THREE VARIABLES NOR/NAND LOGIC WHEN COMPLEMENTED LITERALS ARE AVAILABLE

A Dissertation
submitted in partial fulfilment
of the requirements for the Degree
of

MASTER OF ENGINEERING

in

ELECTRONICS & COMMUNICATION ENGINEERING
(Applied Electronics & Servomechanism)

By

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This is to further certify that he has worked for a period of seven months from December, 1965 to July 1966 for preparing this thesis for Master of Engineering at the University.

July 30 , 1966

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CONTENTS

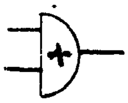
SYNOPSIS

NOR and NAND logics are being preferred in computers for their various advantages. The minimal implementation of these has long been the goal of logic designers.

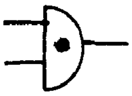
Here an algebraic method of implementation is given, which is very simple to work out.

Also, the catalog of networks for three variable functions (complemented literals being available) is given and the equivalence table is worked out, the various tables which inter-relate the min terms , octal numbers, leader, AND-OR form, NOR form, NAND form etc are given.

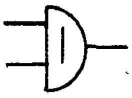
SYMBOLS USED



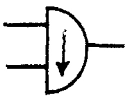
OR Logic



AND Logic



NAND Logic



NOR Logic

INVERTER

A AND B	AB
A OR B	A + B
NOT A Complement of A	$\bar{A}$
A Exclusive OR B or, A ring sum B	A + B
A NOR B	$A \downarrow B$
A NAND B	A|B

CHAPTER I

INTRODUCTION

NOR and NAND logics are extensively used in the arithmetic unit as well as the control unit of the computer. NOR and NAND logics are preferred to other logics like AND OR and NOT in the mass production of Digital Computers for reasons of manufacturing ease and economy.

For lowering the cost and size, it is necessary that the number of gates or blocks used for any purpose be minimum. This minimality is not only from the point of view of gates, but also the inputs and the number of levels used.

This minimal implementation has long been the goal of logic designers. Quite some work has been done to work out the implementation by a regular process, rather then an hit and trial method. John Earle and Maley(1) have given the transform method and the map method of implementation. They have also given the circuits, 78 in number, which are minimal in nature for the 256 functions that occur in the three variables.

Figures written in paranthesis denotes the serial number of References given at the end.

Lee Hollerman[5] of IBM Corporation found the same circuits by feeding the data into a computer. R.A. Smith[2] did the same work by feeding into a computer again , but he assumed that both the complemented and uncomplemented literals were available (while Hollerman's work is only on Uncomplemented literals). Hollerman got 70 circuits and Smith obtained 10 .

Hollerman and Smith got their minimal circuits by feeding the data into the computer, but all these circuits can be got in a simpler way too, i.e., by conversion from AND OR to NOR or NAND . The given function is first simplified on the V-K[4] map, and the simplified function is got in AND - OR (for NAND) or OR AND (for NOR) and the conversion is then from AND-OR logics to NOR and NAND.

In the VII Chapter this method of getting the minimal circuits is proved for each function and the circuits against which they were tallied were the Smith's circuits. Here 10 circuits are used instead of the 10 proposed by Smith. This work is only valid when uncomplemented variables are available.

This method of getting minimal circuits is likely to hold good for the 4 and higher variables.

By changing the input literals the 256 functions in three variables boil down to 70 functions. The table which gives the 70 functions and the permutations to be used to get all the 256 functions is called equivalence table and is given in the Appendix (Table III).

The relation between function number, [(1,2,...256) as used by Earle and Maley] Min terms and Octal numbers is also given in the Appendix (Table II).

Also the transform and map method of implementation has been looked into (Chapter II and III) and the Hellerman's and Smith's approach to the problem of minimality is given in Chapter IV and V.

1.1. NOR and NAND LOGICS

A. NOR (Dagger) Function: $\downarrow$

For two variables, it is completely defined by the truth table given below:

A	B	$A \downarrow B$
0	0	1
0	1	0
1	0	0
1	1	0

Symbolic representation of NOR Gate having three inputs A, B and C is shown in Fig. 1.1(a) also are shown the Boolean Equivalents in, NOR is equivalent to OR followed by an Inverter or Inverters followed by AND.

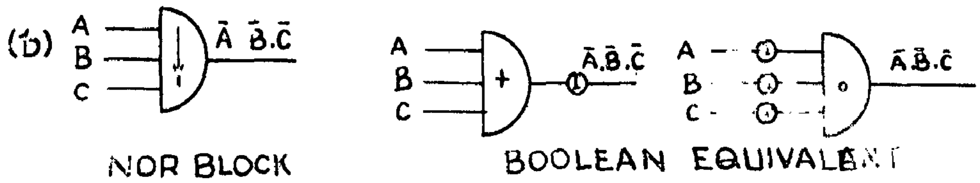


FIG. 1-1

In a logical net work this function is a black box whose output is 1, if and only if all the inputs are 0.

Boolean representation of n variables

$$\overline{A_1+A_2+\ldots\ldots A_n} = \bar{A}_1 . \bar{A}_2 . \bar{A}_3 . \ldots\ldots \bar{A}_n$$

B. <u>NAND (Stroke) Function:</u>

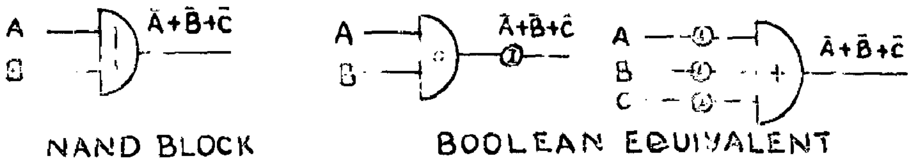
For 2 variables, it is completely defined by the truth table given below :

A	B	A|B
0	0	1
0	1	1
1	0	1
1	1	0

Symbolic representation NAND gate having three inputs A, B & C is shown in Fig. 1.1(b) also are shown the Boolean equivalents i.e. NAND is equivalent to AND followed by inverter or inverters followed by OR.

In a logical net work this function is a black box, whose output is 1, whenever at least one of the inputs is 0.

(c)



Boolean representation is shown below for N variables.

$$\overline{A_1 \cdot A_2 \cdots\cdots A_n} = \bar{A}_1 + \bar{A}_2 + \cdots\cdots \bar{A}_n$$

1.2 PRACTICAL ADVANTAGES OF NOR & NAND LOGICS

These logics are more widely used for the computer i.e., transistor circuit family of NOR and NAND, for they give the following advantages :

1. Each is a universal block, and sets of them can be used to generate any Boolean function.
2. Each block has voltage and current gain.
3. Each block can drive several others directly.
4. The number of inputs can be increased by just connecting the common collectors.
5. They have considerable logic power.(as shall be shown later on. one block can work both as AND and OR).

6. Circuits in various speeds and costs , are available to generate any given function.

7. Testing,repairing, etc., is very simple, for only one type of testing equipment is necessary.

8. For mass production, getting one type of function in integrated circuitry is no problem.

For these and other economic and simplicity reasons, these logics are preferred in computers.

CHAPTER II

TRANSFORM METHOD

Given any Boolean function how do we implement it using NOR or NAND logics and implemented in a way to use minimum number of blocks; inputs and levels.

The first method to be dealt will be transform method, given by Maley and Earle[1].

This transform method makes the logic design with these logic elements (NOR & NAND) as easy as with OR, AND & NOT . In the initial stages the work is same as used for OR , AND & NOT , and when putting it in NAND (or NOR) form, the transform rules are applied. However, to get the minimality certain contraints are placed.

From the figures 2,1 and 2,2 it is seen that NAND is equivalent to AND -OR , and its dual NOR to OR-AND . A three stage NAND will be equivalent to OR - AND - OR while its dual NOR to AND -OR-AND Also when using the transform rules, the variables

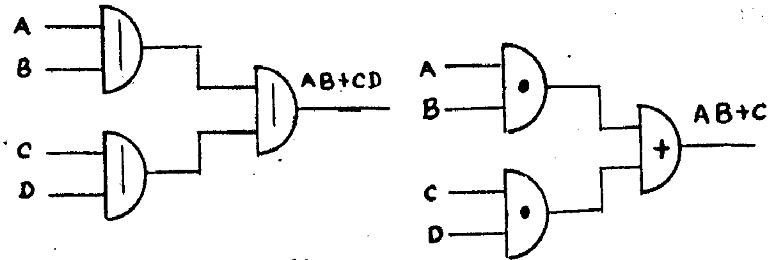


FIG- 2-1

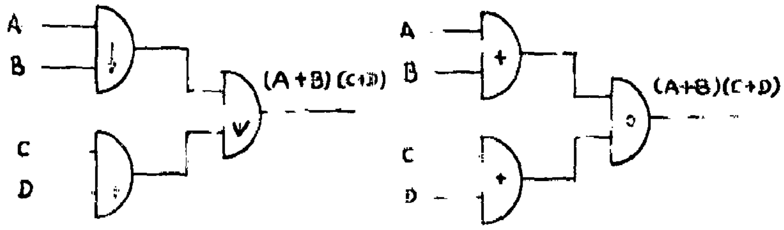


FIG. 2-2

entering the odd levels are complemented. (here the levels are counted from once as we go inwards) (

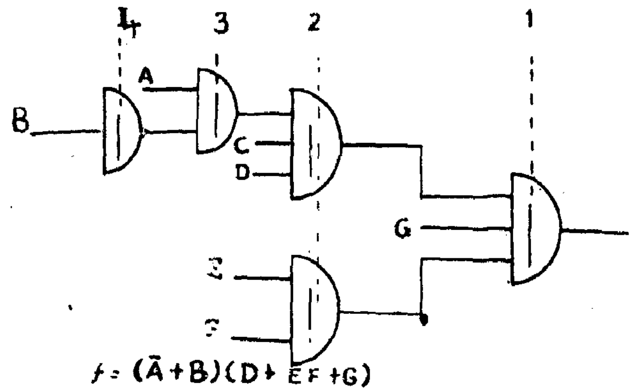


FIG. 2-3

2.2 INVERSE TRANSFORM USING 1.0. FROM NAND / NOR NETWORK TO BOOLEAN FUNCTION :

Now given any NAND or NOR network, how to bring it to the BOOLEAN form ? The following simple rules will answer this question :

a. For NAND Networks - Rules :

1. Write an '+' inside every even block and . for odd block ignoring all inverters (i.e. single input N blocks). If a gate appears as both odd and even, then leave it blank; and all the gates preceding it also as blank.

2. Transform all +'s to OR, and '.'s to AND for the blank gates either way is possible a) split them into two gates, AND driving odd levels, and OR

driving even level leads, b) OR; Treat the blank as if it is driving only odd levels, transform the AND's and insert an inverter for output driving even leads.

3. Complement all variables entering odd levels and any inverter to even levels is same.

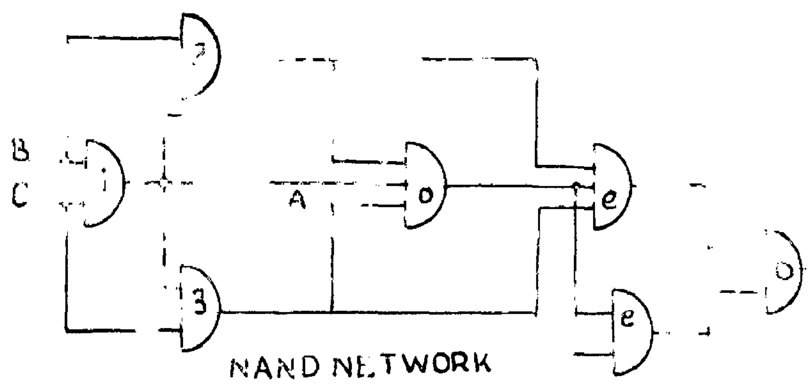
INVERSE TRANSFORM FOR NOR NETWORK

1. Same as Rule 1 for NAND.

2. Here transform all o's to OR's and e's to AND's Transform all blank gates as a) OR - driving odd levels, and AND driving even leads, b) OR , treat the gates as if they drove only odd levels, transform them to OR and for even put an inverter before it.

3. Complement all variables entering odd level or AND's and any inverter to e level or OR remain same.

As one example we shall take an NAND network and find its Boolean Equivalent.



NAND NETWORK
FIG. 2-4 (a)

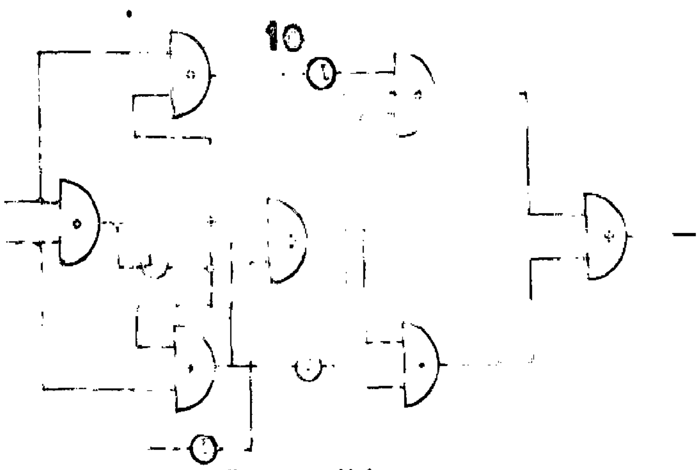


FIG. 2-4(b)

Now for the Boolean equivalent of the network in Fig. 2.4(b) we apply the rules given above i.e. put O's and O's and number the blank gates.

Now convert O's to OR's and O's to AND's and about the blank gates, here we shall put them as AND (driving odd gates) and invert the even inputs

6 Complement the variables entering odd levels.

Hence our Boolean equivalent will look as shown in Fig. 2.4 b.

To get the function we shall follow this circuit and the function, for the above case works out as

$$f = (\bar{A}+B+C)\,(A+BC+\overline{BC})$$

On the same line, but following the rules given for it, the NOR network can be broken down to Boolean form.

2.3 TRANSFORM RULES (From Boolean Function to NAND or NOR)

1 a) NAND : Factor the Boolean equation such that the output is an OR (i.e. get the equation in the form of OR-AND -OR form) Try and get complemented variables on odd levels uncomplemented on even levels.

b) NOR - It is the same rule as above except that output is AND (i.e. the form of equation is AND-OR-AND)

2 Lay out the gating from equations as if we were implementing AND-OR logic the only difference here is that variables at odd levels must be complemented.

2.4 SOME OF THE TRICKS THAT ARE USEFUL TO HELP OUT THE TRANSFORMATION:

To get the equation in the proper form, (i.e. AND-OR-AND for NAND and OR-AND-OR for NOR) or to reduce the number of blocks necessary , or seperating the uncomplemented variables on to even levels and complemented on odd levels, or even for using the output of previous blocks, there are certain tricks used to reduce the work. They are all listed below, with examples : With the help of transform tricks and rules we can get a network (minimal) for any Boolean Equation.

1) Partial Multiplication:

Example : $(\bar{A}+\bar{B})(C+D) = (\bar{A}+\bar{B})C+(\bar{A}+\bar{B})D$

This trick as can be seen easily separates complemented variables on odd levels and uncomplemented to the even ones.

2) Add or multiply a constant :

Example : $(A+B)(C+D) + 0$

or $(AB+CD).1$

If the equation is got in the wrong form, then this trick is used to get the right form, i.e., if to implement NOR , we get the equation as in form 2 i.e., AB+CD , as by transform rules we know that for NOR the equation should be in the form of AND-OR-AND hence by multiplying by 1 , we get the desired effect.

3) Associativity :

Example $D\left[\bar{A}+\bar{B}+C\right] = D\left[(\bar{A}+\bar{B})+C\right]$

This separates the complemented variable from uncomplemented ones, of course this is done to bring them onto the even and odd levels.

4) Bundling

This trick is quite useful if both the outputs of one block are needed. Here if the output of the block is say f, and f and $\bar{f}$ both are needed $\bar{f}$ is supplied not by using an inverter but by taking the inputs to the block (that produce f), OR letting the complement of a signal be represented by a bundle of wires which are inputs to the block. The advantage of this is that it eliminates gates (Note: this can be used for NAND S only)

5) Distributive Law:

Example $AB + \overline{CD}$: $(AB+\bar{C})(AB+\bar{D})$

This is used to seperate variables to proper levels and to factor out common terms.

6) Adding Redundant terms: Like $B\bar{B}$, $A\bar{A}$ ...

a) This is used to share gates or make gates identical; eliminate them or even to get the variables in proper levels.

b) To make factoring possible (Note: This is what causes the blank gates).

7) Getting a complement from Complex Form

$f_1 = \bar{A}BC$ $\qquad f_2 = A(B+D)$ $\qquad f_3 = A(\overline{B+D}).B.C$

CHAPTER IX

IMPLEMENTATION USING MAP METHOD

9.1 GENERAL

In this Chapter the map method of implementation is reviewed, which was proposed by Maley and Earle[1].

This method gives very good results for few variables (upto 4) and using this the full logic power of NOR and NAND is used, which is not the case in Karnaugh tricks. Maley and Earle had used for their factoring purposes Karnaugh map.[6]

9.2. Principle Used:

Here we make use of inhibition i.e. completing the higher order block with some smaller order block which was not to be used in completion.

The example below will explain clearly what we mean by inhibition.

The function given in Fig. 9.1 can be written as B_{AND} NOT ABC or loop C(AND NOT ABC

or $B(\overline{ABC}) + C(\overline{ABC})$

and using the transform tricks we get the network as shown in Fig. 3.2 , which requires four blocks. While any ordinary method of simplification (it simplifies to $B\bar{C}+C(\bar{A}+\bar{B})$) will require 5 blocks. This is because of Redundancy being used in the map method.

FIG. 3-1

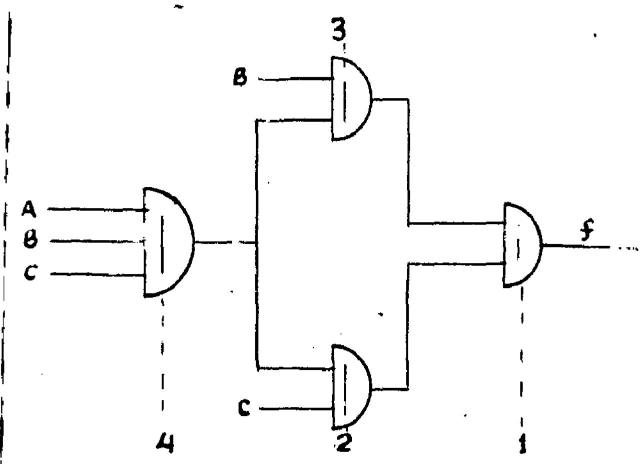


FIG. 3-2

Now we shall approach loops (as through any required blocks that come in even level that on odd levels as zero's (last loop must be of zeros). i.e., the problem could have been worked out this way.

Take loop $\overline{AB}$ (gate 1) , take loop C inhibit ABC from this (gate 2), take loop B and inhibit ABC (gate 3) and since this gives all ones, inhibit these from unity giving all zeros .

As can be seen that only uncomplemented literal loops can be used in this case or else we will have to use inverters, which increases the number of blocks.

In case of NOR , the reverse is true i.e, the permissible loops are only complemented literal loops, for here the uncomplemented ones will have to be put with an inverter.

3.3. The permissible loops for 3 and 4 variables are given , both for NAND and NOR, and the difference can be seen. (Fig. 3.3 and 3.4)

3.4. <u>Method of Map Factoring</u> - Rules

1. Restricting to permissible loops, loop a selection of one's or zeros or both.

2. Using loops already cho sen as inhibitions, try to cover remaining ones or/ zeros.

3. Repeat the above steps till all ones and zeros are covered. For Nand last level of loops must be zeros, (must be an OR) for NOR last loop shiuld be ones (AND).

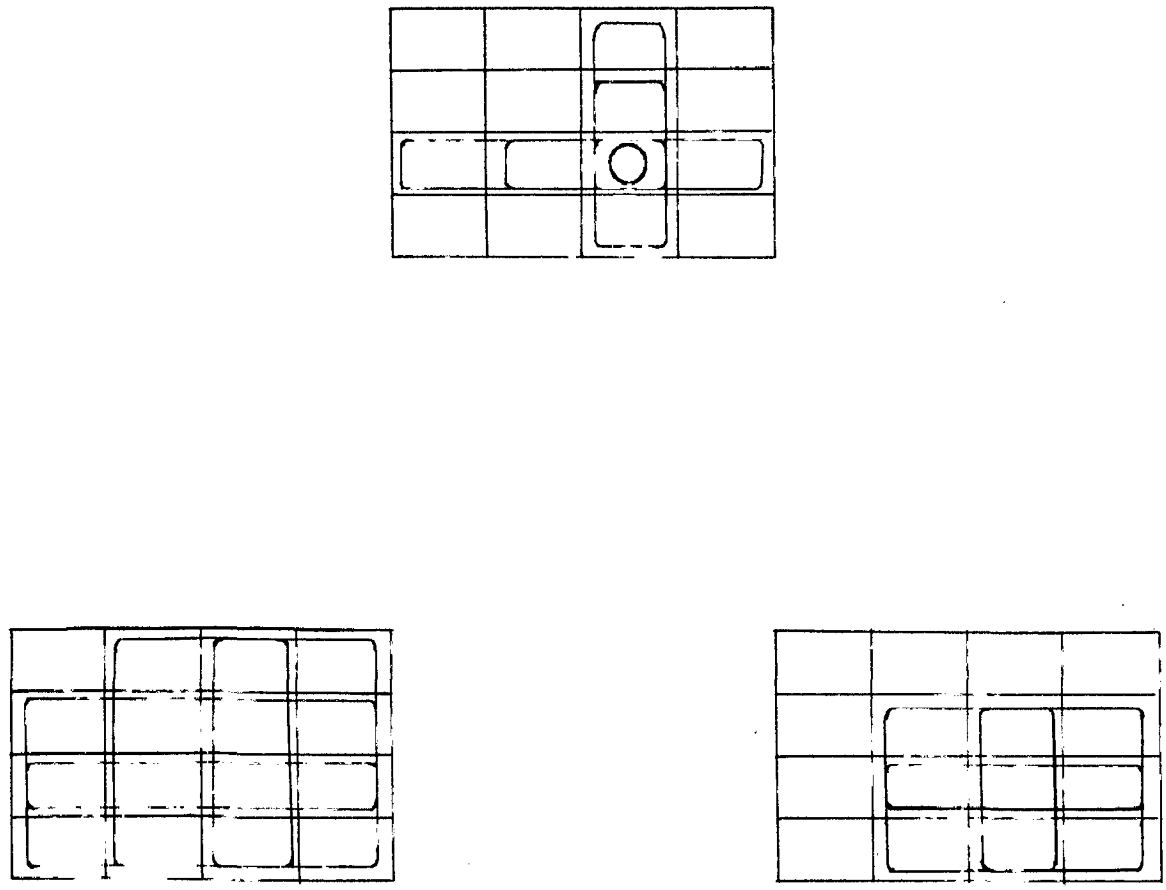
Example : An example is dealt here just to illustrate the principle.

$$f = A\overline{C} + \overline{A}BC + \overline{BC}$$

This is implemented using NA ND blocks. (Refer Fig. 3.5 and 3.6)

Similarly the function could be implemented by NOR logics, we should start in this case with a loop of ones, so that we end up with a loop of ones.

3 Variables

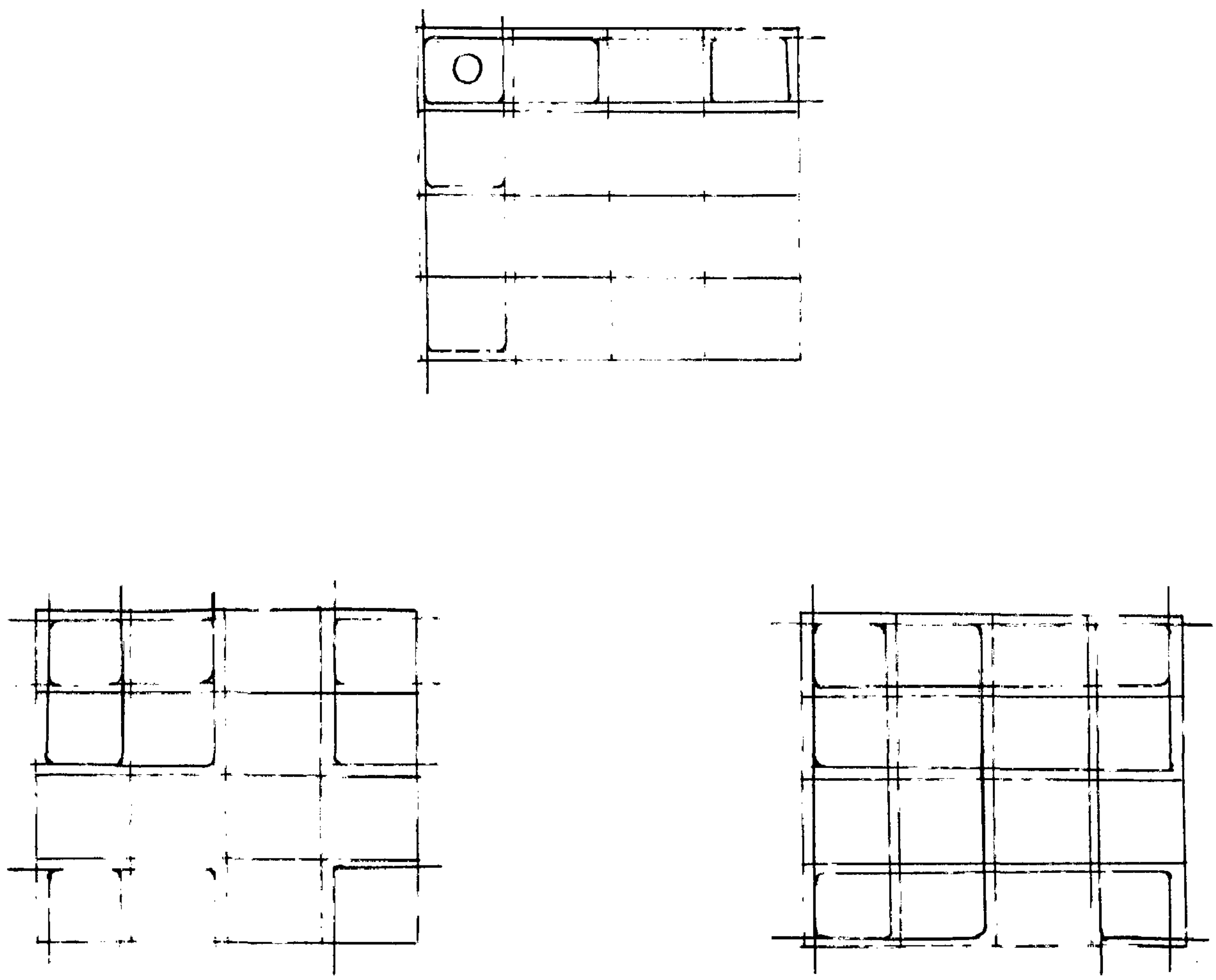


4 Variables

Permissible loops for NAND Logics

FIG. 3-3

3 Variables



4 Variables

Permissible loops for Logics

FIG. 3-4

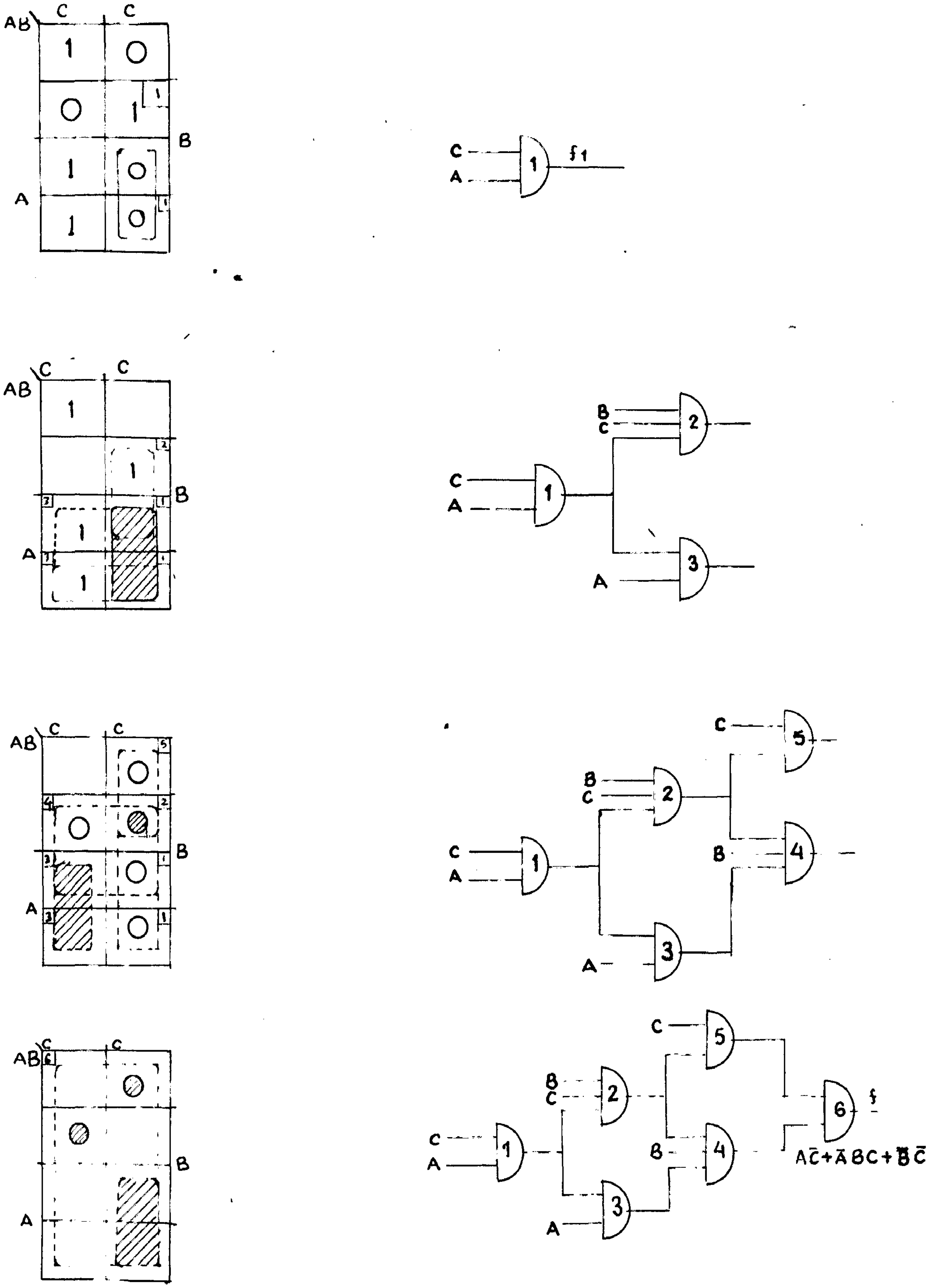


FIG. 3-5

3.5. LOGIC POWER OF NOR AND NAND

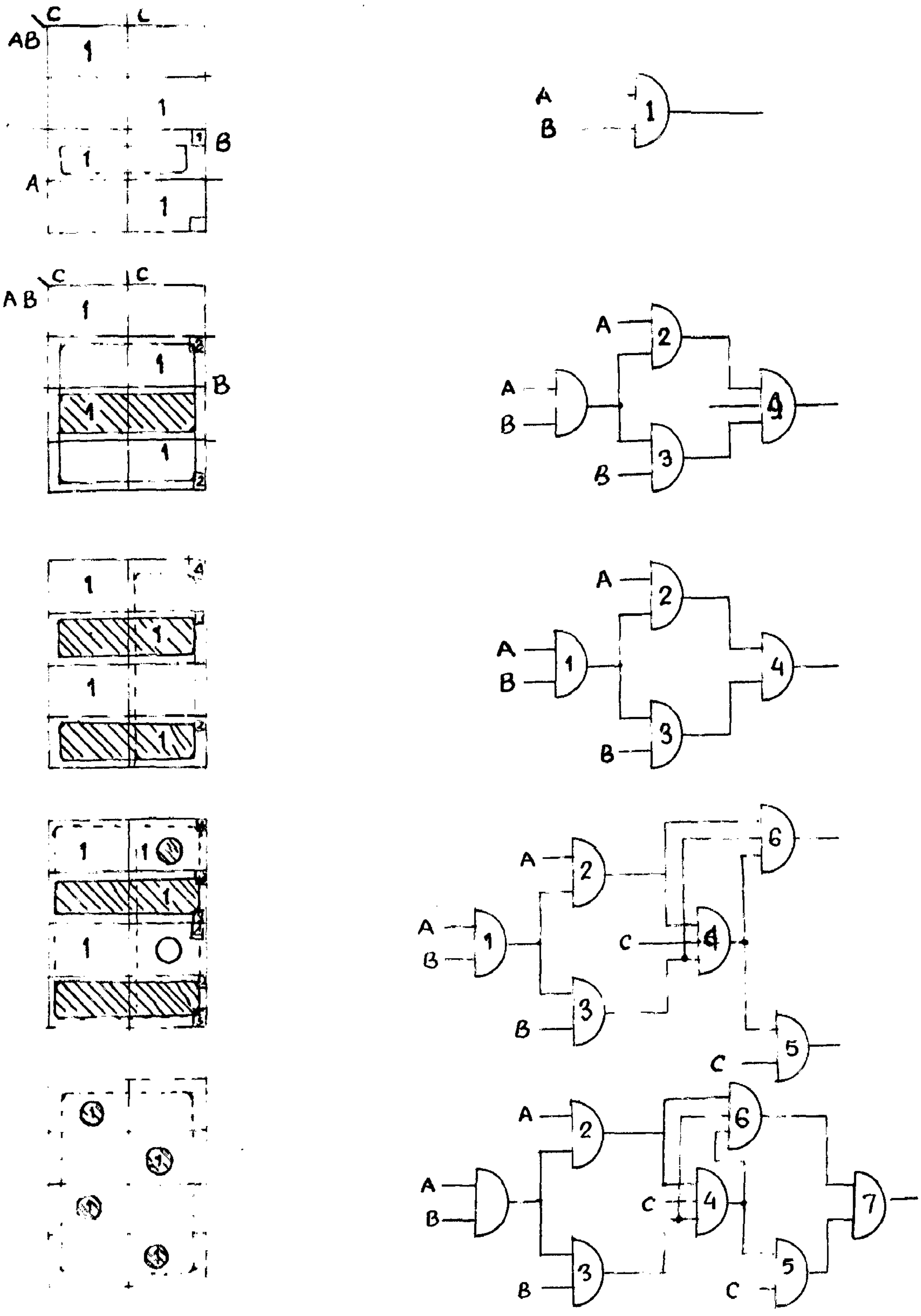
Now we are in a position to exploit the logic power of NAND or NOR . This power is that a single NAND block (or NOR) can act as both AND and OR if it drives both an even and odd levels.

In the previous chapter we went into blank gates, this is where the full logic power of NAND (or NOR) is used.

It is easy to argue out that to be effective in both an even level and odd level, a gate must partake both ones and zeros of function. Indeed blank gate corresponds in the way to a loop which has both ones and zeros. Of course this cannot enter the actual function but it acts as an inhibitor in a resultant loops of zeros and ones.

For picking up initial loops, a good thumb rule is to choose them not from point of view of looping ones and zeros, but for facilitating the generation of loops that will cover ones and zeros.

A simple example will again be dealt here (Fig. 3.7) Here blocks 2 & 3 are blank gates or they are feeding both even and odd levels and hence using full power of their logic.



$f = \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}C + AB\bar{C} = A \oplus B \oplus C$

FIG. 3-7

CHAPTER IV

HELLERMAN'S CATALOG OF THREE VARIABLES

NOR , NAND LOGICS

4.1. Leo Hellerman of IBM has given a catalog which contains the circuits for all the 256 functions which are there in three variable case. His circuits are all minimal, minimal in the sense 1) that the logic blocks required to generate the function are minimum 2) the number of inputs must be minimum, (even total number of inputs must be minimum) , also the circuits satisfy the conditions of fan in and fan out.

Now how do we know whether a circuit is minimal or not? Before Hellerman and Smith set about giving the minimal circuits there were no circuits which were proved to be minimum. Once the table for minimum circuits was formed using a computer, the algebraic method of getting circuits could be formed, (as has been proposed by this thesis).

Before Hellerman, Maley and Earle had given out minimal networks using NAND blocks and they used the map method (described in previous Chapter) to get it. There have been other people who have done work on minimal networks, like Hubert[5] whose minimal criterion was levels, ii. Hellerman whose minimal criterion is cost.

4.2. METHOD USED BY HELLERMAN

This method was exhaustive, i.e., he looked at all the possible combinational networks of NOR blocks, and for each three variable logic function the desired minimum was saved.

This full work of forming functions and comparing them was done by IBM 7090. The input data to this were the 256 logic functions, and the procedure was as follows : Only one block was considered at a time and the various combinations of inputs that can be fed were used and the functions generated.

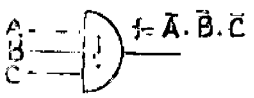


Fig. 4·1

Obviously for the function generated this was the minimum circuit, hence this function was crossed from the list.

Next considering all networks with two blocks in the same way the functions generated were compared with the list. If the function was on the list, the circuit was saved and the function crossed off, if the function had already been crossed off, then the network was crossed off. Then 3, 4 ...... upto 7 blocks were taken which are necessary to get all the functions of three variables. This would give us circuits which are minimal from the point of view of blocks.

Next for the connectives, the universal network as shown in Fig. 3.2 was used. Arrows indicate the output and inputs. There is a possible path from each input to each block and from each block to other. For notation also ij was used, i.e., i was the block being fed and j was the input that was feeding it

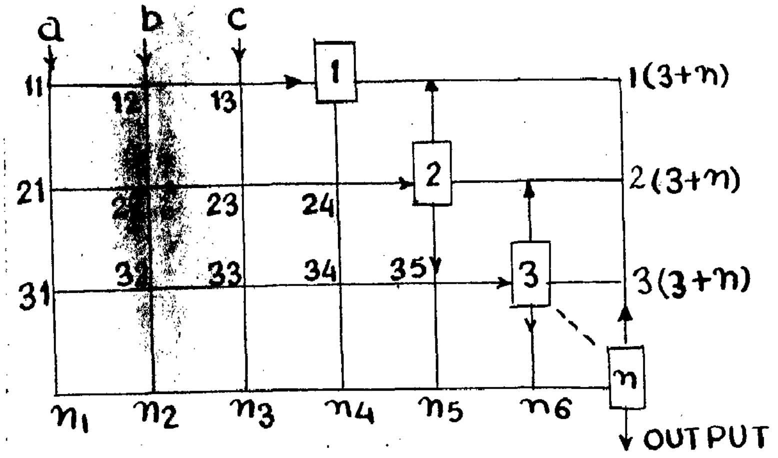
$$ij \begin{cases} = 1 & \text{if } ij \text{ is a connection point} \\ = 0 & \text{if } ij \text{ is not a connection point} \end{cases}$$

This universal network could also be put in the matrix form as shown below :

$$\begin{bmatrix} 11 & 12 & 13 & x & 15 & \ldots\ 1(5+n) \\ 21 & 22 & 23 & 24 & x & \ldots\ldots\ 2(5+n) \\ - & - & - & - & - & ------ \\ n1 & n2 & n3 & (n4)(n4) & (n_5\ ) & x \end{bmatrix}$$

For his catalog he was dealing with single output circuit and in the combinational circuits no feedback was allowed. Hence the universal network reduced to one shown in Fig. 4.3, and the matrix for this also reduces to the one shown below

$$\begin{bmatrix} 11 & 12 & 13 & & \\ 21 & 22 & 23 & 24 & \\ - & - & - & - & - \\ n1 & n2 & n3 & n4 & ------\ n(n+2) \end{bmatrix}$$



UNIVERSAL NETWORK
FIG. 4-2

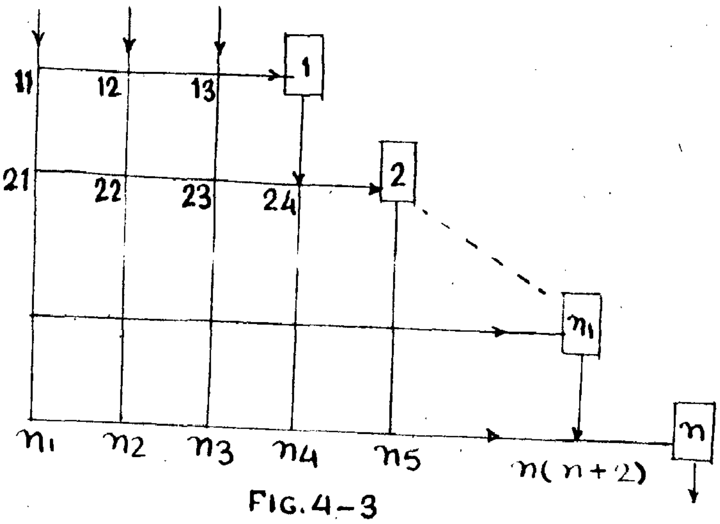


FIG. 4-3

As said earlier the number of blocks used to generate three variable functions is 7, hence the matrix is as follows :

11	12	13						
21	22	23	24					
31	32	33	34	35				
41	42	43	44	45	46			
51	52	53	54	55	56	57		
61	62	63	64	65	66	67	68	
71	72	73	74	75	76	77	78	79

The total number of elements in this matrix are 42 and each element can take the value of 1 or 0 hence the total number of combinational networks are $2^{42} = 4.5 \times 10^{12}$.

In the above matrix, the first three columns specify the connections of the inputs and the rest of the interconnections of the blocks.

4.9. EVALUATION OF EQUIVALENT FUNCTION CIRCUITS

It is well known that the 256 logical functions of three variables can be partitioned into 78 equivalence classes.

Two functions are equivalent and belong to same class if, and only if, one can be obtained from the other by a permutation of input variables. Similar is the equivalence of input line permutation where the input lines are juggled. (The equivalence table has been dealt exhaustively in Chapter VI of this thesis).

Now since implementation of any member of function class serves to implement all the functions in its class, hence it was necessary only to evaluate one network from equivalence class of networks.

For this a simple rule was set up. The columns of the matrix written as a row is

$$(1j)(2j)\ \ldots\ldots(nj) \qquad j = 1,2,3,\ldots\ldots$$

and each one i.e., 1j , 2j etc, is either 1 or 0, depending on the connections. Hence this row is designated by a binary number and is referred to as the magnitude of column j.

Now the simple rule as stated by Hellerman is that

$D_1 \leq D_2 \leq D_3$ . . . to choose a proper network of the class. Also he eliminated some of impossible cases as,

1) Since $D_1 \leq D_2 \leq D_3$ and since the first block must be fed by atleast one input, hence D_1 can never be zero.

2) Block (n-1) always feeds 'n', hence n(n-2) is never equal to zero.

3) Restrictions on fan in and fan out were such that no row or column may contain more than 3 1's.

4) Also each block must be fed, either from input or from some other block, and every block must feed some other block excluding the last one, hence there must be atleast one 1 in any row or column.

5) Any logical expression fed to the lastblock cannot feed any other block.

The catalog that is given by Hellerman contains the equivalence table and table containing octal numbers, simplified functions, circuit number, the number of transistors used. The circuits can be looked up in IEEE June 1963 EC-12, Page 203 and onwards.

CHAPTER V

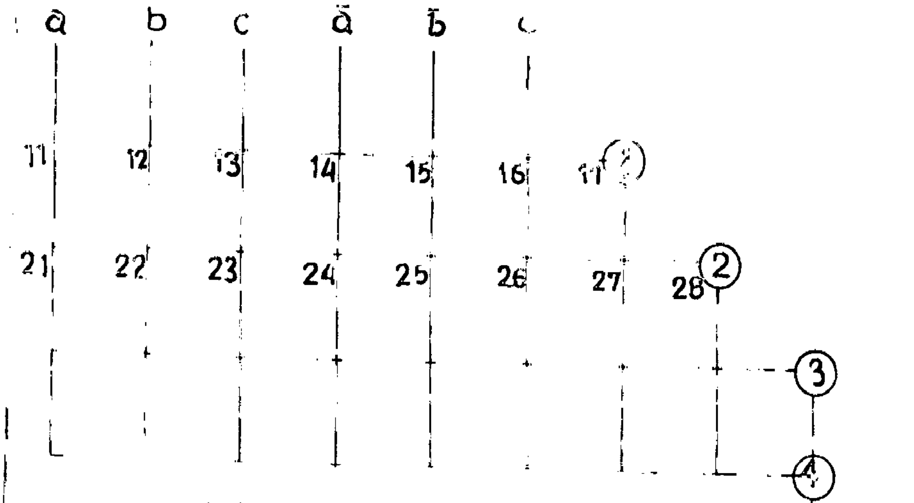
SMITH'S CATALOG OF MINIMAL NOR - NAND LOGICS WITH COMPLEMENTED LITERALS AVAILABLE

5.1. Hellerman gave out his catalog and circuits assuming that only uncomplemented literals were available. R.A. Smith in his thesis for Master's degree has extended the work of Hellerman, in that he has assumed that complemented literals are available.

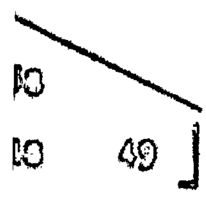
The criterion on which Smith based his minimality is same, i.e. minimum with respect to the number of blocks and the connections.

The method of approach was also the same, but in Smith's case the universal circuit was a bit modified and is as shown in Fig. 5.1.

universal



UNIVERSAL CIRCUIT
FIG. 5-1



This matrix has 30 elements and each element can take on a value between 0 and 1 , hence the possible connection matrixes will be $2^{30} = 10^9$

The program started by generating as many functions as possible from a single NAND block, then the network, the inputs and functions were stored in memory. Then all possible configurations of two NAND elements were looked into, and the functions generated. If the function had been generated earlier with fewer blocks or inputs this was discarded, if not it was retained and so on upto 4 blocks.

After Smith had placed some restrictions on the matrix, the number of circuit configurations was reduced to 5×10^6 .

5.2. RESULTS OF SMITH'S WORK

Not all of the circuits were found for 256 functions, some functions had to be generated using 5 blocks, hence they are minimal from blocks point of view, but may not be so from inputs point of view.

After getting the circuit configuration using NANDS the NOR circuits were directly derived from them and they were minimal as given by the following lemma.

Lemma :

Let $F(x_1, x_2 \ldots\ldots x_n)$ be the logical function performed by NOR function and let $G(x_1, x_2 \ldots\ldots\ldots x_n)$ be the logical function performed by the same circuit but with all NOR blocks replaced by NAND blocks.

Then $F(x_1, x_2 \ldots\ldots x_n) = G'(x'_1, x'_2, x'_3 \ldots\ldots x'_n)$

and $G(x_1, x_2 \ldots\ldots x_n) = F'(x'_1, x'_2, x'_3 \ldots\ldots x'_n)$

Proof: If $S(x_1 \ldots\ldots\ldots x_n)$ is the NOR Sum of $(x_1, x_2 \ldots x_n)$ and if $C(x_1 \ldots\ldots x_n)$ is the NAND, then

$$S(x_1, x_2 \ldots\ldots x_n) = \overline{(x_1 + x_2 + \ldots\ldots x_n)} = \bar{x}_1 . \bar{x}_2 \ldots\ldots\ldots \bar{x}_n$$

$$C(x_1 \ldots\ldots\ldots x_n) = \overline{(x_1 \ldots\ldots x_n)} = \bar{x}_1 + \bar{x}_2 \ldots\ldots \bar{x}_n$$

$$\therefore S(x_1 x_2 \ldots\ldots x_n) = C'(x'_1 \ldots\ldots x'_n)$$

and $C(x_1 \ldots\ldots x_n) = S'(x'_1 \ldots\ldots x'_n)$

While Maley and Earle had listed their functions in min terms form and using function no's (1, 2, ...256). Hellerman and Smith used octal numbers (they shall be dealt in next Chapter) and they have listed the circuits for 78 functions, rest of the 256 could be got using the equivalence table.

CHAPTER VI

EQUIVALENCE TABLE.

In this Chapter the method of getting the equivalence table in octal numbers is given. In the next Chapter, after getting the 78 functions that are leaders, they shall be implemented assuming that complemented variables are available, this implementation will be done by a very simple process i.e. using NAND & NOR implementation.

6.1 As has been said earlier that the 256 logical function (in 3 variables) can be partitioned in 78 equivalent classes.

Definition of Equivalence:

Two functions are said to be equivalent if and only if one function can be obtained from other by permutation of inputs.

Now what is the relation between the min terms if inputs are changed, i.e. if we have min terms $\sum 0,1,4,6$ what will be the equivalent if we change the inputs to BAC, the example below is shown for that purpose.

$$f_{ABC} = \sum 0, 1, 4, 6$$

$$= \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + AB\overline{C}$$

If we now change the x inputs to BAC, i.e., we interchange B and A we get a function

$$f' = \overline{B}\,\overline{A}\,\overline{C} + \overline{B}\,\overline{A}C + B\overline{A}\,\overline{C} + BA\overline{C}$$

$$= \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + AB\overline{C}$$

$$f' = \sum_{ABC} 0, 1, 2, 6$$

∴ It shows that there is no need to have two circuits to obtain the function f and f'. The circuit which gives f, by the input as ABC, will give f', if we change the input to BAC.

<u>Example : 2.</u>

$$f_{ABC} = \sum 0, 1, 2, 7$$

$$= \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + ABC$$

Change to BAC

$$f' = \overline{B}\,\overline{A}\,\overline{C} + \overline{B}\,\overline{A}C + \overline{B}A\overline{C} + BAC$$

$$= \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + ABC$$

$$f' = \sum 0, 1, 4, 7$$

To get the relation between min terms when input literals are changed we form the tables given on page 29 and 30.

	ABC	BAC				
0	$\overline{ABC}$	$\overline{BAC}$	=	$\overline{ABC}$	=	0
1	$\overline{AB}C$	$\overline{BA}C$	=	$\overline{AB}C$	=	1
2	$\bar{A}B\bar{C}$	$\bar{B}A\bar{C}$	=	$A\overline{BC}$	=	4
3	$\bar{A}BC$	$\bar{B}AC$	=	$A\bar{B}C$	=	5
4	$A\overline{BC}$	$B\overline{AC}$	=	$\bar{A}B\bar{C}$	=	2
5	$A\bar{B}C$	$B\bar{A}C$	=	$\bar{A}BC$	=	3
6	$AB\bar{C}$	$B\overline{AC}$	=	$AB\bar{C}$	=	6
7	ABC	BAC	=	ABC	=	7

	ABC	ACB		
0	$\overline{ABC}$	$\overline{ACB}$	$\overline{ABC}$	0
1	$\overline{AB}C$	$\overline{AC}B$	$\bar{A}B\bar{C}$	2
2	$\bar{A}B\bar{C}$	$\bar{A}C\bar{B}$	$\overline{AB}C$	1
3	$\bar{A}BC$	$\bar{A}CB$	$\bar{A}BC$	3
4	$A\overline{BC}$	$A\overline{CB}$	$A\overline{BC}$	4
5	$A\bar{B}C$	$A\bar{C}B$	$AB\bar{C}$	6
6	$AB\bar{C}$	$AC\bar{B}$	$A\bar{B}C$	5
7	ABC	ACB	ABC	7

	ADC	DCA		
0	$\overline{ADC}$	$\overline{DCA}$	$\overline{ADC}$	0
1	$\bar{A}\bar{D}C$	$\bar{D}\bar{C}A$	$A\bar{D}\bar{C}$	4
2	$\bar{A}D\bar{C}$	$\bar{D}C\bar{A}$	$\bar{A}\bar{D}C$	1
3	$\bar{A}DC$	$\bar{D}CA$	$A\bar{D}C$	5
4	$A\bar{D}\bar{C}$	$D\bar{C}\bar{A}$	$\bar{A}D\bar{C}$	2
5	$A\bar{D}C$	$D\bar{C}A$	$AD\bar{C}$	6
6	$AD\bar{C}$	$DC\bar{A}$	$\bar{A}DC$	3
7	ADC	DCA	ADC	7

	ADC	CAD		
0	$\overline{ADC}$	$\overline{CAD}$	$\overline{ADC}$	0
1	$\bar{A}\bar{D}C$	$C\bar{A}\bar{D}$	$\bar{A}D\bar{C}$	2
2	$\bar{A}D\bar{C}$	$\bar{C}\bar{A}D$	$A\bar{D}\bar{C}$	4
3	$\bar{A}DC$	$C\bar{A}D$	$AD\bar{C}$	6
4	$A\bar{D}\bar{C}$	$\bar{C}A\bar{D}$	$\bar{A}\bar{D}C$	1
5	$A\bar{D}C$	$CA\bar{D}$	$\bar{A}DC$	3
6	$AD\bar{C}$	$\bar{C}AD$	$A\bar{D}C$	5
7	ADC	CAD	ADC	7

	ADC	CDA		
0	$\overline{ADC}$	$\overline{CDA}$	$\overline{ADC}$	0
1	$\bar{A}\bar{D}C$	$\bar{C}\bar{D}A$	$A\bar{D}\bar{C}$	4
2	$\bar{A}D\bar{C}$	$\bar{C}D\bar{A}$	$\bar{A}D\bar{C}$	2
3	$\bar{A}DC$	$\bar{C}DA$	$AD\bar{C}$	6
4	$A\bar{D}\bar{C}$	$C\bar{D}\bar{A}$	$\bar{A}\bar{D}C$	1
5	$A\bar{D}C$	$C\bar{D}A$	$A\bar{D}C$	5
6	$AD\bar{C}$	$CD\bar{A}$	$\bar{A}DC$	3
7	ADC	CDA	ADC	7

Now the above tables can be consolidated into one table which gives the equivalent min terms when input permutations are made.

S.No.	ABC	ACB	BAC	BCA	CAB	CBA
1	0	0	0	0	0	0
2	1	2	1	4	2	4
3	2	1	4	1	4	2
4	3	3	5	5	6	6
5	4	4	2	2	1	1
6	5	6	3	6	3	5
7	6	5	6	3	5	3
8	7	7	7	7	7	7

Now using this table we form the equivalence Table. For this we first write down all the 256 functions under the column ABC and under the other heads i.e, ACB , BAC .....etc , we write the corresponding min terms (i.e, the equivalent of that given under ABC (Table I)

6.3. <u>CHANGING INTO OCTAL NUMBERS</u>

Once the equivalence table in min terms is formed the equivalence table in octal numbers is formed, now this is done by converting each min term function to octal numbers.

Changeover :

Each min term present is written down as one and the ones not presented as zero.

Here m_i can take any value 1 and 0 depending on whether the corresponding min term is present or not, then we bunch these 1's and 0's in three's (starting from right to left) and each bunch gives us a number hence we get the number for the whole function as three digit number and this is called the octal number for the functional expression

Example $f = \sum 0,1,2,3,6,7$

m_7	m_6	m_5	m_4	m_3	m_2	m_1	m_0
1	1	0	0	1	1	1	1
3			1			7	

Hence octal number for $f = \sum 0,1,2,3,6,7$ is 317

This type of designation has one to one correspondence, i.e, for any function there is only one octal number, while a function can be written in various ways & expressions, of course expressions are easier to group them numbers . The equivalence table of octal numbers is given in Table X.

6.4 Once the table is formed of octal numbers, the leader of each group of class is chosen. The leader is the one which has the minimum octal number.

Now the functions in the equivalence class are given by any row of our table, and the function having the minimum octal number is taken as the leader. These leaders are 70 in number.

The final equivalence table that is given in Table XXX is then formed. Here we used the prefixes P, Q, R, S, T N, Δ O . All these letters represent particular permutations. Like N is no change at all. P is change B to C and C to B while A is same, and soon , the full legend is given with the table.

For each octal number the leader is given and the permutation to be used is given.

Table XX is a table which gives the relations between the function numbers, the min terms, the corresponding octal number, the leader for that octal number and the permutations (inputs) under which the leader is found.

USAGE OF TABLES:

Given any functional expression , the first job is to put it in min term form , having done that the octal number corresponding to this is looked up from Table XX.

Referring to Table XXX we get the vector and the permutation that is to be used. The circuit is looked up from Table XV (given in Last Chapter) and the proper permutation is used. This gives the minimal circuit for the corresponding function.

CHAPTER VII

A NEW APPROACH

Chapter V gave the way in which Smith got his minimal circuits (using computers) assuming that complemented variables are available. Till now no algebraic method is known to get the minimal circuits.

Here a new method has been proposed and proved for the function in three variables case and this method may be true for higher variables too.

METHOD

The method is very simple in application. Given any functional expression (or min terms) we reduce it using V-K map and factoring (if necessary) then we implement it in OR - AND (for NOR) or AND -OR (for NAND) and these blocks are changed to NOR or NAND blocks by the help of inverters.

It should be remembered that NOR is equivalent to OR followed by an inverter or inverters followed by AND and NAND is equivalent to AND followed by inverter or inverters followed by OR and these principles are used for conversion. The diagrams which prove each circuit (given at the end) will help much in understanding

It should however, be noted that when implementing NAND blocks, the ones on V-K map are simplified so as to get sum of products (AND-OR form) while in case of NOR the zeros are simplified (excluding in some cases where 1's are simplified and then converted to AND-OR - AND form).

Below all the 78 leaders of equivalence class are implemented, here the functions have been simplified first and similar simplified functions are grouped together and the circuit is proved for one case and the rest follow, being the same type of functions.

The functions are taken in the following order First terms like A, B etc., are taken, then A +B or B +C .... then A+B+C ... ... and so on till we reach the term like $(A\bar{B}+A\bar{C}+B\bar{C}+ABC)$

First the functional numbers are given, the corresponding min terms, then the simplified form as got after using V-K map and beneath it the inputs to be put, and for any one function under each class the circuit has been proved (these correspond to the minimal circuits as got by R.A. Smith).

One of the circuits that varies is the circuit number 16, the octal numbers under this head are 227, 159, 351, & 256. For these the circuits have been proved with 8 inputs and not 9 (as given by Smith) though the number of levels increase from 2 to 3.

The fan out limit for all the circuits is 8.

Table XV gives the octal number (only leaders of each class) the simplified function in AND - OR form, the simplified form in a NOR form; and the circuit number to be referred to, b) NAND form and the corresponding circuit number.

Hence making use of Table XI, XII, XV and the circuits, any circuit in three variables case could be looked up.

This method of implementation is very likely to hold good for higher variables even, but it should be remembered that this method can be only used if complemented variables are present.

7.2. A CASE OF [illegible]

There are certain functions, which, when the input variables are changed do not change at all i.e., the function generated remains same for any change in input permutations, and these functions have a strange property that their octal numbers are [illegible] like 26, 27 , 350 , 351 etc. There are total 7 pairs like this in the 256 functions and all of them are listed below .

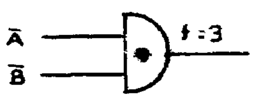
Octal no. 1 & 376 , 200 & 201 , 26 & 27 , 150& 151 226 & 227 , 350 & 351 , 176 & 177.

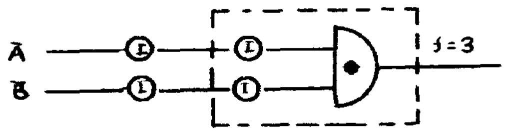
These two properties i.e. occurring inpairs and the function remaining same for all permutations, occurs simultaneously. There is a scope of further investigations in this.

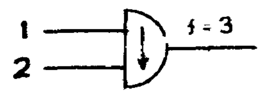
Function number (octal)	17	252
Min terms	(0,1,2,3)	(1,3,5,7)
Simplified Function	$\bar{A}$	C
Inputs	$\bar{A}$	C

Circuit no. 0

Function number (octal)	3	12	210
Min terms	(0,1)	(1,5)	(5,7)
Simplified Function	$\bar{A}\bar{B}$	$\bar{A}C$	BC
Inputs	AB	$A\bar{C}$	$\bar{B}C$

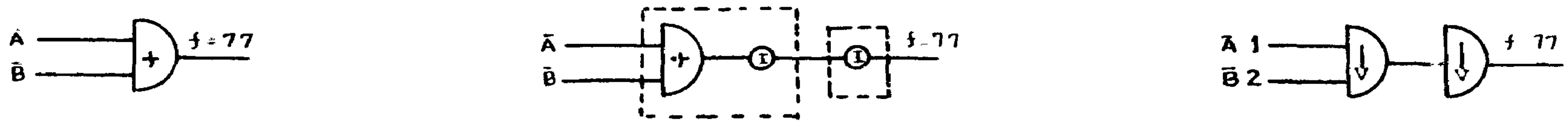






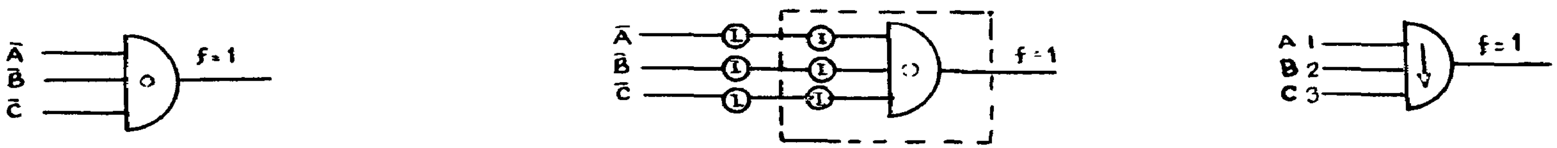
Circuit no. 1

Function number (octal)	77	257	353
Min terms	(0,1,2,3,4,5)	(0,1,2,3,5,7)	(1,2,3,5,6,7)
Simplified function	$\bar{A}+\bar{B}$	$\bar{A}+C$	$B+C$
Inputs	$\bar{A}\bar{B}$	$\bar{A}C$	BC



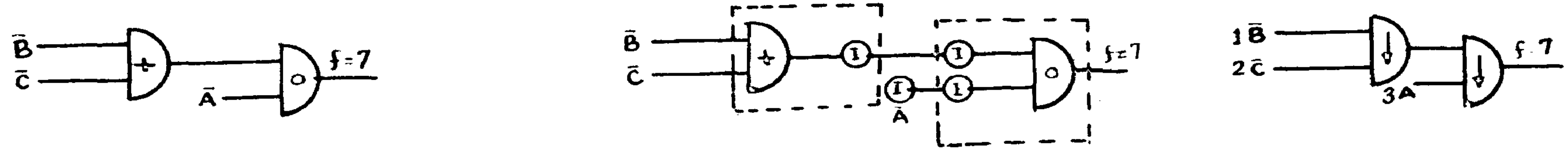
Circuit No. 3

Function number (octal)	1	2	10	200
Min terms	0	1	3	7
Simplified function	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	$\bar{A}BC$	ABC
Inputs	ABC	$AB\bar{C}$	$A\bar{B}\bar{C}$	$\bar{A}\bar{B}\bar{C}$



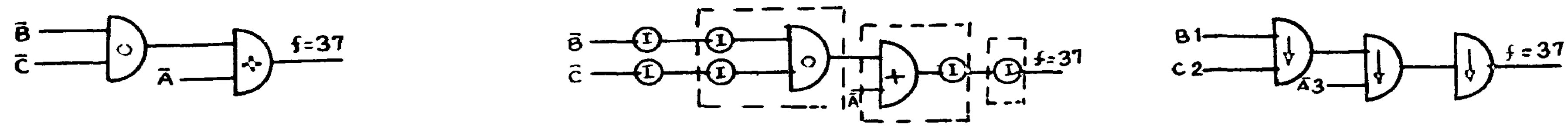
Circuit No. 2

Function number (octal)	7	13	16	52	212	250
Min terms	(0, 1, 2)	(0, 1, 3)	(1, 2, 3)	(1, 3, 5)	(1, 3, 7)	(3, 5, 7)
Simplified function	$\bar{A}(\bar{B}+\bar{C})$	$\bar{A}(\bar{B}+C)$	$\bar{A}(B+C)$	$C(\bar{A}+\bar{B})$	$C(\bar{A}+B)$	$C(A+B)$
Inputs	$\bar{B}\bar{C}A$	$\bar{B}CA$	BCA	$\overline{AB}C$	$\bar{A}\bar{B}C$	ABC



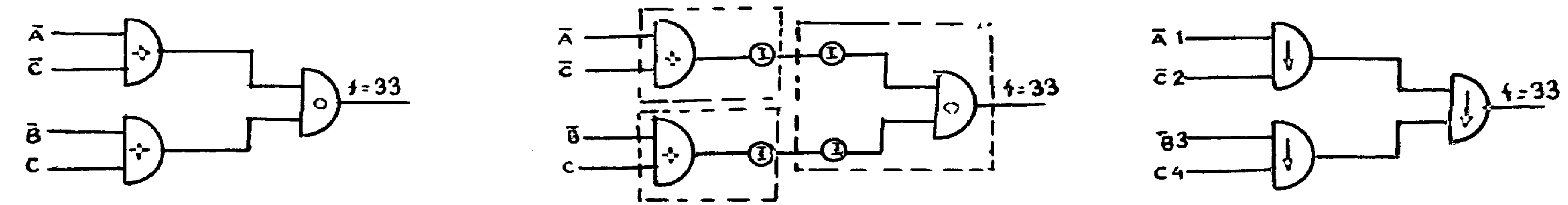
Circuit No. 4

Function number (octal)	37	57	217	253	256	352
Min terms	(0, 1, 2, 3, 4)	(0, 1, 2, 3, 5)	(0, 1, 2, 3, 7)	(0, 1, 3, 5, 7)	(1, 2, 3, 5, 7)	(1, 3, 5, 6, 7)
Simplified function	$\bar{A}+\bar{B}\bar{C}$	$\bar{A}+\bar{B}C$	$\bar{A}+BC$	$C+\bar{A}\bar{B}$	$C+\bar{A}B$	$C+AB$
Inputs	$BC\bar{A}$	$\bar{B}\bar{C}\bar{A}$	$\bar{B}\bar{C}\bar{A}$	[illegible]	$A\bar{B}C$	$\bar{A}BC$



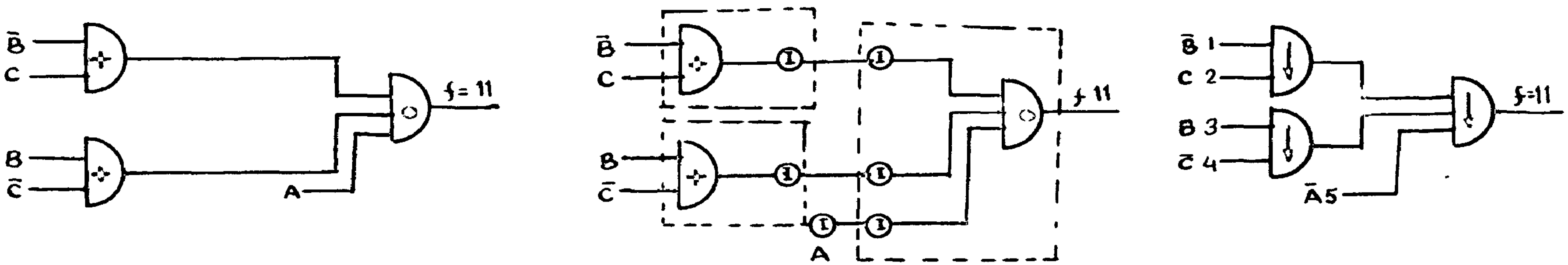
Circuit No. 5

Function number (octal)	33	[illegible]	[illegible]	36		
Min terms	(0,1,3,4)	(0,1,3,7)	(0,3,4,7)	(1,2,[illegible])	[illegible]	[illegible]
Simplified function	$(\bar{A}+\bar{C})(\bar{B}+C)$	$(\bar{A}+B)(\bar{B}+\bar{C})$	$(\bar{B}+C)(B+\bar{C})$	$(B+C)(\bar{A}+\bar{B})$	$(A+B)(\bar{A}+\bar{B})$	[illegible]
Inputs	$\bar{A}\bar{C}\bar{B}C$	$\bar{A}B\bar{B}\bar{C}$	$\bar{B}CB\bar{C}$	$BC\bar{A}\bar{B}$	$AB\bar{A}\bar{B}$	$\bar{A}B\bar{B}C$



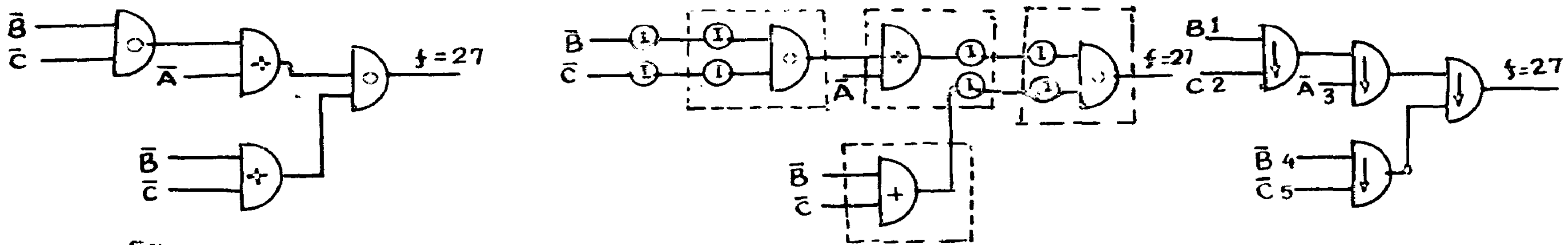
Circuit No. 6

Function number (octal)	11	6	202	50
Min terms	(0,3)	(1,2)	(1,7)	(3,5)
Simplified function	$A(B+\bar{C})(\bar{B}+C)$	$A(B+C)(\bar{B}+\bar{C})$	$C(\bar{A}+B)(A+\bar{B})$	$C(\bar{A}+\bar{B})(A+B)$
Inputs	$\bar{B}CB\bar{C}A$	$BC\bar{B}\bar{C}A$	$\bar{A}BA\bar{B}C$	$\bar{A}\bar{B}ABC$



Circuit No. 7

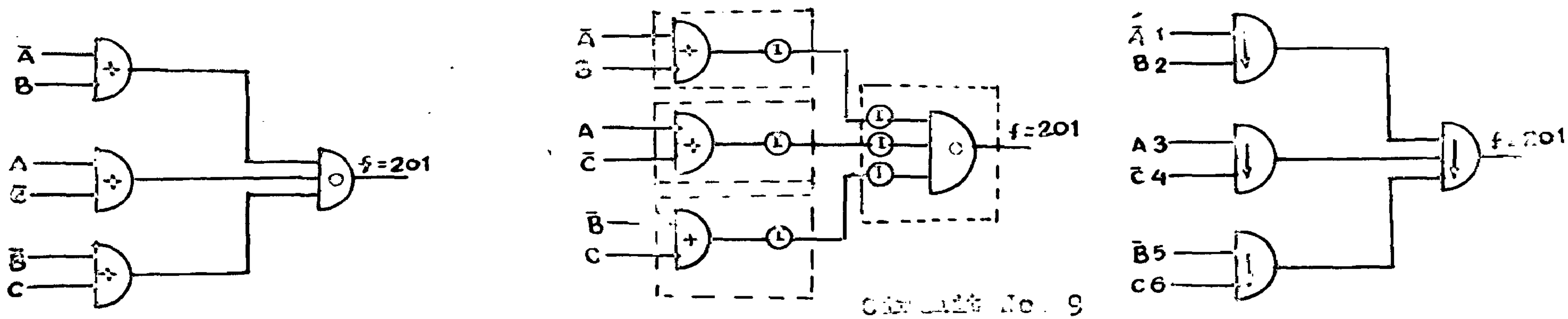
F.N.	[illegible]	20[illegible]	21[illegible]	32	54	230
Min term	(0,3,4)	(0,1,7)	[illegible]	(1,3,4)	(2,3,5)	(3,4,7)
S.F.	[illegible]	[illegible]	[illegible]	[illegible] (A+C)	[illegible]	[illegible]
Input	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]



F.N.	27	53	216	350
Min term	(0,1,2,4)	(0,1,3,5)	(1,2,3,7)	(3,5,6,7)
S.F.	[illegible]	[illegible]	[illegible]	[illegible] (A+C)
Inputs	[illegible]	[illegible]	[illegible]	[illegible]

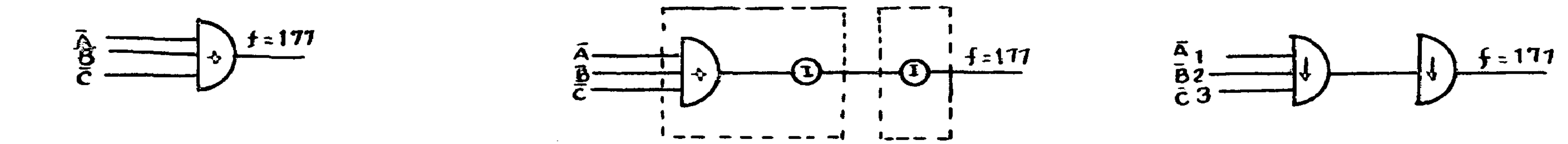
Circuit No. 8

F.N.	201	30
Min term	(0,7)	(3,4)
S.F.	[illegible]	[illegible]
Inputs	[illegible]	[illegible]



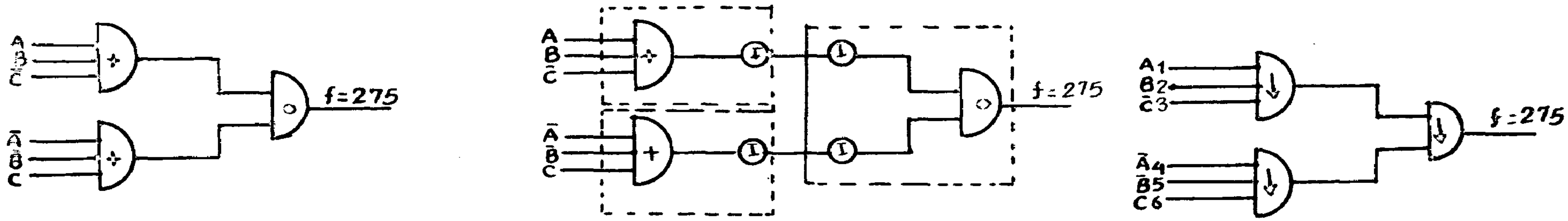
Circuit No. 9

Function number (Octal)	177	277	357	376
Min terms	(0,1,2,3,4,5,6)	(0,1,2,3,4,5,7)	(0,1,2,3,5,6,7)	(1,2,3,4,5,6,7)
Simplified Function	$\bar{A}+\bar{B}+\bar{C}$	$\bar{A}+\bar{B}+C$	$\bar{A}+B+C$	$A+B+C$
Inputs	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	$\bar{A}BC$	ABC



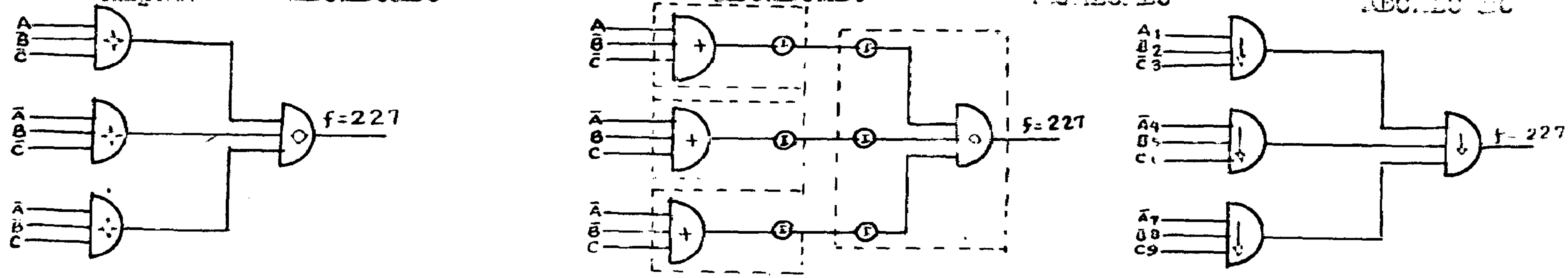
Circuit No. 10

[illegible]	[illegible]	157	353	276	275	176
Min terms	(0,1,2,3,4,7)	(0,1,2,3,5,6)	(0,1,3,5,6,7)	(1,2,3,4,5,7)	(0,2,3,4,5,7)	(1,2,3,4,5,6)
[illegible]	([illegible])$(\bar{A}+\bar{B}+C)$	$(\bar{A}+B+C)(\bar{A}+\bar{B}+\bar{C})$	$(A+\bar{B}+C)(\bar{A}+B+C)$	$(A+B+C)(\bar{A}+\bar{B}+C)$	$(A+B+\bar{C})(\bar{A}+\bar{B}+C)$	$(A+B+C)(\bar{A}+\bar{B}+C)$
Inputs	$\overline{ABC}\,\overline{ABC}$	$\bar{A}BC\,\overline{ABC}$	$\bar{A}BC\,\bar{A}BC$	$ABC\,\bar{A}BC$	$ABC\,\overline{ABC}$	$ABC\,\overline{ABC}$



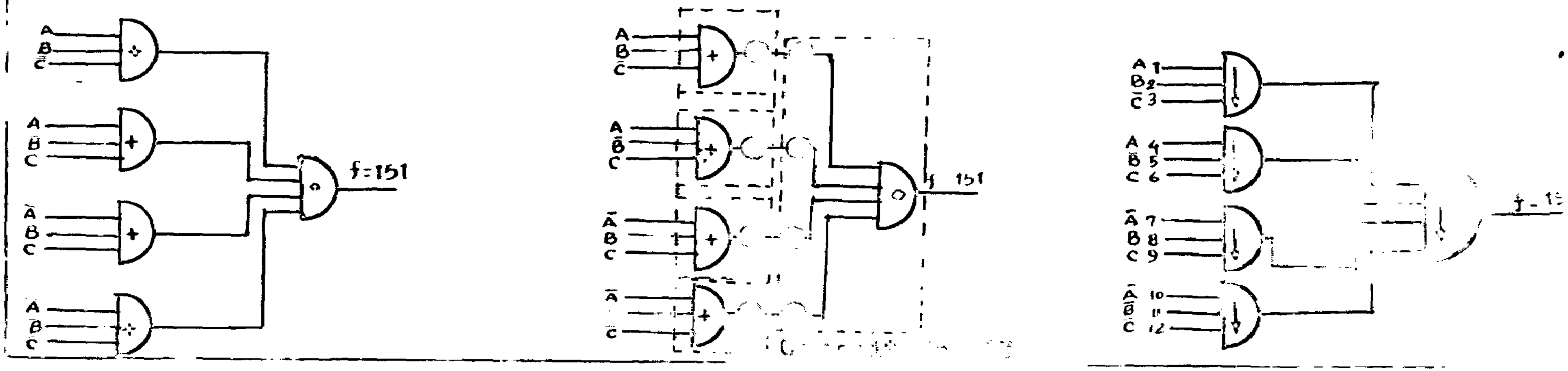
Circuit No. 11

F.N.	227	155	351	233
M.T	(0, 1, 2, 4, 7)	(0, 1, 3, 5, 6)	(0, 3, 5, 6, 7)	(1, 2, 4, ?, ?)
S.F.	$(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$	$(\bar{A}+\bar{B}+\bar{C})(\bar{A}+B+C)(A+\bar{B}+C)$	$(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+B+C)$	$(A+B+C)(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$
Inputs	$A\bar{B}\bar{C}\bar{A}B\bar{C}\bar{A}\bar{B}C$	$\bar{A}\bar{B}\bar{C}\bar{A}BCA\bar{B}C$		$ABC\bar{A}B\bar{C}\bar{A}\bar{B}C$

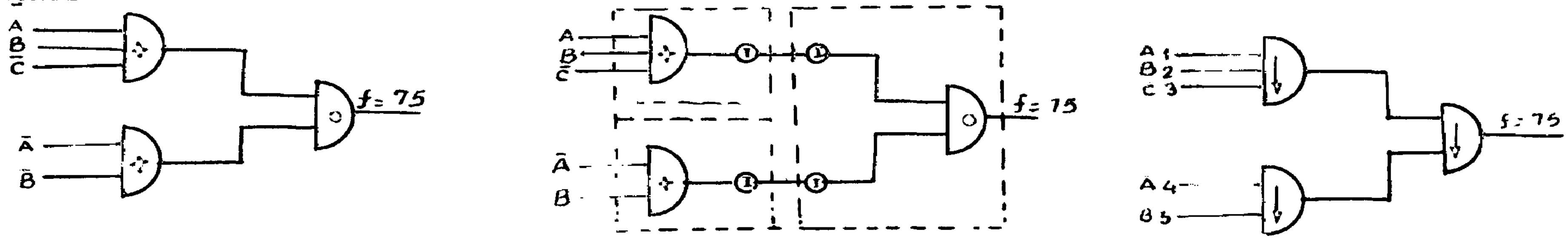


Circuit No. 12

F.N.	151	225
M.T	(0, 3, 5, 6)	(1, 2, 4, 7)
S.F.	$(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+B+C)(\bar{A}+\bar{B}+\bar{C})$	$(A+B+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$
Inputs	$AB\bar{C}A\bar{B}C\bar{A}BC\bar{A}\bar{B}\bar{C}$	$ABC A\bar{B}\bar{C}\bar{A}B\bar{C}\bar{A}\bar{B}C$

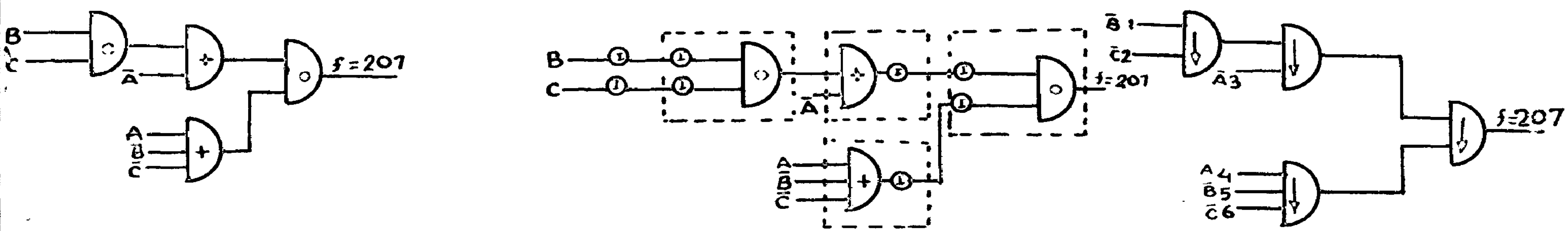


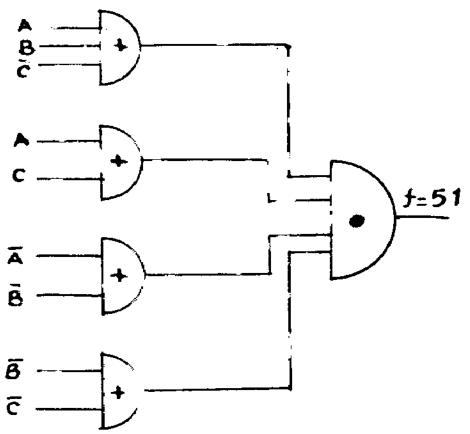
F	75	255	255	76	156	[illegible]
Min terms	(0, 2, 3, 4, 5)	(0, 1, 3, 4, 7)	(0, 2, 3, 5, 7)	(1, 2, 3, 4, 5)	(1, 2, 3, 5, 6)	(2, 3, 4, 5, 7)
S.P.	$(A+B+\bar{C})(\bar{A}+\bar{B})$	$(A+B+\bar{C})(\bar{B}+C)$	$(A+B+\bar{C})(\bar{A}+C)$	$(A+B+C)(\bar{A}+\bar{B})$	$(\bar{A}+\bar{B}+\bar{C})(B+C)$	$(\bar{A}+\bar{B}+C)$ [illegible]
Inputs	$ABC\bar{A}\bar{B}$	$AB\bar{C}\bar{B}C$	$AB\bar{C}\bar{A}C$	$ABC\bar{A}\bar{B}$	$\bar{A}\bar{B}\bar{C}BC$	[illegible]

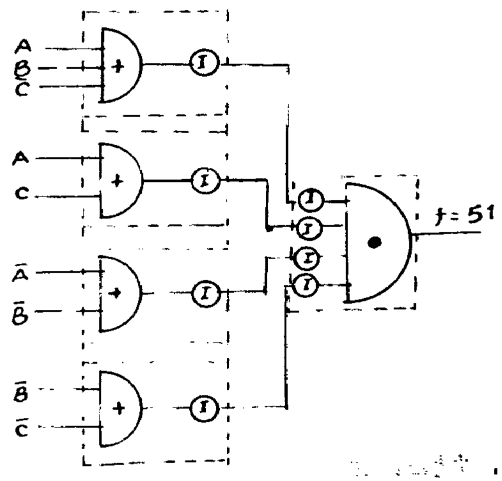


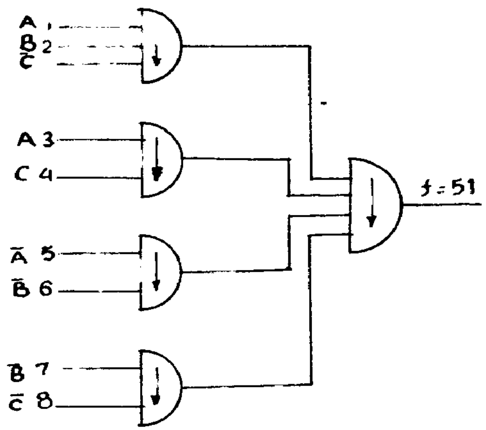
Circuit No. 14

F	207	55	[illegible]	232	56	[illegible]
Min T.	(0, 1, 2, 7)	(0, 2, 3, 5)	(0, 3, 5, 7)	(1, 3, 4, 7)	(1, 2, 3, 4)	[illegible]
S.P.	[illegible] $(\bar{A}+BC)$	$(A+B+\bar{C})$ [illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Input	[illegible]	[illegible]	$ABCAB\bar{C}$	$\bar{A}BC\bar{A}B\bar{C}$	$BC\bar{A}BC$	[illegible]

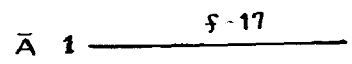



A
B
C̄
A
C
Ā
B̄
B̄
C̄
f=51


A
B
C̄
A
C
Ā
B̄
B̄
C̄
f=51

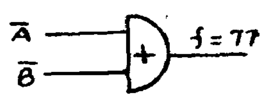

A 1
B 2
C̄
A 3
C 4
Ā 5
B̄ 6
B 7
C̄ 8
f=51

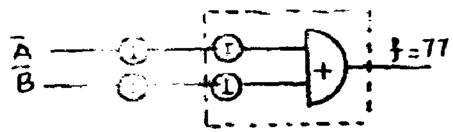
Function number (Octal)	17	252
Min terms	(0,1,2,3)	(1,3,5,7)
Simplified function	$\bar{A}$	C
Inputs	$\bar{A}$	C



Circuit No. 0

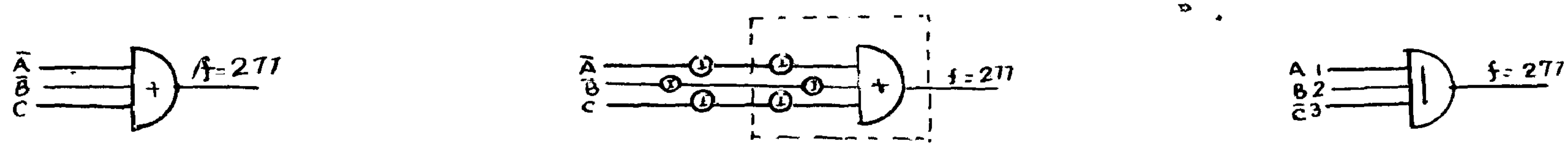
Function number (Octal)	77	257	356
Min terms	(0,1,2,3,4,5)	(0,1,2,3,5,7)	(1,2,3,5,6,7)
Simplified function	$\bar{A}+\bar{B}$	$\bar{A}+C$	B + C
Inputs	AB	$A\bar{C}$	$\overline{BC}$





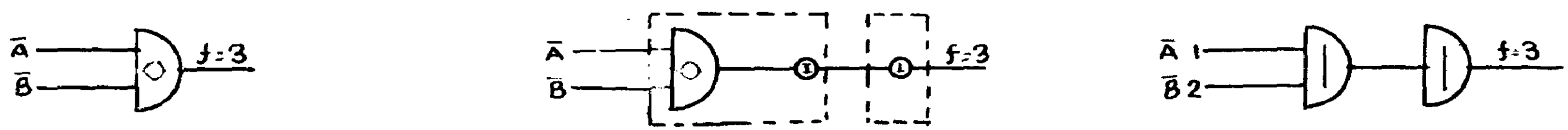
Circuit No. 1

Function number (octal)	277	337	376	377
Min terms	(0,1,2,3,4,5,7)	(0,1,2,3,5,6,7)	(1,2,3,4,5,6,7)	(0,1,2,3,4,5,6)
Simplified function	$\bar{A}+\bar{B}+C$	$\bar{A}+B+C$	$A+B+C$	$\bar{A}+\bar{B}+\bar{C}$
Inputs	$AB\bar{C}$	$A\bar{B}\bar{C}$	$\bar{A}\bar{B}\bar{C}$	ABC



Circuit No. 2

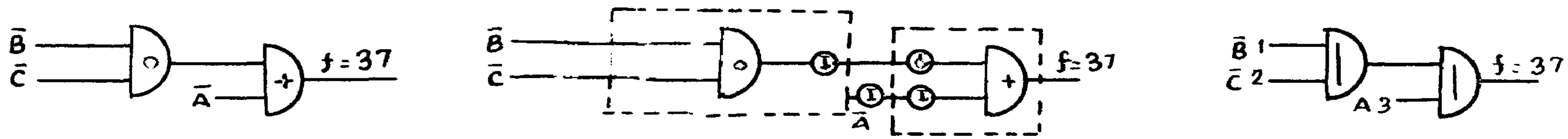
Function number (octal)	3	12	210
Min terms	(0,1)	(1,5)	(3,7)
Simplified function	$\bar{A}\bar{B}$	$\bar{B}C$	BC
Inputs	$\bar{A}\bar{B}$	$\bar{B}C$	BC



Circuit No. 3

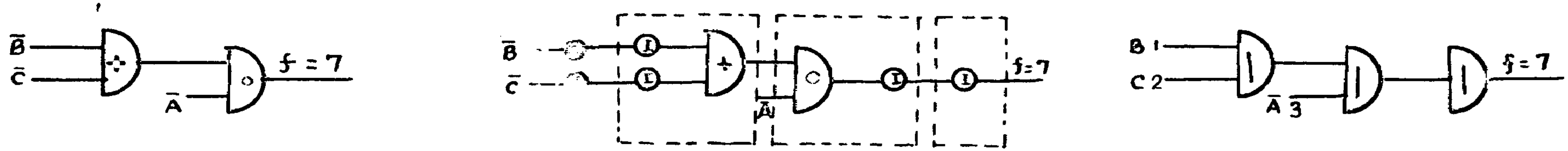
Function Number (Octal)

F.N.	37	57	2[illegible]7	253	256	352
M.T.	(0,1,2,3,4)	(0,1,2,3,5)	(0,1,2,3,7)	(0,1,3,5,7)	(1,2,3,5,7)	(1,3,5,[illegible])
S.F.	$\bar{A}+\bar{B}\bar{C}$	$\bar{A}+\bar{B}C$	$\bar{A}+BC$	$C+\bar{A}\bar{B}$	$C+\bar{A}B$	$C+AB$
Inputs	$\bar{B}\bar{C}\bar{A}$	$\bar{B}C\bar{A}$	$BC\bar{A}$	$\bar{A}\bar{B}C$	$\bar{A}BC$	ABC



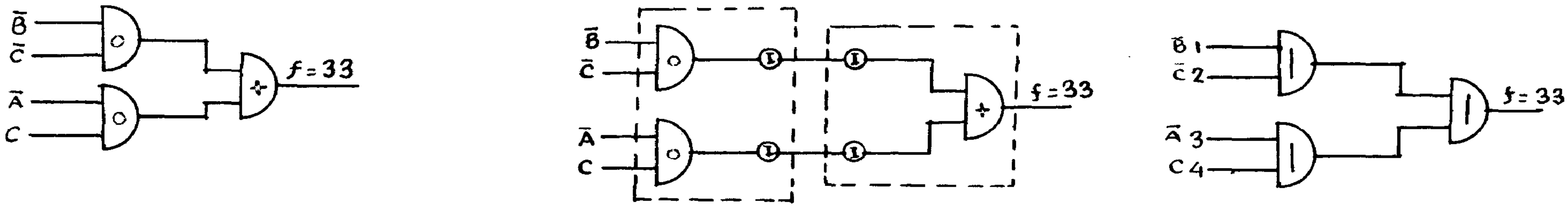
Circuit No. [illegible]

F.N.	7	13	1[illegible]	52	212	250
M.T.	(0,1,2)	(0,1,3)	(1,2,3)	(1,3,5)	(1,3,7)	(3,5,7)
S.F.	$\bar{A}(\bar{B}+\bar{C})$	$\bar{A}(\bar{B}+C)$	$\bar{A}(B+C)$	$C(\bar{A}+\bar{B})$	$C(\bar{A}+B)$	$C(A+B)$
Inputs	$\bar{B}\bar{C}\bar{A}$	$\bar{B}C\bar{A}$	$\bar{B}C\bar{A}$	ABC	$\bar{A}\bar{B}C$	$\bar{A}\bar{B}C$



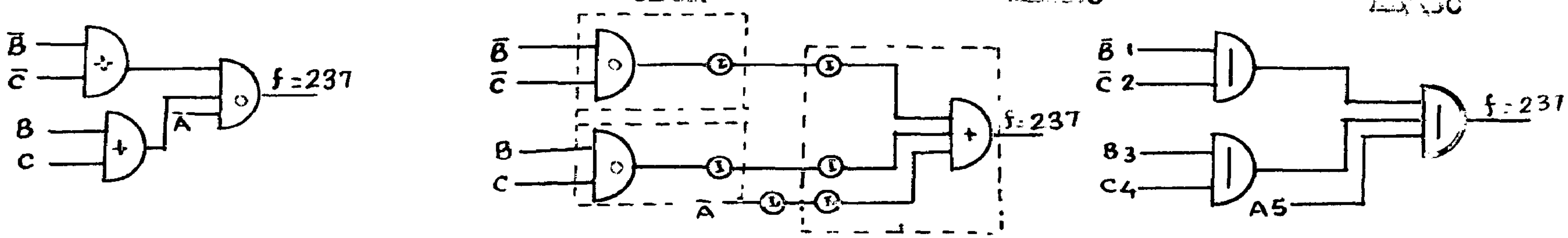
Circuit No. 5

F.N.	33	213	251	56	74	254
M.T.	(0,1,3,4)	(0,1,3,7)	(0,3,4,7)	(1,2,3,5)	(2,3,4,5)	(2,3,5,7)
S.F.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Inputs	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]



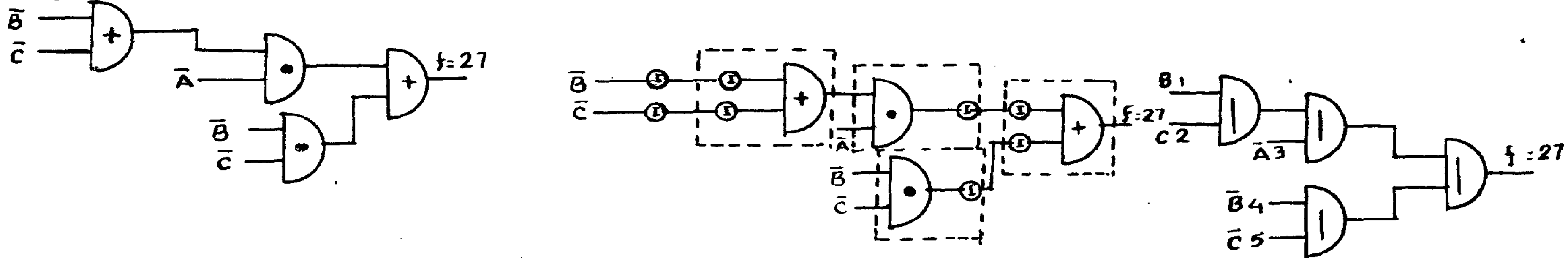
Circuit No. 6

F.N.	237	157	353	276
M.T.	(0,1,2,3,4,7)	(0,1,2,3,5,6)	(0,1,3,5,6,7)	(1,2,3,4,5,7)
S.F.	[illegible]	[illegible]	[illegible]	[illegible]
Inputs	[illegible]	[illegible]	[illegible]	[illegible]



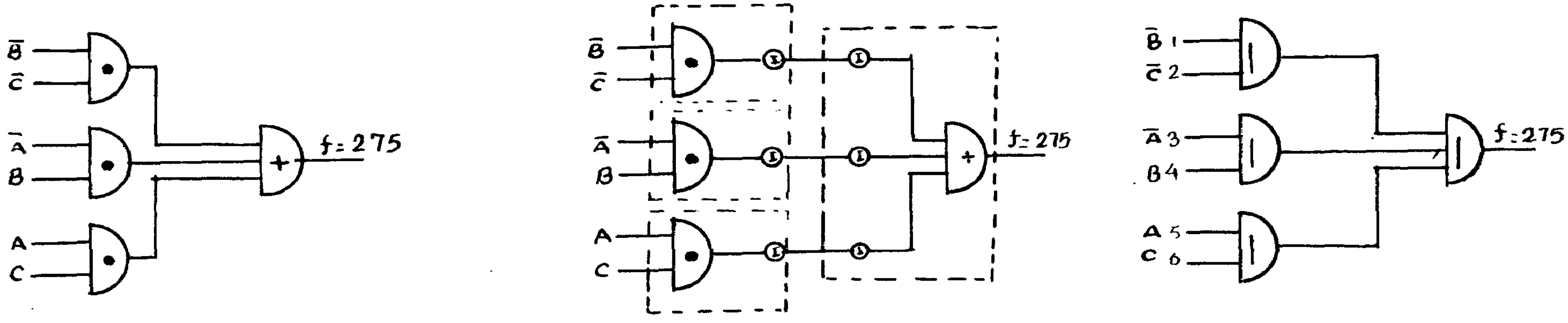
Circuit No. 7

F.	27	53	216	350	75
Min.T.	(0,1,2,4)	(0,1,3,5)	(1,2,3,7)	(3,5,6,7)	(0,[illegible])
S.F.	$\bar{A}(\bar{B}+\bar{C})+\bar{B}\bar{C}$	$C(\bar{A}+\bar{B})+\bar{A}\bar{B}$	$B(\bar{A}+C)+\bar{A}C$	$A(B+C)+BC$	$\bar{A}(B+C)+BC$
Inputs	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Inputs	$\bar{B}A\bar{C}\bar{B}\bar{C}$	$\bar{A}\bar{B}C\bar{A}\bar{C}$	$\bar{C}A\bar{B}\bar{A}B$	$ACB\bar{B}C$	$A\bar{C}BAB$



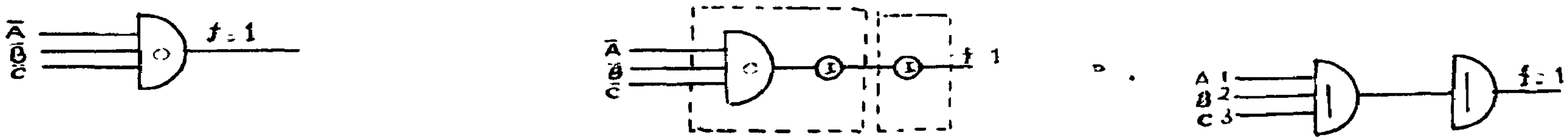
Circuit No. 8

F.	275	176
Min.T.	([illegible],3,[illegible],5,7)	(1,2,3,4,5,6)
S.F.	$(\bar{B}\bar{C}+\bar{A}B+AC)$	$(A\bar{B}+\bar{A}C+B\bar{C})$
Inputs	$\bar{B}\bar{C}\bar{A}BAC$	$A\bar{B}\bar{A}CB\bar{C}$



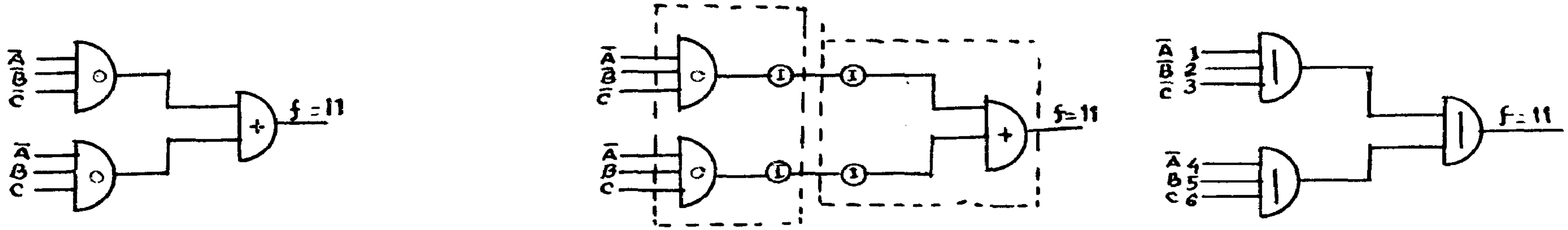
Circuit No. 9

[illegible]	1	2	10	200
[illegible]	0	[illegible]	3	7
[illegible]	ABC	ABC	ABC	ABC
Inputs	ABC	ABC	ABC	ABC



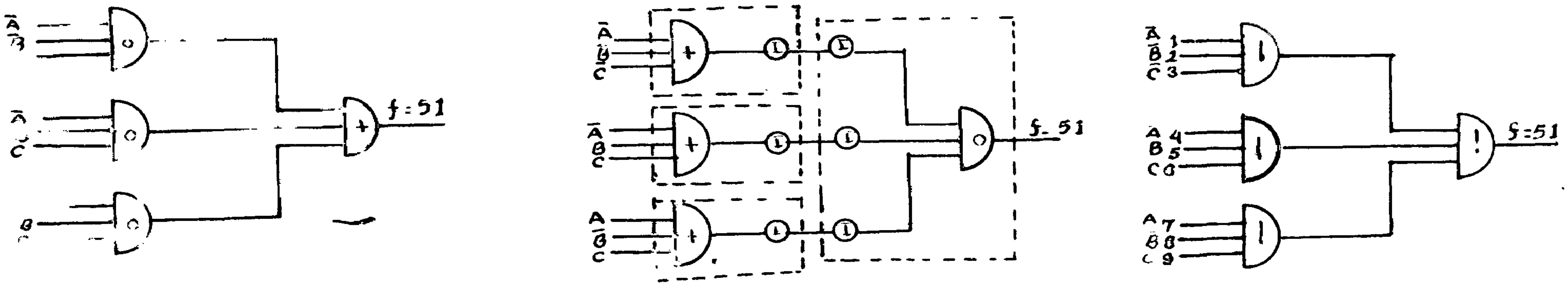
Circuit No. 10

[illegible]	11	201	5	30	202	50
[illegible]	0, 3	0, 7	1, 2	3, 4	1, 7	3, 5
[illegible]	(ABC + ABC)	(ABC + ABC)	(ABC + ABC)	(ABC + ABC)	(ABC + ABC)	(ABC + ABC)
Inputs	ABCABC	ABCABC	ABCABC	ABCABC	ABCABC	ABCABC



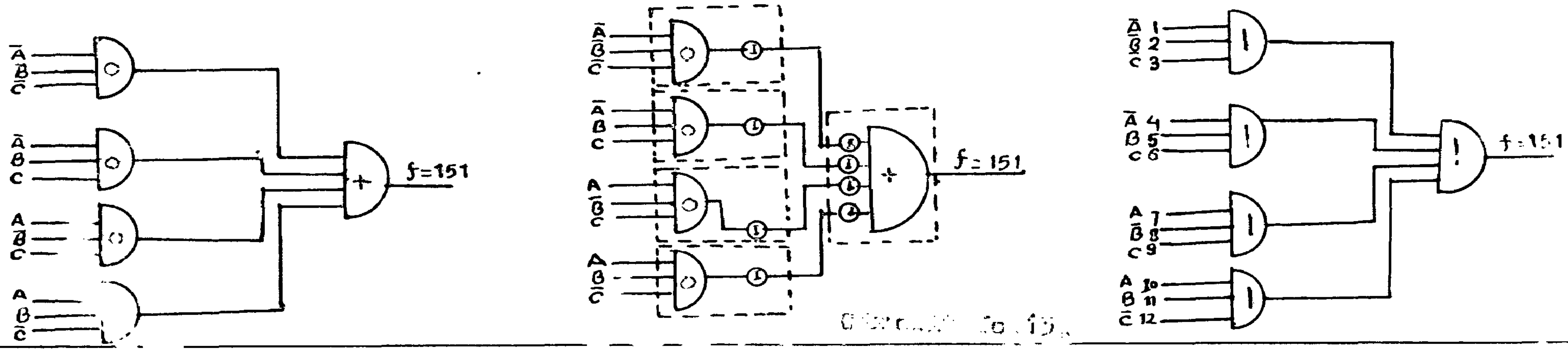
Circuit No. 11

F. No.	5[illegible]	2[illegible]	203	[illegible]50
M. N.	(0, 3, 5)	(1, 2, 4)	(1, 2, 7)	(3, 5, 6)
S. of P.	([illegible])	($\bar{A}\bar{B}C+\bar{A}B\bar{C}+[illegible]$)	($\bar{A}\bar{B}C+\bar{A}B\bar{C}+ABC$)	($AB\bar{C}+A\bar{B}C+\bar{A}BC$)
Inputs	[illegible]	[illegible]	$\bar{A}\bar{B}C\bar{A}B\bar{C}ABC$	[illegible]



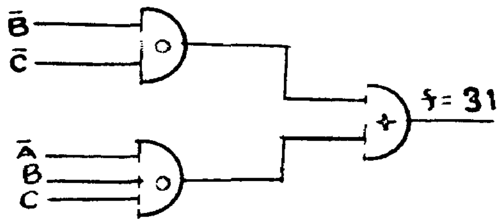
Circuit No. 12

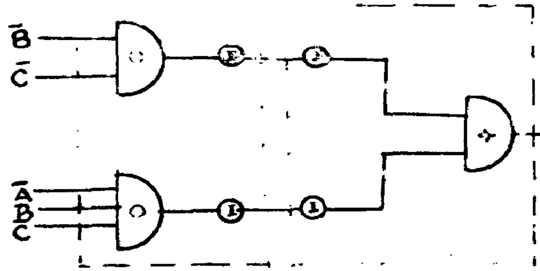
F. No.	15[illegible]	226
M. N.	(0, 3, 5, 6)	(1, 2, 4, 7)
S. of P.	([illegible] $+\bar{A}BC+A\bar{B}C+AB\bar{C}$)	([illegible] $+\bar{A}B\bar{C}+A\bar{B}\bar{C}+ABC$)
Inputs	[illegible]	[illegible]

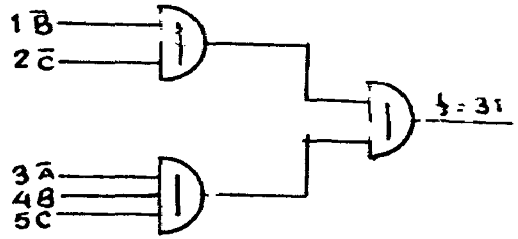


Circuit No. 13

[illegible]	31	203	[illegible]	32	54	230
[illegible]	(0,3,4)	(0,[illegible],7)	(0,3,7)	(1,3,4)	(2,3,5)	(3,4,7)
[illegible]	$\bar{B}\bar{C}+\bar{A}BC$	$\bar{A}\bar{B}+[illegible]$	$BC+\bar{A}\bar{B}\bar{C}$	$\bar{A}C+A\bar{B}\bar{C}$	$\bar{A}B+A\bar{B}C$	$BC+[illegible]$
Inputs	$\bar{B}\bar{C}\bar{A}BC$	$\bar{A}\bar{B}ABC$	$BC\bar{A}\bar{B}\bar{C}$	$\bar{A}CA\bar{B}\bar{C}$	$\bar{A}BA\bar{B}C$	$BC[illegible]$

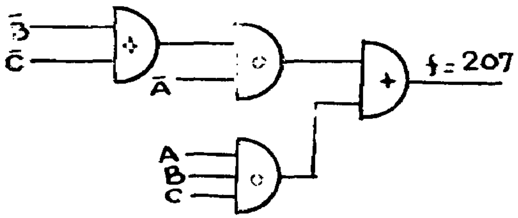


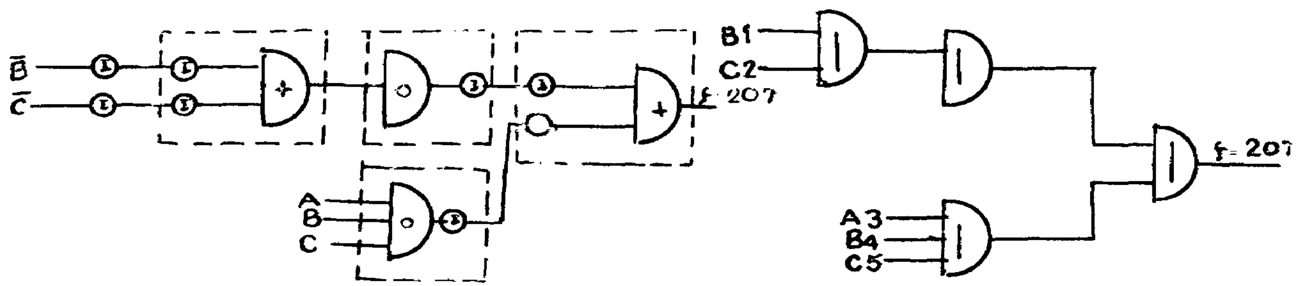




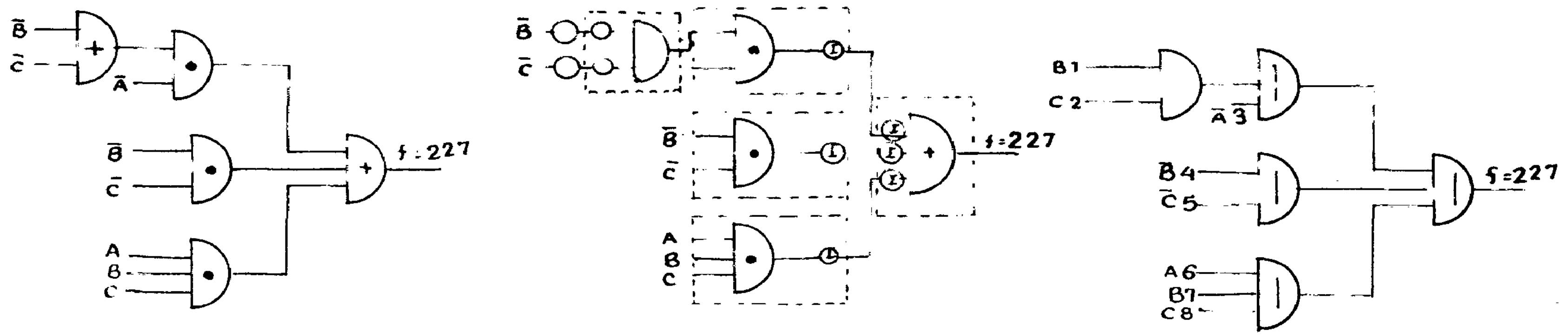
Circuit No. 14.

[illegible]	207	55	251	232	36	152
[illegible]	(0,1,2,7)	(0,2,3,5)	(0,3,5,7)	(1,3,4,7)	(1,2,3,4)	([illegible])
[illegible]	$\bar{A}(\bar{B}+\bar{C})+ABC$	$\bar{A}(B+C)+A\bar{B}C$	$C(A+B)+\bar{A}\bar{B}\bar{C}$	$C(\bar{A}+B)+A\bar{B}\bar{C}$	$\bar{A}(B+C)+A\bar{B}\bar{C}$	$C(\bar{A}+\bar{B})+[illegible]$
Inputs	$\bar{B}\bar{C}\bar{A}BC$	$\bar{B}\bar{C}\bar{A}BC$	$\bar{A}\bar{B}CA\bar{B}C$	$\bar{A}\bar{B}CA\bar{B}C$	$\bar{B}\bar{C}A\bar{A}BC$	$\bar{A}\bar{B}C[illegible]$





Circuit No. 15


f=227
f=227
f=227
B1
C2
A3
B4
C5
A6
B7
C8
A
B
C

CHAPTER VIII

C O N C L U S I O N S

R.A. Smith in his paper on NOR , NAND logics says that " At present there are no known algebraic methods of obtaining absolutely minimal NOR or NAND circuits, or , for that matter, circuits using any other types of logic elements. Methods such as the Karnaugh map method, which has been extended to NOR and NAND circuits give only near to minimal realizations."

Here, an algebraic method of implementation has been given and to prove the validity of this method each circuit has been proved and compared with Smith's minimal circuits which he got by using a computer. This method may hold good for 4 and Higher variables but since there are no proved minimal circuits the method could not be checked.

This method holds good only when complemented variables are available. There is Scope of further investigation to improve or supplement the method so that it may also include those cases where the complemented variables are not available.

R E F E R E N C E S

1 Maley, G.A., and J. Earle. " The Logic Design of Transistor Digital Computers " Englewood Cliff., N.J. Prentice Hall 1963 , Ch. 6.

2 R.A. Smith " Minimal Three Variable NOR AND NAND Logic Circuits" IEEE Trans Feb, 1965 Vl.EC-14 No.1 pp. 79-81.

3 Hellerman, L.A. " Catalog of three variable OR invert and AND - invert logical circuits", IEEE Trans, on Electronic Computers, Vol. EC-12, June, 1963 pp. 198-223.

4 N.N.Biswas " Letter to the Editor" Control , April 1965 p. 185.

5 Shubert , " Logical Design by Regression " Trans., IEEE Vol. 80 pp. 380-383, Sept. 61.

6 Karnaugh M, Map Method for Synthesis of combinational logic circuits" Trans. AIEE, Pt. I , Nov, 1953 p. 593.

APPENDIX

TABLE NO. I

Function number			Partition leader	
Sl. No.	Min. terms	Octal number	Octal number	Permutations under which the leader is found
1	2	3	4	5

N = 1

1	0	1	1	*	*
2	1	2	2	ABC	BAC
3	2	4	2	ACB	BCA
4	3	10	10	ABC	ACB
5	4	20	2	CAB	CBA
6	5	40	10	BAC	CAB
7	6	100	10	BCA	CBA
8	7	200	200	ABC*	*

N = 2

9	0,1	3	3	ABC	BAC
10	0,2	5	3	ACB	BCA
11	0,3	11	11	ABC	ACB
12	0,4	21	3	CBA	CAB
13	0,5	41	11	BAC	CAB
14	0,6	101	11	BCA	CBA
15	0,7	201	201	*	*

N = 2 Contd..

* BREAK CASES
Leader is found under all permutations.

1	2	3	4	5	
16	1,2	6	6	ABC	ACB
17	1,3	12	12	ABC	-
18	1,4	22	6	BAC	CAB
19	1,5	42	12	BAC	-
20	1,6	102	30	BCA	CBA
21	1,7	202	202	ABC	BAC
22	2,3	14	12	ACB	-
23	2,4	24	6	BCA	CBA
24	2,5	54	30	BAC	CAB
25	2,6	104	12	BCA	-
26	2,7	204	202	ACB	BCA
27	3,4	30	30	ABC	ACB
28	3,5	50	50	ACB	BAC
29	3,6	110	50	ACB	BCA
30	3,7	210	210	ABC	ACB
31	4,5	60	6AB		-
32	4,6	120	12	CBA	-
33	4,7	220	202	CAB	CBA
34	5,6	140	50	CAB	CBA
35	5,7	240	210	BAC	CAB
36	6,7	300	210	BCA	CBA

1	2	3	4	5	
			n = 3		
37	0,1,2	7	7	ABC	ACD
38	0,1,3	13	13	ABC	-
39	0,1,4	23	7	BAC	CAB
40	0,1,5	43	13	BAC	-
41	0,1,6	103	31	BCA	CDA
42	0,1,7	203	203	ABC	BAC
43	0,2,3	15	13	ACD	-
44	0,2,4	25	7	BCA	CDA
45	0,2,5	45	31	BAC	CAB
46	0,2,6	105	13	BCA	-
47	0,2,7	205	205	ACD	BCA
48	0,3,4	31	31	ABC	ACD
49	0,3,5	51	51	ABC	BAC
50	0,3,6	111	51	ACD	BCA
51	0,3,7	211	211	ABC	ACD
52	0,4,5	61	13	CAB	-
53	0,4,6	121	13	CBA	-
54	0,4,7	221	203	CBA	CAB
55	0,5,6	141	51	CBA	CAB
56	0,5,7	241	211	BAC	CAB
57	0,6,7	301	211	BCA	CBA
58	1,2,3	16	16	ABC	ACD
59	1,2,4	26	26	°	°

1	2	3	4	5	
60	1,2,5	46	32	BAC	-
61	1,2,6	106	32	DCA	-
62	1,2,7	206	206	ADC	ACD
63	1,3,4	32	32	ABC	-
64	1,3,5	52	52	ADC	DAC
65	1,3,6	112	54	ACB	-
66	1,3,7	212	212	ABC	ACD
67	1,4,5	62	62	BAC	CAD
68	1,4,6	122	32	CDA	-
69	1,4,7	222	206	DAC	CAB
70	1,5,6	142	54	CAD	-
71	1,5,7	242	212	BAC	-
72	1,6,7	302	230	BCA	CBA
73	2,3,4	34	32	ACB	-
74	2,3,5	54	54	ADC	-
75	2,3,6	114	52	ACD	BCA
76	2,3,7	214	212	ACD	-
77	2,4,5	64	32	CAB	-
78	2,4,6	124	16	DCA	CBA
79	2,4,7	224	206	DCA	CBA
80	2,5,6	144	54	CBA	-
81	2,5,7	244	230	DAC	CAD
82	2,6,7	304	212	BCA	-

1	2	3	4	5	
83	3,4,5	70	54	BAC	-
84	3,4,6	130	54	BCA	-
85	3,4,7	230	230	ABC	ACB
86	3,5,6	150	150	*	*
87	3,5,7	250	250	ABC	BAC
88	3,6,7	310	250	ACB	BCA
89	4,5,6	160	52	CBA	CAB
90	4,5,7	260	212	CAB	-
91	4,6,7	320	212	CBA	-
92	5,6,7	340	250	CBA	CAB

<u>N = 4</u>

93	0,1,2,3	17	17	ABC	ACB
94	0,1,2,4	27	27	*	*
95	0,1,2,5	47	33	BAC	-
96	0,1,2,6	107	33	BCA	-
97	0,1,2,7	207	207	ABC	ACB
98	0,1,3,4	33	33	ABC	-
99	0,1,3,5	53	53	ABC	BAC
100	0,1,3,6	113	53	ACB	-
101	0,1,3,7	213	213	A[illegible]	-
102	0,1,4,5	63	17	BAC	CAB
103	0,1,4,6	123	33	CBA	-

1	2	3	4	5	
104	0,1,4,7	225	207	BAC	CAB
105	0,1,5,6	145	55	CAB	-
106	0,1,5,7	245	215	BAC	-
107	0,1,6,7	305	251	BCA	CBA
108	0,2,3,4	35	33	ACB	-
109	0,2,3,5	55	55	ABC	-
110	0,2,3,6	115	53	ACB	BCA
111	0,2,3,7	215	213	ACB	-
112	0,2,4,5	65	33	CAB	-
113	0,2,4,6	125	17	BCA	CBA
114	0,2,4,7	225	207	BCA	CBA
115	0,2,5,6	145	55	CBA	-
116	0,2,5,7	245	231	BAC	CAB
117	0,2,6,7	305	213	BCA	-
118	0,3,4,5	71	55	BAC	-
119	0,3,4,6	131	55	BCA	-
120	0,3,4,7	231	231	ABC	ACB
121	0,3,5,6	151	151	*	*
122	0,3,5,7	251	251	ABC	BAC
123	0,3,6,7	311	251	ACB	BCA
124	0,4,5,6	161	53	CBA	CAB
125	0,4,5,7	201	213	CAB	-
126	0,4,6,7	321	213	CBA	-
127	0,5,6,7	341	251	CBA	CAB

1	2	3	4	5	
128	1,2,3,4	36	36	ABC	ACB
129	1,2,3,5	56	56	ABC	-
130	1,2,3,6	116	56	ACB	-
131	1,2,3,7	216	216	ABC	ACB
132	1,2,4,5	66	36	BAC	CAB
133	1,2,4,6	126	36	BCA	CBA
134	1,2,4,7	226	226	*	*
135	1,2,5,6	146	74	CBA	CAB
136	1,2,5,7	246	232	BAC	-
137	1,2,6,7	306	232	BCA	-
138	1,3,4,5	72	56	BAC	-
139	1,3,4,6	132	74	ACB	BCA
140	1,3,4,7	232	232	ABC	-
141	1,3,5,6	152	152	ABC	BAC
142	1,3,5,7	252	252	ABC	BAC
143	1,3,6,7	312	254	ACB	-
144	1,4,5,6	102	56	CAB	-
145	1,4,5,7	262	216	BAC	CAB
146	1,4,6,7	322	232	CBA	-
147	1,5,6,7	342	254	CAB	-
148	2,3,4,5	74	74	ABC	BAC
149	2,3,4,6	134	56	BCA	-
150	2,3,4,7	234	232	ACB	-

1	2	3	4	5	
151	2,3,5,6	154	152	ACB	BCA
152	2,3,5,7	254	254	ABC	-
153	2,3,6,7	314	152	ACB	BCA
154	2,4,5,6	164	56	CBA	-
155	2,4,5,7	264	252	CAB	-
156	2,4,6,7	324	216	BCA	CBA
157	2,5,6,7	344	254	CBA	-
158	3,4,5,6	170	152	CBA	CAB
159	3,4,5,7	270	254	BAC	-
160	3,4,6,7	330	254	BCA	-
161	3,5,6,7	350	350	*	*
162	4,5,6,7	360	252	CBA	CAB
163		M=5			
163	0,1,2,3,4	37	37	ABC	ACB
164	0,1,2,3,5	57	57	ABC	-
165	0,1,2,3,6	117	57	ACB	-
166	0,1,2,3,7	217	217	ABC	ACB
167	0,1,2,4,5	67	37	BAC	CAB
168	0,1,2,4,6	127	37	BCA	CBA
169	0,1,2,4,7	227	227	*	*
170	0,1,2,5,6	147	75	CBA	CAB
171	0,1,2,5,7	247	233	BAC	-
172	0,1,2,6,7	307	233	[illegible]	-
173	0,1,3,4,5	73	57	BAC	-
174	0,1,3,4,6	133	75	ACB	BCA

1	2	3	4	5	
175	0,1,3,4,7	233	233	ABC	-
176	0,1,3,5,6	153	153	ABC	BAC
177	0,1,3,5,7	253	253	ABC	BAC
178	0,1,3,6,7	313	255	ACB	-
179	0,1,4,5,6	163	57	CAB	-
180	0,1,4,5,7	263	217	BAC	CAB
181	0,1,4,6,7	323	233	CBA	-
182	0,1,5,6,7	343	255	CAB	-
183	0,2,3,4,5	75	75	ABC	BAC
184	0,2,3,4,6	135	57	BCA	-
185	0,2,3,4,7	235	233	ACB	-
186	0,2,3,5,6	155	153	ACB	BCA
187	0,2,3,5,7	255	255	ABC	-
188	0,2,3,6,7	315	253	ACB	BCA
189	0,2,4,5,6	165	57	CBA	-
190	0,2,4,5,7	265	233	CAB	-
191	0,2,4,6,7	325	217	BCA	CBA
192	0,2,5,6,7	345	255	CBA	-
193	0,3,4,5,6	171	153	CBA	CAB
194	0,3,4,5,7	271	255	BAC	-
195	0,3,4,6,7	331	255	BCA	-
196	0,3,5,6,7	351	351	*	*
197	0,4,5,6,7	361	253	CBA	CAB

1	2	3	4	5	
198	1,2,3,4,5	76	76	ABC	BAC
199	1,2,3,4,6	136	76	ACB	BCA
200	1,2,3,4,7	236	236	ABC	ACB
201	1,2,3,5,6	156	156	ABC	ACB
202	1,2,3,5,7	256	256	ABC	-
203	1,2,3,6,7	316	256	ACB	-
204	1,2,4,5,6	166	76	CBA	CAB
205	1,2,4,5,7	266	236	BAC	CAB
206	1,2,4,6,7	326	236	BCA	CBA
207	1,2,5,6,7	346	274	CBA	CAB
208	1,3,4,5,6	172	156	BAC	CAB
209	1,3,4,5,7	272	256	BAC	-
210	1,3,4,6,7	332	274	ACB	BCA
211	1,3,5,6,7	352	352	ABC	BAC
212	1,4,5,6,7	362	256	CAB	-
213	2,3,4,5,6	174	156	BCA	CBA
214	2,3,4,5,7	274	274	ABC	BAC
215	2,3,4,6,7	334	256	BCA	-
216	2,3,5,6,7	354	352	ACB	BCA
217	2,4,5,6,7	364	256	CBA	-
218	3,4,5,6,7	370	352	CBA	CAB

1	2	3	4	5	

N = 6

219	0,1,2,3,4,5	77	77	ABC	BAC
220	0,1,2,3,4,6	137	77	ACB	BCA
221	0,1,2,3,4,7	237	237	ABC	ACB
222	0,1,2,3,5,6	157	157	ABC	ACB
223	0,1,2,3,5,7	257	257	ABC	-
224	0,1,2,3,6,7	317	257	ACB	-
225	0,1,2,4,5,6	167	77	CBA	CAB
226	0,1,2,4,5,7	267	237	BAC	CAB
227	0,1,2,4,6,7	327	237	BCA	CBA
228	0,1,2,5,6,7	347	275	CBA	CAB
229	0,1,3,4,5,6	175	157	BAC	CAB
230	0,1,3,4,5,7	273	257	BAC	-
231	0,1,3,4,6,7	333	275	ACB	BCA
232	0,1,3,5,6,7	353	353	ABC	BAC
233	0,1,4,5,6,7	363	253	CAB	-
234	0,2,3,4,5,6	175	157	BCA	CBA
235	0,2,3,4,5,7	275	275	ABC	BAC
236	0,2,3,4,6,7	335	257	BCA	-
237	0,2,3,5,6,7	355	353	ACB	BCA
238	0,2,4,5,6,7	365	257	CBA	-

1	2	3	4	5	
239	0,2,4,5,6,7	371	353	CBA	CAB
240	1,2,3,4,5,6	176	176	*	*
241	1,2,3,4,5,7	276	276	ABC	BAC
242	1,2,3,4,6,7	336	276	ACB	BCA
243	1,2,3,5,6,7	356	356	ABC	ACB
244	1,2,4,5,6,7	366	276	CBA	CAB
245	1,3,4,5,6,7	372	356	BAC	CAB
246	2,3,4,5,6,7	374	356	BCA	CBA

<u>M = 7</u>

1	2	3	4	5	
247	0,1,2,3,4,5,6	177	177	*	*
248	0,1,2,3,4,5,7	277	277	ABC	BAC
249	0,1,2,3,4,6,7	337	277	ACB	BCA
250	0,1,2,3,5,6,7	357	357	ABC	ACB
251	0,1,2,4,5,6,7	367	277	CBA	CAB
252	0,1,3,4,5,6,7	373	357	BAC	CAB
253	0,2,3,4,5,6,7	375	357	BCA	CBA
254	1,2,3,4,5,6,7	376	376	*	*

TABLE NO. II

S. No.	ABC		ACB		BCA		BAC		CBA		CAB	
	Min terms	Octal no	Min terms	Octal no	Min terms	Octal no	Min terms	Octal no	Min terms	Octal no	Min terms	Octal no
1	2	3	4	5	6	7	8	9	10	11	12	13
					N = 1							
1	0	1	0	1	0	1	ø 0	1	0	1	0	1
2	1	2	2	4	4	20	1	2	4	20	2	4
3	2	4	1	2	1	2	4	20	2	4	4	20
4	3	10	3	10	5	40	5	40	6	100	6	100
5	4	20	4	20	2	4	2	4	1	2	1	2
6	5	40	6	100	6	100	3	10	5	40	3	10
7	6	100	5	40	3	10	6	100	3	10	5	40
8	7	200	7	200	7	200	7	200	7	200	7	200
					N = 2							
9	0,1	3	0,2	5	0,4	21	0,1	3	0,4	21	0,2	5
10	0,2	5	0,1	3	0,1	3	0,4	21	0,2	5	0,4	21
11	0,3	11	0,3	11	0,5	41	0,5	41	0,6	101	0,6	101
12	0,4	21	0,4	21	0,2	5	0,2	5	0,1	3	0,1	3
13	0,5	41	0,6	101	0,6	101	0,3	11	0,5	41	0,3	11
14	0,6	101	0,5	41	0,3	11	0,6	101	0,3	11	0,5	41
15	0,7	201	0,7	201	0,7	201	0,7	201	0,7	201	0,7	201
16	1,2	6	2,1	6	1,4	22	1,4	22	2,4	24	2,4	24
17	1,3	12	2,3	14	4,5	60	1,5	42	4,6	120	2,6	104
18	1,4	22	2,4	24	2,4	24	1,2	6	1,4	22	1,2	6
19	1,5	42	2,6	104	4,6	120	1,3	12	4,5	60	2,3	14
20	1,6	102	2,5	44	3,4	30	1,6	102	3,4,	30	2,5	44
21	1,7	202	2,7	204	4,7	220	1,7	202	4,7	220	2,7	204

1	2	3	4	5	6	7	8	9	10	11	12	13
22	2,3	14	1,3	12	1,5	42	4,5	30	2,6	104	4,6	120
23	2,4	24	1,4	22	1,2	6	2,4	24	1,2	6	1,4	22
24	2,5	44	1,6	102	1,6	102	3,4	30	2,5	44	3,4	30
25	2,6	104	1,5	42	1,3	12	4,6	120	2,3	14	4,5	60
26	2,7	204	1,7	202	1,7	202	4,7	220	2,7	204	4,7	220
27	3,4	30	3,4	30	2,5	44	2,5	44	1,6	102	1,6	102
28	3,5	50	3,6	110	5,6	140	3,5	50	5,6	140	3,6	110
29	3,6	110	3,5	50	3,5	50	5,6	140	3,6	110	5,6	140
30	3,7	210	3,7	210	5,7	240	5,7	240	6,7	300	6,7	300
31	4,5	60	4,6	120	2,6	104	2,3	14	1,5	42	1,3	12
32	4,6	120	4,5	6 0	2,3	14	2,6	104	1,3	12	1,5	42
33	4,7	220	4,7	220	2,7	204	2,7	204	1,7	202	1,7	202
34	5,6	140	5,6	140	3,6	110	3,6	110	3,5	50	3,5	50
35	5,7	240	6,7	300	6,7	300	3,7	210	5,7	240	3,7	210
36	6,7	300	5,7	240	3,7	210	6,7	300	3,7	210	5,7	240
					N = 3							
37	0,1,2	7	0,1,2	7	0,1,4	23	0,1,4	23	0,2,4	25	0,2,4	25
38	0,1,3	13	0,2,3	15	0,4,5	61	0,1,5	43	0,4,6	121	0,2,6	105
39	0,1,4	23	0,2,4	25	0,2,4	25	0,1,2	7	0,1,4	23	0,1,2	7

1	2	3	4	5	6	7	8	9	10	11	12	13
40	0,1,5	43	0,2,6	105	0,4,6	121	0,1,3	13	0,4,5	61	0,2,3	15
41	0,1,6	103	0,2,5	45	0,3,4	31	0,1,6	103	0,3,4	31	0,2,5	45
42	0,1,7	203	0,2,7	205	0,4,7	221	0,1,7	203	0,4,7	221	0,2,7	205
43	0,2,3	15	0,1,3,	13	0,1,5	43	0,4,5	61	0,2,6	105	0,4,6	121
44	0,2,4	25	0,1,4	23	0,1,2	7	0,2,4	25	0,1,2	7	0,1,4	23
45	0,2,5	45	0,1,6	103	0,1,6	103	0,3,4	31	0,2,5	45	0,3,4,	31
46	0,2,6	105	0,1,5	43	0,1,3	13	0,4,6	121	0,2,3	15	0,4,5	61
47	0,2,7	205	0,1,7	203	0 1,7	203	0,4,7	221	0,2,7	205	0,4,7	221
48	0,3,4	31	0,3,4	31	0,2,5	45	0,2,5	45	0,1,6	103	0,1,6	103
49	0 3,5	51	0,3,6	111	0,5,6	141	0,3,5	51	0,5,6	141	0,3,6	111
50	0,3,6	111	0,3,5	51	0,3,5	51	0,5,6	141	0,3,6	111	0,5,6	141
51	0,3,7	211	0,3,7	211	0,5,7	241	0,5,7	241	0,6,7	301	0,6,7	301
52	0,4,5	6 1	0,4,6	121	0,2,6	105	0,2,3	15	0,1,5	43	0,1,3	13
53	0,4,6	121	0,4,5	61	0,2,3	15	0,2,6	105	0,1,3	13	0,1,5	43
54	0,4,7	221	0,4,7	221	0,2,7	205	0,2,7	205	0.1.7	203	0,1,7	203
55	0.5,6	141	0,5,6	141	0,3,6	111	0,3,6	111	0,3,5	51	0,3,5	51
56	0,5,7	241	0,6,7	301	0,6,7	301	0,3,7	211	0,5,7	241	0,3,7	211
57	0,6,7	301	0,5,7	241	0,3,7	211	0,6,7	301	0,3,7	211	0,5,7	241
58	1,2,3	16	1,2,3	16	1,4,5	62	1,4,5	62	2,4,6	124	2,4,6	124
59	1,2,4	26	1,2,4	26	1,2,4	26	1,2,4	26	1,2,4	26	1,2,4	26
60	1,2,5	46	1,2,6	106	1,4,6	122	1,3,4	32	2,4,5	64	2,3,4	34

1	2	3	4	5	6	7	8	9	10	11	12	13
61	1,2,6	106	1,2,5	46	1,3,4	32	1,4,6	122	2,3,4	34	2,4,5	64
62	1,2,7	206	1,2,7	206	1,4,7	222	1,4,7	222	2,4,7	224	2,4,7	224
63	1,3,4	32	2,3,4	34	2,4,5	64	1,2,5	46	1,4,6	122	1,2,6	106
64	1,3,5	52	2,3,6	114	4,5,6	160	1,3,5	52	4,5,6	160	2,3,6	114
65	1,3,6	112	2,3,5	54	3,4,5	70	1,5,6	142	3,4,6	130	2,5,6	144
66	1,3,7	212	2,3,7	214	4,5,7	260	1,5,7	242	4,6,7	320	2,6,7	304
67	1,4,5	62	2,4,6	124	2,4,6	124	1,2,3	16	1,4,5	62	1,2,3	16
68	1,4,6	122	2,4,5	64	2,3,4	34	1,2,6	106	1,3,4	32	1,2,5	46
69	1,4,7	222	2,4,7	224	2,4,7	224	1,2,7	206	1,4,7	222	1,2,7	206
70	1,5,6	142	2,5,6	144	3,4,6	130	1,3,6	112	3,4,5	70	2,3,5	54
71	1,5,7	242	2,6,7	304	4,6,7	320	1,3,7	212	4,5,7	260	2,3,7	214
72	1,6,7	302	2,5,7	244	3,4,7	230	1,6,7	302	3,4,7	230	2,5,7	244
73	2,3,4	34	1,3,4	312	1,2,5	46	2,4,5	64	1,2,6	106	1,4,6	122
74	2,3,5	54	1,3,6	112	1,5,6	142	3,4,5	70	2,5,6	144	3,4,6	130
75	2,3,6	114	1,3,5	52	1,3,5	52	4,5,6	160	2,3,6	114	4,5,6	160
76	2,3,7	214	1,3,7	212	1,5,7	242	4,5,7	260	2,6,7	304	4,6,7	320
77	2,4,5	64	1,4,6	122	1,2,6	106	2,3,4	34	1,2,5	46	1,3,4	32
78	2,4,6	124	1,4,5	62	1,2,3	16	2,4,6	124	1,2,3	16	1,4,5	62
79	2,4,7	224	1,4,7	222	1,2,7	206	2,4,7	224	1,2,7	206	1,4,7	222
80	256	144	1,5,6	142	1,3,6	112	3,4,6	130	2,3,5	54	3,4,5	70

1	2	3	4	5	6	7	8	9	10	11	12	13
81	2,5,7	244	1,6,7	302	1,6,7	302	3,4,7	230	2,5,7	244	3,4,7	230
82	2,6,7	304	1,5,7	242	1,3,7	212	4,6,7	320	2,3,7	214	4,5,7	260
83	3,4,5	70	3,4,6	130	2,5,6	144	2,3,5	54	1,5,6	142	1,3,6	112
84	3,4,6	130	3,4,5	70	2,3,5	54	2,5,6	144	1,3,6	112	1,5,6	142
85	3,4,7	230	3,4,7	230	2,5,7	244	2,5,7	244	1,6,7	302	1,6,7	302
86	3,5,6	150	3,5,6	150	3,5,6	150	3,5,6	150	3,5,6	150	3,5,6	150
87	3,5,7	250	3,6,7	310	5,6,7	340	3,5,7	250	5,6,7	340	3,6,7	310
88	3,6,7	310	3,5,7	250	3,5,7	250	5,6,7	340	3,6,7	310	5,6,7	340
89	4,5,6	160	4,5,6	160	2,3,6	114	2,3,6	114	1,3,5	52	1,3,5	52
90	4,5,7	260	4,6,7	320	2,6,7	304	2,3,7	214	1,5,7	242	1,3,7	212
91	4,6,7	320	4,5,7	260	2,3,7	214	2,6,7	304	1,3,7	212	1,5,7	242
92	5,6,7	340	5,6,7	340	3,6,7	310	3,6,7	310	3,5,7	250	3,5,7	250

N = 4

1	2	3	4	5	6	7	8	9	10	11	12	13
93	0,1,2,3	17	0,1,2,3	17	0,1,4,5	63	0,1,4,5	63	0,2,4,6	125	0,2,4,6	125
94	0,1,2,4	27	0,1,2,4	27	0,1,2,4	27	0,1,2,4	27	0,1,2,4,	27	0,1,2,4	27
95	0,1,2,5	47	0,1,2,6	107	0,2,4,6	123	0,1,3,4	33	0,2,4,5,	65	0,2,3,4	35
96	0,1,2,6	107	0,1,2,5	47	0,1,3,4	33	0,1,4,6	123	0,2,3,4	35	0,2,4,5	65
97	0,1,2,7	207	0,1,2,7	207	0,1,4,7	223	0,1,4,7	223	0,2,4,7	225	0,2,4,7	225
98	0,1,3,4,	33	0,2,3,4	35	0,2,4,5	65	0,1,2,5,	47	0,1,4,6	123	0,1,2,6	107
99	0,1,3,5	53	0,2,3,6	115	0,4,5,6	161	0,1,3,5	53	0,4,5,6	161	0,2,3,6	115
100	0,1,3,6	113	0,2,3,5	55	0,3,4,5	71	0,1,5,6	143	0,3,4,6	131	0,2,5,6	145

	ABC		ACB		BCA		BAC		CBA		CAB	
1	2	3	4	5	6	7	8	9	10	11	12	13
101	0,1,3,7	213	0,2,3,7	215	0,4,5,7	261	0,1,5,7	243	0,4,6,7	321	0,2,6,7	305
102	0,1,4,5	63	0,2,4,6	125	0,2,4,6	121	0,1,2,3	17	0,1,4,5	63	0,1,2,3	17
103	0,1,4,6	123	0,2,4,5	65	0,2,3,4	35	0,1,2,6	107	0,1,3,4	33	0,1,2,5	47
104	0,1,4,7	223	0,2,4,7	225	0,2,4,7	225	0,1,2,7	207	0,1,4,7	223	0,1,2,7	207
105	0,1,5,6	143	0,2,5,6	145	0,3,4,6	131	0,1,3,6	113	0,3,4,5	71	0,2,3,5	55
106	0,1,5,7	243	0,2,6,7	305	0,4,6,7	321	0,1,3,7	213	0,4,5,7	261	0,2,3,7	215
107	0,1,6,7	303	0,2,5,7	245	0,3,4,7	231	0,1,6,7	303	0,3,4,7	231	0,2,5,7	245
108	0,2,3,4	35	0,1,3,4	33	0,1,2,5	45	0,2,4,5	65	0,1,2,6	107	0,1,4,6	123
109	0,2,3,5	55	0,1,3,6	113	0,1,5,6	143	0,3,4,5	71	0,2,5,6	145	0,3,4,6	131
110	0,2,3,6	115	0,1,3,5	53	0,1,3,5	53	0,4,5,6	161	0,2,3,6	115	0,4,5,6	161
111	0,2,3,7	215	0,1,3,7	213	0,1,5,7	243	0,4,5,7	261	0,2,6,7	305	0,4,6,7	321
112	0,2,4,5	65	0,1,4,6	123	0,1,2,6	107	0,2,3,4	35	0,1,2,5	47	0,1,3,4	33
113	0,2,4,6	125	0,1,4,5	63	0,1,2,3	17	0,2,4,6	125	0,1,2,3	17	0,1,4,5	63
114	0,2,4,7	225	0,1,4,7	223	0,1,2,7	207	0,2,4,7	225	0,1,2,7	207	0,1,4,7	223
115	0,2,5,6	145	0,1,5,6	143	0,1,3,6	113	0,3,4,6	131	0,2,3,5	55	0,3,4,5	71
116	0,2,5,7	245	0,1,6,7	303	0,1,6,7	303	0,3,4,7	231	0,2,5,7	245	0,3,4,7	231
117	0,2,6,7	305	0,1,5,7	243	0,1,3,7	213	0,4,6,7	321	0,2,3,7	215	0,4,5,7	261
118	0,3,4,5	71	0,3,4,6	131	0,2,5,6	145	0,2,3,5	55	0,1,5,6	143	0,1,3,6	113
119	0,3,4,6	131	0,3,4,5	71	0,2,3,5	55	0,2,5,6	145	0,1,3,6	113	0,1,5,6	143
120	0,3,4,7	231	0,3,4,7	231	0,2,5,7	245	0,2,5,7	245	0,1,6,7	303	0,1,6,7	303

	ABC		ACB		BCA		BAC		CBA		CAB	
1	2	3	4	5	6	7	8	9	10	11	12	13
121	0,3,5,6	151	0,3,5,6	151	0,3,5,6	151	0,3,5,6	151	0,3,5,6	151	0,3,5,6	151
122	0,3,5,7	251	0,3,6,7	311	0,5,6,7	341	0,3,5,7	251	0,5,6,7	341	0,3,6,7	311
123	0,3,6,7	311	0,3,5,7	251	0,3,5,7	251	0,5,6,7	341	0,3,6,7	311	0,5,6,7	341
124	0,4,5,6	161	0,4,5,6	161	0,2,3,6	115	0,2,3,6	115	0,1,3,5	53	0,1,3,5	53
125	0,4,5,7	261	0,4,6,7	321	0,2,6,7	305	0,2,3,7	215	0,1,5,7	245	0,1,3,7	215
126	0,4,6,7	321	0,4,5,7	261	0,2,3,7	215	0,2,6,7	305	0,1,3,7	215	0,1,5,7	245
127	0,5,6,7	341	0,5,6,7	341	0,3,6,7	311	0,3,6,7	311	0,3,5,7	251	0,3,5,7	251
128	1,2,3,4	36	1,2,3,4	36	1,2,4,5	66	1,2,4,5	66	1,2,4,6	126	1,2,4,6	126
129	1,2,3,5	56	1,2,3,6	116	1,4,5,6	162	1,3,4,5	72	2,4,5,6	164	2,3,4,6	134
130	1,2,3,6	116	1,2,3,5	56	1,3,4,5	72	1,4,5,6	162	2,3,4,6	134	2,4,5,6	164
131	1,2,3,7	216	1,2,3,7	216	1,4,5,7	262	1,4,5,7	262	2,4,6,7	324	2,4,6,7	324
132	1,2,4,5	66	1,2,4,6	126	1,2,4,6	126	1,2,3,4	36	1,2,4,5	66	1,2,3,4	36
133	1,2,4,6	126	1,2,4,5	66	1,2,3,4	36	1,2,4,6	126	1,2,3,4	36	1,2,4,5	66
134	1,2,4,7	226	1,2,4,7	226	1,2,4,7	226	1,2,4,7	226	1,2,4,7	226	1,2,4,7	226
135	1,2,5,6	146	1,2,5,6	146	1,3,4,6	132	1,3,4,6	132	2,3,4,5	74	2,3,4,5	74
136	1,2,5,7	246	1,2,6,7	306	1,4,6,7	322	1,3,4,7	232	2,4,5,7	264	2,3,4,7	234
137	1,2,6,7	306	1,2,5,7	246	1,3,4,7	232	1,4,6,7	322	2,3,4,7	234	2,4,5,7	264
138	1,3,4,5	72	2,3,4,6	134	2,4,5,6	164	1,2,3,5	56	1,4,5,6	162	1,2,3,6	116
139	1,3,4,6	132	2,3,4,5	74	2,3,4,5	74	1,2,5,6	146	1,3,4,6	132	1,2,5,6	146
140	1,3,4,7	232	2,3,4,7	234	2,4,5,7	264	1,2,5,7	246	1,4,6,7	322	1,2,6,7	306
141	1,3,5,6	152	2,3,5,6	154	3,4,5,6	170	1,3,5,6	152	3,4,5,6	170	2,3,5,6	154
142	1,3,5,7	252	2,3,6,7	314	4,5,6,7	360	1,3,5,7	252	4,5,6,7	360	2,3,6,7	314

	ABC		ACB		BCA		BAC		CBA		CAB	
1	2	3	4	5	6	7	8	9	10	11	12	13
143	1,3,6,7	312	2,3,5,7	254	3,4,5,7	270	1,3,6,7	342	3,4,6,7	330	2,5,6,7	344
144	1,4,5,6	162	2,4,5,6	164	2,3,4,6	134	1,2,3,6	116	1,3,4,5	72	1,2,3,5	56
145	1,4,5,7	262	2,4,6,7	324	2,4,6,7	324	1,2,3,7	216	1,4,5,7	262	1,2,3,7	216
146	1,4,6,7	322	2,4,5,7	264	2,3,4,7	234	1,2,6,7	306	1,3,4,7	232	1,2,5,7	246
147	1,5,6,7	342	2,5,6,7	344	3,4,6,7	330	1,3,6,7	312	3,4,5,7	270	2,3,5,7	254
148	2,3,4,5	74	1,3,4,6	132	1,2,5,6	146	2,3,4,5	74	1,2,5,6	146	1,3,4,6	132
149	2,3,4,6	134	1,3,4,5	72	1,2,3,5	56	2,4,5,6	164	1,2,3,6	116	1,4,5,6	162
150	2,3,4,7	234	1,3,4,7	232	1,2,5,7	246	2,4,5,7	264	1,2,6,7	306	1,4,6,7	322
151	2,3,5,6	154	1,3,5,6	152	1,3,5,6	152	3,4,5,6	170	2,3,5,6	154	3,4,5,6	170
152	2,3,5,7	254	1,3,6,7	312	1,5,6,7	342	3,4,57	270	2,5,6,7	344	3,4,6,7	330
153	2,3,6,7	314	1,3,5,7	252	1,3,5,7	252	4,5,6,7	360	2,3,6,7	314	4,5,6,7	360
154	2,4,5,6	164	1,4,5,6	162	1,2,3,6	116	2,3,4,6	134	1,2,3,5	56	1,3,4,5	72
155	2,4,5,7	264	1,4,6,7	322	1,2,6,7	306	2,3,4,7	234	1,2,5,7	246	1,3,4,7	232
156	2,4,6,7	324	1,4,5,7	262	1,2,3,7	216	2,4,6,7	324	1,2,3,7	216	1,4,5,7	262
157	2,5,6,7	344	1,5,6,7	342	1,3,6,7	312	3,4,6,7	330	2,3,5,7	254	3,4,5,7	270
158	3,4,5,6	170	3,4,5,6	170	2,3,5,6	154	2,3,5,6	154	1,3,5,6	152	1,3,5,6	152
159	3,4,5,7	270	3,4,6,7	330	2,5,6,7	344	2,3,5,7	254	1,5,6,7	342	2,3,6,7	314
160	3,4,6,7	330	3,4,5,7	270	2,3,5,7	254	2,5,6,7	344	1,3,6,7	312	1,5,6,7	342
161	3,5,6,7	350	3,5,6,7	350	3,5,6,7	350	3,5,6,7	350	3,5,6,7	350	3,5,6,7	350
162	4,5,6,7	360	4,5,6,7	360	2,3,6,7	314	2,3,6,7	314	1,3,5,7	252	1,3,5,7	252

<u>N = 5</u>

163	0,1,2,3,4,	57	0,1,2,3,4	57	0,1,2,4,5	67	0,1,2,4,5	67	0,1,2,4,6	127	0,1,2,4,6	127
164	0,1,2,3,5	57	0,1,2,3,6	117	0,2,4,5,6	165	0,1,3,4,5 73	163	0,2, ,3,6	135	0,2,3,4,6	135
165	0,1,2,3,6	117	0,1,2,3,5	57	0,1,3,4,5	73	0,1,4,5,6	163	0,2,3,4,6	135	0,2,4,5,6	165

	ABC		ACB		BCA		BAC		CBA		CAB	
1	2	3	4	5	6	7	8	9	10	11	12	13
166	0,1,2,3,7	217	0,1,2,3,7	217	0,1,4,5,7	263	0,1,4,5,7	263	0,2,4,6,7	325	0,2,4,6,7	325
167	0,1,2,4,5	67	0,1,2,4,6	127	0,1,2,4,6	127	0,1,2,3,4	37	0,1,2,4,5	67	0,1,2,3,4	37
168	0,1,2,4,6	127	0,1,2,4,5	67	0,1,2,3,4	37	0,1,2,4,6	127	0,1,2,3,4	37	0,1,2,4,5	67
169	0,1,2,4,7	227	0,1,2,4,7	227	0,1,2,4,7	227	0,1,2,4,7	227	0,1,2,4,7	227	0,1,2,4,7	227
170	0,1,2,5,6	147	0,1,2,5,6	147	0,1,3,4,6	133	0,1,3,4,6	133	0,2,3,4,5	75	0,2,3,4,5	75
171	0,1,2,5,7	247	0,1,2,67,	307	0,1,4,6,7	323	0,1,3,4,7	233	0,2,4,5,7	265	0,2,3,4,7	235
172	0,1,2,6,7	307	0,1,2,5,7	247	0,1,3,4,7	233	0,1,4,6,7	323	0,2,3,4,7	235	0,2,45,7	205
173	0,1,3,4,5	7[illegible]	0,2,3,4,6	135	0,2,4,5,6	165	0,1,2,3,5	57	0,1,4,5,6	163	0,1,2,3,6	117
174	0,1,3,4,6	133	0,2,3,4,5	75	0,2,3,4,5	75	0,1,2,5,6	147	0,1,3,4,6	133	0,1,2,5,6	147
175	0,1,3,4,7	233	0,2,3,4,7	235	0,2,4,5,7	265	0,1,2,5,7	247	0,1,4,6,7	323	0,1,2,6,7	307
176	0,1,3,5,6	153	0,2,3,5,6	155	0,3,4,5,6	171	0,1,3,5,6	153	0,3,4,5,6	171	0,2,3,5,6	155
177	0,1,3,5,7	253	0,2,3,6,7	315	0,4,5,6,7	361	0,1,3,5,7	253	0,4,5,6,7	361	0,2,3,6,7	315
178	0,1,3,6,7	313	0,2,3,5,7	255	0,3,4,5,7	271	0,1,5,6,7	34	0,3,4,6,7	331	0,2,5,6,7	345
179	0,1,4,5,6	163	0,2,4,5,6	165	0,2,3,4,6	135	01,2,3,6	117	0,1,3,4,5	73	0,1,2,3,5	57
180	0,1,4,5,7	263	0,2,4,6,7	325	0,2,4,6,7	325	0,1,2,3,7	217	0,1,4,5,7	263	0,1,2,3,7	217
181	0,1,4,6,7	323	0,2,4,5,7	265	0,2,3,4,7	235	0,1,2,6,7	307	0,1,3,4,7	233	0,1,2,5,7	247
182	0,1,5,6,7	343	0,2,5,6,7	345	0,3,4,6,7	331	0,1,3,6,7	313	0,3,4,5,7	271	0,2,3,5,7	255
183	0,2,3,4,5	75	0,1,3,4,6	133	0,1,2,5,6	147	0,2,3,4,5	75	0,1,2,5,6	147	0,1,3,4,6	133
184	0,2,3,4,6	135	0,1,3,4,5	73	0,1,2,3,5	57	0,2,4,5,6	165	0,1,2,3,6	117	0,1,4,5,6	163
185	0,2,3,4,7	235	0,1,3,4,7	233	0,1,2,5,7	247	0,2,4,5,7	265	0,1,2,6,7	307	0,1,4,6,7	323

1	2 ABC	3	4 ACB	5	6 BCA	7	8 BAC	9	10 CBA	11	12 CAB	13
186	0,2,3,5,6	155	0,1,3,5,6	153	0,1,3,5,6	153	0,3,4,5,6	171	0,2,3,5,6	155	0,3,4,5,6	171
187	0,2,3,5,7	255	0,2,3,6,7	313	0,2,5,6,7	343	0,3,4,5,7	271	0,2,5,6,7	345	0,3,4,6,7	331
188	0,2,3,6,7	315	0,1,3,5,7	253	0,1,3,5,7	253	0,4,5,6,7	361	0,2,3,6,7	315	0,4,5,6,7	361
189	0,2,4,5,6	165	0,1,4,5,6	165	0,1,2,3,6	117	0,2,3,4,6	135	0,1,2,3,5	57	0,1,3,4,5	73
190	0,2,4,5,7	265	0,1,4,6,7	323	0,1,2,6,7	307	0,2,3,4,7	235	0,1,2,5,7	247	0,1,3,4,7	233
191	0,2,4,6,7	325	0,1,4,5,7	263	0,1,2,3,7	217	0,2,4,6,7	325	0,1,2,3,7	217	0,1,4,5,7	263
192	0,2,5,6,7	345	0,1,5,6,7	343	0,1,3,6,7	313	0,3,4,6,7	331	0,2,3,5,7	255	0,3,4,5,7	271
193	0,3,4,5,6	171	0,3,4,5,6	171	0,2,3,5,6	155	0,2,3,5,6	155	0,1,3,5,6	153	0,1,3,5,6	153
194	0,3,4,5,7	271	0,3,4,6,7	331	0,2,5,6,7	345	0,2,3,5,7	255	0,1,5,6,7	343	0,2,3,6,7	313
195	0,3,4,6,7	331	0,3,4,5,7	271	0,2,3,5,7	255	0,2,5,6,7	345	0,1,3,6,7	313	0,1,5,6,7	343
196	0,3,5,6,7	351	0,3,5,6,7	351	0,3,5,6,7	351	0,3,5,6,7	351	0,3,6,7	351	0,3,5,6,7	351
197	0,4,5,6,7	361	0,4,5,6,7	361	0,2,3,6,7	315	0,2,3,6,7	315	0,1,5,7	253	0,1,3,5,7	253
198	1,2,3,4,5	76	1,2,3,4,6	136	1,2,4,5,6	166	1,2,3,4,5	76	1,2,4,5,6	166	1,2,3,4,6	136
199	1,2,3,4,6	136	1,2,3,4,5	76	1,2,3,4,5	76	1,2,4,5,6	166	1,2,3,4,6	136	1,2,4,5,6	166
200	1,2,3,4,7	236	1,2,3,4,7	236	1,2,4,5,7	266	1,2,4,5,7	266	1,2,4,6,7	326	1,2,4,6,7	326
201	1,2,3,5,6	156	1,2,3,5,6	156	1,3,4,5,6	172	1,3,4,5,6	172	2,3,4,5,6	174	2,3,4,5,6	174
202	1,2,3,5,7	256	1,2,3,6,7	316	1,4,5,6,7	362	1,3,4,5,7	272	2,4,5,6,7	364	2,3,4,6,7	334
203	1,2,3,6,7	316	1,2,3,5,7	256	1,3,4,5,7	272	1,4,5,6,7	362	2,3,4,6,7	334	2,4,5,6,7	364
204	1,2,4,5,6	166	1,2,4,5,6	166	1,2,3,4,6	136	1,2,3,4,6	136	1,2,3,4,5	76	1,2,3,4,5	76
205	1,2,4,5,7	266	1,2,4,6,7	326	1,2,4,6,7	326	1,2,3,4,7	236	1,2,4,5,7	266	1,2,3,9,7	236
206	1,2,4,6,7	326	1,2,4,5,7	266	1,2,3,4,7	236	1,2,4,6,7	326	1,2,3,4,7	236	1,2,4,5,7	266

	ABC		ACB		BCA		BAC		CBA		[illegible] CAB	
1	2	3	4	5	6	7	8	9	10	11	12	13
207	1,2,5,6,7	346	1,2,5,6,7	346	1,3,4,6,7	332	1,3,4,6,7	332	2,3,4,5,7	274	2,3,4,5,7	274
208	1,3,4,5,6	172	2,3,4,5,6	174	2,3,4,5,6	174	1,2,3,5,6	156	1,3,4,5,6	172	1,2,3,5,6	156
209	1,3,4,5,7	272	2,3,4,6,7	334	2,4,5,6,7	364	1,2,3,5,7	256	1,4,5,6,7	362	1,2,3,6,7	316
210	1,3,4,6,7	332	2,3,4,5,7	274	2,3,4,5,7	274	1,2,5,6,7	346	1,3,4,6,7	332	1,2,5,6,7	346
211	1,3,5,6,7	352	2,3,5,6,7	354	3,4,5,6,7	370	1,3,5,6,7	352	3,4,5,6,7	370	2,3,5,6,7	354
212	1,4,5,6,7	362	2,4,5,6,7	364	2,3,4,6,7	334	1,2,3,6,7	316	1,3,4,5,7	272	1,2,3,5,7	256
213	2,3,4,5,6	174	1,3,4,5,6	172	1,2,3,5,6	156	2,3,4,5,6	174	1,2,3,5,6	156	1,3,4,5,6	172
214	2,3,4,5,7	274	1,3,4,6,7	332	1,2,5,6,7	346	2,3,4,5,7	274	1,2,5,6,7	346	1,3,4,6,7	332
215	2,3,4,6,7	334	1,3,4,5,7	272	1,2,3,5,7	256	2,4,5,6,7	364	1,2,3,6,7	316	1,4,5,6,7	362
216	2,3,5,6,7	354	1,3,5,6,7	352	1,3,5,6,7	352	3,4,5,6,7	370	2,3,5,6,7	354	3,4,5,6,7	370
217	2,4,5,6,7	364	1,4,5,6,7	362	1,2,3,6,7	316	2,3,4,6,7	334	1,2,3,5,7	256	1,3,4,5,7	272
218	3,4,5,6,7	370	3,4,5,6,7	370	2,3,5,6,7	354	2,3,5,6,7	354	1,3,5,6,7	352	1,3,5,6,7	352
					N = 6							
219	0,1,2,3,4,5	77	0,1,2,3,4,6	137	0,1,2,4,5,6	167	0,1,2,3,4,5	77	0,1,2,4,5,6	167	0,1,2,3,4,6	137
220	0,1,2,3,4,6	137	0,1,2,3,4,5	77	0,1,2,3,4,5	77	0,1,2,4,5,6	167	0,1,2,3,4,6	137	0,1,2,4,5,6	167
221	0,1,2,3,4,7	237	0,1,2,3,4,7	237	0,1,2,4,5,7	267	0,1,2,4,5,7	267	0,1,2,4,6,7	327	0,1,2,4,6,7	327
222	0,1,2,3,5,6	157	0,1,2,3,5,6	157	0,1,3,4,5,6	173	0,1,3,4,5,6,	173	0,2,3,4,5,6	175	0,2,3,4,5,6	175
223	0,1,2,3,5,7	257	0,1,2,3,6,7	317	0,1,4,5,6,7	363	0,1,3,4,5,7	273	0,2,4,5,6,7	365	0,2,3,4,6,7	335
224	0,1,2,3,6 ,7,	317	0,1,2,3,5,7	257	0,1,3,4,5,7	273	0,2,4,5,6,7	363	0,2,3,4,6,7	335	0,2,4,5,6,7	365
225	0,1,2,4,5,6	167	0,1,2,4,5,6	167	0,1,2,3,4,6	137	0,1,2,3,4,6	137	0,1,2,3,4,5	77	0,1,2,3,4,5	77
226	0,1,2,4,5,7	267	0,1,2,4,6,7	327	0,1,2,4,6,7	327	0,1,2,3,4,7	237	0,1,2,4,5,7	267	0,1,2,3,4,7	237

	ABC		ACB		BCA		BAC		CBA		CAB	
1	2	3	4	5	6	7	8	9	10	11	12	13
227	0,1,2,4,6,7	327	0,1,2,4,5,7	267	0,1,2,3,4,7	237	0,1,2,4,6,7	327	0,1,2,3,4,7	237	0,1,2,4,5,7	267
228	0,1,2,5,6,7	347	0,1,2,5,6,7	347	0,1,3,4,6,7	333	0,1,3,4,6,7	333	0,2,3,4,5,7	275	0,2,3,4,5,7	275
229	0,1,3,4,5,6	173	0,2,3,4,5,6	175	0,2,3,4,5,6	175	0,1,2,3,5,6	157	0,1,3,4,5,6	173	0,1,2,3,5,6	157
230	0,1,3,4,5,7	273	0,2,3,4,6,7	335	0,2,4,5,6,7	365	0,1,2,3,5,7	257	0,1,4,5,6,7	363	0,1,2,3,6,7	317
231	0,1,3,4,6,7	333	0,2,3,4,5,7	275	0,2,3,4,5,7	275	0,1,2,5,6,7	347	0,1,3,4,6,7	333	0,1,2,5,6,7	347
232	0,1,3,5,6,7	353	0,2,3,5,6,7	355	0,3,4,5,6,7	371	0,1,3,5,6,7	353	0,3,4,5,6,7	371	0,2,3,5,6,7	355
233	0,1,4,5,6,7	363	0,2,4,5,6,7	365	0,2,3,4,6,7	335	0,1,2,3,6,7	317	0,1,3,4,5,7	273	0,1,2,3,5,7	257
234	0,2,3,4,5,6	175	0,1,3,4,5,6	173	0,1,2,3,5,6	157	0,2,3,4,5,6	175	0,1,2,3,5,6	157	0,1,3,4,5,6	173
235	0,2,3,4,5,7	275	0,1,3,4,6,7	333	0,1,2,5,6,7	347	0,2,3,4,5,7	275	0,1,2,5,6,7	347	0,1,3,4,6,7	333
236	0,2,3,4,6,7	335	0,1,3,4,5,7	273	0,1,2,3,5,7	257	0,2,4,5,6,7	365	0,1,2,3,6,7	317	0,1,4,5,6,7	363
237	0,1,2,3,5,6,7	355	0,1,3,5,6,7	353	0,1,3,5,6,7	353	0,3,4,5,6,7	371	0,2,3,5,6,7	355	0,3,4,5,6,7	371
238	0,2,4,5,6,7	363	0,1,4,5,6,7	363	0,1,2,3,6,7	317	0,2,3,4,6,7	335	0,1,2,3,5,7	257	0,1,3,4,5,7	273
239	0,3,4,5,6,7	371	0,3,4,5,6,7	371	0,2,3,5,6,7	355	0,2,3,5,6,7	355	0,1,3,5,6,7	353	0,1,3,5,6,7	353
240	1,2,3,4,5,6	176	1,2,3,4,5,6	176	1,2,3,4,5,6	176	1,2,3,4,5,6	176	1,2,3,4,5,6	176	1,2,3,4,5,6	176
241	1,2,3,4,5,7	276	1,2,3,4,6,7	336	1,2,4,5,6,7	366	1,2,3,4,5,7	276	1,2,4,5,6,7	366	1,2,3,4,6,7	336
242	1,2,3,4,6,7	336	1,2,3,4,5,7	276	1,2,3,4,5,7	276	1,2,4,5,6,7	366	1,2,3,4,6,7	336	1,2,4,5,6,7	366
243	1,2,3,5,6,7	356	1,2,3,5,6,7	356	1,3,4,5,6,7	372	1,3,4,5,6,7	372	2,3,4,5,6	374	2,3,4,5,6,7	374
244	1,2,4,5,6,7	366	1,2,4,5,6,7	366	1,2,3,4,6,7	336	1,2,3,4,6,7	336	1,2,3,4,5,7	276	1,2,3,4,5,7	276
245	1,3,4,5,6,7	372	2,3,4,5,6,7	374	2,3,4,5,6,7	374	1,2,3,5,6,7	356	1,3,4,5,6,7	372	1,2,3,5,6,7	356
246	2,3,4,5,6,7	374	1,3,4,5,6,7	372	1,2,3,5,6,7	356	2,3,4,5,6,7	374	1,2,3,5,6,7	356	1,3,4,5,6,7	372

	ABC		ACB		BCA		BAC		CBA		CAB	
1	2	3	4	5	6	7	8	9	10	11	12	13
						N = 7						
247	0,1,2,3,4,5,6	177	0,1,2,3,4,5,6	177	0,1,2,3,4,5,6	177	0,1,2,3,4,5,6	177	0,1,2,3,4,5,6	177	0,1,2,3,4,5,6	177
248	0,1,2,3,4,5,7	277	0,1,2,3,4,6,7	337	0,1,2,4,5,6,7	367	0,1,2,3,4,5,7	277	0,1,2,4,5,6,7	367	0,1,2,3,4,6,7	337
249	0,1,2,3,4,6,7	337	0,1,2,3,4,5,7	277	0,1,2,3,4,5,7	277	0,1,2,4,5,6,7	367	0,1,2,3,4,6,7	337	0,1,2,4,5,6,7	367
250	0,1,2,3,5,6,7	357	0,1,2,3,5,6,7	357	0,1,3,4,5,6,7	373	0,1,3,4,5,6,7	373	0,2,3,4,5,6,7	375	0,2,3,4,5,6,7	375
251	0,1,2,4,5,6,7	367	0,1,2,4,5,6,7	367	0,1,2,3,4,6,7	337	0,1,2,3,4,6,7	337	0,1,2,3,4,5,7	277	0,1,2,3,4,5,7	277
252	0,1,3,4,5,6,7	373	0,2,3,4,5,6,7	375	0,2,3,4,5,6,7	375	0,1,2,3,5,6,7	357	0,1,3,4,5,6,7	373	0,1,2,3,5,6,7	357
253	0,2,3,4,5,6,7	375	0,1,3,4,5,6,7	373	0,1,2,3,5,6,7	357	0,2,3,4,5,6,7	375	0,1,2,3,5,6,7	357	0,1,3,4,5,6,7	373
254	1,2,3,4,5,6,7	376	1,2,3,4,5,6,7	376	1,2,3,4,5,6,7	376	1,2,3,4,5,6,7	376	~~0~~,1,2,3,4,5,6,7	376	1,2,3,4,5,6,7	376

LEGENED FOR TABLE III

Sign	Changes
N	No change of inputs
O	Any of the five changes given below
P	Change B to C and C to B (A is same) - BC
Q	Change A to B, B to C , C to A - ABC
R	Change A to B, B to A, C is same - AB
S	A to C, C to A, B is same - AC
T	Change A to C, B to A, C to B - ACB

USAGE OF TABLE

For many a numbers two combinations are given like for , No. 40 , corresponding is Q1OR , hence its leader is 10 and 40 is got by any combination Q or R . Similarly any number could be looked in.

—

TABLE III

	0	1	2	3	4	5	6	7
	-	01	N2R	N3R	P2T	P3T	N6P	N7P
10	N10P	N11P	N12-	N13-	P12-	P13-	N16P	N17P
20	Q2S	Q3S	Q6R	Q7R	S6T	S7T	Q26-	Q27-
30	N30P	N31P	N32-	N33-	P32-	P33-	N36P	N37P
40	Q10R	Q11R	R12-	R13-	Q30R	Q31R	R32-	R33-
50	N50R	N51R	N52R	N53R	N54-	N55-	N56-	N57-
60	Q12-	Q13-	Q16R	Q17R	Q32-	Q33-	Q36R	Q37R
70	Q54-	R55-	R56-	R57-	N74R	N75R	N76R	N77R
100	S10T	S11T	S30T	S31T	T12-	T13-	T32-	T33-
110	P50T	P51T	P54-	P55-	P52T	P53T	P56-	P57-
120	S12-	S13-	S32-	S33-	S16T	S17T	S36T	S37T
130	T54-	T55-	P74T	P75T	T56	T57	P76T	P77T
140	Q50S	Q51S	R54-	Q55-	S54-	S55-	Q74S	Q75S
150	0150-	0151-	N152R	N153R	P152T	P153T	N156P	N157P
160	Q52S	Q53S	Q56-	Q57-	S56-	S57-	Q76S	Q77S
170	Q152S	Q153S	Q156R	Q157R	S156T	S157T	0176-	0177-

Contd...

	0	1	2	3	4	5	6	7
200	O200-	O201-	N202R	N203R	P202S	P203T	N206P	N207P
210	N210P	N211P	N212-	N213-	P212-	P213-	N216P	N217P
220	Q202T	Q203S	Q206R	Q207R	S206T	S207T	O226-	O227-
230	N230P	N231P	N232-	N233-	P232-	P233-	N236P	N237P
240	Q210R	Q211R	R212-	R213-	Q230R	Q231R	R232-	R233-
250	N250R	N251R	N252R	N253R	N254-	N255-	N256-	N257-
260	Q212-	Q213-	Q216R	Q217R	Q232-	Q233-	Q236R	Q237R
270	R254-	R255-	R256-	R257-	N274R	N275R	N276R	N277R
300	S210T	S211T	S230T	S231T	T212-	T213-	T232-	T233-
310	P250T	P251T	P254-	P255-	P252T	P253T	P256-	P257-
320	S212-	S213-	S232-	S233-	S216T	S217T	S236T	S237T
330	T254-	T255-	P274T	P275T	T256-	T257-	P276T	P277T
340	Q250S	Q251S	Q254-	Q255-	S254-	S255-	Q274S	Q275S
350	O350-	O351-	N352R	N353R	P352T	P353T	N356P	N357P
360	Q252S	Q253S	Q256-	Q257-	S256-	S257-	Q276S	Q277S
370	Q352S	Q353S	Q356R	Q357R	S356T	S357T	O376-	

TABLE IV

Partition leader	Minimal form in AND- OR	NAND FORM	Circuit No.	NOR FORM	Ci N
1	$\bar{A}\bar{B}\bar{C}$	$(\bar{A}\mid\bar{B}\mid\bar{C})\mid$	10	$(A\downarrow B\downarrow C)$	2
2	$\bar{A}\bar{B}C$	$(\bar{A}\mid\bar{B}\mid C)\mid$	10	$(A\downarrow B\downarrow\bar{C})$	2
3	$\bar{A}\bar{B}$	$(\bar{A}\mid\bar{B})\mid$	3	$(A\downarrow B)$	1
6	$\bar{A}\bar{B}C+\bar{A}B\bar{C}$	$(\bar{A}\mid\bar{B}\mid C)\mid(\bar{A}\mid B\mid\bar{C})$	11	$(B\downarrow C)\downarrow(\bar{B}\downarrow\bar{C})\downarrow\bar{A}$	7
7	$\bar{A}(B+\bar{C})$	$[(B\mid C)\mid\bar{A}]\mid$	5	$(\bar{B}\downarrow\bar{C})\downarrow A$	4
10	$\bar{A}BC$	$(\bar{A}\mid B\mid C)\mid$	10	$(A\downarrow\bar{B}\downarrow\bar{C})$	2
11	$\bar{A}\bar{B}\bar{C}+\bar{A}BC$	$(\bar{A}\mid\bar{B}\mid\bar{C})\mid(\bar{A}\mid B\mid C)$	11	$(B\downarrow\bar{C})\downarrow(\bar{B}\downarrow C)\downarrow\bar{A}$	7
12	$\bar{A}C$	$(\bar{A}\mid C)\mid$	3	$(A\downarrow\bar{C})$	1
13	$\bar{A}(\bar{B}+C)$	$[(B\mid\bar{C})\mid\bar{A}]\mid$	5	$(\bar{B}\downarrow C)\downarrow A$	4
16	$\bar{A}C+\bar{A}B$	$[(\bar{B}\mid\bar{C})\mid\bar{A}]\mid$	5	$(B\downarrow C)\downarrow A$	4
17	$\bar{A}$	–	0	–	0
26	$(\bar{A}\bar{B}C+A\bar{B}\bar{C}+\bar{A}B\bar{C})$	$(\bar{A}\mid\bar{B}\mid C)\mid(A\mid\bar{B}\mid\bar{C})\mid(\bar{A}\mid B\mid\bar{C})$	12	$(A\downarrow B\downarrow C)\downarrow(\bar{A}\downarrow\bar{C})\downarrow(\bar{B}\downarrow\bar{C})\downarrow(\bar{A}\downarrow\bar{B})$	16
27	$(\bar{A}\bar{B}+\bar{B}\bar{C}+\bar{A}\bar{C})$	$[(B\mid C)\mid\bar{A}]\mid(\bar{B}\mid\bar{C})$	8	$[(B\downarrow C)\downarrow\bar{A}]\downarrow[(\bar{B}\downarrow\bar{C})]$	8
30	$(\bar{A}BC+A\bar{B}\bar{C})$	$(\bar{A}\mid B\mid C)\mid(A\mid\bar{B}\mid\bar{C})$	11	$(A\downarrow B)\downarrow(\bar{A}\downarrow\bar{C})\downarrow(\bar{B}\downarrow C)$	9
31	$(\bar{B}\bar{C}+\bar{A}BC)$	$(\bar{B}\mid\bar{C})\mid(\bar{A}\mid B\mid C)$	14	$[(A\downarrow\bar{B})\downarrow\bar{C}]\downarrow(\bar{B}\downarrow C)$	8
32	$(\bar{A}C+A\bar{B}\bar{C})$	$(A\mid C)\mid(A\mid\bar{B}\mid\bar{C})$	14	$[(\bar{A}\downarrow\bar{C})\downarrow B]\downarrow[\bar{A}\downarrow B]$	8
33	$(\bar{B}\bar{C}+\bar{A}C)$	$(\bar{B}\mid\bar{C})\mid(\bar{A}\mid C)$	6	$(\bar{A}\downarrow\bar{C})\downarrow(\bar{B}\downarrow C)$	6
36	$(\bar{A}C+\bar{A}B+A\bar{B}\bar{C})$	$(\bar{B}\mid\bar{C})\mid\bar{A}\mid(A\mid\bar{B}\mid\bar{C})$	15	$[(B\downarrow C)\downarrow\bar{A}]\downarrow(A\downarrow B\downarrow C)$	15
37	$(\bar{A}+\bar{B}\bar{C})$	$(\bar{B}\mid\bar{C})\mid(A)$	4	$[(B\downarrow C)\downarrow\bar{A}]\downarrow$	5
50	$(\bar{A}BC+A\bar{B}C)$	$(\bar{A}\mid B\mid C)\mid(A\mid\bar{B}\mid C)$	11	$(\bar{A}\downarrow\bar{B})\downarrow(A\downarrow B)\downarrow\bar{C}$	7
51	$(\bar{A}\bar{B}\bar{C}+\bar{A}BC+A\bar{B}C)$	$(\bar{A}\mid\bar{B}\mid\bar{C})\mid(\bar{A}\mid B\mid C)\mid(A\mid\bar{B}\mid\bar{C})$	12	$(A\downarrow B\downarrow\bar{C})\downarrow(\bar{A}\downarrow C)\downarrow(\bar{A}\downarrow\bar{B})\downarrow(\bar{B}\downarrow C)$	16
52	$(\bar{A}C+\bar{B}C)$	$[(A\mid B)\mid C]\mid$	5	$(\bar{A}\downarrow\bar{B})\downarrow C$	4
53	$(\bar{A}\bar{B}+$ [illegible] $C+\bar{B}C)$	$[(A\mid B)\mid C]\mid[\bar{A}\mid\bar{B}]$	8	$[B\downarrow\bar{C}\downarrow\bar{A}]\downarrow[B\downarrow C]$	8
54	$(\bar{A}B+A\bar{B}C)$	$(\bar{A}\mid B)\mid(A\mid\bar{B}\mid C)$	14	$[\bar{C}\downarrow\bar{A}]\downarrow\bar{B}]\downarrow[\bar{A}\downarrow B]$	8
55	$(\bar{A}B+\bar{A}\bar{C}+A\bar{B}C)$	$[(\bar{B}\mid C)\mid\bar{A}]\mid[A\mid\bar{B}\mid\bar{C}]$	15	$[(B\downarrow\bar{C})\downarrow\bar{A}]\downarrow[A\downarrow B\downarrow\bar{C}]$	15
56	$(\bar{A}B+\bar{B}C)$	$(\bar{A}\mid B)\mid(\bar{B}\mid C)$	6	$(B\downarrow C)\downarrow(\bar{A}\downarrow\bar{B})$	6
57	$(\bar{A}+\bar{B}C)$	$(\bar{B}\mid C)\mid(A)$	4	$[(B\downarrow\bar{C})\downarrow\bar{A}]\downarrow$	5
74	$(\bar{A}B+A\bar{B})$	$(A\mid\bar{B})\mid(\bar{A}\mid B)$	6	$(A\downarrow B)\downarrow(\bar{A}\downarrow\bar{B})$	6
75	$(\bar{A}B+\bar{B}\bar{C}+A\bar{B})$	$[(\bar{B}\mid\bar{C})\mid\bar{A}]\mid[\bar{B}\mid\bar{C}]$	8	$[A\downarrow B\downarrow\bar{C}]\downarrow[\bar{A}\downarrow\bar{B}]$	14
76	$(\bar{A}B+\bar{B}C+A\bar{B})$	$[(\bar{C}\mid\bar{A})\mid\bar{B}]\mid[\bar{A}\mid B]$	8	$[A\downarrow B\downarrow\bar{C}]\downarrow[\bar{A}\downarrow\bar{B}]$	14
77	$(\bar{A}+\bar{B})$	$(A\mid B)$	1	$(\bar{A}\downarrow\bar{B})\downarrow$	3
150	$(AB\bar{C}+A\bar{B}C+\bar{A}BC)$	$(A\mid B\mid\bar{C})\mid(A\mid\bar{B}\mid C)\mid(\bar{A}\mid B\mid C)$	12	$(\bar{A}\downarrow\bar{B}\downarrow\bar{C})\downarrow(A\downarrow C)\downarrow(B\downarrow C)\downarrow(A\downarrow B)$	16

1	2	3	4	5	6
151	$(\bar{A}\bar{B}\bar{C}+A\bar{B}\bar{C}+\bar{A}BC+AB\bar{C})$	$(\bar A\mid\bar B\mid\bar C)\mid(A\mid\bar B\mid C)\mid(\bar A\mid B\mid C)\mid(A\mid B\mid\bar C)$	15	$(A\downarrow B\downarrow\bar C)\downarrow(A\downarrow\bar B\downarrow C)\downarrow(\bar A\downarrow B\downarrow C)\downarrow(\bar A\downarrow\bar B\downarrow\bar C)$	13
152	$(\bar AC+\bar BC+AB\bar C)$	$[(A\mid B)\mid C]\mid[A\mid B\mid\bar C]$	15	$(\bar A\downarrow\bar B)\downarrow\bar C\downarrow(\bar A\downarrow\bar B\downarrow\bar C)$	15
153	$(\bar A\bar B+\bar AC+\bar BC+ABC)$	$(\bar A\mid\bar B)\mid(\bar A\mid C)\mid(\bar B\mid C)\mid(A\mid B\mid\bar C)$	16	$(\bar A\downarrow\bar B\downarrow\bar C)\downarrow(\bar A\downarrow B\downarrow C)\downarrow(A\downarrow\bar B\downarrow C)$	12
156	$(\bar BC+B\bar C+\bar AB)$	$[(A\mid C)\mid X]\mid(\bar B\mid C)$	8	$(\bar A\downarrow\bar B\downarrow\bar C)\downarrow(B\downarrow C)$	14
157	$(\bar A+\bar BC+B\bar C)$	$(\bar B\mid C)\mid(B\mid\bar C)\mid A$	7	$(\bar A\downarrow B\downarrow C)\downarrow(\bar A\downarrow\bar B\downarrow\bar C)$	11
176	$(A\bar B+A\bar C+B\bar C)$	$(A\mid\bar B)\mid(A\mid\bar C)\mid(B\mid\bar C)$	9	$[A\downarrow B\downarrow C]\downarrow[\bar A\downarrow\bar B\downarrow C]$	11
177	$(\bar A+\bar B+\bar C)$	$(A\mid B\mid C)$	2	$(\bar A\downarrow\bar B\downarrow\bar C)\downarrow$	10
200	ABC	$(A\mid B\mid C)\mid$	10	$(\bar A\downarrow\bar B\downarrow\bar C)\downarrow$	10 2
201	$(\bar A\bar B\bar C+ABC)$	$(\bar A\mid\bar B\mid\bar C)\mid(A\mid B\mid C)$	11	$(\bar A\downarrow B)\downarrow(A\downarrow\bar C)\downarrow(\bar B\downarrow C)$	9
202	$(\bar A\bar BC+ABC)$	$(\bar A\mid\bar B\mid C)\mid(A\mid B\mid C)$	11	$(\bar A\downarrow B)\downarrow(A\downarrow\bar B)\downarrow\bar C$	7
203	$(\bar A\bar B+ABC)$	$(\bar A\mid\bar B)\mid(A\mid B\mid C)$	14	$[(\bar A\downarrow\bar C)\downarrow\bar B]\downarrow[\bar A\downarrow B]$	8
206	$(\bar A\bar BC+\bar AB\bar C+ABC)$	$(\bar A\mid\bar B\mid C)\mid(\bar A\mid B\mid\bar C)\mid(A\mid B\mid C)$	12	$(A\downarrow\bar B\downarrow\bar C)\downarrow(B\downarrow C)\downarrow(\bar A\downarrow B)\downarrow(\bar A\downarrow C)$	16
207	$(\bar A\bar C+\bar A\bar B+ABC)$	$[(B\mid C)\mid\bar A]\mid[A\mid B\mid C]$	15	$[(\bar B\downarrow\bar C)\downarrow\bar A]\downarrow[A\downarrow\bar B\downarrow\bar C]$	15
210	(BC)	$(B\mid C)\mid$	3	$\bar B\downarrow\bar C$	1
211	$(BC+\bar A\bar B\bar C)$	$(B\mid C)\mid(\bar A\mid\bar B\mid\bar C)$	14	$[(A\downarrow C)\downarrow B]\downarrow[B\downarrow C]$	8
212	$(\bar AC+BC)$	$[(A\mid\bar B)\mid C]\mid$	5	$(\bar A\downarrow B)\downarrow\bar C$	4
213	$(\bar A\bar B+BC)$	$(\bar A\mid\bar B)\mid(B\mid C)$	6	$(\bar A\downarrow B)\downarrow(\bar B\downarrow C)$	6
216	$(\bar AC+\bar AB+BC)$	$[(A\mid\bar C)\mid B]\mid[\bar A\mid C]$	8	$[(C\downarrow A)\downarrow B]\downarrow[(\bar A\downarrow C)]$	8
217	$(\bar A+BC)$	$(B\mid C)\mid A$	4	$(\bar B\downarrow\bar C)\downarrow\bar A$	5
226	$(\bar A\bar BC+\bar AB\bar C+A\bar B\bar C+ABC)$	$(\bar A\mid\bar B\mid C)\mid(\bar A\mid B\mid\bar C)\mid(\bar A\mid\bar B\mid\bar C)\mid(A\mid B\mid C)$	13	$(A\downarrow B\downarrow C)\downarrow(A\downarrow\bar B\downarrow\bar C)\downarrow(\bar A\downarrow B\downarrow\bar C)\downarrow(\bar A\downarrow\bar B\downarrow\bar C)$	13
227	$(\bar A\bar B+\bar A\bar C+\bar B\bar C+ABC)$	$(\bar A\mid\bar B)\mid(\bar A\mid\bar C)\mid(\bar B\mid\bar C)\mid(A\mid B\mid C)$	16	$(A\downarrow\bar B\downarrow\bar C)\downarrow(\bar A\downarrow\bar B\downarrow C)\downarrow(\bar A\downarrow B\downarrow\bar C)$	12
230	$BC+A\bar B\bar C$	$(B\mid C)\mid(A\mid\bar B\mid\bar C)$	14	$[(A\downarrow\bar B)\downarrow\bar C]\downarrow[(B\downarrow C)]$	8
231	$BC+\bar B\bar C$	$(B\mid C)\mid(\bar B\mid\bar C)$	6	$[(\bar B\downarrow C)\downarrow(B\downarrow\bar C)]$	6
232	$(\bar AC+BC+A\bar B\bar C)$	$[(A\mid B)\mid C]\mid[A\mid B\mid C]$	15	$(A\downarrow B\downarrow C)\downarrow(A\downarrow B\downarrow C)$	15
233	$(BC+\bar B\bar C+\bar AC)$	$[(\bar B\mid A)\mid C]\mid[B\mid C]$	8	$[A\downarrow B\downarrow\bar C]\downarrow[B\downarrow C]$	14
236	$(\bar AB+\bar AC+BC+A\bar B\bar C)$	$(\bar A\mid B)\mid(\bar A\mid C)\mid(B\mid C)\mid(A\mid\bar B\mid\bar C)$	16	$(\bar A\downarrow B\downarrow\bar C)\downarrow(\bar A\downarrow\bar B\downarrow C)\downarrow(A\downarrow B\downarrow C)$	12
237	$(\bar A+\bar B\bar C+BC)$	$(\bar B\mid\bar C)\mid(B\mid C)\mid A$	7	$(\bar A\downarrow B\downarrow\bar C)\downarrow(\bar A\downarrow\bar B\downarrow C)$	11
250	AC+BC	$[(\bar A\mid\bar B)\mid C]\mid$	5	$(A\downarrow B)\downarrow\bar C)$	4
251	$(BC+AC+\bar A\bar B\bar C)$	$[(\bar A\mid\bar B)\mid C]\mid[\bar A\mid\bar B\mid\bar C]$	15	$[(A\downarrow B)\downarrow C]\downarrow[A\downarrow B\downarrow\bar C]$	15

			4	5	6	7
		–	0	–	0	
--	$C+\overline{AB}$	$(\overline{A}\mid\overline{B})\mid\overline{C}$	4	$[(A\downarrow B)\downarrow C]\downarrow$	5	
254	$(\overline{A}B+AC)$	$(\overline{A}\mid B)\mid(A\mid C)$	6	$(\overline{A}\downarrow B)\downarrow(\overline{A}\downarrow C)$	6	
255	$(\overline{AC}+AC+BC)$	$[(\overline{A}\mid\overline{B})\mid C]\mid[\overline{A}\mid\overline{C}]$	8	$[A\downarrow B\downarrow\overline{C}]\downarrow[\overline{A}\downarrow\overline{C}]$	14	
256	$(C+\overline{A}B)$	$(\overline{A}\mid B)\mid\overline{C}$	4	$[(A\downarrow\overline{B})\downarrow C]\downarrow$	5	
257	$(\overline{A}+C)$	$(A\mid\overline{C})$	1	$(\overline{A}\downarrow C)\downarrow$	5	
274	$(A\overline{B}+\overline{A}B+BC)$	$[(A\mid\overline{C})\mid B]\mid[A\mid\overline{B}]$	8	$[(\overline{A}\downarrow\overline{B})\downarrow C]\downarrow[A\downarrow B]$	14	
275	$(\overline{A}B+AC+\overline{BC})$	$(\overline{B}\mid\overline{C})\mid(\overline{A}\mid B)\mid(A\mid C)$	9	$(A\downarrow B\downarrow C)\downarrow[\overline{A}\downarrow\overline{B}\downarrow\overline{C}]$	11	
276	$(C+\overline{A}B+A\overline{B})$	$(A\mid\overline{B})\mid(\overline{A}\mid B)\mid\overline{C}$	7	$(A\downarrow B\downarrow C)\downarrow(\overline{A}\downarrow\overline{B}\downarrow C)$	11	
277	$(\overline{A}+\overline{B}+C)$	$(A\mid B\mid\overline{C})$	2	$(\overline{A}\downarrow\overline{B}\downarrow C)\downarrow$	10	
350	$(AC+BC+AB)$	$[(\overline{B}\mid\overline{C})\mid\overline{A}]\mid[B\mid C]$	8	$[\overline{A}\downarrow\overline{C}\downarrow B]\downarrow[A\downarrow C]$	8	
351	$(\overline{ABC}+AB+AC+BC)$	$(A\mid B)\mid(A\mid C)\mid(B\mid C)\mid(\overline{A}\mid\overline{B}\mid\overline{C})$	16	$(A\downarrow B\downarrow\overline{C})\downarrow(A\downarrow\overline{B}\downarrow C)\downarrow(\overline{A}\downarrow B\downarrow C)$	12	
352	$(C+AB)$	$(A\mid B)\mid\overline{C}$	4	$[(\overline{A}\downarrow\overline{B})\downarrow C]\downarrow$	5	
353	$(C+\overline{AB}+AB)$	$(\overline{A}\mid\overline{B})\mid(A\mid B)\mid\overline{C}$	7	$(A\downarrow\overline{B}\downarrow C)\downarrow(\overline{A}\downarrow B\downarrow C)$	11	
356	$(B+C)$	$(\overline{B}\mid\overline{C})$	1	$(B\downarrow C)\downarrow$	3	
357	$(\overline{A}+B+C)$	$(A\mid\overline{B}\mid\overline{C})$	2	$(\overline{A}\downarrow B\downarrow C)\downarrow$	10	
376	$(A+B+C)$	$(\overline{A}\mid\overline{B}\mid\overline{C})$	2	$(A\downarrow B\downarrow C)\downarrow$	10	