

# **FLEXURAL BEHAVIOUR OF CONTINUOUS STEEL CONCRETE COMPOSITE BRIDGE GIRDERS**

**M.Tech THESIS**

*by*

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ROORKEE - 247 667 (INDIA)  
MAY 2019**

# **FLEXURAL BEHAVIOUR OF CONTINUOUS STEEL CONCRETE COMPOSITE BRIDGE GIRDERS**

**A THESIS**

*Submitted in partial fulfilment of the  
requirements for the award of the degree  
of*

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*in*

**Structural Engineering**

*by*

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MAY, 2019**



**INDIAN INSTITUTE OF TECHNOLOGY ROORKEE  
CANDIDATE'S DECLARATION**

I hereby certify that the work which is being presented in this dissertation report entitled **“FLEXURAL BEHAVIOUR OF CONTINUOUS STEEL CONCRETE COMPOSITE BRIDGE GIRDERS”** in the partial fulfilment of the requirement for the award of degree of Master of Technology has been by carried out by me in the Department of Civil Engineering, Indian Institute of Technology Roorkee, under the supervision of Professor **Dr.Akhil Upadhyay**.

The matter presented in this report has not been submitted by me for the award of any other degree in this or other institute.

Place: Roorkee

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**CERTIFICATE**

This is to certify that the above statement made by the candidate is true to the best of my knowledge.

Date: May 2019

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## **Abstract**

Steel concrete composite bridges are becoming very popular as they are efficient and economical and provide a section which is lower in weight as compared to concrete bridge girders of similar span length and subjected to similar loading. The construction time of the continuous composite bridge girders is far less than concrete bridge girders. The continuous composite girder has several advantages over the simply supported girders such as having lower span deflections for similar cross-sectional properties and for similar load conditions. The current practice of neglecting the concrete in the negative bending zone of the girder has been studied and an alternate model has been proposed which takes the tension stiffening of cracked concrete into account. Different spans and interaction ratios were studied, and comparisons were drawn. A lower mid-span deflection and lower moment in steel girders in the negative bending zone was observed in the tension stiffened Steel Concrete Composite girder as compared to the girders which were analysed without considering concrete in the negative bending zone.

A 2-D model was created to simulate and study the behaviour of the steel concrete composite girders. The model was validated against theoretical models which are in practice. The validation was done for a Steel Concrete Composite with rigid connection and for partial interaction with inter layer slip. The model was also validated against the current practice of neglecting concrete in the negative zone of the composite girder. The results so obtained were in accordance with the already present numerical models. The effective moment of inertia was also used to understand the effects of using the residual strength of the concrete after cracking.

To simulate the behaviour of the Steel Concrete Composite girder under negative bending, a cracked model which incorporated the effective moment of inertia of the concrete was developed. The percentage decrease in the span deflections were reported showing a considerable amount of decrease in deflections as compared to sections with complete cracked properties. The moment decrease in the steel girder at the negative moment zone was also reported.

# CONTENTS

## Abstract

<b>Chapter 1. Introduction.....</b>	<b>1</b>
1.1 Shear connectors .....	3
1.2 Interface behaviour .....	4
1.3 Slip.....	6
1.4 Shear connection .....	7
1.6 General behaviour of composite beams .....	8
1.8 Buckling behaviour .....	11
1.9 Concept of composite girders extended to bridges .....	13
1.10 Effect of shear slip on the deflection of composite girder.....	15
1.11 Effect of cracking of concrete on the overall deflection of the girder.....	16
1.12 Gap in the current studies.....	17
1.14 Layout of the thesis.....	18
<b>Chapter 2. Literature Review.....</b>	<b>21</b>
<b>Chapter 3. Methodology.....</b>	<b>31</b>
3.1 Introduction.....	31
3.2 Model formulation.....	31
3.3 Cases considered in the present study.....	33
<b>Chapter 4. Continuous Composite Girder Without Cracked Concrete.....</b>	<b>34</b>
4.1 Introduction.....	34
4.2 Model verification .....	34
4.3 Extension of the numerical method to continuous girders using linear partial interaction theory.....	38
4.3.1 Analytical modelling as per N.A Jasim (1997) .....	39
4.4 Numerical modelling of continuous composite girders.....	41
4.4.1 Validation 1 .....	41
4.4.2 Validation 2.....	43
4.4.3 Parametric studies.....	47
4.4.4 Conclusions .....	52
<b>Chapter 5. Continuous Composite Girders Considering Fully Cracked Concrete... 53</b>	<b>53</b>
5.1 Introduction.....	53
5.2 Direct stiffness method.....	54
5.3 Proposed numerical model.....	55
5.4 Validation.....	55
5.4.1 Results of the validations.....	58
5.4.2 Conclusions .....	63

<b>Chapter 6. Continuous Composite Girders with Effective Moment of Inertia of Concrete.....</b>	<b>65</b>
6.1 Introduction.....	65
6.2 Proposed model in this study .....	68
6.3 Validation.....	70
6.4 Parametric studies.....	73
6.5 Results and comparisons.....	73
6.5.2 Conclusions .....	94
<b>Chapter 7. Modelling of Continuous Composite Girder Bridge Using Grillage Analogy.....</b>	<b>96</b>
7.1 Introduction.....	96
7.2 3-D grillage model .....	97
7.3 Results and comparison with model proposed in previous chapter.....	98
7.4 Conclusions .....	100
<b>Chapter 8. Discussions on Results and Concluding Remarks .....</b>	<b>101</b>
8.1 Results and discussions .....	101
8.2 Scope for the future studies .....	104
<b>References .....</b>	<b>105</b>

## **FIGURES**

<i>Figure 1: General cross section of the composite beam (Oehlers D.J., Bradford M.A., 1999)</i> .....	1
<i>Figure 2: Composite column sections (Oehlers D.J., Bradford M.A., 1999)</i> .....	2
<i>Figure 3: Composite profiled slabs (Oehlers D.J., Bradford M.A., 1999)</i> .....	2
<i>Figure 4: Mechanical Shear Connectors (David Collings, 2005)</i> .....	3
<i>Figure 5: Dowel action of a stud shear connector (Oehlers D.J., Bradford M.A., 1999)</i> .....	4
<i>Figure 6: Interface shear behaviour (Oehlers D.J., Bradford M.A., 1999)</i> .....	5
<i>Figure 7: Transfer of connector force (Oehlers D.J., Bradford M.A., 1999)</i> .....	5
<i>Figure 8: Fig Slip and Strain (Oehlers D.J., Bradford M.A., 1999)</i> .....	7
<i>Figure 9: Deformation of composite beam (Oehlers D.J., Bradford M.A., 1999)</i> .....	8
<i>Figure 10: Degree of shear connection</i> .....	10
<i>Figure 11: Lateral - torsional buckling (Oehlers D.J., Bradford M.A., 1999)</i> .....	12
<i>Figure 12: Lateral - distortional buckle Oehlers D.J., Bradford M.A., 1999)</i> .....	12
<i>Figure 13: Local buckling (Oehlers D.J., Bradford M.A., 1999)</i> .....	12
<i>Figure 14: Span ranges for various bridge types (David Collings, 2005)</i> .....	13
<i>Figure 15: Proposed model analytical layout to simulate the structural behaviour of the composite beam</i> .....	32
<i>Figure 16: Cross-section of the composite girder</i> .....	34
<i>Figure 17: Line diagram of the proposed model (SAP2000)</i> .....	35
<i>Figure 18: Extruded 3-D model (SAP2000)</i> .....	35
<i>Figure 19: Comparison of deflections obtained from Bradford theory and that obtained from model for 0.8 interaction</i> .....	36
<i>Figure 20: Comparison of deflections obtained from Bradford theory and that obtained from model for Full connection</i> .....	36
<i>Figure 21: Load v/s Slip for 10m girder with 0.8 interaction</i> .....	37
<i>Figure 22: Load v/s Slip for 15m girder with 0.8 interaction</i> .....	37
<i>Figure 23: Load v/s Slip for 30m girder with 0.8 interaction</i> .....	38
<i>Figure 24: Internal span of continuous beam (Jasim et al. 1998)</i> .....	39
<i>Figure 25: Slip v/s Span for a 3-span continuous beam subjected to point load</i> .....	42
<i>Figure 26: Deflection v/s Span for a 3-span continuous beam subjected to point load</i> .....	42
<i>Figure 27: Deflection comparison</i> .....	43
<i>Figure 28: Deflection v/s Span (Rigid)</i> .....	44
<i>Figure 29: Deflection comparison (Rigid)</i> .....	44



<i>Figure 30:Slip v/s Span (Flexible shear connection)</i> .....	45
<i>Figure 31:Deflection v/s Span (Flexible shear connection)</i> .....	45
<i>Figure 32:Deflection comparison (Flexible shear connection)</i> .....	46
<i>Figure 33:Percentage error between the numerical model and theory proposed by N.A Jasim</i> .....	46
<i>Figure 34:Deflection v/s Span (7.5m 2-span)</i> .....	47
<i>Figure 35:Deflection v/s Span (7.5m 3-span)</i> .....	48
<i>Figure 36:Deflection v/s Span (7.5m 5-span)</i> .....	48
<i>Figure 37:Deflection v/s Span (15m 2-span)</i> .....	49
<i>Figure 38:Deflection v/s Span (15m 3-span)</i> .....	49
<i>Figure 39:Deflection v/s Span (15m 5-span)</i> .....	50
<i>Figure 40:Deflection v/s Span (30m 2-span)</i> .....	50
<i>Figure 41:Deflection v/s Span (30m 3-span)</i> .....	51
<i>Figure 42:Deflection v/s Span (30m 5-span)</i> .....	51
<i>Figure 43:General arrangement for a 3-span continuous beam subjected to a 10kN/m UDL</i> .....	56
<i>Figure 44:Deflection v/s Span for a 3-span continuous beam subjected to UDL</i> .....	57
<i>Figure 45:Slip-Span for a 3-span continuous beam subjected to UDL</i> .....	57
<i>Figure 46:Moment-Span for a 3-span continuous beam subjected to UDL</i> .....	58
<i>Figure 47:Moment comparison at supports numerical model</i> .....	58
<i>Figure 48:Moment comparison at supports DSM</i> .....	59
<i>Figure 49:Percentage error between moment values for cracked section between DSM and numerical model</i> .....	59
<i>Figure 50:Moment comparison at internal spans numerical model</i> .....	60
<i>Figure 51:Moment comparison at internal spans DSM</i> .....	60
<i>Figure 52:Percentage error between moment values for cracked section between DSM and numerical model</i> .....	61
<i>Figure 53:Deflection comparison at internal spans numerical model</i> .....	62
<i>Figure 54:Deflection comparison at internal spans DSM</i> .....	62
<i>Figure 55:Percentage error between deflection values for cracked section between DSM and numerical model</i> .....	63
<i>Figure 56:Process flow diagram for the proposed model</i> .....	69
<i>Figure 57:Layout of the test setup (recreated from Jiang et al. 2015)</i> .....	71
<i>Figure 58:Deflection v/s Span for SCCCB1 (Jiang et al. 2015) beam numerical model</i> ....	72

<i>Figure 59:Deflection comparison (model and experiment)</i> .....	72
<i>Figure 60:Deflection v/s Span (40m 3-span 0.8 interaction)</i> .....	74
<i>Figure 61 :Deflection v/s Span (40m 3-span 0.5 interaction)</i> .....	74
<i>Figure 62:Deflection v/s Span (40m 5-span 0.8 interaction)</i> .....	75
<i>Figure 63:Deflection v/s Span (40m 5-span 0.5 interaction)</i> .....	75
<i>Figure 64:Deflection v/s Span (30m 3-span 0.8 interaction)</i> .....	76
<i>Figure 65:Deflection v/s Span (30m 3-span 0.5 interaction)</i> .....	76
<i>Figure 66:Deflection v/s Span (30m 5-span 0.8 interaction)</i> .....	77
<i>Figure 67:Deflection v/s Span (30m 5-span 0.5 interaction)</i> .....	77
<i>Figure 68:Deflection v/s Span (20m 3-span 0.8 interaction)</i> .....	78
<i>Figure 69:Deflection v/s Span (20m 3-span 0.5 interaction)</i> .....	78
<i>Figure 70:Deflection v/s Span (20m 5-span 0.8 interaction)</i> .....	79
<i>Figure 71:Deflection v/s Span (20m 5-span 0.5 interaction)</i> .....	79
<i>Figure 72: Percentage difference between deflections (no concrete v/s effective moment of inertia)</i> .....	80
<i>Figure 73: Percentage difference between deflections (no concrete v/s effective moment of inertia)</i> .....	80
<i>Figure 74: Moment-span for 40m 3-span entire Steel Concrete Composite girder with 0.5 interaction</i> .....	81
<i>Figure 75: Moment-span for 40m 3-span steel component of Steel Concrete Composite girder with 0.5 interaction</i> .....	81
<i>Figure 76: Moment-span for 40m 3-span entire Steel Concrete Composite girder with 0.8 interaction</i> .....	82
<i>Figure 77: Moment-span for 40m 3-span steel component of Steel Concrete Composite girder with 0.8 interaction</i> .....	82
<i>Figure 78: Moment-span for 40m 5-span entire Steel Concrete Composite girder with 0.5 interaction</i> .....	83
<i>Figure 79: Moment-span for 40m 5-span steel component of Steel Concrete Composite girder with 0.5 interaction</i> .....	83
<i>Figure 80: Moment-span for 40m 5-span entire Steel Concrete Composite girder with 0.8 interaction</i> .....	84
<i>Figure 81: Moment-span for 40m 5-span steel component of Steel Concrete Composite girder with 0.8 interaction</i> .....	84

<i>Figure 82: Moment-span for 30m 3-span entire Steel Concrete Composite girder with 0.5 interaction.....</i>	<i>85</i>
<i>Figure 83: Moment-span for 30m 3-span steel component of Steel Concrete Composite girder with 0.5 interaction.....</i>	<i>85</i>
<i>Figure 84: Moment-span for 30m 3-span entire Steel Concrete Composite girder with 0.8 interaction.....</i>	<i>86</i>
<i>Figure 85: Moment-span for 30m 3-span steel component of Steel Concrete Composite girder with 0.8 interaction.....</i>	<i>86</i>
<i>Figure 86: Moment-span for 30m 5-span entire Steel Concrete Composite girder with 0.5 interaction.....</i>	<i>87</i>
<i>Figure 87: Moment-span for 30m 5-span steel component of Steel Concrete Composite girder with 0.5 interaction.....</i>	<i>87</i>
<i>Figure 88: Moment-span for 30m 5-span entire Steel Concrete Composite girder with 0.8 interaction.....</i>	<i>88</i>
<i>Figure 89: Moment-span for 30m 5-span steel component of Steel Concrete Composite girder with 0.8 interaction.....</i>	<i>88</i>
<i>Figure 90: Moment-span for 20m 3-span entire Steel Concrete Composite girder with 0.5 interaction.....</i>	<i>89</i>
<i>Figure 91: Moment-span for 20m 3-span steel component of Steel Concrete Composite girder with 0.5 interaction.....</i>	<i>89</i>
<i>Figure 92: Moment-span for 20m 3-span entire Steel Concrete Composite girder with 0.8 interaction.....</i>	<i>90</i>
<i>Figure 93: Moment-span for 20m 3-span steel component of Steel Concrete Composite girder with 0.8 interaction.....</i>	<i>90</i>
<i>Figure 94: Moment-span for 20m 5-span entire Steel Concrete Composite girder with 0.5 interaction.....</i>	<i>91</i>
<i>Figure 95: Moment-span for 20m 5-span steel component of Steel Concrete Composite girder with 0.5 interaction.....</i>	<i>91</i>
<i>Figure 96: Moment-span for 20m 5-span entire Steel Concrete Composite girder with 0.8 interaction.....</i>	<i>92</i>
<i>Figure 97: Moment-span for 20m 5-span steel component of Steel Concrete Composite girder with 0.8 interaction.....</i>	<i>92</i>
<i>Figure 98: Percentage difference between moments in the steel girder (no concrete v/s effective moment of inertia) .....</i>	<i>93</i>

<i>Figure 99: Percentage difference between moments in the steel girder (no concrete v/s effective moment of inertia) .....</i>	<i>94</i>
<i>Figure 100: Moment span diagram for steel girder for a 40m 3-span continuous girder. .</i>	<i>95</i>
<i>Figure 101: Truss idealization of a steel-concrete composite girder (Vayas et al. 2010) ..</i>	<i>97</i>
<i>Figure 102: 3-D view of the loaded Grillage model .....</i>	<i>98</i>
<i>Figure 103: Deflection v/s Span (20m 3-span 0.8 interaction) .....</i>	<i>99</i>
<i>Figure 104: Percentage difference between deflections in the Steel Concrete Composite girder (proposed model v/s 3-D grillage).....</i>	<i>99</i>



## **TABLES**

<i>Table 1: Dimensions and Loadings of the Simply Supported Girder</i> .....	34
<i>Table 2: Dimensions of 20m 3-span continuous composite beam</i> .....	41
<i>Table 3: Dimensions of 10.5m 2-span continuous composite beam</i> .....	43
<i>Table 4: Dimensions of continuous composite beam for parametric studies</i> .....	47
<i>Table 5: Dimensions of a 3-span continuous composite beam for validation</i> .....	56
<i>Table 6: Dimensions of continuous composite beam for validation studies (Jiang et al. 2015)</i> .....	71
<i>Table 7: Dimensions of continuous composite beam for validation studies (Jiang et al. 2015)</i> .....	71
<i>Table 8: Dimensions of continuous composite beam for parametric studies</i> .....	73
<i>Table 9: Dimensions of continuous composite beam for parametric studies</i> .....	98

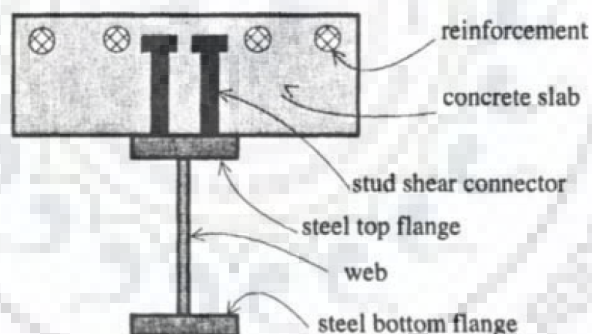


## Chapter 1. Introduction

Composite construction is a method where two or more materials are used together so as to utilise the individual materials to the best of their advantages. The Steel-Concrete Composite structures are widely used in modern bridge and building construction. The composite girder is generally composed of an I-section beam or a Channel section or any other steel section which is strong in bending, is attached to a concrete component like bridge deck or a floor slab. The comparatively high strength of the concrete in compression complements the high strength of the steel in tension.

The steel and concrete components interact with each other through mechanical shear connectors allowing the transfer of longitudinal shear forces in the concrete to the steel and vice versa. The mechanical shear connectors also keep the concrete and steel from separating.

The most common form of mechanical shear connectors, is the stud shear connector, consists of a flat head and a plain shank connected to the steel component by a weld collar.



*Figure 1: General cross section of the composite beam (Oehlers D.J., Bradford M.A., 1999)*

The preceding Figure 1 demonstrates a structural arrangement of a composite beam in which the concrete component is supported on the steel I-beam. The composite behaviour is achieved through the interaction of the concrete and steel elements such that there is migration of the forces between them.

The composite construction in buildings also use concrete filled tubes (Figure 2) the failure of these elements is through squashing, and whose strength is governed by the strength of the cross section. These are steel tubes filled with concrete, the strength of these columns is greatly increased by the steel tube which encases it. The steel tube is subjected to tension

when the concrete is loaded axially. The concrete when loaded imparts outward pressure and the steel absorbs that. Also, this form of construction greatly increases the flexural strength of the cross section as the section modulus for steel tube, as it encases the concrete, is large as compared to having steel rebars of the same area inside of the concrete. This in turn leads to greater resistance to the tensile stresses generated when the section is subjected to bending. This form of composite construction thus leads to a cross section of a compression member which has an enhanced compressive and flexural strength.

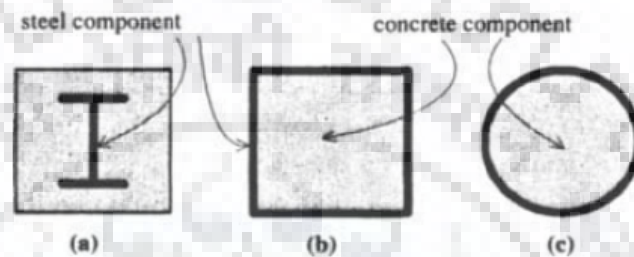


Figure 2: Composite column sections (Oehlers D.J., Bradford M.A., 1999)

The concept of composite construction has also been extended to composite floor systems, in which the concrete slab is cast onto the cold formed profiled steel sheeting of about 1mm thick. Steel acts as a permanent formwork for the concrete slab (Figure 3).

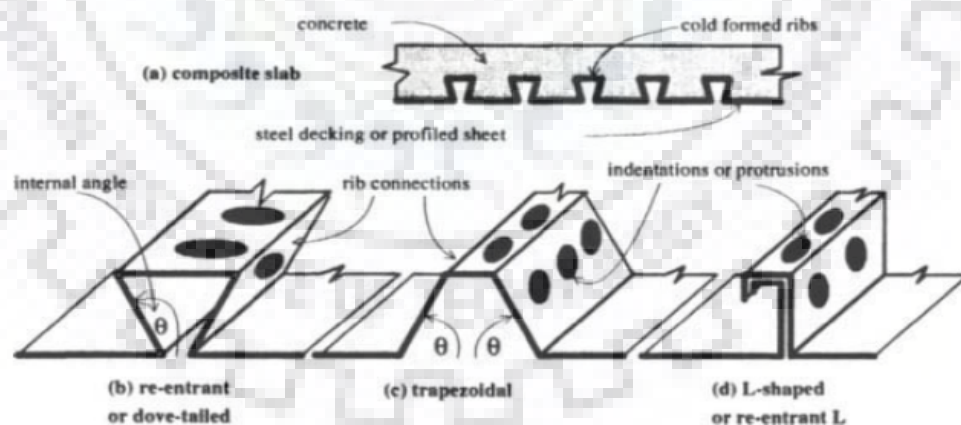


Figure 3: Composite profiled slabs (Oehlers D.J., Bradford M.A., 1999)

The composite action is achieved by the embossments in the steel sheeting. The ribs of the profiled sheeting are in general perpendicular to the centreline of the section which supports it. The studs are welded through the thin sheet into the top flange element of the steel component. The shear connection achieved in the longitudinal direction is in the form of

shear connectors and in the transverse direction by the means of embossments in the profiled sheeting. This action is generally referred to as a double composite action.

## 1.1 Shear connectors

The shear connectors play an important role in ensuring an efficient shear connection between the steel to the concrete that allows for the longitudinal force transfer at the interface which also carries the shear and any coexistent tension between the two materials.

The shear strength of the connectors resists the shear force, and which results in transfer of force. The strength of the connectors is determined by the push-out test where the shear load is directly applied to the shear connection. The two main categories of shear connectors are flexible and rigid connectors (Figure 4).

Flexible type of connectors includes headed stud connectors which behave in a ductile manner at the ultimate limit state. The connectors act as steel dowels embedded in the concrete (Figure 5) to resist the shear flow and permit slip at the interface. The head of the shear connector restrains the detachment of the two components. The specified spacing is determined based on the dowel strength, the strength of the concrete element and the steel element, whichever is smallest. The diameter of the shank of the connector  $d_{sh}$  varies from 13 mm to about 22 mm. The most common size of the shear connector used in buildings and bridges is 19 mm. The head of the shear stud is about  $1.5d_{sh}$  wide and  $0.5d_{sh}$  deep. The collar is  $1.3d_{sh}$  wide and varies in height from 0 to  $0.4d_{sh}$ .

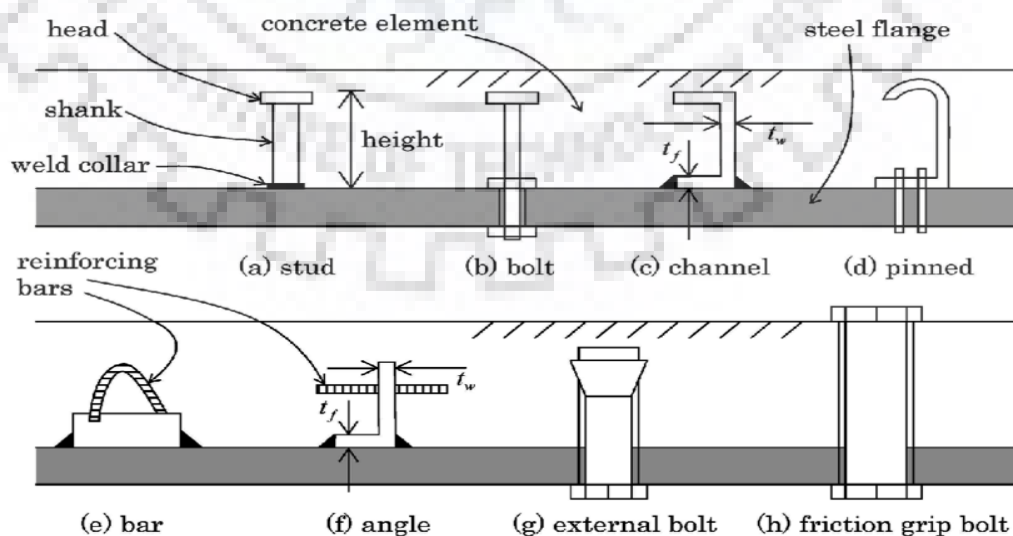


Figure 4: Mechanical Shear Connectors (David Collings, 2005)



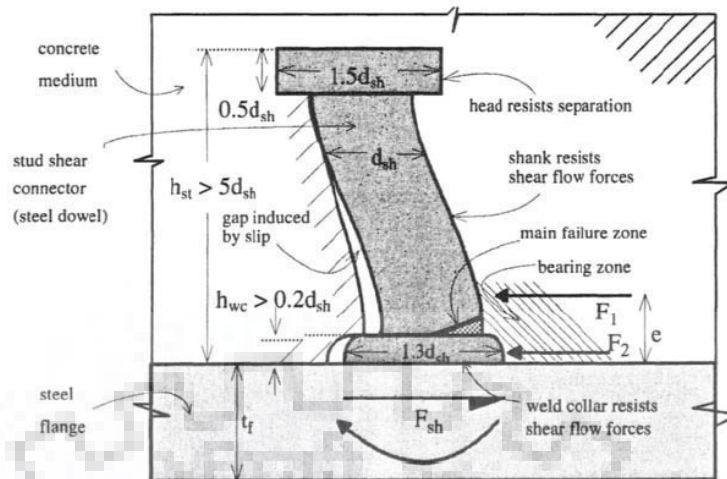


Figure 5: Dowel action of a stud shear connector (Oehlers D.J., Bradford M.A., 1999)

Rigid connectors include fabricated steel plates or blocks, channel or angle sections, perforated or toothed plates. They are more brittle in comparison to the flexible connectors. The failure either by the local crushing of the concrete or by the rupture of the weld connecting the plates to the steel beam. For uniform shear flows, flexible shear stud connectors are feasible. For heavy shear flows, bar or plate connectors may be appropriate.

The choice of shear connector to be employed depends upon the nature of the load applied. For an approximate uniform shear flow, flexible shear connectors shall be used, whereas for cases where there is heavy shear flow, bar or plate type shear connectors shall be used.

## 1.2 Interface behaviour

The deformations, stresses distributions and the different modes of failure of the steel concrete composite beams depend on the behaviour of the shear connection developed between the steel component and the concrete component. This behaviour is represented by a relationship between the shear force and the slip as shown in the Figure 6. The bond behaviour of the interface varies from extremely brittle as depicted in the curve A, to extremely ductile as depicted in the curve B.

Mechanical shear connectors such as stud, bolt and angle connectors exhibit substantial plastic regions, but the failure occurs because of fracture happening at a finite slip as depicted in the curve C. This happens because the slip capacity is controlled by the deformation capacity of the shear connector.

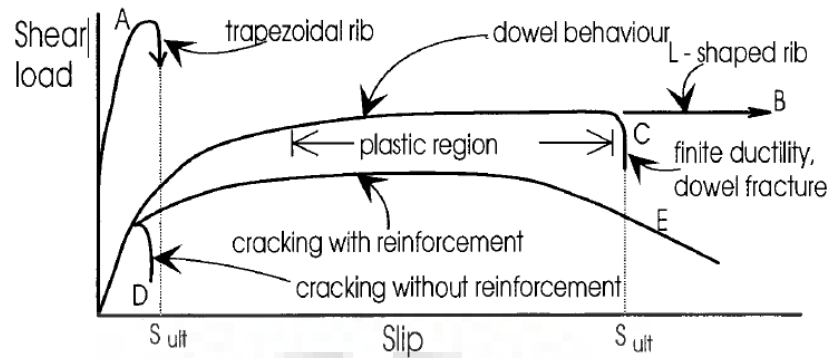


Figure 6: Interface shear behaviour (Oehlers D.J., Bradford M.A., 1999)

Mechanical connectors tend to impart very high concentrations of loads onto the concrete component. This load is transferred from the steel component to the concrete component through the dowel action of the shear connectors. The load is distributed into the concrete component and the action of this distributed force can lead to a phenomenon known as tensile cracking. The tensile cracking occurs due to the shear and spitting actions. These cracks can also occur due to the connector resisting the separation of the components from one another at the interface of the composite beam, these cracks are referred to as embedment cracks as depicted in the following figure.

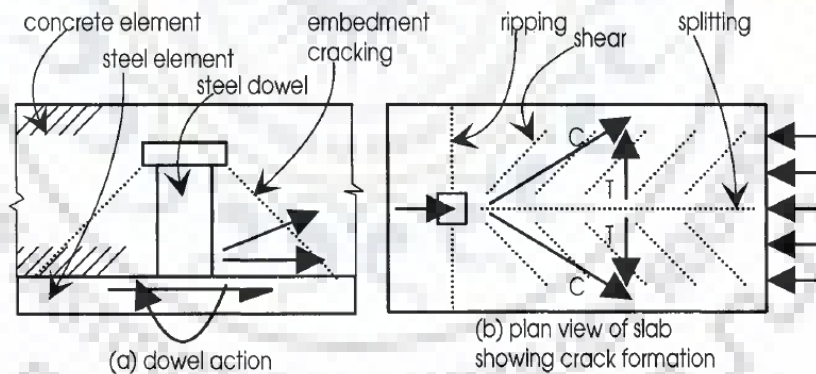


Figure 7: Transfer of connector force (Oehlers D.J., Bradford M.A., 1999)

These four forms of tensile failure of the slab can affect both the dowel strength and the ductility of the shear connectors. This reduction in ductility gets enhanced when there is no reinforcement which leads to immediate cracking and slip capacity of the section gets reduced. The presence of a reinforcing bar across a crack plane makes failure much more ductile as depicted in the curve E of Figure 6, this can also lead to an increase in load carried after cracking.

### 1.3 Slip

The shear studs and its spacing, even though is designed for the longitudinal shear force, will most certainly deform under the shear load causing the concrete and the steel component to slide over each other at the interface of the steel concrete composite girder and this phenomenon is known as slip. Slip is defined as the difference in location of a reference points on the two elements of the steel concrete composite girder, established when the beam is not deformed, to the final location of the reference points after the beam has deformed at the interface.

The behaviour of a composite beam is affected directly by the slip of the shear connection at the steel/concrete interface. The elevation of a simply supported composite beam is shown in the following Figure (a). When the composite beam is not loaded, the sections AB in the concrete component and CD in the steel component are in line and positioned at a certain distance L from a convenient reference axis. When the beam is loaded by a load F, the beam deflects, and the section deforms as shown in (b). The compressive forces developed in the top fibres of the concrete component and steel component cause these fibres to contract, whereas the tensile forces in the bottom fibres of the concrete and steel cause these fibres to expand. There is thus sliding action at the interface, and the relative movement at the interface caused by this sliding action is referred to as the slip 's'.

If the new position of B in the concrete component is at  $L + u_c$  as shown in Figure 8b, and that of C in the steel component is at  $L + u_s$ , then  $s = u_c - u_s$ : This slip is resisted by the longitudinal shear forces. If we now consider the distribution of strains in the concrete and steel components over the length L, as in Figure 8c, then

$$u_c = \int_L \varepsilon_c dx \quad \text{and} \quad u_s = \int_L \varepsilon_s dx \quad (1)$$

and substituting these values into  $s = u_c - u_s$  and upon differentiation,

$$\frac{ds}{dx} = \varepsilon_c - \varepsilon_s \quad (2)$$

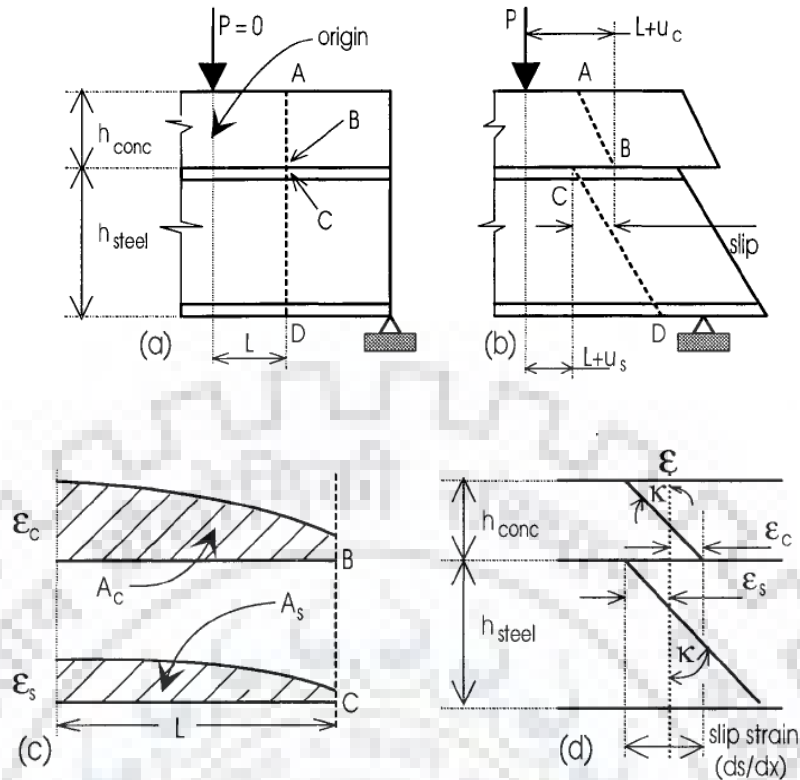


Figure 8: Fig Slip and Strain (Oehlers D.J., Bradford M.A., 1999)

The derivative of the slip  $ds/dx$  is known as the *slip strain* and as can be seen in above Figure, it is the step change between the strain profiles in each component of the steel concrete composite girder.

#### 1.4 Shear connection

Two regularly used terms that describe composite behaviour of the steel concrete composite girder are partial shear-connection and partial-interaction, and these associate with the behaviour of the shear connection between the steel and concrete components. Partial-shear-connection relates to the equilibrium of the forces within a composite member, while partial-interaction concerns compatibility of deformations at the steel/concrete interface. Partial-shear-connection thus represents a strength criterion, while partial interaction represents a stiffness criterion.

#### 1.5 Degree of interaction

A condition of no interaction is achieved when the entire interface at the steel beam and the concrete slab is smooth without any friction, but even so, when the steel and concrete components are in contact with each other and so have the same curvature and deflection.

On the other hand, when the interface is glued or there is no movement at the interface, then  $\epsilon_c = \epsilon_s$  and the slip strain  $ds/dx = 0$ . This condition is referred to as full interaction, and clearly partial interaction is the usual condition encountered between full interaction and no interaction.

The degree of interaction is a stiffness-based property and is not the same as the degree of shear connection as discussed earlier in this section which is based on strength. The degree of shear connection and degree of interaction are related directly and increasing the number of shear connectors, both the degree of interaction and the degree of shear connection increases.

### 1.6 General behaviour of composite beams

The distribution of the forces in the positive or sagging region between the points of contraflexure is completely different from the behaviour in the negative or hogging region between a point of contraflexure and the adjacent support.

Between the points of contraflexure, the composite beam can be idealised as simply supported, as depicted in Figure 9. The net flexural forces in the concrete component of the composite beam is compressive in nature, and the net forces in the steel component is tensile in nature.

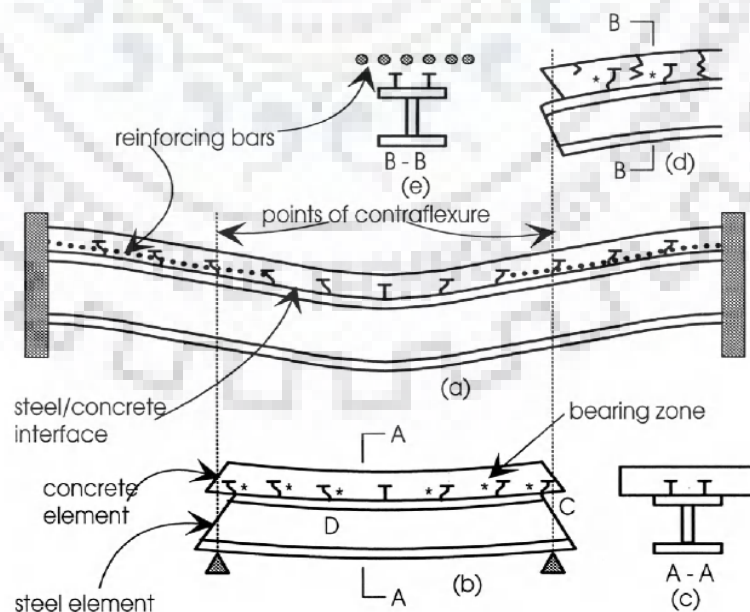


Figure 9: Deformation of composite beam (Oehlers D.J., Bradford M.A., 1999)

The concrete element near the interface tends to expand under the flexural forces, on the other hand the steel component adjacent to the interface will tend to contract under the flexural loads. This deformation distorts the connectors; hence this leads to the connectors bearing into the concrete. The connectors hence apply a thrust onto the concrete directed towards the midspan of the beam, the connectors themselves are subjected to horizontal shear forces.

The flexural distortion of the steel concrete composite girder also tries to impart vertical separation between the steel and concrete components. The separation of these elements is resisted by the tensile strength of the shear connectors.

### **1.7 Determination of degree of shear connection**

The strength of a shear connection is dependent upon which element of the composite beam yields first. That is steel, concrete or the shear studs. If there is a case where the strength of concrete is more than the strength of steel, the steel element must be fully yielded as shown in figure below. Also, since concrete strength is more than that of the steel, only a part of the concrete element will get stressed, hence the neutral axis must lie in the concrete element. Also, the stress in both the components that is the concrete and the steel, will be equal to the max stress carried by the steel component.

It is also necessary to ensure that the concrete component is in equilibrium. For which, the force in the shear connectors shall be equal to the force carried by the steel component. For the equilibrium to be maintained, the force carrying capacity of the shear connectors has to be greater or equal to the force carried by the weaker component. This phenomenon when the shear stud strength is equal or greater than the forces transferred for equilibrium is known as “full interaction”

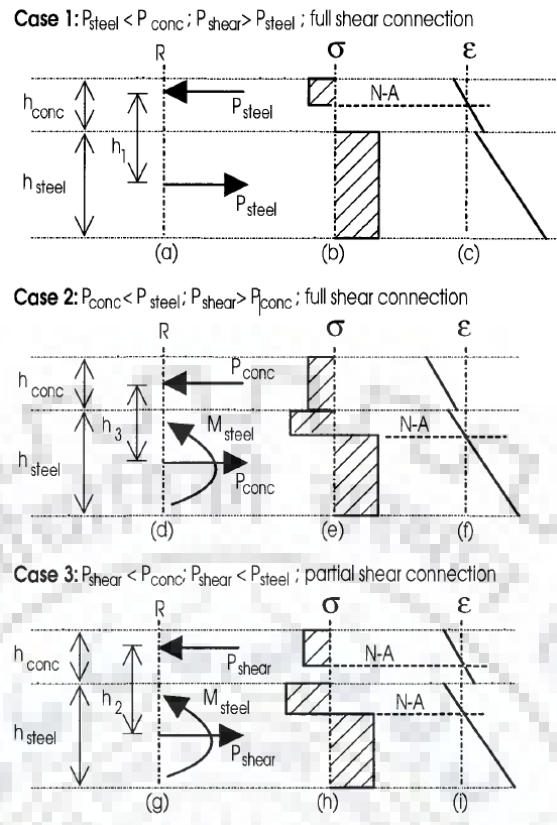


Figure 10: Degree of shear connection

The full interaction case also occurs when the concrete component yields first, resulting in a force equal to the strength of the concrete to take part in the equilibrium condition. In this case, the force in steel component also carries a moment as the neutral axis is in the steel component and the steel will carry tensile and compressive forces both, resulting in a net tensile and a moment component due to the internal lever arm between the forces in steel component.

Even for this case, for steel section to be in equilibrium, the strength of the shear studs has to be greater than the strength of the concrete element. This also leads to a full interaction case. In any case where the shear strength of the shear studs is less than the minimum force required for yielding of either element, will result in a partial shear connection. This phenomenon shall be accompanied by development of two neutral axis, one in the concrete component and one in the steel component.

The degree of shear connection  $\eta$  is often defined as the strength of the shear connection in a shear span, as a proportion of the strength required for full shear connection, so that

## 1.8 Buckling behaviour

### 1.8.1 General

The limiting strength limit state in the design of the structural steelwork is that of buckling, this phenomenon has attracted considerable attention in all international steel standards. Buckling generally is attributed to a structure having thin walls and to large elastic range of steel members.

The buckling modes can be global and local. As the term implies the overall buckling takes place when the whole structure becomes unstable and contorts and shifts sideways to the nearest equilibrium position, the half wavelength of the buckle being the order of the member length.

Local buckling takes place when there are localised deformations of the cross-section shape, the buckling half wavelength being of the order of depth or width of the section.

### 1.8.2 Global buckling

The steel concrete composite tends to buckle during and after the concrete has hardened. Global buckling must be checked for while designing the structure. During the stage when the concrete is wet, the top flange of the girder is susceptible to lateral torsional buckling as it is subjected to compressive stresses in the positive region.

The lateral torsional buckling can also impact the bottom flange of a continuous composite beam as for a continuous composite beam, the bottom flange is subjected to flexural compressive forces at the supports. This needs to be taken into consideration while designing the steel concrete composite. The lateral torsional buckling occurs as and when each cross-section of the steel deforms and twists as a rigid body. This mode of buckling generally is one of the easiest modes to predict and can be taken care of by propping the flanges of the girders or even as bracings. The lateral torsional buckling mode is depicted in Figure 11 (buckling mode of a steel concrete composite with unset concrete), and Figure 12 (buckling mode for a steel concrete composite under negative bending moment).



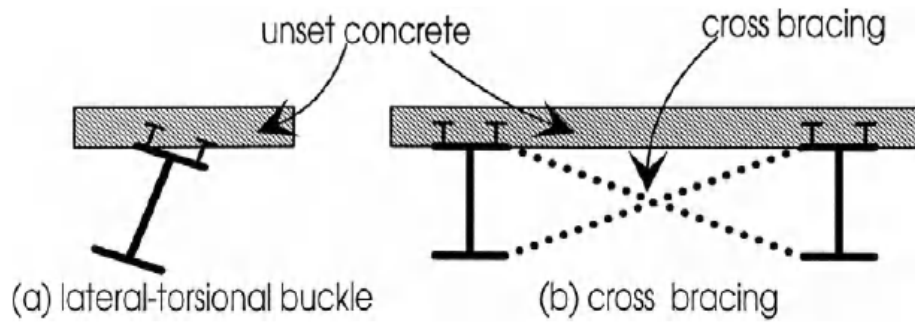


Figure 11: Lateral - torsional buckling (Oehlers D.J., Bradford M.A., 1999)

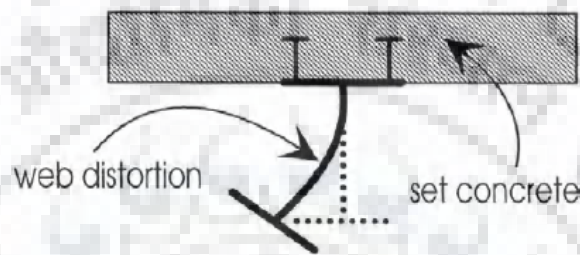


Figure 12: Lateral - distortional buckle Oehlers D.J., Bradford M.A., 1999)

### 1.8.3 Local buckling

in general context, the buckling of the flange and web is an instability possibility which must be checked for. The buckling of the steel cross-section as depicted in the Figure 13, happens as and when the plates of the steel section contorts, but the remaining plates remain straight. The flanges may buckle locally in both the positive bending and negative bending zone.

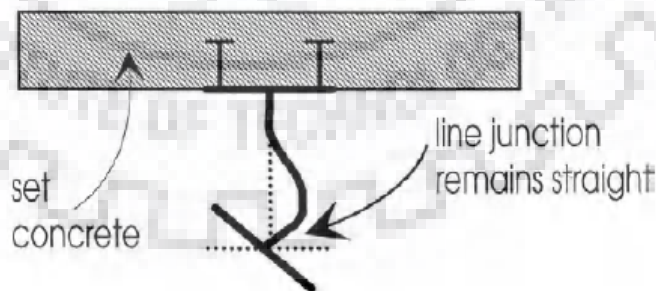


Figure 13: Local buckling (Oehlers D.J., Bradford M.A., 1999)

When the section is under positive, the neutral axis lies in the steel web, the top flange is then subjected to compressive forces. The flange tends to buckle locally which is restrained by the shear connectors, and the presence of solidified concrete, contribute to supressing this

mode of buckling. This kind of moment is unlikely to occur in a number of situations prior to the attainment of the full plastic moment.

The local buckling also occurs in the negative bending zone, it is characterised by the deformation of the web and rotation of the usually stocky flange.

### 1.9 Concept of composite girders extended to bridges

The steel concrete composite bridges have become increasingly common as a method of bridge construction and are competitive with the concrete beam and steel beam bridges from spans of about 20m and generally hold an intermediate position between concrete and steel structures.

For bridges used by railways and for bridges with longer spans, steel truss bridges are generally used. Composite arch bridges and cable stayed have been designed and constructed for situations where the span ranges from 50m to 500m. However, steel structures are preferred for the longer span bridges. The following figure illustrates the typical span ranges for the common bridge forms.

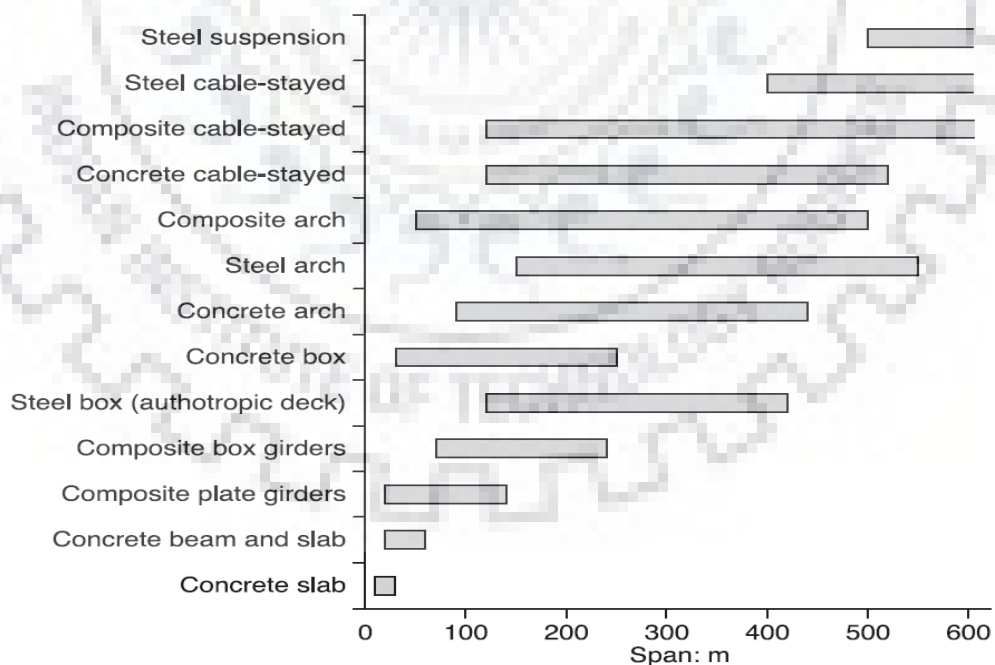


Figure 14: Span ranges for various bridge types (David Collings, 2005)

The strength of the steel concrete composite girder is derived from the shear connection developed between the Reinforced concrete deck and the steel girders through several shear

connectors. If the concrete deck was directly placed on top of the steel girders and the interface between the concrete and the steel was made friction less, the overall deflection and the resulting slip between the two components would be large as compared to a composite cross section having shear studs embedded into the concrete top slab. This would happen because the two components in the first case would act independently and each component would carry its own weight and half of the superimposed load.

The increased stiffness of composite beams allows its depth to be optimised for the same span. As compared to concrete bridges of equivalent span and width composite beams are lighter which reduces the loads sustained by their supports in the event of an earthquake. In normal circumstances, only regular renewal of its corrosion protection and sometimes renewal of its waterproofing course is required.

### **1.9.1 Simply supported steel concrete composite bridge girders**

Simply supported steel beam and concrete slab bridges are a common form of construction for spans to about 50 m. At the lower spans, Universal Beam sections may be used; for spans greater than about 20m a fabricated girder is likely to be more economic. The form of the structure consists of several beams placed side by side (usually parallel and in pairs) at 2m to 4m spacing.

The beams will be linked by bracing to prevent buckling instability during construction. A concrete slab about 200 to 250mm thick is constructed on the beams. The slab carries the highway (or railway) loading and spreads it to the underlying beams. To ensure composite action between the slab and steel girder, shear connectors are provided at the steel–concrete interface.

### **1.9.2 Continuous composite bridge girders**

In comparison to simply supported beams, continuous steel concrete composite girders have many advantages such as higher span/depth ratio, less deflection, and higher fundamental frequency of vibration due to its higher stiffness. However, in negative bending regions near interior supports, tension in concrete is unfavourable and a complicated issue, a detailed study on which has been presented in this work.

The reduction in girder depth and overall dead load results in a decrease in foundation loads of a structure, which results in a more economical construction. The increase in the use of

high-strength concrete and steel has led to shallower sections, deflection may become a critical factor that controls design. For continuous composite beams, the negative bending moment near interior support regions will generate tension in concrete slab and compression in steel, which will result in a complicated case of cracking of concrete in the negative zone. Furthermore, the negative zone bending also causes the compression flange of the steel girder to buckle before reaching yield, this reduces the efficiency of the structure. Another complication that occurs is that a high shear zone and a zone of high flexural moment occur at the intermediate support, requiring a consideration of shear-bending interaction. The strength of the concrete in the negative zone is generally neglected after the concrete in the area crosses the rupture strength. This is a common practice in design of a continuous steel concrete composite beams. This approach is conservative in nature as it neglects the effect of concrete in between the rebar which has not cracked (tension stiffening). This approach yields safe designs but also leads to designs and structures which could be optimised even further.

### **1.10 Effect of shear slip on the deflection of composite girder**

Composite steel construction has gained a lot of popularity in the last several decades. With the use of high-performance steels (HPSs) and high-performance concrete (HPC), composite steel bridges have been designed for much greater span lengths than those possible with ordinary materials, also leading to a reduced total overall cost by 10%. The reduction in overall size of the structures owing to the use of higher strength materials may lead to higher deflections, hence proper deflection calculations must be made in order to ensure a safe and serviceable design.

The composite action between steel and concrete depends on the performance of shear connectors at their interface. The use of HPS and HPC requires more shear connectors for full composite action as HPS and HPC have a higher capacity of load transfer. Due to the limitation of the number of shear connectors that the top flange can accommodate or for an optimal design, a partial composite design may be selected. A partial composite design will result in more slip at the steel–concrete interface, which leads to an additional deflection. Even though a beam is designed as a full composite section, a perfect composite action without any slip cannot be obtained due to the deformation of the flexible shear studs, which are often treated as “ductile” connectors. In the case of composite beams with profiled sheeting construction, there may be no enough space in the troughs to provide enough stud connectors in the negative bending regions. Hence all the composite sections have partial

interaction and shall be designed accordingly to account for the additional deflections in the girders due to slip. For girders with full interaction ignoring deflections due to slip may severely underestimate the deflections of the structure. For full composite beams, slip effects may result in stiffness reduction up to 17% for short span beams.

However, slip effects are ignored in many design specifications that use transformed section method except that American Institute of Steel Construction (AISC) specifications recommend a calculation procedure in the commentary. In the AISC procedure, stress and deflection calculations of partially composite girders are based on effective section modulus and moment of inertia to account for slip, while ignoring slip effects in full composite sections. For full composite sections, the effective section modulus and moment of inertia calculated with the AISC specifications are larger than that of present study, meaning that the specifications are not on the conservative side.

### **1.11 Effect of cracking of concrete on the overall deflection of the girder**

The concrete, as cited in the previous section, is susceptible to cracking under the influence of negative bending moment and hence severely suffers a loss of flexural and axial force transfer capabilities. The deflection of the girders is significantly increased when the concrete cracks under negative bending.

As of now, the most common practice has been to design a continuous steel concrete composite girder as a steel only section in the negative bending zone and completely ignoring the contribution of concrete in the negative bending zone. This assumption though will always yield safer designs as far as the design of the steel girder is concerned and leads to an overuse of the material and a certain increase in the cost of the structure. The design also assumes a rigid connection between the concrete and steel girder, a case which is not possible.

The concrete has a residual strength even after it undergoes cracking, the concrete in between the reinforcing bar still has tensile flexural capacities. The residual strength has not been utilized in any design standard because of the complexities and the time required for the computation of the strength.

## **1.12 Gap in the current studies.**

The current state of the studies as mentioned in the next chapter section outlines the determination of the forces and deflections for a continuous steel concrete composite girder either by completely ignoring the concrete in the negative bending zone of the steel concrete composite or by considering a complete uncracked section of the concrete at the negative bending zone in the steel concrete composite. Even if the procedure does involve the use of concrete in the cracked zone, the method of application is complex and requires iterative procedure for the calculation of the required parameters.

A holistic and simple approach which incorporates the concrete's residual strength after cracking when subjected to negative bending moment is required for the analysis of the continuous steel concrete composite girder which can be used in and implemented design offices using commercially available structural analysis software packages such as STAAD Pro and SAP2000 etc., to name a few, for the optimisation of the material and for designing the steel concrete composite such that the residual strength property of the concrete after cracking can be utilised.

This dissertation aims to propose a simple approach which encapsulates all the requirements as mentioned above in this section.

## **1.13 Objective and Scope of The Study**

### **1.13.1 Objective of the study**

The objective of the current study is to analyse the steel concrete composite girder using numerical methods and already performed experimental works.

The objective of the current study is to come up with a holistic and simple method for the analysis of continuous steel concrete composite girder and propose a simple numerical model on readily available structural analysis software packages such as SAP2000. The steel concrete composite girder shall first be analysed as per the prevalent norms followed by validation of the proposed numerical model arrangement against the prevalent norms.

### **1.13.2 Scope of the study is as follows:**

The objectives of the study have been summarised as follows:

1. Validate the numerical modelling by comparing the deflections by manual calculations using analytical relationships based on simplifying assumptions which have already been developed.
2. Understand the behaviour of slip in a continuous composite girder bridge and the resulting effect on the deflections of the girder through numerical studies.
3. The study shall also cover the effects of cracking in the concrete deck at the negative bending zone and the resulting effect on slip between the concrete and the steel section, the span deflections, the redistribution in component forces as the concrete cracks.
4. The study shall draw comparisons between the deflections and the forces in the components for an uncracked section, a completely cracked section and a section in which residual strength of partially cracked concrete is used for analysis.
5. The concept of residual strength of the cracked concrete shall be extended to grillage modelling of a bridge to understand the effects of residual strength on the lateral buckling of the girder.

The study is currently focussed to develop numerical models using the commercially available finite element tool SAP2000 for the steel concrete composite continuous girders and to study the results obtained by the model and comparing them with the already available experimental and theoretical methods. Simulating the tension stiffening of the concrete in commercially available software package SAP2000 and listing out the variations in deflections and component forces between the tension stiffened model and the models with complete concrete section and the section with completely cracked concrete.

### **1.14 Layout of the thesis**

The work developed in this thesis has been organised into 9 chapters, and their concerns are described below. To facilitate in easy and intuitive reading of this thesis, every chapter starts with an introduction (except for chapters 1,2,8 and 9), where a brief idea about the chapter is given followed by a validation and parametric studies and terminates at conclusions (except for chapters 7,8 and 9).

**Chapter 1** deals with the introduction to the basic concepts of the steel concrete composite girders in general and gives insight to the reader about the current study. while highlighting the gaps in the study and outlining how the present work attempts at addressing it and hence defining definite scope and objectives of the thesis.

**Chapter 2** outlines the publications available in open literature related to the topics covered in the thesis, this intends to provide a framework of the current thesis while highlighting the gaps in the study and outlining how the present work attempts at addressing it and hence defining definite scope and objectives of the thesis. It highlights all the literature that have been referred to in order to accomplish the objectives defined for the current work.

**Chapter 3** deals with the methodology that has been used in order to determine the results and explains the methods used for the implementation of a numerical method into the current study. This chapter lays out the foundation for all the calculations and forms a framework for the numerical modelling ideology.

**Chapter 4** deals with the validation of the proposed modelling ideology as implemented in SAP2000. Since the ideology of the model taken was for a simply supported composite girder, validation and parametric studies have been done in order to validate the use of the numerical model. The proposed model is then extended to the continuous girders and is then followed by validation and parametric studies of the model.

**Chapter 5** deals with the current state of affairs when it comes to the effect of cracking of concrete to overall structural rigidity and structural integrity. It gives an insight to how concrete behaves when it is subjected to negative bending moment at the support region. This chapter also lists out all the theories which relate to concrete subjected to negative bending moment. It incorporates the theory of cracked concrete into the proposed model for which validation and parametric studies have been performed.

**Chapter 6** deals with the attempt to formulate a numerical model in order to incorporate the residual strength of concrete after it has undergone cracking under the influence of negative bending moment. The chapter explores all the different theories and methodology in place to effectively determine the residual strength of the cracked concrete. The modelling of residual strength of the concrete is followed by the validation of the model against an experiment and is followed by parametric study and the conclusions of the results of the parametric study.

**Chapter 7** deals with the spatial modelling techniques. Since all the studies that have mentioned in this work till now are related to the 2-D modelling of the continuous composite girder. The chapter aims to take that theory and modelling technique to the spatial system of modelling of girders, namely 3-D grillage modelling. The basic methodology the spatial



system is discussed followed by a numerical model and its validation against previously determined results in this thesis.

**Chapter 8** deals with the results and conclusions of the entire thesis highlighting the benefits of the proposed work while also laying out the scope for the future studies that can be done in this field of work.



## Chapter 2. Literature Review

All the beams designed have certain degree of partial interaction. As a full interaction composite beam would mean that there is no slip in between the concrete and steel surfaces. This though is not practically possible to achieve. There always is slip in between the interfaces. Hence all the shear interactions transferring the shear force between the two components, i.e. steel and concrete are partial interaction. An extensive amount of study has been done in determination of the ratio of interaction between the two interfaces. The interaction ratio depends on the stiffness of the shear connectors and the spacing at which they are provided at.

The experiments on the composite beams date back to the early 1970s. The results of one of the more famous set of experiments was conducted by Ansourian in the year 1981. He presented 6 composite beams of 9m each. Each beam had a different constraint, either in the form of restraints or in the form of loading condition. Two beams had limited rotation capabilities and were tested for sagging rotation condition. The other four beams were loaded symmetrically and included the failure mode in which buckling occurred. It was proposed that the beams having a buckling factor greater than 1.4 could be designed using the simple plastic theory, with no regard to the loading and interior support. It was noted that for a compact section with adequate slab and shear connection, simple plastic design could be incorporated for continuous beams with a compact section.

Plum and Horne (1976), presented a detailed analysis for beams with two unequal spans subjected to concentrated loads. The beams were assumed to be prismatic, being composed of two elastic materials. The connection between the two materials is assumed to have a linear load-slip characteristic. The importance of linear analysis and the limitations of present practice were summarized. A general solution for composite beams was obtained, and then applied to the case of a two-span continuous beam. Approximate methods for determination of deflection with respect to the degree of shear connection were developed and were compared with exact solutions. The results from the beam tests were compared with the appropriate theoretical values. It was noted that deflection increased, and shear flow force decreased as the interaction decreased from full interaction to a very poor interaction. The derivatives of slip-force and slip-strain increase with a decrease in the interaction. It was also noted that with greater interaction slip strain becomes significant over a shorter length of span.

Girhammar et al. (1993), proposed a method for exact analysis of composite beam-columns with interlayer slip. They presented an exact first and second order analysis of composite beam and columns with PI which were subjected to transverse and axial loading. Closed-form solution for the displacement functions were presented which also included solutions for other actions of the composite beam. The axial loads acting on the composite structure are assumed to be a function of the actual axial load carrying capacity of the structure such that the resultant acts at the centroid of the transformed composite structure. The loads acting along the centroid ensure that during the first order analysis, the axial strain remain constant throughout the depth of the structure and ensures that no bending is induced by the resultant load.

N.A Jasim (1997), proposed a method for calculation of deflections of continuous composite beams. Jasim proposed an interaction theory which incorporated slip. The beam model was assumed to be made of two elastic materials and the connection between them was assumed to have a linear-elastic characteristic. It was proposed as a holistic method for determination of deformations of beams having different spans, span lengths, and even loading. Large number of variables, pertaining to beam geometry, material properties and connector properties, which influence the deflection were grouped together into two dimensionless factors. This helped in formulation of design charts which could be used for determination of deflection.

The method was simplified and published by N.A Jasim and A. Atalla in 1998. The method was based on the principal of superposition and determination of fixed end moments for different span subjected to the respective loading. The continuous beam is represented by series of single span beams deflections for which have already been determined using linear partial interaction theory.

Wang (1998), presented a paper for the calculation of deflection of steel-concrete composite beams with partial integration. The approach was theorised on the basis of the fact the deflection is related to the shear strength of the shear connectors. The stiffness of the shear connectors was based on the basis of a simply supported beam subjected to a uniformly distributed load. The deflections were compared against FEM solutions and a few experimental tests on the composite beam. He also gave a simple method for the determination of shear connector stiffness.

Faella et al. (2002), proposed a FE model for the analysis of the steel concrete composite beams based on displacements and having flexible shear connectors. Stiffness matrix and the nodal force vector were directly determined from the solution of Newmark's equation, this meant that no approximations were introduced in the calculation of the matrix and the vector.

Loh et al. (2003), studied the effects of partial shear connection in the hogging moment regions of composite beams by performing experimental analysis on eight beams subjected to hogging moment and then performing an analytical study on them. The study also discussed partial interaction concepts allowing for interface slip, in conjunction with the inherent equilibrium and compatibility principles. The analytical model was based on successive iterations of slip strain distribution at all cross-sections along the beam. Each solution step required the principles of equilibrium, compatibility and material stress-strain laws to be satisfied at all cross-sections. The complexities of the iteration process were mainly attributed to the interaction amongst the constitutive relationships and the strong nonlinearity of material behaviour. The analytical model had been developed to simulate the behaviour of the composite beam under hogging moment, and the results were compared with the experimental data which were reported in the paper. Since the studies were conducted keeping the Australian conditions in forefront, this was not a holistic method and would not be applicable in areas with higher lateral force intensities.

Seracino et al. (2006), developed simplified equations and practical assessment charts that were used to more accurately assess the remaining strength or endurance of the shear connection in continuous composite beams, the assessment was based on linear elastic partial interaction theory. The technique was validated using a finite element program developed to model the behaviour of composite structures. The method provided for an assessment of the fatigue related residual strength of the shear connection without the need of heavy finite element program, hence making it easier and quicker to use and interpret the results. The approach presented was shown to be applicable to continuous beams of any span, varying span lengths, and varying connector distribution or cross section. The approach also allowed for the incorporation of interface friction.

Higher order beam theory for the analysis of a composite beam with partial interaction was proposed by Chakrabarti et al. 2012. The method took into account the effect of PI between adjacent layers of the composite structure and also the transverse shear deformation of the beam subjected to load. The variation of axial displacement was taken to have a parabolic

variation of shear stress. The method proposed did not require for incorporation of any shear correction factors.

David Leaf (2012), proposed a model for the determination of deflection of a steel concrete composite girder subjected to pressure loading. The model was made up of frame elements for the shear stud and the steel girder but employed shells (shell thin) for the concrete slab. With the use of shell elements in the model, it was easy to predict the shear lag and shear flow around the shear stud as the shear lag showed up as stress contours on the shell element in postprocessing. The vertical shear transfer was done through dummy elements connecting the steel girder and the concrete slab. These elements were provided with large areas and no moment of inertia, which meant that the elements could only transfer the vertical shear and would not impart any flexural rigidity to the model.

Turmo et al. (2015), proposed a finite element model for the analysis of composite beam with partial interaction. The elements in the model were modelled by six types of frame elements. The validation of the proposed method was done against analytical equations available for different boundary and loading conditions.

D.J. Oehlers and M.A. Bradford, in their book “Composite Steel and Concrete Structural Members, Fundamental Behaviour”, have given detail on the relationship between partial interaction and slip and their effect on the overall deformations of the composite girders.

Effect of slip on the deflections of a simply supported composite girder is well researched but the effect of slip on the deflections of a continuous composite beam is still a grey area of research. Though some work has been done to relate the slip in a continuous composite girder and the total resultant deflections. Similarly, there have been few researches relating the deflection of the continuous composite girder and the long-term effects of creep and shrinkage.

These can influence the overall deflection and need to be accounted for during the calculation of overall deflections for serviceability criteria.

Newmark et al. (1951), proposed various equation to account for the shear connection nonlinearity. The method proposed was based off on the finite difference method. The method assumed that the plane cross sections for slab and the profile, it assumed the compatibility of the displacement between the two methods. it assumed the linear relationship between the shear force per unit length and the slip occurring at the interface.

McGarraugh and Baldwin (1971), Six composite beams with lightweight concrete slabs were tested and the results compared with a previously developed theoretical analysis. The theoretical analysis was then used to study the effects of partial shear connection, modular ratio, and unshored construction on the behaviour of composite beams. It was determined that at service load, the actual stiffness of beams with full composite design is about 85–90% of their calculated stiffness where slip is ignored.

Grant et al. (1977), stated that there is a loss in stiffness, and this is because the shear studs used are flexible and can deform locally hence permitting some slip and hence leading to loss of interaction even if there is a complete shear connection. Two conditions were considered; the first condition was when the corrugations of the deck run parallel to the beam. It was assumed that the condition could be modelled as a composite beam with a haunched slab. It was noted that shear connection is not significantly affected by the ribs. A check was warranted to ensure that the shearing of the concrete would not occur on a failure plane over the top of the connectors. They reported the results of tests on 17 composite beams which were conducted at the Lehigh University incorporating steel deck. The research evaluated the shear connector capacity and beam flexural capacity and behaviour and compared such capacity and behaviour with then existing design criterion.

Girhammar and Gopu (1993), presented first and second order analysis for the composite structures with partial interaction. These structures were also subjected to transverse and axial loading. Closed form solutions for the displacement problems were presented. The analysis also gave solutions for other composite beam related problems like the problem of shear connections, i.e. full and non-composite beams. The axial loads acting on the beam were assumed to be proportional to the axial stiffness and capacity of the composite beam. The result of which was that the resultant force caused no bending moment in the transformed composite section.

Girhammar and Pan (1993), presented an exact and approximate analysis of composite structures with partial interaction subjected to dynamic loading. General closed form solutions were presented for both functions of displacement and other internal actions of a composite beam. The solution reduced to well-known values for the extreme cases of shear connection i.e. a fully composite and a completely non-composite beam. Approximate solutions for eigenfunctions were presented for the exact analysis procedure and the results were evaluated by drawing comparisons with the exact solutions.

Nie et al. (2003), presented a study that investigated the effects of shear slip on the deflection of steel concrete composite beams. The corresponding rigidities of steel concrete composite beams using 3 different loading types was first determined based on equilibrium and curvature compatibility, from which a universal formula to take slip effects into consideration was developed. The results were checked against measurements of six steel concrete composite beam tested in the present study and also against other available results simply supported and continuous beams. It was seen that slip at the interface of the steel concrete composite beam for a partially composite beam lead to a considerable addition of deformation in the mid span. It was determined that inclusion of slip effects in the determination of the overall selections greatly improved the accuracy of the results. It was also deduced that even for a fully composite beam, shear slip results in great stiffness reduction of the steel concrete composite beam, the reduction was seen to be as high as 17% of the initial value. The study also drew parallels between the results determined against the AISC recommendations. It was found that effective section modulus and moment of inertia determined by the AISC recommendations for a fully composite section gave rise to an unconservative design. And for a partially composite section, the AISC recommendations gave rise to a more conservative design.

Zhou et al. presented a paper in (2003) in which they performed experiments on 11 continuous composite beams. The paper covered laws of deflection and slip between the concrete component and steel component. Governing differential equations of slip and deflections for simply supported composite beams with various boundary conditions and different loads were established. A set of numerical expressions for slip and deflection of continuous composite beams are determined by the method of superposition. The deflection and slip of 17 steel concrete continuous composite beams were calculated by the numerical expressions so obtained. The results were validated against experimental results and showed great agreement. The numerical results indicated that the slip of the continuous composite beams increased and tended to be equal to the slip of sandwiched panels assumed to have no shear connectors. The study also highlighted that the degree of shear connection does not influence the deflections much at full shear connection and the increment of deflection caused by slip was low (less than 5%) when the degree of shear connection was between 60% to 100%. But as the degree of shear connection reached 80% to 140%, the strain in the lower flange of the steel beam at the internal support gets influenced.

The long-term effects of creep and shrinkage also influence the overall deflection of the girder. Since the effect of shrinkage and creep take some time to show and hence are generally ignored but these may have a significant effect on the final service conditions as the structure might not have been designed to take care of such deformations. Hence when these long-term effects come into play, the serviceability condition might change significantly. Several studies have been done to understand these effects (creep and shrinkage) and their impact on the overall serviceability conditions.

There have been numerous studies which have linked the effect of creep and shrinkage on reinforced and prestressed concrete and the various methods have been devised in order to incorporate the effects of creep and shrinkage in concrete in numerical models and finite element considerations.

One of the early formulations were made by Zdenek P. Bazant (1972), he formulated the refinement of the effective modulus method, the method accounted for concrete aging. The method was extended to allow for the variation of elastic modulus and an unbound final value of creep. He pointed out the errors that are involved in creep analysis of concrete structures. One stemmed from the inaccurate knowledge of creep law and the other stemmed from the simplification of analysis, which the designers introduce to avoid the complexities of an exact analysis. He used the Volterra's integral equation for the determination of the aging coefficient. He theorised that if the attention is restricted to the working stress range and the strain reversals are ignored, creep of concrete may be assumed to be governed by the linear principle of superposition. Hence the stress strain relationships could be defined in terms of functions such as creep function, prescribed stress independent inelastic strain representing shrinkage and thermal movement of the structure. He concluded that the age-adjusted modulus was theoretically similar for any creep related issue in which strain changes linearly with the creep coefficient. The method was also extended to unbounded ultimate values of creep and for the variation of elastic modulus whose omission is lead to a significant error.

Gilbert et al. (1995), documented the behaviour of continuous composite girders under long term loads. the model analysis accounted for the cracking of concrete in the negative bending moment zone at the interior support. The model also took the time dependent properties into consideration and determined the deformations for the effect of creep and shrinkage. The results of the model were compared against full-scale continuous steel concrete composite



girders were done in order to validate the model. The period of the sustained loading was 340 days.

Hyo-Gyoung Kwak and Young-Jae Seo (2000) proposed an analytical model that incorporated the long-term effects of creep and shrinkage of concrete, and the cracking of concrete slab in negative bending moment. The prediction of stresses and strains in the constitutive materials at any point of time is achieved through equilibrium of forces and compatibility of strains with time. To consider the different material properties across the sectional depth, the layer approach in which a section is divided into imaginary concrete and steel layers was adopted. Correlation studies between analytical and experimental results were conducted with the objective to establish the validity of the proposed model.

M. Fragiacommo et al. (2004) developed numerical procedure for studying steel-concrete composite beams with regard to both the collapse analysis and long-term behaviour at the serviceability limit state. The interaction among different parameters that affect the beam response, i.e., connection flexibility, rheological phenomenon of concrete (creep and shrinkage), and nonlinear behaviour of component materials (concrete cracking, nonlinear behaviour of connection, yield of reinforcement, and yield of steel beam). Was adequately considered by uniaxial finite element model. The creep was taken into account using Maxwell's generalised rheological model through step by step time increment procedure.

Concrete is a very non-linear, it is highly heterogenous in nature and the overall strength of material highly depends on numerous factors and hence to expect it to behave as a linear material and incorporating the strength parameters into designs without understanding the effect of change in temperature, initial cracks and moisture content, the amount of fines in the mix design, the relative humidity etc. can severely undermine or overestimate the strength of the concrete and hence several studies have been conducted for the determination of the strength parameters and the non-linear behaviour of concrete.

In a case of a continuous steel concrete composite girder where the girder is subjected to numerous interacting forces such as the maximum shear and bending moment occurs at the support region. These kinds of complexities need an in-depth study. There have been several studies which have presented the non-linearity of concrete in numerical methods. these methods in general are complex and include higher order beam equations to be solved for. Which though gives a more realistic solution to the complex behaviour of concrete and continuous steel concrete composite girders but includes tremendous computational efforts.

Chakrabarti et al. (2012) and N.A Jasim (1997) and Jasim and Atalla (1998), though gave formulations which gave accurate results for a continuous composite beam but fail to incorporate the reduction in stiffness due to cracking in concrete, hence these theoretical and finite element formulation models overestimate the overall strength of the composite section by not considering the concrete cracking in the steel concrete composite girder.

Manfredi et al. (1999) proposed a model of continuous steel concrete composite which incorporated the effect of negative bending moment on the composite beam. They theorised that negative moment which arise at the interior supports of the continuous beam generate tensile stresses in the concrete fibres at the top and compressive stresses near the steel profile. This results in a highly nonlinear mechanical behaviour of the beams even at low stress levels. This non-linearity occurs not only due to the slip occurring at the interface but also due to cracking of the concrete. The model proposed considered the slip at interface and the slip that would occur between the concrete and the rebar. The study though was limited to steel sections which were compact in nature and also the model did not account for the advent of buckling to occur.

Nonlinear models of composite beams with partial interaction by means of direct stiffness have also been developed (Ranzi (2003); Ranzi and Bradford (2007); Ranzi and Bradford (2008)). These models demonstrate the nonlinear behaviour of the continuous steel concrete composite girders. This approach completely neglects the effect of concrete in the negative zone.

Studies on shear connection nonlinearity have been performed and a simplified approach to the determination of the deflections considering connection nonlinearity was proposed (Faella et al. (2003)). The method considers the tension stiffening of the concrete slab after cracking. The method proposed uses CEB Eurocode guidelines for the determination of an effective area of the residual concrete after negative moment cracking has occurred.

Intensive work in the determination of residual strength of concrete and the effect of tension stiffening has been done and there are numerous codal provisions for the incorporation of the same (CEB 1985, ACI:318). These provision along with

Jiang et al. (2015) did experimental and analytical studies on continuous composite beam and partially encased continuous beam. They investigated the mechanical behaviour and the law of moment redistribution of partially encased continuous composite beam. Experiments

were performed on 4 continuous composite beams of which one of them is a continuous composite beam, other three were partially encased continuous composite beams. The beams were subjected to five-point loading and the results were determined in the form of deflection, load-deformation and load-curvature graphs.

Since all the above studies have been done on the composite girder at a very localised level, i.e., only on a generic 2-D structure assuming the deck properties of an equivalent girder.

The structure can also be studied at the global level by modelling of the steel concrete composite beam using the spatial system of modelling.

Vayas et al. (2009), proposed a newer model to overcome the shortcomings of a FEM model and a grillage model. The FEM model though always gave a correct result for almost all problem statements but takes a lot of time and computational power. The grillage analogy is easy to apply and gives sufficiently accurate results for a wide variety of bridge decks. The problem with the grillage analogy is that the transverse beams of the grillage model do not offer an accurate representation of the dispersed behaviour as the bending takes place in both the direction. It also becomes difficult to map the buckling phenomenon of the steel girder when loaded. The diaphragms and cross bracings cannot be represented in the model.

The proposed model overcame the above-mentioned shortcomings and gave a representative model for the steel concrete composite beams. The deck was still modelled as like in the grillage method, but the steel girders were modelled as equivalent trusses. the model was less cumbersome than FEM simulations and was a better representative of the overall steel concrete composite girder.

## **Chapter 3. Methodology**

### **3.1 Introduction**

The general methodology approach taken includes the finite element modelling of steel concrete composite girders on commercially available software SAP2000. Since the data on effect of partial interaction on deflection of simply supported steel concrete composite girders was available, trial models were made, and the results were verified with already present data as for validation of any numerical modelling technique must be done against already validated present data.

The same methodology has been modified and adopted to develop relationships between partial interaction and deflections for a continuous steel concrete composite girder.

The steel girder of the composite beam is modelled as a frame element in SAP2000, with the frame element placed at the centre of the real steel beam.

The concrete deck was modelled as a frame element (as a wide beam) and was modelled at the centre of where the actual deck would've been. The two elements were connected to each other by means of rigid frame members and spring elements (links).

The property of the steel girder, concrete deck, the rigid links and the spring elements were provided to match the existing data model. Spring stiffness was provided equal to the stiffness of the shear studs for the given degree of interaction. The spacing required for the required degree of interaction of the shear studs were calculated as per methods given in the book "Composite Steel and Concrete Structural Members. Fundamental Behaviour".

The links were provided at the spacing that was so determined, and the stiffness of the links were provided to emulate the stiffness of two shear studs in one row on the steel beam. The stiffnesses of the shear studs were calculated as per the geometry of the shear studs. The failure for the stud occurs when the stud breaks off from the top flange of the steel beam.

### **3.2 Model formulation**

In the current study, various finite element models have been created to simulate the response of a composite beam subjected to loading. The modelling technique is based on technique proposed by Turmo et al. (2015), in their paper "Modelling composite beams with partial interaction".

The model proposed was made up of 5 elements as shown in the figure below.

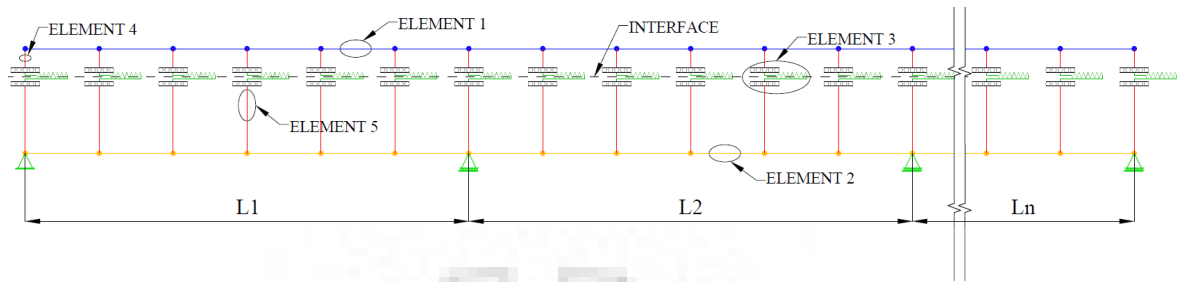


Figure 15: Proposed model analytical layout to simulate the structural behaviour of the composite beam

### 3.2.1 Description of the numerical model

The model presented to study the structural behaviour of composite beams with partial interaction (as the one presented in Figure 15) is based on an intuitive FEM model. In this model, 5 different types of beam elements are used.

The five frame elements of the proposed model are as follows:

1. Element type 1 for concrete slab: these elements model the concrete slab. These elements include the properties of the concrete at its centroid
2. Element type 2 for steel beam: these beam elements model the steel beam and they include the properties of the steel beam at its centroid.
3. Element type 3 for shear connector springs: these elements simulate the effects of the stud-stiffness  $k_q$  of the shear connectors illustrated in the Figure 15. These elements correspond with axial linear springs located at the concrete–steel interface. The shear connector springs have no bending stiffness.
4. Element type 4: these beam elements simulate the distance between the concrete centroid and the steel–concrete interface of the composite beam. These elements connect the concrete nodes with the shear connectors at the concrete–steel interface (nodes of element types 1 and 3). These elements have infinity axial and flexural stiffnesses.
5. Element type 5: these frame elements simulate the distance between the steel centroid and the steel–concrete interface of the composite beam. These elements connect the steel nodes with the shear connectors at the concrete–steel interface (nodes of element types 2 and 3). These elements include infinity axial and flexural stiffnesses. If the shear connectors are uniformly distributed these elements might be spaced a length  $S$  throughout the beam. With  $h_s$  being the height of the steel beam,  $u_s$  being the horizontal

displacement at the steel beam node and  $\theta_s$  its rotation they enable the following horizontal movement  $u_1$  at the concrete–steel interface for a y-symmetric cross section beam:

The connection of the strength is calculated on the basis of longitudinal forces due to bending in the steel and concrete component. The calculations for the spacing of shear studs of certain ultimate strength was made as per methods given in by D.J. Oehlers and M.A Bradford (1995). The ultimate strength ‘Q’ was determined by the methods provided in IS: 11384-1985

### 3.3 Cases considered in the present study

The effect of concrete cracking on the behaviour of the continuous steel concrete composite girders is addressed considering the following three different cases.

- Case 1. Continuous steel concrete girders without considering cracking of concrete in negative bending zone.
- Case 2. Continuous steel concrete girders considering fully cracked concrete in negative bending zone.
- Case 3. Continuous steel concrete girders with effective moment of inertia of cracked concrete in negative bending zone. For the incorporation of the effective moment of inertia of the cracked concrete, a numerical modelling method has been proposed.

## Chapter 4. Continuous Composite Girder Without Cracked Concrete

### 4.1 Introduction

Since the determination of spacing based on degree of shear interaction and was developed for a simply supported steel concrete composite beams, to verify the effectiveness of the numerical model, several models for a simply supported bridge girders were made. The deflection results determined from the model using the stiffness and shear stud spacing as discussed above were in close comparison with those determined by theory given in “Composite Steel and Concrete Structural Members. Fundamental Behaviour”.

### 4.2 Model verification

The models were made for 10m, 15m and 20m spans. With different cross-sectional properties of the composite beam.

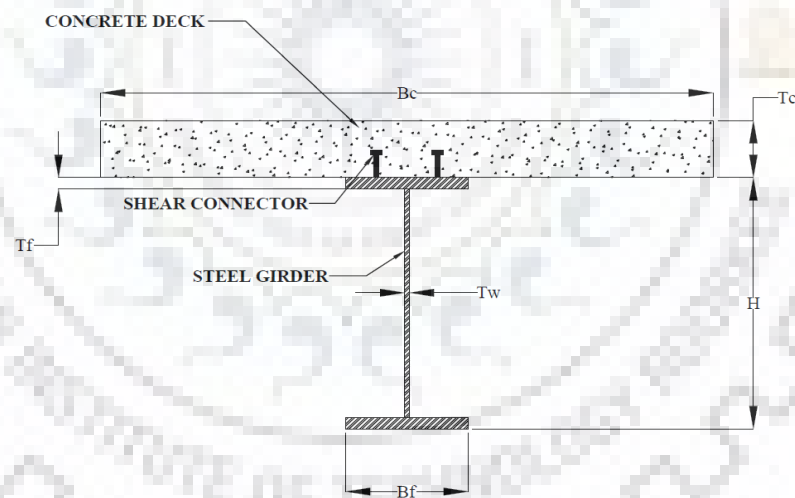


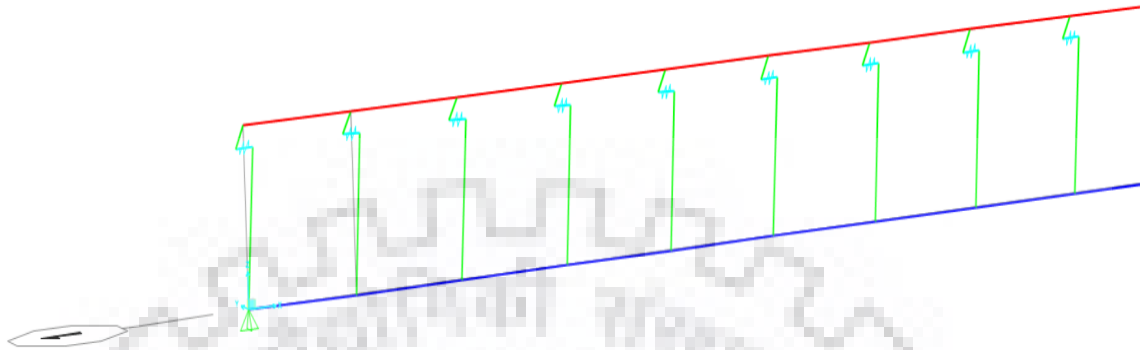
Figure 16: Cross-section of the composite girder

The following table gives the dimensions of the composite girder shown above:

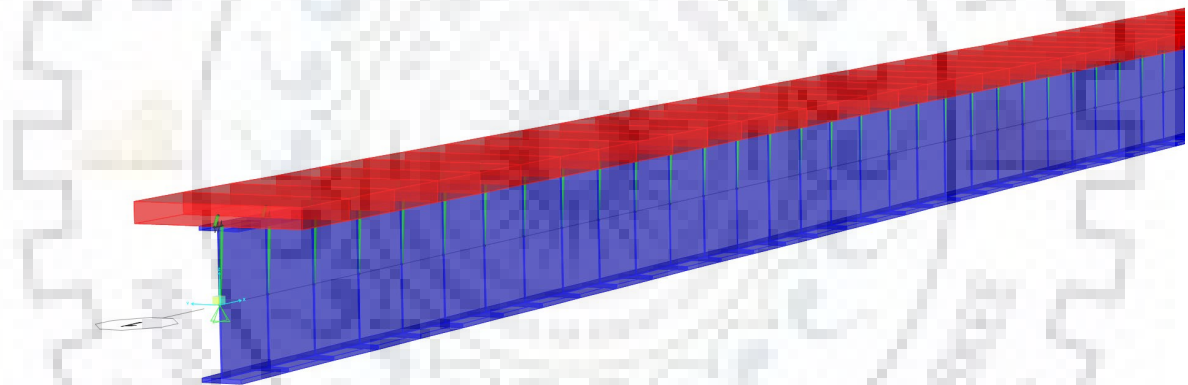
Table 1: Dimensions and Loadings of the Simply Supported Girder

SPAN (m)	LOADING (UDL kN/m)	Dimensions (mm)					
		H	Tf	Tw	Bf	Bc	Tc
10	50	715	20	6	200	2500	250
15	55	950	25	6	250	2500	250
30	70	1590	45	10	450	2500	250

The proposed model ideology has been in SAP2000 and the rendered view of the model is as follows:



*Figure 17: Line diagram of the proposed model (SAP2000)*



*Figure 18: Extruded 3-D model (SAP2000)*

The values of deflections obtained from the theories and those obtained from the model for complete interaction and for partial interaction were in close comparison, this validates the trial model. Some of the results have been represented graphically below



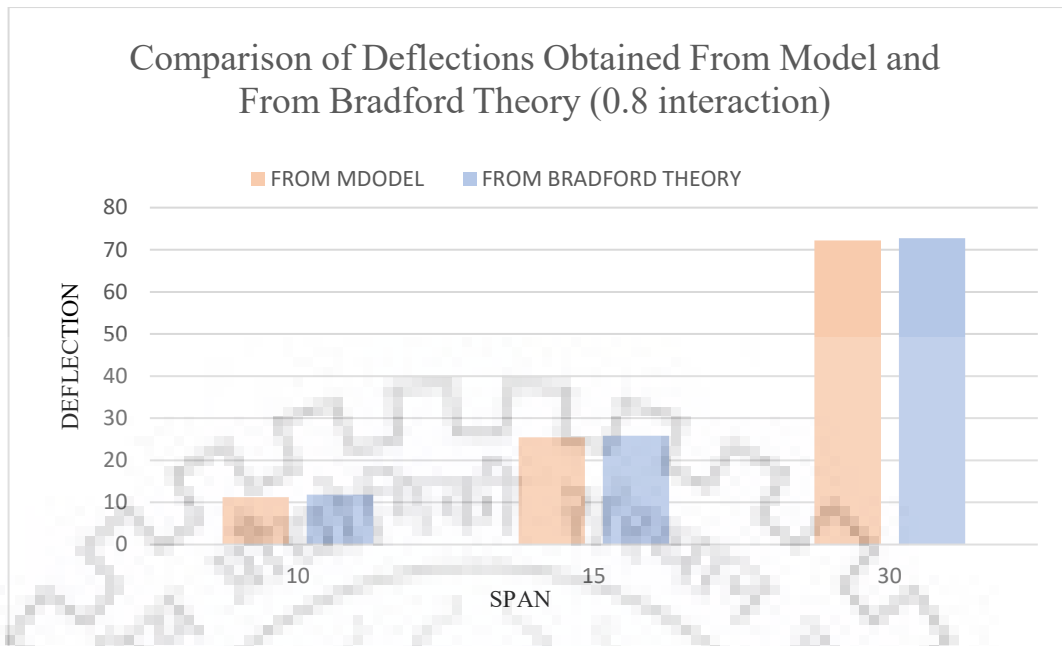


Figure 19: Comparison of deflections obtained from Bradford theory and that obtained from model for 0.8 interaction

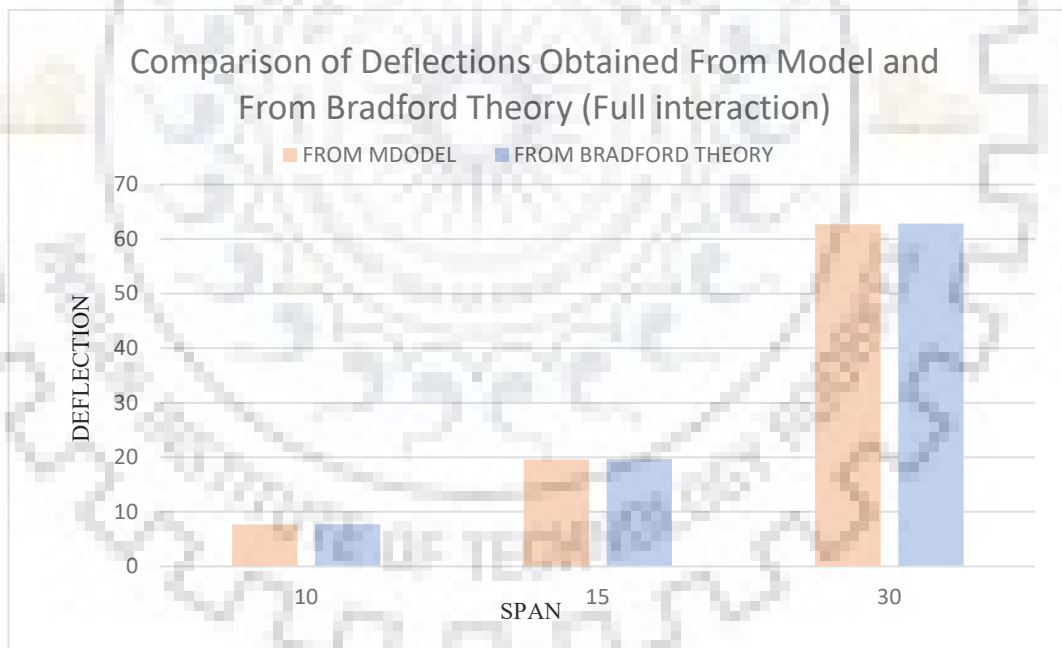


Figure 20: Comparison of deflections obtained from Bradford theory and that obtained from model for Full connection

The slip v/s span for the different spans has also been plotted and found to be in accordance with the trend expected for a simply supported girder. Since the shear force for a simply supported beam is maximum at the supports and that it decreases linearly as becoming zero

at the mid span for a UDL, the slip would also be more at the supports as the slip occurring in the girder is directly proportional to the shear force in the girder.

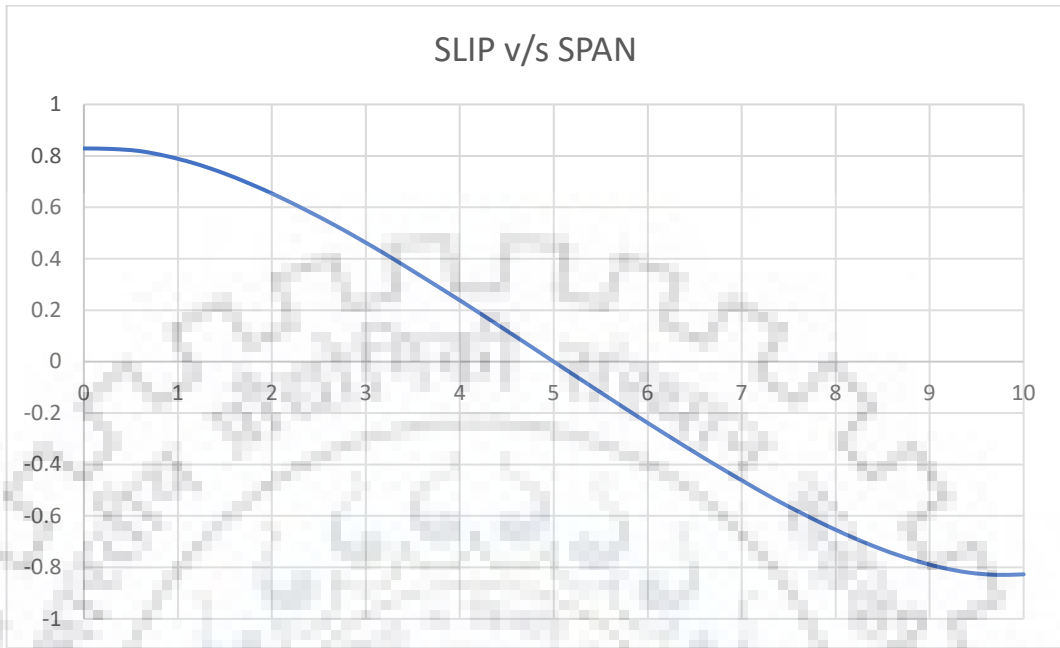


Figure 21: Load v/s Slip for 10m girder with 0.8 interaction

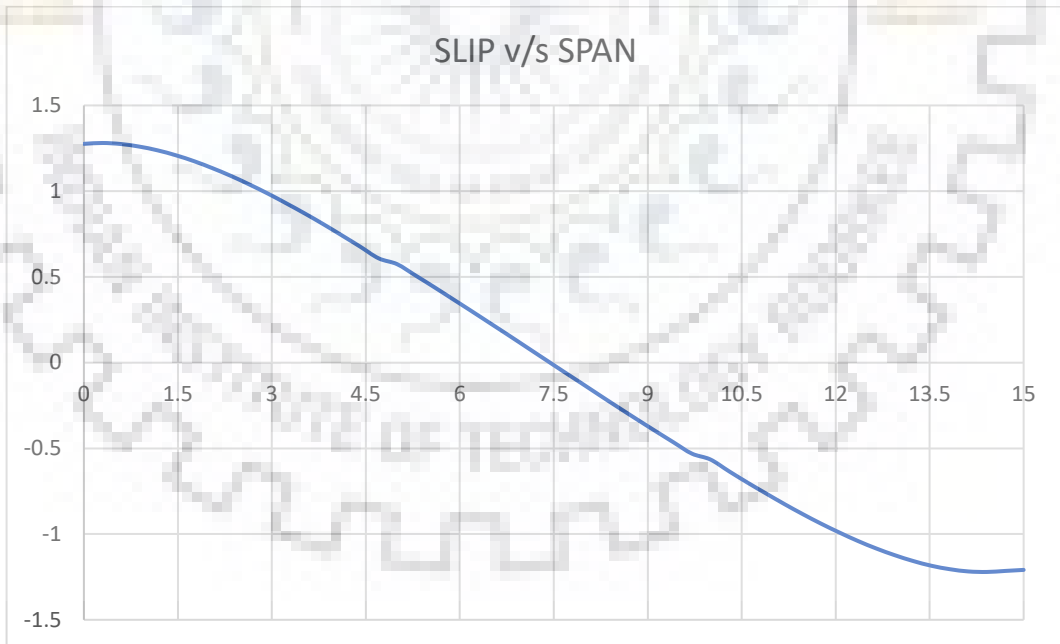


Figure 22: Load v/s Slip for 15m girder with 0.8 interaction

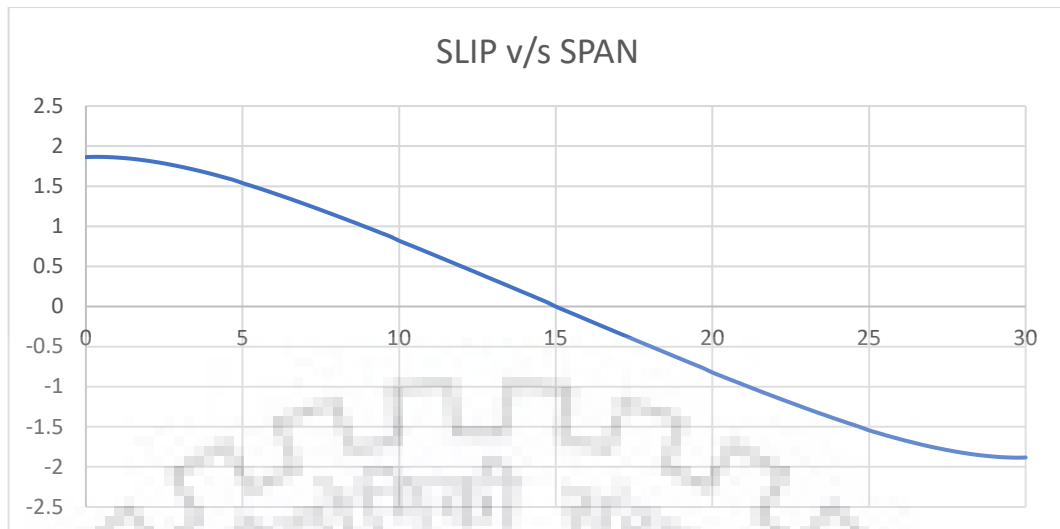


Figure 23: Load v/s Slip for 30m girder with 0.8 interaction

It is also seen that as the span increases, the slip at the supports also increases, this is primarily since as the total shear force at the support is directly proportional to the span of the girder for a girder subjected to a UDL. The greater length of the span means that there would be greater shear force at the support.

#### 4.3 Extension of the numerical method to continuous girders using linear partial interaction theory

There have been many theories which predict the deflection of a simply supported girder with partial interactions but there are far and few theories which can predict the deflection of a continuous composite beam with partial interaction. Plume and Horne (1976), proposed an analysis based on the elastic theory which resulted in a differential equation relating the deflection of the beam to the distance along the beam. The equation contained a derivative of a fourth order in the deflections. It was applied to the case of a continuous composite beams with two unequal spans which were subjected to concentrated loads at mid-span.

The solution of this fourth order differential equation yielded 16 constants of integration in addition to the unknown redundant moments. Which required considerable work and computational effort to obtain the solution. Jasim introduced a method for determination of deflections taking partial interaction into consideration. The composite beam was assumed to be made of two elastic materials. Governing differential equations were employed which were derived from linear partial interaction theory. Design charts were prepared on the basis of the results obtained from the differential equations. Different types of loading and

different configurations, i.e. number and lengths of spans of continuous beams were considered.

### 4.3.1 Analytical modelling as per N.A Jasim (1997)

A general conclusion was drawn that continuous beams can be classified into groups for each of which one design chart may be used. N.A. Jasim (1997) also used the exact solution of the governing differential equation but formulated the equations in such a way that they could be used in everyday design. This made the analysis of continuous composite beam with partial interaction much easier. The method so proposed by Jasim assumed that the moments at the internal supports have same values for both the full interaction and partial interaction. The assumption can easily be justified as it has been seen that the percentage change in the moments for a partial interaction beam from that of a full interaction beam, depending on the position of the section in focus along the length of the beam only varies by about 0-8%.

The procedure starts off with the determination of interior support moments using any method of analysis for a redundant beam. The procedure starts off with the determination of interior support moments using any method of analysis for a redundant beam. The deflections for component beams subjected to a fraction of the loads are determined and method of superposition as depicted in Figure 24 is utilised for determining the final deflection of the actual composite beam.

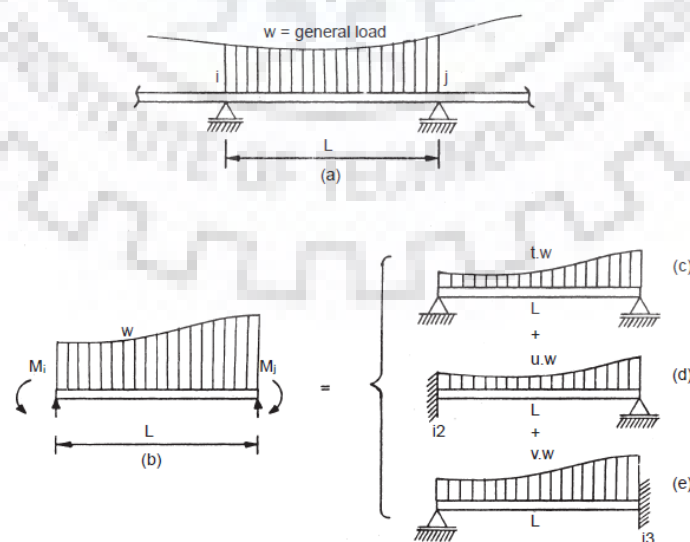


Figure 24: Internal span of continuous beam (Jasim et al. 1998)

The fractions of internal moments for the continuous beam and the fixed end moments of the propped cantilever are determined. Depending upon the properties of the fully composite beam. The factors 'c' and 'K' are determined.

Where 'c' and 'K' are defined as follows:

$$c = \frac{I}{I_m} = 1 + \frac{a^2}{I_m A_m} \text{ and } K = \frac{\sqrt{c_1 L}}{2} \quad (3)$$

Where  $A_m$  and  $I_m$  are the summation of the areas and the moment of inertias of the steel and the concrete component. 'a' is the distance between the centroids of the two components.

$$c_1 = \frac{kn}{pE_2} A_m \left( 1 + \frac{a^2}{I_m A_m} \right) \quad (4)$$

Where k, p and  $E_2$  are the connector stiffness, i.e. load per unit slip, pitch of the connectors and modulus of elasticity of the steel respectively.

The deflections of partially composite beams were defined as a ratio of the deflections of corresponding composite beams with exact same load and orientation but with full interaction.

The ratio of  $y_p/y_f$  for a few different cases have been presented here.

**Case 1:** Two equal span, subjected to a UDL

$$\frac{y_p}{y_f} = \frac{1}{2} (-5 + 3C_N) + \frac{6(c-1)}{K^4 \cosh(2K)} \times [1 + K \sinh(2K) - 2K \sinh(K)] - 2 \cosh(K) + \frac{1}{4} K C_N \{ \sinh(2K) - 2 \sinh(K) \} + (1 - K^2) \cosh(K) \quad (5)$$

**Case 2:** Two equal span, subjected to two concentrated loads at the midspans

$$\frac{y_p}{y_f} = \frac{4}{7} \left[ \left( \frac{9}{4} C_N - 4 \right) + \frac{12(c-1)}{K^3 \cosh(2K)} \left\{ \frac{3}{2} \sinh(2K) - 2 \sinh(K) - \frac{3}{16} C_N [2 \sinh(K)] - \sinh(2K) \right\} - K \cosh(2K) \right] \quad (6)$$

**Case 3:** Three equal span, subjected to a concentrated load at the midpoint of middle span

$$\frac{y_p}{y_i} = \frac{1}{11} \left[ (10 - 21C_N) + \frac{3}{2} \frac{c-1}{K^3} \left\{ (20 + 3C_N) \times \left[ -2K + \frac{3 \sinh(2K) \cosh(K)}{\cosh(3K)} \right] - 2(50 + 3C_N) \times \frac{\sinh(2K)}{\cosh(3K)} + 40 \tanh(3K) \right\} \right] \quad (7)$$

**Case 4:** Two equal span, subjected to a UDL on one span only

$$\frac{y_p}{y_f} = \frac{1}{7} \left[ (-10 + 3C_N) + \frac{6(c-1)}{K^4} \times \{ -4[1 - \cosh(2K)] \times \left[ \frac{2 \sinh(K)}{\sinh(2K)} + \frac{2 \sinh(K) - \sinh(2K)}{2 \cosh(2K) \sinh(2K)} \right] - K(4 + C_N) \times \frac{2 \sinh(K) - \sinh(2K)}{2 \cosh(2K)} + 8[1 - \cosh(K)] - 4K^2 \} \right] \quad (8)$$

Where,  $y_p$  is the deflection of the girder for a partial interaction,  $y_f$  is the deflection of the girder for a complete interaction.

There were certain assumptions that were made in order to reach at the equations. These are listed below:

1. Reinforcement in the slab is neglected
2. Concrete and steel girder are linearly elastic materials with same elastic properties in tension and compression.
3. Shear connection is continuous along the entire length of the continuous composite girder.
4. The concrete and steel components have similar curvature at their respective midplanes.
5. The shear connectors have equal modulus of elasticity and are equally spaced along the entire length of continuous composite beam.

The validation of the proposed model was done against the method proposed by Jasim. The validation was done by modelling two composite beams and comparing the deflections obtained from the model with deflections obtained from application of Jasim’s method.

#### 4.4 Numerical modelling of continuous composite girders

##### 4.4.1 Validation 1

A finite element model for a 3 span (20m) continuous beam subjected to a point load of 1000kN has been created using the 5-element method of modelling as described earlier.

The cross-sectional properties for the continuous composite beam are as follows.

*\*Refer Figure 16 for the variables*

Table 2: Dimensions of 20m 3-span continuous composite beam

SPAN (m)	LOADING (Point kN)	Dimensions (mm)					
		H	Tf	Tw	Bf	Bc	Tc
20	1000	1200	25	10	250	2500	250

The results of the model were compared with the deflections obtained from the method proposed by N.A Jasim. The span v/s slip and span v/s deflection graphs have been shown below. As seen for a simply supported beam, the slip variation along the span of the beam varies as the shear at a point varies along the span.

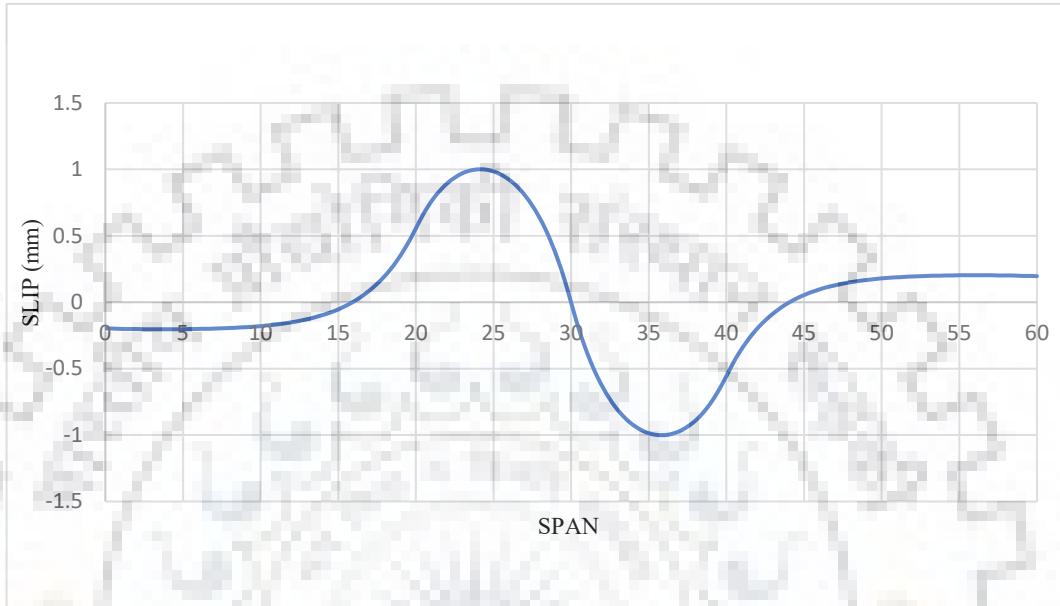


Figure 25: Slip v/s Span for a 3-span continuous beam subjected to point load

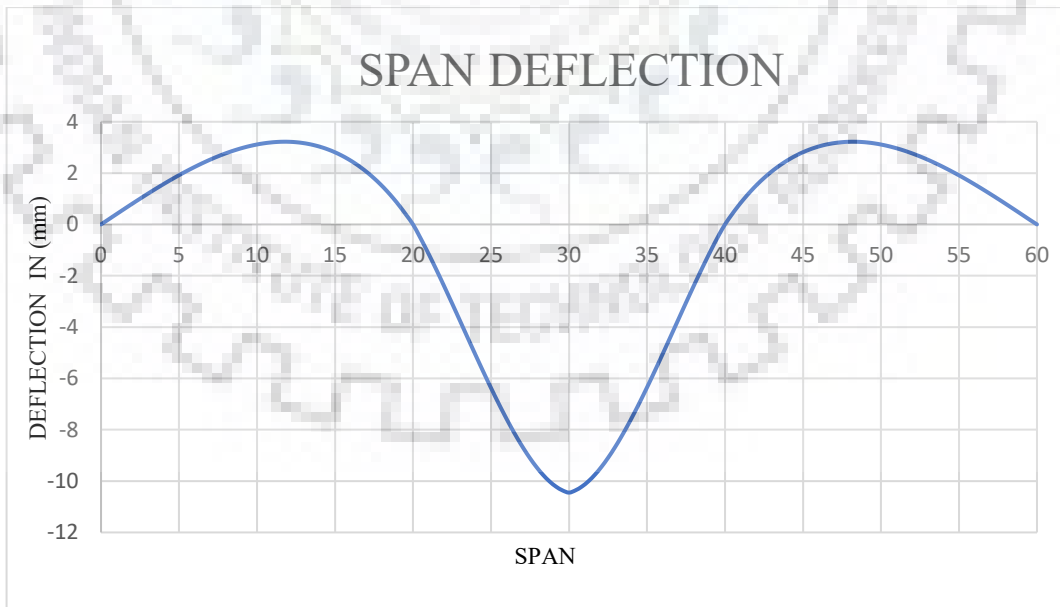


Figure 26: Deflection v/s Span for a 3-span continuous beam subjected to point load

The comparison has been made to the deflection obtained from the method presented by N.A Jasim and A. Atalla and that obtained from the model and are presented in the plot below.

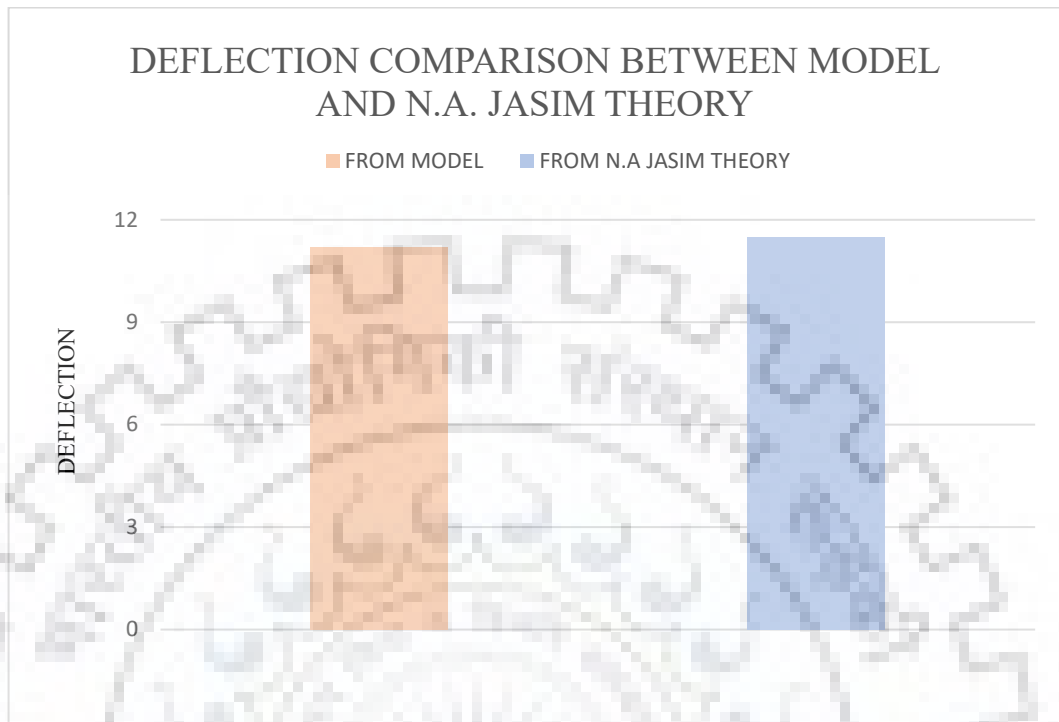


Figure 27: Deflection comparison

#### 4.4.2 Validation 2

Another composite beam configuration was validated, and the results were compared with the method proposed by Jasim.

The cross-sectional properties and the load for which the beam was subjected to has been documented below:

\*Refer Figure 16 for the variables

Table 3: Dimensions of 10.5m 2-span continuous composite beam

SPAN (m)	LOADING (UDL kN/m)	Dimensions (mm)					
		H	Tf	Tw	Bf	Bc	Tc
10.5	45	409.4	14.3	8.8	178.8	1800	150

The beam was first modelled as having a rigid connection and then having a flexible full shear connection having a connection stiffness of 400000kN/m.



The beam was first modelled as having a rigid connection and then having a flexible shear connection having a connection stiffness of 400000kN/m.

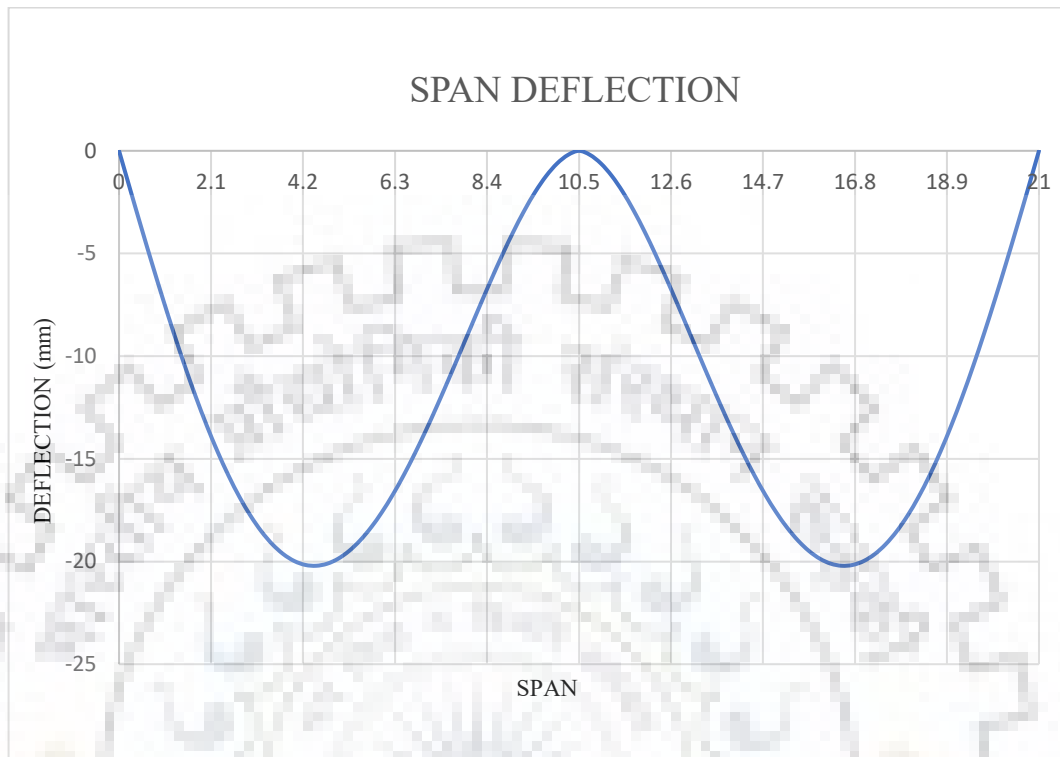


Figure 28: Deflection v/s Span (Rigid)

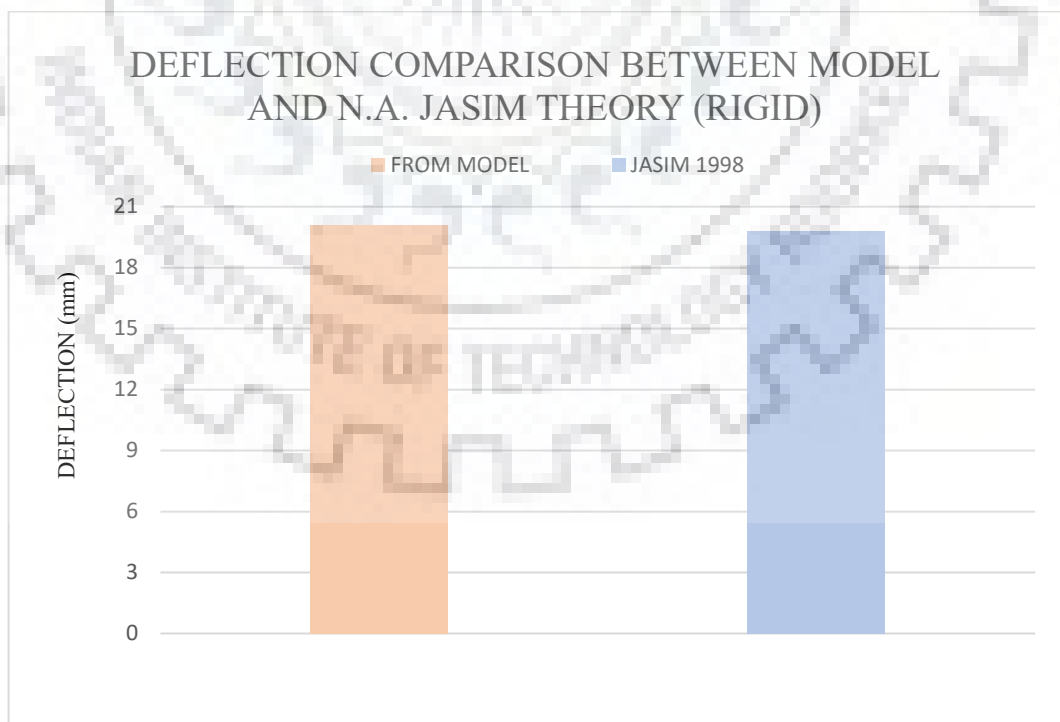


Figure 29: Deflection comparison (Rigid)

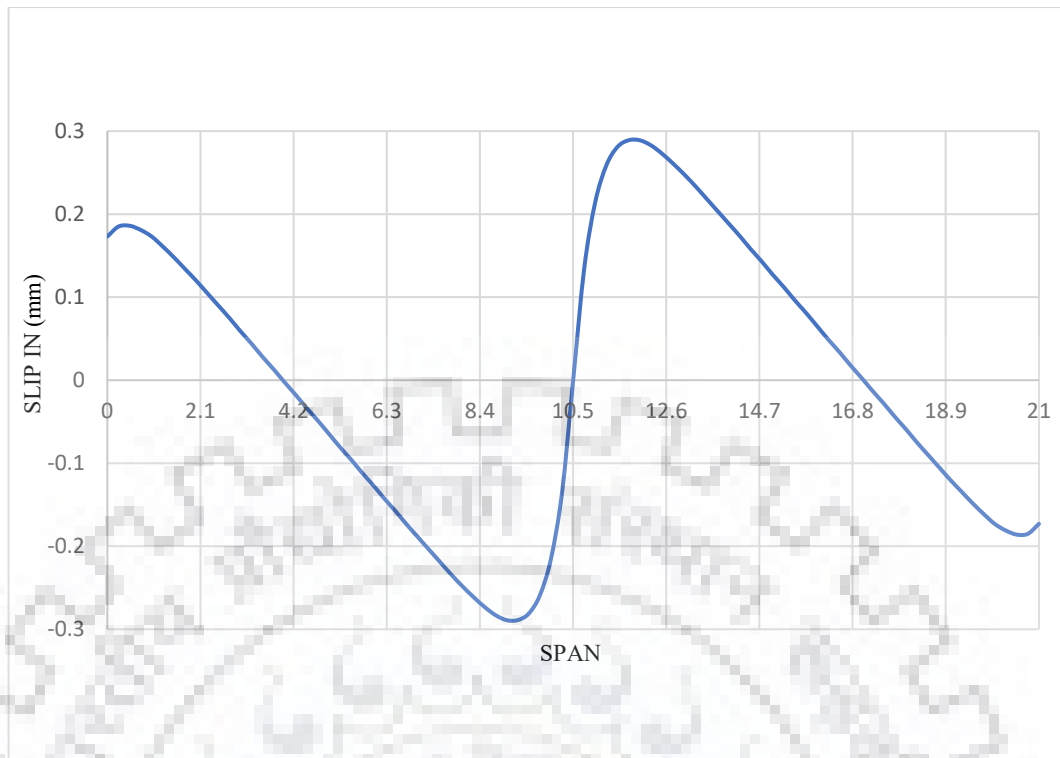


Figure 30: Slip v/s Span (Flexible shear connection)

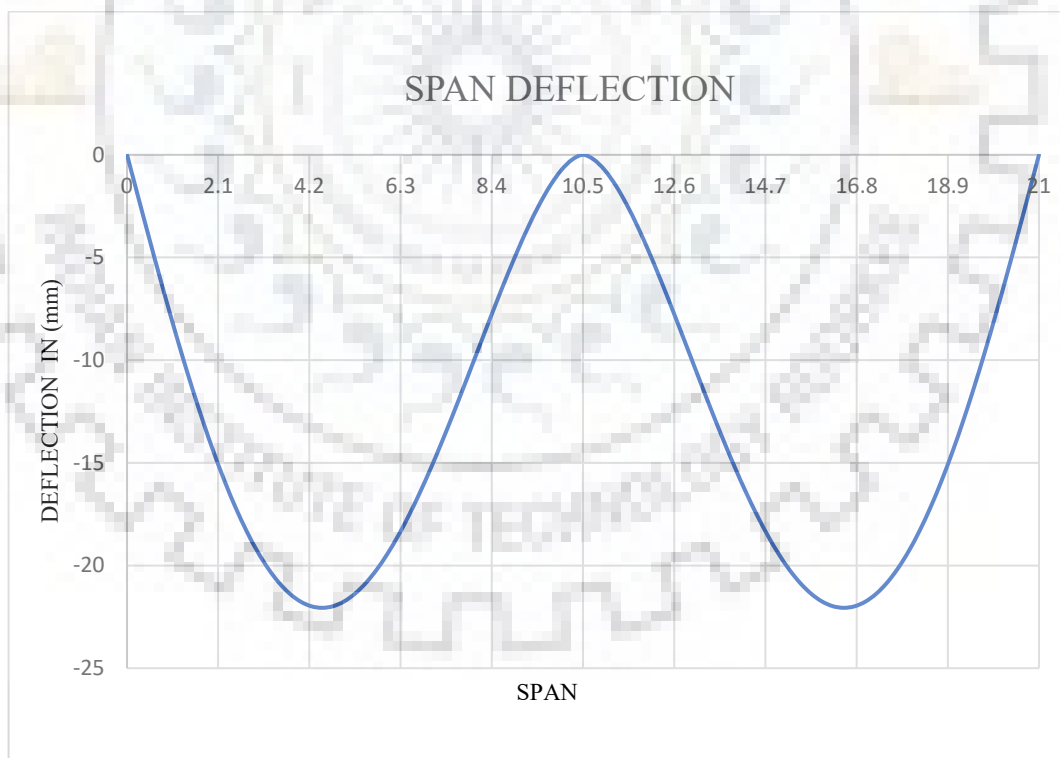


Figure 31: Deflection v/s Span (Flexible shear connection)

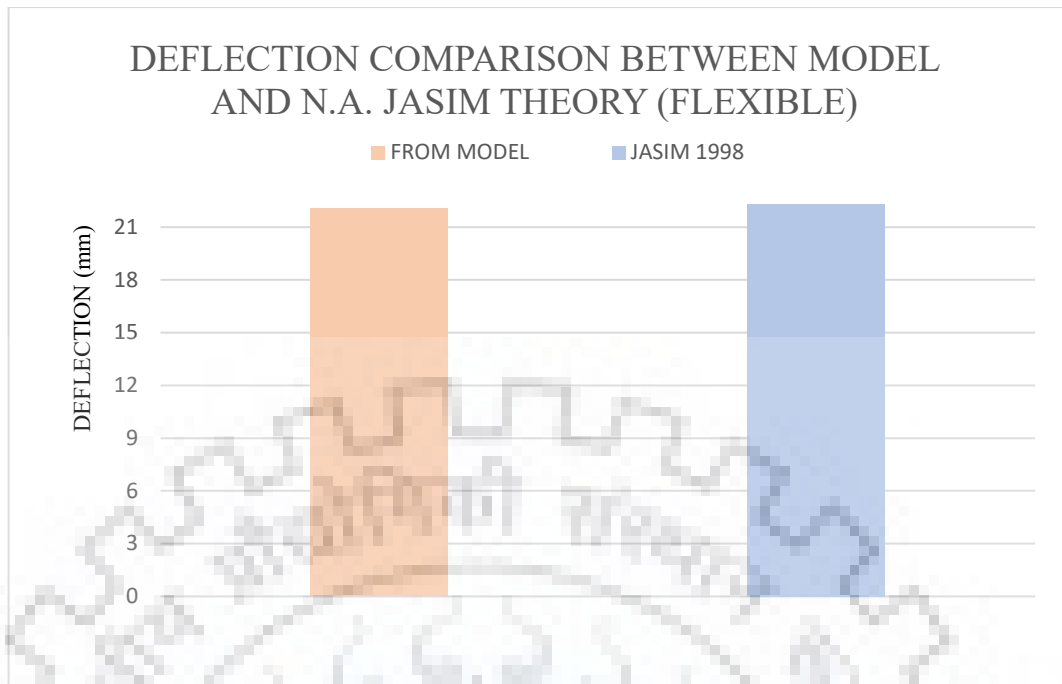


Figure 32: Deflection comparison (Flexible shear connection)

The error percentage between the values of deflections determined by the numerical model and the deflections obtained by the method proposed by N.A Jasim has been plotted in the following figure (Figure 31). The error percentage recorded was 1.36% for rigid connection and 1.08% for a flexible 0.05 partial interaction.

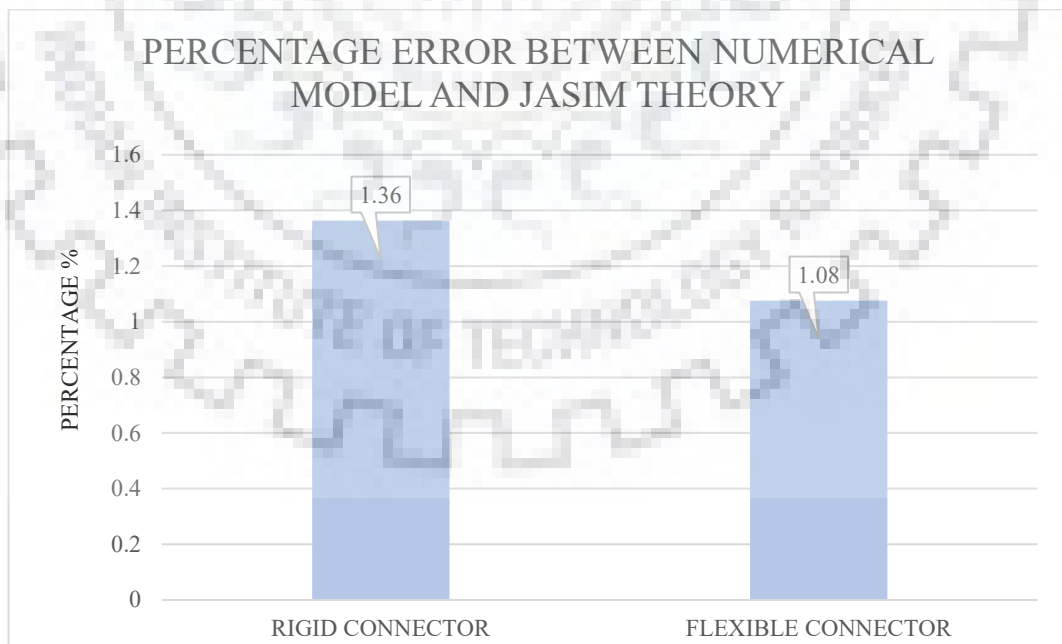


Figure 33: Percentage error between the numerical model and theory proposed by N.A Jasim

### 4.4.3 Parametric studies

After the validation of the model, parametric studies were done for better understanding of the trends that could be expected through the application of the model. The parametric study was done for three spans 7.5m, 15m, 30m the span configuration configurations being 2,3 and 5. Deflections have been documented in the form of span- deflection curves for a rigid connection, full shear connection, 0.8 interaction and 0.5 interaction. The grade of the concrete used is M40 and the grade of steel used is Fe250.

The property dimensions have been documented in the table below:  
 \*Refer Figure 16 for the variables

Table 4: Dimensions of continuous composite beam for parametric studies

SPAN (m)	LOADING (UDL kN/m)	Dimensions (mm)					
		H	Tf	Tw	Bf	Bc	Tc
7.5	10	500	9.8	6.2	120	1300	150
15	15	650	12	6.2	140	1300	150
30	50	1160	30	12	300	1500	150

#### 4.4.3.1 Results of the parametric studies

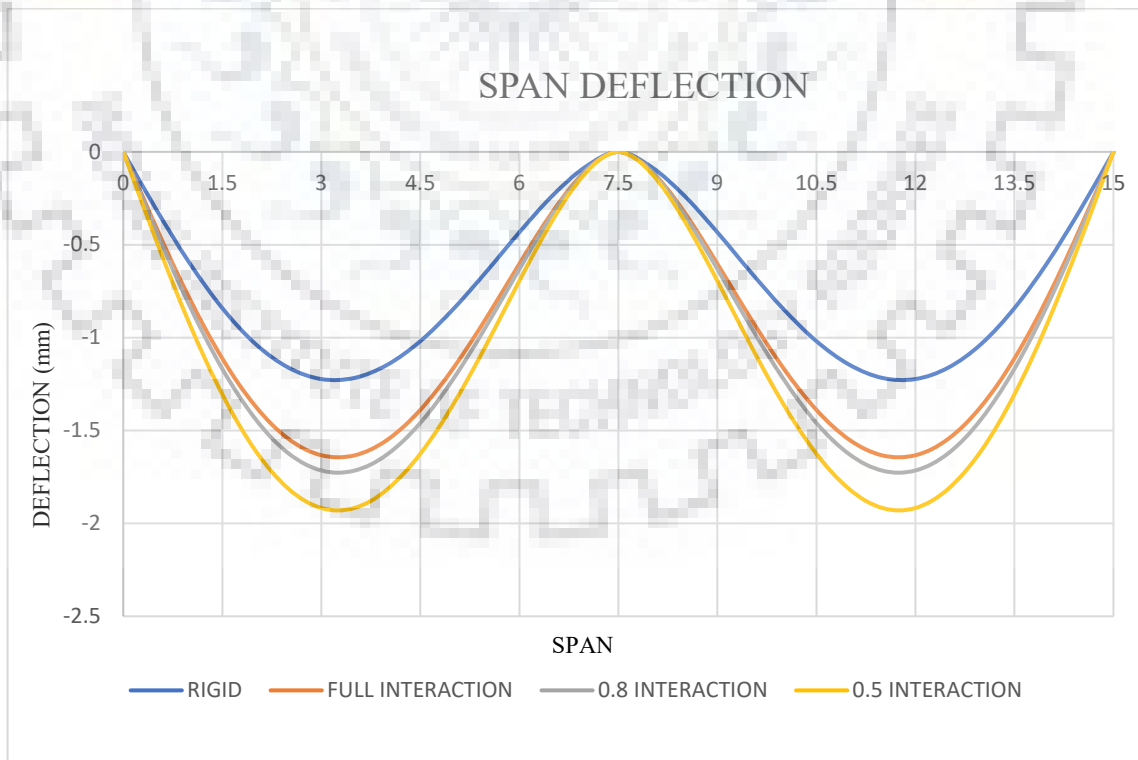


Figure 34: Deflection v/s Span (7.5m 2-span)

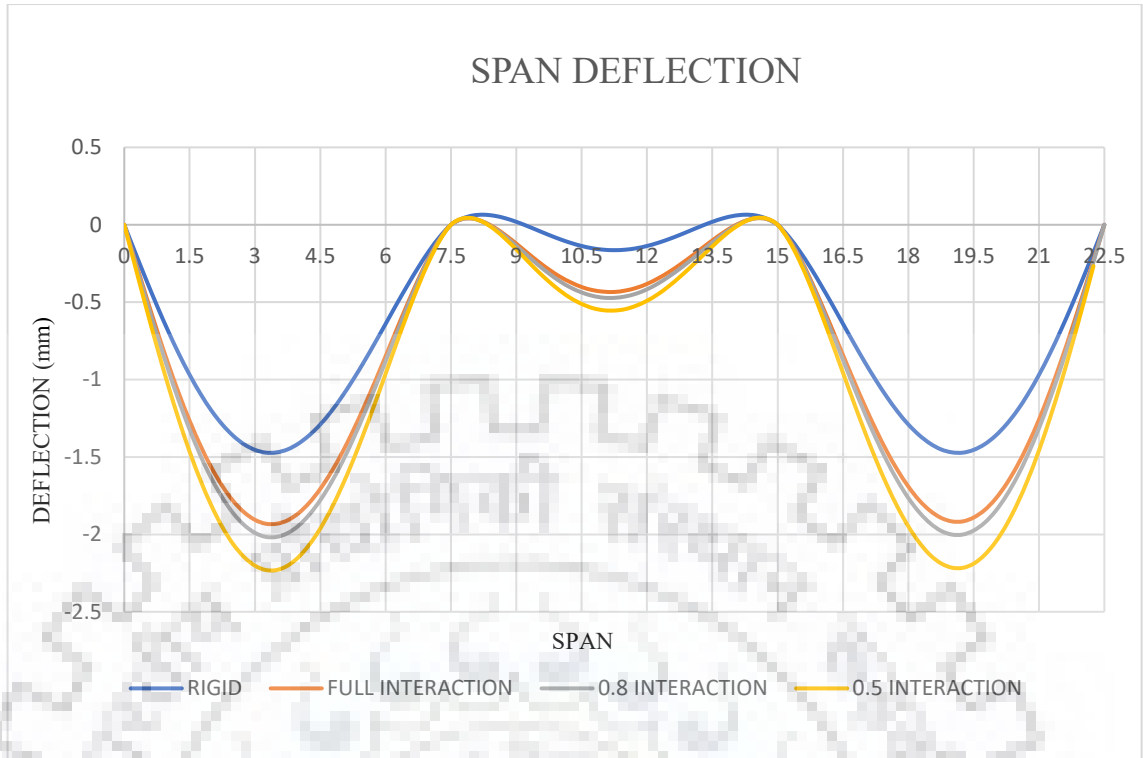


Figure 35: Deflection v/s Span (7.5m 3-span)

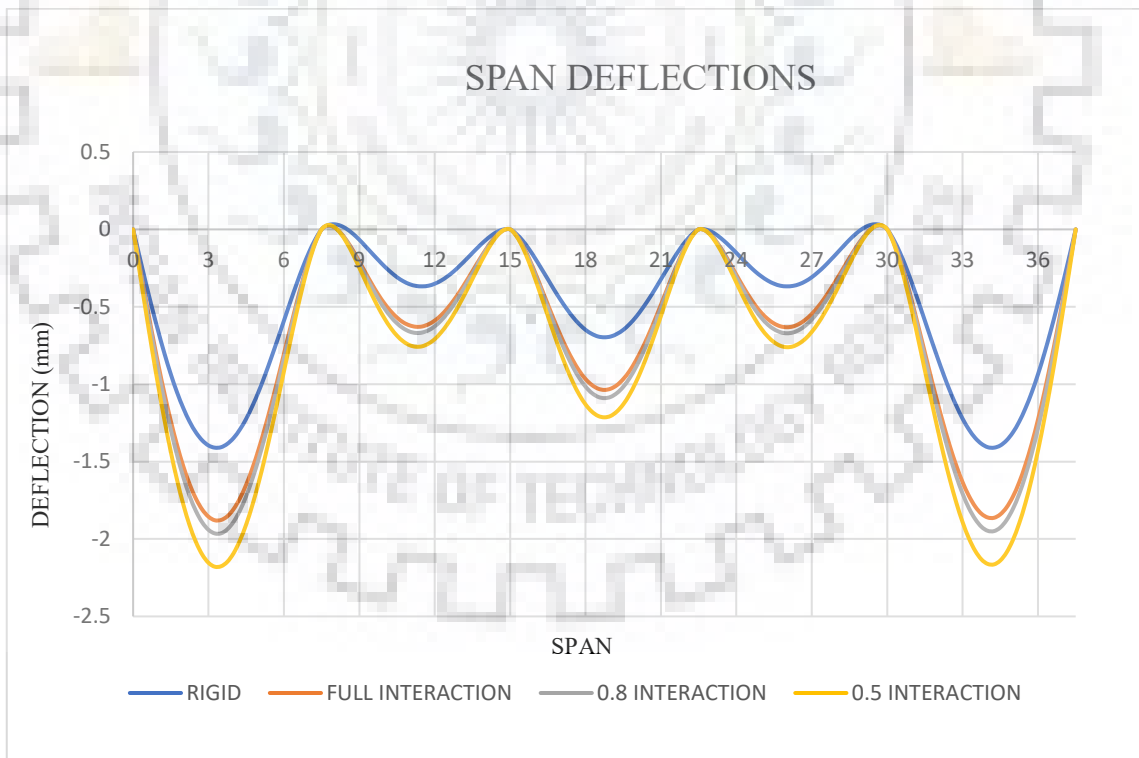
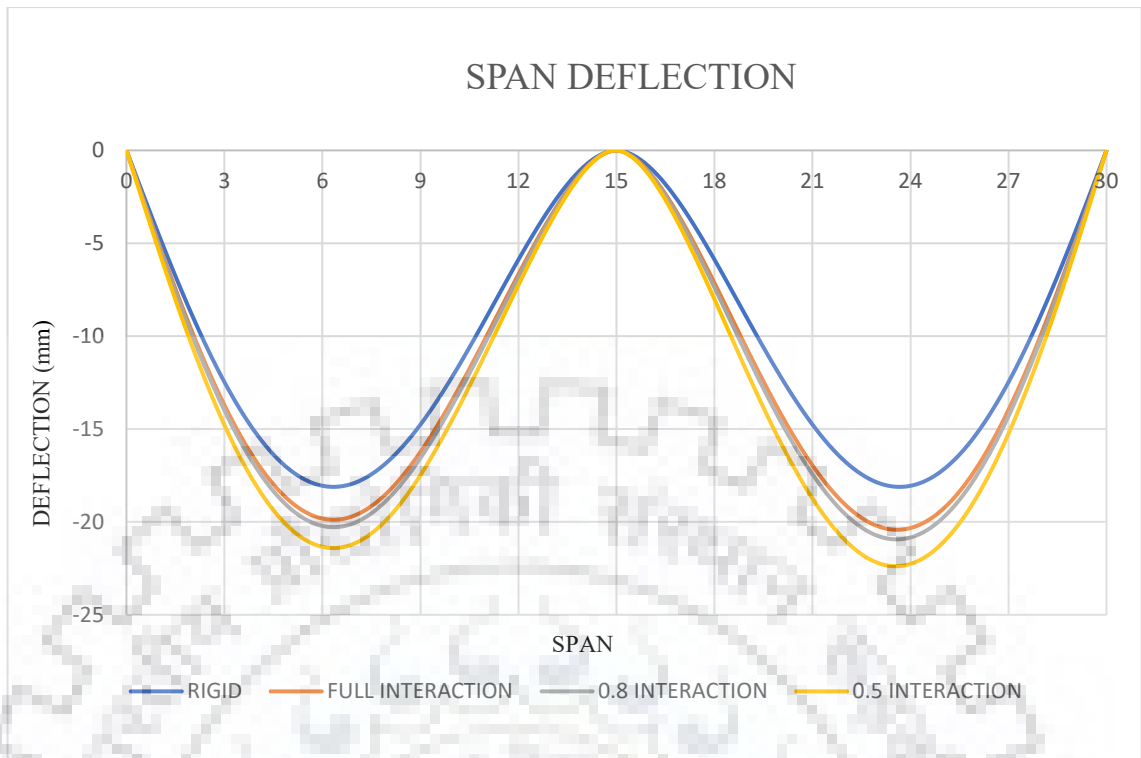
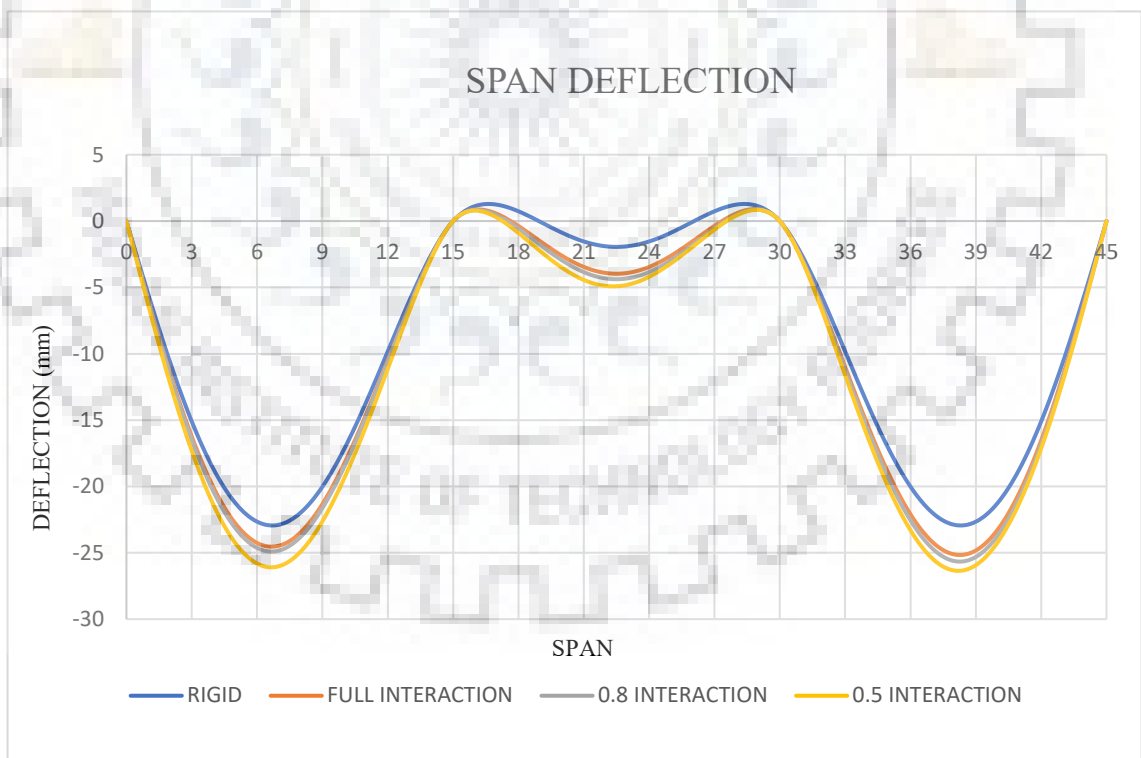


Figure 36: Deflection v/s Span (7.5m 5-span)



*Figure 37: Deflection v/s Span (15m 2-span)*



*Figure 38: Deflection v/s Span (15m 3-span)*

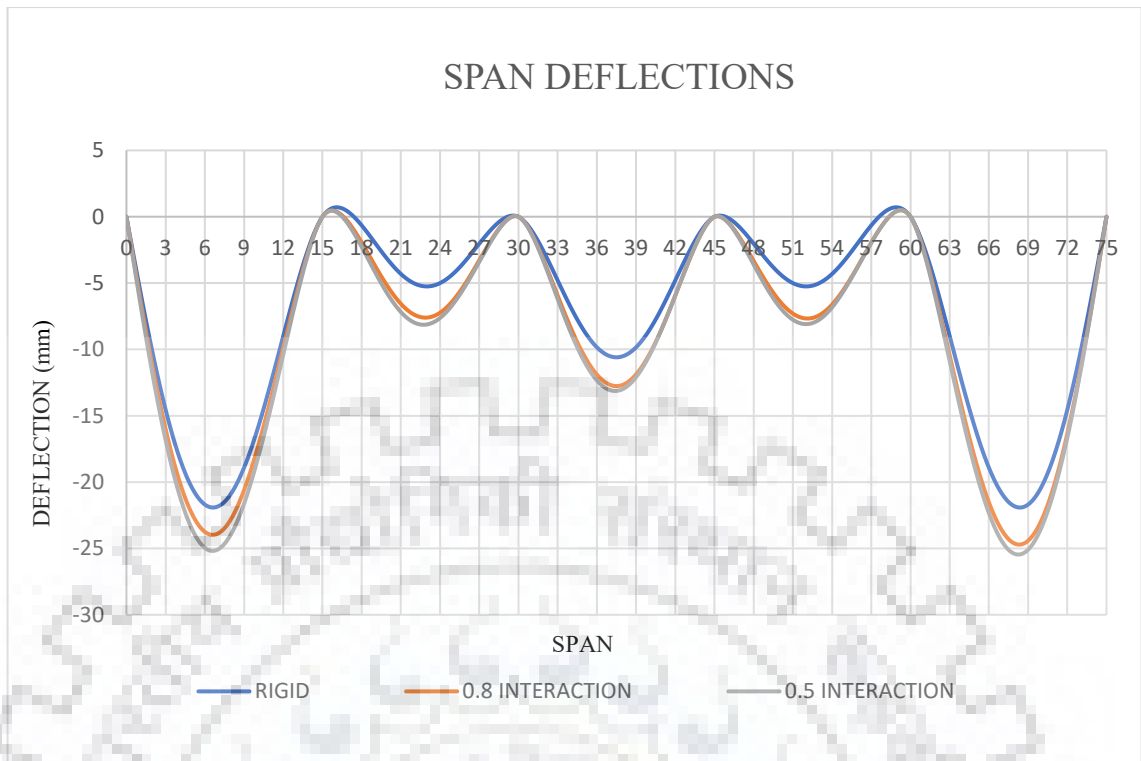


Figure 39: Deflection v/s Span (15m 5-span)

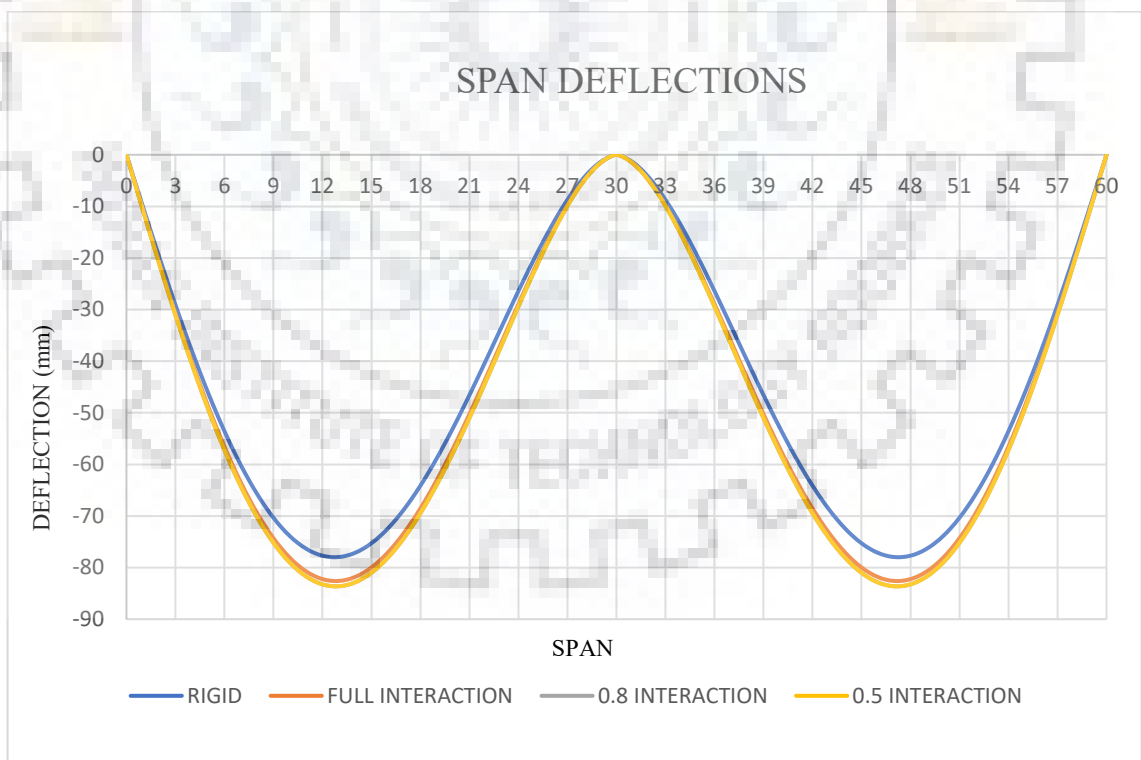


Figure 40: Deflection v/s Span (30m 2-span)

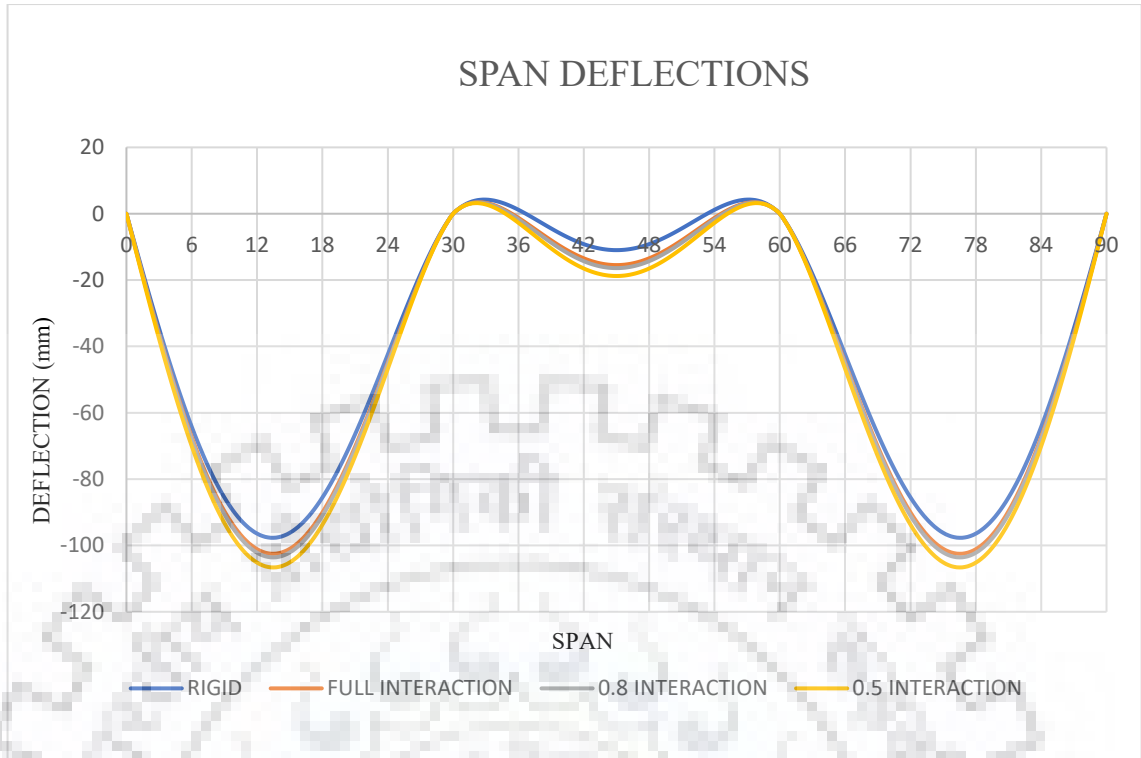


Figure 41: Deflection v/s Span (30m 3-span)

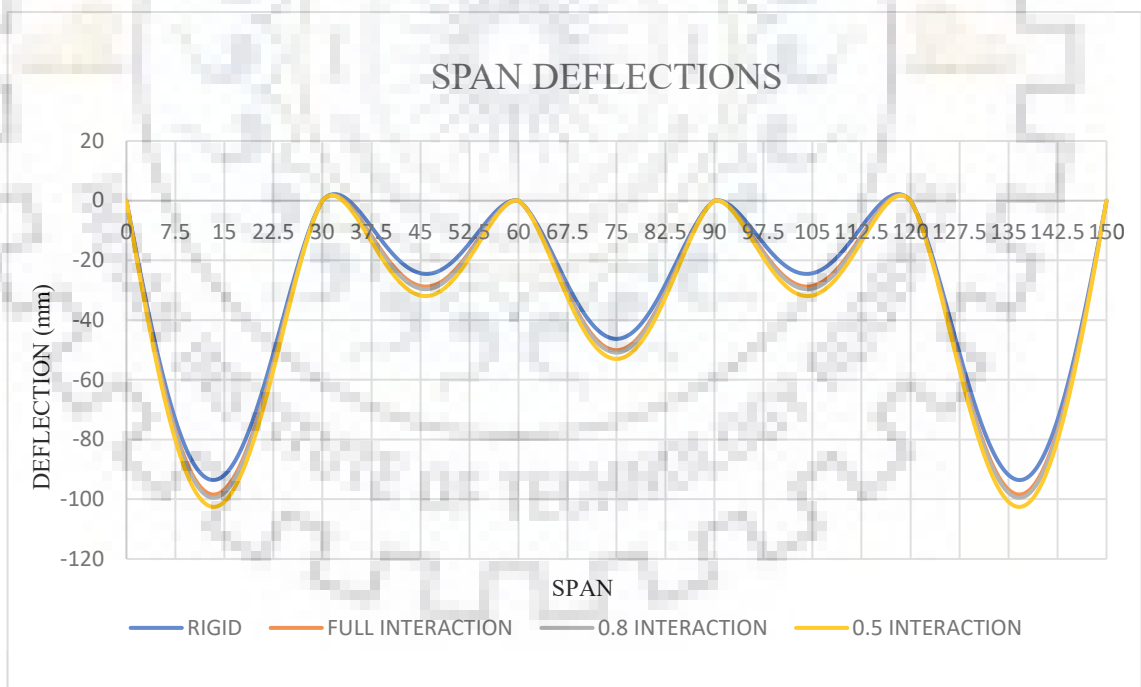


Figure 42: Deflection v/s Span (30m 5-span)



#### 4.4.4 Conclusions

The method proposed by Jasim for the determination of deflections of continuous composite beam with partial interaction. However, this method fails to incorporate certain parameters such as the cracking of the concrete under the influence of the negative bending moment and the buckling of the steel girder, which can affect the results so obtained. The method gives a fair idea of how a continuous composite beam will behave when the degree of shear interaction is varied. The effect of change of shear interaction on the deflection and slip has been discussed.

The parameters such as the degree of shear interaction, the magnitude of the applied moment, the span configuration play an important role in affecting the behaviour of the steel concrete composite girders.

Since this method is based on off an assumption that the two component materials i.e. concrete and steel remain linearly elastic, the results obtained for the deflections and slip is not very well representative of how an actual steel concrete composite beam will behave when it is subjected to loading.

The behaviour of a steel concrete composite beam is better represented by theories which include cracking of concrete at the internal support. The model which incorporate cracking of concrete when subjected to negative bending have been discussed in depth in the next chapter.

# **Chapter 5. Continuous Composite Girders Considering Fully Cracked Concrete**

## **5.1 Introduction**

This chapter attempts to incorporate the cracking of concrete under negative moment in the steel concrete composite girders and a) validate the numerical approach against already published data and b) to study the variation of slip and deflections on various span length and span configuration.

Continuous beams in general will always be subjected to a negative bending moment at the supports. This provides for a case which requires in depth analysis to prevent plastic hinge formation in the steel concrete composite girders at the supports. Adequate negative reinforcement needs to be provided for the beam to be able to resist the formation of plastic hinges.

Generally, cracking of the concrete affects the overall strength of the concrete. Numerous studies have been done in order to predict the behaviour of continuous composite beams after the concrete has cracked under the influence of negative bending moment. Most of the studies consider concrete to be completely cracked and take no consideration of residual strength of concrete. The steel girder at the interior support is designed to carry the entire negative moment at the support. This leads to design of structures which are in general conservative (both in the determination of the strength of the girder and the determination of the deflections), which leads to an increased usage of construction materials leading to greater expenses. This approach also leads to the susceptibility of the structure to attract greater seismic loads as the seismic loads are directly proportional to the weight of the structure.

In general, the amount of study and the computational work required for the determination of the residual strength of cracked concrete under negative bending moment weighs out the conservative design approach that is followed.

Theories proposed by Manfredi et al. 1999, Faella et al. 2003 and even Ranzi in his PhD thesis 2003 and in the papers with M.A Bradford 2006 and 2009, all consider cracking of concrete under negative bending in one way or the other. But the common feature of all these theories is that they all neglect the residual strength of concrete after it has cracked.

Ranzi in his PhD thesis gave an in-depth analysis of composite beams with partial shear interaction. Ranzi used the Direct Stiffness Method for the determination of slip and deflections of the continuous composite beam.

## 5.2 Direct stiffness method.

The direct stiffness method as adopted by Ranzi in his thesis is based on off on methods derived from Newmark's model proposed in the article by Newmark et al.. (1951). The Newmark's model was based on simplifying assumptions in the mechanics of the materials, such as elastic behaviour, the Euler-Bernoulli beam theory of bending for the top and bottom elements respectively and the prevention of vertical separation of the steel and concrete components. The method also highlighted how the partial interaction problem was governed by a linear differential equation of the second order in the longitudinal force resisted by the top element, and the unknowns contained in the equation were the longitudinal force and the expressions for the moment along the beam. In regard to the behaviour in the linear-elastic range of composite beams with two layers, majority of the studies had focussed on the behaviour of steel concrete composite beams assuming that no axial load was taken up by the composite cross section with the exceptions of the finite element models and of the model presented by Girhammar and Gopu (1993). One of the earliest forms of stiffness element method able to carry out linear elastic analyses without the need of appreciated fields was developed by Faella et al.. (1997) and was later re-proposed by Faella et al. (2002), as an element with 6 degrees of freedom, which included the deflections, the rotations and the slips at both element ends. Stiffness analysis carried out using the mentioned element yielded the same results that were obtained by solving the differential equation of the partial interaction problem mentioned in Newmark's article. The assumption to the was that the element ignored the axial load carrying capacity by the cross section of the continuous composite girder.

The matrix for the element was derived by inverting the flexibility matrix obtained for the case of a simply supported beam. The flexibility coefficient of which had already been determined in the paper published by Coenza and Pecce 1997. The inverted degrees of freedom were rotations and the slips at the beam ends. Rotational equilibrium was applied at both ends the additional vertical degree of freedoms were then introduced. Therefore, Ranzi used the linear-elastic range for the derivation of the stiffness element. The element was able to handle axial loading. Hence the element had 8 degree of freedoms.

### 5.3 Proposed numerical model

The model introduced in the previous chapter has been modified in order to incorporate the cracked concrete properties in a similar way the Direct Stiffness Method incorporates the cracked concrete properties.

The numerical model was prepared in a similar way as mentioned in the previous chapter. The moments developed in the concrete element due to the curvature formation of the girder was noted. The lengths of the span where the moment of in the girder was negative (near the internal support and at the fixed end) were provided a zero-pseudo moment of inertia and area for shear interaction. The model was analysed again, and the similar process was repeated till the length for which zero moment of inertia was to be provided became constant.

The results of the resultant model were interpreted. The span-moment, span-deflection and span-slip graphs were generated. For the development of the span-moment graphs, the component forces (both axial and moments) for both steel and concrete were determined, the component moments were added to the moment formed by the couple of the axial forces in the components. The span-deflection and span-slip curves were directly determined from the numerical model.

### 5.4 Validation

Ranzi determined the deflections, slip and the overall bending moment for continuous steel concrete composite girder. The girder chosen by Ranzi for his analysis was a three span continuous beam with one far end fixed and the other simply supported. The span length of the continuous composite beam is 10m each span and the stiffness of the shear connectors assumed was 12000 kN/m<sup>2</sup>.

The general arrangement of the beam used by Ranzi for studying the behaviour of cracking of concrete on overall slip, deflection and moment is depicted in Figure 43.

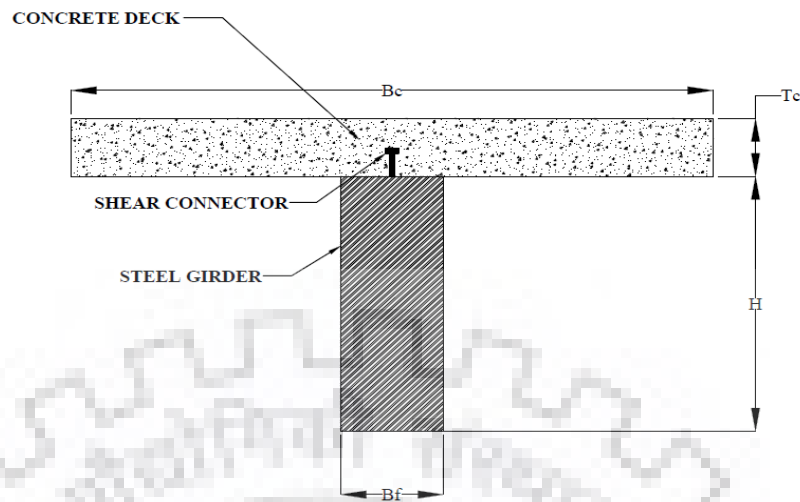


Figure 43: General arrangement for a 3-span continuous beam subjected to a 10kN/m UDL

The dimensions and the loading properties have been tabulated below:

Table 5: Dimensions of a 3-span continuous composite beam for validation

SPAN (m)	LOADING (UDL kN/m)	Dimensions (mm)			
		H	Bf	Bc	Tc
10	10	300	60	600	300

The numerical model was modified from the model used for validation of results with the theory presented by Jasim et al. in the previous chapter. The modification was made in the form of providing property modifiers to the concrete deck frame model. As per the theory proposed by Ranzi, the concrete in the negative zone shall be ignored for all intents and purposes.

The analysis done by Ranzi was by using a direct stiffness method (developed by Ranzi and M.A Bradford (2003)). The method uses an iterative process in order to determine the deflections, slips and the moment in the span at various control points along the length of the girder. The results obtained from the theory proposed by Ranzi and the results determined from the method proposed in this work have been compared and the results have been found to agree with each other. The comparison has been drawn in terms of deflections of the girder, slip between the two components at the interface and the moments carried by the girder at different points along the length of the girder.

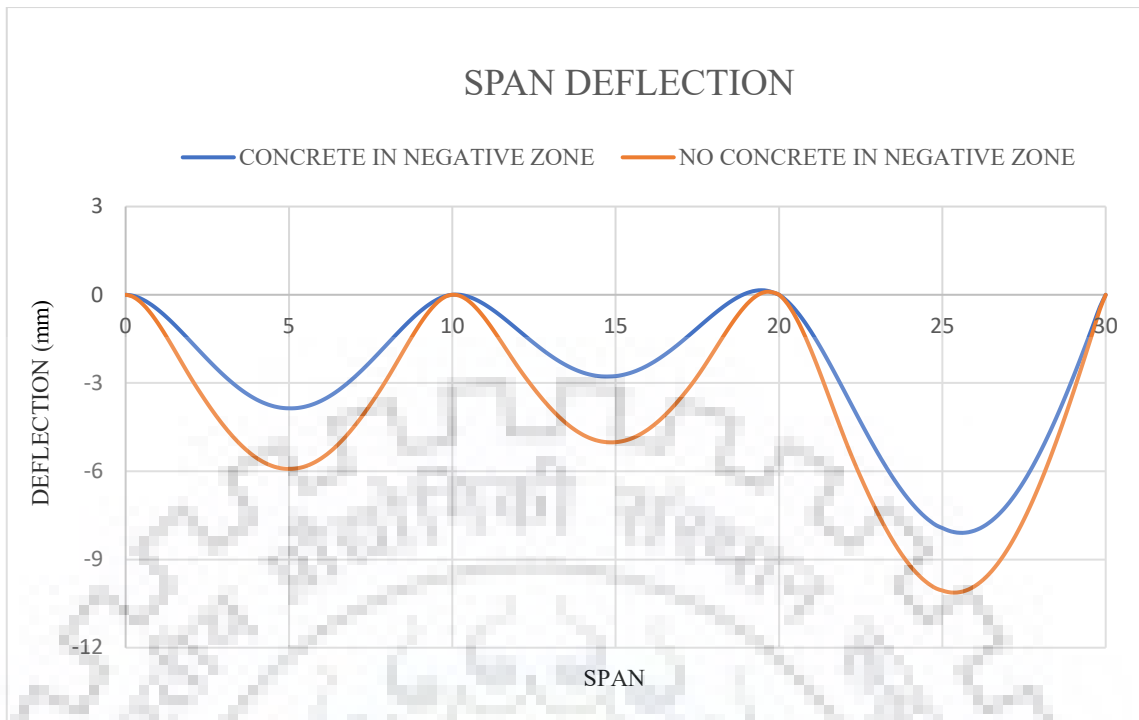


Figure 44: Deflection v/s Span for a 3-span continuous beam subjected to UDL

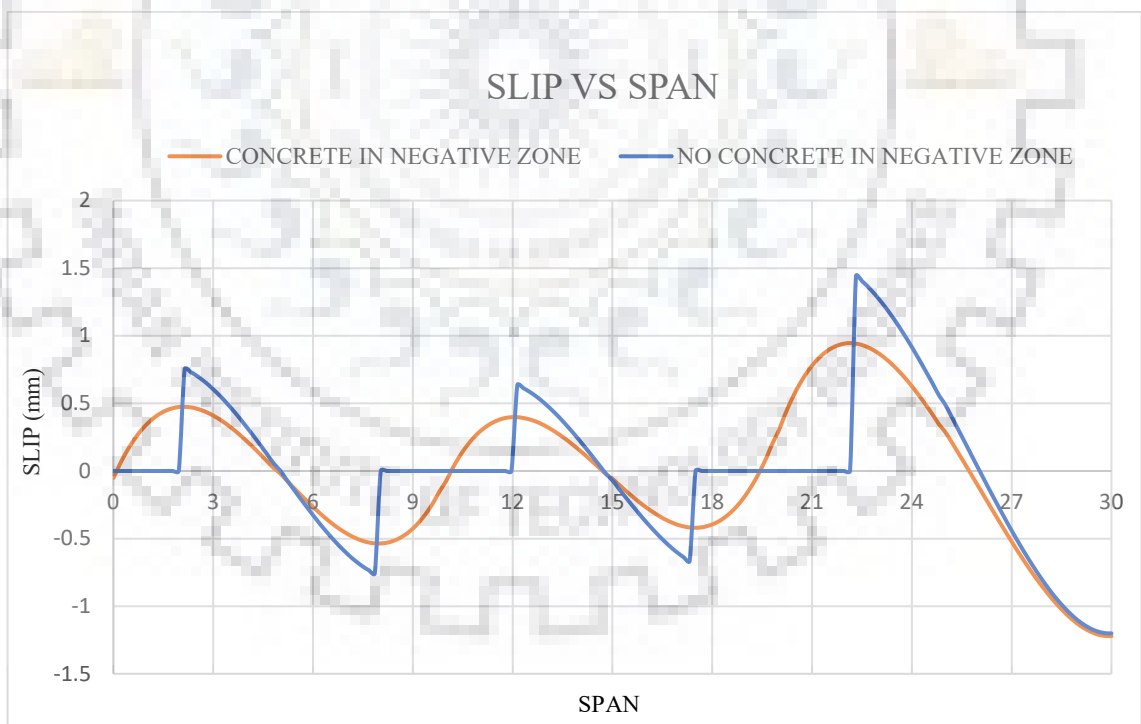


Figure 45: Slip-Span for a 3-span continuous beam subjected to UDL

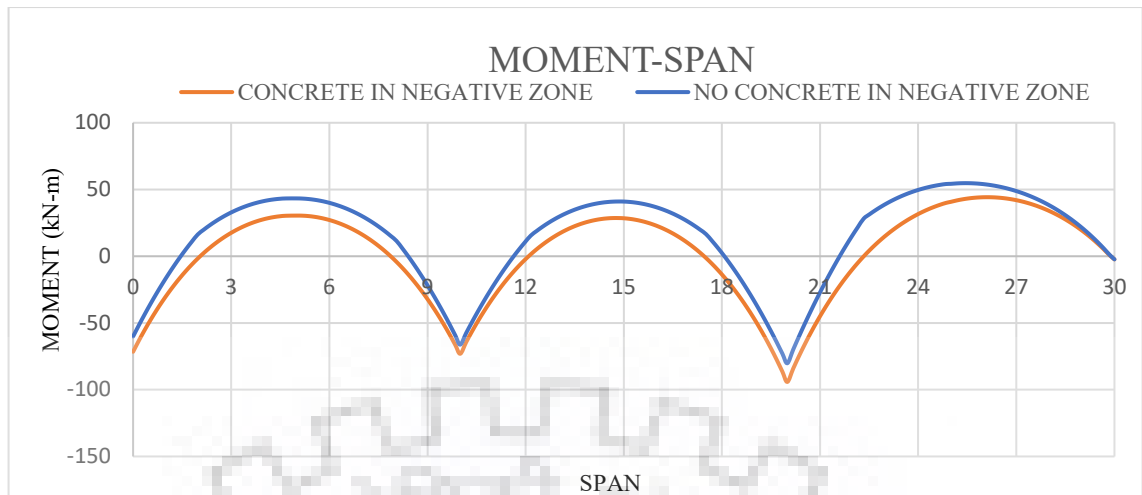


Figure 46: Moment-Span for a 3-span continuous beam subjected to UDL

The span and end moments, deflections and interface slip determined from the numerical method proposed in this study have been matched against the moments, deflections and the interface slip determined from the Direct stiffness method. These results have been presented in the form of bar charts in the following sections.

### 5.4.1 Results of the validations

The results of the validation and the comparison has been presented as follows. The results have been arranged according to the moments determined at the support and at the span. The span deflections have also been arranged in the following section

#### 5.4.1.1 Moment at the supports

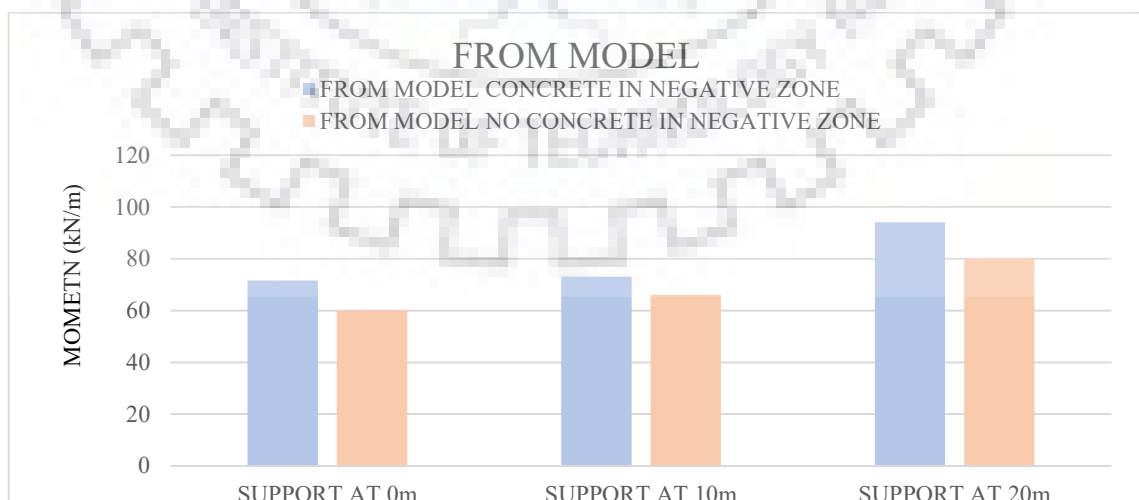


Figure 47: Moment comparison at supports numerical model

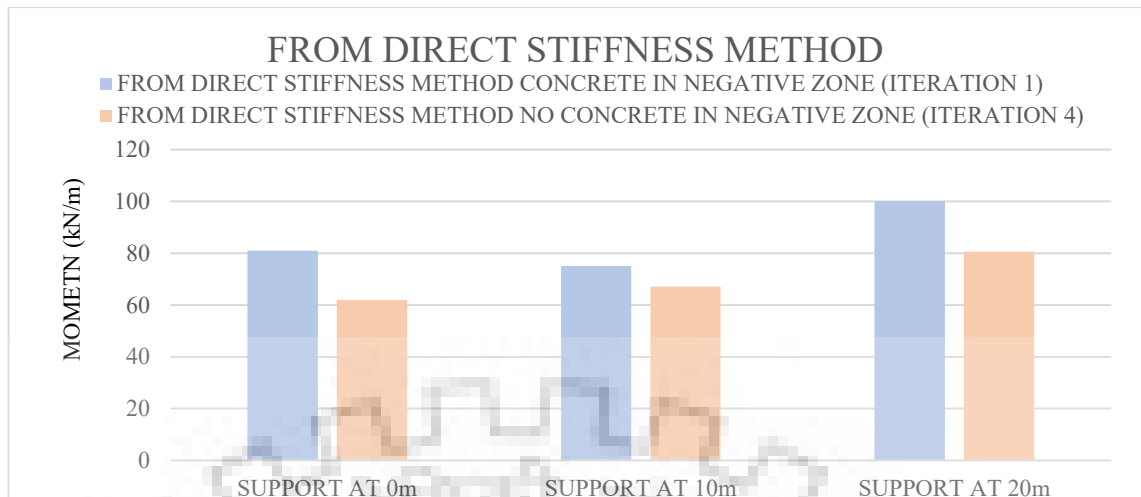


Figure 48: Moment comparison at supports DSM

The error percentage between the values of moments at the support determined by the numerical model and the moments obtained by the Direct stiffness method has been plotted in the following figure (Figure 49). The error percentage recorded was 3.09% for moment at the fixed end, 1.46% at the support 10m from the fixed end and 0.63% at the support 20m from the fixed end. The support at 30m from the fixed end was a rolling support hence there was no moment recorded at the support.

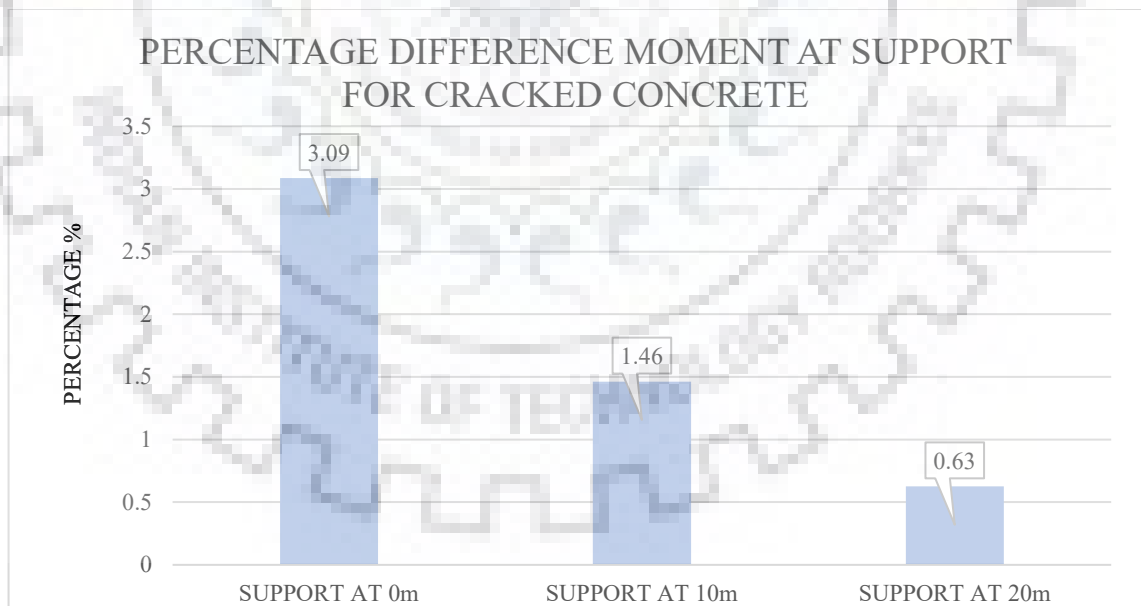


Figure 49: Percentage error between moment values for cracked section between DSM and numerical model



5.4.1.2 Moment in the spans

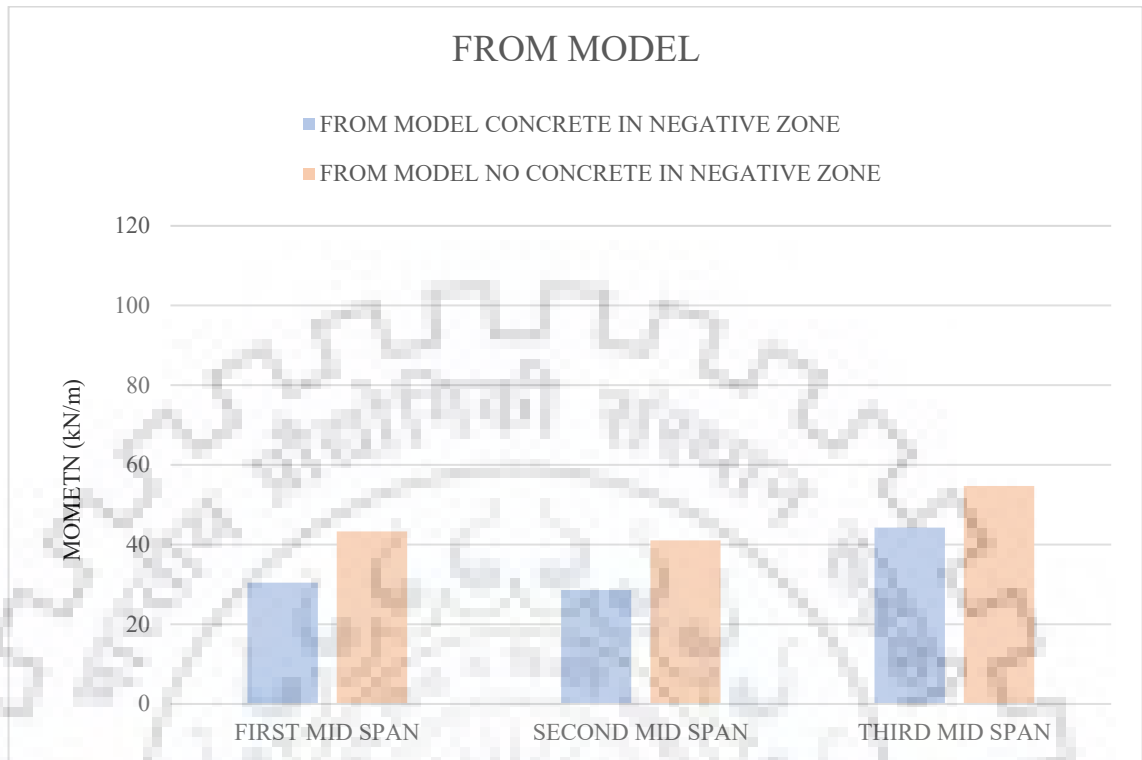


Figure 50: Moment comparison at internal spans numerical model

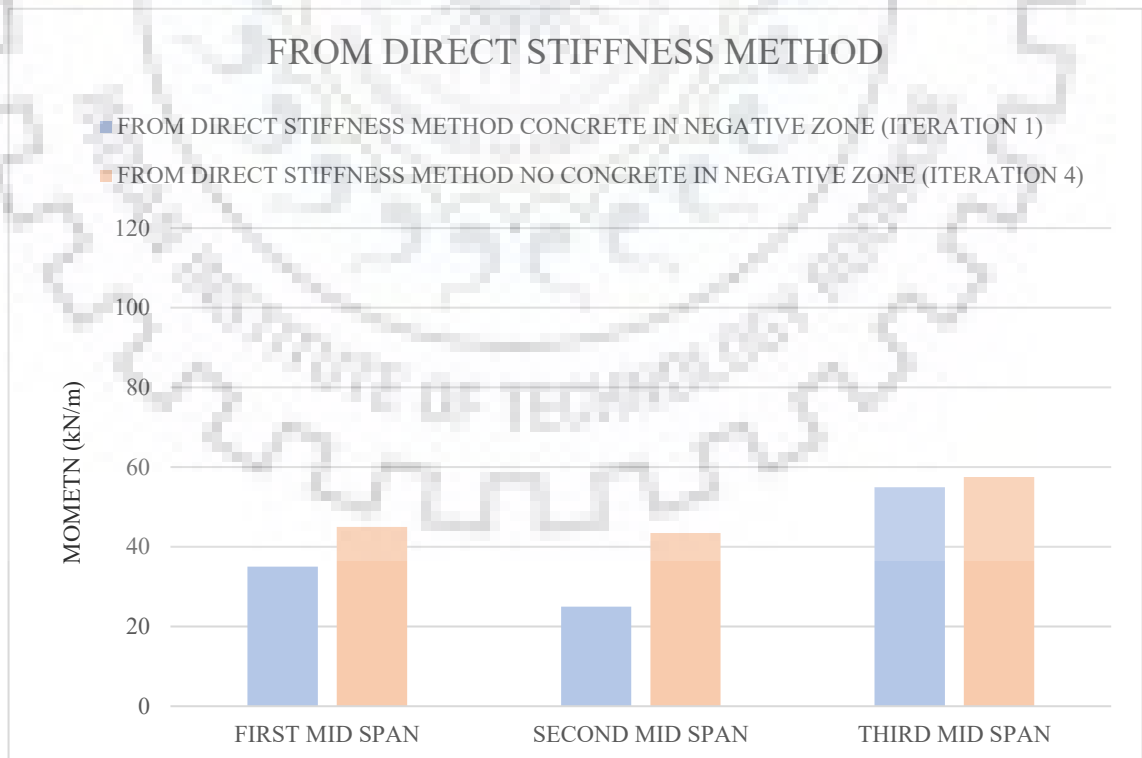
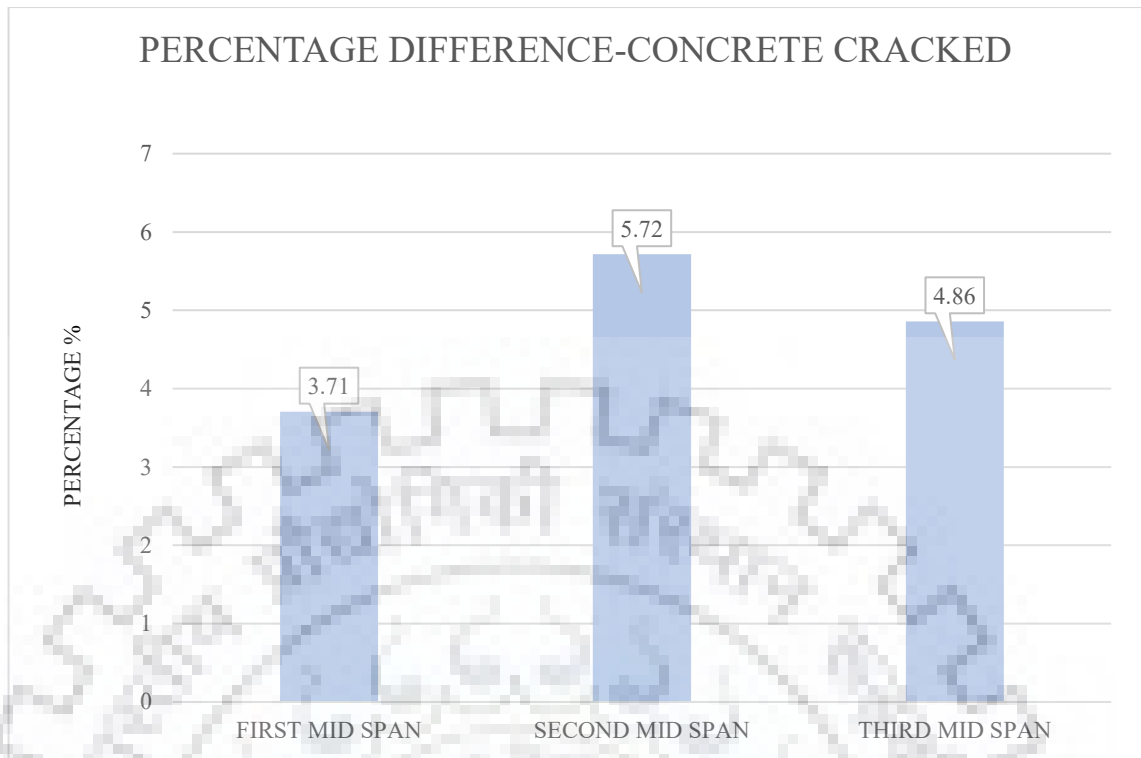


Figure 51: Moment comparison at internal spans DSM



*Figure 52: Percentage error between moment values for cracked section between DSM and numerical model*

The error percentage between the values of moments at the internal span determined by the numerical model and the moments obtained by the Direct stiffness method has been plotted in the following figure (Figure 52). The error percentage recorded was 3.71 % for moment at the first mid span, 5.72% at the second mid span and 4.86 % at the third midspan. The mid span moments recorded for the DSM were slightly greater, the error though was within 10%.

Span deflection studies for obtaining the similar results have been done in the next section. The results of the next section are also presented in the form of bar charts.

5.4.1.3 Span deflections

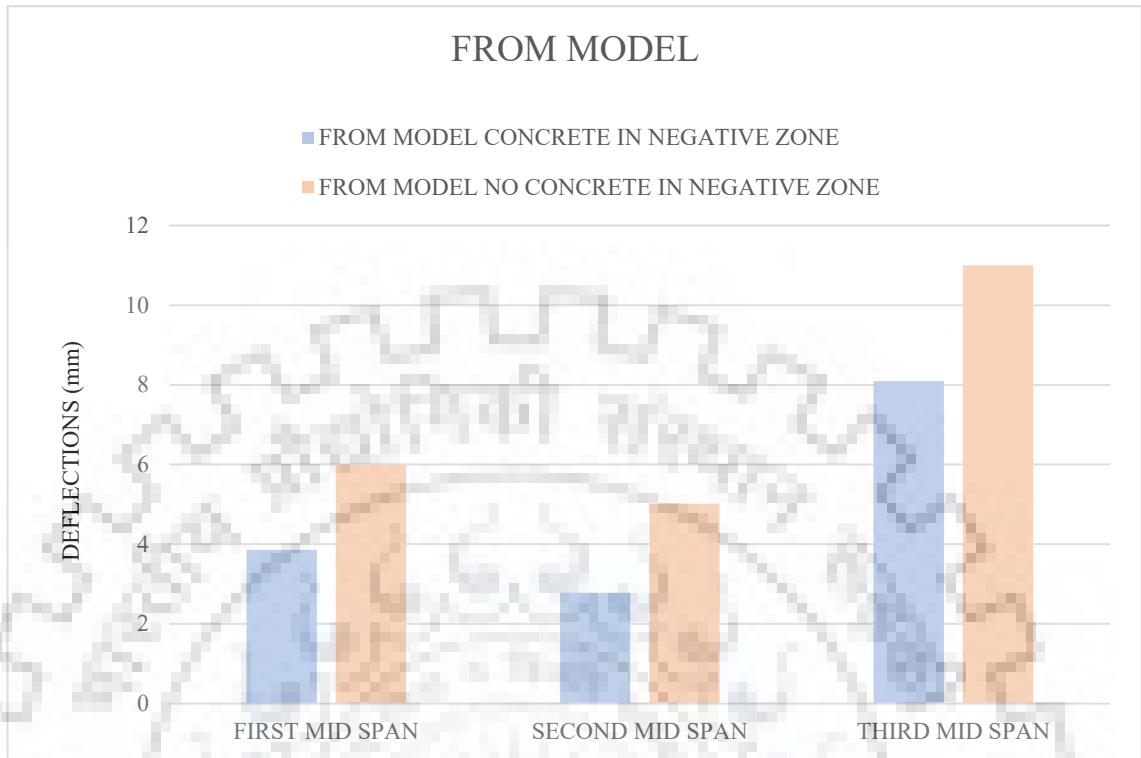


Figure 53: Deflection comparison at internal spans numerical model

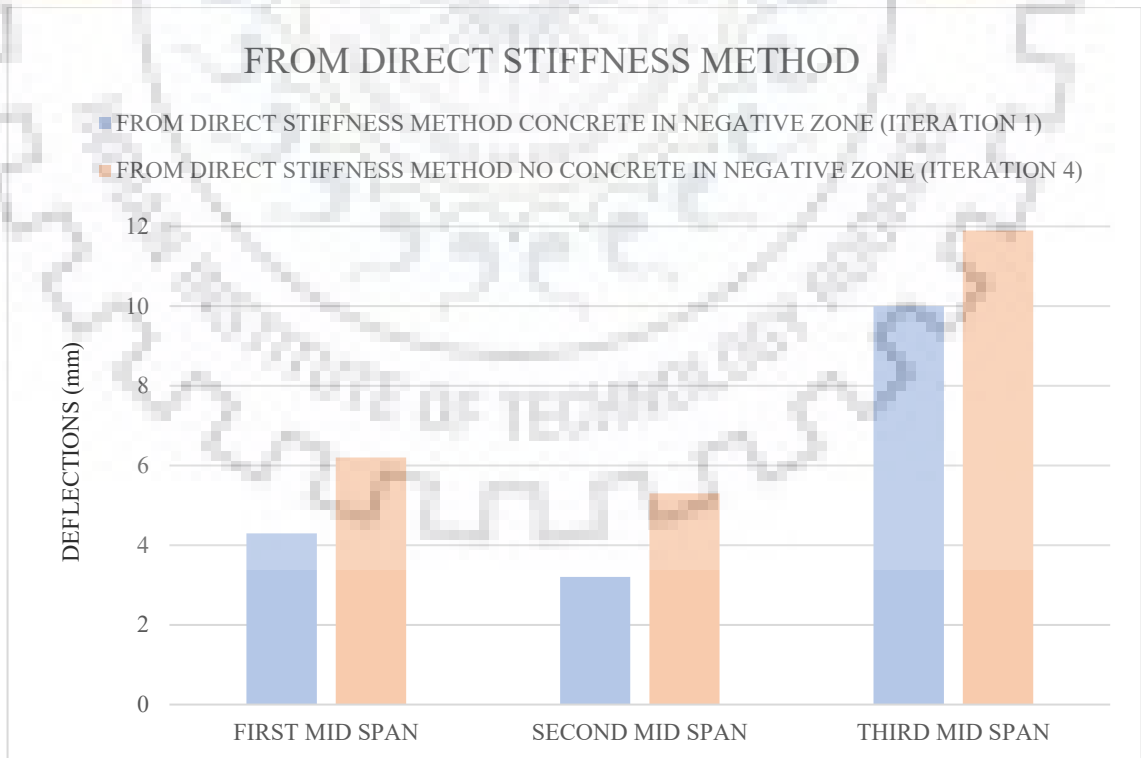
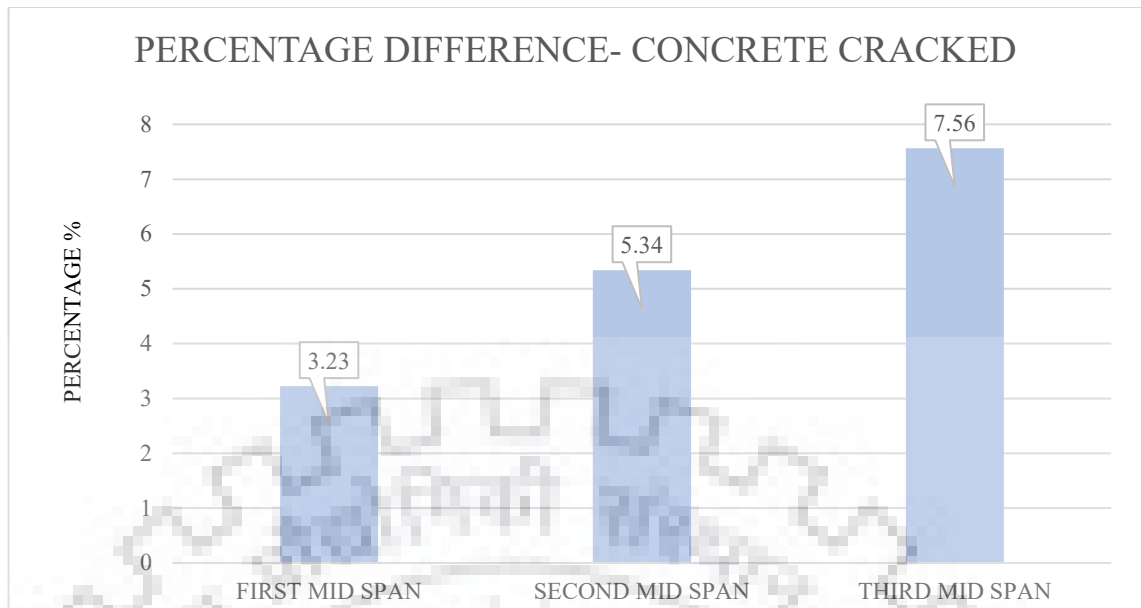


Figure 54: Deflection comparison at internal spans DSM



*Figure 55: Percentage error between deflection values for cracked section between DSM and numerical model*

The error percentage between the values of deflections at the internal span determined by the numerical model and the moments obtained by the Direct stiffness method has been plotted in the following figure (Figure 55). The error percentage recorded was 3.23 % for deflections at the first mid span, 5.34% at the second mid span and 7.56 % at the third midspan. The mid span deflections recorded for the DSM were slightly greater, the error though was within 10%.

#### **5.4.2 Conclusions**

It is seen from the trend of the results that the negative moment at the supports gets reduced as the cracked concrete properties are incorporated. This is effectively due to the fact that the overall flexural rigidity of the structure at that section gets reduced and hence the moment carrying capacity gets reduced as well. The moment redistribution occurs, and the moment is redistributed to the mid span. The increase in moment at the mid span as seen from the bar charts and the moment-span curves in the previous sections is due to the fact that the negative moment at the support reduces and the moment gets redistributed at the mid spans of the steel concrete composite girder.

It is also evident from the slip-span curve trend that the reduction in the effective area of the concrete element at and near the support causes a reduction in slip as the slip is a function of shear flow in the concrete element, since there is no concrete in the negative zone, there is

no slip. The assumption of the DSM proposed by Ranzi was that the concrete in the negative bending zone shall be neglected for all intents and purposes, the shear flow reduces at the area where the area is neglected. This leads to an increase in slip in the vicinity of that area and at the end supports. A comparison has also been made between the model based on the Ranzi's theory and a model which takes into account the effect of cracked concrete in the determination of deflections and span moments.



## **Chapter 6. Continuous Composite Girders with Effective Moment of Inertia of Concrete**

### **6.1 Introduction**

Concrete is in general very weak in tension both direct and flexural. This leads to mild cracking and at times complete destruction of concrete when it is subjected to tensile stresses of any form. Let alone tensile stresses, concrete as a material is highly dependent on the environmental parameters such as relative humidity, temperature at casting, max temperature difference in a day, time of day for casting. Concrete also is dependent on various other parameters such as the amount of cement content, water content, types of aggregates, type and amount of plasticizer if used any, it also depends upon the curing process, curing time, etc. This makes concrete highly heterogeneous and makes it have varying properties for even similar strength grades of concrete.

Concrete not only develops stress related cracks, but also cracks which occur due to the drying process of concrete. Shrinkage both because of drying and because of plasticity of concrete lead to crack formation in concrete. Though at times these cracks are superficial and do not have a depth of more than a fraction of a millimetre, these cracks though become the hot sites for development of structural cracks.

Ignoring effect of concrete at internal supports in composite girders as discussed in the previous chapter leads to a loss of strength and stiffness. Since this phenomenon has already been covered in the previous chapter, we shall only be concerned about the determination of residual strength of concrete and its advantages in the determination of overall strength of the composite girders and this chapter aims to apply the findings on residual strength of concrete to a numerical model, validate the approach against an experiment whose entire work history is known and then perform parametric studies while comparing the results of the studies with results obtained by assuming zero concrete strength in the negative bending one of the continuous composite girder.

This chapter will also include a flow chart to highlight the process behind the determination of residual strength of the concrete.

The only assumption made is that for axial force transfer, only the effective concrete area of the rebar has been considered. Though this also will yield in slightly conservative designs but still will have superior economy when compared to the current design practices.

Background of the process developed in this study is the need for a quantifiable method for considering the residual strength of concrete which would give more economic designs. There have been studies which have been conducted in order to somehow incorporate the strength of cracked concrete in analysis of continuous composite beam. The processes are generally either highly iterative in nature thereby requiring longer computational times or are based off on finite element formulation of the elements at play, which again requires long computational times and computational effort, thereby erasing the offsets of the benefits of using such theories in first place and leading the designer to take the easier path for the design of continuous composite beam (that of neglecting the effect and residual strength of the concrete at negative bending zone).

The high non-linear case of tensile stresses occurring at the top fibre and the compressive stresses arising at the bottom of the fibre near the interface of the concrete and steel components. The is subjected to flexural tension at the interface as the concrete is subjected to flexural compression. As reported by Manfredi et al., this phenomenon is highly non-linear at even very small stresses. The non-linearity is further made complex by the effect of slip occurring between the reinforcement and the concrete along with the slip between the steel and concrete at the interface.

Concrete cracking due to the application of negative bending moment leads to the formation of plastic hinges in re-enforced concrete and is the main cause for the concrete to undergo strain softening and leads to an eventual collapse.

For the incorporation of development of plastic hinge into the model, several avenues were explored.

The CEB-FIP model code 1983 (design manual on cracking and deformations) provides a method for determination of residual strength of concrete in bending or in tension using the approach of tension stiffening. The method is dependent on the deformation of the reinforcement in the tension zone when subjected to normal force. The strains are determined in terms of the ratio of the change in length of the concrete member after the application of load and attainment of equilibrium to the actual length of the specimen. Average strain in

the concrete is determined relative to that in the concrete element. With this the effective stress in the section is determined using the elastic modulus of the concrete and transforming the steel reinforcement to equivalent concrete area with the help of modular ratio.

Mendis (2001), re-examined existing formulas and equations for the determination of plastic hinge and discussed various factors affecting the length of plastic hinge. He determined that, the ACI committee 428 provides for very accurate estimates of plastic hinge lengths for normal and high strength concrete.

Ramm et al. (1996), carried out experimental investigations and parametric studies at the University of Kaiserslautern. The details of the study showed that the formations of the cracks in the concrete component of the steel concrete composite girder is dependent on the percentage of reinforcement present in the slab. The study also highlighted that for load bearing capacities, there exists two kinds of reinforcement requirements, one for the girder to reach plastic state and the other for the plastic analysis applicability to the girder.

Faella et al. (2003), provided a finite element procedure which considered load-slip relationship for shear connectors. The procedure also accounted for cracking of the concrete slab at the negative bending zone and accounted for the tension stiffening effect. The method also assumed different load-slip associations for shear connectors for a cracked concrete specimen. The results of the study on simply supported composite girders were checked for against experimental data of composite simply supported girders. A formula for effective area and effective width of the cracked section is given which is based on the tension stiffening model for concrete members subjected to pure bending and pure axial force which were derived from CEB (1983). The method so proposed is an iterative procedure and leads to a finite element formulation for the member under consideration. Which is a computationally extensive work and requires time apart from computational prowess.

Aref et al. (2007), also proposed a method for determination of effective width of continuous composite girder under negative bending. The formulation was an iterative process and also required a finite element package (ABAQUS) for the determination of effective tensile stresses in concrete at the top and bottom fibre.

ACI 318-8 (building code requirements for structural concrete) provides for calculation of deflections of structures with cracked concrete. This method takes into account the effective modulus of elasticity which is calculated as a function of cracked moment of inertia, gross



moment of inertia, cracking moment for the section of concrete and the applied moment to the concrete element. The equation is as follows.

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \quad (9)$$

Where,  $I_e$  is the effective moment of inertia of the section subjected to moment  $M_a$  whose cracking moment is denoted by  $M_{cr}$ ,  $I_g$  and  $I_{cr}$  are the gross and completely cracked moment of inertia respectively.

Where  $M_{cr}$  is determined as follows:

$$M_{cr} = \frac{f_r I_g}{y_t} \quad (10)$$

Where,  $f_r$  is the rupture strength of concrete in flexure and  $y_t$  is defined as the depth of centroid from the extreme most fibre (half the depth of the section).

IS 456-2000 also provides for a method for determination of effective moment of inertia, the formula provided is:

$$I_{eff} = \frac{I_r}{1.2 - \frac{M_r z}{M d} \left(1 - \frac{x}{d}\right) \frac{b_w}{b}} \quad (11)$$

Where,  $I_r$  is the moment of inertia of the completely cracked section,  $M_r$  is the cracking moment is determined exactly as for ACI code,  $M$  is the applied moment at the section considered,  $x$  is the depth of neutral axis of the concrete section,  $d$  is the effective depth of the concrete section and  $b_w$  and  $b$  are breadth of the web and breadth of the compression face respectively.

## 6.2 Proposed model in this study

After going through all the literature related to concrete subjected to negative bending mentioned above, the method for determination of effective moment of inertia has been finalised as the method for adoption and calculation of residual strength of the concrete element subjected to negative bending.

The proposed modelling procedure in the study as depicted in the flow chart (Figure 56) is a very simple and intuitive procedure.

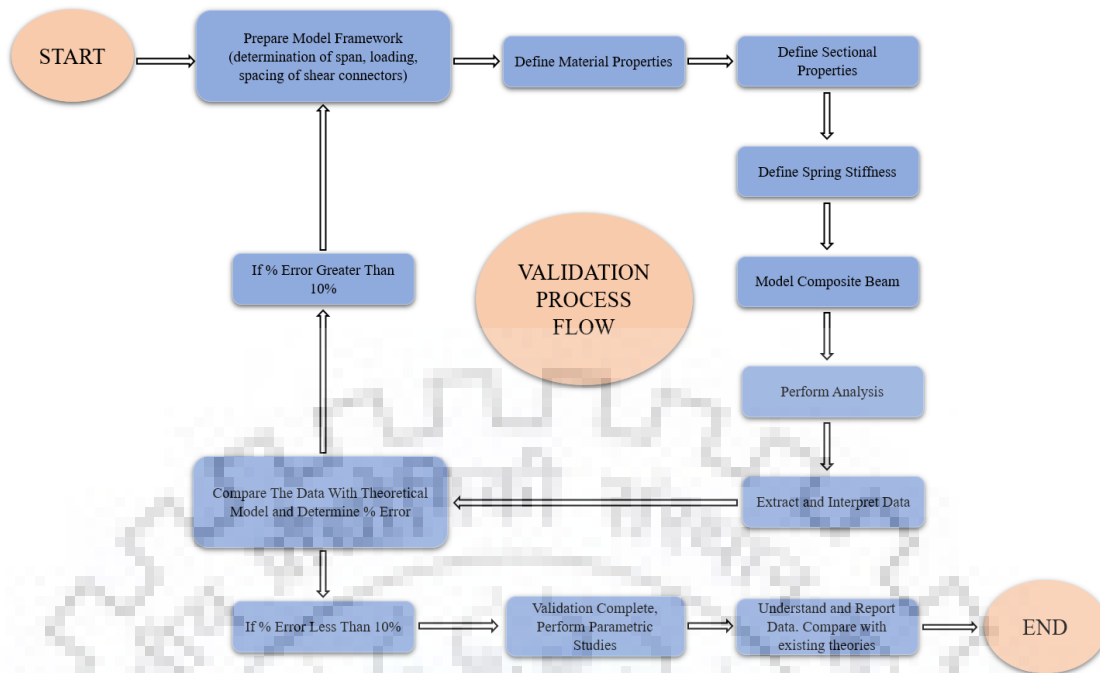


Figure 56: Process flow diagram for the proposed model

The steps for the application of the models are as follows:

1. A model is made-up of frame elements as depicted in Figure 15 with some interaction ratio (full or partial interaction) and was loaded with a UDL.
2. Forces carried by the steel and concrete element were determined from SAP2000.
3. A percentage reinforcement is provided to the concrete. The reinforcement provided was for resisting the maximum moment in the concrete element.
4. Cracked moment of inertia  $I_{cr}$  is determined for the above reinforcement percentage.
5. Cracking moment  $M_{cr}$  was determined for the concrete gross section and the grade of concrete.
6. Modular ratio for was calculated, where modular ratio is defined as  $E_s/E_c$ ; where  $E_s$  is the modulus of elasticity of the grade of reinforcement chosen and  $E_c$  is the modulus of elasticity for the grade of concrete chosen.
7. Bending moment due to the applied moment is determined for every section of the concrete element. This is the moment acting on the concrete and shall be denoted by  $M_a$  hereon forward.
8. The ratio of cracking moment to the moment acting at the section,  $M_{cr}/M_a$ , is determined for every section along the length of the girder.
9. The effective moment of inertia  $I_e$  for remaining concrete subjected to  $M_a$  is determined for every section where  $M_{cr}/M_a$  is less than 1.

10. The ratio of  $I_e/I_g$  is calculated for all the sections for which  $M_{cr}/M_a$  is less than 1.
11. The ratio of  $I_e/I_g$  is provided in the model in the form of moment of inertia property modifiers.
12. The area of residual concrete is assumed to be made up of transformed reinforcement area. The area of concrete in the areas where  $M_{cr}/M_a$  is less than 1 is determined by multiplying the area of reinforcement provided to the modular ratio as defined above. This area shall be denoted as  $A_{eff}$ .
13. The ratio  $A_{eff}/A_g$  is determined, where  $A_g$  is the gross area of the concrete section.
14. The ratio  $A_{eff}/A_g$  is implemented in the model as area property modifier.

Validation of the proposed model has been done against a composite beam published by Jiang et al. 2015. Parametric studies are conducted where number of spans, span length, cross-section, degrees of shear connection are varied. The results so determined from the above parametric studies are compared with the results of the parametric studies done on model defined in the previous chapter.

### 6.3 Validation

The proposed model was validated against experimental data present in open literature. Jiang et al. 2015, performed experimental analysis on a composite girder named SCCCB1. The beam was a two-span continuous beam which was subjected to point loads at the midspans. The girder had equal spans of length 3.9m.

The girder arrangement is shown in Figure 57. Nelson studs of diameter 19mm were used as shear connectors and they were placed along the length of the steel girder at equal spaces.

The connectors were spaced such that a full interaction behaviour is achieved. Since the stiffness of the studs was not mentioned, the stiffness of the shear studs and the spacing was calculated as per methods given in IS:11384 - 1985.

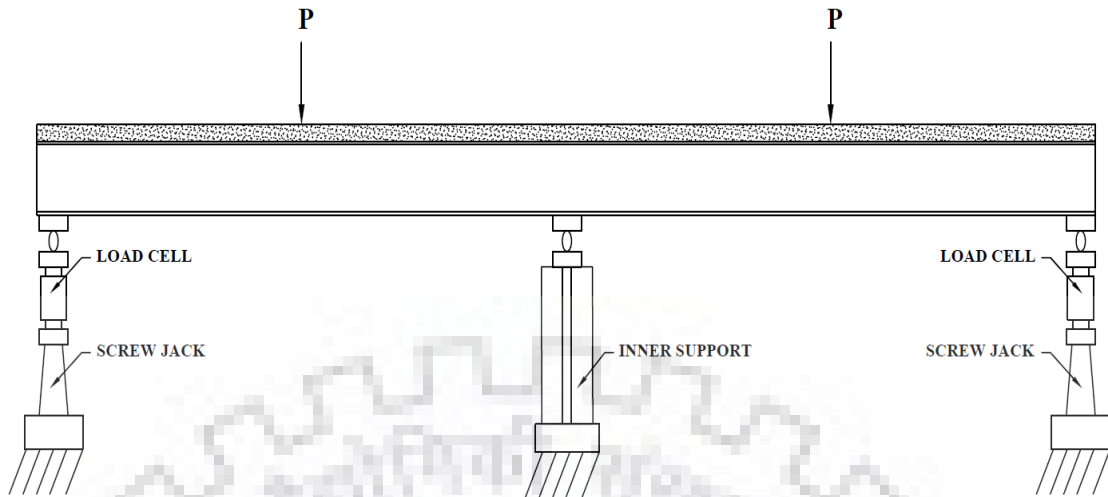


Figure 57: Layout of the test setup (recreated from Jiang et al. 2015)

The loading and material property of the beam are tabulated below:

\*Refer Figure 16 for the variables

Table 6: Dimensions of continuous composite beam for validation studies (Jiang et al. 2015)

SPAN (m)	Loading (Point kN)	Dimensions (mm)					
		H	Tf	Tw	Bf	Bc	Tc
3.9	386.2	294	12	8	200	900	120

Table 7: Dimensions of continuous composite beam for validation studies (Jiang et al. 2015)

Concrete		Steel Girder		Steel Bars (14 dia)	
$f_{cu}$	27.1	$f_y$	270.0	$f_{ry}$	433.0
$f_t$	3.64	$f_u$	447.0	$f_{ru}$	539.0
$E_c$	$3 \times 10^4$	$E_s$	$2 \times 10^5$	$E_s$	$2 \times 10^5$

The results of the model have been presented in the form of span-deflection curves and the deflection obtained from the curves are compared with the results of the experiment.

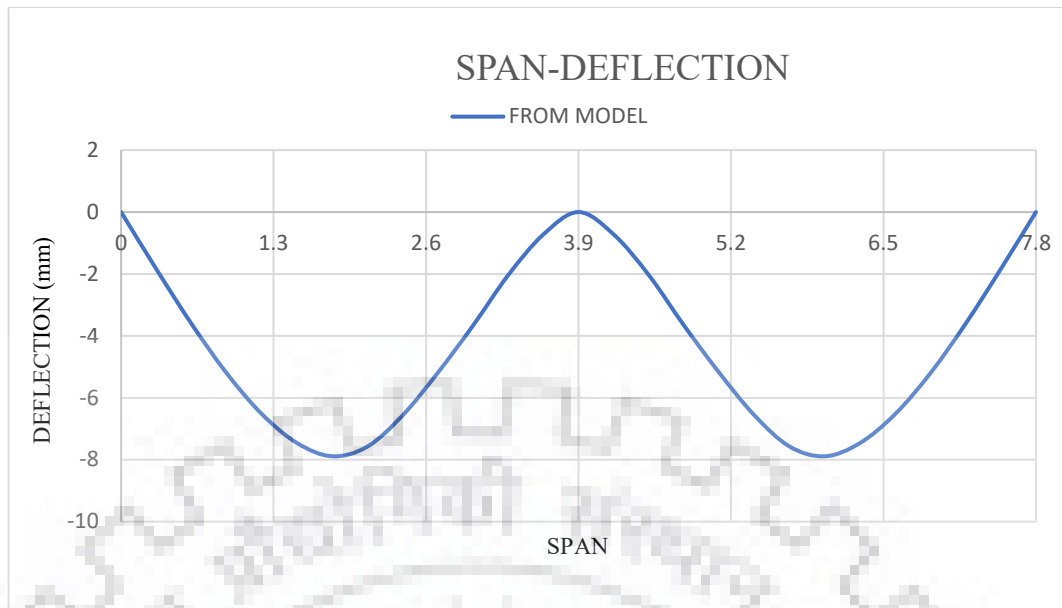


Figure 58: Deflection v/s Span for SCCCB1 (Jiang et al. 2015) beam numerical model

The deflections obtained from the experiment is compared against the values obtained from the numerical modelling. A bar chart was prepared to compare the deflections and a percentage difference was calculated

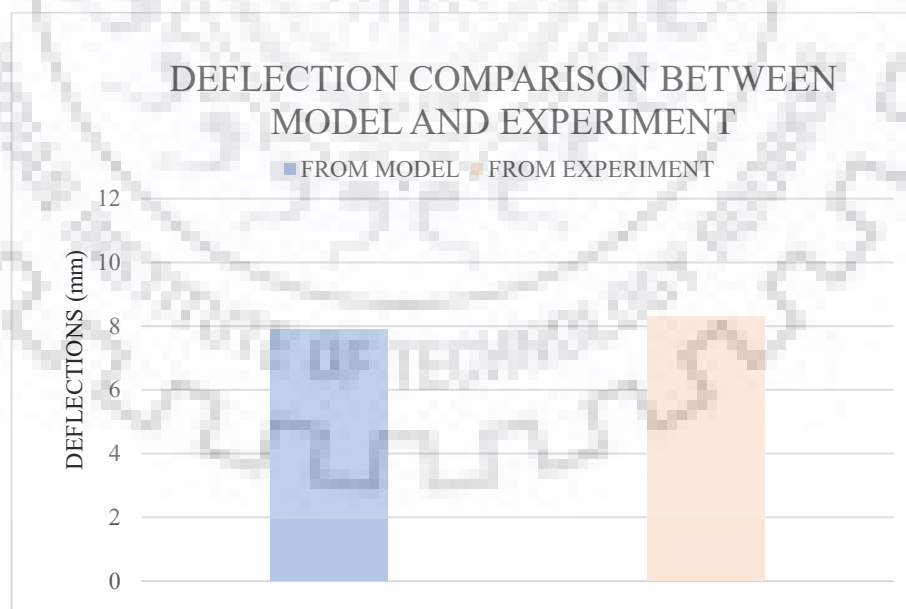


Figure 59: Deflection comparison (model and experiment)

The percentage difference recorded for the difference in deflections between the proposed model and the experiment was 4.96%. the error percentage is less than 5% and hence the model holds good regard with the experiments. The error in deflection may be owed to the fact that the actual stiffness of the 19 mm nelson stud and the stiffness calculated for the stud could be different.

#### 6.4 Parametric studies

The proposed model was used to perform parametric studies so as to get a better idea of the behaviour of continuous composite girders when effective moment of inertia was taken into consideration. The results of the parametric studies were compared against results obtained by ignoring the effect of concrete in the negative zone.

The span length chosen are tabulated below along with the girder cross sectional properties. The number of spans chosen were 2,3 and 5 with degree of interaction ranging from 0.8 to 0.5.\*Refer Figure 16 for the variables

Table 8: Dimensions of continuous composite beam for parametric studies

SPAN (m)	LOADING (UDL kN/m)	Dimensions (mm)					
		H	Tf	Tw	Bf	Bc	Tc
20	50	1060	30	10	300	1300	150
30	50	1160	30	12	300	1500	150
40	50	1300	30	16	400	1800	180

The results of the parametric studies and the comparison with methods discussed in the previous chapter are detailed in the following section.

#### 6.5 Results and comparisons

In the following sections, the deflections and moment span trends have been plotted to help draw the comparisons.

### 6.5.1.1 Span deflection studies

#### 6.5.1.1.1 40 m 3-span

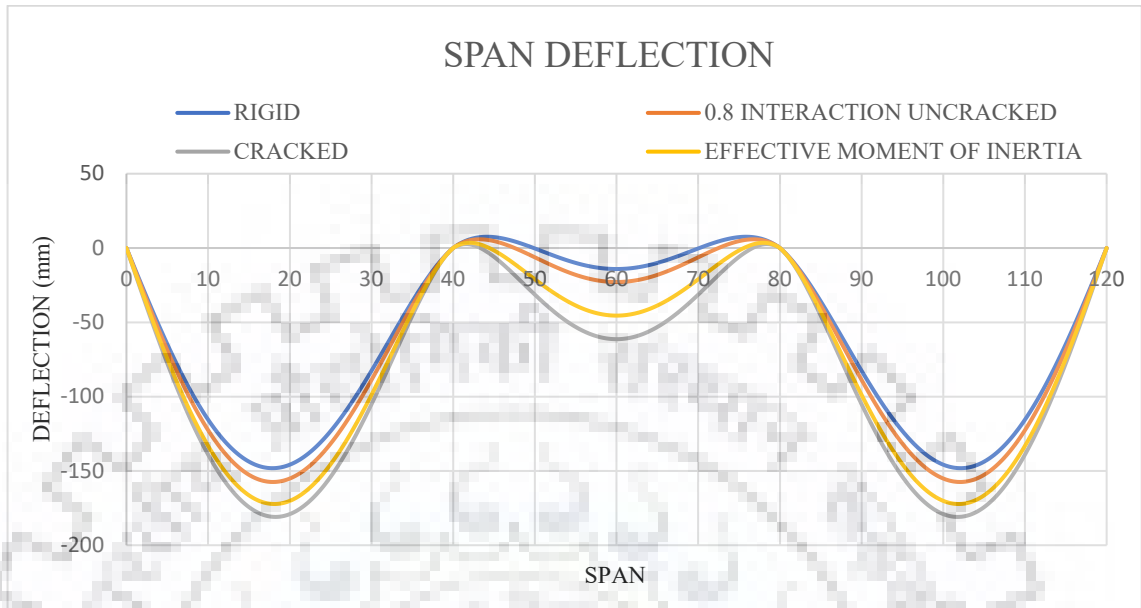


Figure 60: Deflection v/s Span (40m 3-span 0.8 interaction)

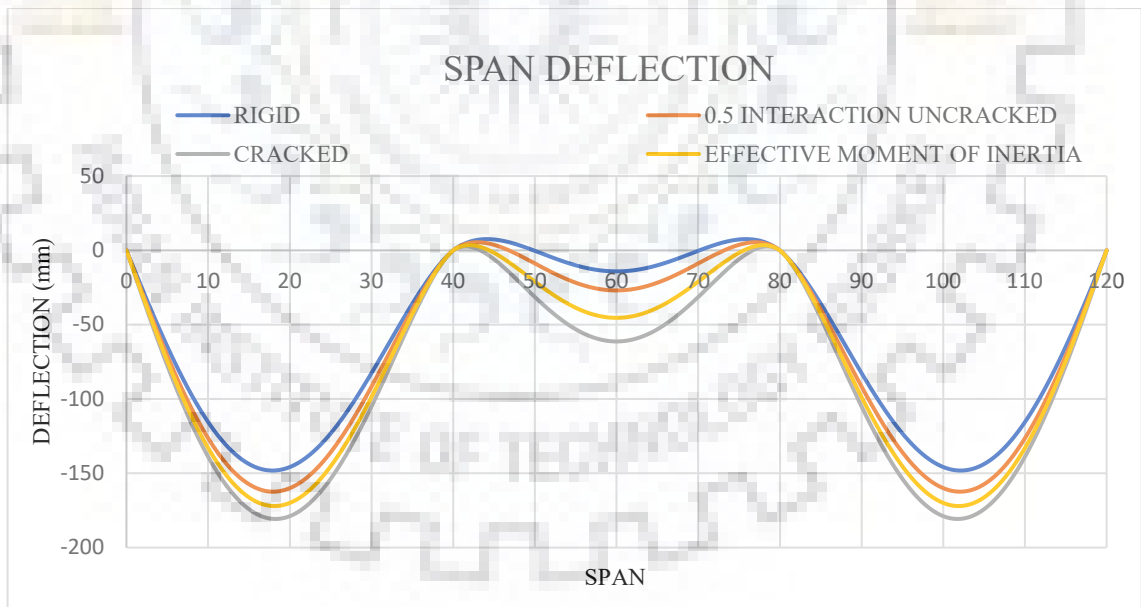


Figure 61 : Deflection v/s Span (40m 3-span 0.5 interaction)

6.5.1.1.2 40 m 5-span

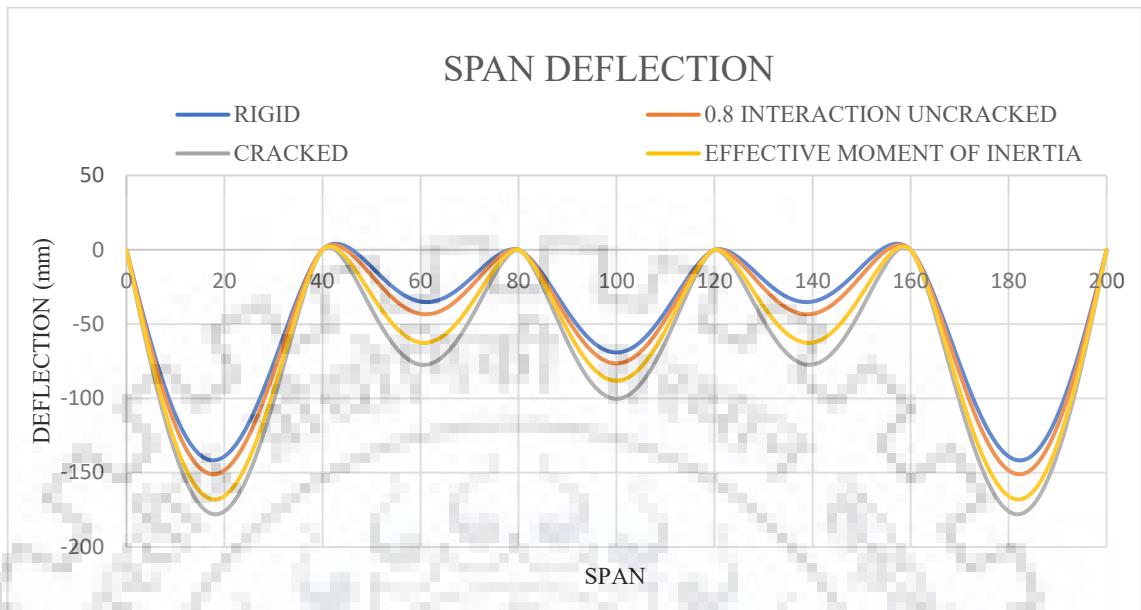


Figure 62: Deflection v/s Span (40m 5-span 0.8 interaction)

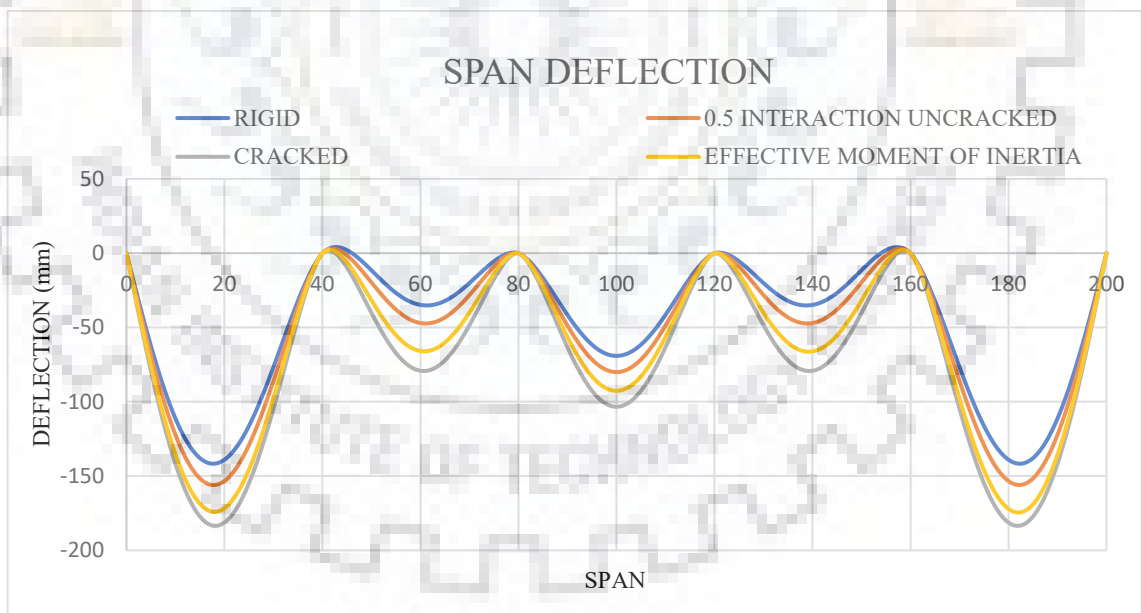


Figure 63: Deflection v/s Span (40m 5-span 0.5 interaction)



6.5.1.1.3 30 m 3-span

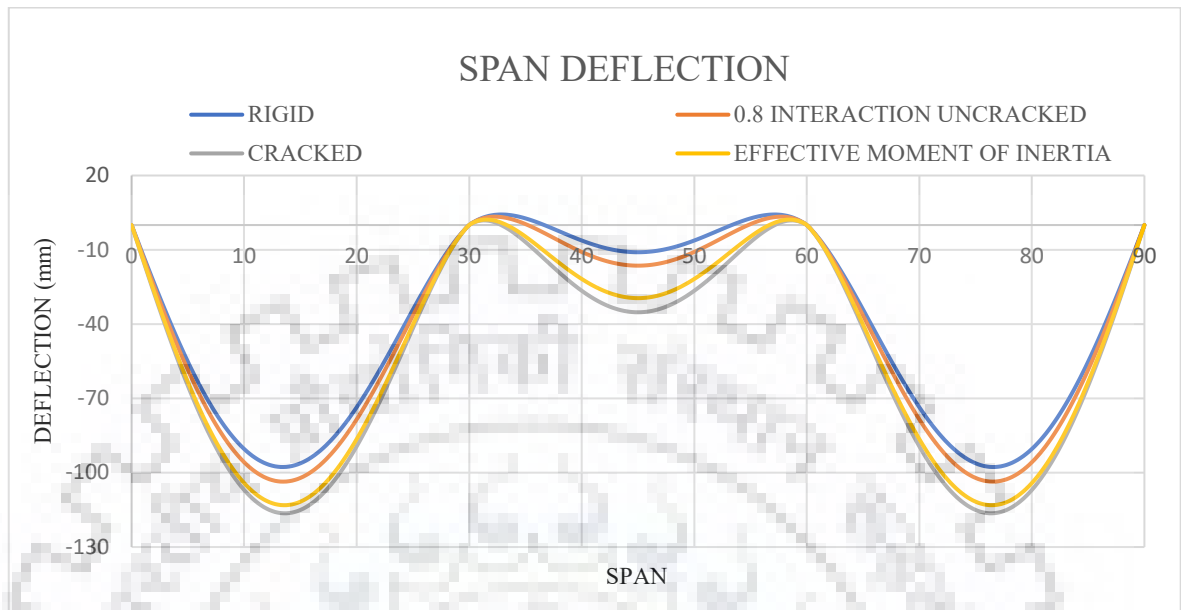


Figure 64: Deflection v/s Span (30m 3-span 0.8 interaction)

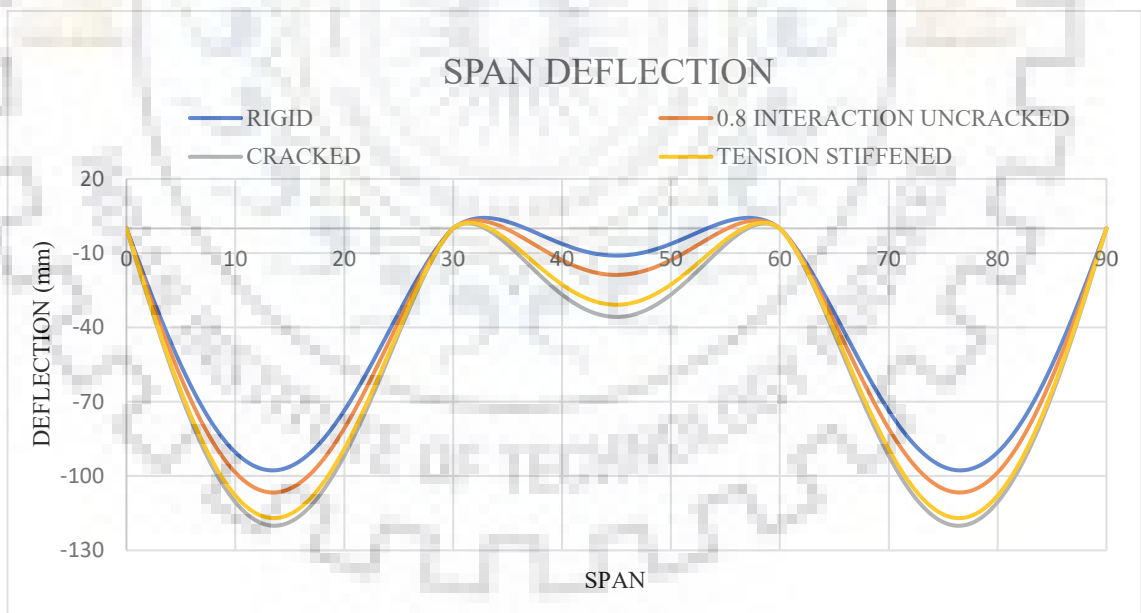


Figure 65: Deflection v/s Span (30m 3-span 0.5 interaction)

6.5.1.1.4 30 m 5-span

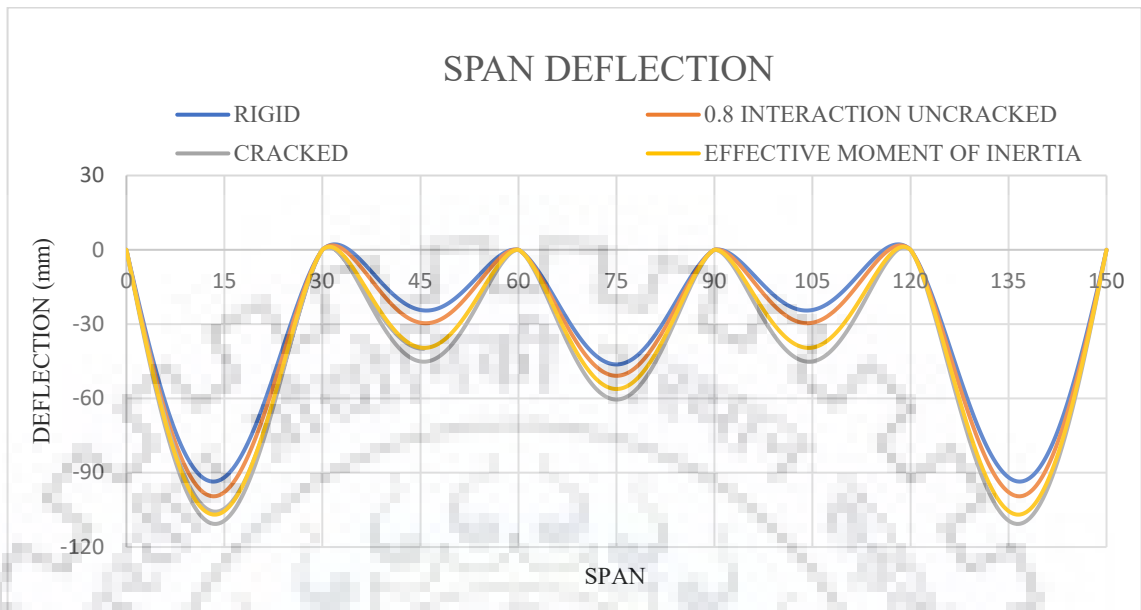


Figure 66: Deflection v/s Span (30m 5-span 0.8 interaction)

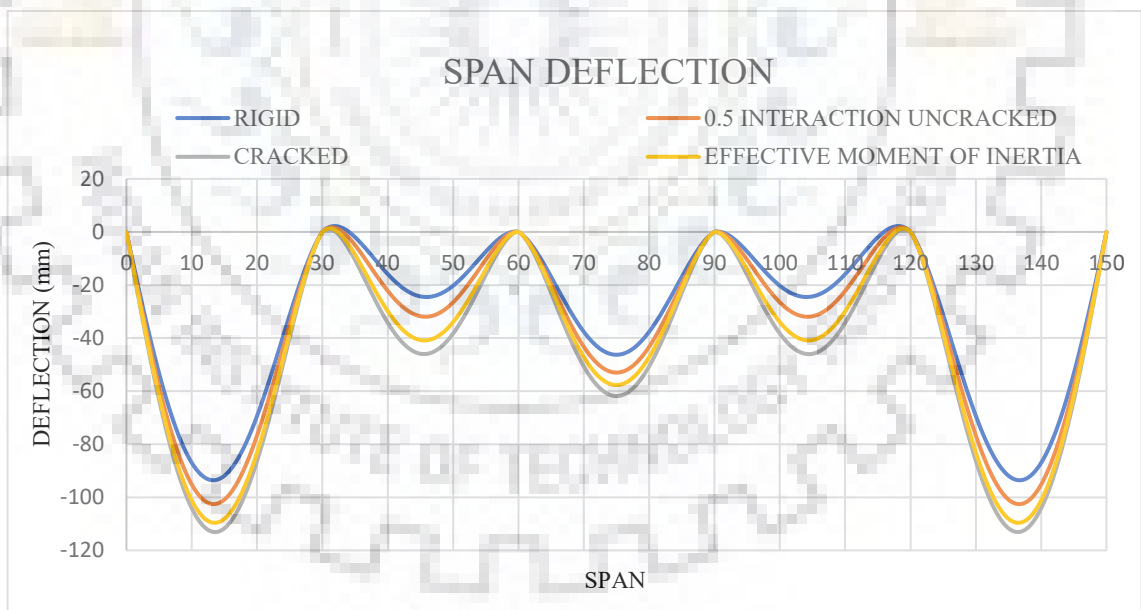


Figure 67: Deflection v/s Span (30m 5-span 0.5 interaction)

6.5.1.1.5 20 m 3-span

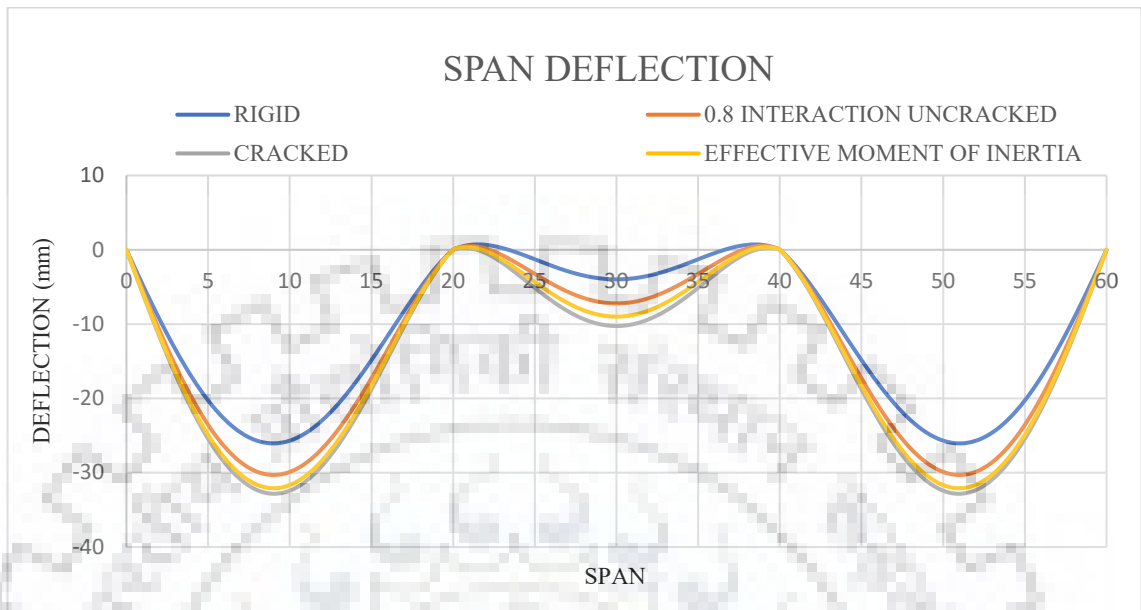


Figure 68: Deflection v/s Span (20m 3-span 0.8 interaction)

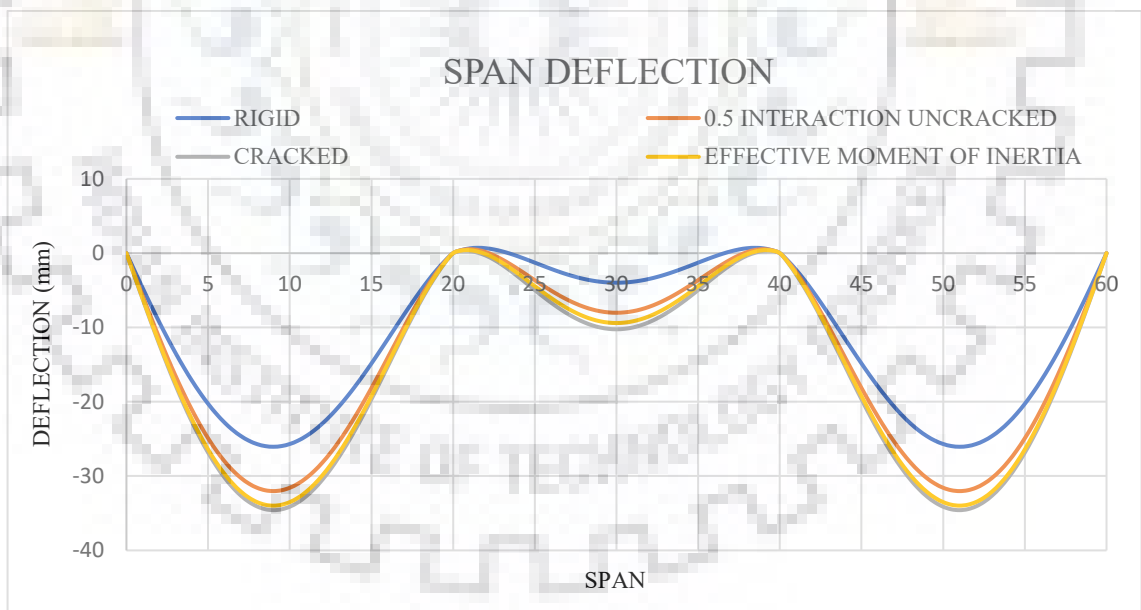


Figure 69: Deflection v/s Span (20m 3-span 0.5 interaction)

6.5.1.1.6 20 m 5-span

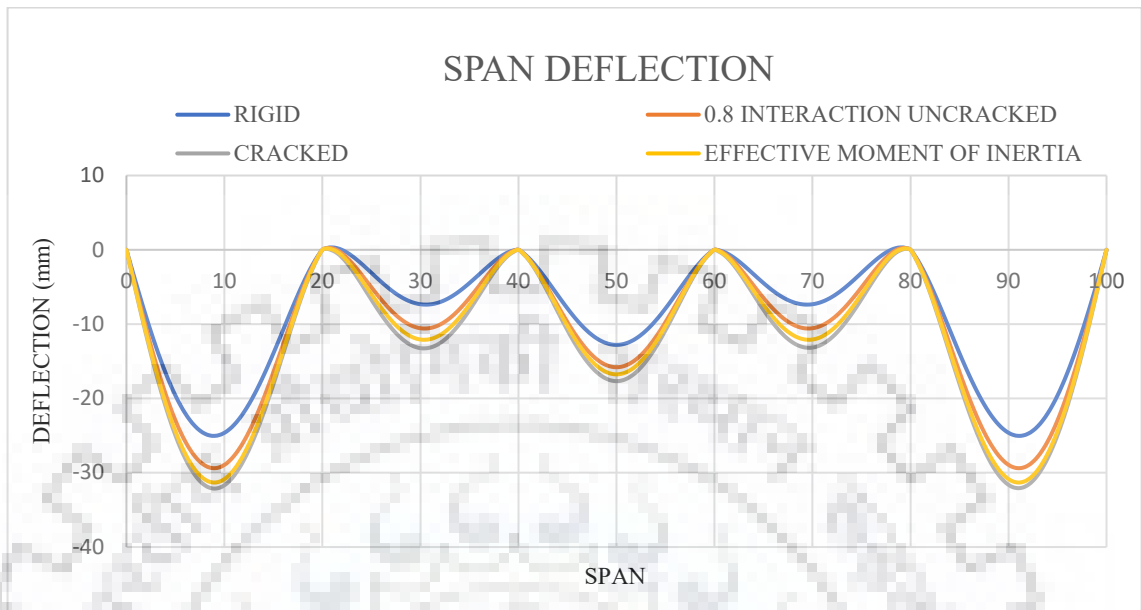


Figure 70: Deflection v/s Span (20m 5-span 0.8 interaction)

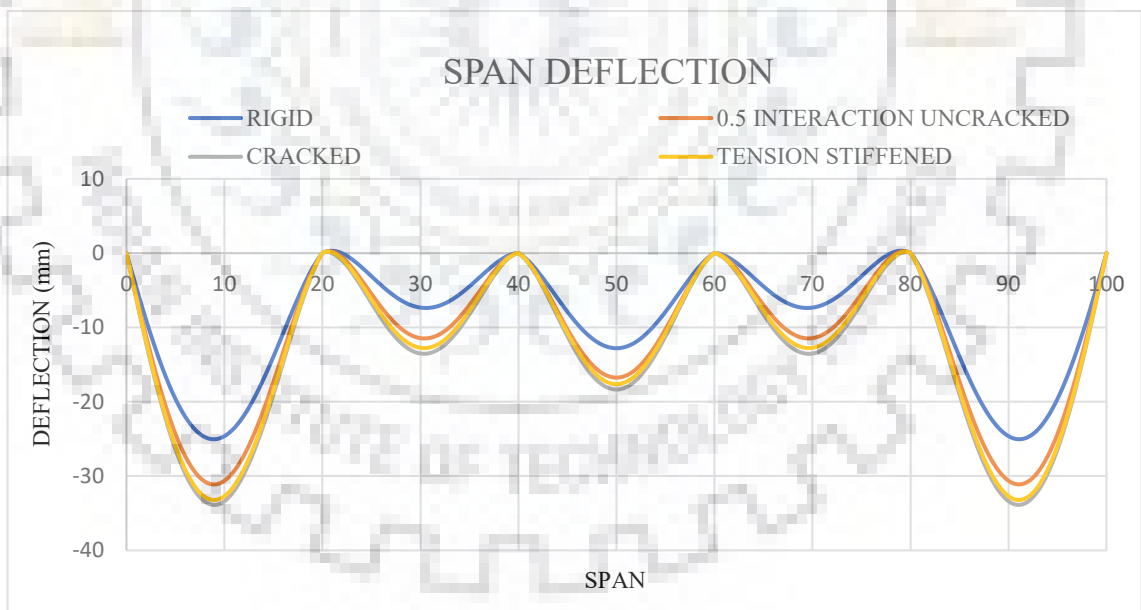


Figure 71: Deflection v/s Span (20m 5-span 0.5 interaction)

**6.5.1.1.7 Percentage change in deflections**

The bar charts to compare percentage change in deflections between no concrete considered at the negative zone vs applying effective moment of inertia at the negative zone. The comparison is made between the different spans, span lengths and degree interaction.

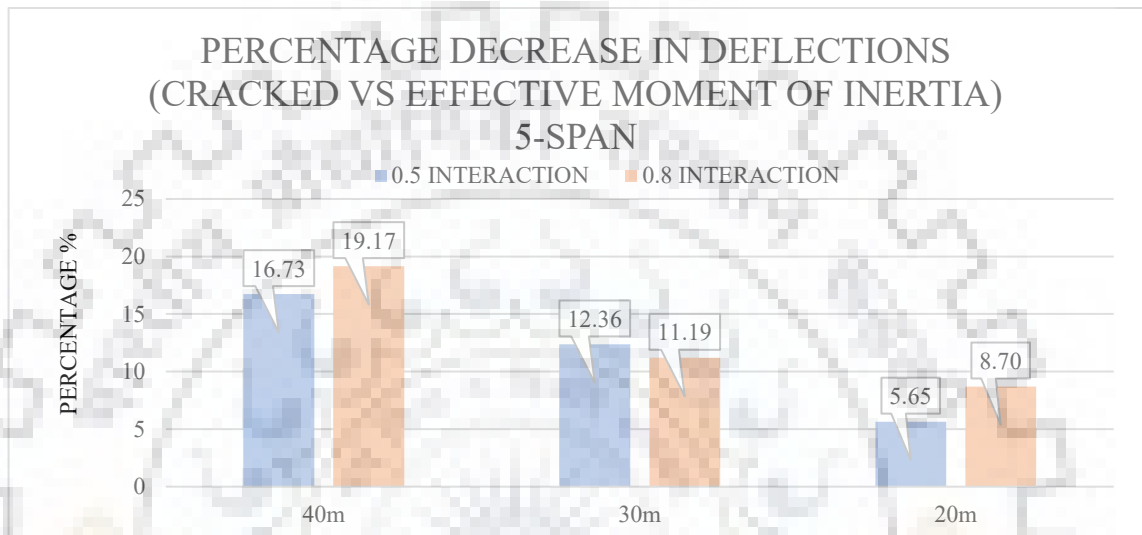


Figure 72: Percentage difference between deflections (no concrete v/s effective moment of inertia)

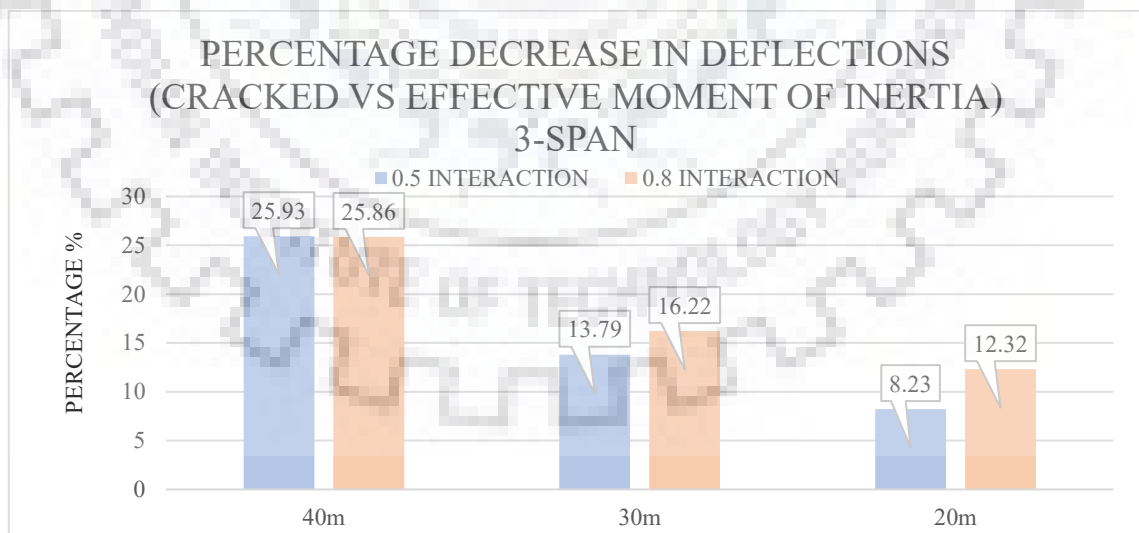


Figure 73: Percentage difference between deflections (no concrete v/s effective moment of inertia)

**6.5.1.2 Moment span studies**

**6.5.1.2.1 40 m 3-span**

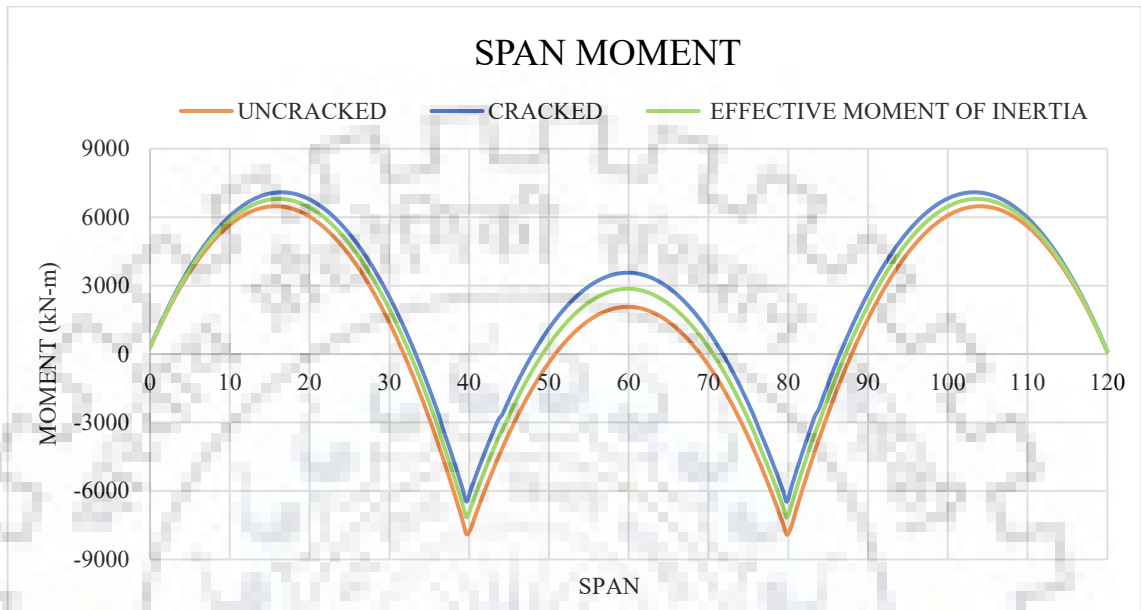


Figure 74: Moment-span for 40m 3-span entire Steel Concrete Composite girder with 0.5 interaction

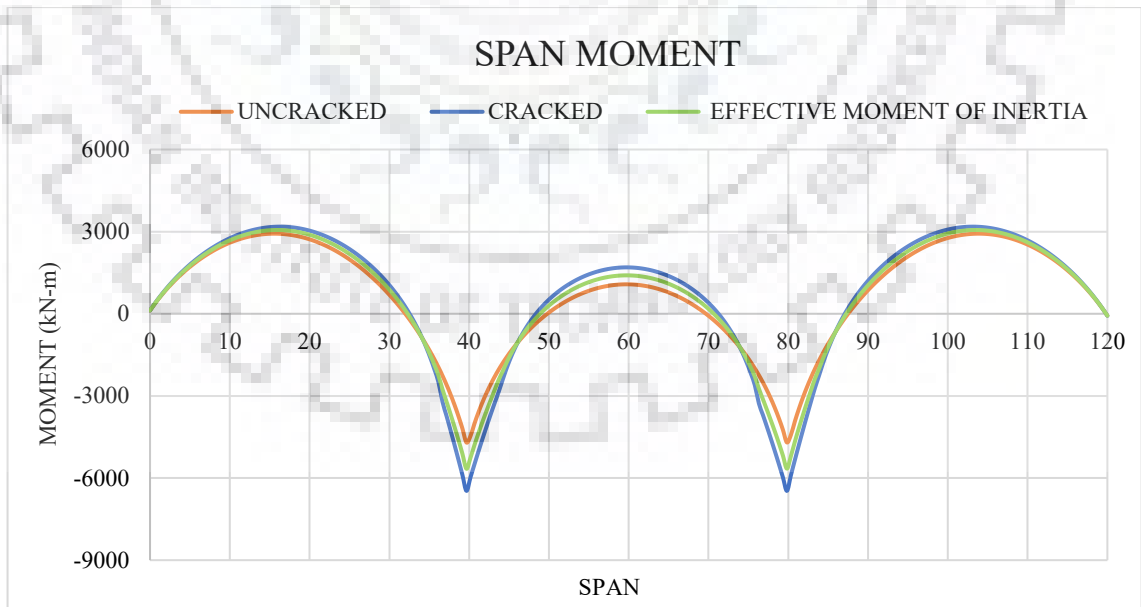


Figure 75: Moment-span for 40m 3-span steel component of Steel Concrete Composite girder with 0.5 interaction

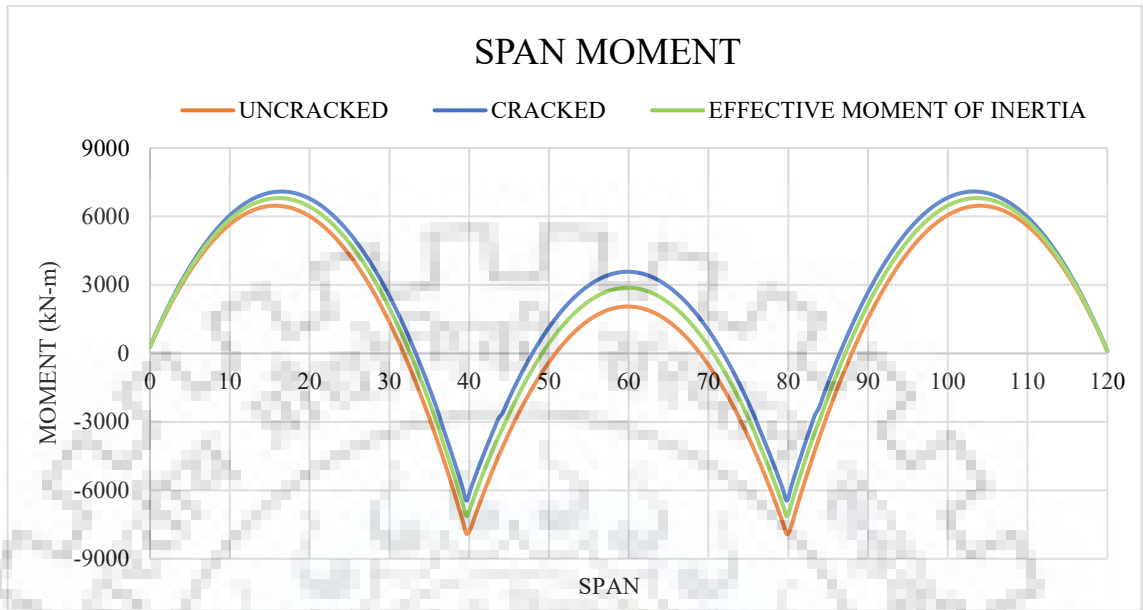


Figure 76: Moment-span for 40m 3-span entire Steel Concrete Composite girder with 0.8 interaction

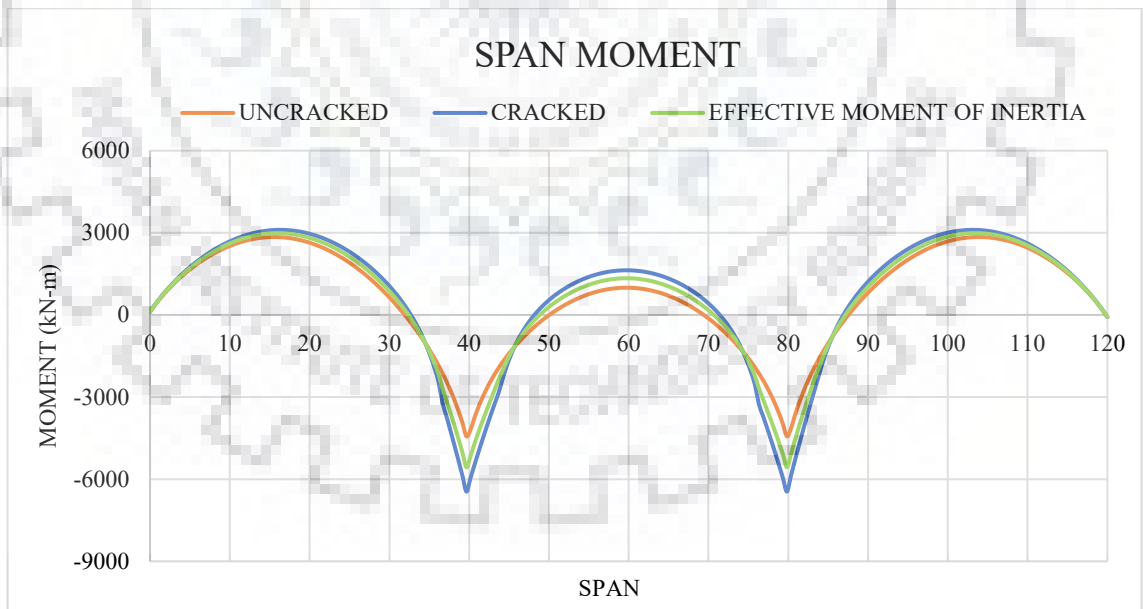


Figure 77: Moment-span for 40m 3-span steel component of Steel Concrete Composite girder with 0.8 interaction

6.5.1.2.2 40 m 5-span

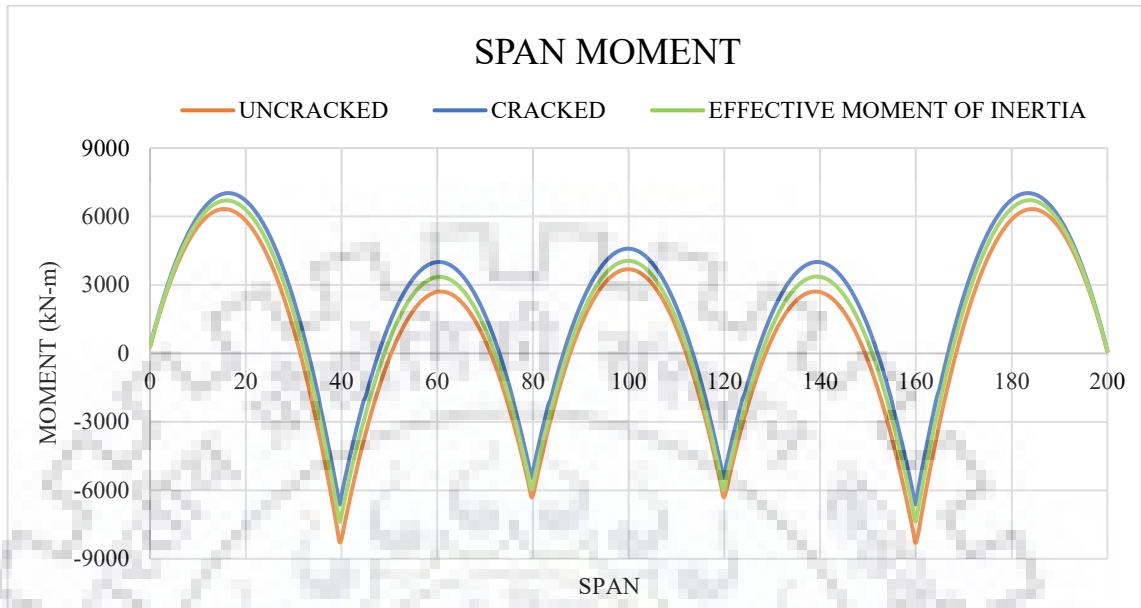


Figure 78: Moment-span for 40m 5-span entire Steel Concrete Composite girder with 0.5 interaction

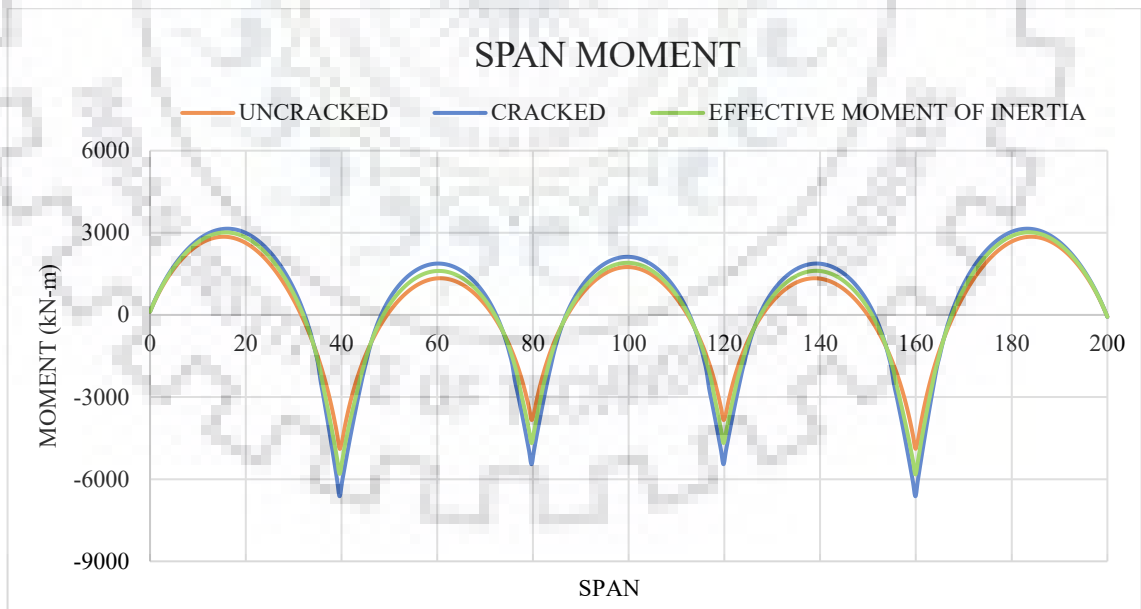


Figure 79: Moment-span for 40m 5-span steel component of Steel Concrete Composite girder with 0.5 interaction



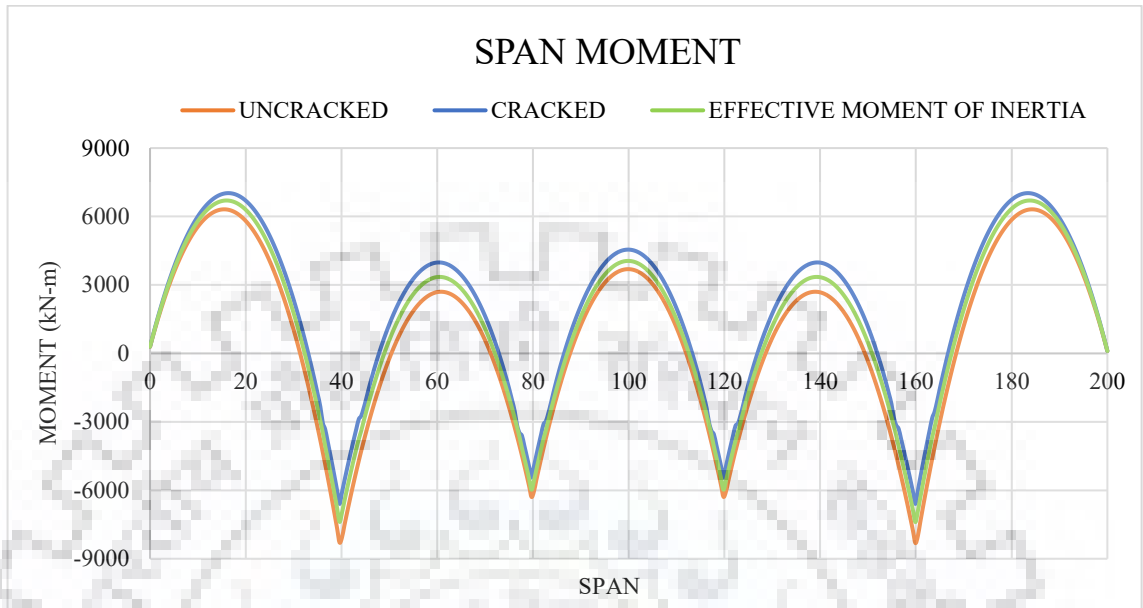


Figure 80: Moment-span for 40m 5-span entire Steel Concrete Composite girder with 0.8 interaction

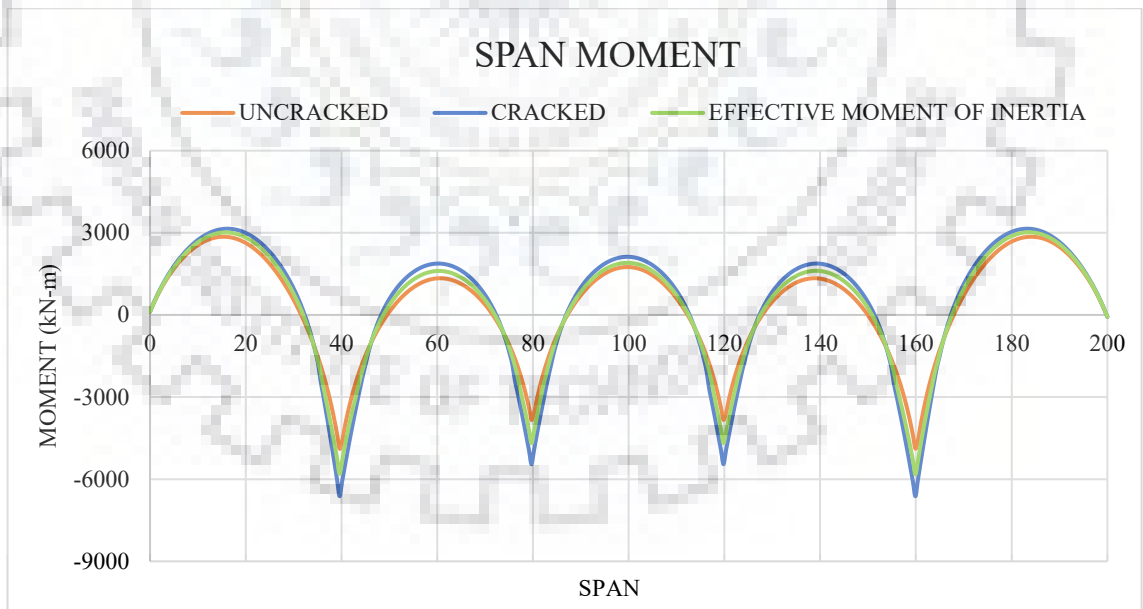


Figure 81: Moment-span for 40m 5-span steel component of Steel Concrete Composite girder with 0.8 interaction

6.5.1.2.3 30 m 3-span

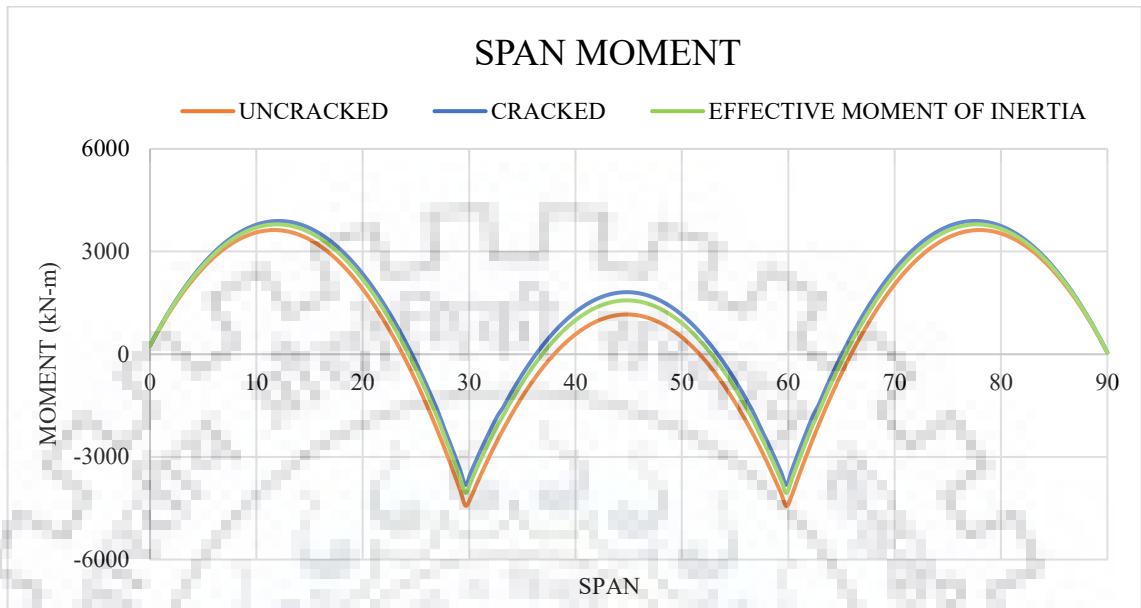


Figure 82: Moment-span for 30m 3-span entire Steel Concrete Composite girder with 0.5 interaction

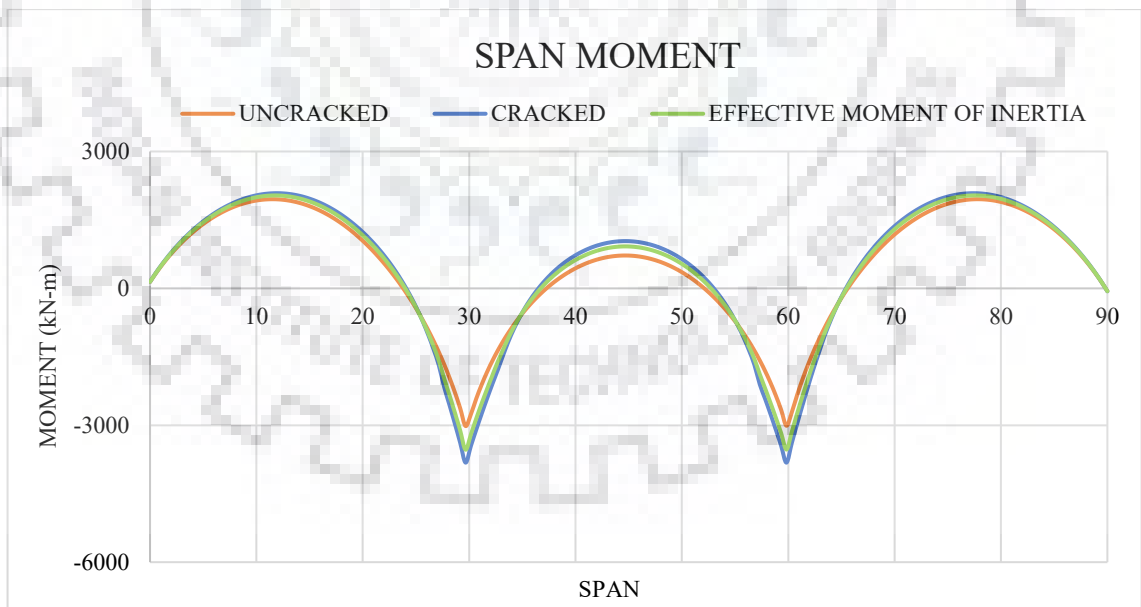


Figure 83: Moment-span for 30m 3-span steel component of Steel Concrete Composite girder with 0.5 interaction

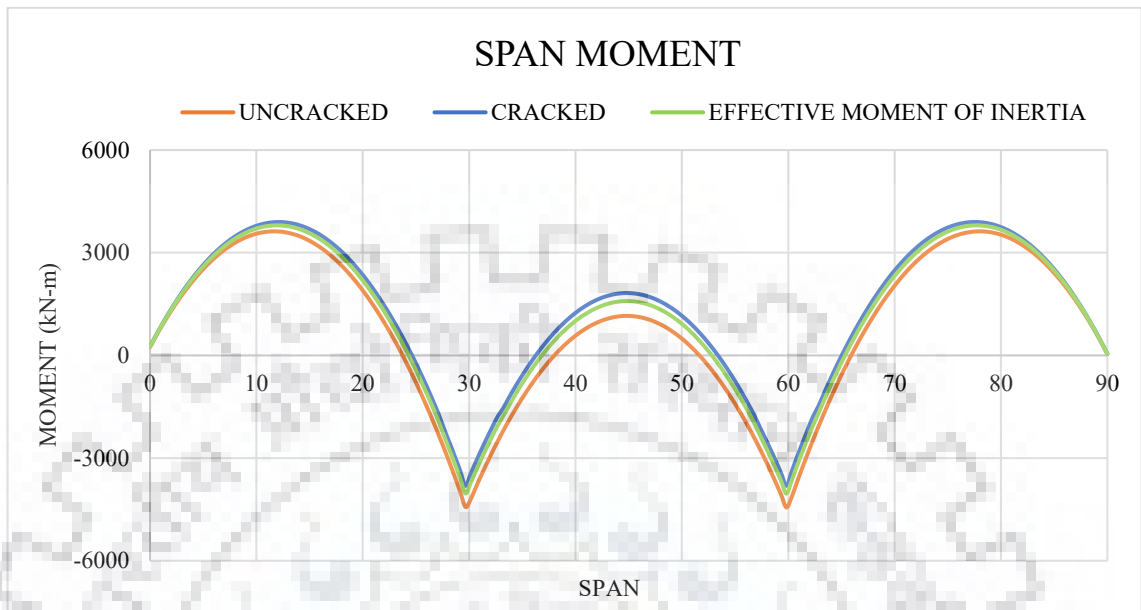


Figure 84: Moment-span for 30m 3-span entire Steel Concrete Composite girder with 0.8 interaction

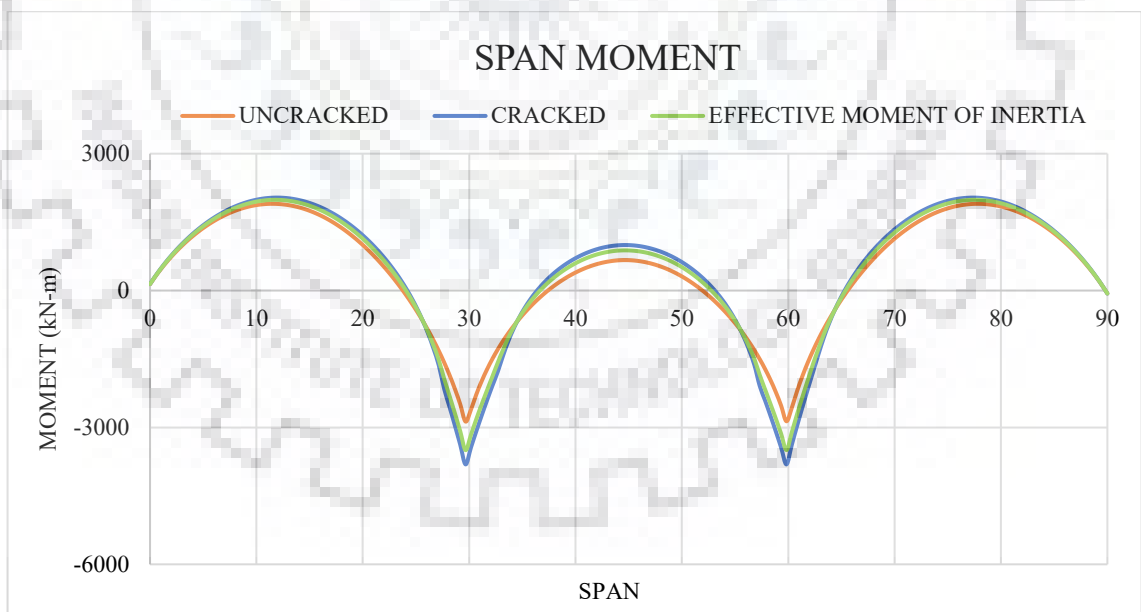


Figure 85: Moment-span for 30m 3-span steel component of Steel Concrete Composite girder with 0.8 interaction

6.5.1.2.4 30 m 5-span

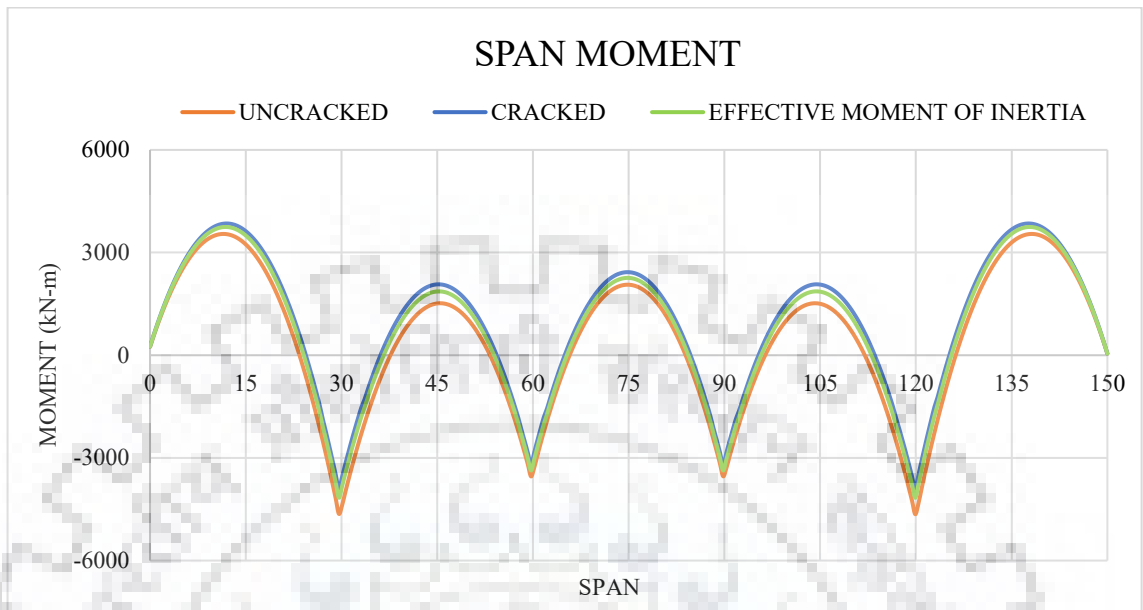


Figure 86: Moment-span for 30m 5-span entire Steel Concrete Composite girder with 0.5 interaction

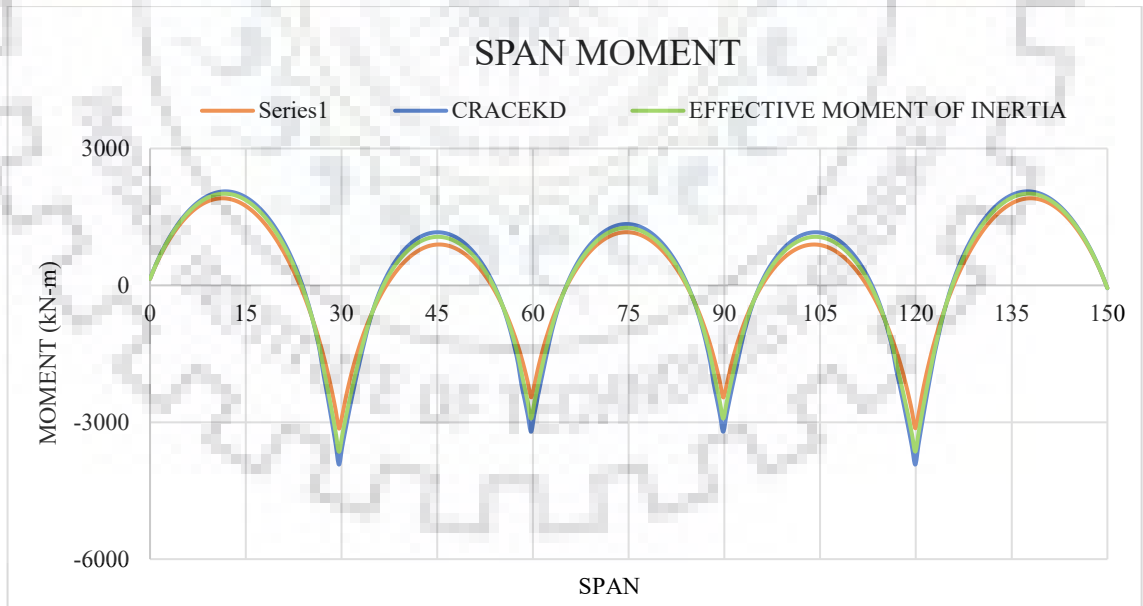


Figure 87: Moment-span for 30m 5-span steel component of Steel Concrete Composite girder with 0.5 interaction

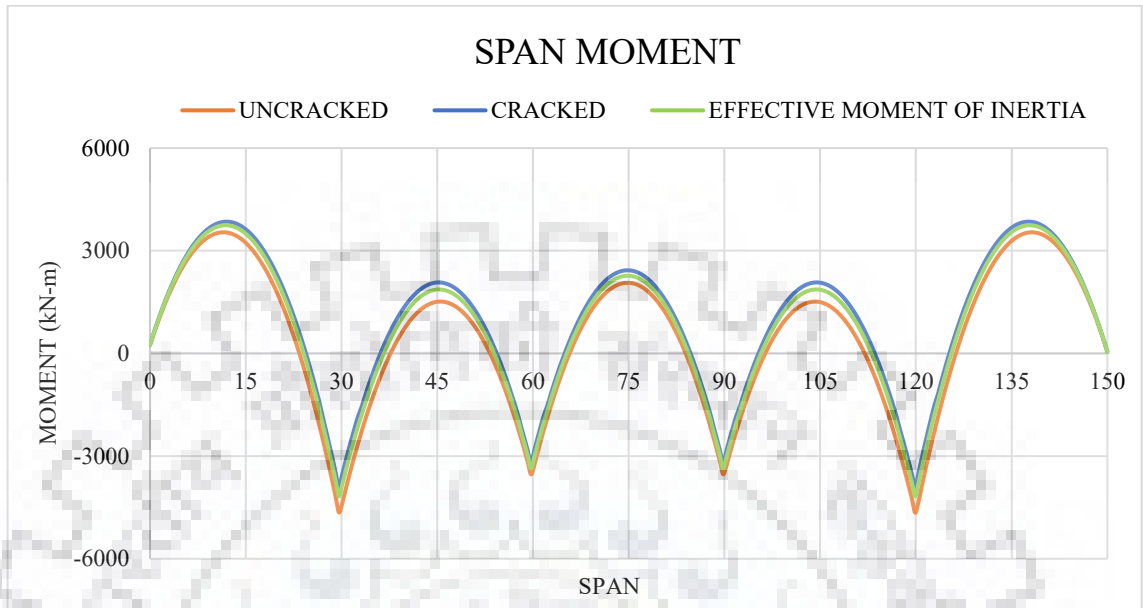


Figure 88: Moment-span for 30m 5-span entire Steel Concrete Composite girder with 0.8 interaction

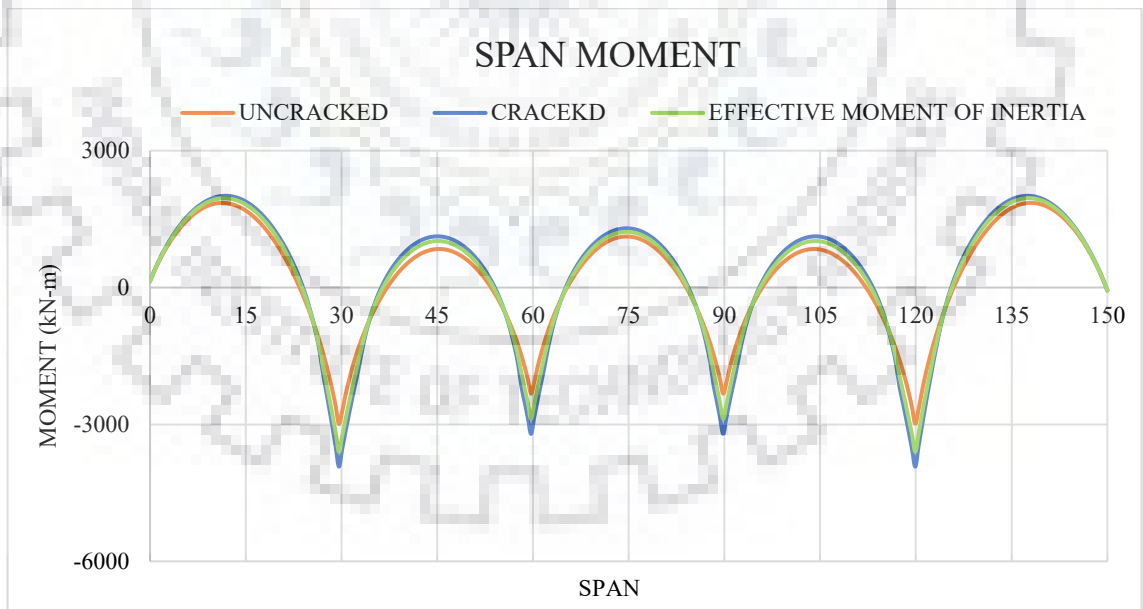


Figure 89: Moment-span for 30m 5-span steel component of Steel Concrete Composite girder with 0.8 interaction

6.5.1.2.5 20 m 3-span

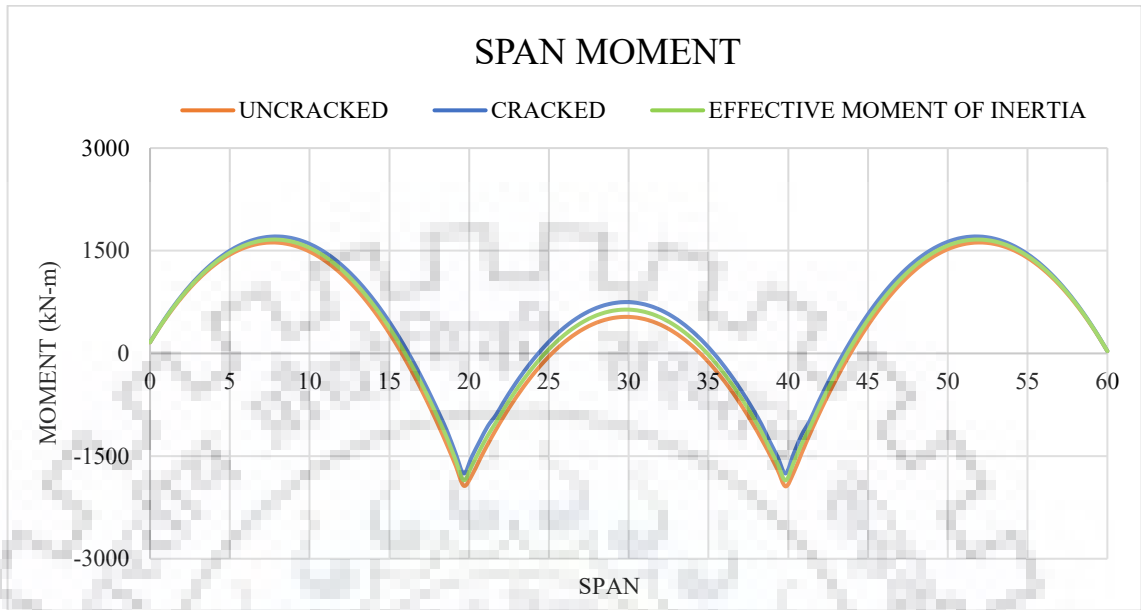


Figure 90: Moment-span for 20m 3-span entire Steel Concrete Composite girder with 0.5 interaction

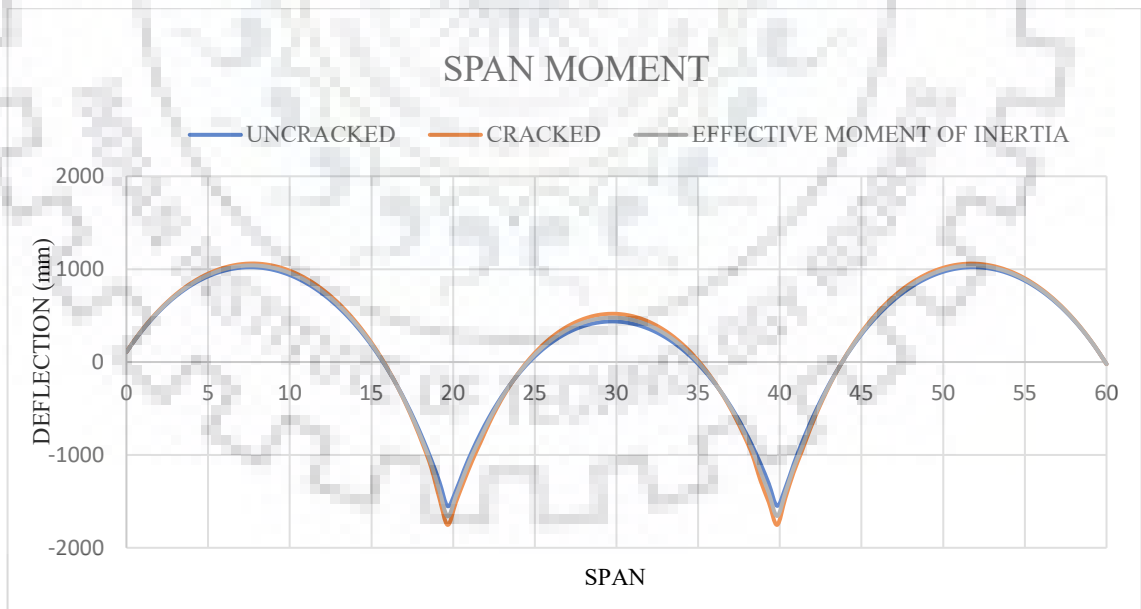


Figure 91: Moment-span for 20m 3-span steel component of Steel Concrete Composite girder with 0.5 interaction

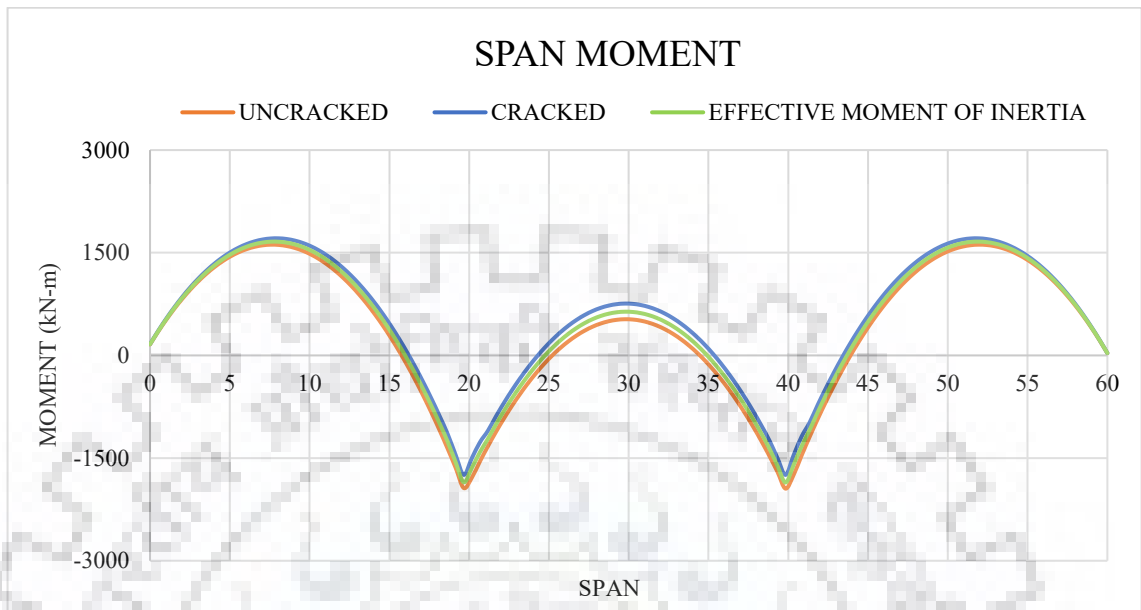


Figure 92: Moment-span for 20m 3-span entire Steel Concrete Composite girder with 0.8 interaction

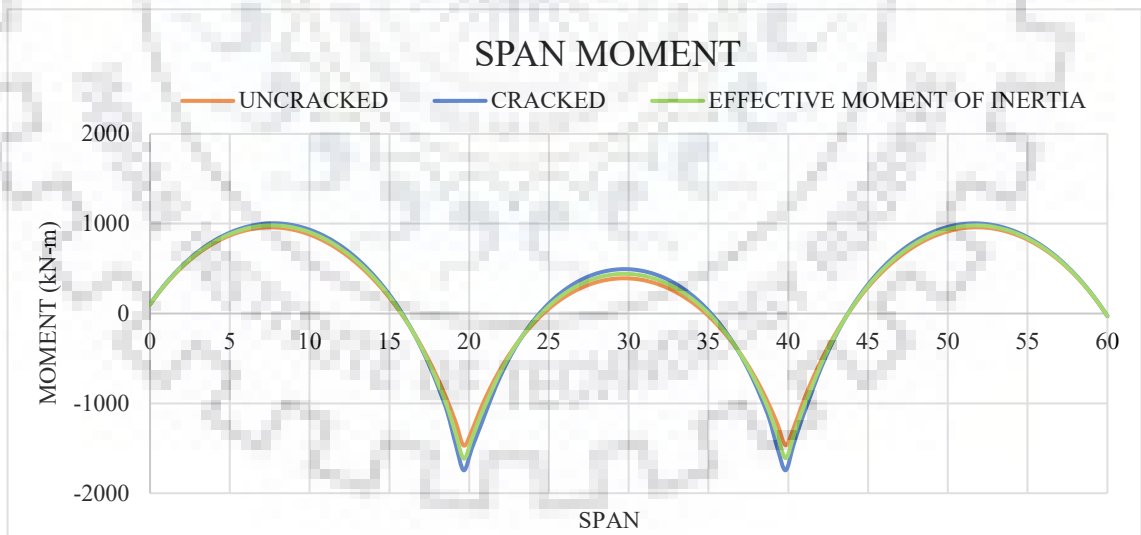


Figure 93: Moment-span for 20m 3-span steel component of Steel Concrete Composite girder with 0.8 interaction

6.5.1.2.6 20 m 5-span

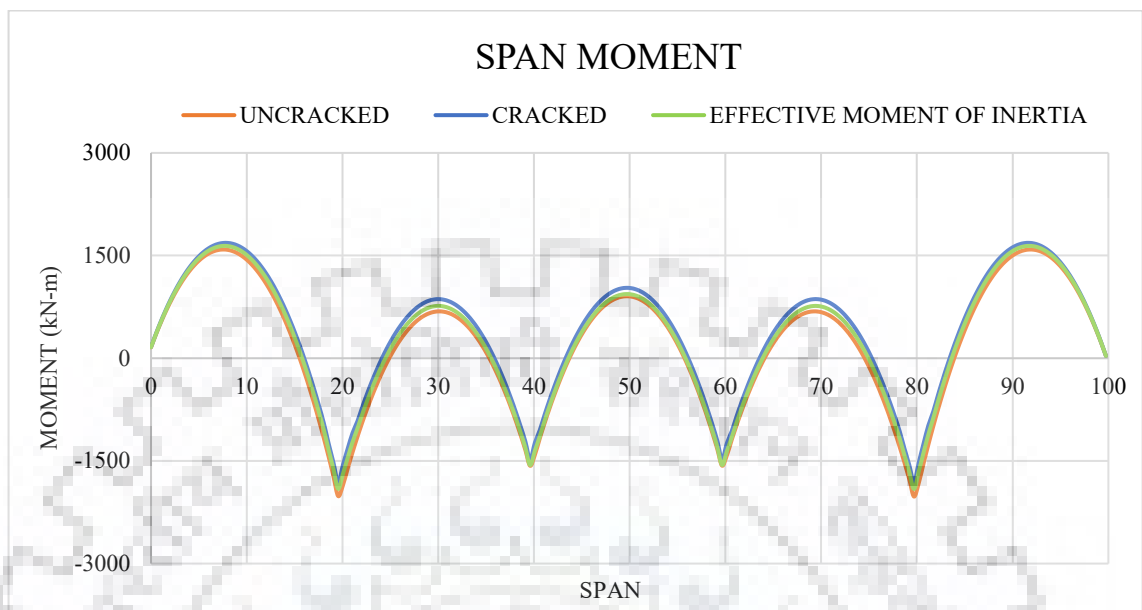


Figure 94: Moment-span for 20m 5-span entire Steel Concrete Composite girder with 0.5 interaction

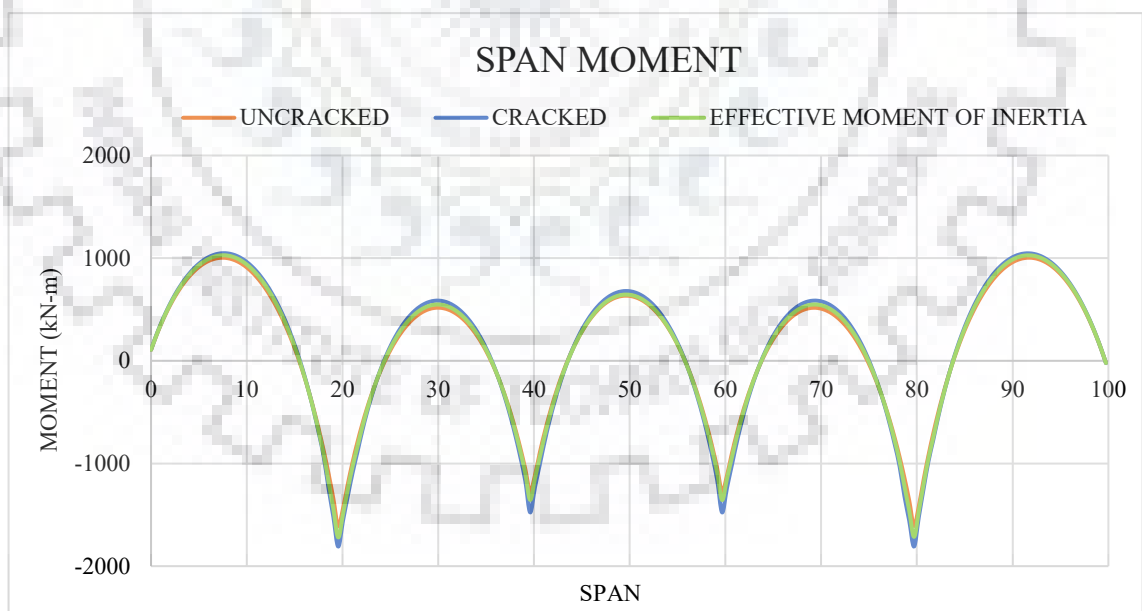


Figure 95: Moment-span for 20m 5-span steel component of Steel Concrete Composite girder with 0.5 interaction



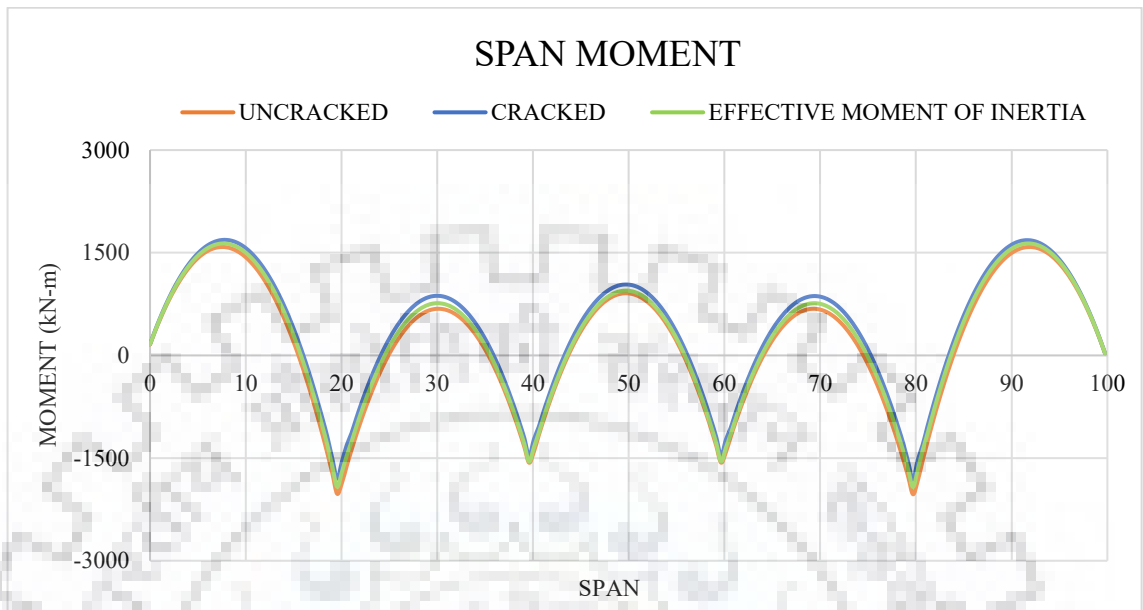


Figure 96: Moment-span for 20m 5-span entire Steel Concrete Composite girder with 0.8 interaction

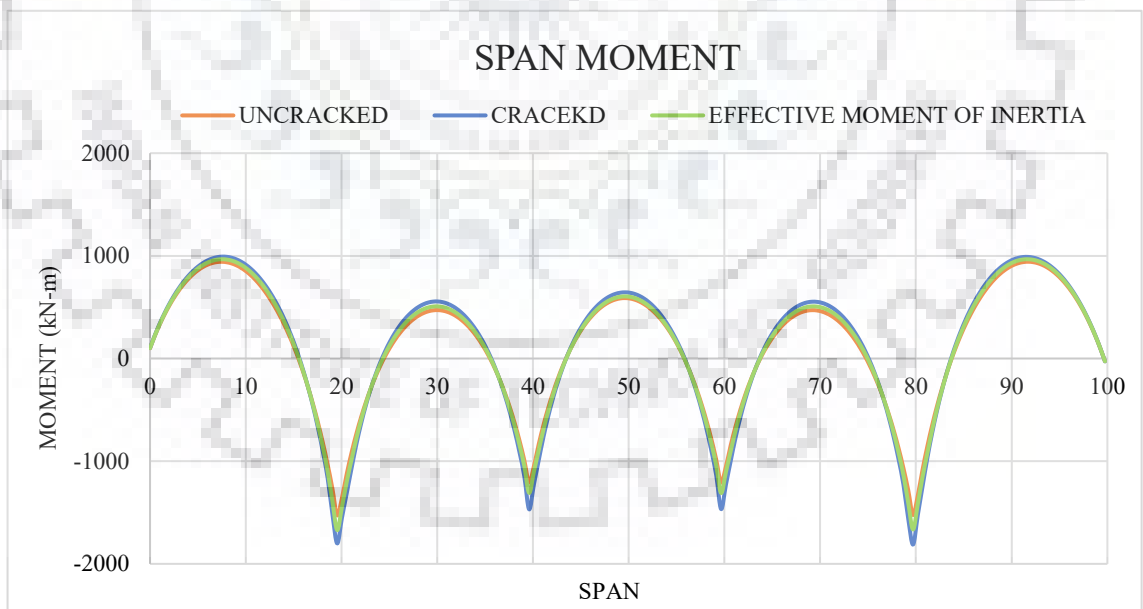


Figure 97: Moment-span for 20m 5-span steel component of Steel Concrete Composite girder with 0.8 interaction

**6.5.1.2.7 Percentage change in moment in the steel girder**

The bar charts to compare percentage change in deflections between no concrete considered at the negative zone vs applying effective moment of inertia at the negative zone. The comparison is made between the different spans, span lengths and degree interaction.

The percentage decrease in the span moment by applying the proposed method for the spans, span length and the degree of shear interactions aforementioned have been presented in the following section. The percentage decrease in the moment depends on several factors such as the type of span, the span length and the type and magnitude of loading used.

This comparison shows the major importance of the study done. The results completely agree with the proposed idea that there will be a reduction in moment at the supports for the steel girder as residual concrete will resist some bending moment.

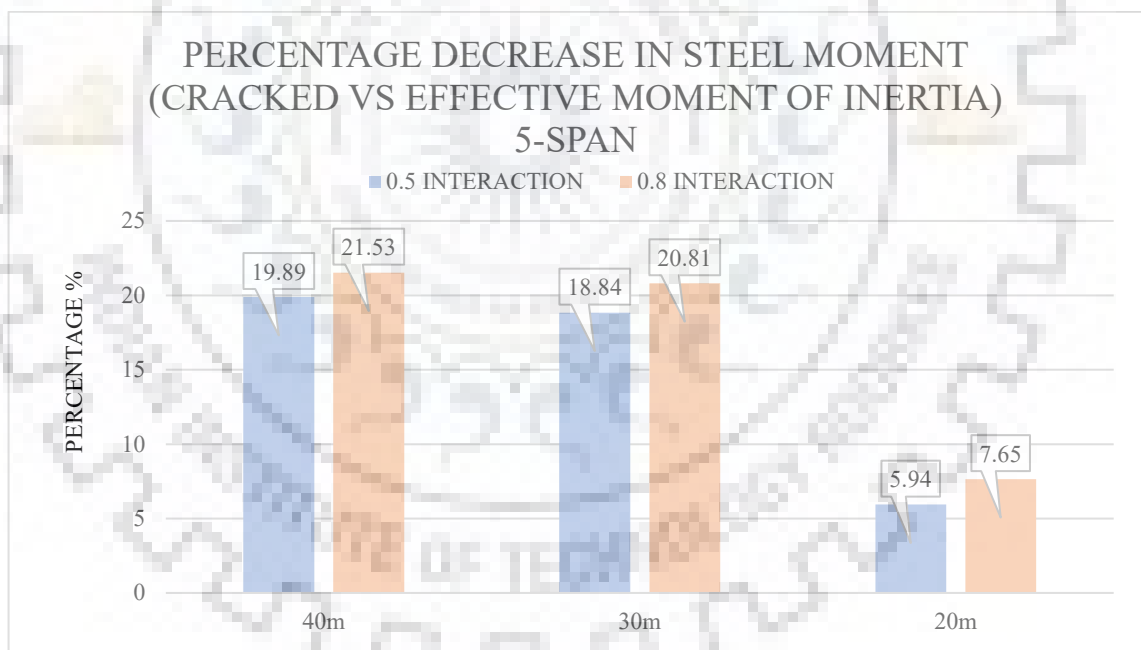


Figure 98: Percentage difference between moments in the steel girder (no concrete v/s effective moment of inertia)

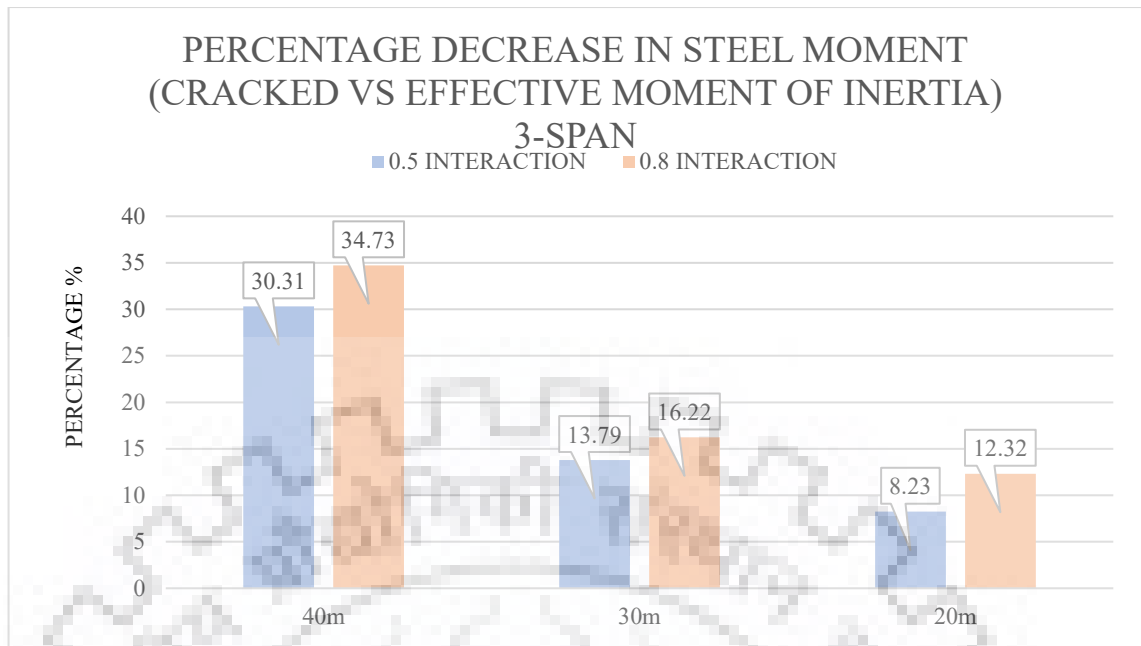


Figure 99: Percentage difference between moments in the steel girder (no concrete v/s effective moment of inertia)

### 6.5.2 Conclusions

As it is seen from the above bar charts, there is a significant decrease in moment taken up the steel girder at the supports when the effective moment of inertia is applied to the concrete. These findings will vary on the span length, type of span and the type of loading.

Since the composite girder is designed as rigid girder (a case which is not practicable as there always will be slip, hence there always will be a change in the moment of inertia of the Steel Concrete Composite girder). The moment at the mid span is assumed to be shared by the two elements assuming that there is a perfect bond between the elements, which generally is not the case. A true scenario exists in which there is slip and partial shear connection. In such scenarios the moment carrying capacity of the components greatly change. The study showed that the steel component of the girder generally takes up more moment than it would have taken had it been a rigid connection. To put that into perspective, the following figure deals with highlighting the change in the mid span steel moment for the rigid as well as the cracked section. It shows the steel girder moment diagram for a 40m 3-span continuous girder subjected to 50kN/m UDL

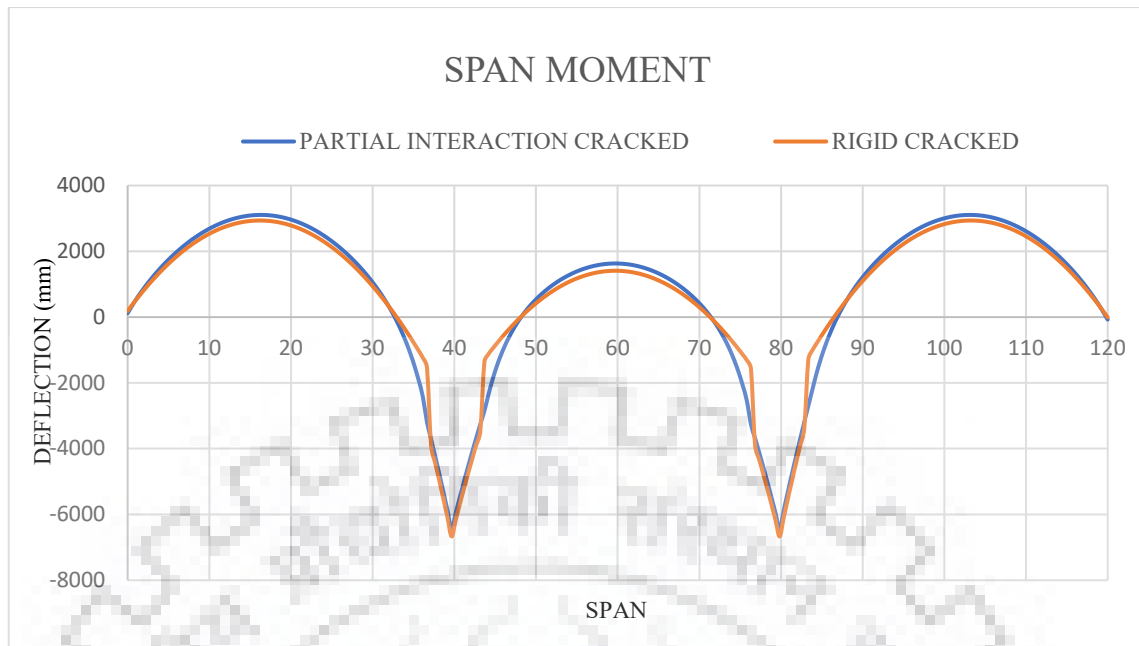


Figure 100: Moment span diagram for steel girder for a 40m 3-span continuous girder.

The change in moment values for the second span when the girder is subjected to similar loading of 50kN/m is 221.053 kNm. Which is a 16% increase in moment from the moment in the steel girder when the interface connection of the composite beam is assumed as rigid. The change in moment will further depend on the type of loading, the magnitude of loading and span length and span configuration.

However, since majority of the designs do not consider any contribution of concrete to resistance of bending moment, the steel girder in the positive bending zone in any case is being designed for higher moments than it would take up.

# **Chapter 7. Modelling of Continuous Composite Girder Bridge Using Grillage Analogy**

## **7.1 Introduction**

The previous chapter deals with the modelling of the continuous composite girder bridge at a very local level. The studies done were for a single girder which spanned for a certain length and had certain spans. This chapter will deal with the analysis of the steel concrete composite girder at a more global level. A very common and widely accepted modelling method for understanding the behaviour of the composite girder bridges is the “Grillage Analogy”. Grillage analogy works around the premise that a solid girder bridge can be represented in the form of truss elements and the deck can be represented in the form of equivalent beam members.

The truss elements of the grillage analogy are provided with equivalent sectional properties of part of the girder which the truss element is representing. The solid steel girder is generally divided into members which represent the flanges and a part of the web (one-third of the web), the web is represented as vertical members which carry the moment of inertia equivalent of parts of web.

This method of analysis is a prevalent method for analysis of simply supported composite girder as this takes far less time than the analysis of a complete solid girder bridge. The method gives accurate results when compared to a FEM model of similar properties.

The grillage analogy has been implemented for the case of a continuous composite girder. The results show a close relationship with the results obtained by the 2-D analysis. Hence rendering the 3-D grillage method more than applicable to be used for analysis of continuous composite girders with partial interaction. The results also were in close agreement with those obtained through the application of effective moment of inertia method. The grillage modelling method can also help in the buckling analysis of the composite beam. Buckling modes can be obtained and the most critical buckling mode and the corresponding load can be determined.

The results of the two methods i.e., the 3-D grillage analogy and the equivalent moment of inertia method have been presented in the subsequent sections of this chapter.

## 7.2 3-D grillage model

Vayas et al. 2010, presented a relatively new method of modelling of composite beams. The model was basically idealised an I-steel beam as equivalent truss elements. The slab was idealised as equivalent bar elements which would basically mimic the actual properties of the slab element.

The diagrammatical representation of the grillage model is shown below.

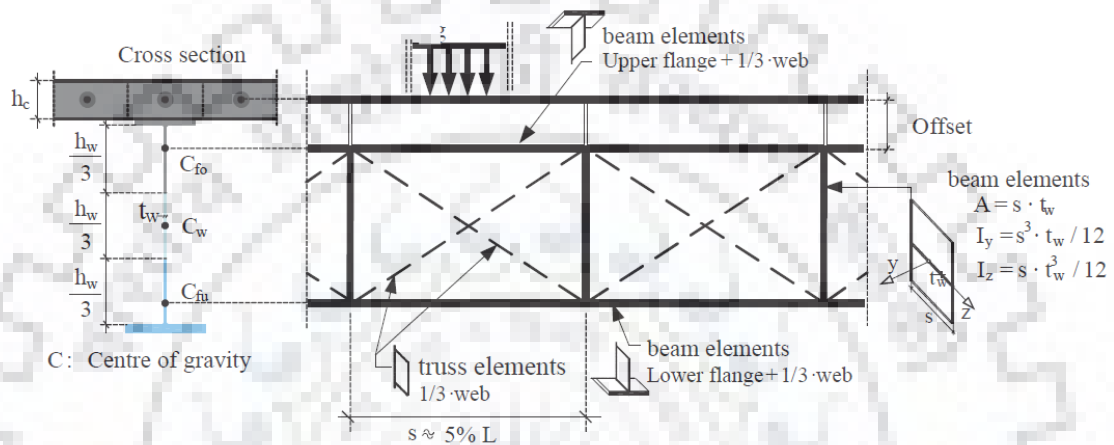


Figure 101: Truss idealization of a steel-concrete composite girder (Vayas et al. 2010)

The steel I-girders is divided into three parts for the ease in modelling. The first and the second part is a T-section comprising of the top flange and bottom flange plus 1/3 of the web respectively. The third portion comprises the remaining 1/3 web. The top and the bottom flanges of the truss are beam elements with the properties of the top and bottom T-section respectively. The vertical members of the truss are beam elements with the cross-sectional properties as follows.  $A = s \cdot t_w$ ,  $I_z = s \cdot t_w^3 / 12$ ,  $I_x = h_w / 3 \cdot t_w^3 / 12$ ,  $I_y = s^3 \cdot t_w / 12$  where  $s$  is the distance between the vertical struts which is taken as equal to or smaller than 5% of the span. The diagonals are truss elements which are assigned the properties of 1/3 of the web. The concrete slab is defined as beam elements with relevant offset to the distance between the centroid of the concrete slab and the upper flange.

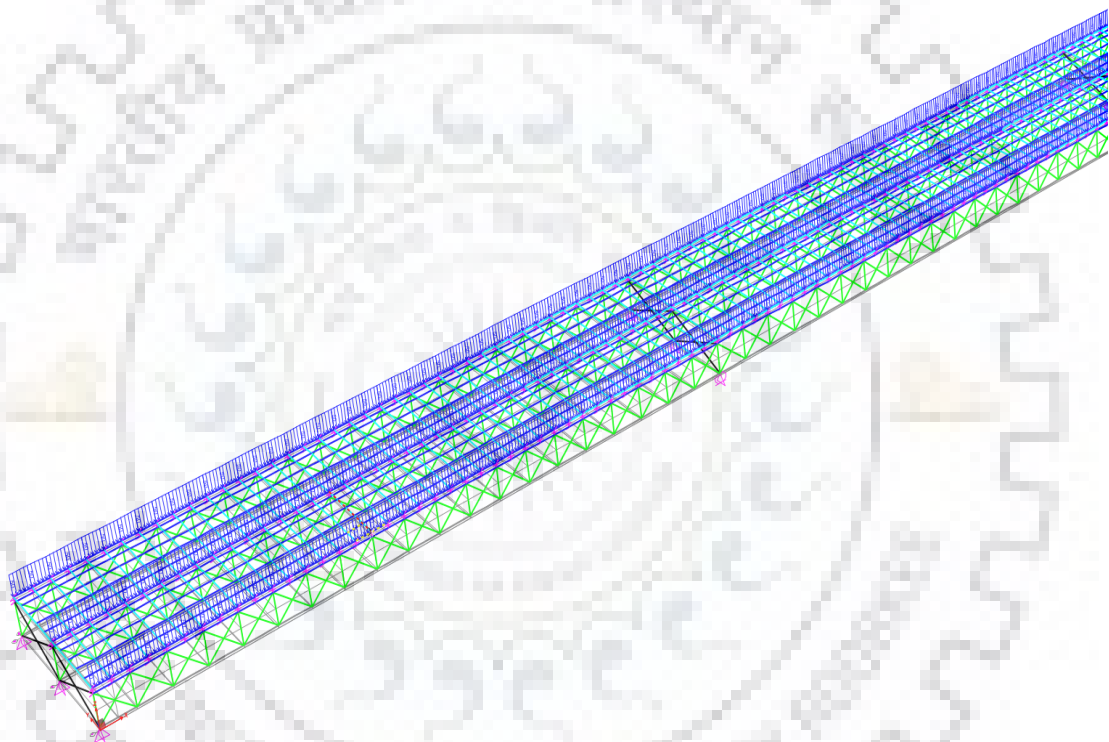
For the current case, the 3-D grillage model was developed for the 20m 3-span continuous bridge girder subjected to a load of 50kN/m on the main girder and 25kN/m on the side girders.

The property of the Steel Concrete Composite girder has been tabulated in the next table:  
*\*Refer Figure 16 for the variables*

*Table 9: Dimensions of continuous composite beam for parametric studies*

SPAN (m)	LOADING (UDL kN/m)	Dimensions (mm)					
		H	Tf	Tw	Bf	Bc	Tc
20	50	1060	30	10	300	1300	150

The rendered 3-D model of the grillage has been presented in the Figure 102.



*Figure 102: 3-D view of the loaded Grillage model*

### **7.3 Results and comparison with model proposed in previous chapter**

After the modelling, the results of the initially uncracked composite girder were extracted. The longitudinal equivalent beam members were then provided with effective moment of inertia at the negative bending zone. The results were extracted, and the comparisons were drawn with the results obtained by the method proposed in the previous chapter.

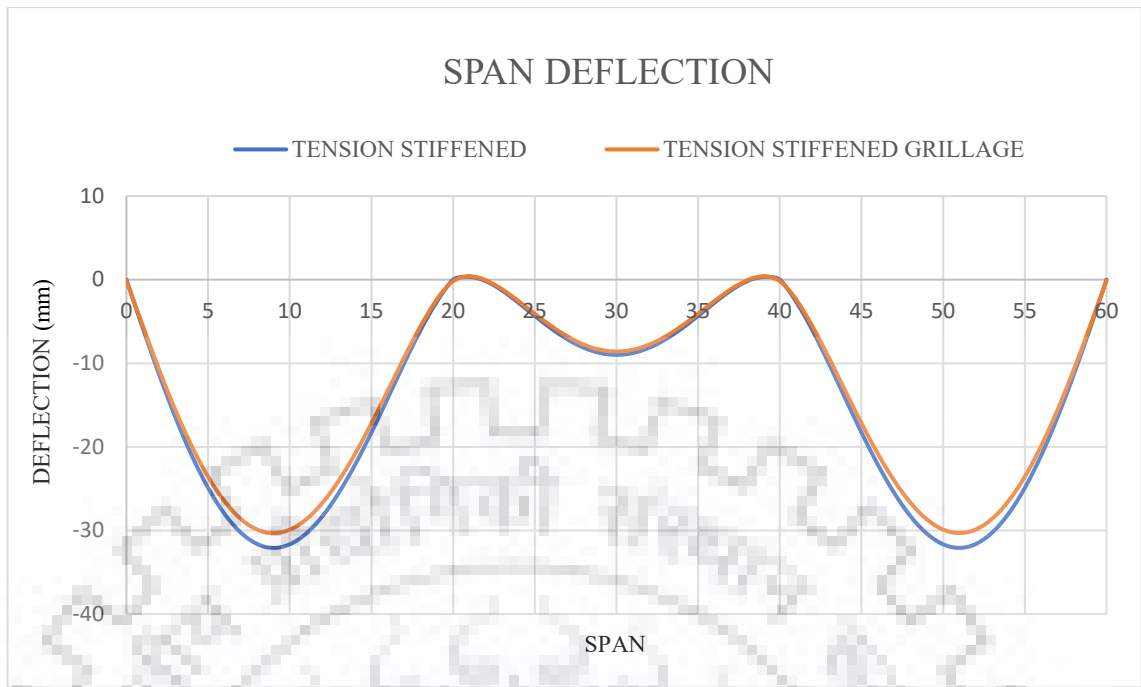


Figure 103: Deflection v/s Span (20m 3-span 0.8 interaction)

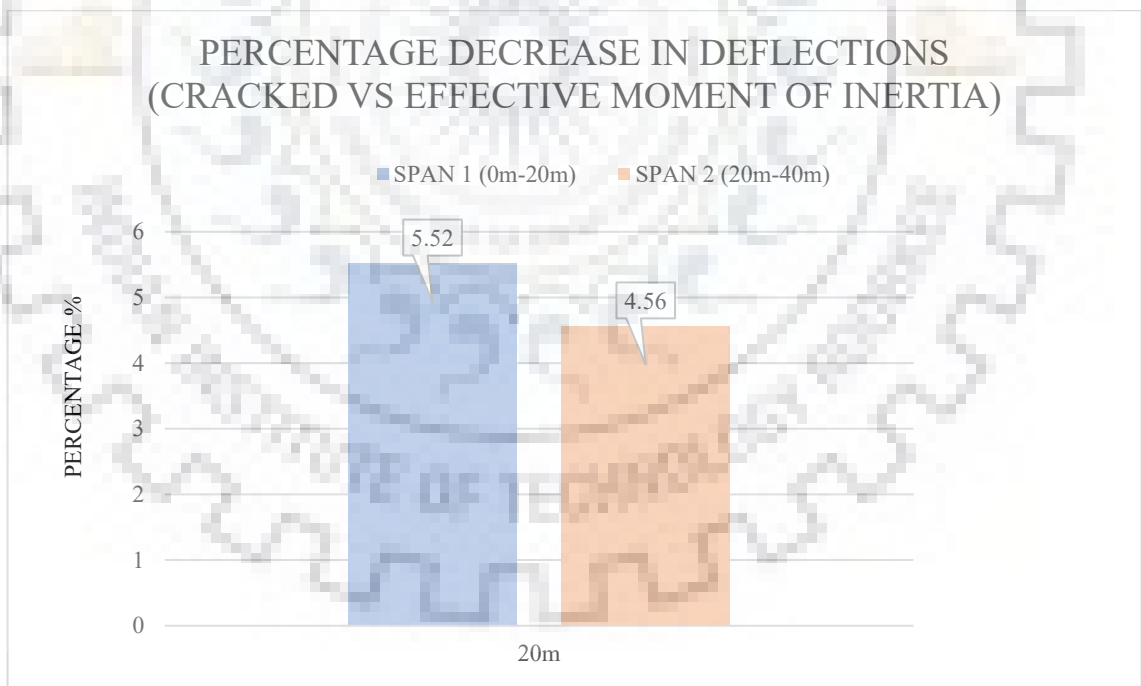


Figure 104: Percentage difference between deflections in the Steel Concrete Composite girder (proposed model v/s 3-D grillage)



## 7.4 Conclusions

As seen from Figure 103 and Figure 104, the difference in the deflections as obtained from the 3-D grillage analogy and the 2-D model proposed in chapter 6 is very small. The percentages of difference in the span deflections for span 1 and span 2 are 5.52 and 4.56 respectively, the deflections recorded by the grillage is slightly less than the deflections determined by the 2-D model because transverse stiffness plays an important role in yielding a more rigid model and hence resulting in reduction of the overall deflections.



## Chapter 8. Discussions on Results and Concluding Remarks

### 8.1 Results and discussions

The present study discusses the behaviour of continuous steel concrete composite girders with partial shear interaction. A numerical study has been carried out considering various influencing parameters such as degree of shear connection, span lengths and span configurations. Since the continuous girders are subjected to negative bending moment, the concrete in the steel concrete composite girder experiences tensile stresses eventually leading to concrete cracking. The effect of concrete cracking on the behaviour of the continuous steel concrete composite girders is addressed considering the following three different cases.

Case 1. Continuous steel concrete girders without considering cracking of concrete in negative bending zone.

Case 2. Continuous steel concrete girders considering fully cracked concrete in negative bending zone.

Case 3. Continuous steel concrete girders with effective moment of inertia of cracked concrete in negative bending zone. For the incorporation of the effective moment of inertia of the cracked concrete, a numerical modelling method has been proposed.

For all the above cases considered a parametric study has been carried out considering aforementioned parameters.

1. It is observed that when the degree of shear interaction decreases the deflection increases with respect full interaction. For an instance, a 60% increase in the deflection of a 7.5m 5-span composite girder subjected to a 10kN/m UDL was observed when the degree of shear connection was varied from full interaction to 0.5-degree of shear interaction. This percentage increase is lesser for longer spans.
2. The deflections in the steel concrete composite girder also changed when the span configuration changes. The change in deflections for a 7.5m composite girder subjected to 10kN/m UDL varied from 20% to 60% for a girder with 3-spans to a girder with 5-span. The percentage increase in the deflections for longer spans with variation of span configuration was lesser.

3. For a similar span configuration, the amount of deflection increases as the length of span increases.
4. Since for a continuous composite girder with a particular span length and span configuration, the forces in the concrete and steel component are not influenced by the change in degree of shear interaction.
5. It is observed that when the degree of shear interaction decreases the deflection increases with respect full interaction with cracked concrete at the supports. For an instance, a 20% increase in the deflection of a 40m 3-span composite girder subjected to a 50kN/m UDL was observed when the degree of shear connection was varied from full interaction to 0.5-degree of shear interaction. This percentage increase is greater for shorter spans.
6. The deflections in the steel concrete composite girder also changed when the span configuration changes. The change in deflections for a 40M composite girder subjected to 50kN/m UDL varied from 10.24% to 20% for a girder with 3-spans to a girder with 5-span. The percentage increase in the deflections for longer spans with variation of span configuration was lesser.
7. For a similar span configuration, the amount of deflection increases as the length of span increases.
8. For a composite girder with cracked concrete in the negative bending moment area, the redistribution of forces in concrete and steel component takes place as the flexural and axial rigidities of the concrete is neglected for areas subjected to negative bending moment. The redistribution of forces for the entire section of the composite girder occurs when compared to a composite girder with uncracked concrete section at the negative bending zone. For instance, an increase of 24.78% moment at mid span and a decrease of 13.83% moment at the internal support of a 40m 3-span composite girder with 0.5-degree shear interaction was recorded. The change in percentage of span moment was greater for shorter spans for similar shear interaction and span configuration whereas the change in percentage of support moment decreased for shorter span length for similar shear interaction and span configuration. The change in steel girder moment was also seen when the composite girder with no concrete in negative zone was compared against composite girder with full concrete in negative zone. An increase in moment of 50.7% was observed at the internal support as and 64.2% increase in moment at the span for a composite 40m

3-span with 0.5-degree shear interaction. This change in moments was smaller for shorter spans with similar degree shear interaction and span configuration.

9. For a composite girder with cracked concrete in the negative bending moment area, the change in moments at supports and at mid span for similar span configuration and span length was observed to be lower for a greater degree of shear interaction.
10. For composite girders with effective moment of inertia for concrete subjected to negative bending moment, a lower deflection was observed when compared to deflections obtained from numerical models with no concrete in the negative bending zone. For instance, there is a decrease in deflections 25.93% for a 40m 3-span continuous girder subjected to 50kN/m UDL and 5.65% for a 20m 5-span continuous girder subjected to 50kN/m UDL. The reduction in span deflection was lower for shorter spans.
11. The reduction in the steel moment at the supports of the composite girder was also observed. For instance, a 34.73% reduction in moment for a 40m 3-span continuous girder subjected to 50kN/m UDL and 5.94% reduction in moment for 20m 5-span continuous girder subjected to 50kN/m UDL. The reduction in moment in the steel girder was lower for shorter spans.
12. The overall moment in the composite girder also lower at the mid spans for girders with effective moment of inertia as compared to composite girders with no concrete in the cracked zone. For instance, a decrease in 19.85% moments was observed at mid span for a 40m 3-span continuous girder with 0.5 shear interaction. The change in moments for different span lengths and shear connection ranged from 10-20%.

From the above findings it is observed that partial shear interaction has a significant influence on the deformation of the composite girder if this is ignored during the design stage it may lead to major serviceability issues.

Effect of using effective moment of inertia increases the overall stiffness of the girder and leads to an overall decrease in the deflections of the continuous composite girder when compared with a fully girder with fully cracked concrete at the support regions. This leads to an overall economic design of the composite girder.

## 8.2 Scope for the future studies

The current study deals with the importance of slip-deformation relationships, the change on deformation with change in the degree of shear interaction, effect of span configuration on the resulting deformation and also deals with the importance of residual strength of concrete after it has undergone cracking due to the application of negative bending moment. The study has conveniently proposed a numerical model for the application of residual strength of concrete in a steel concrete composite girder.

The application of the proposed model theory has been extended to the 3-D spatial system of loading through the application of the grillage analogy of representation of member properties. A partial study on the buckling and stability analysis of the continuous steel concrete composite girders has been done but has not been included in the current study due to the incompleteness of the study of the impact buckling will have when coupled with the proposed model.

While the above-mentioned topics have been addressed in the present study and a viable methodology accompanied by a numerical model simulation mechanism has been proposed, there are still some areas which can be addressed which the current study was unable to address due to the constraint of time.

The following works can be taken up as a future expansion to the current work

1. The incorporation of the long-term effects of creep and shrinkage due to the sustained loading condition and its eventual effect on the deformations of the continuous steel concrete composite girder.
2. The incorporation of a buckling and stability analysis for different types of loading can be taken up and can be coupled with the proposed model to understand the effects of change in loading type and magnitude on the overall stability of the proposed model.
3. The incorporation of the above mentioned to a system level study using grillage methodology to understand the effects of each on the global level.

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