UNIVERSITY OF ROORKEE
ROORKEE

Certified that the attached Dissertation on

"Approach to Elasto-Plastic Analysis and Design of Multi Storeyed Frames of Steel"

was submitted by

Sri H. S. [illegible]

and accepted for the award of Master of Engineering Degree in

Structural Engineering Including Concrete Technology

Vide this office Notification NO. Ex/66/P-65/ [illegible]

Dated Nov. 23, 1961.

[illegible signature]

Assistant Registrar
(Examinations)

26/11/61

Dated: Roorkee Nov. 23, 1961

[illegible signature]

ELASTO-PLASTIC
APPROACH TO ANALYSIS AND DESIGN OF MULTISTOREYED FRAME OF STEEL

BY

ARDAMAN SINGH MADAN

THESIS

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF ENGINEERING

(Structural Engineering including Concrete Technology)

DEPARTMENT OF CIVIL ENGINEERING
UNIVERSITY OF ROORKEE
ROORKEE, INDIA
1961

C E R T I F I C A T E

Certified that the dissertation entitled "Elasto-Plastic Approach to Analysis and Design of Multistoreyed Structures" which is being submitted by Sri Ardaman Singh Madan in partial fulfilment for the award of the Degree of Master of Engineering in Structural Engineering including Concrete Technology of University of Roorkee is a record of student's own work carried out by him under our supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other Degree or Diploma.

This is further to certify that he has worked for a period of 11 months from November 1, 1960 to September 30, 1961 for preparing dissertation for Master of Engineering Degree at the University.

(O.P.JAIN)
ASSOCIATE PROFESSOR
CIVIL ENGINEERING
University of Roorkee
Roorkee

(JAI KRISHNA)
PROFESSOR
CIVIL ENGINEERING
University of Roorkee
Roorkee

Dated 6/10/61

ACKNOWLEDGEMENTS

It is indeed, a pleasant duty for the author to express his thanks to Dr. Jai Krishna, who has been a great source of inspiration and guidance for the author in carrying out the work. He has also been very kind to go through the manuscript and give valuable suggestions.

The author also gratefully acknowledges the immense encouragement received from Dr. O.P.Jain under whose guidance the work has been carried out. His keen interest and valuable suggestions at all stages of work has been a great help to the author.

Thanks are also due to Sri S.H.Moorjani for his help in carrying out the experimental work.

Indebtedness to the council of Scientific and Industrial Research, New Delhi is gratefully acknowledged for enabling the author to under-take this work.

" C O N T E N T S "

		Page
INTRODUCTION	... ...	1
CHAPTER I	Elastic Methods of Analysis	7
CHAPTER II	Plastic Methods of Analysis	24
CHAPTER III	Extent of Redistribution of Moments	50
CHAPTER IV	Instrumentation	102
CHAPTER V	Testing	126
CHAPTER VI	Discussion of test Results and Conclusions	153
APPENDIX A	Analysis of the Frame by Slope Deflection Equations.	157
APPENDIX B	Analysis of the Frame by Kani's Iteration method	166
APPENDIX C	Analysis of the Frame for Collapse load by Mechanism Method ...	172
APPENDIX D	Plastic Analysis of the Frame by Plastic Moment Distribution. ...	186
BIBLIOGRAPHY	... ...	191

INTRODUCTION

The problem of an acute shortage of space due to the need of more commercial houses, greater influx of population into the big cities, undesirability of diverting agricultural lands for habitation and the need for greater open space and greenery, has found its solution in vertical expansion of the cities by building up a series of multistoreyed buildings both for commercial as well as residential purposes. Besides, tall buildings have also been advocated due to variety of other reasons such as ; difficulty of shifting industries to new cities; the demand for rehabilit-ating the slum dwellers in the same area, and the revulsion of the urban dewellers to being uprooted and transfered to rural or newly developed areas.

In designing tall rigid frames the main problem is to find a method which is sufficiently accurate and at the same time avoids heavy design costs. The difficulty is increased by the fact that the designer must steer a course between speed of design on the one hand and loss of accuracy on the other. The best method of design is one which combines in it all the three basic requirements, that is, it should be ;

(i) a true representative of the actual behaviour of the structure.

(ii) rapid and convenient in its application.

(iii) most economical.

Broadly speaking there exist two distinctly different methods of design and analysis of rigid frames they are :

(a) Those based on maximum allowable safe stresses in the
8 various members of the structure, such stresses being within the elastic range.

(b) Those based on the ultimate load carrying capacity of the structure.

The two categories differ in their basic approach to the problem. The former i.e. Elastic methods, is older and more conservative. It is based on the assumption that stable equilibrium exists between internal and external forces and within elastic limit any change in loading conditions does not produce disproportionate increase of stress or elastic distortion of frame. The design is based on the analysis of stresses in members, and providing a section of adequate strength to resist the forces. This method assumes that if local collapse of members can be prevented the structure as a whole is safe.

The second type which has come to vogue recently finds its principal application at present to the design of statically indeterminate or redundant framed structures of mild steel which carry load by virtue of the resistance of their members to bending action. Theoretically a structure may fail in a number of ways. It may reach its limit of usefulness through instability, fatigue, or excessive deflections. Alternatively, if none of these modes of failure occurs the structure will continue to carry load beyond the elastic limit until it reaches its ultimate load through plastic deformations and then collapses. Most of the indeterminate structural steel building frames and similar structures fall into this category. Plastic analysis provides a rational method for basing the design on this most typical mode of failure. In other words it bases the design on the maximum load that the structure can support.

Elastic analysis of all but the simplest of structures is complicated. There are so many secondary effects that deserve attention while working within the elastic range. Simplifying assumptions must be made to the point where the relation between a practical solution and a rigorous Elastic Theory is quite obscure. But it has been shown that at ultimate load the deformations of the structure become so large that the deformations due to the secondary effects are comparatively negligible and hence simplifying assumptions are not necessary.

It would be seen that the two theories look upon a structure in a different way even though the resultant design may be more or less the same. Structures designed on ultimate load theory however take the advantage of the ductility of steel so that when the stresses are higher than the elastic limit the structure can still carry more load because of the two facts, that is,

1) Distribution of stresses at a section, from the highest stressed fibres to the understressed fibres.
2) Redistribution of moments due to the formation of plastic hinges.

OBJECT OF THE PRESENT STUDY

Although considerable work has been done both theoretically and experimentally on the elstic as well as plastic methods of analysis, yet most of it is either only in elastic range in case of multistoreyed structures, or only for smaller structures in the Plastic range of loading. Very little is known about the actual behaviour of a multistoreyed structure near its collapse and hardly any experimental data is available. It was, therefore, proposed to investigate experimentally by conducting a loading test on the model of multistoreyed multibay frame to determine :

1) whether the frame can safely take theload as calculated by simplified Plastic theory.

2) Howfar the concepts of plastic hinges, and the mode of collapse (by the formation of mechanisms) arrived at at theoretically are exhibited actually in the structure.

3) If the required redistribution of moments as assumed in ultimate load theory is actually available or not.

and 4) To study the behaviour of structure by recording its deflections during both elastic and plastic range of loading and to compare them with those calculated theoretically.

With the above objects in view a critical study of various methods in use at present for analysing a multistoreyed structure according to elastic and or plastic theories was undertaken and a brief review of these methods is given in chapters I and II respectively. In Chapter III theoretical investigations into the extent of moment redistribution in a multistoreyed structure has been dealt with. This forms the theoretical part of the thesis.

For experimental investigations, two 6 storeyed - 3 bay frames were separated from mild steel plate, and complete arrangement for loading the test frame was designed and fabricated. The frame was loaded with both vertical and horrizontal loads simultaneously . All the details regarding the test frame and the loading frame along with the loading devices are described in Chapter IV. For the purpose of comparing the experimental results with the theoretical results it was necessary to find the actual stress-strain and bending moment - curvature relationships for the steel used in making the test frame. Tension and bending tests were, therefore, undertaken and are described at length in Chapter V, which also includes all the other test results.

Discussion of the results, conclusion drawn on the basis of this study and suggestions for future study on the subject are given in chapter VI.

Besides these 6 Chapters there are four appendices attached at the end in which is presented the computational work for analysing the test frame according to various methods viz;

1) Slope Deflection Equations]
2) Kani's iteration method] (For elastic analysis)
3) Neal's method of combining the mechanisms.] (For Plastic
4) Horne's plastic moment distribution.] analysis)

It was found after experiments that the actual ultimate load carrying capacity of the frame (provided sufficient lateral stability is provided) is much higher (may be as much as 1.25 to 1.5 times) than that perdicted by the simple plastic theory based on idealised stress-strain curve, and hence its use for multi-storeyed structures is over safe. It is therefore expected that more economy in steel can be affected if the working loads are specified after actual loading tests instead of basing them on idealised plastic theory. However only two frames were tested for present study, it is therefore suggested that more number of frames may be tested under different conditions of loading so as to be sure of the reserved strength of steel. The testing equipment as designed and fabricated for this study proved very satisfactory and it can be recommended that with slight perfection to test bigger size frames and in large number , the equipment may prove very useful for finding out ultimated load carrying capacity of any multistoreyed structure by testing its geometrically similar model frame of steel.

CHAPTER I

"ELASTIC METHODS OF ANALYSIS"

1.1. Elastic methods of analysing multistoreyed frames, based on Hooke's law of linear relationship between stress and strain, can be dated for their origin as far back as the end of the nineteenth century. Since then many methods have been developed by different persons from time to time. Some of them are claimed to be exact but are unfortunately time consuming. Others are quite rapid and easy but approximate only. Third type which is more commonly used consists of "Relaxation or Iteration methods,". These methods lead to fairly accurate results and yet do not involve the tedious calculations required for exact methods. In this chapter a brief review of various methods commonly adopted in practice, is given. Effort has also been made to comment upon their use and limitations. The methods discussed in following pages are :-

1) Slope Deflection Equations.

2) Relaxation and Iteration methods which include
   (i) Moment Distribution method.
   (ii) Distribution of Deformation method.
   (iii) Kani's iteration method.

3) Approximate methods like

(i) Portal Method.

(ii) Cantilever, method.

(iii) Sutherland Bowman's method.

(iv) Factor method.

1.3. METHOD USED OF SLOPE DEFLECTION EQUATIONS.

In 1880 Manderla published a method of computing the secondary bending moments induced in truss members whose joints were rigid rather than pinned. This work was exact in the sense that the effect of the axial forces in the members on these bending moments was taken into account but the resulting equations were too complex to be used for design purposes. As simplified by Winkler and again by Mohr, they have been extensively used in secondary stress calculations but their application in rigid frame analysis seems to be due to Wilson and Maney. In a well known paper published in 1915, these authors showed how to analyse the effect of lateral loads on buildings of the sky-scraper type and in so doing gave designers, for the first time a convenient means of dealing with welded steel and reinforced concrete structures. Allowing for the assumption made in general for Elastic Theory, this method is quite exact, being based on fundamental principles of statics.

The method by itself is very simple to be applied, but it is the solution of simultaneous equations to evaluate unknown rotations and displacements of the joints that makes this method very tedious for multistoreyed buildings. Although this method is now obsolete for the analysis of multistoreyed frames, but it has still a great importance because most of the recent methods are in fact only an attempt to solve these equations by successive approximations. All later developments take this method as the standard for comparing their accuracy. Southwell's relaxation methods of solution of simultaneous equations has proved to be very advantageous especi lly when computing machines are available.

1.2. [illegible] OF ITERATION METHODS :-

As already stated in above paragraph these methods were developed only as a result of efforts made in solving the slope deflection equation. Various methods are discussed below:-

MOMENT DISTRIBUTION METHOD.

Of all the iteration methods put forward, perhaps no other method is so commonly used as the one due to HardiCross. The principles on which this method is based, and the procedure have been omitted here, as they are so well known and understood,

Moment distribution method when first presented in 1932, by Hardy Cross, was essentially meant only for the frames without sway. It has, however, been extended to the multistoreyed frames because of following reasons :-

1. The frames can be analysed for horizontal loads separately and results superimposed.
2. For vertical loads (symmetrical) only, the symmetrical frames do not undergo any translatory displacement.
3. Even the deviation from (2) in practical cases does not cause any appreciable joint displacements so as to affect the results sufficiently.
4. The work could be considerably shortened at the cost of some accuracy, by carrying out analysis floorwise, as the vertical carryover of moments is quite small.
5. No other method, so simple and rapid, was available.

The method was constantly criticised because of its inability to approach the sidesway effects directly. Hardy cross suggested an indirect method which consists of an independent distribution of moments corresponding to each degree of freedom for translation, with inter-relationships established by the solution of a number of simultaneous equations equal to the number of the degrees of freedom.

"Grinter" has suggested many improvements in this cumbersome method from time to time. These include :-

1) A sequence of partial side sways with joints

restrained against rotation which converge to the actual sidesway and form a series of corrections to the process of moment distribution.

2) A reversed technique by which the computed or estimated moments of the joint that accompany side lurch are used to determine the fixed end moments for distribution. This method has however been effective in solving the problem of wind stress analysis for tall buildings.

3) Cantilever moment distribution:-

This is the latest improvement put forward by Grinter in 1953. In the usual process of moment distribution the relaxation or rotation of the joints, where joint translation is restricted, permits, the reduction or loss of shear in members influenced by side lurch. It is therefore evident that a variation of this system of moment distribution would be useful for lessening or preferably preventing shear loss. This cantilever distribution has the characteristics of allowing the member to undergo no change in shear as moment distribution occurs. Complete freedom of sidelurch by this method is, however, possible only in case of structures involving identical columns. If the columns are unlike the solution obtained is only approximate.

Naylor has suggested a method by making use of symmetry and asymmetry of frames. This method reduces the labour involved in performing the moment distribution, but however its applicability is limited only to symmetrical frames.

Cotton has suggested another method of procedure which has some points of similarity with Naylor's method but which is not restricted to symmetrical frames. The idea is to build up distribution operations which can be used to liquidate out of balance joint moments, in the usual manner, but which do not disturb the sway relationships in the process. The distribution operations are synthesised from the basic individual joint rotations and translations appropriate to particular problem under examination.

The moment distribution method is nothing but a relaxation process of solving the slope deflection equations with only difference that the process is one of successive approximations, however, rather than one of successive corrections so that mistakes are perpetuated in moment distribution while they are eliminated in relaxation. The most important characteristic of moment distribution is that the calculations are conducted entirely interms of moments and not in terms of displacements, thus eliminating the labour involved in converting the displacements back into moments. The method of moment distribution also gives a physical picture of the problem and the significance of the figures that the designer is handling , throughout the process. Cantilever moment distributions, and other methods as suggested by Naylor and Hotton are very useful in their range of application. But still the moment distribution becomes more and more complicated and difficult to be handled, when the number of storeys and bays become large and in that case more recent methods like Kloucek's method of distribution of deformation and Kani's iteration process are more convenient to apply . Two cycle process of moment distribution when carried out floor wise is very useful for preliminary analysis and it takes into account the worst conditions of loading.

[illegible].

This new method of structural design based on slope deflection equation was developed by the Czech engineer Dr. C.V.Kloucek. The method however does not require the orthodox solution of equations. Solution of equations and the fundamental conceptions underlying the new theory may be summed up as below:

The local loading of any structure produces waves of deformation which spread throughout the whole structure. The magnitude of these waves is defined by the angles which are successively produced in individual joints. The general term deformation has been used to denote these angles. If we express numerically the successive distribution of deformations from joint to joint, we obtain angles with exactly the same values as would be obtained by the solution of equations.

The following equation, which is only a different form of writing the slope deflection equation is the starting point for understanding this method.

$$M_{AB} = K_{AB} \; (2 \; d_A + d_B) \; - M_{F(AB)}$$

where M_{AB} = Final moment at the end A of a member AB.

K_{AB} = Relative stiffness of the member.

d_A + d_B are the total deformation at A and B respectively.

$M_{F(AB)}$ = F. E. M. at A

Now the application of method consists in finding up of total deformations at a joint which is done in two steps.

i) Determination of the primary deformation of the loaded joint.

ii) Distribution of this deformation to adjacent joints as the secondary deformation.

We have a set of formulae by which we compute the following :-

a) The relative stiffnesses of members

b) Stiffness factors of joints.

c) Stiffness constants of members.

d) Primary deformation of all joints.

e) Carry over factor for secondary deformation.

By a standardised method the total deformations at all joints are calculated by adding the total of carried over secondary deformations to the primary deformation. The exact formulaes are however too cumbersome to be applied in practical designs. But if an error of only 20% (maximum) is allowed for the simplyfied formulaes are most simple to be applied.

The theoretical analysis of this method can be compared to as a deformation counterpart of the well known moment distribution theory. But the two main advantages can be immediately obviated.

1) At any joint where n members meet, there will be n unknown moments involved in the method of moment distribution, but in case of the distribution of deformation method only one unknown viz. the angle of rotation of the joint will be involved.
2) For a frame net work each new joint will require only one distribution of deformation, but four distributions of moments.

In case of whole frame analysis the above method presents no special difficulty. After computing the primary deformations at all joints the carryover factors are computed both horizontally along beams and vertically along columns for calculating the secondary deformation. The really important application of this method is found in the solution for the sidesway of a wind loaded multistorey frame using a substitute, knotted cantilever which has the same lateral displacements and angles of rotations of the knot at each storey of the frame. The solution of the substitute cantilever gives the solution for the whole frame. However an iteration process has to be carried out finaly in order to find out the share of moments taken up by columns in different bays depending upon their stiffness. This iteration is unnecessary for symmetrical single bay frame or the one which is so constructed as to obey the principle of multiples (as called by the author), that is the frame can be split up into parts. Kloucek has however, given various constants which with the help of which the iteration process can be avoided by losing some accuracy.

MODIFICATION [illegible]

The modified method deals with the end moments (instead of rotations or displacements) all the time. It is based on the concept of a "Substituted cantilever" which is substituted for the actual frame for determination of sway only. The properties of substituted cantilever are obtained in the same way as that of Kloucek. The cantilever is however solved by a slightly modified method of moment distribution This directly gives the "fixing moments" in the actual frame if it deflects sideways by the true displacements with joints locked against rotation. The actual frame is then solved by the usual moment distribution.

The advantage of this method over Kloucek's method of substitute cantilever is only that the more laborious iteration process is replaced by an easy moment distribution process, and the conversion of moments into deformations and finally back is eliminated. Otherwise the basic concept is almost similar.

KANI'S METHOD OF ITERATION:

Of all the iteration methods of frame analysis Kani's method has proved to be extremely satisfactory for the analysis of any type of multistoreyed building frame under any type of loading. The method is based on southwell's principle of relaxation and can be classed as absolutely correct if more cycles of iteration are performed. But lesser number of cycles can give as good results as any other method of iteration, with comparatively much more ease and speed.

The basic theory of Kani's method can be understood by the equation:-

$$M_{AB} = M_{F(AB)} + 2 M'_{AB} + M'_{BA} + M''_{AB}$$

Where M_{AB} = Final moment at the end A of a member AB.

$M_{F(AB)}$ = Fixed end moment at end A considering member AB to be fixed at both ends.

M'_{AB} = the moment correction on account of rotation of near end A.

M'_{BA} = the moment correction on account of rotation of the far end B.

M''_{AB} = moment correct due to lateral displacement of ends. In the absence of sidesway this term will be eliminated.

The equation is only a different form of writing the slope deflection equation. Applying the usual conditions of equilibrium i.e. sum of all final moments at a joint is equal to zero and the shear condition, the following relations are obtained easily;

$$M'_{AB} = U_{AB}\left(\bar{M}_A + \sum M'_{BA} + \sum M''_{BA}\right)$$

where $U_{AB} = -\frac{1}{2}\frac{K_{AB}}{\sum K_{AB}}$ (i.e. $-\frac{1}{2}$ x usual distribution factor in ordinary moment distribution)

$\bar{M}_A$ = Unbalanced moment at joint A

= Algebraic sum of fixed end moments on the near ends of all the members meeting at A.

Displacement moment $M''_{AB} = V_{AB}\left(\bar{M}_r + \sum (M'_{AB} + M'_{BA})\right)$

Where $V_{AB} = -3/2 \frac{K_{AB}}{\sum K_{AB}}$

$\bar{M}_r = -\frac{Q_r h_r}{3}$ and is called storey moment.

h_r = Height of columns in a particular storey.

$Q_r = \sum P + \sum H + \sum \bar{H}_{AB}$

ΣP = Sum of all the point loads in horizontal direction above that storey.

ΣU = Sum of all the uniformly distributed loads in the lateral direction upto that particular storey from the top.

$$\bar{H}_{AB} = H^{o}_{AB} + \frac{\bar{M}_{AB} + \bar{M}_{BA}}{h_{AB}}$$

H^{o}_{AB} = shear at the ends of column considering it to be simply supported at the two ends.

In case the columns at a particular storey are of unequal length, then above expression is slightly modified. It may however be noted that Σ in above expression includes only columns in the storey and under consideration, the beams are not to be taken into account.

The procedure of calculations is very simple. The work can be started from any point. All those values which are not there, are assumed to be zero as first approximation and the required correction for the near end is obtained. Then we proceed to next point. Similarly displacement moments can be calculated approximately. The next cycle is repeated with the approximate values obtained in the first cycle. The operation is performed again and again till the results obtained do not change in next cycle, or change is very small.

The method is very simple and can be preformed with ease with the help of a calculating machine. Beside-s it passes following more advantages :-

1) Error is eliminated automatically.
2) Final numeraical values can be checked.

1.4. APPROXIMATE METHODS OF ANALYSIS FOR HORRIZONTAL LOADS ONLY

In the foregoing methods it was realised that it is the sway in the frames which makes the analysis very lengthy and tedious. Fortunately it was found that the sway due to vertical load only, on a more or less symmetrical frame is only a negligible amount and the frame can be solved neglecting the way with errors which can be reasonably allowed. But the horrizontal loads (wind and Earthquake forces) are nevertheless more important and cause significant swaying. So it has been suggested to carry out the analysis in two steps.

a) For vertical loads only by any of usual methods, neglecting the side sway.
b) For horrizontal loads only.

the two effects can be lateron superimposed.

Attempts for analysing the frame for horrizontal loads only, have made considering the problem as a statically determinate by making a number of assumptions. The more common methods adopted for such purpose are :-

1) Cantilever Method:-

The assumptions are :-

i) Points of inflexion are at mid-height of columns and mid-ponts of girders.

ii) The unit direct stress in the column varies as the distance of column from C.G. of the bent. Columns have the same area every where in the same storey.

2) Portal Method:-

The assumptions in this method are :-

i) The points of inflexion are at mid-height of columns and mid-points of girders.

ii) The shear in each column is the same and equals one half of the shear in an interior column.

3) Sutherland and Bowman Method:

Sutherland & Bowman have improved upon the two methods mentioned above. They have suggested different positions of the points of inflexion for different storeyes. The distribution of shear in the columns of a storey consists of two parts. One part is equal in all columns. The remaining shear is divided among the bays inversely as the width of the bays.

4) Factor Method:

This method makes certain assumptions regarding the elastic action of the structure, which makes it possible to have an approximate slope deflection analysis. The previous methods make some assumptions regarding stresses which made a statical solution of the problem possible. In its application it is very mechanical and only gives approximate values.

5) Gottschalk Method:

Otto Gottschalk has presented a simple method of determining the distribution of shear at the column of the bent by taking into account the relative sidesway of the joint. Then assuming the points of inflexion at mid-points or according to sutherland & Bowman determine the moments at the joint. The method is simplified when certain assumptions regarding the rigidity at various joints are made.

It can be broadly stated that the first three approximate methods do not require the knowledge of the elastic properties of the frame whereas the last two involve use of these properties. The first three are method of analysis before the design is done whereas the last two are analysing methods after the design is done we know the areas of x-sections etc.

On a four storied three bay frame loaded with unit load horrizontally at all storeys , when analysis were carried out by the first four approximate methods and the slope deflection equations, the values calculated by approximate methods in terms of slope deflection results were wise:

Cantilever method	:-	various values lies between 60% to 160%
Portal method	:-	70% to 140%
Factor Method	:-	90% to 120%
Sutherland & Bowmen method	:-	90% to 110%.

In other words the last method admits of an error of ± 10% each generally maximum. The location of the point of inflexion as given by Sutherland and Bowman rules is almost exactly the same as given by Slope and Deflection method. No definite conclusions can be based only on the results from one example but this may point a trend for the relative accuracy of these methods.

CHAPTER II

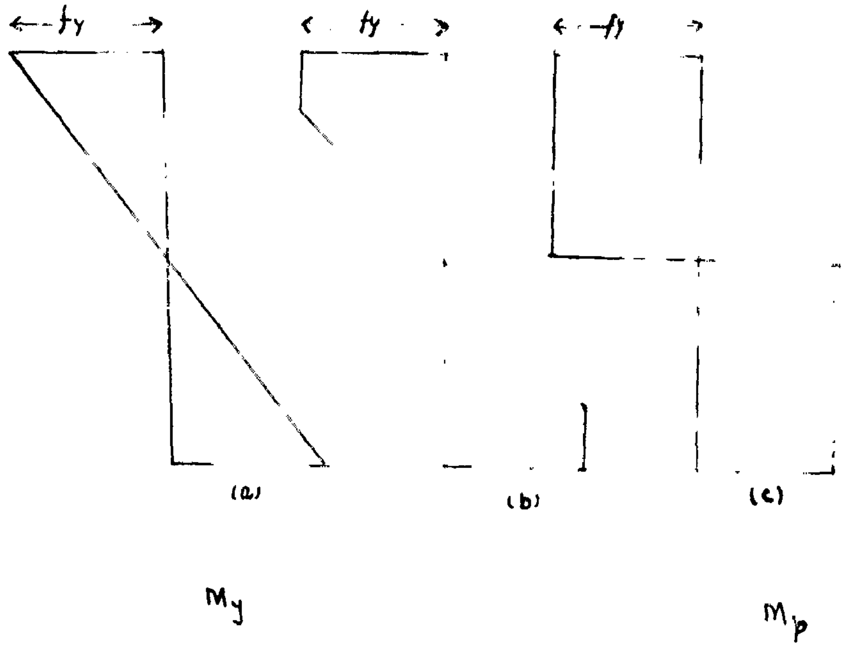
PLASTIC METHODS OF ANALYSIS

2.1. Plastic methods of analysis differ from the ordinarily employed Elastic method of analysis in the sense that they take into account the fact that structural materials like steel, due to its ductility, can withstand strains much larger than those encountered within the elastic limit. Plastic methods assume that if a structure does not reach its limit of usefulness through instability, fatigue, or excessive deflection, then it will continue to carry load beyond the elastic limit until it reaches its ultimate load through plastic deformation and then collapse. It is this ultimate load carrying capacity of the structure on which we allow certain safety factor and base our design. The theory which forms the basis for the calculation for the ultimate strength of a structure is called the plastic theory.

2.2. PLASTIC THEORY.

The theory is nothing more than an extension of the simple elastic theory, according to which the longitudinal stress distribution across any section of a beam, acted upon by a theory holds for all bending moments less than the yield moment My under which the yield stress of the material is reached in the extreme fibres. The beam is capable of sustaining a

FIG. 2.1.



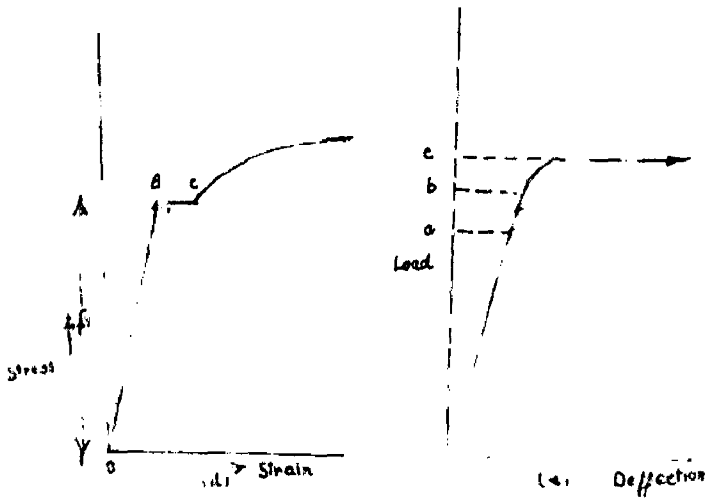


FIG. 2.1 (d) + (e)

bending moment greater than My but to deduce the corresponding stress distribution it is necessary to consider the nature of the stress - strain relationship of the material beyond the yield point. This depends upon the composition and history of the steel, but, for structural mild steel in general, the curve shown in fig. 2.1.(d) is a sufficient close approximation. After yield at B, there is a range BC of pure plastic deformation with no increase of stress, until strain hardening occurs at C at a strain of from 8 to 20 times the strain at yield. The pure plastic range Bc is usually sufficient to enable considerable deformations to take place before strain hardening has any significant effect. It is therefore reasonable to assume that, after the yield stress has been reached in any Fibre of a beam under increasing bending moment the stress in the Fibre will remain constant. After this process has been carried some depth into the beam, the stress distribution will therefore be as shown in fig. 2.1.(b), which corresponds to a moment of a resistance somewhat greater than the yield moment My. The maximum moment of resistance is obtained when the whole section has been strained beyond the yield point Fig 2.1(c), its value then being referred to as the full plastic moment Mp.

The stress distribution in fig 2.1 (c) corresponds to theoratically infinite curvature and therefore when the full plastic moment has been reached at any section of a simply supported beam, the deflections become indefinitely large, as shown in fig. 2.1(c), and collapse is said to occur though if the load were reduced a little, it could be supported safely.

2.3. PLASTIC HINGE:

When at some section of the frame bending moment attains the full plastic moment value Mp, the angular deformation locally becomes indefinitely large, and finite changes of slope can occur over an infinitely small length of the member near this cross section. Thus the behaviour at the cross section where Mp is attained can be described by imagining a hinge to be inserted at the section - the hinge being capable of resisting rotation until Mp is attained and then allowing rotation of any magnitude while Mp remains constant.

2.4. REDISTRIBUTION OF MOMENTS:

This is the property of redundant structures that enables them to carry much higher loads than those calculated by usually adopted elastic methods. It is actually this phenomenon that marks the real use of application of the plastic theory by revealing an enormous reserve of strength of an indeterminate structure beyond the elastic limit. The redistribution takes place as a consequence of the action of plastic hinges. As load is added to a structure eventually the plastic moment is reached at a critical range. As further load is added, this plastic moment value is maintained while the section rotates. Other less highly stressed sections maintained.

equilibrium with the increased load by a proportionate increase in moment. This process of moment transfer due to the successive formation of plastic hinges continues until the ultimate load is reached when the structure collapses by the formation of a mechanism.

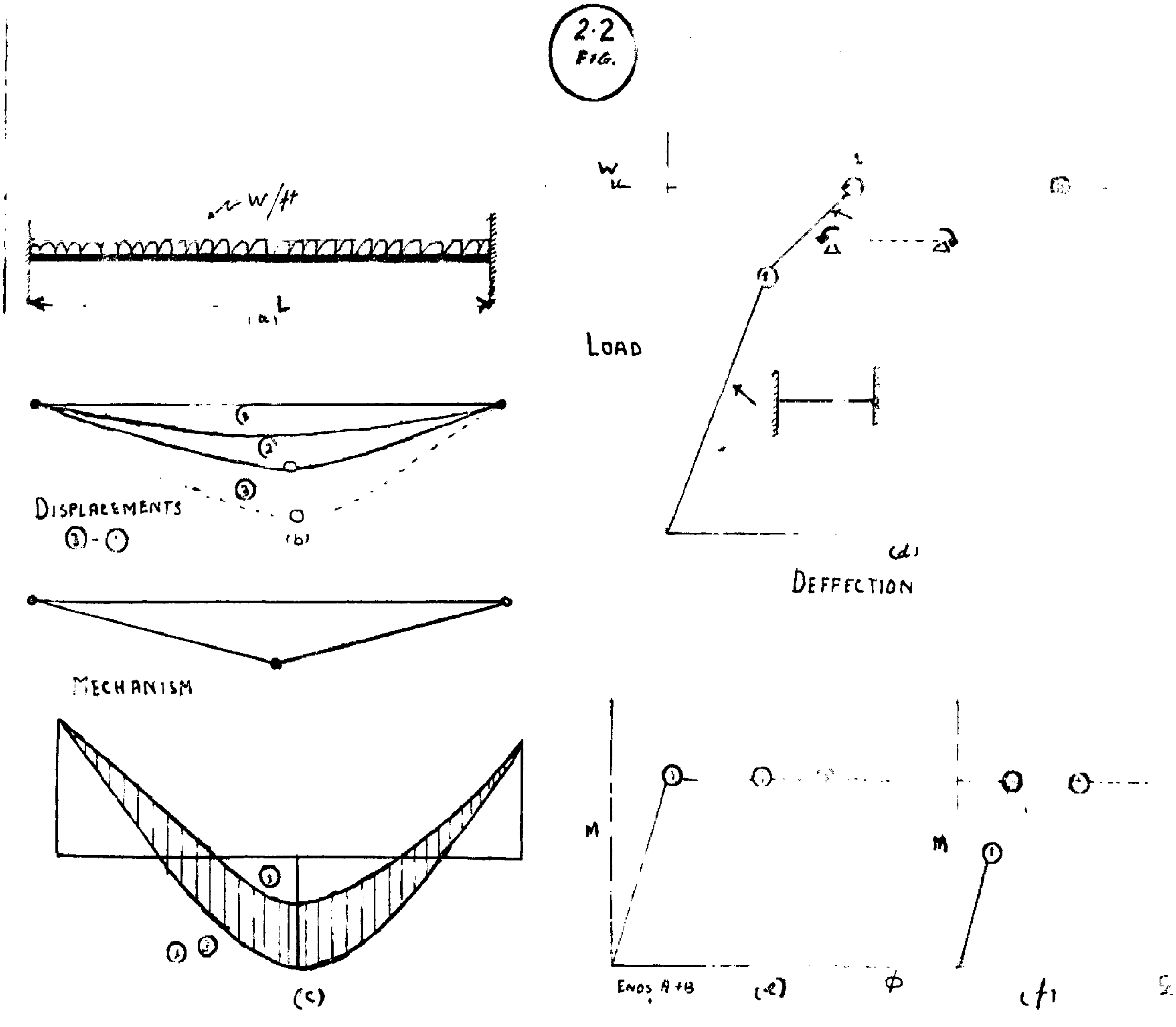
The redistribution of moments can be best illustrated by a fixed ended, uniformly loaded beam of fig. 2.2.(a). In fig. 2.2.(b) are curves of deflected shape; in fig 2.2.(c) are the moment diagrams; in fig. 2.2.(d) is shown the load M - ϕ action at the ends and at the centre, respectively. The members (a), (2), and (3) represent three phases of loading:

(1) Attainment of first yield.

(2) First attainment of computed ultimate load.

(3) An arbitrary deflection obtained by continued straining at the ultimate load.

By an elastic analysis, the moment diagram of sketch (c) of fig. 2.2. can be determined when yielding commences (phase (1)). The centre moment would be $\frac{wl^2}{24}$ and the end moment would be $\frac{wl^2}{12}$. On the load-deflection curve the load has reached point (1). The moment capacity has been used up at the ends; however since at the centre of beam M=½Mp at phase (1), additional moment capacity is still available there. Therefore, as load increases beyond phase (1), "Hinge


2·2
FIG.
w/ft
L
(a)
DISPLACEMENTS
(b)
MECHANISM
(c)
W
LOAD
(d)
DEFFECTION
M
ENDS A+B
(e)
φ
M
(f)

action" will start at the ends and the beam now behaves as if it were simply supported, except that the end moment remains constant at Mp.

At phase (2) the beam reaches its maximum lead since the moment capacity at the centre is also exhausted. Beyond phase (2), beam will continue to deform under constant load (phase (3)) . The shaded portion of sketch (c) represents the simple beam moment diagram that due to redistribution of moment, is superimposed upon the existing moment diagram (phase (1)) and corresponds to the increase of load between phases (1) and (2) fig 2.2. (d).

2.5. MECHANISM AT COLLAPSE:

After a sufficient number of plastic hinges have formed to accomplish all of the redistribution of moment that is possible, any further displacement occurs at constant load. The segments of the beam between plastic hinges are able to move without an increase of load like linkage, and this system of members in called a mechanism. Referring to fig. 2.2(b) mechanism motion occurs between stages (2) and (3); and subtracting the displacements of (2) from those of (3) we get the mechanism shown in lower part of sketch (b). Thus there is no further deformation within the member itself; it all occurs at the plastic hinges. It is this mode of failure which forms the basis for determining collapse load in plastic theory.

2.6. OTHER ASSUMPTIONS IN SIMPLE PLASTIC THEORY.

Besides the idealisation of stress-strain curve and moment curvature relation-ship for steel, we make following more assumptions.

(i) Material obeys Hooks law until the stresses reach the lower yield value while on further straining the stress remains constant at the lower yield value.

(ii) The lower yield stress and modulus of elasticity have the same values in compression, tension as well as in bending.

(iii) Material is homogeneous and isotropic in both the elastic and the plastic states.

(iv) Plane transverse sections remain plane and normal to the longitudinal axis after bending, the effect of shear being neglected.

(v) The cross section is symmetrical about an axis through its centroid parallel to the plane of bending.

(vi) Every layer of material is free to expand and contract longitudinally and laterally under stresses as if separated from the other layers.

(vii) Failure does not occur due to lateral stability or effect of shear.

(viii) Effect of axial forces is only to cause reduction in values of plastic moment of resistant and does not cause any other complication. Ordinarily this reduction in Mp is also neglected.

(ix) The deformations at collapse are sufficiently small to enable the effect of change of geometry on the equations of equilibrium to be neglected.

(x) Residual stresses and stress concentrations have negligible effect on ultimate bending strength of a member.

(xi) The connections provide full continuity so that the plastic moment Mp can be transmitted.

(xii) The loading is propotional.

2.7. FUNDAMENTAL PROPOSITIONS IN PLASTIC THEORY.

For calculating collapse load for any structure, use is made of some fundamental propositions described in following paragraph. For the validity of these theorems reference should be made to (Bib. Ref. No. 28.).

PROSITION 1 :-

8 · If a structure reaches such a state that, whilst it is in equilibrium under an applied system of forces, sufficient plastic hinges are formed for the structure to become a mechanism, the structure is on the point of collapse.

PROSITION 2 :-

As the deflections of a structure on the point of collapse increase, the work done by the external forces in equal to the energy expended in the deformation of the plastic hinges.

CHARACTERISTICS OF BENDING MOMENT DISTRIBUTION AT COLLAPSE.

In elastic theory a structure before it is rendered unserviceable, must satisfy three conditions viz. conditions of continuity, equilibrium and limiting stress. Similarly the Bending moment distribution diagram, in the plastic theory also has to satisfy the three conditions viz;

(i) Mechanism condition :- Moment must be equal to full value of Mp at sufficient number of sections for the structure or part of it to become a mechanism.

(ii) Equilibrium condition: Conditions of statics must be satisfied, i.e. the bending moment distribution must be in equilibrium with the applied loads.

(iii) Yield condition :- Full plastic moments must now here be exceeded.

Whilst it may be difficult to satisfy all three requirements simultaneously, they are much more readily satisfied in pairs. The collapse load thus obtained will not be unique as a result of following two propositions.

PROPOSITION 3 :- MINIMUM PRINCIPLE STATIC THEOREM:

For a given frame and proportional loading - if there exists any distribution of bending moment throughout the frame which is both safe and statically admissible with a set of loads W, the value of W must be less than or equal to the collapse load Wc.

PROPOSITION 4 :- MAXIMUM PRINCIPLE OR KINEMATIC THEOREM:

For a given frame subjected to a set of loads W, the value of W which is found to correspond to any assumed mechanism must be either greater than or equal to the collapse load Wc.

From the above theorems it is apparent that the only value of Wc can be obtained if the two theorems are combined.

PROPOSITION 5:- UNIQUENESS THEOREM:

If for a given frame and loading at least one safe and statically admissible bending moment distribution can be found, and in this distribution the bending moment is equal to the fully plastic moment at enough cross-sections to cause failure of the frame as a mechanism due to rotations of plastic hinges at these sections, the corresponding load will be equal to the collapse load Wc.

2.8. METHODS OF PLASTIC ANALYSIS.

Even though the first systematic method can not be dated back to more than 15 years, yet we see many attempts and revisions to achieve singular results in a convenient manner. Various methods are there to find collapse load for any indeterminate structure, but it is recommended that for multistorey frames the application of basic approach as suggested by Greenberg and Praguer that is to approach the collapse load value by simultaneously using both the principles - static and Kinematic, will be most useful. The upper bound can be established by Neal's method of combining elementary mechanisms while lower bound can be obtained by Horne's plastic moment distribution. These two methods have been discussed in the subsequent paragraphs. A brief account of other methods has also been given, just to understand the principles involved and their limitations for use in practice.

GIRKMANN'S METHOD

His was one of the earliest attempts to design the large frames by plastic theory. His approach was to construct a statically admissible bending moment distribution for the frame and loading under consideration and them to assign each member a fully plastic moment equal to the magnitude of the greatest bending moment in the member. Girkmann however considered only rectangular frames and was content merely to adjust his distribution of bending moment until the maximum sagging and hogging bending moment in each beam were equal . This can be of course improved upon as was later on done by Horne in his method of Plastic moment distribution. In general this method implied that collapse would not occur under the given loads, so that the designs were oversafe and hence uneconomical. This method has been pushedout of practice by Homne's plastic moment distribution which is also based on the same principle that is Minimum Principle.

GENERALIZED HINGE ROTATION METHOD

This method was first suggested by Heyman and Nachbar and is based on minimum principle. It consists of imposing arbitrary hinge rotations at each joint, under each concentrated load and at arbitrary positions under distributed loads. The positions and relative values of rotations are then so adjusted that the load factor as obtained from the corresponding virtual work equation is a minimum. This method becomes cumbersome in complicated frame and is only of academic interest how as the method of "Analysis by Combination of Elementary Mechanisms" which is most widely used in practice, has been derived from this method.

TRIAL AND ERROR METHOD.

This method which is due to Baker is based on Uniqueness theorem. An assumed mechanism is analysed statically to see whether a corresponding safe and statically admissible bending moment distribution for the whole frame can be found. If such a distribution can be shown to exist then the collapse mechanism assumed is the actual one, otherwise a fresh guess to the collapse mechanism is made guided by the results of previous analysis and the process repeated. The method is satisfactory only in case of complete and over complete collapse, but there also it is more time consuming as a statical check has to be made after every trial. In case of partial collapse which is more often the case in multistoreyed frames, its application becomes extremely lengthy and ~~kodi~~ impracticable.

HEYMAN AND NACHBAR'S METHOD BASED ON ADJUSTMENT OF RESTRAINT:

A direct corollarly to static theorem states that if a restraint either internal or external is removed from a structure the load factor at collapse will either decrease or remain unchanged. Following above statement, Heyman and Nachbar developed a method to obtain the lower bound to collapse load, as below.

The frame is split up into many component parts which are either statically determinate or redundant. The hypothetical collapse loads Wc^* for each of the component parts are then determined. The actual value of Wc for collapse of the whole frame cannot be less than the smallest collapse load thus obtained which will be denoted by say W_1. A lower bound Wl on Wc may now be improved by introducing the redundant reactions and bending moments at the cuts. These redundancies are then adjusted systematically so as to increase Wl the lowest value of the collapse loads Wc^* for each of the component parts of the frame.

Simultaneously an upper bound on Wc is established

established by the generalised hing rotation method described already. The calculations for the improvement of the upper and lower bounds are carried out simultaneously and the calculations cease when these bounds are close enough together for the collapse load to be quoted to the desired degree of accuracy.

The method does not lead to a rapid solution of the collapse load and hence is not suitable for exact analysis. It may however be useful to calculate the effect of extensions to existing structures or if two structures are joined.

METHOD OF INEQUALITIES :-

The method of inequalities involving an essentially statical approach was first used for beam problems by Symonds and Neal ( ) . Their method was based on a systematic presentation of the elimination of variables which was first given by Dines ( ) .

The problem is to find a set of moments M_1 , M_2 .... M_n which satisfy (n-r) linear equations of equilibrium, r being the number of redundancies. At least one of these latter will always contain the load W. In general r is not zero, hence the problem as stated so far is not unique. However the moments must also satisfy 2n linear inqualities of the form

$$-Mp \leq M_1 \leq Mp \quad \ldots\ldots\ldots\ldots .$$

$$\text{- - - - - - - - - - -}$$

$$-Mp \leq M_n \leq Mp$$

Now (n-r) of the bending moments can be expressed interms of other r moments and the values of external loads. This results in the formation of 2nd inequalities involving only r bending moments as variables. Since the equations of equilibrium are linear in the moments and the applied loads, the inequalities are all linear.

The r bending moments can be eliminated in turn from the inequalities leaving a number of inequalities on the value of W, it being supposed that each load is expressed as a multiple of a parameter W. Each of these inequalities sets an upper bound on the value of W, and the smallest of these upper bounds is the collapse load Wc.

This method is too lengthy even for simple frames so it is seldom used for multistoreyed frames. Anotner disadvantage is that the conception about the physical behaviour of the frame is not obtained until the calculations are complete. The method is however completely systematic and its only merit is that it might lend itself to machine computation.

METHOD OF COMBINING MECHANISMS.

This method which is due to Neal and Symonds is most widely adopted for the multistoreyed frames. The method is based on kinematic theorem and gives an upperbound to the collapse load. Therefore there is always fear of over estimating unless a statical admissible moment distribution throughout the frame is shown, a step which becomes little tedious in case of partial collapse. But the plastic moment distribution has been great asset to this method for this purpose. The method is very rapid and with little practice the correct collapse load can be calculated without any difficulty.

The essential motion underlying the method of combining mechanism is that for a given frame and loading every possible collapse mechanism can be regarded as some combination of a certain number of independent mechanisms. It is naturally very important to determine the correct number of independent mechanisms and to identify them. The number of such elementary mechanisms which must be conside-red in any particular problem can be shown to be equal to the number of independent equations of equilibrium relevant to the structure in question. To decide upon the number of independent equations of equilibrium it is noted that if the frame has r redundancies and where as the comple bending moment distribution is specified by n values of bending moment at n cross sections, then we have :

Number of independent equations of equilibrium i.e. independent mechanisms (N) = n-r

In general there are only four types of independent mechanisms viz;

1) Beam type Mechanism : These can develop in any member carrying transverse loads.

2) Sidesway mechanism : These can develop whenever sway of members can occur.

3) Joint Rotations : These are possible at any connecting three or more members. These are of course meaningless in itself but are of significance when combined with other mechanism.

and 4) Frame Mechanisms, which are present only in case of pitched roof portals.

Once these independent mechanisms have been identified a work equation can be written down and the corresponding value deduced for each of these mechanisms. Now the actual collapse mechanism is distinguished from amongst all the possible mechanisms by the fact that it has the highest corresponding value of Mp, by the kinematic theorem. According, those independent mechanisms with high corresponding values of Mp are examined to see whether they can be combined to form a mechanism which gives an even higher value of Mp. It is only necessary to examine a few of the more likely combinations in this way in order to arrive at a mechanism which is almost certainly the actual collapse mechanism or a close approximation to it. An independent statical check is then carried out which either verified that the actual collapse mechanism has been found or indicates the minor adjustments that need to be made. By this procedure the apparent necessity for examining every possible collapse mechanism in order to determine which one gives the highest correspon-ding value of Mp is avoided.

The complete procedure can be summarised to consis of following 5 steps.

(i) Determine the number of independent mechanisms.

(ii) Sketch the possible beam, sway and joint mecha-nisms and check that their total number is correct.

(iii) Write down the equilibrium equations correspond-ing to each possible independent mechanism pre-ferably using the virtual work method for this purpose.

(iv) Investigate possible combinations of independent mechanisms, observing that progress towards the correct solution is only made if plastic hinges are eliminated by such combination.

(v) Finally draw the bending moment distribution corresponding to the solution obtained and check that the plastic moment is nowhere exceeded.

The technique of combining the independent mechanisms can be better explained with the help of a numerical example. This has been done in Appendix III, where the collapse load for a six storeyed frame, which was later on tested in the laboratory, has been calculated.

PLASTIC MOMENT DISTRIBUTION METHOD

A quite different technique based upon the lower-bound theorem is that of moment distribution originally developed by Horne (Ref. 29). The basis of this method is to assume any equilibrated moments in the frame, still satisfying the equilibrium equations. At any stage, multiplying the load by the ratio of the yield moment to the maximum moment will provide a lowerbound. Further if the load is multiplied by the ratio of the yield moment to the maximum existing moment in any particular mechanism an upper bound will be obtained. It is a very convenient method and is advantageous in a way because we are always on the safer side and can have a physical picture at any stage of calculations. Moreover it is not necessary to assign the ratio of the fully plastic moments of the members at the outset, when using this method. Instead the fully plastic moments can be selected in turn towards the end of the calculations. The process is therefore truly one of design.

Sign convention adopted for bending moments in this method is different from that used in other methods of plastic analysis. Here a clockwise end moment and sagging moment in centre are taken as positive.

The computations are carried out in the following three stages.

(i) A set of bending moments is written down which satisfies all the equilibrium requirement except those at the joints. Now while there are, of course many ways to do so, the best procedure is to use the set of moments whose magnitudes are those obtained in analysing the independent collapse mechanisms corresponding to equilibrium equations which can be written down very easily from considerations of virtual work applied to the mechanisms.

(ii) The bending moments are adjusted so that all the joints are brought into rotational equilibrium, without violating the other requirements of equilibrium. The balancing here, however, is not automatic as in the elastic moment distribution. The out of balance moments at a joint can be distributed in any member, keeping in mind that the moments carried over to the adjustant sections in order to maintain the equilibrium equations, should be such as to keep all the moments approximately equal. However in first place they may be distributed in the ratio of their full plastic moments.

(iii) Further adjustments are made to the bending moments in such a way that all the equilibrium requirements are still fullfilled.

The plastic moment distribution method has been used to verify the collapse load obtained for the frame analysed by Neal's method, as described earlier. The procedure is explained in Appendix IV which will also clarify the use of this method.

UPPER AND LOWER BOUNDS BY GREENBERG & PRAGER:

Greenberg and Prager's approach to the problem was to use both upper and lower bounds simultaneously for the determination of collapse load. Their method consists in assuming a mechanism of collapse, and from the work equation the corresponding value of load W is found, which is an upper bound by Kinematic theorem. Simultaneously a statical admissible bending moment diagram is drawn such that the condition of yield is reached as sufficient points. If the mechanism assumed is of complete or overcomplete type, the bending moment distribution can be determined throughout the structure from purely statical considerations. The load corresponding to this safe statically admissible bending moment distribution will be a lower bound on the collapse load. Unless the upper and lower bounds coincide the procedure is repeated successively with different assumed collapse mechanisms until coincidence is obtained.

The method had a great draw back in solving frame where partial collapse occurs. But now with the help of plastic moment distribution this difficulty is eliminated. It is now only the basic approach that has been suggested by Greenberg and Prager which is left over. The actual methods adopted are

(1) Combination of Elementary mechanisms due to Neal and Symmonds

(2) Plastic moment distribution by Horne.

The two methods are complimentary to each other. In the former an unsafe estimate of λ is progressively reduced where as in latter a safe estimate is progressively raised. If both methods are used alternately for the same frame, it is possible to obtain the collapse load correctly and rapidly. This method is by far the most popular for the analysis of multistorey frames.

ENGLISH METHOD OF RELAXATION OF HINGES

This method developed by J. Morkey Elglish (R4.24) is very similar to Plastic moment distribution by Horne. It consists of arbitrary distribution of moments to the joints of the structure and then the adjustment of this distribution until the highest ratio of actual moment to the limit moment at any point in the structure reaches a minimum. The location of yield hinges are the points where these minima occur. The advantage claimed by the author lies in the fact that this method accounts for the order of the developments of hinges. Since this order is dependent on the elastic properties of the members, the initial moment distribution as well as the adjustment of moments are done consistent with the stiffness of the members.

LOAD INTERACTION METHOD

All the methods discussed so far can be used only when the entire loading system can be represented by a single parameter say W. In short these methods are valid only for proportional loading, where all the loads beas a constant ratio to each other at any stage of loading. In practice this never occurs and there may be any number of loads varying independently. These methods have a inherent shortcoming in this respect.

Cases in which the entire loading system can be represented by two independent parameters say W & P are quite frequent in practice. A very common example is one where W is due to vertical loads (dead & live) and P is due to wind only. Such cases can be treated only by the method known as "Load Interaction Method". It is in fact a graphical extension of Neal and Symonds. Method based on "Combination of Elementary Mechanisms". This method can be summed up in the following steps:-

(i) All loads are expressed in terms of the two parameters say W & P.

(ii) All probable collapse mechanisms(both elementary and combination of elementary mech.) are investigated by virtual work = ns. Each mech. yields an equation involving two variables W and P and representing straight lines.

(iii) All these =ns of st.lines are plotted with W and P along Y and X axis.

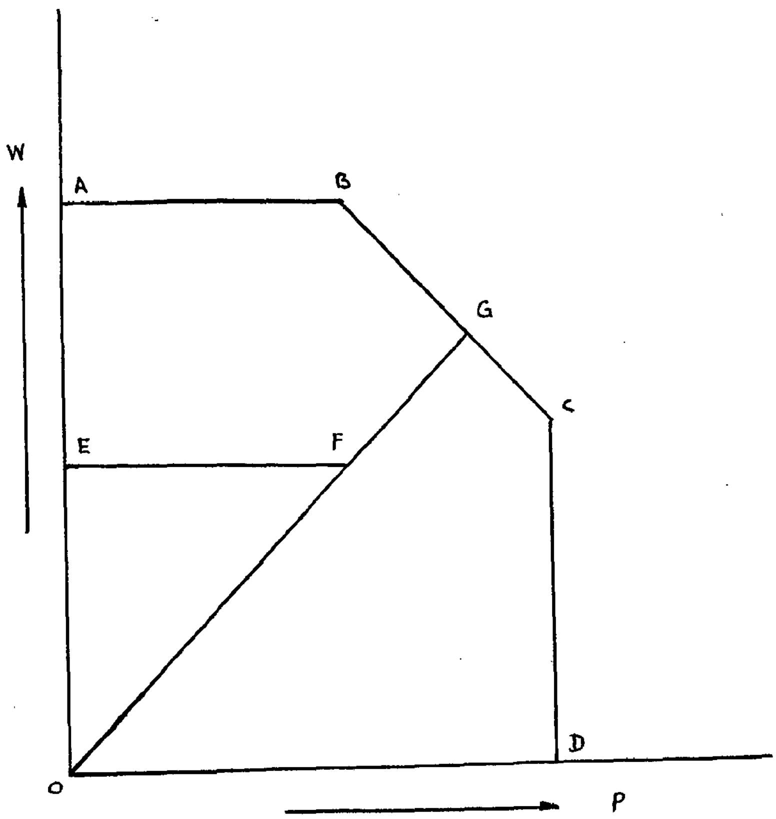
(iv) The net common area bounded by two axes and these straight lines is the actual loading envelope. Referring to fig. say OA BCD represent a loading envelope and say load factor λ is required for particular values of W & P e.g. W_1, and P_1, Let OE represent the value of W_1, and EF represent P_1, then OF represent this loading condition. Produce OF to meet the boundary line of the loading envelope at G. Then $\frac{OG}{OF}$ = the required collapse load factor.

In general the boundary of loading envelope can consist of any number of straight lines. In case the frame is subjected to uniformly distributed loads. The =ns as obtained from the collapse mechs. will represent curves and a portion of the loading envelope will be replaced by an irregular curve.

Cases in which the loading system can be expressed only in terms of three parameters say W,P and Q can be tackled in a similar manner. The collapse mechanisms will give equations of the form of

$$f1\ (W) + F_2\ (P) + F_3\ (Q) + K_1 = 0$$

Where F_1, F_2, and F_3 are three functions and K_1, is a constant. These are =ns of planes. Representing W, P and Q along X, Y and Z axis, this can best be dealt with help of a model either of cardboard or plastic. Similar to the loading envelope, here we will get a certain volume of space; Let this volume be termed as Loading Care". Any loading condition



represented by a point with co-ordinates x, y & z equal to W, P and Q respectively will be safe only if the point lies within this loading core.

Cases in which the loading system can be expressed only in terms of more than 3 parameters cannot be dealt in such a general way.

CHAPTER III

EXTENT OF REDISTRIBUTION OF MOMENTS.

3.1. PURPOSE OF THE INVESTIGATION:-

Based on simple Plastic Theory, the collapse of a redundant structure of steel required the development of sufficient plastic hinges to convert the structure into a mechanism with one degree of freedom. These hinges do not develope simultaneously but are formed in succession. For all additional loads, large angular movements are required at the first formed hinges in order to convert the structure into a collapse mechanism with the result that high strains develope at the hinges. The concept of a structure to form into a collapse mechanism will be true only if the high strains at the plastic hinges do not exceed the ultimate strain of the material. i.e., the structure would not break at some earlier formed hinges.

Here to before it is assumed that the material is perfectly plastic beyond lower yield point. Because of the large assumed ductile behaviour of Mild steel beyond yield point, it has always been taken for granted that required redistribution of moments can take place in a redundant structure without exceeding ultimate strain. To establish the authenticity of this assumption it was proposed to investigate by solving a number of cases, as to whether the ultimate strain is actually exceeded at some plastic hinges or not. To investigate this it is essential to determine the maximum available hinge rotation and the rotation which a particular hinge is called upon to undergo for the required redistribution of moments.

A number of cases were worked out and in each it was found that the stress-strain curve (Fig (3·3a)) which is due to Baker, fails to provide the necessary hinge rotation for the required moment redistribution. In fact practically very little redistribution is possible if strain hardening is neglected. This means that failure will occur due to value of strain exceeding the ultimate strain of the material. Theoretically the collapse load as given by simple plastic theory can never be reached, without exceeding the ultimate strain of the material. Hence the actual stress-strain curve of mild-steel upto the ultimate strain was found out. A relation between moment M and angular deformation Ø was worked out for both rectangular and I-sections upto the ultimate strain. This relation was then used to investigate the load at which ultimate strain is reached in a number of cases of beams and frames. Various examples were worked out. In all cases it was found that it takes about 40 to 50% extra load than what is given by simple Plastic Theory before the ultimate strain is reached. There is no likelihood of strain at any plastic hinge exceeding the ultimate strain due to the effect of strain hardening and the drooping nature of stress-strain curve near yield point.

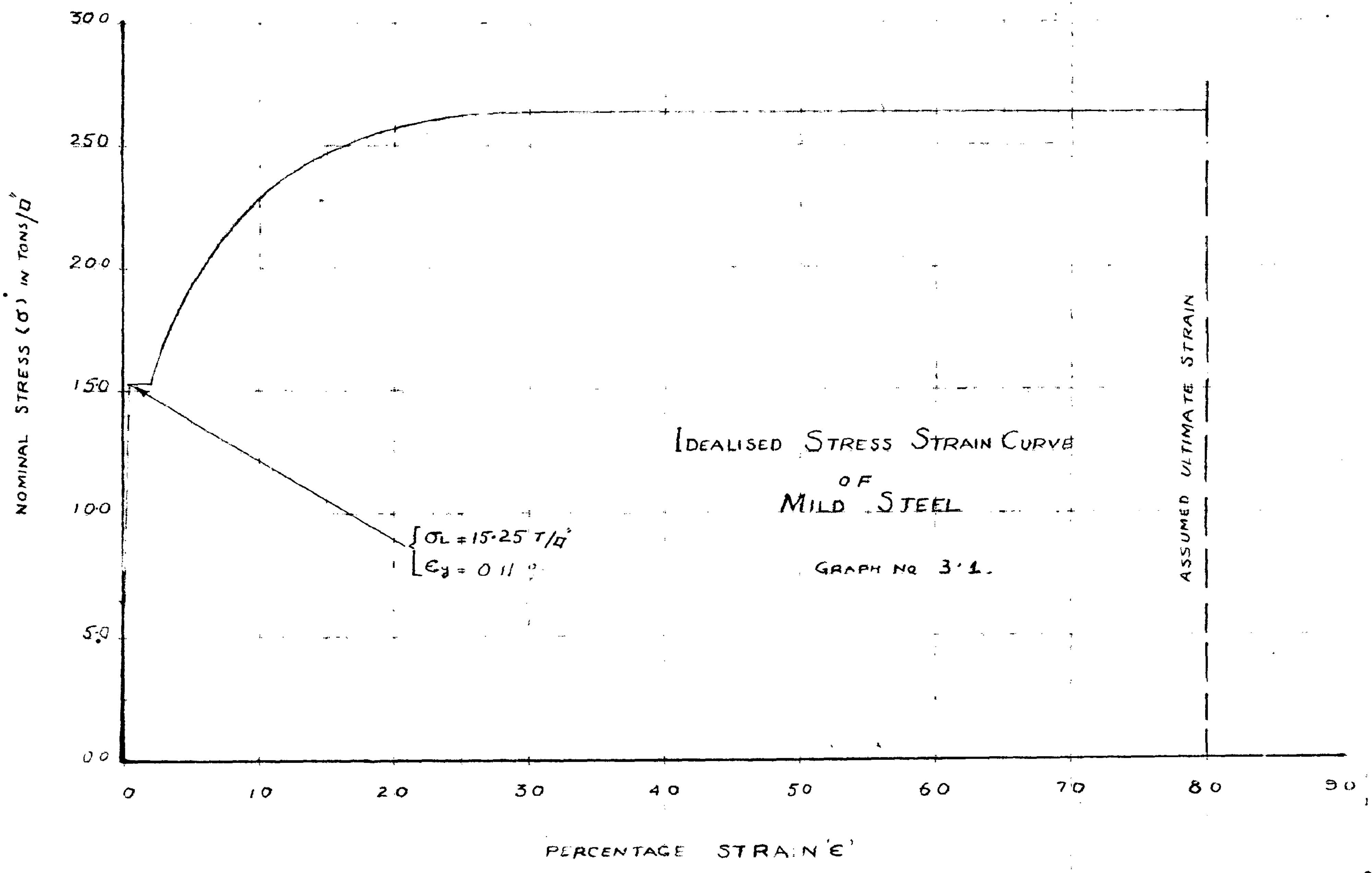
3.2. ASSUMPTIONS MADE:-

For the purpose of this investigation, Baker's simple stress strain curve Fig (3.3a) forming the basis of the simple plastic theory and the actual idealised stress-strain curve (graph 3.1) are employed. In the latter curve, it is assumed that the stress after the formation of necking remains constant upto the ultimate strain which may be as high as 30%. A reference to page 112 of "Testing and Inspection of the Engineering materials by Davis, Troxel, and Wiskocil" would show that the above value of the ultimate strain is safe in absence of the knowledge of the absolute ultimate strain which cannot be determined experimentally. In the calculations that follow, theusual assumptions made are :-

1) Cross-sections plane before bending remains plane after bending.
2) The members in bending are symmetrical about neutral axis.
3) The effect of normal forces, shearing forces and elastic instability are disregarded.

3.3. PLASTIC HINGE FORMATION:

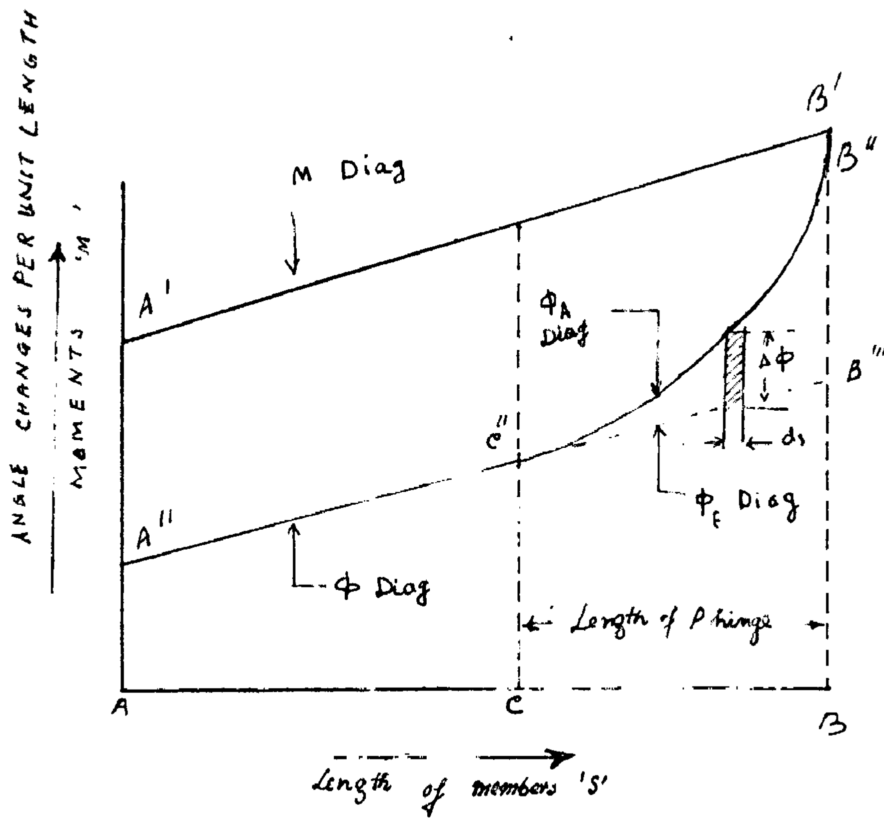
A section when subjected to gradually increasing moments, deforms elastically upto a certain extent, and then plastic deformations accompany it till it reaches the ultimate moment of resistance. This plastic deformation occurs in a certain



IDEALISED STRESS STRAIN CURVE
OF
MILD STEEL
GRAPH No 3·1.
NOMINAL STRESS (σ) IN TONS/□"
PERCENTAGE STRAIN 'ε'
ASSUMED ULTIMATE STRAIN
σL = 15·25 T/□"
εy = 0.11 %
30.0
25.0
20.0
15.0
10.0
5.0
0.0
0
10
20
30
40
50
60
70
80
90

length known as length of plastic hinge. It is equal to the total cumulative rotation in the length of the member which forms the plastic hinge minus the cumulative elastic rotation in the same length, on the assumption that the material remains elastic.

Consider a length AB of a member whose section B has reached its ultimate moment of resistance. In Fig (3.2) . A'C'B' is a moment diagram over this length of the member. A"C"B" is the diagram of the actual angular deformation (ϕ_A) In the elastic range, angular deformations are proportional to the applied moment ($\frac{M}{EI}$) and are indicated by the part A"C" of ϕ_A diagram. Beyond the elastic range, the angle change increase much faster than the bending moments as shown by the part C"B". The length CB over which plastic angle changes occur is called the length of the plastic hinge. ϕ_E is the graph of the angle changes in the length CB assuming elastic behaviour. The extra rotation at any point D due to plasticity is $\Delta\phi = \phi_A - \phi_E$ and the total plastic rotation (total cummulative rotation over CB - cummulative elastic rotation in the same length) i.e. the available rotation of the plastic hinge without overstraining is therefore, equal to $\Sigma_c^B \Delta\phi . ds$. i.e. the area between the curves. To find ϕ_A curve a relation between moment M and angular deformation ϕ is required at all stages uptill the ultimate strain is reached. Hence M - ϕ relation based on actual stress - strain curve will be derived in this chapter.



ANGLE CHANGES PER UNIT LENGTH
MOMENTS 'M'
B'
B''
M Diag
A'
ϕ_A Diag
$\Delta\phi$
B'''
C''
ds
ϕ_E Diag
A''
ϕ Diag
Length of P hinge
A
C
B
Length of members 'S'

FIG. 3·2

3.4 M – φ RELATION BASED ON BAKER'S STRESS STRAIN CURVE :-

(a) RECTANGULAR CROSS SECTION :-

Consider the rectangular section shown in Fig. (3.3) If q is the depth of the section which is in elastic state at any stage, and b , d, are brodth and depth of the section, it can be shown that ,

$$\phi = \frac{2\sigma_L}{qE} \qquad \text{......Eqn} \quad (3.1)$$

and

$$M = \sigma_L b \left(\frac{d^2}{4} - \frac{q^2}{12} \right) \qquad \text{......Eqn.} \quad (3.2)$$

From the Eqn no. (3.1) and (3.2) M – φ relation can be found for the various values of q.

(b) I – SECTION :-

Consider an I-section as shown in fig (3.4) Let q be the depth of the section which remains in the elastic state at any stage.

If q d

$$M = 2 \left(t_1 d_2 + \frac{t_2}{4} (d_1^2 - q^2) + \frac{t_2 q^2}{6} \right) \qquad \text{.... (3.3a)}$$

q d_1

$$M = \left(\frac{b(d^2 - q^2)}{4} + \frac{b q^3 - (b-t) d_1^3}{6 q} \right) \qquad \text{.... Eqn} \quad (3.3b)$$

Value of ϕ as in last case can be found from Eqn (3.1)

RECTANGULAR CROSS SECTION

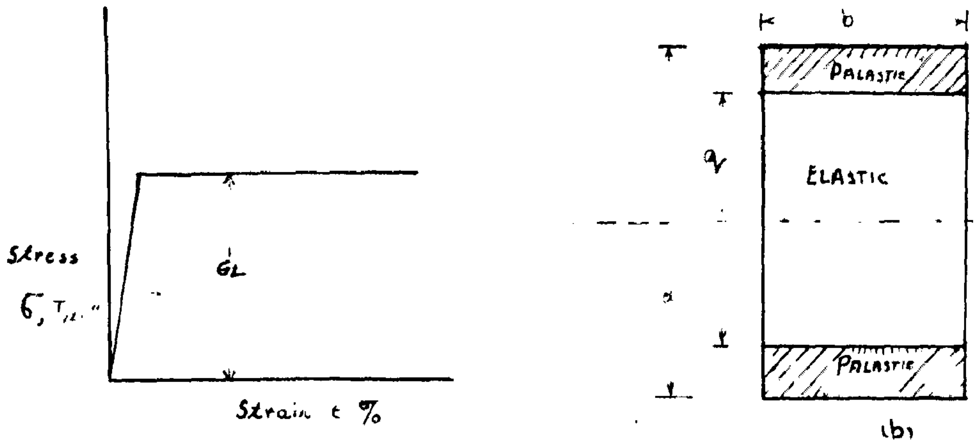


Fig 3.3

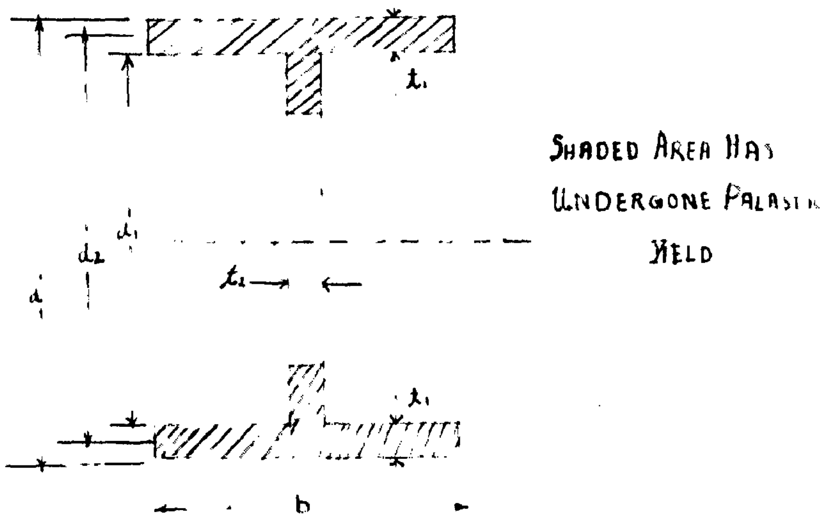


Fig 3.4

3.5 M – φ [illegible] :-

Using actual stress strain curve it is impossible to get an exact mathematical relation between M & φ. Hence a method based on graphical integration will be used. The stress-strain curve used for this is shown in graph No. (3.1).

(a) RECTANGULAR SECTION

In the rectangular beam Fig. (3.7a) the states of strains and stresses are represented by diagrams fig (3.7b) and Fig (3.7c). The stress diagram consists of two equal parts resembling the stress strain curve fig (3.5) of the material upto the point A whose d/2 is proportional to the strain ε corresponding to the point A with the increase in bending moment, the state of extreme fibre stress and strain in the beam correspond to the point B of fig (3.5). The extreme strain is again proportional to d/2.

Considering half the part of the beam with width b and depth d the moment of the stresses in the upper half of the beam about o o is proved as under.

$$\frac{x}{d/2} = \frac{\epsilon}{\epsilon_1}$$

$$x = \frac{\epsilon}{\epsilon_1} \cdot \frac{d}{2}$$

$$dx = \frac{d\epsilon}{\epsilon_1} \cdot \frac{d}{2} = \frac{d}{2} \cdot \frac{d\epsilon}{\epsilon_1} .$$

the moment of the internal forces in the half of

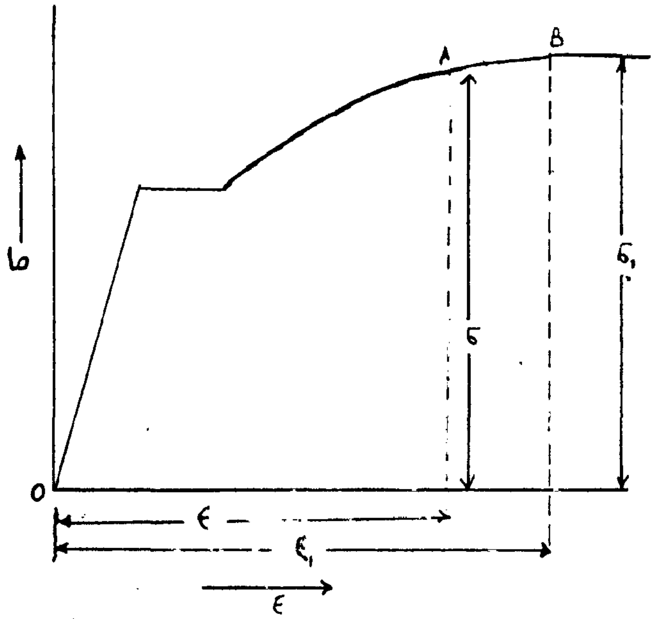


Fig 3.5

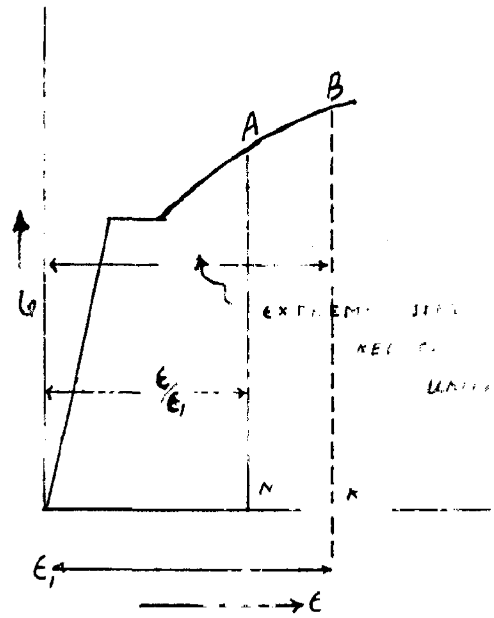


Fig 3.6

RECTANGULAR SECTION:-

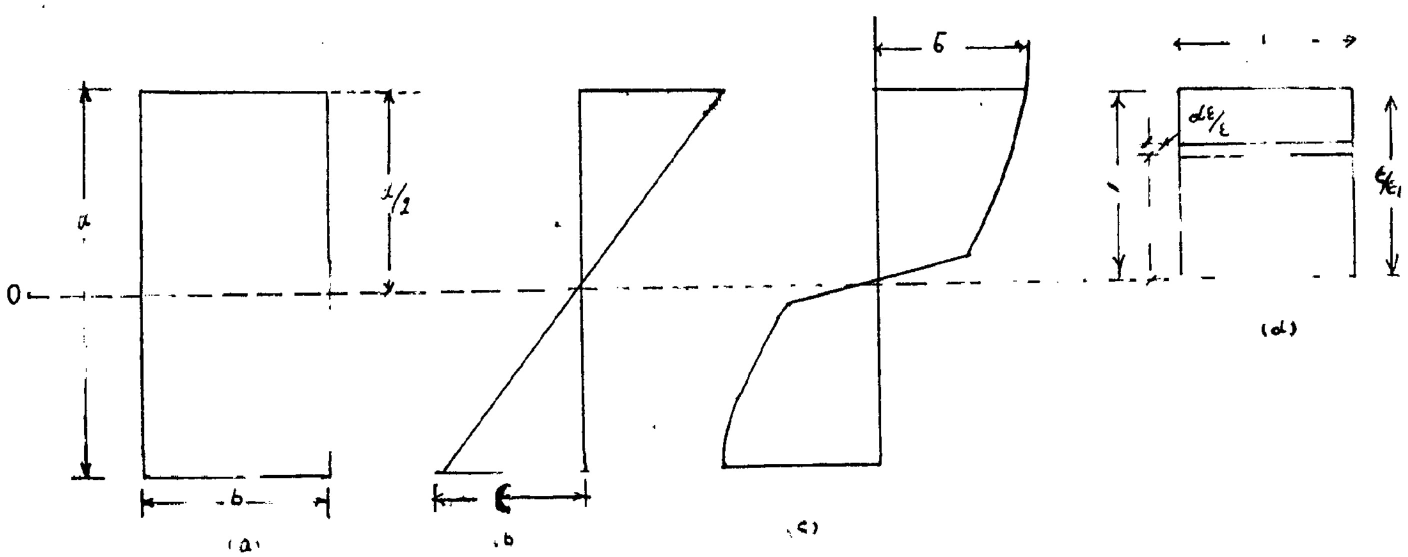


FIG. 3.7

a recourse is taken to be numerical summation.

For various position of B on $\sigma - \epsilon$ curve (graph 3.2) m_1 is found out & tabulated in table No. 3.1

TABLE No. 3.1.

$m_1 - \epsilon$ RELATION FOR RECTANGULAR BEAM

in Percentage.	m_1	Remarks
0	0	
0.11	5.08	yield Point
1.0	7.69	
2.0	7.62	Strain hardening start.
5.0	8.75	
10.0	10.25	
15.0	11.22	
20.0	11.88	
25.0	12.21	
30.0	12.73	
35.0	12.75	
40.0	12.79	
45.0	12.85	
50.0	12.92	
55.0	12.98	
60.0	13.03	
65.0	13.06	
70.0	13.08	
75.0	13.09	
80.0	13.10	Ultimate strain reached.

the beam about the neutral axis,

$$m_1' = \sigma\,(d \times b)\ x.$$

$$= \int b\,\sigma\left(\frac{\epsilon}{\epsilon_1}\, d/2\right) d/2 \,.\, d\epsilon/\epsilon_1$$

$$= \frac{bd^2}{4\,\epsilon_1^2} \int \sigma\,\epsilon \quad d\epsilon \qquad \text{Eqn. (3.4)}$$

If $b = 1$, $d/2 = 1$

$$m_1' = m_1 = \frac{1}{\epsilon_1^2} \int \sigma\,\epsilon\, d\epsilon \qquad \text{Eqn (3.5)}$$

Where the expression, $\int \epsilon\,\sigma\, d\epsilon$, signifies the statical moment of an area under $\sigma - \epsilon$ curve about σ -axis taken to a variable point D.

Hence the moment M of the internal stress about the neutral axis is

$$M = 2\, m_1' = \frac{m_1\, b\, d^2}{2} \qquad \text{Eqn. (3.6)}$$

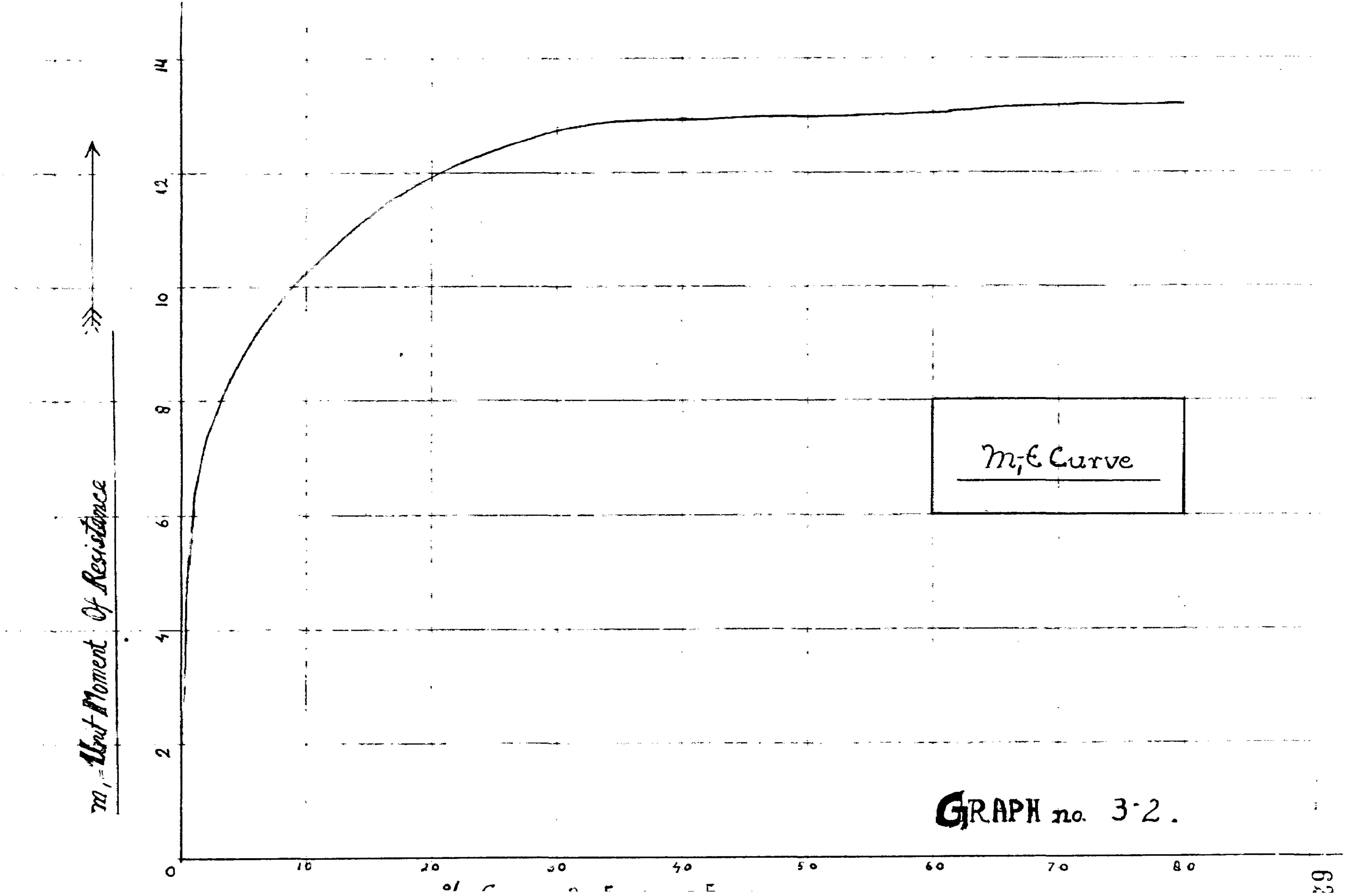
$$= \frac{bd^2}{2\,\epsilon_1^2} \int \epsilon\,\sigma\, d\epsilon$$

The corresponding angle change, $\phi = \frac{2\epsilon_1}{d}$ Eq.n(3.7)

For the solution of equation (3.6) value of m_1

$$m_1 = \int \frac{\epsilon\,\sigma}{\epsilon_1^2}\, d\epsilon$$

is required. Since the relationship between σ & ϵ cannot be expressed explicitly as $\sigma = f(\epsilon)$ for the full range of $\sigma - \epsilon$ curve for the solution of the equation (3.3)


m₁ = Unit Moment Of Resistance
m_1-ε Curve
0
2
4
6
8
10
12
14
10
20
30
40
50
60
70
80

GRAPH no. 3·2.

(b) I-SECTION :-

The I - section is idealised as shown in fig. (3.8) For the sake of convenience in the analysis of the stresses. It is assumed as in the plate girder that the distributionof the stress σ_1 in the flanges is uniform. The error involved in this approximation in the region above the elastic limit is smaller than that involved in the elastic range and can safely be assumed without much loss of accuracy. The area Af of the flange is concentrated at the extremity of the web, which is taken as an area Aw between the centres of the flanges as shown in fig. (3.8) . The accuracy is not much affected as a result of this assumption.

Let $\frac{2Af}{Aw} = K$ (which gives the properties of the section).

for the section shown in fig (3.9) K = Af as Aw/2 = 1 and the unit moment m, is therefore related as

$$m = m_1 + K\sigma_1 = \frac{1}{\epsilon_1^2}\int \sigma\epsilon \, d\epsilon + k\sigma_1 \qquad \text{Equ (3.8)}$$

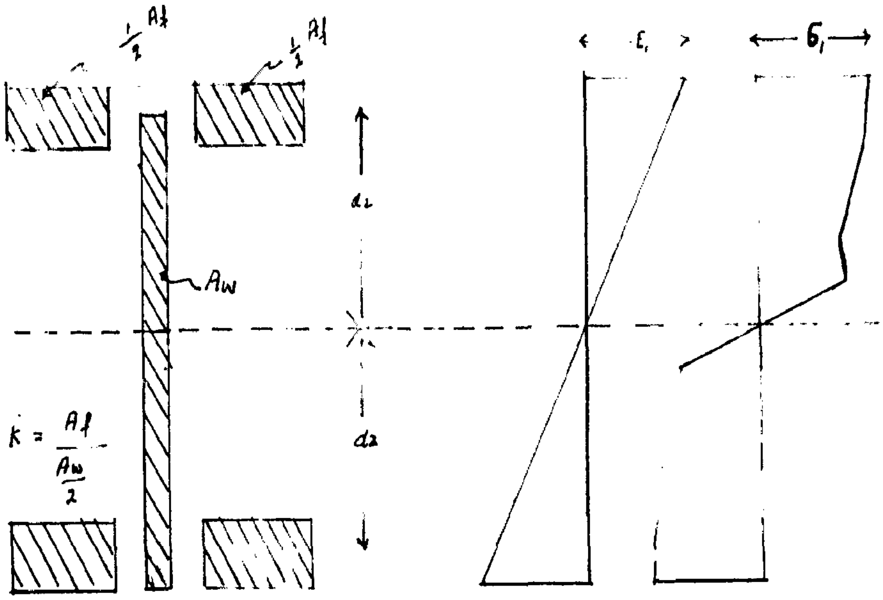
where $m_1 = \frac{1}{\epsilon_1^2}\int \sigma\epsilon \, d\epsilon$

If a beam has a maximum strain of ϵ_1, the moment M is proportional to Awd.

$$M = 2\left(m\frac{Aw}{2}\cdot\frac{d}{2}\right) = m\frac{Awd}{2} = \frac{Awd}{2}(m_1 + K\sigma_1)$$

$$= \frac{Awd}{2}\left(\frac{1}{\epsilon_1^2}\int \sigma\epsilon \, d\epsilon + K\sigma_1\right) \qquad \text{Eqn (3.9)}$$

I SECTION :-



IDEALISED I SECTION (a) STRAIN DIAG. (b) STRESS DIAG. (c)

FIG. 3·8

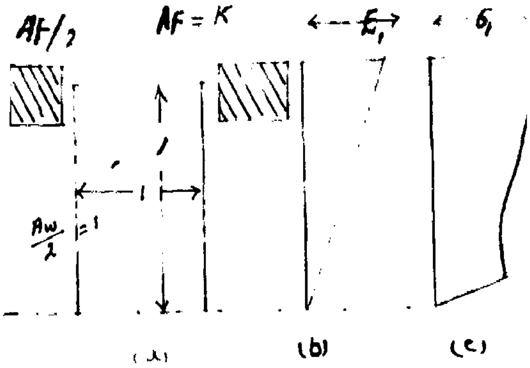


FIG. 3·9

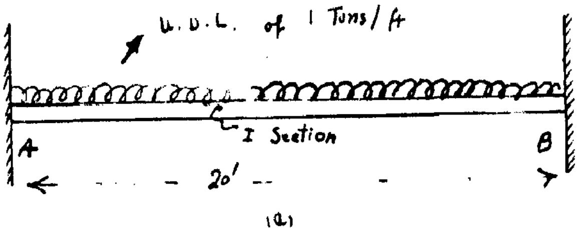
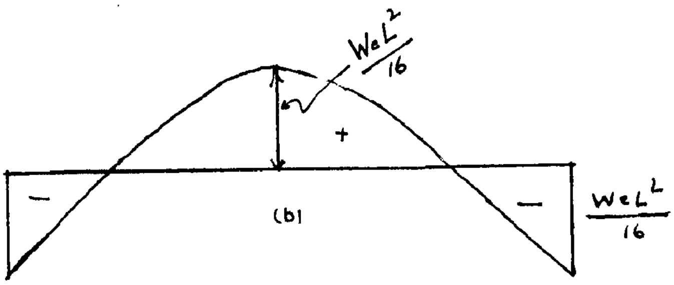
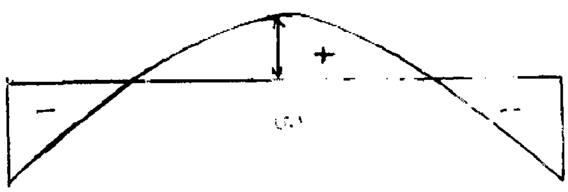


FIG. 3·10



B.M Diag at Collapse



The new function n can be revaluated for the various values of K making use of the table No. 3.2.

Table no. (3.2) gives the value of n for different values of ϵ for four values of K namely 0 , ½ , 1, 1.5. These values of K cover the normal range of I sections. K = 0 corresponds to the case of a rectangular beam. These n – ϵ curves, have been plotted in graph no. (3.3.) For intermediate values of K, either it can be found by interpolation from the above graph or by directly working out from = n (3.3)

Table No. 3.2.

n – ϵ Relation for I beam.

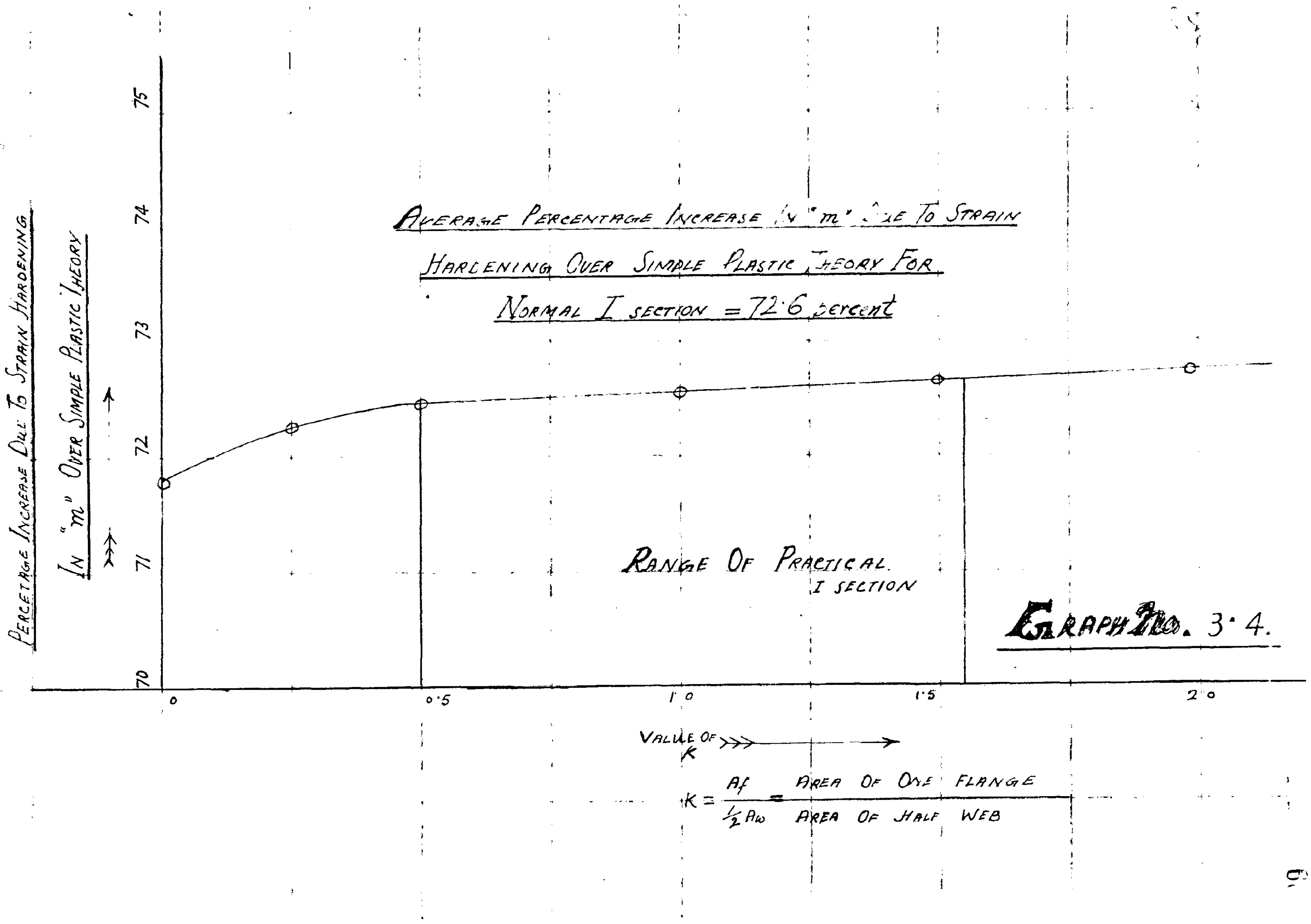
ϵ ‰	n ---- (UNIT OF n = Tons/Inch²)				
	K = 0	K = ½	K = 1	K = 1½	σ_0 in Tons/In²
0	0	0	0	0	0
0.11	5.03	13.7	20.33	27.95	15.25
1.00	7.59	15.21	22.54	30.43	15.25
2.00	7.62	15.24	22.87	30.40	15.25
5.00	8.75	19.45	26.15	37.85	19.40
10.00	10.25	21.75	33.25	44.75	23.00
15.00	11.22	23.63	33.02	49.62	24.00
20.00	11.97	24.94	37.50	50.73	25.94
25.00	12.31	25.23	37.31	51.33	26.10
30.00	12.73	26.03	30.13	52.20	26.40
35.00	12.75	26.05	30.15	51.35	26.40
40.00	12.70	26.00	30.10	52.30	26.40
45.00	12.75	26.05	30.25	51.45	26.40
50.00	12.83	26.12	30.23	52.57	26.40
55.00	12.87	26.17	30.37	52.59	26.40
60.00	13.03	26.23	30.43	53.63	26.40
65.00	13.03	26.23	30.47	53.03	26.40
70.00	13.03	26.28	30.49	53.63	26.40
75.00	13.00	26.30	30.40	53.60	26.40
80.00	13.10	26.30	30.50	53.70	26.40

EFFECT OF STRAIN HARDENING ON VALUE OF m :

Table no. (3.3) gives the %age increase in value of m due to strain hardening over simple plastic theory for different values of K. For rectangular section percentage increase in value of m is 71.8 and for K between 0.5 to 1.5 (this covers the normal range of I-sections) it is = 73.5 to 73.73 . Average value for I section may be taken as 73.5%. Graph no. (3.4) shows a vivid picture of the %age increase in value of m due to strain hardening with respect to ratio K.

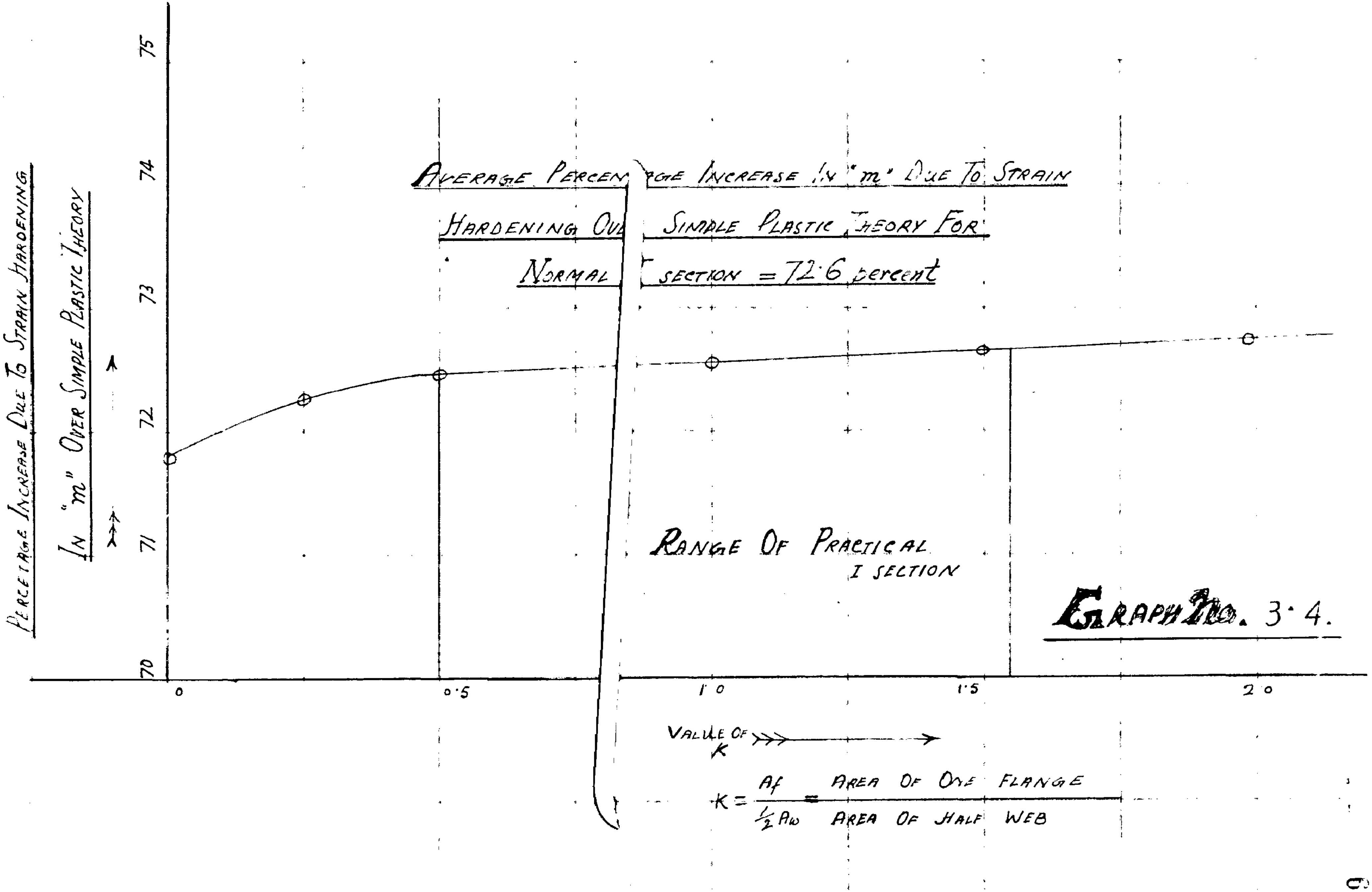
Table No. 3.3.

K	m		% increase in m	REMARKS
	by simple Plastic Theory.	with strain Hardening taken into account		
0	7.625	13.10	71.8	Rectangular Beam
0.5	15.25	26.50	73.5	k = 0 to K = 1.5 covers normal range of I section.
1.0	23.375	40.50	73.6	
1.5	30.90	53.70	73.7	
2.0	38.125	66.20	73.73	[illegible]



AVERAGE PERCENTAGE INCREASE IN "m" DUE TO STRAIN HARDENING OVER SIMPLE PLASTIC THEORY FOR NORMAL I SECTION = 72·6 percent
PERCENTAGE INCREASE DUE TO STRAIN HARDENING IN "m" OVER SIMPLE PLASTIC THEORY
75
74
73
72
71
70
0
0·5
1·0
1·5
2·0
RANGE OF PRACTICAL I SECTION
VALUE OF K
K = Af / ½Aw = AREA OF ONE FLANGE / AREA OF HALF WEB

GRAPH No. 3·4.


PERCENTAGE INCREASE DUE TO STRAIN HARDENING
IN "m" OVER SIMPLE PLASTIC THEORY
70
71
72
73
74
75
0
0·5
1·0
1·5
2·0
AVERAGE PERCENTAGE INCREASE IN "m" DUE TO STRAIN
HARDENING OVER SIMPLE PLASTIC THEORY FOR
NORMAL I SECTION = 72·6 percent
RANGE OF PRACTICAL
I SECTION
VALUE OF K
K = Af / ½ Aw = AREA OF ONE FLANGE / AREA OF HALF WEB

GRAPH No. 3·4.

As an example for finding the extent of moment redistribution possible, we will first take for understanding the problem, a simpler case of a fixed beam as shown in Fig. (3.10a) The span of the beam is 20 ft and section used will be an I-section.

Assuming a load factor of 1.75, the maximum free bending moment is equal to 1050 ton inches. P.M.R. of beam required = 525 ton inches.

$$\text{Required} \quad Z_p = \frac{525}{15.25} = 34.5 \text{ in}^3$$

Assuming a shape factor = 1.14 for I-section

$$Z_0 = \frac{34.5}{1.14} = 30.3 \text{ in}^3$$

An I-section 12" x 5" @ 30 lbs/rft with Z_0 = 34.49 in^3 will be adopted.

This gives a total weight of 600 lbs. The actual collapse load given by simple plastic theory for this section used = 2.0 Tons/rft.

<u>RELATION BETWEEN MOMENT AND DEFORMATION:</u>

For finding M - ϕ relation for 12" x 4" I-Section, equations (3.1) & (3.3) will be used. Referring to fig. (3.4) in present case, b = 5", t_1 = 0.507" ; t_2 = 0.35", d = 12", d_1 = 10.986", d_2 = 11.493". Assuming ultimate strain = 30%, the maximum value of ϕ is equal to

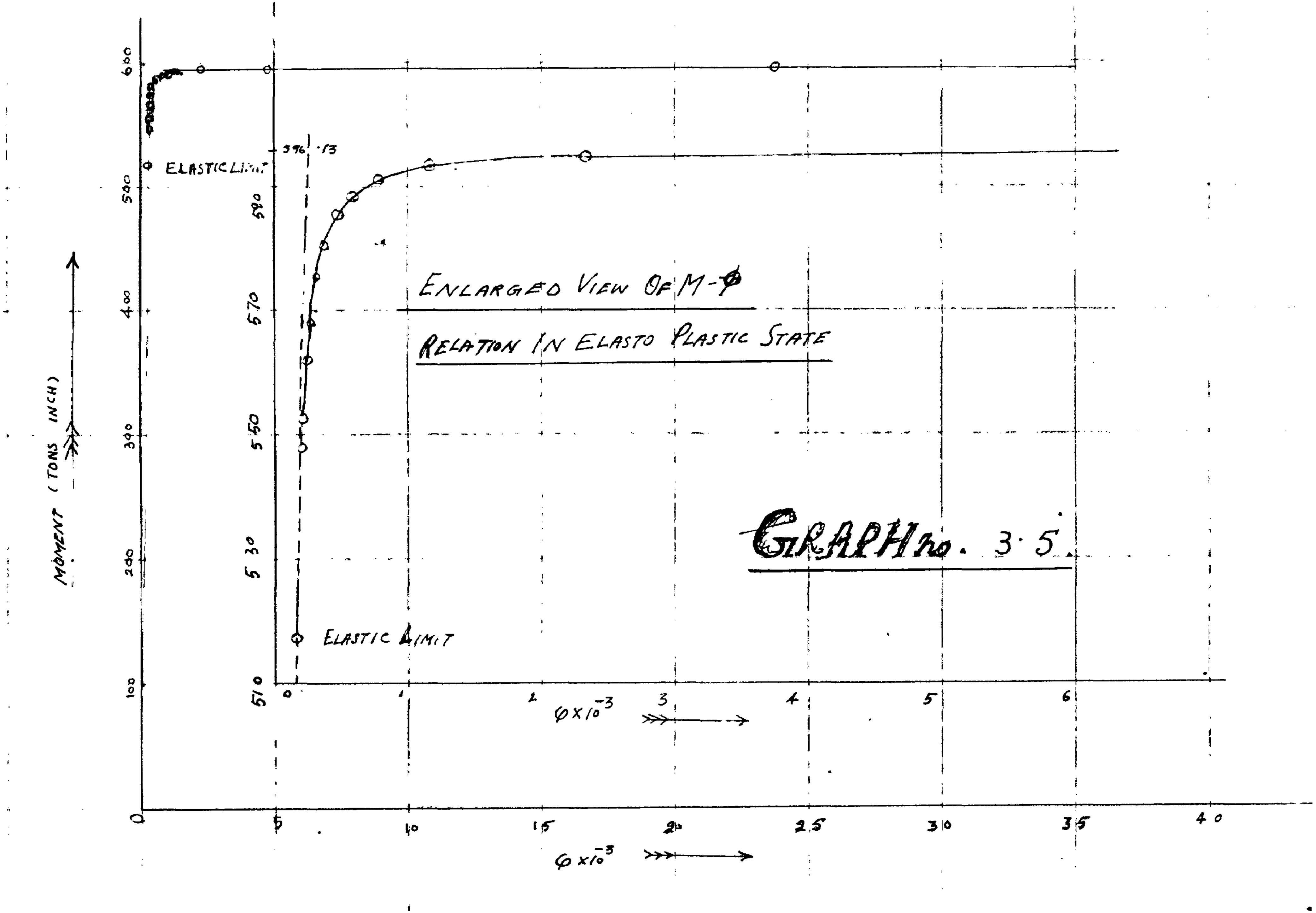
50×10^{-3} radians. The value of q corresponding to this from equation (3.1.) is 0.047" . For values of q less than this, the ultimate strain of material will be exceeded in the extreme fibres. Using equation (3.1) and 3.3 the relation between M and β is found out as given in Table No. (3.4.) . This has been plotted in graph No. (3.5.) Equation (3.3.) is reduced for this section as

$$M = 15.25 (39.083 - 0.0275 q^3).$$

Table No. 3.4.

M - β RELATION FOR 18" x 6" I-BEAM

q (inches)	M (tons)	β ($\times 10^{-3}$ radians)
12.	510.0	0.186
11	540.8	0.214
10	564.4	0.235
9	582.9	0.231
8	590.2	0.294
7	576.0	0.336
6	591.0	0.393
5	596.1	0.470
4	599.6	0.597
3	602.0	0.789
2	604.1	1.175
1	606.0	2.360
0.5	605.90	4.7
0.1	605.98	23.5
0.047	606.19	50.0



ENLARGED VIEW OF M-Φ
RELATION IN ELASTO PLASTIC STATE
GRAPH no. 3·5.
ELASTIC LIMIT
ELASTIC LIMIT
596·13
MOMENT (TONS INCH)
φ x 10^-3
φ x 10^-3

EXTENT OF REDISTRIBUTION POSSIBLE :

At first plastic hinges are formed at the ends and then if load is increased a third hinge is formed in the centre. Let us check the available and required angle of plastic rotations when the moments at ends = P.M.R of the section and the moment at the centre = M_o. The moment diagram at this stage is given by Fig. (3.10)c. The load at this stage as given by statics is 1.863 Tons/oft. Elastic theory gives a moment = 743.2 Tons inch at ends. The ends must develops an angle of discontinuity = θ for the moment to decrease from 743.2 Ton inches to 503.13 ton inches. Using slope deflection equation the angle of discontinuity required is

$$= \frac{1}{2EI} \text{ (Elastic moment - Actual Moment)}$$

Here EI = 23.9 x 10^5 inch^2 Ton.

$$\theta \text{ reqd.} = \frac{20 \times 12}{2 \times 23.9 \times 10^5} \, (743.2 - 503.13)$$

$$= 6.55 \times 10^{-3} \text{ radians.}$$

Available rotation is found out by finding $\Sigma \Delta \phi$ dx with the help of moment diagram and M - ϕ curve of the beam, in a tabular form Table (3.5.).

TABLE NO. 3.5.

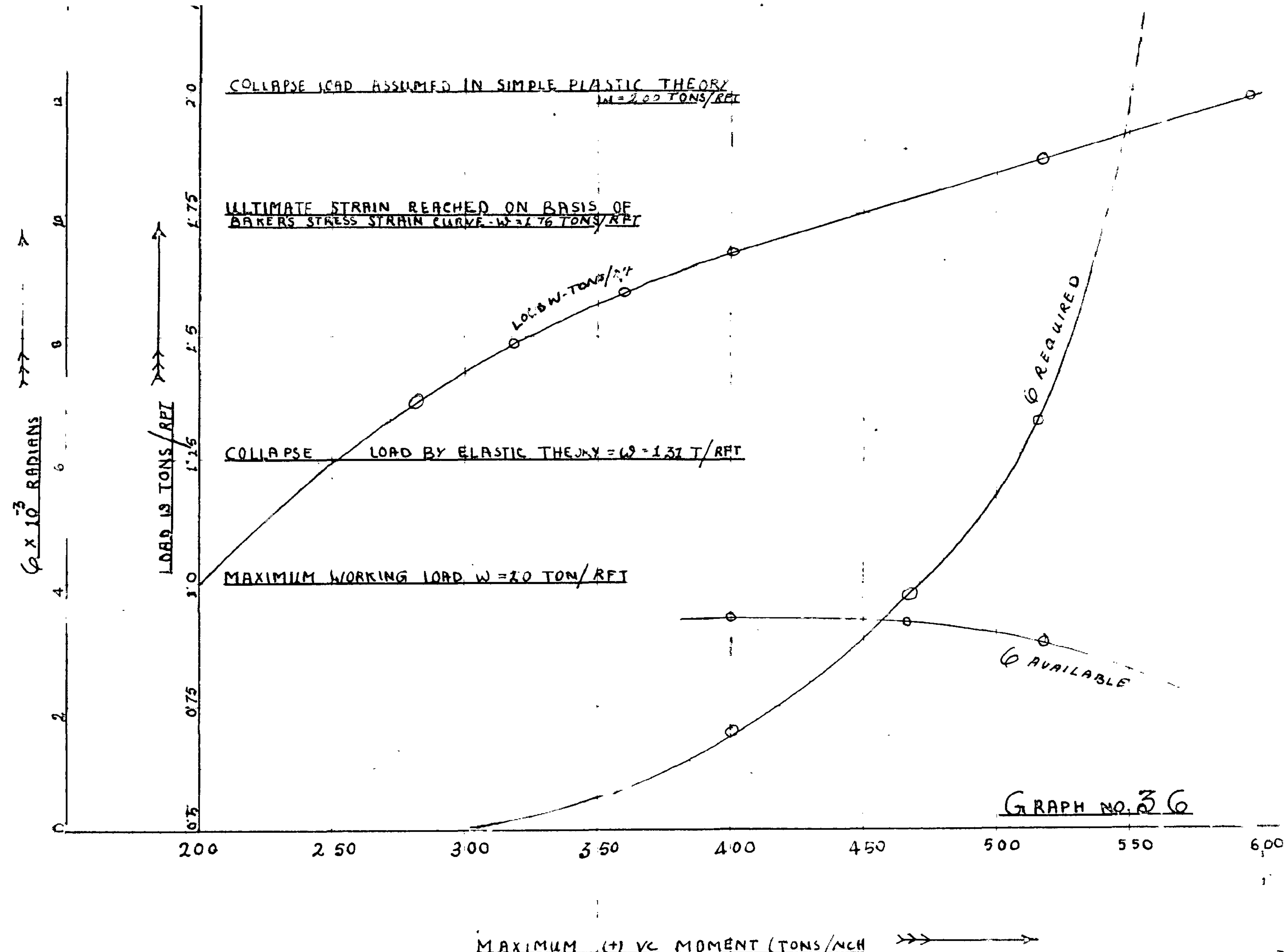
PLASTIC ROTATION AT SUPPORTS.

Distance from support in inches.	Moment Tons Inches.	$\phi_A \times 10^{-3}$ Radians	$\phi_B \times 10^{-3}$ Radians	$\times 10^{-3}$ Radians
0	596.13	50	0.222	49.778
0.1	594.272	1.397	0.221	1.176
0.2	592.417	0.857	0.220	0.637
0.3	590.563	0.591	0.219	0.372
0.4	588.710	0.551	0.218	0.333
0.5	586.859	0.478	0.217	0.261
1.0	577.624	0.337	0.214	0.123
1.5	568.427	0.290	0.211	0.079
2.0	559.236	0.251	0.208	0.043
2.5	550.143	0.215	0.204	0.011
3.0	541.053	0.208	0.201	0.007
3.5	531.870	0.201	0.197	0.004
4.0	522.01	0.195	0.194	0.001
4.8	510.03	0.193	0.193	0.000

Using mean ordinate method

$$\theta_s = 2.059 \times 10^{-3} \text{ radians.}$$

Now the required rotation is = 6.65×10^{-3} radians whereas available is only 2.059×10^{-3} radians. This means even this much of redistribution of moments is not possible theoretically on the basis of Baker's assumed idealised stress - strain curve. The required angle of discontinuity will very rapidly increase after plastic yield has taken place in the positive moment zone. Graph (3.6) has been drawn to show how the required and availabl


COLLAPSE LOAD ASSUMED IN SIMPLE PLASTIC THEORY
W = 2.00 TONS/RFT
ULTIMATE STRAIN REACHED ON BASIS OF
BAKER'S STRESS STRAIN CURVE - W = 1.76 TONS/RFT
LOAD W-TONS/R.F.
COLLAPSE LOAD BY ELASTIC THEORY = W = 1.31 T/RFT
MAXIMUM WORKING LOAD W = 1.0 TON/RFT
θ REQUIRED
θ AVAILABLE
θ x 10⁻³ RADIANS
LOAD IS TONS/RFT
MAXIMUM (+) VC MOMENT (TONS/NCH
GRAPH NO. 36
200
250
300
350
400
450
500
550
600
0.5
0.75
1.0
1.25
1.5
1.75
2.0
0
2
4
6
8
10
12

plastic rotations at supports vary with respect to maximum (+) ve moment. The corresponding values of load has also been plotted. It will be noticed that ultimate strain is reached at X load W_0 = 1.73 Tons/sft., whereas simple plastic theory gives collapse load for this beam = 2.0 tons/sft. This means that failure will occur at a load much less than the collapse load given by simple plastic theory if Baker's stress-strain curve is true, Also the required moment redistribution is not possible on the basis of this curve.

Now the extent of moment redistribution for the same example will be reworked by using actual stress-strain curve.

<u>M-Φ RELATION FOR 12" x 5" I-SECTION.</u>

Using equation (3.7) & (3.9)

$M = 21.83 (m_1 + 1.335)$ Ton inches Equ (3.10)

$\phi = 1.74 (\epsilon \beta) \times 10^{-3}$ radian Equ (3.11)

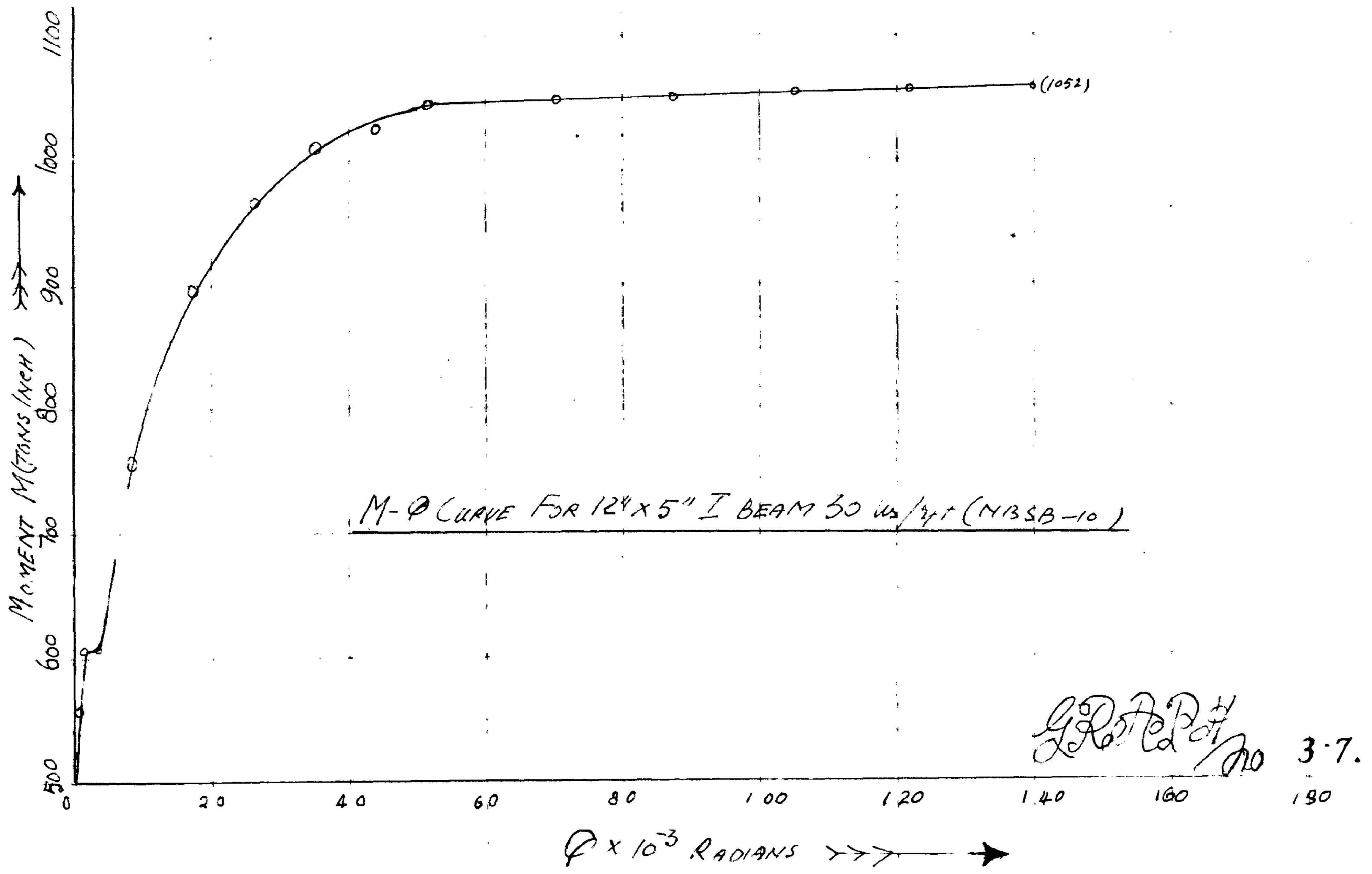
Using these equations with the help of $m - \epsilon$ curve values of m and ϕ are calculated for different values of ϵ. Table 3.6) and Graph No.(3.7).

TABLE No. 3.6

β	$\phi \times 10^{-3}$ radians	Moment ton inches.	Remarks.
0.11	0.21	554	Elastic limit moment Mo
1.00	1.74	605	
2.00	3.48	608	Strain hardening starts.
5.00	8.70	753	
10.00	17.4	898	
15.00	26.1	934	
20.00	34.8	1008	
25.00	43.5	1021	
30.00	52.2	1042	
40.00	69.6	1046	
50.00	87.00	1048	
60.00	104.4	1050	
70.00	121.8	1052	
80.00	139.2	1052	Ultimate strain reached in extreme fibre.

DETERMINATION OF LOAD AT WHICH ULTIMATE STRAIN IS REACHED.

Let us check the hinge rotations when maximum +(ve) moment = 700 ton inches and support moment = 1052 ton inches. By statics load at this stage = 2.82 tons/sft. Elastic moment for this load, at the ends = 1139 ton inches.


M-φ CURVE FOR 12" x 5" I BEAM 30 lbs/ft (NBSB-10)
MOMENT M (TONS INCH)
φ x 10^-3 RADIANS
(1052)
500
600
700
800
900
1000
1100
0
20
40
60
80
100
120
140
160
180
No 3·7.

Angle of discontinuity required at ends A & B in order that the end moments decrease from 1137 to 1053 tons inches.

$$= \frac{20 \times 12}{3 \times 23.0 \times 10^{6}} (1137 - 1053)$$

$$= 5.20 \times 10^{-3} \text{ radians.}$$

Besides this the required angle of discontinuity at support will also be increased due to plastic rotation in (+) ve moment zone. Tables No. (3.7) (3.8) gives the plastic rotations in (+)ve and (-)ve moment zones.

TABLE No. 3.7.

PLASTIC ROTATIONS IN +(VE) MOMENT ZONE.

Distance in inches from supports.	Moment Ton inches.	$\theta A \times 10^{-3}$ radians	$\theta B \times 10^{-3}$ radians.	$\theta \times 10^{-3}$ radians.
86	647	0.21	0.21	0
90	662	0.90	0.23	0.67
96	683	4.0	0.24	3.76
103	688	5.9	0.25	5.65
109	694	6.1	0.23	5.94
114	697	6.8	0.27	6.53
120	700	7.0	0.27	6.73
114	697	6.8	0.27	6.53
109	694	6.1	0.23	5.94
103	688	5.9	0.25	5.65
96	683	4.0	0.24	3.76
90	662	0.9	0.23	0.67
86	647	0.21	0.21	0

Using mean ordinate method

$$\theta = \Sigma \Delta\phi \, ds = 303.01 \times 10^{-3}$$

$\therefore$ Total θ required at support = $(154.0 + 5.20) \times 10^{-3}$

$$= 159.2 \times 10^{-3} \text{ radians.}$$

TABLE NO. 3.8

PLASTIC ROTATION IN (-)VE MOMENT ZONE.

0.00	1052	139.2	0.40	138.80
0.5	1037	48.0	0.39	47.61
1.0	1023	41.8	0.39	41.41
1.5	1009	34.8	0.38	34.42
2.0	994	30.1	0.37	29.73
4.0	939	22.0	0.35	21.65
6.0	881	16.0	0.33	15.67
8.0	826	12.1	0.31	11.79
10.0	772	9.6	0.29	9.31
12.0	720	7.5	0.27	7.23
14.0	668	6.0	0.25	5.75
16.0	617	3.9	0.23	3.69
18.0	567	0.6	0.21	0.39
20.0	554	0.21	0.21	0

$\therefore \Sigma \Delta\phi \, ds = 300.50 \times 10^{-3}$ radians

Hence the available rotation is much more than required It will take some more load.

Now let us check when the maximum +(ve) moment = 980 ton inches and support moment = 1063 ton inches. By statics the load at this stage = 3.17 tons/sft and moments at the ends by Elastic theory for this load = 1239 tons inches. Angle of discontinuity required for the decrease in moment from 1239 to 1063 ton inches

$$= \frac{20 \times 18}{3 \times 30.0 \times 10^3} (1239 - 1063) = 0.7 \times 10^{-3} \text{ radians.}$$

As previously plastic rotations in +(ve) and (-) ve moment zones were calculated and they come out to be:

Plastic rotation at centre = 323 $\times 10^{-3}$ radians.

Plastic rotation at ends = 273 $\times 10^{-3}$ radians.

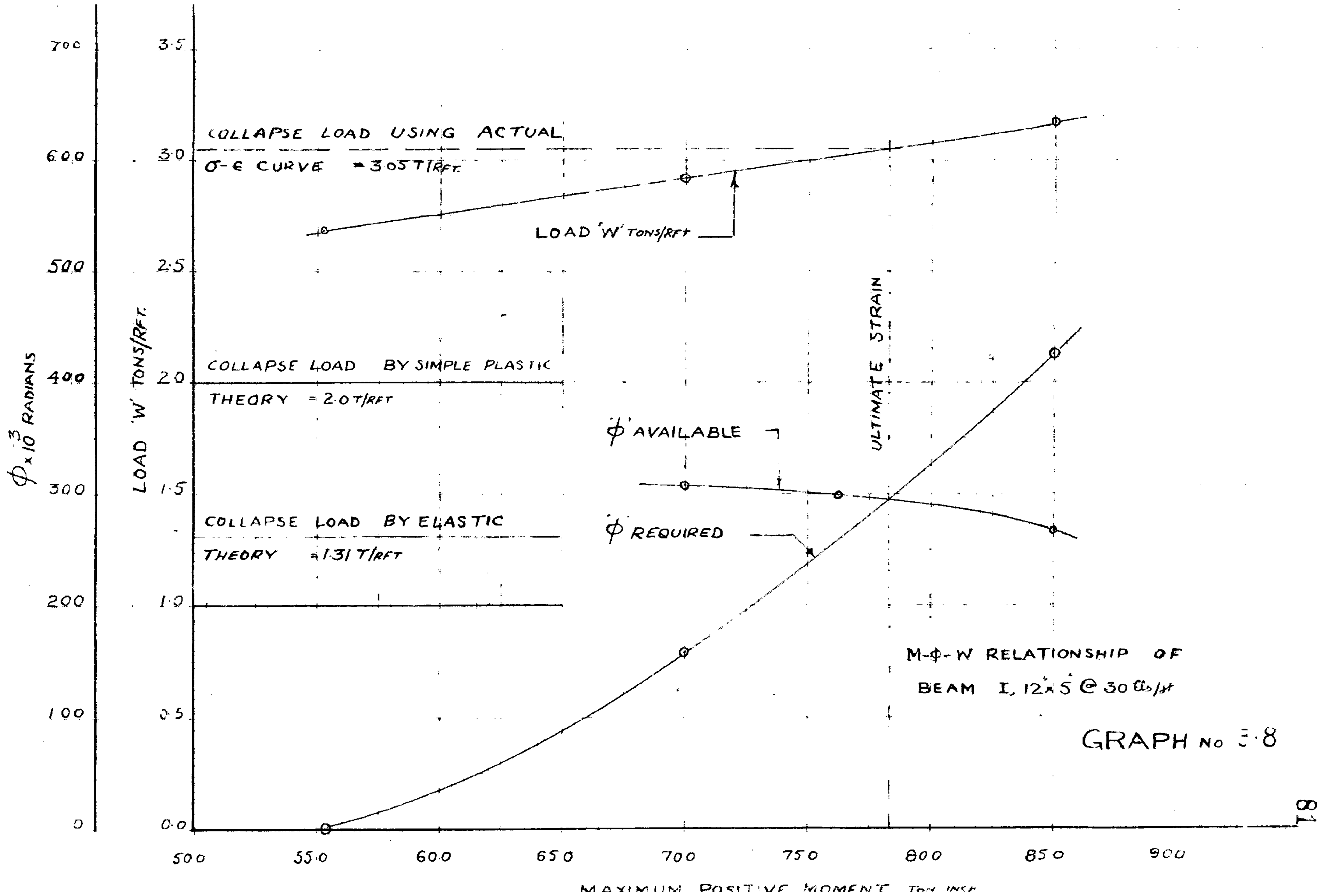
Now required rotation at support = $(425.3 + 0.7) \times 10^{-3}$

$= 426.0 \times 10^{-3}$ radians.

This is much more than that is available. Hence this much redistribution is not possible. Table No. (3.9) gives a summary of how the hinge rotations required and available varies for different values of maximum (+)ve moment. It has also been plotted in graph No. (3.8) From this graph the collapse load is found out to be 3.06 Tons/sft.

TABLE NO. 3.9

Maximum (+)ve Moment Ton inches.	Load Tons/sft.	∅ required $\times 10^{-3}$ radians.	∅ available $\times 10^{-3}$ Radians.
854		required starts from here.	
700	3.03	160.3	330.6
980	3.17	425.7	333.0


COLLAPSE LOAD USING ACTUAL
σ-ε CURVE = 3.05 T/RFT.
LOAD 'W' TONS/RFT
COLLAPSE LOAD BY SIMPLE PLASTIC
THEORY = 2.0 T/RFT
COLLAPSE LOAD BY ELASTIC
THEORY = 1.31 T/RFT
ϕ AVAILABLE
ϕ REQUIRED
ULTIMATE STRAIN
M-ϕ-W RELATIONSHIP OF
BEAM I, 12" x 5" @ 30 lbs/ft
ϕ x 10^-3 RADIANS
LOAD 'W' TONS/RFT.
MAXIMUM POSITIVE MOMENT TON INCH
0
100
200
300
400
500
600
700
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
500
550
600
650
700
750
800
850
900

GRAPH No 3.8

$$\text{Also}\ \frac{\text{Collapse load using actual stress-strain curve}}{\text{Collapse load as given by simple plastic theory.}}$$

$$= \frac{3.05}{2.00}$$

$$= 1.525.$$

8 In the example just considered, we have observed that while required redistribution of moments is not possible if simple plastic theory is taken to be true, but still the ultimate load corresponding to ultimate strain in any section of the member, based on actual stress-strain curve, is much more than that calculated on the basis of simple plastic theory. This means that results given by simple plastic theory are on safer side. For more elementary cases such as (i) a fixed beam of 10ft. span, and of rectangular section, (ii) Rectangular beam continuous over one support, and (iii) Single storey single bay rectangular portal of I-Section, with columns hinged at the base ;

have been worked out, and same conclusion have been drawn.

It was proposed now to investigate a multistoreyed multi-bay portal frame, for the extent of redistribution of moments, because the failure of such frame as a mechanism involves a quite large number of hinges. It is just possible the ultimate load corresponding to ultimate strain based on actual stress-strain theory. It was also proposed that such frame should be tested experimentally to verify the results obtained theoretically.

For this purpose a 6 storeyed, 2 bay frame shown in fig (4·2) was taken. The various dimensions and loading are shown in the figure. For simplicity in calcula-tions and ease in fabrication of the frame, uniform section was adopted through out. Neglecting the effect of axial and shear forces, the plastic moment of resistance will be uniform for all the members.

Collapse load value of W_c according to simple plastic theory as calculated in Appendix C for this frame = 128 lbs.

$$\left[W_c = \frac{1}{1.613}\,\frac{(.165)(.383)^2}{4} \times 15.25 \times 2240 = 128 \text{ lbs} \right].$$

The mode of failure and the number of Hinges formed has all been discussed in Appendix C. We will now proceed on to find the load at which ultimate strain is reached in section A, the most highly stressed point according to the Elastic theory.

Let us take the value of W = 140 lbs. The corresponding moments calculated by Elastic theory, throughout the frame are shown in fig (3.11). The bending moment diagram for the entire frame is shown in fig (3.11).

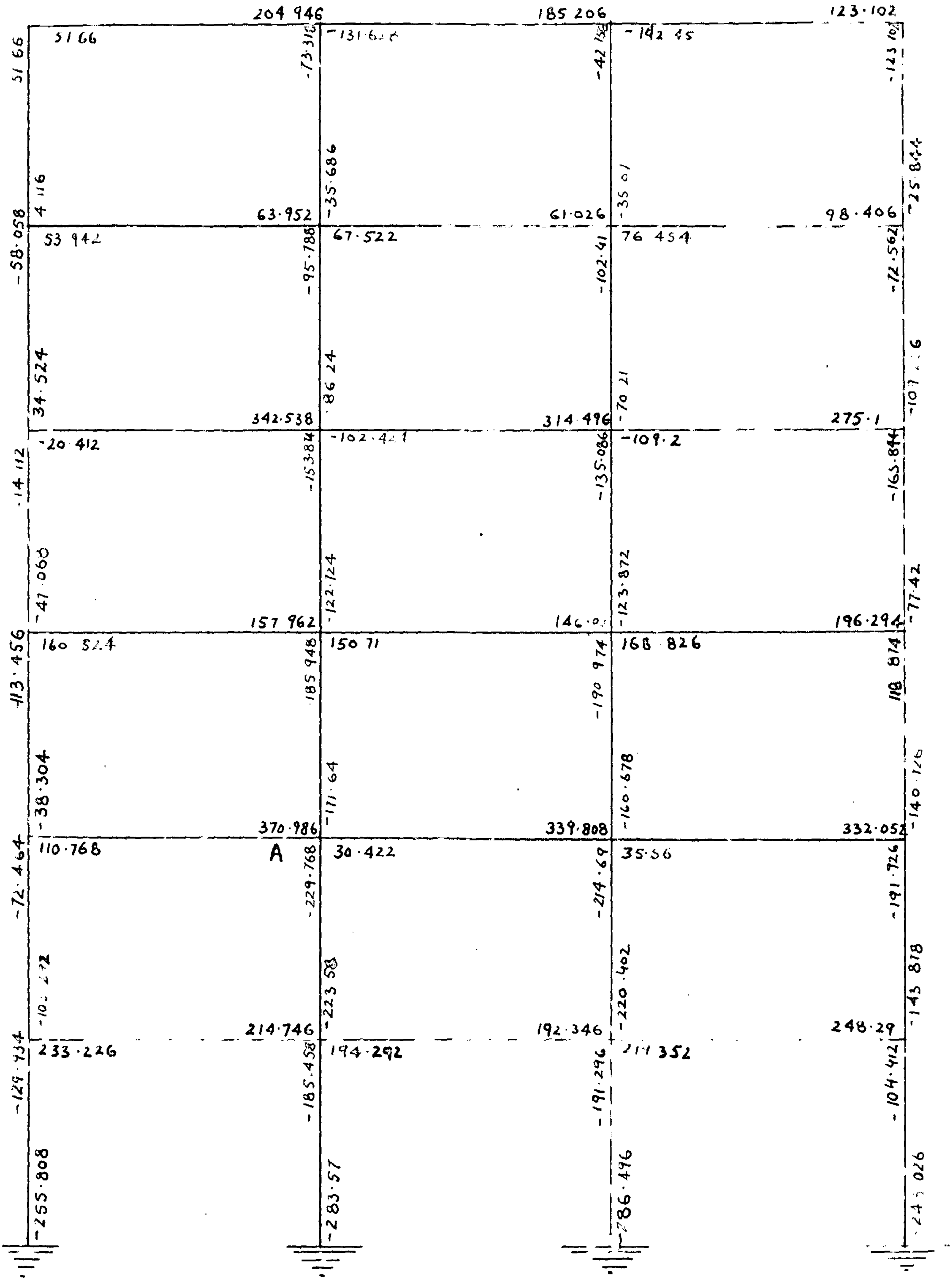
The ultimate moment of resistance of the section

$$= \frac{\sigma_\lambda\, bd^2}{2}$$

$$= \frac{13.1 \times 0.165 \times 0.383^2}{2} \times 2240$$

$$= 355.113 \text{ inch lbs.}$$

Now the elastic moment at section A is 370.823 inch lbs. So angle of discontinuity must develope at this section in order to keep the bending moment at this section



204 946
185 206
123·102
63·952
61·026
98·406
342·538
314·496
275·1
157 962
196·294
370·986
339·808
332·052
A
214·746
192·346
248·29

FIG. 3·11

in order to keep the bending moment at this section equal to 366.116 in. lbs. Let this angle be called δck.

Besides δck, the plastic rotations available at the section A must also cater for the plastic rotations encountered in the positive moment zones.

First of all we will find the value of δck. δck can not be found as easily as was done in previous example. For this purpose use will be made of δik method. This method is more convenient especially for complicated frame like present one. It has been adopted here from a thesis submitted by Dr. O. P. Jain to University of London 1953. (Ref. 30). This method will be explained now.

δik METHOD FOR FINDING ANGLE OF DISCONTINUITY AT PLASTIC HINGES:

For using this method the frame is made statically determinate by introducing sufficient no. of imaginary hinges along with the external moments applied at them equal to the moments existing at these sections. The points at which the angles of discontinuity is required should be included in these hinges. This modification of the frame for purposes of calculation does not make any difference in the distribution of moments, thrusts and shear forces in the frame as any section can be converted into a hinge provided an external moment equal to that existing at that section is applied at the hinge. The frame is now statically determinate and the external of the structure due to a unit force or moment at K. If the force or the moment at K is Q instead of unity then the rotation of the hinge I is $\delta_{IK} = - Q \int M_I M_K \frac{ds}{EI}$.

This point K can be anywhere and even at the hinge I itself. Thus to obtain the total rotation of hinge at I the moments M_K should be calculated for all external loads acting on the frame i.e. vertical loads, horizontal loads and the moments at the assumed hinges including I. The quantity $Q \int M_I M_K \frac{ds}{EI}$

should be evaluated separately for each of the loads and then the results added up algebraically to combine the effects of all loads. If the quantity M_I or M_K is zero at any point, the product is zero and such points may be left from the integration. The sign of the integrated product will be positive if M_I and M_K are both of the same sign. If they are of opposite sign, the integrated product will be negative. It will be seen that the quantity $\int M_I M_K \frac{ds}{EI}$ will work

out to zero for a hinge that is not a plastic hinge or a real hinge and is only an imaginary hinge assumed for these calculations i.e. the angle of discontinuity then will be zero as it should be. The integration of the quantity $\int M_I M_K \frac{ds}{EI}$

for a particular member can be done more easily if at least one of the moments M_I or M_K varies according to the straight line law. Fig. No. (3·12) shows a member of the frame giving the distribution on it of moments M_K and M_I. As these

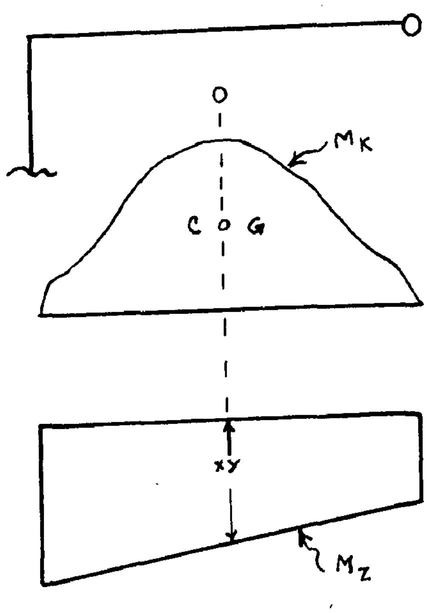


Fig. 3.12

moments are drawn on opposite sides of the members, they indicate opposite sign. Let the moments MI vary according to a straight line law. Let O be the C.G. of MK diagram and XY be the ordinate of the MI diagram at the same point of the member at which the C.G. of the MK diagram lies. Then it can be shown the $\int MK \; MI \; \frac{ds}{EI} = \frac{1}{EI} \times$ (Area of MK diagram X ordinate XY . $\therefore \; \gamma_{AK} = \sum \frac{Q}{EI} \times$ (Area of MK diagram x ordinate XY) (eqn 3.12)

where Q is the magnitude of force on moment at K. Equation (3.12) must be evaluated separately for each of the loads. (external loads and moments at assumed hinges) and then added algebraically.

ANGLE OF DISCONTINUITY AT A

Coming back to our problem in order that the ultimate strain is not exceeded at hinge A, the following conditions must be satisfied.

$\theta_1 \quad \theta_2 + \gamma_{AK}$

Where θ_1 = Plastic rotation available in the +ve moment zone of beam AB.

θ_2 = Plastic rotation required in the +ve moment zone of beam AB.

γ_{AK} = Angle of discontinuity required at A to keep the moment at A = 355.116 inch lbs.

Now let us try to evaluate $\theta_1 - \theta_2$ and δ_{AK} when each vertical load is 140 lbs and correspondingly each horizontal load is 70 lbs. The B.M. at A by elastic theory for these loads is 370.936 inch lbs. The bending moment distribution throughout the frame is shown on page 85 Fig. 3.11 But the ultimate moment is = 355.116 inch lbs. To get the actual B.M. diagram at this load a hinge was introducted at A and a hogging moment of unity was introduced at the hinge A. Corresponding bending moment are shown on page. 92 . Fig 3.13 + 3.14 Bending moments throughout the frame for a sagging moment of 15.87 in lbs. at the hinge C were obtained by simple multiplication and are shown on the page 94. Fig 3.15 The actual B.M. are the algebraic sum of the B.M. diagrams of pages 85 and 94 and are shown on the page 95 . With the help of this diagram and the $M - \theta$ curve for the rectangular section used for the frame (Table No. 3.10 and Graph No. 3-9). The values of θ_1 and θ_2 are calculated in Table No.s (3.11) and (3.12) respectively.

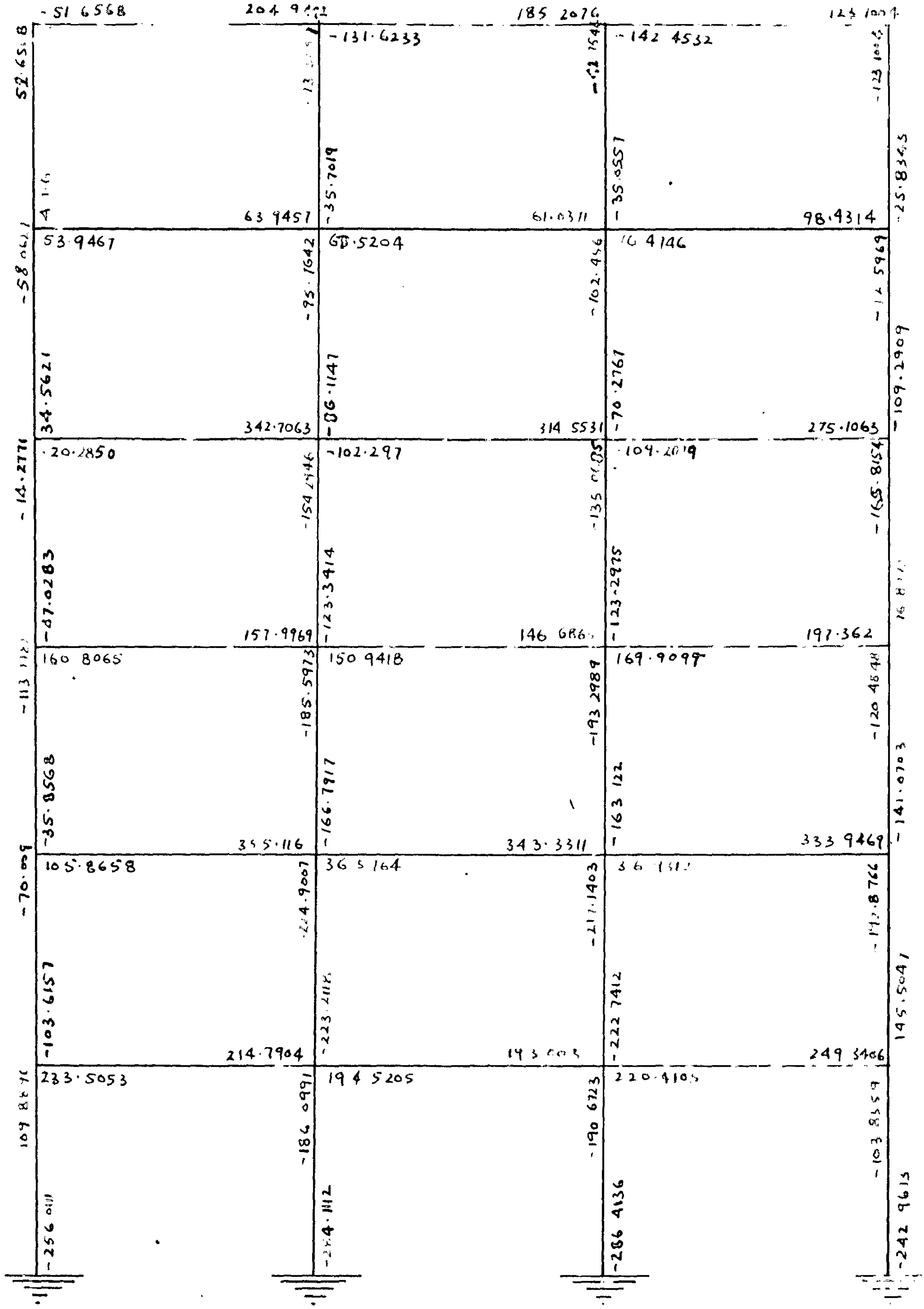


FIG 3.16

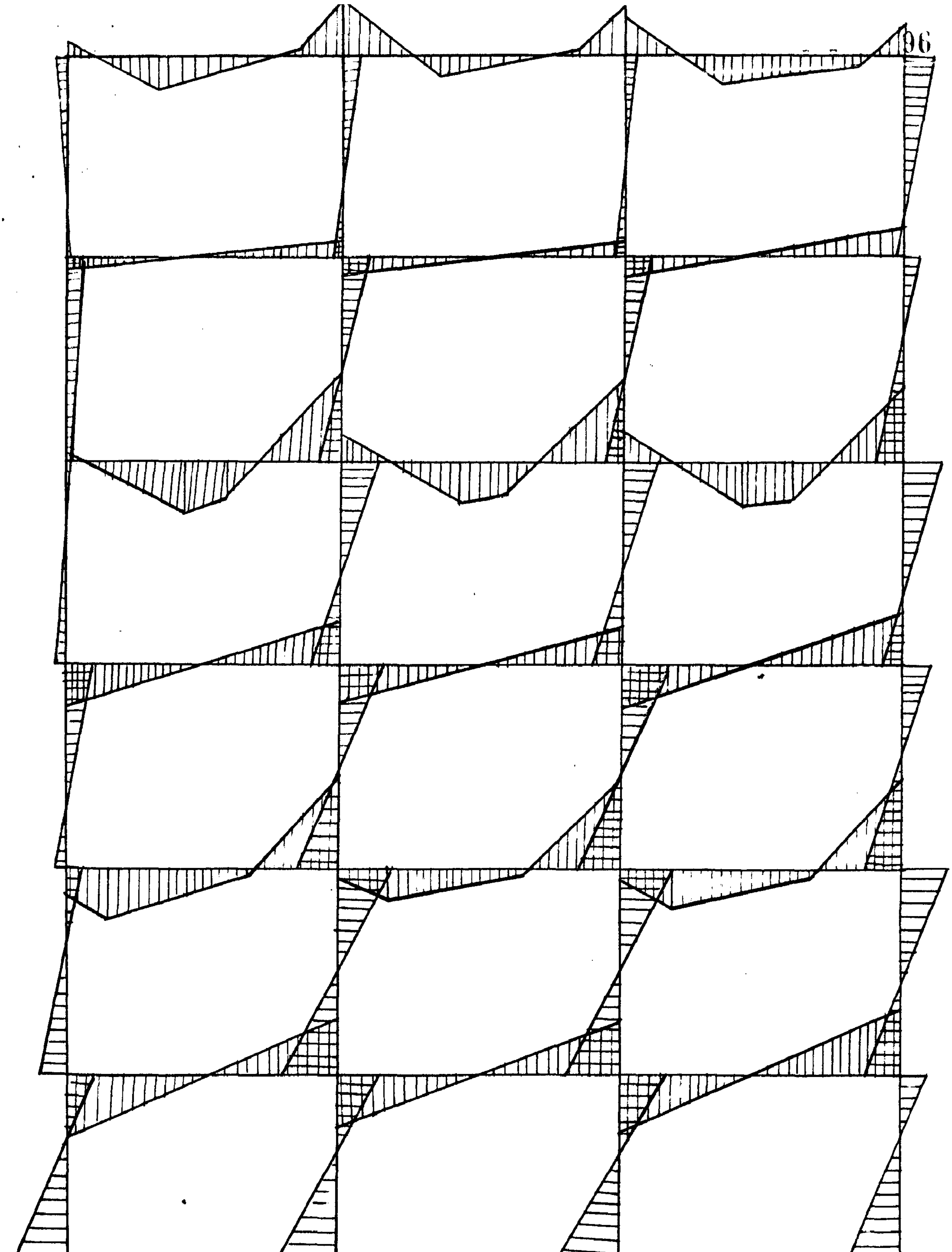
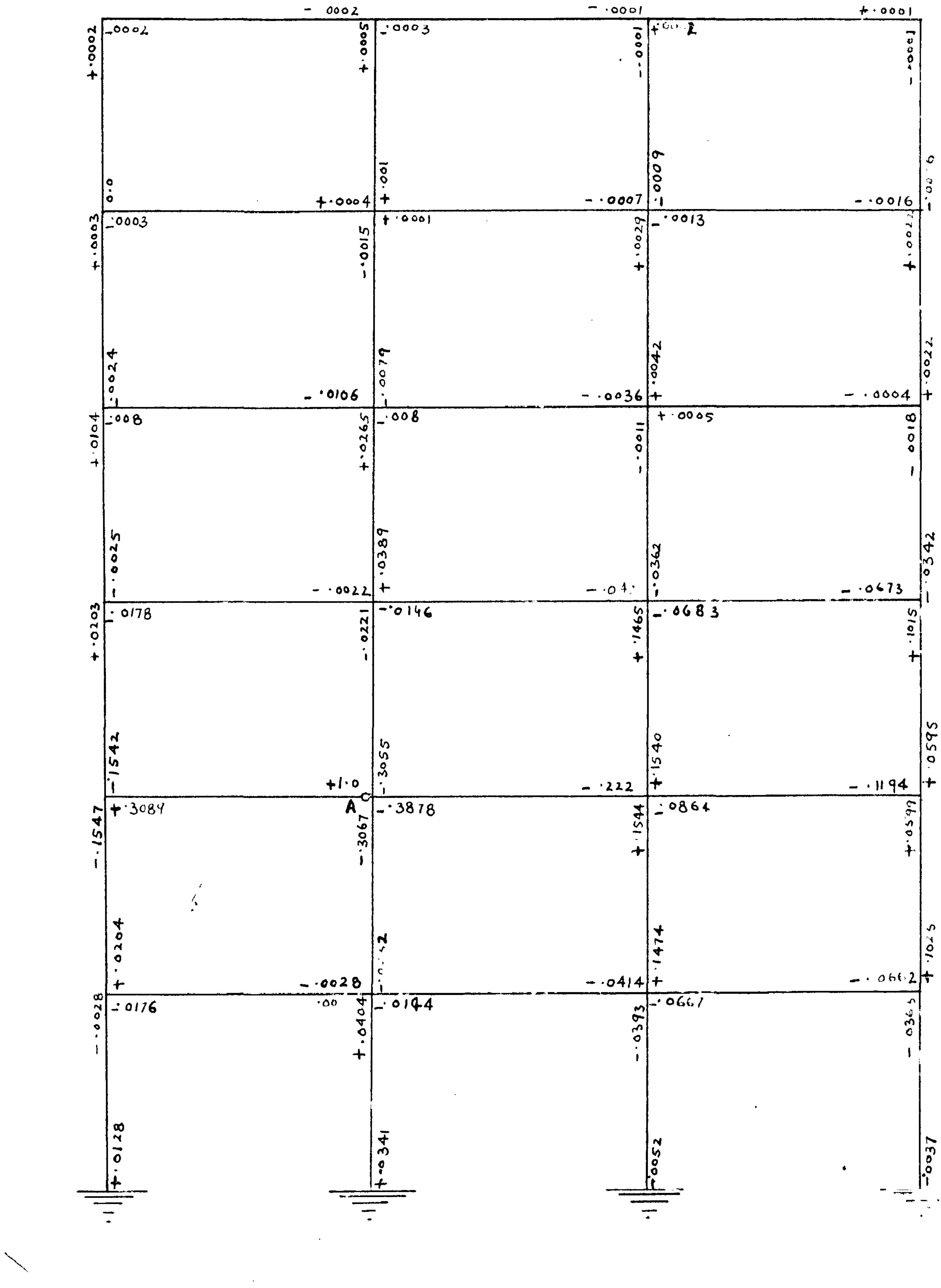


FIG. 3.17



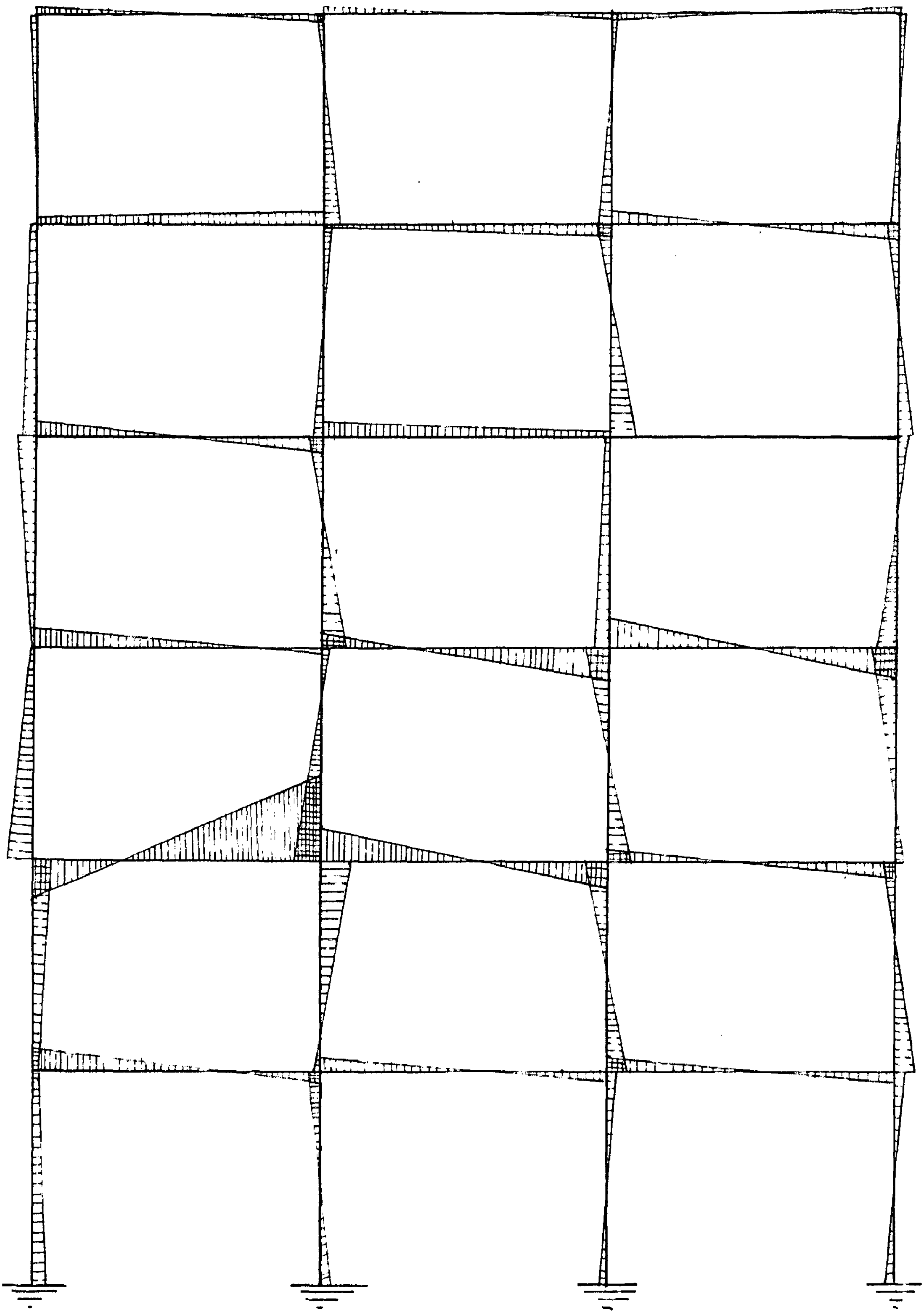


FIG 3 14.

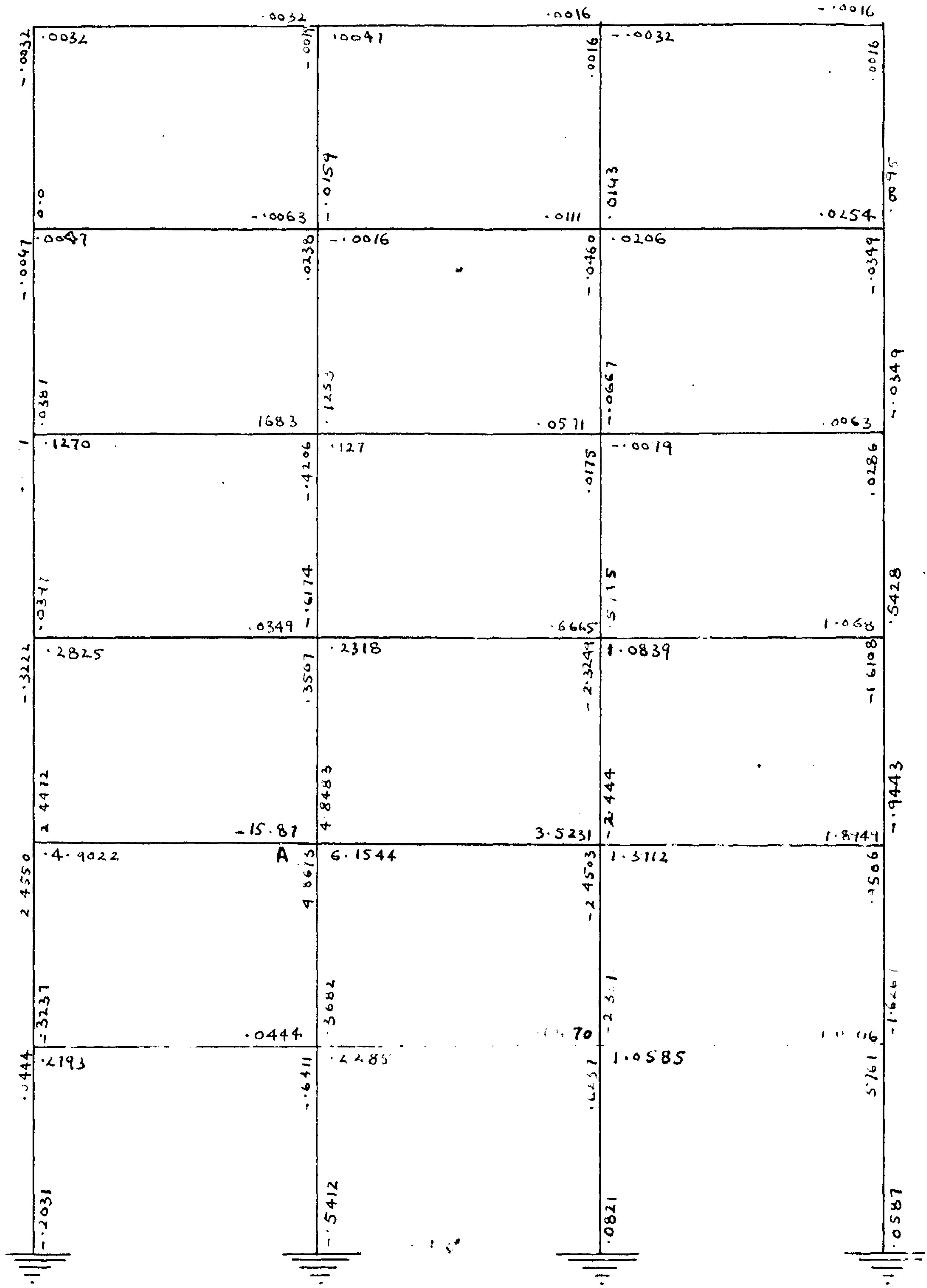
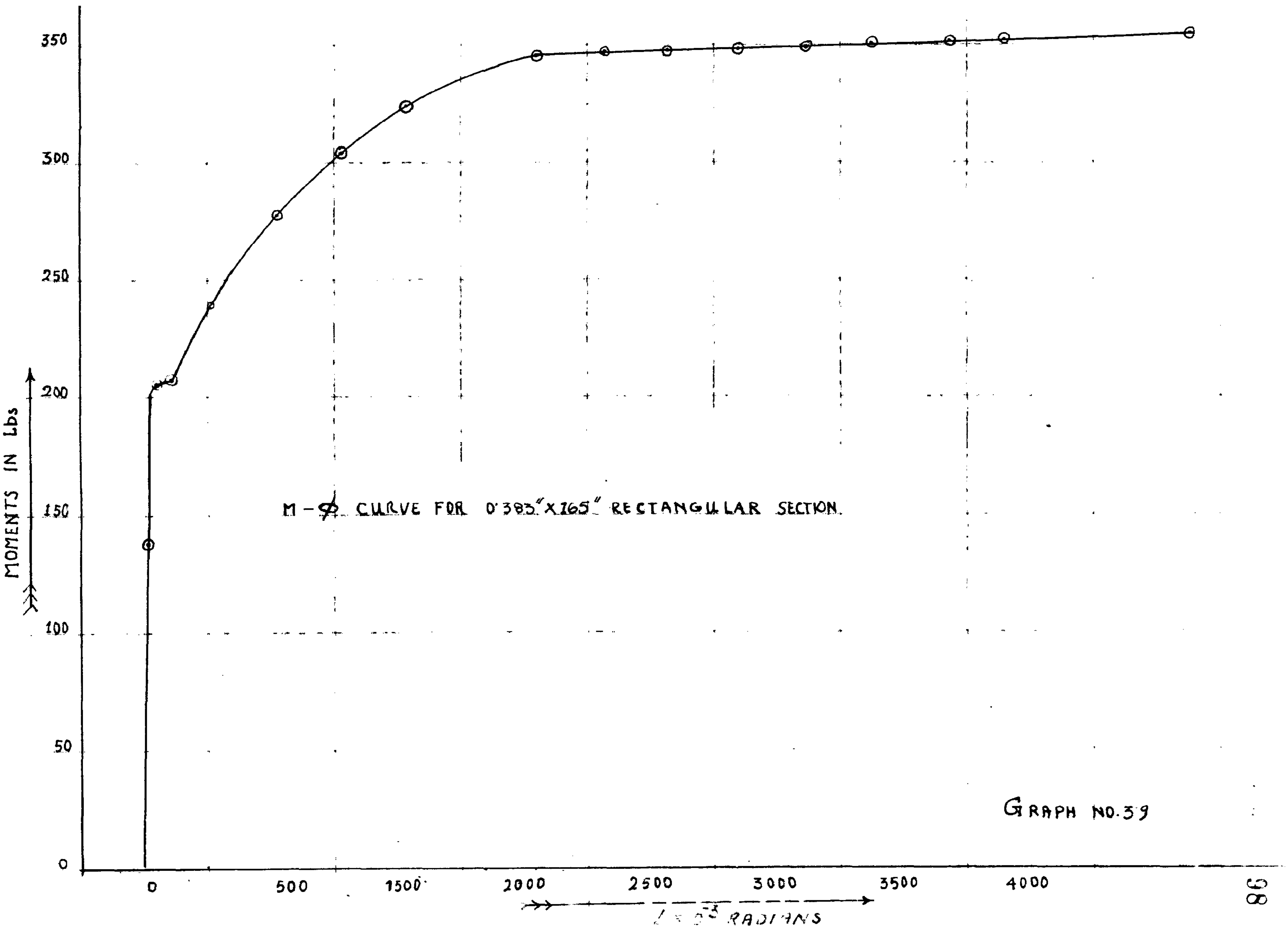


FIG 3.15

TABLE NO. 3.10

M-ϕ RELATION FOR 0.333" x 0.165" SECTION.

ε in %	ϕ x 10^{-3} = $\frac{2\varepsilon}{d}$ Radians.	m	M = $m_1 \frac{bd^2}{2}$ x 2240 in Lbs. Inches.
0	0	0	0
0.[illegible]	5.744	5.07	187.709
1.00	53.310	7.00	[illegible]06.751
2.00	104.490	[illegible]	[illegible]
5.00	[illegible]1.097	[illegible].75	237.103
10.00	522.188	10.36	277.[illegible]
15.00	783.29	11.32	304.103
20.00	1044.386	11.83	322.046
25.00	1305.483	12.21	330.99
30.00	1566.58	12.73	345.093
35.00	1827.676	12.75	345.620
40.00	2088.773	12.79	346.713
45.00	2349.869	12.86	348.239
50.00	2610.966	12.92	350.237
55.00	2872.063	12.97	351.639
60.00	3133.159	13.03	353.210
65.00	3394.256	13.06	354.032
70.00	3655.352	13.08	354.574
75.00	3916.449	13.09	354.835
80.00	4177.546	13.10	355.116



M - Φ CURVE FOR 0·383"X·165" RECTANGULAR SECTION.
MOMENTS IN Lbs
RADIANS
0
50
100
150
200
250
300
350
500
1500
2000
2500
3000
3500
4000

GRAPH NO.39

TABLE NO. 8.11

PLASTIC ROTATION (θ_1) IN -VE MOMENT ZONE.

Distance in inches from the ends.	Moment lbs. in.	$\phi A \times 10^{-3}$ radians	$\phi B \times 10^{-3}$ radians	$\Delta\phi \times 10^{-3}$ radians
0.0	355.113	4177.666	14.805	4163.671
0.1	335.7669	1383.049	13.998	1370.051
0.2	316.417	962.250	13.191	949.059
0.3	297.037	712.938	12.384	700.554
0.4	277.717	521.677	11.577	510.1
0.5	258.339	397.045	10.77	386.275
0.6	239.019	272.706	9.963	262.743
0.7	219.669	171.455	9.156	162.299
0.8	200.319	39.5	8.349	31.151
0.9	180.939	22.0	7.542	14.458
1.0	161.619	12.0	6.735	5.265
1.1	142.269	6.1	5.928	0.172
1.224	137.710	5.744	5.744	0

$\therefore \Sigma \Delta\phi \cdot ds = 648.342 \times 10^{-3}$ radians.

TABLE NO. 3.12

Distance in inches from the ends.	Moment Lbs. in.	ϕ_A x 10^{-3} radians	ϕ_E x 10^{-3} radians	$\Delta \phi$ x 10^{-3} radians
0.632	137.71	5.744	5.744	0
0.6	140.47	5.95	5.87	0.12
0.5	149.12	8.0	6.23	1.77
0.4	157.77	10.7	6.60	4.1
0.3	166.42	14.3	6.96	7.34
0.2	175.07	19.0	7.32	11.68
0.1	183.72	24.1	7.68	16.42
0	192.37	31.2	8.041	23.16
0.1	187.02	27.1	7.82	19.28
0.2	181.67	23.6	7.59	15.01
0.3	176.32	19.4	7.37	12.03
0.4	170.97	16.0	7.15	8.85
0.5	165.62	13.8	6.92	6.88
0.6	160.27	11.6	6.70	4.9
0.7	154.92	9.8	6.47	3.33
0.8	149.57	8.2	6.25	1.95
0.9	144.22	6.3	6.02	0.28
1.0	138.87	5.97	5.79	0.18
1.022	137.71	5.744	5.744	0

$\therefore \quad \Sigma \Delta \phi \; ds = 12.216$

TABLE. 3.13

Distance in inches from the ends.	Moment lbs. in.	ϕ A x 10^{-3} radians	ϕ B x 10^{-3} radians	$\Delta\phi$ x 10^{-3} radians
0.033	137.71	5.744	5.744	0
0.3	140.47	6.06	5.97	0.13
0.5	140.[illegible]	[illegible].0	6.[illegible]	2.77
0.4	137.77	10.7	6.[illegible]	4.1
[illegible].3	1[illegible].[illegible]	14.3	6.[illegible]	7.[illegible]
[illegible]	175.07	20.0	7.23	11.[illegible]
[illegible].1	1[illegible].73	[illegible].1	7.[illegible]	13.[illegible]
0	1[illegible].[illegible]7	[illegible]	[illegible].041	[illegible].[illegible]
0.1	197.03	[illegible]7.1	7.[illegible]	[illegible]
[illegible]	191.07	[illegible].6	7.[illegible]	26.01
0.3	173.[illegible]	10.4	7.57	13.0[illegible]
0.4	170.07	15.0	7.15	[illegible].35
0.5	155.03	12.3	6.[illegible]	6.[illegible]
0.6	160.27	11.3	6.70	4.0
0.7	154.[illegible]	9.3	6.47	3.[illegible]
0.[illegible]	140.07	9.2	6.35	1.05
0.9	144.[illegible]	6.[illegible]	6.[illegible]	0.[illegible]
1.0	1[illegible].97	6.07	5.70	0.17
1.033	137.71	5.744	5.744	0

$\therefore \Sigma \Delta\phi$ δs = 13.325

To find the value of AK the method described just before was employed and the value has been worked out to be 1.69×10^{-3} radians.

Now $\theta_1 = 649.342 \times 10^{-3}$ radians

$\theta_2 =$ 1[illegible].216 $\times 10^{-3}$ radians.

$\delta_{AK} = 1.69 \times 10^{-3}$ radians.

Hence available rotation is much greater than the required. Next a higher load of say 170 lbs. can be tried and the available rotation worked out. The calculations have not been done because it has been verified that the required redistribution for the collapse load which is only 123 lbs. is possible.

CONCLUSIONS:-

(i) In the simple plastic theory using Baker's stress-strain curve, free development of sufficient number of plastic hinges for collapse, is not possible without the strain exceeding the ultimate strain.

(ii) It is found as a result of analysis, using actual stress-strain curve that due to strain hardening, ultimate strain is reached at a load which is much higher than the collapse load given by simple plastic theory.

(iii) It is concluded therefore, that the use of simple plastic theory based on yield stress only is safer and there is no likelihood of the actual strain at the plastic hinges exceeding the ultimate strain of mild steel.

CHAPTER IV

"INSTRUMENTATION "

4.1. It was proposed to investigate by actual testing as to how far the collapse load calculated by the simple plastic theory is a true representation of the actual load that is going to cause the collapse of the structure. It will also be investigated as to how far the concept of plastic hinges and the mode of collapse arrived at theoratically in chapter II at Appendix C are exhibited actually in a structure. This chapter covers a detailed description of the test portal frames, design of loading device and all other equipment used for testing.

4.2. TEST FRAMES - PORTALS.

It is obvious that testing a full size multistoreyed structure is not possible because it involves great labour and cost. It was not even possible to have a model of big size because of the resources available at hand. So it was proposed to test a miniature model frame, the size and other dimensions of which are shown in fig. 4.1. The frame consists of six storeys and three bays and represents the type of structures being built now in India. The height of all the storeys and the span of each bay has been kept constant throughout. Although quite often the height of the first story is somewhat more than that of others, and the central bay is sometimes smaller than the outer ones, the frame adopted here is not outside the usual practice. The height span ratio has been kept as 1 : 1.5 which is very usual. The dimensions 4" for height of each storey and 6" for the width of each bay have been adopted considering the ease of mounting the test frame on the loading frame, and working on the test frame. The section of all the beams and columns has been kept uniform throughout. This is of course not the usual practice but this and all other deviations adopted here are only meant to simplify the theoretical computations. Further the basic idea of testing the frame is only to compare the theoretical results with experiment-al ones in order to test the validity of computational methods. Once their validity is established the behaviour of any other structure can be perdicted very easily.

While making computations for the distribution of moments in the elastic range as well as in plastic range, finding the mode of collapse and the value of collapse load, it is assumed in theory that there is a complete rigidity at a junction of beams and columns. To ensure this condition in the laboratory, the test frame has been cut as one piece from the parent mild steel plate of ½" thickness. The section for all the members was chosen to be 3/16" wide by 3/8" deep, so that value of the collapse load is reduced to facilitate the convenient, application of loads, Moreover the design of all the loading equipment and the size of the cable are also baded on this load. An increase in this load would increase their sizes proportionately.

The model was cut by means of an Oxy-Acetylene flame cutter and brought to final shape first by rought filing and then by smooth filing. Measurements we taken at various sections on the finished model and the average section -- --

adopted is 0.166" x 0.364". An error of only ± .002" has been allowed for. The dimensions at any point, therefore, do not vary by more than .002 inches. from the average values adopted for the calculations. With this variation error involved in the value of plastic moment of resistance of section will be less than ± 1.00%. It was also ensured that all the members of the frame are in one plane, and there were no scratches or stress raisers at any point in the frame. Efforts were also made to reduce the fabrication stresses to minimum by avoiding welding, cold working etc.

The size of second test frame portal was kept .166" x 0.250". The frame was cut from a 3/16" m.s. plate as before and brought to final shape as described above.

4.3. THE LOADS USED AND COMBINATIONS

In general multistoreyed structures are designed for the following three types of loads.

(i) Dead loads.

(ii) Live Loads

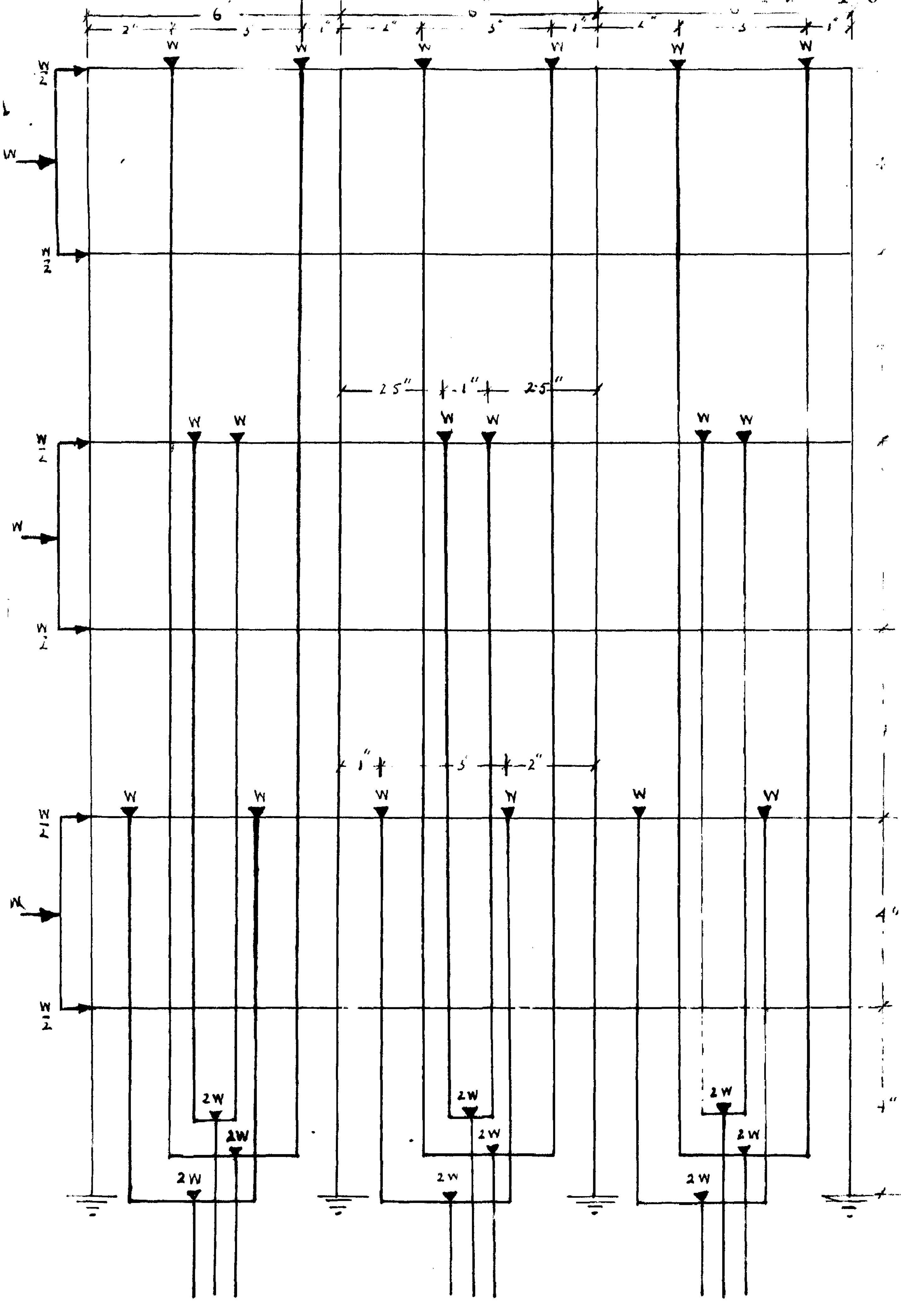
and(iii) Lateral forces due to wind pressure and seismic effects.

The first two categories comprise the total vertical load and is generally taken as uniformly distributed over the span of each bay. Since it was not possible in the laboratory to load the frame with uniformly distributed loads, so the u.d.l. has been replaced by two point loads of equal magnitudes . It was proposed to load only the alternate stories in the first instance. This type of loading is quite a usual case, other loading can however be tried later.

Horizontal loads representing the effect of wind and seismic forces have been applied at the floor level of each storey, with their direction conciding with the axis of the beam of each floor.

All the loads are to act simultaneously . Moreover in plastic analysis as has been said in earlier chapter also, the loading is taken to be proportional, that is, the ratio of the values of all the loads at one stage of loading to the corresponding values at any other stage is to be same throughout or in other words all the loads are function of a single parameter. Here it has been achieved by keeping all the vertical loads equal and all the horrizontal loads equal , but the latter equal to half the value of each vertical load. The increase in loads was carriedout in the same proportions.

One of the important considerations in loading the frame is that the loads applied should be in the plane of the frame to satisfy the assumption made in the theory. To ensure this, it was found necessary to stagger the load points as shown in the Fig. 4.2. the centre of the two points of application of the concentrated loads., in different storeys were staggered by ½" from each other, so that a separate cable could be taken to the load end of the corresponding lever for applying the load. Although this violates the general practice, that a uniformly distributed load should be replaced by two quarter point loads, but this had to be adopted otherwise it was not possible in such a


6"
2"
3"
1"
W
W/2
2.5"
1"
2W
4"

small frame to provide all the loads simultaneously and yet keep them in the plane of the frame. However this change in loading pattern did not cause the fixed end moments at two ends of a bay to different from each other by more than 10 %. Also the shear forces thus caused did not affect the value of fully plastic moment of resistance of the section to any appreciable extent. Hence the arrangement shown in Fig. 4.2. has been thought to be satisfactory.

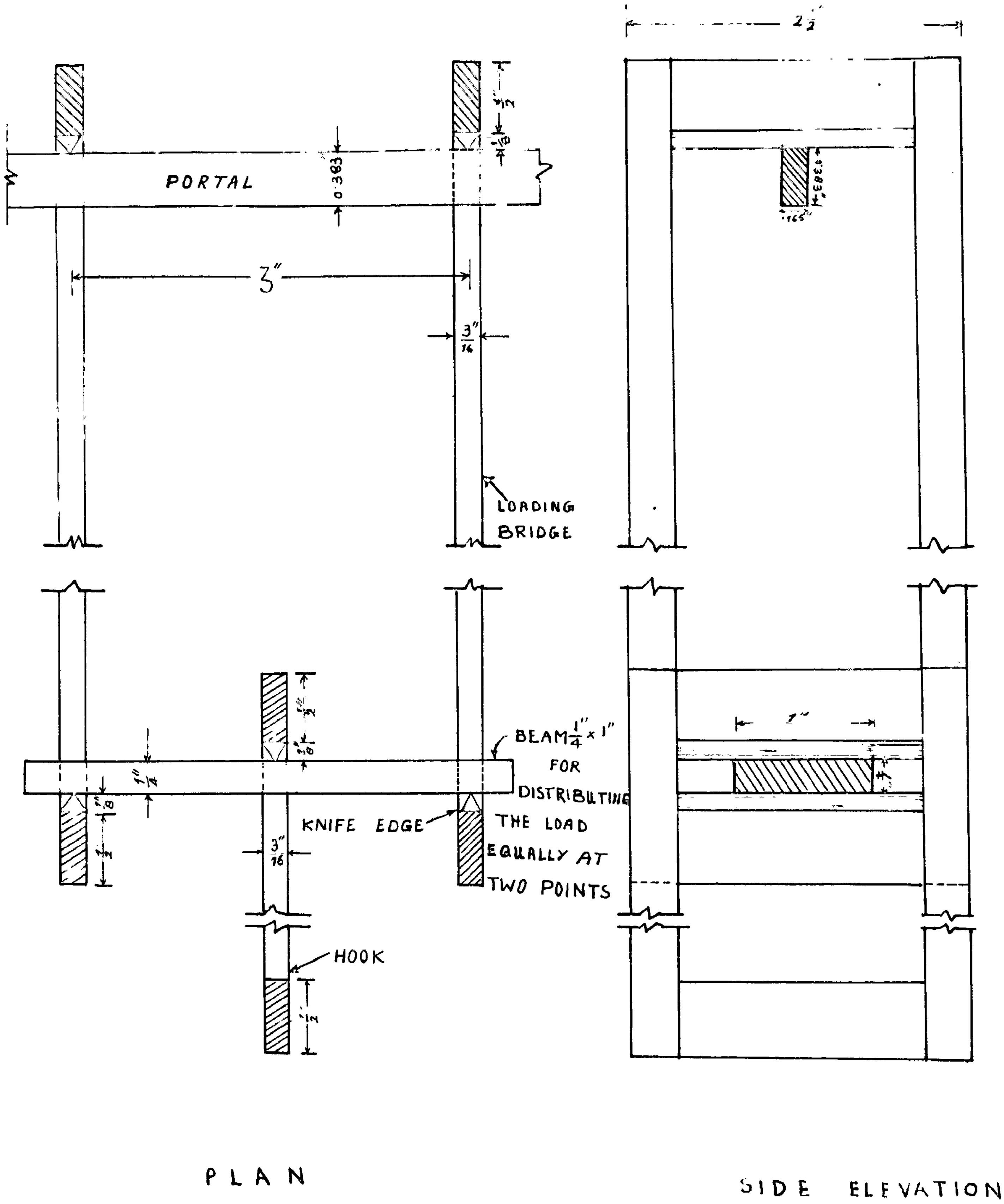
4.4. TWO POINT LOADING DEVICE FOR APPLYING VERTICAL LOADS:

For applying vertical loads to the frame, a two point loading device as shown in fig. 4.3. was used. The essential features of this device are :

(i) Two loads on one span are applied by taking them statically from a beam which is subjected to a central load.

(ii) The loads were applied by means of knife edges, representing the conditions of a point load. The knife edges were case-hardened so that they did not crush under application of heavier loads.

(iii) The beams distributing the point loads to the lower storey caused an obstruction to the application of loads in the upper storeys. This has been suitably overcome by this device by bridging across the obstructing beams as shown in fig.



"TWO POINT LOADING DEVICE"

FIG. 4·3

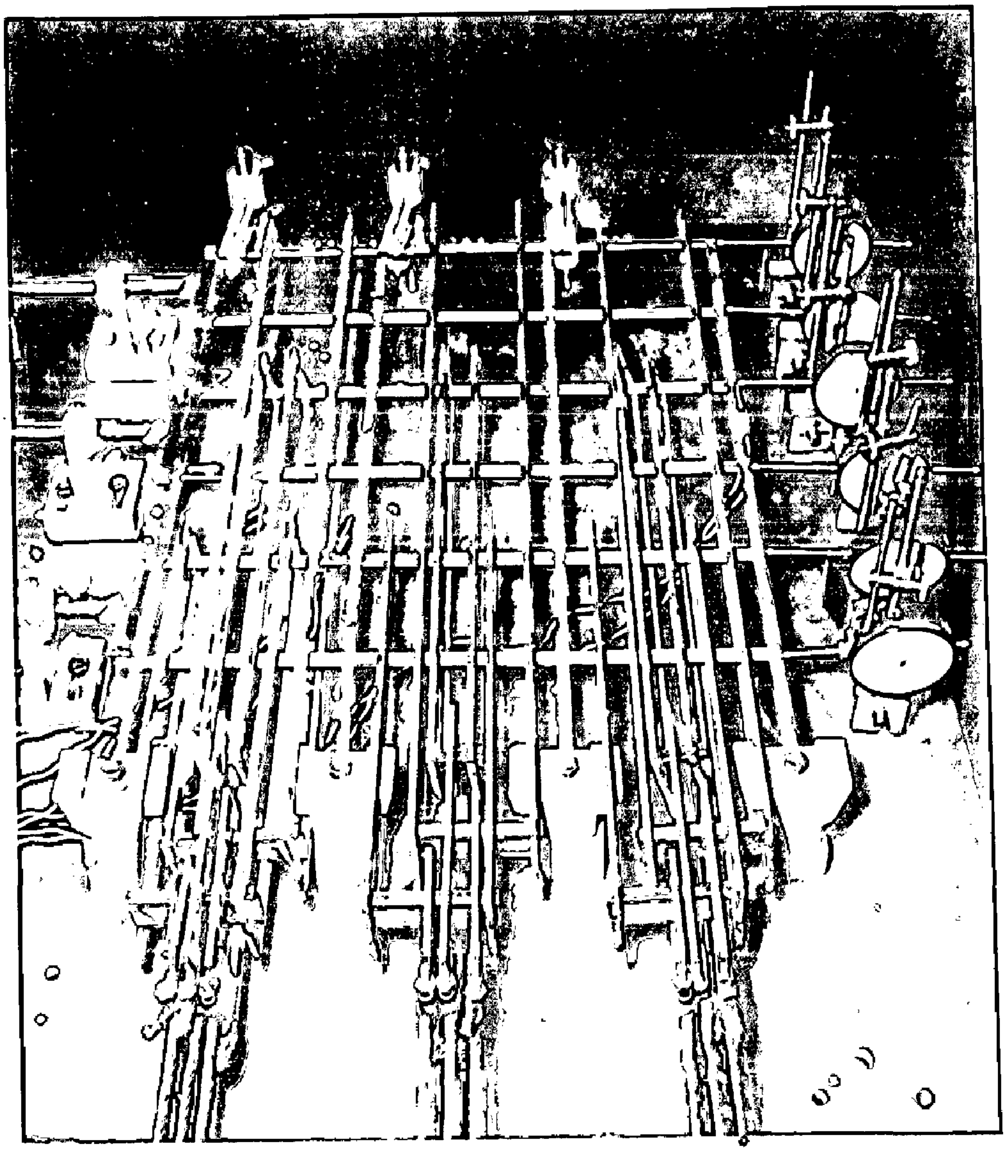
forces on it includes the vertical loads, the horizontal loads and the moments applied at all the assumed hinges. Let it be required to calculate the rotation of hinge I due to a unit force or moment applied at K. Let a unit moment be applied at I also. Let these unit moments at K and I cause bending moments equal to M_k and M_I at any other point P. Let the unit moment at I be applied first and then at K. Now when the moment at K is applied it causes angular deformations to take place throughout the structure and some potential energy is stored in it. This energy due to the moment applied at K will be the same whether there was a moment at I or not. Hence the work done by the unit moment at I and the moments M_I throughout the structure due to the rotations caused by the unit moment at K should be self- balancing. The rotation at any point caused by the unit moment at K is equal to $\frac{M_k\, ds}{EI}$ and also the work done by the previous moment M_I existing at the same point is $M_I M_k \frac{ds}{EI}$.

If the rotation at I due to the unit moment at k is δ_{ik} then the work done by the unit moment at I is $= 1\ \delta_{ik}$. then the Hence the total work done by the moments M_I and the unit moment at I

$$= 1\ \delta_{ik} + \int M_I\ M_k\ \frac{\delta s}{EI} = 0$$

$$\delta_{ik} = - \int M_I\ M_k\ \frac{ds}{EI}$$

The integral has to be evaluated for the entire structure. In the equation M_k was the moment at any point



APPLICATION OF VERTICAL LOADS BY TWO POINT LOADING DEVICES

(iv) The loading is in the plane of the frame.

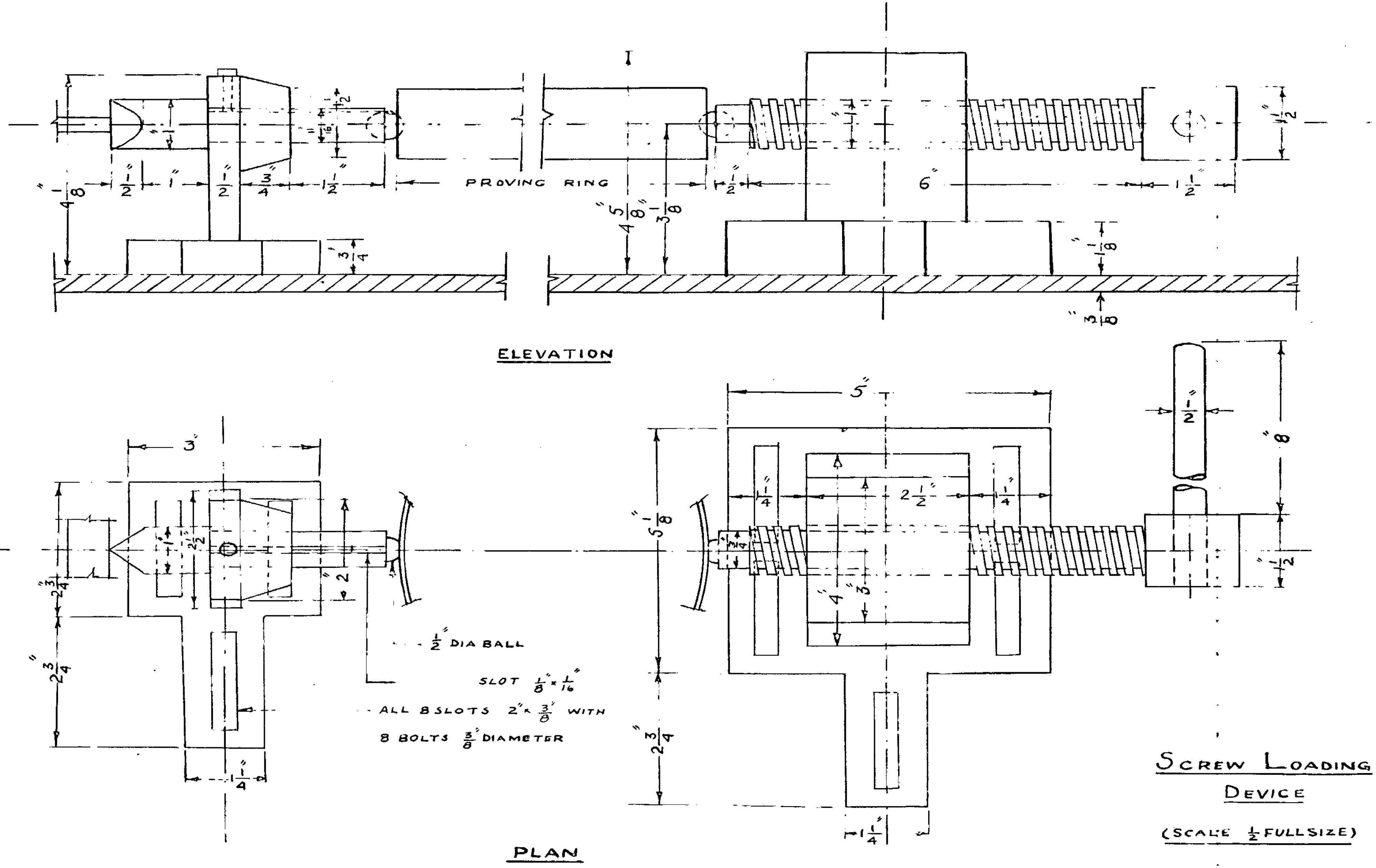
These devices are shown in the photographs on page 109

4.5. SCREW- LOADING DEVICE FOR APPLYING HORIZONTAL LOADS :

For applying horizontal loads, screw type loading device as shown in fig. 4.4. has been used. In this, load applied could be measured at all stages of loading by a proving ring interposed between the screw and the knife edge rod. The dimensions of various parts are shown in the fig. 4.4. For proper loading of the proving ring $\frac{1}{8}$" diameter balls have been fitted into the ends of the screw and the knife edge rod. To allow for adjustments, bolt holes for the screw arrangement were slotted. In order to eliminate any possibility of rotation of proving ring and hence that of knife edge rod, when the screw is turned, as slot was provided in the knife edge rod and a guide pin fixed through the top of the supporting block to guide its movement. The screw loading device in position while working has been shown in photograph on page 113 .

4.6. FIXED END SUPPORTS FOR THE PORTAL FRAME:

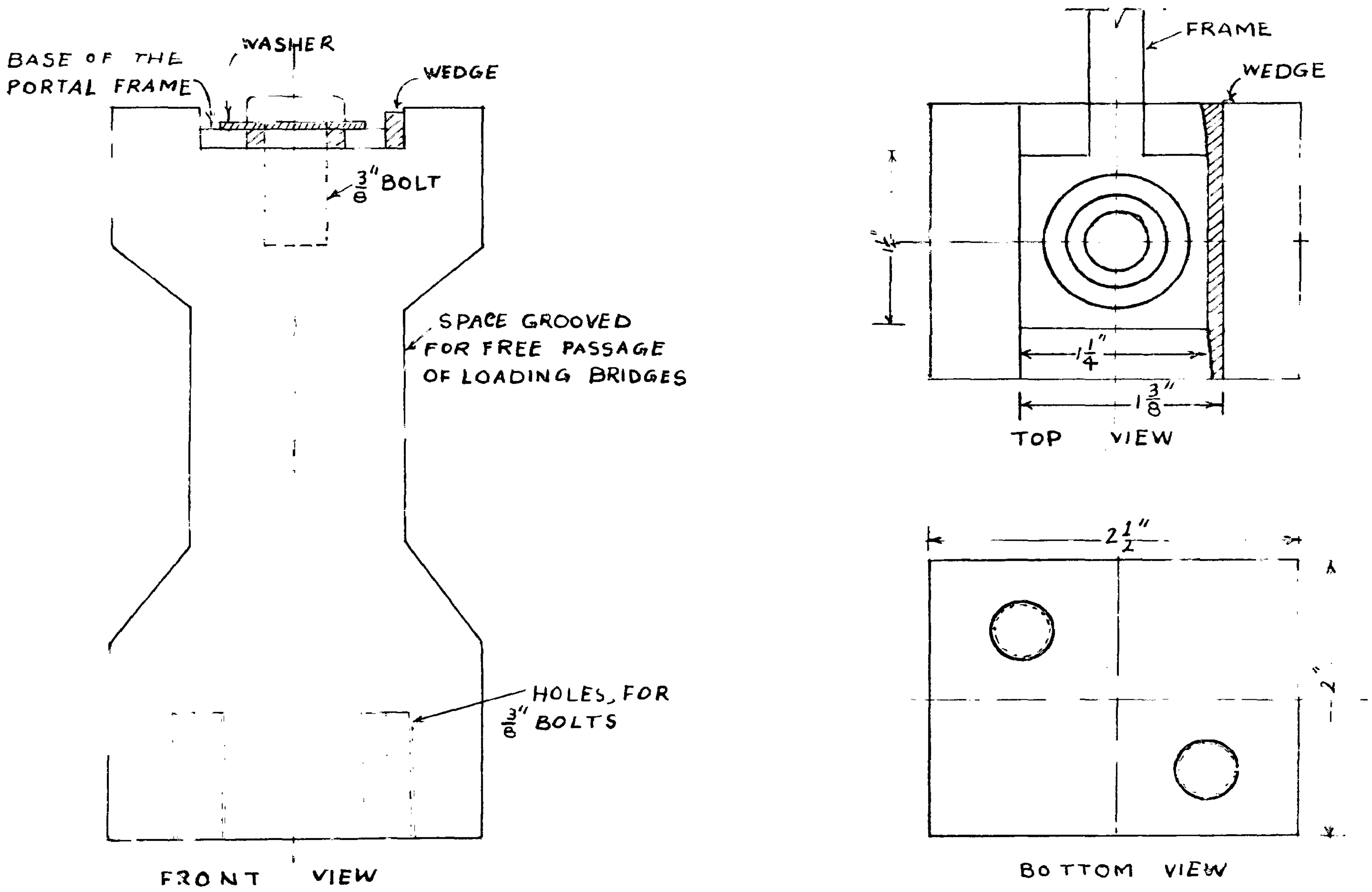
In theoretical computations in Chapter II, for this frame, it was assumed that the columns in the ground floor are fixed in position as well as in direction. In order to simulate the similar conditions in the laboratory the arrangement shown in fig. 4.5. was adopted. The bolt kept the end in its position while the square seat when properly wedged did not allow any end rotation. Photograph on page 109 shows the supports in position.



SCREW LOADING DEVICE

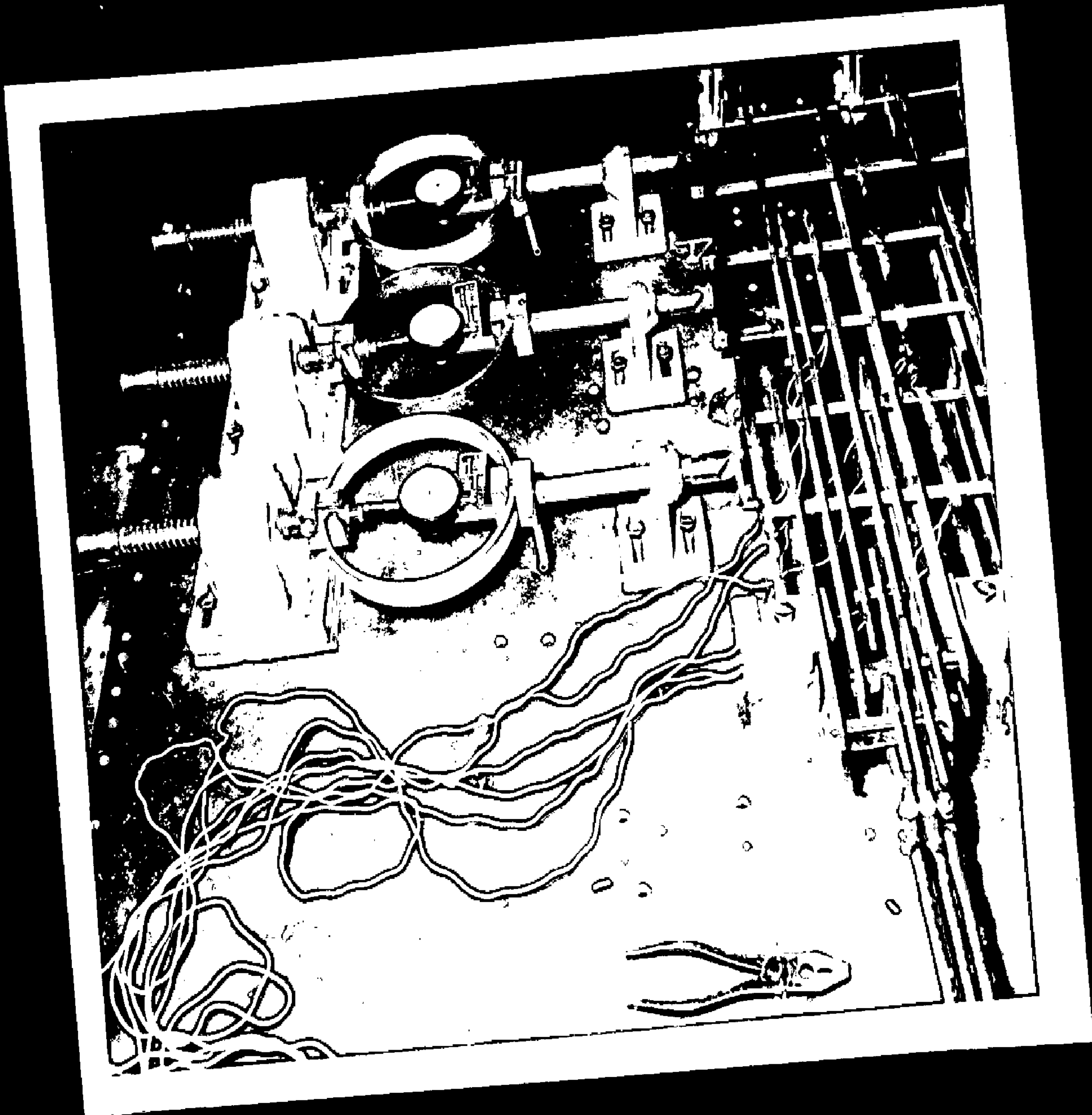
(SCALE 1/2 FULLSIZE)

FIG. 4.4.



SUPPORT FOR THE PORTAL BASE

FIG. 4 · 5.



HORRIZONTAL LOADING DEVICES

4.7. C.I. BALL PADS AND M/S. PIN :

It was feared that during the test the portal may buckle in a direction prependicular to plane of the portal due to the thickness of plate, being too small. In order to prevent this transverse instability it was necessary to support the frame at various points in such a way that transverse movement of the portal frame was prevented while free movement in the horrizontal plane was allowed. For this purpose the arrangement shown in fig. 4.6. has been used. It is convenient with the help of the lower cast iron pad capable of being adjusted at any suitable heihgt to support the portal, level with the top of the support pin, and the upper one prevented the instability. The balls on either side of the portal permitted free movement in the horrizontal plane. To prevent slipping of the balls with excessive movement of the portal frame, recesses with diameter 1/32" greater than the diameter of the ball, were made on the surfaces of the ball pads., with the movement of the portal, the balls will roll in the recess without slipping down. However it is necessary when using these pads that the balls do not get pressed into the pads, because in that case free movement of portal will be prevented. This can be done by tightening the screws only slightly. The pins in use can be seen in the photograph in the page .

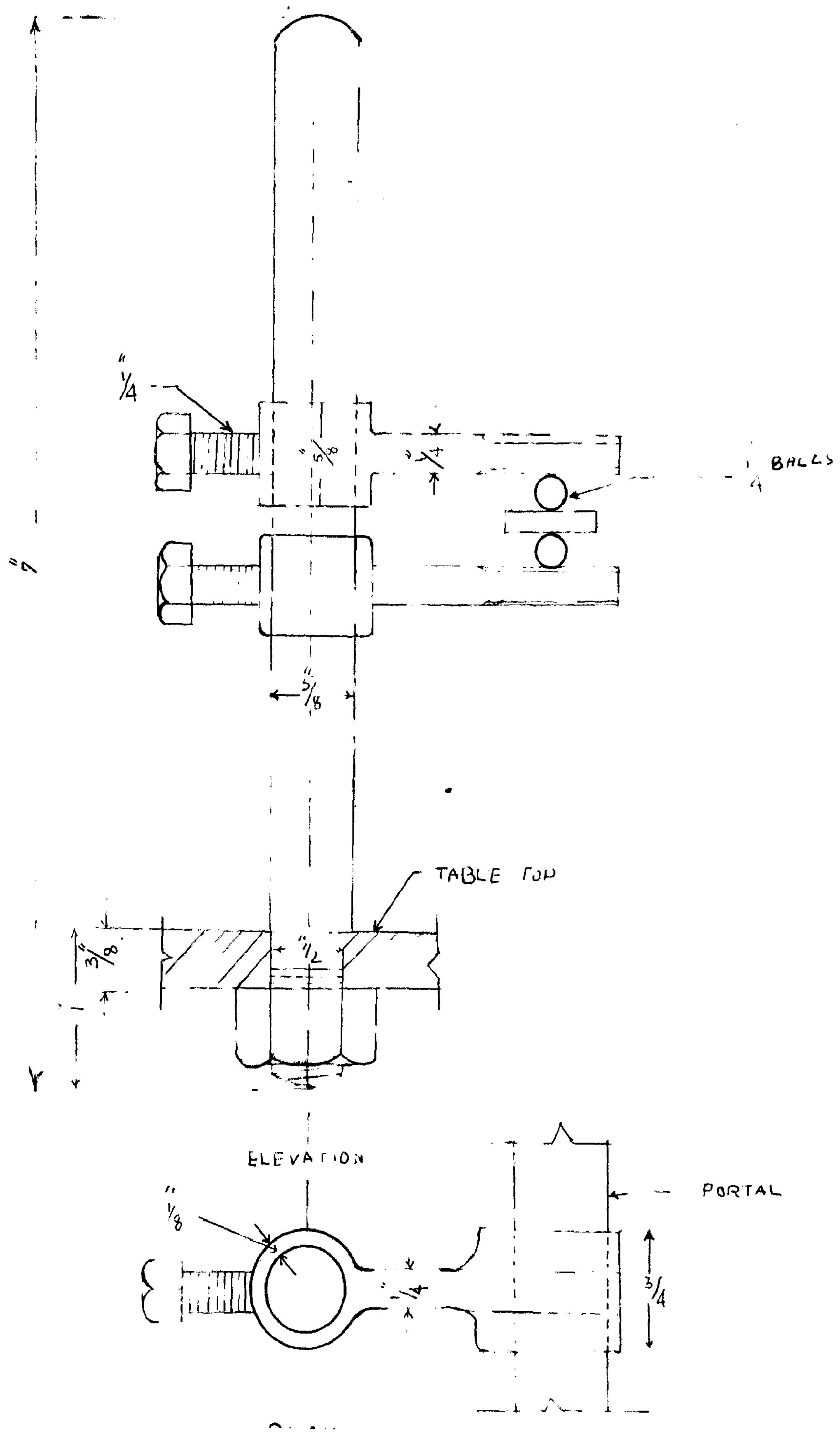
4.8. LEVER SYSTEM FOR LOADING:

4.7. C.I. BALL PADS AND M/S. PIN :

It was feared that during the test the portal may buckle in a direction prependicular to plane of the portal due to the thickness of plate, being too small. In order to prevent this transverse instability it was necessary to support the frame at various points in such a way that transverse movement of the portal frame was prevented while free movement in the horrizontal plane was allowed. For this purpose the arrangement shown in fig. 4.6. has been used. It is convenient with the help of the lower cast iron pad capable of being adjusted at any suitable heihgt to support the portal, level with the top of the support pin, and the upper one prevented the instability. The balls on either side of the portal permitted free movement in the horrizontal plane. To prevent slipping of the balls with excessive movement of the portal frame, recesses with diameter 1/32" greater than the diameter of the ball, were made on the surfaces of the ball pads., with the movement of the portal, the balls will roll in the recess without slipping down. However it is necessary when using these pads that the balls do not get pressed into the pads, because in that case free movement of portal will be prevented. This can be done by tightening the screws only slightly. The pins in use can be seen in the photograph in the page .

4.8. LEVER SYSTEM FOR LOADING:

Loads required for causing collapse of the frame were too arge to be applied directly by means of dead weights. Lever



1/4"
5/8"
1/4"
1/4" BALLS
7"
5/8"
TABLE TOP
3/8"
1/2"
1
ELEVATION
1/8"
1/4"
PORTAL
3/4

system, therefore, has been adopted, which could magnify the effect of dead loads. A lever ratio of 30 has been provided. The arrangement is shown schematically, on photopage 122 . The details of section are shown in fig. 4.7.

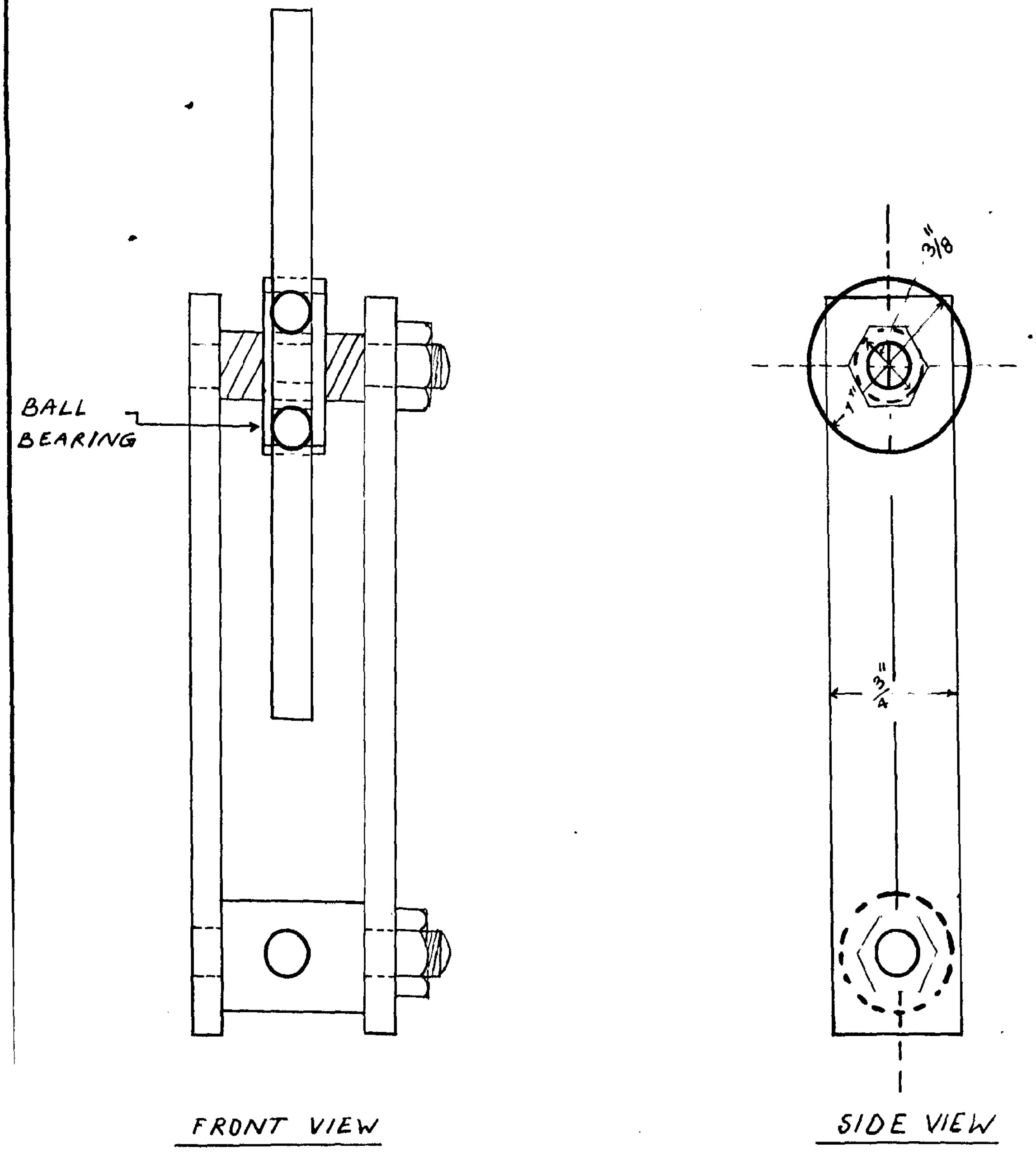
Since one inch deflection of portal frame will cause the loading end of lever to move by 30 inches, it was not practicable to have a knife edge support for the fulcrum. In that case the beam would stop rotating after the v- notch (cut in the beam) has come in contact with the knife edge. A ball bearing of 1" internal diameter has been used for this purpose.

While using lever system, it is essential that the lever ratio should remain constant during application of the loads. This has been ensured by using the device at the load end and the loading end as shown in fig. 4.8. The essential feature of this device is that the hanger by means of which load is to be applied, and the cable by means of which load is to be transfered to the frame will both remain vertical for all positions of the lever, and thus lever ratio will be always same i.e. 30.

Counterbalancing of the levers has been done by extending the lever beams beyond the neutral point, and using heavier sections at that end. Only partial counterbalancing has been done in order to take advantage of self weight of the levers in reducing the amount of dead weights required to load the failure and at the same time saving an appreciable amount of steel required for counterbalancing. The counterbalancing was however so adjusted that the initial loads caused by the levers are much less than the


3/4" ø HOLE
B
A
3/4" ø HOLE
2 1/2" ø HOLE
2"
60"
38"
12"
3
ELEVATION.
1 3/4"
1/2"
1/2"
3 3/4
6/16"
5/16"
1/2"
1 1/4"
1 3/4"
3 3/4
5/16"
1 1/4"
END VIEW B - B
END VIEW A - A.

FIG 4.7



DEVICE FOR CONNECTING CABLE TO THE LOAD END AND HANGER TO THE LOADING END OF THE LEVER.

Fig. 4.8

loads that would cause yield stress anywhere in the frame at the most highly stressed fibre. Supporting arrangement of the levers is shown on photograph 122.

4.9. ARRANGEMENT FOR CALIBRATION OF THE LEVERS.

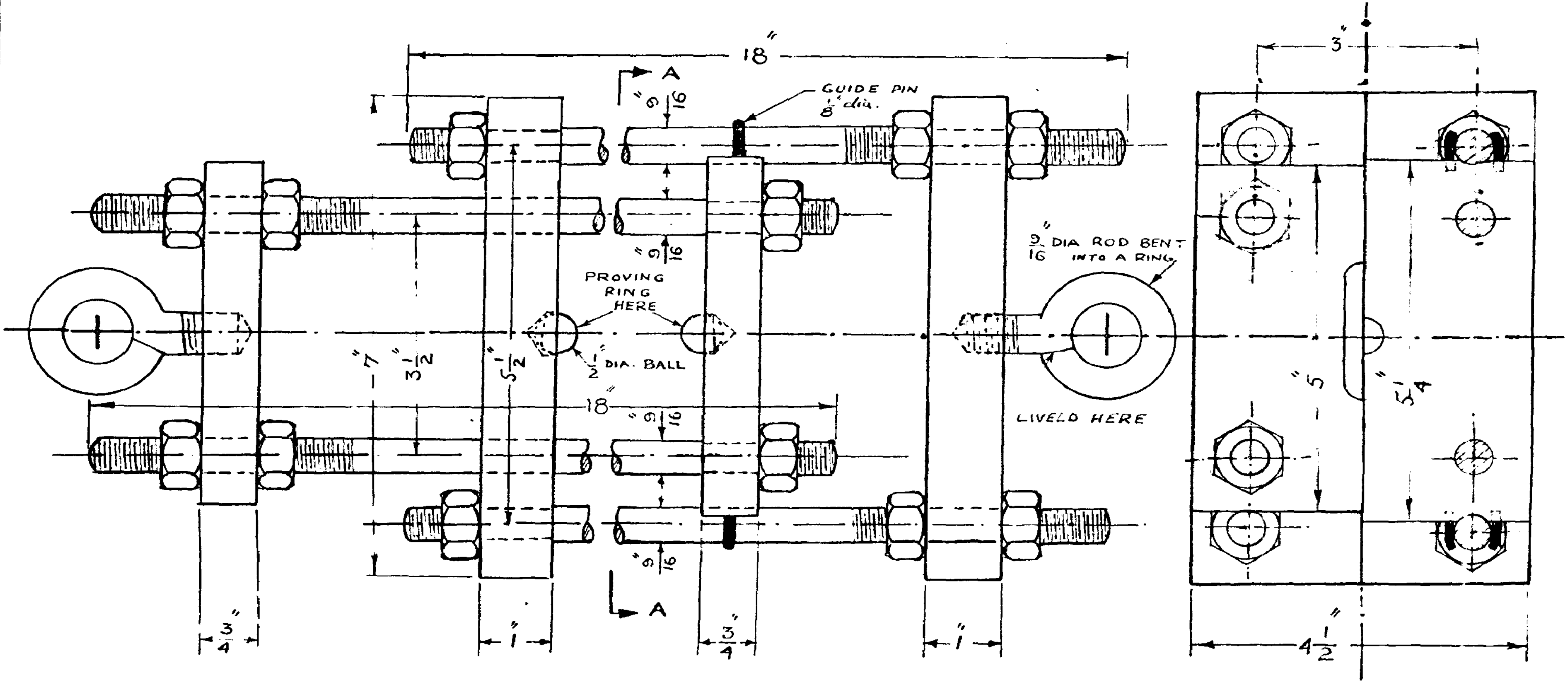
To take into account the effect of partial counterbalancing, and the mechanical efficiency of the whole loading system, exact calibration of each lever was done with the help of apparatus shown in fig. 4.8. As the proving ring available was calibrated for compression, the attachment was so designed that the pull in the cable made the two parts move in opposite direction relative to each other and compress the proving ring in between. To ensure central loading of the proving ring 1/2" diameter balls were embedded in steel plates as shown in the figure 4.9.

In place, it was mounted on the table top (see photograph 125) and one end secured firmly to a 1" diameter steel rod bolted into the table. To the other end was attached a cable from the load end of the loading beam as shown in photograph 125.

Calibration of levers has been explained in detail in Chapter V.

4.10. LOADING TABLE:

The portal frame and all other equipment was mounted on a table. The table top was made of 1/2" thick m.s. plate 4 ft. x 3 ft. Loading table along with all the equipment mounted on it is shown in photograph on page 121.

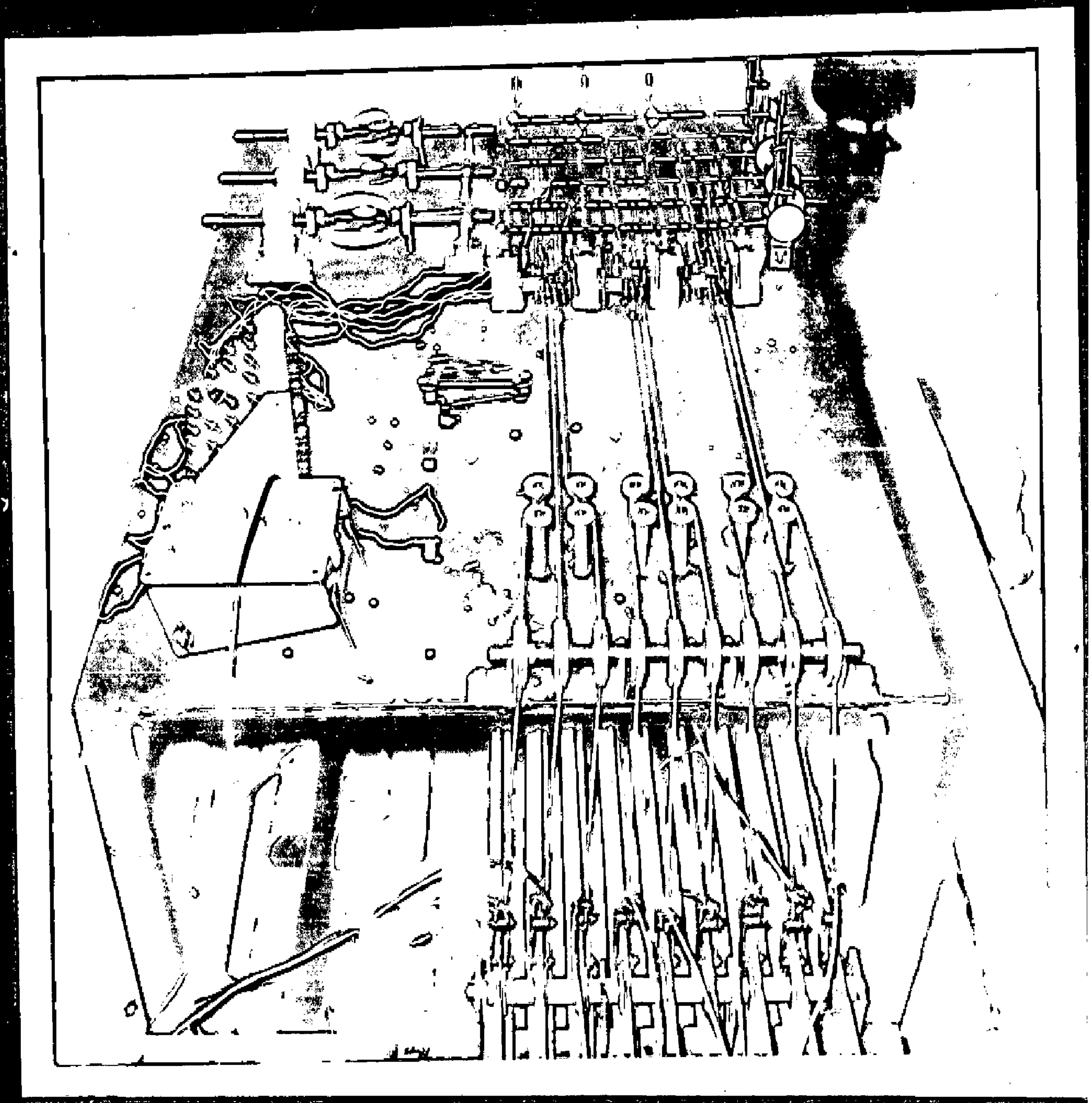


ELEVATION

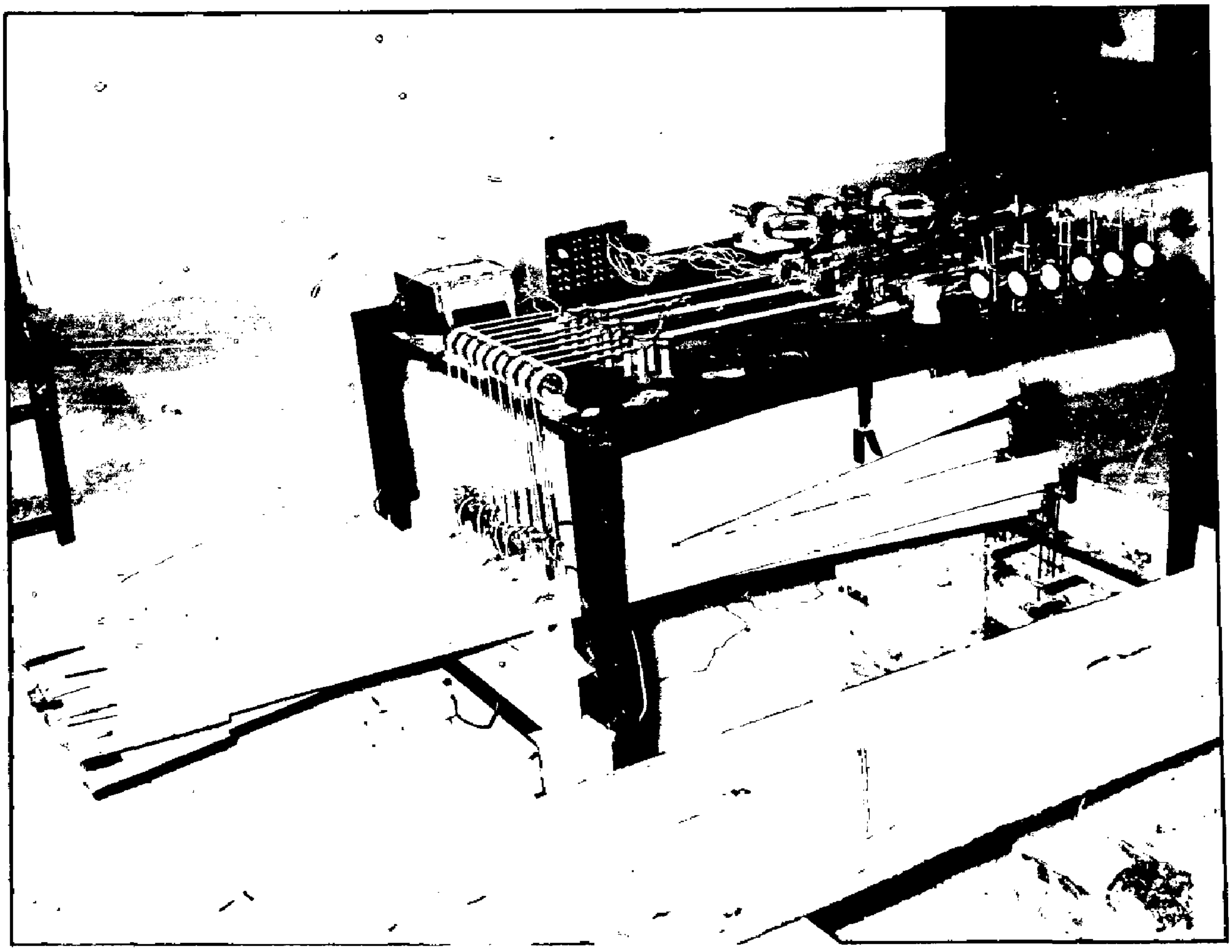
SIDE VIEW WITH RIGHT HALF SEC: AA

M/s PROVING RING ATTACHMENT
FOR
LOADING FRAME
SCALE 1/2 FULLSIZE

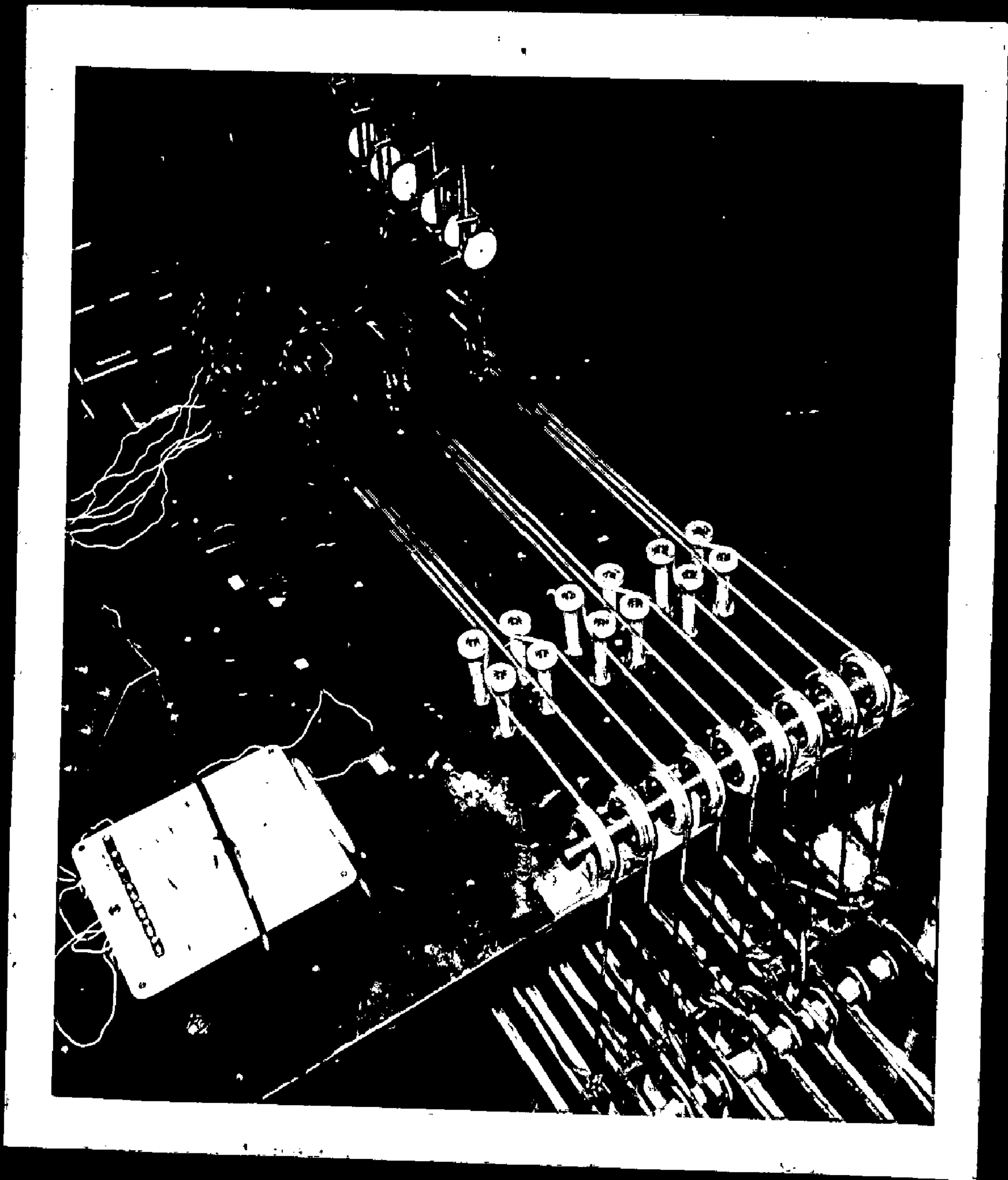
FIG. 4.8.



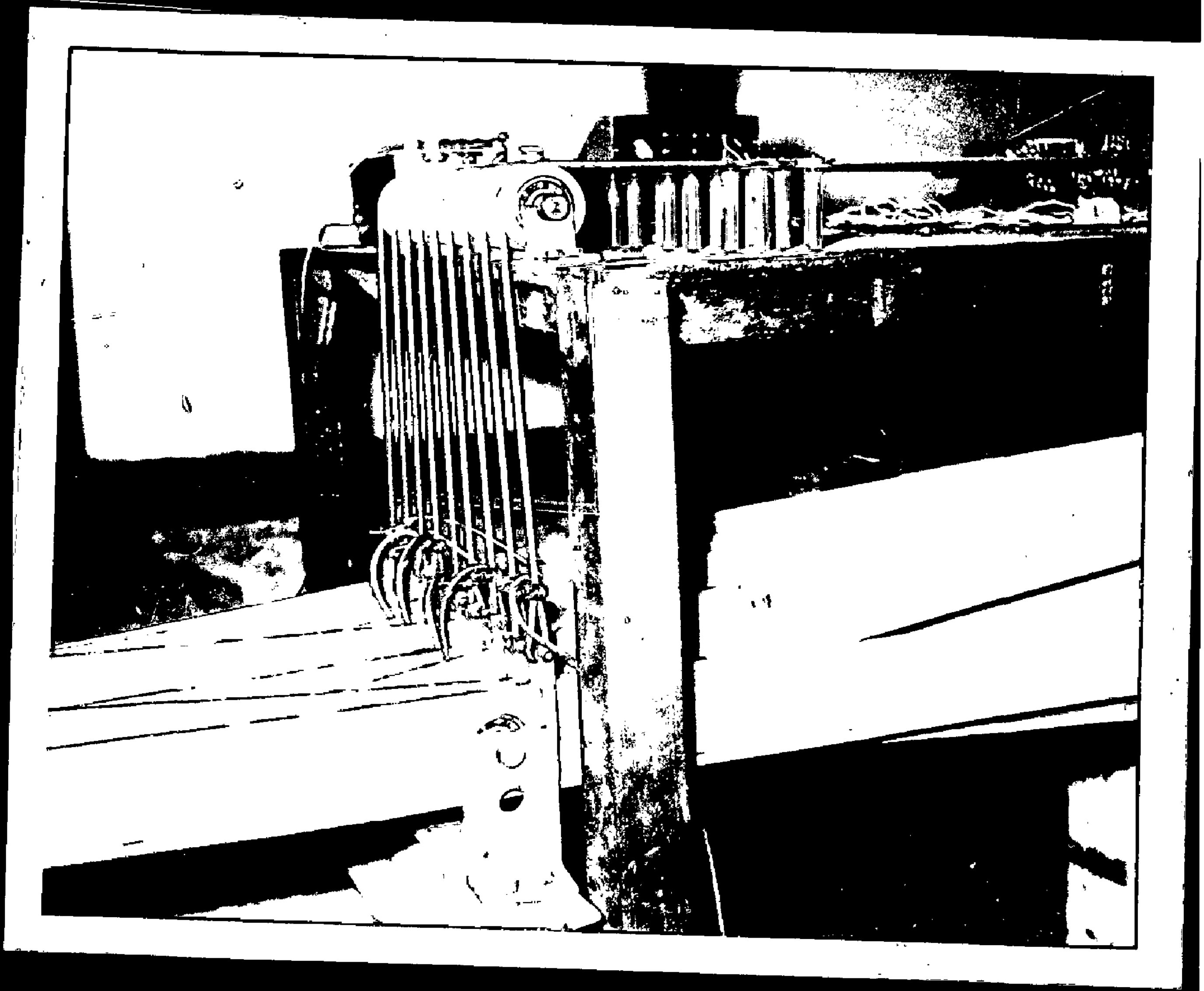
GENERAL VIEW OF LOADING ARRANGEMENT



GENERAL VIEW OF LOADING ARRANGEMENT



DETAILS OF LOADING CABLES



DETAILS OF CONNECTION BETWEEN CABLES AND THE LEVERS

APPARATUS FOR CALIBERATING THE LEVERS

CHAPTER V

" TESTING "

5.1. PROGRAMME AND OBJECTS:

The testing programme was arranged in following order :

1. Tension test on two tension specimens of steel, separated from the same plate from which the frames were separated, in order to find out the stress strain relationship and the lower yield stress from which the plastic moment of resistance shall be calculated.
2. Flexural test on two beam specimens of steel, separated from the same plate from which the frames were separated, in order to find out the ultimate moment of resistance as well as the fully plastic moment of resistance of the section directly, by establishing bending moment curvature relationship.
3. Caliberation of levers for applying loads in such a manner that the loading on the frame is always proportional.
4. Loading test on the test frame.

This chapter covers a brief discription of all these tests, and the test results recorded by observations.

5.2. TENSION TEST :

Tension specimens were separated and finished in the same manner as described for the portal frame in Chapter IV. The dimensions of the specimen are shown in Fig. 5.1. The test was performed on 10 Ton Amsler's Universal Testing Machine. The strains were measured by Lindley's extensometer as well as electrical bonded wire strain gages. Two strain gages were used, one on either side of the specimen for averging the reading. This was done in order to eliminate the effect of any eccentricity which may be due to either any fault in the specimen or any fault in mounting the specimen on testing machine. The test results are shown in Tables No. 5.1 & 5.2 and stress strain relationships are plotted as shown in graph No. 5.1.

SPECIMEN 'A'

Average width = 1.005"

Average thickness = 0.199"

∴ Average area of cross section

= 1.005 x.199

= .2 sq. inches.

Gauge Length of Lindley's extensometer

= 2"

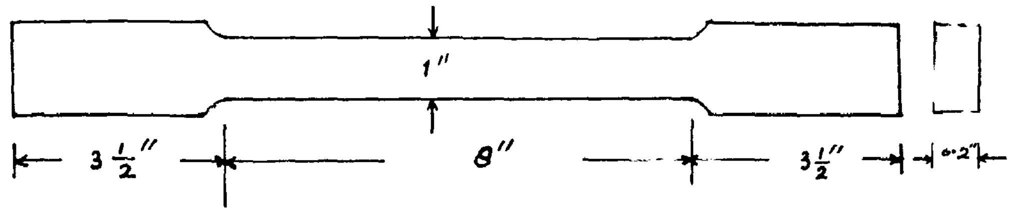
Least count of Lindly's extensometer = $\frac{1}{20000}$ inches.

Specifications of electrical strain gages :-

Type :- Tinsley.

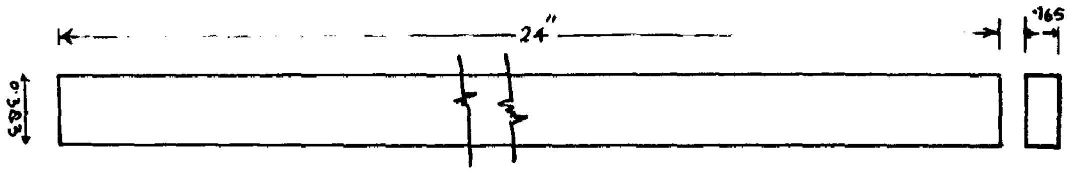
Gage Factor = 2.12.

Resistance = 100.0 ohms.



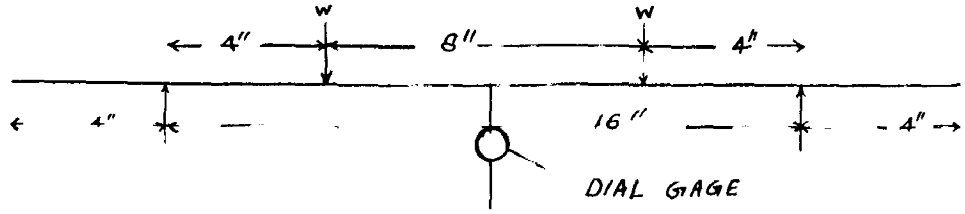
TENSILE TEST SPECIMEN

Fig. 5.1



FLEXURAL TEST SPECIMEN

Fig. 5.2



LOADING ARRANGEMENT OF BENDING TEST

Fig. 5.3

TABLE NO. 5.1.

Load in Tons	Electrical strain gage readings		Lindley extensometer		Stress in Tons/ in^2	Mean Value of strain x 10^{-6}	REMARKS.
	Gage 1	Gage 2	Reading	Strain x 10^{-6}			
0	12380	25930	0	0	0	0	
0.75	12630	26170	10	250	3.75	245	
1.50	12900	26440	21	525	7.50	520	
2.25	13175	26715	32	800	11.25	795	
3.00	13500	26980	44	1100	15.00	1080	
3.50	13760	27190	52	1300	17.50	1280	
3.75	29220	41980	694	17350	18.75	16943	Yielding of specimen.
4.00	32050		792	19800	20.0	19670	
4.25	35540		928	23200	21.25	23180	
4.50	39680	out of range	1096	27400	22.5	27350	
4.75	out of range of measuring bridge	out of range of measuring bridge	1300	32500	23.75	32500	
5.0			1560	39000	25.0	39000	
5.82					29.1	265000	Ultimate strain measured over 8" gage length by steel rule.

Specimen 'B'

Average width = 1.010"

Average thickness = 0.198"

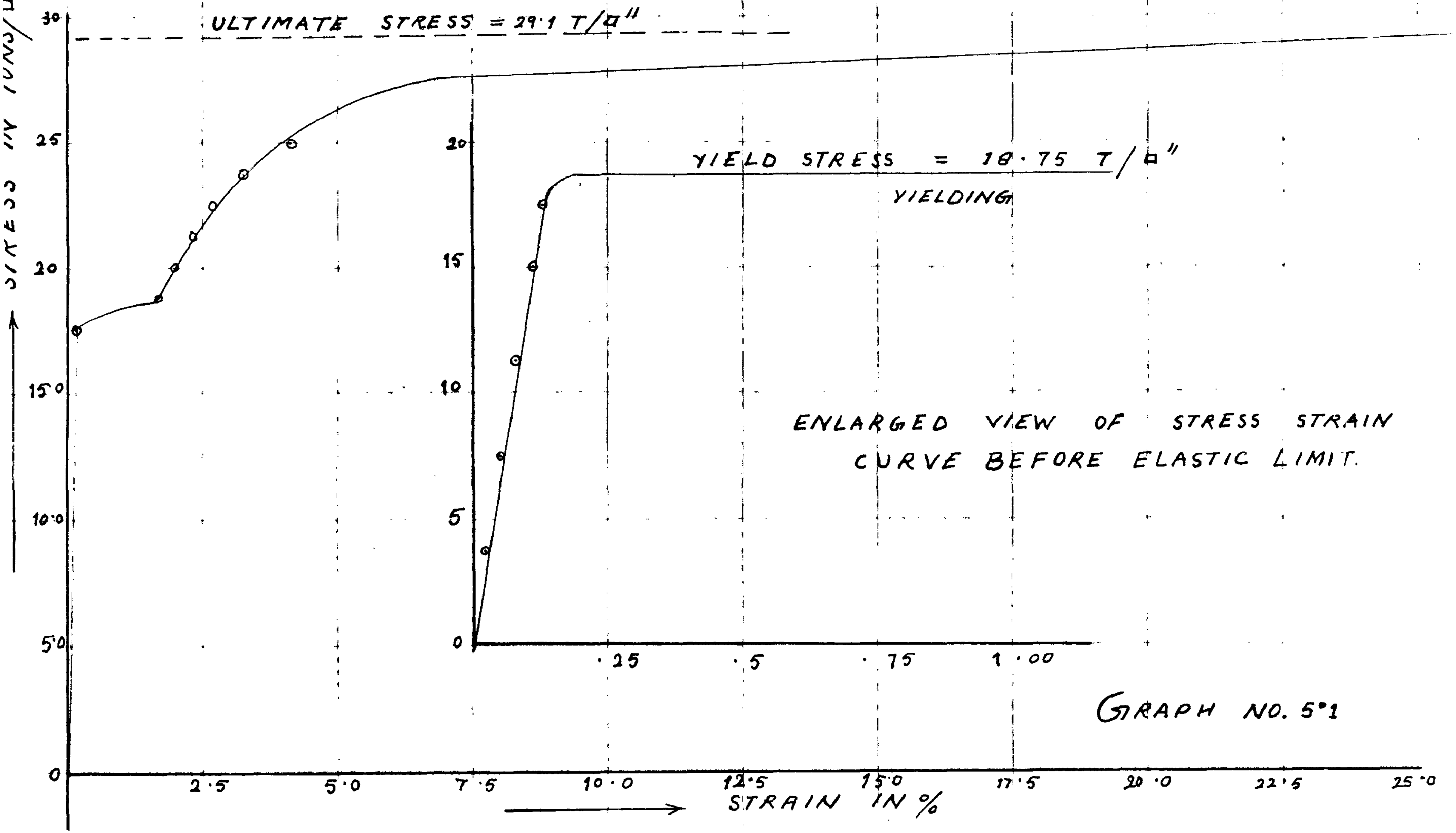
∴ Average area of cross section = 0.2 sq. inches.

Lindley's extensometer and electrical gages same as for specimen "A"

TABLE NO. 5.2.

Load in Tons	Electrical strain gage readings		Lindley extensometer.		Stress in Tons/ in²	Mean value of strain x 10^{-6}	Remarks.
	Gage 1	Gage 2	Reading	Strain x 10^{-6}			
0	12410	11870	0	0	0	0	
0.75	12660	12120	10	250	3.75	350	
1.25	12850	12330	18	450	6.25	445	
1.75	13040	12490	25	625	8.75	625	
2.25	13225	12665	32	800	11.25	805	
2.75	13380	12830	40	1000	13.75	980	
3.00	13530	12980	45	1125	15.00	1120	
3.25	13610	13050	48	1200	16.25	1190	
3.50	13690	13140	52	1300	17.50	1280	
3.75	29280	28720	690	17250	18.75	17000	Yielding starts.
4.00	32110	31530	780	19500	20.00	19650	
4.25	35590	35010	920	23000	21.25	23140	
4.50	39740	39130	1080	27000	22.5	27150	
4.75	Out of range of measuring bridge.		1260	31500	23.75	31500	
5.00			1500	37500	25.00	37500	
5.80					29.0	26200	Ultimate strain measured over 8" gage

STRESS STRAIN CURVE



The stress strain curves as plotted for two specimens were almost similar therefore curve for specimen "A" is only shown in the graph no. 8.1. From the graph

Yield stress = 18.75 Tons per square inch.

Ultimate stress = 20.10 Tons per square inch.

Value of young's modulus i.e. $E = 30.0 \times 10^{6}$ psi.

8.3. FLEXURAL TEST :

Two beam specimens of dimensions given in the fig. 8.3 were separated from the steel plate and finished as usual. The specimens were simply supported with effective span of 16" and two equal loads were applied at quarter points i.e. 4" from either end as shown in the fig. 8.3. Loads were applied by means of screw loading device as described in chapter II. Complete arrangement is shown in the photograph 133. Central deflection which is obviously the maximum was recorded by means of a Mercer's dial gauge. Complete set of readings for two specimens is shown in the Table No. 8.3. Mean Central deflection was plotted as abscissa against the bending moment at centre which is $= \frac{WL}{8}$ as the ordinate in graph no. 8.2.

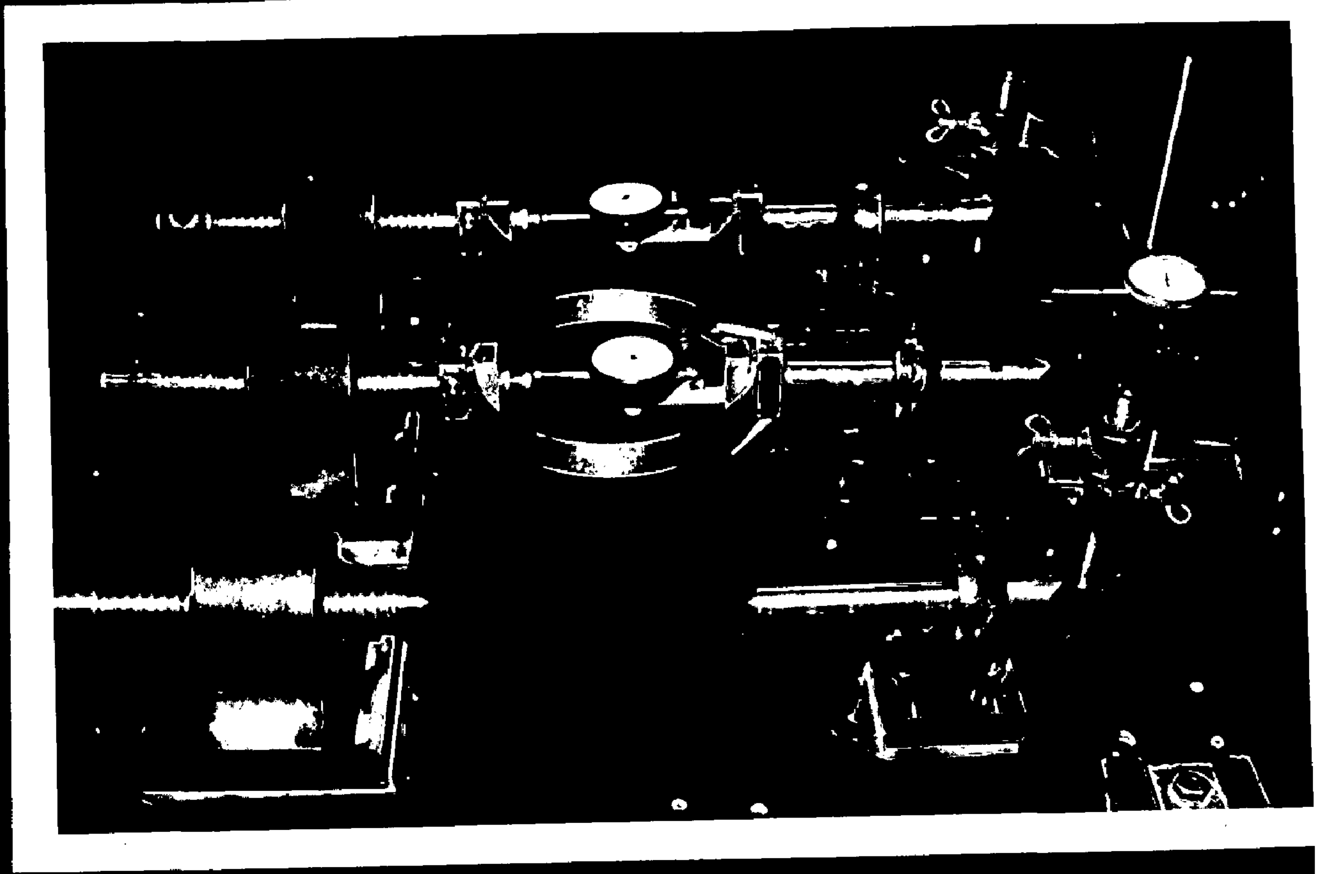
Average width of beam section = 0.185 inches.

Average depth of beam section = 0.273 inches.

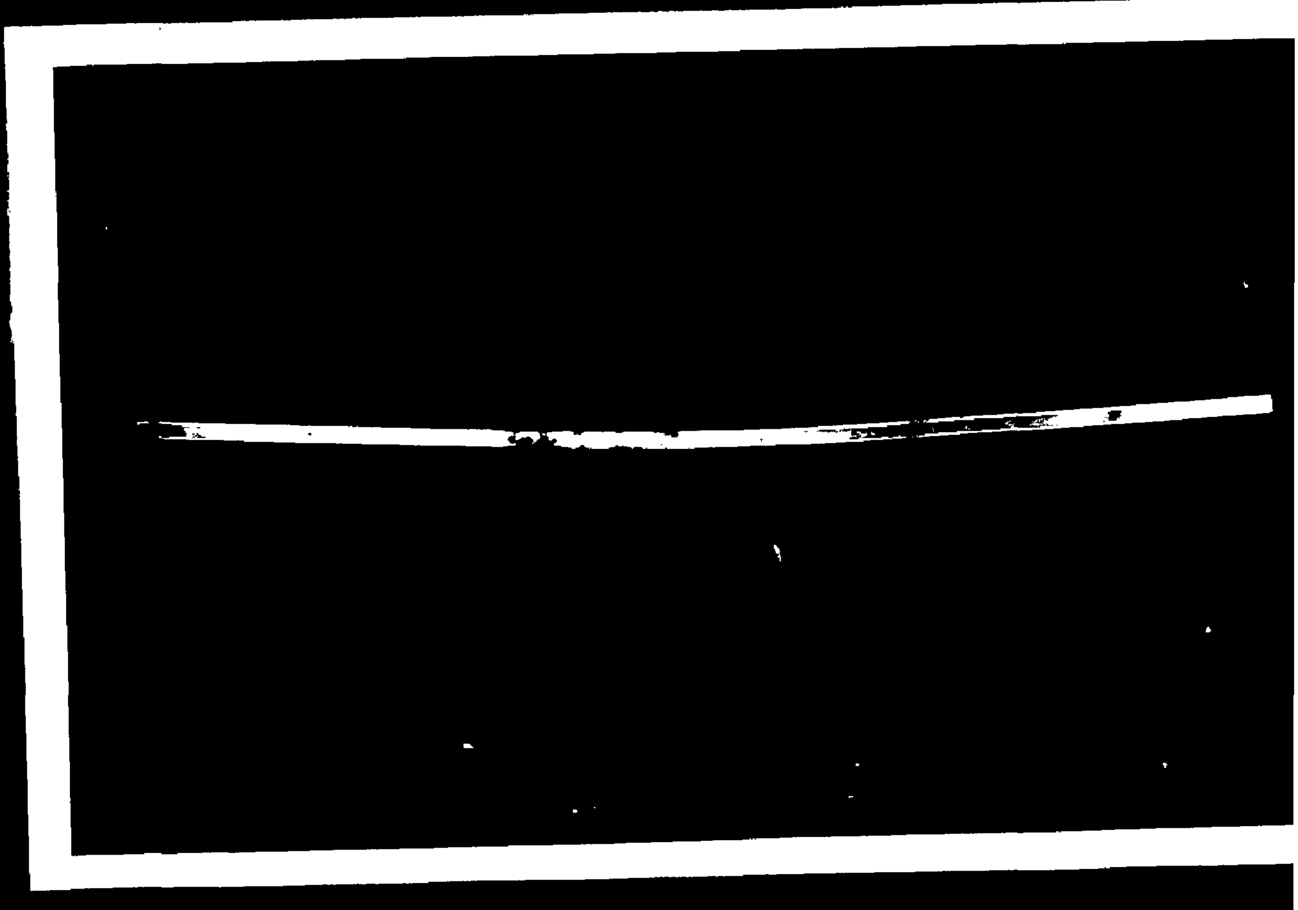
From the graph no. 8.2.

Elastic limit moment = 200 inch lbs. $= f_L \frac{bd^2}{6}$

$\therefore f_L = 23.3$ p.s.i.



FLEXURAL TEST



Corresponding value of Mp = 300 inch lbs.

Ultimate moment of resistance of section = 440 in. lbs.

(from graph) $= Fu \cdot \frac{bd^2}{4}$

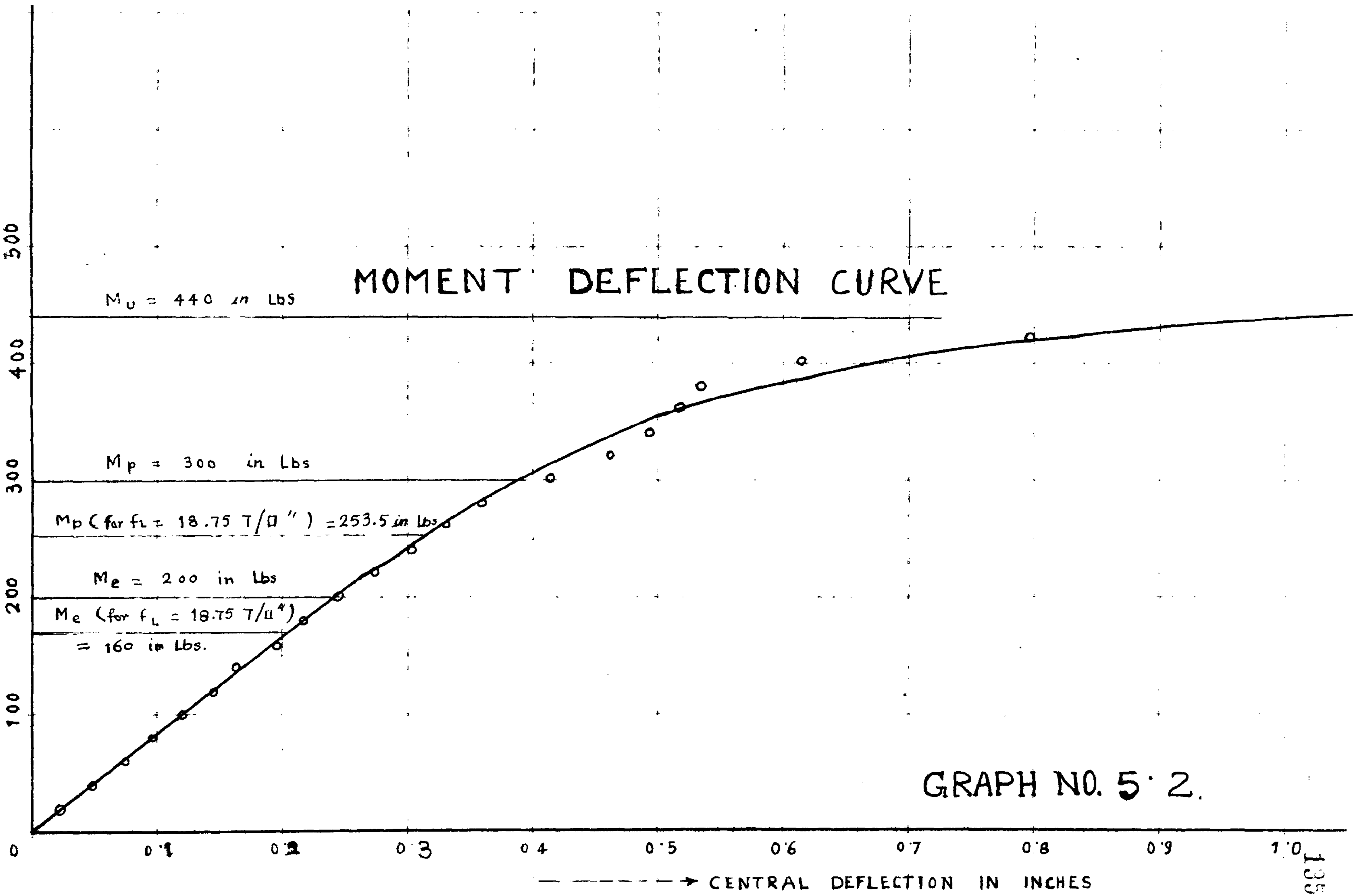
where Fu is ultimate stress and $= \frac{440 \times 4}{bd^2 \times 2240}$

= 32.7 Tons per square inch.

TABLE NO. 5.3.

Each Load in Lbs.	Bending moment at the Centre in inch Lbs.	Deflection at the Centre in inches.			Remarks
		First Specimen	Second Specimen	Average	
0	0	0	0	0	
5	20	.0232	.0235	.02335	
10	40	.046	.049	.0475	
20	80	.0970	.0950	.0960	
25	100	0.121	.119	.120	
30	120	.145	.143	.144	
35	140	.165	.164	.1645	
40	160	.198	.194	.196	
45	180	.221	.215	.218	
50	200	.248	.238	.243	Yielding starts
55	220	.277	.268	.273	
60	240	.315	.295	.305	
65	260	.343	.321	.332	
70	280	.369	.351	.360	
75	300	.425	.407	.416	
80	320	.470	.457	.464	
85	340		.496	.496	
90	360		.518	.518	
95	380		.532	.532	
100	400		.614	.614	
105	420		.799	.799	
110	440		1.068	1.068	Ultimate collapse.

These values of FL and Fu are little toohigh as compared to those found out by the tension test. The values as found out


MOMENT DEFLECTION CURVE
Mu = 440 in Lbs
Mp = 300 in Lbs
Mp (for fL = 18.75 T/□") = 253.5 in Lbs
Me = 200 in Lbs
Me (for fL = 18.75 T/□") = 160 in Lbs.
500
400
300
200
100
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
CENTRAL DEFLECTION IN INCHES
GRAPH NO. 5·2.

by tensile test has however been adopted in the computations for the frame.

The values of elastic limit moment and fully plastic moment of resistance of the section calculated on the basis of F_L and F_u adopted from the tensile test are 169 in lbs and 253.5 in lbs respectively and are also shown on the graph No. 5.2.

5.4. CALIBERATION OF LEVERS

Caliberation of levers was necessary because of variety of reasons. Firstly different cables connecting the load end of respective levers to the corresponding two point loading devices, are passing round different sets as well as different number of pulleys. This would vary the efficiency of different levers. Secondly there could be error in the leverage due to faulty workmanship. Finally the section of all the levers was not uniform due to faulty workmanship and also the levers were not exactly counterbalanced in order to take the advantage of self weight of levers in reducing the amount of weights required at the loading end. For all these reasons it was therefore necessary to caliberate each lever. For this purpose a special apparatus was designed and fabricated. The apparatus has already been described im Chapter IV.

The caliberation apparatus was mounted on the table as shown on photopage 125 . A cable was given requisite initial tension and one end of it connected to the caliberation apparatus, while the other end after passing the cable round the respective set of pulleys was connected one by one inturn to the load ends of the levers. The caliberation apparatus was shifted everytime by an amount equal to the distance between the final positions of the cables, while caliberating the different levers. After having connected the apparatus to the lever, the lever was released and initial reading on the proving ring recorded. The loading end of the lever was loaded with weights of 1 lb. each and corresponding readings recorded.The lever was unloaded and reloaded number of times and mean readings of proving ring were recorded to plot the graph. Similarly graphs between the load at the hanger and the proving ring reading which is the load transfered to the loading points, were plotted for all the levers. As a specimen caliberation curve for the lever no. 1 is shown in the graph no. 8.3. other curves are similar and hence not included here.

Now since there are number of loads to be applied, simultaneously, and increment in each load has to be uniform, it was decided to so adjust the weight of different hangers that initial load applied at the loading points due to the self weight of levers only, is uniform throughout. Further different sets of loads were cast so that by placing them on respective levers, same load is transfered to the loading points throughout, for example it was found out that a load of 1 lb. at the hanger in case of first lever, caused a load of 12.6 lbs. at each loading point, while it was only 14 ozs. at lever no. 2 which when placed at the hanger would cause the same load e.g. 12.6 lbs. at each loading point similarly it was calculated for all the levers. This arrangement greatly facilitated the difficulty of getting proportional loading throughout.

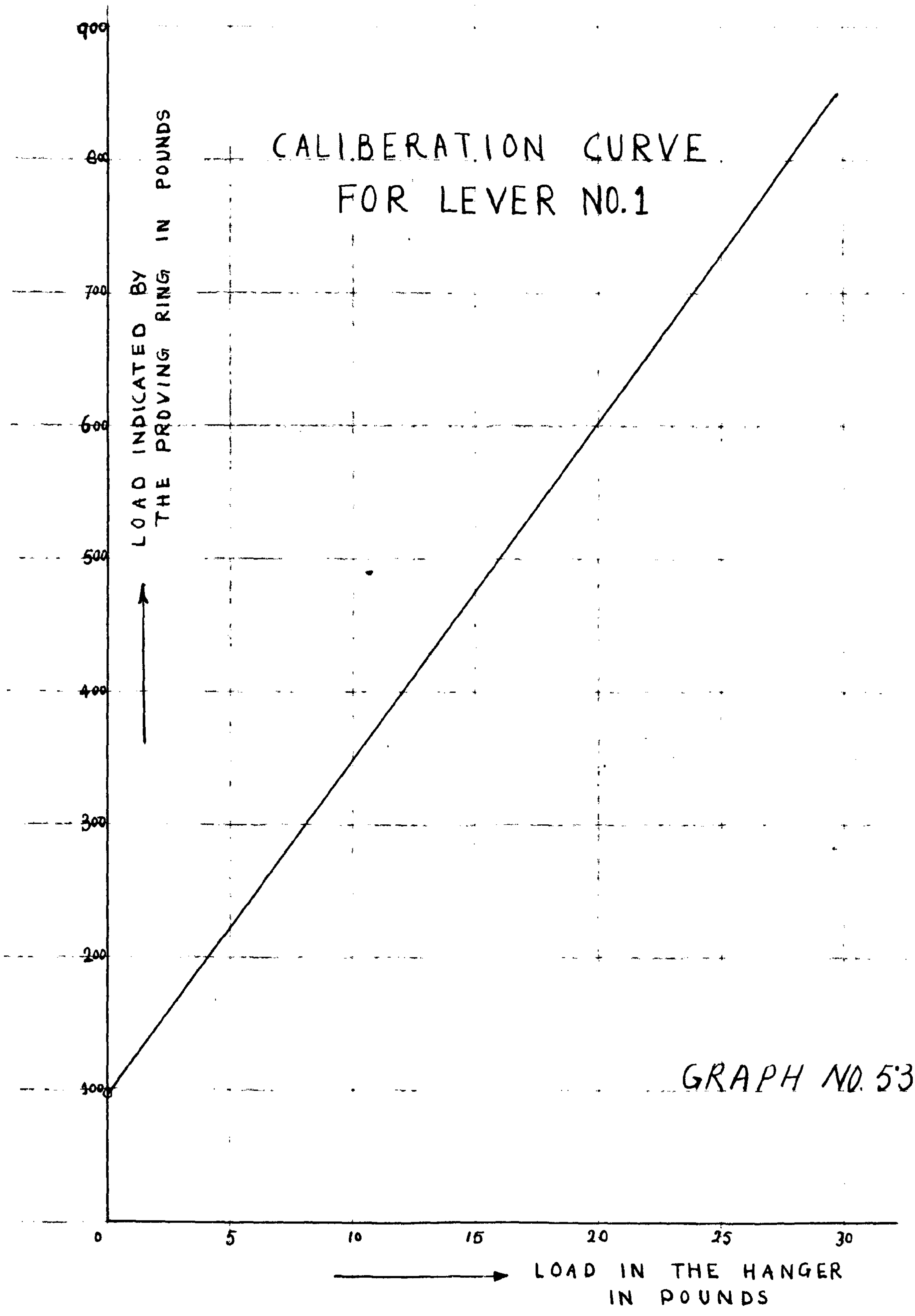

CALIBERATION CURVE
FOR LEVER NO.1
LOAD INDICATED BY THE PROVING RING IN POUNDS
900
800
700
600
500
400
300
200
100
0
5
10
15
20
25
30
LOAD IN THE HANGER IN POUNDS
GRAPH NO. 53

TABLE No. 5.4.

Loads in lbs.		Deflection at (in inches)						
each Vertical load	EACH horrizontal load	First floor level	Second floor level	Third floor level	Fourth floor level	Fifth floor level	Sixth floor level	Remarks
0	0	0	0	0	0	0	0	
48.0	24.0	.009	.025	.039	.048	.066	.072	
54.3	27.15	.012	.030	.046	.055	.063	.079	
60.6	30.3	.014	.035	.054	.062	.072	.086	
66.9	33.45	.015	.039	.062	.072	.082	.097	
73.2	36.6	.017	.043	.068	.083	.094	.110	
85.8	42.9	.019	.049	.074	.096	.109	.126	
98.4	49.2	.022	.055	.085	.112	.126	.141	
111.0	55.5	.025	.062	.101	.128	.148	.163	YIELDING STARTS.
123.6	61.8			.112	.144	.167	.189	
136.2	68.1			.141	.162	.198	.221	
148.8	74.4			.214	.256	.299	.319	
161.4	80.7			.285	.310	.353	.371	
174.0	87.0			.388	.402	.418	.452	
186.6	93.3			.478	.544	.584	.651	
199.2	99.6			.691	.741	.812	.896	

5.5. TESTING OF THE FRAME :

The test frame has already been described in Chapter IV. Before mounting the test frame on the testing table, all the loading bridges were inserted on the frame and their other end was now welded. The frame was mounted on the testing table as shown on photopage 109 . It was carefully levelled by means of a spirit level, by adjusting the c.i. pads on m.s. pins. Although while fabricating various components it was taken care that all the pulleyes are at proper height so that cable is in the plane of the frame, yet it was again checked by means of a level and slight packings inserted where ever found necessary. Supports with steel balls at either side of the frame were now introduced at each joint in order to simulate the existence of beams in a structure and thus eliminating the danger of buckelling of frame in a transverse plane.

After having fixed up the frame the dial gages on magnetic bases were placed against each floor level as shown on photopage and their initial readings recorded. The horrizontal loads were introduced by means of screw loading device as shown in the photopage 113 . The vertical loads were then applied by inserting the hooks in the two point loading device. First reading was taken corresponding to each vertical load equal to 48 lbs. while each horrizontal load equal to 24 lbs, this being due to the self weight of levers only. Subsequent increments to loads were given and deflections recorded as shown in table no. 5.4. and 5.5. for the two frames.

5.5. TESTING OF THE FRAME :

The test frame has already been described in Chapter IV. Before mounting the test frame on the testing table, all the loading bridges were inserted on the frame and their other end was now welded. The frame was mounted on the testing table as shown on photopage 109 . It was carefully levelled by means of a spirit level, by adjusting the c.i. pads on m.s. pins. Although while fabricating various components it was taken care that all the pulleyes are at proper height so that cable is in the plane of the frame, yet it was again checked by means of a level and slight packings inserted where ever found necessary. Supports with steel balls at either side of the frame were now introduced at each joint in order to simulate the existence of beams in a structure and thus eliminating the danger of buckelling of frame in a transverse plane.

After having fixed up the frame the dial gages on magnetic bases were placed against each floor level as shown on photopage and their initial readings recorded. The horrizontal loads were introduced by means of screw loading device as shown in the photopage 113 . The vertical loads were then applied by inserting the hooks in the two point loading device. First reading was taken corresponding to each vertical load equal to 48 lbs. while each horrizontal load equal to 24 lbs, this being due to the self weight of levers only. Subsequent increments to loads were given and deflections recorded as shown in table no. 5.4. and 5.5. for the two frames.

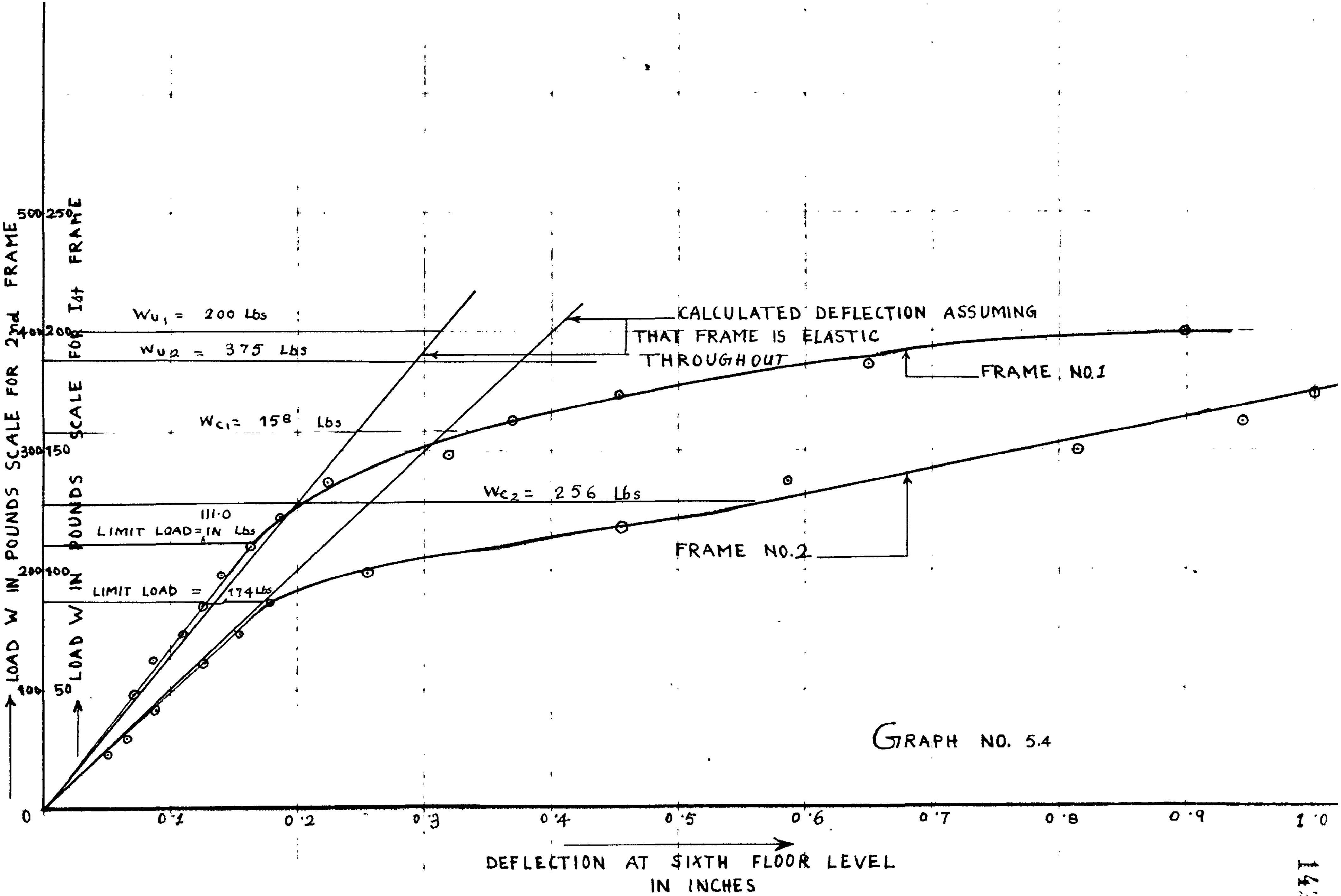
TABLE No. 5.4.

Loads in lbs.		Deflection at (in inches)						
each Vertical load	EACH horrizontal load	First floor level	Second floor level	Third floor level	Fourth floor level	Fifth floor level	Sixth floor level	Remarks
0	0	0	0	0	0	0	0	
48.0	24.0	.009	.025	.039	.048	.056	.072	
54.3	27.15	.012	.030	.046	.055	.063	.079	
60.6	30.3	.014	.035	.054	.062	.072	.086	
66.9	33.45	.015	.039	.062	.072	.082	.097	
73.2	36.6	.017	.043	.068	.083	.094	.110	
85.8	42.9	.019	.049	.074	.096	.109	.126	
98.4	49.2	.022	.055	.085	.112	.126	.141	
111.0	55.5	.025	.062	.101	.128	.148	.163	YIELDING STARTS.
123.6	61.8			.112	.144	.167	.189	
136.2	68.1			.141	.162	.198	.221	
148.8	74.4			.214	.256	.299	.319	
161.4	80.7			.285	.310	.353	.371	
174.0	87.0			.388	.402	.418	.452	
186.6	93.3			.478	.544	.584	.651	
199.2	99.6			.691	.741	.812	.896	

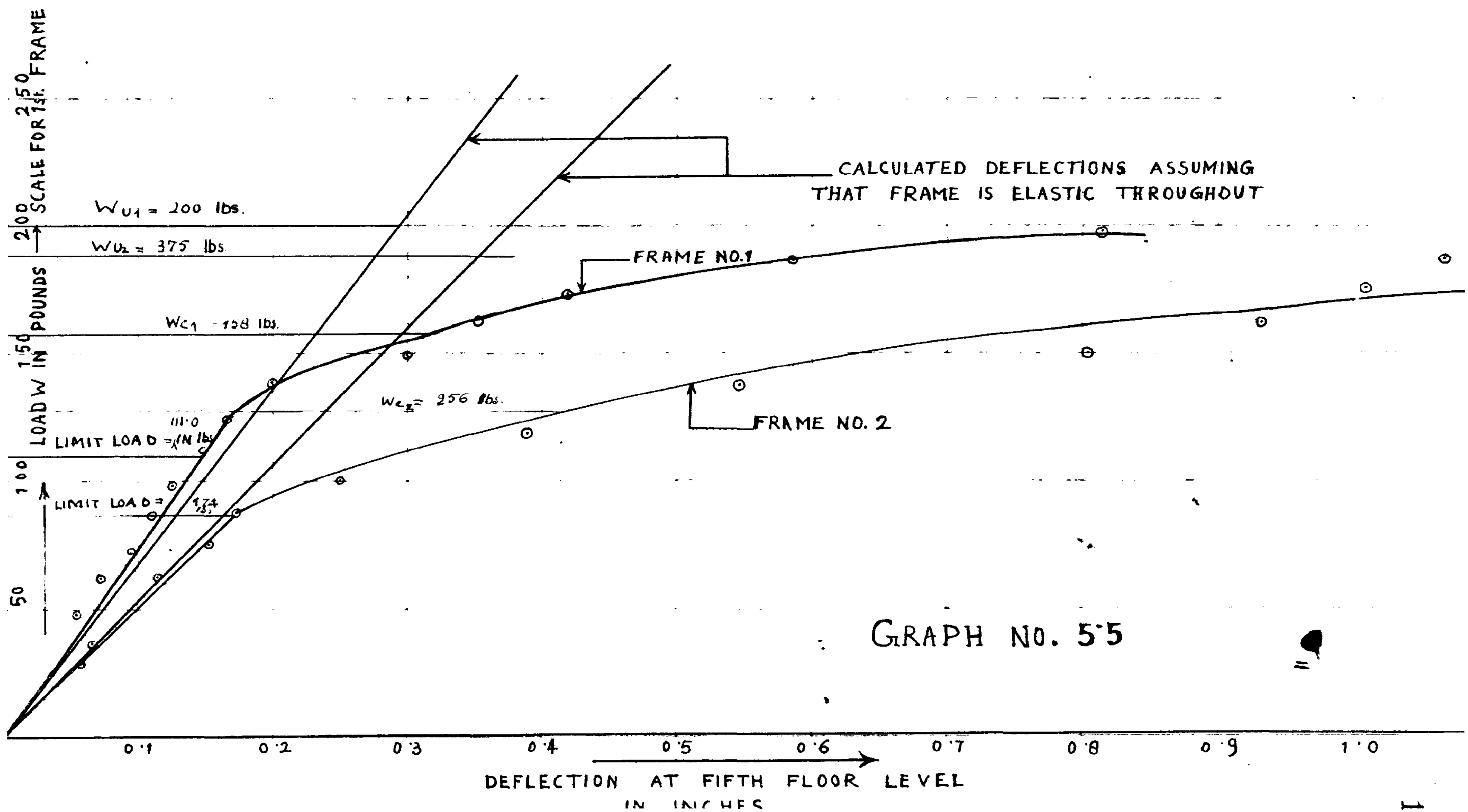
TABLE NO. 5.5.

Loads in lbs.		Deflection at (in inches)						
Each vertical load	Each horriz-ontal load	First floor level	Second floor level	Third floor level	Fourth floor level	Fifth floor level	Sixth floor level	Remarks
0	0	0	0	0	0	0	0	
48.0	24.0	.006	.014	.03	.040	.049	.05	
60.6	30.3	.008	.022	.028	.049	.056	.061	
85.8	42.9	.013	.034	.053	.068	.080	.086	
123.6	61.8	.027	.061	.078	.112	.116	.126	
148.8	74.4	.034	.068	.111	.132	.152	.154	
1.74.0	87.00	.038	.083	.126	.142	.170	.178	
199.2	99.6	.061	.138	.179	.216	.249	.255	Yielding starts.
237.0	118.5	.098	.189	.316	.379	.390	.450	
274.8	137.4	.136	.241	.446	.468	.548	.582	
300.0	150.0	.173	.273	.492	.531	.805	.815	
325.2	162.6	.287	.334	.692	.769	.932	.946	
350.4	175.2	.311	.346	.796	.895	1.008	1.01	
375.6	187.8	.323	.366	.878	.976	1.022	1.033	

Graphs 5.4 to 5.9 were plotted between load parameter and the corresponding deflections at different floor levels. Graphs for one particular floor level for both the frames are shown on the same graph. On the graph are also shown straight lines showing corresponding deflections, had the frame behaved elastically throughout. Comparisón between these strainght lines and actual observed deflection present a vivid picture of --- --- ------

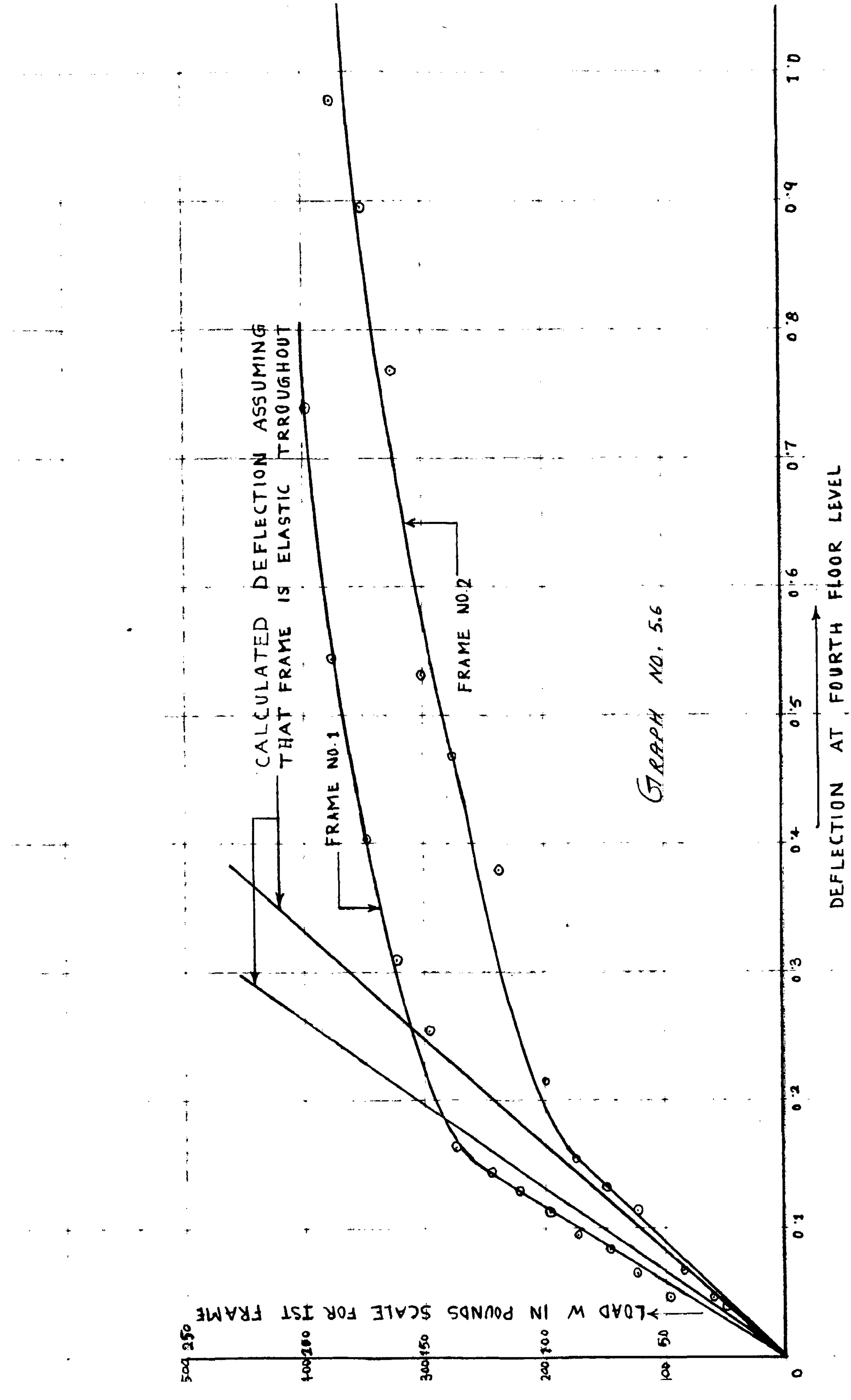

CALCULATED DEFLECTION ASSUMING
THAT FRAME IS ELASTIC
THROUGHOUT
FRAME NO.1
FRAME NO.2
Wu1 = 200 Lbs
Wu2 = 375 Lbs
Wc1 = 158 Lbs
Wc2 = 256 Lbs
LIMIT LOAD = 111·0 IN Lbs
LIMIT LOAD = 174 Lbs
LOAD W IN POUNDS SCALE FOR 2nd FRAME
LOAD W IN POUNDS SCALE FOR Ist FRAME
DEFLECTION AT SIXTH FLOOR LEVEL
IN INCHES
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
50
100
150
200
250
300
400
500

GRAPH NO. 5.4

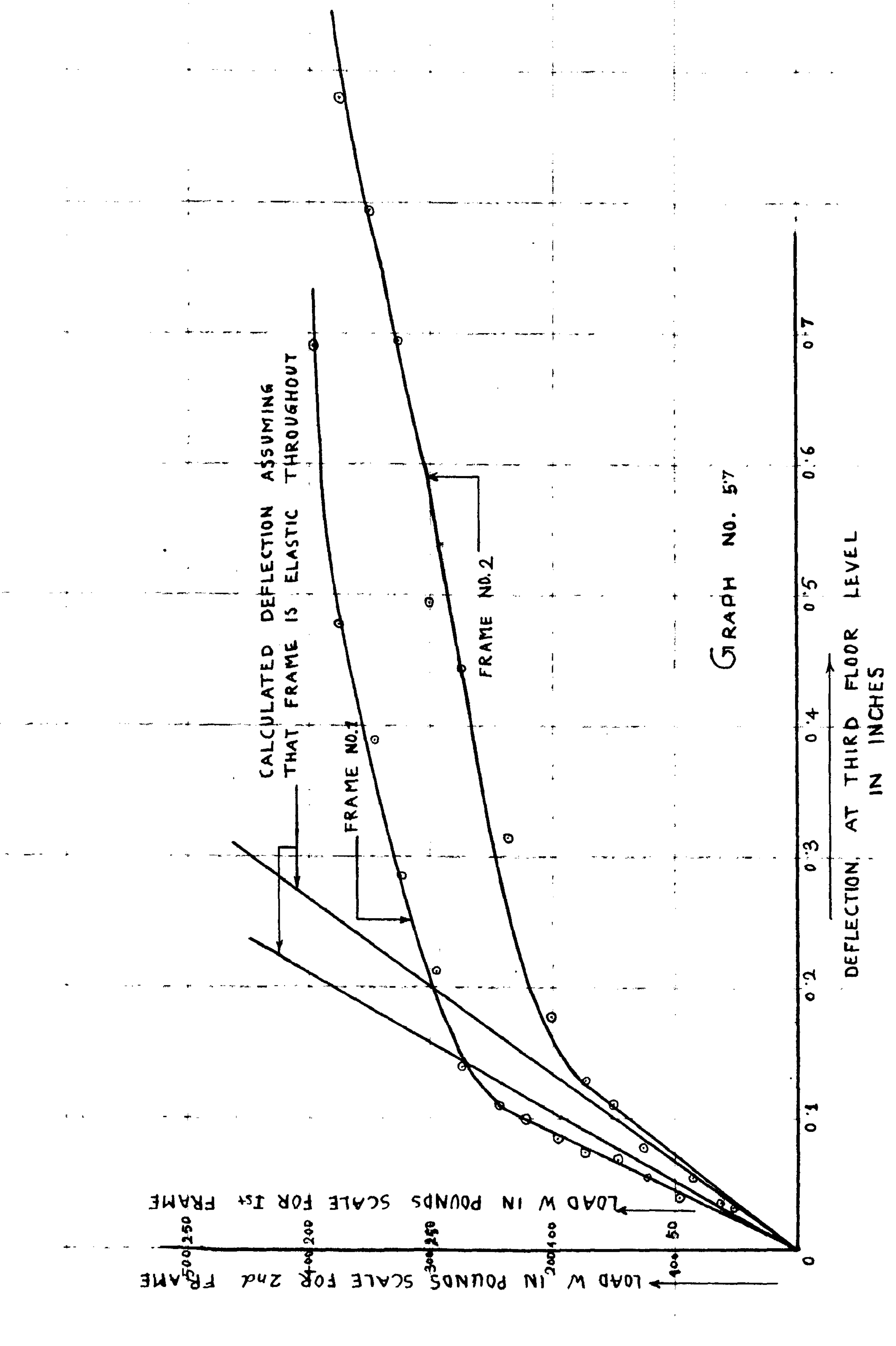


CALCULATED DEFLECTIONS ASSUMING THAT FRAME IS ELASTIC THROUGHOUT
FRAME NO.1
FRAME NO. 2
Wu1 = 200 lbs.
Wu2 = 375 lbs
Wc1 = 158 lbs.
Wc2 = 256 lbs.
LIMIT LOAD
LIMIT LOAD
LOAD W IN POUNDS
SCALE FOR 1st. FRAME
50
100
150
200
250
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
DEFLECTION AT FIFTH FLOOR LEVEL IN INCHES

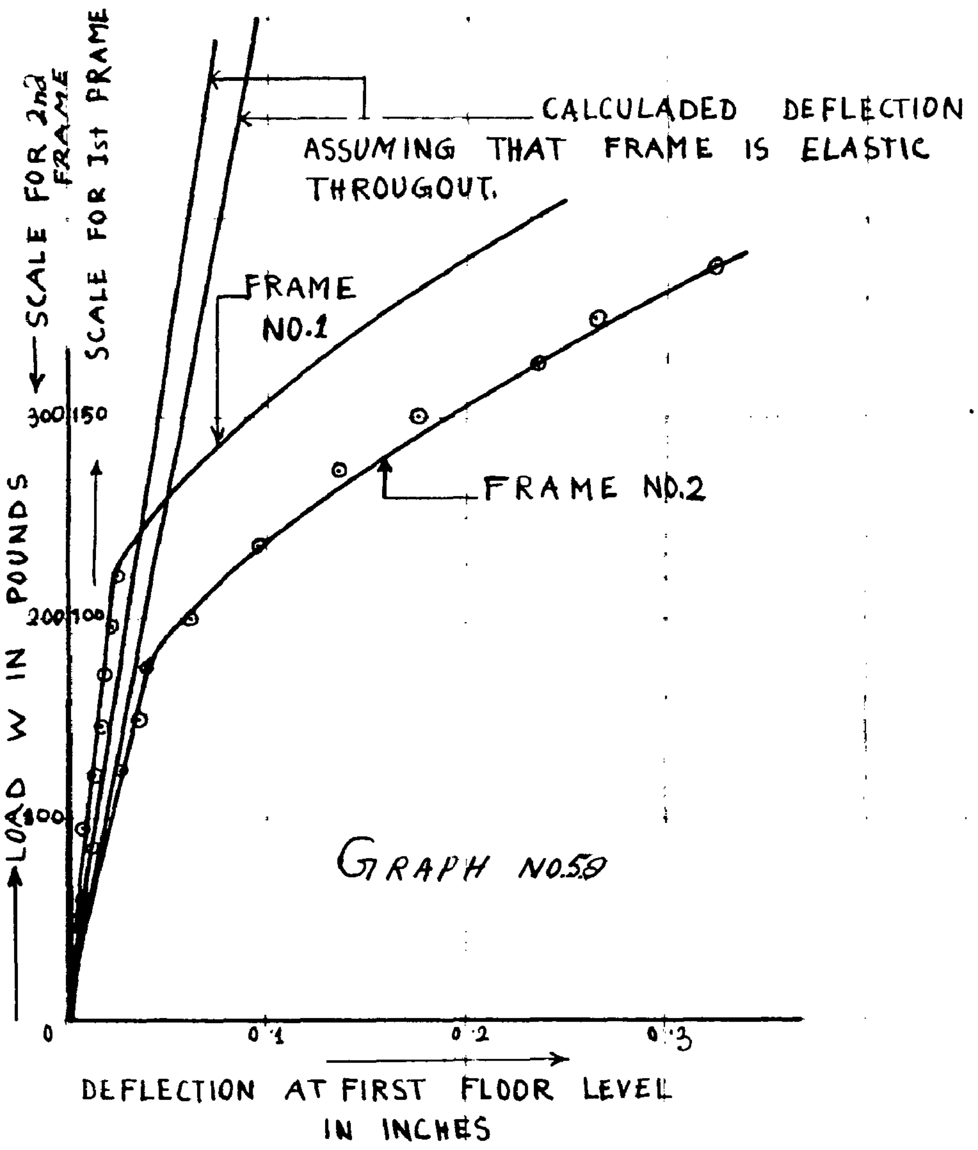
GRAPH No. 5·5



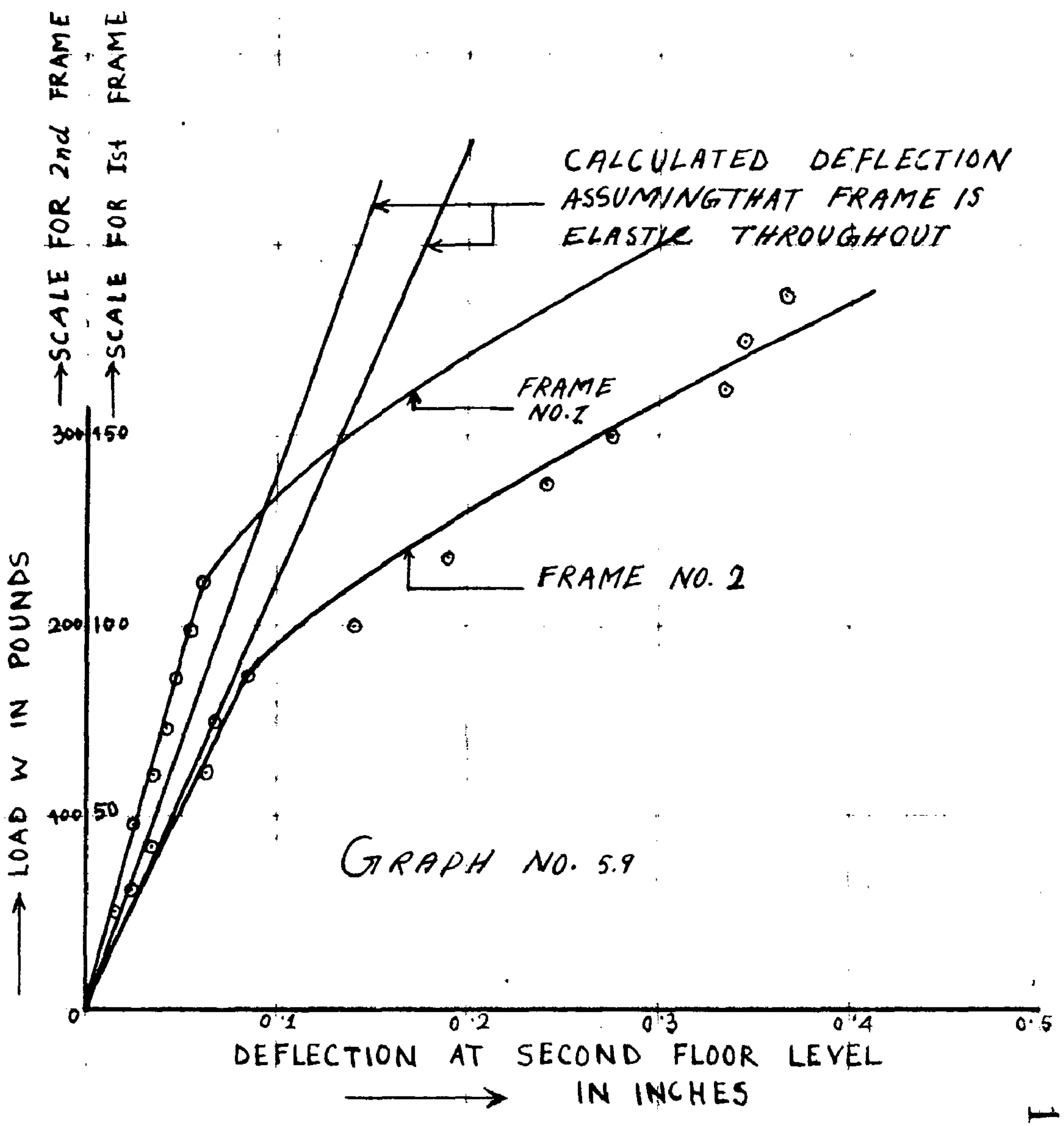
CALCULATED DEFLECTION ASSUMING
THAT FRAME IS ELASTIC TRROUGHOUT
FRAME NO.1
FRAME NO.2
GRAPH NO. 5.6
DEFLECTION AT FOURTH FLOOR LEVEL
IN INCHES
LOAD W IN POUNDS SCALE FOR 1ST FRAME
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
50
100
150
200
250
100
200
300
400
500



CALCULATED DEFLECTION ASSUMING THAT FRAME IS ELASTIC THROUGHOUT
FRAME NO.1
FRAME NO.2
GRAPH NO. 57
DEFLECTION, AT THIRD FLOOR LEVEL IN INCHES
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
LOAD W IN POUNDS SCALE FOR 1st FRAME
50
100
150
200
250
LOAD W IN POUNDS SCALE FOR 2nd FRAME
100
200
300
400
500



CALCULADED DEFLECTION
ASSUMING THAT FRAME IS ELASTIC
THROUGOUT.
FRAME NO.1
FRAME NO.2
SCALE FOR 2nd FRAME
SCALE FOR 1st FRAME
LOAD W IN POUNDS
300 150
200 100
100
0
0·1
0·2
0·3
DEFLECTION AT FIRST FLOOR LEVEL IN INCHES
GRAPH NO.5.8



CALCULATED DEFLECTION
ASSUMINGTHAT FRAME IS
ELASTIC THROUGHOUT
FRAME NO.1
FRAME NO. 2
SCALE FOR 2nd FRAME
SCALE FOR 1st FRAME
LOAD W IN POUNDS
300 150
200 100
400 50
0
0·1
0·2
0·3
0·4
0·5
DEFLECTION AT SECOND FLOOR LEVEL IN INCHES
GRAPH NO. 5.9

how the behaviour of frame changes as loading increases from elastic range to elasto-plastic and finally plastic range.

5.6. TEST RESULTS :

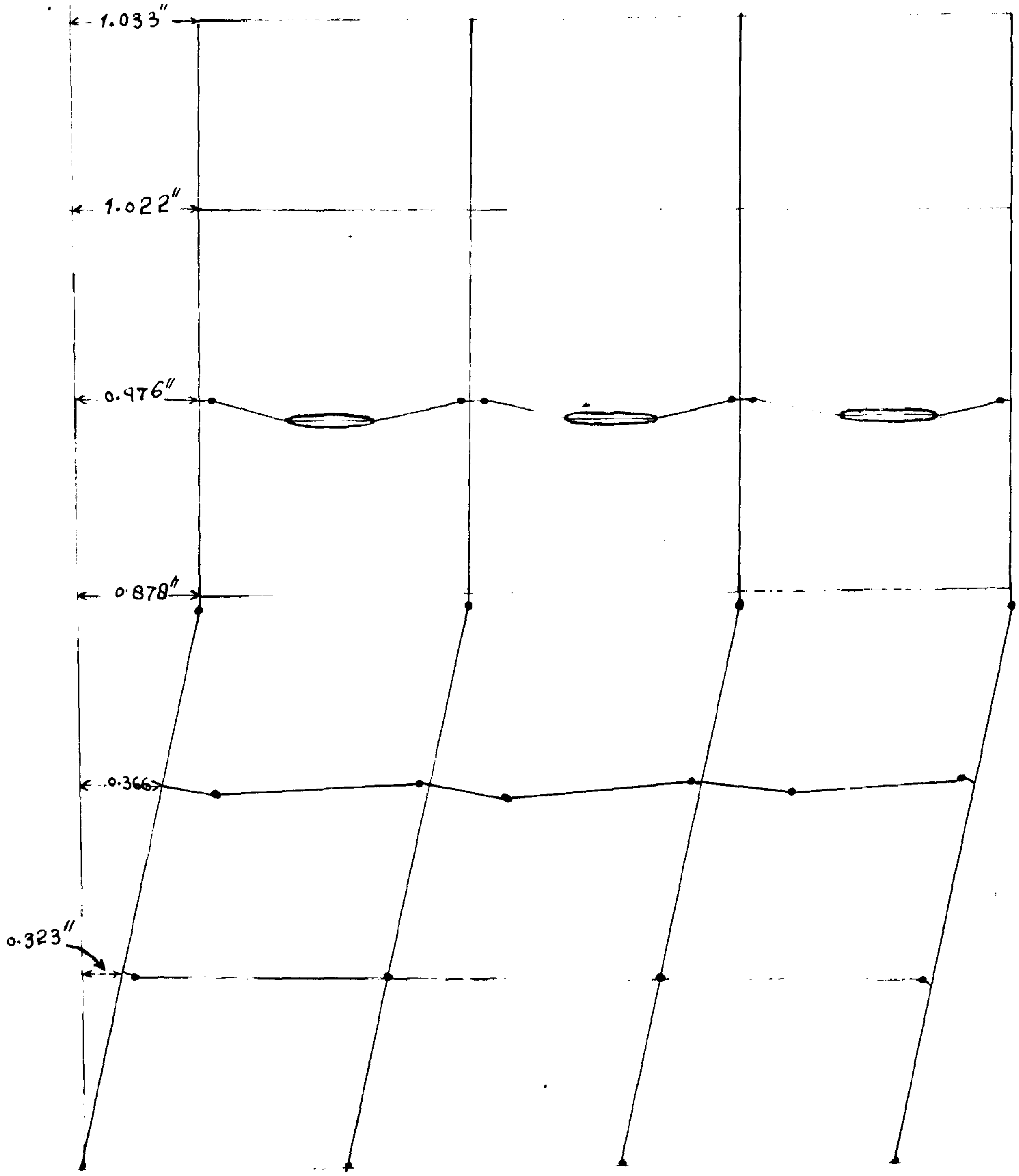
With the help of known value of yield stress of steel from tensile stress-strain curve (Graph 5.1) and the properties of section of the frame, and with the help of graphs 5.4. to 5.9 plotted after actual testing, following results can be arrived at :-

S.No.		Frame 1	Frame 2
1.	Me i.e. elastic moment of resistance of the section when stress in extreme fibre has just reached yield stress $= F_L \frac{bd^2}{6}$	170 in lbs.	276 in. lbs.
2.	M_p i.e. fully plastic moment of resistance of the section $= F_L \frac{bd^2}{4}$	255 in lbs	414 in lbs.
3.	Load at which the stress in the most highly stressed fibre at any section first reaches yield stress $= \frac{M_e}{2.6499} = W_e$	60 lbs.	97 lbs.
4.	Load corresponding to 3 if ultimate stress is taken instead of yield stress $= W_e \times \frac{29.1}{18.75} = W_e'$	93 lbs.	151 lbs.
5.	Collapse load W_c according to idealised plastic theory $= \frac{M_p}{1.613}$	158 lbs.	256 lbs.
6.	Actual observed ultimate load i.e. W_u	200 lbs.	375 lbs.

7. Limit load from the graphs i.e. upto where the deflections are proportional to load parameter i.e. W_L	111.0 lbs.	174.0 lbs.
8. Taking a safety factor of 1.2 on limit load, working load i.e. $W_W = \frac{W_L}{1.2}$	92.5 lbs.	145.0 Lbs.
9. Working load $W_{W'}$ taking a safety factor of 1.2 on W_e	50.0 lbs.	81.0 lbs.
10. Ultimate load/ W_e	3.33	3.86
11. $W_u/W_{e'}$	2.15	2.48
12. W_u/W_e	1.265	1.465
13. W_u/W_W	2.16	2.585
14. W_c/W_e	2.63	2.63
15. $W_e/W_{e'}$	1.7	1.7
16. W_e/W_W	1.71	1.765
17. $W_W/W_{W'}$	1.85	1.79
18. Value of stress $F_{L'}$ if substituted for F_L in the expression of collapse load i.e. $W_c = \frac{1}{1.613} \times \frac{1}{4} F_L \; bd^2$. will give the ultimate collapse load as found out experimentally	23.7 Tons/in^2	27.5 Tons/in^2
19. Strain corresponding to $F_{L'}$ from the stress strain curve.	3.25 %	6.4%

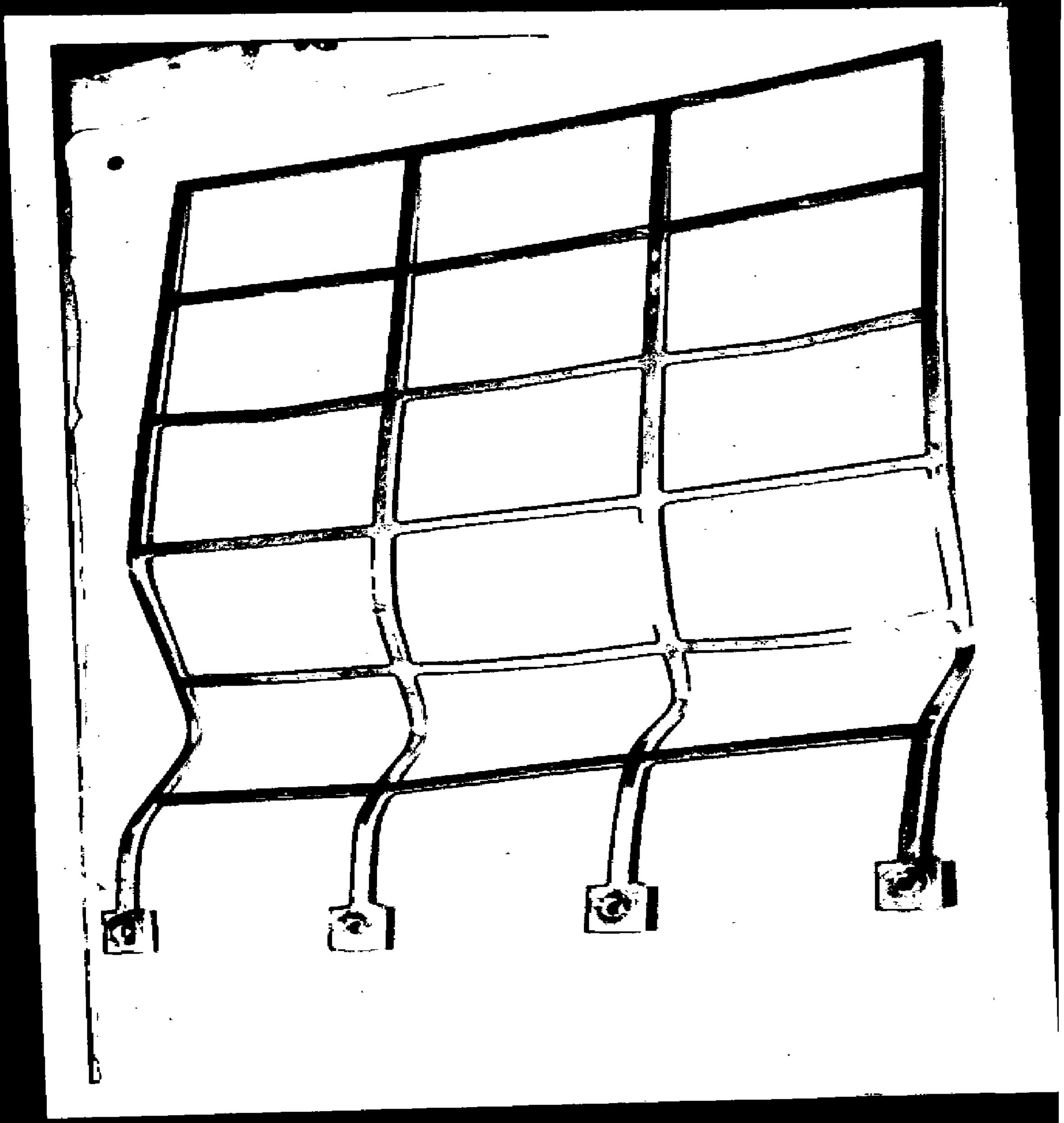
Figure 5.4 and photographs on pages 151 and 152 show the final shape of the frame after testing. Since part of strains are regained back on unloading, deflected shape in the photographs does

not represent the actual shape when fully loaded but idea can be obtained from it about the plastic deformations in the frame. Fig. 5.4. is however not according to scale. This is in order to show more distinctly the type of failure.

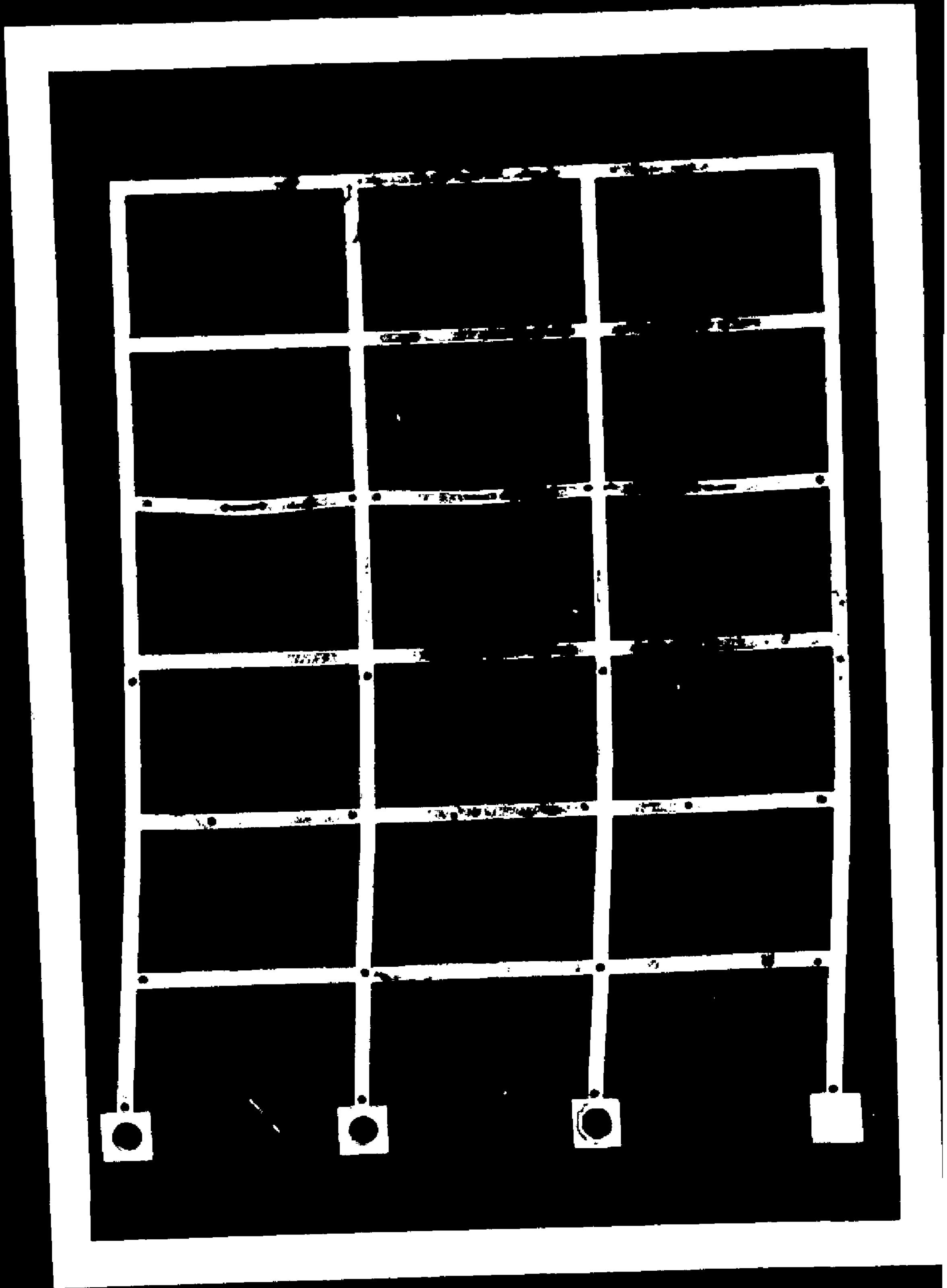


(Dark portions show plastic deformations have occurred at these sections)

Fig. 5.4



FRAME NO. 2 AFTER TESTING



FRAME No. 2 AFTER TESTING

CHAPTER VI

" DISCUSSION OF TEST RESULTS AND CONCLUSIONS "

6.1. The Frames during the test behaved in a fashion very much similar to the one expected according to elastic and plastic theories in their respective range, until the collapse occured by the formation of plastic hinges at various sections, rendering the frame into a sort of mechanism. The final shape of the frames is shown on photopages 151,152. Although the first frame which was thinner than the second bucklled before the loads were removed because of the displacement of the supporting devices whichwere providing transverse stability to the frame, but its behaviour was almost identical to the frame no. 2. The position of various plastic hinges on the frame no. 2 are shown in the photograph on page 152. + fig 5.4. The observations made on the test frames, and a brief discussion on these observations is given in following paragraphs.

6.2. The behaviour of the frames was almost in accordance with Elastic theory during elastic range of loading. In graphs no. 5.4 to 5.9, the observed deflections at different floor levels have been plotted against load parameter i.e. value of each vertical load. Deflections as calculated by elastic theory (Appendix A), for a frame behaving elastic theroughout have also been plotted as a straight lines on the same graphs. The two graphs show a very clear picture of the behaviour of the frame during both the elastic as well as plastic range. The observed deflections differ from the calculated ones only by an amount within ± 10% which shows reasonable accuracy that can be expected from experimental results.

It is also observed from the graphs that deflections vary linearly not only upto the elastic limit load i.e. when stress in any fibre in the frame has just reached the yield stress, but also to some extent beyond it. This may be interpreted as that in a highly redundant structure like a multistoreyed building the formation of first few hinges do not affect elastic behaviour of the structure to any appreciable extent except that the bending moment at the sections where plastic hinges have formed, now increases only at a much smaller rate and does not become altogether constant. This happens because of strain hardening at those sections. Thus it is seen that the deflections immediately after the elastic limit has reached, do not increase as rapidly as it is usually thought but it is only when quite a few sections are in elasto-plastic state that the deflections start increasing very rapidly till they become enormous (in present case it was about 1 inch at top storey which is 24" from base) to be taken as a failure of the structure. This can be clearly seen from the graphs. 5.4 to 5.9

Observations made on the deflected shape of the frame when it was loaded to collapse, revealed that the failure occured due to the formation of a mechanism very much similar to the one arrived at theoretically as explained in the chapter II and appendix C. Of course the rotation at different hinges varies from section to section, some hinges being full hinges while others only semi hinges. This is also true because in theory we take only an idealised curve which is not actually true. The only marked deviation was that of the presence of local failures in the form of beams mechanisms at fourth floor level in addition to the one found out to be most critical by theoretical analysis. The reason may be due to the fact that since the failure of frame has been only due to partial collapse, the frame after having become mechanism according to the one found to be most critical, was undergoing the stage of strain hardening, and during this stage more loads were applied, so the sections where the bending moment was previously very near to the fully plastic moment of resistance, now became plastic thus forming a local mechanism. This is also obvious as the load finally applied to the frame was as much as 1.5 times that of theoretical collapse load as calculated by the simple plastic theory.

The study of test results given on page 147 show that the actual ultimate load may be as much as 1.25 to 1.5 times the calculated collapse load by the idealised plastic theory. This can be explained by the fact that in plastic theory the effect of strain hardening is not taken into account while in actual structure it will always increase the load carrying capacity of the structure. Thus it is seen that instead of taking value of Fy in $M_p = \frac{1}{4} F_y bd^2$ as lower yield stress of material in tensile test, if some higher value is adopted (which in these two cases is found to be corresponding to 3.25 and 6.40 % strain) the ultimate load may be more correctly predicted. It is also observed that the working load as found with the help of tests may be as much as 1.5times the load that can be allowed for according to elastic theory, thus great economy in amount of steel can be expected if we base our designs on actual ultimate load carrying capacity of the frame.

6.3. From the above discussion and the final results it was concluded that :

1. A multistoreyed structure behaves elastically upto stage where elastic limit is reached at any section, after that it will behave in semiplastic state and the deflections do not differ appreciably from those calculated by the elastic theory until quite a few number of semiplastic hinges have developed and finally it will fail in a fashion very much similar to the one expected in accordance with simple plastic theory.
2. The concept of formation of plastic hinges and failure due to the formation of mechanism at ultimate load is quite true in case of multistoreyed structures.
3. The use of simple plastic theory basedon Baker's idealised stress-strain curve is very safe, the actual ultimate load may be as much as 1.25 to 1.5 times that of the collapse load calculated by simple plastic theory.
4. The required rotation at various hinges in order to have complete redistribution of moments till the load is much beyond the value calculated by simple plastic theory will be available in most of the cases in practice.

APPENDIX A.

ANALYSIS OF THE FRAME BY SLOPE DEFLECTION EQUATIONS.

The frame is shown in the fig. AI.1 Various joints have been numbered from 1 to 28.

Fixing Moments.

$$M_{F7-14} = \frac{2 \times 4^2}{6^2} \times W + \frac{5 \times 1^2}{6^2} \times W = 1.0278\ W \text{ lbs in}$$

(anticlockwise, i.e.-)

$$M_{F14-7} = \frac{2^2 \times 4}{6^2} \times W + \frac{5^2 \times 1}{6^2} W = 1.1389\ W \text{ lbs.in}$$

(clockwise i.e. +)

$$M_{F14-21} = M_{F21-28}$$

$$= M_{F7-14} = 1.0278\ W \text{ lbs in (anticlockwise i.e. -)}$$

$$M_{F21-14} = M_{F28-21} = M_{F14-7} = 1.1389\ W \text{ lbs in}$$

(clockwise i.e. +)

$$M_{F6-13} = M_{F13-6} = M_{F13-20} = M_{F20-13}$$

$$= M_{F20-27} = M_{F27-20} = \text{Zero}$$

$$M_{F6-12} = M_{F12-19} = M_{F19-26}$$

$$= W \times \frac{2.5 \times 3.5^2}{6^2} + W \times \frac{3.5 \times 2.5^2}{6^2}$$

$= 1.4583\ W$ (anticlockwise i.e. -)

$$M_{F12-5} = M_{F12-19} = M_{F19-26} = M_{F5-12}$$

$= 1.4583\ W$ (Clockwise i.e. +)

$M_{F_{4-11}} = M_{F_{11-4}} = M_{F_{11-18}} = M_{F_{18-11}} = M_{F_{18-25}} = M_{F_{25-18}}$

= Zero.

$M_{F_{3-10}} = M_{F_{10-17}} = M_{F_{17-24}} = - M_{F_{14-7}}$

= 1.1389 W (Anticlockwise, i.e. -)

$M_{F_{10-3}} = M_{F_{17-10}} = M_{F_{24-17}} = - M_{F_{7-14}}$

= 1.0878 W (Clockwise, i.e. +)

$M_{F_{2-9}} = M_{F_{9-2}} = M_{F_{9-16}} = M_{F_{16-9}} = M_{F_{16-23}} = M_{F_{23-16}}$

= Zero

COMPATIBILITY EQUATIONS :-

$\theta_1 = \theta_8 = \theta_{15} = \theta_{22}$ [illegible]

= Zero.

$\theta_{2-9} = \theta_{2-3} = \theta_{2-1} = \theta_2$

$\theta_{3-10} = \theta_{3-2} = \theta_{3-4} = \theta_3$

and similarly at all other joints.

MEMBER EQUATIONS:-

Writing the slope deflection equations for each end of each member and putting $\frac{I}{L}$ for beam member = K such that $\frac{I}{L}$ for column members then becomes 1.5 K, we have :

$M_{1-2} = 3EK\left(2\theta_1 + \theta_2 - \frac{3\Delta_1}{L}\right)$

$= 3EK\left(\theta_2 - \frac{3\Delta_1}{L}\right)$

Similar expressions can be written for moments at the ends of

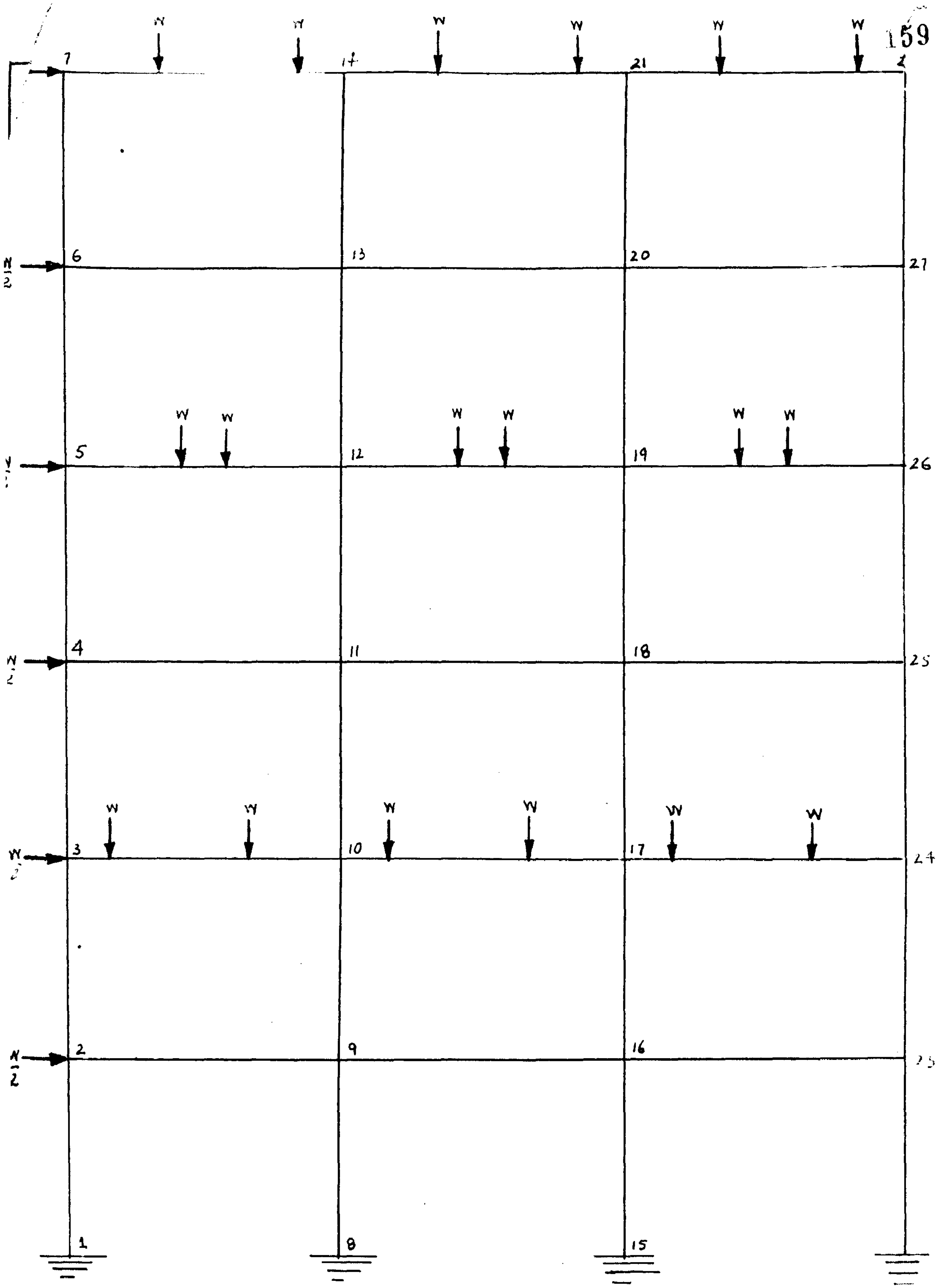


FIG. D.4

JOINT EQUATIONS:-

Joint 2 $\Sigma M_2 = 0$

i.e. $M_{2-1} + M_{2-9} + M_{2-3} = 0$

$$\therefore \; 2EK\left(3\theta_2 - \frac{4.5\Delta 1}{L} + 3\theta_2 + 1.5\,\theta_3 - \frac{4.5\Delta 2}{L} + 2\theta_2 + \theta_9\right) = 0$$

i.e. $8\,\theta_2 + 1.5\,\theta_3 + \theta_9 - 1.125\Delta 1 - 1.125\Delta 2 = 0 \qquad - 1$

Similarly equating $\Sigma M = o$ at all other joints:

$$1.5\theta_2 + 8\theta_3 + 1.5\theta_4 + \theta_{10} - 1.125\Delta 2 - 1.125\Delta 3 - \frac{1.1389W}{2EK} = o \qquad (2)$$

$$1.5\theta_3 + 8\,\theta_4 + 1.5\theta_5 + \theta_{11} - 1.125\Delta_3 - 1.125\Delta_4 = o \qquad (3)$$

$$1.5\theta_4 + 8\theta_5 + 1.5\theta_6 - \theta_{12} - 1.125\Delta 4 - 1.125\Delta 5 = \frac{1.4583W}{2EK} = o \qquad (4)$$

$$1.5\theta_5 + 8\theta_6 + 1.5\theta_7 + \theta_{13} - 1.125\Delta 5 - 1.125\Delta_6 = o \qquad (5)$$

$$1.5\theta_6 + 5\theta_7 + \theta_{14} - 1.125\Delta 6 - \frac{1.0278W}{2\,EK} = o \qquad (6)$$

$$\theta_2 + 10\,\theta_9 + 1.5\theta_{10} + \theta_{16} - 1.125\Delta 1 - 1.125\Delta_2 = o \qquad (7)$$

$$\theta_3 + 10\,\theta_{10} + 1.5\theta_9 + 1.5\theta_{11} + \theta_{17} - 1.125\Delta 2 - 1.125\Delta 3 - \frac{0.1111W}{2EK} = o \qquad (8)$$

$$\theta_4 + 1.5\theta_{10} + 10\theta_{11} + 1.5\theta_{12} + \theta_{18} - 1.125\Delta 3 - 1.125\Delta_4 = o \qquad (9)$$

$$\theta_5 + 1.5\theta_{11} + 10\theta_{12} + 1.5\theta_{13} + \theta_{19} - 1.125\Delta 4 - 1.125\Delta_5 = o \qquad (10)$$

$$\theta_6 + 1.5\theta_{12} + 10\theta_{13} + 1.5\theta_{14} + \theta_{20} - 1.125\Delta 5 - 1.125\Delta 6 = o \qquad (11)$$

$$\theta_7 + 1.5\theta_{13} + 7\theta_{14} + \theta_{21} - 1.125\Delta 6 + \frac{1111W}{2\,EK} = o \qquad (12)$$

$$\theta_9 + 10\theta_{16} + 1.5\theta_{17} + \theta_{23} - 1.125\Delta 1 - 1.125\Delta 2 = o \qquad (13)$$

$$\theta_{10} + 1.5\theta_{16} + 10\theta_{17} + 1.5\theta_{18} + \theta_{24} - 1.125\Delta 2 - 1.125\Delta 3 - \frac{.1111W}{2EK} = o \qquad (14)$$

$$\theta_{11} + 1.5\theta_{17} + 10\theta_{18} + 1.5\theta_{19} + \theta_{25} - 1.125\Delta 3 - 1.125\Delta 4 = o \qquad (15)$$

$$\theta_{12} + 1.5\theta_{18} + 10\theta_{19} + 1.5\theta_{20} + \theta_{26} - 1.125\Delta 4 - 1.125\Delta 5 = o \qquad (16)$$

$$\theta_{13} + 1.5\theta_{19} + 10\theta_{20} + 1.5\theta_{21} + \theta_{27} - 1.125\Delta 5 - 1.125\Delta 6 = o \qquad (17)$$

$$\theta_{14} + 1.5\theta_{20} + 7\theta_{21} + \theta_{28} - 1.125\Delta 6 + \frac{.1111W}{2EK} = o \qquad (18)$$

$$\theta_{16} + 8\theta_{23} + 1.5\,\theta_{24} - 1.125\Delta 1 - 1.125\Delta 2 = o \qquad (19)$$

$$\theta_{17} + 1.5\theta_{23} + 8\theta_{24} + 1.5\theta_{25} - 1.125\Delta 2 - 1.125\Delta 3 + \frac{1.0278W}{2EK} = o \qquad (20)$$

$$\theta_{18} + 1.5\theta_{24} + 8\theta_{25} + 1.5\theta_{26} - 1.125\Delta 3 - 1.125\Delta 4 = o \qquad (21)$$

$$\theta_{19} + 1.5\theta_{25} + 8\theta_{26} + 1.5\theta_{27} = 1.125\Delta_4 - 1.125\Delta_5 + \frac{1.4583\ W}{2\ EK} = 0 \quad (22)$$

$$\theta_{20} + 1.5\theta_{26} + 8\theta_{27} + 1.5\theta_{28} - 1.125\Delta_5 - 1.125\Delta_6 = 0 \quad (23)$$

$$\theta_{21} + 1.5\theta_{27} + 5\theta_{28} = 1.125\,\Delta_6 + \frac{1.1389\ W}{2\ EK} = 0 \quad (24)$$

Equations for side sway :-

Shear

For the 6th storey, we have

$$(M_{7-6} + M_{6-1} + M_{14-13} + M_{13-14} + M_{21-20} + M_{20-21} + M_{28-27} + M_{27-28})$$

$$+ 2\ W = 0$$

i.e. $$3\,\theta_6 + 3\,\theta_7 - \frac{6\Delta_6}{L} + 3\theta_{13} + 3\,\theta_{14} - \frac{6\Delta_6}{L} + 3\theta_{20} + 3\theta_{21}$$

$$- \frac{6\,\Delta_6}{L} + 3\theta_{27} + 3\theta_{28} - \frac{6\,\Delta_6}{L} + \frac{2W}{3\ EK} = 0$$

i.e. $$\theta_6 + \theta_7 + \theta_{13} + \theta_{14} + \theta_{20} + \theta_{21} + \theta_{27} + \theta_{28} - 2\,\Delta_6 + \frac{.4444W}{2\ EK} = 0 \quad (25)$$

Similarly for other storeys :

$$\theta_5 + \theta_6 + \theta_{12} + \theta_{13} + \theta_{19} + \theta_{20} + \theta_{26} + \theta_{27} - 2\Delta_5 + \frac{.8889W}{2\ EK} = 0 \quad (26)$$

$$\theta_4 + \theta_5 + \theta_{11} + \theta_{12} + \theta_{18} + \theta_{19} + \theta_{25} + \theta_{26} - 2\,\Delta_4 + \frac{1.3333W}{2\ EK} = 0 \quad (27)$$

$$\theta_3 + \theta_4 + \theta_{10} + \theta_{11} + \theta_{17} + \theta_{18} + \theta_{24} + \theta_{25} - 2\,\Delta_3 + \frac{1.7778W}{2\ EK} = 0 \quad (28)$$

$$\theta_2 + \theta_3 + \theta_9 + \theta_{10} + \theta_{16} + \theta_{17} + \theta_{23} + \theta_{24} - 2\,\Delta_2 + \frac{2.2222W}{2\ EK} = 0 \quad (29)$$

$$\theta_2 + \theta_9 + \theta_{16} + \theta_{23} - 2\Delta_1 + \frac{2.6667\ W}{2\ EK} = 0 \quad (30)$$

We thus have 30 equations for 30 unknown (24 joint rotations and 6 transtations). Solution of these equations has been done by Southwel's relaxation method of solving simultaneous equations. The operation table and the relaxatiom tables are given on pages 163 and 164

The final values of rotations and deflections in terms of a constant multiplier i.e. $\frac{W}{2EK}$ are

θ_2 = .5994	θ_9 = .4672	θ_{16} = .4533	θ_{23} = .6601
θ_3 = .7461	θ_{10} = .4378	θ_{17} = .4807	θ_{24} = .4316
θ_4 = .38836	θ_{11} = .37	θ_{18} = .33652	θ_{25} = .5374
θ_5 = .5453	θ_{12} = .2217	θ_{19} = .2832	θ_{26} = .11204
θ_6 = .1043	θ_{13} = .1763	θ_{20} = .1298	θ_{27} = .28643
θ_7 = .3308	θ_{14} = - .0028	θ_{21} = .09312	θ_{28} = - .1764

~~xRxx3302~~

Δ_1 = 2.42344 Δ_2 = 3.2489 Δ_3 = 2.75084

Δ_4 = 2.0615 Δ_5 = 1.37404 Δ_6 = .69298

Substituting these values back in the moment equations we get the end moments at various sections. These bending moments at various sections are shown on the table given on page ..165...

Final bending moment diagram after taking into account the sagging moments in various loaded beam members, is shown on page. 96

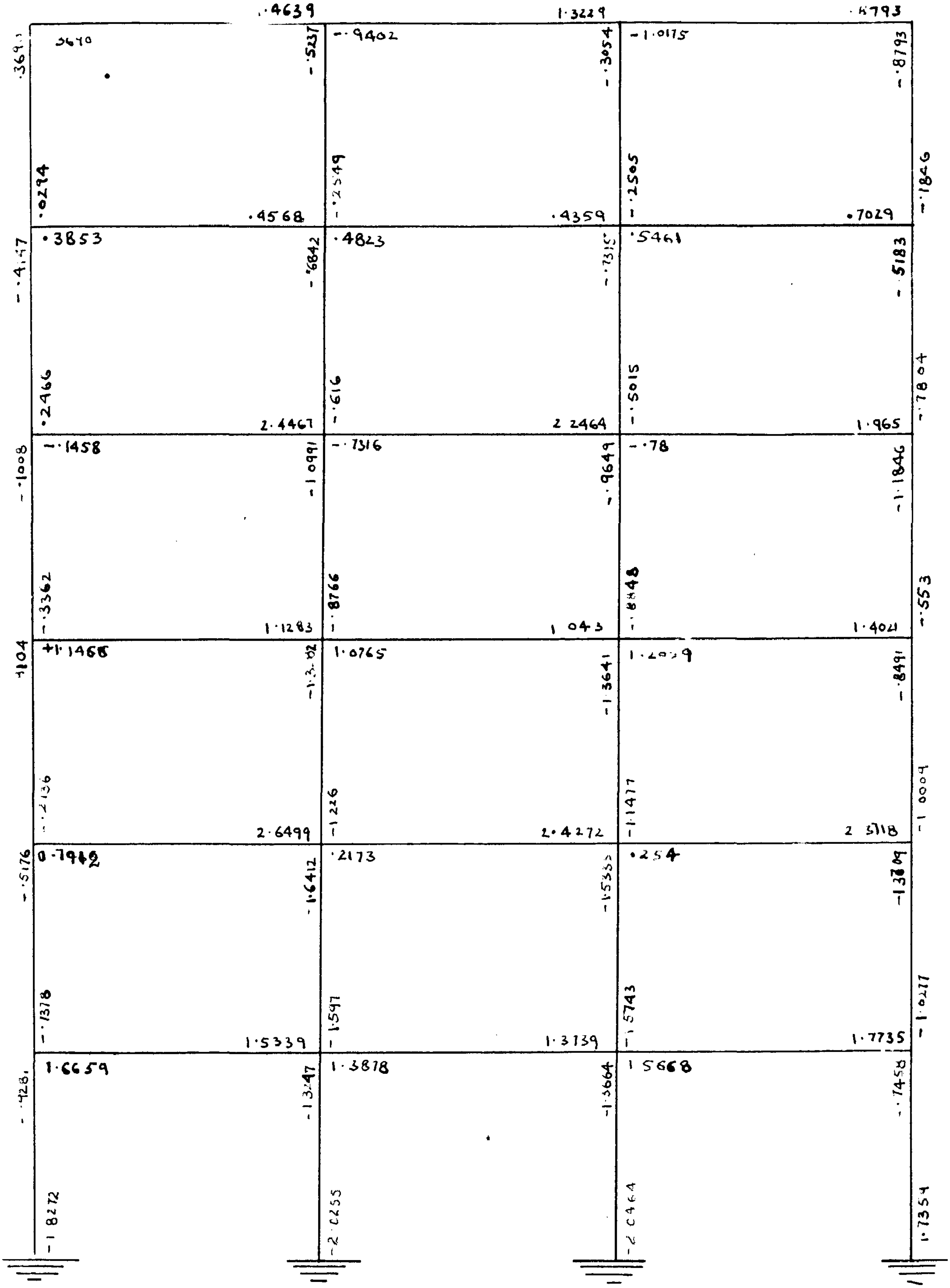
		R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}	R_{11}	R_{12}	R_{13}	R_{14}	R_{15}	R_{16}	R_{17}	R_{18}	R_{19}	R_{20}	R_{21}	R_{22}	R_{23}	R_{24}	R_{25}	R_{26}	R_{27}	R_{28}	R_{29}	R_{30}
1	$\theta_2=1$	[illegible]	1.5					1																						1	1
2	$\theta_3=1$	1.5	8	1.5					1																				1	1	
3	$\theta_4=1$		1.5	8	1.5					1																		1	1		
4	$\theta_5=1$			1.5	8	1.5					1																1	1			
5	$\theta_6=1$				1.5	8	1.5					1														1	1				
6	$\theta_7=1$					1.5	5						1													1					
7	$\theta_9=1$	1						10	1.5					1																1	1
8	$\theta_{10}=1$		1					1.5	10	1.5					1														1	1	
9	$\theta_{11}=1$			1					1.5	10	1.5					1												1	1		
10	$\theta_{12}=1$				1					1.5	10	1.5					1										1	1			
11	$\theta_{13}=1$					1					1.5		1.5					1								1	1				
12	$\theta_{14}=1$						1					1.5	7						1							1					
13	$\theta_{16}=1$							1						10	1.5					1										1	
14	$\theta_{17}=1$								1					1	10	1.5					1								1		
15	$\theta_{18}=1$									1					1.5	10	1.5					1						1			
16	$\theta_{19}=1$										1					1.5	10	1.5					1				1				
17	$\theta_{20}=1$											1					1.5	10	1.5					1		1	1				
18	$\theta_{21}=1$												1					1.5	7						1	1					
19	$\theta_{22}=1$													1						8	1.5									1	1
20	$\theta_{24}=1$																			1.5	8	1.5							1	1	
21	$\theta_{25}=1$															1					1.5	8	1.5					1	1		
22	$\theta_{26}=1$																1					1.5	8	1.5			1	1			
23	$\theta_{27}=1$																	1					1.5	8	1.5	1	1				
24	$\theta_{28}=1$																		1					1.5	5	1					
25	$\Delta_1=1$	-1.125						-1.125												-1.125											
26	$\Delta_2=1$	-1.125	-1.125					-1.125	-1.125											-1.125	-1.125									-2	
27	$\Delta_3=1$		-1.125	-1.125					-1.125	-1.125											-1.125	-1.125							-2		
28	$\Delta_4=1$			-1.125	-1.125					-1.125	-1.125											-1.125	-1.125					-2			
29	$\Delta_5=1$				-1.125	-1.125					-1.125	-1.125											-1.125	-1.125			-2				
30	$\Delta_6=1$					-1.125	-1.125					-1.125	-1.125											-1.125	-1.125	-2					

OPERATION TABLE

RELAXATION

OPERATION NUMBER	MULTIPLIER	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}	R_{11}	R_{12}	R_{13}	R_{14}
ALL VARIABLES ARE ZERO			−1·4583		−1·4583		−1·0278		−·1111				·1111		−·1111
25	2·4232	−2·7261						−2·7261						−2·7261	
		−2·7261	−1·1309		−1·4583		−1·0278	−2·7261	−·1111				·1111	−2·7261	−·1111
26	3·2489	−3·6550	−3·6550					−3·6550	−3·6550					−3·6550	−3·6550
		−6·3811	−4·7939		−1·4583		−1·0278	−6·2811	−3·7661				·1111	−6·3811	−3·7661
1	·5994	4·7952	·8991					·5994							
		−1·5859	−3·8948		−1·4583		−1·0278	−5·7817	−3·7662				·1111	−6·3811	−3·7661
13	·4533							·4533						4·533	·6799
		−1·5859	−3·8948		−1·4583		−1·0278	−5·3284	−3·7661				·1111	−1·8481	−3·0863
19	·6601													·6601	
		−1·5859	−3·8948		−1·4583		−1·0278	−5·3284	−3·7661				·1111	−1·1880	−3·0863
7	·4672	·4672						4·672	·7008					·4672	
		−1·1187	−3·8948		−1·4583		−1·0278	−·6564	−3·0653				·1111	−·7208	−3·0863
14	·4807								·4807					·72105	4·807
									−2·5846					·02025	1·7207
8	·4377		·4377					·65655	4·3770	·65655					·4377
			−·4571					·00015	1·7924	·65655					2·1584
2	·7461	1·11915	5·9688	1·11915					·746						
		+·00045	2·5117	1·11915					2·5385						
20	·4315														·4315
		+·00045	2·5117	1·11915	−1·4583		−1·0278	·00015	2·5385	·65655			·1111	·02025	2·5899
27	2·75084		−3·0947	−3·0947					−3·0947	−3·0947					−3·0947
		+·00045	−·5830	−1·97555	1·4583		−1·0278	·00015	−0·5562	−2·43815			·1111	·00025	−·5048
21	·5329														
15	·3366									·3366					·5049
										−2·10155					[illegible]
9	·37			·37					·5550	3·7	5·550				
				−1·60555					−0·0012	1·59845	·5550				
3	·3884		·5826	3·1072	·5826					·3884					
			−·0004	+1·50165	−·8757					1·98685					
28	2·0685			−2·3192	−2·3192					−2·3192	−2·3192				
				−·81755	−3·1949					·33235	−1·7642				
22	·1114														
16	·2832										·2832				
											−1·4810				
10	·2218				·2218					·3327	2·2100	·3227			
					−2·4731					·00035	·7370	·3227			
4	·5453			·81795	4·3624	·81795					·5453				
				+·0004	·3893	·0745					1·2823				
29	1·37404				−1·5458	−1·5458					−1·5458	−1·5458			
					−·1565	−·72785					−·2655	−1·2131			
23	·2065														
11	·1298											·1298			
												−1·0833			
11	·1763					·1763					·26445	1·763	·26445		
						−·55155					·00095	·6797	·37555		
5	·1043				·15645	·8344	·15645					·1043			
					−·00005	·28285	·87135					·7840			
30	·69248					−·7796	−·7796					−·7796	·7796		
		+·00045	−·0004	·0004	−·00005	−·49675	−1·65095	·00015	−0·0012	·00035	·00095	+·0044	−·40405	·02025	·0001
24	−·1764	·00045	−·0004	·0004	−·00005	·49675	−1·65095	·00015	−·0012	·00035	·00095	+·0044	−·40405		
18	·0932												·0932		
													−·31085		
12	−·0028						−·0028					−·0042	−·0196		
							−1·65375					+·0042	−·33045		
6	·3308				·4962	·4962	1·6540						·3308		
		·00045	·0004	·0004	−·00055	−·00055	·00025	·00015	−·0012	·00035	·00095	+·0002	·00035	·00025	·0001
8	·0001	·00035	·0001					·00015	·0010	·00015					·0001
			−·0003	·0004	−·00055	−·00055	·00025	·00013	−·0002	·0005	·00095	·0002	·00035	·00025	·0002
10	−·0001				−·0001					−·00015	·0010	−·00015			
		·00045	−·0003	·0004	·00015	−·00055	·00025	·0003	−·0002	+·00035	−·00005	·00005	·00035	−·00025	·0002
21	−·00016	·00045	−·0003	·0004	−·00015	−·00055	·00025	·0003	−·0002	+·00035	−·00005	·00005	·00035	·00025	·0002
22	·00014	·045	−·0003	·0004	−·00015	−·00055	·00025	·0003	−·0002	+·00035	−·00005	·00005	·00035	·00025	·0002
23	−·00007														
15	−·00008									−·00008					−·00012
										·00327					·00008
25	+·00024	−·000270						−·000270						+·00027	
		·000180						·00003						−·00002	
18	−·00008	·00018											−·00008		
			−·0003	·0004	−·00015	−·00055	·00025	·00003	−·0002	·00027	·00005	·00005	·00027	−·00002	·00008
2	−·00006	·00018	−·00009	−·00048	−·00009					·00006					
			−·00039	−·00024	−·00024	−·00055	·00025	·00003	−·0002	·00021	·00005	·00005	·00027	−·00002	·00008

R15	R16	R17	R18	R19	R20	R21	R22	R23	R24	R25	R26	R27	R28	R29	R30
			·1111		1·0278		1·4583		1·1389	·4444	1·8889	1·3338	1·7778	2·2222	2·6667
				-2·7261											-4·8464
			·1111	-2·7261	1·0278		1·4583		1·1389	·4444	·8889	1·3338	1·7778	2·2222	-2·1797
				-3·6550	-3·6550									-6·4978	
			·1111	-6·3811	-2·6272		1·4583		1·1389	·4444	·8889	1·3338	1·7778	-4·2756	-2·1797
														·5994	·5994
			·1111	-6·3811	-2·6272		1·4583		1·1389	·4444	·8889	1·3338	1·7778	-3·6762	-1·5803
				·4533										·4533	·4533
			·1111	-5·9278	-2·6272		1·4583		1·1389	·4444	·8889	1·3338	1·7778	-3·2229	-1·1270
				+5·2808	·99015									·6601	·6601
			·1111	-0·6470	-1·63705		1·4583		1·1389	·4444	·8889	1·3338	1·7778	-2·5628	-·4669
														·4672	·4672
			·1111	-0·6470	-1·63705		1·4583		1·1389	·4444	·8889	1·3338	1·7778	-2·0956	·0003
					·4807								·4807	·4807	
					-1·15635								2·2585	-1·6149	·0003
2105													·4377	·4377	
2105													2·6962	-1·1772	·0003
													·7461	·7461	
													3·4423	-·4311	·0003
				·64725	3·4520	·64725							·4315	·4315	
2135			·1111	·00025	2·29565	·64725	1·4583		1·1389	·4444	1·8889	1·3333	3·8738	·0004	·0003
·0947					-3·0947	-3·0947							-5·5168		
31365			·1111	·00025	-·79905	-2·44745	1·4583		1·1389	·4444	·8889	1·3333	-1·62788	·0004	·0003
5327					·79435	+·2632	·79935					·5329	·5329		
79075					·0003	·81575	-·25705					1·8662	-1·9498		
566	·5049					3366						·3366	·3366		
525	·5049					2·15235						2·2028	-·75838		
37												·37	·37		
34525												2·5728	-·38838		
												·3884	·3884		
												2·9612	+·0002		
3192	-2·3192					-2·3192	-2·3192					-4·1230			
2315	1·8143					-·16685	-0·06155					-1·1618			
	·1114					·16785	·8952	·16785			·1114	·1119			
	-1·7024					+·00100	+·03565	·16785			1·0008	-1·0499			
48	2·832	+·149					[illegible]				·2832	·2832			
85	1·1296	+·248					1·11685				1·2840	-·7667			
	·2218										·2218	·2218			
	1·3514										[illegible]	-·5449			
											·5453	·5453			
											[illegible]	·0004			
	-1·5459	-1·545					1·5458	-1·5459			-2·24				
	-·1944	·1210					·42845	-1·37745			-·64698				
		[illegible]					·42995	2·2420	·42475	·2865	·2865				
		·8345					-·0008	·41405	1·56865	·7309	-·41049				
	·1947	1·298	·1947					·1298		·1298	·1298				
	+·0003	·4635	·3058					1·04385		-·0067	-·1076				
		·1763								·1763	·1763				
		·6348								1·0370	-1·0438				
										·1043	·1043				
										1·1413	-·00008				
		-·7796	-·7796					-·7796	-·7796	-1·38596					
85	+·0003	-·1348	-·4738	·00025	·0003	+·0010	-·0008	·26425	·78905	-·24466	-·00008	·0004	+·00002	·0004	·0003
			-·1764					-·2646	-·882	-·1764					
		-·1398	-·6502	·00025				+·00035	-·09295	-·42106					
		·1398	·6524						·0932	·0932					
		·0	·0022						·00025	-·32786					
			-·0028							-·0028					
			-·0006							-·33066					
										·3308					
·085	·0003	0·0000	-·0006	·00025	·0003	·001	-·0008	+·00035	·00025	·00014	-·00008	·0004	+·00002	·0004	·0003
													·0001	·0004	
·085	·0003	·0000	·0006	·00025	·0003	·001	-·0008	+·00035	·00025	·00014	-·00008	·0004	+·00012	·0005	·0003
	-·0001										-·0001	-·0001			
·085	·0002	·0000	·0006	·00025	·0003	·001	-·0008	+·00035	·00025	·00014	-·0018	·0003	+·0012	·0005	·0003
·016					-·000240	-·00128	-·00024					-·00016	-·00016		
·069	·0002	·0000	·0006	·00025	·00060	-·00028	-·00104	+·00035	·00025	·00014	-·00018	·00014	-·00004	·0005	·0003
	·00014					·00021	·00112	·00021			·00014	·00014			
·0069	·00034	—	·0006	·00025	·00006	-·00007	·00008	·00056	·00025	·00014	-·00004	·00028	-·00009	·0005	·0003
		-·0007					·00005	-·00056	-·000105	-·00007	-·00007				
·0069	·00034	-·0007					·000025	—	·000145	·00007	-·00011	-·00008	-·00008		
·008	-·00012					-·00008									
·0011	·00022	-·00007				-·0005						·0002	-·00012		
				-·00027											-·00048
				-·0002											-·00018
		-·00012	-·00056						·00008	-·00008					
·0011	·00022	-·00019	·00004	-·00002	·00006	-·00015	-·000025	—	-·000065	-·00001	-·00011	+·0002	-·00012	·0005	-·00018
												-·00006	·00006		
·0011	-·00022	-·00019	·00004	-·00002	·00006	-·00015	-·00025		·000065	-·00001	-·00011	·00014	-·00018	·0005	-·00018



FINAL END MOMENTS

APPENDIX B.

ANALYSIS OF THE FRAME BY KANI'S ITERATION METHOD

Fixed end moments :-

The fixed end moments for various beam members have been calculated in Appendix A. These are shown above the beam lines in Table no. B.1. The total restraint moment at any joint is the algebraic sum of the fixed end moments in the members coming at this joint. These have been shown in the inner circle at a joint.

For example total restraint moment at 2nd joint from the left end in top storey = 1.1[illegible] - 1.0[illegible] = 0.1111 in Lbs.

Rotation Factors:-

Since section of all the members is same while length of beam members is 1.5 times that of stanchion members, the relative stiffnesses of beam and columns will be K and 1.5 K. Now rotation factor U_{ik} is given by

$$U_{ik} = -\frac{1}{2}\,\frac{K_{ik}}{\sum K_{ik}}$$

Therefore considering for example say second joint from the left end in top storey;

$$\text{Rotation factor for left beam} = -\frac{1}{2}\,\frac{K}{(K+K+1.5K)} = -1/7$$

Rotation factor for right beam $= -\frac{1}{2}\,\dfrac{K}{(k + k + 1.5k)} = -\dfrac{1}{7}$

Rotation factor for lower Column $= -\frac{1}{2}\,\dfrac{1.5\,k}{(k + k + 1.5k)}$

$$= -\frac{1.5}{7}$$

check $\quad -\dfrac{1}{7} - \dfrac{1}{7} - \dfrac{1.5}{7} = -\frac{1}{2}$

Similarly the rotations factors for all other joints were calculated and inserted in the bigger circle as shown in the table.

<u>Storey Moments.</u>

The frame is loaded with horizontal loads also. Therefore there exists, storey moments for each storey.

Now storey moments $\bar{M}_n = \dfrac{Q_r\, h_r}{3}$

Where $Q_r = \frac{1}{2}W + \frac{1}{2}W + \frac{1}{2}W + \frac{1}{2}W = 2W$

$h_r = 4''$

$\therefore \bar{M}_n = \dfrac{2W \times 4}{3} = 2.6667$ in lbs.

Similarly storey moments for all the storeys were calculated and also shown in the rectangles drawn for each storeys moment on the left of frame.

Linear Displacement factors :-

Linear displacement factor $\vee ik$ is given by

$$\vee ik = -\frac{3}{2} \; \frac{Kik}{\sum Kik}$$

Where $\sum Kik$ = sum of the K values of the columns of particular storey.

Considering first storey say:

Linear displacement factor for first column

$$= -\frac{3}{2} \; \frac{K}{4K} = -\tfrac{3}{8}$$

Since all the columns are of same stiffness throughout and there are 4 columns in each storey, the linear displacement factor for all the columns is $-\tfrac{3}{8}$.

Check:- $\sum \vee ik$ for any storey $= -\tfrac{3}{8} \times 4 = -\frac{3}{2}$

The values of linear displacement factors are shown on the left of each column in the centre of each storey.

Iteration Work.

Iteration work was carriedout as explained in Chapter I. The last three cycles of operation are shown on the table B. 1.

Calculation of final moments:-

From the equation $Mik = \bar{M}ik + 2\,M'ik + M'ki + M''ik$. (refer page 17). Mik can be calculated for all the members. This can be done in a much simpler way in the tabular form as shown in Table B.3. The procedure is

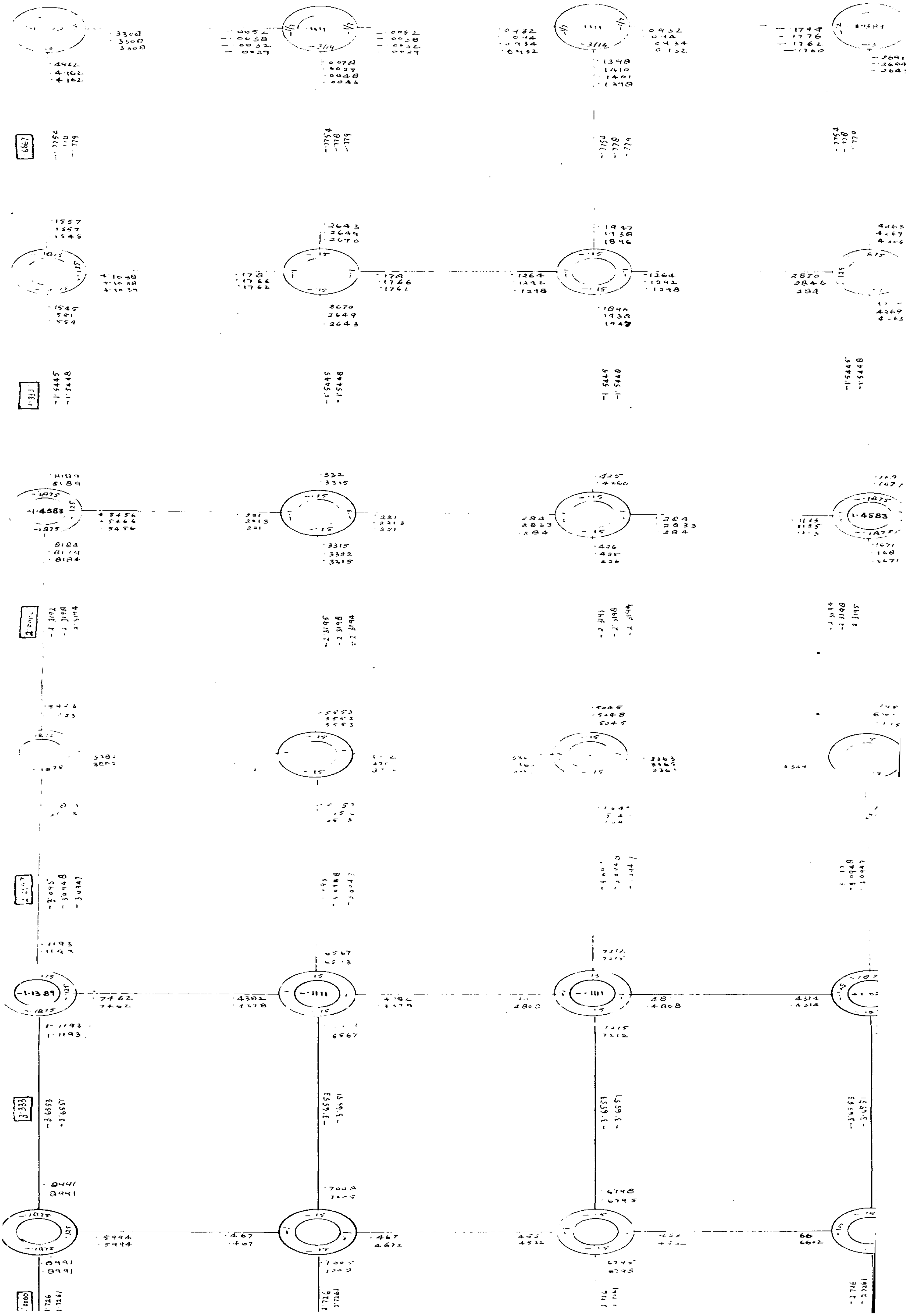
1) Write down with proper signs all the fixed end moment above the line and rotation contributions below the line for beam member while at the ends in case of columns. The linear displacement contribution is written in the centre of each column.

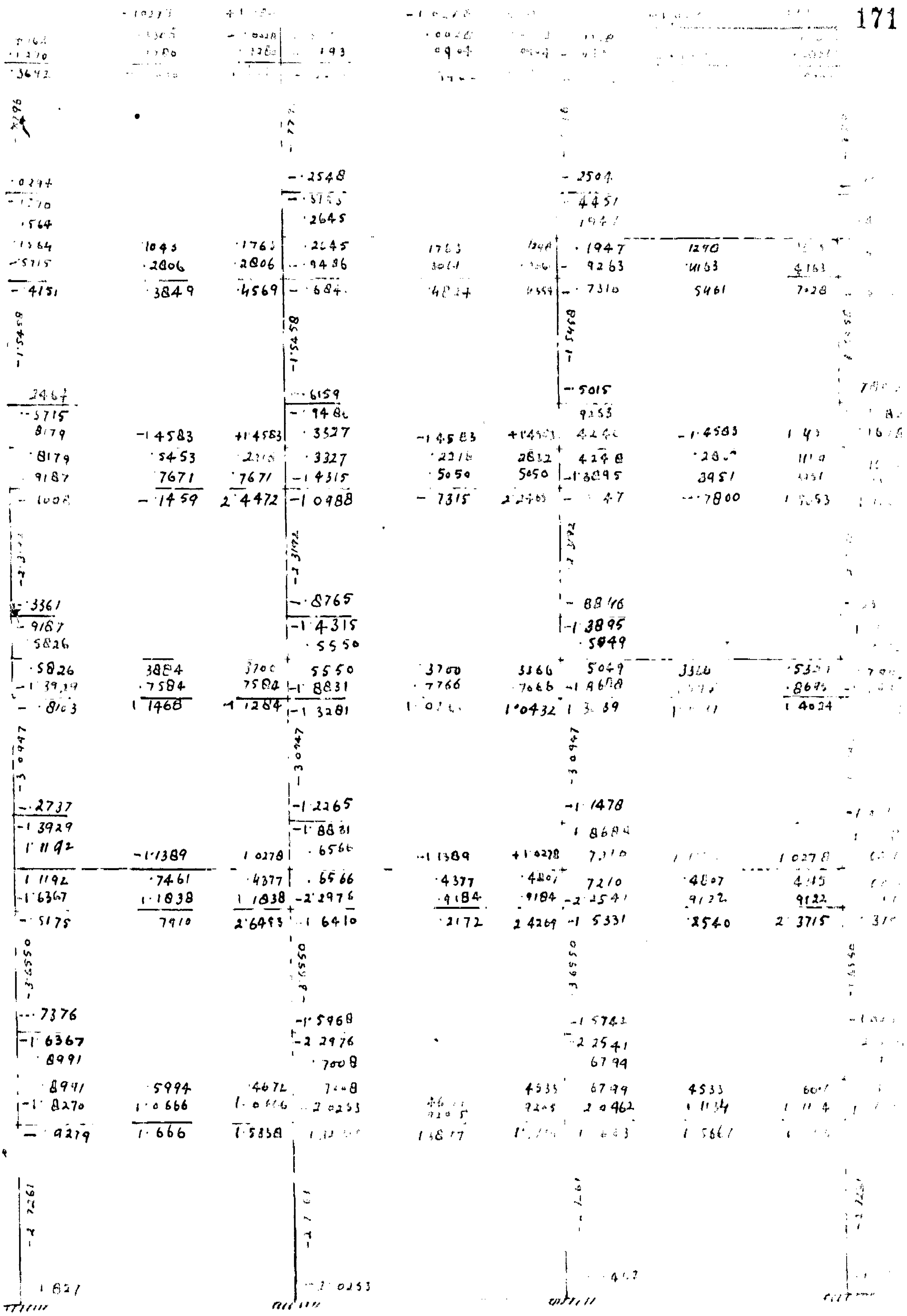
2) Write down the algebraic summation of rotation contributions at the two ends and the linear displacement contribution (in case of columns only), under the rotation contributions already written. Addition of all the terms will give final values of end moments.

Final values of end moments are shown in the sketch on page 170.

ITERATION PROCEDURE

TABLE B.1





-·2548
·2645
·2645
-·9486
-·684
-·2504
-·4451
·1947
·1947
-·9263
-·7310
1270
·4163
5461
7·28
·1045
·2806
·3849
·1763
·2806
·4569
-·4151
-1·5458
-·6159
·3327
·3327
-1·4315
-1·0988
-·5715
·8179
·8179
·9187
-1·4583
·5453
·7671
-1·459
+1·4583
·7671
2·4472
-1·4583
·5050
-·7315
5050
-·5015
4248
-1·3895
-·47
-1·4583
3951
-·7800
-2·3192
-·8765
-1·4315
·5550
·5550
-1·8831
-1·3281
-·3361
-·9187
·5826
·5826
-1·3929
-·8103
3884
·7584
1·1468
3700
7584
-1·1284
·3700
·7766
3366
·7666
1·0432
-·8846
-1·3895
·5049
·5049
-1·3689
·5321
·8695
1·4024
-3·0947
-1·2265
-1·8831
·6566
·6566
-2·2976
-1·6410
-·2737
-1·3929
1·1192
1·1192
-1·6367
-·5175
-1·1389
·7461
1·1838
·7910
1·0278
·4377
1·1838
2·6493
-1·1389
·4377
·9184
·2172
+1·0278
·4807
·9184
2·4269
-1·1478
1·8689
7210
7210
-2·2541
-1·5331
·4807
·9122
·2540
1·0278
·9122
2·3715
-3·6550
-·7376
-1·6367
·8991
·8991
-1·8270
-·9219
-1·5968
-2·2976
·7008
-1·5743
-2·2541
6794
6794
-2·0462
4533
·5994
1·0666
1·666
·4672
1·0666
1·5338
4533
-2·7261
-2·0253

APPENDIX C

ANALYSIS OF THE FRAME FOR COLLAPSE LOAD BY MECHANISM METHOD

The first step is to decide upon the correct number of independent mechanisms. For this purpose it is noted that the number of bending moments n which is required to specify the bending moment distribution throughout the frame = 100. Since the bending moment must vary linearly between those sections numbered from 1 to 100 in fig C.1. The number of redundancies (n) for this frame = 54. Therefore the number of independent equations of equilibrium, and therefore, the number of independent mechanisms, is (n-r) = 100-54

= 46

These 46 independent mechanisms can be readily identified as consisting of

(i) 18 Beam mechanisms as shown in figs C.2 (a to f)

(ii) 6 sidesway mechanism as shown in figs C.3(a to f)

(iii) 22 joint rotations

(At the joints where more than two members are meeting).

The work equations for beam type mechanisms are as given below. It may be noted that there are actually only 6 different work equations for 18 beam type mechanism because of the similarily between various mechanisms.

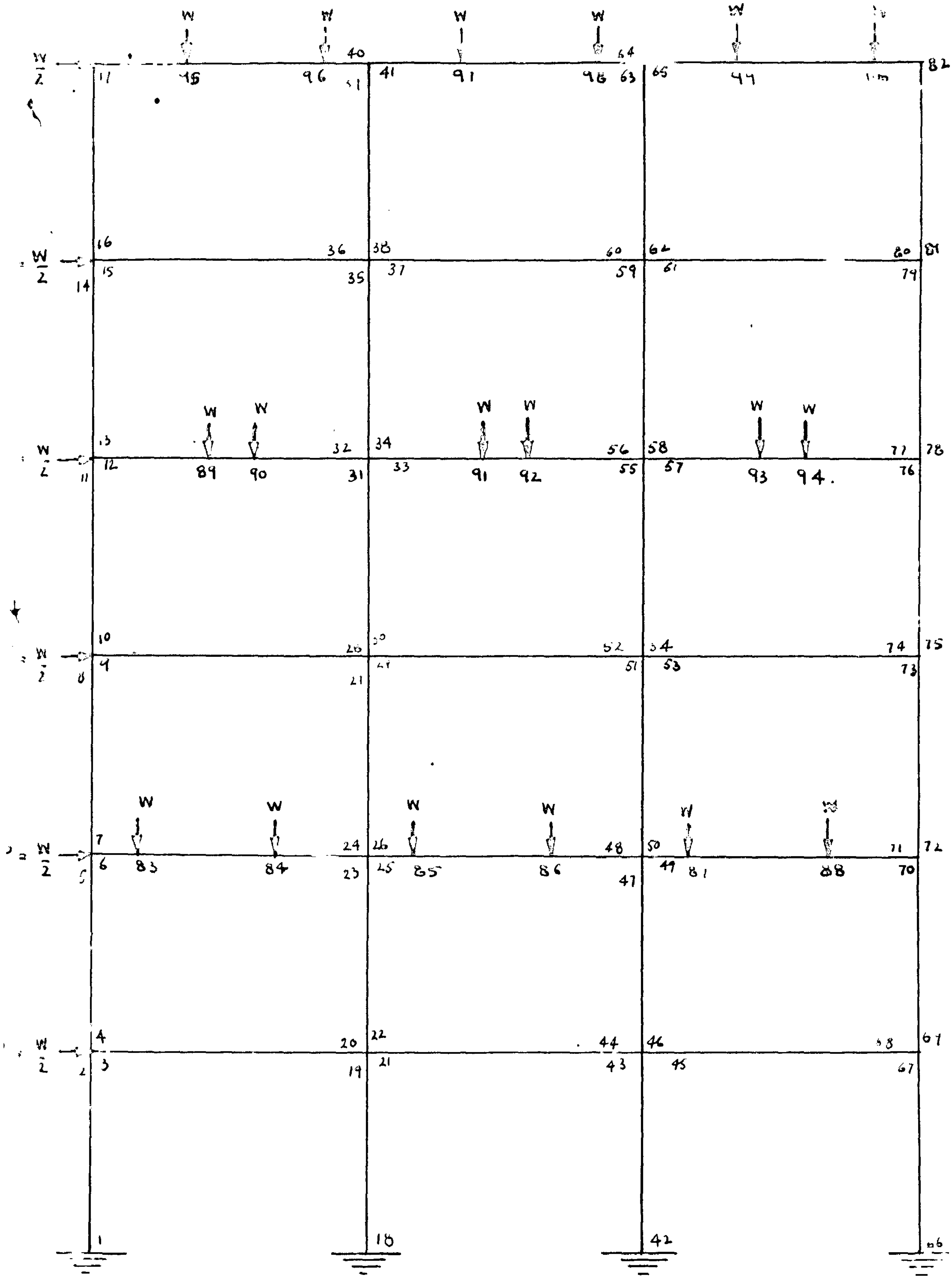


FIG. C. 1.

Mechanism I Beams in 6th storey.

(a) When two hinges are at the ends and third hinge is under the left hand load (Fig.C.2a)

$$M_p(\theta + 2\theta) = W\left(2\theta + \frac{\theta}{2}\right)$$

i.e. $3\,M_p\theta = 2.5\,W\theta$

$\therefore \quad W = 1.2\,M_p$

(b) When two hinges are at the ends while third hinge is under the right hand load (Fig.C.2b)

$$M_p(\theta + \theta + 5\theta + 5\theta) = W(2\theta + 5\theta)$$

$12\,M_p\theta = 7\,W\theta$

i.e. $W = 1.71\,M_p$.

Mechanism II :- Beams in 4th storey.

(a) When two hinges are at the ends while third hinge is under the left hand load (Fig.C.2c)

$$M_p\left(\theta + \theta + \frac{5}{7}\theta + \frac{5}{7}\theta\right) = W\left(2.5\theta + 2.5 \times \frac{5}{7}\theta\right)$$

i.e. $\frac{24}{7} M_p\theta = \frac{30}{7} W\theta$

$\therefore \quad W = 0.8\,M_p$.

(b) When two hinges are at the ends while third hinge is under the right hand load (Fig. C.2d.)

$$Mp\left(\theta + \theta + \frac{7}{5}\theta + \frac{7}{5}\theta\right) = W\left(2.5\theta + 2.5 x \frac{7}{5}\theta\right)$$

$$\text{i.e. } \frac{24}{5} Mp\theta = \frac{30}{5} W\theta$$

$$\therefore W = 0.8 \text{ Mp.}$$

Mechanism III :-

(a) When two hinges are at the ends while third hinge is under the left hand load (Fig. C.2e)

$$Mp\left(\theta + \theta + \frac{\theta}{5} + \frac{\theta}{5}\right) = W\left(\theta \;\; 2 x \frac{\theta}{5}\right)$$

$$\text{i.e. } \frac{12}{5} Mp\theta = \frac{7}{5} W\theta$$

$$\therefore W = 1.71 \text{ Mp.}$$

(b) When two hinges are at the ends while third hinge is under the right hand load (Fig.C.2f)

$$Mp(\theta + \theta + 2\theta + 2\theta) = W(\theta + 2 x 2\theta)$$

$$\text{i.e. } 6 Mp\theta = 5 W\theta$$

$$\therefore W = 1.2 \text{ Mp.}$$

The work equations for sidesway mechanisms are:-

Mechanism IV : When only the 6th storey sways (Fig. C.3a)

$$2Mp\theta = 2 W\theta$$

$$\therefore W = 2 \text{ Mp.}$$

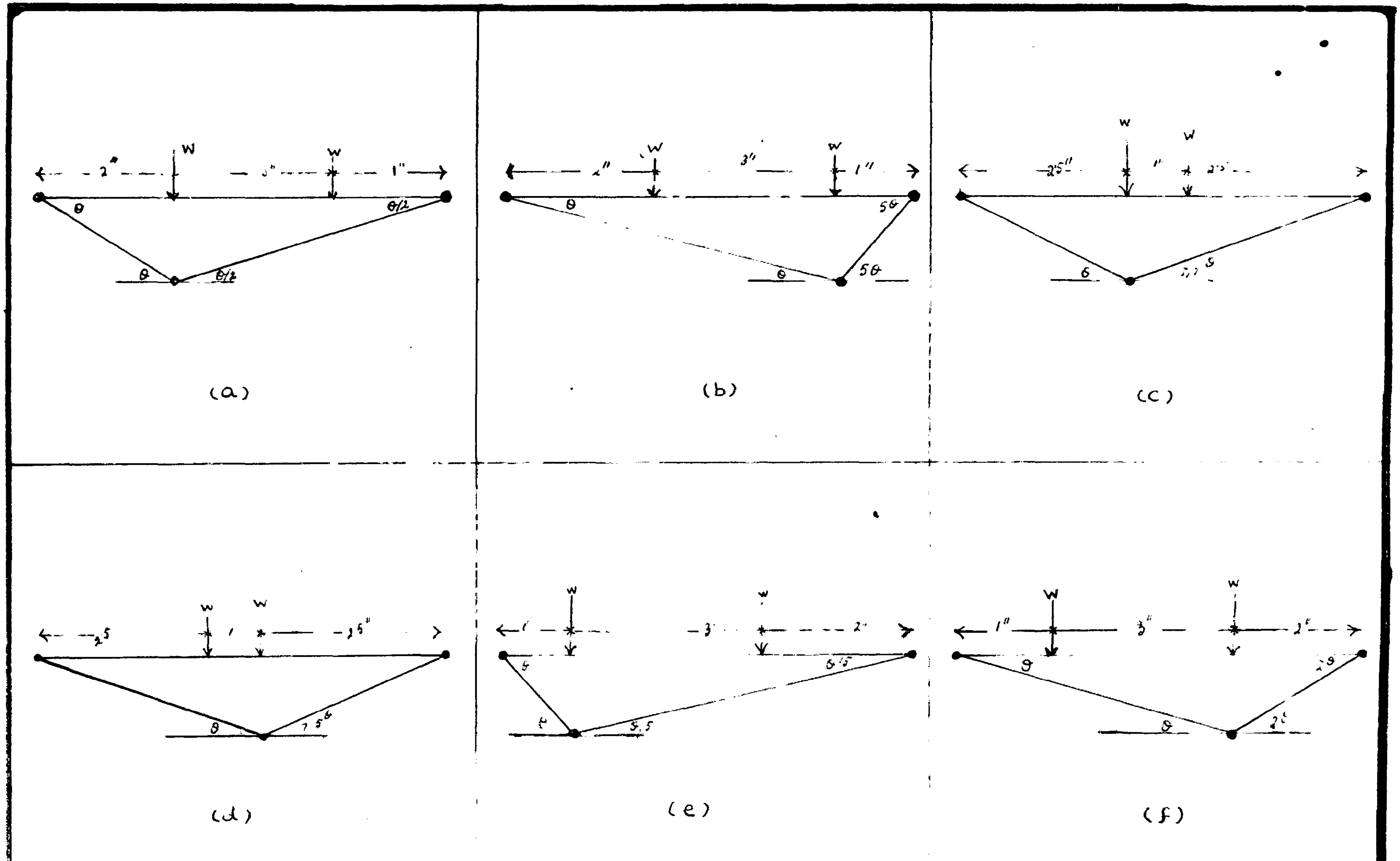


FIG. C 2.

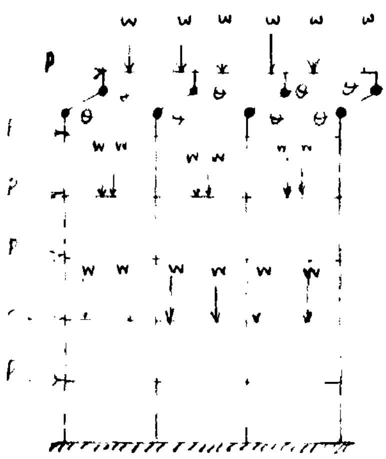


FIG. C.3 a.

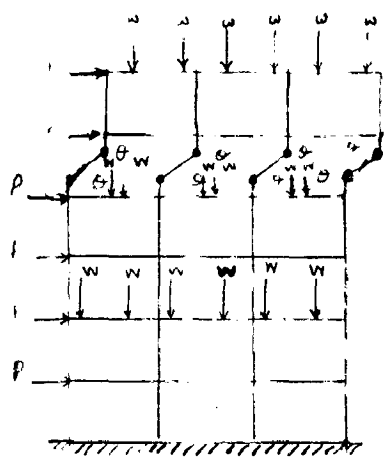


FIG. C.3 b

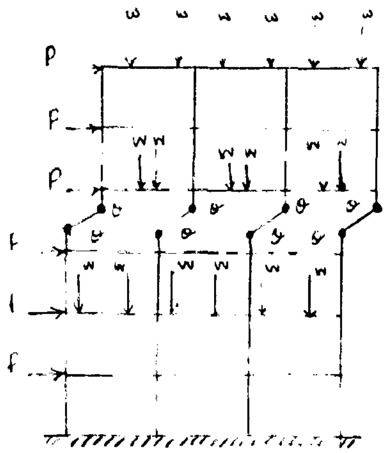


FIG C.3 c

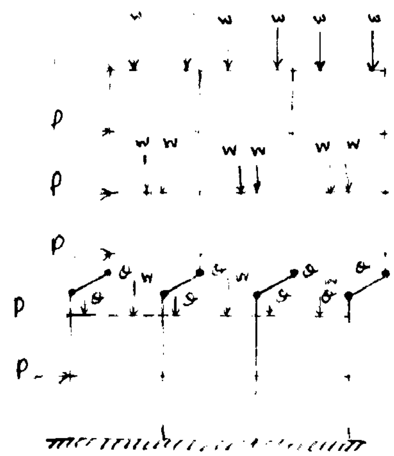


FIG. C.3 d

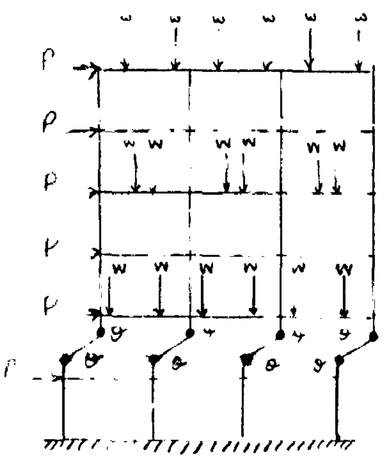


FIG. C.3 e.

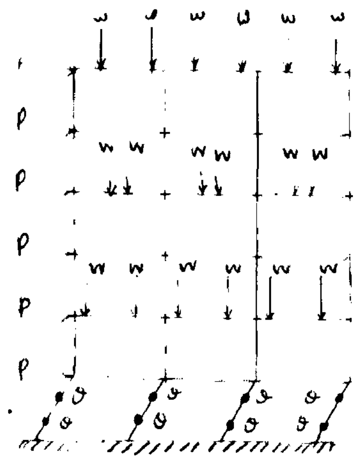


FIG C.3. f

Mechanism V :- When only the 5th storey sways (Fig.C.3b)

$\Sigma M_p\theta = 4W\theta$

$\therefore W = 2\ M_p.$

Mechanism VI :- When only the 4th storey sways (Fig. C3c)

$\Sigma M_p\theta = 6W\theta$

$\therefore W = 1.33\ M_p.$

Mechanism VII :- When only the 3rd storey sways (fig. C.3d)

$\Sigma M_p\theta = 8W\theta$

$\therefore W = M_p.$

Mechanism VIII :- When only the 2nd storey sways (Fig.C.3e)

$\Sigma M_p\theta = 10W\theta$

$\therefore W = 0.8\ M_p$

Mechanism IX :- When only the 1st storey sways (fig C.3f)

$\Sigma M_p\theta = 12W\theta$

$\therefore W = 0.667\ M_p.$

COMBINING OF MECHANISMS

From the various work equations which have been derived above the independent mechanism that gives least value of collapse load is a side sway mechanism of first storey. The next mechanism in order is a side sway mechanism of second storey. A straight forward addition of the dis-

displacements and hinge rotations of these two mechanisms would be pointless, for they have no common hinge whose rotation would be cancelled by the addition and thus reduce the work absorbed in the plastic hinges in the combined mechanism. The mechanism resulting from this simple addition is shown in fig C.4 a. However it will be noticed that in this mechanism there are two plastic hinges, one in each column (top and bottom columns) at all the joints on the first floor level. By rotating all the joints clockwise through an angle θ , these two hinges in the exterior, columns can be replaced by a single hinge in the beams. (Fig. C.4 b) However in central two joints two hinges in the columns are replaced only by two corresponding hinges in the beams, so no cancellation has occured or no reduction in the work absorbed in these plastic hinges, as the Mp for all the sections is same. In nut-shell there has been reduction of only two hinges by the combination of mechanisms no VIII and IX. so we will investigate this combination, if the collapse load decreases or increases.

Now work equation for mechanism No. IX is

$$8\,M_p\theta = 12\,W\theta$$

& Now work equation for mechanism No. IX is

$$8\,M_p\theta = 10\,W\theta$$

∴ Combined mechanism No. X (Fig. C.4 b)

$$16\,M_p\,\theta - 2\,M_p\theta = 22\,W\theta$$

FIG. C.4a

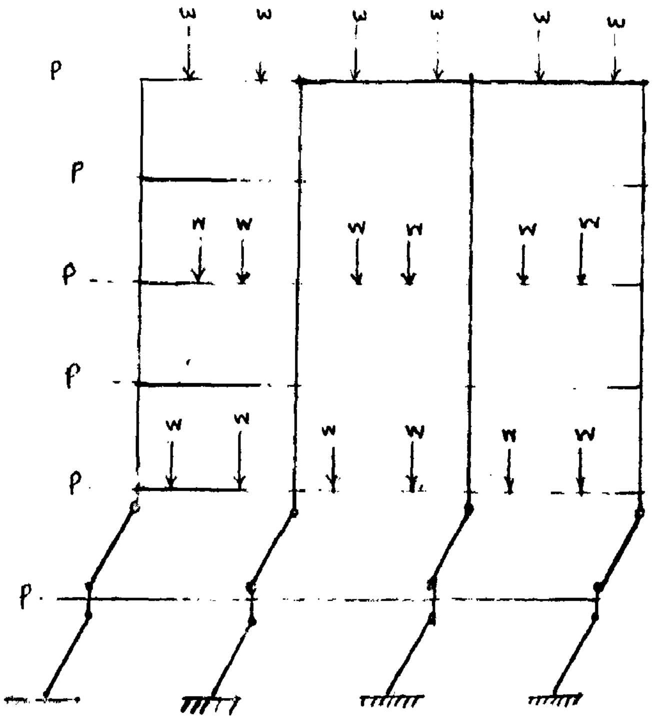
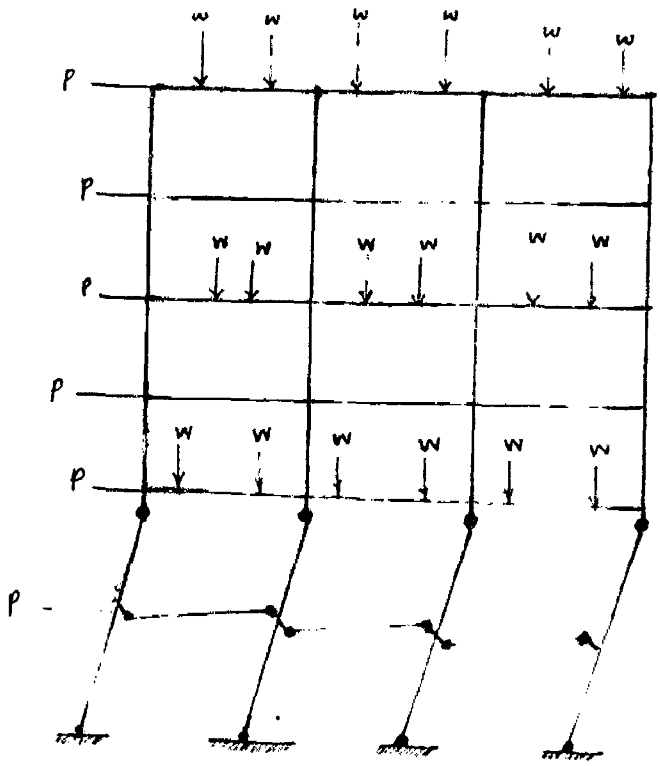


FIG. C.4b



i.e. $14\ M_p\theta = 22\ W\theta$

$$\therefore\ W = \frac{M_p}{1.57}$$

Hence the value of collapse load has decreased.

Next again we will try to combine the sway mechanism of third storey with combined mechanism No. X. This combination plus the joint rotations will yield a mechanism No. XI. (Fig.C4c)

From Mechanism No. X $\quad 14\ M_p\theta = 22\ W\theta$

From Mechanism No. VII $\quad 6\ M_p\theta = 8\ W\theta$

$\therefore$ Combined Mechanism No. XI $= \quad 20\ M_p\theta = 30\ W\theta$

$$\text{or}\ W = \frac{M_p}{1.5}$$

This mechanism in itself does not give least value of collapse load. But this mechanism if combined with beam type mechanism will result in further cancellation of some more hinges. So further reduction in collapse load value is expected. Therefore trying this combination we get,

From mechanism No. XI $\quad 20\ M_p\theta = 30\ W\theta$

From mechanism No. III a $\quad \frac{12}{5}\ M_p\theta = \frac{7}{5}\ W\theta$

$\therefore$ Combined mechanism no XII : $(20 + \frac{12}{5} \times 3 - 6)\ M_p\theta = (30 + 3 \times \frac{1}{5})\ W\theta$

$$\therefore\ W = \frac{M_p}{1.623}$$

(Fig. C.4d)

FIG C.4c

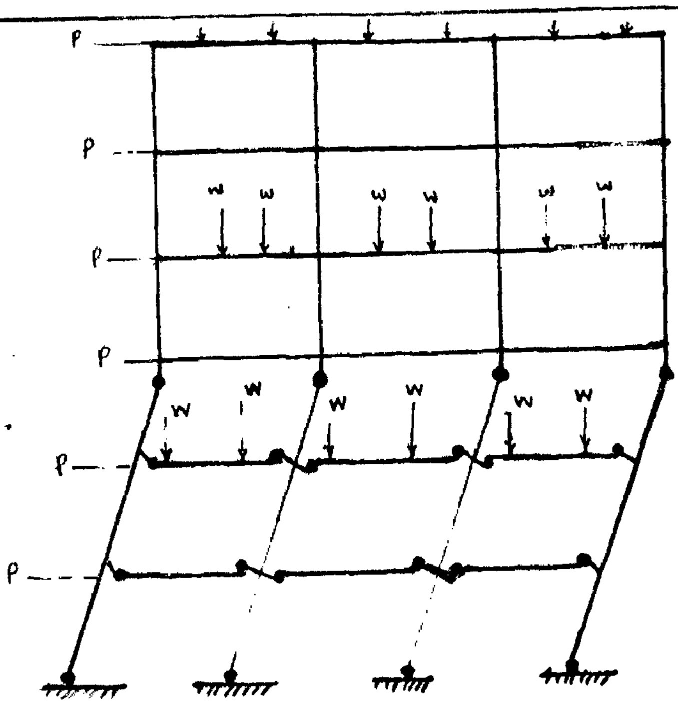
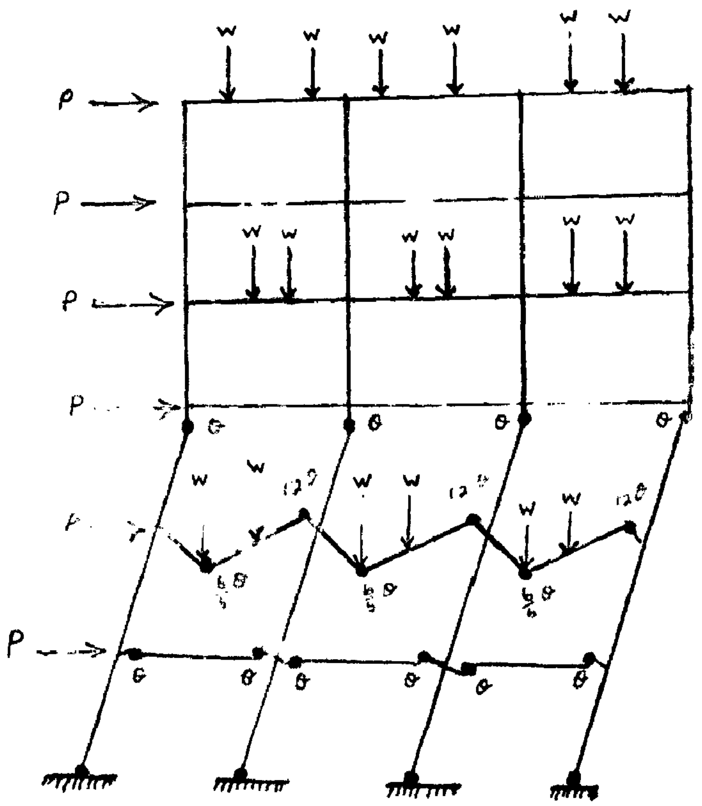


FIG C.4d



Similarly:

From mechanism No. XI $20\ M_p\theta = 30M\theta$

From mechanism No. IIIb $6\ M_p\theta = 8M\theta$

Combine mechanism No. XII

$$(20 + 6 \times 3 - 6)\ M_p\theta = (30 + 3 \times 6)\ M\theta$$

i.e. $33\ M_p\theta = 46\ M\theta$

$$\therefore\ U = \frac{M_p}{1.405}$$

Therefore mechanism no. XIII is the most critical of all the combinations consider so far.

Now again curiousily arises as to combine the mechanisms no. VI, V, & II a with XII . This has been done as below:-

Mechanism No. XII $\frac{100}{6} M_p\theta = \frac{171}{6} M\theta$

Mechanism No. VI $3\ M_p\theta = 6\ M\theta$

$\therefore$ Combined [illegible] No. XIV $\frac{130}{6} M_p\theta = \frac{201}{6} M_p\theta$

i.e. $U = \frac{M_p}{1.47}$

Mechanism No. V $8\ M_p\theta = 4\ M\theta$

$\therefore$ Combined Mechanism No. XV $\frac{136}{6} M_p\theta = \frac{221}{6} M\theta$

$$\text{i.e. } U = \frac{M_p}{1.9[illegible]1}$$

Mechanism No. II a $\quad \frac{24}{7} M_p\theta = \frac{20}{7} W\theta$

$\therefore$ Combined Mechanism No. X/I. $\left(\frac{166}{5} + \frac{3 \times 24}{7} - 6\right) M_p\theta$

$$= \left(\frac{221}{5} + \frac{3 \times 20}{7}\right) W\theta$$

$$\text{i.e.}\quad \frac{1312}{35} M_p\theta = \frac{1907}{35} W\theta$$

$$\therefore \quad W = \frac{M_p}{1.43}$$

Hence this mode of failure is not critical. Further combination are obviously unnecessary. However one possible mechanism has been left over. This is a combination of mechanism No. X, III a and joint rotations.

Mechanism No. X $\quad 24 M_p\theta = 22 W\theta$

Mechanism No. III a $\quad \frac{12}{5} M_p\theta = \frac{7}{5} W\theta$

$\therefore$ Combined Mechanism No. X/III $\left(24 + 3 \times \frac{12}{5} - 8\right) M_p\theta$

$$= \left(22 + 3 \times \frac{7}{5}\right) W\theta$$

$$\text{i.e.}\quad \frac{91}{5} M_p\theta = \frac{131}{5} W\theta$$

$\therefore \quad W = \frac{M_p}{1.43}$ which is again less than the critical.

Hence the value of collapse load $W_c = \frac{M_p}{2.319}$

Substituting the value of Mp for particular section, the collapse load Wc can be calculated. This value of collapse load as obtained above is only an upper bound to the collapse load value. To show that this is the actual value, it is necessary to draw a statically admissible bending moment diagram throughout the frame without violating the yield condition i.e. bending moment does not exceed the value of Mp at any section. Since the failure is of a partial collapse in this problem, it is not so easy to draw the bending moment distribution. For this we have to resort to plastic moment distribution as given in appendix D. Plastic moment distribution fixes the lower bound to the value of collapse load. If two values concide the collapse load as obtained above is correct, otherwise further investigations have to be made. Incidently the value obtained above is correct one.

APPENDIX D

PLASTIC ANALYSIS OF THE FRAME BY PLASTIC MOMENT DISTRIBUTION

First step is to write down equilibrium equations for all the independent mechanisms which are 46 in number. As already stated in Chapter II the joint rotations are meaningless in themselves so they have been left over. Other equations are ;

Equilibrium Equations :-

I. For Beams :-

If M_1 and M_2 represent the moments under the left and right hand loads respectively, the beam equations are :-

$- M_L + 1.5\ M_1 + 0.5\ M_R = 2.5\ W$

i.e. $- 2\ M_L + 3\ M_1 + M_R = 5\ W$ Top storey (i)

and $- M_L + 6\ M_2 + M_{RR} = 7\ W$ " " (ii)

$- M_L + \frac{12}{7} M_1 + \frac{5}{7} M_R = \frac{30}{7} W$

i.e. $- 7\ M_L + 12\ M_1 + 5\ M_{RR} = 30\ W$ 4th storey (iii)

and $- M_L + \frac{12}{5} M_2 + \frac{7}{5} M_R = \frac{30}{5} W$

i.e. $- 5\ M_L + 12\ M_2 + 7\ M_R = 30\ W$ 4 th storey (iv)

$- M_{LL} + \frac{6}{5} M_{11} + \frac{1}{5} M_R = \frac{7}{5} W$

$- 5\ M_L + 6\ M_1 + M_R = 7\ W$ 2nd storey (v)

and $- M_L + 3\ M_1 + 2\ M_R = 5\ W$ 2nd storey (vi)

II. For Panels :-

$- M_{16} - M_{17} - M_{38} - M_{39} - M_{62} - M_{63} - M_{81} - M_{82} = 2\ W$

$-M_{13}-M_{14}-M_{34}-M_{35}-M_{58}-M_{59}-M_{78}-M_{79} = 4\,W$

$-M_{10}-M_{11}-M_{30}-M_{31}-M_{54}-M_{55}-M_{75}-M_{76} = 6\,W$

$-M_{7}-M_{8}-M_{26}-M_{27}-M_{50}-M_{51}-M_{72}-M_{73} = 8\,W$

$-M_{4}-M_{5}-M_{22}-M_{23}-M_{46}-M_{47}-M_{69}-M_{70} = 10\,W$

$-M_{1}-M_{2}-M_{18}-M_{19}-M_{42}-M_{43}-M_{66}-M_{67} = 12\,W$

Now a table is made as shown on page and a set of bending moments is minserted at various sections in accordance with equilibrium equations written above. For example in top storey M_1 and M_2 are kept zero so that $M_2 = -2\,W$ and $M_R = 1\,W$. For convenience W is kept as unity. Similarly in 4th storey $M_L = -2.5W$ $M_R - 2.5\,W$ while $M_1 = M_2 = 0$ and in 2nd storey $M_L = -1$ $M_R = +2$ while $M_1 = M_2 = 0$. In stanchions the moments were distributed equally in top and bottom sections in any particular panel e.g. in top panel the value is $-\frac{2}{8} = -.25$ and similarly $-.5$, $-.75$, -1.00, -1.25 and -1.5 in successive storeys.

Next step is to make a table for carry-over factors, so that any change affected in bending moment at a section is accompanied by corresponding other changes so as to not to disturb the equilibrium equations. For stanchions it is obvious from the above equations that the sum of all the changes in a panel must be zero. For beams the table is given below :-

TABLE NO. D.1.

Operation	Left hand end moment i.e. M_L	Moment under the left hand load i.e. M_1	Moment under the right hand load i.e. M_2	Right hand end moment i.e. M_R
a. Top Storey	1	2/3	1/6	0
b. Top Storey	1	0	0	-½
c. Top Storey	0	-5/6	-1/3	1
d. Fourth Storey	1	7/12	5/12	0
e. Fourth Storey	1	0	0	-1
f. Fourth Storey	0	-7/12	-5/12	1
g. 2nd Storey	1	5/6	1/3	0
h. 2nd Storey	1	0	0	-2
i. 2nd Storey	0	-2/3	-1/6	1

Balancing of joints for rotational equilibrium can be carried out in any manner so long as equilibtium is maintained. Here in this problem the work has been greatly facilitated by the knowledge of position of hinges, and the distribution has been performed in one step only. e.g. at the joint of sections 2, 3 and 4 the out of balance moment is + 2.75. It has been distributed as .339, 1.613 and .798 to sections 2, 3 and 4 respectively. Similarly at other joints in bottom storey, the balancing of joint was carriedout. Now carrying over operation was performed so as to maintain equilibirum equations e.g. the total change in moments at the top of columns in first storey = .339 - .113 - .113 + .339 = 0.452. This has been distributed equally as - 0.113 in all the columns. Similarly the operation was performed for columns in 2nd storey. Algebraic sum-mation will give the final results. Similar procedure is adopted for other storeys as shown in the tables D. 1 and D.2 .

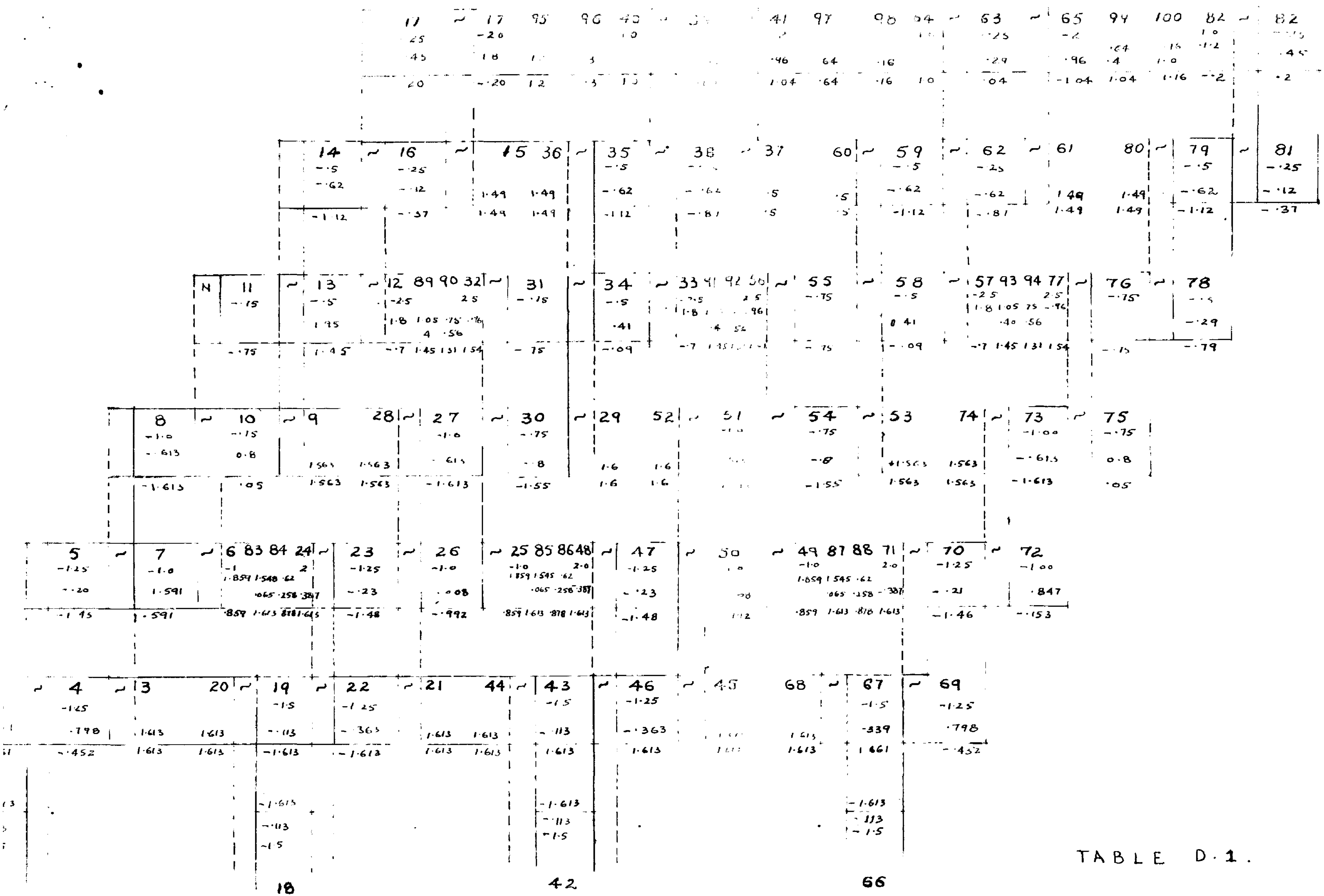


TABLE D.1.

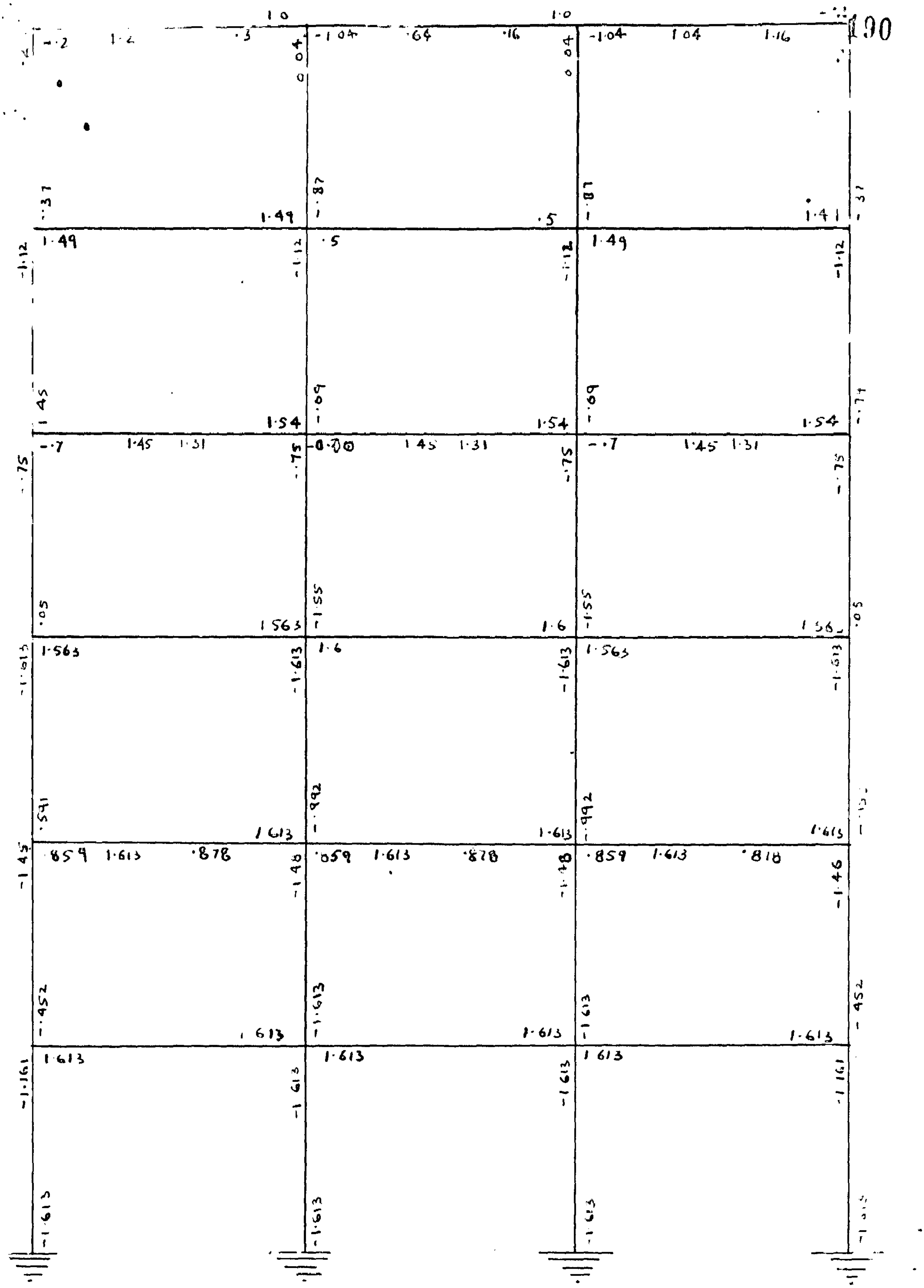

1.0
1.0
-.2
1.2
.3
0.04
-1.04
.64
.16
0.04
-1.04
1.04
1.16
-.37
1.49
-.87
.5
-.87
1.41
-.37
1.49
-1.12
.5
-1.12
1.49
-1.12
-1.12
1.45
1.54
-.69
1.54
-.69
1.54
-.74
-.7
1.45
1.31
-.75
-0.00
1.45
1.31
-.75
-.7
1.45
1.31
-.75
-.75
.05
1.563
-1.55
1.6
-1.55
1.56
.05
1.563
-1.613
-1.613
1.6
-1.613
1.563
-1.613
.591
1.613
-.992
1.613
-.992
1.613
-1.45
.859
1.613
.878
-1.46
.859
1.613
.878
-1.48
.859
1.613
.818
-1.46
-.452
1.613
-1.613
1.613
1.613
1.613
-.452
1.613
-1.161
1.613
-1.613
1.613
-1.613
1.613
-1.161
-1.613
-1.613
-1.613

TABLE D.2

BIBLIOGRAPHY

No.	Author	Title
1.	Bolton A.	A new approach to the elastic analysis of of two dimentional rigid frames. Structural Engineer 1952. P. 1.
2.	Grinter.	Simplified Wind Analysis T.Am. S.C.E. 1934 page 610.
3.	Grinter & Tsao	Joint Translation by Cantilever Moment Distribution P. 1195 Trans. Aim. Soc. C.E. 1954 Vol. 119.
4.	Gottschalk	Simplified Wind Analysis T.Am. Soc. C.E. 1940 P. 1019.
5.	Hardy Cross.	Analysis of Continuous frames by Distributing fixed End Moment T.Am. S.C.E. Vol. 96 (1932) Page I.
6.	John E.Goldberg.	Wind stresses by Slope Deflection and converging Approximations Trans. Am. Soc. C.E. paper No. 1878 p. 962 Vol. 99 1934.
7.	Kani	Analysis of Multi Storeyed frames English Translation by Charles P. Hywan I.C.I. May 1956.
8.	Kloucek C.V.	Distribution of Deformation.
9.	Maney G.A.	Slope Defl^{n} . Method of Wind Stress Analysis Engg. studies No. 1 University of Minnesota 1915.
10.	Naylor N.	Side Swa in Symmetrical building frames. The Structural Engineer April 1950l
11.	Ross A.W. & Moris C.T.	The design of Tall Building Frames to Resist Wind.Proceeding of Am. Soc. C.E. May 1928 P. 1595.
12.	Smith Albert (Portal method)	Wing stresses in the frames of office Buildings Journal of Western Soc. of C.E. April 1915 P. 341.
13.	Sutherland and Bowman.	Structural Theory P. 295.
14.	Wilbur & Norris	Elementary Structural Analysis B. 280.
15.	(Cantilever method)	Wind Bracings with Knee Braces of Gusset Plates Engg. Record Sept. 5, 1908.

16.	Wood R.H.	An economical Design of Rigid Steel Frames for Multistorey Buildings. N.B.S. research paper No. 10 Published by Her Majesty's Stationary office London.
17.	Baker J.F.,	Rational Design of Steel Building Frames-Journal of Institution of Civil Engineers Vol. 3 (1935-36) p. 127.
18.	Baker, J.F.,	Plasticity as a Factor in Design of War-time Structures - The Civil Engineer in War, Vol. 3 - 1948, p. 30.
19.	Baker , J.F.	Recent Investigations into Behaviour of Steel Frames in the Plastic Range - Journal of Institution of Civil Engineers Vol. 31, Jan, 1949, P. 188.
20-	Baker , J.F.,	Short-commings of Structural Analysis - Engineering, Jan. 11, 1952.
21.	Baker , J.F, Horne, M.R., & Heyman, J.	Steel Skeleton (Vol. I) - Elastic Behaviour published by Cambridge University Press - 1956.
22.	do	Steel Skeleton (Vol II) - Elastic Behaviour published by Cambridge University Press - 1956.
23.	Broek, J.A.Vanden,	Theory of Limit Design , Journal of A.Soc. C.E. Paper No. 2334, Vol. 113,1948,P. 213-268.
24.	English , J.Morley,	Design of Frame by Relaxation of Yield Hinges Journal of A.Soc. C.E. Paper No. 2715, Vol. 119, 1954, P. 1143-1153.
25.	Greenberg, H.J., & Prager, W.,	Limit Design of Beams and Frames, Journal of A.Soc. C.E. , Paper No. 2501, Vol. 117, 1952 P. 447-480.
26.	Heyman, J., & Roderick, J.W.	Extension of Simple Plastic Theory to take account of the Strain Hardening Range-Journal of Institutio of Mechanical Engineering , London, Vol. 165, 1951.
27.	Horne, M.R.,	Collapse Load factor for a Rigid Framed Structure Theory of Beams Carrying Distributed Loads in Multistoried, Multibay Frames - Engineering London 1954-Vol. 177 P. 210-212.
28.	Horne, M.R.,	Fundamental Propositions in Plastic Theory of Structures-Journal of I.C.E., April 1950.

29. Horne, M.R., — Moment Distribution method for Analysis and Design of Structures, Proc. of I.C.E., Vol. 3 , No. 1, April 1954.

30. Jain O.P. — Ultimate theory of Redundant Structures of Concrete with Particular Reference to Two Hinged Arches - Vol. I. Thesis submitted for Ph.D. to University of London, 1953.

31. Neal, B.G., & Symonds, P.S., — Rapid calculation of Plastic Collapse Load for a Framed Structure Proc. of I.C.E. London Vol. I, No. 3, April 1952.

32. Neal, B.G. — Plastic Methods of Structural Analysis C.H., 1956.

33. B.C.S.A., Publication No. 5. — Collapse Method of Design - published in 1952.