VIRTUAL MASS EFFECT ON SUBMERGED EARTH AND ROCK-FILL SLOPES UNDER VIBRATIONS

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BY

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CERTIFICATE

CERTIFIED that the thesis entitled " VIRTUAL MASS EFFECT ON SUBMERGED EARTH AND ROCK FILL SLOPES UNDER VIBRATIONS" which is being submitted by Shri Bhim Sen Gupta in partial fulfilment for the award of the Degree of Master of Engineering in Earthquake Engineering (Structural Dynamics) of the University of Roorkee, Roorkee is a record of student's own work carried out by him under our supervision and guidance. The matter embodied in this thesis has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for an effective period of 6 months from January, 1971 to July, 1971 for preparing this thesis for Master of Engineering degree at this University.

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SYNOPSIS

Normally the hydrodynamic pressure on structures in water are evaluated by applying Westergaard's theory or Zanger's theory. These theories consider the structure to be rigid. Same theories are adopted while computing hydrodynamic pressures on earthen and rock fill dams. In case of masonary dams the hydrodynamic pressure acts on the upstream surface. But the action of water is a little different in case of rock fill dams. The action of water is not only confined to the upstream face but has also access into the pores. The reservoir water in this case is able to communicate with the water inside the pores.

In this thesis virtual masses on sand or gravel slopes representing upstream face of earth dams have been found experimentally. A comparison is made with the results obtained by applying Zanger's theory, treating the dam as an impervious rigid body. The author has also tried to determine the effective unit weight of the fill which should be considered while computing the inertia force. Effect of reservoir water on the damping of the structure is also observed. It is observed that the added mass obtained experimentally is lesser than that obtained by Zanger's theory. The

damping however increases with increase in the level of reservoir water.

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C O N T E N T S

NOMENCLATURE

a	Radius of cylinder
a^*	Area of cross section of stand pipe
C	Velocity of sound in water
E	Modulus of Elasticity
e	Void ratio of the soils
f	Frequency of the system
g	Acceleration due to gravity
G_s	Specific gravity of the solids
h	Depth of water in reservoir
i	$\sqrt{-1}$
I	Moment of inertia of the legs of the model
J_0, J_1	Bessel functions of the order 0 and 1
K	Modulus of elasticity of volume of water
K'	Stiffness of the leg of Model
K^*	Coefficient of permeability
L	Length of leg of the model
m	Effective mass of the system
T	Time period of the ground motion
w	Natural circular frequency of ground motion
w_o	Unit weight of water
t	Any instant of time
w_s	Weight of the soil sample taken in the pycnometer.
x	Amplitude of vibration
x_o	Define the displacement of ground at any instant 't'
y	Depth of water in reservoir

Y_0, Y_1	Modified Bessel's functions of the order of 0 and 1
x, y, z	Carterian coordinate axes
r, θ, z	Cylindrical coo rdinate axes
θ'	Inclination of surface of fill to vertical
α_n	Seismic coefficient
ϕ	Velocity potential
σ	Hydrodynamic pressure
(ξ, n)	Moment of water particle in the direction of x and y respectively.
ξ	Damping ratio

CHAPTER - I INTRODUCTION

A body standing in still water is acted upon by static water pressure. But when the water is in motion, some additional water pressure is imposed on the body due to inertia of water. This additional water pressure is called Hydrodynamic Pressure. It is of transient nature. As soon as the oscillation in water ceases, the hydrodynamic pressure disappears. It give rise to virtual and added mass effect which can be better understood from the following considerations:

A body, standing in water, exhibits different dynamic properties than those which are observed when the same body is moving in air. The most affected parameter governing the dynamic behaviour is the natural time period of vibration. It is an observed fact that the time period of a body oscillating in water is elongated. Since the stiffness of the body is not altered, this indicates that the effective inertia or the effective mass of the body is increased. The increase in the mass of the body is due to the effect of water This increase is defined as 'Added Mass' , while the total effective mass of the body oscillating in water is defined as 'Virtual Mass'. Hydrodynamic pressure and the added mass are interrelated. If hydrodynamic pressure distribution is known, added mass can be calculated. The hydrodynamic pressure gives rise to additional forces in the structure. Hence its determination is necessary for the structures which are standing in water like intake structures and bridge piers or those which are standing against water such as dams (Earthen, Rockfill, Masonary) etc.

To compute the hydrodynamic pressure on plane surfaces, Zanger's and Westergaard's theory are usually applied. These theories assume the structure to be rigid. Moreover the reservoir water has very little or no communication with the pore water. These conditions are realised in a masonary dam. But the same may not be applicable to earth and rockfill dams. The voids in rockfill mass of the dams are particularly quite large and the water in voids is probably able to communicate rather freely with the reservoir water.

1.1 OBJECTIVES AND SCOPE OF THE STUDY

The main objective of this study was to investigate, by model experiments, the hydrodynamic effect on the upstream shell material of earth and rock fill dams and to compare the same with the theoretical values as used for concrete or masonary dams.

In the experiments different materials were used for the fill with different slopes to study the effect of physical properties of the material on the magnitude of the added mass. Damping in all the cases was also found to see the effect of reservoir water on the damping.

The fill materials used varied from fine sand with mean grain size 0.70 mm to shingle with average particle size 19.00 mm. The permeabilities of the materials ranged from 2.2 cm/sec. to 11.0 cm/sec. Two upstream slopes of 20° and 25° were used. The slope of the impermeable core was 45° to horizontal. The reservoir level was varied from full to zero. The virtual and added mass values are determined by an experimental set up, specially made for the purpose, by measuring the natural period of vibration by free vibration tests.

1.2 OUTLINE OF THE THESIS

In Chapter II, a brief review of the previous work done on the problem of hydrodynamic pressures and virtual mass is presented. Theoretical as well as experimental work done is described. The method of approach and the assumptions made by various authors to evaluate the hydrodynamic pressure are stated.

Chapter III deals with the physical properties of the materials used in the experiment. Definitions and necessary formulae to determine the properties are also presented.

Chapter IV and V deals with the experimental investigation of the effect of the reservoir water on dynamic characteristics of the model dam.

Details of experimental set up and technique of testing with and without reservoir water is given in detail. The values of the added mass found experimentally are compared with those obtained by Zanger's theory.

Discussion of the results and conclusions arrived at are presented in Chapter VI.

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CHAPTER II REVIEW OF PREVIOUS WORK

2.1 HISTORICAL GLANCE

Most of the theoretical studies on dynamic water pressure on a dam consist in finding out a suitable solution to the equation of fluid motion with respect to given boundary conditions. Usually it is assumed that the upstream face of a dam is vertical and the reservoir water spreads semi-infinitely with a horizontal bottom towards the upstream.

The problem is in reality a three dimensional problem and involves many parameters. Suitable assumptions are made to simplify the problem. There are two points which have been changed, depending on the investigators, in the theoretical studies on the dynamic water pressure acting on a dam or inside a water container. The first is the compressibility of water. Some have considered it and some have ignored it in the analysis. Second is whether the acceleration due to gravity is taken into the equation of motion or not. The boundary conditions in all the cases are more or less the same, (4,5,8,11,13,21,22) namely, the body is rigid i.e. the horizontal movement of the upstream face of the dam is same as that of the ground, the water pressure is zero at the water surface, the vertical component of the movement of water particle is zero at the base. The ground motion due to earthquake is replaced by a simple harmonic motion for convenience of calculation. There are few reports in which the vibrational characteristics of the structure are taken into account (9,10,14,18). In one of the report the irregular motion as the ground motion during an earthquake is considered (26).

Experimental work on this problem is also available. Various shapes of the bodies have been experimentally tested (6,8,18). In one report model dams were constructed on shaking table and the dynamic water pressure acting on a model dam was measured (15). There exists a few reports (20) where virtual masses of bodies of various shapes have been found experimentally.

2.2 OUTLINES OF IMPORTANT STUDIES IN THIS FIELD

I. H.M.WESTERGAARD (22)

He found out the dynamic water pressure by solving the equation of fluid motion and proposed the formula known by his name, after simplifying and approximating his solution. It is probably the first study of the dynamic water pressure on a dam.

Besides the assumption stated above, it was also assumed that the relative movement between the dam and the water particle is small enough so that the derivatives of higher order may be neglected.

Adopting the coordinate system as shown in Fig. (1) the following equations are used as the equations of fluid motion:

$$\left.\begin{aligned} \frac{\partial \sigma}{\partial x} &= \frac{w_o}{g} \frac{\partial^2 \xi}{\partial t^2} \\ \frac{\partial \sigma}{\partial y} &= \frac{w_o}{g} \frac{\partial^2 \eta}{\partial t^2} \end{aligned}\right\} \qquad (2.1)$$

Equation of continuity:

$$\sigma = K \left(\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} \right) \qquad (2.2)$$

The ground motion in an earthquake is assumed as

$$X_o = - \frac{\alpha g T^2}{4 \pi^2} \cos \frac{2 \pi t}{T} \qquad (2.3)$$

Various terms appearing here are,

h depth of water in reservoir

g	=	Acceleration due to gravity
k	=	Modulus of elasticity of volume of water (bulk modulus)
x_0	=	Define the displacement of ground at any instant 't'.
σ	=	Hydrodynamic pressure
η, ξ	=	Movement of water particle in the direction of x and y respectively
T	=	Time period of ground motion
w_0	=	Unit weight of the water

Boundary conditions used are :

i) $$\left|\sigma\right|_{y=0} = 0 \qquad (2.4)$$

ii) $$\left|\eta\right|_{y=h} = 0 \qquad (2.5)$$

iii) $$\left|\xi\right|_{x=0} = -\frac{\alpha g T^2}{4\pi^2} \cos^2 \frac{2\pi t}{T} \qquad (2.6)$$

iv) σ approaches to zero in accordance with increase of x.

Solving the equations and substituting x = 0 into the solution finally we obtained ,

$$\left|\sigma\right|_{x=0} = \frac{8\alpha w_0 h}{\pi^2} \sum_{1,3,5}^{n} \frac{(-1)^{\frac{(n-1)}{2}}}{n^2 C_n} \qquad (2.7)$$

where,

$$C_n = \sqrt{1 - \frac{16 w_0 h^2}{n^2 g K T^2}}$$

The equation was further approximated assuming the hydrodynamic water pressure distribution as parabolic with vertical axis. The approximated solution is,

$$\sigma = C \alpha \sqrt{hy} \qquad (2.8)$$

where,

C is a coefficient depending upon h/T

In the range considered in this theory 'C' varies very slowly,

with $T \geq \frac{1}{3}$ sec

$h < 310$ ft, $C = 0.026$ ton/ft^3

$310 < h < 540$ ft, $C = 0.027$ ton/ft^3

$540 < h < 680$ ft, $C = 0.028$ ton/ft^3

Therefore a constant value of 'C' could be taken as,

$C = 0.02734$.

$w_o = 0.03725$ ton/ft^3

$$\therefore \sigma = 0.02734 \; \alpha \sqrt{hy} \qquad (2.9)$$

Further simplification was done by assuming that a certain body of water is moving with the dam. This body of water considered in the form of ice frozen in horizontal layers (the expansion of ice due to freezing was ignored).

The shape of the body of water or ice was determined such that the inertia forces became equal to the pressures actually exerted by the water due to dynamic action.

Let 'b' denote the dimension of the body in x - direction at a depth 'y' (Fig. 1b).

Therefore, corresponding mass per unit area of the upstream face of dam,

$$= \frac{bw_o}{g}$$

$$\therefore \quad \text{Inertia force} = \frac{b\ w_o}{g} \times \alpha\, g = bw_o\, \alpha \qquad ..(2.10)$$

This should be equal to σ

$$\therefore \quad b = \frac{\sigma}{w_o\ \alpha} \qquad ..(2.11)$$

$$= \frac{0.02734\ \alpha\ \sqrt{hy}}{0.03725\ \alpha} = \frac{7}{8}\sqrt{hy} \qquad ..(2.12)$$

From 2.10 & 2.12

$$\sigma = \frac{7}{8}\ w_o\ \alpha\ \sqrt{hy} \qquad ..(2.13)$$

(II) <u>HATANO'S STUDY</u> (11b)

Hatano had some objections to Westergaard's study. He corrected them and worked out his own solution. The objections were, when $(H/T) < 360$, C_n is imaginary and Westergaard's solution cannot be used. Moreover it is also unreasonable to put $\sigma = 0$ at $x = \alpha$. His solution is as follows:

Take the coordinates system shown in Fig. (2). The equations of the fluid motion are,

$$\left.\begin{aligned} \frac{\partial^2 \xi}{\partial t^2} &= - \frac{g}{w_o} \frac{\partial \sigma}{\partial x} \\ \frac{\partial^2 \eta}{\partial t^2} &= - g - \frac{g}{w_o} \frac{\partial \sigma}{\partial y} \end{aligned}\right] \qquad \cdot\cdot(2.14)$$

$$\sigma = - K \left(\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} \right) - w_o y \qquad \cdot\cdot(2.15)$$

Motion of the dam is $(\alpha g / w^2)$ sin wt in the direction of x where w is the circular natural frequency of the ground motion ($w = 2\pi / T$).

Boundary conditions are,

$$\text{i)} \quad \left| - \frac{\partial f}{\partial y} \right|_{y=-h} = 0 \qquad \cdot\cdot(2.16)$$

$$\text{ii)} \quad \left| - \frac{\partial f}{\partial x} \right|_{x=0} = \frac{\alpha g}{w} \cos wt \qquad \cdot\cdot(2.17)$$

$$\text{iii)} \quad \left| \frac{w_o}{g} \frac{\partial^2 f}{\partial t^2} + w_o \frac{\partial f}{\partial y} \right| = 0 \qquad \cdot\cdot(2.18)$$

where,

$$\frac{\partial \xi}{\partial t} = - \frac{\partial f}{\partial x} \quad \text{and} \quad \frac{\partial \eta}{\partial t} = - \frac{\partial f}{\partial y}$$

Solving:

$$\sigma = \frac{4 \alpha w_o}{J_o} \frac{\sinh K_o h}{\sinh 2K_o h + 2K_o h} \cosh K_o (y+h) \cos(wt - J_o x) +$$

$$+ \sum_{m=1}^{r} \frac{4\alpha w_o}{J_m} \frac{\sin 2K'_m h}{\sin 2K'_m h + 2K'_m H} \cos K'_m (y+h) \cos(wt - J_m X) -$$

$$- \sum_{m=r+1}^{\alpha} \frac{4\alpha w_o}{J'_m} \frac{\sin 2K'_m h}{\sin 2K'_m h + 2K'_m h} \cos K'(y+h) - e^{-J'_m X \sin wt} \qquad (2.19)$$

where K_o = the root of $K \tanh kh = w^2/g$

K'_m = root of $K \tanh K'h = - w^2/g$

$$J_o = \sqrt{C^2 - K_o^2}$$

$$J_m = \sqrt{C^2 - K'^2_m} \quad \text{for } C^2 > K'^2_m$$

$$C^2 = \frac{w_o w^2}{g K}$$

He also carried out model tests and measured the dynamic water pressure acting on the model dam constructed on a vibration table.

(III) ANZO'S STUDY (2)

He solved the two dimensional Laplace equation, in velocity potential (ϕ), subjected to various boundary conditions.

Using the coo rdinates system as in Fig. (3)

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi^2}{\partial y^2} = 0 \qquad (2.20)$$

boundary conditions are,

i) $$\left| - \frac{\partial \phi}{\partial y} \right|_{y=h} = 0 \qquad (2.21)$$

ii) $$\left| - \frac{\partial \phi}{\partial x} \right|_{x=0} = \frac{2\pi a}{T} \cos \frac{2\pi t}{T} = \frac{ag \cos wt}{w} \qquad (2.22)$$

iii) $$\left| \frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial y} \right|_{y=0} = 0 \qquad (2.23)$$

where, a = Amplitude of ground motion

$w = 2\pi/T$

Finally the obtained,

$$\phi = A_0 \sin(wt - C x) \cosh C_0(h+y) + \cos wt \sum_{n=1}^{\infty} A_n e^{-C_n x} \cos C_n(h+y)$$

(2.24)

$$\sigma = \rho \left| \frac{\partial \phi}{\partial t} - g\, y \right|_{x=0} \qquad (2.25)$$

where,

$$A_o = \frac{4 \alpha g \sinh C_o h}{C_o w \,(2 C_o h + \sinh 2 C_o h)}$$

$$A_n = \frac{4 \alpha g \sin C_n h}{C_n w \,(2 C_n h + \sin 2 C_n h)}$$

$$C_o \tanh C_o h = w^2/g$$

$$-C_n \tan C_n h = w^2/g$$

Dynamic water pressure is obtained by equation (2.24) and (2.25).

(IV) ZANGER'S APPROACH (25)

In all the previous studies the upstream face was considered vertical but Zanger established a simple formula, to calculate the dynamic water pressure , applicable to the dams which are having sloping upstream face. The slope of the upstream face was considered to find a coefficient (C) appearing in the formula . He also gave relationships to evaluate shear forces and bending moments. With the help of these formulae added mass can be calculated as given below:

$$\sigma = C \alpha_n w_o h \qquad (2.26)$$

$$C = \frac{1}{2} C_m \left[\frac{y}{h}\left(2 - \frac{y}{h}\right) + \sqrt{\frac{y}{h}\left(2 - \frac{y}{h}\right)} \right] \qquad (2.27)$$

$$Q_{max} = \text{Max. shear force} = 0.726\, \sigma\, y \qquad (2.28)$$

$$\text{Inertia force} = \frac{d\,Q_{max}}{dy} = 0.726\, \sigma$$

$$= 0.726\, C \frac{\alpha}{g} w_o h$$

Added mass = Inertia force/Acceleration = $0.726\ C\ wh/g$

$$\text{Added weight} = 0.726\ Cwh \qquad (2.29)$$

Total increase in weight = added weight intensity x Area of which it acts

= $0.726\ Cwh \cdot A$.

Where,

α_n = seismic coeffiient

α = Acceleration of ground motion.

h = Maximum depth of reservoir

y = Depth from top of reservoir to the point under consideration.

C_m = Depends on slope and is equal to $0.73\left(\frac{\theta}{90^\circ}\right)$,

where θ = Angle in degrees that the slope of dam makes with horizontal.

In most of the studies on dynamic water pressure acting on a dam, the reservoir upstream of the dam was thought to be infinite. But when the upstream of the dam is considered finite, fluid motion in a reservoir is, in principle, similar to that in a container or outside a body when it is surrounded by water. Notable studies on the oscillation of the fluid in a container or around a body are as follows:

(V) L.S. JACOBSEN (13)

Jacobsen published a paper in 1939 on impulsive hydrodynamics of fluid inside a cylindrical tank and of fluid surrounding a cylindrical pier. In his theoretical approach the three dimensional Laplace equation was integrated at the surface of cylinder with the help of known boundary conditions. The ground motion was considered as the simple harmonic motion, $x = f(t) \sin \theta$ where the coordinate system shown in Fig. 4 is adopted. The velocity potential equation in cylindrical coordinates i.e.,

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \qquad (2.30)$$

where ϕ = velocity potential,

(r, θ, z) = cylindrical coordinates, The pressure is then determined by,

$$\sigma = - w_o \frac{\partial \phi}{\partial t} \qquad (2.31)$$

Boundary conditions are similar to those used by Westergaard.

i) $$\left| w' \right|_{z=0} = \left(\frac{\partial \phi}{\partial z}\right)_{z=0} = 0 \qquad (2.32)$$

ii) $$\left| \sigma \right|_{z=h} = 0 \qquad (2.33)$$

iii) Translatory velocity (u) in the direction of x at $r = a$ must be equal to

$$f'(t) = - \frac{\partial x}{\partial t} \cos \theta$$

i.e., $$\left| \frac{\partial \phi}{\partial r} \right|_{r=a} = - \frac{\partial x}{\partial t} \cos \theta \qquad (2.34)$$

iv) Circumferential velocity at A and B is zero

$$\left| \frac{\partial \phi}{r \partial \theta} \right|_{\theta = \frac{\pi}{2}} = 0 = \left| \frac{\partial \phi}{r \partial \theta} \right|_{\theta = -\frac{\pi}{2}} \qquad (2.35)$$

where, a = radius of cylinder

h = depth of water.

w = translatory velocity in the direction of z - axis.

Finally he obtained,

$$\sigma = - w_o \frac{\partial \phi}{\partial t} = - w_o f''(t) \cos\theta \sum_{1,3,5}^{\infty} \cos nKz \cdot A_n J_1(inKr)$$

(2.36)

$$A_n = \frac{4}{\pi K} \frac{1/n^2}{J_o(K_a) - \frac{1}{nK_a} J_1(K_a i)}$$

$$B_n = \frac{4}{-K} \frac{1/n^2}{Y_o(iK_a) + \frac{1}{nKa} Y_1(iKa)}$$

where $i = \sqrt{-1}$

J_o, J_1 = Bessel's function of first kind of orders 0 and 1 respectively.

Y_o, Y_1 = Modified Bessel's function of zero and first order respectively.

n = Integer

(VI) H.GOTO AND K.TOKI (8)

They followed the same approach as by Jacobsen to find the dynamic water pressure on submerged circular cylinderical piers and obtained the following relation to determine dynamic water pressure.

$$\sigma = w_o \alpha_n \lambda a^2 \sum_{m=1}^{\infty} \frac{(-1)^{m-1}}{\alpha_m h} \frac{4}{\alpha_m a} \frac{K_1(\alpha_m a) \cos \alpha_m z}{K_o(\alpha_m a) + K_2(\alpha_m a)}$$

(2.38)

where

$$\tan \alpha_m \times h = -\frac{w^2}{\alpha_m g} \qquad (2.39)$$

K_0, K_1, K_2 are the Bessel's function of the order of 0, 1 and 2 respectively.

(VII) VERNER AND SUNDQUIST (21)

They published the theoretical solution of the dynamic water pressure in various shaped tanks affected by vibrations. The solution is as follows:

Consider a rectangular tank as shown in Fig. (5). Equation of fluid motion and the boundary conditions are exactly the same as Westergaard's case. However, an additional boundary condition is imposed in this case due to the finite width of the reservoir.

$$\text{i.e.,} \quad \left|\xi\right|_{x = \ell} = -\frac{\alpha g\, T^2}{4\,\pi^2} \cos \frac{2\,\pi t}{T} \qquad (2.40)$$

The expression for dynamic water pressure is as fiven here,

$$p = \frac{8h}{\pi^2} \cdot \alpha\, w_o \frac{2\,\pi\, t}{T} \sum_{n=0}^{\infty} \frac{1}{\lambda_h (2n+1)^2} \frac{\cosh \psi x - \cosh \psi(\ell - x)}{\sin \psi\, \ell}$$

$$\cdot \sin (2n+1) \frac{\pi y}{2h} \qquad (2.41)$$

where,

$$\psi = \lambda_n (2n+1) \frac{\pi y}{2h}$$

$$\lambda_n = \sqrt{1 - \left[\frac{4h}{(2n+1)KT}\right]}$$

$$K = \sqrt{\frac{gE}{w_o}}$$

ℓ = length of reservoir or width of tank

The resultant of the dynamic water pressure acting on the wall perpendicular to x- axis is,

$$R = \pm \frac{16 H^2}{\pi^3} \alpha w_o \cos \frac{2\pi t}{T} \sum_{n=0}^{\infty} \frac{1}{\lambda_n (2n+1)^3} \tanh \frac{\psi \ell}{2}$$

(2.42)

(VIII) G.W.HOUSNER (11)

Housner calculated the dynamic fluid pressure in a tank theoretically. His approach was different in the sense that he did not start from the equation of fluid motion. But he divided the actual fluid motion in a tank into fundamental motions and established the equation for each simple motion. Combining the simple equations he obtained the theoretical solution of the equation of fluid motion. The fluid is replaced by an equivalent mass system in his analysis. In the theories reviewed above, cases of dynamic water pressure on cylindrical water container and piers are also included to complete the review work on hydrodynamic pressure studies. Moreover if the reservoir length is considered finite then the case is comparable to those of piers and water containers.

All these theories except Zanger's theory assume the water face of the structure to be perfectly vertical. This assum ption is quite reasonable in masonary dams where the upstream face is vertical or very near to vertical. But in case of earthen and rockfill dams, where the slopes are of the order of 3.5 : 1 to 2.0 : 1, this assumption will result in excessive values of dynamic water pressure and will cause uneconomical designs.

Keeping this point in view the author has used the Zanger's formula for the purpose of comparison of experimental values to theoretical ones. Moreover following points emphasize the use of Zanger's theory.

i. It is the only theory which considers the slope of upstream face in the formula.

ii. I.S. Code for design of shock resistant structures also uses this theory.

iii. The formula is simple and quite accurate.

Reliability of the formula was judged by finding the added weight on an inclined mild steel plate. The result compared well with that given by Zanger's theory.

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CHAPTER III PHYSICAL PROPERTIES OF THE MATERIALS USED IN THE TEST (26)

Physical properties such as specific gravity (G_s), void ratio (e), grain size distribution, permeability (K*) were determined experimentally for the following materials used in the test.

i. Fine sand (Ranipur)

ii. Coarse sand (Badarpur)

iii. Small size gravel.

iv. Large sized gravel.

3.1 SPECIFIC GRAVITY

Specifid gravity of the solids was determined by pycnometer method using the following relationship.

$$G_s = \frac{W_s}{W_s + W_2 - W_1} \qquad (3.1)$$

where,

W_s = Weight of dry soil.

W_1 = Weight of pycnometer + soil + water.

W_2 = Weight of pycnometer full of water.

G_s = Specific gravity of solid particles.

3.2 VOID RATIO

It is defined as the ratio of volume of voids to the volume of solid. It was determined from the following formula

$$e = \left(G_s \frac{V_s}{W_s} - 1\right) \qquad (3.2)$$

where,

V_s = Volume of soil sample

W_s = Weight of soil sample

e = Void ratio.

3.3 GRAIN SIZE DISTRIBUTION

It was determined by sieve analysis method. The results are in the form of curves shown in Fig.(9).

3.4 PERMEABILITY

Coefficient of permeability (K*) is defined as the rate of discharge of water under laminar flow condition through a unit cross sectional area of the soil under unit hydraulic gradient and standard temperature condition.

It was determined by Variable Head Parameter using the following formula:

$$K^* = 2.303 \frac{a^*L}{At} \times \log_{10} \left(\frac{h_1}{h_2} \right) \qquad (3.3)$$

where,

a^*	=	Area of cross section of stand pipe
A	=	Area of cross section of soil sample
h_1	=	Initial head of water.
h_2	=	Head of water after time 't' has elapsed.
K^*	=	Coefficient of permeability
t	=	Time period elapsed.

All the physical properties are given in Table (1).

Note: Permeability of 3/4" gravel will be of the order of 11.0 Cm/Sec.

CHAPTER IV

EXPERIMENTAL SET UP

Experimental set up consisted of rectangular tank made up of 1/8" thick steel sheet Fig.(10). The tank was made in two parts. One part was fixed to the ground with the help of four channel sections. Stiffness of this part was made very large. This part represented the part of reservoir far away from the dam where the water is stationary. The other part was fixed to the ground with the help of four round steel bars, of 13 mm diameter, one at each end. This part was made quite flexible so as to make the time period large enough. In this part at one end a mild steel plate was welded, inside the tank, with an inclination of 45^{o} to horizontal. This plate represented the impervious core of an actual earthen dam or an rock fill dam. The assumption of no seepage through the foundation core or sides is thus realised. The fill was placed against this inclined plate and it represented the upstream shell of an earthen dam. The other end of this part was connected to the part of the tank already fixed, with a flexible rubber membrane, so that it may oscillate freely keeping the continuity of water in the two parts. The water in the movable part of the tank represents the reservoir water near the dam itself. Fill was placed upto 28 cm height and the full water depth was kept at 25 cm.

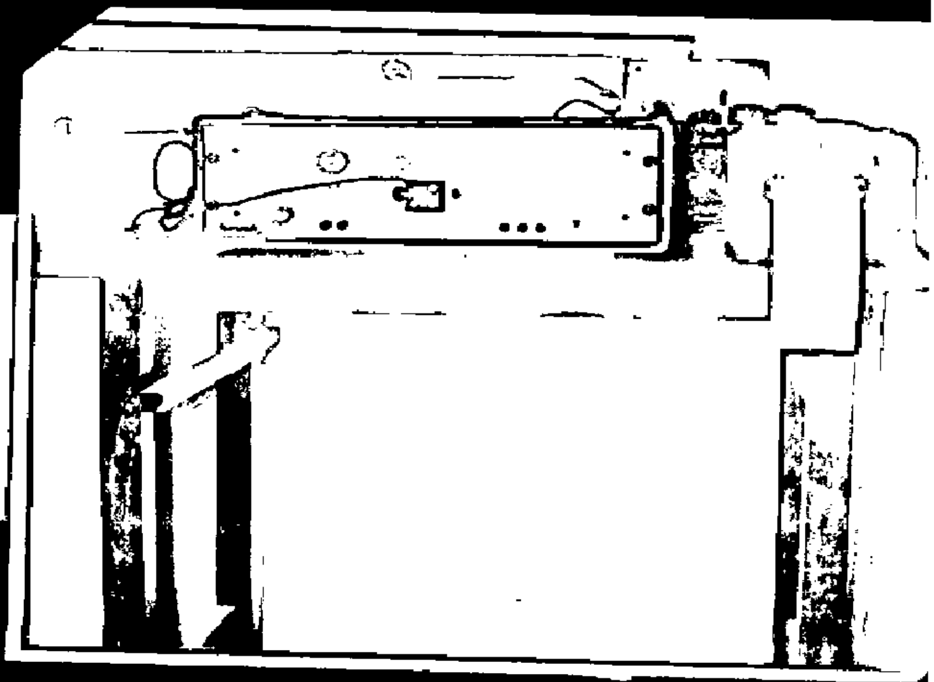
To enable free vibrations of the flexible tank containing the model dam, a realising system was provided to it by introducing a clutch and a thin wire string tied to the tank and carrying a fixed load at the other end. The wire was carried around a small pulley having negligible friction, so that loss of tension due to friction was negligible. At the bottom end of the tanks a wooden block was fixed to accommodate an acceleration pick up. A pen recorder and a universal amplifier were used to record the free vibrations of the system.

To verify the applicability of the single degree freedom system formula for obtaining the effective weight of the materials in the tank, the free vibration test was first carried out with a known quantity of fill placed in the tank. It was seen that the mass obtained by the frequency-stiffness mass relationship was slightly different from the actually measured weight of the fill. The stiffness was assumed as that obtained from static load deflection curves (Fig. 8).

For comparison of virtual mass with the theoretical results, actual mass of the material of fill was obtained from the frequency relationship and this mass was adopted in computations.

Fig.10a.
1. Mild steel plate inclined at 45°
2. Wooden block to accommodate the pickup
3. Flexible rubber Membrane.
4. Fixed part of themodel
5. Movable part of the tank.

Fig. 10b.
rumental Set up.
iversal Amplifier.
celeration pickup
recorder.



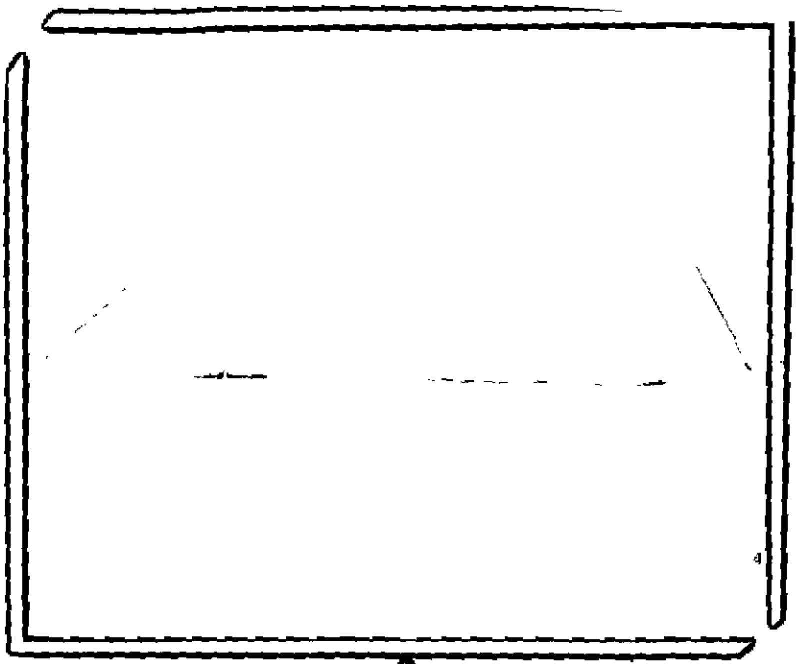
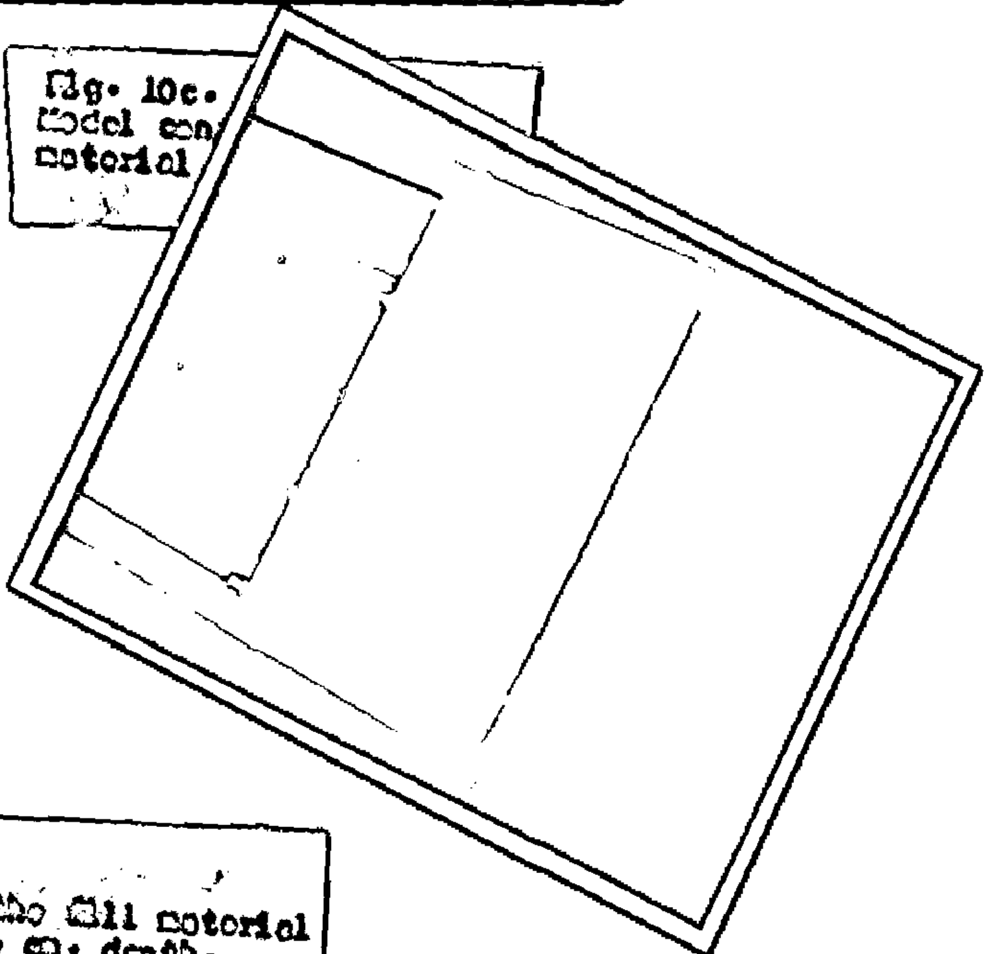


Fig. 10c.
Model con
material



0d.
ontaining the fill material
er up to 20 cm. depth.

CHAPTER V TYPE OF EXPERIMENT

5.1 In each case two types of tests i.e. static and dynamic were carried out. Static test was done to find the load deflection relationship. Then free vibration test was conducted to determine the frequency and damping etc., For this test, the system was pulled to a distance of 1.0 to 1.5 cm (for the system to remain in elastic range) and then suddenly released. The system started oscillating and the direct record was obtained with the help of 'brush' oscillograph.

In the first case, using fine sand as fill, the free vibration test was carried out in the following six stages to ascertain whether the test was repeatable or not under similar conditions:

I No reservoir water.

II Reservoir full.

III Reservoir drained to half of the full level.

IV Reservoir drained completely.

V Reservoir refilled to half of full level.

VI Reservoir full of water.

The results showed that the experiments are repeatable. Therefore, the other materials were tested only in I,II, and IVth stages. The IV th stage was considered to determine the weight of water retained in the material after immediate draining.

Two different slopes were tried i.e. 20^{o} and 25^{o} to horizontal to see the effect of slope on the virtual mass and added mass of the structure.

5.2 BEHAVIOUR OF THE STRUCTURE UNDER FREE VIBRATIONS

5.2.1 Time Period

Time period of the system was elongated when water was filled into the reservoir. A minimum of 6% and maximum of 9% increase in time period was observed. Elongation in time period of a structure vibrating under water as compared to its vibration in air is a general known fact and the same was observed here also.

5.2.2 Damping

It was found to increase when the structure was vibrated under water. The value was of the order of 0.5% for fine materials and of the order of 2.5% for coarse materials. The increase in damping was of the order of 80% to 90%.

5.2.3. Mass:

Effective inertia of the system increased due to increase in effective mass as was apparent from the change in time period.

5.3. LATERAL LOAD DEFLECTION TESTS

Load deflection tests were conducted in order to determine the elastic range and stiffness. Amplitude of the system was kept within this limit while conducting dynamic tests. The results are presented in the form of graphs shown in Fig. (8).

5.4 FREE VIBRATION TESTS

The experimental system behaved as a single degree freedom system having the usual frequency relationship as follows:

$$f = \frac{1}{2\pi}\sqrt{\frac{K'}{m}} \quad \text{c.p.s.} \qquad (5.1)$$

where,

f = frequency of the system.

K' = stiffness

m = effective mass of the system.

Time period , T , is then evaluated by,

$$T = 2\pi\sqrt{\frac{m}{K'}} \quad \text{sec.}$$

Thus to determine the time period, mass and stiffness are needed. The stiffness is calculated as below:

$$K' = 4 \times \frac{12EI}{L^3} \quad \text{Kg/cm.} \qquad (5.2)$$

where,

L = Length of the leg in cm.

E = Modulus of elasticity in Kg/cm^2

I = Moment of inertia of one leg about its diameter.

In the present case,

L = 100 cm.

E = 2.1×10^6 Kg/cm^2

I = $\frac{\pi d^4}{64}$, d = diameter of the bar = 12 mm.

$$K' = \frac{4 \times 12 \times 2.1 \times 10^6 \times (1.2)^4 \times \pi}{(64)\times(1.10)^3 \times 10^6}$$

= 17.3 Kg/cm.

Stiffness was also found from the load deflection curves. This stiffness comes out a little higher than the one calculated about. A slight variation in stiffness exists showing a variation for each material. Experimental stiffness takes into account some other factors which are difficult to account for in the calculations. Hence the experimental stiffness is more reliable. An average value found on the basis of the plots in Fig. 8 is used in all the calculations instead of the theoretical stiffness.

5.5 EFFECTIVE MASS

In computing the weight of the system the unit weight of m-ild steel was taken as 7.85 gm/cc. However it was not possible to take into account the weight of fixtures like nuts and bolts screws and steel strips. Therefore, the weight was also computed experimentally by the known stiffness and frequency of the system. A difference of 1.7 Kg was observed. That could be due to the above mentioned reasons.

$$m = \frac{K' T^2}{4 \pi^2} \quad Kg.sec^2/cm \qquad (5.3)$$

For empty reservoir condition,

K' = 18.66 Kg/cm.

T = 0.347 sec.

m = mass of tank.

w = mg = weight of tank.

Therefore,

$$W = \frac{K' T^2 \cdot g}{4 \pi^2} = \frac{18.66 \times 0.347^2}{4 \pi^2} \times 981$$

$$= 56 \text{ Kg.}$$

The computed weight comes out to be 54.3 Kg. However, after the experimental work was over, the tank was removed from the ground and weighted. It was found to be 55.65 Kg. But the experimental value of 56.0 Kg was considered more pertinent in the method adopted for evaluating all the effective weights by the frequency formula. Hence in further computational work the calculated value of 56 Kg. was used.

Free vibration tests were then carried out for the various materials used to represent the earth dam. Time period and damping were determined from each record. The results are presented in Table (2). Damping was calculated by logarithimic decrement formula.

$$\xi = \frac{1}{2} \log_e \frac{x_1}{x_2} \qquad (5.4)$$

x_1 and x_2 are successive amplitude of vibration.

ξ is the fraction of critical damping.

5.6 CALCULATION OF VIRTUAL AND ADDED WEIGHT

$$T = 2\pi\sqrt{\frac{m}{K}}$$

$$\text{or} \quad T \propto \sqrt{\frac{m}{K}}$$

$$\therefore \quad T_{dry} \propto \sqrt{\frac{m_{dry}}{K_{dry}}}$$

$$T_{sub} \propto \sqrt{\frac{m_{sub}}{K_{sub}}} \qquad (5.5)$$

Taking an average value of K,

$$\frac{T_{dry}}{T_{sub}} = \sqrt{\frac{m_{dry}}{m_{sub}}} \qquad (5.6)$$

$$\text{or} \quad m_{sub} = \left[\frac{T_{sub}}{T_{dry}}\right]^2 m_{dry} \qquad (5.7)$$

$$\text{or} \quad W_{sub} = \left[\frac{T_{sub}}{T_{dry}}\right]^2 w_{dry} \qquad (5.8)$$

where,

m_{dry} = Mass of the system when there is no water in reservoir.

m_{sub} = Mass of the system when the reservoir if full.

W_{dry}, W_{sub} = corresponding weights of system.

T_{dry}, T_{sub} = time period corresponding to the two different stages of reservoir.

W_{sub} contains the weight of the tank plus weight of the material plus added weight due to hydrodynamic action plus effective weight of water in voids. W_{sub} less the weight of the tank gives the virtual weight of the fill.

Added weight = Virtual weight of fill - weight of dry material - weight of water in voids.

Added weight was also calculated by Zanger's theory treating the fill as impervious body. The added weight calculated on the mild steel plate from Zanger's theory is very nearly equal to that obtained by experiment. This theory is applied here to the earthen and rockfill dam models also.

$$P = C \alpha_n w_h$$

$$C = \frac{1}{2} C_m \left[\frac{y}{h} \left(2 - \frac{y}{h}\right) + \sqrt{\frac{y}{h} \left(2 - \frac{y}{h}\right)} \right]$$

The value of C_m is dependent upon the inclination θ

of the surface with the vertical.

$\theta' = 0, 15, 30, 45, 60, 75$.

$C_m = 0.735, 0.625, 0.52, 0.41, 0.30, 0.17, 0.735$.

For the mild steel plate, $\theta' = 45^{o}$, $C_m = 0.41$.

For the fill case (1) $\theta' = 65^{o}$, $C_m = 0.26$

For the fill case (2), $\theta' = 70^{o}$, $C_m = 0.22$

$$y = 25 \text{ cm}$$

$$h = 28 \text{ cm}.$$

$$C_{plate} = \frac{1}{2} \times 0.41 \left[\frac{25}{28} \left(2 - \frac{25}{28} \right) + \sqrt{\frac{25}{28} \left(2 - \frac{25}{28} \right)} \right]$$

$$= 0.395.$$

Added weight $= 0.726\ C\ w\ h \cdot A$

$$= 0.726 \times 0.395 \times 1000 \times 0.25 \times 0.60 \times 0.25$$

$$= 10.78 \text{ Kg}.$$

Experimentally this value comes out to be 11.00 Kg.

Now $C_m \Big| \theta = 65^{o} = 0.30 - \frac{0.30 - 0.17}{15} \times 5 = 0.26$

$C_m \Big| \theta = 70^{o} = 0.30 - \frac{0.30 - 0.17}{15} \times 10 = 0.22$

$$C_\theta = 65^\circ = \frac{1}{2} \times 0.26 \times 1.9265 = 0.25$$

$$C_\theta = 70^\circ = \frac{1}{2} \times 0.22 \times 1.9265 = 0.22$$

Added weight ($\theta = 65^\circ$)

$$= 0.726 \times 0.25 \times 1000 \times 0.25 \times 0.60 \times 0.25$$

$$= 6.82 \text{ Kg}$$

Experimental values comes out to be 60%, 66%, 78% and 85% of the above for fine sand, coarse sand , small gravel and large gravel respectively.

Added weight ($\theta = 70^\circ$) = 5.77 Kg.

Experimental value comes out to be 59%, 76% of the above for coarse sand and large sized gravel respectively.

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CHAPTER VI RESULTS

The observation made in the experimental studies conducted here are as follows:

1. The hydrod ynamic pressure increases with decrease in void ratio. For the case of smallest void ratio the hydrodynamic pressure approach the value obtained from Zanger's theory .(Fig.9)

2. Better the gradation more is the added weight. From the grain size distribution curve (Fig. 6) it is seen that the large size gravel has the best gradation out of all the materials taken in the test. The added weight also is maximum for this case.

3. It is seen that slope of the shell also affects the added weight. Flatter the slope lower is the value of added weight.

4. Damping is indicated to increase with increase in reservoir water level.

DISCUSSIONS

In the experiments conducted in this study the hydrodynamic pressure or added weight in all the cases is lesser than what is obtained by using Zanger's theory except in the case where upstream surface is adopted to be of mild steel plate. In this case the result is very near to that obtained by Zanger's theory. This observation exhibits the difference in behaviour of the system in the two cases. In case of fill the magnitude of added weight is reduced apparently due to the communication of pore water with the reservoir water. It is also supported by experimental study where it is found that the added weight increases with increase in particle size. This is probably due to the fact that in case of large size particles the bigger voids will be filled up by smaller size particles present thereby decreasing the void ratio. The gradation of larger size material is also found to be improved for the materials considered. Hence the communication of pore water with reservoir water would be lesser for soils having lower void ratio. Therefore better the graduation and lower the void ratio more is the added weight. Now if we extrapolate this to the limiting case of no void ratio, we can say that for impervious materials e.g. the case of mild steel plate, added weight is maximum. But in case of fill materials the void ratio will definitely have some value. Thus the presence of voids would allow communication of water to some extent and reduce the added mass.

The effect of communication of reservoir water with the pore water may be viewed as the case of a perforated plate standing in water wherein the hydrodynamic pressure would be lesser than for a solid plate.

CONCLUSIONS

In the present case for the best gradation curve the value of added weight is 85% of that given by applying Zanger's theory. For the poorest gradation it is about 60% of that obtained by Zanger's theory i.e. for the case of fine sand. In an actual earthen or rockfill dam best possible gradation of various size particles is used to obtain maximum compaction and strength. Therefore based on the results of the study, Zanger's theory is recommended to be used for evaluating the dynamic water pressures on earthen and rockfill dams, the pressures being appropriately reduced depending upon the void ratio of the fill material.

Damping apparently increases due to the hydrodynamic action of the reservoir water. This cannot be directly related to the prototype dam where damping is mainly associated with the shear strains in the body of dam. It is however likely that increase in damping due to similar action of hydrodynamic pressures on prototype dam may occur.

TABLE 1

1. Specific Gravity of the solids = 2·66

2. Void ratio = V_v = $\left(\frac{G_s V_s}{W_s} - 1 \right)$

3. Weight of tank only = 56 Kg.

S. no.	Materials taken.	Average grain size. mm	Permeability Cm/cc	Void Ratio %	Volume of material (V_s) cm^3	Volume of voids (V_v) cm^3 (Kg)	Weight of water in voids (Kg)	Weight of taken (Kg)
1	Fine sand	1·15	2·2	69	14400	9750	9·8	30·5
2	Coarse sand	1·26	3·00	66	15730	10900	10·9	42·00
3	Small sized gravel	7·50	5·20	57	19450	10700	10·70	52·00
4	Large sized gravel	10·60		52	22300	11200	11·20	59·50
5	Coarse sand	1·26	3·00	66	25850	17600	17·6	69·00
6	Large sized gravel	10·60		52	30000	15000	18·00	8500

TABLE 2

cific gravity of the solids = 2.66

d ratio = $V_v = \left(\frac{G_s V_s}{W_s} - 1 \right)$

ght of tank only = 56 Kg.

terials taken		Time period			Damping coefficient		Effective wt. of system when full of reservoir water Kg	Virtual weight Kg	Added wt. Kg	Added wt / wt. of fill %
		T_{dry}	$T_{sub.}$	$T_{drained}$	ζ % dry	ζ % sub.				
e sand	94.5	.458	.463	.470	.25	.40	108.4	52.40	4.12	10.90
rse sand	98.00	.462	.500	.480	.35	.60	112.50	56.5	4.50	10.70
ll size vel	111.5	.502	.530	.510	.40	.80	124.0	68.0	5.30	10.20
ge size vel	115.0	.503	.538	.500	1.10	2.58	132.0	76.0	5.8	9.84
ge sized avel	136.0	.555	.580	.560	1.0	2.50	162.9	106.9	3.80	4.50
rse sand	121.0	.530	.560	.547	.30	.51	146.0	88.30	3.40	4.94

$S_1 = 25^{\circ}$

$S_2 = 20^{\circ}$

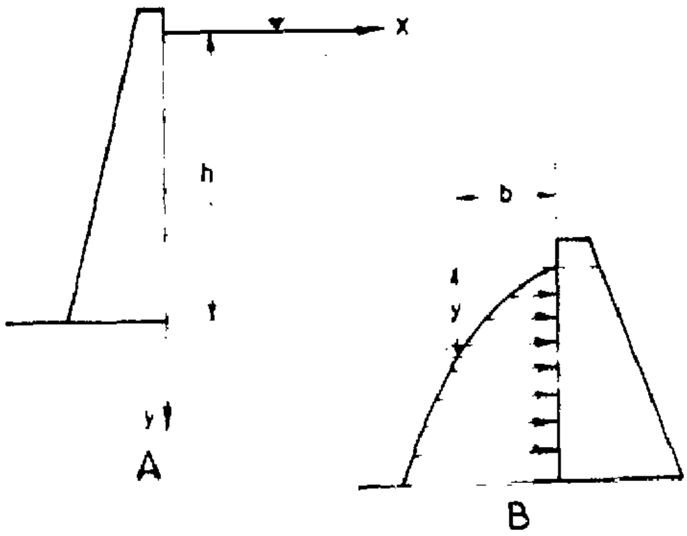

FIG. 1

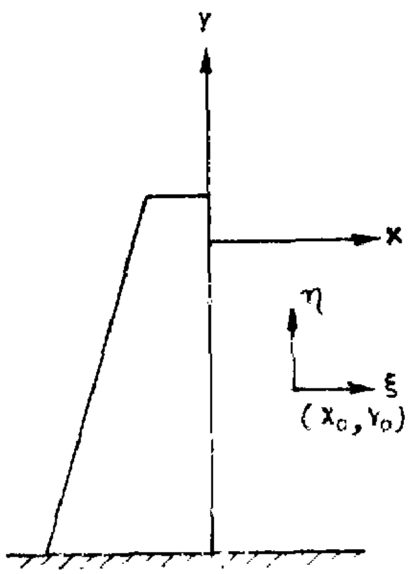

FIG. 2

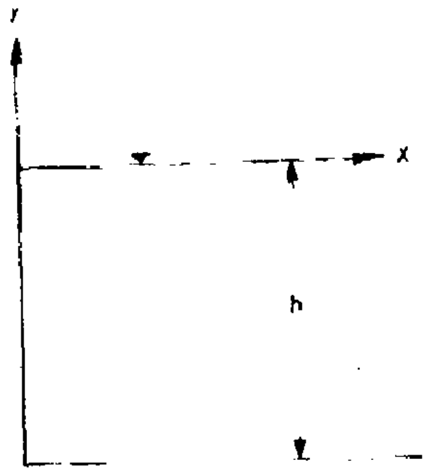

FIG. 3

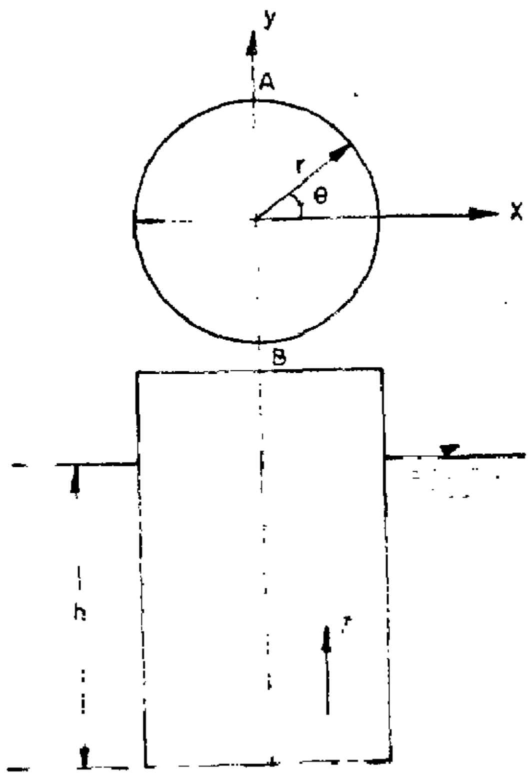

FIG. 4

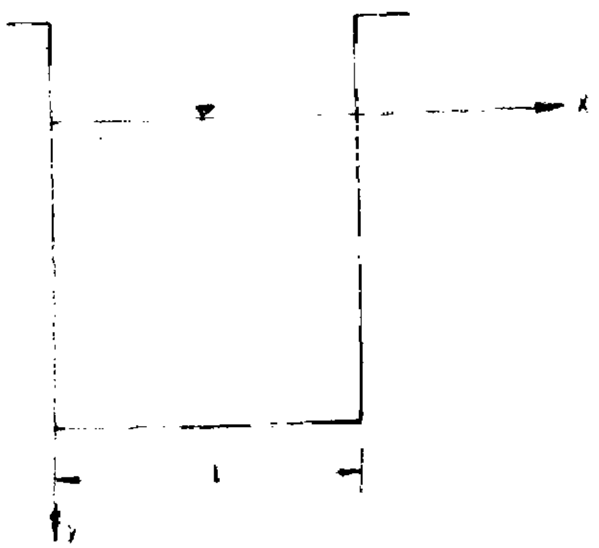

FIG 5

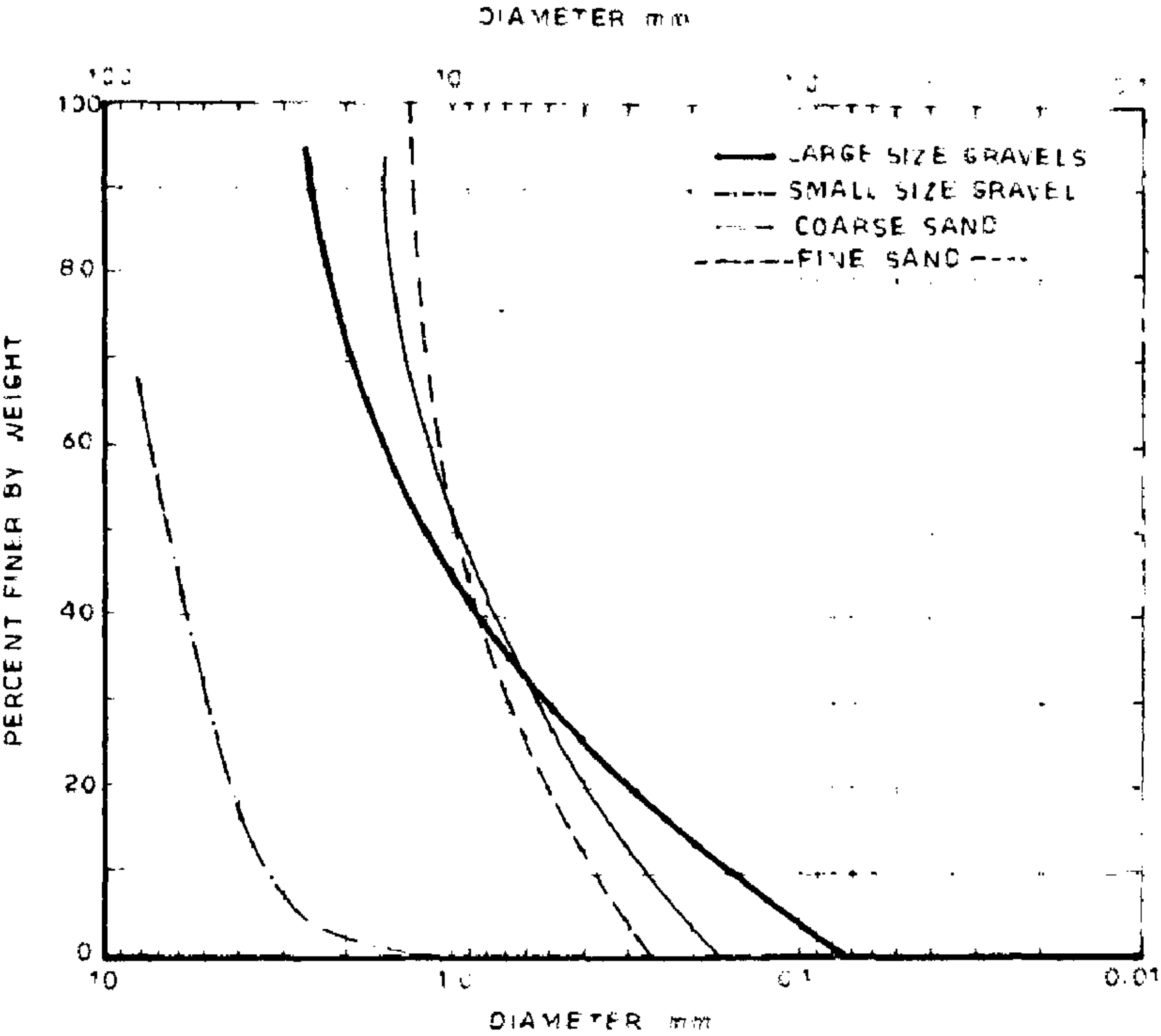

FIG. 6 _ GRAIN SIZE DISTRIBUTION

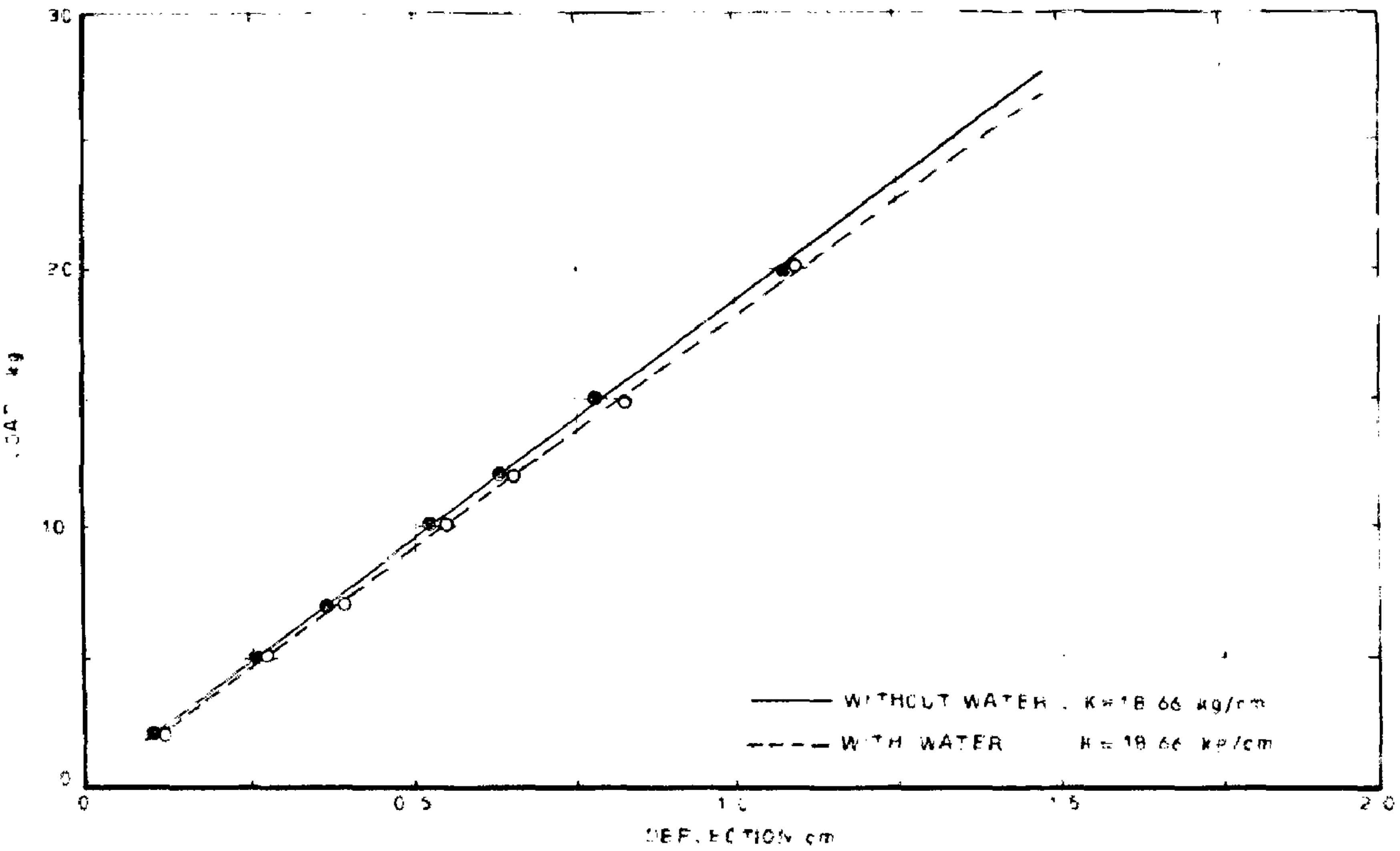

FIG. 8 a _LOAD DEFLECTION RELATIONSHIPS FOR THE MODEL ONLY
(NO FILL IN THE TANK)

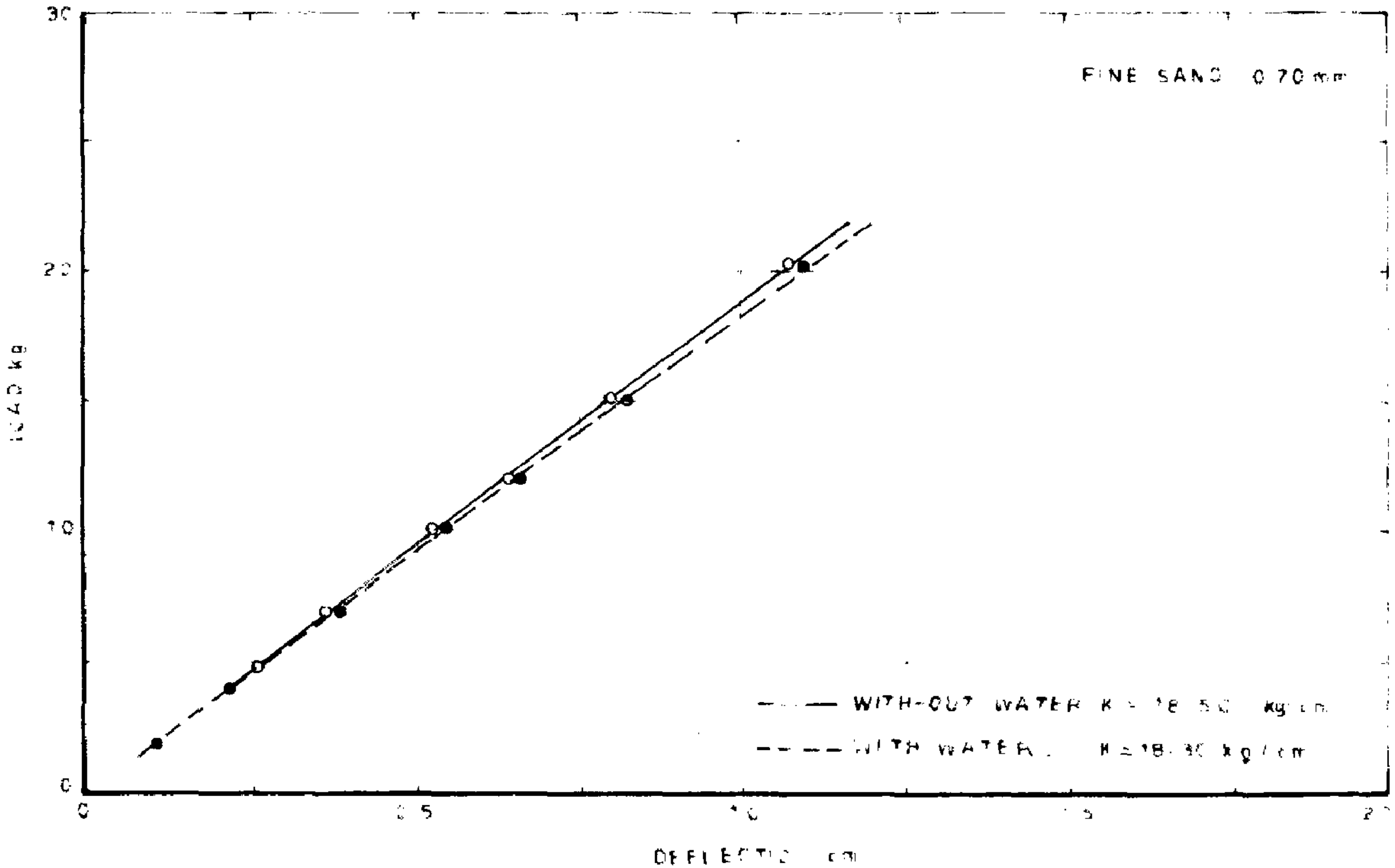

FIG. 8 b _LOAD-DEFLECTION RELATIONSHIP FOR THE MODEL WITH FINE SAND AS THE FILL

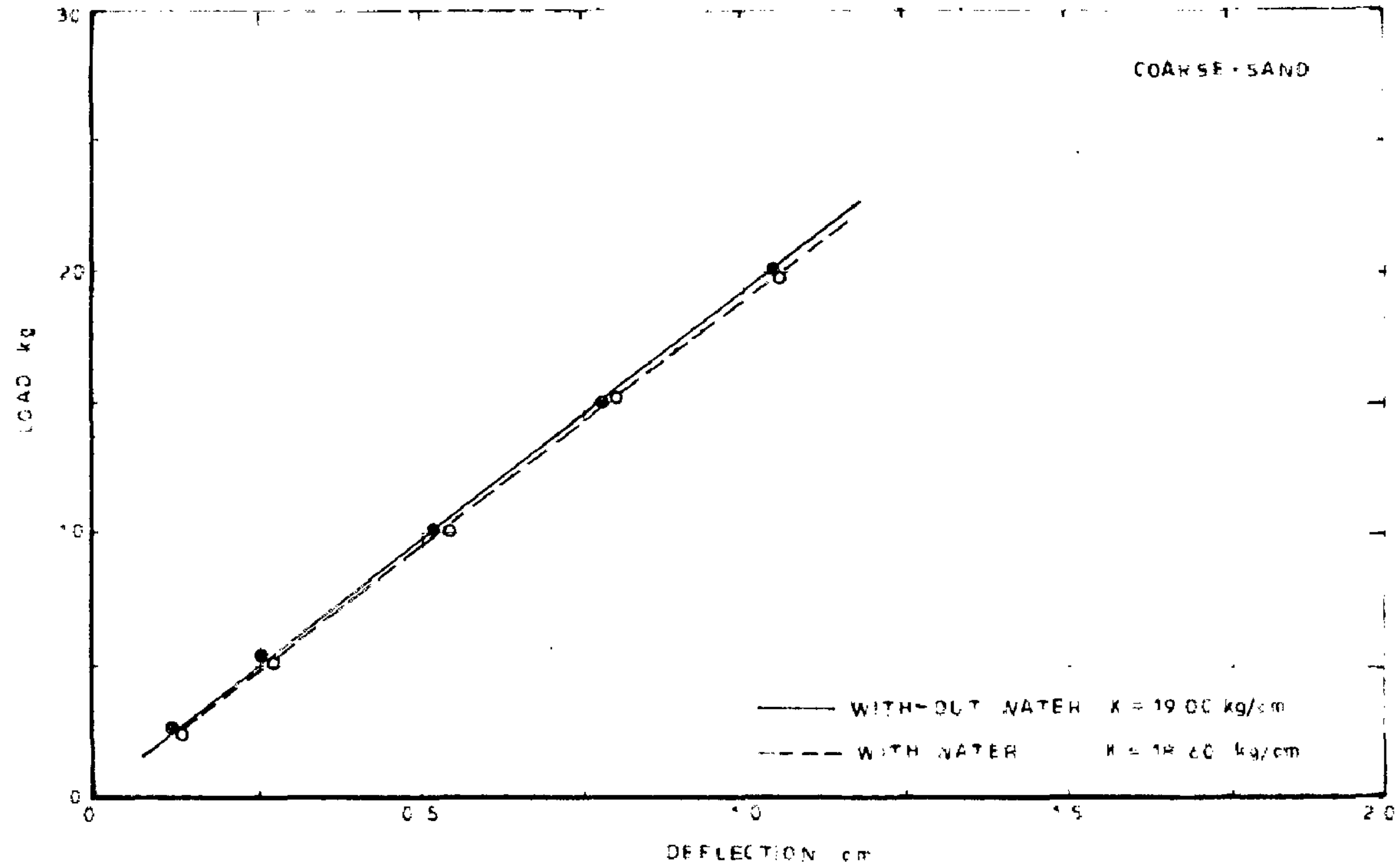

FIG. 8c _ LOAD DEFLECTION CURVE

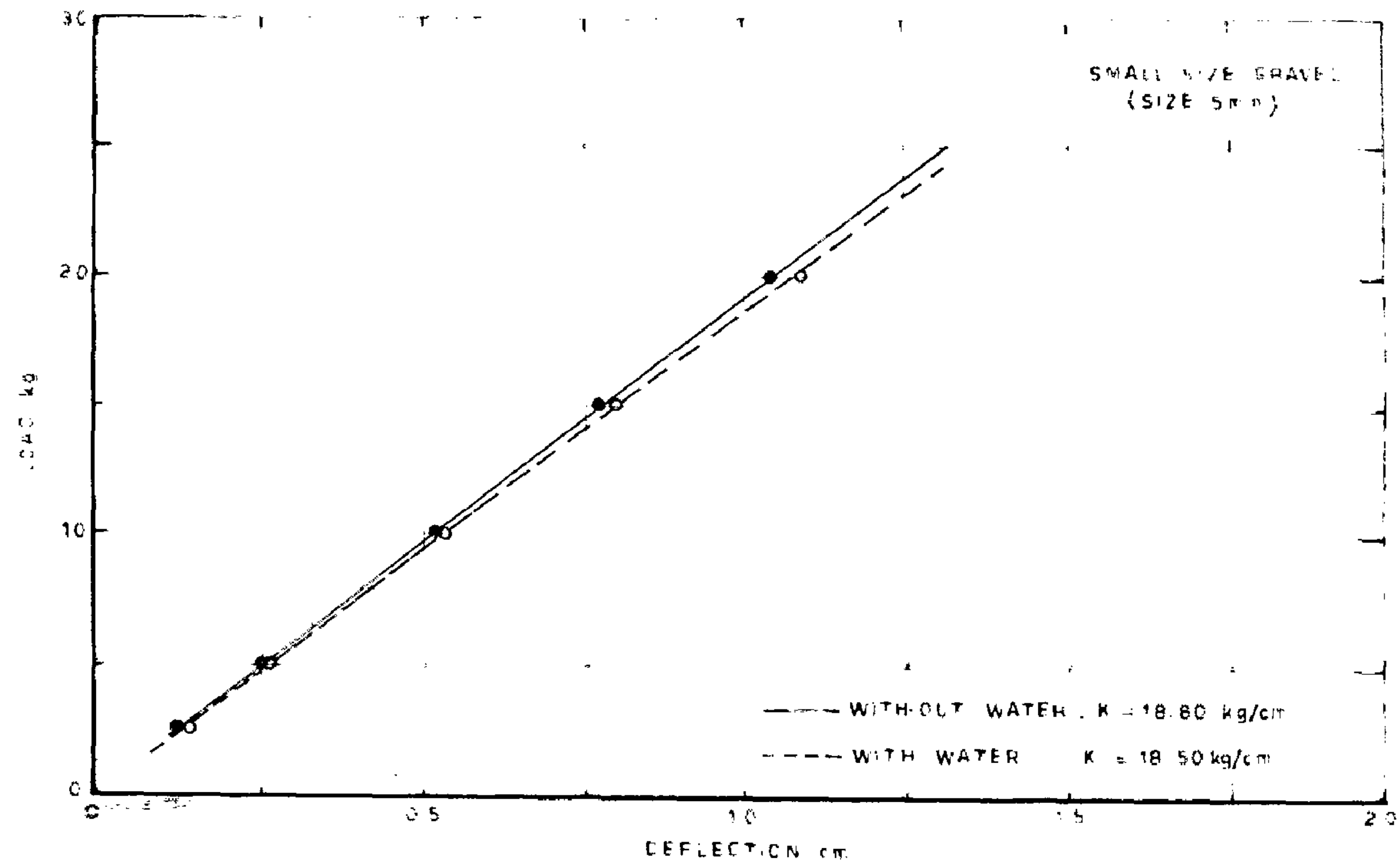

FIG 8d _ LOAD-DEFLECTION CURVE

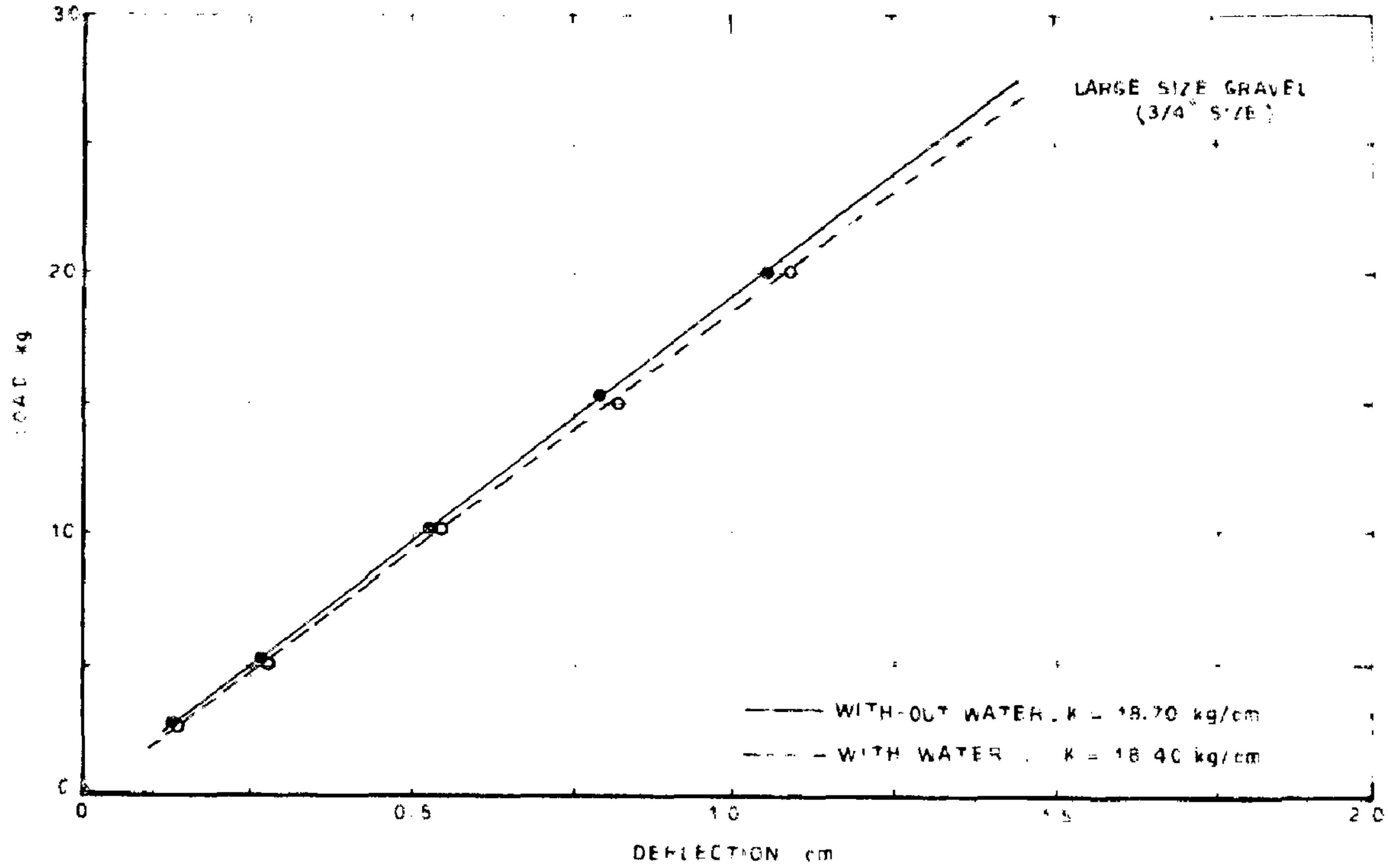

FIG. 8 e _LOAD - DEFLECTION CURVE

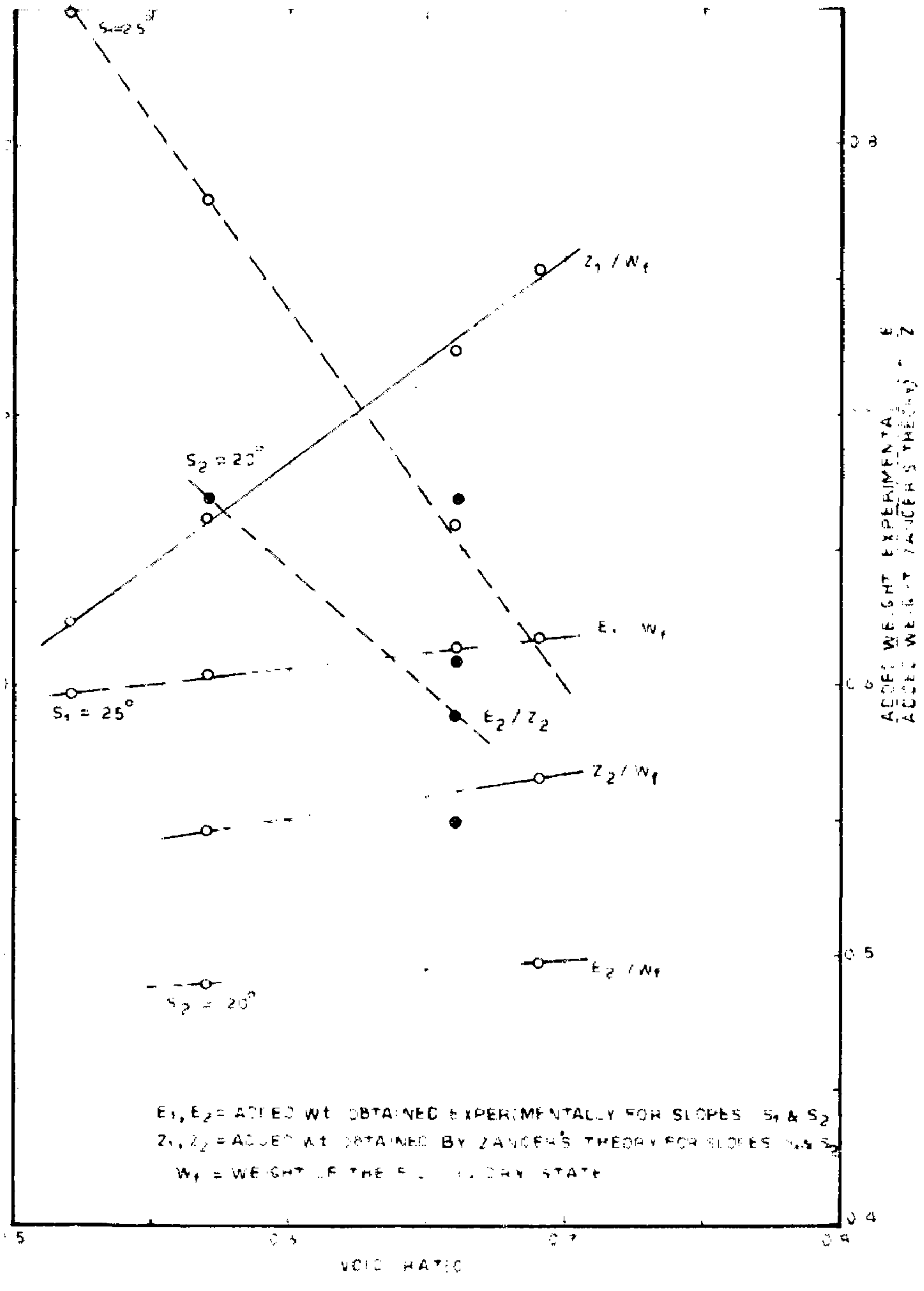
Z1 / Wf
E1 / Wf
E2 / Z2
Z2 / Wf
E2 / Wf
S2 = 20°
S1 = 25°
0.8
0.6
0.5
0.4
VOID RATIO
E1, E2 = ADDED WT OBTAINED EXPERIMENTALLY FOR SLOPES S1 & S2
W1 = WEIGHT OF THE FILL IN DRY STATE

FIGURE . 9

R E F E R E N C E S

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