APPLICATION OF DYNAMIC PROGRAMMING TO ANTENNA ARRAY SYNTHESIS

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By
MADHAVENDRA NATH

DEPTT. OF ELECTRONICS & COMMUNICATION ENGINEERING
UNIVERSITY OF ROORKEE
ROORKEE
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CERTIFICATE

CERTIFIED that the dissertation entitled "A[illegible] OF [illegible] TO ANTENNA ARRAY [illegible]" which is being submitted by Shri RAGHAVENDRA [illegible] in partial fulfilment for the award of the Degree of MASTER OF ENGINEERING in ADVANCED ELECTRONICS, Department of Electronics and Communication Engineering of the University of Roorkee, Roorkee is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or Diploma.

This is further to certify that he has worked for about 8 months from January to August 1966 in preparing this thesis for Master of Engineering at the University.

R K Arora

(R. K. ARORA)
Reader in
Electronics and Communication
Engineering Department
University of Roorkee,
Roorkee U.P.

Roorkee
August 24, 196[illegible].

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ABSTRACT

A significant amount of work has been done since 1940 on equally spaced linear antenna arrays. Note-worthy contributions are by S. A. Schelkunoff and C.L. Dolph. From 1960 work started on unequally spaced linear antenna arrays. Many optimizing techniques were developed. The growth of the subject is reviewed in Part I.

Dynamic programming is studied in Part II as an optimizing technique in synthesizing unequally spaced symmetrical linear antenna arrays. The criterion of optimization is to find an element combination which has the highest side lobe level over a specified angular interval, less than the highest side lobe level of any other combination.

A 9 elements array is synthesized with aperture length 19 λ, and spacing quantization $\lambda/2$. Results obtained are quite encouraging. Two more approaches are studied in order to support the results obtained by dynamic programming technique. The effect of varying region of optimization on the side lobe level is also studied.

The calculations were performed on an IBM 1620 Computer.

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CHAPTER I

INTRODUCTION

Antenna arrays are used when greater directivity is required than can be obtained from a single antenna. An antenna array is constituted of similar antennas, similarly oriented. The main interest in designing an antenna array is to reduce the number of elements or to broaden the band width and the scanning range of the array.

When the element excitations are subjected to optimization, the problem can be solved analytically while if the element position is subjected to optimization, the problem is not susceptible to a simple analytical treatment.

1.1. FUNDAMENTALS OF ANTENNA ARRAY THEORY

In our study of antenna arrays, we will be dealing with linear arrays only.

LINEAR ARRAYS

An array is linear when the elements of the array are spaced equally along a straight line.

Linear arrays are further classified as :

1. Uniform Linear Arrays: A linear array is said to be uniform when the elements are fed with currents of equal magnitude and having a uniform progressive phase shift along the line.

ii. Non Uniform Linear Arrays : Non uniform arrays are those linear arrays in which the currents and phase are governed by some distribution functions. These distribution function may be (a) Binomial (b) Triangular (c) Tschebyscheff.

Both uniform as well as non uniform linear arrays can be arranged in a manner, so as to get either

i. Broad side pattern.

or ii. End fire pattern.

A. Broad Side Arrays: Basically a broad side array consists of a number of radiators equally spaced along a line, and carrying currents of same phase in all radiators. This array has the property of concentrating radiation, in a plane at right angle to the line of the array. The radiation in the other directions is relatively small. The width of the major lobe of the broad side array is less the longer the array. For array lengths exceeding two wave lengths, the width of the major lobe is almost exactly inversely proportional to the array length. The directive gain under these conditions will be proportional to the array length. The spacing between consecutive antenna elements in broad side array has a secondary effect upon the directional characteristics of the array provided the spacing is not too large, when individual antennas are isotropic radiators, varying the spacing from very low values upto $3\lambda/4$

effects only the minor lobe structure, when the spacing is greater than $3\lambda/4$ large secondary lobes appear when the individual antenna is a directive antenna the radiators can be spaced a full wave length apart without large parasitic side lobes being developed.

The side lobes in a broad side array will be eliminated entirely provided,

a. The spacing between adjacent antennas does not exceed $\lambda/2$, and

b. The currents of the various antenna elements starting from the outer edge and going towards the centre are proportional to the coefficients of the successive terms in the binomial series.

$$(a+b)^{n-1} = a^{n-1} + (n-1)a^{n-2}\, b + \frac{(n-1)(n-2)}{2!}\, a^{n-3}\, b^{2} + \ldots$$

where n = number of radiators.

This arrangement eliminates the side lobes, but it widens the width of the main lobe. Therefore to get the main lobe of specified width, and pattern free of side lobes the array length would be larger than if the side lobes are tolerated.

D. End Fire Array: These types of array consist of a number of identical equally spaced antennas arranged along a line, and carrying currents of equal amplitude, but the elements are so excited that there is a progressive phase difference between the adjacent antennas equal in cycles to the spacing between the antennas expressed in wave lengths.

In end fire arrays the radiation pattern is unidirectional and points along the array axis in the direction in which antenna currents become more lagging. The directional characteristic of the end fire array is due to the fact that although a distant point in the direction in which radiation concentrate is at different distances from the individual antennas, the phases of the more distant antennas are just enough leading to compensate for the greater distance. In the directions that are appreciably different, including the opposite direction, the radiation from different elements will not be in phase at a distant point, and as a consequence will more or less cancel each other.

The width of the major lobe decreases as the array length increases just as in broad side array, the only difference being that here the lobe width is inversely proportional to the square root of the array length.

The directional pattern of the end fire array like that of broad side array is substantially independent of the spacing of antenna radiators, provided this spacing of the antenna does not exceed a critical value, which is $3\lambda/8$ for isotropic radiations. However, if individual antennas possess directivity that aids the concentration of the radiation along the axis of the array, then greater spacing is permissible.

The directive gain of an end fire array that has appreciable length is almost exactly twice the gain of the broad side array of the same length when both are composed of isotropic radiators, and the directive gain of the end fire array is proportional to the array length.

The minor lobes in the end fire pattern may be reduced in amplitude or may be eliminated entirely by appropriate tapering of the current distribution along the array, and also in some cases by making the phase difference in cycles between adjacent antennas different from the spacing in wave lengths. Side lobes can also be reduced or eliminated in an end fire array by controlling both the current distribution and phase difference at the same time between adjacent antennas. This will also reduce the width of the main lobe.

(c) Array of Arrays: A desired radiation pattern which is directional in character may often be obtained by assembling a number of individual antennas into an array and then combining two or more such arrays to give the desired over all results.

The various radiation patterns obtained are as follows :

(a) Concentration of the radiation in azimuth can be obtained by arranging radiators (or array) to form a broad side array in which the line of array is horizontal. The direction of concentration will then be at right angles to the line of the array.

(b) Concentration of radiation in vertical plane can be obtained by arranging radiators (or array) to form a broad side array in which the line of array is vertical.

(c) A uni-directional characteristics can be obtained by arranging two radiators (or two arrays) so that they are an odd number of quarter wave lengths apart and are excited an odd number of quarter cycles out of phase.

(d) In systems radiating horizontally polarised wave, the angle of elevation above the ground of the major lobe is determined in large measure by the height above ground of the center of the antenna system.

(e) The directional characteristics of the individual antenna may have a significant effect upon the overall directional pattern.

1.2. RADIATION PATTERN OF ANTENNA ARRAYS

The pattern of antenna arrays discussed above can be obtained by adding vectorially the field intensities due to each of the elements. For a uniform array of non-directional elements the field intensity would be :

$$E = E_0 \left| 1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \ldots\ldots + e^{j(n-1)\psi} \right| \quad (1)$$

where $\psi = \beta d \cos \phi + \alpha$

β = propagation constant

α = progressive phase shift between elements.

d = distance between successive elements.

ϕ = Angle measured from the line of the array.

Maximum value of this expression occurs when $\psi = 0$. This is the principal maximum of the array and occurs at $\cos \phi = -\alpha / \beta d$. If the spacing is equal to or greater than λ, then there may be more than one principal maxima.

For broad side array

$\phi = 90^\circ$ $\therefore$ $\alpha = 0^\circ$

For end fire array, $\phi = 0$, $\alpha = -\beta d$.

Equation (1) can be written as

$$E/E_0 = \left| \frac{\sin n\psi/2}{\sin \psi/2} \right| \qquad .....(2)$$

Nulls of the pattern occurs at

$n \psi/2 = K\pi$ where $K = 1,2,3,......$

Therefore the secondary maxima occurs at

$n \psi/2 = \pm (2m+1) \pi/2$, $m = 1,2,3$ .....

Angle between the principal maximum and the first null is given by

$n \psi_1/2 = \pi$ or $\psi_1 = 2\pi/n$

Radiation pattern of an antenna array can be displayed for quantitative work on C.R.O. An appropriate way of displaying is discussed by R. King[1].

1.3. POWER GAIN OF ARRAYS

Power gain of certain antenna arrays can be calculated from radiation resistance. A method of calculating the power gain of antenna arrays is devised by P. Hammond[2] The general indication of the power gain which is to be expected can be obtained from diffraction pattern of the antenna array. Hammond's method is a direct approach.

1.4. FACTORS AFFECTING THE PERFORMANCE OF LINEAR ARRAYS

(a) The performance of linear arrays which has been designed for low side lobe levels is markedly affected by small variations of element excitation, due to the inaccuracies inherent in the manufacture of the array. However, these variations can be avoided either by

i. Small tolerance in manufacture.

or by ii. Designing the array to compensate for the perturbations introduced by the manufacturing processes.

Figures written in parenthesis denote the serial number of References given at the end of Part I, and II.

The first method results in expensive manufacturing procedures, while the second causes reduced aperture efficiency. E. Dolin and N.J. [illegible][5] discussed these factors in details.

(b) A periodic variation in the field intensity across the radiating aperture may alter the shape of the radiation pattern and reduces the power gain of the aperture.

The usual formula for the power radiated by an aperture is only valid in the absence of these variations of field intensities. Such variations must be considered separately in the amount of power radiated. All these aspects are discussed in detail by J. Brown[4].

(c) The mutual coupling effect in large antenna arrays causes a variation in the input impedance. These variation in the input impedance. These variations can be reduced by making use of compensating structure. This modifies the environmental configuration. These effects were considered by S. Edelberg and A.A. Oliner [9], but they could not determine the optimum environmental configuration.

CHAPTER II

REVIEW OF THE CONTRIBUTIONS TO THE ANTENNA ARRAY THEORY

Major developments in the field of antenna arrays have been witnessed since 1940. The broadside arrays which we have discussed is one of the examples of a large class of linear arrays. First in the field was S.A. Schelkunoff[6] who showed that linear arrays can be represented as polynomials. This concept was utilized in the analysis and synthesis of antenna arrays. Several articles were published on equally spaced arrays. Since 1950[24] work has been done on unequally spaced arrays with space tapering. These arrays were proved to be more economical than the equally spaced arrays.

A broad classification of the arrays can be :

i. Equally Spaced Arrays.

ii. Unequally Spaced Arrays.

2.1. EQUALLY SPACED ARRAYS

S.A. Schelkunoff developed mathematical theory of linear arrays, according to which the relative amplitude of the radiated field intensity of a general linear array of equally spaced elements can be written as :

$$E = \left| A_0 + A_1 z + A_2 z^2 + \ldots\ldots + A_{n-2} z^{n-2} + z^{n-1} \right|$$

where $z = e^{j\psi}$ and $A_n = a_n e^{j\alpha_n}$

$$\psi = \beta d \cos\phi + \alpha, \qquad \beta = 2\pi/\lambda$$

d = Spacing between elements.

α = Progressive phase shift (lead) from left to right.

$\alpha_1, \alpha_2, \ldots\ldots \alpha_n$ = respective deviations from the progressive phase shift.

If any of the coefficients is zero, the corresponding element of the array will be missing and thus the actual separation between adjacent elements can be greater than the apparent separation 'd'.

The fundamental theorems formulated by Schelkunoff are as follows :

(a) Every linear array with commensurable separation between elements can be represented by a polynomial and vice-versa.

(b) There exists a linear array with space factor equal to the product of the space factors of two linear arrays.

(c) The space factor of a linear array of n apparent elements is the product of (n-1) virtual couplets with their null points at the zeros of E.

The coefficients of the corresponding elements are complex conjugate given by :

$$A_n = a_0 \quad A_{-n} = a_n - jb_n \quad A_{+n} = a_n + jb_n$$

$$A_{-n} e^{-n} + A_{+n} e^{n} = a_n(e^{n} + e^{-n}) + jb_n(e^{n} - e^{-n})$$

$$= 2 a_n \cos n\psi - 2b_n \sin n\psi$$

since $e^{n} = e^{jn\psi}$

The expression for $|E|$ can be written as

$$|E| = 2\left[\tfrac{1}{2} a_0 + (a_1\cos\psi + \dots a_n\cos n\psi) - (b_1\sin\psi + \dots b_n\sin n\psi)\right]$$

$$= 2\left[\frac{a_0}{2} + \sum_{n=1}^{m} (a_n \cos n\psi + (-b_n)\sin n\psi)\right]$$

There are $2n + 1$ terms of Fourier series. Any radiation pattern specified as a function of $f(\psi)$ may be expanded as a Fourier series with an infinite number of terms, such a pattern may be approximated to any desired accuracy by means of this finite series.

Schelkunoff's polynomial formulation of the linear arrays was studied from the point of view of potential theory by Thomas T. Taylor and John R. Whinnery(7). They discussed the effects of roots of the polynomials on the magnitude and phase of the space factor of an array. The lack of uniqueness in the solution limits the utility of this technique.

D.R. Rhodes[8] developed a technique to obtain an optimum single beam and fire pattern for a linear array having any given number of elements.

D.K. Cheng and M.T. Ma[9] in 1960 developed a new mathematical approach for a linear array analysis. They considered the amplitude distribution of the currents in the discrete elements of a linear array as a sampled values of a continuous function. The z- transform developed for sample data systems was used to express polynomials in closed form and thus the polynomials of many terms were avoided in the analysis of the important properties of radiation pattern. This approach offers an advantage over conventional methods since the technique of determining the location and level of side lobes, and half power beam width is simpler.

A. Broad side Array

C.L. Dolph[10] first discussed the antenna array synthesis based on Tchebyscheff polynomials. The antenna arrays based on Dolph's technique are called Dolph-Tchebyscheff arrays. By the help of Tchebyscheff polynomials Dolph found current distribution for broad side arrays which optimizes the relationship between beam width and side lobe level.

The Tchebyscheff polynomial of mth degree is given by

$$T_n(x) = \cos\,(n \cos^{-1} x)$$

$$= \cos\,(n\delta\,)$$

where $\delta = \cos^{-1} x$

The nulls of the pattern are given by

$$\delta^{0}_{k} = \frac{(2k-1)\pi}{2n}\;,\quad \text{where } k = 1,\ 2,\ \ldots n)$$

The degree n of the polynomial will be one less than the apparent number of elements.

Dolph found that for side lobes to have equal amplitude the spacings of the nulls on the unit circle for a pattern is given by :

$$\psi^{0}_{k} = \cos^{-1}\left[\frac{\cos \delta^{0}_{k}}{x_0}\right]$$

The value of x_0 is given by the relation

$$x_0 = \tfrac{1}{2}\left[(b + \sqrt{b^2-1})^{1/n} + (b - \sqrt{b^2-1})^{1/n}\right]$$

where b is the amplitude of main lobe.

The advantage of this method over binomial expansion is that it requires less labour in calculating required current distribution from the location of nulls, more over binomial method is not suitable, when large arrays are required, since this distribution results in increased beam width, loss of gain, and large current ratios. Schelkunoff's technique offers many advantages since it is applicable to broad side as well as end fire arrays, but it does not reach into the complete answer to the problem, while in C.L. Dolph's method, when the side lobe level or the position of the first null is specified current distribution can be found out. The distribution is optimum as the beam width is quite narrow for a given side lobe level, and the side lobe level is minimized for a given first null. If the position of the first null or the side lobe level is specified the position of the other side lobes and nulls can be found out by simple calculations. Since all the side lobes are at the same level, the character of the pattern can be known completely.

The improvement in beam width given by the above distribution results from raising the side lobe levels at wide angles to the level of those near the main beam. But from practical point of view it is rather inconsequential for two reasons:

(i) If the side lobes are sufficiently low in level every where, it is of no importance that they fall off with increasing angles.

(ii) The primary pattern of many types of radiating elements fall off with increasing angles, so that final wide angle lobes would be at lower level than those close to the main beam.

Therefore still improvement is to be made such that side lobes increase in magnitude with increasing angles and thus the pattern possesses side lobes at the desired constant level.

Dolph did not consider the validity of his technique, it was Riblet[11] who pointed out that it is only valid when the spacing between the elements is greater than or equal to one half wave length. Riblet generalized the method which removed this limitation.

Robert J. Stegen[12] obtained exact expressions for the excitation coefficients by equating array space factor to a Fourier series, these coefficients are readily calculated. Stegen presented a set of curves of half power beam width v/s antenna length for various side lobe levels.

A further simplification in the calculation of Dolph - Tchebyscheff arrays was done by D.J. Vander Maas[13]. He pointed out that two end elements play a very important role in the regulation of the side lobe level. H.E. Salzer[14] also presented a simple formula for calculating feeding coefficients for the optimum beam pattern. This formula is simpler in form and convenient for computations.

The gain of the Tchebycheff array depends upon the number of elements or the side lobe level. Expression for the gain as a function of the number of elements and the side lobe level was derived by Robert J. Stegen[15]. The techniques of optimization existing until now were for beam width and side lobe levels, while the optimization of gain was untouched. C.T. Tai[16] discussed a synthesis technique which is solely based upon the optimization of gain. This technique was derived for broad side array and then extended to end fire arrays. The current distribution resulting from the optimum gain was impracticable therefore this technique could not be adopted.

(D) End Fire Arrays:

The optimum design method of Dolph and Riblet for the broad side array with an odd number of elements has been modified by DuHamel[17] for end fire arrays. The advantage of this method is that direct control of the side lobe level is easily obtained. Edward K. Reed[18] working on near fields of long end fire dipole arrays pointed out that radiation pattern of an array may be predicted from the knowledge of the amplitude and phase of the field near the array and any improvement in the behaviour of the array must come about as a result of changes in near field.

(c) Pattern Synthesis

V. Maraga, D.S. Hadley and P. Ross[19] discuss the design of arrays with specified radiation patterns with the help of high speed analogue computer and pattern tracer. Later on J.N. Hines, V.H. Rumsey and C.H. Walter[20] derived an exact expression for pattern of an array. It is given as :

$$E = E_0 \left[I_0 + I_1 \exp(j\beta d \sin\theta) + I_2 \exp(j\beta 2d \sin\theta) + \ldots\ldots \right]$$

where I_0, I_1, I_n represent current flowing into the respective elements when all elements of array are energised.

$$E_n = E_0 \exp(j\beta n d \sin\theta)$$

represents radiation pattern when unit current is injected into nth pair of the array terminals, all other terminals are open circuited and the radiation pattern is in the plane containing the array.

A different technique which made use of the generalization of the sampling theorem of the band limited functions to non uniformly distributed sampling points in a synthesis of line source and infinite strip sources was discussed by J.L. Yen[21]. R.A. Tribout[22] extended a mathematical method for increasing the accuracy of the pattern synthesis by using weighted function.

(d) Super Gain Arrays:

Arrays which are capable of attaining very high gains with reasonably small dimensions, are known as super gain arrays.

Nicholas Yaru[29] performed numerical calculations for linear broad side arrays. He showed that as the directivity of gain is increased tremendous currents are required to produce only small radiated field. The exceedingly large currents required caused large ohmic losses with resultant low efficiency.

2.2. UNEQUALLY SPACED ARRAYS

The recent activity in electronically scanned arrays has resulted in means of reducing both the number of elements required for the given size aperture and the number of different types of transmitters which would be necessary in an array using illumination taper. Beams from a space tapered array can be scanned through a large angle without serious deterioration of the pattern. Moreover, an unequally spaced array of uniform amplitude with any desired side lobe level may be designed.

The conventional theories of S.A. Schelkunoff and Dolph Tchebyscheff array cannot be applied to unequally spaced array synthesis, as they have following drawbacks:

1. Since the order of polynomial increases with the number of elements, computations become more and more laborious for a large number of elements.

2. This polynomial method can be applied only to the equally spaced array.

3. These methods cannot be used for an array on curved surface.

The performance of the unequally spaced arrays remains the same with respect to the corresponding equally spaced arrays, because of the following reasons.

1. The side lobe level depends primarily on the number of elements in the array and only very little on the average spacing when the average spacing exceed about two wave lengths.

2. The 3 db beam width of the main lobe depends primarily on the length of the array.

3. The product of band width and steerability can be made much larger than for conventional equally spaced arrays.

SYNTHESIS TECHNIQUES

Various synthesis techniques exist for unequally spaced arrays. Most of the work has been done in the past few years. H. Unz[24] worked on linear arrays with arbitrarily distributed elements. He deduced a matrix relation only between the elements of the array and its far zone pattern and deduced its figure of merit, lower bound of stored energy and Q factor. D.D. King, R.F. Packard and R.K. Thomas[25] gave the requirements for a broad band steerable linear antenna array. They computed the data by means of the formula

$$P = 20 \log_{10} \left[\frac{C + 2 \sum_{k=1}^{n} \cos \left[2 \pi u \frac{x_k}{\lambda} \right]}{2n + C} \right]$$

Where P = the magnitude of the pattern factor in db.

$2n + C$ = Number of elements in the array.

$C = 1$, for an odd number of elements.

$C = 0$, for an even number of elements.

x_k/λ = distance in wave length from the center of the array in wave lengths.

$u = d_{min.} (\sin \theta - \sin \theta_0)$

d_{min} = the smallest of the set of unequal spacings in wave lengths.

θ_0 = The angle to which the beam is steered.

θ = The azimuth angle measured from the broad side.

$2\pi(x_n/\lambda)$ = Arguments of the cosine term for $n = 1, \ldots\ldots N$

They discussed scheme for controlling the cosine arguments, in the radiation pattern formula. The array synthesized was capable of operation over a 2 to 1 band width, beam steerability to $\pm 90^\circ$ with side lobe level -5 db or below.

Sheldon. S. Sandler[26] derived general analytical expressions for unequally spaced arrays. In the analysis he expressed non uniformly spaced array in terms of its equivalent equally spaced array and discussed a synthesis problem of monotonically increasing interelement spacing. A.L. Maffett[27] made use of statistical theory in synthesizing unequally spaced arrays. He described statistically the individual array factor values for a case when element placement is statistically described.

To over come the disadvantages of Dolph-Tchebyscheff arrays M.G. Andreasen[28] designed linear arrays with variable inter-element spacings with the help of analogue and digital computers. The arrays have fewer elements than Dolph-Tchebyscheff arrays with same beam width and side lobe level and have constant excitation.

Robert E. Willey[29] also simulated the amplitude tapered arrays by space tapered arrays with equally excited elements. He made use of graphical techniques and simple mathematics to design predicted gains, beam widths and side lobe levels. Until now the methods used for the design of unequally spaced arrays were based on matrices, computers and perturbation techniques, none of these methods were effective in treating arrays of large number of elements. Akira Ishimaru[30] gave a theory of unequally spaced arrays based on Poissons sum formula, he introduced a new function called 'source position function' By his method the original radiation pattern can be converted into a series of integrals, each of which is equivalent to the radiation from a continuous source distribution, whose amplitude and phase distribution clearly exhibits the effect of unequal spacing.

A synthesis technique for linear data processing linear array for signals with finite spectra was developed by D.K. Cheng [31]. Along with the unequal element spacing he made use of matched filter to get fewer elements, smaller over all dimensions and same performance as that of conventional arrays M.I. Skolnik, G. Nemhauser and J.W. Sherman[32] applied dynamic programming as an optimization technique for the synthesis of 'thinned' arrays with unequally spaced elements. 9 elements and 25 elements linear arrays were synthesized by the help of this technique. Later on M.I. Skolnik, J.W. Sherman and F.C. Ogg[93] considered the design of thinned planar array in which the density of elements located within the aperture is made proportional to the amplitude of the aperture illumination of a conventional "filled" array. The design technique is a statistical method in which suitable model amplitude taper is selected first, which is then utilized as a probability density function for determining whether or not an element should be located at a particular point within the aperture. Santo Calog'o[94] discussed the possibility of designing space tapered linear arrays by representing the element positions by a polynomial and by minimizing the side lobe energy averaged over a finite frequency band. However, his results did not give any convincing improvements. The analytical technique developed by Ishimaru[90] was extended by Ishimaru and Y.S. Chen[95]. This analytical method provides a design formula by which the exact location of each element can be predicted in order to produce prescribed radiation characteristics. The various techniques developed until now did not pay much attention to the size of elements. R.V. Larson, J.A. Dorwin and V.H. Lowe[96] studied the behaviour of unequal size elements non uniformly spaced arrays. They showed that considerable beam steering capability and low grating lobes can be obtained with less complexity of array design. Y.L.Lee and S.V.Lee(37) in a comprehensive study of space taper arrays discussed the statistics of many possible element arrangements.

(A)

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PART II

CHAPTER III

INTRODUCTION TO DYNAMIC PROGRAMMING THEORY

Dynamic programming theory is developed by Richard Bellman[1] . It is one of the various branches of modern mathematics. Dynamic programming technique is simple but a very powerful concept. One of its various applications is in solving multi stage decision problems.

The word dynamic implies that time is an essential variable, and the order of operation must be crucial. Many static processes are also re-interpreted as dynamic processes by artificially introducing time as one of the variables.

The mathematical advantages of dynamic programming are as follows :

a. It reduces the dimensionality of the process to a convenient level, thus making the problem computationally simpler.

b. The reduced form obtained by this technique has a property like "monotonicity of convergence", and therefore well suited to applications.

In dynamic programming technique a very difficult or unsolvable problem is transformed into a class of simpler, solvable problems, which are easy to handle.

An optimum system design problem is visualized as multistage decision problem. These multi-stage decision problems are best solved by means of 'functional equation approach'. (described in 3.1).

3.1. PRINCIPLE OF OPTIMALITY[2]

The functional equation describing a multistage decision process can be derived by making use of the 'Principle of Optimality'. Optimum decisions for a N - stage process can be obtained by the repeated application of the functional equation.

Principle of optimality is a fundamental principle of dynamic programming technique. It states that

> " An optimal policy or control strategy has the property that, whatever the initial state or decision, the remaining decision must form an optimal control strategy with respect to the state resulting from the first decision".

For illustration let us consider that a state of physical system is transformed from x^1 to x^2 by the transformation;

$$x^2 = T (x^1 , d_1) \qquad ..(1)$$

This operation yields an output or return

$$R_1 = r (x^1 , d_1) \qquad ...(2)$$

where D_1 = decision to be taken.

The decision which yields the maximum value of the return or criterion function, is referred to as optimal decision or optimal control strategy.

The maximum return for this one stage decision process is given by

$$f_1(x^1) = \underset{D_1}{\text{Maximum}} \left[r\,(x^1, D_1) \right] \quad ..(3)$$

Considering the case of two stage decision process from first transformation we have

$$x^2 = y\,(x^1, D_1)$$

This is further transformed into x^3

$$x^3 = y\,(x^2, D_2) \quad ...(4)$$

The sequence of operation results in a total return

$$R_2 = r\,(x^1, D_1) + r\,(x^2, D_2) \quad ...(5)$$

And the maximum return is given by

$$f_2(x^1) = \underset{D_1, D_2}{\text{Maximum}} \left[r\,(x^1, D_1) + r\,(x^2, D_2) \right] \quad ...(6)$$

The total result is maximised over the policy (D_1 , D_2) and the policy which maximises R is called optimal policy.

In general for a N - stage decision process, the problem is to choose a N - stage policy.

$$\left[m_1 , m_2 \; .... m_N \right]$$

So as to maximize the total return

$$f_N(x^1) \text{ forms } = \max_{[m_j]} \left[\sum_{j=1}^{N} r (x^j , m_j) \right] \quad ...(7)$$

Where $[m_j]$ forms an N - stage control policy

This functional equation so obtained can be solved by conventional techniques available.

CHAPTER IV

4.1. DERIVATION OF RADIATION PATTERN

The array considered in the synthesis process is symmetrical unequally spaced linear array shown in Fig. 1. The antenna elements are energised so as to get main lobe in any specified direction θ_0. The radiation pattern of a linear array containing an odd number of isotropic elements symmetrically arranged about the center element can be computed as follows :

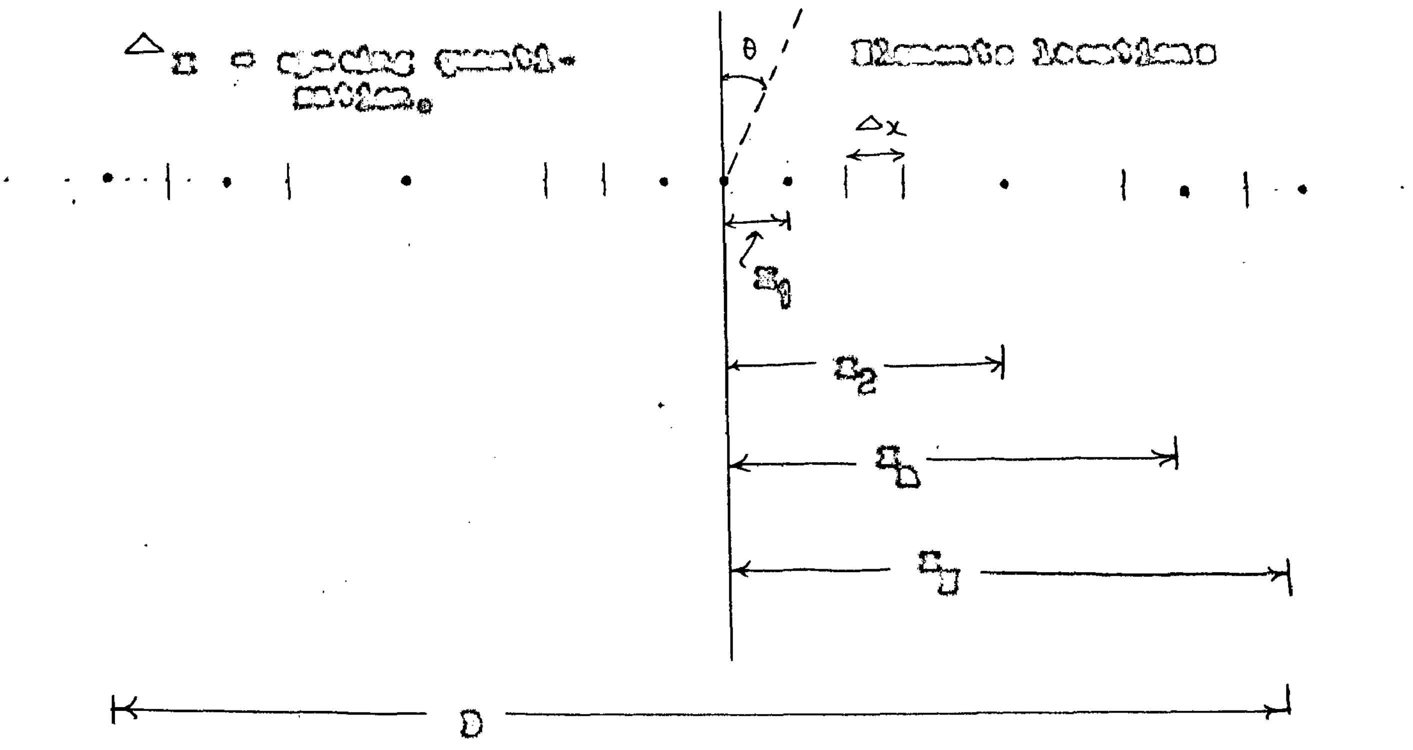


Figure 1.

Let x_n = the distance of the nth pair of elements measured in wave-lengths from the centre of the aperture.

D = Array length

Δx = Spacing Quantization

x_N = Spacing of Nth pair of elements.

θ = Angle with respect to the array normal.

$$D = 2\, x_N$$

The elements are allowed to occupy positions whose location from the array center is an integral number of some prespecified value Δx.

Considering only two elements symmetrically placed about the center element, the radiation pattern is given by

$$D\,(x_i,\ \theta) = A_0 + A_0\, e^{j(x_i \beta \sin\theta + \alpha^{+})} + A_0\, e^{-j(\alpha^{-} + \beta x_i \sin\theta)}$$

Where A_0 = current excitation

$\beta = 2\pi/\lambda$ = propagation constant

λ = wave length

α = progressive phase shift, lead from left to right.

$\alpha^{+} = -\alpha^{-} = \alpha$

x_i = distance of element pair from the centre element.

$$E(x_1, \theta) = A_0\left[1 + e^{j(\alpha + \beta x_1 \sin\theta)} + e^{-j(\alpha + \beta x_1 \sin\theta)}\right]$$

$$= A_0\left[1 + e^{j(\alpha + \beta x_1 \sin\theta)} + e^{-j(\alpha + \beta x_1 \sin\theta)}\right]$$

$$= A_0\left[1 + 2\left\{\cos(\alpha + \beta x_1 \sin\theta)\right\}\right]$$

At beam maximum

$$\cos(\alpha + \beta x_1 \sin\theta) = 1$$

or $\alpha + \beta x_1 \sin\theta_0 = 0$

or $\alpha = -\beta x_1 \sin\theta_0$

$\therefore \alpha = -2\pi/\lambda \; x_1 \sin\theta_0$

On substitution, we get

$$E(x_1, \theta, \theta_0) = A_0\left[1 + 2\cos\left\{\frac{2\pi}{\lambda} x_1 (\sin\theta - \sin\theta_0)\right\}\right]$$

Define $u = \sin\theta - \sin\theta_0$

= angular coordinate

Considering unit current excitation

i.e. $A_0 = 1$

$$E(x_1, u) = 1 + 2\cos(2\pi x_1 u)$$

where x_1 = distance of element pair from center element measured in wave lengths.

For a N element array, we get the radiation pattern as :

$$E\,(x_1\,,\,x_2\,,\;\ldots\ldots x_N\;,\,u) = 1 + 2\sum_{n=1}^{N} \mathrm{Cos}\,(\,2\pi\,x_n\,u)$$

..(1)

4.2. OPTIMIZATION CRITERION

In order to get desirable radiation pattern, criterion for optimization is established. The shape of the main lobe and the maximum intensity remains relatively unaffected by the precise arrangement of a given number of elements within the given size aperture. On the other hand dependence of side lobes on the arrangement of elements is significant. Consideration of main beam parameters as design criterion is therefore of least importance. Thus it seems reasonable to establish the criterion on the basis of the side lobe properties.

The various criteria under consideration may be

(a) To determine current ratio's that will result in the smallest side lobe level for the given beam width of the main lobe. The current distribution that produces such a pattern will be considered as being the optimum. This was achieved by Dolph - Chebyshev[9] in their conventional uniform array synthesis by making all the side lobes of equal amplitude.

(ii) Using some spacing function, placement of elements according to which might give optimum results. Ishimaru and Chen[4] considered spacing function of the type $x = (2 A_0 / \pi) \sin \pi x$, where $A_0 < 1$

(iii) Maintaining a minimum element spacing by space quantization of the aperture into discrete points and selecting as the optimum radiation pattern the one whose highest side lobe peak over a specified angular interval was less than the highest peak of any other pattern.

First criterion is not possible in thinned array of unequally spaced elements because of the lack of sufficient elements to specify completely the radiation pattern at required number of angular coordinates and because it is generally required to maintain a minimum element spacing. While in second criterion it is doubtful that numerical convergence could be retained as more terms are considered for a general spacing function. The criterion which is best suited to our problem is the last one, we shall regard the optimum array as one which among many possible element arrangements, within a given aperture, gives the lowest side lobe level in certain region of u. This is possible since in general beam width is essentially independent of element arrangements. This is a special case of the general criterion of minimizing maximum deviation.

CHAPTER V

5.1. MATHEMATICAL FORMULATION OF DYNAMIC PROGRAMMING AS APPLIED TO UNEQUALLY SPACED ANTENNA ARRAY SYNTHESIS

The problem is to locate the vector $x = (x_1, x_2, .. x_N)$ such that over some region of u, the maximum value of the function

$$D(x_1, x_2, \ldots x_N, u) = 1 + 2\sum_{n=1}^{N} \cos(2\pi x_n u)$$

is a minimum. The technique to be applied is dynamic programming.

The given input conditions are:

i. Each x_n has an upper and lower bound

$$a \leq x_n \leq b$$

which is variable with n, and

ii. The region of u over which the expression is to be evaluated

$$u_{min} \leq u \leq u_{max}$$

With these given values the optimization is carried out, so that the values of the x_n's are determined, which minimize the maximum value of this function over the required value of u.

If x_n^0 denotes optimum values of x_n, then

$$E(x_1^s, x_2^s, \ldots, x_N^s, u) = \max_{x_1 \le x_2 \le x_N} \left[\min_{x_{n-1} \le x \le x_{n+1}} \quad n = 1, 2, \ldots N \quad E(x_1, x_2, \ldots, x_N, u) \right]$$

9.2. SYMMETRY OF RADIATION PATTERN

Since the radiation pattern function is essentially a cosine function, it must be symmetrical about some value of u, say $u = u_0$. If $u = u_0$ be that,

$$E(u_0 + \Delta u) = E(u_0 - \Delta u)$$

i.e. $\cos\left[2\pi x_1(u_0 + \Delta u)\right] = \cos\left[2\pi x_1(u_0 - \Delta u)\right]$

or $\cos 2\pi x_1 u_0 \cos 2\pi x_1 \Delta u - \sin 2\pi x_1 u_0 \sin 2\pi x_1 \Delta u$

$= \cos 2\pi x_1 u_0 \cos 2\pi x_1 \Delta u + \sin 2\pi x_1 u_0 \sin 2\pi x_1 \Delta u$

The equality will hold good only if,

$$\sin 2\pi x_1 u_0 = 0$$

or $2\pi x_1 u_0 = 0, \pm\pi, \pm 2\pi, \ldots\ldots, \pm n\pi$

or $x_1 u_0 = 0, \pm 1/2, \pm 1, \ldots\ldots, \pm n/2$

When x_1 is multiple of $\lambda/2$, then,

$u_0 = 1$ i.e. pattern is symmetrical about $u_0 = 1$

When x_1 is multiple of $\lambda/4$, then $u_0 = 2$

i.e. pattern is symmetrical about $u_0 = 2$.

3.3. SELECTION OF INPUT PARAMETERS

Input parameters play important role in optimization process. One of the parameters is a region over which the antenna pattern is to be optimized. The input parameters are discussed below :

(1) U_{min}: The important point to be considered is that region of optimization should not include the main lobe. The value of U_{min} should be such as to lie close to the first null. By careful observation of various radiation patterns, it was found that for the case of 9 elements array, U_{min} could be safely taken as .1 . It has three fold advantages.

a. It safely excludes the main lobe.
b. It economises some of the computations.
c. It does not miss any of the higher side lobe which fall nearer to the main lobe.

As the number of elements increases main lobe becomes narrower, so the U_{min} is accordingly reduced. A rough idea could be drawn from the case of equally space elements, for which the main lobe beam width is given by λ / D (between the first nulls), where D is the aperture width and λ is the wave length.

(ii) U_{max} : The value of U_{max} depends upon the value of u about which the pattern is symmetrical. In 9 elements array case U_{max} could be taken as 1 and U_{min} = .1 Because of the symmetry the pattern is optimised over the region $.1 \leq |u| \leq 1.9$. This means that $\theta_0 = 90^0$, in other words 90^0 of scanning is possible. If the radiation pattern is to be scanned over a smaller angle or is scanned not at all, the u region over which the pattern is to be optimised can be smaller and consequently the side lobe level is reduced.

U_{max} = 0.90 corresponds to angular region 90^0 to either side of the main beam of an unscanned array.

As the spacing quantization is reduced from $\lambda/2$ to $\lambda/4$, U_{max} increases, since the pattern is now symmetrical about u = 2 instead of 1. U_{max} in this case is taken as 1.90.

U_{min} depends upon the number of elements under consideration while U_{max} is dependent upon the spacing quantization.

(iii) Δx – Spacing quantizations. In systematic optimization process it is essential to establish a minimum distance of consecutive element placement. Quantizing the possible element locations results in a pattern symmetry in the angular coordinate u . The symmetry has a significant effect on the design, since optimizing the pattern over the specified region of u also optimizes the pattern over the corresponding image region. Problem faced is that low side lobes over the image region may not be of interest, at the same time the side lobes in the desired region may not be as low as possible, since the total region of optimization includes the image region, and may be larger than necessary. By judicious selection of spacing quantization it is possible however, to improve the side lobe level in the desired region of optimization.

The computations done in the problem taken up assumed that the elements could be located along the aperture of the array only at spacing quantized in half wave length intervals. Quarter wave length and eighth wave length quantizations are also feasible. With half wave length quantization the applicable interval in which to place elements is 8 λ, with 0.25 λ quantization the interval is 4 λ, with 0.125 λ quantization the allowed interval is reduced to 2 λ.

The region of optimization varies in accordance with the spacing quantization. As mentioned before for 0.25 λ quantization the pattern is symmetrical about u = 2 while for 0.125 λ quantization it is symmetrical about u = 4. Therefore as the spacing quantization is reduced more computations are required.

9.4. [illegible]

Having studied the input parameters suitable values are assigned to them. It is seen before that for each n, x_n assumes only a finite number of discrete values (m) between $x_{n(min)}$ and $x_{n(max)}$, where m is not necessarily same for each n. Also u can assume only a finite number of discrete values L between u_{min} and u_{max}. The evaluation and optimization is carried out for each N with $N_{(min)} \leq N \leq N_{(max)}$. A further requirement is that

$$|x_{n+1} - x_n| \geq a$$

i.e. no x_n is placed closer than a (spacing quantization) to its nearest neighbour.

Because each x_n, (except center and last elements) can assume 'm' values, there are m^n possible combinations of the values of x_n's, where m has been assumed equal for each n. Each combination gives rise to a function of u, from which the 'best' one is to be chosen, such that the

$$q_{min} \leq u \leq q_{max} \quad \overset{max}{} \left[D(x_1, x_2 \ldots\ldots\ldots x_n, u) \right]$$

is least. Because there are p values of u there would be $p\,n^{n}$ combinations to investigate, but the number is reduced if dynamic programme is applied. The number of combinations is reduced to $p.\,(n-1) \times \frac{n(n-2)}{2}$

by the following procedure:

STAGE I.

The various symbols used are defined in Table 1

Compute 2 cos $D(2\pi x_n\, u)$ for all x_n,

$1 \leq n \leq n_{max}$, and all u, $q_{min} \leq u \leq q_{max}$. All these values of cosine function are stored in computer memory. These are $p\,.\,n$ terms. This stage is not contribute to the optimization process.

STAGE II.

Considering elements x_1 and x_2, certain positions of x_2 and x_1 according to figure 2 make it a point that no value of x_1 ever lay or exceed a particular value of x_2 under consideration.

Compute the absolute value of sum

$$F_1(x_2, x_1, u) = (1 + 2\cos(2\pi x_1\, u) + 2\cos(2\pi x_2\, u))$$

for $x_{2(min)} \leq x_2 \leq x_{2(max)}$

$x_{1(min)} \leq x_1 \leq x_{1(max)}$

TABLE I

Quantity	Description
x_n	The position of nth element. The nth element is located between
$x_{n(min)}$	The lower bound
$x_{n(max)}$	The upper bound
n	Positions between $x_{n(min)}$ & $x_{n(max)}$ with
Δx	being the same increment in x $n\Delta x = x_{n(max)} - x_{n(min)}$
$C_{i-1}(x_i, x_{i-1}, u)$	Refers to sum of the cosine function when optimization is done between the ith and i-1 th elements.
$C^{0}_{i-1}(x_i, x_{i-1})$	Equals max ($C_{i-1}(x_i, x_{i-1}, u)$) with $u_{min} \leq u \leq u_{max}$ Determines the peak values of the sum as a function of x_{i-1}, for each x_i and is now independent of u. Next, for each x_i the best value of x_{i-1} is found. This term is
$C^{0}_{i-1}(x_i)$	$\min_{x_{i-1(min)} \leq x_{i-1} \leq x_{i-1(max)}} \left[C^{0}_{i-1}(x_i, x_{i-1}) \right]$ Thus the best combination of x_{i-1} is known for each x_i. The term

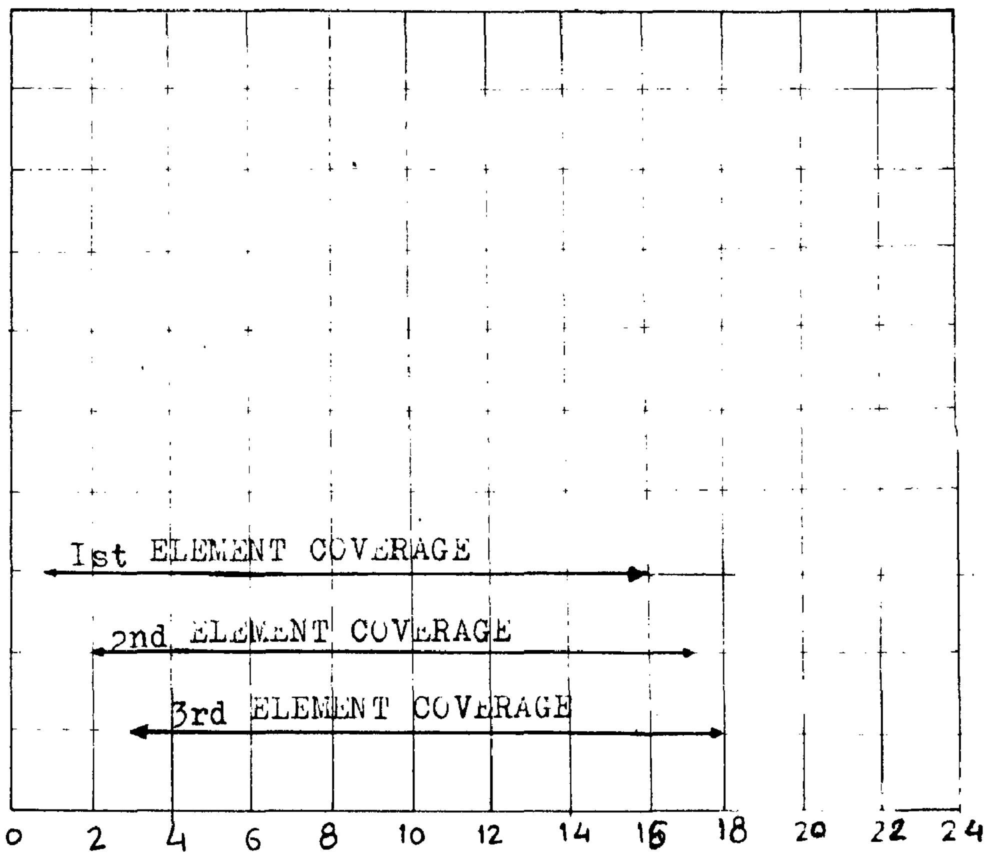
Table X contd...

$U^{o}_{i-1}(x_i, u)$	$U_{i-1}(x_i, x_{i-1}(x_i), u)$ is stored in the memory for use in the next stage. The function includes the best $x_{i-1}(x_i)$ previously determined and hence represents the optimum combination for each x_i to the i th stage of the computation.
u	is the region over which the computation is optimized and is located between
u_{min}	the lower bound
u_{max}	the upper bound
N	location between u_{min} and u_{max}
Δu	being the increment in u $N \Delta u = u_{max} - u_{min}$
U	is the value to which the computation is computed and is bounded by
U_{min}	the lower bound
U_{max}	the upper bound. Optimum results for each $U_{min} \leq U \leq U_{max}$ are punched out of the program.

DECISION NUMBER

(DISPLACEMENT FROM ARRAY ORIGIN IN WAVE-LENGTH)

ELEMENT COVERING AS A FUNCTION OF DECISION NUMBER FOR A LINEAR ARRAY



9 ELEMENTS ARRAY CASE, $\Delta x = .5\lambda$,
ARRAY LENGTH = 19 λ .

(0th ELEMENT AND LAST ELEMENTS ARE FIXED)

FIG. -2.

and $u_{min} \le u \le u_{max}$

The values of the cosine function are stored in the computer memory, they are called for as required.

There will be in all i.e. $n(N-1)/2$ combinations.

Determine maximum of the sum $f(x_2, x_1, u)$ for each x_2 in combination with all x_1's over the u region. That is

$$U_1^o(x_2, x_1) = \max_{u_{min} \le u \le u_{max}} U_1(x_2, x_1, u)$$

There are a maximum of $n(N-1)/2$ such values. From these sums the value of x_1 which minimises the sum $U_1^o(x_2, x_1)$ for each x_2 is selected. That is the 'best' x_1 associated with each x_2.

i.e. $U_1^o(x_2) = \min_{x_{1(min)} \le x_1 \le x_{1(max)}} \left[U_1^o(x_2, x_1) \right]$

From these computations obtained

$$U_1^o(x_2, u) = U_1(x_2, x_1(x_2), u)$$

Where $x_1(x_2)$ denotes the best x_1 associated with each x_2.

There will be n values of $x_1(x_2)$ and corresponding n $U_1^o(x_2, u)$. These values are stored in computer memory.

STAGE III

Having determined $x_1(x_2)$, proceed on to the next step, in which x_3 also comes into the computation. Computations are done to find $x_2(x_3)$. The optimum value of x_1 corresponding to x_2 will follow from stage II. It saves some of the computations, which are already performed in Stage II.

As before Compute.

$$U_2(x_3, x_2 u) = \frac{1}{4}\left(1 + 2\cos(2\pi x_1 u) + 2\cos(2\pi x_2 u) + \cos(2\pi x_3 u)\right)$$

For all $x_{3(min)} \leq x_3 \leq x_{3(max)}$

$x_{2(min)} \leq x_2 \leq x_{2(max)}$

$x_1(x_2)$

and $u_{min} \leq u \leq u_{max}$

There will be maximum of $N. \, n\,(N-1)/2$ such combinations

Determine $U_2^0(x_3, x_2) = \max_{u_{min} \leq u \leq u_{max}} \left[U_2(x_3, x_2\, u)\right]$

There will be maximum of $n(N-1)/2$ such values.

Determine $U_2^0(x_3) = \min_{x_{2(min)} \leq x_2 \leq x_{2(max)}} \left[U_2^0(x_3, x_2)\right]$

Obtain $U_2(x_3, u) = U_2(x_3, x_2(x_3), u)$

where $x_2(x_3)$ denotes the best x_2 associated with each x_3.

There will be n values of $x_2(x_3)$ and corresponding

$n \cdot O_2^0(x_3, u)$.

These values are stored in computer memory.

Stage i (GENERAL STAGE)

Compute $F_{i-1}(x_i, x_{i-1}, u) = 1 + 2 \sum_{n=1}^{i} \cos(2\pi x_n u)$

for all $x_{i(min)} \leq x_i \leq x_{i(max)}$

$$x_{i-1(min)} \leq x_{i-1} \leq x_{i-1(max)}$$

$$x_{i-2}(x_{i-1}),\ x_{i-3}(x_{i-2}) \ldots\ldots x_1(x_2),$$

$$u_{min} \leq u \leq u_{max}$$

As usual there will be ? $n(N-2)/2$ such combinations.

Determine

$$O^0_{i-1}(x_i, x_{i-1}) = \min_{u_{min} \leq u \leq u_{max}} \left[F_{i-1}(x_i, x_{i-1}, u) \right]$$

There will be maximum of $n(N-2)/2$ such values.

Determine

$$O^0_{i-1}(x_i) = \min_{x_{i-1(min)} \leq x_{i-1} \leq x_{i-1(max)}} \left[O^0_{i-1}(x_i, x_{i-1}) \right]$$

Compute

$$U^{o}_{i-1}(x_i , u) = U_{i-1}(x_i , x_{i-1}(x_i) , u)$$

where x_{i-1} (x_i) denotes "best" x_{i-1} associated with x_i and x_{i-2} (x_{i-1}), $x_{i-3}(x_{i-2})$ ......$x_1(x_2)$ are all determined from previous stages.

Stages from II to i are all identical, but each successive stage adds an additional element.

Final Stage.

This stage corresponds to the computation of optimum position of last but one element corresponding to the last (fixed) element. It is repetition of previous stages except that last element remains fixed. The best value found for

$$U_{min} \leq U \leq U_{max}$$

is to be punched out in the computer program.

$$U(x_1^o, x_2^o, ...x_N^o, u) = 1 + 2\sum_{n=1}^{N} Cos(2\pi x_n u)$$

is then calculated for the optimum x_n^o s over the region $0 \leq u \leq u_{max}$ and punched out in the computer program for plotting purposes.

CHAPTER VI

COMPUTATION AND RESULTS

Computer was used as a main tool for materializing dynamic programming in antenna array synthesis. A set of program were written for IBM 1620 computer. The storage capacity of IBM 1620 is 60,000. The storage capacity required by this type of problem especially when array containing large elements is to be designed is much more than 60,000. The execution time also limits the utility of this program. Because of the above mentioned reasons only the array containing 9 elements is considered. The program can compute maximum of 200 different values of u (u are $u = \sin\theta - \sin\theta_0$) in determining particular configuration of elements.

The aperture length considered is 19 λ (where λ is taken as 1 meter) and the various input parameters are $u_{min} = .1$, $u_{max} = 1$, spacing quantization $\Delta x = \lambda/2$ and $\Delta u = 0.01$. The two end elements along with the center element are kept fixed so the remaining elements are 6. Since the array is symmetrical about the center element computations are done to find the optimum locations of three elements. The rest three can be placed symmetrically about the center element.

The array length on each side of the center element is 9.5 λ, therefore the available element locations of the quantization would be 19 on each side. Figure 2 shows the element coverage as a function of decision number for a linear array. The purpose being to economize unnecessary computations. This shows that first element can take up any of the positions from 1 to 16, while second element from 2 to 17 and third element can occupy any of positions from 3 to 18. Location 19th is fixed for the last element. Thus n is equal to 16.

u_{min} and u_{max} are related to each other by the relation

$$u_{max} = u_{min} + \Delta u \times N$$

where N = total number of discreet value of u.

The value of Δu is determined from the analogy to sampling theorem. The u region should be sampled at intervals not larger than $\lambda/2D$.
where D = maximum aperture length for D = 19 λ

$$\Delta u = \lambda/19\lambda = 0.05$$

Still to be on the safer side Δu is taken as 0.01
Since $u_{min} = .1$ and $u_{max} = 1$

$$N = \frac{u_{max} - u_{min}}{\Delta u} = \frac{1 - .1}{.01}$$

i.e. N = 90

Total computations required in in brute force approach for this type of problem are 4032640, but the dynamic programming gives optimum results only in 30720 computations. Thus there is a net saving of 4001920 computations. Even with modern high speed computing devices the brute force approach is generally not practical.

Computations were performed by the help of three approaches.

1. Dynamic programming technique.
2. Space taper approach.
3. Considering all possible combinations.

6.1. [illegible]

Program written for designing unequally spaced array by dynamic programming is given in Appendix 'A' . This program is only applicable for 9 element synthesis problem. The run time of this program is 43 minutes. The spacings measured in wave lengths from the array center, of each pair of elements in 9 elements array as found by computer are $x_1 = .5$ $x_2 = 2.5$, $x_3 = 4$ and $x_4 = 5.5$. The radiation pattern corresponding to the above spacings is shown in Figure 3. The maximum side lobe level is 5.64 db. below the main beam. Region of optimization line of symmetry of pattern are all indicated in the Figure.

BY THE APPLICATION OF "DYNAMIC PROGRAMMING"

(Wave Length, λ = 1 Meter).

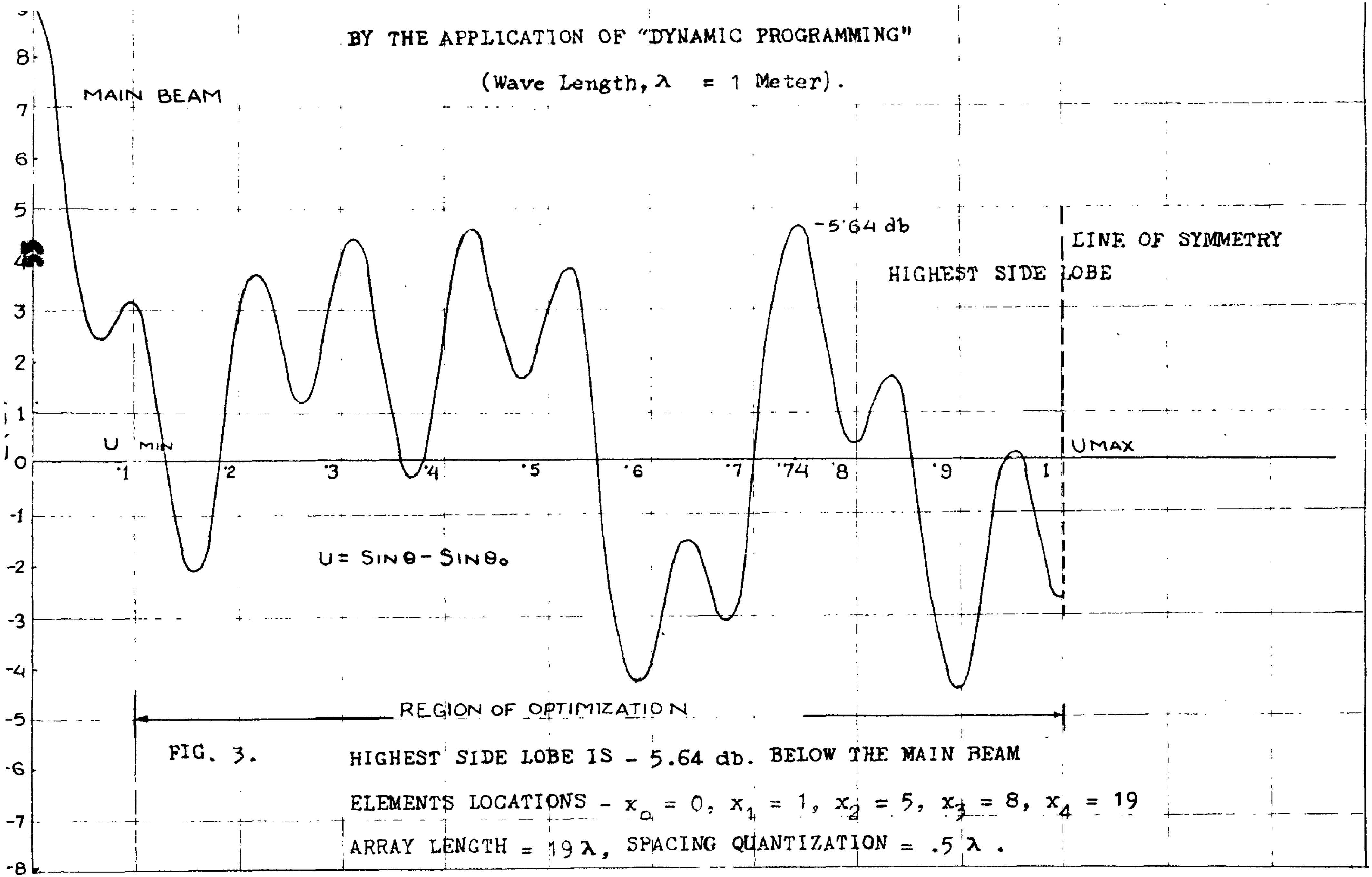


FIG. 3. HIGHEST SIDE LOBE IS − 5.64 db. BELOW THE MAIN BEAM

ELEMENTS LOCATIONS − $x_0 = 0$, $x_1 = 1$, $x_2 = 5$, $x_3 = 8$, $x_4 = 19$

ARRAY LENGTH = 19 λ, SPACING QUANTIZATION = .5 λ .

Figure 4 shows the frequency distribution of the side lobe level for the cases considered in optimization process. It shows that out of 16 possible combinations of elements only two element combinations give low side lobe level.

Since the radiation pattern of Fig. 3 is designed with $u_{max} = 1$ and $u_{min} = 0.1$, it permits the antenna beam to be scanned to $\theta_o = 90°$. With peak side lobe level, no greater than -5.64 db. Other cases considered have region of optimization less than the previous case i.e. $u_{max} = .50$ and $u_{max} = .96$. The radiation pattern of the optimum case when region of optimization is $u_{max} = .50$ and $u_{min} = .1$ is drawn in Figure 5. The side lobe level within the region of optimization is considerably reduced, as the highest side lobe level is 8.76 db below the main beam. But high side lobe occurs out side the region of optimization, the highest side lobe level outside the region of optimization is 1.14 db down the main lobe. The locations of element, pair for the optimum case as measured in wave lengths from the array center are $x_1 = 1.5$, $x_2 = 3$, $x_3 = 4.5$ and $x_4 = 9.5$ This permits the antenna beam to be scanned to $\theta_o = 30°$ with peak side lobe level 8.76 db down the main lobe

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1. DYNAMIC PROGRAMMING TECHNIQUE.

SPACING QUANTIZATION $\Delta x = .5\lambda$

NO. OF CASES CONSIDERED =16

MAIN BEAM CORRESPONDS TO 9 UNITS OF AMPLITUDE.

$\theta_o = 90^o$ Umin. = .1, U max. = 1

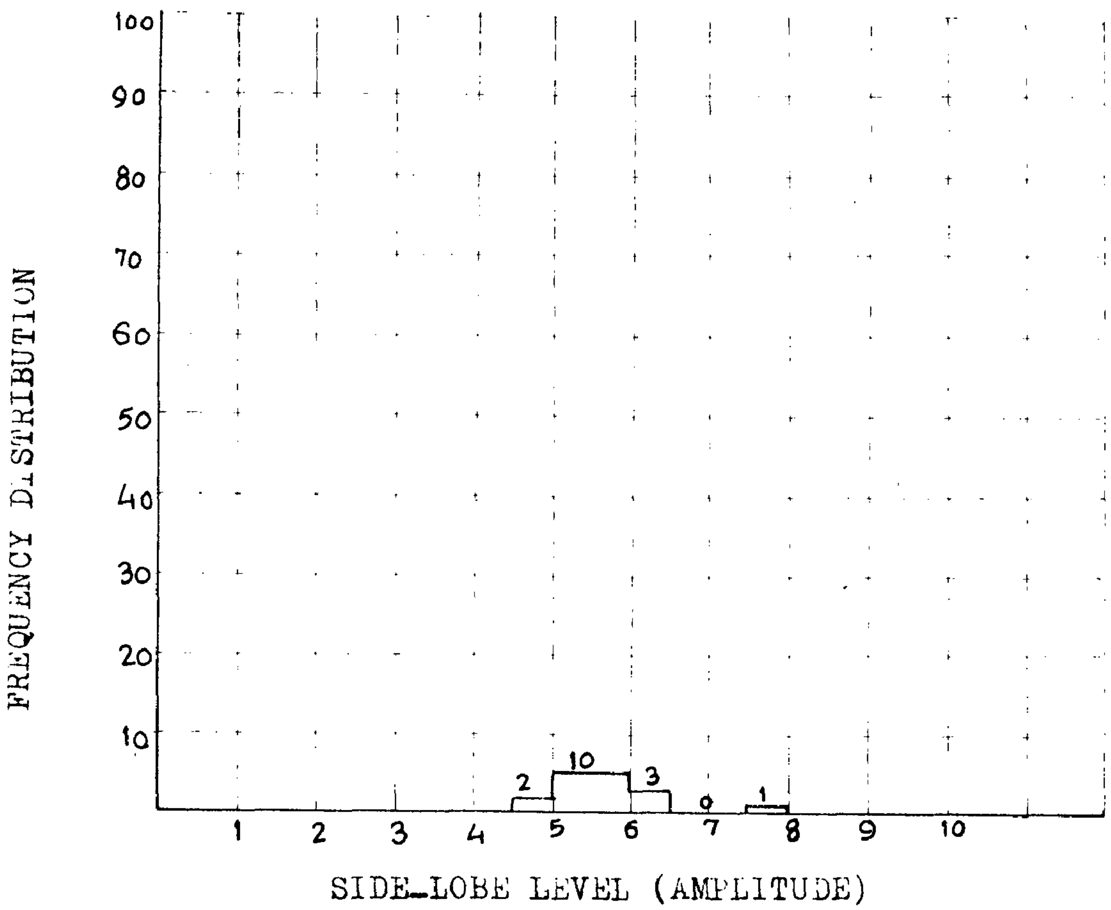


FIG.-4.

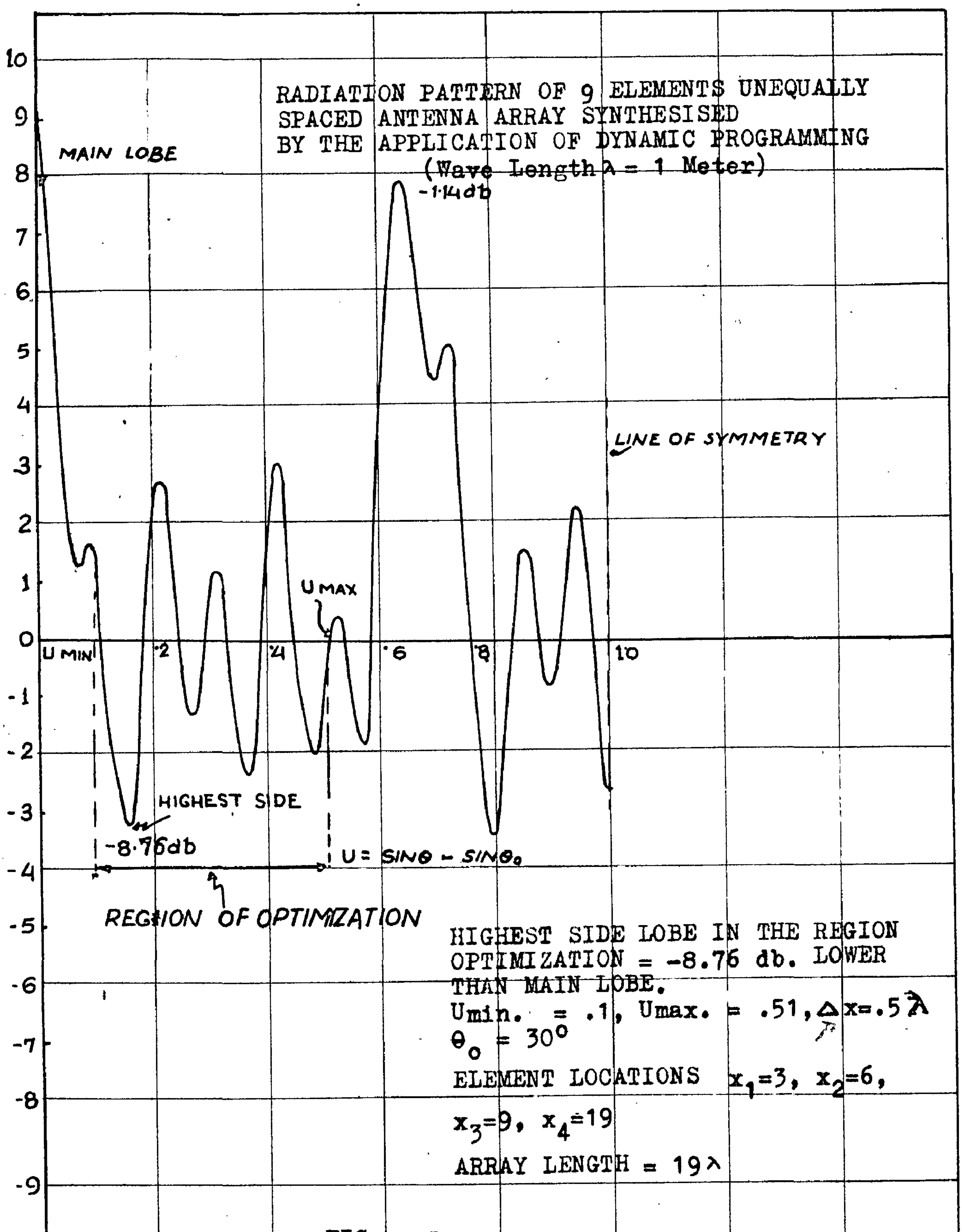

RADIATION PATTERN OF 9 ELEMENTS UNEQUALLY SPACED ANTENNA ARRAY SYNTHESISED BY THE APPLICATION OF DYNAMIC PROGRAMMING
(Wave Length λ = 1 Meter)
MAIN LOBE
-1.14db
LINE OF SYMMETRY
U MAX
U MIN
HIGHEST SIDE
-8.76db
U = SINθ - SINθ₀
REGION OF OPTIMIZATION
E(U)
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
.2
.4
.6
.8
1.0
HIGHEST SIDE LOBE IN THE REGION OPTIMIZATION = -8.76 db. LOWER THAN MAIN LOBE.
Umin. = .1, Umax. = .51, Δx=.5λ
θ₀ = 30°
ELEMENT LOCATIONS x₁=3, x₂=6,
x₃=9, x₄=19
ARRAY LENGTH = 19λ

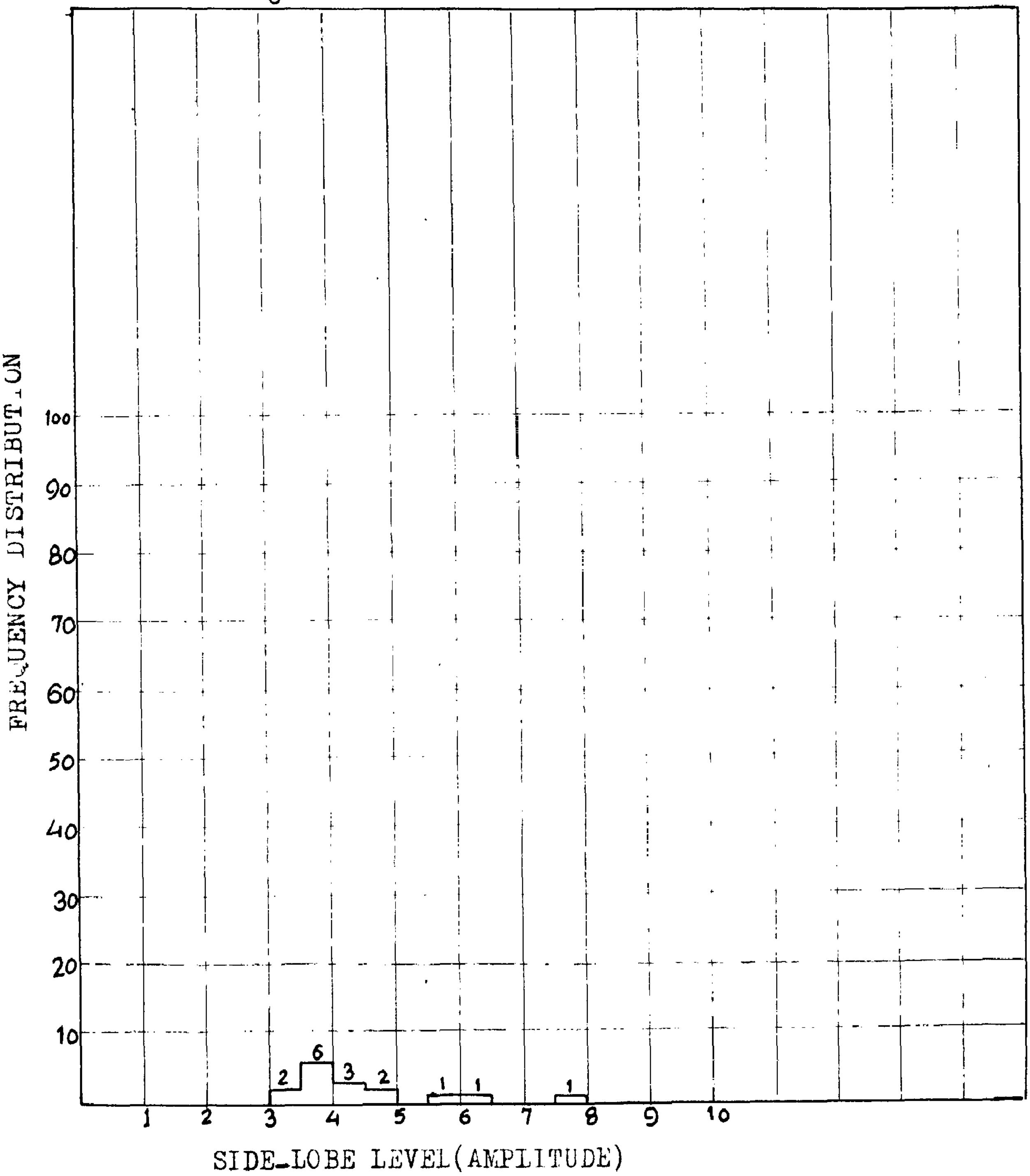
FIG. - 5.

Frequency distribution of side lobe level for the cases considered in optimization process is given in Fig. 6. Two combinations out of 15 combinations considered give the low side lobe level in this particular case. The radiation pattern of the optimum case when region of optimization is $u_{max} = 0.85$ and $u_{min} = .1$ is drawn in Figure 7. The highest side lobe level within the region of optimization is 6.96 db below the main beam., while the highest side lobe level outside the region of optimization is 8.76 db below the main lobe. This permits the main beam to be scanned to $\theta_0 = 60^\circ$ with peak side lobe level 6.96 db below the main beam. Frequency distribution of side lobe level considered in optimization process is given in Figure 8 . One combination out of 16 combinations considered gives the low side lobe level. The spacings, measured in wave lengths from the array center, of each pair of elements are $x_1 = 1$, $x_2 = 2$, $x_3 = 4.5$ and $x_4 = 9.5$

Program given in Appendix 'B' was written to check that the region of optimization excluded the main beam and at the same time no significant side lobe was missed . The radiation pattern of the element combination which gives optimum results (the element locations are obtained from the execution of program of Appendix 'A') can be obtained from the program of Appendix 'C'.

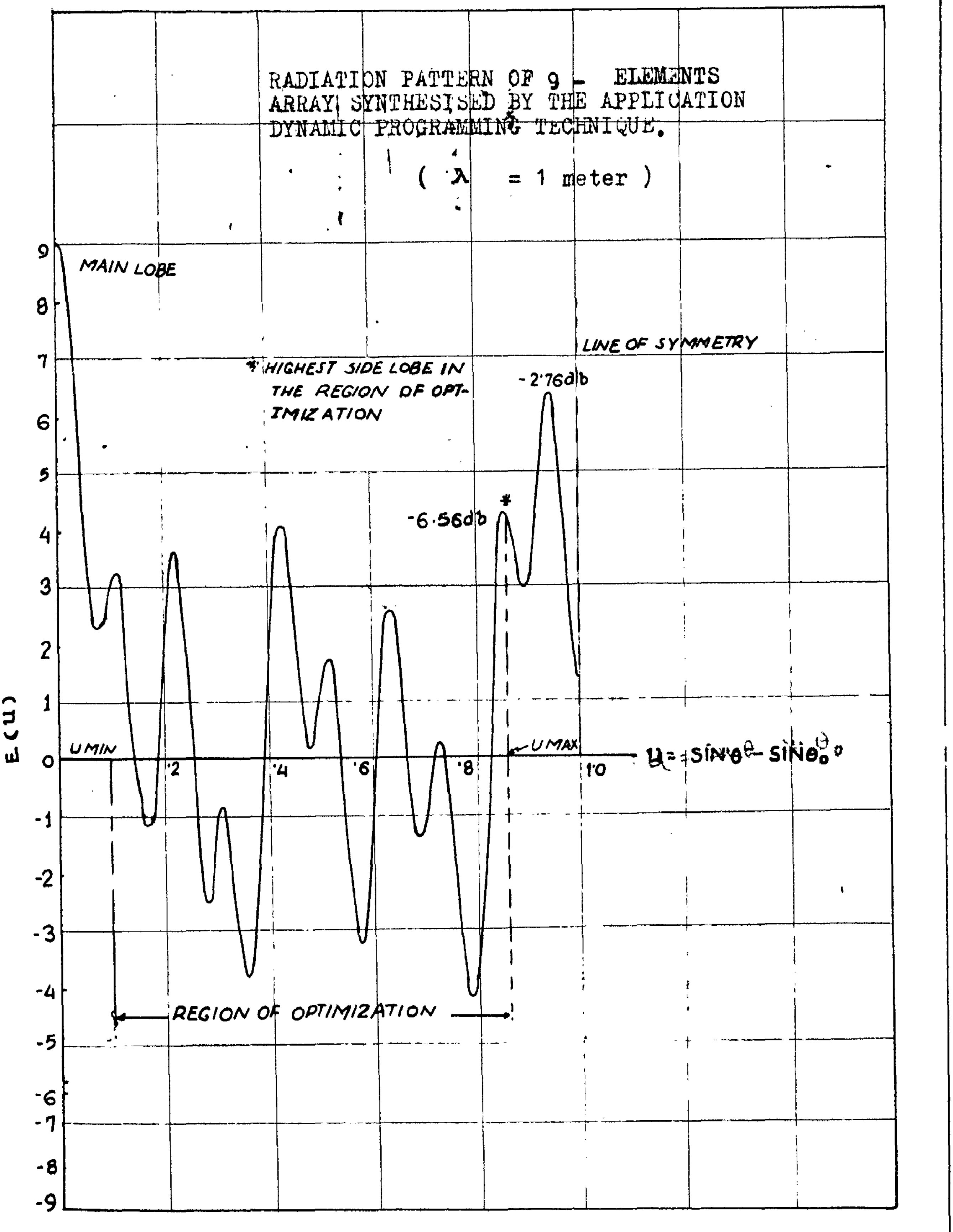
1. DYNAMIC PROGRAMMING TECHNIQUES
SPACING QUANTIZATION = $\lambda/2$
NO. OF CASES CONSIDERED = 15

$\theta_o = 30^\circ$, U min. = .1, U max. = 51



ELEMENT LOCATIONS FOR OPTIMUM CASE, $x_1 = 3$, $x_2 = 6$, $x_3 = 9$, $x_4 = 19$

FIG.-6.



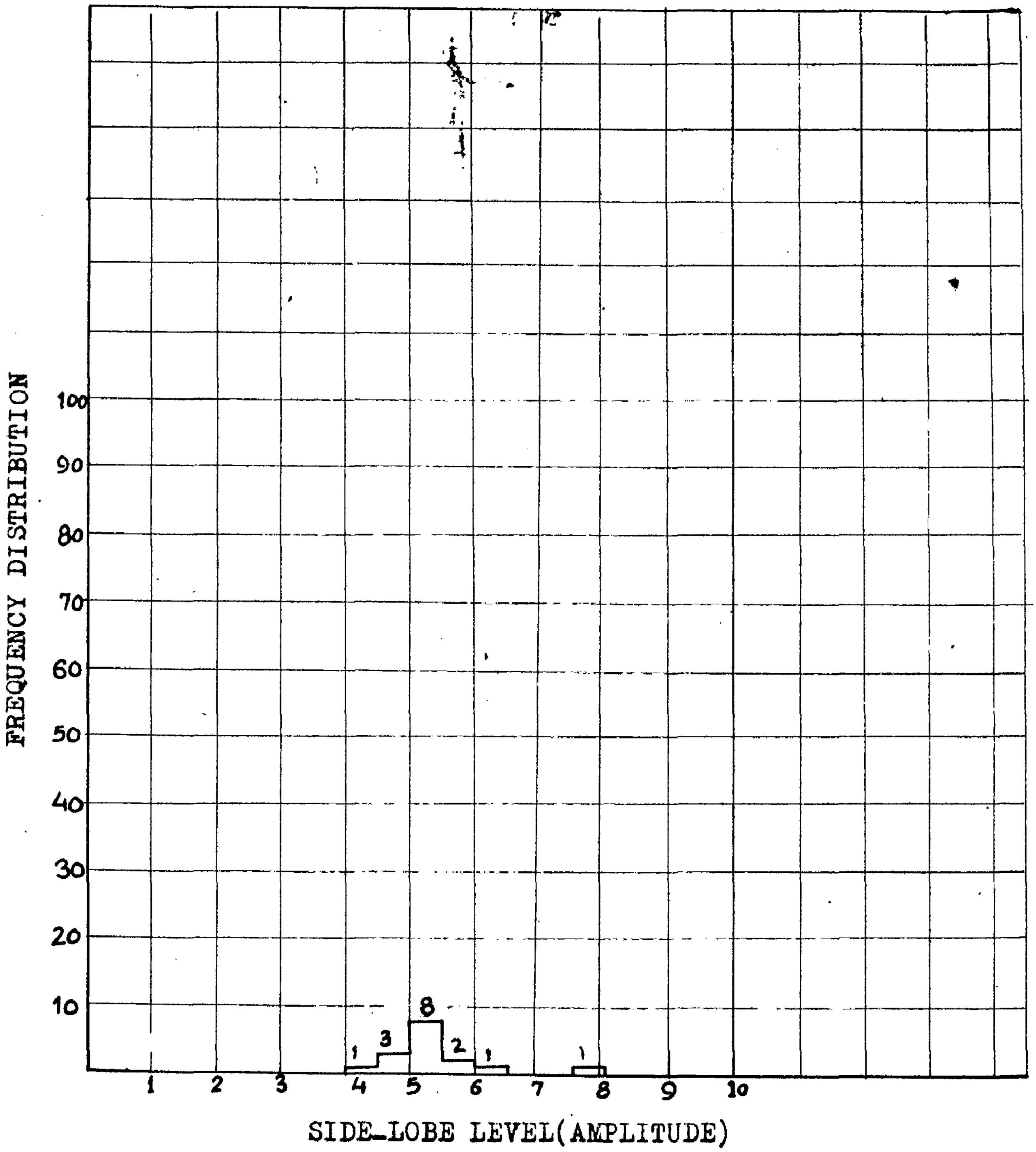
HIGHEST SIDE LOBE IS -6.56 db. BELOW MAIN LOBE
ELEMENTS LOCATIONS - x_1 = 2, x_2 = 4, x_3 = 9, x_4=19
$\Delta x = \lambda/2$, U min. = .1, U max. = .86

FIG. - 7

1. DYNAMIC PROGRAMMING TECHNIQUE $\Delta x = \lambda/2$, U min. = .1
U max. = 78

NO. OF CASES CONSIDERED = 16

$\theta_0 = 60^0$



ELEMENT LOCATIONS FOR OPTIMUM CASE, $x_1 = 2$, $x_2 = 4$, $x_3 = 9$. $x_4 = 19$

FIG.-8.

6.2. [illegible]

A space taper method is developed in which instead of allotting equal positions to each element, they are allotted in ascending order i.e. first element is given three positions, second element six, while the third element is given nine positions. Thus the 18 available positions are divided in the ratio 1 : 2 : 3, the 19th position is fixed for the last element. The optimization process was then carried out with all the five elements (i.e. center, three variable and the last element) present. A program written for this optimizing technique is given in Appendix 'D'. The run time of this program is 33 minutes. The various input parameters remain the same as in dynamic programming technique. Although it saves lot of computations, it does not give optimum results. Computations are done for $\theta_0 = 90^\circ$ and $\theta_0 = 30^\circ$. Frequency distribution are plotted in Figures 9 and 10. The lowest side lobe for $\theta_0 = 90^\circ$ occurring lies between 5.5 to 6 units of amplitude, which is much higher than the lowest side lobe obtained in dynamic programming technique, out of 24 possible combinations only 6 combinations give result in this range. For $\theta_0 = 30^\circ$ the lowest side lobe level lies between 3.5 to 4 units of amplitude, which

2. SPACE TAPER APPROACH $\Delta x = .5\lambda$

NO. OF CASES CONSIDERED = 54

AMPLITUDE OF MAIN BEAM = 9

$\theta_0 = 90^0$, U min. = .1, U max. = 1.

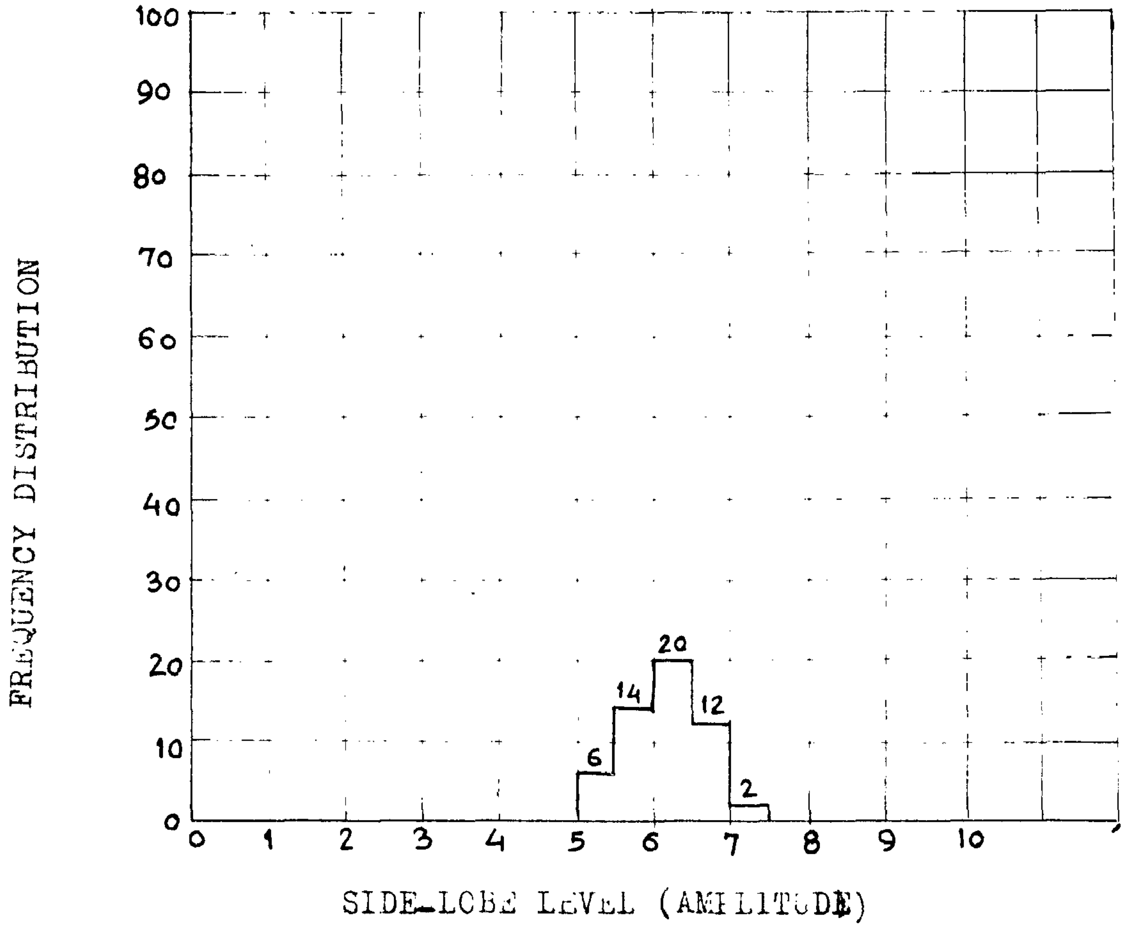


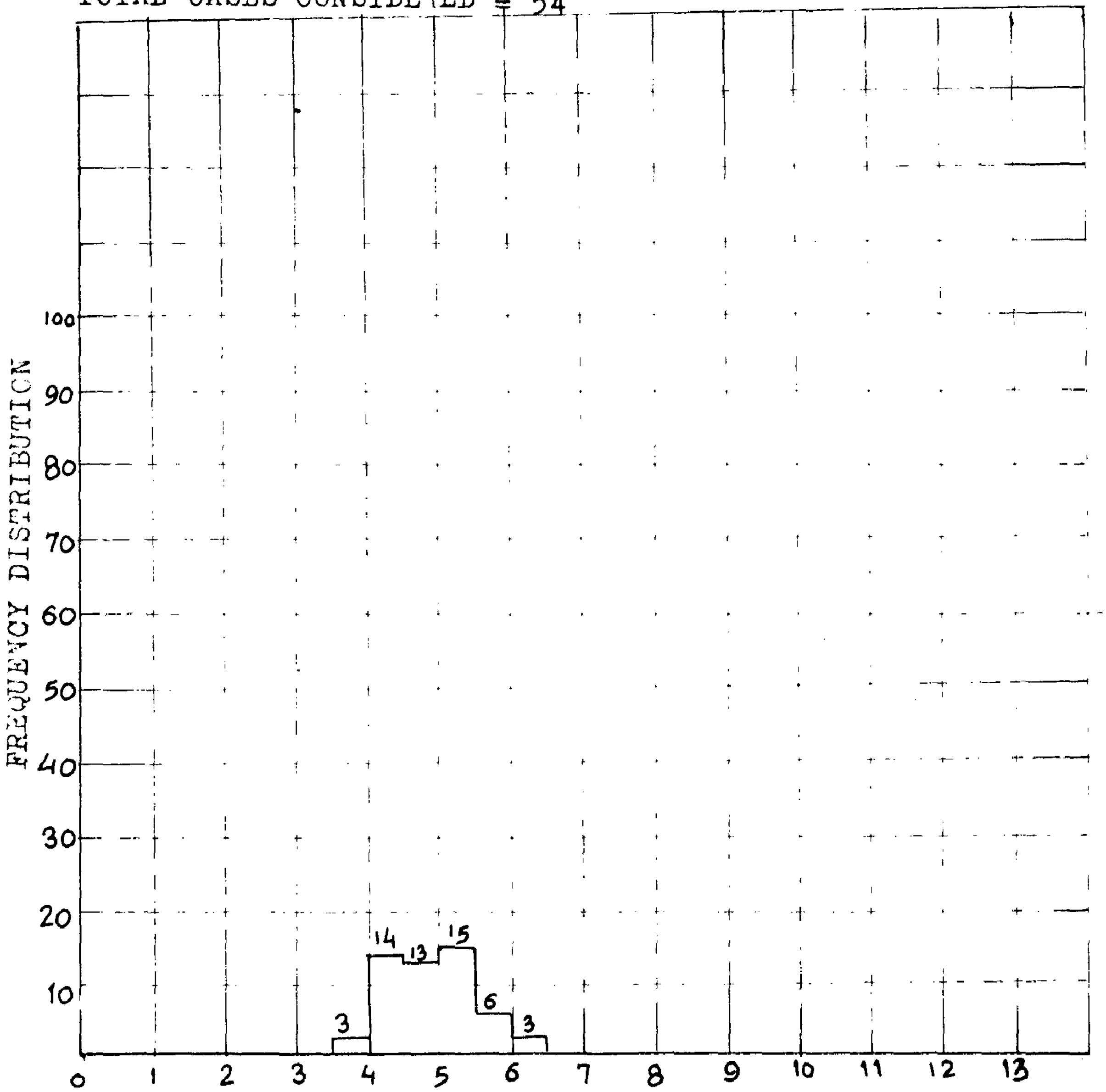
FIG.-9.

SPACE TAPER APPROACH

ANGLE OF SCANNING =30° U min. = .1, U max. = .50

Δx = .5 λ , ARRAY LENGTH = 19 λ

TOTAL CASES CONSIDERED = 54



SIDE_LOBE LEVEL(AMPLITUDE)

ELEMENT LOCATIONS FOR OPTIMUM CASE, x_1 = 3, x_2 = 9, x_3 =16, x_4 =19

FIG.-10.

is also higher than obtained previously, out of 54 possible combinations only 5 combinations give results in this range.

6.3 COMPUTERISED ALL LOW SIDE COMBINATIONS

The main aim of computing all possible combinations and then selecting the combination which gives lowest side lobe level is to check the results obtained by dynamic programming technique. The run time of the program written for this particular case is 5 hours. The program is given in Appendix 'D'. The frequency distribution of the side lobe level given by various combinations is given in Figure 11. The various input parameters taken are the same as used in previous two approaches. 199 combinations are considered out of which 4 combinations give side lobe level belonging to lowest side lobe level group. The element distribution of lowest side lobe level group is given in Figure 12. The lowest side lobe obtained by this method is the same as given by dynamic programming method.

The program written for this approach is a generalised form of program written for quasi Taylor approach. Each element like dynamic programming technique is given 16 positions and in optimisation process all the elements are present. This method confirms that

3. CONSIDERING ALL POSSIBLE COMBINATIONS, $\Delta x = .5\lambda$

NO. OF CASES CONSIDERED = 135

AMPLITUDE OF MAIN BEAM = 9 UNITS

$\theta_0 = 90^\circ$ U min. = -1, U max. = 1.

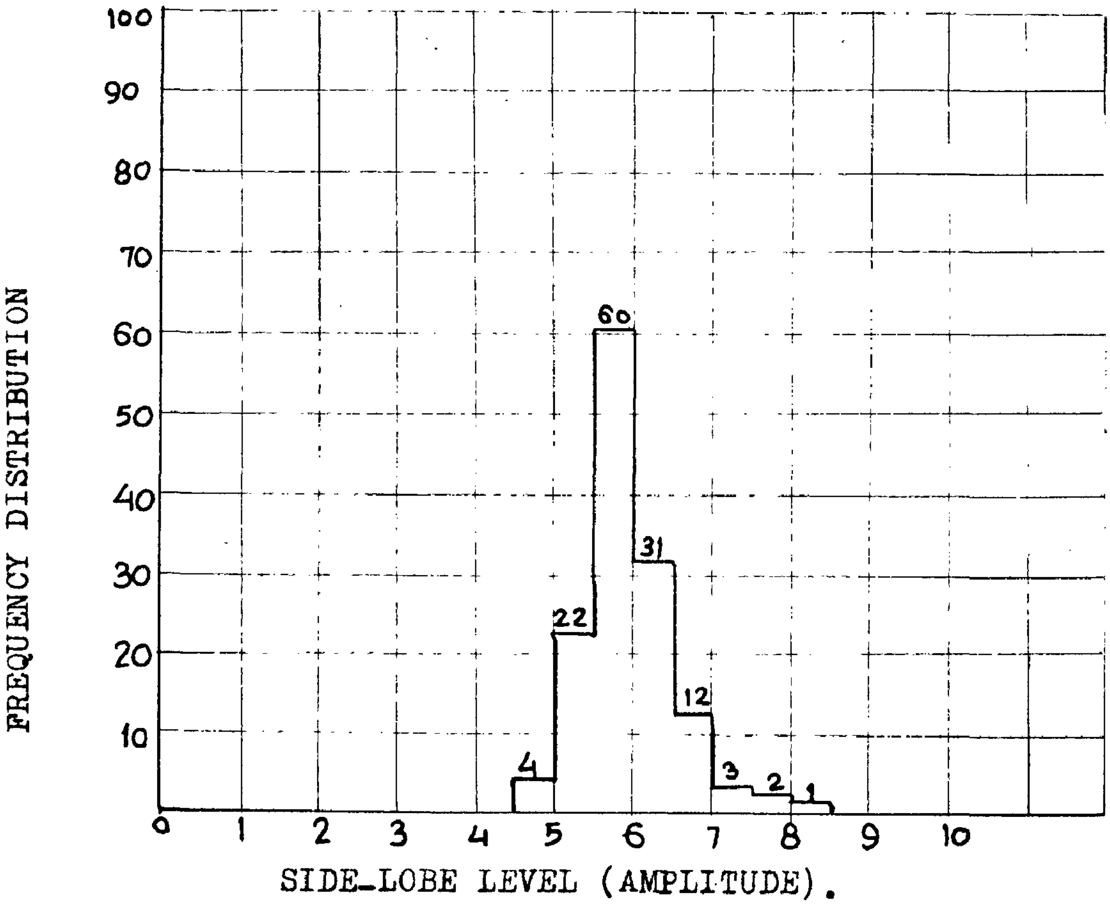
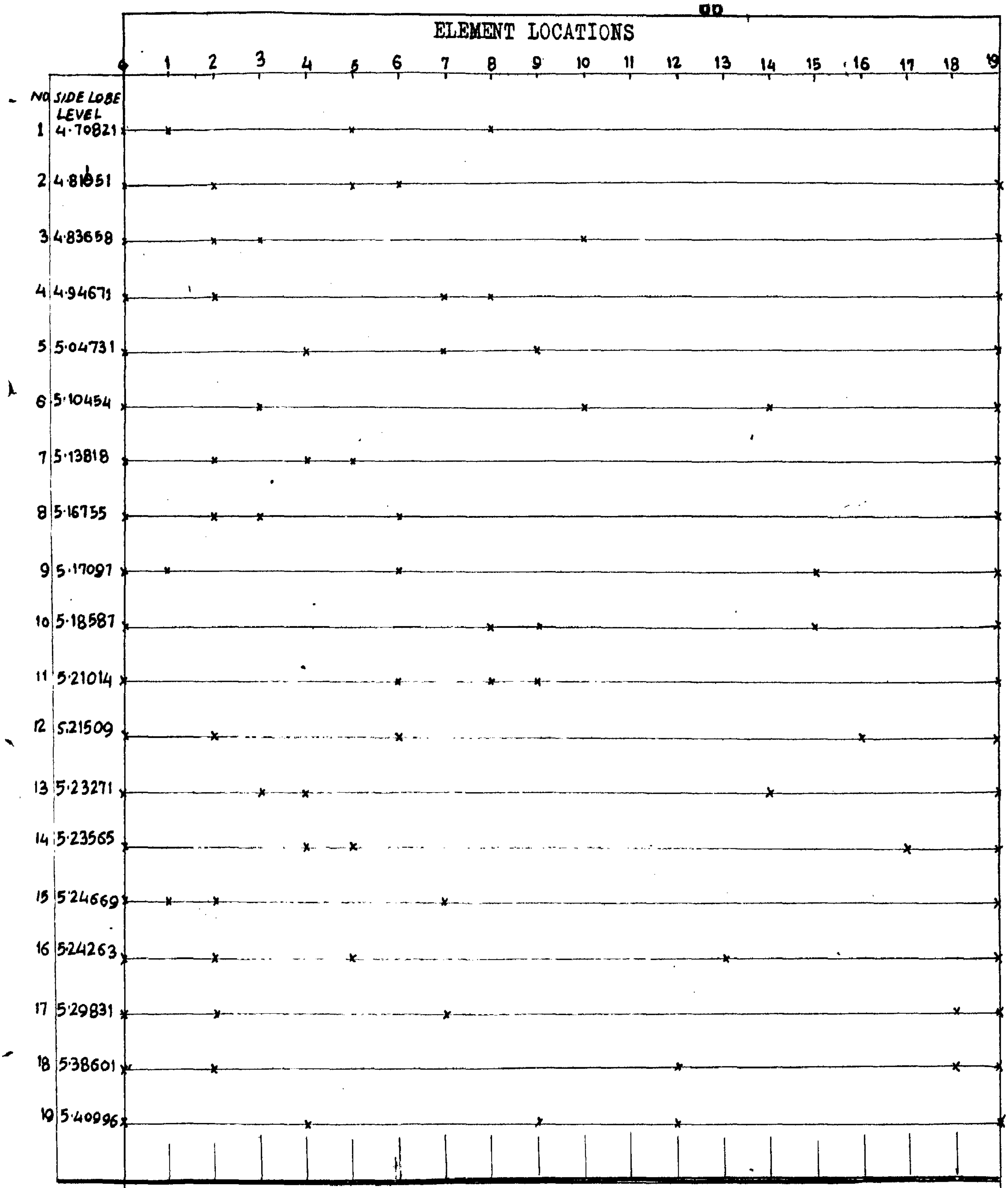


FIG.-11.



ELEMENT POSITIONS SYMMETRICAL WITH RESPECT TO X=0

SPACE QUANTIZATION $\Delta x = .5\lambda$ THE ELEMENT DISTRIBUTION OF THE LOWEST SIDE LOBE LEVEL GROUP. ARRAY APERTURE $=19\lambda$.

FIG.-12

For the case considered the optimum location of 1st & element with respect to the second element remains unchanged whether other elements are present or not, similarly in the case with second and third elements.

TABLE XI

SUMMARY OF PEAK SIDE LOBE LEVEL OBTAINED IN THE [illegible] ARRANGEMENT

θ_0°	Dynamic programming technique. db.	Space taper approach db.	Considering all possible combinations db.
30	-8.76	-7.58	-
60	-6.53	-	-
90	-5.64	-4.08	-5.64

The datas not obtained are shown by dashes.

TABLE III

COMPARISON OF VARIOUS SOLUTIONS FOR 9 ELEMENT ARRAY CASE

Dynamic programming technique as applied by Skolnik et.al (5)	Ishimaru and Chen's formula (4)	Statistical method of Lo (6)	Dynamic programming as applied here.
Peak side lobe level	Peak side lobe level	Peak side lobe level	Peak side lobe level
9.247*	9.220	4.6305 with $\lambda/4$ quantization	4.703*

* Spacing quantization used $\lambda/2$.

Main lobe level (Amp.) = 9 units.

CHAPTER VII

C O N C L U S I O N

From the results obtained one tends to conclude that the relation between the side lobe level and the element arrangement is so complex that it is very difficult to find a general analytical expression which can predict the side lobe level without much computation. Ishimaru and Chen[4] derived an analytical expression for predicting the side lobe level, for a given number of elements but the results obtained by them are not truely optimum. Since the lowest side lobe group consists of only a very small portion of all possible cases, one can hardly expect to obtain an optimum design without knowing the an optimization method. Dynamic programming technique is one of the possible optimization methods which is capable of yielding results better than previously computed. Results obtained by various design techniques are written in table III. This shows that only Lo's[6] statistical method yields slightly better results, but the element locations found out for the optimum case in their method are the same as computed by the dynamic programming technique, the only difference being that Lo used $\lambda/4$ quantization and while in dynamic programming technique $\lambda/2$ quantization is considered.

Most of the space tapered arrays donot yield very low side lobe level, the summary of various approaches considered is written in Table II. It shows that in the three cases considered dynamic programming gives better results than space taper approach.

To check that the optimum case considered by dynamic programming is truely an optimum, approach mentioned in 6.3 has been tried. The optimum case is in the two tables. The element distribution of the lowest side lobe level group obtained from this approach is given in figure 12. A study of it yield that as the optimum distribution is reached the tendency of various elements is to lie close to the center element. It has been observed that when the elements are equally placed near the center element the tendency of highest side lobe is to be near the main beam, the relation between location of highest side lobe and the element arrangement is highly non linear, which as discussed earlier cannot be explained by analytical means. It is only by computing all possible combinations that one can find the combination which gives the desired results for the case considered. It is seen that the dynamic programming technique gives an alternative method of getting this combination in much less time, as well as less computations. In this respect it seems to be an efficient technique. One may expect that this technique may yield the optimum design in other cases as well.

The effect of varying frequency on the radiation pattern can be visualised by considering a general angular coordinate $u = (f_0/f)(\sin\theta - \sin\theta_0)$ where f_0 is the design frequency and f is the new frequency of interest. As the frequency is increased, the value of u about which the pattern is symmetrical will be reduced and the net effect would be an increment in spacing quantization, in other words it would act as if the same number of elements are placed within an electrically larger aperture. On the other hand lowering of the frequency would make the pattern symmetrical about larger value of u and the net effect is reduction in spacing quantization. This is not desirable in practical arrays because of the increased coupling between the elements located in close proximity.

Array containing larger number of elements could not be synthesized by the help of IBM 1620, the limitation being its storage capacity and its slow speed of computations. However, this problem can be handled efficiently on computers having larger storage capacity and high speed of computation such

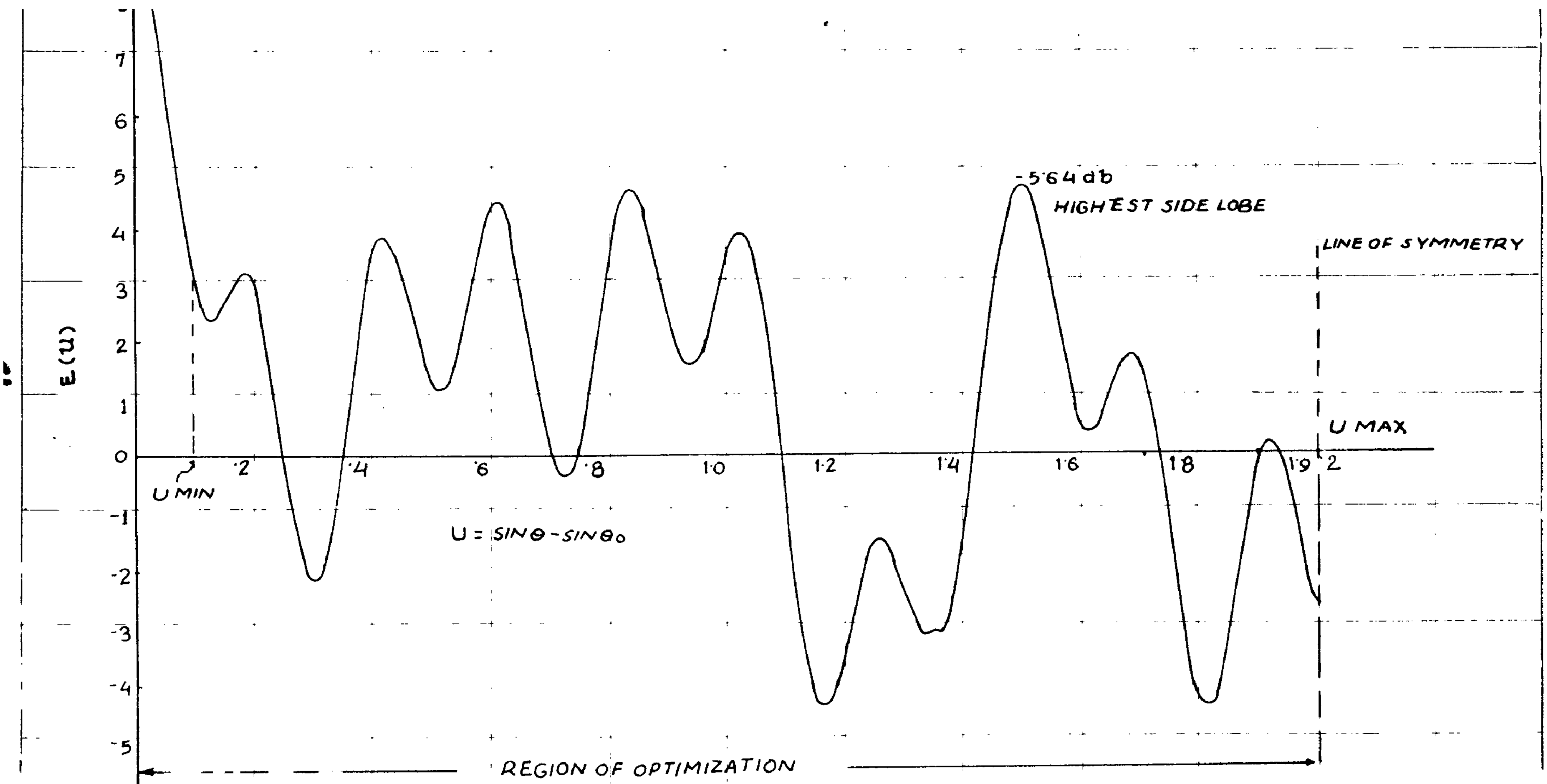


FIG.-13.

HIGHEST SIDE LOBE IS -5.64 db. DOWN THE MAIN BEAM,
ELEMENT LOCATIONS $x_1=1$, $x_2=5$, $x_3=8$, $x_4=19$

ARRAY LENGTH=19 λ , SPACING QUANTIZATION=.25 λ as IBM 7094 or CDC 3600, which are several times faster than IBM 1620. Arrays with $\lambda/4$ and $\lambda/8$ quantization could not be synthesized because of similar reasons. However, radiation patterns is drawn in Figure 19 for the element arrangement obtained in 6.1 computed for $\lambda/4$ quantization. There is no significant difference between this case and the one already considered by spacing quantization $= \lambda/8$ except that the pattern is now symmetrical about $u = 2$

A general program based on dynamic programming is written in Appendix 'C' which can be utilized for synthesizing symmetrical linear arrays having any odd number of elements, only thing required is alteration of dimension and data cards depending upon the problem.

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3. C.L. Dolph. " A current Distribution for Broad side Arrays which Optimizes the Relationship between Beam Width and Side Lobe Level" Proc. IRE 34, No. 6, pp. 335, 1946.

4. Ishimaru and Chen " Thinning and Broad Banding Antenna Arrays by Unequal spacings" pp. 34-42, IEEE Trans. on Ant. and Prop., Vol. AP-13 No. 1 , Jan. 65.

5. M.I. Skolnik et. al. " Dynamic Programming for Unequally Spaced Arrays" pp 35-49 , IEEE Trans. on Ant. Prop. Vol. 8, AP-12, No. 1, Jan. 1964.

6. Y.T. Lo and S.W. Lee " A Study Study of Space Taper Arrays" IEEE Transactions on Antenna and Propagation" Vol. AP-14 No. 1 pp 22-30, January 1966.

APPENDIX-A

```
      PROGRAM DYNAMIC PROGRAMMING TECHNIQUE
     H REFERS TO INCREMENT IN U
     THE VALUE OF HI IS ASSIGNED SUCH THAT K=1 GIVES UMIN.
     AAL REFERS TO LEMDA,LEMDA=1
     JLAST REFERS TO POSITION OF FIXED ANTENNA
     KLAST GIVES THE VALUE OF U ABOUT WHICH THE PATTERN IS SYMMETRICAL
     N CORRESPONDS TO THE POSITIONS ALLOTED TO X2
     NA CORRESPONDS TO THE POSITIONS ALLOTED TO X1
     I CORRESPONDS TO THE POSITIONS ALLOTED TO X3
     VARIATION OF U IS CONTROLLED BY K
     ABE(K)CORRESPONDS TO THE ABSOLUTE SUM
 C    PROGRAM ARRAY SYNTHESIS M.NATH Z
     DIMENSION TX1(19),ABE(100),L(19),A(19),AD(19),B(19),LA(19),AF(19)
     DIMENSION M(19),U(100),YY(19,100),KA(19),KC(19),AH(19),C(19)
     DIMENSION KB(19)
     READ 100,H,HI,AAL,JLAST,KLAST
 00  FORMAT(3F10.0,2I3)
                                      STAGE-1
     ILAST=JLAST-1
     PY=3.14159
     HAAL=AAL/2.
     TP=PY+PY
     TPH=TP*HAAL
      DO61J=1,JLAST
     AJ=J
 1   TX1(J)=TPH*AJ
     DO62K=1,KLAST
     AK=K
 2   U(K)=HI+H*AK
     DO63J=1,JLAST
     DO63K=1,KLAST
     O=COSF(U(K)*TX1(J))
 3   YY(J,K)=O+O
                                      STAGE-2
     DO1N=2,ILAST
     N1=N-1
     DO2NA=1,N1
     DO3K=1,KLAST
     AK=K
     ABE(K)=ABSF(1.+YY(NA,K)+YY(N,K)+YY(JLAST,K))
     AB=ABE(1)
     MA=1
     DO4K=2,KLAST
     IF(AB-ABE(K))5,5,6
     T=AB
     AB=ABE(K)
     ABE(K)=T
      L(NA)=K
     MA=MA+1
     A(NA)=AB
     IF(MA-1)7,7,4
     L(NA)=1
```

```
4       CONTINUE
2       CONTINUE
        PUNCH 302,N,(L(NA),NA=1,N1)
302      FORMAT(2HN=I2,(19I4))
        AC=A(1)
        MB=1
        IF(N1-1)11,11,42
42      DO8NA=2,N1
        IF(AC-A(NA))9,9,10
10      T=AC
        AC=A(NA)
        A(NA)=T
        M(N)=NA
        MB=MB+1
9       AD(N)=AC
        IF(MB-1)11,11,8
11      M(N)=1
8       CONTINUE
         GO TO1
41      AD(N)=A(1)
        M(N)=1
1       PUNCH 200,M(N),N,AD(N)
200      FORMAT(2I10,F10.5)
C                                                  STAGE-3,4
        DO12I=3,ILAST
         AI=1
        I1=I-1
        DO13N=2,I1
        MN=M(N)
        DO14K=1,KLAST
14      ABE(K)=ABSF(1.+YY(MN,K)+YY(N,K)+YY(I,K)+YY(JLAST,K))
        AE=ABE(1)
         MC=1
        DO13K=2,KLAST
        IF(AE-ABE(K))15,15,16
15      T=AE
        AE=ABE(K)
        ABE(K)=T
        MC=MC+1
        LA(N)=K
16      B(N)=AE
        IF(MC-1)17,17,13
17      LA(N)=1
13      CONTINUE
        PUNCH 303,I,(LA(N),N=2,I1)
303      FORMAT(2HI=I2,(19I4))
        AE=B(2)
        MD=1
        IF(I1-1)51,51,52
52      DO18N=3,I1
        IF(I1-2)21,21,44
44       IF(AE-B(N))19,19,20
20      T=AE
        AE=B(N)
        B(N)=T
```

```
      KA(I)=N
      MD=MD+1
 19   AF(I)=AE
      IF(MD-1)21,21,18
 21   KA(I)=2
      AF(I)=B(2)
 18   CONTINUE
      GO TO 53
 51   AF(N)=B(2)
      KA(I)=2
 53   KAI=KA(I)
 12   PUNCH 201,M(KAI),KA(I),I,AF(I)
201   FORMAT(3I10,F10.5)
      DO22J=JLAST,JLAST
      AJ=J
      J1=J-1
      DO24I=3,J1
      KAI=KA(I)
      MKAI=M(KAI)
      DO23K=1,KLAST
      AK=K
23     ABE(K)=ABSF(1.+YY(MKAI,K)+YY(KAI,K)+YY(I,K)+YY(J,K))
       MA=1
      AB=ABE(1)
      DO24K=2,KLAST
      IF(AB-ABE(K))25,25,26
 25   T=AB
      AB=ABE(K)
      ABE(K)=T
      KB(I)=K
      MA=MA+1
 26   C(I)=AB
      IF(MA-1)27,27,24
 27   KB(I)=1
 24   CONTINUE
      AG=C(3)
      MB=1
      PUNCH304,J,(KB(I),I=3,J1)
304   FORMAT(2HJ=I2,(19I4))
      DO28I=4,J1
      IF(J1-3)31,31,45
 45   IF(AG-C(I))29,29,30
 30   T=AG
      AG=C(I)
      C(I)=T
      KC(J)=I
      KCJ=KC(J)
      KAKCJ=KA(KCJ)
       MB=MB+1
 29    AH(J)=AG
      IF(MB-1)31,31,28
 31   KC(J)=3
      AH(J)=C(3)
 28   CONTINUE
      KCJ=KC(J)
```

```
      KAKCJ=KA(KCJ)
22    PUNCH204,M(KAKCJ),KA(KCJ),KC(J),J,AH(J)
204   FORMAT(4I10,F10.5)
      STOP
      END
```

```
       .01        .09        1. 19100
```

APPENDIX-B

```
C       PROGRAM FOR CHECKING SIDE-LOBES NEAR MAIN LOBE
C   C   PROGRAM FOR PUNCHING ABE(K),WHEN L(NA)=1Z
        DIMENSION TX1(19),ABE(91),L(19),A(19),AD(19),B(19),LA(19),AF(19)
        DIMENSION M(19),U(91),YY(19,91),KA(19),KC(19),AH(19),C(19),KB(19)
        DIMENSION UU(91),YYY(19,91),DE(91)
        READ 100,H,HI,AAL,JLAST,KLAST,KL
100     FORMAT(3F10.0,3I3)
        ILAST=JLAST-1
        PY=3.14159
        HAAL=AAL/2.
        TP=PY+PY
        TPH=TP*HAAL
        DO61J=1,JLAST
        AJ=J
61      TX1(J)=TPH*AJ
        DO62K=1,KLAST
        AK=K
62      U(K)=HI+H*AK
        DO63J=1,JLAST
        DO63K=1,KLAST
        O=COSF(U(K)*TX1(J))
63      YY(J,K)=O+O
        DO1N=2,ILAST
        N1=N-1
        DO2NA=1,N1
        DO3K=1,KLAST
        AK=K
3       ABE(K)=ABSF(1.+YY(NA,K)+YY(N,K)+YY(JLAST,K))
        AB=ABE(1)
        MA=1
        DO4K=2,KLAST
        IF(AB-ABE(K))5,5,6
5       T=AB
        AB=ABE(K)
        ABE(K)=T
        MA=MA+1
        L(NA)=K
6       A(NA)=AB
        IF(MA-1)7,7,4
7       L(NA)=1
4       CONTINUE
        IF(L(NA)-1)2,71,2
71      DO72K=1,KL
        AK=K-1
72      UU(K)=H*AK
```

```
      DO73J=1,JLAST
      DO73K=1,KL
      O=COSF(UU(K)*TX1(J))
73    YYY(J,K)=O+O
      DO76K=1,KL
76    BE(K)=ABSF(1.+YYY(NA,K)+YYY(N,K)+YYY(JLAST,K))
      PUNCH 305,NA,(BE(K),K=1,KL)
2     CONTINUE
20    PUNCH 302,N,(L(NA),NA=1,N1)
1     CONTINUE
302   FORMAT(2HN=I2,(19I4))
305   FORMAT(I2,1X,(11F7.4))
      STOP
      END

        .01          .09          1. 19 91 91
```

APPENDIX-C

```
C      PROGRAM FOR RADIATION PATTERN
C   C  PROGRAM ARRAY SYNTHESIS M.NATH  2
       DIMENSION TX(19),ABE(200),U(200),YY(19,200)
       READ 100,H,HI,AAL,JLAST,KLAST
       PY=3.14159
       HAAL=AAL/2.
       TP=PY+PY
       TPH=TP*HAAL
       DO62K=1,KLAST  $  AK=K-1
 62    U(K)=H*AK
 1     READ 101,J
       AJ=J
        TX(J)=TPH*AJ
       DO2K=1,KLAST
       O=COSF(U(K)*TX(J))
 2     YY(J,K)=O+O
        IF(J-19)1,30,30
30      DO3K=1,KLAST
3      ABE(K)=1.+YY(1,K)+YY(5,K)+YY(8,K)+YY(19,K)
         PUNCH 200,(ABE(K),K=1,KLAST)
 100    FORMAT(3F10.0,2I3)
 101      FORMAT(I2)
200    FORMAT(10F8.4)
       STOP
        END
```

APPENDIX-D
AND
APPENDIX-E

```
 * CARDS TO BE USED IN PLACE OF + CARDS FOR APPENDIX E
C     PROGRAM SPACE TAPER APPROACH
      PROGRAM ALL POSSIBLE COMBINATIONS  *
C  C  PROGRAM ARRAY SYNTHESIS M.NATH   Z
      DIMENSION TX1(19),ABE(91),L(19),A(19),AD(19),M(19),AF(19),KA(19)
      DIMENSION U(91),YY(19,91),KE(19),AE(19)
      READ 100,H,HI,AAL,JLAST,KLAST
100   FORMAT(3F10.0,2I3)
      ILAST=JLAST-1
      PY=3.14159
      HAAL=AAL/2.
      TP=PY+PY
      TPH=TP*HAAL
      DO61J=1,JLAST
      AJ=J
61    TX1(J)=TPH*AJ
      DO62K=1,KLAST
      AK=K
62    U(K)=HI+H*AK
      DO63J=1,JLAST
      DO63K=1,KLAST
      O=COSF(U(K)*TX1(J))
63    YY(J,K)=O+O
      DO1N=10,ILAST +
      DO1N=3,ILAST        *
      DO2NA=4,9 +
      N1=N-1        *
      DO2NA=2,N1        *
      DO3I=1,3 +
      NA1=NA-1        *
      DO3I=1,NA1        *
      DO4K=1,KLAST
      AK=K
4     ABE(K)=ABSF(1.+YY(I,K)+YY(NA,K)+YY(N,K)+YY(JLAST,K))
      AB=ABE(1)
      MA=1
      DO3K=2,KLAST
      IF(AB-ABE(K))6,6,7
6     T=AB
```

```
      AB=ABE(K)
      ABE(K)=T
      L(I)=K
      MA=MA+1
7     A(I)=AB
      IF(MA-1)8,8,3
8     L(I)=1
3     CONTINUE
      AC=A(1)
      MB=1
10    DO11I==2,3 +
10    DO11I=1,NA1         *
      IF(AC-A(I))12,12,13
13    T=AC
      AC=A(I)
      A(I)=T
      M(NA)=I
      MB=MB+1
12    AD(NA)=AC
      IF(MB-1)9,9,11
9     M(NA)=1
11    CONTINUE
2     PUNCH 200,M(NA),NA,N,AD(NA)
200   FORMAT(3I10,F10.5)
      AC=AD(2)
      MD=1
      DO18NA=5,9 +
      DO18NA=3,N1         *
      IF(AC-AD(NA))15,15,16
16    T=AC
      AC=AD(NA)
      AD(NA)=T
      KA(N)=NA
      MD=MD+1
15    AF(N)=AC
      IF(MD-1)17,17,18
17    KA(N)=4 +
17    KA(N)=2         *
18    CONTINUE
      KAN=KA(N)
      MKAN=M(KAN)
1     PUNCH 204,M(KAN),KA(N),N,AF(N)
204   FORMAT(3I10,F10.5)
      AG=AF(10) +
      AG=AF(3)        *
      ME=1
      DO19N=11,18 +
      DO19N=4,ILAST         *
      IF(AG-AF(N))20,20,21
21    T=AG
      AG=AF(N)
      AF(N)=T
      KC=N
      ME=ME+1
20    AE(N)=AG
      IF(ME-1)22,22,19
22    KC=10 +
22    KC=3          *
19    AEN=AE(N)
      KAKC=KA(KC)
      PUNCH204,M(KAKC),KA(KC),KC,AEN
      STOP
      END
```

APPENDIX-P

```
      PROGRAM DYNAMIC PROGRAMMING GENERALIZED FORM
C  C  ARRAY SYNTHESIS-GENERAL CASE      Z  20/7/66
      DIMENSION U(200),C(400),LA(12),ABE(200),L(25),M(12,200),A(25)
      COMMON PY,HI,H,P,TPY
C     SENSE SWITCH 1 ON WILL PUNCH OPTIMUM VALUES OF K FOR EACH VALUE OF
C     NA AND N
C     SENSE SWITCH 1 OFF WILL AVOID THE ABOVE PUNCHING
C     SENSE SWITCH 2 ON WILL PUNCH OPTIMUM LOCATIONS OF X1,X2,X3...ETC.
C     FOR EACH VALUE OF N AND FOR EACH OF TWO ANTENNA LOCATIONS
C     SENSE SWITCH 2 OFF WILL PUNCH OPTIMUM LOCATIONS OF X1,X2,X3...ETC.
C     FOR EACH VALUE OF N WHEN LAST TWO ANTENNA LOCATIONS ARE BEING COMP
C     LANT=NO. OF ANTENNAS
C     JLAST=NO. OF ANTENNA LOCATIONS
C     LAST=NO. OF TIMES THE COMPARRISON IS REQUIRED
      READ100,H,HI,AAL,JLAST,KLAST,LANT
100    FORMAT(3F10.0,3I3)
      N2=2
      N3=1
C     LL=1,N2=2 AND N3=1 REFERS TO COMPARISION OF X2 AND X1
C     LL=2,N2=3AND N3=2 REFERS TO COMPARISON OF X3AND X2
C     LL=3,N2=4 AND N3=3 REFERS TO COMPARISON OF X4 AND X 3 AND SO ON
      N4=JLAST-LANT+2
      LAST=LANT-1
      PY=3.14159265
      TPY=PY+PY
      READ101,M1,M2,M3,AM
 101  FORMAT(3I3,F11.0)
      P=PY/AM
      DO31I=M1,M2,M3
      AI=I
      D=COSF(AI*P)
  31  C(I)=D+D
      LL=1
      PUNCH202,N2,N3, LL
150   FORMAT(3I4)
72    IF(SENSE SWITCH 4)12,13
12    PRINT 150,N2,N3,LL
13    M1=N2
      M2=N4
75    DO1N=M1,M2
      GOTO74
73    M1=JLAST
      M2=M1
      PRINT150,N2,N3,LL
      GOTO75
 74   N1=N-1
      LL1=LL-1
       DO2NA=N3,N1
C     LL1= NO. OF TERMS TO BE ADDED
      IF(LL1) 70,70,69
69    DO 64 I=1,LL1
      IF(I-1)67,67,68
 67   NB=NA
      GOTO 69
```

```
 68     ND=LA(I-1)
 69     LX=LL-1
 64     LA(I)=M(LX,ND)
 70     DO3K=1,KLAST
        TERM=0.
        IF(LL1)87,87,88
 88     DO66I=1,LL1
        LD=LA(I)
        CALLO(LD,K,IA)
 66     TERM=TERM+C(IA)
        IF(LL-LAST)87,71,87
 71     CALLO(NA,K,IB)
        CALLO(N,K,IC)
 710    SUM=1.+C(IB)+C(IC)+TERM
        GOTO3
 87     CALLO(NA,K,ID)
        CALLO(N,K,IE)
        CALLO(JLAST,K,IG)
 870    SUM=1.+C(ID)+C(IE)+C(IG)+TERM
  3     ABE(K)=ABSF(SUM)
        AB=ABE(1)
        MA=1
        DO2K=2,KLAST
        IF(AB-ABE(K))5,5,6
  5     T=AB
        AB=ABE(K)
        ABE(K)=T
C       L(NA) REFERS TO THE OPTIMUM POSITION OF K FOR EACH VALUE OF NA AND N
        L(NA)=K
        MA=MA+1
  6     A(NA)=AB
        IF(MA-1)7,7,2
  7     L(NA)=1
  2     CONTINUE
        AC=A(N2)
        NB=1
        IF(N1-N2)41,42,42
 42     DO8NA=N2,N1
        IF(AC-A(NA))9,9,10
 10     T=AC
        AC=A(NA)
        A(NA)=T
        M(LL,N)=NA
        NB=NB+1
  9     AMP=AC
C       M(1,N) IS THE OPTIMUM LOCATION OF X1 FOR A LOCATION OF X2 AT N
C       M(2,N) IS THE OPTIMUM LOCATION OF X2 FOR A LOCATION OF X3 AT N
C       M(3,N) IS THE OPTIMUM LOCATION OF X3 FOR A LOCATION OF X4 AT N AND SO
        IF(NB-1)11,11,8
 11     M(LL,N)=NB
        AMP=A(NB)
  8     CONTINUE
        GO TO 43
 41     M(LL,N)=N2
        AMP=A(N2)
```

```
43     IF(SENSE  SWITCH 1)77,78
 77     PUNCH 200,N,(L(NA),NA=N3,N1)
 78     IF(SENSE SWITCH 2)79,80
80     IF(LL-LAST) 1,79,86
  79    LA(1)=M(LL,N)
C      LA(I),(I=1,LL) REFERS TO THE OPTIMUM LOCATIONS OF X(LL-1),X(LL-2),
C      ........X2,X1 FOR EACH LOCATION OF N
       IF(LL1)81,81,82
 82    DO83I=2,LL
       NB=LA(I-1)
       LX=LL-I+1
  83    LA(I)=M(LX,NB)
  81    PUNCH 201,N,AMP,(LA(I),I=1,LL)
 1     CONTINUE
       N2=N2+1
       N3=N3+1
       LL=LL+1
       N4=N4+1
       IF(SENSE SWITCH 3)85,86
85     PUNCH 202,N2,N3,LL
86     IF(LL-LAST)72,73,84
  84    STOP
 200    FORMAT(2HN=I2,(19I4))
201    FORMAT(2HN=I2,1X4HAMP=F10.6,1X,(15I4))
202    FORMAT(/3HN2=I2,4X3HN3=I2,4X3HLL=I2)
        END
       SUBROUTINE Q(J,K,I)
       COMMON PY,HI,H,P,TPY
       AJ=J
       AK=K
       ANG=AJ*(HI+H*AK)
 23    IF(ANG-2.)21,21,22
22     ANG=ANG-2.
       GOTO23
21     S=ANG*100.
       I=S
       R=I
       IF(S-R-.5)24,24,25
25     I=I+1
24     RETURN
       END
.01       .1        1.        019091004
001200001100.
```