Ph.D. Thesis

State-Space Approach to Network Synthesis

BY

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CERTIFICATE

Certified that the thesis entitled " State-Space Approach to Network Synthesis" which is being submitted by Mr. Harpreet Singh in fulfilment of the requirements for the award of the Degree of Doctor of Philosophy in Electronics and Communication Engineering of the University of Roorkee is a record of the student's own work carried out by him under my supervision and guidance. The matter embodied in this thesis has not been submitted for the award of any other degree.

This is further to certify that he has worked for a period of four years (from February, 1967 to April,1971) at this University for preparing this thesis.

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ABSTRACT

(Harpreet Singh, "State-Space Approach to Network Synthesis", Ph.D. Dissertation, Department of Electronics and Communication Engineering, University of Roorkee, April 1971. Guide and Supervisor : Dr. M.Lal, Professor, Department of Electronics and Communication Engineering, University of Roorkee.)

The state-space approach to network analysis and synthesis has aroused considerable interest during the recent years, primarily, to develop computer-aided analysis and design techniques. This thesis is concerned with the application of this approach to various aspects of network synthesis problem. In particular, state-space interpretation of classical synthesis methods is sought and new techniques for network realization from state-variable or input-output characterization are discussed with a view to evolve improved procedures.

The classical synthesis methods for linear, time-invariant networks are well known. An interesting problem concerning the use of state variables for network synthesis would be interpretation, in state-space terms, of common synthesis procedures such as Foster, Cauer, Brune etc. This problem along with the interpretation of some of the properties of network functions in state-space terms is briefly discussed first. State-space techniques for the determination of impedance matrix from its given even part and a direct method for determining the transfer-function matrix from the given state-space specifications are proposed.

In modern synthesis, many a time, the given information is in terms of state-variable characterization

rather than the input-output characterization. In this case, the natural approach to network synthesis is by state models. Before developing new synthesis procedures, generalized state models for RLC networks have been discussed. As regards synthesis procedures, a realization technique was given by Yarlagadda[86] for state model belonging to n-port LC networks. An improved method for this class has been evolved which is suitable for computerization. The proposed computer algorithm exploits the results reported by Anderson and Newcomb[6] and is free from many problems faced while using Yarlagadda and Tokad[86] procedure. Further, a synthesis procedure is proposed for a class of n-port RLC networks in which there are no cut-sets of inductors only, no loops of capacitors only and there is no coupling between the link resistances and tree-branch conductances. A synthesis procedure for a similar class of LC time-varying networks is also suggested. A procedure for the realization of A-matrix (portless networks) for a more general class of RLC networks in which there is no coupling between link resistances and tree-branch conductances is also given. It may be noted that starting from minimal state model these procedures result in minimal realizations and in case the given set of time-invariant state equations is not minimal, procedures exist for obtaining a minimal set[50]. ~~In this~~ context, for synthesis from a given set of non-minimal time-varying state equations, an interesting algorithm for removing uncontrollable (unobservable) states is proposed.

CHAPTER I

INTRODUCTION AND STATEMENT OF THE PROBLEM

1.1 INTRODUCTION

With the emerging of new levels of sophistication, advent of fast digital computers and the introduction of non-linear and time-variable devices, it has become mandatory to accept state-space approach as a powerful tool for network theory[6], [7], [9], [19], [37], [38], [47], [90]. The fact is supported by the recent trends in literature which evidence a growing interest in the use of this approach in network theory[81], [85]. A fairly large amount of work has been done in network analysis using this approach[9], [38], [66]. However, the application of this approach to network synthesis is only at the beginning stage[37]. The present thesis is devoted to the synthesis problem.

Network synthesis is concerned with the problem of passing from a given information to a description of an interconnection of subnetworks such as resistors, capacitors inductors etc. The synthesis problem can be broadly classified into two sub-problems namely, when the given information is in terms of state-variable characterization or the input-output characterization[37]. As regards state-variable characterization, it is well known that lumped, linear and passive networks can be characterized by the state equations

$$\begin{aligned} \dot{X} &= AX + BU, \\ Y &= CX + DU, \end{aligned} \qquad \ldots (1.1)$$

where X is n-vector, the state, having its components as capacitor voltages and inductor currents. U is m-vector, the input and Y is p-vector, the output. A, B, C and D are n x n, nxm, pxn and pxm dimensional matrices. A is characterized by network topology and element values; B specifies a relation between the input and the state; C gives relation between output and the state; D describes direct input-output relation which is independent of the state. The problem is to find a network which specifies equation(1.1). Sometimes, we are given only A-matrix and the problem is to find the network whose zero-state response is given by A.

As regards input-output characterization, we have

$$Y(s) = \left[C(sI-A)^{-1}B+D\right]U(s) \qquad \ldots (1.2a)$$

and

$$\mathcal{W}(s) = C(sI-A)^{-1}B+D \qquad \ldots (1.2b)$$

The problem is, given the transfer-function matrix $\mathcal{W}(s)$, to determine $\left[A,B,C,D\right]$ which can be further realized by passive networks.

Whether the given information is in state-variable characterization or the input-output characterization in s-domain, our ultimate object is to obtain a passive RLC network using state-space approach.

1.2 STATE-SPACE APPROACH IN NETWORK THEORY

The state of a dynamical system is a set of numbers such that the knowledge of these numbers and the input or forcing function will, with the equations describing the dynamics, provide future state and output of the system. The state variables constitute a set which is written as a column matrix called the state vector and the state space is defined as the set of all possible numbers the state variables can assume and the state-space approach is nothing but the characterization of the system by a set of first-order differential equations[17], [91].

The approach has been recognised by many investigators as a useful tool both in network analysis and synthesis[37], [38]. Some of the advantages of this approach in network theory are indicated as under.

(1) The state-space approach provides a general basis for the analysis and design of not only time-invariant and passive networks but time-variable and non-linear networks also[35], [38], [74].

(2) The state-space methods are especially compatible with the use of digital computer as computational aid as they involve only a few basic manipulations which can be easily programmed[26].

(3) The first-order differential equations can be easily simulated on analog computer. So, once the system is characterised by a set of first-order differential equations, it is considered solved[32].

(4) Once the solution of state equations is found, one knows the instantaneous values of all the elements in a network in terms of minimal set of variables, because the other variables can be

expressed as a linear combination of the chosen state variables[38].

(5) Time responses can be easily found by Taylor-series approximation to the transition matrix instead of inverse transform technique requiring factoring of characteristic polynomial[40].

(6) Network functions can be easily found out by characterizing a network by a set of first-order differential equations because there exist algorithms which when applied to state equations give the network function without the problem of rational matrix inversion. On the other hand, the conventional loop or node method for finding these functions is quite involved[40].

(7) The state-variable technique offers greater scope for extensions to problems such as equivalent network problems. The reason being that once a realization is found, the other equivalent realizations can be determined through a range of non-singular transformation. In this way, optimal realization based on considerations such as sensitivity etc. can be obtained[81].

1.3 STATEMENT OF THE PROBLEM

This thesis is concerned with the problem of passing from a given information to a description of an interconnection of passive subnetworks preferably without gyrators and transformers exploiting the state-space approach. Specifically, the problems considered in this thesis can be stated as follows:

(1) To seek the state-space interpretation of the well-known results in classical network synthesis. This part of the problem deals with the interpretation of already well established results by state-space technique, e.g. the interpretation of the properties of driving-point functions for two-element-kind networks and the common synthesis procedures such as Foster, Cauer and Brune etc. from state-space point of view.

(2) Given information in terms of state-variable characterization, to evolve new realization procedures and algorithms convenient for digital computer studies for different classes of transformerless LC, RLC and portless networks.

(3) Given a positive real symmetric matrix, to evolve synthesis procedures resulting in RLCT networks without gyrators using the state-space approach and modify the state-space synthesis procedures for some common classes of driving-point functions such as minimum biquadratic functions.

It may be mentioned that some aspects of these problems have been considered by a few investigators[3], [11], [19], [36], [85]-[87] and some results are available. For example, as regards the first problem, the necessary and sufficient conditions for positive realness of a matrix[3], realization of LC driving-point functions by Foster Method[86], interpretation of poles, zeros, residues [11],[36] and the multiport Darlington Synthesis from state-space point of view, have already been obtained[5].

As regards the second problem, the synthesis of state models for RC, RL and RLC networks and the realization of restricted classes of A-matrix have been obtained[19], [61], [75], [86], [87], but these methods, due to one reason or the other, cannot be implemented easily on digital computer.

As far as the third problem is concerned, two procedures have recently been proposed by Yarlagadda[85] for the realization of a symmetric positive real matrix without involving gyrators. The methods suggested by him can be modified so as to implement them easily on digital computer.

1.4 ORGANISATION OF THE THESIS

The classical synthesis methods are now a well established discipline and are discussed in several books [25], [27], [79]. But it has been emphasized recently that state-variable approach is more promising especially due to the study of equivalent networks[81] and so an attempt has been made in this thesis to first interpret some of the classical results from state-space point of view.

When the given information is in state-variable characterization of RLC networks, the common synthesis approach is the decomposition of state equations into two parts, one part giving the elements and the topology of the reactive elements and the other giving the element values and topology of the hybrid-resistive network and complementing the topology of reactive network [19], [87]. Simpler procedures for the decomposition and hence realization of state equations for different classes of networks have been sought.

Further, when the given information is in terms of input-output characterization in s-domain, it has been recognised by many investigators that state-model approach to network synthesis is more fruitful, because the state model of the network provides more direct information about the network topology than the network matrices [86]. So, an essential step towards any such synthesis development would be to translate the given specifications in s-domain to the formation of a state model. The state model is then subjected to a non-singular co-ordinate transformation so as to satisfy the constraints of passivity and reciprocity and thus to result in a passive reciprocal RLC network. Improved procedures for the realization from s-domain specifications exploiting state models have been proposed.

For convenience the following arrangement has been adopted in the organisation of this thesis.

The review of existing literature related to network synthesis using state-space approach has been included in the second chapter. This chapter also contains derivations of state models belonging to linear time-invariant, and time-varying networks. The interpretation of various terms such as poles, zeros, etc. and necessary and sufficient conditions for positive realness, from state-space point of view,have been included. Some key properties of passive and reciprocal state models are also recalled with a view to have a clear perspective and for frequent reference in the sequel.

Chapter III discusses the state-space interpretation of well-known properties of networks and their realization. In particular, the properties of LC and RC driving-point functions are discussed using state-variable technique. The state-space interpretation of Cauer procedure for reactance functions and Brune procedure for biquadratic minimum functions are discussed. The methods presented are illustrated by examples. An algorithm for the determination of impedance matrix from specified even part using state-variable technique is presented.

In Chapter IV, the realization of state equations for different classes of networks is discussed. In particular, a new procedure, suitable for computerization for the decomposition of state equations for LC n-port networks is proposed. A digital computer programme and the corresponding flow chart for the proposed method are given. An example considered previously[86] is computerized and the method compared with the existing method. Further, for restricted classes of n-port RLC time-invariant and LC time-varying state models and for a restricted class of A-matrix, simpler procedures are developed. Further, a procedure for removing uncontrollable and/or unobservable modes from non-minimal time-varying state equations is given. The methods are illustrated with the help of examples.

In Chapter V, a synthesis procedure for a symmetric matrix which results in a reciprocal realization is given. A different procedure is also proposed for the realization of a symmetric positive real matrix into a passive RLC networks and without the use of gyrators. The proposed method is simpler than the methods suggested recently [85]. A comparison of the proposed method is made with the existing methodsby way of a numerical example considered previously [85]. The synthesis of minimum biquadratic driving-point functions is also discussed in this chapter.

A summary of the work done has been given in Chapter VI. A brief guide for further work which might lead to more fruitful results has been included in this chapter.

CHAPTER II

CRITICAL REVIEW AND GENERAL CONSIDERATIONS

2.1 INTRODUCTION

Several attempts have been made in utilizing the state-space approach for the various aspects of network synthesis problem in the last decade [1]-[9], [32]-[45], [52]-[67], [69]-[78] and there is an ever-increasing interest in the problem as is evidenced by the abundance of recent papers on the subject [81], [82], [85]. Many useful techniques and results, long recognised in system design and based on state-space approach [91], have begun to illuminate network design problem. The synthesis from both the transfer-function description and the state-equation description has been taken up by various investigators [37]. The validity of transfer-function description is limited only to the representation of linear systems while state-space description is indispensable for non-linear and time-varying systems. As frequency domain description may still be preferred for linear time-invariant systems for many design problems, the translation of one description from the other has also attracted the attention of several investigators so as to bridge the gap between the two characterizations [11], [36]. A critical survey of the work done in this field is embodied in this chapter.

2.2 HISTORICAL REVIEW

The state-variable technique in network theory was not used until 1957 when Bashkow[9] gave a new method of network description by representing the dynamics of RLC networks by a set of first-order differential equations in a mathematical form

$$F_a + \frac{dy_a}{dt} = A_a y_a \qquad \ldots (2.1)$$

This procedure for characterizing the network was based on choosing a 'proper tree' and selecting capacitor tree-branch voltages and link inductor currents as state variables. Following Bashkow, Bryant gave a method to characterize the same in an explicit form[13],

$$\dot{X} = AX + BU , \qquad \ldots (2.2a)$$

$$Y = CX + DU . \qquad \ldots (2.2b)$$

The choice was made on 'normal tree'[12] and again the capacitor -tree-voltages and link inductor currents formed a set of dynamically independent variables. Following the work of Bashkow and Bryant, a number of papers appeared in which the description of the network is made in terms of the state model. Much of the effort was directed towards the formation of state models from given network which is essentially the problem of network analysis.

The synthesis problem, on the other hand, is the determination of a structure of the network giving a topological disposition and the element values of the components from given state equations. The solution of the problem belonging to reactive n-port networks was given by Yarlagadda and Tokad in [86] and its extension to RLC networks by the same authors in [87]. If only portless passive networks are considered, the network equations simply become

$$\dot{X} = AX \; . \qquad \ldots (2.3)$$

The problem of realization of A-matrix for different classes of RLC networks has also been investigated by several authors [19], [57]. Dervisoglu [19] proposed a procedure for the realization of A-matrix belonging to a class of half-degenerate RLC networks. Nordgren and Tokad [57] considered the same problem when A-matrix also admits loops of capacitances and cut-sets of inductances only. Yarlagadda [84] gave a procedure for the realization of A-matrix obtained from a given characteristic polynomial.

Usually, the given information is in terms of input-output characterization in s-domain and therefore, the problem of determining the matrices A,B,C and D from a specified transfer-function matrix has also been widely investigated. The problem was first established in the theory of linear dynamical systems by Gilbert [22] and Kalman [32] who propounded the theorems concerning the decomposition of rational matrices when the matrix has only simple poles, and numerators and denominators of each entry in the matrix are given in factored form. The non-minimal realizations of the cases when the numerators and denominators are not given in factored form has also been discussed in these papers. These realizations can be made minimal by removing uncontrollable and unobservable modes by standard techniques. The realization of transfer-function matrix with multiple poles was also given by Kalman[34] first, using the theory of elementary divisors. Similar methods for minimal and non-minimal realizations for simple as well as multiple poles have been given by various authors from time to time — the suggested procedures having one advantage or the other over the previous methods. Probably the simplest method for computing realization from a transfer-function matrix has been given by Ho and Kalman[28]. The method was evolved from the study of so-called Markov parameters[21], [73]. Although the problem of minimal realization has reached a state of maturity, the research in this direction is still going on for improved methods[39]. Once a set of first-order differential equations is obtained, the realization is said to have been done as this set can be simulated on analog computer[32]. But in passive network synthesis, one is always interested in finding such set of state equations as result in passive RLCT networks. When one such set of A,B,C,D matrices is found out, all others can be determined by applying non-singular transformations to it. Hence, the strategy for passive network synthesis, is to start with any minimal realization of a positive real immittance matrix and then introduce the coordinate transformation on the realization so as to satisfy the constraintsof passivity and reciprocity.

Keeping the above in view, Anderson and Newcomb made successful attempts to establish a synthesis procedure for lossless n-ports in [7] and for a general positive real matrix in [6] resulting in RLC elements, transformers and gyrators. Youla and Tissi [90] considered scattering parameters and have given synthesis procedures from state model point of view, using RLC elements and transformers. Combining the procedures of Anderson and Newcomb, and Youla and Tissi, Yarlagadda [85] has developed procedures for realizing the hybrid matrix through state-space considerations.

Due to the inevitable use of state-variable technique for non-linear systems and the extensive use of transfer function for design techniques in linear systems, it is worthwhile to establish a communication link between the two approaches. In accordance with this spirit, a number of investigators initiated the work to bridge this gap. The state-space interpretation of the common terms used in s-domain synthesis such as poles, zeros, residues etc. has already been given [11], [36], [58]. The positive realness of a matrix and Foster form synthesis etc. have also been investigated from state-space point of view [3], [86].

Recently, a method has been proposed to identify the given system by state equations directly from the record of input and output sequences rather than determining the transfer-function matrix first[24]. The method will prove useful in synthesis especially in the cases when the given input-output data is contaminated with noise.

Having given a brief historical review, some recent work concerning the state-space interpretation of classical concepts, state models and input-output characterization is discussed further in the following sections.

2.3 STATE-SPACE INTERPRETATION

It is well known that state-space techniques have contributed greatly to the modern network and control theory. Yet the importance of frequency domain methods cannot be belittled due to their extensive use in the majority of design problems and the situation is unlikely to change in the near future. This has led the system engineer to communicate both in terms of state-variable characterization and the input-output characterization. Some endeavours have been made to explore the connection between these two representations. Brocket[11] developed expressions for poles and zeros of a system in terms of state matrices. Kuh[36] also obtained the similar relations by signal flow graph representation of the state-space characterization of the linear systems.

Consider a single-input, single-output system

$$\dot{X} = AX + bu ,$$
$$y = cX + du . \qquad \ldots (2.4)$$

It is shown that, for the case d=o, the characteristic polynomial of A gives poles of the transfer function and characteristic polynomial of A_o gives the zeros of transfer function [36] where

$$A_o = (I - \frac{bc}{cb})A ; \qquad \ldots (2.5a)$$

whereas, for the case d≠o, A_o is given as

$$A_o = (I - \frac{bc}{sd+cb})A \qquad \ldots (2.5b)$$

as given in [36], or

$$A_o = A - \frac{bc}{d} \qquad \ldots (2.5c)$$

as given in [67]. The expression for the transfer function is given by So [67] as follows :

$$z(s) = \frac{\det \begin{bmatrix} d & -c \\ b & sI-A \end{bmatrix}}{\det \begin{bmatrix} sI-A \end{bmatrix}} . \qquad \ldots (2.5d)$$

The interpretation of feedback, return difference etc. has been given in [36] for the state model (2.4). However, if we consider linear RLC networks, it is shown in [76] that their state-space representation with independent sources of the network as exclusive component of the input vector does not always exist. When the state variables are chosen as capacitor voltages and inductor currents, the state-space characterization of linear RLC networks involves derivative of sources and in general, for single-input single-output system, the state model is of the form [60], [76]

$$\dot{X} = AX + bu + b_o\dot{u} \,, \qquad y = cX + du + d_o\dot{u} \,. \qquad \ldots (2.6a)$$

The expression (2.5d) becomes [43]

$$z(s) = \frac{\det \begin{bmatrix} s & -1 & 0 \\ d & d_o & -c \\ b & b_o & sI-A \end{bmatrix}}{\det \left[sI-A \right]} \,. \qquad \ldots (2.6b)$$

The various expressions viz. network response, poles, zeros controllability and observability and models for composite systems etc. have been derived in [43] for the model (2.6).

The relationship between state-space and frequency-domain descriptions of the dynamical systems has further led the network theorist to examine the interpretation of classical synthesis in state-space terms. Anderson and

Brockot[5] gave the state-space interpretation of Multiport Darlington method. The interpretation of Classical Foster Method in state-space terms was given by Yarlagadda[86].

2.4 STATE-VARIABLE CHARACTERIZATION

State-variable characterization, though long recognised in system theory, has recently been adopted in network theory. Network theory is inextricably linked with the mathematics of differential equations, so the natural approach for the network models should be in a form compatible with the current mathematical results. The state variables lead us to a set of first-order differential equations known as the state models. The salient steps in the derivation of a state model for n-port RLC network are given below.

Let a normal tree[12] be chosen and the branch voltages and currents be partitioned as follows :

$$V_1 = \begin{bmatrix} V_R \\ V_S \\ V_L \\ V_K \end{bmatrix} \quad \text{and} \quad I_1 = \begin{bmatrix} I_R \\ I_S \\ I_L \\ I_K \end{bmatrix} \qquad \ldots (2.7a)$$

for the links, where the subscripts R, S and L denote link resistance, elastance and inductance and V_K and I_K denote the voltage and current vectors of current sources and

$$V_2 = \begin{bmatrix} V_V \\ V_C \\ V_\Gamma \\ V_G \end{bmatrix} \quad \text{and} \quad I_2 = \begin{bmatrix} I_V \\ I_C \\ I_\Gamma \\ I_G \end{bmatrix} \qquad \ldots (2.7b)$$

for the tree-branches, where the subscripts C, Γ and G denote capacitance, reciprocal inductance and conductance and V_V and I_V denote the voltage and current vectors of the voltage source. Let the form of F expressing the topological relations between links and the branches be chosen as follows[38]:

$$F = \begin{bmatrix} F_{RV} & F_{RC} & 0 & F_{RG} \\ F_{SV} & F_{SC} & 0 & 0 \\ F_{LV} & F_{LC} & F_{L\Gamma} & F_{LG} \\ F_{KV} & F_{KC} & F_{K\Gamma} & F_{KG} \end{bmatrix}, \qquad \ldots (2.8)$$

where F_{RV} expresses the topological relation between resistance links and voltage-source tree-branches and similarly, F_{KV} expresses the topological relation between current-source links and voltage-source tree-branches.

The Kirchhoff voltage and current law equations can be written as

$$\begin{bmatrix} I & F \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 0 , \qquad \ldots (2.9a)$$

and

$$\begin{bmatrix} -F' & I \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = 0 . \qquad \ldots (2.9b)$$

The branch voltage-current relations can be written as follows[38].

$$\begin{bmatrix} I_S \\ I_C \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}\begin{bmatrix} V_S \\ V_C \end{bmatrix}, \qquad \ldots (2.10a)$$

$$\begin{bmatrix} V_R \\ I_G \end{bmatrix} = \begin{bmatrix} R_1 & 0 \\ 0 & G_2 \end{bmatrix}\begin{bmatrix} I_R \\ V_G \end{bmatrix}, \qquad \ldots (2.10b)$$

$$\begin{bmatrix} V_L \\ V_\Gamma \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}\begin{bmatrix} I_L \\ I_\Gamma \end{bmatrix}. \qquad \ldots (2.10c)$$

Combining (2.9) and (2.10) and eliminating the unwanted variables, and letting $\overset{*}{V}_V = V_V, \overset{*}{V}_K = V_K$ and $\overset{*}{I}_V = -I_V$ and $\overset{*}{I}_K = -I_K$ represent the vectors of terminal variables for an n-port RLC network [86] the state model can be obtained in the form (2.2) where

$$X = \begin{bmatrix} V_C \\ I_L \end{bmatrix}, \qquad U = \begin{bmatrix} \overset{*}{V}_V \\ \dot{\overset{*}{V}}_V \\ \overset{*}{I}_K \\ \dot{\overset{*}{I}}_K \end{bmatrix}, \qquad Y = \begin{bmatrix} \overset{*}{V}_K \\ \overset{*}{I}_V \end{bmatrix} \qquad \ldots (2.11)$$

and

$$A = \begin{bmatrix} \mathscr{C}^{-1} & 0 \\ 0 & \mathscr{L}^{-1} \end{bmatrix}\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$= \begin{bmatrix} \mathscr{C}^{-1} & 0 \\ 0 & \mathscr{L}^{-1} \end{bmatrix}\begin{bmatrix} -F'_{RC}\mathscr{R}^{-1}F_{RC} & F'_{LC}-F'_{RC}\mathscr{R}^{-1}F_{RG}R_2F'_{LG} \\ -F_{LC}+F_{LG}\mathscr{G}^{-1}F'_{RG}G_1F_{RC} & -F_{LG}\mathscr{G}^{-1}F'_{LG} \end{bmatrix},$$

$$\ldots (2.12a)$$

$$B = \begin{bmatrix} \mathcal{C}^{-1} & 0 \\ 0 & \mathcal{L}^{-1} \end{bmatrix} \begin{bmatrix} -B_{11} & -B_{12} & -B_{13} & 0 \\ B_{21} & 0 & B_{23} & B_{24} \end{bmatrix}$$

$$\begin{bmatrix} \mathcal{C}^{-1} & 0 \\ 0 & \mathcal{L}^{-1} \end{bmatrix} \begin{bmatrix} -F'_{RC}\,\mathcal{R}^{-1}F_{RV} & -F'_{SC}C_1F_{SV} & -F'_{KC}+F'_{RC}\,\mathcal{R}^{-1}F_{RG}R_2F'_{KG} & 0 \\ F_{LG}\,\mathcal{G}^{-1}F'_{RG}G_1F_{RV}-F_{LV} & 0 & F_{LG}\,\mathcal{G}^{-1}F'_{KG} & L_{12}F'_{K\Gamma}+F_{L\Gamma}-L_{22}F'_{K\Gamma} \end{bmatrix},$$

.. (2.12b)

$$C = \begin{bmatrix} -F_{KC}+F_{KG}\,\mathcal{G}^{-1}F'_{RG}G_1F_{RC}-B'_{24}\,\mathcal{L}^{-1}A_{21} & -B'_{23}+B'_{24}\,\mathcal{L}^{-1}A_{22} \\ B'_{11}-B'_{12}\,\mathcal{C}^{-1}A_{11} & -F'_{LV}+F'_{RV}\,\mathcal{R}^{-1}F_{RG}R_2F'_{LG}+B'_{12}\,\mathcal{C}^{-1}A_{12} \end{bmatrix}$$

.. (2.12c)

$$D = \begin{bmatrix} -F_{KV}+F_{KG}\,\mathcal{G}^{-1}F'_{RG}G_1F_{RV}-B'_{24}\,\mathcal{L}^{-1}B_{21} & 0 \\ F'_{RV}\,\mathcal{R}^{-1}F_{RV}-B'_{12}\,\mathcal{C}^{-1}B_{11} & F'_{SV}C_1F_{SV}-B'_{12}\,\mathcal{C}^{-1}B_{12} \\ F_{KG}\,\mathcal{G}^{-1}F'_{KG}+B'_{24}\,\mathcal{L}^{-1}B_{23} & F_{K\Gamma}L_{22}F'_{K\Gamma}-B'_{24}\,\mathcal{L}^{-1}B_{24} \\ F'_{KV}-F'_{RV}\,\mathcal{R}^{-1}F_{RG}R_2F'_{KG}-B'_{12}\,\mathcal{C}^{-1}B_{13} & 0 \end{bmatrix},$$

.. (2.12d)

where

$$\left.\begin{aligned} \mathcal{R} &= R_1+F_{RG}R_2F'_{RG}\,, \\ \mathcal{G} &= G_2+F'_{RG}G_1F_{RG}\,, \\ \mathcal{C} &= C_2+F'_{SC}C_1F_{SC}\,, \\ \mathcal{L} &= L_{11}+F_{L\Gamma}L_{21}+L_{12}F'_{L\Gamma}+F_{L\Gamma}L_{22}F'_{L\Gamma}\,. \end{aligned}\right] \quad .. \text{ (2.13)}$$

$\mathcal{R}$ is the loop resistance matrix for the fundamental loops defined by the resistance links, $\mathcal{G}$ is the cut-set conductance matrix for the fundamental cut-sets defined by the conductance tree branches, both are positive definite. $\mathcal{C}$ (symmetric and positive definite) is the cut-set capacitance matrix for the normal tree i.e. the cut-set capacitance matrix for those fundamental cut-sets defined by capacitance tree-branches and $\mathcal{L}$ (symmetric positive semi-definite) is the loop conductance matrix for the normal tree, i.e. the loop inductance matrix for those fundamental loops defined by inductance links[38].

It may be noted that the state model given by (2.11) to (2.13) is general enough to encompass all n-port RLC networks and the model appears to be promising in connection with the synthesis of n-port general RLC networks. Similar state models using different approaches have also been derived elsewhere [13], [37], [38], [47], [52], [53], [85].

In order to obtain state-models for linear time-varying systems, eqn. (2.10) becomes (taking $L_{12}=L_{21}=0$)

$$\begin{bmatrix} I_S \\ I_C \end{bmatrix} = \frac{d}{dt} \begin{bmatrix} C_1(t) & 0 \\ 0 & C_2(t) \end{bmatrix} \begin{bmatrix} V_S \\ V_C \end{bmatrix}, \qquad \text{.. (2.14a)}$$

$$\begin{bmatrix} V_R \\ I_G \end{bmatrix} = \begin{bmatrix} R_1(t) & 0 \\ 0 & G_2(t) \end{bmatrix} \begin{bmatrix} I_R \\ V_G \end{bmatrix}, \qquad \text{.. (2.14b)}$$

$$\begin{bmatrix} V_L \\ V_\Gamma \end{bmatrix} = \frac{d}{dt} \begin{bmatrix} L_{11}(t) & 0 \\ 0 & L_{22}(t) \end{bmatrix} \begin{bmatrix} I_L \\ I_\Gamma \end{bmatrix} \qquad \text{.. (2.14c)}$$

and proceeding in the same way as for time-invariant systems, and dropping the (t) for convenience the state model is given by

$$\left.\begin{aligned}
A_{11} &= -F'_{RC}\mathcal{R}^{-1}F_{RC}-\dot{\mathcal{C}}\ , \\
A_{12} &= F'_{LC}-F'_{RC}\mathcal{R}^{-1}F_{RG}R_2F'_{LG}\ , \\
A_{21} &= -F_{LC}+F_{LG}\mathcal{G}^{-1}F'_{RG}G_1F_{RC}\ , \\
A_{22} &= -F_{LG}\mathcal{G}^{-1}F'_{LG}-\dot{\mathcal{L}}\ , \\
\\
B_{11} &= F'_{RC}\mathcal{R}^{-1}F_{RV}-F'_{SC}\dot{C}_1F_{SV}\ , \\
B_{12} &= F'_{SC}C_1F_{SV}\ , \\
B_{13} &= F'_{KC}-F'_{RC}\mathcal{R}^{-1}F_{RG}R_2F'_{KG}\ , \\
B_{14} &= 0\ , \\
\\
B_{21} &= -F_{LV}+F_{LG}\mathcal{G}^{-1}F'_{RG}G_1F_{RV}\ , \\
B_{22} &= 0 \\
B_{23} &= F_{LG}\mathcal{G}^{-1}F'_{KG}-F_{L\Gamma}\dot{L}_{22}F'_{K\Gamma}\ , \\
B_{24} &= F_{L\Gamma}L_{22}F'_{K\Gamma}\ .
\end{aligned}\right] \qquad \text{.. (2.15)}$$

The matrices C and D for the time-varying case will be similar to (2.12c) and (2.12d) for the time-invariant case.

Following the same approach, the state models can be obtained for active linear time-varying networks for which the voltage-current relations for resistive elements including some types of active elements and gyrators, transformers etc. can be described by the hybrid equations[38], [52]

$$\begin{bmatrix} V_R \\ I_G \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} I_R \\ V_G \end{bmatrix} . \qquad .. (2.16)$$

The A-matrix for this case has been derived in [52] and B, C and D are not difficult to obtain. Further, by following the approach discussed in [53] for decomposing **non-linear** time-varying reactive elements, the state-model for the class of non-linear networks considered in [53] can also be obtained. The A-matrix for this class has already been given in [53].

2.4.1 SYNTHESIS FROM STATE MODEL

From the generalised state model obtained for the time-invariant case(eqn. 2.12) and for time-varying case (eqn. 2.15), the state models for different sub-classes of time-invariant and time-varying networks can be derived. For example, by substituting R=0 in (2.12), state model for LC networks can be obtained and by substituting L=0, state model for RC networks can be obtained and so on.

Given the information in terms of state-variable characterization, the state models derived in topological expressions, serve as useful starting point for network synthesis. By comparing the various topological expressions, with the given values, a set of simultaneous equations can be obtained, the solution of which gives the element values as well as the topology of the network which can be tested for realizability by the well-known techniques [49]. The synthesis of LC and RLC time-invariant state equations has been discussed by Yarlagadda and Tokad in [86] and [87]. The realization of active networks from state-equations has been considered by Martens[48].

2.4.2 SYNTHESIS OF A-MATRIX

Many a time, we are given only zero-input response. In such cases, we obtain portless networks i.e. networks without excitations and the state equations (2.2) take the form

$$\dot{X} = AX ,$$

where A can be decomposed into [57],

$$\begin{bmatrix} \mathcal{E}^{-1} & 0 \\ 0 & \mathcal{L}^{-1} \end{bmatrix} \begin{bmatrix} -A_{11} & A_{12} \\ -A_{12}' & A_{22} \end{bmatrix} = D_a^{-1} H_a \qquad \ldots (2.17)$$

such that $\mathcal{E}$ and $\mathcal{L}$ become the terminal matrices of the capacitor and inductor network and H_a is the terminal hybrid matrix of the resistive network. By decomposing the given A as shown above, the element values and partly the topology of capacitor and inductor elements can be obtained from D_a thus reducing the realization problem to the realization of hybrid resistive network given by H_a.

2.5 INPUT-OUTPUT CHARACTERIZATION

When the given information is in terms of input-output characterization in s-domain, a natural approach to network synthesis is by decomposing a given positive real $Z(s)$ into a quadruple $[A, B, C, D]$ given by $Z(s)=C(sI-A)^{-1}B+D$. $[A, B, C, D]$ is known as the realization of $Z(s)$, since by knowing matrices A, B, C and D, the set of first-order differential equations corresponding to the given n-port can be simulated on analog computer. There exists a smallest integer n_o in a minimal (irreducible) set and is known as the complexity [12] of the network. If the order of A is more than n_o, the set of equations obtained is non-minimal (reducible). The number n_o is given by the number of reactive elements less the number of independent capacitor-only loops less the number of independent inductor only cut-sets in a network. Thus the number is related to the degree of a rational matrix, written as $\delta W(s)$ which denotes the minimum number of reactive elements required in any passive synthesis of a positive real impedance matrix. Various definitions of degree have been given from time to time and were reconciled as one by Kalman [34]. Further, given one minimal realization, it was shown in [32] that all others can be obtained choosing a transformation T ranging through the set of non-singular matrices, such that set

$$[T^{-1}AT, \; T^{-1}B \; , \; CT] \qquad .. (2.18)$$

constitutes another minimal realization. It may be noted that D is always $W(\infty)$ irrespective of T. All transfer-function matrices $W(s)$ with $W(\infty)$ finite, possess minimal realizations.

2.5.1 GENERAL PASSIVE NETWORK SYNTHESIS

An interesting approach for passive network synthesis using state variable technique based on reactance extraction is reviewed in this section.

Consider the minimal passive synthesis of given Z(s). The resulting n-port can be divided into non-dynamic elements and dynamic elements of unit inductors. It is reasonably well-known [4] that capacitors can be replaced by gyrators and inductors and the ideal transformers can be used to make all inductors unity. The impedance matrix of the frequency independent portion is given [37] by

$$M_1 = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \qquad .. \ (2.19)$$

where

$$Z(s) = z_{11} - z_{12}(sI + z_{22})^{-1} z_{21}. \qquad .. \ (2.20)$$

Comparing (2.19) with (1.2b) and using (2.18), we get

$z_{11} = D$,

$z_{12} = CT$,

$z_{21} = -T^{-1}B$,

$z_{22} = -T^{-1}AT$

or

$$M_1 = \begin{bmatrix} I & 0 \\ 0 & T^{-1} \end{bmatrix} \begin{bmatrix} D & C \\ -B & -A \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & T \end{bmatrix} . \quad .. (2.21)$$

Therefore, when minimal realization is obtained, the impedance matrix of the frequency-independent network is given by (2.21). The non-singular matrix T gives the flexibility to introduce passivity and reciprocity constraints. The concepts of passivity and reciprocity as related to state-space synthesis are briefly discussed next.

2.5.2 PASSIVITY CRITERION

Passivity as characterized in terms of impedance matrix is given first. If M_1 is the impedance matrix of a frequency-independent network, its symmetric part must be positive semi-definite. The transformation T is thus chosen in such a manner that it makes the symmetric part of M_1 positive semi-definite. It will be worthwhile in this context to state the Anderson's lemmas for positive realness[3] which will lead to the transformation T.

LEMMA 2.1 - Let Z(s) be a matrix of rational functions such that $Z(\infty) = 0$ and Z(s) has poles only in Res<0. Let $[A, B, C]$ be a minimal realization of Z(s). Then Z(s) is positive real if and only if there exists a symmetric positive definite matrix P and a matrix L such that

$$PA + A'P = -L'L ,$$

$$PB = C' . \quad .. (2.22)$$

LEMMA 2.2- Let a positive real $Z(s)$ have all imaginary axis poles with $Z(\infty) = 0$ and let A, B, C be a minimal realization of Z. Then there exists a symmetric positive definite matrix P such that

$$PA + A'P = 0 ,$$
$$PB = C' . \qquad .. (2.23)$$

LEMMA 2.3- Let $Z(s)$ be a matrix of rational transfer functions such that $Z(\infty)$ is finite and $Z(s)$ has poles which lie in $\text{Re } s<0$ or are simple on $\text{Re } s = 0$ and $[A, B, C, D]$ be a minimal realization of $Z(s)$. Then $Z(s)$ is positive real if and only if there exists a symmetric positive definite P and matrices W_0 and L such that

$$PA + A'P = -L'L ,$$
$$PB = C' - L'W_0 ,$$
$$W_0'W_0 = D + D' , \qquad .. (2.24)$$

and there exists a matrix $W(s)$, unique to within left multiplication by a constant orthogonal matrix such that $Z(s) + Z'(-s) = W'(-s)W(s)$.

Reference [4] describes procedures for determining the symmetric positive definite matrix P from which suitable T given by

$$P = T'T \qquad .. (2.25)$$

can be determined, which, when applied on $[A, B, C, D]$, makes the symmetric part of M_1 positive semi-definite and its realization can be achieved by RTG(Resistance, Transformer and Gyrator) network [4].

2.5.3 RECIPROCITY CRITERION

The following theorem is stated here concerning reciprocity. For proof see [85].

Theorem 2.1- Let $Z(s)$ be a $n \times n$ matrix of real rational transfer functions with $Z(\infty)$ finite. $Z(s)$ possesses a state model of the form

$$\dot{X} = AX + BU ,$$

$$Y = CX + DU ,$$

such that

$$(I \dotplus \Sigma)M_1 = \text{symmetric matrix} \qquad .. (2.26)$$

where Σ being a unique ordered diagonal matrix of plus ones and minus ones, $\dotplus$ denotes direct sum and

$$\left[I \dotplus \Sigma\right] = \begin{bmatrix} I & 0 \\ 0 & \Sigma \end{bmatrix} \quad \text{and} \quad M_1 = \begin{bmatrix} D & C \\ -B & -A \end{bmatrix},$$

if and only if $Z(s) = Z'(s)$.

It has been shown in [37] and [90] that there always exists a symmetric T such that

$$M_1' = (I \dotplus T^{-1})\, M_1 (I \dotplus T), \qquad .. (2.27)$$

where T can always be expressed as

$$T = L_1 \Sigma L_1' . \qquad .. (2.28)$$

2.5.4 PASSIVITY AND RECIPROCITY

The flexibility in choosing a transformation T should allow us to introduce both passivity constraints and reciprocity constraints i.e. by Lemma 2.1, 2.2, 2.3 and (2.25), T can be chosen which guarantees passive network while application of (2.27) guarantees reciprocal synthesis. Unfortunately, it is difficult to find a T which satisfies both passivity and reciprocity conditions simultaneously.

It may be mentioned, as has been shown in [37], that all reciprocal realizations for RL or RC impedance matrices are passive. So in case of RL and RC networks, once a transformation T is found which results in reciprocal realization, passivity is automatically guaranteed.

2.6 CONCLUSION

The recent literature available on the use of state-variable technique in network synthesis is an evidence of the growing interest of research workers in this field. Both, state-variable characterization and input-output characterization are being amply used as given specifications for network synthesis. The synthesis procedures from state-variable characterization still require further modifications in order to become amenable to computerization.

Moreover, the synthesis procedures from input-output characterization, have yet to be moulded so as to evolve a satisfactory algorithm for the synthesis of positive real symmetric matrices into passive RLC networks without gyrators.

In other words, the investigations of transformation which, when applied on a state model, makesit satisfy passivity and reciprocity constraints, need be carried out. These problems are discussed in the following chapters with a view to achieve the desired objectives.

CHAPTER III

STATE-SPACE INTERPRETATION

3.1 INTRODUCTION

It cannot be disputed that the state-space techniques have generated a lot of interest in network analysis and synthesis in the past few years. Even then, at present, majority of the design problems are being done using frequency-domain methods and the situation is not likely to change within the coming few years. Hence, it becomes imperative to establish a communication link between the state-space characterization and the frequency domain methods. Although classical synthesis methods are well known, their interpretation in state-space terms will be of great interest and some work has already been initiated in this direction [3], [5], [11], [36]. The present chapter discusses the state-space interpretation of the well-known properties of network functions and the common synthesis procedures such as Foster, Cauer and Brune etc. Besides, the determination of minimum reactance matrix from the given even part specifications and the determination of transfer-function matrix from the given state equations are also discussed.

3.2 STATE-SPACE INTERPRETATION OF SOME OF THE PROPERTIES OF NETWORK FUNCTIONS

Some of the properties of reactance functions are interpreted first in state-space terms.

(1) Property No.1

Let $Z(s)$ be a lossless impedance matrix with $Z(\infty)$ finite and let $[A,B,C,D]$ be a minimal realization for $Z(s)$. Then there exists a symmetric positive definite matrix P such that

$$PA + A'P = 0 \,, \qquad .. \ (3.1a)$$

$$PB = C' \,, \qquad .. \ (3.1b)$$

where A,B,C are the state-matrices.

This property has also been proved in [3] but here it is proved in a more generalized way i.e. from the state model belonging to the LC networks. This proof gives better insight as the state models are the basic building blocks in state-space analysis just as loop and node methods are in classical analysis. The proof can be carried out by taking a general LC network having sources at the ports and writing down its state model in topological entities. This state model can be obtained by substituting R = 0 in eqn.(2.12) and is given by

$$\dot{X} = AX + B_t U + B_o \dot{U} \,,$$

$$Y = CX + D_t U + D_o \dot{U} \,,$$

where,

$$A = \begin{bmatrix} \mathcal{C}^{-1} & 0 \\ 0 & \mathcal{L}^{-1} \end{bmatrix} \begin{bmatrix} 0 & F'_{LC} \\ -F_{LC} & 0 \end{bmatrix} , \qquad .. \ (3.2a)$$

$$B_t = \begin{bmatrix} \mathcal{C}^{-1} & 0 \\ 0 & \mathcal{L}^{-1} \end{bmatrix} \begin{bmatrix} 0 & -F'_{KC} \\ -F_{LV} & 0 \end{bmatrix} , \qquad .. \ (3.2b)$$

$$B_o = \begin{bmatrix} \mathcal{C}^{-1} & 0 \\ 0 & \mathcal{L}^{-1} \end{bmatrix} \begin{bmatrix} -F'_{SC}C_1F_{SV} & 0 \\ 0 & F_L\Gamma L_{22}F'_{K}\Gamma \end{bmatrix}, \qquad .. (3.2c)$$

$$C = \begin{bmatrix} 0 & -F'_{LV}+(F'_{SC}C_1F_{SV})'\mathcal{C}^{-1}F'_{LC} \\ -F_{KC}-(F_L\Gamma L_{22}F'_K\Gamma)'\mathcal{L}^{-1}(-F_{LC}) & 0 \end{bmatrix}, \qquad .. (3.2d)$$

$$D_t = \begin{bmatrix} 0 & F'_{KV}-(F'_{SC}C_1F_{SV})'\mathcal{C}^{-1}(-F'_{KC}) \\ -F_{KV}-(F_L\Gamma L_{22}F'_K\Gamma)'\mathcal{L}^{-1}(-F_{LV}) & 0 \end{bmatrix}, \qquad .. (3.2e)$$

$$D_o = \begin{bmatrix} F'_{SV}C_1F_{SV}-(F_{SC}C_1F_{SV})'\mathcal{C}^{-1}(F_{SC}C_1F_{SV}) & 0 \\ 0 & F_K\Gamma L_{22}F'_K\Gamma-(F_L\Gamma L_{22}F'_K\Gamma)'\mathcal{L}^{-1}(F_L\Gamma L_{22}F'_K\Gamma) \end{bmatrix} \qquad .. (3.2f)$$

Let us consider the transformation

$$X = X_1 + B_oU , \qquad .. (3.3)$$

$$\dot{X} = \dot{X}_1 + B_o\dot{U} . \qquad .. (3.4)$$

Substituting (3.3) and (3.4) in (3.2), we get

$$\dot{X}_1 + B_o\dot{U} = AX_1 + AB_oU + B_tU + B_o\dot{U} .$$

or

$$\dot{X}_1 = AX_1 + (B_t + AB_o)U \qquad .. (3.5a)$$

and

$$Y = CX_1 + (D_t + CD_o)U . \qquad .. (3.5b)$$

Or

$$\dot{X}_1 = AX_1 + BU \,, \qquad \text{.. (3.6a)}$$

$$Y = CX_1 + DU \,, \qquad \text{.. (3.6b)}$$

where

$$B = B_t + AB_o$$

$$= \begin{bmatrix} 0 & \mathcal{C}^{-1}F'_{KC} \\ -\mathcal{L}^{-1}F_{LV} & 0 \end{bmatrix} + \begin{bmatrix} 0 & \mathcal{C}^{-1}F'_{LC} \\ -\mathcal{L}^{-1}F_{LC} & 0 \end{bmatrix} \begin{bmatrix} -\mathcal{C}^{-1}F'_{SC}C_1F_{SV} & 0 \\ 0 & \mathcal{L}^{-1}F_{L\Gamma}L_{22}F'_{K\Gamma} \end{bmatrix}$$

$$= \begin{bmatrix} \mathcal{C}^{-1} & 0 \\ 0 & \mathcal{L}^{-1} \end{bmatrix} \begin{bmatrix} 0 & -F'_{KC}+F'_{LC}\mathcal{L}^{-1}F_{L\Gamma}L_{22}F'_{K\Gamma} \\ -F_{LV}+F_{LC}\mathcal{C}^{-1}(F'_{SC}C_1F_{SV}) & 0 \end{bmatrix}$$

.. (3.7)

and

$$D = D_t + CD_o \,.$$

Let $P = \begin{bmatrix} \mathcal{C} & 0 \\ 0 & \mathcal{L} \end{bmatrix}$, a symmetric positive definite matrix.
.. (3.8)

From (3.2a) we find that

$$PA + A'P = 0$$

and from (3.2d),(3.7) and (3.8), we get,

$$PB = C' \quad .$$

Property No.2- Poles and zeros of LC driving-point functions lie on imaginary axis.

Proof

A driving-point function z(s) can be written as eqn. (2.5d). If it is a proper function i.e. z(s) vanishes at $s = \infty$, it becomes

$$z(s) = \frac{P(s)}{Q(s)} = \frac{\det\begin{bmatrix} 0 & -c \\ b & sI-A \end{bmatrix}}{\det\begin{bmatrix} sI - A \end{bmatrix}} \qquad .. (3.9)$$

If it is not proper, its reciprocal must be proper. Therefore, no generality is lost in starting with a proper LC driving-point function.

For lossless z(s), we have from (3.1)

$$PA + A'P = 0 \ ,$$
$$Pb = c' \ . \qquad .. (3.10)$$

Applying a transformation T such that $T^{-1} = P^{1/2}$ the set given by expression (2.18) becomes

$$A_p = P^{1/2} A \ P^{-1/2} \ ,$$
$$b_p = P^{1/2} b \ , \qquad .. (3.11)$$
$$c_p = c \ P^{-1/2} \ .$$

It may be noted that since, on applying the above transformation, driving-point function remains the same, the poles and zeros are not affected by a similarity transformation. From (3.10) and (3.11) we get

$$A_p + A_p' = 0 \ ,$$
$$b_p = c_p' \ . \qquad .. (3.12)$$

Therefore from (3.9)

$$z(s) = \frac{\det \begin{bmatrix} 0 & -b_p' \\ b_p & sI-A_p \end{bmatrix}}{\det \left[sI - A_p \right]} \qquad \text{.. (3.13)}$$

A_p is a skew symmetric matrix from eqn.(3.12). Numerator of eqn.(3.13) is the determinant of bordered $[sI-A_p]$ matrix by a column and its negative transpose row with the added zero diagonal element. Simple analysis[51] will show that roots of numerator and denominator polynomials are imaginary and hence poles and zeros of LC driving-point function lie on imaginary axis.

Property No.3- Poles and zeros of LC driving-point functions interlace.

Proof

Consider a proper driving-point function as discussed above. In a suitable neighbourhood of infinity, z(s) can be expanded as [28]

$$z(s) = cb\ s^{-1} + cAb\ s^{-2} + cA^2b\ s^{-3} + \ldots \qquad \text{.. (3.14)}$$

As explained in the proof of property no.2, there always exists a transformation T which transforms A, b, c to a form such that new A is skew symmetric and new b is equal to the transpose of new c.

Since new c is a non-zero column vector, the scalar $cc'>0$. Further, since A is skew matrix cAb, which is equal to cAc', a sclar, will always be zero,

(A is skew symmetric, any matrix product cAc' is either skew or a null matrix. In our case as c is a row vector, the product cAc' will be scalar and hence equal to zero.)

and $cA^2c' \leqslant 0$ since $cA^2c' = cAAc' = cA(-A'c') = -(cA)(cA)'$.

$$\left.\begin{aligned} \text{Similarly, } cA^3c' &= 0 \\ cA^4c' &\geqslant 0 \\ cA^5c' &= 0 \\ cA^6c' &\leqslant 0 \text{ and so on.} \end{aligned}\right\} \quad .. (3.15)$$

Therefore

$$z(j\omega) = \frac{cc'}{j\omega} + \frac{cA^2c'}{(j\omega)^3} + \frac{cA^4c'}{(j\omega)^5} + \frac{cA^6c'}{(j\omega)^7} + \ldots, \quad .. (3.16)$$

$$z(j\omega) = jX(\omega),$$

$$= \frac{-jcc'}{\omega} + \frac{jcA^2c'}{\omega^3} + \frac{-jcA^4c'}{\omega^5} + \frac{jcA^6c'}{\omega^7} + \ldots \quad .. (3.17)$$

or

$$X(\omega) = \frac{-cc'}{\omega} + \frac{cA^2c'}{\omega^3} + \frac{-cA^4c'}{\omega^5} + \frac{cA^6c'}{\omega^7} + \ldots \quad .. (3.18)$$

From (3.15) and (3.18) it is obvious that $\frac{dX}{d\omega}$ will always be positive and therefore poles and zeros of reactance function will always interlace. It may be noted that it is possible to interpret and prove various other properties of network functions from state-space point of view. For example,

Property No.4- Poles and zeros of RC driving-point function lie on negative real axis and interlace.

Proof

By substituting L = 0 in the generalized state model given in eqn.(2.12), and letting only a current source at the input we get the state model for RC impedance function as

$$\left.\begin{aligned} \dot{V}_c &= \mathcal{C}^{-1}\left[-F'_{RC}\mathcal{R}^{-1}F_{RC}\right]V_c + \mathcal{C}^{-1}\left[-F'_{KC}+F'_{RC}\mathcal{R}^{-1}F_{RG}R_2F'_{KG}\right]\overset{*}{i}_K \\ \overset{*}{v}_K &= \left[-F_{KC}+F_{KG}\mathcal{G}^{-1}F'_{RG}G_1F_{RC}\right]V_c + F_{KG}\mathcal{G}^{-1}F'_{KG}\overset{*}{i}_K \end{aligned}\right] \quad ..(3.19)$$

where, $F_{KG}\mathcal{G}^{-1}F'_{KG}$ is a scalar and is denoted by d.

Consider the transformation,

$$T^{-1} = \mathcal{C}^{1/2} \quad . \qquad ... (3.20)$$

The new state model becomes

$$\dot{V}_{c1} = \mathcal{C}^{-1/2}\left[-F'_{RC}\mathcal{R}^{-1}F_{RC}\right]\mathcal{C}^{-1/2}V_{c1} + \mathcal{C}^{-1/2}\left[-F'_{KC}+F'_{RC}\mathcal{R}^{-1}F_{RG}R_2F'_{KG}\right]\overset{*}{i}_K$$

$$= AV_{c1}+bu , \qquad .. (3.21)$$

$$\overset{*}{v}_K = \left[-F_{KC}+F_{KG}\mathcal{G}^{-1}F'_{RG}G_1F_{RC}\right]\mathcal{C}^{-1/2}V_{c1}+F_{KG}\mathcal{G}^{-1}F'_{KG} \quad ..$$

$$= cV_{c1}+du . \qquad .. (3.22)$$

Clearly, A is a symmetric negative definite matrix thus having negative real eigen-values. Further from (3.21) and (3.22), b = c' [38] and d is a scalar. So $A - \frac{bc}{d}$ is a symmetric negative semi-definite matrix. Thus zeros of eqn.(2.5c) are negative real. Hence, poles and zeros of RC driving point impedance function lie on negative real axis. As regards interlacing property, matrix A, which gives poles is a real symmetric matrix and the matrix giving zeros (eqn.2.5d) will be bordered $[sI-A]$ matrix by a column and its transpose row with the added diagonal element d. Simple analysis will show that poles and zeros will interlace for this case[29]. By letting a voltage source at the input, we can prove this property for RC admittance functions. Similarly the property can be proved for RL case. Having discussed the state-space interpretation of the various properties of network functions, a similar interpretation for the classical synthesis procedures is sought next.

3.3 STATE-SPACE INTERPRETATION OF CLASSICAL SYNTHESIS METHODS

3.3.1 FOSTER METHOD

The classical Foster method assumes the topology of the network a priori. The method of realization for the case of canonic 1-port LC networks, using state model and without assuming topology in advance, was considered by Yarlagadda[86]. Similar realization for RC case is discussed here.

Consider an RC driving-point impedance function which can be decomposed as

$$z(s) = K_o + \frac{K_1}{s+\sigma_1} + \frac{K_2}{s+\sigma_2} + \dots + \frac{K_n}{s+\sigma_n} \qquad \text{.. (3.23)}$$

where σ's and K's are positive constants. A state model corresponding to z(s) can be written as

$$A = \begin{bmatrix} -\sigma_1 & & & \\ & -\sigma_2 & & \\ & & \ddots & \\ & & & \sigma_n \end{bmatrix}, \qquad b = \begin{bmatrix} \sqrt{K_1} \\ \sqrt{K_2} \\ \vdots \\ \sqrt{K_n} \end{bmatrix}, \qquad \text{.. (3.24)}$$

$$c = \begin{bmatrix} \sqrt{K_1} & \sqrt{K_2} & \ldots & \sqrt{K_n} \end{bmatrix}, \qquad d = K_0 \,.$$

Applying the transformation

$$T = \begin{bmatrix} \frac{1}{\sqrt{K_1}} & & & \\ & \frac{1}{\sqrt{K_2}} & & \\ & & \ddots & \\ & & & \frac{1}{\sqrt{K_n}} \end{bmatrix}, \qquad \text{.. (3.25)}$$

$$\left.\begin{aligned} A_t = T^{-1}AT &= \begin{bmatrix} -\sigma_1 & & & \\ & -\sigma_2 & & \\ & & \ddots & \\ & & & -\sigma_n \end{bmatrix}, \quad b_t = T^{-1}b = \begin{bmatrix} K_1 \\ K_2 \\ \vdots \\ K_n \end{bmatrix}, \\ c_t = T_c c &= \begin{bmatrix} 1 & 1 & \ldots & 1 \end{bmatrix}, \qquad d = K_0 \,. \end{aligned}\right\} \quad \text{.. (3.26)}$$

Comparing with the state model obtained from eqn. (3.19) for the class of RC networks having $F_{RG} = 0$,

$$\begin{aligned} \dot{X} &= AX + bu \\ &= \left[\mathcal{C}^{-1}\right]\left[-F'_{RC}R_1^{-1}F_{RC}\right]V_c + \mathcal{C}^{-1}(-F'_{KC})\overset{*}{i}_K \,, \end{aligned}$$

$$\begin{aligned} \text{and } y &= cX + du \\ \overset{*}{v}_K &= -F_{KC}V_c + F_{KG}R_2F'_{KG}\overset{*}{i}_K \,, \end{aligned} \qquad \text{.. (3.27)}$$

and noting that the entries of F'_{KC} are to be ± 1, 0 etc., we can take

$$\mathcal{C}^{-1} = \begin{bmatrix} K_1 & & & \\ & K_2 & & \\ & & \ddots & \\ & & & K_n \end{bmatrix} . \qquad .. \ (3.28)$$

Hence, eqn. 3.27 becomes

$$\dot{X} = \begin{bmatrix} K_1 & & & \\ & K_2 & & \\ & & \ddots & \\ & & & K_n \end{bmatrix} \begin{bmatrix} -\frac{\sigma_1}{K_1} & & & \\ & -\frac{\sigma_2}{K_2} & & \\ & & \ddots & \\ & & & -\frac{\sigma_n}{K_n} \end{bmatrix} V_c + \begin{bmatrix} K_1 & & & \\ & K_2 & & \\ & & \ddots & \\ & & & K_n \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \overset{*}{i}_K$$

.. (3.29)

$$\overset{*}{v}_K = \begin{bmatrix} 1 & 1 & \dots & 1 \end{bmatrix} V_c + K_o \overset{*}{i}_K .$$

Therefore

$$(-F'_{RC} R_1^{-1} F_{RC}) = \begin{bmatrix} -\frac{\sigma_1}{K_1} & & & \\ & -\frac{\sigma_2}{K_2} & & \\ & & \ddots & \\ & & & -\frac{\sigma_n}{K_n} \end{bmatrix}, \quad F'_{KC} = \begin{bmatrix} -1 \\ -1 \\ \vdots \\ -1 \end{bmatrix},$$

$$F_{RC} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix}, \quad R_1^{-1} = \begin{bmatrix} \frac{\sigma_1}{K_1} & & & \\ & \frac{\sigma_2}{K_2} & & \\ & & \ddots & \\ & & & \frac{\sigma_n}{K_n} \end{bmatrix},$$

and

$$R_1 = \begin{bmatrix} \frac{K_1}{\sigma_1} & & & \\ & \frac{K_2}{\sigma_2} & & \\ & & \ddots & \\ & & & \frac{K_n}{\sigma_n} \end{bmatrix} . \qquad .. (3.30)$$

Obviously F_{KG} will be 1 and R_2 will be the scalar K_0.

$[F \quad I]$ so obtained will be

$$\begin{array}{c} \\ K \\ R_1 \\ R_2 \\ \vdots \\ R_n \end{array} \begin{array}{c} \begin{array}{ccccc} G & C_1 & C_2 & \dots & C_n \end{array} \\ \left[\begin{array}{ccccc|ccccc} 1 & -1 & -1 & \dots & -1 & 1 & & & & \\ 0 & 1 & 0 & \dots & 0 & & 1 & & & \\ 0 & 0 & 1 & \dots & 0 & & & 1 & & \\ \vdots & \vdots & \vdots & & \vdots & & & & \ddots & \\ 0 & 0 & 0 & \dots & 1 & & & & & 1 \end{array}\right] \end{array} , \qquad .. (3.31)$$

and the corresponding graph and the network are as shown in Figs. (3.1a) and (3.1b).

3.3.2 STATE-SPACE INTERPRETATION OF CAUER METHOD

The procedure given below discusses the Cauer realization for one-port LC networks in state-space terms. The procedure discussed here, exploits the graph-theoretic concepts and the topology of the network need not be assumed a priori. Consider a proper reactance function

$$z_1(s) = \frac{a_1 s^{n-1} + a_3 s^{n-3} + \dots + a_{n-1} s}{s^n + b_2 s^{n-2} + \dots + b_n} \qquad .. (3.32)$$

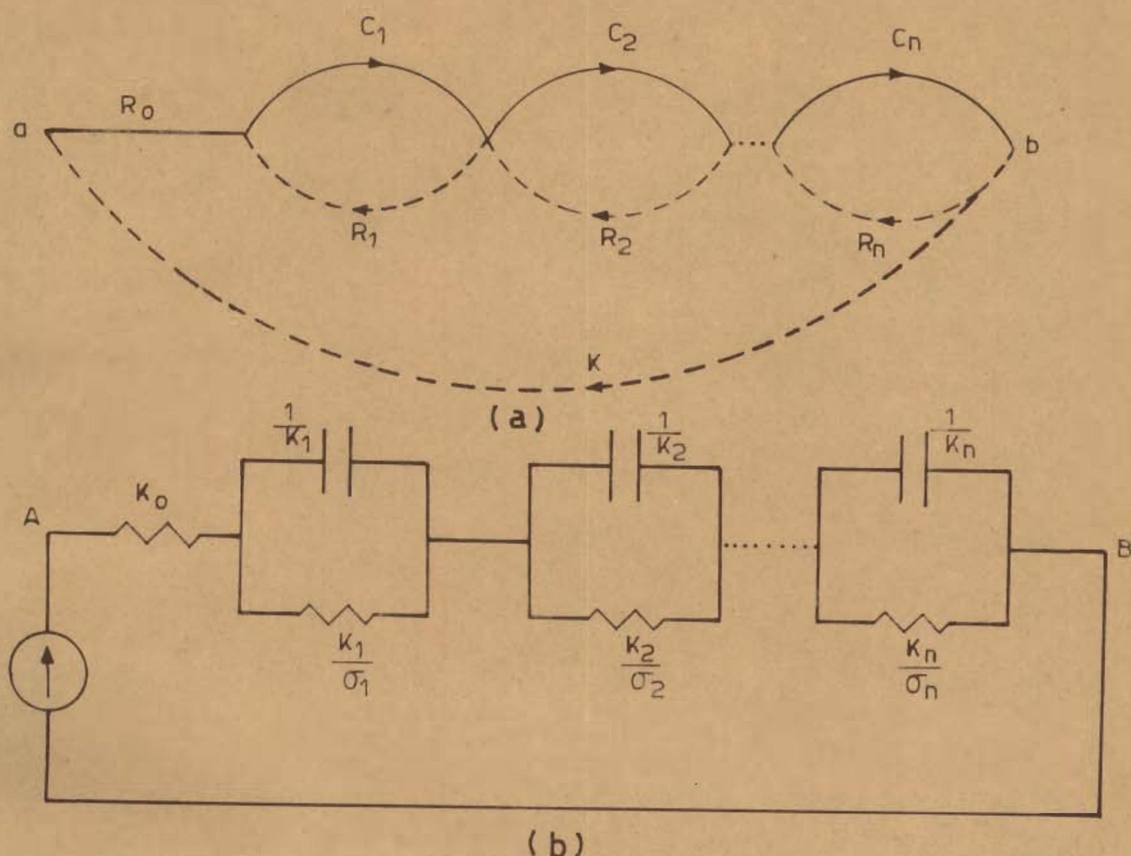


FIG. 3.1 (a)_REALIZATION OF CIRCUIT MATRIX [EQN. 3.31]
(b)_REALIZATION OF z (s) [EQN. 3.23]

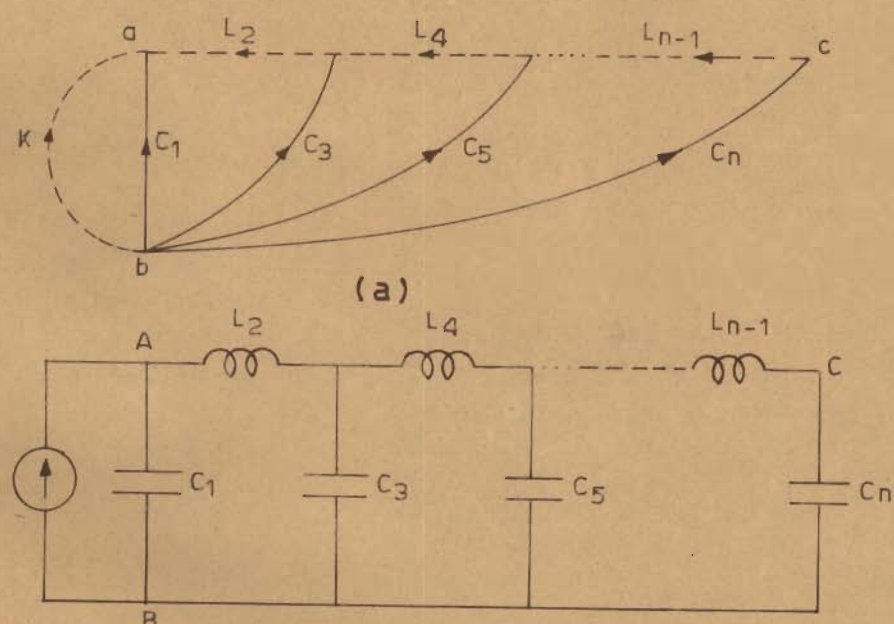


FIG. 3.2 (a)_REALIZATION OF CIRCUIT MATRIX [EQN. 3.42]
(b)_REALIZATION OF $z_1(s)$ [EQN. 3.32]

In order to find out the Hurwitz coefficients, coefficients of numerator and denominator of eqn. (3.32) are written in the usual tabular form as

$$\begin{array}{c|cccc} k_o & 1 & b_2 & \cdots & b_n \\ k_1 & a_1 & a_3 & \cdots & a_{n-1} \\ \vdots & & & & \\ k_n & & & & \end{array} \qquad .. (3.33)$$

where $k_o = 1$, $k_1 = a_1$ etc. i.e. the entries in the first column of eqn. (3.33). The state model for eqn. (3.32) can be obtained in the modified Schwarz form as [46]

$$\begin{bmatrix} \dot{V}_c \\ \dot{I}_L \end{bmatrix} = \begin{bmatrix} 0 & -\frac{k_2}{k_o} & & & & & & \\ 1 & 0 & -\frac{k_3}{k_1} & & & & & \\ & 1 & 0 & -\frac{k_4}{k_2} & & & & \\ & & 1 & 0 & -\frac{k_5}{k_3} & & & \\ & & & 1 & 0 & -\frac{k_6}{k_4} & & \\ & & & & \cdot & \cdot & \cdot & \\ & & & & & 1 & 0 & -\frac{k_n}{k_{n-2}} \\ & & & & & & 1 & 0 \end{bmatrix} \begin{bmatrix} V_c \\ I_L \end{bmatrix} + \begin{bmatrix} \frac{1}{k_o} \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \overset{*}{i}_K ,$$

$$\overset{*}{v}_K = \begin{bmatrix} k_1 & 0 & 0 & 0 & \cdot, \cdot & 0 & 0 \end{bmatrix} \begin{bmatrix} V_c \\ I_L \end{bmatrix} . \qquad .. (3.34)$$

The validity of eqn. (3.34) can also be verified by comparing its Markov parameter [cb, cAb, cA^2b etc.] with those of eqn. (3.32). These should be same as proved in the following Lemma.

Lemma 3.1- The products CB, CAB, CA^2B etc. of all realization corresponding to a given transfer-function matrix should be same.

Proof

If A,B,C is a realization of $\mathcal{W}(s)$, any other realization $[A_1, B_1, C_1]$ is given by $T^{-1}AT$, $T^{-1}B$, CT. Therefore, $C_1B_1 = CB$; $C_1A_1B_1 = CAB$ and so on.

Next, applying the transformation

$$T = \begin{bmatrix} \frac{1}{k_1} & & & & & & \\ & \frac{1}{k_2} & & & & & \\ & & \frac{1}{k_3} & & & & \\ & & & \frac{1}{k_4} & & & \\ & & & & \frac{1}{k_5} & & \\ & & & & & \frac{1}{k_{n-1}} & \\ & & & & & & \frac{1}{k_n} \end{bmatrix} \quad .. (3.35)$$

to eqn. (3.34) we get the state model

$$\begin{bmatrix} \dot{V}_c \\ \dot{I}_L \end{bmatrix} = \begin{bmatrix} 0 & \frac{-k_1}{k_o} & & & & & \\ \frac{k_2}{k_1} & 0 & \frac{-k_2}{k_1} & & & & \\ & \frac{k_3}{k_2} & 0 & \frac{-k_3}{k_2} & & & \\ & & \frac{k_4}{k_3} & 0 & \frac{-k_4}{k_3} & & \\ & & & \frac{k_5}{k_4} & 0 & \frac{-k_5}{k_4} & \\ & & & & \frac{k_{n-1}}{k_{n-2}} & 0 & \frac{-k_{n-1}}{k_{n-2}} \\ & & & & & \frac{k_n}{k_{n-1}} & 0 \end{bmatrix} \begin{bmatrix} V_c \\ I_L \end{bmatrix} + \begin{bmatrix} \frac{k_1}{k_o} \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \overset{*}{i}_K , \quad ..(3.36)$$

$$\overset{*}{v}_K = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_c \\ I_L \end{bmatrix} .$$

Applying the transformation [84]

$$T = \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & \frac{n+1}{2} & \frac{n+3}{2} & & n-1 & n \end{array} \\ \left[\begin{array}{cccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ \\ n-1 \\ n \end{array} \qquad .. (3.37a)$$

when n is odd and

$$T = \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & \frac{n}{2} & \frac{n}{2}+1 & & n-1 & n \end{array} \\ \left[\begin{array}{cccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ \\ n-1 \\ n \end{array} \qquad .. (3.37b)$$

when n is even, we get the state model

$$\begin{bmatrix} \dot{V}_c \\ \hdashline \dot{I}_L \end{bmatrix} = \left[\begin{array}{cccc:cccc} & & & & \frac{-k_1}{k_0} & & & \\ & & & & \frac{k_3}{k_2} & \frac{-k_3}{k_2} & & \\ & & & & & \frac{k_5}{k_4} & \frac{-k_5}{k_4} & \\ & & & & & & \cdots & \frac{k_n}{k_{n-1}} \\ \hdashline \frac{k_2}{k_1} & \frac{-k_2}{k_1} & & & & & & \\ & \frac{k_4}{k_3} & \frac{-k_4}{k_3} & & & & & \\ & & \cdots & & & & & \\ & & \frac{k_{n-1}}{k_{n-2}} & \frac{-k_{n-1}}{k_{n-2}} & & & & \end{array}\right] \begin{bmatrix} V_c \\ \hdashline I_L \end{bmatrix} + \begin{bmatrix} \frac{k_1}{k_0} \\ 0 \\ 0 \\ 0 \\ \hdashline 0 \\ 0 \\ 0 \end{bmatrix} \overset{*}{i}_K$$

$$\overset{*}{v}_K = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \cdots 0 & 0 \end{bmatrix} \begin{bmatrix} V_c \\ I_L \end{bmatrix} . \qquad .. (3.38)$$

The above state model can be written as

$$\begin{bmatrix} \dot{V}_c \\ \dot{i}_L \end{bmatrix} = \left[\begin{array}{ccccc|cccc} \frac{k_1}{k_o} & & & & & & & & \\ & \frac{k_3}{k_2} & & & & & & & \\ & & \frac{k_5}{k_4} & \ddots & & & & & \\ & & & & \frac{k_n}{k_{n-1}} & & & & \\ \hline & & & & & \frac{k_2}{k_1} & & & \\ & & & & & & \frac{k_4}{k_3} & \ddots & \\ & & & & & & & & \frac{k_{n-1}}{k_{n-2}} \end{array}\right] \left[\begin{array}{cccc|ccccc} & & & & -1 & & & & \\ & & & & 1 & -1 & & & \\ & & & & & 1 & -1 & & \\ & & & & & & \ddots & \ddots & \\ & & & & & & & & 1 \\ \hline 1 & -1 & & & & & & & \\ & 1 & -1 & & & & & & \\ & & \ddots & \ddots & & & & & \\ & & 1 & -1 & & & & & \end{array}\right] \begin{bmatrix} V_c \\ I_L \end{bmatrix}$$

$$+ \left[\begin{array}{ccccc|cccc} \frac{k_1}{K_o} & & & & & & & & \\ & \frac{k_3}{k_2} & & & & & & & \\ & & \frac{k_5}{k_4} & \ddots & & & & & \\ & & & & \frac{k_n}{k_{n-1}} & & & & \\ \hline & & & & & \frac{k_2}{k_1} & & & \\ & & & & & & \frac{k_4}{k_3} & \ddots & \\ & & & & & & & & \frac{k_{n-1}}{k_{n-2}} \end{array}\right] \left[\begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \end{array}\right] \overset{*}{i}_K ,$$

$$\overset{*}{v}_K = \begin{bmatrix} 1 & 0 & 0 \cdot 0 & | 0 & 0 \cdot 0 \end{bmatrix} \begin{bmatrix} V_c \\ I_L \end{bmatrix} . \qquad .. (3.39)$$

Or

$$\begin{bmatrix}\dot{V}_c\\ \dot{I}_L\end{bmatrix} = \left[\begin{array}{cccc|cccc} C_1^{-1} & & & & & & & \\ & C_3^{-1} & & & & & & \\ & & C_5^{-1} \ddots & & & & & \\ & & & C_n^{-1} & & & & \\ \hline & & & & L_2^{-1} & & & \\ & & & & & L_4^{-1} & & \\ & & & & & & \ddots & \\ & & & & & & & L_{n-1}^{-1} \end{array}\right] \left[\begin{array}{cccc|cccc} & & & & -1 & & & \\ & & & & 1 & -1 & & \\ & & & & & 1 & -1 & \\ & & & & & & \ddots & \\ & & & & & & & 1 \\ \hline 1 & -1 & & & & & & \\ & 1 & -1 & & & & & \\ & & \ddots & & & & & \\ & & 1 & -1 & & & & \end{array}\right] \begin{bmatrix}V_c\\ I_L\end{bmatrix}$$

$$+ \left[\begin{array}{cccc|cccc} C_1^{-1} & & & & & & & \\ & C_3^{-1} & & & & & & \\ & & C_5^{-1} \ddots & & & & & \\ & & & C_n^{-1} & & & & \\ \hline & & & & L_2^{-1} & & & \\ & & & & & L_4^{-1} & & \\ & & & & & & \ddots & \\ & & & & & & & L_{n-1}^{-1} \end{array}\right] \left[\begin{array}{c} 1\\ 0\\ 0\\ \vdots\\ 0\\ \hline 0\\ 0\\ \vdots\\ 0 \end{array}\right] \overset{*}{i}_K \qquad .. (3.40)$$

$$\overset{*}{v}_K = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 \cdots & 0 & 0 & 0 \cdots & 0 \end{array}\right] \begin{bmatrix}V_c\\ I_L\end{bmatrix},$$

from which various submatrices can be found by comparing with the corresponding model for LC one-port and we get

$$F_{LC} = \begin{bmatrix} -1 & 1 & & \\ & -1 & 1 & \\ & & \ddots & \\ & & -1 & 1 \end{bmatrix},$$

$$F_{KC} = \begin{bmatrix} -1 & 0 & 0 \ldots & 0 \end{bmatrix}. \qquad .. (3.41)$$

Therefore, $\begin{bmatrix} F & I \end{bmatrix}$ can be written as

$$\begin{array}{c} \\ L_2 \\ L_4 \\ \\ L_{n-1} \\ K \end{array}
\begin{array}{c}
\begin{array}{cccc} C_1 & C_3 & C_5 & \cdots C_n \end{array} \\
\left[\begin{array}{cccc:cccc}
-1 & 1 & & & 1 & & & \\
 & -1 & 1 & & & 1 & & \\
 & & \cdots & & & & \ddots & \\
 & & -1 & 1 & & & 1 & \\
-1 & 0 & 0 \cdots & 0 & & & & 1
\end{array}\right]
\end{array} \qquad ..\ (3.42)$$

from which the graph and the corresponding network can be obtained as shown in figs.(3.2a) and (3.2b).

3.3.3 STATE-SPACE INTERPRETATION OF BRUNE METHOD

The classical Brune method requires at the first stage the determination of the frequency at which the real part of the impedance function vanishes and the determination of the imaginary part at that frequency. The imaginary part divided by this frequency gives the element value of series inductance (positive or negative). The removal of the series inductance leaves a function which after inverting is realized as an admittance function and so on. In the following, these steps are discussed by giving them the state-space interpretation. The discussion is, however, limited only to biquadratic minimum functions. Various steps are illustrated by taking a suitable example[79].

Example 3.1

$$\text{Given } z(s) = \frac{\frac{1}{2}s^2 + \frac{1}{2}s + 1}{s^2 + \frac{1}{2}s + \frac{1}{2}} \qquad ...\ (3.43)$$

Therefore

$$z_1(s) = \frac{\frac{1}{2}s^2 + \frac{1}{2}s + 1}{s^2 + \frac{1}{2}s + \frac{1}{2}} - \frac{1}{2} = \frac{\frac{1}{4}s + \frac{3}{4}}{s^2 + \frac{1}{2}s + \frac{1}{2}} \qquad ...\ (3.44)$$

and the corresponding realization is given by [32]

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}, \qquad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$c = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \end{bmatrix}, \qquad d = \begin{bmatrix} \frac{1}{2} \end{bmatrix}. \qquad .. (3.45)$$

Knowing $[A,b,c,d]$ of z(s), $[A_r, b_r, c_r, d_r]$ of Even z(s) can be found as [3]

$$A_r = \begin{bmatrix} A & 0 \\ 0 & -A' \end{bmatrix}, \qquad b_r = \begin{bmatrix} b \\ c' \end{bmatrix},$$

$$c_r = \begin{bmatrix} c & -b' \end{bmatrix}, \qquad d_r = \begin{bmatrix} 2d \end{bmatrix}. \qquad .. (3.46)$$

The zeros of real part are given by finding the eigenvalues of A_{or} [eqn. (2.5)] i.e. by finding the zeros of

$$\det\left[sI - A_{or}\right] = \det\left[sI - (A_r - \frac{b_r c_r}{d_r})\right]$$

where, for the example under consideration

$$A_{or} = \left[A_r - \frac{b_r c_r}{d_r}\right] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{5}{4} & -\frac{3}{4} & 0 & 1 \\ -\frac{9}{16} & -\frac{3}{16} & 0 & \frac{5}{4} \\ -\frac{3}{16} & -\frac{1}{16} & -1 & \frac{3}{4} \end{bmatrix} \qquad .. (3.47)$$

and $\det(sI - A_{or}) = (s^2 + 1)^2$. Therefore, the frequency ω_1 at which real part is zero is 1. It can be easily proved that if $[A, b, c]$ is a realization of $z(s)$ then A_i, b_i and c_i give the realization of $[z(s)-z'(-s)]$ where A_i, b_i, and c_i are determined by

$$A_i = \begin{bmatrix} A & 0 \\ 0 & -A' \end{bmatrix}, \qquad b_i = \begin{bmatrix} b \\ c' \end{bmatrix},$$

$$c_i = \begin{bmatrix} c & b' \end{bmatrix}, \qquad d_i = \begin{bmatrix} 0 \end{bmatrix}. \qquad \text{.. (3.48)}$$

Imaginary part $\frac{1}{2}[z(s) - z'(-s)]$ is given by $\frac{1}{2}\,\frac{c_1 b_i P(s)}{s\,Q(s)}$ [Eqn.16, Ref.36] where

$$P(s) = \det\left[sI - (I - \frac{b_i c_i}{c_i b_i})\, A_i\right].$$

and

$$Q(s) = \det\left[sI - A_i\right]$$

Therefore, for the example under consideration

$$\frac{1}{2}[z(s) - z'(-s)] = \frac{1}{2} \times \frac{1}{2} \times \frac{s^3 - s}{(s^2 + \frac{1}{2})^2 - (\frac{1}{2}s)^2}$$

which equals $(-s)$ on substituting $s^2 = -\omega^2 = -1$.

In the conventional Brune synthesis, this reactance which equals $(-s)$ in this example, is removed from the given impedance function and the resulting impedance function is inverted. In state-space terms, the removal of this reactance corresponds to changing d in eqn.(3.45) to $(\frac{1}{2} + s)$ and the corresponding state model becomes

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}, \qquad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$c = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \end{bmatrix}, \qquad d = \begin{bmatrix} \frac{1}{2} + s \end{bmatrix}. \qquad \text{.. (3.49)}$$

In state-space terms, the inverting of the impedance function in classical Brune synthesis corresponds to the determination of inverse system $[A_v, b_v, c_v, d_v]$ which is given as follows[90]

$$A_v = A - bd^{-1}c ,$$
$$b_v = bd^{-1} ,$$
$$c_v = -d^{-1}c ,$$
$$d_v = d^{-1} \qquad .. (3.50)$$

and for the example under discussion, A_v, b_v, c_v, d_v become

$$A_v = \begin{bmatrix} 0 & 1 \\ \frac{-2s-4}{2(2s+1)} & \frac{-2s-2}{2(2s+1)} \end{bmatrix}, \qquad b_v = \begin{bmatrix} 0 \\ \frac{2}{2s+1} \end{bmatrix},$$
$$c = \begin{bmatrix} \frac{-3}{2(2s+1)} & \frac{-1}{2(2s+1)} \end{bmatrix}, \qquad d_v = \begin{bmatrix} \frac{2}{2s+1} \end{bmatrix}, \qquad .. (3.51)$$

which correspond to the admittance function

$$y_2(s) = \frac{2s^2 + s + 1}{2s^3+2s^2+2s+2} \ . \qquad .. (3.52)$$

Continuing with this procedure, let us realize the admittance function so obtained in equation (3.52).

In general $y_2(s)$ can be written as,

$$y_2(s) = \frac{r_1}{s + q_1} + \frac{r_2 s}{s^2+q_2^2} \ . \qquad .. (3.53)$$

The corresponding state model is given as

$$A = \left[\begin{array}{c|cc} 0 & -q_2 & 0 \\ \hline q_2 & 0 & 0 \\ 0 & 0 & -q_1 \end{array}\right], \qquad b = \left[\begin{array}{c} 0 \\ \hline \sqrt{r_2} \\ \sqrt{r_1} \end{array}\right], \quad .. (3.54)$$

$$c = \left[\begin{array}{c|cc} 0 & \sqrt{r_2} & \sqrt{r_1} \end{array}\right].$$

Choosing the transformation

$$T = \left[\begin{array}{c|cc} \frac{\sqrt{r_2}}{q_2} & & \\ \hline & \frac{1}{\sqrt{r_2}} & \\ & & \frac{1}{\sqrt{r_1}} \end{array}\right] \quad .. (3.55)$$

we obtain the new model as

$$\left[\begin{array}{c} \dot{V}_c \\ \hline \dot{I}_L \end{array}\right] = \left[\begin{array}{c|cc} \frac{q_2^2}{r_2} & & \\ \hline & r_2 & \\ & & r_1 \end{array}\right] \left[\begin{array}{c|cc} 0 & -1 & 0 \\ \hline 1 & 0 & 0 \\ 0 & 0 & -\frac{q_1}{r_1} \end{array}\right] \left[\begin{array}{c} V_c \\ \hline I_L \end{array}\right]$$

$$+ \left[\begin{array}{c|cc} \frac{q_2^2}{r_2} & & \\ \hline & r_2 & \\ & & r_1 \end{array}\right] \left[\begin{array}{c} 0 \\ \hline 1 \\ 1 \end{array}\right] \overset{*}{v}_V , \quad .. (3.56)$$

$$\overset{*}{i}_V = \left[\begin{array}{c|cc} 0 & 1 & 1 \end{array}\right] \left[\begin{array}{c} V_c \\ I_L \end{array}\right].$$

Comparing with the state model(It can be obtained from (2.12) by connecting a voltage source at the input and by assuming that there are no loops of capacitance only, no cut-sets of inductance only and $F_{RG} = 0$)

$$\begin{bmatrix} \dot{V}_c \\ \dot{I}_L \end{bmatrix} = \begin{bmatrix} C_2^{-1} & 0 \\ 0 & L_{11}^{-1} \end{bmatrix} \begin{bmatrix} -F'_{RC}G_1F_{RC} & F'_{LC} \\ -F_{LC} & -F_{LG}R_2F'_{LG} \end{bmatrix} \begin{bmatrix} V_c \\ I_L \end{bmatrix}$$

$$+ \begin{bmatrix} C_2^{-1} & 0 \\ 0 & L_{11}^{-1} \end{bmatrix} \begin{bmatrix} -F'_{RC}G_1F_{RV} \\ -F_{LV} \end{bmatrix} \overset{*}{v}_V , \quad \ldots (3.57)$$

$$\overset{*}{i}_V = \begin{bmatrix} F'_{RV}G_1F_{RC} & -F'_{LV} \end{bmatrix} \begin{bmatrix} V_c \\ I_L \end{bmatrix} ,$$

we get,

$$F_{LC} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \qquad F_{LV} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} .$$

Further

$$F'_{RC}G_1F_{RC} = 0 ,$$

$$F'_{RC}G_1F_{RV} = 0$$

A possible solution is

$$F_{RC} = 0 , \quad F_{RV} = 0 , \quad G_1 = 0$$

and as

$$-F_{LG}R_2F'_{LG} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{-q_1}{r_1} \end{bmatrix} ,$$

hence

$$F_{LG} = \begin{bmatrix} 0 \\ \\ 1 \end{bmatrix}, \qquad R_2 = \frac{q_1}{r_1} \text{ etc.}$$

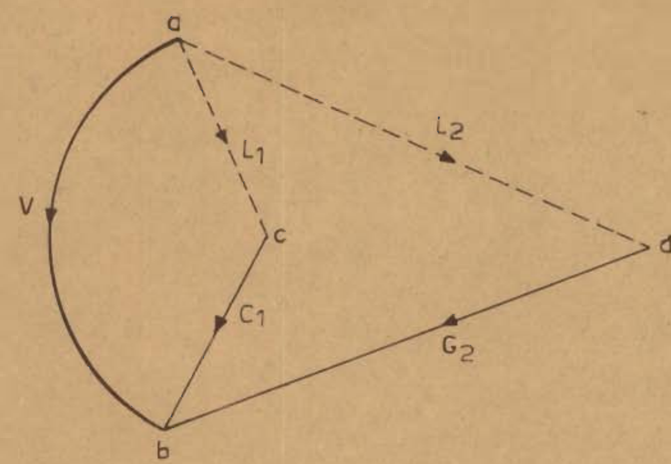
For the example under consideration, eqn.(3.53) becomes

$$y_2(s) = \frac{\frac{1}{2}}{s+1} + \frac{\frac{1}{2}s}{s^2+1}$$

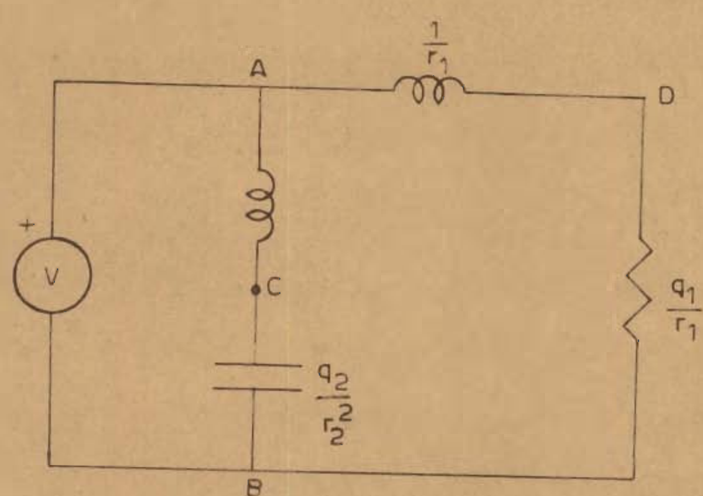
Therefore, $\begin{bmatrix} C_2^{-1} & \\ & L_{11}^{-1} \end{bmatrix}$ from eqn.(3.56) and (3.57) is given by $\begin{bmatrix} 2 & & \\ & 1/2 & \\ & & 1/2 \end{bmatrix}$, the corresponding graph and the network for $y_2(s)$ in (3.52) are shown in Figs. (3.3a) and (3.3b) and the network corresponding to eqn.(3.43) is shown in Fig.(3.3c).

It may be mentioned that the above treatment deals only with the state-space interpretation of various steps performed in classical Brune method and the discussion is, by no means, complete. An elegant state-space interpretation of Brune sections has been given by Newcomb[55]. The synthesis procedure described by him is based on reactance extraction and does not utilise the state models in topological entities. It is hoped that the discussion given here might lead to a synthesis procedure exploiting the graph-theoretic concepts.

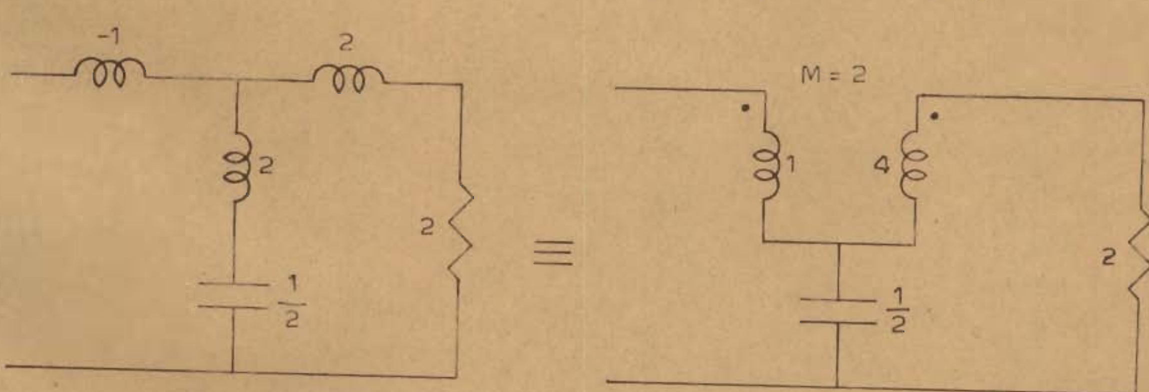
The determination of network functions from the information given in some form is closely related to the synthesis problem. The subsequent portion of the chapter is thus devoted to the use of state-space techniques for the determination of impedance matrix from its given even part and a method for determining the transfer-function matrix



(a) REALIZATION OF CIRCUIT MATRIX CORRESPONDING TO EQN. 3.57



(b) REALIZATION OF $y_2(s)$ [EQN. 3.53]



(c) REALIZATION OF $z(s)$ [EQN. 3.43]

FIG. 3.3

from the given state-space specifications.

3.4 DETERMINATION OF Z(s) FROM GIVEN Z(s) + Z'(-s)

In this section, a state-variable technique for determining the state-model and the impedance matrix $Z(s)$ when given $Z(s) + Z'(-s)$ is suggested. The technique presented here is applicable for impedance matrix of any order n. For ready reference Lemma 2.3 is repeated here.

Given an nxn p.r. $Z(s)$, there always exists an rxn matrix $W(s)$ such that

$$Z(s) + Z'(-s) = W'(-s)\, W(s) \,, \qquad \text{.. (3.58)}$$

where r denotes the rank of $Z(s) + Z'(-s)$.

Furthermore, if $Z(s)$ be a matrix of rational functions such that $Z(\infty) = 0$ and $Z(s)$ has poles which lie in $\text{Re } s < 0$ or are simple on $\text{Re } s = 0$ and if $[A, B, C, D]$ is a minimal realization of $Z(s)$, then $Z(s)$ is positive real if and only if there exists a symmetric positive definite matrix P and a matrix L such that [3]

$$PA + A'P = -L'L, \qquad \text{... (3.59a)}$$

$$PB = C' - L'W_o, \qquad \text{... (3.59b)}$$

$$W_o'W_o = D + D'. \qquad \text{... (3.59c)}$$

Now, given $[Z(s) + Z'(-s)]$, $Z(s)$ can be found as follows.

(i) Determine $W(s)$ as in eqn. (3.58).

(ii) Find $[A, B, L, W_o]$, a minimum realization of $W(s)$ by any of the known methods [32].

(iii) Determine P from eqn.(3.59a), C from eqn.(3.59b) and D from eqn.(3.59c). It may be noted that $D = 0$ for $Z(\infty) = 0$ which implies $W_o(s) = 0$ and further D is not unique except for the case when Z(s) is of order 1.

(iv) A,B,C,D thus obtained, give the minimum realization of Z(s) and Z(s) can be obtained by

$$Z(s) = C(sI-A)^{-1}B + D \qquad .. (3.60)$$

for which well known algorithms exist which do not involve the problem of rational matrix inversion [see next section]

The procedure is illustrated with the help of an example.

Example 3.2

Consider

$$Z(s)+Z'(-s) = \begin{bmatrix} 1 & \frac{s-1}{s+1} \\ \frac{s+1}{s-1} & 1 \end{bmatrix} \qquad .. (3.61)$$

It is required to determine positive real matrix Z(s) having this $Z(s) + Z'(-s)$.

(i) $Z(s) + Z'(-s) = W'(-s)W(s)$,

therefore

$$W(s) = \begin{bmatrix} 1 & \frac{s-1}{s+1} \end{bmatrix}.$$

(ii) $A = \begin{bmatrix} -1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \end{bmatrix}$,

$L' = \begin{bmatrix} -2 \end{bmatrix}$, $W_o = \begin{bmatrix} 1 & 1 \end{bmatrix}$,

$-L'L = -4$.

(iii) From (3.59a) $P = \begin{bmatrix} 2 \end{bmatrix}$ and from (3.59b) C is obtained as $\begin{bmatrix} -2 & 0 \end{bmatrix}$. From (3.59c), D is obtained as

$$\begin{bmatrix} \frac{1}{2} & 1 \\ 0 & \frac{1}{2} \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

(iv) Thus from eqn.(3.60), Z(s) becomes

$$\begin{bmatrix} \frac{1}{2} & \frac{s-1}{s+1} \\ 0 & \frac{1}{2} \end{bmatrix}$$

or

$$\begin{bmatrix} \frac{1}{2} & \dfrac{\frac{1}{2}s - \frac{3}{2}}{s + 1} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}. \qquad .. (3.62)$$

The arbitrariness in Z(s) is obviously due to the non-uniqueness of D in general except when $W_0 = 0$ (in this case $D = Z(\infty) = 0$) or when Z(s) is of order 1.

It is observed that Z(s) obtained by this method will never have poles on the imaginary axis, so Z(s) will always be minimum reactance matrix. The justification for these steps is self-evident.

The conditions on $Z(s) + Z'(-s)$ for the successful implementation of the proposed procedure can be stated as follows,

(i) $Z(s) + Z'(-s)$ is a parahermitean matrix with real coefficients,

(ii) On $j\omega$ axis, $Z(s) + Z'(-s)$ is bounded and is non-negative definite hermitean.

This is obvious in view of the fact that the conditions[88] stated above which are extension of the scalar case[68] ensure factorization given in eqn.(3.58) and the existence of a positive real Z(s). Therefore, a matrix P satisfying (3.59) will exist[3] and the enumerated steps can be followed through successfully.

The interesting feature of the proposed technique, which is based on state-space concepts is its generality of approach compared with other methods [10],[25]. Even for Z(s) of order 1, the technique, if not simpler, can well be compared with the Brune-Gewertz and Bode methods[79]. It is interesting to note that for the case when Z(s) is of order 1 and $Z(s) + Z'(-s)$ is zero at $s = o$, the minimal realization $[A, B, L, (W_0 = 0)]$ can be written in phase canonical form [15] and as the entries of B are $[0, 0, 0, \ldots 1]'$ etc., it can easily be checked that the coefficients of the last column of P give the coefficients of the numerator of Z(s) (denominator of Z(s) being same as that of W(s)) and so eqn.(3.60) is bye-passed in the determination of Z(s). For illustration 1x1 matrix is also considered.

Example 3.3

Consider the Butterworth resistance function [79] $R(\omega) = \dfrac{1}{1 + \omega^6}$. It is required to determine the minimum reactance p.r. Z(s) having this response.

(i) $$\left.\frac{1}{1+\omega^6}\right|_{s=j} = \frac{1}{2}\left[Z(s) + Z'(-s)\right] = \frac{1}{1-s^6} \qquad ..\ (3.63)$$

$$Z(s) + Z'(-s) = \frac{2}{1-s^6} = W'(-s)\,W(s).$$

Therefore,

$$W(s) = \frac{\sqrt{2}}{s^3 + 2s^2 + 2s + 1}.$$

(ii) $$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -2 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$$L' = \begin{bmatrix} \sqrt{2} \\ 0 \\ 0 \end{bmatrix}, \qquad -L'L = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

(iii) From eqn.(3.59a) and (3.59b) P is obtained

$$P = \begin{bmatrix} \frac{8}{3} & \frac{8}{3} & 1 \\ \frac{8}{3} & 5 & \frac{4}{3} \\ 1 & \frac{4}{3} & \frac{2}{3} \end{bmatrix} \quad \text{and } C' = \begin{bmatrix} 1 \\ \frac{4}{3} \\ \frac{2}{3} \end{bmatrix}.$$

(iv) $[A, B, C]$ is the minimal realization of $Z(s)$. As $Z(s)$ is only 1x1 matrix and the configurations of A and B have been taken to be in the phase-variable form. So, coefficients of the last column of P give the coefficients of the numerators of $Z(s)$.

Therefore

$$Z(s) = \frac{\frac{2}{3}s^2 + \frac{4}{3}s + 1}{s^3 + 2s^2 + 2s + 1} \qquad ..\ (3.64)$$

Now A_i, B_i, C_i of the imaginary part of $Z(s)$ can be found as in eqn. (3.48)

$$A_i = \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & & & \\ 0 & 0 & 1 & & & \\ -1 & -2 & -2 & & & \\ \hline & & & 0 & 0 & 1 \\ & & & -1 & 0 & 2 \\ & & & 0 & -1 & 2 \end{array}\right] \quad B_i = \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline 1 \\ 4/3 \\ 2/3 \end{array}\right] \quad C_i = \left[\begin{array}{c} 1 \\ 4/3 \\ 2/3 \\ \hline 0 \\ 0 \\ 1 \end{array}\right] \qquad .. (3.65)$$

The imaginary part, if desired can be obtained by the usual methods by converting $[A_i, B_i, C_i]$ to phase-variable form, the C_i of which will provide twice the value of each coefficient of the numerator of imaginary part. The denominator will be the same as that of the real part.

3.5 DETERMINATION OF TRANSFER-FUNCTION MATRIX FROM THE GIVEN STATE EQUATIONS

Various methods are available for the determination of $(sI - A)^{-1}$ which is required for the evaluation of transfer-function matrix $\mathcal{W}(s) = C(sI - A)^{-1}B + D$ for a given linear time-invariant system described by the state equations (1.1)[40], [91]. A direct method for the determination of transfer-function matrix from given A,B,C,D is suggested. The method is based on the reverse of the approach discussed by Ho and Kalman[28] for determining A,B,C,D from the given transfer-function matrix, and is more interesting than the existing methods as it offers a straightforward proof for its validity besides avoiding the usual difficulty in rational matrix inversion. The method consists of determining det $[sI - A]$ and CB, CAB, ... $CA^{n-1}B$ etc. Det $[sI - A]$ gives the common denominator of each entry of $\mathcal{W}(s)$ while the numerator of each entry is given in terms of the so-called Markov parameters CB, CAB ... CA^nB. The method can be explained as follows:

$$\mathcal{W}(s) = \mathcal{W}_1(s) + D , \qquad .. (3.66)$$

where

$$D = \mathcal{W}(s)\Big|_{s\to\infty}$$

and

$$\mathcal{W}_1(s) = \sum_{n=1}^{\infty} CA^{n-1}B \; s^{-n}. \qquad .. (3.67)$$

The integer n denotes the order of A-matrix and the common denominator q(s) is given by

$$q(s) \doteq \det[sI - A] = s^n + b_1 s^{n-1} + \ldots + b_n \qquad .. (3.68)$$

It has been shown in [28] that the Markov parameters CB, CAB ... $CA^{n-1}B$ etc. are obtained by dividing numerator of each entry of the transfer-function matrix by the common denominator and collecting terms of s^{-1}, s^{-2} ... s^{n-1} etc. So, in the proposed reverse process, CB, CAB ... $CA^{n-1}B$ etc. are found from the given A,B and C and the corresponding terms in s^{-1}, s^{-2} ... s^{-n} etc. are written as entries in a matrix. Multiplying these entries by the common denominator and collecting terms having only positive powers of s should naturally give the entries of the required transfer-function matrix, Just as any transfer-function matrix having a finite value at $s=\infty$ results in a set A,B,C,D; given any conformable set, it will always result in a transfer-function matrix.

The process is illustrated with the help of an example.

Example 3.4- Given

$$A = \begin{bmatrix} 0 & -2 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 2 \end{bmatrix}, \qquad D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad .. (3.69)$$

The transfer-function matrix is determined as follows :

$$n = 3, \quad \det\left[sI - A\right] = s^3 + 2s$$

therefore,

$$b_1 = 0, \quad b_2 = 2, \quad b_3 = 0$$

and

$$CB = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},$$

$$CAB = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}, \quad .. (3.70)$$

$$CA^2B = \begin{bmatrix} -4 & 0 \\ 0 & -2 \end{bmatrix}.$$

Thus $\mathcal{W}_1(s)$ can be written as

$$\frac{s^3 + 2s}{s^3 + 2s} \begin{bmatrix} 2s^{-1}+0s^{-2}-4s^{-3} & 0s^{-1}+2s^{-2}+0s^{-3} \\ 0s^{-1}-2s^{-2}+0s^{-3} & 2s^{-1}+0s^{-2}-2s^{-3} \end{bmatrix} \quad .. (3.71)$$

Collecting terms with positive powers of s, we get

$$\mathcal{W}_1(s) = \frac{1}{s^3+2s}\begin{bmatrix} 2s^2 & 2s \\ -2s & 2s^2+2 \end{bmatrix} \quad .. (3.72)$$

and therefore from equation (3.66)

$$\mathcal{W}(s) = \begin{bmatrix} \frac{s^2+2s+2}{s^2+2} & \frac{2}{s^2+2} \\ -\frac{2}{s^2+2} & \frac{s^3+2s^2+2s+2}{s^3+2s} \end{bmatrix} \quad .. (3.73)$$

The method is attractive for finding the transfer-function matrix from the state equations and is often needed to serve as a check for [A,B,C,D] found from the given transfer-function matrix, in the synthesis procedures. The method is particularly suited for the cases when det $[sI - A]$ can be easily calculated.

3.6 CONCLUSION

The classical synthesis methods such as Foster's Cauer's and Brune's etc. are well established. Computationally, there is not perhaps a great deal to choose between the classical procedures and state-space methods akin to these procedures. However, the state-space technique does offer greater scope for extensions to problems such as equivalent network problems and discussion of these methods from state-space point of view has been taken up in this chapter in this context. Some of the known properties of network functions have been derived in state-space terms. A procedure for determining Z(s) from given $\left[Z(s) + Z'(-s)\right]$ using state-variable technique has been evolved. Based on the reverse process of determining Markov parameters from the transfer-function matrix, a direct procedure for determining the transfer-function matrix from the state equations has been given which does not involve the usual difficulty of inversion of the rational matrix (sI - A).

The modern network synthesis is different from the classical synthesis in the sense that the given information may be either in terms of state-variable characterization or in terms of input-output characterization. The following chapter discusses the realization techniques when the given information is in terms of state-variable characterization.

CHAPTER IV

REALIZATION OF STATE EQUATIONS

4.1 INTRODUCTION

In modern synthesis, many a time, the given information is in terms of state-variable characterization as the transfer-function description is not valid for time-varying and non-linear systems. Further, as has been shown recently[24], state equations can be obtained from a sequence of input-output data, without involving the computation of impulse response. The importance of this characterization is evidenced by a number of papers published recently giving the procedures for network realization from the state equations. In this connection, Yarlagadda and Tokad have given procedures for network realization of state equations for LC[86] and for RLC[87] networks. Dervisoglu[19] and Nordgren and Tokad[57] have considered the realization of A-matrix. In this chapter simpler procedures have been evolved for the realization of state models for LC networks, and a class of each of time-invariant RLC, time-varying LC, and A-matrix for RLC networks. The procedure for the realization of LC networks has been computerized and actually run on IBM 1620.

4.2 REALIZATION OF STATE-MODEL OF n-PORT LC NETWORKS

Realization of state model of n-port LC networks has been discussed earlier[86]. The method consists of obtaining a state model of general LC networks in topological quantities as in eqn.(3.2) in the form

$$\dot{X} = AX + B_tU + B_o\dot{U} \;, \quad \text{.. (4.1a)}$$

$$Y = CX + D_tU + D_o\dot{U} \;. \quad \text{.. (4.1b)}$$

The element values and the topology of the network are determined by comparing these topological expressions given by (4.1) with the known quantities available from the given state equations. Consequently, the method results in seeking solution of a number of equations. The set of matrix equations thus obtained is difficult to solve as the equations are of the type

$$PK_u = Q_u \;,$$

where K_u is known and, in general, is a rectangular matrix, Q_u and P are unknown rectangular and square matrices respectively. Obviously, a search for a satisfactory solution of the set of equations obtained in the method is desirable. Yarlagadda and Tokad [86] suggest yet another method of decomposing (4.1a) in the form

$$\dot{X} = P^{-1}A_rX + P^{-1}B_{tr}U + P^{-1}B_{or}\dot{U} \quad \text{.. (4.2)}$$

as a first step towards the realization of the given state equations, where P is a symmetric positive definite matrix giving element values of capacitances and inductances. The decomposition procedure suggested in [86] is far from satisfactory as, in general, it gives a non-unique solution for P and it is difficult to select the desired symmetric positive definite P.

In view of the above mentioned difficulties, it is worthwhile to obtain a suitable procedure for decomposing (4.1a) into the form (4.2) and subsequent realization of the resulting equations. Further finding an algorithm suitable for computerization of this procedure will be a fruitful problem [56]. In this section, such an algorithm is presented for the realization of state equations of LC networks. The algorithm is especially suitable for computerization.

Consider the state model for LC networks given in (3.2). It needs be emphasised that the state-model given in (3.2) does not satisfy the equation (2.23) for some appropriate P. However, by applying the transformation (3.3) and letting

$$P = \begin{bmatrix} \mathcal{C} & 0 \\ 0 & \mathcal{L} \end{bmatrix}$$

relation (2.23) is satisfied and consequently for LC case under discussion

$$PA + A'P = 0 \ , \qquad \text{.. (4.3a)}$$

$$PB = C' . \qquad \text{.. (4.3b)}$$

From (4.3a) and (4.3b), we find [7]

$$\begin{aligned} PAB &= -A'PB = -A'C' \\ PA^2B &= -A'PAB - (A')^2C' \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ PA^{n-1}B &= (-1)^{n-1}(A')^{n-1}C' . \end{aligned} \qquad \text{.. (4.4)}$$

With the controllability matrix

$$W = \begin{bmatrix} B & AB \ldots & A^{n-1}B \end{bmatrix} \qquad \text{.. (4.5)}$$

and modified observability matrix

$$V = \begin{bmatrix} C' & -A'C' & \ldots & (-1)^{n-1}(F')^{n-1}C' \end{bmatrix}, \quad \ldots (4.6)$$

the equations (4.4), (4.5) and (4.6) give

$$PW = V,$$

or

$$PWW' = VW'. \quad \ldots (4.7)$$

Since W has a right inverse by the minimality of the realization [7],

$$P = (VW')(WW')^{-1}. \quad \ldots (4.8)$$

Therefore

$$P = \begin{bmatrix} P_{11} & 0 \\ 0 & P_{22} \end{bmatrix}$$

can be determined from the given A, B_t, B_o and C, D_t, D_o.

Equation (3.2) can be written as

$$\dot{X} = P^{-1}A_r X + P^{-1}B_{tr}U + P^{-1}B_{or}\dot{U},$$
$$Y = CX + D_t U + D_o\dot{U},$$

where

$$A = P^{-1}A_r, \; B_t = P^{-1}B_{tr}, \; B_o = P^{-1}B_{or} \text{ etc.} \quad \ldots (4.9)$$

Thus, A_r, B_{tr} and B_{or} can be determined. Hence from eqn. (3.2)

$$A_r = \begin{bmatrix} 0 & F'_{LC} \\ -F_{LC} & 0 \end{bmatrix}, \qquad B_{tr} = \begin{bmatrix} 0 & B_{tr12} \\ B_{tr21} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -F'_{KC} \\ -F_{LV} & 0 \end{bmatrix},$$

$$B_{or} = \begin{bmatrix} B_{or11} & 0 \\ 0 & B_{or22} \end{bmatrix} = \begin{bmatrix} -F'_{SC}C_1F_{SV} & 0 \\ 0 & F_{L\Gamma}L_{22}F'_{K\Gamma} \end{bmatrix} \quad .. \; (4.10a)$$

and

$$C = \begin{bmatrix} 0 & -F'_{LV}+(F'_{SC}C_1F_{SV})'\mathcal{C}^{-1}F'_{LC} \\ -F_{KC}-(F_{L\Gamma}L_{22}F'_{K\Gamma})'\mathcal{L}^{-1}(-F_{LC}) & 0 \end{bmatrix},$$

$$D_t = \begin{bmatrix} 0 & D_{t12} \\ D_{t21} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & F'_{KV}-(F'_{SC}C_1F_{SV})'\mathcal{C}^{-1}(-F'_{KC}) \\ -F_{KV}-(F_{L\Gamma}L_{22}F'_{K\Gamma})'\mathcal{L}^{-1}(-F_{LV}) & 0 \end{bmatrix},$$

$$D_o = \begin{bmatrix} D_{o11} & 0 \\ 0 & D_{o22} \end{bmatrix}$$

$$= \begin{bmatrix} F'_{SV}C_1F_{SV}-(F_{SC}C_1F_{SV})'\mathcal{C}^{-1}(F_{SC}C_1F_{SV}) & 0 \\ 0 & F_{K\Gamma}L_{22}F'_{K\Gamma}-(F_{L\Gamma}L_{22}F'_{K\Gamma})'\mathcal{L}^{-1}(F_{L\Gamma}L_{22}F'_{K\Gamma}) \end{bmatrix}$$

.. (4.10b)

Therefore, from comparison of eqn. (4.10a) with the known values of A_r, B_{tr} determined from (4.9), F'_{LC}, F'_{KC} and F_{LV} are uniquely determined. In order to determine the element values C_1, C_2, L_{11} and L_{22} etc. and to determine the various submatrices of $\begin{bmatrix} F & I \end{bmatrix}$ given by

$$\begin{bmatrix} F_{SV} & F_{SC} & 0 & \vdots & 1 & & \\ F_{LV} & F_{LC} & F_{L\Gamma} & \vdots & & 1 & \\ F_{KV} & F_{KC} & F_{K\Gamma} & \vdots & & & 1 \end{bmatrix} \qquad .. (4.11)$$

the following procedure is adopted.

Construct the matrices

$$Y_c = \begin{bmatrix} F'_{SV}C_1F_{SV} & F'_{SV}C_1F_{SC} \\ F'_{SC}C_1F_{SV} & C_2+F'_{SC}C_1F_{SC} \end{bmatrix} \qquad .. (4.12)$$

and

$$Z_L = \begin{bmatrix} I_{22}+F_{L\Gamma}L_{11}F'_{L\Gamma} & F_{L\Gamma}L_{11}F'_{K\Gamma} \\ F_{K\Gamma}L_{11}F'_{L\Gamma} & F_{K\Gamma}L_{11}F'_{KT} \end{bmatrix}, \qquad .. (4.13)$$

where Y_c and Z_L are given in terms of the various known quantities as

$$Y_c \triangleq \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{bmatrix}$$

$$= \begin{bmatrix} D_{o11}+B'_{orl1}P_{11}^{-1}B_{orl1} & -B'_{orl1} \\ -B_{orl1} & P_{11} \end{bmatrix} \qquad .. (4.14)$$

and

$$Z_L \triangleq \begin{bmatrix} Z_{11} & Z_{12} \\ Z'_{12} & Z_{22} \end{bmatrix} = \begin{bmatrix} P_{22} & B_{or22} \\ B'_{or22} & D_{o22}+B'_{or22}P_{22}^{-1}B_{or22} \end{bmatrix}. \quad .. (4.15)$$

Applying Cederbaum's factorization to Y_c and Z_L, and rearranging rows and columns as in [86], the submatrices F_{SV}, F_{SC} and $F_{L\Gamma}$, $F_{K\Gamma}$ and element values of C_1, C_2, L_{11}, L_{22} can be determined. The only undetermined submatrix F'_{KV} is determined from eqn. (4.10b) as

$$\begin{bmatrix} 0 & F'_{KV} \\ -F_{KV} & 0 \end{bmatrix} = \begin{bmatrix} 0 & D_{t12} \\ D_{t21} & 0 \end{bmatrix} + \begin{bmatrix} B'_{o11} & 0 \\ 0 & B'_{o22} \end{bmatrix} \begin{bmatrix} P_{11} & 0 \\ 0 & P_{22} \end{bmatrix} \begin{bmatrix} 0 & B_{t12} \\ B_{t21} & 0 \end{bmatrix}, \quad .. (4.16)$$

where terms on the right hand side are known. So, $[F \quad I]$ is known which can be tested for realizability by the well-known methods [49].

4.3 COMPUTER ALGORITHM FOR THE PROPOSED METHOD

The computer algorithm for the proposed method for realization of n-port LC state equations is described in the following steps.

(i) Read n and m, n_a and m_a where n is the order of A matrix. Matrix B_t and B_o are nxm matrices, n_a denotes number of capacitor voltages as state variables and m_a denotes number of output currents.

(ii) From the given A, B_t, B_o, C, D and D_o (state model (3.2)) (B_t+AB_o) and (D_t+CD_o) are determined in order to obtain the state model in the form (3.5).

(iii) Determine P as in equation (4.8).

(iv) Find A_r, B_{tr} and B_{or} from equation (4.9). So F_{LC}, F_{KC} and F_{LV} are known. If the entries of these matrices are other than +1, -1 or zero (computational errors are to be accounted) the method fails, i.e. realization of the given state model is not possible.

(v) For determining Y_c (eqn. (4.14)) and Z_L (eqn. (4.15)) Y_{12}, Y_{22}, Z_{11} and Z_{12} are available from step (iv) above. In order to determine Y_{11} and Z_{22}, calculate

$$D_o + B'_{or} P^{-1} B_{or} = D_o + B'_o P B_o \quad . \qquad .. (4.17)$$

The form of these matrices gives Y_{11} and Z_{22}, i.e.

$$\begin{bmatrix} D_{o11} & 0 \\ 0 & D_{o22} \end{bmatrix} + \begin{bmatrix} B'_{or11} & 0 \\ 0 & B'_{or22} \end{bmatrix} \begin{bmatrix} P_{11}^{-1} & 0 \\ 0 & P_{22}^{-1} \end{bmatrix} \begin{bmatrix} B_{or11} & 0 \\ 0 & B_{or22} \end{bmatrix}$$

$$= \begin{bmatrix} D_{o11} & 0 \\ 0 & D_{o22} \end{bmatrix} + \begin{bmatrix} B'_{o11} & 0 \\ 0 & B'_{o22} \end{bmatrix} \begin{bmatrix} P_{11} & 0 \\ 0 & P_{22} \end{bmatrix} \begin{bmatrix} B_{o11} & 0 \\ 0 & B_{o22} \end{bmatrix}$$

$$= \begin{bmatrix} D_{o11}+B'_{or11}P_{11}^{-1}B_{or11} & 0 \\ 0 & D_{o22}+B'_{or22}P_{22}^{-1}B_{or22} \end{bmatrix}$$

$$= \begin{bmatrix} D_{o11}+B'_{o11}P_{11}B_{o11} & 0 \\ 0 & D_{o22}+B'_{o22}P_{22}B_{o22} \end{bmatrix}$$

$$= \begin{bmatrix} Y_{11} & 0 \\ 0 & Z_{22} \end{bmatrix} . \qquad .. (4.18)$$

From the above simplification, it is clear that the inverse of P is not needed although appeared in steps of calculating various entries of Y_c and Z_L in eqn.(4.14) and (4.15).

(vi) Y_c and Z_L are decomposed by Cederbaum factorization into the form $A_c D_c A_c'$. For computerization into this form, the procedure due to Winter[83] has been chosen. If diagonal D_c does not have positive entries, the matrices Y_c and Z_L are not realizable.

(vii) From the submatrices found in steps (v) and (vi) above, $\begin{bmatrix} F & I \end{bmatrix}$ can at once be written which can be checked for realizability by any of the available computer-algorithms [31].

4.3.1 SPECIAL FEATURES OF THE PROGRAMME

The programme (actual listing given in appendix) accepts the various entries of the matrices A, B_t, B_o, C, D_t and D_o of eqn.(4.1) and punches the various submatrices of $\begin{bmatrix} F & I \end{bmatrix}$ which can be tested for realizability. The programme is quite general and can realize the state equations of any order considering limitations of time and storage.

Machine reads various entries of the matrices A, B_t, B_o, C, D_t, D_o etc. column-wise, the order of the matrices, the number of capacitor voltages in the state vector and the number of current outputs in the output vector. Calculation of symmetric positive definite matrix P(eqn.4.8) requires the inversion of matrix WW'(eqn.4.5) for which a subroutine INVERT has to be called. The programme then determines A_r, B_{tr} etc. from which submatrices F_{LC}, F_{KC} and F_{LV} are found out. The entries of these submatrices are to be ± 1 or zero but due to the computational errors these may not be exactly ± 1 or zero and so, these are punched in FLOATING POINT. To determine the other submatrices of $[F \quad I]$, Y_c and Z_L of eqns. (4.12) and (4.13) need be factorized. The process of Cederbaum (for programming, Winter's method [83] of factorization has been chosen) has been used as a separate subroutine which may be called twice in the process of realization. The entries in A_c of $K_c = A_c D_c A_c'$ are punched in fixed point while diagonal D_c is punched in FLOATING POINT. The procedure fails if entries of submatrices of $[F \quad I]$ so obtained are different from ± 1, 0 (taking into account computational errors) or elements of diagonal D_c are not positive or $[F \quad I]$ is not realizable, otherwise a network can always be drawn.

4.3.2 INPUT-OUTPUT

INPUT

CARD NO	CONTENTS	COMMENTS
1.	n,m	Punch as a five digit number with no decimal point. Each of n and m takes 5 column spaces one after the other
2.	n_a, m_a	Punch as a five digit number with no decimal point. Each of n_a, m_a takes 5 column spaces one after the other.

CARD NO	CONTENTS	COMMENTS
3.	A_{11}, $A_{21} \cdots A_{n1}$ A_{12}, $A_{22} \cdots A_{n2}$	Punch as 10 digit number, each of A_{11}, A_{21} ... takes 10 column spaces one after the other. If no decimal point is punched, it will be placed as XXXXXXXX.YY
4.	B_{t11}, $B_{t21} \cdots B_{n1}$ B_{t12} ... ... B_{tnm}	Punch as 10 digit number, each of B_{t11} B_{t21} ... takes 10 column spaces one after the other. If no decimal point is punched it will be placed as XXXXXXXX.YY
	For B_o, C, D_t and D_o	The same way as A and B_t above etc.

OUTPUT

The machine will punch intermediate results, the values of submatrices F_{LC}, F_{KC} AND F_{LV} of $\begin{bmatrix} F & I \end{bmatrix}$ and matrix A_c ($K_c = A_c D_c A_c'$) obtained from applying Cederbaum factorization to Y_c and Z_L of eqns.(4.14) and (4.15). By rearranging A_c and D_c as in [86], the remaining submatrices of $\begin{bmatrix} F & I \end{bmatrix}$ and the element values of capacitances and inductances are known.

A simple problem for the realization of state equations is given next. The same problem as has been given in [86] is chosen for comparison.

Example 4.1

Given the state model of the form (4.1)

$$\begin{bmatrix} \dot{V}_{c1} \\ \dot{V}_{c3} \\ \dot{I}_{L7} \\ \dot{I}_{L8} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1/4 & 1/4 \\ 0 & 0 & 0 & 1/4 \\ 1 & 0 & 0 & 0 \\ -1/2 & -1/2 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{c2} \\ V_{c3} \\ I_{L7} \\ I_{L8} \end{bmatrix} + \begin{bmatrix} 0 & 1/4 \\ 0 & 0 \\ 0 & 0 \\ 1/2 & 0 \end{bmatrix} \begin{bmatrix} \overset{*}{V}_V \\ \overset{*}{I}_K \end{bmatrix}$$

$$+ \begin{bmatrix} 1/4 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\overset{*}{V}}_V \\ \dot{\overset{*}{I}}_K \end{bmatrix},$$

$$\begin{bmatrix} \overset{*}{I}_V \\ \overset{*}{V}_K \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/4 & 3/4 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{c2} \\ V_{c3} \\ I_{L7} \\ I_{L8} \end{bmatrix} + \begin{bmatrix} 0 & 3/4 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \overset{*}{V}_V \\ \overset{*}{I}_K \end{bmatrix}$$

$$+ \begin{bmatrix} 23/4 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} \dot{\overset{*}{V}}_V \\ \dot{\overset{*}{I}}_K \end{bmatrix}. \qquad \text{.. (4.19)}$$

Now using transformation (3.3), we get the state model (3.6) for which

$$A = \begin{bmatrix} 0 & 0 & -1/4 & 1/4 \\ 0 & 0 & 0 & 1/4 \\ 1 & 0 & 0 & 0 \\ -1/2 & -1/2 & 0 & 0 \end{bmatrix}, \qquad AB_o = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/4 & 0 \\ -1/8 & 0 \end{bmatrix}$$

and

$$B = B_t + AB_o = \begin{bmatrix} 0 & 1/4 \\ 0 & 0 \\ 1/4 & 0 \\ 3/8 & 0 \end{bmatrix}. \qquad \text{.. (4.20)}$$

Using eqn.(4.5)

$$W = \begin{bmatrix} 0 & .25 & .09 & 0 & 0 & -.09 & -.02 & 0 \\ 0 & 0 & 0 & 0 & 0 & -.03 & -.01 & 0 \\ .25 & 0 & 0 & .25 & .03 & 0 & 0 & -.09 \\ .37 & .03 & 0 & -.12 & -.06 & 0 & 0 & .06 \end{bmatrix}$$

.. (4.21)

and using eqn.(4.6)

$$V = \begin{bmatrix} 0 & 1 & .12 & 0 & 0 & -.37 & -.09 & 0 \\ 0 & 0 & .37 & 0 & 0 & -.12 & -.06 & 0 \\ .25 & 0 & 0 & .25 & .03 & 0 & 0 & -.09 \\ .75 & 0 & 0 & -.25 & -.12 & 0 & 0 & .12 \end{bmatrix}$$

.. (4.22)

and using eqn. (4.8), we get

$$P = \begin{bmatrix} 4 & & & \\ & 4 & & \\ & & 1 & \\ & & & 2 \end{bmatrix}, \qquad \text{.. (4.23)}$$

Therefore, from (4.8)

$$A_r = \left[\begin{array}{cc:cc} 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \\ \hdashline 1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 \end{array}\right], \quad B_{tr} = \left[\begin{array}{c:c} 0 & 1 \\ 0 & 0 \\ \hdashline 0 & 0 \\ 1 & 0 \end{array}\right], \quad B_{or} = \left[\begin{array}{c:c} 1 & 0 \\ 0 & 0 \\ \hdashline 0 & 0 \\ 0 & 0 \end{array}\right],$$

$$C = \left[\begin{array}{cc:cc} 0 & 0 & 1/4 & 3/4 \\ \hdashline 1 & 0 & 0 & 0 \end{array}\right], \quad D_t = \left[\begin{array}{c:c} 0 & 3/4 \\ \hdashline -1 & 0 \end{array}\right], \quad D_o = \left[\begin{array}{c:c} 23/4 & 0 \\ \hdashline 0 & 3 \end{array}\right],$$

from which we find .. (4.24)

$$F_{LC} = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}, \quad F_{LV} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad F_{KC} = \begin{bmatrix} -1 & 0 \end{bmatrix}. \qquad \text{.. (4.25)}$$

Now,

$$D_o + B_o'PB_o = \begin{bmatrix} 23/4 & 0 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 1/4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 4 & & & \\ & 4 & & \\ & & 1 & \\ & & & 2 \end{bmatrix} \begin{bmatrix} 1/4 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 \\ 0 & 3 \end{bmatrix}. \qquad \text{.. (4.26)}$$

Using eqns.(4.14), (4.26) and applying Cederbaum factorization

$$Y_c = \left[\begin{array}{c|cc} 6 & -1 & 0 \\ \hline -1 & 4 & 0 \\ 0 & 0 & 4 \end{array}\right]$$

$$= \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 & & & \\ & 4 & & \\ & & 1 & \\ & & & 5 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \qquad \ldots (4.27)$$

and using eqn.(4.15) and (4.26) and applying Cederbaum factorization

$$z_L = \left[\begin{array}{cc|c} 1 & & \\ & 2 & \\ \hline & & 3 \end{array}\right] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 & & \\ & 1 & \\ & & 2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}. \qquad \ldots (4.28)$$

From eqns.(4.27) and (4.28), element values of capacitances, inductances and submatrices F_{SV}, F_{SC}, $F_{L\Gamma}$ and $F_{K\Gamma}$ can be found out.

Further

$$D+B_o'PB_t = \begin{bmatrix} 0 & 3/4 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 1/4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 4 & & & \\ & 4 & & \\ & & 1 & \\ & & & 2 \end{bmatrix} \begin{bmatrix} 0 & 1/4 \\ 0 & 0 \\ 0 & 0 \\ 1/2 & 0 \end{bmatrix} \qquad \ldots (4.29)$$

$$= \left[\begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array}\right]$$ from which submatrix F_{KV} is found.

From above, $\begin{bmatrix} F & I \end{bmatrix}$ as given in (4.11) can be written at once as

$$\begin{bmatrix} F & I \end{bmatrix} = \left[\begin{array}{cccc|ccccc} -1 & 1 & 0 & 0 & 1 & & & & \\ 1 & 0 & 0 & 0 & & 1 & & & \\ 0 & -1 & 0 & 0 & & & 1 & & \\ -1 & 1 & 1 & 0 & & & & 1 & \\ 1 & -1 & 0 & 1 & & & & & 1 \end{array}\right] \quad \ldots (4.30)$$

This is realizable as a circuit matrix resulting in the graph and the network given in Figs.(4.1a) and (4.1b)

It may be noted that the resulting network by the proposed method turns out to be the same as in [86]but the main point besides the novelty of approach which needs be emphasized is the ease of computerization compared to Yarlagadda[86] method.

4.3.3 PROGRAMME DETAILS

(a) Language: Fortran II

(b) Number of Variables:- 30

(c) Special Word Length Required: None

(d) Number of Statements

(i) in actual programme: 200 approximately

(ii) in subroutine CEDBUM :- 60 approximately

(e) Additional relevant information :- It is assumed that computer library contains subroutines for the inversion of matrix.

4.3.4 PERFORMANCE GUIDE

(a) Computer used :- IBM 1620

(b) Core size :- 60 K

(c) Input medium :- Card Reader

(d) Output medium :- Punched cards.

(e) time taken:

(i) Compilation time: 5 mts. approximately

(ii) Execution time : 1 mt. 40 sec. (This time is noted from the moment the input-data has been entered.

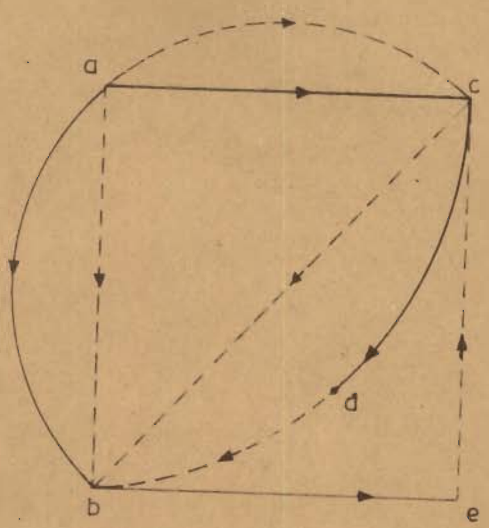
(f) Additional relevant information: None.

4.3.5 FLOW CHART

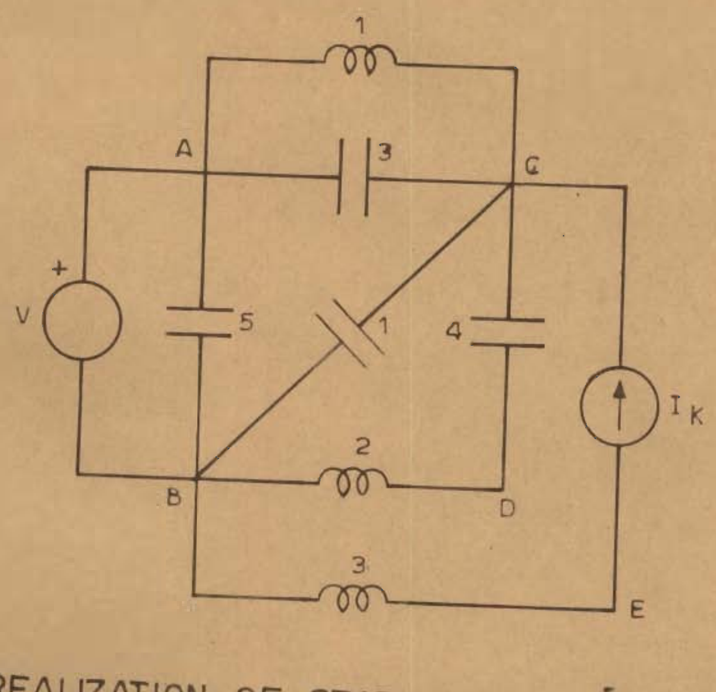
The flow chart for the programme is given on page 87.

4.4 SYNTHESIS OF A CLASS OF n-PORT RLC NETWORKS

Yarlagadda and Tokad [87] have given a synthesis procedure for RLC n-port networks based on state model approach. The technique given by them is quite cumbersome firstly because the decomposition of the given A-matrix is not unique and secondly a large number of equations given in theorem 1 of Ref. [87] have to be solved for which no satisfactory algorithm has been proposed in [87]. In this section, a synthesis

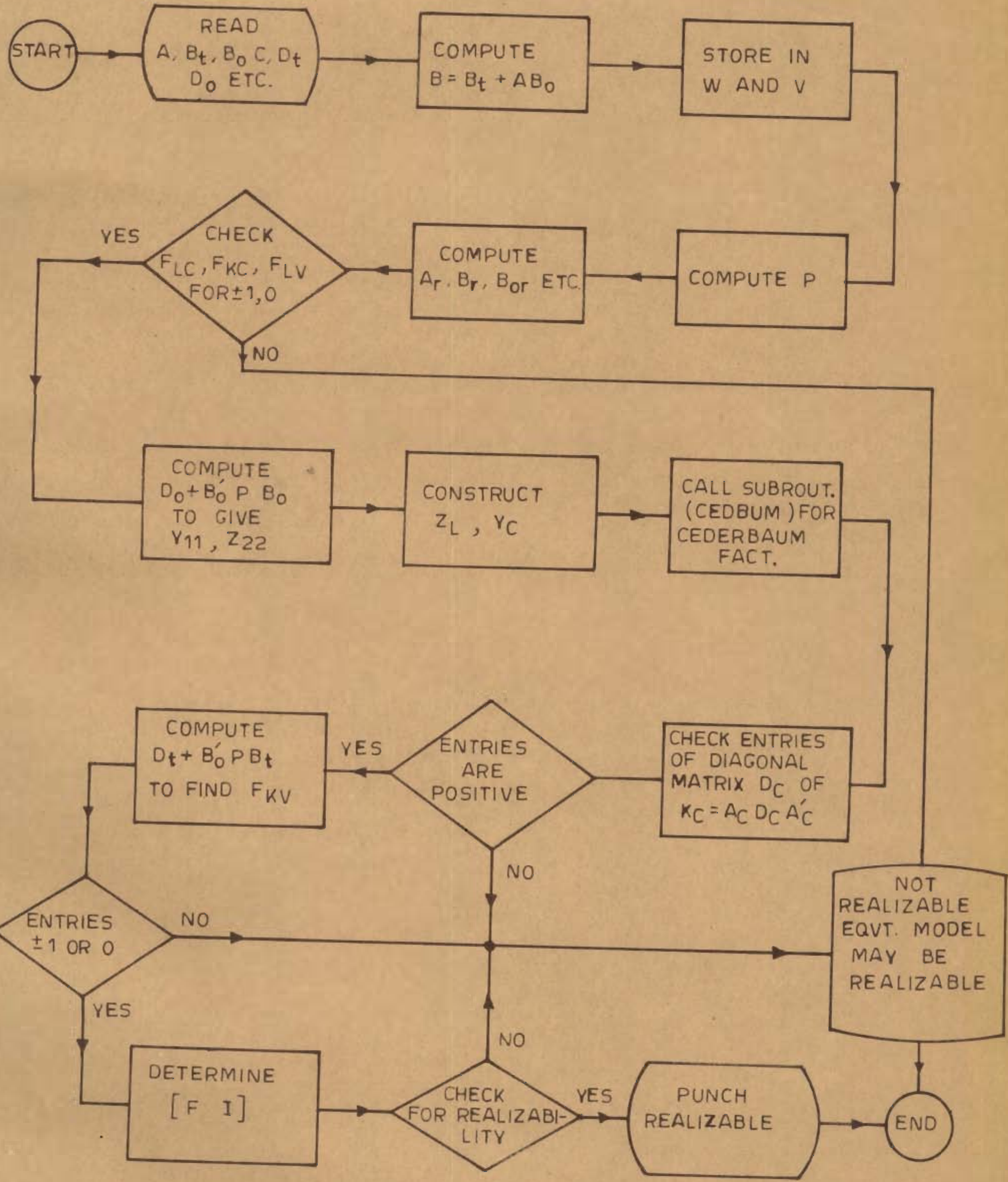


(a) REALIZATION OF CIRCUIT MATRIX [EQN. 4.19]



(b) REALIZATION OF STATE MODEL [EQN. 4.30]

FIG. 4.1



4.3.5. FLOW CHART

procedure is evolved for the class of half-degenerate n-port RLC networks without mutual inductance and having no coupling between the link resistances and the tree-branch conductances. A state model for this class of RLC networks is derived in the form

$$\dot{X} = AX + BU ,$$

$$Y = CX + DU ,$$

The basic idea in the realization procedure is to decompose the A and B matrices of the above equation in the form $A = \Lambda A_1$ and $B = \Lambda B_1$. The matrix Λ gives the element values of reactive elements and A_1, B_1, C and D give the topology of the network and the values of the resistances. For the class under discussion, whereas the A-matrix can be decomposed merely by inspection, the other unknowns are determined by factorizing a proposed specialised decomposition using Cederbaum factorization.

Consider a class of half-degenerate n-port RLC networks which contain no circuits of capacitorswith or without the voltage sources, and no cut-sets of inductorswith or without current sources. The networks, however, may contain cut-sets of capacitors with the voltage sources only and circuitsof inductors with the current sources only. Such a network may be called a canonic RLC network. Further, the network does not contain mutual inductances and there is no coupling between link resistances and tree-branch conductances in the network ($F_{RG} = 0$, page 684 ref. 38).

The theorem concerning the realization of a network belonging to this special class is given as follows.

THEOREM 4.1

A state model is realizable as an n-port RLC network belonging to the class defined in this section if (i) the matrices A and B admit the factorization ΛA_1 and ΛB_1 respectively where Λ is a diagonal matrix with positive entries (ii) a solution exists for the set of equations(4.35) or more appropriately the decomposition (4.36) exists for its right hand side where the matrices G_1 and R_2 are positive definite and (iii) $[F \quad I]$ so obtained is realizable as a fundamental circuit matrix.

PROOF

Clearly for this class of networks, a proper tree can always be drawn and under the above mentioned restrictions, we have

$$F_{SV} = F_{SC} = F_{L\Gamma} = F_{K\Gamma} = 0 \qquad \ldots (4.31)$$

$$L_{22} = 0 \qquad L_{12} = L_{21} = 0 \text{ and } F_{RG} = 0$$

and F expressing the topological relation between links and tree branches for this class becomes

$$F = \begin{bmatrix} F_{RV} & F_{RC} & 0 \\ F_{LV} & F_{LC} & F_{LG} \\ F_{KV} & F_{KC} & F_{KG} \end{bmatrix}, \qquad .. (4.32)$$

and substituting above assumptions in (2.12) the state model for this special class of RLC networks can be obtained as

$$\dot{X} = \Lambda A_1 X + \Lambda B_1 U,$$
$$Y = \Lambda C_1 X + \Lambda D_1 U, \quad \ldots (4.33)$$

or
$$\begin{bmatrix} \dot{V}_c \\ \dot{I}_L \end{bmatrix} = \begin{bmatrix} C_2^{-1} & \\ & L_{11}^{-1} \end{bmatrix} \begin{bmatrix} -F'_{RC} G_1 F_{RC} & F'_{LC} \\ -F_{LC} & -F_{LG} R_2 F'_{LG} \end{bmatrix} \begin{bmatrix} V_C \\ I_L \end{bmatrix}$$
$$+ \begin{bmatrix} C_2^{-1} & 0 \\ 0 & L_{11}^{-1} \end{bmatrix} \begin{bmatrix} -F'_{RC} G_1 F_{RV} & -F'_{KC} \\ -F_{LV} & F_{LG} R_2 F'_{KG} \end{bmatrix} \begin{bmatrix} \overset{*}{V}_V \\ \overset{*}{I}_K \end{bmatrix}, \quad \ldots (4.34a)$$

$$\begin{bmatrix} \overset{*}{V}_K \\ \overset{*}{I}_V \end{bmatrix} = \begin{bmatrix} -F_{KC} & -F_{KG} R_2 F'_{LG} \\ F'_{RV} G_1 F_{RC} & -F'_{LV} \end{bmatrix} \begin{bmatrix} V_c \\ I_L \end{bmatrix}$$
$$+ \begin{bmatrix} -F_{KV} & F_{KG} R_2 F'_{KG} \\ F'_{RV} G_1 F_{RV} & F'_{KV} \end{bmatrix} \begin{bmatrix} \overset{*}{V}_V \\ \overset{*}{I}_K \end{bmatrix}. \quad \ldots (4.34b)$$

It can be seen that ΛA_1 in (4.33) and (4.34) is a symmetric skew-symmetric (hybrid) matrix[19]. The synthesis procedure for this class of n-port RLC network is self-evident from the form of (4.34a) and (4.34b) and can be easily implemented since the decomposition of A can be done by the method given by Dervisoglu and by direct comparison of a given A and B with the form shown in (4.34a) and (4.34b). Appropriate decomposition of B is also straightforward. C_2 and L_{11} can be obtained by direct comparison. Further, it is clear from (4.34a) and (4.34b) that F_{LC}, F_{KC}, F_{LV} and F_{KV} are uniquely determined. To find the remaining undetermined submatrices, G_1, R_2, F_{RV}, F_{RC}, F_{LG} and F_{KG} of the equation, one has to solve the following set of matrix equations,

$$
\begin{aligned}
F'_{RC}G_1F_{RC} &= P_e \\
F_{LG}R_2F'_{LG} &= Q_e \\
F'_{RC}G_1F_{RV} &= R_e \\
F_{LG}R_2F'_{KG} &= S_e \\
F_{KG}R_2F'_{LG} &= T_e \\
F'_{RV}G_1F_{RC} &= U_e \\
F_{KG}R_2F'_{KG} &= V_e \\
F'_{RV}G_1F_{RV} &= W_e
\end{aligned}
\qquad .. (4.35)
$$

where P_e, Q_e, R_e, S_e, T_e, U_e, V_e, W_e are known constant matrices ($R_e = V'_e$, $S'_e = T'_e$).

The system of equations (4.35) can be written as

$$
\begin{bmatrix} F'_{RC} & 0 \\ 0 & F_{LG} \\ F'_{RV} & 0 \\ 0 & F_{KG} \end{bmatrix}
\begin{bmatrix} G_1 & \\ & R_2 \end{bmatrix}
\begin{bmatrix} F_{RC} & 0 & F_{RV} & 0 \\ 0 & F'_{LG} & 0 & F'_{KG} \end{bmatrix}
=
\begin{bmatrix} P_e & 0 & R_e & 0 \\ 0 & Q_e & 0 & S_e \\ U_e & 0 & W_e & 0 \\ 0 & T_e & 0 & V_e \end{bmatrix} .
\qquad .. (4.36)
$$

The eqn.(4.36) is of special interest since the known matrix of its right hand side can be decomposed by Cederbaum's algorithm. Rearranging rows and columns[86] the decomposed form can always be reduced to the specialised one given on left hand side of eqn.(4.36)[14]. Obviously, the block partitioned matrix on the far left of eqn.(4.36) contains entries ± 1 and 0. The matrices F_{RC}, F_{LG}, F_{RV}, F_{KG}, and the diagonal matrices G_1 and R_2 can thus be found by comparison. This decomposition is essentially unique owing to the nature of this algorithm. If the elements of G_1, R_2 are positive and if $\begin{bmatrix} F & I \end{bmatrix}$ is a circuit matrix, the state model is realizable. $\begin{bmatrix} F & I \end{bmatrix}$ can be tested for realizability by well known techniques[49].

It may, however, be noted that if the conditions in the theorem are not satisfied, it does not imply that the network is not realizable as an n-port belonging to the class specified in the theorem. An equivalent state model obtained by a similarity transformation on the given model may be realizable.

The procedure is illustrated with the help of an example.

Example 4.2

Consider the state model

$$\begin{bmatrix} \dot{V}_{c1} \\ \dot{V}_{c2} \\ \hline \dot{I}_{L1} \\ \dot{I}_{L2} \\ \dot{I}_{L3} \end{bmatrix} = - \left[\begin{array}{cc|ccc} 4 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 1/2 & 0 \\ \hline 0 & 0 & 4 & 0 & 0 \\ -1/2 & -1/2 & 0 & 0 & 0 \\ -1/4 & 0 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} V_{c1} \\ V_{c2} \\ \hline I_{L1} \\ I_{L2} \\ I_{L3} \end{bmatrix} - \left[\begin{array}{c|c} 0 & -2 \\ 0 & 0 \\ \hline 1 & 0 \\ 1/2 & 0 \\ 1/4 & 0 \end{array}\right] \begin{bmatrix} \overset{*}{V}_V \\ \hline \overset{*}{I}_K \end{bmatrix},$$

.. (4.37a)

$$\begin{bmatrix} \overset{*}{V}_K \\ \overset{*}{I}_V \end{bmatrix} = \left[\begin{array}{cc:ccc} 1 & 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & -1 & -1 & -1 \end{array}\right] \begin{bmatrix} V_{c1} \\ V_{c2} \\ I_{L1} \\ I_{L2} \\ I_{L3} \end{bmatrix} + \left[\begin{array}{c:c} -1 & 0 \\ \hdashline 0 & 1 \end{array}\right] \begin{bmatrix} \overset{*}{V}_V \\ \overset{*}{I}_K \end{bmatrix}. \quad .. (4.37b)$$

The matrices A and B can be easily decomposed in the form $A = \Lambda A_1$ and $B = \Lambda B_1$ by comparing the given A with the form given in (4.34a) and noting that the entries λ's of the diagonal matrix must be positive and entries in F_{LC}, F_{KC} and F_{LV} of equation (4.34) must be -1, 0 or +1, these being the entries of matrix to be realized as fundamental circuit matrix. Using Dervisoglu's method Λ can be found as

$$\Lambda = \begin{bmatrix} 4\lambda_1 & & & & \\ & \lambda_1 & & & \\ & & \lambda_2 & & \\ & & & \lambda_1 & \\ & & & & 1/2\lambda_1 \end{bmatrix}, \quad .. (4.38)$$

and comparing as described above $4 x \lambda_1 = 2$ i.e. $\lambda_1 = \frac{1}{2}$ and from $B = \Lambda B_1$ we get $1 = \lambda_2 x 1$ etc. Therefore

$$A = -\left[\begin{array}{cc:ccc} 2 & & & & \\ & 1/2 & & & \\ \hdashline & & 1 & & \\ & & & 1/2 & \\ & & & & 1/4 \end{array}\right] \left[\begin{array}{cc:ccc} 2 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 4 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{array}\right], \quad .. (4.39a)$$

and

$$B = -\left[\begin{array}{cc|ccc} 2 & & & & \\ & 1/2 & & & \\ \hline & & 1 & & \\ & & & 1/2 & \\ & & & & 1/4 \end{array}\right]\left[\begin{array}{c|c} 0 & -1 \\ 0 & 0 \\ \hline 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{array}\right] . \qquad \text{.. (4.39b)}$$

Comparing A and B in (4.39a) and (4.39b) with (4.37a) we have

$$C_2 = \begin{bmatrix} 1/2 & \\ & 2 \end{bmatrix}, \qquad L_{11} = \begin{bmatrix} 1 & & \\ & 2 & \\ & & 4 \end{bmatrix},$$

$$P_e = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, \; Q_e = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; R_e = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; S_e = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

$$T_e = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}, \; U_e = \begin{bmatrix} 0 & 0 \end{bmatrix}, \; V_e = \begin{bmatrix} 0 \end{bmatrix}, \quad W_e = \begin{bmatrix} 0 \end{bmatrix}.$$

Therefore matrix on the right hand of (4.36) becomes

$$M_e = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

which is decomposed by Cederbaum algorithm into

$$M_\epsilon = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & \\ & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix},$$

therefore

$$\begin{bmatrix} G_1 & \\ & R_2 \end{bmatrix} = \begin{bmatrix} 2 & \\ & 4 \end{bmatrix}.$$

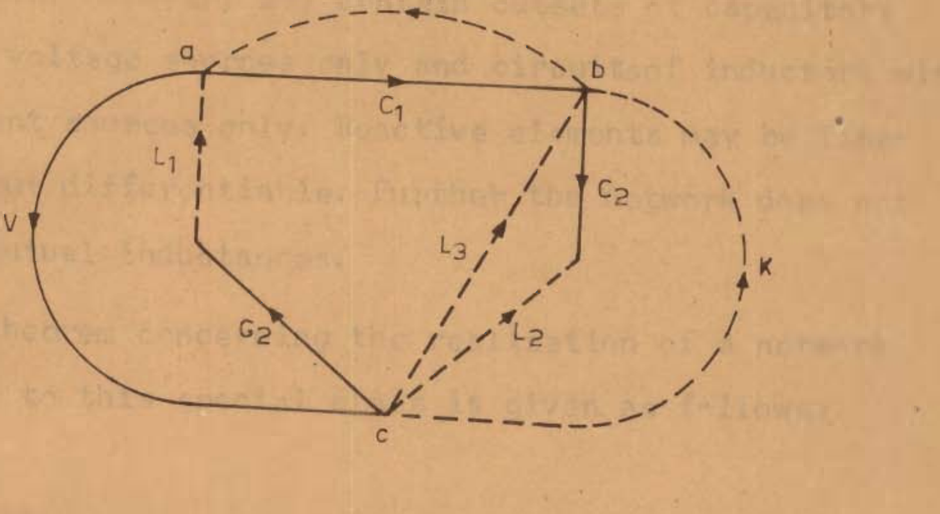
and the various submatrices in eqn.(4.33) can be written at once. The matrix to be realised as fundamental circuit matrix is given by

$$\begin{bmatrix} F & I \end{bmatrix} = \begin{array}{c} \\ R_1 \\ L_1 \\ L_2 \\ L_3 \\ I_K \end{array} \begin{array}{c} \begin{array}{ccccccccc} V_V & C_1 & C_2 & G_2 & R_1 & L_1 & L_2 & L_3 & I_K \end{array} \\ \left[\begin{array}{cccc:ccccc} 0 & 1 & 0 & 0 & 1 & & & & \\ 1 & 0 & 0 & 1 & & 1 & & & \\ 1 & -1 & -1 & 0 & & & 1 & & \\ 1 & -1 & 0 & 0 & & & & 1 & \\ 1 & -1 & 0 & 0 & & & & & 1 \end{array}\right] \end{array}, \quad \text{.. (4.40)}$$

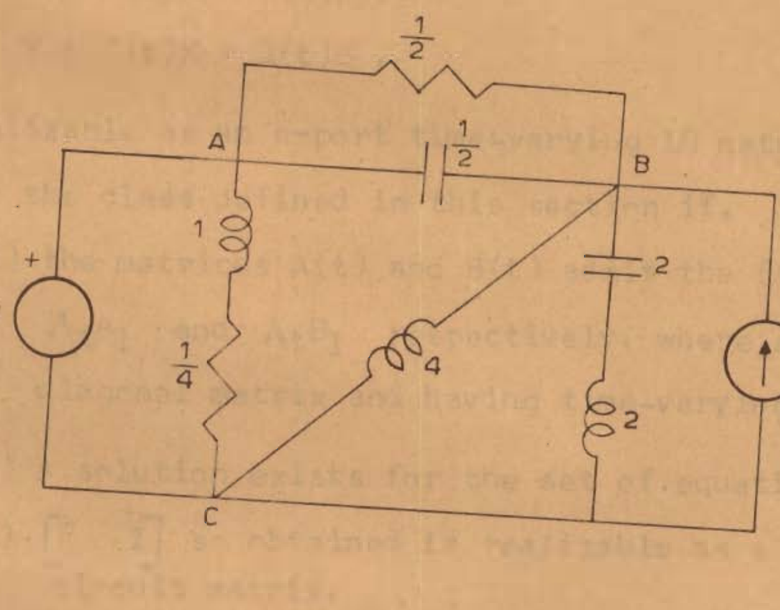
which is realizable, the graph and the corresponding network are shown in Figs.(4.2a) and (4.2b) respectively.

4.7 SYNTHESIS OF A CLASS OF n-PORT LC TIME-VARYING NETWORKS

Consider a class of n-port LC networks which contain no circuits of capacitors with or without the voltage sources,



(a) REALIZATION OF CIRCUIT MATRIX [EQN. 4.40]



(b) REALIZATION OF STATE MODEL [EQN. 4.37]

FIG. 4.1

and no cut-sets of inductors with or without current sources. The network, however, may contain cut-sets of capacitors with the voltage sources only and circuitsof inductors with the current sources only. Reactive elements may be time-varying but differentiable. Further the network does not contain mutual inductances.

The theorem concerning the realization of a network belonging to this special class is given as follows:

THEOREM 4.2

A state model of the form

$$\dot{X} = A(t)X + B(t)U \,,$$
$$Y = C(t)X + D(t)U \,, \qquad .. (4.41)$$

is realizable as an n-port time-varying LC network belonging to the class defined in this section if,

(i) the matrices A(t) and B(t) admit the factorization $\Lambda_t A_1$ and $\Lambda_t B_1$ respectively, where Λ_t is a diagonal matrix and having time-varying entries.

(ii) A solution exists for the set of equations(4.43)

(iii) $\begin{bmatrix} F & I \end{bmatrix}$ so obtained is realizable as a fundamental circuit matrix.

PROOF

Proceeding in the same way as for the proof of the theorem in Section (4.6), we get

$$F = \begin{bmatrix} F_{LV} & F_{LC} \\ F_{KV} & F_{KC} \end{bmatrix}, \qquad .. (4.42)$$

for this class and the state model derived from eqn.(2.15.) after making the above assumptions becomes

$$\begin{bmatrix} \dot{V}_c \\ \dot{I}_L \end{bmatrix} = \begin{bmatrix} C_2^{-1}(t) & 0 \\ 0 & L_{11}^{-1}(t) \end{bmatrix} \begin{bmatrix} \dot{C}_2(t) & F'_{LC} \\ -F_{LC} & \dot{L}_{11}(t) \end{bmatrix} \begin{bmatrix} V_c \\ I_L \end{bmatrix} + \begin{bmatrix} C_2^{-1}(t) & 0 \\ 0 & L_{11}^{-1}(t) \end{bmatrix} \begin{bmatrix} 0 & -F'_{KC} \\ -F_{LV} & 0 \end{bmatrix} \begin{bmatrix} \overset{*}{V} \\ \overset{*}{I}_K \end{bmatrix},$$

$$\begin{bmatrix} \overset{*}{V}_K \\ \overset{*}{I}_V \end{bmatrix} = \begin{bmatrix} -F_{KC} & 0 \\ 0 & -F'_{LV} \end{bmatrix} \begin{bmatrix} V_c \\ I_L \end{bmatrix} + \begin{bmatrix} -F_{KV} & 0 \\ 0 & F'_{KV} \end{bmatrix} \begin{bmatrix} \overset{*}{V}_V \\ \overset{*}{I}_K \end{bmatrix}. \quad \ldots (4.43)$$

The synthesis procedure for this class is self-evident from the form of (4.43). F_{KC}, F_{LV}, F_{KV} are uniquely determined. As $C_2^{-1}(t)$ and $L_{11}^{-1}(t)$ are diagonal, their values can be determined by inspection keeping in mind that entries F_{LC}, F_{KC} and F_{LV} are to be ± 1 or 0. For a consistent solution, it is necessary that the values of $C_2(t)$ and $L_{11}(t)$ determined in this way should always satisfy $\dot{C}_2(t)$ and $\dot{L}_{11}(t)$. $[F \quad I]$ obtained can be tested for realizability by well-known conditions[49].

4.6 REALIZATION OF A CLASS OF A-MATRIX

With the introduction of A-matrix as a new method of network description by Bashkow in 1957, there has been a considerable interest in the realization of this matrix. In particular, Dervisoglu[19] considered the realization of a class of half-degenerate RLC networks i.e. when the network under consideration has no cut-sets of inductors only and no circuits of capacitors only. Nordgren and Tokad [57] gave a procedure for the realization of a more general class of A-matrix than given by Dervisoglu [19] i.e. the network may have cut-sets of inductors only and circuits of capacitors only. This section considers the realization of a class of A-matrix in which the netowrk may have cut-sets of inductors only and circuits of capacitors only but there should be no coupling between the link resistances and tree-branch conductances in the network ($F_{RG} = 0$, page 684 Ref.38).

The A-matrix for RLC networks can be written from eqn. (2.12a) as

$$A = \begin{bmatrix} \mathcal{C}^{-1} & 0 \\ 0 & \mathcal{L}^{-1} \end{bmatrix} \begin{bmatrix} -\mathcal{Y} & \mathcal{A} \\ -\mathcal{A}' & -\mathcal{Z} \end{bmatrix}, \qquad \text{.. (4.44)}$$

where

$$\mathcal{C} = C_2 + F'_{SC} C_1 F_{SC},$$

$$\mathcal{L} = L_{11} + F_{L\Gamma} L_{22} F'_{L\Gamma},$$

$$\mathcal{Y} = F'_{RC} \mathcal{R}^{-1} F_{RC},$$

$$\mathcal{Z} = F_{LG} \mathcal{G}^{-1} F'_{LG},$$

$$\mathcal{A} = F'_{LC} - F'_{RC} \mathcal{R}^{-1} F_{RG} R_2 F'_{LG},$$

$$\mathcal{R} = R_1 + F_{RG} R_2 F'_{RG},$$

$$\mathcal{G} = G_2 + F'_{RG} G_1 F_{RG},$$

$$R_2 = G_2^{-1} \text{ and } G_1 = R_1^{-1}, \qquad \text{.. (4.45)}$$

If A-matrix in eqn.(4.44) is restricted to a class such that $F_{RG} = 0$, it becomes

$$A = \begin{bmatrix} \mathscr{C}^{-1} & 0 \\ 0 & \mathscr{L}^{-1} \end{bmatrix} \begin{bmatrix} -F'_{RC}G_1F_{RC} & F'_{LC} \\ -F_{LC} & -F_{LG}R_2F'_{LG} \end{bmatrix} . \qquad .. (4.46)$$

For the same class, the matrix K_a defined in [57] is given as

$$K_a = \begin{bmatrix} \mathscr{C}^{-1} & 0 \\ 0 & \mathscr{L}^{-1} \end{bmatrix} \begin{bmatrix} F'_{RC}G_1F_{RC} & F'_{LC} \\ F_{LC} & F_{LG}R_2F'_{LG} \end{bmatrix}$$

$$= D_a^{-1}A_1 \qquad .. (4.47)$$

The K_a matrix is decomposed by the method given in [57]. By comparing the elements of A_1 found in this way with the topological expressions in (4.47) various submatrices can be obtained. The value of the arbitrary parameter γ in D_a^{-1} of [57] is chosen such that entries in F_{LC} and F'_{LC} are +1, -1 or zero. It is clear from eqn.(4.47) that F_{LC} is determined uniquely and G_1, R_2, F_{RC} and F_{LG} are found by applying Cederbaum algorithm to $F'_{RC}G_1F_{RC}$ and $F_{LG}R_2F'_{LG}$. $\mathscr{C}$ and $\mathscr{L}$ can be decomposed by Cederbaum's algorithm. Rearranging rows and columns gives,

$$\mathscr{C} = \begin{bmatrix} I & F'_{SC} \end{bmatrix} \begin{bmatrix} C_2 & \\ & C_1 \end{bmatrix} \begin{bmatrix} I \\ F'_{SC} \end{bmatrix}$$

$$\mathscr{L} = \begin{bmatrix} I & F_{Lt} \end{bmatrix} \begin{bmatrix} L_{11} & \\ & L_{22} \end{bmatrix} \begin{bmatrix} I \\ F'_{Lt} \end{bmatrix}$$

from which C_2, C_1, L_{11}, L_{22}, F_{SC} and $F_{L\Gamma}$ can be determined. Fundamental circuit matrix $[F \quad I]$ can be obtained and tested for the realizability by the well-known techniques of Gould or Mayeda [49].

EXAMPLE 4.3

In order to illustrate the procedure discussed above, the example considered previously [57] has been chosen. It may be noted that the example belongs to the class of RLC networks discussed in this section.

Let

$$\dot{X} = AX = \begin{bmatrix} \dot{V}_{c1} \\ \dot{V}_{c2} \\ \hline \dot{I}_{L3} \end{bmatrix} = \left[\begin{array}{cc|c} -3/5 & 1/5 & 2/5 \\ 1/5 & -2/5 & 1/5 \\ \hline -1/8 & -1/8 & -1/4 \end{array}\right] \begin{bmatrix} V_{c1} \\ V_{c2} \\ \hline I_{L3} \end{bmatrix}$$

$$= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} V_{c1} \\ V_{c2} \\ I_{L3} \end{bmatrix} \qquad \ldots (4.48)$$

and let K be $= \begin{bmatrix} -A_{11} & A_{12} \\ -A_{21} & -A_{22} \end{bmatrix} = \begin{bmatrix} \mathcal{C}^{-1} & 0 \\ 0 & \mathcal{L}^{-1} \end{bmatrix} \begin{bmatrix} Y & H \\ H' & Z \end{bmatrix}$

$$= D_a^{-1} A_1$$

$$= D_a^{-1/2} E_a D_a^{1/2}$$

$$\text{where } E_a = \begin{bmatrix} \mathcal{E}^{-1/2} Y_a \mathcal{E}^{-1/2} & \mathcal{E}^{1/2} H_a \mathcal{L}^{-1/2} \\ \mathcal{L}^{-1/2} H_a \mathcal{E}^{-1/2} & \mathcal{L}^{-1/2} Z_a \mathcal{L}^{-1/2} \end{bmatrix}$$

is a symmetric matrix which can be diagonalised into $P^{-1} \Lambda_a P$

$$= D_a^{-1/2} P_a^{-1} \Lambda_a P_a D_a^{-1/2}$$

$$= Q_a^{-1} \Lambda_a Q_a \qquad \text{where } Q_a = P_a D_a^{1/2}$$

and $D_a = Q_a' Q_a$ has been calculated to be equal to

$$15\alpha^2 \left[\begin{array}{cc|c} 2 & 1 & 0 \\ 1 & 2 & 0 \\ \hline 0 & 0 & 8 \end{array}\right]$$, where α is an arbitrary parameter[57]

$$\text{Thus} \quad A_1 = D_a K_a = 15\alpha^2 \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \\ \hline 1 & 1 & 2 \end{array}\right] .$$

As the entries of F_{LC}, F'_{LC} in eqn.(4.47) are to be +1, -1, or zero, α^2 is chosen as $\frac{1}{15}$ and therefore

$$A_1 = \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \\ \hline 1 & 1 & 2 \end{array}\right] .$$

Now comparing various entries of A_1 with eqn.(4.47) and using Cederbaum decomposition procedure, we get

$$F'_{LC} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad L_{11} = \begin{bmatrix} 8 \end{bmatrix},$$

$$F'_{RC} G_1 F_{RC} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} .$$

Therefore

$$F_{RC} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad G_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, F'_{RC} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} . \qquad .. \ (4.49)$$

Also, we get

$$F_{LG} R_2 F'_{LG} = \begin{bmatrix} 2 \end{bmatrix},$$

therefore

$$F_{LG} = \begin{bmatrix} 1 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 2 \end{bmatrix}, \quad F'_{LG} = \begin{bmatrix} 1 \end{bmatrix}$$

and further, as we get

$$C_2 + F'_{SC} C_1 F_{SC} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= \left[\begin{array}{cc:c} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}\right] \left[\begin{array}{cc:c} 1 & & \\ & 2 & \\ \hdashline & & 1 \end{array}\right] \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \\ \hdashline 1 & 1 \end{array}\right], \qquad .. \ (4.50)$$

therefore

$$C_2 = \begin{bmatrix} 1 & \\ & 2 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 \end{bmatrix}, \quad F_{SC} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} . \qquad .. \ (4.51)$$

$\begin{bmatrix} F & I \end{bmatrix}$ obtained from above is given by

$$\begin{array}{c} \\ R_1 \\ R_2 \\ L_1 \\ S_1 \end{array} \begin{array}{c} \begin{array}{ccc:cccc} C_1 & C_2 & G_1 & R_1 & R_2 & L_1 & S_1 \end{array} \\ \left[\begin{array}{ccc:cccc} 1 & 0 & 0 & 1 & & & \\ 0 & 1 & 0 & & 1 & & \\ 1 & 1 & 1 & & & 1 & \\ 1 & 1 & 0 & & & & 1 \end{array}\right] \end{array}, \qquad .. \ (4.52)$$

which is realizable, the graph and the corresponding network being given in Figs.(4.3a) and (4.3b).

The procedure evolved, though applicable to a restricted class of networks is simpler than that proposed by Nordgren and Tokad[57]. This is mainly because the procedure discussed here gives a simpler technique for realizing the hybrid matrix and the topology of the network compared to that discussed in [57].

4.7 MINIMAL REALIZATIONS OF STATE EQUATIONS

The realization procedures discussed earlier in this chapter are meant for realizations of minimal state equations. If the given set of state equations is not minimal, proceduresexist to obtain the one which is minimal, This problem of minimal realizations of uncontrollable and/or unobservable state equations has been investigated by several authors for time invariant[28], [50] and time-varying [8] state equations. In this section, a simpler algorithm is given by constructing minimal realization of time-varying systems.

The state equations for a time-varying system can be written

$$\dot{X}(t) = A(t)X(t) + B(t)u(t),$$
$$Y(t) = C(t)X(t) \, , \qquad .. (4.53)$$

where A(t), B(t) and C(t) are $n \times n$, $n \times m$ and $p \times n$ matrices respectively with possibly time-varying elements and where A(t), B(t) and C(t) and their (n-2), (n-1) and (n-1) derivatives are continuous functions. The controllability and observability matrices for such a system, are given as

$$Q_c(t) = \left[P_o(t) \,\vdots\, P_1(t) \,\vdots\, \ldots \,\vdots\, P_{n-1}(t)\right] \qquad \ldots (4.54a)$$

where

$$P_{k+1}(t) = -A(t)P_k(t) + \dot{P}_k(t), \qquad P_o(t) = B(t)$$

and

$$Q_o(t) = \left[R_o(t) \,\vdots\, R_1(t) \,\vdots\, \ldots \,\vdots\, R_{n-1}(t)\right] \qquad \ldots (4.54b)$$

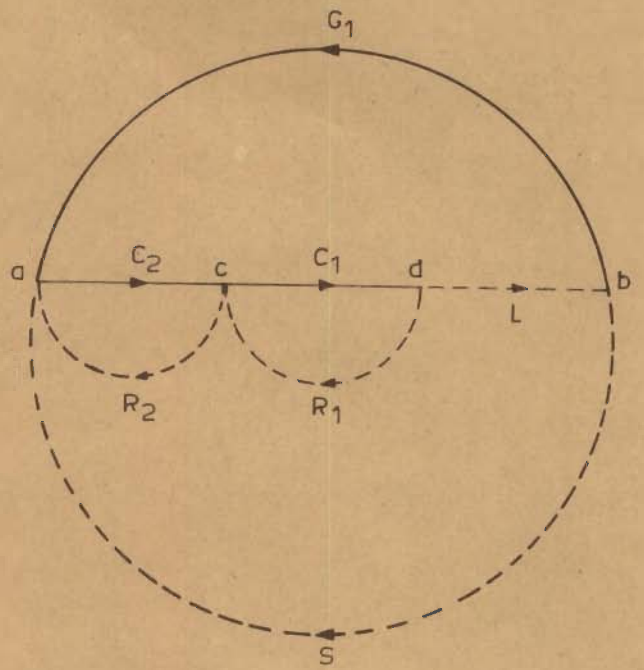
where

$$R_{k+1}(t) = A'(t)\, R_k(t) + \dot{R}_k(t), \qquad R_o(t) = C'(t).$$

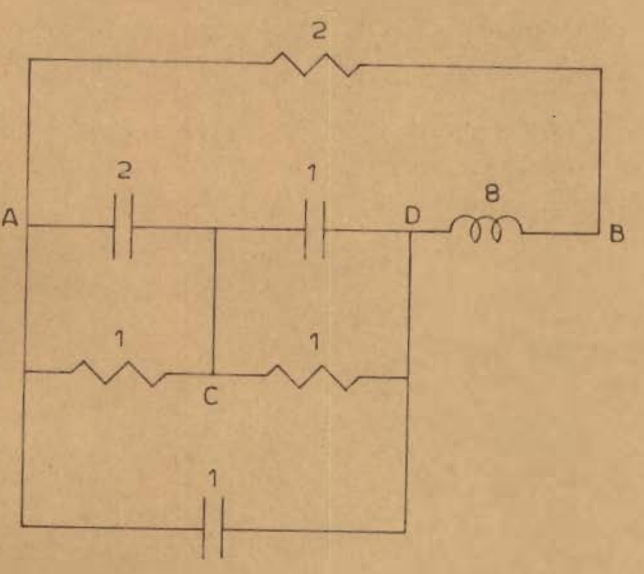
The procedure discussed here is an extension of the method given by Mayne [50] and although it is applicable to remove uncontrollable as well as unobservable modes, only the removal of uncontrollable modes is considered here. In order to remove such modes, the following algorithm is suggested.

Algorithm

(1) Construct an $n \times n_k$ matrix S(t) by the procedure described in [45] or [50] using eqn. (4.54), where n and n_k are the dimensions of uncontrollable and controllable sub-space. Let $s_1(t), s_2(t), \ldots, s_{nk}(t)$ denote the independent column vectors in S(t); $b_1(t), b_2(t), \ldots, b_m(t)$, the column vectorsin B(t); $p_o^1, p_o^2, \ldots, p_o^m$, the column vectors in $P_o(t)$; $p_1^1, p_1^2, \ldots, p_1^m$, the column vectors of $P_1(t)$ and



(a) REALIZATION OF CIRCUIT MATRIX [EQN. 4.52]



(b) REALIZATION OF STATE EQUATIONS [EQN. 4.48]

FIG. 4.3

and so on.

Start with the vector $S_1(t) = p_o^1(t) = b_1(t)$.
Now select the next vector

$$p_{k+1}^1(t) = -A(t)p_k^1(t) + \dot{p}_k^1(t), \qquad k=o \text{ to } n-1$$

If the selected vector is linearly independent of all the previous vectors, retain it in the columns of S(t) otherwise omit it. Next, proceed with the vector $b_2(t)$ and repeat the preceding to find $p_{k+1}^2(t)$ and so on (k=o to n-1). Proceed till n independent vectors are formed. If there are uncontrollable modes in the given system, the above process will terminate at n_k where $n_k \leqslant n$.

(ii) Find any n_k x n matrix V(t) such that

$$V(t)S(t) = I_{nk} \qquad .. (4.55)$$

(iii) The controllable part

$$R_k(t) = \left[A_k(t), B_k(t), C_k(t) \right]$$

of given realization is found by

$$A_k(t) = V(t)\,A(t)\,S(t),$$
$$B_k(t) = V(t)\,B(t),$$
$$C_k(t) = C(t)\,S(t)\,. \qquad .. (4.56)$$

The proof of the algorithm is similar to that given in [50] for time-invariant systems. The use of the algorithm is illustrated with the help of an example considered previously [8], [23], [70].

Example 4.4

Let

$$A(t) = \begin{bmatrix} t-1 & 0 & -t+2 \\ t-2 & 1 & t+2 \\ t & 0 & -t-1 \end{bmatrix}, \quad B(t) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix},$$

$$C(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \qquad .. (4.57)$$

By step (i) we find

$$S(t) = \begin{bmatrix} 1 & 1-t \\ 1 & 1+t \\ 0 & -t \end{bmatrix}. \qquad .. (4.58)$$

Using step (ii)

$$V(t) = \begin{bmatrix} 1+\frac{1}{2t} & -\frac{1}{2t} & -1 \\ -\frac{1}{2t} & \frac{1}{2t} & 0 \end{bmatrix}. \qquad .. (4.59)$$

Finally, using step(iii) we find

$$A_K(t) = V(t)A(t)S(t) = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \quad B_K(t) = V(t)B(t) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix},$$

$$C_K(t) = C(t)S(t) = \begin{bmatrix} 1 & 1-t \\ 1 & 1+t \\ 0 & -t \end{bmatrix}. \qquad .. (4.60)$$

The method suggested here is simpler than that given in [8] because S(t) is of lower order than $Q_c(t)$ of eqn.(8) in [8], having retained the independent columns only. Consequently, further manipulations are with the lower order matrix. The algorithm is more attractive especially when the number of uncontrollable/(unobservable) modes is large.

4.9 CONCLUSION

Considerable interest has been shown by various investigators on the realization of state equations for n-port LC, RLC and portless RLC networks. However, the methods used by these investigators are quite involved as the decomposition of A-matrix of the state equations and the realization of resulting hybrid matrix are quite cumbersome. Simpler algorithms have been presented in this chapter.

In particular, the state-space representation for a general LC network with independent sources of the network as exclusive component of the input vector does not always exist. When the state variables are chosen as capacitor voltages and inductor currents, the state-space characterization of LC networks involves derivatives of sources, so in order to transform the state model (3.2) to state model (3.6) a transformation (3.3) has been chosen. Once the state model is in the form (3.6), it is plausible that its decomposition in the form (4.2) can be carried out, simplifying considerably the rest of the procedure. Further, the available methods proposed for RLC n-port state equations are quite tedious. But if we restrict the RLC state equations to a class described in Section (4.4) the method of decomposition of state equations and the realization of resulting hybrid matrix becomes quite easy. Similar procedure has also been proposed for LC time varying case. Further, interest has also been shown in literature on the realization of A-matrix. The available methods are quite tedious. Again, a procedure though applicable to a restricted class, given in Section (4.6), is simpler than the existing methods.

Some times the given state equations are not minimal. However, they can be made minimal by the well-known techniques available for time-invariant linear state equations. An algorithm has been presented by which uncontrollable (unobservable) states can be removed for the given state equations for linear time-varying systems. The algorithm which is an extension of the technique given by Mayne[50] for linear time invariant systems makes use of time-varying controllability and observability matrices and appears to be simpler than the existing methods.

In the next chapter, the other facet of the problem is touched upon i.e. the synthesis from input-output characterization in s-domain is discussed.

CHAPTER V

SYNTHESIS FROM INPUT-OUTPUT CHARACTERIZATION IN s-DOMAIN

5.1 INTRODUCTION

Quite often, the given information is in terms of input-output specifications in s-domain. A natural approach to the synthesis, as has been recognised by many investigators, is by means of state model since the state model of the network provides more direct information about the network and its topology than the network matrices. Several attempts have been made to realize the network by this approach [6]. The state models derived in these cases necessitate the use of RLC elements, transformers and gyrators. Youla and Tissi [90] gave a procedure for obtaining a network without gyrators from scattering parameters. Recently, by combining the techniques of Youla and Tissi [90] and Anderson and Newcomb [6], Yarlagadda has proposed synthesis procedures from hybrid parameters of an RLC network, which eliminate the use of gyrators. This chapter discusses the improved and systematic synthesis procedures from the input-output specifications in s-domain. In particular, the chapter presents (i) minimal reciprocal realizations from a given symmetric matrix, (ii) minimal passive reciprocal synthesis from a given positive real hybrid matrix, and (iii) some aspects of synthesis of minimum biquadratic functions from state-space point of view.

5.2 MINIMAL RECIPROCAL REALIZATION FROM A GIVEN SYMMETRIC MATRIX

The problem of minimal realizations from a transfer-function matrix has been widely investigated during recent years, but the method given by Ho and Kalman has been acclaimed to be the simplest one available [28]. The method essentially consist of generating "Hankel matrix" S_r, where

$$S_r = \begin{bmatrix} Y_o & Y_1 & \cdots & Y_{r-1} \\ Y_1 & Y_2 & \cdots & Y_r \\ \vdots & \vdots & & \vdots \\ Y_{r-1} & Y_r & \cdots & Y_{2r-2} \end{bmatrix} \qquad \ldots (5.1)$$

is built of Markov parameters [21] Y_k's determined by dividing numerator polynomials of each entry of the transfer-function matrix by common denominator. Non-singular matrices P_r and Q_r are then found by well-known computing techniques [17] such that

$$P_r S_r Q_r = \begin{bmatrix} I_s & 0 \\ 0 & 0 \end{bmatrix} = J_r , \qquad \ldots (5.2)$$

where I_s is $s \times s$ unit matrix and suffix s equals rank of S_r, and J_r is idempotent.

If we choose E_p a block matrix $\begin{bmatrix} 1_p & 0_p & \ldots & 0_p \end{bmatrix}$ and ulh means the operator which picks out the upper left hand block in block matrices, then the minimal realization A,B,C is given by

The importance of such realizations is attributed to the fact that they result in reciprocal networks, and further it has been proved in [37], [90] that all reciprocal realizations for RC and RL cases are passive. Hence, in the following, the algorithm by Ho and Kalman is modified so as to determine such P_r and Q_r as further result in $[A, B, C]$ satisfying eqn.(5.5).

Now, since the given matrix is symmetric, S_r of eqn.(5.1) will obviously be symmetric. Therefore, a non-singular P_r can always be found such that

$$P_r S_r P_r' = \begin{bmatrix} \Sigma_1 & & \\ & \Sigma_2 & \\ & & 0 \end{bmatrix}, \qquad ..\ (5.6)$$

where

$$\begin{bmatrix} \Sigma_1 & \\ & \Sigma_2 \end{bmatrix} = \text{rank of } S_r.$$

Multiplying both sides of the above equation by

$$\begin{bmatrix} \Sigma_1 & & \\ & \Sigma_2 & \\ & & 0 \end{bmatrix}$$

we get

$$P_r S_r P_r' \Sigma = \begin{bmatrix} I_s & 0 \\ 0 & 0 \end{bmatrix}$$

$$= I_r \qquad ..\ (5.7)$$

$$A = \text{ulh}\left[J_r P_r (\uparrow S_r) Q_r J_r\right],$$

$$B = \text{ulh}\left[J_r P_r S_r E_m'\right], \qquad \text{.. (5.3)}$$

and

$$C = \text{ulh}\left[E_p S_r Q_r J_r\right],$$

where

$$\uparrow S_r = \begin{bmatrix} Y_1 & Y_2 & \cdots & Y_r \\ Y_2 & Y_3 & \cdots & Y_{r+1} \\ \vdots & \vdots & & \vdots \\ Y_r & Y_{r+1} & \cdots & Y_{2r-1} \end{bmatrix}. \qquad \text{.. (5.4)}$$

For a particular S_r, determination of P_r and Q_r satisfying eqn.(5.2), is a well-known problem in matrix algebra. For a particular S_r, there can result innumerous P_r's and Q_r's such that eqn.(5.2) is satisfied. Each set of P_r and Q_r will give a different realization $[A, B, C]$.

In many problems we are given symmetric matrix $\mathcal{W}(s)$ and we are interested in finding a realization $[A, B, C, D]$ such that

$$\begin{bmatrix} I & \\ & \Sigma \end{bmatrix} \begin{bmatrix} D & C \\ B & A \end{bmatrix} \qquad \text{.. (5.5)}$$

is symmetric, where

$$\Sigma = \begin{bmatrix} \Sigma_1 & \\ & \Sigma_2 \end{bmatrix},$$

when Σ_1 = number of +1's on diagonal matrix, and

Σ_2 = number of -1's on diagonal matrix.

Therefore

$$A = \text{ulh}\left[J_r P_r (\Upsilon S_r) P_r' \Sigma J_r \right],$$

$$B = \text{ulh}\left[J_r P_r S_r E_m' \right],$$

$$C = \text{ulh}\left[E_p S_r P_r' \Sigma J_r \right]. \quad .. (5.8)$$

It may be seen that this realization will satisfy (5.5) as is illustrated in the following examples.

Example 5.1

Consider a symmetric matrix [50]

$$W(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{1}{s+1} \\ \frac{1}{s+1} & \frac{1}{s+1} \end{bmatrix}. \quad .. (5.9)$$

From eqn. (5.1)

$$S_r = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

Therefore

$$\Upsilon S_r = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}.$$

From eqn. (5.6),

$$P_r = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, J_r = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}. \quad .. (5.10)$$

Therefore from eqn.(5.8), we get

$$A = \begin{bmatrix} -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \qquad ..(5.11)$$

It can be seen that eqn.(5.5) is satisifed.

Example 5.2

$$\text{Given a symmetric } \mathcal{W}(s) = \frac{\frac{1}{8}s^2 + \frac{2}{8}s + \frac{9}{8}}{s^2 + 2s + 1}. \qquad ..(5.12)$$

We get

$$\mathcal{W}_1(s) = \frac{\frac{1}{8}s^2 + \frac{2}{8}s + \frac{3}{8}}{s^2 + 2s + 1} - \frac{1}{8}$$

$$= \frac{1}{s^2 + 2s + 1}$$

$$= s^{-2} - 2s^{-3} + 3s^{-4} - 4s^{-5} + \ldots$$

Hankel Matrix

$$S_r = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} \text{ and } \uparrow S_r = \begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix}. \qquad ..(5.13)$$

As S_r is symmetric, it can always be decomposed in the form (5.6)

where

$$P_r = \begin{bmatrix} \sqrt{2} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 1 & \\ & -1 \end{bmatrix}, \quad J_r = \begin{bmatrix} 1 & \\ & 1 \end{bmatrix},$$

$$E'_m = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad E_b = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

Using (5.8), we get

$$A = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{3}{2} \end{bmatrix}, \qquad B = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix},$$

$$C = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} \frac{1}{8} \end{bmatrix},$$

Therefore, we get

$$\begin{bmatrix} D & C \\ B & A \end{bmatrix} = \begin{bmatrix} \frac{1}{8} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{3}{2} \end{bmatrix}.$$

It can be seen that eqn.(5.5) is satisfied.

The proposed method is a modification of the method given by Ho and Kalman [28] for the case of symmetric matrices and has its novelty because of eqn.(5.7) which according to author's knowledge, has not been considered earlier. The method is better than that given by Youla and Tissi [90] as the additional labour of finding inverse of matrices (Eqn. I-29 [90]) is avoided.

5.3 MINIMAL PASSIVE RECIPROCAL SYNTHESIS FROM A GIVEN SYMMETRIC POSITIVE REAL HYBRID MATRIX

In the synthesis procedure, flexibility in choosing a transformation T in Section (2.5) allows us to introduce both passivity and reciprocity constraints. But, unfortunately, both the constraints cannot be met simultaneously and as such it is difficult to choose a T which makes the state model both passive and reciprocal. To remedy this, two methods have recently been given by Yarlagadda [85]. The salient steps involved in these methods are recapitulated first as follows. In method 1, passivity constraints are satisifed first and then a transformation is chosen which satisfies reciprocity but maintains passivity. In method 2, reciprocity is satisfied first and then a transformation is applied which maintains reciprocity but satisfies passivity. These methods are briefly discussed first with a view to obtain an improved synthesis method.

Method 1

Let H(s) represent hybrid parameters of an RLCT network having no pole at infinity. The method for arriving at the desired state model is explained in the following steps.

(i) From given H(s), obtain any state model

$$\dot{X}_1 = A_1X_1 + B_1U,$$
$$\bar{U} = C_1X_1 + D_1U, \qquad .. (5.14)$$

such that $H(s) = D_1 + C_1(sI - A_1)^{-1}B_1$.

(ii) The necessary and sufficient conditions for H(s) to be positive real have been given in lemma 3 of Section (2.5.2). So, there exists a symmetric positive definite P and matrices W_o and L such that eqn. (2.24) is satisfied. Determine the positive definite matrix P and $P^{1/2}$.

(iii) Choose a transformation,

$$X_2 = P^{-1/2} X_1 \qquad \text{.. (5.15)}$$

and obtain the new state model

$$\dot{X}_2 = A_2 X_2 + B_2 U, \qquad \text{.. (5.16a)}$$

$$\bar{U} = C_2 X_2 + D_2 U. \qquad \text{.. (5.16b)}$$

It has been shown in [85] that matrix M_2 generated from (5.16)

$$M_2 = \begin{bmatrix} D_2 & C_2 \\ -B_2 & -A_2 \end{bmatrix} \qquad \text{.. (5.17)}$$

is positive semi-definite.

(iv) The next step is to apply eqn.(2.27) to (5.17) i.e. select the non-singular transformation T_1 such that

$$(I \dotplus T_1)M_2 = M_2'(I \dotplus T_1) . \qquad \text{.. (5.18)}$$

T_1 is also symmetric and it can be represented as [85]

$$T_1 = S_1 E = E S_1 , \qquad \text{.. (5.19)}$$

where S_1 is a symmetric positive definite matrix and E is symmetric and an orthogonal matrix and can be written as

$$E = Q_1 \Sigma Q_1', \qquad \text{.. (5.20)}$$

where Q_1 is an orthogonal matrix, and Σ is a diagonal matrix with $\pm$ 1's.

(v) Choose a transformation

$$X_2 = Q_1 X_3 \qquad .. (5.21)$$

and apply on eqn.(5.16). Multiply the first equation thus obtained by $Q_1' S_1$ to obtain the state model

$$Q_1' S_1 Q_1 \dot{X}_3 = Q_1' S_1 A_3 Q_1 X_3 + Q_1' S_1 B_3 U,$$

$$\bar{U} = C_3 Q_1 X_3 + D_3 U. \qquad .. (5.22)$$

The state model (5.22) can be realized using RLC elements, and transformers, the proof being given in [85].

Method 2

In this method reciprocity conditions are satisfied first. The method consists of the following steps.

(i) Obtain any state model

$$\dot{X}_4 = A_4 X_4 + B_4 U,$$

$$\bar{U} = C_4 X_4 + D_4 U, \qquad .. (5.23)$$

such that

$$H(s) = D_4 + C_4 (sI - A_4)^{-1} B_4.$$

(ii) Applying theorem 2.1 in Section (2.5.3), determine T satisfying eqn.(2.27) and Σ and L_1 given by eqn.(2.28).

(iii) Applying transformation $T = L_1$ obtain the new state model

$$\dot{X}_5 = A_5X_5 + B_5U,$$

$$\bar{U} = C_5X_5 + D_5U, \qquad .. (5.24)$$

which will satisfy the symmetry condition, i.e.

$$\begin{bmatrix} I & \\ & \Sigma \end{bmatrix} \begin{bmatrix} D_5 & C_5 \\ -B_5 & -A_5 \end{bmatrix}$$

is a symmetric matrix. .. (5.25)

(iv) From (5.24), obtain symmetric positive definite P as discussed in [85].

(v) The matrix P

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P'_{12} & P_{22} \end{bmatrix}$$

can always be written as [85]

$$\begin{bmatrix} I & -Q \\ -Q' & I \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P'_{12} & P_{22} \end{bmatrix} \begin{bmatrix} I & -Q \\ -Q' & I \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{12} & K_{22} \end{bmatrix} \qquad .. (5.26)$$

where $K_{12} = 0$.

(vi) Choose the transformation

$$T = \begin{bmatrix} I & -Q \\ -Q' & I \end{bmatrix} \qquad .. (5.27)$$

where, T^{-1} is given by

$$T^{-1} = \begin{bmatrix} (I-QQ')^{-1} & 0 \\ 0 & (I-Q'Q)^{-1} \end{bmatrix} \begin{bmatrix} I & Q \\ Q & I \end{bmatrix}$$

such that

$$\Sigma\, T\, \Sigma T = D_q \qquad .. (5.28)$$

and

$$\Sigma\, D_q \Sigma = D_q$$

(vii) Using the transformation $X_6 = T^{-1}X_5$ given by eqn. (5.27) and eqn. (5.28) we obtain the new state model as

$$D_q \dot{X}_6 = \Sigma\, T\, \Sigma\, A_6 T\, X_6 + \Sigma\, T\, \Sigma\, B_6 U ,$$

$$\bar{U} = C_6 T\, X_6 + D_6 U . \qquad .. (5.29)$$

It is proved in [85] that state model in (5.29) can be realized using RLCT elements.

5.4 PROPOSED METHOD SUITABLE FOR COMPUTERIZATION

The first step in both the methods given above is to determine any state model corresponding to the given specifications. Suitable constraints of passivity or reciprocity are then applied. As in synthesis problem, we are given a symmetric positive real matrix and our object is to construct an RLCT realization without gyrators, the above procedures can be modified by exploiting the symmetry of the given specifications. The proposed synthesis procedure is given in the following steps.

(i) Construct $[A, B, C, D]$ as follows. Determine the Hankel Matrix from the Markov parameters of the given positive real matrix. As the given matrix is symmetric, the Hankel matrix will also be symmetric. A symmetric matrix can always be decomposed in the form $M\Sigma M'$ [17], where Σ can be uniquely determined by H(s) and is the associated reactance matrix. Now, set

$$M = M$$

and

$$N = \Sigma M'$$

where

$$M = \begin{bmatrix} M_o \\ M_1 \\ \vdots \\ M_{r-1} \end{bmatrix} \quad \text{and } N = \begin{bmatrix} N_o & N_1 & \cdots & N_{r-1} \end{bmatrix}. \qquad \text{.. (5.30)}$$

We get [90]

$$C = M_o ,$$

$$B = \Sigma M_o' ,$$

$$A = M^{-1} \Omega M = (N \Omega' N^{-1}) , \qquad \text{.. (5.31)}$$

where Ω is the 'generalized' companion matrix given by eqn.I-14 of [90] as

$$\Omega = \begin{bmatrix} O_m & I_m & O_m & \cdots & O_m \\ O_m & O_m & I_m & \cdots & O_m \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ O_m & O_m & O_m & \cdots & I_m \\ -b_r I_m & -b_{r-1} I_m & \cdots & \cdots & -b_1 I_m \end{bmatrix}$$

and

$$D = H(\infty)$$

and also as proved in [90]

$$\Sigma A = A'\Sigma$$

and

$$\begin{bmatrix} I \dotplus \Sigma \end{bmatrix} \begin{bmatrix} D & C \\ B & A \end{bmatrix}$$

is a symmetric matrix.

(ii) Calculate $W(s)$ by spectral factorization [88] and determine Hankel matrix formed from the Markov parameters of $W(s)$. It has been shown in [6] that if $H(s)$ has a realisation $[A,B,C]$, $W(s)$ has a realization $[A,B,L]$. For $H(s)$ of order 1x1, $[A,B,L]$ can be easily determined as is shown in [6], but for $H(s)$ of higher order and when $[A,B,C]$ of $H(s)$ are determined from Markov parameters, the author feels that there is no specific procedure available to find the realization of $W(s)$ such that $[A, B]$ of $H(s)$ and that of $W(s)$ are same [1]. To do this, we use the derivation in [58] used for determining the residues of poles of transfer function from given state equations. This result can be stated in the form of the following Lemma.

LEMMA 5.1 If the sequence CB, CAB, ... , $CA^{n-1}B$ and the sequence B, AB, ... , A^{n-1} B of a minimal realization are given, then C is determined uniquely by

$$C = \begin{bmatrix} CB & CAB & CA^{n-1}B \end{bmatrix} \begin{bmatrix} B & AB & A^{n-1}B \end{bmatrix}^{-1} \quad .. (5.32)$$

For proof see [58].

Now in our case, $\begin{bmatrix} CB, & CAB \ldots & CA^{n-1}B \end{bmatrix}$ are determined as the Markov parameters of W(s) and if $\begin{bmatrix} B & AB \ldots & A^{n-1}B \end{bmatrix}$ is known, matrix C which in our case of W(s) is L, can be determined from eqn.(5.32).

It is interesting to note that $\begin{bmatrix} B & AB & \ldots & A^{n-1}B \end{bmatrix}$ of W(s) which is also equal to $\begin{bmatrix} B & AB & \ldots & A^{n-1}B \end{bmatrix}$ of H(s) is nothing but the matrix M determined already in the decomposition of Hankel matrix found from Markov parameters of H(s)[90]. It is seen that M is often required for finding the intermediate expressions in the rest that follows and so can be stored separately for computerization of the procedure. So knowing L, L'L (Eqn.2.24) can be obtained.

(iii) The next step is the determination of P. It can be obtained from equation (2.24a). The method as given in [6] requires the solution of nx(n+1) / 2 simultaneous equations which becomes quite complicated as n, the order of A, increases as it would require the determination of inverse of a large matrix. In the following, a method for the determination of P is suggested. The method is especially suited for computerization as P can be obtained in terms of the already available expressions in the procedure or their multiplications etc. For simplicity the case when $H(\infty) = 0$ is considered here although similar expressions can be derived for the case when $H(\infty)$ is finite. Equation (2.22) is rewritten as

$$PA + A'P = -L'L ,$$
$$PB = C' . \quad .. (5.33)$$

So we can write

$$PA = -A'P - L'L$$

$$PAB = -A'PB - L'LB = -A'C' - L'LB = (-1)A'C'+(-1)L'LB$$

$$PA^2B = -A'PAB - L'LAB = -A'\left[-A'C'-L'LB\right] - L'LAB$$
$$= (-1)^2A'^2C'+(-1)^2A'L'LB - L'LAB$$

$$PA^3B = -A'PA^2B - L'LA^2B$$
$$= -A'\left[(-1)^2A'^2C' + (-1)^2A'L'LB - L'LAB\right] - L'LA^2B$$
$$= (-1)^3A'^3C' + (-1)^3A'^2L'LB + (-1)^2A'L'LAB - L'LA^2B$$

$$PA^4B = -A'PA^3B - L'LA^3B$$
$$= -A'\left[(-1)^3A'^3C' + (-1)^3A'^2L'LB + (-1)^2A'L'LAB-L'LA^2B\right] -L'LA^3B$$
$$=(-1)^4A'^4C'+(-1)^4A'^3L'LB+(-1)^3A'^2L'LAB+(-1)^2A'L'LA^2B-L'LA^3B$$

... ...

... ...

$$PA^{n-1}B = (-1)^{n-1}A'^{n-1}C'+(-1)^{n-1}A'^{n-2}L'LB+(-1)^{n-2}A'^{n-3}L'LAB +\dots +(-1)^{n-3}L'LA^{n-3}B-L'LA^{n-2}B$$

.. (5.34)

Therefore

$$P\left[B \quad AB \quad A^2B \quad \ldots \quad A^{n-1}B\right]$$

$$=\Big[C' \quad (-1)A'C'-L'LB \quad (-1)^2A'^2C'+(-1)^2A'L'LB-L'LAB$$

$$(-1)^3A'^3C'+(-1)^3A'^2L'LB+(-1)^2A'L'LAB-L'LA^2B \quad \ldots$$

$$(-1)^{n-1}A'^{n-1}C'+(-1)^{n-1}A'^{n-2}L'LB+\ldots+(-1)^{n-3}L'LA^{n-3}B$$

$$-L'LA^{n-2}B\Big] \qquad \ldots (5.35a)$$

$$\Rightarrow \left[C' \quad (-1)A'C' \quad (-1)^2A'^2C' \ldots (-1)^{n-1}A'^{n-1}C'\right]$$

$$+ (-1)\,L'L\left[0 \quad B \quad AB \quad A^2B \quad \ldots \quad A^{n-2}B\right]$$

$$+(-1)^2A'L'L\left[0 \quad 0 \quad B \quad AB \quad \ldots \quad A^{n-3}B\right]$$

$$+(-1)^3A'^2L'L\left[0 \quad 0 \quad 0 \quad B \quad \ldots \quad A^{n-4}B\right]$$

$$\ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots$$

$$+(-1)^{n-1}A'^{n-2}L'L\left[0 \quad 0 \quad 0 \quad 0 \quad \ldots \quad B\right]$$

$$= Q_m$$

or $\qquad \ldots (5.35b)$

$$P = \left[B \quad AB \ldots A^{n-1}B\right]^{-1}\left[\,Q_m\,\right]$$

The inverse of $\left[B \quad AB \quad A^2B \quad \ldots \quad A^{n-1}B\right]$ is already known from step (ii) above. So the terms on the right hand side of eqn.(5.35) are known or can be easily manipulated and therefore P can be determined without taking inverse of any matrix once more, which otherwise would have been required in the method as given in [6] or [85].

(iv) The transformation (5.27) is next determined from eqn.(5.26) and the rest of the procedure is identical to that discussed in Method 2 described by Yarlagadda[88].

Further, if H(s) has a pole at infinity, it is always possible to write[6]

$$H(s) = sL + H_1(s)$$

where, $L = L'$ is non-negative definite and $H_1(s)$ is positive real[6]. The synthesis of matrix sL can be accomplished by transformers and coupled inductors[6]. So, the problem of synthesis of H(s) can be reduced to the synthesis of $H_1(s)$.

The algorithm suggested above is more suitable for computerization than the method suggested in [85] because the reciprocity constraint in the suggested algorithm is achieved by determining the appropriate realization directly from the Markov parameters instead of determining it by first finding any realization and then seeking for a transformation which when applied on this realization transforms it to a reciprocal realization. Besides, the suggested algorithm presents a unified approach for determining the realization of W(s) required in the synthesis procedure by making use of Lemma 5.1. Further, the determination of symmetric positive definite matrix P required for passivity constraint is also achieved in terms of the intermediate expressions found in the algorithm.

In order to illustrate the procedure suggested above, an example is considered below. For comparison, the same example, as has been chosen by Yarlagadda [85], is discussed.

Example 5.3 Synthesise the positive real admittance function

$$Y(s) = \frac{\frac{1}{8}(s^2+2s+9)}{s^2+2s+1} . \qquad .. (5.36)$$

Solution :

Step (i)

$$Y_1(s) = \frac{\frac{1}{8}(s^2+2s+9)}{s^2+2s+1} - \frac{1}{8}$$

$$= \frac{1}{s^2+2s+1}$$

$$= s^{-2}-2s^{-3}+3s^{4}-4s^{-5}+ \dots + \dots \qquad .. (5.37)$$

Hankel Matrix

$$S_r = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}, \qquad .. (5.38)$$

which can be decomposed in the form

$$M\Sigma M' = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} -1 & \\ & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\sqrt{2} \\ -\frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$= MN = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{2}} & \sqrt{2} \\ \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} M_o \\ M_1 \end{bmatrix} \begin{bmatrix} N_o & N_1 \end{bmatrix} = \begin{bmatrix} C \\ CA \end{bmatrix} \begin{bmatrix} B & AB \end{bmatrix} \qquad .. (5.39)$$

Therefore

$$C = M_o = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

and

$$B = N_o = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} .$$

Further,

$$A = M^{-1} \Omega M$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\sqrt{2} & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\sqrt{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{3}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} . \qquad .. (5.40)$$

As a check A can also be found as

$$A = N \Omega' N^{-1}$$

$$= \begin{bmatrix} -\frac{1}{\sqrt{2}} & \sqrt{2} \\ \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{2}} & \sqrt{2} \\ \frac{1}{\sqrt{2}} & 0 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} -\frac{3}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} .$$

Therefore

$$\begin{bmatrix} D & C \\ B & A \end{bmatrix} = \begin{bmatrix} \frac{1}{8} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{3}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} . \qquad .. (5.41)$$

It can be seen that

$$\begin{bmatrix} I & \\ & \Sigma \end{bmatrix} \begin{bmatrix} D & C \\ B & A \end{bmatrix}$$

is symmetric.

Step (ii)

By spectral factorization

$$W(s) = \frac{1}{2} \cdot \frac{s^2+3}{s^2+2s+1} \qquad .. (5.42)$$

$$= W_1(s) + W(\infty) ,$$

or

$$W_1(s) = \frac{-s+1}{s^2+2s+1} . \qquad .. (5.43)$$

Therefore

$$S_r = \begin{bmatrix} -1 & 3 \\ 3 & -5 \end{bmatrix} . \qquad .. (5.44)$$

From eqn. (5.32)

$$L = \begin{bmatrix} CB & CAB \end{bmatrix} \begin{bmatrix} B & AB \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} CB & CAB \end{bmatrix} \begin{bmatrix} N \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} -1 & 3 \end{bmatrix} \begin{bmatrix} 0 & \sqrt{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{3}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} , \qquad .. (5.45)$$

$$L'L = \begin{bmatrix} \frac{9}{2} & \frac{3}{2} \\ \frac{3}{2} & \frac{1}{2} \end{bmatrix} . \qquad .. (5.46)$$

(iii) Now considering the case when $W(\infty)$ is not zero, eqn.(2.24) is

$$PA + A'P = -L'L \ , \qquad .. (5.47a)$$

$$PB = C' - L'W_o \ , \qquad .. (5.47b)$$

$$W_o'W_o = W(\infty) + W(\infty) \ . \qquad .. (5.47c)$$

So, we can get

$$PA = -A'P - L'L \ ,$$

$$PAB = -A'PB - L'LB$$

$$= -A'(C'-L'W_o) - L'LB$$

$$= -A'C' + A'L'W_o - L'LB \ . \qquad .. (5.48)$$

Therefore

$$P = \begin{bmatrix} C'-L'W_o & -A'C'+A'L'W_o-L'LB \end{bmatrix} \begin{bmatrix} B & AB \end{bmatrix}^{-1} \qquad .. (5.49)$$

Now $W(\infty) + W'(\infty) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$.

Therefore, from eqn.(5.47c)

$$W_o = \frac{1}{2} \ .$$

Further

$$L'W_o = \begin{bmatrix} \frac{3}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \end{bmatrix} ,$$

$$LL'B = \begin{bmatrix} \frac{9}{2} & \frac{3}{2} \\ \frac{3}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} -\frac{3}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix},$$

$$A'L'W_o = \begin{bmatrix} -\frac{3}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{3}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \end{bmatrix} = \begin{bmatrix} -\frac{5}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \end{bmatrix}. \quad .. (5.50)$$

From eqn.(5.49)

$$P = \begin{bmatrix} \frac{1}{\sqrt{2}} - \frac{3}{2\sqrt{2}} & \sqrt{2} - \frac{5}{2\sqrt{2}} + \frac{3}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}} & 0 + \frac{1}{2\sqrt{2}} + \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 0 & \sqrt{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{5}{4} \end{bmatrix}. \quad .. (5.51)$$

(iv) To determine transformation T corresponding to eqn.(5.27), the procedure identical to the one given in [85] is adopted. For the matrix P_1 given by [85]

$$P_1 = \begin{bmatrix} -\frac{5}{4} & \frac{3}{4} \\ -\frac{3}{4} & \frac{5}{4} \end{bmatrix},$$

the eigen-values are given by

$$(\lambda + \frac{5}{4})(\lambda - \frac{5}{4}) - \frac{9}{16} = 0$$

or taking the positive value $\lambda = 1$, the corresponding eigen-vector is

$$\begin{bmatrix} \bar{X}_1 \\ \bar{X}_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ 1 \end{bmatrix}.$$

therefore, T (eqn.5.27) and T^{-1} are given by

$$T = \begin{bmatrix} 1 & -\frac{1}{3} \\ -\frac{1}{3} & 1 \end{bmatrix} \quad \text{and} \quad T^{-1} = \begin{bmatrix} \frac{9}{8} & 0 \\ 0 & \frac{9}{8} \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{3} \\ \frac{1}{3} & 1 \end{bmatrix}.$$

Therefore

$$C_1 = CT = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{3} \\ -\frac{1}{3} & 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{2}{3} & \frac{2}{3} \end{bmatrix}, \qquad \ldots (5.52a)$$

$$B_1 = \begin{bmatrix} \frac{9}{8} & 0 \\ 0 & \frac{9}{8} \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{3} \\ \frac{1}{3} & 1 \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9}{8} & 0 \\ 0 & \frac{9}{8} \end{bmatrix} \begin{bmatrix} -\frac{\sqrt{2}}{3} \\ \frac{\sqrt{2}}{3} \end{bmatrix}, \qquad \ldots (5.52b)$$

$$A_1 = T^{-1}AT = \begin{bmatrix} \frac{9}{8} & 0 \\ 0 & \frac{9}{8} \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{3} \\ \frac{1}{3} & 1 \end{bmatrix} \begin{bmatrix} -\frac{3}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{3} \\ -\frac{1}{3} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9}{8} & 0 \\ 0 & \frac{9}{8} \end{bmatrix} \begin{bmatrix} -\frac{16}{9} & \frac{8}{9} \\ -\frac{8}{9} & 0 \end{bmatrix}. \qquad \ldots (5.52c)$$

Therefore, we have the state model

$$\begin{bmatrix} \frac{8}{9} & 0 \\ 0 & \frac{8}{9} \end{bmatrix} \begin{bmatrix} \dot{V}_c \\ \dot{I}_L \end{bmatrix} = \begin{bmatrix} -\frac{16}{9} & \frac{8}{9} \\ -\frac{8}{9} & 0 \end{bmatrix} \begin{bmatrix} V_c \\ I_L \end{bmatrix} + \frac{1}{\sqrt{2}} \begin{bmatrix} -\frac{2}{3} \\ \frac{2}{3} \end{bmatrix} U ,$$

$$\bar{U} = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{2}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} V_c \\ I_L \end{bmatrix} + \frac{1}{8} U . \qquad \text{.. (5.53)}$$

Therefore

$$\begin{bmatrix} D & C \\ B & A \end{bmatrix} = \begin{bmatrix} \frac{1}{8} & \frac{\sqrt{2}}{3} & \frac{\sqrt{2}}{3} \\ -\frac{\sqrt{2}}{3} & -\frac{3}{2} & \frac{1}{2} \\ \frac{\sqrt{2}}{3} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} . \qquad \text{... (5.54)}$$

It can be checked that

$$\begin{bmatrix} I & \\ & \Sigma \end{bmatrix} \begin{bmatrix} D & C \\ B & A \end{bmatrix}$$

is a symmetric matrix and the state model (5.53) comes out to be same as eqn.D-18 of [85]. Therefore, the network (Fig.5.1) will also be identical to the one given in [85]. Obviously due to the flexibility in decomposing the symmetric Hankel matrix obtained from Markov parameters, many equivalent realizations can be obtained.

5.5 SYNTHESIS OF MINIMUM BIQUADRATIC FUNCTIONS

The synthesis of minimum functions has been a challenge to network theorists for quite a long time until Brune gave the synthesis procedure for these functions. Later,

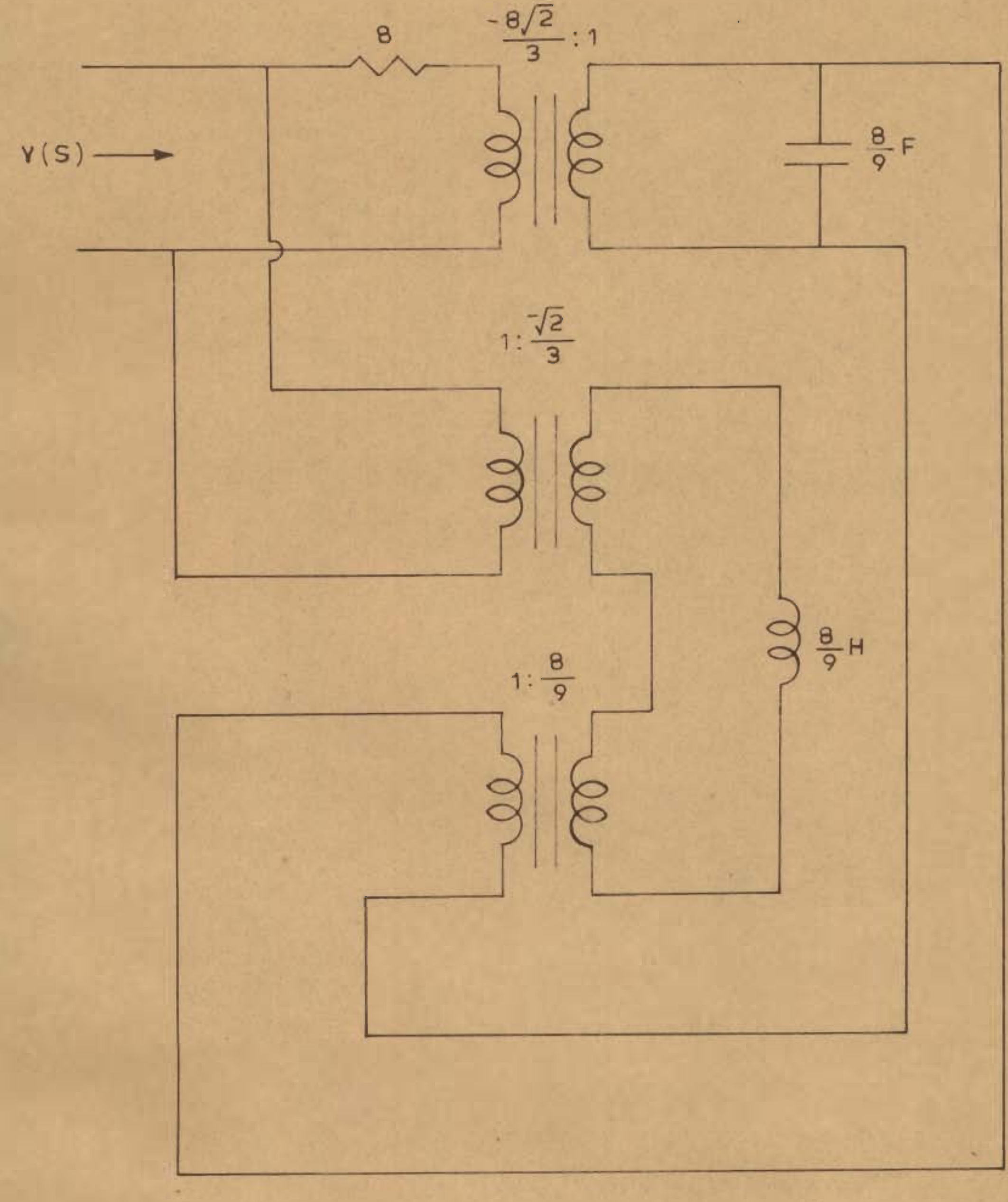


FIG. 5.1 - REALIZATION OF Y(s) [EQN. 5.36]

Bott-Duffin and several others attacked the same problem, the different methods giving different number of elements, Brune method, however, gives the minimum number of elements. In order to get minimum number of reactive elements, one can advantageously go in for state-space technique, as a minimal state model always results in a minimum number of reactive elements, the number being given by the degree of the given matrix in rational polynomials. The problem can also be tackled to get minimum number of resistive elements (reactive elements not necessarily minimum) based on the approach recently given by Vongpanitilerd and Anderson [81]. Further, as the state-space technique offers greater scope for extensions to problems such as equivalent networks, the synthesis of these functions using this technique will be fruitful. Knowing one, a number of equivalent networks can be obtained, all having the minimum reactive elements. The problem of realizing biquadratic functions, although sufficiently tackled, still remains interesting [80] and useful due to its application in cascade synthesis methods. In Section (5.3), various synthesis procedures using state-model approach have been given for positive real matrices. As a bye-product of these procedures, it is interesting to note that for biquadratic minimum functions, by making use of the property given in eqn. (5.63) some intermediate expressions in these procedures become very simple and can be obtained directly in terms of numerator and denominator coefficients of the function. For example a simple relationship expressing P , required for synthesis, directly in terms of the coefficients of biquadratic function can be established as follows :

Consider a driving-point biquadratic minimum function

$$z(s) = \frac{s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0} . \qquad .. (5.55)$$

(for simplicity, scalar constant is assumed to be unity)

Because

$z_1(s) = z(s) - z(\infty)$,

therefore

$$z_1(s) = \frac{(a_1-b_1)s + (a_0-b_0)}{s^2 + b_1 s + b_0} . \qquad .. (5.56)$$

The steps required are :

(i) Determine a state model given by

$$A = \begin{bmatrix} 0 & 1 \\ -b_0 & -b_1 \end{bmatrix} , \qquad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix} ,$$

$$c = \begin{bmatrix} (a_0-b_0) & (a_1-b_1) \end{bmatrix} , \qquad d = \begin{bmatrix} 1 \end{bmatrix} . \qquad .. (5.57)$$

(ii) Determine W(s)

W(s) is found as follows :

From eqn. (2.24), we get

$$z(s) + z'(-s) = W'(-s)W(s) = \text{Even}\left[z(s)\right] \qquad .. (5.58)$$

Let z(s) be

$$\frac{m_1 + n_1}{m_2 + n_2}.$$

therefore

$$\text{Even}\left[z(s)\right] = 2\,\frac{m_1 m_2 - n_1 n_2}{m_2^2 - n_2^2}$$

$$= 2\,\frac{(s^2+a_o)(s^2+b_o)-a_1 b_1 s^2}{(s^2+b_o)^2-(b_1 s)^2}$$

$$= 2\,\frac{s^4+(a_o+b_o-a_1 b_1)s^2+a_o b_o}{(s^2+b_o)^2-(b_1 s)^2}$$

$$= 2\,\frac{(s^2+\sqrt{a_o b_o})^2 + (a_o+b_o-a_1 b_1-2\sqrt{a_o b_o})s^2}{(s^2+b_o+b_1 s)(s^2+b_o-b_1 s)}$$

$$= 2\,\frac{(s^2+\sqrt{a_o b_o})^2 - \left[(a_1 b_1+2\sqrt{a_o b_o}-a_o-b_o)^{1/2} s\right]^2}{(s^2+b_o+b_1 s)(s^2+b_o-b_1 s)}.$$

Therefore, from (5.58)

$$W(s) = \sqrt{2}\,\frac{s^2+\sqrt{a_o b_o}+\left[a_1 b_1+2\sqrt{a_o b_o}-a_o-b_o\right]^{1/2} s}{s^2+b_o+b_1 s},$$

$$W(o) = \sqrt{2}. \qquad .. (5.59)$$

(iii) Find L'L

$W_1(s)$ is given by

$$W_1(s) = \frac{\sqrt{2}\ s^2+\sqrt{2a_o b_o}+\left[2a_1b_1+4\sqrt{a_o b_o}-2a_o-2b_o\right]^{\frac{1}{2}}\ s-\sqrt{2}\ s^2-\sqrt{2}b_1 s-\sqrt{2}b_o}{s^2+b_1 s+b_o}$$

.. (5.60)

$$= \frac{\left[(2a_1b_1+4\sqrt{a_o b_o}-2a_o-2b_o)^{\frac{1}{2}}-\sqrt{2}\ b_1\right]s+\left[\sqrt{2a_o b_o}-\sqrt{2}\ b_o\right]}{s^2+b_1 s+b_o}.$$

.. (5.60)

Therefore

$$L = \left[\sqrt{2a_o b_o}-\sqrt{2}b_o \qquad (2a_1b_1+4\sqrt{a_o b_o}-2a_o-2b_o)^{\frac{1}{2}}-\sqrt{2}\ b_1\right] \qquad ...\ (5.61)$$

and

$$L'L = \begin{bmatrix} \bar{L}_{11} & \bar{L}_{12} \\ \bar{L}_{21} & \bar{L}_{22} \end{bmatrix} \qquad ..\ (5.62)$$

where

$$\bar{L}_{11} = 2b_o\left[a_o+b_o-2\sqrt{a_o b_o}\right],$$

$$\bar{L}_{21} = \bar{L}_{12} = \left[\sqrt{2a_o b_o}-\sqrt{2}\ b_o\right]\left[(2a_1b_1+4\sqrt{a_o b_o}-2a_o-2b_o)^{\frac{1}{2}}-\sqrt{2}\ b_1\right],$$

$$\bar{L}_{22} = \left[2a_1b_1+4\sqrt{a_o b_o}-2a_o-2b_o\right] +2b_1^2-2\sqrt{2}\ b_1\left[2a_1b_1+4\sqrt{a_o b_o}-2a_o-2b_o\right]^{\frac{1}{2}}.$$

As z(s) in eqn.(5.55) is biquadratic minimum, the following relation holds good

$$a_1b_1 = (\sqrt{a_o}-\sqrt{b_o})^2 = a_o+b_o-2\sqrt{a_o b_o} \quad . \qquad ...\ (5.63)$$

Substituting (5.63) in (5.62) we get

$$\bar{L}_{11} = 2b_o a_1 b_1 ,$$

$$\bar{L}_{21} = \bar{L}_{12} = -2b_1\sqrt{a_1 b_1 b_o} ,$$

$$\bar{L}_{22} = 2b_1^2 . \qquad ..\ (5.64)$$

(iv) Find P

As a symmetric positive definite matrix always exists satisfying equation

$$PA + A'P = -L'L ,$$

take

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} \qquad ..\ (5.65)$$

and A for this case obviously would be

$$A = \begin{bmatrix} 0 & 1 \\ -b_o & -b_1 \end{bmatrix} . \qquad ..\ (5.66)$$

Solving for P, we get

$$P_{11} = b_o a_1 + a_o b_1 ,$$

$$P_{12} = a_1 b_1,$$

$$P_{22} = a_1 + b_1 .$$

Therefore

$$P = \begin{bmatrix} b_o a_1 + a_o b_1 & a_1 b_1 \\ a_1 b_1 & a_1 + b_1 \end{bmatrix} \qquad ..\ (5.67)$$

which is given in terms of the numerator and denominator coefficients of the minimum function.

Hence, by making use of the property (5.63) of minimum function, we can find the realization A,b,c,d and the corresponding P needed in the synthesis procedure by inspection and directly in terms of the coefficients of biquadratic function. For rest of the procedure, Method I given by Yarlagadda [85] can be applied. This result concerning the biquadratic function may also find interesting application in cascade synthesis using state-space approach.

5.6 CONCLUSION

When the given specifications are in terms of input-output characterization in s-domain, state-model approach is considered as most useful tool for the synthesis problem. Many results pertaining to the identification of systems in terms of state equations are available. A modified method based on the approaches of Ho and Kalman [28] and Youla and Tissi [90] is presented here when the given matrix is symmetric. The realization thus obtained satisfies the reciprocity constraints. Further, two procedures have been recently proposed by Yarlagadda [85] for obtaining the state model which satisfies reciprocity constraints together with passivity constraints. An improved procedure which is particularly suitable for computerization has been developed in this chapter. The elegance of the proposed method is attributed to the determination of the realization A,B,C and A,B,L of H(s) and W(s) from the symmetric Hankel matrices obtained from their respective Markov parameters. The symmetric positive definite matrix P is determined in terms of the matrices already found in the algorithm rather than solving a set of simultaneous equations as in [85] and so the difficulty in determining the inverse of matrices is circumvented. Further, by exploiting a well known property of the biquadratic minimum functions, results for some intermediate simple expressions e.g. P etc., have been derived in terms of numerator and denominator coefficients of the function which facilitate the procedure for the synthesis of minimum biquadratic functions.

CHAPTER VI

SUMMARY AND CONCLUSIONS

6.1 INTRODUCTION

The application of state-space techniques to system synthesis, particularly networks, is a significant recent development especially with the advent of fast digital computers. New methods suitable for computer-aided design are being developed. The present work is essentially concerned with this problem and proposes new and improved methods regarding the application of state-variable technique to modern network synthesis. The procedures embodied here deal with the synthesis of networks when the given specifications are in either state-variable characterization or input-output characterization in s-domain. Attempt has been made to bridge the gap between the twin concepts of these characterizations. This chapter, after summarising the results derived in the earlier ones, gives a number of challenging problems still open for investigation in this field.

6.2 SUMMARY OF THE RESULTS

A critical review presenting the various phases of the use of state-space approach in network synthesis has been given first. Various significant results scattered in recent publications have been collected in the form of an historical sketch with a view to acquaint the reader with the importance of this powerful tool in the hands of network designer. Generalized state models for RLC networks have been derived and the extension of these models for time-variable and non-linear cases has been considered. The role of the state models and other various results, e.g. constraints of reciprocity and passivity, in arriving at passive-reciprocal synthesis has been examined.

State-variable technique has become inevitable, especially, for non-linear systems for which transfer-function description does not exist while the latter still carries popularity in many of the design problems in linear systems. So, attempt has been made to seek the state-space interpretation of the well-known results in classical synthesis which will prove to be significant in the study of equivalent networks. In particular, the state-space interpretation of classical Foster, Cauer and Brune methods has been given. A new procedure for the determination of impedance matrix Z(s) from given $Z(s) + Z'(-s)$ is proposed which makes use of the well-known Anderson Lemma for positive real matrices. Further, based on the reverse of Ho and Kalman method for determining state equations, a method for determining the transfer-function matrix from given state equations is discussed which does not involve the determination of the rational matrix inversion $(sI-A)^{-1}$. The method is especially suitable for the cases in which $\det\left[sI-A\right]$ can be easily determined.

When the given information is in terms of state-variable characterization, synthesis is achieved by comparing the state model in topological entities with the known quantities and then solving the corresponding set of equations. Such recently available proceduress are quite cumbersome and so a satisfactory solution of the problem is desirable. In the present work, the generalized state models for time-invariant and time-varying cases are restricted to various classes of networks for which synthesis procedures are given. In particular, a procedure suitable for computer implementation for the synthesis of state equations belonging to n-port LC networks is discussed. The procedure has been programmed in Fortran II and has been actually run on the available IBM 1620. An easy algorithm for the synthesis of n-port RLC state equations belonging to a class defined in Section (4.4) is also given. Similarly, a procedure for a class of time-varying LC state-equations given in Section (4.5) is also considered. Further, there has been quite a bit of interest in the realization of portless networks from given A-matrix. Again the existing methods are far from satisfactory. An improved procedure for the synthesis of A-matrix belonging to a class defined in Section(4.6)is given. Sometimes the given state equations are not minimal and procedures exist for making the set minimal for time-invariant and time-varying state equations. An improved computational procedure for determining a minimal set of time-varying state-equations is given. The technique is based on the computational procedure given by Mayne for time-invariant systems and makes use of time-variable controllability and/or observability matrices.

When the given specifications are in terms of input-output characterization in s-domain, the natural approach as agreed by many investigators recently, is through the use of state models. Several procedures are available for finding state models from the given transfer-function matrix. An improved procedure for finding the state equations is proposed for the case when the given matrix is symmetric. The resulting state equations satisfy reciprocity conditions. If reciprocity is to be satisfied together with passivity, the determination of the suitable transformation becomes a difficult problem and so in the procedures given earlier, gyrators could not be eliminated. Recently, by combining the approaches of Anderson and Newcomb and Youla and Tissi, Yarlagadda gave two procedures for the realization of given symmetric positive real matrix, without the use of gyrators. A relatively improved procedure suitable for computerization is given in the present work. Further, by making use of a well-known condition for biquadratic minimum functions, it has been found that in synthesis procedures using state-model approach, some intermediate expressions can directly be determined in terms of coefficients of numerator and denominator polynomials of the given function.

6.3 SOME PROBLEMS FOR FURTHER INVESTIGATIONS

The state-space approach to network synthesis has been reviewed and applied to the classical synthesis methods and to the modern synthesis methods when the given information is in terms of state-variable characterization or the input-output characterization in s-domain. There are a number of problems still remaining in the use of state variables in network synthesis.

1. The state-space methods, despite involving comparatively more manipulations because of generalized approach, are being used to interpret classical synthesis procedures owing to the extremely important problem of equivalence of networks. The classical synthesis methods have been well recognised, while their interpretation in state-space terms is being investigated recently. The interpretation of some of the one-port synthesis methods has been presented in this thesis. The interpretation of some of the remaining one-port and two-port methods is worth investigating, e.g. Bott-Duffin procedure and Guilemin Method etc. The interpretation for 2-port methods may also indicate some possible approach to n-port synthesis. Besides, the interpretation of one-port Foster, Cauer and Brune methods, in terms of state space, also indicates the possibility of n-port synthesis of Foster, Cauer and Brune networks.

2. Inspite of the extensive use of state-space techniques in modern network and control theory, a majority of design problems are being solved using frequency-domain methods.

Therefore, it will be desirable to bridge the gap between the state-space and frequency-domain description of dynamical systems and establish a firm connection between these twin concepts. The interpretation of poles, zeros, residues, positive real matrices has been done. It will be worthwhile to give the state-space interpretation of some other common concepts in network synthesis, e.g. removing a pole, shifting a zero and other known properties of networks.

3.A well-known property for reactance functions given in [3] has been proved in Section (3.2) in a different way i.e. from the general state model of LC networks. The state models are the basic building blocks in state-space terms just as the conventional loop and node methods in classical network theory. In order to have a deeper insight, it will be worthwhile to prove other known results for RLC cases, e.g. Anderson's Lemmas given in Section (2.5.2) from the state models. Further, study of state-space methods in this context, may also reveal certain interesting properties of n-port networks.

4. A technique has been presented for the determination of $Z(s)$ from given $Z(s) + Z'(-s)$ in Section(3.4). For lossless $Z(s)$, the algebraic process gives $Z(s) + Z'(-s) = 0$. It has been shown in [25] that $\mathrm{Re}\, z(j\omega)$ for the lossless scalar function consists of sum of impulses located at the pole positions which the usual algebraic process fails to

detect. The reverse process i.e. the determination of $z(s)$ from such a Re $z(j\omega)$ has also been discussed in [25]. It is desirable to extend these results of [25] to the matrix case exploiting the technique discussed in Section (3.4).

5. The synthesis procedures for a class of n-port RLC networks and portless RLC networks have been given. It will be worthwhile to evolve the necessary and sufficient conditions such that the state equations belong to the class defined in Section (4.4) of Chapter IV.

6. The realization of state equations belonging to time-invariant linear networks have been sufficiently stressed. Little has been done to realize the state equations for time-varying, active and non-linear networks. The state models for these networks can be obtained and so it should be interesting to evolve the synthesis of time-varying, active and non-linear state equations.

7. Combining the techniques of Yarlagadda and Tokad and Anderson and Newcomb, a new algorithm has been proposed for the realization of LC n-port state-equations. The extension of the results reported in Section (4.2) to n-port RLC state-equations obtained in [87] will be very fruitful.

8. The synthesis of a class of A-matrix for non-degenerate networks when the network is assumed to have connected resistive part has been given by Dervisoglu [19]. Based on the decomposition of A, given by Nordgren and Tokad [57] and the one proposed in Section (4.6), the synthesis

procedure for the realization of A-matrix for degenerate networks having connected resistive part can also be done.

9. By making use of time-variable controllability and observability matrices, Mayne's [50] computational procedure has been extended to time-varying case in Section(4.7). It may be possible to extend a number of other results such as Anderson's Lemmas given in Section(2.52) for time-invariant cases to the impulse response matrices for the time-varying cases.

10. Currently the growing interest is towards sensitivity methods as they constitute a vital link between the discipline of system analysis and system design. Sensitivity-state models based on graph-theoretic concepts have been obtained for linear systems[65]. The method can be easily extended to time-varying and non-linear cases based on the derivation of these models for such cases discussed in Section (2.4).

11. In Chapter V, procedures are given to realize the state model by RLCT elements. The problem of state-model realization ultimately reduces to the problem of R-network synthesis which need be solved in order to have a transformerless realization[92]. Further the ideas proposed in [30] may also prove useful in obtaining transformerless realization.

12. By exploiting a well-known property of scalar minimum functions, the symmetric positive definite matrix P required in the realization procedure has been obtained in terms of the coefficients of numerator and denominator of the given function. The results should be extended to minimum matrix case which may facilitate the synthesis of Brune n-port sections [54], [55].

13. The algorithm presented in Section (5.3) results in minimum number of reactances, the number being equal to the degree of the given symmetric positive real matrix, while the procedure suggested in [81] gives minimum number of resistors. There is yet no available procedure for the realization of positive real matrices which results in minimum number of reactive as well as resistive elements [4]. The problem is an open challenge.

In conclusion, it may be said that with the advent of digital computer, because of the generalized approach, deeper insight and the importance of equivalent networks, the state-space approach is being advantageously used in network problems and much more can be done to utilise this approach in network synthesis. Once the synthesis of lumped, linear finite reciprocal, passive, time-invariant networks based on state-variable technique is thoroughly investigated, it will not be difficult to extend this approach to extremely interesting cases of active, time-variable and non-linear networks especially in view of the fact that the state-variable description provides a general basis for the study of such networks.

APPENDIX

```
C  C  SYNTHESIS OF LC NETWORKS - A STATE MODEL APPROACH   HSS Z
      DIMENSION A(4,4), B(4,4), B0(4,4), C(4,4), D(4,4), D0(4,4)
      DIMENSION  B1(4,4), W(8,8),V(8,8),B2(4,4),DD(4,4),E(4,4)
      DIMENSION CC(4,4),AA(4,4),FDKV(4,4)
      DIMENSION AR(4,4),BR(4,4),B0R(4,4),P(4,4)
      DIMENSION XX(4,4),YY(4,4),Y(4,4),Z(4,4),XA(4,4),XB(4,4)
      DIMENSION G(4,4),GG(4,4),NAC(4),PRD(4,4),AN(4)
      COMMON A,B,B0,C,D,D0,B1,W,V,B2,DD,E,P,CC,AA,FDKV
      COMMON AR,BR,B0R,   XX,YY,Y,Z,XA,XB,G,GG,NAC,PRD,AN
  100 READ 99,N,M
      READ 99,NA,MA
      READ 40,((A  (I,J), I=1,N), J=1,N)
      READ 40,((B  (I,J), I=1,N), J=1,M)
      READ 40,((B0(I,J), I=1,N), J=1,M)
      READ 40,((C  (I,J), I=1,M), J=1,N)
      READ 40,((D  (I,J), I=1,M), J=1,M)
      READ 40,((D0(I,J), I=1,M), J=1,M)
   40 FORMAT(7F10.2)
   99 FORMAT (2I5)
C     COMPUTE W,V
      DO 41 I=1,N
      DO 41 J=1,M
      S=0.
      DO 42 K=1,N
   42 S=S+A(I,K)*B0(K,J)
      S=B(I,J)+S
      B1(I,J)=S
   41 W(I,J)=S
      J1=0
      N1=N-1
      DO 47 KK=1,N1
      J1=J1+M
      DO 44 I=1,N
      J2=J1
      DO 44 J=1,M
      J2=J2+1
      S=0.
      DO 45 K=1,N
   45 S=S+A(I,K)*B1(K,J)
      B2(I,J)=S
   44 W(I,J2)=S
      DO 46 I=1,N
      DO 46 J=1,M
   46 B1(I,J)=B2(I,J)
   47 CONTINUE
      DO 48 I=1,M
      DO 48 J=1,N
      CC(J,I)=C(I,J)
   48 V(J,I)=C(I,J)
      DO 49 I=1,N
      DO 49 J=1,N
   49 AA(J,I)=-A(I,J)
      J1=0
      DO 50 KK=1,N1
      J1=J1+M
      DO 51 I=1,N
      J2=J1
```

```
      DO 51 J=1,M
      J2=J2+1
      S=0.
      DO 52 K=1,N
   52 S=S+AA(I,K)*CC(K,J)
      B2(I,J)=S
   51 V(I,J2)=S
      DO 53 I=1,N
      DO 53 J=1,M
   53 CC(I,J)=B2(I,J)
   50 CONTINUE
      J1=J1+M
      PUNCH 40,((W(I,J),  I=1,N), J=1,J1)
      PUNCH 40,((V(I,J),  I=1,N), J=1,J1)
C     COMPUTE W*WD
      DO 54 I=1,N
      DO 54 J=1,N
      S=0.
      DO 55 K=1,J1
   55 S=S+W(I,K)*W(J,K)
   54 DD(I,J)=S
      PUNCH 40 ,((DD(I,J), I=1,N), J=1,N)
      CALL INVERT(DD,N,4)
      PUNCH 40,((DD(I,J), I=1,N), J=1,N)
C     COMPUTE  P
      DO 56 I=1,N
      DO 56 J=1,N
      S=0.
      DO 57 K=1,J1
   57 S=S+V(I,K)*W(J,K)
   56 E(I,J)=S
      PUNCH 40,((E (I,J), I=1,N), J=1,N)
C     COMPUTE AR,BR,BOR, ETC.
      DO 58 I=1,N
      DO 58 J=1,N
      S=0.
      DO 59 K=1,N
   59 S=S+E(I,K)*DD(K,J)
   58 P(I,J)=S
      PUNCH 40,((P (I,J), I=1,N), J=1,N)
      NC=N-MA
      MC=M-MA
      DO 101 I=1,N
      DO 101 J=1,N
      S=0.0
      DO 102 K=1,N
  102 S=S+P(I,K)*A(K,J)
  101 AR(I,J)=S
      DO 103 I=1,N
      DO 103 J=1,M
      S=0.0
      DO 104 K=1,N
  104 S=S+P(I,K)*B(K,J)
```

```
  103 BR(I,J)=S
      DO 105 I=1,N
      DO 105 J=1,M
      S=0.0
      DO 106 K=1,N
  106 S=S+P(I,K)*BO(K,J)
  105 BOR(I,J)=S
      PUNCH 107,((AR  (I,J), I=1  ,NA), J=1  ,NA)
      NB=NA+1
      PUNCH 107,((AR  (I,J), I=1  ,NA), J=NB,N  )
      PUNCH 107,((AR  (I,J), I=NB,N  ), J=1  ,NA)
      PUNCH 107,((AR  (I,J), I=NB,N  ), J=NB,N  )
      PUNCH 107,((BR  (I,J), I=1  ,NA), J=1  ,MA)
      MB  =MA+1
      PUNCH 107,((BR  (I,J), I=1  ,NA), J=MB,M  )
      PUNCH 107,((BR  (I,J), I=NB,N  ), J=1  ,MA)
      PUNCH 107,((BR  (I,J), I=NB,N  ), J=MB,M  )
      PUNCH 107,((BOR(I,J), I=1  ,NA), J=1  ,MA)
      PUNCH 107,((BOR(I,J), I=1  ,NA), J=MB,M  )
      PUNCH 107,((BOR(I,J), I=NB,N  ), J=1  ,MA)
      PUNCH 107,((BOR(I,J), I=NB,N  ), J=MB,M  )
  107 FORMAT (7F10.4)
      DO 110 I=1,M
      DO 110 J=1,M
      S=0.0
      DO 111 K=1,N
  111 S=S+BO(K,I)*BOR(K,J)
  110 XX(I,J)=S
C     CONSTRUCT YC
      DO 112 I=1,MA
      DO 112 J=1,MA
  112 Y(I,J)=XX(I,J)+DO(I,J)
      II=MA
      DO 113 I=1,NA
      II=II+1
      DO 113 J=1,MA
      Y(II,J)=-BOR(I,J)
  113 Y(J,II)=-BOR(J,I)
      II=MA
      DO 114 I=1,NA
      II=II+1
      JJ=MA
      DO 114 J=1,NA
      JJ=JJ+1
  114 Y(II,JJ)= P(I,J)
      PUNCH 107, ((Y(I,J), I=1,II), J=1,JJ)
      NNN=II
      CALL CEDBUM (Y,NNN)
C     CONSTRUCT ZL
      II=NA
      DO 115 I=1,NC
      II=II+1
      JJ=NA
```

```
      DO 115 J=1,NC
      JJ=JJ+1
  115 Z(I,J)= P(II,JJ)
      II=NC
      DO 117 I=1,MC
      II=II+1
      JA=I+MC
      DO 117 J=1,NC
      IA=J+NC
      Z(II,J)=BOR(IA,JA)
  117 Z(J,II)=BOR(IA,JA)
      DO 116 I=1,MC
      II=I+NC
      IA=I+MA
      DO 116 J=1,MC
      JJ=J+NC
      JA=J+MA
  116 Z(II,JJ)=DO(IA,JA)+XX(IA,JA)
      PUNCH 107, ((Z(I,J), I=1,II), J=1,JJ)
      NNN=II
      CALL CEDBUM (Z,NNN)
C     COMPUTE FDKV
      DO 118 I=1,N
      DO 118 J=1,M
      S=0.0
      DO 119 K=1,N
  119 S=S+P(I,K)*B(K,J)
  118 XA(I,J)=S
      DO 120 I=1,M
      DO 120 J=1,M
      S=0.0
      DO 121 K=I,N
  121 S=S+BO(K,I)*XA(K,J)
  120 XB(I,J) =S
      DO 122 I=1,MA
      DO 122 J=1,MC
      JJ=J+MA
  122 FDKV (I,J)=D( I,JJ)+XB( I,JJ)
      PUNCH 107, ((FDKV(I,J),I=1,MA), J=1,MC)
      GO TO 100
      END
```

```
      SUBROUTINE CEDBUM (G,N)
      DIMENSION A(4,4), B(4,4), B0(4,4), C(4,4), D(4,4), D0(4,4)
      DIMENSION  B1(4,4), W(8,8),V(8,8),B2(4,4),DD(4,4),E(4,4)
      DIMENSION CC(4,4),AA(4,4),FDKV(4,4)
      DIMENSION AR(4,4),BR(4,4),BOR(4,4),P(4,4)
      DIMENSION XX(4,4),YY(4,4),Y(4,4),Z(4,4),XA(4,4),XB(4,4)
      DIMENSION G(4,4),GG(4,4),NAC(4),PRD(4,4),AN(4)
      COMMON A,B,B0,C,D,D0,B1,W,V,B2,DD,E,P,CC,AA,FDKV
      COMMON AR,BR,BOR,  XX,YY,Y,Z,XA,XB,G,GG,NAC,PRD,AN
      DO 200 I=1,N
      DO 200 J=1,N
  200 GG(I,J)=G(I,J)
      N1=N-1
  290 DO 201 I=1,N1
      K=I+1
      DO 201 J=K,N
      IF (G(I,J)) 202,201,202
  202 AT=ABS(G(I,J))
      I1=I
      II=I
      JJ=J
      GO TO 203
  201 CONTINUE
      GO TO 220
  203 DO 204 I=I1,N1
      K=I+1
      DO 204 J=K,N
      IF (G(I,J)) 205,204,205
  205 IF (ABS(G(I,J))-AT) 206,204,204
  206 AT=ABS(G(I,J))
      II=I
      JJ=J
  204 CONTINUE
      PUNCH 295,AT,II,JJ
  295 FORMAT (F10.3,2I5)
      IF (G(II,JJ)) 207,208,208
  207 DO 209 I=1,N
  209 GG(I,JJ)=-G(I,JJ)
  208 DO 210 I=1,N
      IF (G(I,II)) 211,215,212
  211 IF (GG(I,JJ)) 213,215,215
  213 NAC(I)=-1
      GO TO 210
  212 IF (GG(I,JJ)) 215,215,216
  216 NAC(I)=1
      GO TO 210
  215 NAC(I)=0
  210 CONTINUE
      PUNCH 299, (NAC(I), I=1,N)
  299 FORMAT (14I5)
```

```
      DO 231 I=1,N
  231 AN(I)=NAC(I)
      DO 217 I=1,N
      DO 217 J=1,N
  217 PRD(I,J)=AN(I)*AN(J)*AT
      PUNCH 298,((PRD(I,J), I=1,N), J=1,N)
  298 FORMAT (6F12.3)
      DO 218 I=1,N
      DO 218 J=1,N
  218 G(I,J)=G(I,J)-PRD(I,J)
      GO TO 290
  220 DO 221 I=1,N
      IF (G(I,I)) 222,221,221
  222 PUNCH 223
  223 FORMAT(22H MATRIX NOT REALISABLE)
  221 CONTINUE
      PUNCH 297,(G(I,I), I=1,N)
  297 FORMAT(5F 14.3)
      RETURN
      END
```

INPUT DATA

4	2					
2	1					
0.0	0.0	1.0	-0.5	0.0	0.0	0.0
-0.5	-.25	0.0	0.0	0.0	0.25	0.25
0.0	0.0					
0.0	0.0	0.0	0.5	0.25	0.0	0.0
0.0						
0.25	0.0	0.0	0.0	0.0	0.0	0.0
0.0						
0.0	1.0	0.0	0.0	0.25	0.0	0.75
0.0						
0.0	-1.0	0.75	0.0			
5.75	0.0	0.0	3.0			

RESULTS

0.00	0.00	.25	.37	.25	0.00	0.00
0.00	.03	.09	0.00	0.00	0.00	0.00
.25	-.12	0.00	0.00	.03	-.06	-.09
-.03	0.00	0.00	-.02	-.01	0.00	0.00
0.00	0.00	-.09	.06			
0.00	0.00	.25	.75	1.00	0.00	0.00
0.00	.12	.37	0.00	0.00	0.00	0.00
.25	-.25	0.00	0.00	.03	-.12	-.37
-.12	0.00	0.00	-.09	-.06	0.00	0.00
0.00	0.00	-.09	.12			
.07	0.00	0.00	0.00	0.00	.01	0.00
0.00	0.00	0.00	.13	.05	0.00	0.00
.05	.16					
14.50	-9.02	0.00	0.00	-9.02	105.51	0.00
0.00	0.00	0.00	8.58	-2.86	0.00	0.00
-2.86	7.04					
.29	.02	0.00	0.00	.02	.04	0.00
0.00	0.00	0.00	.13	.10	0.00	0.00
.05	.32					
3.99	0.00	0.00	0.00	0.00	3.99	0.00
0.00	0.00	0.00	1.00	0.00	0.00	0.00
0.00	1.99					
0.0000	0.0000	0.0000	0.0000			
-.9999	0.0000	.9999	.9999			
1.0000	-.9999	0.0000	-.9999			
0.0000	0.0000	0.0000	0.0000			
0.0000	0.0000					
.9999	0.0000					
0.0000	.9999					
0.0000	0.0000					
.9999	0.0000					
0.0000	0.0000					

0.0000	0.0000					
0.0000	0.0000					
5.9999	-.9999	0.0000	-.9999	3.9999	0.0000	0.0000
0.0000	3.9999					
.999	1	2				
1	-1	0				
.999	-.999	0.000	-.999	.999	0.000	
0.000	0.000	0.000				
5.000	3.000	3.999				
1.0000	0.0000	0.0000	0.0000	1.9999	0.0000	0.0000
0.0000	3.0000					
0.000	1	2				
1	1	0				
0.000	0.000	0.000	0.000	0.000	0.000	
0.000	0.000	0.000				
1.000	1.999	3.000				
.9999						

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