DEVELOPMENT OF FEEDBACK LOOP ALGORITHM FOR PSEUDO-DYNAMIC TESTING

A DISSERTATION

Submitted in partial fulfilment of the

requirements for the award of the degree

of

MASTER OF TECHNOLOGY

in

EARTHQUAKE ENGINEERING

(With Specialisation in Structural Dynamics)

By

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JUNE, 2019

CANDIDATE'S DECLARATION

I hereby declare that the work carried out in this end term evaluation report entitled **Development of Feedback Loop Algorithm for Pseudo-Dynamic Testing**, is being submitted to the department of Earthquake Engineering, IIT Roorkee in partial fulfilment for the requirements of the award of degree of Master in Earthquake Engineering with specialization in STRUCTURAL DYNAMICS. This is an authentic report of my work prepared by myself during the period of May, 2018 to June, 2019 under expert guidance of the supervisor **Dr. Manish Shrikhande**, Professor, Department of Earthquake Engineering, IIT Roorkee.

The matter embodied in this report has not been submitted by me for the award of any other degree or diploma.

Place: Roorkee — Ajmera Shreyansh Manojkumar

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CERTIFICATE

This is to certify that the above statement made by the candidate is correct to best of my knowledge and belief.

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ABSTRACT

In understanding the dynamic behaviour of many structures where analytical modelling does not suffice and in the development of the codal provisions, experimental testing has remain the common thing to rely on. Many experimental testing procedures have been developed in the past and is still being updated based on the requirement of the testing. Pseudodynamic testing procedure is one of the many testing methods for large scale testing which has the simplicity of quasi-static testing and the reliability of the shake table testing method. Here, various testing methods has been described and then the development of pseudodynamic testing method is presented. The testing procedure, numerical techniques and the error in the testing method is discussed further. For the development of this testing facility in pseudodynamic laboratory at IIT Roorkee, this study is done. For the analytical simulation of pseudodynamic testing, a code is developed which updates the stiffness of the non-linear member during dynamic analysis. The code is verified with a solved example and then is used for the analytical simulation of a steel cantilever column as a SDOF system. The simulation is done for three different earthquakes and the results are presented thereafter.

ACKNOWLEDGEMENT

I express my utmost respect to my supervisor **Dr. Manish Shrikhande**, Professor, Department of earthquake Engineering, IIT Roorkee, for his expert guidance, co-operation and encouragement for research throughout my stay in IIT Roorkee. I would also like to thank him to introduce me with Finite Element Method and Numerical Methods for Dynamic Systems in the first semester.

I would also like to thank Dr. Pankaj Agarwal, Professor and Head, Department of Earthquake Engineering, IIT Roorkee. I express my gratefulness to other faculty members as well as departmental staffs.

I would like to thank my friends and seniors for extending their immediate helping hands among whom special mentions are Mr. Arka Maity and Mr. Kaushal Patel.

Finally, I would like to thank my Parents who always stood by me irrespective of my failure or success.

Ajmera Shreyansh Manojkumar

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Chapter 1

Introduction

Earthquake is a rare phenomenon but whenever an earthquake of high intensity comes also brings on devastating effects. The principal goal of the designer is to design a structure so that no loss of life should occour. The energy during a seismic event that a structure is subjected to, needs to be dissipated in some form. Either we can design our structure to respond elastically which makes it highly uneconomical or to allow our structure to respond inelastically and hence accounting for economy also. Various performances were defined according to the usage of the structures immediately after a seismic event. The structure must correspond to that performance level after going through the inelastic deformations. However inelastic behaviour of the structure is very complex and depends upon various factors such as material properties, detailing of the reinforcements, workmanship employed during construction, and also on the seismic characteristics of the event. Understanding the inelastic behaviour of the structure is extremely important to keep a check on the performance of the structure during any seismic event.

To rely on the inelastic deformation of the structure to dissipate the energy coming on the structures, we need analytical as well as experimental validation. Techniques to evaluate the structure analytically are available but are based on one or other assumptions and mathematical idealizations to simplify the analysis. Although design codes have been developed for dynamic analysis, actual behaviour of the structure during the seismic event cannot be assesed completely with confidence based only on the analytical evaluation of the structure. Hence experimental analysis of the structure or a part of it is necessary in cases where we cannot place confidence on the analytical evaluation. Experimental testing remains the most reliable means to evaluate the inelastic behaviour of the structural systems and to devise the structural details to improve structural performance [8]. Various testing methods are available for the assesment of the structural behaviour and Pseudo-Dynamic Testing (On-Line testing method or Hybrid method) [9] is a new testing method emerged in early 1970's and is reliable for most cases and hence will be discussed in this work.

1.1 Various Testing Methods

Testing the structure experimentally remains most reliable means to assess the structure. Various testing methods such as Shake Table test, Forced Vibration tests, Quasi-Static testing, Pseudo-Dynamic Testing and Real-Time hybrid method are available with their advantages and disadvantages altogether.

In Shake Table testing, the structural model is prepared and is mounted on the table where the earthquake records are applied to test the model in real time and hence the dynamic and rate-dependent behaviour is completely incorporated. Facility available at E-Defence laboratory in Japan can test full scale structures but is a costly affair hence, structural model which is placed on the table are usually scaled down for the testing due to the limitations of the table itself. The model which is to be tested is restricted in size and weight and installing a new table or increasing the capacity of the existing table is a very costly option. To drive the actual full scale model requires huge power and cost. Also due to this scaling down of the structure, material and dynamic similitude problem arises in the structure. Various size based effects such as crack propogation, shear and bond in RCC members and buckling in steel members are the issues with shaking table test method. Due to these limitations various other testing methods were developed.

In Quasi-Static testing, predefined loading histories or displacement histories are applied to the structure in a quasi static (i.e. slow rate) manner which is easy to conduct in most of the structural engineering laboratories with the existing hardware infrastructure. During testing, the structure can be monitored well and the testing can be stopped anytime to analyse the condition of the structure. The predefined displacement histories that are to be applied are determined from the analytical computation and then the displacements computed at different nodes are used to control the experiment but again the idealizations make these displacement histories much more idealistic and hence are not likely to be realistic. Generally these tests are performed with cyclic displacement histories or loading to assess the following: (i) assessing the the effect of different structural details on the inelastic behaviour of the structures by subjecting different specimens to identical loading histories; and (ii) studying the basic mechanisms that affect the inelastic behaviour of a particular structure by varying the amplitude, rate or pattern of the applied deformation histories [8].

Hence for testing the structure in a more rational and reliable manner a Computer-

Actuator On-Line system was developed in Japan in early 1970's for the study of the inelastic behaviour of the structure. This testing method has the realism of shaking table test along with the versatality and economy of the quasi-static testing. Here more realistic displacement histories can be applied quasi-statically along with incorporating the dynamic effects in it and with the available hardwares in most structural engineering laboratories. This testing method is also called as Pseudo-Dynamic testing method or Hybrid Testing method. This unique technique combines both the numerical techniques and the experimental testing and hence has the benifits of both.

The pseudodynamic testing method or hybrid method combines both the numerical technique and the experimental testing for testing the structure. It is a displacement control method which uses the feedback response from the structure. While in quasi-static testing the displacement or force histories that were applied to the structures were cyclic in nature, here in pseudodynamic testing method, the dispalcement to be applied to the structure is calculated in each step and then a feedback from the system is taken back to the computer to calculate the next displacmement which is to be applied. This feedback depicts the stiffness properties of the structure and the dynamic properties are taken into account using the computer, making this testing method more reliable and rational.

The testing can be done on a full scale model where the mass and damping properties are modelled analytically and the stiffness comes from the feedback during the testing. Displacement calculated using predefined inetrial and damping properties and the ground mortion input is applied to the structure and then a feedback is taken from the load transducers, which represents the restoring force of the structure, is then used to calculate the next step displacement using the time integration schemes and the process goes on.

Chapter 2

Development of Pseudo-Dynamic Testing Method

The development of the pseudodynamic testing method started with the testing of the SDOF systems in Japan. Initial series of tests were done on the single story single bay reinforced concrete and steel frames [9]. Takanashi et al. showed that the results obtained using the pseudodynamic testing were well corelated with the shaking table test and analytical test results. In 1980, Okada, T. et. al. preformed the pseudodynamic testing of RC frames to bi-directional ground motion [6]. Problems associated with the errors in experimental testing, numerical techniques and the structural idealisations were also reported. EERC in 1980's published many reports on the development of the pseudodynamic testing facility [8], error propogation effects in experiments [7], hybrid solution techniques [10] and the substructuring techniques [3] in pseudodynamic testing method. Modified Newmark Method was proposed by Shing and Mahin to compensate for the spurious growth of higher frequency responses due to the errors during experiments in MDOF systems.

The testing of the structure as a whole requires a high capacity equipments and can be costly too. To test the structure on a reduced scale model again raises an issue of similitude and the actual behaviour of the local critical elements can not be attained. Moreover, out of the whole structure only certain members undergoes inelastic deformation whose analytical formulation requires nonlinear modelling. Hence, a new technique was developed in which the only a part of the structure is actually tested in the laboratory and the remaining part of the structure is analytically analysed on the computer. This technique is called substructuring in pseudodynamic testing. In ths technique, the whole structure is divided into two parts (i) physical substructure and (ii) numerical substructure. The equation of motion of the combined system is solved on the computer where the restoring force characteristics are fed back from the experimental substructure. Since the testing of a MDOF system raises an issue of the stability of the numerical procedure which is bounded by the limit to $\omega \Delta t$.

Since for the large scale testing, of MDOF systems, an implicit-explicit integration algorithm was developed, where the physical sybstructure was solved using explicit schemes and the numerical substructure was solved using implicit schemes [3]. The substructuring loop is shown in the Figure 2.1 where the numerical substructure command the actuator to apply the computed displacements and then the feedback is sent back from the physical substructure to the numerical substructure [11].

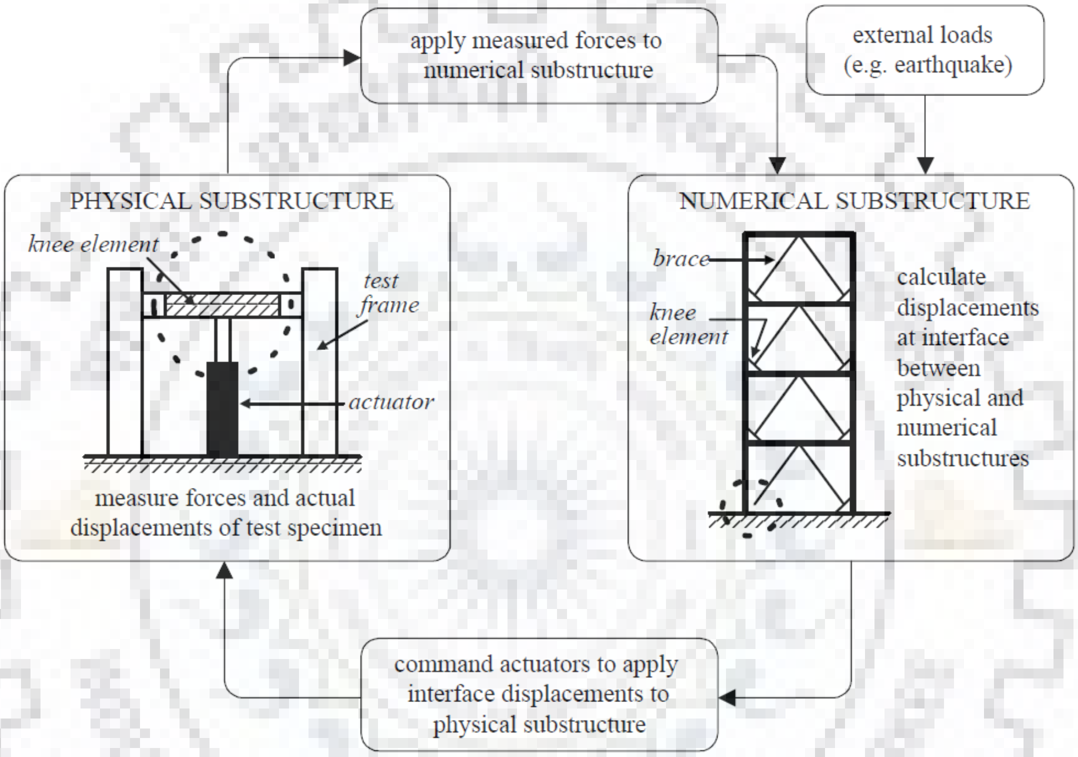


Figure 2.1: Schematic of Substuructuring Pseudodynamic Testing Loop [11]

In pseudodynamic testing, since the loading is applied in a quasi-static manner, the structures with the material sensitive to the rate dependent effects cannot yield reliable results. As explained in the Section 3.4.1, the pseudodynamic test is not applicable for structures with such materials. In order to overome this problem, Takanashi and Ohi in 1983 developed an algorithm for fast testing. In pseudodynamic testing, the actuator stops when the target displacement is achieved but in fast testing the actuator moves continuously which raises the issue of overshooting [4]. During the feedback and computation of the next target displacement, the actuator doesnot stops and continues to apply the extrapolated displacement till the next displacement is calculated. This started the development of the Real Time Hybrid Testing.

Chapter 3

The Pseudo-Dynamic Test Method

3.1 Testing Procedure

Pseudodynamic testing is a combination of experimental setup and analytical computations done together. The foremost assumption here is that the response analysis can be done upto certain accuracy using discrete parameter system with finite number of degrees of freedom. Hence, for analytical computations, the structure is modelled as a discrete parameter model (lumped mass model) in the computer with finite number of nodes and degrees of freedom. Damping matrix is formulated using standard models and modal damping properties. Mass matrix and damping matrix are analytically computed and fed to the computer model. A physical substructure or a structure as a whole is built, upon which testing is to be done. Actuators are attached at the nodes where displacement response is to be applied quasi-statically, according to the degrees of freedom to be provided. A time stepping algorithm fit for the test is used in the computer to calculate the next response which is to be applied on the physical specimen. This displacement response is computed based on the previous responses, initially defined mass and damping matrix, and the restoring force feedback from the physical specimen and again forms the basis of the next displacement response which is to be applied quasi-statically to the physical specimen. This testing is done on an extended time scale usually goes upto 100 times the actual earthquake duration. Figure 3.1 shows a schematic diagram of pseudodynamic testing procedure.

Here, the stiffness property of the physical specimen is measured experimentally and hence the uncertainities or difficulties associated with the modelling of the material properties and hysteresis loop are removed. This helps in improving the analytical models available for modelling the non-linear behaviour of the material and members to represent more realistic behaviour.

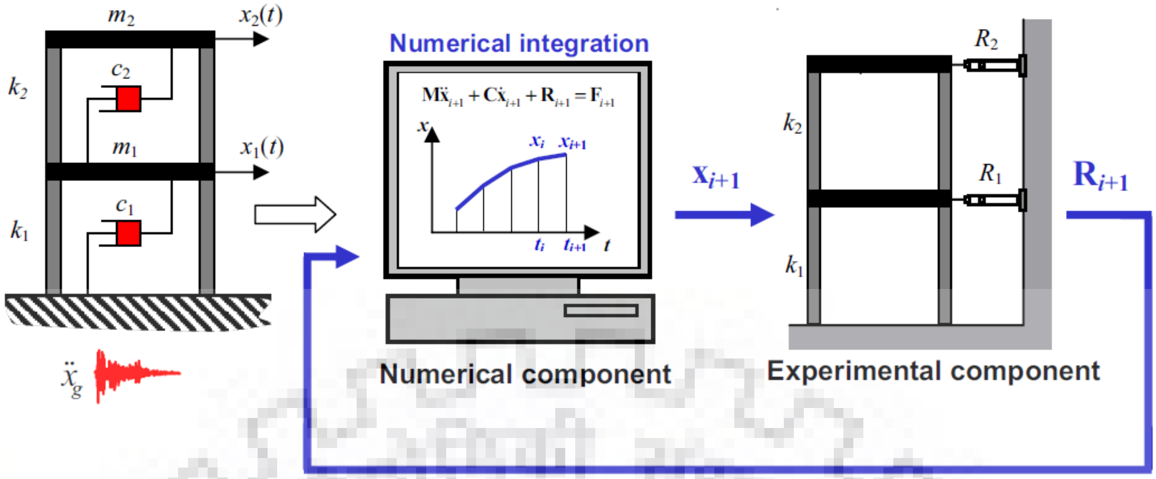


Figure 3.1: Schematic diagram of pseudodynamic test method [2]

3.2 Discrete Parameter Structural Model

In pseudodynamic testing method, the displacement responses are imposed at certain nodes on the physical structure. Imposition of the displacement response on the whole continnum is not possible in this method. For this, the modelling of the structure in computer is done as a discrete parameter model. The actual structure has mass distributed throughout and has infinite degrees of freedom. Such system involves the solution of partial differntial equation which is a difficult task to do. This discretization leads to the simplify the calculation of the response of the strucure. The mass is assumed to be concentrated at certain nodes where the displacement response is to be applied. The equation of motion of this discrerte parameter model is given as follows:

$$mx2_{i+1} + cx1_{i+1} + fs_{i+1} = p_{i+1} \tag{3.1}$$

where m and c are mass and damping respectively. $x2_{i+1}$ and $x1_{i+1}$are the acceleration and velocity response quantities respectively at time $(i+1)\,\Delta t$, where Δt represents the time step for the integration scheme. The fs_{i+1} represents the restoring force quantity and is measured experimentally during the testing. $p_{i+1} = -ma_g$, where a_g is the earthquake ground acceleration, represents the external excitation force (earthquake) for which the structure is to be tested. The mass matrix is formed by lumping the mass at certain nodes. Damping matrix is formed using idealized modal damping properties and can be assumed to be mass or stiffness proportional daming or both. In lumping the masses at certain nodes, the continuum structure is discretized and hence, only those number of modes of the structure is considered and the higher mode effects are not considered. Since modal contribution factor is comparitively less for higher mode responses, this discretization is valid and provides sufficiently accurate results.

3.3 Numerical Integration Scheme

The pseudodynamic method requires the calculation of the displacement response which is to be applied quasi-statically to the physical specimen. For this response, the equation of motion for the structure is solved using well established numerical integration schemes based on previous responses. These integration schemes are broadly classified into two: (i) implicit and (ii) explicit integration schemes. Schemes which require the information from previous steps only are called explicit schemes whereas the schemes which requires information of the previous steps and of the current step are called implicit schemes. Some of the examples of explicit and implicit schemes are given below:

Explicit Schemes

1. Central difference method
2. Newmark's explicit method
3. Modified Newmark's method

Implicit Schemes

1. Newmark's method
2. α-Operator Splitting method
3. Generalized alpha method

The main requirement of these integration schemes are its accuracy and stability criteria which forms the criteria for the selection of an integration method . Accuracy refers to the closeness of the analytical result with the actual result while a method is stable if it doesnot give abrupt result and grows out of bound for any given initial condition [1]. Stability criteria plays an important role in selecting the time step for the time integration methods.

There are some unconditionally stable methods where the method remains stable for any value of $\omega\Delta t$ conditionally stable methods are those which remains stable for the value of $\omega\Delta t$ below certain limit. When the value of $\omega\Delta t$ goes beyond a certain range, the solution becomes unstable. Still, we usually prefer explicit schemes as we have small degrees of freedom implying to not so big ω value, hence Δt is not so small.

A family of time-stepping method was proposed by N.M. Newmark in 1959 [5], which is given as show below:

$$x1_{i+1} = x1_i + [(1-\gamma)\,\Delta t]\,x2_i + (\gamma\Delta t)\,x2_{i+1} \tag{3.2}$$

$$x_{i+1} = x_i + (\Delta t)\,x1_i + \left[(0.5-\beta)\,(\Delta t)^2\right]x2_i + \left[\beta\,(\Delta t)^2\right]x2_{i+1} \tag{3.3}$$

Here, β and γ defines the variation of acceleration in the given time step. For $\gamma = 1/2$, there is no numerical damping observed in the solution, while for $\gamma < 1/2$, there is negative numerical damping which subsequently increases the response of the system while for $\gamma > 1/2$, there is positive numerical damping introduced in the system. With proper selection of β and γ value, we can achieve desired stability and accuracy. With keeping $\gamma = 1/2$ and varying the β value, we can get differernt integration methods. For $\beta = 1/4$, we get implicit and unconditionally stable constant average acceleration method. By putting $\beta = 1/6$, we get implicit and conditionally stable linear acceleration method. For $\beta = 0$, the $x2_{i+1}$ terms in the Equation 3.2 and 3.3 vanish off and the method becomes explicit and then is a single step method, called as Newmark Explicit scheme.

3.3.1 Numerical Accuracy and Stability Analysis

For a linear elastic SDOF system, the accuracy and stability analysis is given. The direct integration scheme can be represented in a recursive matrix form as shown in Equation 3.4.

$$\hat{x}_{i+1} = A\hat{x}_i + Lf_{i+v} \tag{3.4}$$

where $\hat{x}_i$ is a vector of solution quantities at time i. The parameter v can be either 0 or 1 depending upon the type of solution techniques used. The vector L is a load vector and the matrix A is amplification matrix. $\hat{x}_{i+1}$ is the solution quantities at time $i+1$. The scalar f_{i+v}

is the external applied force. The numerical analysis is done for free vibration response, we have

$$\hat{x}_n = A\hat{x}_{n-1} = A^n \hat{x}_0 \tag{3.5}$$

where $\hat{x}_0$ is the initial solution vector. For stable intergation technique, the above must yield a bounded solution response for any arbitrary initial solution vector.

For Newmark Explicit method, the inital solution vector is given as:

$$\hat{x}_0 = \begin{Bmatrix} x_0 \\ x1_0 \\ x2_0 \end{Bmatrix} \tag{3.6}$$

and

$$A = \begin{bmatrix} 1 & \Delta t & \dfrac{\Delta t^2}{2} \\ \dfrac{-\omega^2 \Delta t}{2} & 1 - \dfrac{\omega^2 \Delta t^2}{2} & \dfrac{\Delta t}{2} - \dfrac{\omega^2 \Delta t^3}{4} \\ -\omega^2 & -\omega^2 \Delta t & \dfrac{-\omega^2 \Delta t^2}{2} \end{bmatrix} \tag{3.7}$$

The matrix A has 3 distinct eigen values λ_1, λ_2 and λ_3, and there exist a diagonal matrix J such that

$$J^n = \Phi^{-1} A^n \Phi \tag{3.8}$$

where $\Phi = [\phi_1, \phi_2, \phi_3]$ and $J = diag\,(\lambda_1, \lambda_2, \lambda_3)$, and the vectors ϕ_i are the eigen vectors of A corresponding to the eigen values λ_i. Hence by using Equation 3.5 and 3.8, we can get

$$x_n = c_1 {\lambda_1}^n + c_2 {\lambda_2}^n + c_3 {\lambda_3}^n \tag{3.9}$$

where x_n is the displacement value at step n and is a part of $\hat{x}_n$ and c_1,c_2 and c_3 are the constants based on initial conditions. Out of the three eigen solution of the matrix A, two eigen solutions $\lambda_{1,2}$ should be a complex conjugate and $|\lambda_3| < |\lambda_{1,2}| \leq 1$. The method will be stable if the given conditions are satisfied. λ_3 is called the spurious root as it doesnot represent the realistic numerical solution of free vibration. By solving $(A - \lambda I) = 0$, for eigen solutions, we get

$$\lambda_{1,2} = A \pm \iota B = e^{\left(-\bar{\zeta} \pm \iota\right)\bar{\Omega}} \tag{3.10}$$

where,

$$A = 1 - \frac{\omega^2 \Delta t^2}{2} \tag{3.11}$$

and

$$B = \frac{\sqrt{4 - \left(\omega^2 \Delta t^2 - 2\right)^2}}{2} \tag{3.12}$$

For the response to be stable, the condition $(A^2 + B^2) \leq 1$ should be satisfied and B should be real. The value $(A^2 + B^2)$ will always be equal to 1 and the B should be real, hence the stability condition,

$$\left(\omega^2 \Delta t^2 - 2\right)^2 \leq 4 \tag{3.13}$$

leads to

$$0 \leq \omega \Delta t \leq 2 \tag{3.14}$$

For $\omega \Delta t = 2$, the response will not be accurate but stable.

The eigen solutions can be represented as given in the Equation 3.10 where $\bar{\zeta}$ and $\bar{\Omega}$ is defined as:

$$\bar{\zeta} = -\frac{ln\left(A^2 + B^2\right)}{2\bar{\Omega}} \tag{3.15}$$

$$\bar{\Omega} = arctan\left(\frac{B}{A}\right) \tag{3.16}$$

substituting Equation 3.10 in the Equation 3.9, we get

$$x_n = e^{-\bar{\zeta}\bar{\omega}\Delta tn}\left(c_1 \cos \bar{\omega}\Delta tn + c_2 \sin \bar{\omega}\Delta tn\right) + c_3 {\lambda_3}^n \tag{3.17}$$

where $\bar{\omega} = \bar{\Omega}/\Delta t$. The reponse of the underdamped free vibration of a SDOF system is given as

$$x\left(t\right) = e^{-\zeta\omega t}\left(c_1 \cos \omega_D t + c_2 \sin \omega_D t\right) \tag{3.18}$$

where ζ is the viscous damping ratio and ω_D is the damped natural frequency of the system. Comparing the Equation 3.17 with the response of a free vibration of a underdamped SDOF system, we can say that $\bar{\omega}$ and $\bar{\zeta}$ are the numerical frequency and damping corresponding to that of the system respectively. If $\bar{\zeta} \neq 0$, we get numerically induced damping. Also, the difference between the numerical frequency $\bar{\omega}$ and the natural frequency of the system ω gives the distortion in the frequency in the numerical technique.

Putting the value of A and B from the Equation 3.11 and 3.12 in the Equation 3.15, we get $\bar{\zeta} = 0$ which shows that the Newmark Explicit method doesnot have numerical damping. Also, substituting Equation 3.11 and 3.12 in the Equation 3.16, we get

$$\bar{\omega} = \frac{1}{\Delta t} arctan \left(\frac{\sqrt{4 - \left(\omega^2 \Delta t^2 - 2\right)^2}}{2 - \omega^2 \Delta t^2} \right) \tag{3.19}$$

The difference in the natural frequency and the numerical frequency will give the amount of frequency distortion in the system. The plot of percentage of period distortion $\left(T - \bar{T}\right)/T$ and $\Delta t/T$ shows that the we get a reasonably accurate solution when $\Delta t/T$ is less than 0.05 and the period distortion vanishes when Δt goes to zero.

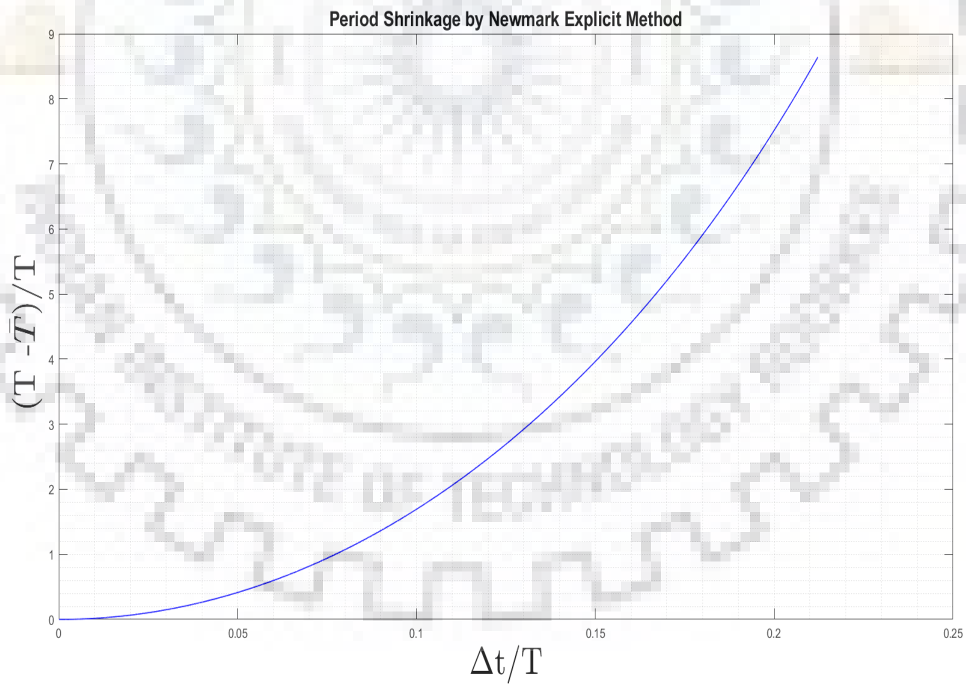


Figure 3.2: Period Shrinkage by Newmark Explicit Method

The stability requirement of this method is $\omega \Delta t \leq 2$, which is bounded by the highest value of ω in case of a MODF system. To ensure the accuracy of this method, the value of

Δt should be less than or equal to $2/\omega$ .Putting $\beta = 0$ and $\gamma = 1/2$ in the Equation 3.2 and 3.3, we get the following equation:

$$x1_{i+1} = x1_i + (x2_i + x2_{i+1})\,\frac{\Delta t}{2} \tag{3.20}$$

$$x_{i+1} = x_i + (\Delta t)\,x1_i + \frac{\Delta t^2}{2} x2_i \tag{3.21}$$

Using Equation 3.20 and 3.21 in the Equation 3.1, and rearranging the terms for $x2_{i+1}$ gives:

$$x2_{i+1} = \left[m + \frac{\Delta t}{2} c\right]^{-1} \left[p_{i+1} - fs_{i+1} - cx1_i - \frac{\Delta t}{2} cx2_i\right] \tag{3.22}$$

Using the Equations 3.21, 3.20 and 3.22, algorithm for Newmark Explicit scheme is shown below in Figure 3.3:

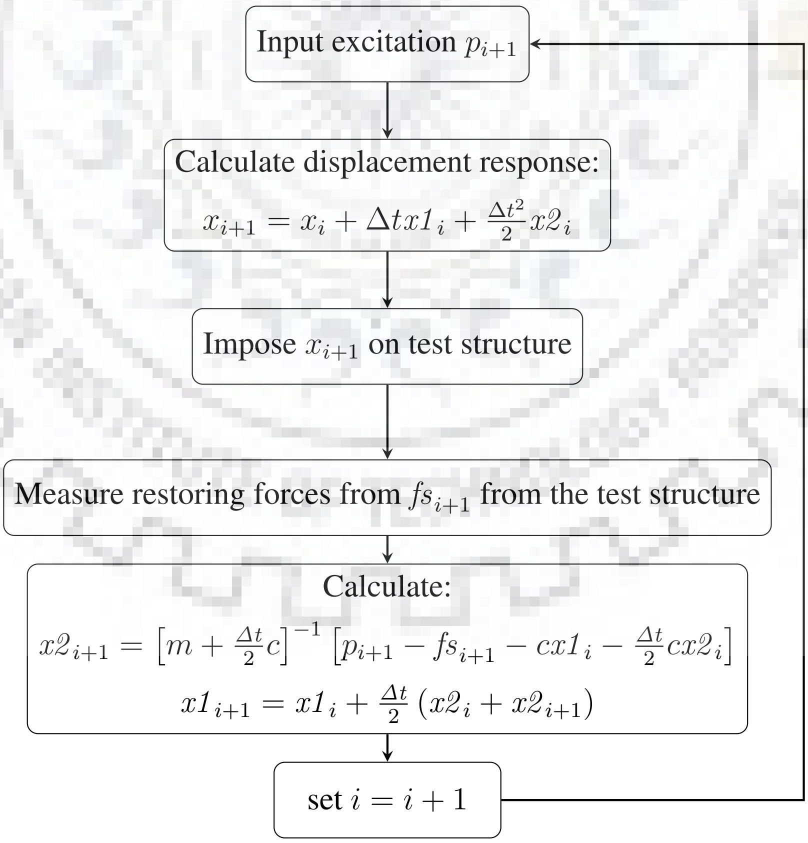


Figure 3.3: Newmark Explicit Scheme flowchart

3.4 Errors in the testing method

Errors are inevitable in experimental testing methods and in this testing method, the errors can occour from the given three sources and are explained in detail in the subsequent section.

1. Errors due to the idealisation of structure

2. Errors manifested in numerical techniques

3. Errors due to the measurements in experiment

3.4.1 Errors Due to the Idealisation of Structure

The test model idealised as a discrete parameter model should represent the actual dynamic behaviour of the test structure itself. The idealisation of dicrete parameters involves the formation of analytical mass and damping matrices. The actual structure is a continuous system which is idealised as a discrete system due to which the higher mode effects are lost and the lower modes shows the distortion in frequencies. In most of the cases where the lumped mass is more than 80%, the lumped mass model gives reliable results. For more number of nodes or degrees of freedom, the frequency error in the higher modes are high but the mode participation factor for higher modes are insignificant hence, does not impose a problem while the mode participation for lower modes are high and the frequency error is low. Hence, the discrete parameter model is relaible if the concentrated mass constitutes 80% or more of the total mass [8].

Various damping mechanisms are present in the actual structure and modelling of all of the damping properties is not an easy task. The type of damping includes viscous damping, coulomb damping and hysteretic damping. While coulomb and hysteretic damping are automatically accounted for in the pseudodynamic testing in measuring the restoring forces from the physical specimen, hence, the viscous damping is not possible to determine in a free vibration test. The coulomb damping exists due to the friction between the contact surfaces in the joints and connections in the structure. This damping remains constant throughout the test and its effect becomes negligible if the amplitude of the displacement response is comparitively larger [8]. The hysteretic damping is present in the system due to the inelastic behaviour of the material. The energy is dissipated due to the large inelastic deformation of the physical specimen and reflects in the measurement of the restoring forces which is feeded back in the computer. Hence the coulomb and hysteretic damping is not needed to be included in the modelling of damping matrix. However the determination of viscous damping is required which can be found out by the difference in the reponse of the free-vibration pseudodynamic simulation with the free vibration test response of the specimen. During the inelastic response, the viscous damping does not have a significant effect in the response quantities as hysteretic damping dominates in the system. Hence, the determination of the equivalent viscous damping does not plays much significant role in the errors in pseudodynamic testing.

In pseudodynamic testing, the physical specimen is loaded quasi-statically. Due to this the strain rate effects are not included in the response analysis. The rate of loading influences the yield and ultimate strength of the material. For large strain rate, the material shows higher strength. Hence, the rate of loading should be controlled in the testing. If the rate of loading is slow, stress relaxation may occur while if high rate of loading is applied, the dynamic effects will come in in the feedback from the physical specimen.

3.4.2 Error Manifested in Numerical Techniques

Errors in numerical integration often leads to the distortion in frequency, energy dissipation and the growth of spurious root as explained in the Section 3.3.1. If implicit integration schemes are used, then the error due to the calculation of tangent stiffness is also incorporated in the response analysis other than the the errors due to the numerical schemes itself. In explicit scheme, the problem associated with the stiffness calculation does not exist. The error in numerical integration scheme depends on the time step selected. If the time step is too large, the errors will lead to the instability and inaccuracy of the numerical scheme and if the time step selected is too small, the computation effort requires for the method will increase significantly. Hence an appropriate time step should be chosen based on the stablility and accuracy requirement of the method.

3.4.3 Errors Due to the Measurements in Experiment

Other than the errors mentioned above, errors are also introduced due to the incorrect feedback used into the calculation of the response of the system. This leads to the incorrect displacement calculation which again leads to the measurement of incorrect feedback and this will result into the cumulative error and the test results will be unreliable. The errors are generally incorporated due to the inaccuracy of the experimental equipments. The displacment response computed cannot be exactly applied to the physical specimen. This is due to the sensitivity of the actuator controller system. Again the displacements applied will render incorrect feedback measurement. This feedback measurement will also be affected by the friction in the actuator connections. Hence, the erroneous force feedback and displacement will again generate an erroneous response and hence the test results will be not be reliable.

Chapter 4

Methodology

For the development of pseudodynamic testing facility, a feedback loop algorithm is to be build for MOOG actuators available in IIT Roorkee Pseudodynamic Testing Labratory. The pseudodynamic testing requires a feedback of the restoring force value from the physical specimen which is possible to get only during the experiment. Here, the analytical simulation of the pseudodynamic testing is done where the restoring force value is updated from the static pushover curve generated using OpenSees (Open System for Earthquake Engineering Simulation). As mentioned earlier in Section 3.3, this simulation is done using Newmark Explicit scheme, algorithm of which is shown in the figure 3.3. For the simulation purpose, a program is prepared in MATLAB which takes the pushover curve of the model as input and performs the nonlinear time history analysis where the restoring forcce vallue is updated using the current displacement and velocity of the mass of the system. This program is given in Appendix A. The code is verified with the example given in the NPTEL couse on "*Introduction to Earthquake Engineering*" for non-linear seismic response of structures with bi-linear force deformation curve.

For an elasto-plastic SDOF system having mass 1 kg, elastic stiffness 39.478 N/m and damping constant 0.251 N.sec/m, the response of the system under El-Centro, 1940 motion for yield displacement of 0.05 m and 0.025 m given in the example 7.1 of the couse Introduction to Earthquake Engineering which is solved using $\beta = 1/4$ i.e. average acceleration method is compared with the result for the same input paramters feeded in the pseudodynamic simulation code given in Appendix A. The results of the comparison between the results given in the course and using the simulation program is shown below in the Figure 4.1.

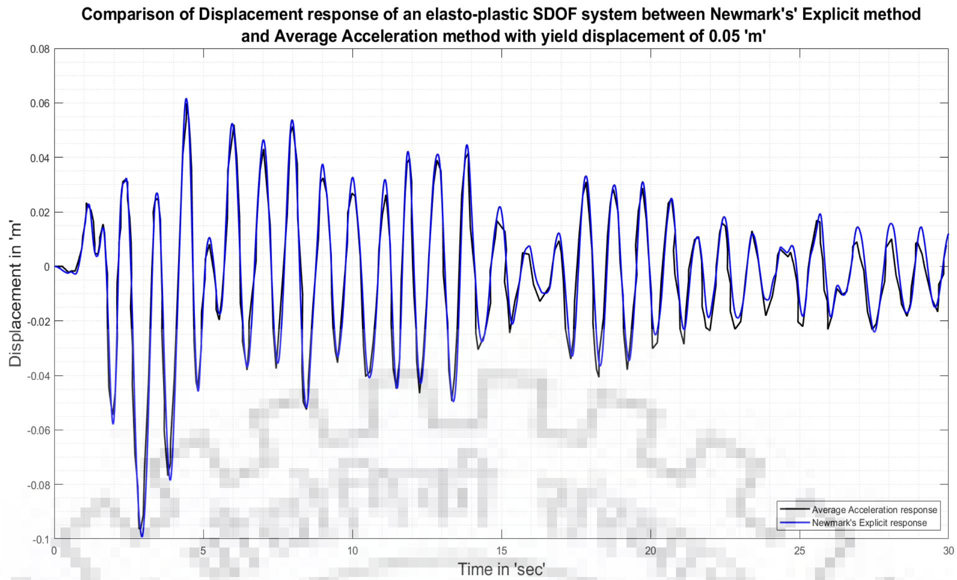


Figure 4.1: Comparison of Displacement response of an elasto-plastic SDOF system between Newmark's Explicit method and Average Acceleration method with yield displacement of 0.05 m

The above results verify the code given in Appendix A and hence is valid for the pseudodynamic simulation. The pseudodynamic simulation for a SDOF system subjected to different earthuake records is done and the results are shown in the Chapter 5.

This code uses the force value from the pushover curve based on the displacement measured after reloading takes place in the member. No degradation in the stiffness property and no hardening in the hysteresis curve is assumed. During unloading, the stiffness of the member remains same as initial displacement and remains constant till the reloading takes place.

Chapter 5

Results

The pseudodynamic simulation of a single degree of freedom system is done using Newmark Explicit Algorithm of a cantilever steel column fixed at the base. First, the force displacement relationship is generated using OpenSees (Open System for Earthquake Engineering Simulation) and a push-over curve is plotted. This force displcement curve serves as a basis for the updation of restoring force value in simulation of pseudodynamic testing. The simulation is done for three different earthquakes. All three ground motion datas selected are of different magnitude and the simulation is done and the output is recorded to see the variation in each of them. Then the comparison is shown between the responses using Newmark Explicit method and Linear Acceleration method.

5.1 Simulation of a SDOF system

The system is modelled in OpenSees with Steel01 which is a uniaxial bilinear steel material object. The material properties are given below:

- Yield Strength $Fy = 250\,\mathrm{MPa}$
- Initial Elastic Tangent $E0 = 2.1 \times 10^5\,\mathrm{MPa}$
- Strain Hardening Ratio $b = 0.01$

The stress strain curve for Steel01 material is shown in the Figure 5.1:

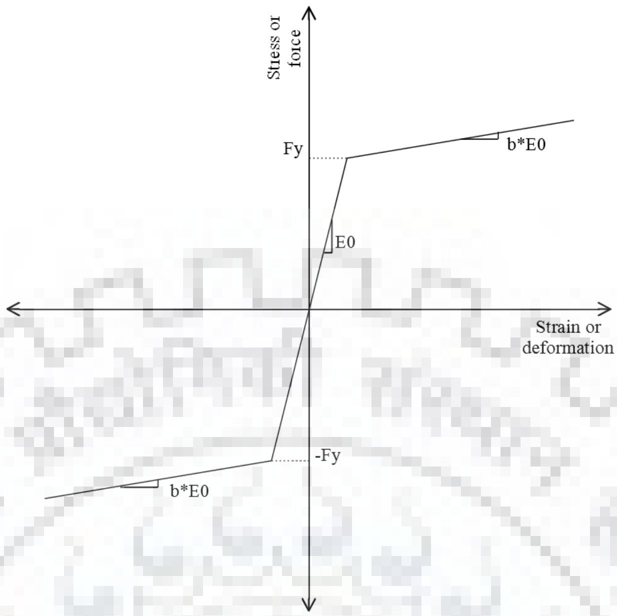
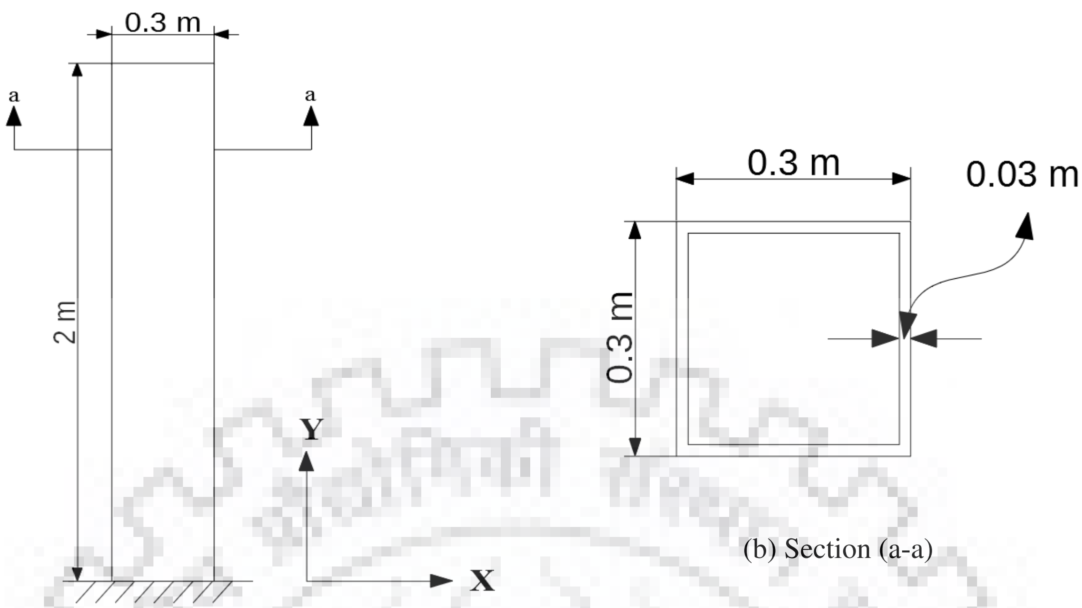


Figure 5.1: Stress-Strain curve for Steel01 material

The cantilever column and its section properties are shown in the Figure 5.2. The column is fixed at node 1 and is free at node 2. The column has a hollow tube section of $300mm \times 300mm$ and having a thickness of $30mm$ as shown. The actual stiffness of the section is reduced by **100 times** to ensure the non-linear behaviour of the system when subjected to the different earthquake records. This pushover curve as shown in the Figure 5.3 is generated including the P-Δ effects as the gravity loads were made contant during the analysis in OpenSees.

The initial input parameters for the analysis is given as under:

- mass $m = 2543.4\,\mathrm{kg}$
- initial stiffness $k_{ini} = 312.559\,\mathrm{kN/m}$
- initial displacement $x(0) = 0$
- initial velocity $x1(0) = 0$
- damping ratio $z = 0.05$
- natural frequency $\omega = \sqrt{\frac{k_{ini}}{m}} = 11.0856\,\mathrm{rad/s}$



(b) Section (a-a)

(a) Steel cantilever column

Figure 5.2: SDOF system model for pseudo-dynamic simulation

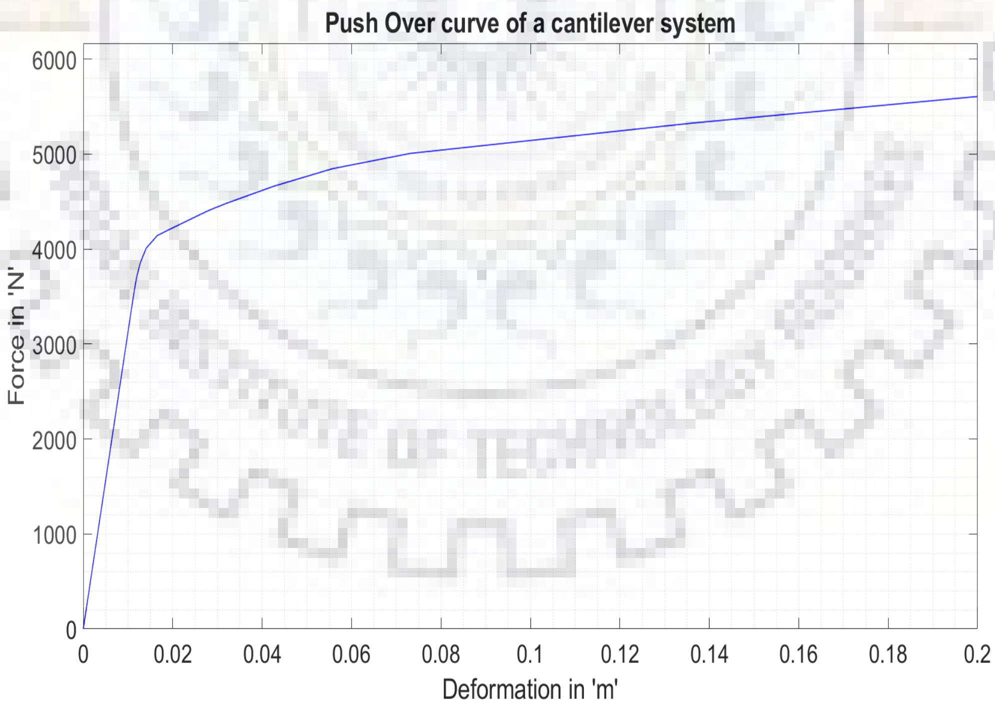


Figure 5.3: Pushover curve for cantilever model

The three different ground motion used for the analysis are:

1. El-Centro Earthquake:

   Record Date: 18/05/1940

   Magnitude: 6.9

   PGA: $0.32\,\mathrm{g}$

   Time step: $0.02\,\mathrm{sec}$

2. Northridge Earthquake:

   Record Date: 17/01/1994

   Magnitude: 6.8

   PGA: $0.57\,\mathrm{g}$

   Time step: $0.01\,\mathrm{sec}$

3. Kobe Earthquake:

   Record Date: 16/01/1995

   Magnitude 6.9

   PGA: $0.34\,\mathrm{g}$

   Time step: $0.01\,\mathrm{sec}$

The stability criteria for using the Newmark Explicit Scheme, $\omega\Delta t$ is 0.2217 for El-Centro earthquake and 0.111 for Kobe and Northridge earthquake data which is less than the upper limit of 2.

5.1.1 Response to El-Centro Earthquake

Figure 5.4, 5.5, 5.6 and 5.7 shows the displacement, velocity, acceleration response and the force deformation characteristics respectively when the model is subjected to El-Centro earthquake.

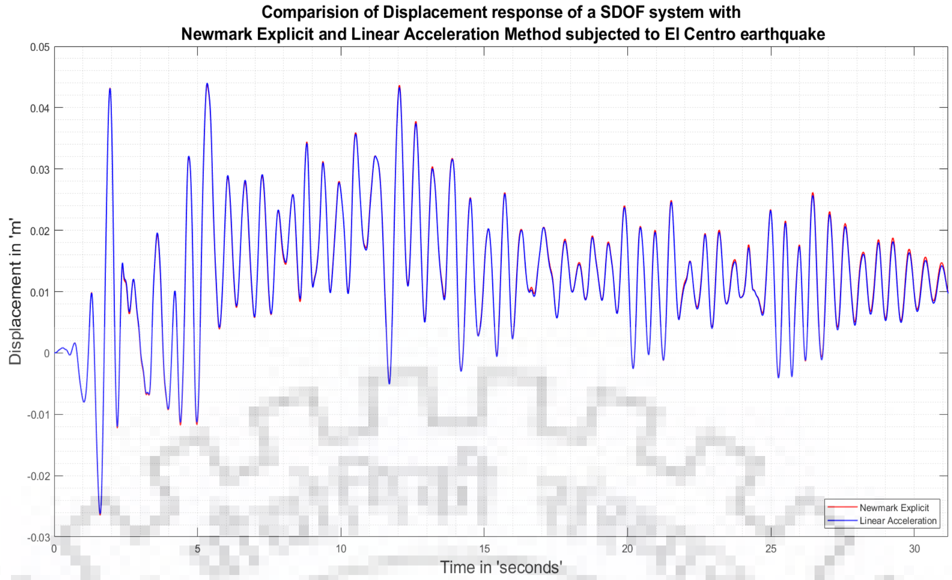


Figure 5.4: Comparison of Displacement response of a SDOF system with Newmark Explicit scheme and Linear Acceleration method subjected to El-Centro earthquake

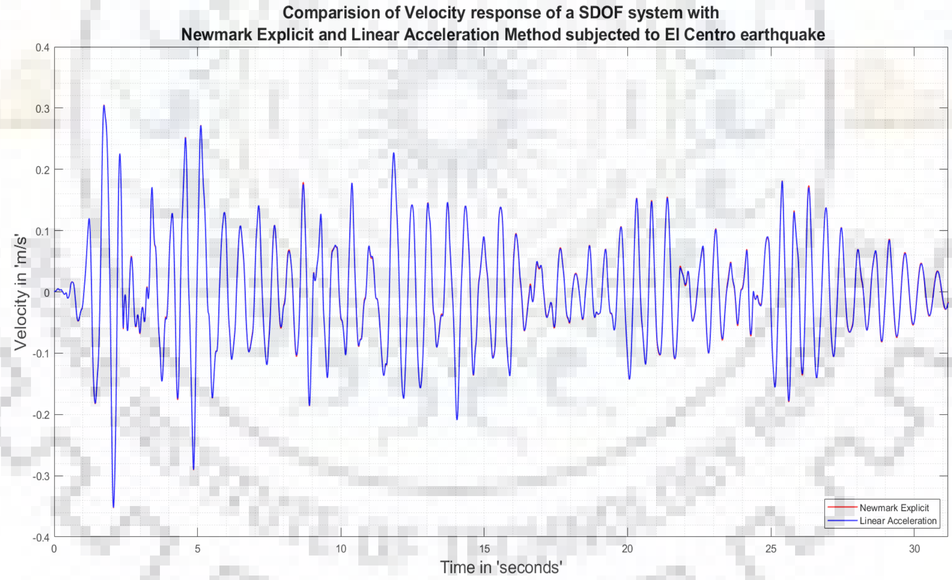


Figure 5.5: Comparison of Velocity response of a SDOF system with Newmark Explicit scheme and Linear Acceleration method subjected to El-Centro earthquake

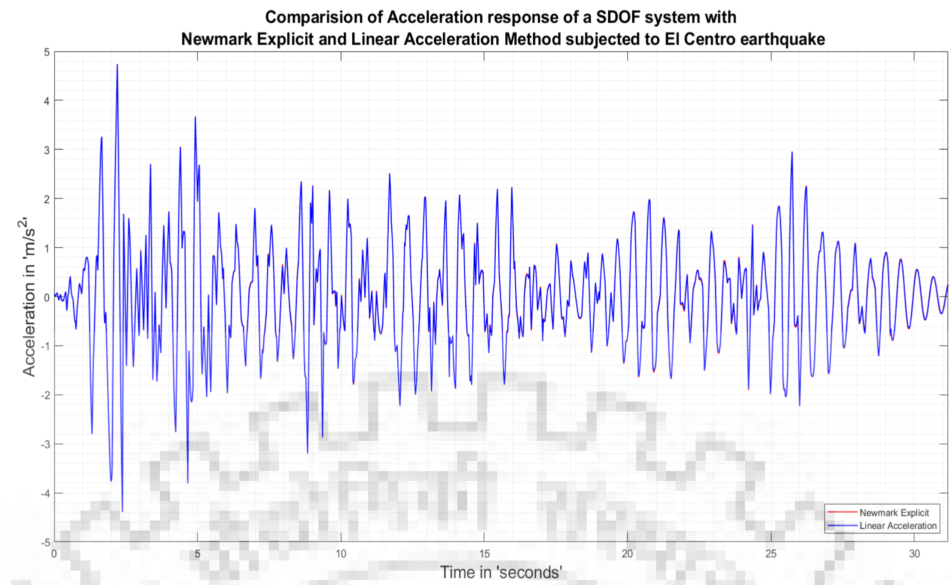


Figure 5.6: Comparison of Acceleration response of a SDOF system with Newmark Explicit scheme and Linear Acceleration method subjected to El-Centro earthquake

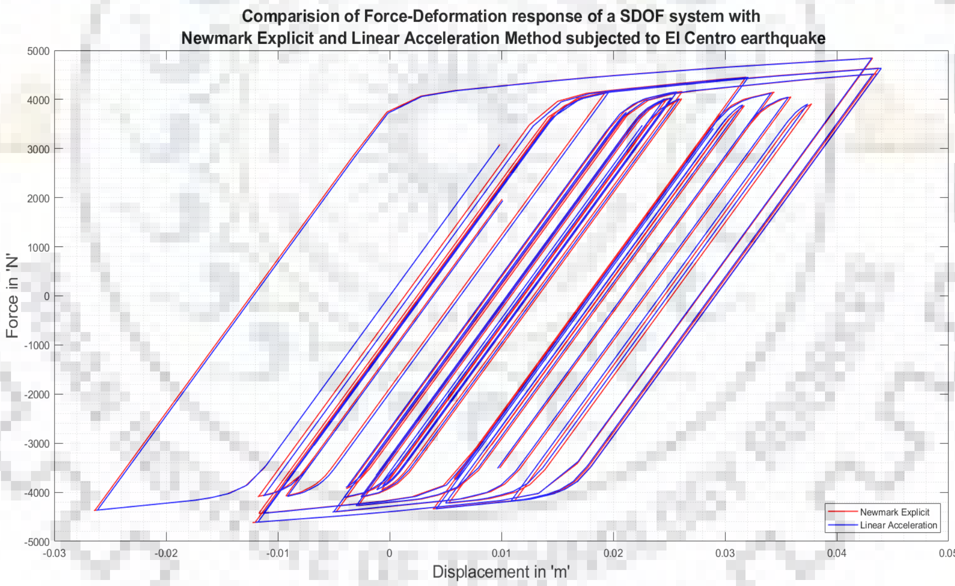


Figure 5.7: Comparison of Force-Deformation response of a SDOF system with Newmark Explicit scheme and Linear Acceleration method subjected to Kobe earthquake

5.1.2 Response to Northridge Earthquake

Figure 5.8, 5.9, 5.10 and 5.11 shows the displacement, velocity, acceleration response and the force deformation characteristics respectively when the model is subjected to Northridge earthquake.

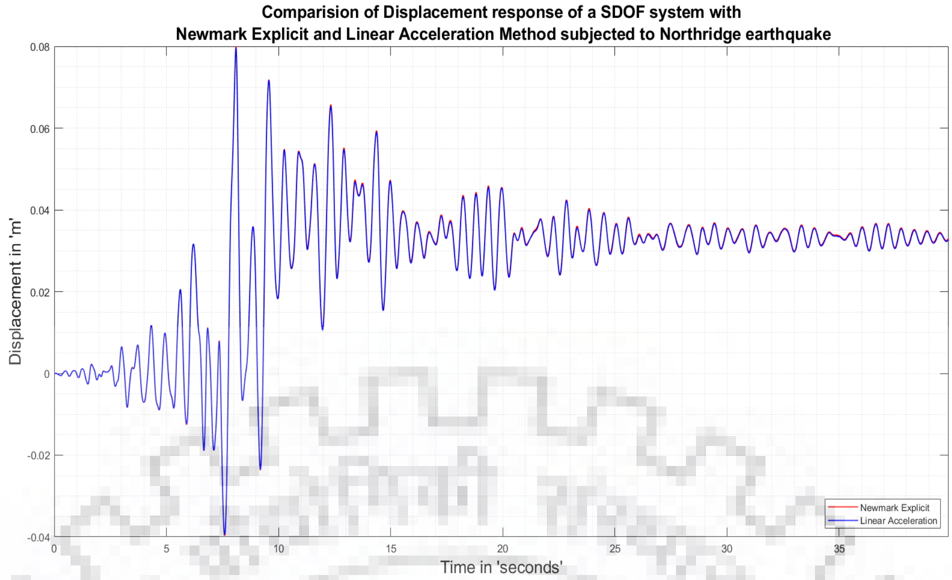


Figure 5.8: Comparison of Displacement response of a SDOF system with Newmark Explicit scheme and Linear Acceleration method subjected to Northridge earthquake

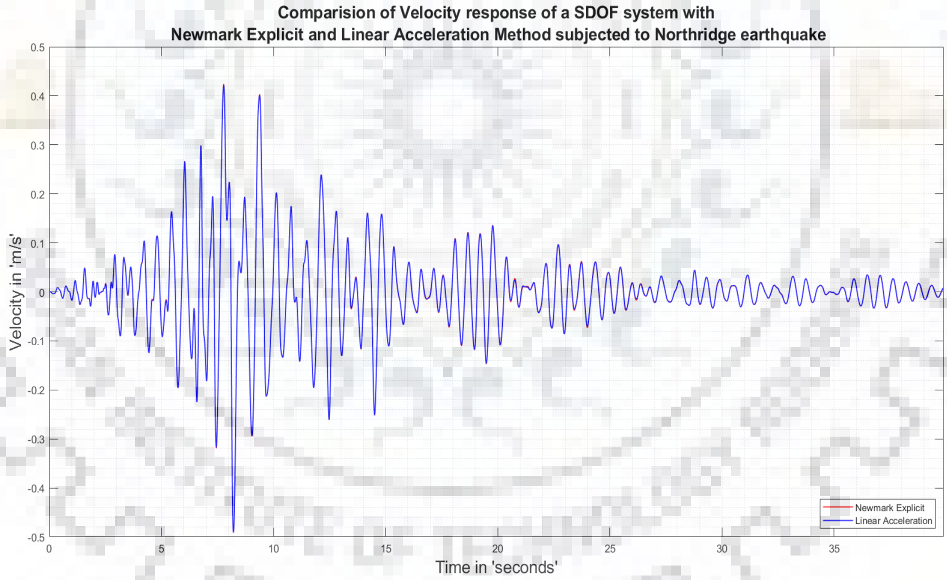


Figure 5.9: Comparison of Velocity response of a SDOF system with Newmark Explicit scheme and Linear Acceleration method subjected to Northridge earthquake

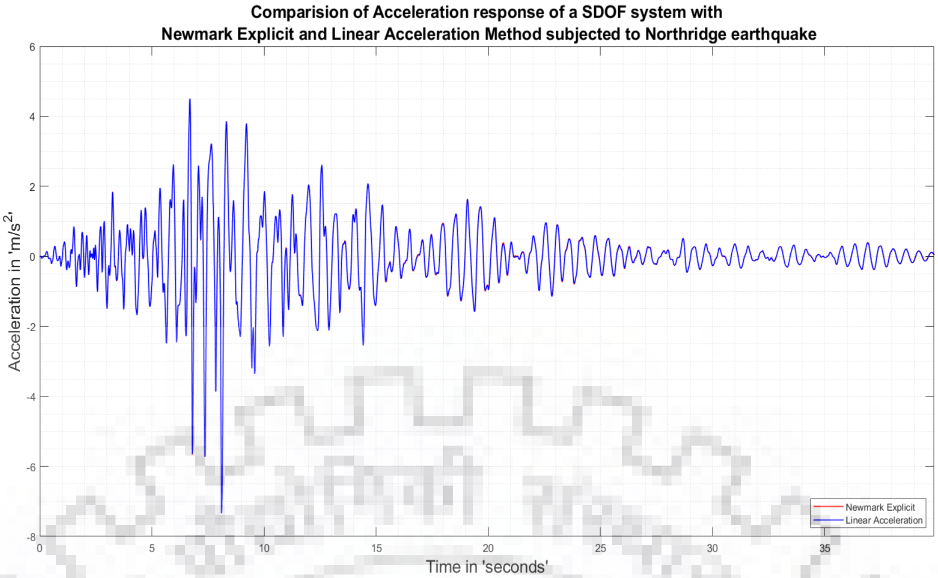


Figure 5.10: Comparison of Acceleration response of a SDOF system with Newmark Explicit scheme and Linear Acceleration method subjected to Northridge earthquake

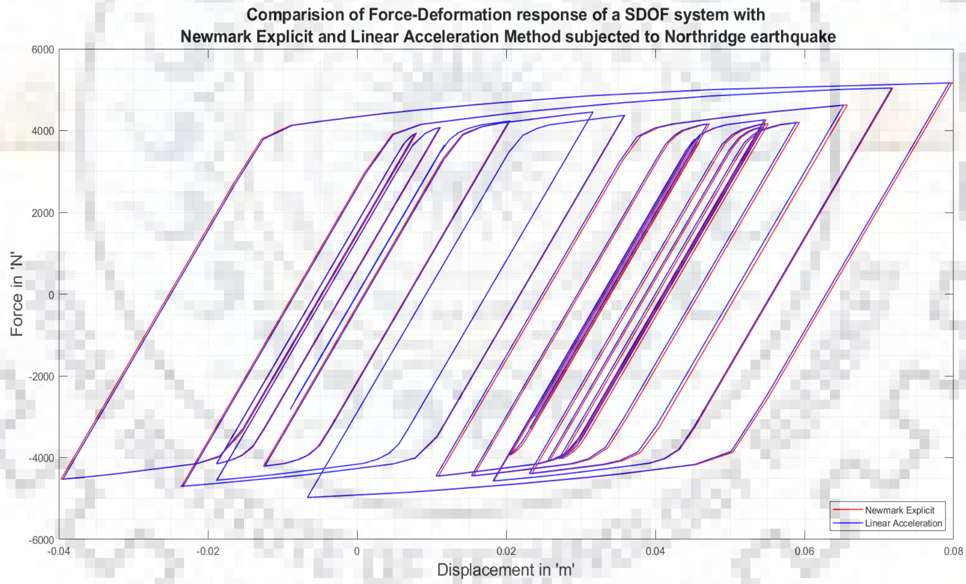


Figure 5.11: Comparison of Force-Deformation response of a SDOF system with Newmark Explicit scheme and Linear Acceleration method subjected to Kobe earthquake

5.1.3 Response to Kobe Earthquake

Figure 5.12, 5.13, 5.14 and 5.15 shows the displacement, velocity, acceleration response and the force deformation characteristics respectively when the model is subjected to Kobe earthquake.

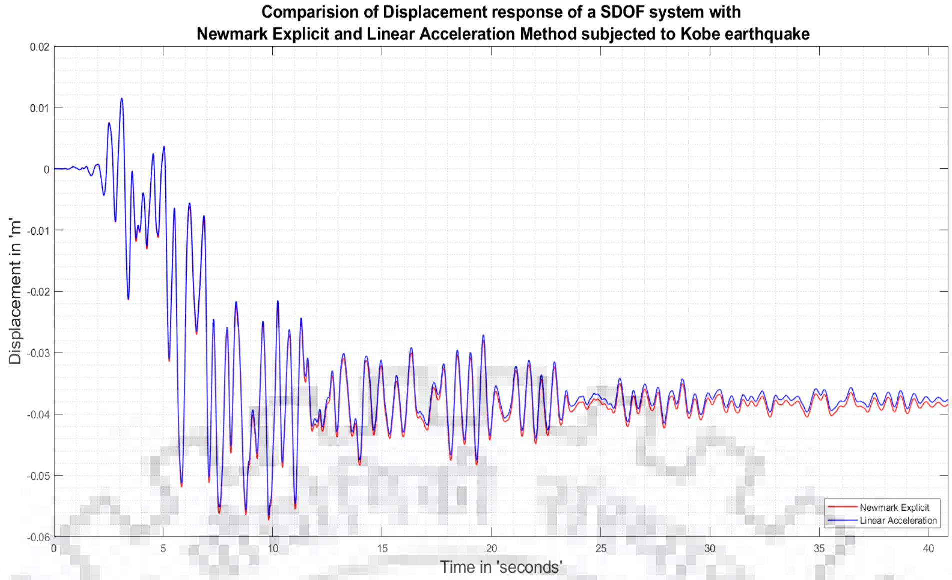


Figure 5.12: Comparison of Displacement response of a SDOF system with Newmark Explicit scheme and Linear Acceleration method subjected to Kobe earthquake

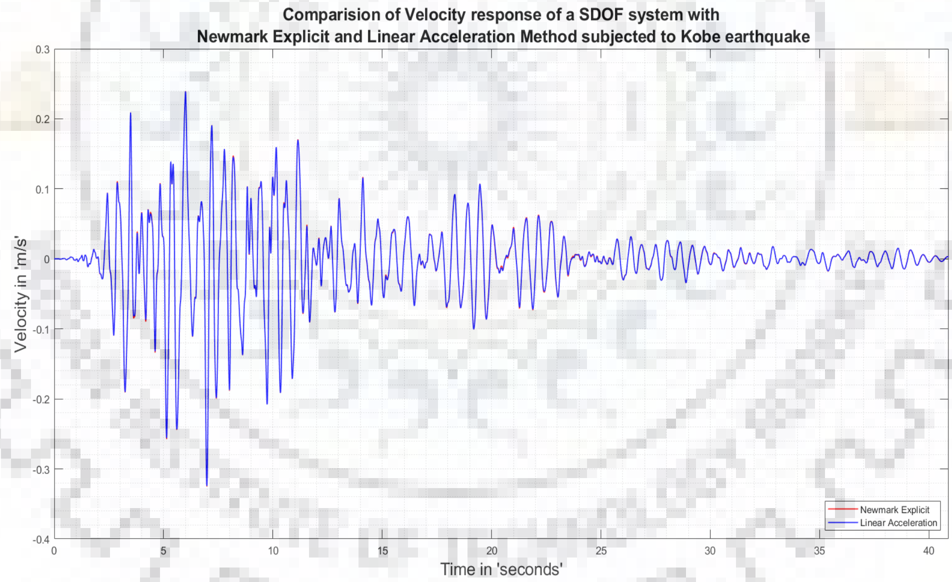


Figure 5.13: Comparison of Velocity response of a SDOF system with Newmark Explicit scheme and Linear Acceleration method subjected to Kobe earthquake

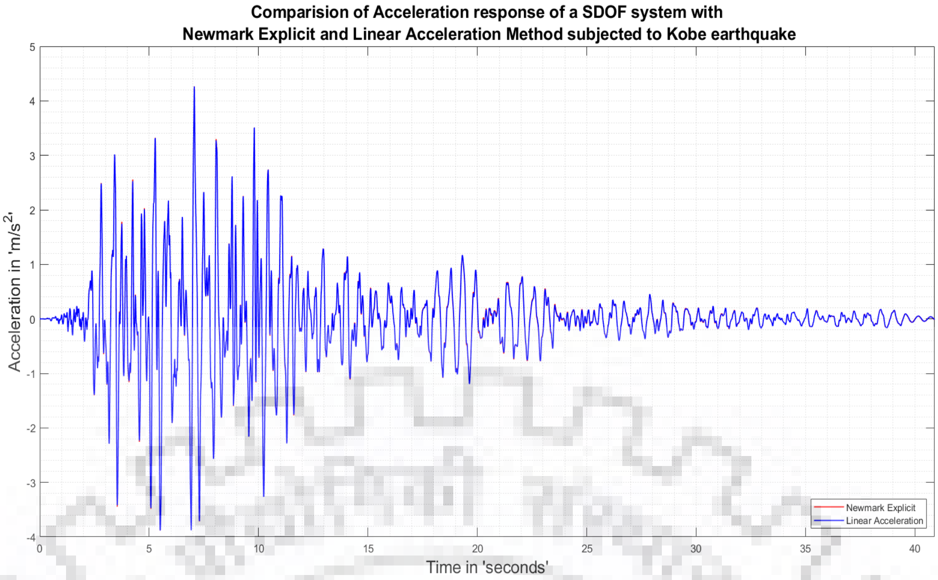


Figure 5.14: Comparison of Acceleration response of a SDOF system with Newmark Explicit scheme and Linear Acceleration method subjected to Kobe earthquake

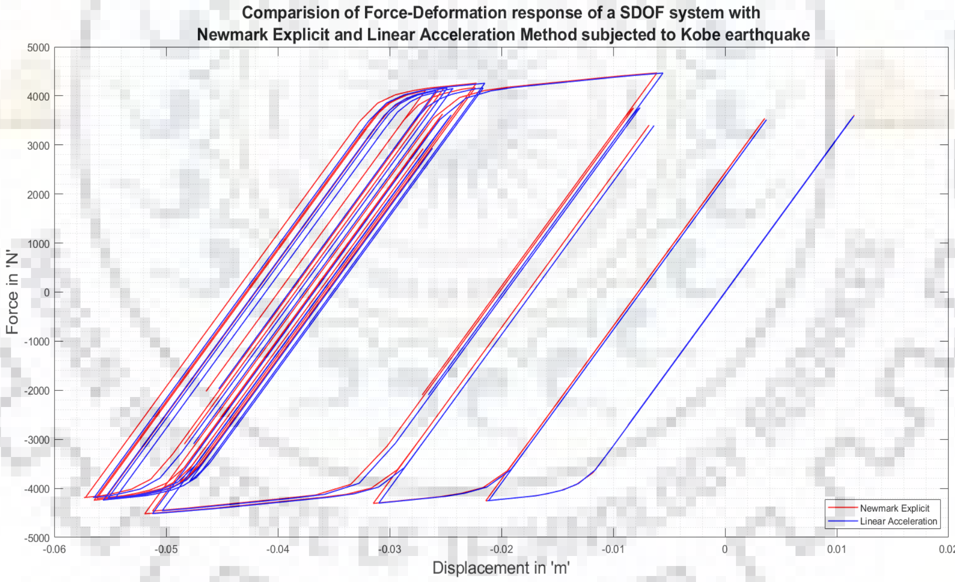


Figure 5.15: Comparison of Force-Deformation response of a SDOF system with Newmark Explicit scheme and Linear Acceleration method subjected to Kobe earthquake

Chapter 6

Conclusion

Pseudodynamic testing method is a powerful technique for testing the large scale structures under dynamic loading. It has the advantage of the conventional methods such as the realism of shake table testing and the economy and control of quasi-static testing and hence, has gained the interest of many researchers.

Numerical techniques plays an important role in the reliability of pseudodynamic testing and hence the stability and accuracy analysis is shown and the Newmark Explicit method is used. Pseudodynamic testing is a hybrid testing method where the numerical component computes the target displacement and is applied to the structure and a feedback is obtained which is used to calculate the next target displacement. In this report, the simulation of the pseudodynamic testing is presentes based on the restoring force characteristics obtained from the pushover curve of the system. A response of a SDOF system is simulated using a code prepared in MATLAB and is presented in the results and discussions.

The results gives the variation of the displacement, velocity and acceleration of the structure with time and the force deformation curve is plotted. This simulation is carried out for three different earthquakes. The Department of Earthquake Engineering, IIT Roorkee has developed the pseudodynamic testing facility to test the large structures where the strong floor and wall and the hardwares for testing are available. This report gives the basis of the code which is to be written for the MOOG controllers using the software development kit of the controller for the actuators.

Newmark Explicit algorithm is used in this report since it is an explicit scheme and hence the iterations associated with the implicit scheme to predict the tangent stiffness to calculate the next response is eliminated. Since this scheme is conditionally stable, the stability requirements are also fulfilled in the case of SDOF system. Other explicit methods such as Modified Newmark algorithm should be used for the anlaysis of MDOF system which has a numerical dissipative properties.

Chapter 7

Future Scope

This work has been carried out for a SDOF system and can be extended to MODF system with bi-directional loadings. Here, the Newmark Explicit scheme is used for the simulation of pseudodynamic testing and there are other explicit and implicit methods available with their advantages and disadvantages. The other integration method can also be explored and a suitable integration scheme can be used for the software which is required to be developed for controlling the MOOG actuators in the Pseudodynamic Testing Laboratory at IIT Roorkee. Also, since the testing of the structure as a whole is not always feasible in the case where the nonlinearity is concentrated at certain parts of the structure, substructuring technique can also be developed in the future.For substructuring concept, since the actual structure will have a large degrees of freedom, to satisfy the stability condition will not be possible and in such case implicit integration schemes can be used. The strain rate effects are also predominant in some type of structures, and hence, this pseudodynamic testing will not render reliable results. The development of Real-Time hybrid test facility for testing such kind of structures can be developed in the future.

Appendix A

Simulation Code

This appendix contains the codes written for doing the pseudodynamic simulation in MATLAB. It uses Newmark Explicit method for simulating the response. Datas from the pushover curve is the input to the code.

```
% Pseudodynamic Algorithm for a SDOF system using Newmark Explicit Method

clear all
clc

Reaction = load('RBase.txt');
Displacement = load('DFree.txt');

% Removing gravity load anlysis data

R = zeros(length(Reaction)-9,1);
D = zeros(length(Reaction)-9,1);

D(2:length(D)) = Displacement(11:length(Reaction), 2);
R(2:length(D)) = -Reaction(11:length(Reaction), 2);
R = R/50; % Reducing stiffness value

m = 2543.4; % Mass of the system
kini = (R(2)-R(1))/(D(2)-D(1)); %Initial Stiffness
z = 0.05; %Damping Ratio
w = sqrt(kini/m); %natural frequency of the system
c = 2*m*w*z; %Viscous damping coefficient
x(1) = 0; x1(1) = 0; %Initial conditions
dt= 0.01; %Time step
loop = 1; xpivot1 = 0;
```

```
kk = 1;

%Reading the earthquake file

PP = load('el_centro.txt');
[row, column] = size(PP);
if column == 2 && PP(1, 1) == 0.00
  pp = -PP(:, 2)*9.81*m;
else
  if column > 2
    for rr = 1:row
      for cc = 1:column
        pp(kk,1) = -m*9.81*PP(rr, cc);
        kk = kk + 1;
      end
    end
  else
    pp = -m*9.81*PP(:, 2);
  end
end
p = pp;

% Reducing time step to 0.01 sec

p = zeros(2*length(pp) - 1, 1);
for ii = 1:length(pp)-1
  p(2*ii) = (pp(ii)+pp(ii+1))*0.5;
end
for ii = 1:length(pp)-1
  p(ii*2 - 1) = pp(ii);
end
t = [0:dt:dt*(length(p)-1)];
if p(1) < 0
  p = -p;
end

%Initial acceleration and restoring force

x2(1) = (p(1) - c*x1(1) - kini*x(1))/m;
```

```
fs(1) = kini*x(1);


%Loop arrangement for calculating the restoring force value

for i = 1:length(p)-1
  if loop == 1
    x(i+1) = x(i) + dt*x1(i) + x2(i)*dt*dt*0.5;
    xx = x(i+1) - xpivot1;
    [r] = force(D, R, xx);
    fs(i+1) = r;
    x2(i+1) =  (p(i+1) - fs(i+1) - c*x1(i) - dt*0.5*c*x2(i))/(m + dt*0.5*c);
    x1(i+1) = x1(i) + dt*0.5*(x2(i) + x2(i+1));
    if x1(i+1) <= 0
      loop = 2;
      k = kini;
      xmax = x(i+1);
      fmax = fs(i+1);
      xpivot = xmax - fmax/kini;
    end
  elseif loop == 2
    x(i+1) = x(i) + dt*x1(i) + x2(i)*dt*dt*0.5;
    r = fmax - kini*(xmax - x(i+1));
    fs(i+1) = r;
    x2(i+1) =  (p(i+1) - fs(i+1) - c*x1(i) - dt*0.5*c*x2(i))/(m + dt*0.5*c);
    x1(i+1) = x1(i) + dt*0.5*(x2(i) + x2(i+1));
    if x(i+1) <= xpivot && x1(i+1) >= 0
      loop = 4;
      xx = xpivot - x(i+1);
      r = force(D, R, xx);
      fs(i+1) = -r;
      xmin = x(i+1);
      fmin = fs(i+1);
      k = kini;
      xpivot1 = xmin - fmin/kini;
    elseif x(i+1) <= xpivot
      loop = 3;
```

```
      xx = xpivot - x(i+1);
      r = force(D, R, xx);
      fs(i+1) = -r;
    elseif x1(i+1) >= 0
      loop = 5;
      xp = x(i+1);
      fp = fs(i+1);
    end
  elseif loop == 3
    x(i+1) = x(i) + dt*x1(i) + x2(i)*dt*dt*0.5;
    xx = (xpivot - x(i+1));
    r = force(D, R, xx);
    fs(i+1) = -r;
    x2(i+1) =  (p(i+1) - fs(i+1) - c*x1(i) - dt*0.5*c*x2(i))/(m + dt *0.5*c);
    x1(i+1) = x1(i) + dt*0.5*(x2(i) + x2(i+1));
    if x1(i+1) >= 0
      loop =  4;
      xmin = x(i+1);
      fmin = fs(i+1);
      xpivot1 = xmin - fmin/kini;
    end
  elseif loop == 4
    x(i+1) = x(i) + dt*x1(i) + x2(i)*dt*dt*0.5;
    r = fmin + kini*(x(i+1) - xmin);
    fs(i+1) = r;
    x2(i+1) =  (p(i+1) - fs(i+1) - c*x1(i) - dt*0.5*c*x2(i))/(m + dt *0.5*c);
    x1(i+1) = x1(i) + dt*0.5*(x2(i) + x2(i+1));
    if x(i+1) >= xpivot1 && x1(i+1) <= 0;
      loop = 2;
      xx = x(i+1) - xpivot1;
      r = force(D, R, xx);
      fs(i+1) = r;
      xmax = x(i+1);
      fmax = fs(i+1);
      xpivot = xmax - fmax/kini;
    elseif x(i+1) >= xpivot1
      loop = 1;
```

```
      xx = x(i+1) - xpivot1;
      r = force(D, R, xx);
      fs(i+1) = r;
    elseif x1(i+1) <= 0
      loop = 6;
      xp1 = x(i+1);
      fp1 = fs(i+1);
    end
  elseif loop == 5
    x(i+1) = x(i) + dt*x1(i) + x2(i)*dt*dt*0.5;
    r = fp + kini*(x(i+1) - xp);
    fs(i+1) = r;
    x2(i+1) =  (p(i+1) - fs(i+1) - c*x1(i) - dt*0.5*c*x2(i))/(m + dt
       *0.5*c);
    x1(i+1) = x1(i) + dt*0.5*(x2(i) + x2(i+1));
    if x(i+1) >= xmax && x1(i+1) <= 0
      loop = 2;
      xx = x(i+1) - xpivot1;
      r = force(D, R, xx);
      fs(i+1) = r;
      xmax = x(i+1);
      fmax = fs(i+1);
      xpivot = xmax - fmax/kini;
    elseif x(i+1) >= xmax
      loop = 1;
      xx= x(i+1) - xpivot1;
      r = force(D, R, xx);
      fs(i+1) = r;
    elseif x1(i+1) <= 0
      loop = 2;
    end
  elseif loop == 6
    x(i+1) = x(i) + dt*x1(i) + x2(i)*dt*dt*0.5;
    r = fp1 - kini*(xp1 - x(i+1));
    fs(i+1) = r;
    x2(i+1) =  (p(i+1) - fs(i+1) - c*x1(i) - dt*0.5*c*x2(i))/(m + dt
       *0.5*c);
    x1(i+1) = x1(i) + dt*0.5*(x2(i) + x2(i+1));
    if x(i+1) <= xmin && x1(i+1) >= 0
```

```
      loop = 4;
      xx = xpivot - x(i+1);
      r = force(D, R, xx);
      fs(i+1) = -r;
      xmin = x(i+1);
      fmin = fs(i+1);
      xpivot1 = xmin - fmin/kini;
    elseif x(i+1) <= xmin
      loop = 3;
      xx = xpivot - x(i+1);
      r= -force(D, R, xx);
      fs(i+1) = r;
    elseif x1(i+1) >= 0
      loop = 4;
      r = fmin + kini*(x(i+1) - xmin);
      k = kini;
      xpivot1 = xmin - fmin/kini;
    end
  end
end

%Plotting the response curves

figure(1)
plot(t, x2, "linewidth",1.5, 'color', 'k')
xlim([0 max(t)])
title("Acceleration response of a SDOF system subjected to El Centro
   earthquake", 'fontsize',16);
xlabel('Time␣in␣''seconds''', 'fontsize',16)
ylabel('Acceleration␣in␣''m/s^2''', 'fontsize',16)
grid minor;

figure(2)
plot(t, x1, "linewidth",1.5, 'color', 'k')
xlim([0 max(t)])
title("Velocity response of a SDOF system subjected to El Centro
   earthquake", 'fontsize',16);
xlabel('Time␣in␣''seconds''', 'fontsize',16)
ylabel('Velocity␣in␣''m/s''', 'fontsize',16)
```

```
grid minor;

figure(3)
plot(t, x, "linewidth",1.5, ’color’, ’k’)
xlim([0 max(t)])
title("Displacement response of a SDOF system subjected to El Centro
    earthquake", ’fontsize’,16);
xlabel(’Time␣in␣’’seconds’’’, ’fontsize’,16)
ylabel(’Displacement␣in␣’’m’’’, ’fontsize’,16)
grid minor;

figure(4)
plot(x, fs, "linewidth",1.5, ’color’, ’k’)
title("Force Deformation curve of a SDOF system subjected to El Centro
    earthquake", ’fontsize’,16);
xlabel(’Displacement␣in␣’’m’’’, ’fontsize’,16)
ylabel(’Force␣in␣’’N’’’, ’fontsize’,16)
grid minor;

%Function to give the restoring force value using push-over curve

function [r] = force(D, R, disp)
  for j = 1:length(R)-1
    if D(j) <= disp && D(j+1) >= disp
      r = R(j) + (((disp)-D(j))/((D(j+1) - D(j))))*(R(j+1) - R(j));
    end
  end
end
```

Appendix B

Notations

m = mass matrix

c = viscous damping matrix

fs = nodal restoring force vector

a_g = earthquakeground acceleration

p = external force excitation vector

x = nodal displacement vector

$x1$ = nodal velocity vector

$x2$ = nodal acceleration vector

ω = angular natural frequency of the structure

Δt = integration time step

$\bar{\omega}$ = numerical frequency

$\bar{\zeta}$ = numerical damping

$\lambda_1, \lambda_2, \lambda_3$ = eigenvalue of amplification matrix

β, γ = Newmark algorithm parameter

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