# DYNAMIC ANALYSIS OF RAIL-WHEEL INTERACTION FOR FREIGHT WAGON

## A DISSERTATION

*Submitted in partial fulfilment of the*

*Requirements for the award of the degree*

Of

MASTER OF TECHNOLOGY

IN

MECHANICAL ENGINEERING

(With Specialization in Cad Cam & Robotics)

*by*


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**ROORKEE–247667**


**MAY, 2019**

# CANDIDATE’S DECLARATION

I hereby declare that the work carried out in this report entitled, “**DYNAMIC ANALYSIS OF RAIL-WHEEL INTERACTION FOR FREIGHT WAGON**”, is presented on behalf of partial fulfilment of the requirements for the award of degree of “Master of Technology” in Mechanical Engineering with specialization in Cad Cam & Robotics, submitted to the Department of Mechanical and Industrial Engineering, Indian Institute of Technology, Roorkee, under the guidance of **Prof. S.P.Harsha** and **Prof. Satish C. Sharma**, department of Mechanical and Industrial Engineering. I have not submitted the record embodied in this report for the award of any other degree.

Date: 17th MAY 2019 PULKIT MAHERA

Place: Roorkee **Enrolment no: 17538007**

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# CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of my knowledge and belief.

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# ABSTRACT

*This report deals with the dynamic analysis of rail-wheel interaction. The model has been developed for dynamic analysis of freight railway wagon. A solid model has been developed using solid works on the basis of standard rail profile (UIC-60) and wheel profile (S-1002). The freight wagon incorporated is widely used BOXN25 wagon and the bogie which is used is CASNUB bogie which is mainly used for freight wagons. Finite Element analysis (FEA) of the solid model has to be done using ANSYS software. In this anaysis, The values of natural frequencies has been calculated using Block Lancoz solver which is a recurssive solver. Mode shapes and the values of the maximum and minimum deformation has also been calculated. During the analysis the body of the wagon has been considered flexible.*

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# 1. INTRODUCTION

The performance of rail vehicles is essentially linked to small areas where the wheels meet the rail. This highly stressful interface is the root of almost all research areas of interest in vehicle-track interaction - from the dynamic performance of the vehicle to the maintenance of infrastructure. Therefore, a better understanding of the contact phenomenon between wheel and rail is important for designing a customized railway system of transport. From the very beginning, wheel-rail links have been an important interest for railway engineers. In fact, the idea of reducing traction interface in a small area using difficult materials like steel is only on the point that separates the railway from road transport. By doing so, due to the rolling resistance the energy loss is greatly reduced. However, the introduction of such drastic changes raises to face other issues and challenges. The research area, which includes wheel-rail links, is vast. The mobility of the rail vehicles depends on the forces applied on the wheels through this interface. Nowadays, multi-body dynamic simulation of rail vehicles is an important part of their design process. Such simulations require a precise accurate estimation of wheel-rail contact forces. Without a proper understanding of wheel-rail interaction, stability and derailment required safety analysis or Ride Conflict Analysis are disputed. Today, with the increasing demands of the market for heavy pivot load and high speed, expensive maintenance of both infrastructure and rolling stock is among the top items in concern list of all train operators and track owners. A large part of these maintenance costs stems from the events happening in the infamous wheel-rail interface. Two of these incidents are flaws of contact and rolling contact fatigue (RCF). In recent years, a great deal of dedicated research has been done to understand and predict these events. However, the primary step which enables such a system to be examined, clearly knows the contact size and shape of the patches and the distribution of stress within it. This is what the solution to the wheel-rail contact problem can provide us with. In addition, environmental aspects such as energy usage of the railway transport are closely connected to the wheel-rail interaction. The growing interest in studying rail vehicle generated noise and wear particle emissions necessitates a more detailed investigation of the contact phenomena. The use of energy of rolling stock is dependent on the adhesion levels in the wheel-rail interface. Adoption status modeling in this interface is also important for security issues regarding breaking performance. However, modeling friction for an open system under different sources of contamination and changing environmental conditions is still a current challenge for researchers. Generally, the treatment of wheel-rail contact can be divided into two parts: a geometric (kinematic) part, which aims at the detection of the contact points, and an elastic (elasto-plastic) part, which solves the contact problem from a Solid Mechanics point of view. The geometric part requires the geometrical data for track and wheelset together with the kinematical data of the wheelset. Wheel and rail profiles, track geometry and irregularities, and wheelset geometry together with its lateral displacement and yaw angle are all inputs to the contact geometric solver. Detection of point of contact can be done by assessing wheel and rail as hardening or accounting for their flexibility. Crawling is also done to calculate wheels contact and the velocity vector points. The production of geometric part is the necessary input for elastic solver. The elastic problem is to basically find the area formed by the bodies in contact (i.e. contact patch), and this is to indicate the distribution of contact pressure (normal compressed stress) with the distribution of tangent traction within the patch. After traction distribution, a wheel can calculate the total friction force known as the crawling force in the rail contact.

## 1.1 Importance of Rail wheel Interaction

Freight wagon comprises track layout, three piece bogie, open wagon. Open wagon comprises wagon, underframe, primary suspension, secondary suspension, wheel-set axle etc. All vertical load passes through the wheel into rail via contact patches, therefore, contact patches is very much important under dynamic condition.

The freight vehicle progression is one of the troublesome errands to study. It is extremely perplexing system since it has numerous level of opportunity and there are numerous non-linearity created in the framework .the vehicle load is exchanged from wheel to track. The contact arrangement amongst haggle is fundamentally affected by the complex geometry of the wheel string and rail, their material properties and natural condition which add greater nonlinearity to the framework. Furthermore, entire framework is represented by suspension and damper which assume vital part to make the framework under control.

The digression track circumstance, there are dynamical circumstance and issue to be considered.

It is found that at the slower speed it confronts shake and move issue happens. Today request of expanding recurrence and hub stack, it is found that vehicle may chase and bob seriously at higher speed and can crash from track .while moving over a bent track because of high horizontal powers, wheel tends to climb the rail and may entire vehicle gets the move over.

There is numerous dynamic issue which is in charge of debased execution and security of freight. It is a matter to manage the vehicle development on track legitimately or it is digression track or bent. The insufficient direction the parallel vibration prompts wear because of this unbalance strengths made.

For a steady ride of the vehicle is vital to accomplishing adjust compel like contact constraint, idleness powers, suspension –damper strengths and gravitational powers which for the most part impacts the progression of freight vehicle .the connection of haggling is extremely basic part to consider. Wheel-rail contact is a point contact.The contact area is very few square $cm^2$. In rail-wheel interaction nonlinearity has taken in consideration.

The fundamental character of the contact has to comprehend, which is get affected many components like grating, geometry and material conduct in the contact zone. These elements are difficult to control amid running yet with legitimate design, suspension and damper the directing of the vehicle can be controlled to abstain from unbalancing powers. To accomplish at fast cargo vehicle having abnormal state of steadiness and abnormal state of ride quality.

Table – 1.1 Load carrying capacity of Freight wagons. Source: (Indian Railway white book)

**PERMISSIBLE CARRYING CAPACITY OF 8 WHEELER Freight WAGONS (in tonnes)**

| Type of wagons | Excepted CC+6 routes | Universalized "CC+6 routes | CC+8 routes | | Loading tolerance |
|---|---|---|---|---|---|
| | | | For ores, gypsum, limestone & dolomite, stones, clinker, cement, all types of coal, slag, | All other commodities | |
| **OPEN WAGONS** | | | | | |
| **BOXN** | | | | | 1 |
| **BOXNCR** | | | | | 1 |
| **BOXNHS** | 64 | 66 | 68 | 66 | 1 |
| **BXNHSM1** | | | | | 1 |
| **BOXNM1** | | | | | 1 |
| **BOXNHL** | 66 | 68 | 70 | 66 | 1 |
| **BOXNHA** | 63 | 65 | 67 | 65 | 1 |
| **BOXNLW, BOXNLWM** | 66 | 68 | 70 | 66 | 1 |
| **BOXNR** | 65 | 67 | 69 | 69 | 1 |
| **BOXNEL** | 64 | 65 | 67 | 65 | 1 |
| **BOX, BOXT** | 59 | 60 | 60 | 60 | 1 |
| **BOXC, BOXR** | 60 | 60 | 60 | 60 | 1 |

About table showed the axle loading capacity per 8 wheeler wagon. Nowadays maximum load carrying capacity 60 tonnes to 70 tonnes. If we calculate per wheel load found that 75 KN. Simulation of static loading has been done 200 KN load per wheel and dynamic simulation has been done under application of vertical loading at 50 KN, 100 KN, and 200 KN.

## 1.2. Railway Wheel

As the wheel is directed over the track the entire elements of vehicle-reliant on it. There has been much research has been done to comprehend the essential way of rail wheel connection, the state of a profile, material properties, and contact arrangement.Basically, the wheel is assembling process .precision of assembling procedure ought to be high generally any sort of running disappointment.

Wheels are fundamentally sorted into strong, tire and gathered sort. A strong wheel Fig [7] many constitute three sections: the edge, the plate, and the center for the various wheel diverse profile, breadth, and state of the circle.

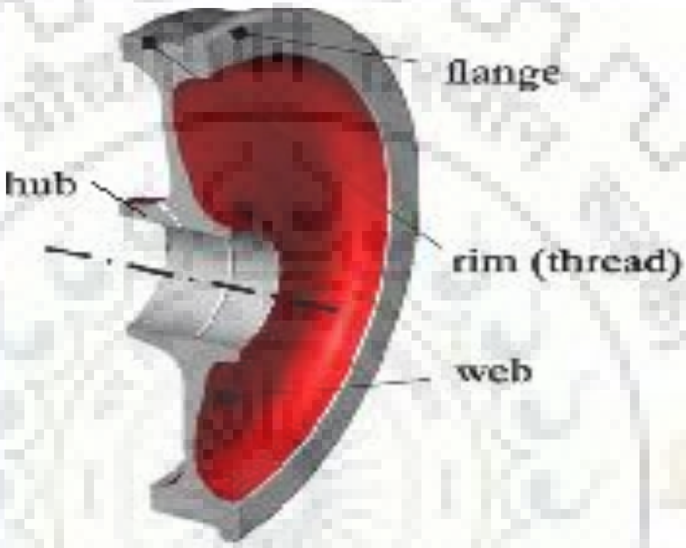



Figure- 1.1 Wheel profile S-1002 Indian Railway. Source: (Google images)

## 1.3 Rail Geometry

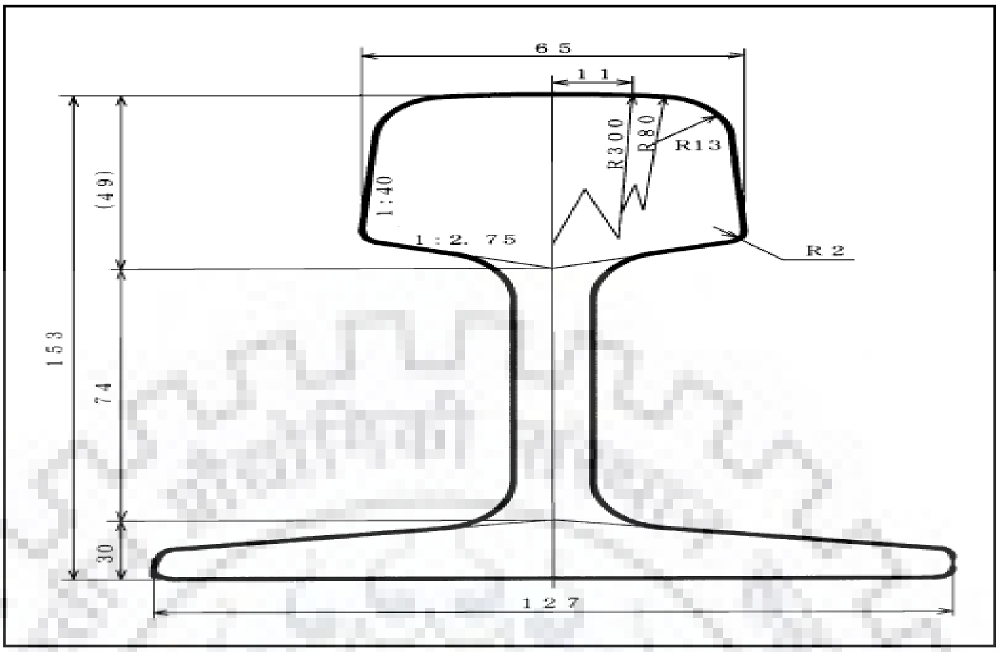



Figure –1.2 Rail profile.(UIC-60 Indian Railway std. ). Source: (RDSO Drawings)

## 1.4 BOXN 25 Wagon

The model of the BOXN 25 wagon has been developed using the standard drawing provided by RDSO, Lucknow. This model has been developed using SOLIDWORKS software, which is as shown assembled with underframe:-

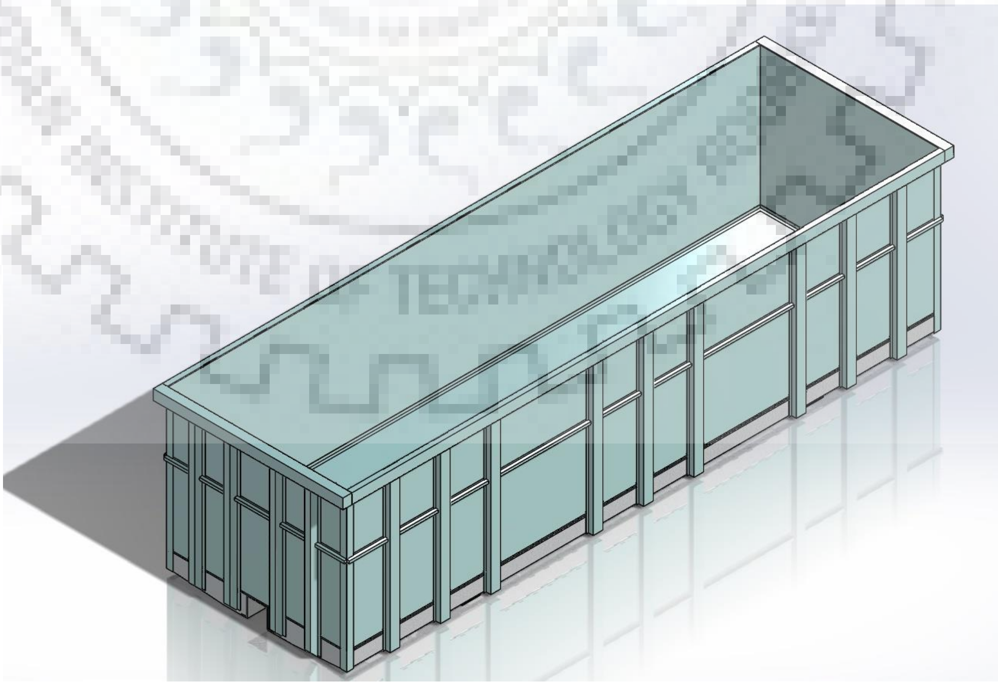


Fig:-1.3 BOXN 25 Wagon with underframe.

The drawing provided by indian railways of BOXN25 wagon is as shown below.

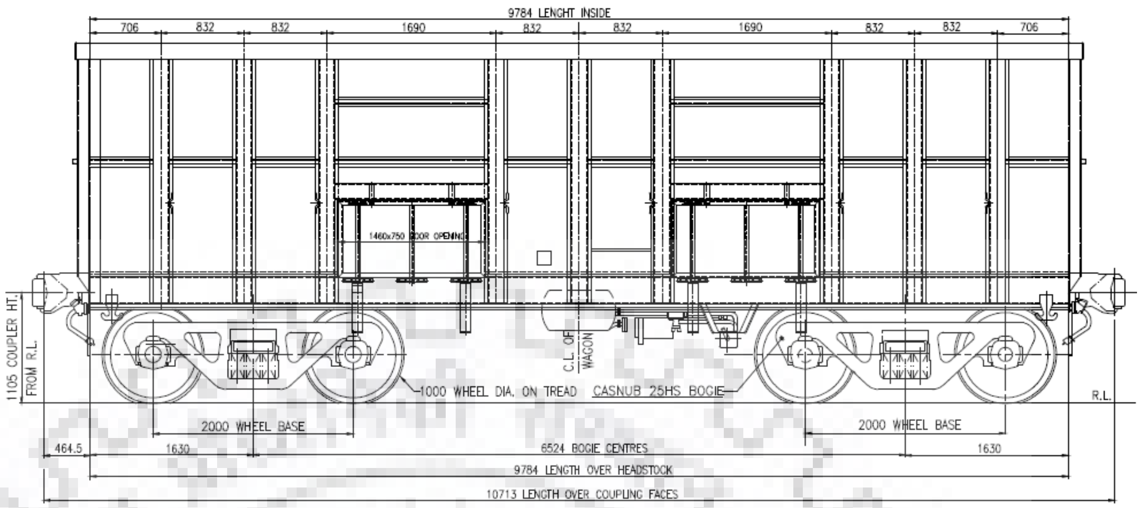



Fig:- 1.4 BOXN25 Wagon dimensions. Source: (Maintenance Manual of Wagons, Indian Railways)

## 1.5 CASNUB BOGIE

CASNUB Bogie is widely used for freight wagons. In this bogie there is only primary suspension system is used. It has two cast side frames and a floating bolster. The side frames of a CASNUB bogie are connected through a spring plank. This spring plank is made up mild steel. The floating bolster is connected to the spring plank by two groups of springs. These springs also provides the necessary frictional damping. It is mainly fitted to BOXN, BCN, BCNA, BRN, BTPN, BTPGLN wagons. It starts hunting at the speed of 90 km/hr. The main characteristics of these bogies are that these bogies are shorter in length, light in weight, high reliability, high capacity and easier maintenance. A photograph of a CASNUB bogie is shown as

Fig:-1.5 CASNUB bogie. Source: (Google Images)

Some of the features of these bogies are as shown,

| | |
|---|---|
| Wheel Diameter: | 1000mm (new), 925mm (condemning) |
| Wheel base: | 2000mm |
| Type of pivot: | IRS spherical |
| Type of roller bearing: | Standard AAR tapered cartridge bearing |
| Anti rotation features: | Anti rotation lugs have been provided between bogie bolster and side-frame |

## 1.6 Various versions of CASNUB Bogie

There are various versions of CASNUB bogie which are being used by the Indian Railways. They are as follows,

1. CASNUB-22W
2. CASNUB-22W(Retrofitted)
3. CASNUB-22W(M)
4. CASNUB-22NL
5. CASNUB-22NLB
6. CASNUB-22HS(At 100 km/h)

In the analysis, the version of CASNUB bogies used is CASNUB-22W (M). This version is an improved version of the previous ones. In this version elastomeric pads are provided at the bearer level and the axle box level. These pads reduces the tendency of hunting of the bogie. This also reduces the biased wear of the wheel flanges.

## 1.7 Suspension Characteristics of a CASNUB Bogie

It consists of two side frames which are made of cast steel and are connected by a mild steel riveted spring plank. The suspension which is used in CASNUB bogies is bolster level suspension. There is no primary suspension system used. Therefore, the spring plank is subjected to torsion and bending when the wheels try to adjust for negotiating track twist. The floating bolster made of cast steel is supported on the side frames through two nests of springs. The springs contain friction snubber for oscillation control. However, these snubber springs are not designed to take vertical load of the wagon.

Each bogie is provided with four snubbers. The function of snubbers is to provide damping to the vibrations induced.

## 1.8 Arrangement of springs

The springs are arranged in a bogie as shown in the figure below,

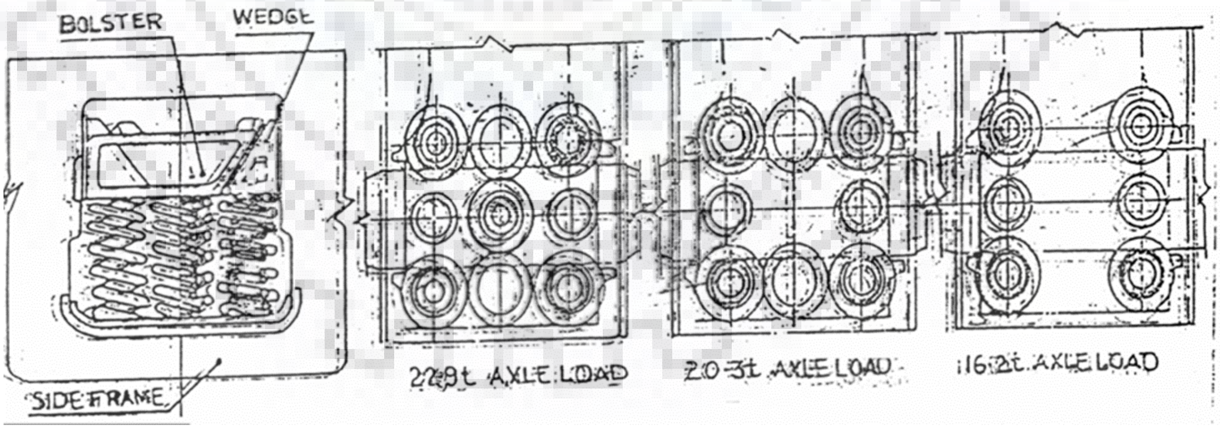



Fig:- 1.6 Arrangement of springs in suspension system

The number of springs provided in each of the bogie is different in case of different axle loads as clear from the diagram shown above. The description of the number of springs for different axle load is shown as below,

Table 1.2 Details of springs for different axle load. Source: (Indian Railways Whitebook)

| | AXLE LOAD | No. of springs required | | |
|---|---|---|---|---|
| | | Outer | Inner | Snubber |
| 1) Snubbing arrangement & snubber Springs are same for 22.9t, 20.3t & 16.21 Axle Load | 22.9t | 7 | 3 | 2 |
| 2) Details of the springs are as per Drg,No.WD-83069-S/1 | 20.3t | 6 | 4 | 2 |
| | 16.2t | 4 | 4 | 2 |

**DETAIL OF SNUBBING, SRING GROUP ARRANGEMENT CASNUB BOGIES (EXCEPT CASNUB-22 HS BOGIE)**

The description of free length and the condemning free lengths of the various springs used in the suspension system are described as below in table 1.3,

Table 1.3:- Free lengths of the springs. Source: (Indian Railways Whitebook)

| Spring | Free height nominal (mm) | Recommended condemning free height (mm) |
|---|---|---|
| Outer load spring | 260 | 245 |
| Inner load spring | 262 | 247 |
| Snubber spring | 294 | 279 |

Matching of both load and snubber spring is important. The springs should not have free height variation more than 3mm, assembled in the same group.

## 1.9 Assembly of Wagon and a Pair of Bogies

Each wagon consists of a pair of bogies. Each bogie has a pair of axles over which wheels are mounted. Hence, each wagon has eight wheels which roll over the track. The assembly of the bogies and the wagon is done in SOLIDWORKS, which is as shown:-

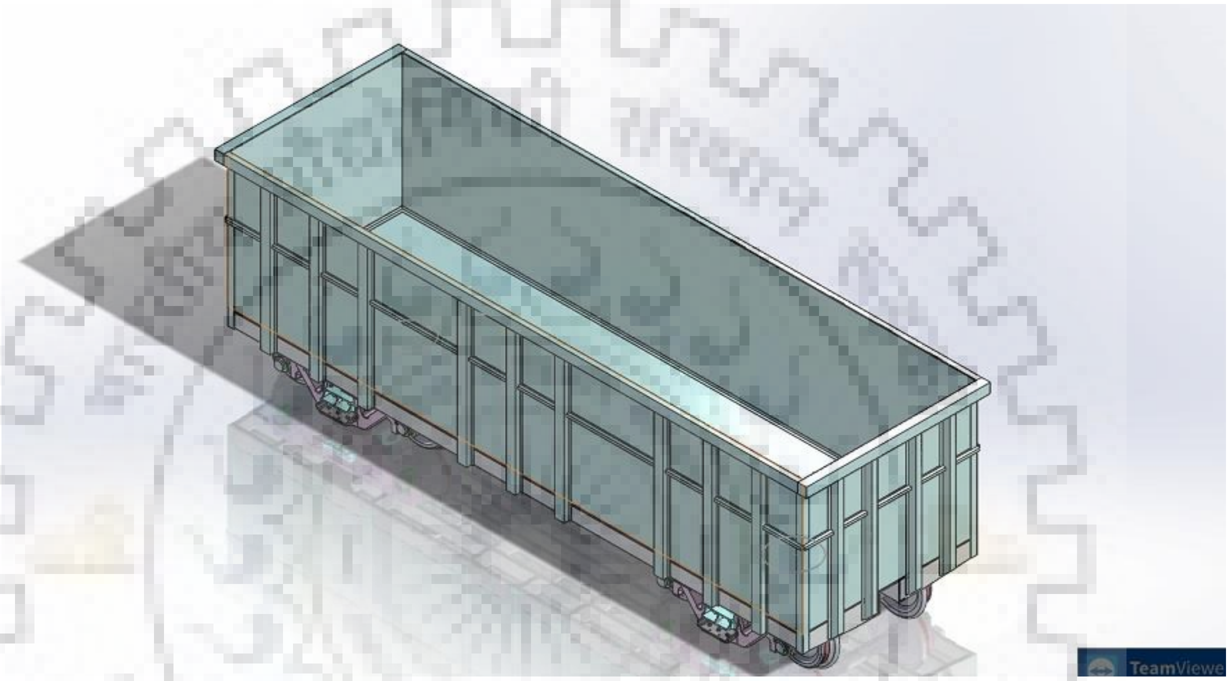


Fig:- 1.7 Assembly of BOXN 25 Wagon and a pair of CASNUB bogies.

# 2. Literature Review

## 2.1 Introduction

Rail wheel interaction is an important research field for developing Freight wagon for Indian Railways. This is directly influencing dynamics of rail-wheel, Stress concentration between Rail-wheel, Deformation zone, nature of Contact patches under the dynamic condition, the failure of wheel thread, wear etc.

In article of (smith, 2014) Finite Element method (FEM) study focus on pressure distribution when two contacting bodies under pure rolling. In that analysis involve seventeen point interface between S1002 wheel profile and Rail 60E2 profile .study concern on approximation on the basis on seventeen empirical equation and find out the close result of pressure distribution. In this study, it has been clear that Maximum stress distribution and lateral displacement on of Rail –wheel interaction.

(Telliskivi T., 2000) Study focus on the difference between Analytical methods and FE method when spinning rolling and sliding has been included along with kinematic Harding. The result between above approach was close. The simulation was done by traditional software which is based Kalker's theory (Kalker J. J., 1982) has given the combined approach of FE and tradition method on the basis of this two approach he has given that tradition method can give only when main contact should be more as compare to the dimension of the contact area. (Wiest M., 2008) Has study on noise when wheel running on crossing and without crossing with the help of numerical technique. As Telliskivi et al. study have been a focus on contact area analysis, pressure distribution with rigid body approach and has given the theory assuming that rail-wheel material should be elastic –plastic in nature.

(Zhao X., 2011) Has study taking in consideration that rail-wheel profile in simplified geometry. Analysis has been done by FE Tool and has been finding out pressure distribution, shear stress distribution, sticking area and sliding area in the contact zone. Compare his result to CONTACT and analytical Hertz solution. According to his study, he has been finding that simplified geometry give closer result in rail-wheel interaction either material may be elastic or plastic. (Pau M., 2002) Has given an ultrasonic technique for evaluating contact pressure. On application ultrasonic reflection coefficient matrix has been obtained. By analysis, the load finds out contact pressure. The result has been obtained verified the Experimental result as well as Hertz's theory.

My aim is to simulate Rail-wheel interaction, contact pressure distribution under a static Quasi-Static& dynamic condition with help of ANSYS 17 workbench.

## 2.2 Contact Patches

Freight wagon dynamical system is governed by force in this study aerodynamic forces only consider the external forces. The rail-wheel interaction in very complex and imperfection. As wheel moves over Rail three type of motion longitudinal, lateral and yaw motion forms contact patches of different size Creep forces and moments that arise are highly nonlinear in nature as shown below.

In previous research, many assumptions have been put for simulation the contact patches Generally, we assume that material of wheel-rail are to be same with this assumption Hertz gave theory in 1881 for finding a contact. Due to the geometric feature of the wheel tread and rail top surface, the wheel/rail contact is not a geometrically symmetrical problem. Therefore, wheel/rail contact is a 3-D nonlinear contact problem. 3-D model can simulate the complex motion states, including sliding, rolling, and transverse movement and spinning motion. And it can get the detailed characteristics of the contact area. At the same time, the temperature and the shape of contact patch on the rail surface can be obtained by 3-D model. However, the data gotten by the 2-D model cannot solve the problems, such as the shape and size of the contact zone, the distribution laws of temperature and strain fields on the contact surface. Therefore, using a 2-D plan model to solve the problems is inappropriate in some aspects. Because there are difficulties in computing, the relevant research reports on the 3-D direct coupling nonlinear contact model of wheel/rail are still scarce.

## 2.3 Rail- Wheel Interaction

Numerical analysis was performed before 1970 and study of contact patches have done by mathematical modeling after 1970 British Rail has been developed Medyna software by research Deutsche luftund and Raumfarhrt in the USA. (Andersson, 2000) Has been studying how to reduce time in the simulation of contact analysis.(Gensys, 1999). Has developed software for the commercial purpose. It was a user-friendly software for dynamic simulation MBS (Multi -body system).

Simpack, and Dads have been developed named multi- body analysis with whole rail body simulation with different speed and coefficient of friction. Adams, Gensys, No cars, Simpack, and Vampire have recently been benchmarked (Iwnicki, 1998) has been bench march of existing study on rail-wheel contact .before that many existing benchmarks but all were very complex in modeling and understanding .has taken a complex wheel rail section and simulation has been done result were close to mathematical modelling. (Knothe, 1994).has done a simulation of wheel-rail contact using FE analysis. Dispersion technique has been used for free vibration to 15 kHz frequency. He has applied dispersion technique in different model compare the result of all the model of rail-wheel interaction.

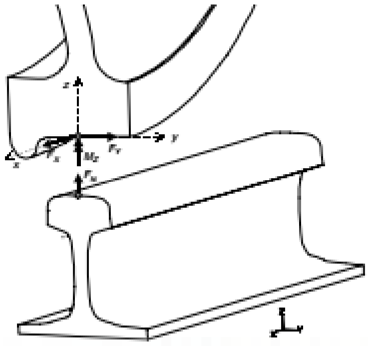


Figure – 2.1 Contact Forces from MBS simulation (*reference 1*)

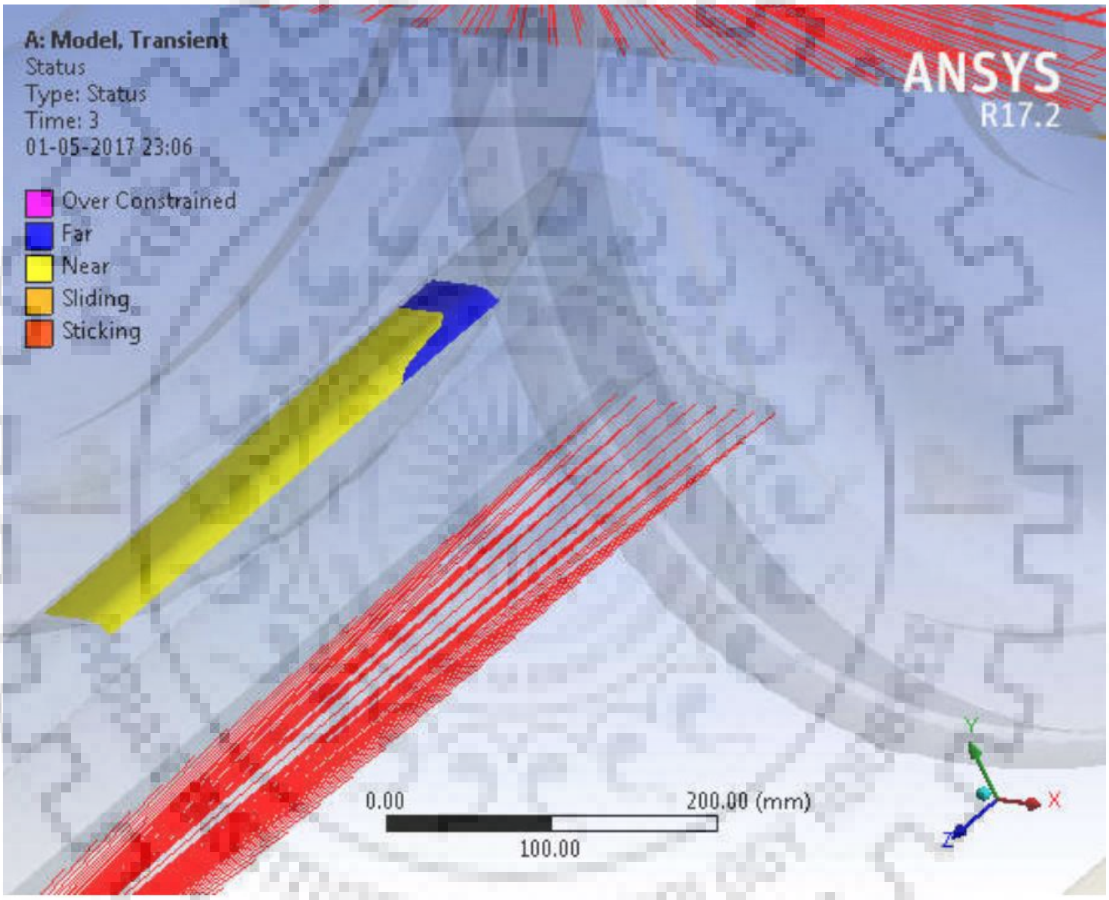



Figure – 2.2 Rail-Wheel Contact Patches

Damage mechanisms such as surface cracks, plastic deformation and wear, see for example (Knothe K., 1999) for a survey, it has been studied micro cracks propagation with the help of multibody analysis and resulting stress has more as compare to yield stress of material due to addition effect like initial cracks. This has to lead the negative aspect of rolling noise and rider comfort. Till now five theory were developed for an understanding of rail-wheel interaction.

The due complexity of rail-wheel geometry it's required complex mechanisms behind it. (Carter, 1926), the linear theory (Kalker, 1967), the complete contact theory (Kalker, "Two algorithms for the contact problem inelaststatics", 1983), the theory of

(Shen, 1984), and the simplified theory. All of these theories have limitations and they can be viewed as complementary. Continuum rolling contact theory started with a publication by (Carter, 1926) has given two-dimensional theory in this theory he has assumed that wheel like cylinder and rail like infinite half space. He has given the concept of creepage .actual velocity has not attainable some sliding has been occurring. (Kalker, 1967), has considered contact mechanics and find out how the force transmitted in the rail-wheel interaction. (Shen, 1984) Has taken a parametric approach for solving rail-wheel interaction.
μ=Coefficient of friction $F_t$= tangential force $F_n$= Normal force.

Carter's theory has given approx. result when the wheel was driven, and calculate the frictional loss. But his result was not accurate in the case of a vehicle in motion. Lateral force and tangential force must take into account for getting the exact result. (Kalker, "Wheel-rail rolling contact theory",, 1991). (Johnson, 1958) Have summarized the theory developed in last 60 years. Generalized Carter's has studied about longitudinal creepage lateral creepage. (Vermeulen, 1964) Given the study that tangential force transmitted in an asymmetric way some part of sticking region some of the sliding region the contact patches which has been generated elliptical in nature .find out semi-major, semi-minor axis for getting pressure distribution. (Shen Z. H., 1984) Has given for improvement of creepage coefficient for getting an approx result which has given by Vermeulen and Johnson. In these theories, which are all Hertzian-based, semi-major axis (a) semi- minor axis (b) and lateral displacement for getting the area of contact .it has been found that contact pressure only depends on (a),(b) value. Ratio a/b dependent on the curvature of the rail-wheel profile. Has study contact only depends on $F_n$ that is normal load but it is independent on $F_t$ that is a tangential load. (Zefeng Wen, 2011) Implemented advanced plasticity theory using FEM for study contact area pressure and tangential traction. (Cristopher P. Ward, 2011) Developed a technique to use real-time fault detection to estimate creep forces of rail-wheel contact. (Rovira, 2011) Tried to investigate wear with analytical model and validate with the experimental result. (Nicola Bosso, 2013) Have great effort review previous theories and application on dynamic condition of railway wheel contact. (Hyun wood lee .corina sandu, 2011) Invested a new 3D nonlinear friction coefficient for dry rail condition with help of mass spring damper. (Krishnaraja, 2010) Crack tip plastic zone analysis in 3D FEM

For getting exact result used FE (Finite Element Analysis) has been used. In this case, we can use a number of constraining but in the traditional approach, it depends on various limitation. ANSYS Tool based on the geometry of wheel-rail define material properties, Meshing parameter and boundary condition which we have applied.

For analysis of worn wheel, he has been kinematic constraints with ANSYS 17.2.He has taken the material as an elastic-plastic model along with kinematic hardening. By applied various boundary condition many results have been generated. Static and quasi-static simulation can be used in this rail wheel intersection with the help of FE analysis. In software interface it could be added sliding, rolling and spinning. The exact result can be calculated by FE simulation. Super element technique can be used for reducing simulation time.

## 2.4 Contact Pressure

Railway engineer and researcher have given the theoretical numerical which were based on hertz (Normal contact theory).Before studying Rail-wheel interaction starting point should be Hertz's theory. Verified the static and dynamic analysis of rail-wheel interaction. But we can introduce the influencing parameter. Simulation has been done with help of computer software. (Paul, 1981). Has been finding out contact patches, internal stress, shear stress distribution on rail wheel surface, slippage region, an adhesive region in contact patches. In the dynamic simulation, this all depend on all relative forces which developed between rail-wheel interfaces. (J.B. Johnson, 1894) (Andrews, 1958). (W. Poole, 1986). They have a study on friction roller and find out contact pressure, contact stress, contact zone .and (J. Haines, 1963) (E. Ollerton, 1963-64)or optical techniques (Y. Zanhou, 1992). Has study robust experiment method for finding out static analysis of rail-wheel contact.

## 2.5 Static Analysis

The stress developed in rail wheel profile tensile &compressive in nature. In order to study Stress analysis, we should find out critical stress which leads to failure and fracture of the wheel set. At the state of initial state small crack generated when stress developed in contact zone exceed the critical stress and this small crack will propagate and lead to fracture and failure of the material of wheelset. The main objective of this paper to find out critical stress and contact patches at the point of rail-wheel interaction. (Smith, 1953) Studied normal as well as tangential load on an elastic solid and find out stress developed in the contact zone. (Haines, 1963) Studied contact stress distribution on an elliptical shape which was subjected to radial and tangential force. (Sackfield, 1983) Gave the appropriate result which was based on theory normal contact problem. (Sladkowski, 2005) FE software was used for finding out contact stress on rail-wheel interaction. (Wiest M., 2008) Has found out contact pressure generated between in Rail-wheel /switch contact (Donzella, 2010).has given failure diagram which is subjected to rolling.

(Monfared, 2011) Find out the analytical theory of stress and how the crack propagation in rotating body and compared result with FEM (Finite element model) in a static state. (Peter Tamas Zwierczyk, 2014) Analysis the pressure distribution in elliptical, rectangular, circular surface with help of ANSYS tool. (Mehnal Ali Arslan, 2012) Have been finding stress statin in right loading on 3-D FE model would give a better result as compared to 2-D axis –symmetric FEA Result.

## 2.6 Description of Track Geometry

Track geometry is the main parameter to excite the moving railway vehicle over the rail. Track geometry has following four types of irregularities:-

### 2.6.1 Gage

Gage is defined as the horizontal distance between the tracks. It is the distance between the heads of the rail and it is measured perpendicular to the rails. It is measured in a plane 5/8 in. below the top the rail head. Mathematically, we can write,

$$\text{Gage} = (Z_L - Z_R) / 2$$

### 2.6.2 Cross-level

Both the rails of the track are not equally elevated. The difference between the elevations of both the rails. Mathematically, we can write,

$$\text{Cross-level} = (Y_L - Y_R) / 2$$

### 2.6.3 Vertical surface profile

Vertical surface profile is the average of the elevation of the rails. Mathematically, we can write that,

$$\text{Vertical profile} = (Y_L + Y_R) / 2$$

### 2.6.4 Alignment

The average of the lateral positions of the two rails is called the alignment. The main reason of the lateral vibrations are the alignment and the cross-level variations. Mathematically, we can write,

$$\text{Alignment} = (Z_L + Z_R) / 2$$

The pictorial description of the above track irregularities is shown in the following diagrams. In the following diagrams, (a) shows the typical track geometry, (b) shows the alignment and the gage and (c) shows the cross-level and the vertical track profile.

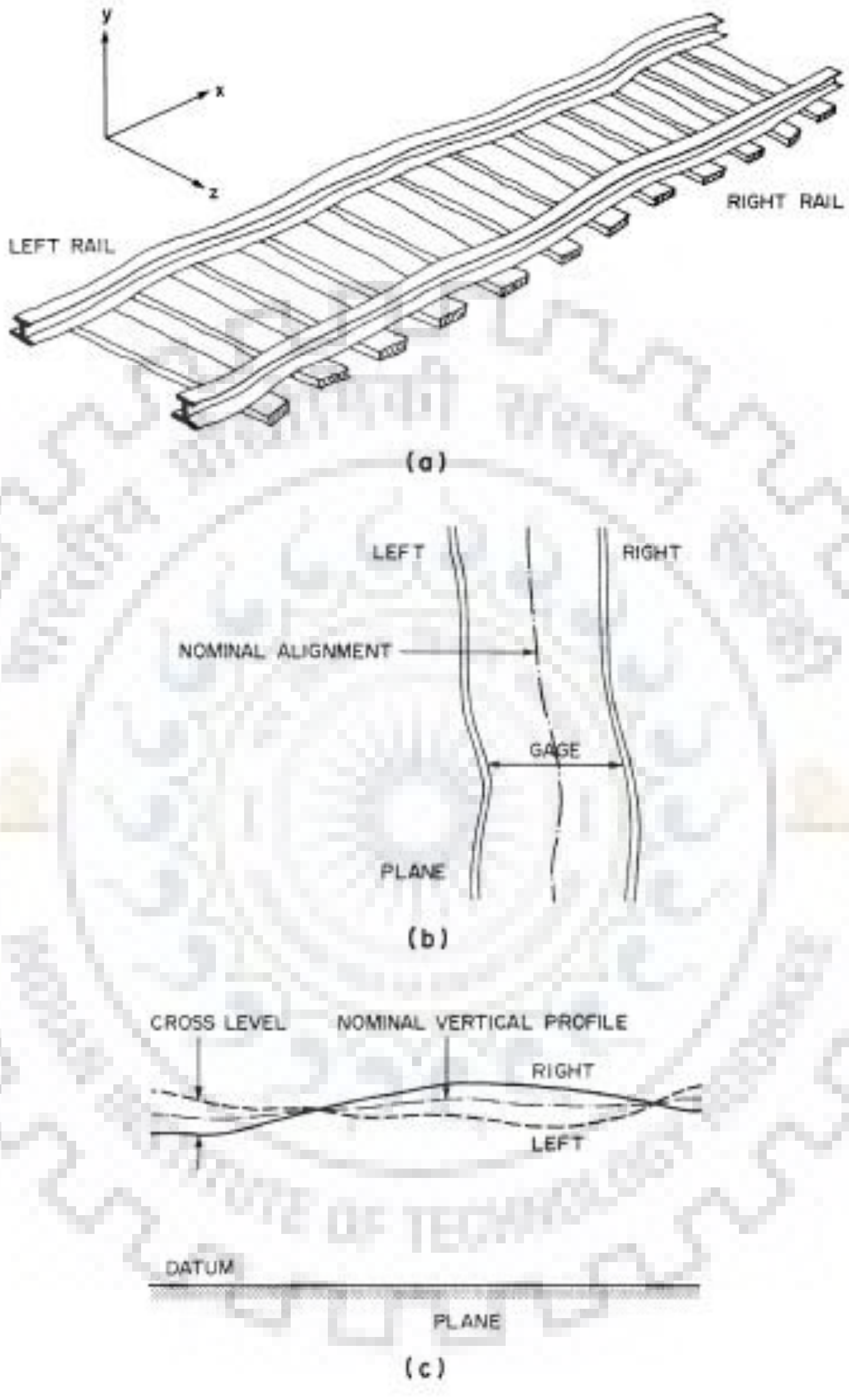



Fig 2.3:- Definitions of track irregularity parameters, (a) Typical track, (b) gage and alignment,(c) cross level and nominal vertical profile.( *Reference 9*)

## 2.7 Modal Analysis using Finite Elements Method

Finite elements method is an approximate numerical method to solve complex vibrational problems. It is an approximate but accurate method. For finite elements analysis of three dimensional objects, stiffness matrix, mass matrix and the force matrix are derived. Further, the elements matrices are assembled in order to find the modal analysis results of overall solid object which has been discretized into small and simple in geometry elements. Then the boundary conditions are incorporated. Then the set of algebraic equations is solved to find the modal frequencies and the mode shapes of the vibration. Modal analysis is an eigenvalue problem. Eigenvalues of the $[M^{-1}K]$ matrix represents the natural frequencies while the eigenvector corresponding to each eigenvalue represents the corresponding mode shape.

## 2.8 Elements for finite element analysis:-

As all the physical systems have three dimensions. Therefore, it is not possible to discretize the physical systems into 2D or 1D elements. Therefore, we have to discretize the physical systems into three-dimensional elements in which the field variables are dependent of all the three dimensions x, y and z. Some the variables in the analysis can be in any direction of the space, for example, force can be applied in direction in the space on the system. It is not necessary that the 3D solid under analysis is having some definite shape. They can have any shape and can be made of any material. These solids can be subjected to any boundary and initial condition. All the components of a vector and tensor quantities have to be taken into account. Force can have three components in the space. Therefore, all the three components of a force should be taken into account. All the components of the stress (three normal and three shear) should also considered. A three-dimensional finite element can have any of the following shape- tetrahedron and hexahedron. They can have either flat or curved surface. Each node of a 3D element is free to move in any of the three directions. So, the element can have deformation in any direction in space.

A three-dimensional element is the most general form of a solid element. There are some special cases of three-dimensional solid elements which are (1) truss (2) beam (3) 2D solid and shell elements. 3D elements are the most general elements that they can be used to model any physical system like railway vehicle, beam, bridges, trusses. It can be used to mesh any complex geometry, which is the importance of a 3D solid element. The main problem associated with the 3D elements is that it takes a lot computational efforts. Therefore, a smart analyst will always try to model his system using 1D and 2D elements as much as possible to reduce the computational efforts and the computational time. Approximating the elements using 1D and 2D elements is sometimes necessary and very easy task. 3D solid elements are used when there is no possible choice. When an object is discretized using a 3D elements then it is a very easy and quick task for a computer as the 3D element is an upgraded version of a 2D element. All the mathematical techniques and the procedures followed in 2D elements can be followed in 3D elements. But, in case of a 3D element, the field variables can vary in x, y and z directions.

### 2.8.1 Tetrahedral Element

Consider a beam which is in the form of a cuboid. Let us discretize the cuboid into tetrahedral elements as shown in the fig 2.4.

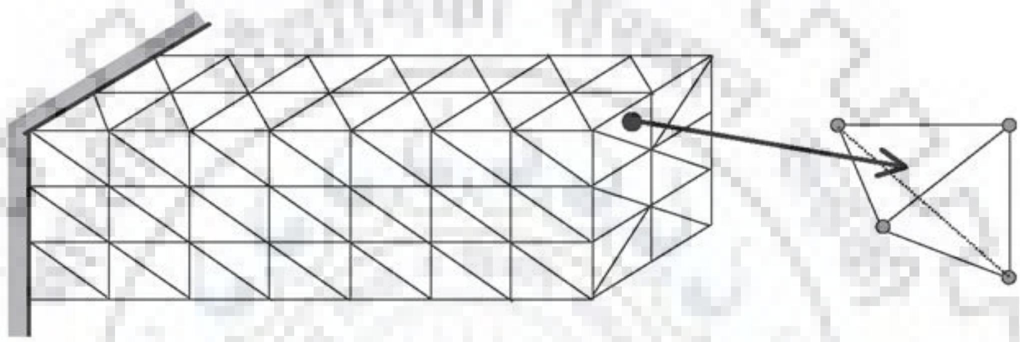


Fig 2.4:- Cuboidal solid block divided into tetrahedral elements. (*Reference 33*)

A tetrahedron element has four nodes, each having three DOFs (u, v and w), making the total DOFs in a tetrahedron element twelve, as shown in Figure 2.5.

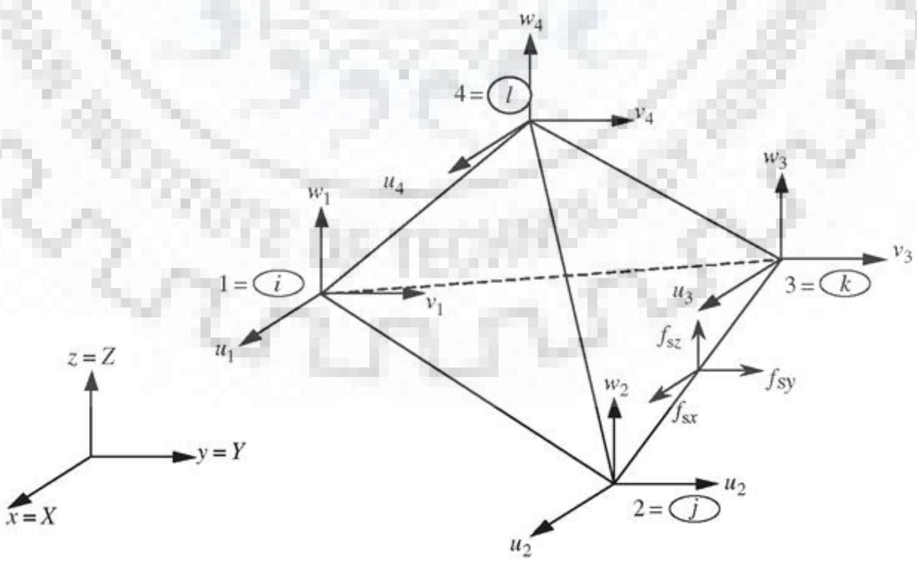



Fig 2.5:- A tetrahedral element (*Reference 33*)

The nodes are numbered 1, 2, 3 and 4 by the right-hand rule. The local Cartesian coordinate system for a tetrahedron element can usually be the same as the global coordinate system, as there are no advantages in having a separate local Cartesian coordinate system. In an element, the displacement vector U is a function of the coordinate x, y and z, and is interpolated by shape functions in the following form, which should by now be shown to be part and parcel of the finite element method:

$$U^h(x, y, z) = N(x, y, z)d_e$$

Where the nodal displacement vector, $d_e$, is given by

$$\mathbf{d}_e = \left\{ \begin{array}{c} u_1 \\ v_1 \\ w_1 \\ u_2 \\ v_2 \\ w_2 \\ u_3 \\ v_3 \\ w_3 \\ u_4 \\ v_4 \\ w_4 \end{array} \right\} \begin{array}{l} \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{displacements at node 1} \\ \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{displacements at node 2} \\ \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{displacements at node 3} \\ \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{displacements at node 4} \end{array}$$

The matrix of shape function is as shown:-

$$\mathbf{N} = \begin{bmatrix} \overbrace{N_1 \quad 0 \quad 0}^{\text{node 1}} & \overbrace{N_2 \quad 0 \quad 0}^{\text{node 2}} & \overbrace{N_3 \quad 0 \quad 0}^{\text{node 3}} & \overbrace{N_4 \quad 0 \quad 0}^{\text{node 4}} \\ 0 \quad N_1 \quad 0 & 0 \quad N_2 \quad 0 & 0 \quad N_3 \quad 0 & 0 \quad N_4 \quad 0 \\ 0 \quad 0 \quad N_1 & 0 \quad 0 \quad N_2 & 0 \quad 0 \quad N_3 & 0 \quad 0 \quad N_4 \end{bmatrix}$$

To develop the shape functions, we make use of what is known as the volume coordinates, which is a natural extension from the area coordinates for 2D solids. The use of the volume coordinates makes it more convenient for shape function construction and element matrix integration. The volume coordinates for node 1 is defined as

$$L_1 = \frac{V_{P234}}{V_{1234}}$$

Where Vp234 and Vi234 denote, respectively, the volumes of the tetrahedrons P234 and 1234, as shown in Figure 2.6. The volume coordinate for node 2-4 can also be defined in the same manner:

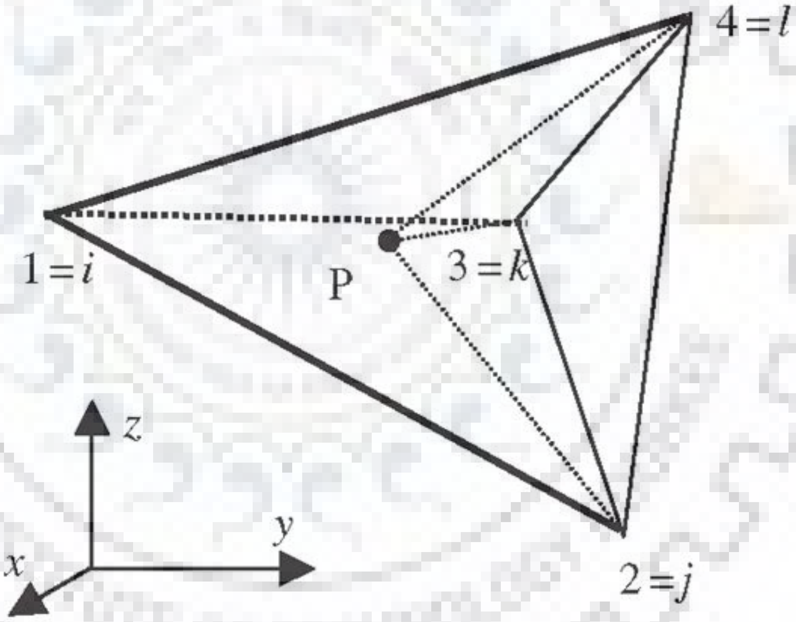



Fig 2.6:- Volume coordinates for tetrahedron elements.(*Reference 33*)

Hence,

$$L_2 = \frac{V_{P134}}{V_{1234}}$$

$$L_3 = \frac{V_{P124}}{V_{1234}}$$

$$L_4 = \frac{V_{P123}}{V_{1234}}$$

It can be easily verified that,

$$L_1 + L_2 + L_3 + L_4 = 1$$

### 2.8.2 Element Matrix

Once the strain matrix has been obtained, the stiffness matrix $k_e$ for 3D solid elements can be obtained. Since the strain is constant, the element strain matrix is obtained as

$$K_e = \int_{V_e} B^T cB dV = V_e B^T cB$$

Where the strain matrix [B] is given by,

$$\mathbf{B} = \frac{1}{2V}\begin{bmatrix} b_1 & 0 & 0 & b_2 & 0 & 0 & b_3 & 0 & 0 & b_4 & 0 & 0 \\ 0 & c_1 & 0 & 0 & c_2 & 0 & 0 & c_3 & 0 & 0 & c_4 & 0 \\ 0 & 0 & d_1 & 0 & 0 & d_2 & 0 & 0 & d_3 & 0 & 0 & d_4 \\ c_1 & b_1 & 0 & c_2 & b_2 & 0 & c_3 & b_3 & 0 & c_4 & b_4 & 0 \\ 0 & d_1 & c_1 & 0 & d_2 & c_2 & 0 & d_3 & c_3 & 0 & d_4 & c_4 \\ d_1 & 0 & b_1 & d_2 & 0 & b_2 & d_3 & 0 & b_3 & d_4 & 0 & b_4 \end{bmatrix}$$

And the matrix [c] is given by,

$$c_i = -\det\begin{bmatrix} y_j & 1 & z_j \\ y_k & 1 & z_k \\ y_l & 1 & z_1 \end{bmatrix}$$

And the mass matrix can similarly be obtained as,

$$\mathbf{m}_e = \int_{V_e} \rho \mathbf{N}^T \mathbf{N}\, \mathrm{d}V = \int_{V_e} \rho \begin{bmatrix} \mathbf{N}_{11} & \mathbf{N}_{12} & \mathbf{N}_{13} & \mathbf{N}_{14} \\ \mathbf{N}_{21} & \mathbf{N}_{22} & \mathbf{N}_{23} & \mathbf{N}_{24} \\ \mathbf{N}_{31} & \mathbf{N}_{32} & \mathbf{N}_{33} & \mathbf{N}_{34} \\ \mathbf{N}_{41} & \mathbf{N}_{42} & \mathbf{N}_{43} & \mathbf{N}_{44} \end{bmatrix} \mathrm{d}V$$

Where,

$$
\mathbf{N}_{ij} = \begin{bmatrix} N_i N_j & 0 & 0 \\ 0 & N_i N_j & 0 \\ 0 & 0 & N_i N_j \end{bmatrix}
$$

We can evaluate the above integral to get,

$$
\mathbf{m}_e = \frac{\rho V_e}{20} \left[ \begin{array}{cccccccccccc}
2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
 & 2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
 & & 2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
 & & & 2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
 & & & & 2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
 & & & & & 2 & 0 & 0 & 1 & 0 & 0 & 1 \\
 & & & & & & 2 & 0 & 0 & 1 & 0 & 0 \\
 & & & & & & & 2 & 0 & 0 & 1 & 0 \\
 & & & & & & & & 2 & 0 & 0 & 1 \\
 & & & sy. & & & & & & 2 & 0 & 0 \\
 & & & & & & & & & & 2 & 0 \\
 & & & & & & & & & & & 2
\end{array} \right]
$$

## 2.9 Vehicle Vibration Modes

Vehicle Vibration modes and its importance is described under following headings.

### 2.9.1 Need for Modal Analysis

Frequency analysis is having two major analyses in frequency domain i.e. (1) Eigenvalue analysis and (2) Spectral density analysis. Therefore, for the analysis of the characteristics of all the bodies of vehicle and the track i.e. mass, stiffness, damping, eigenvalue analysis is more valid. By eigenvalue analysis of the system, natural frequencies of the system can be found and the stability of the system can be investigated by using the natural frequencies of the vehicle. One more important result which is obtained by the eigenvalue analysis is the eigenmode. Hence, one can find out the sensitive resonance element and investigate. Like, one can find the effect of track irregularities on the stability of the system and can find its response. The eigenvalue analysis can be performed on a linear system. Therefore, for the eigenvalue analysis of a non-linear system, the system is linearized and then the eigenvalue analysis is performed.

### 2.9.2Modal Analysis of a Continuous System

Using D'Alembert's principle and discretizing the continuous system for finite elements, following equation can be obtained.

$$[M]\{\ddot{u}(t)\} + [c]\{\dot{u}\} + [K]\{u(t)\} = \{F(t)\}$$

Where:
$[M]$ = Structural mass matrix
$[C]$ = Structural damping matrix
$[K]$ = Structural stifness matrix
$\{\ddot{u}(t)\}$ = Nodal acceleration vector
$\{\dot{u}(t)\}$ = Nodal velocity vector
$\{u(t)\}$ = Nodal displacement vector
$\{F(t)\}$ = Applied load vector

Modal analysis is used to determine the vibrational characteristics of a system. This analysis is carried during the design process. For carrying out modal analysis of a system to find out natural frequencies, the right hand side of the above equation has to be zero. Hence, the resulting equation will be,

$$[M]\{\ddot{u}(t)\} + [K]\{u(t)\} = 0$$

The free vibration response of the system will be harmonic. Hence,

$$\{u\} = \{\emptyset\}_i \cos(\omega_i t)$$

After substituting in the above equation,

$$(-\omega^2{}_i[M] + [K])\{\emptyset_i\} = 0$$

$\{\phi_i\}$=0 will be a trivial solution of the problem. Therefore, the solution of the above equation will be.

$$[K] - \omega_i^2[M] = 0$$

Hence, the natural frequencies will be the eigenvalue of the $[M^{-1}K]$ matrix and the corresponding eigenmodes will be the mode shapes. The modal analysis forms the basis for the more advanced dynamic analysis like response of the system under dynamic loading or transient loading using the mode superposition method. The system should be made linear before the modal analysis or the software will ignore any kind of non-linearity present in the system.

Following assumptions has been made for the modal analysis:-

- System is linearly elastic.
- There is no non-linearity in the system.
- Small deflection theory has been utilized.
- No damping is included.
- Mode shapes are relative.
- The unknown modal displacement varies with time.

### 2.9.3 Solution Algorithm

By default the Subspace Method uses the Frontal Solver to obtain the first natural frequencies of a structure. This solver works efficiently for small models of up to 50,000 active degrees of freedom. However, if models consist mainly of solid elements with more than 50,000 active degrees of freedom, the Subspace Method combined with the PCG-Solver should be the preferred solution method. In ANSYS, the combination of the Subspace Method together with the PCG-Solver is called Power dynamics Method. For large models of up to 10,000,000 degrees of freedom, this method significantly reduces solution time.

## 2.10 Harmonic Response analysis

A different module of ANSYS is used for the harmonic analysis of the systems. In Modal analysis, there is no external force acting on the system while in case of the harmonic analysis, a time varying harmonic force acts on the system. In case of a railway vehicle moving over a track, the main cause of this harmonic force is the track irregularities. Considering these irregularities, the equation of motion using D'Alemberts' principle and discretizing the continuous system in finite elements can be written as,

$$[M]\{\ddot{u}(t)\} + [c]\{\dot{u}\} + [K]\{u(t)\} = \{F^a\}$$

Where,

$F^a$ = Applied Force (Varies sinusoidally)

All the system is moving with the same frequency but with different phase angle. Therefore, the displacement can be written as,

$$\{u\} = \{u_{max} e^{i\phi}\} e^{i\Omega t}$$

Where,

$U_{max}$ = Maximum displacement

$\Omega$ = Imposed frequency

$\phi$ = Phase angle

Hence, the equation can be written as,

$$\{u\} = (\{u_r\} + i\{u_{img}\}) e^{i\Omega t}$$

Where,

$U_r$ = real displacement vector

$U_{img}$ = imaginary displacement vector

In the same manner, the forces can be written as,

$$\{F\} = (\{F_r\} + i\{F_{img}\}) e^{i\Omega t}$$

Where,

$F_{r,img}$ = real and imaginary force vector

Substituting the above two equations in the equation of the motion,

$$(-\Omega^2[M] + i\Omega[C] + [K])(\{u_r\} + i\{u_{img}\}) e^{i\Omega t} = (\{F_r\} + i\{F_{img}\}) e^{i\Omega t}$$

After removing the time dependency,

$$([K] - \Omega^2[M] + i\Omega[C])\{u_r\} + i\{u_{img}\} = \{F_r\} + i\{F_{img}\}$$

The equation obtained can be easily solved using Gauss Elimination approach. In Harmonic analysis, we obtain the structural deformation for the whole range of the frequency while in case of modal analysis, we particular frequency at which system will make resonance with the external force.

### 2.10.1 Solution Method

In Harmonic analysis, there are two types of solution methods.

(1) Full analysis
(2) Mode-Superposition analysis

In both the methods, the damping ratio is treated differently. In full analysis, the damping matrix is calculated explicitly. In Mode-Superposition method, the damping matrix is calculated using the definition itself.

# 3. METHODOLOGY

In general, a computer model of a railway vehicle can be constructed and run in a virtual environment, and a range of possible designs or parameter changes can be investigated. Also the outputs from the model can be set up to provide accurate predictions of the dynamic behaviour of the vehicle and its interaction with the track. The inputs are normally provided at each wheelset of the model. The typical inputs are deviations in gauge and cross level and vertical and lateral track irregularities. A generalization of the simulation process was presented by Polach et al. (2006) is shown as below.

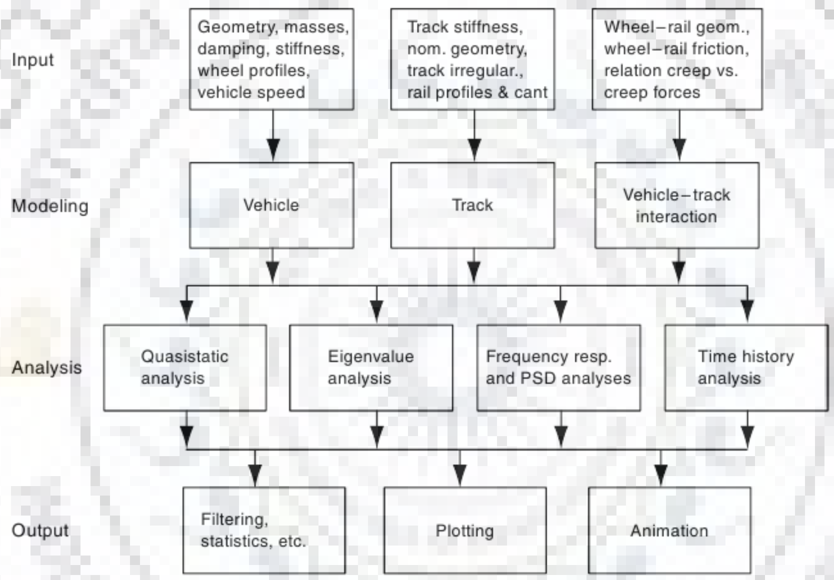
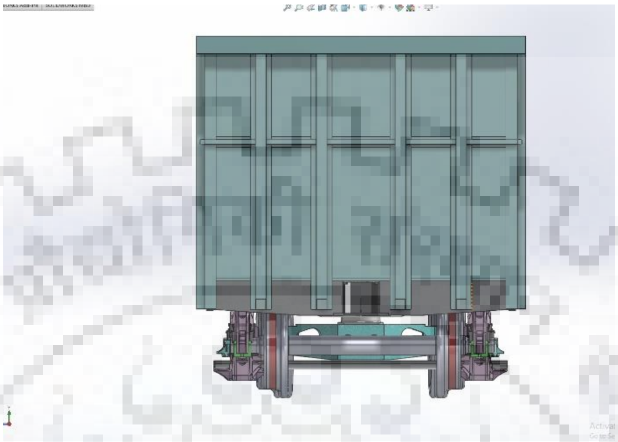



Fig 3.1:- Simulation Process of Railway Vehicles (*Reference 7*)

## 3.1. CAD MODEL

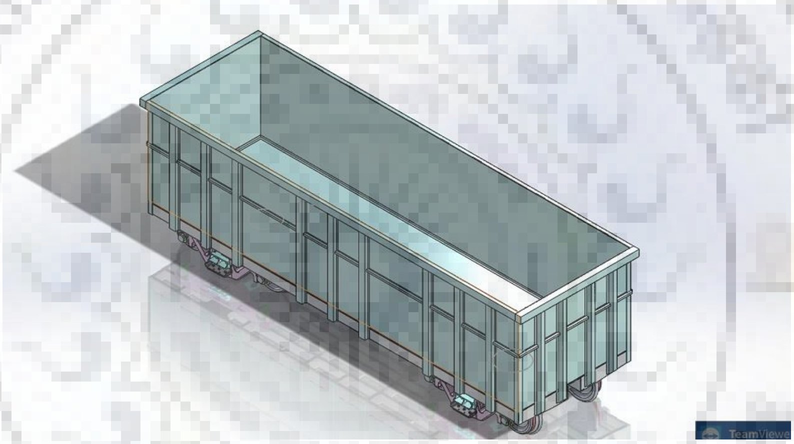
First of all the CAD model of CASNUB bogie, BOXN 25 Wagon, s1002 wheel, UIC 60 track has been modeled using solidworks software. Various parts of above mentioned bodies has also been assembled using the same software. The material used for this model is Indian Railway standard steel which is IRSM-44 is used. The suspension system has been modeled using ANSYS software. Springs during modeling has not been given any mass as the total mass of all the springs is less than the 1% of total mass of the system. Hence, this assumption will not affected the results considerably. The model which has been generated using the SOLIDWORKS software has been shown as below with its side, top and front views.



(a) Front view of assembly

(b) Side view of assembly

(c) Top view of assembly



(d) Isometric view of the assmbly

Fig 3.2 Top, Front, Side and Isometric view of Assembly of BOXN 25 wagon and a pair of CASNUB bogies.

## 3.2 IMPORTING THE MODEL IN ANSYS

As the file format of SOLIDWORKS (i.e. SLDPRT and SLDASM) is not recognizable by the ANSYS software. Therefore, before importing the assembly CAD file into ANSYS, it has been converted to the STEP (STandard for the Exchange of

Product model data) and then imported to the solidworks design modeler. After importing the file to the ANSYS the assembly is read by this as follows.

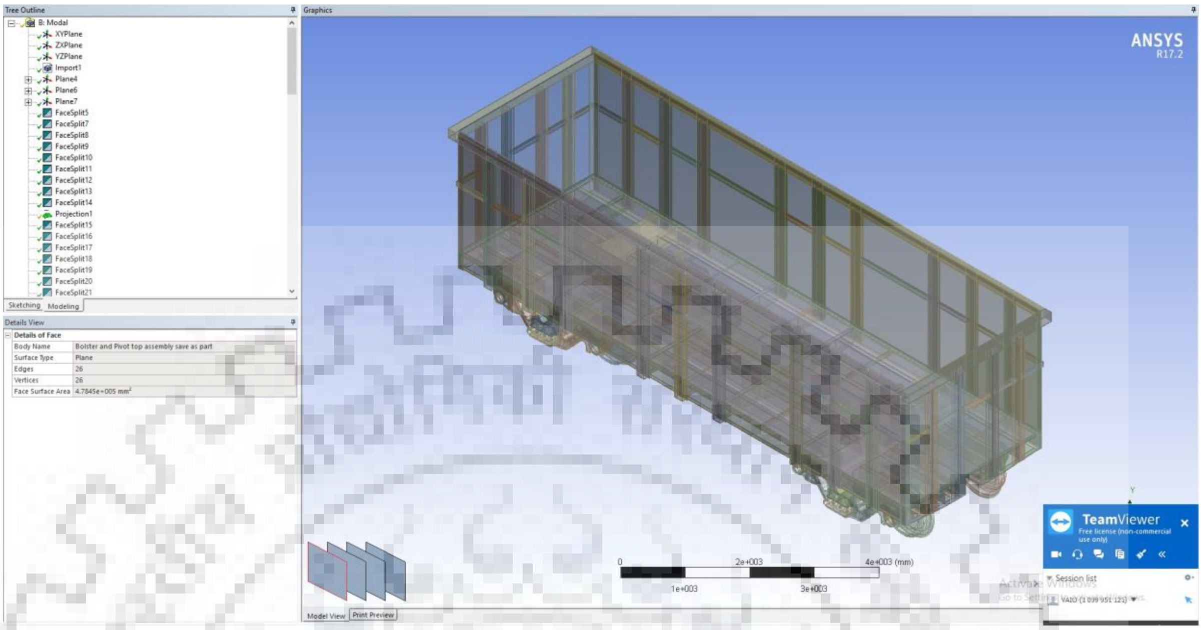


Fig 3.2 CAD model imported to design modeler of ANSYS

## 3.3 MESHING:-

After importing the assembly into ANSYS it is necessary to break the body into elements to carry out the desired analysis. Element control is by program during the discretization. Overall volume of the body is 0.75648 $m^3$ and overall mass of the body is 5938.4 kg as the density of the ferric steel is 8050 $kg/m^3$. Total number of bodies is 271 if not assembled. While active bodies are 47 as the assembly has been broken about the axis of symmetry and the body of the wagon has been modeled as rigid mass. Total number of elements are 353558 and the total number of nodes are 627361. Total number of connections in the body are 938 and the active connections are 120. The parts which are moving with respect to each other has been given bonded contacts with MPC (Multi-point contact) formulation. The meshed body is shown as.

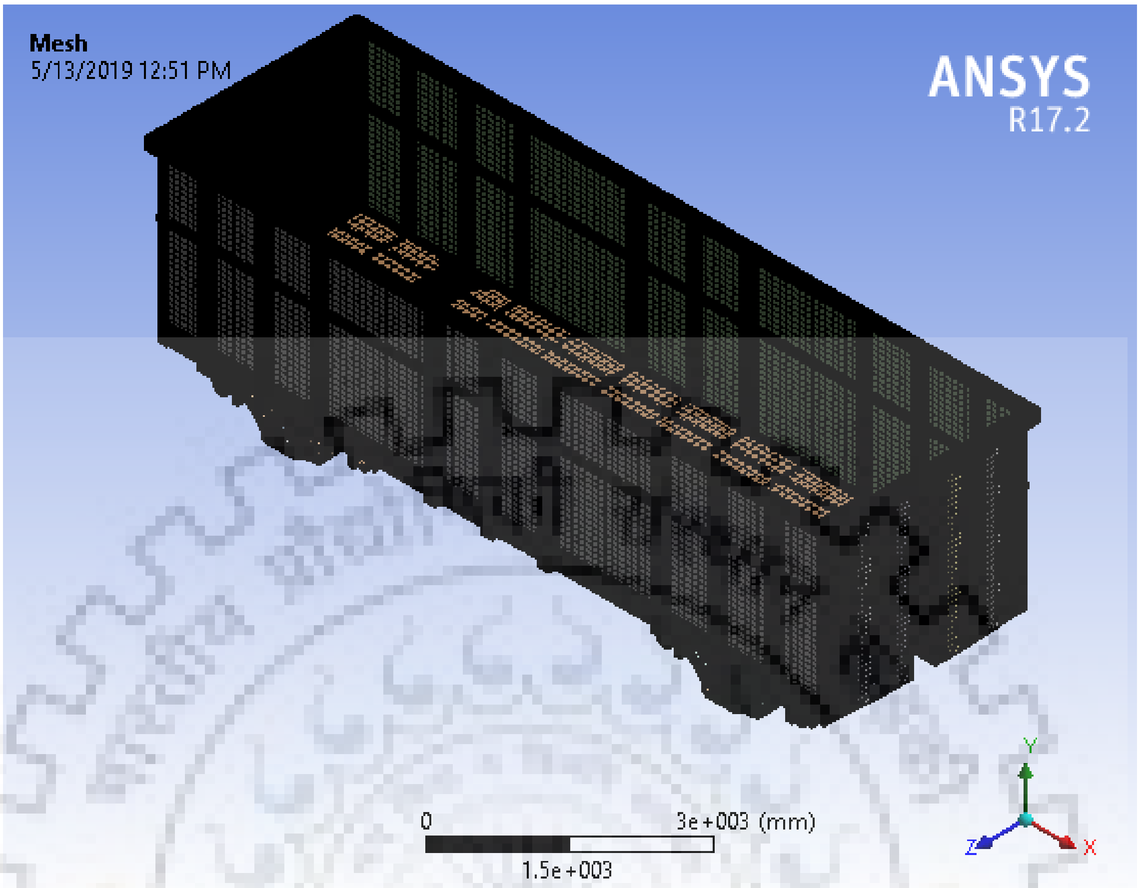



Fig 3.3:- Meshing of the system.

## 3.4 MODELING OF SPRINGS

The suspension system of the springs are modeled in the ANSYS. Each bogie consists 14 inner springs, 14 outer springs and 4 snubber springs. The specifications of the springs are shown in the table below.

| SPRING DATA | | | |
|---|---|---|---|
| DESCRIPTION | OUTER SPRING | INNER SPRING | SNUBBER SPRING |
| FREE HEIGHT mm. | 253 mm. | 225 mm. | 304 mm. |
| SOLID HEIGHT mm. | 152 mm. | 152 mm. | 152 mm. |
| WIRE DIA. mm. | 21.5 mm. | 16.5 mm. | 16.5 mm. |
| OUTER DIA. OF COIL mm. | 136 mm. | 85 mm. | 104 mm. |
| LOAD AT SOLID HEIGHT kg | 2410.26 kg | 2077.9 kg | 2074.8 kg |
| WINDING | LEFT HAND | RIGHT HAND | RIGHT HAND |
| No.OF EFFECTIVE COILS | 5.97 | 8.112 | 8.112 |
| TOTAL No OF COILS | 7.47 | 9.612 | 9.612 |
| SCRAG TEST LOAD | 2410.76 Kg | 2077.9 Kg | 1774.5 |
| HEIGHT AT TEST LOAD mm. | 152 | 152 | 174 |
| VERTICAL STIFFNESS | 23.864 Kg/mm | 28.451 Kg/mm. | 13.65 Kg/mm. |
| WORKING HEIGHT mm | 191.5 | 191.5 | 191.5 |
| LOAD AT WORKING HEIGHT kg | 1467.5 | 953 | 1535.5 |

Table 3.1 Spring Data

The spring modeled are shown as below.

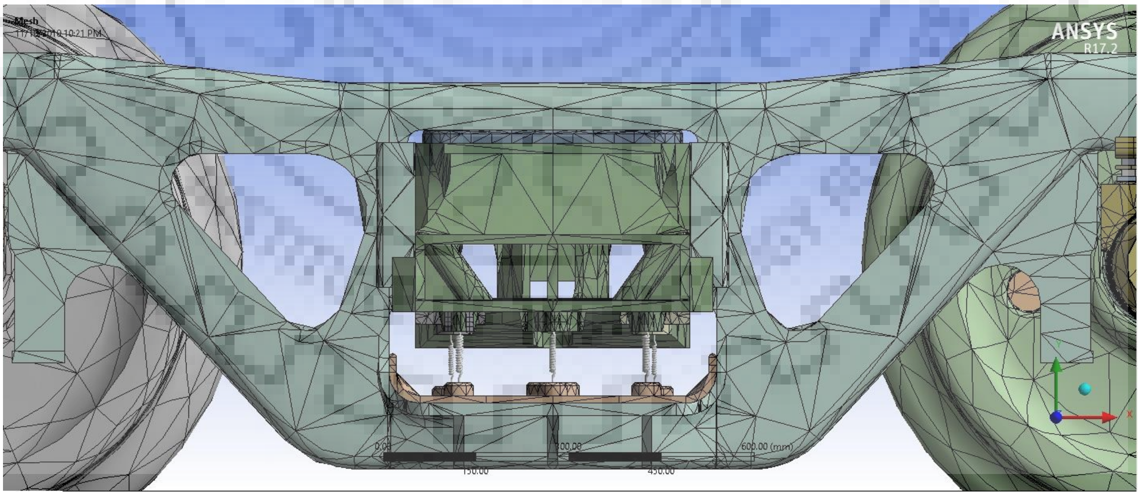



Fig 4.4 Modeling of springs in ANSYS

# 4. RESULTS

The finite elements model and the modal frequencies represents the railway vehicle. The deformation mainly is mainly influenced in the elastic underframe and the sidewalls. The mode shapes are also taken into the consideration. Mode shapes affects the comfort of the passengers and the safety of the laden weight. In this analysis the modal frequencies below 25 Hz has been taken into the account. The minimum frequency obtained is 6.94 Hz. The maximum frequency obtained which is below 25 Hz is 23.21 Hz. There are five modal frequencies below 20 Hz. The mode shapes corresponding to these frequencies can be described as the torsion, lateral swaying of the sidewalls.

## 4.1 NATURAL FREQUENCIES

At natural frequencies, the system oscillates without any external disturbance. The values of the natural frequencies calculated using Block Lancoz solver in ANSYS are as follows.

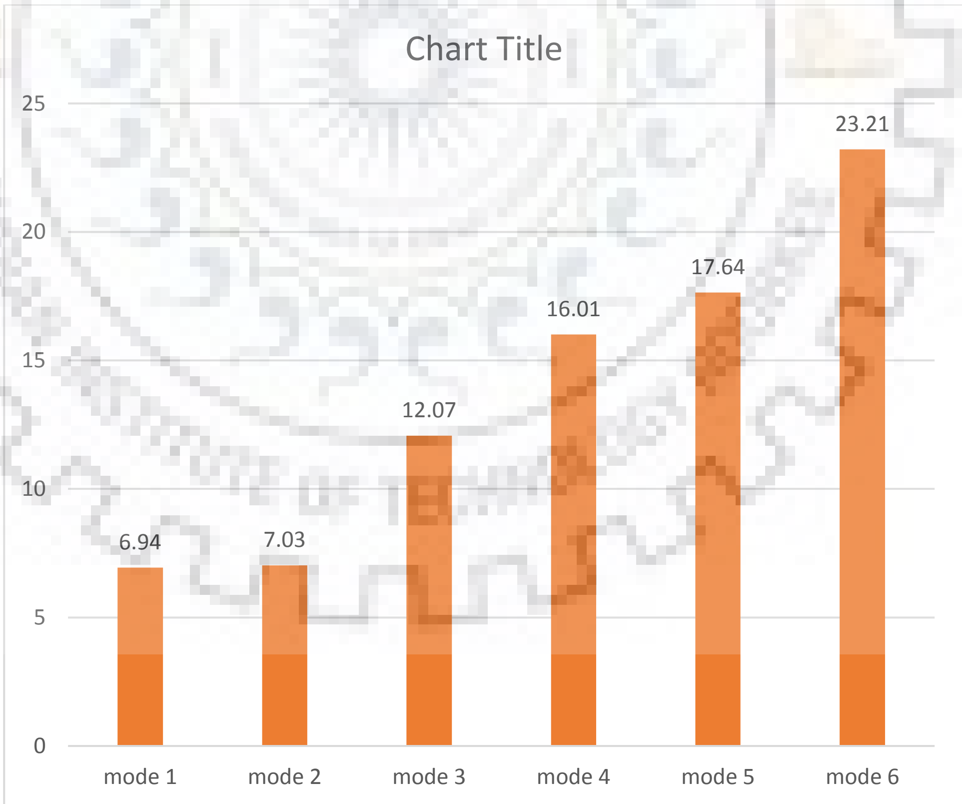
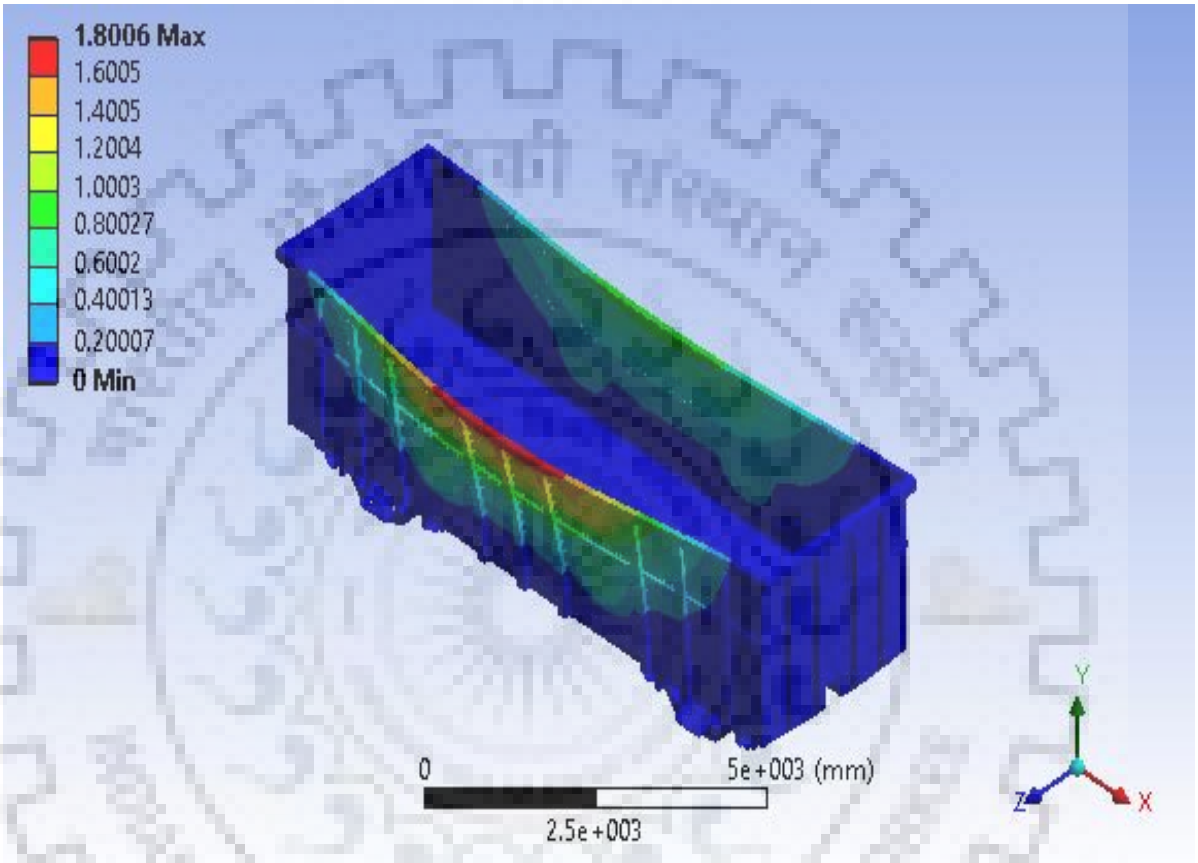


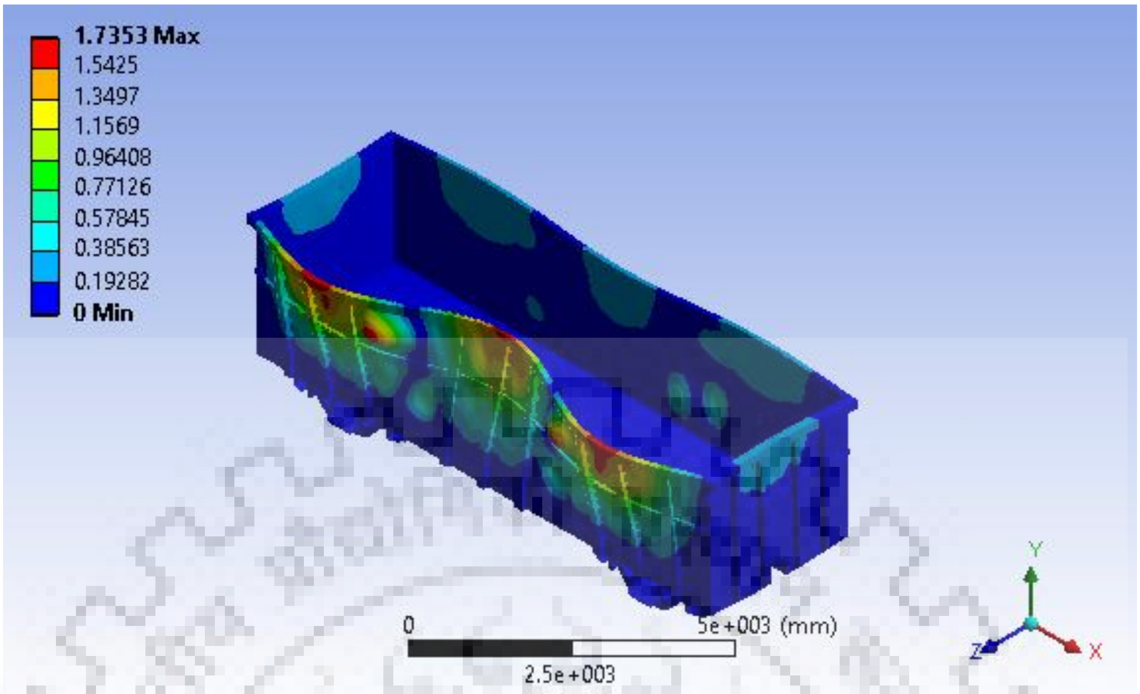

Table 4.1 Natural Frequencies

## 4.2 MODE SHAPES

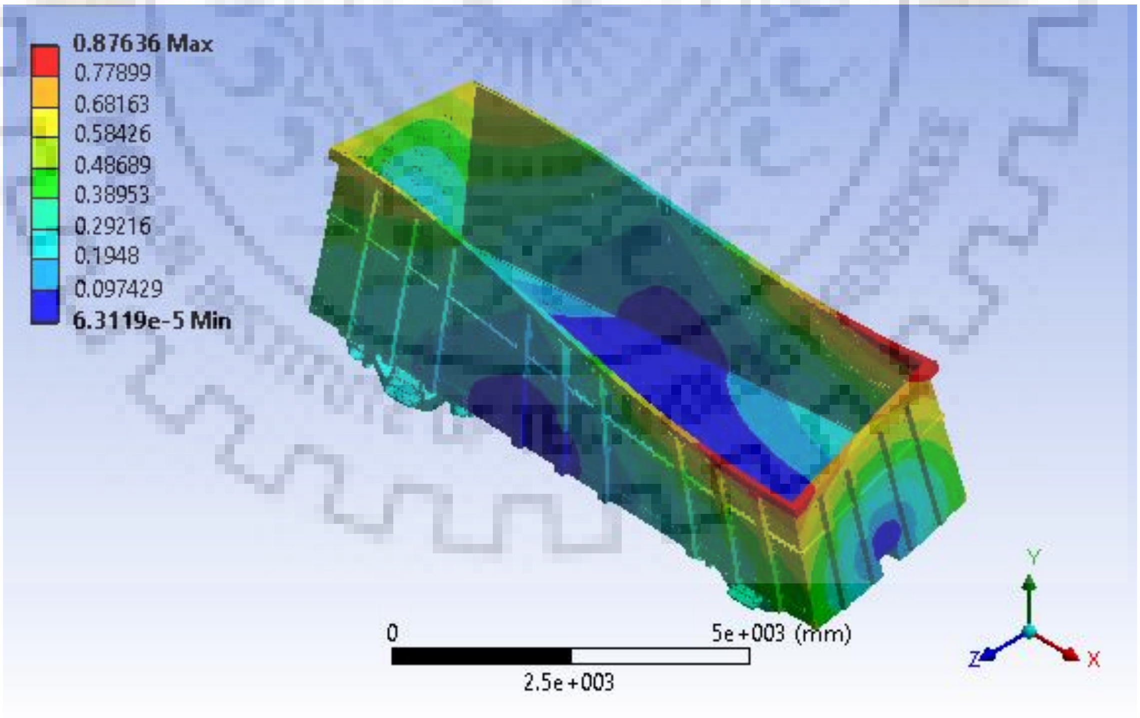
Corresponding to every frequency there is a different mode is obtained which is shown as below.



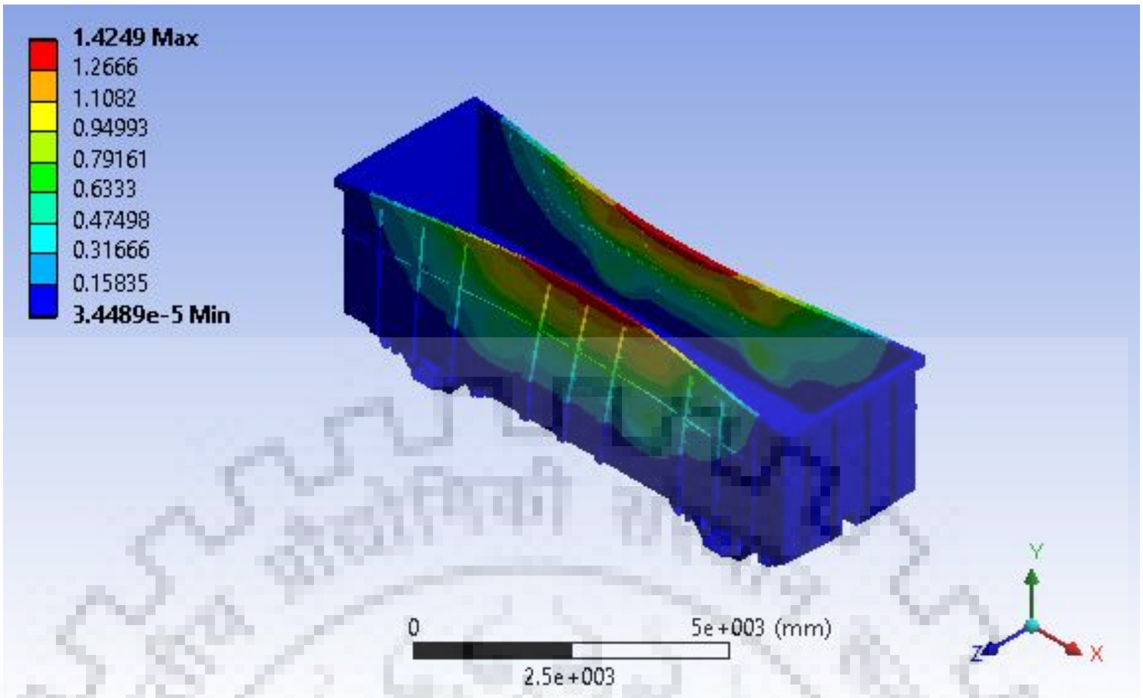

(a) First Mode (6.94 Hz)



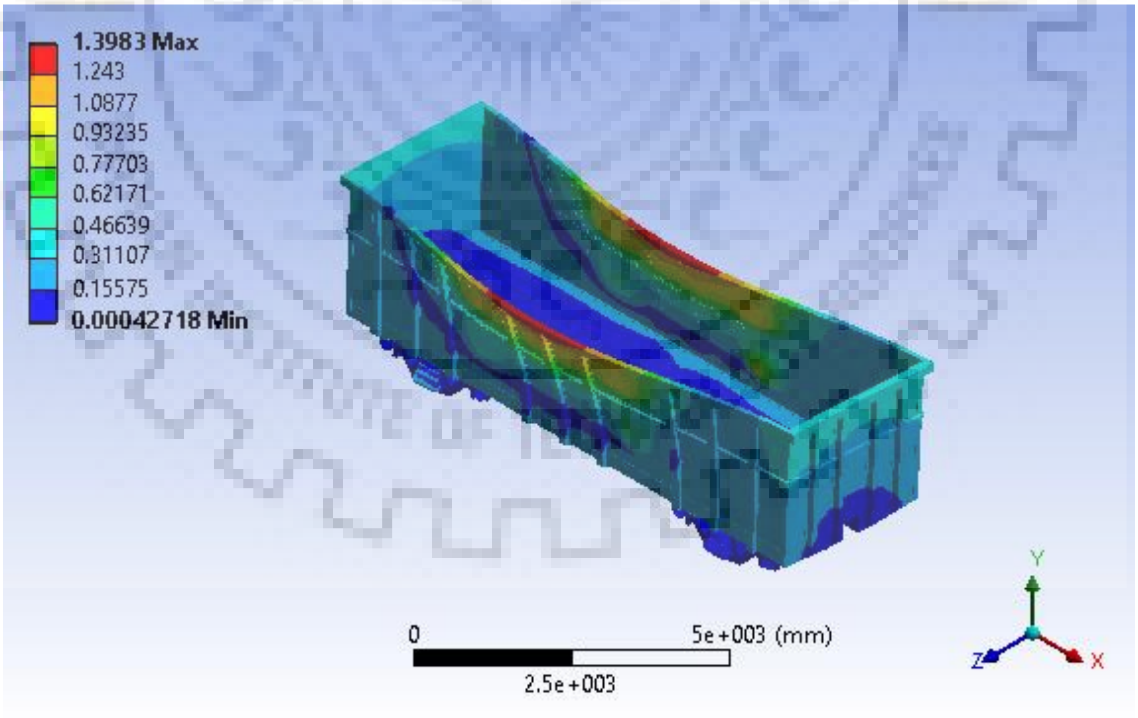

(b) Second Mode (7.03 Hz)



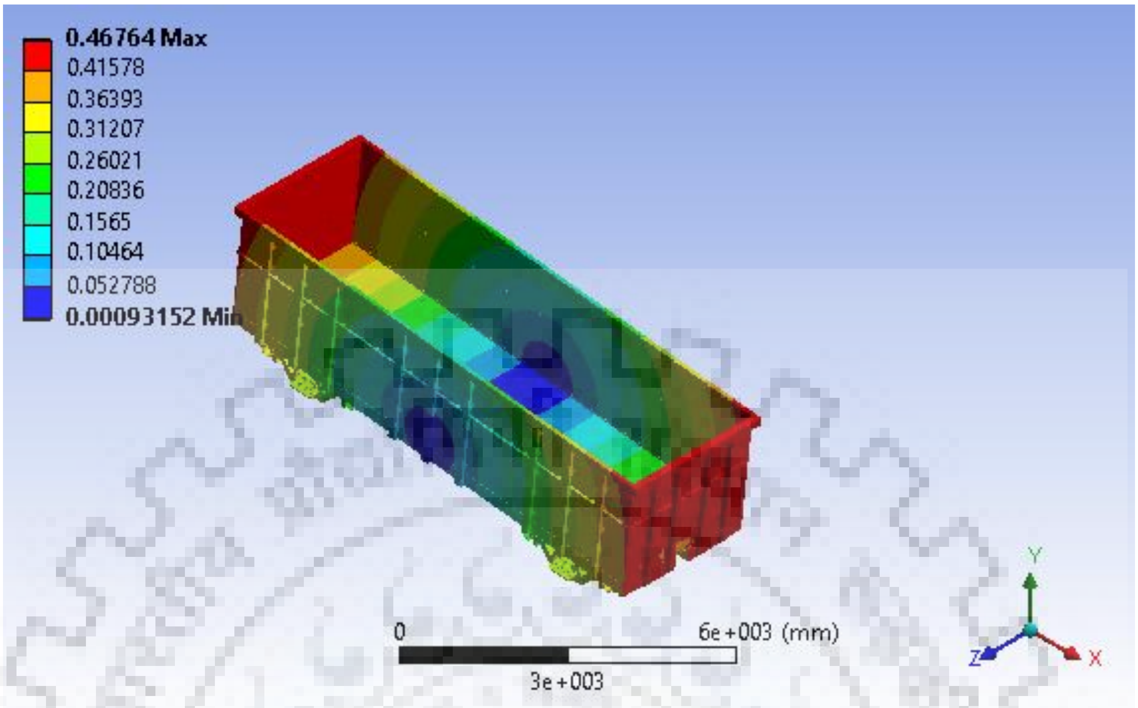

(c) Third Mode (12.07 Hz)




(d) Fourth Mode (16.01 Hz)




(e) Fifth Mode (17.64 Hz)




(f) Sixth Mode (23.21 Hz)

Fig 4.1 Mode Shapes

# 5. CONCLUSION

1. Solid model of BOXN 25 wagon, CASNUB bogie, UIC 60 rail-track, s1002 wheel has been modeled in SOLIDWORKS.

2. Configuration of freight wagon and its parts have been studied and their detailed drawings have also been taken into account.

3. Modal frequencies and mode shapes has been evaluated in ANSYS. If there is any matching frequency with the modal frequencies of the system then the instability of the car body will increase. In some cases, the excitation may cause derailment of the vehicle.

4. There are track irregularities present like vertical profile, gage, cross level etc. these irregularities are also present in the form of sinusoidal wave form. If any of these sinusoidal wave frequency coincides with the modal frequency of the system then it may cause excessive deformation in the system which is cause of the discomfort of passengers and the damage of the laden goods.

5. The results obtained by the modal analysis can be used for the harmonic, transient and power spectral density analysis using the method of mode superposition. The harmonic analysis gives the results for a wide range of frequency.

6. The modal analysis results should be considered during the design process of the system.

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