unbalance operation of a motor results in the presence of negative sequence current in the phase windings in addition to normally existing positive sequence component of current. In most cases , temperature rise as a consequence of additional heating due to negative sequence current imposes a restriction on the allowable load under unbalance operation. For a Schrage motor, further difficulties may arise due to commutation. To account for reduction in output, a 'rerating factor' may be defined as 'the ratio of the permissible output of the motor under unbalance operation without violating service conditions (thermal or commutational), to the rated output under normal operation.'

The rerating of an induction motor fed with unbalance supply voltage was first suggested by Tracy[37], who indicated that the rating should be reduced below name plate value in the light of heating considerations. He also deduced an approximate but simple expression for allowable output in terms of negative sequence voltage and impedance. It was pointed out later by Gafford, Duesterhoft and Masher[52] that the presence of negative sequence current results in unbalanced spatial distribution of heat in the winding and therefore, the additional heating of a winding should be taken proportional not only to the square of the magnitude of the negative sequence current I_n but to kI_n^2 where $k>1$. Since then number of attempts have been made to determine allowable output of an induction motor from thermal considerations, theoretically as well as experimentally[53-56]. All the approaches suggested, so far, are either very approximate or relate rerating with current unbalance. The degree of current unbalance is different at different loads even for the same voltage unbalance. The voltage unbalance, on the other hand, is independent of motor parameters and is determinable even before switching on the motor. Therefore, for any meaningful results, evidently , it is more realistic to express re-rating in terms of voltage unbalance.

5.2 Thermal considerations

For an induction motor, the rerating factor has been invariably obtained on the basis of stator heating. It is argued that the temperature rise of the rotor is not likely to pose thermal restrictions because of good conduction and better ventilation available to it, though the copper loss due to negative sequence component in rotor is more than in the stator. Further, it is assumed that the rotor heating does not influence stator temperature rise because of airgap.

As pointed out by RamaRao and JyothiRao[56], the methods used in earlier papers fail to experimental corroboration due to some valid reasons put forward by them. In their opinion, truly, it is the lack of proper accounting of the contribution of neighbouring phases to the heating of worst affected phase, which is responsible for the failure of these methods. They have also suggested two methods. First is for short pitch windings and accounts to some extent the heat transfer to adjacent phases. Obviously this method is not valid for full pitch windings. Second method in which heating is assumed to be proportional to $(I_{1p}^2 + I_{1n}^2 + C\, I_{1p} I_{1n})$, takes a better account of the copper losses in all the phases. The factor C is a constant and is stated to be dependent upon the voltage rating and the class of insulation. As a matter of fact C should also depend upon the winding details, temperature difference between hot and cold spots and speed of the rotor. Therefore, it may be widely different for the various machines even when they belong to the same voltage and insulation class.

It has been shown by Roy[57] that the temperature rise of a phase winding may be estimated by the following expression with reasonable accuracy,

$$\theta_T = C_L P_{CL} + C_M P_{CM} + \theta_K \qquad .. (5.1)$$

where C_L is the temperature rise of a winding per unit copper loss P_{CL} in the winding itself, C_M is the temperature rise of a phase winding per unit copper loss P_{CM} in neighbouring phases, and θ_K is the temperature rise due to core loss and heat conduction from other member of the machine. The coefficients C_L, C_M and θ_K are substantially constant for a machine subjected to moderate degree of voltage unbalance. The temperature rise of a stator phase winding, therefore, when carrying normal full load current I_N is given by

$$\theta_T = I_N^2 (C_L + 2C_M) r_1 + \theta_K \qquad .. (5.2)$$

while the temperature rise of the worst affected phase winding i.e. the winding carrying two sequence currents in same time phase, under unbalance operation is given by

$$\theta_T' = C_L(I_{1p} + I_{1n})^2 r_1' + 2C_M(I_{1p}^2 + I_{1n}^2 - I_{1p}I_{1n})r_1'' + \theta_K \quad .. (5.3)$$

To restrict the temperature rise under unbalance operation to the normal temperature rise, θ_T' should be equal to θ_T. It is to be noted that in general, the temperature rise given by eqn.(5.3) is not equal to that given by eqn.(5.2). Therefore, the effective stator resistance under unbalanced conditions would be different for various phases. However, as the current in the two phases other than the worst affected one is same, their resistance may be taken to be equal. For the imposed condition of the same temperature rise $r_1 \simeq r_1' \simeq r_2''$. Therefore, from equation (5.2) and (5.3),

$$I_N^2 = I_{1p}^2 + I_{1n}^2 + 2\left(\frac{C_L - C_M}{C_L + 2C_M}\right) I_{1p}I_{1n} \quad .. (5.4)$$

It may be immediately recognised that the factor $2(C_L-C_M)/(C_L+2C_M)$ is similar to the factor C. Therefore,

$$C = 2\left(\frac{C_L - C_M}{C_L + 2C_M}\right) \quad .. (5.5)$$

As the values of C_L and C_M depend upon the winding configuration, conduction to neighbouring phases, class of insulation, ventilating details etc.,[57] all these factors are automatically accounted for in C if determined from eqn.(5.5). The value of C calculated as above for a number of machines of the same voltage rating but having different types of winding and other constructional features are tabulated in Table 5.1 (C_L and C_M were determined experimentally, App. 5.1). It follows therefrom that C cannot be taken same for all machines belonging to like voltage and insulation class. It is interesting to note that C = 0.5 as taken in reference (55) is a good mean value.

Table 5.1

1. Squirrel cage, 10 hp, 400 V, 1440 rpm	C= 0.464
2. Squirrel cage, 7.5 hp. 400 V, 940 rpm	0.473
3. Slip ring, 7.5 hp, 400 V, 1440 rpm.	0.382
4. Slip ring, 5 hp, 400 V, 1430 rpm	0.64
5. Cage rotor, 5 hp. 400 V, 1440 rpm	0.49

It seems, therefore, that the approach suggested by Roy and applied as above is expected to yield better and more realistic results.

In a Schrage motor, the primary winding is housed on rotor along with tertiary winding. Therefore, the rotor heating will be caused by primary as well as secondary current flowing through a portion of tertiary winding. The number of effective turns of the tertiary winding change with brush separation, consequently in some of the slots heating would be more than in other slots occupied by that portion of tertiary winding which does not carry secondary circuit current for a particular value of β. This may result in hot spots on rotor. But a better ventilation is available to rotor in general and tertiary winding in particular, being nearer to air gap. Both in primary as well as tertiary, the negative and positive sequence currents being of the same frequency, produce equal amount of heating per amphere. The effective resistance of the windings for both sequence currents is also same.

The stator of a Schrage motor, on the contrary, carries secondary winding for which conditions are more severe. It has been shown in Chapter IV that $I_{2n}/I_{2p} > I_{1n}/I_{1p}$ and even for a small degree of primary voltage unbalance, the negative sequence current in secondary may become prohibitive depending upon the value of β and ρ. Therefore, for a Schrage motor, it is the secondary winding heating which shall determine the derating of the motor from thermal point of view under unbalance operation. However, it is to be noted that it is true only when ventilating conditions are good and machine is not subjected to frequent start-stop cycle.

The problem of estimating temperature rise and rerating is somewhat complicated in Schrage motor, being an adjustable speed motor. In case of an induction motor any moderate unbalance does not influence the speed of the motor to an extent effecting the cooling of stator. The temperature rise of an induction motor, thus could be presumed proportional to the effective copper loss under balanced as well as unbalanced conditions. For a Schrage motor, speed is a function of β and varies widely in its range of operation.The validity of any method used to predetermine its temperature rise is to be ensured for all operational limits.

In order to ascertain the applicability of the eqns. (5.1) to (5.5) to Schrage motor, comprehensive tests were carried out for the determination of the values of C_L , C_M and θ_K at different speeds. The test results obtained on an induction motor and the stator of a Schrage motor revealed no change with speed in the value of these constants. Therefore, the temperature rise of a stator phase of a Schrage motor operating at any speed with balanced voltages is given by

$$\theta_T = I_2^2(C_L + 2C_M) R_2 + \theta_K \qquad .. (5.6)$$

With unbalanced supply voltages, the secondary current shall comprise of the positive as well as negative sequence components. As the frequency of the two components is not equal viz. sf for positive sequence, (2-s)f for negative sequence, the effective value of the current in any phase winding is $(I_{2p}^2 + I_{2n}^2)^{1/2}$, irrespective of the relative time phase between I_{1p} and I_{1n}. The temperature rise of a phase winding is, therefore,

$$\theta_T' = (I_{2p}^2 R_{2p} + I_{2n}^2 R_{2n})(C_L+2C_M)+\theta_K \qquad .. (5.7)$$

As before , to restrict the temperature rise under unbalance operation to the normal temperature rise

$$I_{2N}^2 = I_{2p}^2 + I_{2n}^2 \left(\frac{R_{2n}}{R_{2p}} \right) \qquad .. (5.8)$$

The eqn. (5.8) is independent of heating constants unlike the similar condition in an induction motor, eqn.(5.5). Moreover eqn.(5.7) shows that the heating of a phase winding always remains same for a given value of I_{2p} and I_{2n}, whatever may be the phase angle between them. This is an interesting result, as this not only obviates the necessity of considering the temperature rise of the worst affected phase but shows that stator is always uniformly heated.

As it is desired that the heating under unbalance operation should not exceed the normal permissible value, I_{2N} in eqn.(5.8) may be taken as the normal secondary current for a particular setting of brushes. The full load current of a Schrage motor with balanced supply voltages changes with β and ρ. The input current is more for crossed brushes. It has already been shown that I_{2p}/I_{1p} is always greater than unity in usual operating range. The secondary winding of a Schrage motor is capable of carrying current corresponding to the highest value of rated primary current at highest speed setting. Therefore, secondary circuit may carry only this much maximum current whatever be the value of β and ρ. Consequently in eqn.(5.8) I_{2N} may be assumed to be fixed and equal to the maximum permissible secondary current I_{2m}. Now, the rerating of a Schrage motor from thermal considerations may be estimated by the following condition,

$$I_{2p}^2 + I_{2n}^2 \frac{R_{2n}}{R_{2p}} = I_{2m}^2 \qquad .. (5.9)$$

5.3 Commutational Considerations

The a.c. commutation of a polyphase machine depends upon the reactance and rotational e.m.f of the short circuited coil undergoing commutation. These e.m.f's appear continuously between the heel and toe of the brush though resulting from a succession of different coils.

The reactance e.m.f is directly proportional to the r.m.s.value of the current in the conductors of the commutating winding. Under unbalance operation, tertiary winding has to carry both positive and negative sequence currents. The reactance e.m.f., therefore, may be conceived to be having two components each proportional to one of the sequence currents. These e.m.fs are in time phase with corresponding brush currents and act in such a direction round the short circuited coil as to oppose the respective current change in it. The reactance voltage alternate sinusoidally at the frequency of the currents being commutated, in the present case sf and (2-s)f. The total voltages induced due to current commutation are therefore given by

$$\left.\begin{aligned} e_{xbp} &\propto I_{2p} \sin s\omega t \\ e_{xbn} &\propto I_{2n} \sin(2-s)\omega t \end{aligned}\right] \quad .. (5.10)$$

and

$$E_{xb} \propto \sqrt{I_{2p}^2 + I_{2n}^2} \quad .. (5.11)$$

It can be seen that the magnitude of reactance voltage E_{xb} is independent of the relative phase of I_{2p} and I_{2n}. On the other hand the frequency of the two sequence components of tertiary current is equal to that of supply; therefore the r.m.s value of resultant current depend upon the magnitude and phase of I_{2p} and I_{2n}. Thus, the tertiary current in a particular phase may increase inordinately over the normal value but the current commutation is simply dependent on the value of $(I_{2p}^2 + I_{2n}^2)^{1/2}$. Evidently, the two currents are likely to be in same time phase only when s = 1 i.e., standstill. Therefore, at all speeds, more so with normal operating speeds, the two currents will not be in same phase and the r.m.s. value of the total current through tertiary conductors may be taken approximately equal to $(I_{2p}^2 + I_{2n}^2)^{1/2}$.

It follows that if the secondary circuit current is restricted to the maximum permissible value I_{2m}, it would automatically account for any possible deterioration in current commutation under unbalance conditions.

The amplitude of the rotational voltage is proportional to the magnitude and speed of the airgap flux. The two sequence fluxes move past the commutating winding at synchronous speed irrespective of rotor speed, though they rotate in opposite direction with respect to each other. Furthermore, the speed of negative sequence field relative to brushes is $(2-s)n_s$ whereas that of positive sequence field is sn_s. Therefore, e.m.fs induced in short-circuited coil are given by

$$\left.\begin{aligned} e_{rbp} &= 2\pi\ f\ T_C\ \phi_p \sin s\omega t \\ e_{rbn} &= 2\pi\ f\ T_C\ \phi_n \sin(2-s)\omega t \end{aligned}\right] \quad .. (5.12)$$

If the effect of leakage reactance of primary winding is ignored, the two sequence fluxes are approximately equal to

$$\left.\begin{aligned} \phi_p &= V_p / \sqrt{2}\ \pi\ f\ T_1 \\ \phi_n &= V_n/\sqrt{2}\ \pi\ f\ T_1 \end{aligned}\right] \quad .. (5.13)$$

T_1 is the effective number of primary winding turns per phase, f is the supply frequency and T_C is the number of turns of the short-circuited coil.

The r.m.s. value of rotational voltages at brushes are therefore,

$$\left.\begin{aligned} E_{rbp} &= V_p\ T_C / T_1 \\ E_{rbn} &= V_n\ T_C/T_1 \end{aligned}\right] \quad .. (5.14)$$

As these voltages are of different frequencies, the resultant e.m.f. across the coil undergoing commutation would be equal to $E_{rb} = V_p(T_C/T_1)\sqrt{1 + \lambda_v^2}$, irrespective of the relative phase of E_{rbn}, with E_{rbp}. The presence of negative sequence voltage, thus, increases the rotational voltage by $\sqrt{1 + \lambda_v^2}$. For satisfactory commutation E_{rb} should not exceed 3 to 4 volts. In usual design, an allowance is made for marginal increase in supply voltage. Even if λ_v is 50 percent, E_{rb} exceeds its normal value only by about 11 percent. In general, a machine is expected to be capable of withstanding this small increase in E_{rb}.

In the light of above discussion, it may be concluded that the allowable output of a Schrage motor under unbalance operation may be estimated with reasonable practicability in accordance with thermal considerations. Any commutational limitation is not likely to offset the results so obtained.

5.4 Method of Calculation

A method of calculating rerating factor or permissible output under unbalance supply voltages is evidently more satisfactory if it relates the desired quantity with degree of voltage unbalance rather than current unbalance. The sequence components of the voltage are independent of the machine parameter and simply depend upon the three line voltages available at the motor terminals. On the other hand, the current unbalance depend not only upon machine parameters but rotor speed also. Therefore, degree of current unbalance is different at various loads even for the same degree of voltage unbalance. Hence, for any meaningful result, it is desirable to express permissible output of the machine in terms of voltage unbalance factor.

The relationship between secondary circuit current of a Schrage motor with applied voltage to primary has already been established. Thus, from eqn.(5.9),

$$I_{2m}^2 = \left(\frac{V_p}{X_{11}} \right)^2 \frac{k_e}{R_o} \left[f_1(s, \beta, \rho_1) + \lambda_v^2 f_2 \left\{ (2-s), \beta, \rho_2 \right\} \right] \quad .. (5.15)$$

where f_1 and f_2 are functions of slip, brush separation, brush axis shift and machine constants. With known values of machine parameters and assumed value of slip, λ_v may be calculated for a given value of V_p for any brush setting from eqn.(5.15). The value of k_n is substituted corresponding to assumed values of slip. The magnitude of I_{2m} may be taken either from the nameplate data of the motor supplied by the manufacturers or determined experimentally.

Fig. 5.1 shows a family of curves between λ_v and slip for different per unit value of V_p. The calculations, satisfying eqn. (5.15), were carried out for a number of values of β as recorded in Fig.5.1. For any degree of voltage unbalance and magnitude of V_p or given values of V_p and V_n, s may be read off this figure satisfying the criteria of allowable output. Then,

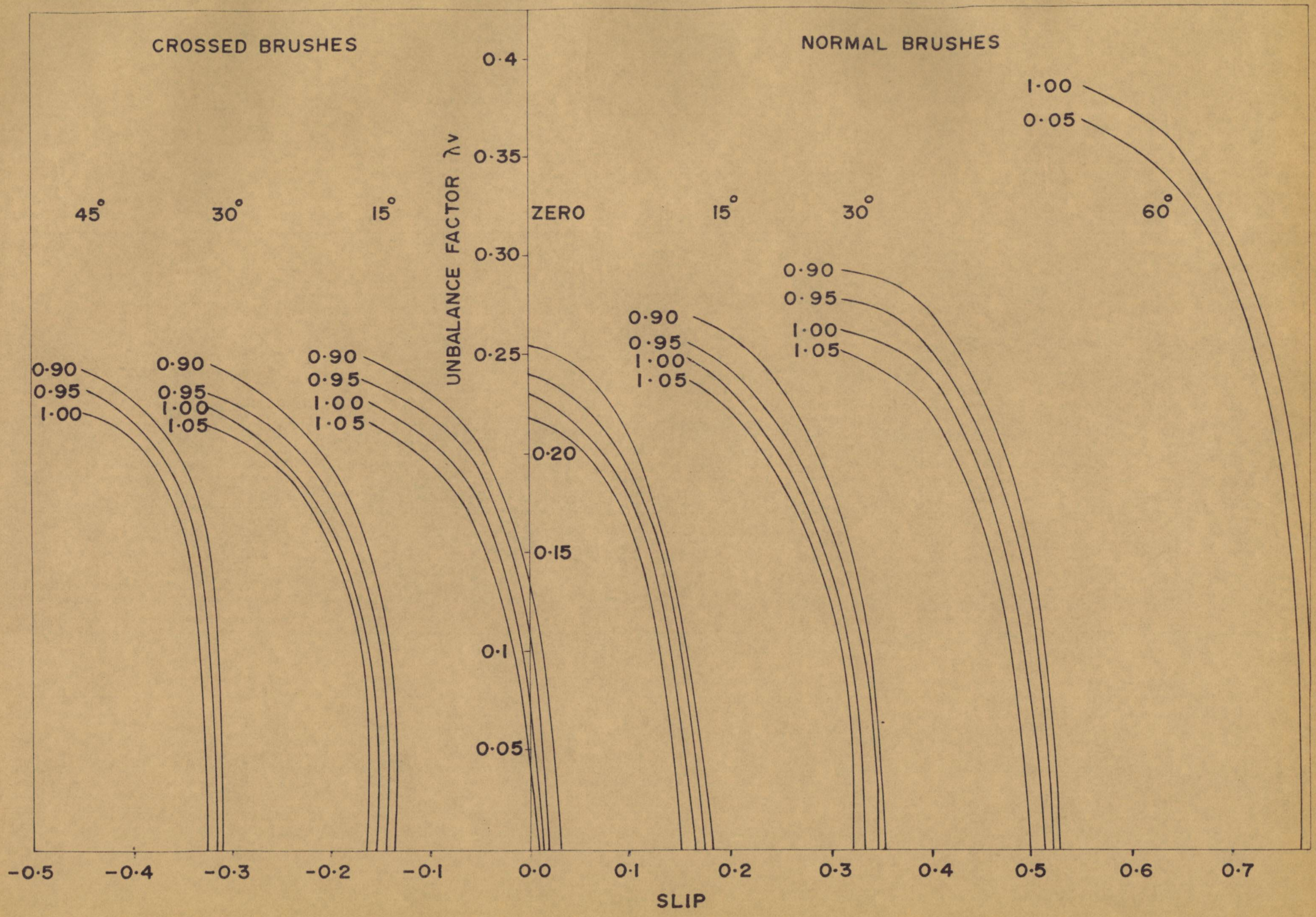


FIG. 5·1 ALLOWABLE VOLTAGE UNBALANCE FACTOR WITH SLIP FOR DIFFERENT VALUES OF β.

the permissible loading of the motor is calculated for this value of slip, λ_v and V_p from eqn.(4.8).

5.5 Discussion

The calculated results for the machine under test are plotted in Figs. 5.2, 5.4 and 5.5. No direct verification of calculations is possible. But corroboration of the analysis was obtained by loading the motor under test to a load given by Fig. 5.4 and adjusting V_p, λ_v, β etc. accordingly. The temperature rise of the stator, for different degrees of unbalance and operational settings, was recorded. This temperature rise was found to be approximately same in all cases and equal to that obtained with simulated operating conditions of the motor i.e. balanced voltage, rated load and maximum speed setting. To check the heating of rotor corresponding to allowable load on the motor, the temperature rise was estimated by measuring change in primary winding resistance. But, as pointed out earlier, in some of rotor slots, heating would be more than in others. Therefore, primary winding resistance measurement yields only average temperature rise and not of hot spots. Uneven spatial distribution of heat can be detected only by inserting thermocouples appropriately. However, average temperature rise of primary was found to vary with brush separation even with balanced operation. Under unbalance voltages and rerated loading, the temperature rise was usually a little more than that obtained with maximum permissible primary current, balanced voltage and zero brush separation.

As expected, Schrage motor is capable of sustaining more severe unbalances with normal brush settings compared to crossed brush settings. At no load, the safe degree of voltage unbalance increases rapidly with increase of brush separation under normal setting. It decreases in case of crossed brush setting, though only marginally. This behaviour follows the nature of I_{2n} with β. Figure 5.3 which shows the plot of I_{2n} at no load speed(pertaining to a given brush separation) with β, reveals that I_{2n} decreases sharply with normal brush setting while its increase with crossed brushes is not so rapid. Therefore, motor can handle more negative sequence current with normal brushes and consequently more voltage unbalance.

The allowable output of the motor is influenced not only by the degree of voltage unbalance but by the value of positive sequence voltage as well. If positive sequence voltage is reduced, the capability of a Schrage motor to withstand unbalance voltages increases. This increase is, however, more or less same for different crossed brush setting but increases for normal brush setting with increase of brush separation(Fig.5.1). Since with crossed brush setting I_{2p} increases with slip while I_{2n} is practically constant, permissible λ_v decreases sharply on increasing load. In contrast,

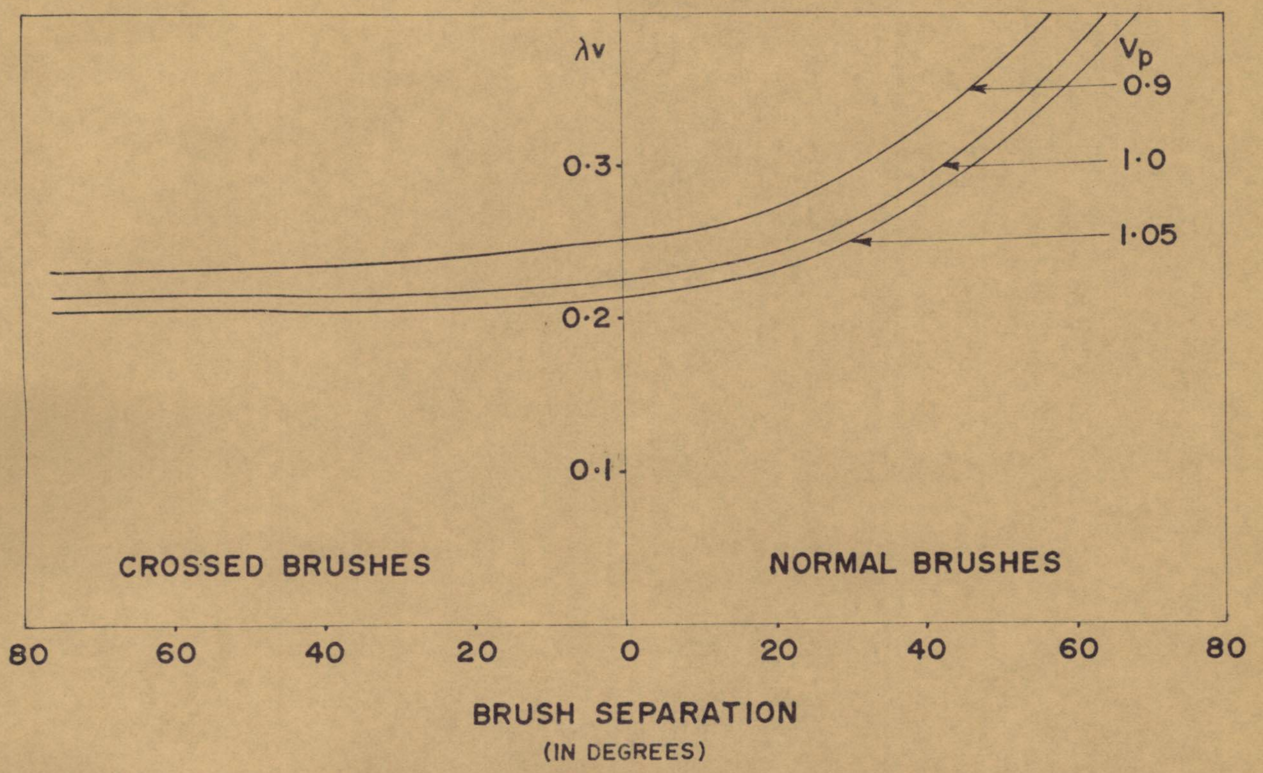


FIG. 5·2 MAXIMUM PERMISSIBLE UNBALANCE FACTOR AT NO LOAD WITH RESPECT TO β.

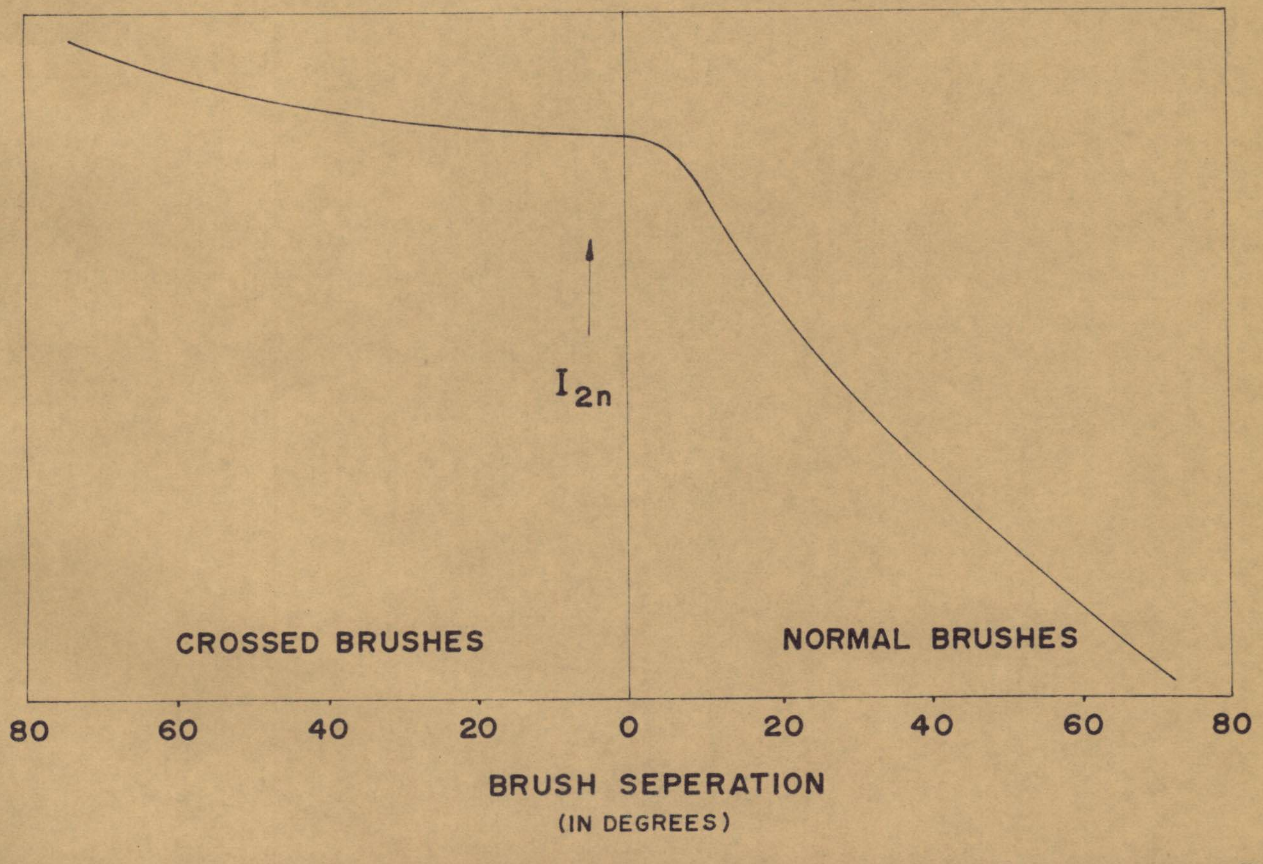


FIG. 5·3 INFLUENCE OF BRUSH SEPARATION ON NO LOAD VALUE OF I_{2n}.

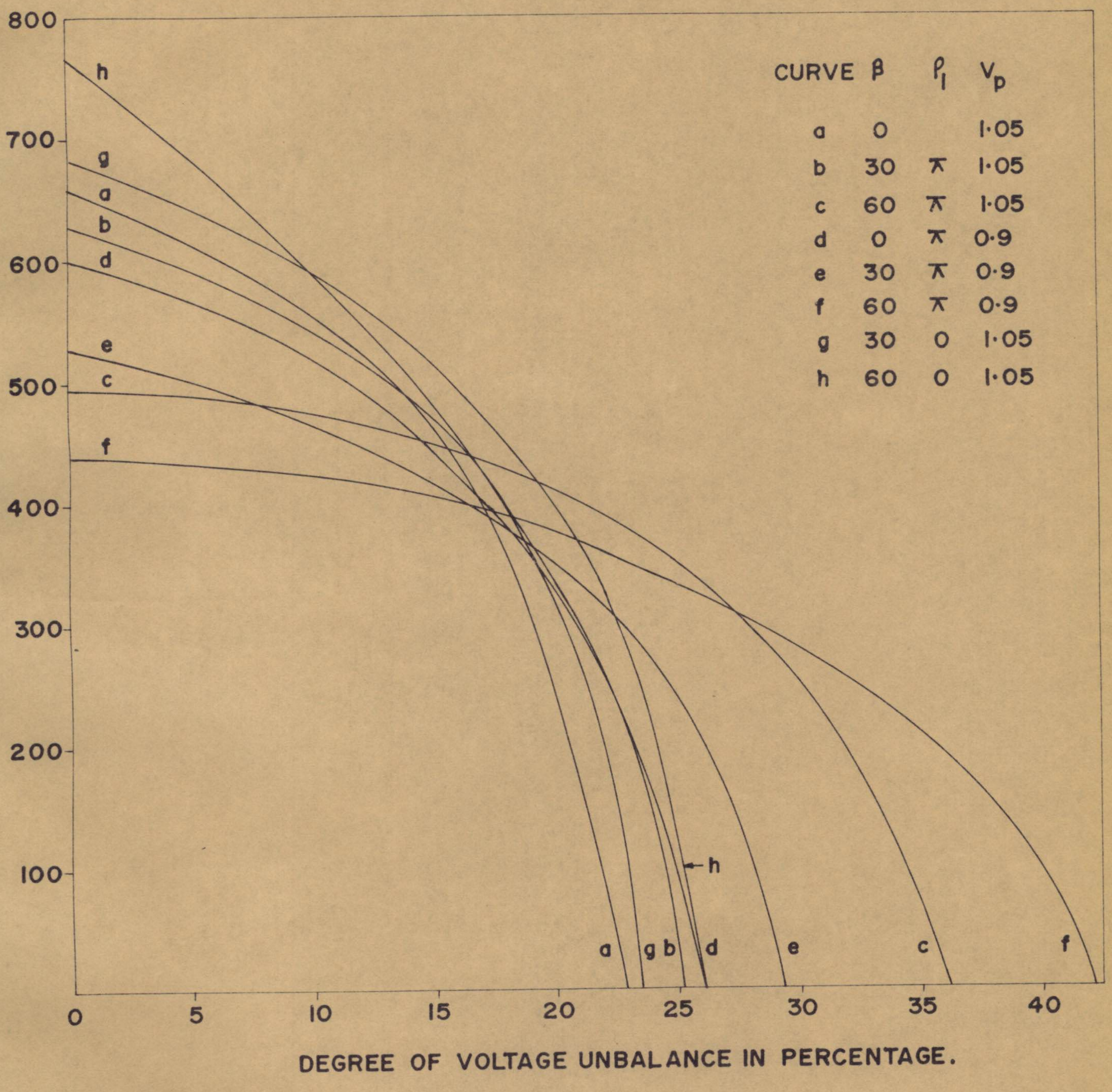


FIG.5·4 ALLOWABLE OUTPUT Vs VOLTAGE UNBALANCE CURVES.

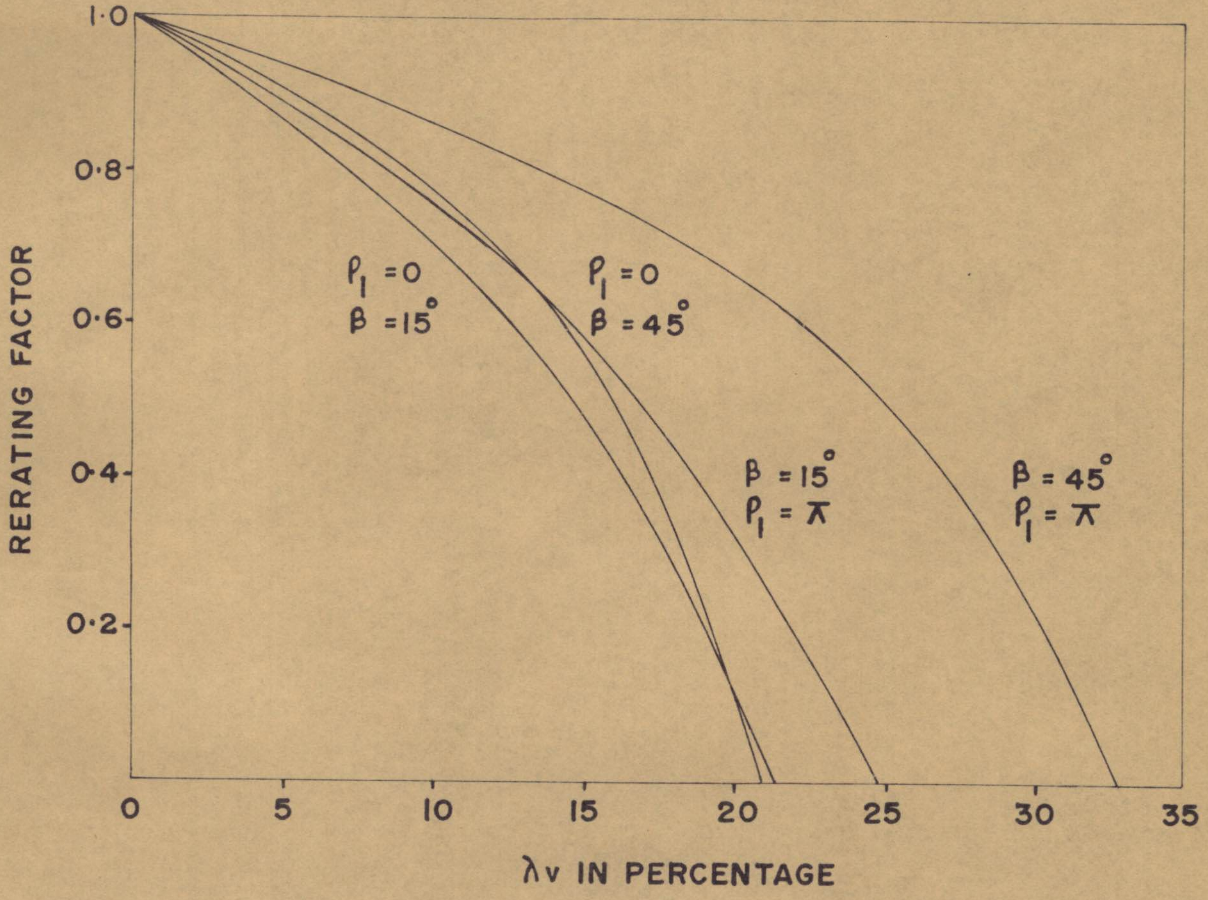


FIG.5·5 VARIATION OF RERATING FACTOR WITH VOLTAGE UNBALANCE λv FOR V_p = 1.p.u.

I_{2n} decreases with slip when brushes are normal. Therefore, the variation of λ_v with slip is relatively less steep.

The current in the worst affected phase of primary is also calculated and shown in Fig. 5.6. For balanced supply, the current for any brush setting does not exceed the maximum rated value, although it is more than normal full load value with normal setting, since permissible value of secondary current I_{2m} is more than usual I_{2p} with $\rho_1 = \pi$. The current is more than 1 pu as λ_v increases. But as the temperature rise of the rotor is dependent on the current of all the three phases, increase in the value of current of the worst affected phase is not a detrrent.

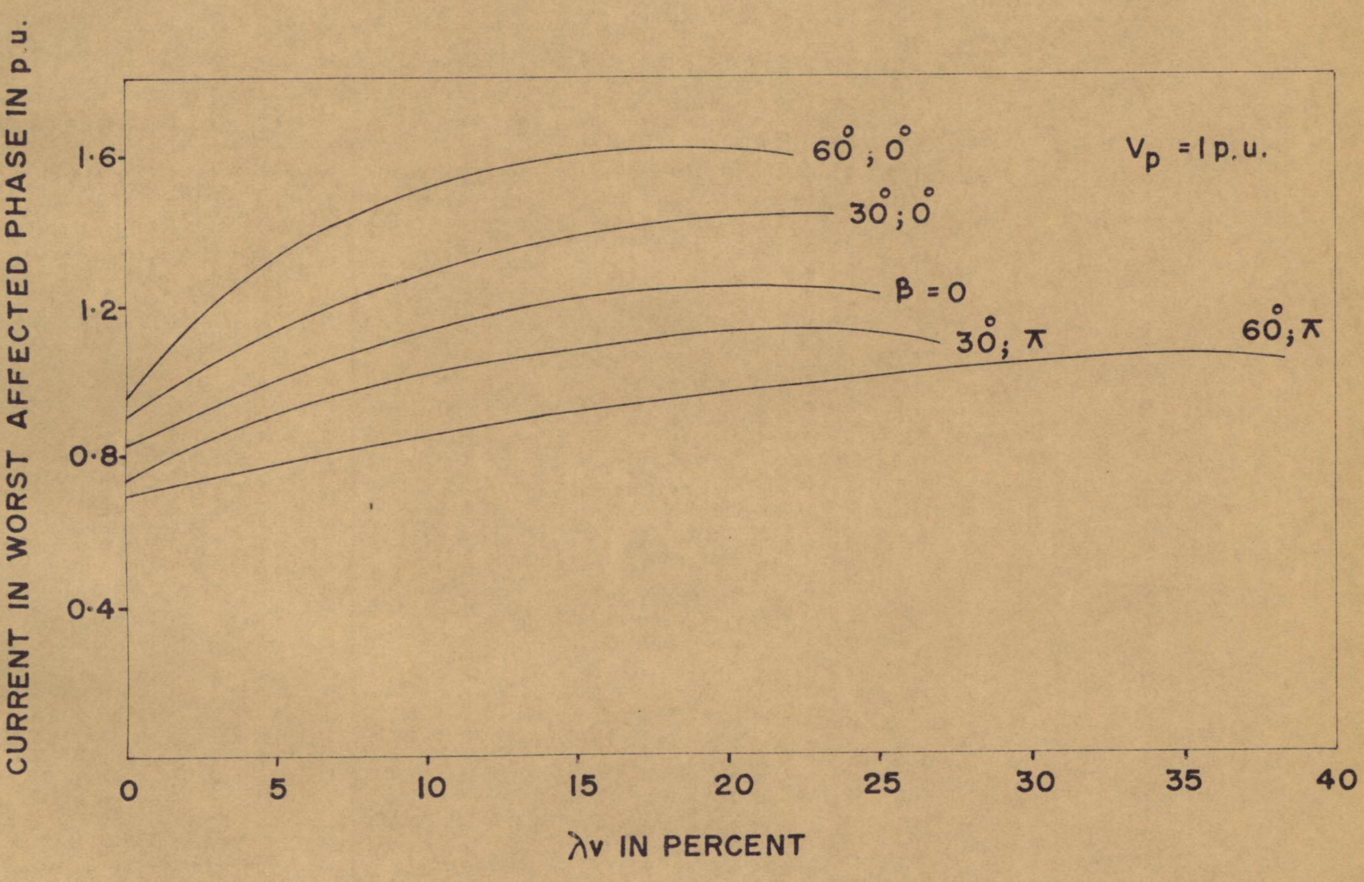


FIG. 5·6 VARIATION OF CURRENT IN WORST AFFECTED PHASE OF PRIMARY WITH VOLTAGE UNBALANCE λv.

CHAPTER VI

Single Phase Operation

6.1 General

The single phasing of a polyphase machine either at starting or during running is of usual occurrence. It is well known that such an operation of an ordinary induction motor is characterized by lack of starting torque, though running is possible otherwise. All split phase motors are incorporated with some or other static phase convertors to develop torque at starting. As mentioned in Chapter IV, a Schrage motor may develop torque at standstill with single phase supply under favourable conditions of brush adjustments without any external phase convertor. This remarkable quality was first reported by Kohli and Ray[58] recently.

In this chapter, analysis of single phase operation of a Schrage motor has been presented. Behaviour of the motor has been discussed with regard to starting as well as running under this condition. Investigation has been extended to explore suitable combination of brush axis shift and brush separation so that motor torque at standstill may be sufficient to overcome static retarding inertia.

6.2 Analysis of Performance

The performance characteristics of the motor may be calculated for any operational adjustment from the sequence voltage and current relationships pertaining to applied voltage conditions. For single phase supply to a three phase machine, the inspection equations[38] may be written and solved to determine the sequence voltages in terms of machine parameters. It can be easily shown that zero sequence is absent under single phasing and the sequence voltages of star and delta connected windings are related by a factor of $\sqrt{3}$. However, there is a phase shift also, but it is of no consequence as far as magnitude of torque developed by the motor is concerned. Thus, for the present, it is necessary to consider only one of the connections and in what follows, attention is confined to delta connected machine.

For a delta connected machine fed with single phase source, the voltage and current relationship are given by the following:

$$\left.\begin{aligned} V_p &= \frac{Z_p}{Z_p+Z_n} V \\ V_n &= \frac{Z_n}{Z_p+Z_n} V \\ V_o &= 0 \end{aligned}\right] \qquad \text{.. (6.1)}$$

Line current

$$I_L = \frac{3V}{Z_p + Z_n} \qquad \text{.. (6.2)}$$

The complete performance of the motor can now be synthesized from eqns.(6.1) and (6.2) through (3.1), (3.2), (4.4) and (4.7).

6.3 Starting Performance

6.3.1 Starting torque

At standstill, slip is unity. Substitution of this condition in eqns.(3.1), (3.2) and (6.1), shows that positive and negative sequence voltages are equal to each other. Therefore, from eqn.(4.8), the starting torque is given by

$$T_s = \left(\frac{V}{2X_{11}}\right)^2 \frac{X_{12}k_e}{R_o} \cdot \left[\frac{\pm 2X_{13}\sin\beta\sin\rho\left\{(T'_{2s}-X_{12}T'_{\rho s}/X_{13})+\sin^2\beta\,(T'_{3s}-X_{13}T'_{\rho s}/X_{12})\right\}}{\left[\frac{R_1}{X_{11}}-k_e\left\{(T'_{3s}\sin\beta\pm T_{\rho s}\cos\rho)\sin\beta+(T'_{2s}\pm T'_{\rho s}\sin\beta\cos\rho)\right\}\right]^2+\left[1+\frac{R_1}{X_{11}}k_e\left\{(T'_{3o}\sin\beta\pm T'_{\rho o}\cos\rho)\sin\beta+(T'_{2o}\pm T'_{\rho o}\sin\beta\cos\rho)\right\}\right]^2} \right] \qquad ..\ (6.3)$$

In this equation, positive sign is to be taken for crossed brushes and negative for normal brush separations. For qualitative analysis, those terms of the denominator which are multplied by (R_1/X_{11}) may be neglected. The eqn.(6.3) then reduces to

$$T_s = \left(\frac{V}{X_{11}} \right)^2 \frac{X_{12} k_e}{2R_o} \left[\frac{\pm X_{13} \sin\beta \sin\rho \left[(T'_{2s} - X_{12} T'_{\rho s}/X_{13}) + \sin^2\beta (T'_{3s} - X_{13} T'_{\rho s}/X_{12}) \right]}{1 + k_e^2 (T'_{2s} + T'_{3s} \sin^2\beta \pm 2T'_{\rho s} \sin\beta \cos\rho)^2} \right] \quad .. \ (6.4)$$

It is evident from qn.(6.4) that the torque direction is governed by brush setting. If the brushes are separated for subsynchronous operation, the torque developed with single phase voltage, would tend to run the motor in a direction opposite to the forward direction under normal operation. It is to be noted that the backward torque is consequent to the shifting of the brushes opposite to the normal direction of rotation of the rotor (necessary for power factor improvement with subsynchronous setting). If the brush axis shift ρ is made negative i.e. brushes are shifted along the normal direction of rotation, the starting torque would be in forward direction. Similarly with crossed brushes, the single phase starting torque is positive for positive values of ρ and negative with negative values of ρ. The following table sums up the relative direction of torque with different combinations of brush adjustments. The reference for comparison is the normal direction of rotation under balanced 3-phase conditions.

Brush Separation	Brush axis shift	Direction of standstill torque on single phase
(i) Normal	along normal rotation $(-\rho)$	Forward
(ii) Normal	opposite to rotation (ρ) (p.f. improvement)	Backward
(iii) Crossed	along normal rotation (p.f. improvement, $-\rho$)	Backward
(iv) Crossed	opposite to rotation (ρ)	Forward

(a) For fixed values of β, the starting torque expression may be written as

$$T_s = \frac{\pm A \sin \rho}{1+(B \pm D \cos \rho)^2} \qquad .. (6.5)$$

Since $T'_{\rho s}$ is fairly small compared to T'_{2s}, D is negligible in comparison with B. It follows, now, that the starting torque is approximately proportional to $\sin \rho$. Therefore, for a given brush separation, the standstill torque is maximum when brush axis is shifted by $\pi/2$. Further, it is obvious that for the same value of β and ρ, T_s, although of opposite nature, is more with normal brushes compared to that obtained with crossed brush separations since denominator is reduced. Consequently, a single phase Schrage motor, like polyphase one, should be started with normal brush separations as far as starting torque consideration is concerned.

Larger the brush separation, more would be the torque.

The variation of starting torque of a Schrage motor, fed with single phase supply, with brush axis shift is shown in Fig. 6.1. The torque, for a fixed value of β, increases as brush axis shift is made larger, but no substantial improvement is noticed beyond 60° value of ρ with subsynchronous setting. Also, comparatively torque is more with normal brushes. Therefore, large brush separation with about 60° brush axis shift is expected to give sufficiently high starting torque.

As most Schrage motors of usual design are provided with only about 10° adjustment of brush axis shift, special gear arrangement would be necessary for obtaining large values of ρ .

(b) The ratio of <u>T_s to the starting torque T_{1s}</u> under balanced 3-phase conditions is given by the equation:

$$\frac{T_s}{T_{1s}} = \frac{\pm\, a\, X_{13}\sin\beta\, \cos\rho}{X_{12} \pm X_{13}\sin\beta(\cos\rho + a\,\sin\rho)} \qquad .. \; (6.6)$$

where, 'a' is independent of brush setting and is a dimensionless parameter. This ratio is maximum when

$$\cos\rho = \mp\, \frac{X_{13}\sin\beta}{X_{12}} \qquad .. \; (6.7)$$

and corresponding value of the torque ratio is given by

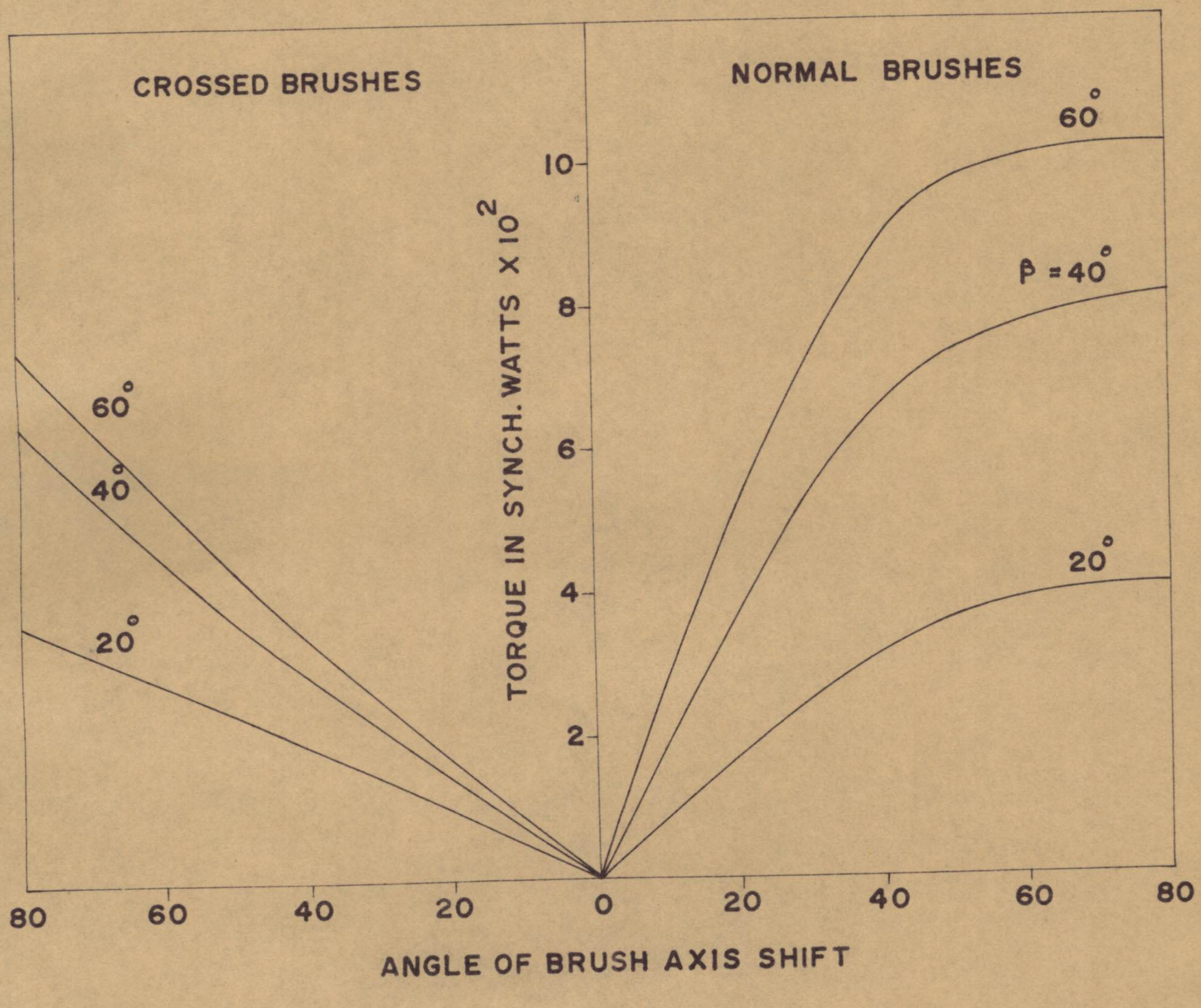


FIG. 6·1 STARTING TORQUE WITH BRUSH AXIS SHIFT WITH FIXED VALUES OF BRUSH SEPARATIONS.

$$\left.\begin{aligned}\frac{T_s}{T_{1s}} &= \frac{X_{13}\sin\beta(P+Q\sin^2\beta)}{(X_{12}^2 - X_{13}^2\sin^2\beta)^{1/2}} \\ &= \frac{s_o(P + Q\sin^2\beta)}{(1 - s_o^2)^{1/2}}\end{aligned}\right] \quad .. (6.8)$$

Equation (6.7) shows that the torque ratio is maximum when the brush shift is adjusted equal to an angle whose cosine is equivalent to no load slip of the neutral set motor when operated on balanced supply with given brush separation. The torque ratio is, however, unaffected by brush setting i.e. the magnitude remains unaltered whether the brushes are crossed or normal. Larger the brush separation, more is s_o and so is (T_s/T_{1s}).

As the starting torque is approximately proportional to $\sin\rho$, the highest value is possible when $\rho = \pi/2$. But from the foregoing analysis, it seems that reasonably high torque at standstill is possible provided single phase supply is switched on with large separation β, normal setting and brush axis shift $\rho = \cos^{-1}(X_{13}\sin\beta/X_{12})$. This value of ρ is operationally possible since s_o is usually kept 1/2 with lowest speed setting, so that ρ is about 60^o only. This result was concluded earlier also.

6.3.2 Starting Current

The starting current can readily be expressed in terms of the corresponding balanced three phase current I_b.

By substituting the values of sequence impedances in eqn.(6.2),

$$I_L = \frac{\sqrt{3}}{2} \; I_b \qquad .. (6.9)$$

while the current through the phase winding directly across the supply is I_b and through other two phases in series is $I_b/2$. Thus, the starting current with single phase supply bears a definite ratio with the starting current drawn by the machine with same brush separation and brush axis shift but balanced 3-phase supply. The current in the worst affected phase is equal to its normal starting current while the line current is only half of the usual value. Thus, if brush separation is large with normal setting, the single phase starting current of a phase winding would be equal to the normal starting current of the motor. In general, the magnitude of single phase current is governed by all those considerations which control the current magnitude under 3-phase operation.

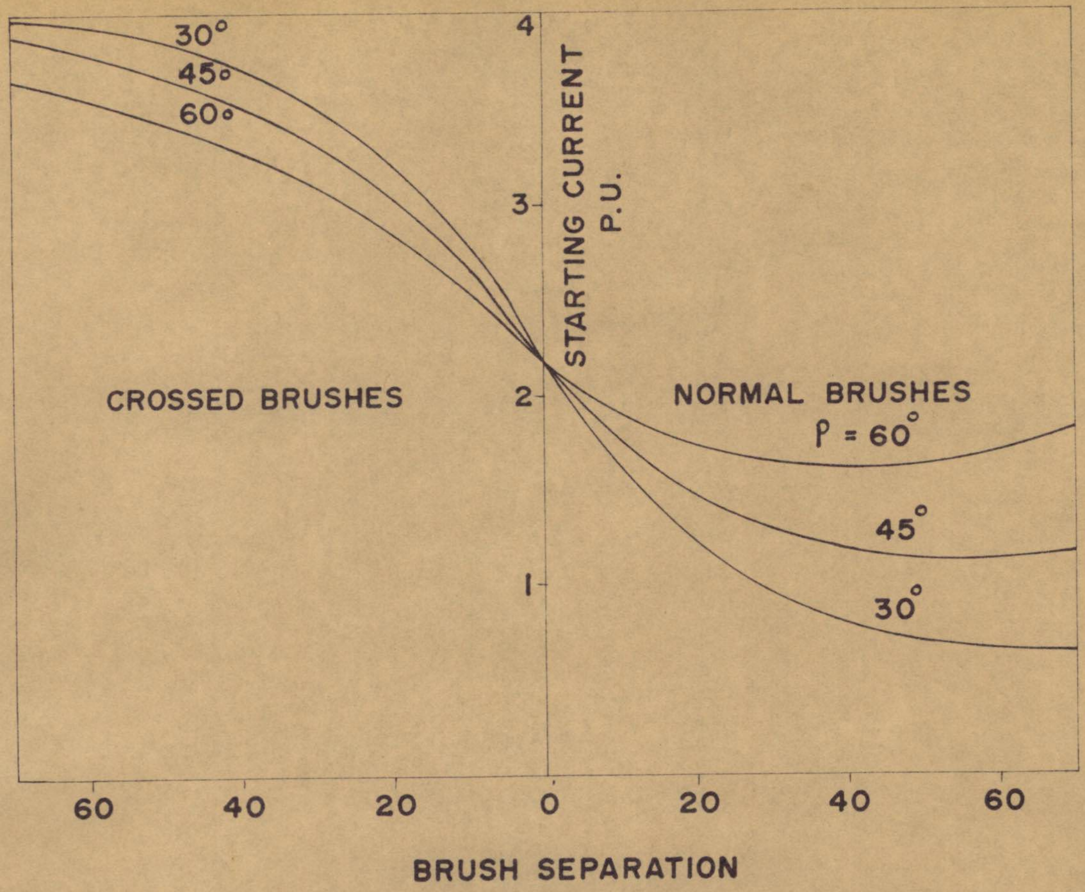
The starting quality of a single phase motor is a measure of the optimization of the desired starting characteristics of the motor. Usually three different starting criteria are popular among designers of single phase motors. Some designers consider that the best use of material is to obtain maximum starting torque per ampere of line current while some others prefer starting torque[59] per ampere squared as a basis for comparison of machines. The argument in favour of latter approach rests with the rating of the machine which is determined by the square of the current. Still another starting quality based on heating considerations is to calculate the torque per unit copper loss in all the three phases of the machine[60].

Whatever criterion is choosen, any comparison of single phase starting with polyphase one, at once reveals that starting current does not influence the quality of starting. It is only the torque expression which is to be optimized. Therefore, the best condition for starting would be to minimize the current with as large starting torque as possible. Fortunately, in single phase operation of a Schrage motor, the two requirements are simultaneously achieved by the same operational adjustment.

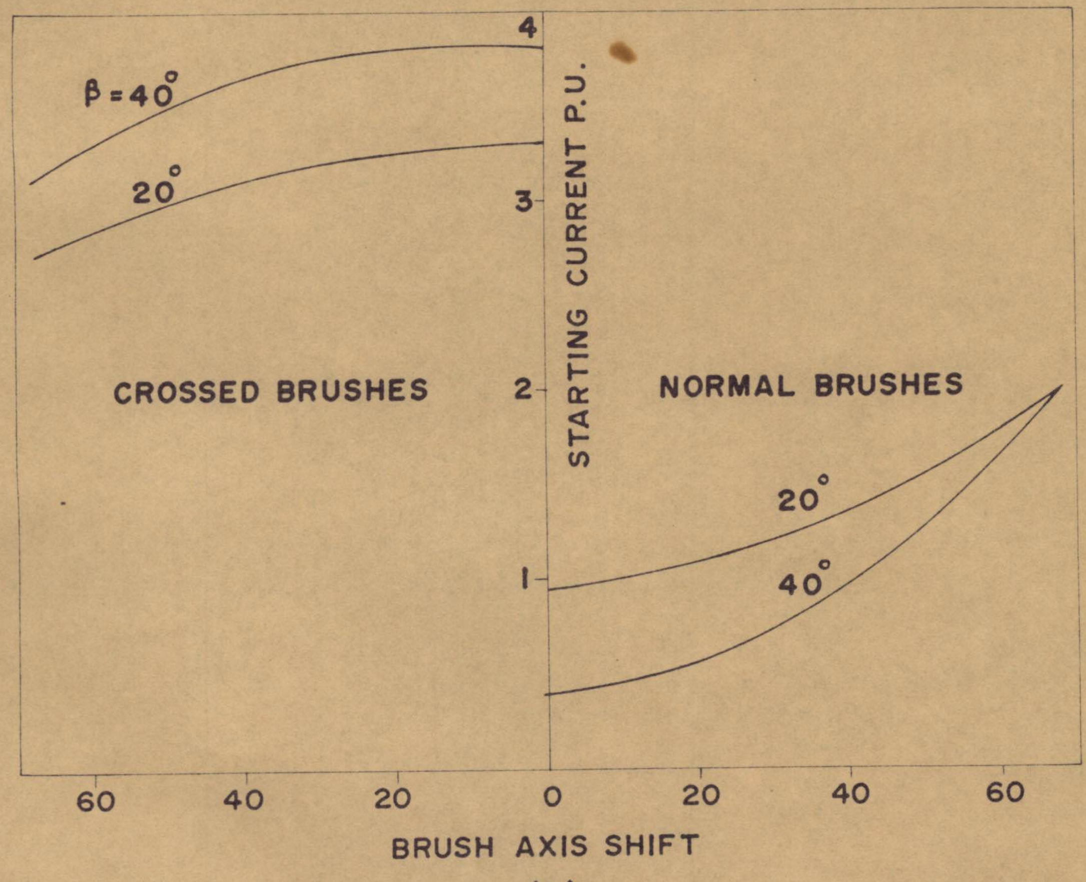
The starting current drawn by the motor decreases if the brush separation is increased with normal brush setting, whereas if the brush separation is increased with crossed brushes, the current increases. On the other hand, with normal brush separations, the single phase starting current increases on increasing brush axis shift ρ (Fig. 6.2). In contrast, with crossed brushes, as ρ is increased, the starting current decreases though not sharply.

From the above discussion, the following conclusions may be drawn:

(i) In order to obtain high starting torque, large brush axis shift ρ as well as brush separation β is required. If the brush separation is normal, large values



(a)



(b)

FIG.6·2 VARIATION OF STARTING CURRENT WITH BRUSH of β reduces the starting current, but brush axis shift tends to increase this value. In contrast, with crossed brushes, large separation increases the primary current whereas brush shift tries to reduce its magnitude. But the current magnitude is comparatively large with crossed brushes for a given combination of β and ρ .

Therefore, it is recommended that at starting, brush separation as well as brush axis shift should be large with subsynchronous setting.

(ii) The direction of starting torque should be reversed only by changing the direction of brush axis shift, for, it would restrict the starting current without decreasing the torque. If the backward torque is obtained by crossed brushes retaining the setting of brush axis shift, comparatively lower starting torque with higher starting current will result.

6.4 Running Performance

6.4.1 Torque-speed Characteristic

If a Schrage motor is fed with three phase balanced supply, the direction of rotation of the motor is determined by the phase sequence of the supply. The torque is said to be in forward direction irrespective of the values of brush separation and brush axis shift. If the motor is switched on to a single phase supply, the direction of torque depends upon the combination of β and ρ . If, for example, brush separation is normal and axis shift is along the direction of forward rotation (with reference to three phase operation), the single phase torque would also be in forward direction. Therefore, motor will pick up speed provided starting torque is high enough to accelerate the motor against inertia, friction torque and load on the motor shaft. The steady operation will depend upon the torque slip characteristics of the motor.

The operation of the motor is categorically different under two possible mode of operating contingencies: (a) During starting and subsequent running, motor is on single phase supply, and (b) Motor is running initially on three phase supply but one phase wire of supply is accidentally opened or fuse of one of the phases is blown off. Under former operation, the torque characteristic is given by the equations (6.1) to (6.2) through (3.1), (3.2), (4.4) and (4.7). Appropriate substitution of the parameters and operational adjustments determine the magnitude and direction of torque. Figs. 6.3 and 6.4 show the torque-slip characteristics for different combinations of β and ρ , respectively for normal and crossed brush separations. The brush axis shift, in all the curves drawn in these figures, has been taken of a sign which gives forward rotation. The curves are equally applicable to the case when the torque developed is backward, except for the fact that the motion is now in opposite direction.

If the motor is started on single phase supply and one of the supply phases is opened to simulate single phase operation, the behaviour of the motor depends upon the combination of different factors viz. brush separation,

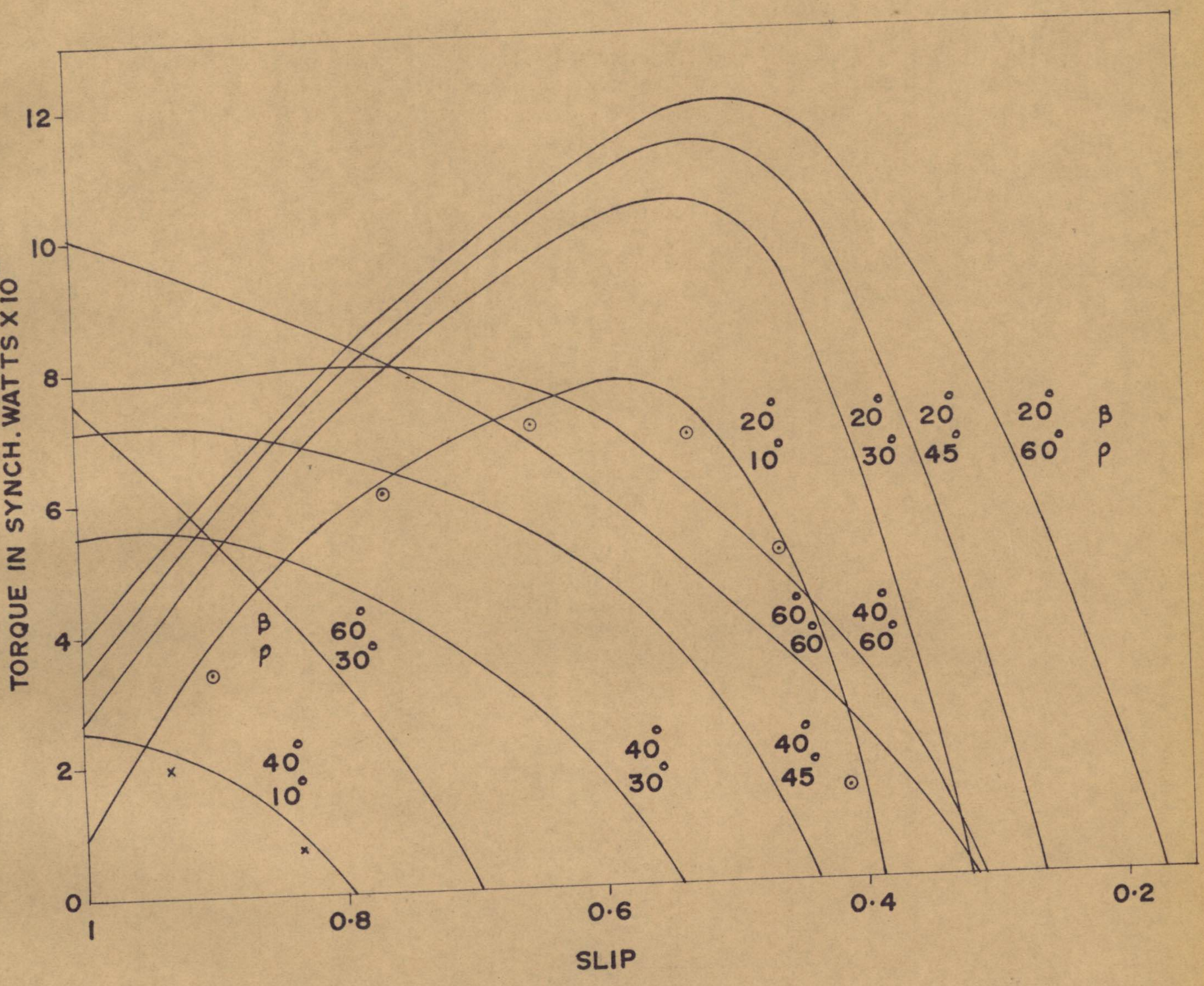


FIG. 6·3 TORQUE—SLIP CHARACTERISTIC OF A SINGLE—PHASE SCHRAGE MOTOR WITH SUBSYNCHRONOUS BRUSH SETTING.

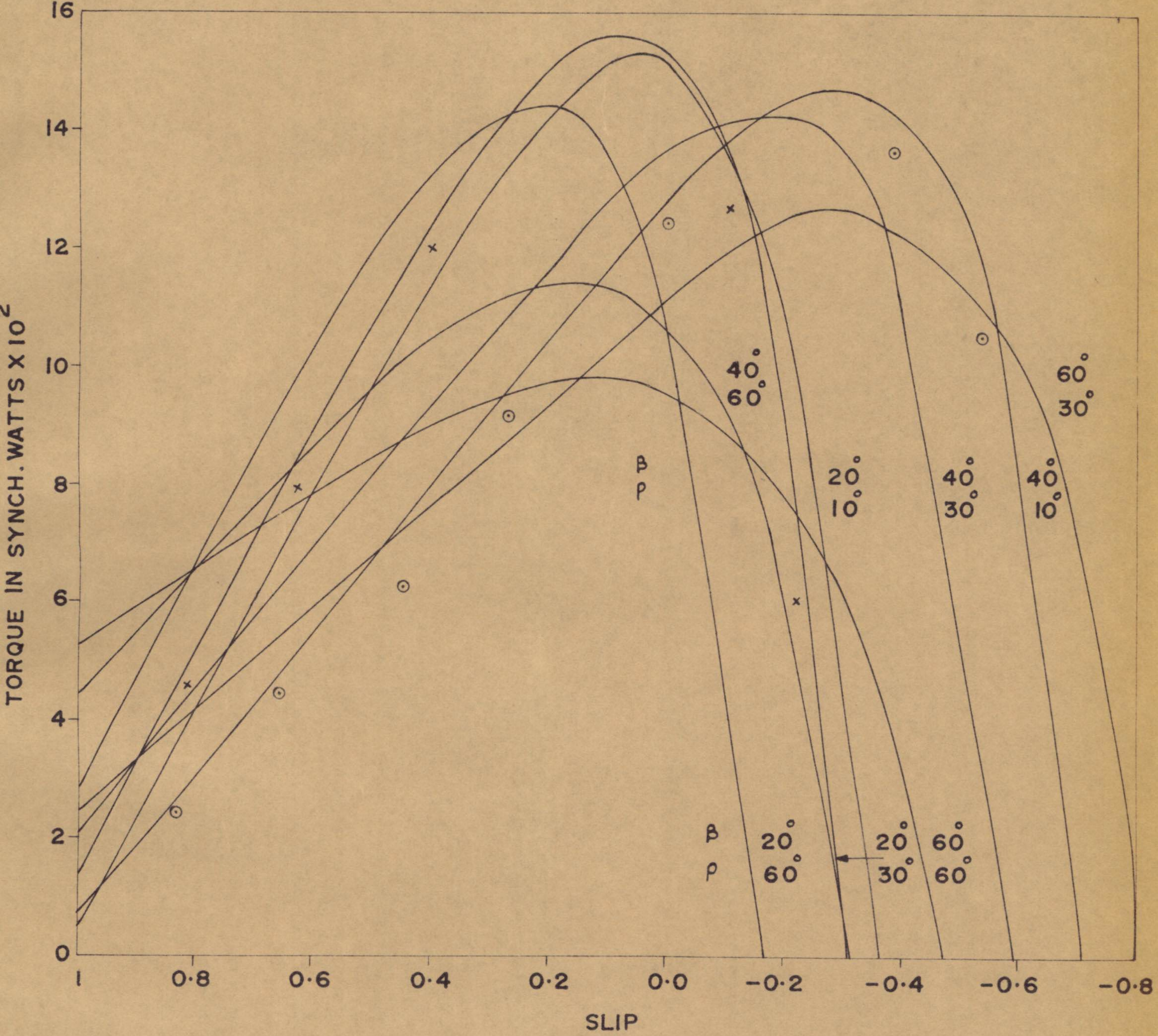


FIG.6·4 TORQUE–SLIP CHARACTERISTIC OF A SINGLE–PHASE SCHRAGE MOTOR WITH SUPERSYNCHRONOUS BRUSH SETTING.

brush setting (normal or crossed), brush axis shift and magnitude of the load torque. The motor may continue to run in the same direction at a reduced speed, deaccelerate to standstill or reverse.

6.4.1.2 Single phase starting as well as running

As has been discussed , the motor when fed with single phase supply will start under favourable conditions of β and ρ combination. The motor speed is likely to stabilise at a speed at which the torque developed by motor is equal to the load torque. A general qualitative picture of the dependence of torque on brush settings may be obtained from Figs. 6.3 and 6.4.

The operation of the motor is confined to subsynchronous zone only when brushes are separated normally similar to polyphase operation. The similarity is further continued so much as the dependence of speed on brush separation is concerned. Larger the brush separation, reduced is the no load as well as running speed for a fixed torque on the shaft. However, if the brush axis shift is altered, the torque characteristics shift bodily in the region of stable operation. For a given value of β, the shift is towards higher speed as the axis shift is increased. Moreover, the breakdown torque is also increased.

In general, the pattern of torque-speed characteristics is same as for polyphase operation except that the running speed for a given torque is reduced and pull out torque is lowered. This is a feature inherent with a single phase running of a polyphase induction motor. The characteristics are still shunt in nature and have a tendency to be steeper as β is increased. Therefore, lesser ~~is~~ the brush separation, better is the speed regulation.

The no load speed of the motor may be adjusted to any desired value from almost zero to twice the synchronous speed by suitably adjusting β and ρ.

If the brushes are crossed, motor can run steadily at supersynchronous speed. In this case, as for polyphase operation, no load speed increases as β is advanced but decreases if brush axis shift is made larger. Therefore, smaller the value of ρ, better is the running performance of the motor.

In the light of the above discussion and inferences, the following recommendations may be made regarding the running of a single phase Schrage motor.

(i) Motor should always be started with normal brush separations, β and ρ as large as possible, whatever may be the desired speed or direction of the rotation. The shift ρ should be adjusted in accordance with the required direction of motion.

(ii) If the subsynchronous operation is required, β may be suitably decreased without altering ρ to obtain any desired speed. However, for a finer speed control, ρ may be adjusted accordingly.

(iii) If the supersynchronous speed is desired, first ρ should be brought to zero, once the motor has picked up speed. Then β is to be reduced to zero, brushes crossed and separation increased. It is to be noted that if brushes are crossed either without bringing ρ equal to zero or after increasing ρ in other direction, the torque reversal takes place and motor may coast to standstill.

The brush axis shift may be increased now, if a smooth and finer speed control is required.

(iv) It is possible to brake the motor to standstill or even to reverse the rotation by either changing the direction of angle ρ or brush setting. If the initial brush separation is normal, it is to be changed to cross and vice-versa to affect braking. The dynamic braking or plugging action is possible only with finite values of ρ. If brush axis shift is zero, the operation would be just similar to a polyphase induction motor with single phase supply. Brush separation will result only in speed modification. Braking to standstill is not possible.

6.4.1.2 Three phase starting, single phase running

If a Schrage motor is running on three phase supply, its response on the occurrence of single phasing, depends upon the brush separation, brush axis shift and load torque versus speed characteristic. Under crossed brush setting, if ρ is positive (opposite to the direction of forward rotation) or under normal brush setting, if ρ is negative, the motor will continue to run in the forward direction on the occurrence of single phasing, but the speed will reduce. These combinations of β and ρ are rare in practice. Usually, the operation of a three phase Schrage motor is either with $\rho = 0$ or with a value of ρ to give power factor improvement i.e. positive for normal brush separations and negative for crossed ones.

In case brush axis shift is zero i.e. neutral set motor, the motor will continue to run steadily at a new speed dictated by load torque and the torque developed by the motor. The operation is identical to a polyphase induction motor made to run on single phase supply.

With usual combination of β and ρ, the torque developed by the motor on single phasing, tends to give backward rotation. Therefore, under these conditions, motor will come to standstill. It may start rotating in reverse direction depending upon the load torque and the starting torque developed by the motor corresponding to the operating values of β and ρ. If the machine picks up speed in reverse direction, the operation may stabilise as for a motor running on single phase supply.

The entire behaviour of the motor consequent to opening of one of the supply phases is characterized by the fact that the magnitude and direction of the negative sequence torque may be less, equal or more than positive sequence torque. This is a result in contrast to the behaviour of a polyphase induction motor under similar conditions. Whereas the negative sequence torque due to single phasing of an induction motor while running, is always less than positive sequence torque, it may be more in Schrage motor resulting in the braking and reversal of motor.

In general, the starting of a single phase Schrage motor is similar to a single phase repulsion motor, since the starting torque is approximately proportional to sine of the angle of brush axis shift. Whereas the running characteristic is shunt in nature in contrast to repulsion motor series characteristics.

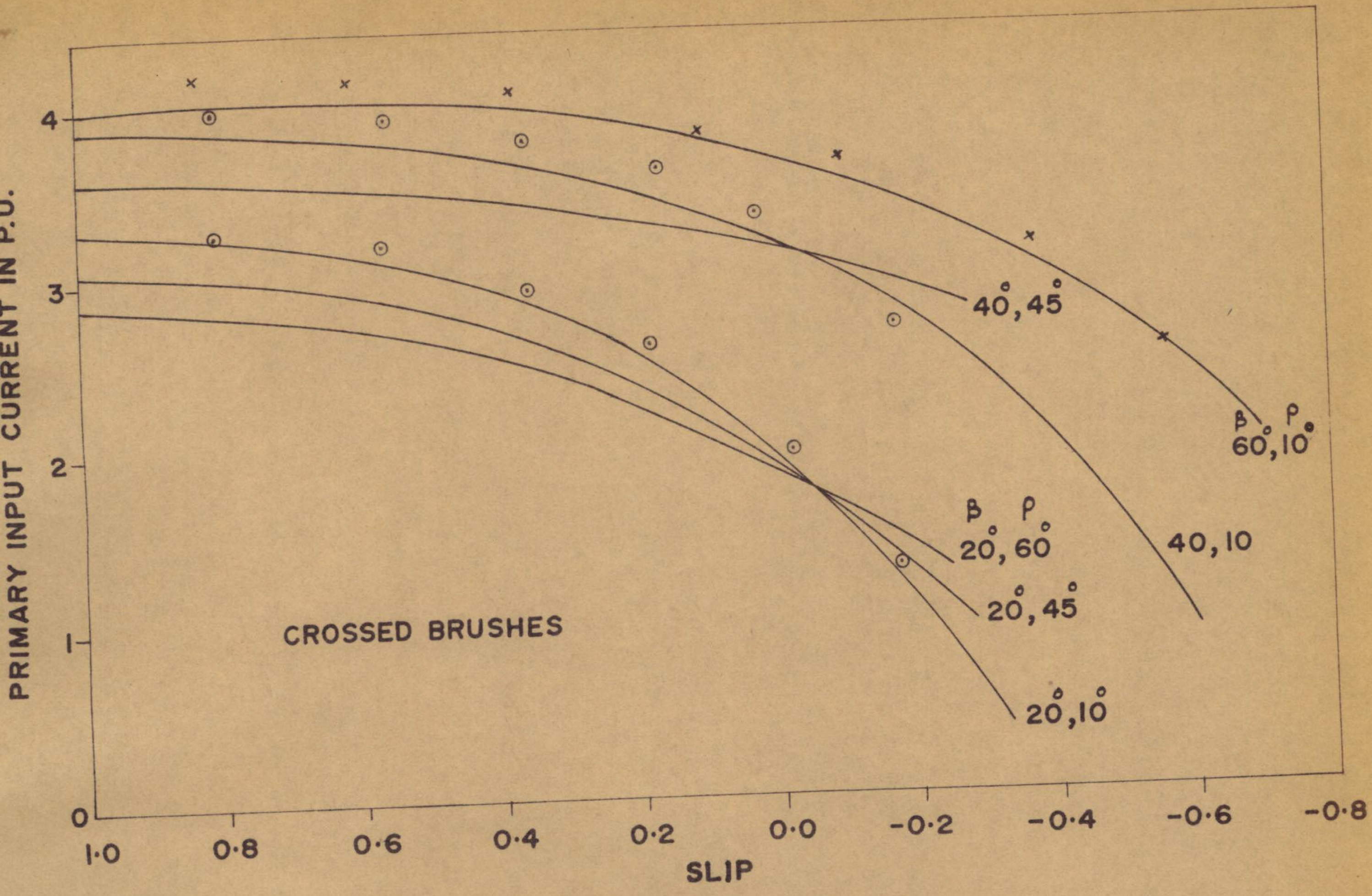
6.4.2 Primary Current

The current drawn by a Schrage motor under single phase operation is given by eqn. (6.2). It is obvious from this expression that similar to an induction motor, the current would be more compared to balanced three phase operation. In order to obtain a general overall picture of the variation of current with speed for various brush settings, Fig. 6.5 was plotted.

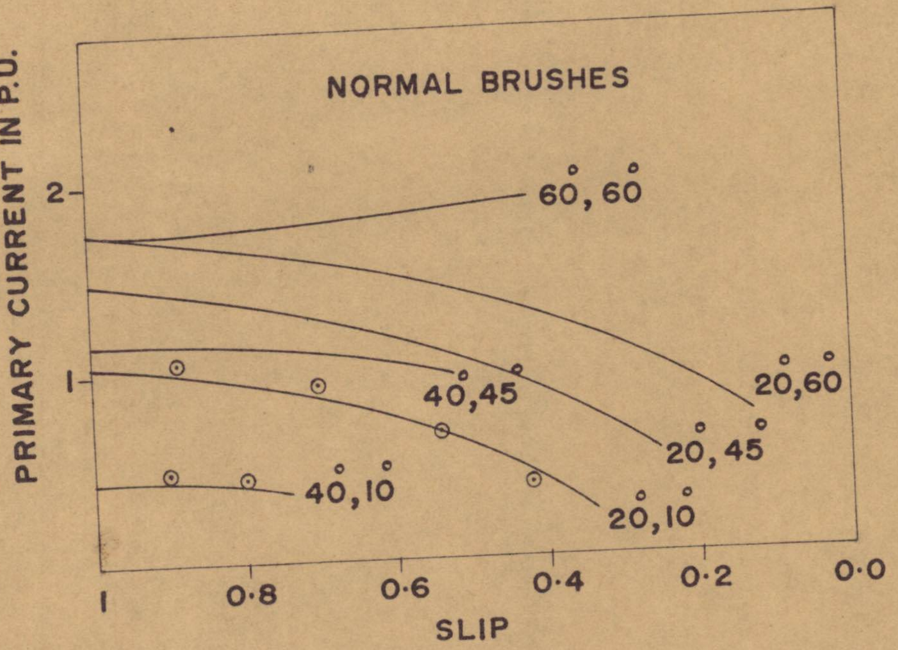
It may be seen from Fig. 6.5(a), that the current variation follows more or less same pattern as with three phase supply, on changing brush separation. As the value of β is increased, the effective impedance decreases thereby increasing the current. It has already been shown (Chapter III) that both positive as well as negative sequence impedances of a Schrage motor decrease with increase in crossed brush separations. Therefore, the current drawn by the motor is more with large values of β. However, it may be noted that unlike the operation on balanced three phase supply, the light load current is not same for all brush separations because negative sequence impedance is different with different values of β though positive sequence impedance is uneffected, its value being approximately equal to that obtained at no load under corresponding 3-phase operation.

The effect of increase in brush axis shift, with fixed value of β, is to decrease the starting current as in case of polyphase operation; but it leads to larger steady state current value. The change in current magnitude is, however, not very sharp.

If the brush separation is normal to obtain sub-synchronous operation, once more the current is governed by the considerations valid for polyphase operation. As β is increased, Z_p and Z_n increase, consequently the current decreases, Fig. 6.5(b). Further, the current increases on increasing brush axis shift for a given brush separation.



(a)



(b)

FIG. 6·5 VARIATION OF INPUT PRIMARY CURRENT WITH SPEED FOR DIFFERENT SETTINGS.

To sum up, the input current of a single phase Schrage motor is comparatively more with crossed brushes than with normal brushes. Increase in the value of ρ, reduces the starting current magnitude when brush are crossed and reverse is the case under normal brush setting.

It has already been concluded that the single phase starting and running is possible only when brush axis shift is of opposite nature from its normal one in three phase operation. For example, with subsynchronous setting, ρ is against the direction of rotation to affect power factor improvement. To accomplish single phase starting without altering the direction of rotation of the motor, brush axis is to be shifted along this direction. If the setting of ρ is not disturbed and motor is switched on to single phase supply, it will run in reverse direction thereby making the brush axis shift along the direction of rotation. Therefore, the power factor under single phase operation is expected to be worse than that obtained with single phase operation of 3-phase induction motor, since a negative value of ρ deteriorates the power factor.

Thus, it seems that running power factor of a single phase motor can best be equal to that obtained with $\rho = 0$ for a given value of β. Therefore, while ρ should be large for higher starting torque, it should be as low as possible for better powerfactor during running.

6.5 Experimental Work

Tests were carried out on a 6 pole, 50 c/s, 3-phase, 230/400, delta/star, 8/0 h.p, 23/13 Ampere Schrage motor. The motor was provided with coplaner brush arrangement with an additional gear to affect brush axis shift in either direction. For test purposes, the machine was coupled to a calibrated d.c. machine whose speed could be controlled by means of a Ward Leonard arrangement.

The parameters of Schrage motor were determined as for a neutral set motor. All other tests necessary to determine losses, variation of secondary winding resistance with speed etc. were carried out.

The test results of single phase operation are recorded on the same figures as the corresponding theoretical results i.e. on Figs. 6.3, 6.4 and 6.5. The brush axis shift for which observations could be recorded was restricted to 10° only. A reference to these figures shows that there is sufficiently close agreement between theory and experimentation to provide complete confirmation of the theory deduced. The slight difference in the calculated and experimental results may be attributed to (i) the use of fixed value of brush contact resistance corresponding to full load value (ii) saturation, although tests were conducted on low voltage such that current did not exceed rated value under any operating condition, and (iii) all other uncertainties inherent with the determination of machine parameters.

CHAPTER VII

Analysis with Unbalance Secondary Injected Voltages (Primary Symmetrical and fed with) balanced voltages

7.1 General

The effect of secondary circuit unbalance on the operation of a 3-phase induction motor was first investigated by Georges[61]. It was shown that the motor develops steady torque at approximately half the normal speed with single phase rotor. Since then, various aspects of unbalance in secondary circuits have been the subject of interest for many investigators, e.g. (Ref. 63-70).

The practical feasibility of Georges' phenomenon has been exploited as a possible means of operating induction motors at half speed with unbalanced external impedances[66] as well as with unbalance connections of the secondary winding[70]. Intentional secondary unbalance has also been suggested as a means of speed control over wider range[67].

The likelihood of secondary circuit unbalance in Schrage motor is more than in an induction motor, while it is either bad contact at slip rings or unbalance in secondary circuit external resistors, which results in induction motor secondary unbalance, the Schrage motor secondary unbalance may follow as a consequence of bad brush contacts, faulty brush gear and wrong secondary connections. In most cases, this gives rise to unbalance injection of voltages into secondary, complicating the situation. To the best of author's information, no published literature is available dealing with the study and analysis of performance of a Schrage motor with secondary circuit unbalance.

In this chapter, an attempt has been made to present a method of analysing this aspect of Schrage motor operation.

7.2 Qualitative Analysis

It is worthwhile to visualize physically the behaviour of the motor with unbalance in secondary circuit before analysing mathematically with the aid of symmetrical component theory.

Let a positive sequence voltage V_{1p} of normal frequency be impressed on to the primary of the motor. The resulting positive sequence current produces a rotating field running past the primary and tertiary windings at synchronous speed. This field induces an e.m.f in the secondary as well as in tertiary winding. If either the secondary circuit is unbalanced or the voltage injected in phases through brushes are unequal or otherwise unsymmetrical, unbalanced currents in secondary circuit would result. As shown in Appendix 7.1, secondary current will consist of zero, positive and negative sequence components since each phase of secondary is individually closed unlike the case of slip ring induction motor.

The secondary voltage and current are of slip frequency sf when the rotor is running at a speed $n_r = (1-s)n_s$ in the forward direction. The positive sequence component of secondary current produces a magnetic field rotating relative to secondary at sn_s but stationary with primary positive sequence field. The interaction of these two fields results in a forward direction torque T_1. The negative sequence component also develops a field rotating with respect to secondary at a speed sn_s but opposite to the direction of positive sequence field. As the speed of this field relative to rotor is $(2s-1)n_s$, voltages are induced in primary as well as tertiary windings. The tertiary winding voltage is injected back to secondary at the correct frequency sf, for, the speed of the field past brushes is sn_s. The primary induced voltage forces a current of frequency (2s-1)f in its own circuit.

The primary current of (2s-1)f frequency sets up a rotating magnetic field in airgap in a direction opposite to that of positive sequence field but stationary with respect to negative sequence field of secondary. The voltage induced in tertiary due to this m.m.f is once more injected in secondary. A negative sequence torque similar to positive sequence Schrage motor torque is developed. This torque T_2 will aid T_1 if $s>\frac{1}{2}$ and oppose if $s<\frac{1}{2}$.

The m.m.f due to secondary circuit zero-sequence current is pulsating in nature. Moreover primary circuit is open for any zero-sequence operation. Therefore, no zero-sequence component is possible in line currents. However, a zero-sequence current shall exist in closed tertiary winding, resulting in a small amount of torque similar in nature to a single phase induction motor torque. In fact multiple reflections as in the case of input zero-sequence phenomenon, between secondary and tertiary will result giving rise to large number of harmonics in secondary current and ripples in net torque.

Besides the steady torque developed by positive and negative sequence components of secondary current, pulsating torques are also produced due to the interaction of primary and secondary fields which are not stationary with respect to each other. For example, the positive sequence field of primary interacting with negative sequence m.m.f. of secondary or negative sequence field of primary interacting with positive sequence m.m.f. of secondary result in torques pulsating at the relative slip frequency of the two components. The magnitude of this alternating torque is dependent upon the product of the magnitudes of the components producing it. These torques would be responsible for the vibration in motor as in the case of an induction motor under similar circumstances. However, there is no change in the net output of the motor due to pulsating torques , since their mean value is zero.

7.3 Analysis with unequal injected voltages in three secondary phases.

Let the brush separations be β_1, β_2, and β_3, and corresponding injected voltages in three phases be E_{3a} , E_{3b} and E_{3c} respectively. The schematic representation of secondary and tertiary circuit is as shown in Fig. 7.1. Z_a, Z_b and Z_c consists of tertiary winding impedance and mutual reactance between secondary and tertiary. The voltages existing across the secondary terminals (A,B,C and N in Fig. 7.1) are given by

$$\left.\begin{aligned} V_a &= I_a Z_a + E_{3a} \\ V_b &= I_b Z_b + E_{3b} \\ V_c &= Z_c Z_c + E_{3c} \end{aligned}\right] \qquad ..(7.1)$$

$I_a Z_a$ etc.,are the effective impedance drops of tertiary winding accounting for sequence components of current. Therefore,

$$\left.\begin{aligned} I_a Z_a &= I_{2p} Z_{ap} + I_{2n} Z_{an} + I_{2o} Z_{ao} \\ I_b Z_b &= a^2 I_{2p} Z_{bp} + a I_{2n} Z_{bn} + I_{2o} Z_{bo} \\ I_c Z_c &= a I_{2p} Z_{cp} + a^2 I_{2n} Z_{cn} + I_{2o} Z_{co} \end{aligned}\right] \qquad ..(7.2)$$

In eqn.(7.2), the sequence impedance for each phase is different because of different brush separation.

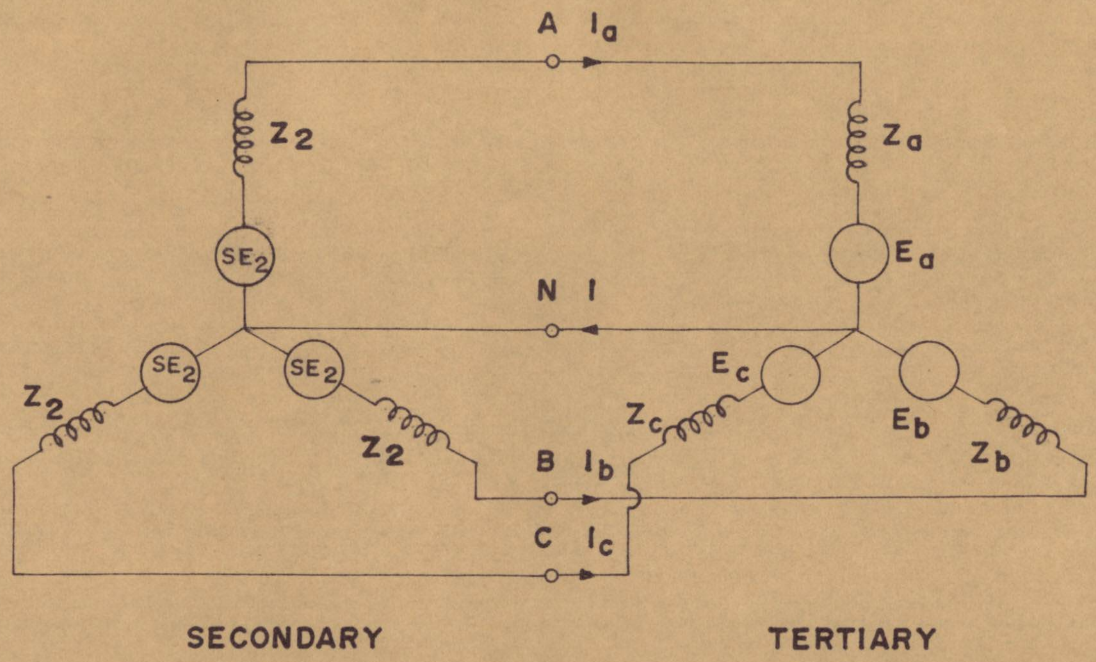


FIG. 7·1 SCHEMATIC REPRESENTATION OF SECONDARY AND TERTIARY WINDING CIRCUIT WITH UNEQUAL BRUSH SEPARATIONS.

The values of positive and negative sequence components in terms of machine parameters may be obtained from the following:

$$Z_{ap} = r_3\beta_1 + j\left[X_{33}\sin^2\beta_1 + X_{23}\sin\beta_1\epsilon^{j\rho_1} + sX_{23}\sin\beta_1\epsilon^{-j}\right]$$

$$Z_{an} = r_3\beta_1 + j\left[(2s-1)X_{33}\sin^2\beta_1 + (2s-1)X_{23}\sin\beta_1\epsilon^{j\rho_2} + sX_{23}\sin\beta_1\epsilon^{-j\rho_2}\right]$$

$$Z_{bp} = r_3\beta_2 + j\left[X_{33}\sin^2\beta_2 + X_{23}\sin\beta_2\epsilon^{j\rho_1} + sX_{23}\sin\beta_2\epsilon^{-j\rho_1}\right]$$

$$Z_{bn} = r_3\beta_2 + j\left[(2s-1)X_{33}\sin^2\beta_2 + (2s-1)X_{23}\sin\beta_2\epsilon^{j\rho_2} + sX_{23}\sin\beta_2\epsilon^{-j\rho_2}\right]$$

Similarly for Z_{cp} and Z_{cn} .. (7.3)

The frequency of negative sequence current in tertiary is (2s-1)f while in secondary it is still sf, therefore for negative sequence component of impedance, tertiary winding reactance is to be multiplied by (2s-1). It is not easy to express the zero-sequence impedance of tertiary along with mutual effect between secondary and tertiary winding in mathematical terms as in case of zero-sequence impedance of primary. This follows from the fact that the effective magnetic circuit for zero-sequence flux is radically different from that of positive or negative sequence fields. Therefore, its value is to be determined experimentally.

The voltages across secondary winding given by eqn.(7.1) may be resolved into their sequence components in accordance with symmetrical component theory. Therefore,

$$\left.\begin{aligned} V_{2p} &= I_{2o}Z_{op} + I_{2p}Z_{po} + I_{2n}Z_{nn} + E_{3p} \\ V_{2n} &= I_{2o}Z_{on} + I_{2p}Z_{pp} + I_{2n}Z_{no} + E_{3n} \\ V_{2o} &= I_{2o}Z_{oo} + I_{2p}Z_{pn} + I_{2n}Z_{np} + E_{3o} \end{aligned}\right] \qquad (7.4)$$

where,

$$\left.\begin{aligned} Z_{po} &= \tfrac{1}{3}\left(Z_{ap} + Z_{bp} + Z_{cn} \right) \\ Z_{pp} &= \tfrac{1}{3}\left(Z_{ap} + aZ_{bp} + a^2Z_{cp} \right) \\ Z_{pn} &= \tfrac{1}{3}\left(Z_{ap} + a^2Z_{pp} + aZ_{cp} \right) \\ Z_{no} &= \tfrac{1}{3}\left(Z_{an} + Z_{bn} + Z_{cn} \right) \quad \ldots \text{ etc.} \end{aligned}\right] \qquad .. \ (7.5)$$

and

$$\left.\begin{aligned} E_{3p} &= \tfrac{1}{3}\left(E_{3a} + aE_{3b} + a^2E_{3c} \right) \\ E_{3n} &= \tfrac{1}{3}\left(E_{3a} + a^2E_{3b} + aE_{3c} \right) \\ E_{3o} &= \tfrac{1}{3}\left(E_{3a} + E_{3b} + E_{3c} \right) \end{aligned}\right] \qquad .. \ (7.6)$$

The voltage injected by tertiary into secondary, say in phase a is E_{3a}. This voltage is induced across the tertiary winding turns separated by brushes $2\beta_1$ electrical degrees apart. There are two fluxes namely positive sequence and negative sequence in the airgap due to primary current as described in Section 7.2. The corresponding voltages induced in tertiary conductors are of different frequencies but as far as secondary injection is concerned, their frequency is same. Therefore the two voltages may be combined together to obtain resultant voltage. Thus,

$$\left.\begin{aligned} E_{3a} &= X_{13}\sin\beta_1 \left[I_{1p}\epsilon^{j\rho_1} + I_{1n}\epsilon^{j\rho_2} \right] \\ E_{3b} &= X_{13}\sin\beta_2 \left[I_{1p}\epsilon^{j\rho_1} + I_{1n}\epsilon^{j\rho_2} \right] \\ E_{3c} &= X_{13}\sin\beta_3 \left[I_{1p}\epsilon^{j\rho_1} + I_{1n}\epsilon^{j\rho_2} \right] \end{aligned}\right] \quad .. (7.7)$$

As the secondary circuit acts as a source of positive, negative and zero-sequence voltages, the performance equations for the motor are to be written separately for each sequence operation. The currents, torques etc., are then to be determined from these equations. For positive sequence operation, the performance equations are

$$\left.\begin{aligned} V_1 &= I_{1p}\,(R_1 + jX_{11}) + I_{2p}(jX_{12}) + E'_{3p} \\ -V_{2p} &= I_{1p}\,(jsX_{12}) + I_{2p}(R_2 + jsX_{22}) \end{aligned}\right] \quad .. (7.8)$$

For negative sequence

$$\left.\begin{aligned} 0 &= I_{1n}\left[R_1 + j(2s-1)X_{11}\right] + I_{2n}(2s-1)jX_{12} + E'_{3n}(2s-1) \\ -V_{2n} &= I_{1n}\,(jsX_{12}) + I_{2n}(R_2 + jsX_{22}) \end{aligned}\right] \quad .. (7.9)$$

and for zero-sequence

$$-V_{2o} = I_{2o}Z_{2o} \quad .. (7.10)$$

In eqn.(7.8), E'_{3p} represents the effect of secondary current through tertiary as felt by primary, such that

$$E'_{3p} = E'_{3a} + aE'_{3b} + a^2E'_{3c} \qquad .. (7.11)$$

where,

$$E'_{3a} = X_{13}\sin\beta_1 \left[I_{2p}\epsilon^{-j\beta_1} + I_{2n}\epsilon^{-j\beta_2} \right] \qquad .. (7.12)$$

The value of E'_{3n} may be found out in a similar manner.

The total torque is obtained by combining the sequence torques. As discussed earlier, the torque due to zero-sequence currents does not contribute to steady value but results in vibration only.

In order to advance experimental support to the results obtained in this section, knowledge of secondary circuit zero-sequence impedance is essential.It is not possible to determine zero-sequence impedance of closed data connected tertiary winding experimentally unless it is opened at one corner. Although, it may be calculated with the help of design data[41], but this data was not available for the Schrage motor under test. Therefore, experimental verification could not be undertaken

CHAPTER VIII

Behaviour with Non-sinusoidal Supply Voltages

8.1 General

Usually the harmonic content in supply voltage is quite small. According to the standard practice followed, the magnitude of the time harmonics should not exceed 5 percent of the fundamental. But if the power supply is obtained from a solid state power source instead of conventional generators, the voltage wave is far from being sinusoidal, as in fact these sources supply periodically reversed d.c. voltage. Consequently the so-called a.c. voltage is not sinusoidal although the wave shape of three-phase voltages is cyclic. Further, if the voltage impressed to a motor is controlled by delaying the firing angle of a silicon controlled rectifier in the supply circuit, the terminal voltage across the motor is never sinusoidal. Therefore, in the light of present trend of high voltage d.c. transmission using invertors at the supplier end and use of solid state devices in static frequency changers, consideration of time harmonics in supply to induction type rotating machines has gained importance. It is interesting to note that contrary to the harmonics which may exist in conventional power source voltages, the voltage wave in this case may have any harmonic including even harmonics and harmonics of the order of (3n + 3); n being any integer.

In this chapter, an attempt has been made to analyse the behaviour of a Schrage motor supplied with non-sinusoidal voltages of the above type. In particular, the effect on torque-slip characteristic with normal and crossed brush positions and heating of stator and rotor have been discussed.

8.2 Analysis

8.2.1 Nature of Harmonic Fields

The resultant armature m.m.f. in the airgap of a 3-phase electric machine with respect to any reference axis, due to harmonic excitation of the nature of,

$$i_m = \sum_h I_h \sin\left[h\omega t + \theta_h - h\frac{2\pi}{3}(m-1)\right] \quad .. (8.1)$$

is given by[77],

$$M = F \sum_h \sum_p \frac{K_{\omega p}}{p} I_h \sin\left[hwt + \theta_h \pm p(x - x_o)\frac{\pi}{\tau}\right] \quad .. (8.2)$$

where, i_m is the current in the mth phase, θ_h is the phase angle of the hth time harmonic and p is the space harmonic such that $h = 3n+p$.

The expression (8.2) represents a system of travelling waves having the speeds,

$$\frac{\pi}{\tau}\frac{dx}{dt} = \omega_o - \frac{h\omega}{p} \text{ for } h = 3n+p \quad .. (8.3)$$

$$\frac{\pi}{\tau}\frac{dx}{dt} = \omega_o + \frac{h\omega}{p} \text{ for } h = 3n-p \quad .. (8.4)$$

where, $\omega_o = \left(\frac{\pi x_o}{\tau} \right)$ is the armature speed,

$\omega = 2\pi f$, and

f = the frequency of the fundamental of the voltage wave.

For a Schrage motor considering the space distribution of mmf as purely sinusoidal, the mmf wave due to time harmonics of the order of $h_F = (3n+1)$ will always be rotating in forward direction with a speed of $(h_F N_s - N)$ in space, while if the harmonic order is $h_B = (3n+2)$, the field rotates backward at a speed of $(h_B N_s + N)$. The voltage wave time harmonics of the order of $(3n+3)$ are in the same time phase in all the three phases (Eqn.8.1), and their effect is analogous to that of the zero sequence voltages. In the absence of a return path for inphase components, there is no mmf due to these harmonics irrespective of their origin. The slip with respect to stator for hth harmonic field in terms of fundamental field slip s is given by

$$s_h = 1 \mp (1-s) / h \qquad \text{.. (8.5)}$$

8.2.2 <u>Operation with hth harmonic voltage</u>

If a 3-phase balanced sinusoidal voltage of frequency hf and magnitude V_h is impressed on the primary of the motor, the corresponding airgap field rotates at a speed of $s_h N_s$ with respect to stator. The voltage induced in the stator winding is of the frequency $hs_h f$ and the same frequency voltage is injected by the tertiary winding owing to the fact that the relative speed of the airgap field with respect to both stator and brushes is the same, irrespective of its speed with respect to rotor. If the deep bar and skin effects due to higher order time harmonics are neglected, the currents in primary and secondary circuits are given by

$$I_{1h} = \frac{V_h}{jhX_{11}} \left[\frac{1+jhk\left[(T'_{3o}\sin\beta + T'_{\rho o}\epsilon^{j\alpha})\sin\beta + s_h(T'_{2o} + T'_{\rho o}\epsilon^{-j\alpha}\sin\beta)\right]}{1+jhk\left[(T'_{3s}\sin\beta + T'_{\rho s}\epsilon^{j\alpha})\sin\beta + s_h(T'_{2s} + T'_{\rho s}\epsilon^{-j\alpha}\sin\beta)\right]} \right] \qquad ..\ (8.6)$$

$$I_{2h} = \left(\frac{kV_h}{X_{11}R_e}\right) \frac{s_h X_{12} + X_{13}\sin\beta\,\epsilon^{j\alpha}}{\left[1+jhk\left[(T'_{3s}\sin\beta + T'_{\rho s}\epsilon^{j\alpha})\sin\beta + s_h(T'_{2s} + T'_{\rho s}\epsilon^{-j\alpha}\sin\beta)\right]\right]} \qquad ..\ (8.7)$$

In these equations α is the effective brush axis shift such that $\alpha = \rho_1$ for $h=(3n+1)$ and $\alpha=\rho_2$ for $h=(3n+2)$. The factor k accounts for the change in the secondary circuit resistance due to brush separation as well as frequency effects. Since $s_h = 1 \mp \frac{(1-s)}{h}$, it approaches unity as the order of harmonic increases even if the motor is running at supersynchronous speed. Therefore, for every high order harmonics i.e. $h >> 1$, the expressions (8.6) and (8.7) reduce to

$$I_{1h} = \frac{V_h}{jhX_{11}} \frac{1+jhT'_s k}{1+jhT'_s k} \qquad ..\ (8.8)$$

$$I_{2h} = \frac{kV_h}{X_{11}R_e} \frac{X_{12}+X_{13}\sin\beta}{1+jhT'_s k} \qquad ..\ (8.9)$$

where, $T'_o = T'_{2o} + T'_{3o}\sin^2\beta + 2T'_{\rho o}\sin\beta\cos\alpha$

$$T'_s = T'_{2s} + T'_{3s}\sin^2\beta + 2T'_{\rho s}\sin\beta\cos\alpha$$

$$T'_{s1} = (T'_{3s}\sin\beta + T'_{\rho s}\cos\alpha)\sin\beta$$

$$T'_{s2} = T'_{2s} + T'_{\rho s}\sin\beta\,\cos\alpha$$

The torque developed by the motor with hth harmonic voltage is given by

$$T_h = \frac{V_h^2 k}{X_{11}^2 R_e}\,\frac{\left[\begin{array}{l}(s_h X_{12}^2 + X_{12}X_{13}\sin\beta\cos\alpha) + \\ hkX_{12}X_{13}\sin\beta\sin\alpha(s_h T'_{2s} + T'_{3s}\sin^2\beta)\end{array}\right]}{\left[\begin{array}{l}\left[(1 - hk^2 T'_{\rho s}\sin\beta\sin\alpha(1-s_h)\right]^2 + h^2k^2\left[(T'_{3s}\sin\beta + \right. \\ \left. T'_{\rho s}\cos\alpha)\sin\beta + s_h(T'_{2s} + T'_{\rho s}\sin\beta\cos\alpha)\right]^2\end{array}\right]}$$

.. (8.10)

For higher order harmonics:

$$\text{Torques} \simeq \left(\frac{V_h}{X_{11}}\right)^2 \frac{kX_{12}}{R_e}\,.$$

$$\left[\frac{(X_{12} - X_{13}\sin\beta\cos\alpha) + hkX_{13}\sin\beta\sin\alpha(T'_{2s} + T'_{3s}\sin^2\beta)}{1 + h^2k^2T'^2_s}\right]$$

.. (8.11)

$$\text{Primary copper loss} :\simeq \left(\frac{V_h}{hX_{11}}\right) \frac{1 + h^2k^2T'^2_o}{1 + h^2k^2T'^2_s} R_1 \qquad .. (8.12)$$

$$\text{Secondary circuit copper loss} \simeq \left(\frac{V_h}{X_{11}}\right)^2 \frac{k}{R_e}\,\frac{\left[\begin{array}{l}X_{12}^2 + X_{13}^2\sin^2\beta \\ 2X_{12}X_{13}\sin\beta\cos\alpha\end{array}\right]}{1 + h^2k^2T'^2_s} \qquad .. (8.13)$$

The above expressions are independent of the speed of the motor. Therefore, the torque developed by the motor and corresponding electrical losses are all constant in magnitude and simply depend upon the amplitude and frequency of the harmonic voltage. The input to the motor is given by

$$P_h = \left(\frac{V_h}{X_{11}}\right)^2 \frac{1}{1+h^2k^2T_s'^2}\left[\frac{R_1}{h}(1-h^2k^2T_o'^2) + k\,\frac{2X_{12}^2+X_{13}^2\sin^2\beta+3X_{12}X_{13}\sin\beta.\cos\alpha+hkX_{12}X_{13}\sin\beta\sin\alpha(T_{2s}'+T_{3s}'\sin^2\beta)}{R_e}\right]$$

Also, since $h^2k^2T_o'^2 >> 1$ and $h^2k^2T_s'^2 >> 1$,

$$P_h = \left(\frac{V_h}{hX_{11}T_s'}\right)^2\left[R_1hT_o'^2 + \frac{2X_{12}^2+X_{13}^2\sin^2\beta+X_{12}X_{13}\sin\beta\left[3\cos\alpha + hk\sin\alpha(T_{2s}'+T_{3s}'\sin^2\beta)\right]}{k\,R_e}\right] \quad .. (8.14)$$

If the copper losses in the two circuits of the motor at standstill with sinusoidal impressed voltage of fundamental frequency are:

Primary copper loss: $W_{c1} = \frac{V_1^2}{X_{11}^2}\left(\frac{1+T_o'^2k^2}{1+T_s'^2k^2}\right)R_1$

Secondary copper loss $W_{c2} = \frac{V_1^2}{X_{11}^2}\left(\frac{X_{12}^2+X_{13}^2\sin^2\beta+2X_{12}X_{13}\sin\beta\cos\rho_1}{1+T_s'^2k^2}\right)\frac{R_{2e}k}{R_e}$

then, assuming that k for hth harmonic is approximately equal to that with h=1, s=1,

$$\text{Primary copper loss} = \left(\frac{V_h}{hV_1}\right)^2 \left(\frac{1+T_s'^2k^2}{T_s'^2}\right)\left(\frac{T_o'^2}{1+T_o'^2k^2}\right) W_{c1} \quad .. (8.15)$$

$$\text{Secondary copper loss} = \left(\frac{V_h}{hV_1}\right)^2 \frac{1+T_s'^2k^2}{k^2T_s'^2} W_{c2} \quad .. (8.16)$$

Also, the torque due to hth harmonic in terms of the fundamental voltage torque at standstill is given by,

$$T_h = \left(\frac{V_h}{V_1}\right)\left(\frac{1+T_s'^2}{h^2T_s'^2}\right)\left[\frac{(X_{12}^2+X_{12}X_{13}\sin\beta\cos\alpha)+ hkX_{13}\sin\beta\sin\ (T_{2s}'+T_{3s}'\sin^2\beta)}{(X_{12}^2+X_{12}X_{13}\sin\beta\cos\rho_1)+ X_{13}\sin\beta\sin\rho_1(T_{2s}'+T_{3s}'\sin^2\beta)k}\right] \quad .. (8.17)$$

It follows from the above deductions that to include the effects of higher order harmonics of voltage wave, it is not necessary to consider the effect at different speeds. A constant magnitude term, corresponding to each harmonic present, is sufficient to describe the desired effect (torque as well as copper loss).

8.2.3 Operation with non-sinusoidal supply

For analysis of the behaviour of an electrical machine under harmonic excitation, it is customary for the sake of simplification to neglect the saturation effects. Therefore, the performance may be described by proper summation of effects of each harmonic present in the impressed voltage, considered individually. As discussed in section 8.2.1 harmonics of the order of (3n+1) result in forward rotating field while those of the order of (3n+2) produce backward rotating field. Consequently the nature of the torque developed corresponding to (3n+2)th harmonic is opposite to that for (3n+1)th harmonic. Therefore, it should be more convenient to sum up the effects of the two groups of harmonic separately and then to combine them. Now due to all harmonics of the order of $h_F = (3n+1)$, the primary current is given by

$$I_{1h_F} = \sum \frac{V_{h_F}}{jh_F X_{11}} \left[\frac{1+jk\left[h_F(T'_{3o}\sin\beta + T'_{\rho o}\epsilon^{j\rho_1})\sin\beta + (h_F-1+s)(T'_{2o}+T'_{\rho o}\epsilon^{-j\rho_1}\sin\beta)\right]}{1+jk\left[h_F(T'_{3s}\sin\beta + T'_{\rho s}\epsilon^{j\rho_1})\sin\beta + (h_F-1+s)(T'_{2s}+T'_{\rho s}\epsilon^{-j\rho_1}\sin\beta)\right]} \right] \quad ..(8.18)$$

The r.m.s. value of the current may be obtained from the expression

$$\left|I_{1h_F}\right| = \sqrt{\left[\sum_{h=o}^{\infty} \frac{V_{h_F}}{h_F X_{11}} \frac{\left[1-(1-s)kT'_{\rho o}\sin\beta\sin\rho_1\right]^2 + k^2\left[(3n+1)\right.}{\left[1-(1-s)kT'_{\rho s}\sin\beta\sin\rho_1\right]^2+k^2\left[(3n+1)\right.} \frac{(T'_{3o}\sin\beta+T'_{\rho o}\cos\rho_1)\sin\beta+(3n+s)}{(T'_{3s}\sin\beta+T'_{\rho s}\cos\rho_1)\sin\beta+(3n+s)} \frac{\left.(T'_{2o}+T'_{\rho o}\sin\beta\cos\rho_1)\right]^2}{\left.(T'_{2s}+T'_{\rho s}\sin\beta\cos\rho_1)\right]^2}\right]} \quad ..(8.19)$$

If it is assumed that except for $n = 0$, the harmonic order is high enough to make $s_{hF} \approx 1$, the current is given by

$$\left|I_{1h_F}\right| = \sqrt{\left[\left(\frac{V_1}{X_{11}}\right)^2 \cdot \frac{\left[1-(1-s)kT'_{\rho o}\sin\beta\cos\rho_1\right]^2+k^2\left[(T'_{3o}\sin\beta+T'_{\rho o}\cos\rho_1)\sin\beta+s(T'_{2o}+T'_{\rho o}\sin\beta\cos\rho_1)\right]^2}{\left[1-(1-s)kT'_{\rho s}\sin\beta\sin\rho_1\right]^2+k^2\left[(T'_{3s}\sin\beta+T'_{\rho s}\cos\rho_1)\sin\beta+s(T'_{2s}+T'_{\rho s}\sin\beta\cos\rho_1)\right]^2} + \sum_{n=1}^{\infty}\left(\frac{V_{h_F}}{(3n+1)X_{11}}\,\frac{T'_o}{T'_s}\right)^2 \right]} \quad ..(8.20)$$

Similarly the secondary circuit current is

$$\left|I_{2h_F}\right| = \sqrt{\left[\left(\frac{V_1 k}{X_{11}R_e}\right)^2 \cdot \frac{s^2X_{12}^2+X_{13}^2\sin^2\beta+2sX_{12}X_{13}\sin\beta\cos\rho_1}{\left[\left[1-kT'_{\rho s}\sin\beta\sin\rho_1(1-s)\right]^2+k^2\left[(T'_{3s}\sin\beta+T'_{\rho s}\cos\rho_1)\sin\beta+s(T'_{2s}+T'_{\rho s}\sin\beta\cos\rho_1)\right]^2\right]} + \sum_{n=1}^{\infty}\left(\frac{V_{h_F}}{X_{11}R_e}\right)^2 \frac{X_{12}^2+X_{13}^2\sin^2\beta+2X_{12}X_{13}\sin\beta\cos\rho_1}{(3n+1)^2} \right]} \quad ..(8.21)$$

The approximation taken for eqns.(8.20) and (8.21) is valid only in subsynchronous zone, since the harmonic field slip does not approach unity unless n is large when the machine is running in supersynchronous region for which slip is negative. Therefore, for supersynchronous zone it would be more appropriate to approximate for $n>2$ only, both for

h_F as well as h_B. With this consideration, expressions for primary and secondary current may be written similar to eqns. (8.20) and (8.21), if the order of harmonics is h_B.

The total current in the two circuits may be obtained from -

$$I_1 = \sqrt{(I^2_{1h_F} + I^2_{1h_B})}$$
$$I_2 = \sqrt{(I^2_{2h_F} + I^2_{2h_B})} \qquad ..(8.22)$$

The total torque developed with all the harmonics considered is given by

$$T = T_{h_F} - T_{h_B} \qquad ..(8.23a)$$

where,

$$T_{h_F} = \frac{kX_{12}}{X^2_{11}R_e}\left[\sum_{n=0}^{\infty} V^2_{h_F} \frac{\left(\frac{(3n+s)}{(3n+1)} X_{12}+X_{13}\sin\beta\cos\rho_1 + (3n+1)X_{13}\sin\beta\sin\rho_1\right).\left(T'_{2s}\frac{3n+s}{3n+1} + T'_{3s}\sin^2\beta\right)}{\left[1-kT'_{\rho s}\sin\beta\sin\rho_1(1-s)\right]^2 + k^2\left[(T'_{3s}\sin\beta+T'_{\rho s}\cos\rho_1)(3n+1).\sin\beta+(3n+s)(T'_{2s}+T'_{\rho s}\sin\beta\cos\rho_1)\right]^2}\right]$$

..(8.23b)

$$T_{hB} = \frac{kX_{12}}{X_{11}^2 R_e} \left[\sum_{n=o}^{\infty} V_{hB}^2 \frac{\begin{array}{c}(\frac{3n+3-s}{3n+2})X_{12} + X_{13}\sin\beta\cos\rho_2 + (3n+2)X_{13}\sin\beta \\ \sin\rho_2(T'_{2s}\frac{3n+3-s}{3n+2} + T'_{3s}\sin^2\beta)\end{array}}{\begin{array}{c}[1+kT'_{\rho s}\sin\beta\sin\rho_2(1-s)]^2 + \\ k^2[(T'_{3s}\sin\beta + T'_{\rho s}\cos\rho_2) . \\ (3n+2)\sin\beta + (3n+3-s). \\ (T'_{2s} + T'_{\rho s}\sin\beta\cos\rho_2)]^2\end{array}} \right] \qquad ..(8.23c)$$

8.3 Discussion

8.3.1 Torque developed

The general expression for the torque developed at any value of brush separation and brush axis shift, with non-sinusoidal supply voltage is given by Eqn.(8.23). This expression may be simplified further for a particular mode of operation of the motor. As for example, if the power factor control is not desired, both ρ_1 and ρ_2 are π or zero for normal and crossed brush settings respectively. Thus, for a neutral set motor, the torque is given by-

(i) For normal brush setting

$$T = \frac{kX_{12}}{X_{11}^2 R_e} \left[V_1^2 \frac{sX_{12} - X_{13}\sin\beta}{1+k^2(T'_{s1} + sT'_{s2})^2} - V_2^2 \frac{\frac{3-s}{2}X_{12} - X_{13}\sin\beta}{1+4k^2(T'_{s1} + \frac{3-s}{2}T'_{s2})^2} + \frac{X_{12} - X_{13}\sin\beta}{T'^2_s k^2} \sum_1^{\infty} (\frac{V_{hF}}{3n+1})^2 - (\frac{V_{hs}}{3n+2})^2 \right] \qquad ..(8.24)$$

(ii) For crossed brush setting

$$T = \frac{kX_{12}}{X_{11}^2 R_e}\left[V_1^2 \frac{sX_{12}+X_{13}\sin\beta}{1+k^2(T'_{s1}+sT'_{s2})^2} - V_2^2 \frac{\frac{3+s}{2}X_{12}+X_{13}\sin\beta}{1+4k^2(T'_{s1}+\frac{3-s}{2}T'_{s2})^2} + \sum_{1}^{\infty} \left(\frac{V_{hF}}{3n+1}\right)^2 \frac{\frac{3n+s}{3n+1}X_{12}+X_{13}\sin\beta}{k^2(T'_{s1}+\frac{3n+s}{3n+1}T'_{s2})^2} - \left(\frac{V_{hB}}{3n+2}\right)^2 \frac{\frac{3n+3-s}{3n+2}X_{12}+X_{13}\sin\beta}{k^2(T'_{s1}+\frac{3n+3-s}{3n+2}T'_{s2})^2}\right] \quad ..(8.25)$$

It is obvious from eqn.(8.24), that the magnitude of the torque due to forward rotating as well as backward rotating harmonic field is almost constant irrespective of the speed of the motor. Moreover both components decrease rapidly as the order of harmonic increases and equals to the torque developed by the motor at standstill under that particular harmonic excitation. Therefore, when a Schrage motor is made to run at subsynchronous speed, its behaviour is similar to an induction motor under the same operating conditions, but in the present case, the decrease in the net torque reduces as β is increased. In the crossed brush position, ρ_1 is zero and slip is negative. This change in operating condition necessitates that s_h is not taken equal to unity, although from the denominator of the torque expression, unity may be neglected compared to $k^2h^2(T'_{s1}+T'_{s2}s_h)^2$. Consequently the torque expression reduces to that of Eqn.(8.25). It may be deduced easily that in supersynchronous zone, the reduction in torque due to time harmonics, in terms of the torque developed by the fundamental frequency voltage is more compared to the subsynchronous case. However, as before the change in torque is influenced by the value of β and reduction in torque increases with increase in brush separation.

Figure 8.1 shows the calculated speed torque curve for a Schrage motor supplied with a non-sinusoidal supply having the following harmonic contents in p.u.

$V_1 = 1$	$V_2 = 0.21$
$V_4 = 0.104$	$V_5 = 0.446$
$V_7 = 0.67$	$V_8 = 0.045$
$V_{10} = 0.93$	$V_{11} = 0.0051$
$V_{12} = 0.284$	

It is obvious from the curves drawn for different values of β, that the general nature of the torque is more or less the same as discussed above. The torque reduction is more as the speed decreases, being maximum at standstill. Although from the curves it appears that the decrease in the magnitude of the torque is not substantial in the usual operating range, but in actual practice it may not be so due to increase in resistance as a consequence of skin effect.

8.3.2 Stator and rotor heating

The primary and secondary circuit copper loss may be determined with the help of Eqns.(8.6) and (8.7).

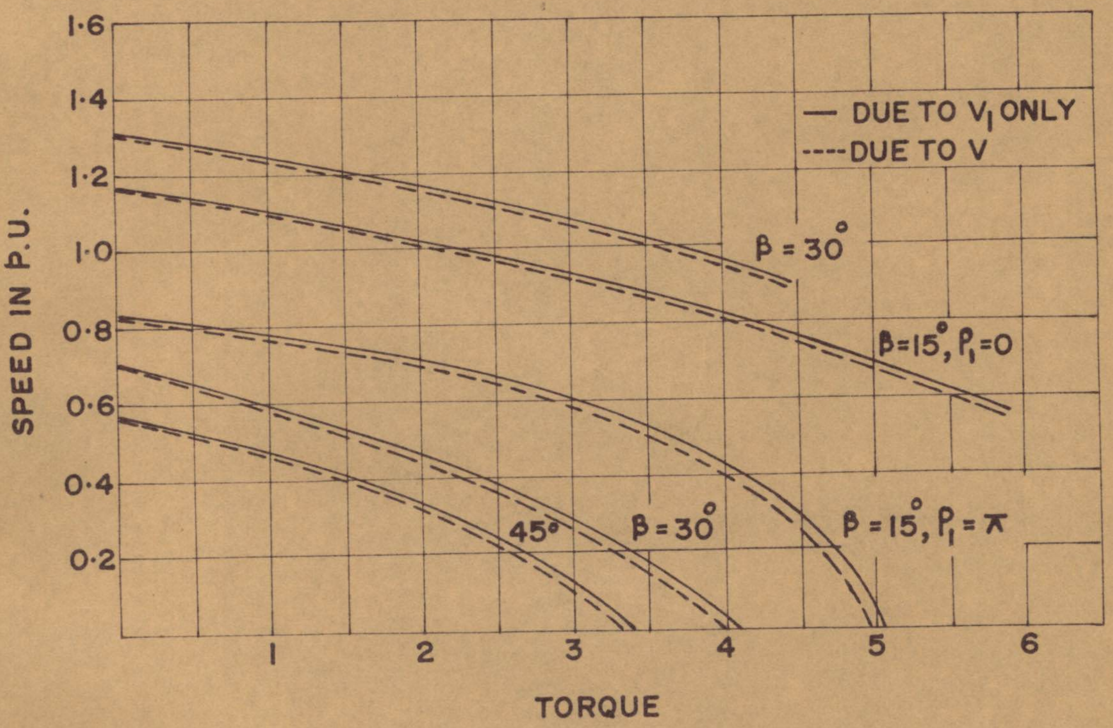


FIG.8·1 SPEED TORQUE CHARACTERISTIC OF A SCHRAGE MOTOR WITH NON-SINUSOIDAL SUPPLY VOLTAGE.

Accordingly,

Primary copper loss =

$$\frac{R_1}{X_{11}^2}\sum_{o}^{\infty}\left(\frac{V_h}{h}\right)^2\frac{\left[1-kT'_{\rho o}\sin\beta\sin\alpha(1-s_h)\right]^2+k^2h^2\,(T'_{o1}+s_hT'_{o2})^2}{\left[1-kT'_{\rho s}\sin\beta\sin\alpha(1-s_h)\right]^2+k^2h^2(T'_{s1}+s_hT'_{s2})^2}$$

Stator copper loss =

$$\frac{k^2R_{2e}}{R_e^2X_{11}^2}\sum_{o}^{\infty}V_h^2\,\frac{(s_hX_{12}+X_{13}\sin\beta\cos\alpha)^2+X_{13}^2\sin^2\beta\sin\alpha}{\left[1-kT'_{\rho s}\sin\beta\sin\alpha(1-s_h)\right]^2+k^2h^2(T'_{s1}+T'_{s2}s_h)^2}$$

..(8.26)

For a neutral set motor

$$\text{Primary copper loss} \simeq \frac{R_1}{X_{11}^2}\left[V_1^2\,\frac{1+k^2(T'_{o1}+sT'_{o2})^2}{1+k^2(T'_{s1}+sT'_{s2})^2}+V_2^2\,\frac{(T'_{o1}+\frac{3-s}{2}\,T'_{o2})^2}{(T'_{s1}+\frac{3+s}{2}\,T'_{s2})^2}+\left(\frac{T'_o}{T'_s}\right)^2\sum\left(\frac{V_h}{h}\right)^2\right] \quad ..(8.27a)$$

$$\text{Stator copper loss} \simeq \frac{R_{2e}k}{R_e^2X_{11}^2}\left[V_1^2\,\frac{(sX_{12}+X_{13}\sin\beta\cos\alpha)^2}{1+k^2(T'_{s1}+sT'_{s2})^2}+V_2^2\,\frac{(\frac{3-s}{2}\,X_{12}+X_{13}\sin\beta\cos\alpha)^2}{4k^2(T'_{s1}+\frac{3-s}{2}\,T'_{s2})^2}+\left(\frac{X_{12}+X_{13}\sin\beta\cos\alpha}{T'_s}\right)^2\sum\left(\frac{V_1}{h}\right)^2\right]$$

..(8.27b)

The rotor which carries two windings will he heated up by the copper-losses of both primary and tertiary windings. Therefore, the rotor copper losses are given by

$$\text{Rotor copper loss} \approx \text{Primary copper loss} + \frac{R_3(\beta)}{R_{2e}} \text{ Stator copper loss} \qquad ..(8.28)$$

The Eqns.(8.27) and (8.28) show that the copper loss in both the members of the motor is approximately constant at all speeds for values of $h>>1$. Interestingly the extra copper losses in both the members due to time harmonics are more or less in the same proportion i.e. $\Sigma\ (V_h/h)^2$. In terms of the standstill copper loss of different windings with sinusoidal voltage, the extra loss with non-sinusoidal supply is given by

$$\text{Stator copper loss} = \frac{1+k^2T_s'^2}{k^2T_s'^2}\left[\Sigma\ (\frac{V_h}{h})^2\right] W_{c2} \qquad ..(8.29)$$

$$\text{Rotor copper loss} = \frac{1+k^2T_s'^2}{T_s'^2}\left[\Sigma\ (\frac{V_h}{h})^2\right]\left(W_{c1}\ \frac{T_o'^2}{1+k^2T_o'^2} + W_{c3}\right) \qquad ..(8.30)$$

It may be easily seen that the total copper loss with sinusoidal voltage of the same r.m.s. value as non-sinusoidal voltage is more than with non-sinusoidal voltage, approximately in the proportion of $\Sigma(V_h)^2/\Sigma(V_h/h)^2$. However, all these deductions are valid only if the skin effect is ignored. In fact, actual copper losses with non-sinusoidal supply are expected to be more than calculated one both in stator as well as rotor due to increased effective resistance of the respective windings not only as a consequence of high frequency but stray load losses also.

The iron loss with non-sinusoidal supply is more than with sinusoidal supply of the same r.m.s. value particularly in primary winding which is embedded deep in the slot. The hysteresis loss would increase in proportion to the harmonic order while eddy current loss to the square of the harmonic order i.e. hysteresis loss would be more approximately in the ratio of $(\Sigma h V_h^2)/(\Sigma V_h^2)$, while eddy current loss is more in the ratio of $(\Sigma h^2 V_h^2)/(\Sigma V_h^2)$. Moreover the degree of saturation augmented by deep bar effect would be more with purely sinusoidal voltage than with any individual harmonic of non-sinusoidal supply. Therefore, it may be concluded reasonably that heating of the machine would be practically the same under both supply voltages.

To sum up, the nature of harmonic torque depends upon the order of the harmonic. The torque magnitude is not only a function of p.u. harmonic content and harmonic order but it is also influenced by the brush separation and brush axis shift of the motor. It has been shown that reduction in torque magnitude for a neutral set motor for given harmonic contents, is different with normal and crossed brush positions. This harmonic torque expressed as percentage of the torque developed with fundamental component of voltage wave, increases with brush separation for normal brush setting. If the motor is set to run at supersynchronous speed with crossed brushes, the reduction in torque is more than with normal setting for a particular brush separation.

The copper losses of a Schrage motor are more with sinusoidal voltage supply than with non-sinusoidal supply of the same r.m.s. value. But due to increased iron losses and effective resistances of the different windings, it is expected that there would hardly be any noticeable change in heating of the motor under two conditions of operation. It follows, therefore, that whereas the torque developed by the motor is reduced approximately in proportion to the square of the ratio of the effective value of fundamental and r.m.s. value of impressed voltage, the thermal requirement is unaltered. Consequently, the allowable output of the motor is restricted from heating considerations and is less with non-sinusoidal supply of the same r.m.s. value as rated voltage. Therefore, it is desirable that the percentage of fundamental in a non-sinusoidal voltage should be as large as possible for more economical operation of the motor.

CHAPTER IX

Main Conclusions and Scope for further work

9.1 Conclusions

A polyphase motor is likely to be subjected to abnormal conditions of operation during its normal working. Prediction and calculation of the performance of such an operation is an important problem. The investigation reported in this thesis was undertaken to enable determination and analysis of the behaviour of a Schrage motor under some of the abnormal conditions. The basic purpose was to investigate operational features and limitations,if any, or otherwise, to ascertain feasibility of improvement in performance and to make necessary recommendations to meet likely contingencies. The following abnormal operating conditions were considered:-

(i) Unbalanced supply voltages,

(ii) Single phase supply,

(iii) Balanced supply but voltage injection to secondary unbalanced, and

(iv) Non-sinusoidal supply to the motor.

In order to study these conditions , a general and comprehensive method of analysis has been developed. It is also applicable to induction motor as a boundry value case. The main features of the present work are summarized in the following.

The steady state behaviour of a Schrage motor under various conditions is completely represented by the performance equations deduced in Chapter II. The equations are in terms of measurable machine parameters. The methods for experimental determination of constants have been described. Later on, substitution of derived parameters of time constant nature in these equations offers an elegant approach rendering simple and comprehensive expressions which permit calculation of performance characteristics for the entire range of operation.

The negative sequence impedance for any value of brush separation and brush axis shift may also be calculated by same set of equations which describe the positive sequence impedance of a Schrage motor. However, resistance of secondary circuit is to be properly accounted for frequency effects. It has been shown that positive sequence as well as negative sequence impedance is similar in nature to that of an induction motor. If the brushes are adjusted for subsynchronous operation, both Z_p and Z_n are more than that for supersynchronous setting. The value of Z_n decreases with speed for normal brush setting. But with crossed brushes, variation in Z_n is very little. Larger the brush separation, lesser is the variation in Z_n with speed. Therefore, negative sequence impedance of a Schrage motor may be assumed to be constant and equal to corresponding standstill value provided the brushes are crossed.

It is difficult to relate zero-sequence impedance of a Schrage motor with machine parameters. But it has been found that its magnitude remains more or less uneffected by different brush separations, although characterized by sharp peaks at $n_s/3$ and $2n_s/3$. For analytical purposes, it is sufficient to assume a constant value of Z_o.

The Schrage motor characteristics may be looked upon as those of an induction motor but modified by both voltage as well as current injection in its secondary circuit. It is argued that to and fro transfer of the slip power betweentertiary and secondary windings of a Schrage motor is responsible for bringing out the main changes in its characteristics and governs the no load speed, magnitude of torque and region of stable operation. The current injection also contributes to the magnitude of the torque. The pull out and starting torque may be looked upon as that of an induction motor with changed secondary winding resistance, duly modified by current injection. It seems feasible to adjust the design of the motor to obtain high starting torque with subsynchronous setting.

The behaviour of a Schrage motor under unbalance supply voltages is governed not only by the degree of unbalance but brush adjustments also. The torque-speed characteristics for entire range of operation have been deduced. It is found that the reduction in the net torque developed by the motor is more with normal setting than with crossed brush settings. But for moderate unbalance the torque reduction is small and the full load speed is not likely to be affected seriously.The power flow concept renders the necessary physical explanation to this behaviour.

The current unbalance of secondary circuit always exceeds that of primary. The total secondary current under unbalance supply voltages may reach prohibitive proportions when Schrage motor is operated with crossed brushes. Threfore, it is desirable that the load on the motor is restricted so as not to damage it.

The possibility of limiting the output of a Schrage motor under unbalance supply voltages has been examined from thermal as well as commutational considerations. It is expected that unlike an induction motor, the heating due to secondary current imposes the thermal restrictions in case of a Schrage motor. Commutation also gets deteriorated but it is the heating which derates the motor. A new conduction coefficient has been proposed to account for the temperature rise of the machine, but it is found that its value is unaffected by degree of unbalance and speed of the motor. This makes the allowable heating of the stator of a Schrage motor independent of conduction coefficient. The desirability of relating rerating with voltage unbalance has been stressed. Accordingly, the criterion, method of calculation and typical results regarding rerating are discussed. It has been shown that a Schrage motor operation is comparatively safer on unbalanced voltages with subsynchronous brush setting than with crossed brush setting. The rerating factor which decreases with increase in voltage unbalance depends upon the positive sequence voltage. Even a small voltage unbalance may result in substantial reduction in allowable load subjected to brush adjustments.

The general performance equations are applicable to the analysis of the behaviour of the machine when connected to single phase supply. Investigations on single phase operation of a Schrage motor revealed one very interesting feature, that it may develop starting torque provided the brush axis shift is other than zero. The magnitude of starting torque is approximately proportional to the sine of the angle by which brush axis is shifted. For the same brush separation and axis shift, the torque is comparatively more with normal separations than with crossed separations. However, larger the brush separation, more is the torque. It is shown that a large brush separation with normal brush setting and axis shift by an angle whose cosine is equal to the no load slip pertaining to the adjusted brush separation, results in sufficiently high starting torque. The direction of the starting torque depends upon the combination of brush axis shift and brush separation. If a Schrage motor is allowed to run on single phase supply, its direction of rotation will always be such as to make brush axis shift opposite to one, needed for power factor improvement under three phase operation. Assuming that the direction in which machine runs when all the three phases are energized is the forward direction, brush axis shift should be along the direction of rotation with normal separations while it should be opposite to the direction of rotation with crossed brushes to obtain forward rotating torque of a Schrage motor under single phasing.

The starting current with single phase supply bears a definite ratio with the current drawn by the machine when connected to three phase supply without altering brush adjustments. Therefore, single phase starting current would be comparatively less in magnitude with normal brush setting. To restrict the starting current and to obtain simultaneously high starting torque, it is suggested that a Schrage motor ought to be switched on to a single phase supply with large normal brush separations and corresponding axis shift. The direction of rotation should be reversed by changing the axis shift, not by crossing over the brushes retaining axis shift undisturbed.

The running characteristics of a Schrage motor on single phase supply conform to shunt nature. The no load as well as running speed may be adjusted to any desired value within design limits.

The response of a Schrage motor on the occurrence of single phasing while already running on three phase supply is governed by the combination of brush setting, separation and axis shift. If axis shift in conjunction with brush separation is such as to allow the motor to develop torque in forward direction, motor will continue to run provided the developed torque is sufficient to meet load demand. If brushes are crossed and axis shift is opposite to the direction of rotation, the current input would be excessive, although forward direction torque results. If initial value of axis shift is zero, the operation is similar to an induction motor on single phase supply. On the other hand, if axis shift is such as to affect power factor improvement under three phase operation, the net torque developed on the occurrence of single phasing is in reverse direction. The motor decelerates to standstill. It may run in opposite direction provided starting torque corresponding to the adjusted brush setting is sufficient to overcome shaft load. It is desirable to run the motor with as little axis shift as practicable, best being zero, in order to restrict power factor deterioration.

A method to analyse secondary circuit unbalance with symmetrical primary conditions has been presented. It is shown that under such condition, the secondary circuit becomes a source of negative and zero sequence voltages. While the negative sequence component contributes to the net torque developed, the zero sequence torque is mainly pulsating. The performance equations obtained are not amenable to physical interpretation but it may be easily visualized that the presence of zero sequence will cause vibrations and large number of secondary current harmonics. The usual Georges phenomenon, however, is always present whenever secondary circuit becomes a source of negative sequence voltage, in induction class of machines.

The theory of a Schrage motor under non-sinusoidal impressed voltages has been discussed. Methods have been suggested to calculate torque, current, losses etc. The torque developed by time harmonic voltages is small in comparison with fundamental torque but the losses of the motor supplied with non-sinusoidal voltages may be appreciably different from normal operation losses. The actual performance depends upon the waveform and the amplitude of impressed voltage. It has been concluded that the harmonic torque expressed as percentage of the fundamental component torque increases with brush separations, this increase being comparatively more when brushes are crossed.

The losses of the motor with or without harmonics in voltage wave are expected to be approximately same, while the output of the motor is reduced with non-sinusoidal supply of the same r.m.s. value as rated voltage. Therefore, it is recommended that the voltage waveform should have low harmonic content or the loading of the motor should be restricted.

9.2 Scope for further work

The abnormal conditions investigated in this thesis are the most common one. The present method of analysis may be extended to cover more complicated cases such as double unbalance, asymmetrical arrangement of the three rotor or stator windings.

In the text of this work, the possibility of obtaining high starting torque under normal operation, by suitably adjusting design parameters is mentioned. The implications of modifying machine dimensions on running characteristics has not been looked into in detail and may need further investigation.

The rerating of a Schrage motor under unbalance supply voltages has been determined on the basis of stator heating , assuming that rotor heating is not a deterrent. But the likelihood of the presence of severe hot spots on rotor can not be ruled out and a further check may be carried out. However, the method suggested may be usefully exploited to ascertain the permissible loading of the motor on reduced voltage condition.

The capability of developing starting torque on single phase supply by a Schrage motor provides not only a interesting study but also a practically possible proposition for the development of a new single phase drive. A machine having single winding primary, two or three phase secondary and a corresponding tertiary winding, may prove to be an economical, efficient and flexible arrangement for single phase drives of low h.p. rating. An application for the grant of patent of such a motor has been moved and further work is being taken up.

The analysis pertaining to secondary unbalance was confined to the theoretical formulations only. More information and investigation is necessary to embrace peculiarities of the behaviour of a Schrage motor near synchronous speed caused by secondary unbalance.

In this thesis, only an approximate estimate of the characteristics and losses of a Schrage motor fed with non-sinusoidal voltages has been presented. Although, it is sufficient to give fairly good idea of the response of the motor under this condition but a more detailed investigation need be done to examine precisely, the nature and variation of its losses.

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APPENDIX 2.1

Power developed by a Schrage Motor

The motional impedance for a rotating electric machine is obtained by retaining only those terms of transient impedance matrix which correspond to speed voltages and omitting all others. For Schrage motor, eqn.(2.2) gives,

$$
[Z''(\nu)] = \begin{array}{c|cccccc}
 & d_1 & d_3 & d_2 & q_1 & q_3 & q_2 \\
\hline
d_1 & & & & & & \\
d_3 & & & & & & \\
d_2 & & & & M_{21}\nu & M_{23}\sin\beta\,\epsilon^{j\beta_1} & L_{22}\nu \\
q_1 & & & & & & \\
q_3 & & & & & & \\
q_2 & -M_{12}\nu & -\nu M_{23}\cdot\sin\beta\,\epsilon^{-j\beta_1} & -L_{22}\nu & & &
\end{array}
$$

The self-impedance terms in the motional impedance matrix do not contribute any torque since the airgap is uniform and there can be no reluctance torque. Elimination of these terms and division by common factor ν gives the torque matrix as

			M_{21}	$M_{23}\sin\beta\,\epsilon^{j\beta_1}$	
$-M_{12}$	$-M_{23}\sin\beta\,\epsilon^{-j\beta_1}$				

Since the steady state power and torque are required, $i_{d1} = I_1$, $i_{q1} = -jI_1$, $i_{d2} = i_{d3} = I_2$ and $i_{q2} = i_{q3} = -jI_2$. The electrical power is then equal to

$$P_e = 3\nu\,\mathrm{Re}\left[I_2^*(-jM_{12})I_1 + I_2^*(-jM_{23}\sin\beta\,\epsilon^{-j\beta_1})I_1\right]$$

APPENDIX 2.2

Experimental Check on the Validity of Power Flow Concept

To ascertain the relative magnitudes of secondary induced voltage and that injected by tertiary winding at no load, the no load speed of a Schrage motor was recorded for different brush separations in its entire speed range. Measurement of open circuit voltage across brushes as a function of β and secondary winding voltage was carried out. The above two records were co-ordinated to plot the secondary induced voltage and the voltage across the brush halves of each phase with respect to no load speeds in Fig. A2.1. It may be seen that sE_2 is greater than injected voltage for subsynchronous setting and vice-versa for supersynchronous setting. The difference between the two voltages increases with brush separation.

To check the validity of power flow concept, two tests were performed.

Test 1- A wattmeter was inserted between secondary and tertiary windings such that it reads positive if the power is conducted from tertiary to secondary windings. Wattmeter observations were recorded with a dead load on the shaft of the motor. It is evident from the observations reproduced below that power flows from secondary to tertiary with normal brushes and from tertiary to secondary with crossed brushes. The magnitude increases with an increase in separation.

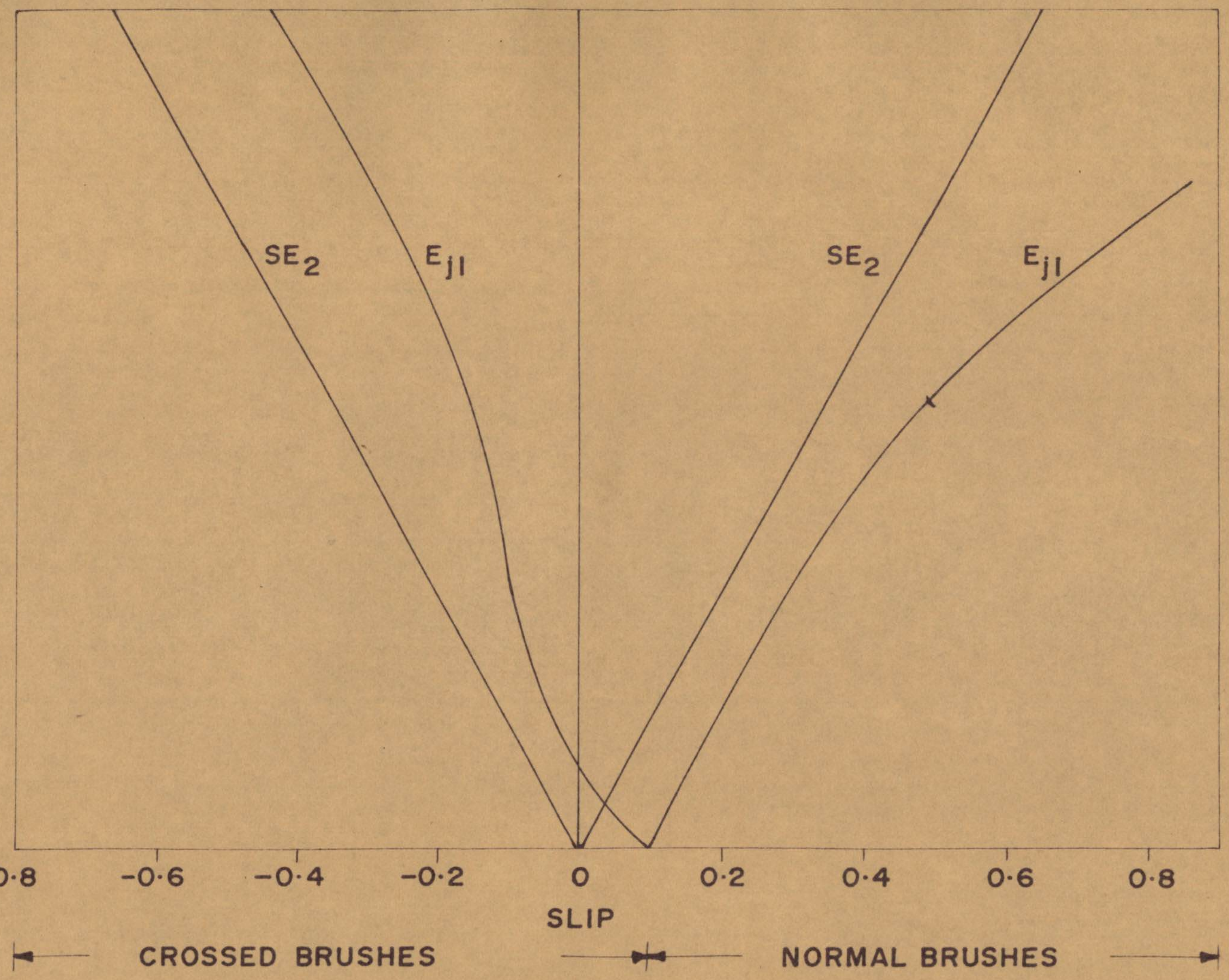
SE2
Ej1
SE2
Ej1
−0·8
−0·6
−0·4
−0·2
0
0·2
0·4
0·6
0·8
SLIP
CROSSED BRUSHES
NORMAL BRUSHES

FIG. A2.

Brush setting	Brush separation in degrees	Wattmeter reading in Watts
Normal	60	-190
	45	-155
	30	-120
	0	- 20
Crossed	4	0
	30	+160
	45	270
	60	380
	80	460

Test 2- The power input to the machine was recorded for various speed settings maintaining the ratio (power output/speed) constant. The input power was calculated with a knowledge of airgap power, various losses and conceived power handling by tertiary winding. The table given on page 140 shows typical values for three brush settings, compared with observed input power. A close agreement between the two confirms the power flow concept.

No.	Brush Setting	Input power in watts	
		calculated	observed
1.	Induction motor operation (β=o)	1715	1740
2.	Normal brush separation (subsynch. speed)	1457	1475
3.	Crossed brush separation (supersynch. speed).	2079	2090

APPENDIX 3.1

Experimental details

Tests were conducted on a 220/380 V, 5.5-8/3.2-4.9A, 0.5 - 2 KW, 50 c/s, 4 pole, 600-2200 rpm Schrage motor to verify the analytic deductions. For this purpose, motor under test was coupled to a calibrated d.c. machine.

A3.1 No load losses

The various losses of a Schrage motor vary widely in its operating speed range. Due to somewhat complex nature and peculiar power flow in this machine, different losses are supplied from different sources under test conditions during its abnormal operation. Therefore, a precise knowledge of the magnitude, variation with slip and source of these losses is essential. The following losses are recognized in this motor at no load:

(i) Mechanical loss, P_m - (a) Bearing friction, slip ring brushes friction and windage loss, P_f ,

(b) Brush friction loss at commutator, P_b,

(ii) Iron loss, P_c - (c) Rotor hysteresis and eddy current loss (independent of rotor speed) $P_{c1} = P_{h1} + P_{e1}$,

(d) Stator hysteresis loss, P_{h2},

(e) Stator eddy current loss, P_{e2},

(iii) Parasitic loss, P_p - (f) Surface loss, P_{sp},

(g) Loss in short circuited coil (tertiary winding), P_{sc}.

The method to determine and separate these losses is essentially the one used by Alger and Eksergian[45]. The following test procedure is adopted:

Test 1

A separately excited d.c. driving motor is run at no load with a constant field current. The armature is supplied with a variable voltage through a Ward-Leonard arrangement and excitation adjusted to obtain speed of driving motor equal to the highest speed of Schrage motor, with rated armature voltage. The no load input power is recorded and variation of no load losses with speed after correcting for copper loss is plotted, [curve 0, Fig.A3.1a]. In this as well as in subsequent tests, the armature current of d.c. machine is small enough to enable the effect of armature reaction flux to be ignored.

Test 2

The driving motor is coupled to Schrage motor under test, and the power drawn by it to drive the combination with unexcited Schrage motor is measured for different speeds. Now the test motor is fed with normal rated voltage with secondary circuit open. The phase sequence of a.c. is ensured to develop rotating field in the same direction as that of the set. The input power to driving motor and to the Schrage motor at different speeds is recorded once more. The corresponding curves after due correction for copper loss are respectively 1, 2 and 5 in Fig. A3.1(a).

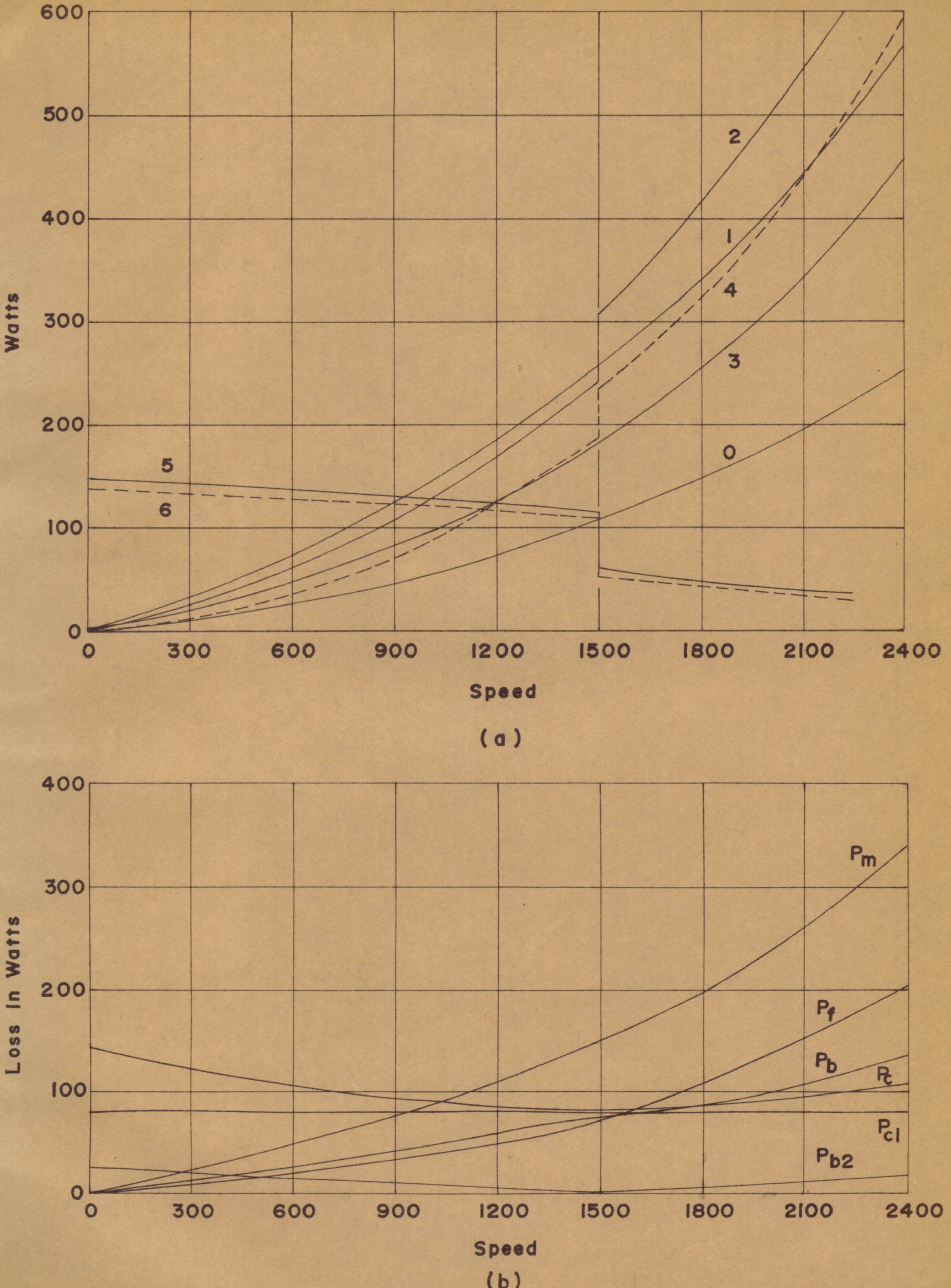

FIG. A3·1 DETERMINATION AND SEPARATION OF LOSSES IN A SCHRAGE MOTOR.

Test 3

The test 2 is repeated with Schrage motor unexcited as well as excited but brushes lifted from commutator (not from slip rings through which power is fed to the primary of test motor). The curves 3 and 4 show the d.c. motor power needed to drive the combination and curve 6, the input to Schrage motor corresponding to curve 4.

The various losses from these seven curves are determined as follows:

(i) The bearing friction, slip ring brushes friction and windage loss, P_f, is obtained by subtracting curve 0 from curve 3.

The cummutator brush friction loss is given by the difference of curves 1 and 3. The variation of total mechanical loss P_m, with speed is shown in Fig.A3.1(b).

(ii) The rotor iron loss, P_{cl} is independent of speed and simply depends upon the magnitude and frequency of the applied voltage to primary. It equals to the mid-point of the vertical portion of the curve 6 at synchronous speed. The abrupt decrease in a.c. power of the test motor at synchronous speed is owing to the reversal of the direction of the hysteretic torque. Since the slip of the primary field with respect to secondary is zero at this speed, there is no secondary eddy current loss. Therefore, the average value of curve 6 at synchronous speed corresponds to P_{cl}. Since curve 5 includes P_{sc}, which is not absent even at synchronous speed, P_{cl} cannot be obtained as the average value of this curve at synchronism unlike a stator fed motor[46].

The half of the amount of decrease in power at synchronous speed of curve 6 is equal to P_{h2} at standstill because the hysteretic torque is constant in magnitude upto that speed and equals to the hysteresis loss of secondary at supply frequency[49] (slip unity). At any other speed, a portion, proportional to slip, of the hysteretic power developed constitutes hysteresis loss while the balance appears as mechanical power. The loss is positive irrespective of the sign of slip and is directly proportional to it.

The a.c. power loss at standstill with brushes lifted from commutator (given by the curve 6) equals the total standstill value of iron loss P_c, since parasitic losses are zero when rotor is stationary. Therefore, stator eddy current loss corresponding to supply frequency is given by the ordinate of curve 6 at speed zero minus P_{cl} and P_{h2} determined above. P_{e2} at any other speed of the motor may be obtained easily as it is proportional to the square of the slip although an empirical formula has also been suggested to determine it[47]. The experimentally determined value of P_{e2} at standstill is 31 Watts. Its variation is not shown in Fig. A3.1(b) for the sake of clarity.

(iii) The surface losses consist of tooth pulsation loss, true surface loss occurring in thin layer of iron at the slotted periphery of rotor and stator and the losses caused by harmonics due to winding distribution[50]. These losses are absent at standstill and vary in some proportion with speed. In test 3, as the brushes are lifted, loss in the coils short circuited by brushes, P_{sc} is absent. The total power input to the d.c. - a.c. motor set must equal to $(P_o+P_m+P_e+P_{sp})$. Therefore, the variation of P_{sp} with speed is given by

$$P_{sp}(s) = (\text{curve } 6 + \text{curve } 4) - (\text{curve } 0 + P_{c1} + P_m + P_{c2})$$

or $$P_{sp}(s) = (\text{curve } 6 + \text{curve } 4) - (\text{curve } 3 + P_c)$$

The loss in short circuited coil, P_{sc}, is given directly by the difference of curve 6 from 5. This loss is dependent upon the voltage induced in each coil of the tertiary winding, number of coils short circuited by brushes and the resistance of these coils[50]. As the tertiary and primary windings are on the same member of the motor, the coil voltage is same irrespective of the speed of the rotor and is proportional approximately to the applied primary voltage. Thus P_{sc} is more or less constant at all speeds for a particular excitation, as is evident from experimental results and equals to 9 Watts.

A3.2 Determination of k_n

It is well known that the effective resistance of a conductor, embedded in iron slot, changes as the frequency of the alternating current flowing through it increases. Therefore, the secondary winding resistance to negative sequence current is higher than that offered to the positive one. In order to determine this effective resistance, a variable frequency test was conducted. The stator of the Schrage motor was supplied with variable frequency voltage obtained from the slip rings of an induction machine run by a d.c. motor. The rotor circuit of the motor under test was left open and brushes lifted from commutator. The input power and current were recorded for different frequencies from 2 c/s to 100 c/s. The effective resistance was calculated after applying due correction for iron loss of stator as determined in Section A3.1. To ensure constant flux throughout the test, V/f ratio was kept constant and normal value of secondary current was maintained.

The results of this test are recorded in Fig.3.2, in somewhat unconventional form. Instead of plotting the variation of R_{2n} as a function of frequency, the ratio R_{ep}/R_{en} is shown with respect to slip, so that the factor k_n is readily available corresponding to any forward field slip.

It is to be noted that for torque calculation, a prior knowledge of secondary current is essential to properly account for variation of brush resistance R_b and k_n. It has been found, however, that if R_b as well as k_n corresponding to full load value of I_2 are substituted, the results are fairly close. Therefore in Fig.4.2, k_n has been plotted taking full load current value of R_b.

A3.3 Experimental determination of machine parameters

The parameters of the motor under test were determined as described in Chapter II.

A very high order of accuracy is to be maintained in the determination of self and mutual reactances. Even a small error in the measurement of any of the parameters may lead to substantially wrong results. In order to ensure the values of constants corresponding to the actual conditions during test, all parameters as well as subsequent experimental verification was carried out under unsaturated conditions.

APPENDIX 4.1

Typical calculation and experimental verification of unbalance operation of a Schrage motor

A Schrage motor with normal brush separation of $\beta = 40^{\circ}$ was fed with unbalance voltage through three single phase auto-transformers. The magnitude of Line voltages was

$$V_{AB} = 182V \; ; \; V_{BC} = 180V \; ; \; V_{CA} = 216V \; .$$

The values of $V_p = 192\underline{/6.67^{\circ}}$; $V_n = 23\underline{/69.3^{\circ}}$ and $\lambda_V = 12.4\,\%$; were calculated in accordance with the method given by Williams. The following table lists the calculated and experimentally observed values of impedances, currents and torque.

Quantity	Calculated value	Experimental
Z_p	58.5 ohm	60.4 ohm
Z_n	23.6 ohm	24.8 ohm
I_{1p}	$3.28\underline{/-58.83^{\circ}}$ A	
I_{1n}	$1.01\underline{/9^{\circ}}$ A	
I_A	$-6.9\,\underline{/79.7^{\circ}}$ A	6.4A
I_B	$-4.0\underline{/-33.1^{\circ}}$ A	3.7A
I_c	$-6.5\underline{/45.1^{\circ}}$ A	6.3A
T_p	350 syn.W	
T_n	14.4 syn.W	
T	461 W	480W

APPENDIX 5.1

Determination of the heating coefficients

In order to determine the values of C_L and C_M, the following tests are carried out.

(i) The motor under test is driven at the desired speed by an auxiliary motor and one of its phases is fed with direct current. The magnitude of the current and steady state temperature rise of each phase on the stator (or rotor as the case may be) is recorded. The temperature rise is measured by noting the resistance of a winding by the help of a Kelvin bridge.

(ii) All the three phases are connected in series in same sense across a d.c. source and motor is driven at the speed of the above test. The current and steady state temperature rise in each phase is recorded. The test is repeated for different values of current.

From the results of test (i), the value of $(C_L - C_M)$ is determined. If phase a is fed with d.c., the temperature rise of phases a and b is given by

$$\theta_a = C_L P_{CL} + \theta'$$

$$\theta_b = C_M P_{CM} + \theta'$$

Since P_{CL} is equal to P_{CM}, $(C_L - C_M)$ is easily calculated. In calculating $P_{CL} (= I^2 R)$, R should correspond to hot value pertaining to the final steady temperature as recorded under test.

For each current in test (ii), the temperature rise can be written as

$$\theta = (C_L + 2C_M)P_C + \theta_K' \,.$$

A plot of θ vs. P_C gives the value of (C_L+2C_M). The values of C_L and C_M or C can now be calculated.

APPENDIX 7.1

Sequence voltages across unbalance impedances connected to balanced 4-wire supply voltages

If three impedances Z_1, Z_2 and Z_3 are connected in star and I_1, I_2 and I_3 are line currents due to balanced 4-wire supply to the impedances, the voltage drop across each impedance is given by

$$V_1 = I_1 Z_1 = (I_p + I_n + I_o) Z_1$$
$$V_2 = I_2 Z_2 = (a^2 I_p + a I_n + I_o) Z_2$$
$$V_3 = I_3 Z_3 = (a I_p + a^2 I_n + I_o) Z_3$$

Therefore, sequence components of the voltages acting across unbalanced impedances are obtained as,

$$V_p = \frac{1}{3} (V_1 + aV_2 + a^2 V_3)$$
$$V_n = \frac{1}{3} (V_1 + a^2 V_2 + aV_3)$$
$$V_o = \frac{1}{3} (V_1 + V_2 + V_3)$$

If combination of impedances is defined as follows:

$$\frac{1}{3} (Z_1 + Z_2 + Z_3) = Z_o$$
$$\frac{1}{3} (Z_1 + aZ_2 + a^2 Z_3) = Z_p$$

and $$\frac{1}{3} (Z_1 + a^2 Z_2 + aZ_3) = Z_n$$

then $$V_p = I_o Z_p + I_p Z_o + I_n Z_n$$
$$V_n = I_o Z_n + I_p Z_p + I_n Z_o$$
$$V_o = I_o Z_o + I_p Z_n + I_n Z_p$$

Therefore, unbalance impedances fed with balanced 4-wire supply are equivalent to a source of negative and zero-sequence voltages.