ANALYSIS OF TWO-UNIT WARM STANDBY SYSTEM UNDER TWO TYPES OF FAILURE AND TWO TYPES OF REPAIRS

A DISSERTATION

Submitted in partial fulfilment of the
requirements for the award of the degree
of
MASTER OF PHILOSOPHY
in
MATHEMATICS

By

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JANUARY, 1989

IN
THE MEMORY OF
MY
LATE FATHER
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CERTIFICATE

Certified that the dissertation entitled **"ANALYSIS OF TWO UNIT WARM STANDBY SYSTEM UNDER TWO TYPES OF FAILURE AND TWO TYPES OF REPAIRES"**. Which is being submitted by **Mr. ATUL KUMAR VARSHNEY** in partial fulfilment for the award of **MASTER OF PHILOSOPHY in MATHEMATICS, UNIVERSITY OF ROORKEE, ROORKEE** is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

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CONTENTS

ACKNOWLEDGEMENTS

First of all I have indeed a great pleasure in expressing my deep sense of gratitude to my supervisor **Prof. K. MURARI, DEPARTMENT OF MATHEMATICS, UNIVERSITY OF ROORKEE, ROORKEE** for his invaluable guidance and encouragement from time to time during my investigations.

I am thankful to **Dr. S.N. PANDAY Prof. and Head, DEPARTMENT OF MATHEMATICS, UNIVERSITY OF ROORKEE, ROORKEE** for providing the necessary facilities.

I whole heartedly thank the faculty members of the department of mathematics. I shall be failing in my duty if I don't thank all my colleagues and friends ,specially to Mr. NAVEEN K. AGARWAL Research Scholar in Mathematics , who have helped me in my work.

I also wish to bring it on record that whatever I have achieved upto this stage, could not have been possible without blessings of the almighty and **my parents**.

I also thank **Er. R.R. Pal** for his invaluable help during my investigations.

DATED :JAN.,27 1989

(ATUL KUMAR VARSHNEY)

ABSTRACT

In this dissertation we discuss two different models. In the first model we have analysed a system consisting of two cold standby units with two types of failure and two types of repairs. Either of unit can be in operation or in partial failure mode or in total failure mode. An operating unit can fail partially or totally direct or totally through partial failure. A partially failed unit can be repaired during operation since partially failed unit remains operative with reduced capacity. A totally failed unit may require two types of repairs viz., major repair or minor repair. The failure of one unit is statistically independent of failure of the other unit. The system fails only when both the units fail totally.

In the second model we have two unit warm standby system. An operative unit can not fail totally without failing partially as we have assumed in the first model.

For both the models we have assumed all the failure and repair time distributions as general distributions. MTSF is derived by using techniques of regenerative process.

and other remains standby. As soon as operating unit fails the standby unit starts operation. The failure of a unit may be of one of the following types.

(a) Partial Failure : Failures resulting from deviations in characteristics beyond specified limits but not such as to cause complete lack of required function. In other words the unit works at a reduced capacity and can be repaired during operation.

(b) Total (Complete) Failure : Failure resulting from deviations in charactristics beyond specified limits such as to cause total lack of the required functions.

In 1975 Procter and Singh first studied a single unit system with three modes viz., operating, partial failure and total failure. Gupta et al. have studied a two-unit redundant system with these three modes of operation.

Goel, Kumar and Rastogi [2] have worked on two-unit redundant system under partial failure and two types of repairs with the assumption that an operative unit cannot go directly to total failure mode i.e. an operative unit first goes to partial failure mode and then to total failure mode.

This thesis deals with two different models. In the model 1 we have analysed two - unit cold standby system under two types of failure and two types of repairs assuming that an operative unit can fail partially or totally direct or totally through partial failure. The model 2 has two unit

INTRODUCTION

Conventional intutive approaches to the evaluation of system adequacy are not sufficient in modern engineering applications and are gradually being replaced by consistent quantitative techniques. A basic and common requirement in any quantitative procedure is the development of a suitable mathematical model to describe the system. One of these models is reliability.

Reliability is a new concept. The general area of reliability theory is extremely wide and infact encompasses all aspects of engineering technology. In modern technology, space landing programmes, exploration to Mars and communication systems in the world are a few eloquent examples of present day technology. It has been possible for the first time to demonstrate a very high reliability (99.9999 %) through a maze of complex systems and procedures in an environment with a high degree of variability. The reliability, a measure of quality, is an essential element at each stage of the equipment manufacturing procedure through design and production to final delivery to the user. The increasing sophistication of electronic and telemetric equipments needed for defence, space research programmes, satellites and guided missile systems demand a high degree of reliability. So during the second world war reliability was considered to be one of the pressing needs in order to study the behaviour of various systems used by the military.

In the real life it is seen that many a reliability systems have two or more units in which only one works at a time and other remains standby. As soon as operating unit fails the standby unit starts operation. The failure of a unit may be of one of the following types.

(a) Partial Failure : Failures resulting from deviations in characteristics beyond specified limits but not such as to cause complete lack of required function. In other words the unit works at a reduced capacity and can be repaired during operation.

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This thesis deals with two different models. In the model 1 we have analysed two - unit cold standby system under two types of failure and two types of repairs assuming that an operative unit can fail partially or totally direct or totally through partial failure. The model 2 has two unit warm standby system under two types of failure and two types of repairs. Either unit can fail partially or totally through partial failure.

Before discussing the above models we have revision of basic points in reliability, transforms etc.

RELIABILITY

The concept of reliability has been interpreted in many different ways. ELECTRONICS INDUSTRIES ASSOSIATION (EIA), USA, has defined reliability as

" Reliability is the probability of a device performing adequately for the period of time intended under operating conditions encountered". In the other words, it is the measure of the capacity of an equipment to operate without failure when put into service.

We write reliability R(t) of a device as -

$$R(t) = Pr \{ \text{The system works satisfactorily in the interval } (0, t] \}$$

R(t) is the non increasing function between two extremes 0 and 1. If reliability is R(t) and unreliability is $\bar{R}(t)$ then we have

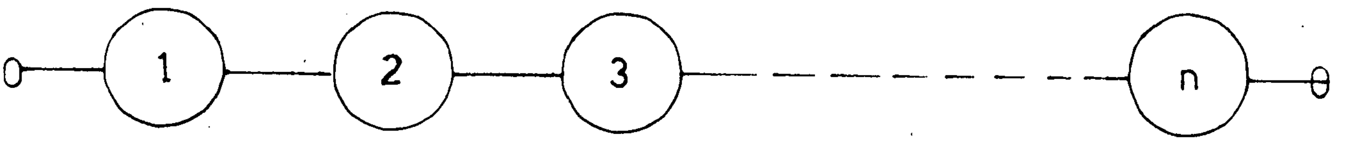
$$R(t) + \bar{R}(t) = 1$$

SYSTEM

A system consisting of N components can be considered to be in series, parallel and a combination of series and parallel. In addition we also have standby systems and K out of N systems.

SERIES SYSTEM :

A series system has configuration as given in the following figure -



A series system fails if one of the components fails i.e. the series system works satisfactorily if and only if all the components work satisfactorily.

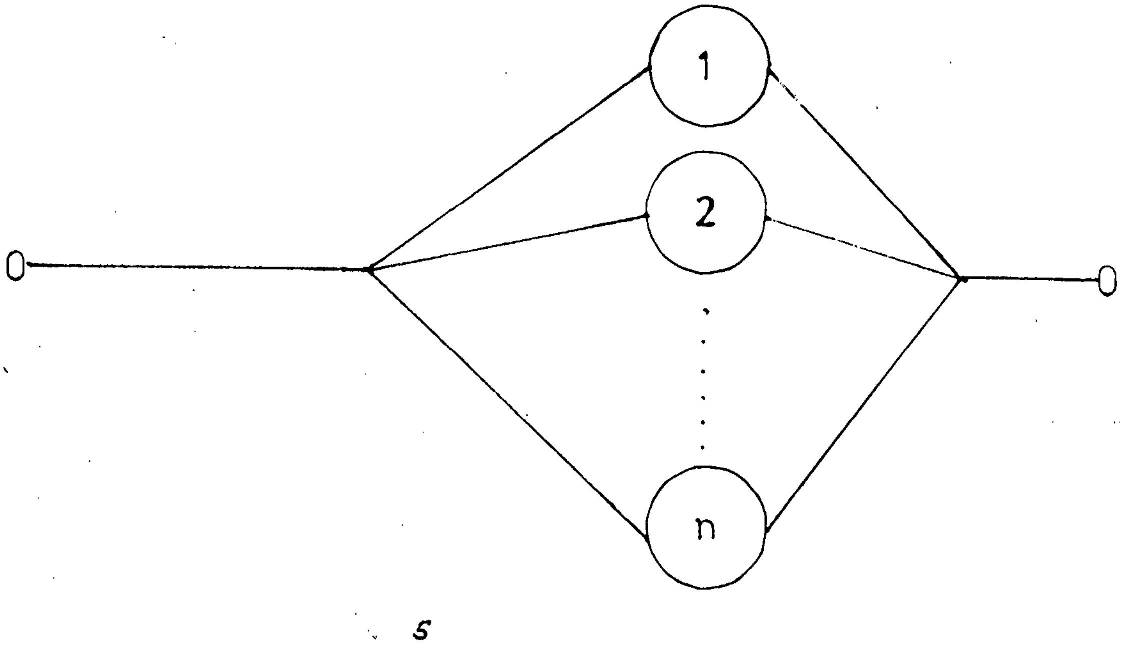
Reliability R(t) of a series system having n units is given by

$$R(t) = R1(t).R2(t).........Rn(t) = \prod_{i=1}^{n} Ri(t)$$

where $Ri(t)$, $1 \le i \le n$, is the reliability of the i^{th} component.

PARALLEL SYSTEM

A parallel system has configuration as given in the following figure -

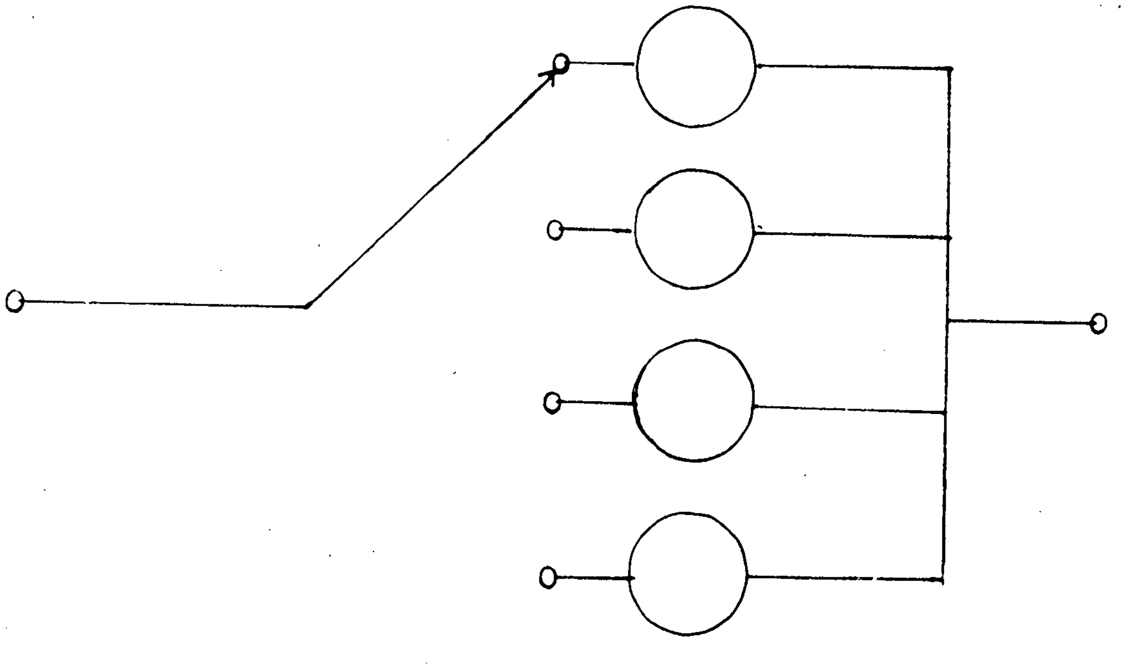


In this case the system can function properly when at least one of the components is working i. e. the system fails only when all the components fail. The reliability of a parallel system consisting of n units is given as

$$R(t) = 1 - \prod_{i=1}^{n} \{ 1 - Ri(t) \}$$

STANDBY SYSTEM :

In this type of system only one unit works at a time. If this unit fails a standby unit is immediately switched on-line and failed unit is taken off-line. The system functions till every unit fails. Configuration is given in the following figure.



MAINTAINED SYSTEM :

This type of system can be maintained i.e. the failed part (or parts) of the system can be repaired or replaced while the system is in working state or in failed state.

NON MAINTAINED SYSTEM : There is no repair facility for repair of the failed part of the system.

AVAILABILITY

INSTANTENEOUS AVAILABILITY : It is the probability that the system will be available at any random time t. We denote it by A(t). Some times we may be interested in the average up - time for some definite period T. It is defined as proportion of time in a specified interval (0, T] that the system is available for use.

$$A(T) = \frac{1}{T} \int_0^T A(t)\, dt \qquad \ldots\ldots\ldots\ldots 1.1$$

STEADY STATE AVAILABILITY : The proportion of time that the system is available for use when the time interval considered is very large , is known as steady state availability and is defined as

$$A(\infty) = \lim_{T \to \infty} \frac{1}{T} \int_0^T A(t)\, dt \qquad \ldots\ldots\ldots\ldots 1.2$$

TRANSFORMS

LAPLACE TRANSFORM (LT) : If f(t), t > 0 is continuous function of t then the Laplace Transform of f(t) is defined as

$$L[f(t)] = f^*(s) = \int_0^\infty e^{-st}.f(t)\,dt \quad , \text{ for real } s \geq 0 \qquad \ldots\ldots\ldots\ldots 1.3$$

LAPLACE STIELTJES TRANSFORM (LST) :

LST of a continuous function F(t) is defined as

$$LST[F(t)] = F^{**}(s) = \int_0^\infty e^{-st}.dF(t) \qquad \ldots\ldots\ldots 1.4$$

LAPLACE CONVOLUTION :

If f(t) and g(t) are continuous functions of t then convolution of f(t) and g(t) denoted by f(t).(c).g(t), is defined as

$$f(t).(c).g(t) = \int_0^t f(u).g(t-u)\,du \qquad \ldots\ldots\ldots 1.5$$

LAPLACE STEILTJES CONVOLUTION :

Let F(t) and G(t) be two continuous function of t then the LSC of F(t) and G(t) is defined as

$$F(t).(s).G(t) = \int_0^t F(t-u).dG(u) \qquad \ldots\ldots\ldots 1.6$$

By the convolution theorem we also have

$$LT[f(t).(c).g(t)] = f^*(s).g^*(s)$$

$$LST[F(t).(s).G(t)] = F^{**}(s).G^{**}(s)$$

MEAN TIME TO SYSTEM FAILURE (MTSF)

It is the expected time, the system is in operation before it fails completely. If F(t) is the failure distribution of the system, f(t) the density function and R(t) is the reliability of the system then

$$MTSF = \int_0^\infty t.f(t)dt = \int_0^\infty R(t)dt \qquad ....1.7$$

CHAPTER 2

DESCRIPTION OF SYSTEM UNDER MODEL 1 : In this chapter we consider a two unit system in which each unit can be in either of the following three modes -

(a) Operative / Normal O

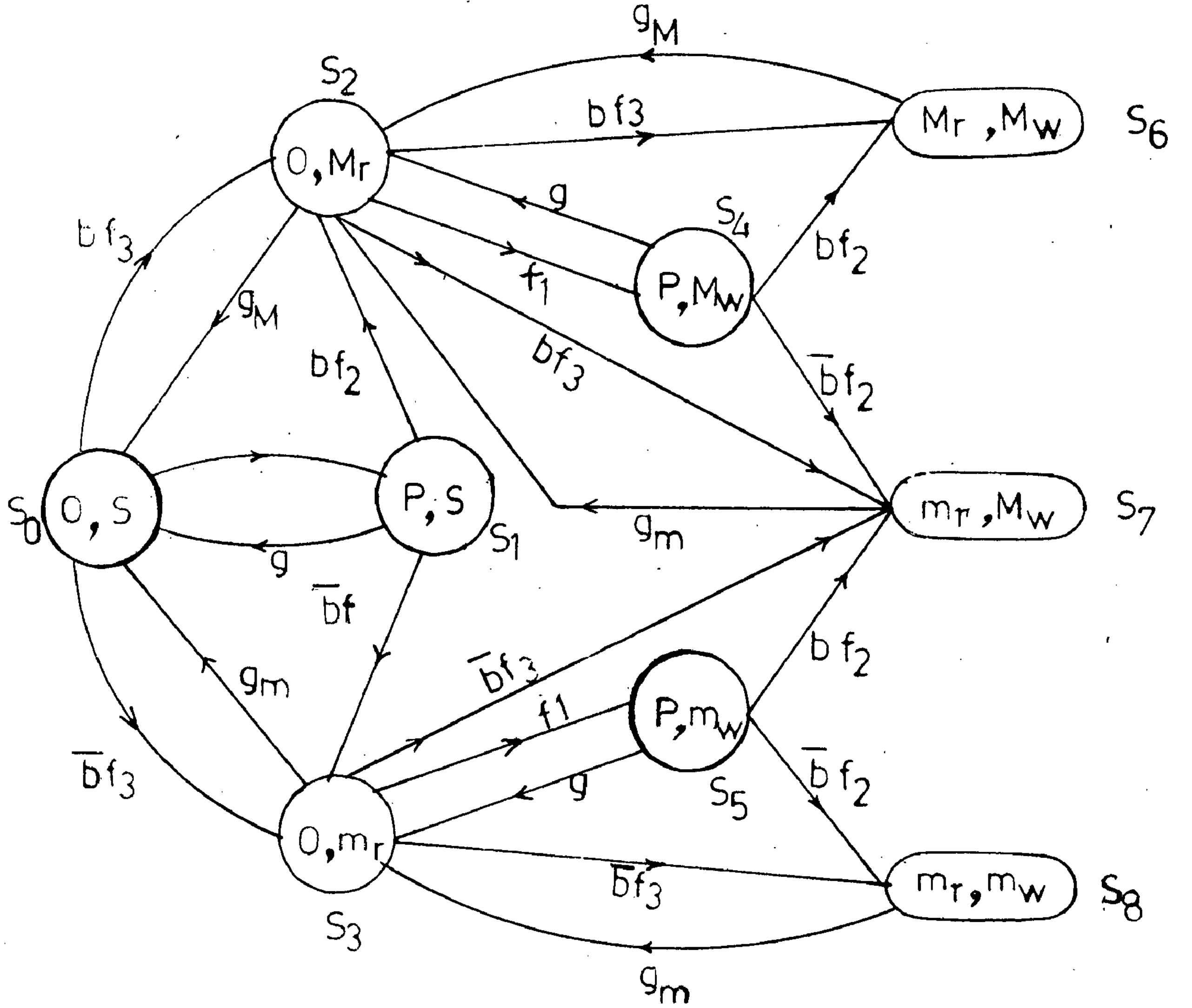
(b) Partial Failure P

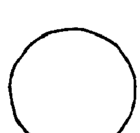
(c) Total Failure F

i.e. a unit can fail partially or totally. When a unit fails totally it might have to undergo either major or minor repair . Assuming general distribution for failure and repair times, we obtain distribution of time to system failure, availability and MTSF by using the technique of regenerative process. In the description of the system we have following points -

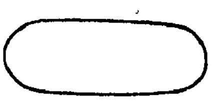
1. Two similar units operate in cold standby configuration and at time t = 0 one unit is operating and other unit is cold standby.
2. An O-mode unit can go to P-mode or direct to F-mode or to F-mode through P-mode.
3. Switching of standby unit in operation is instanteneuos and perfect.
4. The failure and repair time distributions are assumed to be arbitrary with different cdf's in each case.
5. In F-mode a unit may need major or minor repair with probabilities b and $\bar{b}$ such that $b + \bar{b} = 1$.
6. Single repair man is available to repair a P- or F- unit.

TRANSITION DIAGRAM OF MODEL I





UP STATE



DOWN STATE

7. After repair in F- mode the unit goes directly to O- mode.

8. The time to failure and repair of unit one is statistically independent of the time to failure and repair of other unit.

9. In case an O- unit enters P- mode while the other unit is in F-mode and under repair, the repair of F-unit is stopped and the P-unit is taken up for repair in preemptive repeat fashion.

NOTATIONS :

O	operative mode
P	partial failure
S	standby
M_r, m_r	under major or minor repair after total failure
M_w, m_w	waiting for major or minor repair after total failure.

The system may be in of the following states at any instant

Up states S_0 (O,S) , S_1(P,S), S_2(O,M_r)

S_3(O,m_r), S_4(P,M_w), S_5(P,m_w)

Down states S_6(M_r,M_w), S_7(m_r,M_w), S_8(m_r,m_w)

We are also using some following notations

$f_1(t)$, $F_1(t)$	pdf and cdf of time to pass from O to P-mode
$f_2(t)$, $F_2(t)$	pdf and cdf of time to pass from P to F-mode
$f_3(t)$, $F_3(t)$	pdf and cdf of time to pass from O to F-mode
$f_s(t)$, $F_s(t)$	pdf and cdf of time to pass from S to P-mode
$g(t)$, $G(t)$	pdf and cdf of time to repair of a P-unit
$g_M(t)$, $G_M(t)$	pdf and cdf of time for major repair of a F-unit
$g_m(t)$, $G_m(t)$	pdf and cdf of time for minor repair of a F-unit

$q_{ij}(t), Q_{ij}(t)$	pdf and cdf of time for transition from state S_i to S_j
b	Prob. of F-unit requiring major repair
$*$	Laplace transform
$**$	Laplace steiltjes transform
(c)	Laplace convolution
(s)	Laplace steiltjes convolution
μ_i	mean sojourn time in state $S_i = \sum_j m_{ij}$
m_{ij}	mean sojourn time in state S_i given that transition is to state S_j

$$= \int t\,[dQ_{ij}(t)]$$

$$= \int t\, e^{-st}\, d\,Q_{ij}(t) \Big|_{s=0} = -\,Q_{ij}^{**\prime}(0)$$

U_i	time to system failure when it starts from state S_i
V_i	$Pr\{\, U_i \le t \,\}$
$M_i(t)$	Pr {system starting from state S_i is up at time t without transiting into any other regenerative state }

DENSITY FUNCTION OF TRANSITION TIMES

$$q_{01}(t) = f_1(t)\,\bar{F}_3(t)$$

$$q_{02}(t) = b.f_3(t)\,\bar{F}_1(t)$$

$$q_{03}(t) = \bar{b}.f_3(t)\,\bar{F}_1(t)$$

$$q_{10}(t) = q_{42}(t) = q_{53}(t)$$

$$= g(t)\,\bar{F}_2(t)$$

$$q_{12}(t) = q_{48}(t) = q_{57}(t)$$

$$= b.f_2(t)\,\bar{G}(t)$$

$q_{13}(t) = q_{47}(t) = q_{58}(t)$

$= \bar{b}.f_2(t)\ \bar{G}(t)$

$q_{24}(t) = f_1(t)\ \bar{F}_3(t)\ \bar{G}_M(t)$

$q_{35}(t) = f_1(t)\ \bar{F}_3(t)\ \bar{G}_m(t)$

$q_{28}(t) = b.f_3(t)\ \bar{F}_1(t)\ \bar{G}_M(t)$

$q_{27}(t) = \bar{b}.f_3(t)\ \bar{F}_1(t)\ \bar{G}_M(t)$

$q_{37}(t) = b.f_3(t)\ \bar{F}_1(t)\ \bar{G}_m(t)$

$q_{20}(t) = g_M(t)\ \bar{F}_3(t)\ \bar{F}_1(t)$

$q_{30}(t) = g_m(t)\ \bar{F}_3(t)\ \bar{F}_1(t)$

$q_{38}(t) = \bar{b}.f_3(t)\ \bar{F}_1(t)\ \bar{G}_m(t)$

$q_{82}(t) = g_M(t)$

$q_{72}(t) = g_m(t)$

$q_{83}(t) = g_m(t)$

$q_{ij}(t) = 0$ otherwise

Note:- $P = [(p_{ij})]$ denotes the transition probability matrix

$= [Q_{ij}\ (\infty)] = [Q_{ij}^{**}\ (0)]$

TIME TO SYSTEM FAILURE

Now we have

$V_0(t) = Q_{01}(t)(s)\ V_1(t) + Q_{02}(t)(s)\ V_2(t) + Q_{03}(t)(s)\ V_3(t)$

$V_1(t) = Q_{10}(t)(s)\ V_0(t) + Q_{12}(t)(s)\ V_2(t) + Q_{13}(t)(s)\ V_3(t)$

$V_2(t) = Q_{20}(t)(s)\ V_0(t) + Q_{24}(t)(s)\ V_4(t) + Q_{26}(t) + Q_{27}(t)$

$V_3(t) = Q_{30}(t)(s)\ V_0(t) + Q_{35}(t)(s)\ V_5(t) + Q_{37}(t) + Q_{38}(t)$

$V_4(t) = Q_{42}(t)(s)\ V_2(t) + Q_{46}(t) + Q_{47}(t)$

$V_5(t) = Q_{53}(t)(s)\ V_3(t) + Q_{57}(t) + Q_{58}(t)$

Taking Laplace Stieltjes transform and solving we obtain the LST of distribution functions of the failure times assuming that system was initially in state S_0. Then we have

$$V_0^{**}(s) = Q_{01}^{**}(s)\ V_1^{**}(s) + Q_{02}^{**}(s)\ V_2^{**}(s) + Q_{03}^{**}(s)\ V_3^{**}(s)$$

$$V_1^{**}(s) = Q_{10}^{**}(s)\ V_0^{**}(s) + Q_{12}^{**}(s)\ V_2^{**}(s) + Q_{13}^{**}(s)\ V_3^{**}(s)$$

$$V_2^{**}(s) = Q_{20}^{**}(s)\ V_0^{**}(s) + Q_{24}^{**}(s)\ V_4^{**}(s) + Q_{28}^{**}(s) + Q_{27}^{**}(s)$$

$$V_3^{**}(s) = Q_{30}^{**}(s)\ V_0^{**}(s) + Q_{35}^{**}(s)\ V_5^{**}(s) + Q_{37}^{**}(s) + Q_{38}^{**}(s)$$

$$V_4^{**}(s) = Q_{42}^{**}(s)\ V_2^{**}(s) + Q_{46}^{**}(s) + Q_{47}^{**}(s)$$

$$V_5^{**}(s) = Q_{53}^{**}(s)\ V_3^{**}(s) + Q_{57}^{**}(s) + Q_{58}^{**}(s)$$

If we consider $V_0^{**}(s) = \dfrac{N_2\ (s)}{D_2\ (s)}$ Then we can write

$$N_2(s) = (1-Q_{35}^{**}(s)\ Q_{53}^{**}(s))\ [Q_{01}^{**}(s)\ Q_{12}^{**}(s) + Q_{02}^{**}(s))$$
$$\{Q_{24}^{**}(s)(Q_{46}^{**}(s)+ Q_{47}^{**}(s)) + Q_{28}^{**}(s) + Q_{27}^{**}(s)\}]$$
$$+ (1-Q_{24}^{**}(s)\ Q_{42}^{**}(s))\ [(Q_{01}^{**}(s)\ Q_{13}^{**}(s)$$
$$+ Q_{30}^{**}(s))\{Q_{35}^{**}(s)(Q_{57}^{**}(s) + Q_{58}^{**}(s)) + Q_{37}^{**}(s)$$
$$+ Q_{38}^{**}(s)\}]$$

$$D_2(s) = (1-Q_{24}^{**}(s)\ Q_{42}^{**}(s))(1-Q_{35}^{**}(s)\ Q_{53}^{**}(s))\ x$$
$$(1-Q_{01}^{**}(s)\ Q_{10}^{**}(s))$$
$$-(1-Q_{35}^{**}(s)\ Q_{53}^{**}(s))\ [Q_{01}^{**}(s)\ Q_{12}^{**}(s)\ Q_{20}^{**}(s)$$
$$+ Q_{20}^{**}(s)\ Q_{02}^{**}(s)]$$
$$-(1-Q_{24}^{**}(s)\ Q_{42}^{**}(s))\ [Q_{01}^{**}(s)\ Q_{13}^{**}(s)\ Q_{30}^{**}(s)$$
$$+ Q_{03}^{**}(s) + Q_{30}^{**}(s)]$$

Now we have to find out mean time to system failure (MTSF)

i.e. $$E\ [\ U_0\ ] = \frac{-d}{ds} V_0^{**}(s)\Big|_{s=0}$$

i.e. $$MTSF = \frac{D_2'(0) - N_2'(0)}{D_2(0)} \qquad ....2.1$$

In the following expressions we shall be writing Q_{ij}^{**} for $Q_{ij}^{**}(s)$.

$$N_2'(s) = (1-Q_{35}^{**}\ Q_{35}^{**})\ (Q_{01}^{**}\ Q_{12}^{**} + Q_{02}^{**})\ [Q_{24}^{**'}(Q_{46}^{**}$$

$$+ Q_{47}^{**}) + Q_{24}^{**}(Q_{46}^{**'} + Q_{47}^{**'} + (Q_{26}^{**'} + Q_{27}^{**'})]$$

$$+ [(1-Q_{35}^{**}\ Q_{53}^{**})(Q_{01}^{**}\ Q_{12}^{**'} + Q_{01}^{**'}Q_{12}^{**} + Q_{02}^{**'})$$

$$- (Q_{35}^{**'}Q_{53}^{**} + Q_{35}^{**}\ Q_{53}^{**'})(Q_{01}^{**}\ Q_{12}^{**} + Q_{02}^{**})]$$

$$\times\ [Q_{24}^{**}(Q_{46}^{**} + Q_{47}^{**}) + Q_{26}^{**} + Q_{27}^{**}]$$

$$+ (1-Q_{24}^{**}\ Q_{42}^{**})(Q_{01}^{**}\ Q_{13}^{**} + Q_{03}^{**}) \times$$

$$[Q_{35}^{**'}(Q_{57}^{**} + Q_{58}^{**}) + Q_{35}^{**}(Q_{57}^{**'} + Q_{58}^{**'})$$

$$+(Q_{37}^{**'} + Q_{38}^{**'})]$$

$$+ [(1-\ Q_{24}^{**}\ Q_{42}^{**})(Q_{01}^{**}\ Q_{13}^{**'} + Q_{01}^{**'}Q_{13}^{**} +$$

$$Q_{03}^{**'}) - (Q_{24}^{**'}Q_{42}^{**} + Q_{24}^{**}\ Q_{42}^{**'}) \times$$

$$(Q_{01}^{**}\ Q_{13}^{**} + Q_{03}^{**})]\ [Q_{35}^{**}\ (Q_{57}^{**} + Q_{58}^{**}) +$$

$$(Q_{37}^{**} + Q_{38}^{**})]$$

.........2.2

$$D_2'(s) = -(Q_{24}^{**'}Q_{42}^{**} + Q_{24}^{**}\ Q_{42}^{**'}\ (1-Q_{35}^{**}\ Q_{53}^{**}) \times$$

$$(1-Q_{01}^{**}\ Q_{10}^{**}) - (1-Q_{24}^{**}\ Q_{42}^{**})\ (Q_{35}^{**'}Q_{53}^{**}$$

$$+ Q_{35}^{**}\ Q_{53}^{**'})(1-Q_{10}^{**}\ Q_{01}^{**})-(1-Q_{24}^{**}Q_{42}^{**})\times$$

$$(Q_{10}^{**'}\ Q_{01}^{**} + Q_{10}^{**}\ Q_{01}^{**'})\ (1-Q_{35}^{**}\ Q_{53}^{**})$$

$$-(1-\ Q_{35}^{**}\ Q_{53}^{**})\ [Q_{01}^{**}\ Q_{12}^{**}\ Q_{20}^{**'} +$$

$$Q_{01}^{**}\ Q_{12}^{**'}Q_{20}^{**} + Q_{01}^{**'}Q_{12}^{**}\ Q_{20}^{**} +$$

$$Q_{02}^{**'}Q_{20}^{**} + Q_{02}^{**}\ Q_{20}^{**'}]$$

$$-(1-\ Q_{24}^{**}\ Q_{42}^{**})\ [Q_{01}^{**}\ Q_{13}^{**}\ Q_{30}^{**'} +$$

$$Q_{01}^{**}\ Q_{13}^{**'}Q_{30}^{**} + Q_{01}^{**'}Q_{13}^{**}\ Q_{30}^{**} +$$

$$Q_{03}^{**'}Q_{30}^{**} + Q_{03}^{**}\ Q_{30}^{**'}]$$

$$+(Q_{35}^{**'}Q_{53}^{**} + Q_{35}^{**}\ Q_{53}^{**'}) \times$$

$$[Q_{01}^{**}\ Q_{12}^{**}\ Q_{20}^{**} + Q_{02}^{**}\ Q_{20}^{**}]$$

$$+(Q_{42}^{**'}Q_{24}^{**} + Q_{42}^{**}\ Q_{24}^{**'}) \times$$

$$[Q_{01}^{**}\ Q_{13}^{**}\ Q_{30}^{**} + Q_{03}^{**}\ Q_{30}^{**}]$$

............2.3

$$D_2'(s)-N_2'(s)$$

$$= -(1-Q_{35}^{**}Q_{53}^{**})(1-Q_{24}^{**}Q_{42}^{**})[(Q_{01}^{**'}+Q_{02}^{**'}+Q_{03}^{**'})+Q_{01}^{**}(Q_{10}^{**'}+Q_{12}^{**'}+Q_{13}^{**'})]$$

$$-(1-Q_{35}^{**}Q_{53}^{**})[(Q_{20}^{**'}+Q_{24}^{**'}+Q_{26}^{**'}+Q_{27}^{**'})(Q_{01}^{**}Q_{12}^{**}+Q_{02}^{**})+(Q_{42}^{**'}+Q_{46}^{**'}+Q_{47}^{**'})(Q_{02}^{**}Q_{24}^{**}+Q_{01}^{**}Q_{12}^{**}Q_{24}^{**})$$

$$-(1-Q_{24}^{**}Q_{42}^{**})[(Q_{30}^{**'}+Q_{35}^{**'}+Q_{37}^{**'}+Q_{38}^{**'})(Q_{01}^{**}Q_{13}^{**}+Q_{03}^{**})+(Q_{53}^{**'}+Q_{57}^{**'}+Q_{58}^{**'})(Q_{03}^{**}Q_{35}^{**}+Q_{01}^{**}Q_{13}^{**}Q_{35}^{**})]$$

............2.4

Substituting $s = 0$ we have $Q_{ij}^{**} = p_{ij}$ and we get

$$MTSF = \frac{D_2'(0) - N_2'(0)}{D_2(0)}$$

Where

$$D_2'(0) - N_2'(0)$$

$$= (1-p_{35}p_{53})(1-p_{42}p_{24})[\mu_0 + p_{01}\mu_0]$$

$$+(1-p_{42}p_{24})[(\mu_3 + p_{35}\mu_5).(p_{03} + p_{01}p_{13})]$$

$$+(1-p_{35}p_{53})[(\mu_2 + p_{24}\mu_4).(p_{02} + p_{01}p_{12})]$$

........2.5

and

$$D_2(0) = (1-p_{35}p_{53})(1-p_{42}p_{24})(1-p_{01}p_{10})$$

$$-p_{30}(1-p_{42}p_{24})(p_{03}+p_{01}p_{13})$$

$$-p_{20}(1-p_{35}p_{53})(p_{02}+p_{01}p_{12})$$

.........2.6

POINTWISE AVAILABILITY AND STEADY STATE AVAILABILITY

Probability that the system is up at time t starting from state S_i without transiting to any other regenerative state is given by $M_i(t)$ and for different i's we have

$$M_0(t) = \bar{F}_3(t)\,\bar{F}_1(t)$$

$$M_1(t) = \bar{G}(t)\,\bar{F}_2(t)$$

$$M_2(t) = \bar{F}_3(t)\,\bar{F}_1(t)\,\bar{G}_M(t)$$

$$M_3(t) = \bar{F}_3(t)\,\bar{F}_1(t)\,\bar{G}_m(t)$$

$$M_4(t) = \bar{G}(t)\,\bar{F}_2(t)$$

$$M_5(t) = \bar{G}(t)\,\bar{F}_2(t)$$

In the beginning of this chapter we have mentioned that we shall apply technique of regenerative process. Applying that theory, the pointwise availabilities $A_i(t)$ of system starting from a regenerative point are given by the following relations.

$$A_0(t) = M_0(t) + q_{01}(t)(c)\,A_1(t) + q_{02}(t)(c)\,A_2(t) + q_{03}(t)(c)\,A_3(t)$$

$$A_1(t) = M_1(t) + q_{10}(t)(c)\,A_0(t) + q_{12}(t)(c)\,A_2(t) + q_{13}(t)(c)\,A_3(t)$$

$$A_2(t) = M_2(t) + q_{20}(t)(c)\,A_0(t) + q_{24}(t)(c)\,A_4(t) + q_{26}(t)(c)\,A_6(t) + q_{27}(t)(c)\,A_7(t)$$

$$A_3(t) = M_3(t) + q_{30}(t)(c)\,A_0(t) + q_{35}(t)(c)\,A_5(t) + q_{37}(t)(c)\,A_7(t) + q_{38}(t)(c)\,A_8(t)$$

$$A_4(t) = M_4(t) + q_{42}(t)(c)\,A_2(t) + q_{46}(t)(c)\,A_6(t) + q_{47}(t)(c)\,A_7(t)$$

$$A_5(t) = M_5(t) + q_{53}(t)(c)\,A_3(t) + q_{57}(t)(c)\,A_7(t) + q_{58}(t)(c)\,A_8(t)$$

$$A_6(t) = q_{62}(t)(c)\,A_2(t)$$

$$A_7(t) = q_{72}(t)(c)\,A_2(t)$$

$$A_8(t) = q_{83}(t)(c)\,A_3(t)$$

Taking Laplace Transform and solving for $A_0^*(s)$ we get

$$A_0^*(s) = \frac{N_A(s)}{D_A(s)} \qquad \ldots\ldots\ldots 2.7$$

Where { In the following expression we are writing q^* for $q^*(s)$}

$$N_A(s) = \{1- q_{38}^* q_{83}^* -q_{35}^*(q_{53}^* +q_{58}^* q_{83}^*)\} x$$

$$\{1- [q_{24}^* q_{42}^* +q_{62}^* q_{26}^* +q_{72}^* q_{27}^* - q_{24}^* x$$

$$x(q_{48}^* q_{82}^* +q_{47}^* q_{72}^*)\} [M_0^* + q_{01}^* M_1^*]$$

$$+ \{1- q_{38}^* q_{83}^* -q_{35}^*(q_{53}^* +q_{58}^* q_{83}^*)\} x$$

$$\{ q_{01}^* q_{12}^* + q_{02}^* \} [M_2^* + q_{24}^* M_4^*]$$

$$+ \{1- [q_{24}^* q_{42}^* +q_{62}^* q_{26}^* +q_{72}^* q_{27}^* - q_{24}^* x$$

$$x(q_{48}^* q_{82}^* +q_{47}^* q_{72}^*)\} \{q_{01}^* q_{13}^* + q_{03}^* \}x$$

$$[M_3^* + q_{35}^* M_5^*]$$

$$+ (q_{01}^* q_{13}^* +q_{03}^*) (q_{35}^* q_{57}^* q_{72}^* + q_{37}^* q_{72}^*) \qquad ....2.8$$

$$D_A(s) = \{1- q_{38}^* q_{83}^* -q_{35}^*(q_{53}^* +q_{58}^* q_{83}^*)\} x$$

$$x\{1- [q_{24}^* q_{42}^* +q_{62}^* q_{26}^* +q_{72}^* q_{27}^* - q_{24}^* x$$

$$x(q_{48}^* q_{62}^* +q_{47}^* q_{72}^*)\}$$

$$- \{1- q_{38}^* q_{83}^* -q_{35}^*(q_{53}^* +q_{58}^* q_{83}^*)\}$$

$$x\{q_{01}^* q_{12}^* q_{20}^* + q_{02}^* q_{20}^* \}$$

$$- \{1- [q_{24}^* q_{42}^* +q_{62}^* q_{26}^* +q_{72}^* q_{27}^* - q_{24}^* x$$

$$x(q_{48}^* q_{62}^* +q_{47}^* q_{72}^*)\}\{q_{01}^* q_{13}^* q_{30}^* + q_{03}^* q_{30}^* \}$$

$$- q_{01}^* q_{13}^* \{q_{35}^* q_{57}^* q_{72}^* + q_{37}^* q_{72}^* \} q_{20}^*$$

Hence the steady state availability

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} s.A_0^*(s) = \frac{N_A(0)}{D_A'(0)} \qquad ....2.9$$

Where

$$N_A(0) = p_{20} [p_{30} + p_{37} + p_{35} p_{57}] [\mu_0 + p_{01} \mu_1]$$

$$+ [\mu_2 + p_{24} \mu_4] [p_{01} p_{12} p_{30} + p_{35} p_{57} (p_{12} + p_{13})+$$

$$p_{30} p_{02}] +$$

$$+ [\mu_3 + p_{35} \mu_5] [p_{20} (p_{03} + p_{01} p_{13})] \qquad ....2.10$$

And

$$
\begin{aligned}
D_A'(0) = {} & p_{20} [p_{30} + p_{37} + p_{35} p_{57}] [\mu_0 + p_{01} \mu_1] \\
& + [\mu_2 + p_{24} \mu_4] [p_{01} p_{12} p_{30} + p_{35} p_{57} (p_{12} + p_{13}) + \\
& \quad p_{30} p_{02}] + \\
& + [\mu_3 + p_{35} \mu_5] [p_{20} (p_{03} + p_{01} p_{13})] \\
& + \{ [p_{26} + p_{48} p_{24}] \mu_8 + [p_{27} + p_{47} p_{24}] \mu_7 \} x \\
& x [p_{01} p_{12} p_{30} + (p_{12} + p_{13}) p_{35} p_{57}] \\
& + \{ [p_{38} + p_{35} p_{58}] \mu_8 + [p_{37} + p_{35} p_{57}] \mu_7 \} x \\
& x \; p_{01} p_{13} p_{20} \\
& + \{ [p_{38} + p_{35} p_{58}] \mu_8 \; p_{20} p_{03} \\
& + \{ [p_{27} + p_{47} p_{24}] \mu_7 \; p_{02} p_{30} \qquad \ldots\ldots.2.11
\end{aligned}
$$

CHAPTER 3

DESCRIPTION OF SYSTEM UNDER MODEL 2 : In this chapter we consider a two unit system with all those assumptions which we have considered in the previous chapter along with the following ones

1. The units are warm standby in place of cold standby.
2. The operative unit can go from O- mode to P- mode and then to F- mode i.e. O-mode unit can not go directly to the F-mode and the standby unit can fail only totally.
3. Standby unit after total failure requires two types of repairs viz. major and minor with probabilities α and $\bar{\alpha}$ respectively such that $\alpha + \bar{\alpha} = 1$.
4. $F_s(t)$ and $f_s(t)$ are the cdf and pdf to time to failure a standby unit totally.

DENSITY FUNCTION OF TRANSITION TIMES

$q_{01}(t) = f_1(t)\,\bar{F}_s(t)$

$q_{02}(t) = \alpha . f_s(t)\,\bar{F}_1(t)$

$q_{03}(t) = \bar{\alpha} . f_s(t)\,\bar{F}_1(t)$

$q_{10}(t) = g(t)\,\bar{F}_2(t)\,\bar{F}_s(t)$

$q_{42}(t) = q_{53}(t)$

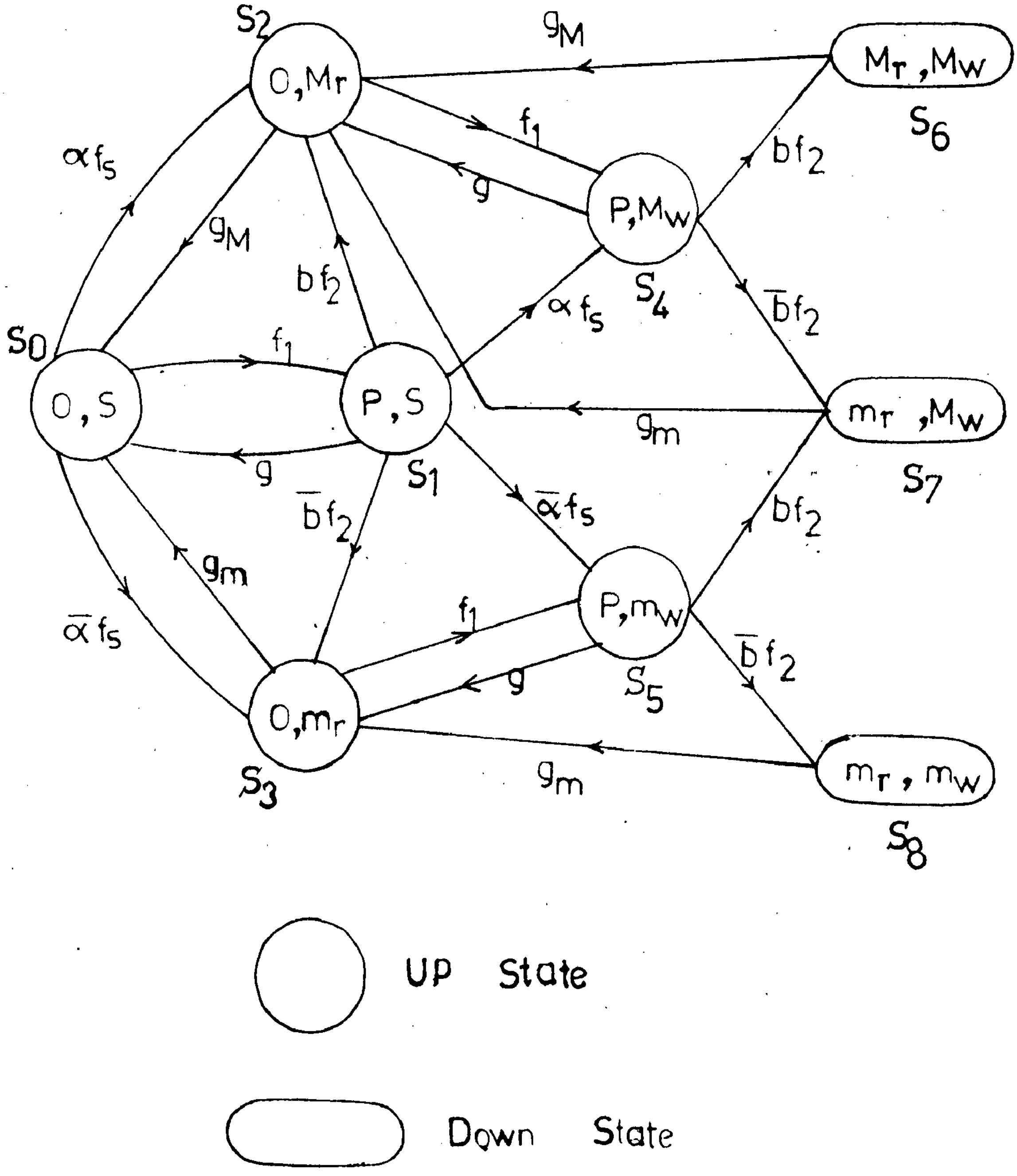
$= g(t)\,\bar{F}_2(t)$

$q_{12}(t) = b . f_2(t)\,\bar{G}(t)\,\bar{F}_s(t)$

$q_{46}(t) = q_{57}(t)$

$= b . f_2(t)\,\bar{G}(t)$

TRANSITION DIAGRAM OF MODEL 2



$q_{13}(t) = \bar{b}.f_2(t)\ \bar{G}(t)\ \bar{F}_8(t)$

$q_{47}(t) = q_{58}(t)$

$= \bar{b}.f_2(t)\ \bar{G}(t)$

$q_{24}(t) = f_1(t)\ \bar{G}_M(t)$

$q_{35}(t) = f_1(t)\ \bar{G}_m(t)$

$q_{20}(t) = g_M(t)\ \bar{F}_1(t)$

$q_{30}(t) = g_m(t)\ \bar{F}_1(t)$

$q_{14}(t) = \alpha.f_8(t)\ \bar{F}_2(t)\ \bar{G}(t)$

$q_{15}(t) = \bar{\alpha}.f_8(t)\ \bar{F}_2(t)\ \bar{G}(t)$

$q_{62}(t) = g_M(t)$

$q_{72}(t) = g_m(t)$

$q_{83}(t) = g_m(t)$

$q_{ij}(t) = 0$ otherwise

Note:- $P = [(p_{ij})]$ denotes the transition probability matrix

$= [Q_{ij}\ (\infty)] = [Q_{ij}^{**}\ (0)]$

TIME TO SYSTEM FAILURE

We have following renewal process arguments :-

$V_0(t) = Q_{01}(t)(s)\ V_1(t) + Q_{02}(t)(s)\ V_2(t) + Q_{03}(t)(s)\ V_3(t)$

$V_1(t) = Q_{10}(t)(s)\ V_0(t) + Q_{12}(t)(s)\ V_2(t) + Q_{13}(t)(s)\ V_3(t) +$

$Q_{14}(t)(s)\ V_4(t) + Q_{15}(t)(s)\ V_5(t)$

$V_2(t) = Q_{20}(t)(s)\ V_0(t) + Q_{24}(t)(s)\ V_4(t)$

$V_3(t) = Q_{30}(t)(s)\ V_0(t) + Q_{35}(t)(s)\ V_5(t)$

$V_4(t) = Q_{42}(t)(s)\ V_2(t) + Q_{46}(t) + Q_{47}(t)$

$V_5(t) = Q_{53}(t)(s)\ V_3(t) + Q_{57}(t) + Q_{58}(t)$

Taking Laplace Stieltjes transform and solving we obtain the LST of distribution functions of the failure times assuming that system was initially in state S_0 and if we consider

$$V_0^{**}(s) = \frac{N_4(s)}{D_4(s)}$$ Then we have from the above set

$$N_4(s) = \{1 - Q_{35}^{**}(s)\, Q_{53}^{**}(s)\}[Q_{01}^{**}(s)\, \{ Q_{12}^{**}(s) + Q_{14}^{**}(s) Q_{42}^{**}\} + Q_{02}^{**}(s)(Q_{46}^{**}(s) + Q_{47}^{**}(s))Q_{24}^{**} + \{Q_{14}^{**}(s)\ (Q_{46}^{**}(s) + Q_{47}^{**}(s)) + Q_{15}^{**}(s)\ (Q_{57}^{**}(s) + Q_{58}^{**}(s)\}\ \{1-Q_{24}^{**}(s)\ Q_{42}^{**}(s)\}] + \{1-Q_{24}^{**}(s)\ Q_{42}^{**}(s)\}\ (Q_{57}^{**}(s) + Q_{58}^{**}(s))\ [\ Q_{01}^{**}(s)\{\ Q_{13}^{**}(s) + Q_{15}^{**}(s)\ Q_{53}^{**}(s)\ \} + + Q_{03}^{**}(s)\ ]\ Q_{35}^{**}(s) \qquad \ldots\ldots\ldots 3.1$$

$$D_4(s) = (1-Q_{24}^{**}(s)\ Q_{42}^{**}(s))(1-Q_{35}^{**}(s)\ Q_{53}^{**}(s))\ \times (1-Q_{01}^{**}(s)\ Q_{10}^{**}(s)) -\{1-Q_{35}^{**}(s)\ Q_{53}^{**}(s)\}\ [\{Q_{12}^{**}(s) + Q_{14}^{**}(s)\ Q_{42}^{**}(s)\}\ \{Q_{02}^{**}(s)+Q_{01}^{**}(s)\}]\ Q_{20}^{**}(s) -\{1-Q_{24}^{**}(s)\ Q_{42}^{**}(s)\}\ [\{Q_{13}^{**}(s) + Q_{15}^{**}(s)\ Q_{53}^{**}(s)\ \{Q_{03}^{**}(s)+Q_{01}^{**}(s)\}]\ Q_{30}^{**}(s) \qquad \ldots\ldots\ldots\ldots 3.2$$

Now we have to find out mean time to system failure (MTSF)

i.e. $$E[\,U_0\,] = \frac{-d}{ds} V_0^{**}(s)\big|_{s=0}$$

i.e. $$\text{MTSF} = \frac{D_4'(0) - N_4'(0)}{D_4(0)}$$

In the following expressions we are writing Q_{ij}^{**} for $Q_{ij}^{**}(s)$

After differentiation of 3.1 and 3.2 we can find $D_4'(s) - N_4'(s)$ and substituting $s = 0$ we have $Q_{ij}^{**} = p_{ij}$ and

$D_4'(0) - N_4'(0)$

$$= (1-p_{35}\ p_{53})(1-p_{42}\ p_{24})[\mu_0 + p_{01}\{\mu_0 + p_{14}\ \mu_4 + p_{15}\ \mu_5\}]$$

$$+(1-p_{42}\ p_{24})[(\mu_3 + p_{35}\ \mu_5) . (p_{03} + p_{01}\ p_{13} + p_{01}\ p_{15}\ p_{53})]$$

$$+(1-p_{35}\ p_{53})[(\mu_2 + p_{24}\ \mu_4) . (p_{02} + p_{01}\ p_{12} + p_{01}\ p_{14}\ p_{42})] \quad \ldots\ldots\ldots\ldots 3.4$$

and also

$$D_4(0) = (1-p_{35}\ p_{53})(1-p_{42}\ p_{24})(1- p_{01}\ p_{10})$$

$$- p_{30}(1-p_{42}\ p_{24})[p_{13} + p_{15}\ p_{53}]\{p_{01} + p_{03}\}$$

$$- p_{20}(1-p_{35}\ p_{53})[p_{12} + p_{14}\ p_{42}]\{p_{01} + p_{02}\} \quad \ldots\ldots 3.5$$

For conveniency we have omitted the steps in the above calculations.

Hence by assuming varios types of distribution we can find MTSF.

Probability that the system is up at time t starting from state S_i without transiting to any other regenerative state is given by $M_i(t)$ and for different i's we have

$$M_0(t) = \bar{F}_8(t)\ \bar{F}_1(t)$$

$$M_1(t) = \bar{G}(t)\ \bar{F}_2(t)\ \bar{F}_8(t)$$

$$M_2(t) = \bar{F}_1(t)\ \bar{G}_M(t)$$

$$M_3(t) = \bar{F}_1(t)\ \bar{G}_m(t)$$

$$M_4(t) = \bar{G}(t)\ \bar{F}_2(t)$$

$$M_5(t) = \bar{G}(t)\ \bar{F}_2(t)$$

POINTWISE AVAILABILITY AND STEADY STATE AVAILABILITY

Applying the same procedure as we have done in the previous chapter, we get the pointwise availabilities $A_i(t)$ of system starting from a regenerative point as follows :

$$A_0(t) = M_0(t) + q_{01}(t)(c)\,A_1(t) + q_{02}(t)(c)\,A_2(t) + q_{03}(t)(c)\,A_3(t)$$

$$A_1(t) = M_1(t) + q_{10}(t)(c)\,A_0(t) + q_{12}(t)(c)\,A_2(t) + q_{13}(t)(c)\,A_3(t) + q_{14}(t)(c)\,A_4(t) + q_{15}(t)(c)\,A_5(t)$$

$$A_2(t) = M_2(t) + q_{20}(t)(c)\,A_0(t) + q_{24}(t)(c)\,A_4(t)$$

$$A_3(t) = M_3(t) + q_{30}(t)(c)\,A_0(t) + q_{35}(t)(c)\,A_5(t)$$

$$A_4(t) = M_4(t) + q_{42}(t)(c)\,A_2(t) + q_{48}(t)(c)\,A_8(t) + q_{47}(t)(c)\,A_7(t)$$

$$A_5(t) = M_5(t) + q_{53}(t)(c)\,A_3(t) + q_{57}(t)(c)\,A_7(t) + q_{58}(t)(c)\,A_8(t)$$

$$A_6(t) = q_{62}(t)(c)\,A_2(t)$$

$$A_7(t) = q_{72}(t)(c)\,A_2(t)$$

$$A_8(t) = q_{83}(t)(c)\,A_3(t)$$

Taking Laplace Transform of the above set and solving for $A_0^*(s)$ we get

$$A_0^*(s) = \frac{N_B(s)}{D_B(s)}$$

Where

$$N_B(s) = \{1 - q_{24}(q_{42}\,q_{46}\,q_{62} + q_{47}\,q_{72})\}\{[1 - q_{35}(q_{53} + q_{58}\,q_{83})][M_0 + q_{01}(M_1 + q_{15}\,M_5 + M_4\,q_{14})] + (M_3 + q_{35}\,M_5)[q_{01}(q_{13} + q_{15}(q_{53} + q_{58}\,q_{83})) + q_{03}]\}[1 - q_{35}(q_{53} + q_{58}\,q_{83})](M_2 + q_{24}\,M_4)[q_{01}(q_{12} + q_{14}(q_{46}\,q_{62} + q_{47}\,q_{72}) + q_{15}\,q_{57}\,q_{72}) + q_{02}\} + (M_2 + q_{24}\,M_4)[q_{01}(q_{13} + q_{15}(q_{53} + q_{58}\,q_{83})) + q_{03}]\,q_{35}\,q_{57}\,q_{72}$$

$$D_B = \{1 - (q_{01}\,q_{10})[1 - q_{35}(q_{53} + q_{58}\,q_{83})] - q_{03}\,q_{30} - [q_{01}(q_{13} + q_{15}(q_{53} + q_{58}\,q_{83}))]\,q_{30}\} \times \{1 - q_{24}(q_{42}\,q_{46}\,q_{62} + q_{47}\,q_{72})\}$$

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