STATE-VARIABLE REALIZATION
OF
LUMPED NETWORKS & DYNAMICAL SYSTEMS

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The state-variable approach, because of its inherent importance, has aroused considerable interest in the study of systems and networks during the past decade. This thesis is, primarily, concerned with the problem of state-variable realization of linear time-invariant dynamical systems and its application to lumped networks, with a view to evolve new synthesis procedures suitable for integrated circuit fabrication.

The problem of minimal reciprocal realization of linear, time invariant dynamical systems is investigated. Two simplified algorithms for constructing minimal reciprocal realization from a given symmetric transfer function matrix and symmetric impulse response matrix have been proposed. Both the methods exploit the symmetry of the given transfer-function matrix and impulse response matrix and require determining the Hankel matrix, the first from the Markov-parameters and the second from the moments of the impulse response matrix, the latter being preferable in the presence of noise. The order of

the Hankel matrices required in the procedure of both the algorithms is much smaller than the existing methods, thereby reducing significantly the computing time and memory storage required. The realizations obtained by the proposed algorithms result in reciprocal networks. Further, utilizing these results, a passive reciprocal (gyratorless) synthesis of symmetric positive real immittance matrices is given.

Since the classical synthesis methods for linear, time-invariant networks are well-known, it is quite important to establish a communication link between the state-variable characterization and the input-output description. Some endeavours have already been initiated in this direction. Here, a state-space interpretation of classical Foster synthesis of multiport lossless network has been discussed. Well-known Cauer driving point synthesis and active RC filter design using coefficient matching technique are also revisited in state-space terms using observability matrix as a canonical transformation.

Various synthesis techniques, which realize an arbitrary rational function matrix of a multiport active RC network, have been developed during recent years. But, the upper bound on the number of active elements required in these methods is quite large and in some cases, the number of resistors used in the realization is also more. In this thesis, a simple and systematic synthesis procedure, based on a state-variable approach and the reactance extraction principle, has been presented whereby any arbitrary rational function matrix can be realized as an immittance matrix of an active RC multiport network with a minimum number of grounded capacitors having unity capacitance spread. The proposed technique reduces the upper bound on the number of active elements and can be reasonably expected to require fewer resistors. Besides, the structure of the realized circuits in terms of the minimum number of elements and grounded ports make it particularly desirable for integrated-circuit fabrication.

Finally, some suggestions for further investigations in this area are also included.

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CHAPTER I

INTRODUCTION AND STATEMENT OF THE PROBLEM

1.1 INTRODUCTION

During recent years, there has been a growing interest in the application of state-variable techniques for the study of dynamical systems and networks. This may be attributed to the fact that the state-variable approach is computationally more attractive, especially in terms of computer aided design (CAD) [79]. Moreover, the approach is more general than the classical Laplace and Fourier transform theory and hence is applicable to many systems for which transform theory breaks down [153]. Since the approach is in time-domain, it is equally applicable to both non-linear and time-varying systems in addition to the time invariant linear systems [77]. Apart from providing a more general representation of a physical process, a very important advantage of this technique lies in its flexibility in generating "equivalent" canonical representations which are very useful in system analysis. Another important contribution of this approach is that it permits problems in networks and systems to be treated in an unified manner [10]. Besides, the technique is particularly useful in multiport network synthesis [104] and consequently new synthesis procedures using this approach are being developed [6], [8] - [11], [18], [35], [57], [69], [71], [75], [96], [108], [122], [139], [147], [150],

[151], [156]-[158], [160].

This thesis is concerned with the state-variable realization of dynamical systems and its application to the synthesis of finite, lumped, linear, time-invariant, passive and active multiport networks.

It is well-known that a state-variable characterization of finite, lumped, linear, time-invariant, passive p-port network is given by the dynamical equations or state equations

$$\dot{X} = AX + BU,$$
$$Y = CX + DU, \qquad \ldots (1.1)$$

where X is n-vector, the state, having its components as capacitor voltages and inductor currents, U is p-vector, the input, and Y is q-vector, the output. The matrices A, B, C, and D are real constant matrices of dimensions n x n, n x p, q x n and q x p, respectively.

In network synthesis, we are mainly concerned with the realization of a passive or active network that has a prescribed immittance or transfer function matrix G(s); whereas the system realization problem is to pass from an input-output description of a system in the form of an impulse response matrix G(t), or a transfer function matrix, G(s), to a state-space description of the type (1.1). Thus, system realization problem is intimately related to modern network synthesis [28].

Once the dynamical equation (1.1) is known for a system, the system can be easily simulated on an analog computer. Further, transfer function is an input-output description of a system, whereas a dynamical equation describes not only the input-output relation but also the internal structure of a system. Thus, the realization problem may also be considered as an "identification problem", a problem of identifying the internal structure of a system from the knowledge obtained through direct measurements at the input and output terminals[28]. Because of its wide applications, the realization problem has been actively considered over the past decade by several investigators and consequently, a well developed theory of realization is now available in the technical literature[66], [67], [127], [137].

In the field of network synthesis, the first-step is to determine a minimal realization $\{A, B, C, D\}$ of a given input-output description. Since the realization is minimal, the number of dynamic or reactive elements and integrators needed to synthesize a network will be minimum, which is desirable for reasons of economy and sensitivity. If a given state-model $\{A, B, C, D\}$ satisfies Anderson's positive real lemma[5], a synthesis of the network using only passive elements is possible. Further, the realization set $\{A, B, C, D\}$ satisfying reciprocity criterion due to Yarlagadda[156] will lead to reciprocal network realizations.

Modern system theory concepts have also been exploited to give state-space interpretation of some of the well-known classical synthesis procedures, and properties of network functions. Recently, the state-variable technique has also found application in evolving novel active RC multiport network synthesis methods suitable for integrated circuit fabrication [18], [57], [69], [71], [96], [98], [122], [123], [158].

Thus, the problem of realization of dynamical systems assumes great significance because of its manifold applications in studying problems of various engineering disciplines such as optimal control, system theory and network theory.

Having introduced the problem of state-variable realization for linear, time-invariant dynamical systems and discussed its implications, the specific problems considered in the present thesis are stated in the next section.

1.2 STATEMENT OF THE PROBLEM

The work embodied in this thesis can be broadly classified in three sections:

1. State-variable realization of linear, time-invariant dynamical systems,
2. State-space interpretation of some classical synthesis procedures, and
3. Multiport active RC network synthesis with a minimum number of capacitors.

Specifically, the following problems are considered in this thesis:

(1) New algorithms are developed for obtaining minimal reciprocal realization from a given symmetric transfer-function matrix and symmetric impulse response matrix. The algorithm for symmetric transfer-function matrix uses Markov-parameters and gives a simpler procedure, whereas, moments of the impulse response are used for the reciprocal realization of impulse response matrix. The minimal reciprocal realization is useful for passive reciprocal network synthesis of symmetric positive real (SPR) immittance matrices.

(2) In order to establish a link between state-variable characterization and specifications in s domain, a state-space interpretation of Foster multiport LC network synthesis, Cauer driving point (dp) synthesis, and well-known coefficient matching technique of active RC filter design, is presented.

(3) An active RC multiport network synthesis procedure, suitable for integrated circuit fabrication, is evolved. Specifically, the proposed procedure is applied to the synthesis of short-circuit admittance matrix, open-circuit impedance matrix, and transfer-impedance matrix using operational amplifiers.

(4) Given a symmetric positive real (SPR) immittance matrix, a synthesis procedure, based on the preceding

results, is stated for passive-reciprocal multi-port realization using RCT(resistor, capacitor and Ideal Transformer) network.

It is worthwhile to mention that some aspects of these problems have been studied by many authors[89], [116], [115], [59], [18], [98], [151], [156], [160] and some results are available. The work reported in [89] and [116] is concerned with the first problem where the minimal realization of a symmetric matrix is obtained by modifying the well-known Ho-Kalman algorithm[56]. The procedures proposed in the present thesis reduces the computations considerably by requiring Hankel matrices of lower order.

The second problem i.e. state-space interpretation has been considered in [5], [9], [20], [59], [74], [87], [115], [139] where the classical synthesis procedures and network properties have been revisited via state-space characterization with a view to bridge the gap between the synthesis procedures in s domain and state-space.

As regards the third problem, the procedures due to Bickart and Melvin[18], [98], Mann and Pike[96], and Huang[57] are available. But the upper bound on the number of active elements required in these methods is quite large. In some cases, the number of resistors required in the realization is also more.

The fourth problem i.e. passive reciprocal multiport synthesis has been investigated by Youla and Tissi[160], Vongpanitlerd and Anderson[150], [151] and Yarlagadda[156] using RLCT network. The proposed procedure in this thesis is for passive reciprocal multiport RCT network and requires a minimum number of capacitors.

1.3 ORGANISATION OF THE THESIS

Having stated the problem in the preceding section, the organization of the remaining part of the thesis is given below.

In Chapter II, the problem of state-variable realization of linear, time-invariant dynamical systems is introduced. Having given some system theory preliminaries, a historical review of some selective literature on minimal realization methods of linear dynamical systems, state-space interpretation of some well-known network properties and synthesis procedures, general state-space passive network synthesis based on reactance extraction technique, and multiport active RC network synthesis procedures, is presented. The well-known Ho-Kalman algorithm[56] is also discussed because of its importance and use in the subsequent work in this thesis.

The minimal reciprocal realization algorithms, for symmetric transfer-function matrix, and symmetric impulse response matrix using moments, are developed in Chapter III. The use of moments of the impulse response is advantageous in the presence of noise. The proposed procedures are simpler and computations are considerably reduced as they require Hankel matrices of lower dimensions. Superiority of the realization techniques evolved here in regard to simplicity and elegance is amply illustrated with the help of suitable examples.

Chapter IV is devoted to seeking state-space interpretation of Foster multiport LC network synthesis, Cauer driving point synthesis, and coefficient matching technique for active RC second order filter design.

In Chapter V, new active RC multiport network synthesis procedure, with a minimum number of capacitors and suitable for integrated circuit fabrication, is discussed. The proposed approach of active RC multiport network synthesis is first outlined. Subsequently, the synthesis of short-circuit admittance matrix, open-circuit impedance matrix, and transfer-impedance matrix using operational amplifiers is considered. The proposed method is illustrated with the help of suitable examples. Also, based on the above approach, a passive reciprocal multiport synthesis procedure using RCT network for SPR immittance matrices is briefly described. Examples are given to illustrate the procedure.

Chapter VI contains a summary of the results presented in this thesis. Some suggestions, for further investigations in this field which might lead to some interesting results, have been included at the end of this chapter.

CHAPTER II

REVIEW AND GENERAL CONSIDERATIONS

2.1 INTRODUCTION

The characterization of a real dynamical system by a suitable mathematical model is considered to be one of the most important and interesting problems in the study of systems and networks. Once a mathematical description is obtained, it can be used to optimize, control or predict future behaviour of a physical process.

The problem of determining a minimal state-model from input-out or external description has received considerable attention in the recent years as is evidenced by the abundance of technical papers[2],[19],[21],[24],[43],[45],[46],[63],[117], and consequently a fairly complete realization theory has been evolved. State-model realization has assumed great importance in network theory also because of the modern trend of carrying out network synthesis via state-space [8]-[11],[75],[85],[104],[108],[139],[151]. As the frequency domain methods may still be preferred for linear, time-invariant systems for many design problems, the interpretation of one description from the other has also attracted the attention of several authors and some well-known classical synthesis methods have been re-examined in state-space terms[5],[9],[20],[59],[70],[74],[87],[88],[133]. Recently, state-variable techniques have been exploited to develop new active RC multiport network synthesis procedures suitable for integrated circuit fabrication [18],[69],[71],[81],[98],[107],[122],[123],[147]. Also, state-variable representation is most convenient for time-varying and non-linear systems and networks [73],[86],[105],[108],[121],[136],[142].

This chapter surveys some selective literature on minimal realization of linear, time-invariant dynamical systems, and passive and active network synthesis procedures. The realization algorithm due to Ho-Kalman [56] is discussed in some detail because of its importance in the subsequent work in this thesis.

2.2 STATE-VARIABLE DESCRIPTION OF LINEAR DYNAMICAL SYSTEMS AND NETWORKS

A linear, time-invariant, multi-variable, finite-dimensional dynamical system may be described in many different ways. However, there are two standard forms in which a precise definition can be given to the system. The first is by means of input-output or external description and the second is by means of internal or state-variable description. In the first case, the system is characterized by a qxp rational transfer-function matrix, G(s), which relates the Laplace transform of the input p-vector U(s) to the Laplace transform of the output q-vector Y(s) through

$$Y(s) = G(s)\ U(s) \qquad \ldots (2.1)$$

Another input-output description of the dynamical system is by means of a qxp impulse response matrix

$$G(t) = \left[g_{ij}(t)\right], \qquad i=1,2,\ldots,q, \quad j=1,2,\ldots,p,$$

where $g_{ij}(t)$ is the impulse response between the jth input terminal and the ith output terminal.

Thus G(s) or G(t) yield an external description (input-output mapping) of a dynamical system.

In the case of state-variable representation, the system is governed by the canonical state-space equations of the form

$$\begin{aligned} \dot{x}(t) &= A\ x(t) + B\ u(t) \\ y(t) &= C\ x(t) + D\ u(t) \end{aligned} \qquad \ldots (2.2)$$

where the output q-vector y(t) and the input p-vector u(t) are related via an abstract intermediate vector variable, the state n vector x(t), which is a vector function of time. The matrices A, B, C, and D are real constant matrices of dimensions nxn, nxp, qxn and qxp respectively.

It is easy to see that the transfer function matrix G(s) is related to the state-variable description (2.2), by

$$G(s) = D + C(sI - A)^{-1} B \qquad \ldots (2.3)$$

It may be noted that while characterizing a system by the equation (2.2), we have assumed that G(s) has no pole at infinity, i.e. G(∞) is finite. In fact if G(s) possesses a pole at infinity, the associated state-space equations (2.2) have extra terms involving the first derivative of the sources (i.e. $\dot{u}(t)$ terms are present) [99], [144]. As will be seen later, it is sufficient in our work to assume that the canonical state-space equations (2.2) applies.

It is clear that any quadruple {A,B,C,D} determines a G(s) with G(∞) finite. The converse, however, is not obvious immediately. It is known, however, [28], [29], [56], [63], [137], [160], that any G(s) does determine an infinity of triples {A, B, C} such that (2.3) holds with D = G(∞) . The methods of constructing the triples are discussed in these references, the most significant ones being the algorithm due to Ho-Kalman [56], which will be discussed later.

Definition 2.1

Any quadruple {A,B,C,D} satisfying (2.3) is called a realization of G(s); a realization for which A has the smallest dimension is termed an irreducible or a minimal realization.

The minimal dimension of A is the smallest dimension of a state vector which is sufficient to describe the dynamics of the system, and this dimension is called the degree of G(s), denoted by $\delta[G(s)]$.

The concept of degree has appeared in many publications in the field of network and control theory and various definitions to it have been given from time to time [10,p.116], [51]. Tellegen defines the order of a network as the maximum number of natural frequencies obtained by embedding the given network in an arbitrary passive network. His order definition agrees with McMillan's definition of the degree which denotes the minimum number of reactive elements required in any passive synthesis of a positive real impedance matrix Z(s). Kalman [65] has shown that these other definitions are the same as that of definition 2.1, provided that poles at infinity are properly accounted for.

Several important properties of minimal realizations may be noted as follows:

(i) Minimal realizations are determined by G(s) <u>uniquely</u> to within arbitrary state-space co-ordinate transformations. In other words, if $\{A, B, C, D\}$ is a minimal realization, all possible equivalent minimal realizations are given by, [63],

$$\hat{A} = T_o^{-1} A\, T_o, \quad \hat{B} = T_o^{-1} B, \quad \hat{C} = C\, T_o, \quad \hat{D} = D \qquad \ldots (2.4)$$

where T_o ranges over all non-singular constant matrices. Equations (2.4) result from the change of state-space basis $\hat{x} = T_o x$; for a given non-minimal realization, others are generated by an arbitrary non-singular matrix T_o in

(2.4); however not all non-minimal realizations can be so generated [7].

(ii) The dimension of a minimal realization of a transfer function G(s) is called the degree, $\delta[G(s)]$, of G(s), and is related to the minimal number of energy-storage or reactive elements in any passive network synthesis of a realizable matrix G(s).

(iii) If $\{A, B, C, D\}$ is minimal, the realization is completely controllable and observable [63], which means, that

$$\text{rank}[B, AB, A^2B, \ldots, A^{n-1}B] = \text{rank}[C', A'C', \ldots, (A^{n-1})'C'] = n \qquad \ldots (2.5)$$

where A is nxn and prime denotes matrix transposition.

It is clear, on observing property (i), that the properties of controllability and observability are independent of the particular choice of basis in the state-space.

It may be noted that the matrix sequence $[C', A'C', \ldots, (A^{n-1})'C']$ is known as an observability matrix [92], [109], which will be used in Chapter IV as a non-singular canonical transformation matrix.

In the next section, a historical review of various minimal realizations methods and network synthesis procedures is outlined.

2.3 REVIEW

In this section, a historical development, of various minimal realization algorithms, state-space passive, and active synthesis methods and state-space interpretation of some classical synthesis procedures, is briefly discussed.

2.3.1 Historical Review of Minimal Realization Methods

The problem of deriving a minimal realization of a linear dynamical system was first introduced by Kalman[63] in 1963, who gave an algorithm for determining a triple of matrices $\{A, B, C\}$, which describes the system behaviour in the usual state-space, from the knowledge of any other given characterization. At the same time, Gilbert [43] also gave a method for computing state-variable differential equations from a transfer-function matrix. Both the above methods heavily rely on the dual concepts of controllability and observability. In 1965, Kalman, employing the classical theory of elementary divisors and the language of modules, proposed a new algorithm for constructing the state-equations from a given transfer-function matrix having multiple poles[65]. This algorithm exhibits the canonical form, under equivalence, of a rectangular polynomial matrix[42]. Based on Kalman's approach[65], a minimal realization method was suggested by Raju[119]. He obtains the order of the system and the state matrix A by following the method of [65], and the matrices B and C are obtained by drawing signal flow graph. In 1965, Ho and Kalman [56], based on the study of Markov parameters [42], evolved an irreducible realization algorithm, which is considered to be one of the most useful and computationally simpler one. The impulse response data of the system, which is assumed to have zero initial state, can be given in the time or the s-domain in the form of Markov parameters. Ho and Kalman algorithm hinges on "the generalized Hankel matrix" constructed from the Markov parameters. An interesting procedure, for computing a state-variable model, in the canonical form of Bucy [23], from the given matrix of impulse response sequences of a finite-dimensional discrete-time linear constant dynamical system, was proposed by Ackermann and Bucy [2] in 1971. The construction is an alternate to the Ho-Kalman algorithm [56] in which two transformation matrices P and Q must be found. Since P and Q in [56] are not unique, the realization obtained by Ho and Kalman is not in any special canonical form. Albertson and Womack [4] also gave an algorithm for computing the dimension of and constructing irreducible realization of a prescribed system transfer-function. Their procedure is simple and provides more insight into the physical significance of the problem. In 1969, Gopinath [45] suggested a new method for computing the parameters which determine the differential equations governing a linear time-invariant multivariable system. Unlike earlier approaches,this method[45] does not involve computation of the impulse response. One of the main advantages of the method[45] is its easy generalization to the case when the given data is contaminated with noise. Based on Gopinath approach[45], a direct procedure for obtaining a minimal realization from input-output observations was presented by Budin[24] which improves the computational requirements. One of the important by-products of this procedure is a well-defined structure for the realization. In 1969, Wolovich and Falb[154] stated and proved a structure theorem for time-invariant multivariable linear systems, which is then applied to obtain an algorithm analogous to that of Mayne[97] for solving the problem of realization. Another interesting method for the determination of state-space representation for linear multivariable systems has been given by Wolovich[155]. Bruni et al.[21] also proposed a realization method based on the moments of the impulse response matrix. Although, this procedure[21] utilizes the Ho-Kalman algorithm[56], but the Hankel matrix in this case is constructed from the moments in place of Markov parameters. As discussed in [21], in the presence of noise, computation of moments is preferable to that of Markov parameters which are the local time-derivatives of the impulse response matrix.A new method for realizing a rational transfer-function matrix into an irreducible Jordan canonical form state-equation was presented by Kuo[80]. Youla and Tissi[160], and Silverman[137], also gave procedures, based on the Hankel matrices, to obtain minimal realizations from input-output data. Recently, an interesting irreducible realization algorithm was proposed by Chen and Mital[29]. The procedure is a simplification of the methods based on Hankel matrices[56],[137],[160]. Compared with the existing methods, this algorithm[29] uses Hankel matrices of smaller order thereby reducing significantly the computing time and memory storage required.

Apart from the above procedures of direct computation of minimal realizations from input-output description, there are some other methods which are based upon the reduction of sub-optimal realizations to minimal ones, e.g. [44],[97],[128].

During recent years, the minimal partial realization problem of multivariable linear constant dynamical system when only finite input-output data is available, has been studied by Kalman[67],Tether[143], Ackermann[1], and Dickinson et al.[36].

Recently minimal reciprocal realization from symmetric transfer-function matrix and impulse response matrix have been obtained by Lal and Singh[89],Lal et al. [83], Puri and Takeda[116]. Essentially, the methods[89], [116] are the modifications of Ho-Kalman algorithm[56]. These realizations are important in passive network synthesis as they result in reciprocal networks.

It is worthwhile to mention that the discrete case is analogous to the continuous one and the methods of continuous time solutions are equally applicable to the theory of discrete time minimum realizations.

In the next section the minimal realization algorithm due to Ho-Kalman[56] is given.

2.3.2 The Ho and Kalman Algorithm

Here we discuss briefly the well-known Ho-Kalman algorithm[56] to obtain a minimal realization of a linear constant dynamical system from its input-output specifications given in the form of Markov-parameters[42]. The realization problem can be stated as follows:

"Given a sequence of qxp constant matrices, Y_k (Markov parameters), k=0,1,2,..., find a triple $\{A, B, C\}$ of constant matrices such that

$$Y_k = C\, A^k\, B \qquad k=0,1,2,\ldots \text{ ''}. \qquad \ldots (2.6)$$

The sequence Y_k has a finite dimensional realization if and only if there is an integer r and constants $\nu_o, \nu_1, \ldots, \nu_{r-1}$ such that

$$Y_{r+j} = \sum_{i=1}^{r} \nu_i \, Y_{r+j-i} \qquad \text{for all } j \geqslant 0 \qquad \ldots (2.7)$$

where the degree r of the annihilating polynomial[42] of A_{min} is assumed to be known. Later, a method to determine r is also given. Now, we describe the procedure of constructing a minimal realization.

The algorithm begins by generating the rxr block matrix (Generalized Hankel matrix) built out of the

Markov parameters,

$$S_r = \begin{bmatrix} Y_o & Y_1 & \cdots & Y_{r-1} \\ Y_1 & Y_2 & \cdots & Y_r \\ \vdots & \vdots & & \vdots \\ Y_{r-1} & Y_r & \cdots & Y_{2r-2} \end{bmatrix} = \begin{bmatrix} Y_{i+j-2} \end{bmatrix} \qquad \ldots (2.8)$$

If Y_k has a finite dimensional realization, then

$$n_o = \text{rank } S_r .$$

The following steps yield an irreducible realization.

Step I Generate the matrix S_r.

Step II Find non-singular matrices P and Q such that

$$P\, S_r\, Q = \begin{bmatrix} I_n & \vdots & 0 \\ \cdots & \vdots & \cdots \\ 0 & \vdots & 0 \end{bmatrix} = J \qquad \ldots (2.9)$$

where I_n is an nxn unit matrix, n = rank S_r, and J is an idempotent.

Step III Let E_q be the block matrix $[\, I_q \; 0_q \; , \ldots , \; 0_q \,]$ and let ulh denote the operator which picks out upper left-hand block. Then a minimal realization of Y_k is given by

$$A = \text{ulh}[\, J\, P\, (\tau\, S_r)\, Q\, J \,] , \qquad \ldots (2.10a)$$

$$B = \text{ulh}[\, J\, P\, S_r\, E_p' \,] , \qquad \ldots (2.10b)$$

and $$C = \text{ulh}[\, E_q\, S_r\, Q\, J \,] , \qquad \ldots (2.10c)$$

where τ is a constant and

$$\tau S_r = \begin{vmatrix} Y_1 & Y_2 & \cdots & Y_r \\ Y_2 & Y_3 & \cdots & Y_{r+1} \\ \vdots & \vdots & & \vdots \\ Y_r & Y_{r+1} & \cdots & Y_{2r-1} \end{vmatrix} \qquad \ldots (2.11)$$

The procedure described above makes only one assumption, namely a knowledge of the integer r. In order to determine r, it is given the values 1,2,...,etc. . For each value of r, the rank of S_r is determined. That value of r is chosen, when

$$\text{rank } S_r = \text{rank } S_{r+1} \text{ .}$$

In the next section, the state-space interpretation of well-known network properties and classical synthesis procedures is briefly reviewed.

2.3.3 State-Space Interpretation

As mentioned earlier, the state-variable techniques have recently emerged as a powerful tool in the field of modern network and control theory. However, the importance of frequency domain methods can not be disparaged because of their applications in the majority of design problems of linear, time-invariant dynamical systems and networks. Therefore, it is quite important to establish communication links between the state-variable characterization and the input-output description of networks. Some endeavours have already been made in this direction. The state-space interpretation of the common terms such as poles and zeros etc. has been given by Brocket[20]. Kuh[74] also derived the similar expressions for the poles and zeros by signal flow graph representation of the state-space description of linear systems. Techniques for evaluating the poles and zeros of a scalar transfer-function from the state-equations of the systems were developed by Sandberg and So[133].Further, Anderson and Brocket[9] gave a state-space interpretation of multiport Darlington synthesis. The positive realness of a matrix [5], and determination of an impedance function from its even[88],[113], and odd[114] parts have also been investigated from state-space point of view. Recently, Lal and Singh[87] have derived some well-known properties of LC and RC networks etc. in state-space terms and have also given the state-space interpretation of classical Foster and Cauer synthesis procedures.

The state-space interpretation of Foster synthesis method for driving point immittance function of LC network has been given by Puri and Takeda[115], and Jain[59]. Puri and Takeda[115] realize the LC Foster canonical network by finding the residues associated with the partial fraction expansion of the lossless network function to be synthesized in terms of the Markov-parameters which are deduced from a knowledge of the $\{A, b, c\}$ matrices in the state-space formulation. On the other hand, Jain[59] uses non-singular observability matrix[92] as a transformation for a canonical state-model representation of the Foster network which is then compared with a similar canonical state model written directly in terms of the coefficients of the network function. Thus the element values are determined, via state-space characterization, in terms of the coefficients of the network function to be synthesized.

The notions of passivity and reciprocity useful for state-space passive synthesis are briefly discussed next.

Passivity Criterion

The interpretation of the positive real constraint — usually viewed as a frequency domain constraint— in terms of a state-space realization of a prescribed positive real matrix was given by Anderson[5] and is generally known as passivity criterion or Anderson's system theory criterion, or Anderson's positive real lemma, which is stated as follows:

Lemma 2.1

Let $Z(s)$ be a matrix of rational functions such that $Z(\infty)$ is finite and $Z(s)$ has poles which lie in $\mathrm{Re}\ s < 0$, or are simple on $\mathrm{Re}\ s = 0$, and $\{A, B, C, D\}$ be a minimal realization of $Z(s)$. Then $Z(s)$ is positive real if and only if there exists a symmetric positive definite matrix P and real matrices W_o and L such that

$$\left.\begin{aligned} PA + A'P &= -L'L \quad , \\ PB &= C' - L' W_o \quad , \\ W_o' W_o &= D + D' \quad , \end{aligned}\right\} \qquad \ldots (2.12)$$

and there exists a matrix W(s), unique to within left multiplication by a constant orthogonal matrix such that

$$Z(s) + Z'(-s) = W'(-s)\ W(s). \qquad \ldots (2.13)$$

W(s) is found by using a lemma on spectral factorization, due to Youla[10].

Reciprocity Criterion

The following theorem due to Yarlagadda[156] concerning reciprocity is stated below.

THEOREM 2.1 Let Z(s) be an pxp matrix of real rational transfer-functions with $Z(\infty)$ finite, and let Z(s) possesses a state-model of the form

$$\begin{aligned} \dot{X} &= A\,X + B\,U \,, \\ Y &= C\,X + D\,U \,, \end{aligned} \qquad \ldots (2.14)$$

such that

$$(I \dot{+} \Sigma)\, M_1 = M_1'(I \dot{+} \Sigma) \qquad \ldots (2.15)$$

where I is an unit matrix, Σ is an unique diagonal matrix of ± 1's, $\dot{+}$ denotes direct sum, and

$$M_1 = \begin{bmatrix} D & C \\ B & A \end{bmatrix}$$

if and only if

$$Z(s) = Z'(-s)$$

It may be noted that it is rather difficult to satisfy both passivity and reciprocity conditions simultaneously. However, it has been shown in [75] that all reciprocal realizations for RL and RC impedance matrices are passive.

2.3.4 General Passive Network Synthesis

In this section, we review an interesting approach to passive network synthesis using state-variable technique based on the reactance extraction principle. The concept of reactance extraction method was first introduced by Youla and Tissi [160] and extended further in [8], [10], [35], [108].

Consider the synthesis of p-port passive network from its multiport description, say an admittance matrix Y(s). Y(s) being the short-circuit admittance matrix of a passive network will be a positive real one. Further, we can assume Y(s) to be regular at $s = \infty$. If it is not so, we can split the given positive real Y(s) matrix which can have at most one pole at $s = \infty$, as

$$Y(s) = s\,[\underset{\sim}{C}] + Y_1(s) \qquad \ldots (2.16)$$

where $[\underset{\sim}{C}]$ is non-negative definite symmetric matrix, and $Y_1(s)$ is rational positive real with $Y_1(\infty) < \infty$. Now

Y(s) can be realized as the parallel connection of transformer-coupled capacitors and a network having an admittance matrix $Y_1(s)$. Thus the assumption that Y(s) is regular at infinity will involve no loss in generality. Now we can obtain a minimal realization $\{A, B, C, D\}$ associated with Y(s) using Ho-Kalman algorithm such that

$$Y(s) = D + C(sI-A)^{-1}B \qquad \ldots (2.17)$$

and A has the smallest dimension equal to the degree of the matrix Y(s). Since the realization is minimal, the number of reactive elements needed for any network realizing Y(s), and also the number of integrators needed for analog computer simulation of Y(s), will be minimum.

Let us assume that the multiport network realizing Y(s) matrix contains capacitors (C), resistors (R), ideal transformers (T), gyrators(Γ), and inductors (L). Since an inductor can always be replaced by a gyrator loaded by a capacitor, we can assume without loss in generality that there will not be any inductor in the network realization. Also, as the value of a capactor can be increased or decreased by terminating it in an ideal transformer, it can be assumed that all the capacitors are of unit magnitude. Further, we know that a minimum of n reactive elements $\{$where $n = \delta[Y(s)]\}$ is needed for any realization for Y(s) and n dynamic elements are enough if there is no degeneration in the network. Hence we can assume that there will be n capacitors in the network realizing Y(s). Thus, it is concluded that, if a realization is possible for Y(s) using R, C, T, Γ and L, then yet another realization for Y(s) can be found using R, C, T and Γ only with the minimum possible capacitors (n) of unit value.

Now we can consider the network N realizing Y(s) to consist of a (p+n) port network N_r of resistors, transformers and gyrators, loaded by another n-port subnetwork N_c of n capacitors as shown in Fig.2.1. The network N_r is frequency independent or non-dynamic and its (p+n) port admittance matrix $\bar{G}$ is a positive real matrix with all entries real.

If we partition $\bar{G}$ with respect to the first p-ports, we have,

$$\bar{G} = \begin{bmatrix} \bar{G}_{11} & \bar{G}_{12} \\ \bar{G}_{21} & \bar{G}_{22} \end{bmatrix} .$$

The p-port admittance matrix Y(s) of N in terms of the sub-matrices of $\bar{G}$ is given by

$$Y(s) = \bar{G}_{11} - \bar{G}_{12}(sI_n + \bar{G}_{22})^{-1} \bar{G}_{21} \qquad \ldots (2.18)$$

Comparing (2.18) with (2.17), it is clear that any quadruple $\{A, B, C, D\}$ realizing Y(s) can be identified with the matrices $\{-\bar{G}_{22}, -\bar{G}_{21}, \bar{G}_{12}, \bar{G}_{11}\}$. Further, as we noted earlier, given any minimal realization $\{A, B, C, D\}$, we can get innumerable equivalent realizations given by $\{T_o^{-1}A\ T_o,\ T_o^{-1}B,\ C\ T_o,\ D\}$ where T_o is a non-singular arbitrary matrix. From this we can easily infer that a realization for Y(s) is possible if we are able to obtain a

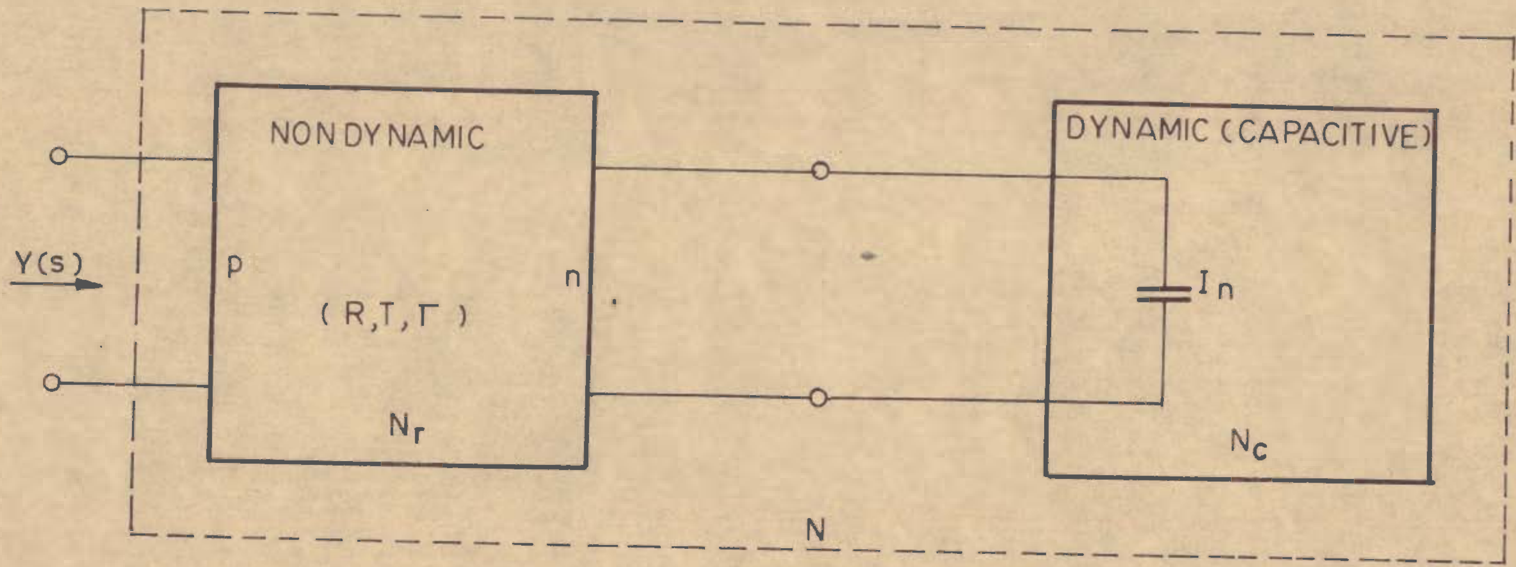

FIG. 2.1- A REALIZATION SCHEME FOR Y(s) USING n-CAPACITORS.

quadruple $\{T_o^{-1} A\ T_o,\ T_o^{-1} B,\ C\ T_o,\ D\}$ which when identified with the corresponding submatrices of $\overline{G}$, gives a positive real matrix $\overline{G}$. It has been shown [8] that once a decomposition $\{A, B, C, D\}$ is obtained for a given positive real matrix Y(s), a non-singular matrix T_o can be found out so that the matrix

$$\overline{G} = \begin{bmatrix} D & C\ T_o \\ -T_o^{-1} B & -T_o^{-1} A\ T_o \end{bmatrix}$$

is positive real.

Once $\overline{G}$ is obtained, the network realizing Y(s) is obtained by realizing $\overline{G}$ with a (p+n) port network N_r of resistors, gyrators, and ideal transformers, and then loading the last n-ports of N_r by capacitors of unit value.

2.3.5 Multiport Active RC Network Synthesis

Active RC network synthesis has experienced a tremendous growth in the last two decades for essentially two reasons. First, active RC networks are particularly suitable for low frequency applications where inductors and crystals are not satisfactory. Secondly, the availability of small, reliable, precise resistors, capacitors and transistors make these networks attractive for micro-circuit applications. In the following, the historical development of some active RC multiport network synthesis procedures is briefly discussed.

Synthesis of active RC networks from their multiport description, i.e., from a given matrix of real rational functions of the complex frequency variable s, has attracted the attention of many research workers since 1961, when Sandberg[131] proposed a method to realize a p-port active RC network from its short-circuit admittance matrix Y(s) description. He established the fact that a realization using p-controlled sources is possible. Subsequently, Sandberg[132], also showed that a realization using p negative converters (NIC) is possible. Since then many papers have appeared in the technical literature. Barranger[15] has given a method to realize a given rational function matrix as the voltage transfer function matrix of an active RC network using current negative immittance converters. Hazony and Joseph [52] suggested procedures to synthesize transfer matrices of active RC networks. Joseph and Hilberman[61] have dealt with the synthesis of immittance matrices of active RC networks. Subsequently, Hilberman[54] advanced a method for the realization of active RC multiports with common ground from given rational transfer and admittance matrices, and using unity gain voltage amplifiers. Mitra[101] has dealt with the synthesis of voltage transfer function matrices using operational amplifiers. A method for the synthesis of arbitrary transfer function matrix using RC one port and operational amplifiers was given by Bhattacharya[17]. Procedures for realizing admittance matrices of active RC multiports using voltage controlled voltage sources have been given by Even[39], Goldman and Ghausi[48], and Hilberman[53]. The synthesis technique developed by Goldman and Ghausi[48] requires not more than 2 p common-ground voltage-controlled voltage sources, of which p have differential outputs and p have positive gains. The significance of this technique derives from the fact that an active subnetwork of common-ground voltage-controlled voltage sources is easily implemented. A bound on the number of operational amplifiers required to synthesize active RC networks from the voltage transfer matrix description was given by Kim and Su[72]. Recently, Yarlagadda and Ye[158], Ramamoorthy et al.[122] gave methods to realize short-circuit admittance matrices by active RC multiports using nullator-norator pairs with a common end as the active elements.

All the above papers with the exception of [53] and [158] either (a) deal with a restricted class of rational functions or (b) use active elements that are not readily available, or (c) require the use of excessive number of capacitors with possibly some of them floating.

Mann and Pike[96] have shown that, by using the state-space techniques and the reactance extraction principle, it is possible to realize active RC networks using a minimum number of capacitors having their one end common and grounded, a desirable feature for integrated circuit fabrication. Subsequently, Melvin and Bickart[98] elaborated this approach and gave an interesting method to realize active RC network from its short-circuit admittance matrix description. Later, they [18] extended their results to the synthesis of multiport active RC networks from other types of descriptions such as voltage gain matrix, current gain matrix, impedance matrix etc. Based on the approach of [98], a synthesis procedure to realize a voltage transfer matrix using operational amplifiers was suggested by Ramamoorthy et al.[123]. Huang[57] also gave a method to realize a transfer-admittance matrix of a p-port active RC network using a Hamiltonian state-space model. Recently, Lal and Khan[81] proposed a synthesis procedure, for a short-circuit admittance matrix Y(s) when Y(∞) is either hyperdominant or has all non-negative entries, which reduces the upper bound on the number of active elements and can be reasonably expected to require fewer resistors while retaining all the advantages of [98].

2.4 CONCLUSION

It is obvious that there is plenty of literature available on state-variable realization techniques. The available information or input-output data, from which an irreducible realization is to be derived, may be in the form of a rational transfer-function matrix, impulse response matrix, Markov parameters, or moments of the impulse response. Recently, there have been several attempts to find minimal reciprocal realization from the given symmetric transfer-function matrix and impulse response matrix. The existing methods need further modifications, which may improve the computational requirements. Although a fairly complete theory of realization of linear dynamical systems is available, state-space synthesis procedures for active and passive networks are still being developed.

The following chapters deal with some new techniques of minimal reciprocal realization of linear time-invariant dynamical systems, state-space interpretation of some classical synthesis methods, and development of new synthesis procedures for multiport active RC and passive reciprocal networks with a minimum number of capacitors.

CHAPTER-III

MINIMAL RECIPROCAL REALIZATION OF LINEAR TIME-INVARIANT DYNAMICAL SYSTEMS

3.1 INTRODUCTION

In the past decade, there has been considerable interest in the problem of computing minimal (or irreducible) realizations of real finite dimensional linear time-invariant dynamical systems from their input-output specifications in the form of either rational transfer-function matrices, or impulse response matrices. This problem, being one of the basic problems in linear system theory, was first introduced by Gilbert [43] and Kalman [63] in 1963, and it is still an interesting area of research both for theoretical implications and for the role the state-space representation plays in the development of unitary and efficient algorithms for analysis and synthesis purposes [137]. Specially, such a realization is useful in analog computer simulation [28], operational amplifier circuit synthesis [107], filtering and system identification [66]. Another important advantage of the realization theory is that it provides a better insight into the relationship between input-output and the state-models of the dynamical systems. In this chapter, some new and simplified algorithms have been developed for obtaining minimal reciprocal realizations from a given symmetric transfer-function matrix and impulse response matrix. The attractive feature of the proposed methods is that computations are considerably reduced, as they require Hankel matrices of smaller order.

3.2 MINIMAL RECIPROCAL REALIZATION FROM A GIVEN SYMMETRIC TRANSFER FUNCTION MATRIX

The minimal realization problem, as mentioned earlier, has been extensively considered in recent years. Various techniques have been evolved [4], [21], [28], [29], [43]-[44], [46], [56], [63], [126], [137], [159], [160], for determining minimal realization $\{A, B, C\}$ such that

$$C \exp(At) B = G(t) \qquad \ldots (3.1a)$$

$$C[sI - A]^{-1}B = G(s) \qquad \ldots (3.1b)$$

where G(t) is the impulse response matrix of a linear time-invariant finite dimensional strictly proper system, and G(s) is its Laplace transform.

Out of the above methods for determining minimal realization, those based on Hankel matrices are most suitable for computerization [56], [137], [160]. However, the order of the Hankel matrices required in these methods, are sometimes unnecessarily large. Recently, Chen and Mital [29] proposed a simplification of the methods based on Hankel matrices. Compared with the existing methods, their algorithm [29] uses Hankel matrices of smaller order and thus the computing time and memory storage required are reduced significantly.

Chen and Mittal determine a minimal realization $\{A, B, C, D\}$ from a qxp rational transfer-function matrix G(s) by first determining the matrices Ω and $\bar{\Omega}$ each of order $\alpha \times \beta$, defined as

$$\Omega \triangleq \begin{bmatrix} H_{11}(\alpha_1,\beta_1) & H_{12}(\alpha_1,\beta_2) & \cdots\cdots & H_{1p}(\alpha_1,\beta_p) \\ H_{21}(\alpha_2,\beta_1) & H_{22}(\alpha_2,\beta_2) & \cdots\cdots & H_{2p}(\alpha_2,\beta_p) \\ \vdots & \vdots & & \vdots \\ H_{q1}(\alpha_q,\beta_1) & H_{q2}(\alpha_q,\beta_2) & \cdots\cdots & H_{qp}(\alpha_q,\beta_p) \end{bmatrix} \quad \ldots (3.2)$$

and

$$\bar{\Omega} \triangleq \begin{bmatrix} \bar{H}_{11}(\alpha_1,\beta_1) & \bar{H}_{12}(\alpha_1,\beta_2) & \cdots\cdots & \bar{H}_{1p}(\alpha_1,\beta_p) \\ \bar{H}_{21}(\alpha_2,\beta_1) & \bar{H}_{22}(\alpha_2,\beta_2) & \cdots\cdots & \bar{H}_{2p}(\alpha_2,\beta_p) \\ \vdots & \vdots & & \vdots \\ \bar{H}_{q1}(\alpha_q,\beta_1) & \bar{H}_{q2}(\alpha_q,\beta_2) & \cdots\cdots & \bar{H}_{qp}(\alpha_q,\beta_p) \end{bmatrix} \quad \ldots (3.3)$$

where $H_{ij}(\alpha_i,\beta_j)$ is the Hankel matrix constructed from Markov parameters of $g_{ij}(s)$; $G(s) = [g_{ij}(s)]$; α_i denotes the degree of the least common denominator of the ith row of G(s); β_j denotes the degree of the least common denominator of the jth column of G(s), and

$$\alpha = \sum_{i=1}^{q} \alpha_i \qquad \text{and} \qquad \beta = \sum_{j=1}^{p} \beta_j \qquad \ldots (3.4)$$

$\bar{\Omega}$ is obtained from $H_{ij}(\alpha_i, \beta_j+1)$ by deleting the first column. Then, according to the algorithm [29], an irreducible realization of G(s) is given by

$$\left.\begin{aligned} A &= P_n \, \bar{\Omega} \, Q_n \\ B &= P_n \, W \\ C &= Z \, Q_n \\ D &= G(\infty) \end{aligned}\right\} \qquad \ldots (3.5)$$

where P_n is the matrix consisting of the first n rows of the P matrix and Q_n is the matrix consisting of the first n columns of the Q matrix, n being the rank of the Ω matrix.

The non-singular matrices P and Q are obtained by using elementary transformations on matrix Ω such that

$$P \, \Omega \, Q = \begin{bmatrix} I_n & \vdots & \Lambda \\ \cdots & \cdots & \cdots \\ 0 & \vdots & 0 \end{bmatrix} \qquad \ldots (3.6)$$

where

I_n an nxn unit matrix

Λ an $n x(\beta - n)$ arbitrary matrix

$W \triangleq \Omega$ (by setting $\beta_i = 1$, $i = 1,2,\ldots,p$) ... (3.7a)

$Z \triangleq \Omega$ (by setting $\alpha_j = 1$, $j = 1,2,\ldots,q$) ... (3.7b)

The order of the matrices W and Z are $\alpha x p$ and $q x \beta$ respectively.

Determination of P and Q for a particular Ω satisfying eqn.(3.6) is a well-known problem in matrix algebra. For a given Ω, there can result innumerous P's and Q's such that eqn.(3.6) is satisfied. Each set of P and Q will give a different realization $\{A, B, C\}$.

In many problems, the given transfer-function matrix G(s) is symmetric and we are interested in finding a realization $\{A, B, C, D\}$ such that

$$(I \dotplus \Sigma) M_1 \text{ is symmetric.} \qquad \ldots (3.8)$$

where Σ_1 is the number of +1's on the diagonal matrix,

Σ_2 is the number of -1's on the diagonal matrix, and

$\dot{+}$ denotes direct sum.

The importance of such realization is due to the fact that they result in reciprocal networks and further it has been proved in [75], [160] that all reciprocal realizations for RC and RL cases are passive. Hence, in the following, by exploiting the symmetry of the given rational matrix, the algorithm of Chen and Mital [29] is modified in order to determine such P and Q as further result in {A, B, C, D} satisfying (3.8).

Since the given transfer-function matrix G(s) is symmetric having order pxp, the matrices Ω and $\bar{\Omega}$ each of order $\alpha x \alpha$ obtained by using (3.2) and (3.3) will obviously be symmetric, and

$$\alpha = \sum_{i=1}^{p} \alpha_i = \sum_{j=1}^{p} \beta_j \qquad \text{for symmetric } G(s) \qquad \ldots (3.9)$$

where α_i denotes the degree of the least common denominator of the ith row or column of G(s).

Therefore, a non-singular transformation P can always be found by the well-known technique in matrix algebra [32], such that

$$P \ \Omega P' = (\Sigma \dot{+} 0) \ , \qquad \ldots (3.10)$$

the order of Σ being equal to the rank of Ω matrix and prime denotes matrix transposition.

Multiplying both sides of (3.10) by ($\sum$ + 0), we get

$$P \Omega P' \sum = (I_n \dot{+} 0) = J \qquad \ldots (3.11)$$

where J is an idempotent.

Therefore, from (3.5), (3.6) and (3.11), the minimal realization for symmetric transfer-function matrix G(s) becomes

$$\begin{aligned} A &= P_n \bar{\Omega} P_n' \sum \\ B &= P_n W \\ C &= W' P_n' \sum \\ D &= G(\infty) \end{aligned} \qquad \ldots (3.12)$$

It is obvious that the realization so obtained will satisfy (3.8), and hence will result in reciprocal networks. The versatility of the procedure is demonstrated with the help of examples.

Example. 3.1, [128]

Consider a symmetric transfer-function matrix

$$G(s) = \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \\ \frac{1}{s^2} & \frac{1}{s^3} \end{bmatrix} .$$

Obviously,

$$D = G(\infty) = \begin{bmatrix} 0 \end{bmatrix}.$$

Here, $\alpha_1 = 2$, $\alpha_2 = 3$ and so $\alpha = 5$,

and thus Ω is of order 5x5 and is given by,

$$\Omega = \left[\begin{array}{cc:ccc} 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \hdashline 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right],$$

having rank n as 3.

Since Ω is symmetric, it can always be decomposed in the form (3.10). For the present example, we can get

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 & 1/2 \\ 0 & 1/2 & 1/2 & 0 & -3/2 \\ 0 & -1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\sum = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix},$$

and

$$J = \left[\begin{array}{ccc:c} 1 & & & \\ & 1 & & 0 \\ & & 1 & \\ \hdashline & 0 & & 0 \end{array}\right].$$

Further

$$P_n = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 & 1/2 \\ 0 & 1/2 & 1/2 & 0 & -3/2 \end{bmatrix},$$

$$W = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

and

$$\bar{\Omega} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} .$$

Using (3.12), we get

$$A = \begin{bmatrix} 0 & 1/2 & -1/2 \\ 1/2 & 0 & 0 \\ 1/2 & 0 & 0 \end{bmatrix} ,$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} ,$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} ,$$

and

$$D = \begin{bmatrix} 0 \end{bmatrix} .$$

Therefore, we get

$$\begin{bmatrix} D & C \\ B & A \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1/2 & -1/2 \\ 0 & 1 & 1/2 & 0 & 0 \\ 0 & -1 & 1/2 & 0 & 0 \end{bmatrix}$$

It can be seen that (3.8) is satisfied.

In the example considered above, the order of the matrices Ω and $\bar{\Omega}$ is 5x5 as against 6x6 required in the technique given in [56] or [89].

Example 3.2

Given a symmetric matrix G(s) of order 3x3 as

$$G(s) = \begin{bmatrix} \frac{2s+3}{s+1} & \frac{s}{s+1} & \frac{s+2}{s+1} \\ \frac{s}{s+1} & \frac{3s+4}{s+1} & \frac{s+1/2}{s+1} \\ \frac{s+2}{s+1} & \frac{s+1/2}{s+1} & \frac{2s+3}{s+1} \end{bmatrix} .$$

Obviously,

$$D = G(\infty) = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix} .$$

Therefore,

$$G_1(s) = G(s) - G(\infty)$$

$$= \begin{bmatrix} \frac{1}{s+1} & \frac{-1}{s+1} & \frac{1}{s+1} \\ \frac{-1}{s+1} & \frac{1}{s+1} & \frac{-1/2}{s+1} \\ \frac{1}{s+1} & \frac{-1/2}{s+1} & \frac{1}{s+1} \end{bmatrix} .$$

Here, $\alpha_1 = \beta_1 = 1$

$\alpha_2 = \beta_2 = 1$

$\alpha_3 = \beta_3 = 1$

Therefore, $\alpha = \beta = 3$

and thus matrices Ω and $\bar{\Omega}$ of order 3x3 are given by

$$\Omega = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1/2 \\ 1 & -1/2 & 1 \end{bmatrix}$$

and

$$\bar{\Omega} = \begin{bmatrix} -1 & 1 & -1 \\ 1 & -1 & 1/2 \\ -1 & 1/2 & -1 \end{bmatrix} .$$

Ω having rank n as 3.

Since Ω is symmetric, it can always be decomposed in the form (3.10). For the present example, one can get

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & -1 \end{bmatrix} ,$$

$$\Sigma = \begin{bmatrix} 1 & & \\ & 1 & \\ & & -1 \end{bmatrix} .$$

Obtaining P_n from P, and W from Ω simply as stated in the procedure, and substituting in (3.12), we get

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} ,$$

$$B = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1/2 & 1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix} ,$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1/2 & 1/2 \\ 1 & 1/2 & -1/2 \end{bmatrix} ,$$

$$D = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix} .$$

Therefore,

$$\begin{bmatrix} D & C \\ B & A \end{bmatrix} = \left[\begin{array}{ccc:ccc} 2 & 1 & 1 & -1 & 0 & 0 \\ 1 & 3 & 1 & -1 & 1/2 & 1/2 \\ 1 & 1 & 2 & 1 & 1/2 & -1/2 \\ \hdashline 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 & -1 & 0 \\ 0 & -1/2 & 1/2 & 0 & 0 & -1 \end{array}\right] .$$

It can be seen that (3.8) is satisfied.

It may be noted that for the above example, the order of the Hankel matrix required in [56] or [89] is 9x9, while the order of the matrices Ω and $\tilde{\Omega}$ used here is 3x3 only. Since the memory required and the number of operations depend upon the number of entries in the matrices, the proposed method will significantly reduce the computing time and memory storage required for many examples. Further, additional labour of finding inverse of matrices as required in [160, Eqn.I-29] is avoided.

In the next section, minimal reciprocal realization from a given symmetric impulse response matrix using moments is considered.

3.3 MINIMAL RECIPROCAL REALIZATION FROM A GIVEN SYMMETRIC IMPULSE RESPONSE MATRIX USING MOMENTS

In recent years, some interest has been generated in constructing a minimal realization from a given symmetric rational matrix [89],[116]. Such a realization was obtained in [89] by modifying the technique given by Ho-Kalman and was required to determine Markov parameters as a first step of the algorithm. Puri and Takeda[116] obtain such a realization by exploiting the method given by Bruni et al. [21], and thus used moments of the impulse response matrix instead of Markov-parameters for the purpose because of their preference in the presence of noise, as discussed in [21]. Both [89] and [116] require determining the Hankel matrix, the former from Markov-parameters and the latter from moments.

Recently it has been shown by Chen and Mital[29] that the order of the Hankel matrix required in [56] can be reduced considerably, and thus the computing time and memory storage required are reduced. Based on their algorithm, a method for determining the minimal reciprocal realization from a given symmetric transfer-function matrix has been developed in the preceding section. In the following, a simplified algorithm for computing an irreducible reciprocal realization from a given symmetric impulse response matrix, G(t), using moments is presented.

The procedure for the minimal realization of a given symmetric transfer-function matrix G(s) discussed in [116] requires determining moments by first expanding G(s) in a positive power series according to

$$G(s) = \sum_{k=o}^{\infty} C_k \, s^k \; . \qquad \ldots (3.13)$$

This series converges in a suitable neighbourhood of the origin, and it can be analytically continued on the whole plane except for the singularities of G(s)[21]. Consequently, the sequence $\{C_k\}$ identifies uniquely the G(s). Each C_k is uniquely connected to the corresponding moments of the impulse response matrix $\{G(t) = [g_{ij}(t)]\}$ by the relation,[21],

$$M_k = (-1)^k \, k! \, C_k \qquad \ldots (3.14)$$

where

$$M_k = \int_o^{\infty} t^k \{g_{ij}(t)\} \, dt, \quad k=0,1,2,\ldots, \; . \qquad \ldots (3.15)$$

Since G(t) is symmetric, its moments are also symmetric as is clear from (3.14) and (3.15). Then the Hankel matrix Ω constructed from the moments will also be symmetric. Therefore, a nonsingular transformation matrix P can again be found such that (3.10) is satisfied. The procedure of the previous Section (3.2) can then be applied without any modification to the Hankel matrix constructed from M_k^* in place of Markov parameters, where [21]

$$M_k^* = \frac{(-1)^k}{(k-1)!} M_{k-1} \; , \qquad k = 0,1,2,\ldots \qquad \ldots (3.16)$$

and

$$M_o^* = [\, G(t) \,]_{t=o} = \lim_{t \to \infty} s \, G(s) \qquad \ldots (3.17)$$

The quantities thus introduced are connected with the matrices A, B, C according to the relation

$$M_k^* = C\,A^{-k}\,B, \qquad k = 0,1,2,\ldots . \qquad \ldots (3.18)$$

Then, in light of the methods given in [21], [82] and [116], the minimal realization from a symmetric impulse response matrix becomes, [83],

$$\begin{aligned} A^{-1} &= P_n \bar{\Omega}\, P_n' \Sigma \\ B &= P_n\, W \\ C &= W' P_n' \Sigma \\ D &= G(\infty) \end{aligned} \qquad \ldots (3.19)$$

where P_n is the matrix consisting of the first n rows of the P matrix, n being the rank of the Ω matrix.

The steps for the proposed algorithm are as follows:

I. Calculate M_k^* for the given G(t) using (3.14) to (3.17).

II. Determine α_i and β_j, as defined in (3.9).

III. Construct symmetric matrices Ω and $\bar{\Omega}$ of each of order $\alpha \times \alpha$ from M_k^*, as given by (3.2) and (3.3).

IV. Decompose Ω in the form (3.10) as discussed in the preceding section.

V. Obtain P_n from P, and W from Ω using (3.7a).

VI. Determine {A, B, C} using (3.19).

It is obvious that this realization also satisfies (3.8). At the end of the calculations, it is necessary, of course, to invert the matrix A^{-1} to obtain A. For reciprocal RC and RL networks, G(t) will be asymptotically stable, and from [21], A is then non-singular.

The following example illustrates the procedure.

Example 3.3

Consider the asymptotically stable impulse response matrix

$$G(t) = \begin{bmatrix} e^{-t} & e^{-t} \\ e^{-t} & te^{-t} \end{bmatrix}.$$

(i) M_k^* for the entries of first row and first column of G(t) are

$$M_0^* = 1,\ M_1^* = -1,\ M_2^* = 1,\ M_3^* = -1,\ M_4^* = 1,\ M_5^* = -1, \ldots$$

and for the entry te^{-t} of G(t),

$$M_0^* = 0,\ M_1^* = -1,\ M_2^* = 2,\ M_3^* = -3,\ M_4^* = 4,\ M_5^* = -5, \ldots$$

(ii) A procedure for determining α_i and β_j directly from the impulse response matrix is suggested below-

Construct a mode matrix M of G(t) as

$$M = \begin{bmatrix} 1 & 1 \\ 1 & (1)^2 \end{bmatrix}.$$

It may be noted that the entries in the mode matrix M correspond to the distinct poles of G(s) and their multiplicities; the entries such as $(1)^2$ etc. correspond to $t\ e^{-t}$ or a double pole at s = 1. Reduce M to M_r by row combination or M_c by column combination [4].

Here $M_r = M_c = \begin{bmatrix} 1 & - \\ - & (1)^2 \end{bmatrix}$ (since for symmetric M, $M_r = M_c$)

giving $\alpha_1 = \beta_1 = 1$ and $\alpha_2 = \beta_2 = 2$; thus $\alpha = \beta = 3$.

(iii) Symmetric matrices Ω and $\bar{\Omega}$ of order 3x3 constructed from M_k^* are given by

$$\Omega = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 0 & -1 \\ -1 & -1 & 2 \end{bmatrix}, \text{ and } \bar{\Omega} = \begin{bmatrix} -1 & -1 & 1 \\ -1 & -1 & 2 \\ 1 & 2 & -3 \end{bmatrix}.$$

Ω having rank n as 3.

(iv) For the present example, we can get

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}, \text{ and } \sum = \begin{bmatrix} 1 & & \\ & 1 & \\ & & -1 \end{bmatrix}.$$

(v) $P_n = P$ (in this case), and

$$W' = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}.$$

(vi) From (3.19), we get the minimal realization

$$A^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & -1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \text{ and } D = \begin{bmatrix} 0 \end{bmatrix}.$$

As discussed in [21] and [116], it is necessary, of course, to invert the matrix A^{-1} to obtain A, at the end of the calculations. Thus, with

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & -2 \end{bmatrix}$$

it can be seen that

$$\begin{bmatrix} I & + & \Sigma \end{bmatrix} \begin{bmatrix} D & C \\ B & A \end{bmatrix}$$

is symmetric as discussed in [83], [89], [116].

In the above example, the order of the matrices Ω and $\bar{\Omega}$ is 3x3 as against 4x4 required in the technique given in [116] or [21]. Obviously the computing time and memory storage required will be reduced significantly for many examples as discussed earlier.

3.4 CONCLUDING REMARKS

Two simplified algorithms have been developed, in this chapter, for obtaining minimal reciprocal realization from a given symmetric transfer-function matrix and impulse response matrix, based on the approach of Chen and Mital [29]. The proposed methods require Hankel matrices of smaller order than the one used in [89], and [116], and thus the computing time and memory storage required are significantly reduced. The algorithm presented in Section 3.3 is based on the computation of moments of the impulse response matrix instead of Markov-parameters used in Section (3.2). Both the algorithms yield minimal realizations which satisfy the reciprocity criterion (2.15) due to Yarlagadda[156]. These results will be utilized in evolving a passive reciprocal multiport synthesis of SPR immittance matrices using RCT network in Chapter V.

However, before discussing the multiport active RC network synthesis procedure, the state-space interpretation of some classical synthesis methods is presented in the next chapter.

CHAPTER IV

STATE - SPACE INTERPRETATION

4.1 INTRODUCTION

In the past few years, it has been recognised by several network theorists that an elegant approach to network analysis and synthesis is by means of state-models, as the network state-model provides more direct information about the network topology than the conventional network matrices. Consequently, there is an increasing emphasis on network synthesis via state-space techniques in the current literature [8], [10], [11], [75]-[76], [85], [108], [135]-[136], [139], [147], [156]-[158]. Since the frequency domain methods are still being used for the majority of design problems, it is quite interesting and useful to establish some communication link between state-variable characterization and the specifications in s-domain. Several authors have put in endeavours in this direction [5], [9], [20], [59], [70], [74], [87], [88], [115].

The present chapter discusses the state-space interpretation of classical Foster and Cauer synthesis procedures. In particular, the interpretation of Foster p-port LC, 1-port RC and RL synthesis, Cauer driving point synthesis, and the well-known coefficient-matching technique of second order active RC filter design, is done via state-space characterization.

4.2 STATE-SPACE INTERPRETATION OF FOSTER SYNTHESIS METHOD

Recently, an interesting procedure to realize Foster 1-port LC network in state-space terms was given by Puri and Takeda[115]. In this section, the state-space interpretation of Foster 1-port RC and RL networks, and n port LC network is given by exploiting the technique of [115].

Let the impedance matrix Z(s) has a state-space representation

$$\dot{X} = A X + B U \qquad \text{... (4.1a)}$$

$$Y = C X + D U \qquad \text{... (4.1b)}$$

such that

$$\left.\begin{aligned} Z(s) &= C(sI-A)^{-1}B + D \\ &= Z_1(s) + Z(\infty) \end{aligned}\right\} \qquad \text{... (4.2)}$$

where X is an n-dimensional state-vector, U is p-dimensional input-vector, Y is q-dimensional output vector, and A, B, C, D are real constant matrices of dimensions nxn, nxp, qxn, and qxp respectively.

In a suitable neighbourhood of infinity, $Z_1(s)$ can be expressed as

$$\left.\begin{aligned} Z_1(s) &= CB\, s^{-1} + CAB\, s^{-2} + CA^2B\, s^{-3} + \ldots + CA^{n-1}B\, s^{-n} + \ldots \\ &= Y_o\, s^{-1} + Y_1\, s^{-2} + Y_2\, s^{-3} + \ldots + Y_{n-1}\, s^{-n} + \ldots \end{aligned}\right\} \qquad \text{... (4.3)}$$

where Y_o, Y_1, Y_2 etc. are called Markov-parameters[56] and are determined by dividing the numerator polynomial of each entry of the transfer-function matrix by the common denominator.

For the scalar case, (4.1) is represented as

$$\dot{X} = A X + b u \qquad \ldots (4.4a)$$

$$y = c X + d u \qquad \ldots (4.4b)$$

such that

$$\left.\begin{aligned} z(s) &= c(sI-A)^{-1} b + d \\ &= z_1(s) + z(\infty) \end{aligned}\right\} \qquad \ldots (4.5)$$

where c and b become row and column vectors respectively, and d a scalar. Further, $z_1(s)$ can be expanded as

$$\left.\begin{aligned} z_1(s) &= cb\, s^{-1} + cAb\, s^{-2} + cA^2b\, s^{-3} + \ldots + cA^{n-1}b\, s^{-n} + \ldots \\ &= y_o s^{-1} + y_1 s^{-2} + y_2 s^{-3} + \ldots + y_{n-1} s^{-n} + \ldots \end{aligned}\right\} \qquad \ldots (4.6)$$

From the given $\{A, b, c\}$ Puri and Takeda[115] determine the realization of 1-port LC Foster form by solving a set of equations obtained by comparing y_o, y_1, y_2, etc. with similar expressions obtained by expanding $z_1(s)$ written in partial fraction form. The procedure[115] with some modification can be applied to RC and RL cases as explained below.

(i) <u>Foster 1-Port RC and RL Synthesis</u>

Consider first a proper RC driving point impedance function $z_1(s)$, $\{z(\infty) = 0$ i.e. $d = 0$; a non-zero d will result in a series resistive element$\}$

$$z_1(s) = \frac{k_o}{s} + \sum_{i=1}^{\hat{n}} \frac{k_i}{s + \sigma_i} \qquad \ldots (4.7)$$

Expanding $z_1(s)$, we get

$$z_1(s) = k_o s^{-1} + \sum_{i=1}^{\hat{n}} k_i (s^{-1} - \sigma_i s^{-2} + \sigma_i^2 s^{-3} - \sigma_i^3 s^{-4} + \ldots) \quad \ldots (4.8)$$

where k_o, k_i are positive and real constants, and $\sigma_i (i=1,2, \ldots,\hat{n})$, the poles of (4.7) are the eigen values of the characteristic equation of matrix A and

$\hat{n} = n$ if A has all non-zero eigen values.

$= n-1$ if one of the eigen values of A is zero.

Comparing (4.6) and (4.8), an infinite set of equations can be written, of which the first $(\hat{n}+1)$ equations in matrix form are

$$\begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \\ 0 & \sigma_1 & \sigma_2 & \ldots & \sigma_{\hat{n}} \\ 0 & \sigma_1^2 & \sigma_2^2 & \ldots & \sigma_{\hat{n}}^2 \\ 0 & \sigma_1^3 & \sigma_2^3 & \ldots & \sigma_{\hat{n}}^3 \\ \vdots & \vdots & & & \vdots \\ 0 & \sigma_1^{\hat{n}-1} & \sigma_2^{\hat{n}-1} & \ldots & \sigma_{\hat{n}}^{\hat{n}-1} \end{bmatrix} \begin{bmatrix} k_o \\ k_1 \\ k_2 \\ k_3 \\ \vdots \\ k_{\hat{n}} \end{bmatrix} = \begin{bmatrix} y_o \\ -y_1 \\ y_2 \\ -y_3 \\ \vdots \\ (-1)^{\hat{n}} y_{\hat{n}} \end{bmatrix} \quad \ldots (4.9)$$

The coefficient matrix of (4.9) by virtue of its nature is non-singular, and so k_o, k_1 .. etc., the residues of (4.7), can be evaluated and hence Foster canonical form of RC network can be easily drawn.

The RL driving point case can similarly be interpreted by taking a proper z(s), {an improper function having a pole at infinity will result in a series inductance} , and expanding $\frac{z(s)}{s}$, instead of z(s), such that

$$\left.\begin{aligned}\frac{z(s)}{s} &= \frac{k_o}{s} + \sum_{i=1}^{\hat{n}} \frac{k_i}{s+\sigma_i} \\ &= k_o s^{-1} + \sum_{i=1}^{\hat{n}} k_i(s^{-1} - \sigma_i s^{-2} + \sigma_i^2 s^{-3} - \sigma_i^3 s^{-4} + \ldots)\end{aligned}\right\} \quad \ldots (4.10)$$

or

$$\begin{aligned}z(s) = &(k_o+k_1+k_2+\ldots+k_{\hat{n}}) - (k_1\sigma_1+k_2\sigma_2+\ldots+k_{\hat{n}}\sigma_{\hat{n}})s^{-1} + \\ &(k_1\sigma_1^2+k_2\sigma_2^2+\ldots+k_{\hat{n}}\sigma_{\hat{n}}^2)s^{-2} - (k_1\sigma_1^3+k_2\sigma_2^3+\ldots+k_{\hat{n}}\sigma_{\hat{n}}^3)s^{-3} \\ &+ \ldots\ldots \qquad \ldots (4.11)\end{aligned}$$

where $\hat{n} = n$.

Proceeding as above, the expression corresponding to (4.9) becomes,

$$\begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \\ 0 & \sigma_1 & \sigma_2 & \ldots & \sigma_{\hat{n}} \\ 0 & \sigma_1^2 & \sigma_2^2 & \ldots & \sigma_{\hat{n}}^2 \\ \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot \\ 0 & \sigma_1^{\hat{n}} & \sigma_2^{\hat{n}} & \ldots & \sigma_{\hat{n}}^{\hat{n}} \end{bmatrix} \begin{bmatrix} k_o \\ k_1 \\ k_2 \\ \cdot \\ \cdot \\ \cdot \\ k_{\hat{n}} \end{bmatrix} = \begin{bmatrix} d \\ -y_o \\ y_1 \\ \cdot \\ \cdot \\ \cdot \\ (-1)^{\hat{n}} y_{\hat{n}-1} \end{bmatrix} \quad \ldots (4.12)$$

from which the residues k's etc. can be evaluated and thus, the Foster form of 1-port RL network can be easily constructed.

(ii) Foster p-Port LC Network Synthesis

The treatment given in the preceding for state-space interpretation of FOSTER synthesis method of 1-port networks is extended to p-port LC network as follows.

Consider a lossless positive real (PR) impedance matrix Z(s) in partial fraction form as [106]

$$\left.\begin{aligned} Z(s) &= \xi_\infty + s\,\Theta_\infty + \frac{1}{s}\Theta_0 + \sum_{i=1}^{\hat{n}} \frac{s\,\Theta_i + \xi_i}{s^2 + \omega_i^2} \\ &= \quad Z(\infty) \quad + \quad Z_1(s) \end{aligned}\right\} \quad \ldots (4.13)$$

where each term is separately lossless positive real; all the Θ matrices are symmetric and positive semi-definite, while all the ξ matrices are skew symmetric. ξ_∞ is the constant term at infinity which must be skew symmetric. Actually $\hat{n}$ is finite and if any diagonal element of Θ_i is zero, the corresponding row and column of Θ_i and ξ_i are both zero, for finite i. These properties, of course, follow from the fact that the residue matrix K_i at $s=j\omega_i$ satisfies $K_i \geq 0$ and

$$\left.\begin{aligned} \Theta_i &= (K_i + K_i') \qquad i = 0,1,\ldots,\hat{n}\,,\ \infty \\ \xi_i &= j\omega_i(K_i - K_i') \qquad i = 1,\ldots,\hat{n} \end{aligned}\right\} \quad \ldots (4.14)$$

where prime denotes matrix transposition.

In state-space terms, $Z(\infty)$ of (4.13) corresponds to matrix D of (4.1). The realization of the first two terms of (4.13) is simple and is achieved by using congruence transformations [106]. The realization of the remaining two terms i.e. $Z_1(s)$ is discussed here using state-variable technique.

Thus,

$$Z_1(s) = \frac{1}{s}\,\Theta_o + \sum_{i=1}^{\hat{n}} \frac{s\,\Theta_i + \xi_i}{s^2 + \omega_i^2} \qquad \ldots (4.15)$$

$$= (\Theta_o+\Theta_1+\Theta_2+\ldots+\Theta_{\hat{n}})s^{-1} + (\xi_1+\xi_2+\ldots+\xi_{\hat{n}})s^{-2} - (\Theta_1\omega_1^2+\Theta_2\omega_2^2+\ldots+\Theta_{\hat{n}}\omega_{\hat{n}}^2)s^{-3} - (\xi_1\omega_1^2 + \xi_2\omega_2^2 +\ldots+ \xi_{\hat{n}}\omega_{\hat{n}}^2)s^{-4} + \ldots \qquad \ldots (4.16)$$

where $\Theta_{\hat{n}}$ and $\xi_{\hat{n}}$ are residue matrices (4.14) and $\omega_1^2, \omega_2^2, \ldots, \omega_{\hat{n}}^2$ are the eigen values of the characteristic equation of A-matrix which are positive and real $(\omega_i^2 \neq \omega_j^2)$, and

$$\hat{n} = \begin{cases} \frac{n}{2} & (n \text{ even}) \\ \frac{n-1}{2} & (n \text{ odd}) \end{cases}$$

Comparing (4.3) with (4.16), we get

$$\begin{aligned} Y_o &= \Theta_o+\Theta_1+\Theta_2+ \ldots +\Theta_{\hat{n}} \\ Y_1 &= \xi_1+\xi_2+\xi_3+ \ldots +\xi_{\hat{n}} \\ -Y_2 &= \Theta_1\omega_1^2+\Theta_2\omega_2^2+\ldots +\Theta_{\hat{n}}\omega_{\hat{n}}^2 \\ -Y_3 &= \xi_1\omega_1^2+\xi_2\omega_2^2+\ldots +\xi_{\hat{n}}\omega_{\hat{n}}^2 \\ &\vdots \end{aligned} \qquad \ldots (4.17)$$

Writing $(\hat{n} + 1)$ equations in matrix form for $\Theta_{\hat{n}}$ and $\xi_{\hat{n}}$ separately, we get

$$\begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \\ 0 & \omega_1^2 & \omega_2^2 & \ldots & \omega_{\hat{n}}^2 \\ 0 & \omega_1^4 & \omega_2^4 & \ldots & \omega_{\hat{n}}^4 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & \omega_1^{2\hat{n}-2} & \omega_2^{2\hat{n}-2} & \ldots & \omega_{\hat{n}}^{2\hat{n}-2} \end{bmatrix} \begin{bmatrix} \Theta_o \\ \Theta_1 \\ \Theta_2 \\ \vdots \\ \Theta_{\hat{n}} \end{bmatrix} = \begin{bmatrix} Y_o \\ -Y_2 \\ Y_4 \\ \vdots \\ (-1)^{\hat{n}+1} Y_{2\hat{n}-2} \end{bmatrix} \qquad \ldots (4.18a)$$

and

$$\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ \omega_1^2 & \omega_2^2 & \omega_3^2 & \cdots & \omega_{\hat{n}}^2 \\ \vdots & \vdots & \vdots & & \vdots \\ \omega_1^{2\hat{n}-2} & \omega_2^{2\hat{n}-2} & \omega_3^{2\hat{n}-2} & \cdots & \omega_{\hat{n}}^{2\hat{n}-2} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_{\hat{n}} \end{bmatrix} = \begin{bmatrix} Y_1 \\ -Y_3 \\ \vdots \\ (-1)^{\hat{n}+1} Y_{2\hat{n}-1} \end{bmatrix} \quad \ldots (4.18b)$$

For lossless networks, all the poles of $Z_1(s)$ lie on the $j\omega$-axis. Therefore, $\omega_1^2, \ldots, \omega_{\hat{n}}^2$ are all positive. Then the coefficient matrix of (4.18) will obviously be non-singular. Thus, residue matrices $\Theta_{\hat{n}}$ and $\xi_{\hat{n}}$ having been evaluated from (4.18), p-port Foster form can be easily drawn. The procedure is illustrated with an example.

Example 4.1

Given the state-model for a lossless impedance function $Z(s)$ as

$$\dot{X} = \begin{bmatrix} 0 & 1/2 & 1 \\ 0 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} X + \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix} U \quad \ldots (4.19a)$$

$$Y = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} X + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} U + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \frac{dU}{dt} \quad \ldots (4.19b)$$

From (4.3), the Markov-parameters are

$$Y_o = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \qquad Y_1 = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix},$$

$$Y_2 = \begin{bmatrix} -4 & 0 \\ 0 & -2 \end{bmatrix}, \quad Y_3 = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix}, \ \ldots\ldots \ .$$

Here, the order of the matrix A is 3. This gives $\hat{n} = 1$. The characteristic equation of A-matrix is

$$s\,(s^2 + 2) = 0$$

which gives the eigen values $s = 0$, and $s^2 = -2$. Then, $\omega_1^2 = 2$. Using (4.18), the residue matrices can be obtained as

$$\Theta_0 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \ \Theta_1 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \ \xi_1 = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} .$$

Thus

$$Z_1(s) = \frac{1}{s}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + \frac{1}{s^2 + 2}\left\{\begin{bmatrix} 2s & 0 \\ 0 & s \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}\right\} \quad \ldots (4.20)$$

and from (5.19b)

$$Z(s) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + s\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \frac{1}{s}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + \frac{1}{s^2 + 2}\begin{bmatrix} 2s & 2 \\ -2 & s \end{bmatrix}$$

... (4.21)

which results in non-reciprocal Foster form shown in Fig.4.1, [70], [106].

It may be noted that the treatment discussed above is quite general and is not restricted to scalar LC case as implied in [115]. It is shown that the technique is applicable to RC, RL, and LC p-port networks.

In the next section, a state-variable approach for Cauer driving point synthesis is discussed.

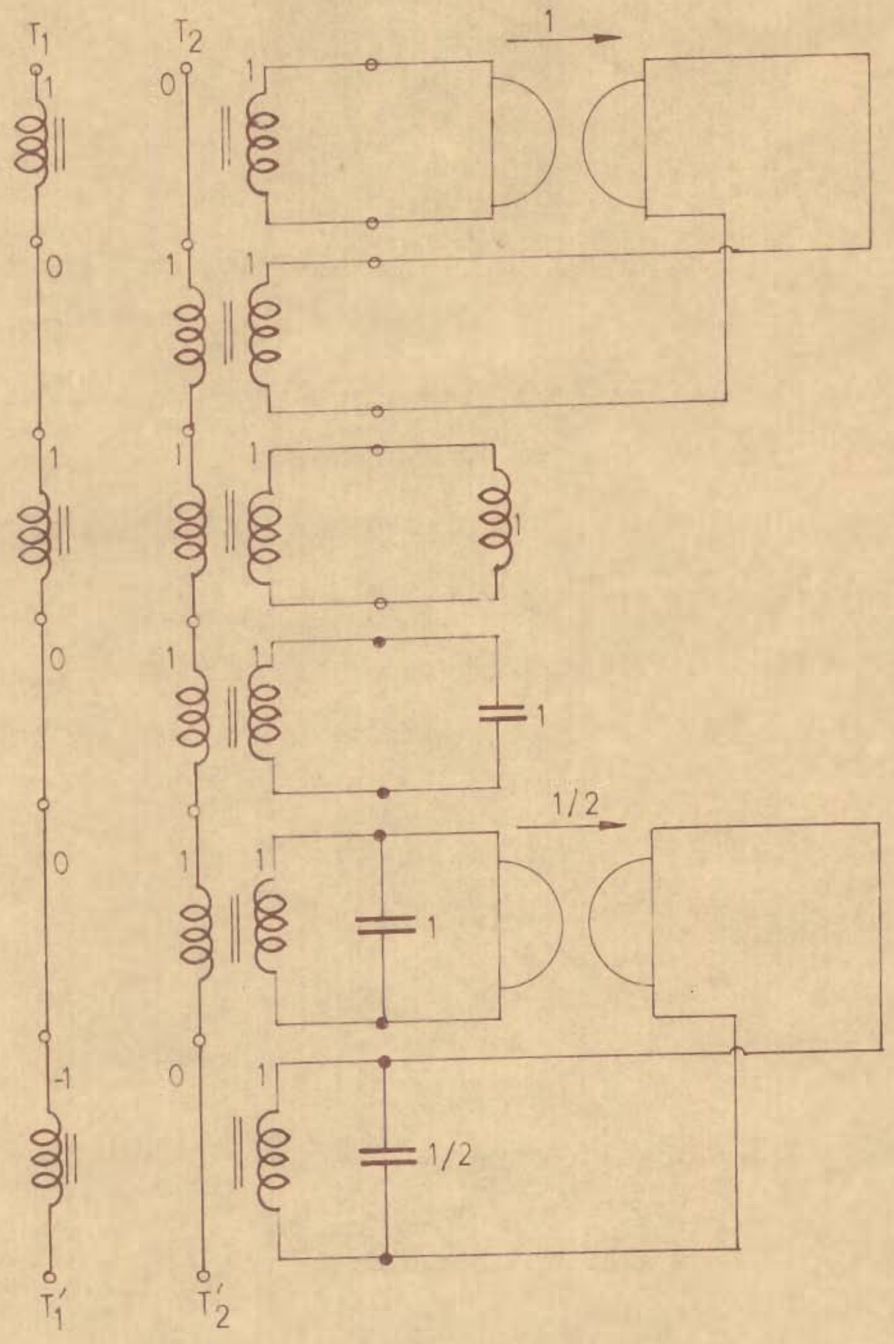

FIG. 4·1· EXAMPLE 4·1

Realization of Eqn.(4.21)

4.3 STATE-VARIABLE APPROACH FOR CAUER 1-PORT SYNTHESIS

The concepts of controllability and observability define some fundamental characteristics of linear systems and have been widely used in optimal control, estimation and identification problems. Recently, the observability matrix[92] has been employed in network synthesis as a non-singular canonical transformation to realize Foster and Brune 1-port networks[59].

In this section, the Cauer synthesis method for driving point immittance function is re-examined via state-space characterization exploiting the technique of [59].

Cauer First Form of RC Network

Consider RC driving point impedance function z(s) which may be written as

$$z(s) = \gamma + z_1(s) \qquad \ldots (4.22)$$

where $\gamma = z(\infty) = d$,

and

$$z_1(s) = \frac{b_1 s^{n-1} + b_2 s^{n-2} + \ldots + b_n}{s^n + a_1 s^{n-1} + \ldots + a_{n-1} s} \qquad \ldots (4.23)$$

The first Cauer form for the RC driving point impedance function z(s) is shown in Fig.4.2(a) where $z_1(s)$ is represented by the enclosed dotted line. The normal tree is indicated by the thick lines. Assuming port excitation to be a current driver I_o and choosing the voltages across the capacitors $\{V_{c_1}, V_{c_2}, \ldots , V_{c_n}\}$ as the state-variables, the following state-model for Fig.4.2(a) can be easily obtained [28], [10].

$$\begin{bmatrix} \dot{V}_{c_1} \\ \dot{V}_{c_2} \\ \dot{V}_{c_3} \\ \vdots \\ \dot{V}_{c_n} \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1C_1} & \frac{1}{R_1C_1} & & & \\ \frac{1}{R_1C_2} & -(\frac{1}{R_1C_2}+\frac{1}{R_2C_2}) & \frac{1}{R_2C_2} & & \\ & \frac{1}{R_2C_3} & -(\frac{1}{R_2C_3}+\frac{1}{R_3C_3}) & \frac{1}{R_3C_3} & \\ & & \ddots & \ddots & \ddots \\ & & & & \frac{1}{R_{n-1}C_n} \\ & & & \frac{1}{R_{n-1}C_n} & -(\frac{1}{R_{n-1}C_n}+\frac{1}{R_nC_n}) \end{bmatrix} \begin{bmatrix} V_{c_1} \\ V_{c_2} \\ V_{c_3} \\ \vdots \\ V_{c_n} \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{1}{C_1} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} I_o \qquad \ldots (4.24a)$$

Let $-E$ be the output.

$$y = -E = [1 \ \ 0 \ \ 0 \ldots 0][V_{c_1} \ \ V_{c_2} \ldots V_{c_n}]' \qquad \ldots (4.24b)$$

Symbolically

$$\dot{X} = A X + b u \qquad \ldots (4.25a)$$

$$y = c X \qquad \ldots (4.25b)$$

The observability matrix T_{ob}^{-1} given by, [92],

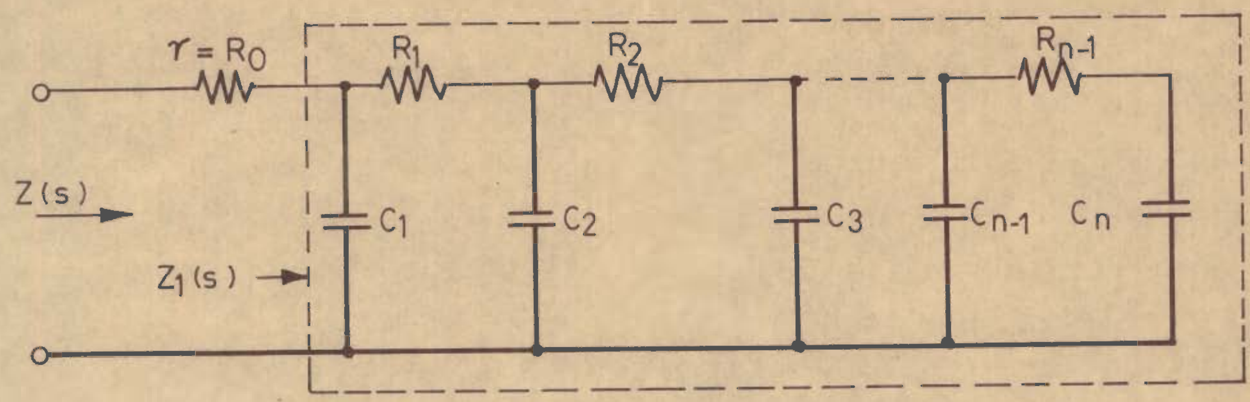

FIG.4·2(a)- FIRST CAUER RC NETWORK.

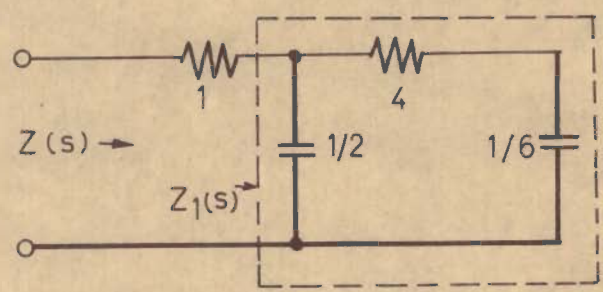

FIG. 4·2(b)-EXAMPLE 4·2.

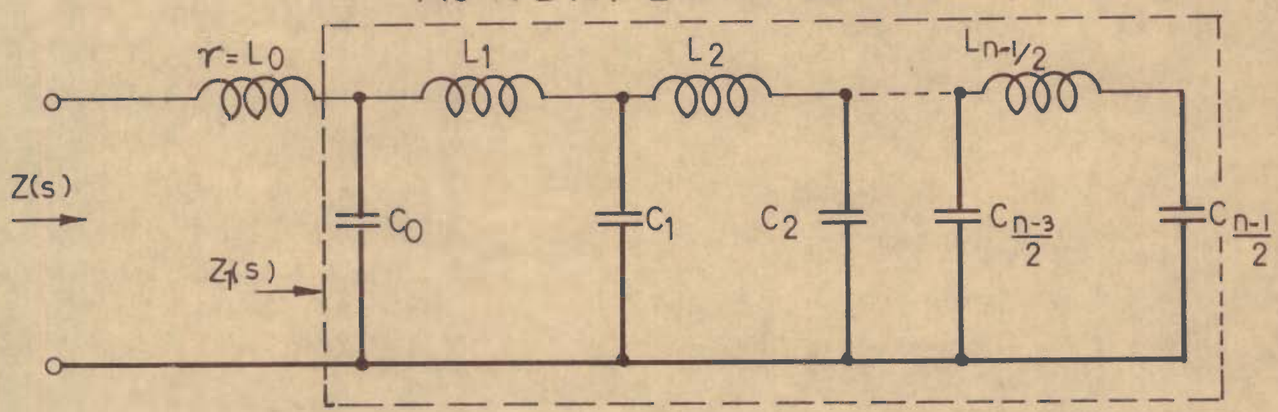

FIG. 4·3(a)-FIRST CAUER LC NETWORK.

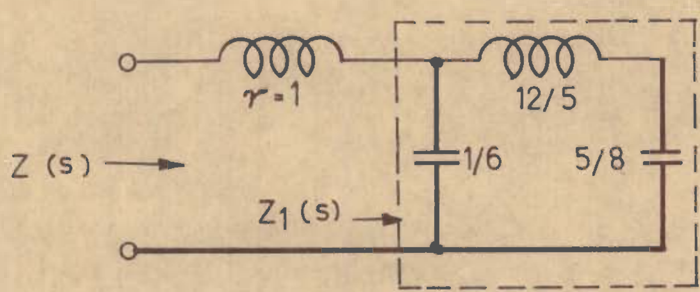

FIG. 4·3(b)-EXAMPLE 4·3.

$$T_{ob}^{-1} = \begin{bmatrix} C \\ C\ A \\ C\ A^2 \\ \vdots \\ C\ A^{n-1} \end{bmatrix} \qquad \ldots (4.26)$$

is used as a non-singular transformation for obtaining a canonical state-model representation of the Cauer network.

Applying the transformation $\hat{X} = T_{ob}^{-1} X$, the canonical state-model is given by

$$\frac{d\hat{X}}{dt} = T_{ob}^{-1}\ A\ T_{ob}\hat{X} + T_{ob}^{-1}\ b\ u = \hat{A}\ \hat{X} + \hat{b}\ u \qquad \ldots (4.27a)$$

$$y = C\ T_{ob}\ \hat{X} \qquad = \hat{C}\ \hat{X} \qquad \ldots (4.27b)$$

Also the phase-canonical state-model, in terms of the coefficients of the given immittance function (4.23) can be directly written as [32]

$$\frac{d\hat{X}}{dt} = \begin{bmatrix} 0 & 1 & & & \\ 0 & 0 & 1 & & \\ \vdots & \vdots & \vdots & \ddots & \\ 0 & & & & 1 \\ 0 & -a_{n-1} & -a_{n-2} & \ldots\ldots & -a_1 \end{bmatrix} \hat{X} + \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_{n-1} \\ h_n \end{bmatrix} u \qquad \ldots (4.28a)$$

$$y = \begin{bmatrix} 1 & 0 & 0 & \ldots\ldots & 0 \end{bmatrix} \hat{X} \qquad \ldots (4.28b)$$

where,

$$h_1 = b_1 - a_1 b_o$$
$$h_2 = b_2 - a_2 b_o - a_1 h_1$$
$$h_3 = b_3 - a_3 b_o - a_2 h_1 - a_1 h_2$$
$$\vdots \qquad \ldots (4.29)$$
$$h_k = b_k - a_k b_o - a_{k-1} h_1 \cdots\cdots - a_1 h_{k-1}$$

The canonical state-models (4.27) and (4.28) are identical in the sense that $z_1(s)$ for both the cases is the same. Thus the element values of R_i and C_i can be easily determined by comparing the corresponding entries of (4.27) and (4.28).

Cauer First Form of LC Network

The first Cauer form of LC network can also be re-examined via state-space characterization on the same lines discussed above and the canonical state-model can again be achieved by using the observability matrix as a non-singular transformation as follows.

Consider the LC driving point impedance function z(s) of the form

$$z(s) = \gamma s + \frac{b_1 s^{n-1} + b_3 s^{n-3} + \ldots + b_n}{s^n + a_2 s^{n-2} + \ldots + a_{n-1} s} \qquad \ldots (4.29)$$
$$= \gamma s + z_1(s) .$$

where n is odd.

The first Cauer form of LC network is shown in Fig.4.3(a) where thick lines indicate the normal tree. The state-model for this network can also be constructed easily [28].

Taking the voltages across the capacitors and currents through the inductors as state-variables ($V_{c_o}\ V_{c_1} \ldots V_{c_{\frac{n-1}{2}}} \vdots I_{L_1}\ I_{L_2} \ldots\ldots I_{L_{\frac{n-1}{2}}}$),

the state model of Fig.4.3(a) can be obtained as given below.

$$\dot{X} = \left[\begin{array}{cccc|cccc} & & & & -\frac{1}{C_o} & & & \\ & & & & \frac{1}{C_1} & -\frac{1}{C_1} & & \\ & & & & & \frac{1}{C_2} & -\frac{1}{C_2} & \\ & & & & & & \ddots & \\ & & & & & & & \frac{1}{C_{\frac{n-1}{2}}} \\ \hline \frac{1}{L_1} & -\frac{1}{L_1} & & & & & & \\ & \frac{1}{L_2} & -\frac{1}{L_2} & & & & & \\ & & \ddots & & & & & \\ & & \frac{1}{L_{\frac{n-1}{2}}} & -\frac{1}{L_{\frac{n-1}{2}}} & & & & \end{array}\right] X + \begin{bmatrix} \frac{1}{C_o} \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} u \qquad \ldots (4.30a)$$

$$y = \begin{bmatrix} 1 & 0 & 0 & \ldots\ldots & 0 \end{bmatrix} X \qquad \ldots (4.30b)$$

where,

$$X = \left[V_{c_o}\ V_{c_1} \ldots\ldots V_{c_{\frac{n-1}{2}}} \vdots\ I_{L_1}\ I_{L_2} \ldots\ldots I_{L_{\frac{n-1}{2}}} \right]$$

$$u = I_o$$

Now, the phase canonical state-model can be obtained using the observability matrix T_{ob}^{-1} given by (4.26) as a non-singular transformation, which is then compared with the one given by (4.27) and the element values L_i and C_i are determined in the same way as discussed in the preceding.

The above procedure for both the cases is illustrated with the help of examples [149].

Example 4.2. [149,p.151]

Given RC driving point impedance function z(s) as

$$z(s) = \frac{s^2 + 4s + 3}{s^2 + 2s} \qquad \ldots (4.31)$$

$$= 1 + \frac{2s + 3}{s^2 + 2s}$$

Obviously,

$$\gamma = 1 = z(\infty) = d$$

and
$$z_1(s) = \frac{2s + 3}{s^2 + 2s} \qquad \ldots (4.32)$$

The state-model obtained using (4.24) is given by

$$A = \begin{bmatrix} -\frac{1}{R_1C_1} & \frac{1}{R_1C_1} \\ \frac{1}{R_1C_2} & -\frac{1}{R_1C_2} \end{bmatrix}, \quad b = \begin{bmatrix} \frac{1}{C_1} \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}. \qquad \ldots (4.33)$$

Observability matrix T_{ob}^{-1} is obtained from (4.26) as

$$T_{ob}^{-1} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{R_1C_1} & \frac{1}{R_1C_1} \end{bmatrix} \qquad \ldots (4.34)$$

The canonical realization, thus obtained from (4.33) and (4.34) is

$$\hat{A} = T_{ob}^{-1}\, A\, T_{ob} = \begin{bmatrix} 0 & 1 \\ 0 & -\left(\frac{1}{R_1 C_1} + \frac{1}{R_1 C_2}\right) \end{bmatrix} \qquad \text{... (4.35)}$$

$$\hat{B} = T_{ob}\, B = \begin{bmatrix} \frac{1}{C_1} \\ -\frac{1}{R_1 C_1^2} \end{bmatrix}$$

Now $\hat{A}$ and $\hat{B}$ of the given function (4.32) are obtained by inspection as,

$$\hat{A} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}, \qquad \hat{B} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}. \qquad \text{... (4.36)}$$

Comparing the corresponding entries of the matrices of (4.35) and (4.36), the element values are found as

$C_1 = \frac{1}{2}$, $R_1 = 4$, $C_2 = \frac{1}{6}$ and $R_o = \gamma = 1$.

The corresponding Cauer first form of RC network is shown in Fig.4.2(b).

Example 4.3, [149, p.128]

Given LC driving point impedance function

$$z(s) = \frac{s^4 + 10s^2 + 9}{s^3 + 4s} \qquad \text{... (4.37)}$$

$$= s + \frac{6s^2 + 9}{s^3 + 4s}$$

Obviously,

$$\gamma = 1$$

and

$$z_1(s) = \frac{6s^2 + 9}{s^3 + 4s} \qquad \ldots (4.38)$$

The state-model obtained from (4.30) is given by

$$\dot{X} = \begin{bmatrix} 0 & 0 & -\frac{1}{C_o} \\ 0 & 0 & \frac{1}{C_1} \\ \frac{1}{L_1} & -\frac{1}{L_1} & 0 \end{bmatrix} X + \begin{bmatrix} \frac{1}{C_o} \\ 0 \\ 0 \end{bmatrix} u \qquad \ldots (4.39a)$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} X \qquad \ldots (4.39b)$$

Observability matrix T_{ob}^{-1} is obtained from (4.26) as

$$T_{ob}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{C_o} \\ -\frac{1}{L_1 C_o} & \frac{1}{L_1 C_o} & 0 \end{bmatrix} \qquad \ldots (4.40)$$

The canonical state-model thus obtained from (4.39) and (4.40) is given by

$$\frac{d\hat{X}}{dt} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -(\frac{1}{L_1 C_o} + \frac{1}{L_1 C_1}) & 0 \end{bmatrix} \hat{X} + \begin{bmatrix} \frac{1}{C_o} \\ 0 \\ -\frac{1}{L_1 C_o^2} \end{bmatrix} u \qquad \ldots (4.41a)$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \hat{X} \qquad \ldots (4.41b)$$

Also, the canonical state-model for the given function (4.38) can be written by inspection as

$$\frac{d\hat{X}}{dt} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & 0 \end{bmatrix} \hat{X} + \begin{bmatrix} 6 \\ 0 \\ -15 \end{bmatrix} u \qquad \dots (4.42a)$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \hat{X} \qquad \dots (4.42b)$$

Thus comparing (4.41) and (4.42), the network element values are determined as

$$C_o = \frac{1}{6}\ , \ L_1 = \frac{12}{5}\ , \ C_1 = \frac{5}{18} \text{ and } L_o = \gamma = 1$$

as shown in Fig.4.3(b).

It may be noted that the procedure given above can similarly be applied to Cauer second form also.

4.4 STATE-VARIABLE APPROACH FOR ACTIVE RC FILTER DESIGN USING COEFFICIENT MATCHING TECHNIQUE

The coefficient matching technique is a very efficient and practical method of designing second order active RC filters which form the basic building blocks in cascade realization of higher order transfer-functions [102]. The inherent advantage of this type of design procedure is that the network topology is known a priori. Therefore, the procedure of the preceding section can be easily applied to this case also, as discussed below.

Consider an active RC low pass filter section, Fig.4.4, proposed by Sallen and Key [130]. The thick lines indicate the normal tree. Choosing the voltages across the capacitors as state-variables the state-model of Fig.4.4 can be easily written as

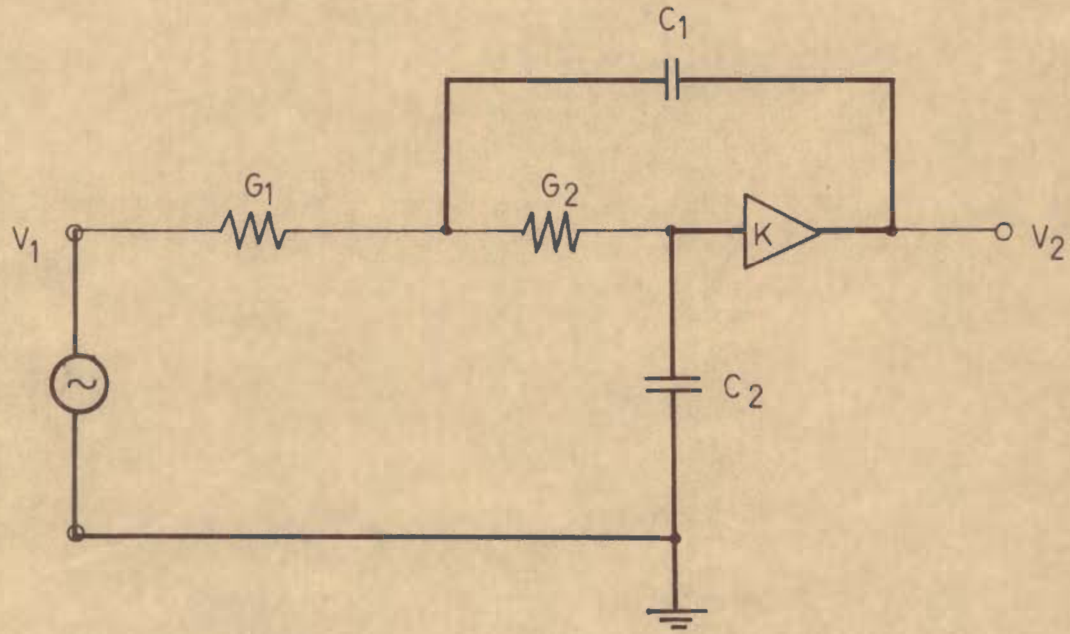

FIG.4·4 - A SECOND ORDER LOW PASS ACTIVE R C FILTER SECTION.

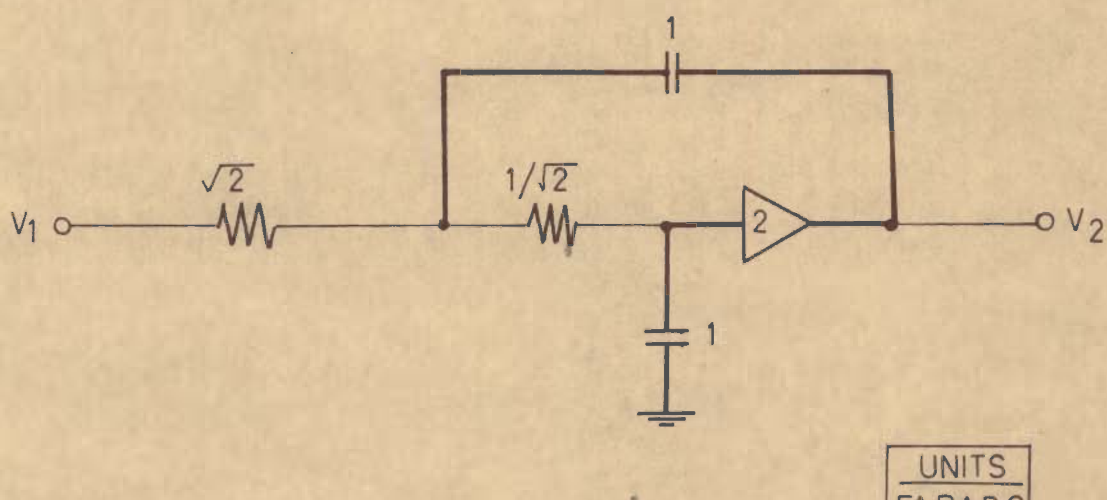

FIG. 4.5 - Example 4.4: REALIZATION OF Eqn.(4.46)

$$\begin{bmatrix} \dot{V}_{c_1} \\ \dot{V}_{c_2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{C_1}(G_1+G_2) & \frac{G_2}{C_1} \\ \frac{G_2}{C_2} & -\frac{G_2}{C_2} \end{bmatrix} \begin{bmatrix} V_{c_1} \\ V_{c_2} \end{bmatrix} + \begin{bmatrix} \frac{G_1}{C_1} \\ 0 \end{bmatrix} V_1 \quad \text{... (4.43a)}$$

$$Y = V_2 = \begin{bmatrix} 0 & k \end{bmatrix} \begin{bmatrix} V_{c_1} \\ V_{c_2} \end{bmatrix} \quad \text{... (4.43b)}$$

The observability matrix T_{ob}^{-1}, from (4.26), is obtained as

$$T_{ob}^{-1} = \begin{bmatrix} 0 & k \\ k\frac{G_2}{C_2} & -k\frac{G_2}{C_2} \end{bmatrix} \quad \text{... (4.44)}$$

The canonical state-model is then achieved from (4.43) and (4.44) as

$$\hat{A} = T_{ob}^{-1}\, A\, T_{ob} = \begin{bmatrix} 0 & 1 \\ -\left\{\frac{G_1G_2}{C_1C_2} + G_2^2\left(\frac{1}{C_1C_2} - \frac{1}{C_2^2}\right)\right\} & -\left\{\frac{(G_1+G_2)}{C_1} - \frac{G_2}{C_2}\right\} \end{bmatrix} \quad \text{... (4.45a)}$$

$$\hat{b} = T_{ob}^{-1}\, b = \begin{bmatrix} 0 \\ k\frac{G_1G_2}{C_1C_2} \end{bmatrix} \quad \text{... (4.45b)}$$

$$\hat{c} = c\, T_{ob} = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad \text{... (4.45c)}$$

The procedure is illustrated with an example.

Example 4.4, [102, p.330]

The following second order low pass Butterworth filter is to be realized with the help of the above procedure.

$$\frac{V_2}{V_1} = \frac{2}{s^2 + \sqrt{2}\, s + 1} \qquad \text{... (4.46)}$$

The canonical state-model of the given transfer function (4.46) is written by inspection as

$$\hat{A} = \begin{bmatrix} 0 & 1 \\ -1 & -\sqrt{2} \end{bmatrix}, \qquad \text{... (4.47a)}$$

$$\hat{b} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \qquad \text{... (4.47b)}$$

$$\hat{c} = \begin{bmatrix} 1 & 0 \end{bmatrix} . \qquad \text{... (4.47c)}$$

Comparing the corresponding entries of (4.45) and (4.47), the element values are obtained (assuming $C_1 = C_2 = 1$) as

$$G_1 = \sqrt{2}\ , \qquad G_2 = \frac{1}{\sqrt{2}} \text{ and } k = 2.$$

The realization of (4.46) is indicated in Fig.4.5.

4.5 CONCLUSION

In this chapter, the state-space interpretation of some well-established classical synthesis procedures such as Foster's and Cauer's methods is given. The well-known coefficient matching technique of active RC filter design is also re-examined in state-space terms. It may be remarked that computationally there is not, perhaps, a great deal to choose between the classical procedures and state-space methods akin to these procedures. But the state-space technique does offer greater scope for extension to problems such as the equivalent network problem [34] and discussion of these methods from state-space point of view has been taken up here.

The next chapter is devoted to develop new active RC multiport network synthesis procedure for the realization of immittance matrices.

CHAPTER V

MULTIPORT ACTIVE RC NETWORK SYNTHESIS WITH A MINIMUM NUMBER OF CAPACITORS

5.1 INTRODUCTION

With the publications of Sandberg [131] - [132] in 1961, multiport active RC network synthesis has received much attention during the past decade [15], [39], [52], [48], [53], [54], [101], [72]. These papers, with the exception of Hilberman [53], use more number of capacitors (with possibly some of them floating) than the minimum number which is equal to the degree of the given matrix [65].

In the early years efforts were directed towards reducing the number of active elements; but with the advent of integrated circuit technology, the trend is towards reducing the number of passive elements, particularly capacitors and having their one end common and grounded, even if it results in an increase in the number of active devices [103]. It was shown by Mann and Pike [96] that, with the help of a state-variable approach and the reactance extraction principle, it is possible to realize active RC networks using a minimum number of capacitors. Subsequently Melvin and Bickart [98], exploiting the technique of [96], proposed an interesting synthesis procedure to realize active RC network from a given admittance matrix Y(s) using voltage-controlled voltage sources. Later they extended their results [98] to the synthesis of other types of multiport network functions [18].

This chapter presents a simple and systematic synthesis procedure for the active RC realization of immittance matrices using a similar approach due to Melvin and Bickart [98]. The structures of the realized circuits in terms of the minimum number of elements and grounded ports make them particularly attractive for integrated circuit fabrication.

First, the proposed synthesis approach is briefly discussed in Section (5.2). Then, synthesis of short-circuit(s.c.) admittance matrix, open-circuit (o.c.) impedance matrix, and transfer-impedance matrix using operational amplifiers is considered. Later utilizing this approach and the results of Chapter III, a new passive reciprocal synthesis procedure for SPR immittance matrices using RCT network is evolved.

5.2 PROPOSED APPROACH TO ACTIVE RC MULTIPORT NETWORK SYNTHESIS

The general idea in the proposed approach is to realize a given multiport network function with the help of capacitive and resistive sub-networks and, by introducing suitable active elements, to force the short-circuit conductance matrix of the resistive sub-network to be hyperdominant which can be easily realized, while ensuring that the state-model of the realized network corresponds to the state-model obtained from the given network function.

Let N be a multiport network excited at p of its ports by voltages and/or currents which are elements of the p-vector u(t). Let the responses, the voltages and/or currents of q of the ports, be elements of the q-vector y(t). If N is excited at a port from which a response is derived, then at that port, if the excitation is a voltage (current), the response must be a current(voltage). Let T(s) be a qxp matrix of real rational functions of the complex variable s such that

$$Y(s) = T(s)\, U(s)$$

where $U(s) = \mathcal{L}[u(t)]$ and $Y(s) = \mathcal{L}[y(t)]$; then T(s) is said to be a multiport network function.

A synthesis procedure, based on the above idea, is to be developed by which T(s) may be realized as an immittance matrix of an active RC multiport network with a minimum number of grounded capacitors $n = \delta[T(s)]$ and at the most (p+2n) inverting, grounded voltage amplifiers.

In the synthesis method to be presented, T(s) is assumed to be regular at $s = \infty$. If it is not so, it can be made regular at infinity by invoking Möbius transformation[55]

$$s = \frac{\hat{s}\, z}{1-z} \qquad \ldots (5.1)$$

where $-\hat{s}$ is a point of regularity of T(s) on the negative real axis. The synthesis procedure to be discussed is applied to the newly formed T(z) matrix which is of the same degree as T(s). The realization for the original matrix is then obtained by inverse transformation i.e. the final network is obtained by replacing each capacitor of value c in the realization of T(z) by a capacitor and a resistor in series having admittance $\frac{c\, s}{s+\hat{s}}$.

A minimal realization set $\{A, B, C, D\}$ associated with $T(s)$ can be easily obtained by applying Ho-Kalman algorithm (Section 2.3.2) to give thc state equations of the form

$$\dot{X} = A X + B U \qquad \ldots (5.2)$$
$$Y = C X + D U$$

such that

$$T(s) = D + C(sI-A)^{-1} B$$

and A has the minimum dimension n equal to the degree $[T(s)]$ and $D = T(\infty)$.

The network that realizes T(s) will be the inter-connection of a n port grounded capacitive sub-networks, N_C, and a (p+n) port grounded resistive sub-network, N_R, as shown in Fig.5.1. Let e_2 and i_2 denote respectively the n-vectors of voltages and currents at the ports common to N_R and N_C sub-networks. The relationship imposed by N_C on e_2 and i_2 is

$$i_2 = -\mathcal{C}\dot{e}_2 \qquad \ldots (5.3)$$

where $\mathcal{C}$ is an nxn nonsingular matrix and can be assumed to be diagonal with positive entries only, resulting in a capacitive sub-network in the form of a star of n-capacitors as shown in Fig.5.2, thereby ensuring that no more than n capacitors are needed in the realization. That the realization requires at least n capacitors follows from the fact that $\mathcal{C}$ would be singular if the sub-network N_C contained fewer than n-capacitors.

Assuming the structure of the sub-network, N_R, consisting of resistors and active elements as shown in Fig.5.3(a,b),

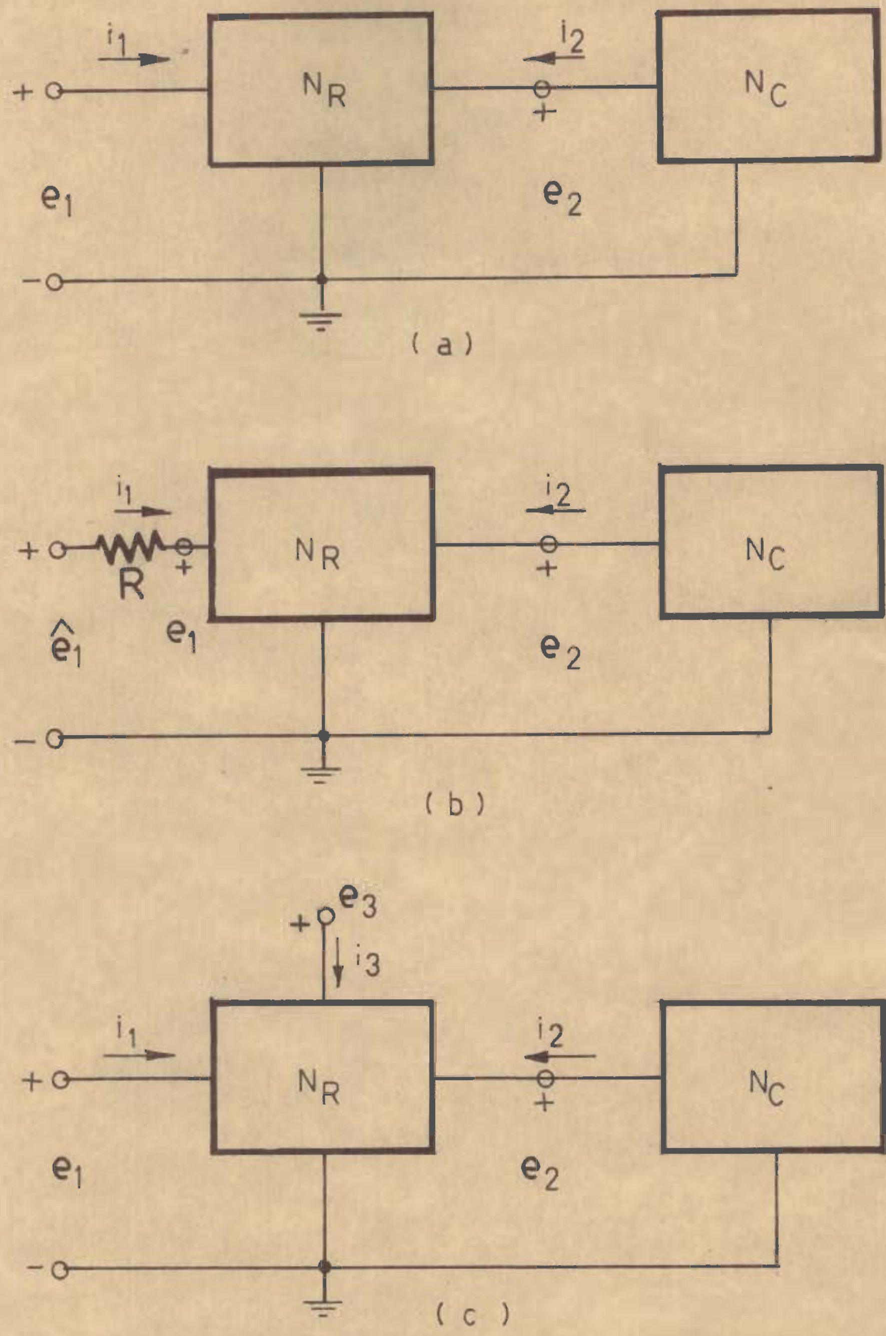

FIG. 5.1- NETWORK BLOCK DIAGRAMS.

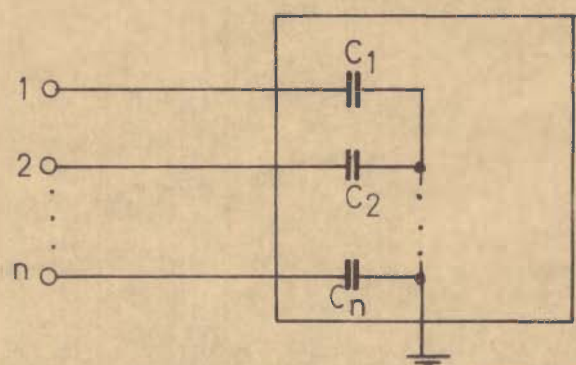

FIG-5-2- CAPACITIVE SUB NETWORK N_c.

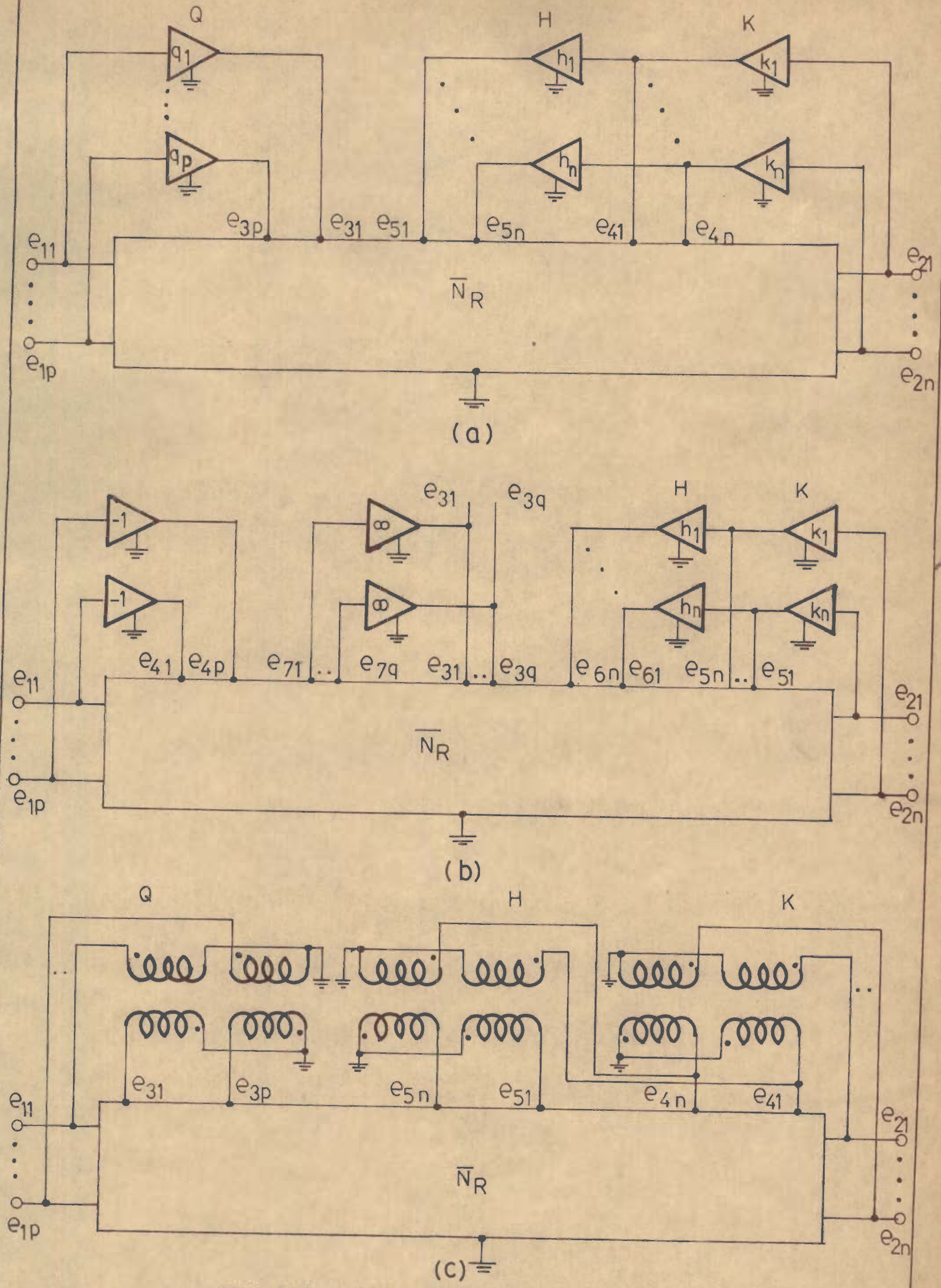

FIG. 5.3- BLOCK DIAGRAMS OF N_R.

the short-circuit parameter equations of $\bar{N}_R$ may be written as

$$I = \bar{G}\, E \qquad \text{... (5.4)}$$

where $\bar{G}$, is the short-circuit conductance matrix, of a common ground resistive network, which will be forced to be hyperdominant* by incorporating suitable active elements.

With the help of (5.3), (5.4) and the constraints imposed by the active elements [Fig.5.3(a,b)] , the state-model {A, B, C, D} of the given structure for each case is obtained in terms of the sub-matrices of $\bar{G}$ and the gains of the active elements. Thus, the problem of realization of any multiport network function T(s) is reduced to that of specifying the various sub-matrices of $\bar{G}$ subject to the condition that it is hyperdominant.

First, the synthesis of a short-circuit admittance matrix is discussed.

5.2.1 Short-Circuit Admittance Matrix Synthesis

The result established in this section can be enunciated as the following theorem:

<u>Theorem 5.1</u>

Any pxp matrix T(s), of real rational functions of

---

* A matrix is called 'hyperdominant' if it is dominant and all the off-diagonal entries are non-positive. Further, a real matrix is defined to be 'dominant' if each of its main diagonal entries is not less than the sum of the absolute values of all the other entries in the same row [25], [140].

the complex frequency variable s when $T(\infty)$ is the sum of a strictly hyperdominant matrix plus a non-negative matrix, can be realized as the short-circuit admittance matrix of a p-port active RC network using a minimum number of n capacitors with a unity capacitance spread, $n = \delta[T(s)]$ and at the most $(p + 2n)$ inverting, grounded voltage amplifiers. All the capacitors, active elements and ports will have the ground as a common terminal.

The proof of the theorem is a logical consquence of the realization procedure for T(s) given as follows:

As T(s) is assumed to be a pxp short-circuit admittance matrix Y(s), it is implied that p-vector U is the vector of network port voltages e_1 and that Y is the p-vector of corresponding port currents i_1 (Fig.5.1a). Thus the state equations (5.2) become,

$$\dot{e}_2 = A\, e_2 + B\, e_1$$
$$i_1 = C\, e_2 + D\, e_1 \qquad \ldots (5.5)$$

where e_2, an n-vector of state variables, is the port voltages at the ports common to N_R and N_C subnetworks (Fig.5.1a). Assuming the subnetwork, N_R, consisting of resistors and inverting, grounded voltage amplifiers to have a structure shown in Fig.5.3(a), where $\overline{N}_R$ is a $(2p + 3n)$ port common terminal resistive network. The short-circuit parameter equations of $\overline{N}_R$ can be written as

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} & g_{15} \\ g_{21} & g_{22} & g_{23} & g_{24} & g_{25} \\ g_{31} & g_{32} & g_{33} & g_{34} & g_{35} \\ g_{41} & g_{42} & g_{43} & g_{44} & g_{45} \\ g_{51} & g_{52} & g_{53} & g_{54} & g_{55} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix} = \bar{G} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix} \quad \ldots (5.6)$$

where i_1, i_3, e_1 and e_3 are each p-vectors, i_2, i_4, i_5, e_2, e_4 and e_5 are each n-vectors; and, the elements of $\bar{G}$ are the submatrices with $g_{ij} = g'_{ji}$, where prime denotes matrix transposition. $\bar{G}$ being the short-circuit conductance matrix of a common ground resistive network has to be hyperdominant, a necessary and sufficient condition for the (2p + 3n) port common terminal resistive $\bar{N}_R$ to be realizable without internal nodes[140].

The following constraints are imposed due to active elements (Fig.5.3a)

$$\left.\begin{aligned} e_3 &= Q\, e_1 \\ e_4 &= K\, e_2 \\ e_5 &= H\, K\, e_2 \end{aligned}\right\} \quad \ldots (5.7)$$

where the matrices Q, K and H are

$Q = \text{diag.}\{q_1, \ldots, q_p\}$, with $q_i \lessgtr 0$ for all i,

$K = \text{diag.}\{k_1, \ldots, k_n\}$, with $k_j \lessgtr 0$ for all j, and

$H = \text{diag.}\{h_1, \ldots, h_n\}$, with $h_j \lessgtr 0$ for all j.

From (5.3), (5.6) and (5.7), we obtain the state equations in the form (5.5) and {A, B, C, D} can be expressed as

$$A = \zeta^{-1}(g_{22} + g_{24} K + g_{25} H K) \quad \ldots (5.8a)$$

$$B = -\zeta^{-1}(g_{21} + g_{23} Q) \quad \ldots (5.8b)$$

$$C = (g_{12} + g_{14} K + g_{15} H K) \quad \ldots (5.8c)$$

$$D = (g_{11} + g_{13} Q) \quad \ldots (5.8d)$$

Thus the problem of realization of T(s) has been reduced to that of specifying the g sub-matrices associated with $\overline{N}_R$ and given by (5.8), subject to the condition that $\overline{G}$ is hyperdominant. The existence of such a realization is evident from the following steps in the procedure for specifying the various submatrices of $\overline{G}$.

Step I

Since D is the sum of a strictly hyperdominant matrix plus a non-negative matrix, while g_{11} is hyperdominant and g_{13} Q is non-negative, we can select suitable values for g_{11} and Q such that g_{13}, as specified in (5.8d), has only non-positive entries.

Step II

As (5.8b) contains both non-negative and non-positive elements, therefore, by a suitable choice of ζ, the sub-matrices g_{21} and g_{23} are obtained such that these have non-positive entries.

Step III

From (5.8c), we have

$$g_{14}K + g_{15} H K = C - g_{12} = P + M \quad \ldots (5.9)$$

where $P = \left[p_{ij} \right]_{pxn}$ contains all the non-negative entries of $C - g_{12}$, and

$M = \left[m_{ij} \right]_{pxn}$ contains all the non-positive entries of $C - g_{12}$.

Thus from (5.9) we have

$$g_{14} K = P \qquad \text{... (5.10a)}$$

$$g_{15} H K = M \qquad \text{... (5.10b)}$$

Obviously, by taking the $|k_j|$ and the $|h_j|$ sufficiently large, the non-zero elements of g_{14} and g_{15} can be made as small in magnitude as is necessary to make the rows of $[g_{11}\ g_{12}\ g_{13}\ g_{14}\ g_{15}]$ hyperdominant. Later it will be shown that it is always possible to select such amplifier gains. If $H^+ [K^+]$ denotes the pseudoinverse [31] of $H[K]$, then (5.10) yields

$$g_{14} = P K^+ \qquad \text{... (5.11a)}$$

provided the consistency condition $P[I - K^+ K] = 0$ is satisfied, and

$$g_{15} = M K^+ H^+ \qquad \text{... (5.11b)}$$

provided the consistency conditions,

$M [I - K^+ K] = 0$, and

$M K^+ [I - H^+ H] = 0$, are satisfied.

Step IV

Rewriting (5.8a) as

$$g_{24} K + g_{25} H K = - \mathcal{C} A - g_{22} = P_1 + M_1 \qquad \text{... (5.12)}$$

where $P_1 = [p_{ij}]_{nxn}$ contains all the non-negative entries of $-\zeta A - g_{22}$ and

$M_1 = [m_{ij}]_{nxm}$ contains all the non-positive entries of $-\zeta A - g_{22}$.

Thus, from (5.12),

$$g_{24} K = P_1 \qquad \text{... (5.13a)}$$

$$g_{25} H K = M_1 \qquad \text{... (5.13b)}$$

Since K and H are already known, the submatrices g_{24} and g_{25} can be obtained from (5.12) and (5.13) with a suitable choice of g_{22}. It may be noted that g_{22} may be selected such that the modulus of the sum of the rows corresponding to it is just equal to zero, thereby reducing the number of resistors required in the realization.

Thus, having determined the g's that appear in (5.8), the remaining entries of the $\overline{G}$ matrix can be filled in arbitrarily, however, a maximum number of zero entries, such that the hyperdominant nature of $\overline{G}$ is retained, is advantageous.

Amplifier Gain Selection

The above synthesis procedure was developed on the observation that amplifier gains (K and H) exist such that the equations in (5.10) have solutions corresponding to which the rows of $[g_{11}, \ldots, g_{15}]$ are hyperdominant. Now some criteria for choosing the amplifier gains will be described.

From (5.8d), g_{11}, g_{13} and Q are found. g_{21} and g_{23} are obtained from (5.8b).

Let g_{11} be a matrix with only positive diagonal entries g_{ii} and S_i be the sum of the magnitude of entries in the ith row of g_{12} and g_{13}. For dominance,

$$g_{ii} > S_i \ .$$

Let $$g_{ii} - S_i > \pi_i \quad \text{where } \pi_i > 0.$$

From step III, we have

$$g_{14}\ K = P \ , \text{ and } \quad g_{15}\ H\ K = M$$

Let $$g_{14} = [d_{ij}]_{pxn} \text{ where } d_{ij} \leq 0$$

$$g_{15} = [e_{ij}]_{pxn} \text{ where } e_{ij} \leq 0$$

From (5.10), we get

$$d_{ij} k_j = p_{ij} \qquad \ldots (5.10c)$$

$$e_{ij} h_j k_j = m_{ij} \qquad \ldots (5.10d)$$

Choosing k_j in (5.10c) such that

$$|k_j| > \max_i \left\{ \frac{p_{ij} . 2n}{\pi_i} \right\} ,$$

then $$d_{ij} < \frac{\pi_i}{2n} < g_{ii} \ .$$

Hence $$\sum_{j=1}^{n} |d_{ij}| < \frac{\pi_i}{2} \ .$$

Similarly by considering (5.10d), h_j is chosen such that

$$| h_j | > \max_i \left\{ \frac{m_{ij} \ . \ 2n}{k_j \ . \ \pi_i} \right\}$$

which ensures

$$\sum_{j=1}^{n} |e_{ij}| < \frac{\pi_i}{2} \ .$$

Therefore $\sum_{j=1}^{n} |d_{ij}| + \sum_{j=1}^{n} |e_{ij}| < \pi_i$.

Since $g_{12}, g_{13}, g_{14}, g_{15}$ are the only submatrices (corresponding to the rows denoted by g_{12} to g_{1n}) in the various expressions, it is clear that the rows of $[g_{11}\ g_{12}\ g_{13}\ g_{14}\ g_{15}]$ will be hyperdominant. Similarly it can be easily shown that the rows of $[g_{21},\ldots,g_{25}]$ will also be hyperdominant, thus ensuring that it is always possible to construct a hyperdominant matrix $\bar{G}$ of $\bar{N}_R$ for the proposed structure,Fig.5.3(a). This completes the specification of the $\bar{G}$ and hence the theorem.

Example 5.1

The example of Melvin and Bickart [98] is taken for illustration.

$$T(s) = Y(s) = \begin{bmatrix} \frac{2s+1}{s+1} & \frac{s}{s+1} \\ \frac{s+1/2}{s+1} & \frac{2s-1}{s+1} \end{bmatrix} .$$

Obviously,

$$D = T(\infty) = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} .$$

Using the Ho-Kalman algorithm an irreducible realization is obtained; thus:

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & 0 \\ -1/2 & -5/2 \end{bmatrix} .$$

Select

$$g_{11} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad \text{and} \quad Q = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

From (5.8d), we obtain

$$g_{13} = \begin{bmatrix} 0 & -1/2 \\ -1/2 & 0 \end{bmatrix}$$

Choosing $\zeta = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$, $K = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$,

$$H = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}, \text{and } g_{21} = \begin{bmatrix} -1/2 & -1/2 \\ 0 & -1/2 \end{bmatrix};$$

from (5.8c) and (5.8b), the sub-matrices obtained are

$$g_{12} = \begin{bmatrix} -1/2 & 0 \\ -1/2 & -1/2 \end{bmatrix}, \quad g_{14} = \begin{bmatrix} 0_2 \end{bmatrix}, \quad g_{15} = \begin{bmatrix} -1/8 & 0 \\ 0 & -1/2 \end{bmatrix},$$

and $g_{23} = \begin{bmatrix} 0_2 \end{bmatrix}$.

It is easily verified that the rows of $\begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} & g_{15} \end{bmatrix}$ are hyperdominant.

Then select g_{22} in the manner discussed earlier, thus;

$$g_{22} = \begin{bmatrix} 7/6 & 0 \\ 0 & 1/2 \end{bmatrix}.$$

From (5.8a), set

$$g_{24} = \begin{bmatrix} 0_2 \end{bmatrix} \quad \text{and} \quad g_{25} = \begin{bmatrix} -1/6 & 0 \\ 0 & 0 \end{bmatrix}.$$

The remaining entries of $\overline{G}$ matrix can be filled in arbitrarily such that it remains hyperdominant. A suitable choice for $\overline{G}$ is given below.

$$\overline{G} = \begin{bmatrix} 2 & 0 & -1/2 & 0 & 0 & -1/2 & 0 & 0 & -1/8 & 0 \\ 0 & 2 & -1/2 & -1/2 & -1/2 & 0 & 0 & 0 & 0 & -1/2 \\ -1/2 & -1/2 & 7/6 & 0 & 0 & 0 & 0 & 0 & -1/6 & 0 \\ 0 & -1/2 & 0 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ -1/2 & 0 & 0 & 0 & 0 & 1/2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1/8 & 0 & -1/6 & 0 & 0 & 0 & 0 & 0 & 7/24 & 0 \\ 0 & -1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 \end{bmatrix}$$

Having obtained the $\overline{G}$-matrix of $\overline{N}_R$, the network can be easily constructed as shown in Fig.5.4. It may be noted that the $\overline{G}$ matrix obtained above has all zero entries in two rows and two columns, meaning thereby, that the nodes corresponding to them (in this case e_{41} and e_{42}) will disappear; so the cascaded active elements can be combined as shown in Fig.5.4. Further, only 9 resistors are needed in the above realization as against 12 used in [98], while the numbers of capacitors and active elements remain same. In general, this procedure will require at the most p+2n active elements compared to at the most 2p+2n elements required by Melvin and Bickar [98] for the Y(s) under consideration. It may be noted that the network $\overline{N}_R$ (Fig.5.4) which realizes the hyperdominant $\overline{G}$ has no internal nodes as indicated earlier.

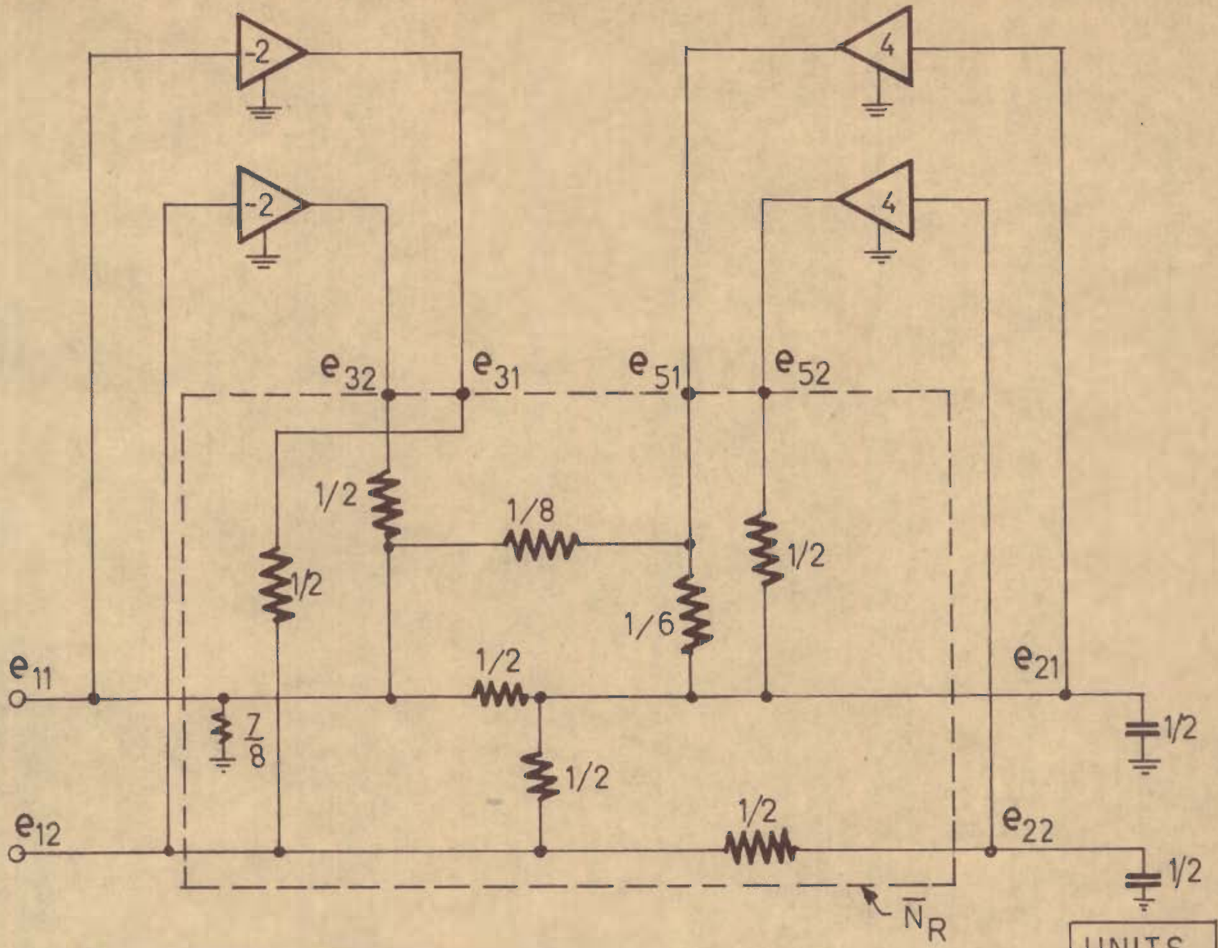

FIG.5.4-Example 5.1: REALIZATION OF Y(s).

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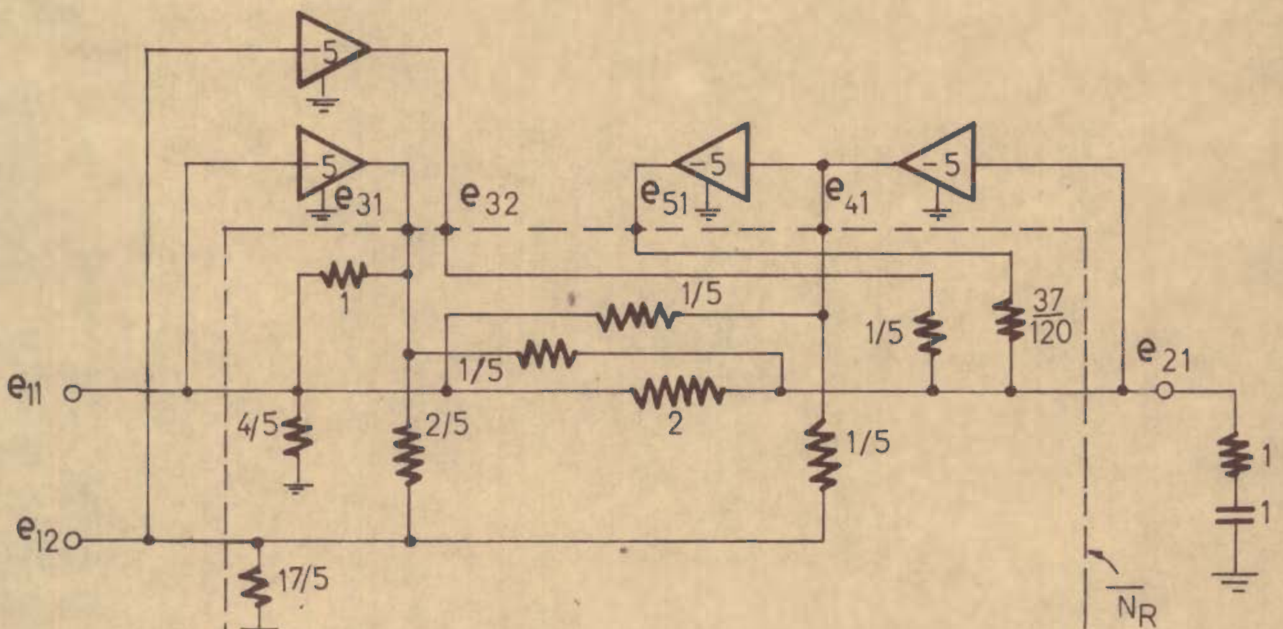

FIG. 5.5-Example 5.2: REALIZATION OF Z(s).

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5.2.2 Open-Circuit Impedance Matrix Synthesis

When T(s) is assumed to be a pxp open-circuit impedance matrix Z(s), it is implied that the p-vector U is the vector of network port currents i_1 and that Y is the p-vector of corresponding port voltages $\hat{e}_1$.

Consider the network block diagram in Fig.5.1(b), where N_R and N_C are specified as in the previous case. $\hat{e}_1$, e_1 and i_1 are related through the resistor R as

$$\hat{e}_1 = e_1 + R\, i_1 \qquad \ldots (5.14)$$

The state equations (5.2) in this case become

$$\dot{e}_2 = A\, e_2 + B\, i_1 \qquad \ldots (5.15a)$$

$$\hat{e}_1 = C\, e_2 + D\, i_1 \qquad \ldots (5.15b)$$

We may choose the same structure of N_R [Fig.5.3(a)] as in the preceding section, then (5.6) and (5.7) will remain unaltered. From (5.3), (5.6), (5.7), and (5.14), the state-equations in the form (5.15) are obtained and {A, B, C, D} can be expressed as:

$$D = R + (g_{11} + g_{13}\, Q)^{-1} \qquad \ldots (5.16a)$$

$$B = -\mathcal{C}^{-1}(g_{21} + g_{23}\, Q)(g_{11} + g_{13}\, Q)^{-1} \qquad \ldots (5.16b)$$

$$C = -(g_{11} + g_{13}\, Q)^{-1}(g_{12} + g_{14}\, K + g_{15}\, H\, K) \qquad \ldots (5.16c)$$

$$A = -\mathcal{C}^{-1}\{(g_{22} + g_{24}\, H + g_{25}\, H\, K) - (g_{21} + g_{23}\, Q)(g_{11} + g_{13}\, Q)^{-1}(g_{14}\, K + g_{15}\, H\, K)\} \qquad \ldots (5.16d)$$

Thus the problem of realization of Z(s) has been reduced to that of identifying the various terms of g-submatrices of $\overline{N}_R$ given by (5.16) subject to the condition that $\overline{G}$ is hyperdominant. The existence of such a realization is evident from the following steps in the procedure for specifying the various submatrices of $\overline{G}$.

Step I.

In (5.16d), if D is non-singular, R can be set as $R = [0]$; otherwise the elements of R can always be specified such that (D-R) is non-singular. Assuming suitable values of g_{11} and Q, g_{13} can be obtained such that it has all negative entries or zeros.

Step II.

Since g_{11}, g_{13} and Q are fixed, then by suitable choice of $\mathcal{C}$, g_{21} and g_{23} can be obtained from (5.16b) such that these have non-positive entries.

Step III.

On substituting the values of g_{11}, g_{13}, Q and g_{12} in (5.16c), we get

$$-(g_{11} + g_{13}\, Q)\, C - g_{12} = g_{14}\, K + g_{15}\, H\, K \quad \ldots (5.17)$$

The right hand side of (5.17) can be split up into matrices P and M where P[M] contains all the non-negative [non-positive] elements of $-(g_{11} + g_{13}\, Q)\, C - g_{12}$. Thus

$$g_{14}\, K = P$$
$$g_{15}\, H\, K = M$$

Suitable values for K and H can be assumed such that g_{14} and g_{15} are as small in magnitude as is necessary to make the rows of $[g_{11}\ g_{12}\ g_{13}\ g_{14}\ g_{15}]$ hyperdominant.

Step IV.

Now (5.16d) can be written as

$$g_{24}K+g_{25}HK = -\zeta A-g_{22}+(g_{21}+g_{23}Q)(g_{11}+g_{13}Q)^{-1}(g_{14}K+g_{15}HK) \quad \ldots (5.18)$$

Substituting in (5.18) the values of the terms obtained earlier, the only unknowns to be determined are g_{22}, g_{24}, and g_{25}. Thus with suitable choice of diagonal sub-matrix g_{22}, g_{24} and g_{25} can be found from (5.18) such that these have negative or zero entries. It may be noted that we may select g_{22} such that the modulus of the sum of the rows corresponding to it is just equal to zero; thereby reducing the number of resistors required in the realization.

Thus having determined the terms appearing in (5.16), the remaining entries of the $\overline{G}$ matrix can be filled in arbitrarily; however keeping maximum zero entries such that the hyperdominant nature of $\overline{G}$ is retained is advantageous.

Now, the main result established above can be stated as the following theorem:

<u>Theorem 5.2</u>

Any pxp matrix T(s), of real rational functions of the complex frequency variable s, can be realized as the open-circuit impedance matrix Z(s) of a p-port active RC network using a minimum number of n capacitors having unity capacitance spread, $n = \delta[T(s)]$ and at the most (p + 2n) inverting, grounded voltage amplifiers. All the capacitors and amplifiers will share a common ground.

The procedure described above is illustrated with the help of the same example as in [18] and it is shown that the realization is possible with only four active elements instead of six required in [18].

Example 5.2 [18]

The following 2x2 matrix is to be realized as the open-circuit impedance matrix of a two port:

$$T(s) = Z(s) = \begin{bmatrix} s & s+1 \\ s-1 & s \end{bmatrix}.$$

Since T(s) is not regular at $s = \infty$, a Möbius transformation can be invoked to make T(s) regular at $s=\infty$.

Let $$s = \frac{z}{1-z},$$

then $$T(z) = \begin{bmatrix} \frac{z}{1-z} & \frac{1}{1-z} \\ \frac{-1+2z}{1-z} & \frac{z}{1-z} \end{bmatrix}$$

will be realized; lateron each capacitor having admittance cz will be replaced by a capacitor and a resistor, having admittance $\frac{cs}{s+1}$.

Applying Ho and Kalman algorithm, a minimal realization set $\{A,B,C,D\}$ of T(z) is obtained as

$$A = [1], \quad B = [1 \quad 1], \quad C = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \quad D = T(\infty) = \begin{bmatrix} -1 & 0 \\ -2 & -1 \end{bmatrix}.$$

Since D is non-singular, R can be set as $R = [0]$.

From (5.16a), with $\bar{g}_{11} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ and $Q = \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$,

g_{13} is obtained as $g_{13} = \begin{bmatrix} -1 & 0 \\ -\frac{2}{5} & -1 \end{bmatrix}$.

Choosing $\zeta = [1]$, from (5.16b), we obtain

$$g_{21} = [-2 \quad 0] \text{ and } g_{23} = \left[-\tfrac{1}{5} \quad -\tfrac{1}{5}\right].$$

From (5.16c) and (5.17), with $K = [-5]$, $H = [-5]$, we find,

$$g_{14} = \begin{bmatrix} -1/5 \\ -1/5 \end{bmatrix} \text{ and } g_{15} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

It is easily verified that the rows of $[g_{11}\ g_{12}\ g_{13}\ g_{14}\ g_{15}]$ are hyperdominant. Now substituting the values determined above in (5.16d) or (5.18), we get

$$g_{24}K + g_{25}HK = -5 - g_{22}.$$

Selecting g_{22} in the manner discussed earlier, thus with

$$g_{22} = \left[\tfrac{65}{24}\right],$$

we obtain,

$$g_{24} = [0] \text{ and } g_{25} = \left[-\tfrac{37}{120}\right].$$

The remaining entries of $\bar{G}$ can now be filled in arbitrarily such that it remains hyperdominant. A suitable choice for $\bar{G}$ is given below

$$\bar{G} = \begin{bmatrix} 4 & 0 & -2 & -1 & 0 & -1/5 & 0 \\ 0 & 4 & 0 & -2/5 & 0 & -1/5 & 0 \\ -2 & 0 & \frac{65}{24} & -1/5 & -1/5 & 0 & -\frac{37}{120} \\ -1 & -2/5 & -1/5 & 16/5 & 0 & 0 & 0 \\ 0 & 0 & -1/5 & 0 & 1/5 & 0 & 0 \\ -1/5 & -1/5 & 0 & 0 & 0 & 2/5 & 0 \\ 0 & 0 & -\frac{37}{120} & 0 & 0 & 0 & \frac{37}{120} \end{bmatrix} .$$

Having obtained the $\bar{G}$-matrix of $\bar{N}_R$, the network can now be constructed as shown in Fig.5.5. The number of active elements used in the realization is four instead of six required in [18]. It may be noted that a capacitor and a resistor are in series at each capacitive port of $\bar{N}_R$, as Möbius transformation has been invoked in this example.

5.2.3 Transfer-Impedance Matrix Synthesis using Operational Amplifiers

This section presents a synthesis procedure to realize T(s) as a q x p O.C. transfer-impedance matrix of a multiport active RC filter using commercially available operational amplifiers (OA), and inverting, voltage-controlled voltage sources (VCVS), which can be easily constructed from OA, as active elements. The main result established in this section can be given as the following theorem:

<u>Theorem 5.3</u>

Any qxp matrix T(s), of real rational functions of the complex frequency variable s, can be realized as the

O.C. transfer-impedance matrix of a (q + p) port active RC network containing a minimum number of n capacitors with unity capacitance spread, $n = \delta[T(s)]$, q operational amplifiers (OA), and at the most (p + 2n) inverting, common ground voltage-controlled voltage sources (VCVS). All the capacitors, ports and active elements will have the ground as a common terminal.

The following proof incorporates a step by step realization procedure for T(s).

By assuming that T(s) is a q x p O.C. transfer-impedance matrix, it is implied that the p-vector U is a vector of source port currents i_1, the q-vector Y is a vector of response port voltages e_3, with the response ports open i.e. $i_3 = 0$ [Fig. 5.1(c)].

The state equations (5.2), in this case, become

$$\begin{aligned} \dot{e}_2 &= A\, e_2 + B\, i_1 \\ e_3 &= C\, e_2 + D\, i_1 \end{aligned} \qquad \ldots (5.19)$$

Assuming the sub-network, N_R, consisting of the resistors, OA and inverting VCVS to have a structure shown in Fig.5.3(b), where $\overline{N}_R$ is a (2p + 2q + 3n) port grounded sub-network of resistors. The short circuit parameter equations of $\overline{N}_R$ can be denoted as

$$I = \overline{G}\, E \qquad \ldots (5.20)$$

where,

$$I = [i_1 \;\; i_2 \;\; i_3 \;\; i_4 \;\; i_5 \;\; i_6 \;\; i_7]' ,$$
$$E = [e_1 \;\; e_2 \;\; e_3 \;\; e_4 \;\; e_5 \;\; e_6 \;\; e_7]' ,$$

and $\bar{G} = [g_{ij}]_{(2p+2q+3n)x(2p+2q+3n)}$

where i_1, i_4, e_1 and e_4 are each p-vectors, i_3, i_7, e_3 and e_7 are each q-vectors, i_2, i_5, i_6, e_2, e_5 and e_6 are each n-vectors and the elements of $\bar{G}$ are the submatrices with $g_{ij} = g'_{ji}$; $\bar{G}$ being the short circuit conductance matrix of a common ground resistive network has to be hyperdominant, a necessary and sufficient condition for the (2p+2q+3n) port common terminal network $\bar{N}_R$ to be realizable without internal nodes [140].

The active elements [Fig.5.3(b)] impose the following constraints:

$$\left.\begin{aligned} e_4 &= -e_1 \\ e_5 &= K\ e_2 \\ e_6 &= HK\ e_2 \\ e_7 &= i_7 = 0 \end{aligned}\right\} \qquad \text{... (5.21)}$$

where the matrices K and H are

$K = \text{diag.}\{k_1, \ldots, k_n\}$, with $k_i \leqslant 0$ for all i and
$H = \text{diag.}\{h_1, \ldots, h_n\}$, with $h_i \leqslant 0$ for all i.

From (5.3), (5.20), and (5.21), we obtain the state equations in the form (5.19), and $\{A, B, C, D\}$ can be expressed as:

$$D = -g_{73}^{-1}(g_{71}-g_{74})(g_{11}-g_{14})^{-1} \qquad \text{... (5.22a)}$$

$$B = -\zeta^{-1}(g_{21}-g_{24})(g_{11}-g_{14})^{-1} \qquad \text{... (5.22b)}$$

$$C = -g_{73}^{-1}\{(g_{75}K+g_{76}H\ K)-(g_{71}-g_{74})(g_{11}-g_{14})^{-1}(g_{12}+g_{15}K+g_{16}H\ K)\} \qquad \text{... (5.22c)}$$

$$A = -\zeta^{-1}\{(g_{22}+g_{25}K+g_{26}H\ K)-(g_{21}-g_{24})(g_{11}-g_{14})^{-1}(g_{12}+g_{15}K+g_{16}HK)\} \qquad \text{... (5.22d)}$$

Thus the problem of realization of T(s) has been reduced to that of specifying the g submatrices associated with $\bar{N}_R$ and given by (5.22), subject to the condition that $\bar{G}$ is hyperdominant. The existence of such a realization is evident from the following steps in the procedure for identifying the various submatrices of $\bar{G}$.

Step I.

Since g_{73} is assumed non-singular with non-positive entries, while g_{11} is hyperdominant and g_{14} is non-positive, we can select suitable values for g_{73}, g_{11} and g_{14} such that g_{71} and g_{74} as specified in (5.22a) have only non-positive entries.

Step II.

As g_{11}, g_{14} and g_{73} are fixed, then by a suitable choice of $\mathcal{C}$, the submatrices g_{21} and g_{24} are obtained from (5.22b) such that these have non-positive entries.

Step III.

From (5.22c), assuming $g_{75} = g_{76} = 0$, we get

$$g_{15}K + g_{16}HK = (g_{71}-g_{74})^{-1}(g_{11}-g_{14})g_{73}C-g_{12} = P + M \quad \ldots (5.23)$$

where,

$P = [p_{ij}]_{pxn}$ contains all the non-negative entries of $(g_{71}-g_{74})^{-1}(g_{11}-g_{14})g_{73}C-g_{12}$ and

$M = [m_{ij}]_{pxn}$ contains all the non-positive entries of $(g_{71}-g_{74})^{-1}(g_{11}-g_{14})g_{73}C-g_{12}$.

$$\text{Thus,}\quad g_{15}K = P \quad \ldots (5.24a)$$

$$g_{16}H\,K = M \quad \ldots (5.24b)$$

It is obvious that, by making $|k_j|$ and the $|h_j|$ sufficiently large, the non-zero elements of g_{15} and g_{16} can be made as small in magnitude as is necessary to make the rows of $[g_{11}, \ldots, g_{17}]$ hyperdominant. It will be shown later that it is always possible to select such amplifier gains. If $K^+ [H^+]$ denotes the pseudoinverse [31] of $K[H]$, then (5.24) yields

$$g_{15} = P K^+ \qquad \ldots (5.25a)$$

provided the consistency condition $P [I - K^+ K] = 0$ is satisfied, and

$$g_{16} = M K^+ H^+ \qquad \ldots (5.25b)$$

provided the consistency conditions $M [I - K^+ K] = 0$, and $M K^+ [I - H^+ H] = 0$ are satisfied.

Step IV.

From (5.22d), we get

$$g_{25}K+g_{26}HK = -\phi A+(g_{21}-g_{24})(g_{11}-g_{14})^{-1}(g_{12}+g_{15}K+g_{16}HK)-g_{22} \qquad \ldots (5.26)$$

Thus g_{25} and g_{26} can be found from (5.26) by selecting a suitable value of g_{22} such that these have non-positive entries. We may also select g_{22} in such a way that the modulus of the sum of the rows corresponding to it is just equal to zero, thereby reducing the number of resistors required in the realization.

Thus having determined the g's that appear in (5.22), the remaining entries of $\bar{G}$ matrix can be filled in arbitrarily; however, a maximum number of zero entries such that the hyper-dominant nature of $\bar{G}$ is retained, is advantageous.

Amplifier Gain Selection

The above synthesis procedure was developed on the observation that amplifier gains (K and H) exist such that the equations in (5.24) have solutions corresponding to which the rows of $[g_{11}, \ldots, g_{17}]$ are hyperdominant. Now some criteria will be given for choosing the amplifier gains.

From steps I and II, the various submatrices appearing in (5.22a) and (5.22b) are obtained. Let g_{11} be a matrix with only positive diagonal entries g_{ii} and S_i be the sum of the magnitude of entries in the ith row of g_{12} and g_{13}. For dominance,

$$g_{ii} > S_i \ .$$

Let $g_{ii} - S_i > \pi_i$ where $\pi_i > 0$.

From step III, we have

$$g_{15}K = P \text{ and } g_{16}HK = M$$

Now let $g_{15} = [d_{ij}]_{pxn}$, where $d_{ij} < 0$

and $g_{16} = [e_{ij}]_{pxn}$ where $e_{ij} < 0$

From (5.24) we get,

$$d_{ij}k_j = p_{ij} \qquad \ldots (5.24c)$$

and

$$e_{ij}h_jk_j = m_{ij} \qquad \ldots (5.24d)$$

Choosing k_j in (5.24c) such that

$$|k_j| > \max_i \left\{ \frac{p_{ij} \cdot 2n}{\pi_i} \right\}$$

then

$$d_{ij} < \frac{\pi_i}{2n} < g_{ii} .$$

Hence $\sum_{j=1}^{n} |d_{ij}| < \frac{\pi_i}{2}$.

Similarly by considering (5.24d), h_j is chosen such that

$$|h_j| > \max_i \left\{ \frac{m_{ij}.2n}{k_j.\pi_i} \right\}$$

which ensures

$$\sum_{j=1}^{n} |e_{ij}| < \frac{\pi_i}{2}$$

Therefore, $\sum_{j=1}^{n} |d_{ij}| + \sum_{j=1}^{n} |e_{ij}| < \pi_i$.

Since g_{12}, g_{14}, g_{15}, g_{16}, g_{17}, are the only submatrices (corresponding to the rows denoted by g_{12} to g_{1n}) in the various expressions, it is clear that with the choice of $g_{13}=0$, the rows of $[g_{11},\ldots,g_{17}]$ will be hyperdominant. Similarly it can be shown that the rows of $[g_{21},\ldots,g_{27}]$ will also be hyperdominant; thus ensuring that it is always possible to construct a hyperdominant matrix $\bar{G}$ of $\bar{N}_R$ for the proposed structure [Fig.5.3(b)]. This completes the specification of the $\bar{G}$ and hence the theorem.

Example 5.3

To illustrate the above result, the following 2x2 matrix will be realized as an open-circuit transfer impedance matrix of a two port:

$$T(s) = \begin{bmatrix} \frac{2s-1}{s+1} & \frac{s+\frac{1}{2}}{s+1} \\ \frac{-1}{s+1} & \frac{2s+1}{s+1} \end{bmatrix} .$$ Here $p = q = 2$ and $n = 2$.

Since T(s) is regular at $s = \infty$, a Möbius transformation will not be needed. Using the Ho-Kalman algorithm an irreducible realization of T(s) is obtained; thus:

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 1/6 \\ 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} -3 & 0 \\ -1 & -5/6 \end{bmatrix}, \qquad D = T(\infty) = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}.$$

Select

$$g_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad g_{73} = \begin{bmatrix} -1/10 & 0 \\ 0 & -1/10 \end{bmatrix} \text{ and } g_{14} = \begin{bmatrix} 0_2 \end{bmatrix}.$$

From (5.22a), we obtain

$$g_{74} = \begin{bmatrix} -2/10 & -1/10 \\ 0 & -2/10 \end{bmatrix} \text{ and } g_{71} = [\,0_2\,].$$

Choosing $\mathcal{C} = \begin{bmatrix} 1/10 & 0 \\ 0 & 1/10 \end{bmatrix}$, the submatrices obtained from (5.22b) are

$$g_{21} = \begin{bmatrix} -1/10 & -1/60 \\ 0 & -1/10 \end{bmatrix} \text{ and } g_{24} = [\,0_2\,].$$

Select $K = \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$ and $H = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

From (5.23) and (5.24), we obtain

$$g_{15} = \begin{bmatrix} -27/100 & 0 \\ -31/300 & -31/300 \end{bmatrix} \text{ and } g_{16} = \begin{bmatrix} 0 & -1/24 \\ 0 & 0 \end{bmatrix}.$$

It is easily verified that the rows of $[g_{11}, g_{12}, g_{13}, g_{14}, g_{15}, g_{16}, g_{17}]$ are hyperdominant. Then select g_{22}

in the manner discussed earlier; thus:

$$g_{22} = \begin{bmatrix} \frac{1586}{11520} & 0 \\ 0 & \frac{133}{960} \end{bmatrix} .$$

From (5.26), the submatrices obtained are,

$$g_{25} == \begin{bmatrix} 0 & -\frac{1}{288} \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad g_{26} = \begin{bmatrix} -\frac{16244}{57700} & 0 \\ -\frac{1}{100} & -\frac{76}{4800} \end{bmatrix}$$

Therefore, the following $\overline{G}$ composed according to the guide-lines set forth earlier will be hyperdominant.

$$\overline{G} = \begin{bmatrix}
1 & 0 & -\frac{1}{10} & 0 & 0 & 0 & 0 & 0 & -\frac{27}{100} & 0 & 0 & -\frac{1}{24} & 0 & 0 \\
0 & 1 & -\frac{1}{60} & -\frac{1}{10} & 0 & 0 & 0 & 0 & -\frac{31}{300} & -\frac{31}{300} & 0 & 0 & 0 & 0 \\
-\frac{1}{10} & -\frac{1}{60} & \frac{1586}{11520} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{288} & -\frac{16244}{57700} & 0 & 0 & 0 \\
0 & -\frac{1}{10} & 0 & \frac{133}{960} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{100} & -\frac{76}{4800} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{10} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{10} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{10} & 0 & 0 & 0 & 0 & 0 & -\frac{2}{10} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{10} & 0 & 0 & 0 & 0 & -\frac{1}{10} & -\frac{2}{10} \\
-\frac{27}{100} & -\frac{31}{300} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{112}{300} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{31}{300} & -\frac{1}{288} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{769}{7200} & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{16244}{57700} & -\frac{1}{100} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{16821}{57700} & 0 & 0 & 0 \\
-\frac{1}{24} & 0 & 0 & -\frac{76}{4800} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{276}{4800} & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{10} & 0 & -\frac{2}{10} & -\frac{1}{10} & 0 & 0 & 0 & 0 & \frac{4}{10} & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{10} & 0 & -\frac{2}{10} & 0 & 0 & 0 & 0 & 0 & \frac{3}{10}
\end{bmatrix}$$

The realization of T(s) based on the realization of $\overline{G}$ is shown in Fig.5.6. Note that two differential output CA are

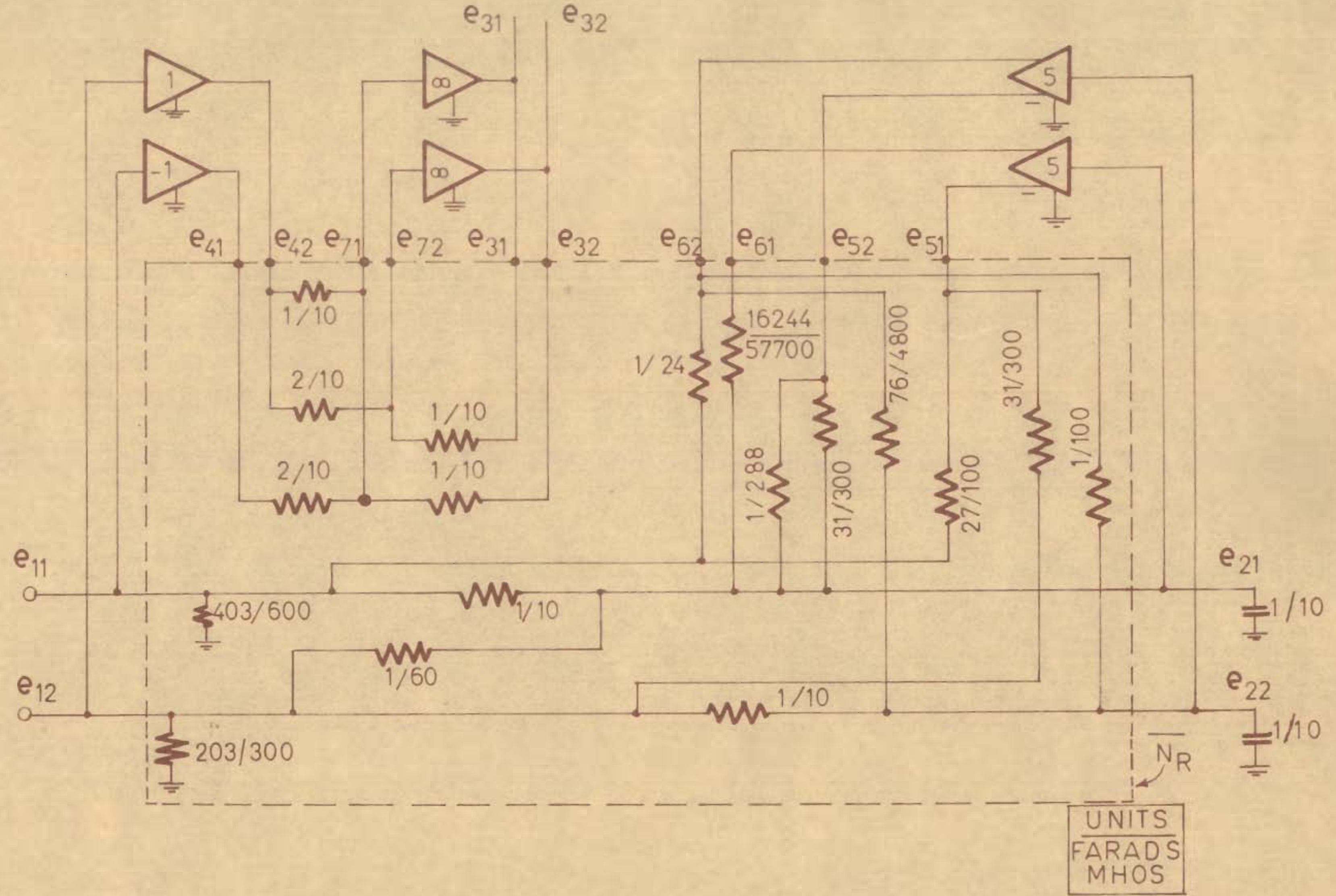

FIG. 5.6- Example 5.3: REALIZATION OF T(s).

shown rather than the cascade of two with gains $k_1=k_2=-5$ and $h_1=h_2=-1$. Thus the total number of active elements used is 6. Further, the network $\overline{N}_R$ (Fig.5.6) which realizes the hyperdominant $\overline{G}$ has no internal nodes as indicated earlier.

It may be noted that the proposed method has two distinct advantages over the one due to Bickart and Melvin[18] for the case of C.C. transfer-impedance matrix. First, it uses commercially available CA instead of voltage amplifiers and secondly, it will usually require fewer resistors, because the submatrix g_{22}, as discussed in the procedure and illustrated in the example, can always be chosen to be hyperdominant.

5.3 PASSIVE RECIPROCAL RCT MULTIPORT NETWORK SYNTHESIS

The problem of realizing a given SPR immittance matrix with passive reciprocal RLCT multiport network (without gyrators) is one of the important and interesting problems in network synthesis and has been studied via state-variable approach by several authors during recent years[10], [11], [139], [150], [131], [156], [160].

In this section, a new synthesis procedure to realize a given SPR immittance matrix using passive reciprocal RCT multiport network with a minimum number of capacitors is presented. The method is essentially an extension of the technique of active RC multiport synthesis discussed in the preceding section and uses ideal transformers in place of active elements. By selecting suitable transformation ratios [149], the hyperdominant matrix $\overline{G}$ is again constructed.

Of course, this synthesis is only possible if the minimal realization set $\{A, B, C, D\}$, associated with the given SPR immittance function, satisfies the following constraints.

$$\text{(i)}\quad M_1 + M_1' \geq 0 \qquad \ldots (5.27)$$

$$\text{(ii)}\quad (I \dot{+} \sum)M_1 = M_1'(I \dot{+} \sum) \qquad \ldots (5.28)$$

where $M_1 = \begin{bmatrix} D & C \\ B & A \end{bmatrix}$, and $\sum$ is an unique diagonal matrix of $\pm$ 1's as defined earlier.

More precisely, the first condition (5.27) says that M_1 possesses a passive synthesis, while the second condition (5.28) guarantees reciprocal realization [10], [75], [160].

A minimal reciprocal realization, of a given SPR immittance matrix, that fulfills the above conditions can be constructed with the help of the algorithms proposed in Chapter III. Once such a realization is in hand, a passive reciprocal synthesis of the given immittance function can be obtained by using an identical procedure as given in Section 5.2.

In the following, the main result of the passive reciprocal synthesis of SPR short-circuit admittance matrix Y(s) and SPR open-circuit impedance matrix Z(s) is stated in the form of the following theorem:

Theorem 5.4

Any pxp SPR matrix T(s), of real rational functions of complex frequency variable s, can be realized as the immittance matrix [Y(s) or Z(s)] of a p-port passive reciprocal RCT network using a minimum number of grounded capacitors $n = \delta[T(s)]$ and at the most (p+2n) ideal transformers. In addition, the ports will be grounded.

Note: If T(s) is a short-circuit admittance matrix, then $T(\infty)$ must be the sum of a strictly hyperdominant matrix plus a non-negative matrix.

Since proof of the theorem follows identically to the one given in Section (5.2.1) when T(s) is Y(s) with the above constraint on D, and in Section (5.2.2) when T(s) is Z(s), the synthesis procedure is illustrated with the help of suitable examples for both Y(s) and Z(s) respectively. It may be noted that the entries of the diagonal matrices Q, K and H, in this case, will correspond to the transformation-ratios rather than the gains of the amplifiers. Further, the network block diagram is same as shown in Fig.5.1(a,b) whereas, Fig.5.3(c) depicts the block diagram of N_R consisting of ideal transformers and resistors.

First, the synthesis of SPR Y(s) is illustrated with the help of the following example:

Example 5.4

The following 2x2 SPR matrix T(s) is to be realized as the short-circuit admittance matrix Y(s) of a 2-port RCT network:

$$T(s) = Y(s) = \begin{bmatrix} \frac{2s+3}{s+1} & \frac{s+2}{s+1} \\ \frac{s+2}{s+1} & \frac{2s^2+4s+3}{(s+1)^2} \end{bmatrix} . \quad p = 2,\ n = 3.$$

$$\text{Obviously, } D = T(\infty) = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} .$$

$$T_1(s) = T(s) - D$$

$$= \begin{bmatrix} \frac{1}{s+1} & \frac{1}{s+1} \\ \frac{1}{s+1} & \frac{1}{(s+1)^2} \end{bmatrix} .$$

A minimal reciprocal realization $\{A, B, C, D\}$ associated with $T_1(s)$ satisfying (5.27) and (5.28) is obtained by the algorithm presented in Chapter III, [83]. Thus

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & -2 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \text{ and } \quad D = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Now we realize the 2-port RCT network corresponding to this state-model using the procedure of Section (5.2.1).

Select, $\quad g_{11} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad Q = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$,

from (5.8d), g_{13} is obtained as

$$g_{13} = \begin{bmatrix} 0 & -1/4 \\ -1/4 & 0 \end{bmatrix} .$$

Choosing $\mathcal{C} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$, the submatrices obtained

from (5.8b) are

$$g_{23} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -1/8 \end{bmatrix} \text{ and } \quad g_{21} = \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} .$$

Select $K = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix}$, $H = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$,

from (5.8c), we obtain

$$g_{14} = \begin{bmatrix} -3/8 & 0 & 0 \\ -3/8 & 0 & -1/4 \end{bmatrix} \text{ and } g_{15} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Then select g_{22} in the manner discussed earlier; thus:

$$g_{22} = \begin{bmatrix} 7/6 & 0 & 0 \\ 0 & 1/6 & 0 \\ 0 & 0 & 2/5 \end{bmatrix}.$$

From (5.8a), the submatrices obtained are

$$g_{24} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1/8 & -3/20 \end{bmatrix} \text{ and } g_{25} = \begin{bmatrix} -1/6 & 0 & 0 \\ 0 & -1/24 & -1/8 \\ 0 & 0 & 0 \end{bmatrix}.$$

The remaining entries of $\bar{G}$ matrix can be filled in arbitrarily such that it remains hyperdominant. A suitable choice for $\bar{G}$ is given below.

$$\bar{G} = \begin{bmatrix}
2 & 0 & -\frac{1}{2} & 0 & 0 & 0 & -\frac{1}{4} & -\frac{3}{8} & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & -\frac{1}{2} & 0 & 0 & -\frac{1}{4} & 0 & -\frac{3}{8} & 0 & -\frac{1}{4} & 0 & 0 & 0 \\
-\frac{1}{2} & -\frac{1}{2} & \frac{7}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{6} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{24} & -\frac{1}{8} \\
0 & 0 & 0 & 0 & \frac{2}{5} & 0 & -\frac{1}{8} & 0 & -\frac{1}{8} & -\frac{3}{20} & 0 & 0 & 0 \\
0 & -\frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{4} & 0 & 0 & 0 & -\frac{1}{8} & 0 & \frac{3}{8} & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{3}{8} & -\frac{3}{8} & 0 & 0 & 0 & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{8} & 0 & 0 & 0 & \frac{1}{8} & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{4} & 0 & 0 & -\frac{3}{20} & 0 & 0 & 0 & 0 & \frac{2}{5} & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{24} & 0 \\
0 & 0 & 0 & -\frac{1}{8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{8}
\end{bmatrix}$$

The realization of Y(s) based on the realization of $\bar{G}$ is shown in Fig.5.7. Note that the network $\bar{N}_R$ which realizes the hyperdominant $\bar{G}$ has no internal nodes as indicated earlier.

Next the passive reciprocal synthesis of SPR Z(s) is illustrated.

Example 5.5

The following 3x3 SPR matrix T(s) is to be realized as the O.C. impedance matrix of a 3-port RCT network:

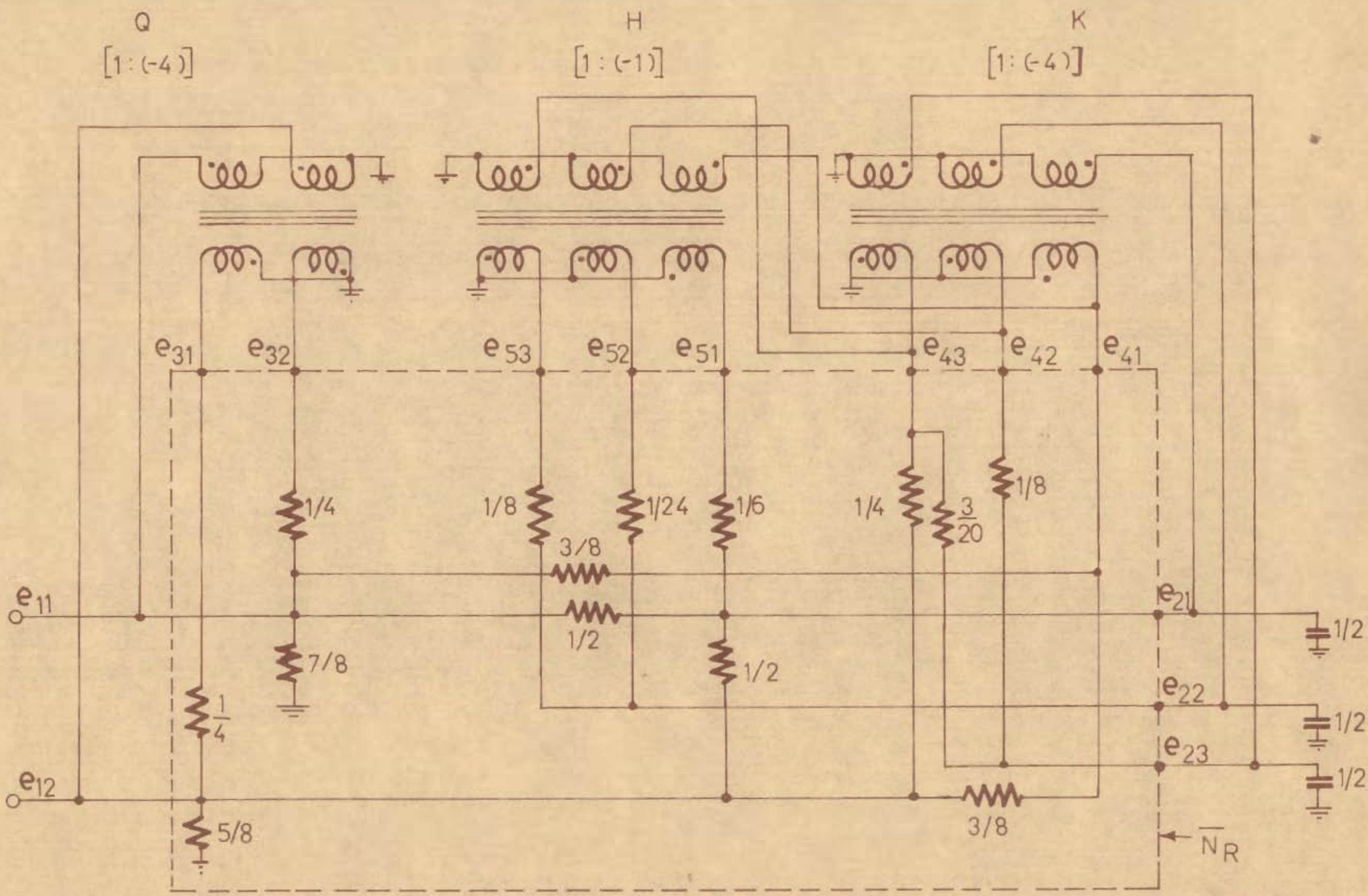

FIG. 5.7-Example 5.4: REALIZATION OF SPR Y(s).

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$$T(s) = Z(s) = \begin{bmatrix} \frac{2s+3}{s+1} & \frac{s}{s+1} & \frac{s+2}{s+1} \\ \frac{s}{s+1} & \frac{3s+4}{s+1} & \frac{s+1/2}{s+1} \\ \frac{s+2}{s+1} & \frac{s+1/2}{s+1} & \frac{2s+3}{s+1} \end{bmatrix}$$

A minimal reciprocal realization set $\{A, B, C, D\}$ of the above function has been constructed in Chapter III (Example 3.2); thus

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1/2 & 1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1/2 & 1/2 \\ 1 & 1/2 & -1/2 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}.$$

It is verified that this realization satisfies both passivity and reciprocity conditions i.e. (5.27) and (5.28). Therefore, we can proceed to realize it as an o.c. impedance matrix of a 3-port RCT network following the procedure of Section 5.2.2, and using Fig.5.1(b) and Fig.5.3(c).

Since D is nonsingular, R is set as $R = [0]$.

Select

$$g_{11} = \begin{bmatrix} 5/7 & -1/7 & -2/7 \\ -1/7 & 3/7 & -1/7 \\ -2/7 & -1/7 & 5/7 \end{bmatrix} \quad \text{and} \quad Q = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix},$$

g_{13} is obtained from (5.16a) as

$$g_{13} = [0_3].$$

Choosing
$$\mathcal{G} = \begin{bmatrix} 1/50 & 0 & 0 \\ 0 & 1/50 & 0 \\ 0 & 0 & 1/50 \end{bmatrix},$$

from (5.16b), the submatrices obtained are

$$g_{21} = \begin{bmatrix} -2/50 & 0 & -2/50 \\ -1/50 & -2/50 & -3/100 \\ 0 & 0 & -1/100 \end{bmatrix}, \quad g_{23} = \begin{bmatrix} 0 & -1/100 & 0 \\ 0 & 0 & 0 \\ 0 & -1/100 & 0 \end{bmatrix}.$$

Select

$$K = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix}, \text{ and } H = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix},$$

from (5.16c) and (5.17), we obtain

$$g_{14} = \begin{bmatrix} 0 & -13/1400 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -277/2800 \end{bmatrix},$$

and
$$g_{15} = \begin{bmatrix} -93/2800 & 0 & -1/224 \\ -5/112 & -9/1400 & -1/56 \\ -93/2800 & -197/11200 & 0 \end{bmatrix}.$$

It can easily be seen that the rows of $[g_{11}\ g_{12}\ g_{13}\ g_{14}\ g_{15}]$ are hyperdominant.

Now select g_{22} in the manner discussed earlier; thus:

$$g_{22} = \begin{bmatrix} \frac{90961}{105000} & 0 & 0 \\ 0 & \frac{83452}{525} & 0 \\ 0 & 0 & \frac{25118}{1050} \end{bmatrix},$$

from (5.16d) or (5.18), the submatrices obtained are

$$g_{24} = \begin{bmatrix} 0 & -\frac{93}{140000} & 0 \\ -\frac{2132}{35000} & 0 & -\frac{133}{140000} \\ 0 & 0 & 0 \end{bmatrix},$$

$$g_{25} = \begin{bmatrix} -\frac{224}{} & 0 & -\frac{217}{112000} \\ 0 & -\frac{249}{21} & 0 \\ -\frac{359}{14000} & -\frac{7}{40000} & -\frac{795}{672000} \end{bmatrix}.$$

The remaining entries of $\overline{G}$ can now be filled in arbitrarily such that its hyperdominant nature is retained.

A suitable choice of $\overline{G}$ is given below.

$$
\bar{G} =
\begin{bmatrix}
\frac{5}{7} & -\frac{1}{7} & -\frac{2}{7} & -\frac{2}{50} & -\frac{1}{50} & 0 & 0 & 0 & 0 & 0 & -\frac{13}{1400} & 0 & -\frac{93}{2800} & 0 & -\frac{1}{224} \\
-\frac{1}{7} & \frac{3}{7} & -\frac{1}{7} & 0 & -\frac{2}{50} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{5}{112} & -\frac{9}{1400} & -\frac{1}{56} \\
-\frac{2}{7} & -\frac{1}{7} & \frac{5}{7} & -\frac{2}{50} & -\frac{3}{100} & -\frac{1}{100} & 0 & 0 & 0 & 0 & 0 & -\frac{217}{2800} & -\frac{93}{2800} & -\frac{197}{11200} & 0 \\
-\frac{2}{50} & 0 & -\frac{2}{50} & \frac{90.961}{105} & 0 & 0 & 0 & -\frac{1}{100} & 0 & 0 & -\frac{93}{14}\times10^{-4} & 0 & -\frac{2.24}{42} & 0 & -\frac{2.17}{1120} \\
-\frac{1}{50} & -\frac{2}{50} & -\frac{3}{100} & 0 & \frac{83.352}{525} & 0 & 0 & 0 & 0 & -\frac{2.132}{35} & 0 & -\frac{133}{14}\times10^{-4} & 0 & -\frac{2.49}{21} & 0 \\
0 & 0 & -\frac{1}{100} & 0 & 0 & \frac{25.118}{1050} & 0 & -\frac{1}{100} & 0 & 0 & 0 & 0 & -\frac{35.9}{14}\times10^{-3} & -\frac{7}{40}\times10^{-3} & -\frac{7.95}{6.72}\times10^{-3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{100} & 0 & \frac{1}{100} & 0 & \frac{2}{100} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{2.132}{35} & 0 & 0 & 0 & 0 & \frac{2.132}{35} & 0 & 0 & 0 & 0 & 0 \\
-\frac{13}{1400} & 0 & 0 & -\frac{93}{14}\times10^{-4} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1393}{14}\times10^{-4} & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{217}{2800} & 0 & -\frac{133}{14}\times10^{-4} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{10.983}{140} & 0 & 0 & 0 \\
-\frac{93}{2800} & -\frac{5}{112} & -\frac{93}{2800} & -\frac{2.24}{42} & 0 & -\frac{35.9}{14}\times10^{-3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7.013}{42} & 0 & 0 \\
0 & -\frac{9}{1400} & -\frac{197}{11200} & 0 & -\frac{2.49}{21} & -\frac{7}{40}\times10^{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{15.02}{11.76} & 0 \\
-\frac{1}{224} & -\frac{1}{56} & 0 & -\frac{2.17}{1120} & 0 & -\frac{7.95}{6.72}\times10^{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{12.126}{336}
\end{bmatrix}
$$

Once $\overline{G}$ matrix of $\overline{N}_R$ is obtained, the network consisting of R, C and T can be easily constructed as shown in Fig.5.8. Further, the network $\overline{N}_R$ (Fig.5.8) which realizes the hyperdominant $\overline{G}$ has no internal nodes as indicated earlier.

6.4 CONCLUDING DISCUSSIONS

A simple and systematic synthesis procedure, based on a state-variable approach and the reactance extraction principle, has been developed whereby any qxp matrix T(s), of real rational functions of the complex frequency variable s can be realized as (i) a s.c. admittance matrix, (ii) an o.c. impedance matrix, and (iii) a transfer-impedance matrix of an active RC multiport network with the ports grounded. The realized network, in each case, contains a minimum number of n grounded capacitors having unity capacitance spread, $n = \delta[T(s)]$, and at the most (p + 2n) inverting VCVS. Of course, in the case of qxp transfer-impedance matrix synthesis q OA are also needed. The facts that all the minimum number of capacitors have same value and that all the active elements, capacitors and ports are grounded, are very much desirable if the network is to be fabricated as an integrated circuit. The distinct advantages of the proposed method over the one due to Bickart and Melvin[18] are that it requires, in general, less number of active elements and resistors while retaining all the advantages of their method. Also, for the case of transfer-impedance matrix synthesis, this method uses commercially available operational amplifiers in place of finite gain

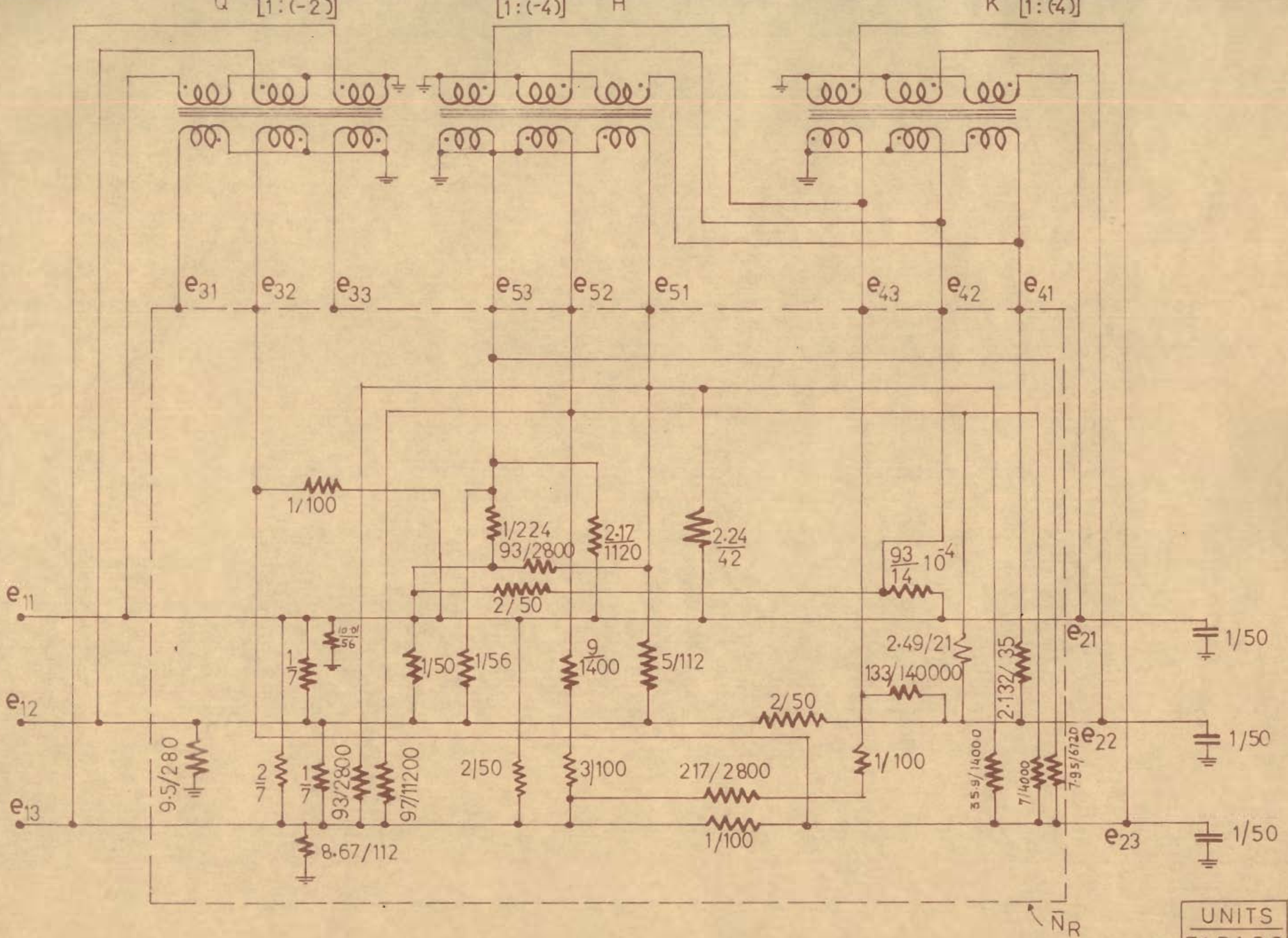

FIG. 5.8 - Example 5.5: REALIZATION OF SPR Z(s).

voltage amplifiers used in [18].

Based on the synthesis approach in Section (5.2) and the result of Chapter III, a new passive reciprocal synthesis of a SPR immittance matrix using RCT multiport network with a minimum number of capacitors has been evolved.

Since a minimum number of capacitors is used, it is conjectured that the realization of T(s) will be relatively insensitive to capacitance variations. It is hoped that further investigations of this synthesis procedure will provide a quantitative assessment of the sensitivity of selected network attributes and validity of the conjecture. Finally, the procedure can be reduced to a digital computer program.

CHAPTER VI

SUMMARY AND SUGGESTIONS FOR FURTHER WORK

6.1 INTRODUCTION

The problem of realization of a rational transfer-function matrix into an irreducible (controllable and observable) dynamical equation is one of the fundamental problems in linear system theory. In this thesis, the problem of minimal reciprocal realization from a given symmetric transfer-function matrix and symmetric impulse response matrix has been discussed. New methods for the design of multiport active RC, and passive reciprocal networks using state-variable techniques are evolved. Some endeavours are also made to re-examine some of the well-known classical synthesis procedures via state-space characterization. The present chapter gives the summary of the various results obtained in this thesis, along with some suggestions for new research problems to be pursued for further investigations in this area.

6.2 SUMMARY OF THE RESULTS

A mathematic description of linear time-invariant dynamical systems and networks in the input-output form and state-variable vector differential equation form is reviewed first. Having stated some system theory preliminaries, the problem of state-variable realization of linear, time-invariant dynamical systems and networks is discussed with a view to have a clear understanding of the subsequent results obtained in this thesis.

The problem of minimal reciprocal realization of linear time-invariant dynamical systems is considered. Two new and simplified algorithms have been evolved for obtaining minimal reciprocal realization from a given symmetric transfer-function matrix and symmetric impulse response matrix, one using the Markov-parameters and the other requiring moments of the impulse response matrix. Both the methods exploit the symmetry of the given transfer-function matrix or the impulse response matrix. In both the algorithms, the order of the Hankel matrices required in the procedure is much less than the existing methods and consequently, the computations and memory storage required are considerably reduced. The methods are essentially a modification of the Chen and Mital algorithm [29]. The realizations obtained by the proposed algorithms result in reciprocal networks. Further, a method based on the computation of the moments of the impulse response matrix is preferable when a realization is to be constructed from an empirically obtained data of G(t) which may be contaminated with noise.

An attempt has been made to establish yet another link between state-space and frequency domain methods. A state-space interpretation of the classical Foster multiport synthesis method for LC network has been presented. The proposed method is essentially an extension of the one given by Puri and Takeda [115] for 1-port Foster LC network realization.

State-variable techniques are also exploited to re-examine the well-known Cauer driving point synthesis of RC and LC networks, and active RC filter design using coefficient matching technique. A non-singular observability matrix has been employed as a canonical transformation to convert the state-model representation of the Cauer network or the active RC filter section into a canonical state-model.

A new and systematic synthesis procedure, based on a state-variable approach and the reactance extraction principle, has been developed whereby any qxp matrix T(s), of real rational functions of the complex frequency variable s, can be realized as an active RC multiport network with the ports grounded. Specifically, the proposed procedure is applied to the active synthesis of a pxp short-circuit admittance matrix Y(s) when $Y(\infty)$ is the sum of a strictly hyperdominant matrix plus a non-negative matrix, a pxp open-circuit impedance matrix, and a qxp transfer-impedance matrix with operational amplifiers. The realized network contains a minimum number of n grounded capacitors with unity capacitance spread, n being the McMillan's degree of T(s), and at the most (p+2n) inverting, grounded voltage amplifiers. Of course, in the case of qxp transfer-impedance matrix synthesis q-operational amplifiers are also required. The facts that all the minimum number of capacitors have the same value, and that all the active elements, capacitors and ports are grounded, are very much desirable if the network is to be fabricated as an integrated circuit.

The proposed method is essentially a modification over the one given by Bickeart and Melvin [18]. The modification reduces the upper bound on the number of active elements from (2p + 2n) to only (p + 2n). Also, it will usually require fewer resistors because the sub-matrix g_{22}, as discussed in the procedure and illustrated in the examples, can always be chosen to be hyperdominant. Moreover, in the case of transfer-impedance matrix synthesis, the proposed method uses commercially available operational amplifiers instead of voltage amplifiers as used in [18]. The other advantages of [18] are retained. Further, it is conjectured that the realization of T(s) will be relatively insensitive to capacitance variations because of their minimum number used in the network.

Based on the approach of multiport active RC network synthesis, considered here, and the results of minimal reciprocal realization from a given symmetric rational matrix, a new method for passive reciprocal multiport synthesis of a SPR immittance matrix using RCT network with a minimum number of capacitors, has been evolved. Since the given immittance matrix is symmetric positive real, the minimal realization set $\{A, B, C, D\}$, obtained with the help of the algorithm discussed earlier, will satisfy both reciprocity and passivity constraints, a necessary and sufficient condition for $\{A, B, C, D\}$ to be realizable with passive and reciprocal network elements [10].

6.3 SUGGESTIONS FOR FURTHER INVESTIGATIONS

The state-variable approach to linear systems realization, and passive and active network synthesis has been reviewed and applied to minimal reciprocal realization of linear time-invariant dynamical systems, classical synthesis methods, and modern active as well as passive multiport network synthesis procedures. Based on the research contribution of the thesis, some suggestions are given for further investigations in the following paragraphs:

1. Algorithms for obtaining minimal state-models $\{A, B, C, D\}$, satisfying reciprocity constraint, from a given symmetric rational transfer-function matrix and a symmetric impulse response matrix have been given in Chapter III. These state-models in general do not result in any canonical structure. It would be worthwhile to develop a method by which the minimal reciprocal state-model is in some standard canonical form, such that they can be realized further by standard techniques. In this connection, the references [19], [23], [28], [80], [100] and [161] will be useful.

2. In network problems, usually the given transfer-function matrix or impulse response matrix are symmetric. By exploiting the symmetry of the positive real immittance matrices, a passive reciprocal synthesis procedure using multiport RCT network with a minimum number of capacitors has been presented in Section (5.3). A passive reciprocal synthesis method using a minimum number of resistors was proposed in [151]. Investigations leading to a passive reciprocal synthesis procedure, from a symmetric positive real matrix, resulting in minimum number of reactive as well as resistive elements will be quite useful.

3. From a given symmetric impulse matrix, a method for constructing a minimal reciprocal realization using moments of

the impulse response has been given in Section (3.3). It would be interesting to extend this technique for time-varying impulse response matrices.

4. Because of some interest in the problem of sub-optimal approximation of a linear system of large dimension by one of the smaller dimension, a method has been recently given in [3] for obtaining a sub-optimum reduced model from the given input-output data in the form of Markov-parameters. It would be worthwhile to further reduce the order of Hankel-matrices used in [3] by exploiting the technique given in Chapter III in order to reduce the computation time and memory storage required.

5. The existing state-space techniques for the synthesis of positive real functions result either in RLC networks with transformers [10] or transformerless active networks. The equivalence of even simple transformerless procedures such as Bott-Duffin method [152] etc. in state-space has not been done so far [10]. It will be worthwhile to give state-space interpretation to such simple classical techniques possibly resulting in transformerless RLC synthesis.

6. A multiport active RC network synthesis procedure with a minimum number of capacitors for the realization of immittance matrices has been given in Section (5.2). The procedure reduces the upper bound on the number of active elements from (2p + 2n) required in [18], to only (p+2n). Investigations leading to further reduction of upper bound on the number of active elements, of course, with a minimum number of capacitors,

will be quite useful.

7. The network structure proposed in Section (5.2) is for the active RC realization of immittance functions with a minimum number of capacitors. Since the number of capacitors is minimum, it is conjectured that the realization of immittance matrices will be relatively insensitive to capacitance variations, However, further investigations are required in order to provide a quantitative assessment of the sensitivity of the selected network attributes and validity of the conjecture.

8. An approach of multiport active RC network synthesis presented in Section (5.2) is applied to the realization of a short-circuit admittance matrix, open-circuit impedance matrix, and transfer-impedance matrix. It may be extended to the active synthesis of other multiport network functions such as current gain matrix, transfer-admittance matrix, etc.

9. A passive reciprocal multiport RCT network synthesis of SPR immittance matrices is given in Section (5.3). It is worthwhile to extend this technique to the realization of other SPR multiport network functions such as voltage gain matrix, current gain matrix, transfer-impedance matrix etc. which are often available as given specifications in network synthesis.

10. Synthesis procedures described in this thesis are limited to linear networks and systems. Hardly any work has been done in the synthesis of non-linear networks. Recently, state-variable formulation of Lagrangian and Hamiltonian

equations for nonlinear networks has been proposed by Chua and McPherson [27]. It is hoped that a break through in the synthesis of nonlinear networks would be possible in the light of the procedure suggested in [27].

In conclusion, the theory of state-variables has opened new vistas in the realization of dynamical systems and lumped networks. It is hoped that the applications of the new techniques suggested in this thesis will help in solving more fascinating practical problems of nonlinear and distributed networks and systems.

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4. 'A simplified minimal realization algorithm for symmetric impulse response matrix using moments', IEEE Trans.Automatic Control, Vol.AC-18, No.6, Dec.1973, pp.683-84.
5. 'Pseudorandom pulse generator-design and applications', Presented and published in the Proc. of the Advanced Seminar on Instrumentation, held at R.E.C. Kurukshetra(Haryana), Dec.1970, pp.68-81.
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