SETTING UP A MOMENT ANALOGY COMPUTOR

FOR MULTI STOREYED FRAMES

BY

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Thesis for the Degree of Master of Engineering
in Structural Engineering including
Concrete Technology.

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C E R T I F I C A T E

This is to certify that the thesis entitled " Setting up a moment analogy computer for Multi-storeyed frames", which is being submitted by Sri T. Kishan Rao as a partial fulfilment for the degree of Master of Engineering in Structural Engineering including Concrete Technology of University of Roorkee, is a record of bonafide work carried out by him under my supervision and guidance. He has worked for a period of 7 months from July 15, 1959 to February 15, 1960 at Structural Engineering laboratory of University of Roorkee, Roorkee. The results embodied in this thesis have not been submitted for the award of any other degree or diploma.

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CONTENTS

CHAPTER 1

SYNOPSIS

The labour involved in the analysis of Indeterminate Structures may be greatly reduced by the use of Electrical Analogs, especially in those cases, where the many different dimensions have to be investigated. The basis of these analogs is the similarity between the structure and its analog. Which often permits the construction of the analog without regard to the governing equations.

Part one of the Dissertation recapitulates the previous relevant work which has been done during past two and half decades, on the structural problems of beams, trusses, frames and plates, under static as well as dynamic loads, utilising the equivalence between the properties of an electrical net work and the elastic properties of the structure.

Second part presents a working model of the Analyser for the analysis of multi storeyed building frames. This consists of a net work of circuits, in which each member of the structure is represented by a group of three resistors. Current is fed in to the circuit to simulate the effect of load on the structure, and the resultant distribution in the net work is recorded by meters. These meter readings multiplied by an appropriate scale factor gives the magnitude of the bending moment in the corresponding parts of the structure.

Experiments have been conducted for a three bay and

seven storeyed multi storeyed building frame for vertical and horizontal loads. Experimental results that are accurate to within 5.0% of the maximum theoretically calculated values have been obtained.

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CHAPTER 2

INTRODUCTION

The experimental analysis of indeterminate structure is a useful method which often gives results within the limits of accuracy required in practice in a very short time. Two methods of experimental analysis which generally used are:

1. Mechanical models, may be examined as with a shaking table usually with the aid of principles of dynamic similitude,
2. Electrical analogous circuit, which is governed by the same equations as govern the structure.

The analysis of simpler types of rigid rectangular frame work may be under taken quite satisfactorily by employing such methods as those of slope deflection and moment distribution. When, however, it is necessary to analyse the usual size of the building frame, these methods prove to be impracticable owing to excessive amount of computation involved. This difficulty has been overcome in the conventional method of design by making use of simplyfying assumptions which considerably ease the burden of computation. Unfortunately, stresses calculated on the basis of this method bear little relation to those occuring in practice, and the designs produced can not beregarded as really economic. But by the application of electric circuit models we can have a rigorous analysis with a drastic reduction in the amount of time spent in the design of intricate structure.

In the past twenty five years several electrical analogies have been deviced, which can be used in the solution of problems concerning beams, trusses, plates and frames to a greater extent. Some of these can easily be used in the solution of problems with static loads, while the others can be used in the normal mode analysis, and still others can be used in the solution of transient problems.

The basic idea of the treatment of mechanical systems by means of measurements on analogous electrical circuits appear in several places in the literature. The general scheme is to produce an electrical net work governed by the same set of differential or algebric equations as the mechanical system, and subject to the same system of constraints, to set up the analogy between mechanical and electrical quantities, with the convenient conversion factors, and then to measure electrical quantities in order to obtain the value of corresponding mechanical quantities.

The development of electrical analogy gave such a spectacular demonstration of its application to various types of structures, that engineers every where have grown to recognize this new tool as some thing very practical and ready for immediate use for their time consuming problems.

PART ONE

CHAPTER 3

REVIEW OF LITERATURE

Vannevar Bush (1934)

In the year 1934 V. Bush has devised electrical net works which are equivalent to pin connected and rigid joint structural frames. A special circuit is available for the solution of each of the following problems

3.01 Pin connected statically determinate structures

At each pinned joint of a plane frame work the sum of horizontal component of force must vanish, and the sum of vertical components of force must vanish. In Bush's analogy, current corresponds force. Thus the equilibrium conditions at a joint are represented by Kirchoff's current law. The two net works are required, one for horizontal force and one for vertical forces. Horizontal members appear only in one net work, and vertical members only in other. Since a diagonal member has both horizontal and vertical components, it appears in both net works.

To construct the electrical analog of the pratttruss shown in Fig.1 first lay out 12 terminals representing the 12 Pannel Points. One set of terminals is for horizontal forces and the other set is for vertical forces. In the horizontal net work connect each pair of terminals, which correspond to pannel points, that are connected by a horizontal member in the structure. Similarly in the vertical net work connect each pair of terminals which correspond to panel points that are connected by a vertical member. In both net works connect each pair of terminals which correspond to the end of diagonal member, but interpose a transformer with its primary coilin the horizontal net work and its secondary coil in the vertical net work.The transformer ratio is made equal to the tangent of the angle (α) of inclination of diagonal member with the horizontal.

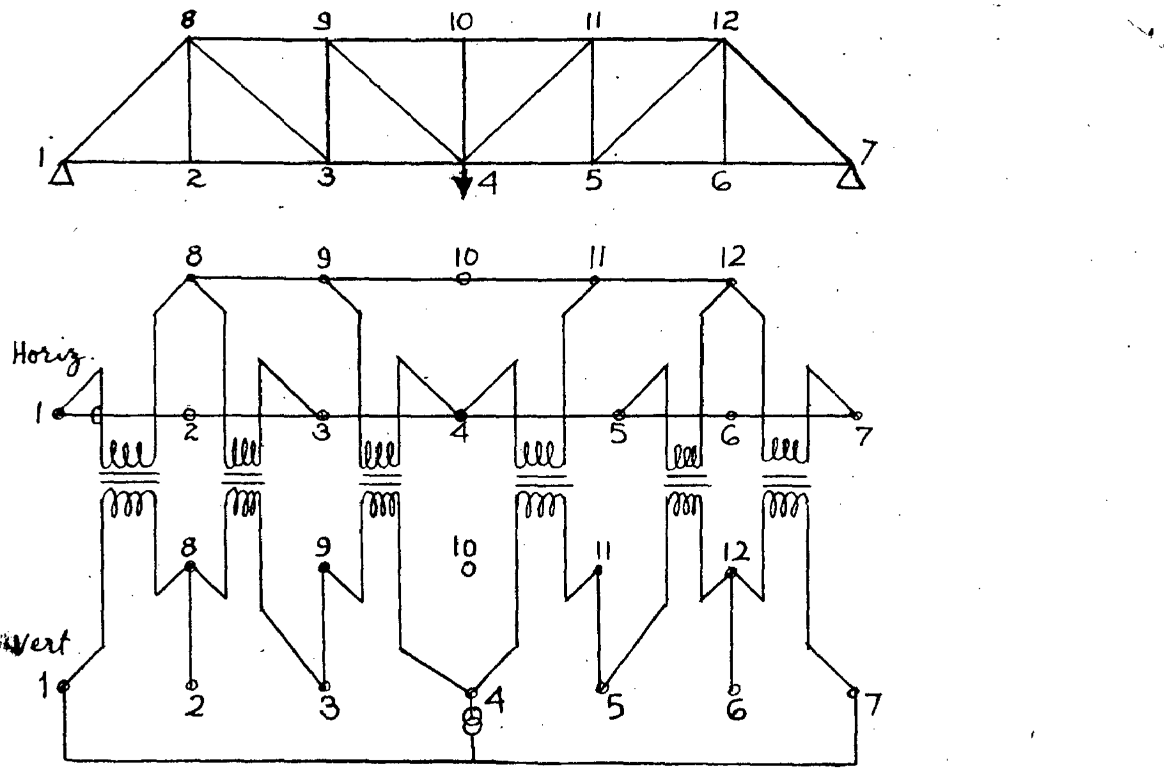


FIG.1 PIN CONNECTED STATICALLY DETERMINATE STRUCTURE AND BUSHE'S EQUIVALLENT CIRCUIT

Forces are applied by inserting alternating currents at the load and support points. There are the terminals 1,4 and 7 in the vertical net work, since the forces are vertical and applied at panel points 1,4 and 7. The currents should be proportional to the external forces and should be all in phase. If the ratio of external force to the current inserted is(r), the force in the horizontal or vertical member is equal to (r) times the current measured between the appropriate terminals. The forcd in a diagonal member is obtained by measuring the current between the corresponding terminals in the vertical or horizontal net work and multiplying by (r) Cosec (α) or (r) Sec (α), respectively.

If there are several loads, then there will be several sources alternating current, and each current must be adjusted until its magnitude represents to scale the corresponding load. If there is no symetry, then a separate source must be used to adjust the reactions. All the sources must be adjusted so that the currents supplied are strictly in phase. If there are horizontal loads, or horizontal components of loads, they are handled by sources connected to the horizontal net work.

If the problem is a three dimensional one, three net works are necessary. All panel points appear in each net work diagram, although, since there is no need that they be placed in their correct geometrical relationship, they can all appear in a plane for convenience. Connection are made as before. A member located in a principal plane is treated as above. When a member is inclined w.r.t.all three axes the corresponding electrical connections are interrelated by transformer in such a manner as to preserve the correct relation-ship between all three components of stress. Two transformer being necessary for this purpose. The rest of the procedure is self-evident.

3.02 Equilibrium of Pin-connected frames with Redundant Member.

If a pin-connected structure has redundant members the force in the members are influenced by their changes in length. In a single member the relation between axial force P and change of length Δ is

$$\Delta = \frac{Pl}{AE} \qquad \ldots\ldots\ldots \quad 1$$

To form the analogy, force, as before is taken proportional current . $P = r_1 I$ 2

Where r_1 is scale factor, change of length of a member is taken proportional to voltage drop (V) between terminals.

$$\Delta = r_2 V \qquad \ldots\ldots\ldots \quad 3$$

The analogus of equation(1) is Ohm's Law

$$V = I R$$

$\therefore$ Electrical resistance R corresponds to compliance $1/AE$

$$R = \frac{r_1 l}{r_2 AE} \qquad \ldots\ldots\ldots \quad 4$$

The circuit is constructed exactly as in the statically determinate case except that, whereever two terminals were connected before the appropriate resistance (given by abve equation) is inserted between them.

Equations from (1) to (4) represents the analogy for a single member. The extension to a frame work follows from the energy considerations.

If the potential energy stored in a member is P^2L/AE If the subscripts h,v,d refer to horizontal, vertical, and diagonal members, respectively, the total energy of deformation of structure is,

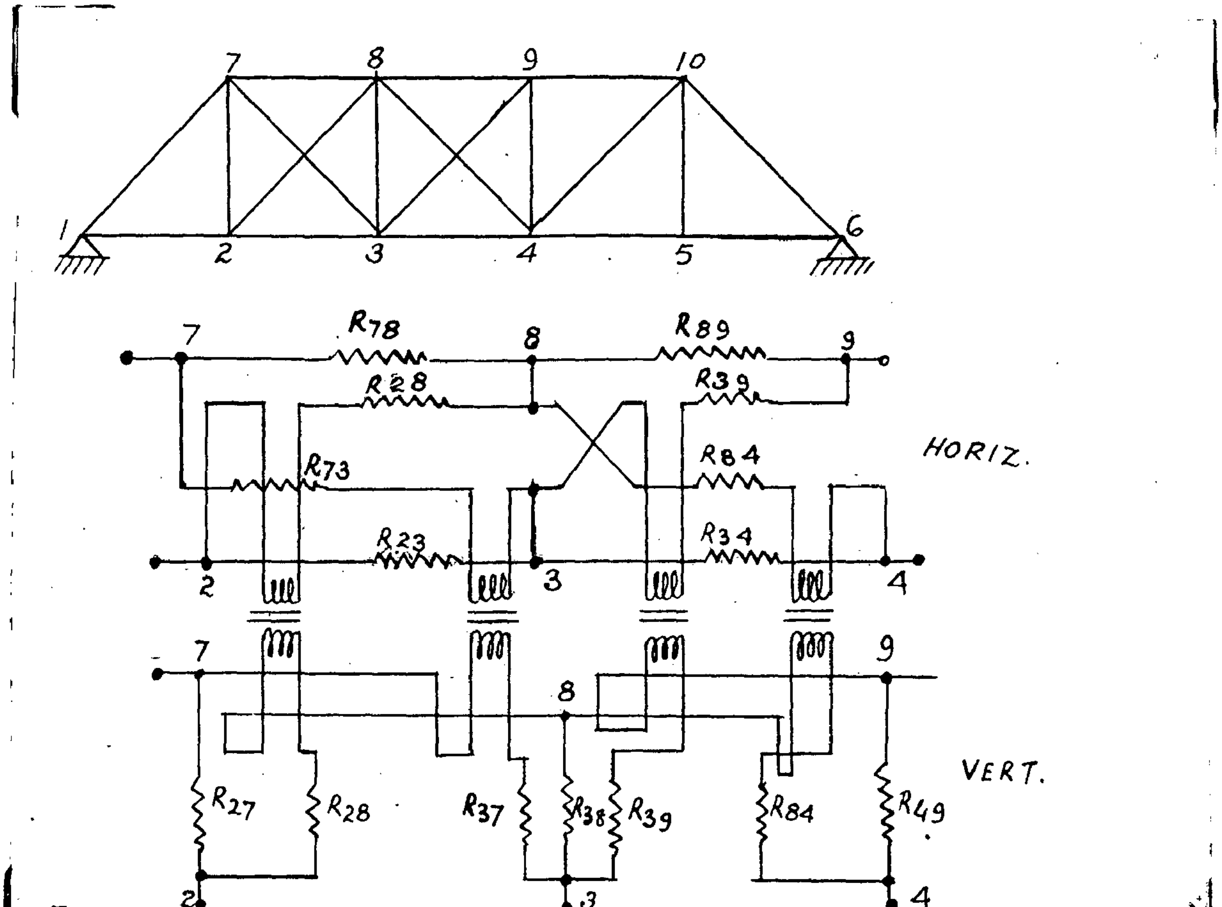


FIG – 2. BUSHS CIRCUIT EQUIVALLENT OF THE TWO INDETERMINATE PONDS OF THE TRUSS SHOWN ABOVE.

$$\Sigma_h \frac{P_h^2\, l_h}{2\, A_h\, E_h} + \Sigma_v \frac{P_v^2\, l_v}{A_v\, E_v} + \Sigma_d \frac{P_d^2\, l_d}{A_d\, E_d} \qquad \ldots\ldots\ldots \qquad 5$$

The forces in the members will assume such values as to make the total energy of deformation a minimum. Hence,

$$\partial\left\{\Sigma_h \frac{P^2 l}{AE} + \Sigma_v \frac{P^2 l}{AE} + \Sigma_d \frac{P^2 l}{AE}\right\} = 0 \qquad \ldots\ldots\ldots \qquad 6$$

In the electric circuit the power dissipated in an element is I^2R. The total power dissipation in the horizontal net work will be a minimum,

$$\partial\left(\Sigma_h I_h^2 R_h + \Sigma_d I_{h_d}^2 R_d\right) = 0 \qquad \ldots\ldots\ldots \qquad 7$$

Where I_{h_d} is the current in the dth diagonal element of the horizontal net work.

Similarly, the total power dissipated in the vertical net work will be minimum,

$$\partial\left(\Sigma_v I_v^2 \; R_v + \Sigma_d I_{vd}^2 \; R_d\right) \quad = 0 \quad \ldots\ldots \quad 8$$

Where I_{vd} is the current in the dth diagonal element of the horizontal net work.

Now
$$r_1 \, I_h = P_h$$
$$r_1 \, I_v = P_v$$
$$r_1^2 \left(I_{hd}^2 + I_{vd}^2\right) = P_d^2$$
$$\frac{r_2 \, R_h}{r_1} = \frac{L_h}{A_h \, E_h}$$
$$\frac{r_2 \, R_v}{r_1} = \frac{L_v}{A_v \, E_v} \quad \text{and} \quad \frac{r_2 \, R_d}{r_1} = \frac{L_d}{A_d \, E_d}$$

Hence substituting in the equation 7 and 8, adding the two equations and cancelling the scale factors, we arrive at equation 6. The analogy is then complete.

3.03 Equilibrium of Rectangular frames

Consider a member AB of length L acted on by moments M_A and M_B at the ends A and B, respectively. Both moments are taken positive clock-wise.

To be in equilibrium under moments and transverse force applied only at its ends, a shear force S is required such that,

$$M_A + M_B + S\,L \quad = \quad 0 \qquad \ldots\ldots\ldots \quad 9$$

and

$$\theta_A - \theta_B = \frac{M_A\,L}{E\,I} + \frac{S\,L^2}{2\,E\,I} \qquad \ldots\ldots\ldots \quad 10$$

$$\triangle A - \triangle B = \theta_B\,L + \frac{M_A\,L^2}{2\,E\,I} \pm \frac{S\,L^3}{3\,E\,I} \qquad \ldots\ldots\ldots \quad 11$$

where θ_a and θ_B are the rotations of the tangents to the elastic curve at A and B, respectively, both positive clock-wise.

$\Delta A - \Delta B$ is the relative transverse displacement of the two ends of the member, considered positive for clock wise rotation of the line joining the ends of the member. The above equations are called slope deflection equation.

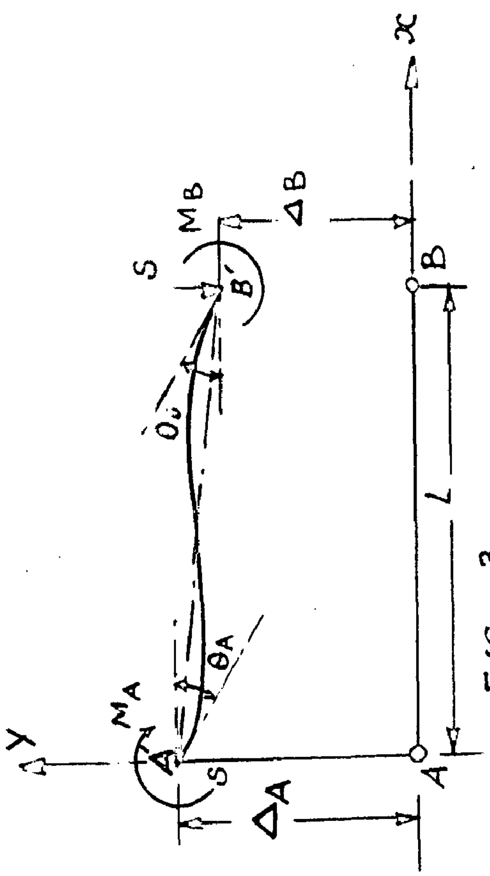
In the Bush's analogy, force, moment, rotation, and displacements are all represented by currents. Calling r_1, r_2, r_3, r_4 the scale factors between current and shear, moment, rotation and displacement, respectively, we write

$$S = r_1 I_s \qquad \theta_B = r_3 I_{\theta B}$$
$$M_A = r_2 I_{ma} \qquad A = r_4 I_{\Delta A}$$
$$M_B = r_2 I_{mb} \qquad B = r_4 I_{\Delta B}$$
$$\theta_A = r_3 I_{\theta A}$$

Substituting the above quantities in equations 9, 10 and 11, we have the electrical equivalents of the slope deflection equations.

$$I_{MA} + I_{MB} + (r_1 L/r_2) I_s = 0 \qquad \ldots\ldots\ 12$$
$$I_{\theta A} - I_{\theta B} = (r_s L/r_3 E I) I_{MA} + (r_1 L^2/2 r_3 E I) I_s \quad \ldots 13$$
$$I_{\Delta A} - I_{\Delta B} = \left(\frac{r_3 L}{r_4}\right) I_{\theta B} + \left(\frac{r_2 L^2}{2 r_4 EI}\right) I_{MA} + \left(\frac{r_1 L^3}{3 r_4 EI}\right) I_s \quad \ldots 14$$

Bush's equivalent circuit for member AB is shown in the fig. 4 in which four upper terminals represent the end A, and the four lower terminals represent the end B. Positive shear is represented by current I_s entering at A and leaving at B.



FIG—3.

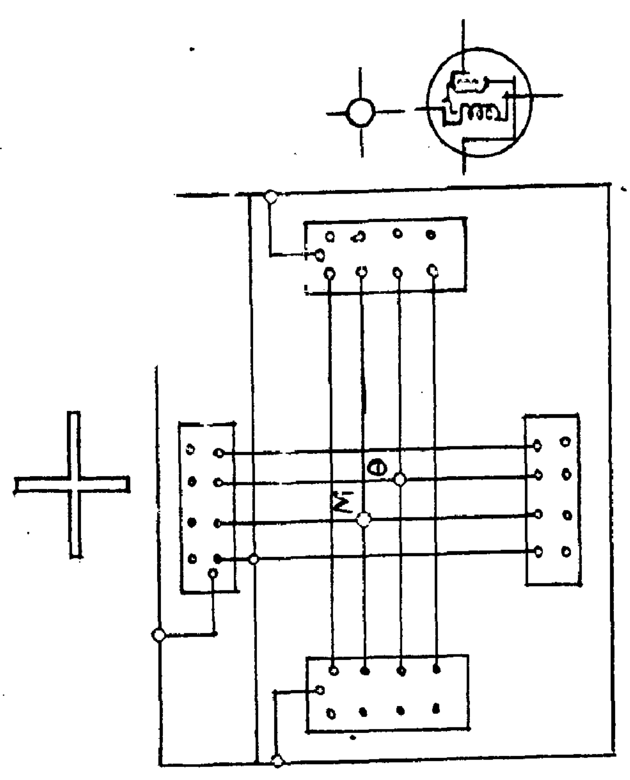
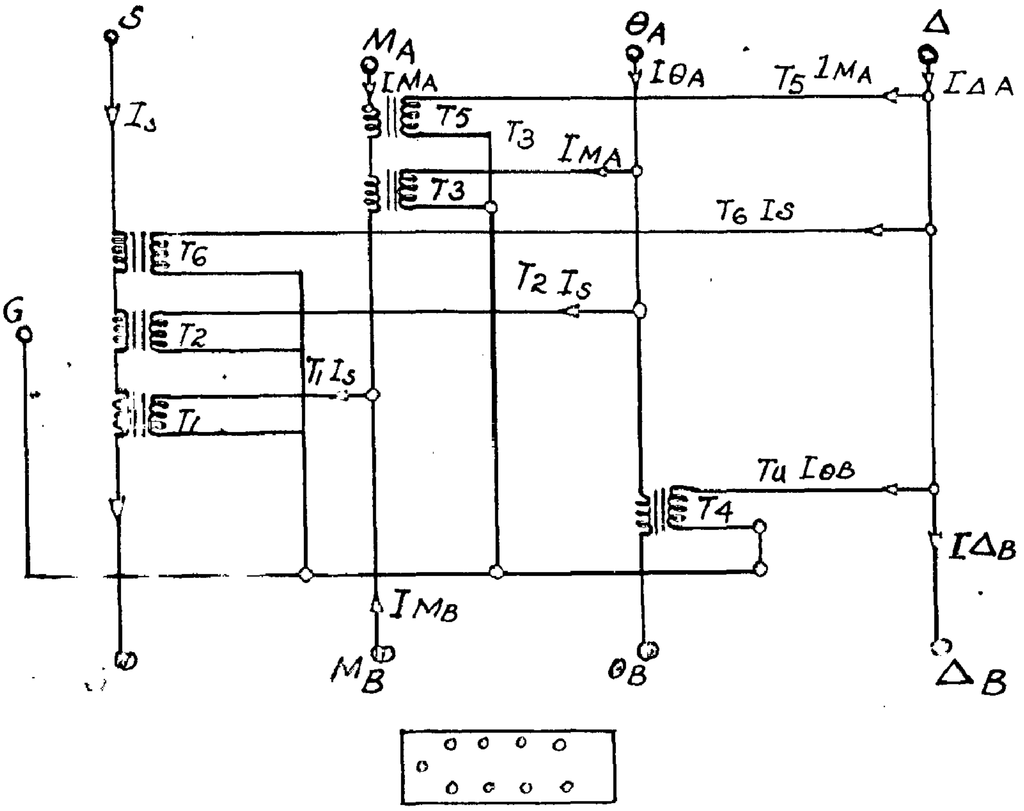


FIG. 5 CONNECTION REQUIRED IN BUSH'S EQUIVALENT CIRCUIT WHEN FOUR MEMBERS MEET AT A REGID JOINT.



F.. 4 BUSH'S CIRCUIT EQUIVALENT OF SLOPE DEFLECTION EQUATION.

Positive moment, represented by currents I_{MA} entering at A and I_{MB} entering at B.

Positive rotation-represented by currents $I_{\theta A}$ entering at A and $I_{\theta B}$ leaving at B.

Positive transverse displacement-by currents $I_{\Delta A}$ entering at A and $I_{\Delta B}$ leaving B
so that $I_{\Delta A} - I_{\Delta B}$ represents positive relative transverse displacement (that is clock wise rotation of the member)

Six current transformers are used, with ratios $T_1 \ldots T_6$ given by the six quantities in parentheses in equations 12 to 14.

Then

$$T_1 = \frac{r_1 L}{r_2} \qquad T_4 = r_3 L/r_4$$

$$T_2 = \frac{r_1 L^2}{2 r_3 E I} \qquad T_5 = r_2 L^2/2 r_4 E I$$

$$T_3 = \frac{r_2 L}{r_3 E I} \qquad T_6 = r_1 L^3/3 r_4 E I$$

with these transformer ratios, it may be verified readily, by applying Kirchhoff's current laws that the circuit satisfies the equivalent slope deflection equations 12 to 14.

The nine terminals in Fig. 4 are used for connecting the members to adjacent members, G being a common terminal. Thus if terminals G and θ_B are connected to terminals G' and θ'_A of an adjacent member, the current $I_{\theta B}$ is forced to be equal to $I'_{\theta A}$, and then the rotation θ_B is made equal to the rotation θ'_A, as required by continuity.

3.04 - Consider the four members meeting at a rigid joint. The equivalent circuit is shown in the figure 5.

The sum of allthe moments acting on the joint is made equal to zero by joining the four corresponding terminals at M. The rotations of all members meeting at the joint are made equal by using a 1:1 transformer connection at θ. The actual connecting circuit at θ is shown at the right. The transverse displacements of adjoining vertical members are made equal by series connections, and the same is done for adjoining horizontal members, but the two connections are not joined, since the displacements are at right angles and not necessarily equal. The shear connections depend on the distribution of applied loads in complete structure.

3.05 - A problem of wind loads on a nine-panel bent is shown in fig.6.

In the circuit the sum of the moments at each joint is madeequal to zero by inter-connection as described previously. Rotation of adjoining members are made equal by series connections in the case of two members, and by transformer connections in the case of three or four. Where two vertical members meet, the transverse displacements are made equal by series connection. The transverse displacement of all the horizontal members must be zero; so the corresponding circuit of members 1-9 is left open. At each floor level the lateral displacements of all vertical members are made equal by transformers connections. The displacements and rotation of the lower ends of the first storey columns are assumed to be zero, and so the corresponding terminals are left unconnected. The applied loads are represented by adjustable source of alternating current, F_1,F_2 and F_3 held strictly in phase. The currents which appear at R_1,R_2,R_3 and R_4 are proportional to

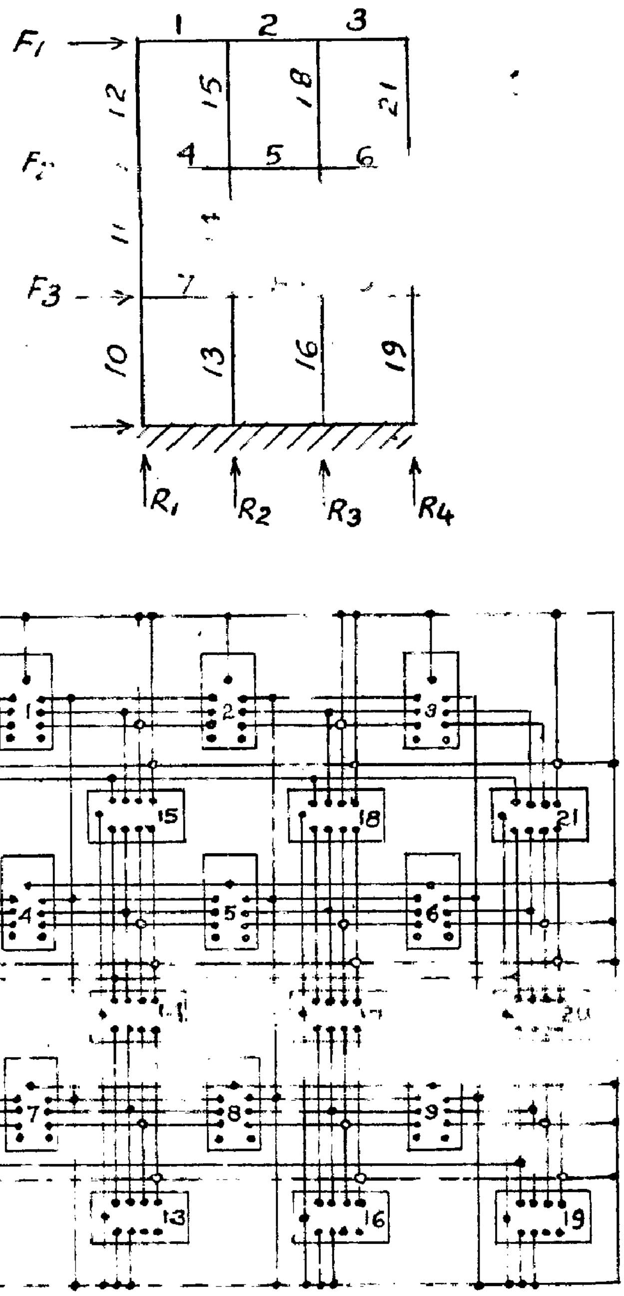


FIG. 6. NINE-PANEL BENT UNDER WIND LOAD AND WIND LOAD AND BUSH'S EQUIVALENT CIRCUIT.

the foundation reactions.

The sum of the shears in members 12, 15, 18 and 21 is made equal to the load F_1 by connecting the five terminals together. Similarly, for reaction R_1 and the shears in 1, 4, 7. The sum of the shears in 11, 14, 17, 20 minus the sumof the shears in 12, 15, 18, 21 is made equal to F_2 by connecting the all the terminals representing three forces.

The load F_3 and the reactions R_2 and R_3 are treated similarly. Attention must be made to algebric signs in making connections.

3.06 - Ideal transformers appear in all of Bush's equivalent circuits.They may be constructed to the required degree of accuracy by using nickel iron alloy cores and closely coupled windings. The flux density should be low and the primary turns sufficient so that the existing current is negligible in comparison with the work current. Frequency should be high enough to make the size of the transformer small, but not so high that currents flowing through distributed capacities will be appreciable in comparison with the work current. The care should be taken to produce a minimum of leakage reactance.

In measuring currents in the circuit, a low resistance shunt and a vacuum - tube amplifier were used. Great care had to be taken to ensure low-resistance connections as the resistance every-where in the circuit had to be low.

The final results for shear, moments, displacements, and rotation had errors ranging from 0 to 30 per cent, but the larger percentage errors were always associated with the smaller magnitudes.

The individual members of mechanical structures have in general not one degree of freedom but six (elongation in three dimensions and rotation in three). So each elastic member in a frame structure is considered as a six wire transmission line, and a net work of elastic members is considered as a similar net work of inter connected six wire transmission system subjected to un-balance loading at various points along the line. This theory serves as an introduction to further extension of the electrical analogy to general dynamical systems, in which elastically and in elastically connected rigid bodies have relative rotation and translation with respect to each other.

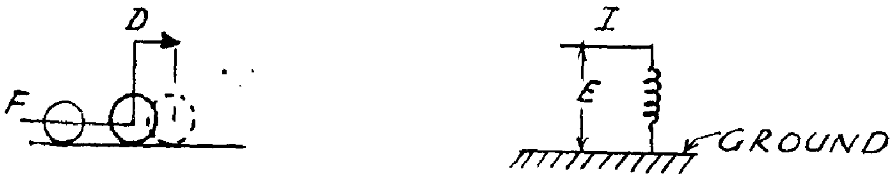
3.07 - Simple bar as a transmission line.

An elastic structure is assumed to consist of masses concentrated in parts, and inter-connected by elastic links consisting of long thin bars, to be called beams. The most general elements of the structure are found in these steps:

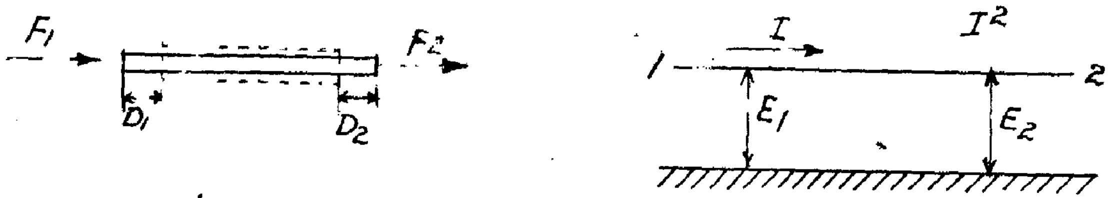
1. All inter connections between beams are removed, leaving a number of isolated but spatially oriented long, thin beams and a number of rigid bodies.

2. Each beam and the rigid body is rotated to lie along the X axis with these three principal axes oriented along the x, y, z axes.

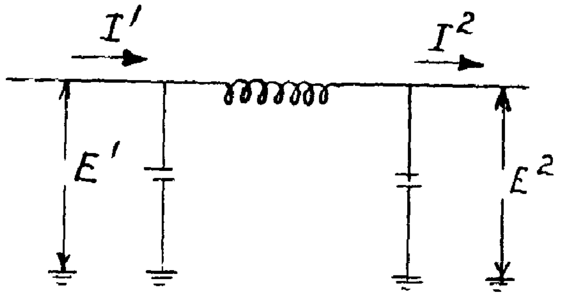
3. Since at each end of the beam actually six forces act, for the preliminary study it will be assumed that only



(a) MASS POINT AND ITS ELECTRICAL ANALOGUE.



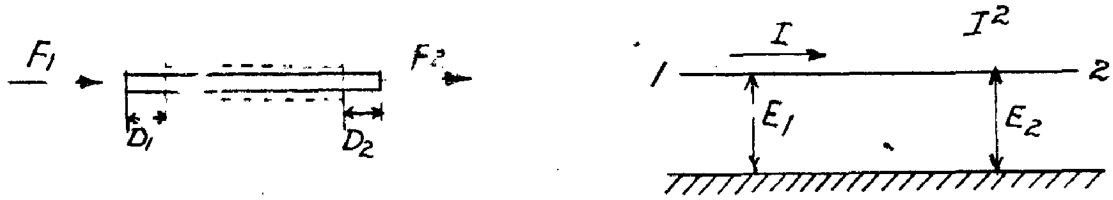
(b) ELASTIC BAR AND ITS ELECTRICAL ANALOGUE



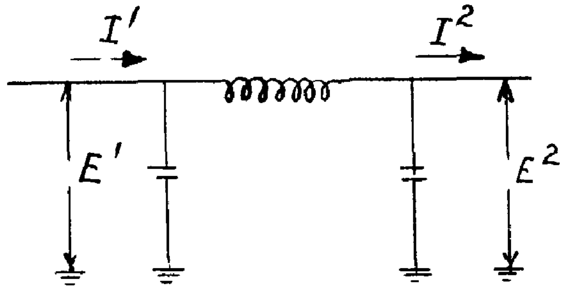
(C) CIRCUIT REPRESENTATION OF TRANSMISSION LINE

FIG-7.

(a) MASS POINT AND ITS ELECTRICAL ANALOGUE .



(b) ELASTIC BAR AND ITS ELECTRICAL ANALOGUE



(C) CIRCUIT REPRESENTATION OF TRANSMISSION LINE

FIG - 7.

one force acts at each end of the beam and only one force acts on the point mass.

Fig. 7 represents,

(a) Mass point and its electrical analogue,

(b) Elastic bar and its electrical analogue,

(c) Circuit representation of transmission line.

3.08 - Equation of a coil

The currents I will be assumed to represent force F and the voltages E will be assumed to represent displacement D. The equations of a mass point and an electrical coil will be assumed to be,

$$F = \frac{M\,d^2 D}{dt^2} = M\,p^2 D \qquad I = \frac{M\,d^2 E}{dt^2} = M\,p^2 E \quad \ldots\ 1$$

$$F = Y\,D \qquad I = Y\,E \quad \ldots\ 2$$

where $P = d/dt$

3.09 - Equations of transmission line

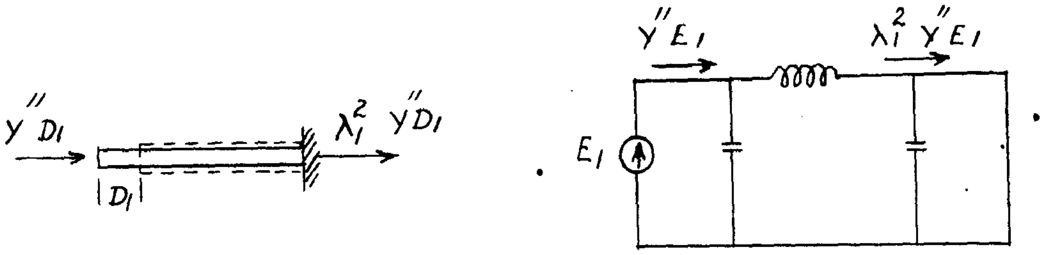
The sending and receiving ends of a transmission line are denoted by 1 and 2. The currents will correspond to forces and the voltages to the displacements of the beam.

First let end 2 of the beam be fixed (Fig. 8a) and a displacement D_1 be applied at end 1. The stored elastic energy is

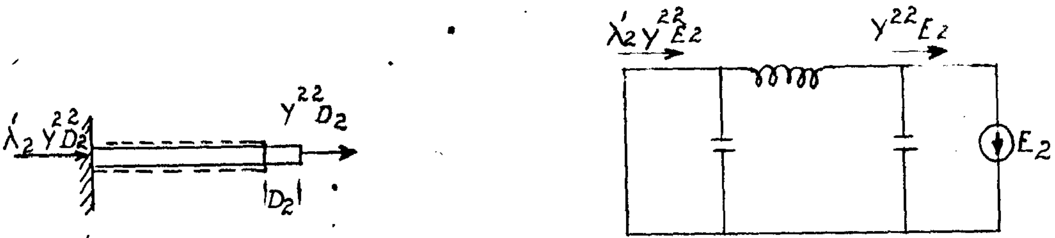
$$U = 1/2\ \frac{AE}{L}\ F^2 \quad \ldots\ 3$$

and the displacement of end 1 is

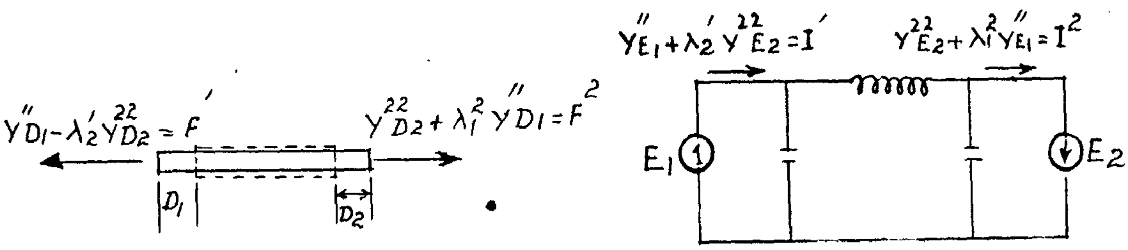
$$D_1 = \frac{\partial U}{\partial F} = \frac{AE}{L}\ F = Z'_{11}\, f \quad \ldots\ 4$$



(a) DISPLACEMENT APPLIED ON END 1.



(b) DISPLACEMENT APPLIED ON END 2.



(C) DISPLACEMENT APPLIED ON BOTH ENDS

FIG. 8.

$$\text{or } F = (Z'_{11})^{-1} D_1 = Y^{11} D_1 = \frac{L}{AE} D_1 \qquad \ldots\ldots\ldots \quad 5$$

It is also known that the force on end 2 is

$$- F = (Z'_{11})^{-1} D_1 = \lambda_1^2 F = \lambda_1^2 Y^{11} D_1$$

$$\text{where } \lambda_1^2 = \frac{\text{force on end 2}}{\text{force on end 1}} = -1 \qquad \ldots\ldots\ldots \quad 6$$

The transmission line analogy consists of short circuiting end 2 (Fig.8a) and applying a voltage E_1 at end I. If the short circuit admittance of the line measured from end 1 is Y^{11} the currents at the two ends of the line are shown in the (Fig.8a).

The second set of measurements is made by inter-changing the role of two ends as shown in (Fig.8b). These measurements define Z_{22}, $Y^{22} = (Z_{22})^{-1}$, also $D2$, E_2 and $\lambda_2^1 = -1$, it should also be noted that $\lambda_1^2 \lambda_2^1 = 1$

If now both ends of the beam are subjected to displacements simultaneously, the resultant forces on each end of the beam are the sum of the previous cases. (Fig.8c)

for the beam

$$\left.\begin{array}{l} F' = Y^{11} D_1 + \lambda_2^1 Y^{22} D_2 \\ F^2 = \lambda_1^2 y^{11} D_1 + Y^{22} D_2 \end{array}\right\} \qquad \ldots\ldots \quad 7$$

If now mutual admittances are defined as

$$Y^{12} = \lambda_1^2 Y^{22} \text{ and } Y^{21} = \lambda_1^2 Y^{11} \qquad \ldots\ldots\ldots \quad 8$$

the equation of the beam become

$$\left.\begin{array}{l} F^1 = Y^{11} D_1 + Y^{12} D_2 \\ F^2 = Y^{221} D_1 + Y^{22} D_2 \end{array}\right\} \qquad \ldots\ldots\ldots \quad 9$$

Similarly for the transmission line

$$\left.\begin{array}{l} I^1 = Y^{11} E_1 + Y^{12} E_2 \\ I^2 = Y^{21} E_1 + Y^{22} E_2 \end{array}\right\} \qquad \ldots\ldots\ldots \quad 10$$

3.10- The Un-oriented Beam

In general, each end of the beam has six degrees of freedom instead of one, namely,

1. Three linear displacements D_x, D_y, D_z ;
2. Three angular displacements D_p, D_q, D_r.

As the first set of measurements, let end 2 of the beam be fixed and end 1 be subjected to linear and angular displacements. The stored elastic energy is

$$U = 1/2 \int_0^L \left(\frac{(F_x)^2}{EA} + \frac{(F_q + r F_z)^2}{B_1} + \frac{(F_r - r F_y)^2}{B_2} + \frac{(F_p)^2}{C} \right) dr \qquad \ldots\ldots 11$$

Where B_1 = flexural rigidity about axis Y,

B_2 = flexural rigidity about axis Z,

C = torsional rigidity.

Then by differentiating U with respect to each of the forces six equations may be written as,

$$D_x = (L/EA) F_x$$

$$D_y = + (L^3/3B_2) F_y \qquad - (L^2/2B_2) F_r$$

$$D_z = + (L^3/3B_1) F_z + (L^2/2B_1) F_q$$

$$D_p = + (L/c) F_p$$

$$D_q = + (L^2/2B_1) F_z + (L/B_1) F_q$$

$$D_r = - (L^2/2B_2) F_y \qquad + (L/B_2) F_r$$

or in terms of matrices

$$D_1 = Z'_{11} F' \text{ or } (D_1) = (Z'_{11})(F') \qquad \ldots\ldots 12$$

	X_1	Y_1	Z_1	P_1	Q_1	R_1
D_1 =	D_{x1}	D_{y1}	D_{z1}	D_{p1}	D_{q1}	D_{r1}

$$F^1 = \begin{array}{cccccc} X_1 & Y_1 & Z_1 & P_1 & Q_1 & R_1 \\ \hline F^{x1} & F^{y1} & F^{z1} & F^{p1} & F^{q1} & F^{r1} \end{array}$$

$$Z'_s = \begin{array}{c|cccccc|} & X_1 & Y_1 & Z_1 & P_1 & Q_1 & R_1 \\ \hline X_1 & L/EA & & & & & \\ Y_1 & & L^3/3B_2 & & & & -L/2B_2 \\ Z_1 & & & L^3/3B_1 & & L^2/2B_1 & \\ P_1 & & & & L/C & & \\ q_1 & & & L^2/2B_1 & & L/B_1 & \\ r_1 & & -L^2/2B_2 & & & & L/B^2 \end{array}$$

In the electrical analogy there are now six transmission lines and one ground. End 2 of all transmission lines is short circuited and voltages E_{x1}, E_{y1} ...., are impressed on end 1 in succession while all the other end 1 terminals are kept open circuited (Fig.9). The matric equation of the transmission line is

$$E_1 = Z'_s \; I'$$

$$E_1 = \begin{array}{cccccc} X_1 & Y_1 & Z_1 & P_1 & q_1 & r_1 \\ \hline E_{x1} & E_{y1} & E_{z1} & E_{p1} & E_{q1} & E_{r1} \end{array}$$

$$I' = \begin{array}{cccccc} X_1 & Y_1 & Z_1 & P_1 & q_1 & R_1 \\ \hline I^{x1} & I^{y1} & I^{z1} & I^{p1} & I^{q1} & I^{r1} \end{array}$$

..... 13

For further analysis the inverse equation is needed representing the reaction of beam in which both ends are rigidly connected to neigbouring beam. The inverse equation is,

$$F' = (Z'_{n})^{-1} D_1 = Y'' D_1 \;,\; I' = (Z_n)^{-1} E_1 = Y'' E_1 \qquad \text{------ 14}$$

$Y'' =$

	X_1	Y_1	Z_1	P_1	q_1	r_1
X_1	EA/L					
Y_1		$12\,B_2/L^3$				$6B_2/L^2$
Z_1			$12B_1/L^3$		$-6B_1/L^2$	
P_1				C/L		
q_1			$-6B_1/L^2$		$4B_1/L$	
r_1		$6B_2/L^2$				$4B_2/L$

In the electrical analogy Y'' represents the self and mutual admitances of end 1 terminals measured while all other end 1 and end 2 terminals are short circuited (Fig.9b).

The relation between the forces at the two ends of the beam are,

$$F^{x2} = -F^{x1}$$
$$F^{y2} = -F^{y1}$$
$$F^{z2} = -F^{z1}$$
$$F^{p2} = -F^{p1}$$
$$F^{q2} = -L\,F^{z1} - F^{q1}$$
$$F^{r2} = L\,F^{y1} - F^{r1}$$

In terms of matrices these may be written as

$$F^2 = \qquad F^1 = \qquad Y''\, D_1 \qquad \text{--------- 15}$$

	X_2	Y_2	Z_2	P_2	q_2	r_2
$F^2 =$	F^{x2}	F^{y2}	F^{z2}	F^{p2}	F^{q2}	F^{r2}

λ_1^2 =

	X_1	Y_1	Z_1	p_1	q_1	r_1
X_2	-1					
Y_2		-1				
Z_2			-1			
p_2				-1		
q_2			-L		-1	
r_2		L				-1

It should be noted that horizontal and vertical unit vectors of λ_1^2 belong to different ends of the beam.

For the second set of measurements, end 1 of the beam is fixed and displacements are applied on end 2. In the elastic energy U, Equation (11), the parameter r is now represented by -r, giving $D_2 = Z'_{22}\ F^2$ where

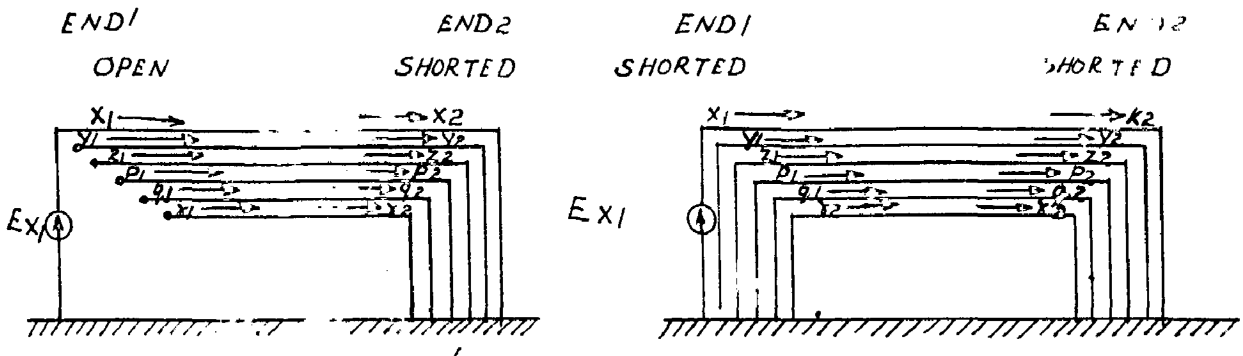
Z'_{22} = ---------16

	X_2	Y_2	Z_2	p_2	q_2	r_2
X_2	L/EA					
Y_2		$L^3/3B_2$				$L^2/2B_2$
Z_2			$L^3/3B_1$		$-L^2/2B_1$	
p_2				L/C		
q_2			$-L^2/2B_1$		L/B_1	
r_2		$L^2/2B_2$				L/B_2

The inverse equation is $F^2 = (Z'_{22})^{-1}\ D_2 = Y^{22}\ D_2$

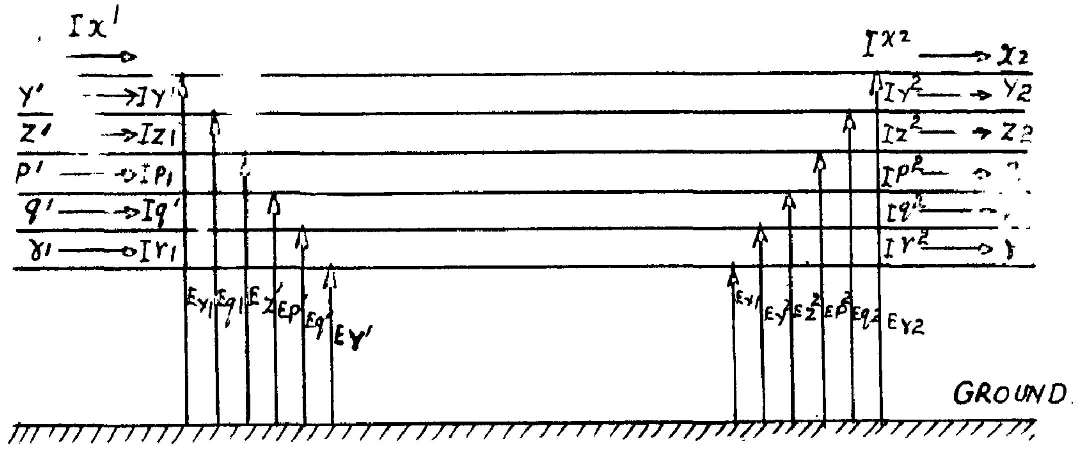
Y^{22} =

	X_2	Y_2	Z_2	p_2	q_2	r_2
X_2	EA/L					
Y_2		$12B_2/L^3$				$-6B_2/L^2$
Z_2			$12B_1/L^3$		$6B_1/L^2$	
p_2				C/L		

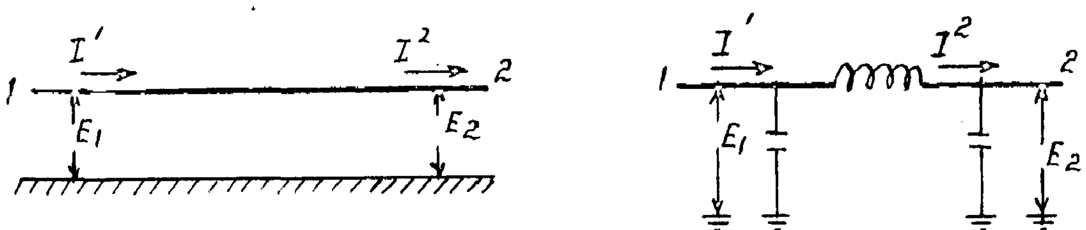


(a) MEASUREMENTS OF $(\bar{Z}'_{11})$ (b) MEASUREMENTS OF $\bar{Y}_{11}$

FIG- 9.



a SIX WIRE TRANSMISSION LINE.



(b) COMPOUND TRANSMISSION LINE
AND ITS COMPOUND CIRCUIT REPRESENTATION

FIG. 10 REPRESENTATION OF A COMPLICATED NET WORK
BY SIMPLE "COMPOUND" NET WORK.

q_2			$6B_1/L^2$		$4B_1/L$	
r_2		$-6B_2/L^2$				$4B_2/L$

..... 17

The forces on end 1 are $\lambda_2^1 F^2 = \lambda_2^1 Y^{22} D_2$ where $\lambda_2^1 = (\lambda_1^2)^{-1}$ and is found from λ_1^2 by changing the sign of L.

$\lambda_1^2 =$

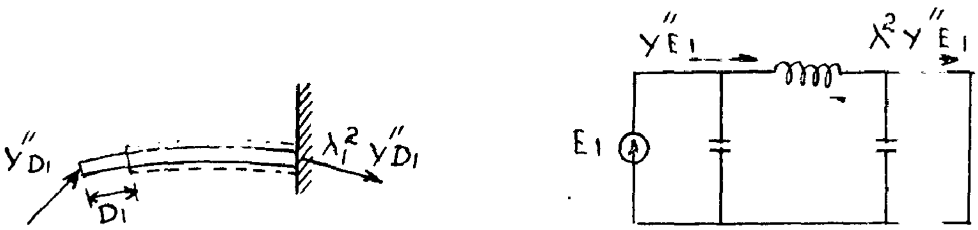
	X_2	Y_2	Z_2	P_2	q_2	r_2
X_1	-1					
Y_1		-1				
Z_1			-1			
P_1				-1		
q_1			L		-1	
r_1		-L				-1

.... 18

<u>3.11 Compound net works</u>

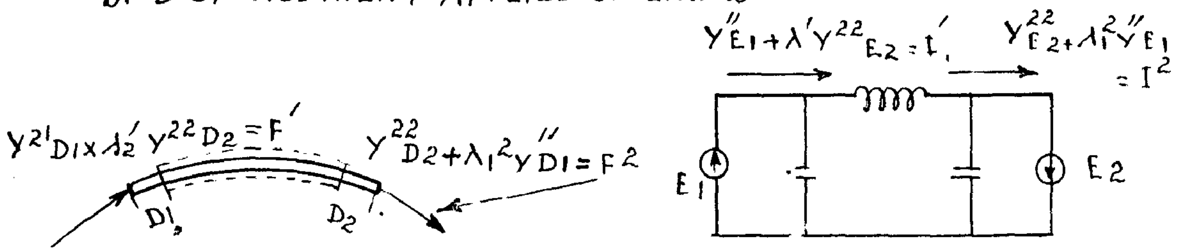
In analogy to the matric representation of any of numbers by one bold face letter, a similar representation of diagrams will be used in which one arrow will represent six currents, one mesh will represent six meshes, etc. Such diagrams will be drawn with heavy lines. A compound net work must be so designed, as to give the correct relations between the various matrices, just as the conventional net work gives the correct relations between the scalars.

Fig. 10 the representation of a complicated net work by simple "Compound" net work and Fig. 11 represents Matric equations of a long thin beam.



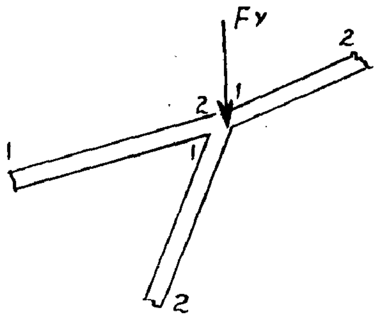
a. DISPLACEMENT APPLIED ON END 1

b. DISPLACEMENT APPLIED ON END 2.

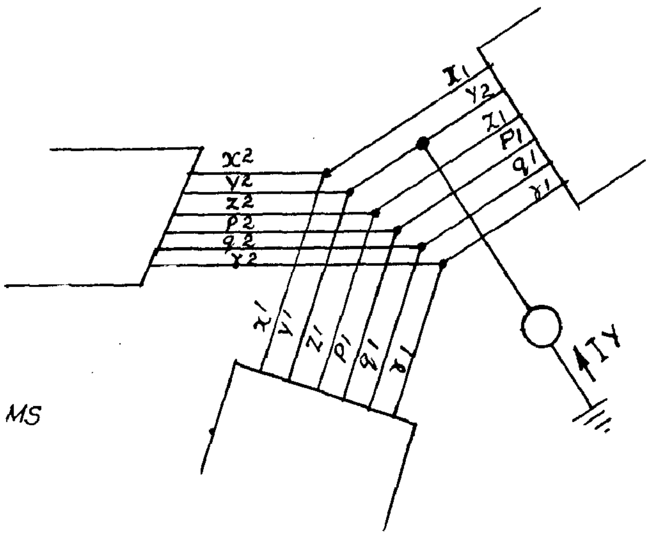


C. DISPLACEMENT APPLIED ON BOTH ENDS

FIG-11. MATRIX EQUATIONS OF A LONG THIN BEAM.



(a) INTERCONNECTION OF BEAMS



(b) INTERCONNECTION OF EQUIVALENT CIRCUIT OF BEAM.

FIG-12.

3.12 Matric equations of un-oriented beam

When the both the ends are subjected to displacements simultaneously, the resultant forces are the sum of the previous cases. That is for the beam

$$F^{1} = Y^{11} D_1 + \lambda^1_2 \; Y^{22} D_2,$$
$$F^2 = \lambda^2_1 Y^{11} D_1 + Y^{22} D_2$$

If now mutual admittance matrices are defined as

$$Y^{12} = \lambda^1_2 Y^{22} \text{ and } Y^{21} = \lambda^2_1 \; Y^{11} \qquad \ldots\ldots \quad 19$$

$Y^{12} =$

	x_2	y_2	z_2	p_2	q_2	r_2
x_1	$-EA/L$					
y_1		$-12B_2/L^3$				$6B_2/L^2$
z_1			$-12B_1/L^3$		$-6B_1/L^2$	
p_1				$-C/L$		
q_1			$6B_1/L^3$		$2B_1/L$	
r_1		$-6B_2/L^2$				$2B_2/L$

$Y^{21} =$

	x_1	y_1	z_1	p_1	q_1	r_1
x_2	$-EA/L$					
y_2		$-12B_2/L^3$				$-6B_2/L^2$
z_2			$-12B_1/L^3$		$+6B_1/L^2$	
p_2				$-C/L$		
q_2			$-6B_1/L^2$		$2B_1/L$	
r_2		$6B_2/L^2$				$2B_2/L$

The equations of the beam and transmission line become,

$$F^1 = Y'' D_1 + Y^{12} D_2 \quad \Big| \quad I' = Y'' E_1 + Y^{12} D_2 \quad --$$
$$F^2 = Y^{21} D_1 + Y^{22} D_2 \quad \Big| \quad I^2 - Y^{21} E_1 + Y^{22} E_2 \quad \ldots\ldots 20$$

While solving these equations it should be noted that if Z'_{11} and λ_1^2 are given, all other matrices may be found by the following formulae:

$$Y^{11} = (Z'_{11})^{-1} \quad \Big| \quad Y^{21} = \lambda_1^2 \, Y'' \quad \Big| \quad Y^{22} = (\lambda_1^2)_t \, Y'' \lambda_1^2$$
$$\lambda_2^1 = (\lambda_1^2)^{-1} \quad \Big| \quad Y^{12} = (Y^{21})_t. \quad \Big| \quad Z'_{22} = (\lambda_2^1) \quad Z'_{11} \lambda_2^1 \quad \ldots\ldots. 21$$

Also $Y^{11} = (Y'')_t$ and $Y^{22} = (Y^{22})_t$

3.13. Equivalent circuit of a structure

To represent a whole rigid structure it should be assumed that the equivalent circuit of each beam lies in space along the whole length of the beam. Thereby the compound electrical net work has the same configuration as the elastic net work. The steps are as follows:

1. Assume all inter-connections of the beam as removed.
2. Calculate for each beam in its actual spatial positi(the admittance coefficients.
3. Set up the equivalent circuit of each beam with all terminals open circuited. Assume the circuit to lie along the beam.
4. The inter-connections of beams is equivalent to inter-connecting the corresponding terminals of the lines (Fig.12).
5. If the end of a beam is fixed against a wall, all the corresponding terminals are shorted to the ground.

3.14 - In this analogy the problem is formulated in finite difference terms and the solution of the difference equation is carried out on electric analogy computer.

Problems involving deflections under constant load, transient vibrations or normal loads can be solved in this way.

3.15 Electrical Analogy for Bending of Beams

The differential equation for the bending of a beam is

$$\frac{\partial^2}{\partial x^2} \left(E\,I \frac{\partial^2 w}{\partial x^2} \right) = q \qquad \ldots\ldots\ 1$$

where W is the deflection

q the load intensity

for a uniform beam E I is constant, and the finite difference equivalent of Equation (1) may easily be derived.

$$W_{n+2} - 4\ W_{n+1} + 6\ W_n - 4\ W_{n-1} + W_{n-2} = \frac{q\ \Delta x^4}{E\ I} \quad \ldots\ 2$$

An electrical ladder net work can be constructed in which Kirchhoff's current law for nth node is identical with Equation (2). This net work requires the use of negative impedances which is serious disadvantage for the solution of normal mode and transient - vibration problems.

A less direct analogy will be used. If the slope and shear are defined as

$$\theta = \frac{\partial W}{\partial x} \qquad \ldots\ldots\ 3$$

and

$$Q = -\partial/\partial x \left(E\ I \frac{\partial \theta}{\partial x} \right) \qquad \ldots\ldots\ 4$$

then Equation (1) becomes

$$- \frac{\partial Q}{\partial x} = q \qquad \ldots\ldots\ldots \quad 5$$

When E I n t constant, it is simpler to write the finite difference equations corresponding to Equations (3),(4), and (5) separately than to write the finite - difference equations corresponding to Equation(I).

The equivalent finite difference equations are, respectively.

$$\theta_{n+1/2} = \frac{W_{n+1} - W_n}{\Delta x} \qquad \ldots\ldots \quad 6$$

$$- Q_{n+\frac{1}{2}} = (\theta_{n+3/2} - \theta_{n+\frac{1}{2}}) \frac{E I}{\Delta x^2} + (\theta_{n-\frac{1}{2}} - \theta_{n+\frac{1}{2}}) \frac{EI_n}{\Delta x^2} \ldots \quad 7$$

$$- \frac{(Q_{n+\frac{1}{2}} - Q_{n+\frac{1}{2}})}{\Delta x} = q_n \qquad \ldots\ldots\ldots \quad 8$$

The subscript $n+\frac{1}{2}$ refers to the mid-point between X_n and X_{n+1}, the points at which the deflection is defined.

Equations (7) and (8) may be regarded as expression of Kirchhoff's current laws in an electrical net work.

The circuit in the Fig. 13 satisfies all three of these equations if w and Q are regarded as voltages.

Equation (6) expresses the relationship between the primary and secondary voltages of a transformer where the turns ratio is Δ_x. Equation (7) is Kirchhoff's law for the sum of the currents entering the node $\theta_{n+\frac{1}{2}}$. The impedance the branch connecting $\theta_{n+\frac{1}{2}}$ and $\theta_{n+3/2}$ is $\frac{(\Delta x^2)}{(EI_{n+1})}$. The

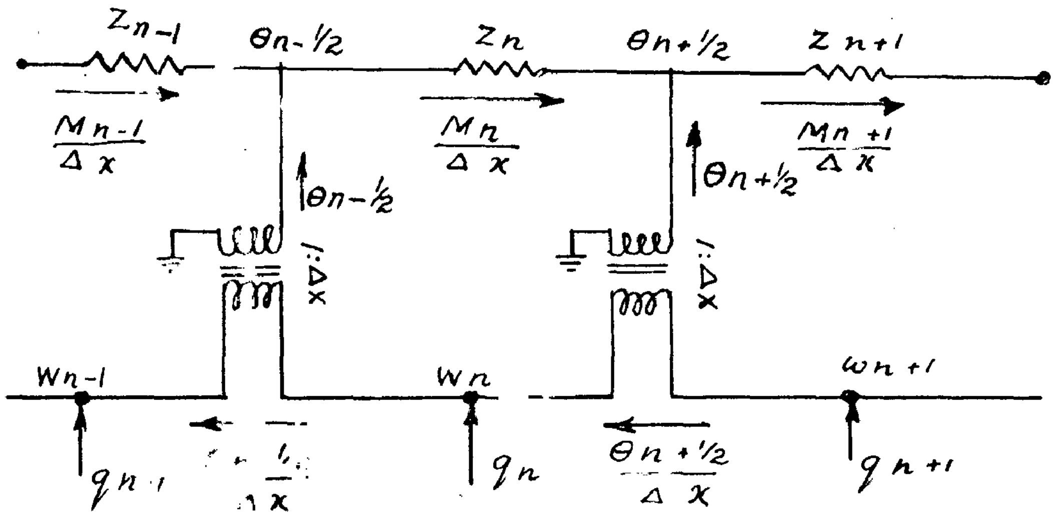


FIG. 13. DYNAMIC ANALOGY FOR BENDING OF A BEAM $Zn = \frac{\Delta x^2}{E\,In}$

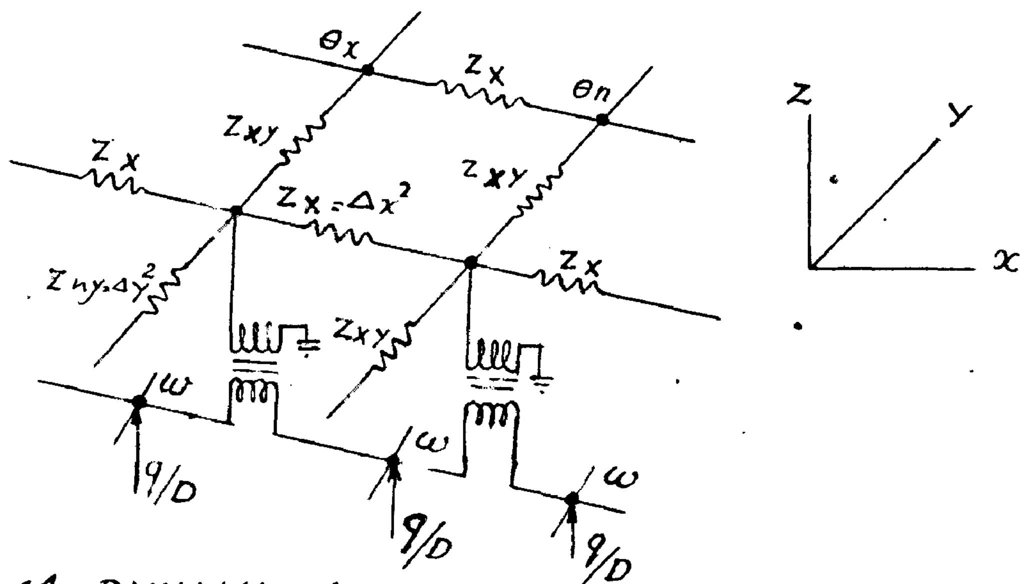


FIG. 14. DYNAMIC ANALOGY FOR THE FIRST TERM OF EQUATION.

current flowing in the secondary of the transformer is the turns ratio times the current in the primary, i.e., $Q_{n+\frac{1}{2}}$.

Equation (8) is Kirchhoff's law for the sum of current entering the node W_n. The bending moment is the current flowing

$$M = - EI \frac{\partial^2 W}{\partial x^2} \qquad \cdots\cdots\cdots\cdots 9$$

in the slope circuit, multiplied by Δ_x as shown. For static-loading problems, the impedences Z_n may be purely resistive, while for transient and normal node problems they must be inductive.

3.16 Dyhamic Analogy for Constant thickness plate

The differential equation for the deflection of a constant thickness plate is

$$\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^2 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4} = \frac{q}{D} \qquad \cdots\cdots\cdots 10$$

where D is the flexural rigidity of the plate.

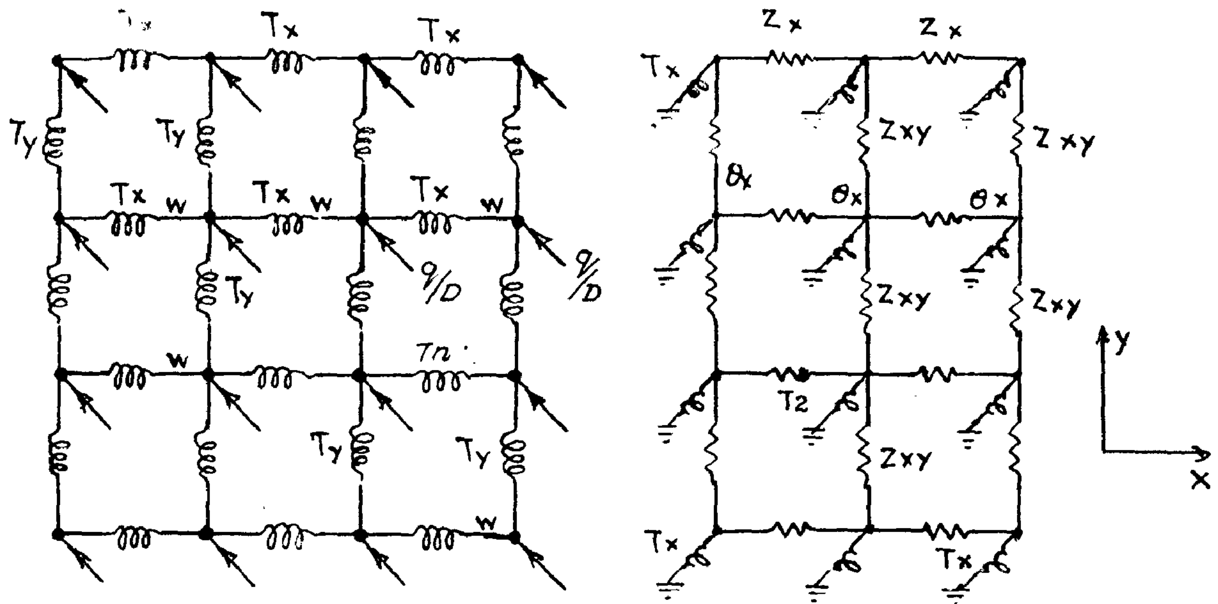
Equation (10) can be written in the following form

$$\frac{\partial}{\partial x}\left\{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y_2}\right\} + \frac{\partial}{\partial y}\left\{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right\}\frac{\partial W}{\partial y} = \frac{q}{D} \quad \cdots 11$$

The equation for a uniform beam is

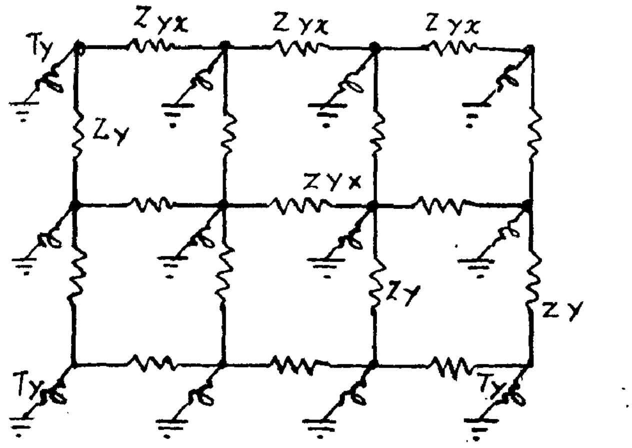
$$\frac{\partial}{\partial x}\left\{\left(\frac{\partial^2}{\partial x^2}\right)\frac{\partial W}{\partial x}\right\} = \frac{q}{EI} \qquad \cdots\cdots\cdots 12$$

For the beam analogy it was shown that the operator $\partial^2/\partial x^2$ is represented by the impedences in θ circuit connecting neighbouring nodes. For the plate, $\partial^2/\partial x^2 + \partial^2/\partial y^2$ is represented by



(a) W CIRCUIT.

(b) θx CIRCUIT.



(C) θy CIRCUIT

$$Z_x = \Delta x^2$$
$$Z_{xy} = \Delta y^2$$
$$Z_y = \Delta y^2$$
$$Z_{yx} = \Delta x^2$$

FIG. 15 – DYNAMIC ANALOGY FOR CONSTANT-THICKNESS PLATE.

of the Equation (1) is shown in fig.14. by definition the slope in the x direction is

$$\theta_n = \frac{\partial W}{\partial x}$$

The additional impedences Zny oriented prallel to the y-axis represents the term $\partial^2/\partial y^2$ consequently a two dimensional grid of impedences connects nodes in the θ_x circuit.

The second term of Equation (1) is represented by a second net work entirely similar to that in fig.14. It has two dimensional grid of impedences connecting nodes in the θ_y circuit. The primaries of its transformers which are oriented parallel to y- axis are joined at the nodes of the w-circuit with the x- transformers so that a two dimensional grid of transformer primary winding is formed. Thus the complete analogy consists of three separate two- dimensional grids in which the voltages to the ground are, respectively.

$$W; \quad \theta_x = \frac{\partial W}{\partial x} \quad \text{and} \quad \theta_y = \frac{\partial W}{\partial y}$$

The coupling between these three net works is by means of the magnetic fields of the transformers only. Plan views of these net works are shown in Fig. 15.

3.17 Dynamic Analogy for variable thickness plate.

The equations given for the bending moments by Timoshenko are,

$$M_x = - D \left(\frac{\partial \theta_x}{\partial x} + r \frac{\partial \theta_y}{\partial y} \right) \quad \ldots\ldots\ldots \quad 13$$

$$M_y = - D \left(\frac{\partial \theta_y}{\partial x} + v \frac{\partial \theta_x}{\partial x} \right) \quad \ldots\ldots\ldots \quad 14$$

$$M_{yx} = - D \; (1-v) \; \frac{\partial \theta_x}{\partial y} = - D \; (1 - v) \frac{\partial \theta_y}{\partial x} \quad \ldots \quad 15$$

The shears are given by

$$Q_n = \frac{\partial M_x}{\partial x} + \frac{\partial M_{yx}}{\partial y} \quad \ldots\ldots\ldots \quad 16$$

$$Q_y = \frac{\partial M_y}{\partial y} + \frac{\partial M_{yx}}{\partial x} \quad \ldots\ldots\ldots \quad 17$$

The static equilibrium of the plate requires that

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = 0 \quad \ldots\ldots\ldots \quad 18$$

combining Equations (13) through (18), where D need not be constant, the equation of the plate may be written as

$$\frac{\partial}{\partial x} \left\{ \frac{\partial}{\partial x} \left[D \left\{ \frac{\partial \theta_x}{\partial x} + v \frac{\partial \theta_y}{\partial y} \right\} \right] + \partial/\partial y \left[D \left\{ (1-v) \frac{\partial \theta_x}{\partial x} \right\} \right] \right\}$$
$$+ \frac{\partial}{\partial y} \left\{ \partial/\partial y \left[D \left\{ \frac{\partial \theta_y}{\partial y} + v \frac{\partial \theta_x}{\partial x} \right\} \right] + \partial/\partial x \left[D \; (1-v) \frac{\partial \theta_y}{\partial x} \right] \right\} = q \quad \ldots\ldots \quad 19$$

The only term that cannot be represented by the net work in Fig.? are those involving

$$V \frac{\partial \theta_y}{\partial y} \quad \text{and} \quad v \frac{\partial \theta_x}{\partial x}$$

The other terms can be represented by the net work with

$$Z_x = \frac{\Delta x^2}{D} \qquad Z_{xy} = \frac{\Delta y^2}{D(1-v)}$$

$$Z_y = \frac{\Delta y^2}{D} \qquad Z_{yx} = \frac{\Delta x^2}{D(1-v)}$$

The terms $v\,(\partial\theta_y/\partial y)$ and $v\,(\partial\theta_x/\partial x)$ represents a mutual coupling between the θ_x and θ_y circuits. By super imposing the net works in Fig.15. It will be seen that the Z_x and Z_y branches are at the same geometrical points. A mutual admittance between the two branches is required where,

$$Y_x = \frac{v\,D}{\Delta x\,\Delta y}$$

This mutual admittance requires an essentially perfect transformer for its realisation, bringing to three the number of such transformers required per cell. In practice it is expedient to convert this mutual admittance to a mutual impedence with accompanying modifications in Z_x and Z_y. The impedances required are

$$Z_x = \frac{1}{1-v^2}\;\frac{\Delta x^2}{D}$$

$$Z_y = \frac{1}{1-v^2}\;\frac{\Delta y^2}{D}$$

$$Z_m = \frac{v}{1-v^2}\;\frac{\Delta x\,\Delta y}{D}$$

$$Z_{xy} = \frac{1}{1-v}\;\frac{\Delta y^2}{D}$$

$$Z_{yx} = \frac{1}{1-v}\;\frac{\Delta x^2}{D}$$

Fig.16 shows the net work for a variable thickness plate. In this net work the current in the slope circuits are proportional to the moments, and the currents in the W circuit are proportional to shears.

Current is $Z_x = \dfrac{M_x}{\Delta x}$

Current in $Z_y = \dfrac{M_y}{\Delta y}$

Current in $Z_{xy} = \dfrac{M_{yx}}{\Delta y}$

Current in $Z_{yx} = \dfrac{M_{yx}}{\Delta x}$

Current in the x- branch of w- circuit $= \dfrac{Q_x}{\Delta x}$

Current in the y- branch of w-circuit $= \dfrac{Q_y}{\Delta y}$

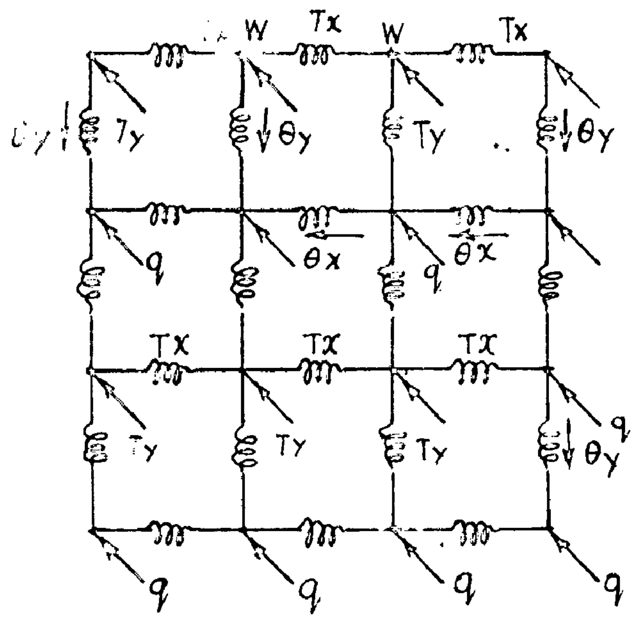
We see that all of the quantities of physical interest are available in electrical analogy.

The variable thickness plate analogy can be easily extended to include anisotropic plates. For such a plate four constants instead of two are needed to define the elastic properties of the material. In terms of these four constants the equation of the plate may be written as:

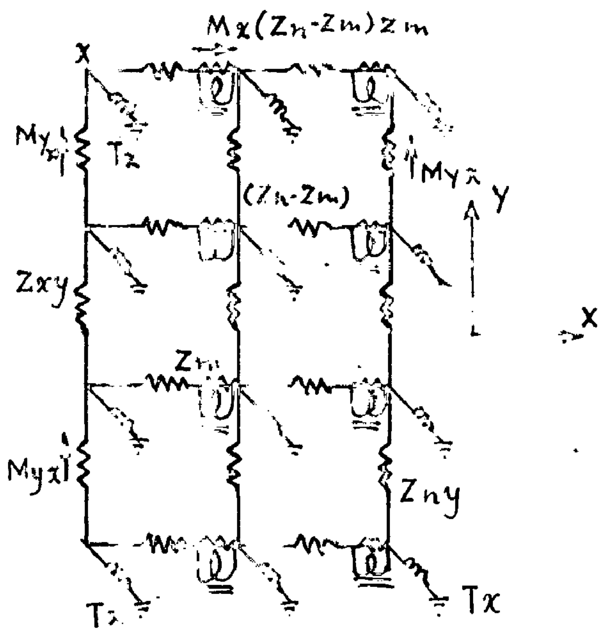
$$\partial/\partial x \left\{ \frac{\partial}{\partial x} \left(D_x \frac{\partial \theta_x}{\partial x} + D_1 \frac{\partial \theta_y}{\partial y} \right) + \partial/\partial y \left(2 D_{xy} \frac{\partial \theta_x}{\partial y} \right) \right\}$$
$$+ \partial/\partial y \left\{ \partial/\partial y \left[D_y \frac{\partial \theta_y}{\partial y} + D_1 \frac{\partial \theta_x}{\partial x} \right] + \partial/\partial x \left(2 D_{xy} \frac{\partial \theta_y}{\partial x} \right) \right\} = q$$

This equation is entirely similar inform to 20

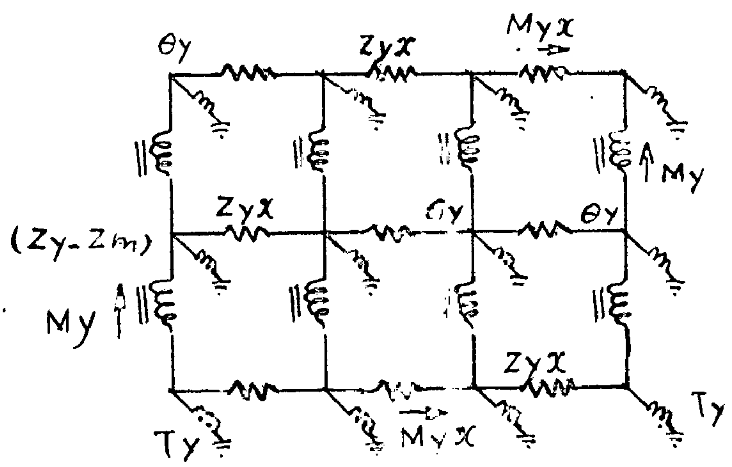
Equation (19). Referring to Fig.(15) we see that self- admittance of the Z_x branches are $D_x/(\Delta x^2)$ and $D_y/(\Delta y^2)$ respectively, that the self admittance of Z_{xy} and Z_{yx} branches are $2D_{xy}/(\Delta y^2)$ and $2 D_{xy}/(\Delta x^2)$, respectively, and that the mutual admittance



(a) W CIRCUIT



(b) θx CIRCUIT



(c) θy CIRCUIT

$$Z_n = \frac{1}{1-\nu^2} \quad \frac{\Delta x^2}{D}$$

$$Z_y = \frac{1}{1-\nu^2} \quad \frac{\Delta y^2}{D}$$

$$Z_m \cdot \frac{\nu}{1-\nu^2} \cdot \quad \Delta x\,\Delta y$$

$$Z_{xy} = \frac{1}{1-\nu} \quad \frac{\Delta y^2}{D}$$

$$Z_{yx} = \frac{1}{1-\nu} \quad \frac{\Delta x^2}{D}$$

FIG. 16 DYNAMIC ANALOGY FOR VARIABLE THICKNESS PLATE.

between the Z_x and Z_y branches is $D_1/(\Delta x \Delta y)$

Method of Applying Boundary Conditions

3.18 - Two boundary conditions must be applied along each edge of elastic plate. A general result of application of the principle of virtual work is that along any edge one condition from each of the following two groups must apply:

Group 1

$$W = W_o \quad \text{- - - - - - - - - - - - (a)}$$

$$Q_x + \frac{\partial M_{tn}}{\partial t} = R_o \quad \text{- - - - - - - - (b)}$$

Group 2

$$\frac{\partial W}{\partial x} = \theta_o \quad \text{- - - - - - - - (c)}$$

$$\partial M_x = M_o \quad \text{- - - - - - - (d)}$$

where M_{tn} = twisting moment

n is the co-ordinate normal to the edge and t is the co-ordinate parallel to the edge.

The combination of (a), (c) gives the boundary condition of damped edges, while combination (a), (d), with $M_o = o$ refers to a simply supported edge. For an edge that is in sufficiently damped a linear combination of (c) and (d) can be applied in addition to (a). The boundary condition of a free or elastically supported edge are given by the combination of (b) and (d).

The quantities involved in group 1 are represented by currents and voltages in deflection net work, while the quantities in group 2 are analogous to the currents and voltages in the slope circuit.

For the discussion of boundary conditions the net work impedences must be regarded as distributed elements. For example, along an edge where θ_x is zero, all impedences in θ_x circuit should be grounded at the points where they cross the edge. A part of an impedence element is retained proportionally to the portion of its length interior to theplate. The transformer primaries of the deflection circuit should similarly considered as distributed coils and grounded at the point where the coil crosses the edge. The part of the winding extending beyond the edge cannot be omitted.

If the edge ix is damped, the constraints require that w=o and θ_x and θ_y are zero. These conditions can be imposed on the net work by grounding the deflection and slope net works where they cross the boundary.

If the edge x is simply supported the mathematical boundary conditi ns are.

$$W=o$$

$$M_x = -D \left(\frac{\partial^2 W}{\partial x^2} + v \frac{\partial^2 W}{\partial y^2} \right) = o \quad \ldots\ldots \quad 21$$

Therefore $\frac{W}{x} = o$, $\frac{\partial^2 W}{\partial y^2} = o$ and $\frac{\partial^2 W}{\partial x^2} = o$

In the constant thickness plate analogy, the current Z_x is proportional to $\frac{\partial^2 W}{\partial x^2}$. Consequently a Z_x branch that crosses a simply supported edge is left open. The W and θ_y circuits are grounded when they cross the simply supported edge.

Fredrick L. Ryder 1953

3.19 - In the electrical circuit force and moment are simulated by current, and deflection and slope by voltage drop. The analogy is based on the fact that castigliano's theorem and the principle of learnt work have their counterparts in certain electrical net works. The theory is developed for the two dimensional and three dimensional truss, and two dimensional rigid frames and truss frame combinations, including strusses with secondary stresses.

3.20 Electrical Energy Theorems

Principle of learnt power:- In a circuit of resistors and ideal transformers that is supplied with given external alternating currents a single frequency and phase angle, each internal current is a minimum subject to the requirements of currents continuity. This is analogous to the principle of learnt work, which states that each internal force and moment in a structure is a minimum, subject to the requirements of equilibrium.

Castigliano's theorem for electrical circuits:- In the circuit previously described, P denotes the power dissipated in the circuit; C is a supplied current, considered + ve when leaving the circuit; and V denotes the voltage (measured below ground) of the point at which the C leaves the circuit-

$$V = \frac{1}{2} \frac{\partial P}{\partial C} \qquad \ldots\ldots\ldots\ldots \quad (1)$$

Equation (1) is analogous to cartigliano's theorem for

Photo.1. Moment Analogy Computor

structures which state that $\delta = \frac{\partial W_d}{\partial F}$ .... (2)

and $\theta = \frac{\partial W_d}{\partial M}$ .... (3)

where W_d = is the work of deformation

F and M are applied force and bending moment.

δ and θ are the deflection and slope, respectively measured at the point of application of F and M.

The absence of the factor $\frac{1}{2}$ in Equation 2 and 3 arises from the fact that electrical power equals the product of current and voltage, whereas the work of deformation is equal to one half the product of force and (deflection) or of moment and slope.

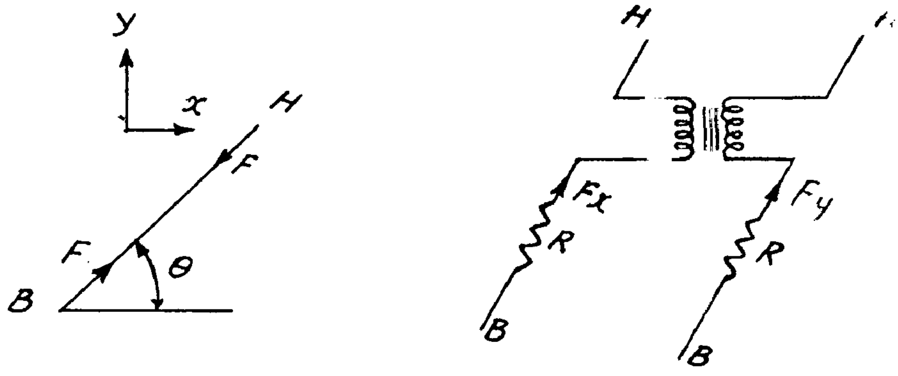
Therefore the following parameters may be considered

analogous	Force	------------	current
	Moment	------------	current
	Deflection	------------	voltage
	Slope	------------	voltage
	work	----------	One half the powe

3.21 Pin connected Truss

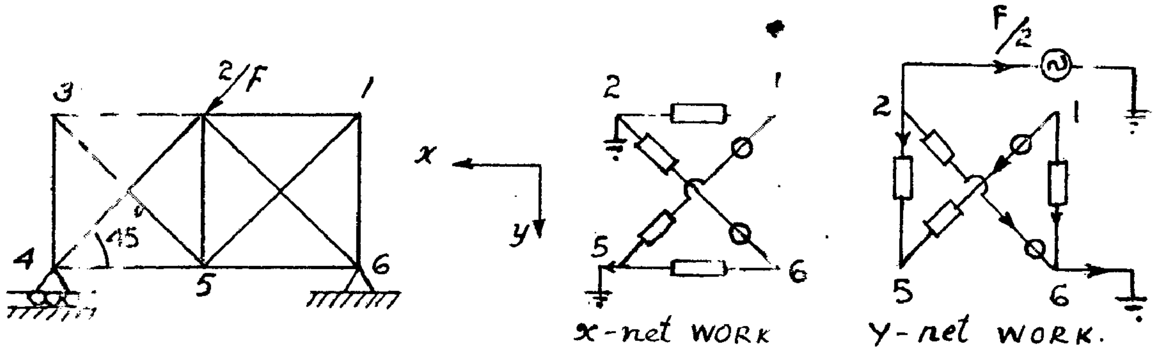
Two dimensional case-

B H is a two dimensional truss member, its' analog consists of X and Y branch connected by ideal transformer as shown in Fig (17b). The arbitrary oriented cartesian axes are indicated, as well as the angle of inclination, θ, the latter being measured from the X axis to the direction of member B.H. The reference direction for θ is that in which Y-axis is



(a) MEMBER (b) ANALOG

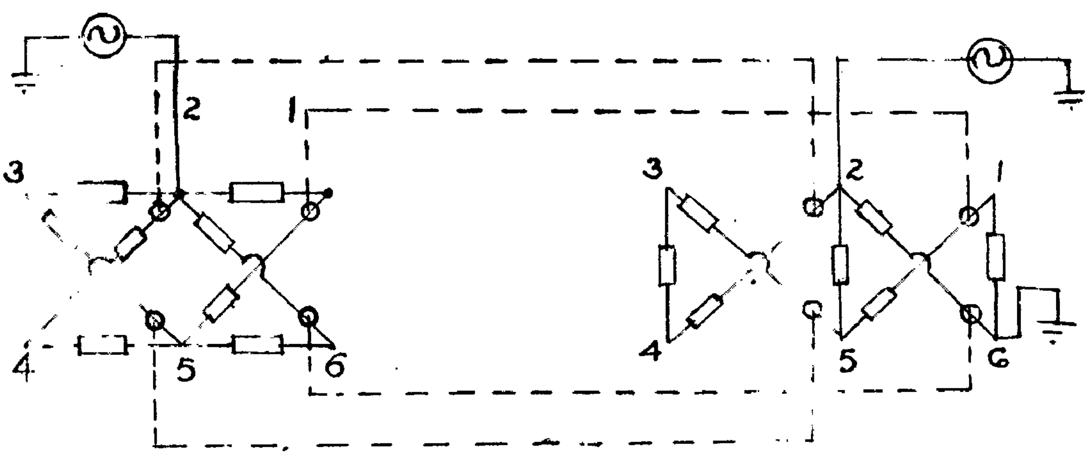
FIG. NO. 17



(a) TRUSS

x-net WORK Y-net WORK.

(C) ANALOG OF RIGHT HALF



X-NETWORK Y-NET WORK

FIG. 18 (b) ANALOG.

rotated 90° from the x- axis.

The force in branch B-H is designated as F, and is considered + ve if tensile. The alternating currents in the electrical branches are designated as Fx and F_y respectively and are at the same frequency and phase, the transformer turns ratio being such that these currents are in the same ratio as the X and Y components of force F, measured in the branch direction chosen for the currents. To achieve this the following must be satisfied:

$$\frac{N_x}{N_y} = - \tan \theta \qquad \ldots\ldots\ldots (4)$$

in which N_x and N_y are the number of turns in x- and y windings of the transformer, respectively. The resultant current whose rectangular components are F_x and F_y is designated as F, since it is analogous to branch force. This resultant is not explicit in the analog.

The analog of entire truss of which branch B-H is a member consists of a separate x and y net work each consisting of one branch for each truss member, the branches being connected in the same way as the truss members. The separate net works are inter-connected magnetically through transformer cores. The x and y components of external forces, including both loads and reactions are simulated by appropriately supplied x and y currents, at the frequency and phase.

To compare the equations of equilibrium and current continuity, the sum of the force, applied to joint B in the truss and the currents leaving joint B in the analog are equated to zero. In the former case,

$$F'_x + F \cos \theta = 0, \qquad \ldots\ldots\ldots \quad (5a)$$

and

$$F'_y + F \sin \theta = 0, \qquad \ldots\ldots\ldots \quad (5b)$$

in which F'_x and F'_y are respectively, the resultant X and Y components of all the forces applied to joint B by branches other than member B H and by the external forces, which include with loads and reactions. In the electrical case, equation 5 will be satisfied if F'_x is the total current, other than F_x) leaving joint B in the x- net work, and can be composed of branch currents and supplied currents. A similar result is obtained for F'_y.

The force and current summations at joint H are similar to those at joint B, except that all the terms are perfixed by a - ve sign.

The work of deformation of each branch is $\frac{F^2 L}{2\, AE}$, as the work of deformation is ½ the electrical power, the branch resistance R in Fig. (17b) equals the compliance $\frac{L}{AE}$, in which case the power for the branch equals

$$P = (F^2_x + F^3_y)\ R = \frac{F^2 L}{AE} \qquad \ldots\ldots\ldots \quad (6)$$

If its imagined that the x- and y - deflection of joints of a typical truss are computed by a conventional use of Castigliano's theorem, the corresponding computation for the analog would be similar, but with applied x or y current substituted for the load, and with deflection replaced by voltage. Hence the x- and y - components of deflection at any joint must be simulated by the corresponding voltages, measured below a datum corresponding to points of zero deflection, in the x- and y- net works.

This leads to two interesting conclusions. First it is unnecessary to maintain at predetermined values the currents corresponding to reaction components, whether the reactions are redundant or not; if electrical joints at which the corresponding deflection component is zero are grounded, the currents leaving the joints must correspond to appropriate components of reaction force applied to the truss by its supports. Only the currents corresponding to load components, therefore must be held at predetermined values.

Second, considering the inter-pretation of the voltage drops in Fig. (17.b), the drops across the resistors in the direction BH are $\frac{F_x L}{AE}$ and $\frac{F_y L}{AE}$ which correspond to the respective components of deflection associated with branch elongation. Since the total drop from B to H in each electrical branch must equal the corresponding components of total branch deflection, the transformer voltage drops must equal the approximate components of branch deflection caused by rotation.

In the special case in which truss branch is parallel to x-axis, the corresponding transformers and y- branch in the analog are omitted, the x- branch being shorted part its transformer terminals. This assures that the analog of the y-component of branch force is zero. A similar arrangement is used if the branch is parallel to y-axis.

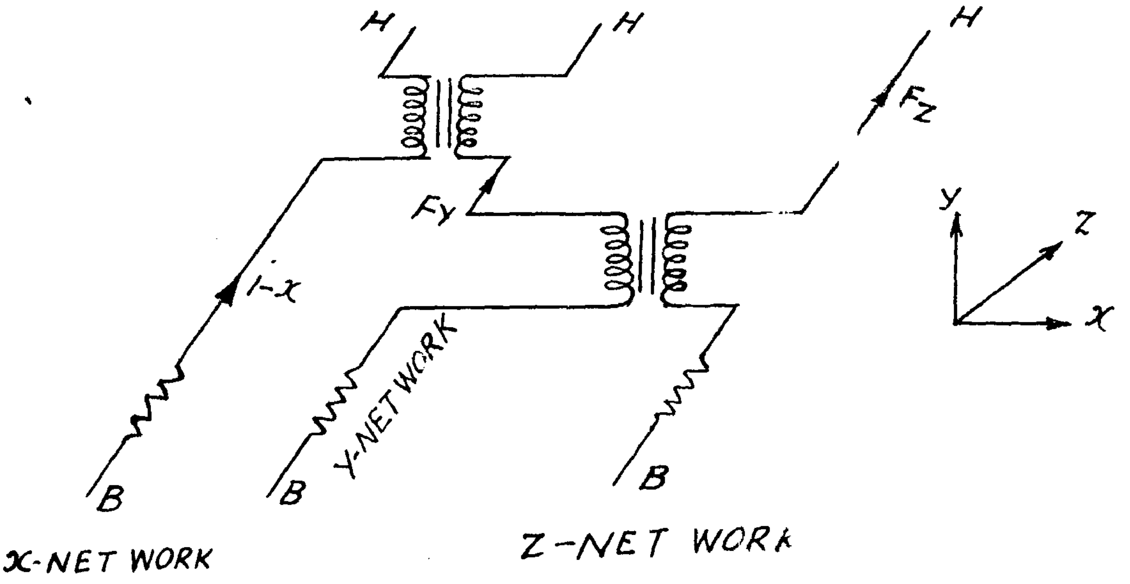
3.22 - Consider the truss of Fig. (18) which is indeterminate to the second degree, F_{1-5} amd F_{3-5} are suitable choices for redundants. In the analog, the horizontal bars appear only in the x-net work, and the vertical branches only in the y -net work. The inclined bars appear in both the net works, the appropriate x- and y- net works are connectd by transformers, the separate winding of which are represented symbolically by circles, and the mutual care by a doted line in a manner similar to that used by Mr. Bush. For branches 1-5 and 2-4 the value of N_x/N_y is - 1 and for branches 2-6 and 3-5 is +1 by use of equation 4. A resistance corresponding to branch compliance is placed in each electrical branch.

To simulate the constraints imposed on the truss by its supports, joint 6 is earthed in x and y- net works, and joint 4 in y-net work. Appropriate alternating currents at the same frequency and phase are with drawn from joint 2 to simulate the load components.

In the special case in which the load in y is vertical the analog can be simplified by the consideration of symetry, and the analog of only right half of the truss need be considered, as shown in fig. (18.c). The resistance in the branch 2-5 must be equal twice the associated, compliance, while the currents equals the half the corresponding branch force, since the member was cut in half by the plane of symetry.

3.23 Three Dimensional Case

The analog of a typical three dimentional branch BH is shown in Fig.(19). The transformers relate F_x and F_2 to



THREEDIMENTIONAL TRUSS MEMBER

FIG. 19

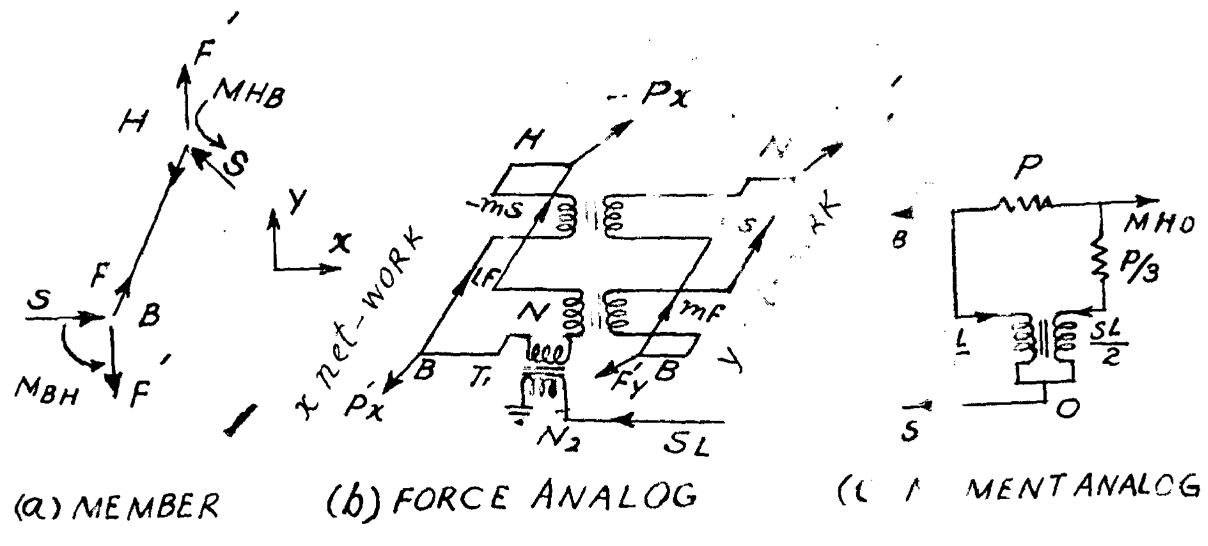


FIG. 20

F_y, keeping the currents in the same ratio as x -, y-, and z- components of the branch force. Alternately these transformers could be replaced by transformers relating F_y and F_x to F_z, or F_z and F_y to F_x. In any case, for transformers relating x- and y- currents, y and z currents, and Z- and x- currents respectively, the transformer turns requirements are

$$\frac{N_x}{N_y} = - \frac{m}{l} , \quad \frac{N_y}{N_z} = - \frac{n}{m} , \quad \frac{N_z}{N_x} = - \frac{l}{n}$$

in which l, m and n are direction Cosines = of branch B-H. In the special case in which the branch is perpendicular to one of the co-ordinate axes, the electrical branch corresponding to that branch is omitted, and only one transformer is needed. If the branch is parallel to one of the coordinate axes, the electrical branches corresponding to the other axes are omitted, and no transformers are required.

The connections of the various electrical branches to form a complete analog, and the selections of grounds and withdrawal of load currents, are governed by the same procedure as in the two-dimensional case. The resistances equal the compliances L/(AE), theresistor and transformer voltage drops corresponding to appropriate components of deflection caused by branch elongation and rotation respectively.

3.24. Analog of the Rigid frame

This presentation will be limited to rigid frames and truss frame combinations in which all loads and members are in one plane. The general scheme of the analogs is to replace force and moment by current and to arrange electrical circuits so that the equations of equilibrium and twice the work of deformation are replaced by the equations of current continuity and power dissipation respectively.

3.25 Straight Member of constant cross section, Loaded at End points only.

The typical member BH of lengthof L is shown in Fig. (20a), The parameters being F, the axial force applied to point B by to member BH, and considered positive when tensile; S, the resultant shear applied to B by loads and by adjoining members, and considered +ve when it tends to rotate BH in the direction of +ve slope, (which is the direction in which y- coordinate is 90° removed from x- coordinate), and M_{BH} and M_{HB} are the resultant moments considered positive in the +ve direction of slope. Denoting by F'_x and F'_y the components of resultant force applied to B by the loads and members other than BH, and noting that the resultant is also the resultant of S and - F,

$$F'_x = m^S - l_F \qquad \text{-------} \qquad (9a)$$

$$F'_y = l_s - m_F \qquad \text{------} \qquad (9b)$$

Moment summation yields

$$M_{BH} + M_{HB} + S\,L = 0 \qquad \text{-----} \qquad (10)$$

The x- and y- net works of an electrical analog for the forces are shown in Fig.(20) current leaving a joint is analogous to a force component applied to the corresponding structural joint. Ideal transformers are used to fix the ratio of the currents, the required turns ratio being, for axial face $\frac{N_x}{N_y} = -\frac{m}{I}$, and for shear force $\frac{N_x}{n_y} = \frac{1}{m}$

The equations of current continuity at joint B and H simulate Equation 9 of force summation. If the tensile and shear work is appreciable, it can be simulated by inserting suitable resistors in the branches. By analogy with tensile case the shear resistance may be written $\frac{\alpha L}{A G}$ in which G is the shear modulus, and is the ratio of shear stress at the centroidal plane to the average shear stress S/A. It can be shown by Castigliano's theorems that the branch voltage drops simulate the corresponding components of total branch deflection. The resistor voltage drops in the tension and direct shear branches simulate deflection components caused by direct tension and direct shear, respectively.

The moment relationships can be obtained, the analog indicated in Fig. (20c) with transformer turns ratio equal to -1, The SL - current can be obtained by the indicated transformer tie- in with the x- shear branch, the turns ratio being,

$$\frac{N_1}{N_2} = \frac{L}{m} \qquad \text{.........} \qquad (11)$$

in which the units for L are arbitrary currents learning B and H simulate moments applied to the corresponding structural joints. The equation of current continuity is identical to Equation 10 for moment equilibrium, while the power dissipation is

$$P = \rho \left[M^2_{BH} + M_{BH}\, SL + \left(\frac{SL}{3}\right)^2 \right] \qquad \text{.........} \qquad (12)$$

in which P is resistance analogous to bending compliance. The work of bending a straight beam is

$$W_b = \int_0^L \frac{M_s^2 \, d_s}{2E\,I} \quad \ldots\ldots\ldots \quad (13)$$

in which MS - is the moment at a distance S from the joint B, Substituting for Ms its value in terms of M_{BH}, S and S, and integrating, it is found that twice the work of deformation equals the power dissipation if

$$P = \frac{L}{EI} \quad \ldots\ldots\ldots \quad (14)$$

By Castigliano's theorems, it can be shown that the voltage drop from B to H in Fig. (20c) represents the change of slope between B and H caused by bending. Also, the voltage drop from B to O represents $\frac{1}{L}$ times the bending deflection of H from the line tangential to the beam at B, in the direction of +ve shear; and correspondingly for the voltage, drop from H to O.

To construct the analog for an entire rigid frame, points B and H for all branches are inter-connected according to the configuration of the structure, a separate net work being required for x- force, y - force, and moments. Electrical joints corresponding to zero x- and y- deflections or slope are grounded, and alternating currents of same frequency and phase are with drawn to simulate the applied force components and moments.

3.26 Some special members

The general method of approach is to simulate the

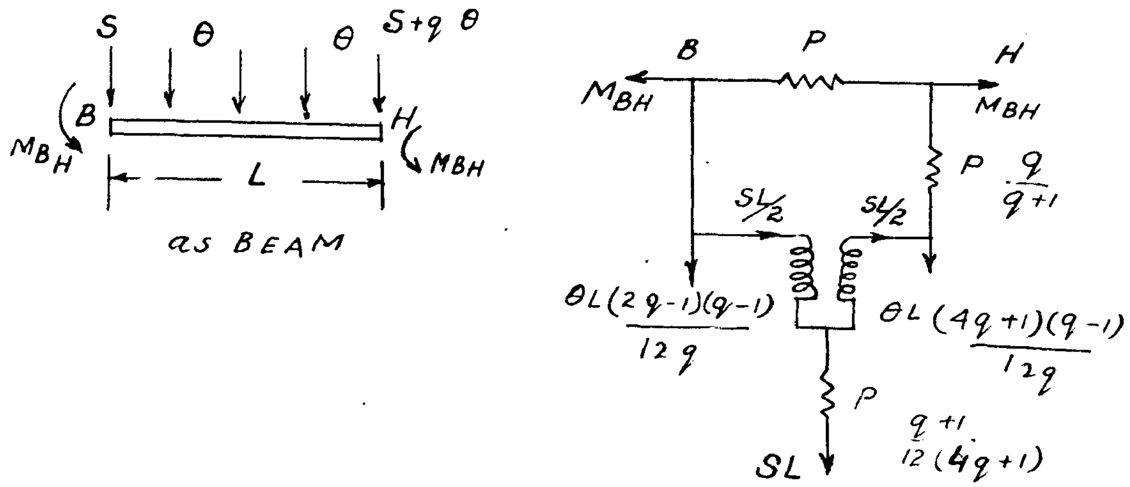


FIG. 21 BEAM SUBJECTED TO UNIFORMLY SPACED LOADS.

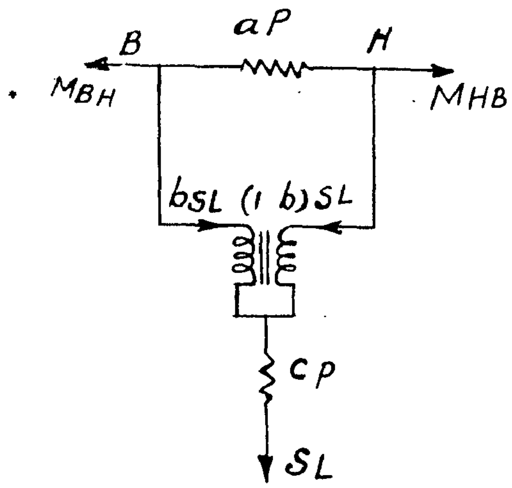


FIG. 22 ANALOG OF A STRAIGHT MEMBER WITH VARYING MOMENT OF INERTIA.

W.T. Russell and R.H. Mac Neal 1953

This analogy is suitable for the solution of static load problems, vibration problems and transient load problems of beams.

3.27 Derivation of concentrated load analogy

The usual assumptions made in deriving the differential equations of a beam will be made here. Inaddition it will be assumed that the distributed loads acting on the beam can be replaced by concentrated loads acting at a finite number of stations. At each station a jump in the shear will occur due to concentrated loads.

Consider a segment of beam, unloaded between a pair of stations, with end forces and displacements. The segment of length r is assumed to be in a condition of equilibrium under the action of bending moments and shears applied at its ends.

Then the shear at any point

$$V_x = V_a = V_b \quad \ldots\ldots\ldots \quad 1$$

and the bending moment at any point

$$M_x = M_b + (r-x)\,V_b \quad \ldots\ldots\ldots \quad 2$$

The following equations give the deflection of beam any point λa, and the differences in slopes at the two ends.

$$w\,\lambda a = w_a + \lambda a \phi_a + \int_0^x \frac{M_x}{EI}\,(\lambda a - x)\,d_x \quad \ldots\ldots \quad 3$$

conditions of moment equilibrium and the bending work of deformation. By Castigliano's theorems, voltage drop between the joints which represent the ends of the member is analogous to the change of slope.

(1) Beam subjected to uniformly spaced loads

q is the number of equal spaces between the loads. If the load Q exists at B it may be added to S. The work associated with Q alone is not correctly simulated, but this work has a derivative only with respect to Q, and is therefore of no interest in the differentiations associated with Castigliano's theorem and the principle of least power. The analog may be reduced to the case of a uniformly distributed load Q', by letting the number of loads equal infinity (Fig.21).

(2) Analog of a straight member with varying moment of inertia

In this case the beam is loaded at the end points, with a moment of inertia of I varying linearly from I_B at B to I_H at H. The resistances are always positive irrespective of the relative magnitudes of I_B and I_H (Fig. 22).

$$a = \log_e i \qquad \ldots\ldots 15 \qquad (a)$$

$$b = \frac{1}{\log_e i} - \frac{1}{i-1} \qquad (b)$$

$$C = \tfrac{1}{2} + \frac{1}{i-1} - \frac{1}{\log_e i} \qquad (c)$$

and

$$P = \frac{L}{E\,(I_H - I_B)}, \qquad (d)$$

where $i = \dfrac{I_H}{I_B}$.

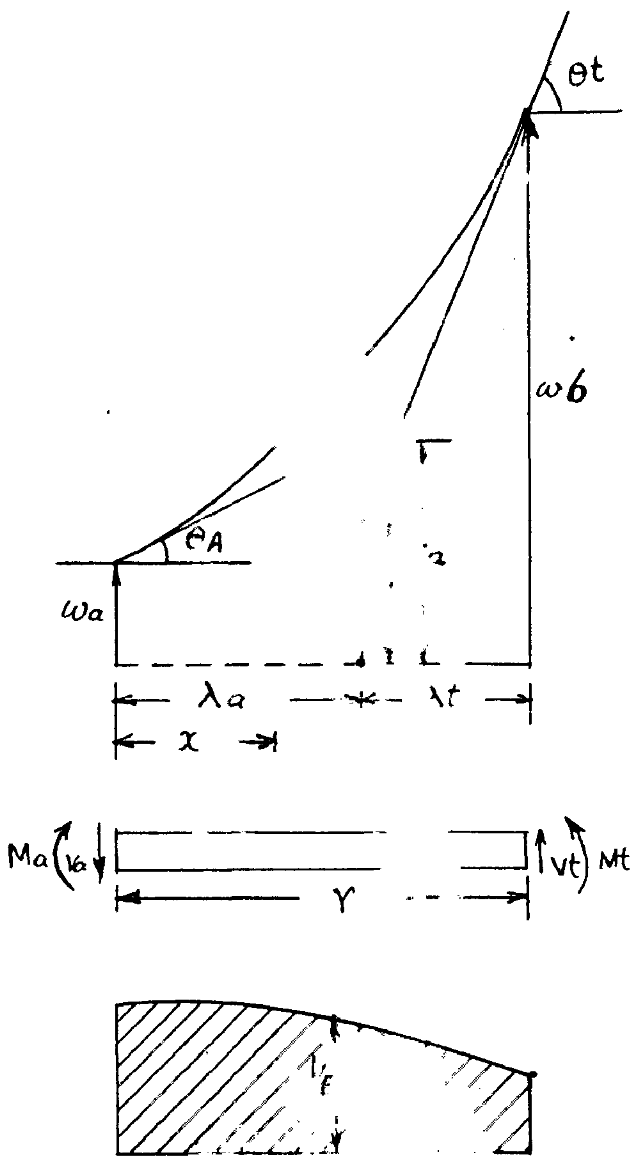


FIG-23.

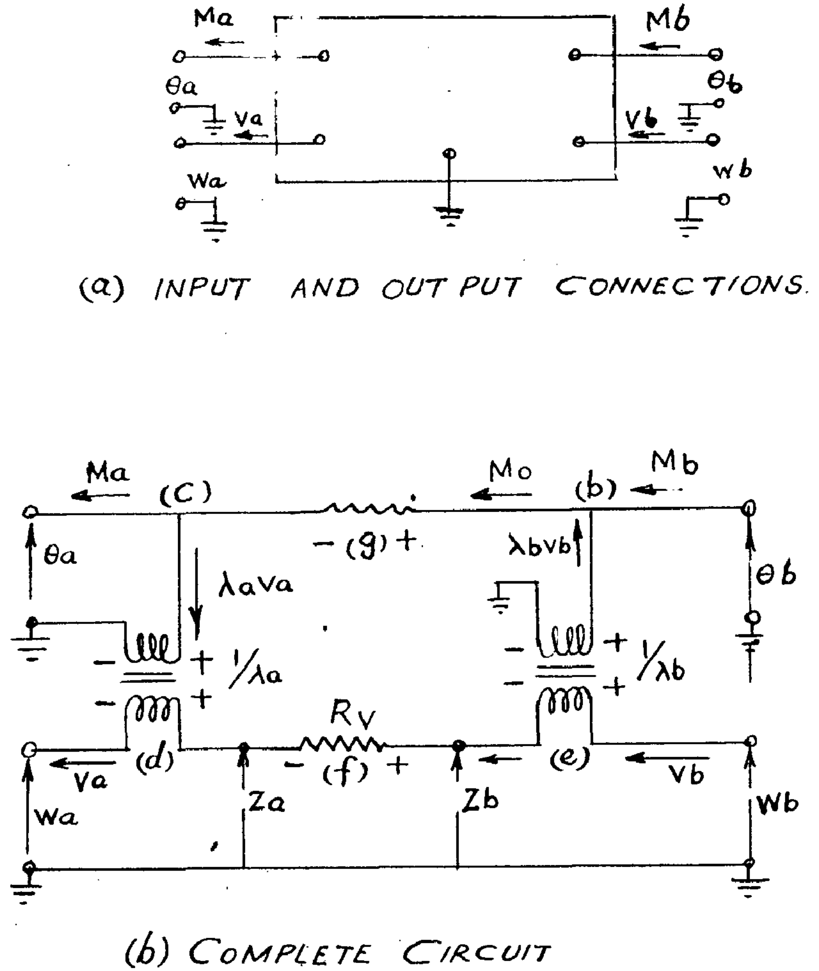


FIG. 24. CONCENTRATED-LOAD BEAM ANALOGY.

$$w_{\lambda a} = w_b - \lambda b\, \theta_b + \int_x^r \frac{M_x}{EI} (x-\lambda a)\, d_x \quad \cdots\cdots \quad 4$$

$$\theta_b - \theta_a = \int_0^r \frac{M_x}{EI}\, d_x \quad \cdots\cdots \quad 5$$

subtract equation number 3 from equation 4

$$w_b - w_a = \lambda_a \theta_a + \lambda b\, \theta_b - \int_0^r \frac{M_x}{EI} (x-\lambda a)\, d_x \quad \cdots\cdots \quad 6$$

The deflections of the extended tangents from points A and B at x = a are from Fig. (23)

$$Z_a = w_a + \lambda a\ \theta_a \quad \cdots\cdots \quad 7$$

$$Z_b = w_b + \lambda b\ \theta_b \quad \cdots\cdots \quad 8$$

Hence by substitution in to equation (6)

$$Z_b - Z_a = - \int_0^r \frac{M_x}{EI} (x-\lambda a)\ d_x \quad \cdots\cdots \quad 9$$

Insert thevalue of M_x given by equation (2) in to this equation

$$Z_b - Z_a = - M_b \int_0^r \frac{x - \lambda a}{EI}\ d_x + V_b \int_0^r \frac{(x-\lambda a)(x-r)}{EI}\ d_x \cdots 10$$

If a is so chosen that x= λa is at the centre of gravity of 1/EI diagram, the first integral of the equation (10) vanishes and in the second integral (x-r) may be replaced by (x- λ a). This choice of λa has the effect of uncoupling the equations for deflection and slope.

$$Z_b - Z_a = V_b \int_0^r \frac{(x-x_a)^2}{EI}\ d_x \quad \cdots\cdots \quad 11$$

Hence $Z_b - Z_a$ is equal to shear multiplied by the moment of inertia of 1/EI about the C.G. of 1/EI diagram -

Similarly the difference in slopes at the two ends is obtained by substituting the value of M_x from equation (2) in to equation (5)

$$\theta_b - \theta_a = \int_0^r \frac{1}{EI} \left\{ M_b + V_b \left\{ \lambda_b + (\lambda_a - x) \right\} \right\} d_x \qquad \ldots \quad 12$$

Since $x = \lambda a$ in the C.G. of 1/EI

$$\theta_b - \theta_a = M_o \int_0^r 1/EI \; d_x \qquad \ldots\ldots \quad 13$$

where $M_o = M_b + V_b \; \lambda b$

M_o is the bending moment at the centre of gravity of 1/EI.

The equations which are essential to the development of the electrical circuit are summarised as follows:

$$V_a = V_b \qquad \ldots\ldots a.14$$

$$M_o = M_b + \lambda_b \; V_b \qquad \ldots\ldots b.14$$

$$M_a = M_a + \lambda a \; V_a \qquad \ldots\ldots c.14$$

$$Z_a = w_a + \lambda a \; Q_a \qquad \ldots\ldots d.14$$

$$Z_b = w_b + \lambda b \; \theta_b \qquad \ldots\ldots e.14$$

$$Z_b = z_a = V_b \, R_V \qquad \ldots\ldots f.14$$

$$\theta_b - \theta_a = M_o \, R_M \qquad \ldots\ldots g.14$$

where $R_M = \int_0^r \frac{d_x}{Ei}$

$$R_V = \int_0^r \frac{(x - \lambda a)^2}{EI} d_x$$

In the electrical analogy displacement quantities (deflection and slopes) will be represented by voltages to ground and force quantities (shear and B.M) will be represented by currents.

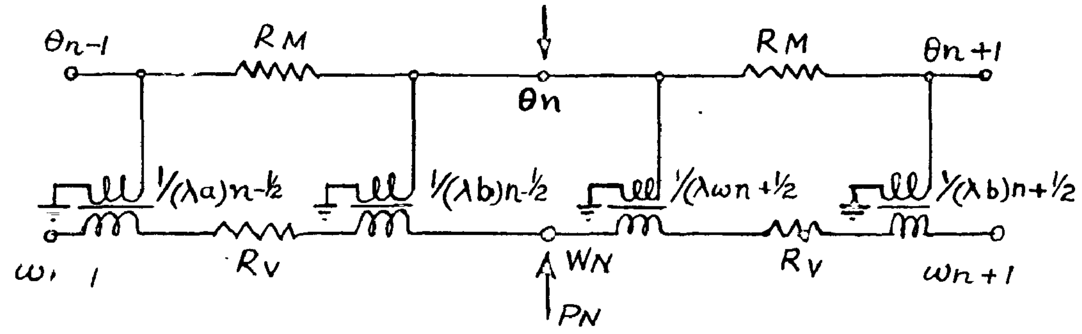
3.28

Fig (24a) shows the in put and out put connections while the (24.b) shows the complete analogous circuit and the places where each equation is satisfied. The properties of ideal transformers awe used in this circuit. For example the current flowing in the lower winding of the transformer at the right is equal to V_b. Since the turns ratio of the transformer is $\lambda b:1$, the current flowing in the upper winding is equal to $V_b \ \lambda b$. This current adds to give M_b to give M_o, thus satisfying the equation (14b). Since the voltage across the upper winding is equal to θ_b, the voltage across the lower winding is equal to $\lambda b \ \ \theta_b l$ and this voltage subtracted from w_b gives Z_b.

Equations (14.c) and (14.d) are similarly satisfied by the transformer at the left. Equations (14.f) and (14.g) applied to the circuit are an expression of ohm's law for resistors whose resistances are respectively, R_V and R_M. Equation (14.a) is satisfied because no branches occur in the circuit connecting W_a and W_b.

When several such circuits are connected together they form the analogy for a complete beam.

Concentrated loads are reoresented by currents inserted in to the deflection circuit at the points of inter-section



(a) CIRCUIT WITH TWO TRANSFORMERS PER SECTION.

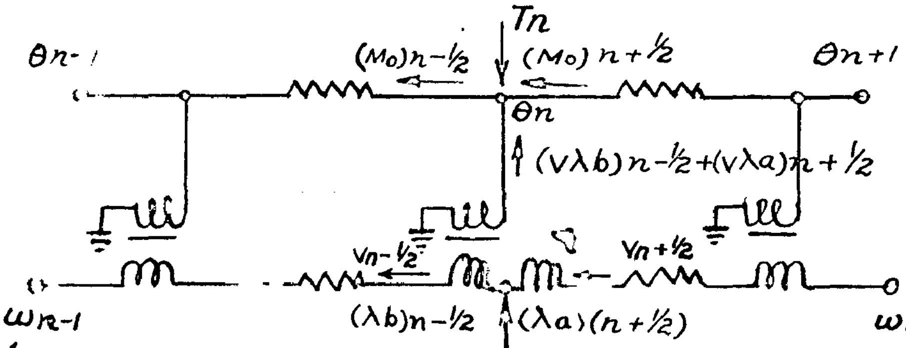
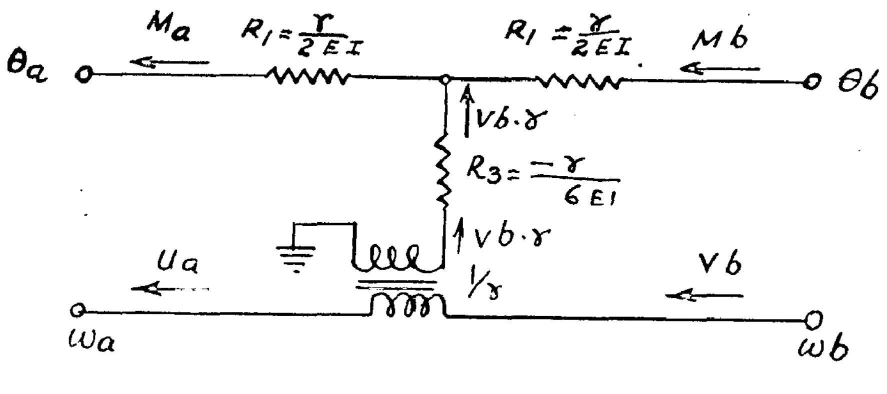


FIG25.(b) CIRCUIT WITH ONE TAPPED TRANSFORMER PER SECTION.



(a) CIRCUIT WITHOUT NEGATIVE RESISTANCE.

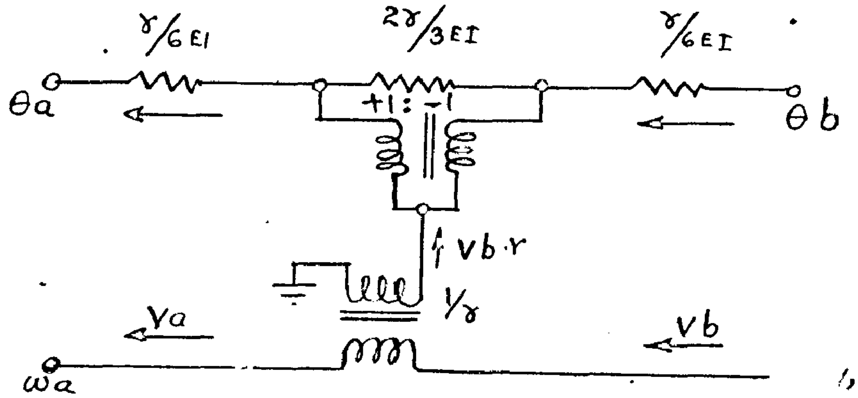


FIG.26. CIRCUIT WITHOUT NEGATIVE RESISTANCE

Currents representing concentrated moment loads also may be inserted into the slope circuits at these points.

The two transformers on either side of the nth node may be replaced by a simple transformer with a lapped winding as shown in the fig. (25b). The bending moments at the ends of each segment no larger can be measured. However, M_o the bending moment at the centre of gravity of 1/EI diagram for each segment should provide sufficient information.

In the static stress analogy flexibility is represented by the electrical resistance and strain energy is analogous to electrical power. In vibration problems with distributed mass loads the same form of the circuit is used but the resistors are replaced by inductors having same numerical values, capacitors, having numerical values equal to the mass of each section, are connected between the nodes of deflection circuit and the ground.

3.29 An equivalent form

This circuit shown in the fig (26) may be derived by setting $\lambda a = r$ and substituting $M_x = M_a - x V_b$ in equation 3.

Then
$$w_b = w_a + r \left\{ \theta_a + M_a \, 1/r \int_0^r \frac{r-x}{EI} \, d_x \right\} - V_b \int_0^r x \frac{(r-x)}{EI} \, d_x \qquad \ldots\ldots 15$$

Define-
$$R_1 = 1/r \int_0^r \frac{r-x}{EI} \, d_x \qquad \ldots\ldots a.\ 16$$

$$R_2 = 1/r \int_0^r \frac{x}{EI} \, d_x \qquad \ldots\ldots b.\ 16$$

$$R_3 = - 1/r^2 \int_0^r x \frac{(r-x)}{EI} d_x \qquad \ldots\ldots c.16$$

$$\text{Then } w_b - w_a = r\{(\Theta_a + M_a R_1) + V_b r R_3\} \qquad \ldots\ldots 17$$

$$\text{From equation (2) } M_a = M_b + rv_b \qquad \ldots\ldots 18$$

If equation (2) is substituted into equation(5) and V_b is eliminated by equation (18) then the equation for the difference in the slopes at the two ends of the segment is,

$$\Theta_b = \Theta_a = M_a R_1 + M_b R_2 \qquad \ldots\ldots 19$$

From the electrical circuit equation (19) is obtained by summing up the voltage drops across R_1 and R_2. Equation (17) by adding Θ_a to the voltage drops across R_1, and R_3 and multiplying bytransformer turns ratio. Equation (18) is satisfied by the currents at the junction of R_1 and R_2.

The negative resistance R_3 can be replaced by realisable circuit containing a transformer as shown in fig.(26). The circuit of the figure 26a is more practical as it requires only one transformer compared to circuit (26b). However, it is believed that circuit in Fig.(26b) will be useful in developing a more accurate analogy for thinplates, and for thin multi cell shells where distributed twisting moment act on the sides of idalised beams.

<u>G.W. Riesz and B.J. Swain 1954</u>

3.30 - This analogy utilises the equivalence between stored energy in a net work of resistors and capacitors and strain energy in a structure for the solution of indeterminate structural problems. The method makes use of a small number of cheap and easily obtainable parts.

3.31 - 1. Mathematical theory

The potential distribution along a sting of resistors connected in series when currents are introduced at several points is analogous to the bending moment, diagram due to loads applied at similar points on a straight beam.

Consider a beam which is in equilibrium under the action of several transverse loads and reactions, w_1, w_2, --- w acting at distances x_1, x_2 ------ x_x from the left end of the beam. Then since the beam is in equilibrium

$$\Sigma W = 0 \qquad \text{-----(1)}$$

$$\Sigma M = 0 \qquad \text{-----(2)}$$

The second equation may be written as

$$M_L = M_R \qquad \text{-----(3)}$$

where W = vertical fòrce as reaction

M_L = Bending moment evaluated by consideration of the loads and reactions to the left of the section.

M_R = Bending moment evaluated by consideration of loads and reactions to the right of a section

At a section x

$$M_L = M_R = w_1 (x-x_1) + w_2 (x-x_2) + \text{----} + w_k (x-x_k)$$

where there are K loads applied to the left of the section x.

Consider now a string of resistors into which several currents i_1, i_2 -----i_x, are fed at a points where the total resistance from the left end to each node is r_1,r_2---r_x, also shown in the figure (27).

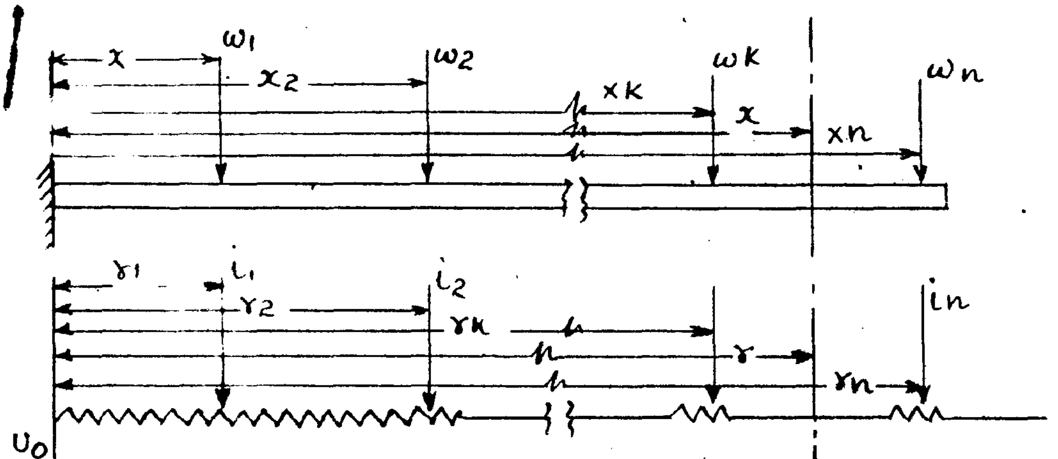


FIG-27 ANALOGY BETWEEN VOLTAGE AND BENDING MOMENT.

Applying Kirchhoff's law;

$$\Sigma i = o \qquad \text{-------} \qquad (5)$$

at each node, and taking currents entering the string of resistors as +ve, the total current at each section is given by the sum of the currents i_1, i_2---- to the left of that section.

At a point where the total resistance from the left end is r and there are currents fed into the resistor string to the left of the section under consideration, the voltage V may be obtained by the i_r drops in each section.

If the beam is loaded or supported at the left end, the distance r_1 will be both zero. If in addition the left end is clamped there will be a fixed moment which is represented in the analog by the potential V_o theyoltage V is given by

$$V = U_o - (r_2 - r_1)\, i_1$$
$$- (r_3 - r_2)\,(i_1 + i_2)$$
$$- (r_4 - r_3)\,(i_1 + i_2 + i_3)$$
$$- (r - r_k)\,(i_1 + i_{k-1} + i_k) \qquad \text{------ (6)}$$

This reduces to

$$V_o - V = i_1\,(r - r_1) + i_2\,(r - r_2) + \text{--} + i_k\,(r - r_k) \quad \text{-(7)}$$

Also at a section $V_L = V_R$ ------ (8)

It will be seen that equations (5) (7) and (8) are analogous to equations 1, (4) and (3) respectively. The behaviour of circuit is therefore analogous to that of beam, resistance being analogous to length and current to transverse load. It should be noted that correct scale factors are maintained in the analog.

3.32 - This simple circuit is therefore all thatis required to solve bending problems of statically determinate beams. But if the solution of statically indeterminate problems is to be attempted, then an analogy to the elastic properties of the beam must be introduced. The theorem of castigliano states that in a structure where there is a initial strain, the redundants have such a magnitude that the rate of change of strain energy with respect to each of the redundants is equal tothe corresponding deflection of that redundant. If the corresponding deflection is zero, the theorem may be stated in another familiar form by saying that the total strain energy is a minimum.

Now the strain energy of bending of an element of beam is given by the expression

$$d_u = \frac{M^2}{2EI} d_s \qquad \text{-------} \qquad (9)$$

This is analogous to the expression for the electrical energy stored in a capacitance which is,

$$U = \tfrac{1}{2} C V^2 \qquad \text{-------} \qquad (10)$$

where M is represented by V, and $\frac{ds}{EI}$ is represented by C.

It has been shown above that an analogy of voltage to bending moment may be obtained by the use of resistors. By combining this with the analogy of stored energy in a capaitance to strain energy in a beam element, a circuit can be obtained in which currents representing the redundants are adjusted until the stored energy is minimum, thus solving the corresponding statically in-determinate structural problem. It should be noted that the assumptions made in the derivation of this analogy are the same as those usually made in the solution of statically indeterminate problems, namely the Bernoulli assumption for flexure, and that of the shear strain energy may be neglected.

G. Sved 1956

3.33 - The members of a rigid frame work are represented by resistance net works, the resistance value being selected to confirm to the stiffness ratios, and the carry over factors of the structural members. If the currents representing the fixed end moments are fed in at the joints then the distribution of current in the net work will represent the distribution of moment in the actual frame work.

The calculation of net work resistance is based on the concept of stiffness K and carry over factor C.

The following formulae apply quite generally including the cases of non-uniform beams and/or semi rigid joints.

(Either T or Π or any equivalent 3 terminal net work can be used (Fig. 28).

For a T net work,

$$R_A = 1/K_A \quad \frac{1-C_B}{1-C_A C_B} \; ; \quad R_B = \frac{1}{K_B} \quad \frac{1-C_A}{1-C_A C_B}$$

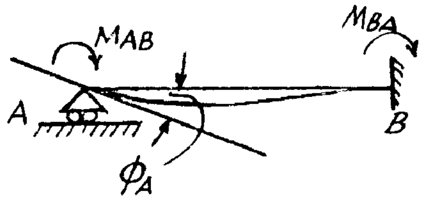
$$R = \frac{1}{K_A} \quad \frac{C_B}{1-C_A C_B} = \frac{1}{K_B} \quad \frac{C_A}{1-C_A C_B}$$

whereas for a Π net work,

$$r_A = \frac{1}{K_A} \frac{1}{1-C_A} \; ; \quad r_B = \frac{1}{K_B} \quad \frac{1}{1-C_B}$$

$$r = \frac{1}{C_A K_A} = \frac{1}{C_B K_B}$$

FOR END A OF THE BEAM

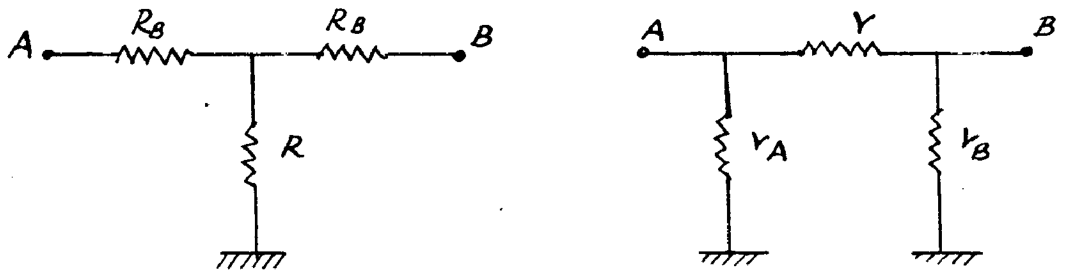


$$K_A = \frac{M_{AB}}{\phi_A}$$

$$C_A = \frac{M_{AB}}{M_{AB}}$$

SIMILAR DEFINITIONS APPLY TO QUANTITIES K_B AND C_B

1.



2.　　　　3.

FIG-28.

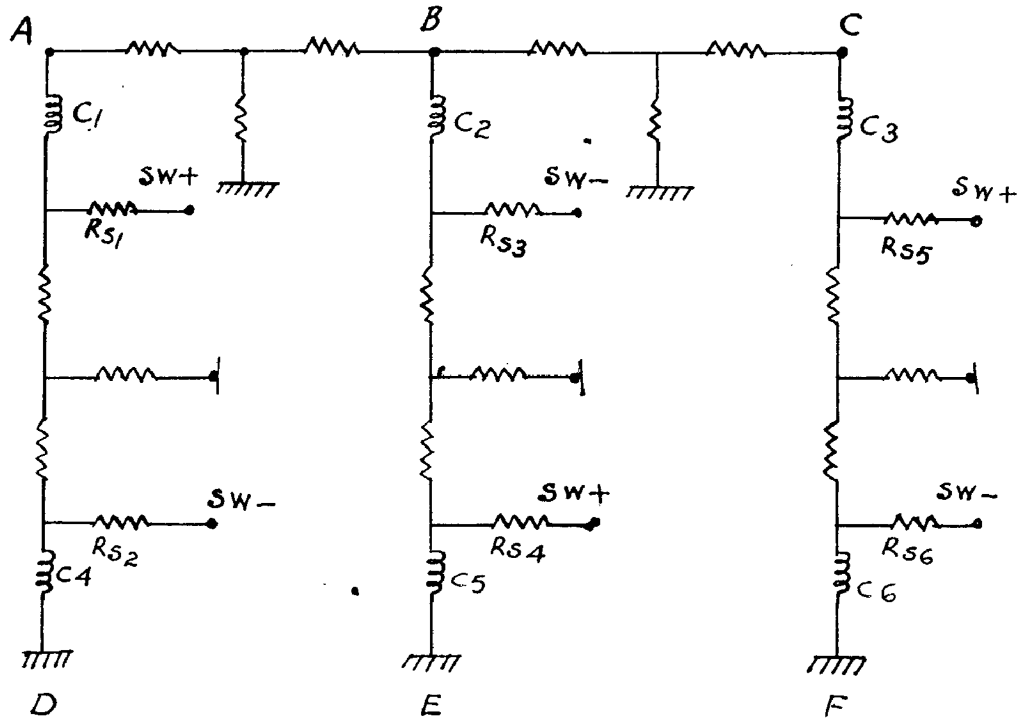


FIG — 29

Naturally all resistance values may be multipiied by an arbitrary constant.

For a beam of constant section and rigid cross section

$$C_A = C_B = 1/2 \quad \text{and} \quad \bar{K}_A = \bar{K}_B = K, \text{ hence}$$

$$R_A = R_B = R = 2/3 \ \ 1/K$$

$$r_A = r_B = r = 2 \ \ 1/K$$

3.34 Construction of analogue

The analogue set up for a two bay building is shown in Fig. (29), the extension to any other structure is straight forward.

Six of the seven coils of multi-coil meter are used to measure the sum of the currents corresponding to column end moments (See C_1 to C_6). The polarity of those near A, E, and C is opposite to those near D, B_1 and F. If the columns are of the same length, then all coils will have same number of turns; if they are different, either the number of turns of the coil is adjusted or shunt resistances are connected across the coils effected. The seventh coil is used to "bias" the meter if a sway load is acting in the structure.

(For columns of different length the sway check equation is,

$$V = \frac{M_{AD} + M_{BA}}{L_{AD}} + \frac{M_{BA} + M_{ED}}{L_{BE}} + \frac{M_{CF} + M_{FC}}{L_{CF}})$$

If the multi coil meter shows a sway unbalance, this can be corrected by feeding currents through approximate resistances R_{s1} to R_{s6} from two sources S_{W+} and S_{W-} to the current feed points. The potentials of the two sources are adjusted by equal currents in opposite directions relative to ground potentials until the meter registers balance.

Since in the actual structure the sway - corrections are not equally distributed if the second moments of area and/or the lengths of the columns differ, the correction currents have to be distributed accordingly by modifying the sway supply resistors (R_s), in inverse proportion to the appropriate I/L^2 values.

G. Browwer and S. Van Der Meer - 1957

3.35 - The analogue system consists of resistor net works and feed back amplifiers. The bending moments, shear force translations, and rotations resulting from a static load are represented by potentials and currents. The influence of change in load, flexural rigidity of members, or shape of the structure in the distribution of bending moments may readily be derived from the model experiments. This requires simply resulting of necessary potentiometers and measuring the new potential distribution.

3.36 The general principle of analogue.

A continuous bea may be considered to an assembly of separate rigid bodies connected by elastic joints. A continuous load may also be split up in to concentrated loads. This principle is applied to the electric circuit model of a beam.

The deflected beam shown in Fig. (30) is divided into 9 equal sections of length 2L and two sections of length L. Stations spaced 1 apart are numbered consecutively 0,1---- 20. These numbers will serve as locational indices to various quantities considered.

A load V_x is acting upon the beam as well as reactive moments M, and reactive forces V. The local flexural rigidity is denoted by the symbal K_x. The position of deflected beam is determined by the vertical displacement V_x of the mark n with respect to unloaded state. The slope of the deflected curve is denoted by m_n. The sign convention is indicated in figure (31). The slope of the beam and vertical displacements are drawn to an exaggerated scale but both are assumed to be small.

The following relations hold inFig. (30a)

$$V_o + V_x + V_{2o} = 0 \qquad (1a)$$

$$M_{zo} - M_o = V_{on}1 - V_{zo}\,(20-x)1 \qquad (2a)$$

$$M_n = -\,(K_n/4\,1^2)\,(V_{n+2} - 2V_x + V_{x-2}) \qquad (3a)$$

of which the first two describe the equibrium condition and last are deflections of beam in terms of difference equations, in the latter the distance between two elastic joints is 21 except at the edges where the length of the rigid part is 1. In this case bending moments due to horizontal loads are neglected. The net work analogue is drawn in Fig. (30b). Two horizontal rows of resistors are shown, which are inter-connected by vertical resistors R_x. The horizontal rows are

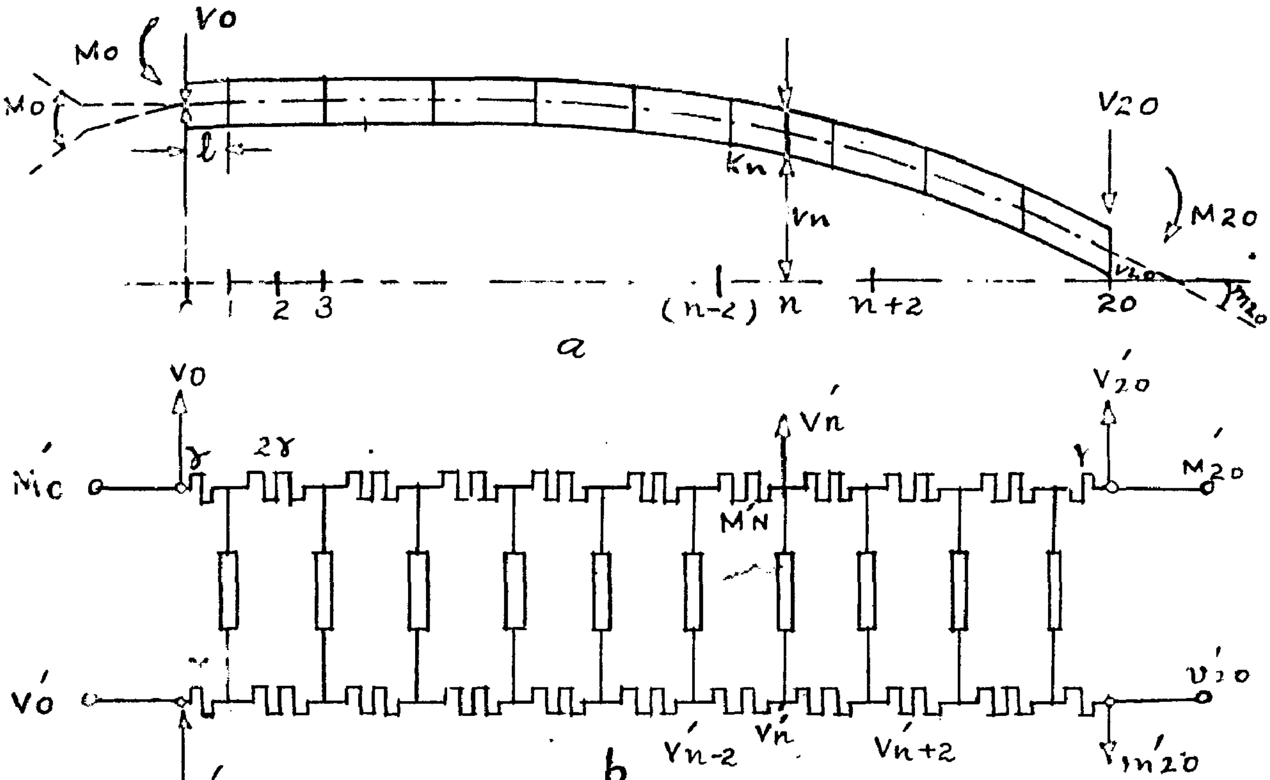


FIG- 30 ANALOGUE OF STATICALLY LOADED BEAM

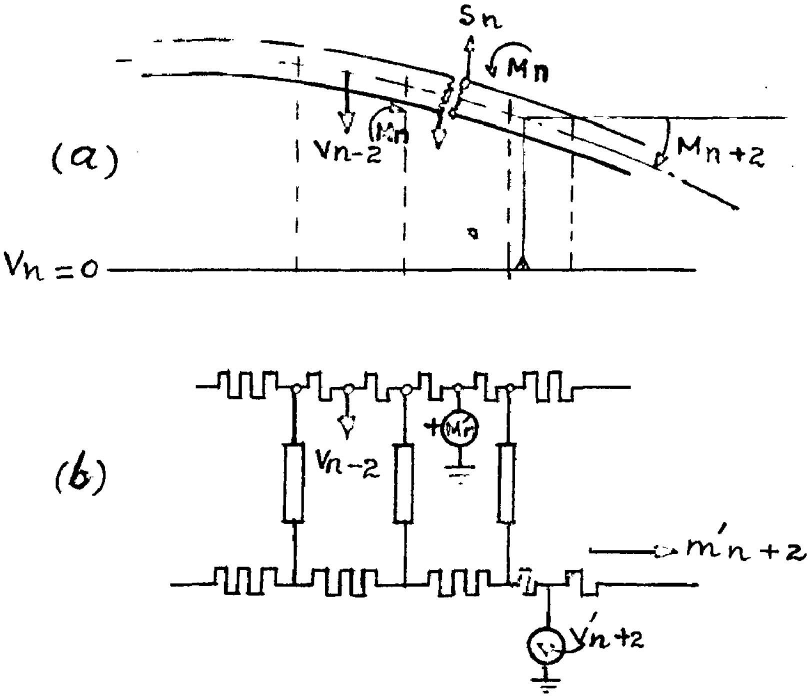


FIG-31. SIGN CONVENTION

sub divided into 9 resistors 2_r and 2 resistors r at the edges, and are tapped between adjacent resistors. The top row potentials are indicated by M', the bottom row potentials by V', the current drawn from the top row by V', the current flowing in the bottom row, from left toright by m'. R_n is chosen to be large compared to r and automatically V' is always large compared to m'. The potential distribution M' is almost independent of rest of work and may be computed or if the vertical resistors were replaced by open circuit. More over the potentials v' are negligible compared to M' and so the current flows in vertical resistors are set completely by the top row potentials. Taking these considerations into account and applying Kirchhoff's laws to the electrical net work we get, with fairly good accuracy,

$$V'_o + V'_n + V'_{zo} = 0 \qquad (1b)$$

$$M'_{zo} - M'_o = V'_o \, nr - V'_{zo} (20-n)r \qquad (2b)$$

$$M'_n = - (R_n /2r) (v'_{n+2} + 2 v'_n + v'_{n-2}) \qquad (3b)$$

The relations between the mechanical quantities are of a nature similar to those between the electrical quantities. The net work analogue will give equivalent results provided the model law is satisfied and boundary

$$R_n/2_r = K_n/ 41^2 \qquad (4)$$

conditions correspond to those present in the beam.

Alternating currents are to be applied rather than direct currents, because it is more convenient to a-c amplifiers.

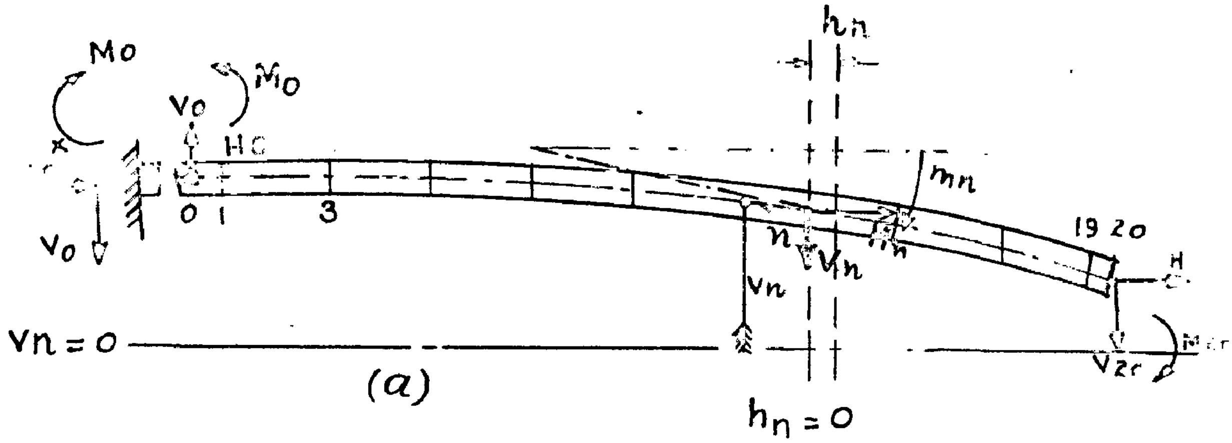
3.37 Analog of a cantilever beam with built in supports

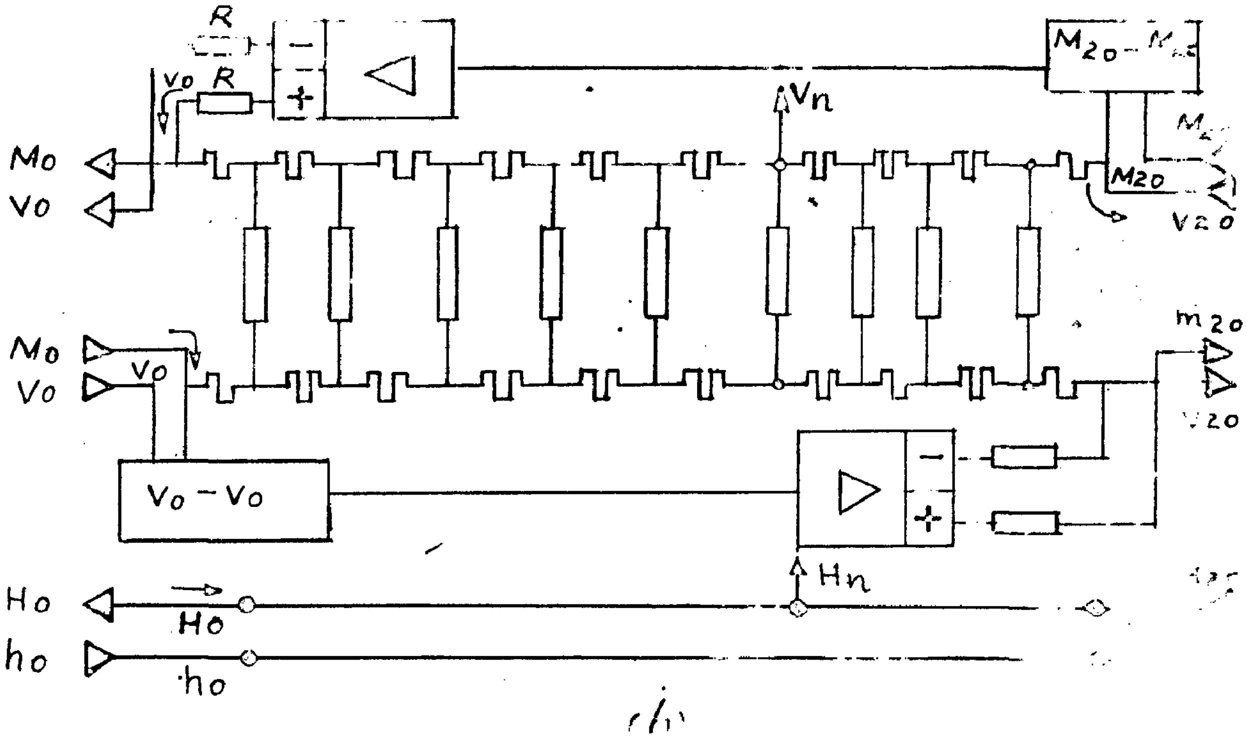
In the figure (32a) a cantilever beam is shown which is loaded by a force V_n and which is clamped at the left edge. The damping condition is given by the angle and the vertical displacement at the supporting side.

The analog is drawn in Fig. (32b). A number of boundary conditions have to be satisfied.

First equilibrium is considered. A matching of the vertical forces is achieved in the analog by keeping the total of all currents drawn from the top row zero. This is done by drawing an appropriate current from the supporting end of top row by means of feed back control. Assuming that the steady state of flow of currents is established then the equilibrium of moments is achieved automatically. This equilibrium state is achieved in the following way: The prescribed potention M_{20}^* is compared with the value M_{20} of the top row by means of a difference device with large in put impudences. The output signal of latter is fed to a large gain amplifier with both positive and negative out puts. The positive out put signal is fed back to the supporting edge of top row via a resistor R which is large compared to r. Now if M_{20}^* is larger than M_{20}, then a current is flowing into the left edge of top row, which increases M_{20}, the potential difference between M_{20} and M_{20}^* will be reduced to zero with an accuracy depending on the gain. From the negative out put signal of the amplifier the reactive force V_0^W on the adjacenent element is derived.

ANALOG OF A CANTILEVER BEAM WITH BUILT IN SUPPORTS.





In order to determine the position of beam two boundary conditions have to be satisfied, corresponding to the angle and vertical location of clamped edge. The first one is introduced by a current of prescribed magnitude entering bottom row at the supporting end. In the steady state the total of all currents entering the bottom row should be equal to zero. This is done bydrawing a current from the supported end by means of a second feed back control. Then the potential V_o is stabilised at the prescribed value V_o^* in a similar way as in the case of equilibrium V_o^* and V_o are compared in a difference device the out put of which is connected to a large - gain amplifier with both positive and negative out put. The negative part in fed back to the un-supported end of the bottom row Via a resistor R. As in the equilibrium case the difference between V_o^* and V_o is reduced to zero.

Horizontal forces and displacements may also be incorporated into the Analog by the following way. As the deformations of a rod as a result of longitudinal stresses are usually negligible, the translations h in the horizontal directions are the same anywhere in the beam. This corresponds to a short circuit. The matching of horizontal force H is achieved automatically by the introduction of another short circuit, from which the currents H_H may be drawn.

The analog of Fig (32b) has two sets of six terminals of which the in puts are marked by triangles pointing inwards and out puts by triangles pointing outwards. The quantities belonging to the beam are indicated by the usual symbols and those of adjacent elements are distinguished by a star.

The analog treated above is valid only if the left leg is clamped. When the right edge is clamped, the symetrical counterpart is required. The transfer of forces and moments is always in the direction from the free end to the supported side and the transfer of deflection in the opposite direction.

3.38 Analog of a statically determinate rigid joint

The joint has four branches. The lower branch is fixed in this case the forces and moments acting upon the unsupported part of the structure are to be transferred to the fixed member and the deflection of latter in the opposite way. The following relations have to be satisfied:

$$H_o + H_1 = H_2 + H_3 = 0 \quad (5)$$

$$V_o + V_1 + V_2 + V_3 = 0 \quad (6)$$

$$-M_o + M_1 + M_2 + M_3 = 0 \quad (7)$$

$$h_o = h_1 = h_2 = h_3 \quad (8)$$

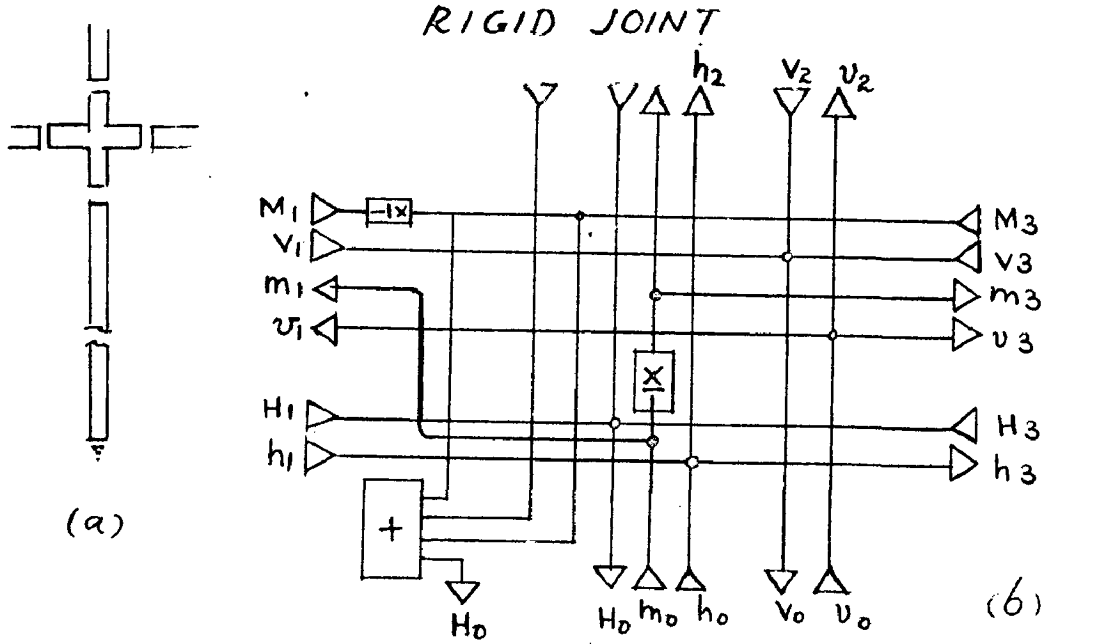
$$v_o = v_1 = v_2 = v_3 \quad (9)$$

$$m_o = m_1 = - m_2 = - m_3 \quad (10)$$

The electric circuit model (Fig.33) contains four sets of terminals which may be connected to the corresponding terminals which-m of the beams or column. The various terminals are distinguished by the usual symbols.

The matching of horizontal forces is achieved simply by connecting all the terminals H. The same applies to vertical forces. The matching of moments asks for a summing

FIG—33 ANALOG OF A STATICALLY DETERMINATE RIGID JOINT



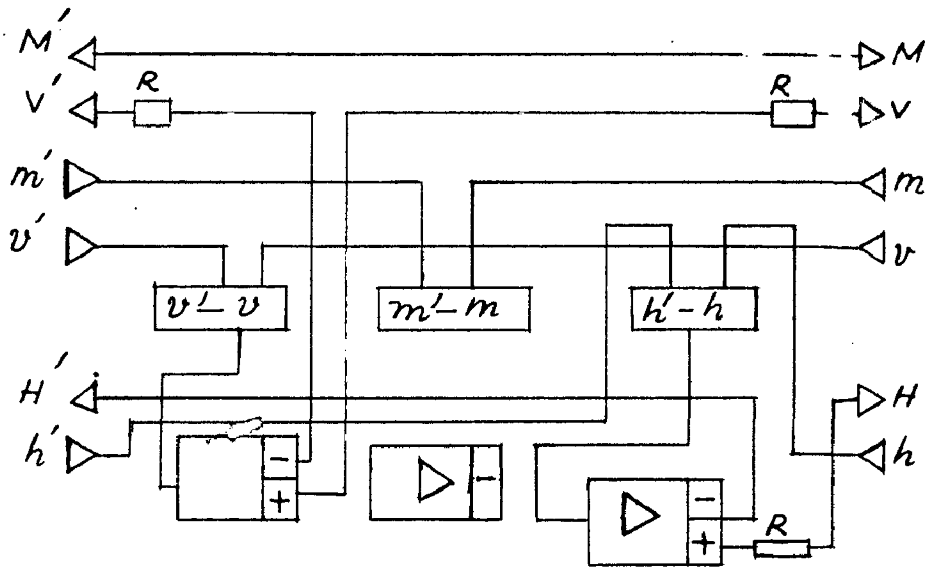


FIG. 34.

invertor in the case A.C. is employed. The horizontal displacements are equalised by connecting all terminals h. The same applies to vertical displacements. The transfer of angular rotation m_o is accomplished by means of short circuits and a phase invertor shown in figure (32b).

If, instead of the lower branch, another part of the joint is fixed some modifications of signs are indispensable.

3.39 Analog of a statically indeterminate joint.

There are however situations inwhich the reactions between two adjacent parts of the system are coupled to the deflections. Such a coupling is present when for instance, an additional attachment is made in a statically indeterminate structure. In order to solve this another element is to be introduced into the system of analogs, which will be called statically indeterminate joint. This element must satisfy the continuity of structure at the additional connection.

The electric analog of the element (Fig.34) has two sets of terminals, denoted by the usual symbols of which the primed ones belong to the left set. These terminals fit into those of the adjacent elements. The vertical displacement of the left element r' is compared with v at the other side by means of a difference device with large in put impedances, and the out put signal is fed to a large gain amplifier with both positive and negative out put. Via a large resistor R the negative out put is fed back to V' and the positive out put to V- Now suppose that v' is larger than v, then a force is exerted upon the adjacent element on the left hand side and an equally large force in the opposite directions upon the neighbouring right element which will cancel the original difference between v' and v. Independently horizontal continuity is obtained in the same way.

In the matching of angles the currents m' and m are fed to a difference device. This out put is connected to a large gain amplifier with negative out put only, which is connected to M' and M straight sway. Otherwise the principle is same.

3.40 Analog of a curved beam

The curved beam of Fig. (35) is clamped at the left edge and a vertical force V_{1p} and a horizontal force H_{2p}? are exerted at (X_p, Y_p). The total length L, of the girder is measured along its curvature, is divided into sections 2 1 and 1 or before. The sections are projected on the x and y axes. The deflections of the un-supported end are governed by the relations.

$$V_o - V_{20} = \int_o^L M/_K (x_{20} - x_n)\, ds \approx \sum_{n\, odd} \frac{M_x}{K_x} (x_{20} - x_z)\ 2\ 1 \quad -($$

$$h_o - h_{20} = \int_o^L \frac{M}{K} (y_{20} - y_o)\, ds \approx \sum_{n\, odd}^{19} \frac{M_n}{K_n} (y_{20} - y_x) 21 \quad ($$

$$m_{20} - m_o = \int_o^L \frac{M}{K}\, ds \approx \sum_{n\, odd}^{19} \frac{M_n}{K_n}\ 2\ 1 \quad (1$$

in which ds is the element of beam and M is bending moment due to the combined action of the vertical and horizontal forces.

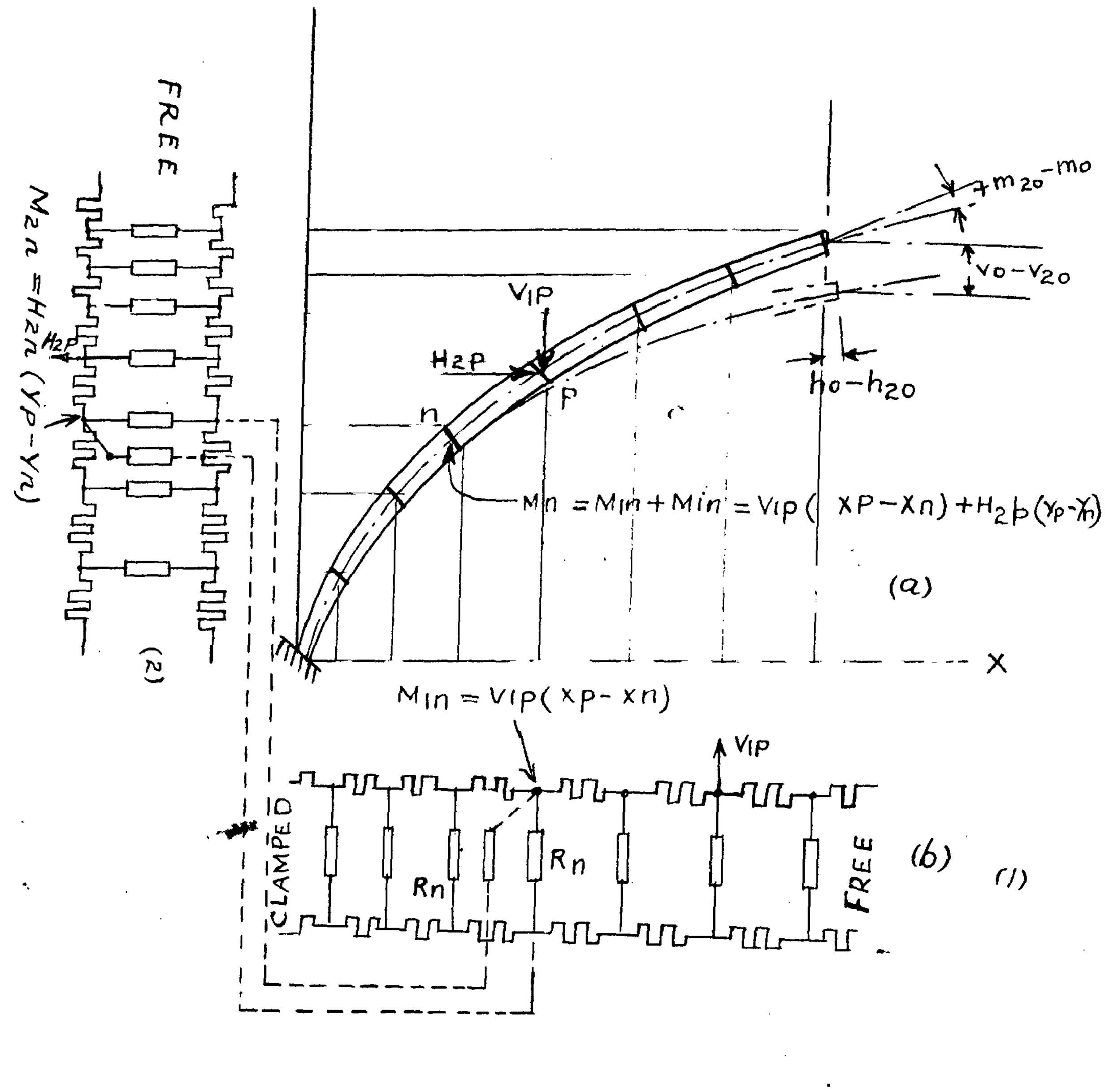


FIG 35 — ANALOG OF A CURVED BEAM

The analog of Fig. (35)b is built up from already known elements; a beam 1 in horizontal position clamped at the left edge, and a separate built in column 2. The sections of the both members are non uniform and the resistances are proportional to the projection of the sections 1and 2 1 of the actual beam on the x- and y - axes. The vertical force V_{1p} is exerted on the beam only, resulting in a B.M, M_{1n}. Similarly a moment M_{2n} results from the action of V_{2p} on the built in support. The deflection of the curved beam are caused by the combined effect of M_{1n} and M_{2n}. Now the dashed connections are introduced between both the elements, containing resistors R_n corresponding to local flexural rigidity K_n of the curved beam. By repeating the inter-connections at all sections the total B. M. $M_n = M_{1n} + M_{2n}$ influences the deflection of both bottom rows and the relations (11)- (13), are satisfied. The top row only contain information about the partial bending moment, being either M_{1n} or M_{2n}; the bottom rows gives the complete deflections.

If the shape of the curved beam has a maximum w.r.t. x or y axis, it should be cut then and both parts should be treated as separate curved beams, connected by a rigid joint.

3.41 Generalisation of three dimensional case

The additional problems ariving in case of three dimensions have to do with double bending and tension.

Double bending may be seen as simultaneous, independent bending of the beam in the planes of principal axes of the cross section. This phenanena may be treated in exactly the same way as in section 3.37.

Torsion problems may be treated in the same fashion on bending and an analogous net work applies to it.

Then the complete electric circuit model of a straight beam exists, and is made up of four separate, similar net works, two of which represent the bending action, are the torrian, and the last one the compression of the beam.

As a match of fact the joints will be somewhat more complicated than in the two dimensional case, but the same principles hold.

<u>Dr. J.W. Bray 1957</u>

Using the equivalence between the slope deflection equations of a beam, and the equations of an electric circuit, containing a group of three resistors, the electrical net works for beams, trusses and frames, have been devised.

<u>3.42</u> - Consider a beam, the ends of which have been rotated under load through angles θ_A and θ_B and are maintained inequilibrium by fixing moments M_{AB} and M_{BA}

Relation between the moments and rotation is

$$M_{AB} = M'_{AB} - \frac{2\,EI}{L} \quad (2\,\theta_A + \theta_B) \qquad (1)$$

$$M_{BA} = M'_{BA} - \frac{2\,EI}{L} \quad (2\,\theta_B + \theta_A) \qquad (2)$$

In the circuit shown (Fig 36) the ends A and B are maintained at voltages V_A and V_B and currents i'_{AB} and i'_{BA}

ARE FED into the circuit as shown. As a result current i_{AB} and i_{BA} flow towards the nodes A and B. These two type of currents will be referred to as feed and net work currents respectively. It can be shown that,

$$i_{AB} = i'_{AB} - 1/R\ (2\ v_A - v_B) \qquad \dots\dots \quad (3)$$

$$i_{BA} = i'_{BA} - 1/R\ (2\ v_B - v_A) \qquad \dots\dots \quad (4)$$

To establish an analogy between the elastic and electrical systems, the quantities must be proportional to each other, that is

$$\left.\begin{array}{ll} i_{AB} = +\,p\,M_{AB} & i_{BA} = -\,p\,M_{BA} \\ i_{AB} = +\,p\,M'_{AB} & i'_{BA} = -\,p\,M'_{BA} \\ V_A = +\,q\,\theta_A & v_B = q\,\theta_B \end{array}\right\} \dots \quad (5)$$

where p and q are arbitrary constants. When the beam is continuous over a simple support, (Fig. 37).

The conditions are expressed by the equation

$$\left.\begin{array}{l} M_{BA} + M_{BC} = o \\ \theta_{BA} = \theta_{BC} \end{array}\right\} \dots \quad (6)$$

If the expressions given in the equation 5 are substituted into these equations the result is

$$\left.\begin{array}{l} i_{BA} + i'_{BC} = o \\ V_{BA} = V_{BC} \end{array}\right\} \dots \quad (7)$$

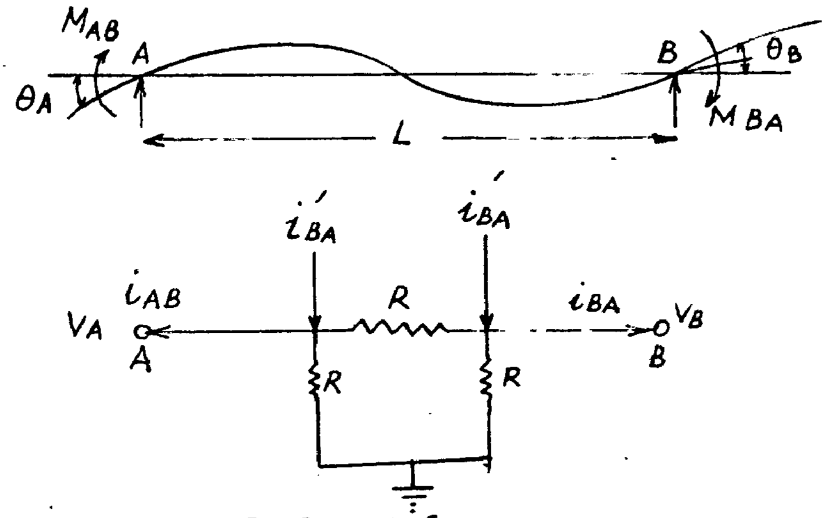


FIG – 36.

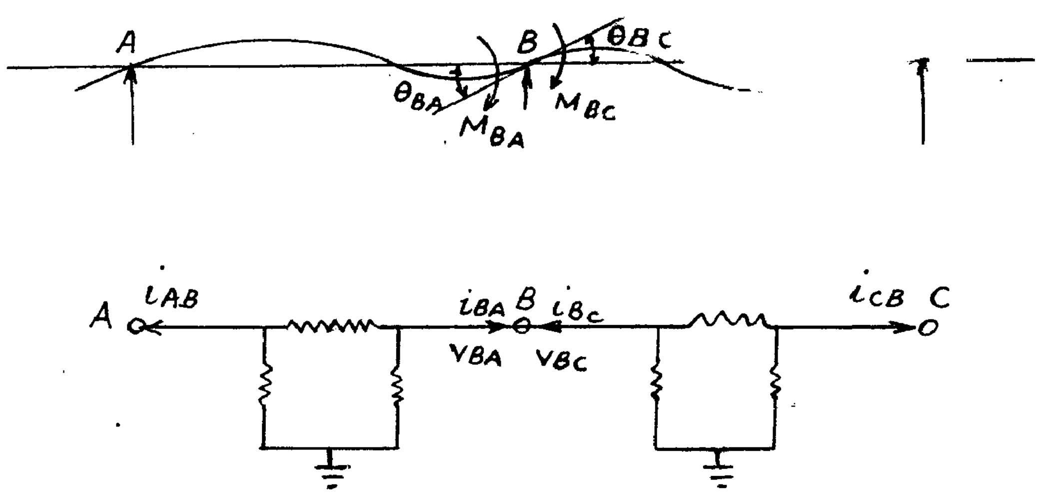


FIG – 37.

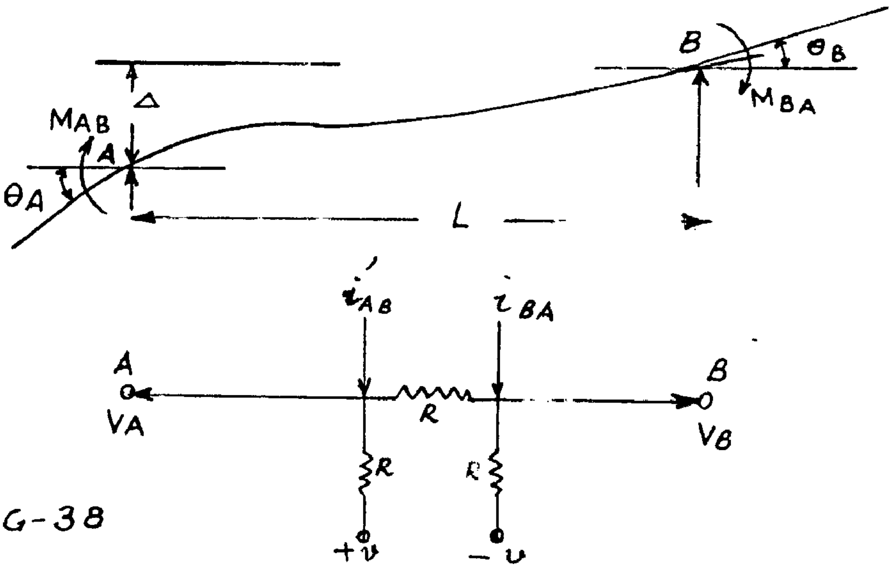


FIG-38

In this analogy the feed current corresponds f.e.m., voltages to rotation and resistance to $L/2_{EI}$ ratio.

As it is seen from the Equation No.5, that +ve signs occur in all the quantities concerning A and -ve signs occur in case of B, it is necessary to attach alternate +ve and - ve signs to the nodes of the net work.

The angle slope of beam over supports can be found out by measuring voltages at A, B and C

$$R = q/P \quad \frac{L}{2\ EI}$$

$$q = R_P \quad \frac{2\ EI}{L}$$

Dividing these voltages by the scale factor q and by their respective node signs we obtain the corresponding angles of rotation.

3.43 - If one end of the beam is built in so that no rotation is allowed, the corresponding point in the net work should be at zero potential.

(b) In continuous beams, cantilever part is statically determinate one. It has no effective stiffness and hence no resistors are needed to represent it in the model.

(c) If one of the support sinks by an amount $\triangle$, under load, the feed current at the load point should include the amount of moment caused due to sinking of supports.

3.44 Alternate way of dealing with settlement

Here the feed currents are due to applied loads only, the allowance for settlement is made by raising the corresponding potential points by an amount $V = \frac{3\Delta}{L} q$

Consider the beam shown in Fig. (38a) when the beam sinks by an amount , the fixing moments are given by,

$$M_{AB} = M'_{AB} - \frac{2\,EI}{L} \left(2\,\theta_A + \theta_B + \frac{3\Delta}{L} \right) \quad \text{-----} \quad (8)$$

$$M_{BA} = M'_{BA} - \frac{2\,EI}{L} \left(2\,\theta_B + \theta_A + \frac{3\,\Delta}{L} \right) \quad \text{----} \quad (9)$$

From the diagram of electric circuit the corresponding currents are

$$i_{BA} = i'_{AB} - 1/R\,(2\,v_A - v_B - v) \quad \text{----} \quad (10)$$

$$i_{BA} = i'_{BA} - 1/R\,(2\,v_B - v_A + v) \quad \text{----} \quad (11)$$

Thus a complete analogy is obtained if in addition to the usual conditions of terminal voltage v are made proportional to the quantity $\frac{3\Delta}{L}$

$$v = q\ \frac{3\Delta}{L} \quad \text{----} \quad (12)$$

3.45 Axial load

The above Equations (8) and (9) are only applicable when the axial load on the member is negligible. When this is not so, the equations have the modified for

$$M_{AB} = M'_{AB} - \frac{EI}{L} \left\{ C_2\,\theta_A + C_1\,\theta_B + (C_1 + C_2)\,\Delta/2 \right\} \text{-} (13)$$

$$M_{BA} = M'_{BA} - \frac{EI}{L} \left\{ C_1\,\theta_A + C_2\,\theta_B + (C_1 + C_2)\,\Delta/2 \right\} \text{-} (14)$$

where $c_1 = \frac{(2\alpha\ \mathrm{Cos}\ 2\alpha - 1)}{\alpha - \tan}$

$$c_2 = \frac{(1 - 2\alpha\ \mathrm{Cot}\ 2\alpha)}{\alpha - \tan\alpha}$$

and

$$= \frac{L}{2}\sqrt{\frac{P}{EI}}$$

P is the axialload.

The current equations are

$$i_{AB} = i'_{AB} - 1/R \left\{(1/a + 1/b)\, V_A - 1/a - 1/b\ ^{u}\right\} \quad ..(15)$$

$$i_{BA} = i'_{BA} - 1/R \left\{(1/a + 1/b)\, V_B - 1/a\, V_A + 1/b^{u}\right\} \quad (16)$$

Where $= 2/C_1$, $b = \frac{2}{C_2 - C_1}$, and $U = \frac{C_1 + C_2}{C_2 - C_1} \frac{q}{L}$

Since the axial loads are not initially known, however a method of successive approximation must be applied, the procedure being as follows:

a. Ignore the axial load, set the model to determine the moments in the usual way and from the values calculate the approximate axial loads.

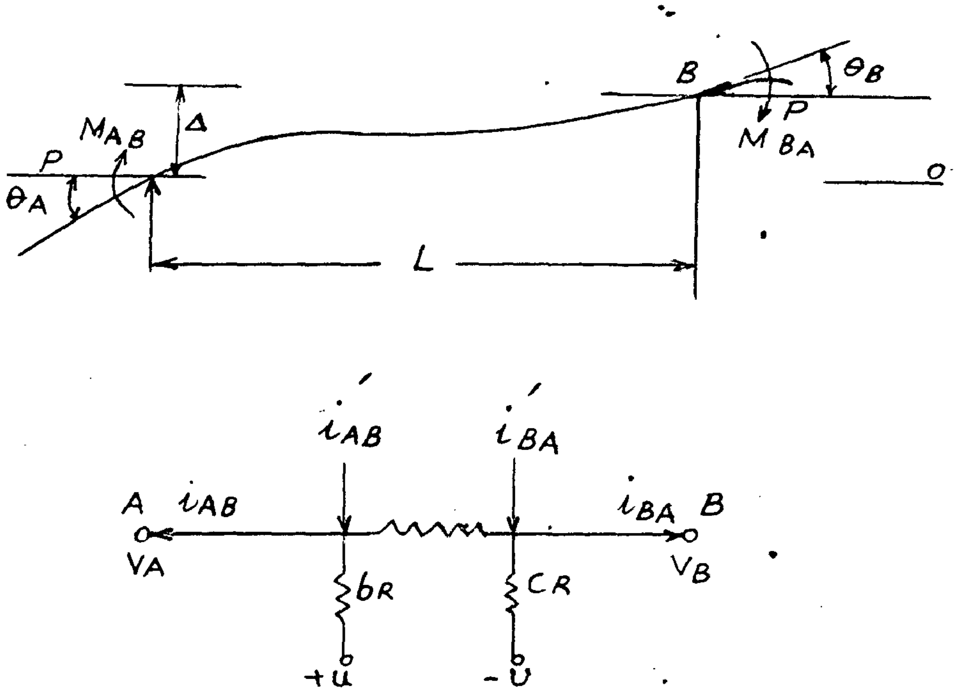
b. Modify net work resistances to allow for these loads and from the new current distribution redetermine moments and axial loads. Repeat this process until the required accuracy is obtained. The results usually converge fairly rapidly.

3.46 Member of non uniform section

When considering the deformation of a beam of non-uniform section, as in the Fig (40),

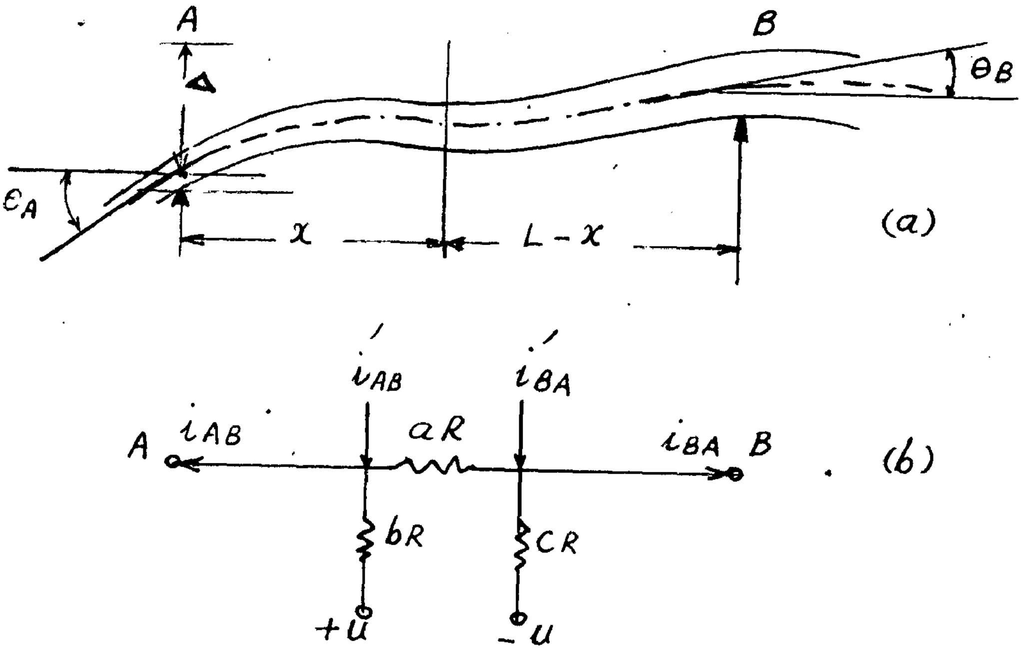
It is necessary to introduce three parameters involving the geometry of the beam, these being defined as follows,

$$\alpha = \frac{I_o}{L^3}\int_o^L \frac{x^2}{I}\, d_x \; ; \quad \beta = \frac{I_o}{L^3}\int_o^L \frac{x\,(L-x)}{I}\, d_x$$



B
θB
P
M BA
MAB
Δ
P
θA
L
i'AB
i'BA
A
iAB
iBA
B
VA
VB
bR
CR
+u
-u

FIG- 39.



A
Δ
B
θB
εA
x
L - x
(a)
i'AB
i'BA
A
iAB
aR
iBA
B
(b)
bR
CR
+u
-u

FIG -40

$$\text{and } r = \frac{I_o}{L^3}\int_0^L \frac{(L-x)^2}{I}\, d_x$$

where I_o is the second moment of area of the section of the beam at same standard reference point, such as at the centre of span. It can be shown that

$$M_{AB} = M'_{AB} - \frac{EI_o}{L}\left\{ C_2\,\theta_A + C_1\theta_B + (C_1+C_2)\,\frac{\Delta}{L}\right\} \text{ ---- (17)}$$

$$M_{BA} = M'_{BA} - \frac{EI_o}{L}\left\{ C_1\,\theta_A + C_3\,\theta_B + (C_1+C_3)\,\frac{\Delta}{L}\right\} \text{--- (18)}$$

$$\text{where } C_1 = \frac{\alpha}{\alpha r-\beta^2};\ C_2 = \frac{\beta}{\alpha r-\beta^2};\ C_3 = \frac{r}{\alpha r-\beta^2}$$

In the equivalent electric circuit the resistances of the group will be represented by aR, bR and cR, where R= P/q $\frac{L}{2\,EI}$

Analysis of the circuit yields the equations

$$i_{AB} = i'_{AB} - 1/R\left(\left(\frac{1}{a}+\frac{1}{b}\right) V_A - 1/a\, V_B\ 1/b\ u \right) \text{ ---- (19)}$$

$$i_{BA} = i'_{BA} - 1/R\left(\left(\frac{1}{a}+\frac{1}{b}\right) V_B - 1/a\, V_A + 1/b^{v} \right) \text{ ----- (20)}$$

Then for the system to be analogous

$$\left.\begin{aligned} a &= \frac{2}{C_1} ;\ b = \frac{2}{C_2-C_1} ;\ C = \frac{2}{C_3-C_1} \\ u &= \frac{C_2+C_1}{C_2-C_1}\, q\, \frac{\Delta}{L} ;\ v = \frac{C_3+C_1}{C_3-C_2}\, q\, \frac{\Delta}{L} \end{aligned}\right\} \text{---(21)}$$

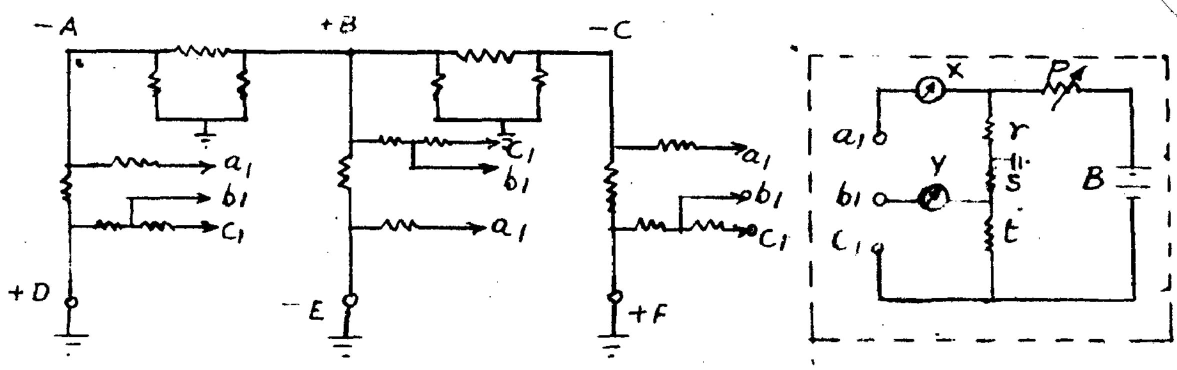
3.47 Rectangular frame work

In case of rectangular frame work construct the net work, with resistors R being proportional to L/I' so that the dis-position of mode points corresponds to the position of joints in structure. Supply currents to the net work proportional to thevalues of f.e.ms. and measure the currents in various links of the net work, and convert them to fixing moments by using the proper scale factor.

3.48 Sway

In order to include the effect of sway the circuit must be modified by providing a fourth resistor for each column and a shear circuit for each storey. This circuit has got two functions:

1. To simulate the effect of sway in the storey,
2. To provide means of measuring the amount of horizontal shear in the storey.



ELECTRICAL CIRCUIT FOR A TWO BAY FRAME

SHEAR CIRCUIT

FIG—41.

The shear circuit (Fig 41) consists essentially of a string of resistors r, s and t, supplied with current by a batt B, through a potentiometer P. The junctions between r, s is earthed and milliameters are inserted in the leads to terminals a and b. Since r and s are equal resistances the voltage induced at (a) and (b) by the current flowing from the battery must be equal in magnitude and opposite in sign, the actual voltage depends on the setting of the potentiometer.

The voltage $\pm$ v applied to the end resistors of any group is equivalent to a relative displacement Δ between the ends of corresponding column, whereas

$$V = q \frac{3\Delta}{L}$$

the shear circuit applies the same voltage $\pm$ v to all column groups of a given storey, it gives the effect of sway of amount Δ for the storey, r and s are kept low in comparison with the main work resistors, to minimize the effect of current flowing through r and s from the main net work.

The second function of the shear circuit is made clear by the following relation.

Consider the circuit diagram of fig.41. The current flowing to terminal (a) from the resistor group A_D is given by

$$2/3\ i_{AD} + 1/3\ i_{DA} + 2/3\ v/R$$

There are similar expressions for the currents flowing from the groups E_B and C_F and hence the total current flowing to terminal a_1 is

$$2/3\ (i_{AD} + i_{EB} + C_F) + 1/3\ (i_{DA} + i_{BF} + i_{FC}) + 2V/R$$

This is also the reading of meter x.

Similarly the current flowing from the main net work to terminal b_1 is

$$2/3\,(i_{DA} + i_{BE} + i_{FC}) + 1/3\,(i_{DA} + i_{EB} + i_{CF}) + 2V/R$$

This is however a current flowing from B to C through the extra column resistors, as the potential of C is $\frac{4v}{3}$ below that of b, the magnitude of this current will be $\frac{4V/3}{R} \times 3 = \frac{4V}{R}$. Hence the resultant current flowing to terminal b_1

$$= 2/3\,(i_{DA} + i_{BE} + i_{FC}) + 1/3\,(i_{DA} + i_{EB} + i_{CF}) - 2\,V/R$$

This is the reading of y

The difference δ is therefore

$$= 1/3(i_{DA} + i_{EB} + i_{CF}) - 1/3\,(i_{DA} + i_{EB} + i_{CF})$$

$$= P/3\,(M_{AD} + M_{EB} + M_{CF} + M_{DA} + M_{BE} + M_{FC})$$

$$\therefore \; \delta 1 = P/3 \; S \; L \qquad \text{------} \qquad (22)$$

where P = moment current scale factor

S = Horizontal shear in the storey

L = Length of columns

3.49 Wind load

The effect of the wind load on a frame is to induce horizontal shear in the various storeys of the frame, and these shears may be simulated in the analogue by means of shear circuit. Find the values of $\frac{P}{3}$ SL for each storey and adjust the potentiometer till the shear meter shows the required value.

3.50 Settlement of foundation

This is similar to that of sway. Whereas in sway there is horizontal moment, which is same for all columns in same storey. With the settlement the moment is vertical and is same for all the beams in a given bay.

Hence it is obvious that shear circuit can also be used here, connections being made to groups of resistors representing beams instead of columns. Let any portion of the frame sinks by an amount Δ - with the scale factor q, $\frac{2\ EI\ RP}{L}$ find out the amount of voltage required to produce at the terminals a and b of both the shear circuits, which are on either side of settled portion, by using the following equation,

$$v = \frac{2\ EI\ R\ P}{L} \times \frac{3\Delta}{L}$$

The setting of potentiometers is varied until this condition is reached.

3.51 Semi-rigid connections

When a riveted joint is loaded it suffers an appreciable amount of deformation so that the members meeting at the joint rotate relatively each other. If the curvature of the characteristic curves obtained by plotting values of angle of rotation on a base of applied moment, is ignored and the curves replaced by suitable straight lines, the rigidity of any given joint, defined by the ratio $\frac{\text{moment}}{\text{rotation}}$ can be taken as a constant for the joint, being represented hereby the symbol r. It will be seen that r is zero for a hinge and infinite for a perfectly rigid joint.

Semi rigid joints are represented in the computer by inserting additional resistors between the resistors groups and adjacent node pointsinstead of making the connections directly.

The amount of resistance $\eta = \frac{p}{q} \quad \frac{1}{r}$

$$\eta = 2 \frac{EIR}{Lr} = \text{in ohm.}$$

If desirable the resistances can be made variable so that the true curved characteristic of the connections can be followed, instead the linear approximation used here. The potentiometer must be set to a resistance corresponding to the value of θ/M at the point of operation

i.e. $\eta = P/q \quad \theta/M$ ------- (23)

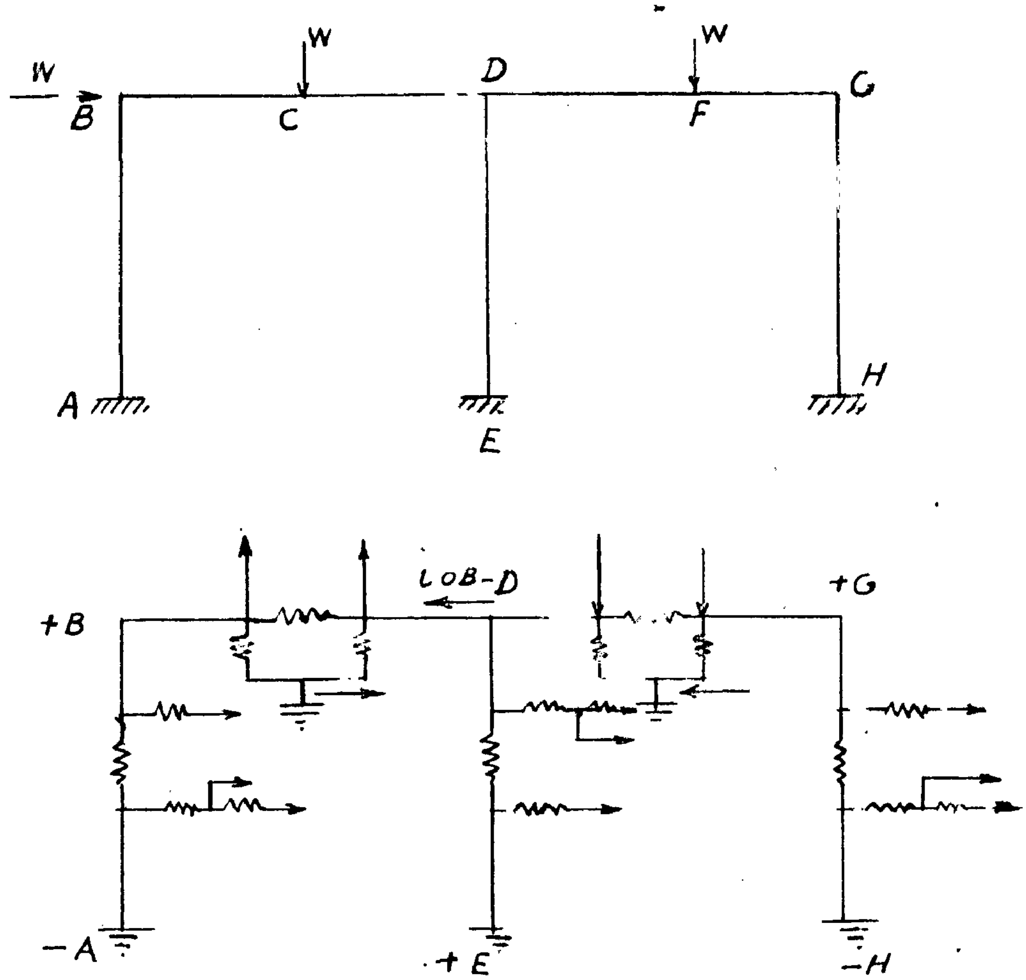
3.52 Plastic hinges

In the conventional method of design, a structure is taken to have failed when the stress at any point exceeds the yield stress. The modern method of design recognises the fact that in practice failure occurs only when a sufficient number of plastic hinges are formed to render the structure unstable.

In determining the collapse load of a structure it is necessary to establish the position of plastic hinges and the mode of collapse. This is easily done with electric analyser.

Consider a two bay fixed base portal with full plastic moment of M, (fig.42).

(a) LOADED STRUCTURE



(b) ELECTRIC CIRCUIT

FIG – 42.

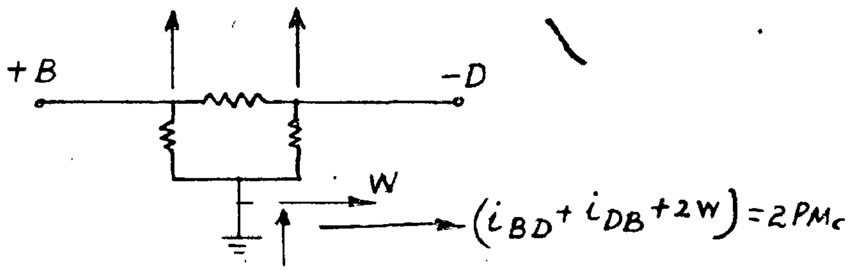


FIG – 43.

In operating the analyser provision must be made to ensure that the corresponding net work currents do not exceed the limiting value Z_{ma} where Z_{m-a} represents M to scale. Plastic hinges may occur at any of the nodes and under the concentiated load also. If a current of amount w is applied to earth lead of the resistor group BD as shown in fig (43), the resultant current in this lead will represent to scale twice the amount at point C.

Hence the current must be limited to $2Z_{ma}$. The same remarks apply to the resistor group DG.

To simulate the effect of increasing load the feed and the shear currents are gradually increased until one of the net work currents reaches the limiting values of $\bar{Z}_{m.a}$. Say the first current to reach the value is i_{DB}. To simulate the plastic hinge formed at the point the connection between group BD and node D is broken and currents and + Z and -z ma are fed in to the circuit on either side of break. The feed and shear currents may now be increased until the next current reaches its limiting value. This applies no less to the current in the earth lead of group BD, which reaches its maximum value of $2z_{m.a}$ when the load is P . Final stage is that where it is found impossible to produce any further increase in shear current, which is completely independent of shear voltage at this stage. By this we can find out mode of collapse and collapse load factor.

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PART TWO

Photo.1. Moment Analogy Computor

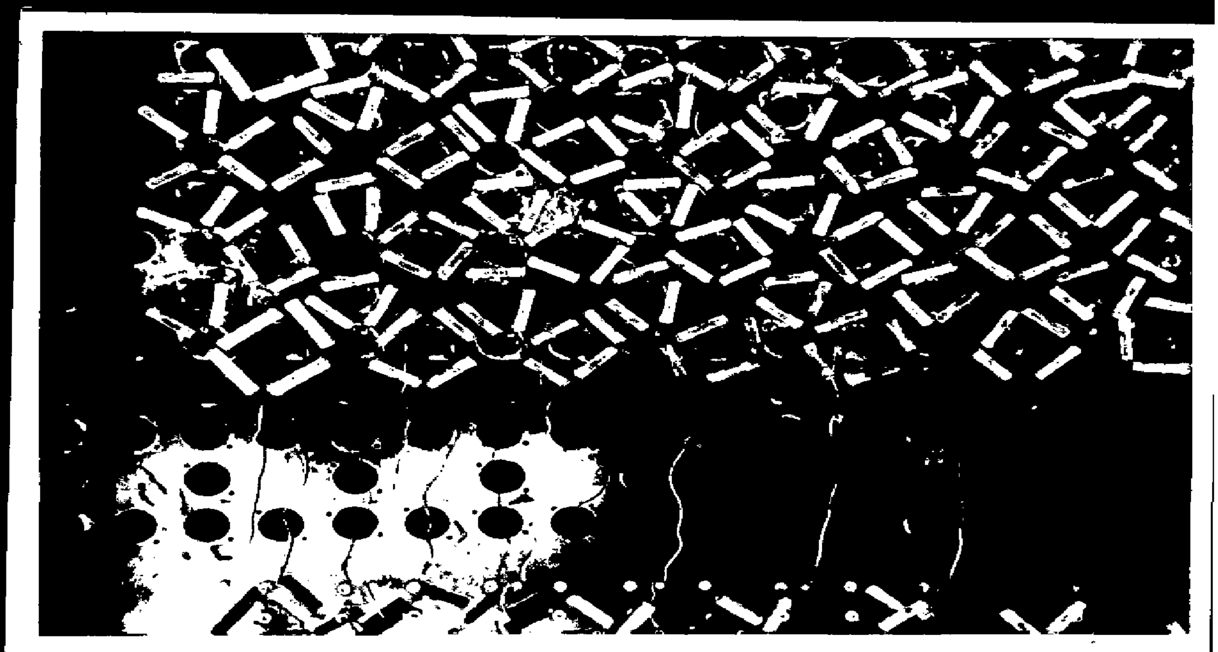

Photo.2. Registor Groups

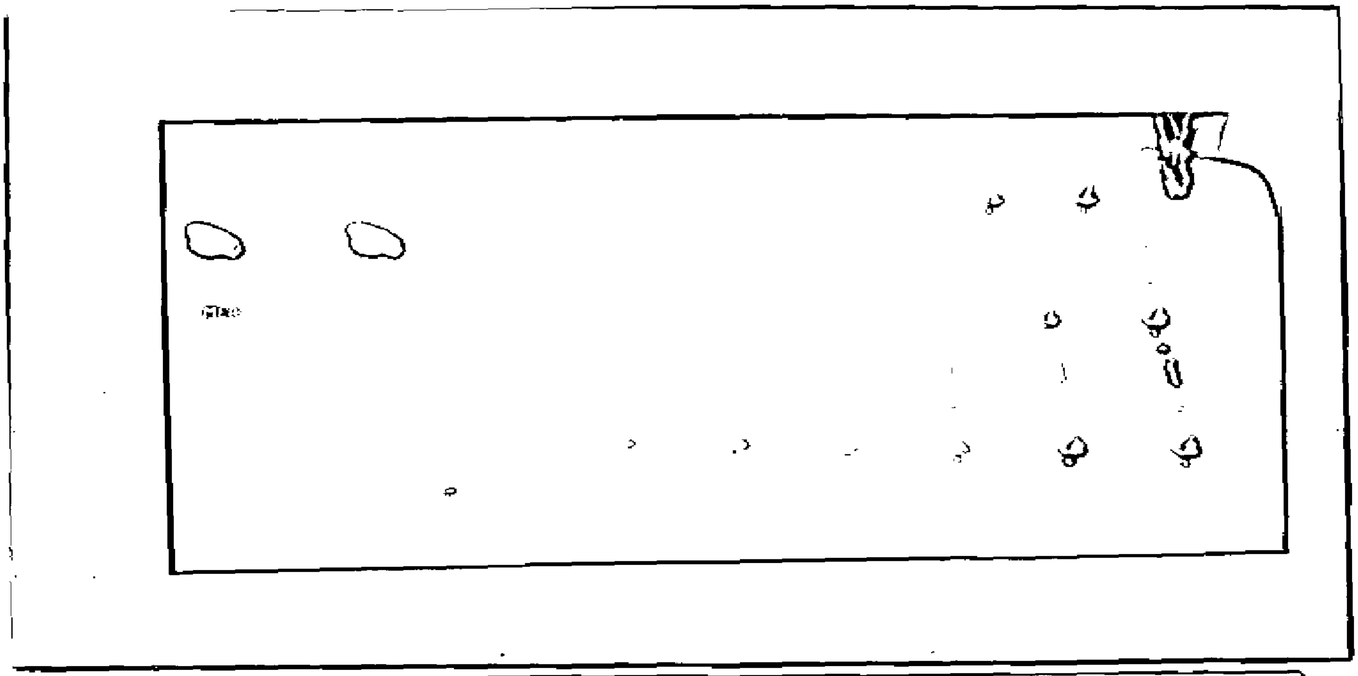

Photo.3. Feeding and Measuring Circuit

Photo.4. Shear and Main circuit power units

CHAPTER 4.

Setting up of a Moment Analogy Computer for Multi-Storeyed Frames.

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4.01 - The construction of multi-storeyed frames is of a greater importance in places where there is a shortage of land. The development in steel and concrete technology have made the construction of framed structures easier and advantageous.

In modern times the construction of all multi-storeyed buildings is done in steel or reinforced concrete. These materials permit of skeleton frames which can be designed and constructed to suit particular requirements because of their high strength in compression for concrete and in tension or compression for steel.

The design of these skeletons can be accomplished either by analytical methods or by experimental methods. The former one needs a lot of labourand much of time. But among the experimental methods Electrical Analyser provides an easy solution with in a short time and gives the results within the limits of accuracy in practice.

4.02-Practical form of the Analyser:

The experimental Analyser consists essentially of a (40") x (24") base board on which is mounted an array of sockets and switches. The Analyser is five units wide and ten units deep.

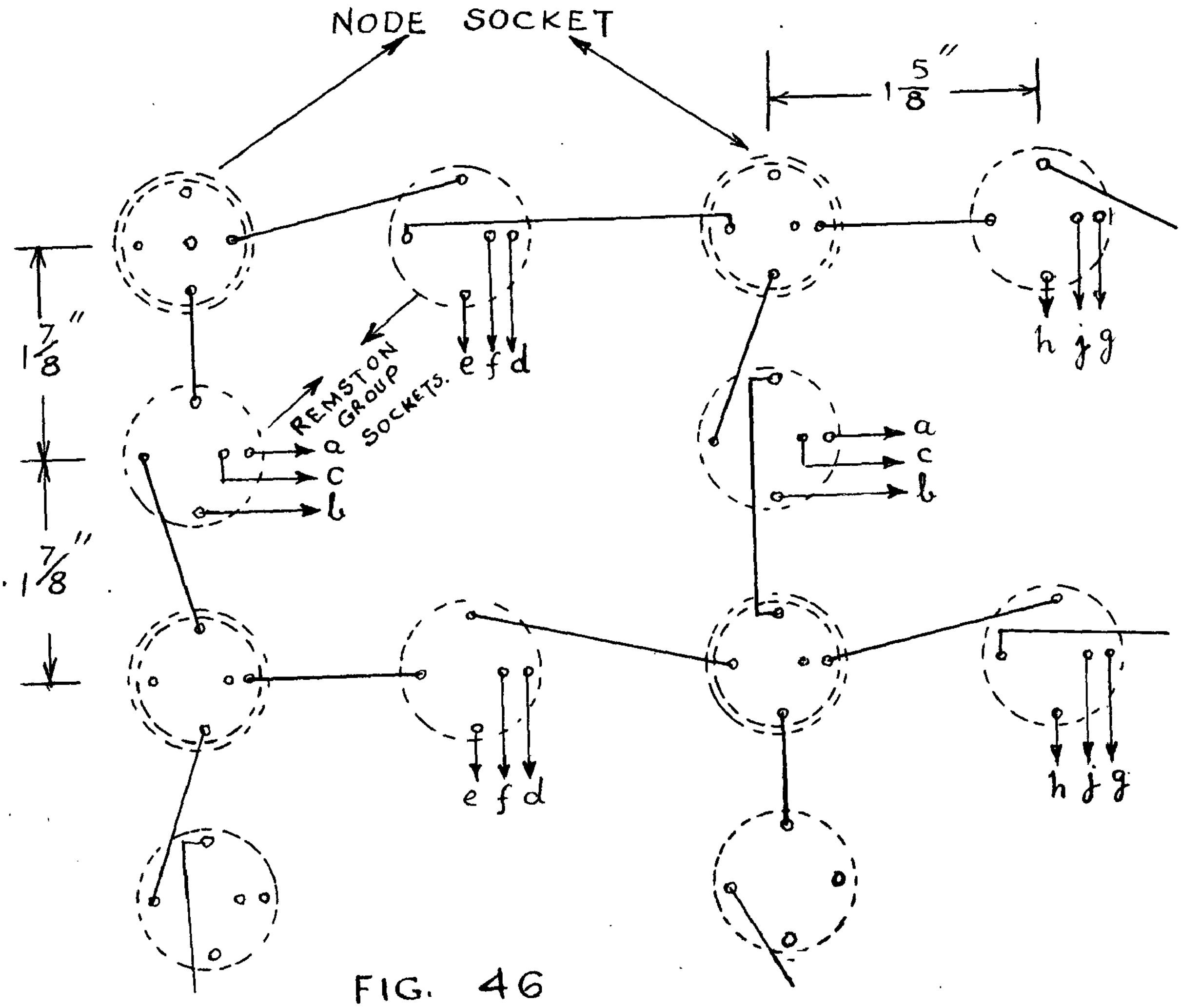

NODE SOCKET
1 5/8"
1 7/8"
1 7/8"
REMSTON GROUP SOCKETS.
a
c
b
e f d
h j g
a
c
b
e f d
h j g

FIG. 46

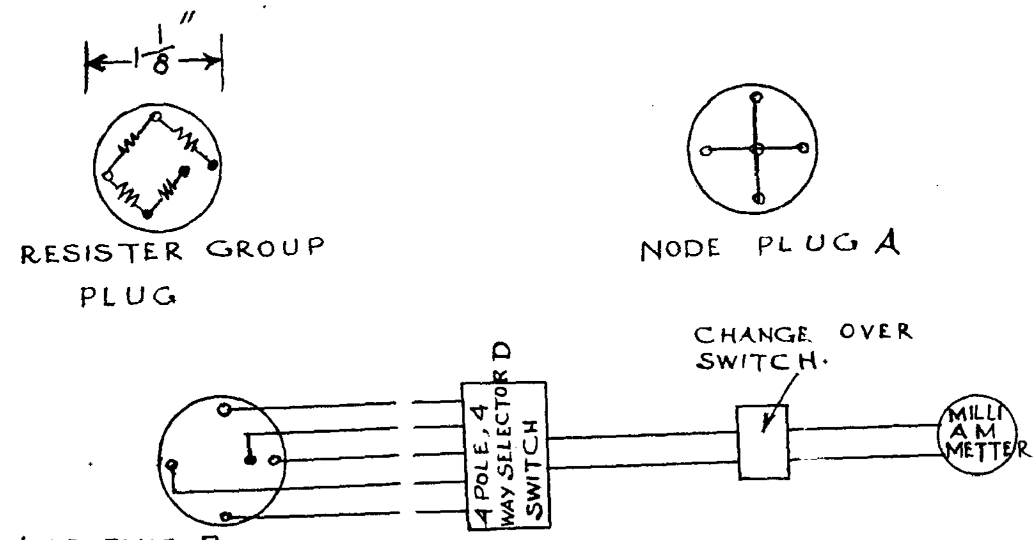

1 1/8"
RESISTER GROUP PLUG
NODE PLUG A
4 POLE, 4 WAY SELECTOR SWITCH
D
CHANGE OVER SWITCH.
MILLI AM METTER
NODE PLUG B.

FIG-47.

Base board is 1/8 in bakelite sheet, on which no. of eight pin sockets are mounted.

Fig. 47 shows the construction details of Resistor group plug, Node plug A, Node plug B, and that of Selecter switch. At the start each node socket is fitted with a node plug of type A. When measuring current at any node, the node plug B is substituted for A and meter connected in turn to each leg of the circuit, by means of a selector switch.

A separate board of 1/8" thick bakelite sheet of size (28") x (19") carries the feeding circuit connections having potentiometers and fuses fitted to it. And carries switches for measuring current in each leg of the circuit.

These boards are fitted in a wooden frame as shown in the photograph no.1. The bottom has got a box type arrangement, which is fitted with necessary transformers and rectifiers for feeding the main and shear circuit.

4.03- Specimen frame:

The specimen frame for which this analyser has been constructed consists of three bays and seven storeys as shown in fig. 44. The numbers written in the middle of the beam or column indicate the stiffness factors, and the no. s in circles refer to the no. of the corresponding joint.

Length of each beam 12'.
Each storey height 18'.

The frame is to be tested for the following loading conditions:

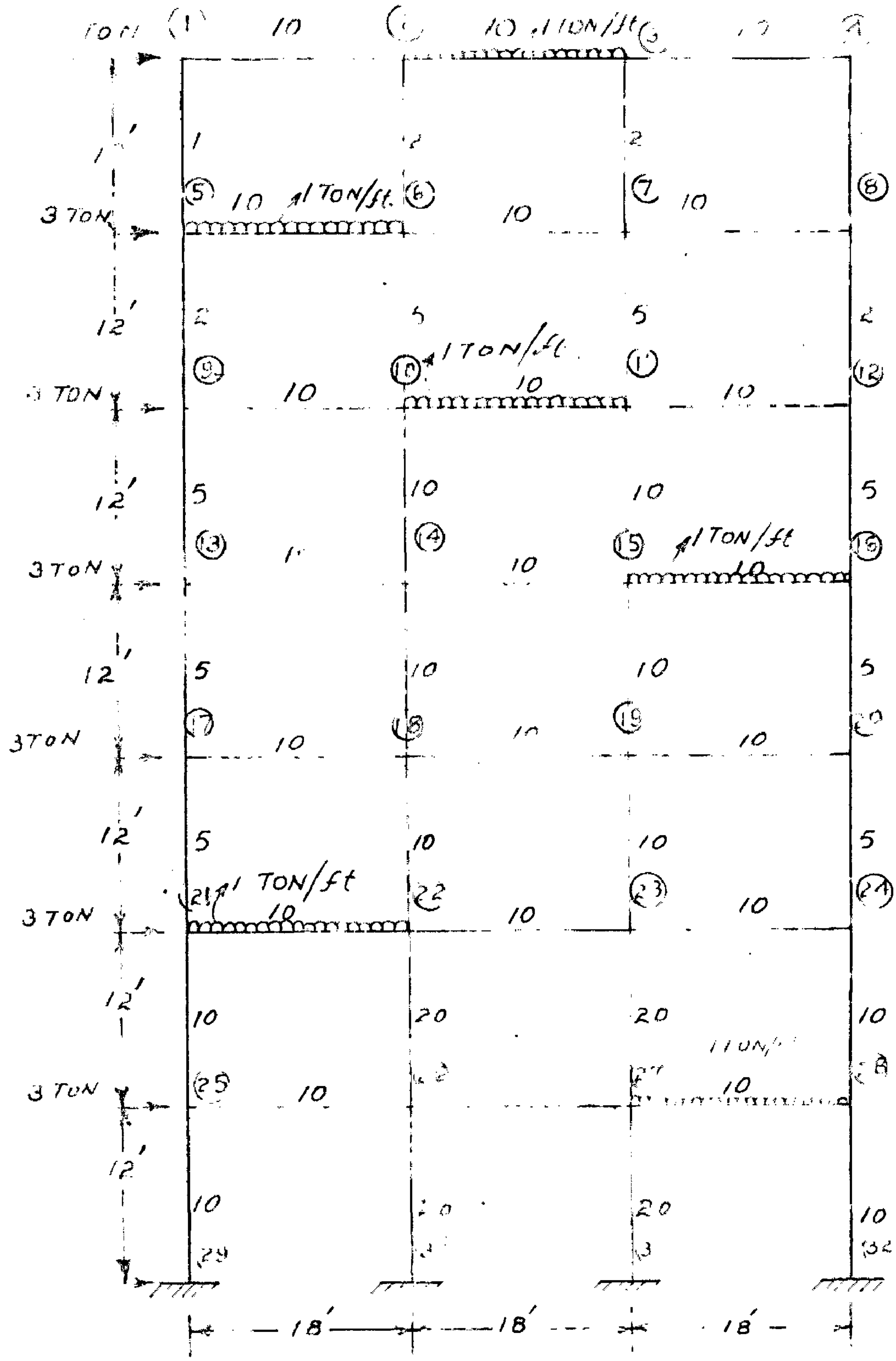


FIG - 44.

1. Vertical loads.

It consists of uniformly distributed load of 1-ton per foot run on six beams as shown in fig. 44.

2. Horizontal loads.

This consists of horizontal loads of 3 tons at every floor level.

In case of vertical loads there are two cases. One is to find out moments without sway considerations, and the second case is to find the moments with sway effect.

4.04- Construction of Analyser:

Resistances corresponding to members.

Resistance R is proportional to L/I ratio or 1/K, where K is the stiffness factor.

i.e. $R = 1/K$

or $R = A\ 1/K$

Where A is an arbitrary constant.

The following table shows resistances corresponding to the parts when the A value is taken as 10000.

Table 1

Part	Stiffness factor	Resistance
All beams	10	1000 Ohms
(Columns)		
1-5, 4-8	1	10000 Ohms
2-6, 5-9, 8-12, 3-7	2	5000 ohms
9-15, 12-16, 13-17, 6-10, 16-20, 17-21, 20-24, 2-11	5	2000 Ohms
10-14, 11-15, 14-18, 15-19, 18-22, 19-23, 21-25, 24-28, 25-29, 28-32	10	1000 Ohms
22-26, 23-27, 26-30, 27-31	20	500 Ohms

Fig. 45 shows the details of the circuit diagram and fig. 46 gives out the some of the construction details of the base board.

The beams and columns are represented by π group resisters. The column group is provided with a fourth resister and an auxiliary circuit for each storey, called the shear circuit to include the effect of sway and to provide a means of measurin the amount of horizontal shear in the storey. The leads a, b and c of each column group of a storey are connected to the terminals a, b and c of corresponding shear circuit. Connections of a and b are made alternately to upper and lower resistors of adjacent groups. The extra resistance supplied to each column group has the same resistance as the original three members of the group and is inserted between the column resistors and the terminal c of the shear circuit. Again the connections are made alternately to the upper and lower resistor of adjacent groups. At the fixed ends of the frame there will be no rotation. So they are earthed in the circuit diagram.

All earth points of the main circuit are inter-connected, and finally they are connected to the negative lead of the feed current power unit.

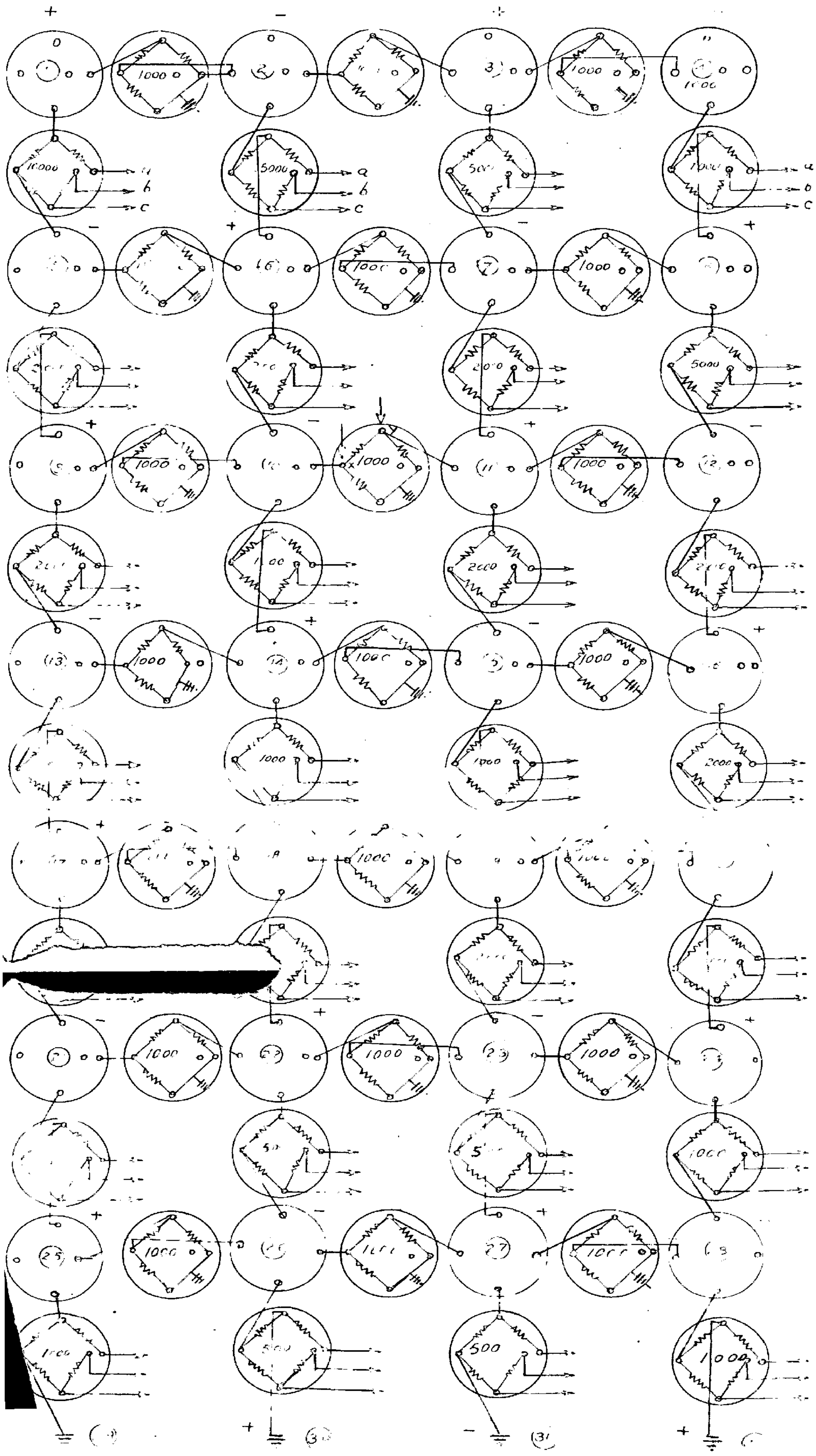
The earthed points of each shear circuit are connected to the negative lead of the shear circuit powers.

Shear circuit potentiometers are of 500 K.ohms capacity.

4.56- Feeding Circuit:

Fig. 45 shows general feeding circuit connections. This

CIRCUIT DIAGRAM



SHEER CIRCUIT

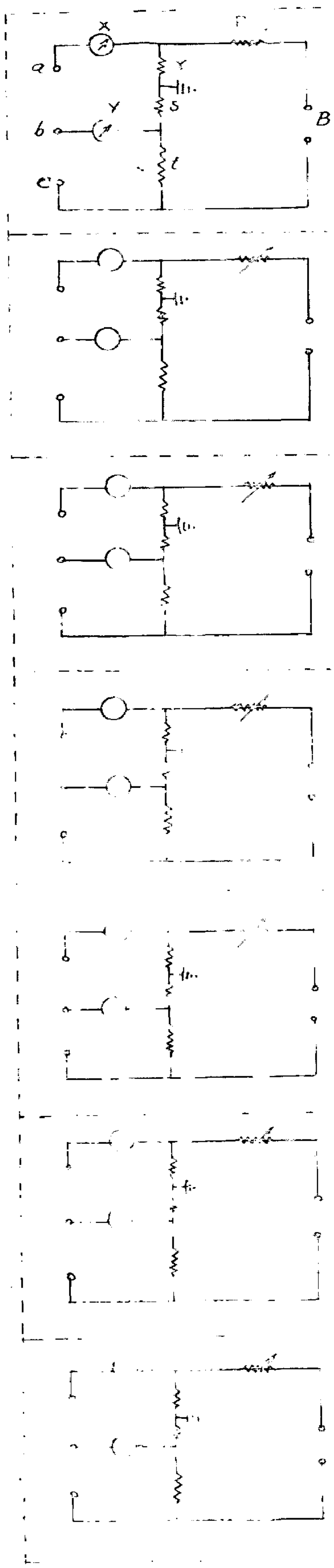
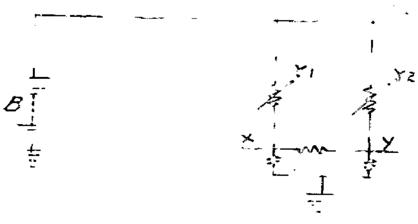


FIG. 45



FEEDING CIRCUIT

Fig. 45 shows the details of the circuit diagram and fig. 46 gives out the some of the construction details of the base board.

The beams and columns are represented by Π group resisters. The column group is provided with a fourth resister and an auxiliary circuit for each storey, called the shear circuit to include the effect of sway and to provide a means of measuring the amount of horizontal shear in the storey. The leads a, b and c of each column group of a storey are connected to the terminals a, b and c of corresponding shear circuit. Connections of a and b are made alternately to upper and lower resistors of adjacent groups. The extra resistance supplied to each column group has the same resistance as the original three members of the group and is inserted between the column resistors and the terminal c of the shear circuit. Again the connections are made alternately to the upper and lower resistor of adjacent groups. At the fixed ends of the frame there will be no rotation. So they are earthed in the circuit diagram.

All earth points of the main circuit are inter-connected, and finally they are connected to the negative lead of the feed current power unit.

The earthed points of each shear circuit are connected to the negative lead of the shear circuit powers.

Shear circuit potentiometers are of 500 K.ohms capacity.

<u>4.66- Feeding Circuit:</u>

Fig. 45 shows general feeding circuit connections. This consists of variable resistances r_1 and r_2 of the capacity of 100 k.ohms each. They can be varried to feed the required amount current at the points x and y of a beam.

4.06- Power:

Two power units of 100 volts and 50 m. amp. capacity are used for positive and negative fedd currents. One power unit of 10 volts and 1 amp. capacity for each shear circuit, for feeding the shear circuit are needed.

The mains are of a. c.supply of 220 volts.

For measuring the current and voltage two Avo meters are used. These are very precise ones, and the current can be measured up to 0.01 m. amp.

All the power units are provided with transformers and rectifiers to bring the current to required voltage and to change it from a.c to d.c.

4.07- Non Sway Moments:

Fixed end moments.

Consider any one of the loaded beam AB with uniformly distributed load of 1 tonper foot run, and of length 18'.

$$M_{AB} = WL^2/ 12$$
$$= - 27 \text{ ft. tons.}$$

and

$$M_{BA} = + 27 \text{ ft. tons.}$$

All the six of the loaded beams carry same amount of uniformly distributed load. So the fixed end moments at the ends of the all the loaded beams will be 27 ft. tons with the negative sign at the left side and positive sign at the right.

Chosing a scale factor P to be 1 m.amp. to 10ft. tons, the amount of feed current is 2.7 m. amp.

It will be seen from the equations given on page (67) that the positive signs occur in all the quantities concerning A, and negative signs occur inall the quantities concerning B. Where A and B are the ends of a beam. This makes it necessary to attach alternate positive and negative signs to the nodes of the net work as shown in the fig.44.

Signs:-

Positive,

Clockwise moments,

Anti clockwise rotations,

Current directed towards node,

Voltage above earth potential.

Sign of current = (Sign of f.e.m.) X (Sign of node)

Sign of fixing moment. = (Sign of net work current)X(Sign of node)

Sign of rotation= (Sign of voltage) X (Sign of node)

From the consideration of node signs it is seen that the feed current is positive at both the ends of a beam.

Initially the node plugs of type A are fitted at every node and the potentiometer in the each shear circuit is set at maximum resistance, so that the current flowing from the power unit through the circuit is negligible and terminals a and b are at earth potential. When the feed current operates

Node	Sign	Subscript	Feed current in m amp	Net work current in m amp	Fixing moment in ft ton	Calculated fixing moment
		5-1	-	-0.22	+2.20	+2.31
5	-	5-6	2.7	+0.73	-7.30	-7.50
		5-9	-	-0.51	+5.10	+5.17
		6-2	-	-0.13	-1.30	-1.64
		6-7	-	-1.45	-14.5	-15.31
6	+	6-10	-	-0.55	-5.5	-6.04
		6-5	2.7	+2.13	+21.3	+23.10
		7-3		+0.06	-0.60	-0.76
		7-8		-0.40	+4.00	+4.26
7	-	7-11		+0.01	-0.10	-0.19
		7-6		+0.32	-3.20	-3.32
		8-4		+0.01	+0.10	+0.13
8	+	8-12		+0.002	+0.02	+0.04
		8-7		-0.01	+0.10	-0.15
		9-5		+0.17	+1.70	+1.76
9	+	9-10		+0.05	+0.50	+0.56
		9-15		-0.22	-2.20	-2.30
		10-6		-0.15	+1.5	+1.63
10	-	10-11	2.7	+1.98	-19.8	-21.13
		10-14		-1.00	+10.00	+10.36
		10-9		-0.83	+8.30	+9.32

Node	Sign	Subscript	Feed current in m amp	Net work current in m amp	Fixing moment in ft ton	Calculated fixing moment
		11-7		-0.46	-4.60	-4.73
11	+	11-12		-0.90	-9.00	-9.18
		11-15	2.7	+2.02	+20.20	+20.67
		12-8		-0.09	+0.90	+0.98
12	-	12-16		+0.10	-1.00	-1.17
		12-11		-0.01	+0.10	+0.18
		13-9		+0.035	-0.35	-0.50
13	-	13-14		-0.007	+0.07	+0.10
		13-17		-0.028	+0.28	+0.33
		14-10		+0.25	+2.50	+2.64
14	+	14-15		+0.215	+2.15	+2.28
		14-18		-0.232	-2.32	-2.48
		14-13		-0.232	-2.32	-2.48
		15-11		-0.47	+4.70	+4.94
15	-	~~15-16~~	~~2.7~~	~~+2.31~~	~~-23.10~~	~~-24.58~~
		15-19		-0.96	+9.60	+10.19
		15-14		-0.88	+8.80	+9.43
		16-12		-0.68	-6.80	-7.11
16	+	16-20		-0.79	-7.90	-8.11
		16-15	2.7	~~+0.060~~ +1.47	+14.70	+15.28

Node	Sign	Subscript	Feed current in m amp	Net work current in m amp	Fixing moment in ft ton	Calculated fixing moment
17	+	17-13		-0.060	-0.60	-0.69
		17-18		-0.13	-1.30	-1.38
		17-21		+0.190	+1.90	+2.08
18	-	18-14		-0.011	+0.11	+0.144
		18-19		-0.079	+0.79	+0.87
		18-22		+0.160	-1.60	-1.71
		18-17		-0.060	+0.60	+0.66
19	+	19-15		+0.350	+3.50	+3.70
		19-20		-0.060	-0.60	-0.67
		19-23		-0.210	-2.10	-2.25
		19.28		-0.080	-0.80	-0.97
20	-	20-16		+0.29	-2.90	-3.16
		20-24		-0.155	+1.55	+1.66
		20-19		-0.135	+1.35	+1.45
21	-	21-17		-0.57	+5.70	+5.88
		21-22	2.7	-1.70	-17.00	-17.62
		21-25		-1.13	+11.30	+11.74
22	+	22-18		-0.61	-6.10	-6.12
		22-23		-0.71	-7.10	-7.21
		22-26		-1.30	-13.00	-15.12
		22-21	2.7	+2.62	+26.20	+26.42

Node	Sign	Subscript	Feed current in m amp	Net work current in am amp	Fixing moment in ft amp	Calculated fixing moment
23	-	23-19		+0.16	-1.60	-1.70
		23-24		-0.05	+0.50	+0.56
		23-27		-0.45	+4.50	+4.59
		23-22		+0.34	-3.40	-3.90
24	+	24-20		+0.15	+1.50	+1.54
		24-28		-0.31	-3.10	-3.23
		24-23		+0.16	+1.60	+1.69
25	+	25-21		+0.39	+3.90	+4.15
		25-26		-0.17	-1.70	-1.84
		25-29		-2.20	-0.22	-2.30
26	-	26-22		+0.51	-5.10	-5.18
		26-27		-0.35	+3.50	+3.59
		26-30		-0.17	+1.70	+1.84
		26-25		+ .01	-0.10	-0.23
27	+	27-23		+1.02	+10.20	+10.32
		27-28	2.7	-2.63	-26.30	-26.77
		27-31		+1.04	+10.40	+10.70
		27-26		-0.57	+5.70	+5.80

Node	Sign	Subs-cript	Feed current in m amp	Net work current in am amp	Fixing moment in ft amp	Calculated fixing moment
		28-24		-0.91	-9.10	-9.28
28	-	28-32		+1.00	-10.00	-10.22
		28-27	2.7	-1.91	+19.10	+19.45
29	-	29-25		+0.105	-1.05	-1.15
30	+	30-26		+0.08	+0.80	+0.92
31	-	31.27		-0.51	+5.10	+5.35
32	+	32-28		-0.48	-4.80	-5.11

The analytical method of solving for non sway moments and final calculation are given in Appendix 'A'.

The percentage error in the readings obtained over the Analyser is about 5% in case of maximum moments, and as the moments become small the error introduced is more.

the case corresponds to that of without sway. Now the current in every branch at the node points multiplied by the scale factor P and by node sign gives the B.M. in the corresponding branch.

For measuring current remove the node plug A from the required node and put the plug B, which is being connected to 4 pole 4 way selector switch, change over switch and finally to AVO meter.

By operating the above two switch we can measure the current in one branch at a time by keeping the connections to other three branches on.

Table No.2

Sign	Subscript	Feed current in m amp	Net work current in m amp	Fixing moment in ft ton	Calculated fixing moment
+	1-2	-	-0.033	-0.33	-0.39
	1-6	-	+0.033	+0.33	+0.39
-	2-3	2.7	+1.65	-16.10	-16.77
	2-6	-	-0.21	+2.10	+ 2.18
	2-1	-	-1.40	+14.0	+10.58
+	3-4	-	-1.40	-14.0	-14.51
	3-7	-	-0.30	- 3.0	- 3.26
	3-2	2.7	+1.70	+17.90	+17.91
-	4-8	-	-0.06	+0.60	+00.71
	4-3	-	+0.06	-0.60	-0.61

4.08- Sway moments:

Shear circuit which is described on page (72) stimulates the effect of sway in the storey. This c nsists of astring of resistors r, s and t, which are of 10, 10 and 13.3 ohms respectively. These resistors are kept low compared to the main circuit resistors in order to minimize the effect of current flowing through r and s from the main net work.

When the feed current operates the readings of X and Y in each of the shear circuit are taken. They will be found to differ by a certain amount, and the difference is in fact proportional to the horizontal shear in the corresponding storey. The effect of sway is then introduced by varying the setting of the potentiometer P until the meter readings in each circuit are equal. As the shear circuit does not operate independently it is necessary to adjust the potentiometers in turn two or three times before all differences in readings are finally eliminated. When the condition is reached, the horizontal shear in effect has been reduced to zero. Now the distribution of the current at every node multiplied the node sign and scale factor P gives the B.M. inclusive of sway effect.

Table - 3

Node	Sign	Subscript	Feed current m amp.	Net work current m amp.	Fixing moment in ft ton	Calculated fixing moment
1	+	1-2		-0.035	-0.35	-0.40
		1-6		+0.035	+0.35	+0.40

Node	Sign	Subscript	Feed current m amp.	Net work current m amp.	Fixing moment inft am ton	Calculated fixing moment
		2-3	2.7	+1.60	-16.00	-16.80
2	-	2-6		-0.20	+2.00	+2.14
		2-1		-1.4	+14.00	+14.50
		3-4		-1.405	-14.05	-14.45
3	+	3-7		-0.305	-3.05	-3.26
		3-2	2.7	+1.71	17.10	+18.00
4	-	4-8		-0.062	+0.62	+0.71
		4-3		-0.062	-0.62	-0.71
		5-1		-0.21	+2.10	+2.30
5	-	5-6	2.7	+0.73	-7.30	-7.70
		5-9		-0.51	+5.10	+5.29
		6-2		-0.15	-1.50	-1.60
6	+	6-7		-1.48	-14.80	-15.42
		6-10		-0.55	-5.50	-5.78
		6-5	2.7	+2.18	+21.80	+23.00
		7-3		+0.070	-0.70	-0.79
7	-	7-8		-0.38	+3.80	+4.04
		7-4		-0.007	+0.07	+0.09
		7-6		+0.330	+3.30	-3.54

Node	Sign	Subs-cript	Feed current m amp.	Net work current m amp.	Fixing moment in ft ton	Calculated fixing moment
		8-4		+0.011	+0.11	+0.13
8	+	8-12		+0.015	+0.15	+0.17
		8-7		-0.026	-0.26	-0.26
		9-5		+0.17	+1.70	+1.90
9	+	9-10		+0.040	+0.40	+0.43
		9.15		- .211	-2.11	-2.28
		10-11	2.7	+2.00	-20.00	-21.30
10	-	10-14		-1.00	+10.00	+10.30
		10-9		-0.900	+9.00	+9.14
		11-7		-0.420	-4.20	-4.41
11	+	11-12		-0.905	-9.05	-9.38
		11-15		-0.650	-6.50	-6177
		11-10	2.7	+1.975	19.75	+20.55
		12-8		-0.100	-1.00	+1.11
12	-	12-16		+0.109	-1.09	-1.19
		12-11		-0.010	-0.10	-0.16
		13-9		+0.043	-0.43	-0.46
13	-	13-14		-0.007	+0.07	+0.08
		13-17		-0.036	+0.36	+0.38

Node	Sign	Subscript	Feed current m amp.	Net work current m amp.	Fixing moment in ft ton	Calculated fixing moment.
14	+	14-10		+0.250	+2.50	-2.68
		14-15		+0.210	+2.10	+2.22
		14-18		-0.230	-2.30	-2.45
		14-13		-0.230	-2.30	-2.47
15	-	15-11		-0.480	+4.80	+4.99
		15-16	2.7	+2.350	-23.5	-24.58
		15-19		-0.970	+9.7	+10.17
		15-14		-0.905	+9.05	+9.45
16	+	16-12		-0.65	-6.50	7.10
		16-20		-0.820	-8.20	-8.52
		16-15	2.7	+1.47	+14.70	+15.16
17	+	17-13		0.060	-0.60	-0.73
		17-18		-0.140	-1.40	-1.49
		17-21		+0.209	+2.09	+2.19
18	-	18-14		-0.005	+0.05	+0.09
		18-19		-0.070	+0.70	+0.77
		18-22		+0.125	-1.25	-1.48
		18-17		-0.050	+0.50	+0.56
19	+	19-15		+0.346	+3.46	+3.64
		19-20		-0.068	-0.68	-0.76
		19-23		-0.170	-1.70	-1.83
		19-28		-0.098	-0.98	-1.07
20	-	20-16		+0.31	-3.10	-3.58
		20-24		-0.17	+1.70	+1.79
		20-19		-0.14	+1.40	+1.39

Node	Sign	Subscript	Feed current m amp.	Net work current m amp.	Fixing moment in ft ton	Calculated fixing moment
		21-17		-0.60	+6.00	+5.99
21	-	21-22	2.7	+1.75	-17.50	-18.70
		21-25		-1.150	+11.50	+11.68
		22-18		-0.560	-5.60	-5.89
22	+	22-23		-0.700	-7.00	-7.29
		22-26		-1.25	-12.5	-13.19
		22-21	2.7	+2.530	+25.3	+26.34
		23-19		+0.097	-0.97	-1.07
23	-	23-24		-0.042	+0.42	+0.48
		23-27		-0.418	+4.18	+4.42
		23-22		+0.363	-3.63	-3.96
		24-20		+0.150	+1.50	+1.65
24	+	24-28		-0.310	-3.10	-3.32
		24-23		+0.160	+1.60	+1.59
		25-21		+0.38	+3.80	+4.11
25	+	25-26		-0.15	-1.50	-1.82
		25-29		-0.23	-2.30	-2.32
		26-22		+0.49	-4.90	-5.16
26	-	26-27		-0.335	+3.35	+3.60
		26-30		-0.17	+1.70	+1.86
		26-25		+0.015	-0.15	-0.19
		27-23		+0.97	+9.70	+10.12
27	+	27-28	2.7	-2.56	25.60	-26.83
		27-31		+1.00	+10.00	+10.52
		27-26		+0.59	+5.90	+5.76

Node	Sign	Subs-cript	Feed current m amp.	Net work current m amp.	Fixing moment in ft ton	Calculated fixing moment
		28-24		+0.900	-9.00	-9.33
28	-	28-32		+0.98	-9.80	-10.20
		28-27	2.7	-1.68	+18.8	+19.42
29	-	29-25		+0.105	-1.05	-1.17
30	+	30-26		+0.08	+0.80	+0.91
31	-	31-27		-0.50	+5.0	+5.24
32	+	32-28		-0.48	-4.8	-5.09

The analytical method for the solution of sway moments and the final calculations of moment are given in the Appendix 'B'.

4.09- Horizontal or Wind loads:

The effect of the wind load on a frame is to induce horizontal shear, and these shears may be simulated in the analogue by means of shear circuit.

From theequation No.(22), given on page (74), the amount of i for each shear circuit can be evaluated.

$$i = P/3 \times SL$$

Where

i = Difference of the shear circuit meter readings.

P = Moment current scale factor.

S = Horizontal shear in a storey.

L = Length of the column

Table No-4

Storey	1	2	3	4	5	6	7
Shear	21	18	15	12	9	6	3
Length	12' for all the storeys						
i	8.4	7.2	6.0	4.8	3.6	2.4	1.2

Operate the potentiometer P of each shear circuit till the difference i of the meter readings X and Y correspond to the values given in the table No.(4).As the shear circuit does not operate independently it is necessary to adjust the potentiometers in turn two or three times before all the values of i correspond to the values required. Now measure the current in every branch at the node points. The value of the current multiplied by the scale factor and the node sign gives the required bending moment.

Table-5

Node	Sign	Subs-cript	Net work current m amp.	Fixing moment in ft ton	Fixing moment calculated
1	+	1-2	+0.26	+2.6	+3.09
		1-5	-0.26	-2.6	-3.10
		2-3	-0.26	+2.6	+3.09
2	-	2-6	+0.53	-5.3	-6.2
		2-1	-0.27	+2.7	+3.09
		3-4	+0.26	+2.6	+3.09
3	+	3-7	-0.53	-5.3	-6.20
		3-2	+0.27	+2.7	+3.09
4	-	4-8	-0.26	+2.6	-3.10
		4-3	+0.26	-2.6	+3.09
		5-1	+Ø25	-2.5	-3.00
5	-	5-6	-0.80	+8.0	+9.12
		5-9	+0.55	-5.5	-6.10
		6-2	-0.52	-5.2	-6.00
6	+	6-7	+0.82	+8.2	9.12
		6-10	-1.16	-11.6	-12.20
		6-5	+0-86	Ø8.6	9.12
		7-3	+0.52	-5.2	-6.00
7	-	7-8	-0.82	+8.2	9.12
		8-11	+1.16	-11.6	-12.20
		7-6	-0.86	+8.6	+9.12
		8-4	-0.25	-2.5	-3.00
8	+	8-12	-0.55	-5.5	-6.10
		8-7	+0.80	+8.0	+9.12
		9-5	-0.41	-4.1	-4.9
9	+	9-10	+1.47	+14.7	15.05
		9-15	-0.96	-9.6	-10.08

Node	Sign	Subscript	Net work current m amp.	Fixing moment in ft ton	Fixing moment calculated
10	-	10-6	+0.95	-9.5	-9.8
		10-4	-1.46	+14.6	+15.05
		10-14	+1.92	-19.20	-20.16
		10-9	-1.41	+19.1	+15.05
11	+	11-7	-0.95	-9.5	-9.8
		11-12	+1.46	+14.6	+15.05
		11-15	-1.92	-19.20	-20.16
		11-10	+0.95	-9.5	+15.05
12	-	12-8	+0.41	-4.1	-4.9
		12-16	+0.96	-9.6	-10.08
		12-11	-1.47	+14.70	+15.05
13	-	13-9	+0.87	-8.7	-9.20
		13-14	-1.92	+19.2	+20.20
		13-17	+1.05	-10.5	-10.97
14	+	14-10	-1.76	-17.6	-18.40
		14-15	+1.94	+19.4	+20.20
		14-18	-2.09	-20.90	-21.94
		14-13	+1.91	-19.1	+20.20
15	-	15-11	+1.77	-17.7	-18.40
		15-16	-1.94	+19.40	+20.20
		15-19	+2.09	-20.9	-21.94
		15-14	-1.92	+19.20	+20.20
16	+	16-12	-0.88	-8.8	-9.20
		16-20	-1.04	-10.4	-10.97
		16-15	+1.92	+19.20	+20.20

Node	Sign	Subs-cript	Net work current m amp.	Fixing moment inft ton	Fixing moment calculated
		17-13	-0.94	-9.40	-9.94
17	+	17-18	+2.47	+24.7	+26.0
		17-21	-1.53	-15.3	-16.0
		18-14	+1.88	-18.8	-19.88
		18-19	-2.46	+24.6	+26.0
18	-	18-22	+3.07	-30.7	-32.0
		18.17	-2.49	+24.9	+26.0
		19-15	-1.87	18.7	-19.88
		19-20	+2.48	+24.8	+26.0
19	+	19-23	-3.09	-30.9	-32.0
		19-28	+2.48	+24.8	+26.0
		20-16	+ .92	-9.2	-9.94
20	-	20-24	+1.53	-15.3	-16.00
		20-19	-2.45	+24.5	+26.00
		21-17	+1.33	-13.3	-14.05
21	-	21-22	-3.10	+31.0	+32.2
		21-25	+1.77	-17.7	-18.2
		22-18	-2.69	-26.9	-28.10
		22-23	+3.10	+31.0	+32.2
22	+	22-26	-3.48	-34.8	-36.4
		22-21	+3.07	+30.7	+32.2
		23-19	+2.69	-26.9	-28.10
		23-24	-3.10	+31.0	+32.2
23	-	23-27	+3.48	-34.8	-36.4
		23-22	-30.7	+30.7	+32.2

Node	Sign	Subs-cript	Net work current m amp.	Fixing moment in ft ton	Fixing moment calculated
		24-20	13.5	-13.5	-14.05
24	+	24-28	17.3	-17.3	-18.2
		24-23	+3.08	+30.8	+32.2
		25-21	-1.69	-16.9	-17.83
25	+	25-26	+3.16	+31.6	+33.36
		25-29	-1.470	-14.70	-15.50
		26-22	-3.47	-34.7	-35.66
		26-27	+3.19	+31.9	+33.36
26	-	26-30	-2.92	-29.2	-31.00
		26-25	+3.20	+32.0	+33.36
		27-23	-3.43	-34.3	-35.66
		27-28	+3.19	+31.9	+33.36
27	+	27-31	-29.5	-2.95	-31.00
		27-26	+3.19	+31.9	+33.36
		28-24	+1.72	-17.2	-17.83
28	-	28.32	+1.48	-14.8	-15.50
		28-27	-3.200	+32.00	+33.36
29	-	29-25	+2.58	-25.8	-26.75
30	+	30-26	-2.59	-25.9	-26.75
31	-	31.27	+2.59	-25.9	-26.75
32	+	32-28	-2.600	-26.00	-26.75

The analytical method for the solution of horizontal loads is given in the Appendix 'C'.

4.10 Points to be considered before making the investigations on the Analyser:-

1. A large number of group resistors would be necessary to cover tho range of members likely to obtain in any given structure. This can be obvitated to a large extent by arranging for resistor groups to be joined in parallel.For this purpose a five pin valve holder is to be soldered to the top of the each valve base, and it is then possible to plug one resistor group in to the top of the another. With this arrangement it is better to work in terms of conductance rather than resistance, and this has the advantage that the stiffness is directly proportional to coductance. A con venient procedure is to stamp a stiffness number on each resistor group, t where

Stiffness number = 10,000/ Resistance say.

Then a group of 1000 ohms resistance has a stiffness no. of 10. To represent a stiffness of for example 15.5 in^3., the groups having stiffness number of 10, 5 and 0.5 are pluged together and the whole unit is pluged into the base board. The sockets of the top resistor group can be used for making connections for feed current and voltage measurements.

2. Use of fixed resistors is better than the use of potentiometers. No net work resistance should be less than 500 ohms, as otherwise arrors due to the resistance of meters and shear circuit may become appreciablo.

3. For simplicity in operations the two meters X and Y of shear circuit may be replaced by a single meter which re-

o

cords the current difference directly. This will be called shear meter.

4. Settlement of foundation problem can be solved on the same Analyser, by making the connections to the groups of resistors representing beams instead of columns similar to that of shear circuit. The theory about this is given on page (75).

4.11 Precautions to be taken while using the Analysers:-

1. See that the connections of main circuits, shear circuit and feeding circuit are as shown in the circuit diagram and are properly made.

2. Check up the power units offeeding and shear circuit for the required voltage and current capacity.

3. While feeding the current at number of positions, it is necessary to adjust the potentiometers in turn two or three times, to get the required amount at every position. This is necessary as the power unit is common for all the feeding circuits.

4. As the current carrying capacity of potentiometers used in feeding as well as shear circuit are of 0.5 amp. capacity, care should be taken to not allow the current to exceed 0.5 amp. even for a short time. If it is required to feed more than 0.5 amp. the use of the higher capacity potentiometers is essential. Otherwise the potentiometer connections will burn away.

CHAPTER - 5

CONCLUSIONS

The introduction of Electrical Analogies in Structural field made the solution of the complicated problems as multi-storeyed frames and plates etc. rapid and simpler. The essential advantages claimed for Electrical Analogies are, they are accurate, simple, rapid and flexible.

The use of the analogies is not only restricted to the problems of beams, trusses, frames and plates, because they can serve as building blocks in the construction of analogous circuits for such complicated structures as airplanes, ship-hulls and gasturbines etc.

The accuracy of the method depends of course on the accuracy of different components of the analyser viz. the transformers, resistors, amplifiers, meters and whatever we use. On this Analyser the results that are accurate to within 5.0% of the maximum theoretically calculated values have been obtained. If the components of the analyser of good standard the values be accurate to within 2.0%.

The methods are simple and rapid to apply as it consists in merely measuring the current and voltage distribution in net works. Of the total time required to solve a given problem a fair proportion is spent in just writing down the results. Owing to the ease with which the analysis of a given structure is carried out, it is possible to investigate the effect of number of different load systems to determine the worst case for each member; and to consider a number of alternative designs or modifications to the original design, in order to decide which would produce the most economic structure.

The method is so flexible that it can be applied to the various structural problems ranging simple beams to the complicated structures as multi-storeyed building frames. Even it is possible to simulate the behaviour of rectangular frames is three dimensions.

APPENDIX -A

Gasper Kani's method of analysis of structures with non-translatory joints

If the structure deforms under a given loading, without the action of any restraint moments, then each joint undergoes a certain rotation, that is, if we fix our attention on an arbitrary bar i-k, we notice that the end i has rotated by an amount T_i and the end k by an amount T_k. We can conceive that this final deformation of the bar i-k, which was produced by the loading on this bar the rotations of the bar ends, is due to a super-position of the following three steps of deformation (Fig. 1):

I. The ends being fixed, the bar i-k deforms under the action of the given loading (beam, clamped at both ends).

2. The end i undergoes a rotation T_k, while the end i does not rotate.

We see that the end moment for each bar end may be thought of as being composed of three contributions (components).

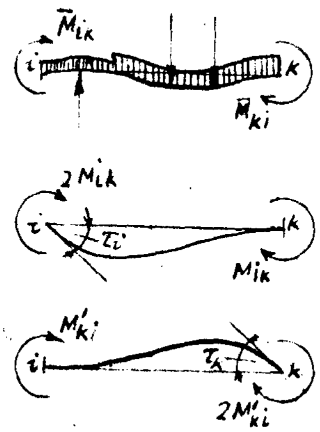


FIG. 1.

Thus, for example, the end moment for the end i of the bar i-k is composed of

contribution $\bar{M}_{ik}$- produced by the given external loading (fixed-end moment).

contribution $2M'_{ik}$- produced by the rotation of its own end i,

contribution M'_{ki} - produced by the rotation of the other end k of the bar.

Thus, the following expression for the end moment of the end i of the bar i-k may in general be written

$$M_{ik} = \bar{M}_{ik} + 2M'_{ik} + M'_{ki} \qquad \text{--------} \qquad (1)$$

The contribution M'_{ik}, which was produced by the rotation T_i of the end i, is proportional to the rotation TI and the corresponding K-value of the bar and will be referred to kn the future as the rotation contribution of the end i[2]. Similarly, the contribution M'_{ki}, which is proportional of the end i[2]. to the angle of rotation T_k and to the K-value of the bar, will be designated as the rotation contribution of the end k.

If these contributions are known, then in accordance with expression (I), the corresponding end moment M_{ik} is also known, that is,it is obtained by the summation of

the fixed-end moment,

the double of the rotation contribution of its own end i, and the single rotation contribution of the other end of the same bar.

The rotation contributions, as will be shown, are obtained by the repetition of a single calculation. Proceeding from joint to joint in an arbitrary sequence, we repeat this simple calculation until we reach at each joint the desired degree of accuracy, whereby any degree of accuracy is possible.

The calculation of end moments consists of the following parts:

1. For the given loading, the fixed-end moments $\bar{M}_{ik}$ are calculated and are indicated at the corresponding bar ends. The restraint moments are obtained by calculating at each joint i the sum of the corresponding fixed-end moments.

$$\bar{M}_i = \sum_{(i)} \bar{M}_{ik}$$

which are written down at the centers of the joints.

2. The rotation factors are obtained by distributing the value $\frac{1}{2}$ at each joint to the connecting bar ends in proportion of the K - values ($K = \frac{I}{l}$). For a bar end i, k of the joint i we have

$$\mu_{ik} = - \frac{1}{2} \frac{K_{ik}}{\sum K_{ik}} \qquad \text{---------} \qquad (2)$$

after entering the rotation factors into the calculating scheme, the sum of the rotation factors at each joint is calculated as a verification and should yield $- \frac{1}{2}$:

$$\sum \mu_{ik} = - \frac{1}{2} \qquad \text{(control)}$$

3. The calculation of the rotation contributions M'_{ik} is carried out by repeated application of the basic operation.

$$M'_{ik} = \mu_{ik} \left(\bar{M}_i + \sum_{(i)} M'_{ki}\right) \qquad \text{--------} \qquad (3)$$

proceeding from joint to joint in arbitrary sequence, until the desired accuracy is reached at each joint.

4. The final end moments are obtained from the fixed-end moments Mik and the rotation contributions by addition. For a bar end i, k we have

$$M_{ik} = \bar{M}_{ik} + 2 M'_{ik} + M'_{ki} \qquad \text{---------} \quad (1)$$

The calculation of the final end moments for the specimen frame fig.(44) is indicated in fig.(2). First, all the fixed-end moments and final rotation contributions are noted at the corresponding bar ends. In addition to the fixed end moment and the rotation contribution of its own end, the sum of the rotation contribution of both ends of the bar is also indicated. In each case the sum of these three values (for column ends only two because there are no fixed end moments) yields the end moments, which is entered at each bar end, below and above summation sign respectively.

5.17
+0.35
2.47
5.04
+27.00
+5.55
24.58
+27.00
8.70
+27.00
127.00
+44
+10.45

APPENDIX - B

Gasper Kani's method of analysis of structures with joints movable horizontally. (Ref. 11)

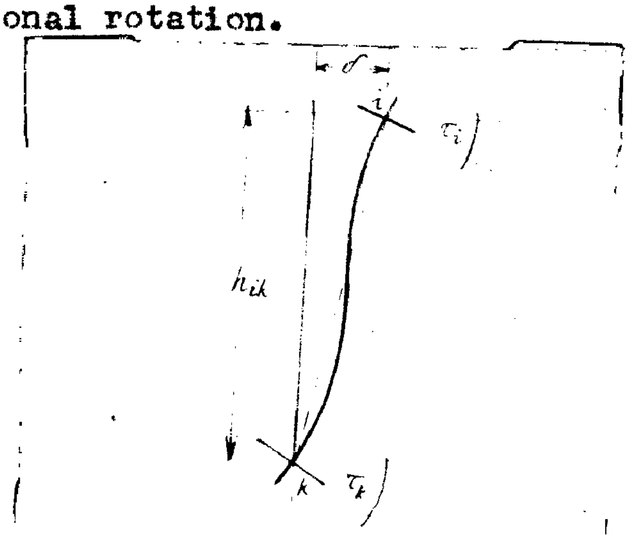
If the structure is designed in such a manner that during deformation the joints of the structure, that is, the ends of the bars, may not only rotate but may also be mutually displaced (Fig. 9), then here too, in accordance with the line of thought adopted above, the deformation of a bar may be considered as being produced as follows:-

1. Under a given loading, the bar i-k deforms without the bar ends being subjected to any rotations or relative displacement (fixed- end state).

2. The bar end i is subjected to a rotation Ti (without displacement of the bar end and without rotation of the end k).

3. The bar end k rotates by an amount Tk (without linear displacement of the bar end and without additional rotation of the end i).

4. The ends of the bar i-k are displaced with respect to each other by an amount (Fig. 3), whereby the bar ends are not subjected to any additional rotation.



Since the first three partialstates of deformation are precisely the same as in the calculation of frames with non-translatory joints, the expression (1) used above for the end moment Mik will receive an additional contribution M"ik, which is due to the displacement of the bar .

After these four contributions to the end moment are calculated, the end moment itself may be obtained again by simple addition:

$$M_{ik} = \bar{M}_{ik} + 2\,M'_{ik} + M'_{ki} + M''_{ik} \quad \text{--------(1a)}$$

The contribution M"ik due to the linear displacement of the bar shall be referred to henceforth as the linear-displacement contribution. If the expression (1a) is used again to formulate the condition of equilibrium for some joint i,

$$\sum_{(i)} M_{ik} = 0$$

the rule for the basic operation to determine the rotation contributions will be thesame as in the case of non-translatory joints.[6]

With the same rotation factors as before, we again obtain the rotation contributions here from the sum of all contributions, which now also contain, however, in addition to the restraint moment $\bar{M}_i$ and the rotation contributions of the far bar end, the displacement contribution M"ik of all bars of the corresponding joint, that is, from the sum

$$\bar{M}_i + \sum_{(i)} (M'_{ki} + M''_{ik}).$$

For straight bars with a constant cross section, as is assumed here, we have M" ik = M" ki. For each bar there exists, therefore, only one number that represents the displacement contribution, which is written down, conveniently, at the center of the bar.

Vertical Loading

We cut horizontally through all the columns of some story r. From consideration of equilibrium it follows that the sum of all the shear forces at the columns of this story is equal to zero.

$$\sum_{(r)} Q_{ik} = 0.$$

This equilibrium condition, which must be satisfied at each story (if horizontal displacements of the joints are possible) and which is satisfied by horizontal relative displacement of the girders, serves to determine the linear displacement contributions. If the story r contains only columns of equal length, which is assumed, then this equilibrium condition, together with equation (1a), yields, by substituting the expression for the shear force of a column i-k,

$$Q_{ik} = - \frac{M_{ik} + M_{ki}}{h_{ik}}$$

$$\sum_{(r)} Q_{ik} =$$

$$= - \frac{1}{h_{ik}} \left(2 M'_{ik} + M'_{ki} + M''_{ik} + 2M'_{ki} + M'_{ik} + M''_{ki}\right) = 0$$

and from this

$$\sum_{(r)} M''_{ik} = - \frac{3}{2} \sum_{(r)} (M'_{ik} + M'_{ki})$$

The sum of the displacement contributions of all columns of a story r can, therefore, be determined from the rotation contributions of the column ends of this story).

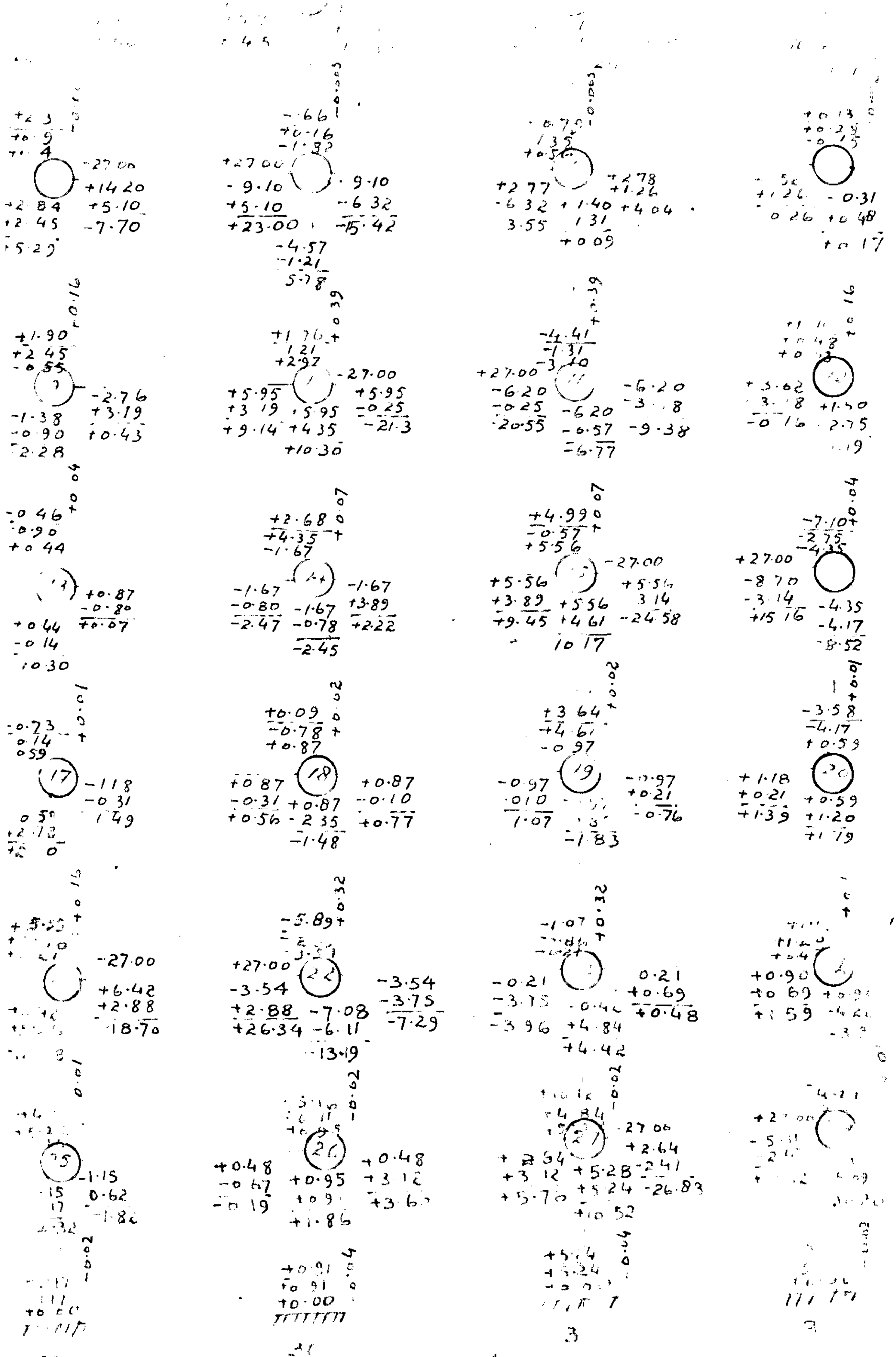
To make the calculation more convenient, and in analogy to the rotation factors, we may now introduce the displacement factors, which are obtained by distributing the value $-\frac{3}{2}$, in proportion to the bar values K among all the columns of the story.

Thus the same convenient analytical operation is obtained for the determination of the linear - displacement contributions as before for the determination of the rotation contributions.

Sum the rotation contributions of all column ends of the story in question. Multiply the sum so obtained by the linear- displacement factors of the column one after the other, to obtain the linear-displacement contributions.

From the rotation contributions we calculate the displacement contributions, and from these again the rotation contribution of the following approximation, etc., until results of an arbitrary accuracy are reached.

The calculation of the final end moments for the specimen frame fig.(44) is indicated in fig.(4). First all the fixdd end moments and final rotation contributions are noted at the corresponding bar ends and displacement contributions along the each column at the middle. In addition to the fixed end moment and the rotation contributions of its own end, the sum of the rotation contribution of both ends



-27.00
+14.20
+5.10
-7.70
+2.84
+2.45
+5.29
+27.00
-9.10
+5.10
+23.00
-9.10
-6.32
-15.42
-4.57
-1.21
5.78
+2.77
-6.32
3.55
+2.78
+1.26
+1.40
+4.04
1.31
+0.09
-0.31
-0.26
+0.48
+0.17
+1.90
+2.45
-0.55
-2.76
+3.19
+0.43
-1.38
-0.90
-2.28
+1.76
1.21
+2.97
-27.00
+5.95
+3.19
+9.14
+5.95
+4.35
+10.30
-0.25
-21.3
-4.41
-1.31
+27.00
-6.20
-0.25
-20.55
-6.20
-6.57
-6.77
-9.38
+3.62
+1.50
-0.16
-0.46
-0.90
+0.44
+0.87
-0.80
+0.07
+0.44
-0.14
+0.30
+2.68
+4.35
-1.67
-1.67
-0.80
-2.47
-0.78
-2.45
+3.89
+2.22
+4.99
-0.57
+5.56
-27.00
+3.89
+9.45
+5.56
+4.61
10.17
3.14
-24.58
-7.10
-2.75
-4.35
+27.00
-8.70
-3.14
+15.16
-4.17
-8.52
17
-1.18
-0.31
1.49
+0.09
-0.78
+0.87
18
+0.87
-0.31
+0.56
-2.35
-1.48
-0.10
+0.77
+3.64
+4.61
-0.97
19
-0.97
0.10
1.07
+0.21
-0.76
-1.83
-3.58
-4.17
+0.59
20
+1.18
+0.21
+1.39
+0.59
+1.20
+1.79
-27.00
+6.42
+2.88
-18.70
-5.89
+27.00
-3.54
+2.88
+26.34
22
-7.08
-6.11
-13.19
-3.54
-3.75
-7.29
-1.07
-0.21
-3.75
-3.96
+4.84
+4.42
0.21
+0.69
+0.48
+0.90
-0.69
+1.59
25
-1.15
0.62
-1.82
26
+0.48
-0.67
-0.19
+0.95
+1.86
+3.12
+4.84
27
-27.00
+2.64
+3.12
+5.76
+5.28
+5.24
+10.52
-26.83
+27.00
+0.91
+0.00
21
3

FIG - No 4

in case of beam and sum of the rotation contributions of both ends and displacement contribution in case of columns is also indicated. In each case the sum of these three values (for columns ends only two because there are no fixed end moments) yields the end moments which is entered at each bar end, below and above summation sign respectively.

APPENDIX - C

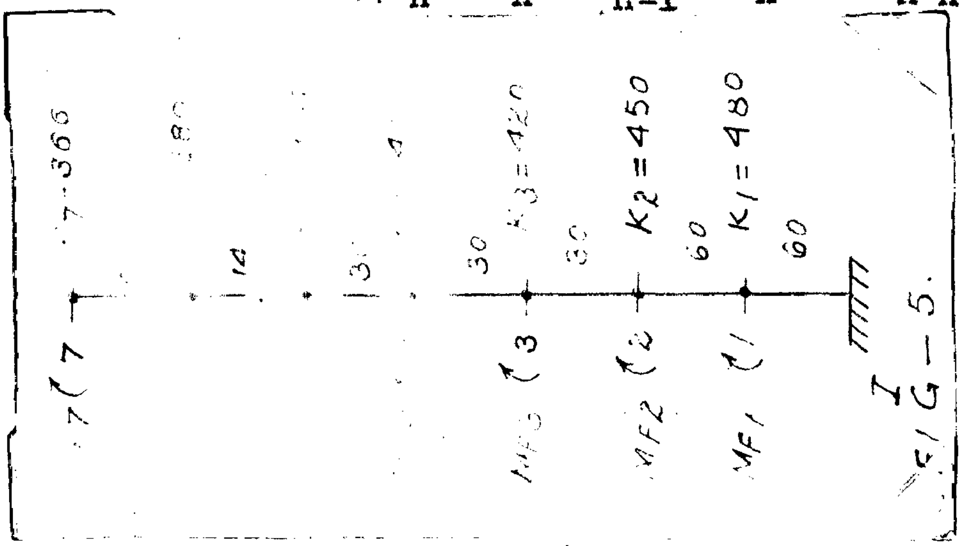
The solution of the substitute cantilever by deformation distribution - (Ref. 12)

The calculation of the end moments for horizontal loads of the frame shown in fig.(44) by Dr. C.V. Klonocck's method of deformation distribution, consists of the following parts:

1. Find out the stiffness factors of the substitute cantilever using the following equations:

$$\text{Column stiffness } K_n^C = \sum_{1}^{n} (K_n)$$

$$\text{Knot stiffness, } K_n^c = K_n^C + K_{n=1}^C + A_n \quad tK_{n-n}$$



2. For the given loading calculate the values of S and M_F, where

$$S_1 = \frac{\sum_{1}^{7} (V)L}{3} \quad \text{and } M_{F1} = 3(S_1 + S_2)$$

where V = Horizontal face at joints

and L = Storey height.

3. Calculate the values of a_{1-2}, a_{2-3} and d_1, d_2 ----- by with the help of the following equation

$$a_{1-2} = K_{1-2}^2 / K_1 K_2$$

and $d_1 = \frac{M_{F1}}{K_1\ (1-a_{1-2})}$

Then find out the distributed values of d_1, d_2 -------
by ignoring the values of 'a' so that

$$d_{A-B} = d_A^o \ \frac{K_{A-B}}{K_B}$$

4. After calculating actual distributed values of d_1, d_2 ... calculate the values of u_1, u_2----

where $U_1 = -\ 3/2 \left\{ \frac{S_1}{K_{1-I}} + d_1 + d_I \right\}$

Then find out the values of column end moments

$$M_{I-1}\ (2\ d_I + d1 + u_1)$$

There are the values for the cantilever. The actual and frame moments can be found by distributing the above calculated values in the proportion of column stiffness.

5. The beam moments are obtained as follows

$$M_{1-2} = K_{1-2}\ (2d_1 + d_2) = K_{1-2} \times 3\ d_2 \quad \text{Since } d_1 = d_2$$

Fig.5 in the substitute cantilever for the specimen Frame for the analysis of horizontal loads.

The table No.1 gives the values of S, M_F, a, d and V values for the substitute frame of the Fig. 5.

Table No.1

	S	M_F		a		d (distributed		V	
S_1=	84	M_F1	468	a_{I-1}	o	d_1	1.112	V_1	-3.17

	S	M_F		a		d (distributed		V	
S_2	72	M_{F2}	396	a_{1-2}	0.0167	d_2	1.073	V_2	-5.08
S_3	60	M_{F3}	324	a_{2-3}	0.0048	d_3	.895	u_2	-5.95
S_4	48	M_{F4}	252	a_{3-4}	.0051	d_4	.696	u_4	-4.465
S_5	36	M_{F5}	180	a_{4-5}	.0053	d_5	.504	u_5	-3.623
S_6	24	M_{F6}	108	a_{5-6}	.0013	d_6	.304	u_6	-3.79
S_7	12	M_{F7}	36	a_{6-7}	.00027	d_7	0.103	u_7	-3.61

R E F E R E N C E S

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APPENDIX - D

Notation:-

For the purpose of this paper and unless otherwise defined in the text, the following letter symbols shall have the meaning indicated against each:-

A = Cross sectional Area

D = Flexural Rigidity

E = Young's Modulus

F = Applied force

I = Moment of Inertia

L = Length of the member

M = Bending Moment

N = Number of turns in the transformer winding

P = Power

Q = Shear

q = Intensity of Loading

R = Resistance

V = Voltage

i = Current

v = Poisson's Ratio

U = Stored energy

Δ = Displacement

δ = Deflection

θ = Slope.