ANALOG MODEL STUDIES OF GROUND WATER FOR DAHA AREA USING RESISTOR CAPACITOR NETWORK

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By
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CERTIFICATE

Certified that the dissertation entitled "ANALOG MODEL STUDIES OF GROUNDWATER FOR DAHA AREA USING RESISTOR - CAPACITOR NETWORK" which is being submitted by Shri R. C. Desai in partial fulfillment of the requirements for the award of the Degree of Master of Engineering in Hydrology of the University of Roorkee, Roorkee is a record of the candidates own work carried out by him under my supervision and guidance. To the best of my knowledge the matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is to further certify that Sri R. C. Desai has worked for January 1977 to November 1977 at Gujarat Engineering Research Institute, Baroda and from December 12, 1977 to March 31, 1978 at the University in the preparation of this dissertation under our guidance.

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SYNOPSIS

Ground Water has come to be developed at a fast rate. Over exploitation of this valuable resource can cause severe complications in the near future. Systematic studies are therefore necessary for proper evaluation of the resource and assessment of possible effects of alternative schemes of development in conjunction with surface water. Such studies are greatly facilitated by ground water modelling by resistor capacitor network analog.

Dehm area presents a case for such studies. This area located between two great canal systems has seen rapid increase in tubewells. The seasonal pumpages vary from 4500 ha.m. to 10650 ha.m. for against rainfall recharge ranging from 3000 ha.m. to 5400 ha.m. The lowering of water tables is less than proportional and large quantities of water seem to flow into the area from the adjoining irrigated areas.

An R.C.network analog model has been designed set up and operated for study of Dehm area aquifers. The results are encouraging and show that quite accurate results can be obtained by introducing some improvements.

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CHAPTER - 1

INTRODUCTION

A phenomenal increase in demand for water has been created during the recent years due to several reasons. The increase in population as well as per capita consumption of food have made it necessary to grow more food. High-yielding varieties of improved seeds are developed, but their success depends largely on availability of water for irrigation. Industrial development and municipal growth have also added to the demand for water.

The stupendous efforts made by the central and state governments in developing surface water resources have been supplemented with development of ground water by government as well as private sector. There are statutory bodies like the Central Board of Irrigation & Power, the Central Water Commission, Central Ground Water Board and the design organisations of the state governments who regulate the development of surface and ground water. As there is no suitable legislation to control activities in ground water, any individual or a corporate body with sufficient finances, borrowed or otherwise, can drill a private tubewell on its land, and start pumping out water. This has resulted in a large number of private tubewells in addition to the increasing number of state tubewells. The actual utilisation of ground water for irrigation to the end of the fourth Five Year Plan is only about 105 million m^3, as against the estimated ultimate potential of 275 million m^3 (Verma, 1977). This development is not related to the potential in each area and this has already started showing effects of over development in certain areas.

Ground-water is a resource which can be a boon to the economy of a region only if it is developed in a region proper manner. Uncontrolled development can create more problems than it solves. Moreover it is being realised that an integrated development of ground-water with surface water is much more desirable than one sided development of either resource, so that the two resources can interact and the integrated development will help boost the economy of the region.

An essential prerequisite for any serious attempt at evolving an optimal utilization plan is that the system be analysed in as great details as possible and responses of the system to alternative schemes of management be predicted with a fair degree of accuracy (Walton, 1970). The approach may be to study the surface water and the ground water systems separately and then study their interaction, or to study the behaviour of both the systems simultaneously. Study of the surface water system itself is out of scope here and attention is concentrated on ground-water system.

Evaluation of a ground-water system and its behaviour involves several complications. Some of these arise out of the properties of aquifer such as heterogenity, anisotropy, irregular and complex boundary conditions etc. Others may be present due to spatial and temporal variations of inputs and outputs. The pattern of recharge due to rainfall in the outcrop areas and due to irrigation varies from time to time and place to place. The contributions of streams is a variable along the length of river with respect to time and also as regards direction. An influent river may become an effluent river are a short distance away or after some time. The horizontal inflows and outflows from adjoining aquifers also poses similar variability.

The mathematical methods available for study of response of groundwater to a certain steady or non-steady input output pattern are applicable for the ideal aquifers or to aquifers which can be approximated as idealised aquifers. The aquifer is assumed to be homogeneous, isotropic and of infinite extent. If there are recharge or barrier boundaries, then the image well theory is used to be able to apply the formulae derived for infinite aquifers (Walton, 1970). Although the mathematical models give reliable results in cases where they are appropriate, they cannot be resorted to where known aquifer heterogenities exist or recognised gross departures from ideal conditions exist (Prickett, 1975). In Indian conditions, where there is a multitude of tubewells located in no systematic pattern and operated in an unregulated manner the mathematical analysis can render at the best only rough indications of the aquifer behaviour Particularly, when dealing with problems of regional scale, it becomes imperative to resort to other methods of analysis.

As alternatives to the mathematical approach, one can resort to digital computer studies, sand models or to analog models. The digital models start with the fundamental differential equations of flow of ground-water and solve them by numerical methods of finite difference or finite elements (Remson, et al, 1977). Digital models can yield very useful results and have their own advantages as well as disadvantages. A high-speed computer of adequate core storage and/or peripherals has to be available within a reasonable distance. Even a minor change also requires a re-run and one must go to the computer centre. The model gives exactly that information which has been asked for, no less, but at the same time, no more, unlike other modelling techniques which can reveal a behaviour not anticipated by the analyst (Prickett, 1979). The major consideration is that the digital model bears no physical resemblance to the problem under study. This deprives the analyst of the physical association with the system. One would therefore turn to the direct method of problem solving in the form of model studies.

The physical models can be either true models or analog models. Sand tank models constitute the former. These models are 'true' in the sense that same laws governing the flow of water apply to both the model and the aquifer. They are scaled down representations of the aquifers (Thomas, 1979). They are suitable for local problems but not for the regional flow problems involving aquifers of large horizontal areal extent, due to restrictions imposed by scale factors. For practical size model, the vertical depth would be small, thus giving rise to a very small time scale which is very difficult to instrumentate (Prickett, 1975).

Analog models, in ~~constant~~ contrast to the true models, use an altogether different physical system for representation of the aquifer. Many types are possible, falling into main categories such as viscous fluid models, stretched membrane model, thermal model, electrical model etc. All these types of modelling techniques take advantage of the fact that the fundamental partial differential equation of ground-water flow is the diffusion equation which also applies to the physical system used for the model, i.e. the systems are mutually analogous. Of all the systems, the electrical system is more popular because of the ease with which the electrical quantities viz voltage and current can be measured without affecting the behaviour of the models. Unlike in the other systems, in the electrical systems it is possible to ensure that the measuring probes do not vitiate the flow conditions around them. (Karplus , 1958).

The electric analog models can be further sub-divided as continuous system models and discrete system models. The conducting paper analogs and electrolyte tank models are examples of continuous system electrical analog. They are found to give good results but are applicable to steady state conditions only, i.e. for solving problems where Laplace equation governs the flow, and are very useful for plotting flow nets. (Vitkovitch, 1966). For time-variant inputs their use is not possible. The discrete system models can be the resistor network or resistance-capacitance network. The former is suitable for the steady-state conditions whereas the latter are used for study of unsteady state flow conditions.

The resistance-capacitance network analog is a versatile tool for simulation of ground-water flow. The aquifer variables are represented by analogous physical quantities and pieces of equipment. Thus, the pulse generator is equivalent to a large-scale pump, whereas the oscilloscope or the pen-recorder serves the same function as that of a water-level recorder. These and many other analogies which can be drawn, enable one to gain a closer understanding of the system under study. This is a very important advantage of the r.c. network analog. Moreover it is advantageous when aquifer conditions require large number of nodes for the solution. R.C.network can also be extended to three-dimensional case. It is a continuous-time simulator which is advantageous particularly when non-linear boundaries are included, as in the case of Daha aquifer for which the analog model under report is prepared.

Daha area aquifer is located in the doab between rivers Krishná and Hindon in parts of Meerut and Muzaffarnagar Districts of Uttar Pradesh (Fig. 1). The area is almost circular in shape with river Krishná flowing along north-west, west, and southwest boundaries, and Hindon

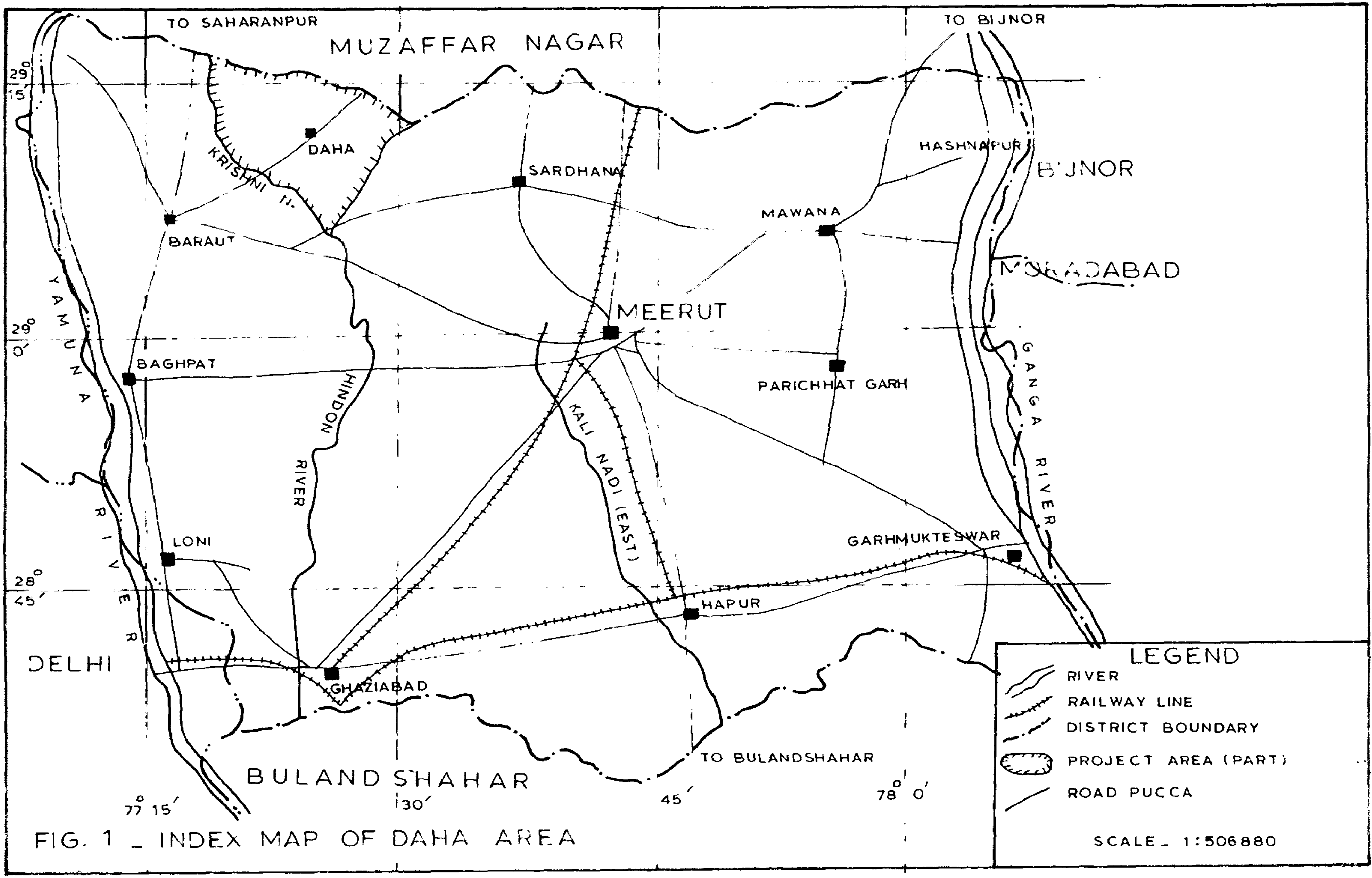


FIG. 1 _ INDEX MAP OF DAHA AREA

along north-east, east and southeast boundaries. To the south there is a confluence of the two rivers. Out of a total length of nearly 91 km, as many as about 80 km of the boundary is formed by the rivers, only about 11 km being an arbitrary boundary to the north. This characteristic shape of the aquifer is such that the method of image wells cannot be applied even for a rough approximation, network not withstanding the multitude of wells existing in the area. The inputs as well as outputs are variable with respect to time as well as space. The land is very fertile. Irrigation facilities are not available although two great canal systems exist just across the rivers. The green revolution has brought in its wake an increasing awareness on the parts of the farmers about the need of irrigation by tubewell water. The development of ground water by private tube-wells is proceeding at a very fast rate. There were 1180 private tubewells in 1972. The number had ~~a~~ increased to 1663 in 1975. Nevertheless, there has been a decline not only in the total number of pumping hours but also in the discharges of many state tubewells. The latter could be due, atleast partially, to the trend of lowering of water table.

The situation has not yet taken an alarming form, but can all the same ~~determinate~~ deteriorate considerably in not-too-distant future if the development goes on taking place unscientifically. Here is therefore an interesting case to study on a r.c.network analog model, with a view to understand the geo-hydrology of the aquifer for ultimate application to study the management alternatives.

CHAPTER - 2

MODELLING TECHNIQUE

2.0 INTRODUCTORY

The resistor-capacitor network analog modelling is a very versatile technique, which can be adopted for a wide variety of problems. It is based on the analogy of flow of water in the aquifer and the flow of electricity in a network consisting of resistors and capacitors. The various aspects of the technique are discussed in this chapter. Their application to the actual problem on hand will be discussed in a subsequent chapter.

2.1 THEORY

Water Balance, Lumped Model :-

The theory of ground-water flow should be discussed prior to the theory of the modelling technique.

The theory of groundwater flow, is based essentially on the water balance equation which can be written as under -

$$R_r + R_c + R_i + I_G + S_i$$

$$= S_o + O_G + E_t + T_p + \Delta_s \qquad (1)$$

Where

R_r = recharge due to rainfall

R_c = recharge due to canal seepage

R_i = recharge from irrigation water

I_G = groundwater inflow into the basin

S_i = influent seepage from streams

S_o = effluent seepage to the streams

O_g = ground water outflow from the basin

E_t = evapotranspiration losses from ground water

T_p = pumpage from ground water

Δ_s = change in storage in the aquifer.

The equation is for a given time interval, Δt, The various terms are all in flow rate ~~in~~ units. They can be classified in three groups.

(1) Terms relating to horizontal movement of water, governed by Darcy's law. (Q_h) expressed as

$$Q = K A \frac{\Delta h}{\Delta L}$$

Where

K = permeability

A = cross-sectional area perpendicular to the direction of flow

ΔL = length of flow

Δh = head loss over the length of flow

Q = discharge rate

$= I_g \mp O_g + S_i - S_o$

(2) Terms relating to vertical movement of water viz.

$$Q_v = R_r + R_c + R_i - E_t - T_p$$

(3) Terms pertaining to release of water from the pores, or storage of water into the pores, i.e. Δs.

<u>Water balance-distributed model:-</u>

The water balance can be studied for the entire basin as a lumped model or at a given point in the aquifer as a distributed parameter model. Equation (1) expresses the water balance as a lumped model. Whereas a proper assessment of the water balance, using equation (1), is an essential prerequisite for any meaningful hydrological investigation, it need not necessarily give a unique solution to the problem on hand. Not all of the terms can be estimated accurately. Errors in one term will cause an equivalent error in another term. Even if the solution is error-free, it would not give the picture of the pattern of ground water movement within the aquifer.

Distributed parameter modelling provides a solution to the above problem. The basic concept of water balance is applied to the conditions existing at just one point in the continuous system that the aquifer is. The flow of water in the horizontal direction, which is governed by Darcy's law can be expressed in its final form as

$$Q_h = T \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right) \qquad ..(2)$$

Where

Q_h = horizontal flow of water

$= I_g + S_i - O_g - S_o$

T = transmissibility = K. B

(where B = thickness of aquifer)

h = head

x, y = coordinates

The vertical flow of water is lumped into one term Q (expressed as depth of water per unit time) and is given by

$$Q = R_r + R_o + R_i - E_t - T_p \qquad ..(3)$$

The change in storage Δs is given, by definition, by the equation

$$\Delta_{\square} = S \frac{\partial h}{\partial t} \qquad ..(4)$$

Now, the water balance equation for a point can be written as

$$T\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right) = S \frac{\partial h}{\partial t} + Q \qquad ..(5)$$

Equation (4) is the fundamental partial differential equation of ground water flow. It can also be derived by combining the continuity equation and Darcy's law. The brackets on the left hand side would have one, two or three terms, depending upon whether the flow is one, two or three dimensional one.

Analogy with electrical system

The flow of electrical charge in a conducting medium is governed by two fundamental laws. The first is the continuity equation, and is based on the principle of conservation. It is also expressed in the form of Kirchoff's laws. The second law is the Ohm's law expressed as

$$I = \sigma A_{c\square} \frac{\Delta V}{\Delta L_{\square}} \qquad (6)$$

Where

I = electrical current

σ = electrical conductivity

$A_{c_{\square}}$ = cross-sectional area of the material perpendicular to the direction of current flow

ΔV = voltage drop across the material

$\Delta L_{\square}$ = length of flow

These two laws and the definition of capacitance together lead to the fundamental partial differential equation governing the flow of electricity as

$$\frac{1}{R}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) = C\,\frac{\partial v}{\partial t} + I \qquad ..(7)$$

Where

$$R = \frac{\Delta L_m}{\sigma A_{c_m}}$$

= resistance of the material

C = capacitance

The bracket on left hand side can have one or three terms if the flow happens to be one or three dimensional.

Equation (5) and (8) are comparable, term by term, and are therefore analggous. In fact, Darcy's law and Ohm's law are also analogous. Equation (5) and (7) extend the analogy to the general case and establish the analogy of the two systems vis. ground water flow and the electrical flow. If proper scale n factors are selected, then the results obtained in one system can be applied to the other. The electrical system is a very convenient one to work with. It can be made compact, fast and inexpensive. It can be easily altered at will. Measurement and recording of voltage and current at different points in the system are easy, reliable and accurate. It is therefore quite advantageous to work with the ~~with~~ electrical system.

Equation (5) and (7) can be directly applied if one is dealing with continuous system. They can not, however, be applied directly to the resistor-capacitor network analog model because the model is a space-discretised system representing the finite difference approximations. One has therefore to work with the finite difference approximations of Equations (5) and (7). There are two approaches available for deriving the finite difference approximations. One is the mathematical approach whereas the other is the physical one. Both lead to identical results. (Karplus, 1958). The mathematical derivation makes use of Taylor series expansions in which the terms of second and higher order derivatives are dropped and form the truncation error. The error can therefore be quantified and analysed. Inthe physical approach the error term cannot be quantified. Here the mathematical approach will be used for deriving the finite difference approximations for equation 5 for being able to analyse the error term later on. The physical approach will be illustrated while deriving finite difference approximations for equation (7).

The first step in the both the approaches is to superimpose a coordinate grid on the field. The grid may be one, two or three dimensional, depending upon the nature of the problem. It may have uniform spacing or even non-uniform spacing. Although derivations are generally made for uniform spacing grid, these results can hm also be applied to non-uniform grids. After superimposing a grid, attention is concentrated on one particular node point and those near it. For example, consider the uniform spacing square grid of Fig.2 superimposed on a two dimensional field, andone particular node of the grid as shown therein. The head at the central

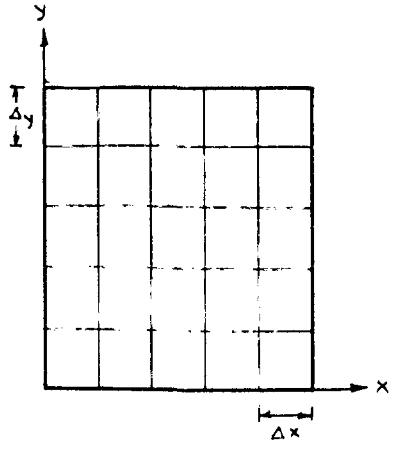


FIG 2 _ FINITE DIFFERENCE GRID AND TYPICAL NODE

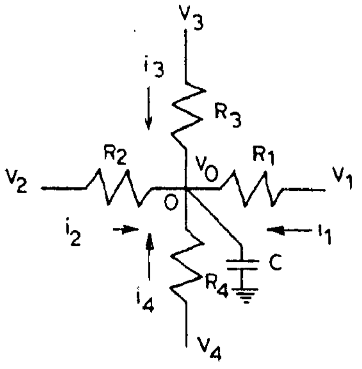


FIG. 3 _R-C NETWORK

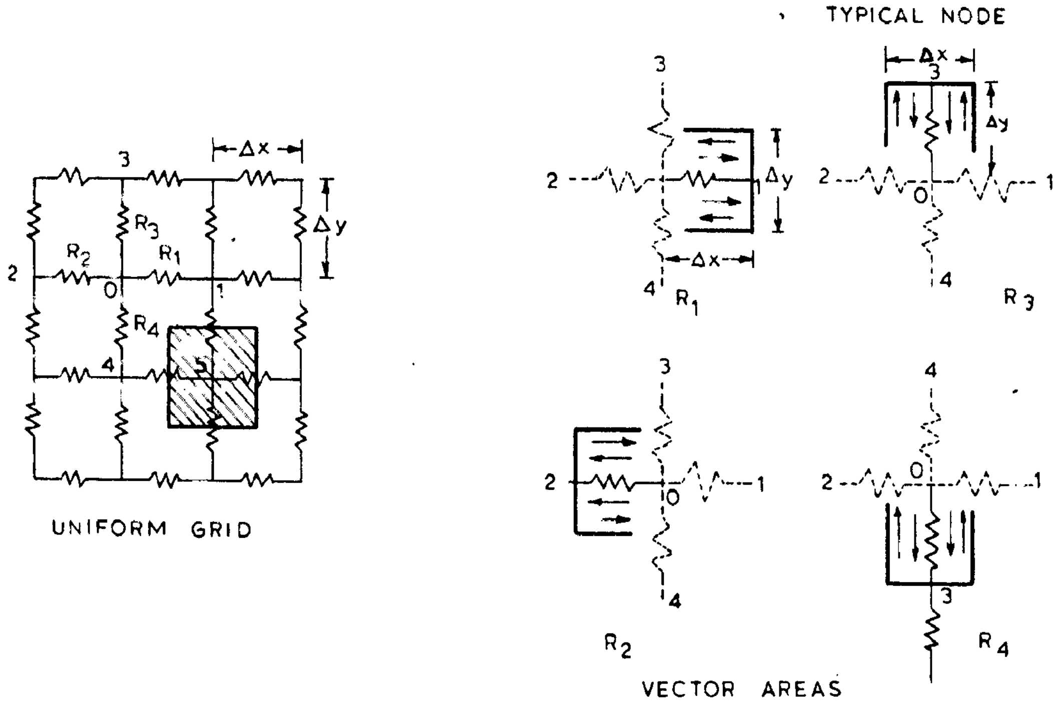


FIG 4 _ VECTOR AREAS REPRESENTED BY EACH RESISTOR OF A UNIFORM GRID. SHADED AREA SHOWS SPECIFIC PORTION OF NODE 5

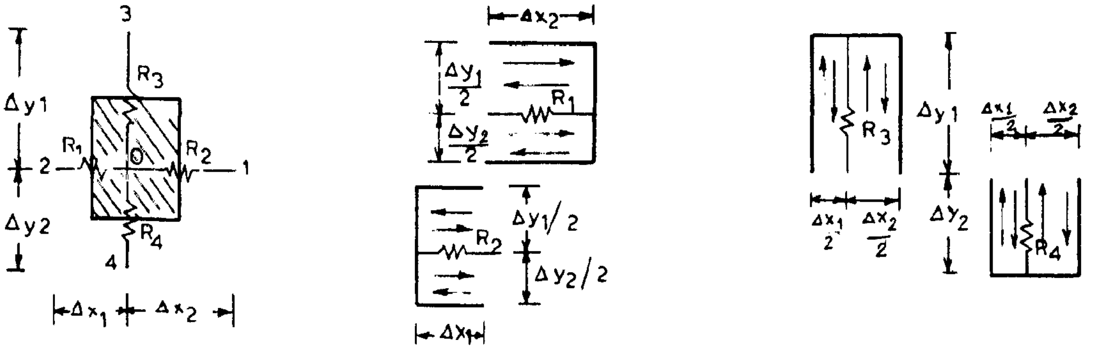


FIG. 5 _ VECTOR AREAS AND SPECIFIC PORTIONS FOR NONUNIFORM GRID. SHADED AREA SHOWS SPECIFIC PORTION OF NODE O node point can be evaluated in terms of the heads of the points, 1,2,3 and 4. From Taylor series expansion, one gets,

$$\frac{\partial^2\phi}{\partial x^2} = \frac{1}{\Delta x^2}\left[\phi_1 + \phi_2 - 2\phi_o - \frac{\Delta x^4}{12}\left(\frac{\partial^4\phi}{\partial x^4}\right)_o - \frac{2\Delta x^6}{6!}\left(\frac{\partial^6\phi}{\partial x^6}\right)\right] \qquad \ldots(8)$$

The higher order terms of the RHS are lumped into a single term E defined as

$$E = -\frac{1}{\Delta x^2}\left[\frac{\Delta x^4}{12}\left(\frac{\partial^4\phi}{\partial x^2}\right) + \frac{2\Delta x^6}{6!}\left(\frac{\partial^6\phi}{\partial x^6}\right) \ldots\ldots\right] \qquad \ldots(9)$$

Equation (8) is therefore written as

$$\frac{\partial^2\phi}{\partial x^2} = \frac{1}{\Delta x^2}\left[\phi_1 + \phi_2 - 2\phi_o\right] + E \qquad \ldots(10)$$

Similarly for the y coordinate one can write

$$\frac{\partial^2\phi}{\partial y^2} = \frac{1}{\Delta y^2}\left[\phi_3 + \phi_5 - 2\phi_o\right] + E \qquad \ldots(11)$$

Substituting equation (10) and (11) in equation (5), putting $\Delta x = \Delta y$, and dropping the error terms,

$$T\left[\phi_1 + \phi_2 + \phi_3 + \phi_4 - 4\phi_o\right] = S\Delta x^2 \frac{\partial h}{\partial t} + Q \qquad \ldots(12)$$

Equation (11) is the finite difference approximation of equation (4). It is significant to note that the RHS contains the derivative term without any approximation, except that the term Δx^2 is introduced to account for the limited area represented by the nodal point. The pair of terms $S\Delta x^2$ is together known as storage factor.

In case of the RC network analog it is not necessary to discretise the time variable.

If the nature of the potential function is such that the derivatives higher than the third order are zero, then the E term will become zero and equation (7) gives exact results. (Karplus 1958). In ground water problems, there is no way of ascertaining whether this condition is fulfilled or not. It is safer to presume that the condition does not exist and accept the error terms with proper evaluation. The error term can be made negligible by taking sufficiently small values of Δx and Δy.

The physical derivative of the finite difference approximation of equation (1) leads to the same equation (7), with the exception that the error term E is not quantified. The physical approach is illustrated with reference to the electrical system.

Consider a typical node of an RC network as shown in Fig.3. The voltages at nodes 0,1,2,3 and 4 are V_o, V_1, V_2, V_3 and V_4, and the resistors in the four limbs are R_1, R_2, R_3 and R_4. Kirchoff's law states that the algebraic sum of all currents flowing into a node is zero. The current i_1, between nodes 0 and 1 is given by Ohm's law as

$$i_1 = \frac{V_1 - V_o}{R_1} \qquad \ldots(13)$$

Currents for other limbs can be expressed by similar terms. Combining all such terms, one gets

$$\frac{V_1-V_o}{R_1} + \frac{V_2-V_o}{R_2} + \frac{V_3-V_o}{R_3} + \frac{V_4-V_o}{R_4} = C\,\frac{\partial V}{\partial t} + I \qquad \ldots(14)$$

The RHS of equation (14) is based on definition of capacitance and is not discretised because in the electrical system, the capacitor acts on continuous time basis only

If $R_1 = R_2 + R_3 = R_4 = R$ then one gets

$$\frac{1}{R}\left[V_1 + V_2 + V_3 + V_4 - 4\,V_0\right] = \cancel{k}C\,\frac{\partial V}{\partial t} + I \qquad (15)$$

Comparing equation (12) and equation (15) one finds that they contain equal number of terms of the same order. The two equations are therefore analogous and so are the systems governed by them. The following basic analogies are obvious.

Ground water flow	Current flow
$1/T$	R
$S \quad x^2$	C
ϕ (or h)	V
Q	I

Thus the resistance is analogous to the reciprocal of the transmissibility, capacitance to storage factor(product of storage coefficient and cell area) and voltage to head. Current is analogous to rate of flow.

Two assumptions were made in the above derivations. The first is $\Delta x = \Delta y$. Second is $R_1=R_2=R_3=R_4=R$. These two assumptions are made to keep the forms of equation (12) and equation (15) simple enough so that the analogy can be made conspicuous. Nevertheless the analogy is not invalidated if the assumptions are not adhered to. A more rigorous derivation also leads to analogous forms of Eq.12 and 15, but the expression for the error term becomes somewhat complicated. Since in the final forms the error term is neglected, no useful purpose would be served by attempting a rigorous derivation for unequal mesh spacings and resistors.

Strictly speaking the above analogy can not be applied to unconfined aquifers for which equation (4) does not hold good. The nonlinearity due to varying transmissibility with the variable thickness of saturated aquifer introduces complications. Nevertheless, the analogy can be applied to model unconfined aquifer provided the drawdowns are small enough compared to the saturated thickness of aquifer (Herbert, 1970).

The advantage of the analogy ~~as~~ brought out above is taken in the following manner. A rectangular network of uniform or nonuniform spacing as required is superimposed on a map of the aquifer. The values of 1/T for areas between pairs of adjacent node points of the grid are worked out with the help of the vector area concept to be discussed in a subsequent section. Using suitable scale factors, the resistances required to represent the 1/T values are worked out. These are connected between the node points. Similarly the storage factors ($S\Delta x^2$ in case of uniform grid, but $S\Delta x\ \Delta y$ in case of nonuniform grid) are worked out for each node point. The capacitances required to simulate these storage factors are calculated using appropriate scale factors and are connected to each node point. The other end of the capacitor is connected to the system ground. Boundary conditions are imposed on the model in accordance with the principles to be outlined later. The model then is a scaled down discretised version of the aquifer and bears a good resemblance to the prototype. The excitations in the forms of recharge and/or draft are applied to the model. The response is observed in the form of temporal variation of voltage at node points corresprototype observed data.

Model studies are done in two phases. In the first phase historical conditions of recharge and withdrawal are imposed on the model. Its response observed and compared to observed field data. If the comparison is not good the model is adjusted by intelligent trial and error till the responses tally reasonably with the observed water levels. This is the calibration (or proving) phase ~~phone~~. In second prediction phase~~s~~, alternative schemes of future development are simulated to examine their effects on the aquifer response.

There is another way of describing the model which relates to the numerical method of relaxation for solving the Laplace equation. Equation (11) and (14) would reduce to Laplace equations if the right hand terms were zero. In that case the model can be considered as a device ~~derived~~ for carrying out relaxation (Vine, 1960). There would, however, be two special advantages. Firstly, it is not necessary to make initial guesses ~~quaries~~ of head values at the numerous node points. Secondly, the error residue reduces to zero instantaneously. In case when right hand side of equation (11) and (14) are not zero, then also, the above comparison is applicable. In the numerical method one would solve the Laplace equation for one instant of time and repeat the exercise at certain time intervals. (Herbert, 1968 b). In RC model the equation goes on getting solved continuously for all instants of time.

2.2. SCALE FACTORS

The analogy between ground water flow and flow of electricity would be of practical use only if quantitative correlationships are established between the corresponding pairs of analogous variables. This is done with the help of suitable scale factors. There are following scale factors.

1. Basic scale factors
   - (a) Voltage scale
   - (b) Resistance scale
   - (c) Capacitance scale
2. Derived scale factors
   - (a) current scale (ampere scale)
   - (b) time scale
   - (c) quantity scale

The above scale factorsare used to convert measurements or observations of one system to corresponding parameters in the other and are described below (Rushtom and Bannistor 1970)

1(a) Voltage scale (S_v)

This is defined as

$$S_v = \frac{V}{h} \qquad ..(16)$$

It correlates the voltages of electrical system to heads in the aquifer system. It is chosen from the considerations of range of head variations in the aquifer and the voltage that can be applied to the model so as to be compatible with the equipment.

1(b) Resistance scale (S_r)

This scale is defined as

$$S_r = \frac{R}{1/T} \qquad (17)$$

This is used to convert reciprocal of transmissibility to the resistances and vice-versa.

Although the definition is simple, one has to evaluate the vector area associated with each resistor while working out the resistances. This aspect is discussed at length in a subsequent section. Choice of S_r depends on S_1 also and are discussed together in a subsequent paragraph.

1(c) Capacitance scale ~~S_1~~ S_c

This is defined as the ratio of the capacitance to the storage factor of the node

$$S_c = \frac{C}{s\,\Delta x^2} \quad \text{for uniform grid} \qquad (18a)$$

or

$$S_c = \frac{C}{s\,\Delta x\,\Delta y} \quad \text{for non-uniform grid} \qquad (18b)$$

The terms Δx^2 or $\Delta x\ \Delta y$ in the quotient give the area represented by the node. This area multiplied by the storage coefficient gives the storage factor.

Before discussing the choice of S_c it would be desirable to discuss the derived scales.

2(a) Current (ampere) scale S_a

The Ohm's law is expressed as

$$I = V/R \qquad (19)$$

Substitution of the basic scales in equation (18) gives

$$S_a = \frac{S_v}{S_r} \qquad ..(20)$$

This scale is very useful in calculating the currents to be applied for simulation of wells, boundaries etc. by the relationship

$$I = S_a \cdot Q \qquad ..(21)$$

2(b) Time scale

If one substitutes the resistance scale and the capacitance scale in equation (12) and (15) one gets

$$t_e = S_r \cdot S_i \cdot t_a \qquad ..(22)$$

Where

t_e = time in the electrical system

t_a = time in the aquifer

The product $S_r \cdot S_c$ is given the symbol S_t as it represents the time scale. Thus

$$S_t = S_r \cdot S_c \qquad ..(23)$$

2(c) Quantity scale ~~S_Q~~ S_q

The volume of water (m^3) in aquifer system can be related to the electrical change (Coulambs) by the quantity scale Sq, which can be shown to be given by

$$S_q = S_v S_c \qquad ..(24)$$

Then

$$Q_e = S_q \cdot V_w \qquad ..(25)$$

Where

Q_e = electrical charge

S_q = Quantity scale
V_w = volume of water (m^3)

The choice of the above scale factors for simulating a given aquifer system has to proceed by trials as they are interrelated amongst themselves and with the grid spacing. The first decision should be about the time scale because it affects the nature of the model, which canbe either slow-time model or fast time model. Altogether different types of equipment are required for slow or fast time models which are discussed in subsequent paragraph. In the former aquifer simulation takes a few minutes to complete ~~whether~~ whereas the latter accomplish it within a few milliseconds.

The time scale can also be shown to be given by

$$S_t = \frac{R.C.}{\Delta x^2} \cdot \frac{T}{S} \qquad \ldots(26)$$

For a given aquifer T and S are constants. The time scale therefore depends on the product RC which is the time constant of the electrical system and Δx^2 which is the cell area of each node. For a slow time analogy RC should be large and/or Δx should be small whereas for a fast time analogy RC should be small and/or Δx should be large. The three terms R, C and Δx are therefore decided in such a way that they together give a suitable time scale for the type of analog desired.

The voltage scale appears to be independent. But it affects the current scale S_a. If there are limitations to the capacity of the current generators, or if there are limitations of the measuring/recording device, then voltage scale is required to be adjusted suitably.

The quantity scale S_q is dependent on the voltage and capacitance scales. Once the latter are decided from the above considerations, the former gets decided automatically and there is no choice left for this scale factor.

The features of the slow time and the long time analogs may be usefully discussed here because the choice of type of analog governs the selection of the scales. These are discussed at length by Paschkiss (1949), Karplus (1958) and by Rustom and Bannister (1970) and are summarised here.

The fast time analog is also known as short-time analog or as repetitive analog. The solution on this type of model is achieved within a few milliseconds, making it impossible to take manual readings. An oscilloscope becomes essential for measurement of the time variant voltage at a selected node point. To enable the human eye to perceive the voltage variations on the cathode ray tube of the oscilloscope, it is necessary to repeat each cycle in quick successions. This is done with the help of electronic devices. Permanent record of the voltage pattern can be taken by photographing the CRT traces. The excitation of the model must be carefully synchronised with the sweep frequency of the oscilloscope. About 20 percent of the cycle time is reserved for permitting the capacitors to get discharged and came back to the same initial conditions as at the beginning of each cycle. The capacitors should be of low value. Each node is sensed separately. The oscilloscope cannot offer accuracy better than 5 percent. Low magnitude network resistors are required. As a result the errors due to leakage through insulation etc. are relatively small.

On the other hand, it takes about 2 to 15 minutes. On the slow time analog(also known as single shot or slow time analogs) to achieve the solution. This facilitates use of electromechanical devices for measurement and/or record of voltage patterns at one or more node points simultaneously with accuracies which may be as high as 0.01 percent (Rustom and Bannister 1970). Some delay in reading the full scale deflections due to inertial effects of moving parts are however unavoidable.The product RC must be larger or Δx must be small to give a suitable time scale. If R or C or both are made large, leakage currents are large and so are errors due to them. Large value capacitors of high quality are not available commercially. One has to rest content with the electrolyte type of capacitors which have low leakage resistance and do not last long. Attempts to circumvent this difficulty by adopting a small Δx, do not necessarily succeed. If they do, then a large number of components are required. Nevertheless slow-time analogs offer a very good advantage when the problem involves variable parameters which can be simulated with relatively simple and inexpensive equipment than is possible with the short time analog.

Relative advantages and disadvantages of analog types are summarized in table. on next page

One has to consider the above aspects and first take a decision whether to design the modal as a fast time or a slow time analog before proceeding with the selection of the scale factors.

TABLE

Item	Short time	Medium time	Long time
Instrumentation			
Type	Oscilloscope or Oscillograph	single point high speed recorders	Recording instruments
Accuracy	Low	Fair	High
Manipulation			
Constant boundary conditions	No difference between types		
Varying boundary conditions	Special input circuit for every different boundary conditions		Variation [illegible] continuous or in steps with no special equipment required.
Voltage dependent parameters.	Resistors and capacitors replaced by electronic circuits which have to be different for each different function of property		Manual by means of switches or relays no special equipment required
Cost			
Input device	High	High	Low
Circuit elements, constant parameters	Low	Low	High
Circuit elements varying paramenters	Very high	Very high	High
Indicating measuring devices	Low	......	Low
Recording measuring devices	Very high	High	Low
Solution time	0.1 sec	1 to 10 sec	2-15 min.

2.3. VECTOR AREA AND SPECIFIC PORTION CONCEPTS

The vector area and specific portion concepts are very useful in calculating the values of the resistors and capacitors at boundaries or at changes in grid spacings at internal nodes. These concepts are related to the process of discretisation of the aquifer.

The aquifer is a continuous system in which the ground water flow takes place along the direction in which the gradient of head is maximum. The direction of flow need not conform to any system of coordinate axes. It varies spatially as well as with respect to time. The RC analog model is a discretised scaled down version of the aquifer in which flow of electrons can take places only along the coordinate axes along which the resistors are aligned. Only reversal of direction is possible.

If the ground water flow in the aquifer is at an auxillary direction, it will be simulated in the model by two currents flowing along the principal axes. One component will flow in the clockwise half loop of a cell whereas the other component will flow in the anticlock wise half loop of the cell. Their magnitudes will be proportional to vector components of the flow in the respective directions, provided the resistors in each loop are proportioned correctly. The vector area concept is very useful for working out the correct values of the resistors.

The process of discretisation of an aquifer can be visualised by imagining that the continuous system is replaced by a network of tubes oriented along the desired coordinate axes, so that flow can now take place along those axes or parallel to them. If this network is to represent the flow conditions faithfully, each segment between two node points must offer the same resistance as the aquifer material represented by it. In other words, for identical potential gradient, it should conduct the same flow as would the aquifer material. In electrical system it should conduct current in proportion to current scale.

Let us first consider the square grid as shown in Fig.4, and concentrate our attention on the four resistors R_1, R_2, R_3 and R_4 connected to the node point 0 between nodes 1,2,3, and 4 respectively. The resistor R_1 represents aquifer material in the area between the ordinates at 0 and 1 and half the grid spacing above R_1 and half the grid spacing below R_1 as shown separately in the right hand portion of the figure. Since the flow can take place only along the directions indicated by the arrows, the area is called ' vector area'. In the aquifer, let there be a difference Δh between the heads at nodes 0 and 1.

The gradient is then $\Delta h/\Delta x$, whereas width of aquifer material is Δy. The flow is then given by

$$\begin{aligned} Q &= T.i.l. \\ &= T_x . \frac{\Delta h}{\Delta x} . \Delta y \qquad (27) \end{aligned}$$

Where T_x is the transmissibility in x direction.

In the electrical system Δh will correspond to Δv, T_x to $1/R_x$ and Q to I., and will be given by

$$I = \frac{\Delta v}{R_x} \qquad (28)$$

Combining ~~Higx~~ Equation (27) and (28) with the help of the scale factor S_r, we get,

$$R_1 = \frac{S_r \cdot \Delta x}{T_x \cdot \Delta y} \tag{29}$$

By similar reasoning

$$R_3 = \frac{S_r \cdot \Delta y}{T_y \cdot \Delta x} \tag{30}$$

Where T_y = transmissibility in y direction.

Since the network has a uniform spacing, we get $R_1 = R_2$ and $R_3 = R_4$. Placing $\Delta x = \Delta y$ and $T_x = T_y$ we get $R_1 = R_2 = R_3 = R_4 = R$ (say). However equation (29) and (30) are useful when either $\Delta x \neq \Delta y$ in which case unequal resistors will have to be provided.

The concept can readily be extended to non-uniform grids, as shown in Fig. 5, where the resistors will work out as -

$$R_1 = \frac{S_r \cdot \Delta x_2}{0.5\,(\Delta y_1 + \Delta y_2) \cdot T_{x_1}}$$

$$R_2 = \frac{S_r \cdot \Delta x_1}{0.5\,(\Delta y_1 + \Delta y_2) \cdot T_{x_2}}$$

$$R_3 = \frac{S_r \cdot \Delta y_1}{0.5\,(\Delta x_1 + \Delta x_2) \cdot T_{y_3}}$$

$$R_4 = \frac{S_r \cdot \Delta y_2}{0.5\,(\Delta y_1 + \Delta y_2) \cdot T_{y_4}}$$

Where

T_{x_1}, T_{x2}, T_{y_1} and T_{y2} are transmissibilities in the four directions.

An additional advantage of the above equations is that that enable simulation of anisotropic conditions, by adopting unequal values of T_{x_1} , T_{x_2}, T_{y_1} and T_{y_2}. Moreover they also enable calculation of resistances on the boundary. If the node is on a y boundary, then either $\Delta x_1 = 0$ or $\Delta x_2 = 0$. Substituting this value in the equation (31) one can readily get the correct resistance value. As a special case, consider uniform grid on homogeneous isotropic aquifer. Then

$$R_3 = \frac{S_r \cdot \Delta y}{0.5\ \Delta x \cdot T}$$

$$= \frac{S_3 \cdot \Delta y}{\Delta x \cdot T}$$

$$= 2R \qquad (32)$$

But, on a corner, one would get

$$R_4 = \frac{S_r \times 0.5\ \Delta y}{0.5\ \Delta x \cdot T}$$

$$= R \qquad (33)$$

Equation 33 brings out another important feature of the network design theory that in case of a uniform square grid representing an isotropic homogeneous aquifer the resistance value is independent of the grid size.

Evidently, the design cannot be entirely independent of the grid size which must appear somewhere. If it does not appear in resistance it must appear in the capacitor design. It does. This is done through the ' specific portion ' concept. (The word ' portion ' is used to permit inclusion of one and three dimensional cases also. For two dimensional cases one may refer to in an ' specific area '. The specific portion extends to half the grid spacing on all sides except across boundaries. This is shown by shaded area around node no.5 in Fig.4 and around node N_0 0 in Fig.5.

If the change in head at a node point, during a time interval Δt, is Δh, then the definition of storage coefficient yield, $\Delta V_a = A \cdot \Delta h \cdot S$. (34)

Where

Δv_a = change in volume of water stored in the portion of aquifer represented by the node.

A = area of the portion

$= \left(\frac{\Delta x_1}{2} + \frac{\Delta x_2}{2}\right)\left(\frac{\Delta y_1}{2} + \frac{\Delta y_2}{2}\right)$ in case of nonuniform grid, ~~or~~

~~uniform different grid spacings along each direction, i.e.~~

~~$\Delta x_1 = \Delta x_2, \Delta y_1 = \Delta y_2$, but $\Delta x = \Delta y$~~

~~or~~

$$= \Delta x \cdot \Delta y \quad \text{if} \quad \begin{aligned} \Delta x_1 &= \Delta x_2 = \Delta x_1 \\ \Delta y &= \Delta y_2 = \Delta y_1 \end{aligned}$$

$$\text{but} \quad \Delta x \neq \Delta y$$

or

$$= \Delta x^2 \quad \text{if } \Delta x = \Delta y$$

In the electrical system, Δv_a corresponds to Δv_e and Δh corresponds to Δv_0. The relationship is given by definition of C as

$$\Delta Q_e = C\ \Delta V_e \tag{35}$$

Correlating equation 34 and 35 through the capacitance scale S_c, one gets

$$C = S_1 . S.A. \qquad (36)$$

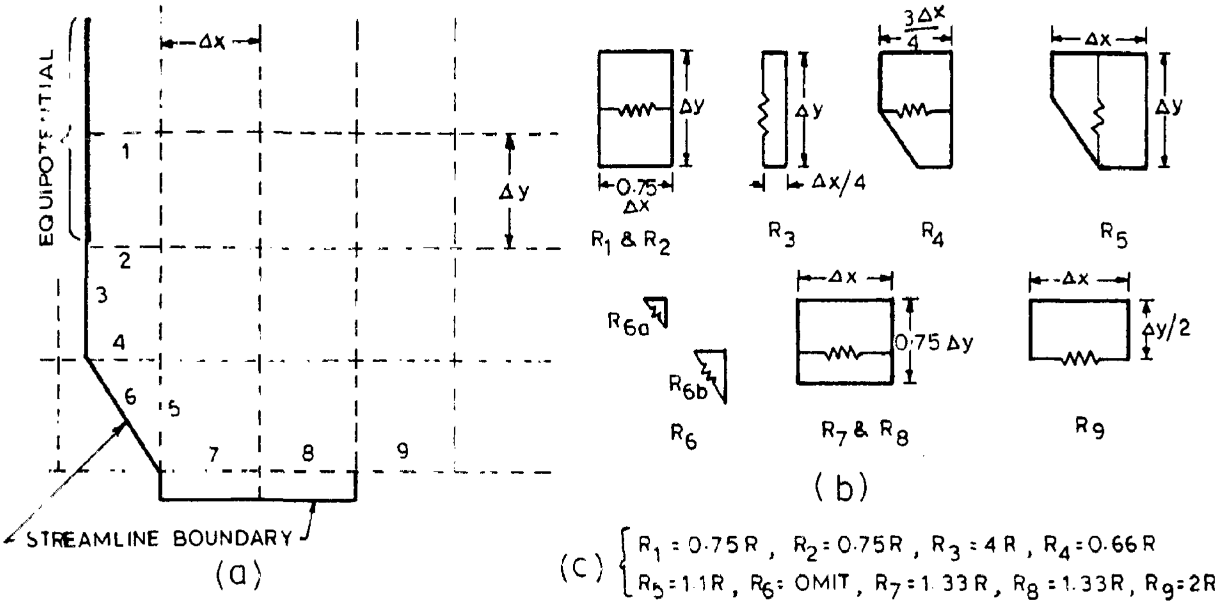
The concepts of vector area and specific portions as outlined above have special significance when calculating the values of resistors and capacitors to nodes n and near irregulator boundaries. Typical examples are shown in Fig.6 and 7.

2.4. NETWORK SPACING

Network spacing of a finite difference grid superimposed on an aquifer is a very important parameter because it governs the following aspects of the analog design.

1. Capacitor magnitudes,
2. time scale,
3. errors, and
4. economy as well as ease of operation of the model

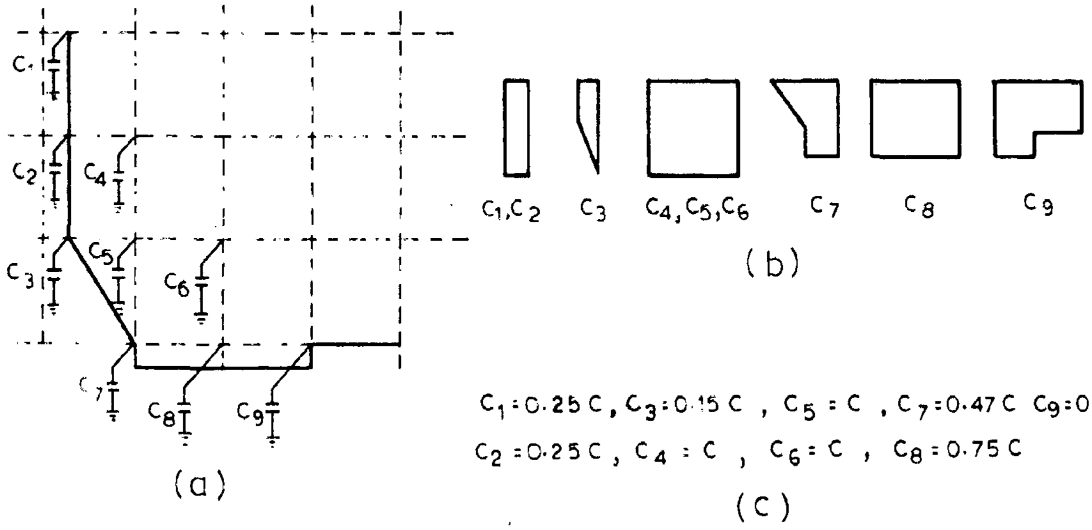
Referring to equation 36 one finds that the capacitance is dependent on the area A and on the scale S_1. If the network spacing is large the area of each node will be large and so will be the capacitances, leading to a larger time scale. If it is desired to have a fast time analog. Δx should not be smaller than a certain value. For slow-time analog it should not be larger than a certain value. In either case network spacing is an important factor to be considered. Even if the chosen spacing is acceptable from the point of view of time scale, one has to ensure that the capacitances



(c) $R_1 = 0.75R$, $R_2 = 0.75R$, $R_3 = 4R$, $R_4 = 0.66R$
$R_5 = 1.1R$, R_6: OMIT, $R_7 = 1.33R$, $R_8 = 1.33R$, $R_9 = 2R$

a _ PORTION OF UNIFORM GRID WITH IRREGULAR BOUNDARY
b _ VECTOR AREAS ASSOCIATED WITH RESISTORS
c _ RESISTOR VALUES FOR $\Delta x = \Delta y : R_x = R_y = R$

FIG. 6



$C_1 = 0.25C$, $C_3 = 0.15C$, $C_5 = C$, $C_7 = 0.47C$ $C_9 = 0.62C$
$C_2 = 0.25C$, $C_4 = C$, $C_6 = C$, $C_8 = 0.75C$

(c)

a _ PORTION OF TWO DIMENSIONAL NETWORK
b _ AREA SIMULATED BY EACH CAPACITOR
c _ CAPACITOR MAGNITUDES

FIG 7 _ RELATIVE MAGNITUDES OF NETWORK RESISTORS IN A TWO-DIMENSIONAL RC NETWORK. RESISTORS NOT SHOWN required conform with the commercially available denominations. If necessary for this purpose, the network spacing may have to be adjusted within the range permissible from time scale point of view.

A vital factor concerning the grid spacing is the error term E, which must be kept as low as possible. Equation 9 gives the error term for the second order partial derivating along the x direction. Similar term for derivative along the y direction (and z direction, if any) can be written. The terms Δx, Δy (and Δz), appear explicitly in the equation, indicating that the error is directly proportional to the network spacing. It would therefore be desirable to keep the network spacing as low as possible. However, there exists a certain limit below which reduction in the grid spacing does not improve the accuracy of the solution. (Stallman, 1963a). Moreover reduction in grid spacing requires ~~in~~ an increase in the number of nodes and therefore the number of components.

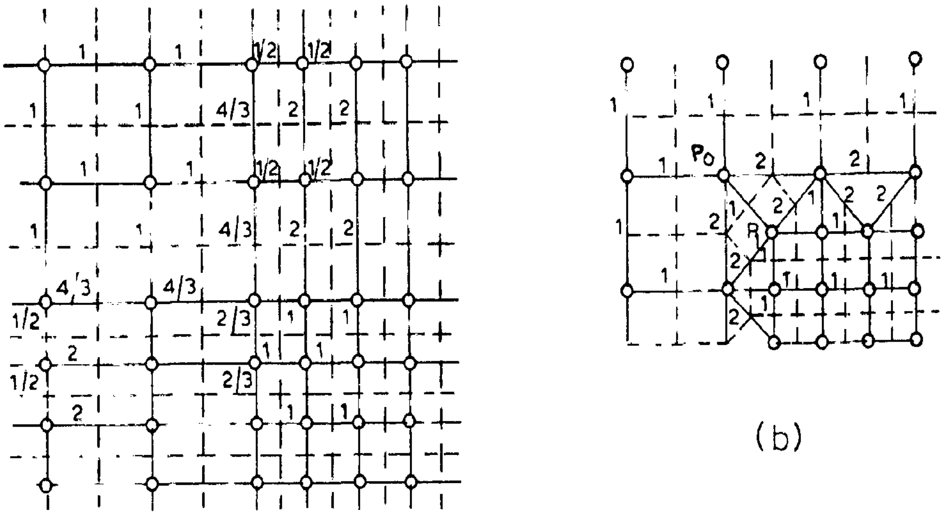
Too many components not only make the model expensive, but also make it unweildy in construction and operation. From this point of view there is a lower limit to the grid size. Experience indicates that the workable number of node points ranges from 250 to 1000 (Rustom et al. 1966, Baturic-Ruber, 1966). It would therefore be desirable to adopt a grid size which gives node points within this range consistent with the other requirements.

Non-Uniform grid spacing

Uniform grid spacings are very convenient to work with, However, non-uniform grid spacings do offer certain advantages as regards minimising the errors.

In certain problems the interest in achieving accurate results may be localised around some nodes although one must simulate the aquifer at large. In such cases one may accept larger errors in the general analog solution provided more accurate solutions are obtained in regions of specific interest. Even if there are no such regions of special interest, the potential gradients may call for variation in the accuracy of results. In those regions where the gradients are steep it is desirable to ensure more accurate solutions, whereas larger errors may be permitted in regions where the gradients are mild. Such a variation in the accuracy of the results can be accomplished by providing small grid spacing (finer mesh) in the regions of interest. Whereas a large grid spacing (coarse mesh) may be provided for the remaining regions, thereby achieving economy in components. This however makes it necessary to pay special attention to the design of resistors and capacitors in the regions of change in grid spacing.

The concepts of vector area and specific portion prove to be very useful in deciding the resistances and capacitances on node points at which the grid spacing gets changed. Karplus (1958) and Vine (1966) have given methods of achieving transition from a fine to a coarse grid and vice versa. Two such examples for doubling the mesh size are reproduced in Fig.8. Fig.8a shows relative magnitudes of resistors, if rectangular coordinates are adhered to. Leibman (1954) and Stallman (1963a) have shown the the errors at the interface of the two regions are quite large. With a view to minimising these errors, dia-



(a)

(b)

FIG 8 _ METHODS OF DOUBLING MESH SIZE

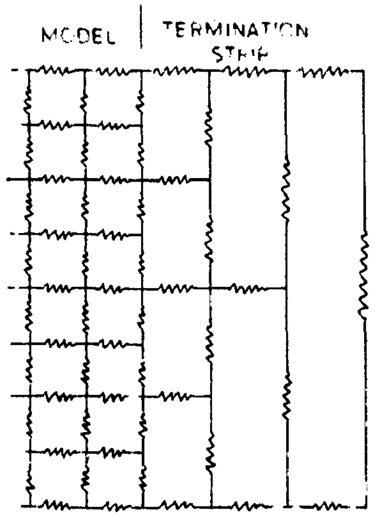


FIG.10_ TERMINATION STRIP

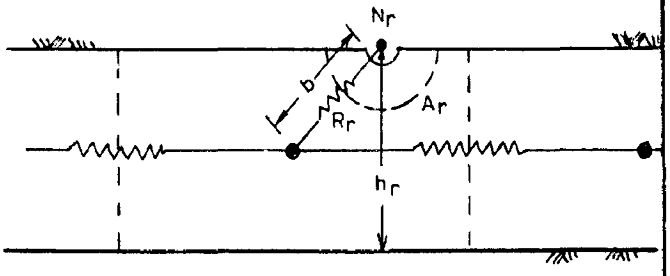


FIG.9 _ MODELLING RIVER BY RIVER RESISTER gonal resistances may be connected as shown in Fig.8b. The values of the capacitors, in either case, can be readily calculated from the specific portion concept.

Stream Aquifer Interaction

The hydrological cycle links the aquifer with streams and neither can operate in isolation. For faithful simulation of the geohydrological conditions, stream-aquifer interaction should also be reproduced to a reasonable degree of accuracy. The following broad categories of cases are encountered.

1. Confined Aquifer
   - (a) stream is located some distance away from the aquifer and is a source of recharge
   - (b) stream is located either at or within the boundary, has incised the confining layer and has
     - i) fairly constant water levels
     - ii) non-steady water levels
2. Unconfined Aquifer
   - (a) Stream penetrates the minimum water table, i.e. it is in hydraulic continuity with the aquifer at all times
     - i) with fairly steady water levels,
     - ii) with non-steady water levels
   - (b) stream not in contact with water table but provides recharge due to induced infiltration,
     - i) without upper limit to rate of infiltration,
     - ii) with upper limit to rate of infiltration.

The simulation of stream aquifer interaction under above cases can be accomplished by methods outlined below.

Confined Aquifer

Case (a) in which the stream is located some distance away but provides recharge to the aquifer is a fairly simple one. The recharge in this case would be in the form of an almost horizontal flux, crossing the aquifer boundary towards the stream. The boundary can therefore be treated as a recharge boundary and simulated as discussed in a subsequent section.

If the stream is located at or within the boundary and is in hydraulic continuity with the aquifer then it may be simulated as a source of recharge. If the water level fluctuations in the stream are small, then the simulation of the stream is only a matter of connecting the node points along the stream to a source of constant voltage corresponding to a constant average water level. The flow of current to or from adjoining nodes will depend upon the voltage at those nodes. If long periods of aquifer activity are simulated, then the effects of inaccuracies due to assuming constant stream water levels will get evened out without introducing serious errors in the solution.

Non-steady conditions of stream water levels are very difficult to simulate. This may be necessary if short duration aquifer behaviour is to be studied vis-a-vis an interacting stream. The discharges in a stream vary not only with time but also along the length of the stream. A flood wave takes its own time to travel to downstream sections, and also gets attenuated due to stream channel routing unless additional quantities of water get added. In the last case, the flood discharge increases along the length the river. The velocity of flow of water is an important parameter in the analysis of streamflow. Unfortunately an equivalent of velocity of water does not exist in the electrical system.This difficulty is circumvented by reckoning the inertial effect of flow of water and taking its analog as inductances. (Einstein and Harder, 1959, Harder et al, 1961, 62, 66). The stream model can then be composed of resistors, capacitors and inductors. But use of inductors make it necessary to use alternating current. whereas R.C. analog models are operated on direct current. This poses formidable difficulties in simulation of interaction of an aquifer with a non-steady state stream flow. No satisfactory solution to this problem has yet been reported.

An unconfined aquifer in hydraulic continuity with a stream with fairly contant water levels can be simulated in a manner identical to the corresponding case of confined aquifer i.e. 1(b) (i). On the other hand the nonsteady case presents the same difficulties as outlined above.

Streams providing an induced recharge due to enhanced infiltration can be simulated in different manners. The simplest method is that due to Watton and Prickott (1963). It can be adopted to those situations in which the flow from the streambed to the aquifer can be assumed to be in an almost vertical direction, thereby permitting its treatment as a two-dimensional flow. The situation then becomes similar to that of flow through an aquitard to a leaky confined aquifer which will be discussed in a subsequent section. Each node point along the stream is connected to a source of constant voltage corresponding to the stream water level through a resistor calculated by (Walton ~~Cheafton~~ and Akroyd, 1966.)

$$R_s = \frac{S_r}{I_s \cdot A_s} \qquad \ldots (31)$$

where,

R_s = stream simulation resistance

S_r = resistance scale

I_s = infiltration rate of stream bed expressed as a flow rate per unit area per unit head loss.

A_s = Area of streambed represented by the resistor.

The above method of simulation presumes that the leakage through the stream bed is unlimited. However, there may exist situations in which there is an upper limit to the leakage. For this purpose it is necessary to use specialised circuits. Such circuits are reported by Skihitzke (1963), Walton et.al. (1967) and Prickett (1970).

The leakage from a stream to an aquifer may have a three-dimensional nature. In this case the shape of the river channel should also be considered. Herbert (1970) has reported how to calculate the resistors to be connected between the river nodes and the aquifer nodes. He has not discussed the simulation of the river itself. He presumes that the river channel can be approximated as a semicircle, as shown in Fig. 9. He also makes a debatable assumption that the aquifer nodes are situated at the centre of the aquifer depth. To represent the river, a special node N_r is created. The river head is applied to this node. The flow is assumed to cross the river perimetre radially. The resistor R_r joining the river node N_r to the nearest aquifer node is used to represent the flow crossing a semi-circular mid-section A_r and is given by

$$R_r = S_r \ \log_e \left| \frac{(b+r_r)}{r_r} \right| / \pi Kl \qquad \ldots.(38)$$

Where

R_r = river resistance (ohms)

S_r = resistance scale

b = distance between the aquifer node and the river perimeter (m)

r_r = radius of river section (m)

k = permeability (m/sec.)

l = length of river segment represented by the river node.

The stream aquifer interaction simulation by above methods is possible only for uniform flow conditions of the stream. If nonsteady streamflow must be simulated serious difficulties as explained earlier are encountered. Perhaps it may be possible to work with a carrier wave superimposed on an alternating current. This will however require complicated electronic circuitry for modulating and demodulating the signals. Till such time as this becomes feasible stream aquifer simulation by differential analyser using operational amplifiers integrated circuits etc. will have to be resolved. One such model is reported by Riley et al (1966). Such models however are out side the scope of RC analogs.

2.6. BOUNDARY CONDITIONS

Boundary conditions govern the flow conditions to a very large extent, making it very essential to ensure that they are properly represented on the model. The following types are encountered.

1. Infinite boundaries
2. Finite boundaries
   (a) barrier boundary
   (b) recharge boundary
      i) constant head boundary (Dirichlet condition)
      ii) constant flux boundary (Neumann conditions)
   (c) streamline boundary

Infinite boundary

The infinite boundary is the most difficult to simulate because the analog cannot be made infinite. A finite representation of an infinite boundary introduces unavoidable errors. One can only minimise them by some method or the other. One method is to provide termination strips (Karplus, 1958) between an arbitrary boundary, of the aquifer within which detailed simulation is desired, and the infinity. One such example of termination strips is shown in Fig.10. As will be seen from there it is nothing but a progressively and rapidly coarsening grid. In this way the total field region is made several times as large as the region of interest. The errors due to mesh oise transitions are confined to the coarse region (Vine, 1966). The total area simulated should be about ten times the area of actual interest.

Another method of representing the infinite boundary is to adopt conformal transformations technique. The infinite boundary can then be brought to a finite shape or point. The region inside the area of actual interest is represented on the un-transformed portion of the model, whereas the outside area is simulated on the transformed model and link isprovided between the two. Calculations of components in the transformed model and the linkdages becomes very complicated. So does the interprotation of the results. (Rastogi, 1973, Karplus 1958)

Barrier Boundary

The barrier boundary is very easy to simulate. There is no ground water flow across the boundary. For the purpose of the aquifer being modelled this tantamounts to zero T and S values outside the boundary. Equation 31 and 34 give R and C values as infinity and zero respectively. The barrier boundary can therefore be simulated by the absence of resistors and capacitors (open circuit) outside the corresponding field boundary (Stallman, 1961, Prickett 1975).

Recharge Boundary

A recharge boundary provides a lateral inflow or outflow across the boundary. In contrast to rainfall recharge to unconfined aquifer or evapotranspiration losses, which affect all the node points of the model, flow across the recharge boundary affects only the node points on it. The flow may be caused by a constant head source such a perennial stream, or may be constant flux source as in a confined aquifer getting recharge from distant outcrop areas. The former can be simulated by connecting the boundary nodes to a source of constant voltage corresponding to the constant head. In the other case the nodes are connected to a source of constant current. In case the sources of aquifer recharge are known to have a definite time pattern of variation of voltage or current, then appropriate voltage function generation or current function generators may replace the constant voltage and constant current sources respectively. In any case the component values are worked out using the concepts presented in section 2.3. For uniform grid spacings the boundary resistors are shown to be 2R (Eq.25).

Streamline boundary

A stream line boundary is one across which there is no flow, but along itself flow may take place according to the potential gradients. The area outside the boundary may be treated as in the case of a barrier boundary. However resistors are connected between the node points along the boundary for simulation of the potential drops. The node points are, however, not connected to any external source.

Boundary shapes

It would be in order to discuss the representation of the boundary shapes. The boundary configuration seldom, if ever, conforms to the network. Irregular shapes are encountered along at least some sections of the boundary. Unless it is essential to achieve accurate solutions at these node points also, the boundary shape may be approximated with jagged lines along the grid itself. For reasonably small net spacings, the errors caused by such approximations are not unacceptable compared to the accuracy of the overall solution (Karplus, 1958) For more accurate solutions, the shape of the boundary may be retained, but then the components along the boundary must be calculated according to concepts of Sec. 2.3 Typical examples are given in Fig.6 and 7.

2.7. INITIAL CONDITIONS

The proper simulation of the initial conditions is very important for ensuring the reliability of the results. There are four conditions that are used. (Rushtom and Wedderburn, 1973).

(a) The heads within the aquifer are all zero.

(b) The heads correspond to a steady state solution due to inflow and other conditions which apply at the starting point of calculation.

(c) The heads resulting from a steady state solution with average values of inflows and outflows are used.

(d) The heads are in a state of dynamic balance.

Rushtom and Wedderburn (1973) have shown that for most aquifers (d) is the only satisfactory conditions from which to start the analysis. All the same it the most difficult condition too.

With fast time (repetitive) RC analogs condition (d) has often been used. For slow time analogues it is usual to start from (a), (b) or (c). The authors have recommended that where (d) can not be satisfied, (c) should be adopted but there should be a time interval t preceeding the actual start of the analysis. The value of t is given by

$$T = \frac{L^2S}{T} \qquad \ldots(39)$$

where

t = duration of presimulation period of initial conditions.
L = effective length of the aquifer
S = Storage coefficient
T = Transmissibility

If it is necessary to adopt condition (a) only, then the pre-simulation time period should be 2.5 times that given by equation 39.

On the model, simulation of condition (a) requires no action for slow time analog. For fast time analog the heads are brought to zero with the help of a line discharge unit, and enough time is allowed for the capacitors to discharge fully before resuming the repeat run of simulation.

Condition (b) or (c) can be simulated with the help of adequate number of constant voltage supply units. The water level contours for (b) or (c) are drawn. For each contour, there is one source supplying constant voltage corresponding to the denomination of the contour. The node points on or near the contour are connected individually to the voltage source. All the voltage sources are set to the required voltages. The switch for starting the simulation run puts the voltage sources off, when it puts the other devices on. Return flow from the model to the source is prevented by introducing diodes, otherwise the contour configuration of starting instant will be maintained unaltered except for the change in denomination.

2.8. SPECIAL CONDITIONS

There are certain situations which require additional details for simulation. These are as under -

1) Leaky confined aquifer with leakage from one or two sides.
2) Three dimensional cases
3) Detailed well simulation

Leaky confined aquifer

Leakage through a semi-pervious layer can be simulated by providing an additional resistor of each node point. The magnitude of this resistor is given by (Walton and Prickett, 1963),

$$R_o = \frac{S_r}{(p'/m').\,A} \qquad \ldots (40)$$

Where

R_e = resistance of the resistor simulating leakage,
S_r = resistance scale factor
p' = vertical permeability of aquitard
m' = saturated thickness of the aquitard
A = area of specific portion of the confining bed represented by the node.

The other end of the resistor is connected to a source of constant voltage corresponding to the head in the source bed across the aquitard. If there is another aquitard on the other side of the aquifer, then another set of resistors may be connected to each node. Alternating one resistor may be made to simulate both the aquitards by connecting it to source of average voltage.

Three Dimensional flow

The three dimensional case may be of two types.

a) Multi-layer aquifer

b) Single aquifer exhibiting three dimensional variations.

The simulation of a multilayered aquifer is an extension of that for the leaky confined aquifer. Each stratum of the aquifer is designed as for the two dimensional case. However the corresponding node joints are connected to each other through appropriate leakage resistors. The other details would be almost the same as for the two dimensional case.

Three dimensional variation of aquifer characteristics can be simulated by a three -dimensional model. The basic principles are the same. However, in this case ' vector volume' will replace ' vector area' and ' specific volume ' will replace the ' specific portion'. Equation 31 and 36 get modified suitably.

Well characteristics

Well simulation in details is also possible. The heads at the node points of the analog give the nonpumping static water levels (SWL) of an observation well. If there also exists a pumping well at the node point, then the observed head corresponds to a point at a distance $\Delta x/4.81$ (Prickett, 1967). The model therefore gives an underestimate of the draw-downs occuring at and near the well. If it is necessary to estimate the heads at the well itself, additional resistors, for simulating the material in the vicinity of the well must be introduced at the node point. It ispossible to determine the heads at different points, e.g. inside well, just outside well, just outside the gravel pack, effects of partial penetration etc. by a potentiometric divider developed and described by Prickett (1967). Details are not given here because Indian conditions seldom necessitate such simulations.

2.9. ERRORS

Neither the finite difference approach on which the resistor-capacitor network analog model is based, nor the model itself, can be considered to be a precise tool for solving the fundamental partial differential equation governing the flow of ground water. For that matter, even the rigorous mathematical solution also involves some errors such as those involved in interpolating the value of the well function for a given value of u. Even the well function itself is expressed as an infinite series involving truncation errors. These would of course be very small compared to those of the analog model.

The accuracy of the data obtained from an RC network model is highly dependent on the accuracy ~~of~~ with which the aquifer is represented on the model. Several sources of errors (Stallman, 1963) must be considered in evaluating the accuracy of the analogy solution. The most frequent errors are -

(b) observational errors

(c) erross due to instability of the components and accessories.

(d) errors due to inaccurate proportion between the aquifer properties and the model components.

(e) errors due to ~~recharge~~ leakage of compacitors and in insulating parts of the circuits.

Fortunately, errors due to discretisation of time are not present in the analog model.

Truncation errors are inherent in the theory of the model and cannot be avoided.

For a two dimensional uniform grid the error term is given by

$$E = \frac{\Delta x^2}{12} \left(\frac{\partial^4 \phi}{\partial x^4} \quad \frac{\partial^4 \phi}{\partial y^4} \right)_o + \frac{2\Delta x^4}{6!} \left(\frac{\partial^6 \phi}{\partial x^6} \quad \frac{\partial^6 \phi}{\partial y^6} \right)_o \qquad \ldots\ldots (41)$$

Where Δx and Δy are grid intervals along x and y directions.

If desired, the order of this error term can be reduced to $\frac{\partial^8 \phi}{\partial x^8}$ and $\frac{\partial^8 \phi}{\partial y^8}$, by introducing diagonal resistors. This, however, require twice as many componentsas in ordinary square grid. The extra expense would be justified only in special cases.

It can be readily seen that the error depends substantially as the grid spacing, and can be minimised by reducing the spacing. There exists, however, a limit below which reduction in the grid interval does not serve the purpose. Truncation errors are systematic in nature, and do not destroy the smoothness of the readings, so that they may not be quite objectionable.

Observational errors depend on the sensing equipment used. Measurement errors can be reduced to 0.01 percent with the help of precision instruments, but may be as high as 5 percent of full scale value if an oscilloscope is used. The measuring accuracies are given in a table in a subsequent section.

The errors due to the electrical components can be of two types. The first is that due to manufacturing tolerances, the second due to the change in the value of a component with temperature, humidity and time. On the whole these errors can be considered to be of statistical nature mutually cancelling out. Walton and Prickett (1963) used resistors and capacitors having manufacturered tolerances of ± 10 percent and obtained highly satisfactory results.

It often happens that the components of the exact value as calculated from the equations 31 and 36 do not conform with the commercially manufactured range of components. The components of required value can be created by series and/or parallel combination of the commercially available components, or else components of value nearest to the desired one may have to be utilised. These adjustments introduce same inaccuracies for which there can be no way of estimating. Usually attempts are made to avoid such difficulties by adjusting the scale factors or the grid size, or both, so that very few components of inaccurate value are used.

Some errors also creep in due to inaccurate representation of the boundary conditions. If an infinite boundary is represented, by a finite boundary, even with the termination strip, some errors are unavoidable. These will depend upon the geometry and cannot be estimated. If very accurate results are desired, the model should cover an area ten times as large as that of the actual area of interest (Vitkovitch, 1966) which makes the model very costly.

In spite of all the above errors involved, the RC network analog models have given fairly reliable results. The results obtained are quite satisfactory compared to the accuracy of the basic field data usually available.

2.9. EQUIPMENTS AND MODEL SET UP

The set up of equipments is designed to suit the requirements of the problem, finances available and equipment on hand. The general set up is shown in Fig.10, which includes the following components.

(a) Model.

The model itself is designed in light of the principles outlined above. If it is prepared for a specific problem then it can have soldered connections. A map of the aquifer on a suitable scale is drawn or pasted on an insulating board such as masonite. A grid as per design is superimposed on it. Holes are drilled into the board at the locations of the node points and brass studs or eyes are inserted. Resistors are soldered on one side of the board whereas the capacitors are soldered on the other side of the board. The other ends of the capacitors are connected to ground of the model. In case of general purpose models, intended for solving several problems at different times sockets are provided on the board. The components are soldered to pins which canbe plugged into the sockets. This ensures flexibility of model setting and interchangeability of components at ease. The model must be diligently checked to eliminate all loose joints, short circuits and leakage paths. Using useful guidelines are given by Parchkin (1968).

(b) Input devices

The inputs may be positive (recharge) or negative (pumpage, evapotranspiration losses etc.). They may be concentrated at a few points or may be uniformly spread over one or more zones of the aquifer. They may be constant or time variant. The input devices will depend the above considerations.

The recharge to an unconfined aquifer, due to rainfall, and evapotranspiration losses are spatially distributed parameters. The net effects of parameters may be reproduced. It would be desirable to apply appropriate currents to each node separately through feed in resistors and diodes.(Prickett and Loniquistt, 1968). Similarly pumpage may be simulated by negative currents and applied to the respective node points through feed in resistors and diodes. If pumpage is also uniformly distributed then a single current source producing current proportionate to this algebraic sum of recharge and pumpage may be applied for reduction in number of equipments. However separate simulation of rainfall pattern has the advantage that the percentage of water table can be easily adjusted with the help of potentiometers without disturbing the other conditions.

Inputs to the boundaries are already discussed earlier. Some important devices are discussed below.

(i) Pulse Generator

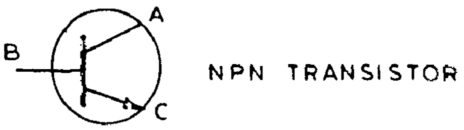
This equipment is a source of electrical current simulating the pumpage from one or more wells and is therefore analogous to a pump. This can be a constant current equipment if the pumpage can be taken to be constant. If the draft has taken place intermittently, provision is made for putting the equipment on and off at appropriate instants of time, in the form of step function pulses. The current is adjusted to value computed from Eq.(19) and is mdistributed to the well nodes through a bank of feed in resistors. It different flow rates in different wells and/or portions of the model are necessary, correspondingly larger number of pulse generators are required.

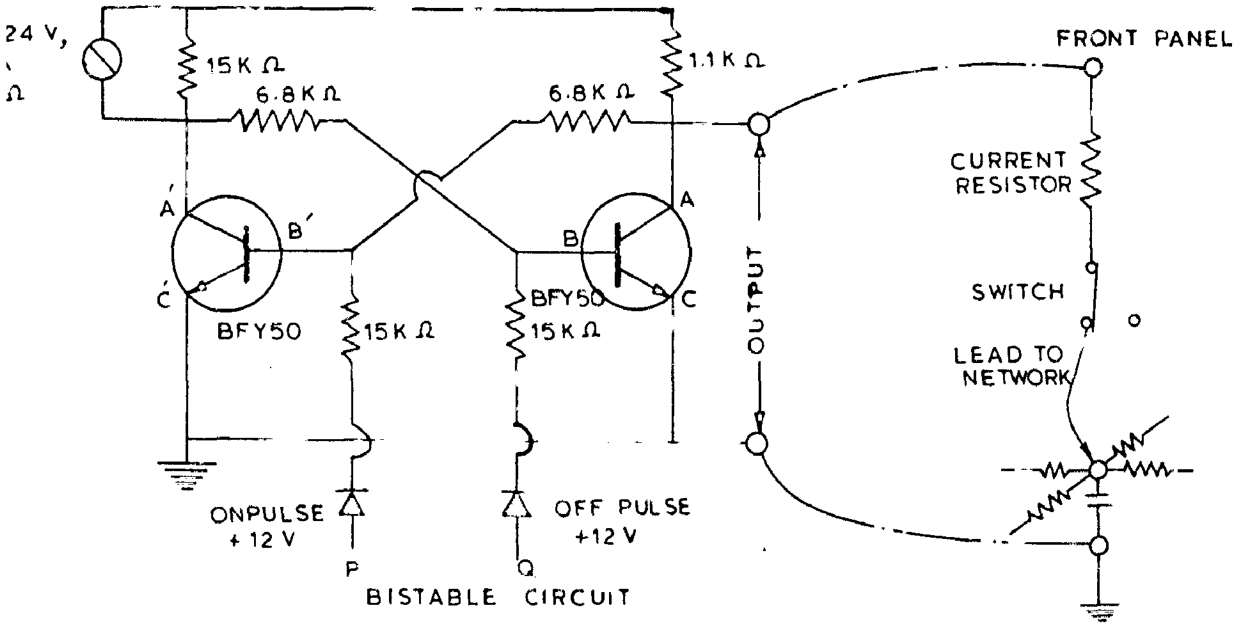
The pumpage may be variable with time, and it may be desirableto simulate its variations. This can be done in two ways. The first is to discretise the pumpage pattern into series of step functions, which can be produced with the help of bistables and as shown in Fig.11. The bistables act as switches for putting ' on ' or ' off ' the respective pulse generators.

If it is desired to simulate the pumpage pattern more accurately it may be broken up in a series of straightline segments with variable slopes. A complex wave form synthesiser can be used for generating such a pattern. The equipment is, however, very expensive and has a very involved circuitry.

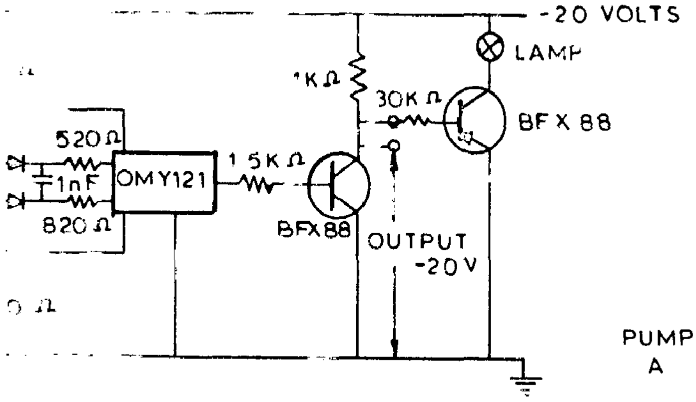
(ii) Wave Form Generator

This equipment is useful in two ways. The first is to synchronise the output of the pulse generator with the sweep of the oscilloscope. This is necessary for observing the signal simultaneously with the pulse generated current. Secondly,

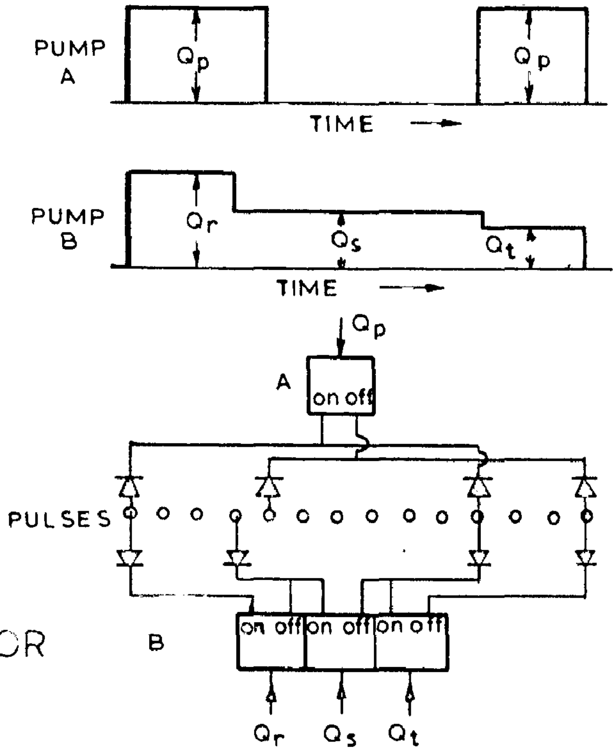




BISTABLE CIRCUIT



EGRATED CIRCUIT FOR NEGATIVE TABLE



METHOD OF REPRESENTING PUMPING PATTERNS

11 . USE OF BISTABLES FOR GENERATING PUMPING PATTERNS

model is designed to be a repetitive one. The duration of each cycle is made longer than the time required for running the model so as to allow sufficient time for the capacitors to get discharged and come to the initial condition. The repetition rates may vary from 0.1 to 10^4 H depending upon the problem requirement.

Output devices

The output of an analog is in the form of a ~~line~~ time voltage or a time current graph. The most conspicuous output is given by the oscilloscope, which can be used for fast time repetitive analogs only. For slow time analogs one has to use recorders or plotters.

(i) Oscilloscope

The oscilloscope is useful in two ways. Before simulation, it enables one to examine and adjust the patterns produced by the input devices. During actual simulation it acts like a water level recorder installed on an observation well. The luminous trace on the oscilloscope is analogous to the well hydrograph and can be photographed for permanent record. It can be converted to well hydrograph through the scale factors, and can be compared with the observed well hydrograph to check whether the model simulation is proper or not. It is important to synchronise and adjust the sweep frequency of the oscilloscope with the other equipments. The scope measurements have an error tolerance of about ± 5 percent. For more accurate results other devices must be used.

(ii) Recorder and plotters

The output of a slow time analog can be obtained on one of the following devices which work as recorders or plotters.

(1) Data logger with digital voltmeter

(2) Ultra violet recorder

(3) Ultra-violet recorder with preamplifier

(4) X-Y plotter

(5) X-Y plotter with preamplifier

(6) Long persistence oscilloscope or storage oscilloscope.

The important characteristics of these instruments are discussed by Rushtom and Daunister (1970) and are summarised in Table No.1. They have recommended the ultra-violet recorder as ' the most suitable instrument because of its accuracy and its ability to record a number of results simultaneously. If the changes in head are not too rapid the X-Y recorder can be used. When a large number of results are required involving contour plots a data logger should be used. Since the ultraviolet recorder is not readily available indigeneously and is quite expensive, one has to rest content with an X-Y plotter.

(iii) Other measuring instruments

These are as under -

i) Multimeter

ii) digital voltmeter

Their characteristics are also given in Table No.1. They are useful when the analog time is slow enough to permit manual observation and recording.

(d) Voltage supply

In addition to the boundary conditions, the initial conditions are also required to be simulated on the model. If a confined aquifer is modelled, then it may suffice to bring the voltage at each node to zero before commencement of each run. The oscilloscope or recorder then gives the time-drawdown curve instead of time-water level curve. Quite often one is interested in drawdowns rather than in the actual water levels so that the above method may serve the purpose. In this case a line-discharge unit is needed to discharge all capacitors after the simulation. Since this takes some time the cycle time for repetitive analog is kept sufficiently longer to include this.

It may be necessary to get the time-level graphs rather than the time-drawdown graphs, either because the problem requirements are such or because the aquifer is an unconfined ones. In such cases the initial condition of water levels must be modelled by ensuring that appropriate voltages occur at the node points just prior to the simulation run. This can be done by having an adequate number of constant voltage sources. The node points lying on a particular water level contour are connected to one voltage source, through diodes those on another contour are connected to another source and so on. The voltage supplies are switched ' off ' simultaneously with the putting 'on ' of the simulation equipments with the help of a multi-point switch. The diodes ensure that the current does not flow in the reverse direction. Otherwise the initial configuration of contours will be maintained unaltered with only increase or decrease in the values. The switching off and on is done automatically on a repetitive analog whereas manually on a single shot analog.

Sl.No.	nt	Whether in India	Approximate cost In Rs. for Indian make In Dollar for foreign make
1.	Multime cord	Yes	Rs.700/-
2.	Digital y,	Yes	Rs.3000/-
3.	Data lo d voltmet	Yes	Rs.20,000/-
4.	Ultra v d	No	Dollar 1600/-
5.	Ultravi d with pr	No	Dollar 1600+ 100 per channel
6.	X-T plo can l)	Yes	Rs.15,000/-
8.	X-T Plo amplife osed)	Yes	Rs.25,000/-
9.	Long pe oscillo an be (storag scope)	Yes	Rs.50,000/-
10	Pen rec ur- ot	Yes	Rs. 35,000/-

TABLE 1

DETAILS OF MEASURING INSTRUMENTS

Sl.No.	Name	Leakage current (measuring 1 volt) Microamperes	Input impedence Megaohms	Accuracy measuring 1 volt (percent)	Speed of reading	Type of output	Whether in India	Approximate cost In Rs. for Indian make In Dollar for foreign make
1.	Multimeter	50	0.2/volt	2	-	Pointer, no record	Yes	Rs.700/-
2.	Digital volt-meter	0.0001	10,000	0.01	50 per sec.	Digital display, no record	Yes	Rs.3000/-
3.	Data logger with digital voltmeter	0.0001	10,000	0.01	10 per sec.	Output on pumped tape	Yes	Rs.20,000/-
4.	Ultra violet recorder	20	0.30	1	80 per sec.	Graphical record	No	Dollar 1600/-
5.	Ultraviolet recorder with preamplifier	0.01	100	1	80 per sec.	Graphical record	No	Dollar 1600+ 100 per channel
6.	X-T plotter	1	1	2	4 in per sec. 0.05 sec. for full scale deflection	Plotted graphs (further plots can be superimposed)	Yes	Rs.15,000/-
8.	X-T Plotter with pre-amplifer	0.01	100	2	4 in per sec. 0.5 sec. for full scale deflection	Plotted graphs (further plots can be superimposed)	Yes	Rs.25,000/-
9.	Long persistence oscilloscope (storage oscilloscope)	0.1	10	5	Fully variable	Trace which can be photographed	Yes	Rs.50,000/-
10	Pen recorder	1	11	2	Variable	plotted graph further plots cannot be superimposed	Yes	Rs. 35,000/-

CHAPTER - 3

BASIN CHARACTERISTICS AND DATA

LOCATION

The aquifer for which the R.C. analog model studies are planned lies around village Daha and is therefore known as Daha Area Aquifer. It is a part of the main doab between Yamuna and Ganga, and is bounded by rivers Krishni to west and Hindon to the east. The northern boundary is an arbitrary boundary formed by a metalled road. To the south, there is a confluence of the two rivers. The area covers 33963 ha. in parts of Meerut and Muzaffarnagar Districts. There are 64 villages belonging to Binauli, Budhana and Kandhla blocks. The lattitudes vary from 29° 05' North to 29° 17' North. The longitudes range from 77° 19' East to 77° 32' East. The maximum width along east-west is 24.3 km whereas along north-south it is 21.12 km.

PHYSIOGRAPHY

The ground water levels in Daha area show a falling trend from the north to the south, and vary from 238.05 m in the north to about 235.05 m in the south in a total length of 21.12 kms. The average slope works out to 1 in 1606. The main drainages also flow from north to south. (Hans Kumar, 1974). The shape of the area is almost a circle.

CLIMATE

Very hot summers and cold winters characteristics the climate of this area. Rainfall is mainly in the monsoon from middle of June to end of September. Occasional water winter rains also take place but are erratic and scanty. The rainfall data are given in Table No.2, whereas data of pan evaporation observed at the nearest station viz Meerut are presented in Table No.3.

IRRIGATION FACILITIES

It is perhaps are irony of geography that the area is deprived of irrigation facilities although two well known irrigation systems viz Eastern Yamuna Canal and Ganga Canal extend upto the opposite banks of the rivers bounding the area. As there is no surface water irrigation, the farmers have taken recourse to ground water. The average annual rainfall of 96.5 cm being inadequate for meeting the irrigation needs, supplementary sources in the form of tubewells, rahats etc. have been developed, This is necessary for growing sugarcane which takes the lions' share in the cropping pattern of the area. There were 121 state tubewells, 1180 private tubewells, 15 pumping sets, 1130 masonry wells with rahat and 1890 drinking water well in 1974. The number of private tubewells increased to 1663 in 1975 which shows the pace of ground water development in the area. The distribution of state tubewells is shown in Fig. 12, and shows that rather than being concentrated around certain locations the tubewells are almost uniformly spread over the area.

GEOLOGY

The area in a part of the Indo-Gangatic plain, and is likewise formed of unconsolidated fluviatile formations

TABLE NO.2

MONTHLY RAINFALL PATTERN IN DANA AREA

(All figures are in Millimeter)

Raingauge Station	Year	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total
DUDHIA	1972	2.5	40.2	4.7	4.3	0.0	19.0	249.4	249.17	70.4	16.0	0.0	2.6	643.8
	1973	3.0	5.2	1.2	0.0	12.0	84.0	290.2	168.4	27.8	93.0	0.0	3.0	667.8
	1974	0.0	0.0	0.0	0.0	28.0	19.0	132.0	176.2	16.2	0.0	0.0	8.0	379.4
	1975	24.0	20.0	7.0	0.0	0.0	40.7	141.8	154.2	38.2	11.1	0.0	0.0	436.7
KANDLA	1972	0.0	34.0	9.0	28.0	0.0	59.0	275.0	190.4	172.1	41.1	25.0	2.0	835.5
	1973	4.0	7.0	12.0	0.0	0.0	8.0	230.8	206.4	55.0	16.3	0.0	0.0	539.5
	1974	5.5	2.2	0.0	0.0	12.0	22.0	156.0	113.0	135.0	5.0	0.0	7.0	456.7
	1975	20.0	18.2	1.2	0.0	0.0	46.0	121.0	165.0	374.0	16.0	0.0	0.0	782.4
SARDHANA	1972	3.2	83.0	13.2	8.0	0.0	28.8	244.6	188.0	101.2	6.8	11.4	5.6	693.8
	1973	11.6	5.2	0.0	1.0	11.2	99.0	152.6	69.4	44.6	41.2	0.0	0.0	435.24
	1974	0.0	0.0	0.0	0.0	4.8	16.2	187.05	165.78	34.4	5.2	0.0	8.8	443.24
	1975	32.0	2.8	8.2	0.0	0.0	79.0	302.05	282.44	253.0	52.2	0.0	0.0	1112.70

TABLE NO.3

Normal Pan Evaporation (Class A Pan) & Potential Evapotranspiration of Doha Area

(All Figures are in millimeter)

S.No.	Month	Pan evaporation	Potential Evapotranspiration
1.	January	61.0	53.1
2.	February	100.0	75.1
3.	March	173.0	127.1
4.	April	259.1	174.7
5.	May	322.0	222.2
6.	June	273.1	225.3
7.	July	180.2	163.0
8.	August	152.0	142.1
9.	September	135.0	142.2
10.	October	120.0	111.3
11.	November	82.0	65.9
12.	December	57.0	49.4
	TOTAL	1914.0	1545.4

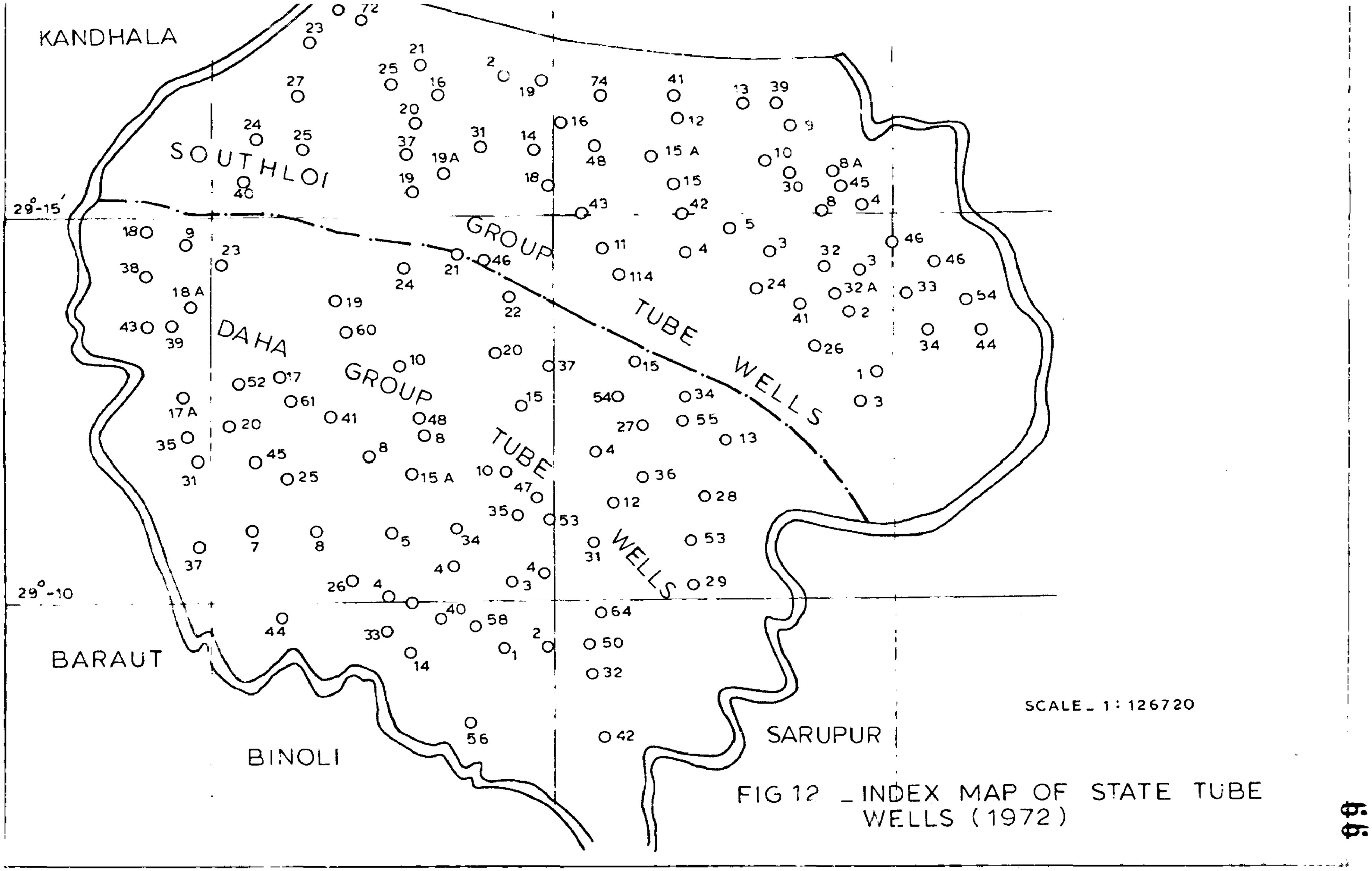


FIG 12 _INDEX MAP OF STATE TUBE WELLS (1972)

comprising of sand, silt, clay and kankar. The thickness of alluvium in the Indo-Gangetic plain is known to be of the order of 2500 to 5000 meters. Geologically the sediments are favourably embedded for the occurrence of ground water. (Rama Kumar, 1974). The area is formed of Pleistocene to Recent alluvial river deposits of Indo-Gangetic plains, formed by unconsolidated fluvial deposits comprising sand bed with intermediate lenses of silt, clay and kankar. On regional scale the aquifers are interconnected but in localised pockets at some places show semi-unconfined and leaky confined condition. Average depth of water is about 12.5 m and fluctuation is about 1 m.

There are no data of deep drilling in the area available. The state tube wells penetrate upto about 150 m. The subsurface geology upto a depth of 150 m can be studied from the lithology of the state tubewells in the area.

AQUIFER CHARACTERISTICS

Detailed test pumping analysis by various methods of the data for one pumping test in the area were carried out earlier (School of Hydrology, 1974-75). The aquifer parameters chosen accordingly are as under. Transmissibility = 1771.2 m^2/day, storage coefficient ~~0.118.~~ = 0.118

There are no other test data available. As a result the above values are adopted for the entire aquifer thereby assuming it to be homogeneous and isotropic.

DATA AVAILABLE

The data available and useful for hydrologic modelling are listed below :-

(1) Monthly rainfall figures for the three raingauge stations at Budhana, Kandhla and Sardhana. Data for Baghpat are available but are not useful as the Thiessen polygon of that station does not reach upto the area.

(2) Data of monthly pan evaporation figures at Meerut could be gathered from IMD Publication.

(3) Monthly spring level observations of water table of open wells for the year 1972 to 1975.

(4) Villagewise area statistics.

(5) Villagewise irrigated area and irrigation works(1972-75).

(6) Yearly average discharge and running hours of state tube wells.

(7) Results of sample survey of private irrigation works, (1972-1975).

(8) Pump test data

(9) Existing cropping pattern - blockwise.

(10) Water requirements for crops.

PROCESSING OF DATA

Sub-Division into Regions:

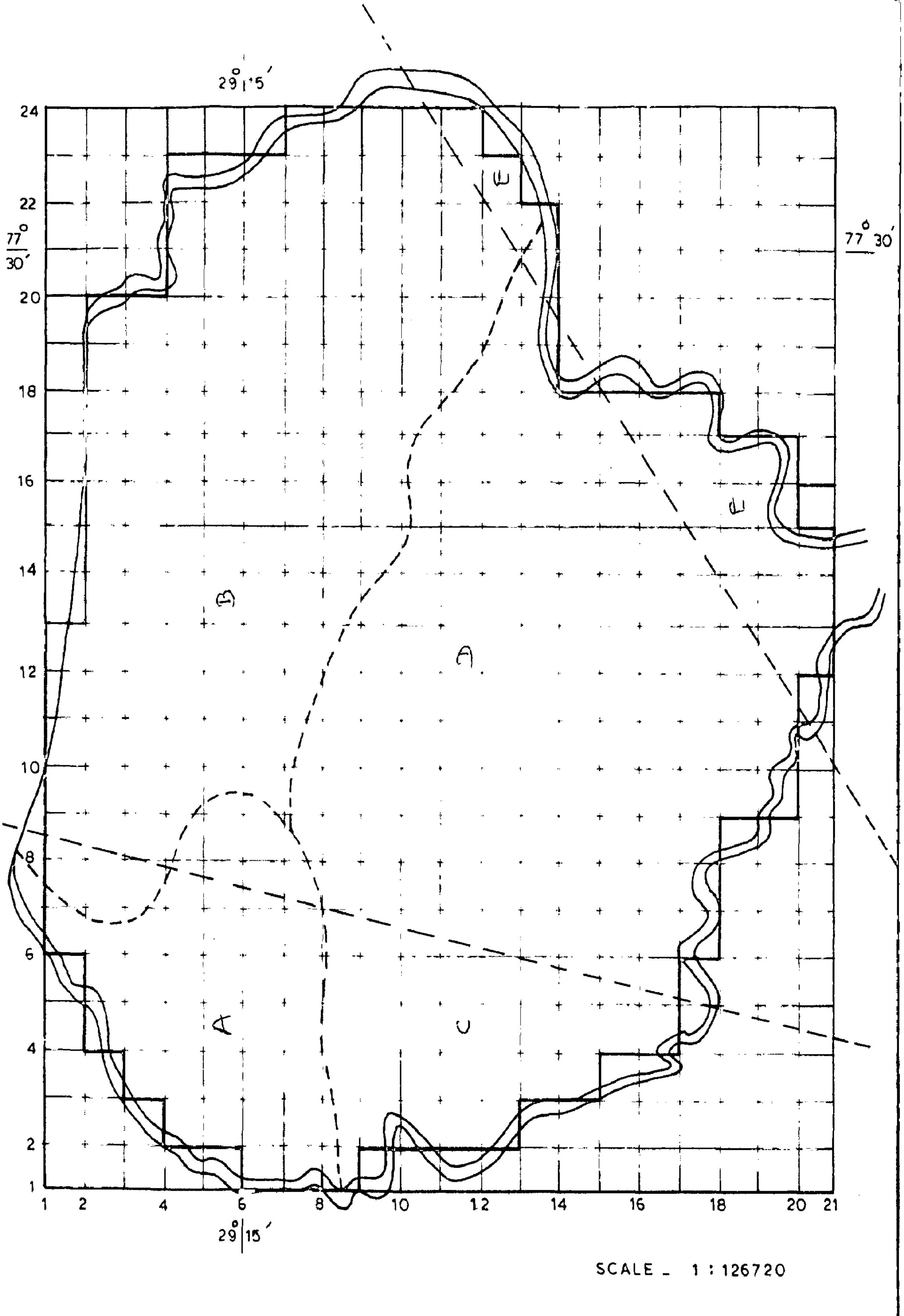
For the purpose of processing the data the aquifer is divided into five regions, designated as A, B, C, D and E as under.

Raingauges → Blocks ↓	Kandhla	Budhana	Sardhana
Budhana	A	B	E
Binauli	C	D	

There are four raingauges around the area at Kandhla, Budhana, Sardhana and Baghpat. Thiessen polygons were drawn for these four raingauges. It is found that no part of the area falls under the polygon for Baghpat. This gives three divisions on the basis of rainfall pattern.

From the point of view of crop pattern, the area gets divided into two parts. One falls under under Budhana block and also covers parts of Kandhla Block. The other falls in the Binauli block. The Thiessen polygon area of Sardhana raingauge gets divided into two noncontiguous parts, one falling under the Budhana block, the other under Binauli block. The two parts individually are very small, and are therefore considered as one common region, with crop pattern composed of weighted average of the two blocks. The divisions of the aquifer into five regions is shown in Fig. 15 by dotted lines.

For purpose of simulation of recharge due to rainfall, regions A and C are grouped together, regions B and D are combined and region E is treated separately.



SCALE _ 1 : 126720

FIG. 13 _ FINITE DIFFERENCE GRID SUPERIMPOSED ON PROJECT AREA

Withdrawals

Withdrawals from the aquifer are in the form of pumpage, evapotranspiration losses, out flow to streams or adjoining aquifer etc.

The data of pumpages are available on an annual basis. An average rate of pumpage round the year can not be presumed as the agricultural practice necessitates substantial temporal variations. The monthwise pumping patterns for each region are worked out separately, on the basis of consumptive use concept, as detailed below.

Consumptive Use Requirements:

Consumptive use is the term applied by the agronomists to the quantum of water consumed in the plant physiological processes, and indicates the water requirements of the crops. If this requirement is not fulfilled by natural precipitation, then the quantity falling short should be supplied by irrigation. Class A pan evaporation data give an overall picture of the various meteorological factors and can be used for assessment of the consumptive use. Multiplication of the pan data by the consumptive use coefficients for the type of crop, give the water requirement of the crop.

The crop pattern for each of the five regions was assessed from the data available and is presented in Table No.4.

The monthwise crop water requirement for each crop in each region are worked out separately. If rainfall had

TABLE NO. 4

CROPPING PATTERN OF DAHA AREA

(All areas are in Hectares)

Area	Wheat	Sugarcane	Rice	Other Kharif	Other Rabi	Total	Forest & Orchard
KANDHLA							
A	1989	1238	133	1213	595	5168	Nil
B	1683	1159	39	922	458	4241	6
Total	3672	2397	172	2135	1053	9409	6
BUDHANA							
C	5768	3604	387	3529	1751	15039	114
D	6894	4766	161	3796	1803	17420	160
Total	12682	8370	548	7325	3554	32459	274
SARDHANA							
E	1082	759	29	600	287	2757	492
GRAND TOTAL	17436	11505	749	10060	4894	44605	792

taken place in a particular month, then the corresponding effective rainfall is worked out, by reference to standard tables giving values of effective rainfall as a function of the normal monthly rainfall and the average monthly consumptive use. The net irrigation requirement (NIR) was computed as a difference between the monthly consumptive use and the effective rainfall. These figures gave the quantum of water that should have been applied at the field. However the irrigation efficiency is presumed to be 65%. To account for this the NIR figures were divided by 0.65 giving thereby the Field Irrigation Requirement. The delivery efficiency was assumed to be 90%, and the FIR figures were divided by 0.90 to get the figures of Gross Irrigation Requirements (GIR).

The GIR figures worked out as above are in mm of water depth. They were multiplied by the area under each crop for each region separately to arrive at the volume of water required for irrigation.

Pumpage Data :

Data of pumpage for different irrigation works are available in different forms. For state tubewells, for each year the figures of average discharge and total hours of running for each well are available separately. Since the location of each tubewell is known, the group to which it belonged could be ascertained. The volume of water pumped by each well in a particular year were worked out ~~and~~ and added separately for each group to get the regional subtotals. The grand totals were also worked out therefrom.

For private tubewells, masonry wells with Rahats, pumping sets and drinking water wells, detailed data are not available. The number of these units in each cell are known and their average running hours and discharge is ascertained from results of sample survey. The regionwise distribution of these works was worked out and the volume of water pumped by them were worked out as a lumped model for each region.

All pumpage volumes as worked out above were added together and compared with the irrigation requirements as calculated from the consumptive use pattern. The pumpages are found to be very low compared to the crop water requirements. However they are factual data, whereas the crop water requirements as worked out on the basis of consumptive use are theoretical figures. The latter however are useful in giving at least some idea of the monthwise variations of irrigation water demand. The monthwise distribution of the pumpages was therefore worked out on the basis of the pattern indicated by the crop water requirement, separately for each region. The drinking water well pumpages were assumed uniform over the year.

Villagewise area statistics are available from which the regionwise average of forests and orchards was worked out. The pan evaporation figures were applied to these figures for working out the volume of ground water lost as evapotranspiration every month.

The total draft consisting of irrigation pumpage, drinking water and evapotranspiration losses, was thus worked out for each month for each region, and then converted to flow rates expressed as m^3/day. These flow rates, when multiplied by the current scale S_a gives the electrical current required for simulating the pumpage (draft).

WATER BALANCE CALCULATIONS:

No hydrological analysis can proceed unless the water balance of the area has been assessed. The quantitative statement of the periodic water balance of an aquifer can be expressed by Eq. (1), discussed under Sec. 2.1 earlier.

Water balance studies are carried out for each season separately. Monsoon season is reckoned to last from June to October, whereas nonmonsoon is taken from November to May of the next year. The water balance studies are carried out for the entire area as a lumped model and not separately for each region as the region boundaries are quite arbitrary. The basis for estimation of the various parameters are discussed below :-

Rainfall Recharge (R_r)

A number of empirical formula were put forward by various workers. They are :

(1) $R = 2.0 (P - 15)^{0.4}$ (Chaturvedi)

(2) $R = 2.50(\phi - 16)^{0.5}$ (Amritsar)

(3) $R = 1.35 (\phi - 14)^{0.5}$ (IRI)

Datta et al (1977) have reported results of tritium studies on right bank of Yamuna and have estimated 20% as the recharge rate.

Rather than relying upon the above formulae, the recharge rates have been worked out from the water balance studies. For monsoon season satisfactory water balance is achieved if the rainfall recharge is reckoned at 22%. For non-monsoon period, rainfall less than 1 mm/day was ignored. The additional rainfall, if any, was assumed to give 14% recharge provided antecedent rainfall had occurred.

The percentage rainfall recharge can however be easily adjusted on the model with the help of a potentiometer. It can therefore be made one of the variables whose effects can be studied on the model.

Recharge from Canal Seepage (R_c)

This component does not exist in the area because there is no canal in the area. The small percolation through the ground water irrigation channels is included in the R_i.

Irrigation Recharge (R_i)

Irrigation efficiency is assumed to be 65% whereas delivery efficiency is assumed to be 90%. This means that out of each cubic meter of water applied at the head of the irrigation channel, $0.65 \times 0.9 = 0.585$ m^3 of water is useful for irrigation. The rest i.e. 40.5% is lost as seepage. For groundwater, this is a gain in the form of return flow.

Ground Water Inflow (I_G)

This factor is usually applicable at the arbitrary boundaries. The quantity of water crossing such boundary is given as

$$I_G = T\,i\,L\,.\,P$$

where

Q = inflow (m^3)

T = transmissibility (m^3/day)

i = head gradient (quadrant)

L = length of boundary

P = Period (days)

The boundary on the north has a length of 11.2 km. The T value is constant. The gradients were worked out from water level contours for May and November and average gradient for each season worked out therefrom. Calculation of I_G for each period was then a matter of simple multiplications.

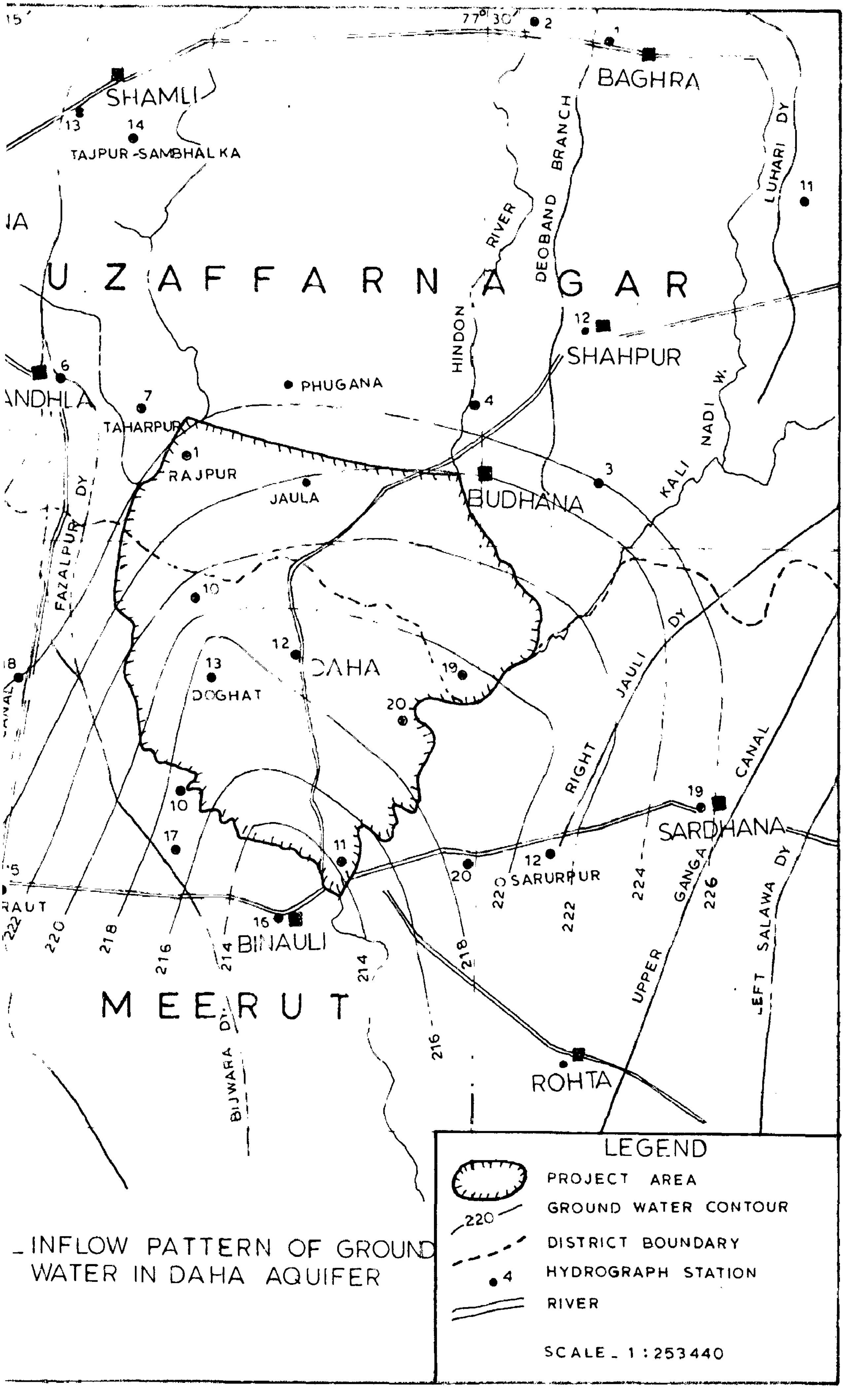
Influent Seepage

The existence of two rivers along more than 50% of the boundary lead to expectation of influent or effluent seepage. It is very difficult to estimate these components even under normal circumstances. In the present case, it is all the more difficult, because discharge data are not available. The only discharge data available were taken at Galota located about 10 km downstream of the confluence of the two rivers. Moreover the two rivers have large catchment areas to the upstream of the study area.

With a view to getting at least a qualitative idea of the conditions at the area it was considered necessary to study the pattern of water level in the region outside the study area also. The data of water levels in the Meerut and Muzaffarnagar districts were procured by special efforts. On plotting the water-level contours, a very interesting feature was noticed.

The water level contours are shown in Fig. 14. It can be seen that the contours are shaped like a horse shoe indicating that there exists a ground water trough in the Doab area inducing considerable inflow from the irrigated areas just across the rivers. This behaviour is noticed for monsoon as well as non-monsoon periods. The positions of the contours were found to fluctuate between narrow bands. This peculiar situation is possible because Pasalpur distributary and the Dim ~~Bijnore~~ Bijwara distributary of the Eastern Yamuna Canal were almost parallel to the river Krishni at a distance of about 9 km to the west. On the eastern boundary, the Deoband branch and the Right Jaulli distributary of the Upper Ganga Canal come close to the river Hindon.

Under the special situation described above the inflow of water from the east and west boundaries need not be classified as to whether it is influent seepage or whether it is ground water inflow. For the purpose of the analog, it would be adequate to simulate an equipotential boundary along the approximate location of the contour of R.L.226.00 m and treat it as an equipotential boundary, disregarding the existence of the river.



_INFLOW PATTERN OF GROUND WATER IN DAHA AQUIFER

For the purpose of water balance studies, the net effect of inflow across the boundary was reckoned and calculated. This may be termed as combination of influent seepage and groundwater inflow.

Effluent Seepage (S_e)

The above discussion indicated that there is no net effluent seepage to the rivers from the Doha aquifer. The stream discharges during non-monsoon period got all their water from the irrigated area outside study area and no water from the study area. S_e is therefore taken to be zero.

Ground Water Outflow (O_g)

Ground water outflow takes place along the small length southern boundary. The quantities were worked out in a manner similar to that adopted for I_g.

Evapotranspiration (E_t)

The assessment of this component has been described earlier under section of withdrawals. It is based on pan evaporation data of Meerut applied to the areas of forests and orchards.

Ground Water Withdrawal (T_p)

This component is a major one. How it was calculated for each month has been described earlier under withdrawals. For purposes of water balance the monthly figures were totalled for 5 and 7 months for monsoon and non-monsoon period respectively.

Change in Storage (ΔS)

Water level contours for each month were plotted. For months of June and November, the actual storages above R.L. 210.00 m. selected as datum were calculated for each contour and added up to get the total figures. These volumes were then multiplied by the storage coefficient (= 0.118) to get the actual volume of water in the voids. The change in storage during any period is then given by the difference between storage at the end and at the beginning of the period.

Overall Balance

The water balance computations are shown in Table No.5. These figures have ~~to~~ two uncertainties. ~~Finally~~ Firstly the percentage of rainfall recharge may be different from that assumed. Secondly the quantum of inflow of water from adjoining areas may not be correct. The first condition can be easily made a variable on the analog. The second condition can be examined but with a little difficulty. The operation of the analog therefore becomes a challenging, and therefore an interesting one. After all the very purpose of modelling is to be able to assess such otherwise inaccessible parameters in a fairly reliable manner. And it is under such conditions that the analog model proves its worth because the operator can play around (purposefully of course) with the model and get a physical insight into the probable behaviour of the aquifer.

TABLE No. 5

GROUND WATER BALANCE OF DAHA AREA

Item	1972-73		1973-74		1974-75	
	1.6.72 to 31.10.72	1.11.72 to 31.5.73	1.6.73 to 31.10.73	1.11.73 to 31.5.74	1.6.74 to 31.10.74	1.11.74 to 31.5.75
T_P	5858.3713	10099.4163	3944.3192	8300.3450	4372.4393	6501.2740
E_T	574.1950	557.7874	574.1950	557.7874	574.1950	557.7874
O_g	24.8887	30.7019	17.2426	32.8268	30.0196	38.1740
ΔS_g	211.7935	-1714.6273	2377.4557	-2701.5167	-927.8128	-2492.1788
$T_P+E_T+O_g+\Delta S_g$	669.1485	8973.2783	6913.2125	6189.4425	4048.8411	4605.0566
R_I	1845.3554	3181.3160	1242.4605	2614.6087	1377.3184	2047.9013
R_C	0.0	0.0	0.0	0.0	0.0	0.0
$I_g(N)$	189.6019	256.7676	153.6412	170.2837	152.8759	278.0911
$I_g(E+W)$	58.4536	5510.1312	939.5593	3404.5501	287.2892	2279.0642
$R_I+R_C+I_g(N)$ $+E_{(g)}(E+W)$	2093.4109	8948.2148	2335.6610	6189.4425	1817.4835	4605.0566
R_1	4575.7376	25.0635	4577.5515	0.0	2231.3576	0.0
Percent of rainfall	22~~2~~	14	22	0	18	0

Note - All units are hectare meter

CHAPTER - 4

DESIGN OF MODEL & EXPERIMENTAL SETUP

The analog model for Daha aquifer is designed keeping in view the availability of the components, equipment and the desirability of keeping the grid spacing as small as practicable. After a few trials the following combination of components, scales and spacing is found to be workable.

COMPONENTS

Resistors (for internal nodes)	100 kiloohms
Capacitors	15 Micro farads

GRID SPACING & SCALE

Uniform grid of 1084 m spacing is adopted because the study is a regional one and there is no area of specific interest. The hydraulic gradients are also more or less uniform.

VOLTAGE SCALE

The maximum water level is observed to be 226 m whereas the minimum is about 214. We may provide for variation from 210 to 230 m, i.e. a range of 20 m. Considering that the recorder can take a maximum of $\overset{4V}{2V}$, the voltage scale is chosen as 0.2 volts/m.

RESISTANCE SCALE

It is proposed to use resistors 100 kilo ohms. The transmissibility the aquifer is found to be 1771.2 m^2/day. Thus we get

$$S_x = \frac{R_x T_x \cdot \Delta y}{\Delta x}$$

$$= \frac{10^5 \times 1771.2 \times 1013.76}{1013.76}$$

$$= 1.7712 \times 10^8 \text{ ohms.m}^2/\text{day.}$$

CAPACITANCE SCALE

Capacitors of 15 microfarads are to be used. The storage coefficient is 0.118. These values give

$$S_c = \frac{C}{\Delta x.\Delta y \cdot S}$$

$$= \frac{15 \times 10^6}{0.118 \times 1013.76 \times 1013.76}$$

$$= 12.3691 \times 10^{-11} \text{ Farads/m}^2$$

These scale factors give the following auxillary scales as wider.

Time Scale:

$$S_t = S_r \;\; S_c$$

$$= 1.7712 \times 10^8 \times 12.3691 \times 10^{-11}$$

$$= 2.1908 \times 10^{-2} \text{ sec/day}$$

$$= 7.99649 \text{ sec/yr.}$$

$$= 8 \text{ sec/yr approximately.}$$

Current Scale:

$$S_a = \frac{S_v}{S_r}$$

$$= \frac{0.2}{1.7712 \times 10^8}$$

$$= 1.1292 \times 10^{-9} \text{ amp. per (m}^3/\text{day)}$$

$$= 1.1292 \times 10^{-3} \text{ microamp per (m}^3/\text{day)}$$

Volume Scale:

$$S_q = S_v \cdot S_c$$
$$= 0.2 \times 12.3691 \times 10^{-11}$$
$$= 2.47382 \times 10^{-11} \text{ coulombs/m}^3$$

The current scale S_c was used to convert the recharge and draft calculated in m^3/day per region, from water balance studies, to microamperes of current for the purpose of design and adjustment of the current generators. These figures are given in Table No.6.

Model Set Up:

The aquifer of Doha area has been discretised in the form of a uniform square grid of 1013.76 m spacing Fig. 13. Each internal node represents an area of 102.77093 ha. The boundary and area corner nodes represent smaller areas. There are total 330 cells. The nodes are distributed as under:

	Nodes	Cells
Internal nodes	288	288.0
Convex corners	22	5.5
Boundary corners	46	23.0
Concave corners	18	13.5
Total	374	330.0

This ensures that the total area represented by the model tallies fairly well with the actual area of the aquifer.

ERU3

Sl. No.	Month	Currents for ciculation of inflow out flow N E W	S
1	6/72	+26.0	-1.6
2	7/72	+14.8	-1.6
3	8/72	+14.6	-1.6
4	9/72	+18.0	-1.6
5	10/72	+18.4	-1.6
6	11/72	+212.8	-1.6
7	12/72	+160.4	-1.6
8	1/73	+130.4	-1.6
9	2/73	+340.4	-1.6
10	3/73	+515.6	-1.6
11	4/73	+318.0	-1.6
12	5/73	+412.4	-1.6
13	6/73	+102.4	-1.2
14	8/73	+23.0	-1.2
15	8/73	+45.0	-1.2
16	9/73	+107.4	-1.2
17	10/73	+50.0	-1.2
18	11/73	+109.2	-1.2
19	12/73	+80.0	-1.2
20	1/74	+ 87.0	-1.8
21	2/74	+234.2	-1.8
22	3/74	+229.4	-1.8
23	4/74	+133.8	-1.8
24	5/74	+166.4	-1.8
25	6/74	+56.4	-2.2
26	7/74	+25.4	-2.2
27	8/74	+20.6	-2.2
28	9/74	+32.4	-2.2
29	10/74	+30.4	-2.2
30	11/74	+105.4	-2.0
31	12/74	+50.0	-2.0
32	1/75	+34.6	-2.0
33	2/75	+107.2	-2.0
[illegible]	[illegible]	[illegible]	-2.0
[illegible]	[illegible]	[illegible]	-2.0
36	5/75	+135.4	-2.0

TABLE-6 CURRENTS REQUIRED TO SIMULATE RECHARGE AND DRAFT PATTERNS

(All currents are in micro amperes)

Sl. No.	Month	currents for regions A and B	Pumpage ~~regions~~ C and D regions	Simulation Region E	Currents for rainfall recharge simulation			Currents for simulation of inflow out flow	
					Regions A And C	Regions B and D	Region E	N E W	S
1	6/72	-185.4	-409.6	-12.4	+ 25.2	Nil	+5.4	+26.0	-1.6
2	7/72	-39.2	-77.8	-38.8	+172.2	+539.0	+44.6	+14.8	-1.6
3	8/72	-36.2	-63.8	-37.4	+119.2	+539.6	+34.2	+14.6	-1.6
4	9/72	-94.6	-172.8	-39.4	+111.4	+157.2	+19.0	+18.0	-1.6
5	10/72	-69.8	-131.6	-32.8	+25.6	+34.6	+1.2	+18.4	-1.6
6	11/72	-76.6	-136.2	-22.6				+212.8	-1.6 -1.6
7	12/72	-45.8	-102.0	-14.2				+160.4	-1.6
8	1/73	-58.8	-80.8	-17.4				+230.4	-1.6
9	2/73	-149.6	-128.4	-30.4				+340.4	-1.6
10	3/73	-347.0	-227.2	-56.0				+515.6	-1.6
11	4/73	-143.2	-148.2	-49.4				+318.0	-1.6
12	5/73	-194.0	-204.0	-61.4				+412.4	
13	6/73	-190.4	-207,8	-62.4	Nil	+187.6	+18.6	+182.4	-1.2
14	7/73	-40.2	-38.4	-33.4	+142.4	+627.2	+27.8	+23.0	-1.2
15	8/73	-37.2	-32.4	-32.2	+229.2	+364.0	+12.6	+45.0	-1.2
16	9/73	-97.4	-88.8	-33.8	+35.6	+62.0	+8.4	+107.4	-1.2
17	10/73	-71.2	-66.6	-28.2	+14.2	+201.0	+7.6	+50.0	-1.2
18	11/73	-67.8	-72.6	-15.2	-	-	-	+109.2	-1.2
19	12/73	-40.2	-54.2	-9.6	-	-	-	+80.0	-1.2
20	1/74	-52.4	-104.4	-13.6	Nil	-	-	+ 87.0	-1.8
21	2/74	-133.0	163.8	-23.8	-	-	-	+234.2	-1.8
22	3/74	-219.6	-293.2	-43.6	-	-	-	+229.4	-1.8
23	4/74	-127.4	-181.2	-38.6	-	-	-	+133.8	-1.8
24	5/74	-172.4	-2.634	-47.8	-	-	-	+166.4	-1.8
25	6/74	-183.0	-272.6	-54.2	+14.2	+42.4	-	+56.4	-2.2
26	7/74	-38.6	-50.4	-31.0	+97.6	+285.2	+34.0	+25.4	-2.2
27	8/74	-35.8	-42.4	-28.0	+74.0	+380.8	+34.0	+20.6	-2.2
28	9/74	-93.4	-116.4	-29.44	+87.4	+36.2	+6.4	+32.4	-2.2
29	10/74	-68.4	-87.2	-24.6	-	-	+1.0	+30.4	-2.2
30	11/74	-36.2	-38.4	-14.8	-	-	-	+106.4	-2.0
31	12/74	-21.6	-25.0	-9.4	-	-	-	+50.0	-2.0
32	1/75	-28.0	-48.0	-13.2	-	-	-	+34.6	-2.0
33	2/75	[illegible]	-76.2	[illegible]	-	-	-	+107.2	-2.0
34	3/75	[illegible]	[illegible]	[illegible]	-	-	-	[illegible]	-2.0
35	4/75	[illegible]	[illegible]	[illegible]	-	-	-	[illegible]	-2.0
36	5/75	-92.2	[illegible]	-47.0	-	-	-	[illegible]	[illegible]

The boundaries are approximated in a jagged pattern as shown in Fig. No.13. Since the grid spacing is reasonably small this type of approximation is not likely to induce serious errors in the solution. The values of resistors and capacitors for the nodes on the boundary are worked out in accordance with Eq.(24) and(27) The values are as under:

Resistors along boundary	200 kilo ohms
Capacitors along boundary nodes	7.5 micro farads
Capacitors on convex node points	3.75 Micro farads
Capacitors on concave node points	11.25 micro farad

Capacitors of the last two types are not in the standard range of manufacture. They are therefore prepared by a judicious combination of the available components so that the actual value is as close to the required value as possible.

The resistors of 200 K.ohms are not readily available. Those of 220 kiloohms are therefore required to be used. All the components have manufactured tolerance of ± 10%.

STREAM AQUIFER INTERACTION

Methods of representation of stream aquifer interaction have been discussed in section 2.5. Since more than 80 percent of the boundary of Doho aquifer is formed of rivers, stream aquifer interaction commanded special attention. On deeper examination, it was however found that the rivers have very little effect on the aquifer behaviour. Water balance studies discussed earlier have revealed that substantial quantities are coming across the boundary from the adjoining areas. This is also substantiated by the shape of the contours within and in the vicinity of the area. The contours are shaped like a horse-shoe indicating a ground water trough in the Doab area. The rivers at best serve as minor leakages in the flux of water flowing underneath them at a skew angle. It is therefore considered that the rivers need not be simulated.

The horizontal inflow into the area is represented by an equipotential boundary corresponding to water level of 226.00. This boundary is located 9 grid spacings away from the geographical boundary. They roughly coincide with the Bijawara and Pasalpur distributaries of the Eastern Yamuna Canal, and Deoband branch and Right Jauli distributary of the Upper Ganga Canal.

The area between the geographical boundary and the equipotential boundary is simulated by a network of only resistors, of 100 kilo ohms each. During the operation of the model, these will be adjusted by trial and error so as to ensure correct levels at nodes corresponding to the observation wells near the boundaries. This adjustment consists of changing the shape and location of the equipotential boundary, or changing the intervening resistors, or both.

There is a very small outflow across the southern boundary which has a length of only 9 grid spacings. It is therefore simulated as a constant (negative) current boundary.

INPUTS

The excitation of the model is required to be done by means of input and output devices.

The input is in two forms. The first is the recharge due to rainfall. This is presumed to be uniformly distributed over the Theissen polygon of each raingauge. The regions A and C fall in the influence of raingauge at Kandhla. The regions B and D cover the raingauge at Budhana. Region E is associated with the raingauge Sardhana. Three different pulse generators yielding currents reproducing the pattern of rainfall as recorded by the three raingauges, are set up. The recharge current is fed into the model at selected node points through diodes to prevent return flow during non-monsoon period. The mode of selecting the feedin points is discussed subsequently.

Another input is the inflow through the boundary and is discussed earlier.

The major draft from the aquifer is the withdrawal, including pumpage, evapotranspiration losses etc. The method of calculating the monthly draft figures has been discussed earlier. These figures are converted into the currents using the scale factor S_a. With a view to keeping the number of equipments low, only three current generators are set up. For this purpose, the regions A and B are grouped together as they are having essentially the same crop pattern. For same reason, regions C and D are combined together. Region E is treated separate. The withdrawal currents are or fed to the model in a fashion similar to that for the recharge currents.

As an alternative to the above arrangement, the net withdrawal pattern can be worked out by algebraic summation of the recharge and withdrawal. The resulting pattern can then be reproduced by the current generators. This could have resulted into five equipments as against six equipments required if recharge and withdrawal are simulated separately. The latter, however, is ~~preparation~~ preferable on two accounts. The current does not change sign (direction). This is important because the diodes prevent flow of current in the opposite direction. Secondly, if the rainfall is a reproduced separately then the percentage of recharge can be adjusted easily but not so in the net withdrawal/recharge pattern.

The input and output currents are fed into certain node joints selected according to following considerations. The number of such node points should be neither too large nor too small compared to the total number of node points. They should be arranged in a uniform pattern to the extent possible. These points should not overlap with the observation well nodes. Minimum number of these points should fall on the boundary. A uniform grid with spacing three times that of the analog grid with origin at (1.0) is found to satisfy the above criteria. The recharge and withdrawal node points are arranged in a staggerred pattern, as shown in Fig. No.13.B

Feed in resistors were calculated for each point of recharge/draft. For this purpose, weightages of each such point were worked out proportionate to the area served by them. The current to be fed to each point was calculated on the basis of these weightages. The voltage at each point was assessed. The feed in resistor was then calculated by the formula :

$$R_f = \frac{V}{I_f} \qquad (42)$$

Where

R_f = feed in resistance (ohms)

V = voltage with reference to that of current generator (Units)

I_f = feed in current (amp.)

The values of the feed in resistors are presented in Table 7 and 8.

Diodes were connected to each feed in resistor, so as to ensure that the feed in points may remain at different voltages.

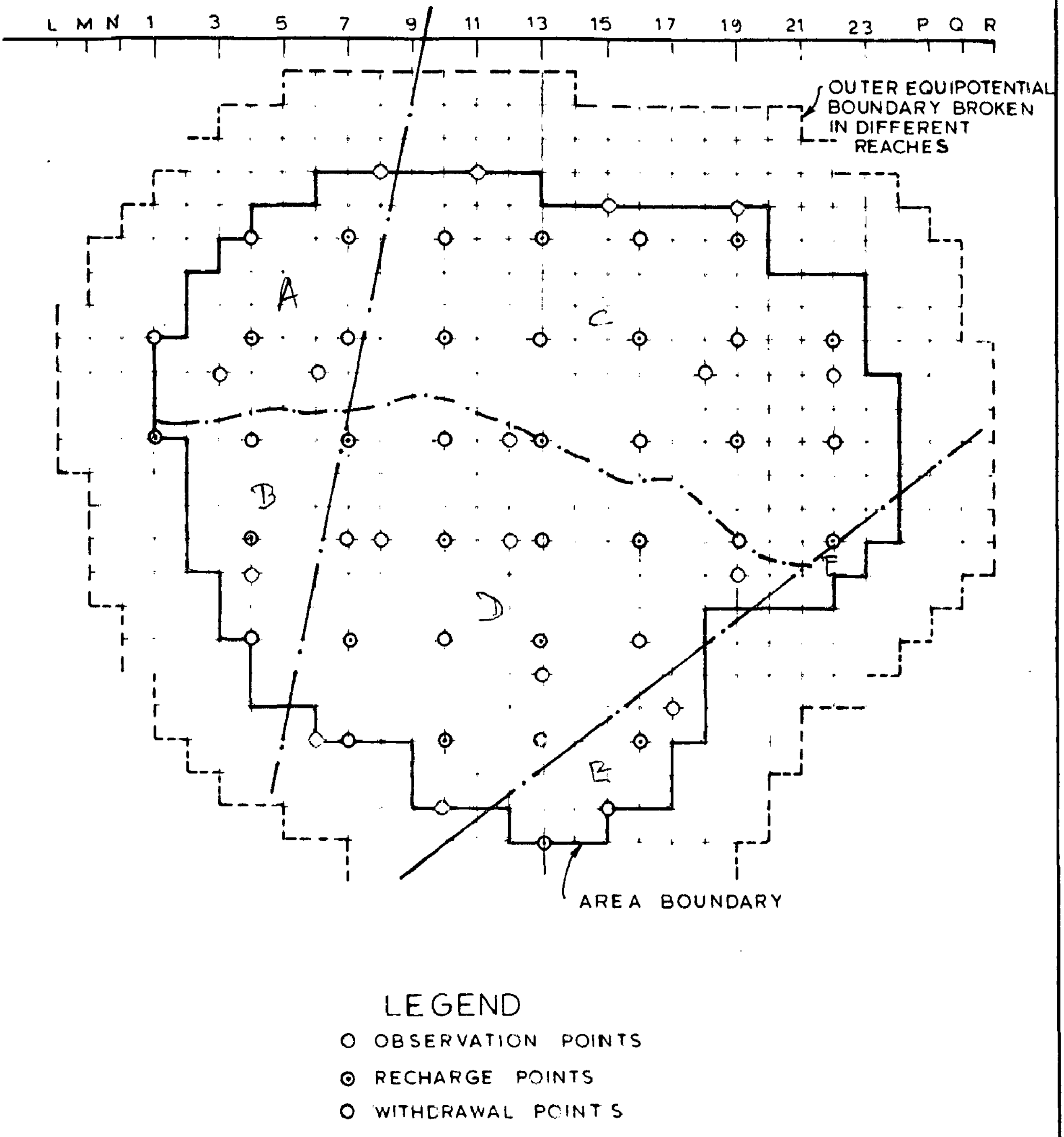


FIG 13B FINITE DIFFERENCE GRID SUPERIMPOSED ON DAHA AREA AQUIFER

TABLE NO. 7

CALCULATIONS FOR FEED IN RESISTORS FOR RECHARGE CURRENTS

Sl. No.	Region	Node Coordinate	Weightage of node	Initial water level	Condition Voltage	Current ~~and~~ micro-~~ohms~~ amps	Resistor kilo-ohms	Remarks
1	A	3,7	1.0	222.5	2.50	46.28	76	Reference Voltage=6V
2	A	6,4	1.1	221.0	2.20	50.91	78	
3	C	9,1	0.3	220.3	2.06	13.88	282	
4	C	12,4	0.9	219.6	1.92	41.65	98	
5.	B	3,13	0.8	223.6	2.72	27.67	118	
6.	B	3,19	0.6	222.5	2.50	20.76	169	
7.	B	6,10	1.0	221.9	2.26	34.59	103	
8.	B	6,16	1.0	221.9	2.38	34.59	105	
9.	B	6,22	0.7	221.6	2.32	24.22	152	
10.	B	9,7	1.0	220.1	2.02	34.59	126	
11.	B	9,13	1.0	219.8	1.96	34.59	117	
12.	D	9,19	1.0	22.4	2.28	34.59	108	
13.	D	12,10	1.0	218.8	1.76	34.59	123	
14.	D	12,16	1.2	220.2	2.04	41.51	55	
15.	D	12,22	0.9	220.8	2.16	31.13	123	
16.	D	15,7	1.0	217.9	1.58	34.59	128	
17.	D	15,13	1.0	217.6	1.52	34.59	130	
18.	D	18,10	0.8	217.2	1.208	27.67	164	
19.	E	18,16	0.8	216.2	1.24	29.69	161	
20.	E	21,13	0.3	215.3	1.06	11.14	44	

TABLE NO. 8

CALCULATIONS FOR FEED-IN RESISTORS FOR PUMPAGE CURRENTS

Sl. No.	Region	Node Coordinates	Node Weight	Initial Conditions		Current Micro-amp.	Resistor kilo-ohms	Remarks
				Water Level	Voltage			
1	(A,B)	3,4	0.8	221.7	2.34	19.06	123	Reference voltage 0V (ground)
2.	"	6,1	0.4	220.7	2.14	9.53	225	
3.	"	6,7	1.0	220.3	2.06	23.8	87	
4.	"	3,10	0.9	223.4	2.68	21.4	125	
5.	"	3,16	0.7	222.7	2.54	16.7	152	
6.	"	6,13	1.0	221.3	2.26	27.8	95	
7.	"	6,19	1.2	221.9	2.38	28.6	83	
8.	"	9,16	1.0	220.9	2.18	23.8	92	
9.	"	9,22	1.1	221.3	2.26	26.2	86	
10.	C ∪ D	9,0	0.9	220.3	2.06	27.5	75	
11.	"	15,4	0.7	219.0	1.80	21.4	84	
12.	"	12,7	1.0	218.9	1.78	30.55	58	
13.	"	12,13	1.0	219.2	1.84	30.55	60	
14.	"	12,19	0.9	220.7	2.14	27.5	78	
15.	"	15,10	1.0	217.8	1.56	30.55	51	
16.	"	15,16	0.9	218.5	1.70	27.5	62	
17.	"	18,7	0.5	217.6	1.52	15.27	100	
18.	"	18,13	1.2	216.0	1.20	36.66	33	
19.	"	9,10	1.0	219.6	1.92	30.55	63	
20.	E	14,21	0.35	220.5	1.10	19.95	105	
21.	E	20,15	0.65	214.9	0.98	37.05	26	

SIMULATION OF TIME VARIANT EXCITATIONS

The pattern of recharge as well as that of withdrawal in the study area are time-variant ones. It is not unusual to simulate the draft and recharge by constant average currents (Anderson, 1968, 1972, Bedinger et al 1970, Cuffin, 1970, Emery, 1966, Glover, 1967, Patten, 1965, Phillips et al, 1969, Schicht, 1964, Skinitzke and Da Costa, 1962, Speiker, 1968, Walton, 1970, White and Handt 1965). These have been reported to give satisfactory results. In the study area, the variation of these inputs and outputs is large so that it is considered desirable to feed the analogous currents in a time-variant pattern as close to their hydr~~aulic~~ologic equivalents as possible.

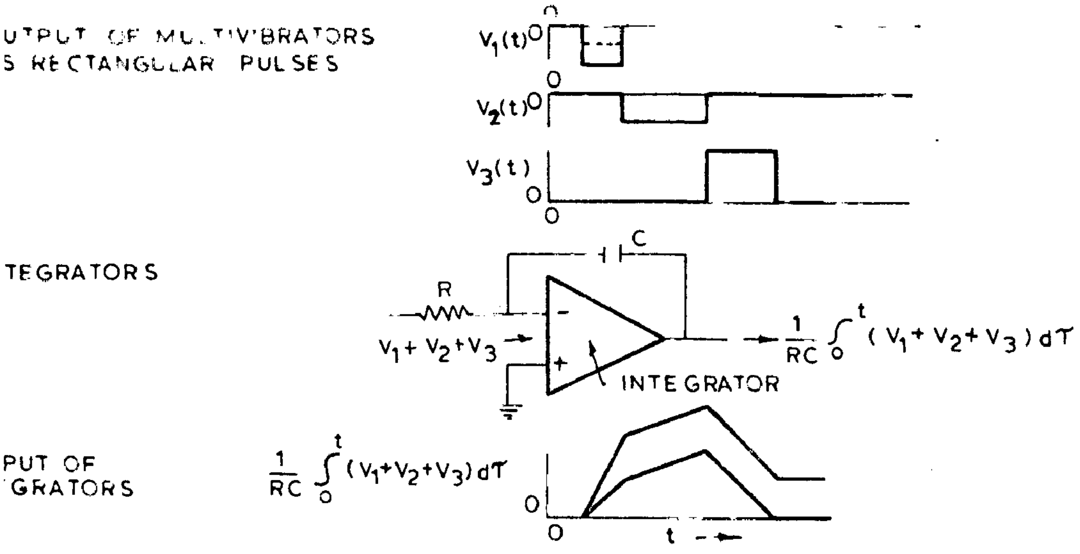
Generation of special functions of input and output currents can be easily done with the help of commercially available complex wave form synthesisers such as that used by Shah et al (1975). Such equipments are however prohibitively expensive, particularly when a large number of different patterns are encountered, for one would need one wave form generator for each different pattern. Attempts were therefore made to fabricate some function generators from simple readily available components. Three alternatives were considered :

(1) Synthesising wave form from linear segments, using multivibrators.

(2) Generation of step function using combination logic in digitally controlled amplifier.

WAVE FORM GENERATOR

The first alternative uses integration of rectangular pulses to produce voltage ramps as shown in Fig. 15. The principle is to approximate the curve by a series of straight lines, i.e. the voltage ramps. The slope and direction of each ramp can be controlled by the amplitude and polarity of the rectangular pulse, whereas the length of the ramp is controlled by the width of the pulse. One-shot multivibrators of type ~~4~~ 121 are connected as shown in the lefthand side block of Fig. 15 for producing the rectangular input pulses to the integrators shown on the right hand side block of the figure. The vibrators are arranged in series in such a way that the complimentary output. $\bar{Q}$ of one unit serves as the trigger for second unit and so forth. A sequence of positive rectangular pulses of varying amplitudes is generated by this arrangement. The duration of each pulse is determined by the timing resistor and capacitor of its one shot and the amplitude of each pulse is set by the potentiometer of the output of one shot. If a ramp with negative slope is desired then the positive pulse may be fed directly to the operational amplifier type 741. For producing positive slopes, an inverter has to be introduced in between. Prevention of drift is accomplished by providing an additional one shot unit and the 2N2481 transistor as shown in the figure. They hold the integrator output ~~of~~ at zero when there is no pulse, thereby preventing integration of offset voltages.



WAVE FORM GENERATION

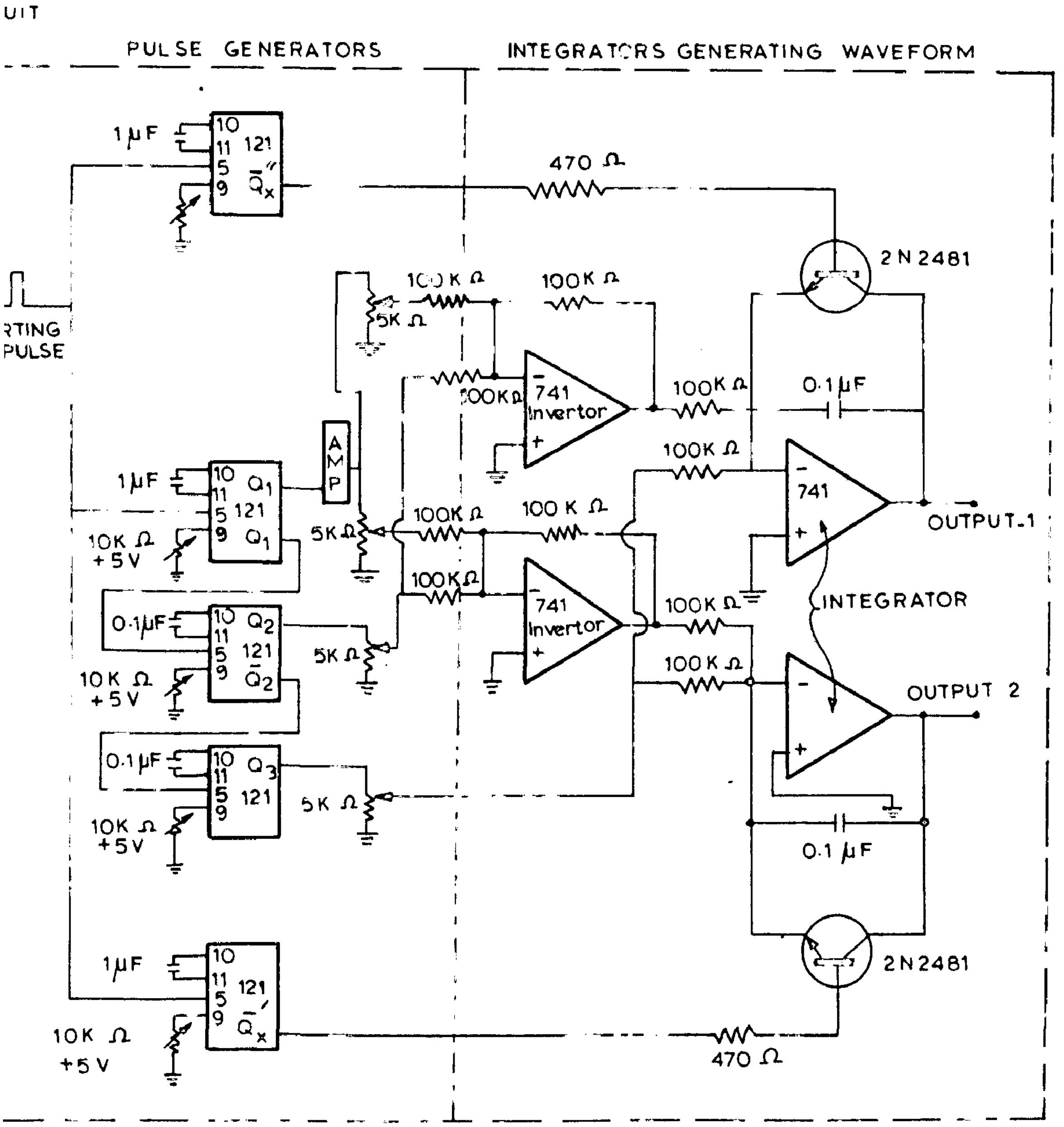


FIG.15 _WAVE FORM GENERATION BY INTEGRATION

The above arrangement is particularly useful if more than one wave forms are required to be generated provided that the slope changes are to be accomplished simultaneously. For instance, the withdrawal patterns of different regions may have different shapes, but being composed of monthly data, have a common time scale. In this case output from the one shot units may be fed to different potentiometers corresponding to each different pattern to be generated. These are then connected to separate integrator circuits for producing the different wave shapes as required. As an illustration, this is shown in the case of the first one shot unit, to show how two different wave shapes can be produced. The output of any one shot unit may have to be amplified before feeding to the respective pots if loading effects of the potentiometers cause problems.

Fig. 15 illustrates how two different wave shapes can be produced with the help of five one shot vibrators type 121, two intervers and two integrators. The output Q_1 of the first one shot is tapped by two inverters feeding to separate integrators producing two different voltage ramps rising from zero at time $t = 0$, to two different values at time $t = t_1$. The complementary output of the first one shot triggers the second unit, which provide rectangular pulse of equal amplitude and duration to the two inverters. They therefore produce two voltage ramps of identical slope and duration, but located at two different levels starting from the end points of the first two ramps. Similarly the third one shot produces two negative ramps because they are fed directly to the integrators. They start from the end points of the previous ramps.

Each integrator is provided with an additional pair of one-shot unit and transistor to serve as anti-drift control.

The above circuit can be extended to produce different wave shapes with the help of common set of one-shot units providing the rectangular pulses of required amplitude and duration.

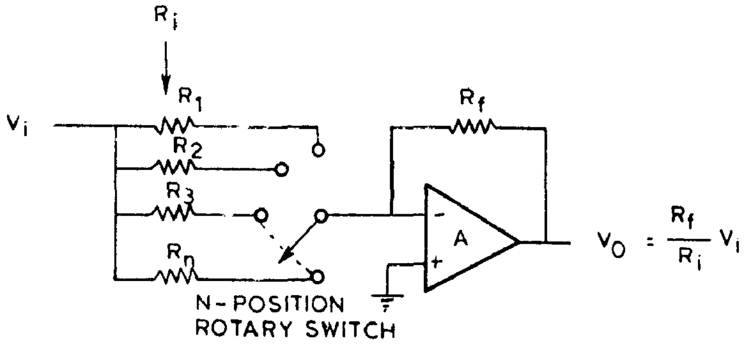
STEP FUNCTION GENRATOR

This unit can produce voltages which have the shape of step functions. The required pattern is discretised into a series of levels changing abruptly at the end of given time intervals. More often than not, the data available are basically in the nature of step functions (or histograms). For example the withdrawal pattern is worked out on monthly basis. The variation within a month is either not known or is ignored. A sloping pattern, if plotted, is only a process of smoothing out the abrupt changes by presuming a gradual variation. However, since the basic data available are in discrete form, it would not introduce serious inaccuracies, if the input/output currents are produces in a step function for.

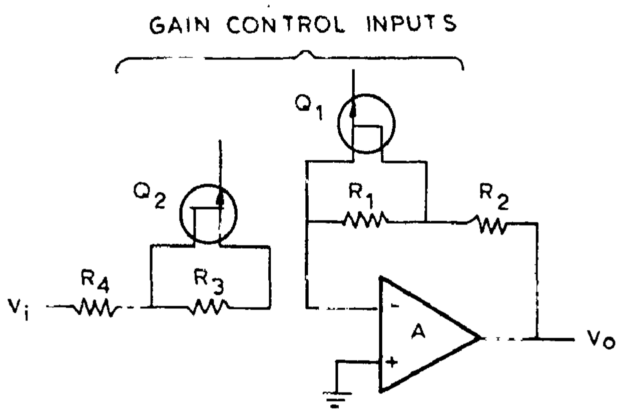
Step function wave forms can be produced by digitally controlled amplifier, using combination logic to minimise the number of components (Metz, 1977). The arrangement is shown in Fig. 16. It uses the simple basic principle that the gain of an inverting operational amplifier stage is equal to the ratio of feed back resistance R_f to the input resistance R_i. If one can change the ratio by changing R_f or R_i or both, the gain can also be changed accordingly. What is required in a suitable switching device. Field effect transistors (FET) can be advantageously utilised to serve as these switches.

Two resistors R_1 and R_2 are provided in the feed-back path. An FET is connected in parallel to R_1. If the FET is on, it serves to short circuit R_1, so that the effective resistance in the feed back loop is only R_2. When FET is in the off state, the effective feed back resistance is $R_1 + R_2$. Similar arrangement on the input side given input resistance, equal to R_4 when the FET connected across R_3 is on, and equal to $R_3 + R_4$ when the FET is in the off state. Thus there are four combinations available yielding gains as tabulated below :

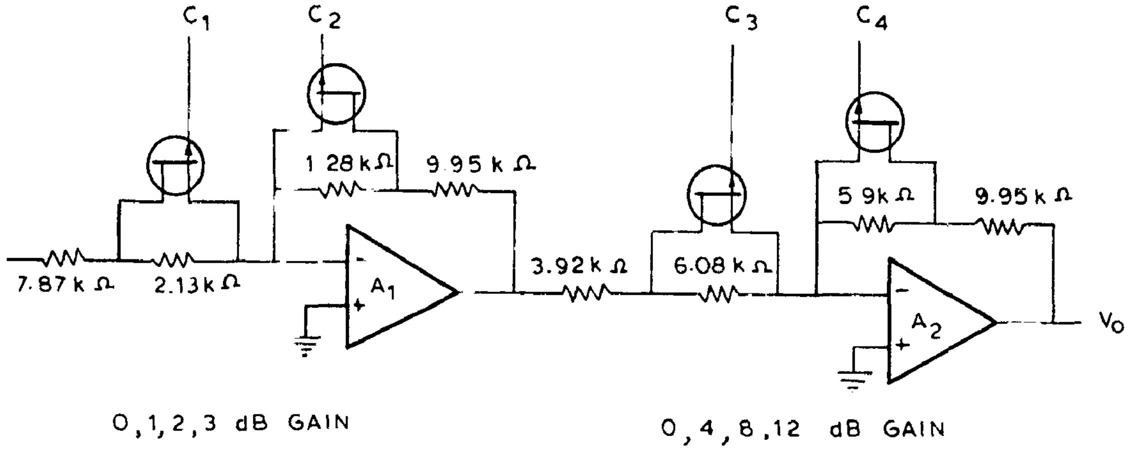
Input FET	FEEDBACK FET OFF	ON
~~IN~~ ON	$\frac{R_1+R_2}{R_4} = 10^{a/20}$	$\frac{R_2}{R_4} = 10^{c/20}$
OFF	$\frac{R_1+R_2}{R_3+R_4} = 10^{b/20}$	$\frac{R_2}{R_3+R_4} = 10^{d/20}$



Controllable gain



Combinations control



Cascade generating adjustable gain from 0 to 15dB in 16 1dB STEPS

FIG. 16 _ DIGITALLY CONROLLED AMPLIFIER

Here a, b, c and d gains. However, they are not arithmetic gains but dB gains, output voltage V_o is proportionate to antilogarithms of a, b, c or d depending upon the on-off state combination of the FETs. Moreover it has been shown that

$$a + d = b + c$$

The gains obtainable therefore form a symmetrical set such as 0, 1, 2, 3, dB or 5, 8, 12, 15.

The above principle can be applied for producing patterns which can be approximated by four stages such that they can be represented by decibel gains satisfying Eq. If it is desired to produce the pattern as a combination of 18 levels, it can be done by cascading two stages as shown in Fig. 16 . Any desired set of 4^s symmetrically spaced dB steps of amplification/attenuation can be accomplished using a cascade of s stage controlled by only 2s digital inputs. This drastically reduces the number of resistors required.

The above circuit can be gainfully used where the required voltages vary widely, and representation of smaller amplitude signals would be inaccurate if some arithmatic scale is adopted. It is however necessary that the required values are such that they can be fitted to a symmetric set of decibel levels. It is also necessary to have digital inputs for putting the FETs on or off in the required sequence of combinations. Moreover, for producing more than one patterns, common portions of circuits cannot be utilised.

The two circuits as described above produce voltage waveform. For conversion of the voltage waveform to the current wave a simple circuit consisting of a resistor and a transistor. The voltage waveform is applied, through a resistor to the base of a transistor. The emitter of the resistor is grounded. The output at the collector is the required current waveform.

The third alternative is the one actually adopted for the present studies. Several constraints such as non-availability of some components, shortage of time, etc. prevented adoption of one of the two circuits desired described above. A commercial function generator was borrowed from another laboratory. This unit provided the basic pulse, which was fed to six different circuits. Each circuit was composed of combination of resistors and capacitors designed to modified the input pulse to the required amplitude and duration of pulses, and individually adjusted and calibrated.

CHAPTER-5

OPERATION OF THE MODEL AND DISCUSSION OF RESULTS

The model for Daha area aquifer, designed and set up as discussed in Chapter-4, was operated, initially for the period June 1972- June 1973. This period covers the first two seasons for which water-balance studies were carried out.

Operation

The first step towards operation of the model required establishment of the initial conditions. The ground water level contours of June 1972 (Fig.17) provided the basis for this adjustment. There are 11 contours at intervals of 1 m each and ranging from El.215 m. to El.225 m. Only six constant voltage supplies were available. It was therefore possible to simulate only six contours at the interval of 2m each and ranging from 215m to 225 m i.e. odd value contours were simulated. As the contour of 225 m covers only about two cells, this was supplied voltage tapped and stepped down, from the supply for the equipotential boundary of 226 m. The following equation was used for calculating the voltage.

$$V_i = 0.2\ (\ H_i - 210.0) \qquad \ldots (43)$$

Where

V_i = voltage to be applied to a particular contour

H_i = contour denomination

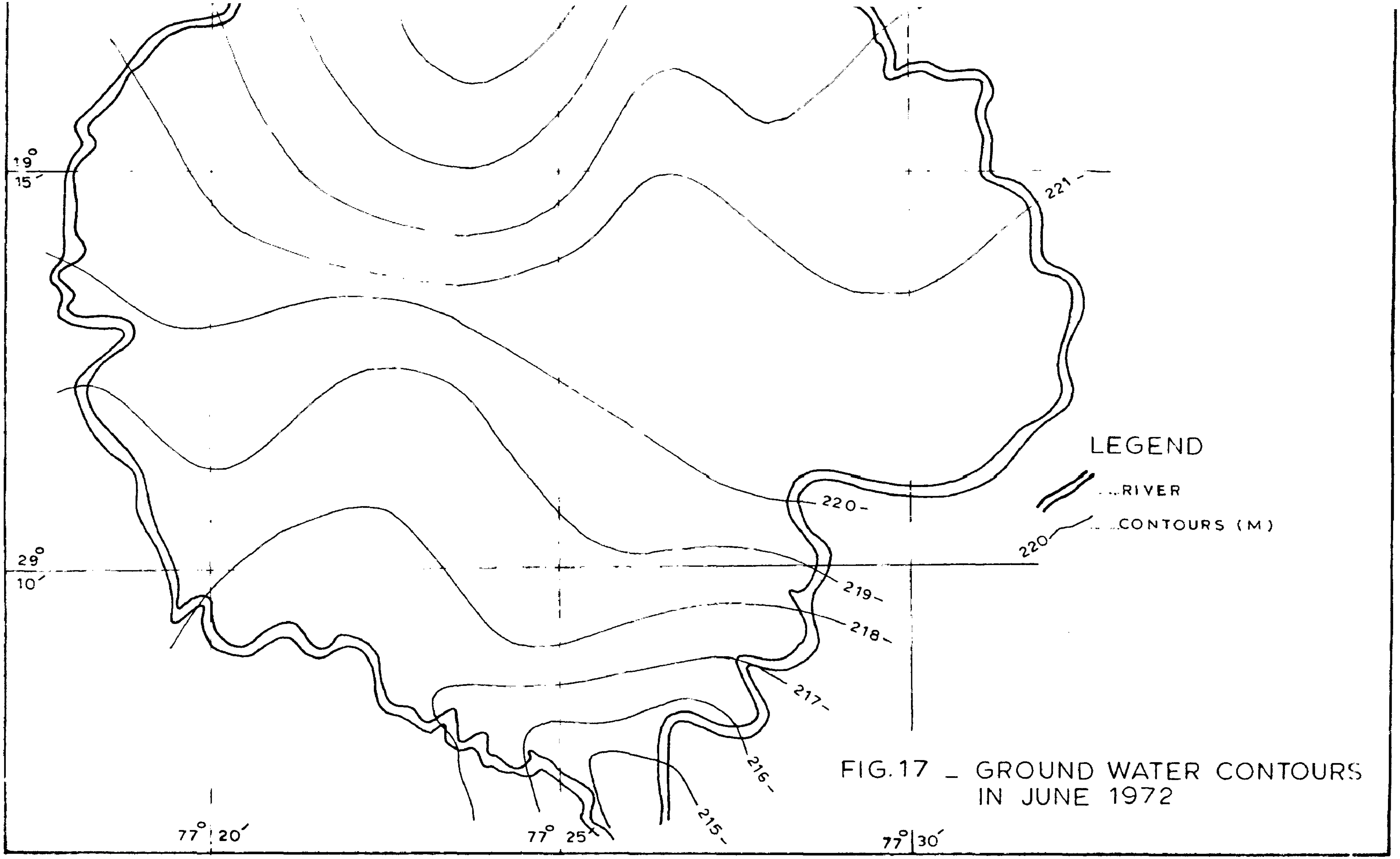


FIG.17 _ GROUND WATER CONTOURS IN JUNE 1972

Simulation of the contours is done by connecting selected node points on a particular contour, or nearby it, to the respective voltage supply through diodes. To keep the number of diodes small, so as to be able to use a medium size terminal board, the node ~~firsts~~ points connected were as under.

Contour m.	Voltage v	No. of nodes connected
215	1.0	2
217	1.4	4
219	1.8	7
221	2.2	10
223	2.6	4
225	3.0	2
	Total	29

Adjustment of the initial conditions necessitated trials and errors because the outer boundary as well as the contours interacted amongst themselves. It was found necessary to divide the outer boundary into four segments with supplied voltage equal that applied to the contour to the north of the segments. This approximated the fall of water levels along the Fazalpur and Bijwanra Distributaries on the west and the right Jaulli distribut~~ing~~aries on the east. Supplying the same voltage of 3.2 v to the outer boundary all along used to result in reverse biasing of the diodes connected to the contours of 215m and 217 m. This could be avoided by breaking up the outer boundary in reaches at different potentials.

The next step was to adjust the sweep frequency of the oscilloscope, so as to synchronise with the time scale of 8 seconds per year. This was done by putting the oscilloscope to the lowest sweep frequency and adjusting the calibration knob. The speed of travel of the luminous spot was adjusted to 10 seconds per sweep. The function generators were all connected, through their respective feed in resistors to the various points on the analog. Schematic diagram of of the model set up is shown in Fig.18. Photograph 1 shows the whole set up, with connection being made to the feed in resistors. Photograph 2 shows the connection being made to the analog.

Magnetic relay switches are provided. Normally closed contacts of those switches were connected to the voltage supplies except those for the outer boundaries which were kept on throughout the operation. Normally open contacts were connected to the function generator circuits. Operation of the magnetic swtiches would put the former off and simultaneously put the latter on. This was done when the trace on the CRO had travelled by 1 cm from left to right. This started the pumpages. As in June 1972, there was very little rainfall, the rainfall generator was put on after 2/3 seconds i.e. at the beginning of July 1972. 8 seconds after the start of pumpage the magnetic switch was operated to put the current generators off and the voltage supplies on.

The records of the temporal variations of the voltages at the selected node points were obtained on a ' Omniscribe ' chart recorder. It was connected in parallel with the oscilloscope through a potential divider, so as to reduce the current drawn by the recorder and also to change the scale from 1 volt

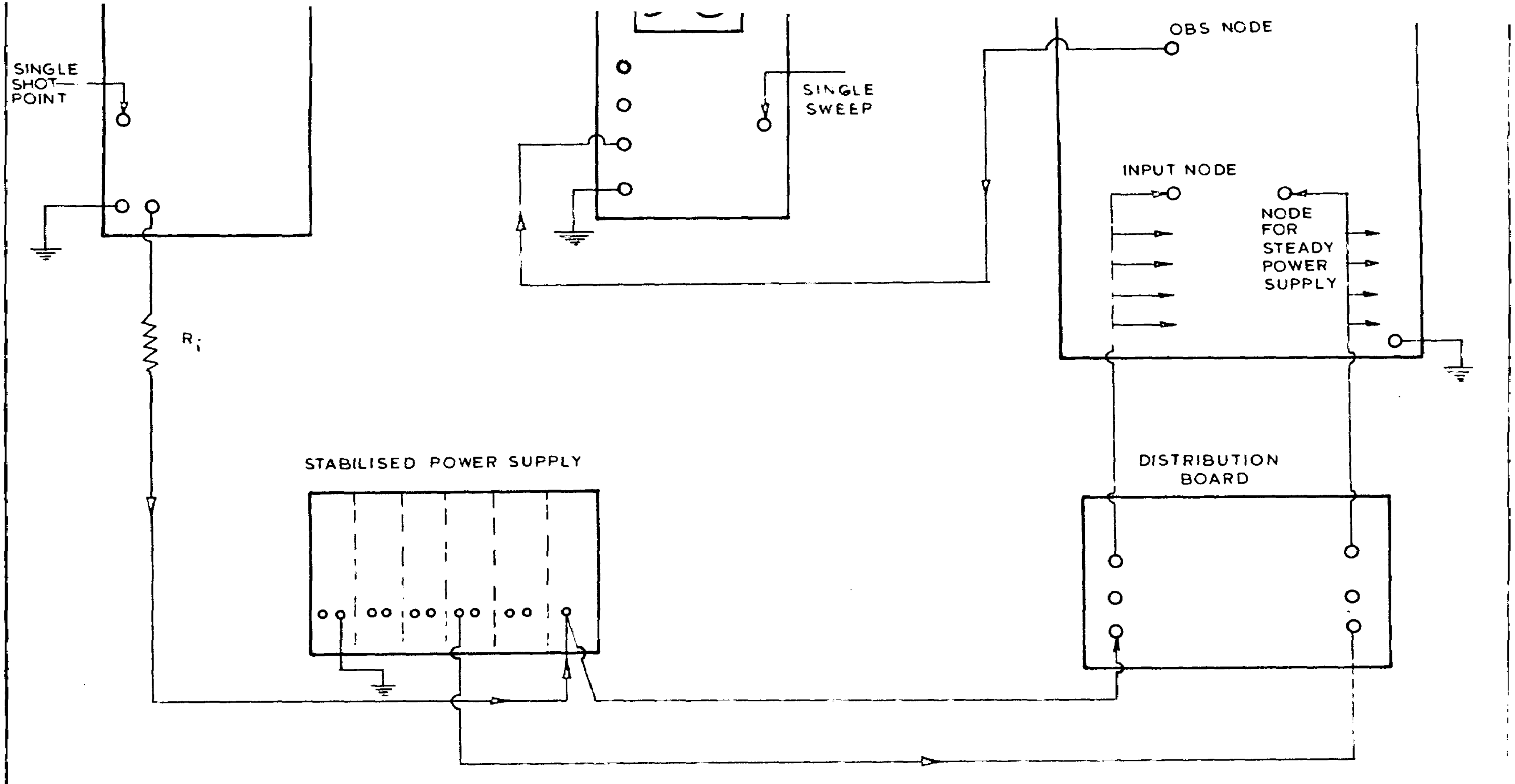
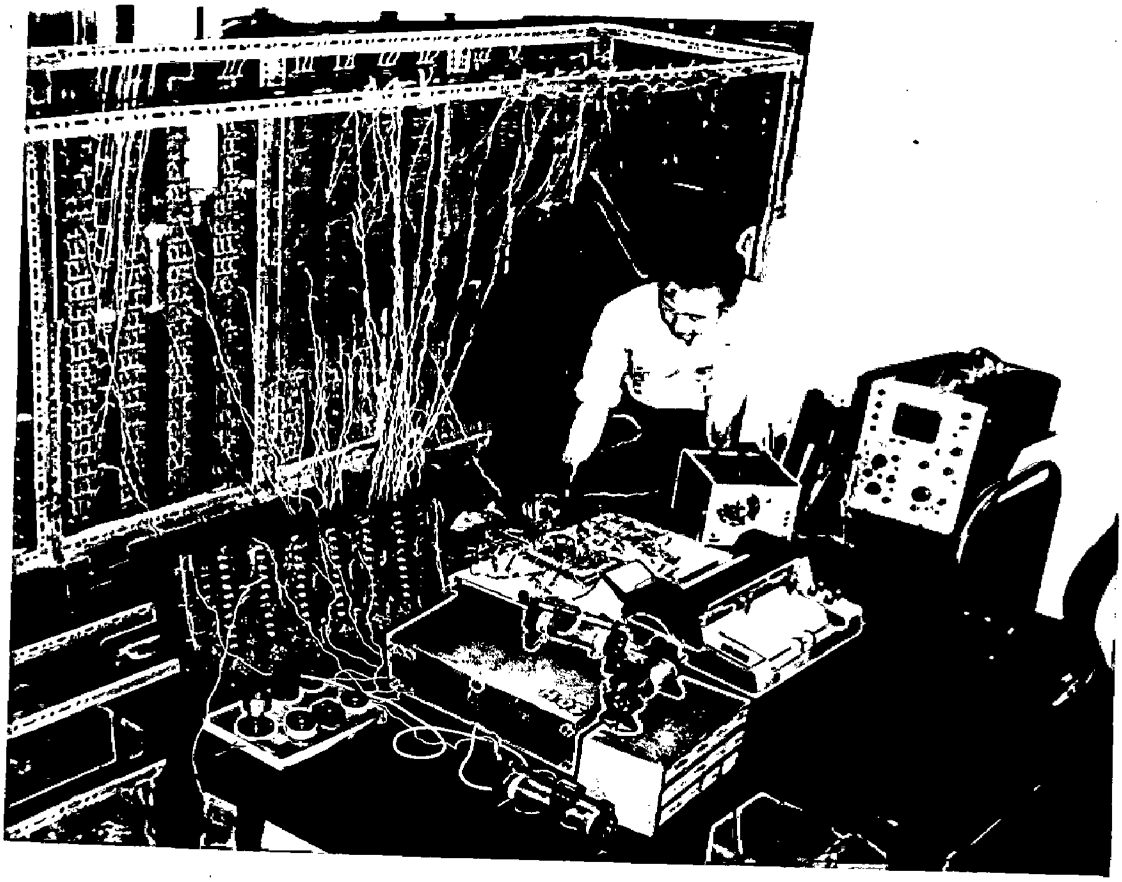
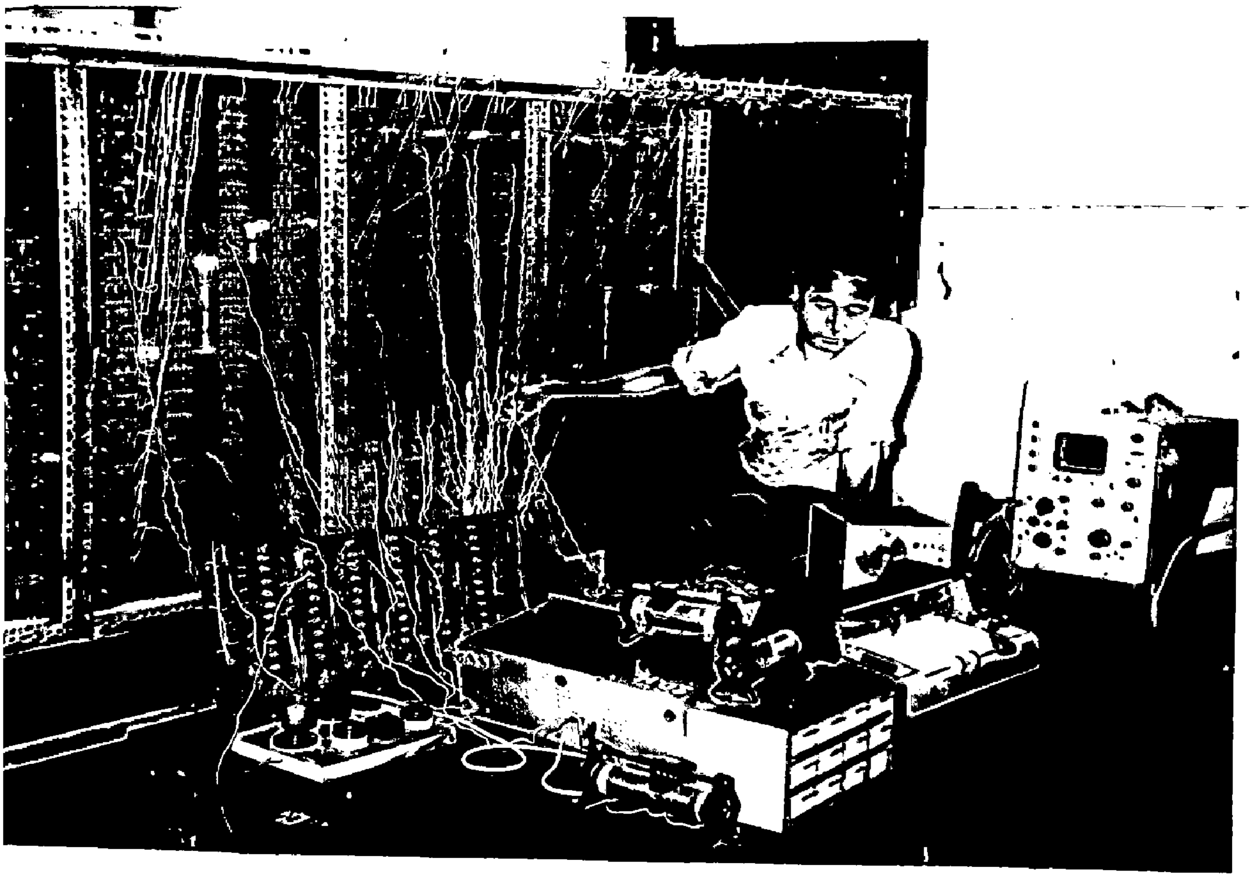


FIG.18 _MODEL SET UP



TOGRAPH 1 Experimental Setup showing connection being made to feed in resistors.



PHOTOGRAPH 2 Connection being made to analog

per inch to 1/3 volt per inch. This, however, made it necessary to adjust the zero of the recorder for each observation separately. The initial condition voltages for each observation point were therefore measured with the help of the oscilloscope and recorded ~~normally~~ manually on the chart. Shortage of time did not permit repeating the operation of the model for two more years or trials with different conditions.

Results

Observations were taken at 32 nodes. Out of these 21 nodes were those selected only because of their proximity to the field observation wells. Although care was taken to ensure that the recharge pumpage points do not coincide with the observation wells, many such nodes were only one resistor away from the observation nodes. Results indicated that recharge/discharge points located only one resistor away did distort the water level patterns adversely. Eleven additional points, lying at least two resistors away from recharge /discharge points were selected and their time voltage variations recorded.

Results of the 32 node points are presented in Table No.9, which gives a comparison of the field data with the analog data. The field data for the eleven additional nodes were interpolated from the relevant ground water level contour maps. It is seen that there are differences between the field and analog water levels. The differences in build ups or drawdowns are relatively smaller. These values are summarised as under.

Range of difference in build up or draw-down m.	No. of cases		
	(a)	(b)	Total
Less than 0.1m	1	5	6
0.1 to 0.2 m	4	2	6
0.2 to 0.3 m	3	1	4
0.3 to 0.4 m	Nil	1	1
0.4 to 0.5 m	1	3	4
0.5 to 0.6 m	2	1	3
0.6 to 0.7 m	Nil	1	1
0.7 to 0.8 m	1	Nil	1
0.8 to 0.9 m	1	Nil	1
0.9 to 1.0 m	1	Nil	1
1.0 to 1.46 m	4	Nil	4
Total	18	14	32

The figures under column (a) are for node points which are affected due to its being on the boundary or due to a recharge/discharge point on a neighbouring node. Figures under column (b) are for nodes which are not affected in either way.

The results indicate that in group (a) 50 percent of the nodes showed a departure less than 0.5 m from the observed value. In group (b) 50 percent of the nodes exhibited a difference of only 0.2 m. All points taken together, 50 percent points gave departure less than 0.3 m. The maximum departure was 1.46 m and 0.65m for groups (a) and (b) respectively.

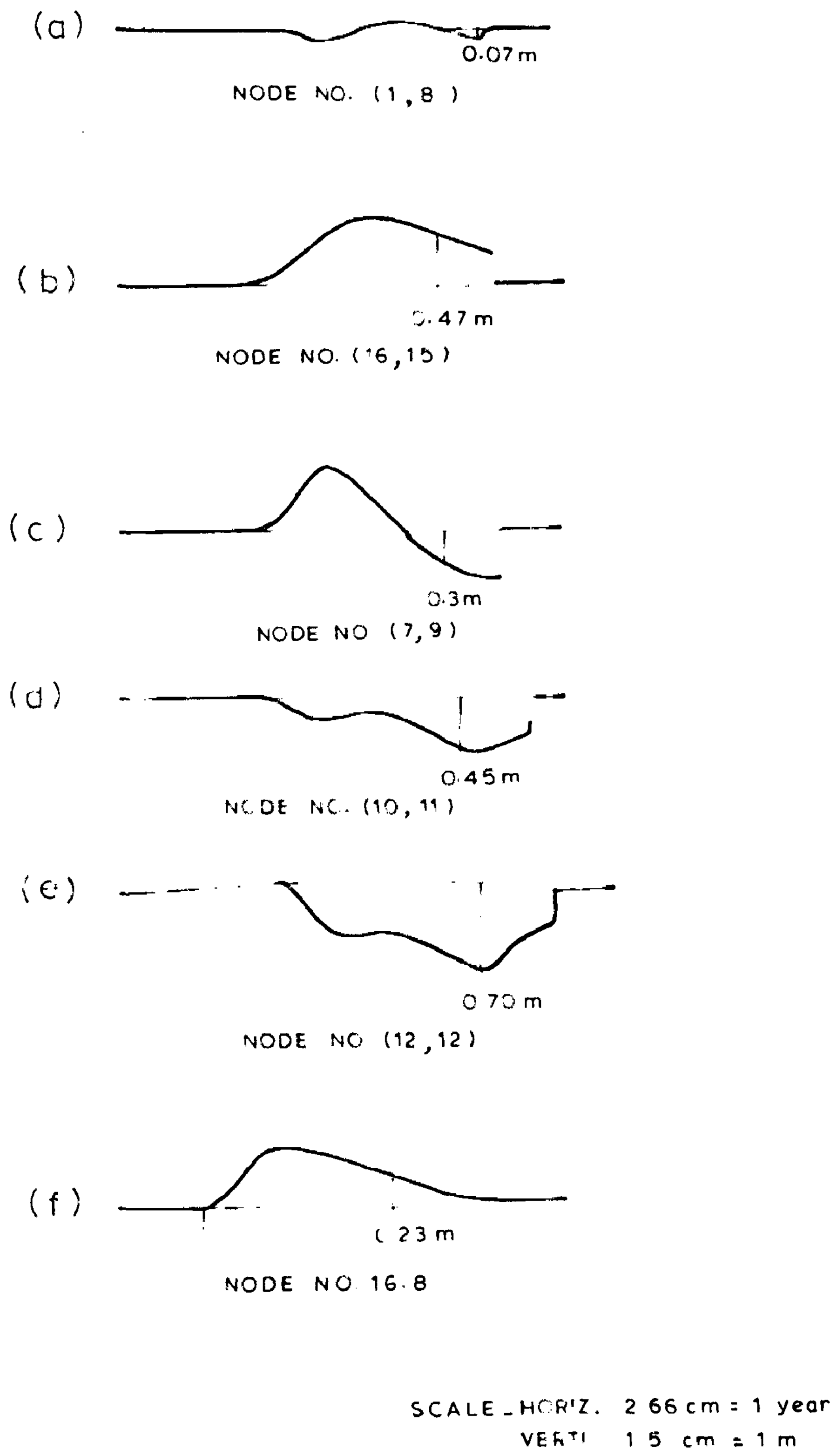


FIG 19 - TYPICAL WELL HYDROGRAPHS RECORDED BY ANALOG MODEL

EL RESULTS

Sl. No.	Nod	model +(build up) (drawdown)		Remarks
		73	Change m	
1.	7,3	7	-0.57	Discharge point nearby Recharge point bear by
2.	13,4	9	+1.220	
3.	5,5		-0.20	Additional node
4.	10,5		+0.43	Additional node
5.	7,6	4	-0.48	
6.	18,6	3	-0.970	Discharge point nearby
7.	1,8	3	-0.07	
8.	12,8	8	-1.030	Discharge point nearby
9.	16,8		+0.08	Additional node
10.	7,9		+0.10	Additional node
11.	20,1	5	+0.85	Boundary point
12.	1,11	3	-0.59	Boundary node
13.	5,11		-0.30	Additional node
14.	10,1		+0.10	Additional node
15.	17,1		+0.18	Additional node
16.	8,12	3	-0.45	
17.	12,1	5	-0.12	Discharge point nearby
18.	14,1		+0.65	Additional node
19.	2,13	5	+0.29	Recharge point nearby
20.	8,14		+0.40	Additional node
21.	21,1		+1.26	Boundary node
22.	2,15		-0.48	Boundary node
23.	16,1		-0.03	Additional node
24.	17,1		+1.46	Boundary node
25.	7,16	7	+0.55	
26.	10,1		-0.73	Capacitor defective
27.	14,1	5	-0.21	Boundary node
28.	13,1		+0.28	Recharge point nearby
29.	16,1		-0.03	
30.	7,22	9	+0.16	Recharge point nearby
31.	13,2	9	-0.12	Boundary point
32.	12,2		+0.18	Boundary point

TABLE-9

COMPARISON BETWEEN FIELD OBSERVATIONS OF GROUND WATER LEVELS WITH ANALOG MODEL RESULTS

Sl. No.	Node No.	Field Data +(build up) Water levels-(drawdown)			Analog model data +(build up) water levels -(drawn)			Difference between model +(build up) and field data - (drawdown)			Remarks
		June'72 m	June'73 m	Change m	June'72 m	Jun'73 m	Change m	June'72 m	June'73 m	Change m	
1.	7,3	220.437	220.507	+0.07	220.75	220.25	-0.50	+0.213	-0.257	-0.57	Discharge point nearby Recharge point bear by
2.	13,4	219.501	219.231	-0.27	219.50	220.45	+0.95	-0.001	+1.219	+1.220	
3.	5,5	221.50	221.60	+0.10	221.0	220.90	-0.10	-0.50	-0.70	-0.20	Additional node
4.	10,5	220.00	219.40	-0.60	219.50	219.33	-0.17	-0.50	-0.07	+0.43	Additional node
5.	7,6	221.534	221.564	+0.03	221.50	221.05	-0.45	-0.034	-0.914	-0.48	
6.	18,6	217.037	217.357	+0.32	219.00	218.35	-0.65	+1.963	+0.993	-0.970	Discharge point nearby
7.	1,8	223.473	223.443	-0.03	223.50	223.40	-0.10	+0.027	-0.043	-0.07	
8.	12,8	218.322	218.402	+0.08	219.50	218.50	-1.00	+1.178	+0.098	-1.080	Discharge point nearby
9.	14,8	217.65	217.80	+0.15	217.75	217.98	+0.23	+0.10	+0.18	+0.08	Additional node
10.	7,9	220.70	220.30	-0.40	218.00	217.70	-0.30	-2.70	-2.60	+0.10	Additional node
11.	20,10	217.325	217.175	-0.5	217.00	217.70	+0.70	-0.325	+0.525	+0.85	Boundary point
12.	1,11	225.493	225.483	-0.01	224.00	223.40	-0.60	-1.493	-2.083	-0.59	Boundary node
13.	5,11	221.25	220.55	-0.70	220.50	219.50	-1.0	-0.75	-1.05	-0.30	Additional node
14.	10,11	219.40	218.85	-0.55	219.00	218.55	-0.45	-0.40	-0.3	+0.10	Additional node
15.	17,11	217.10	217.45	+0.35	217.00	217.53	+0.53	-0.10	+0.08	+0.18	Additional node
16.	8,12	219.387	219.757	+0.37	221.50	221.420	-0.08	+2.113	+1.663	-0.45	
17.	12,12	219.335	218.755	-0.58	219.00	218.30	-0.70	-0.335	-0.455	-0.12	Discharge point nearby
18.	14,12	218.15	217.40	-0.75	218.00	217.90	-0.10	-0.15	+0.50	+0.65	Additional node
19.	2,13	223.425	223.635	+0.21	221.25	221.75	+0.50	-1.175	-1.885	+0.29	Recharge point nearby
20.	8,14	220.40	219.90	-0.50	219.00	218.90	-0.10	-1.40	-1.00	+0.40	Additional node
21.	21,14	214.360	214.200	-0.16	215.00	216.10	+1.10	+0.64	+1.90	+1.26	Boundary node
22.	2,15	222.700	222.680	-0.02	223.50	223.00	-0.50	+0.80	+0.32	-0.48	Boundary node
23.	16,15	217.25	217.80	+0.55	217.25	217.72	+0.47	Nil	-0.08	-0.08	Additional node
24.	17,17	216.35	215.190	-0.16	216.50	217.80	+1.30	+0.15	+1.61	+1.46	Boundary node
25.	7,18	221.687	221.537	-0.15	219.00	219.40	+0.40	-2.687	-2.137	+0.55	
26.	10,18	221.05	220.85	-0.20	220.00	219.07	-0.93	-1.05	-1.78	-0.73	Capacitor defective
27.	14,18	220.725	220.885	+0.16	219.00	218.95	-0.05	-1.725	-1.935	-0.21	Boundary node
28.	13,19	220.41	219.73	-0.68	218.50	218.10	-0.40	-1.90	-1.63	+0.28	Recharge point nearby
29.	16,13	216.85	216.02	-0.83	217.00	216.20	-0.80	+0.15	+0.18	-0.03	
30.	7,22	221.229	221.299	+0.07	218.25	218.02	+0.23	-2.979	-3.279	+0.16	Recharge point nearby
31.	13,23	220.009	220.019	+0.01	219.25	219.12	-0.13	-0.759	-0.899	-0.12	Boundary point
32.	12,24	220.346	220.360	+0.02	219.25	219.45	+0.02	-1.096	-0.91	+0.18	Boundary point

Although these results are not very good, all the same they are not very bad if one considers that the effects of the proximity of recharge or discharge point vitiated results of eight points. Whereas eight more points were lying on the boundaries. The former affect the results because the recharge/discharge points, at which the recharge/discharge-Currents of 9 nodes are lumped in one, act as sources/sinks, resulting into non-uniform distribution of these currents. In the latter case, the boundaries affect the results in two ways. As has been brought out in Chapter-2 representation of boundaries by segments of straight lines falling on grid lines, i.e. jagged boundaries does entail unavoidable errors which cannot be assessed reliably. Secondly, in this particular case, the lumping of recharge/ discharge current at one node out of nine results in situation in which these points across the boundary are missing thereby resulting into more non-uniform distribution of current. Moreover, perhaps the absence of capacitors at node points between the outer boundary and the aquifer boundary also contributes to the errors.

Another source of error is the fact that the heads in the initial condition were not in dynamic balance but correspond to steady state condition (sec.2.7). The criteria of Eq.39, suggested by Rushton and Wedderburn (1973), are for one dimensional aquifer. Similar criteria for the present two dimensional case are not available. Even if one tries to apply Eq.39, the assessment of the effective length of the aquifer is a matter of judgement rather than quantitative analysis. It would require considerable trials for deciding the duration of the presimulation period of initial conditions. Unfortunately equipment malfunction and related difficulties deprived this worker of an opportunity to work in this direction.

Notwithstanding the above limitations, results presented in Table-9 establish the possibilities of achieving very good results through intelligent trials and adjustments of the model and equipment.

CHAPTER - 6

CONCLUSIONS AND RECOMMENDATIONS

Ground water resources of any region can play a very vital role in its development. The fruits of development can be greater and better if conjunctive utilisation of ground water with surface water is planned on a scientific basis. The complex situation encountered preclude simple analytical methods, and necessitate ground water modelling.

Many methods of ground water modelling are available. Ranking high among them is the resistor - capacitor network analog modelling which, besides being versatile, enables the analyst to develop a feel of the system due to direct physical resemblance with the prototype.

An R.C.Analog has been designed and set up for the study of ground - water of Daha aquifer in keeping with the principles of the modelling technique outlined in Chapter 2. Several constraints of components, equipments, funds and time were operative.

The geohydrology of Daha area was studied first as a lumped model by seasonal water balance computations and then modelled for study as distributed parameter model. The water balance studies revealed a few important features of the system.

(1) The withdrawals from the system range between about 4500 ha. m. to about 10690 ha m. and are in excess of the rainfall recharge ranging between about 3000 to 9000 ha.m. This recharge is due to rainfall only as there is no canal system in the area.

(2) The fraction of rainfall water reaching the aquifer varies from 22% in monsoon of 1972 to 18% in monsoon of 1974.

(3) There is a trend of lowering of ground water levels in the area. The quantity of water released from storage varies from about 1000 ha.m. to 2700 ha.m. which is less than the difference between the recharge and withdrawal. This indicates that there is inflow from the adjoining basins.

(4) Inflow from the northern boundary is of the order of 250 ha.m. and is inadequate to meet the shortfall. Substantial quantities of water must be coming across the east and west boundaries, ranging between about 3000 ha.m. to 5000 ha.m.

(5) The ground water level contours of the area and its surrounding show a trough centred around Raha, indicating that large quantities of water are flowing into the aquifer from the adjoining irrigated areas where the water table elevations are higher than in the Raha area. The irrigation distributaries located about 3 km away from the aquifer boundary seem to be major contributing sources of water rather than the influent seepage from the rivers.

The above situation provides an interesting case for study on the R.C.analog. The model is designed to simulate the above conditions. Since the model is a distributed parameter system spatial variability of recharge and discharge pattern were identified and the area was divided into five zones, for which different continuous patterns were computed.

The model is designed, set up and operated initially for one year viz 1972-73. The following points are noticed:-

(1) The model boundary configuration requires breaking it up in different reaches held at different potentials.

(2) The lumping of recharge/withdrawal currents at node points are located at 3 km spacing. These affect water level pattern upto about 1 km. from the node. The pattern in the intermediate locations are relatively less affected.

(3) The water level fluctuations in the area as observed at 21 observation wells range between 0.23 m to 2.77 m. The errors of the analog results ranges between 0.03 m to 0.35 m at the intermediate points and 0.37 m to 1.46 m at the boundary and near recharge and discharge points.

90% of the results show an error of less than 0.2 m at intermediate points and less than 0.3 m at the other points. The maximum error at the intermediate points is 16.0% of the fluctuations and at other nodes 62.3%. The reasons for these errors are analysed. As has been brought out under section 2.5, errors at boundary nodes are larger than for the internal nodes. It is possible to reduce these errors by overcoming some of the limitations. The possible measures for doing this are as under.

RECOMMENDATIONS FOR IMPROVEMENTS

(1) Aquifer Constants:

Aquifer constants were available for only one pump test, so that the model had to be designed as homogeneous isotropic aquifer model. If more data become available, aquifer may be treated as heterogeneous or anisotropic or both.

(ii) Boundary Conditions:

Non-availability of river discharge data, combined with other difficulties in matching currents, made it necessary to simulate the boundaries as equipotential ones at the locations of the distributaries across the rivers. It was found necessary to break up this boundary into four reaches and apply different voltages. More trials with different configurations of the outer boundary may improve the results. Capacitors may be provided at the intervening node points. The values of resistors and capacitors between the outer boundary and the aquifer boundary may be changed suitably by trial and error, to ensure proper head distributions within the aquifer. For lowering the errors at boundary nodes, where quite a number observation wells are located, it may be necessary to increase the coverage of the model to several times that of the aquifer. [illegible] (1966) suggests a ratio of 10 which may perhaps be impracticable. However, shifting the outer boundary to about 6 km from the actual boundary would be possible on the existing board and can be tried if adequate components can be procured.

At an appropriate stage, simulation of stream-aquifer interaction may be attempted.

(iii) Initial Conditions:

Initial conditions have been shown to have important effects on the results (Rushton and Bannister, 1973). A few trials runs may be made to assess the period of presimulation run for establishing a dynamic balance of heads, as quantitative approach for two dimensional case is not yet available.

(iv) Feeding in of Currents:

The recharge and discharge currents were fed at a few points. Currents for about 18 nodes were lumped at one node and fed to it. This is done in a staggered pattern having a spacing of about 3 km. This was done to reduce the number of diodes required, and was advantageous in the sense that at least a few points unaffected by proximity of recharge/discharge points were available, which would not be possible if 2 km spacing is provided. It would be desirable to feed the recharge/discharge currents at each node point separately to ensure uniform distribution.

(v) Equipment:

(a) The recharge/discharge currents may be generated by one of the two alternatives discussed in Chapter 4, in preference to the simple circuit set up in the present studies.

(b) A digital voltmeter may be used to measure the initial condition voltages.

(c) Impedance matching circuits may be introduced between the recorder and the model.

The above suggestions, if implemented can improve the model substantially and yield very reliable results.

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