ELECTRICAL ANALOGY AS APPLIED TO SUBSOIL FLOW STUDIES WITH SPECIAL REFERENCE TO THREE - DIMENSIONAL PROBLEMS

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By

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AN ABSTRACT

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by

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Adviser : Prof. R.S. Chaturvedi

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The main purpose of this thesis is to present the electrical analogy investigations in the field of three-dimensional subsoil flow, with most of which the author has been associated or is currently associated. An attempt has also been made to concisely present the available information concerning the history, theory, technique, applications and the recent developmentsof the various electrical analogy methods, viz. the two-dimensional, the sector, and the three-dimensional model methods. The several difficulties and limitations encountered in the various methods have been discussed in the appropriate contexts. Apart from the applications to the subsoil flow problems which have been dealt with in greater detail, some important hydraulic applications involving surface flow phenomenon have also been discussed and typical results presented. The review of research work already performed by the analogy methods has been given, and exhaustive but typical results have been presented.

Investigations in the field of subsoil flow under hydraulic structures on permeable foundations, with which the author has been associated, have indicated that almost all such problems require a three-dimensional approach, except for a very limited number. The need for change of the usual two-dimensional concept in tackling such problems has, therefore, been indicated. Design considerations for the underground parts of hydraulic structures on permeable foundations may be improved by the use of the three-dimensional technique in general. Particularly, for simple three-dimensional structures like syphons and falls, where boundary conditions are not too complicated, rational criteria for the design of foundations with respect to subsoil flow may be obtained by this technique. Such criteria for canal syphons have been presented typically.

Problems of simulation of a tubewell, design of dewatering systems for deep foundations, and design of relief wells for hydraulic structures have been discussed and typical results presented. It has been concluded as a result of some basic studies carried out recently, that the relative performance of a properly designed filter, and of properly designed fully penetrating relief wells, provided to relieve pressures at some intermediate reach in case of long escaping structures is almost the same. Certain problems for further research and experimentation have been indicated, and the desirability of observations for the study of model and prototype conformity has been stressed.

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'The Technique and Application of Three-Dimensional Electrical Analogy in the Design of Hydraulic Structures' University of Roorkee, Research Journal Vol. II, 1959 — In Pocket at the End of the Thesis

ELECTRICAL ANALOGY AS APPLIED TO SUBSOIL FLOW STUDIES WITH SPECIAL REFERENCE TO THREE-DIMENSIONAL PROBLEMS

CHAPTER I

INTRODUCTION

1.1. HISTORICAL DEVELOPMENT: The electrical analogy method is a comparatively recent technique for the solution of many important problems in various branches of Science, Engineering, and Technology, and has been very extensively used during the last three decades or so. Undoubtedly, it is a very efficient tool in the hands of research workers and designers today. In the field of Hydraulics, the first application of the electrical analogy to be published was done by N.N.Pavlovsky[1,2] in Russia in 1922, for the investigation of problems relating to seepage flow under dams; although H.B. Muckleston and E.E.Sands used it even earlier to make laboratory investigations of the flow through earth-fill dams. Other subsoil flow problems, essentially two-dimensional in nature, were studied later on in the thirties using this method, by several other research workers notable amongst whom are, C.G.J. Vreedenburgh and O. Stevens,[3,4] R.D.Wyckoff and D.W.Reed[5], M.Muskat[6], E.W.Lane, F.B.Campbell, and W.H.Price[7], L.F. Harza[8], V.I.Vaidhianathan and Gurdas Ram[9]. M.A.Selim[10] used the two-dimensional technique in 1947, to study the uplift forces acting on a dam, built on porous media having different conditions of substrata, for various combinations of simple horizontal floors and cut-offs. Numerous research workers have, since then, used the two-dimensional method to ~~ha~~ solve a variety of subsoil flow problems.

The first successful use of the three-dimensional electrical analogy technique was made by B.F.Reltov[11] in 1933, who used the method for the study of seepage flow patterns under complex structures built on pervious heterogeneous foundations. Subsequently, the method was used in 1936, by the engineers of the U.S.Bureau of Reclamation,[12] for investigating the head loss in the trash-rack structures of the intake towers of the Boulder Dam. The use of sector model technique was made in these studies. H.Rouse and M.M.Hassan[13] carried out interesting studies on sector models in 1948, for design of cavitation free inlets and conduit contractions Almost during the same period, H.E.Babbitt and D.H.Caldwell[14] used pressed carbon sector models of gravity wells, to simulate the free surface around the well by a method of systematic trials. Sector models, and full three-dimensional electrical analogy models, were used by P.G.Hubbard[15] in 1949, to investigate problems of boundary pressures for flow confined by various surfaces of revolution, and of flow patterns around bodies in parallel uniform flow. Important advance in the technique of sector models was made by P.G.Hubbard and S.C.Ling[16] in 1952, who investigated the problem of boundary pressures for prototype transitions, in which the absence of an axis of symmetry makes a mathematical analysis impractical or virtually impossible. At about the same time, the problem of study of horizontal earthquake shock on dams was admirably tackled by the electrical analogy method by C.N.Zangar and R.J.Haefeli[17], whose results are of direct application in calculations of stability of gravity dams.

F.W.Opsal[18] in 1954, used a new conductive material consisting of gelatin, glycerin, water, and salt, for carrying out tests to determine the potential distribution around a gravity well for different penetrations of the well, and also to determine the free surface around the gravity well. An important advance in this direction was made by C.H.Zee, D.F.Peterson, and R.O.Bock[19] in 1955, who combined the electrical analogy for hydraulic flow with the membrane analogy for the free surface, in order to study the problem of radially symmetrical unconfined flow to a well, and obtained empirical relationships between the flow and the geometric variables of the system.

H.A.Johnson[20] in 1955, using both two - and three - dimensionlal models, investigated the seepage forces in a concrete gravity dam and its foundations, by means of the electrical analogy, assuming flow to occur through both the concrete and the rock foundation.

Apart from the development of the linear electrical analogy mentioned above, non-linear electrical analogy has developed considerably during the last one decade or so. Non-linear electrical analogy has made possible the solution of unsteady and periodic phenomenon in the field of Hydraulics and allied branches. H.M. Paynter[21] studied cases of pressure transients in a uniform pipe, and surges in a tank, by the use of electronic analogs and computers. M.S.McIlroy[22] used electric network analysers comprising of special non-linear resisters to obtain solutions for pipe networks, comparable with results of rigorous algebraic solutions. Electronic analogs for stream-flow routing have been used, amongst others, by M.A. Kohler[23] for preparation of river stage forecasts. Studies on an analog computer model have also been carried out by R.E.Glover, D.J.Hebert, and C.R.Daum[24], for finding out flow distribution patterns, and estimation of water level stages, in a network of channels with or without tidal effects. Recently, L.Bernell and R.Nilsson[25] in 1957, used an electrical analog, consisting of about 400 capacitors, to solve non-stationery two-dimensional flow problems, such as consolidation processes and pore pressures in foundations and earth dams.

The development of the electrical analogy method in India, started with the pioneering work of V.I.Vaidhianathan and Gurdas Ram[9,26,27] in the early thirties, at the Punjab Irrigation Research Institute, in connection with the design of weirs and similar structures on permeable foundations. Since then, the method has been used extensively for solution of two-dimensional problems of subsoil seepage at various Research Institutes of India. The first successful application of the three-dimensional electrical analogy technique in India was made in 1955, at the U.P.Irrigation Research Institute, Roorkee, for the case of remodelling of a syphon[28] on a major canal of Uttar Pradesh. This technique has been used at an increasing rate at this research station ever since, and today, several problems of subsoil flow of basic as well as applied nature are being tackled by this technique. Recently, in 1958, the three-dimensional technique was also used at the Central Water and Power Research Station[29], Poona, for seepage studies of the foundations of a major barrage by use of wax models.

1.2. FIELDS OF APPLICATION: Of all the experimental techniques, electrical analogy holds almost the widest field of application in various branches of Science, Engineering and Technology. Some of the important fields of its application, along with a few typical problems in each, are mentioned below :-

(1) Magnetism and Electrostatics - Problems of field plotting in various cases.

(2) Thermodynamics - Problems of heat-transfer in the design of furnaces and internal combustion engines.

(3) Mechanical Engineering - Problems of torsion of shafts and stress analysis in machin parts.

(4) Structural Engineering - Problems of design of framed structures.

(5) Electrical Engineering - Problems of design of electrical distribution systems.

(6) Municipal Engineering - Problems of solution of pipe net-works and water-hammer phenomenon.

(7) Hydraulic Engineering - Problems of irrotational fluid flow and subsoil seepage flow.

(8) Petroleum Technology - Problems of oil recovery in drilled wells.

The types of hydraulic problems which can be solved by the electrical analogy technique, may be classified as those pertaining to slow flow of water through porous media, and irrotational motion of water in surface flow. The most important application of the first category lies in the estimation of uplift pressures and exit-gradients, under hydraulic structures on permeable foundations[30]. Other applications of this class are the study of pore pressures[31] under dams on permeable foundations, study of seepage flow in earth dams and embankments, tubewells, galleries and coffer dams;[32] computation of rapid draw-down effects in earthen dams;[33] investigation of seepage forces in a concrete gravity dam and its foundations;[20] and design of well point systems for dewatering of deep foundations[34]. The applications of the second category can be enumerated to be, design of cavitation-free inlets for sluices in dams;[13] cavitation around stream-lined bodies[15], design of transitions for conduit contractions[35], determination of negative pressures developed in overfall spillways[36], and the determination of velocity and pressure distributions in a liquid jet deflected by a normal boundary[37]. The analogy also finds an application in the study of horizontal earthquake shock on dams[38]. In fact, this method has been developed in many of its important aspects, and has become a very powerful and reliable method for the solution of pressure and flow problems in various structures.

The applications of the non-linear electrical analogy are also numerous and important, and a few applications in Hydraulics and allied branches have already been mentioned in Section 1.1. Wide use of electronic analogs and computers in various fields of Engineering and Technology bears recognition to the importance of the non-linear analogy. The scope of this thesis does not permit any detailed discussion of the technique and applications of the non-linear electrical analogy.

Apart from the detailed discussion of the technique and applications of the electrical analogy as applied to subsoil flow studies, its technique and applications for certain surface flow and other important studies in Hydraulics have been included in brief in this thesis, because of their interesting nature and relative importance in broadening the conceptions of the electrical analogy method.

1.3. OTHER METHODS OF SOLUTION OF SEEPAGE FLOW PROBLEMS: In approaching the solution of any problem of seepage, the graphical representation of flow, known as the flow-net, is of great assistance. Hence, the problem of subsoil flow boils down essentially to one of determination of the flow-net, under the given boundary conditions. Apart from the electrical analogy method, various other methods have been developed for this purpose. The most widely used of such other methods is the graphical solution by trial sketching, more where bound condn are known

popularly known as the Forchheimer Solution[39]. It requires little equipment, and it gives the investigator valuable understanding or feel of the flow pattern and the way in which the pattern would change were the boundaries to be revised slightly. The solution is obtained by systematic trials[40], based on certain recommended guiding principles and past experience of working by the method. A second method of solution is the theoretical or mathematical solution. In this method the boundary conditions of the problem are expressed by equations, and solutions are obtained by mathematical procedures. The classical work of W. Weaver[41] by this method for some simple cases of floors on porous media illustrates the complexity of the Mathematics involved. The mathematical solution becomes sufficiently complicated even for ordinary problems, and hence the method is largely of academic interest only. The third method of solution through the use of sand or hydraulic[27,30] models has been applied widely for the determination of paths of seepage or flow lines. Another method less commonly applied is the capillary flow analogy[42] or the viscous fluid model method, based on the analogy of capillary flow between two closely spaced parallel plates to the two-dimensional flow through soils. The method is in many ways similar to the sand model method, and has been used mostly as a check on other methods of solution.

Most of the two-dimensional subsoil flow problems can be solve admirably by one or more of the methods given above besides the electrical analogy method. Other analogies which can be used conveniently in certain two-dimensional cases are the slab analogy[43], membrane analogy[44], soap film analogy[43], and elastic analogy[45]. The three-dimensional subsoil flow problems are, however, difficult and most often impossible to be solved by the methods and analogies given above, except by the electrical analogy method. A limited application of hydraulic models[28, 46], and membran~~e~~e analogy cum electrical analogy models[19], has been done in such three-dimensional problems.

1.4. ADVANTAGES AND LIMITATIONS: The chief advantages of the method are that the laboratory equipment is cheap and handy, and the analysis of many complicated flow problems can be done speedily and with a very reasonable accuracy. Both three- and two-dimensional systems can be studied by the electrical method, the former in particular being much more readily investigated in this way than in any other, owing to the possibility of probing the interior of the system even when it has no significant symmetry characteristics. The three dimensional electrical analogy in many cases represents the only means to obtain a correct solution of problems connected with the design of underground parts of hydraulic structures. The irregularities of the boundary forms of the prototype add almost no extra labour for the solutions by the electrical analogy method. This advantage is in sharp contrast to the drawback of the analytical methods, which are easily applicable only for a few geometrical boundary forms. The scope of the electrical analogy method, as already outlined in Section 1.2, is wider than that of any other analogy techniques or computational methods.

The general practice in the solution of most of the hydraulic problems in the past has been to take recourse to hydraulic models, but it has now been established, that in cases where the problem is amenable to solution by electrical analogy technique too, as is the case with almost all subsoil flow problems, it is always more desirable to use this method[46]. While, results of practical accuracy can be obtained from either of the two models, there are other definite advantages in the electrical method, besides those given in the above paragraph, which make the electrical analogy method more preferable. Models of much smaller scales can be used with electrical set-ups, while capillary forces will vitiate the results, if the scale is reduced very much in hydraulic models. Also, the pressure pipe piezometers used in a hydraulic model cannot be placed exactly in the required position, due to their having a certain optimum diameter to minimise capillary effect in the piezometers to a negligible value; and as such, the pressure read by the piezometers will not correspond exactly to that at the point at which it is required. In electrical models such a difficulty is not encountered due to the use of very fine probes for potential observations. Other disadvantages[27] present in the hydraulic models, which do not come up in electrical models, are that the difference of levels in the piezometers being generally small is difficult to measure accurately, effects of surface tension vitiate the readings somewhat in the usual size models, stable conditions are achieved in the model after a considerably long period of running, and ends of piezometer tubes in sand get choked at times. The electrical model has a very important advantage[46] over its hydraulic counterpart, in as much as, it is much easier to study the effects of changes in any part of the structure, the only thing required being to drain off the electrolyte and then introduce the necessary changes wherever desired, while the hydraulic model should be dismantled first to enable the changes to be incorporated in it.

The electrical analogy method has its limitations also, but if they are recognised, results of practical value will be obtained. The greatest disadvantage of this method is that the top flow lines or the phreatic lines must be located by trial, or use made for their location of other methods usually mathematical or sand model methods. In regions of high velocity subsoil flow, such that Darcy's law becomes inapplicable, the results of analogy method become unreliable. Similarly, in irrotational surface flow problems, such as flow around submerged bodies of revolution, if separation occurs in the hydraulic model, the results of analogy method no longer become applicable[47]. Capillary forces, which often affect the seepage flow considerably in prototypes, are not amenable to simulation by the analogy. Also, in the analogy method, since the electrical resistance and permeability coefficient of soils cannot be related together, there is no direct method of assessing the amount of seepage through or below a structure. Electrical analogy results must be supplemented by data of permeability, obtained separately, for such a purpose. Although, in two-dimensional flow cases the seepage can thus be determined rather accurately, yet for three-dimensional cases it can be determined only approximately[11].

In most of the problems, usually in the three-dimensional ones, assumption has to be made for the subsoil to be homogeneous and isotropic upto the depth of the impermeable layer, since it becomes rather difficult to represent stratification of subsoil in the models. That such conditions of homogeneous and isotropic subsoil do not exist in the field has to be kept in mind. But, fortunately for most of the problems, such an assumption will not be very wrong[46]. And, after all, most of the calculation procedures, even for two-dimensional work, are based on the above assumptions. However, even in three-dimensional electrical analogy work, it is possible to simulate stratification with a properly developed technique.

* * *

CHAPTER II

THEORY OF THE ELECTRICAL ANALOGY

2.1. LAWS OF SIMILARITY: The various laws of similarity entering into the scope of electrical analogy as applied to hydraulic problems are mentioned below :-

2.1.1. Subsoil Flow Through Homogeneous Pervious Medium:

It is well known that the slow flow of water through subsoil obeys the Darcy's law, which may simply be represented by the equation,

$$V = K. \frac{H}{L} \qquad \dots \quad \dots \qquad (1)$$

where, V represents the velocity of flow through the subsoil bed, the proportionality factor K represents the permeability of the medium and is known as the coefficient of permeability, H represents the head causing the flow, and L represents the thickness of the bed measured in the direction of flow.

The above equation can be written in the following differential form:

$$V = -K \frac{\partial H}{\partial L} \qquad \dots \quad \dots \qquad (2)$$

where, $\frac{\partial H}{\partial L}$ is the average hydraulic gradient along the path of flow, ∂L being the elementary length along the path of flow.

If we consider three-dimensional flow i.e. the flow of water not only in two directions, but in all the three directions, as is the usual case of subsoil flow, the easiest way of studying the problem will be to resolve it into three components along the three cartesian coordinates. Thus, if v_x, v_y, and v_z, denote the velocities parallel to x, y, and z, axes respectively, the general vector law of

Darcy, equation (2), can be written for isotropic and homogeneous subsoils as:

$$v_x = -K \frac{\partial H}{\partial x}\,; \quad v_y = -K \frac{\partial H}{\partial y}\,; \; v_z = -K \frac{\partial H}{\partial z} \qquad \ldots \quad (3)$$

Since, the flow of water in a soil is not different from the flow of a hypothetical incompressible fluid, which we may suppose to replace both the water in the soil and the soil itself,[48] the equation of continuity,

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \qquad \ldots \quad (4)$$

will hold good, which expresses that the volume of water flowing in a small elemental volume of soil during a certain interval of time must be equal to the volume flowing out of it during the same interval of time.

Substituting in equation (4) the values of v_x, v_y, & v_z, as given above in equation (3), the following equation is obtained.

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} = 0 \qquad \ldots \quad (5)$$

This equation is the well known LaPlace's equation governing the subsoil flow. The pressure head function H which satisfies this partial differential equation is called the potential function. This potential function determined from the equation (5), will be subject to the boundary conditions present in each particular problem. The function is finite, continuous, and single valued, at all points of the medium, and the general methods of solution usual in potential theory will apply.

For two-dimensional flow, the equation (5) reduces to:

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = 0 \qquad \ldots \quad (6)$$

2.1.2. Steady Irrotational Flow of a Fluid:

Using the same notations as applied in Section 2.1.1, the following conditions (Cauchy-Riemann equations) for irrotational flow of a fluid in three dimensions may be written:

$$\frac{\partial v_x}{\partial z} = \frac{\partial v_z}{\partial x} \; ; \; \frac{\partial v_z}{\partial y} = \frac{\partial v_y}{\partial z} \; ; \; \frac{\partial v_y}{\partial x} = \frac{\partial v_x}{\partial y} \qquad \ldots \quad (7)$$

Considering the incompressibility of the fluid, the equation of continuity (4) holds good. Further, the steady flow requires that the velocity at any point must be constant. If ϕ is the velocity potential of the field, the velocities in the three directions are equal to the gradient of ϕ in the corresponding directions, as expressed by

$$v_x = \frac{\partial \phi}{\partial x} \; ; \; v_y = \frac{\partial \phi}{\partial y} \; ; \; v_z = \frac{\partial \phi}{\partial z} \qquad \ldots \quad (8)$$

Substituting in equation (4) the values of v_x, v_y, and v_z, given in equation above, the following LaPlace's equation is obtained,

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \qquad \ldots \quad (9)$$

which, for two-dimensional flow becomes,

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad \ldots \quad (10)$$

2.1.3. Flow of Electricity Through a Homogeneous Conductor:

The flow of electricity through a homogeneous and isotropic conductor or electrolyte follows the well known Ohm's law:

$$I = \frac{E}{R} \qquad \ldots \quad (11)$$

where, I is the current flowing through the conductor of resistance R, under the applied voltage E. If ix, iy, and iz, are the components of current parallel to x, y, and z, axes respectively, the Ohm's law (11) can be expressed in the following differential form:

$$i_x = \frac{1}{R}\frac{\partial E}{\partial x}\ ;\quad i_y = \frac{1}{R}\frac{\partial E}{\partial y}\ ;\quad i_z = \frac{1}{R}\frac{\partial E}{\partial z} \qquad \ldots \quad (12)$$

where, $\frac{\partial E}{\partial x}$, $\frac{\partial E}{\partial y}$, and $\frac{\partial E}{\partial z}$, are voltage gradients at any point x, y, z. Furthermore, for the steady flow of current in the electrodynamic system, the equation of continuity given below must be satisfied.

$$\frac{\partial i_x}{\partial x} + \frac{\partial i_y}{\partial y} + \frac{\partial i_z}{\partial z} = 0 \qquad \ldots \quad (13)$$

Substituting in equation (13) the values of i_x, i_y, and i_z, given in equation (12), the following La Place's equation is obtained.

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = 0 \qquad \ldots \quad (14)$$

For two-dimensional flow, this differential equation reduces to,

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} = 0 \qquad \ldots \quad (15)$$

2.1.4. Analogy Between Darcy's and Ohm's Laws:

The forms of the La Place's equations (5),(9), and (14), or (6),(10), and (15), are identical respectively, and thus clearly indicate the analogy between the slow flow of water through a porous medium or irrotational flow of a fluid, and the flow of electricity in homogeneous and isotropic conductors.

The important analogy between the subsoil flow and the electric flow can be more directly visualised by writing down the two corresponding laws of flow as below, to bring out the ~~simuilatiry~~ similarity between the two.

	DARCY'S LAW	OHM'S LAW
	$Q = \frac{K\,A\,H}{L}$	$I = \frac{K'A'V}{L'}$ ... (16)
where,	Q = rate of flow of water	I = current (rate of flow of electricity)
	K = coefficient of permeability of soil	K' = conductivity coefficient of the electrolyte.

A = cross-sectional area for the flow of water

A' = cross-sectional area for the current flow.

H = head producing flow

V = voltage producing current.

L = length of path of percolation

L' = length of path of current

Moreover, by writing the two laws as above, the analogous or the comparable quantities involved in the analogy become obvious. A knowledge of such analogous quantities is essential for the correct simulation of boundary conditions in the analogy model, and the projection of the results obtained on the model to the prototype.

2.2. THE FLOW-NET:

The flow-net is a means of representing graphically the nature of the flow of a fluid, and gives a vivid picture of the flow conditions in a region specified by definite boundary conditions. It permits the accompanying velocities and pressures to be computed for many conditions, which were otherwise usually determined by measurement on hydraulic models.

2.2.1. The Stream Surfaces and the Equipotential Surfaces:

The La Place's equations (5), (9), and (14) derived in Section 2.1, all represent two sets of surfaces, the stream surfaces and the equipotential surfaces represented generally by ψ (x, y, z) = constant and ϕ (x, y, z) = constant respectively. These surfaces are mutually orthogonal for isotropic conditions of the medium, and comprise the three-dimensional flow-net. For the case of a two-dimensional problem, these surfaces become lines - ψ (x, y) = constant being the stream lines, and ϕ (x, y) = constant, the equipotential lines. For the case of flow of water through subsoils, the streamlines are the paths which a particle of water follows in its course of seepage through the saturated soil mass. The equipotential lines determine points where the total pressure head of water is constant.

The same quantity of water is assumed to flow betweem each pair of consecutive stream lines in a two-dimensional net. The velocity, therefore, must be inversely proportional to the spacing of these lines at each point. Since the drop in pressure is directly proportional to the velocity, the spacing of the equipotential lines must everywhere bear a constant ratio to the spacing of the stream lines at that point. It is convenient to make the spacing of the two systems of lines equal and each figure an approximate square with the same average breadth and length.

2.2.2. Effect of Boundary Conditions and Methods of Solution:

Since the La Place's equation has a unique solution, one and only one flow-net is possible for a given set of boundary conditions. The flow-net gets changed by changing one or more of the boundary conditions. This possibility of obtaining a unique solution under any given condition strikes a note of caution, since a flow-net can always be obtained for any assumed form of boundaries, even if these are incorrectly reproduced for obtaining the solution by any of the methods. But, the results will not naturally be applicable to the prototype unless the boundary conditions are simulated accurately.

The solution of the La Place's equation for drawing the flow-nets can be found by various procedures. Analytically, the solution can be obtained by solving the equations for known boundary conditions. As an illustration, such a solution for the case of an impervious dam with a flush horizontal base resting on a pervious infinite foundation is shown in Drawing 1. The stream lines are a system of confocal ellipses, whose limiting form is a straight line, being the base of the dam itself, with centre at O (see Drawing 1) having the equation,

$$\frac{x^2}{(\frac{b}{2}\cosh\frac{\pi}{H}\psi)^2} + \frac{y^2}{(\frac{b}{2}\sinh\frac{\pi}{H}\psi)^2} = 1 \qquad \ldots \quad (17)$$

where, the notation used and quantities entering in the equation are the same as indicated on Drawing 1. The equipotential lines are given by

$$\frac{x^2}{(\frac{b}{2}\cosh\frac{\pi}{H}\phi)^2} + \frac{y^2}{(\frac{b}{2}\sinh\frac{\pi}{H}\phi)^2} = 1 \qquad \ldots \quad (18)$$

and are shown on the Drawing 1. ψ and ϕ, as already mentioned, are the stream function and the potential function respectively. The uplift pressure, h, along the base of the dam has also been shown on the Drawing 1 in terms of H, the head acting. The equation for this line obtained mathematically is

$$h = \frac{H}{\pi} \cos^{-1} \frac{2x}{b}. \qquad \ldots \quad (19)$$

Apart from the electrical analogy and analytical methods, graphical methods of trial sketching[40], relaxation method[49,50], and method of sources and sinks[7], are the other methods that can be applied. However, as already discussed in Section 1.3, electrical analogy method is very preferable for such a purpose, moreso in three-dimensional problems, which become far more tedious and often impossible to be tackled by other methods.

2.2.3. Uses and Limitations of the Flow-Net:

A correct knowledge of the flow-net in the field of subsoil flow problems, enables the computation of loss of water through dams and their foundations by seepage, uplift pressures on the foundations of hydraulic structures with which the seepage water comes in contact, and the exit-gradient and safety against piping. Examples of its uses in other flow problems are to be found in prediction of cavitation in steady irrotational flow of water over overfall crests of or eddy losses are more likely to set in, and the results are apt to be less reliable. On parallel reasoning, it can be seen that the flow through suddenly expanding sections cannot be accurately analysed, since eddies are very likely to be caused in such a flow. The accuracy of the agreement of the flow-net analysis with the flow of the water in the actual case, will depend upon the closeness with which the flow of water approaches the action of a perfect fluid. This has to be kept in mind in deciding on the probable reliability of a flow-net analysis.

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spillways, and in flow through inlets and conduit contractions. It is also applied in the studies to eliminate or minimise the effects of cavitation, in the boundary design of hydraulic machinery like turbine runners and ship propellors.

It is worthwhile to stress that the flow-net loses most of its significance when applied to three-dimensional flow cases. In a flow-net for two-dimensional flow the velocity at any point varies inversely as the width of the stream tube, as pointed out in Section 2.2.1. But, although the discharge through a stream tube still remains constant, the cross-sectional area of the stream tubes in the three-dimensional net is not a function of the width of the stream tube alone, but also of its depth, which is a variable. As such, it is not possible to obtain the velocities directly from the three-dimensional net, as can be done in the case of a two-dimensional net. It is possible, however, to obtain velocities in these three-dimensional nets by a simple but laborious method.[12] The main purpose, therefore, in drawing three-dimensional nets is to indicate the direction of flow.

It is important to keep in mind the various limitations of the flow-net solutions, since at times it might be tempting to extend the method at the expense of factors which cannot be considered in the flow-net theory. Thus, the flow-net solution is inapplicable where friction or impact losses are appreciable, in so far as, such a behaviour of water detracts from its assumption as a perfect fluid, which is the basic assumption in the theory of flow-net as applied to hydraulic problems. Where water is accelerating as in the case of flow over a weir or a dam crest, through an orifice or under a gate, the flow-net analysis can be used with considerable accuracy; but where water is decelerating, impact or eddy losses are more likely to set in, and the results are apt to be less reliable. On parallel reasoning, it can be seen that the flow through suddenly expanding sections cannot be accurately analysed, since eddies are very likely to be caused in such a flow. The accuracy of the agreement of the flow-net analysis with the flow of the water in the actual case, will depend upon the closeness with which the flow of water approaches the action of a perfect fluid. This has to be kept in mind in deciding on the probable reliability of a flow-net analysis.

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CHAPTER III

ELECTRICAL ANALOGY TECHNIQUE

3.1. TWO-DIMENSIONAL SET-UP:

The basic principles and technique used in two-dimensional electrical analogy work are the same for the various types of problems tackled by this method, and are presented hereunder:

3.1.1. Simulation of Boundary Conditions:

The first step in the construction of the electrical analogy model is the simulation of the boundary conditions accurately. This is a very vital and important part of experimentation, and often a correct representation of the boundaries is the most difficult job of the experimenter. To explain the various details, the case of flow beneath an impervious earth dam with a central sheet pile cut-off resting on permeable foundations is considered. Referring to Drawing 2, the boundaries to be simulated are :

(a) the upstream face A B, where seepage flow enters,

(b) the downstream face E F where seepage finds its exit,

(c) the intervening boundary B C D E of the impermeable base and the sheet pile, and

(d) the subsurface rock stratum G H.

At the upstream and downstream ends theoretically the boundaries are to extend to infinity, but for practical purposes it is considered sufficient to simulate regions of length of about five times the depth of the pile. These equipotential surfaces are simulated by copper strips or other perfectly conducting strips, and the stream line boundaries by non-conducting strips such as ebonite, pyralin, plexiglass, perspex or other plastics, seasoned wood, or any other perfectly non-conducting material. The physical dimensions of the model and the prototype are kept in proportion, and the various strips are secured in position in a flat insulated tray of suitable dimensions by non-conducting adhesives such as plasticiene, wax, or durofix, care being taken to keep the copper plates flush with the surface at the tray sides.

3.1.2. Simulation of the Resisting Medium:

In addition to the simulation of the boundaries as mentioned in Section 3.1.1, a conductive material through which electric current can flow is required to be substituted for the granular material through which water percolates in the prototype. The requirement for the conductive material~~s~~ is that it be homogeneous so that the electrical resistance is constant. Both solid and liquid conductive materials have been used. The following two solid resisting media are commonly used:

i) tin foil of about 0.005 inch thickness pasted on stiff manila paper, and

ii) heavy firm paper sprayed with a number of coats of graphite colloid, called 'aquadag' paper. Thin metal sheets other than tin have also been occasionally used for the purpose.

In case of liquid resisting media water is used usually with an electrolyte solution of sodium chloride, ammonium chloride, or copper sulphate. Even ordinary tap water is found to be quite satisfactory.

With conducting sheet models D.C. is generally used, while with electrolyte solution A.C. only should be used to obviate polarisation effects. The conduction paper method possesses the advantage that impermeable barriers, such as sheet piling, is particularly simple to represent. All that is needed is to cut out from the conduction sheet figures geometrically similar to the barrier being studied. Also, this method is free from shock hazards. However, liquid conductive materials are versatile to use, since they are the most satisfactory medium for three-dimensional work also. The conductive material to be used in a particular model is determined by the problem required to be solved.

3.1.3. Simulation of Stratification:

Problems of stratification in subsoil can be handled with either type of conductors. It is not so easy with solid resisting media, but in case of liquid media all that we have to do is to keep the depths of solution at respective places in the ratio of their permeability coefficients. Thus variations of the permeability can be expressed by variations in the depth of the electrolyte, brought about by proper stepped construction of the model. In order to satisfy the condition of two-dimensional electric flow in the electrolyte where the depth changes suddenly, it is necessary to fasten small metal strips or wires vertically on the boundaries of the stepped section of the model, particularly if the ratio of the depths is large. Another way of simulating permeabilities is to use electrolyte solutions having proportional conductivities in the required places in the model. Sometimes, when the ratio of permeabilities of the strata to be represented is too large, recourse is taken to a combination of the two methods, i.e. varying the depth as well as the conductivities at the same time. In cases with extreme differences of permeability, a good conductive material, such as copper, may be used for the extremely permeable layer.

Quite frequently, the permeability at any point is the same in all directions in the prototype but diminishes with depth. This condition is simulated by tilting the tray and allowing the depth of solution to be proportional to the permeability at respective places in the model.

A common type of non-homogeneity occurs due to stratification of the foundation material, when the permeability in a horizontal direction is greater than in a vertical direction. The horizontal coefficient of permeability may often be as much as ten times as large as the vertical value. If the ratio, k, of horizontal permeability to vertical permeability is constant at all points, the condition may be simulated by studying a model of the structure that has had its horizontal dimensions reduced by the ratio $\frac{1}{\sqrt{k}}$. To get the actual flow-net in such a case, the flow-net obtained with the compressed model is expanded to the proper scale by plotting vertical ordinates of the flow-net obtained for the compressed model under the corresponding points of the natural-shaped, non-reduced structure. As a rule, equipotential lines and stream lines corresponding to this case are not mutually normal.

The technique is still not fully developed to enable simulation of complex cases of anisotropy.

3.1.4. Determination of Potential Distribution:

The Wheatstone's Bridge circuit principle is used to determine the potential distribution along and inside the boundary of the model. The apparatus consists essentially of a source of electric supply, a potential divider, and a null detector. It has been found that for work on electrolyte models, A.C. at a high frequency of 1000-1200 cycles/second is more suitable. Accordingly, an audio oscillator is incorporated in the set-up. The other equipment needed for an accurate work can be an amplifier and a head-phone, or an amplifier with a built-in speaker, e.g. a signal tracer. Visual methods of null detection may also be used, and in that case an oscillograph is used in the circuit[51,52]. The complete set-up using an audio oscillator is shown in Drawing 2 for the example referred in Section 3.1.1. Referring to this drawing, the two electrodes AB and EF representing 0% and 100% potentials respectively are connected in parallel with a simple 10-wire potentiometer, which is fed with the output of the audio oscillator. One terminal of the null indicator is connected to a probe which is a thin conducting wire of very small dimensions projecting from a shielded insulated sleeve, and the other terminal is connected to a jockey which is moved on the potentiometer wire. To make the set-up more sensitive an amplifier can be incorporated, as shown in the drawing, in series with the null indicator. The connections are completed as shown in the Drawing 2. The general lay-out of a two-dimensional model, with the electrical set-up used as described above, is shown in Photograph 1.

Water mixed with weak solution of an electrolyte is filled in the tray to the required depth to simulate the depth of homogeneous subsoil, when the current establishes its own path and distribution. The potential at any point can then be found by just probing the point, and moving the sliding contact of the jockey on the potentiometer wire to hear a null sound in the indicator. Knowing the percentage potential at every point, the equipotential lines can be easily drawn. Alternatively, by keeping the contact at a fixed percentage on the potentiometer wire, the equipotential lines can be obtained directly by moving the probe at points in the model boundary to hear the null sound in the indicator. A pantograph may be incorporated to directly trace the points probed in the boundary on a map[10].

In A.C. circuits an electronic vacuum tube voltmeter[10,16] can also be used in null detection. A sensitive A.C. galvanometer can likewise be used. However, it has been the experience of the author that the best set-up for null detection is the one using an amplifier with a built-in speaker. This set-up enables the determination of null points with accuracy and speed. In case of visual methods one can not attain that speed. The use of head-phones also is not much recommended, since the experimenter gets a dizziness in his head due to all sorts of shrill sounds coming in the ears directly for a long period of time in a work done at a stretch.

The accuracy of measurement depends to a certain extent on the electrolyte, the electrode materials and the frequency of supply. The electrodes require to be sand-papered often to minimise the effect of a rust film which forms on copper plates rather early. Errors may also creep in due to the electrode plates not being flush with the surface at the sides. Also, the walls of the electrolytic tray usually introduce slight errors due to reflection of current paths, which can be overcome by som choosing the dimensions of the tray[10] as to minimize the boundary effects below a certain value which can be neglected.

3.1.5. Analysis of Experimental Data:

Having determined the potentials at all conveniently chosen grid-points within and along the boundaries of the electrical analogy model, the results are analysed in accordance with the demands of the problem being tackled. For example, in the case of determination of uplift pressure distribution under weirs on permeable foundations, a knowledge of potential distribution along the weir foundation profile alone will suffice.

It is essential to have a clear conception of hydrostatic potential and hydrostatic pressure for a correct evaluation of the results. Hydrostatic potential is given by the elevation of water level in a piezometer tube inserted at that point, measured with respect to a certain fixed datum; whereas, the hydrostatic pressure is the depth of the water in the piezometer tube erected at that point. Mathematically, this can be expressed by the relation:

$$p = \phi \mp y \qquad \ldots \quad (20)$$

where, p is the pressure - head, ϕ the potential, and y the elevation of the point under consideration with respect to certain fixed datum. Every care should be taken to convert the hydrostatic potential to pressure-head in any design calculation.

In most of the problems, the flow-net is required to be drawn. The net is easily drawn by first drawing the equipotential lines by interpolation from the observed potentials, and then drawing the stream lines orthogonal to the equipotential lines guided by the various rules that have been developed for the purpose[40]. The stream lines may, however, be constructed electrically by changing the terminals[7]. Thus, referring to the example shown in Drawing 2, the foundation profile of dam and sheet piling, BCDE, are formed of metal strips to act as one terminal, and the base of tray and its side edges similarly formed to act as the other terminal, the former two terminals AB and EF being replaced by a non-conductor. With these new terminals, the set of potential lines would be identical with the flow lines of the actual case. Having drawn the flow-net, computation of additional design data may be carried out from it. The exit-gradient is calculated from the formula :

$$i_s = \frac{H}{Nd.\Delta S} \qquad \ldots \quad (21)$$

where i_s is the exit-gradient, H is the total head of water inducing flow, Nd is the number of equipotential drops in the net, and ΔS is the length of the end-square on the discharge face. The total quantity of seepage flow underneath the structure is calculated from :

$$Q = K.H. \frac{Nf}{Nd} \qquad \ldots \quad (22)$$

where, Q is the quantity of seepage flow, and Nf is the number of flow tubes in the flow-net. It may be remembered, that a flow-net consisting of squares is to be used for the calculations.

Often, the construction of the flow-net may be obviated for problems of estimation of seepage loss. This is done by direct determination of the 'shape-factor', which is the quantity $\frac{Nf}{Nd}$ depending only on the shape of the boundary profile. The shape-factor is deduced from the electrical analogy model by using the formula[47]:

$$\frac{Nf}{Nd} = \frac{\rho}{R} \qquad \ldots \quad (23)$$

where, ρ is the specific resistance, and R is the total resistance of the electrolyte measured between the electrodes (AB and EF in Drawing 2). The total resistance of the electrolyte is measured by the Wheatstone's Bridge circuit, and the specific resistance of the electrolyte is measured in the usual way for a sample siphoned out from the tray.

3.2. TWO-DIMENSIONAL SET-UPS OF VARIOUS RESEARCH WORKERS:

The fundamental requirements of electrical analogy may be obtained in a number of ways. To give a comparative idea of such arrangements, the different devices used by the various research workers at different times have been briefly described, together with certain typical results obtained by each.

3.2.1. Pavlovsky's Apparatus:

The scheme of N.N.Pavlovsky's apparatus is shown in Drawing 3(a), where P is a tested plate of staniol, R a rheostat, R' an agometer, Acc an accumulator, G a galvanometer, A an ampermeter, I and II are buses, and e is a needle for plotting points. Drawings 3(b) and (c) give an example of the construction of equipotential and stream lines for a dam with three sheet pilings of equal length, with only one symmetrical half of the permeable region having been reproduced on the drawings above-mentioned.

3.2.2. Lane's Apparatus:

The apparatus used by E.W.Lane[7] is shown in Drawing 4(a). The tray consists of a shallow trough with plate glass bottom and sides, provided with means for accurate levelling, and filled to a depth of say ½ inch with an electrolyte, usually a solution of common salt. The model is made of a non-conductor, and is fixed in position along with two 1/8 inch copper plates simulating the pervious reaches. The terminals are connected to a 110 volt alternating current through a lamp bank which reduces the voltage drop across the terminals to about 18 volts. At the opposite side of the tray, over a scale is a high-resistance wire 1 metre long, each end of which is connected to one of the terminals by a heavy copper wire. On the high resistance wire in question, a sliding contact connects through a set of head-phones to a probing pencil, to determine the potential distribution at the required points.

The Drawing 4(a) also shows a typical flow-net obtained by Lane with this apparatus, for the case of a dam with one sheet pile, resting on pervious subsoil.

3.2.3. Harza's Set-Up:

The electrical analogy set-up used by L.F.Harza is shown in Drawing 4(b). The tray is 24 by 46 inch in size, using a salt solution about one inch deep. Copper terminals, X and Y, permit simulating the base of a dam 20 inch or less in width, AB, of any desired sectional design. With Switch E closed to F, the head-phones P permit the determination of the proportionate voltage drop at C along the base by selecting point D to produce silence. Likewise, to determine the resistance of the solution between the copper strips X and Y, Switch E is closed to F' through the known resistance R with head-phones connected to C'. To obtain unit resistance of the solution for parallel flow, a bakelite frame temporarily immersed in the tray having two corresponding electrodes X' and Y' is used, as shown in the drawing.

Typical results for base pressure obtained by Harza with this set-up for the case of a dam on sand with no cut-offs, and having a base width b of twice the head H are shown in Drawing 4(c). The drawing also shows the theoretically computed values of uplift pressures, and the upward escape gradient at the sand surface.

3.2.4. Vaidhianathan's Set-up:

The details of the experimental arrangement used by V.I. Vaidhianathan are shown in Drawing 5. The set-up illustrated is for the case of a simple floor flush with the upstream and downstream surface of the porous strata. AB and CD are two thick conducting plates of copper each 1.5 feet long, representing the surfaces of the porous strata. BC is an ebonite plate 1 foot long and flush with the conductors, representing the impervious base of the floor. The outer boundary ARD of the tank formed of 1/16 inch thick ebonite strips is made semi-elliptical, with AD as the major axis, and B and C as the foci, on consideration that elliptical boundary conforms to the theoretical condition that the boundary is at an infinite distance away. The ground-glass-bottomed tray S is filled with a dilute solution of ammonium chloride. P is a probe made up of a piece of platinum wire fused into a glass tube, with the end of the wire alone exposed. P_1 is a jockey moving on a ten-metre accurately calibrated potentiometer wire X X_1, and T is a head-phone. The electrical connections are as shown in the Drawing 5.

The A.C. source was obtained from a Neon Oscillator with an amplifier. This arrangement has been shown completely in the drawing, and is a very convenient source of oscillating current giving a sharp null point in the phone.

A typical result obtained by Vaidhianathan for the case of a sheet pile at the end of a flush floor is shown in Drawing 6, which also contains the results obtained by the potential theory method, hydraulic model method, Bligh's Plain Creep theory, and Lane's Weighted Creep theory, for a comparison.

3.2.5. Selim's Set-Up:

The experimental set-up used by M.A.Selim[10] is shown in Drawing 7(a). The experimental tray consisted of a wooden tank (40½" by 31" x 2½" deep) coated inside with a waterproof non-conductive paint. The model floor and cut-off walls were constructed of 1/32 inch bakelite plates when the potential lines were mapped, and of 1/32 inch copper plates when flow lines were mapped. The electrolyte ¾ inch deep was a solution of copper sulphate acidulated with sulphuric acid. A vacuum tube voltmeter was used to detect the equipotential lines instead of the usual Wheatstone's Bridge and head-phones. The instrument was equipped with six accurately calibrated scales covering various ranges to a maximum of 16 volts. It was also equipped with a 10,000 ohm resistance, and could be used to measure the voltage at any point in the tank without using an appreciable amount of current. The equipotential lines were detected by means of a probe attached to a pantograph. The arrangement of the pantograph, equipped with probe and pencil, and the experimental tank are shown in Drawing 7(a), for mapping the equipotential lines under a dam with a horizontal floor.

The general wiring diagram of the apparatus is shown in Drawing 7(b), which is self explanatory. Drawing 7(c) shows the uplift pressure diagram for the case of a dam with horizontal floor only, investigated by Selim with his apparatus for various cases of depth to impermeable boundary. The uplift diagrams according to Bligh's Theory and Weaver's theoretical formula[41] are also given in Drawing 7(c) for a comparison.

3.3. SECTOR MODEL TECHNIQUE: In cases of electrical analogy investigations concerning bodies having axial symmetry about a line, or even those having one or more planes of symmetry[16], it is not essential to make full models. The purpose in such cases is amply served by constructing sector models, since the flow pattern in the model is representative of the entire one.

As an illustration of the technique, a cross-section of the assembly of the sector model of a three-dimensional boundary contraction has been shown in Drawing 8(b), the analogy set-up for the corresponding two-dimensional boundary contraction having been illustrated in Drawing 8(a). The segmental electrolytic tray is formed confined between an inclined sheet of plate glass, two terminal plates, and a flexible strip of some plastic like 'Pyralin' to represent the boundary profile. On filling an electrolyte into the space thus provided, until the horizontal plane of the free surface intersects the plate glass at the predetermined axis of symmetry, the space occupied is a representative segment of the entire volume. The segmental portion simulated may have such/a small central angle, that the outer plastic boundary need be curved in only one plane. Since, the interest of the investigation is mostly limited to the potential distribution along the boundary, and also since the required accuracy of measurement can not be obtained by the customary probe, a series of fixed electrodes are accurately located along the inner face of the flexible boundary strip for potential measurements. The potentials are measured, as usual, by the Weatstone's Bridge principle. A circuit diagram[13] used in such studies is given in Drawing 8(c). A precision decade potentiometer is connected in parallel with the terminal plates, through which a constant voltage is applied to the electrolyte. A sensitive null-indicator is connected between the potentiometer and each electrode along the boundary, and a variable range condenser is used in parallel with the pontentiometer to minimize the effect of capacitance and thereby increase the sensitivity of the apparatus.

It is essential that the models should be thoroughly insulated and water-tight, because any leakage of the electrolyte will upset the readings. Other precautions that should be observed are, that the electrolyte should be clean and homogeneous, alternating current at a low voltage should be used to minimize polarization and avoid heating of the electrolyte, the electrochemical reaction between anodes and electrolyte should be reversible, models should be sufficiently large to maintain a close relative tolerance in dimensions, and the size and spacing of the fixed electrodes should be sufficiently small to measure the desired trends - the smallest spacing being necessary where rapid acceleration and high pressure gradients exist. A precisely finished paraffin wall at the centre line of the model should be provided, so that the electrolyte approaches a zero depth without capillary irregularities.

Apart from liquid medium sector models, solid, and semi-solid medium sector models have also been used in some studies. Pressed carbon sector models were used by H.E.Babbitt and D.H. Caldwell[14], and gelatin models were used by F.W.Opsal[18]. Fundamentally, the technique of experimentation with these mediums is almost similar to the one described above for liquid medium models.

The segmental method of construction is not readily applicable[15] to study of cases in which flow is external to the boundary, such as various head forms for submerged bodies, because the least accuracy would be obtained where the greatest accuracy is required, due to the fact that the model would be located in the region where the electrolyte approaches zero depth, and the measurements would, consequently, be less reliable. The outer boundary would also necessarily be placed at a large distance from the model or bent to the form of a stream line.

No useful application of the sector model technique can be made for decelerating flow around tail forms, or through expansions, or outlets, that involve boundary-layer separation[16]. Moreover, the analogy may not represent the actual pattern even for generally accelerating flow, if regions of local deceleration exist that tend to cause local separation. The model will indicate the existence of such conditions, however, and one of its most valuable uses is in testing and modifying designs to obtain the most economical separation - free form.

3.4. THREE - DIMENSIONAL SET-UP: The problem of flow of water through a granular material is essentially a three-dimensional one, and hence three-dimensional electrical analogy models are more a necessity than a choice in case of most of the problems of subsoil flow. However, it would be difficult to lay any standard practice for three-dimensional electrical analogy modelwork, due to complicated and varying boundary conditions in each problem, and hence every individual problem has to be tackled separately according to the special conditions and demands imposed by it. Simulation in the strict sense of the term is very difficult in three-dimensional experimentation, and several simplifying assumptions and approximations have to be made in each problem. The technique of model construction in such cases is much different from that in two-dimensional models, although the underlying principles for both are the same.

3.4.1. Preparation of Model and Electrolytic Tray:

A typical three-dimensional set-up for the case of a hydraulic structure on permeable foundations is shown in Drawing 9, for the study of uplift pressure distribution on the foundations. The three-dimensional model is constructed by engraving the bottom profile of the whole structure undistorted, to a suitable scale accurately on well seasoned wood, below a conveniently fixed datum elevation. The types of wood recommended for the purpose are Burmah Teak, C.P.Teak, or Andaman Padauk, since they give the least trouble in warping after seasoning and maintain their **form** accurately over long periods of time. Other non-conductors like various types of plastics can also be used, but it has been the experience of the author that they are not satisfactory for use in tropical climates due to an easy susceptibility to warping.

The wooden model thus prepared is then well baked in hot molten wax, so as to make it a perfect non-conductor. The surface of the model is then thoroughly cleaned and finally rubbed with cotton waste soaked in kerosene oil, in order to remove any extra sticking wax from the surface. As a further precaution to ensure non-conductivity and water-tightness of the model surface, it is given a thin coat of water-proof varnish.

The electrolytic tray may be an ordinary bath of wood, masonry, or steel, of suitable dimensions to hold the model, and provide enough extra space for simulation of other boundaries to a satisfactory degree. The sides and bottom of the tray must be perfectly insulated by applying repeated coats of hot molten wax, and the entire tray should be protected from earthing effects. Three layers of 700 gauge 'alkathene' film installed at the very base of the tray provides necessary insurance against earthing.

3.4.2. Fitting-Up of the Model:

The completed and treated model as described in Section 3.4.1. is placed upside down in the insulated tray, so that the lowest point in the foundation profile of the prototype is actually the highest in the model. At this stage hot molten wax is used to fix the model in position, and to bring the copper plates representing the surfaces at which the water enters the subsoil or leaves the same under the prototype structure, to the desired elevation. The surface of the model and copper plates are cleaned to remove any dirt or extra wax.

3.4.3. Determination of Potentials:

An electrolyte solution, or even ordinary tap water, is filled in the tank to represent the pervious material under the prototype structure. The top of the electrolyte gives the bottom of the pervious material, or the location of the impermeable layer in the prototype. A vacuum tube oscillator with an amplifier and a signal tracer are used, to provide the necessary current source, and for the determination of null points on a potentiometer. The electrical connections and the plan of a typical model are shown in the Drawing 9. The different copper plates are, as usual, given the percentage potentials corresponding to the water head acting on the prototype at the corresponding areas. Spot potentials at conveniently fixed grid-points on the foundation profile are determined by the help of an insulated probe, which makes contact only at its tip. The equipotential contours can then be drawn by interpolation.

The general lay-out of a typical three-dimensional electrical analogy model is shown in Photograph 2.

The usual assumptions and limitations of the technique have already been discussed in Section 1.4. Some other assumptions and limitations which especially come up in certain specific type of problems have been enumerated in appropriate contexts later.

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CHAPTER IV

APPLICATIONS TO TWO-DIMENSIONAL PROBLEMS

Some important applications of the electrical analogy method to two-dimensional hydraulic problems, with special emphasis on subsoil flow problems, are described in this chapter.

4.1. HYDRAULIC STRUCTURES ON PERMEABLE FOUNDATIONS:

The two-dimensional models for analysing subsoil flow under hydraulic structures on permeable foundations are easily constructed, the essential technique for which purpose has already been discussed in Section 3.1. Applications of the method to certain problems of this category, and the typical results obtained have been presented briefly hereunder:

4.1.1. Weirs:

Typical results of potential distribution[27] under a flush floor with an upstream and downstream sheet pile and an intermediate sheet pile of the dimensions indicated thereon, are shown in Drawing 10. In fact, such results can easily be obtained for any form of the weir profile.

The determination of uplift pressure distribution under the floors of weirs founded on pervious homogeneous subsoil of infinite extent can be dealt theoretically, by the well-known procedure given in C.B.I. Publication No. 12 by A.N.Khosla and others[30]. Uplift pressures and exit gradients have been worked out for various standard cases of weir profile, e.g., a single sheet pile, with, or without fall, a simple flush floor, a depressed floor, a floor with a sheet pile at upstream, or downstream, or at some intermediate point, etc. As an example, for the case of a sheet pile at the end of a flush floor, the distribution of pressure on the base of the floor, and exit-gradient have been shown to be given by the relation,

$$P = \frac{H}{\pi} \operatorname{Cos}^{-1} \frac{d - \lambda d + \sqrt{d^2 + x^2}}{\lambda d} \quad \ldots \quad (24)$$

and, $$G \text{ Exit} = \frac{H}{d} \times \frac{1}{\pi\sqrt{\lambda}} \quad \ldots \quad (25)$$

respectively, where $\lambda = \frac{1+\sqrt{1+\alpha^2}}{2}$, $\alpha = \frac{b}{d}$, b being the length of the floor, and d the depth of the sheet pile, H is as usual the total head acting, and x is the distance of the point under consideration measured from the upstream end. Similar results for other standard cases have been presented in form of handy curves, which directly give the result, without the botheration of solution of such equations. The pressure distribution for an actual weir profile is obtained by considering it as composed of two or more standard forms, and consulting the curves given in the publication. Suitable corrections are then applied to the results, e.g., that for slope, thickness of floors, interference of piles, etc. The solution thus obtained is quite satisfactory.

In cases of subsoil having different horizontal and vertical permeabilities, or in cases of impermeable boundary occurring at a short depth below the floors, or a clay stratum occuring in a localised area between the foundations and the impermeable boundary, the only solution possible is by electrical analogy methods. Investigations[53] have been carried out by this method for the case of a simple flush floor, to study the effect on the path of flow lines and equipotential lines of various combinations of layered subsoils of different permeabilities. A typical flow-net below the elementary flush floor, when flow occurs through three layers, having permeability coefficients in the ratios of 1:3:6 respectively, is shown in Drawing 11(a). It is found that the course of seepage flow lines passing from a more permeable to a less permeable layer, and vice versa, is comparable with the path of light rays passing from a rarer to a denser medium, and vice versa, in so far as, the direction of flow lines and light rays after crossing the line of demarcation of the two media, are of the same type. The uplift pressure distribution along the base of the flush floor, for two typical cases of stratification have been shown in Drawing 11 (b), alongwith the results for a single uniform layer, for comparison.

Investigations[54] of uplift pressures for the case of a flush floor having a horizontal or sloping impervious layer in the foundation at a finite depth have been carried out, and the results are shown in Drawing 12 (a) and (c), for the depth to impervious layer of 1/4 ℓ and ℓ respectively, where ℓ is the length of the floor. Drawing 12(b) shows the percentage change in pressures at various points along the flush floor, due to different locations of the horizontal impervious layer below the floor. The percentage change in pressure indicated is the quantity $(\phi_h - \phi_o)$ expressed as a percentage of ϕ_o, where ϕ_h is the uplift pressure at any point on the floor for the horizontal impervious layer case, and ϕ_o is the pressure for the normal case of homogeneous subsoil medium of infinite depth. The typical results presented in the Drawing 12, alongwith other results similarly obtained, for a simple horizontal floor[54], give a good general idea of the changes in uplift pressures from the normal case of homoeneous infinite subsoil that can be expected due to a finite location of an impermeable layer in the foundation. These results can be applied with reasonable accuracy to obtain the uplift pressures in any specific case of a weir or barrage having an impervious layer in the foundation, if the uplift pressures for the normal case are known, either from electrical models, or from theoretical methods like Khosla's method of independent variables. However, such an application of the simplified results of a flush floor to the actual cases is rather of theoretical interest, since it is not at all difficult to simulate the impervious layer in the two-dimensional model for an actual structure, as already mentioned earlier, to obtain the desired results.

Investigations[55] have also been carried out to study the efficiency of sheet piles, for cases when stratification exists in the subsoil. Some important results of such investigations are shown in Drawing 13. The increase in pressures on the middle point of the upstream face of a sheet pile at the upstream end of a horizontal floor of length l, for different depth ratios $n = \frac{T}{S}$, and ratios $m = \frac{S}{l}$, where S is the depth of the sheet pile and T is the depth to the impervious layer, are given in Drawing 13(a). It is seen that for any fixed value of m (say 0.1 to 0.4) the pressures on the upstream face of the sheet pile show a general increase all through, and that the increase in pressure becomes more and more as n decreases. Drawing 13(b) shows the decrease in pressures along the same floor profile typically for the key point at a distance of $\frac{1}{4}$ l measured from the upstream end of the floor for different values of n. For the case of a floor with two sheet piles of equal depth at the upstream and downstream, the variation in pressures along the floor at important points like $\frac{1}{4}$ l, $\frac{1}{2}$ l, $\frac{3}{4}$ l, and l, measured from the upstream end, due to the change in position of the impervious layer, are shown plotted in Drawing 13(c). A general increase in uplift on the first half, and decrease in the second half of the floor, for values of n from 15 to 2 is noted. For n below 2 the curves show a typical variation.

Investigations for a floor with three sheet piles have also been carried out[55].

It is advisable to study the foundation designs of important weirs and barrages, located on stratified subsoils, by actual electrical analogy models rather than other methods. Certain specific examples of this type can be found in References 56 and 57.

4.1.2. Earth Dams:

The construction of the electrical analogy models of earth dams and embankments are somewhat complicated by the presence of a phreatic surface in the prototype, which is the upper limit in the dam-medium above which no water flows. In an electrical system there is inherently no such limit, because the electrical current flowing through a model has only the limitation of the edges of the conductive medium. It is, therefore, necessary to establish in the model an insulating boundary at a location corresponding to that of the phreatic line, and thus exclude the flow of electricity e from that area in which water will not flow in the prototype.

The shape of the phreatic line is easily determined by experimentation on a hydraulic model, or by graphical methods, and in simple cases by mathematical calculations also. The mathematical solution for determining the shape of the phreatic line is exact for the case of an earth dam composed of homogeneous material, located on a foundation of imperveious material, having a very small angle of the discharge slope, and having no special drainage provisions. The phreatic line for such a case is fundamentally a parabola as established by J.S.Kozeny having the equation :

$$x = \frac{y^2 - y_0^2}{2\,y_0} \qquad \ldots \quad (26)$$

The focus and origin of this parabola both lie at the extreme downstream end of the base line, and y_0 is the intercept of the parabola on the perpendicular at its focus which can easily be determined graphically for any specific case[58]. A.Casagrande[59] has developed the theoretical method for location of the phreatic line even in actual complex sections of earth dams, and has supplemented them with results obtained from graphical studies by flownets.

Even without all this procedure the location of the electrical boundary simulating the phreatic line is established by a process of trial and error on the electrical model itself. Since, the phreatic line is at atmospheric pressure, the only type of head that can exist along it is the elevation head. Therefore, there must be a constant drop in elevation between the points at which successive equipotentials meet the top flow line, i.e. the potential along the top boundary of the model should vary linearly with the vertical coordinate. The electrical model can be made of a semi-conductor like 'aquadag' paper, or more preferably liquid resisting medium model is made which permits the representations of zoning in dams easily as already mentioned in Section 3.1.3. The boundary simulating the phreatic line is made of plasticiene, which permits of easy changes in its shape to enable fixation of correct boundary by trial and error. Once the phreatic line is simulated in the model, the other boundaries are simulated very easily, and potential observations are taken in the usual way using the Wheatstone's Bridge principle. The pressure-net can be drawn easily from the equipotentials thus obtained, by the use of equation (20), for use in accounting pore-water pressures in analysing the stability of slopes for earth dams. A typical flow- and pressure-net for an earth dam has been shown in Drawing 14.

The electrical analogy can also be successfully employed to the study of rapid draw-down of the reservoir in earth dam problems. The draw-down is considered to be instantaneous. In such a case, the phreatic line is first established in the model for a full reservoir condition as mentioned in the paragraph above. Next, the conducting strip at the upstream is taken out and replaced with a number of copper strips, at the same time representing the upstream face below the lowered water surface as an equipotential surface. Such strips are also distributed along the phreatic line established for the full reservoir case. All these strips are given corresponding potentials, which are easily ascertained, feeding them from a separate potential divider. The other electrical connections are clear from the set-up shown schematically in Drawing 15(a). The equipotential lines can be surveyed in the usual manner, and the flow-net completed. From the equipotential-net, the pressure-net can also be drawn in the usual manner. An example of the flow-net and the pressure-net for an instantaneous drawdown case is given in Drawing 15(b).

Lot of investigations on earthen dams, both for steady seepage case and drawdown case, have been carried out using the electrical analogy technique, by various research stations, notable amongst which is the U.P.Irrigation Research Institute,Roorkee[60, 61].

4.1.3. Galleries and Coffer Dams:

Infiltration galleries constructed in permeable soils below river beds mainly with a view to supply water during the dry months, and coffer dams built across river beds for dewatering purposes, are capable of simulation by electrical analogy. Experiments can be conducted to assess the best location of the gallery with respect to the underlying rock. The Drawing 16(a) shows a simple set-up for studies on flow through a gallery[32], and the equipotential and stream lines for flow with the gallery resting directly over an impermeable rock. Drawing 16(b) shows the flow-net for flow through the same gallery when it is raised above the impermeable rock. The two drawings clearly indicate that when a gallery is raised above the impermeable rock the discharge increases.

In case of coffer dams, the simulation is almost similar to that described for earth dams in Section 4.1.2., and the equipotential and stream lines are easily obtained. Here also, the free surface is obtained by either a hydraulic model, or a trial and error method on the model itself. The essential technique of experimentation is about the same. However, it should be noted, that in many cases, the coffer dams do not run right across the river and act as a kind of diversion. In such cases, the flow may no more be two-dimensional.

A typical flow-net obtained by electrical analogy[32] for flow through a coffer dam across a river has been illustrated in Drawing 16(c).

4.1.4. Seepage into Drain Tubes:

Application of the electrical analogy method has been made[62] to study the case of flow into drain tubes embedded in subsoil, especially when the subsoil is stratified. The analog of the problem which can be clearly visualised from Drawing 17(a), is shown in Drawing 17(b). Drawing 17(a) is a vertical section of the subsoil stratum containing the drain tubes, with ponded water above soil surface. D_1 and D_2 are the horizontal circular drain tubes of 6 inches diameter. Other dimensions and details are given clearly on the drawing. The drain tubes are simulated by using diagonally cut cylinders of wood of corresponding dimensions coated with wax, with the semi-circular portion of each wrapped with a thin copper sheet thereafter. Simulation of other boundaries and electrical connections can be easily understood from the Drawing 17(b).

The complete flow-net for a typical case of flow through drain tubes is shown in Drawing 17(C). The numbers on the stream lines are such that the difference between any two of them gives directly the percentage of flow entering the drain between these lines. A number on an equipotential line indicates the percentage of undissipated hydraulic head at points on the line. The results obtained by the analogy have checked well with the corresponding theoretical results of D.Kirkham[63].

4.1.5. Tubewells:

The flow through a tubewell installed in a homogeneous sand stratum is radial, and is symmetrical in all vertical planes passing through it. As such, the flow cannot be classified as two-dimensional. Nevertheless, the principles and the fundamental studies of tubewell simulation have been introduced here for convenience sake, being of an elementary nature. The exact simulation is complicated by the presence of a free surface called the 'cone of depression' in such a system due to the gravitational force. As explained in case of earth dams, the process for determining the free surface is rather a tedious one of trial and error. It has been investigated[32], however, that comparative results for the yield of tubewells can be obtained without serious errors, even if the simulation of this free surface be neglected. The exact simulation of a tubewell has been discussed later in Section 6.2.

based on the technique described in Section 3.1. Even the investigation of flow pattern around three-dimensional bodies in parallel uniform flow is not difficult, and has therefore been mentioned here for convenience sake. The technique of simulation and experimentation is similar to that for the two-dimensional case, and only the construction and erection of the model assembly as shown in Drawing 19(a) is somewhat complicated. Such a set-up used by P.G.Hubbard[15] consists of a large haxagonal plate-glass tank filled with an electrolyte, in which Luceti models of various bodies to be investigated are immersed. Since, the interest of study is mostly confined to conditions along the boundary, fixed electrodes flush with the surface of the model are used for potential measurement, their spacing being measured with the requisite accuracy, usually with a machinist's precision surface gauge or by optical magnification and comparison with a standard scale. The arrangement for holding the model in position in the bath is provided as shown in the Drawing 19(a). A.C. voltage source is connected to the electrolyte by means of sheet copper anodes, the lower anode resting on the bottom of the tank, and an angle iron frame supporting the upper one just below the surface of the electrolyte. The electrical measurements are made by means of a simple potentiometer circuit, using a high impedance milli-voltmeter for null detection. The circuit diagram is almost similar to that shown in Drawing 8(c). It is necessary to make the models large enough, so that small errors in construction would be relatively unimportant, and also the tank should be large enough so that the effect of the outer boundary of the tank would be negligible.

To find out the pressure distribution from the measurements, the spacing ΔS of the electrodes and the corresponding potential difference $\Delta\phi$ are found out. Since, the velocity V is proportional to the rate of change of velocity potential, the following equality

A simple set-up for such experimentation as used by V.I. Vaidhianathan^{32} is shown in Drawing 18. A silver wire held in the centre of a hollow metallic tray with an wooden base and filled with an electrolyte, represents the strainer, any portion of which can be blinded easily by enclosing the desired portion in a rubber sheath. The cylinder boundary is kept at 100% potential and the silver wire at 0% potential, and the current passing through the electrolyte in the tank is measured. The current gives the relative values of yields with different lengths of strainer. The results obtained can give an idea of economy of length of strainer^{32} in relation to its yield. It is essential that the conductivity of the electrolyte should not vary during the course of experiments. Further, the temperature of the electrolyte should not be allowed to increase due to the current passing for a long time. A voltage stabilizer may be introduced in the circuit to keep the voltage to the rated fixed value.

Combination of wires separated apart, simulating a system of wells, can be studied similarly. The actual distance in field at which interference begins to play part, can not be simulated in such experiments. Comprehensive work in this field has been done by Vaidheianathan^{32}.

4.2. BODIES IN PARALLEL UNIFORM FLOW: The analogy for the steady irrotational flow of a fluid having been already established in Section 2.1.4, it is possible to develop electrical analogy models for any such flow under given boundary conditions. The investigation of flow pattern around, or within, a two-dimensional body in parallel uniform flow is very simple, since the analogy model having its boundaries geometrically similar to that of the flow system under consideration is easily constructed and studied, based on the technique described in Section 3.1. Even the investigation of flow pattern around three-dimensional bodies in parallel uniform flow is not difficult, and has therefore been mentioned here for convenience sake. The technique of simulation and experimentation is similar to that for the two-dimensional case, and only the construction and erection of the model assembly as shown in Drawing 19(a) is somewhat complicated. Such a set-up used by P.G.Hubbard[15] consists of a large haxagonal plate-glass tank filled with an electrolyte, in which Luceti models of various bodies to be investigated are immersed. Since, the interest of study is mostly confined to conditions along the boundary, fixed electrodes flush with the surface of the model are used for potential measurement, their spacing being measured with the requisite accuracy, usually with a machinist's precision surface gauge or by optical magnification and comparison with a standard scale. The arrangement for holding the model in position in the bath is provided as shown in the Drawing 19(a). A.C.voltage source is connected to the electrolyte by means of sheet copper anodes, the lower anode resting on the bottom of the tank, and an angle iron frame supporting the upper one just below the surface of the electrolyte. The electrical measurements are made by means of a simple potentiometer circuit, using a high impedance milli-voltmeter for null detection. The circuit diagram is almost similar to that shown in Drawing 8(c). It is necessary to make the models large enough, so that small errors in construction would be relatively unimportant, and also the tank should be large enough so that the effect of the outer boundary of the tank would be negligible.

To find out the pressure distribution from the measurements, the spacing ΔS of the electrodes and the corresponding potential difference $\Delta\phi$ are found out. Since, the velocity V is proportional to the rate of change of velocity potential, the following equality may be written :

$$V = \frac{\Delta\phi}{\Delta S} \qquad \ldots \quad (27)$$

Further, the following dimensionless form of the Bernoulli equation may be written for the change in piezometeric head Δh, considering the head loss in flow being negligible.

$$\frac{\Delta h}{\frac{V_o^2}{2g}} = 1 - \left(\frac{V}{V_o}\right)^2 \approx 1 - \left(\frac{\Delta\phi}{\Delta S}\right)^2 \Big/ \left(\frac{\Delta\phi}{\Delta S}\right)_o^2 \qquad \ldots \quad (28)$$

The quantity $\left(\frac{\Delta\phi}{\Delta S}\right)_o$ is evaluated from measurements between electrodes placed in a region of parallel flow. Pressure Distribution curves are obtained, by plotting the values from equation (28) midway betwen points corresponding to the electrode locations. Typical results showing pressure distribution around a hemispherical head form with a cylindrical afterbody[15] are shown in Drawing 19(b). The results of actual pressure distribution obtained from a water tunnel are also shown, indicating a remarkable coincidence with the electrical analogy results.

For the study of bodies in parallel uniform flow, large-tank electrical analogy testing as described above is most suited, and the sector model technique is not readily applicable, as already discussed in Section 3.3.

4.3. NEGATIVE PRESSURES ON SPILLWAY CRESTS: In the proper design of spillway crests it is essential to determine the negative pressures developed for heads greater than the design head. The electrical analogy technique is possible to be applied for such a case[36], since the flow over a spillway dam with a constant water level in the reservoir is essentially a steady non-uniform two-dimensional irrotational flow, which satisfies the usual La Place's equation. As in the case of earth dams, a free surface in the form of the upper nappe, which is not known before-hand, has to be simulated by trial and error in the model. The procedure followed can be understood from Drawing 19(c).

A geometrically similar model ABC of the spillway dam made from a non-conducting sheet is fixed flat in an electrolytic tray, having a section sheet exposed against its transparent bottom. Thickness CD of the nappe at a point C on the crest is calculated from

$$CD = \frac{q}{\sqrt{2gHc}} \qquad \ldots \quad (29)$$

where, q is the discharge per foot length of the spillway, and Hc is the depth of C below reservoir level. A straight copper strip CD is provided as one terminal. The other terminal EA is provided as an arc of a circle, where E is a point on the water surface of the reservoir at a distance equal to about thrice the head over the crest H, from the upstream face of the dam. The simplified circuit connections are shown in the Drawing 19(c). The upper nappe ED has to be determined by trial and error method by adjusting a very thin perspex sheet passing through E and D, such that equal potential drops along ED are recorded at equal depths below the reservoir level. the observation being facilitated from the section sheet exposed below. Thus, ensuring correctness of the shape of upper nappe, the flow-net can be obtained in the usual way, and the pressures on the spillway profile easily computed. An illustration of a flow-net obtained and the computed pressures on the spillway profile is presented in the Drawing 19(c).

4.4. HORIZONTAL EARTHQUAKE EFFECTS ON DAMS: A recent application of the electric analogy[17, 38] has been made in determination of the magnitude and distribution of the water pressure changes, due to a horizontal earthquake shock on a dam having a face of any shape.

The analogy is made possible because the differential equation of the change in pressure P at any point having the rectangular coordinates x and y, can be expressed as a La Place's equation :

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0 \qquad \ldots.(30)$$

considering water as incompressible, and the flow of water as two-dimensional.

The electric analogy tray model[17] is constructed, as shown in Drawing 20(a), using a sheet plastic electrolytic tray whose length is kept at least three times the width to simulate an infinitely long reservoir. A geometrically similar plastic boundary wound with nichrome wire is used to represent the face of the dam. The bottom of the reservoir is simulated in the tray by installing a copper strip to provide a constant potential. The tray is filled with an electrolyte, and the stream lines are surveyed by means of the usual Wheatstone's Bridge. Equipotential lines are drawn orthogonal to the stream lines to form a flow-net composed of curvilinear squares.

A typical flow-net obtained for such a study is shown in Drawing 20(b), which also shows the values of the pressure coefficient C used in the computation of the pressure change due to earthquake effect at any point from the formula :

$$P = C.\alpha.\ w.h. \qquad \ldots\ldots.(31)$$

where, α is the earthquake intensity, w is the unit weight of water, and h is the total reservoir depth. The pressure coefficient C is determined directly from the nominal value of the equipotential line intersecting the dam. It may be noted, that ~~th~~ in this analogy, the electric potentials represent the streamlines of the prototyped problem, unlike the equipotentials of the usual problems.

On the basis of a series of such experiments with dams having faces of the usual varying shapes, C.N. Zangar[38] has given the values of the pressure coefficients in form of Curves. The values of the pressure coefficients at the base, and the maximum values of the pressure coefficients on the slope of dams having constant sloping faces, are shown in Drawing 20(c).

These studies have been very useful in accounting for the earthquake effects in the design of dam sections.

4.5. FLOW THROUGH SYPHON SPILLWAYS: It is possible to apply two-dimensional electrical analogy to the studies of flow through syphons, since it is essentially a steady non-uniform irrotational flow. Strictly speaking, the flow through syphons is a three-dimensional one, and hence either sector or full three-dimensional models should be used. Nevertheless, a two-dimensional study gives satisfactory information about the flow-net.

Extensive two-dimensional studies on volute syphons have been carried out at the Indian Institute of Science, Bangalore,[51,52] and the lay-out for conducting these studies is shown in Drawing 21(a). The geometrically similar model of volute syphon is made of treated teak-wood and is varnished before installation in a flat electrolytic tray. The full reservoir level end and the outlet end are simulated by copper electrodes. Electrical connections are made as shown in the Drawing 21(a), to correspond to the usual Wheatstone's Bridge net-work. Equipotential lines, and therefrom, the flow lines can easily be drawn and the flow-net completed. The flow-net patterns, thus obtained, enable the determination of pressure and velocity conditions at different points. A typical flow-net obtained by the electrical analogy is shown in Drawing 21(b).

Different conditions of working of syphons and their design features can be simulated and studied with the set-up described above. Interference studies on a battery of syphons can also be carried out, and the approximate safe spacing determined so that each may function efficiently without causing any considerable intereference of flow. Interesting results have been obtained[52], regarding the proportion of the flow into the syphon from the river side and from the dam side, stagnation areas, optimum height of drum, correct shape of volutes, flow in funnels past volutes, etc. On the basis of such studies, the most efficient shape for a volute syphon can be suggested.

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CHAPTER V

STUDIES WITH SECTOR MODEL TECHNIQUE

5.1. DESIGN OF INLETS AND CONDUIT CONTRACTIONS: It has been shown in Section 3.3 that solution of flow problems in three-dimensions having an axis or plane of symmetry, can easily be obtained by taking recourse to sector models. The cavitation-free designs of inlets and conduit contractions have been very much simplified by the use of this technique[13].

Since axial symmetry exists in the flow passage of a usual inlet or conduit contraction, a segmental model for the studies may be easily constructed as described in Section 3.3, with a series of fixed flush electrodes accurately located along the inner face of the flexible strip representing the boundary, for measurement of potentials.

If V denotes the velocity, and E denotes the electric potential, the following approximate equality may be written on the basis of the analogy,

$$\frac{V}{Vo} \approx \frac{\Delta E}{\Delta Eo} \qquad \ldots \quad (32)$$

Also, in a well designed inlet or conduit contraction the loss in head is negligible, and hence the dimensionless form of Bernoulli's equation, given as equation (28) in Section 4.2, is applicable, wherein Δh stands for $(h-h_o)$ - the piezometric head h being equal to the pressure head plus the elevation head. The electrode spacing being exactly known, the determination of the voltage between each successive pair of electrodes, yields through equation (32), the relative velocity, and hence through equation (28) the relative pressure at the mid-point between each pair.

It is important, that all boundary proportions, curves, and tolerances should be carefully controlled to obtain accurate results. Other precautions already enumerated in Section 3.3 should be observed. Also the entire electrolyte must be stirred frequently to avoid concentration gradients because of non-uniform temperature, and each electrode must be cleaned with a fine glass rod before its reading is taken to prevent any corrosion film that might develop on the electrode.

Typical results obtained by H.Rouse and M.M.Hassan[13] for an elliptical inlet profile are given in Drawing 8(d), and those for a cubical-arc contraction profile are given in Drawing 8(e). A series of tests were carried out for families of elliptical boundary curves for inlets and cubical-arc boundary curves for contractions. Based on the results of these tests, design criteria have been provided in form of curves, to enable the selection of transition characteristics for cavitation-free design of inlets or conduit contractions for any given case[13].

An example of the application of the sector model technique to represent a design problem conspicuous by the absence of an axis of symmetry has been given by P.G.Hubbard and S.C.Ling[16], as that of a transition using cylindrical sections from a reservoir to a conduit of square cross-section having four planes of symmetry dividing it into eight segments, as shown in Drawing 22(a). The flow pattern in the entire prototype is determined by a study of the pattern in any one of these segments, bounded by two adjacent planes of symmetry and a section of the transition boundary. In the analogy model, a sheet of plate glass inclined at an angle of 45° to the horizontal, represents a plane passing through the centre-lines of opposite sides of the conduit. The section representing the transition boundary and the equipotential surfaces in the conduit and the reservoir, are simulated in the usual way, and small flush electrodes are inserted in an orthogonal pattern in the plastic section representing the boundary for potential measurement. An electrolyte is poured into the formed sector, its free surface intersecting the glass along a line representing the centre-line of the transition, and intersecting the plastic boundary along a line representing the juncture of the conduit sides. Although the radius R of the cylindrical section [Refer Drawing 22(a)] remains fixed, its ratio to the conduit width B may be varied at will over a wide range simply by adjusting the elevation of the free surface.

Typical results[16] obtained with such a model have been shown in Drawings 22(b) and 23. Drawing 22(b) shows isobars (or isovels) deduced from the potential measurements, together with curves showing the pressure distributions along the centre of a side and along the juncture between two sides, for one particular relative radius of curvature. The curves show quite clearly that the most critical region is along the juncture just upstream from the point at which the transition curve becomes tangent to the straight conduit. Drawing 23(a) gives a family of curves showing the variation in pressure distribution along the juncture for various relative radii of curvature, indicating quantitatively the benefits achieved from decreasing the curvature. This trend is more clearly brought out from Drawing 23(b), in which the minimum pressures along the juncture and along the centre-line of a side are plotted against the relative radius of curvature. This drawing indicates, that tendency towards cavitation decreases indefinitely with increasing radius of curvature.

5.2. STUDY OF FLOW THROUGH INTAKE TOWER OF BOULDER DAM: One of the very first applications of the sector model technique was done by the engineers of the U.S.B.R.[12], for investigating the head-loss in the trash-rack structures of the intake towers of the Boulder Dam. No practical way was found to do this on a hydraulic model of reasonable size.

The layout for the electrical analogy sector model is shown in Drawing 24(a), which also shows a section on the centre-line of the tower. A radial section of the tower is constructed of a perfectly non-conducting material, and is cemented to the bottom of an electrolytic tray which is mounted on levelling screws. A salt solution representing the water space of the prototype is filled in the tray, and by adjusting the levelling screws to tilt the tray, solution depth of zero value at the centre-line of the tower is secured as shown in section AA of Drawing 24(a). Copper plate boundary electrodes simulating the 100% potential and 0% potential surfaces are easily installed, except for the case when functioning of both gates is simulated. In such a case, the relative proportion of discharge through the gates in the prototype should be simulated truly in the model. The required data of the distribution of flow through the two gates is collected from a hydraulic model. The simulation in the model is accomplished by a corresponding division of the electric current, secured by adjusting the position of the electrode D [Refer Drawing 24(a)] by trial and error. The equipotential lines are traced as usual by the Wheatstone's Bridge principle. From the pattern of equipotential lines, flow-nets may also be drawn. The main purpose of drawing the flow lines is to know the direction of flow, since the three-dimensional flow-net loses the usual significance of a two-dimensional net as already discussed in Section 2.2.3, wherein it has also been mentioned that the velocities in these three-dimensional nets are possible to be determined by a simple but laborious method[12]. The greatest source of error in obtaining velocities in a three-dimensional flow-net of this type is not in the computations but in the construction of the net. It is usually necessary to make one or two assumptions before attempting to draw the net, and after these are made and the net commenced it is still necessary to use a certain amount of judgement.

Two typical results obtained[12] with the sector model of the Intake Tower as above described have been shown in Drawings 24(b) and (c) respectively. Experiments were conducted with different lengths of trash-racks, and the potential distributions obtained were studied, which finally enabled the adoption of an economic trash-rack length.

5.3. STUDY OF FLOW THROUGH WELLS: Investigations[14] have been carried out regarding the shape of the free surface or the 'draw-down curve' in the case of unconfined radial flow into a well. It is usual to assume that the functional relationship derived by J.Dupuit, as given below, connects the water level inside the pumped well to the normal groundwater table.

$$Q = \frac{\pi K}{2.3} \times \frac{h_e^2 - h_w^2}{\log \frac{r_e}{r_w}} \qquad \ldots \quad (33)$$

where, Q is the discharge, K the permeability coefficient, h_e the depth of water bearing aquifer at the outer boundary, h_w the depth of water in the well, r_e the radius of circle of influence, and r_w the radius of well—h_e and ~~xxx~~ r_e being considered as terms applicable for any general point for plotting. However, the investigations have revealed, that the actual free surface is different from that given by the Dupuit's relation in a certain small zone near the well.

As mentioned earlier in Section 1.4, there is no force which acting on flowing electricity will produce a free surface, as the action of gravity does on flowing water in gravity systems. By some device, the potential should be made to vary linearly in the electrical model at the free surface. This is accomplished by the usual trial and error location of the boundary of the free surface.

A pressed carbon sector model, machined to the required dimensions, was used by H.E. Babbit and D.H.Caldwell[14]. A 6-volt battery for supplying the current for testing, rheostats for adjusting the current, an ammeter, and a potentiometer for measuring the equipotential lines, complete the set-up as shown in Drawing 25(a). The inflow equipotential line is represented by the conducting strip DF, and the constant potential outflow line by the conducting strip C.H. A resistance strip CK is placed along the face of the well above the elevation of the water in the well, the terminals of which are kept at the potentials of the two boundary strips respectively. The rheostat X_2 is provided for adjustment to give sufficient current through the model to produce a voltage drop of about 0.2 volts. The other rheostat X_1 is used to adjust the potential at K to be of same value as that of D. ~~xRx~~ [Refer Drawing 25(a)]. The total voltage drop is measured, and equipotentials plotted on one long surface of the wedge-model, as usual. From this first plot, approximation of the free surface is found by marking the point on each equipotential line which indicates the same fraction of the total drawdown as the corresponding voltage of the equipotential line is of the total voltage drop. By cutting the model along the line so obtained, and repeating the process until no further change is warranted, the final free surface is located. The final free surface can be located in every case in three approximations at the most.

The actual shapes of the free surface curves may thus be obtained for various drawdown and for various ratios of the depth of water stratum to the diameter of the circle of influence. Two typical examples of free surfaces[14] located by this method are shown in Drawing 25(b).

A new conducting material made of gelatin, glycerin, and water, with salt added to make it an electrical conductor, was used by F.W.Opsal[18] to determine the free surface around a gravity well partially penetrating a saturated sand layer. The material consisting of 15% gelatin and 15% of glycerine by weight of water used in the mix with beta naphthol added to prevent decay, was used to form the wedge-shaped model in a specially made box. The experimentation and results obtained are similar to those of pressed carbon models described above. An advantage in the gelatin models is that the surface of the model can be cut very easily with a hot wire to simulate the free surface. Also, the material is easy to mix and form and is very economical, because it can be remelted and poured in new models several times.

Tests were also made with this gelatin material using a simple set-up[18] to determine the potential distributions around an artesian well, varying the penetration of the well,—a typical result for a 6 inch well - probe penetration in a total depth of homogeneous medium of 10.25 inch having been shown in Drawing 25(c). Large potential loss close to the well is observed for all cases. It is observed that for a very shallow well penetration in the layer, the equipotential curves close to the well are spherical, which become increasingly straightened out as the distance from the well increases. Also, for full penetration, the equipotential lines are straight and vertical.

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CHAPTER VI

APPLICATIONS TO THREE-DIMENSIONAL PROBLEMS

Applications of the electrical analogy using full three-dimensional models are given in this chapter. In these applications, the simplification of using section or part-models is not possible to be introduced, as such, they represent the most important applications of the electrical analogy technique.

6.1. HYDRAULIC STRUCTURES ON PERMEABLE FOUNDATIONS: In the case of a long structure, like a weir or a barrage across a river, the subsoil flow is mainly in one direction only, as the flow in a direction parallel to the axis of the works is negligible—being confined only near the two ends. The problem in such cases only can be treated as two-dimensional, excepting for the two end regions. In all other cases, mostly encountered with hydraulic structures, the subsoil flow is a three-dimensional one, and hence only three-dimensional electrical analogy technique is applicable.

The author has been associated with most of the results and studies described in this section.

6.1.1. Specific Example of Jagbura Syphon:

The details of constructing and setting up a full three-dimensional electrical analogy model of a hydraulic structure having already been presented in Section 3.4, an illustration of the results obtained in a specific case of Jagbura Syphon[28] across Sarda Main Canal in the North—Central part of Uttar Pradesh are presented hereunder. Incidentally, the study was the first of its kind to be conducted in India by the three-dimensional electrical analogy.

The salient features of the Jagbura Syphon have been shown in Drawing 26. During three-dimensional electrical analogy model investigations, it was found that almost all the subsoil flow in normal conditions of running, took place from the canal into the river on the two sides of the structure – there being no portion having the so-called two-dimensional flow conditions. Drawing 27 gives the equipotential contours and the flow-net obtained by two-dimensional electrical analogy technique, and Drawing 28 gives the equipotential contours considering the problem as three-dimensional. It was found that the two-dimensional approach gave considerably higher uplift pressures, than those determined by the three-dimensional method. The structure was found to be absolutely unsafe when checked by normal procedures of design, and the fact that the structure had stood the test of time suggested marked existence of three-dimensional flow conditions, and the necessity of three-dimensional model experiments rather than the two-dimensional ones. The total uplift pressure obtained by two-dimensional and three-dimensional approaches was found to be 80.04 million pounds and 55.85 million pounds respectively, against 65.59 million pounds total downward load of the main barrel portion of the structure. Thus, in this case the total uplift pressure found by three-dimensional approach was 32% less than that obtained by two-dimensional approach. Suitable suggestions for remodelling the structure to pass an increased discharge of the canal were framed based on such three-dimensional model studies[28].

The application of the three-dimensional electrical analogy to the probelm of remodelling of Jagbura Syphon described above, clearly brought out the usefulness of this technique.

6.1.2. Importance of Representation of Subsoil Water Mouand

While working on the problem of Jagbura Syphon described in Section 6.1.1, it was observed that the boundary conditions of the subsoil water mound affected the uplift pressure distribution considerably. This mound gradually builds up in the subsoil between the bottom of the structure and the groundwater table which exists there; its shape and size depending on local conditions, calling for certain assumptions to be made for its representation on the model.

An approximate method has been used by the author for the location of the boundaries of the saturated subsoil mound, since it is very difficult to locate phreatic surfaces accurately on three-dimensional models. For this purpose, the effects of the subsoil water mound are neglected first, and observations are taken on the model as such to obtain the residual uplift pressures at salient points on the boundaries of the model nearest which the subsoil water mound boundary is to be represented. Next, assuming the excess head of water (with respect to the groundwater table level there) to be dissipated, at a gradient varying from 1:6 to 1:12 according to the soil conditions of the prototype, the required boundary is obtained, beyond which copper plates are installed in the model to represent the groundwater table. If desired, the boundary of the subsoil water mound can be simulated accurately on the model by systematic trials as above, usually numbering not more than three, using in the next trial the boundary as determined approximately in the previous trial. However, such an accuracy in representation of the subsoil water mound boundaries is not called for, in view of the results of investigations made to study the effect of size of subsoil water mound on the uplift pressures.

It was found[64], that so long as subsoil water mound is depicted on the models with certain reasonable assumptions for its side slopes (e.g. 1:8 for loamy soils), it does not matter appreciably whether the side slopes as existing in the field are somewhat different from those represented. The essential condition is the desirability to depict the subsoil water mound boundaries on the model with certain reasonable assumptions for its side slopes, as its effect is considerable.

6.1.3. Necessity of Three-Dimensional Approach in Design:

The investigations conducted on Jagbura Syphon model[28] struck a note of caution against treating all problems of subsoil flow under hydraulic structures, as two-dimensional. The designer of today, usually treats all hydraulic structures on permeable foundations as only two-dimensional problems. It, therefore, became necessary to point out the inherent weaknesses of the approach. With that aim in view, studies on four important canal headworks[46] of Uttar Pradesh were carried out. Results of studies on Ganga Canal Headworks, Sarda Canal Headworks, and Proposed Ramganga Feeder Channel Headworks, have only been illustrated here.

The Drawings 29 and 30 give the salient features of the Ganga Canal Headworks, and Drawings 31 and 32 show the equipotential contours obtained for the two extreme cases of normal pond in the river with the canal running at full sypply, and high flood in river with the canal closed, respectively. From Drawings 31 and 32, visualising flow lines orthogonal to the equipotential lines, it is clear that the flow of seepage water in the undersluices area and the adjoining canal head regulator area is essentially a three-dimensional one, and not a simple two-dimensional one taking place from upstream to downstream, as considered usually for design purposes. Uplift pressure contours obtained by two-dimensional theoretical considerations (Khosla's method) for the undersluices area have been shown in Drawing 31, for affording a ready comparison with the uplift pressures obtained by three-dimensional electrical analogy. It will be observed from this drawing, that the uplift pressures on section MM are the maximum as obtained experimentally for the area. The uplift pressure lines as obtained by the two approaches for this section MM, have been shown in Drawing 30. Even for this section, the uplift pressures obtained by the three-dimensional approach are approximately 10% lesser at the upstream toe of the floor, 4% lesser at the middle of the floor, and 7% lesser at the downstream toe of the floor, than those obtained by the Khosla's method. At other places in the undersluices area the extent of release of pressures is still more, as can be easily seen from Drawing 31. Thus, the uplift pressures obtained for this region by Khosla's method, or any other two-dimensional approach, is bound to be erroneous.

Similar results for the Sarda Canal Headworks, whose salient features are presented in Drawings 33 and 34, are shown in Drawings 35 and 36 respectively. Also, similar typical results obtained for the Proposed Ramganga Feeder Cahannel Headworks, whose plan and sections are shown in Drawings 37 and 38 respectively, are given in Drawings 39 and 40 respectively. The results presented for these two structures also corroborate the general conclusions regarding uplift pressures and the three-dimensional nature of seepage flow for the end-region, derived from the results of studies on Ganga Canal Headworks model, as discussed in the foregoing paragraph.

These studies conclusively proved that the hydraulic structures on permeable foundations that are built in connection with various canal projects are all three-dimensional problems excepting for long weir or barrage portions, as far as subsoil flow conditions are concerned; and that considerable savings in floor thicknesses of normal canal structures are possible if designed according to this approach.

6.1.4. A Case Involving Adverse Subsoil Water Mound Boundaries:

In cases of structures where adversely high subsoil water mound boundaries are encountered, the floor thicknesses obtained on the basis of three-dimensional approach are greater than those obtained by the normal two-dimensional approach, since, broadly speaking, it is not possible to account for the subsoil water mound effects in the two-dimensional model. An interesting investigation[6] of this nature was carried out in testing the tentative design of the Nanak Sagar Barrage situated in District Naini Tal of Uttar Pradesh. The details of the tentative design tested are given in Drawing 41, and a typical result of the studies is given in Drawing 42 for the case when reservoir is full, the intermediate filter provided in the structure is fully effective, and the downstream water level is at the spring level elevation in the vicinity of the structure. The general lay-out of the three-dimensional electrical analogy model of the barrage is shown in Photograph 3.

Drawings 43(a) and (b) show the uplift pressure distribution on a section through the centre-line of the structure for all the studies conducted on the model. The uplift pressures based on two-dimensional approach for the design of the structure have also been shown on the two Drawings 43(a) and (b), for sake of comparison with the experimentally obtained values. These two drawings clearly show that the two-dimensional approach applied to this structure has given lower values, than those obtained experimentally on the three-dimensional model. It was found, that the floor thicknesses provided in the tentative design in a certain reach of the structure, on consideration of two-dimensional theories, are likely to become unsafe w when the adversely high spring level is established in the vicinity of the structure due to the filling up of the reservoir to the pond level.

The behaviour of uplift pressures in this and other similar studies with[66,67,68] which the author has been associated, was found to be contrary to that in normal canal structures with low groundwater table in the vicinity of the structure, as described in Sections 6.1.1. and 6.1.3, where two-dimensional methods give uplift pressures that are considerably higher than those obtained by actual three-dimensional models. Thus, all hydraulic structures like syphons, silt extractors, super-passages, regulators, falls, and power houses etc., built along any canal system can be categorised[46] under the following tw ~~te~~ design conditions :

i) Where the general groundwater table (or the maximum subsoil water saturation elevation) is sufficiently low in the vicinity of the structures, resulting in lower uplift pressures under the floors than normally calculated. Majority of the structures would fall in this category.

ii) Where the general groundwater table (or the maximum subsoil water saturation elevation) on one or both sides, is higher than the full supply level of the canal or the off - taking channel. A few special cases would fall in this category, and the effect would be of higher uplift pressures under the floors in certain portions than normally calculated.

Both the above categories call for a three-dimensional approach, one giving savings in floor thicknesses, while the other requiring increased thicknesses.

6.1.5. Subsoil Flow Criteria for Design of Barrel Portion of Syphons:

Model experimentation with the three-dimensional electrical analogy technique can be justified for use in design of important structures, but for simpler structures like canal syphons having more or less clearly defined boundary conditions, a great necessity is bound to be felt to have a method by which the uplift pressure distribution could be calculated expeditiously and in a rational manner. As such, extensive studies[69] were carried out on three-dimensional models of canal syphons with a view to prepare a simple design criterion of calculating these pressures for normal sizes of structures encountered in actual practice, and to improve the understanding of uplift pressure distribution under the main barrel portion of such structures.

Drawing 44 shows a comparison of the simplified models used in the studies and the prototype structures. The general lay-out of a typical Syphon model designed is shown in Photograph 4. Typical uplift pressure contours obtained for the two extreme cases of short and long structures, with overall length L divided by width of structure W ratios of 2 and 4, are given in Drawings 45 and 46 respectively. It will be observed from these drawings, that the pressures are rapidly released into the flanks of the structure in the river bed from both upstream and downstream canal sides. If the designer, working with the two-dimensional approach, is able to appreciate the possibility of this release of pressures on the river sides of the structure, he would most probably consider an imaginary two-dimensional structure having a foundation profile same as that along the Canal-River centre-line, for the application of the method. The Canal-River centre-line is the line PQLM, which goes along the canal centre-line upto a point equidistant from both ends of the structure, and then turns at right angles towards the river. The uplift pressures obtained by the application of Khosla's method for the foundation profile along this Canal-River centre-line have been indicated in the Drawings 45 and 46 for the resepective cases, for comparison with the actual uplift pressure line obtained by the three-dimensional approach. The uplift pressures obtained by the three-dimensional approach are on an average, 8% and 25% less respectively than those obtained by two-dimensional approach, for the two extreme cases of short and long structures mentioned above. For other sizes of structures lying within this range, the percentage releas will have some intermediate value.

Many such results obtained from the three-dimensional models were analysed, and a dimensionless grid of upto twenty one design points was proposed, as shown in Drawing 47(a), and a family of design curves provided for each of these points. Design curves giving values of uplift pressures at two typical points H and N of the proposed grid have been shown in Drawing 47(b) and 48 respectively. With the help of dimensionless ratios obtained by dividing the overall length of the structure L, and the head acting H, by a reference dimension W – the width of the barrel portion, expected pressures can be very easily worked out ~~by~~

by means of these design curves for all the proposed grid points distributed on the barrel area of any canal syphon. Thus, a complete picture of the uplift pressure distribution can be obtained by the designer, without taking recourse to otherwise costly experimentation. Drawing 49 gives a typical solved example based on these design curves[69]. Considerable savings are possible, if the syphons are designed on the basis of uplift pressures obtained from these design curves.

6.4.6. Dams and Appurtenant Works on Complex Foundations.

The first difficulty that confronts in setting-up of such a model is to obtain a suitable material, which can be moulded into any shape of required dimensions, and at the same time possess adequate resistance and sufficient stability of form. The resistance must be constant and homogeneous throughout the model. In addition to this, the degree of conductivity and uniformity should not change during the course of experimentation. A mixture of a conductor and a non-conductor may solve this difficulty, but such a mixture is not easy to be designed as even a small layer of non-conductor around a conducting particle in which it is suspended alters the conducting nature, and the mixture becomes a non-conductor and vice versa. In U.S.S.R.,[11] a method had been developed of dispersing fine graphite of almost colloidal dimensions in talcum powder, powdered marble, or fine quartz, using a just sufficient quantity of distilled water to make the material cohesive. The proper moisture content, the optimum quantity of graphite to be added, the variation of resistance by compaction, the estimation of the uniformity of the mass, evaporation and setting of the mass, correct plasticity to maintain the free surface, and conductivity of the material, have to be investigated and properly tested. The actual technique of making this material, and preparation and moulding of the three-dimensional models therefrom, are quite tedius.

The model is made of the material indicated in the foregoing paragraph, having electrical properties corresponding to the hydraulic properties of the prototype material, taking a fixed datum elevation. A number of electrical piezometers are inserted at various points in the plastic mixture to measure the electric potential, using circuit connections similar to that of other three-dimensional models as already described in Section 3.4.3. The piezometers used are insulated wires sharp and bare at the tip. In such models the current has to pass through the whole mass and not at the surface only. Hence, high frequencies of currents are not desirable as they will try to pass at the surface only and thus introduce varying resistance. Low frequency A.C. at a potential of 15-20 volts is used, therefore, in the three-dimensional mass.

Flow characteristics under dams on complex foundations can be investigated in this manner, provided a suitable material for setting-up the model can be made. Such a model can give for any condition, a complete and quantitative picture of the flow and pressure lines, and further it can give the seepage through the dam knowing the permeability. An approximate theory for this purpose has been worked out by B.F.Reltov[11], and he gives the formula :

$$Q = \frac{K.H.n.\rho.I_n}{E_n} \qquad \ldots \quad (34)$$

where, K is the coefficient of permeability,

H is the head causing seepage,

n is the scale number of the model,

ρ is the specific conductivity of the mass,

I_n is the current flowing through the mass,

and, E_n is the electric potential causing the flow of current in the mass.

A number of such three-dimensional model work had been carried out at the Scientific Research Institute of Hydrotechnics, Leningrad (U.S.S.R), for various percolation studies in homogeneous as well as heterogeneous soils. The main purpose of these investigations[11] consisted in the determination of the general characteristics of groundwater flow at the site of the complex structures studied. The method has also enabled the estimation of the effect caused by a variety of geological conditions e.g. layers of varying permeability, presence of artesian water, etc..

Recently, a similar work has been done at the Central Water and Power Research Station[29], Poona, in connection with the seepage studies of Kotah Barrage foundation, using the simpler wax models rather than the more complex graphite models.

Drawing 50(a) shows the features of the Kotah Barrage earth dam and its foundations. An interesting feature of the foundation of the barrage is the presence of a thick clay tongue of an assumed semi-infinite extent both upstream and downstream, protruding into the gravel bed as shown in the drawing. The design of the barrage provides a heavy clay blanket, a generous rock toe, and a system of bleeder wells, as shown in Drawing 50(b).

The three-dimensional model is constructed by carving out the rock contours of the foundation in a wax - mass. Masonry core walls, rolled clay, and clay tongue, are suitably represented.[29] Filter wells are simulated by copper wires of corresponding lengths. The rest of the model set-up is as usually provided for the three-dimensional models, as described in Section 3.4. Fixed piezometers for the portion of foundation covered by clay tongue, and movable probe for the portion not covered by the clay tongue, are used for potential measurements in the usual way.

A typical flow pattern obtained from the experiments on the model has been shown in Drawing 50(c). The extent of uplift pressure release and the corresponding patterns of seepage flow, above and below the clay tongue, were studied for various pressure relief arrangements. It was observed that the potential drop in all cases is mainly concentrated in the limited region of the tunnel formed by the clay tongue and the partial cut-off. Comparative idea of the total amount of seepage under various conditions was had by corresponding current measurements. Important results for the proper design of devices for relieving the uplift pressures in the foundations, and reducing the seepage flow were obtained from the model.

6.2. SIMULATION OF A TUBEWELL, AND WELLS FOR DEWATERING FOUNDATIONS:

A tubewell can be simulated simply by a thin conducting wire, usually a silver wire, as already described in Section 4.1.5. This simple simulation does not attempt the representation of the cone of depression, and hence is only approximate.

6.2.1. Membrane Analogy in Tubewell Studies.

Simulation of the free surface developed in case of a tubewell can be attempted rather exactly, by the use of membrane analogy[44]. It ~~is~~ has been shown, that for small slopes, the surface of a membrane can be expressed in cylindrical coordinates as

$$\frac{d^2z}{dr^2} + \frac{1}{r}\frac{dz}{dr} + \frac{W}{T} = 0 \qquad \ldots (35)$$

where dz is the deflection at a radial distance dr from a central deflecting point, W is the weight of the membrane per unit area, and T is a uniform membrane tension. In cylindrical coordinates, the La Place's equation can be expressed as:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} + \frac{1}{r^2}\frac{\partial^2 h}{\partial \theta^2} + \frac{\partial^2 h}{\partial z^2} = 0 \qquad \ldots \quad (36)$$

where, h is the hydraulic head, and (r, θ, z) are the usual cylindrical coordinates of any point. In cases of steady axially symmetrical flow of an incompressible fluid, this equation reduces to :

$$\frac{d^2h}{dr^2} + \frac{1}{r}\frac{dh}{dr} = 0 \qquad \ldots \quad (37)$$

The equation (37) is applicable to well flow in an ideal confined aquifer, and is a good approximation for the flow in an unconfined aquifer if the drawdown of the well is small. Considering T to be large in comparison with W, the third term of equation (35) becomes very small, and the membrane equation approximates the equation (37) making the analogy operative between the two systems.

For studying the shape of a free surface around a well, a rubber membrane is clamped under uniform tension over a circular boundary in a horizontal position, and a central probe representing a pumping well is used to deflect the membrane. Meausred deflections closely approximate the theoretical semi-logarithmic drawdown relationship, although small systematic deviations due to gravity are noted. The membrane profile agrees very well with the actual free surface profile, where the drawdown is not large.

6.2.2. Combined Membrane and Electrical Model of a Tubewell:

C.H.Zee, D.F.Peterson and R.O.Bock[19] studied the radially symmetrical unconfined flow to a well, using the membrane analogy for the free surface in conjunction with the electrical analogy for hydraulic flow.

The basic apparatus used by Zee and others consists of an open-topped metallic cylindrical tank, a rubber membrane assembly, and a simulated well, as shown in detail in Drawing 51(a). The metallic tank has a non-conducting floor of plywood waterproofed with paraffin. The uniform and homogeneous membrane consisting of ordinary nursery rubber sheeting is mounted on a metallic ring. The rubber sheet is clamped to the ring under tension, providing cushioning with strip rubber. The membrane assembly is then suitably supported from a cross-frame mounted over the tank as shown in the Drawing 51(a), enabling the ring and membrane to be raised and lowered, as desired. To eliminate any difference of pressure between the two sides of the depressed membrane, the electrolyte is placed on both sides taking care that the current from the tank walls may not flow through the solution above the membrane. The surface profile of the depressed membrane is accurately measured by a gauge, to know the shape of the cone of depression of the prototype.

The well is fabricated of a small diameter plastic rod, on which a helix of nichrome wire of suitable size and specific resistance is wound-the wire being seated in threads cut on the rod allowing certain clearance between the adjacent turns of the wire. This arrangement makes possible the potential to drop uniformly along the length of the rod. Prior to winding the wire on the plastic rod, a narrow slot is milled lengthwise in the rod, and a fitting brass tube is placed in the slot which can be slided in the slot to a height representing the water level in the well. By thus shorting across the winding, the constant potential section of the well is reproduced.

For the measurement of the effective radius of the well model constructed as described in paragraph above, a simple indirect method is used. For this purpose, constant potential is maintained along the full length of the well, and a radially symmetrical polar potential field is imposed in the surrounding electrolyte. It is known that such a field has logarithmic distribution radially. By measuring the potential along a radius, this distribution is obtained, and the results extrapolated to correspond to a radius having the same potential as the well. This gives the effective radius.

Electrical connections are made as shown in Drawing 51(b). In the circuit two power sources are required. One source provides the current I_w, through the adjustment of resistance R_1, to maintain the required potential distribution along the well. The other source provides the current I_T, which passes through the electrolyte, and is analogous to the hydraulic discharge. The potential drop between any two points is measured with the help of a potentiometer, as usual. Values of I_w and I_T are calculated from measurements of potential drop across the resistance R_2 and R_4 respectively. The potential field in the electrolyte is surveyed using an insulated probe with its tip only open, inserted through holes in the bottom of the tank fitted with copper seal caps to prevent leakage.

A typical example of a radial flow-net obtained with the set-up has been shown in Drawing 51(c). Also shown on the drawing is the location of the free water surface, as computed using the Dupuit's equation (33). It is clear that the agreement of the experimentally obtained free surface with the theoretical one given by Dupuit, is good at fairly large distances, while larger deviations are observed near the well. Data for a number of simulated cases was readily obtained[19] using the set-up described above.

The results in conjunction with the field data collected have been used to develop empirical curves, showing the general relationships between geometric and flow variables expressed in dimensionless form.

The membrane analogy is particularly adaptable to studies of multiple well systems[70] simulated by a group of probes, and of complex boundary conditions controlled by the membrane frame. A large amount of information may be obtained rapidly and inexpensively using this technique.

6.2.3. Design of a Well System for Dewatering Foundations:

The approximate simulation of a tubewell simply by the use of a thin conducting wire as mentioned in 4.1.5, enables a satisfactory determination of a well system which will effect the desired lowering of water table in a certain area for the purpose of laying the foundations of important structures.

For the design of such a well system, a three-dimensional model is set-up in a well insulated tray of circular or elliptical shape depicting all the boundary conditions, e.g. the presence of clay layer in the subsoil, the impermeable floors of any adjoining structure, etc. The size of strainers and any interposing blind pipes to be represented are fixed by taking into consideration the log charts of the subsoil of the locality, and taking an assumed value, based on experience, of depression of the groundwater table at the tubewell due to pumping. The determination of the well system required in a certain dewatering problem being essentially a process of trial and error, the first approximation of the number of tubewells and the locations to be tried in the model are determined theoretically on simplified assumptions.

For the simple case of a number of wells placed on the circumference of a circle of radius A, mathematical treatment has been given by F.L. Plummer and S.M. Dore[71], who give the following formula for the determination of the number of wells :

$$h_0 = \sqrt{y^2 + \left(s_0 r_0 \log_e \frac{A}{n r_0}\right)^2} - r_0 s_0 \log_e \frac{A}{n r_0} \quad \ldots\ldots(38)$$

where, h_0 = depth of immersion in each well in ft.,

y = H - L, where H is the total depth from base of immersion to the initial spring level, and L is the depth to be dewatered,

s_0 = the slope of hydraulic gradient just outside the well,

r_0 = radius of the well,

and, n = number of wells required.

Also, they have given the following formula for estimating the approximate total discharge, Q, to be handled in the dewatering operations:

$$Q = n.X.\left[\sqrt{y^2 + \left(s_0 r_0 \log_e \frac{A}{n r_0}\right)^2} - r_0 s_0 \log_e \frac{A}{n r_0}\right] \quad \ldots\ldots(39)$$

where, X is equal to the capacity of a well per unit of immersion depth. For the case of wells not actually situated on the circumference of a circle, the only approximation possible is to take ~~a~~ A as the radius of a circle whose area is equal to the plan area inside of the wells. It may be remembered that the mathematical solution for any specific dewatering problem encountered at site, using a non-geometrical pattern of arrangement of wells, is only very approximate.

The well system determined theoretically, as mentioned in the foregoing paragraph, is simulated in the model by silver wires of the proper gauge shielded by rubber valve tube to represent the blind pipe. The groundwater table is represented by ~~proving~~ providing a vertical copper plate at the perimeter of the model tray, kept well insulated from the masonry sides. The boundary plate is given

100% potential, and the silver wires representing the tubewells are given 0% potential, or other intermediate values of potential depending on the elevation of the assumed water level in each well. The tray is filled with tap water, as usual, to a depth representing the depth of the homogeneous water bearing medium. The potentials at suitable grid-points of the model, distributed in the area corresponding to the one to be dewatered in the prototype, are measured out using the Wheatstone's Bridge principle, and the equipotentials drawn to see whether the desired dewatering has been effected by that arrangement of wells or not. In case the desired dewatering has not been effected, the arrangement of well points is changed to some other suitable arrangement guided by past experience of working on such type of problems, and the experiment repeated till ultimately we get all equipotentials in the area to be dewatered of a lower value than that required for the desired dewatering. The general disposition of the set-up for solving the dewatering problems is shown in Drawing 52.

In order to get an idea of the probable discharge likely to be handled in any dewatering problem, it is essential that the current flowing through the electrical model be calibrated with the help of an actual field pumping test. This field pumping test enables the estimation of field permeability, and therefrom, the current flowing in the electrical analogy model can be calibrated to prepresent the pumped discharge.

It will be noted, that no attempt is made to represent the various cones of depression, and thus the representation is only approximate. Furthermore, the assumed value of depression head in each tubewell in the system may be appreciably different from the actual value obtained in the field. Also, as usual, the water bearing medium simulated by water (the electrolyte) in the model is assumed perfectly uniform and homogeneous, which is rarely the case in the prototype. Inspite of these limitations, it has been found by observing the prototype behaviour in seoveral cases, that the system of wells designed with this approximate representation have come off to be very satisfactory. In fact, an attempt can be made to represent the various cones of depression by using membrane analogy as mentioned in Section 6.2.2, but this simulation complicates the experimental set-up too much and is not worth all the trouble in view of the experience gained so far with the approximate representation. However, from an academic view point, this aspect leaves an interesting field of experimentation and research.

As a typical example, an arrangement for dewatering the barrage bays Nos. 1, 2, and 3, of the Proposed Ramganga Feeder Channel Headworks[72] has been shown in Drawing 54, the foundation plan of the headworks having been shown in Drawing 53, and the general plan and section in Drawings 37 and 38 respectively. Drawing 54 also shows the expected groundwater level contours in the area, that shall be obtained when the proposed dewatering system is put into operation.

6.3. DESIGN OF RELIEF WELLS: Performance of pressure relief wells or pipes installed in hydraulic structures can be studied with the electrical analogy technique, or vice versa a system of such relief wells can be proposed to minimise the uplift pressures to a safe value in case of any hydraulic structure on permeable foundations subjected to excessive uplifts otherwise. The essential technique of experimentation is the same as described for tubewells in Section 6.2.

Recently, interesting three-dimensional studies were carried out[73] for the design of a system of relief wells for Sarda Sagar

Dam of Uttar Pradesh. Three-dimensional electrical analogy model was made of ½ inch thick perspex sheets, representing three different strata of the foundations by three absolutely watertight compartments filled separately with electrolytes of the corresponding conductivities, as shown in Drawing 55(a). These compartments were interconnected electrically, by means of a number of copper leads 1 inch apart passing through each plastic sheet. The seepage face was represented by a copper sheet fixed at the bottom of the top plastic plate, and the borrow pits on the upstream by the continuation of this copper sheet into the layer. The wells were represented by 17 gauge copper wire, the blind portions being represented by insulating corresponding lengths of these wires. Observation points installed in the three different layers at the desired depths were brought to the top of the uppermost plate for recording poetntials using a simple circuit based on Wheatstone's Bridge principle. Potentials were measured for arrangement of wells of different percentage penetrations, and different well spacings. Based on the results of these experiments[73], a relief well system was designed for the Sarda Sagar Dam, which has since been installed at site and has been found to function very satisfactorily.

As a matter of interest, studies were also carried out for a wide range of permeabilities in the different pervious strata of the subsoil. Comparing the results obtained with those calculated by available theoretical design criteria, it was found that the pressure relief as shown by the electrical models is less for smaller well spacings and more for greater spacings than the pressure relief calculated theoretically - the difference being more pronounced for decreasing well penetrations and for greater difference in permeability of the pervious strata.

Apart from the above studies, investigations[74] were carried out subsequently on the same model with different well diameters, to determine the effect of percentage penetration, spacing, and well diameter, on the pressures midway between wells. Since screen effects and drainage conditions at exit and entry could not be accounted for in the electrical models, recourse was also taken to hydraulic models, enabling thereby the verification of validity of the electrical analogy results also. Typical results of investigations are shown in Drawing 55(b), which are self-explanatory. It is easy to conclude from the results presented in Drawing 55(b) that the size of well has little effect on the percentage increase of midway pressures as a result of change in spacing or penetration, and that closer spacing is more effective in reducing midway residual pressures than increasing penetration. Lesser penetration is also desirable from economic point of view, since the cost of boring increases as it is carried deeper.

6.4. SEEPAGE FLOW PATTERNS THROUGH AND UNDER GRAVITY DAMS: Investigations of seepage flow patterns in a concrete gravity dam and its foundations have been carried out by the electrical analogy technique[20], using both two-dimensional and three-dimensional models. In such studies, flow is assumed to occur through both the concrete and the rock foundation, which is contrary to the usual assumption of no flow in concrete or rock. A series of electrical analogy models were constructed by H.A.Johnson, to show the effects of the various measures taken to prevent uplift in a concrete gravity dam and its foundation, such as grouting and drainage, on seepage forces. Both the concrete and the foundation of the dam are assumed homogeneous, and isotropic materials, but with not necessarily the same permeabilities. While these assumptions may appear to be far from the actual conditions, it is believed that the results do agree well with the actual measurements made on existing dams, and do show the relative effects of certain measures.

The two-dimensional model for these studies is easily constructed of a perfectly non-conducting material like Bakelite, with copper sheets introduced wherever it is necessary to impress a potential. The potentials to be impressed are calculated in the usual way. The foundation drain is represented by a solid copper sheet connected to the gallery, which is also simulated by a copper sheet, and are set to a potential to correspond to the gallery elevation. The foundation is represented to a depth equal to the maximum upstream head of the reservoir, since it is considered sufficient depth to make the boundary effects negligible. The phreatic line in the dam section is introduced by using modeling clay, and is finally established by a little trial and error. Also, potentials are impressed on the downstream face suitably to simulate the assumed seepage face. These potentials are placed in convenient increments at the required elevations, to force the equipotential lines to intersect the downstream face at the proper points. The method of introducing the phreatic line and the seepage face is similar to the one described for the case of earth dams in Section 4.1.2.

A special electronic hook-up was used by Johnson between the probe and the plotting point on the pantograph of the null detection unit, which was arranged so that when the probe reached a balance point in the model, a high voltage spark would be discharged from the pantograph tracing point to an aluminium ground plate under the paper burning a small hole in the paper and marking the point automatically.

In the two-dimensional model described above, only a continuous drain curtain could be simulated alongwith the drainage gallery. Drainage holes in the concrete section were not simulated in the two-dimensional model, since their effect could more satisfactorily be studied on three-dimensional models, described subsequently.

Results were obtained with the two-dimensional model for different ratios of concrete permeability to foundation permeability, with and without, either or both, of drainage measures and grout cut-off. It was found that the effect of foundation treatment on the potentials observed in the dam section is small. Also regardless of the ratio of permeability of concrete to foundation, the effect of this ratio upon potentials in the foundation is very small. Further, the tests show, that a drain functioning with full effectiveness is much more capable of reducing potentials than a grout cut-off. However, it is seen that without a grout cut-off, the potential lines around the drain are crowded very close together, thus indicating a very high flow into the drain tubes with possible deleterious effects.

A three-dimensional model is more helpful in such studies. It is constructed in a way similar to that of two-dimensional model, with slight changes. To simplify matters, same permeability is simulated for the foundations as for concrete, moreso because the effect of different permeability ratios is easier to study on the two-dimensional model. The gallery is easily represented as in a two-dimensional case, and where used the foundation drain is represented by a copper wire of proper diameter soldered to the gallery copper block. The vertical drain in the concrete is made by stretching a fine string, at the proper depth in the electrolyte, and attaching to it at 10% potential intervals spirals of fine metallic wire. These spirals are connected to the proper potentials, and are shifted back and forth until the equipotential lines intersect the drain at the proper elevations. The phreatic line is introduced, as usual, by trial and error with modeling clay. Details of the model are shown in Drawing 56(a). The model constructed in this way, represents a vertical slice of the dam. A diffferent model representing a horizontal slice through the dam is easily constructed, since the drains in this model may be represented by ordinary conducting wires at the proper spacing, and the length to the phreatic line is obtained from the phreatic line simulated on the vertical model described above. In cases where effect of open joints in relieving potentials is required to be studied, they are considered free draining and are simulated by a copper strip, which has no potential impressed.

A typical result with the vertical three-dimensional model is shown in Drawing 56(b), when there is no cut-off and no drain in the foundations. The general conclusions drawn from the two-dimensional model, as mentioned above, were further confirmed from the results obtained with the three-dimensional model. Further, it was found that the effect of measures taken to reduce foundation potentials, on potentials in the concrete, becomes all the more insignificant with drains in the concrete. Similarly, it was found that the effect of varying ratio of permeability of concrete to foundation upon potentials in the foundation becomes lesser with the presence of drains in the concrete. Also, it was found that drains in the concrete are very effective in reducing potentials in the concrete, but at the same time, they have almost negligible effect on the potentials in the foundation. A comparison of the corresponding results of the two-dimensional model and the three-dimensional vertical model shows that the error in making the drain solid in the former is not great.

Similar experiments were carried out by the engineers of the U.S.Bureau of Reclamation[75], on two-dimensional models, to study the action of drain in the foundations of the Boulder Dam spillway portion. Pressures beneath the dam were computed from the equipotentials, for testing the possibility of lifting up of the structure. The quantity of flow which may be discharged by the drain beneath the dam was also found, to enable the provision of a sufficient number of ejectors to take care of this flow.

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CHAPTER VII

FURTHER RESEARCH PROGRAMME

A description of the current research work, with most of which the author is associated, and certain new suggestions for further research on subsoil flow using the electrical analogy technique are contained in this chapter.

7.1. EFFECT OF TRAY BOUNDARIES ON UPLIFT PRESSURE DISTRIBUTION ON ELECTRICAL ANALOGY MODELS: Generally, the lateral boundaries that are required to be simulated in electrical analogy models of the usual hydraulic structures on permeable foundations are located at infinity. It is, therefore, not possible to simulate these boundaries, except by using unnecessarily large and uneconomic electrolytic trays. Consequently, it becomes obligatory to simulate the boundaries in the model at finite distances. Such a simulation of the boundaries not representative of the actual site conditions, naturally, vitiates the results obtained from the models. Experimenters[10,30] in the past have recognised this source of error, but no satisfactory work in this direction has been done so far, especially for three-dimensional model work. Systematic work has therefore been started to enable specifications being framed concerning the shape and relative location of the model tray boundaries in three-dimensional work, in order to bring about only an inappreciable vitiation of results. It is believed that similar results for two-dimensional work may be concluded from those obtained for three-dimensional work.

Since the usual hydraulic structures cannot be reduced to ones having simple geometric profiles, only typical canal structures can be taken up for study. Even these structures will be simulated in a considerably simplified form to save time in model construction, moreso, because there are many variables in the features of the foundations of a real structure. One representative example of each of canal headworks, barrages, syphons, and falls, shall be taken up for study. However, in case of syphons and falls, since the structure is a simple one with rather definite boundary conditions, a dimensionless model may be designed and constructed, and the need for taking a specific case for study will not arise.

For the study, a combination of the technique of making models at different scales using the same effective electrolytic tray, and of using the same models for different sized trays, is being used. One of the trays is required to have theoretically infinite dimensions, but due to practical limitations, it has been constructed of 20'x 16' size, on consideration that models greater than 5'x 4' size are seldom used in electrical analogy investigations of canal structures, and four times the overall dimensions of the model is considered a sufficient ratio for the location of the boundary walls of the tray, in order to exert inappreciable effect on the potentials observed on the three-dimensional models. Two dimensionless canal syphon models have been designed and constructed, having overall length L to width W ratio of 2 and 4 respectively, and the studies have been started using rectangular and elliptical boundaries at various relative locations with respect to the model boundaries, as shown in Drawing 57. The effects on potential distribution at various strategic points of the foundation of each of the four categories of canal structures will be studied through a long series of experiments for various relative locations of model tray boundaries, as already mentioned above.

The results expected from the study will be in the form of percentage change in potentials observed on the three-dimensional models by a shift in the model tray boundaries. These percentages will be of different values for the four types of canal structures. These results will ultimately help in framing specifications for location of model tray boundaries for the four types of canal structures, so that no appreciable effect of these boundaries on uplift pressures are obtained.

7.2. RELIEF OF UPLIFT PRESSURES BELOW LONG BARRAGE FLOORS:

A novel method of affecting economies in the construction costs of escaping structures of low height founded on permeable subsoils has been evolved recently by the provision of 'Intermediate Pressure Relief System', to reduce uplift pressures all along the structure foundations. Provision of such pressure relief systems has started to be made in the case of Nanak Sagar Barrage and Dhora Barrage of Uttar Pradesh, of which the former has already been completed and is functioning satisfactorily. The pressure relief arrangement can be an inverted filter, or a system of pressure relief wells, or a combination of the two.

7.2.1. Basic Two-Dimensional Study:

Two-dimensional basic studies have been taken up, with a view to study the most effective location and arrangement of the pressure release system for a given floor length and head acting. A typical dimensionless model of barrage floors has been prepared representing an impervious clay layer at a depth equal to the maximum upstream head H acting on the structure. The features of the dimensionless model are representative of the designs of the Nanak Sagar and Dhora Barrages of Uttar Pradesh. The details of the typical model designed, and the electrical analogy set-up used, are shown in Drawing 58. To facilitate study and interpretation of the results, all lengths and relative levels involved in the structure have been expressed in dimensionless ratios of H. Only two-dimensional models are being used for the study, since a study on the three-dimensional models will be very time consuming; and moreover, the uplift pressures observed on the two-dimensional models will bear certain fixed ratios to those observed on the corresponding three-dimensional models under certain given boundary conditions.

A long series of experiments are being performed, under various conditions of effectiveness of the pressure release system and its locations, and for certain modifications in the features of the foundation profile. However, the results of four studies already completed[76] are presented in Drawing 59. Drawings 59(a) and (b) give the equipotential contours for the model without sheet piles, for the cases of pressure relief trench only effective, and for the intermediate filter only effective, respectively. Similar results for the model incorporating three sheet piles, as shown thereon, are shown in Drawing 59(c) and (d). Drawings 60(A) and (B) give the comparison of uplift pressure lines for the cases of pressure relief trench only effective and intermediate filter only effective, for the model without sheet piles, and the model with sheet piles, respectively. It will be observed from these drawings, moreso from Drawing 60(B), that the uplift pressures with the 100% cut-off trench only effective are somewhat lesser than the uplift pressures with the intermediate filter only effective. For the case of actual 100% penetration relief wells, which cannot be simulated in two-dimensional models, the two uplift pressure lines will tend to more or less overlap each other - almost parallel conclusion having been drawn by H.A.Johnson[20] as mentioned in Section 6.4 for the case of simulation of foundation drains in gravity dams. This conclusion is also corroborated by the results given in Section 7.2.2.

The entire study on the several models designed, when completed, is expected to yield results to indicate the most effective arrangement of the pressure release system, consistent with economy, in case of long escaping structures of low height, and their most effective locations.

7.2.2. A Three-Dimensional Study on Barrage Floors:

Experiments[68] were carried out on a three-dimensional model of the modified design of Dhora Barrage of Uttar Pradesh, to determine the uplift pressure distribution under the foundations of the structure, under various conditions of effectiveness of the intermediate pressure release system provided in it. The uplift pressures on the centre-line of the barrage, as obtained from the three-dimensional model[76], are shown in Drawings 60(C) and (D), for conditions of the downstream water level being at spring level elevation and being drained at the river bed elevation, respectively. Each of these drawings shows the corresponding uplift presure lines for the cases of pressure release wells only effective, and for the intermediate filter only effective. The length of the intermediate filter provided in the structure is 17 ft., and relief wells provided are 2 inch in diameter at 20 feet centres. Other details of the structure are shown in the section drawing itself. The two uplift pressure lines shown in each of the two Drawings 60(C) and (D) very much overlap, providing further proof of the conclusion arrived at in Section 7.2.1, that the relative performance of pressure relief by intermediate filter and relief wells is almost the same.

Thus, it appears safe to conclude from the results obtained so far, that the relative performance of a properly designed inverted filter, and of properly designed fully penetrating pressure relief wells, provided to relieve pressures at some intermediate reach in case of long escaping structures, is almost the same, provided unusual foundation conditions are not encountered; and only considerations of economy should be the guiding factors for the choice of one or the other system.

7.3. EFFECT OF STRATIFICATIONS ON UPLIFT PRESSURES: The assumption of homogeneous and isotropic subsoil conditions in electrical analogy experiments is generally speaking not correct, as already discussed in Section 1.4. Studies are, therefore, being taken up[76] to ascertain the effect of soil stratifications on uplift pressures observed on three-dimensional models of hydraulic structures on permeable foundations, and to develop model technique for taking this into account while performing the three-dimensional electrical analogy experiments.

It is believed that investigations of this nature in the field of three-dimensional electrical analogy have not been carried out so far. Certain systematic experiments in the two-dimensional field of this nature had been carried out at the Central Water and Power Research Station Poona,[54,55] as briefly described in Section 4.1.1, but they too are not exhaustive;

and moreover, only a simple case of flush horizontal floor with and without sheet piles had been investigated.

An elaborate electrical analogy model tray has been constructed, with provision to simulate stratifications of different thicknesses and of different permeabilities, in watertight compartments of plastic having suitable electrical interconnections. A long series of experiments is proposed to be carried out, using simplified dimensionless models of various types of canal structures.

7.4. INTERFERENCE AND GROUPING OF TUBEWELLS: It is generally recognised that in any groundwater exploitation project, a large number of shallow wells are more efficient than deep wells, so far as the discharge is concerned. However, siting of tubewells, if done too close together, causes interference between them, lowering the discharge of each, and consequently making the project uneconomical. It is, therefore, proposed to study the interference caused by the grouping of a given number of wells, and to determine their optimum spacing to give the maximum discharge, for aquifers of different characteristics.

Such studies[32] have been carried out for a number of wells arranged in different patterns in an isotropic and homogeneous medium, and relative discharges for various groupings have been studied, neglecting the cone of depression or free surface developed at each well. These results give useful information regarding well arrangement in a system, but only relative significance could be attached to the results since the simulation of various free surfaces had been neglected.

An electrolytic tray has been prepared for experimentation, wherein silver wires will be used to simulate tubewells, as described in Section 4.1.5. Corresponding lengths of the silver wires, painted with non-conducting paint, will simulate blind pipes. It is proposed to use membrane analogy as discussed in Section 6.2.1, or a simpler wax method, to simulate the various cones of depression. The latter method consists in filling wax in the volume, corresponding to one in which no flow takes place in the prototype, and shaping it suitably, so as to represent the free surface after a certain few number of trials - the technique of making the trials being essentially the same as described by H.E.Babbit and D.H.Caldwell[14], and discussed in Section 5.3, for the case of carbon sector model of a well. Different strengths of electrolyte will be used to simulate different permeabilities of the water bearing formations, as usual. Current and potential measurements shall be taken under various conditions of grouping of wells, to indicate quantitatively the presence of interference.

It is expected that the results of the studies will help in the proper location of a group of tubewells to yield greater discharges in the field most economically.

7.5. SUBSOIL FLOW CRITERIA FOR DESIGN OF FLOORS OF FALLS:

Having applied the three-dimensional technique satisfactorily for evolving rational criteria for design of barrel portion of canal syphons with respect to subsoil flow, as discussed in Section 6.1.5, it appears probable to obtain parallel design criteria for floors of other similar structures where boundary conditions are not too complicated[69]. A structure of this category is the canal fall.

Subsoil flow in most of the width of canal falls is simple two-dimensional, taking place from upstream to downstream. However, the two end-regions on the flanks have a positive three-dimensional flow, and this effect may extend to a considerable percentage of the total width in case of narrow falls with small ratios of overall width of the structure to its length. It is, therefore, advisable to provide for this three-dimensional effect in the design of floors of falls. Normally, the effect would be favourable, in so far as lesser floor thicknesses than normally calculated by two-dimensional approach would be required. But, occasionally in cases of adverse groundwater boundaries obtaining in the vicinity of the structure, greater floor thicknesses will be required.

Thus, the major variable entering the problem is the size and elevation of the groundwater mound in the vicinity of the structure. A long series of electrical analogy experiments may, therefore, be carried out on full dimensionless simplified models of usual canal falls, taking into account varying conditions of the groundwater table in the vicinity of the structure. On basis of the results obtained, design curves giving uplift pressures on design points of a proposed dimensionless grid for the foundation of the structure may be provided. The design curves may easily be consulted to obtain uplift pressure distributions under the foundations of a proposed fall, thus enabling appropriate floor thicknesses to be provided to counteract the uplift pressures.

7.6. STUDIES ON MODEL AND PROTOTYPE CONFORMITY: The limitations of the electrical analogy, discussed is Section 1.4 and elsewhere in the appropriate contexts, often make it difficult to simulate one or more boundary conditions accurately. Several simplifying approximations and assumptions are required to be it is just possible that a boundary condition may not be simulated at all or may be simulated only very approximately, due to ignorance or lack of complete information about the same in the prototype. This is especially true of minor geological details in the foundation subsoil, e.g. lenticles of clay sizeable boulders, etc. Hence, it is necessary to observe the behaviour of uplift pressures under actual prototype structures.

Pressure observation pipes or piezometers should be installed in the structures in sufficient numbers while they are under construction, so that studies may be made on the prototype to observe the prevailing uplift pressures under various operating head conditions. Such studies will be helpful to ascertain the extent to which the model is able to reproduce the field conditions, and may establish fuller confidence in the results obtained from the models. Further, the actual prototype results may be helpful in understanding the factor or factors that might be responsible for the variance of the model results, if so, and may help to improve the simulation in the model to reproduce field conditions in a closer way.

Comprehensive work in this direction had been done in the early thirties by A.N.Khosla[30] and his associates, who observed close conformity between the model and prototype behaviour in respect of structures like Panjnad Weir, Khanki Weir, Lloyd Barrage, and Deg Escape Head of Punjab. Since then, there appear to have been done little systematic work in this direction. It is essential to continue observations on important prototype structures for reasons given in the foregoing paragraph. Such prototype observations will be all the more helpful in cases of structures like Nanak Sagar and Dhora Barrages of Uttar it is just possible that a boundary condition may not be simulated at all or may be simulated only very approximately, due to ignorance or lack of complete information about the same in the prototype. This is especially true of minor geological details in the foundation subsoil, e.g. lenticles of clay sizeable boulders, etc. Hence, it is necessary to observe the behaviour of uplift pressures under actual prototype structures.

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Pradesh wherein an intermediate pressure release system has been provided, since the studies will also help in judging the performance of the pressure release system. Arbitrary percentage effectiveness of such pressure release system is assumed in the designs at present, in absence of prototype observations so far on such type of long escaping structures of low height.

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CHAPTER VIII

CONCLUSIONS

The electrical analogy method is simple, speedy, economical, and accurate, and can analyse many types of problems in various branches of Science, Engineering, and Technology. The method has helped immensely in the solution of many hydraulic and groundwater flow problems. In many cases, it represents the only means to obtain a correct solution of problems connected with the design of foundations of hydraulic structures with respect to uplift pressures due to subsoil seepage.

Extensive use of two-dimensional electrical analogy is found in subsoil flow studies connected with hydraulic structures like dams and weirs. It has also been applied in problems of surface flow over spillway crests and through syphons, and for an interesting study of horizontal earth-quake effects on dams. The two-dimensional technique is quite simple.

Sector model electrical analogy technique has made possible some complicated problems of design of hydraulic structures to be tackled simply and expeditiously, and has been of use in research in some fields of Fluid Mechanics. Uses of the method are found in studies connected with flow through penstocks, gates, and intake towers, and in preparation of criteria for cavitation-free designs of inlets and conduit contractions. A very useful application of the sector models is in the study of flow through wells.

Almost all the problems connected with subsoil flow require a three-dimensional electrical analogy approach, except for a very limited number. The fact has amply been brought out in this thesis, that considerable savings in floor thicknesses of normal canal structures are possible, if designed according to this approach. The extent of percentage pressure release as compared to that obtained by two-dimensional approach is a variable factor depending upon various boundary conditions, but an average value of such percentage for a normal canal structure resting on permeable foundations (with usual ground-water boundary conditions) may broadly be estimated to lie between 10 and 30. The technique of three-dimensional electrical analogy has several of its own limitations, still it has immense possibilities.

Various percolation studies in homogeneous as well as heterogeneous soils under complex structures are possible to be carried out with a properly developed technique and model-medium. Design of an economical system of tubewells for dewatering deep foundations of hydraulic structures can be done by the use of the three-dimensional technique. Tubewells can be simulated accurately in three dimensions by a combined use of electrical analogy for flow and membrane analogy for the free surface. A system of relief wells can also be designed by the three-dimensional technique to reduce the uplift pressures below a hydraulic structure to a safe limit. It is possible to obtain design curves for design points of a dimensionless grid on simple three-dimensional structures like canal syphons and falls, where boundary conditions are not too complicated; thus enabling the designer to determine uplift pressures more rationally than provided by any of the two-dimensional methods.

It is believed that electrical analogy offers a vast field for further research and experimentation. Certain problems for this purpose have been indicated in this thesis, and work on some of them, with which the author is associated, has already been started. A result obtained thus far is that the relative performance of a properly designed inverted filter, and of properly designed fully penetrating pressure relief wells, provided to relieve pressures at some intermediate reach in case of long escaping structures is almost the same. More such important conclusions can be drawn with a properly planned research work using the analogy method. There is scope for further perfection in simulation of even complicated boundaries by proper development of instrumentation and technique. One such advancement in the technique may be the working from full - to part - models, if suited to the purpose of the particular problem under investigation. Certain requisite boundary data for simulation in the part-models may be obtained from earlier worked full-models, constructed at smaller scales. Very accurate results may be obtained from these part-models, although the boundary data simulated may have been obtained as a rather approximate result from the full-models. This technique is especially considered useful in sector or two-dimensional models of sluice inlets in dams, and in certain models for studying seepage flow pattern, e.g. subsoil flow from a canal situated above a normal water table.

It is hoped that greater development of the electrical analogy technique for solution of even complicated problems of certain surface and sub-surface flows will take place in the near fututure, and that more and more research and design organisations dealing with hydraulic and groundwater problems will realise the utility of the method, and adopt it as an essential tool for solution of such problems. The essentiality of the method, moreso the three-dimensional one, is especially true for determination of uplift pressure distribution under foundations of major hydraulic structures.

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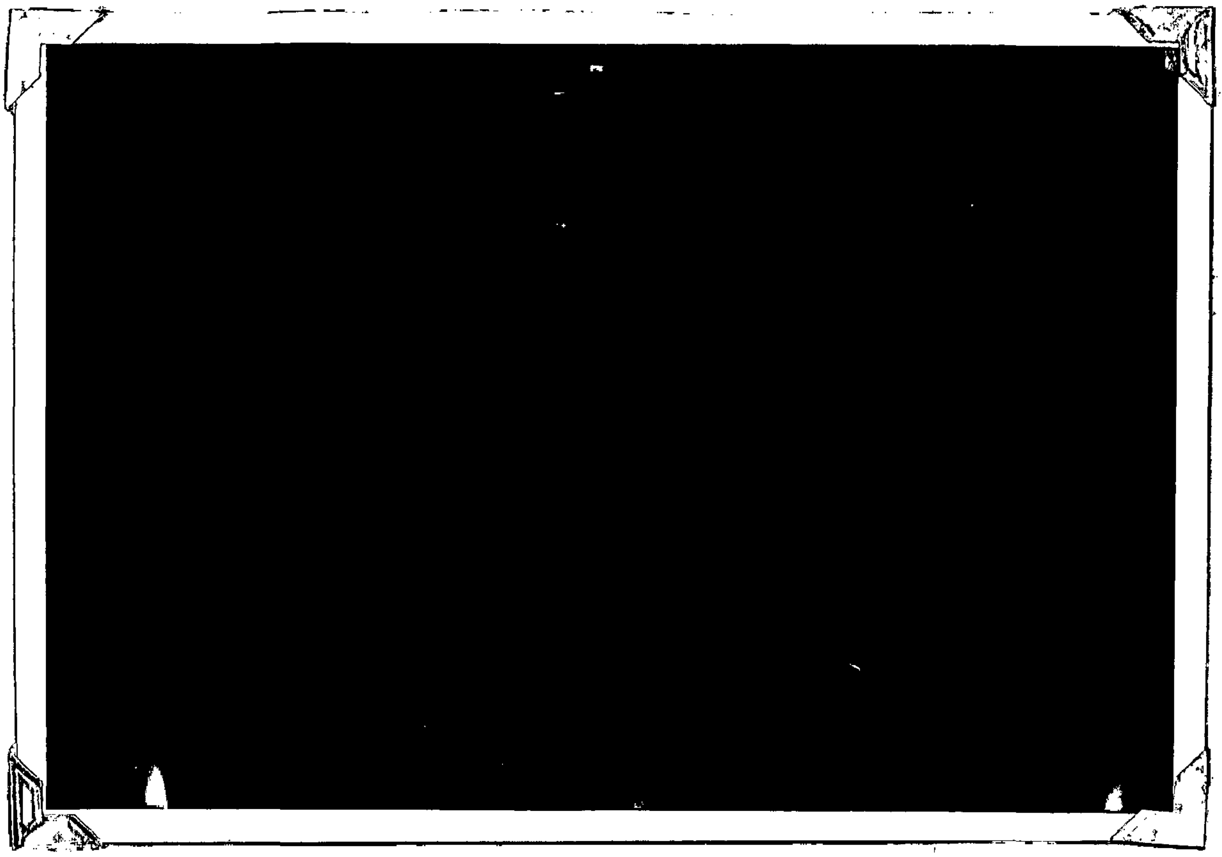
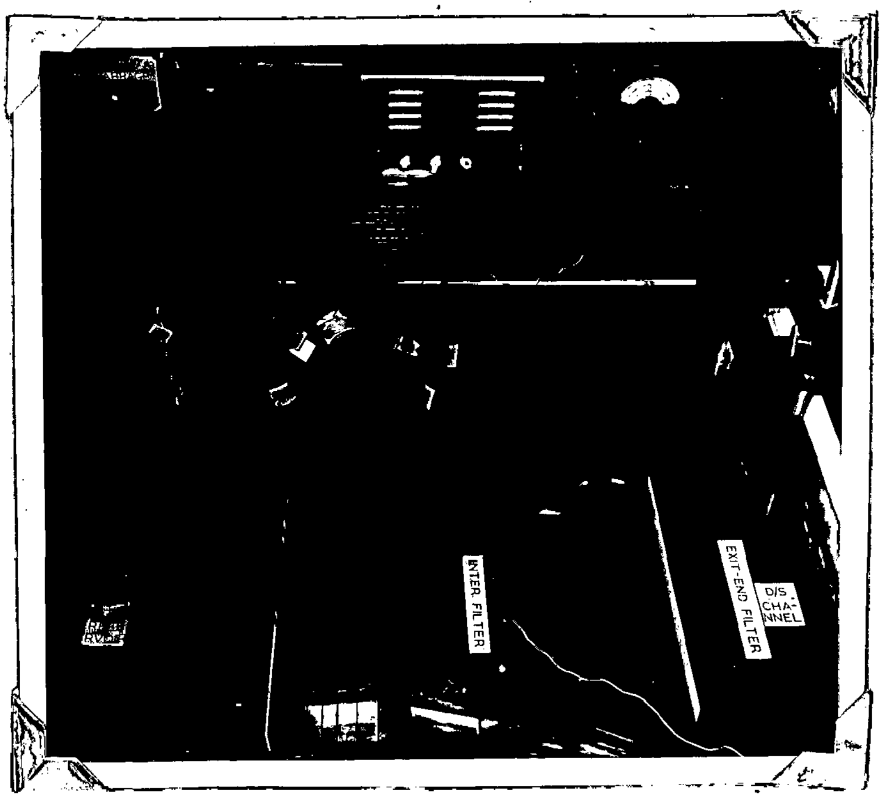
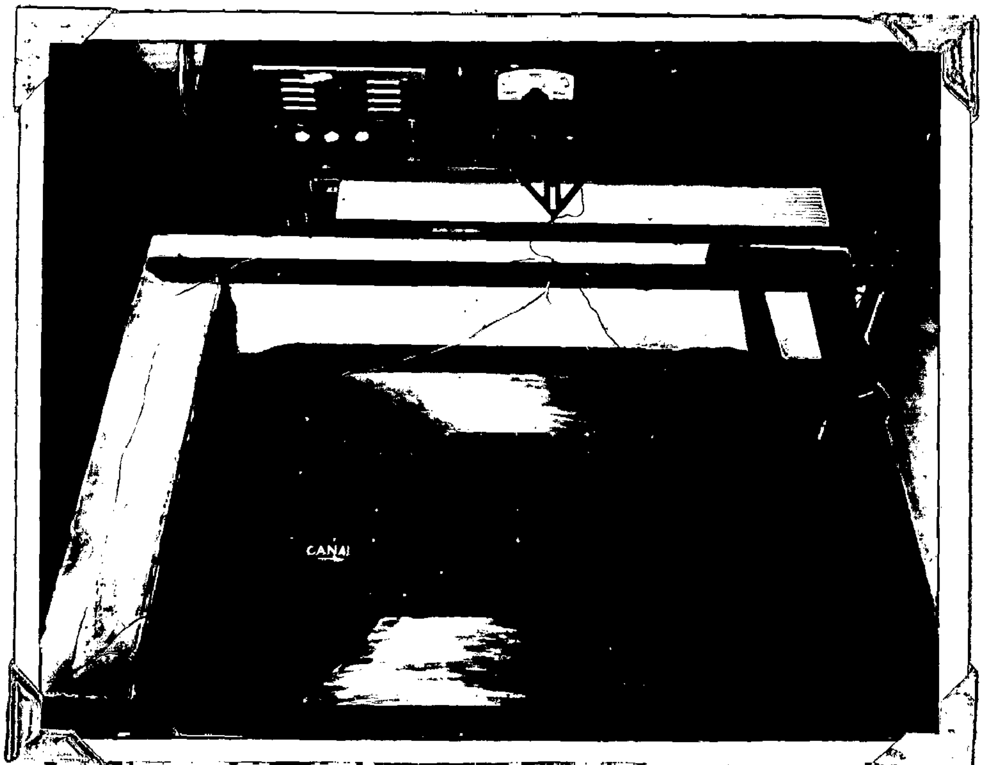


PHOTOGRAPH.1.
SHOWING THE GENERAL LAY-OUT OF A TYPICAL TWO-DIMENSIONAL ELECTRICAL ANALOGY MODEL



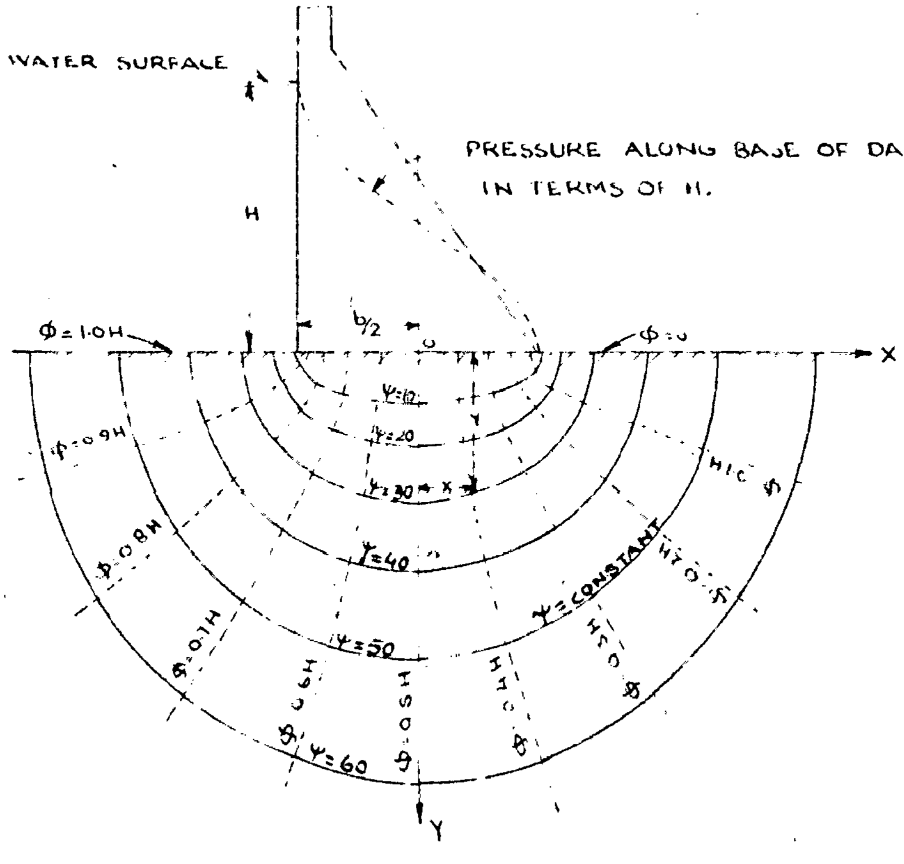
PHOTOGRAPH.2.
SHOWING THE GENERAL LAY-OUT OF A TYPICAL THREE-DIMENSIONAL ELECTRICAL ANALOGY MODEL
(BY COURTESY OF DIRECTOR, U.P. IRRIGATION RESEARCH INSTITUTE, ROORKEE)

PHOTOGRAPH. 3
SHOWING THE GENERAL LAY-OUT OF THE THREE-DIMENSIONAL
ELECTRICAL ANALOGY MODEL OF THE NANAK SAGAR BARRAGE



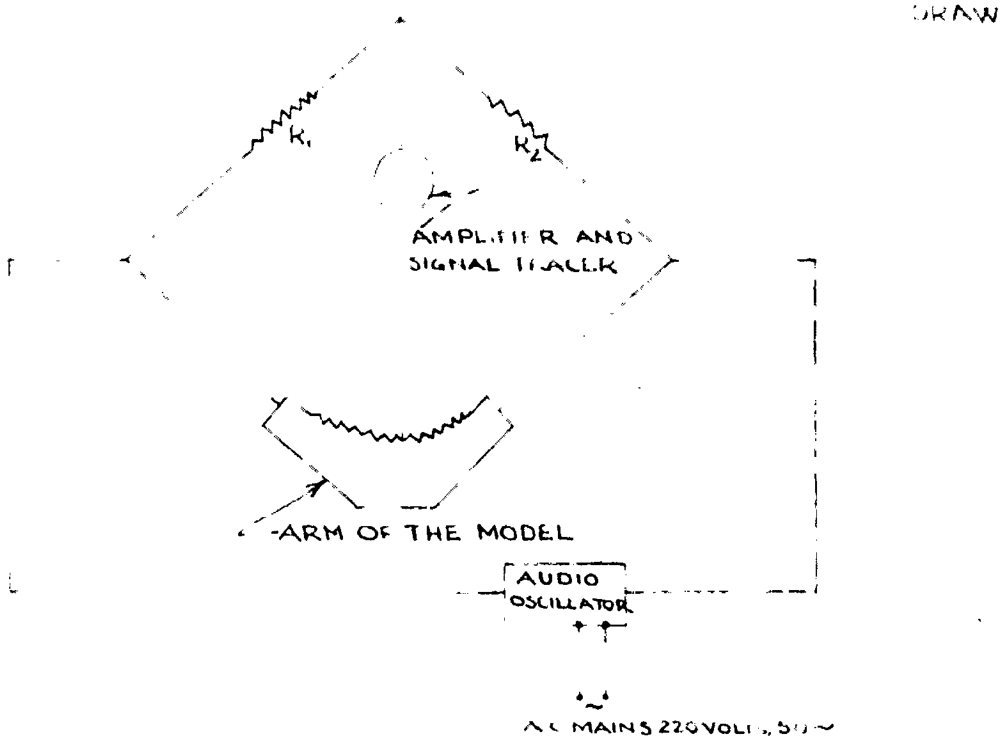
PHOTOGRAPH. 4
SHOWING THE GENERAL LAY-OUT OF A TYPICAL THREE-
DIMENSIONAL ELECTRICAL ANALOGY CANAL SYPHON MODEL
(BY COURTESY OF DIRECTOR, U.P. IRRIGATION RESEARCH
INSTITUTE, ROORKEE)

DRAWING.1

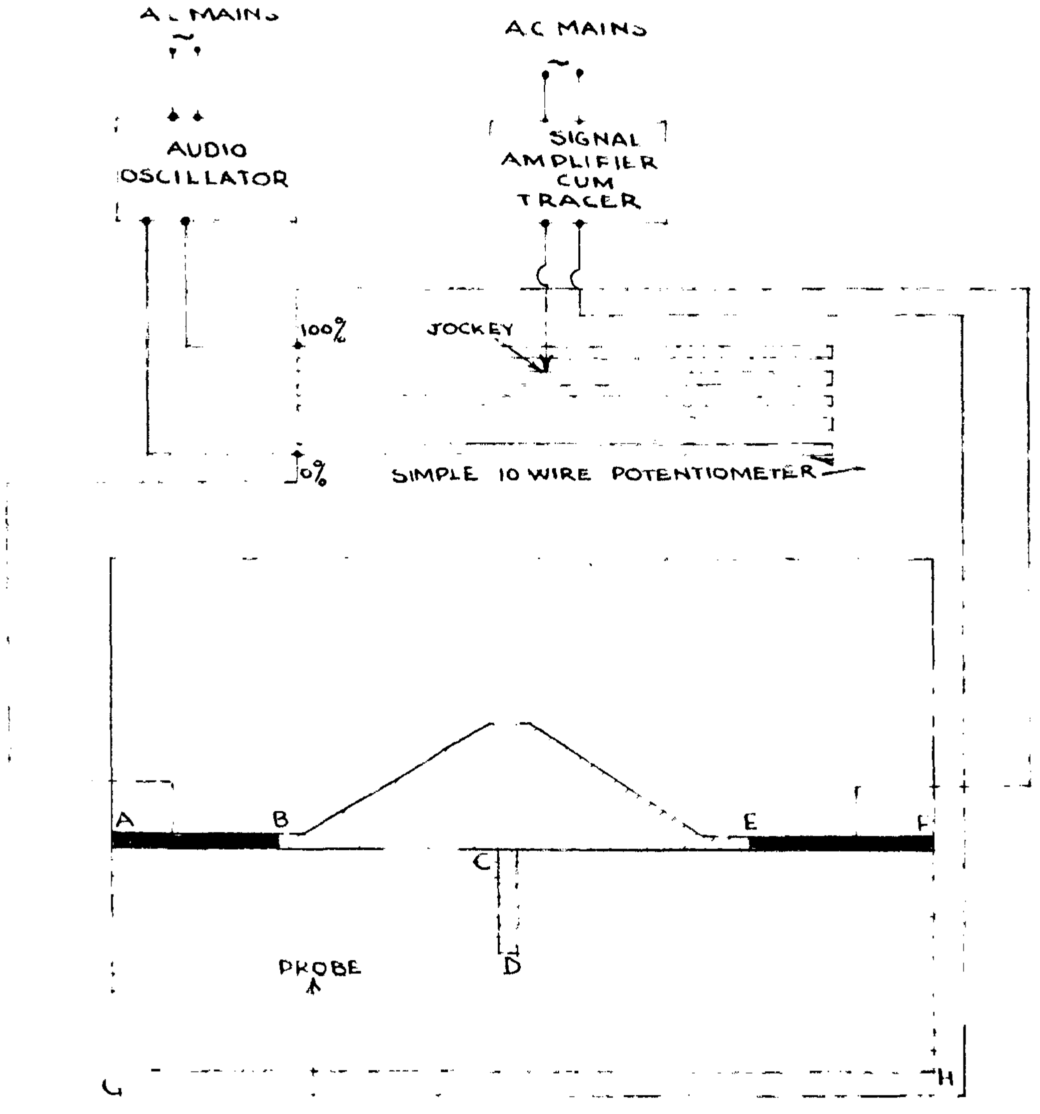


FLOW BENEATH AN IMPERVIOUS DAM
ON A PERVIOUS FOUNDATION
(REPRODUCED FROM ENGINEERING MONOGRAPH NO. 8, U.S.B.R.)
REFERENCE 35

DRAWING 2

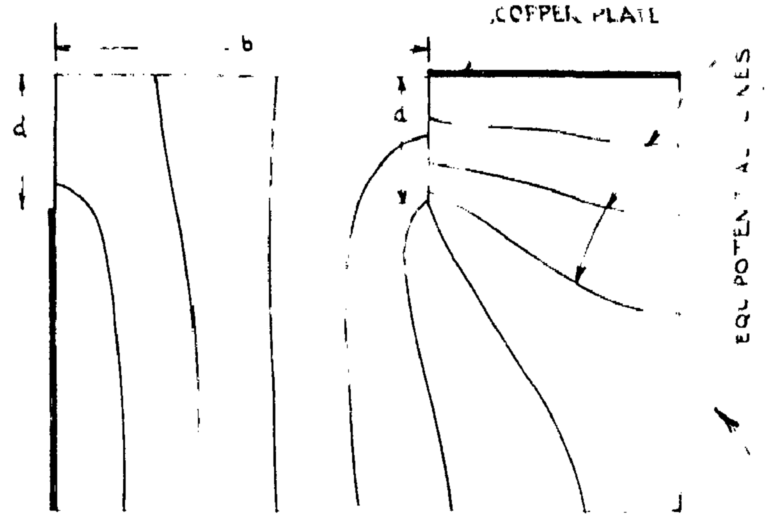


SIMPLE WHEATSTONE BRIDGE PRINCIPLE APPLIED



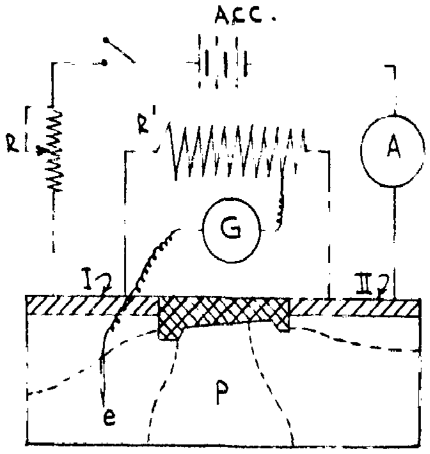
WO-DIMENSIONAL ELECTRICAL ANALOGY MODEL OF AN IMPERVIOUS DAM WITH A CENTRAL SHEET PILE CUT OFF

DRAWING 3



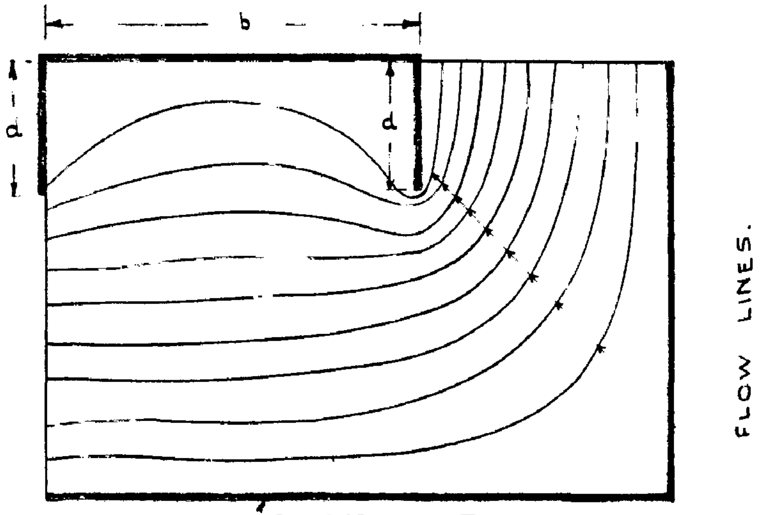
(b)

TYPICAL EQUIPOTENTIAL LINES AND STREAM LINES FOR A DAM WITH THREE SHEET PILINGS OF EQUAL LENGTH STUDIED BY PROF. PAVLOVSKY [ONLY ONE SYMMETRICAL HALF OF THE PERMEABLE REGION REPRODUCED]



(a)

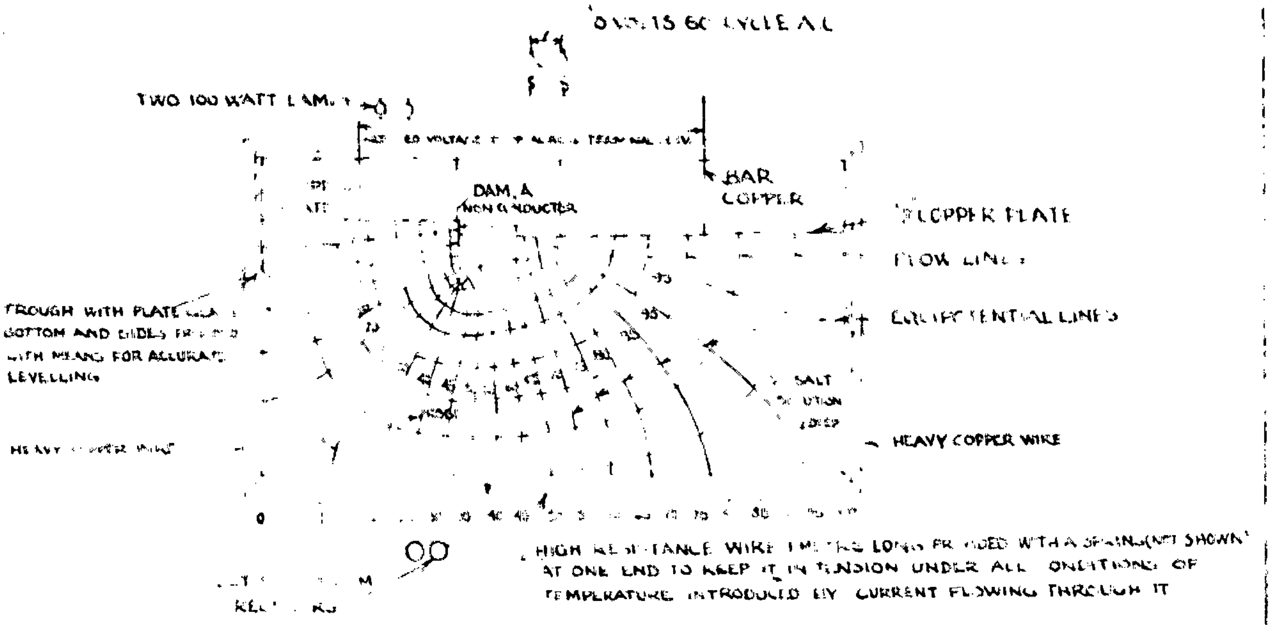
SCHEME OF PROF. PAVLOVSKY'S APPARATUS.



(C)

(FROM REFERENCE 2)

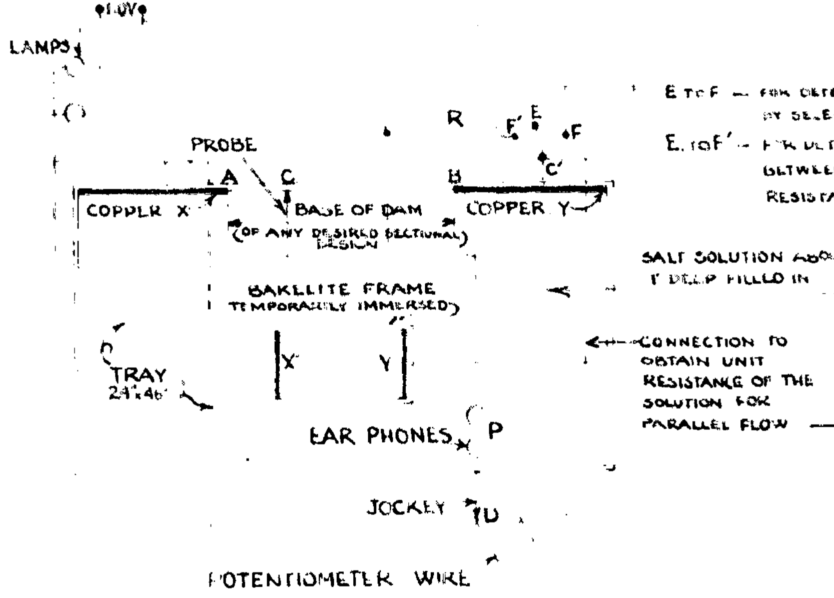
DRAWING 4



(a)

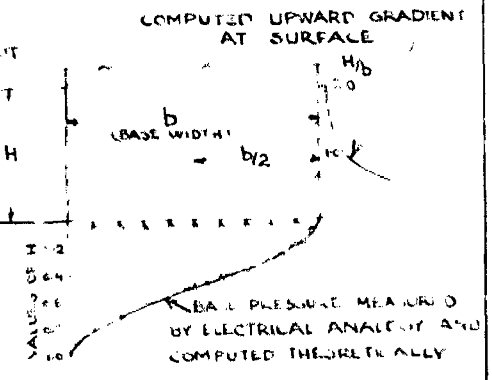
LANES TWO DIMENSIONAL ELECTRICAL ANALOGY APPARATUS

(FROM REFERENCE 7)



(b)

HARZA'S TWO DIMENSIONAL ELECTRICAL ANALOGY APPARATUS



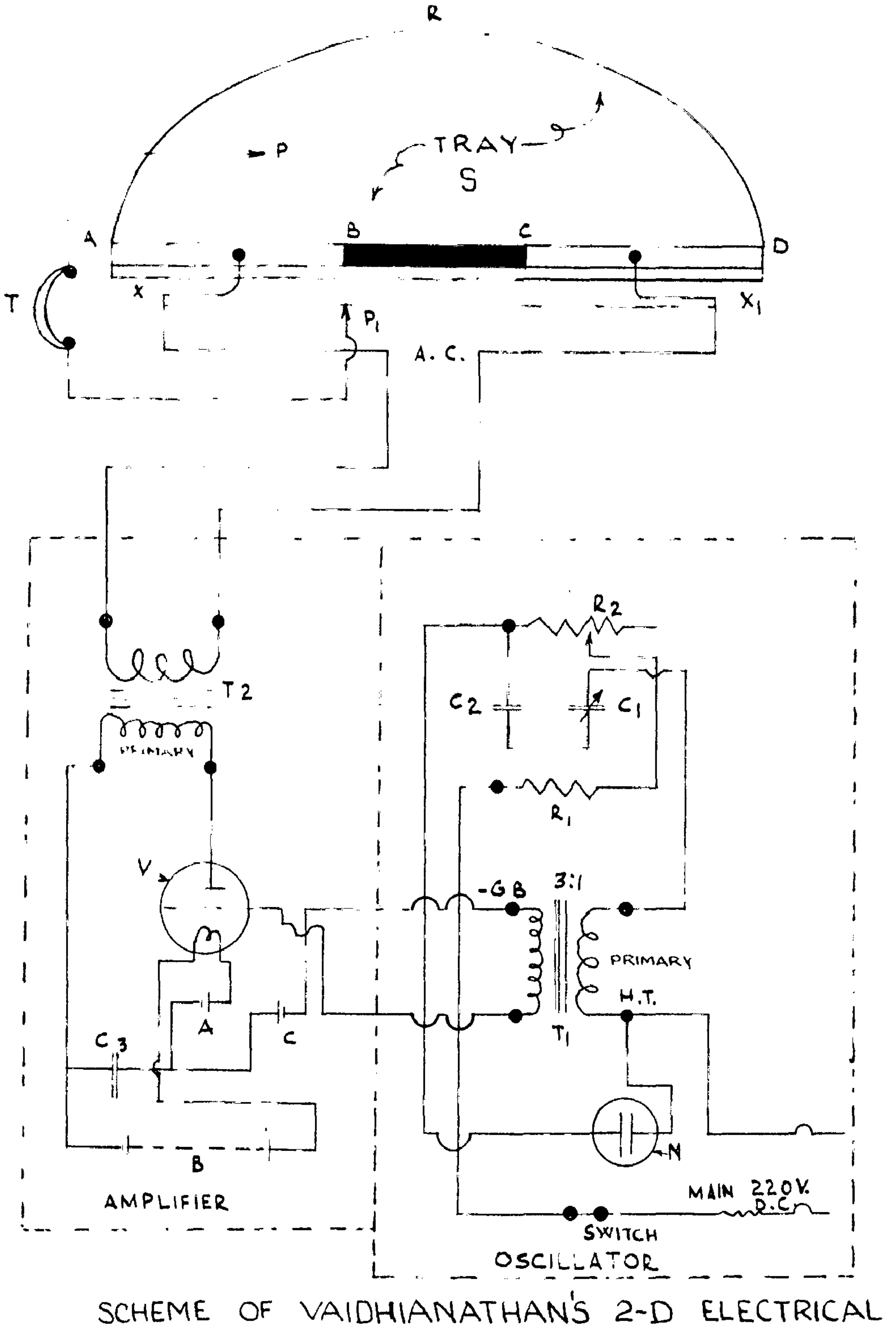
(c)

BASE PRESSURE OBTAINED BY ELECTRICAL ANALOGY FOR DAM ON SAND—NO CUT-OFFS

(AFTER HARZA)

(REFERENCE 6)

DRAWING. 5

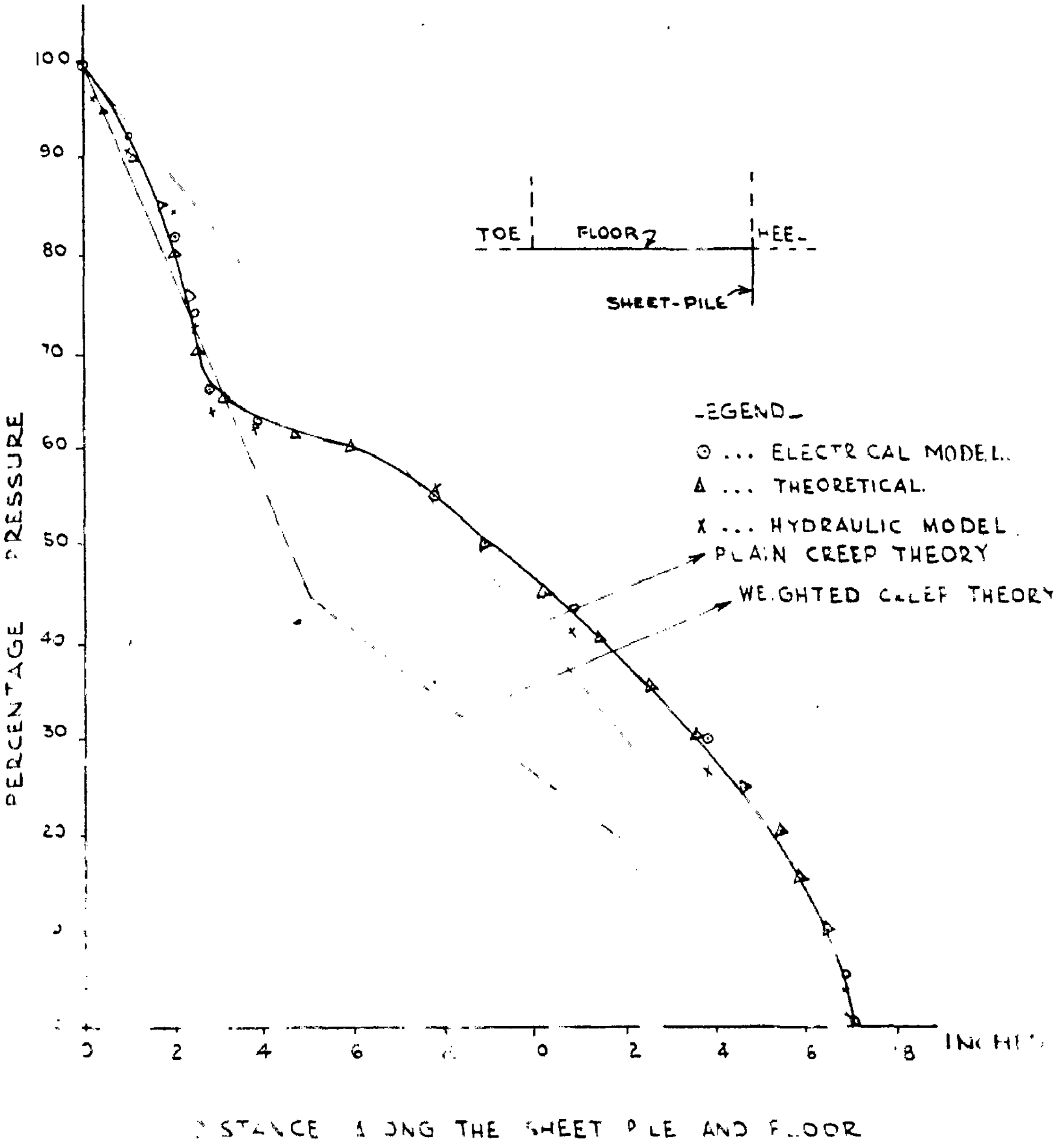


SCHEME OF VAIDHIANATHAN'S 2-D ELECTRICAL ANALOGY SET-UP
(FROM REFERENCE 27)

LEGEND

NEON BULB	N
TRANSFORMER	T T_2
CONDENSER	C C_2 C_3
RESISTANCES ...	R_1 R_2
TRIODE VALVE ...	V
FILAMENT BATTERY	A
PLATE BATTERY	B
GRID BATTERY	C

DRAWING 6.

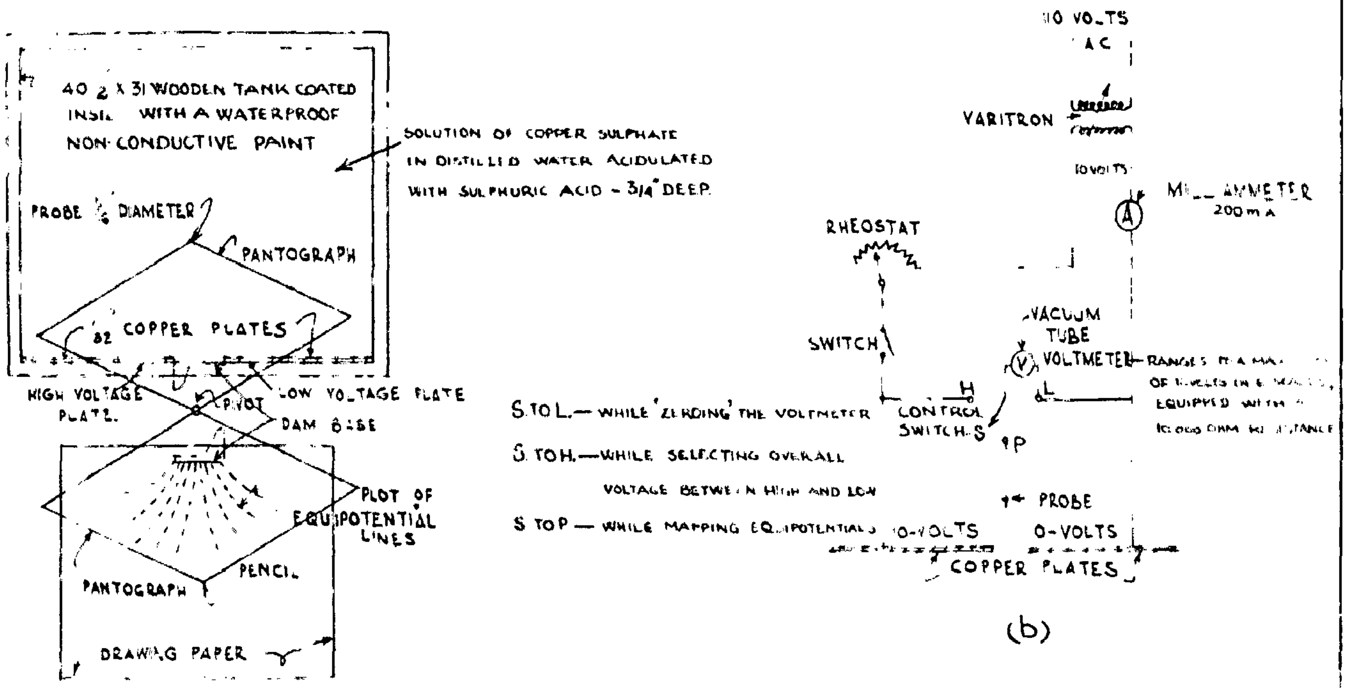


PRESSURE DISTRIBUTION ALONG THE

SHEET PILE AND FLOOR --

[illegible]HIANATHAN REFERENCE [illegible])

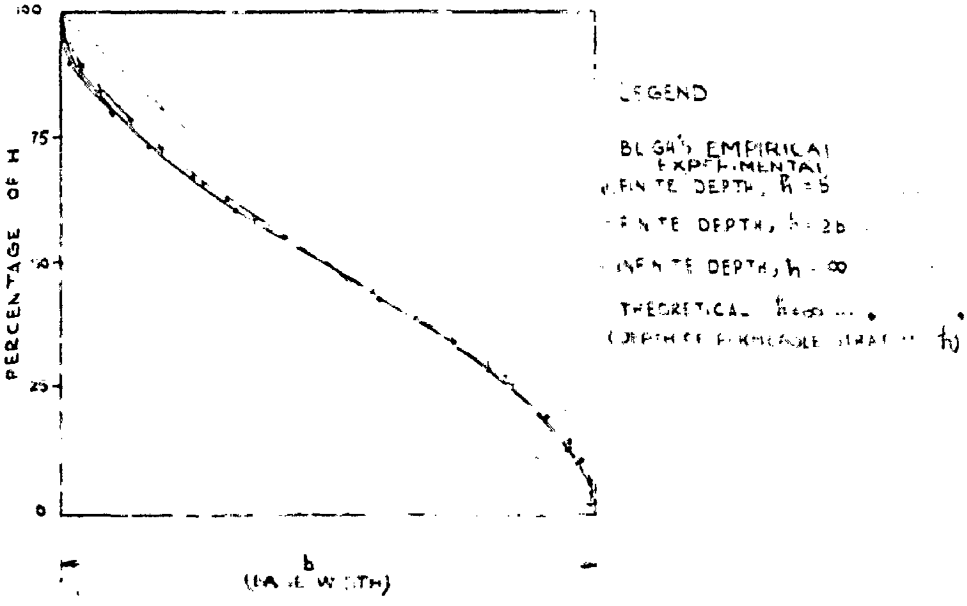
DRAWING 7



(a)
APPARATUS

WIRING DIAGRAM

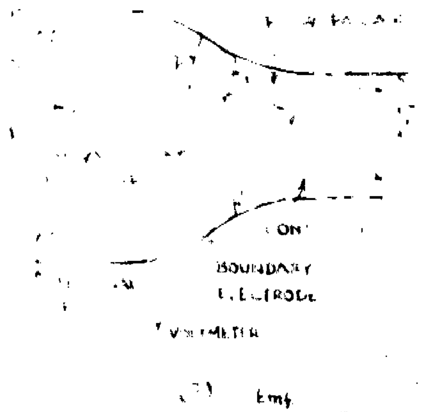
EXPERIMENTAL SET-UP USED BY SELIM FOR
TWO DIMENSIONAL ELECTRICAL ANALOGY WORK
(FROM REFERENCE 10)



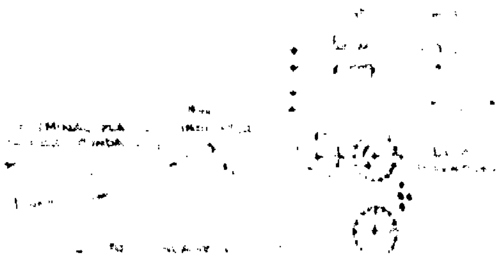
(C)

SHOWING UPLIFT DIAGRAM FOR A DAM
WITH HORIZONTAL FLOOR ONLY —
(AFTER SELIM)
REFERENCE 10

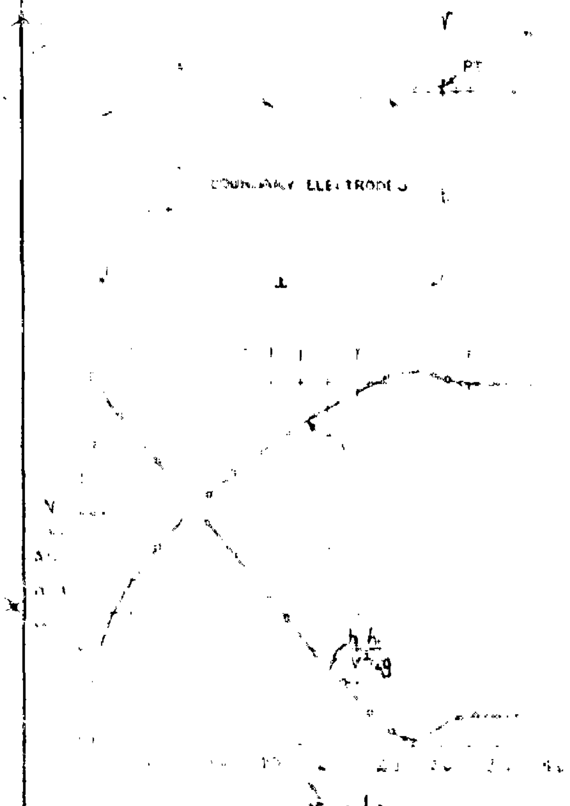
(b)
CROSS SECTION OF ELECTROLYTIC BATH SIMULATING LONGITUDINAL SEGMENT OF THREE-DIMENSIONAL COUNTERPART OF FIG.(a)



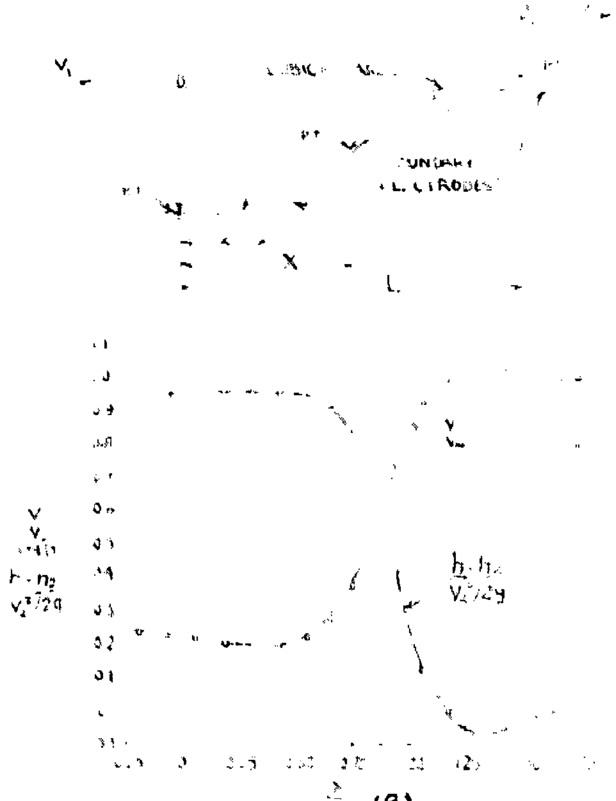
(a)
SCHEMATIC REPRESENTATION OF ELECTRICAL ANALOGY FOR IRROTATIONAL FLUID FLOW FOR A TWO-DIMENSIONAL BOUNDARY CONTRACTION



(c)
CIRCUIT DIAGRAM USED IN THE STUDIES

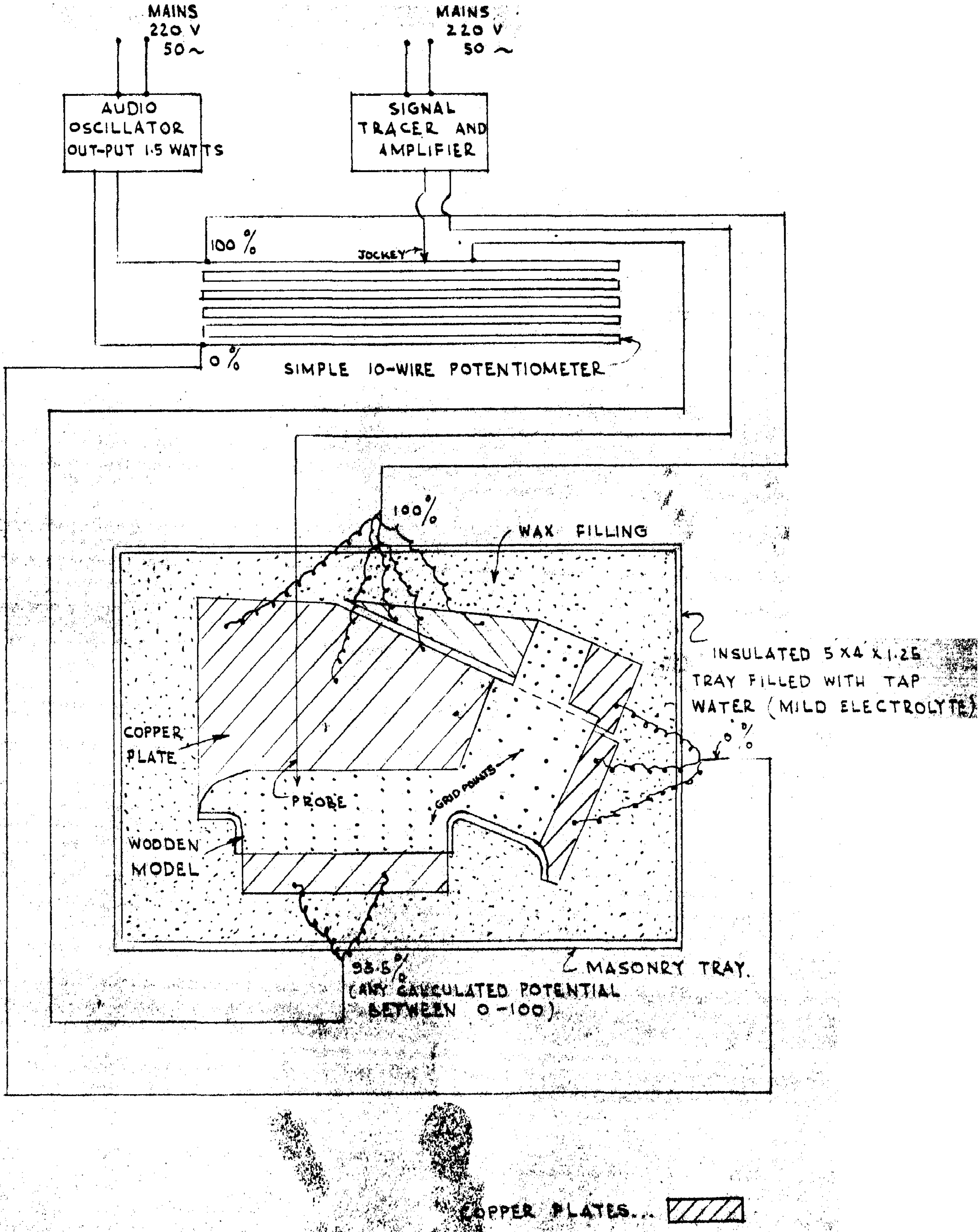


(d)
CO-ORDINATE NOTATION AND TYPICAL RESULTS FOR ELLIPTICAL INLET PROFILE



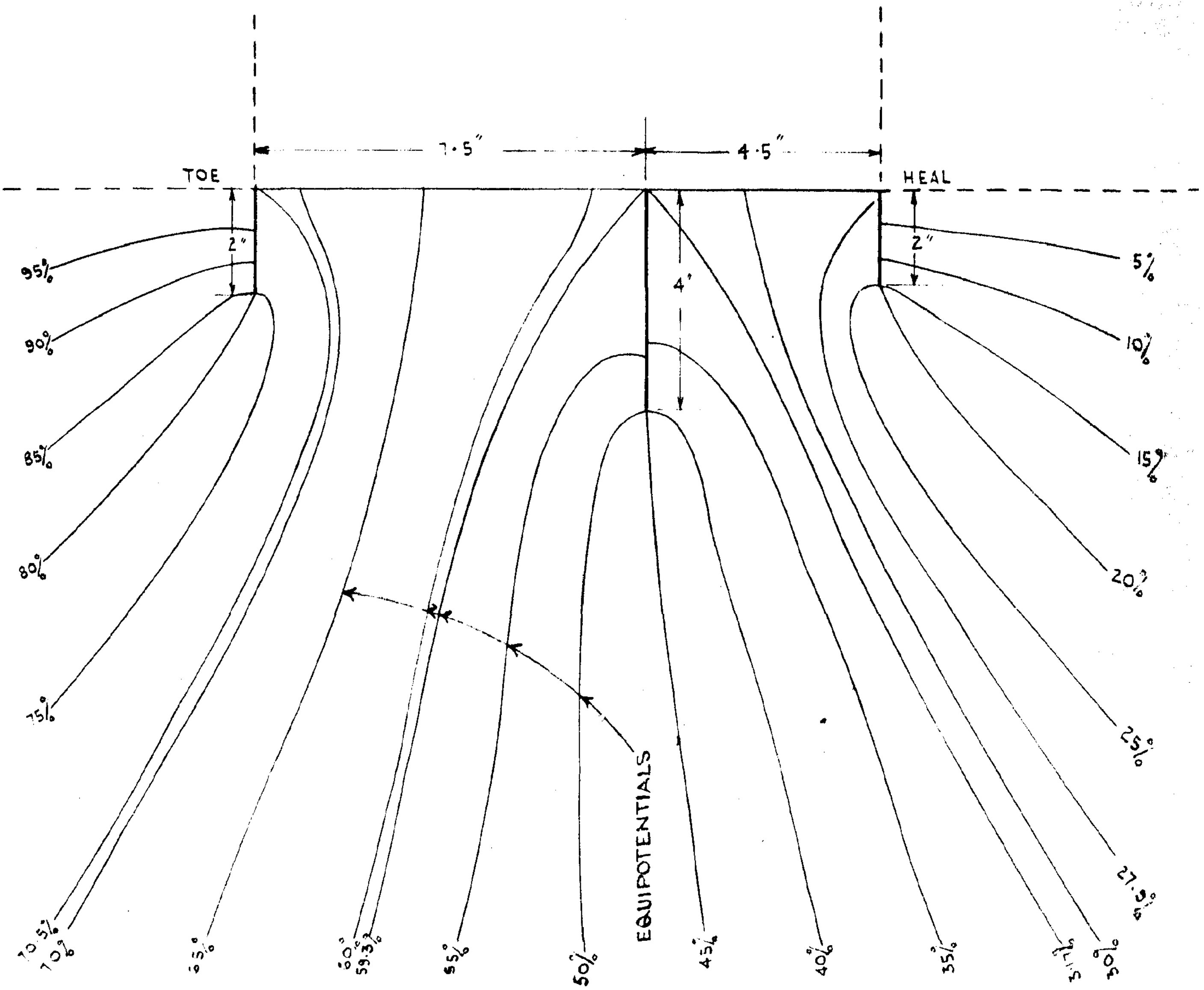
(e)
CO-ORDINATE NOTATION AND TYPICAL RESULTS FOR CUBICAL ARC CONTRACTION PROFILE

(AFTER ROUSE AND HASSAN)
REFERENCE [illegible]



TYPICAL ELECTRICAL CONNECTIONS WITH THREE-DIMENSIONAL ELECTRICAL ANALOGY MODEL OF A HYDRAULIC STRUCTURE ON PERMEABLE FOUNDATIONS

DRAWING.10



TYPICAL EQUIPOTENTIALS
FOR A FLUSH FLOOR WITH THREE SHEET PILES
(AFTER GURDAS RAM AND VAIDHIANATHAN)
REFERENCE 27

DRAWING.11

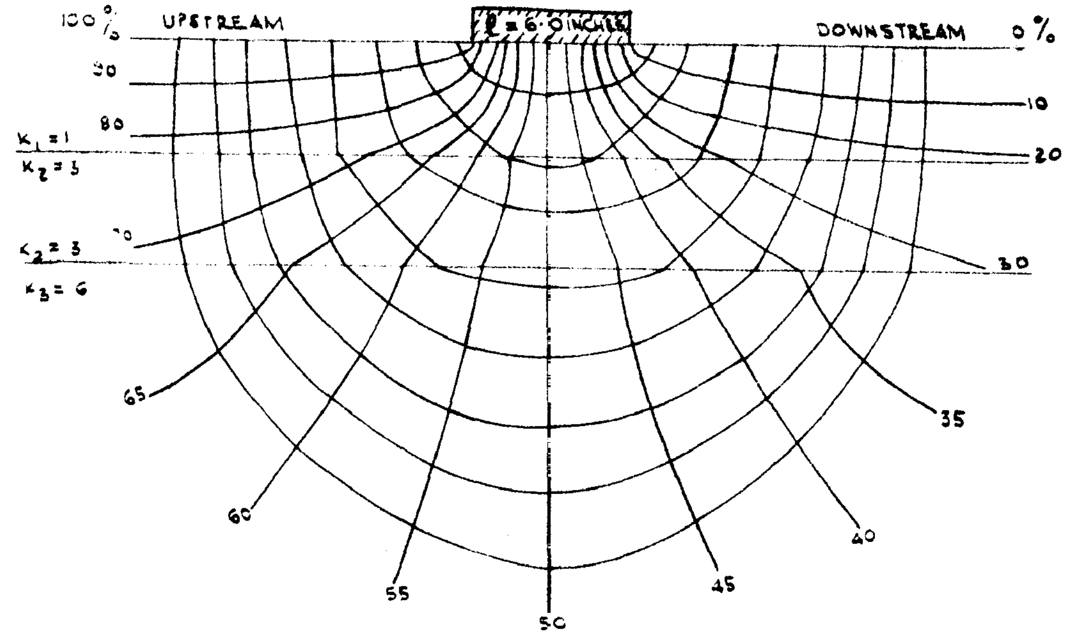
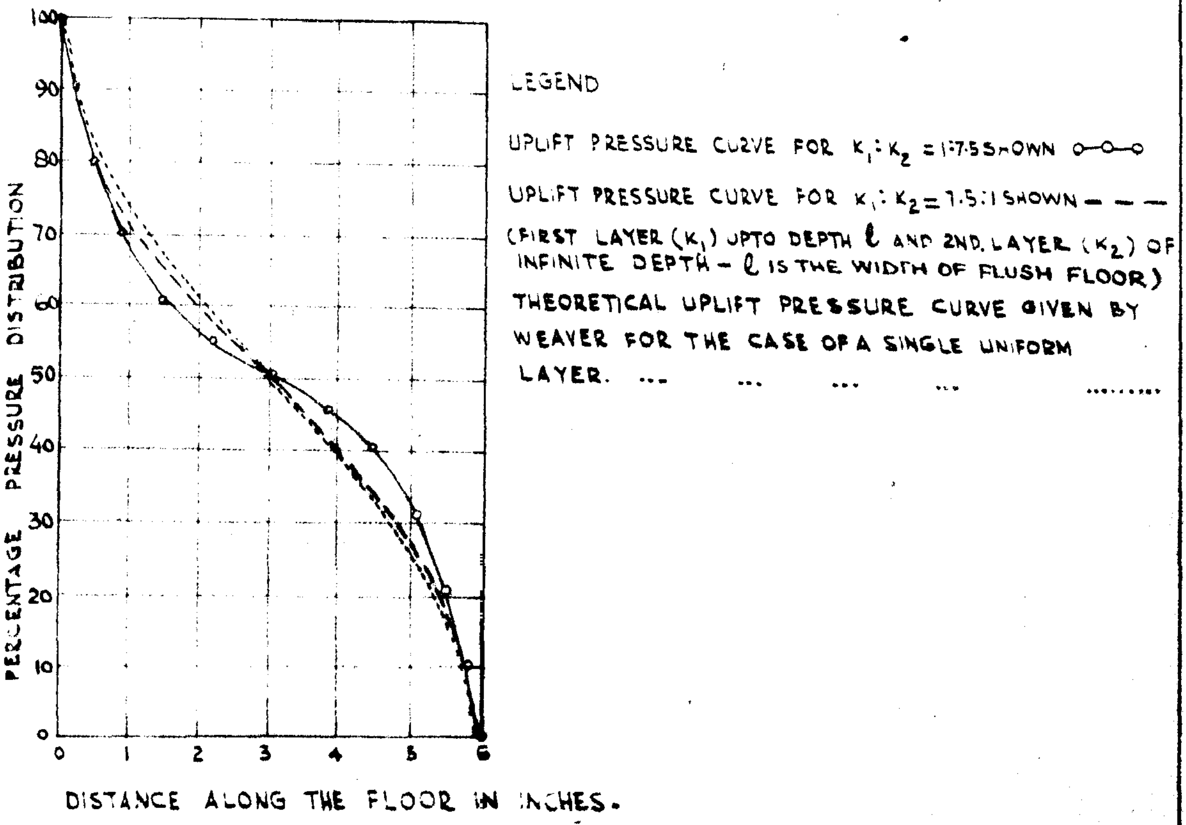


FIGURE SHOWING FLOWNET WHEN $K_1 : K_2 : K_3 = 1:3:6$

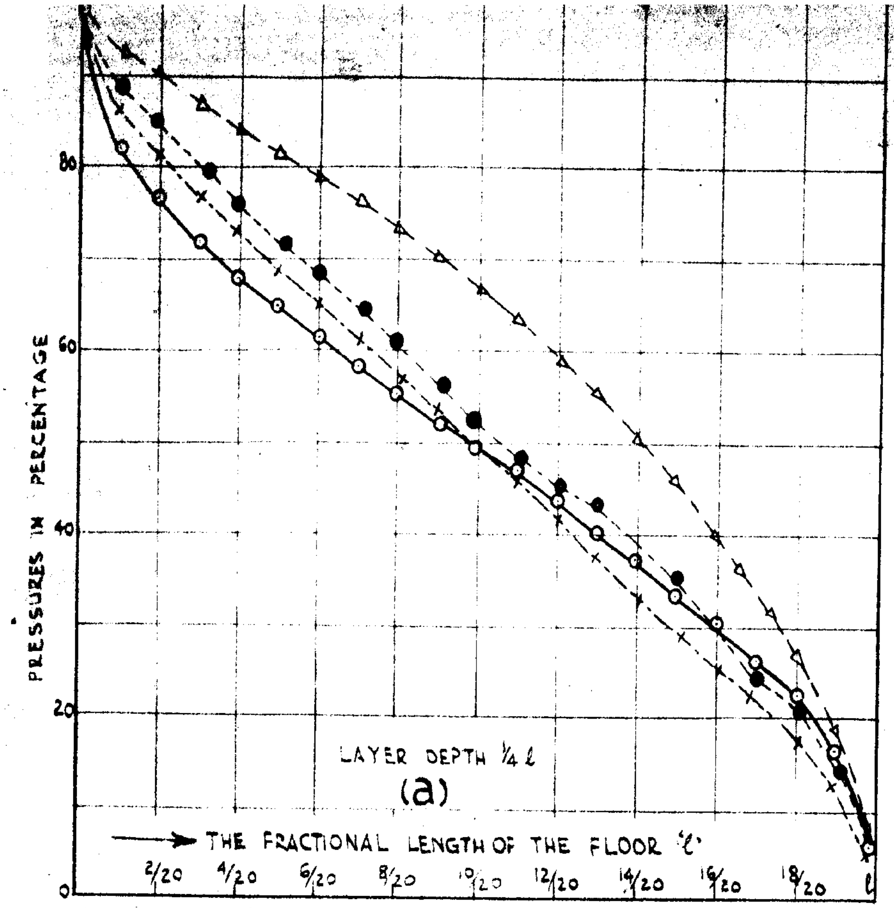
(a)



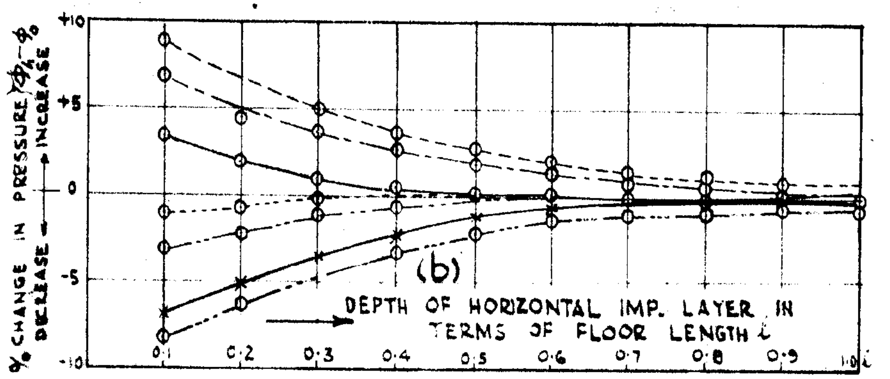
(b)

FIGURE SHOWING THE PRESSURE DISTRIBUTION ALONG THE BASE OF THE FLUSH FLOOR

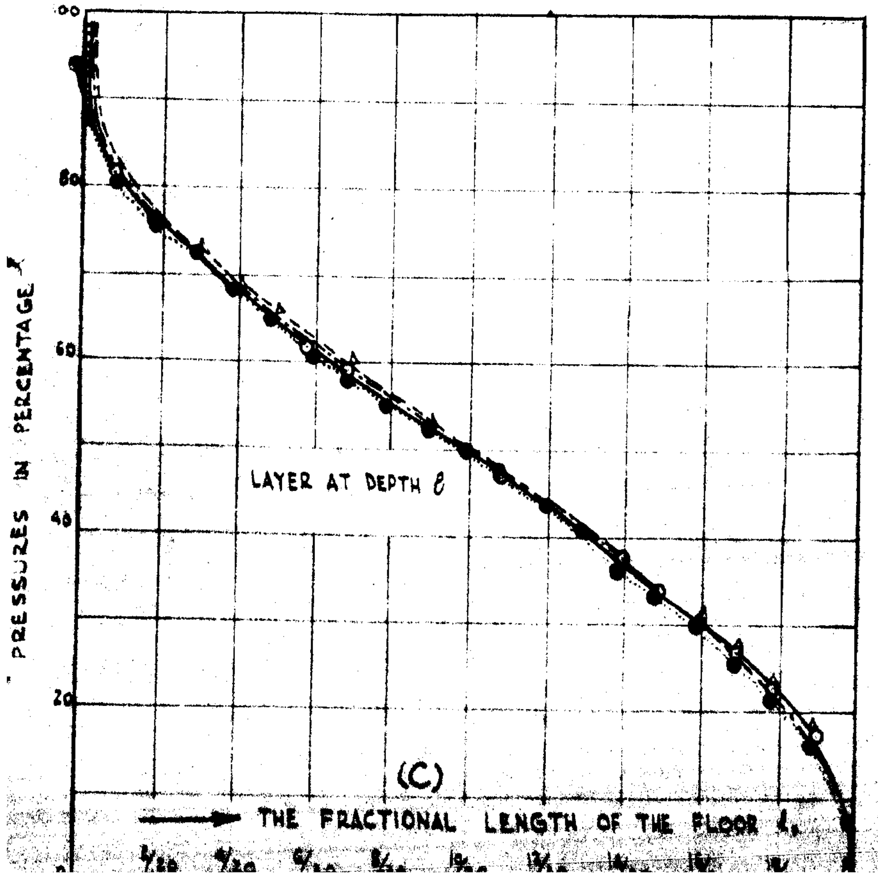
(REPRODUCED FROM REFERENCE 53)



SHOWING PRESSURE AT DIFFERENT POINT ON THE HORIZONTAL FLOOR DUE TO IMPERVIOUS LAYER AT DEPTH 1/4 l

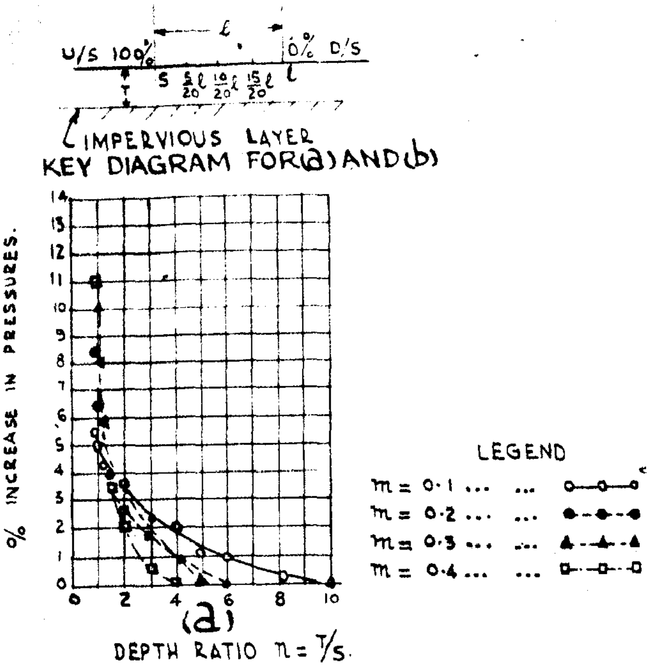


SHOWING CHANGE IN PRESSURE AT DIFFERENT POINTS ON THE HORIZONTAL FLOOR DUE TO HORIZONTAL IMPERVIOUS LAYERS AT VARIOUS DEPTHS.

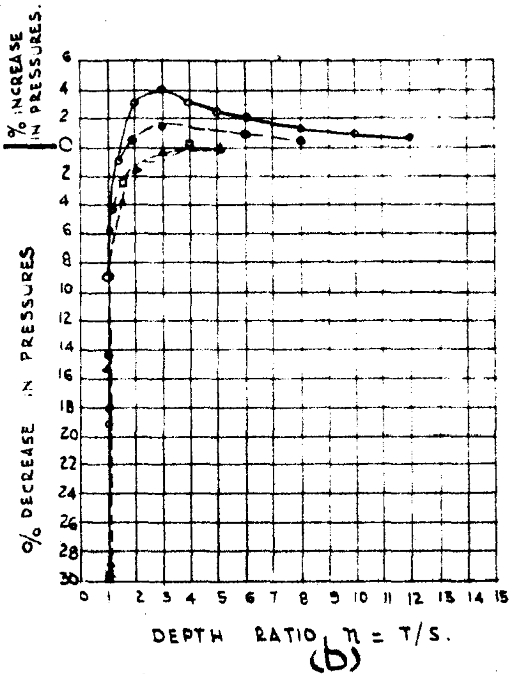


SHOWING PRESSURES AT DIFFERENT POINTS ON THE HORIZONTAL FLOOR DUE TO IMPERVIOUS LAYER AT DEPTH l.
(REPRODUCED FROM REFERENCE 54)

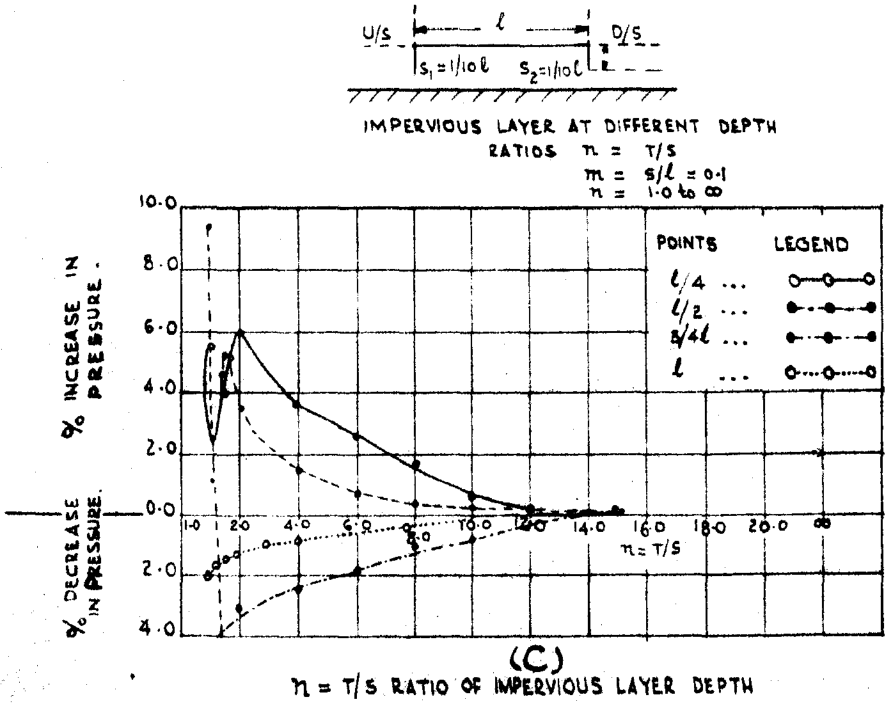
DRAWING.13



SHOWING INCREASE IN PRESSURES ON MIDDLE POINT OF UPSTREAM FACE OF THE SHEET PILE FOR DIFFERENT VALUES OF n = T/S.



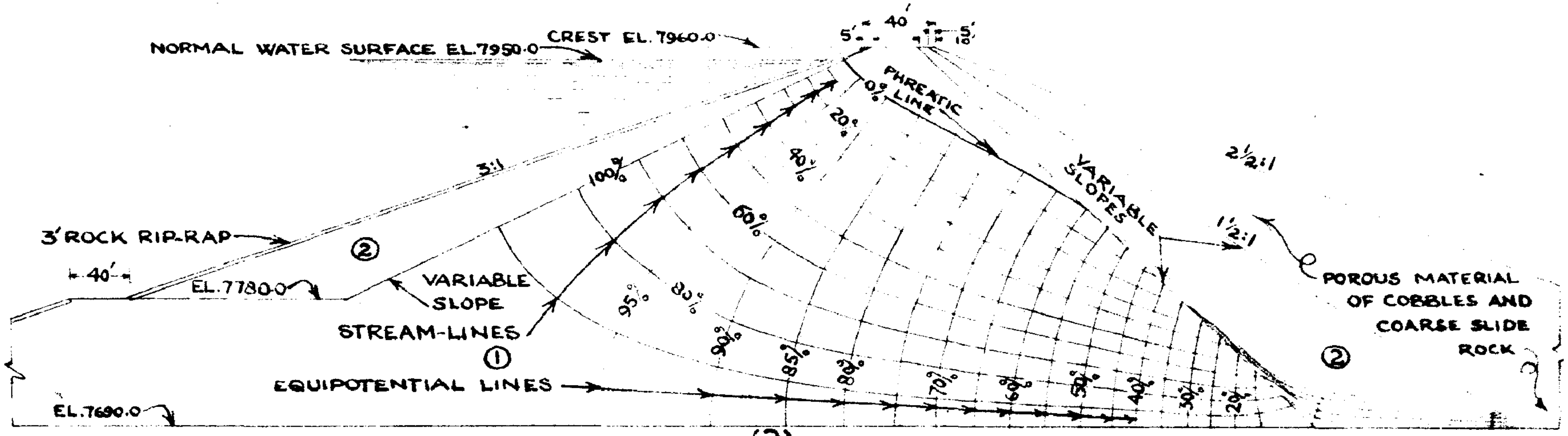
SHOWING DECREASE IN PRESSURES ALONG THE FLOOR AT POINT ¼ℓ FOR DIFFERENT VALUES OF n = T/S.



n = T/S RATIO OF IMPERVIOUS LAYER DEPTH TO SHEET PILE DEPTH.

SHOWING THE VARIATION IN PRESSURES FOR DIFFERENT VALUES OF n = T/S, ALONG THE FLOOR AT IMPORTANT POINTS AS ℓ/4, ℓ/2, 3/4ℓ, AND ℓ.

(REPRODUCED FROM REFERENCE 55)

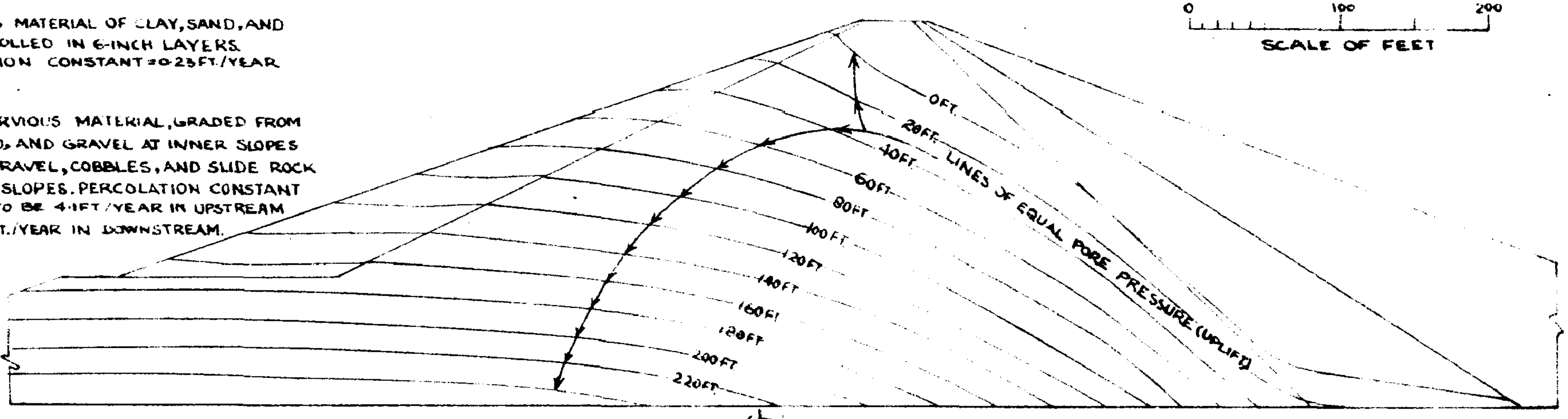


(a)

TYPICAL FLOW-NET FOR AN EARTH DAM (GREEN MOUNTAIN DAM, COLORADO, U.S.A.) OBTAINED, BY TWO-DIMENSIONAL ELECTRIC ANALOGY

① IMPERVIOUS MATERIAL OF CLAY, SAND, AND GRAVEL ROLLED IN 6-INCH LAYERS. PERCOLATION CONSTANT = 0.23FT./YEAR

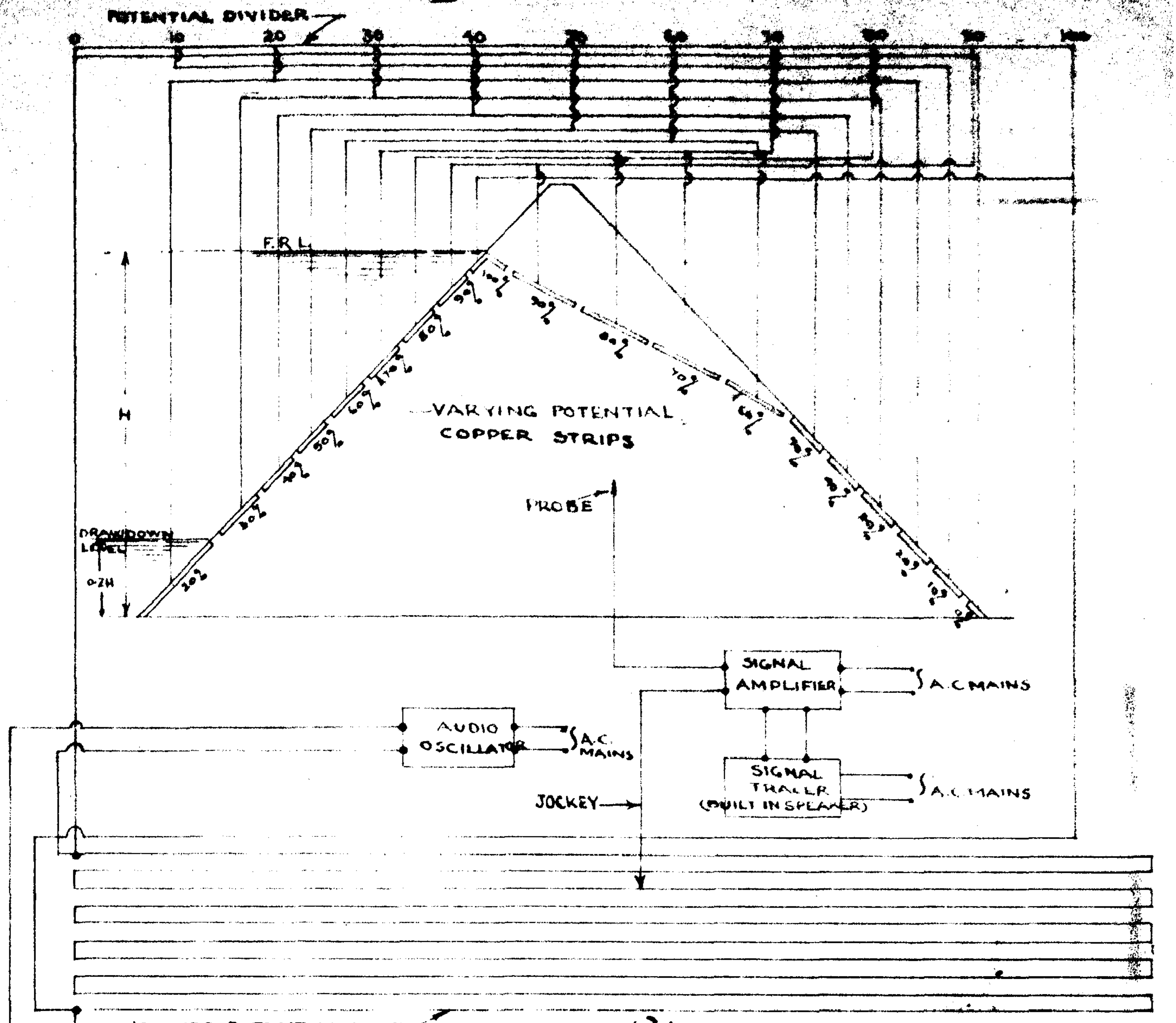
② SEMI-IMPERVIOUS MATERIAL, GRADED FROM CLAY, SAND, AND GRAVEL AT INNER SLOPES TO SAND, GRAVEL, COBBLES, AND SLIDE ROCK AT OUTER SLOPES. PERCOLATION CONSTANT ASSUMED TO BE 4.1FT./YEAR IN UPSTREAM ZONE, 9.2FT./YEAR IN DOWNSTREAM.



(b)

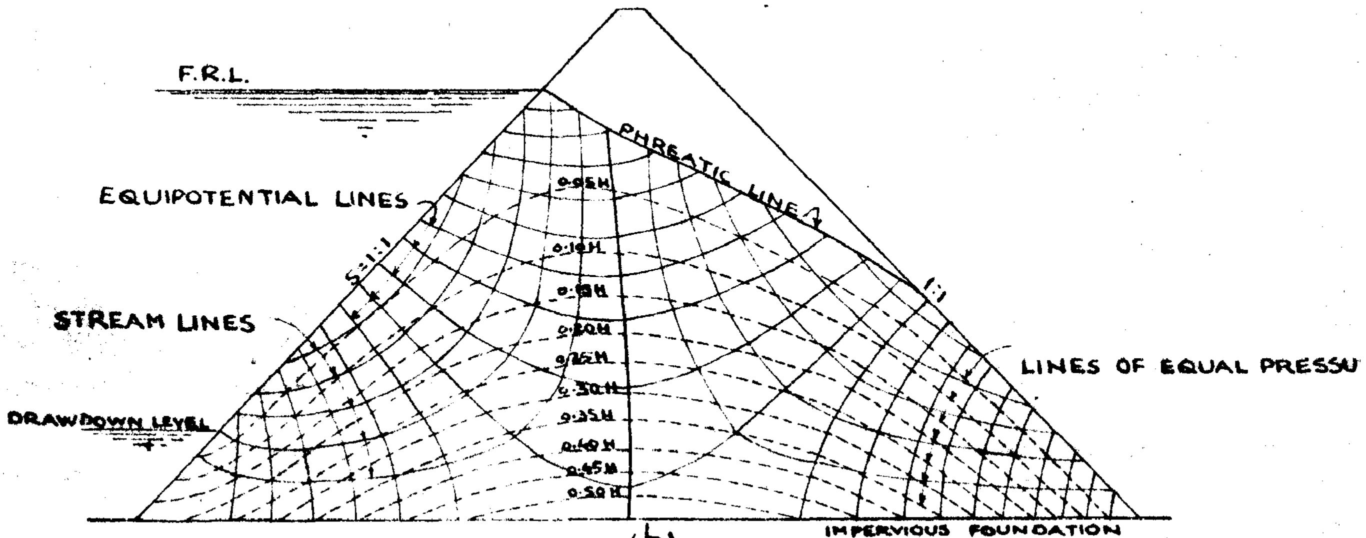
PRESSURE-NET FOR THE GREEN MOUNTAIN DAM, COLORADO, U.S.A., COMPUTED FROM ABOVE FLOW-NET

(REPRODUCED FROM ENGINEERING MONOGRAPH NO.8, U.S.B.R.)
REFERENCE 33



(a)

TWO-DIMENSIONAL ELECTRICAL ANALOGY MODEL FOR SOLUTION OF A DRAWDOWN PROBLEM OF AN EARTH DAM

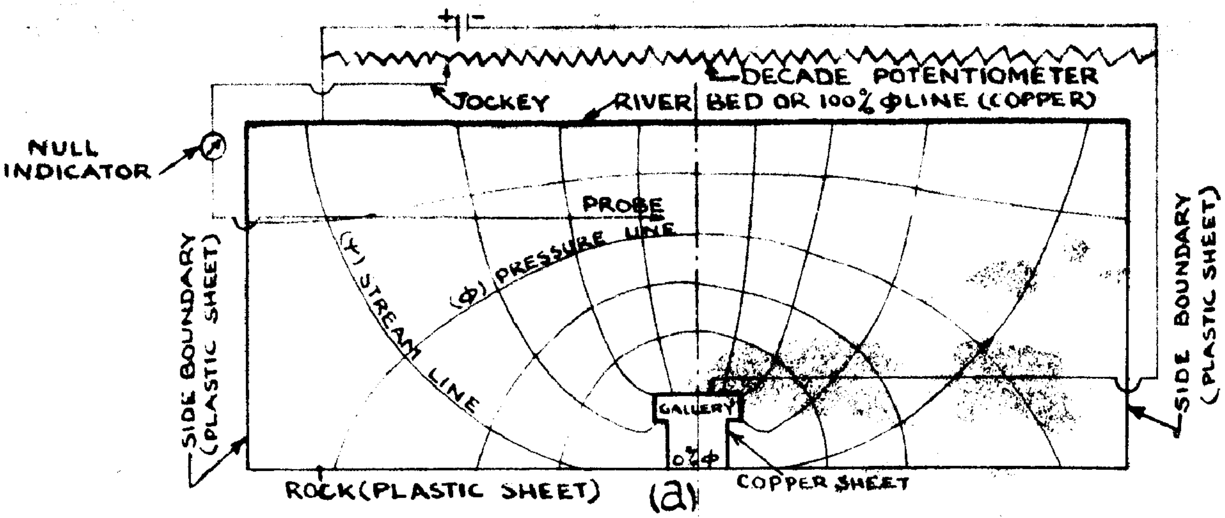


(b)

RAPID DRAWDOWN FLOW AND PRESSURE NETS FOR 1:1 UPSTREAM AND 1:1 DOWNSTREAM SLOPES, HOMOGENEOUS AND ISOTROPIC MATERIAL

(REPRODUCED FROM ENGINEERING MONOGRAPH NO.8, U.S.B.R.)

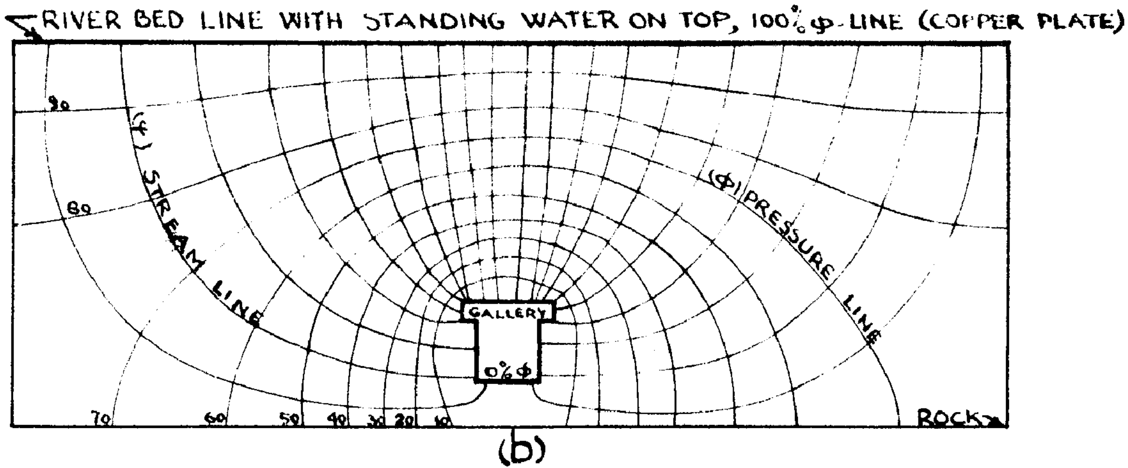
REFERENCE 33



(a)

2-D ELECTRICAL ANALOGY SET UP FOR STUDYING FLOW THROUGH A GALLERY RESTING DIRECTLY OVER AN IMPERMEABLE ROCK.

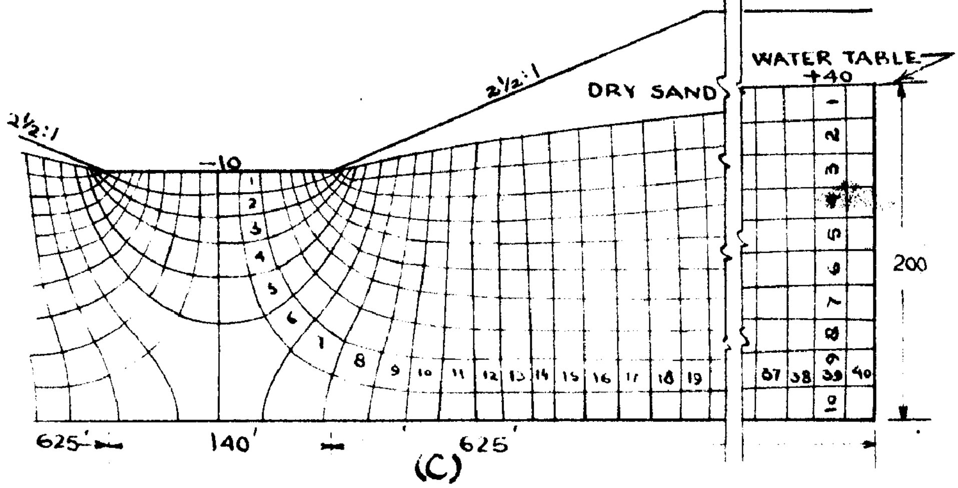
SHAPE FACTOR($N_\psi \div N_\phi$) = 1·52



(b)

FLOW-NET FOR FLOW THROUGH GALLERY WHEN IT IS RAISED ABOVE THE IMPERMEABLE ROCK.

SHAPE FACTOR($N_\psi \div N_\phi$) = 1·94

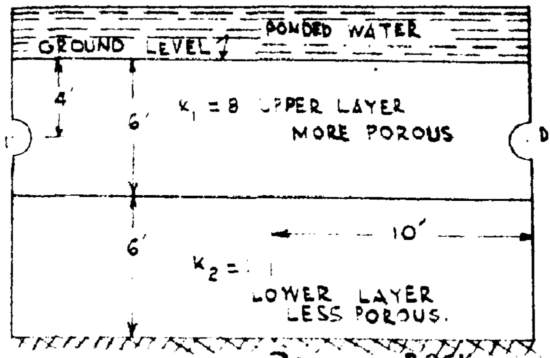


(C)

FLOW-NET OBTAINED BY ELECTRICAL ANALOGY FOR FLOW THROUGH COFFER DAM ACROSS A RIVER

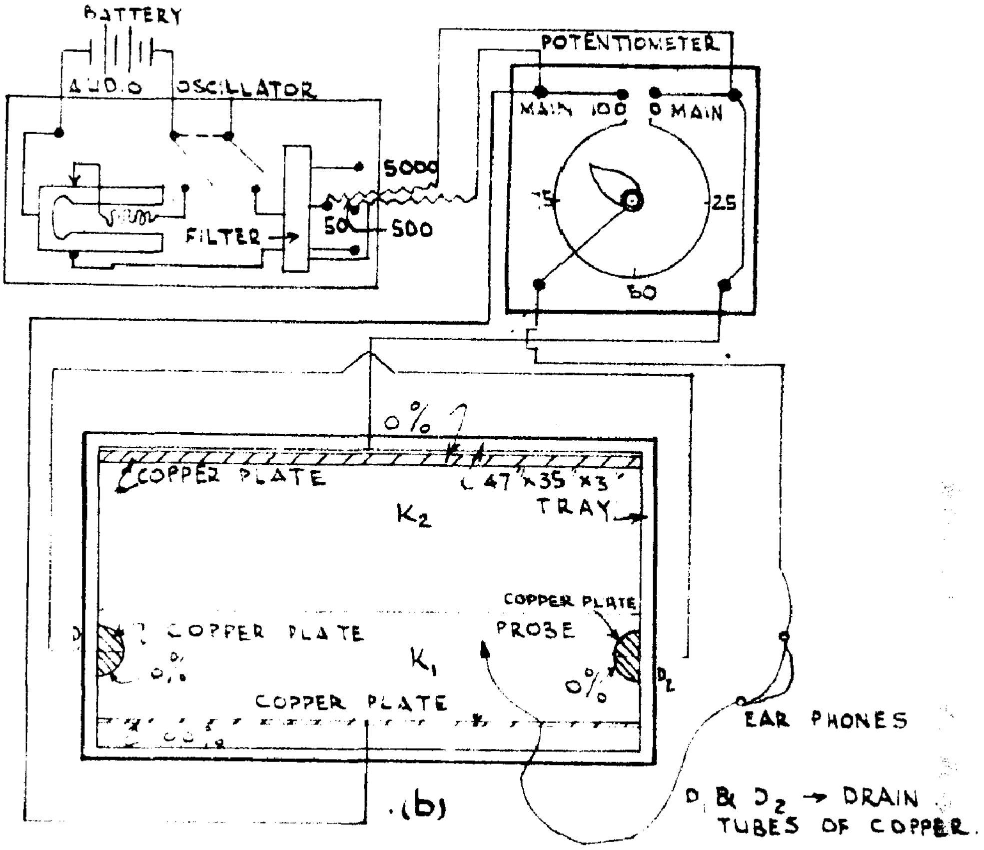
SHAPE FACTOR ($N_\psi \div N_\phi$) = 0·25
(REPRODUCED FROM C.B.I.& P., INDIA PUBLICATION NO.55)
REFERENCE 32

DRAWING.17



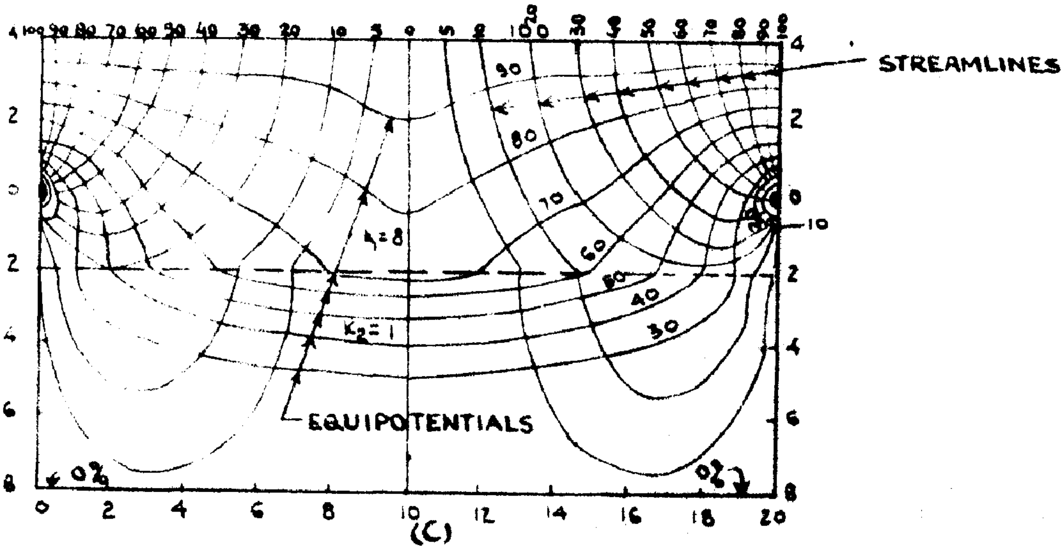
(a)

SHOWING VERTICAL SECTION OF FIELD WITH PONDED WATER AND DRAIN TUBES.



(b)

SHOWING THE EXPERIMENTAL SET-UP AND THE ELECTRICAL ANALOGY TRAY

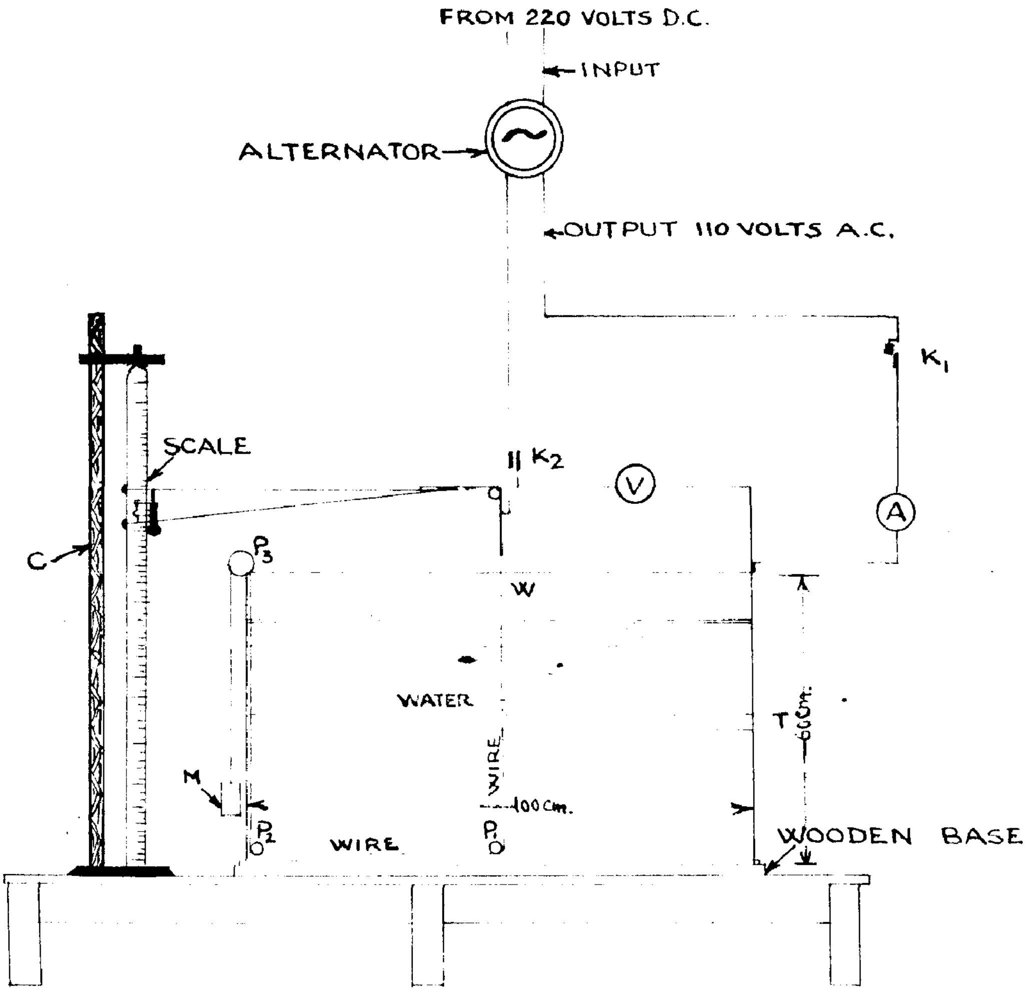


(C)

SHOWING COMPLETE FLOWNET, WHEN $K_1 : K_2 = 8:1$

(REPRODUCED FROM REFERENCE 62)

DRAWING.18



VAIDHIANATHAN'S ELECTRICAL ANALOGY

SET-UP FOR COMPARATIVE

STUDIES ON TUBEWELLS.

(FROM REFERENCE 32)

LEGEND

A.C. VOLTMETER – V

A.C. AMMETER – A

KEYS – K_1, K_2

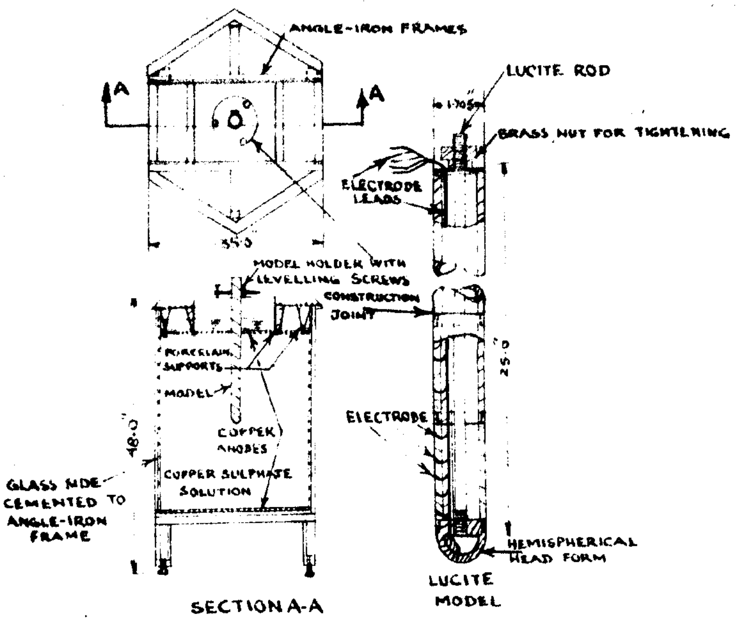
PULLEYS – P_1, P_2, P_3

SILVER WIRE – W

CLAMP – C

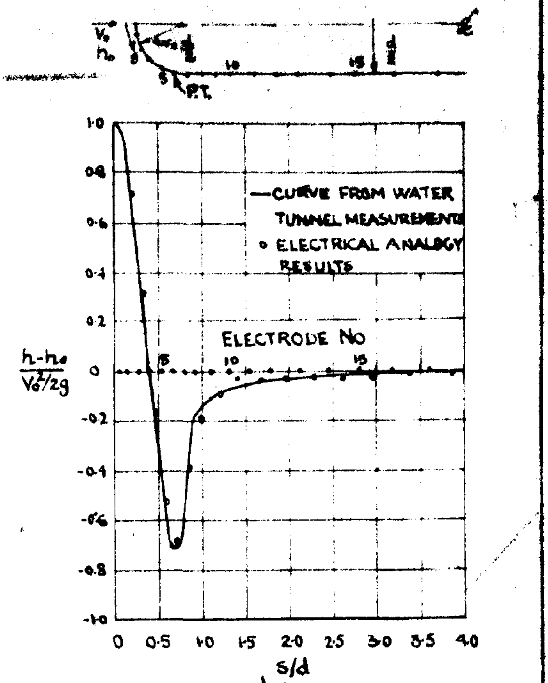
COUNTER-WEIGHT – M

DRAWING.19



(a)

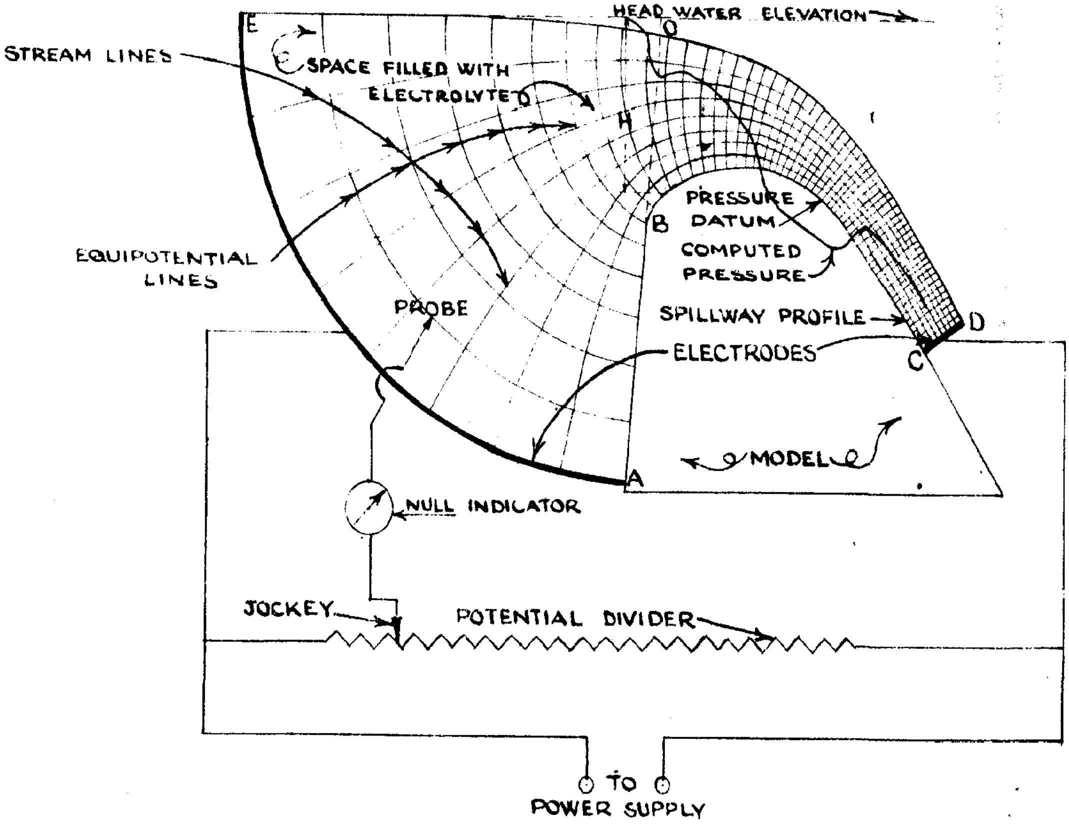
DETAILS OF AN ELECTRICAL ANALOGY TANK AND A TYPICAL MODEL TO STUDY FLOW PATTERN AROUND BODIES IN PARALLEL UNIFORM FLOW



(b)

TYPICAL RESULT SHOWING PRESSURE DISTRIBUTION AROUND A HEMISPHERICAL HEAD FORM WITH CYLINDRICAL AFTERBODY

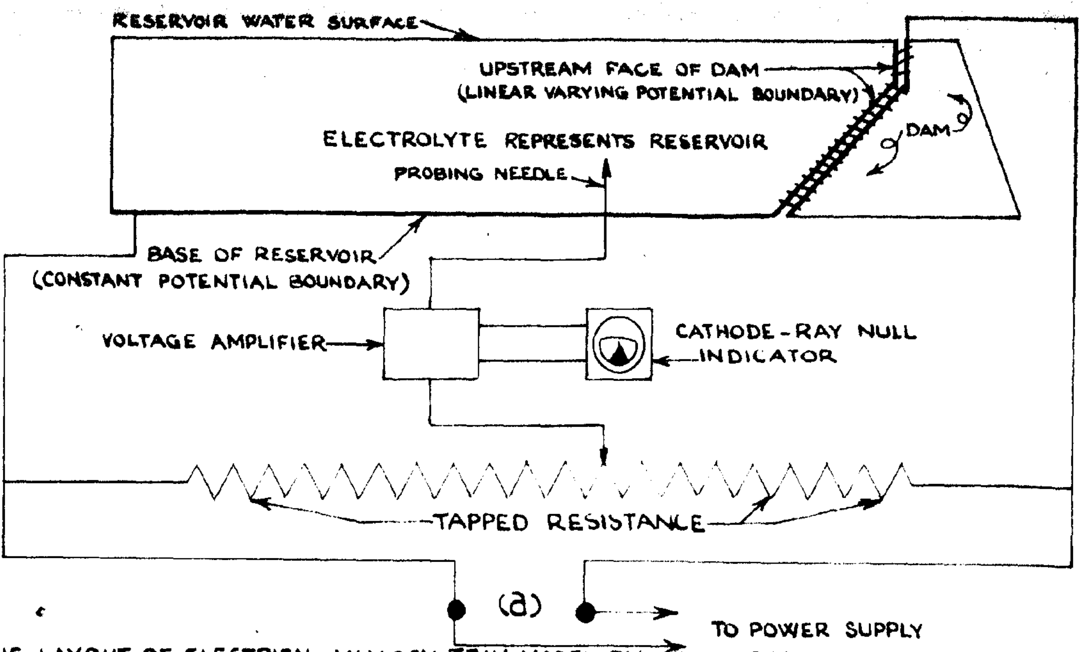
(AFTER P.G.HUBBARD)
REFERENCE 15



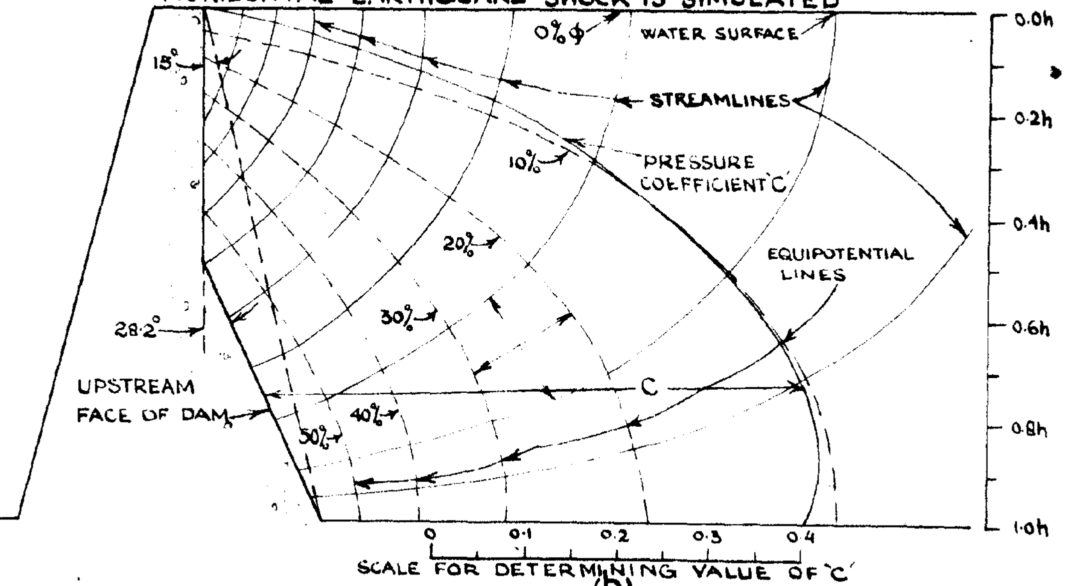
(C)

SHOWING THE PRINCIPLE OF ELECTRICAL ANALOGY APPLIED TO THE STUDY OF CAVITATION AT THE CRESTS OF SPILLWAY DAMS, AND A TYPICAL RESULT OBTAINED

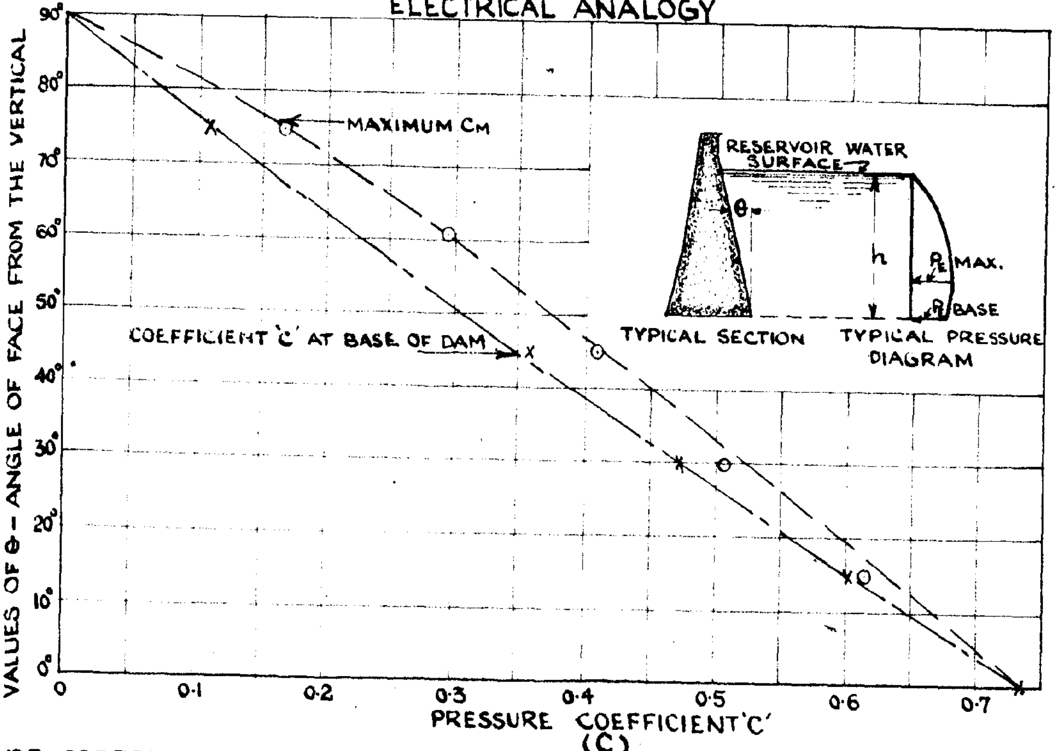
DRAWING.20



SHOWING LAYOUT OF ELECTRICAL ANALOGY TRAY MODEL BY WHICH PRESSURE ON FACE OF DAM DUE TO HORIZONTAL EARTHQUAKE SHOCK IS SIMULATED

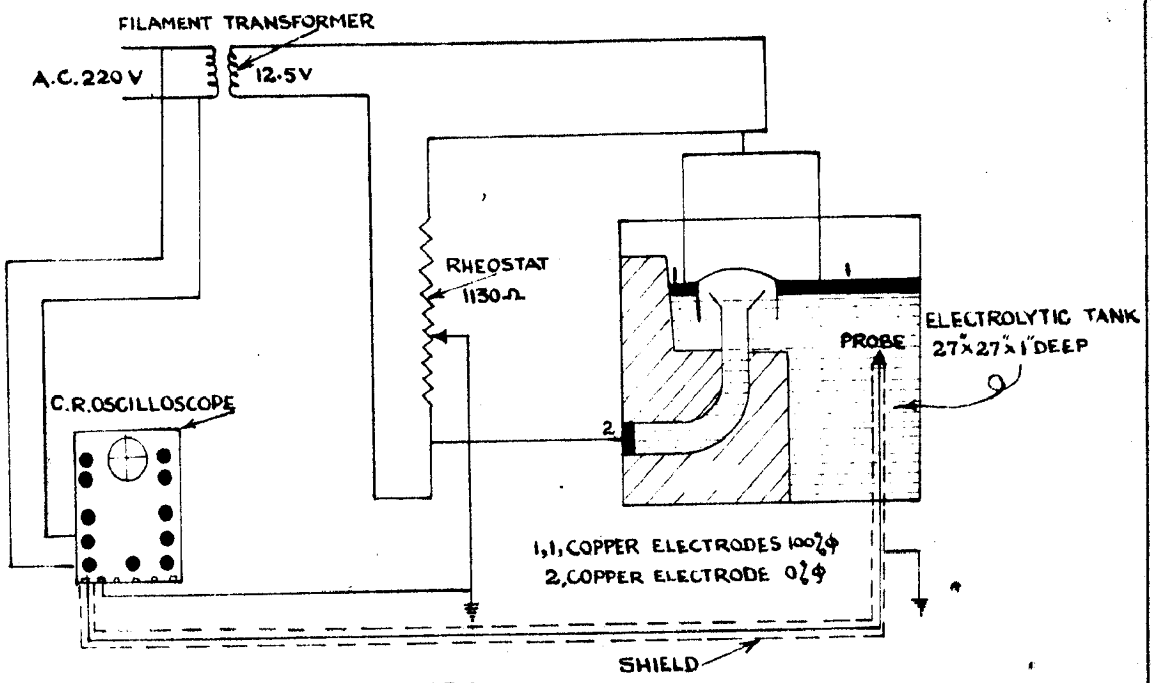


TYPICAL FLOW-NET OBTAINED FOR A HORIZONTAL EARTHQUAKE STUDY BY ELECTRICAL ANALOGY



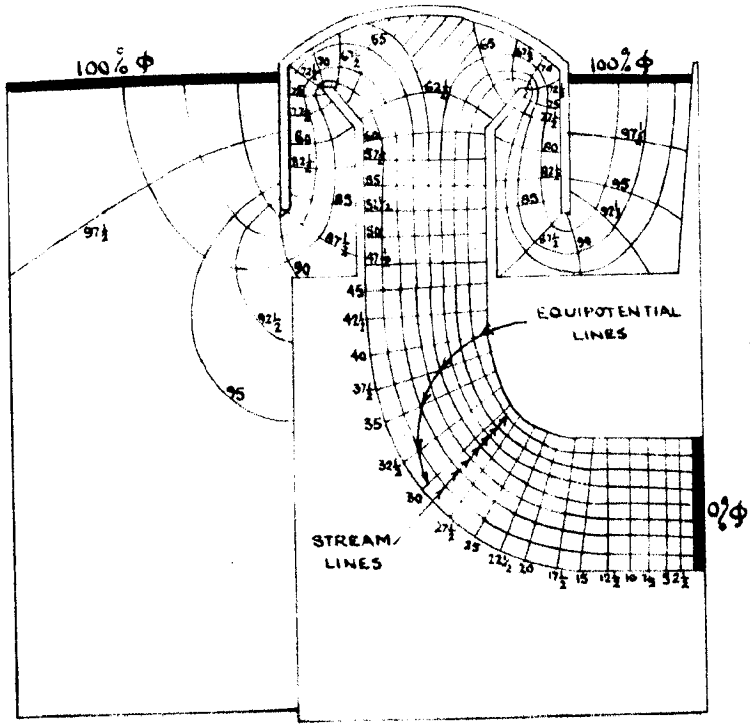
PRESSURE COEFFICIENTS FOR CONSTANT SLOPING FACES BY ELECTRICAL ANALOGY METHOD

DRAWING.21



(a)

TWO-DIMENSIONAL ELECTRICAL ANALOGY SET-UP FOR STUDY OF FLOW THROUGH VOLUTE SYPHONS

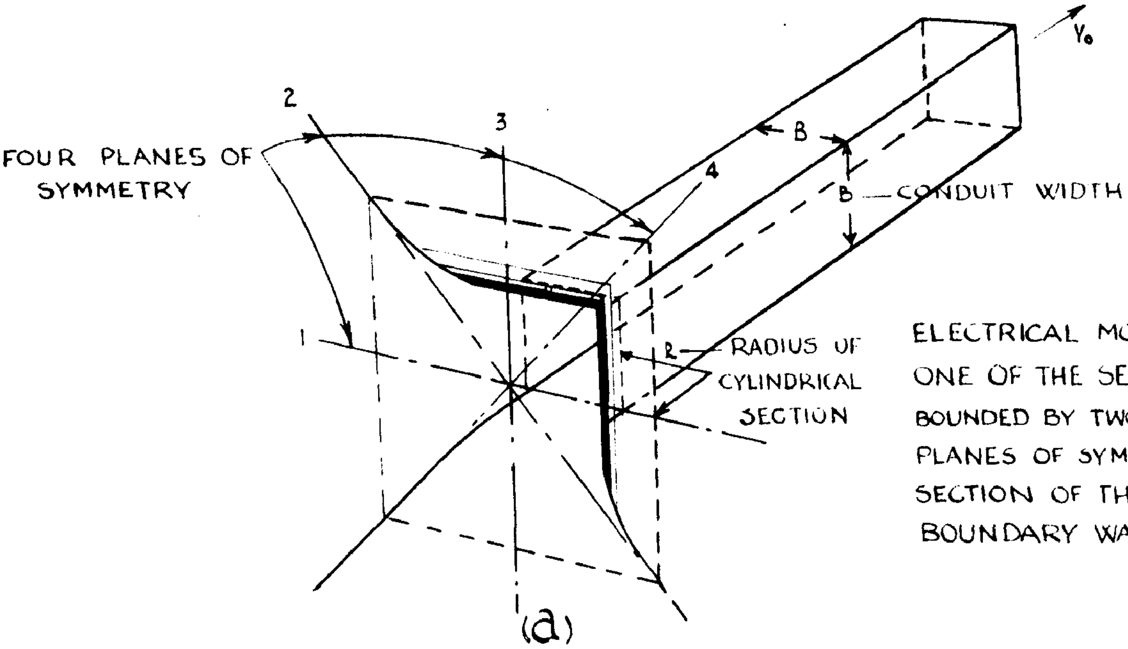


(b)

TYPICAL FLOW-NET FOR FLOW THROUGH A VOLUTE SYPHON BY ELECTRICAL ANALOGY

(FROM INDIAN INSTITUTE OF SCIENCE, BANGALORE)
ANNUAL REPORTS, 1952-1953
REFERENCES 51-52

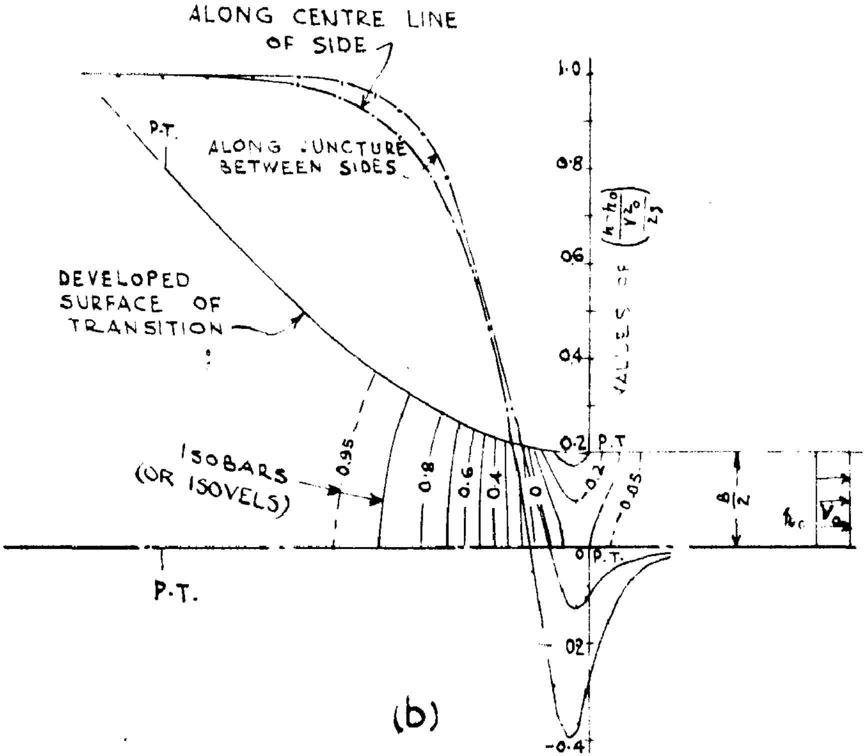
DRAWING.22



ELECTRICAL MODEL OF ANY ONE OF THE SEGMENTS BOUNDED BY TWO ADJACENT PLANES OF SYMMETRY AND A SECTION OF THE TRANSITION BOUNDARY WAS USED

(a)

SCHEMATIC DIAGRAM OF INLET



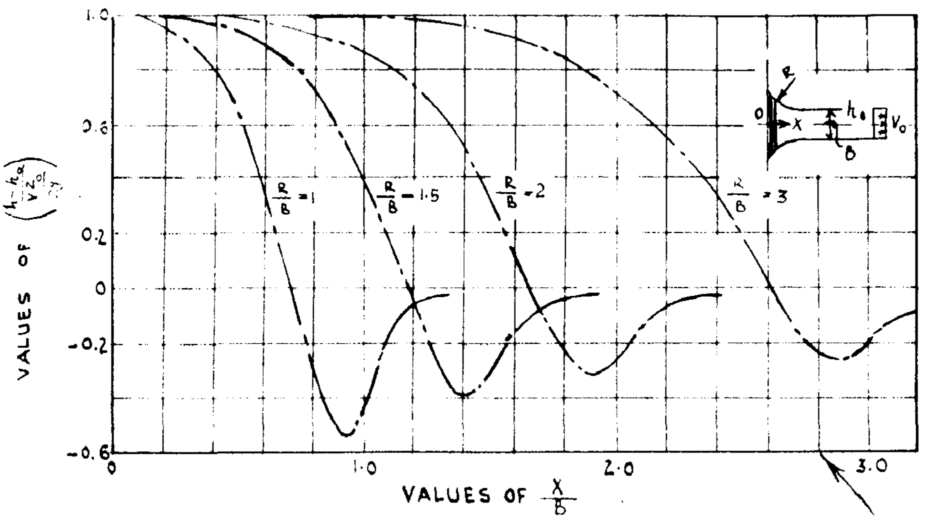
(b)

TYPICAL PATTERN OF PRESSURES ALONG BOUNDARY OF ROUNDED TRANSITION FROM A RESERVOIR TO A SQUARE CONDUIT (R/B = 1.5)

(AFTER HUBBARD AND LING)

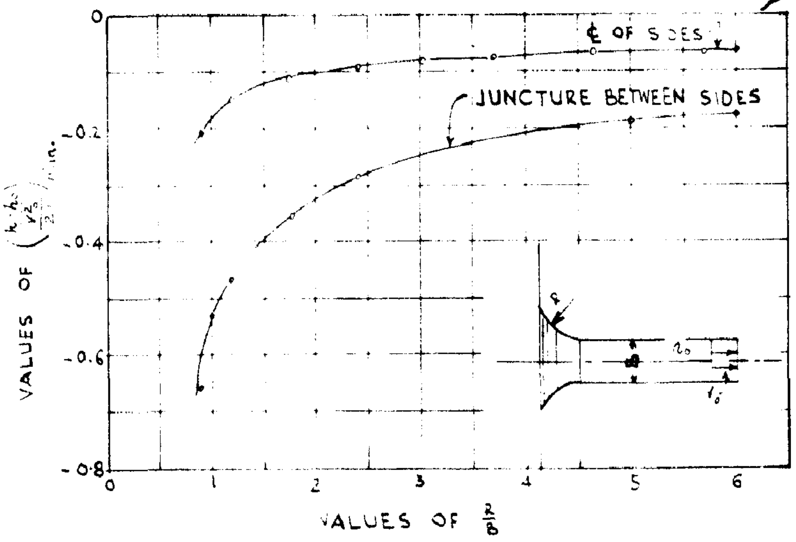
REFERENCE 16

DRAWING.23



EFFECT OF RELATIVE CURVATURE ON PRESSURE DISTRIBUTION ALONG JUNCTURE OF SIDES.

INDICATES QUANTITATIVELY THE BENEFITS ACHIEVED FROM DECREASING THE CURVATURE

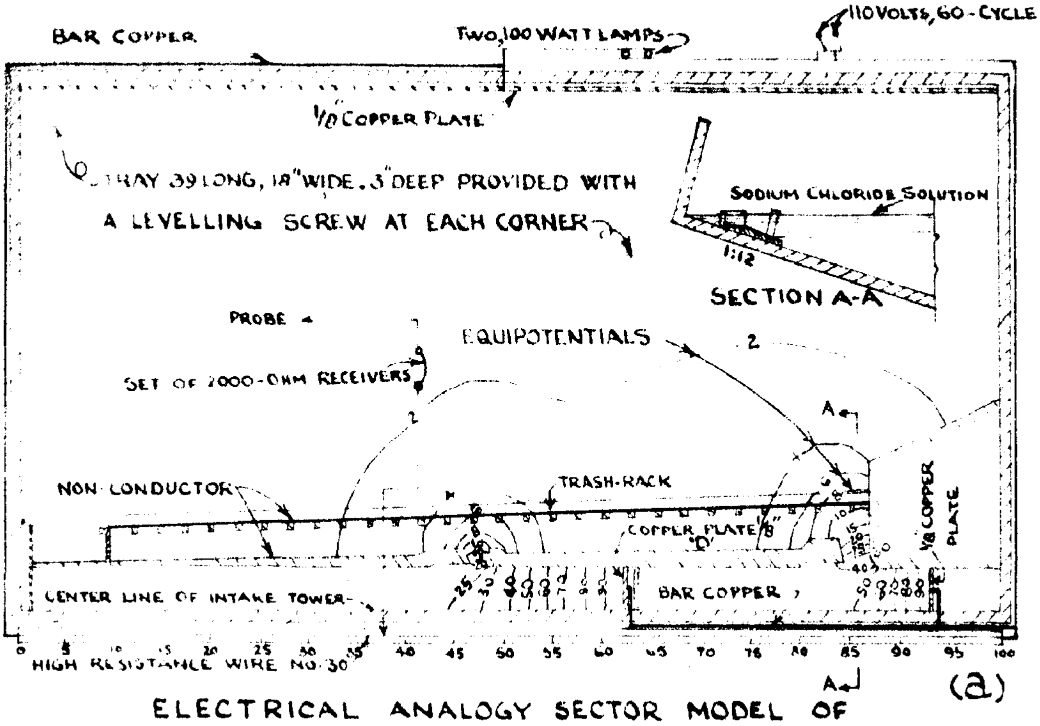


MINIMUM PRESSURE AS A FUNCTION OF RELATIVE CURVATURE.—

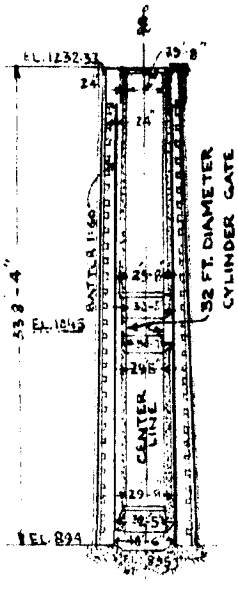
INDICATES TENDENCY TOWARD CAVITATION DECREASES INDEFINITELY WITH INCREASING RADIUS OF CURVATURE

(REPRODUCED FROM REFERENCE 16)

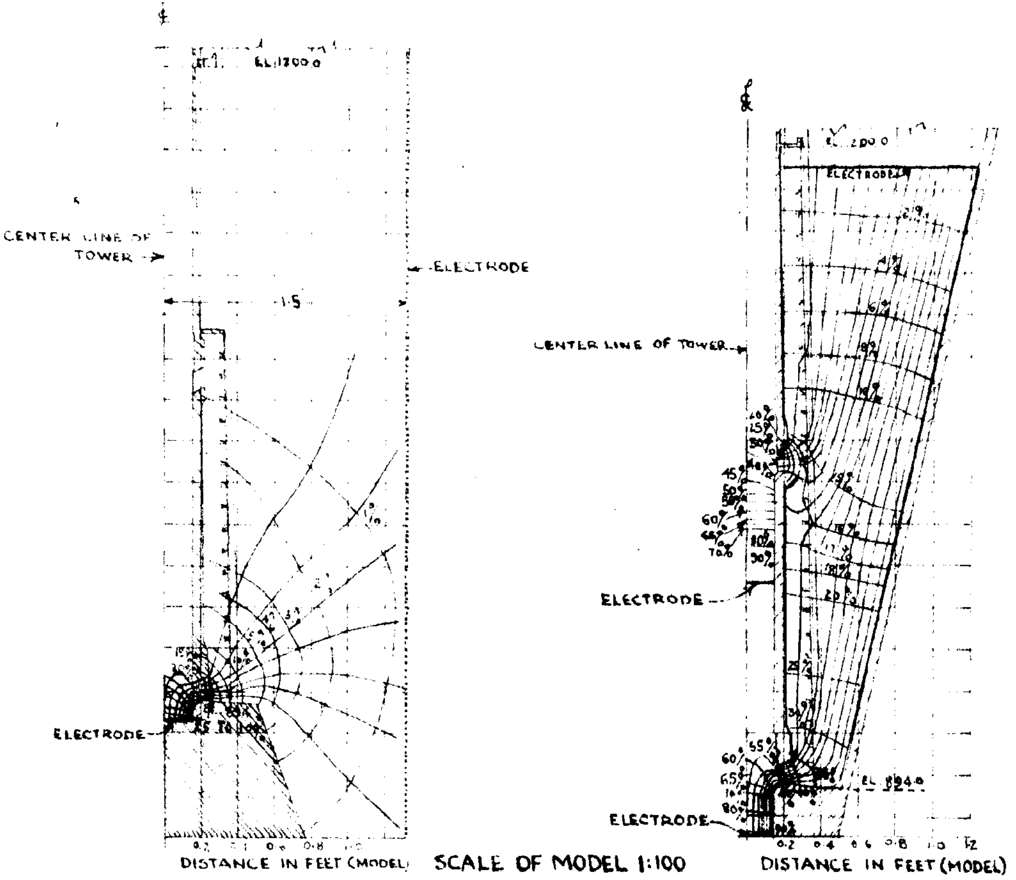
DRAWING.24



ELECTRICAL ANALOGY SECTOR MODEL OF INTAKE TOWER OF BOULDER DAM.



SECTION ON CENTER LINE OF TOWER



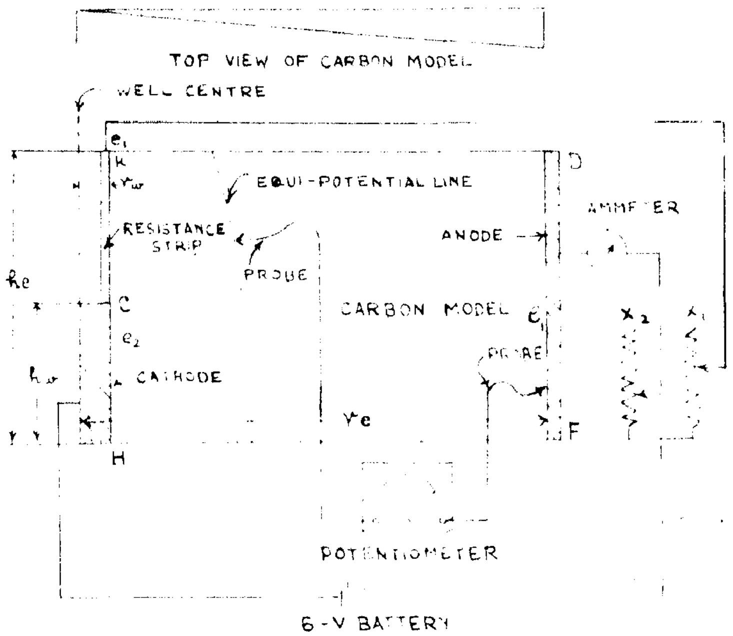
TYPICAL ELECTRICAL ANALOGY RESULTS FROM SECTOR MODEL OF IN-TAKE TOWER OF BOULDER DAM

(b) ON RADIAL SECTION OF TOWER NEAREST TO THE RIVER, UPPER GATE CLOSED, 182 FT. LENGTH OF TRASHRACK

(C) ON RADIAL SECTION OF TOWER NEAREST THE CANYON WALL, BOTH GATES OPEN, 306 FT. LENGTH OF TRASHRACK

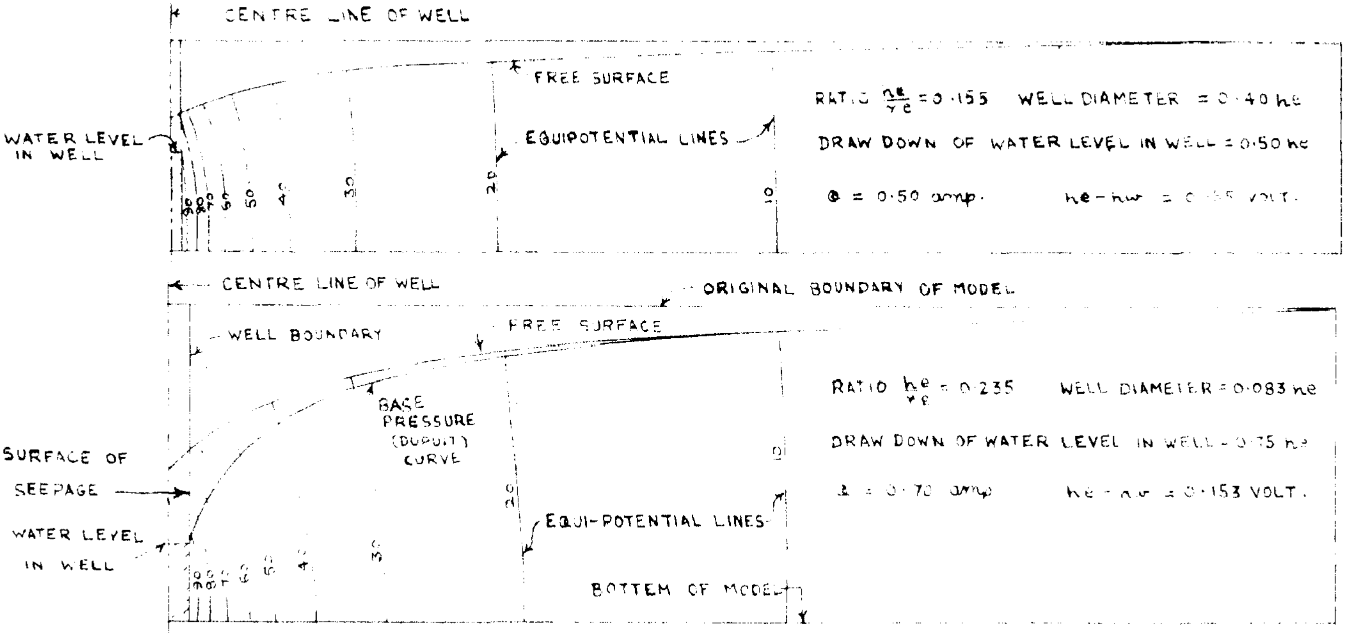
(FROM BOULDER CANYON PROJECT FINAL REPORTS, U.S.B.R.)
REFERENCE 12

DRAWING No. 25



(a)

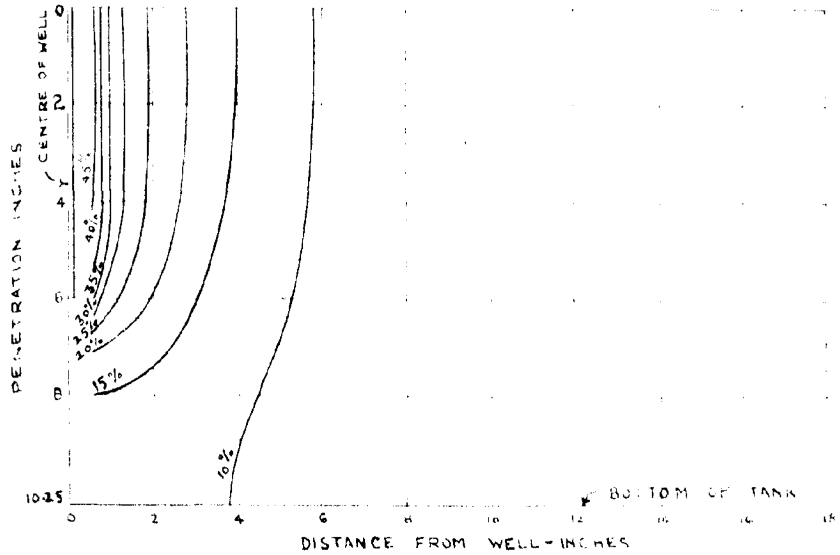
CARBON MODEL SHOWING ELECTRICAL CIRCUIT AND CONNECTIONS TO STUDY UNCONFINED RADIAL FLOW TO A WELL



(b)

TYPICAL FREE SURFACES DEVELOPED IN TESTS BY ELECTRICAL MEASUREMENTS

(REPRODUCED FROM REFERENCE 14)

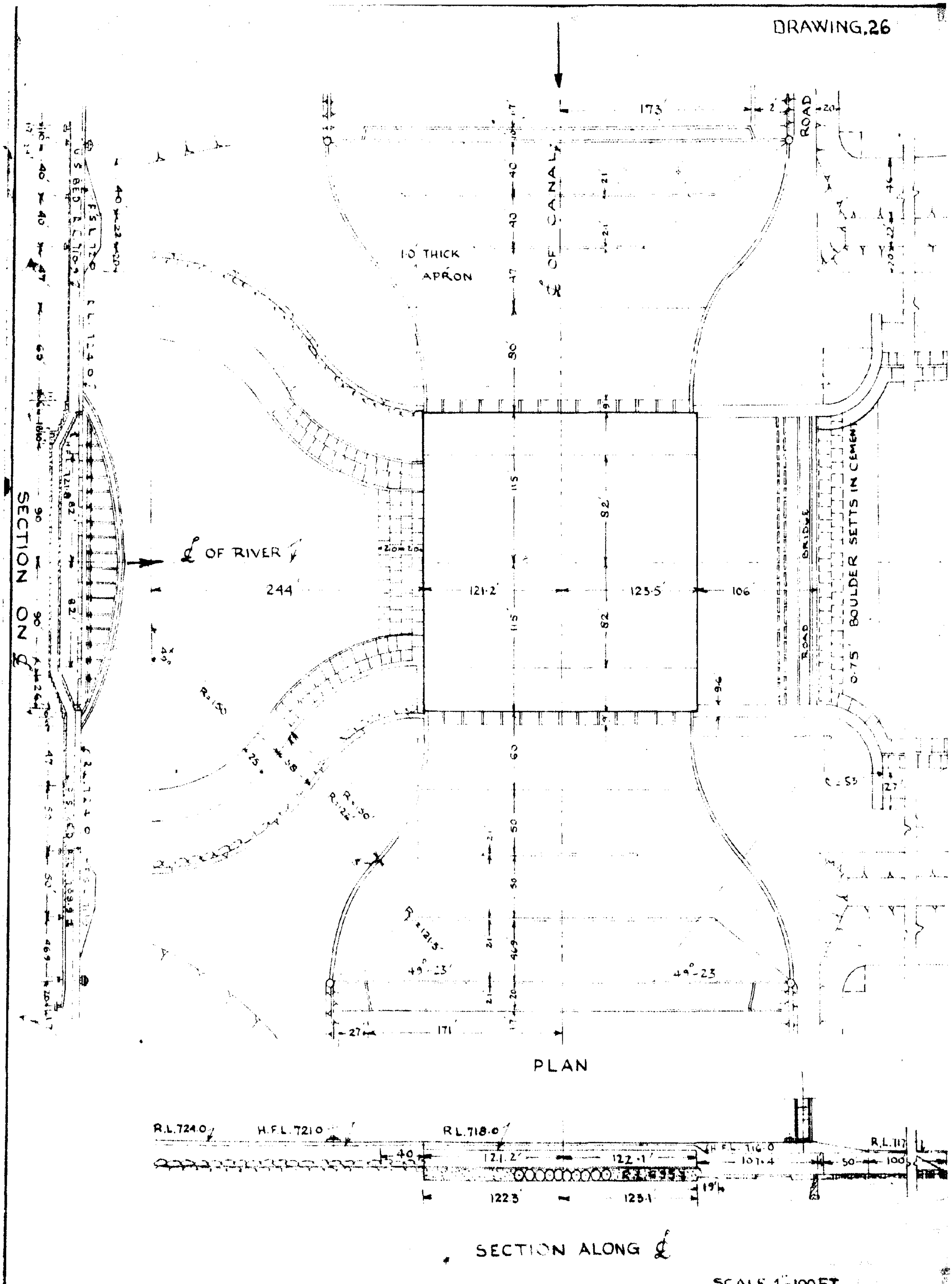


(C)

EQUIPOTENTIAL CURVES FOR SIX-INCH PENETRATING ELECTRODE

(REPRODUCED FROM REFERENCE 18)

DRAWING.26



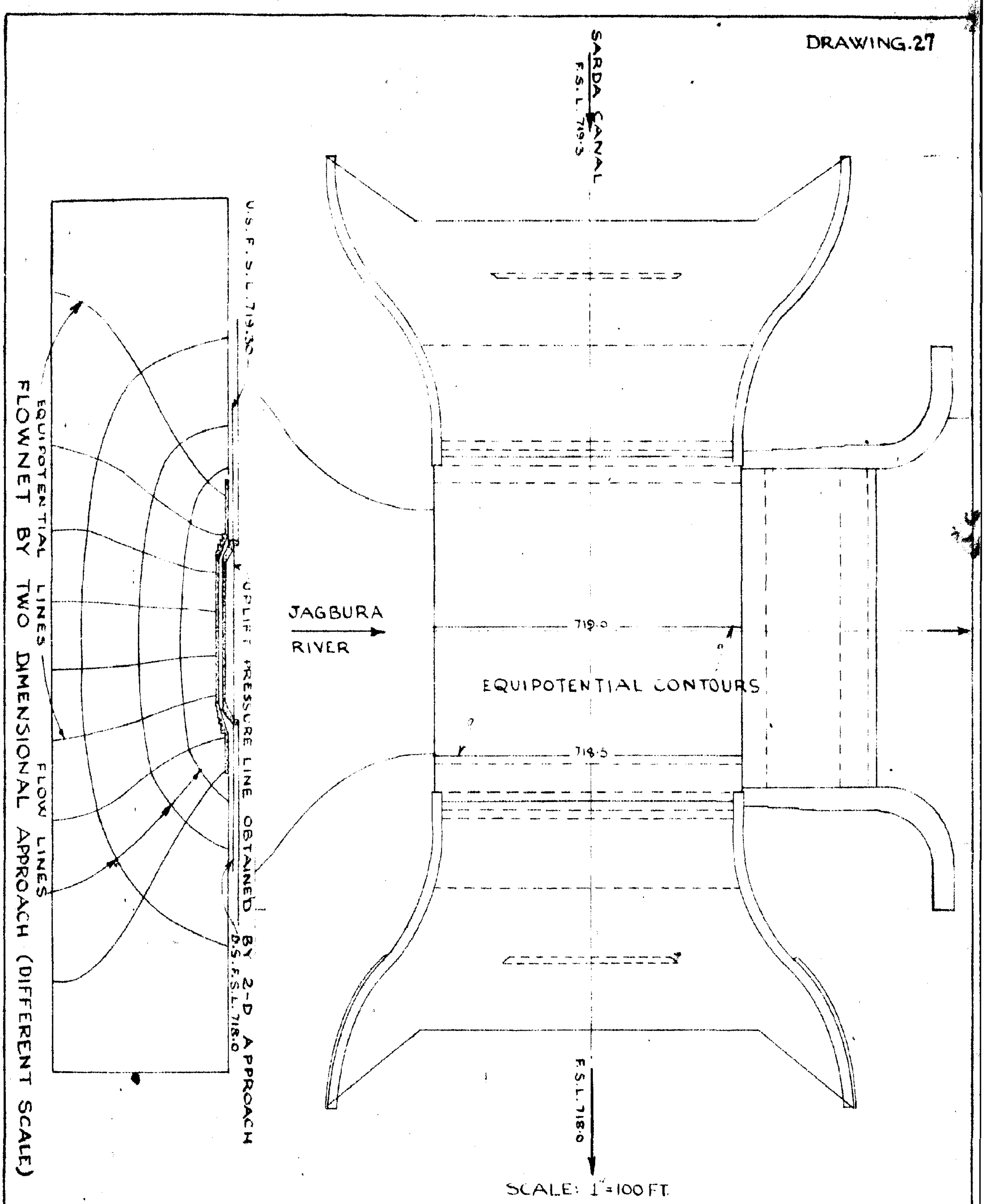
(REPRODUCED FROM TECH. MEMO. 25, I.R.I. ROORKEE)
REFERENCE 28

SCALE 1"=100 FT.

JAGBURA SYPHON

GENERAL PLAN AND SECTIONS

DRAWING.27



FLOWNET BY TWO DIMENSIONAL APPROACH (DIFFERENT SCALE)

(REPRODUCED FROM TECH.MEMO.25, I.R.I. ROORKEE)
REFERENCE 28

TOTAL DOWNWARD LOAD OF MAIN PORTION OF THE STRUCTURE —65.59 MILLION LBS.

TOTAL UPLIFT OBTAINED BY 2-D CALCULATIONS ... ... —80.04 MILLION LBS.

JAGBURA SYPHON

UPLIFT PRESSURE DISTRIBUTION BY TWO DIMENSIONAL APPROACH