Inverted pendulum control using Adaptive Control

A DISSERTATION

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Submitted by

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CANDIDATE'S DECLARATION

I hereby certify that this dissertation work titled **Inverted pendulum control using Adaptive Control** submitted in the fulfilment of the requirement for the award of the degree of **Master of Technology** in **Systems And Control** at **Department of Electrical Engineering, Indian Institute of Technology Roorkee**, is an authentic record of my own work carried out under the supervision and guidance of **Dr. Indra Gupta,** Associate Professor, Department of Electrical Engineering, Indian Institute of Techonology Roorkee, Roorkee (India).

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CERTIFICATE

This is to certify that the above statement made by the candidate is correct to best of my knowledge.

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BHIM SINGH KRIVAL

ABSTRACT

Control of inverted pendulum is an extremely challenging task. The dynamics of pendulum are nonlinear and unstable. In this thesis, an attempt has been made to accomplish the control of an cart inverted pendulum system using linear quadratic regulator (LQR) approach. Linear quadratic regulator is an optimal control scheme that employs a performance index which involves assigning weights to states and the control inputs. First, the mathematical model of the cart inverted pendulum system is derived and then the LQR approach is used for its control. To demonstrate the effectiveness and efficacy of the LQR control scheme, it has been compared with proportional integral derivative (PID) control scheme with respect to the step response. In this thesis the logic is developed to co-ordinate between the different controller such as two LQR to obtain adaptive controller, So that the optimum desired results for plant are obtained. The developed control scheme for CIPS is also validated using simulation. It is observed that the adaptive controller is more effective than individual controller

Contents

List of figures:

CHAPTER -1 : INTRODUCTION

1.1 Introduction

Do you remember when we were a child and we tried on our palm or index finger of our hand to sustain the balance of a broom-stick? By adjust the position of hand constantly to keep that thing upright. In an inverted pendulum (IP) control same thing we have to do . However, Limitation is that now we are adjusting inverted pendulum only for one direction, while our hand which can move up, down , right, left anywhere.

Same as the broom-stick, an Inverted Pendulum having unstable properties. Force must be properly in proper direction need to be extended to keep the IP in upright position. To get this, a developed control system is required. The IP has been a important way to evaluate and compare different control theories.

Without applying some external force it is virtually difficult to balance a IP in upright position. The Carriage Balanced Inverted-Pendulum (CBIP) system, described in this report, allows a stipulated control force on cart. This Carriage Balanced Inverted-Pendulum (CBIP) system provide the control force to the cart by a DC motor . The outputs from this m (CBIP) system are cart position, cart velocity(Only cart position in our case), angle of pendulum and angular velocity of pendulum (in our case only pendulum angle , cart position). The inverted pendulum angle and the cart position is given feedback to Controller that control the motor. The aim is to getting stable the pendulum in a way using position and velocity of the cart on the track accurately and quickly so that the IP is always standing firmly in its upright position in these movements.

This controlling involves a cart, moves forwards and backwards, a pendulum which is mounted with cart through a massless rod at the bottom of the cart in such a way that the pendulum has the ability to rotate in the same plane as the carriage, see Fig 1. In other words pendulum hinged on the bottom of the cart is free to fall along the cart's axis of movements. This IP system has to be controlled so that the pendulum may stand balanced and upright position, and may resist to a step- disturbance.

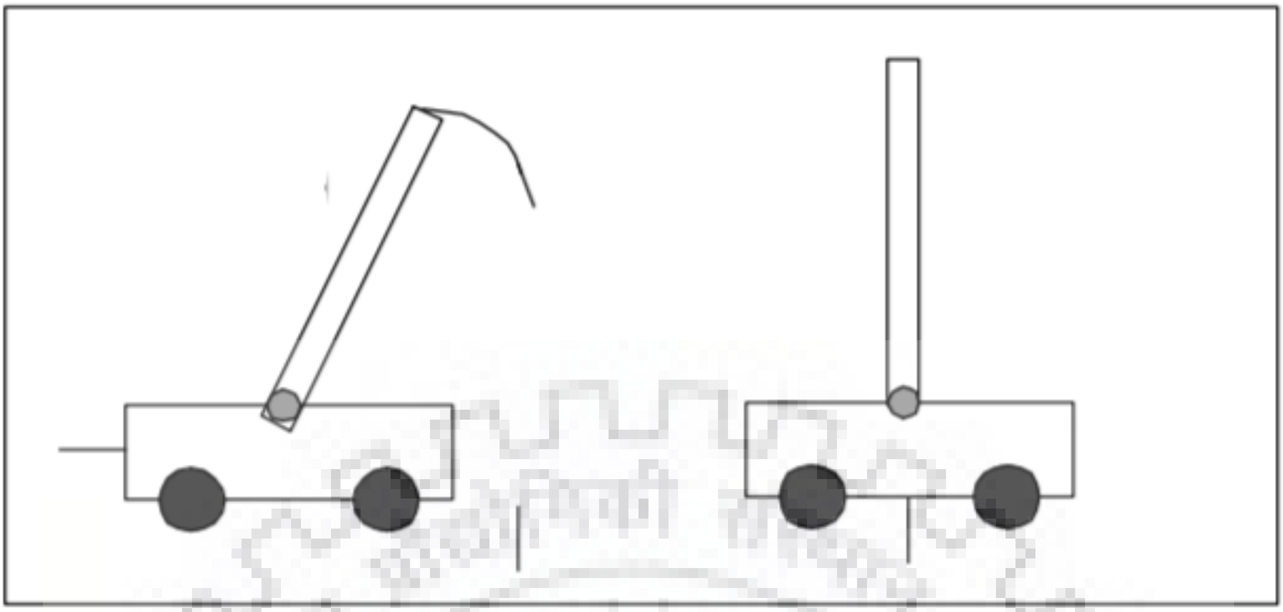


Figure 1.1 Schematic diagram of cart based inverted pendulum

This IP problem consisting a simple coupled system. As the inverted- pendulum looms from the centre, due to gravity it will try to fall. As this pendulum is being coupled with carriage, this carriage will start to move in the reverse direction of pendulum, This moving of the cart will be the cause of falling of pendulum. As per newton's 3rd law change in the one part of the system will react for other part also. Whatever controlling seems easy on first glance is not easy. This is the reason that this kind of problem is oftenly used for fuzzy control demonstration.

The IP cart runs along a track and is pulled by a belt connected to an DC motor. Cart position is measure by a potentiometer and angle of the pendulum is also measured by another potentiometer connected with cart.

Let's assume that the output of the angle which is related with vertical axis comes positive or negative, One can estimate that the system is going to become un-stable, as the pendulam will start to fall if we disturb it with a small angle.

this Inverted Pendulum is unstable in open loop condition .In this problem, we manually keep the pendulum in upright position that is unstable equilibrium position, or some initial displacement (position) also can be given. After this we switch in the controller for balancing the pendulum and in case of disturbances it can maintain its balance. A light lap may work as simple disturbance on pendulum. A gusts of wind may create a complex disturbance for pendulum. This model can be used to understand the behaviour of open-loop unstable systems. It is a demonstration of the stabilizing advantages of feedback control. A Lots of controlling techniques form simplest phase advance compensator to neural net controller can be applied.

1.2 Mathematical Modelling

IP has been a classic control problem. As this system is unstable and nonlinear with several outputs and single input signal. Our aim is that pendulum should be balanced vertically on wagon which is motor driven.

The figure shown below belongs to inverted pendulum problem. What we want is that wagon should go for some desired distance without falling of inverted pendulum. A DC motor is used for driving of wagon. Here pendulum angle θ and position of wagon X are measured and given as states to controller. $F_{DISTURBANCE}$ i.e. a disturbance force can be exerted on top of the pendulum.

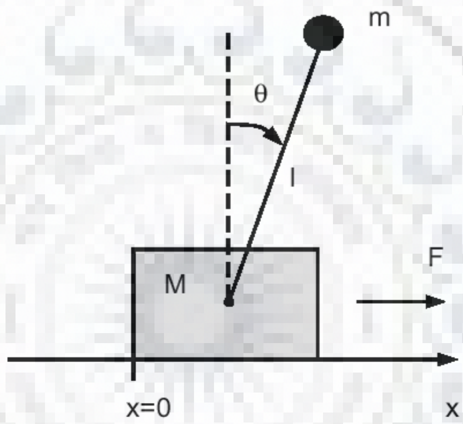


Figure 2 cart based inverted pendulum

A mathematical model is being presented having input as force applied to motor resulting as pendulum angle and cart position.

1.2.1 Setup Description

A IP is hinged on a cart. Cart's translational motion is being controlled by a DC motor using mechanism of a belt/pulley. Potentiometers are being used to feedback the angle of pendulum as well as cart position. This feedback are given to controller so that actuating signal by DC motor can be measured. This motor is derived by servo electronics having electronic circuits. Error signal is being processed by controller circuit, which drives the cart using the DC motor and driving pulley/belt mechanism. For keeping the pendulum in upright position cart has to do To-fro motion.

1.2.2 System Equations of Inverted pendulum

See the following 2 FBD of CBIP system. These free body diagram are being used to derive the equation of motion. An IP mounted on top of the Cart is given fig. 3. The Pendulum mass (m) is assumed to be concentrated at the Centre of Mass. The Pendulum rod has an uniform length (L) from the hinged point to the Centre of mass of pendulum rod. The Pendulum angle(θ) . Mass of the Cart (M) , Cart position is represented as (x). The friction on CIP is neglected. The control force applied is (F). "I" denotes the moment of inertia of pendulum about its centre of mass. The force exerted by cart on the pendulum in vertical direction is assumed as P while in horizontal direction it is N. Only two dimensional problem is considered here

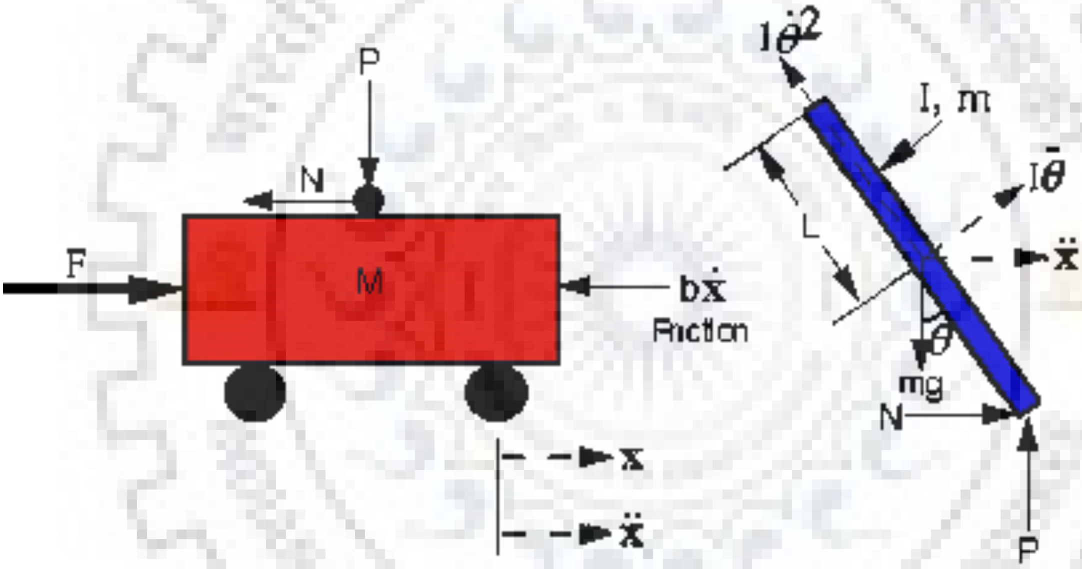


Figure 3 FBD of cart based inverted pendulum

As we do sum of all horizontal forces acting upon cart in the FBD of cart, results in the following equation.

$$M\ddot{x} + b\dot{x} + N = F \quad \text{……………………………(1.1)}$$

For the cart we can sum up all the forces in the vertical direction but as we know that cart is not moving in vertical position so not a useful information for our purpose could be gained. It is understood that cart reaction force in vertical direction would be balanced by earth reaction.

Tangential force exerted perpendicular on the pendulum will be used for getting the force in the horizontal direction:

$$\tau = l * F = I\ddot{\theta} \ldots\ldots (1.2)$$

$$F = \frac{I\ddot{\theta}}{r} \ldots\ldots (1.3)$$

$$= \frac{ml^2\ddot{\theta}}{l} \qquad = ml\ddot{\theta} \ldots\ldots (1.4)$$

This tangential force's Component in the direction of N is $ml\ddot{\theta}\cos\theta$.

While the centripetal force acting in the longitude outward direction of pendulum:

$$F = \frac{I\dot{\theta}^2}{l} \ldots\ldots (1.5)$$

$$= \frac{ml^2\dot{\theta}^2}{l} \qquad = mI\dot{\theta}^2 \ldots\ldots (1.6)$$

Centripetal force's component in the direction of N is $ml\dot{\theta}^2 \sin\theta$

Summing all the forces which are in the horizontal direction of the pendulum free body diagram., so the equation for N:

$$N = m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2 \sin\theta \ldots\ldots (1.7)$$

Now putting value of N from equ. (2)into equ (1).

$$F = (M + m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2 \sin\theta \ldots\ldots (1.8)$$

This Equation (3) is called first equation of motion of cart based inverted pendulum system.

Now let's do the vectoral sum of forces acting perpendicularly of the pendulum. This will lead to second equation of motion of cart based inverted pendulum system. Just for simplifying the complexity of mathematics this axis has been chosen. If we solve equation along this axis a lot of algebra calculations can be avoided. So like the previous equation vertical component need to be added, shown in the following equation.

$$P \sin\theta + N \cos\theta - mg \sin\theta = ml\ddot{\theta} + m\ddot{x} \cos\theta \ldots (1.9)$$

Here P & N terms in the above equation are unknow so to remove them around the centroid of pendulum sum all of the moments. By summing we get the following equation:

$$Pl\sin\theta + Nl\cos\theta = -I\ddot{\theta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.10)$$

Combining equation (1.9) and equation (1.10)

$$(ml^2 + I)\ddot{\theta} = mlg\sin\theta - ml\ddot{x}\cos\theta \ldots\ldots\ldots\ldots\ldots\ldots (1.11)$$

This equation is called second equation of motion of cart based inverted pendulum system

The nonlinear equations of motion are

$$F = (M+m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta \ldots\ldots\ldots\ldots (1.12)$$

$$(ml^2 + I)\ddot{\theta} = mlg\sin\theta - ml\ddot{x}\cos\theta \ldots\ldots\ldots\ldots\ldots (1.13)$$

For linearize a using Taylor series expansion around the equilibrim point. In vertical plane($\theta = 0$) is equilibrium point of IP . This gives $\sin\theta = \theta$, $\cos\theta = 1$ and as $\dot{\theta}^2 = 0$

$\dot{\theta}^2 = 0$; **cos** θ=1 ; **sin** $\boldsymbol{\theta} = \theta$

Substitute in equation 3&6; we get

$$\boldsymbol{F = (M+m)\ddot{x} + ml\ddot{\theta}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.14))$$

$$\boldsymbol{(ml^2 + I)\ddot{\theta} = mlg\theta - ml\ddot{x}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.15)$$

Taking Laplace Transform of (1.14) & (1.15)

$$F(s) = (M+m)s^2X(s) - mls^2\theta(s) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.16)$$

$$-mls^2X(s) + (I + ml^2)s^2\theta(s) = mlg\theta(s) \ldots\ldots\ldots\ldots\ldots\ldots (1.17)$$

Form the above equation (1.16) & (1.17)

$$\frac{\theta(s)}{F(s)} = \frac{ml}{[[(I+ml^2)(M+m)]]s^2] - [(M+m)mgl]} \ldots\ldots (1.18)$$

$$\frac{X(s)}{F(s)} = \frac{1}{[[(M+m)s^2]} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.19)$$

These are the transfer function for pendulum and the cart

Parameter for CIP

Mass of cart ;M	2.4 Kg
Pendulum mass ; m	0.23 Kg

Pendulum length from Centre to pivot ;l	0.4 m
Acceleration due to gravity	9.8 m/s^2
Moment of inertia , I	0.099

Figure 4 Parameter for CIP

Let's for simplicity

$$F_1(s) = \frac{\theta(s)}{F(s)} = \frac{T_2}{S^2 - Z_2} \quad \text{......(1.20)}$$

$$F_2(s) = \frac{X(s)}{F(s)} = \frac{T_1}{S^2} \quad \text{......(1.21)}$$

1.3 State space model

From equation 7&8 state space model will be

$$x_1 = X$$
$$\dot{x_1} = \dot{X} = x_2$$
$$\dot{x}_2 = \ddot{x}$$
$$x_3 = \theta$$
$$\dot{x}_3 = \dot{\theta} = x_4$$
$$\dot{x}_4 = \ddot{\theta}$$

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{M+m}{ml} g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{m}{M} g & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{Ml} \\ 0 \\ \frac{1}{M} \end{bmatrix} u \quad \text{......(1.22)}$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} \quad \text{......(1.23)}$$

1.4 Open loop step response of the system

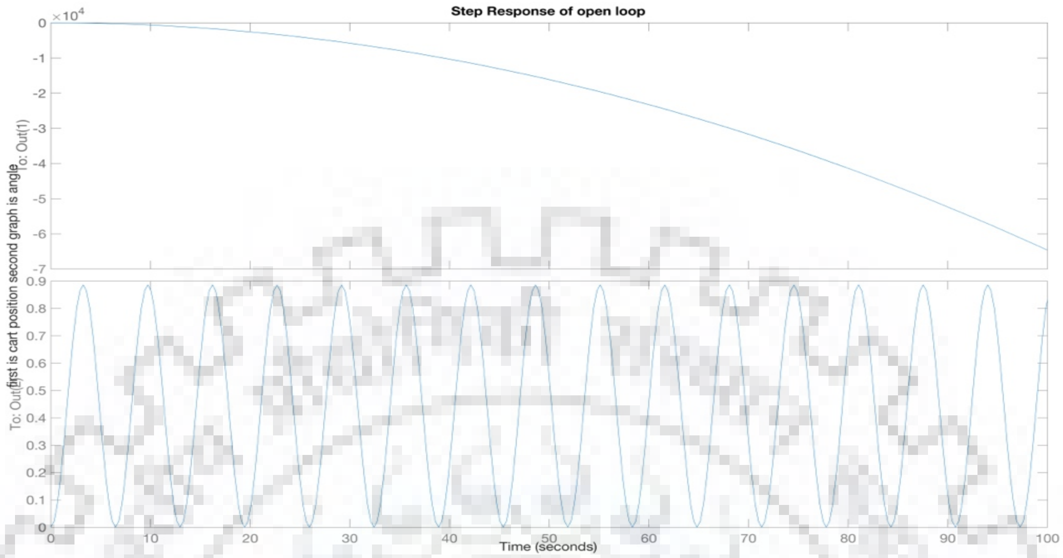


Figure 5 Open loop step response of Cart Angle and position

CHAPTER 2: CONTROLLER DESIGN

2.1 LQR controller design

As we do loop shaping in SISO system do not extend to (MIMO) multivariable system. As MIMO system having transfer matrices rather than transfer function. The notion of optimality is closely tied to MIMO control system design. Optimal controllers are controllers do the best possible, according to some figure of merit, turn out to generate only stabilizing controllers for MIMO plants. In this sense, optimal control solutions provide an automated design procedure – we have only to decide what figure of merit to use. Linear quadratic regulator (LQR) is a famous design technique that give practical feedback gain .

J is cost functional or performance index (PI)

$$JU(t) = \frac{1}{2}\int_{-\infty}^{\infty}(X^T(t)Q(t)X(t) + U^T(t)R(t)U(t))dt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.1)$$

where Q = Semi definite matrix, R = Positive definite matrix.

Riccati equation

$$PA + A^T P + Q - PBR^{-1}B^T P = 0\text{......................}(2.2)$$

Where Q and R are positive definite matrix. Solution of this equation will result in P matrix.
So the feedback

$$u = -KX\text{..........................}(2.3)$$

So the original system

$$\dot{X} = AX + BU\text{.....................}(2.4)$$
$$Y = CX + D\text{.................}(2.5)$$

Will become as

$$\dot{X} = (A - BK)X\text{.............}(2.6)$$
$$Y = CX + DU\text{.............}(2.7)$$

$$K = R^{-1}B^T P\text{...............}(2.8)$$

2.1.1 Script to get the LQR response

```
A=[0 1 0 0;0 0 -26.84 0;0 0 0 1;0 0 -0.939 0];
B=[0;-1.041;0;0.41];
C=[1 0 0 0;0 0 1 0];
R = [10000]
sys=ss(a,b,c,d)
[num, den] = ss2tf(a,b,c,d,1)
step(sys)
K = lqr(sys,Q,R)
```

Calculation of MATLAB attached as annexure#1

2.1.2 LQR controlled step response of cart position and angle of pendulum

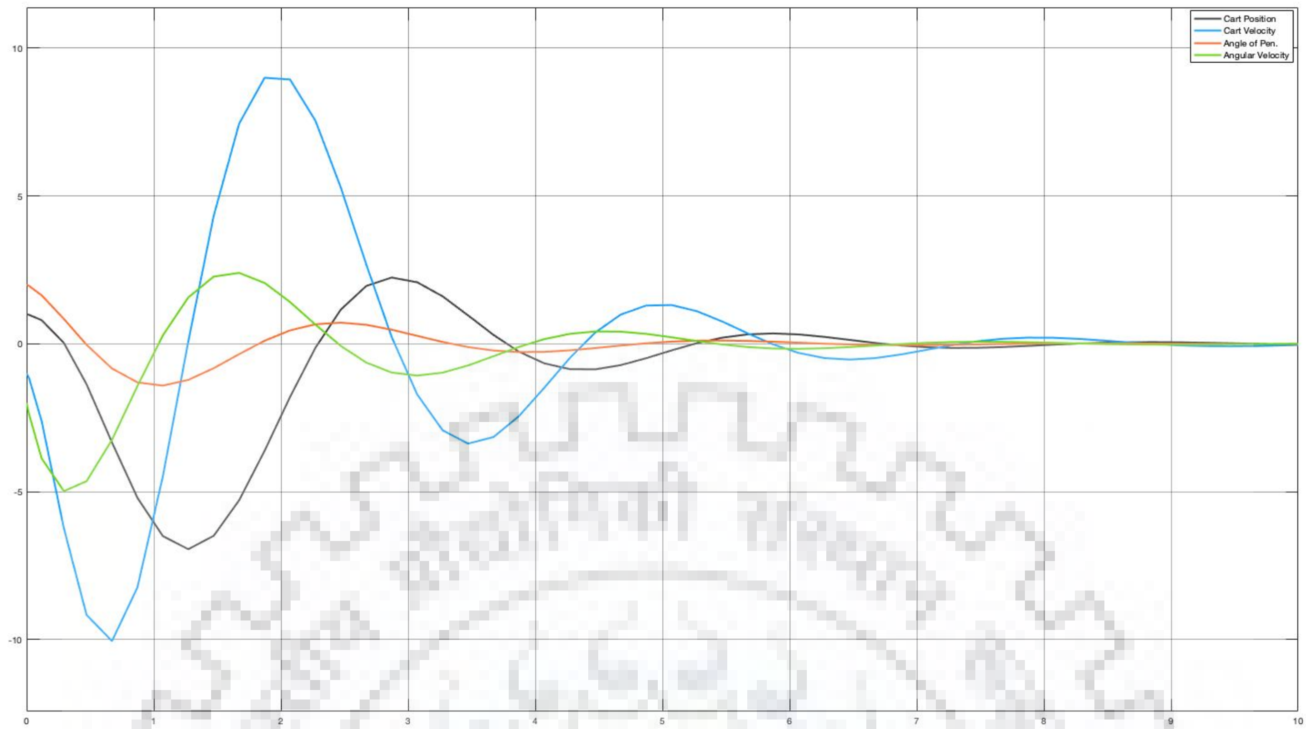


Figure 6 LQR controlled step response of cart position and angle of pendulum

2.1.3 Simulink model of LQR controller

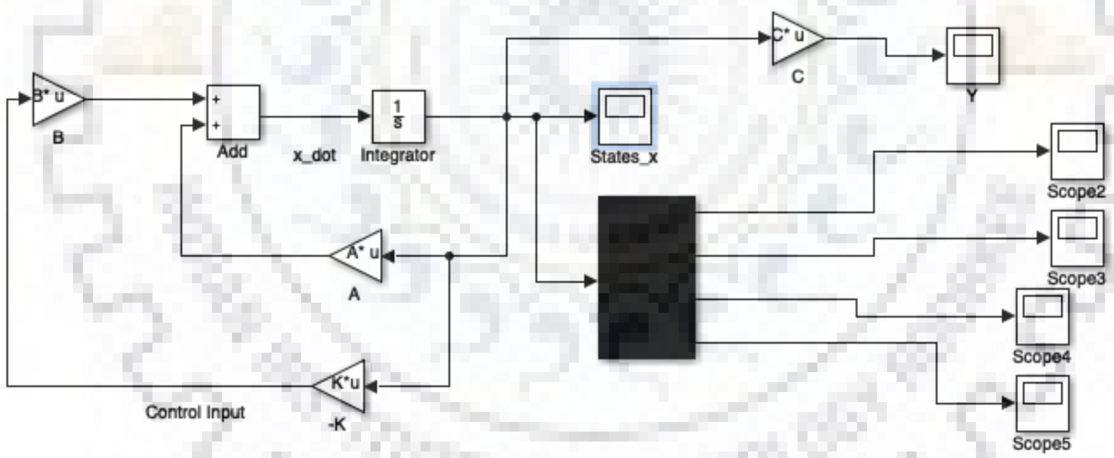


Figure 7 Simulink model of LQR controller

2.2 PID Control of Inverted Pendulum

Let’s tune a PID for our desired characteristic

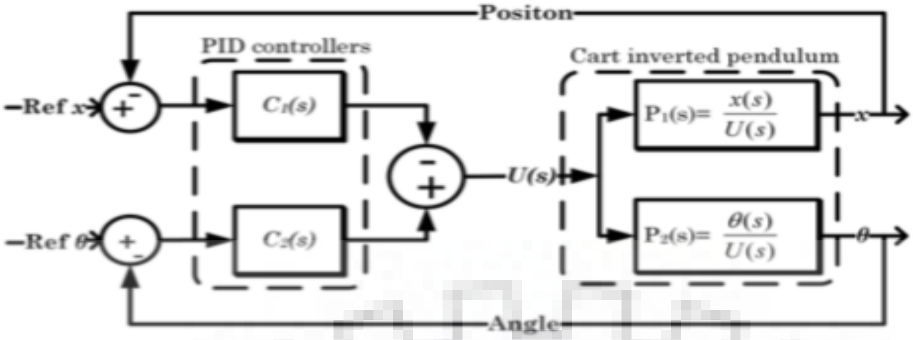


Figure 8 Schematic diagram of PID controller [2]

By trial and error K_{P1} =30, $K_{I1} = 23$, $K_{D1} = 56$
$K_{P2} = 56, K_{I2} = 67, K_{D2} = 67$

$$\frac{\theta(s)}{F(s)} = \frac{.747}{S^2 - 19.276}$$

$$\frac{X(s)}{F(s)} = \frac{.380}{S^2}$$

2.2.1 Step response of PID controlled inverted pendulum

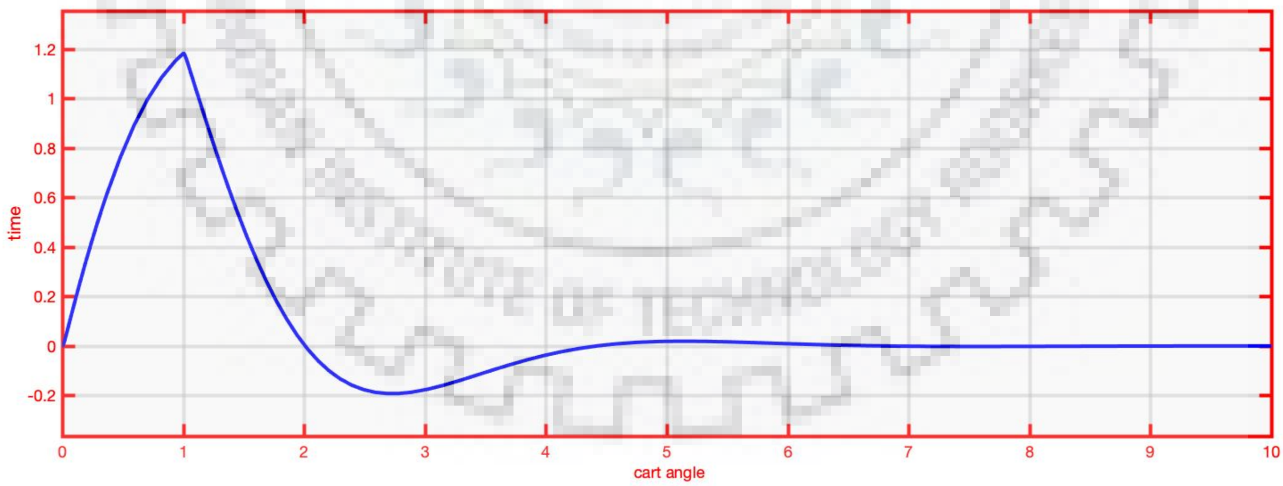


Figure 9 Step response of angle of pendulum using PID controller

CHAPTER 3 : ADAPTIVE CONTROL

3.1 Introduction

Adaptive presents the meaning of changing or turn itself as per requirement. Adaptive is a natural phenomenon in environment as all the living being changes themselves as per environmental requirement. While in adaptive field of controlling the term adaptive can be traced back to 1936s, a US patent was filed on self-stabilizing control mechanism by Nikolai[2] where it was designed hardware mechanism to control the magnitude of variable conditions temperature of the motor. Although in 1947 and 1954, two US patents were filed on the control system for automatic response adjustment and self-adjusting control apparatus, respectively, the word 'adaptive' was first coined in 1958 in the survey paper. Since than many scholars for example, Whitaker et al. gave design for autopilot which can be used for different altitudes and different speed. Researcher Kalman also suggested adaptive pole placement technique in a similar fashion based on LQR. In Control literature many adaptive has been developed till now since older time. For parameter estimation and readjustment adaptive control techniques has always been used to get desired results. Various researcher has researched many ways to make a simple design as well as to get better and faster response.

Whatever control laws reported in adaptive field of control literature can be classified in two types

1) Identifier based
2) Non- identifier based

Identifier based method of designing controller require online parameter estimation. While Non-identifier based controller uses deterministic search algorithms to determine the best configuration of controller. Gain scheduling technique which is non-identifier based technique used in power system operation. Gain scheduling basically deals with a controller block composed of multiple numbers of controllers, each possessing its own characteristics, and a switching logic block which, on the basis of some search algorithm technique, determines which controller to be preferred the most for the current scenario.

Cart inverted pendulum is a very good control engineering problem, one of the classic problem, is solved by adaptive as well as non-adaptive control methods. The hurdles appears due to unstable non-linear , under-actuated and non-minimum phase function. That's the reason control engineer face challenge to design perfect controller which can perform best as well as should be simple in design. Addressing the stabilization problem of CIPS, numerous work has been done in this field.

The weights are updated by online gradient descent learning algorithm. The design procedure is simplified by selecting these two types of controllers which not only reduce the mathematical intricacies but also pro- vide good performance of the system, both in the presence and absence of matched disturbances.

The rest of the article is organized as follows: section 'Proposed adaptive control logic scheme' describes the adaptive control logic proposed in this article, for a generalized plant. A methodical description on how to implement the proposed technique is discussed in section 'Implementation of the proposed method for CIPS'. In section 'Simulation-based analysis of CIPS under different operating conditions', simulation of CIPS with the proposed control law is carried out using MATLAB and Simulink, both in the presence and absence of external disturbances. In addition, a short note is also provided for improvement in system performance in spite of any changes in input gain following the controller output.

3.2 Adaptive Control Versus Conventional Feedback Control

The performance of the control system is degraded by the unmeasurable and unknown variation of process parameter. Similarly to the disturbances acting upon the controlled variables, one can consider that the variations of the process parameters are caused by disturbances acting upon the parameters (called parameter disturbances) . The disturbance in the parameter affects performance of system. Therefore the disturbances acting upon a control system can be classified as follows:

(a) Disturbances acting upon the controlled variables
(b) (parameter) disturbances acting upon the performance of the control system.

Feedback is basically used in conventional control systems to reject the effect of disturbances upon the controlled variables and to bring them back to their desired values according to a certain performance index. To achieve this, one first measures the controlled variables, then the measurements are compared with the desired values and the difference is fed into the controller which will generate the appropriate control.

A similar conceptual approach can be considered for the problem of achieving and maintaining the desired performance of a control system in the presence of parameter disturbances. We will have to define first a *performance index* (IP) for the control system which is a measure of the performance of the system (ex: the damping factor for a closed-loop system characterized by a second-order transfer function is an IP which allows to quantify a desired performance expressed in terms of "damping"). Then we will have to measure this IP. The *measured* IP will be compared to the *desired* IP and their difference (if the measured IP is not acceptable) will be fed into an *adaptation mechanism*. The output of the *adaptation mechanism* will act upon the parameters of the controller and/or upon the control signal in order to modify the system performance accordingly.

3.3 Adaptive control technique:

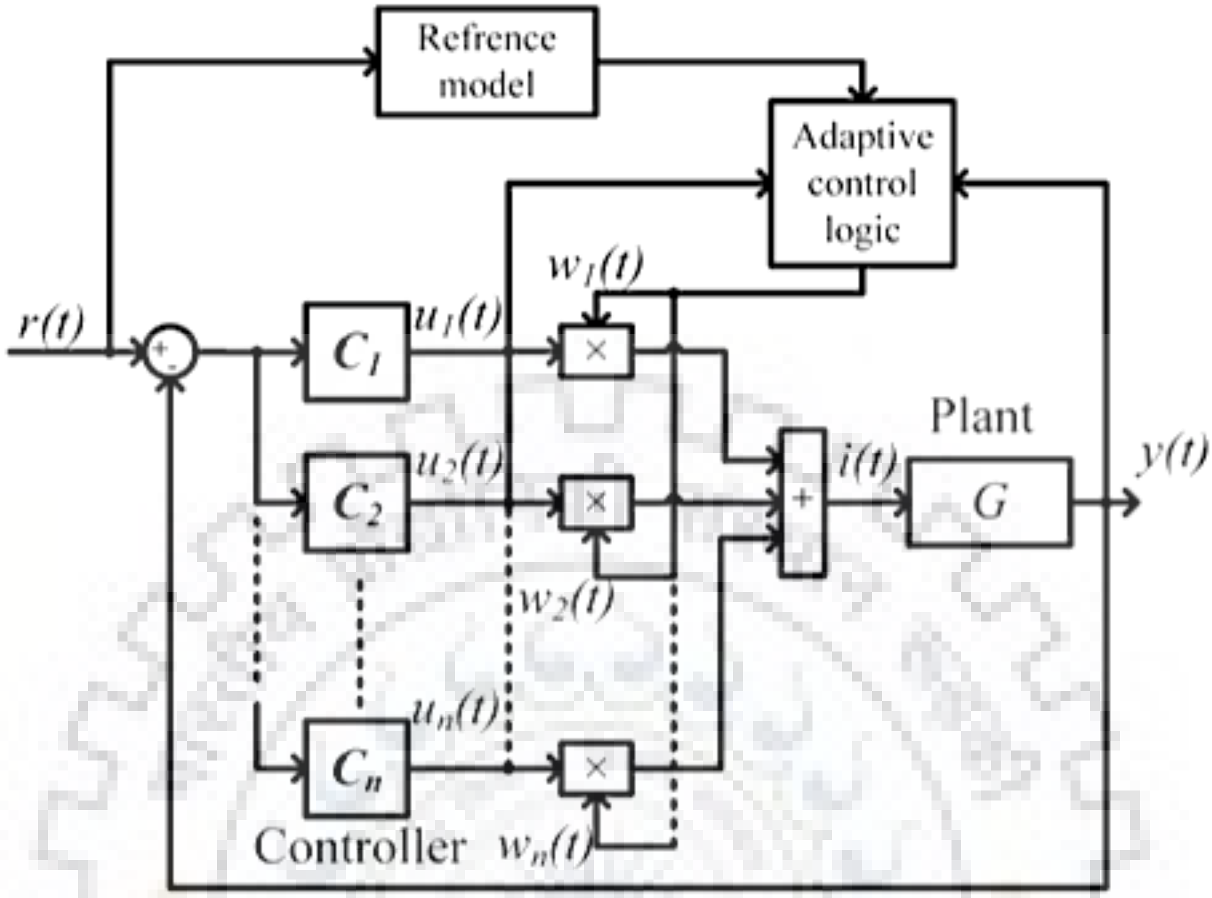
For controlling of system, a supervisory system which select controller from many controller which are already designed. Here supervisory system means adaptive controller but in supervisory system means controlling or supervising should have prior knowledge of system on the basis of that supervisory system chooses correct controller and further it could learn for future aspect.

But in all cases correct data set cannot be provided to controller for every possible scenario or some uncertain behaviour. So Gradient descent technique can be used to obtain online estimation of parameters. While working in real time algorithm get immediate feedback of prediction and use this feedback to improve its future prediction and performance. Figure 9 shows generalized block diagram to control plant G , a set of n number of controller have been designed. The output from the nth controller (where $n \epsilon N$) at time t is denoted by u_n(t). This output is multiplied by weight w_n(t) before it is feeding as input to the plant. So the net control input to the plant at time t can be written as follows

$$U(t) = \sum_{n=1}^{n=N} w(t)u(t)$$

$$\omega(t) = [\omega_1(t)\ \omega_2(t) \quad \omega_3(t) \quad \ldots\ldots \omega_n(t)\ ]^T$$

$$u(t) = [u_1(t)\ u(t) \quad u_3(t) \quad \ldots\ldots u_n(t)]^T$$



Fig

Figure 10 Adaption scheme [2]

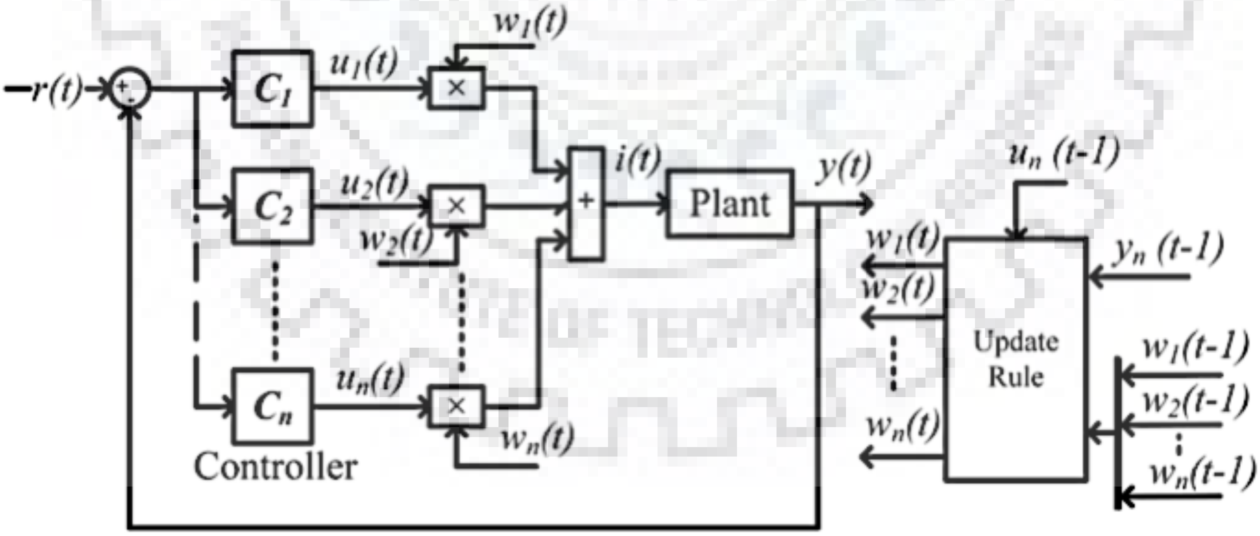


Figure 11 Weight updating rule [2]

As per schematic diagram

$$U(s) = w_1 u_1(s) + w_2 u_2(s) \quad \text{.................................(3.1)}$$

$$U(s) = w_1 c_1(s) e(s) + w_2 c_2(s) e(s) \quad \text{........................(3.2)}$$

Error function

$$e(s) = r(s) - y(s) \quad \text{...(3.3)}$$

$$U(s) = (w_1 c_1(s) + w_2 c_2(s))(r(s) - y(s)) \quad \text{..............(3.4)}$$

$$\mathrm{G(s)} = \frac{\sum_{i=1}^{i=m} g_i s^i}{\sum_{i=1}^{i=n} f_i s^i} \quad \text{.......................................(3.5)}$$

Where $m < n$

So we have to choose weights (equ. 3.1) such that the error function e(t) can be minimized at any time t. So for doing that weights are iteratively updated using gradient descent rule and updated weight can be written as-

Gradient descent rule

$$\omega_n(t+1) \leftarrow \omega_n(t) - \alpha \frac{\partial e(t)}{\partial \omega_n(t)} \quad \text{..............................(3.6)}$$

Here α is learning rate $\alpha \in (0,1)$

We can write $\frac{\partial e(t)}{\partial \omega_n(t)}$ like the followed for calculation purpose

$$\frac{\partial e(t)}{\partial \omega_n(t)} = \frac{\partial e(t)}{\partial y(t)} \frac{\partial y(t)}{\partial u_n(t)} \frac{\partial u_n(t)}{\partial \omega_n(t)} \quad \text{..........................(3.7)}$$

Where $n \in \{1,2,3 \ldots \ldots k\}$

For getting the value of $\frac{\partial e(t)}{\partial y(t)}$, we know that

let us define an error function as follows , ISE function

$$e(t) = \frac{1}{2}\int_0^{\infty}(r(t) - y(t))^2 \quad (3.8)$$

where r(t) is the reference input to the plant at time t and y(t) is the actual output.

So,

$$\frac{\partial e(t)}{\partial y(t)} = -\{r(t) - y(t)\} \quad (3.9)$$

For calculating $\frac{\partial u_n(t)}{\partial \omega_n(t)}$, as per schematic diagram

$$U(t) = w_1 u_1(t) + w_2 u_2(t) \quad (3.10)$$

So

$$\frac{\partial u_n(t)}{\partial \omega_n(t)} = u_n(t) \quad (3.11)$$

Now for getting the value of $\frac{\partial y(t)}{\partial u_n(t)}$ the follow procedure is obtained

$$\frac{y(s)}{u(s)} = \mathrm{G(s)} = \frac{\sum_{i=1}^{i=m} g_i s^i}{\sum_{i=1}^{i=n} f_i s^i} \quad (3.12)$$

$$\frac{y(s)}{u(s)} = \frac{g_m s^m + g_{m-1} s^{m-1} + \cdots .... + g_1 s^1 + g_o}{f_n s^n + f_{n-1} s^{n-1} + \cdots .... + f_1 s^1 + f_o} \quad (3.13)$$

Cross multiplication

$$y(s) f_n s^n + y(s) f_{n-1} s^{n-1} + \cdots .... + y(s) f_1 s^1 + y(s) f_o = u(s) g_m s^m + u(s) g_{m-1} s^{m-1} + \cdots .... + u(s) g_1 s^1 + u(s) g_o \quad (3.14)$$

Writing in time form

$$f_n \frac{d^n y}{dt^n} + f_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \cdots ..... + f_1 \frac{dy}{dt} + f_o y(t) = g_m \frac{d^m u}{dt^m} + g_{m-1} \frac{d^{m-1} u}{dt^{m-1}} + \cdots ..... + g_1 \frac{du}{dt} + g_o u(t) \quad (3.15)$$

Differentiate both side by d/du

$$f_n \frac{d}{du}\left\{\frac{d^n y}{dt^n}\right\} + f_{n-1} \frac{d}{du}\left\{\frac{d^{n-1} y}{dt^{n-1}}\right\} + \cdots ..... + f_1 \frac{d}{du}\left\{\frac{dy}{dt}\right\} + f_o \frac{dy(t)}{du} = g_m \frac{d}{du}\left\{\frac{d^m u}{dt^m}\right\} + g_{m-1} \frac{d}{du}\left\{\frac{d^{m-1} u}{dt^{m-1}}\right\} + \cdots ..... + g_1 \frac{d}{du}\left\{\frac{du}{dt}\right\} + g_o \frac{du(t)}{du} \quad (3.16)$$

$$f_o \frac{dy(t)}{dt}\frac{dt}{du} + f_1 \frac{d}{dt}\left\{\frac{dy}{dt}\right\}\frac{dt}{du} + \cdots \ldots .. + f_{n-1}\frac{d}{dt}\left\{\frac{d^{n-1}y}{dt^{n-1}}\right\}\frac{dt}{du} + f_n \frac{d}{dt}\left\{\frac{d^n y}{dt^n}\right\}\frac{dt}{du}$$

$$= \ g_o \frac{du(t)}{dt}\frac{dt}{du} + g_1 \frac{d}{dt}\left\{\frac{du}{dt}\right\}\frac{dt}{du} + \cdots \ldots . + g_{m-1}\frac{d}{dt}\left\{\frac{d^{m-1}u}{dt^{m-1}}\right\}\frac{dt}{du}$$

$$+ g_m \frac{d}{dt}\left\{\frac{d^m u}{dt^m}\right\}\frac{dt}{du} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .. (3.17)$$

$$f_o \frac{dy(t)}{du} = \left(g_o \frac{du(t)}{dt}\frac{dt}{du} + g_1 \frac{d^2u}{dt^2}\frac{dt}{du} + \cdots \ldots . + g_{m-1}\frac{d^m u}{dt^m}\frac{dt}{du} + g_m \frac{d^{m+1}u}{dt^{m+1}}\frac{dt}{du}\right)$$

$$-\left(f_1 \frac{d^2y}{dt^2}\frac{dt}{du} + \cdots \ldots + f_{n-1}\frac{d^n y}{dt^n}\frac{dt}{du} + f_n \frac{d^{n+1}y}{dt^{n+1}}\frac{dt}{du}\right) \ldots\ldots\ldots\ldots\ldots\ldots .. (3.18)$$

$$\frac{dy}{du} = \frac{1}{f_o\frac{du}{dt}}\left[\sum_{i=0}^{i=m} g_i u'^{(i+1)} - \sum_{i=1}^{i=n} f_i y'^{(i+1)}\right] \ldots\ldots\ldots\ldots (3.19)$$

From Gradient descent rule

$$\omega_n(t+1) = \omega_n(t) - \alpha \frac{\partial e(t)}{\partial \omega_n(t)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.20)$$

$$\omega_n(t+1) = \omega_n(t) - \alpha \frac{\partial e(t)}{\partial y(t)}\frac{\partial y(t)}{\partial u_n(t)}\frac{\partial u_n(t)}{\partial \omega_n(t)} \ldots\ldots\ldots\ldots\ldots\ldots (3.21)$$

So the final equation comes for solution is

$$\omega_n(t+1) = \omega_n(t) - \left\{\alpha(-(r(t) - y(t))u_n(t)\frac{1}{f_o\frac{du}{dt}}\left\{\sum_{i=0}^{i=m} g_i u'^{(i+1)} - \sum_{i=1}^{i=n} f_i y'^{(i+1)}\right\}\right\} \ldots\ldots\ldots (3.22)$$

This is the equation of generalized weight updation rule. In figure 10 it is shown systematically.

3.4 Formulation of rule updation using gradient descent rule:-

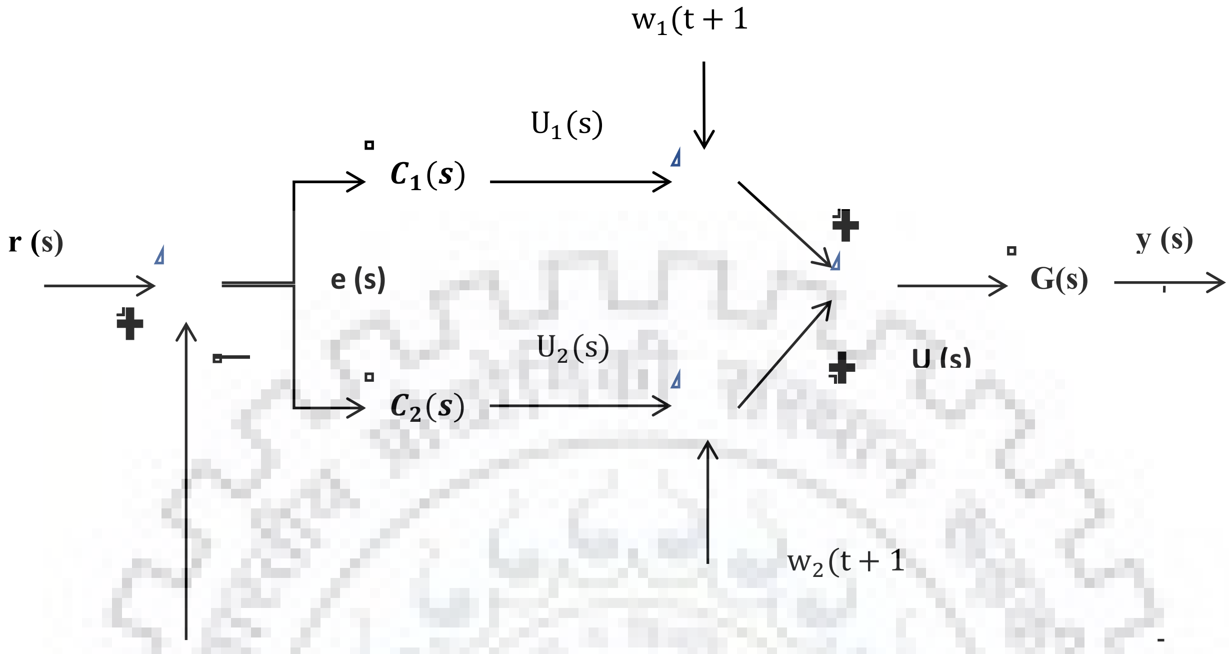


Figure 12 Two LQR controller used in adaptive control formulation

To control cart-inverted pendulum we have to minimize error in cart angle θ and cart position X. Therefor the objective function to control inverted pendulum can be written as

$$e(t) = \frac{1}{2}\int_0^{\infty} (\theta(t)^2 + x(t)^2)\, dt \text{……………………………3.23}$$

Where θ is the cart position measured from reference angle $\theta = 0$ and x is cart position measured from reference distance $x = 0$. Weight updation rule will be

$$\omega_1(t+1) \leftarrow \omega_1(t) - \alpha \frac{\partial e(t)}{\partial \omega_1(t)} \text{…………………………3.24}$$

$$\omega_2(t+1) \leftarrow \omega_2(t) - \alpha \frac{\partial e(t)}{\partial \omega_2(t)} \text{…………………………3.25}$$

$\frac{\partial e(t)}{\partial \omega_1(t)}$ and $\frac{\partial e(t)}{\partial \omega_2(t)}$ can be written in the form of chain derivatives

$$\frac{\partial e(t)}{\partial \omega_1(t)} = \frac{\partial e(t)}{\partial \theta(t)}\frac{\partial \theta(t)}{\partial u(t)}\frac{\partial u(t)}{\partial \omega_1(t)} + \frac{\partial e(t)}{\partial x(t)}\frac{\partial x(t)}{\partial u(t)}\frac{\partial u(t)}{\partial \omega_1(t)} \text{……………3.26}$$

$$\frac{\partial e(t)}{\partial \omega_2(t)} = \frac{\partial e(t)}{\partial \theta(t)}\frac{\partial \theta(t)}{\partial u(t)}\frac{\partial u(t)}{\partial \omega_2(t)} + \frac{\partial e(t)}{\partial x(t)}\frac{\partial x(t)}{\partial u(t)}\frac{\partial u(t)}{\partial \omega_2(t)} \text{……………3.27}$$

linear equation of motion equ.(1.12) and equ.(1.13)

$$u_1 = (M+m)\ddot{x} + ml\ddot{\theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.28))$$

$$(ml^2 + I)\ddot{\theta} = mlg\theta - ml\ddot{x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.29)$$

From equation (3.28) and (3.29)

$$u_j(t) = (M+m)g\theta - \left(\frac{\gamma}{ml}\right)\ddot{\theta} \ldots\ldots\ldots\ldots\ldots\ldots(3.30)$$

Where $\gamma = I + ml^2$

Differentiating with respect to θ

$$\frac{du_j(t)}{d\theta(t)} = (M+m)g - \left(\frac{\frac{\gamma}{ml}(d^3\theta/dt^3)}{\frac{d\theta}{dt}}\right) \ldots\ldots\ldots\ldots(3.31)$$

Now required $\frac{d\theta(t)}{dx(t)}$

$$\frac{d\theta(t)}{dx(t)} = \frac{d\theta(t)}{du_j(t)}\frac{du_j(t)}{dx(t)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.32)$$

Using equation (3.29)

$$\frac{d\theta(t)}{dx(t)} = \frac{(I+ml^2)(d^3\theta/dt^3) + ml((d^3x/dt^3)}{mgl(\frac{dx(t)}{dt})} \ldots\ldots\ldots\ldots(3.33)$$

So now using equation (3.24), (3.25), (3.26), (3.27), (3.32), (3.33) weight updation can be calculated as below

$$\omega_1(t+1) \leftarrow \left\{\omega_1(t) - \alpha u_1(t)\left(\frac{2\theta}{\frac{du_1(t)}{d\theta(t)}} + \frac{2x}{\frac{du_1(t)d\theta(t)}{d\theta(t)\ dx(t)}}\right)\right\} \ldots\ldots\ldots\ldots\ldots\ldots(3.34)$$

$$\omega_2(t+1) \leftarrow \left\{\omega_2(t) - \alpha u_2(t)\left(\frac{2\theta}{\frac{du_2(t)}{d\theta(t)}} + \frac{2x}{\frac{du_2(t)d\theta(t)}{d\theta(t)\ dx(t)}}\right)\right\} \ldots\ldots\ldots\ldots\ldots\ldots(3.35)$$

Example:-

Here C1 and C2 control methods are LQR methods in which different Q and R matrices has been used , Which resulted in K matrices as

$$k1 = [0.3162 \quad 0.8798 \quad 19.9732 \quad 11.8682]$$

And $$k2 = [3.1623 \quad 4.4543 \quad 66.8323 \quad 28.3375]$$

As per simulation the result are given below

3.4.1 Adaptive simulation result of cart position

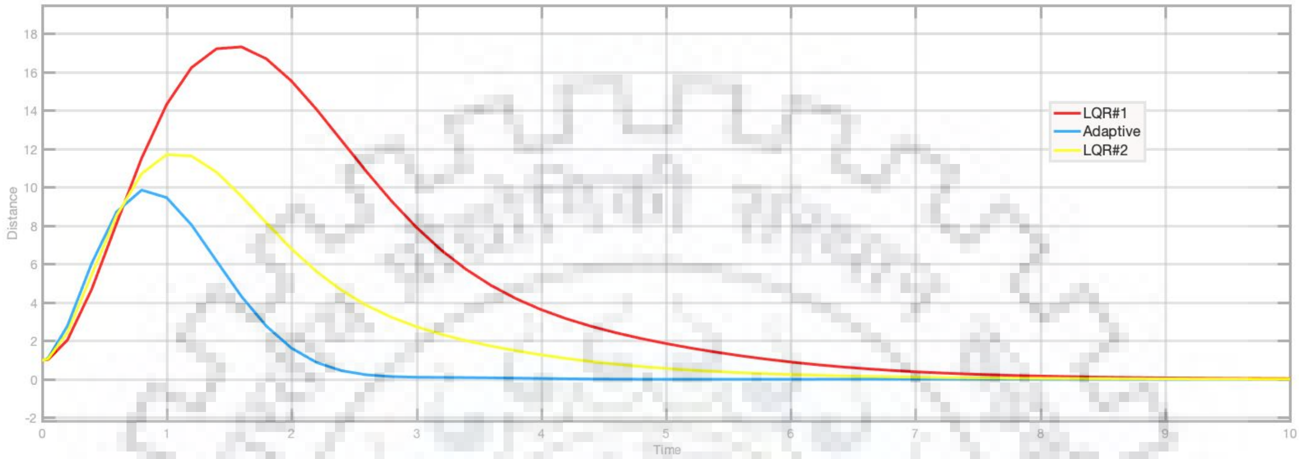


Figure 13: Adaptive distance of cart

3.4.2 Adaptive simulation result of Pendulum angle

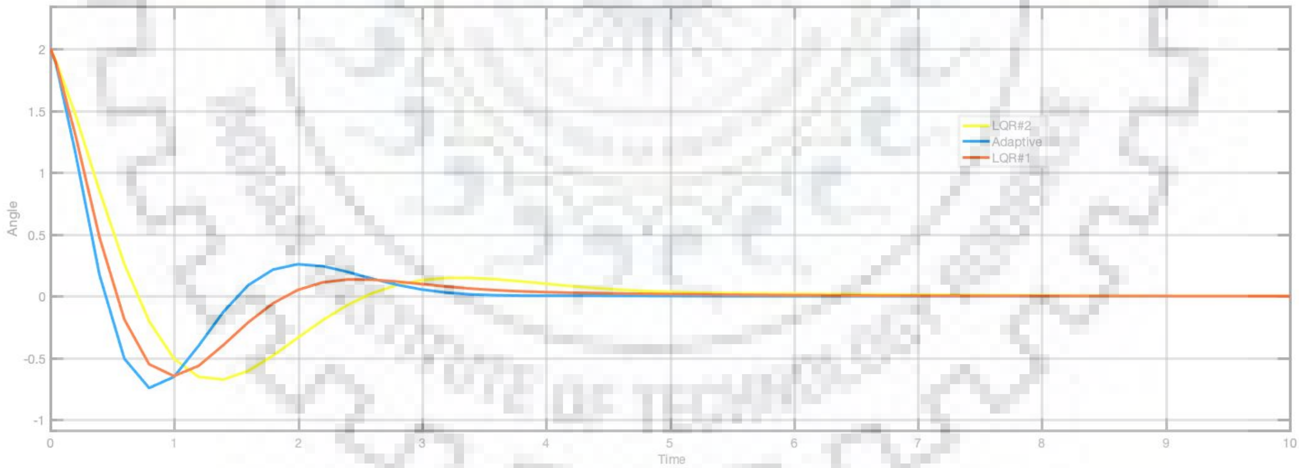


Figure 14 Adaptive Angle of Pendulum

- By observation of cart position it is observed that adaptive controller's response is much good comparing to LQR#1 and LQR#2, Overshoot is minimum and settling time also very less with respect to LQR#1 and LQR#2.
- While observing pendulum angle adaptive controller's settling time is minimum with respect to both LQR#1 and LQR#2.

So the technique is quite useful. If we are having multiple control techniques than with respect to minimum error adaptive result will be the outcome.

Conclusion:-

The main attribution of this thesis is to implement and design a control logic under specific condition to choose appropriate controller of a plant. The objective is to co-ordinate both LQR controller in order to realize optimum result suitable for the plant. The general process is used to realize adaptive control for CIPS and is validated using simulation. It has been observed that adaptive control depicts better result than individual controllers. Further research can be extended to improve the methodology and widen up the applicability to control engineering such as robotics, etc.

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