ANALYSIS AND DESIGN OF PIPE NETWORKS

A DISSERTATION

submitted in partial fulfilment of the
requirements for the award of the degree
of
MASTER OF ENGINEERING
in
MECHANICAL ENGINEERING
(MACHINE DESIGN)

By

NAIK RAJIV J.

DEPARTMENT OF MECHANICAL & INDUSTRIAL ENGINEERING
UNIVERSITY OF ROORKEE
ROORKEE-247 667 (INDIA)

MAY, 1987

CERTIFICATE

Certified that the dissertation entitled, 'ANALYSIS AND DESIGN OF PIPE NETWORKS', which is being submitted by Mr. RAJIV NAIK., in partial fulfilment of the requirement for the award of the degree of MASTER OF ENGINEERING in Mechanical Engineering (Machine Design), of the University of Roorkee, Roorkee, is a record of student's own work carried out by him under our supervision and guidance. The matter embodied in this thesis has not been submitted for the award of any other degree.

This is to further certify that he has worked for a period of ten months from July, 1986 to April, 1987 for preparing this thesis, at this University.

(S.C.Jain)
Reader

Mechanical and Industrial
Engineering Department,
University of Roorkee,
Roorkee, India

(Surendra Kumar)
Reader

Chemical Engineering
Department,
University of Roorkee,
Roorkee, India.

ROORKEE
MAY 25 , 1987.

ACKNOWLEDGEMENT

The author expresses his sincere gratitude to Dr. S.C.Jain, Reader, Mechanical and Industrial Engg. Department and Dr. Surendra Kumar, Reader, Chemical Engg. Department, University of Roorkee, Roorkee, for their expert guidance and encouragement accorded to him.

The author is also thankful to Dr. P.C.Pandey, Professor and Head, Department of Mechanical and Industrial Engg., for providing the necessary facilities to carry out this work.

He is thankful to his friend Mr. Sandeep Kumar Garg for his constant help, encouragement and inspiration to reach the final stage of this Thesis.

(RAJIV NAIK)

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ABSTRACT

Pipe networks are used to transport liquids and gases from the sources to various locations of their utility. Analysis and design of pipenetworks, in general, involves the solution of a set of nonlinear simulataneous equations. Research efforts are naturally made in order to either improve the efficiency/convergence of existing methods or to reduce the total number of equations in the given set of nonlinear equations, used to simulate the network.

There are four types of formulations of pipe network ~~problem,~~ which are described in detail in the literature review alongwith their advantages and disadvantages. ~~In the~~ present work, a new method of formulation, called Mixed method of formulation, is proposed to solve the pipe network problems. Its storage requirements are low and at the same time it is computationally efficient. In essence it has the advantages of both the formulation-1 and formulation-3.

Efficient software packages have been developed to analyse the pipe networks by using existing methods or by the proposed mixed method of formulation. These packages are also capable of carrying out sensitivity analysis of the network, which may be of help in identifying critical nodes or to plan future expansion of the network.

In software packages Newton Raphson method has been used to solve the set of nonlinear equations due to its quadratic convergence. During development of the computer

programs, it is observed that this method is sensitive to initial guess of values of parameters. A convergence scheme is proposed to reduce its dependence on initial guess.

Lastly a computer-aided loop selection method has also been developed and programmed, which selects the set of independent loops of pipes in a given network. This set of independent loops is required in the Formulation-4 and any error in the selection of this set of independent loops, which is likely during manual inspection of the large networks, results in the divergence of the solution.

NOMENCLATURE

A,B	=	constants of pump characteristics curve,
[C]	=	fundamental cycle matrix
C_{HW}	=	Hazen Williams constant for pipe material
$C_{i,j}$	=	constant obtained using Hazan Williams equation, for pipe connecting node i and j
D	=	diameter of pipe in c.m.
d	=	distance between nodes
ELE	=	elevation of node in meter
e	=	equivalent roughness in pipes
ε	=	error tolerance
f	=	friction factor
f(x)	=	function of x (vector)
{F}	=	column vector of function f(x)
g	=	acceleration due to gravity
G	=	graph
[H]	=	approximation of Jacobian [J] in Broydens method.
H_i	=	head at node i in meter
h_f	=	frictional head loss
h_p	=	supply head of the pump
Ho	=	constant obtained for pump
i	=	node i
J	=	number of nodes
[J]	=	jacobian matrix
$K_{i,j}$	=	exponent in Hazen Williams equation for pipe connecting nodes i and j.

L	= Length of pipe in meter.
L_i	= distance from node i.
ℓ	= number of loops
[M]	= incidence matrix
N	= number of nodes
n	= number of equations
P	= number of pipes in network
p_i	= pressure at node i
$p_{i,j}$	= pressure in pipe connected to node i and j
p_k	= pressure in pipe k
Q_k	= flow in pipe k in cubic meter
$q_{i,j}$	= flow in pipe connected to node i and j
q_i	= consumption at node i
ΔQ	= corrective flow rate in loop
Re	= Reynolds number
τ	= damping factor
τ_k	= damping factor in iteration k in case of Broydens method
u	= external variable
V_{Ai}	= set of pipes, which brings flow towards node i
V_{Bi}	= set of pipes, which takes away flow from node i
V	= velocity of fluid in pipe
x	= function variable
$\{y\}$	= solution vector
y_k	= vector of change of residuals in successive iterations in Broyden method.
Z_k	= revised function variable in Broyden method
π	= function of x in Wolfe's method
σ	= vector of pressure drop.

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CHAPTER - I

INTRODUCTION

Pipe networks are common appearances in mechanical, chemical and other process industries. The term 'pipe network' means, mesh of pipes along with other piping elements such as booster pumps, valves etc. The purpose of pipe network is to transport incompressible or compressible fluid from one location such as reservoirs and storage tanks to various locations of its use, under steady state or unsteady state conditions. Various examples of pipe networks are, distribution of lubricants from a storage tank to various machines, supply of process fluids to various process units such as heat exchangers, reactors etc., transportation of crude petroleum from oil wells to various refineries, the municipal water supply and the distribution of cooking gas in big cities.

Analysis and design of simple pipe network (Number of pipes ≤ 5) is simple and can be carried out by usual methods of fluid mechanics. However complex networks (shown in figure (1.1)) pose various problems to designer and computational engineer, and offer wide scope for innovative research. Various aspects associated with analysis and design of pipe network are shown in figure (1.2) (Block diagram). From block diagram one can easily imagine the complexity and magnitude of the problem and it is obvious now that the analysis and design of pipe network is not simply calculation of flow and pressures.

A study of literature review described in chapter-II, indicates that there are more than one formulations of pipe network problem. These formulations in general results in a set of algebraic and transcendental equations, which are solved by suitable numarical methods. In this thesis the methods of solution of these formulations have been studied from the point of view of convergence and computational efficiency and the modifications in existing methods are proposed. Finally efficient software packages for these formulations of pipe network problem are developed.

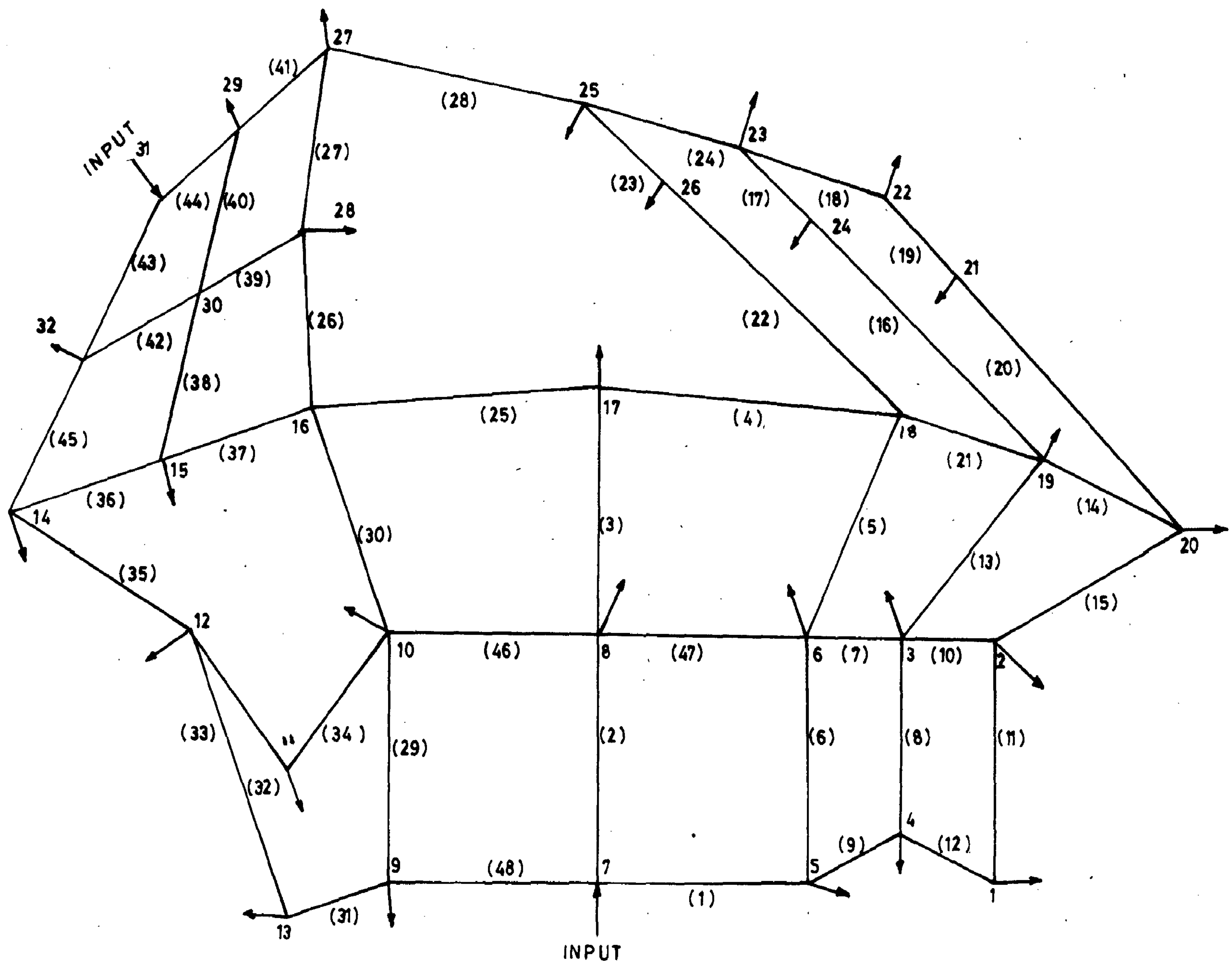
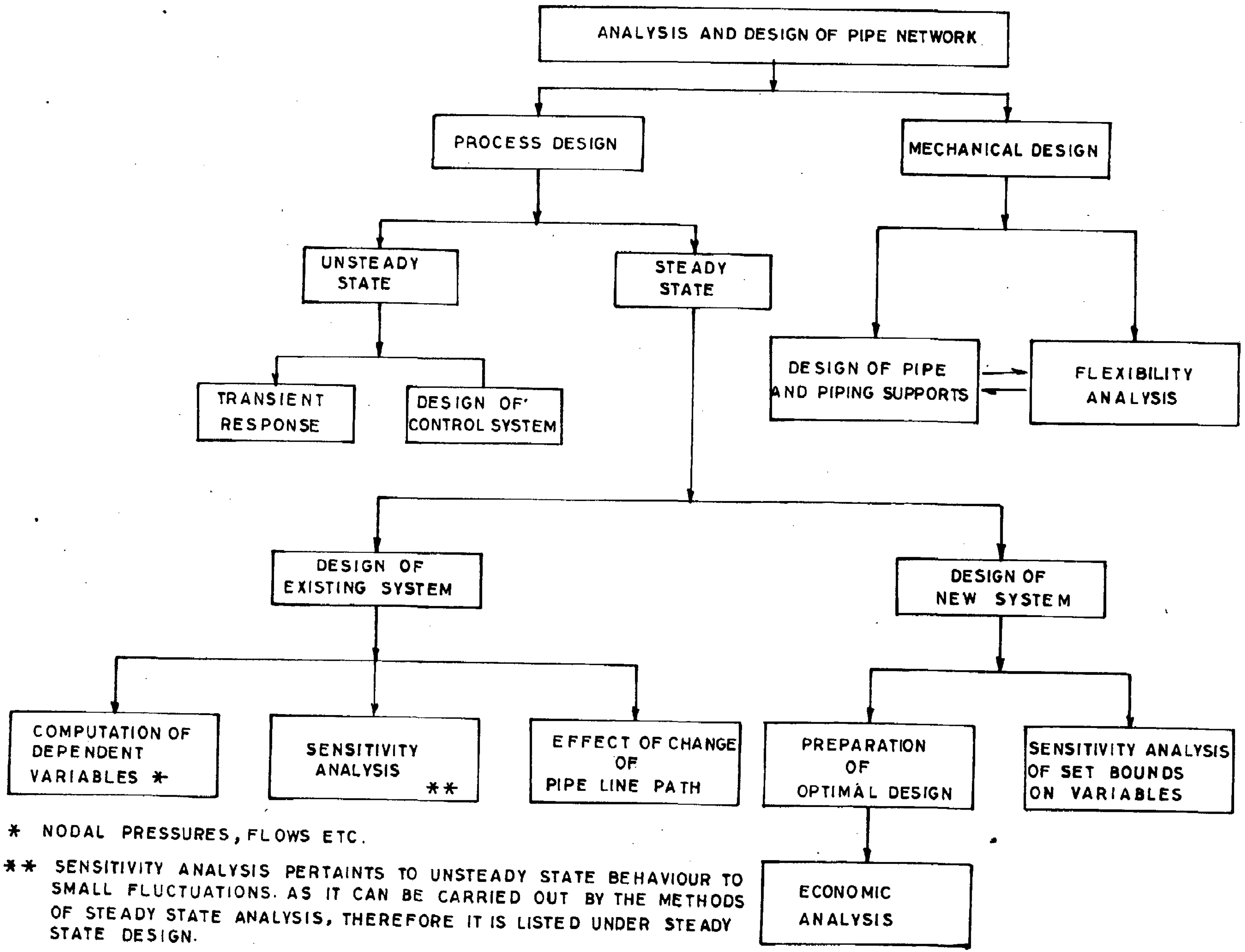


FIG.1.1 A TEST PROBLEM



* NODAL PRESSURES, FLOWS ETC.

** SENSITIVITY ANALYSIS PERTAINTS TO UNSTEADY STATE BEHAVIOUR TO SMALL FLUCTUATIONS. AS IT CAN BE CARRIED OUT BY THE METHODS OF STEADY STATE ANALYSIS, THEREFORE IT IS LISTED UNDER STEADY STATE DESIGN.

FIG. 1·2 DESIGN OF PIPE- NETWORK

CHAPTER - II

LITERATURE REVIEW

In this chapter existing literature in the area of steady state process design of pipe network for transporting incompressible fluids has been reviewed.

2.1.1 FLUID MECHANICS CORRELATIONS

Here few correlations are described which are required for analysis of pipe networks. These correlations are used to calculate frictional head losses.

There are three main equations which are used for calculation of frictional head loss for flow of fluid through pipes.

i) Hazen Williams equation - This is most commonly used equation for flow of water through pipes which is given in Jeppson [1].

$$h_f = \frac{10.7\ L}{C_{HW}^{1.852}\ D^{4.87}}\ Q^{1.852} \qquad \ldots(2.1)$$

where,

h_f = Head loss in meter.

L = Length of pipe in meter

D = Diameter of pipe in cm.

Q = Flow rate in cubic meter

C_{HW} = Hazen Williams Coefficient for pipe material given in table (2.1).

ii) <u>Manning equation</u>

$$h_f = \frac{10.29 \quad n^2 L}{D^{5.333}} \; Q^2 \qquad \ldots(2.2)$$

n = Manning constant for pipe material given in table (2.1)

However results obtained by Manning's equation are not accurate. So as far as possible the use of Manning's equation is avoided.

iii) <u>Darchy Weisbach Equation</u>

$$h_f = f \, LV^2 / \, 2g.D \qquad \ldots(2.3)$$

f = friction factor

g = acceleration due to gravity.

Here for determination of friction factor, accurate values of flow rate in pipes are needed to calculate Reynold's number. Hence above equation cannot be used as accurate flow rates are not initially known. The table (2.2) gives some expressions, which are used to calculate friction factor. In which e/D is the ratio of wall roughness to the diameter of pipe.

TABLE 2.1

Values of the Hazen Williams Coefficient (C_{HW}) and Manning's n for Common Pipe Materials

Material	C_{HW}	n
PVC pipe	150	0.008
Very smooth pipe	140	0.011
New cast iron or welded steel	130	0.014
Wood, concreat	120	0.016
Clay, new riveted steel	110	0.017
Old cast iron, brick	100	0.020
Badly corroded cast iron or steel	80	0.035

2.2.1 METHODS OF FORMULATION

There are more than one way of formulating the pipe network problem. The computational efforts required for the solution naturally vary from formulation to formulation There are two methods generally used for formulating the problem.

(a) PROBLEM FORMULATION BY HAZEN-WILLIAMS EQUATION

This is Simplest method of formulation described by Shamir and Howoard [4]. They have used only Hazen Williams equations given in equation (2.1) for formulation. Here

TABLE 2.2

Expressions for friction factor (f)

No.	Region of Reynolds Number	Formula	Source
i.	Laminar flow $Re \leq 2100$	$f = 64./Re$	Jeppson [1]
ii.	Transition flow $2100 \leq Re \leq 4000$	– Implicit equation in f – $\frac{1}{\sqrt{f}} = -4.0 \log \left(\frac{e/D}{3.7} + \frac{1.255}{Re\sqrt{f}} \right)$	Corebrook [2]
		– Explicit equation in f – $\frac{1}{\sqrt{f}} = -4.\log \frac{e/D}{3.7} - \frac{5.02}{Re} \log \left(\frac{e/D}{3.7} - \frac{5.02}{Re} \log \left(\frac{e/D}{3.7} + \frac{13.0}{Re} \right) \right)$	Zigrang and Sylvester [2].
iii.	Turbulent flow $Re \geq 4000$	– Implicit equation in f – = For smooth pipes $\frac{1}{\sqrt{f}} = 4.0 \log (Re/\sqrt{f}) - 0.4$	Karman [3]
		= For Rough pipes	Karman
	$Re/f\ e/D=100$	$\frac{1}{\sqrt{f}} = 4.0 \log (1/e/D)+2.88$	
		= Explicit equation in f –	Round [3]

contd...

Table 2.2 contd....

No.	Region of Reynolds Number	Formula	Source
	For $4000 \leq Re \leq 10^8$	$1/\sqrt{f}=3.6 \log(Re/0.135 (Re\ e/D+6.5))$	Round [3]
iv.	Generalized equation	$f=2((B/Re)^{12}+1/(A+B)^{1.5})^{\frac{1}{12}}$ were $A=(2.475 \ln(1/(7/Re)^{0.9} +0.27\ (e/D))^{1.16}$ $B=(\frac{37530}{Re})^{16}$	Churchill [5]

the equations of pressure drop in loops or equations of nodal flows are directly written down. Detailed discussion regarding this type of formulation is given in Chapter- 3.

(b) PROBLEM FORMULATION USING GRAPH THEORY

This formulation is same as described in (a), only the graph theory facilitates in writting down equations of nodal flows or pressure drops in loops.

In graph theory topology of network is represented by suitable matrices. Consider figure (2.1) with J number of nodes and l loops with P pipes connecting J nodes. The graph is set of J and P. First of all each node, pipe and loops are labeled properly. Usually nodes are labled with numbers (1,2,...,J), pipes with number enclosed in paranthesis ((1), (2),...P) and loops with Roman numbers (I,II,...ℓ). This method is followed by Epp and Fowler [5]. Such graph is called a 'labled graph'. This labeled graph provides important informations like degree of node i.e. number of pipes connected to that node, path of graph which is a sequence of pipes such as any two consecutive pipes have common node. In case a path has a common node then graph is cyclic. The graph which does not contain any cycles or loops, is called a 'tree'.

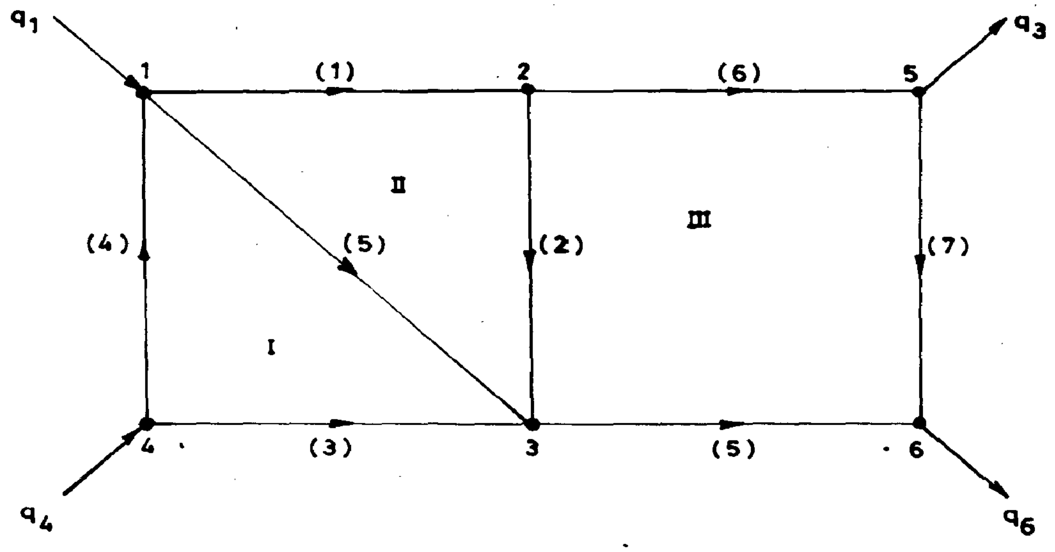


FIG. 2.1 AN EXAMPLE NETWORK

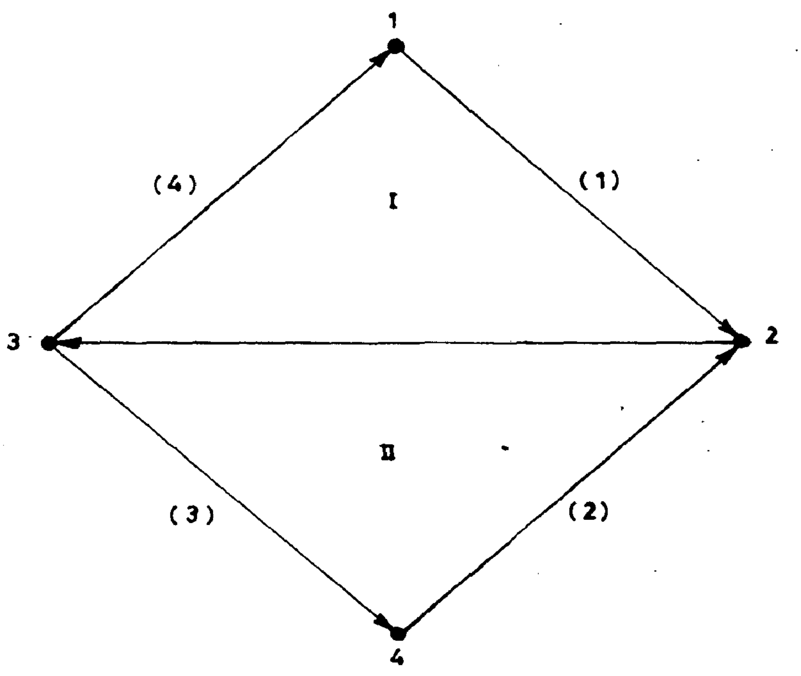


FIG. 2.2 A DIGRAPH

In most of design applications the direction of flow is either known or it is convenient to assume. If direction of flow is mentioned on pipe then graph is called 'digraph' and pipe is called 'arc'.

i) Matrix representation of digraph

Representation of the digraph in terms of matrices containing only elements 0,1 and -1 is called a 'matrix representation'. Further there are two laws, which governs the flows in pipes. These are analogous to Krichoff's laws for an electrical circuits. These laws are:

The algebraic sum of flows at each node must be zero.

The algebraic sum of pressure drops around close loop must be zero.

For any network with J nodes and P pipes, there are (J-1) independent equations based on first law and l(=P-(J-1)) equations based on second law.

For example consider a network shown in Figure (2.2). According to first law there are J equations. If[M]is the matrix (P x J) formed by the rule which states 'Pipe that bring fluid towards node is represented by 1 and that takes away from node by -1', then this matrix for example is given by

$$[M'] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{matrix} (1) & (2) & (3) & (4) & (5) \end{matrix} \\ \begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & -1 & 1 \\ 0 & -1 & 1 & 0 & 0 \end{bmatrix} \end{array} \quad \ldots(2.4)$$

Similarly based on second law matrix C is written for loops

$$[C] = \begin{array}{c} \\ I \\ II \end{array} \begin{array}{c} \begin{matrix} (1) & (2) & (3) & (4) & (5) \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array} \quad \ldots(2.5)$$

(M') is called the 'indidence matrix'. But actually there are only (J-1) independent rows in (M'). Hence there must be one row which is a linear combination of remaining (J-1) rows. So by omitting one of rows of matrix (M') the matrix (M) is formed which is of same-rank of that of matrix (M').

Now from first law

$$[M]\ \{q\} = 0 \qquad \ldots(2.6)$$

and from the second law

$$[C]\ \{p\} = 0 \qquad \ldots(2.7)$$

where {q} is column vector of nodal flows, and

{p} is column vector of pressure drop.

From the above methods and two laws it is possible to formulate the problem of analysis of pipe network in four ways.

Now by using first law it is possible to write the equation of nodal flow balance for (J-1) nodes.

$$\sum_{j \varepsilon V_{Ai}} q_{ij} - \sum_{k \varepsilon V_{Bi}} q_{ik} = q_i \qquad \ldots(2.8)$$

$$i = 1, 2, \ldots J-1$$

where V_{Ai} is a set of nodes associated with the incidence pipes directed towards 'i' and V_{Bi} is a set of nodes with pipes directed away from the node 'i', and q_i is consumption at node i.

By using fluid mechanics correlation for a set of pipes 'P' joining nodes i and j there are 'p' number of equations. For k^{th} pipe joining nodes i and j

$$p_i - p_j = \sigma_k (q_k) \qquad \ldots(2.9)$$

and $\sigma_{ij} = \sigma_k$ is vector of pressure drop which depends up on the type of correlation chosen.

By using second law equations of pressure drop is written down for each loop by calculating pressure drop in each pipe of loop.

$$\sum_{k \varepsilon C_i} \sigma_k(q_k) = 0 \qquad \ldots(2.10)$$

$$i = 1, 2, \ldots l$$

When flow is unknown the equation 2.8 can be written down by knowning nodal flows for J-1 nodes.

$$\sum_{j\epsilon V_{Ai}} (p_j-P_i)/\sigma_{ij} - \sum_{k\epsilon V_{Bi}} (p_i-p_k)/\sigma_{ik}=q_i \qquad ...(2.11)$$

Using these four types of equations the four methods of formulations may be developed. They are listed in table (2.3).

TABLE 2.3

Table for different formulation methods

Method of Formulation	Equations No.	Variables	No. of equa - tions	Source
1.	2.8 and 2.9	Nodal pressures and flow rates in pipes	J-1+P	Carnahan and Christensen [6]
2.	2.8 and 2.10	Flow rates	$J-1+\ell=p$	Mah[7] Williams[8] Jeppson [9]
3.	2.11	Nodal pressures or heads	J-1	Shamir [4] Lam and Wall[10] Gay and Middleton[11]
4.	2.10	Mesh flow by knowing flow in pipes	ℓ	Epp and Fowler [5] Gay [11] and Middleton

2.2.2 FORMULATION WITH OTHER COMPONENTS IN PIPE NETWORK

Other components in the pipe network constitute reservoirs, constant discharge pumps, pressure reducing valves (PRV), check valves, booster pumps, etc. Usually these components are incorporated in the network by choosing their appropriate models [29], which are derived from Hazen-Williams equations. Table (2.4) explains the procedure.

Figure (2.3) gives the values of the constant for different types of valves, pumps, contractions and enlargements. Using these values of constants the components are incorporated in the formulation in the same way as the pipes. For pumps the pump characteristics curves are drawn and then these curves are approximated by a quadratic equation. These quadratic equations are incorporated in the formulation as explained in chapter -3. It is to be noted that.

- The check valve is one which allows the flow in one direction only.
- The PRV is one which maintains constant pressure regardless how large the upstream pressure is. But upstream pressure is less than the valve setting the valve has no effect. It down-stream pressure is greater than the valve setting then it acts are check valve.

Figure 2.3

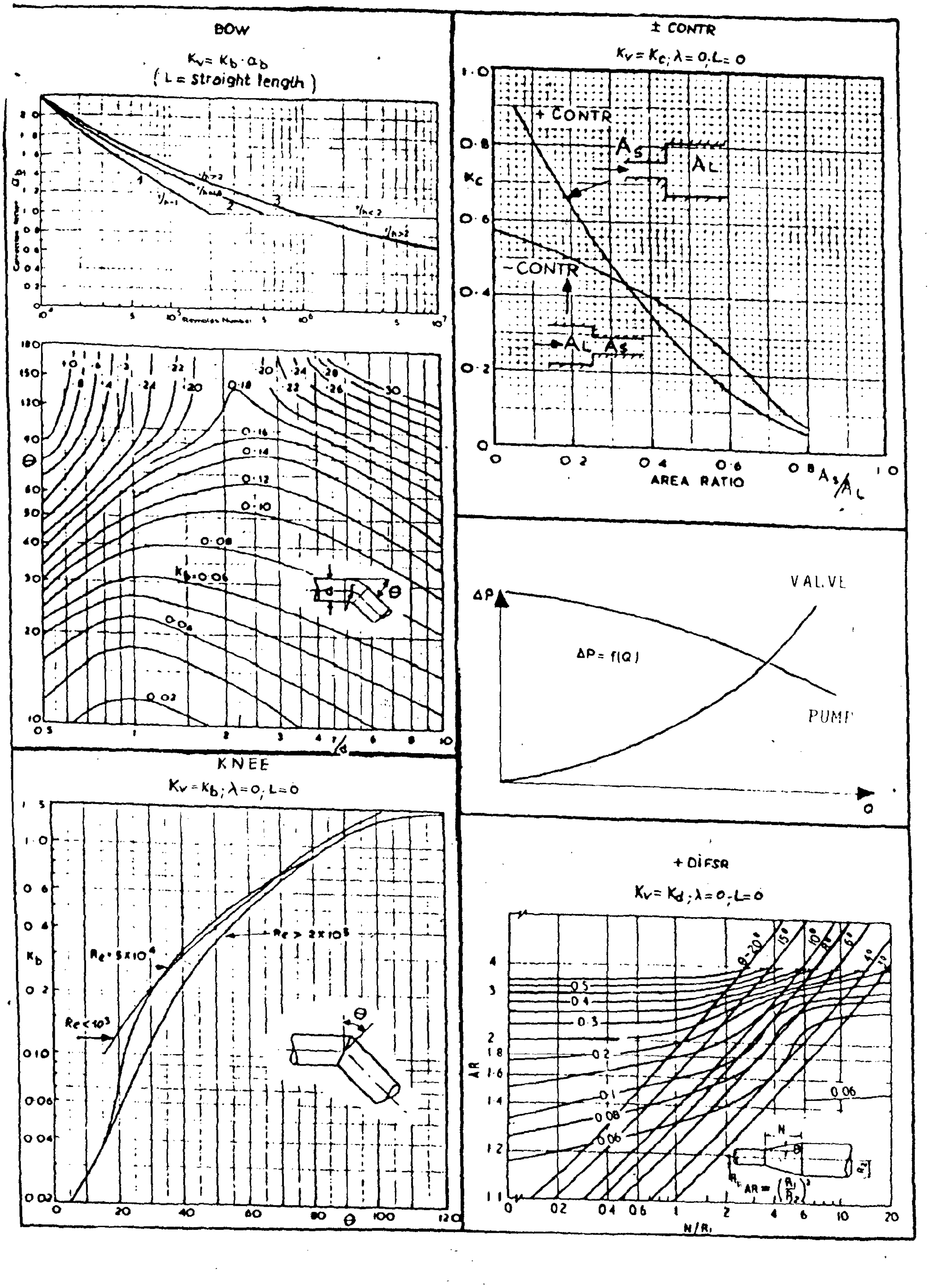

BOW
Kv= Kb·ab
(L = straight length)
± CONTR
Kv= Kc; λ= 0, L= 0
+ CONTR
- CONTR
AREA RATIO
VALVE
PUMP
ΔP= f(Q)
KNEE
Kv= kb; λ=0, L=0
+ DIFSR
Kv= Kd; λ=0, L=0

Table 2.4 Component characteristics for water-distribution system

NAME	SYMBOL	GRAPH MODEL	TEST CONDITIONS	TERMINAL EQUATION	ADMITTANCE VALUE
RESERVOIR # i	H_r, i, DATUM	i, i, DATUM		$H_i = H_r$	
CONSTANT DISCHARGE PUMP # i	q, i	i, i, DATUM		$q_i = -q$	
CONSUMPTION # i	i, q	i, i, DATUM		$q_i = q$	
PIPE # k	i, j	i, k, j		$q_k = K_k h_k^{0.54}$	K_k
BOOSTER PUMP	i, k, j	i, k, j, q_0, q_0, DATUM		$q_k = q_0 - a h_k^{b}$	$-a$
CHECK VALVE # k	i, k, j	i, k, j	$H_i > H_j$	$q_k = K_k h_k^{0.54}$	K_k
		i, k, j	$H_i \leqslant H_j$	$q_k = 0$	0
PRV i, e, j	i, e, j	i, k, j	$H_e < H_{set}$ $H_j < H_i$	$q_k = K_k h_k^{0.54}$	K_k
i, k', H_{set}, e, j, k, DATUM		i, e, k, j, k', e, DATUM	$H_e > H_{set}$ $H_i < H_j$	$\bar{q}_k = K'_k \bar{h}_k^{0.54}$ $\bar{q}_{k'} = \bar{q}_k$ $H_e = H_{set}$	K'_k
	i, j, k	i, j, k	$H_j > H_i$	$q_k = 0$	0

2.3.0 METHODS OF SOLUTION

Each formulation of the pipe network problem results in a set of algebraic and transcendental equations which may be solved by suitable numerical methods. Various methods which are used to solve the equations, are given below:

i) Newton Raphson method
ii) Hardy Cross method
 (a) Method of balancing heads
 (b) Method of balancing flows
iii) Linear theory method
iv) Generalized secant methods
 (a) Broyden's method
 (b) Wolfe's method

The selection of a method for given formulation dependes up on the

(a) type of the network
(b) size of the network
(c) degree of accuracy needed in solution
(d) cost of computation
(e) importance of the network project
(f) computer storage available

i) Newton Raphson Method

This is iterative scheme which starts with initial guess. This method is based on the Taylor's series expansion of function f(x) about kth iterate X_k. For further details please see chapter (3).

Advantages of this method are, faster convergence, less storage requirements and easy to implement in computer algorithm. And the main disadvantage is its sensitivity to the initial guess.

ii) Hardy Cross Method

This method has been developed long before the advent of computers to carry out hand calculations. Its development is based on the Newton Raphson method [12]. In this method iterative corrections are applied to each equation before proceeding to next equation in the iterative manner.

This method is used for small networks using hand calculations and there is no need of calculating partial derivatives as in the case of Newton Raphson method. This method is not suitable for large networks mainly because of its poor convergence.

If the problem formulation is done with 2nd law (with loop pressure drop equations), then the method of balancing heads is applied where-as if formulation is done with first law (Nodal flow equations), then the method of balancing flows is applied.

iii) <u>Linear theory method</u>

In this method the system of nonlinear equations are converted into the system of linear equations. By this method ℓ nonlinear loop equations are converted into linear equations by approximating head in each pipe.

$$h_f = (K_i \; q_i(0)^{n-1})q_i = K_i \, q_i \qquad \ldots(2.12)$$

where $q_i(0)^{n-1}$ is initial estimate of flow rate in ith pipe. Hence these ℓ number of non linear equations are solved by the method of linear algebra [1] till convergence is reached.

This method does not need appropriate initial guesses and hence suitable for large networks. But convergence of this method is poor and storage requirements are comparatively very large.

iv) <u>Generalized secant methods</u>

In these methods the partial derivatives are evaluated by linear approximations using 'secants' rather than 'tangents'.

(a) <u>Broyden's method</u>

This method is called as quasi-Newton's method [13]. Here the inverse of Jacobian (J) is approximated by -H and successive approximations are generated by

$$H_{k+1} = H_k - (\tau_k Z_k + H_k Y_k) Z_k^T H_k / Z_k^T H_k Y_k \qquad \ldots(2.13)$$

where $Z_k = H_k \, f(x_k)$

and $Y_k = f(x_{k+1}) - f(x_k)$

To start the initial guess of H is taken as identity matrix.

This method does not need any initial guess hence best suited for large networks. However the advantage offered by this method may only be marginal, for small network, since (H) is a fairly dense matrix and many multiplications are involved in generating an updated (H) matrix.

(b) Wolfe's method

In this method one dimensional secant approximation is used along with Taylor's series expansion of f(x) as

$$0 = f(x) = f(x_2) + \frac{f(x_2) - f(x_1)}{x_2 - x_1}(x - x_2) \qquad \ldots(2.14)$$

The nonlinear $(x-x_2)$ terms are neglected

Let $\pi_1 = \dfrac{x_2 - x}{x_2 - x_1}$

$\pi_2 = \dfrac{x - x_1}{x_2 - x_1}$

In this method n+1 trial solutions are required to solve n dimensional equations

$$\sum_{j=1}^{n+1} \pi_j = 1$$

$$\sum_{j=1}^{n+1} \pi_j \, f_i(x_j) = 0 \qquad i=1,2,\ldots n$$

And new approximations of x is generated by

$$\bar{x} = \sum_{j=1}^{n+1} \pi_j \, x_j$$

To complete the iteration of new set of trial solution is formed by replacing one of the trial solutions x_k, say by $\bar{x}$.

TABLE 2.5

No.	Method	Formulation	Largest network solved J	Largest network solved P	Source
1.	Newton	2	22	38	Mah [7]
	Raphson	3	70	100	Shamir and Howard [4] Stoner[14], Donachie [15]
		4	170	307	Epp and Fowler [5]
2.	Broyden	3	30	50	Lam and Wolla [10]
3.	Wolfe	1	28	35	Carnahan and Cristensen [6]
4.	Hardy Cross Balancing Heads	4	289	544	Daniel [16], Williams [8]
	Balancing flows	3	20	33	Jeppson[1]
5.	Linearization	1	22	38	Jeppson[1]
		2	46	57	Jeppsonand Tavallaee [9]

TABLE 2.6

Table for comparison of CPU time
(J = 22, P = 38)

No.	Method	Formulation.	Number of iterations	CPU time, in seconds CDC Cyber 73
1.	Newton Raphson	2	4	8.2
2.	Hardy Cross Balanacing Heads	4	59	2.4
3.	Linearization	2	7	3.43
		4	17	1.03

2.4.0 TECHNIQUES FOR LARGE NETWORK

Usually in solving large network, problem of large computer storage requirements and convergence arrise. This problem can be tackled by taking advantage of problem structure. Some of the methods are given below.

2.4.1 LOOP LABELLING ALGORITHM

If loops are randomly labeled the storage requirements greatly increases. As most of the stored matrices (incidence) are sparse, one can reduce the band width to minimum. To find bandwidth consider a pipe which is atleast a part of two loops. From the loop number of largest value substract loop number of smallest value. This is done for all pipes, which are part of atleast two loops. Maximum of such difference is half the bandwidth 'b'. To reduce the band width the loop labelling algorithm given by Epp and Fowler [5] is followed so that the bandwidth of space matrix will be minimum.

2.4.2 LOOP SELECTION ALGORITHM

Formulation 2 and 4 require a set of independent loops. These independent loops are usually not unique Mah [7] and Epp and Fowler [5] says that there is significant influence of the loop selection on the convergence. The procedure of loop selection is given in chapter -4.

2.4.3 SPARSE COMPUTATION TECHNIQUE

For large networks the incidence matrix (M) is usually sparse. Moreover this matrix is banded i.e., all the nonzero elements lie within a band about the diagonal of the matrix. Some of the methods are proposed by Stoner [14] to store only nonzero elements of the matrix in appropriate positions to reduce the storage requirements.

2.4.4 TEARING AND DIAKOPTICS

This method is adopted for reducing number of iterations required and discussed by Ladet and Himmelbau [11].

In this method one of the values of variables are initially assumed and remaining equations are solved sequentially (As discussed in Carnahan and Christensen [6]) one at a time. Then we left with one model equation at last, which must be satisfied. This equation is called tearing equation. If the initial guess is correct then this tear equation will be satisfied and if not correct then iterative process will be required. This method is used only when number of tearing variables required are small in number and tearing equations are easy to solve. However the time taken by computer may be greater than that of with the original set of equations.

Diakoptics is special tearing technique and investigated by Gay and Middleton [11]. In this method a large network is transformed into small intermediate network so that their solutions can be found and these solutions are transformed into solution of the given network. In this way the computation is speeded up and amount of storage is reduced. However, the over all Computational performance of this method as reported by Gay and Middleton (M) has not been impressive. But in any case substantial reduction in storage is achieved.

2.5.0 DESIGN OPTIMIZATION IN PIPE NETWORK

So far different types of formulation and numerical solution of steady state pipe line network problem has been discussed. But in actual practice problem posed to the design engineers is entirely different. He is always interested in the optimal design of the problem in some sense. For example, given a set of requirements and specifications the ultimate design goal is to produce an optimal network that will meet all constraints at minimum cost or maximum profit. Similarly for an operating network he may be interested in minimizing the operating costs while meeting external demands or optimal expansion of existing facilities to meet anticipated future demands.

Several types of formulations of pipe line network problem from optimization point of view have been studied. These references are grouped in two ways. First on the basis of field of application and the second on the basis of the method of optimization. These are given in Table- 2.7 and Table-2.8

TABLE 2.7

Based on Application

Application	Source
Agricultural Engg.	Karmeli et al [17], Liang [18] Yang et al [19].
Gas distribution	Brameller et al [20]
Oil and gas transmission	Zimmer [21].
Pressure-relieving network	Cheng and Mah [22]
Sanitary Engg.	Cembrowicz and Harrington [23]
Water distribution	Lam and Walla [13,10], Watanatada [24], Donachie [15].

TABLE 2.8

Based on Optimization Methods in Pipeline Networks

Method	Source
Conjugate gradient	Watanatada [24]
Discrete merging	Chang and Mah [22]
Dynamic programming	Lang [18], Zimmer [21]
Geometric programming	Cheng and Mah [22]
Gradient projection	Murtagh [25]
Linear programming	Brameller et al [20]
Zontendijk's method of feasible direction	Cembrowicz and Harrington [23].

This study of optimization is classified according to problem requirement as follows -

i) Sensitivity analysis

ii) Design optimization

iii) Synthesis

It should be noted that the different classifications listed above are not independent of each other. For example, design optimization based on sensitivity information can be classified as either design optimization or sensitivity analysis. Likewise optimal pipe line routing may be classified as either design optimization or synthesis.

2.5.1 SENSITIVITY ANALYSIS

Sensitivity analysis is primarily concerned with the behaviour of the network to small fluctuations in variables with reference to the design conditions. If these fluctuations are large and numerous, then it will be more convenient to recompute the network as a whole. But in many cases it will be sufficient to determine the approximate behaviour. For more details one may refer chapter -3.

2.5.2 DESIGN OPTIMIZATION

Here the point of view is broad, that is here whole feasible space is included. For a given problem, the decision variables may be either continuous (i.e. nodal pressures) or discrete (i.e. pipe diameters) provided network configuration remains same. As the design optimization in some cases (as reported by [24]) may not achieve reliability constraints, so the design optimization process is further subdivided into single branch tree [18] and many branch tree [17,25,19,21].

2.5.3 SYNTHESIS

In pipe network synthesis, network configuration is not specified, but only flows and pressures required are specified. Synthesis is optimal design from functional requirement and specifications. Here neither the network nor its elements are given, only output and input conditions which are required are given. As an example of synthesis a study of optimal pipe line routing is reported in the literature (Shamir [4]).

In optimal pipeline routing even though topology of single pipeline is trivial one must take into consideration the factors like carridor through which pipeline passes, type of soil, tree cover, water and rivers, roads and railway lines etc.

It is worth while to mention that the synthesis of new pipe network is an area which is yet to be explored. One does not know that in future what type of general algorithm will emerge for synthesising pipe networks.

CHAPTER - III

PROBLEM FORMULATION

From literature review it is clear that the Newton Raphson method is best suited for most of the pipe network problems. According to Lam and Wolla [10] the Broyden's method does not need accurate initial guesses, but this method needs lot of matrix multiplications and hence lot of computer storage. Moreover it is a slow converging technique and its sensitivity towards improper initial guess can not be ruled out. Estimation of initial guess for Newton Raphson method is not very difficult job especially for formulation 4. But for formulation 3 with improper guess the solution fails to converge. Most of the times this occurs because of the oscillations in solution.

3.1 PROBLEM FORMULATION BY HAZEN WILLIAMS EQUATIONS

There are four types of formulations possible for any network problem as described in Table-2.3. First two methods are less sensitive to initial guess, but requires lot of storage even though these require less number of iterations as reported by Mah [7].

Formulation 3 is suitable when pressure or head at a particular node is known. This method is based on Krichoff's first law. Here equations of flow balance are written for each node whose head is unknown in following manner using Hazen Williams equation. Here the nodal flow balance is obtained with the nodal head as unknown variable using equation (2.8).

$$\sum_{j \varepsilon V_{Ai}} q_{ij} - \sum_{k \varepsilon V_{Bi}} q_{ik} = q_i \qquad \ldots(3.1)$$

$$i = 1, 2, \ldots J-1$$

The formulation 4 is useful when input (s) and outputs (s) flows to the network is given. This method makes use of Krichoff's second law. Here the balance of pressure drop around a closed loop is obtained by writting down the equations of head loss for each pipe in terms of flow rates for each loop using equation (2.10).

$$\sum_{k=1} \sigma_k (q_k) = 0 \qquad \ldots(3.2)$$

More detailed discussion of these two formulations is done with the example network shown in Figure (3.1).

The equations of the flow balance are written using formulation 3 for this network. Let head be known at node 1, then there is no need to write down the equation for node, 1. If the equation is represented by F then for node 2 the equation is

$$F_2 = -\left(\frac{H_1 - H_2}{C_{12}}\right)^{K_{12}} + \left(\frac{H_2 - H_5}{C_{25}}\right)^{K_{25}} + \left(\frac{H_2 - H_3}{C_{23}}\right)^{K_{23}} = 0 \qquad \ldots(3.3)$$

The sign convention followed here is that the pipe which brings the flow towards the node is -ve and that takes always the flow is +ve. C is factor of correlation which is given by Hazen Williams equation, which is as follows.

$$C = 10.7 \times L/D^{4.87} \times C_{HW}^{1.852}$$

where,

L = Length of pipe in meter

D = Diameter of the pipe in c.m.

C_{HW}=Hazen Williams constant for pipe material.

K = 0.54

Similarly for other nodes the equations of flow balance are written down as above.

In formulation 3 the effect of elevation has to be considered. For the given network elevation of each node is computed from fixed datum and following modification is done in equation (3.3).

$$F_2 = -\left(\frac{H_2-H_1+ELE_1-ELE_2}{C_{12}}\right)^{K_{12}} + \left(\frac{H_2-H_5+ELE_2-ELE_5}{C_{25}}\right)^{K_{25}} + \left(\frac{H_2-H_3+ELE_2-ELE_3}{C_{23}}\right)^{K_{23}} \quad \ldots(3.4)$$

where ELE = Elevation of the node from fixed datum.

In case of formulation 4 the equation of head loss in closed loops are written for each loop. For loop 1 the equation is written as

$$F_1 = C_1(Q_1+\Delta Q_1)^{K_1} + C_2(Q_2+\Delta Q_1-\Delta Q_2)^{K_2}$$
$$- C_3(Q_3-\Delta Q_1)^{K_3} - C_4(Q_4-\Delta Q_1)^{K_4} \quad \ldots(2.5)$$

where,

Subscripts indicate pipe numbers.

Q = Flow rate in pipes

K = 1.852

C = Hazen Williams constant for pipe material

ΔQ = Corrective flow for loop.

In this case the equations are nonlinear so corrective flow rate ΔQ for each loop is added in equations. The sign convention followed here is that the clockwise direction of flow is +ve, while anticlockwise direction of flow is -ve.

3.2 SOLUTION OF EQUATIONS

As above equations (3.3) and (3.5) are nonlinear these connot be solved directly. So some iterative technique has to be employed to solve them. One of the power full iterative method is Newton Raphson method. This method needs initial guess. as other iterative techniques require.

This method is based on Taylor series expansion of function f(x) about k^{th} iterate x_k. Here the function vector f(x) is represented by

$$\left.\begin{array}{l} f_1(x_1,x_2\ldots x_n) = f_1(x) = 0 \\ f_2(x_1,x_2\ldots x_n) = f_2(x) = 0 \\ \text{-----------------------} \\ f_n(x_1,x_2\ldots x_n) = f_n(x) = 0 \end{array}\right] = f(x) \quad \ldots(3.6)$$

Taylor series expansion of above function vector $f(\underline{x})$ is

$$0 = f(x) = f(x_k) + [\partial f_i/\partial f_j]\ (x-x_k) + \qquad \ldots(3.7)$$

By neglecting nonlinear higher order terms in $(x-x_k)$ and replacing x by x_{k+1} one can write

$$x_{k+1} = x_k - [\partial f_i/\partial x_j]^{-1}.f(x) \qquad \ldots(3.8)$$

Here the matrix of partial derivatives $[\partial f_i/\partial x_j]$ is called as Jacobian matrix of order N x N and denoted by [J]. It has to be evaluated at $x = x_k$ in each iterations. In this method approximations of x are generated as $x_1, x_2 .. x_k$ And this method converges only when,

$$\lim_{k \to \infty} x_k = x \qquad \ldots(3.8)$$

As exact value is immpossible to attain an error tolerance is generally provided for termination of the method.

$$|f(x)| \leq \varepsilon$$

i.e.

$$|(x_k - x_{k-1})/x_k| \leq \varepsilon \qquad \ldots(3.10)$$

In some cases the solution of f(x) oscillates. To avoide this the step size in each iterations is damped or reduced by factor τ whenever oscillations occur. Where τ should assume any value between 0 and 1. The Jacobian for the example shown in Figure (3.1) for formulation 3 using equation (3.3) is evaluated as follows.

The set of equations for node with unknown heads are written first. The partial derivatives of these equafions with respect to each unknown nodal head is found i.e. for equation (3.3) one can obtain

$$\frac{\partial F_2}{\partial H_2} = \frac{K_{12}}{C_{12}} \left(\frac{H_1-H_2}{C_{12}} \right)^{K_{12}-1} + \frac{K_{25}}{C_{25}} \left(\frac{H_2-H_5}{C_{25}} \right)^{K_{25}-1} + \frac{K_{23}}{C_{23}} \left(\frac{H_2-H_3}{C_{23}} \right)^{K_{23}-1}$$

$$\frac{\partial F_2}{\partial H_3} = - \frac{K_{23}}{C_{23}} \left(\frac{H_2-H_3}{C_{23}} \right)^{K_{23}-1}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots \qquad \ldots(3.11)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\frac{\partial F_2}{\partial H_6} = 0$$

Hence the Jacobian matrix

$$[J] = \begin{bmatrix} \frac{\partial F_2}{\partial H_2} & \frac{\partial F_2}{\partial H_3} & \cdots\cdots & \frac{\partial F_2}{\partial H_6} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial F_6}{\partial H_2} & \frac{\partial F_6}{\partial H_3} & \cdots\cdots & \frac{\partial F_6}{\partial H_6} \end{bmatrix}_{5x5}$$

and vector $f(x) = F = \begin{Bmatrix} F_2 \\ F_3 \\ \vdots \\ F_6 \end{Bmatrix}_{5x1}$ $\qquad \ldots(3.12)$

Similarly for the formulation 4 partial derivatives of equations (2.5) with respect to corrective flow for each loop are written as follows.

$$\frac{\partial F_1}{\partial \Delta Q_1} = C_1K_1(Q_1+\Delta Q_1)^{K_1-1} + C_2K_2(Q_2+\Delta Q_1-\Delta Q_2)^{K_2-1}$$

$$+ C_3K_3(Q_3-\Delta Q_1)^{K_3-1} + K_4(Q_4-\Delta Q_1)^{K_4-1} \cdot C_4$$

$$\frac{\partial F_2}{\partial \Delta Q_2} = - C_2K_2(Q_2+\Delta Q_1-\Delta Q_2)^{K_2-1}$$

The Jacobian matrix [J] can be written as

$$[J] = \begin{bmatrix} \frac{\partial F_1}{\partial \Delta Q_1} & \frac{\partial F_1}{\partial \Delta Q_2} \\ \frac{\partial F_2}{\partial \Delta Q_1} & \frac{\partial F_2}{\partial \Delta Q_2} \end{bmatrix}_{2x2}$$

and function vector f(x)

$$f(x) = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}_{2x1} \qquad \ldots(3.14)$$

Using equation (3.8) the value of x (head or flow) in each iteration is found and procedure is repeated till convergence is attained.

3.3 SENSITIVITY ANALYSIS

Sensitivity analysis is primarily carried out for small fluctuations in one or two variables with reference to design conditions. If these fluctuations are large and numerous then it is better to recompute the network as a whole.

For formulation3, sensitivity analysis is carried out with given head as external variable, and other nodal heads as state variables. The primary aim of the sensitivity analysis is to find out the effect of unit change in given head (one at a time) on heads of remaining nodes in order to determine the node which is less sensitive to changes in given head. The analysis is helpfull in the future expansion of the network.

If u is external variable and x is the state variable then

$$f(u,x) = 0 \qquad \ldots(3.15)$$

The effect of varying u on x is given by

$$df = (\partial f/\partial x)_u \, dx + (\partial f/\partial u)_x \, du$$

$$= f_x \, dx + f_u \, du \qquad \ldots(3.16)$$

or,

$$(\partial f/\partial x)u\,(\partial x/\partial u) = -\,(\partial f/\partial u)_x$$

If Newton Rapheson method is used, the matrix $(\partial f/\partial x)_u$ is already available i.e. Jacobian [J] and $(\partial x/\partial u)$ is to be determined and called as sensitivity matrix.

To determine the sensitivity matrix $(\partial x/\partial u)$ one need the partial derivatives of vector function $f(x)$ with respect to u. external variables. For example network shown in Figure (3.1) one can write for equation (3.3) with H_1 as an external variable u.

$$\frac{\partial f_2}{\partial H_1} = - \frac{K_{12}}{C_{12}} \left(\frac{H_1 - H_2}{C_{12}} \right)^{K_{12}-1} \qquad \ldots (3.17)$$

Hence the sensitivity matrix is column matrix. Using this matrix, the matrix $\{\partial f/\partial x\}_u$ is found out, which in turn gives the effect of varying the given head H_1 on remaining heads H_2, H_3, ... H_6.

3.4 INCLUSION OF PUMPS AND RESERVOIRS IN NETWORK

In formulation 4 is applicable to networks, in which the external flows are assumed known. The amount of flow being supplied from different reservoirs and pumps, as suggested by Jeppson [1], will depend upon heads and flows through out the network. Hence each pump and each reservoir, from which flow enters or leaves the network introduces a additional unknowns. Thus making problem more complex as input and output to the network is not directly known.

As seen in table (2.4) the pump characteristics can be found out by drawing graph of head v/s flow for pump and the polynomial assumed for this curve as quadratic. The constants of this curve are A and B.

In this problem flow in pipe connecting pumps and reservoirs are additional unknowns, while elevation of the reservoir, head at the inlet of the pump and pump characteristics are known parametdrs. To solve these unknowns one need additional equations. These additional equations are written by constructing pseudo loops in network. The problem is explained with the help of figure (3.2).

The pseudo loop is as shown in figure (3.2). This loop is noncyclic. To write down the energy equation, head loss around a pseudo loop must be zero, which can be written as

$$H_2 - C_5 Q_5^{K_5} - C_4 Q_4^{K_4} + C_3 Q_3^{K_3} + C_1 Q_1^{K_1} - h_p = H_1 \qquad \ldots(3.18)$$

where,

h_p = head produced by pump

H_1 and H_2 = Elevations of reserviors.

This equation can be rewritten as

$$-C_1 Q_1^{K_1} - C_3 Q_3^{K_3} + C_5 Q_5^{K_5} + C_4 Q_4^{K_4} + h_p = H_2 - H_1 \qquad \ldots(3.19)$$

Here $H_2 - H_1$ = Difference of elevations in reservoir.

h_p can be written in terms of flow as

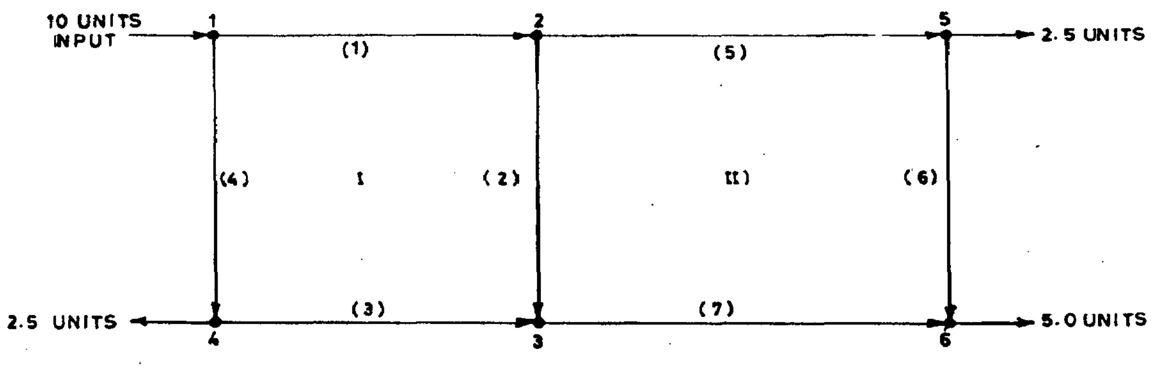


FIG. 3.1 AN EXAMPLE NETWORK

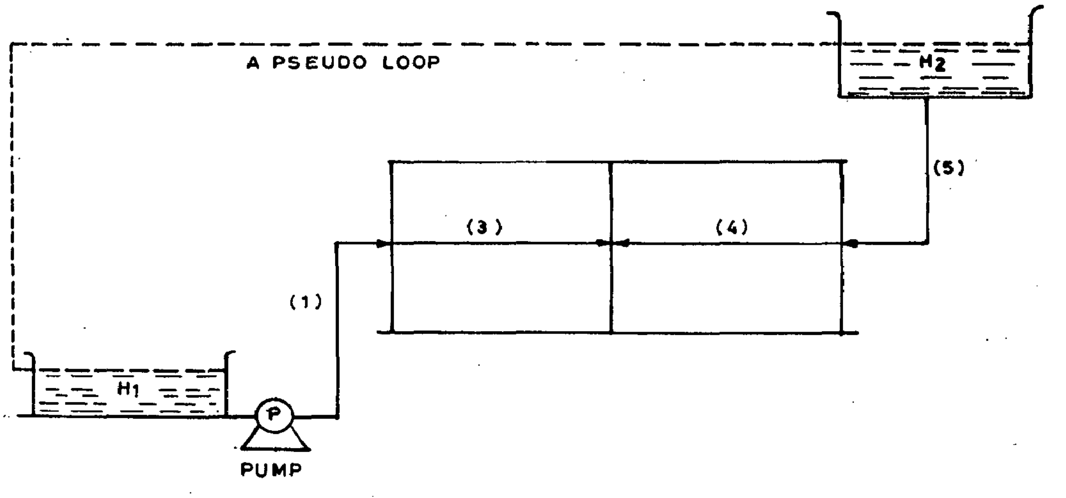


FIG. 3.2 A PSEUDO LOOP

$$h_p = AQ^2 + BQ + H_o$$

where A, B and H_o are constants obtained from pump characteristics curve.

These additional equations with the main set of equations as explained in (3.1) for formulation 4, are used to solve the network with the help of Newton Raphson method.

TABLE 3.1

Summary of formulation

S.No.	Method	Method of solution	Variables	Equation Numbers	Number of equations	Size of Jacobian [J]	Out put results
1.	Formulation 3	Newton Raphson Method	Nodal head in meter	3.1, 3.3 and 3.12	N-M	(N-M)x(N-M)	Head at each node and flow rates in each pipes.
2.	Formulation 4	Newton Rephson method	Flow rate in each pipe in cubic meter.	3.2, 3.4 and 3.14	ℓ	$\ell \times \ell$	Flow rate and head looss in each pipe
3.	Sensitivity Analysis	Newton Raphson Method	Nodal head in meter	3.16 and 3.17	(N-M)x(M)	(N-M)x(N-M)	Effect of change in head at given node on heads of remaining nodes
4.	Inclusion of pumps and reservoirs.	Newton Raphson method	Flow rate in cubic meter	3.18 and 3.19	ℓ+NSL	(ℓ+NSL)(ℓ+NSL)	Flow rates and head loss in each pipe

where,

N = Number of nodes

M = Number of nodes at which head is known

ℓ = Number of loops

NSL = Number of pseudo loops.

CHAPTER - IV

LOOP SELECTION

For formulation 4, set of independent loops are required. These independent loops are usually not unique. Mah [7] and Epp and Fowler [5] have reported that there is significant influence of loop selection on the convergence of solution in terms of computer time for each iterations, or total number of iterations required for convergence. A method of loop selection provides the least number of pipes involved in independent loops. More over if network is complex due to large number of pipes and loops, an efficient and reliable method of the loop selection is required because it is difficult to select the number of independent loops manually. At the same time there is possibility of making errors and an incorrect loop may result in a poor convergence of the solution.

A loop selection algorithm has been suggested by Epp and Fowler [5]. In this chapter this algorithm has been described in short. This algorithm requires the use of an another algorithm for finding shortest path between any two nodes. This algorithm is difficult to use in form of programming point of view. Therefore the use of a new algorithm is proposed for finding the shortest path between two nodes. The proposed algorithm is incorporated in the loop selection method of Epp and Fowler [5].

4.1 LOOP SELECTION ALGORITHM DUE TO EPP AND FOWLER [5]

This algorithm is as follows:

Step 1 - Find the degree of each node i.e. number of pipes connected to that node.

Step 2 - If node of degree 1 is present, then remove tail pipe form node with degree 1 and go to step 1. Otherwise go to step 3.

Step 3 - Start with node of degree 2. It is called the key node.

Step 4 - Go to the other end nodes of the pipes connected to key node.

Step 5 - Remove pipes connected to key node.

Step 6 - Find the shortest path between these two end nodes and assign the loop.

Step 7 - If all the loops are selected then stop. Otherwise go to step 1 and repet the procedure with next key node.

For more details of the algorithm and also of shortest path algorithm, employed by Epp and Fowler [5] reder is advised to refer the original paper. However, the use of this algorithm is explained below for the pipe network shown in figure (4.1).

Step 1: Degree of node is in following table.

Node Number	Degree of Node
1	3
2	2
3	1
4	2
5	3
6	3
7	3
8	2
9	3
10	2

Step 2: Since node 3 is of degree 1, therefore pipe -(1) is removed. Due to this node 2 becomes a node of degree-1. Therefore pipe (2) is also removed. Now node 1 is of degree 2.

Step 3: Node 1 has now become a key node.

Step 4: Node 4 and 7 are at the other ends of pipes, which are connected to node.

Step 5: Pipes (3) and (5) are removed.

Step 6: Shortest path between 4 and 7 is found and it is through nodes 5 and 6. It is denoted as loop I.

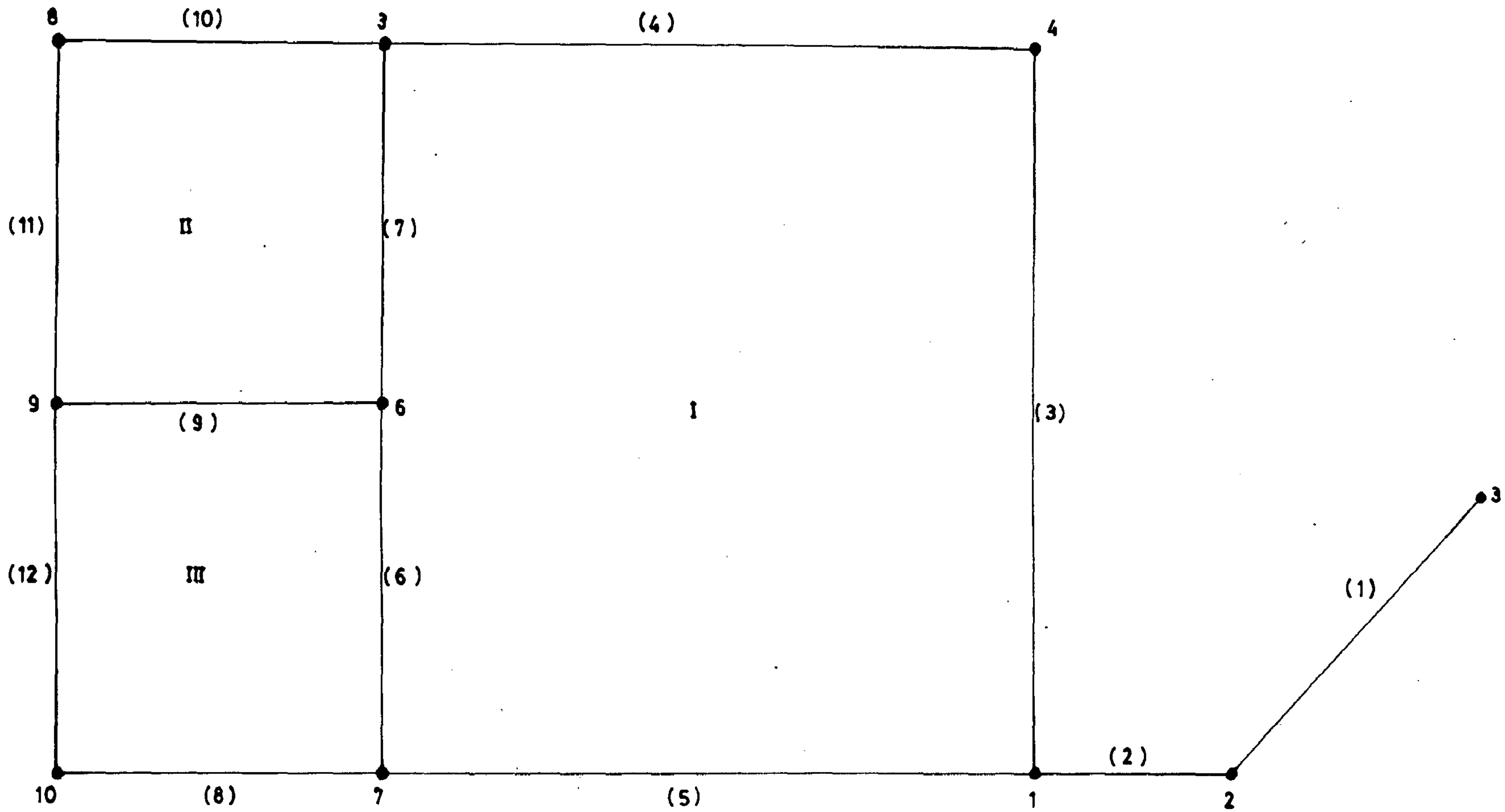


FIG. 4.1 AN EXAMPLE NETWORK TO DEMONSTRATE LOOP SELECTION

Step 7: In this way all the independent loops are found, which are as follows:

Loop Number	Node numbers in Loop	Pipe numbers in Loop
1.	1,4,5,6,7	(3),(4),(7),(6),(5)
2.	5,8,9,6	(10),(11),(9),(7).
3.	6,9,10,7	(9),(12),(8),(6).

4.2 PROPOSED ALGORITHM FOR FINDING SHORTEST PATH BETWEEN NODES

Dijkstra [27,28] have developed an algorithm for finding the shortest path between two nodes for general network flows. This is now a days used for shortest route problems in Operation Research. This algorithm is based upon the principales of graph theory. It is proposed to use this algorithm for finding shortest path between two nodes of a pipe network. If there is no pipe between two nodes, then distance is taken as infinite. Following nomenclature is required for describing this algorithm.

d_{ij} = Distance between node i and j

L_{sk} = Shortest distance between source node s to node K.

L'_{sk} = The shortest distance between source node s to node k using tree arcs and atmost one non tree arc. For all chains from source node s to some node k that require more than one non tree arc, $L'_{sk} = \infty$.

Dijkstra's algorithm.

Step 1: Let $L'_{sk} = d_{sk}$ initially for source node s to only node in the tree and $L_{ss} = 0$

Step 2: L_{sr} = minimum of $L'_{sk} = L_{sj} + d_{jr}$. Here k are neighbour nodes of current tree.

Step 3: Make arc (j,r) a tree arc.

Step 4: If number of tree arcs are N-1, terminate the algorithm. Otherwise go to step 5.

Step 5: L'_{sk} = Minimum $(L'_{sk}, L_{sr}+d_{rk})$. Go to step 2.

The detail description of algorithm is explained with network shown in figure (4.2). The node numbers and distances are shown in Figure. For convinence label of type (L,i) is given to each node, where $L=L'_{sk}$ or L_{sk} and i refers to last node on shortest chain from source node s to node k. The labels are of two types. If $L= L'_{sk}$ then label is temporary and if $L=L_{sk}$, then it is permanent.

In given example, to start with node 1 as source node. The neighbour nodes are 3 and 2. Here

$L'_{1k} = d_{1k}$

i.e.

$L'_{1,2} = d_{1,2} = \{4,1\}$

$L'_{1,3} = d_{1,3} = \{6,1\}$

min $L'_{1,k} = L'_{1,2} = (4,1) = L_{1,2}$ a permanent label.

The neighbour nodes of two node tree 1-2 are 3,4 and 5.

$L'_{1,3} = \min(L'_{1,3}, L_{1,2}+d_{2,3}) = \min(6, 4+\infty) = 6$

$L'_{1,4} = \min(L'_{1,4}, L_{1,2}+d_{2,4}) = \min(\infty, 4+5) = 9$

$L'_{1,5} = \min(L'_{1,5}, L_{1,2}+d_{2,5}) = \min(\infty, 4+5) = 9$

Shortest path is $L'_{1,3} = L_{1,3} = (6,1)$ now becomes permanent label.

The neighbour nodes of tree 1-2-3 are 4 and 5.

$L'_{1,4} = \min(L'_{1,4}, L_{1,3}+d_{3,4}) = \min(9, 6+4) = 9$

$L'_{1,5} = \min(L'_{1,5}, L_{1,3}+d_{3,5}) = \min(9, 6+7) = 9$

Here tie is orbitrarily broken anc 5 is selected as permanent label, i.e. $L_{1,5} = 9 = (9,2)$. Now tree is 1-2-5 and its neighbouring node is 6.

$L'_{1,6} = \min(L'_{1,6}, L_{1,5}+d_{5,6}) = (\infty, 9+1) = 10$

Now 6 is node of destination. The shortest path is as shown in Figure (4.3) and shortest distance is 10 units.

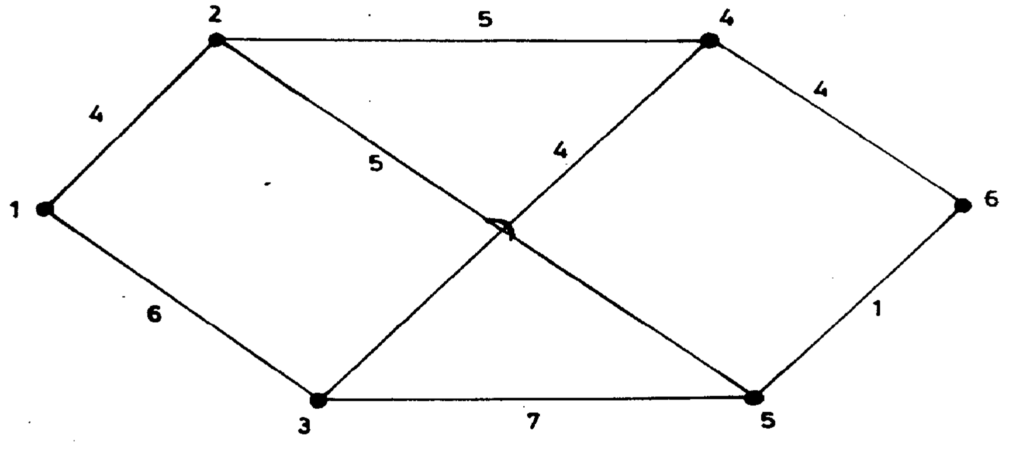


FIG. 4.2 AN EXAMPLE NETWORK TO DEMONSTRATE DIJKSTRA'S ALGORITHM

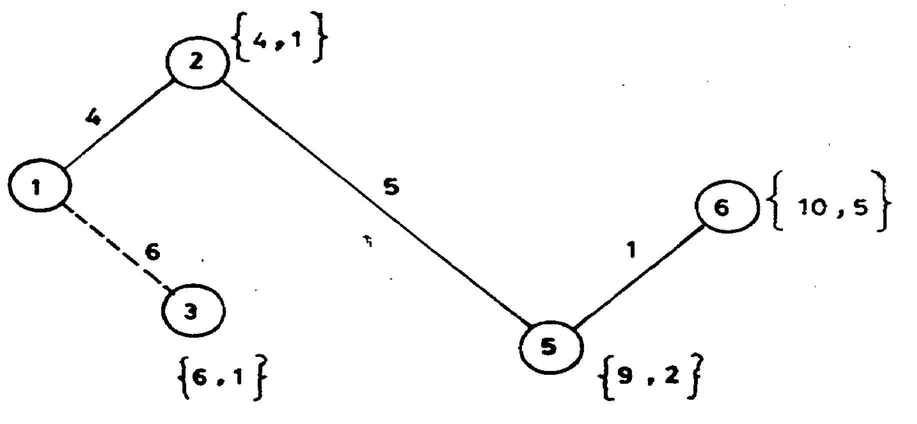


FIG. 4.3 SHORTEST PATH

In this chapter loop selection algorithm due to Epp and Fowler has been modified by incorporating Dijkstras shortest path algorithm. This modified loop selection algorithm shall be used in conjunction with Formulation 4.

CHAPTER - V

SOFT WARE PACKAGES

Three software packages have been developed to solve the pipe network problem. These are described below.

5.1 PROGRAM FOR SOLVING THE PIPE NETWORK USING FORMULATION 3

This program solves the network using the formulation 3. Here the heads at few nodes, called input nodes, are known and the heads of remaining nodes are obtained by using Newton Rephson method. Knowing head at each node the flow rate in each pipe may be calculated. Sensitivity analysis of the network is also performed, by which the effect of unit change in given head at any one of the input nodes on heads of remaining nodes is predicted. Flow chart for this programme is given in Figure (5.1), and details of programme are as given below.

Name of programme : MNRM.FOR

Subroutines used : PNRM, SENSE, SOLVE

Input Variables :

N = Number of nodes

NP= Number of pipes

ITMAX= Maximum number of iterations allowed.

ERR = Error tolerance.

P = Initial estimate of head at each node in meter.

JPGIV = Code, which mentions given head node

1 if head is known and 0 if head is not known.

QN = Nodal input or output flow in cubic meter .
+ve for output flow and -ve for input flow.

H = Elevation of each node in meter.

D = Diameter of pipe in centimeter.

PL = Length of pipe in meter.

CHW = Hazen Williams constant for pipe material

Output variables:

P = Head at each node in meter after convergence.

Q = Flow in each pipe in cubic meter after convergence.

HL = Head loss in each pipe in meter.

Y = Sensitivity of each node.

Intermediate variables of the programe are as given below:

INCID= Matrix defining topology of the network. If value of the element is 1 then there is the pipe present, and 0 when pipe is absent.

ITR = Iteration number.

C = Hazen Williams constant for pipe

AA = Jacobian matrix. (J)

AB = Column vector of the function f(x) i.e. for the nodal flow balance equations.

Y = Solution vector

DS = Sensitivity matrix.

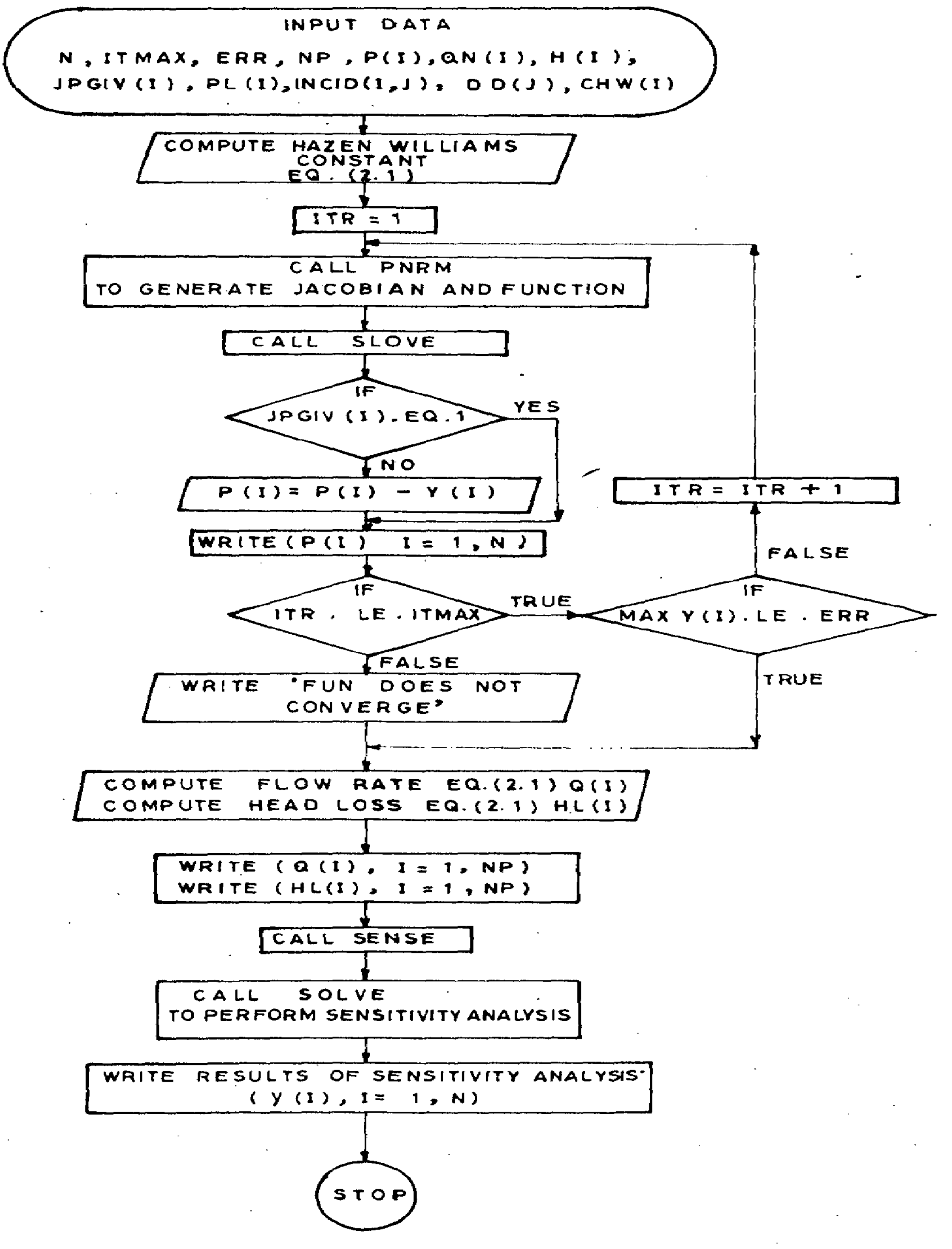


FIG. 5.1 FLOW CHART FOR PROGRAM MNRM.FOR

5.2 PROGRAM FOR SOLVING PIPE NETWORK USING FORMULATION 4

This program solves the network using formulation 4 and Newton Raphson method. This also uses computer aided loop selection procedure for selecting the independent loops in the network. Input flow(s) to the network should be known as the input data. The program results in the values of flow rates in each pipe, which may be subsequently used to calculate the head losses in each pipes. Flow chart for this program is given in figure (5.2) and details of program are as given below:

Name of the program : QNRM.FOR

Subroutines used : LOOP, SHOP, SOLVE

Input Variable:

N, NP, ITMAX, ERR = same as discussed in section (5.1)

NPUMP = Number of pumps

NSL = Number of pseudo loops.

CODE = Code for loop selection. Its value is 1, if loops are provided and 0 when computer aided loop selection has to be done.

D, PL, CHW = Same as explained in section (5.1)

QI = Initial estimate of flow rate in each pipes in cubic meter.

JLP = Number of pipes in the loop.

LP = Pipe numbers in the each loop.

A1, B1 = Constants A and B for pump.

HO = Supply Head of the pump in meter.

LNP = Pipe numbers in the each pseudo loop

NL = Number of loops in the network.

DEL = Elevations of reservoirs.

Output Variables:

QI = Flow rates in each pipes in cubic meter after convergence.

HL = Head loss in pipes in meter .

Intermediate variables of the program are as given below:

FL = Matrix storing the distance between nodes.

INCID = Same as explained in section (5.1).

LPP = Matrix storing pipe numbers between nodes.

NN = Column matrix storing number of pipes in each loop.

DR = Jacobion matrix (J).

RD = Column vector of function f(x) i.e. loop pressure drop equations.

Y = Solution vector.

DQ = Corrective flow ΔQ for each loop.

NCT = Number of iteration.

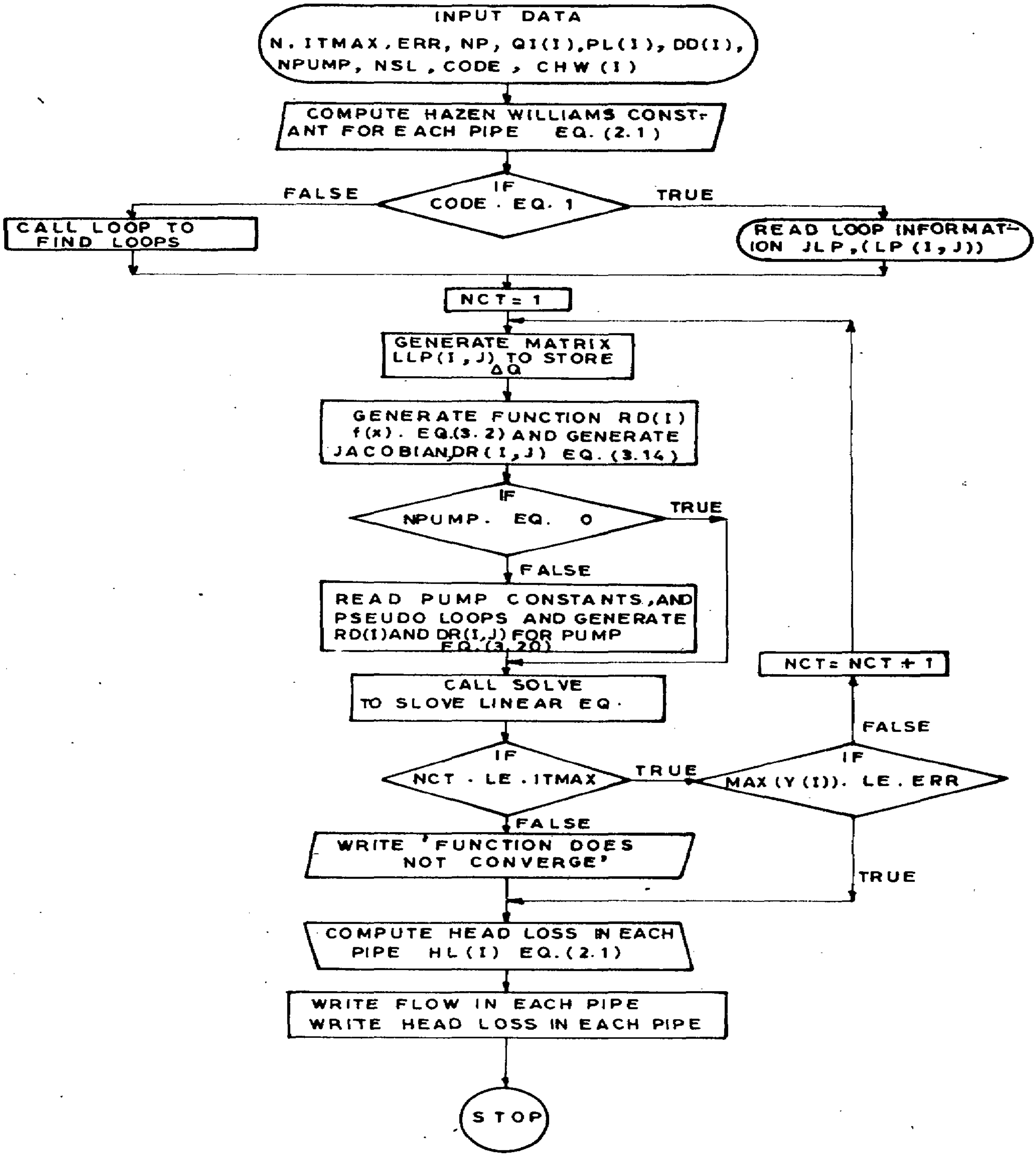


FIG. 5.2 FLOW CHART FOR PROGRAM QNRM. FOR

5.3 PROGRAM FOR SOLVING PIPE NETWORK USING MIXED METHOD OF FORMULATION

In this method of formulation the problem is first started with usual formulation 3. As soon as flow balance at input node(s) is achieved, the problem is solved with formulation 4. By adopting this procedure the in flow to the network remains fixed, while flow in other pipes changes and there by it stablizes the head at the nodes, which are away from the input node(s). After stablization, formulation 3 is used once again to obtain the final solution. The algorithm for mixed method is as follows:

Step 1 - Read Input.

Step 2 - Read pipe number(s) connected to input node(s)

Step 3 - Start iterative procedure using formulation 3.

Step 4 - Check for stablity of inflow in each iteration. If they are stablized proceed, otherwise go to step 3.

Step 5 - Perform iterative procedure using formulation 4 till convergence is attained.

Step 6 - Compute head losses in pipes and there by oompute head at each node.

Step 7 - Again use formulation3 to achieve final solution.

Flow chart for this program is given in figure (5.3) and detailed of this program are as given below.

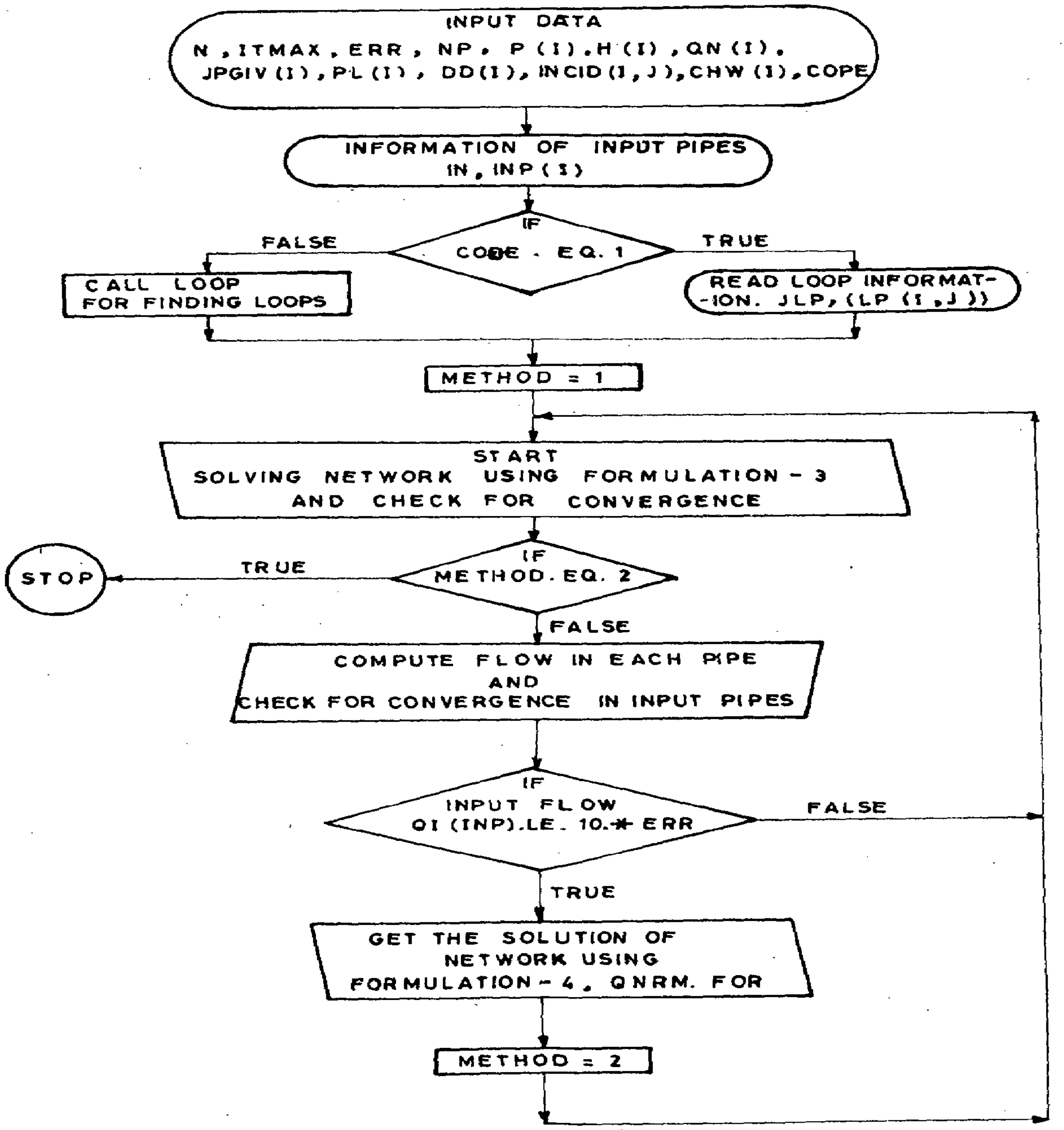


FIG. 5.3 FLOW CHART FOR PROGRAM MIX. FOR

Name of the program : MIX. FOR

Subroutines used : PNRM, LOOP, SHOP, SOLVE, SENSE

Input Variables :

Same as that of MNRM.FOR, which has been explained in section (5.1). Additional data is as given below.

INP : Pipe numbers connected to input node(s)

NIP : Number of pipe(s) connected to input node(s)

Output Variables:

Same as that of MNRM. FOR, which has been explained in section (5.1).

Legends of the programe are same as that of MNRM.FOR and QNRM.FOR explained in section (5.1) and section (5.2) respectively.

5.4 DESCRIPTION OF THE SUBROUTINES

Detail description of the Subroutines used in above programs are as given below.

5.4.1 SUBROUTINE PNRM (N,P,C,H,QN,AA,AB,JPGIV,INCID)

This subroutine develops the nodal flow equations and Jacobian matrix [J] for solution of the network problem by Newton Raphson method using formulation 3. Flow charts for this subroutine is given in Figure (5.4) and details are as given below:

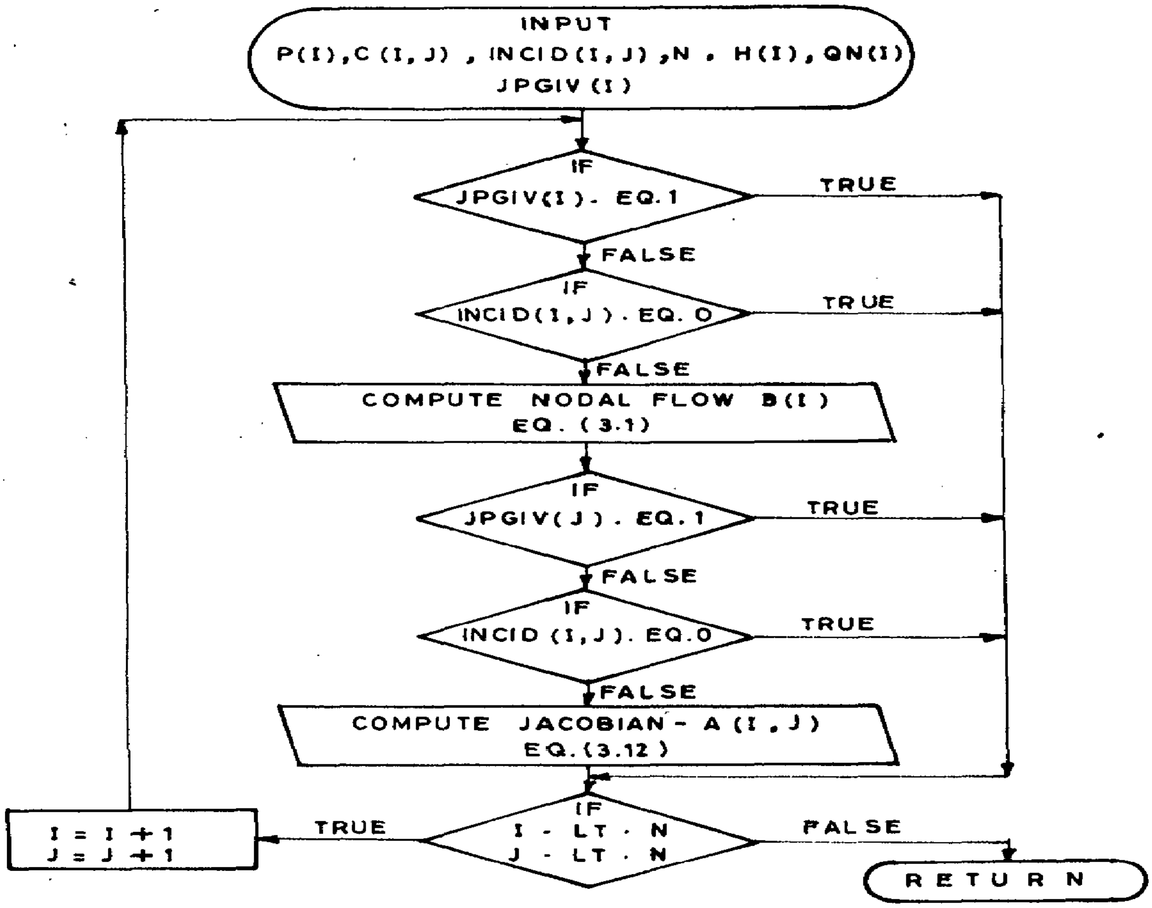


FIG. 5.4 FLOW CHART FOR SUBROUTINE PNRM

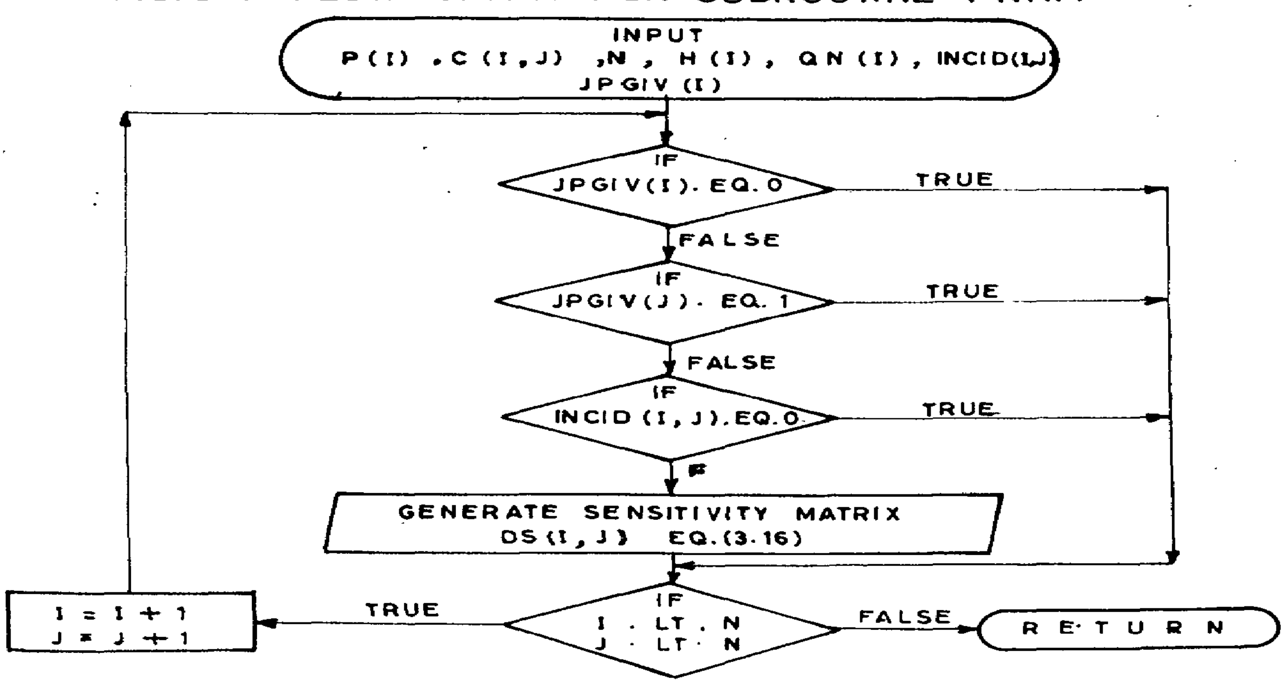


FIG. 5.5 FLOW CHART FOR SUBROUTINE SENSE

Input Variables

N, C, P, H, QN, INCID, JPGIN

Same as explained in section (5.1).

Output Variables :

AA = Jacobian matrix [J]

AB = Function f(x)

5.4.2 SUBROUTINE SENSE (N, P, C, H, JPGIV, INCID, QN, DS)

This subroutine performs sensitivity analysis by generating sensitivity matrix DS. Flow chart for this subroutine is given in figure (5.5) and details are as given below:

Input variables :

N, C, P, H, QN, INCID, JPGIV

Same as explained in section (5.1).

Output Variables:

DS = Sensitivity matrix.

5.4.3 SUBROUTINE SHOP (AM, FL, N, JS, AP)

This Subroutine finds shortest path between any two nodes using Dijkstara's algorithm which has been discussed in Chapter 4.

Flow chart for this subroutine is given in Figure (5.6) and details are as given below.

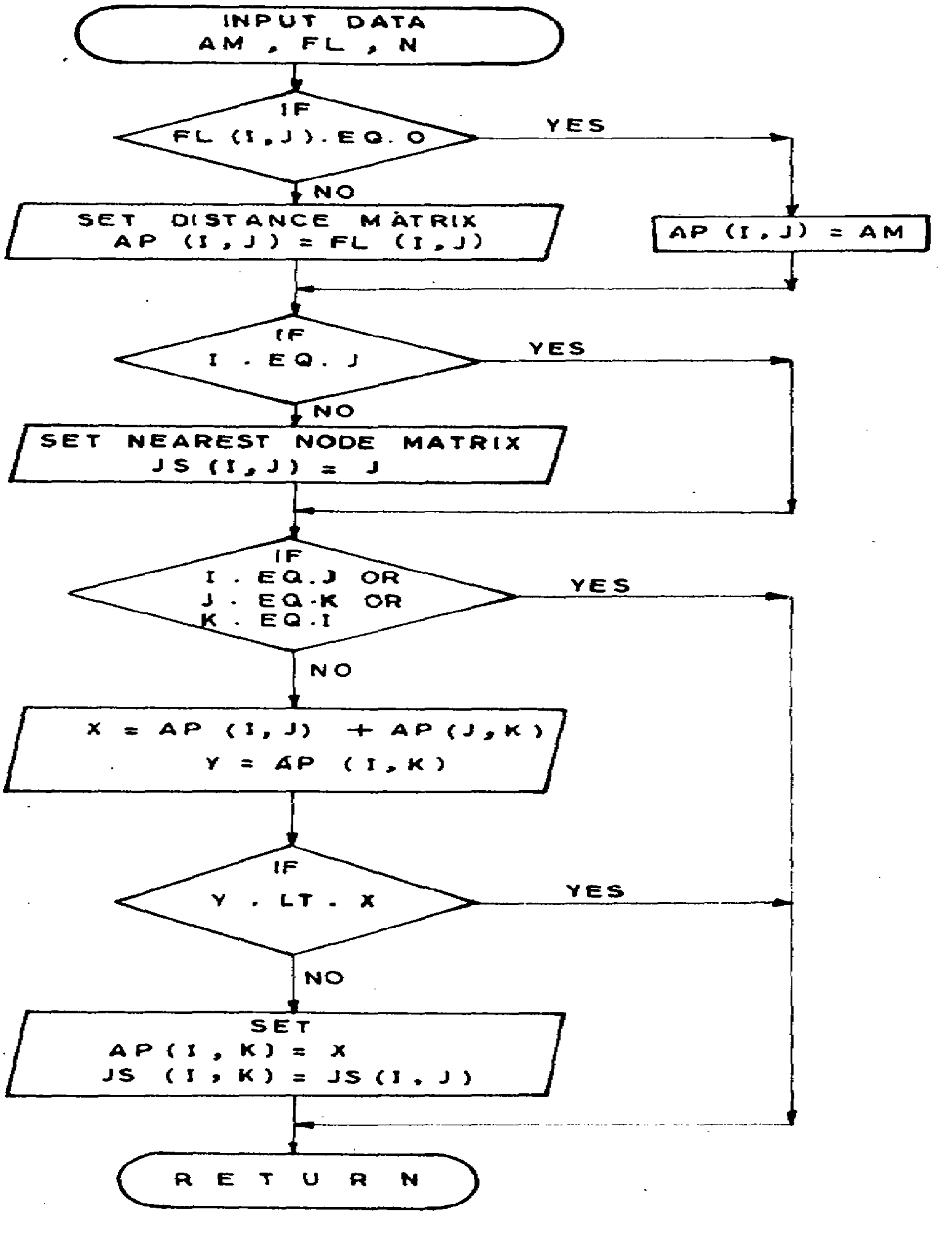


FIG. 5. 6 FLOW CHART FOR SUBROUTINE SHOP

Input Variables :

AM = A very large number (99999.) => ∞.

FL and N Same as explained in section (5.2).

Output Variables:

JS = Matrix which gives the nearest node to each node,

AP = Matrix which gives the shortest distance between any two nodes.

5.4.4 SUBROUTINE LOOP (INCID, FL,N,NP,LP,NN,NL)

This subroutine gives the set of independent loops in the network by making use of shortest path algorithm. This shortest path between any two nodes is made available by subroutine SHOP. Flow chart for this subroutine is given in Figure (5.7), and details are as given below.

Input Variables:

INCID, FL, N,NP

Explained in section (5.2).

Output Variables:

NL, NN, LP

Same as explained in section (5.2).

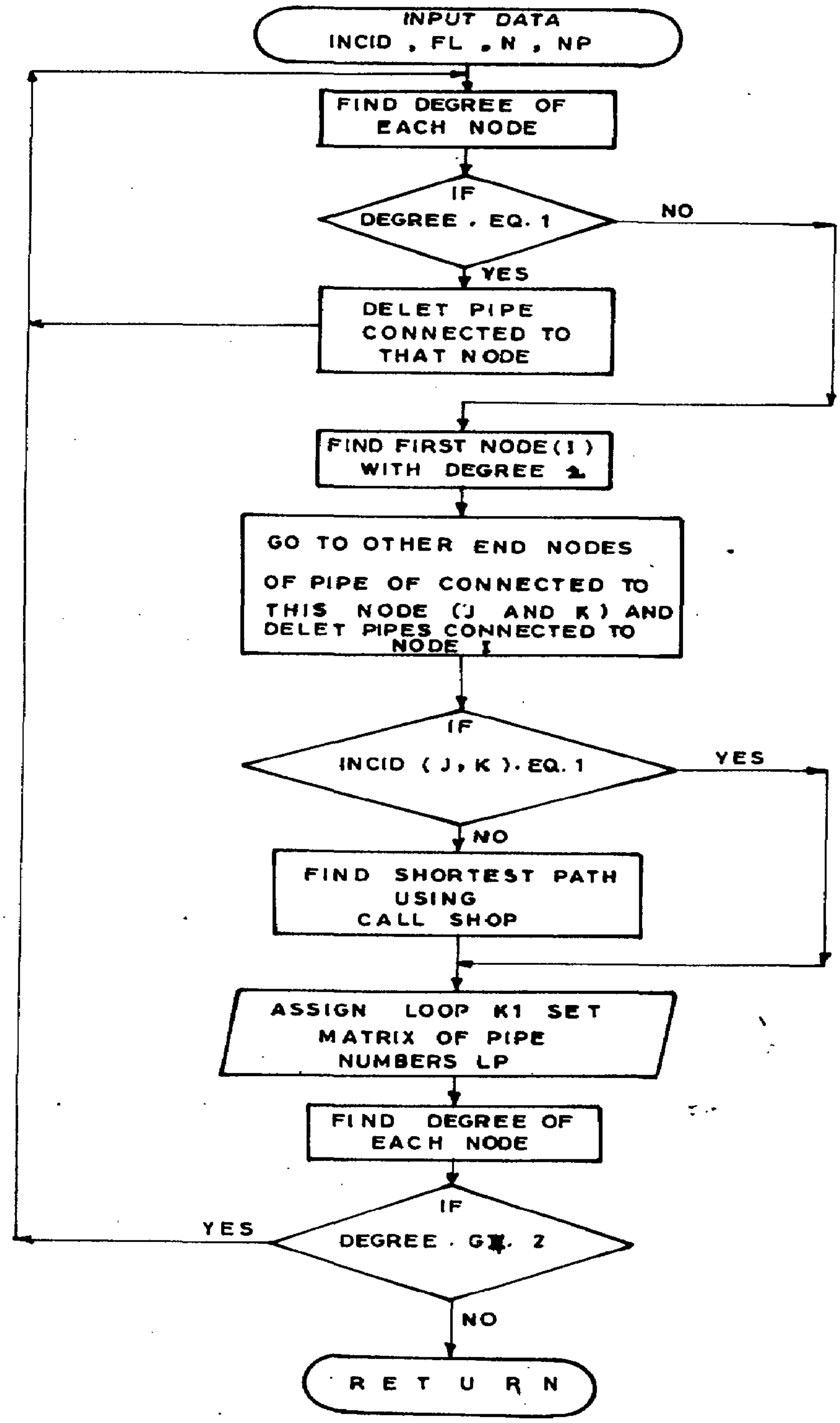


FIG. 5.7 FLOW CHART FOR SUBROUTINE LOOP

5.4.5 SUBROUTINE SOLVE

This subroutine solves set of linear equations with help of Gauss elimination method with Pivot selection.

Input Variables:

A = Jacobian matrix

B = Column vector of function f(x)

N = Number of equations or number of rows in Jacobian matrix.

Output Variables :

Y = Column vector of solutions of each variable.

CHAPTER - VI

RESULTS AND DISCUSSION

In this chapter results pertaining to the convergence problem associated with the Newton Raphson method of solving pipe network problems are reported and the new scheme is proposed to avoid it. Further a new method of formulation of pipe network problem called mixed method of formulation is suggested and the results about its computational effi - ciency are presented. Lastly advantages of computer aided loop selection method is discussed.

It is well known fact that Newton Raphson method is sensitive to initial guess. This aspect has been studied in detail and modifications in the existing algorithms are proposed. It is necessary to mention at this stage that there is no clear cut distinction between a proper guess and an improper guess but the following points should be taken into consideration for deciding the initial guess.

- Order of the values of head at nodes should be in accordance with the topology of the network.
- The values of the head at nodes which are far away from the input node(s) are more sensitive to initial guess. So more care should be taken while supplying guess values at these nodes.

- The values of heads at nodes with lower degree are more sensitive as compared to the nodes with higher degree. Accordingly precaution should be taken.

6.1 EFFECT OF DAMPING FACTOR ON CONVERGENCE

It is observed that with improper initial guess the values of head at nodes in case of formulation 3 oscillate. If these oscillations continue for number of iterations, the convergence can never be attained. It is also to be noted that these oscillations usually starts at nodes which are far away from input node(s). And due to this the head at remaining nodes also starts oscillating This is shown in Figure (6.2) for network in Figure (6.1). In the considered problem, node (25) is far away from the input node (7), while node (1) is comparatively nearer to node (7). To an improper initial guess, oscillations have been observed in the values of head at both nodes. These oscillations persist even if the number of iterations increases. However the approach to the convergence is faster at node (1) as compared to node (25). This is because the node 1 is nearer to input node (7). Thus the improper initial guess require large number of iterations for obtaining final solution.

Next the solution has been attempted with relatively proper values of initial guess. Effect on convergence at node (1) and node (25) has been shown in Figure (6.3).

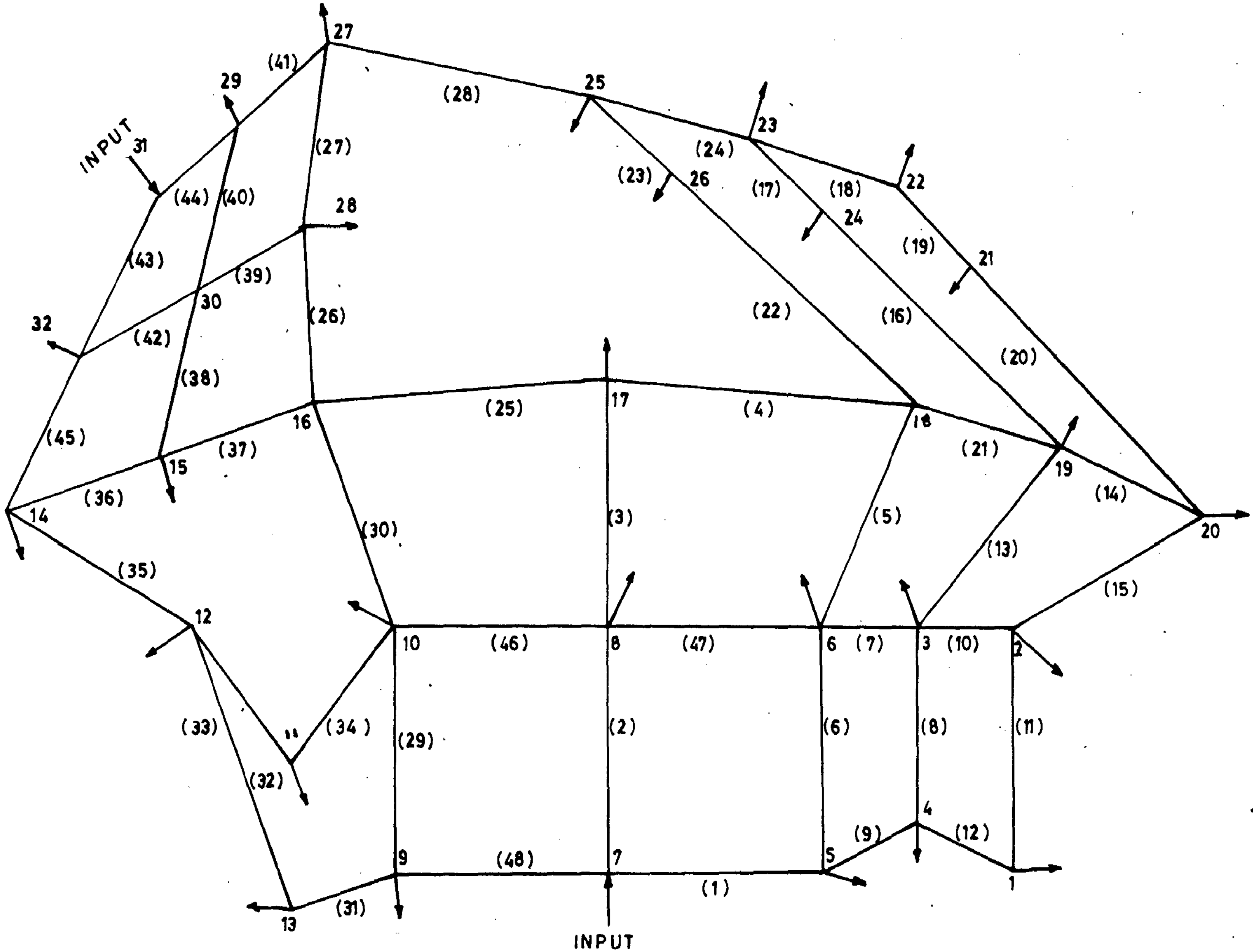


FIG. 6.1 A TEST PROBLEM

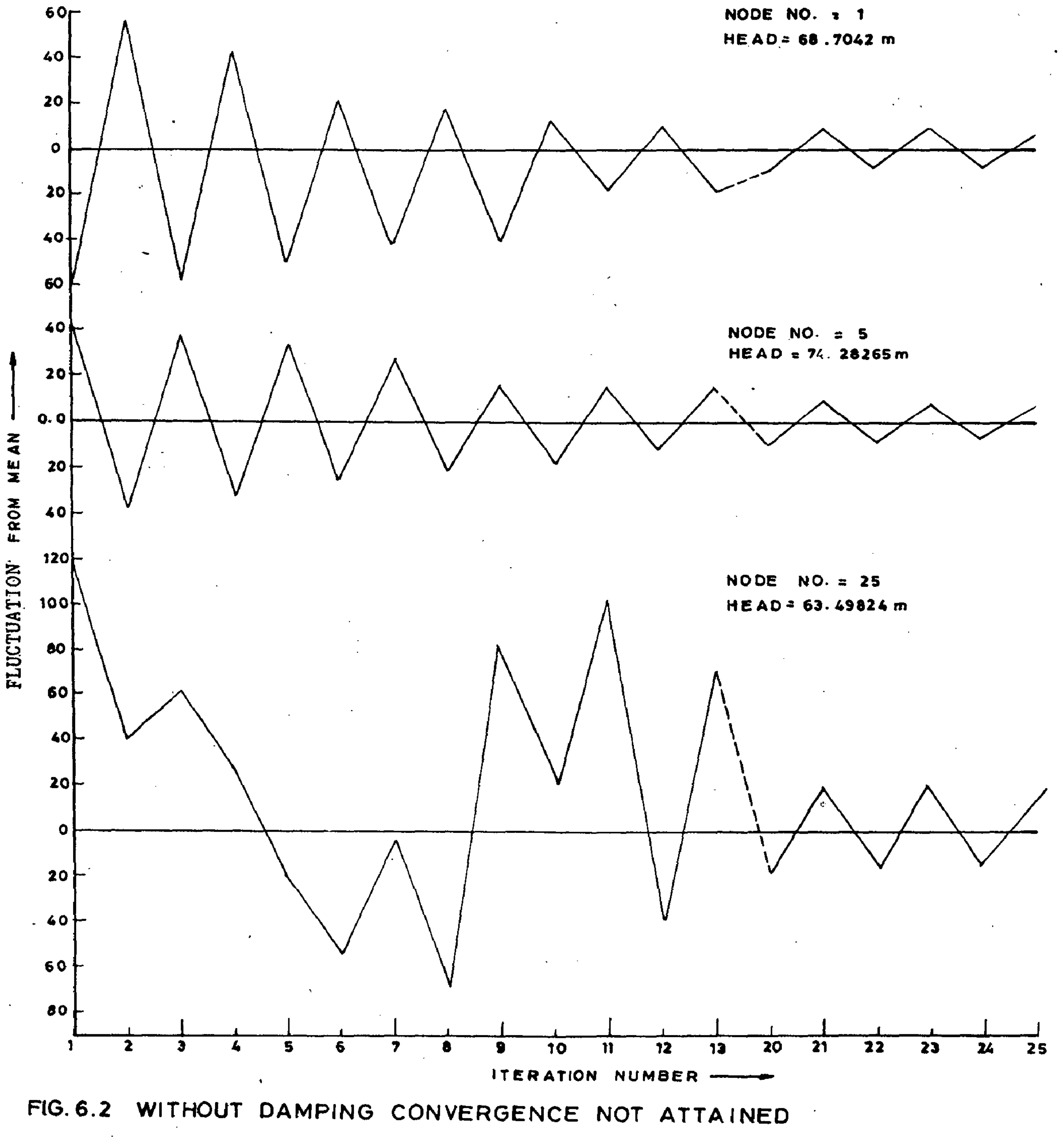


FIG. 6.2 WITHOUT DAMPING CONVERGENCE NOT ATTAINED

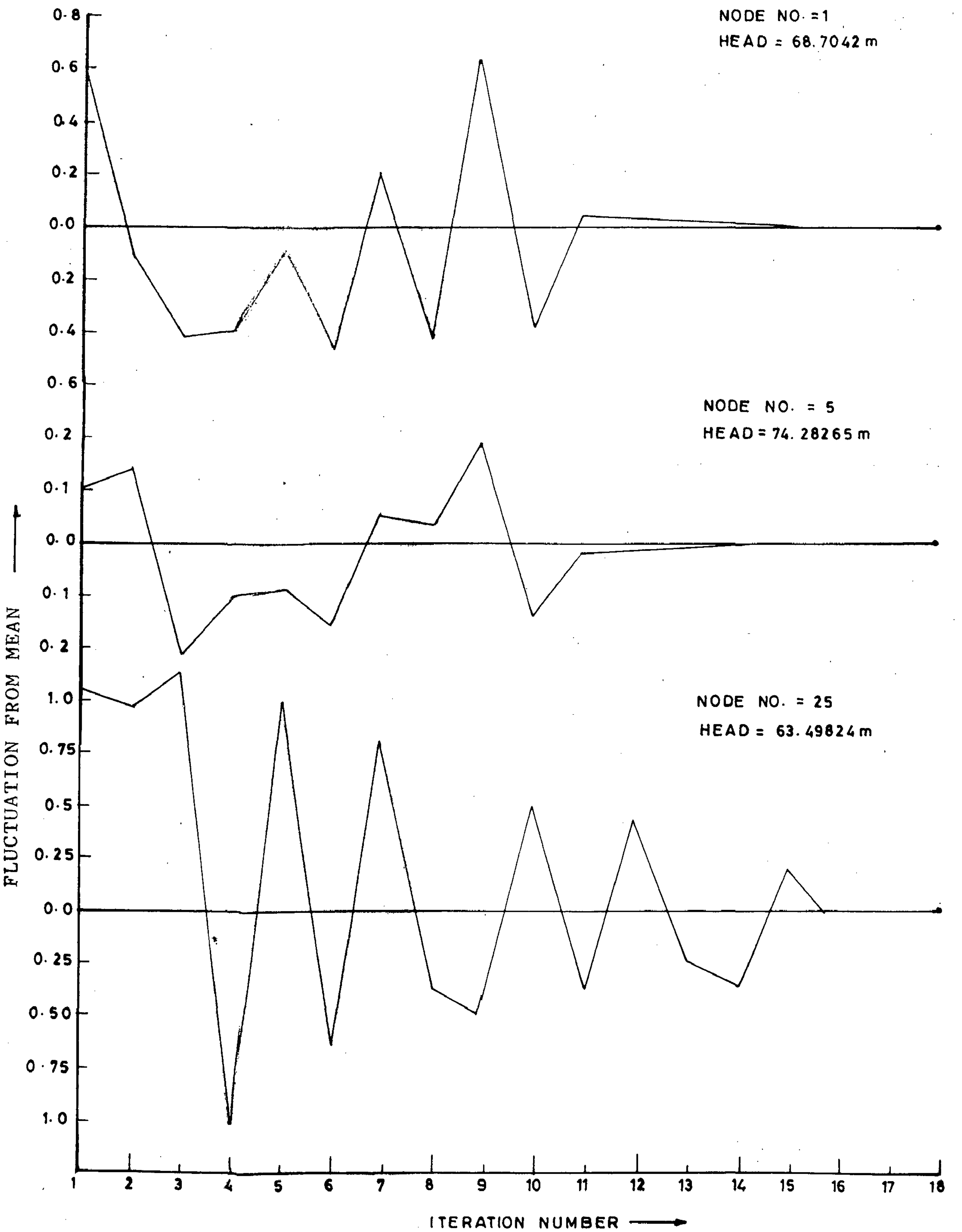


FIG. 6.3 CONVERGENCE ATTAINED WITHOUT DAMPING

It is clear that the magnitude of oscillations at both the nodes is less in comparision to previous case, and also number of iterations required are less.

It is proposed that the solution vector during iterations should be damped by factor τ to improve the rate of convergence. The value of τ varies between 0 and 1 and the exact value depends upon the problem being solved. Value of τ taken here is 0.5. It means that $x_k = x_{k-1} + 0.5\, y_k$. It is observed that the damping accelrates the rate of convergence even with an improper guess. This fact is illustrated in Figure(6.4) Number of iterations required for these cases are tabulated in table (6.1).

TABLE -6.1

Results of the network shown in Figure (6.1) by using formulation -3.

S.No.	Initial guess	Method used	Number of iterations taken for convergence
1.	Improper	With out damping	Terminated after 25 iterations
2.	Proper	Without damping	18
3.	Improper	With damping factor 0.5	12
4.	Proper	With damping factor 0.5	11

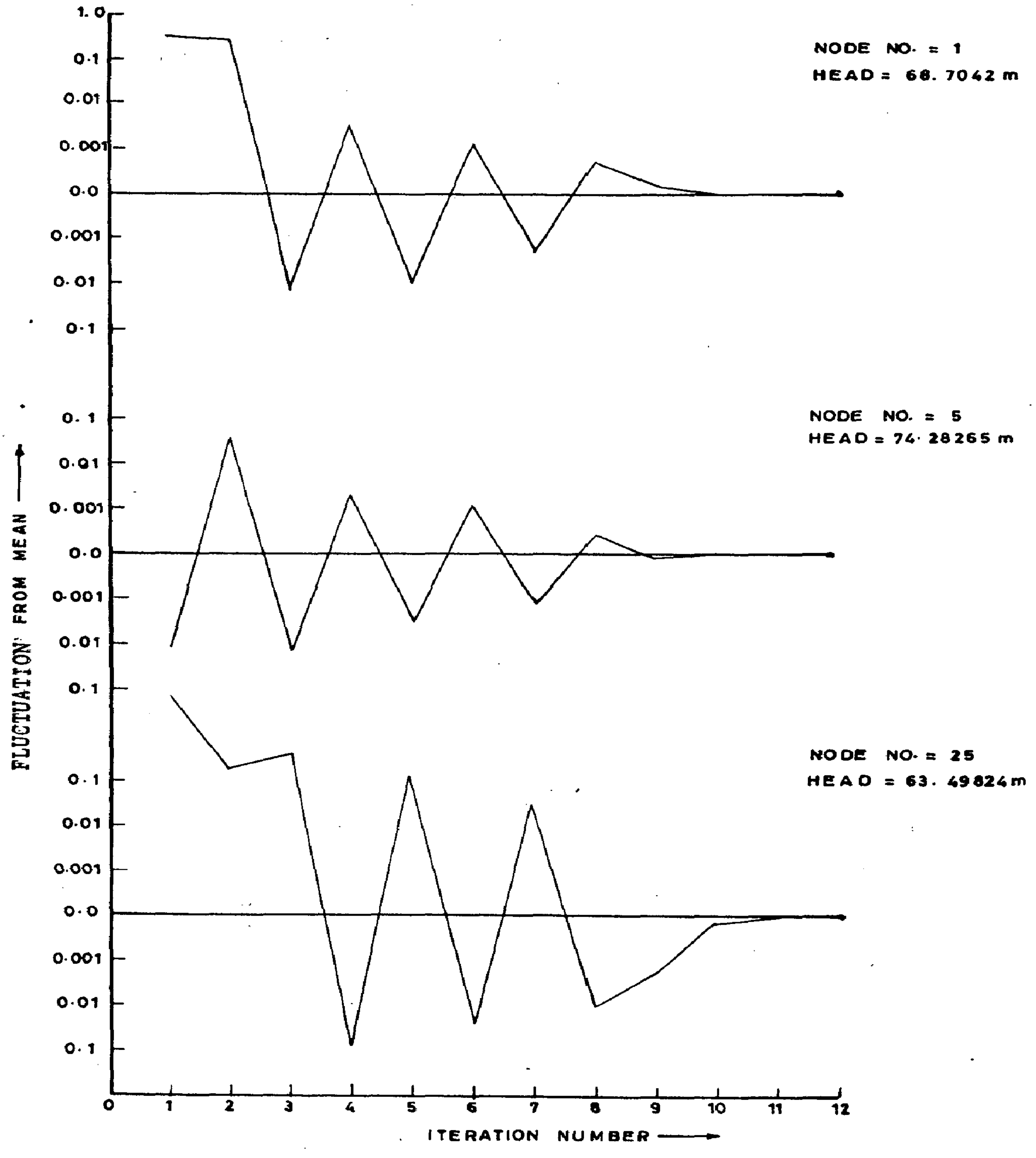


FIG. 6.4 CONVERGENCE PATTERN WITH DAMPING FACTOR 0·5

6.2 CONVERGENCE IMPROVEMENT BY MIXED METHOD OF FORMULATION

A care full study of graphs in Figure (6.4) shows that for formulation3, the values of heads at the node which are nearer to input node(s), converge faster than the values of head at the nodes which are away from it. Due to this fact a new method of formulation, called mixed of formulation, has been proposed which is already discussed in chapter 5.. However, one might think that this method is same as the formulation 1. But this is not true. In fact in this method the number of equations solved are either (J-1) (in case of formulation 3) or ℓ (in case of formulation 4) and hence the amount of storage required is much smaller than in the case of formulation

1. The results of the test problems Figure(6.1) and Figure (6.6))by using mixed method of formulation are listed in Table (6.2).

It is noted that that the switch over from formulation 3 to formulation 4 is done after 4 iterations in case of network in Figure (6.1) and after 5 iterations in case of network in Figure (6.6). At this stage flows in pipes, connected to input nodes, are closely balanced first time during solution. If the switch over is carried out after this stage in either case, the total number of iterations required for obtaining the final solution increases.

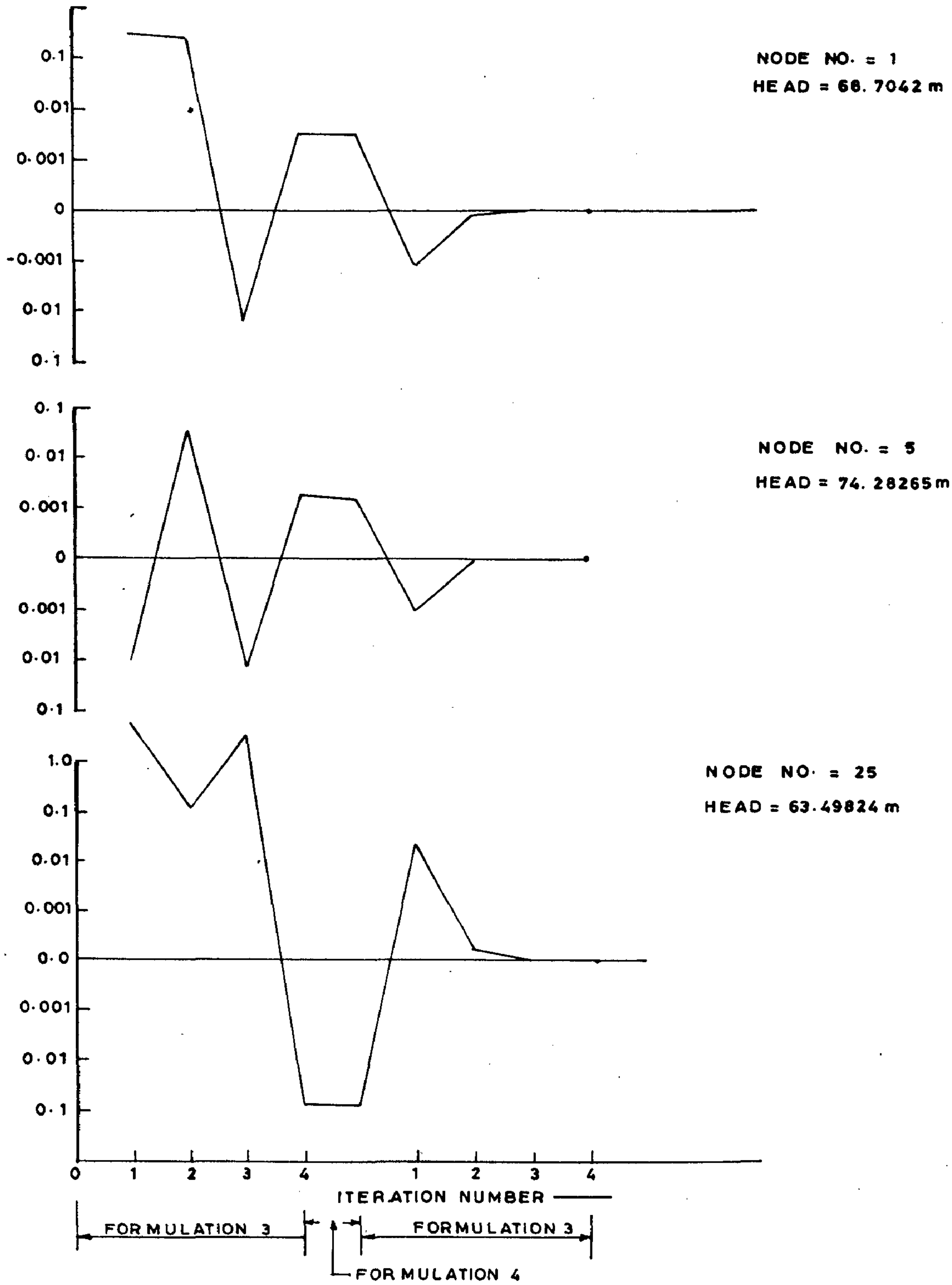


FIG. 6.5 CONVERGENCE PATTERN OF MIXED METHOD OF FORMULATION

From the Figure (6.5) it is observed that the method converges faster than the usual method of formulation 3. Also one notes that the advantage offered by this method for small network is limited. Hence, it is suggested that this method should be used for large networks.

TABLE 6.2

Results of Mixed Method of Formulation

S. No.	Method of Formulation	Number of iterations after which formulation 3 Terminated	Number of iteration in Formulation 4	Number of iterations by formulation 3 for final solution	Total Number of iterations
	For pipe network shown in Figure 6.1				
1.	Mixed method of formulation	3	1	7	11
		4	1	4	9
		5	1	5	11
		6	1	4	11
		7	1	5	13
		8	1	4	13
2.	By direct method of formulation 3	-	-	12	12
	For pipe network shown in Figure (6.6)				
1.	Mixed method of formulation	3	1	7	11
		4	1	6	11
		5	1	[illegible]	10
		6	1	4	11
		7	1	3	11
		8	1	2	[illegible]
2.	By direct method of formulation 3	-	-	11	-

6.3 ADVANTAGE OF COMPUTER AIDED LOOP SELECTION

Use of independent loops in a pipe network assists in reducing the number of equations to be solved for that problem. The loops may be selected either by inspection or with the help of computer using suitable algorithm. One may commit an error in selecting the independent loops in network by inspection. For example, one may identify that there are seven loops in the network shown in Figure (6.7) as listed in table (6.3), while infact there are only six independent loops. If solution is tried with seven loops, it starts diverging from the beginning Therefore it is proposed to select the loops with help of computer by an algorithm which has been discussed in chapter-4.

From table 6.3 it is also clear that the number of iterations taken by problem for convergence also depends on the way in which these six loops are selected. As the computer aided loop selection algorithm makes use of the shortest path algorithm the set of loops selected results into minimum number of iterations for convergence of the network problem. In any case this algorithm saves the mannual labour required in identifying the set of independent loops.

TABLE 6.3

Results of Loop selection algorithm

Set No.	Pipe numbers in set of the loops selected for network shown in figure (6.7).					Number of iterations	Is convergence attained ?
1.	Loops selected with help of computer.						
	i)	11	12	9			
	ii)	9	10	8		25	Yes
	iii)	10	14	13			
	iv)	5	4	15	6		
	v)	2	7	8	6		
	vi)	7	8	15	3		
	Loops selected with mannual inspections						
2.	i,ii,iii same as above						
	iv)	7	3	15	8		
	v)	2	5	4	3	27	Yes
	vi)	6	15	4	5		
3.	i,ii,iii same as above						
	iv)	2	3	4	5		
	v)	7	8	15	3	35	No
	vi)	5	4	15	6		
	vii)	2	7	8	6		

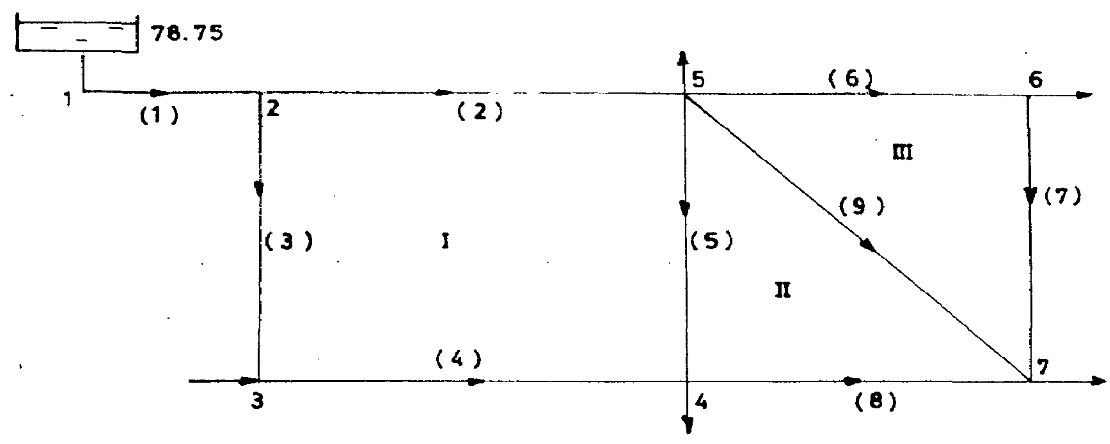


FIG. 6.6 A TEST PROBLEM

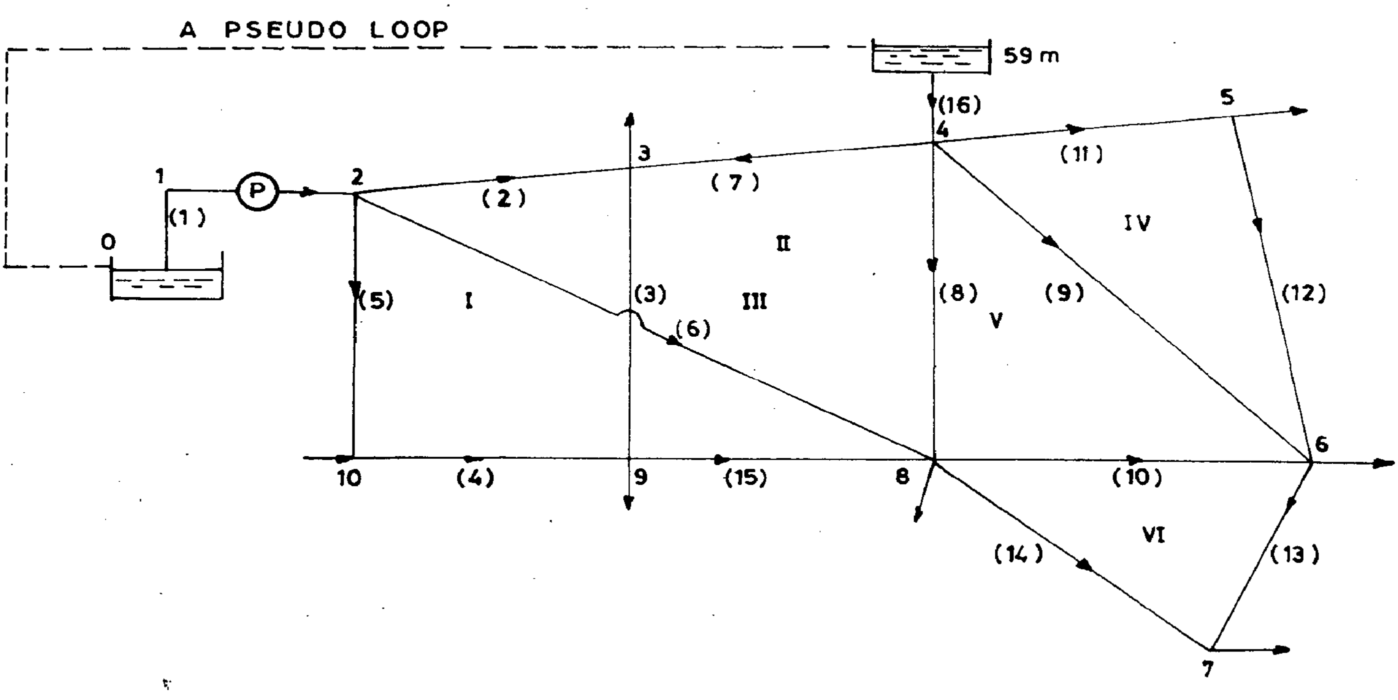


FIG. 6.7 A TEST PROBLEM FOR LOOP SELECTION

CHAPTER - VII

CONCLUSIONS

Main conclusions are given below:

[A] A new method, called Mixed method of formulation, is proposed to analyse the pipe networks. This method is computationally efficient and requires less computer storage. Although this method is tested on few large pipe networks, but it still requires more test on complex networks to prove its worthiness.

[B] Efficient software packages are developed to analyse the pipe networks using the existing methods and the mixed method of formulation. During the development of computer programs following observations about the nature of the solution are made.

- The values of heads at nodes which are aways from the input nodes are more sensitive to initial guess.
- The values of heads at lower degree nodes are more sensitive to initial guess as compared to the values at the higher degree nodes.
- During the solution, the values of heads at nodes near to the input nodes converges faster than the remaining nodes.

- Newton Raphson method is sensitive to initial guess of values of parameters. Its sensitivity can be reduced by damping the solution vector by a factor 0.5. A convergence scheme is also proposed for this purpose.

[C] A computer-aided loop selection method is proposed in the present work, which selects the optimal set of independent loops in a given pipe network. This procedure should be employed in comparision to the mannual inspection of network as it is suceptible to errors.

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APPENDIX

Results of the test problem shown in Figure (6.1) by Mixed method of Formulation.

Input Data

Number of nodes N = 32

Number of pipes NP = 48

Maximum number of iterations ITMAX = 25

Error tolerence ERR = 0.0001

Hazen Williams constant for all pipes = 120.0

Node No.	Initial guess of head P, meter	JPGIV(I) Switch	Nodal Flows QN cubic meter	Elevation of node H
1	2	3	4	5
1.	108.0	0	0.05665	0.0
2.	101.0	0	0.02832	0.0
3.	102.0	0	0.1416	0.0
4.	109.0	0	0.08496	0.0
5.	113.5	0	0.05665	0.0.
6.	67.0	0	0.05665	0.0
7.	78.74	1	0.0	0.0
8.	73.0	0	0.08496	0.0
9.	79.5	0	0.08496	0.0
10.	80.0	0	0.05665	0.0
11.	79.0	0	0.11238	0.0
12	82.5	0	0.05665	0.0

contd...

1	2	3	4	5
13.	80.25	0	0.05665	0.0
14.	86.25	0	0.08496	0.0
15.	86.0	0	0.08496	0.0
16.	83.25	0	0.0	0.0
17.	77.0	0	0.11328	0.0
18.	67.0	0	0.0	0.0
19.	65.75	0	0.08496	0.0
20.	61.0	0	0.05	0.0
21.	60.5	0.	0.05665	0.0
22.	60.0	0	0.025	0.0
23.	99.0	0	0.11328	0.0
24.	59.5	0	0.05665	0.0
25.	63.25	0	0.0708	0.0
26.	62.5	0	0.11328	0.0
27.	89.0	0	0.08496	0.0
28.	86.5	0	0.11328	0.0
29.	93.0	0	0.05665	0.0
30.	87.5	0	0.11328	0.0
31.	96.98	1	0.0	0.0
32.	98.5	0	0.05665	0.0

Pipe No.	From node No.	To node No.	Diameter c.m.	Length m.
1.	7	5	60.96	609.6
2.	7	8	60.96	609.6
3.	8	17	60.96	609.6
4.	17	18	30.48	914.4
5.	6	18	60.96	609.6
6.	6	5	30.48	609.6
7.	6	3	30.48	304.8
8.	4	3	30.48	457.2
9.	5	4	45.72	304.8
10.	3	2	30.48	304.8
11.	1	2	30.48	609.6
12.	4	1	45.72	304.8
13.	3	19	30.48	609.6
14.	19	20	30.48	457.2
15.	2	20	60.96	609.6
16.	19	24	30.48	914.4
17.	23	24	30.48	304.8
18.	22	23	30.48	457.2
19.	21	22	45.72	304.8
20.	20	21	60.96	914.4
21.	18	19	30.48	457.2
22.	18	26	30.48	914.4
23.	25	26	30.48	304.8

contd...

Pipe No.	From node No.	To node No.	Diameter c.m	Length m.
24.	25	23	30.48	457.2
25.	16	17	30.48	914.4
26.	28	16	30.48	457.2
27.	27	28	30.48	457.2
28.	27	25	30.48	762.0
29.	10	9	45.72	609.6
30.	16	10	45.72	609.6
31.	13	9	60.96	304.8
32.	12	13	60.96	762.0
33.	12	11	30.48	457.2
34.	10	11	30.48	457.2
35.	14	12	60.96	609.6
36.	15	14	45.72	457.2
37.	15	16	45.72	457.2
38.	30	15	60.96	457.2
39.	30	28	45.72	365.76
40.	29	30	45.72	457.2
41.	29	27	45.72	304.8
42.	32	30	45.72	365.76
43.	31	32	60.96	457.2
44.	31	29	60.96	304.8
45.	32	14	60.96	457.2
46.	10	8	30.48	609.6
47.	8	6	30.48	609.6
48.	9	7	60.96	609.6

- Output Results -

Head at each node after 9 iterations

Node No.	Head in meter
1.	68.7042
2.	61.8298
3.	63.1411
4.	69.7137
5.	74.2826
6.	67.2495
7.	78.74
8.	77.4144
9.	79.9311
10.	80.3167
11.	79.909
12.	82.8874
13.	80.5901
14.	86.3564
15.	86.3679
16.	83.7413
17.	77.0904
18.	67.1443
19.	61.88240
20.	61.4228
21.	60.9422
22.	60.7065

contd....

Node No.	Head in meter
23.	60.1441
24.	60.0655
25.	63.4982
26.	62.9243
27.	89.0104
28.	86.8803
29.	93.0480
30.	87.6126
31.	96.98
32.	89.7262

Flow rates in each pipe after convergence

Pipe No.	Flow rate in each pipe cubic meter
1.	0.5562
2.	0.2890
3.	0.1350
4.	0.1113
5.	0.0735
6.	-0.1149
7.	0.125
8.	0.1294
9.	0.3845
10.	0.0674
11.	0.1135
12.	0.1701
13.	0.0453
14.	0.0307
15.	0.1527
16.	0.0444
17.	0.0121
18.	0.0350
19.	0.0775
20.	0.1342
21.	0.1148
22.	0.0700
23.	0.0431
24.	0.0903

contd....

Pipe No.	Flow rate in each pipe cubic meter
25.	0.0896
26.	0.0868
27.	0.0704
28.	0.2043
29.	0.0696
30.	0.2263
31.	0.2880
32.	0.3446
33.	0.0844
34.	0.0288
35.	0.4857
36.	0.0121
37.	0.2291
38.	0.3262
39.	0.1296
40.	0.3393
41.	0.3597
42.	0.2298
43.	0.8450
44.	0.7557
45.	0.5585
46.	0.0712
47.	-0.1402
48.	0.2727