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Certified that the attached dissertation on "INVESTIGATIONS OF DRAG C.P ROTOR MOTOR"

was submitted by

V. K. Jain

and accepted for the award of Degree of Master of Engineering in

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Assistant Registrar (Exam.)
S.S.
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INVESTIGATIONS OF DRAG CUP ROTOR MOTOR

A Dissertation
submitted in partial fulfilment
of the requirements for the Degree
of
MASTER OF ENGINEERING
in
ADVANCED ELECTRICAL MACHINES

by
V.K. JAIN

DEPARTMENT OF ELECTRICAL ENGINEERING
UNIVERSITY OF ROORKEE
ROORKEE
September, 1966

C E R T I F I C A T E.

Certified that the dissertation entitled 'Investigations on Drag Cup Rotor Motor' which is being submitted by Sri V.K. Jai in partial fulfilment for the award of the Degree of Master of Engineering in Advanced Electrical Machines of the University of Roorkee, is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for a period of 8 months from January '66 to August '66 for preparing dissertation for Master of Engineering Degree at the University.

Roorkee 30th Sept. 1966

(P. Mukhopadhyay)
Associate Professor,
Electrical Engg. Deptt.,
University of Roorkee,
Roorkee.

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(V.K. Jain)

I N D E X.

ABSTRACT.

In the present work an attempt has been made to develop three dimensional analysis (taking field to be three dimensional) of drag-cup rotor machine. Expressions for combined impedance of airgaps, rotor and central iron core in terms of primary or stator using both rectangular and cylindrical co-ordinate systems have been derived. Radially directed component of magnetic vector potential or rotor currents has been neglected. Effect of rob sections of rotor have been neglected. A comparision of results of two dimensional analysis both in rectangular and cylindrical co-ordinates and those of three dimensional analysis in rectangular co-ordinates as well as the experimental ones has been done. It has been shown that for the machine tested that curvature has little effect except at low values of slips. Further the case of mechanical transients has been studied with an eye to non-linearity of torque speed curve.

LIST OF SYMBOLS

Unless otherwise specified MKS system of units has been used.

The symbols defined below have been used without defining them. They have the meaning defined below unless otherwise specified.

A	=	magnetic vector potential
	=	area under one pole
B	=	flux density vector wb/m^2
B_1	=	$N_1 Kw/D$
B_g	=	air gap flux density
D	=	average rotor diameter
E	=	electric field intensity volts/m
E_g	=	air gap voltage
F_1, F_2, F_3		are function of x, y & z only
H	=	magnetic field intensity Amp/m
I_1, I_2	=	peak value of stator current
Kw	=	$K_d \cdot K_p$ = winding distribution factor
N_1, N_{ph}	=	No. of turns in series per phase
P	=	No. of poles pairs
R_2	=	resistive part of $Z_{gap\text{-}rotor}$
R_1	=	stator winding resistance
τ	=	pole pitch
Z_x, Z_z	=	turn distribution per phase in x and z directions respectively.
V_1, V_2	=	stator terminal voltage on reference & control phases respectively.
X_1	=	Stator leakage reactance
X_2	=	reactive part of $Z_{gap\text{-}rotor}$

A_1, B_1, A_2, B_2, C_1, D_1, D, & A_m, B_m are constants of integration.

a = $p/D = \pi/\tau$

c_1, c_2, c_3 —— are constants of integration.

d = Gap thickness meters

f = frequency in C/S

i = current density Amp/m^2

j = $\sqrt{-1}$

k = π/τ

n = no. of air gap

n = order of harmonic

g n = no. of air gap

p = no. of poles

s = slip in per unit

t = time seconds

ω = angular velocity

μ = permeability of rotor material

= $\mu_r \mu_0$

σ = electrical conductivity of rotor material 1/ohm-m

ρ = resistivity of rotor material ohm - m

λ = machine length in axial direction

CHAPTER - 1

INTRODUCTION

The actuator requirements of a.c. servomechanisms are most satisfactorily met by two-phase induction motors. When used as servomotors, one of the phase windings is connected to a fixed a.c. voltage, the reference voltage, and the other winding supplied by a variable control voltage, in space quadrature with the fixed voltage. The direction of rotation is governed by the polarity of the control voltage. Torque developed is a function of the magnitudes of both the voltages.

The two phase servomotor is a specially designed induction motor with a high ratio of rotor resistance to reactance so as to obtain a sloping torque speed characteristic. The two phase servomotors have the unique and most important feature of high torque to weight ratio and quick response. The requirement of high torque to inertia ratio is best met by drag-cup motors.

The drag-cup motor probably is named after the tachometer devices normally used in automatic speed meters. The stator has a standard distributed winding to obtain nearly sinusoidal space distribution. The rotor of such motors consists of two parts; the central iron core, which is stationary and the rotor conductor (drag-cup) which rotates. The drag-cup fits into the air space between the stator winding and the stationary central iron core, the clearances being kept as small as possible. The revolving field is produced by the two-phase winding on the stator and the magnetic circuit is completed through the central core which is independent of moving system. Torque is produced by the interaction of eddy currents flowing in the cup and the flux. Such machines are used as servomotors, tachogenerators and accelerometers.

The analysis of such devices is complicated due to the presence of two air gaps and the consideration of skin effect which cannot be neglected if the cup is thick as in higher rating machines. Further as the path of eddy currents is not simple, validity of circuit theory approach is doubtful. An accurate analysis must, therefore, be based on electromagnetic field theory.

1.2. REVIEW OF EARLIER WORK

Fillmore(1)(1956) made a study of eddy current paths in the drag-cup induction motor rotors. He assumed cup to be very thin and, therefore, neglected skin effect. He obtained expressions for rotor resistance and average power dissipated in the cup.

When the thickness of the cup is increased as in higher rating machines, skin effect plays an important role and therefore cannot be neglected. Cuilford(2)(1962) considered skin effect and obtained a rigourous mathematical analysis using the vector potential concept. He neglected the circumferentially directed component of magnetic vector potential on the ground that it makes no contribution to energy flow as measured by Poynting vector. Radial component of vector potential has little effect on power transfer except for fringing effects at ends and has been neglected. He obtained expressions for induced voltage in the stator winding and the developed torque. Cuilford's analysis is for a sleeve-rotor machine which is a single air gap machine.

Wood and Concordia have analysed the solid rotor machine. They have studied the effect of curvature(3)(1959) and indicated that with non-magnetic rotors the results may be deviating from the actual ones. Rectangular co-ordinates give fairly good results when rotor used is magnetic and has a large radius. They have considered the finite length effect[4] (1959). They have considered both the circumferential and axial components of vector potential.

In an excellent work by Angst[5] (1962) solid rotor machine has been analysed by field theory. He considers fully both the circumferential and axial component of vector potential. He has derived a so-called end effect factor of the machine.

As the tangential components of rotor currents are loss producing components, they have been taken into account by Koch[6] (1966). He neglects both axial and circumferential components of flux density. He derives equivalent circuit with transformer elements of the drag-cup machine.

The characteristic problem in the analysis of drag-cup rotor motors by field theory is that the presence of sleeve and web sections of rotor result in field equations with differential co-efficients and co-ordinates. The effect of web section has been taken into account by several authors. It is well taken into account by Koch[6]. He has solved for both sections separately and synthesised to obtain the result. A classical work of calculating the resistance of the drag-cup in parts and then synthesising though by circuit theory approach has been done by Fuller and Frickey[7] (1962).

In a most recent work by Blachford[8] (1965) expressions for rotor resistance and reactance, stator to rotor mutual inductance have been derived for a drag-cup machine. Here a comparision of distributed and lumped parameter equations has been done in order to obtain the results. The equivalent circuit representation is simple. He considers field to be two dimensional and magnetic vector potential to be axially directed. He also accounts for rotor end effects using the Fuller and Trickey's method of calculating the resistance of the cup in two parts.

All the above mentioned work except (3) by Wood and Concordia has been done using rectangular co-ordinate system rather than the natural cylinderical co-ordinate system. As indicated earlier the results may be deviating when non-magnetic rotors with small radius of curvature are employed. With this point in view Mukhopadhyay[9] (1965) presented an analysis of drag-cup machine in cylinderical co-ordinate system. He considers field to be two dimensional and neglects end effect. An expression for combined ampere turn of the rotor, air gaps and central core has been found. From this the combined equivalent impedance of gaps, rotor and central core can be obtained. Beauty of the approach is that the equivalent circuit of the machine becomes extremely simple making the performance calculation easier.

CHAPTER - 2

2.0 DIMENSIONAL ANALYSIS IN RECTANGULAR CO-ORDINATES [illegible] ON [illegible] CONCEPT

2.1.1. Geometry of the Machine-

Fig.2.1. shows the geometry of the machine with drag-cup rotor. r_1, r_2, r_3 and r_4 are radii at stator bore, rotor outer and inner surfaces and rotor core respectively. Y axis dimensions are as indicated in fig.

2.1.2. Assumptions-

Idealised machine shown in fig. 2.1. is used for the purpose of analysis. The following assumptions are made:

(i) The stator iron and rotor core have infinite permeability.

(ii) Hysteresis and saturation effects are neglected.

(iii) Stator winding produces only forward travelling field.

(iv) Stator and rotor core can be developed into flat, infinitely long bodies.

(v) Mumfor's method(10) of representing the stator winding by thin axially directed current sheets against a smooth stator surface is used. Effect of slotting is taken in account by Carter's co-efficient.

(vi) The field is two dimensional, i.e., magnetic intensity is independent of co-ordinate Z and electric intensity has component only in Z direction.

2.1.3. Field Equations

Maxwell's equations for electromagnetic field for case in the medium are, in M.K.S. units;

$$\text{Curl } H = J \qquad \ldots \qquad (2.1)$$

$$\text{Curl } E = -\frac{\partial H}{\partial t}\cdot\mu = -\frac{\partial B}{\partial t} \qquad \ldots \qquad (2.2)$$

$$\text{div } B = 0 \quad (\text{div } H = 0) \quad \ldots \quad (2.3)$$

$$\text{div } E = 0 \quad (\text{div } i = 0) \quad \ldots \quad (2.4)$$

In equation (2.1) the displacement current density term is dropped for the induction machine since the magnetic energy storage in the air gap is several orders of magnitude greater than electric storage.

The magnetic vector potential A is defined by-

$$\text{curl } A = B \quad \ldots \quad (2.5)$$

In the conducting region ohm's law holds.

$$E = \rho i \quad \ldots \quad (2.6)$$

Combining (2.1) and (2.5),

$$\frac{1}{\mu} \text{curl } (\text{curl } A) = i$$

From the vector identity-

$$\text{Curl } (\text{curl } A) \equiv \nabla (\nabla . A) - \nabla^2 A,$$

one obtains-

$$\nabla (\nabla . A) - \nabla^2 A = \mu i.$$

Now since div A = 0, the above equation simplifies to-

$$\nabla^2 A = \mu \sigma \frac{\partial A}{\partial t} \quad \ldots \quad (2.7)$$

Equation (2.7) is Poisson's equation applicable to rotor conductor. For air gap, since the conductivity $\sigma = 0$, we have the Laplace's equation-

$$\nabla^2 A = 0 \quad \ldots \quad (2.8)$$

Equations similar to (2.7) and (2.8) hold for other quantities such as B and E as well.

where V_1 is the stator terminal voltage.

This combined equivalent impedance is a function of machine constants and slip. The behaviour of the machine can therefore be readily assessed from equivalent circuit given in fig.2.2.

This equivalent circuit representation though extremely simple is not general since R_2 and X_2 both are functions of slip.

2.2. THE DRAG-CUP MACHINE IN CYLINDRICAL CO-ORDINATE SYSTEM

Mukhopadhyay(9) carried out an analysis of drag-cup rotor machine using cylindrical co-ordinate system. Assumptions are as in analysis in rectangular. Co-ordinates. He obtained the following expression of combined impedance term of the rotor, air gap and core-

$$Z = -j\,\frac{\omega_0 r_1}{p\,\mu_0}\cdot\frac{a_{11} r_1^{2p} + 1}{a_{11} r_1^{2p} + 1}$$

where,

$$a_{11} = \frac{r_2^{\,p-1}\,(a_6\,a_9 + a_{10}) - \mu_r r_2^{\,p-1}(a_6\,a_7 + a_8)}{\mu_r r_2^{\,-p-1}(a_6 a_7 + a_8) + r_2^{\,-p-1}\,(a_6\,a_9 + a_{10})}$$

$$a_{10} = j\,\frac{\alpha}{\omega}\,\left(Y_{p-1}(\alpha\, r_2) - Y_{p+1}\,(\alpha\, r_2)\right)$$

$$a_9 = j\,\frac{\alpha}{\omega}\left(J_{p-1}(\alpha\, r_2) - J_{p+1}\,(\alpha\, r_2)\right)$$

$$a_8 = \frac{\alpha}{\omega}\,\left(Y_{p-1}(\alpha\, r_2) + Y_{p+1}\,(\alpha\, r_2)\right)$$

$$a_7 = \frac{\alpha}{\omega}\,\left(J_{p-1}(\alpha\, r_2) + J_{p+1}\,(\alpha\, r_2)\right)$$

$$a_6 = \frac{a_5(r_3^{2p} - a_1) - \mu_r(r_3^{2p} + a_1)\,a_3}{\mu_r a_2\,.\,(r_3^{2p} + a_1) - a_4(r_3^{2p} - a_1)}$$

a_5, a_4, a_3, a_2 = Values of a_{10}, a_9, a_8 and a_7 respectively with r_2 replaced by r_3.

2.1.4. Boundary conditions-

At the boundary between second air gap and rotor iron H_x vanishes. At the boundaries between conductor and air gap By and H_x are continuous. Continuity in H_x is destroyed at the stator surface where H_x is equal to surface current density.

At $y = \pm g_3$, $H_x = 0$ ... (2.9)

$$\text{At } y = \pm g_2,\quad \left.\begin{aligned} H_{x\,air} &= H_{x\,rotor} \\ B_{y\,air} &= B_{y\,rotor} \end{aligned}\right\} \quad \ldots \quad (2.10)$$

$$\text{At } y = \pm g_1,\quad \left.\begin{aligned} H_{x\,air} &= H_{x\,rotor} \\ B_{y\,air} &= B_{y\,rotor} \end{aligned}\right\} \quad \ldots \quad (2.11)$$

At $y = \pm g_0$, $H_x = I_0$ ... (2.12)

2.1.5. Analysis-

R.M.K.S. system of units is used.

Laplace's equation-

$$\nabla^2 B_x = \frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2} = 0 \quad \ldots \quad (2.13)$$

holds for air gap regions.

Divergence equation-

$$\nabla . B = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0 \quad \ldots \quad (2.14)$$

holds every where.

For rotor Poisson's equation-

$$\nabla^2 E_z = \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} = \mu\sigma \frac{\partial E_z}{\partial t} \quad (2.15)$$

holds good.

Solution of equations (2.13), (2.14) and (2.15) can be effected by the method of separation of variables.

Air gap- In order to solve (2.13) let us assume a solution of the form-

$$B_x = F_1(x) \quad F_2(y)$$

where,

$F_1(x)$ and $F_2(y)$ are functions of x & y only respectively.

Substituting this in (2.13) one obtains-

$$F_1'' F_2 + F_2'' F_1 = 0$$

or,

$$\frac{F_1''}{F_1} = - \frac{F_2''}{F_2} = - k^2 \quad \text{where } k = \text{constant.}$$

Solution for F_1 and F_2 will be written as-

$$F_1 = c_1 \cos kx$$

$$F_2 = c_2 \operatorname{Exp}(ky) + c_3 \operatorname{Exp}(-ky)$$

since the variation in x direction is sinusoidal and that in y direction is exponential. Further,

$$k = \frac{\pi}{\tau}, \; \tau \text{ being the pole pitch.}$$

The complete solution of B_x is given by-

$$B_x = \cos kx \, (A_n \operatorname{Exp}(-ky) + B_n \operatorname{Exp}(ky)) \qquad (2.16a)$$

From (2.14)-

$$B_y = - \int \frac{\partial B_x}{\partial x} dy = \sin kx \, (B_n \operatorname{Exp}(ky) - A_n \operatorname{Exp}(-ky)) \quad \ldots \quad (2.16b)$$

n in the above equations represent the number of air gap.

Rotor core - In rotor core flux density should not become infinite at the centre. Solution for rotor core, therefore becomes,

$$B_x = A_3 \cos kx \operatorname{Exp}(-ky) \quad \ldots \quad (2.17a)$$

$$B_y = -A_3 \sin kx \operatorname{Exp}(-ky) \quad \ldots \quad (2.17b)$$

and $A_1 = a_3 B_1$ where $a_3 = \dfrac{m-kq}{\exp(2kg_1)(m+kq)}$

and $q = \dfrac{1+a_2 \cdot \exp(2mg_1)}{1-a_2 \cdot \exp(2mg_1)}$

At stator surface,

B_x (air gap) $= (A_1 + B_1)\cos kx$

B_y (air gap) $= \sin kx\,(B_1 - A_1)$

If N represents the peak value of combined ampere turn of air gap, rotor and rotor core, we have-

$AT = N \sin kx$

$\therefore B_x = \mu_0 \dfrac{\partial (AT)}{\partial x} = \mu_0 N k \cos kx$

Variation of flux density in y direction is sinusoidal, we have therefore,

$B_y = B_0 \sin kx$, B_0 being the peak value of air gap flux density.

From these one obtains,

$$N = \frac{1+a_3}{1-a_3} \cdot \frac{B_0}{\mu_0 k}$$

Air gap flux density B_0 is related to air gap voltage by the expression-

$$E_1 = 4.44\, K_w\, f\, B_0\, T_{ph} \cdot A$$

where symbols have usual meaning. Thus-

$$N = \frac{1+a_3}{1-a_3} \cdot \frac{E_1}{4.44\, K_w\, f\, T_{ph}\, \mu_0\, k \cdot A}$$

From this combined equivalent impedance of the rotor, air gaps and core in terms of primary can be found as-

$$Z_{eq} = \frac{V_1 \cdot T_{ph}}{N}$$

where V_1 is the stator terminal voltage.

This combined equivalent impedance is a function of machine constants and slip. The behaviour of the machine can therefore be readily assessed from equivalent circuit given in fig.2.2.

This equivalent circuit representation though extremely simple is not general since R_2 and X_2 both are functions of slip.

2.2. TWO DIMENSIONAL ANALYSIS IN CYLINDRICAL CO-ORDINATE SYSTEM

Mukhopadhyay(9) carried out an analysis of drag-cup rotor machine using cylindrical co-ordinate system. Assumptions are as in analysis in rectangular. Co-ordinates. He obtained the following expression of combined ampere turn of the rotor, air gaps and core-

$$M = -j \frac{B_a r_1}{P \mu_0} \cdot \frac{a_{11} r_1^{2P} + 1}{a_{11} r_1^{2P} + 1}$$

where,

$$a_{11} = \frac{r_2^{P-1}(a_6 a_9 + a_{10}) - \mu_r r_2^{P-1}(a_6 a_7 + a_8)}{\mu_r r_2^{-P-1}(a_6 a_7 + a_8) + r_2^{-P-1}(a_6 a_9 + a_{10})}$$

$$a_{10} = j \frac{\alpha}{\omega} \left(Y_{P-1}(\alpha r_2) - Y_{P+1}(\alpha r_2)\right)$$

$$a_9 = j \frac{\alpha}{\omega} \left(J_{P-1}(\alpha r_2) - J_{P+1}(\alpha r_2)\right)$$

$$a_8 = \frac{\alpha}{\omega} \left(Y_{P-1}(\alpha r_2) + Y_{P+1}(\alpha r_2)\right)$$

$$a_7 = \frac{\alpha}{\omega} \left(J_{P-1}(\alpha r_2) + J_{P+1}(\alpha r_2)\right)$$

$$a_6 = \frac{a_5(r_3^{2P} - a_1) - \mu_r(r_3^{2P} + a_1)\, a_3}{a_2 \mu_r (r_3^{2P} + a_1) - a_4(r_3^{2P} - a_1)}$$

a_5, a_4, a_3, a_2 = Values of a_{10}, a_9, a_8 and a_7 respectively with r_2 replaced by r_3.

$$a_1 = -\frac{\mu_r' - 1}{\mu_r' + 1} r_4^{2P}$$

and $\alpha^2 = j\omega s \mu \sigma$

From this expression for ampere turn performance calculation is done in the manner indicated earlier.

CHAPTER - 3

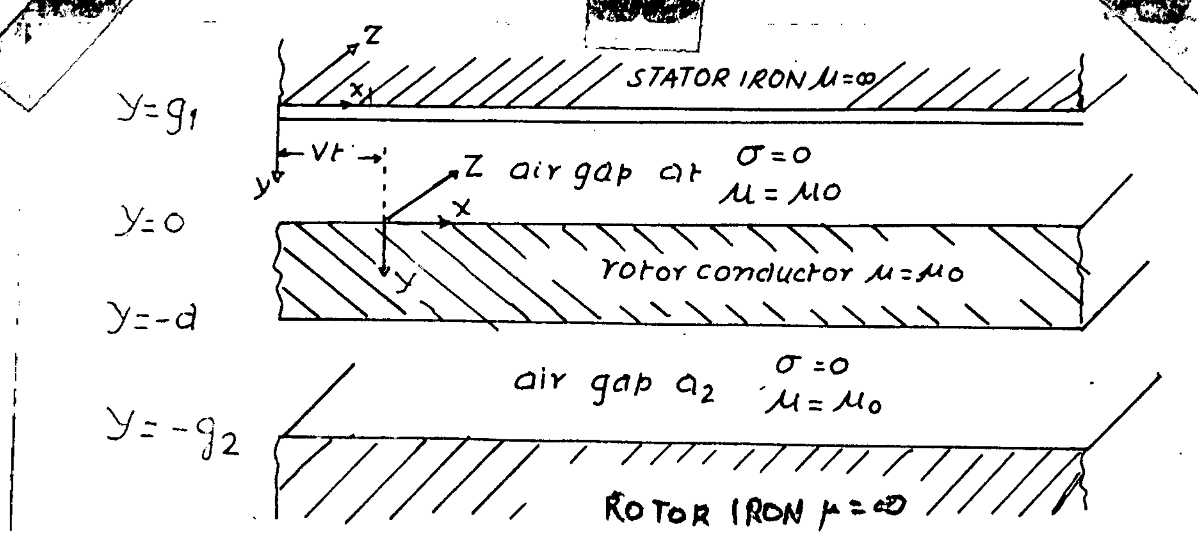
THREE DIMENSIONAL ANALYSIS IN RECTANGULAR COORDINATE SYSTEM

3.1.1. INTRODUCTION:

The previous chapter dealt with a two-dimensional analysis of the machine. This chapter deals with the three-dimensional analysis of the machine. The electromagnetic approach is based on vector potential concept. The radial component of vector potential or rotor current has been neglected. It has been shown that the equivalent circuit representation is complex as it requires summation of a series and the rotor resistance and reactance are both functions of slip.

3.1.2. GEOMETRY OF THE MACHINE:

Fig. 3.1 shows the geometry of the machine rolled out flat. The co-ordinate system (x_1, y, z) is attached to the stator. The co-ordinate system (x, y, z) is attached to moving cup. δ is the thickness of cup.



IDEALISED DRAG-CUP MACHINE

FIG - 3 1

3.1.3. Assumptions:

Assumptions (i) to (v) of article 2.1.2. are applicable here also.

(vi) Radial component of magnetic vector potential or rotor currents have been neglected. This would in turn mean neglecting the fringing effects which take place at the ends.

3.1.4. [illegible]

The wave due to current sheet at stator surface rotates at an angular velocity . Motion of rotor is given by-

$$[illegible] \int \frac{\omega(1-s)}{p} dt \; .$$

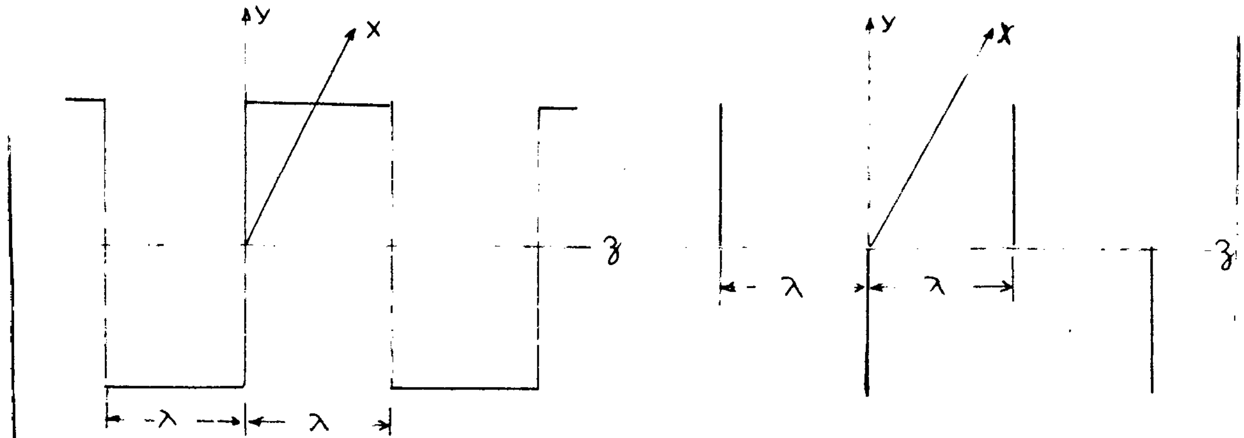
$$\therefore x_1 = x - \frac{D}{P}\int (1-s)\,\omega\, dt$$

$$= x - (1-s) \cdot \frac{\omega t}{a} \qquad \ldots \qquad (3.1)$$

where, $a = \frac{P}{D} = \frac{\pi}{\tau}$

The actual stator winding is replaced by linear current sheet in circumferential direction. This has a sinusoidal variation in circumferential direction and is of constant amplitude in axial direction. For the purpose of analysis the machine may be considered to repeat itself axially with alternate polarity. The current sheet in axial direction may then be expressed by Fourier series. Now since circumferential current sheet is also present it must also have periodic distribution expressible by Fourier series. This can be seen from Kirchhoff's law $\nabla \cdot I = 0$. The circumferential current sheet is solely confined to the end surfaces of the machine. Fig.3.2a and fig.3.2b show the two current sheets.

Writing mathematically, axial current sheet,



AXIAL CURRENT SHEET
FIG-3.2a

TANGENTIAL CURRENT SHEET
FIG 3.2 b.

$$I_{cz} = \frac{4}{n\pi}\left(\frac{3}{2} B_1 I_1\right) \exp[j(\alpha x_1 + \omega t)] \sum_{n=1,3..} \sin\frac{n\pi z}{\lambda} \quad \ldots \quad (3.2)$$

where, B_1 = amplitude of sinusoidal winding distribution

$= N_1 k_1/D$.

From divergence relation $\nabla \cdot I_c = 0$, tangential current sheet,

$$I_{cx} = -\int \frac{\partial I_{cz}}{\partial z} dx$$

$$= j\frac{4}{\alpha\lambda}\left(-\frac{3}{2} B_1 I_1\right) \exp[j(\alpha x_1 + \omega t)] \sum_{n=1,3..} \cos\frac{n\pi z}{\lambda} + C_n(z) \quad \ldots \quad (3.3)$$

where,

$C_n(z)$ is a function of z.

Because of symmetry it can be shown that $C_n(z) = 0$.

Solution for air gap- As shown in article 2.1.3., we have for air gap the Laplace's equation-

$$\nabla^2 A_{cx_1} = \frac{\partial^2 A_{cx1}}{\partial x_1^2} + \frac{\partial^2 A_{cx1}}{\partial y^2} + \frac{\partial^2 A_{cx1}}{\partial z^2} = 0 \quad (3.4)$$

$$\nabla^2 A_{Gz} = \frac{\partial^2 A_{Gz}}{\partial x_1^2} + \frac{\partial^2 A_{Gz}}{\partial y^2} + \frac{\partial^2 A_{Gz}}{\partial z^2} = 0 \quad (3.5)$$

$$A_{Gy} = 0 \quad \ldots \quad (3.6)$$

A solution to (3.4) can be obtained by the method of separation of variables. Assuming a solution for A_G

$$A_{Gz}(x, y, z, t) = \exp(j\omega t)\, X(x)\, Y(y)\, Z(z)$$

Putting it in (3.4),

$$\frac{X''}{X} + \frac{Y''}{Y} = -\frac{Z''}{Z} \quad \ldots \quad (3.7)$$

Variation of A_G is periodic in z direction and sinusoidal in x direction.

$$\therefore \frac{Z''}{Z} = -\frac{n^2\pi^2}{\lambda^2} \quad \ldots \quad (3.8)$$

$$\frac{X''}{X} = -a^2 \quad \ldots \quad (3.9)$$

Solutions to (3.8) and (3.9) are–

$$Z = c_1 \sin \frac{n\pi z}{\lambda}$$

and $X = c_2 \exp(jax_1)$

From (3.7), (3.8) and (3.9),

$$\frac{Y''}{Y} = \frac{n^2\pi^2}{\lambda^2} + a^2 = \beta_n^2 \text{ (say)}$$

Its solution is–

$$Y = c_3 \cosh \beta_n y + c_4 \sinh \beta_n y$$

Complete solution for A_{Gz} is

$$A_{Gz} = \exp\{j(ax_1 + \omega t)\} \{\sum_{n=1,3..} \{A_{nm} \cosh\beta_n y + B_{nm} \sinh\beta_n y\} \sin \frac{n\pi z}{\lambda} \quad \ldots \quad (3.10)$$

From $\nabla \cdot A_G = \frac{\partial A_{Gx1}}{\partial x_1} + \frac{\partial A_{Gz}}{\partial z} = 0$, we have

$$A_{cx_1} = - \int \frac{\partial A_{zm}}{\partial z} \, dx_1$$

$$= \sum_{n=1,3..} \frac{jn\pi}{a\lambda} \exp \left[j(ax_1 + \omega t) \right] (A_{nm} \cosh \beta_n y + B_{nm} \sinh \beta_n y) \, \mathrm{Cos} \, \frac{n\pi z}{\lambda} + D_n(x) \qquad (3.11)$$

the constant of integration $D_n(x)$ reducing to zero because of symmetry. m (=1,2) in above equations represents a number of airgap.

<u>Solution for rotor</u>: As shown in article 2.1.3. Poisson's and divergence equations are applicable for rotor.

$$\nabla^2 A_{rz} = \frac{\partial^2 A_{rz}}{\partial x^2} + \frac{\partial^2 A_{rz}}{\partial y^2} + \frac{\partial^2 A_{rz}}{\partial z^2} = \mu\sigma \frac{\partial A_{rz}}{\partial t} \qquad (3.12)$$

$$\nabla^2 A_{rx} = \frac{\partial^2 A_{rx}}{\partial x^2} + \frac{\partial^2 A_{rx}}{\partial y^2} + \frac{\partial^2 A_{rx}}{\partial z^2} = \mu\sigma \frac{\partial A_{rx}}{\partial t} \qquad (3.13)$$

$$\nabla \cdot A_r = \frac{\partial A_{rx}}{\partial x} + \frac{\partial A_{rz}}{\partial z} = 0 \quad \ldots \qquad (3.14)$$

$$A_{ry} = 0 \quad \ldots \qquad (3.15)$$

A solution to (3.12) may be obtained by the method of separation of variables. Assuming a solution of the form

$$A_{rz} \, (x, y, z, t) = \exp \, (j\omega st) \; X(x) \; Y(y) \; Z(z)$$

Substituting in (3.12),

$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = j \omega s \mu \sigma \quad \ldots \qquad (3.16)$$

Variation of A_{rz} in x and z directions is sinusoidal and periodic respectively.

$$\therefore \; \frac{X''}{X} = - a^2 \quad \ldots \qquad (3.17)$$

$$\frac{Z''}{Z} = - \frac{n^2 \pi^2}{\lambda^2} \quad \ldots \qquad (3.18)$$

Solutions to (3.17) and (3.18) are-

$$X = c_5 \exp(jax)$$

$$Z = c_6 \sin \frac{n\pi z}{\lambda}$$

From (3.16), (3.17) and (3.18),

$$\frac{Y''}{Y} = j\omega s\mu\sigma + a^2 + \frac{n^2\pi^2}{\lambda^2} = \gamma_n^2 \text{ (say)}$$

$$\therefore Y = c_7 \cosh \gamma_n y + c_8 \sinh \gamma_n y$$

complete solution for A_{rz} is-

$$A_{rz} = \sum_{n=1,3..} \exp\{j(ax + \omega st)\}\{(C_n \cosh \gamma_n y + D_n \sinh \gamma_n y) \times \sin \frac{n\pi z}{\lambda} \quad \ldots \quad (3.19)$$

$$A_{rx} = -\int \frac{\partial A_{rz}}{\partial z}\, dx, \text{ from (3.14)}$$

$$= \sum_{n=1,3..} j\frac{n\pi}{\lambda a} \exp\{j(ax + \omega st)\}(C_n \cosh \gamma_n y + D_n \sinh \gamma_n y) . \cos \frac{n\pi z}{\lambda} \quad \ldots \quad (3.20)$$

The constants of integration $A_{nm}(m = 1,2)$, B_{nm} $(m = 1,2)$, C_n and D_n must be determined from boundary conditions.

<u>Boundary conditions:</u> At the boundaries between conductor and airgaps H_x and B_z are continuous. At the rotor core surface H_x vanishes and at the stator surface discontinuity in H_x is equal to stator surface current density.

(a) At $y = -c_2$, $H_{x\,air} = 0$ ... (3.21)

(b) At $y = -d$, $H_{x\,air} = H_{x\,rotor}$ ... (3.22a)

$A_{z\,air} = A_{z\,rotor}$ ... (3.22b)

(c) At $y = 0$, $H_{x\,air} = H_{x\,rotor}$ ... (3.23a)

$A_{z\,air} = A_{z\,rotor}$ ... (3.23b)

(d) At $y = c_1$, $H_{x\,air} = I_{cs}$ ... (3.24)

Now from relationship- curl $A = B$, we have

$$H_x = \frac{1}{\mu}(\text{curl}_x A) = \frac{1}{\mu}\frac{\partial A_z}{\partial y} \quad \ldots \quad (3.25)$$

From (3.21), (3.25) and (3.10),

$$\beta_n(-A_{n2}\sinh\beta_n c_2 + B_{n2}\cosh\beta_n c_2) = 0$$

$$\therefore \frac{A_{n2}}{B_{n2}} = a_1 \text{ (say) } = \coth\beta_n c_2 \quad \ldots \quad (3.26)$$

From (3.22a), (3.22b), (3.25), (3.10), (3.19) and (3.26),

$$\frac{C_n}{D_n} = a_3 \text{ (say) } = \frac{a_2\sinh\gamma_n d + \cosh\gamma_n d}{\sinh\gamma_n d + a_2\cosh\gamma_n d} \quad (3.27)$$

where $a_2 = \frac{\beta_n}{\gamma_n}\cdot\frac{-a_1\sinh\beta_n d + \cosh\beta_n d}{a_1\cosh\beta_n d - \sinh\beta_n d}$ ... (3.28)

From (3.23a), (3.23b), (3.25), (3.10), (3.19) & (3.27),

$$\frac{A_{n1}}{B_{n1}} = a_4 \text{ (say) } = \frac{\beta_n}{\gamma_n}\cdot a_3 \quad \ldots \quad (3.29)$$

From (3.24), (3.25), and (3.19) and (3.29),

$$B_{n1} = \frac{\lambda\mu}{n\pi\beta_n}\left(\frac{3}{2}B_1 I_1\right)\cdot\frac{1}{a_4\sinh\beta_n c_1 + \cosh\beta_n c_1} \quad \ldots \quad (3.30)$$

$$A_{n1} = a_4\cdot B_{n1}$$

Having evaluated all the constants of integration, vector potential at stator surface can be written as-

$$(A_{zs})_{y=c_1} = \sum_{n=1,3..} \exp[j(ax_1 + \omega t)][A_{n1}\cosh\beta_n c_1 + B_{n1}\sinh\beta_n c_1]\sin\frac{n\pi a}{\lambda}$$

$$= \sum_{n=1,3..} \exp\{j(ax_1 + \omega t)\} \frac{4\mu}{n\pi\beta_n}\left(\frac{3}{2} B_1 I_1\right).a_5 \sin\frac{n\pi z}{\lambda},$$

from (3.29) and (3.30)

$$\text{where } a_5 = \frac{a_4 \cosh\beta_n g_1 + \sinh\beta_n g_1}{a_4 \sinh\beta_n g_1 + \cosh\beta_n g_1} \qquad (3.31)$$

and,

$$(A_{gx1})_{y=g_1} = \sum_{n=1,3..} \frac{jn\pi}{a\lambda} \exp\{j(ax_1 + \omega t)\} \frac{4\mu}{n\pi\beta_n}\left(\frac{3}{2} B_1 I_1\right). a_5 \cos\frac{n\pi z}{\lambda} \quad ... \qquad (3.32)$$

Electric force field is given by,

$$E_g = -\frac{\partial A_z}{\partial t}$$

$$\therefore (E_{gz})_{y=g_1} = \sum_{n=1,3..} -j\omega \exp\{j(ax_1 + \omega t)\} \frac{4\mu a_5}{n\pi\beta_n}\left(-\frac{3}{2} B_1 I_1\right). \sin\frac{n\pi z}{\lambda}$$

$$(E_{gx1})_{y=g_1} = \sum_{n=1,3..} \frac{n\pi\omega}{a\lambda} \exp\{j(ax_1 + \omega t)\} \frac{4\mu a_5}{n\pi\beta_n}\left(\frac{3}{2} B_1 I_1\right). \cos\frac{n\pi z}{\lambda}$$

Taking real part of periodic function in x_1 and ,

$$(E_{gz})_{y=g_1} = \sum_{n=1,3..} \sin(ax_1 + \omega t).\frac{4\mu a_5}{n\pi n}.\left(\frac{3}{2} B_1 I_1\right)\sin\frac{n\pi z}{\lambda}$$

$$(E_{gx1})_{y=g_1} = \sum_{n=1,3..} \frac{n\pi\omega}{a\lambda} \cos(ax_1 + \omega t).\frac{4\mu a_5}{n\pi\beta_n}.\left(\frac{3}{2} B_1 I_1\right). \cos\frac{n\pi z}{\lambda}$$

The real part of turn distribution per phase is, from (3.2) and (3.3),

$$T_z = \sum_{n'=1,3..} \frac{4}{n'\pi} B_1 \cos ax_1 \sin\frac{n'\pi z}{\lambda}$$

$$T_{x1} = \sum_{n'=1,3..} \frac{4}{a\lambda} B_1 \sin ax_1 \cos\frac{n'\pi z}{\lambda}$$

Winding voltages: The induced voltage in a conductor is the integral along the conductor length of the total electric field. The conductor voltages accumulated over all the conductors of the winding, yield the winding induced voltage. Voltage induced in a conductor may be written as-

$$v(\text{cond}) = \int_0^{\lambda} \left\{ (T_z \; E_{Gz})_{y=G_1} + (T_{x1} \; E_{Gx1})_{y=G_1} \right\} dz$$

Voltage induced in a phase of the stator winding is-

$$V_1 = p \int_0^{\lambda} v\;(\text{cond}) \quad dx_1$$

It will be observed that n' = n because of the following identity

$$\int_0^{\lambda} \sin \frac{n\pi z}{\lambda} \sin \frac{n'\pi z}{\lambda} \, dz = \int_0^{\lambda} \cos \frac{n\pi z}{\lambda} \cos \frac{n'\pi z}{\lambda} \, dz = 0$$

when $n' \neq n$.

$$V_1 = \frac{12\,\omega p^2 \mu a_5 \lambda\, B_1^2 I_1}{n\pi \beta_n} \left[\int_0^{\lambda} \left\{ \frac{\sin\,(ax_1 + \omega t)\cos(ax_1)}{n} + \frac{n\pi}{a^2 \lambda^2} \cos\,(ax_1 + \omega t).\sin\,(ax_1) \right\} dx_1 \right]$$

$$= \frac{12\,\omega p^2 \mu a_5 \; B_1^2 I_1 \lambda}{n\pi\beta_n} \left(\frac{1}{n\pi} + \frac{n\pi}{a^2 \lambda^2} \right) . \frac{T}{2} \sin \omega t$$

$$= \frac{6\,\omega p^2 \mu \lambda \beta_n \; B_1^2}{n^2 \; a^3} . a_5 \; . \; I_1 \sin \omega t$$

This is the airgap voltage induced in one phase stator winding. This is in quadrature to the winding referred to. Airgap voltage induced in the same winding is-

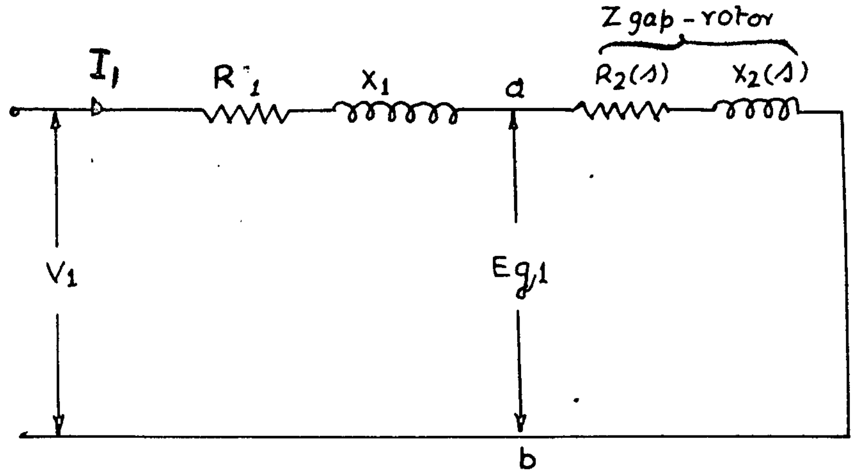
$$V_1 = \sum_{n=1,3..} j \; \frac{6\,\omega p^2 \mu \lambda \beta_n \; B_1^2}{n^2 \; a^3} . \; a_5 \; I_1 \sin \omega t \tag{3.33}$$

Equivalent impedance of rotor, airgap and central core in terms of stator is given by-

$$Z_{gap\text{-}rotor} = \frac{\bar{V}_1}{\lambda_1 \sin \omega t}$$

$$= \sum_{n=1,3\ldots} j\,\frac{6\,\omega\,p\,\mu\,\lambda\,\beta_n\,L_1^2\,a_s}{n^2\,a^3\,\pi} \qquad (3.34)$$

Equations (3.33) and (3.34) may be used to represent the equivalent circuit of the machine. Equation (3.34) gives the impedance of the machine as seen from terminals a & b (Refer fig.3.3).



EQUIVALENT CIRCUIT OF DRAG CUP MACHINE.
FIG 3.3.

The equivalent circuit representation as indicated by equation (3.33) is not simple since more than one term of the series must be used.

The above analysis takes into account tangential component of vector potential or rotor currents. It may be interesting to study the effect of neglecting this component of vector potential. A detailed study of this has been done in Appendix I.

3.2. THREE DIMENSIONAL ANALYSIS IN CYLINDRICAL CO-ORDINATE SYSTEM

3.2.1. INTRODUCTION:

Analysis of earlier section neglected the effect of curvature by developing the stator and rotor into flat infinitely long bodies. With non-magnetic rotor having a small radius of curvature the effect of curvature may be pronounced. This section deals with the effect of curvature by considering the natural cylindrical co-ordinate system.

3.2.2. GEOMETRY OF MACHINE:

Fig.3.4 shows the geometry of the machine. The whole machine may be considered to consist of three cylinders namely stator, moving cup and rotor core. The co-ordinate system (r, θ_1, z) is attached to stator. The co-ordinate system (r, θ, z) is attached to moving cup.

3.2.3. ASSUMPTIONS:

All assumptions except (iv) of article 3.1.3. are applicable here.

3.2.4. ANALYSIS:

Relative Motion: Mmf wave due to current sheet at stator surface rotates with an angular velocity . Motion of rotor is given by $\int (1-s)\omega \, dt$.

$$\therefore \theta_1 = \theta - \int (1-s)\omega \, dt$$

$$= \theta - (1-s)\omega t$$

Current sheets: The stator winding can be replaced by two current sheets, one in circumferential direction and other in axial direction. The axial current sheet is -

I

$$I_{cs} = \sum_{n=1,3} \frac{4}{n\pi} \left(\frac{3}{2} B_1 I_1\right) \exp\left(j(P\theta_1 + \omega t)\right) \sin \frac{n\pi s}{\lambda} \quad \ldots \quad (3.35)$$

Tangential current sheet is, from divergence equation $\nabla . I_s = 0$,

$$I_{c\theta_1} = -\int r \frac{\partial I_{cs}}{\partial s} \, d\theta_1$$

$$= \sum_{n=1,3} \frac{j4r_1}{P} \left(\frac{3}{2} B_1 I_1\right) \exp\left(j(P\theta_1 + \omega t)\right) \cos \frac{n\pi s}{\lambda}, \quad \ldots \quad (3.36)$$

the constant of integration reducing to zero because of symmetry.

__Solution for airgap:__ For airgap regions we have-

$$\nabla^2 A_{g\theta_1} = \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial A_{g\theta_1}}{\partial r}\right) + \frac{1}{r^2} \frac{\partial^2 A_{g\theta_1}}{\partial \theta_1^2} + \frac{\partial^2 A_{g\theta_1}}{\partial s^2} = 0 \quad \ldots \quad (3.37)$$

$$\nabla^2 A_{gs} = \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial A_{gs}}{\partial r}\right) + \frac{1}{r^2} \frac{\partial^2 A_{gs}}{\partial \theta_1^2} + \frac{\partial^2 A_{gs}}{\partial s^2} = 0 \quad \ldots \quad (3.38)$$

$$\nabla \cdot A_g = \frac{1}{r} \frac{\partial}{\partial r}\left(r A_{gr}\right) + \frac{1}{r} \frac{\partial A_{g\theta_1}}{\partial \theta_1} + \frac{\partial A_{gs}}{\partial s} = 0 \quad \ldots \quad (3.39)$$

and $A_{gr} = 0$

A solution to (3.38) can be obtained by the method of separation of variables. Assuming a solution for A_{gs},

$$A_{gs}(r, \theta, s, t) = \exp(j\omega t) \, F_1(\theta) \, F_2(r) \, F_3(s),$$

where F_1, F_2, F_3 are purely functions of θ, r & s respectively.

Substituting it in (3.38),

$$\frac{F_3''}{F_3} = -\frac{F_2''}{F_2} - \frac{1}{r} \frac{F_2'}{F_2} - \frac{1}{r^2} \frac{F_1''}{F_1} \quad \ldots \quad (3.40)$$

Variation of A_{gz} is periodic in z direction and sinusoidal in θ direction.

$$\therefore \quad \frac{F_3''}{F_3} = -\frac{n^2\pi^2}{\lambda^2} \quad \ldots \quad (3.41)$$

$$\frac{F_1''}{F_1} = -P^2 \quad \ldots \quad (3.42)$$

Solutions to (3.41) and (3.42) are -

$$F_3 = c_1 \sin \frac{n\pi z}{\lambda}$$

$$F_1 = c_2 \exp (jP\,\theta_1)$$

From (3.40), (3.41) and (3.42),

$$r^2F_2'' + r\,F_2' + F_2\left(-\frac{n^2\pi^2 r^2}{\lambda^2} - P^2\right) = 0 \qquad (3.42)$$

Equation (3.43) is Bessel's equation whose solution may be written as-

$$F_2 = c_3\, I_P(\beta r) + c_4\, K_P(\beta r)$$

where $\beta = \frac{n\pi}{\lambda}$

and, I_P and K_P are modified Bessel functions of purely imaginary arguments.

Complete solution for A_{gz} is

$$A_{gz} = \sum_{n=1,3..} \exp\left(j\,(P\theta_1 + \omega t)\right)\left\{A_{nm}\, I_P(\beta r) + B_{nm}\, K_P(\beta r)\right\}. \sin\frac{n\pi z}{\lambda} \qquad (3.44)$$

From (3.39),

$$A_{g\theta_1} = -\int r\,\frac{\partial A_{gz}}{\partial z}\, d\theta_1$$

$$= \sum_{n=1,3..} \frac{jn\pi r}{P\lambda} \exp\left(j\,(P\theta_1 + \omega t)\right)\left\{A_{nm}\, I_P(\beta r) + B_{nm} K_P(\beta r)\right\}. \cos\frac{n\pi z}{\lambda} \qquad (3.45)$$

m in above equations represent the number of gap.

<u>Solution for rotor:</u> For moving conductor we have -

$$\nabla^2 A_{rz} = \frac{\partial^2 A_{rz}}{\partial r^2} + \frac{1}{r}\frac{\partial A_{rz}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 A_{rz}}{\partial \theta^2} + \frac{\partial^2 A_{rz}}{\partial z^2}$$

$$= \mu\sigma \cdot \frac{\partial A_{rz}}{\partial t} \qquad \ldots \qquad (3.46)$$

$$\nabla^2 A_{r\theta} = \frac{\partial^2 A_{r\theta}}{\partial r^2} + \frac{1}{r}\frac{\partial A_{r\theta}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 A_{r\theta}}{\partial \theta^2} + \frac{\partial^2 A_{r\theta}}{\partial z^2}$$

$$= \mu\sigma \cdot \frac{\partial A_{r\theta}}{\partial t} \qquad \ldots \qquad (3.47)$$

$$\nabla \cdot A_r = \frac{1}{r}\frac{\partial}{\partial r}(r A_{rr}) + \frac{1}{r}\frac{\partial A_{r\theta}}{\partial \theta} + \frac{\partial A_{rz}}{\partial z} = 0 \qquad \ldots \qquad (3.48$$

and $A_{rr} = 0$

Solution to Poisson's equation (3.47) can be obtained by the method of separation of variables. Assuming a a solution of the form-

$$A_{rz}(r, \theta, z, t) = \exp(j\omega_s t)\, F_1(\theta)\, F_2(r)\, F_3(z)$$

$$\frac{F_2''}{F_2} + \frac{1}{r}\frac{F_2'}{F_2} + \frac{1}{r^2}\frac{F_1''}{F_1} + \frac{F_3''}{F_3} = \alpha^2 \qquad (3.49)$$

where, $\alpha^2 = j\omega_s\mu\sigma$

Variation in θ direction is sinusoidal and that in z direction is periodic.

$$\therefore \frac{F_1''}{F_1} = -p^2 \qquad \ldots \qquad (3.50)$$

$$\frac{F_3''}{F_3} = -\frac{n^2\pi^2}{\lambda^2} \qquad \ldots \qquad (3.51)$$

Solutions to (3.50) and (3.51) are,

$$F_1 = c_5 \exp(jP\theta)$$

and $F_3 = c_6 \sin \frac{n\pi z}{\lambda}$

From (3.49), (3.50) & (3.51),

$$F_2'' + \frac{1}{r} F_2' - (\alpha^2 + \frac{n^2\pi^2}{\lambda^2} + \frac{P^2}{r^2}) F_2 = 0$$

This is a Bessel's equation whose solution may be written-

$$F_2 = c_7 J_P(\gamma r) + c_8 Y_P(\gamma r)$$

where $^2 = -(\alpha^2 + \beta^2)$

and J_P and Y_P are Bessel's functions of first and second kind of order P respectively.

Complete solution for A_{rz} is-

$$A_{rz} = \sum_{n=1,3..} \exp(j(P\theta + \omega st)) \left\{ C_n J_P(\gamma r) + D_n Y_P(\gamma r_3) \right\} . \sin \frac{n\pi z}{\lambda} \quad \ldots \quad (3.52)$$

From (3.48),

$$A_{r\theta} = -\int r \frac{\partial A_{rz}}{\partial z} d\theta$$

$$\sum_{n=1,3..} \frac{jn\pi r}{P\lambda} \exp(j(P\theta + \omega st)) \left\{ C_n J_P(\gamma r) + D_n Y_P(\gamma r) \right\} . \cos \frac{n\pi z}{\lambda} \quad \ldots \quad (3.53)$$

The constants of integration A_{nm} ($m = 1,2$), B_{nm} ($m = 1,2$), C_n and D_n are to be determined from boundary conditions.

<u>Boundary conditions:</u> At the boundaries between conductor and airgaps H_θ and A_z are continuous. At the rotor core surface H_θ vanishes and at the stator surface H_z is equal to stator surface current density.

(a) At $r = r_4$, $H_{\theta\, air} = 0 \quad \ldots \quad (3.54)$

(b) At $r = r_3$, $H_{\theta\ air} = H_{\theta\ rotor}$ (3.55a)

$A_{z\ air} = A_{z\ rotor}$ ... (3.55b)

(c) At $r = r_2$, $H_{\theta\ air} = H_{\theta\ rotor}$ ... (3.56a)

$A_{z\ air} = A_{z\ rotor}$ ... (3.56b)

(d) At $r = r_1$, $H_{\theta\ air} = I_{zs}$ ... (3.57)

From curl relationship curl A = B,

$$H_\theta = \frac{1}{\mu}(\text{curl}_\theta\ A) = -\frac{1}{\mu}\frac{\partial A_z}{\partial r} \quad \ldots \quad (3.58)$$

and
$$H_r = \frac{1}{\mu}(\text{curl}_r\ A) = \frac{1}{\mu}\left(\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right) \quad (3.59)$$

From (3.54), (3.58), (3.44),

$$A_{n2} / B_{n2} = a_1(\text{say}) = -\frac{K_p'(\beta r_4)}{I_p'(\beta r_4)} \quad (3.60)$$

From (3.55a), (3.55b), (3.58), (3.44), (3.52) and (3.60);

$$\frac{C_n}{D_n} = a_3(\text{say}) = \frac{a_2 Y_p(\gamma r_3) - Y_p'(\gamma r_3)}{J_p'(\gamma r_3) - a_2 J_p(\gamma r_3)} \quad (3.61)$$

where
$$a_2 = \frac{\beta}{\gamma}\,\frac{a_1 I_p'(\beta r_3) + K_p'(\beta r_3)}{a_1 I_p(\beta r_3) + K_p(\beta r_3)}$$

From (3.56a), (3.56b), (3.58), (3.44), (3.52) and (3.61),

$$\frac{A_{n1}}{B_{n1}} = a_5\ (\text{say}) = \frac{a_4 K_p(\beta r_2) - K_p'(\beta r_2)}{I_p'(\beta r_2) - a_4 I_p(\beta r_2)} \quad (3.62)$$

where
$$a_4 = \frac{\gamma}{\beta}\cdot\frac{a_3 J_p'(\gamma r_2) + Y_p'(\gamma r_2)}{a_3 J_p(\gamma r_2) + Y_p(\gamma r_2)}$$

From (3.57), (3.58), (3.44), (3.35) and (3.62)-

$$B_{n1} = -\frac{\Delta\mu}{n\pi\beta}\left(\frac{3}{2} B_1 I_1\right)\cdot\frac{1}{a_5 I_p'(\beta r_1) + K_p'(\beta r_1)} \quad (3.63)$$

$$A_{n1} = a_5 \cdot B_{n1}$$

All the constants of integration have now been evaluated. Vector potential at stator surface may be written as-

$$(A_{gz})_{r=r_1} = \sum_{n=1,3..} B_{n1} \exp\left(j\left(P\theta_1 + \omega t\right)\right)\left\{a_5 I_P(\beta r_1) + K_P(\beta r_1)\right\} \sin\frac{n\pi z}{\lambda}$$

and

$$(A_{g\theta_1})_{r=r_1} = \sum_{n=1,3..} B_{n1} \cdot \frac{jn\pi r_1}{P\lambda} \exp\left(j\left(P\theta_1 + \omega t\right)\right)\left\{a_5 I_P(\beta r_1) + K_P(\beta r_1)\right\} \cdot \cos\frac{n\pi z}{\lambda}, \text{ from (3.44) \& (3.45)}$$

Let- $a_6 = a_5 \, I_P(\beta r_1) + K_P(\beta r_1)$

and $a_7 = a_5 \, I_P'(\beta r_1) + K_P'(\beta r_1)$

Electric field in airgap is given by-

$$E_g = -\frac{\partial A_g}{\partial t}.$$

Therefore,

$$(E_{gz})_{r=r_1} = -\sum_{n=1,3...} j\omega B_{n1} \exp\left(j\left(P\theta_1 + \omega t\right)\right) a_6 \sin\frac{n\pi z}{\lambda}$$

and

$$(E_{g\theta_1})_{r=r_1} = \sum_{n=1,3...} \frac{n\pi r_1 \omega}{P\lambda} B_{n1} \exp\left(j\left(P\theta_1 + \omega t\right)\right) a_6 \cdot \cos\frac{n\pi z}{\lambda}$$

Taking real part of periodic function in θ_1 and

$$(E_{gz})_{r=r_1} = \sum_{n=1,3...} \omega B_{n1} \cdot a_6 \sin\frac{n\pi z}{\lambda}\left(\sin(P\theta_1 + \omega t)\right)$$

$$(E_{g\theta_1})_{r=r_1} = \sum_{n=1,3...} \frac{n\pi r_1 \omega B_{n1}}{P\lambda} \cdot a_6 \cos\frac{n\pi z}{\lambda} \cdot \left(\cos\left(P\theta_1 + \omega t\right)\right)$$

Real part of turn distribution per phase from (3.35) and (3.36)as-

$$T_z = \sum_{n'=1,3..} \frac{4}{n'\pi} B_1 \cos P\theta_1 \sin\frac{n'\pi z}{\lambda}$$

$$T_{\theta_1} = \sum_{n'=1,3..} \frac{4 r_1}{P\lambda} B_1 \sin P\theta_1 \cos\frac{n'\pi z}{\lambda}$$

where $B_1 = \frac{K_w \cdot T_{ph}}{D}$

Winding Voltage: Voltage induced in a conductor is the integral along the conductor length of the total electric field.

$$V_{(cond)} = \int_0^{\lambda} \left\{ (T_z \, E_{Gz})_{r=r_1} + (T_{\theta_1} \cdot E_{G\theta_1})_{r=r_1} \right\} dz$$

$$= \frac{4}{n'\pi} \cdot B_1 \cos P\theta_1 \, \omega \, B_{n1} \cdot a_6 \sin (P\theta_1 + \omega t) \cdot$$

$$\int_0^{\lambda} \sin \frac{n'\pi z}{\lambda} \cdot \sin \frac{n\pi z}{\lambda} \, dz + \frac{4r_1}{P\lambda} B_1 \sin P\theta_1 \cdot$$

$$\frac{n\pi r_1 \omega B_{n1}}{P\lambda} \cdot a_6 \cos (P\theta_1 + \omega t) \cdot$$

$$\int_0^{\lambda} \cos \frac{n'\pi z}{\lambda} \cos \frac{n\pi z}{\lambda} \, dz$$

The integrals $\int_0^{\lambda} \sin \frac{n'\pi z}{\lambda} \cdot \sin \frac{n\pi z}{\lambda} \, dz$ and $\int_0^{\lambda} \cos \frac{n'\pi z}{\lambda} \cdot \cos \frac{n\pi z}{\lambda} \, dz$ reduce to zero when $n \neq n'$. Hence $n = n'$.

$$\therefore V_{(cond)} = \frac{4}{n\pi} B_1 \, \omega \, B_{n1} \, a_6 \frac{\lambda}{2} \cos P\theta_1 \sin (P\theta_1 + \omega t) +$$

$$\frac{4r_1}{P^2 \lambda^2} B_1 \, n\pi r_1 \omega \, B_{n1} \, a_6 \cdot \frac{\lambda}{2} \cdot \sin P\theta_1 \cos (P\theta_1 + \omega t)$$

Voltage induced in one phase of the stator winding-

$$V_1 = D \int_0^{\pi} V_{(cond)} \, d\theta_1$$

$$= B_{n1} \, \omega \, B_1 \, a_6 \, \frac{D}{P} \left(\frac{P\lambda}{n} + \frac{n^2 \pi^2 \cdot r_1^2}{P\lambda} \right) \sin \omega t \, . 2P$$

Substituting the value of B_{n1} from (3.63),

$$V_1 = \sum_{n=1,3..} - 12 \frac{B_1^2 \mu \omega \lambda r_1^2}{n^2 \pi \beta P} \left(\frac{P^2}{r_1^2} + \beta^2 \right) \cdot \frac{a_6}{a_7} I_1 \sin \omega t \, . 2P$$

This is the voltage induced in quadrature to winding under consideration. Induced voltage in the same winding, is, therefore,

$$V_1 = \sum_{n=1,3..} j \frac{12 B_1^2 \mu \omega \lambda r_1^2}{n^2 \pi \beta P} \left(\frac{P^2}{r_1^2} + \beta^2 \right) \cdot \frac{a_6}{a_7} I_1 \sin \omega t \, , 2P \quad \dots \quad (3.64)$$

Equivalent combined impedance of airgaps, rotor and central core, in terms of primary is,

$$Z_{gap-rotor} = \frac{V_1}{I \sin\omega t}$$

$$= \sum_{n=1,3..} \frac{12\, B_1^2 \mu\omega\lambda\, r_1^2 \, . \, 2\rho}{n^2 \pi \beta p} \left(\frac{p^2}{r_1^2} + \beta^2\right) \quad (3.65)$$

Equations (3.64) and (3.65) can be used to draw the equivalent circuit. $Z_{gap-rotor}$ is the impedance as seen from terminals a and b of fig. 3.3.

$$Z_{gap-rotor} = R_2 + jX_2,$$

R_2 being the rotor resistance in terms of primary and X_2 being the rotor reactance inclusive of magnetizing reactance in terms of primary.

CHAPTER - 4

MECHANICAL TRANSIENTS

In this chapter expressions for speed rise and acceleration of a servomotor have been derived taking into consideration the effect of nonlinearity of the torque speed characteristic.

Fig.4.1 shows a typical torque speed curve of the machine. It is nonlinear to take the effect of nonlinearity into account square term of speed will be considered. Torque speed curve can

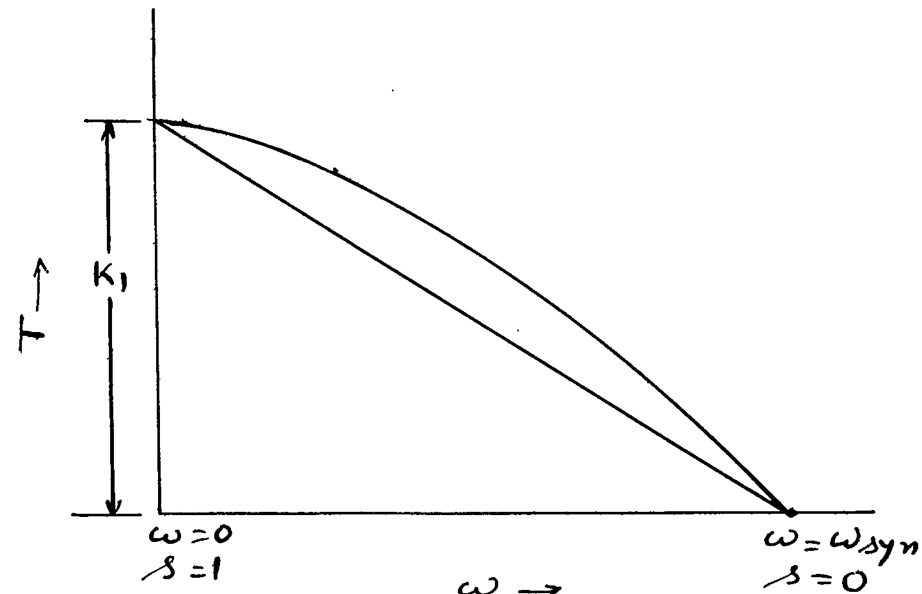


Fig 4.1

be approximated by

$$T = k_1 + k_2\omega + k_3\omega^2 \qquad \ldots \qquad (4.1)$$

where T= developed torque

k_1=stalling torque

and k_2, k_3 are constants to be determined from the actual torque speed curve.

Writing mechanics equation,

$$J\frac{d\omega}{dt} + k_4\omega = T \qquad \ldots \qquad (4.2)$$

where J = M.I. of rotating parts

k_4 = viscous friction of motor and load.

From (4.1) and (4.2),

$$K_1 + K_2\omega + K_3\omega^2 - J\frac{d\omega}{dt} - K_4\omega = 0$$

$$\therefore \; t = J\int \frac{d\omega}{k_1 + k_5\omega + k_3\omega^2}$$

$$= \frac{-2J}{q^{\frac{1}{2}}}\tanh^{-1}\frac{2k_3\omega + k_5}{q^{\frac{1}{2}}} + C \qquad (4.3)$$

where, $k_5 = k_2 - k_4$

$q^{\frac{1}{2}} = k_5^2 - 4k_1k_3$

and C = constant of integration.

Applying the initial condition $\omega = 0$ at $t = 0$, one obtains

$$C = \frac{2J}{q^{\frac{1}{2}}}\tanh^{-1}\frac{k_5}{q^{\frac{1}{2}}} \quad \ldots \qquad (4.4)$$

$$\therefore \; t = -\frac{2J}{q^{\frac{1}{2}}}\left(\tanh^{-1}\frac{2k_3\omega + k_5}{q^{\frac{1}{2}}} - \tanh^{-1}\frac{k_5}{q^{\frac{1}{2}}}\right),$$

from (4.3) & (4.4)

or,
$$\omega(t) = \frac{q^{\frac{1}{2}}}{2k_3}\tanh\left(-\frac{q^{\frac{1}{2}}}{2J}t + \frac{1}{2}\log_e\frac{q^{\frac{1}{2}} + k_5}{q^{\frac{1}{2}} - k_5}\right) - \frac{k_5}{2k_3} \quad \ldots \qquad (4.5)$$

This is the expression for speed rise similarly an expression for speed fall can be obtained by applying the initial condition $\omega = \omega_{max}$ at $t = 0$.

Eqn.(4.5) may be applied to determine the speed response of a servomotor provided its torque speed characteristic and moment of inertia are known.

ACCELERATION:

Since acceleration is time derivative of speed a curve for acceleration may be obtained either by exact or numerical differentiation.

Differentiating (4.5) the expression for speed with respect to time there results an expression for acceleration-

$$\text{Acceleration} = \frac{d\omega(t)}{dt}$$

$$= \frac{-q}{4Jk_3} \operatorname{sech}^2 \left(- \frac{q^{\frac{1}{2}}t}{2J} + \frac{1}{2} \log_e \frac{q^{\frac{1}{2}} + k_5}{q^{\frac{1}{2}} - k_5} \right)$$

CHAPTER - 5

EXPERIMENTAL TESTS & RESULTS

Tests were performed on two motors which were built in different sizes $\frac{1}{2}$ h.p. and 3 h.p. The smaller motor was built with two rotors one with copper cup and the other having aluminium cup. The copper cups of two different thickness were used. Design consideration and principal design data of these motors are given in Appendix III.

TORQUE SPEED CURVE:

A few torque-speed curves both under balanced and unbalanced applied voltages were obtained experimentally by means of a d.c. motor coupled to test motor. Theoretical curves were obtained by (a) two dimensional analysis both in rectangular and cylindrical co-ordinates of Chapter 2 and (b) three dimensional analysis of Chapter 3. Use of a digital Computer was made to obtain the theoretical curves. The Computer programmes are given in Appendix IV. However in three dimensional analysis the axial component of vector potential was neglected.

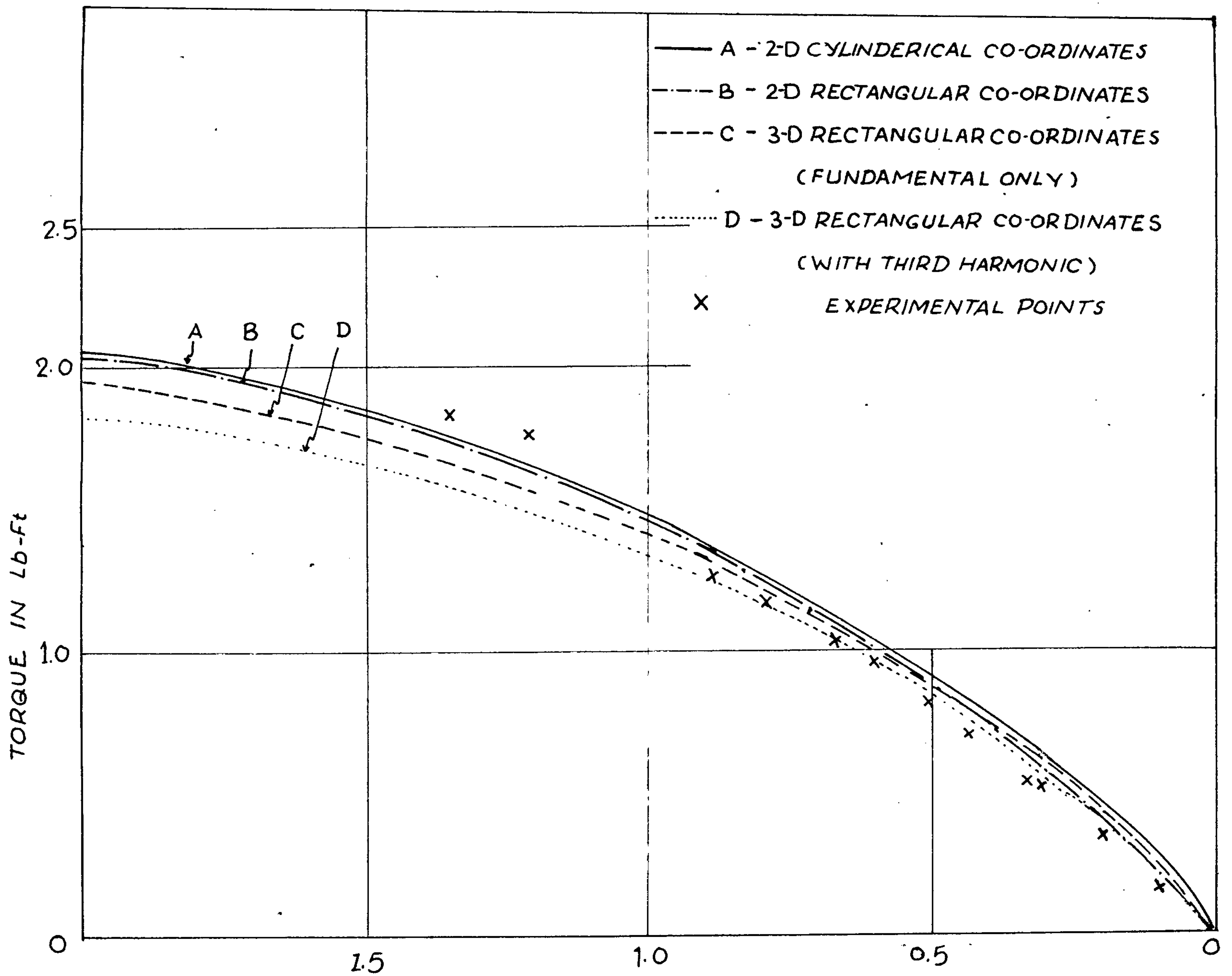
Some relations of Bessel functions for use in analysis of sections 2.2 & 3.2 are given in Appendix II.

Figs. 5.1, 5.2, 5.3 and 5.4 show the torque speed curves both for balanced and unbalanced applied voltages for various rotors: Figs. 5.1, 5.2, 5.3 and 5.4 refer to copper thin cup, copper thick cup, Aluminium cup and large motor. The first three figs. refer to smaller machines

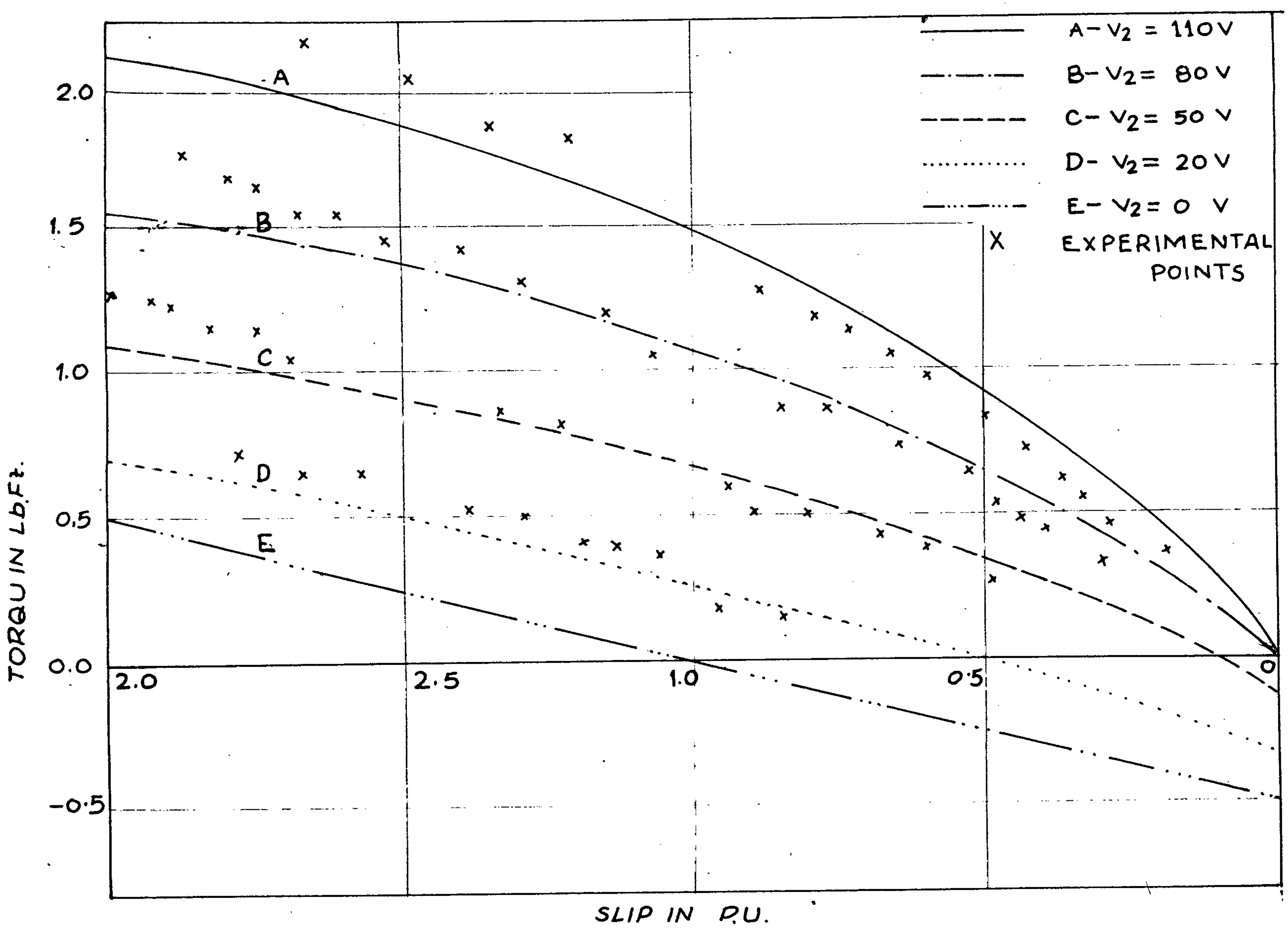
MECHANICAL TRANSIENTS:

A small tachogenerator was coupled to test motor for obtaining oscillograms of speed rise. The current transients were also recorded. Figs. 5.5~~(a)~~, ~~5.5(b)~~, 5.6(a), 5.6(b) and

5.7(a&b) show these curves for various rotors. Figs. 5.5~~(a) and~~ ~~5.5(b)~~ refers to thick copper cup. Figs. 5.6(a), 5.6(b) refer to thin copper cup and fig.5.7(a&b) refers to aluminium cup rotor. It was observed that the aluminium cup rotor gives much faster response than the copper cup of the same size. The aluminium cup rotor when run at no load rises to full speed in 5 to 10 cycles of 50 c/s wave whereas the electrical transients die out in about 3 cycles. For this case the electrical time constant is quite comparable to mechanical one and therefore it is not justifiable to neglect it in transfer function. The copper thick takes much more to attain the full speed. When started at no load the time taken for speed to gain steady value is large and may be more than 40-50 cycles of a 50 c/s wave. As the servomotor is always connected to load the electrical time constant will be small in comparison to mechanical time constant and can therefore, be neglected in transfer function of the motor.

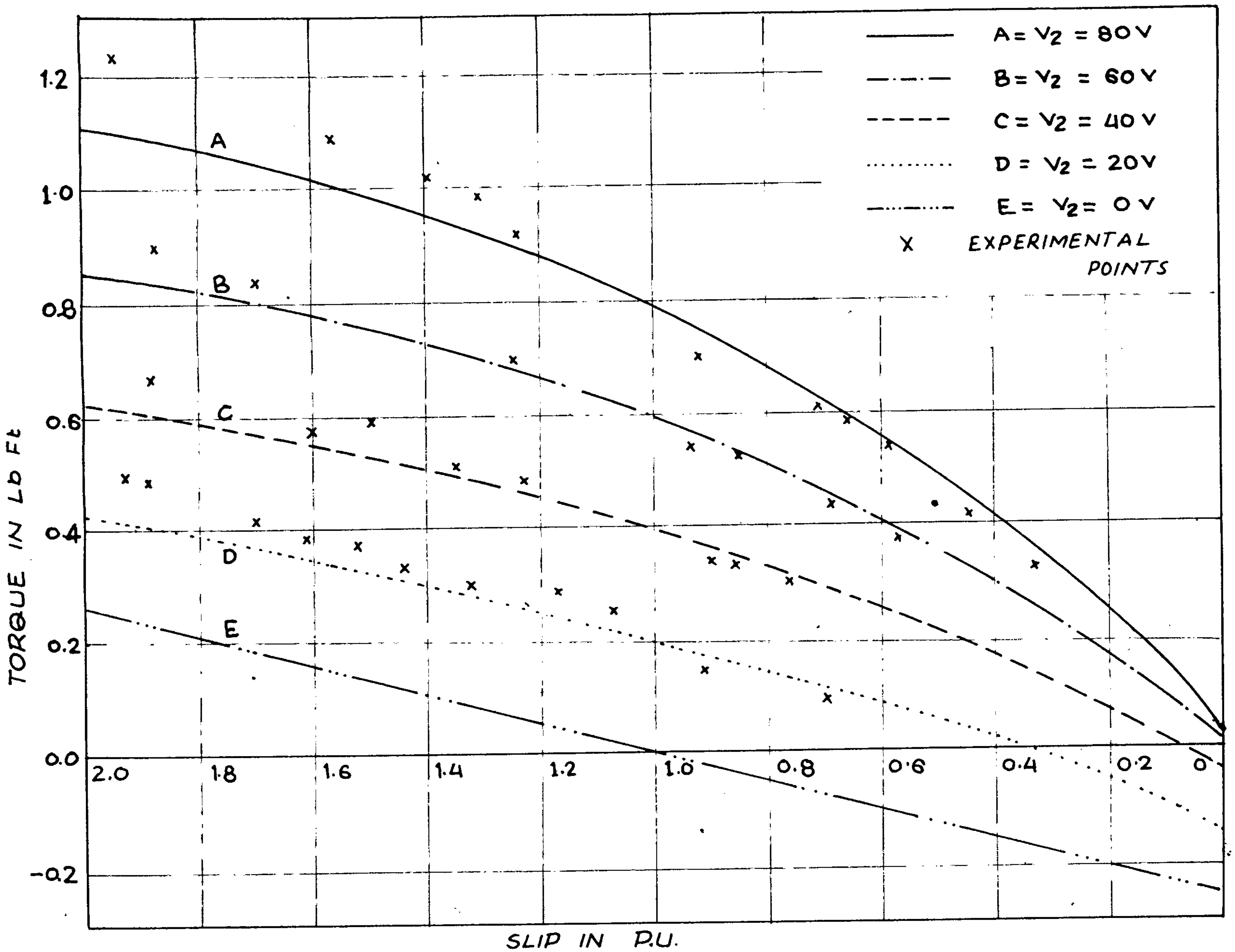


TORQUE SPEED CURVE (COPPER THIN CUP)
WITH BALANCED APPLIED VOLTAGES $V_1 = V_2 = 110$



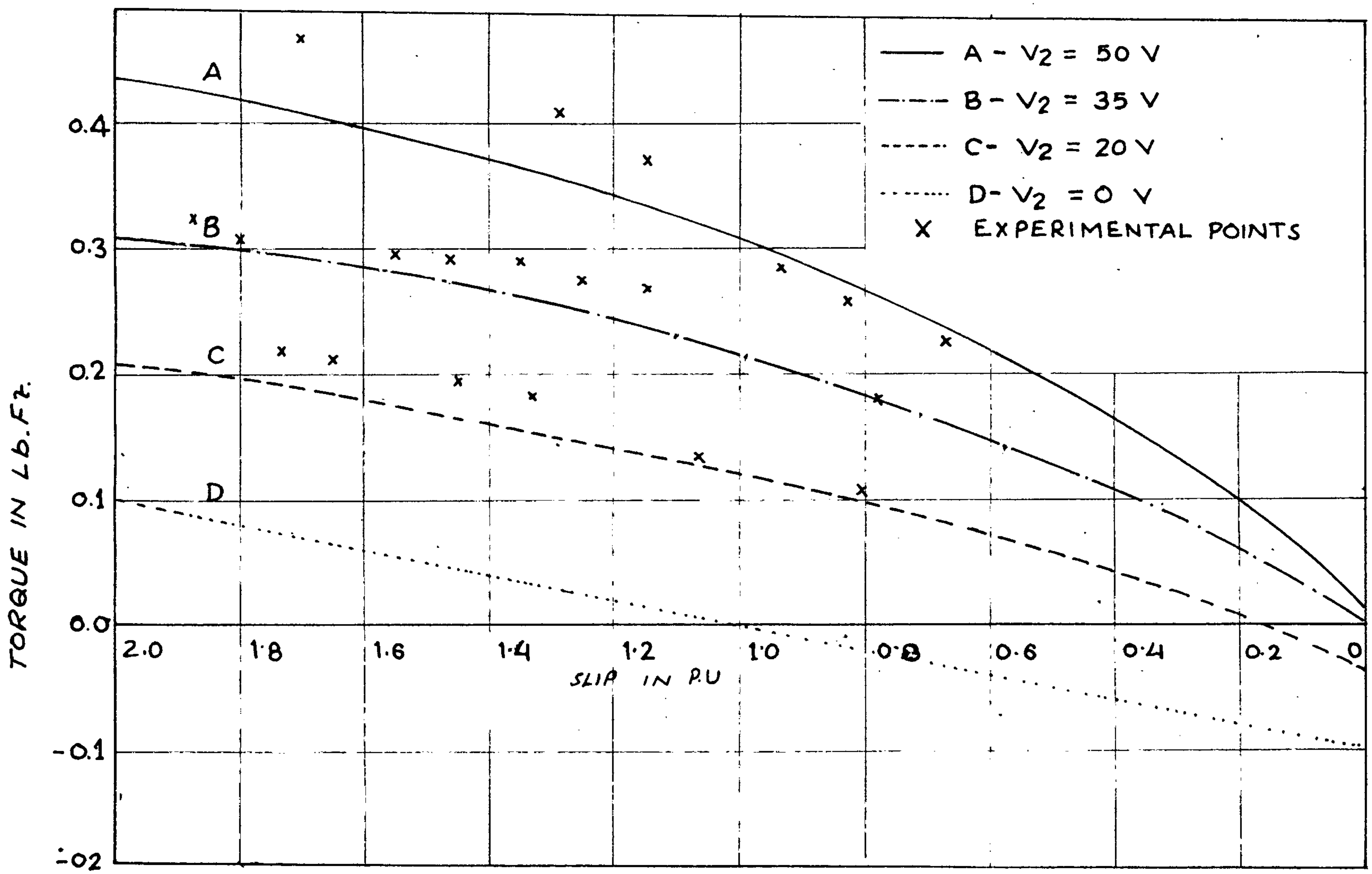
TORQUE-SPEED CURVE (COPPER THIN CUP) WITH UNBALANCED APPLIED VOLTAGES V_1 = 110 V

FIG 5.16



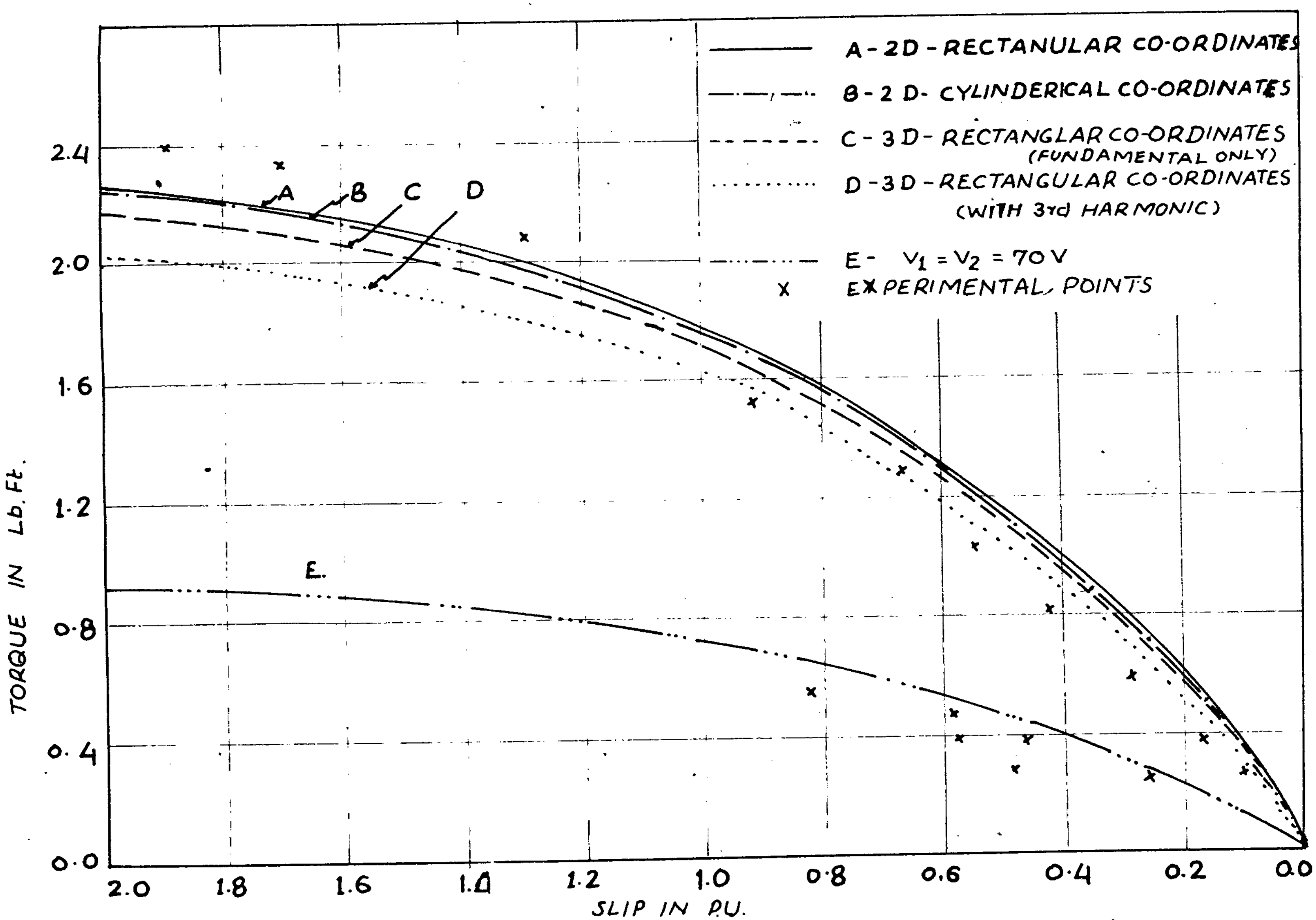
TORQUE - SPEED CURVE (COPPER THIN CUP) WITH UNBALANCED APPLIED VOLTAGE $V_1 = 80V$

FIG. 5.1C



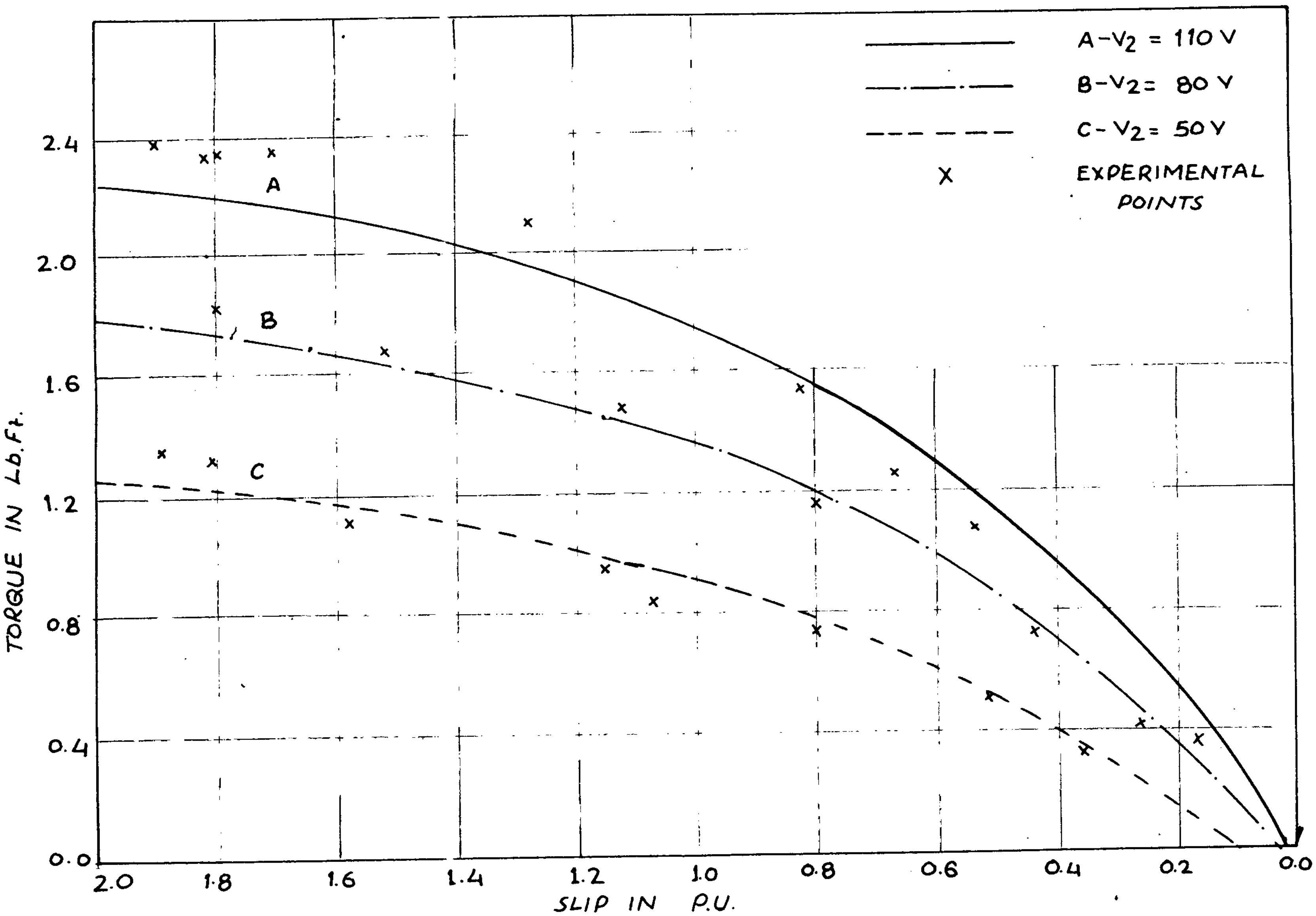
TORQUE - SPEED CURVE (COPPER THIN CUP)
WITH UNBALANCED APPLIED VOLTAGES (V_1= 50 V)

FIG. 5.1d



TORQUE - SPEED CURVE (COPPER THICK CUP) WITH BALANCED APPLIED VOLTAGES $V_1 = V_2 = 110V$

FIG.52 a



SPEED - TORQUE CURVE (COPPER THICK CUP)
WITH UNBALANCED APPLIED VOLTAGES V= 110V.

FIG-5.2b

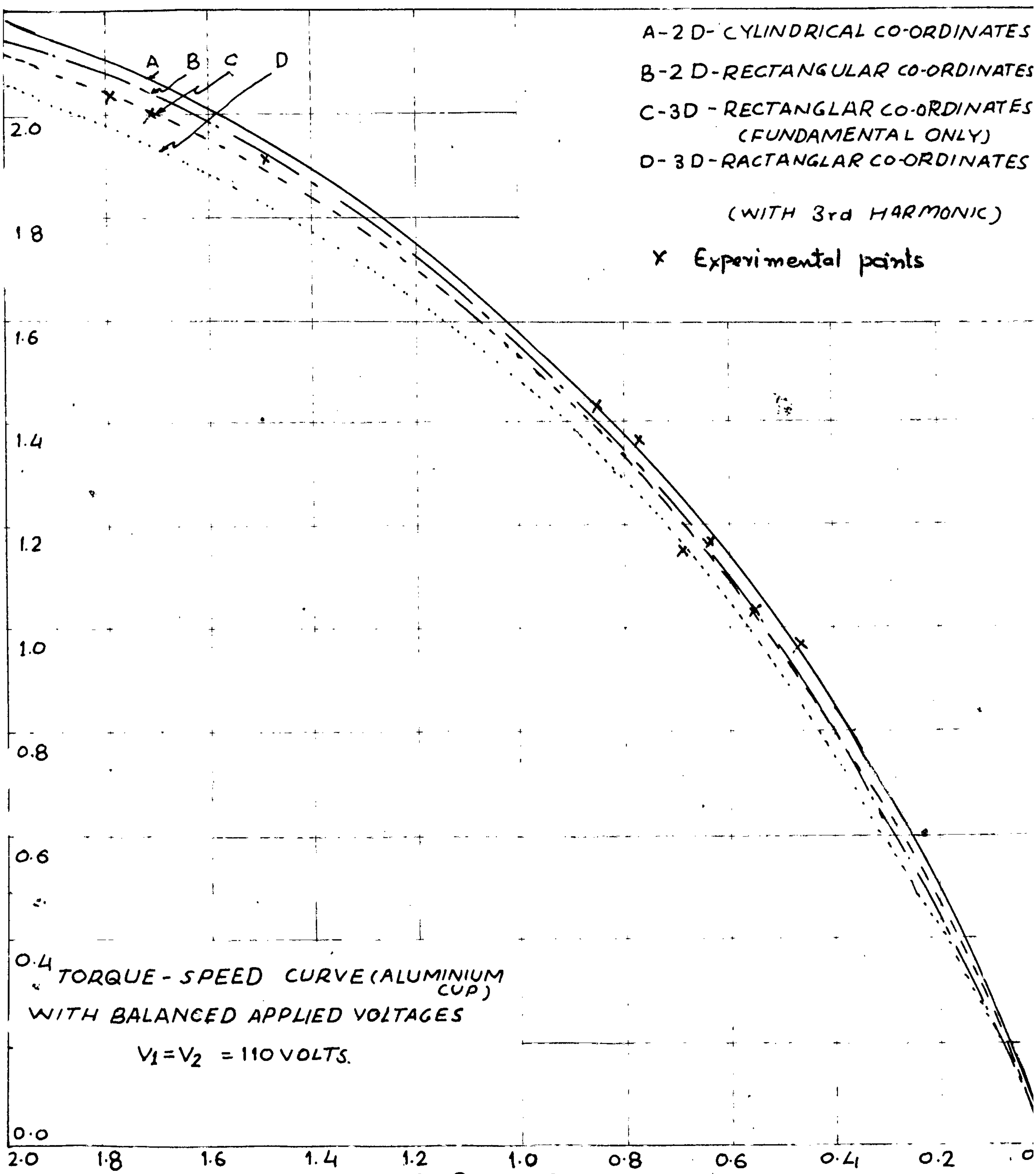
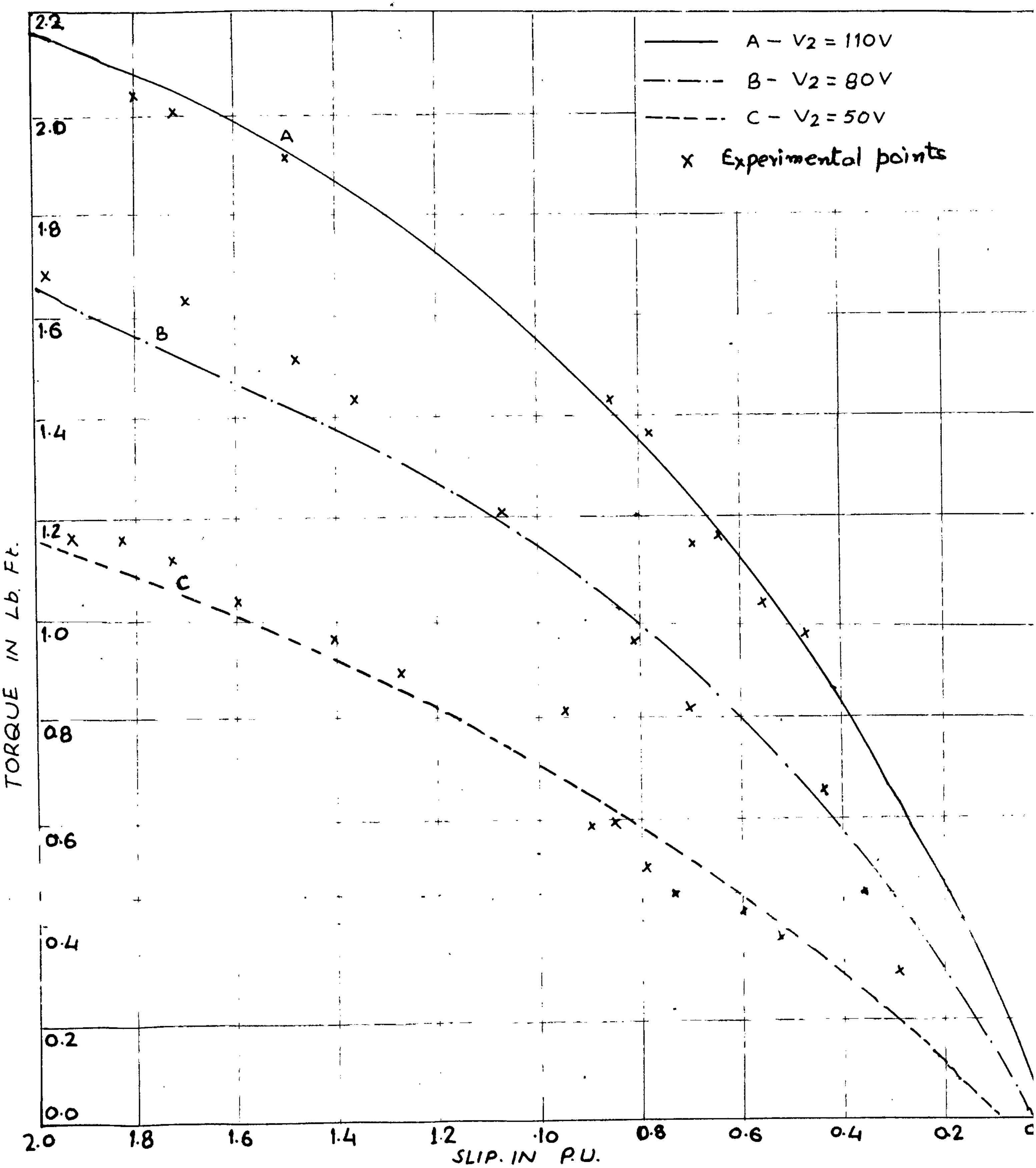
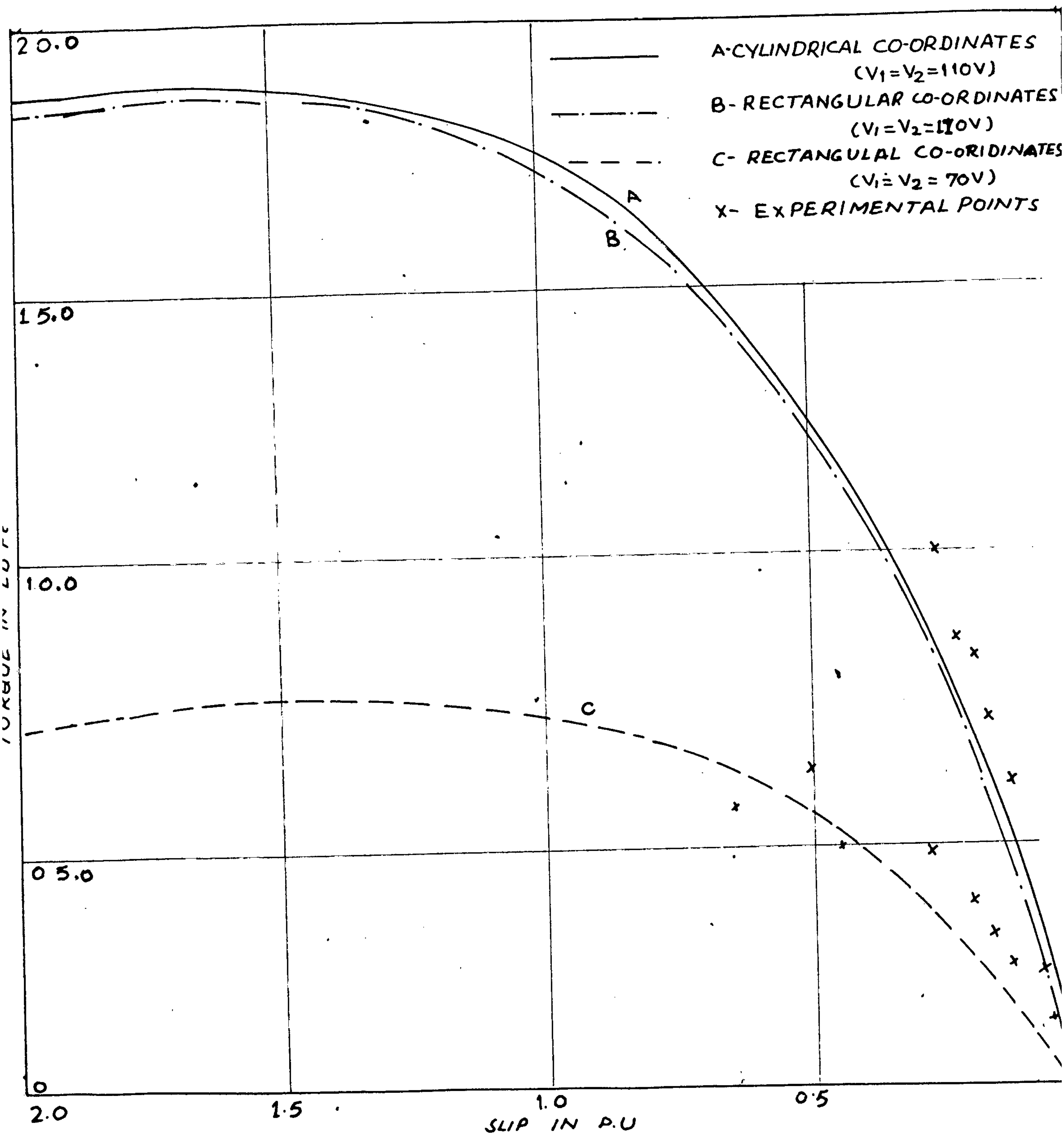

A-2 D- CYLINDRICAL CO-ORDINATES
B-2 D-RECTANGULAR CO-ORDINATES
C-3D - RECTANGLAR CO-ORDINATES (FUNDAMENTAL ONLY)
D- 3 D-RACTANGLAR CO-ORDINATES (WITH 3rd HARMONIC)
X Experimental points
A B C D
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.0
TORQUE - SPEED CURVE (ALUMINIUM CUP)
WITH BALANCED APPLIED VOLTAGES
$V_1 = V_2$ = 110 VOLTS.
2.0 1.8 1.6 1.4 1.2 1.0 0.8 0.6 0.4 0.2 0
SLIP IN P.U.

FIG. 53a



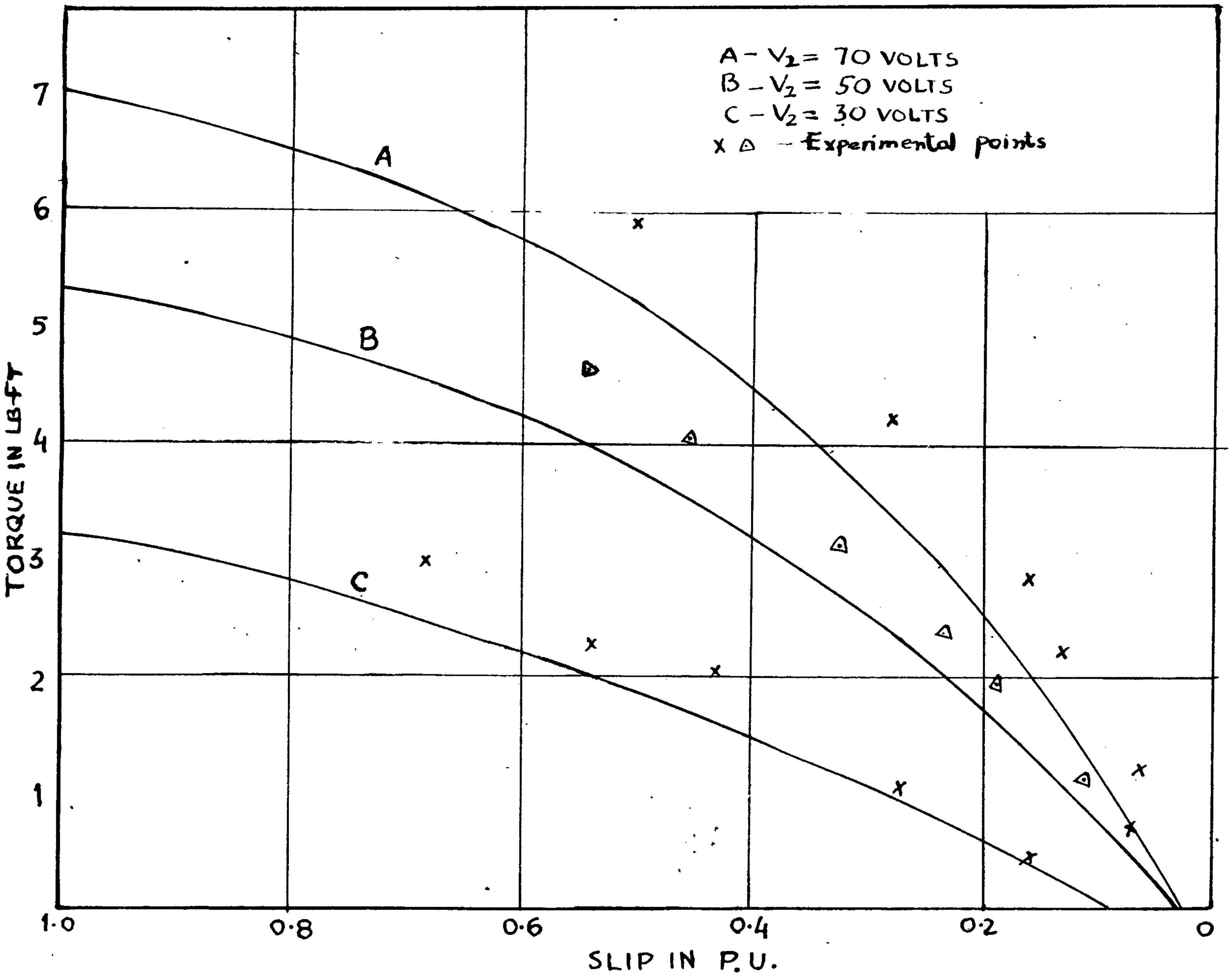
TORQUE SPEED CURVE (ALUMINIUM CUP) WITH UNBALANCED APPLIED VOLTAGES V_1 = 110V.

FIG - 5.3b

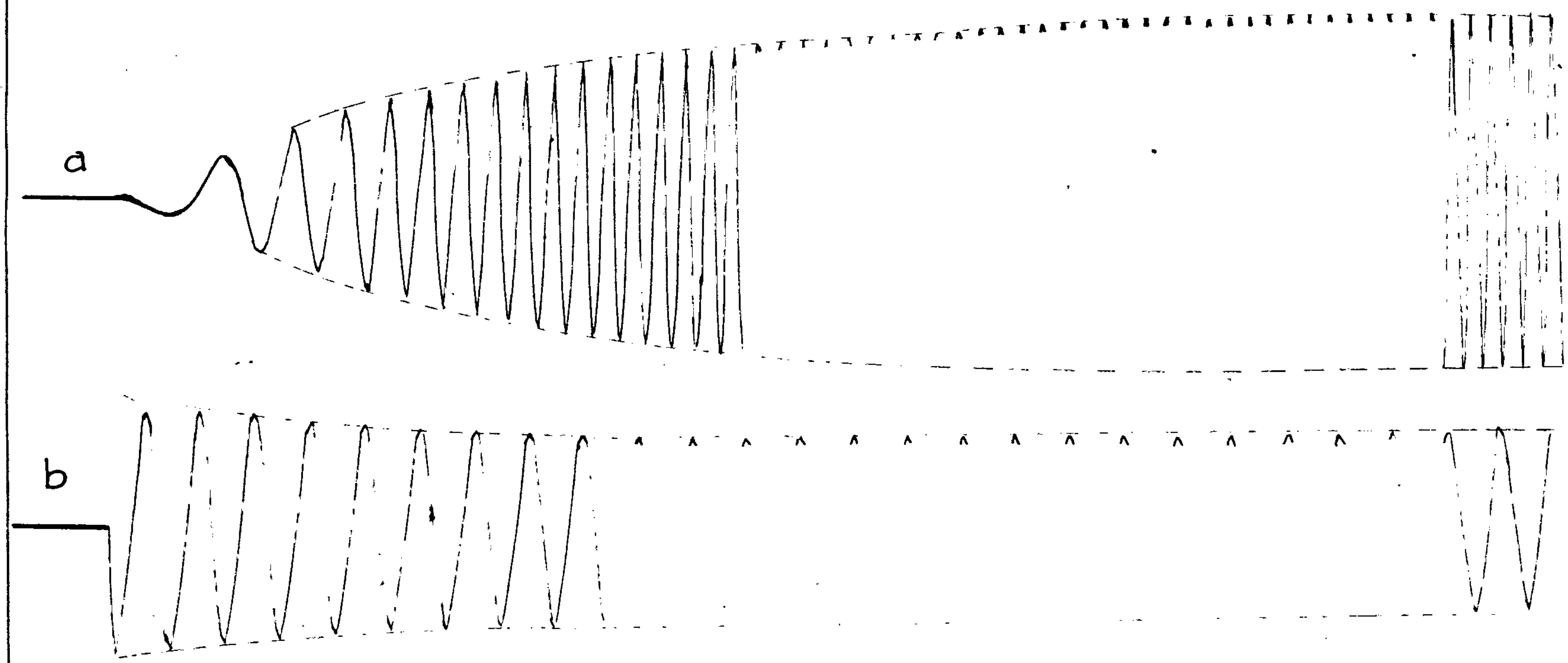


TORQUE-SPEED CURVE LARGE MOTOR
WITH BALANCED APPLIED VOLTAGES

FIG 5.4a



V_1 = 70 VOLTS
TORQUE-SPEED CURVE (LARGE MOTOR)
WITH UNBALANCED APPLIED VOLTAGES
Fig 5.4 b



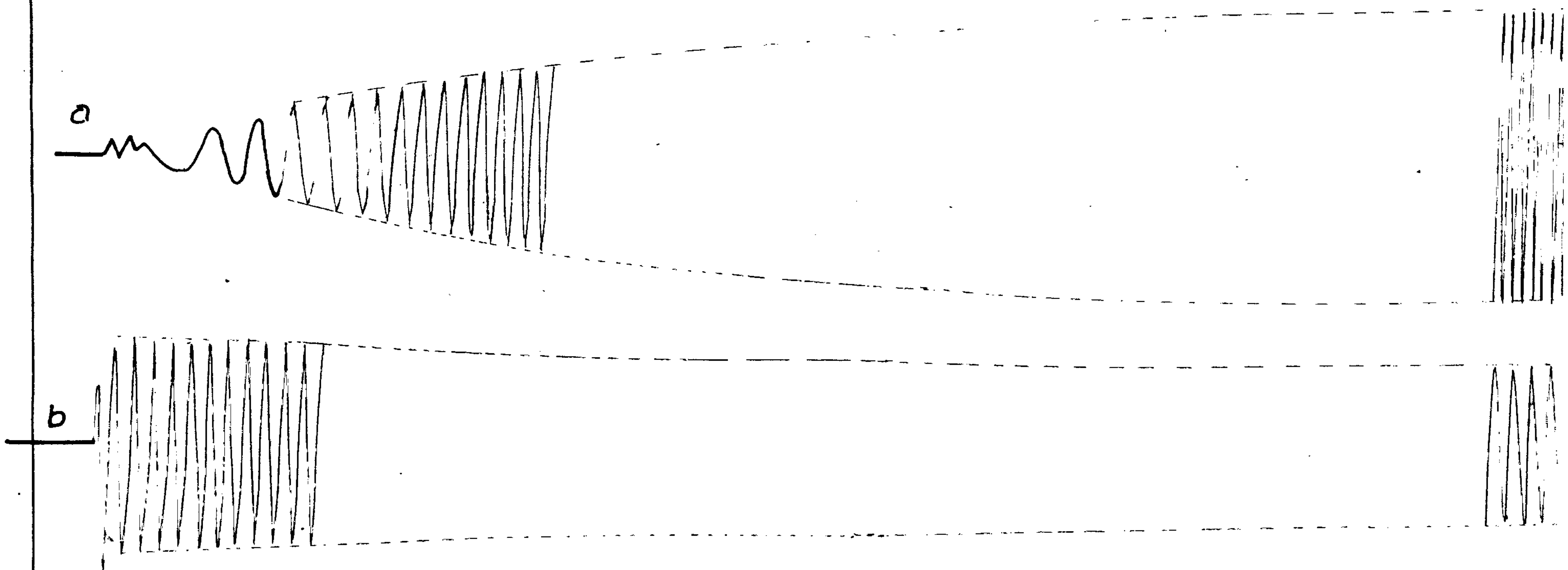
$V_1 = V_2 = 110V$

FINAL SPEED = 1350 R.P.M

a - SPEED TRANSIENT   b - CURRENT TRANSIENT

SPEED RISE (COPPER THICK CUP) AT NO LOAD

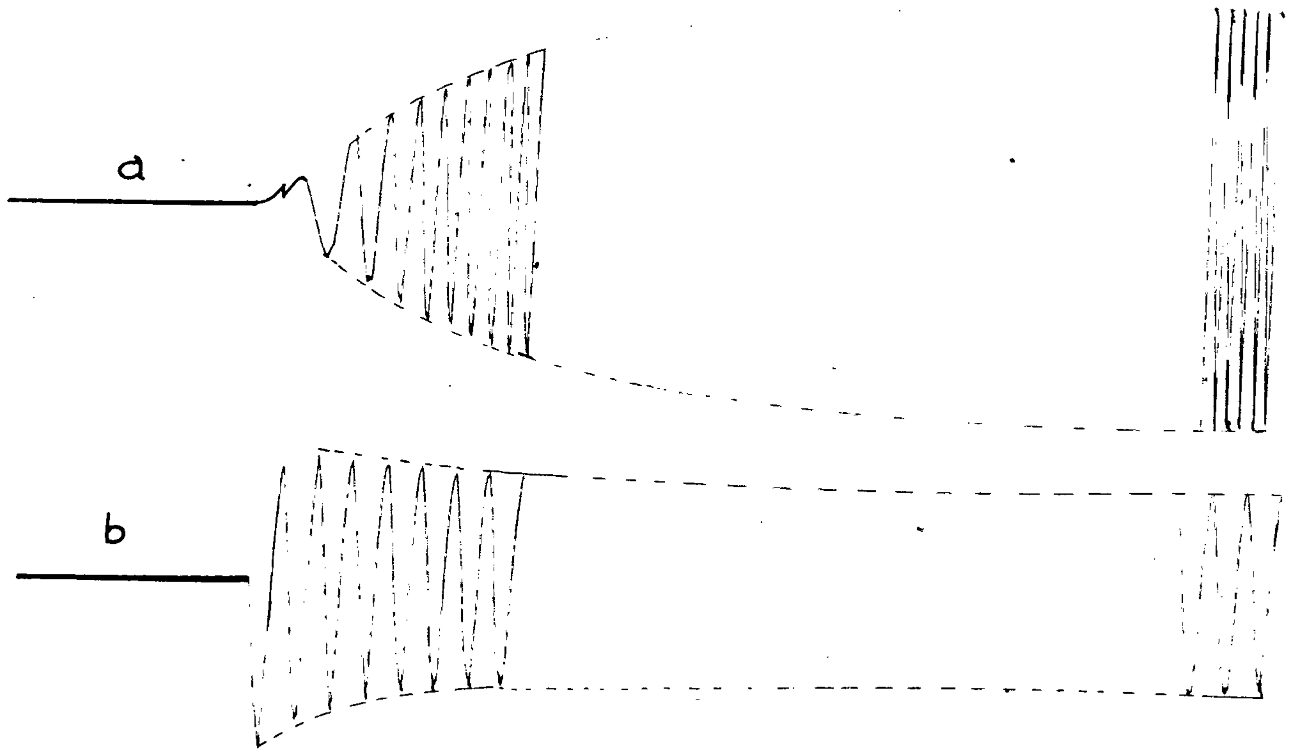
FIG. 5.5



$V_1 = V_2 = 110V$
FINAL SPEED = 1000 R.P.M
a- SPEED TRANSIENT b-CURREN TRANSIENT

SPEED RISE (COPPER THIN. CUP) AT LOAD

FIG. 5.6b



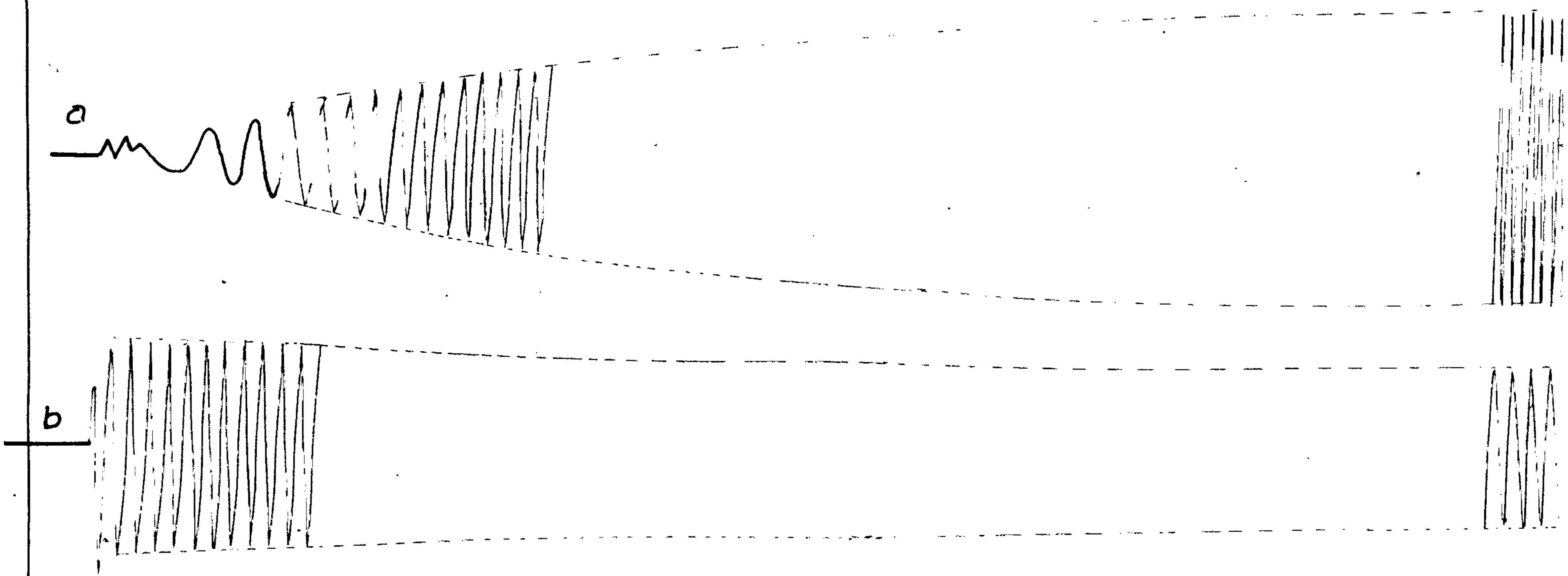
$V_1 = V_2 = 110V$

FINAL SPEED = 1360 RPM

a SPEED TRANSIENT b- CURRENTRANSIENT

SPEED RISE (COPPEER THIN CUP) AT NO LOAD

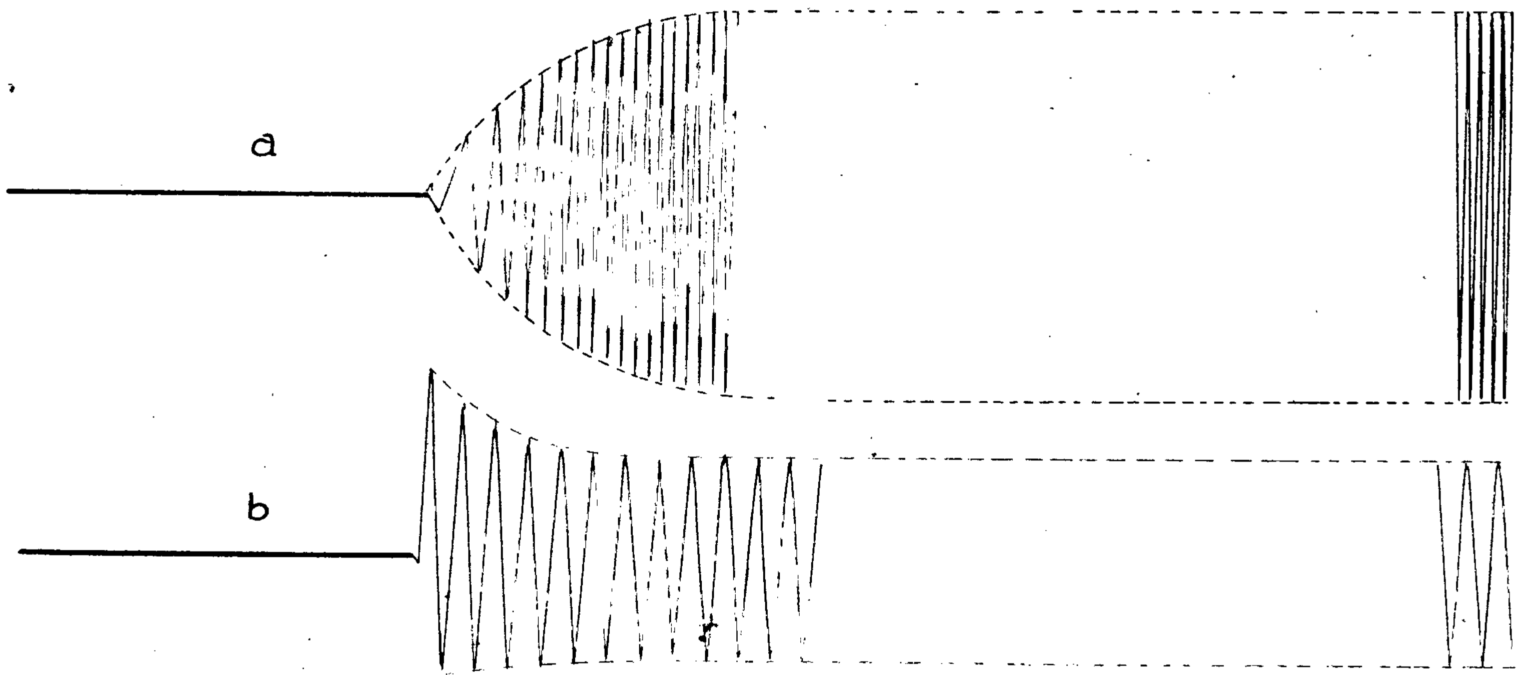
FIG. 5.6a



$V_1 = V_2 = 110V$
FINAL SPEED = 1000 R.P.M
a- SPEED TRANSIENT b-CURREN TRANSIENT

SPEED RISE (COPPER THIN. CUP) AT LOAD

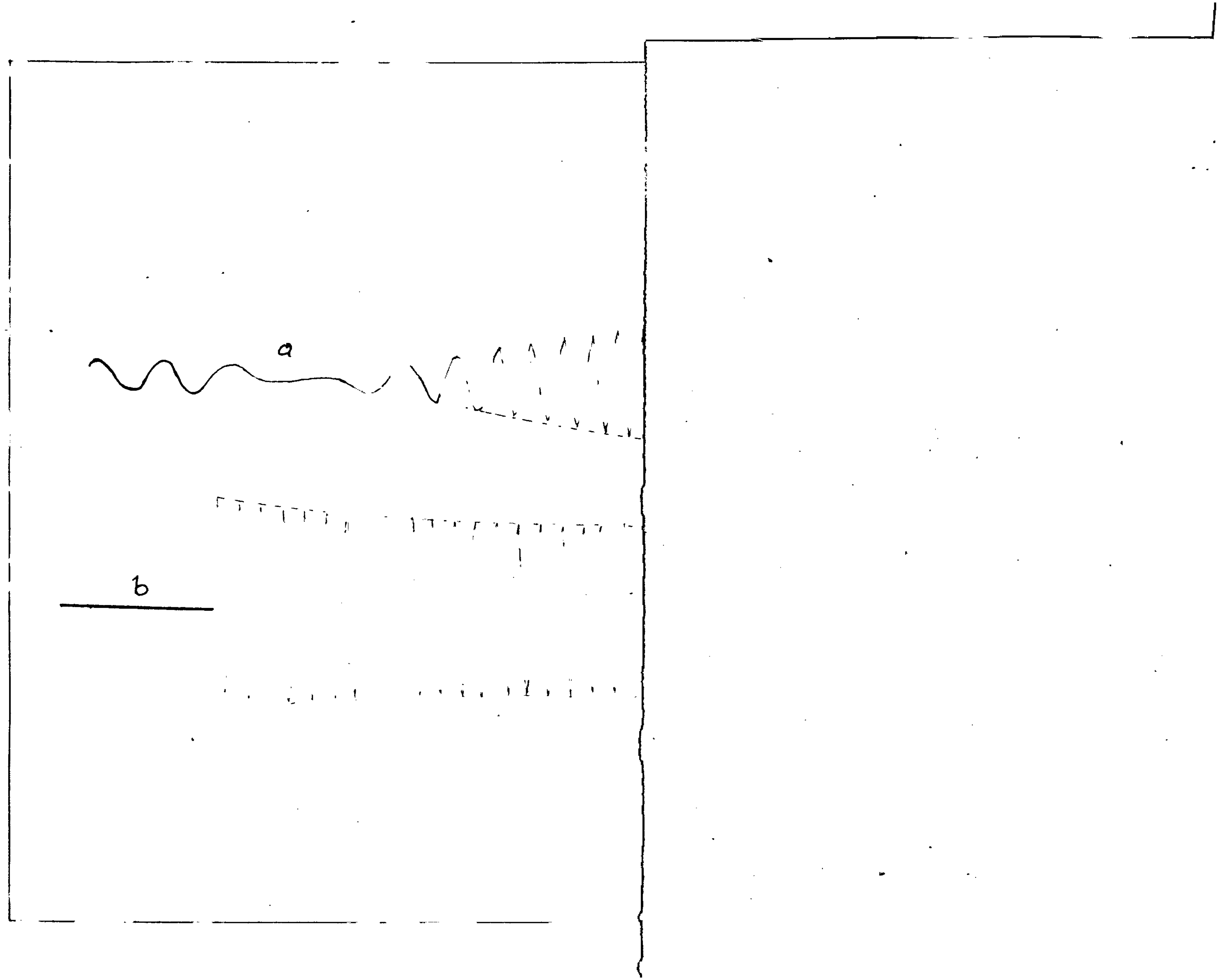
FIG. 5.6b



$V_1 = V_2 = 110V$
FINAL SPEED= 1380 R.P.M
a- SPEED TRANSIENT b- CURRENTTRANSIENT

SPEED RISE (ALUMINIUM CUP) AT NO LOAD

FIG. 5.7a


a
b

C O N C L U S I O N S.

In the present work it has been shown that the curvature of the machine has little effect on torque-speed characteristic except at low values of slip. The results of rectangular co-ordinates are in close agreement with those of cylindrical ones. The three dimensional analysis gives better results over two dimensional analysis in a restricted region of torque speed curve. Results differ significantly at higher slips. Effect of harmonics is pronounced particularly when tangentially directed component of vector potential is taken into account.

The so-called $Z_{gap-rotor}$, the combined impedance of rotor airgaps and central core leads to equivalent circuit which is by no means simple as the rotor resistance and reactance are both functions of slip and the number of harmonics of Fourier series.

APPENDIX I

I.1. EFFECT OF NEGLECTING TANG. COMPONENT OF VECTOR POTENTIAL:

When tangential component of vector potential or rotor current is neglected we have-

$$A_{gx1} = 0$$

Now from the analysis of Chapter 3, section 3.1.4 from expression for voltage induced in a conductor we have-

$$v_{(cond)} = \int_0^{\lambda} (T_z \; E_{gz})_{y=g_1} dz$$

Voltage induced in a phase of stator winding is-

$$V_1 = p \int_0^T v_{(cond)} \; d_{x1}$$

Substituting the values of T_z and E_{gz} in expression for v(cond) and carrying out the required integrations one obtains-

$$V_1 = \frac{3}{8} \frac{p \lambda \mu \omega T}{\beta_n \times 2} \cdot \left(\frac{4}{n\pi}\right)^2 \; B_1^2 \; a_5 . I_1 \sin \omega t$$

Dividing by $I_1 \sin \omega t$,

$$Z_{gap-rotor} = \frac{6 p \lambda \omega \mu \; . \; B_1^2}{\beta_n \pi \; n^2 a} . a_5 \qquad \ldots \qquad (I-1)$$

I.2. ANALYSIS BASED ON AMPERE TURN CONCEPT:

At stator surface $y = g_1$, ampere turn has sinusoidal variation in x direction and periodic variation in z direction. Therefore,

$$AT = \frac{4}{n\pi} M \sum_{n=1,3..} \sin \frac{n\pi z}{\lambda} \exp \left(j \left(\omega t + \frac{x\pi}{T}\right)\right)$$

where M is the peak value of ampere turn.

$$H_x = \frac{\partial (AT)}{\partial x}$$

$$= \frac{\pi}{T} . \sum_{n=1,3..} \frac{4}{n\pi} M . \sin \frac{n\pi z}{\lambda} \exp\left(j\left(\omega st + \frac{\pi x}{T}\right)\right) \qquad \ldots \qquad (I.2)$$

Flux density variation in x direction is sinusoidal and periodic in z direction. We may therefore write,

$$H_y = \sum_{n=1,3..} \frac{A}{n\pi} \frac{B_o}{\mu_o} \exp\left(j\left(\omega t + \frac{\pi x}{T}\right)\right)\left(\sin \frac{n\pi z}{\lambda}\right) \quad \ldots (I.3)$$

From (3.25) and (3.10) of Chapter 3,

$$H_x = \sum_{n=1,3..} \frac{\beta_n}{\mu_o} \exp\left(j\left(\omega t + \frac{\pi x}{T}\right)\right)(A_{n1} \sinh \beta_n c_1 + B_{n1} \cosh \beta_n c_1).$$

$$\sin \frac{n\pi z}{\lambda} \quad \ldots \quad \ldots (I.4)$$

and $H_y = -\sum_{n=1,3..} j \frac{1}{\mu_o} \frac{\pi}{T} \exp.\left(j\left(t\omega + \frac{\pi x}{T}\right)\right) (A_{n1} \cosh \beta_n c_1 + B_{n1} \sinh \beta_n c_1)$

$$\ldots (I.5)$$

For equations (I.2), (I.4) and (I.3), (I.5) to be identical one must have—

$$\frac{\pi}{T} \cdot \frac{A}{n\pi} M = \frac{\beta_n}{\mu_o}(A_{n1} \sinh \beta_n c_1 + B_{n1} \cosh \beta_n c_1)$$

$$\frac{A}{n} \frac{B_o}{} = \frac{-j}{\mu_o} \frac{\pi}{T}(A_{n1} \cosh \beta_n c_1 + B_{n1} \sinh \beta_n c_1)$$

From the a—

$$M = j \frac{\beta_n}{\mu_o} \frac{\omega a^2}{\pi^2 \; c_5}$$

Therefore,

$$Z_{gap-rotor} = \frac{\text{volts x turns}}{M}$$

$$\text{Now volts} = \frac{4.442\, T_{ph} K_r}{p} \; D.$$

Therefore, $Z_{gap-rotor} = \dfrac{4.442\, T_{ph}^2 K_r \pi \; D \lambda \mu_o a^2 a_s}{p \; \beta_n}$ (I.6)

Now since $\beta_n^2 = \dfrac{n^2\pi^2}{\lambda^2} + a^2$ a comparision of (3.44) and (I.6) indicates that these two expressions will be identical if $\beta_n^2 = a^2$. So neglecting tangential component of vector potential will give accurate results only if $\beta_n \approx n$ that means the term $\frac{n\pi}{\lambda}$ is small. This would in turn mean that λ, the machine length in z direction must be large for this assumption to hold. So the consideration of tangential component of vector potential is necessary if machine length is not very large.

APPENDIX II

SOME RELATIONS OF BESSEL FUNCTIONS

Bessel function of 1st kind and order n is given by-

$$J_n(z) = \sum_{m=0} \frac{(-1)^m (z/2)^{n+2m}}{m!\,(n+m)!} \qquad \dots \qquad (1)$$

where z may be real or complex.

Bessel function of 2nd kind and order n is given by-

$$Y_n(z) = 2J_n(z)\left(\log_e \frac{z}{2} + \gamma\right) - \sum_{m=0} \frac{(-1)^m (z/2)^{n+2m}}{m!\,(n+m)!}\left(\sum_{r=1}^{n+m} \frac{1}{r} + \sum_{r=1}^{n} \frac{1}{r}\right) - \sum_{m=0}^{n-1} \frac{(z/2)^{-n+2m}\,(n-m-1)!}{m!} \qquad \dots \qquad (2)$$

where γ = Euler's constant. $= 0.5772157$

When argument is purely imaginary it is convenient to use modified Bessel functions which are defined by-

$$I_n(x) = \sum_{m=0}^{\infty} \frac{(x/2)^{n+2m}}{m!\,(n+m)!} \qquad \dots \qquad (3)$$

$$K_n(x) = \frac{1}{2}\sum_{m=0}^{\infty} \frac{(-1)^m\,(n-m-1)!\,(x/2)^{-n+2m}}{m!} + \frac{(x/2)^{n+2m}}{n!\,(n+m)!}\left\{\log_e \frac{x}{2} + \gamma - \frac{1}{2}\sum_{r=1}^{m} \frac{1}{r} - \frac{1}{2}\sum_{r=1}^{n+m} \frac{1}{r}\right\} \qquad \dots \qquad (4)$$

Expressions (1) to (4) above are slowly convergent and are not suited for computation of functions of large arguments. Use is made of asymptotic expansions when arguments are large. They are defined as under-

$$I_n(x) = \frac{e^{x}}{(2\pi x)^{\frac{1}{2}}} \sum_{m=0}^{\infty} \frac{(-1)^m (n, m)}{(2x)^m} +$$

$$\frac{e^{-x + j(n+\frac{1}{2})\pi}}{(2\pi x)^{\frac{1}{2}}} \cdot \sum_{m=0} \frac{(n, m)}{(2x)^m}$$

where, $(n,m) = \dfrac{(4n^2-1^2)\,(4n^2-3^2)\,\ldots\,(4n^2-(2m-1)^2)}{2^{2m}\,\lfloor m}$

$$K_n(x) = \left(\frac{\pi}{2x}\right)^{\frac{1}{2}} \exp(-x) \cdot \left[1 + \frac{4n^2-1^2}{\lfloor 1(8x)} + \frac{(4n^2-1^2)(4n^2-3^2)}{\lfloor 2(8x)^2} + \frac{(4n^2-1^2)(4n^2-3^2)(4n^2-5^2)}{\lfloor 3\,(8x)^3} + \ldots\right]$$

$$J_n(z) \sim \left(\frac{2}{\pi z}\right)^{\frac{1}{2}} \left[\left\{1 - \frac{(4n^2-1^2)(4n^2-3^2)}{\lfloor 2(8z)^2} + \frac{(4n^2-1^2)(4n^2-3^2)(4n^2-5^2)(4n^2-7^2)}{\lfloor 4\,(8z)^4} - \ldots\right\}\cos\phi - \left\{\frac{4n^2-1^2}{\lfloor 1(8z)} - \frac{(4n^2-1^2)(4n^2-3^2)(4n^2-5^2)}{\lfloor 3\,(8z)^3} + \ldots\right\}\sin\phi.\right]$$

$$Y_n(z) \sim \left(\frac{2}{\pi x}\right)^{\frac{1}{2}} \left[\left\{1 - \frac{(4n^2-1^2)(4n^2-3^2)}{\lfloor 2\,(8z)^2} + \frac{(4n^2-1^2)(4n^2-3^2)(4n^2-5^2)(4n^2-7^2)}{\lfloor 4(8z)^4} - \ldots\right\}\sin\phi + \left\{\frac{4n^2-1^2}{\lfloor 1(8z)} - \frac{(4n^2-1)(4n^2-3^2)(4n^2-5^2)}{\lfloor 3(9z)^3} + \ldots\right\}\cos\phi\right]$$

where $\phi = z - (n+\frac{1}{2})\frac{\pi}{2}$

and $|z| \gg 1$, $|z| \gg |n|$

$$I'_\nu(x) = \frac{1}{2}\left(I_{\nu+1}(x) + I_{\nu-1}(x)\right)$$

$$K'_\nu(x) = -\frac{1}{2}\left(K_{\nu+1}(x) + K_{\nu-1}(x)\right)$$

$$J'_\nu(z) = \frac{1}{2}\left(J_{\nu-1}(z) - J_{\nu+1}(z)\right)$$

$$Y'_\nu(z) = \frac{1}{2}\left(Y_{\nu-1}(z) - Y_{\nu+1}(z)\right)$$

APPENDIX-III

DESIGN CONSIDERATIONS

The design of servomotors differs from that of conventional motors in several aspects. This is because of requirements demanded in control system applications. These are quick response, ability to operate over a wide range of speed, linearity of torque speed characteristic, maximum torque at stall, ability to withstand the continuously applied reference phase voltage and not to single phase when error signal is zero.

In order to achieve low inertia which is necessary for low motor time constant a small diameter rotor is employed while at the same time employing as long a rotor as practicable to maintain the torque capability. To further reduce the inertia the thickness of cup should be small. This results in a quick response motor but at the same time leads to high rotor resistance and, therefore, limiting the output of the machine. In higher rating machines a thick cup must be used. This not only increases the acceleration time constant but results in an excessive input current which may not be easy to handle and at the same time the linearity of the torque speed characteristic is worsened. This is because of an excessively large airgap which is created by the thick cup which forms a part of the airgap. The length of the airgap may be several times higher than that in conventional induction machines. One more trouble that may arise because of sufficiently thick cup is that the torque speed characteristic may become drooping at higher slips and thereby limiting the stability region of the motor. In a balanced design an agreement must therefore, be attained between the thickness of cup, maximum power output and the linearity of the torque speed characteristic. A compromise is also necessary in the degree of linearity that can be achieved. The straighter the torque speed curve, the less stall torque available, which means smaller initial accelerations. In most of the servomotors a compromise is made so that the maximum torque occurs between slips of 1.5 and 2 per unit. This gives reasonable linearity and stalled torque.

It is essential that such a motor does not 'single-phase'. That is, when the control field voltage is reduced to zero, it should not develop a positive torque at any speed. This requirement is met if the torque speed characteristic peaks at a negative speed. It has been shown[20] that a motor will not single-phase provided that the secondary resistance is smaller than the airgap reactance.

In order to keep the exciting current low the clearances between the moving cup and the stationary parts should be limited only by mechanical considerations, allowance being made for uneven heating and gyroscopic torques.

The temperature rise is much pronounced in servomotors. This is because of the continuous excitation of one phase and large exciting current. This may be a problem in large motors. Unlike ordinary rotary machines the temperature cannot be lowered by increased machine dimensions the specific load is determined by the time constant condition as well as the capacity. Consequently this problem must be solved by heat resistant and insulating materials and cooling systems. For large capacities a separately driven blower may be necessary to cool the machine. In still bigger sizes it is necessary to add a circuit which will lower excitation voltage at the zero position to restrain the temperature rise.

DESIGN:

The design of test motor was carried out by the conventional method of induction motor design technique except for rotor. Some of the points earlier discussed were taken into consideration to modify the conventional motor design. The stator winding was designed to carry the extra excitation current required by the large airgap. Ratio of length to diameter was kept high in order to the reduce the moment of inertia of rotor.

Design of cup rotor was done somewhat arbitrarily. Torque speed curve was predetermined and a value of cup thickness selected so as to meet the requirement of power output and linearity of torque speed curve.

ACTUAL DESIGN DATA - [illegible] h.p. motor.

Stator frame:

No. of slots = 24

Tooth pitch = 0.49"

Pole pitch = 2.95"

Average dia = 3.75"

Stack length = 3"

Stator winding:

No. of coils = 24, frequency = 50 c/s.

coil pitch = 6, No. of poles = 4

No. of turns per coil = 40

No. of turns in series per phase = 552

Each coil of 2 strands of 24 SWG super enamelled copper wire.

Double layer two phase winding.

Stator resistance at 20°C = 8.0 ohms

Stator resistance at 50°C = 10.3 ohms

Stator leakage reactance = 7.5 ohms

Rotor core-

14 mil transformer iron sheet laminations.

Rotor cup-

1. Copper mixed with 10% tin. Resistivity of rotor material at 60°C = 4.5×10^{-8} ohm meter.

2. Commercial aluminium. Resistivity at 60°C = 3.58×10^{-8} ohm meter.

   Dimensions are as per fig. 1.

PRINCIPAL DESIGN DATA - 3 h.p. motor:

Stator frame:

No. of slots	= 36
Tooth pitch	= .48"
Mean slot opening	= .125"
Average diameter	= 5.5"
Stack length	= 6.5"

Stator winding:

No. of coils	= 36
Coil pitch	= 4, 5 wound so as to obtain minimum of unbalance.
No. of poles	= 4
Frequency	= 50 c/s
No. of turns per coil	= 9

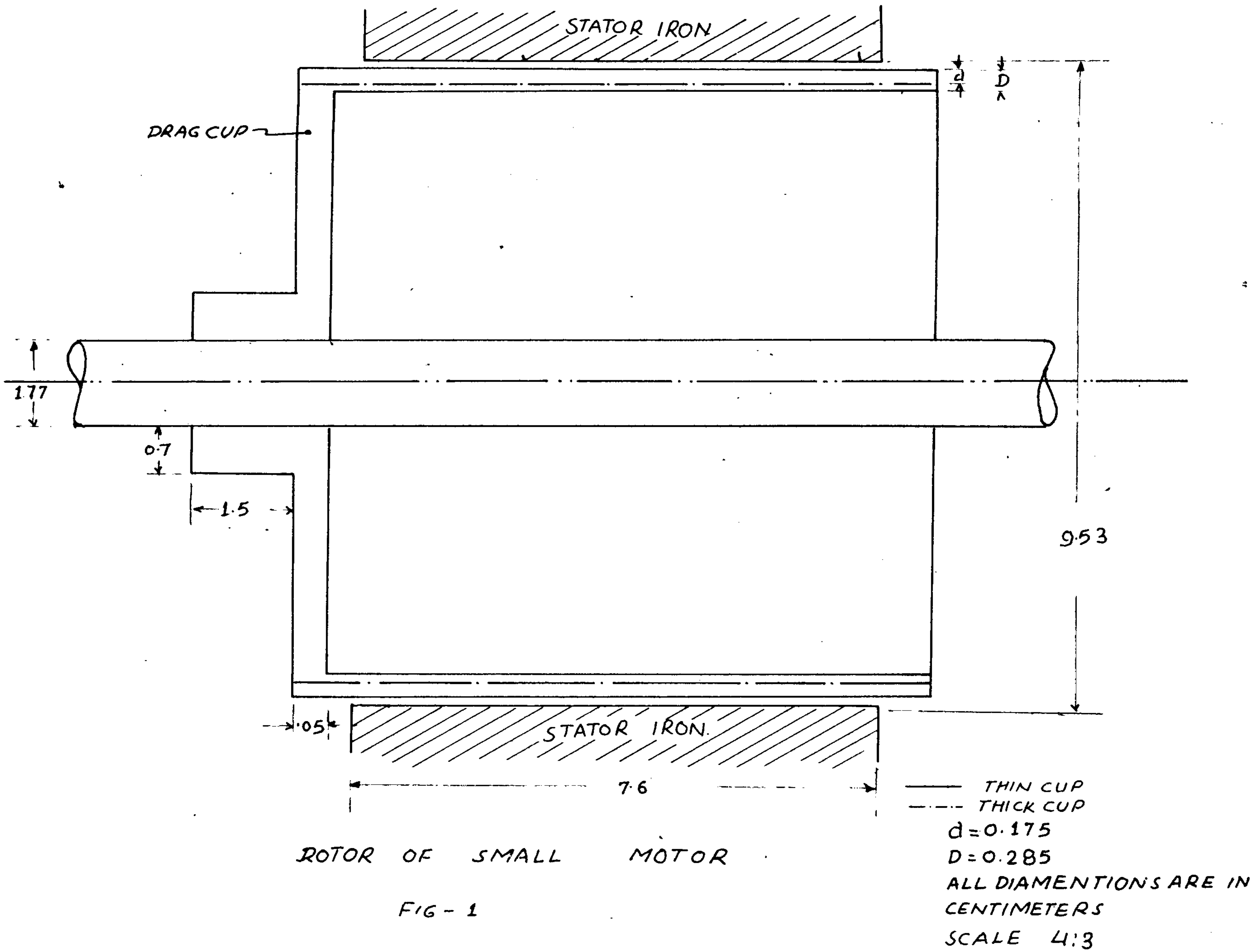
Each coil of 4 strands, 3 strands of 18 SWG and 1 strand of 19 SWG super-enamelled copper wire.

Double layer two phase winding.

No. of turns in series per phase = 162.

Stator resistance at 50°C = 10 ohms

Stator leakage reactance = 1.5 ohms



ROTOR OF SMALL MOTOR

FIG - 1

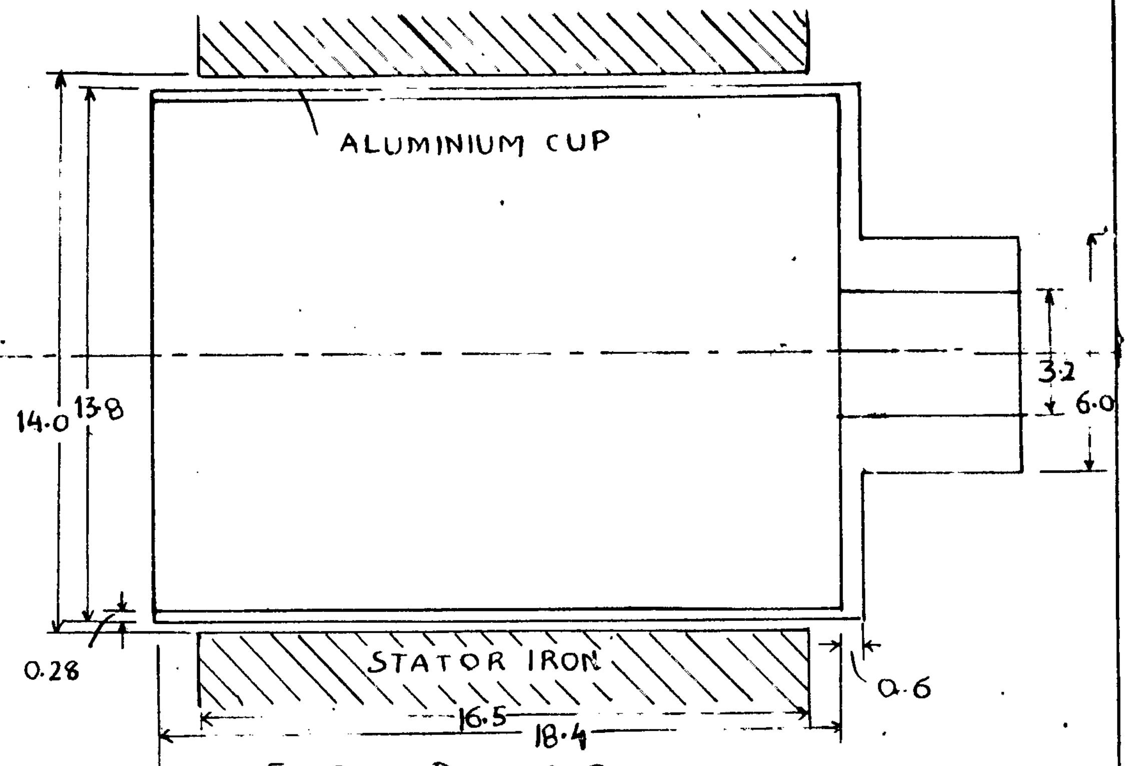


Fig. 2 a DRAG CUP

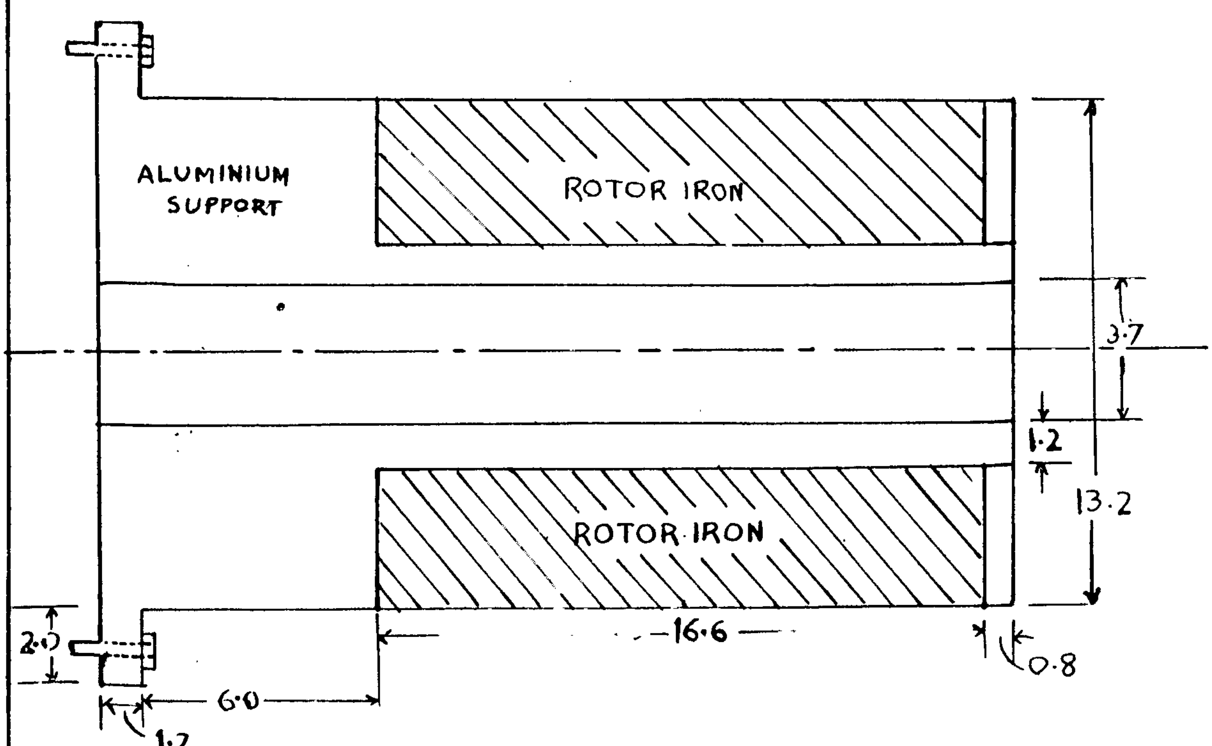


Fig 2 b CENTRAL CORE

ROTOR OF LARGE MOTOR

FIG. 2.

(ALL DIMENSIONS ARE IN CM.)

SCALE – HALF FULL SIZE

Rotor core:

18 mil transformer iron sheet laminations.

Rotor cup:

Commercial aluminium.

Resistivity at 60°C = 3.58 x 10^{-8} ohm meter

Dimensions are as per fig.2.

PROGRAM NO. 1

APPENDIX IV

LIST OF NOTATIONS

```
  QUANTITIES IN INPUT LIST
AMU=PERMEABILITY OF FREE SPACE
OMEGA=ANGULAR FREQUENCY
G1,G2,G3=MACHINE DIMENSIONS IN Y AXIS IN METERS
DIA=DIAMETER IN INCHES AT STATOR BORE
BO=AIRGAP FLUX DENSITY IN WEBERS PER METRE AT DESIGNED VOLTAGE
VOLTS =DESIGN TERMINAL VOLTAGE
TURNS =NO. OF TURNS PER PHASE
SYN=SYNCHRONOUS SPEED
R1=STATOR RESISTANCE
X1=STATOR LEAKAGE REACTANCE
RHO=RESISTIVITY OF CUP MATERIAL IN OHM METERS
I1,I2,I3,XY ARE QUANTITIES USED TO CONTROL THE VARIATION OF SLIP
V1=REFERENCE PHASE VOLTAGE
V2=CONTROL PHASE VOLTAGE
L=NO. OF TIMES THE OPERATION IS REPEATED WITH DIFFERENT VALUES OF V1AND V2
  QUANTITIES IN OUTPUT LIST
ZEQ=EQUIVALENT IMPEDANCE OF ROTOR,ROTOR CORE AND AIRGAPS
R2+JX2=ZEQ
TORQ=TORQUE IN LB-FT
```

```
C   C   TORQUE SPEED CURVEBY TWO DIMENSIONAL ANALYSIS IN RECT. CO-OR.  Z
        DIMENSION SLIP(20),R2(20),X2(20),ZEQ(20)
        PUNCH101
101     FORMAT(5X50HTORQUE SPEED CHARACTERISTIC  2D CASE  RECT. COORD.)
        READ30,L
30      FORMAT(I2)
        LL=1
312     READ50,AMU,OMEGA,G1,G2,G3
50      FORMAT(E20.5,4F10.0)
        READ20,DIA,BO,VOLTS,TURNS,POLES,SYN
20      FORMAT(7F10.0)
        READ60,R1,X1
60      FORMAT(2F10.0)
        READ40,RHO
40      FORMAT(F20.5)
        READ102,I1,I2,I3,XY
102     FORMAT(3I2,F10.0)
        PUNCH300,RHO,G1,G2,G3
300     FORMAT(/4HRHO=E12.5,4X3HG1=F7.5,4X3HG2=F7.5,4X3HG3=F7.5)
        PY=3.14159
        TA=PY*DIA*.0254/POLES
        BBB=180./PY$ CCC=1./BBB$ DDD=2.*7.04/1500.
        AK=PY/TA$ TAK=AK+AK$ A=EXPF(TAK*G3)$ A1=-1./A
        B=EXPF(TAK*G2)$ G=EXPF(TAK*G1)$ X=AK*AK
        DO1I=I1,I2,I3$  AI=I$   SLIP(I)=AI/XY $  S=SLIP(I)
        Y=S*OMEGA*AMU/RHO$ RH=(X*X+Y*Y)**.25
        FI=ATANF(Y/X)$ FIH=FI/2.$ AMR=RH*COSF(FIH)$ AMI=RH*SINF(FIH)
        AA=2.*G2*AMR$ BB=2.*G2*AMI$ CC=EXPF(AA)$ CR=CC*COSF(BB)
        CI=CC*SINF(BB)$ DD=2.*G1*AMR$ EE=2.*G1*AMI$ GG=EXPF(DD)
        DR=GG*COSF(EE)$ DI=GG*SINF(EE)$ AB=A1*B$ F=(1.-AB)/(1.+AB)
        E=F*AMR$ H=F*AMI$ O=E-AK$ P=(E+AK)*CR-CI*H$ Q=CI*(E+AK)+CR*H
        R=P*P+Q*Q$ AR2=(O*P+H*Q)/R$ AI2=(H*P-O*Q)/R$ T=AR2*DR-AI2*DI
        U=AR2*DI+AI2*DR$ V=1.+T$ W=1.-T$ USQ=U*U$ Z=W*W+USQ
        PR=(V*W-USQ)/Z$ PI=(V*U+U*W)/Z$ HH=AK*PR$ OO=AK*PI$ PP=AMR-HH
        QO=AMI-OO$ RR=(AMR+HH)*G$ SS=(AMI+OO)*G$ TT=RR*RR+SS*SS
```

```
      AR3=(PP*RR+QQ*SS)/TT$ AI3=(QQ*RR-PP*SS)/TT$ UU=1.+AR3$ VV=1.-AR3
      XX=AI3*AI3$ WW=VV*VV+XX$ YY=(UU*VV-XX)/WW$ ZZ=(UU*AI3+VV*AI3)/WW
      AAA=BO/(AMU*AK)$ ATR=YY*AAA$ ATI=ZZ*AAA
      ANGLE=ATANF(ATI/ATR)*BBB$ AT=SQRTF(ATR*ATR+ATI*ATI)
      ZEQ(I)=VOLTS*TURNS/AT$   ANG=(90.-ABSF(ANGLE))*CCC
      R2(I)=ZEQ(I)*COSF(ANG)$   X2(I)=ZEQ(I)*SINF(ANG)
1     CONTINUE
31    READ150,V1,V2 $   VF=(V1+V2)/2.$  VB=(V1-V2)/2.
150   FORMAT(2F10.0)
      PUNCH250,V1,V2
250   FORMAT(3HV1=F5.0,5X3HV2=F5.0)
      VFSQ=VF*VF$   VBSQ=VB*VB
      DO2I=I1,I2,I3 $   CFSQ=VFSQ/((R1+R2(I))**2+(X1+X2(I))**2)
      IF(VB)22,21,22
21    CBSQ=0. $    GOTO51
22    CBSQ=VBSQ/((R1+R2(I2-I))**2+(X1+X2(I2-I))**2)
51    TF=CFSQ*R2(I)$     TB=CBSQ*R2(I2-I)$     TORQ=(TF-TB)*DDD
2     PUNCH200,SLIP(I),ZEQ(I),R2(I),X2(I),TORQ
200   FORMAT(F4.1,4F17.3)
      LL=LL+1
      IF(LL-L)31,31,32
32    STOP
      END
```

SAMPLE DATA

```
 1
          .12566E-05314.      .00055     .00365     .00395
5.5       .36       220.      162.       4.
1.        1.5
3.58E-08
 120 1 10.
110.      110.
```

PROGRAM NO. 2

```
  QUANTITIES IN INPUT LIST
NPH=NO. OF PHASES
Y1=STATOR RESISTANCE
Z1=STATOR LEAKAGE REACTANCE
V1-STATOR TERMINAL VOLTAGE
SPEED=SYNCHRONOUS SPEED IN RPM
M1,M2,M3,AAM ARE THE FACTORS TO CONTROL THE VARIATION OF SLIP
R1,R2,R3,R4 ARE RADII AT STATOR BORE,ROTOR OUTER,ROTOR INNERAND ROTOR COREI
TURNS=NO. OF TURNS IN SERIES PER PHASE
VOLT=DESIGN TERMINAL VOLTAGE
BO=AIRGAP FLUX DENSITY IN WB/METRE SQUAREAT DESIGNED VOLTAGE
  QUANTITIES IN OUTPUT LIST
S=SLIP IN PER UNIT
ANGLE=ANGLE OF AMPERE-TURN
ZEQ=EQUIV. IMPENDANCE OF ROTOR,AIRGAPS,ROTOR CORE
Y2+JZ2=ZEQ

C  C  TORQUE SPEED  CURVE BY TWO DIMENSIONAL ANALYSIS IN CYLIN. COORD. Z
      READ 150,NPH
 150  FORMAT (I1)
```

```
      READ 100,Y1,Z1,V1,SPEED
  100 FORMAT(4F10.0)
       READ 50,M1,M2,M3,AAM
   50  FORMAT (3I2,F10.0)
      READ 10,OMEGA,AMEW,RHO,R1,R2,R3,R4
   10 FORMAT(F6.0,E13.6,E11.4,4F10.0)
      READ20,TURNS,VOLT,BO
   20 FORMAT(3F10.0)
      RT=SQRTF(2.)$ TRT=2.*RT$ RTO=RT*OMEGA$ RTSIX=RT*6.$ RRR=R3
      TG=1.154432$ PY=3.14159$ AA=PY*RT$ DD=PY/12.$ EE=PY/2.
      AMEWA=0.125664E-05$ XA=R1**4$ XB=R3**4$ XC=R2**4$ A1=-R4**4
      XD=XB-A1$ XE=XB+A1$ OJ=180./PY$ OH=BO*R1/(2.*AMEWA)$ ANPH=NPH
      OK=VOLT*TURNS$ B=OMEGA*AMEW/RHO$ DDD=ANPH*7.04/SPEED
       DO 1I=M1,M2,M3$ CONS=I$ S=CONS/AAM$ R3=RRR
C     CALCULATION FOR A2,A4,A7,A9 BEGINS
      DO 2J=1,2$ GO TO (3,4),J
    4 R3=R2$ GO TO 3
    3 AL=SQRTF(S*B)$ AO=AL/RTO$ X=AL*R3/2.$ CC=2.*LOGF(X)$ GG=X**3
      XSQN=-X*X$ XX=XSQN$ YY=XX$ SIGN=XSQN*X
      D1=1.$ K=1$ SR1=X/RT$ SI1=SR1$ SR3=-GG/RTSIX$ SI3=-SR3
    6 AK=K$ D1=D1*AK*(AK+1.)$ Q=1.+2.*AK$ U=Q*.7854$ T=SIGN/D1
      TR=T*COSF(U)$ TI=T*SINF(U)$ SR1=SR1+TR$ SI1=SI1+TI$ K=K+1
      SIGN=SIGN*XSQN$ IF(ABSF(TR)-.000001)5,6,6
    5 IF(ABSF(TI)-.000001)7,6,6
    7 L=1$ SIGN=XSQN*GG$ D3=6.
    9 ZL=L$ D3=D3*ZL*(ZL+3.)$ R=3.+2.*ZL$ V=R*.7854$ W=SIGN/D3
      WR=W*COSF(V)$ WI=W*SINF(V)$ SR3=SR3+WR$ SI3=SI3+WI$ L=L+1
      SIGN=SIGN*XSQN$ IF(ABSF(WR)-.000001)8,9,9
    8 IF(ABSF(WI)-.000001)11,9,9
   11 A2R=AO*(SR1+SR3-SI1-SI3)$ A2I=AO*(SR1+SR3+SI1+SI3)
      A4R=AO*(SR3-SR1+SI3-SI1)$ A4I=AO*(SR1-SR3-SI1+SI3)
C     CALCULATION FOR A3,A5,A8,A10 BEGINS
      AR=X/AA$ AI=AR$ KK=1$ D11=1.$ DR=CC-1.+TG$ DI=EE
   14 BK=KK$ CK=BK*EE$ D11=D11*BK*(BK+1.)$ SA=0.0
      DO 12M=1,KK$ AM=M
   12 SA=SA+1./AM$ AM1=AM+1.$ SB=SA+1./AM1$ BB=XX/D11
      BR=BB*COSF(CK)$ BI=BB*SINF(CK)$ CR=CC-SA-SB+TG$ CI=EE
      ER=BR*CR-BI*CI$ FI=BR*CI+BI*CR$ DR=DR+ER$ DI=DI+EI$ KK=KK+1
      XX=XX*XSQN
      IF(ABSF(ER)-.000001)13,14,14
   13 IF(ABSF(EI)-.000001)15,14,14
   15 FR=1./(AA*X)$ FI=-FR$ GR=AR*DR-AI*DI$ GI=AR*DI+AI*DR
      YR1=GR-FR$ YI1=GI-FI
C     CALCULATION FOR 3RD ORDER BEGINS
      HR=-GG/AA$ HI=-HR$ QR=(CC-1.83333+TG)/6.$ QI=DD$ D33=6.$ N=1
   19 CN=N$ DN=CN*EE$ D33=D33*CN*(CN+3.)$ SC=0.$ SD=0.$ N3=N+3
      DO 16M=1,N$ BM=M
   16 SC=SC+1./BM$ DO17M=1,N3$ CM=M
   17 SD=SD+1./CM$ O=YY/D33$ OR=O*COSF(DN)$ OI=O*SINF(DN)
      PR=CC-SC-SD+TG$ PI=EE$ RR=OR*PR-OI*PI$ RI=OR*PI+OI*PR
      YY=YY*XSQN
      QR=QR+RR$ QI=QI+RI$ N=N+1$ IF(ABSF(RR)-.000001)18,19,19
   18 IF(ABSF(RI)-.000001)21,19,19
   21 PP=RT/GG$ QQ=1./(X*RT)$ ZZ=X/TRT$ SR=(-PP+QQ+ZZ)/PY
      SI=(-PP-QQ+ZZ)/PY$ ZR=HR*QR-HI*QI$ ZI=HR*QI+HI*QR$ YR3=ZR-SR
      YI3=ZI-SI$ A3R=AO*(YR1+YR3-YI1-YI3)$ A3I=AO*(YR1+YR3+YI1+YI3)
      A5R=AO*(YR3-YR1+YI3-YI1)$ A5I=AO*(YR1-YR3-YI1+YI3)$ GO TO(23,22),J
```

```
23 B2R=A2R$ B2I=A2I$ B4R=A4R$ B4I=A4I$ B3R=A3R$ B5R=A5R$ B5I=A5I
   B3I=A3I$ GOTO2
22 A7R=A2R$ A7I=A2I$ A9R=A4R$ A9I=A4I$ A8R=A3R$ A8I=A3I$ A10R=A5R
   A10I=A5I
 2 CONTINUE
C     CALCULATON FOR A6,A11 BEGINS
      UR=B5R*XD-B3R*XE$ UI=B5I*XD-B3I*XE$ DDR=B2R*XE-B4R*XD
      DDI=B2I*XE-B4I*XD$ XG=DDR*DDR+DDI*DDI$ A6R=(UR*DDR+UI*DDI)/XG
      A6I=(UI*DDR-UR*DDI)/XG$ XH=A6R*A7R-A6I*A7I+A8R
      XI=A6I*A7R+A6R*A7I+A8I$ XJ=A6R*A9R-A6I*A9I+A10R
      XK=A6I*A9R+A6R*A9I+A10I$ XL=XH+XJ$ XM=XI+XK$ XN=XJ-XH$ XO=XK-XI
      XP=XN*XN+XO*XO$ XQ=XC*XP$ A11R=(XL*XN+XM*XO)/XQ
      A11I=(XM*XN-XL*XO)/XQ$ OA=A11R*XA$ OB=OA+1.$ OC=A11I*XA
      OD=OA-1.$ OE=OD*OD+OC*OC$ OF=(OB*OD+OC*OC)/OE
      OG=(OC*OD-OB*OC)/OE$ ATR=OG*OH$ ATI=-OF*OH
      AT=SQRTF(ATR*ATR+ATI*ATI)$ OI=ATI/ATR$ ANGLE=ATANF(OI)*OJ
      ZEQ=OK/AT$ ANG=(90.-ABSF(ANGLE))/OJ$ Y2=ZEQ*COSF(ANG)
      Z2=ZEQ*SINF(ANG)$ C1=V1/((Y1+Y2)*(Y1+Y2)+(Z1+Z2)*(Z1+Z2))
      C1R=C1*(Y1+Y2)$ C1I=-C1*(Z1+Z2)$ SDR=C1R*Y1-C1I*Z1
      SDI=C1I*Y1+C1R*Z1$ EOR=V1-SDR$ EOI=-SDI$ PIN=EOR*C1R+EOI*C1I
      TORQ=DDD*PIN
1     PUNCH 200,S,ANGLE,ZEQ,Y2,Z2,TORQ
200   FORMAT(F5.1,F11.2,4F14.3)
       STOP    $      END
```

SAMPLE DATA

```
2
1.        1.5        220.        1500.
 120 1 10.
314.   0.125664E-05 0.3580E-07.06985    .0693     .0662     .0659
162.      220.       .36
```

THE ABOVE PROGRAM IS ONLY FOR A MACHINE HAVING 4 POLES. FOR A MACHI HAVING A DIFFERENT NO. OF POLESA SIMILAR PROGRAM CAN BE WRITTEN USING THE PROPER ORDER BESSEL FUNCTIONS OR A GENERAL PROGRAM CAN BE WRITTEN ON THE LIN OF PROGRAM NO. 4

PROGRAM NO. 3

```
C   C   TORQUE SPEED CURVE  BY THREE DIMENSIONAL ANALYSIS IN RECT. COOR. Z
        DIMENSION G(31),GSQ(31),S(40),BC(20),BD(20),R2(31),X2(31) ,X(31)
        READ100,B,RL,P,TPH,WDF$ READ101, R,G1,D,G2
        READ102,I1,I2,I3,N1,N2,N3,N4,AII    $ READ103,R1,X1,SYN,V1
100     FORMAT(7F10.0)
101     FORMAT(F20.5,5F10.0)
102     FORMAT(7I2,5F10.0)
103     FORMAT(7F 10.0)
C       SENSE SWITCH 1 ON WILL PUNCH THE VALUES OF R2(N) AND X2(N) FOR VARIOUS
C       VALUES OF N
        W=314.
        U=.12566E-05
        PY=3.14159265$ B=B*.0254$ RL=RL*.0254$ DD=2.*7.04*V1*V1/SYN
        C=U*W/R$ B1BAR=TPH*WDF/B$ A=P/B$ ASQ=A*A$ E=PY*PY/(RL*RL)
        CC=6.*P*RL*W*U*B1BAR*B1BAR*P /(PY*ASQ*A)
500     DO1N=N1,N2,N3$ SQN=N*N$ GSQ(N)=ASQ+SQN*E$ G(N)=SQRTF(GSQ(N))
        GN=G(N)$ F=EXPF(GN*G2)$ H=1./F$ A1=(F+H)/(F-H)$ O=EXPF(GN*D)
        Q=1./O$ T=(O-Q)*.5$ V=(Q+O)*.5$ X(N)=GN*(-A1*T+V)/(A1*V-T)
501     BA=EXPF(GN*G1)$ BB=1./BA$ BC(N)=.5*(BA-BB)$ BD(N)=.5*(BA+BB)
1       CONTINUE
        DO2I=I1,I2,I3$ AI=I$ S(I)=AI/AII$ Y=C*S(I)
        DO3N=N1,N2,N3$ GSQN=GSQ(N)$ GN=G(N)$ SQN=N*N $ Z=X(N)
502     CALL SQRT(GSQN,Y,ZR,ZI)$ CALL DIV(Z,0.,ZR,ZI,A21,A22)
        CALL SICO(ZR*D,ZI*D,AA,AB,AC,AD)$ CALL MUL(A21,A22,AA,AB,AE,AF)
        CALL MUL(A21,A22,AC,AD,AG,AH)
        CALL DIV(AE+AC,AF+AD,AA+AG,AB+AH,A31,A32)
        CALL DIV(GN*A31,GN*A32,ZR,ZI,A41,A42)$ BCN=BC(N)$ BDN=BD(N)
503     CALL DIV(A41*BDN+BCN,A42*BDN,A41*BCN+BDN,A42*BCN,A51,A52)
        DA=CC*GN/SQN$ DB=DA*A51$ DC=DA*A52$ CALL MUL(0.,1.,DB,DC,DE,DF)
        R2(N)=DE$ X2(N)=DF
3       CONTINUE
        IF(SENSE SWITCH1)11,12
11      PUNCH200,S(I),(N,R2(N),X2(N),N=N1,N2,N4)
12      RR=0.$ XX=0.$ DO4N=N1,N2,N3$ RR=RR+R2(N)
4       XX=XX+X2(N)$ TORQ=DD*RR/((R1+RR)**2+(X1+XX)**2)
2       PUNCH201,S(I),TORQ,RR,XX
200     FORMAT(2HS=F4.1,2X,(2HN=I2,2F7.1,2HN=I2,2F7.1,2HN=I2,2F7.1,2HN=I2,
       12F7.1))
201     FORMAT(2HS=F4.1,5X7HTORQUE=F7.3,5X3HR2=F7.2,5X3HX2=F7.2)$ STOP
        END

        SUBROUTINE SQRT(A,B,C,D)$   E=(A*A+B*B)**.25$ TH=ATANF(B/A)*.5
        C=E*COSF(TH)$ D=E*SINF(TH)$  RETURN $  END

        SUBROUTINE DIV(A,B,C,D,E,F)$ H=C*C+D*D$ E=(A*C+B*D)/H
        F=(B*C-A*D)/H$    RETURN $ END

        SUBROUTINE MUL(A,B,C,D,E,F)$ E=A*C-B*D$ F=A*D+B*C$  RETURN $ END

        SUBROUTINE SICO(A,B,C,D,CC,DD)$  E=EXPF(A)$ F=1./E
        G=(E+F)*.5$  H=(E-F)*.5$ P=SINF(B)$ Q=COSF(B)
        C=Q*H$ D=P*G$ CC=Q*G$ DD=P*H$    RETURN$   END
```

SAMPLE DATA

```
3.7          3.5         4.          552.        .9
        4.50000E-08.00027            .00175      .0019
 120 1 1 7 2 2 10.            4.
10.3          7.5        1500.        110.
```

PROGRAM NO.4

LIST OF NOTATIONS

QUANTITIES IN INPUT LIST
RHO = RESISTIVITY OF ROTOR MATERIAL IN OHM METERS
RL = MACHINE LENGTH IN METERS
R1, R2, R3, R4 ARE RADII AT STATOR BORE, ROTOR OUTER, ROTOR INNER AND ROTOR CORE RESPECTIVELY.
RS = STATOR RESISTANCE
XS = STATOR LEAKAGE REACTANCE
TURNS = NO. OF TURNS IN SERIES PER PHASE
WK = WINDING DISTRIBUTION FACTOR
NP = NO. OF POLE PAIRS
I1, I2, I3, AII ARE THE FACTORS TO CONTROL THE VARIATION OF SLIP
N1, N2, N3, N4 ARE THE FACTORS TO CONTROL VARIATION OF NO. OF HARMONICS UPTO WHICH CALCULATION IS DESIRED.
DIA = AVERAGE ROTOR DIAMETER AND METERS.

QUANTITIES IN OUTPUT LIST
XA = ROTOR RESISTANCE (AN HARMONIC CONSIDERED)
XB = ROTOR REACTANCE (N HARMONIC CONSIDERED)
TOR(N) = TORQUE IN LB.FT. WITH AN HARMONIC CONSIDERED.

PROGRAM NO. 4

```
C   C   TORQUE SPEED CURVE  BY THREE DIMENSIONAL IN CYLIN. CO-OR.  Z
        COMMON PY,ACC
        DIMENSION B(31),C(31),D(31),E(31),F(31),G(31),H(31),O(31),P(31)
        DIMENSION Q(31),RR(31),XR(31),PD(31),XA(31),XB(31),RC(31),RD(31),T
       1OR(31)
        READ100,RHO,RL,R1,R2,R3,R4
 100    FORMAT(E10.2,6F10.0)
        READ101,V1,RS,XS,TURNS,WK,NP
 101    FORMAT(5F10.0,I2)
        READ102,I1,I2,I3,N1,N2,N3,N4,AII,DIA,SYN ,ACC
 102    FORMAT(7I2,5F10.0)
        NA=NP+1
        NB=NP-1
        PY=3.14159265
        W=314.
        R1SQ=R1*R1
        PYSQ=PY*PY
        PN=NP
        U=.12566E-05
        A=W*U/RHO
        B1BAR=TURNS*WK/DIA
        B1SQ=B1BAR*B1BAR
        PA=12.*B1SQ*U*W/PY*2.*P
        PB=PYSQ*R1SQ/(PN*RL)
        PC=PN*RL
        DDD=2.*7.04*V1*V1/SYN
 501    DO1N=N1,N2,N3
        AN=N
        B(N)=AN*PY/RL
        BN=B(N)
        BR3=BN*R3
        BR4=BN*R4
        CALL KNX(NA,BR4,AA)
        CALL KNX(NB,BR4,AB)
        CALL INX(NA,BR4,AC)
        CALL INX(NB,BR4,AD)
 502    A1=(AA+AB)/(AC+AD)
        CALL INX(NA,BR3,AE)
        CALL INX(NB,BR3,AF)
        CALL KNX(NA,BR3,AG)
        CALL KNX(NB,BR3,AH)
        CALL INX(NP,BR3,AP)
        CALL KNX(NP,BR3,AQ)
        C(N)=B(N)*(A1*(AE+AF)-AG-AH)/(2.*(A1*AP+AQ))
 503    BR2=BN*R2
        BR1=BN*R1
        CALL KNX(NP,BR2,DA)
        CALL KNX(NA,BR2,DB)
        CALL KNX(NB,BR2,DC)
        CALL INX(NA,BR2,DE)
        CALL INX(NB,BR2,DF)
        CALL INX(NP,BR2,DG)
        D(N)=DA
        E(N)=-(DB+DC)*.5
        F(N)=(DE+DF)*.5
```

```
504   G(N)=DG
      CALL INX(NP,BR1,EA)
      CALL KNX(NP,BR1,EB)
      CALL INX(NA,BR1,EC)
      CALL INX(NB,BR1,ED)
      CALL KNX(NA,BR1,EE)
      CALL KNX(NB,BR1,EF)
505   H(N)=EA
      O(N)=EB
      P(N)=.5*(EC+ED)
      Q(N)=-.5*(EE+EF)
      PD(N)=PA*(PC/AN+PB*AN)/(BN*AN)
1     CONTINUE
      IF(SENSE SWITCH 1)10,11
10    PUNCH201,(PD(N),N=N1,N2,N3)
201   FORMAT(5E16.8)
11    DO2I=I1,I2,I3
      AI=I
      S=AI/AII
      CX=A*S
      RA=0.
      RB=0.
506    DO3N=N1,N2,N3
      BN=B(N)
      AN=N
      BNSQ=BN*BN
      CALL SQRT(-BNSQ,-CX,G1,G2)
      B1=R3*G1
      B2=R3*G2
      CALL JNZ(NP,B1,B2,BA,BB)
      CALL YNZ(NA,B1,B2,BC,BD)
      CALL YNZ(NB,B1,B2,BE,BF)
      CALL JNZ(NA,B1,B2,BG,BH)
      CALL JNZ(NB,B1,B2,BI,BJ)
507   CALL YNZ(NP,B1,B2,BK,BL)
      CN=C(N)
      CALL DIV(CN,0.,G1,G2,A21,A22)
      CALL MUL(A21,A22,BA,BB,BM,BQ)
      CALL MUL(A21,A22,BK,BL,BO,BP)
      CALL DIV(BO-.5*(BE-BC),BP-.5*(BF-BD),.5*(BI-BG)-BM,.5*(BJ-BH)-BQ,A
     131,A32)
      C1=R2*G1
      C2=R2*G2
      CALL JNZ(NA,C1,C2,CA,CB)
      CALL JNZ(NB,C1,C2,CC,CD)
      CALL YNZ(NA,C1,C2,CE,CF)
508   CALL YNZ(NB,C1,C2,CG,CH)
      CALL JNZ(NP,C1,C2,CI,CJ)
      CALL YNZ(NP,C1,C2,CK,CL)
      CALL MUL(A31,A32,.5*(CC-CA),.5*(CD-CB),CO,CP)
      CALL MUL(A31,A32,CI,CJ,CQ,CR)
      CALL MUL(G1/BN,G2/BN,CO+.5*(CG-CE),CP+.5*(CH-CF),CS,CT)
      CALL DIV(CS,CT,CQ+CK,CR+CL,A41,A42)
      FA=A41*D(N)-E(N)
      FB=A42*D(N)
      FC=F(N)-A41*G(N)
      FD=-A42*G(N)
```

```
509    CALL DIV(FA,FB,FC,FD,A51,A52)
       A61=A51*H(N)+O(N)
       A62=A52*H(N)
       A71=A51*P(N)+Q(N)
       A72=A52*P(N)
       CALL DIV(A61,A71,A72,QA,QB)
       CALL MUL(0.,1.,QA,QB,QC,QD)
       RR(N)=PD(N)*QC
       XR(N)=PD(N)*QD
       XA(N)=QC
       XB(N)=QD
       RA=RA+RR(N)
       RB=RB+RR(N)
       RC(N)=RA
       RD(N)=RB
       TOR(N)=DDD*RA/((RS+RA)**2+(XS+RB)**2)
3      CONTINUE
       IF(SENSE SWITCH 2)12,20
12     PUNCH202,(XA(N),XB(N),N=N1,N2,N4)
202    FORMAT(2X,(6E13.6))
20     PUNCH200,(S,RR(N),XR(N),N=N1,N2,N4)
200    FORMAT(F4.1,6X,(10F7.3))
2      PUNCH203,(N,TOR(N),N=N1,N2,N4)
203    FORMAT(2X,(I5,F8.3,I5,F8.3,I5,F8.3,I5,F8.3,I5,F8.3,I5,F8.3))
       STOP
       END

       SUBROUTINE INX(N,X,R)
       COMMON PY,ACC
       AN=N
       A=.5*A
       ASQ=A*A
       FACT=1
       DO1I=1,N
       A1=1
1      FACT=FACT*AI
       R=B/FACT
       M=0
11     M=M+1
       AM=M
       B=B*ASQ
       FACT=FACT*AM*(AN+AM)
       TERM=B/FACT
       R=R+TERM
       IF(TERM-ACC)10,10,11
10     RETURN
       END

       SUBROUTINE KNX(N,X,R)
       COMMON PY,ACC
       N1=N-1
       IF(N1)31,32,31
```

```
32      IF(X-7.)33,33,34
31      IF(X-10.)33,33,34
34      FNSQ=N*N*4
        W=FNSQ-1.
        U=W*(FNSQ-9.)
        V=U*(FNSQ-25.)
        EX=8.*X
        EXS=EX*EX
        EXC=EXS*EX
        R=SQRTF(PY/(X+X))*EXPF(-X)*(1.+W/EX+U/(EXS+EXS)+V/(6.*EXC))
        GOTO23
33      GAMA=.5772157
        AN=N
        N2=N+1
        AN1=N1
        A=.5*X
        ASQ=A*A
        B1=A**N
        B2=1./B1
        C=LOGF(A)+GAMA
        R=0.
        M=0
        SIGN1=1.
        SIGN2=(-1.)**N2
        FACTN=1.
        FACT1=1.
        FACN1=1.
        DO1I=1,N
        AI=I
1       FACTN=FACTN*AI
        FACT3=FACTN
        IF(N1)21,22,21
21      DO2I=1,N1
        AI=I
2       FACN1=FACN1*AI
22      FACT2=FACN1
        F=0.
        DO3I=1,N
        AI=I
3       F=F+1./AI
        G=0.
24      AM=M
        ANM=N+M
        IF(M)17,18,17
17      G=G+1./AM
        F=F+1./ANM
        B1=B1*ASQ
        FACT1=FACT1*AM
        FACT3=FACT3*(AN+AM)
        FACT2=1.
        NM1=N1-M
        IF(NM1)9,10,7
7       DO4I=1,NM1
        AI=I
4       FACT2=FACT2*AI
10      B2=B2*ASQ
18      P=SIGN1*B2*FACT2/FACT1
```

```
      GOTO19
9     P=0.
19    D=C-.5*(G+F)
190   Q=B1*D/(FACT1*FACT3)
      M=M+1
      SIGN1=-SIGN1
      R=R+.5*P+SIGN2*Q
      IF(ABSF(Q)-ACC)23,23,24
23    RETURN
      END

      SUBROUTINE JNZ(N,X,Y,S1,S2)
      COMMON PY,ACC
      AN=N
      D=X*X+Y*Y
      Z=SQRTF(D)
      ASQ=D*.25
      A=.5*Z
      TH=ATANF(Y/X)
      IF(N-2)21,21,22
21    IF(Z-10.)23,24,24
22    IF(Z-15.)23,24,24
24    FNSQ=4*N*N
      DA=FNSQ-1.
      DB=DA*(FNSQ-9.)
      S=DB*(FNSQ-25.)
      T=S*(FNSQ-49.)
      EZ=8.*Z
      EZS=EZ*EZ
      EZC=EZS*EZ
      TTH=TH+TH
      THTH=TTH+TH
      FTH=THTH+TH
      SIR=X-(AN+.5)*PY*.5
      SII=Y
      CA=EXPF(SII)
      CB=1./CA
      COSH=(CA+CB)*.5
      SINH=(CA-CB)*.5
      COS=COSF(SIR)
      SIN=SINF(SIR)
      COSR=COS*COSH
      COSI=-SIN*SINH
      SINR=SIN*COSH
      SINI=COS*SINH
      W=SQRTF(2./(PY*Z))
      AC=.5*TH
      CR=W*COSF(AC)
      CI=-W*SINF(AC)
      U=DB/(2.*EZS)
      V=T/(24.*EZC*EZ)
      AA=DA/EZ
      AB=S/(6.*EZC)
      PR=1.-U*COSF(TTH)+V*COSF(FTH)
      PI=U*SINF(TTH)-V*SINF(FTH)
      QR=AA*COSF(TH)-AB*COSF(THTH)
      QI=-AA*SINF(TH)+AB*SINF(THTH)
      BR=PR*COSR-PI*COSI-QR*SINR+QI*SINI
```

```
      BI=PI*COSR+PR*COSI-QR*SINI-QI*SINR
      S1=CR*BR-CI*BI
      S2=CI*BR+CR*BI
      GOTO12
 23   B=A**N
      PHI=TH*AN
      TTH=TH+TH
      T1=0.
      T2=0.
      S1=0.
      S2=0.
      SIGN=1.
      M=0
      FACT1=1.
      FACT2=1.
      DO1I=1,N
      AI=I
 1    FACT2=FACT2*AI
 11   C=SIGN*B/(FACT1*FACT2)
      T1=C*COSF(PHI)
      T2=C*SINF(PHI)
      M=M+1
      AM=M
      SIGN=-SIGN
      B=B*ASQ
      PHI=PHI+TTH
      FACT1=FACT1*AM
      FACT2=FACT2*(AN+AM)
      S1=S1+T1
      S2=S2+T2
      IF(ABSF(T2)-ACC)10,10,11
 10   IF(ABSF(T1)-ACC)12,12,11
 12   RETURN
      END

      SUBROUTINE YNZ(N,X,Y,S1,S2)
      COMMON PY,ACC
      AN=N
      N1=N-1
      GAMA=.5772157
      ASQ=.25*(X*X+Y*Y)
      A=SQRTF(ASQ)
      Z=A+A
      TH=ATANF(Y/X)
       IF(N-1)51,51,52
 51    IF(Z-9.)23,24,24
 52    IF(Z-12.)23,24,24
 24   FNSQ=4*N*N
      DA=FNSQ-1.
      DB=DA*(FNSQ-9.)
      S=DB*(FNSQ-25.)
      T=S*(FNSQ-49.)
      EZ=8.*Z
      EZS=EZ*EZ
      EZC=EZS*EZ
      TTH=TH+TH
 501  THTH=TTH+TH
      FTH=THTH+TH
```

```
      SIR=X-(AN+.5)*PY*.5
      SII=Y
      CA=EXPF(SII)
      CB=1./CA
      COSH=(CA+CB)*.5
      SINH=(CA-CB)*.5
      COS=COSF(SIR)
      SIN=SINF(SIR)
502   COSR=COS*COSH
      COSI=-SIN*SINH
      SINR=SIN*COSH
      SINI=COS*SINH
      W=SQRTF(2./(PY*Z))
      AC=.5*TH
      CR=W*COSF(AC)
      CI=-W*SINF(AC)
      U=DB/(2.*EZS)
      V=T/(24.*EZC*EZ)
503   AA=DA/EZ
      AB=S/(6.*EZC)
      PR=1.-U*COSF(TTH)+V*COSF(FTH)
      PI=U*SINF(TTH)-V*SINF(FTH)
      QR=AA*COSF(TH)-AB*COSF(THTH)
      QI=-AA*SINF(TH)+AB*SINF(THTH)
      BR=PR*SINR-PI*SINI+QR*COSR-QI*COSI
      BI=PI*SINR+PR*SINI+QR*COSI+QI*COSR
      S1=CR*BR-CI*BI
      S2=CI*BR+CR*BI
      GOTO120
23    B=A**N
      R=1./B
      PHI=TH*AN
      ALPHA=-PHI
      TTH=TH+TH
      F1=0.
      F2=0.
      SE1=0.
      SE2=0.
      T1=0.
      T2=0.
      S1=0.
      S2=0.
      SIGN=1.
      M=0
      FACT1=1.
      FACN1=1.
      FACTN=1.
      DO1I=1,N
      AI=I
1     FACTN=FACTN*AI
      FACT2=FACTN
      C=.5*LOGF(ASQ)+GAMA
      Q1=C+C
      Q2=TTH
```

```
      F=0.
      G=0.
      DO2I=1,N
      AI=I
2     F=F+1./AI
      IF(N1)21,22,21
21    DO3I=1,N1
      AI=I
3     FACN1=FACN1*AI
22    FACT3=FACN1
15    COS=COSF(PHI)
      SIN=SINF(PHI)
      E=SIGN*B/(FACT1*FACT2)
      FT1=E*COS
      FT2=E*SIN
      P=E*(F+G)
      ST1=P*COS
      ST2=P*SIN
      IF(N1-M)43,42,42
42    Q=R*FACT3/FACT1
      TT1=Q*COSF(ALPHA)
      TT2=Q*SINF(ALPHA)
      GOTO44
43    TT1=0.
      TT2=0.
44    F1=F1+FT1
      F2=F2+FT2
      SE1=SE1+ST1
      SF2=SE2+ST2
      T1=T1+TT1
      T2=T2+TT2
      IF(ABSF(FT1)-ACC)10,10,11
10    IF(ABSF(ST1)-ACC)13,13,11
13    IF(ABSF(FT2)-ACC)14,14,11
14    IF(ABSF(ST2)-ACC)16,16,11
16    IF(ABSF(TT1)-ACC)17,17,11
17    IF(ABSF(TT2)-ACC)12,12,11
11    M=M+1
      AM=M
      NM1=N-M-1
      SIGN=-SIGN
      B=B*ASQ
      R=R*ASQ
      FACT1=FACT1*AM
      FACT2=FACT2*(AN+AM)
      FACT3=1.
      IF(N1-M)34,34,33
33    DO4I=1,NM1
      AI=I
4     FACT3=FACT3*AI
34    PHI=PHI+TTH
      ALPHA=ALPHA+TTH
      G=G+1./AM
      ANM=N+M
      F=F+1./ANM
      GOTO15
```

```
12     S1=(F1*Q1-F2*Q2-SE1-T1)/PY
       S2=(F1*Q2+F2*Q1-SE1-T2)/PY
120    RETURN
          END

       SUBROUTINE SQRT(A,B,C,D)$    E=(A*A+B*B)**.25$ TH=ATANF(B/A)*.5
       C=E*COSF(TH)$ D=E*SINF(TH)$  RETURN $  END

       SUBROUTINE DIV(A,B,C,D,E,F)$ H=C*C+D*D$ E=(A*C+B*D)/H
       F=(B*C-A*D)/H$   RETURN $ END

       SUBROUTINE MUL(A,B,C,D,E,F)$ E=A*C-B*D$ F=A*D+B*C$  RETURN $ END
```

SAMPLE DATA

```
 3.58E-08.115       .0476      .04735     .0445      .04435
110.       10.3       7.5        .36        552.       220.       .9         2
 12010 1 3 2 210.         .0952       1500.     .00001
```

PROGRAM NO. 5

LIST OF NOTATIONS

QUANTITIES IN INPUT LIST
N=NO. OF T,S
T(I)=VARIOUS VALUES OF TIME IN SECONDS
C1=K1,C2=K2,C3=K3,C4=K4
CJ=MOMENT OF INERTIA
QUANTITIES IN OUTPUT LIST
T(J)=TIME IN SECONDS
OMEGA(J)=SPEED

```
C   C   SPEED TRANSIENT   Z
        DIMENSION T(50),OMEGA(50)
        READ100,N
100     FORMAT(I2)
        READ103,(T(I),I=1,N)
103     FORMAT(7F10.0)
104     READ101,C1,C2,C3,C4,CJ
101     FORMAT(F10.0,4E10.4)
102      FORMAT(E20.5)
        PUNCH300,C1,C2,C3,C4,CJ
300     FORMAT(3HK1=F6.3,4H K2=E11.4,4H K3=E11.4,4H K4=E11.4,3H J=E11.4)
11      C5=C2-    C4$ Q=4.*C1*C3-C5*C5
        O=SQRTF(-Q)$ P=O/(2.*CJ)$ S=2.*C3$ R=C5/S$ U=O/S
        V=LOGF((O+C5)/(O-C5))/2.$ DO1NN=1,N$ TH=-P*T(NN)+V
        IF(TH)12,13,12
13      X=1.$ GOTO14
12      X=EXPF(TH)
14      Y=1./X$ TANH=(X-Y)/(X+Y)$ OMEGA(NN)=U*TANH-R $ L=L+1
1       CONTINUE
        PUNCH2000,(T(J),OMEGA(J),J=1,L)
2000    FORMAT(F6.3,F8.0,F7.3,F8.0,F6.3,F8.0,F7.3,F8.0,F6.3,F8.0)
   16   GO TO 104
         END
```

SAMPLE DATA

```
19
.01      .02        .03      .04     .05     .06     .07
.08      .09        .1       .12     .15     .17     .2
.3       .5         .7       .8      1.
1.48     -.146E-3   -.1E-8   .6E-4   .4E-5
```

R_E_F_E_R_E_N_C_E_S.

1. Fillmore, R.L.- "Calculation of Eddy Current Paths in Drag-cup Induction Motor Rotors". Trans. A.I.E.E. Pt. III, Vol.75, pp.922-926, Oct. 1956.

2. Guilford, E.C.- "Theory of Induction Machine with Conducting Sleeve Rotor" Trans. A.I.E.E. Pt.III, Vol.80, pp.1129-1136, Feb. 1962.

3. Wood, A.J. & Concordia, C.- "Analysis of Solid Rotor Machines" Pt.II- The Effect of Curvature" Trans. A.I.E.E. Pt. III, 1960, pp.1666-1673.

4. Wood, A.J. & Concordia, C.- "Analysis of Solid Rotor Machines Pt.III- Finite Length Effect" Trans. A.I.E.E. Pt. III, 1960, pp.21-26.

5. Angst, G.- "Polyphase Induction Motor with Solid Rotor, Effects of Saturation and Finite Length" Trans. A.I.E.E. Pt.III, Feb.1962, pp.902-10.

6. Koch, W.H.- "Equivalent Circuits with Transformer Elements for Eddy Current Rotor Induction Motors derived from the Field Equations, II- Drag-cup Motors" I.E.E. E.Trans. on Power Apparatus and Systems, Vol.83, pp.576-583, June 1964.

7. Fuller, E.L., and Trickey- "Equivalent Drag-cup Resistance" Trans. A.I.E.E.Vol.81, Pt.III, pp.285-291, August 1962.

8. Blachford, C.E.- "Induction Cup Parameters from Electromagnetic Field Theory and Experimental Analysis" I.E.E.E. Trans. on Power Apparatus and Systems, Vol.84, Nov.1965.

9. Mukhopadhyay, P.- "On Drag Cup Rotor Machine" Research Journal, University of Roorkee, Roorkee, India, Vol.VIII, 1965.

10. Saunders, R.M.- "Electromechanical Energy Conversion in Double Cylindrical Structures" I.E.E.E. Trans. on Power Apparatus and Systems, Vol.82, pp.631-638, Oct. 1963.

11. Russell, R.L.and Norsworthy, K.H.- "Eddy Currents and Wall Losses in Screened Rotor Induction Motors" Proc. I.E.E. 1958, Pt.C, pp.163-175.

12. Robinson, R.C.- "Calculation of Can Losses in Canned Motors" Trans. A.I.E.E. Pt.III, January-1957.

13. West, J.C. and Hoomondhalgh, D. .- "Analysis of Thick Wall Cylinder Induction Motor" Proc. I.E.E. Pt.C, Nov. 1961, pp.173-181.

14. Tudbury- "Electric Conditions inside an Induction Heated Workpiece" Trans. A.I.E.E. Pt.II , pp.79, May- 1959.

15. Frazier- "Analysis of a Drag-cup A.C. Tachometer" Trans. A.I.E.E. Vol. 70, pp.1894, 1951.

16. Koopman- "Operating Characteristics of Two-phase Servo Motors" Trans. A.I.E.E. Pt. I, 1949, pp.319-28. vol 68.

17. Hopkin- "Transient Response of Small Two Phase Servomotors" Trans. A.I.E.E. Vol. 70, pp.881-886, 1951.

18. Mukhopadhyay, P.- "Airgap Permeance of a Rotating Machine" Journal of Technology, India, Vol.VIII, No.1, 1963.

19. Watson- "A Treatise on Bessel Functions"(book)

20. "Induction torque and servomotor design" Trans A.I.E.E. pt 3 oct 55 pp 809-815

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