Gamma Radiation Detection and Inverse Problem Modeling

A DISSERTATION

Submitted in partial fulfilment of the

requirements for the award of the degree

Of

MASTER OF TECHNOLOGY

In

Solid State Electronic Materials

(S.S.E.M)

Submitted By

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MAY, 2018

CANDIDATE'S DECLARATION

I hereby declare that the work, which is being presented in the dissertation report entitled, **“Gamma Radiation Detection and Inverse Problem Modeling”**, in partial fulfillment of the requirements for the award of the degree of **Master of Technology** with specialization in **Solid State Electronic Materials**, submitted in Physics Department, Indian Institute of Technology Roorkee, India, is an authentic record of my own work carried out during the period from July 2017 to May 2018 under the supervision of **Dr. Mayank Goswami**, Assistant Professor, Department of physics, IIT Roorkee. The matter presented in this thesis has not been submitted by me for the award of any other degree of this institute or any other institute.

Date:

Place: Roorkee (SHEKHAR RAY)

CERTIFICATE

This is to certify that the above statement made by the candidate is true to the best of my knowledge and belief.

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Acknowledgement

The gratitude is the memory of the heart and in carrying out this dissertation work, persistent inspiration, unflinching support and encouragement of countless persons have served as the driving force.

I would like to take this opportunity to extend my heartless gratitude to my guide and mentor **Dr. MAYANK GOSWAMI,** Assistant Professor, Department of Physics, Indian Institute of Technology Roorkee, for his trust in my work, his invaluable guidance, and constant encouragement throughout this dissertation work. I would state that the present dissertation work would not have been in the present shape without his inspiration support. His gratitude, analytical and methodical way of working inspired me so much. I consider myself fortunate to have done my dissertation work under his guidance.

I also extend my sincere thanks to **Dr. K.L. YADAV**, Professor and Head, Department of Physics for providing me facilities for the work.

I would like to thank all of my friends and well-wishers for their suggestions, help and corporation in completion of present dissertation work.

Finally I would like to say that I am indebted to my parents for everything that they have done for me. All of this would have been impossible without their constant support

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Abstract

Following work has been done under this Thesis: MATLAB code is written for linear wave propagation inside a heterogeneous medium without considering scattering phenomenon. The goal is to investigate if including scattering the final expression can be linearized and if the difference between both expressions is insignificant. Beer Lambert law (as a starting point) is used in first case. While in later case complex expression is derived. It is found that wave propagation length under a discretized area cannot be made independent with medium dependent parameters such as refractive index, scattering cross-section or attenuation constant thus desired expression could not be achieved. Future work is to find this derivation and code it.

CHAPTER 1

Introduction

1.1 Introduction

The mathematical basis for all tomography disciplines was introduced almost a century ago in the pioneering work of the Austrian mathematician Johann Radon. In his paper from 1917, Radon proved that any two-dimensional object can be reconstructed uniquely from an infinite set of its line-integrals, the so-called projections. This result, known as the Radon transform, laid down the key concept needed for the tomographic imaging of scalar functions. The result has been independently rediscovered a number of times since then by other mathematicians, radio astronomers, electron microscopists, workers in optics, and medical radiologists. The first practical reconstructions of this kind were accomplished by the radio astronomer Ronald Bracewell in 1956. The same mathematical problem later arose in electron microscopy and in various optical applications. It is in the area of medical diagnosis, however, that the most excitement has been generated around the Radon transform. The first practical, clinically oriented solution to reconstructive tomography appeared in the early 1960, after Alan Cormack began to popularize and extend the work of Radon. Cormack received the 1979 Nobel Prize for medicine, jointly with Godfrey Hounsfield, who developed the first EMI Scanner, involving X-ray scanning and digital computing. This first scanner generated images of isolated slices of the brain with excellent tissue contrast. In 1974, a whole body scanner was developed by Ledley. Since then, a number of different companies have manufactured body scanners, and a broadly based competitive market has by now been developed. A standard treatise on the mathematics behind scalar tomography is given by F. Natterer. Although the X-ray scanner was the spark that ignited the greatest excitement in the tomography field, other kinds of radiation have proved to be successful as well. The early work in acoustic tomography was based on algorithms that were developed for X-ray tomography. Later, it became evident that more accurate models of the physical mechanisms generating the image were needed for high resolution and correct reconstruction. This point will be of particular interest in this thesis.

When electromagnetic waves encounter an inhomogeneity sample along their path of propagation, the path of the waves are not in straight line. Consider a beam of radiation bombarded onto a sample. It can get scattered, transmitted, absorbed. At the molecular and atomic level Absorption and scattering takes place. For energy from light to be absorbed, it must match available energy states in the atoms or molecules, or it can scatter from the molecule, atom, or electrons, (like billiard balls).. The wave propagation in anisotropic media shows complex behaviour in the form of beam distortion, beam skewing etc.

Mode conversion, beam skewing, critical angle phenomenon, reflection, refraction and the analytical -numerical solutions of wave propagation in anisotropic materials have been studied [1-5].

Based on the solution of the elastic wave equation, Ogilvy [6] presented an Ultrasonic ray tracing method known as RAYTRAIM.

Distinct steps were taken in the ray path along the direction of group velocity. Using the boundary conditions and the properties of the incident ray the reflection and refraction at the grain boundary were determined. Further, Ogilvy [7] extended her work to identify regions of materials to which no ultrasound penetrates and to predict directions of low inspection. Harker et al [8] made a contrast between a ray tracing model and a two-dimensional finite difference model and it was shown that both proceed toward to give similar and consistent results.

To keep track of the ray path through the base materials and weldments Schwitz et al. [9] developed a three-dimensional ray tracing algorithm. The artificial interface was selected to be perpendicular to the actual direction of the ray group velocity in this algorithm. Spies [10] gave rudimentary relationship concerning the propagation of the elastic waves in homogeneous and layered general transversely isotropic media by applying a co-ordinate free approach.

Using a mass spring lattice model, Yim and Choi [11] simulated the ultrasonic waves in transversely isotropic materials

Wang and Klinc [12] presented a ray tracing algorithm for anisotropic materials in presence of inhomogeneity. In inhomogeneous materials, it is required to identify an interface between two dissimilar regions, so that the refraction can be simulated with accuracy

In my thesis I will have to model the linear wave propagation inside a heterogeneous medium without considering scattering phenomenon. I will try to write Matlab code for linear wave propagation. I will have to try to model the wave propagation inside a heterogeneous medium considering scattering phenomenon, reflection and refraction theory.

CHAPTER 2

Radiations

2.1 Radiation

The transmission of energy from a source in the form of waves or particles which travel through the medium and can pass through most of the material is called radiation. The best example of radiation is the sun radiating ultraviolet light (sun rays) on earth. There are also radioactive elements like Plutonium (Pu) and Uranium (U) which radiate waves in the same way that the sun does.

Radiation is divided into two types:

1. Nonionizing radiation

2. Ionizing radiation

2.1.1 Nonionizing Radiation:

Non-ionizing radiation refers to any type of electromagnetic radiation. It does not carry enough energy per quantum (photon energy) to ionize atoms or molecules or to completely remove an electron from an atom or molecule.

Nonionizing radiations are further subdivided:

Blackbody Radiation: Thermal radiation is emitted by an opaque object.

Thermal Radiation: Heat transfer mechanism.

- *Extremely Low Frequency:* The frequency of the radiation is from 300 Hz to 3 KHz.
- *Very Low Frequency:* The range of the radiation frequency is from 3 Hz to 30 KHz.
- *Radio Waves:* Frequency range is from 3 KHz to 300 GHz. Radio waves have wavelength more than infrared light in electromagnetic spectrum.
- *Microwave:* Electromagnetic waves wavelength is from 1 mm to 1 m with frequency between 300 MHz and 300 GHz.

- *Infrared:* The electromagnetic radiation wavelength of infrared light (IR) is 0.7 to 300 micrometers with wavelength range between 1 THz to 430 THz.
- *Visible Light:* Visible light are narrow range of electromagnetic radiation which is visible to human eye of a wavelength range 380 nm to 750 nm.

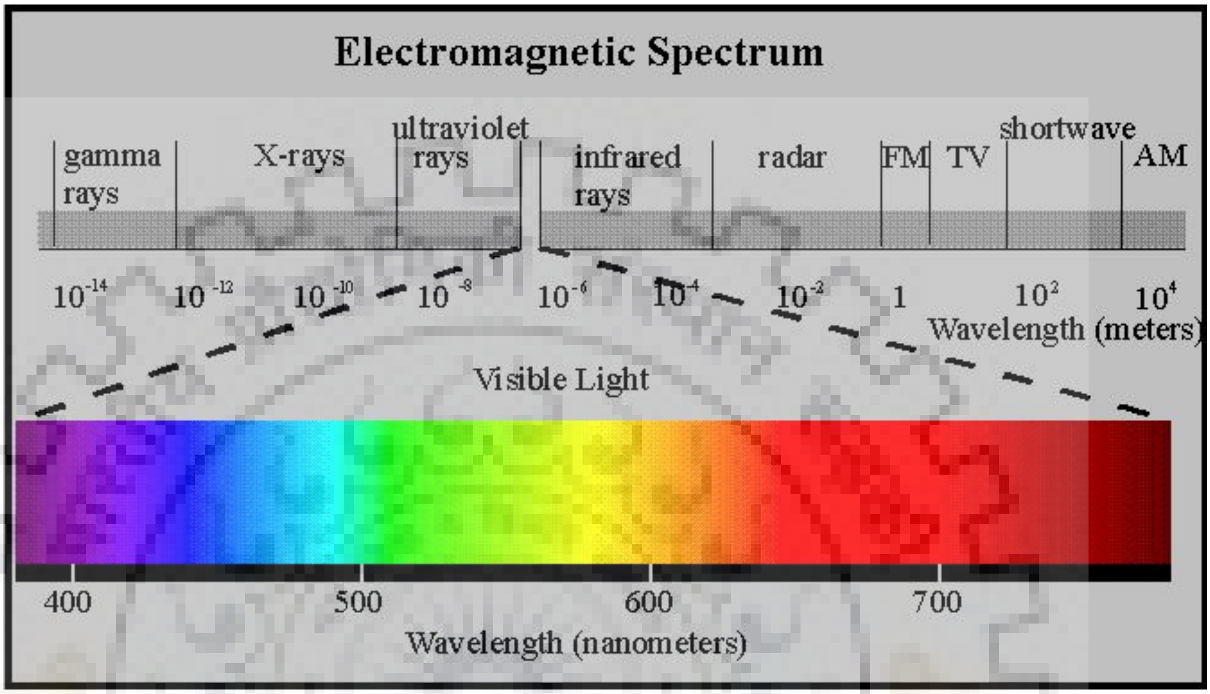


Figure 1 Electromagnetic Spectrum

2.1.2 Ionizing Radiation:

The word ***ionizing*** refers to the ability of the radiation to ionize an atom or a molecule of the medium it traverses or the process in which an atom gains or loses electron Ionization radiation carries with it particles, X–ray, or gamma rays with enough energy so it will ionize an atom within the medium through which it passes.

Ionizing radiation contain following type of radiation:

- *Ultraviolet Ray:* Ultraviolet Radiation (UV) is an electromagnetic radiation with a wavelength from 10 nm (30 PHz) to 400 nm (750 THz) which is shorter than that of visible light but longer than X-rays.
- *X–Rays:* The wavelength ranging of x–rays is from 0.01 to 10 nanometers. The wavelength of x–rays lies between that of UV rays and gamma rays. These are harmful human body.

- *Gamma Radiation:* Two different types of natural radioactivity are *alpha radiation* and *beta radiation* is in the forms of particle. Gamma rays have much energy than any other electromagnetic radiation, with a very-very short wavelength of about less than one tenth of one nanometer. Gamma radiation is *radioactive atoms* products.

2.2 Significance of Radiation

The radiation dose absorbed by a person (that is, the amount of energy deposited in human tissue by radiation) is measured using the SI unit gray (Gy) or conventional unit rad. The biological risk of exposure to radiation is measured using the SI unit Sievert (Sv) or the conventional unit rem.

Following common detectors are used to detect radiation:

- **Geiger-Müller Detector**
- **Scintillation Detector**
- **Semiconductor Detector**

2.2.1 Scintillation Detector:

A Scintillator is a material that shows scintillation when excited by ionizing radiation. The scintillation is the property of luminescence,. There is a property of luminescent materials that when they struck by an incoming particle, it absorb its energy and scintillate. In another word they re-emit its energy in the form of light.

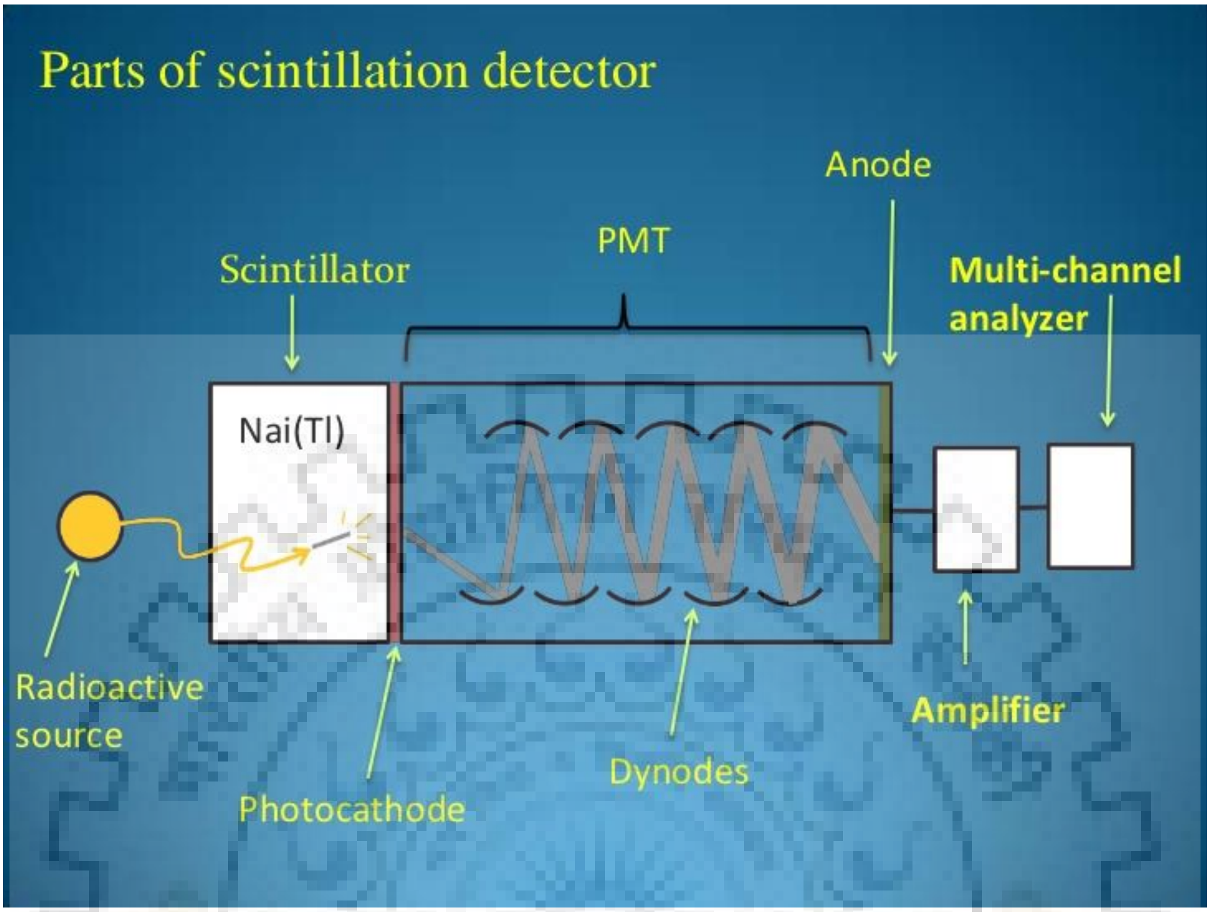


Figure 2 Scintillation Detector

2.2.2 Principle of Operation:

A scintillation detector or scintillation counter is obtained when a scintillator is coupled to an electronic light sensor such as a photomultiplier tube (PMT), photodiode, or silicon photomultiplier. The light emitted by the scintillator absorbed by photomultiplier tube. Photomultiplier tube re-emits it in the form of electrons via the photoelectric effect. The subsequent multiplication of those electrons (sometimes called photo-electrons) results in an electrical pulse which can then be analyzed and yield meaningful information about the particle that originally struck the scintillator. Vacuum photodiodes are similar but do not amplify the signal while silicon photodiodes, on the other hand, detect incoming photons by the excitation of charge carriers directly in the silicon. Silicon photomultipliers consist of an array of photodiodes. These photodiodes are reverse-biased with sufficient voltage so that they can operate in avalanche mode, enabling each pixel of the array to be sensitive to single photons.

Operation in Photomultiplier Tube:

- Absorption of incident radiation raising electron to excited states
- After subsequent de-excitation, the scintillation emits a photon in the visible light range
- The light emitted from the scintillation interacts with the photocathode of a photomultiplier tube releasing electrons
- Electrons are guided, with the help of an electric field, towards the first dynode
- Secondary electrons from the first dynode move towards the second and so on…
- Final amplification is of about 10^6 or higher

2.2.3 Application for Scintillators:

Scintillators can also be used in computed tomography and gas exploration, particle detectors, nuclear cameras, X-ray security,. Other applications of scintillators include CT scanners and gamma cameras in medical diagnostics, and screens in older style CRT computer monitors and television sets.

We find that there is a wide use of the scintillator with a photomultiplier tube in hand held survey meters. These are used for measuring and detecting radioactive contamination and monitoring nuclear material. Scintillators generate light in fluorescent tubes, to convert the ultra-violet of the discharge into visible light. Scintillation detectors are also used as detectors for Gamma ray logs in the petroleum industry.

2.2.4 Types of Scintillators:

Organic Crystals: Organic scintillators consist with arene or aryl hydrocarbon. It contain benzene ring structures and that are interlinked in various ways. Their luminescence typically decays within a few nanoseconds.

Some organic scintillators are pure crystals. The most common types are anthracene ($C_{14}H_{10}$, decay time ≈30 ns), stilbene ($C_{14}H_{12}$, 4.5 ns decay time), and naphthalene ($C_{10}H_8$, few ns decay time).

- **Organic Liquids*:*** The liquid solution of one or more organic scintillators in an organic solvent is called Organic liquids. The typical solutes are fluors such as *p-*

terphenyl ($C_{18}H_{14}$), PBD ($C_{20}H_{14}N_2O$), butyl PBD ($C_{24}H_{22}N_2O$), PPO ($C_{15}H_{11}NO$), and wavelength shifter such as POPOP ($C_{24}H_{16}N_2O$). The solvents such as toluene, xylene, benzene, phenylcyclohexane, triethylbenzene, and decalin are widely used.

- **Plastic Scintillators*:*** The scintillating material in which the primary fluorescent emitter is affected in the base. The primary fluorescent emitter is known as floor. The scintillating material is called plastic scintillators.

- **Bases*:*** Aromatic plastics, polymers with aromatic rings as pendant groups along the polymer backbone are the most commonly used bases in plastic scintillators. Polyvinyl toluene (PVT) and polystyrene (PS) are the most prominent.

- **Fluors*:*** it is also known as luminophors. The scintillation of the base is absorbed by these compounds and then emit at larger wavelength. The ultraviolet radiation of the base is converted into the more easily transferred visible light by using it. Common fluors are polyphenyl hydrocarbons, oxazole and oxadiazole aryls.

Inorganic Crystals: Lanthanum chloride doped with cerium ($LaCl_3$ (Ce),), as well as lanthanum bromide doped with cerium,($LaBr_3$ (Ce)). They are both very hygroscopic. It will damage when it exposed to moisture in the air but offer excellent light output and energy resolution. Bismuth germanium oxide or Bismuth Germinate $Bi_4Ge_3O_{12}$ (BGO) is also an inorganic crystal used as a scintillator.

- **Gaseous Scintillators*:*** Gaseous scintillators are basically consist with nitrogen and noble gases such as helium, argon, krypton, and xenon, specially helium and xenon. The scintillation process is due to the de-excitation of single atoms excited by the passage of an incoming particle.

- **Glasses*:*** The most commonly used glass scintillators are cerium-activated lithium or boron silicates. Since both lithium and boron have large neutron cross-sections, glass detectors are particularly well suited to the detection of thermal (slow) neutrons.

CHAPTER 3

Measurement of Linear Attenuation Cofficient

3.1 Introduction

Gamma-ray transmission method utilizes the application of Lambert-Beer law for the measurement of the linear attenuation coefficient (μ) of the sample under investigation. But the accuracy of the method demands that the sample thickness should to be known precisely. However many samples have irregular shapes. The precise thickness of such samples cannot be measured with a micrometer or a Vernier caliper. It limits the direct application of Lambert-Beer law for the determining of their linear attenuation coefficient. I tried a solution to this problem. This method utilizes standard Lambert-Beer law in such a way that thickness of sample under study is not required. The accuracy of the method depends upon the difference in the values of linear attenuation coefficient of the pair of media used. Thus larger the different in the values of linear attenuation coefficient, greater the accuracy of method. Secondly, the media should be preferably homogenous. Keeping in view of above suggestion, Elias (2003) theoretically proposed a simple procedure to introduce some new combinations by considering air as one of the media. This procedure not only simplifies experimental work, but at the same time, it also allows a greater number of repetitions as well as introduces larger difference in the values of attenuation coefficient of the pair of media used. Further, the resulting linear attenuation coefficient value is an absolute value and not a relative one referred to the linear attenuation values of the media

3.2 Theoretical Formulation of Method

A collimated beam of gamma-ray having initial intensity Io is attenuated in absorber of thickness ‘x’ of absorber according to the Lambert-Beer law:

$$I=I_o\, e^{-\mu x} \tag{3.1}$$

Where I is transmitted bean intensity of unaffected primary photons.

Using this standard Lambert-Beer law we can measure linear attenuation coefficient of any shaped sample without using its thickness. In this method, the gamma-ray transmission intensity through the sample under study is measured by immersing it, turn wise, into two different media with known linear attenuation coefficients. In this method sample of regular shape and unknown thickness is placed inside an acrylic box of known internal dimensions. We have to be measured the linear attenuation coefficients of the sample. The empty space inside the box and around sample is filled with some medium of known attenuation coefficient. Gamma ray beam intensity is measured through that medium. This procedure is repeated for at least two media with known but different attenuation coefficients. Following mathematical expression is obtained for resultant transmitted beam intensity, when the sample under study was immersed in medium 1 which is air in present case:

$$I_1' = I_o (e^{-\mu x})(e^{-\mu_a a}) \qquad (3.2)$$

Where I_1' represents transmitted beam intensity by the assembly of sample, medium 1 and acrylic box, Io is incident beam intensity. μ And μ_a represent linear attenuation coefficient of sample and acrylic box respectively. While 'a' represents total thickness of acrylic box. Now expression of resultant beam intensity without sample becomes:

$$I_1 = I_o (e^{-\mu_a a}) \qquad (3.3)$$

I_1 is transmitted beam intensity of medium 1 and acrylic box similarly by immersing the sample in medium 2 we get:

$$I_2' = I_o (e^{-\mu 2(D-x)}) (e^{-\mu x}) (e^{-\mu aa}) \qquad (3.4)$$

Where I'_2 represents transmitted beam intensity of radiation by the assembly of sample, medium 1 and acrylic box, μ_2 is the linear attenuation coefficient of medium 2, 'D' is the internal dimension of acrylic box and other parameters have the same meaning as describes above. In the absence of sample expression (4.4) can be rewritten as:

$$I_2 = I_o (e^{-\mu 2D}) (e^{-\mu aa}) \qquad (3.5)$$

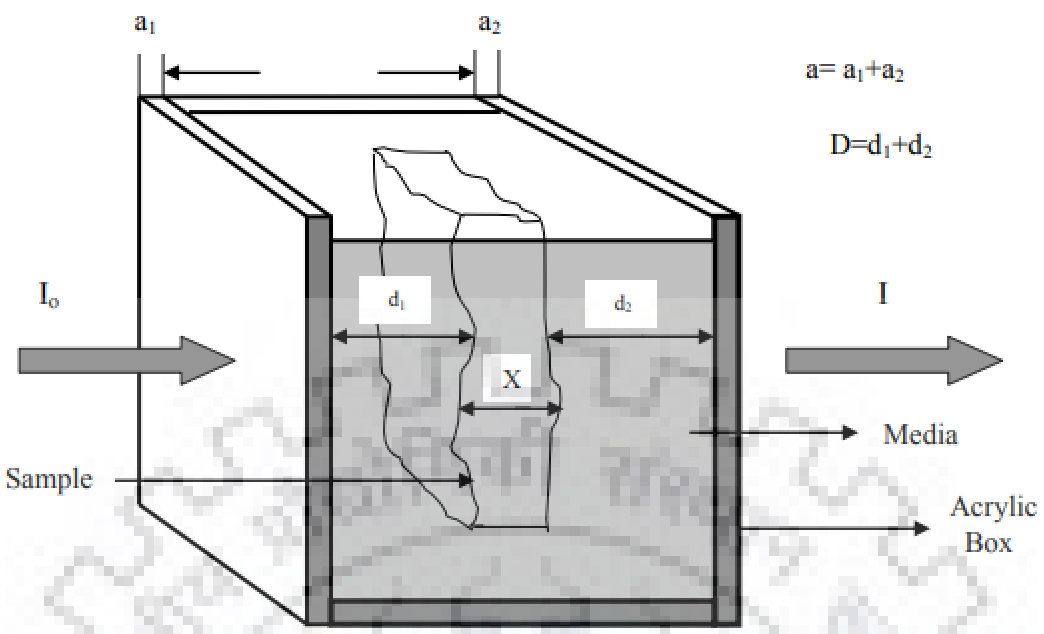


Figure 3 Two Media Method

D - Internal width of acrylic box

a - total width of walls of acrylic box

X - Unknown thickness of sample

where, I_2 is transmitted beam intensity of medium 2 and acrylic box Performing the proper substitutions of above equations, we get the following equation which determine linear attenuation coefficient of irregular shape sample of unknown thickness

$$\mu = \frac{\mu_2}{1 - \left(\frac{\ln(C_2)}{\ln(C_1)}\right)} \, (\mathrm{cm}^{-1}) \tag{3.6}$$

Where, $C_1 = I_1'/I_1$ and $C2 = I_2'/I_2$.

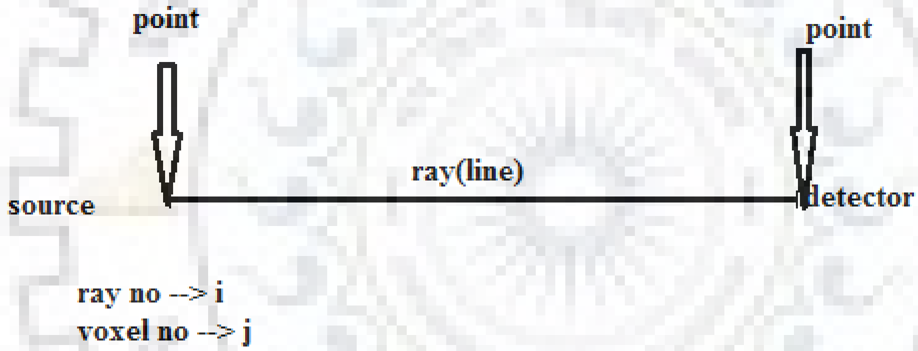
We get the equation that determines the linear attenuation coefficient of sample even without the knowledge of its thickness.

Chapter-4

Results and Discussions

4.1 Mathematical Analysis of Mart Algorithm

In my thesis I used a word voxel that means 3D Cartesian grid of volume element into the region that has to be reconstructed.



Length of interaction of i^{th} ray and j^{th} voxel denoted by T_{ij} for i=1, 2…N and j=1, 2…M represent the contribution of j^{th} voxel to the total attenuation along the i^{th} ray. The attenuation is assumed to be constant say μ_i throughout the j^{th} voxel. Total attenuation denoted by λ_i of i^{th} ray; represent the line integral of the attenuation function along the path of ray. In this discretized model, the line integral takes the form of finite sum and the model may be described by the system of linear equation as in matrix notation.

$$\lambda_i = \sum_{j=1}^{N} T_{ij}\mu_j \tag{4.1}$$

where, i=1,2,…………N

$$[T_{ij}]\,[\mu_i] = [\,\lambda_i] \tag{4.2}$$

The weight function T_{ij} is known for a particular geometry and the projection λ are obtained from experimental measurement.

Thus the problem of reconstruction is of inverting the matrix [T_{ij}] to find the unknown [μ_j].

The problem is now reducing to inversion of matrix.

The algorithm consists of two main parts.

In first part,

The value of single measurement for the current estimate is computed i.e.

$$\lambda_i = \sum_{j=1}^{N} T_{ij}\mu_j$$

This step is called forward projection.

The second step is back projection in which the difference between the value of actual measurement λ_i and value computed in the forward projection λ_i is used to update μ_j^{k-1} subject to the value of relaxation parameter. The major steps in the implementation of the mart family of algorithm are

Step 1. For each iteration k

Step 2 for each iteration i

$$\lambda_i = \sum_{j=1}^{N} T_{ij}\mu_j$$

Calculate the approximate projection λ_i .

Step 3 for each voxel j

Modify the value of the j^{th} voxel for all j=1,2,……..M as

$$\lambda_j^k = c*\mu_j^{k-1}$$

Initialization of λ^o is arbitrary

End step 3

End step2

Check for the convergence is

$$abs\left(\frac{\lambda k{+}1 - \lambda k}{\lambda k}\right) * 100 \leq \in \tag{4.3}$$

Here $\in$ is the suitable stopping criterion.

End step 1

4.2 Implementation of MART Algorithm

First of all I tried a 2*2 square box shaped sample. I assumed a point source which is at a distance. I know the size of the sample. I assumed it is in square form and assumed that it is uniform in nature and divided in four part having different attenuation coefficients.

I assumed I have three point detectors which are odd in number. One detector is in front of source and others are at same distance form it. I know the distance from source to object.

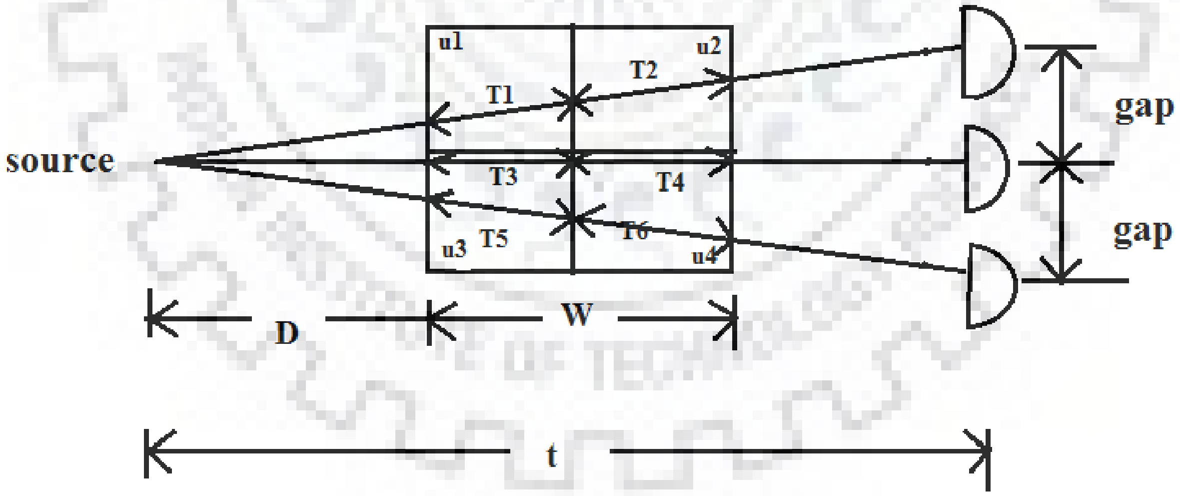


Figure 4 2*2 Pixel Sample

In the above fig (4) I have the size of object, distance between source and object and distance between source and detector. I assumed that source is at (0, 0) coordinate. Now my object has 4 pixels having same size of the entire pixel.

Then I find out the coordinates of all the pixels name as x_p (i,j) where i=1,2,3 and j=1,2,3. Then I found the coordinates of the point at which line intersect at the boundary of pixels of the object.

In the above figure (4)

We can find out the angle of line of detector with the line of source using some calculation.

$$\text{Slope}=m=\text{Tan (theta)} = \text{gap}/t \tag{4.4}$$

Using, $y=mx+c$

We can find out the coordinates of interaction of lines of detectors with the boundary of pixels. Once I found the coordinates of pixels and coordinates of interaction then I found the T1, T2, T3, T4 …T9 for every line of detector using the distance formula as

$$T = \text{sqrt}\ ((x1-x2).\hat{}2+(y1-y2).\hat{}2) \tag{4.5}$$

Then I check which line is going to which pixel using some conditions. Finally I found a matrix for T.

We got the equation like this –

$$T_{11}u_1+T_{12}u_2+T_{13}u_3+T_{14}u_4 = P_{D1}$$

$$T_{21}u_1+T_{22}u_2+T_{23}u_3+T_{24}u_4 = P_{D2}$$

$$T_{31}u_1+T_{32}u_2+T_{33}u_3+T_{34}u_4 = P_{D3} \tag{4.6}$$

Where $P_D = \log (I_0/I)$ for different detectors.

u1, u2, u3, u4 are the attenuation coefficients.

This can be written as:

$$[T][u] = [P] \tag{4.7}$$

My next task is little tricky. Now I want to rotate my object upto an angle with an interval so we have no of views. I found this using some calculations.

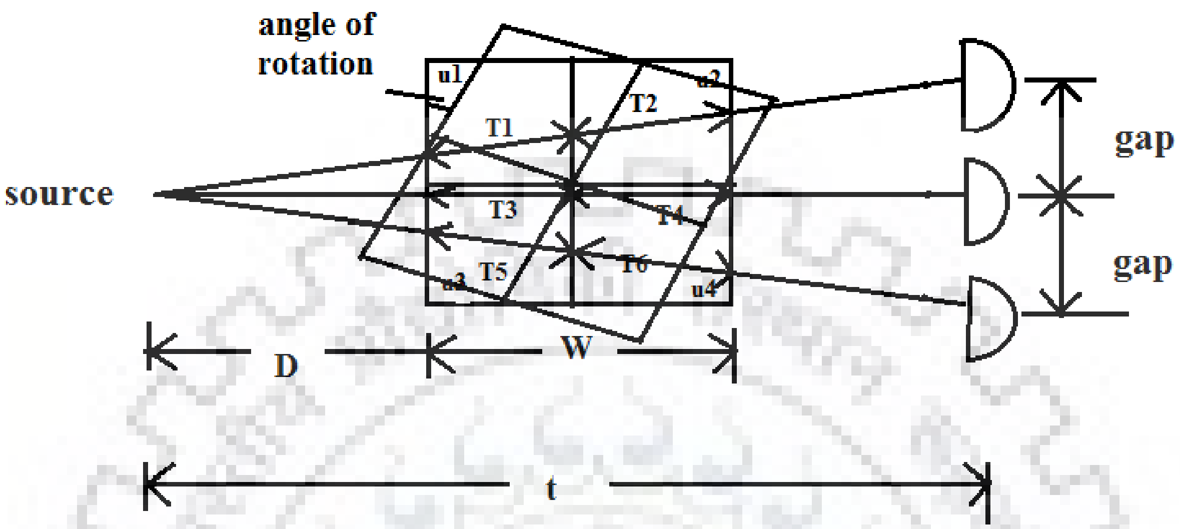


Figure 5 2*2 Sample with Rotation

In this fig (5) I rotate my object. Now my coordinates of pixel and coordinates of interaction are changed. So that T will be changed.

Finally I got the equations as-

For Θ_0

$$T_{11} u_1+T_{12}u_2+T_{13}u_3+T_{14}u_4 = P_{D1}$$

$$T_{21}u_1+T_{22}u_2+T_{23}u_3+T_{24}u_4 = P_{D2}$$

$$T_{31}u_1+T_{32}u_2+T_{33}u_3+T_{34}u_4 = P_{D3} \tag{4.8}$$

For Θ_1

$$T_{11}u_1+T_{12}u_2+T_{13}u_3+T_{14}u_4 = P_{D1}$$

$$T_{21}u_1+T_{22}u_2+T_{23}u_3+T_{24}u_4 = P_{D2}$$

$$T_{31}u_1+T_{32}u_2+T_{33}u_3+T_{34}u_4 = P_{D3} \tag{4.9}$$

Upto total no of views.

Generally I can write-

$$[T]_{m*n}[u]_{n*1} = [P]_{m*}1 \quad (4.10)$$

Where m= no of detectors * no of views

n=no of pixels

My next was to write code the general code for the varying no of detectors and varying no of pixels

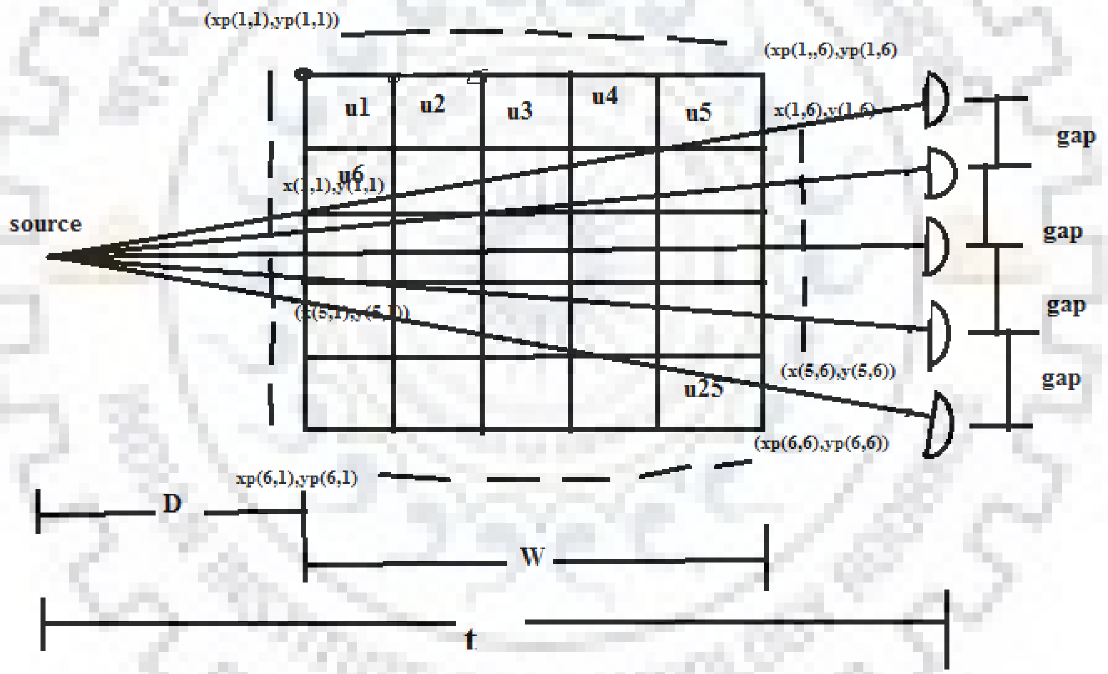


Figure 6 sample with 25 Pixel

Finally I got the equations as-

For Θ_0

$$T_{11}\,u_1+T_{12}u_2+T_{13}u_3+T_{14}u_4 = P_{D1}$$

$$T_{21}u_1+T_{22}u_2+T_{23}u_3+T_{24}u_4 = P_{D2}$$

$$T_{31}u_1+T_{32}u_2+T_{33}u_3+T_{34}u_4 = P_{D3}$$

For Θ_1

$$T_{11}u_1+T_{12}u_2+T_{13}u_3+T_{14}u_4 = P_{D1}$$

$$T_{21}u_1+T_{22}u_2+T_{23}u_3+T_{24}u_4 = P_{D2}$$

$$T_{31}u_1+T_{32}u_2+T_{33}u_3+T_{34}u_4 = P_{D3}$$

.

.

.

Upto total no of view.

Generally I can write-

$$[T]_{m*n}[u]_{n*1} = [P]_{m*1}$$

Where m= no of detectors * no of views

n=no of pixels

Where m can vary due to detector and no of view can vary

And n can vary

Test case 1

```
gap between the detectors,gap=1
enter the no of detector,p=9
enter the final angle of rotation of object=80
enter the interval in angle of rotation of object, interval=20
enter the number of pixel, n=9
enter the value of distance from source to object, D=1.5
enter the value of size of object w=12
enter the distance between source and detector=15

xn =

    1.5000    5.5000    9.5000   13.5000
    1.5000    5.5000    9.5000   13.5000
    1.5000    5.5000    9.5000   13.5000
    1.5000    5.5000    9.5000   13.5000
    1.5000    5.5000    9.5000   13.5000
    1.5000    5.5000    9.5000   13.5000
    1.5000    5.5000    9.5000   13.5000
    1.5000    5.5000    9.5000   13.5000
```

```
    1.5000    5.5000    9.5000   13.5000
    1.2727    4.6667    8.0606   11.4546
    1.3069    4.7921    8.2772   11.7624
    1.3411    4.9175    8.4939   12.0702
    1.3753    5.0429    8.7105   12.3780
    1.4095    5.1683    8.9271   12.6859
    1.4437    5.2937    9.1437   12.9937
    1.4779    5.4191    9.3603   13.3015
    1.5121    5.5445    9.5769   13.6093
    1.5463    5.6699    9.7935   13.9171
    0.8920    3.2705    5.6490    8.0276
    0.9562    3.5062    6.0561    8.6061
    1.0205    3.7419    6.4632    9.1846
    1.0848    3.9776    6.8703    9.7631
    1.1491    4.2132    7.2774   10.3416
    1.2133    4.4489    7.6845   10.9201
    1.2776    4.6846    8.0916   11.4986
    1.3419    4.9203    8.4987   12.0771
    1.4062    5.1560    8.9058   12.6556
    0.4036    1.4798    2.5561    3.6323
    0.4902    1.7974    3.1046    4.4117
    0.5768    2.1149    3.6530    5.1912
    0.6634    2.4325    4.2015    5.9706
    0.7500    2.7500    4.7500    6.7500
    0.8366    3.0675    5.2985    7.5294
    0.9232    3.3851    5.8470    8.3088
    1.0098    3.7026    6.3954    9.0883
    1.0964    4.0202    6.9439    9.8677
   -0.1335   -0.4893   -0.8452   -1.2011
   -0.0350   -0.1282   -0.2215   -0.3147
    0.0635    0.2329    0.4022    0.5716
    0.1620    0.5940    1.0259    1.4579
    0.2605    0.9551    1.6497    2.3443
    0.3590    1.3162    2.2734    3.2306
    0.4574    1.6773    2.8971    4.1169
    0.5559    2.0384    3.5208    5.0032
    0.6544    2.3994    4.1445    5.8896

yn =

    0.4000    1.4667    2.5333    3.6000
    0.3000    1.1000    1.9000    2.7000
    0.2000    0.7333    1.2667    1.8000
    0.1000    0.3667    0.6333    0.9000
    0.0000    0.0000    0.0000    0.0000
   -0.1000   -0.3667   -0.6333   -0.9000
   -0.2000   -0.7333   -1.2667   -1.8000
   -0.3000   -1.1000   -1.9000   -2.7000
   -0.4000   -1.4667   -2.5333   -3.6000
    0.3394    1.2444    2.1495    3.0546
    0.2614    0.9584    1.6554    2.3525
    0.1788    0.6557    1.1325    1.6094
    0.0917    0.3362    0.5807    0.8252
    0.0000    0.0000    0.0000    0.0000
   -0.0962   -0.3529   -0.6096   -0.8662
   -0.1971   -0.7225   -1.2480   -1.7735
   -0.3024   -1.1089   -1.9154   -2.7219
   -0.4124   -1.5120   -2.6116   -3.7112
```

```
    0.2379    0.8721    1.5064    2.1407
    0.1912    0.7012    1.2112    1.7212
    0.1361    0.4989    0.8618    1.2246
    0.0723    0.2652    0.4580    0.6509
    0.0000    0.0000    0.0000    0.0000
   -0.0809   -0.2966   -0.5123   -0.7280
   -0.1703   -0.6246   -1.0789   -1.5331
   -0.2684   -0.9841   -1.6997   -2.4154
   -0.3750   -1.3749   -2.3749   -3.3748
    0.1076    0.3946    0.6816    0.9686
    0.0980    0.3595    0.6209    0.8823
    0.0769    0.2820    0.4871    0.6922
    0.0442    0.1622    0.2801    0.3980
    0.0000    0.0000    0.0000    0.0000
   -0.0558   -0.2045   -0.3532   -0.5020
   -0.1231   -0.4513   -0.7796   -1.1078
   -0.2020   -0.7405   -1.2791   -1.8177
   -0.2924   -1.0720   -1.8517   -2.6314
   -0.0356   -0.1305   -0.2254   -0.3203
   -0.0070   -0.0256   -0.0443   -0.0629
    0.0085    0.0310    0.0536    0.0762
    0.0108    0.0396    0.0684    0.0972
    0.0000    0.0000    0.0000    0.0000
   -0.0239   -0.0877   -0.1516   -0.2154
   -0.0610   -0.2236   -0.3863   -0.5489
   -0.1112   -0.4077   -0.7042   -1.0006
   -0.1745   -0.6399   -1.1052   -1.5705

xpn =

    1.5000    5.5000    9.5000   13.5000
    1.5000    5.5000    9.5000   13.5000
    1.5000    5.5000    9.5000   13.5000
    1.5000    5.5000    9.5000   13.5000
   -0.6426    3.1162    6.8750   10.6337
    0.7255    4.4843    8.2430   12.0018
    2.0936    5.8523    9.6111   13.3699
    3.4617    7.2204   10.9792   14.7380
   -2.7077    0.3565    3.4207    6.4849
   -0.1365    2.9277    5.9918    9.0560
    2.4346    5.4988    8.5630   11.6272
    5.0058    8.0700   11.1341   14.1983
   -4.4462   -2.4462   -0.4462    1.5538
   -0.9821    1.0179    3.0179    5.0179
    2.4821    4.4821    6.4821    8.4821
    5.9462    7.9462    9.9462   11.9462
   -5.6484   -4.9538   -4.2592   -3.5646
   -1.7091   -1.0146   -0.3200    0.3746
    2.2301    2.9247    3.6193    4.3139
    6.1693    6.8639    7.5585    8.2531

ypn =

    6.0000    6.0000    6.0000    6.0000
    2.0000    2.0000    2.0000    2.0000
   -2.0000   -2.0000   -2.0000   -2.0000
   -6.0000   -6.0000   -6.0000   -6.0000
    6.1512    7.5193    8.8873   10.2554
```

2.3924	3.7605	5.1286	6.4967
-1.3664	0.0017	1.3698	2.7379
-5.1251	-3.7570	-2.3890	-1.0209
5.5604	8.1316	10.7027	13.2739
2.4963	5.0674	7.6386	10.2097
-0.5679	2.0032	4.5744	7.1455
-3.6321	-1.0609	1.5102	4.0814
4.2990	7.7631	11.2272	14.6913
2.2990	5.7631	9.2272	12.6913
0.2990	3.7631	7.2272	10.6913
-1.7010	1.7631	5.2272	8.6913
2.5191	6.4583	10.3976	14.3368
1.8245	5.7637	9.7030	13.6422
1.1299	5.0691	9.0084	12.9476
0.4353	4.3746	8.3138	12.2530

T=

0	1.0672	0	0	3.1526	0	5.9956	0	0
0	0.8944	0	0	3.6878	0	0	6.8469	0
0	0.9339	0	0	4.2668	0	0	7.7184	0
0	1.1643	0	0	4.8740	0	0	8.6041	0
0	1.5000	0	0	5.500	0	0	9.5000	0
0	1.8856	0	0	6.1391	0	0	10.403	0
0	2.2961	0	0	6.7877	0	0	11.3126	0
0	2.7203	0	0	7.4431	0	0	12.2262	0
0	3.1526	0	0	8.1038	0	0	0	13.1434
0	0.9055	0	0	2.6749	0	5.0872	0	0
0	0.7793	0	0	3.2132	0	0	5.965	0
0	0.8350	0	0	3.8149	0	0	6.9010	0
0	1.0675	0	0	4.4689	0	0	7.8891	0
0	1.4095	0	0	5.1683	0	0	8.9271	0
0	1.8149	0	0	5.9089	0	0	10.0132	0
0	2.2624	0	0	6.6878	0	0	11.1462	0
0	2.7423	0	0	7.5034	0	0	12.3252	0
0	3.2500	0	0	8.3542	0	0	0	13.5495
0	0.6346	0	0	1.8746	0	0	3.5652	0
0	0.5702	0	0	2.3509	0	0	4.3648	0
0	0.6354	0	0	2.9029	0	0	5.2512	0
0	0.8420	0	0	3.5248	0	0	6.2224	0
0	1.1491	0	0	4.2132	0	0	7.2774	0
0	1.5253	0	0	4.9659	0	0	8.4153	0
0	1.9557	0	0	5.7814	0	0	9.6355	0
0	2.4336	0	0	6.6586	0	0	10.9376	0
0	2.9554	0	0	7.5970	0	0	0	12.3213
0	0.2871	0	0	0.8482	0	0	1.6132	0
0	0.2923	0	0	1.2052	0	0	2.2375	0
0	0.3591	0	0	1.6407	0	0	2.9680	0
0	0.5149	0	0	2.1556	0	0	3.8053	0
0	0.7500	0	0	2.7500	0	0	4.7500	0
0	1.0517	0	0	3.4240	0	0	5.8024	0
0	1.4132	0	0	4.1776	0	0	6.9626	0
0	1.8313	0	0	5.0107	0	0	8.2307	0
0	2.3044	0	0	5.9234	0	0	9.6070	0
0	0.0949	0	0	0.2805	0	0	0.5334	0
0	0.0209	0	0	0.0860	0	0	0.1596	0
0	0.0395	0	0	0.1807	0	0	0.3268	0
0	0.1257	0	0	0.5264	0	0	0.9292	0
0	0.2605	0	0	0.9551	0	0	1.6497	0

0	0.4512	0	0	1.4691	0	0	2.4896	0
0	0.7002	0	0	2.0699	0	0	3.4498	0
0	1.0082	0	0	2.7585	0	0	4.5311	0
0	1.3754	0	0	3.5354	0	0	5.7340	0

Verification of Code

To verify the code I have a simple technique in which I set the position of source and detectors. Then I started to rotate the object with interval of 45° upto 360°. So I have 9 views at angles 0,45,90,135,180,225,270,315,360. We will have to get the images of the object like this. The images should be symmetric. And the matrix T should be same T angle 0&180 degree and 45&225 degree and 90 &270 degree and 135&315 degree.

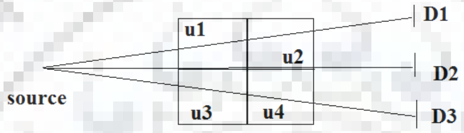


Figure 7 View on 0 degree

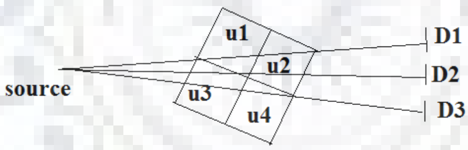


Figure 8 View on 45 degree

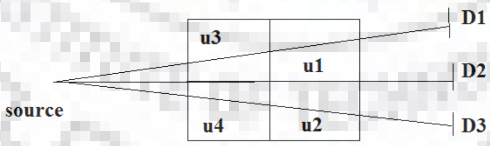


Figure 9 View on 90 degree

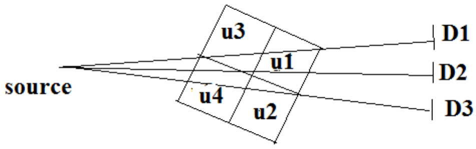


Figure 10 View on 135 degree

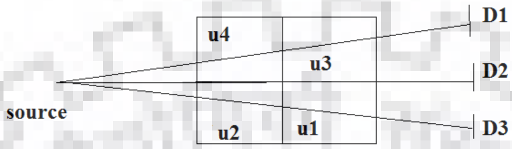


Figure 11 View on 180 degree

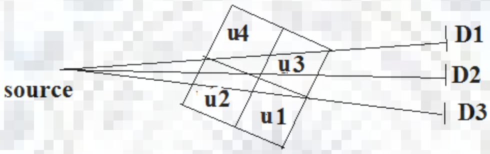


Figure 12 View on 225 degree

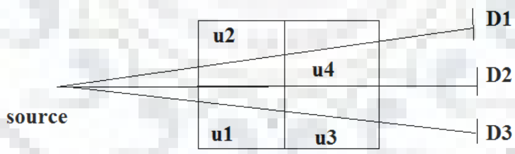


Figure 13 View on 270 degree

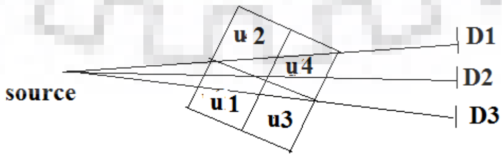


Figure 14 View on 315 degree

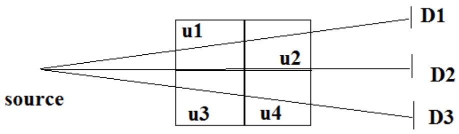


Figure 15 View on 360 degree

```
gap between the detectors,gap=1
enter the no of detector,p=3
enter the final angle of rotation of object=360
enter the interval in angle of rotation of object, interval=45
enter the number of pixel, n=4
enter the value of distance from source to object, D=1.5
enter the value of size of object w=12
enter the distance between source and detector=15
T =

    1.0770         0    6.6121         0
    1.5000    1.5000    7.5000    7.5000
         0    2.0396         0    8.4095
    0.7108         0    4.3638         0
    1.0607    1.0607    5.3033    5.3033
         0    1.5384         0    6.3429
         0    0.0718         0    0.4408
         0         0         0         0
         0    0.1360         0    0.5606
         0    0.8123         0    4.9872
    1.0607    1.0607    5.3033    5.3033
    1.3461         0    5.5500         0
         0    1.0770         0    6.6121
    1.5000    1.5000    7.5000    7.5000
    2.0396         0    8.4095         0
         0    0.7108         0    4.3638
    1.0607    1.0607    5.3033    5.3033
    1.5384         0    6.3429         0
    0.0718         0    0.4408         0
         0         0         0         0
    0.1360         0    0.5606         0
    0.8123         0    4.9872         0
    1.0607    1.0607    5.3033    5.3033
         0    1.3461         0    5.5500
    1.0770         0    6.6121         0
    1.5000    1.5000    7.5000    7.5000
         0    2.0396         0    8.4095

>> k=mat2gray(T)

k =

    0.1281         0    0.7863         0
    0.1784    0.1784    0.8918    0.8918
         0    0.2425         0    1.0000
    0.0845         0    0.5189         0
    0.1261    0.1261    0.6306    0.6306
```

```
         0    0.1829         0    0.7542
         0    0.0085         0    0.0524
         0         0         0         0
         0    0.0162         0    0.0667
         0    0.0966         0    0.5930
    0.1261    0.1261    0.6306    0.6306
    0.1601         0    0.6600         0
         0    0.1281         0    0.7863
    0.1784    0.1784    0.8918    0.8918
    0.2425         0    1.0000         0
         0    0.0845         0    0.5189
    0.1261    0.1261    0.6306    0.6306
    0.1829         0    0.7542         0
    0.0085         0    0.0524         0
         0         0         0         0
    0.0162         0    0.0667         0
    0.0966         0    0.5930         0
    0.1261    0.1261    0.6306    0.6306
         0    0.1601         0    0.6600
    0.1281         0    0.7863         0
    0.1784    0.1784    0.8918    0.8918
         0    0.2425         0    1.0000

>> imshow(k)
```

I got the symmetry in the matrix of intersection distance of rays (line) in each pixel.

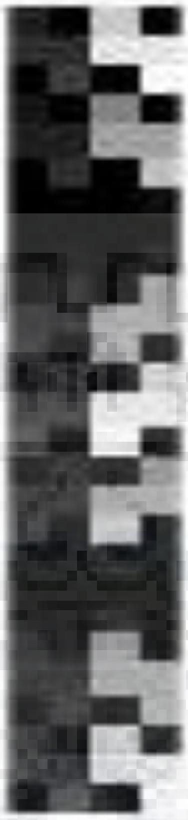


Figure 16 image of matrix T

According to fig 7 to fig 15 I analyzed that the matrix should be symmetric. Interaction distance of rays (line) should be same for angle 0 & 180 & 360 and 45& 225 and 90 &270 and etc. I got the image of matrix T.

4.3 Non-Linear Wave Propagation Including Scattering and Reflection Theory

In the previous section I modelled the problem of linear wave propagation excluding scattering and reflection. I have written the code to find out the intersection distance in each pixel for different wave (line) depends on the no of detectors

PROBLEM

Now my problem is to model the problem of non-linear wave propagation including scattering and reflection theory and write the code for this problem.

In my previous code I assume that line has same angle throughout the sample. There is no scattering, refraction and reflection. But in this case I have different angle of refraction at each interface of the pixel for different waves (line) depend on the no of detectors. I used Snell's law for my problem to find out the angle of refraction at each interface.

Approach 1

First of all I tried a 2*2 square box shaped sample. I assumed a point source which is at a distance. I know the size of the sample. I assumed it is in square form and assumed that it is uniform in nature and divided in four part having different refractive index.

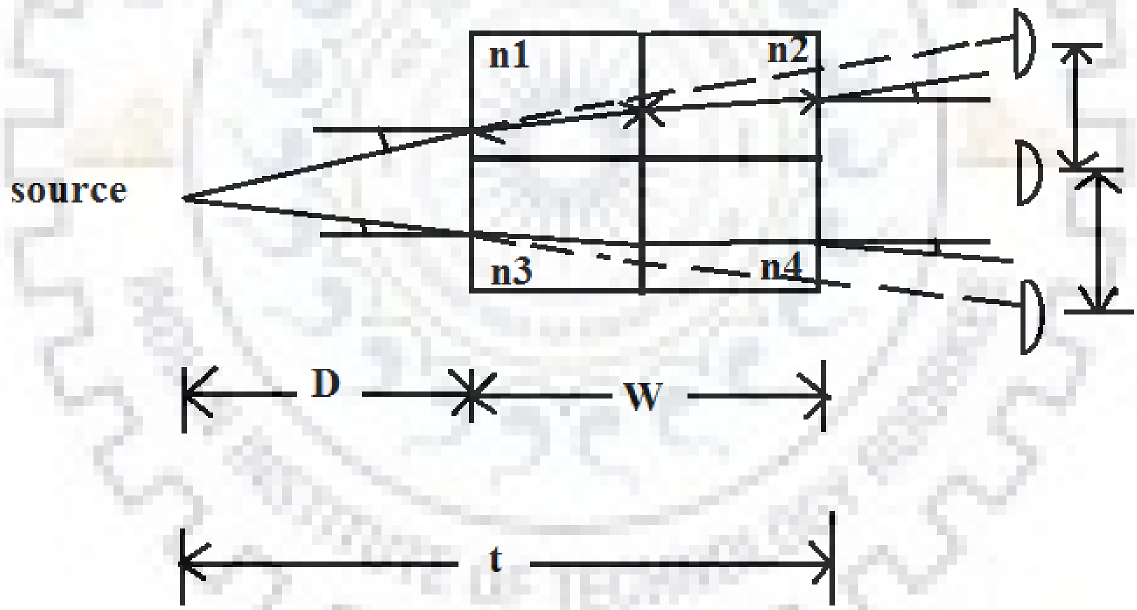


Figure 17 Sample with 4 Pixels with Refraction

In the above figure a radiation emit from a source which is at distance D. I assumed there is three detectors. For a single detector a radiation line intersect at angle $i_1$with the normal and refracted at angle r_1 due to different medium. Again it intersect at angle i_2 (r_1) with the normal and refracted at angle r_2. Again it intersect at angle i_3 (r_2) with the normal and it refracted with angle r_3.

Using Snell law

$$\frac{\sin i}{\sin r} = \frac{n2}{n1} \tag{4.11}$$

We find that incident angle at object and final angle from object is same. Now our task is to find out the angle of intersection at each interface of pixels.

To find out the interaction points we have to know the slope at every point of intersection

We know that

$$\frac{\sin i1}{\sin r1} = n_1 \tag{4.12}$$

We can say that $m=\tan(r_1)$

$=\tan(\sin^{-1}(1/n_1(\sin i_1)))$

And

$$\frac{\sin i2}{\sin r2} = n_2/n_1$$

We know $i_2=r_1$

We can say that $m=\tan(r_2)$

$=\tan(\sin^{-1}(1/n_2(\sin i_1)))$

And

$$\frac{\sin i3}{\sin r3} = 1/n_2$$

We know $i_3=r_2$

We can say that $m=\tan(r_3)$

$=\tan(\sin^{-1}(\sin i_1))$

Generally we can write

$$\mathbf{m = \tan(\sin^{-1}(1/\text{refractive index of pixel}\ (\sin i_1)))} \tag{4.13}$$

Where, i_1 is the angle of incident at the interface of sample.

Finally I got the generalized slope for each wave (line) at each of the interface of the pixel. Using this we can find out the coordinate of the intersection point at each interface of pixel .using equation Y =m. X

Failure

In the above equation (4.13) refractive index is unknown for me to solve the problem

Then I tried to find out how Snell's law behaves in the medium with absorption

Law of Refraction with Absorption

I want to find out the formula for a law of refraction in an absorbing medium.

For different component the Snell law is different i.e. angle of refraction is different.

For s-component i.e. perpendicular to the plane of incident

The Snell law is

$$\frac{sini1}{sinr1} = \frac{n2}{n1}$$

For p-component i.e. parallel to the plane of incident there is some effect in the angle of refraction

The Snell law is

$$\frac{sini1}{sinr1} = \frac{n2(1 - \frac{k2^2}{n2^2})}{n1}$$

Where, k_2 is the absorption coefficient for the second medium. Here we assume first medium is transparent.

Yet I assumed one medium is transparent of pixel, if both the pixels have absorbing medium.

$$\frac{sini1}{sinr1} = \frac{n2(1 - \frac{k2^2}{n2^2})}{n1(1 - \frac{k1^2}{n1^2})}$$

In the above equation we assume k lies below n.

When radiation passes through a medium some part of it will always attenuated.

So we can say that by defining a complex refractive index

$$\underline{n}=n+ik$$

Here real part n is the refractive index

And imaginary part k is the attenuation coefficient.

In another word n indicate the phase velocity.

k indicate the amount of attenuation when the electromagnetic wave propagate through a material.

If k >0 means light is absorbed

If k=0 means light travels forever without loss

If k<0 means amplification of light

When I substitute it in the Snell law

$$\frac{sin i1}{sin r1} = \frac{n2 + ik2}{n1}$$

We get by solving it

$$\sin(\mathrm{r1}) = \frac{\sqrt{\sin(i1)}}{\sqrt{(n2^2 - k2^2 + \sin^2 i1 + \sqrt{(n2^2 - k2^2 - sin i1^2)^2 + 4n2^2k2^2})}} \quad (4.14)$$

if I found n we can find the optical path length which is equal to the product of the geomtric length d of the path light follow through a system and the refractive index of thye medium through with it propagate

$$OPL= nd$$

Suggestion

According to my point of view I tried so much to find out the equation like it

$$m= f(x, y\ldots) \quad (4.15)$$

Where, variables of rhs are known. In my equation (4.13) one variable is unknown .so try to remove that variable from my equation using updated version of Snell's law for absorbing medium

Linear	Non-Linear
• I assumed there is no reflection or refraction	• I assumed there is scattering ,reflection and refraction
• In this assumption I found slope simply as tan ϴ that you can find out using ratio of gap between detectors and distance from source to detector	• In this assumption you can use eq (4.13) to find out the slope for each interface of pixels. You can also use (4.14)
• When I got m, I simply simplified the coordinate of interaction points.	• If you got m in a manner like eq(4.15) then you can put it in my code and you can find out all the intersection point and finally you will got the matrix of T using rotation of sample.

I got the expression of m in eq (4.13). If you got a correct form of slope at each interface of pixels then you can use it in my code to find out the coordinate of intersection.

Eq (4.14) can help you to find out the slope having known variables at rhs.

Approach 2

Mathematical Modeling in 2-D

We take a source placed at the origin of coordinate system. The detectors are placed at a distance of D_2 from the source, which are placed with a distance of D_4 from each other starting from the origin. D_1 is the length from the source to the midpoint of the object and D3 is the length of each square pixel. For the purpose of simplicity the object is square shaped placed uniformly with the center aligned with the origin. We consider the object to be divided into 4 parts of equal size; hence four square shaped pixels (refer the images).

Direct Ray without Refraction and Reflection

First we consider a simple case that the ray neither deviates nor gets attenuated. We try to trace the path of the un-deviated light ray through object

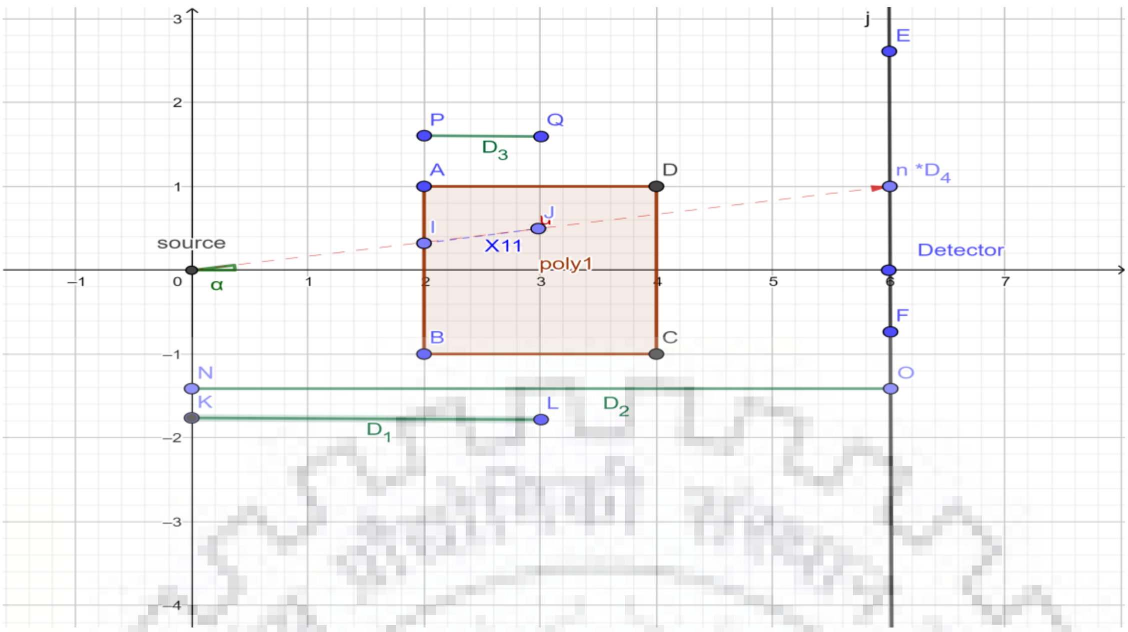


Figure 18 Basic View of the Problem in 2D

By using 2 point form of equation of a line

$$y - y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1) \tag{4.16}$$

With reference to the electromagnetic wave (line) in fig 18 starting from the source to the detector at the n^{th} position. We compute the path length of the ray through individual pixels. The segment of ray in a pixel m with the ray reaching to n^{th} is X_{nm}.

The equation of the X_{nm} is:

$$X_{nm} = \frac{D_3}{D_2}\sqrt[2]{(D_2)^2 + (nD_4)^2} \tag{4.17}$$

Similarly taking a 3*3 matrix we get:

$$X_{nm} = \frac{D_3}{D_2}\sqrt[2]{(D_2)^2 + (nD_4)^2} \tag{4.18}$$

Considering the Refraction Occurring

We now consider that as the light rays pass through the object it suffers change in path due to refraction, also some of the light gets absorbed or scattered by the media. The residual transmitted light is shifted and it reaches a different detector.

We try to analyse and reconstruct the path traced by the light ray.

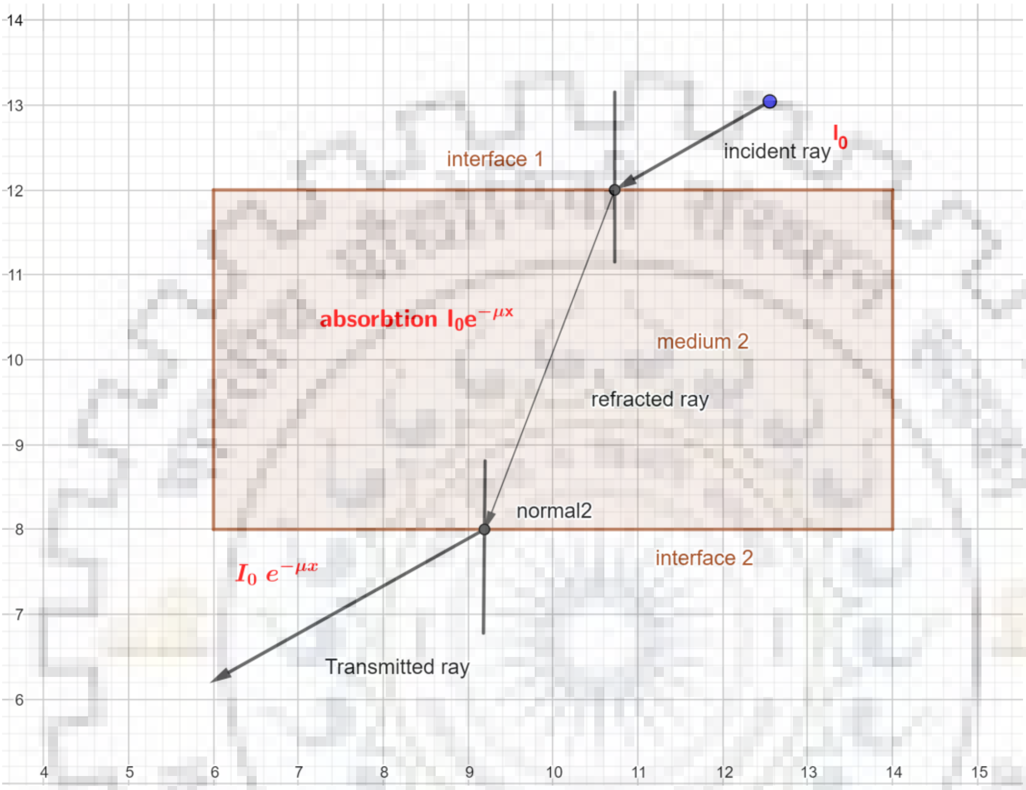


Figure 19 Refraction and Attenuation

It will well know that the light suffers refraction at the interface of the two media. Hence we try to take up the problem by adding boundaries step by step.

First we consider the upper half of the object to have same material, leading to refraction through the material. We know which detector it reaches after refraction (n_f), therefore we treat the media in upper half to have a refractive index of μ_{avg} and the path travelled accounts to X_{avg} .

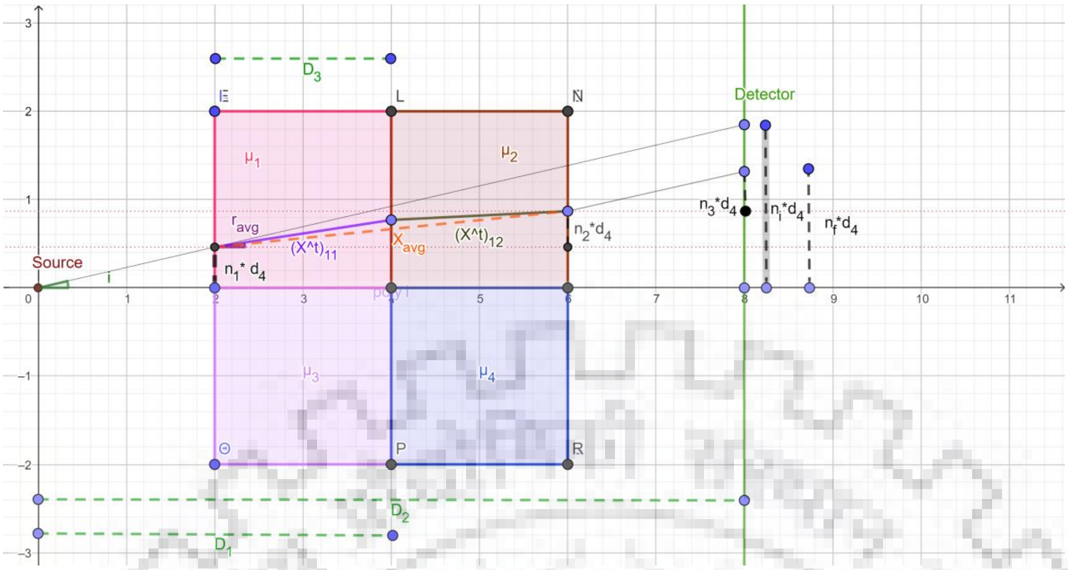


Figure 20 View of the Problem in 2D with Refraction

With reference to Figure 20, n_i is the detector position for the non-refracted ray and n_1 is the distance from the axis passing horizontally through the centre of object to the point of entrance of the light ray into the object. Whereas n_2 refers to the vertical distance travelled by the ray inside the object, n_3 is the vertical shift the ray suffers as it exits the object to reach the detector and the following relation holds:

$$n_2 d_4 = (n_f - n_1 - n_3)\, d_4 \tag{4.19}$$

However we know that the rays outside the object are the side of the triangles that are similar to the case where we do not consider refraction.

$$n_1 = \frac{n_i}{d_2}\left(d_1 - \frac{d_3}{2}\right)$$
$$n_3 = \frac{n_i}{d_2}\left(d_2 - \frac{d_3}{2} - d_1\right) \tag{4.20}$$

Now we substitute eqs (4.20) in eq (4.19) to get the value of n_2.

X_{avg} is the hypotenuse of the right-angle triangle inside the object giving us:

$$Xavg = \sqrt{}(d4n2)^2 + (2d3)^2 \tag{4.21}$$

As we know that by Snell's law

$$\mu = \frac{sini}{sinr}$$

$$\mu_{avg} = \frac{n_i}{\sqrt[2]{(d_2)^2 + (n_i)^2}} \frac{X_{avg}}{n_2} \tag{4.22}$$

Further with reference from Optical path length (OPL) we get:

$$OPL = \mu * (geometrical\ distance)$$

$$\mu_{\mathrm{m}} * X_{\mathrm{nm}}{}^{\mathrm{t}} = X_{nm}$$

Now considering the boundary dividing the two pixels we take the 1st pixel to have refractive index - μ_1 and the actual path the refracted ray takes is the $X^t{}_{11}$.

$$OPL_{\mathrm{average}} = OPL_1 + OPL_2$$

$$\mu_{\mathrm{avg}} * X_{\mathrm{avg}} = \mu_1 X_{11}{}^{\mathrm{t}} + \mu_2 X_{12}{}^{\mathrm{t}}$$

Until now we have avoided the case of attenuation, when we consider attenuation the Snell's law takes the form:

$$\frac{sin\alpha}{sin\beta} = \frac{\mu_1}{\mu_2}(1 - [\frac{k_2}{\mu_2}]^2) \tag{4.23}$$

Here, k_2 is the attenuation coefficient of the second medium.

Conclusion

My project work during master of technology is focused on Gamma Radiation Detection and Inverse Problem Modeling. Firstly I analysed that how detectors works and what are the operating range so that it can give reliable and accurate readings. So in this direction I tried to implement the mart code for linear wave propagation inside a heterogeneous medium without considering scattering phenomenon. I written the code in Matlab to find out the Matrix for the distance travelled by the rays in pixels. This is the big hurdle to find out this matrix with different view of object. I analyzed the wave propagation inside a heterogeneous medium with considering scattering and refraction. It is found that wave propagation length under a discretized area cannot be made independent with medium dependent parameters such as refractive index, scattering cross-section or attenuation constant thus desired expression could not be achieved.

Future scope

There is a future scope is that anyone will try to derive for the non-linear wave propagation including scattering and reflection theory and also try to find out the matrix and code it in Matlab.

Chapter-6

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Appendix A

Code for Mart Algorithm

```
%% mart code
%detector must be odd in number
%middle detector is fixed in front of source
close all;
clear all;
clc;
gap=input('gap between the detectors,gap=');
p=input('enter the no of detector,p=');
thetaf=input('enter the final angle of rotation of object=');
interval=input('enter the interval in angle of rotation of object,interval=');
n=input('enter the number of pixel,n=');
D=input('enter the value of distance from source to object, D=');
w=input('enter the value of size of object w=');
t=input('enter the distance between source and detector=');

z=(thetaf-0)./interval;
x=0;
theta(1)=0;
for i=2:(z+1)*p
   if rem(i-1,p)==0
      x=x+interval;
   end
      theta(i)=x;

end
gap1=((p-1)/2).*gap;
m(1)=(gap1./t);
for i=2:p
   m(i)=m(i-1)-(gap/t);
end
for i=p+1:(z+1)*p
t=rem(i,p);
if t==0
   m(i)=m(p);
else
   m(i)=m(t);
```

```
end
end
for k=1:p*(z+1)
   x(k,1)=D;
end
for i=1:p*(z+1)
   for j=2:sqrt(n)+1
      x(i,j)=x(i,j-1)+(w/sqrt(n));
   end
end

for i=1:p*(z+1)
   for j=1:sqrt(n)+1
      y(i,j)=m(i).*x(i,j);
   end
end

for i=1:p*(z+1)
   for j=1:sqrt(n)+1
      xn(i,j)=x(i,j).*cosd(theta(i))-y(i,j).*sind(theta(i));
      yn(i,j)=m(i).*xn(i,j);
   end
end
xn
yn
for i=1:(sqrt(n)+1)
   for j=1:(sqrt(n)+1)
      xp(i,j)=D+(j-1)*(w/sqrt(n));
      yp(i,j)=(w/2)-(i-1)*(w/sqrt(n));
   end
end
xp
yp

for k=1:(z+1)
   for i=1:sqrt(n)+1
      for j=1:sqrt(n)+1
      xpn(i+((k-1)*(sqrt(n)+1)),j)=xp(i,j).*cosd((k-1)*interval)-yp(i,j).*sind((k-1)*interval);
      ypn(i+((k-1)*(sqrt(n)+1)),j)=yp(i,j).*cosd((k-1)*interval)+xp(i,j).*sind((k-1)*interval);

   end
end
end
xpn
```

```
ypn
for k=1:p*(z+1)
   q=1;
for i=1:(sqrt(n))
   for j=1:(sqrt(n))
      if (ypn(j+1,i)<=yn(k,i)) && (yn(k,i)<=ypn(j,i))
         T(k,q)=sqrt((yn(k,i)-yn(k,i+1)).^2+(xn(k,i)-yn(k,i+1)).^2);
      else T(k,q)=0;
      end
      q=q+1;
      end
end
end
T
```