ANALYSIS OF CONVECTIVE HEAT TRANSFER FOR FLUIDS FLOWING THROUGH VERTICAL TUBES

THESIS
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MASTER OF ENGINEERING
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C E R T I F I C A T E.

Certified that the thesis entitled "ANALYSIS OF CONVECTIVE HEAT TRANSFER FOR FLUIDS FLOWING THROUGH VERTICAL TUBES", which is being submitted by Shri ARVIND SURANGE in partial fulfilment for the award of degree of MASTER OF ENGINEERING in Applied Thermodynamics - Refrigeration and Air Conditioning, of University of Roorkee, is a record of student's own work carried out by him under my supervision and guidance. The results embodied in this thesis have not been submitted for the award of any other degree or Diploma.

This is further to certify that he has worked for a period of about 4½ (four and a half)months from Ist May 1963 to 20th September,1963 for preparing thesis for Master of Engineering Degree at this University.

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A C K N O W L E D G E M E N T

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C O N T E N T S

INTRODUCTION

An experimental analysis of convective heat transfer with forced flow of fluids through an electrically heated, long vertical tube is reported here.

The range of flow in the present work varied from laminar to turbulent and the fluids used were water and air. For low flow rates with water the influence of free convection (usually neglected when considering pure forced flow) is considerable and the case becomes that of the combined forced and free convection. The transfer of heat by such a flow through vertical tube (more generally in vertical closed channels) with a flow parallel to the direction of the body force i.e. gravity or centrifugal), is quite common but it is only recently that investigations have been made in determining the characteristics of such a system. Possible uses in nuclear reactor, turbine blade cooling and certain heat exchange apparatus have created new interests in this field of heat transfer.

The apparatus was designed to fulfil the following objectives:

1. To determine the average heat transfer coefficient for water and air flow through the tube at varying flow rates;
2. To study the effect of free convection forces on the forced flow; and
3. To corolate from the test results the dimensionless Nusselt number, Reynolds number and Prandtl number for the type of flow considered.

SURVEY OF LITERATURE

The case of forced flow heat transfer has been theoretically and experimentally investigated by a number of researchers (1 to 6).

THEORETICAL INVESTIGATION

O.Reynolds, L. Prandtl, T Von Karman, L. Graetz, W.Nusselt, M.A. Leveque, A.P. Colburn, E.N. Sieder, G.E. Tate and W.F. Cope are some of the early workers in the field of convective heat transfer. They have contributed considerably to the understanding of forced flow phenomena. Reynold's analogy (1874) was improved by Prandtl and the additional refinements were made by Von Karman (1939), Boelter (1941), Martinelli (1947) and most recently by Deissler (1951 and 1954). The mathematical theory for the type of flows in the present work has been presented by R.C. Martinelli and Boelter, G.A. Ostroumov, S. Ostrach, M.J. Lighthill, W.M. Kays(4), W. Nusselt and others. A remarkable paper reviewing the main developments which have resulted in the formulation of heat transfer across turbulent incompressible boundary layers has been recently presented by J. Kostin & P.D. Richardson (5).

EXPERIMENTAL INVESTIGATIONS

Information about experiments with forced flow has been given by E.R.G. Eckert and A.J. Diaguila (2), J.A.Clark and W.M. Rohsenow, E.N. Sieder and C.E.Tate, T.M.Hallman(3), A.J. Ede(1) and others.

Reference 2 has given a review of available information about the experiments on mixed flow conducted at NACA laboratories and else-where. Hallman (3) has contributed a good deal of information on the combined forced and free convection through vertical tubes in his Ph.D. work at Purdue University. Recent experiments on forced heat transfer conducted by A.J. Ede (1) at National Engineering Laboratory show some agreement with the results of the present work.

References of Desmon and Sam's recent experiments on air flow, Bernardo and Eian's results for water and other liquids flowing through electrically heated tubes have been given by Mc Adams(10).

THEORETICAL BACKGROUND

The fluid motion can be induced by two processes. The fluid may be set in motion as a result of density differences or buoyancy due to temperature variation, the mechanism being called free or natural convection. When the motion is caused by some external agency such as pump or blower, the heat transfer is caused by forced convection. Actually such buoyancy forces are always present in the forced flow heat transfer as well. Usually they are of smaller order of magnitude than the external forces and may be neglected. In certain engineering applications, however, this cannot be done. It was for instance marked quite early that the heat exchange in oil coolers etc. where laminar flow is employed, is affected markedly by free convection currents superposed to forced flow. Free convection cannot be neglected in such cases where low flow velocities are employed. It is sometimes economically necessary to accept low flow velocities in heat exchange apparatus even with a smaller heat transfer coefficient to reduce the pumping power requirements. A forced flow with high velocities is of course very common in engineering applications.

FREE CONVECTION

Nusselt seems to have first applied (1909) the principle of dimensional similarity to the field of heat transfer in his two papers, one devoted to forced convection and the other to free convection in general.

Applying his analysis, a relation for free convection involving three important dimensionless numbers is obtained.

$$Nu = \phi\,(Gr,\ Pr) \qquad \ldots\ldots \qquad (1)$$

This may be put in the form

$Nu = C.\ Gr^{a}.\ Pr^{b}$ where a, b, c are constants to be determined experimentally for a particular geometry of the system.

FORCED CONVECTION

In engineering practice, the Nusselt number for the flow in closed channels is usually evaluated from emperical equations based on experimental results, although in recent years semi-analytic methods of approach have made considerable strides towards an understanding of the basic principles of forced convection in tubes etc. From the dimensional analysis for the case of forced convection, the experimental results obtained can be corelated by an equation of the form.

$$Nu = \phi\,(Re)\ \theta\,(Pr) \qquad \ldots\ldots \qquad - \quad (2)$$

$$\text{or} \quad Nu = K.Re^{m}.Pr^{n} \qquad \ldots\ldots \qquad - \quad (2a)$$

Here the Reynolds number plays a similar influencing role to the Grashof number in free convection. In both the cases Prandtl number, a property of the substance has an additional influence. Constants k, m & n are different for turbulent and laminar range.

FLOW MECHANISM

It is known that, like the velocity boundary layer in a fluid flow due to a body surface, there is a thermal boundary layer comprising a liquid region within a small distance of the solid-liquid interface which is responsible for the transfer of heat from the solid to the fluid.

Within this liquid layer, the temperature changes from a value t_w at the interface to a value t_b in the fluid bulk.

G. Kroujiline (1936) was apparently the first to use the following heat flow equation of the thermal boundary layer for heat transfer calculations:

$$\frac{d}{dx}\int_0^l (t_b - t)\, u\, dy = \alpha\left(\frac{dt}{dy}\right)_w \qquad \ldots\ldots \qquad (3)$$

This equation holds for laminar as well as turbulent flows and has been derived under the assumption of constant transport properties of the fluids* It can be used where the variation of transport properties with temperature is small; however the range of its validity can be extended by introducing of suitably chosen mean values. A fair treatment of boundary layer heat flow equation is given by Eckert (11).

LAMINAR FLOW EQUATIONS

For a fully developed flow in a tube an approximate calculation of heat transfer may be made by assuming the temperature profile to be a cubic parabola in the form.

$$\theta = a_1 y + b_1 y^2 + c_1 y^3 \qquad \ldots\ldots\ldots \qquad (4)$$

Where y is the distance between the tube wall and an element considered in the fluid bulk and a_1, b_1 & c_1 are constants.

Considering the heat transfer coefficient based on the temperature difference between the average temperature of the fluid bulk (t_b) and the wall temperature (t_w) Eckert (11) derives a simple relation viz.

* Transport properties of importance are viscosity and thermal conducdivity.

$$Nu = \frac{hd}{k} = 4.12 \quad \ldots\ldots\ldots \quad (5)$$

Heat transfer from the walls of the tube was also calculated by Gratz (1889, 1895), Callendar (1902), and Nusselt with Navier Stokes equations, continuity equation and energy equation. With this procedure, for a uniform heat flux from the wall (q) and constant fluid properties (μ, k) and a temperature difference (θ_m) between that of average fluid bulk and the wall, a simple relation results viz.

$$Nu = 4.36 \quad \ldots\ldots\ldots \quad (6)$$

Most experiments conducted in this field with certain fluids do not agree well with the results from the above equation due to:

1) the effect of variation of viscosity on the velocity and temperature profiles;
2) eddies caused by the free convection effects at low flow velocities; and
3) absence of fully developed flow.

Theoretically, the Nusselt number is related to (Re, Pr, d/l, μ_b/μ_w) by a complicated expression which may be represented, approximately, by an infinite series. Reference (19) describes for laminar flow of Newtonian fluids an emporical equation viz.

$$Nu = a' \left(Re.\ Pr.\ d/l \right)^{b'} \left(\mu_b/\mu_w \right)^{c'} \quad \ldots\ldots\ldots \quad (7a)$$

An emporical equation suggested by Sieder and Tate has also been widely used for liquids;

$$Nu = 1.86 \left(Re \times Pr \cdot d/l \right)^{0.33} \left(\frac{\mu_b}{\mu_w} \right)^{0.14} \quad \ldots \quad (7b)$$

TURBULENT FLOW EQUATIONS

A theoretical equation of heat transfer for a fully developed turbulent flow as derived by L. Prandtl & G.I. Taylor is

$$St = \frac{Nu}{Re.Pr} = \frac{0.0384\ (Re)^{-\frac{1}{4}}}{1+A(Re)^{-\frac{1}{8}}(Pr-1)} \qquad \ldots\ldots \quad (8)$$

E. Hoffman gives a relation for A in the above equation; $A = 1.5\ Pr^{-1/6}$.

Because of the complicated distribution of temperature in turbulent flow a precise functional relation of the form of equation (2) cannot be obtained. Reference(12) mentions, in a semi-theoretical manner an equation of the form:

$$Nu = K\ Re^{m}.\ Pr^{n} \qquad \ldots\ldots \quad (2a)$$

Besides theoretically founded relations emperical formulas have also been presented. Dittus and Boelter* gave a well known relation for long pipes. viz.

$$Nu = 0.023\ Re^{0.8}.\ Pr^{0.4} \qquad \ldots\ldots \quad (9)$$

for $10,000 < Re < 120,000$, $0.7 < Pr < 120$, $\frac{L}{d} > 60$.

The physical properties have been evaluated at the mean bulk temperature.

For gases Pr has values ranging from 0.665 to 0.90 and the equation (9) takes an approximate form

$$Nu = 0.02\ Re^{0.8} \qquad \ldots\ldots \quad (10)$$

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* Eckert(11) mentions a constant of 0.0243 in equation (9). Some other investigators have suggested a constant of 0.021 and 0.018 for gases and 0.023 and 0.027 for liquids.

Fishenden and Saunders (9) have taken a value of 0.75 for Pr. for gases and given

$$Nu = 0.023\,(Re.Pr)^{0.8}$$

$$= 0.023\,(Pe.)^{0.8} \quad .... \quad (11)$$

Strictly, for similarity, the ratio of pipe diameter to length (d/l) should be included in above equations but the comparison of results for different values shows that the power of (d/l) is only about 0.05 so that for the ratio l/d above above 10 it can, for practical purposes be neglected.

The famous Martinelli equation for uniform heat flux is of the following form:

$$\frac{h}{C_p.G} = \frac{\sqrt{f/2}}{\dfrac{t_w-t_b}{t_w-t_c}\left(Pr + \ln(1+5\,Pr.) + 0.5\,NDR^{-1}\ \ln\dfrac{Re}{60}\sqrt{\dfrac{f}{2}}\right)} \quad ....(12)$$

where f is the friction factor and NDR is the ratio of diffusivities.

Attempt will not be made to compare all of these relations with the results of the present work; however one or two representative relations may be considered.

<u>MIXED FLOW EQUATIONS</u>

In this case, with upward flow of fluids through the tube the Nusselt number obtained is greater than the pure forced convection value of <u>4.36</u> indicating that the upward flow, tends to increase the heat transfer.

Mixed flow region is usually defined as the region in which the heat transfer differs by more than 10% from the one obtained with pure forced flow or free convection relation. McAdam's proposed to calculate the heat transfer coefficient both for forced convection and free convection separately and used the larger value. Measured values obtained for the flow through vertical tubes did not deviate by more than 25% from the values calculated by McAdam's rule.

Dimensional analysis for mixed flow gives a relation:

$$Nu = \phi\,(Re,\ Pr,\ Gr) \qquad \ldots\ldots \qquad (13)$$

Hallman's (3) analysis involves the product of Prandtl and Grashof numbers (Gr x Pr) known as Rayleigh number (Ra) which is a measure of the extent of free convective effects. He proposed a relation for Nusselt number as a function of Rayleigh number, viz.

$$Nu = 1.40\ (Ra)^{0.28} \qquad \ldots\ldots \qquad (14)$$

$$\text{for } 100 < Ra < 10{,}000$$

but remarks that the experimental data of others fall below that obtained from equation (14).

Two more recent experiments by Gross in his Ph.D. work were designed for pure forced convection, but Rayleigh numbers were high enough and showed significant effect of free convection. Gross's data show a large scatter from the value of Nu of 4.36.

Reference (2) mentions the equations distinguishing the free and forced flow regions.

For the limit between free and mixed flow;

$$Re = 7.39\ (Gr)^{0.35} \qquad \ldots\ldots \qquad (15)$$

and between forced and mixed flow;

$$Re = 19.64\ (Gr)^{0.35} \qquad \ldots\ldots \qquad (16)$$

These equations were offered by S. Ostrach.

A.J.Ede(1) suggests;

$$Nu = 4.36\ (1+0.06\ Gr^{0.3}) \qquad \ldots\ldots \qquad (17)$$

for laminar flow of water.

THE PRESENT WORK

The present thesis is aimed at studying the forced convective heat transfer for the fluids (water and air) flowing through an electrically heated, externally insulated vertical tube by evaluating Re, Pr, Nu and Gr for the varying flow rates of the fluids, applying varying heat fluxes. The heat transfer coefficients are based on the solid-liquid interface area and the mean wall to fluid bulk temperature, the transport properties being taken at the mean bulk temperature.

ASSUMPTIONS MADE

1. The fluid motion is steady (laminar, transient, or turbulent).
2. No radial or tangential velocity components exist.
3. Temperature profiles within the fluid are fully developed.
4. Variation in physical properties of the fluids with temperature are very small.
5. No volume heat sources are present in the fluid.
6. The heat flux applied is uniform for one set of readings.
7. The temperature gradient in the fluid bulk along the length of the tube is constant.

EXPERIMENT

OBJECT -

The object of the experiment was to obtain data to make an analysis of the forced convection heat transfer for fluids flowing upward through a vertical tube. The fluids used were water and air.

The apparatus was instrumented to determine:

(1) the flow rate of the fluid;

(2) temperature variation along the wall of the tube;

(3) heat input to the tube; and

(4) inlet and outlet fluid bulk temperatures.

DESCRIPTION OF THE APPARATUS

A schematic flow diagram appears as Fig.1(a) and a photographic view as Fig.1(b). It consisted of an electrically heated thin walled stainless steel test length (tube) and the associated equipment.

The total length of the tube was 9' 1½". The test section length was 7' 9"+. The tube was ½" OD with .0338" wall thickness°. Length to diameter ratio for the test section was 223. This ratio (L/D) to establish the fully developed flow ~~was~~ ~~also~~ ahead of test section was 12. But as the connecting pipe was also of the same diameter as the tube, it was assumed that the flow was already fully developed.

°In fact the total tube length was obtained by connecting two pieces of stainless steel tube by a brass bush, see Fig.2. The length covered by the brass bush was not considered in the test length, because of its negligible resistance as compared to stainless steel.

+ Effective length of the test section.

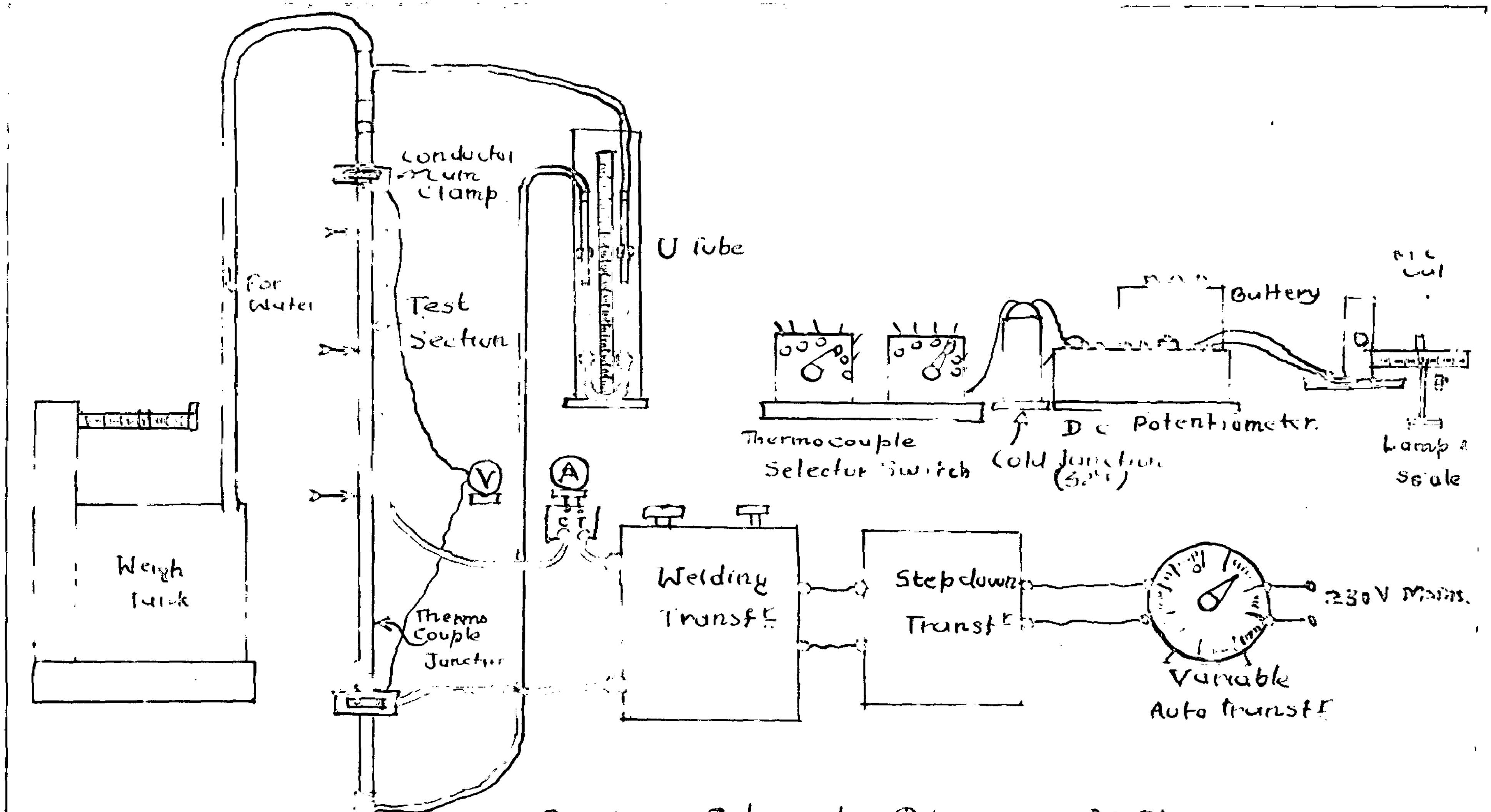


Fig-1 Schematic Diagram Of The Experimental Set-Up

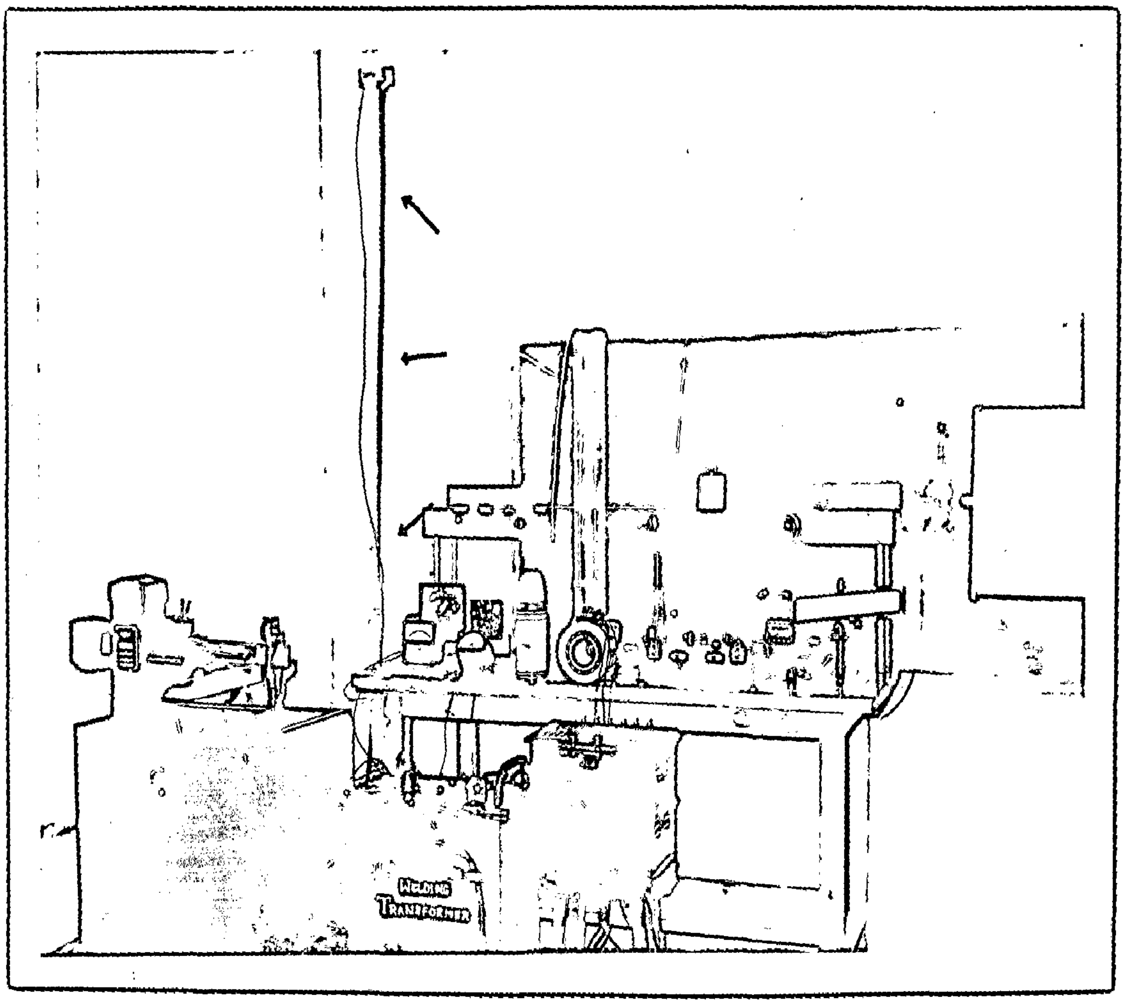


Fig. 1(b) A view of the complete experimental set up

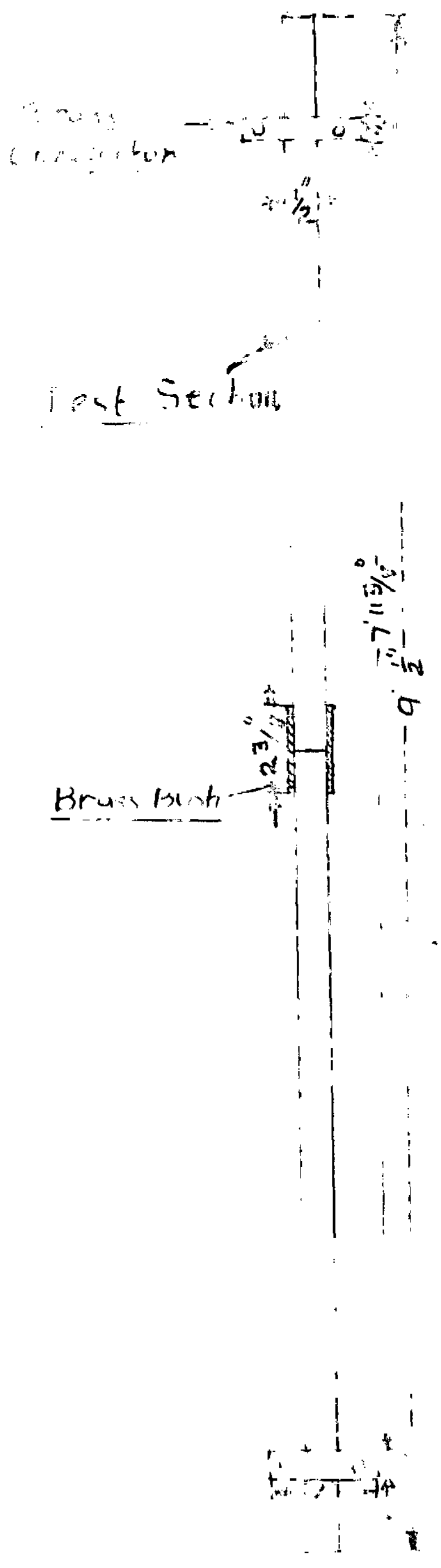


Fig. 3(a) Test Section Details

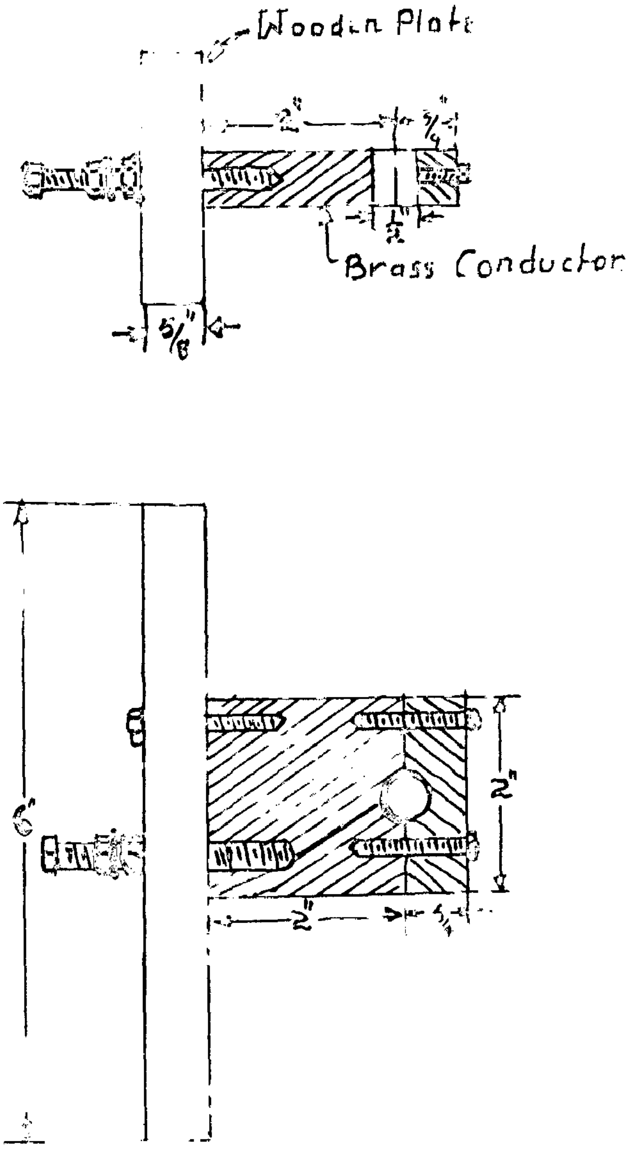


Fig. 4 Brass Conductor Clamp

The test section and the upper part of the tube was well insulated by ½" asbestos rope insulation to reduce the heat loss to the surroundings to a minimum.

WATER AND AIR FLOW SYSTEM

Water supply was made from the tap fitted near the experimental set up. Two valves in series were used to regulate the water flow. Plastic tubing ½" ID was connected from the tap to the tube at inlet and from the tube to the weigh tank at outlet. Flow rate of water was measured by an Avery weigh tank which was accurate enough for practical purposes.

Air supply was made by an air compressor (0-100 psi) at moderate pressures (20 psi) and flow regulated by a valve in the line°. Air was allowed to discharged from the upper end of the tube.

HEAT SUPPLY SYSTEM

The tube was heated by the passage of electrical current along the test length. See figure 3(a) for electrical diagram. A photographic view is given in figure 3(b). Since a single transformer to produce low voltage and stand high current was not available, three transformers of the specifications given below were used in series.

No.(1) Auto transformer: Primary - 230 Volts
Secondary variable - 0 - 270 Volts
Current capacity - 8 Amps.

No.2. Step down transformer: Primary - 230 Volts, 13 Amps.
Secondary - 110, 55, 20, 15, 10, 5 Volts.
Current capacity - 27.5 Amps.

° A U-tube put across the test section (filled with mercury) served as a check in regulating the fluid flow.

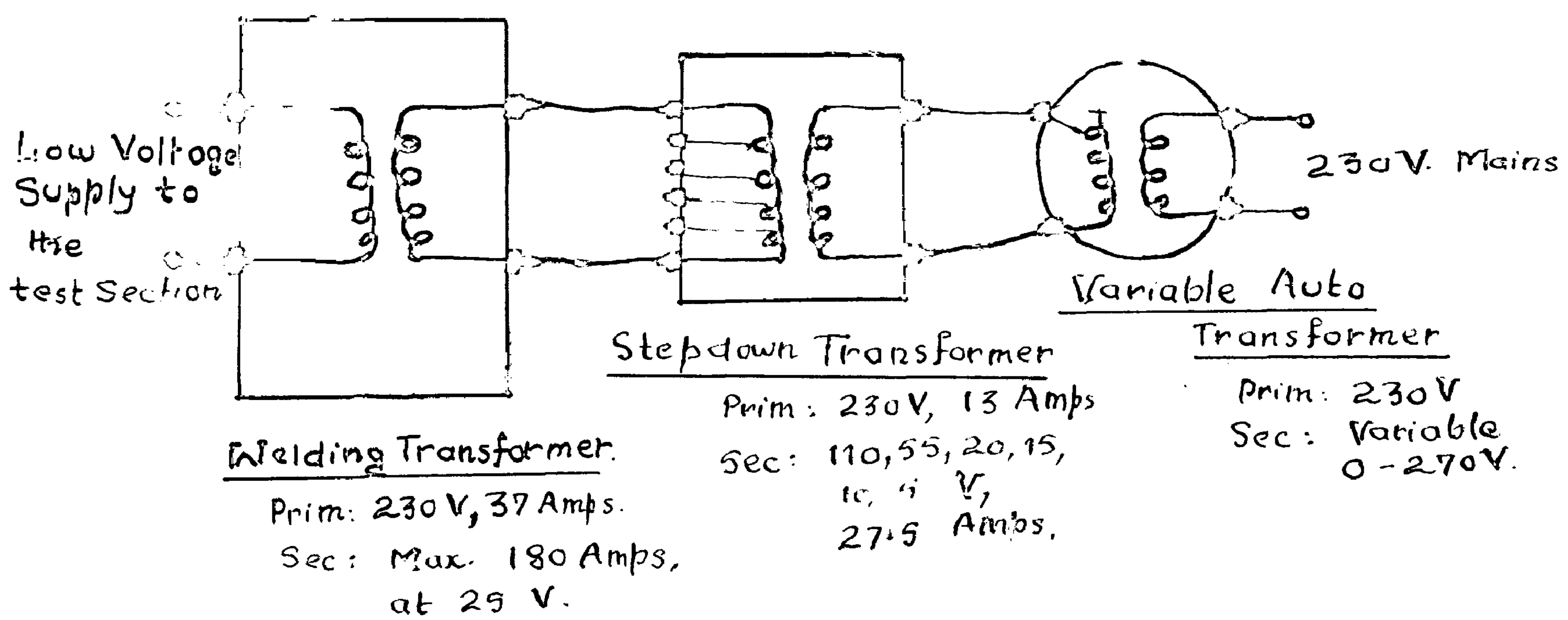


Fig 3 (a) Power Supply System

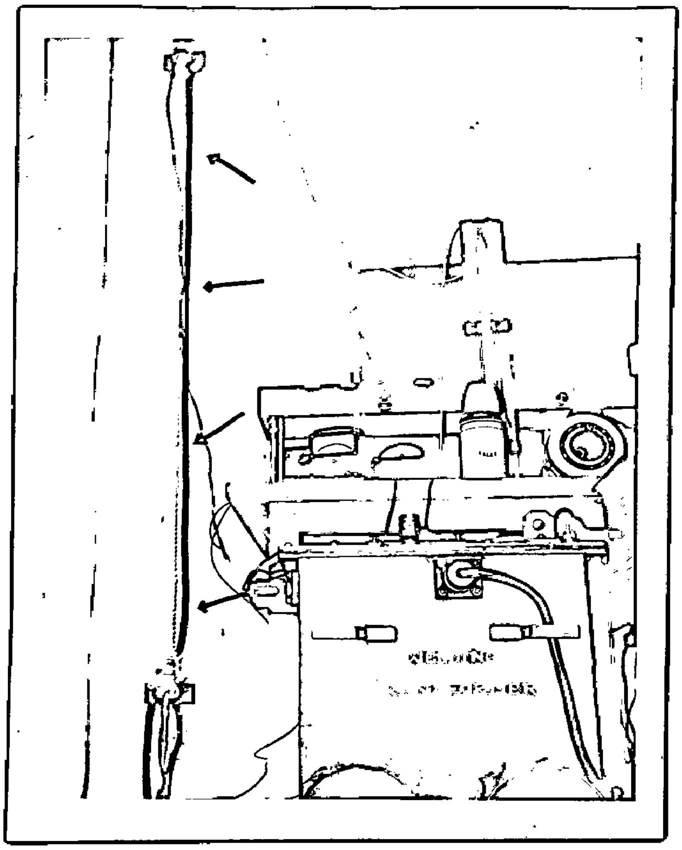


Fig. 2(b) Shows the insulated test section. The arrows indicate the locations of thermocouple junctions.

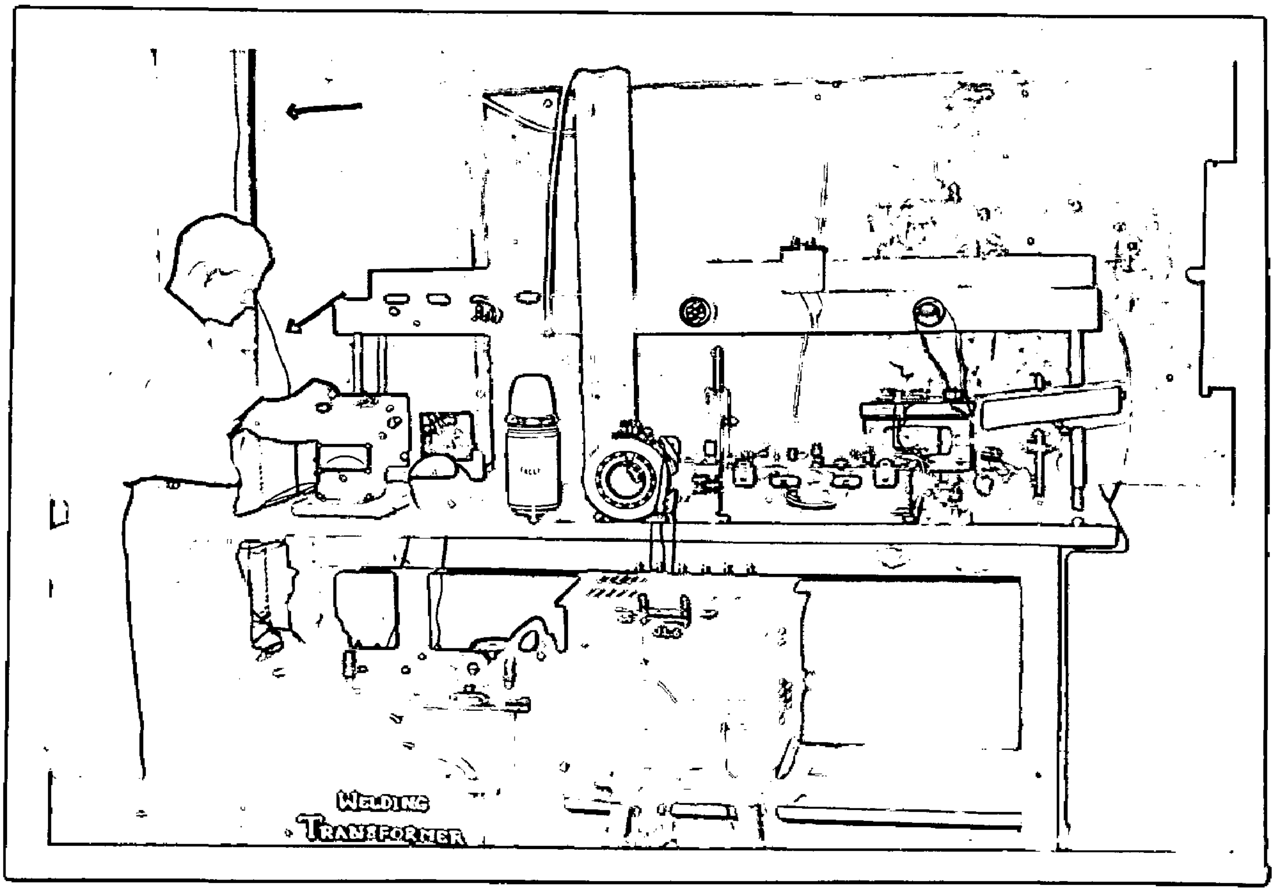


Fig.3(b) A view showing the power supply system and thermo electric circuit.

No.3 Welding transformer: Primary - 230 Volts, 37 Amps.
Secondary - Max. current capacity 1[illegible]0 Amps. at 25 Volts.

This arrangement facilitated to give a voltage upto a maximum of 6 volts on the test section. Higher voltages could not be obtained because of the limited current carrying capacities of the transformers number 1 and 2. Heat fluxes were varied by adjusting voltages to 6 volts, 4.5, 3 and 1.5 volts for runs with water and 4, 3, 2 and 1 volt for runs with air.

To measure the power supply an A.C. voltmeter (0 - 10V) was put across the test section. The test section resistance including the conductors was found to be 0.065 Ohms. This was measured on a double bridge in the electrical measurement lab. It was assumed that the resistance does not vary within the temperature ranges involved. A check was made on the measurement of power supply by introducing an ammeter (0 - 20 Amps.) connected to the secondary of a current transformer, the primary of which was connected in series with the test section. It was observed that the values calculated by voltmeter readings (V^2/R) and those calculated by ammeter's (I^2R) readings were practically the same. ~~Brass~~

Brass conductors were designed to supply the power to the test section as well as to support the tube at the upper and the lower end of the test section. The arrangement has been shown in Fig. 4.

MEASUREMENT OF TEMPERATURE

Copper constantan thermocouples were used to measure the wall temperatures on the test section and inlet and outlet fluid bulk temperatures. All the wires were 30 gauge size.

A conventional thermocouple circuit consisted of

1) D.C. potentiometer (Kaycee)

2) Moving coil galvanometer with lamp and scale arrangement.

3) Standard cell 1.0183V.

4) Accumulator - 6V.

5) Thermocouple selector switch.

6) A thermosflask containing ice, as the cold junction (32°F).

7) Keys etc.

E.m.f.s. of the thermocouples were measured by Null method in which the readings are taken with no current flowing through the circuit and thus they are independent of the lead length and of the resistance in the thermocouple circuit. A calibration under these conditions is dependent only on the compositions of the metals used, the temperature of the junctions, accuracy of the standard cell and the uniformity of the resistance wire.

Four thermocouples connections were made on the test section as shown in Fig.2. The test section was divided in four equal parts of 1' 11½" each and the thermocouple junction was put at the centre of each such part. One thermocouple junction was put on the wall of the tube 3" below from the starting of the test section and one 4" above the end of the test section. For all practical purposes, these thermocouples indicated the temperatures of the inlet and outlet fluid bulk. In case of air flow, however, the thermocouple junction was inserted in the tube to a depth of above 4" from the upper end of the tube to measure the leaving bulk temperature more precisely.

The thermocouples were calibrated for the temperature ranges encountered in the experiments by using a hot water tank as the hot junction. A photograph of such a calibration appratus is shown in Fig.5. It was observed that the values of e.m.f.s. agreed fairly with the values given in NBS circular 561 (17).

EXPERIMENTAL PROCEDURE

The data were taken in the following manner:

1) First the flow rate was established. In case of water the quantity was measured in the weigh tank for 30 minutes period.

2) For a particular flow rate the maximum heat flux was applied to the tube. Redings were taken for all the thermocouples after the steady state was established. Normally the time required for steady state to be established was about 30 minutes. The heat flux was then reduced and the procedure repeated. In this manner the data were obtained with different flow rates and heat fluxes.

In case of water Reynolds number ranged from 270 to 15,100. Higher Reynold numbers were not obtained because of the limitations on the heat flux value. For air the Reynolds numbers ranged from 4,400 to a maximum of 60,000. With air at low Reynolds numbers the corrections were of comparable magnitudes to the true heat transfer and consequently no worthwhile results were obtained. Reynolds numbers higher than 60,000 could not be obtained because of the incapacity of the compressor to give a steady flow at high flow rates.

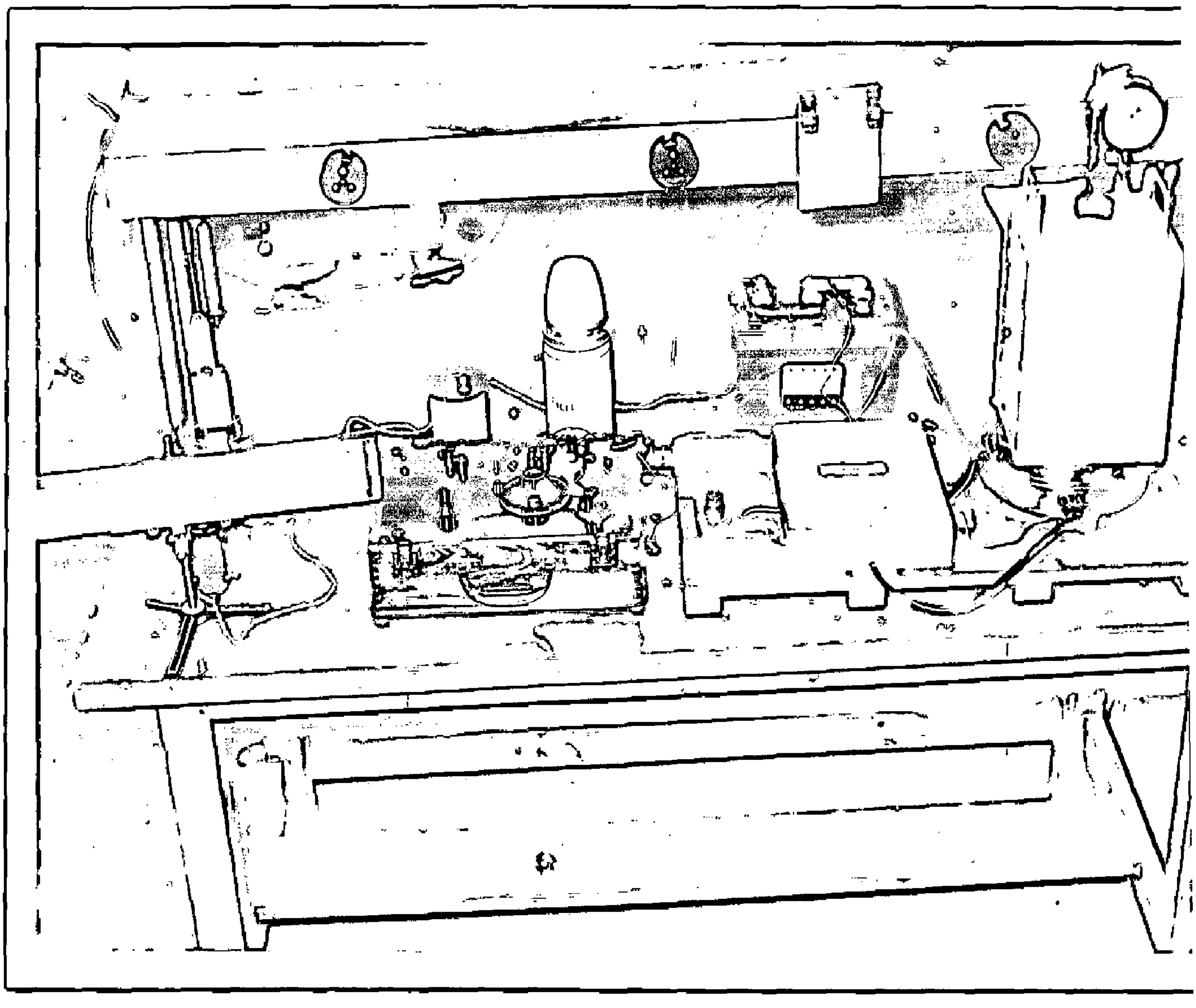


Fig. 5 Shows the apparatus for thermocouple calibration.

DISCUSSION OF RESULTS

The results of the experiment and their discussion will be presented in the following paragraphs. The results have been tabulated in Appendix B and a sample calculation has been given in Appendix C.

The results could be best interpreted by studying the following curves drawn from the test results:

1) A plot between $Nu/Pr^{0.4}$ Vs. Re. both for water and air for the entire flow ranges involved in the experiment;

2) A plot between $\frac{Nu \times 8.75}{Pr \times Gr^{0.1}}$ and Re for laminar flow of water.

3) A plot between Nu Vs. X/d and θ Vs. X/d to show the variation of heat transfer along the pipe length.

Fig. (6) has been plotted to show the variation of $Nu/Pr^{0.4}$ with Re for the entire flow ranges that could be covered during the experiments. To obtain suitable relations in the form:

$Nu = K\ Re^{n}\ Pr^{m}$ for water and air flow, the convention is to plot Nu/Pr^{m} against Re on a log - log paper.

A study of the results of the experiments conducted by other investigators suggested the choice of m as 0.4. Figure 6 reveals that the method of corelation in the form mentioned above is reasonably successful.

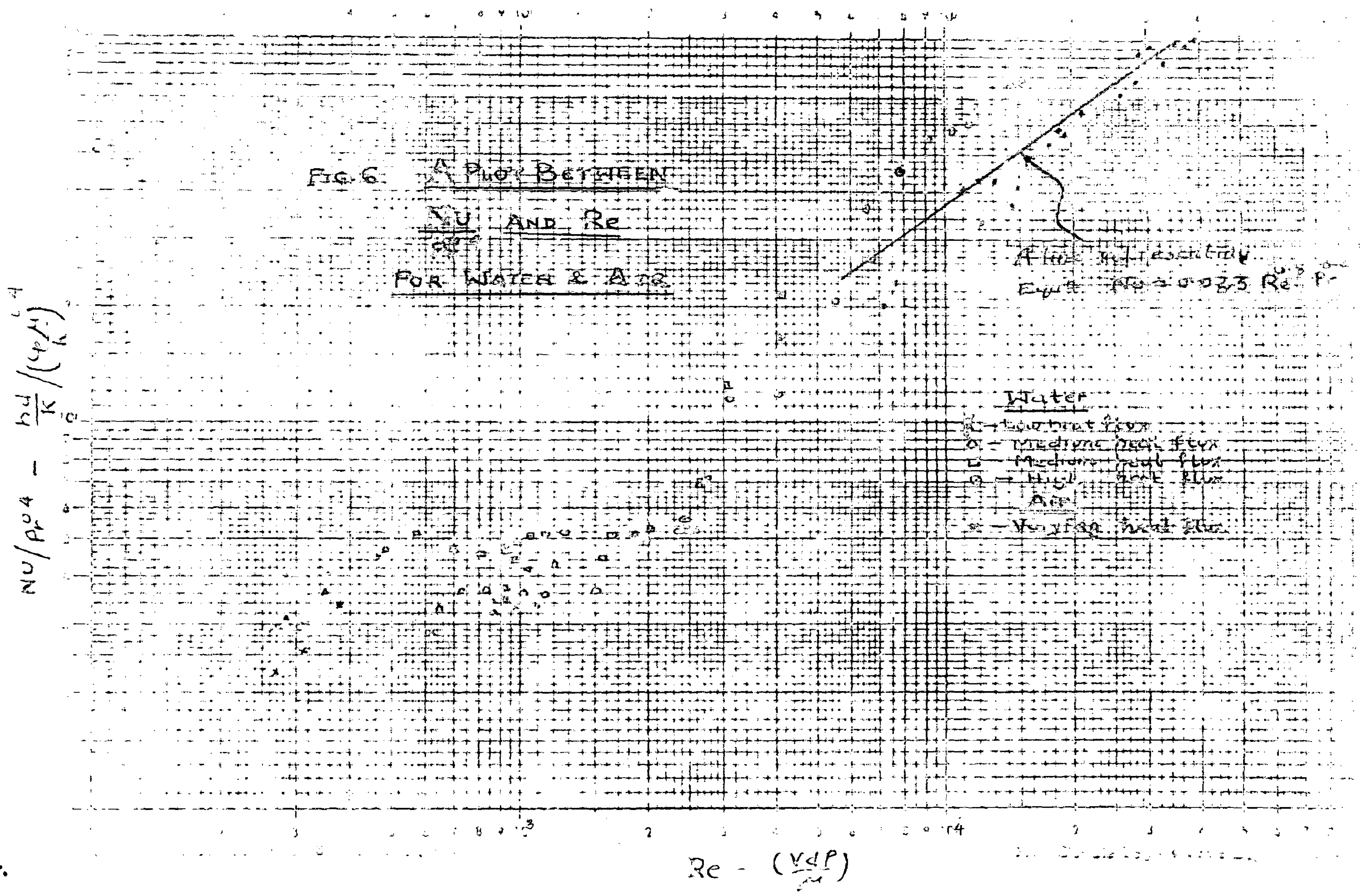


Fig. 6 A Plot Between Nu And Re For Water & Air

In the laminar range for water, a significant scatter can be marked at low values of Re. This was due to the influence of free convection due to the variation in heat fluxes. This influence is discussed later. Data for various heat fluxes have been distinguished in the Fig.

In the transient region (Re = 2300 to 8000) for water, $Nu/Pr^{0.4}$ seems to increase at a much higher rate and some scatter is marked in this region indicating that there is no consistant variation of $Nu/Pr^{0.4}$ with Re.

In the turbulent range, both forr water and air, the scatter in the data is much less, and straight lines could be drawn, averaging the data to obtain the suitable corelations. It can be marked that the data for water fall above those for air indicating that in case of water the effect of free convection may be significant.

Fig. 7 is a plot of $\frac{Nu \times 0.75}{Pr \times Gr^{0.1}}$ Vs Re for laminar range for water. The chief interest to study this region lies in the investigation of influence of free convection effects on forced flow heat transfer. It can be noticed that Nu is always greater than 4.36, the theoretical value for the case of pure forced convection. In these experiments the minimum value of Nu obtained was 5.0 Hallman(3) obtained a minimum value of Nu of 4.62 in his experiments.

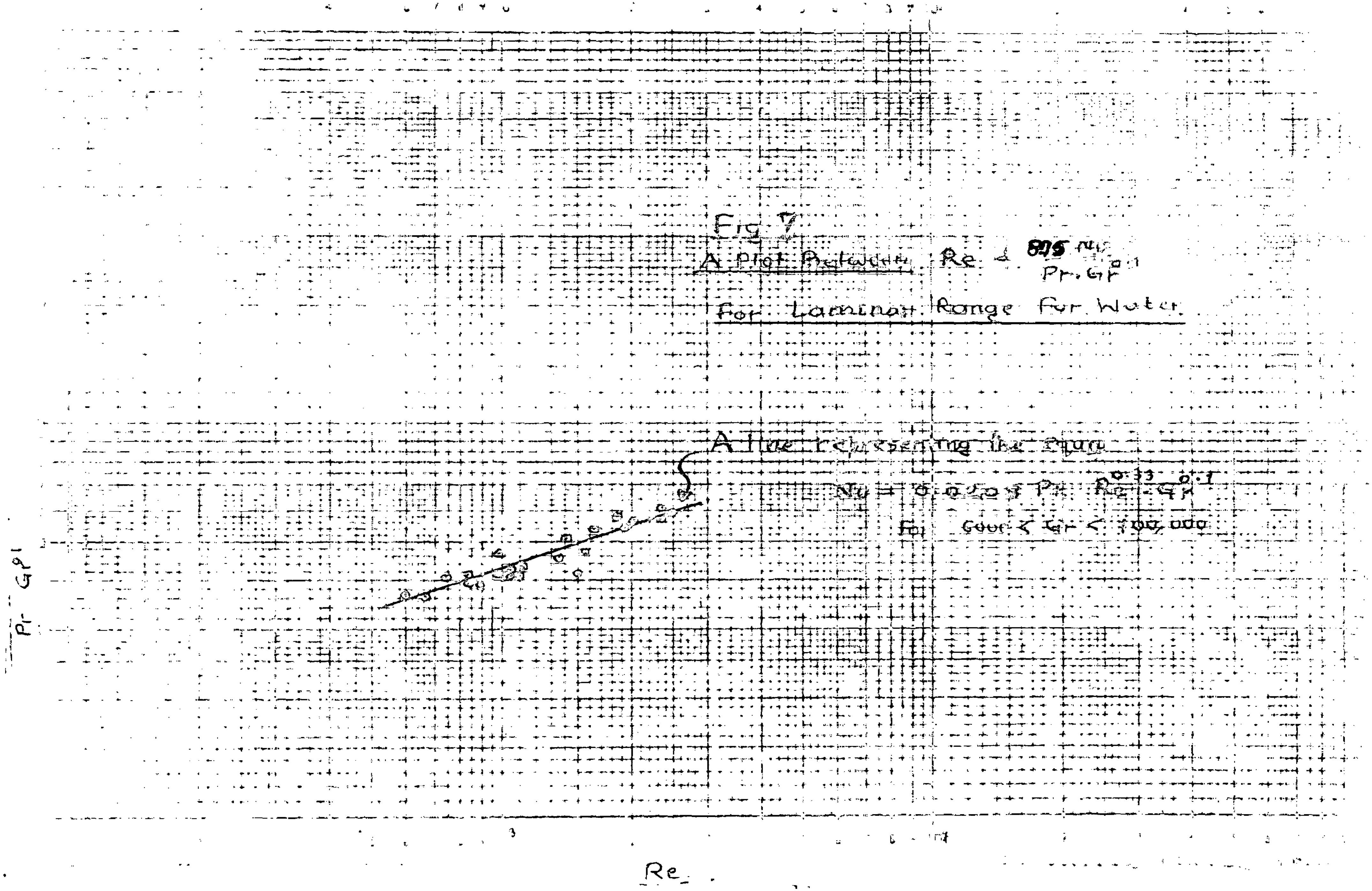


Fig 7
A plot between Re & $\frac{Nu}{Pr.Gr^{0.1}}$
For Laminar Range For Water
A line representing the equn
$Nu = 0.0207\ Pr\ Re^{0.33}\ Gr^{0.1}$
For 6000 < Gr < 700,000
3
Re
Pr Gr

These facts lead us to the conclusion that in practice pure forced convection can hardly be obtained and what is really obtained is the mixed forced and free flow convection. The intensity of free convection effect can be estimated from the values of Gr for the flow.

The data of Eckert and Diaguila (2), Hallman(3) and Edo(1) suggest a corelation between Nu & Gr for the mixed flow. Unfortunately, no consistent variation of Nu with Gr is noticeable in the results of this thesis. This is probably due to the low values of Gr obtained in the present work. On the other hand variation of $Nu/Pr^{0.4}$ with Re seems more reasonable, but still fails to give a direct corelation between $Nu/Pr^{0.4}$ and Re because of the large scatter, due to free convection effects. Lack of variation of Nu with either Re or Gr suggests, that for the type of the flow involved in the present experiments, a better coreation might be expected by considering the influence of both Re and Gr on the heat transfer. Such an expectation is well supported theoretically, but the author has not come across an empirical relation in this form. After long trials, the author has been successful in getting a corelation which involves Nu, Re, Gr and Pr for the mixed flow region.

Fig. 7 is a result of these trials and reveals that the attempt is reasonably successful. The data in figure 7 can be corelated by an equation:

$$Nu = 0.0288\ Pr.\ Re^{0.33}.\ Gr^{0.1} \quad \ldots\ldots\ldots \quad (20)$$

for $6{,}000 < Gr < 100{,}000$

This equation exibits that the influence of Re, for the type of flow involved on the heat transfer, is more pronounce

It can be seen from Fig. 6 that for turbulent range (Re > 8000) for water, the results are closely corelated by the equation:

$$Nu = 0.0246\, Re^{0.84}\, Pr^{0.4} \quad \ldots\ldots \quad (18)$$

but the data obtained with air are not reconciled with those for water.

For air the corelation obtained for turbulent range is:

$$Nu = 0.0242\, Re^{0.79} . Pr^{0.4}$$

$$\text{or} \quad Nu = 0.02095\, Re^{0.79} \quad \ldots\ldots \quad (19)$$

As already mentioned, there a number of formulas which can be compared with these results e.g. the familiar equation:

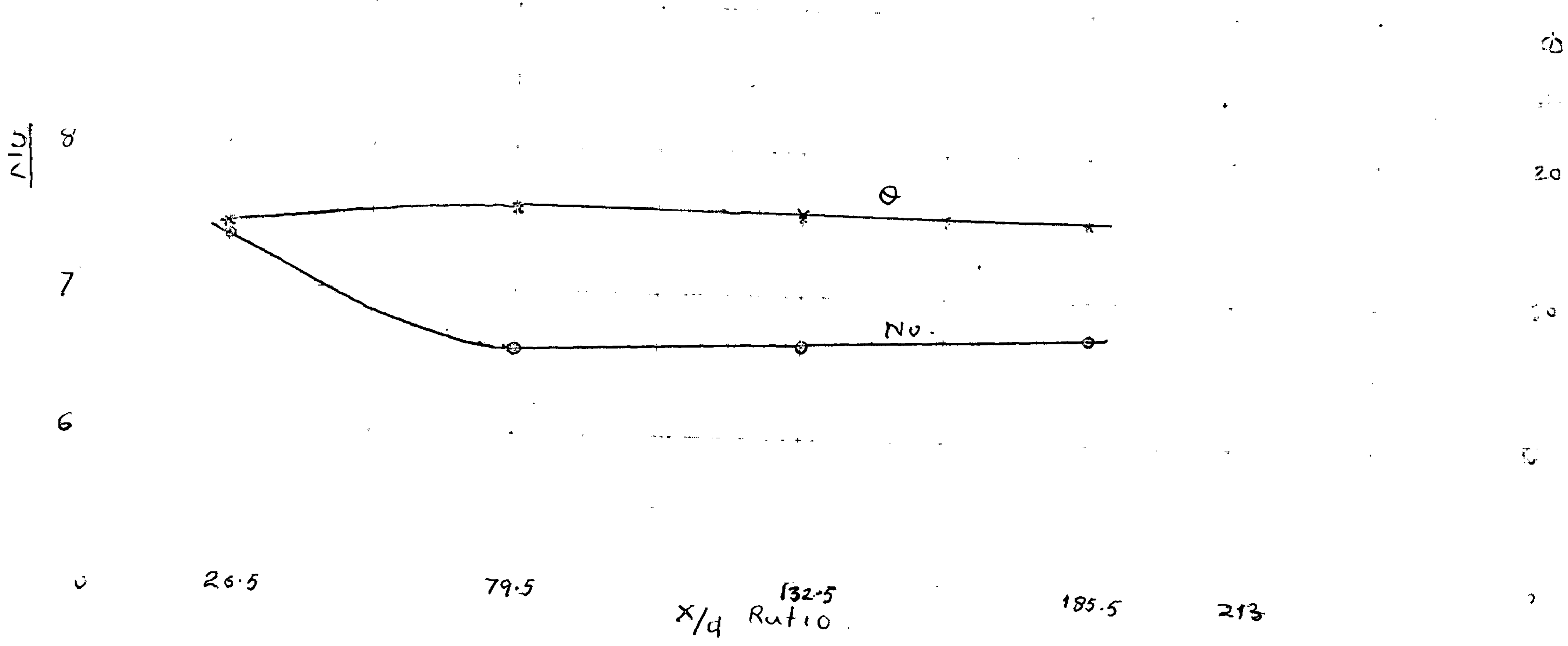
$$Nu = 0.023\ Re^{0.8} .\ Pr^{0.4} \quad \ldots\ldots \quad (9)$$

inwhich the physical properties have been taken at the mean bulk temperature. A line corresponding to this equation is drawn in Fig. 6. This equation serves as a comprise between the data for water and air but does not represent either precisely. The slope of this line is somewhat different but the discrepancy is not serious. It should be mentioned that this equation was recommended for highly turbulent range only.

The higher values of $\frac{Nu}{Pr^{0.4}}$ for water may be due to the effect of free convection prevalent even in the turbulent range involved in the present experiment. This statement could be supported by the fact that even at the highest flow rate (Re = 15,100) the magnitude of the Gr. was of the order of 4000 for water.

Fig. 8 Variation of Nu & θ along the tube length.

[Re - 1600]
Water



In case of air flow the magnitude of Gr are small and the effect of free convection is negligible; the data for air thus fall below the line for equation (9).

Fig. 8 represents the variation of Nu ~~along~~ and θ along the length of the tube for a particular run using water. A little higher value of Nu at the beginning of the session may be due to the entrance effects in the tube. For the rest of the length a very small increase in Nu is noticed. This may be explained by the fact that there is a decrease in the viscosity of water as the temperature along the length of the tube increases, which in turn increases Re and hence the heat transfer coefficient. A corresponding water to wall temperature difference (θ) curve is also shown in the same Fig. For lower heat fluxes, of course, this variation becomes insignificant.

In case of air also the variation of Nu and θ shows the same trend as in case of water inspite of an increase in viscosity of air with the temperature. This interesting fact can however be justified by noticing the faster increase in the value of conductivity of gases as compared to the viscosity as temperature increases.

CONCLUSION

The data have been obtained for the upward flow of water and air through an electrically heated, externally insulated vertical tube.

The heat transfer coefficient for water in the laminar range, varied from 56 to 100. In the turbulent range it varied from 800 to 1500. For air, these coefficients ranged from about 5 to 60.

In the laminar range for water the data is corelated by an equation

$$Nu = 0.0238 \; Pr. \; Re^{0.33} . \; Gr^{0.1} \qquad .... \quad (20)$$

For turbulent range of water the equation obtained is:

$$Nu = 0.0246 \, Re^{0.84} . \; Pr^{0.4} \qquad .... \quad (18)$$

For turbulent range of air the equation obtained is:

$$Nu = 0.02095 \, Re^{0.79} \qquad .... \quad (19)$$

The results show a significant effect of free convection, particularly in laminar range of water. Equations 18 and 19, compare favourably with the well known equation 9.

APPENDIX - A.

NOMENCLATURE

l - Length of the test section - ft.

d - Inside dia. of the tube - ft.

D - Outside dia. of the tube - ft.

a - Cross section area of the tube - ft.^{2} ~~square~~

A - Surface area of the test section - ft.^{2}

h - Heat transfer coefficient BTU/hr-deg.F. - ft.^{2}

k - Thermal conductivity of the fluid BTU/hr-deg.F - ft.

C_p - Specific heat of the fluid ~~lb_m/ft.hr~~ BTU/lb_m-deg.F.

μ - Dynamic viscosity of the fluid lb_m/ft.hr.

ν - Kinematic viscovity of the fluid ft.^{2}/hr.

V - Average fluid velocity ft./sec.

ρ - Mass density of the fluid lb_m/ft.^{3}

β - Coefficient of volumetric expansion 1/deg.F.

α - Thermal diffusiévity = $K/\rho . c_p$

g - Acceleration due to gravity ft./sec.^{2}

t - Any temperature deg.F.

t_b - Temperature of the fluid bulk deg.F.

t_{b_1} and t_{b_o} - Inlet and outlet fluid bulk temperature

t_w - Temperature of inside wall surface

t_{w_o} - Temperature of outside wall surface

W - Weight flow rate lb/hr.

x - Axial distance along the tube - ft.

q - Wall heat flux - BTU/hr. - ft.^{2}

q_{th} - Thermal energy rise of the fluid passing through the test section - BTU/hr. or min.

V - Voltage across the test section.

I - Current passing through the test section - Amps.

R - D.C. Resistance of the test section.

q_e - Net electrical energy input - BTU/hr.

q_1 - Heat loss to the surroundings - BTU/hr.

θ - Fluid to wall temperature difference

θ_m - Mean fluid to wall temperature difference
[Temp. of outer & inner tube surface is practically same]

Re - Reynolds number $= \frac{Vd\rho}{\mu}$

Pr - Prandtl number $= \frac{C_p \mu}{k}$

Nu - Nusselt number $= \frac{hd}{k}$

Gr - Grashof number $= \frac{g\beta\rho^2 d^3 \theta}{\mu^2}$

Ra - Rayleigh number $=$ (Gr x Pr.)

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15.		30.	

APPENDIX - B.

RESULTS FOR WATER

Set No.	Flow rate lb/min.	q	θ_m	Re	Pr	h	Nu	Gr	$\frac{Nu}{Pr^{0.4}}$	$\frac{8.75\ NU}{Pr.Gr^{0.1}}$
1.	0.221	514.0	9.0	283.0	4.58	57.0	5.80	54,500	3.17	
		140.5	3.0	270.0	5.15	43.5	4.42	10,000	2.28	
2.	0.275	520.0	8.0	349.0	4.61	65.0	6.49	44,500	3.62	
		131.0	2.7	313.0	5.27	48.5	4.92	9,650	2.53	
3.	0.34	1169.0	14.0	480.0	4.07	83.4	8.35	126,600	4.75	
		130.0	2.0	375.0	5.36	65.0	6.71	7,460	3.42	
4.	0.42	1180.0	12.5	564.0	4.36	98.5	9.42	99,500	5.2	
		517.0	8.20	498.0	5.0	63.0	6.45	40,000	3.39	
		130.6	1.5	464.0	5.43	86.8	8.98	5,500	4.55	
5.	0.5625	1165.0	13.75	700.0	4.73	86.2	8.8	75,500	4.75	0.52
		519.0	8.25	641.0	5.26	60.5	6.29	34,900	3.3	0.367
		131.2	2.5	608.0	5.65	56.0	6.70	7,700	2.85	0.37
6.	0.654	1990.0	24.0	882.0	4.73	82.7	8.4	180,000	4.68	0.505
		1165.0	13.25	800.0	5.25	86.8	8.75	63,000	4.6	0.515
		515.0	7.75	735.0	5.65	68.5	7.23	29,800	3.64	0.404

Set No.	Flow rate lb/min.	q	θ_m	Re	Pr	h	Nu	Gr	$\frac{Nu}{Pr^{0.4}}$	$\frac{8.75\ Nu}{Pr \cdot Gr^{0.1}}$
7.	0.7625	2000.0	21.25	981.0	4.45	88.5	8.99	140,000	4.93	0.54
		1165.0	13.0	910.0	4.91	88.8	8.45	62,200	4.75	0.534
		503.0	7.5	844.0	5.4	68.8	7.13	27,000	3.62	0.414
8.	0.82	2010.0	22.0	1048.0	4.6	90.0	9.3	131,000	5.05	0.545
		1166.0	15.5	930.0	5.07	77.6	8.47	75,000	4.42	0.475
		503.0	7.5	905.0	5.45	66.3	6.85	27,600	3.47	0.394
		131.5	2.1	868.0	5.71	62.6	6.52	6,800	3.42	0.415
9.	0.925	2020.0	21.0	1155.0	4.71	95.2	9.7	117,000	5.20	0.563
		1175.0	15.0	1058.0	5.2	59.3	8.05	60,600	4.16	0.43
		507.0	7.75	1015.0	5.5	68.5	7.13	26,400	3.6	0.408
		131.5	2.1	980.0	5.75	62.5	6.42	6,680	3.17	0.405
10.	1.05	2030.0	21.5	1285.0	4.81	94.5	9.65	111,000	5.15	0.552
		1174.0	14.5	1200.0	5.2	81.0	8.35	61,000	4.30	0.454
		520.0	7.25	1144.0	5.52	68.5	7.12	26,000	3.59	0.41
		131.5	2.0	1105.0	5.76	65.7	6.85	6,200	3.39	0.433
11.	1.3875	2000.0	21.0	1640.0	5.04	95.4	9.75	98,500	5.10	0.54
		1176.0	14.0	1560.0	5.32	82.4	8.70	52,200	4.45	0.474
		514.00	7.0	1496.0	5.58	71.0	7.35	24,400	3.70	0.39

Set No.	Flow rate lb/min.	q	θ_m	Re	Pr	h	Nu	Gr	$\frac{Nu}{Pr^{0.4}}$	$\frac{8.75\ Nu}{Pr \cdot Gr^{0.1}}$
12.	1.737	2020.0	20.0	2000.0	5.2	100.0	10.30	87,000	5.3	0.56
		1160.0	12.25	1920.0	5.4	94.1	9.75	45,600	4.97	0.542
		514.0	5.0	1860.0	5.63	99.5	10.28	20,400	5.18	0.6
13.	2.115	2025.0	12.0	2430.0	5.27	105.0	10.75	77,000	5.6	0.585
		1177.00	11.5	2410.0	5.45	101.5	10.40	41,100	5.25	0.575
		510.0	4.8	2340.0	5.69	105.6	10.52	17,800	5.27	0.61
14.	2.40	2025.0	15.30	2700.0	5.31	133.7	13.85		7.12	
		1190.0	8.25	2650.0	5.5	149.0	14.40		7.30	
		510.0	4.75	2560.0	5.69	120.0	12.50		6.22	
15.	2.80	2030.0	0.25	3110.0	5.31	215.0	22.50		11.52	
		1165.0	5.0	3030.0	5.56	231.0	24.9		12.70	
		500.0	1.5	2980.0	5.69	333.0	34.9		17.10	
16.	3.675	2030.0	9.05	4080.0	5.37	224.0	23.2		11.8	
		1165.0	3.25	4020.0	5.49	355.0	36.8		16.25	
17.	4.95	2030.0	5.50	5500.0	5.46	365.0	37.8		20.8	
18.	5.80	2030.0	5.80	6380.0	5.58	351.0	36.0		18.2	
19.	6.17	2030.0	3.40	6700.0	5.50	536.0	55.5		27.9	
20.	7.25	2031.0	2.25	7800.0	5.59	893.0	92.5			

Set No.	Flow rate lb/min.	q	θ_m	Re	Pr	h	Nu	Gr	$\frac{Nu}{Pr^{0.4}}$	$\frac{8.75\ NU}{Pr.Gr^{0.1}}$
21.	8.90	2030.0	1.90	9590.0	5.60	1058	108.5		55.0	
22.	9.70	2035.0	1.825	10,400	5.60	1110	115.0		57.5	
23.	11.0	2040.0	1.75	11,650	5.72	1160	120.0		60.0	
24.	13.1	2040.0	1.40	14,220	5.72	1450	151.0		75.2	
25.	14.25	2041.0	1.35	15,100	5.73	1506	153.0	4,200	76.5	

RESULT FOR AIR

Set No.	Flux, q BTU/hr.ft²	θ_m	Re	Pr	h	Nu	$\frac{Nu}{Pr^{0.4}}$
1.	51.6	6.0	7,600	0.7 Constant for the temp. range involved.	8.6	19.4	23.0
2x	44.5	7.0	4,100		6.44	14.6	19.9
	49.0	6.2	7,200		7.93	17.35	20.0
2.	210.0	17.9	13,200		11.78	35.5	41.0
	48.3	3.3	11,000		5.15	36.7	42.5
3.	52.0	4.3	12,300		12.1	28.3	32.8
	211.0	15.3	14,500		13.7	31.6	33.5
4.	53.1	3.5	15,000		15.2	36.0	41.5
5.	492.0	22.0	18.900		22.3	49.7	57.4
	226.0	11.1	17,800		20.4	46.5	53.6
	53.6	2.5	19,250		21.5	49.8	57.5
6.	493.0	18.0	26,000		27.4	62.2	72.0
	208.5	8.9	21,300		24.5	56.2	65.0
7.	502.0	14.1	29,000		35.6	80.6	93.0
	225.0	7.5	28,700		29.7	68.5	78.8

Set No.	Flux, q_2 BTU/hr.ft^2	θ_m	Re	Pr	h	Nu	$\frac{Nu}{Pr^{0.4}}$
8.	685.0	20.35	33,300		33.8	73.0	87.6
	503.0	13.4	35,600	0.7	37.9	87.6	99.8
	225.0	6.25	30,500		35.7	83.3	96.4
9.	690.0	18.4	37,700		37.4	83.7	96.7
	510.0	11.6	42,000		44.0	100.0	115.7
	224.0	5.9	34,700		37.9	88.0	101.5
10.	712.0	14.0	46,500		49.0	115.0	127.3
11.	710.0	12.7	56,200		56.0	127.8	141.5
	510.0	9.4	54,000		54.2	124.5	144.0
12.	517.0	8.5	61,500		60.8	140.0	161.0
	231.0	4.05	55,300		57.0	139.4	160.8

APPENDIX - C.

A SAMPLE CALCULATION.

The test section length = 7.75 ft.(effective)

Outside dia. of the tube = 0.0416 ft.

Inside dia. of the tube = 0.03645 ft.

Inside surface area of the tube = 0.888 sq.ft.

Cross sectional area of the tube = 0.1503 sq.inches.

1. CALCULATIONS FOR WATER (Set No.10)

Flow rate = 1.05 lb/min.

Average velocity of the flow $= \frac{1.05}{62.1} \times \frac{1}{60} \times \frac{144}{0.1503}$

$= 1.05 \times 0.2575 = 0.27$ ft./sec.

(i) Average data(run No.1)

Test section voltage = 5.9 V.

$t_{b_1} = 82.8^oF$; $t_{b_2} = 111.5^oF$

Average bulk temperature $= t_{b_m} = 97.15$

Thermal rise $q_{th} = 1.05 \times 1 \times (111.5 - 82.8)$

$= 1.05 \times 28.7 = 30.1$ BTU/Min.

Heat flux $= \frac{q = q_{th} \times 60}{A} = \frac{30.1 \times 60}{0.888}$

$= 30.1 \times 67.5 = 2030$ BTU/hr. ft^2

$Re = \frac{Vd}{\nu} = \frac{0.27 \times 0.03645}{0.765} \times 10^5 = 1285$

$Pr = \frac{C_p \mu}{k} = 1 \times \frac{1.715}{0.3565} = 4.81$

$h = \frac{q}{\theta_m} = \frac{2030}{21.5} = 94.5$ BTU/hr - ft^2. deg.F.

$Nu = \frac{hd}{k} = \frac{94.5 \times 0.3645}{0.3565} = 9.65$

$$Gr = \left(\frac{\beta g \rho^2}{\mu^2}\right) d^3\theta = 107 \times 10^3 \times 21.5 \times (0.03645)^3$$

$$= 107 \times 21.5 \times 48.4 = 111{,}000$$

(ii) Local Data

(a) $h = \dfrac{\text{heat flux}}{\theta_{local}} = \dfrac{2030}{21.25} = 95.5$ $(x/d = 26.5)$

$$Nu = \frac{95.5 \times 0.03645}{.3515} = 9.90$$

(b)

$x/d = 79.5$

$$h = \frac{2030}{24} = 84.7$$

$$Nu = \frac{84.7 \times 0.03645}{0.354} = 8.71$$

(c)

$x/d = 132.5$

$$h = \frac{2030}{22.3} = 91.0$$

$$Nu = \frac{91.0 \times 0.03645}{0.358} = 9.27$$

(d)

$x/d = 185.5$

$$h = \frac{2030}{21} = 96.8$$

$$Nu = \frac{93.8 \times 0.03645}{0.361} = 9.77$$

(Average Re = 1285)

Note:- The transport properties K and μ have been evaluated at the mean bulk temperature. The values have been taken from the curves drawn for these properties from the tables given in Ref.(12) and (16).

CALCULATIONS FOR AIR

$$Re = \frac{Vd\rho}{\mu} = \left(\frac{Vd\rho\, n}{\mu \cdot d}\right) = \frac{4md}{\pi 60 \mu d^2} = \frac{4}{60\pi \mu d}\ \frac{n}{1} = 0.582\ \frac{m}{\mu}$$

$$\text{Now } m = \frac{q_{th}}{C_p(t_{bo}-t_{b1})}$$

where q is the net heat energy gained by the fluid.

For set No.5 - Run No.1

Test section voltage - 3V.

$t_{b1} = 87.5°$ $t_{b2} = 157.50$ °F

A bulk temperature = 122°F

Net heat energy input /min.

$$= \frac{3.413}{60}(V^2/R) \text{ — heat loss through the insulation}^{*}$$

$$= \frac{9}{0.065} \times \frac{3.413}{60} - 0.56$$

$$= 7.87 - 0.56 = 7.31 \text{ BTU/min.}$$

$$\therefore m = \frac{7.31}{0.24(157.5-86.5)} = 0.43 \text{ lb/min.}$$

$$Re = 0.582 \times \frac{0.43}{1.32} \times 10^5 = 18,900$$

Pr = 0.7 Almost constant for the temperature range involved.

$$q = \frac{q_{th}}{A} = \frac{7.31 \times 60}{0.888} = 7.31 \times 67.5 = 492 \text{ BTU/hr. ft}^2.$$

$$h = \frac{q}{\theta_{lm}} = \frac{492}{22} = 22.3$$

$$Nu = \frac{hd}{k} = \frac{22.3 \times 0.03645}{0.01638} = 49.7$$

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*Notes:- The heat losses through the insulation have been approximately found by comparing the electrical energy input and the thermal rise for data obtained for water.

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