DESIGN OF FRACTIONAL ORDER PID CONTROLLER FOR SELECTED TYPE OF SYSTEMS

A DISSERTATION

Submitted in partial fulfilment of the requirement for the award of the degree

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ELECTRICAL ENGINEERING

(With Specialization in Systems and Control Engineering)

By

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CANDIDATE'S DECLARATION

I hereby certify that this dissertation which is being presented in the progress report titled **"DESIGN OF FRACTIONAL ORDER PID CONTROLLER FOR SELECTED TYPE OF SYSTEMS"** in partial fulfilment for the requirement of award of Degree of **Master of Technology** in **Electrical Engineering** with specialization in **Systems And Control**, submitted to the Department of Electrical Engineering, Indian Institute of Technology, Roorkee, India is an authentic record of the work carried out during a period from July 2018 to November 2018 under the supervision of **Dr. INDRA GUPTA** Department of Electrical Engineering, Indian Institute of Technology, Roorkee.

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Place: Roorkee

Date: Antima Kumari

TABLE OF CONTENTS

ABSTRACT

LIST OF FIGURES

LIST OF TABLES

ABBREVIATION

ADRC	Active Disturbance Rejection Control
CRONE	Commande Robuste d’Ordre Non-Entier
ESO	Extended State Observer
FOIMC	Fractional Order Internal Model Control
FOMCON	Fractional Order Modeling and Control
GADRC	Generalised Active Disturbance Rejection Control
GUI	Graphical User Interface
IAE	Integral Absolute Error
IMC	Internal Model Control
IO	Integer Order
IOPID	Integer Order Proportional Integral Derivative
ISE	Integral Square Error
ITAE	Integral Time Absolute Error
LADRC	Linear Active Disturbance Rejection Control
MATLAB	Matrix Laboratory
PID	Proportional Integral Order
PSO	Particle Swarm Optimization

ABSTRACT

Fractional order calculus is a natural generalization of integer order calculus having the orders of the differential and integral operations as non-integers, which could be real as well as complex numbers. Over the past few decades, the concepts of fractional calculus have been integrated into various academic disciplines especially control systems branch of electrical engineering. In this work, an attempt has been made to explore and formulate two widely different control techniques, namely fractional order internal model control (FOIMC) and active disturbance rejection control (ADRC) approach. The proposed fractional order IMC scheme incorporates the concept of CRONE principle and fractional order filter to formulate a fractional order PID controller, that can be expressed as a series combination of integer or fractional order PID controller and fractional order low pass filter. To further enhance the control performance, a novel scheme namely active disturbance rejection control technique is extended to fractional and integer order time delayed systems. The mathematical formulae for the proposed fractional order ADRC are derived for a generalized time delayed fractional order system for the first time in this thesis. The proposed fractional order active disturbance rejection control(FOADRC) uses a limited plant information, i.e. high frequency gain and the relative order and treats everything else as a generalized disturbance. It incorporates an extended state observer (ESO) to measure the states of the plant as well as the disturbance and subsequently designs a control law that can effectively reject the disturbance and ensure an efficient set point tracking. Further, the proposed FOADRC has an added advantage that it does not require any approximation of time delayed term, which is traditionally adopted for PID based control techniques. ADRC inherits the quality of PID like error driven control law, easy to tune with extra advantages of handling capability of nonlinear disturbance, tracking error, noise degradation in derivative control. .To demonstrate the superiority and effectiveness of the proposed FOIMC and FOADRC scheme, four different numerical examples have been taken from the literature. An extensive qualitative and quantitative comparison has been undertaken in time domain. The robustness of the proposed design criteria is verified via the scrutiny of system response upon perturbation of system parameters. The simulation results indicate the strength and efficacy of the proposed control scheme.

CHAPTER 1

INTRODUCTION

Control systems are ubiquitous in everyday life. A control system is a mathematical law that enables the achievement of desired characteristics. Among many such laws, one of the most popular ones is a PID controller. PID controller has continued to be the widely used process control technique since many decades. It is indispensable for 90% of control industry owing to its simplicity, easy operability and wide area applicability. It performs well for wide class of processes and gives robust performance for a wide range of parametric uncertainty in the system.

Over the past two decades, Fractional order PID controller has received widespread attention since it has two more degrees of freedom in comparison to the conventional PID controllers. For the first time, it was Oustaloup, who proposed the idea of fractional order controller for control of dynamic systems. Later, Podlubny gave generalization of PID controller involving fractional integrator and differentiator. It is also a very good technique to find exact response of higher order integer order by converting it into lower order fractional order system.

Usually, low pass filter is chosen for the IMC controller. IMC is based on crone principle for this method. This is based on robustness that says that phase margin should be constant for a particular gain margin range it is called iso-damping property.

Ideally open loop transfer function for the filter –

$$\mathrm{f}_{\mathrm{open(s)}} = \frac{1}{\tau_c s^{\lambda}} \quad , \ \lambda \in R$$

Where

$$\tau_c^{-\frac{1}{\lambda}} = \omega_{\mathrm{gc}} = \text{gaincrossover frequency}$$

$$\left|\mathrm{f}_{\mathrm{open(s)}}(\omega_{\mathrm{gc}})\right| = 1$$

λ = slope of the magnitude curve on log–log scale and can be integer or non-integer.

If $\lambda > 0$

f(s) = fractional order differentiator

If $\lambda < 0$

f(s) = fractional order integrator

Amplitude and phase curve of f(s) are straight line of constant slope -20λ dB/sec and horizontal line at –λπ/2 rad respectively.

Closed loop system f(s) that insensitive to gain change -

$$f(s) = \frac{1}{1+\tau_c s^{\lambda}}$$

f(s) = filter model for tuning the controller C(s).

It has infinite gain margin with constant phase margin that depends only on λ. So f(s) is robust to process gain change and step response shows iso damping property. As for f(s), λ determines overshoot and τ_c determines settling time. Thus to obtain closed loop's properties ,controller C(s) is computed such that open loop C(s)G(s) is close to f(s).It is called crone principle that all the fractional order controller tuning's proposed method try to obtain in frequency domain.

1.1 BODE'S IDEAL LOOP TRANSFER AS REFERENCE SYSTEM

System's response ranges from relaxation to oscillation corresponding to first and second order systems as particular cases. This system is taken as a reference system. It is starting point of CRONE control. This function can be given as fractional order integrator with gain A and order α and response can be considered as it's closed loop transfer function. Open loop transfer function is same as above f_{open} system but for understanding the characteristics, the general form is taken for any system and it can be given as –

$$F(s) = \frac{A}{s^{\alpha}} \qquad , 0 < \alpha < 2$$

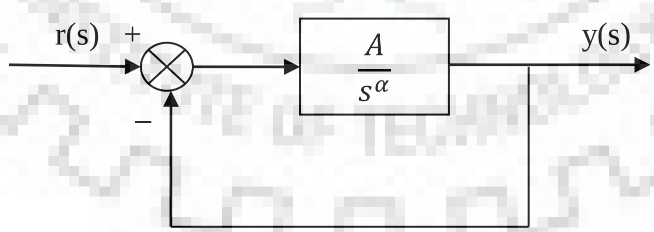


FIgure 1.1 Reference Model

The Bode plot of this transfer function is called ideal loop transfer function.

Closed loop transfer function of F(s) is –

$$G(s) = \frac{A}{A+s^{\alpha}} \qquad , \quad 0 < \alpha < 2$$

1.2 GENERAL CHARACTERSTICS OF OPEN LOOP IDEAL TRANSFER FUNCTION

- Constant slope of Magnitude curve = -20α dB/dec
- Gain cross over frequency ω_{gc} depends on gain A and gain margin is infinite.
- Phase plot is horizontal line at $-\alpha\pi/2$. Nyquist plot starts from origin with angle $-\alpha\pi/2$ and it is straight line.

Phase margin is constant and depends only on α. It has value of $\emptyset_m = \pi\left(1 - \frac{\alpha}{2}\right)$

CHAPTER 2

FRACTIONAL ORDER SYSTEM

Fractional order controller should be stsble and robust toward changes in parameters. So stability can be found out by stability criteria and robustness can be found out by changing parameters[3].

General form of Fractional order system can be given as –

$$a_n D^{\alpha_n} y(t) + a_{n-1} D^{\alpha_{n-1}} y(t) + \ \ldots . + a_0 D^{\alpha_0} y(t) = \ b_m D^{\beta_m} y(t) + b_{m-1} D^{\beta_{m-1}} y(t) + \ \ldots \ldots \ldots + b_0 D^{\beta_0} y(t)$$

Another laplace form of this equation can be given as –

$$G(s) = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} \ + \ \ldots \ldots \ldots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \ \ldots . + a_0 s^{\alpha_0}} = \frac{Q(s)}{P(s)}$$

There a_i and b_j ϵ R and α_i and β_j ϵ R^+

$$i = 0,1 \ldots . n$$

$$j = 0,1, \ldots . m$$

A LTI system can be of two types one of integer and other is non integer.Non integer system can be classified into commensurate and incommensurate.

Commensurate system – If the laplace variable's order of the transfer function is integer multiple of base order or if the common difference of powers of α and β exist that is arithmetical progression,system is called commensurated system.

$$G(s) = \frac{\sum_{k=0}^{m} q_i (s^\gamma)^k}{\sum_{k=0}^{n} p_i (s^\gamma)^k}$$

It can be reduced as – $$G(d) = \frac{\sum_{k=0}^{m} q_i (d)^k}{\sum_{k=0}^{n} p_i (d)^k}$$

It is called pseudo rational function having n pseudo order.

Incommensurate system – If the system doesn't have any common power that is can't be converted into integer order system then system is called incommensurate system.

2.1 ROBUSTNESS ANALYSIS

Parameters of system don't remain constant in real time they vary from minimum range to maximum range. It happens due to environment change, ageing and non linearity. In this case main aim of controller is that it should work in these conditions too. For this type of system uncertainites is defined as follows -

$$G(s) = \frac{Ke^{-\theta s}}{1+\tau s^{\alpha}}$$

Where uncertainities are

$$k' = k \pm \Delta k$$

$$\theta' = \theta \pm \Delta\theta$$

$$\alpha' = \alpha \pm \Delta\alpha$$

$$\tau' = \tau \pm \Delta\tau$$

2.2 FOMCON TOOLBOX FOR FOPID CONTROLLER DESIGN

FOMCON toolbox is used to fractional order system's simulation [8]. Main advantage of this method is that it provides gradual and smooth response of MATLAB commands like step, impulse, bode etc along with this it provides time and frequency domain simulation, GUI(graphical user interface) and useful simulink blocks. It also can validate real time models by FOMCON. Some command that are used in it are as follows –

fotf - it is used to represent the fractional order transfer function. Syntax of this command is as follows -

G(s) = fotf([pole],[pole's power],[zero],[zero's power]);

If input delay is considered in transfer function then

G(s) = fotf([pole],[pole power],[zero],[zero power] ,input dealy);

fracpid – it is used to generate fractional order pid controller using it's parameters Kp, Ki, Kd, λ and υ. Syntax of this is as follows –

f(s) = fracpid(Kp,Ki, λ,Kd, μ);

Step – it is used to get step response of fractional system as it is done in simple transfer function.

[y,t] = step(sys, tin: tint: tend)

Where y= output vector which gives amplitude of step response

t= time

sys =system

tin= initial time

tend = final or end time

tint = time interval of initial to end time

Bode - It is used to get frequency response in frequency domain. Its syntax is as follows -

[mag, phase] = bode(sys,wmin,wmax)

It shows magnitude and phase response in particular frequency range that is wmin to wmax in radians/time unit. If the frequency in degree then this syntax is used –

[mag, phase] = bode(sys, W) and [mag, phase, W] = bode(sys)

Isstable – to find the stability of fractional order system this command is used. In integer order System it is very easy to find stability by routh Hurwitz method or if the roots are at left hand side of s plane then system is stable but it is not such easy in fractional system. If the value of issstable command is 1 then system is stable or if it is 0 then system is unstable. Its syntax is –

a=isstable(sys)

If a =1 ; system is stable

a=0 ; system is unstable

Apart from all these commands FOMCON also provides graphical user interface namely FOTF viewer that can be used for deletion, addition fractional order transfer function objects and analysis of system can be done by it.

CHAPTER 3

SELECTED CONTROL SYSTEMS

Some systems are found to see the response on time domain, closed loop system's stability and robustness. In this thesis, first order time delay system is common in all type of systems. For analysis integer, non-integer and non-minimum integer type systems are added with it.

On these systems proposed method is applied. So different calculations have been done for different systems. IMC based method is applied on system and found out the expression for fractional order PID controller by using a gain pass filter and gain crossover frequency in which λ and μ are used in particular range .Then ,this controller is imposed on original system and found the response.

3.1 INTEGER TYPE FIRST ORDER TIME DELAY SYSTEM

It is a temperature control system.[7]

$$G(s) = \frac{9.87e^{-0.18s}}{(35.22s+1)}$$

3.2 FRACTIONAL TYPE FIRST ORDER TIME DELAY SYSTEM

This fractional order system is related to process control [4]

$$G(s)=\frac{0.99932\, e^{-0.1922s}}{1.0842s^{1.0132}+1}$$

3.3 INTEGER TYPE FIRST ORDER TIME DELAY NON MINIMUM SYSTEM

$$G(s) = \frac{(8-s)e^{-0.05s}}{(10s+1)}$$

3.4 FRACTIONAL TYPE FIRST ORDER TIME DELAY NON MINIMUM SYSTEM

$$G(s) = \frac{(1-0.02s^{0.1})e^{-0.12s}}{(1+15s^{0.2})}$$

CHAPTER 4

PROPOSED METHOD BASED ON IMC CONTROLLER

IMC was developed by Morari and coworkers. It is direct synthesis method that is used in conventional feedback. IMC provides model uncertainity and tradeoffs between performance and robustness[2]. IMC is shown in fig.1 in which G(s) is original plant and $G_m(s)$ is process model plant.

Fig.4.2 shows conventional feedback control. $Q_{IMC}(s)$ is IMC based controller.If $Q_{fb}(s)$ and $G_m^-(s)$ is equal then blocks are identical so $Q_{IMC}(s)$ is equivalent to standard feedback controller $Q_{fb}(s)$.

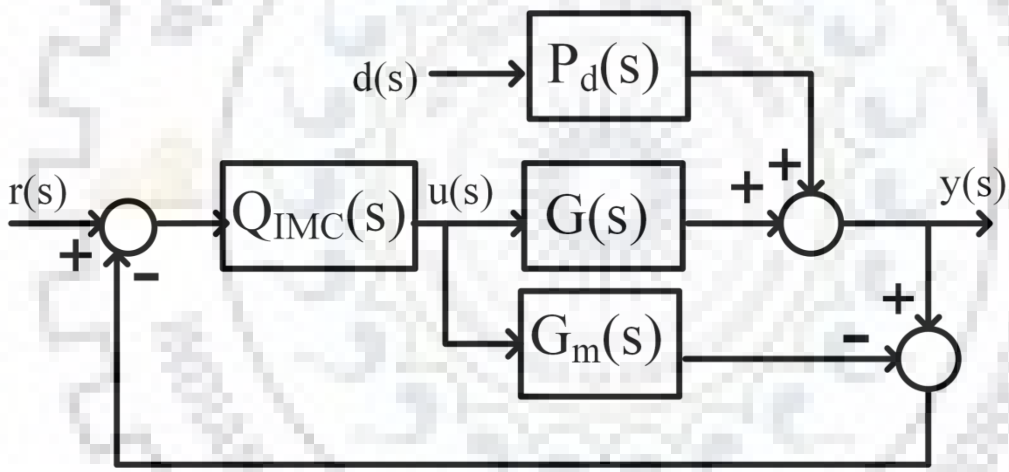


Figure 4.1 Internal model control

Output y is given when $G_m(s)$ and G(s) are equivalent.It can be factorised original tranfer function into two parts one is nonminimum $G_m^+(s)$ and other is minimum $G_m^-(s)$. In calculation of IMC based controller it is used only minimum part of transfer function that is represented by $G_m^-(s)$.

Controller expression is found out by IMC formula of controller by using gain pass filter[1]. Gain pass filter depends on two parameters η and ς. These are in a fixed range for stable closed loop system's response.

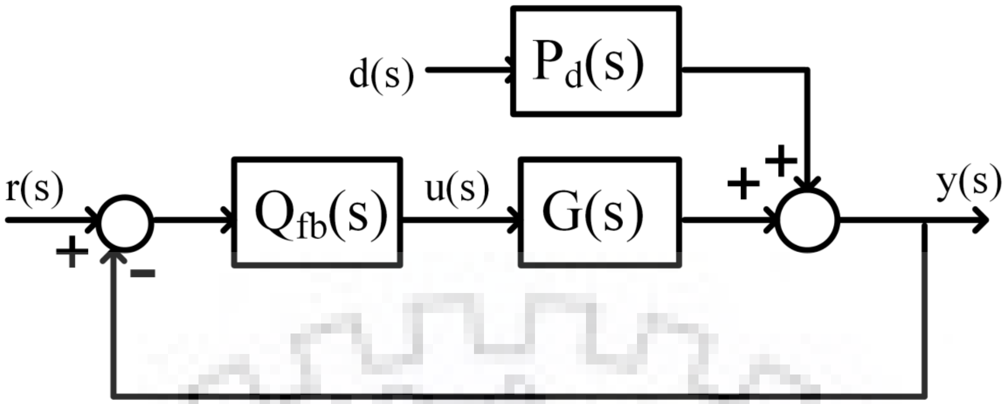


Figure 4. 2 Conventional feedback control system

Above two block diagrams will be identical if $Q_{IMC}(s)$ = $Q_{fb}(s)$

It will possible only when below equation will satisfy -

$$Q_{fb}(s) = \frac{Q_{IMC}(s)}{1-Q_{IMC}(s)G_m(s)} \tag{1}$$

Where $G_m(s)$ is process model that is approximation form of original transfer function.In this thesis approximation is done for time delay term by Taylor's series or all pole approximation[6].

If standard feedback controller is equivalent to IMC controller then output y's expression can be given as follows -

$$y(s)=\frac{Q_{IMC}(s)G(s)}{1-Q_{IMC}(s)(G(s)-G_m(s))}\,r(s) + \frac{1-Q_{IMC}(s)G_m(s)}{1+Q_{IMC}(s)(G(s)-G_m(s))}\,d(s)$$

For the special case $G_m(s)$ =G(s)

$$y(s)=Q_{IMC}(s) * G(s) * r(s) + (1 - Q_{IMC}(s) * G_m(s))d(s)$$

The process model is devided as –

$G_m(s) = G_m^+(s)G_m^-(s)$, $G_m^+(s)$contains zeros of right half plane.Steady state gain of $G_m^+(s)$ should be one.

Controller is defined as –

$$Q_{IMC}(s)=\frac{\mathrm{f(s)}}{G_m^-(s)}$$

f(s) is low pass filter with steady state gain one.

$$\mathrm{f(s)} = \frac{1}{(1+\tau_c s)^r}$$

where τ_c = desired closed-loop time constant

r = positive integer (chosen 1)

The transfer function of FOPID controller is given as –

$$C(s) = k_p\left(1 + \frac{1}{\tau_i s^{\upsilon}} + s^{\mu}\tau_d\right)$$

Where k_p = proportional gain

τ_i = Integration time constant

τ_d = differetiation time constant

υ = integration order

μ = differentiation order

Gain crossover frequency, flat phase around the gain crossover frequency and phase margin specifications are used to calculate the parameter of FOPID controller.

Phase margin specification :

$$\mathrm{Arg}(Q_{fb}(j\omega c)\, G_m(j\omega c)) = -\pi + \phi_m \tag{2a}$$

Where ϕ_m = phase margin of the original system

Flat phase specification :

Open loop system's bode response is flat around the gain crossover frequency that's why closed loop system is robust (it's overshoot of step response doesn't change) to gain change.

Gain crossover frequency specification :

$$|\, Q_{fb}(j\omega c)\, G_m\, (j\omega c)| = 1 \tag{2b}$$

Infinite gain margin specification :

static gain of the closed-loop should be equal to 1 for zero steadystate error.

Above four requirement will fullfill if closed loop transfer function is closed to –

$$f(s) = \frac{1}{1 + \varsigma s^{\eta+1}} \tag{3}$$

where ς is time contant of low pass filter and η is non integer and varies $0 < \eta < 1$.

let transfer function is –

$$G(s) = \frac{Ke^{-\Theta s}}{1+\tau s^{\alpha}}$$

Where Θ = time delay of system

τ = time constant of system

α = order of the system (can be integer or fractional)

This transfer function is approximated by taylor's series to eliminate the nonlinear term of time delay.

$$e^{-\theta s} = \frac{1}{e^{\theta s}}$$

$$e^{\theta s} = 1 + \Theta s + \frac{(\Theta s)^2}{2!} + \frac{(\Theta s)^3}{3!} + \cdots$$

$$e^{-\theta s} \approx \frac{1}{1+\Theta s} \quad \text{(approximate value of nonlinear term)}$$

So approximate process model -

$$G_m(s) = \frac{K}{(1+\tau s^{\alpha})(1+\Theta s)} = G_m^+(s) G_m^-(s) \tag{4}$$

Where $G_m^+(s) = 1$ (no right half zeros)

$$G_m^-(s) = \frac{K}{(1+\tau s^{\alpha})(1+\Theta s)} \tag{5}$$

$$G_m\,(\mathrm{j}\omega\mathrm{c}) = \frac{K}{(1+\tau (j\omega)^{\alpha})(1+j\omega\Theta)} \tag{6}$$

$$Q_{IMC}(s) = \frac{\mathrm{f(s)}}{G_m^-(s)}$$

$$Q_{IMC}(s) = \frac{(1+\tau s^{\alpha})(1+\Theta s)}{K(1+\varsigma s^{\eta+1})} \tag{7}$$

Where $\mathrm{f(s)} = \frac{1}{1+\varsigma s^{\eta+1}}$ ..... by equ(3)

$$Q_{fb}(s) = \frac{Q_{IMC}(s)}{1-Q_{IMC}(s)G_m(s)} \quad \text{.....by equ(1)}$$

After solving equ (4) and (7)

$$Q_{fb}(s) = \frac{(1+\tau s^{\alpha})(1+\Theta s)}{K\varsigma s^{\eta+1}} \tag{8}$$

$$Q_{fb}(\mathrm{j}\omega) = \frac{(1+\tau (j\omega)^{\alpha})(1+j\omega\Theta)}{K\varsigma(\mathrm{j}\omega)^{\eta+1}} \tag{9}$$

By equ (2a) -

$$\mathrm{Arg}(Q_{fb}(\mathrm{j}\omega\mathrm{c})\, G_m\,(\mathrm{j}\omega\mathrm{c})) = -\pi + \phi_m$$

By equ (6) and (9) -

$$-\frac{\pi}{2}(\eta+1) = -\pi + \phi_m$$

$$\eta = \frac{\pi - \phi_m}{\frac{\pi}{2}} - 1 \tag{10}$$

by equ (2b) –

$$|\, Q_{fb}(j\omega c)\, G_m\, (j\omega c)| = 1$$

$$\left|\frac{1}{\varsigma(j\omega_{gc})^{\eta+1}}\right| = 1$$

$$\varsigma = \frac{1}{(\omega_{gc})^{\eta+1}} \qquad (11)$$

Where ω_{gc} = gain crossover frequency of the original system

Controller expression is cascaded form of FOPID with fractional filter to get the closed loop specification.Orders of differentiation and integration are related to open loop system.It can be written as folllows –

$$Q_{fb}(s) = H(s) * \{ k_p \left(1 + \frac{1}{\tau_i s^{\upsilon}} + s^{\mu}\, \tau_d\right)\} = H(s) * F(s)$$

Where H(s) = fractional filter

F(s) = fractional order PID filter

4.1 PROPOSED METHOD CALCULATION FOR INTEGER TYPE FIRST ORDER TIME DELAY SYSTEM

antiwindup for temprature profile control system-

$$G(s) = \frac{9.87e^{-0.18s}}{(35.22s+1)}$$

Where K = 9.87

$\Theta = 0.18$

$\tau = 35.22$

$\alpha = 1$

approximation of non linear time delay term -

$$e^{-0.18s} = \frac{1}{e^{0.18s}} \approx \frac{1}{1+0.18s}$$

$$G(s) = \frac{9.87}{(35.22s+1)(1+0.18s)}$$

$$f(s) = \frac{1}{1+\varsigma s^{\eta+1}}$$

where $\eta = \frac{\pi-\phi m}{\frac{\pi}{2}} - 1$

$$\varsigma = \frac{1}{(\omega_{gc})^{\eta+1}}$$

Let , ϕ_m = phase margin of system =70°

So , η = 0.22

Let ,ω_{gc} = gaincrossover frequency = 1

So , ς = 1

By equ (8)

$$Q_{fb}(s) = \frac{(1+\tau s^{\alpha})(1+\Theta s)}{K\varsigma s^{\eta+1}}$$

$$Q_{fb}(s) = \frac{1}{K\varsigma s^{\eta}}\left(\frac{1+\tau s^{\alpha}+\Theta s+\Theta\tau s^{\alpha+1}}{s}\right)$$

$$Q_{fb}(s) = \frac{1}{K\varsigma s^{\eta}}\left(\Theta + \frac{1}{s} + +\tau s^{\alpha-1} + \Theta\tau s^{\alpha}\right) \tag{12}$$

Put the values of η and ς

$$Q_{fb}(s) = \frac{0.101}{s^{0.22}}\left(35.4 + \frac{1}{s} + 6.3396s\right)$$

Above expression is for fractional order PID controller for given system.

Where $H(s) = \frac{0.101}{s^{0.22}}$ = fractional filter

$F(s) = 35.4 + \frac{1}{s} + 6.3396s$ = PID controller for integer type system

4.2 PROPOSED METHOD CALCULATION FOR FRACTIONAL TYPE FIRST ORDER TIME DELAY SYSTEM

Now take tractional type example to see the result of proposed method

$$G(s)=\frac{0.99932\, e^{-0.1922s}}{1.0842s^{1.0132}+1}$$

Where K = 0.99932

Θ = 0.1922

τ =1.0842

α = 1.0132

approximation of non linear time delay term -

$$e^{-0.1922s} = \frac{1}{e^{0.1922s}} \approx \frac{1}{1+0.1922s}$$

$$G(s) = \frac{0.99932}{(1.0842s^{1.0132}+1)(1+0.1922s)}$$

$$f(s) = \frac{1}{1+\varsigma s^{\eta+1}}$$

$$\text{where } \eta = \frac{\pi-\phi m}{\frac{\pi}{2}} - 1$$

$$\varsigma = \frac{1}{(\omega_{gc})^{\eta+1}}$$

Let , ϕ_m = phase margin of system =70°

So , η= 0.22

Let , ω_{gc} = gaincrossover frequency = 1

So , $\varsigma = 1$

By equ (8)

$$Q_{fb}(s) = \frac{(1+\tau s^{\alpha})(1+\Theta s)}{K\varsigma s^{\eta+1}}$$

$$Q_{fb}(s) = \frac{1}{K\varsigma s^{\eta}}\left(\frac{1+\tau s^{\alpha}+\Theta s+\Theta\tau s^{\alpha+1}}{s}\right)$$

$$Q_{fb}(s) = \frac{1}{K\varsigma s^{\eta}}\left(\Theta + \frac{1}{s} + +\tau s^{\alpha-1} + \Theta\tau s^{\alpha}\right) \tag{12}$$

Put the values of η and ς

$$Q_{fb}(s) = \frac{1.0006}{s^{0.22}}\left(0.1922 + \frac{1}{s} + 0.20838\, s^{1.0132} + 1.0842\, s^{0.0132}\right)$$

Above expression is for fractional order PID controller for given system.

Where $H(s) = \frac{1.0006}{s^{0.22}}$ = fractional filter

$F(s) = \left(0.1922 + \frac{1}{s} + 0.20838\, s^{1.0132} + 1.0842\, s^{0.0132}\right)$

= PID^{μ} controller for given fractional type system

4.3 PROPOSED METHOD CALCULATION FOR INTEGER TYPE FIRST ORDER TIME DELAY NON MINIMUM SYSTEM

let transfer function is –

$$G(s) = \frac{K\left(1-\tau' s^{\beta}\right)e^{-\Theta s}}{1+\tau s^{\alpha}} \tag{13}$$

Where Θ = time delay of system

τ and τ' = time constants of system

α and β = orders of the system (can be integer or fractional)

this transfer function is approximated by taylor's series to eliminate the nonlinear term of time delay.

$$e^{-\Theta s} = \frac{1}{e^{\Theta s}}$$

$$e^{\Theta s} = 1 + \Theta s + \frac{(\Theta s)^2}{2!} + \frac{(\Theta s)^3}{3!} + \cdots$$

$$e^{-\Theta s} \approx \frac{1}{1+\Theta s}$$ (approximate value of nonlinear term)

So approximate process model -

$$G_m(s) = \frac{K\left(1-\tau' s^{\beta}\right)}{(1+\tau s^{\alpha})(1+\Theta s)} = G_m^{+}(s)G_m^{-}(s)$$

Where $G_m^{+}(s) = \left(1-\tau' s^{\beta}\right)$ (right half zeros)

$$G_m^{-}(s) = \frac{K}{(1+\tau s^{\alpha})(1+\Theta s)}$$

$$Q_{IMC}(s) = \frac{f(s)}{G_m^{-}(s)}$$

$$Q_{IMC}(s) = \frac{(1+\tau s^{\alpha})(1+\Theta s)}{K(1+\varsigma s^{\eta+1})}$$

Where $f(s) = \frac{1}{1+\varsigma s^{\eta+1}}$

$$Q_{fb}(s) = \frac{Q_{IMC}(s)}{1-Q_{IMC}(s)G_m(s)}$$

After solving equ (4) and (7)

$$Q_{fb}(s) = \frac{(1+\tau s^{\alpha})(1+\Theta s)}{K\varsigma s^{\eta+1}+K\tau' s^{\beta}}$$

$$Q_{fb}(s) = \frac{1}{K\varsigma s^{\eta}+K\tau' s^{\beta-1}}\left(\Theta + \frac{1}{s} + +\tau s^{\alpha-1} + \Theta\tau s^{\alpha}\right) \tag{14}$$

This is the expression of fractional order PID controller for integer type time delay first order nonminimum system.It is also cascade of fractional filter and fractional/integer type PID controller (fractional for the fractional type of system and integer for integer type of system).

$$G(s) = \frac{(8-s)e^{-0.05s}}{(10s+1)}$$

To make in standard form as equ (13)

$$G(s) = \frac{8(1-0.125s)e^{-0.05s}}{(10s+1)}$$

Where K = 8

Θ = 0.05

τ =10

τ'= 0.125

α and β = 1

approximation of non linear time delay term -

$$e^{-0.05s} = \frac{1}{e^{0.05s}} \approx \frac{1}{1+0.05s}$$

$$G(s) = \frac{8(1-0.125s)}{(10s+1)(1+0.05s)}$$

Where $G_m^+(s) = (1-0.125s)$ (right half zeros)

$$G_m^-(s) = \frac{8}{(10s+1)(1+0.05s)}$$

$$f(s) = \frac{1}{1+\varsigma s^{\eta+1}}$$

where $\eta = \frac{\pi-\phi_m}{\frac{\pi}{2}} - 1$

$$\varsigma = \frac{1}{(\omega_{gc})^{\eta+1}}$$

Let , ϕ_m = phase margin of system = 60.3°

So , η = 0.33

Let , ω_{gc} = gaincrossover frequency = 1

So , $\varsigma = 1$

From equ (14)

$$Q_{fb}(s) = \frac{1}{K\varsigma s^{\eta} + K\tau' s^{\beta-1}}\left(\Theta + \frac{1}{s} + +\tau s^{\alpha-1} + \Theta\tau s^{\alpha}\right)$$

Put the values of η and ς

$$Q_{fb}(s) = \frac{1}{8s^{0.33}+0.1}\left(10.05 + \frac{1}{s} + 0.5\ s\right)$$

Above expression is for fractional order PID controller for given system.

Where H(s) = $\frac{1}{8s^{0.33}+0.1}$ = fractional filter

F(s) = $\left(10.05 + \frac{1}{s} + 0.5\ s\right)$

= PID controller for given integer type system

4.4 PROPOSED METHOD CALCULATION FOR FRACTIONAL TYPE FIRST ORDER TIME DELAY NON MINIMUM SYSTEM

Consider a fractional order non minimum system as-

$$G(s) = \frac{(1-0.02s^{0.1})e^{-0.12s}}{(1+15s^{0.2})}$$

Where K = 1

$\Theta = 0.12$

$\tau = 15$

$\tau' = 0.02$

$\alpha = 0.2$

$\beta = 0.1$

approximation of non linear time delay term -

$$e^{-0.12s} = \frac{1}{e^{0.12s}} \approx \frac{1}{1+0.12s}$$

$$G(s) = \frac{(1-0.02s^{0.1})}{(1+15s^{0.2})(1+0.12s)}$$

Where $G_m^{+}(s) = (1-0.02s^{0.1})$ (right half zeros)

$$G_m^{-}(s) = \frac{1}{(1+15s^{0.2})(1+0.12s)}$$

$$f(s) = \frac{1}{1+\varsigma s^{\eta+1}}$$

where $\eta = \frac{\pi-\phi_m}{\frac{\pi}{2}} - 1$

$$\varsigma = \frac{1}{(\omega_{gc})^{\eta+1}}$$

Let , ϕ_m = phase margin of system = 80.1°

So , η = 0.11

Let , ω_{gc} = gaincrossover frequency = 1

So , ς = 1

From equ (14)

$$Q_{fb}(s) = \frac{1}{K\varsigma s^{\eta} + K\tau' s^{\beta-1}}\left(\Theta + \frac{1}{s} + +\tau s^{\alpha-1} + \Theta\tau s^{\alpha}\right)$$

Put the values of η and ς

$$Q_{fb}(s) = \frac{1}{s^{1.23}+0.02}\left(\frac{1}{s^{0.1}} + 0.12\, s^{0.9} + 15\, s^{0.1} + 1.8\, s^{1.1}\right)$$

Above expression is for fractional order PID controller for given system.

Where H(s) = $\frac{1}{s^{1.23}+0.02}$ = fractional filter

F(s) = $\left(\frac{1}{s^{0.1}} + 0.12\, s^{0.9} + 15\, s^{0.1} + 1.8\, s^{1.1}\right)$

= Fractional order PID controller for given integer type system

CHAPTER 5

SIMULINK MODELS AND RESULTS

Simulation models of every example are presented here and responses in time domain, robustness and control energy are represented in figures and tables with the respective errors.

5.1 TIME DOMAIN ANALYSIS

For example 1 simulation model is given is as follows with comparision by existing methods. Errors are found out by these formula -

$$\mathrm{ISE} = \int_0^{\infty} (\mathrm{y} - \mathrm{r})^2 \mathrm{dt}$$

$$\mathrm{IAE} = \int_0^{\infty} |(\mathrm{y} - \mathrm{r})|\, \mathrm{dt}$$

$$\mathrm{ITAE} = \int_0^{\infty} \mathrm{t}|(\mathrm{y} - \mathrm{r})|\, \mathrm{dt}$$

Where y is represented for output and r is represented for reference input. So it can be said difference of output and reference input is error 'e' i.e.

$$e = y - r$$

For example 1-

Errors / Methods	IAE	ISE	ITAE
Proposed method	1.457	0.5929	6.838
Monje approach[13]	9.876	2.866	208.9
IMC scheme[1]	1.655	0.7068	8.719
Vu's FOPI[5]	2.138	0.7127	28.24

Table 5.1.1 Errors for proposed FOIMC and existing methods in example 1

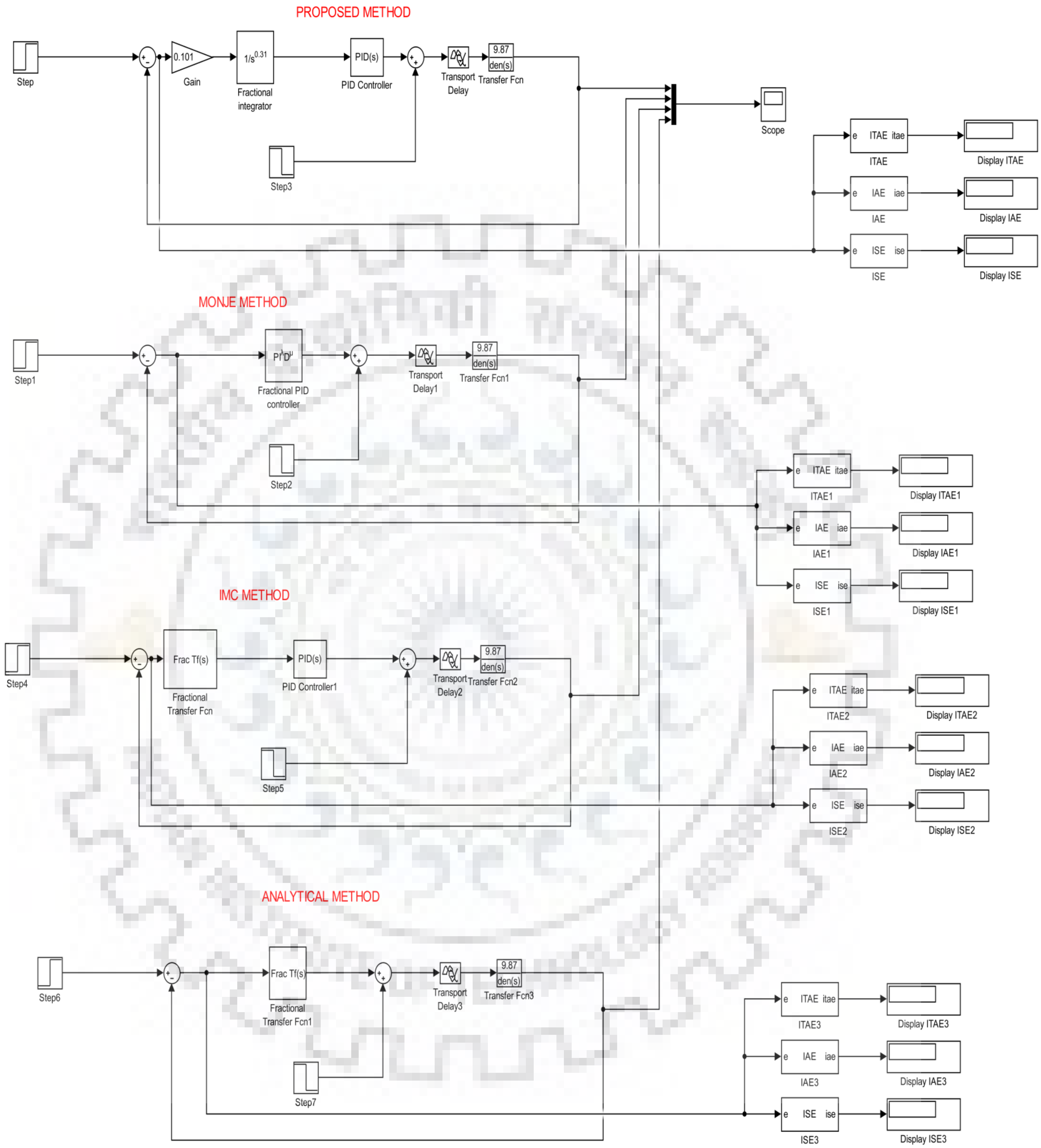


Figure 5.1.1 Proposed FOIMC method with existing methods for comparison

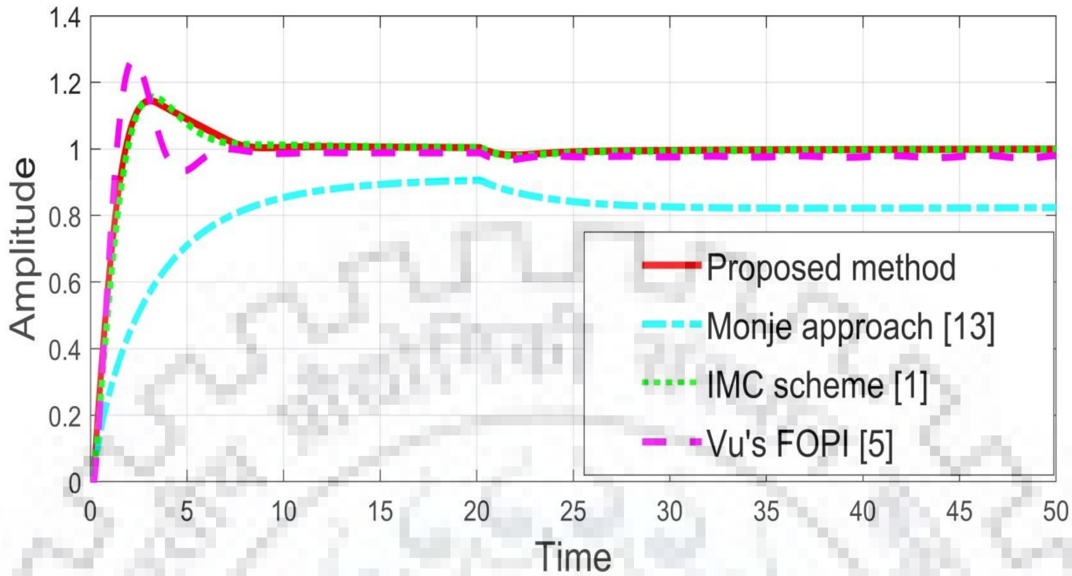


Figure 5.1.2 Example 1 with proposed FOIMC and comparing methods

For example 2 -

Errors / Methods	IAE	ISE	ITAE
Proposed method	0.8846	0.3193	7.818
Monje approach[13]	6.981	1.483	147.4
Vu's FOPI[5]	1.96	0.8044	19.96
Das method[4]	4.971	0.9166	115.2

Table 5.1.2 Errors for proposed FOIMC and existing methods in example 2

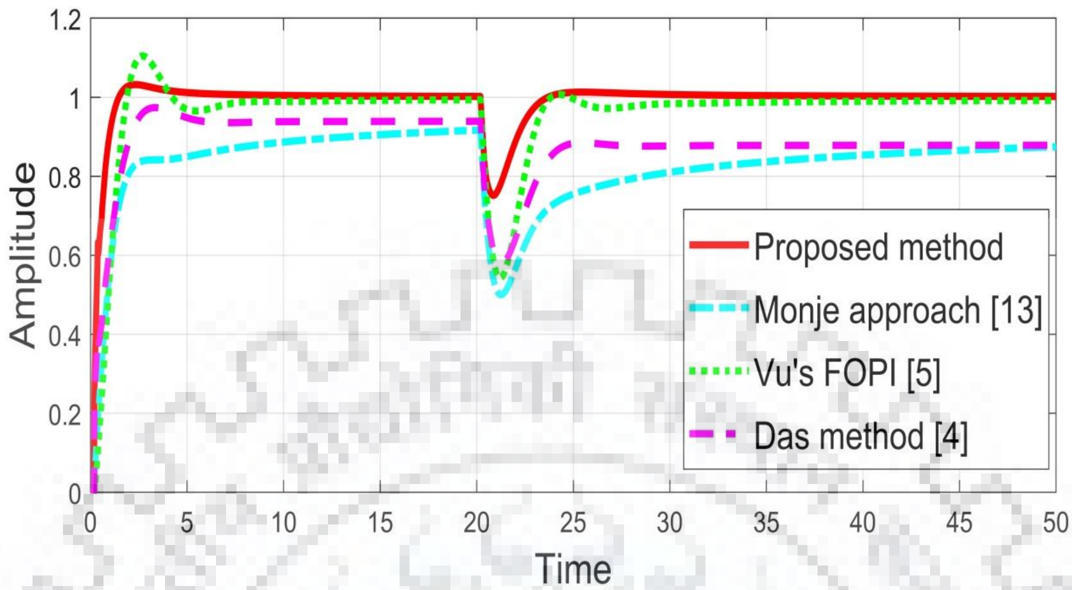


Figure 5.1.3 Example 2 with proposed FOIMC and comparing methods

For example 3 -

Errors / Methods	IAE	ISE	ITAE
Proposed method	1.522	0.6609	9.283
Monje approach[13]	1.958	0.6667	27.33
PSO scheme[12]	1.712	0.706	24.73
Cuckoosearch method[11]	1.894	0.8731	24.43

Table 5.1.3 Errors for proposed FOIMC and existing methods in example 3

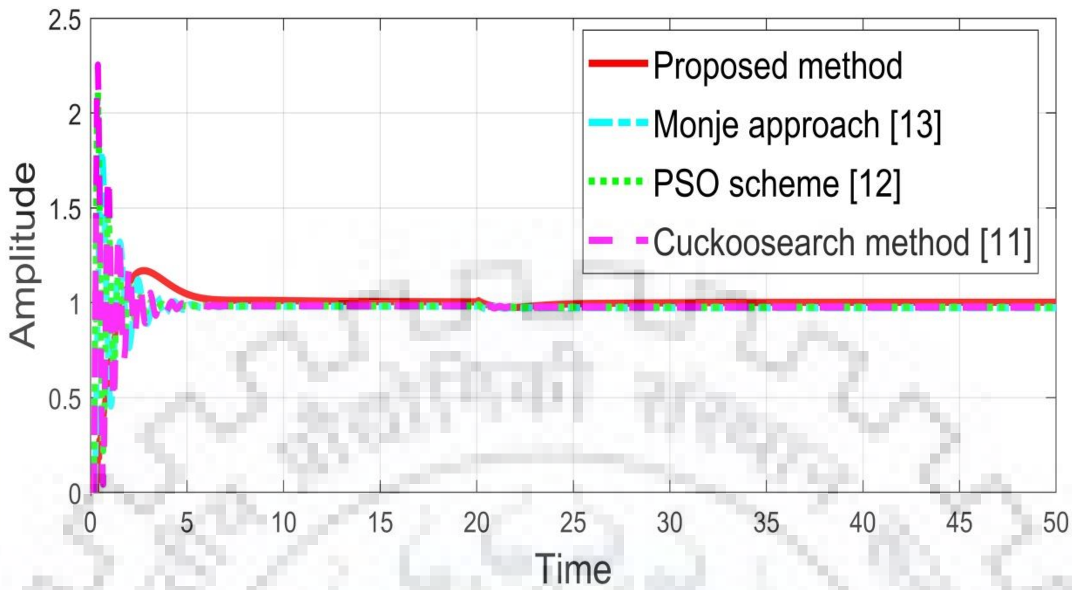


Figure 5.1.4 Example 3 with proposed FOIMC and comparing methods

For example 4 -

Methods \ Errors	IAE	ISE	ITAE
Proposed method	1.49	0.54	17.26
Monje approach[13]	13.63	3.86	323.1
PSO scheme[12]	8.65	1.79	190.7
cuckoosearch method[11]	6.00	1.09	126.2

Table 5.1.4 Errors for proposed FOIMC and existing methods in example 4

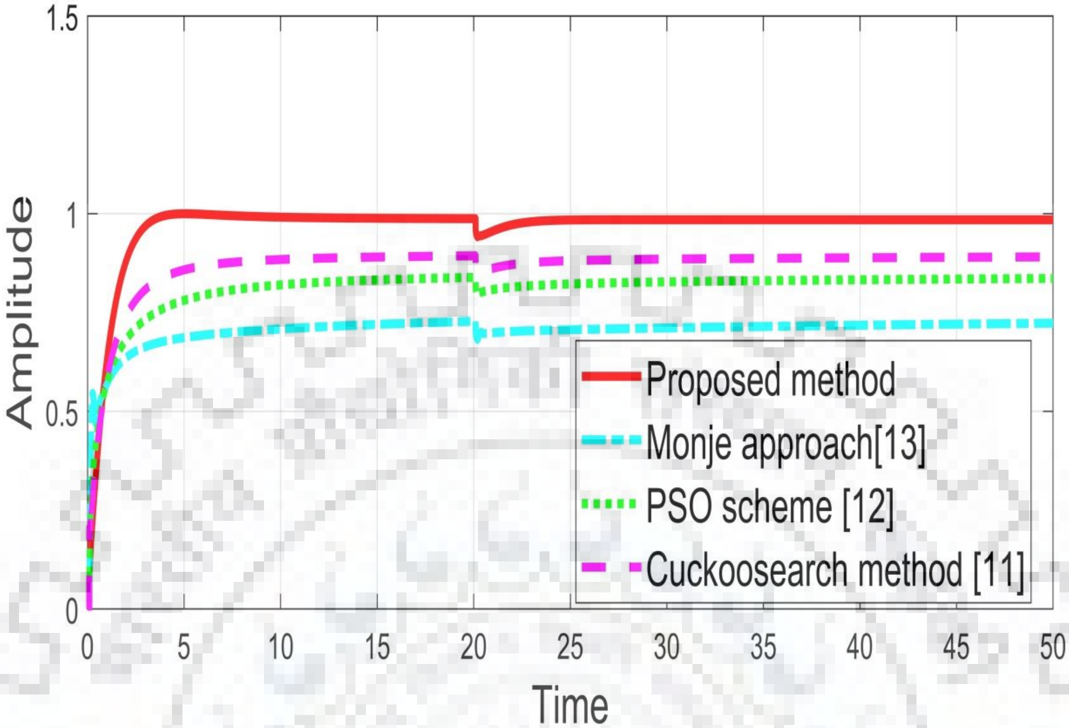


Figure 5.1.5 Example 4 with proposed FOIMC and comparing methods

From the above results, it can be concluded that proposed method gives flexible, good simulation and fast disturbance rejection.

5.2 ROBUSTNESS ANALYSIS

Robustness analysis is done for example 1 in which parameters will be changed $\pm 20\%$. In this situation system with proposed method gives better response as compared to other existing methods that is system is robust in uncertainity too.general form of transfer function is –

$$G(s) = \frac{Ke^{-\theta s}}{1+\tau s^{\alpha}}$$

Where uncertainities are $k' = k \pm \Delta k$

$$\theta' = \theta \pm \Delta\theta$$

$$\alpha' = \alpha \pm \Delta\alpha$$

$$\tau' = \tau \pm \Delta\tau$$

For example 1 -

1) $G(s) = \dfrac{9.87e^{-0.18s}}{(35.22s+1)}$

There $k' \in [7.896, 11.844]$

$\theta' \in [0.144, 0.216]$

$\tau' \in [28.176, 42.264]$

constant term 1 also can vary from 0.8 to 1.2.

Methods \ Errors	ISE		IAE		ITAE	
	-20%	+20%	-20%	+20%	-20%	+20%
Proposed method	0.5645	0.6235	1.428	1.488	6.791	6.856
Monje approach[13]	2.851	2.883	9.881	9.882	209.2	208.9
IMC scheme[1]	0.6758	0.7397	1.623	1.691	8.669	8.752
Vu's FOPI[5]	0.665	0.7653	2.064	2.231	28.04	28.56

Table 5.2.1 Errors for robustness by proposed FOIMC and existing methods in example 1

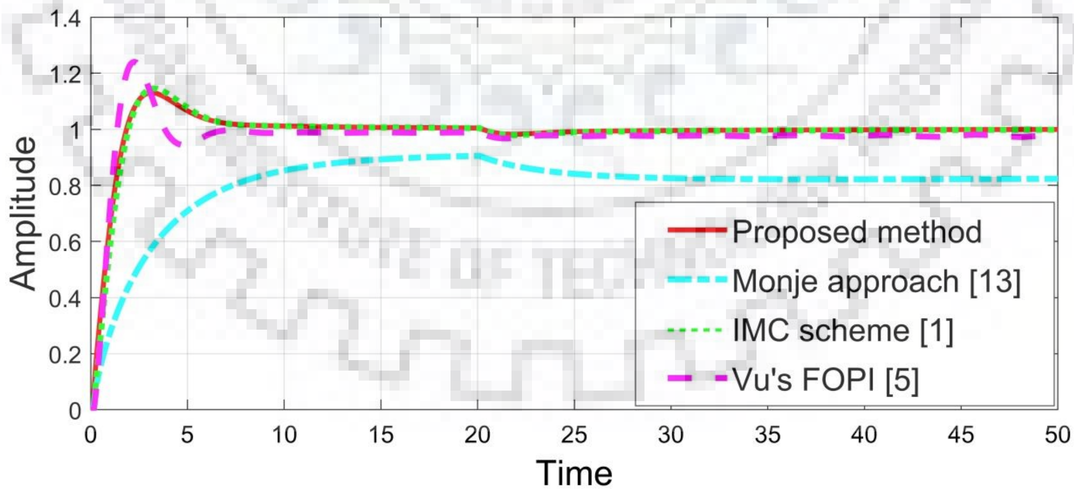


Figure 5.2.1 Robustness analysis for -20% for example 1

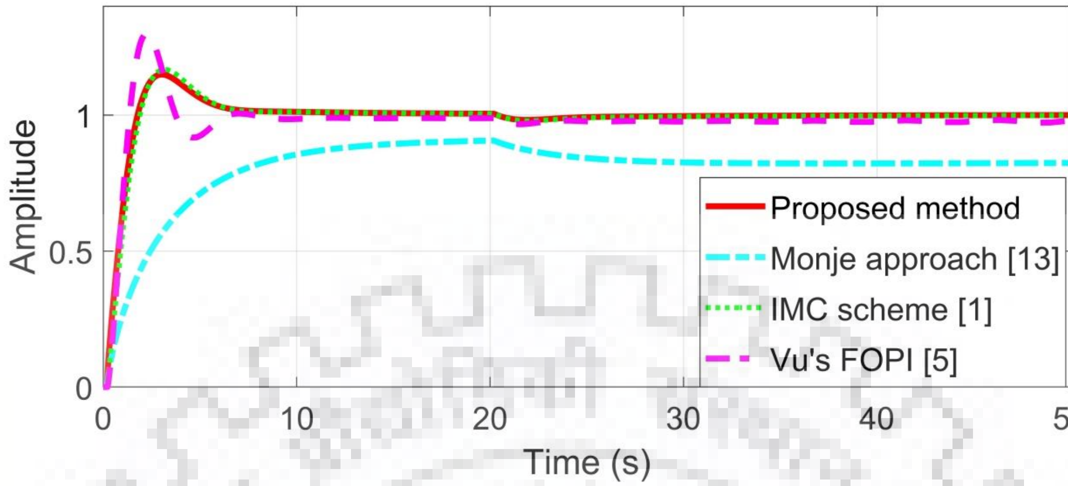


Figure 5.2.2 Robustness analysis for +20% for example 1

For example 2 -

1) $G(s)=\frac{0.99932\, e^{-0.1922s}}{1.0842s^{1.0132}+1}$

There k’ ∈ [0.799456, 1.199184]

θ' ∈ [0.15376, 0.23064]

α ∈ [0.881056, 1.321584]

τ' ∈ [0.86736, 1.30104]

constant term 1 also can vary from 0.8 to 1.2.

Errors / Methods	ISE		IAE		ITAE	
	-20%	+20%	-20%	+20%	-20%	+20%
Proposed method	0.3784	0.4093	0.9109	0.9697	7.927	7.871
Monje approach[13]	1.479	1.542	7.086	6.945	147.7	147.9
Vu’s FOPI[5]	0.7161	1.009	1.778	2.582	18.79	25.17
Das method[4]	0.9009	0.9718	5.06	4.958	116	115.3

Table 5.2. 2 Errors for robustness by proposed FOIMC and existing methods in example 2

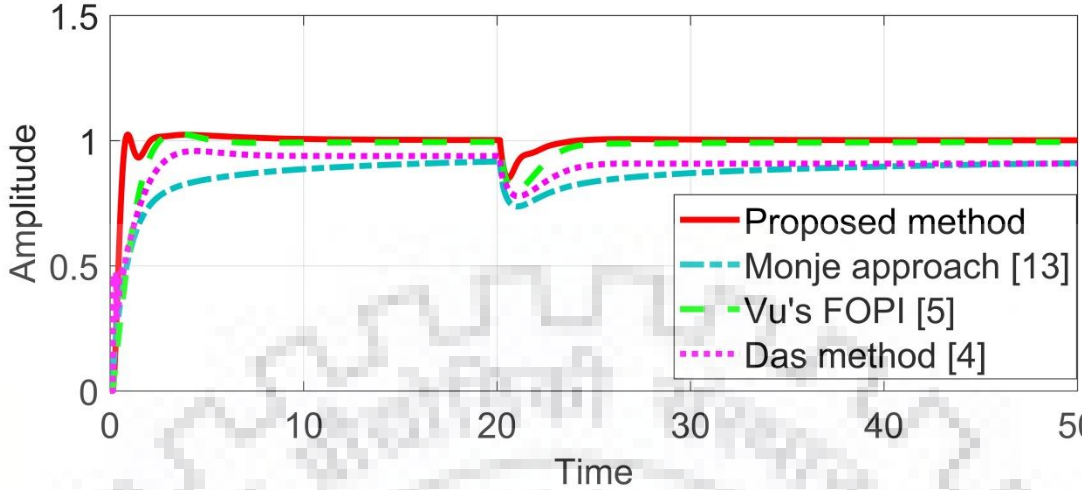


Figure 5.2.3 Robustness analysis for -20% for example 2

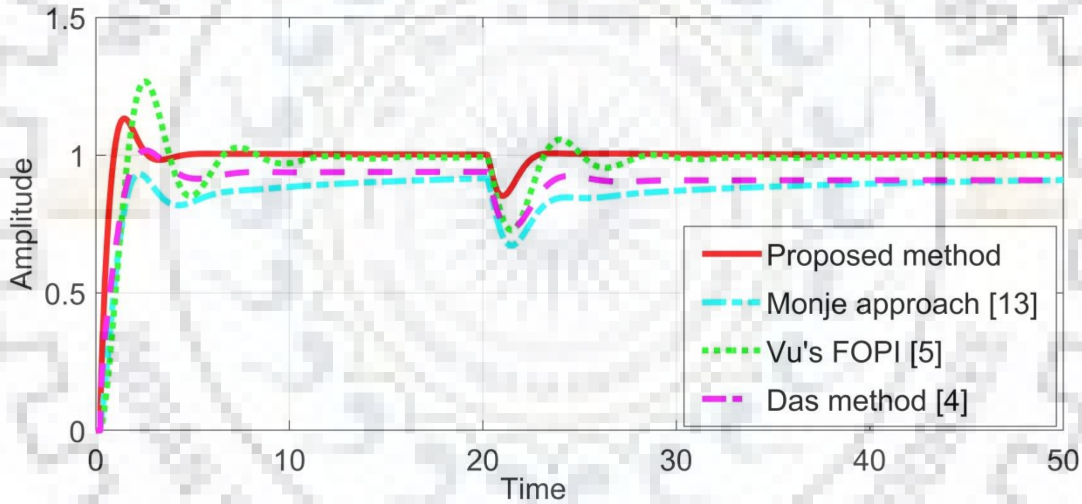


Figure 5.2. 4 Robustness analysis for +20% for example 2

For example 3 -

2) $G(s) = \frac{(8-s)e^{-0.05s}}{(10s+1)}$ can be written in this form

$$G(s) = \frac{(T1-k1s)e^{-\theta s}}{(T2s+1)}$$

There T1' ∈ [6.4, 9.6]

k1' ∈ [0.8, 1.2]

$\theta' \in [0.04, 0.06]$

T2' ∈ [8, 12]

constant term 1 also can vary from 0.8 to 1.2

Errors / Methods	ISE		IAE		ITAE	
	-20%	+20%	-20%	+20%	-20%	+20%
Proposed method	0.5502	0.6696	1.461	1.482	6.296	6.313
Monje approach[13]	0.5516	0.7753	1.939	2.345	32.62	33.38
PSO scheme[12]	0.5841	0.9207	1.679	2.212	29.24	29.96
cuckoosearch method[11]	0.6803	1.242	1.762	2.59	28.4	29.85

Table 5.2.3 Errors for robustness by proposed FOIMC and existing methods in example 3

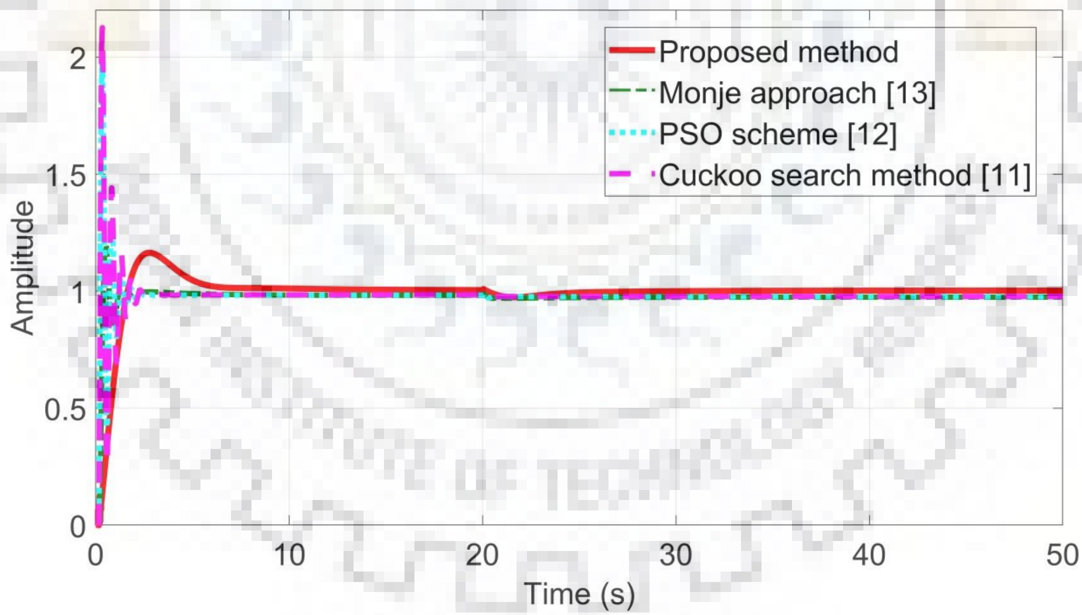


Figure 5.2.5 Robustness analysis for -20% for example 3

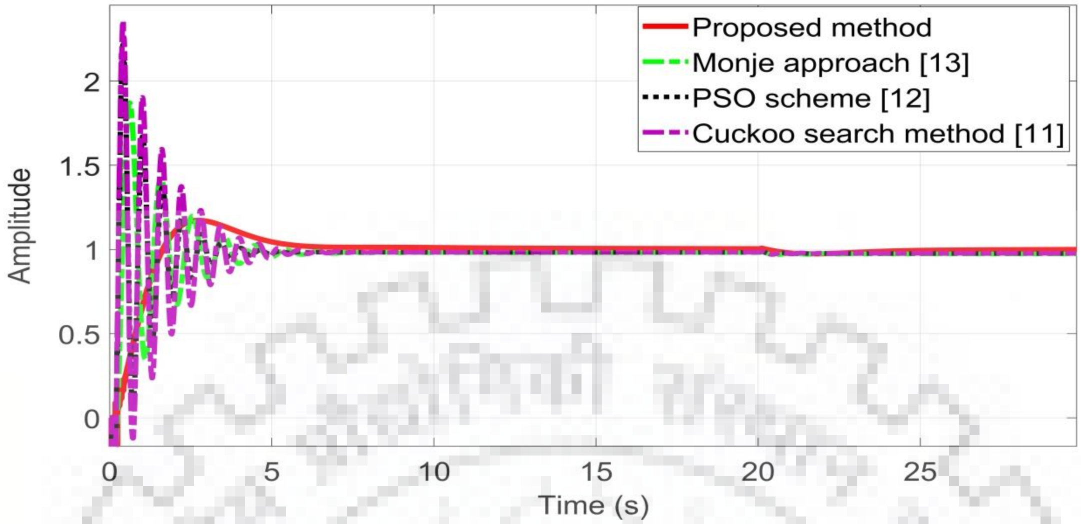


Figure 5.2.6 Robustness analysis for +20% for example 3

For example 4 -

$$G(s) = \frac{(1-0.02s^{0.1})e^{-0.12s}}{(1+15s^{0.2})}$$

There T1’ ∈ [0.8, 1.2]

k’ ∈ [0.016, 0.024]

θ' ∈ [0.096, 0.144]

T2’ ∈ [12, 18]

α ∈ [0.16,0.24]

β ∈ [0.08,0.12]

constant term 1 also can vary from 0.8 to 1.2.

Errors / Methods	ISE		IAE		ITAE	
	-20%	+20%	-20%	+20%	-20%	+20%
Proposed method	0.5264	0.5592	1.622	1.448	20.37	14.99
Monje approach[13]	4.527	3.312	14.85	12.5	356.4	292.5
PSO scheme[12]	2.093	1.563	9.552	7.831	214.3	169.6
Cuckoosearch method[11]	1.232	0.9838	6.685	5.4	143.3	111.2

Table 5.2.4 Errors for robustness by proposed FOIMC and existing methods in example 4

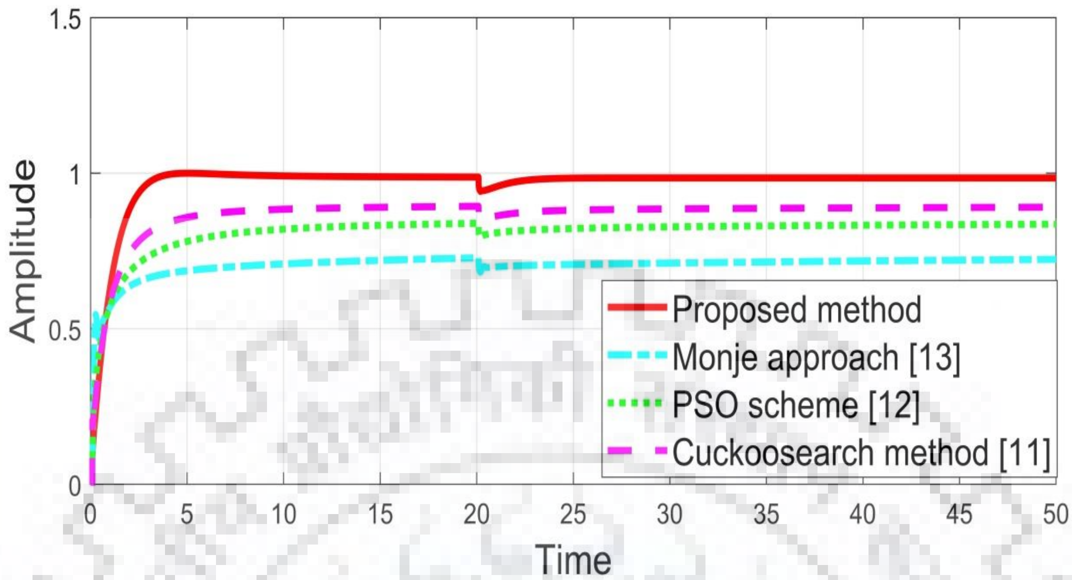


Figure 5.2.7 Robustness analysis for -20% for example 4

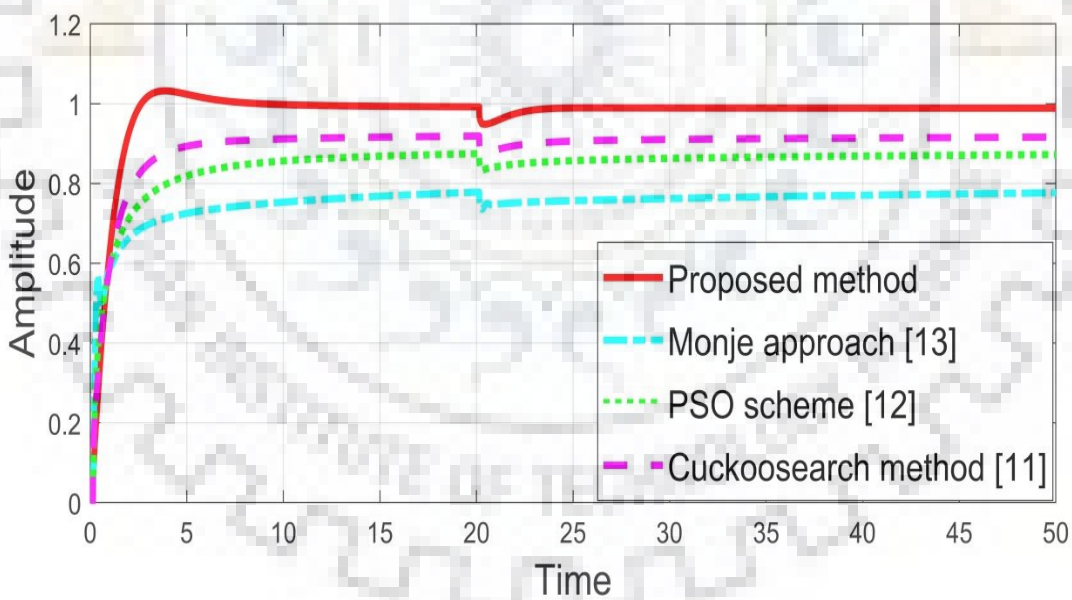


Figure 5.2.8 Robustness analysis for +20% for example 4

Above results shows the best results for all the systems after perturbation in the parameters of the system.

5.3 CONTROL ENERGY ANALYSIS

For any system analysis given energy to operate system should be minimum that is operating energy should be minimum for controller but for fraction it is also future research point to decrease it’s operating energy but in this thesis energy analysis of controller is done.

For numerical examples –

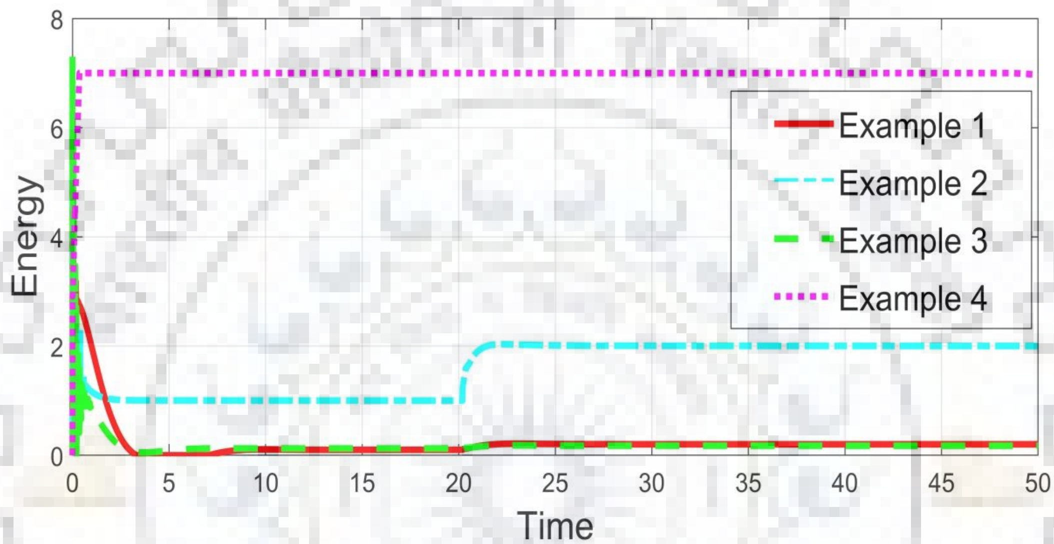


Figure 5.2.9 control energy analysis for proposed FOIMC

Above plot shows the energy value of the controllers. Energy increases with good response ,for the best results there is need to provide more energy to controller. Sometimes more energy is given when system is complex like non minimum system or fractional sytem. Fractional PID controller consumes more energy than integer PID controller because fractional PID controller gives better result so it is main point to decrease control energy in fractional controller.

CHAPTER 6

DESIGN OF ADRC (ACTIVE DISTURBANCE REJECTION RATIO) CONTROLLER FOR FRACTIONAL ORDER TIME DELAY SYSTEMS

6.1 INTRODUCTION AND LITERATURE REVIEW

From last decades Proportional integral derivative (PID) controller is most popular and widely used controller in the industries and process control system because of its simple tuning and response but it can not handle the nonlinear disturbance, external noise/disturbance, tracking error and set point error [20].

Active Disturbance Ration Control (ADRC) is a control technique that is first introduced by Prof. Han [19]-[20]. Main idea of this technique is to consider internal uncertainities and external disturbance as a generalised disturbance F, in real time ESO(extended state observer) is used to estimate the disturbance and feedback controlller is used to compensate. ADRC inherits from proportional–integral–derivative (PID). It embraces the power of nonlinear feedback and puts it to full use. It is a robust control method that is based on extension of the system model with an additional and fictitious state variable, representing everything that the user does not include in the mathematical description of the plant. This virtual state (sum of internal and external disturbances, usually denoted as a "total disturbance") is estimated online with a extended state observer (ESO) and used in the control signal in order to decouple the system from the actual perturbation acting on the plant. This disturbance rejection feature allows user to treat the considered system with a simpler model, since the negative effects of modeling uncertainty are compensated in real time. As a result, the operator does not need a precise analytical description of the system, as one can assume the unknown parts of dynamics as the internal disturbance in the plant. Robustness and the adaptive ability of this method makes it an interesting solution in scenarios where the full knowledge of the system is not available. Structure of conventional ADRC is complex and has multiple of tuning parameters that is difficult to deal in practice but the proposed approach has only two tuning parameters (controller bandwidth and observer bandwidth). ADRC can be simplified to linear ADRC via linear ESO

[21] that makes it simple and practical[22]-[23].ESO is the disturbance estimator [24]-[30] and state space approach that requires minimum plant informations. ESO was first introduced by Prof Han in the context of ADRC [25]-[28].ADRC via ESO is used to estimate not only external disturbance but also plant dynamics in addition ADRC widely used for various applications [31]-[42].

Linear version of ADRC (LADRC) [18] in which ESO and state feedback both are linear and number of parameters are reduced to two and these are related to the performance of closed loop system[15]. For non minimum and unstable system generalised adrc (GADRC) [12] is used. It uses all the information of the plants in control to achieve better performance. Idea of GADRC is to appropriate under amathching condition assumption.

In previous work, fractional order PID controller is used to deal with time delay fractional and integer type systems that have minimum and non minimum phase. When FOPID controller is applied on higher order delay system then it is unable to deal with it, so to remove this high nonlinear disturbance from the system FOADRC (fractional order active disturbance rejection control) technique is applied. It observes the disturbance of the system by extended state observer and then eliminated signal is sent to the system[14].

In the thesis ADRC is modified for fractional order time delayed system so it is known as modified fractional order ADRC (FOADRC) that applicable in wide process industries where time delay is high[14], system is unstable and integrating. Modified FOADRC performs better to track setpoint and to estimate disturbance.Simulation analysis is also done for fractional order examples and it exhibits least disturbance effect, fast rejection control minimum set point error and minimum tracking error. It can be said by robust analysis that system is robust after perturbation in the system parameters.

6.2 MOTIVATION

In previous work, fractional order PID controller is designed for integer and fractional type of systems that have minimum and non minimum phase. It achieves good responses for these examples. Now, to improve the results of FOIMC controller, by combining the idea of fractional with a new technique ADRC that deals with unstable and higher order nonlinear delay systems without approximating the delay term. In process control industries and real time systems delay is the existing term that affects the system's efficiency. When delay is minimum it can be controlled by simple PID controller or FOPID control but higher order and nonlinearity and external disturbance/noise make the system's response unstable then it can not be used these controllers. Delay term also minimizes the phase margin of the system. As delay increases, phase margin decreases and system's performance becomes poor.

Literature works are done for integer order time delay and fractional order without time delay systems via linear ADRC[14],[16]-[17]. In this thesis ADRC controller is designed for fractional order time delayed system that is a new proposed technique that is not done in literature. The controller that designed for fractional order time delayed system is modified ADRC controller in which there is no need to approximate time delay term so it is called fractional order modified ADRC (or FOADRC) that doesn't need more informtion of plant and gives best performance with farctional time delay, higher order nonlinearity and unstable systems.

FOADRC has some advantages over FOIMC-

1) FOADRC can deal with higher order time delay term without approximating it.
2) System's order remains same because approximating delay term doesn't appear in denominator.
3) FOADRC can be used for system with wide range of dynamics like unstable, time delayed , etc..
4) Only a limited information about the plant is sufficient to design a FOADRC controller. In contrast, FOIMC requirws thje complete model of plant.
5) Total disturbance (= internal uncertainty + external disturbance) is actively estimated. In contrast, in FOIMC, the controller acts on the effct of disturbance after the disturbance has occurred.
6) Control scheme is easy to tune and simple to implement since it entails the usage of integer order tools.
7) In FOIMC, fractional order system is to be converted in higher order integer system but in FOADRC there is no such need.
8) FOADRC is integer order controller so easy to implement and can deal with fractional order time delay systems.

FOADRC has above these advantages that motivate to design it with earlier existing fractional order time delay systems. In literature ADRC is not designed for fractional order time delay systems that is done in this thesis and position of poles of controller and observer are taken at the different location that was not done in the literature survey then found out the results in these conditions that makes the result flexible and simulation is also done that shows ADRC can be designed for fractional order time delay systems and for different pole location. These simulation results are compared with respective FOPID controller results and other existing methods.

CHAPTER 7

PROPOSED APPROACH TO DESIGN ADRC CONTROLLER

ADRC also based on the principle of internal model control as FOPID. ADRC is designed for fractional order time delay system so calculation is done for general plant. ADRC doesn't need more information of the plant except the gain (B) and relative order (P). Controlled plant can be generalized as follows –

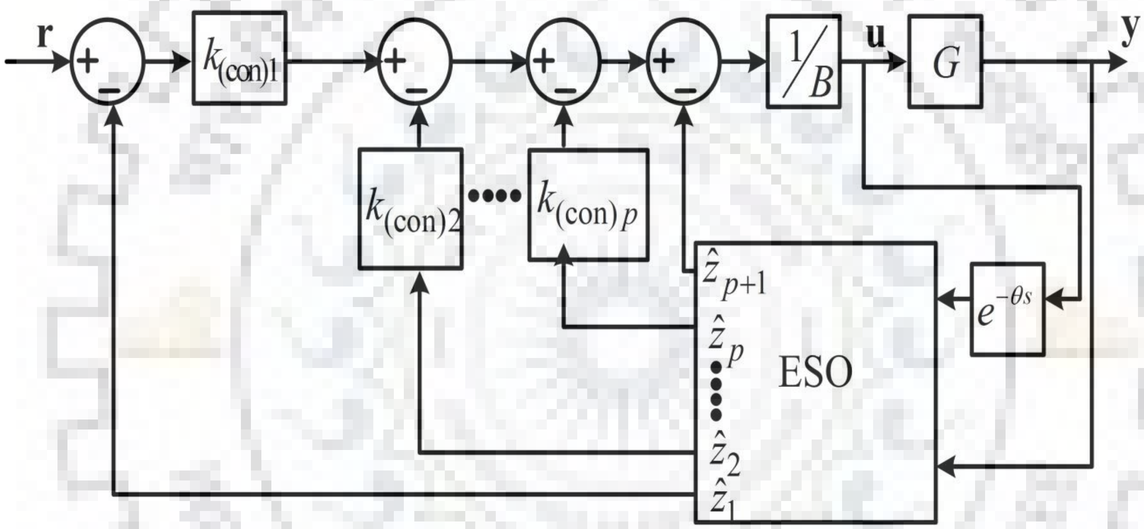


Figure 7.1 Structure of ADRC controller for fractoional order time delay system

General form of Fractional order time delay system can be given as –
$P_n y^{\alpha_n}(t) + P_{n-1} y^{\alpha_{n-1}}(t) + \ldots . + P_1 y^{\alpha_1}(t) + P_0 y(t) = Q_m u^{\beta_m}(t-\Theta) + Q_{m-1} u^{\beta_{m-1}}(t-\Theta)$
$+ \cdots + Q_1 u^{\beta_1}(t-\Theta) + Q_0 u(t-\Theta)$

Another laplace form of this equation can be given as –

$$G(s) = \frac{Q_m s^{\beta_m} + Q_{m-1} s^{\beta_{m-1}} + \ldots \ldots \ldots + Q_1 s^{\beta_1} + Q_0}{P_n s^{\alpha_n} + P_{n-1} s^{\alpha_{n-1}} + \ldots . + P_1 s^{\alpha_1} + P_0} = \frac{y(s)}{e^{-\theta s} u(s)}$$

Where $\alpha_n > \alpha_{n-1} > \cdots > \alpha_1 > 0$, $\beta_m > \beta_{m-1} > \ldots > \beta_1 > 0$ and $\alpha_n > \beta_m$

There P_i and $Q_j \in R^+$

$$i = 0,1 \ldots\ldots n$$

$$j = 0,1, \ldots\ldots m$$

If system has the external disturbance D with the input signal then above equation can be written as-

$$P_n y^{\alpha_n}(t) + P_{n-1} y^{\alpha_{n-1}}(t) + \ldots\ldots + P_1 y^{\alpha_1}(t) + P_0 y(t)$$
$$= Q_m u^{\beta_m}(t-\Theta) + Q_{m-1} u^{\beta_{m-1}}(t-\Theta) + \ldots + Q_1 u^{\beta_1}(t-\Theta) + Q_0 u(t-\Theta)$$
$$+ \mathrm{D} \qquad (1)$$

Equation (1) can be expressed as follows –

$$y^{\alpha_n}(t) + (P_{n-1}/P_n) y^{\alpha_{n-1}}(t) + \ldots + (P_1/P_n) y^{\alpha_1}(t) + (P_0/P_n) y(t) = (Q_m/P_n) u^{\beta_m}(t-\Theta) + (Q_{m-1}/P_n) u^{\beta_{m-1}}(t-\Theta) + \ldots + (Q_1/P_n) u^{\beta_1}(t-\Theta) + (Q_0/P_n) u(t-\Theta) + (D/P_n)$$

Above equation can be written as-

$$y^P(t) = \begin{matrix} y^P(t) - y^{\alpha_n}(t) - (P_{n-1}/P_n) y^{\alpha_{n-1}}(t) - \ldots - (P_0/P_n) y(t) + (Q_m/P_n) u^{\beta_m}(t-\Theta) + \\ (Q_{m-1}/P_n)\ u^{\beta_{m-1}}(t-\Theta) + \cdots + (Q_0/P_n) u(t-\Theta) + (D/P_n) - B\, u(t-\Theta) + B\, u(t-\Theta) \end{matrix}$$

$$y^P(t) = [\begin{matrix} y^P(t) - y^{\alpha_n}(t) - (P_{n-1}/P_n) y^{\alpha_{n-1}}(t) - \ldots - (P_0/P_n) y(t) + (Q_m/P_n) u^{\beta_m}(t-\Theta) + \\ (Q_{m-1}/P_n) u^{\beta_{m-1}}(t-\Theta) + \ldots + (D/P_n) + (Q_0/P_n - B) u(t-\Theta)] + B u(t-\Theta) \end{matrix}$$

$$y^P(t) = F(y(t), u(t-\Theta), D(t)) + B u(t-\Theta) \qquad (2)$$

Where

$$F\big(y(t), u(t-\Theta), D(t)\big) =$$
$$[y^P(t) - y^{\alpha_n}(t) - (P_{n-1}/P_n) y^{\alpha_{n-1}}(t) - \ldots - (P_0/P_n) y(t) + (Q_m/P_n) u^{\beta_m}(t-\Theta) + (Q_{m-1}/P_n) u^{\beta_{m-1}}(t-\Theta)$$
$$\ldots + (D/P_n) + (Q_0/P_n - B) u(t-\Theta)$$
$$]$$

$F\big(y(t), u(t-\Theta), D(t)\big)$ is the effect of external disturbance and unknown dynamics. So, to eliminate the effect of disturbance B as (Q_0/P_n) is chosen.

To estimate the disturbance extended state observer(ESO) is used. As it is known ADRC does not need the whole information of the plant, there is need of the order of controller(P) and gain of the system(B). To represent this in state space form–

$$z_1 = \dot{y}$$

$$z_2 = \ddot{y}$$
$$z_P = y^{P-1}$$
$$z_{P+1} = F\big(y(t), u(t-\Theta), D(t)\big)$$

$F\big(y(t), u(t-\Theta), D(t)\big)$ can be assumed differentiable and it's derivate can be taken as $\dot{F}\big(y(t), u(t-\Theta), D(t)\big) = H$

$$\dot{z} = A_{ESO} z + B_{ESO} u + E_{ESO} H$$
$$y = C_{ESO} z$$

Where $z = [z_1 \ z_2 \ldots z_P \ z_{P+1}]'$

$$A_{ESO} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{(P+1)*(P+1)} \qquad B_{ESO} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ B \\ 0 \end{bmatrix}_{(P+1)*1} \qquad E_{ESO} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{(P+1)*1}$$

$$C_{ESO} = [1 \quad 0 \quad 0 \quad \cdots \quad 0]_{1*(\mathrm{P}+1)}$$

A full-order Luenberger state-observer can be designed as -

$$\dot{\hat{z}} = A_{ESO}\hat{z} + B_{ESO} u + L_{obs}(y - \hat{y})$$
$$\hat{y} = C_{ESO}\hat{z}$$

Where L_{obs} is the observer gain vector –

$L_{obs} = [\varepsilon_1 \quad \varepsilon_2 \quad \cdots \quad \varepsilon_p \quad \varepsilon_{P+1}]^T$

When $A_{ESO} - L_{obs} C_{ESO}$ is asymptotically stable then y(t) and it's derivatives upto P-1 will be approximated by $\hat{z}_1, \ldots \hat{z}_P$ and $\hat{z}_{P+1}$ will approximate the value of generalised disturbance $F\big(y(t), u(t), D(t)\big)$ that is used to reject it faster.

let the control law as –

$$u(t) = \frac{-\hat{z}_{P+1} + u_c(t)}{B} \tag{3}$$

Now equation (2) can be written as –

$$y^P(t) = F(y(t), u(t-\Theta), D(t)) + -\hat{z}_{P+1} + u_c(t)$$

ESO is designed such that –

$$F\big(y(t), u(t-\Theta), D(t)\big) \approx \hat{z}_{P+1}$$

So,

$$y^P(t) \approx u_c(t)$$

This final plant can be controlled as following feedback state law –

$$u_c(t) = k_{(con)1}\big(r(t) - y(t)\big) + k_{(con)2}\big(\dot{r}(t) - \dot{y}(t)\big) + \ldots + k_{(con)P}(r^{P-1}(t) - y^{P-1}(t))$$

Since $\hat{z}_1 , \ldots \hat{z}_P$ approximate $y(t) \ldots . y^{P-1}(t)$ so from equation (3) u(t) can be written as follows–

$$u(t) = \frac{-\hat{z}_{P+1}}{B} + \frac{k_{(con)1}\big(r(t) - \hat{z_1}(t)\big) + \ldots + k_{(con)P}\big(r^{P-1}(t) - \widehat{z_{P+1}}(t)\big)}{B}$$

$$u(t) = k_{con}(\hat{r}(t) - \hat{z}(t))$$

Where

$$\hat{r}(t) = [r(t) \quad \dot{r}(t) \quad \cdots \quad r^{P-1}(t) \quad 0]^T$$

$$k_{con} = \frac{[k_{(con)1} \quad k_{(con)2} \quad \cdots \quad k_{(con)P} \quad 1]}{B}$$

Now final ADRC controller for time delay system is –

$$\dot{\hat{z}}(t) = A_{ESO}\hat{z}(t) + B_{ESO}u(t - \theta) + L_{obs}(y(t) - \hat{y}(t))$$

$$u(t) = k_{con}(\hat{r}(t) - \hat{z}(t))$$

Where θ = time delay of the system

Consider the comparison of the characteristic equation of ESO and pole location for which it is assumed all observer poles are placed at different desired observer bandwidths locations like $\omega_{01} \ldots . \omega_{0(P+1)}$ to derive the L_{obs} –

$$|sI - (A_{ESO} - L_{obs}C_{ESO})| = (s + \omega_{01})(s + \omega_{02}) \ldots . \big(s + \omega_{0(P+1)}\big)$$

$$\left| s * \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} + [\varepsilon_1 \quad \varepsilon_2 \quad \cdots \quad \varepsilon_p \quad \varepsilon_{P+1}]^T[1 \quad 0 \quad 0 \quad \cdots \quad 0] \right|$$

$$= (s + \omega_{01})(s + \omega_{02}) \ldots . \big(s + \omega_{0(P+1)}\big)$$

$$s^{P+1} + \varepsilon_1 s^P + \varepsilon_2 s^{P-1} \ldots + \varepsilon_P s + \varepsilon_{P+1} = s^{P+1} + \sum_{a_1=1}^{P+1} \omega_{oa_1} s^P + \sum_{a_1;a_2=1}^{P+1} \omega_{oa_1}\omega_{oa_2} s^{P-1} + \ldots + \sum_{a_1;a_2 \ldots a_P=1}^{P+1} \omega_{oa_1}\omega_{oa_2} \ldots \omega_{oa_P} s + \omega_{oa_1}\omega_{oa_2} \ldots \omega_{oa_{P+1}}$$

$$\varepsilon_j = \begin{cases} \sum_{a_1;a_2\ldots a_{(j-1)}}^{P+1} \prod_{i=1}^{j} \omega_{oa_i} & j = 1,2,3 \ldots P \quad ; a_m \neq m \,\forall\, m \in (0, j-1) \\ \prod_{i=1}^{P+1} \omega_{ca_i} & j = P+1 \end{cases}$$

To derive the value of k_{con} the controller bandwidths ω_{cj} is considered at the place of controller bandwidths. If the disturbance is absent and $\dot{\hat{z}}_i$ $(i = 1..P)$ are also accurate estimations of states then final state feedback control system is –

$$|sI - (A_{ESO} - B_{ESO}k_{con})| = s(s + \omega_{c1})(s + \omega_{c2}) \ldots .(s + \omega_{cP})$$

$$\left| s * \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ B \\ 0 \end{bmatrix} \left(\frac{[k_{(con)1} \quad k_{(con)2} \quad \cdots \quad k_{(con)P-1} \quad k_{(con)P} \quad 1]}{B} \right) \right|$$

$$= s(s + \omega_{c1})(s + \omega_{c2}) \ldots .(s + \omega_{cP})$$

$$s[s^P + k_{(con)P}s^{P-1} + k_{(con)(P-1)}s^{P-2} \ldots + k_{(con)2}s + k_{(con)1}] =$$

$$s[s^P + \sum_{a_1=1}^{P} \omega_{ca_1} s^{P-1} + \sum_{a_1;a_2=1}^{P} \omega_{ca_1}\omega_{ca_2} s^{P-2} + \ \ldots + \sum_{a_1;a_2\ldots a_{(P-1)}=1}^{P} \omega_{ca_1}\omega_{ca_2} \ldots \omega_{ca_{(P-1)}} s + \omega_{ca_1}\omega_{ca_2} \ldots \omega_{ca_P}]$$

$$k_{(con)j} = \begin{cases} \sum_{a_1;a_2\ldots a_{(P-i+1)}}^{P} \prod_{i=1}^{P-i+1} \omega_{ca_i} & j = 2,3 \ldots P \quad ; a_m \neq m \,\forall\, m \in (0, P-i+1) \\ \prod_{i=1}^{P} \omega_{ca_i} & j = 1 \end{cases}$$

CHAPTER 8

VALIDATION OF PROPOSED ADRC APPROACH

First order time delayed system is taken from previous work. To estimate the disturbance of the system a new technique ADRC is applied in which there is no need of all parameters of the system except gain and relative order. Now this proposed method is applied on these numerical examples. These are first order system so to simplify the above calculation of generalised ADRC it is approximated for first order so $P = 1$ so second order ADRC is designed for these example to estimate the disturbance.

By equation (2)

$$\dot{y} = F\big(y(t), u(t-\theta), D(t)\big) + Bu(t-\theta)$$

So it can be written

$$z_1(t) = y(t)$$

$$z_2(t) = F\big(y(t), u(t-\theta), D(t)\big)$$

$$\dot{z_1} = z_2 + Bu(t-\theta)$$

$$\dot{z_2}(t) = H$$

$$\dot{\hat{z}}(t) = A_{ESO}\hat{z}(t) + B_{ESO}u(t-\theta) + L_{obs}(y(t) - \hat{y}(t))$$

$$\hat{y} = C_{ESO}\hat{z}$$

$$u(t) = k_{con}\big(\hat{r}(t) - \hat{z}(t)\big)$$

In matrix form –

$$\begin{bmatrix} \dot{\hat{z}}_1(t) \\ \dot{\hat{z}}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{z}_1(t) \\ \hat{z}_2(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t-\theta) + [\varepsilon_1 \quad \varepsilon_2]^T \left(y(t) - \begin{bmatrix} 1 \\ 0 \end{bmatrix} [\hat{z}_1(t) \quad \hat{z}_2(t)] \right)$$

$$\dot{\hat{z}}_1(t) = \hat{z}_2(t) + Bu(t-\theta) + \varepsilon_1\big(y(t) - \hat{z}_1(t)\big)$$

$$\dot{\hat{z}}_2(t) = \varepsilon_2\big(y(t) - \hat{z}_1(t)\big)$$

Where

$$\varepsilon_1 = \omega_{01} + \omega_{02}$$

$$\varepsilon_2 = \omega_{01} * \omega_{02}$$

As such there will be only one value of k_{con} for first order system and second order ADRC.

$$k_{con} = \omega_c$$

8.1 PROPOSED APPROACH CALCULATION FOR INTEGER TYPE FIRST ORDER TIME DELAY SYSTEM

anti-windup for temperature profile control system-

$$G(s) = \frac{9.87e^{-0.18s}}{(35.22s+1)}$$

Relative order (P) and system's gain (B)

$$P = 1$$

$$B = (Q_0/P_n) = 9.87/35.22 = 0.280$$

Let

$$\omega_c = 2$$

$$\omega_{01} = 20$$

$$\omega_{02} = 250$$

8.2 PROPOSED APPROACH CALCULATION FOR FRACTIONAL TYPE FIRST ORDER TIME DELAY SYSTEM

Now take tractional type example to see the result of proposed method

$$G(s) = \frac{0.99932\, e^{-0.1922s}}{1.0842s^{1.0132}+1}$$

Relative order (P) and system's gain (B)

$$P = 1$$

$$B = (Q_0/P_n) = 0.99932/1.0842 = 0.916$$

Let

$$\omega_c = 2$$

$$\omega_{01} = 10$$

$$\omega_{02} = 20$$

8.3 PROPOSED APPROACH CALCULATION FOR INTEGER TYPE FIRST ORDER TIME DELAY NON MINIMUM SYSTEM

let transfer function is –

$$G(s) = \frac{(8-s)e^{-0.05s}}{(10s+1)}$$

Relative order (P) and system's gain (B)

$$P = 1$$

$$B = (Q_0/P_n) = 8/10 = 0.8$$

Let

$$\omega_c = 1$$

$$\omega_{01} = 2$$

$$\omega_{02} = 100$$

8.4 PROPOSED APPROACH CALCULATION FOR FRACTIONAL TYPE FIRST ORDER TIME DELAY NON MINIMUM SYSTEM

$$G(s) = \frac{(1-0.02s^{0.1})e^{-0.12s}}{(1+15s^{0.2})}$$

Relative order (P) and system's gain (B)

$$P = 1$$

$$B = 1$$

Let

$$\omega_c = 5$$

$$\omega_{01} = 20$$

$$\omega_{02} = 30$$

CHAPTER 9

SIMULATION RESULTS

Simulation models of every example are shown here and their responses in time domain, robustness and control energy are represented in figures and tables with the respective errors.

9.1 TIME DOMAIN ANALYSIS

Errors are found out by these formula -

$$\mathrm{ISE} = \int_0^\infty (y - r)^2 dt$$

$$\mathrm{IAE} = \int_0^\infty |(y - r)|\, dt$$

$$\mathrm{ITAE} = \int_0^\infty t|(y - r)|\, dt$$

Where y is represented for output and r is represented for reference input. So the difference of output and reference input is error 'e' i.e.

$$e = y - r$$

For example 1 -

Methods \ Errors	IAE	ISE	ITAE
FOIMC approach	1.457	0.5929	6.838
IMC scheme[1]	1.655	0.7068	8.719
Vu's FOPI[5]	2.138	0.7127	28.24
Proposed ADRC method	0.536	0.378	0.8305

Table 9.1.1 Errors for proposed FOADRC and existing methods in example 1

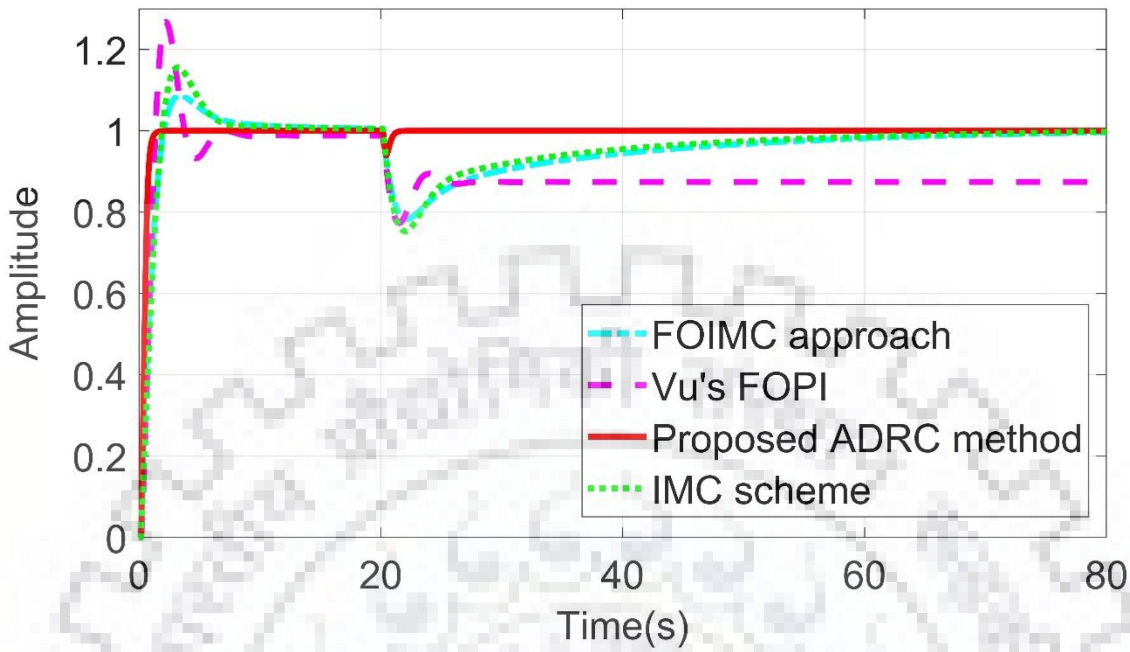
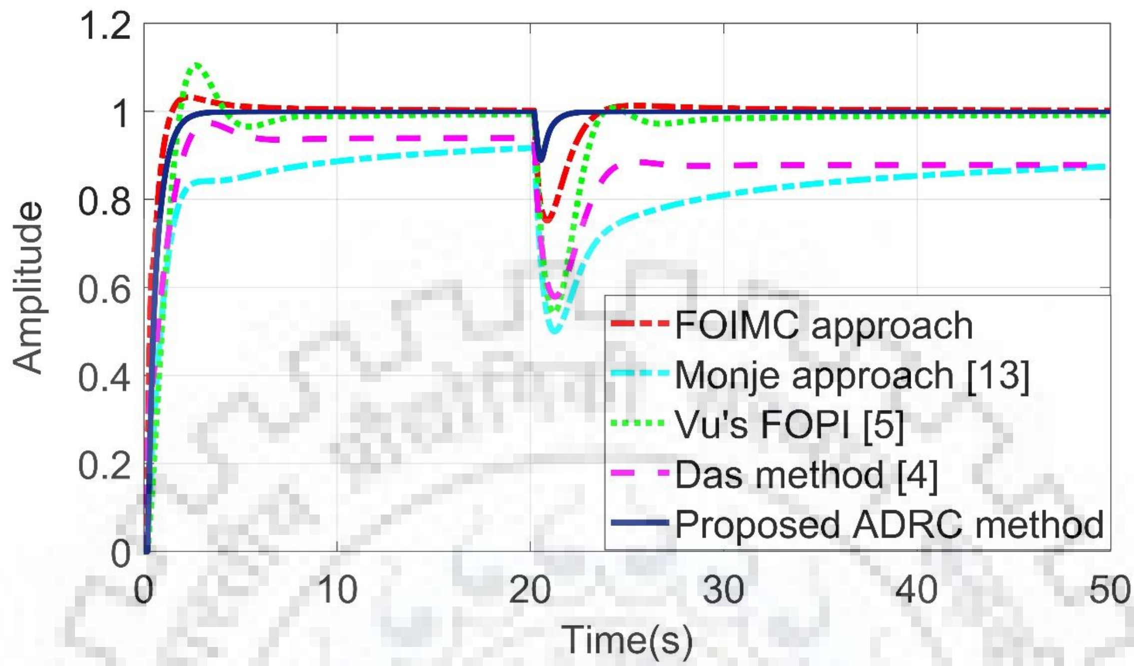


Figure 9.1.1 Example 1 with proposed FOADRC and comparing methods

For example 2 -

Errors / Methods	IAE	ISE	ITAE
FOIMC approach	0.8846	0.3193	7.818
Monje scheme[13]	6.981	1.483	147.4
Vu’s FOPI[5]	1.96	0.8044	19.96
Das method[4]	4.971	0.9166	115.2
Proposed ADRC method	0.7366	0.4228	1.992

Table 9.1.2 Errors for proposed FOADRC and existing methods in example 2



9.1.2 Example 2 with proposed FOADRC and comparing methods

For example 3 -

Methods \ Errors	IAE	ISE	ITAE
FOIMC approach	1.522	0.6609	9.283
Monje scheme[13]	1.958	0.6667	27.33
PSO scheme[12]	1.712	0.706	24.73
Cuckoosearch method[11]	1.894	0.8731	24.43
Proposed ADRC method	1.159	0.5816	3.19

Table 9.1.3 Errors for proposed FOADRC and existing methods in example 3

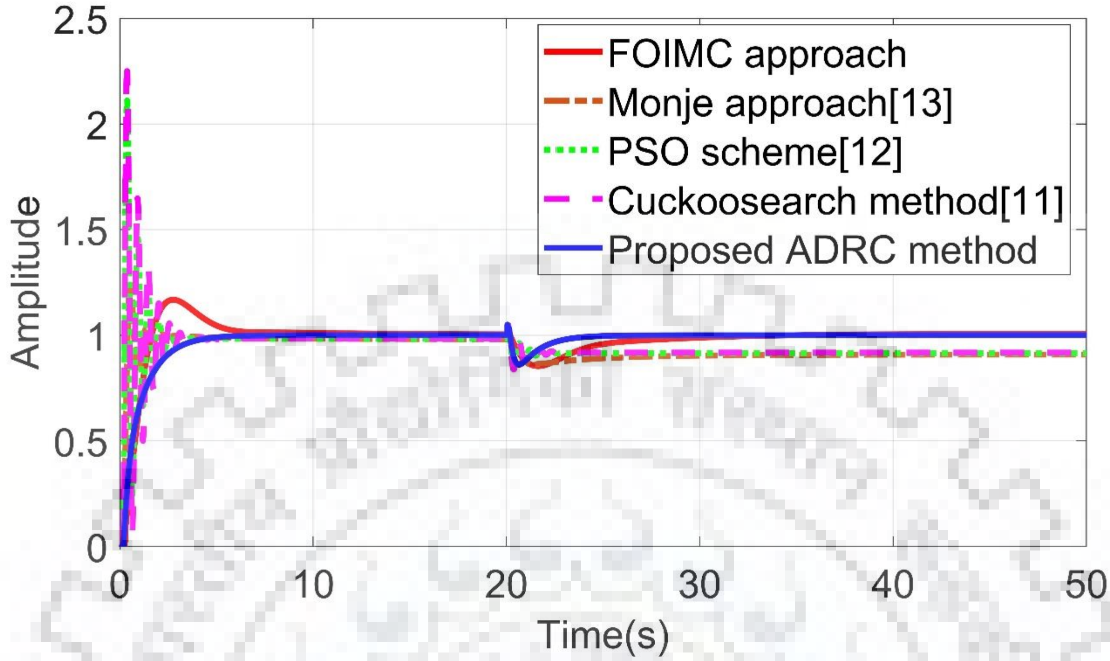


Figure 9.1.3 Example 3 with proposed FOADRC and comparing methods

For example 4 -

Errors / Methods	IAE	ISE	ITAE
FOIMC approach	1.522	0.6609	9.283
Monje approach[13]	1.958	0.6667	27.33
PSO scheme[12]	1.712	0.706	24.73
Cuckoosearch method[11]	1.894	0.8731	24.43
Proposed ADRC method	1.026	0.4425	1.044

Table 9.1.4 Errors for proposed FOADRC and existing methods in example 4

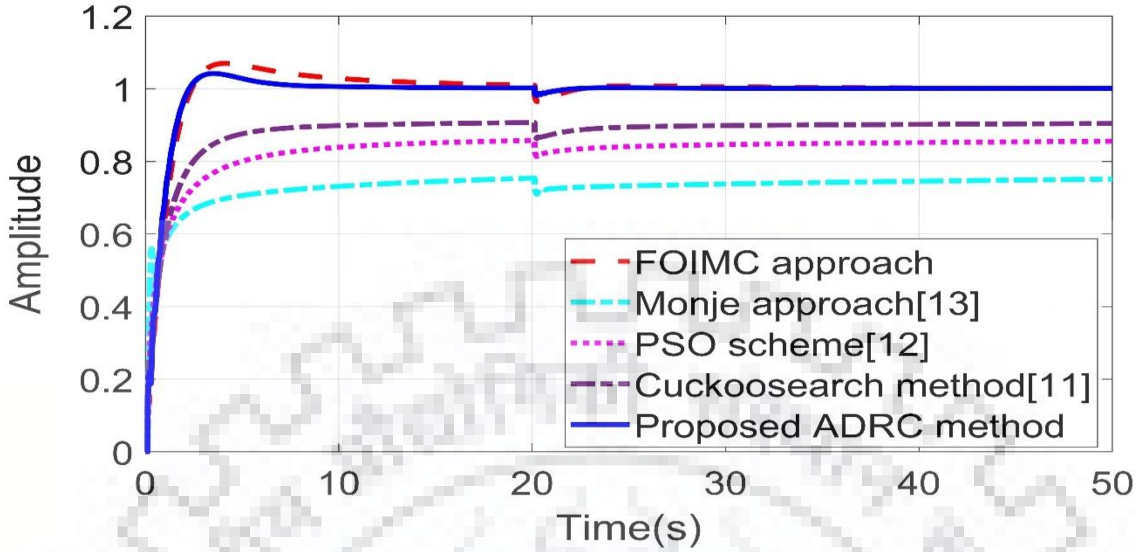


Figure 9.1.4 Example 4 with proposed FOADRC and comparing methods

From the above results, it can be concluded that proposed method gives flexible, good simulation, best estimation and rejection of disturbance and fast tracking set point error.

9.2 ROBUSTNESS ANALYSIS

Robustness analysis is done for example 1 in which parameters will be changed $\pm 20\%$. In this situation system with proposed method gives better response as compared to other existing methods that is system is robust in uncertainity too general form of transfer function is –

$$G(s) = \frac{Ke^{-\theta s}}{1+\tau s^{\alpha}}$$

Where uncertainities are $k' = k \pm \Delta k$

$\theta' = \theta \pm \Delta\theta$

$\alpha' = \alpha \pm \Delta\alpha$

$\tau' = \tau \pm \Delta\tau$

For example 1 -

$$G(s) = \frac{9.87e^{-0.18s}}{(35.22s+1)}$$

There k’ ∈ [7.896, 11.844]

$\theta' \in [0.144, 0.216]$

$\tau' \in [28.176, 42.264]$

constant term 1 also can vary from 0.8 to 1.2.

Errors / Methods	ISE		IAE		ITAE	
	-20%	+20%	-20%	+20%	-20%	+20%
FOIMC approach	0.5645	0.6235	1.428	1.488	6.791	6.856
IMC scheme[1]	0.6758	0.7397	1.623	1.691	8.669	8.752
Vu's FOPI[5]	0.665	0.7653	2.064	2.231	28.04	28.56
Proposed ADRC method	0.3366	0.3933	0.5305	0.5473	0.7519	0.9461

Table 9.2.1 Errors for robustness by proposed FOADRC and existing methods in example 1

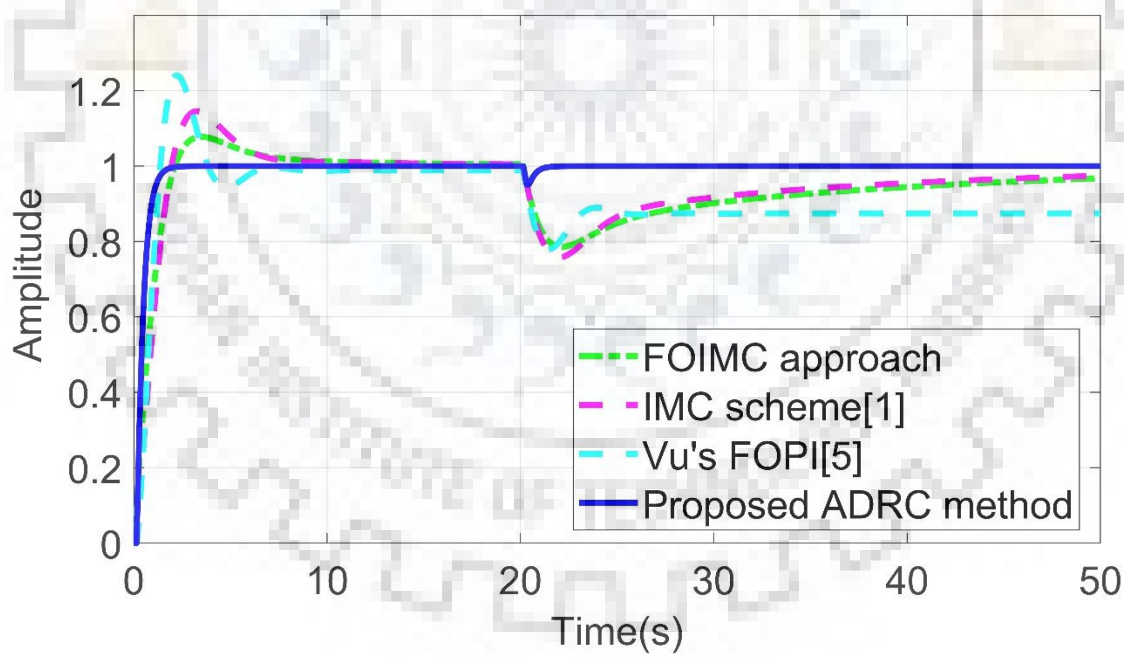


Figure 9.2.1 Robustness analysis for -20% for example 1

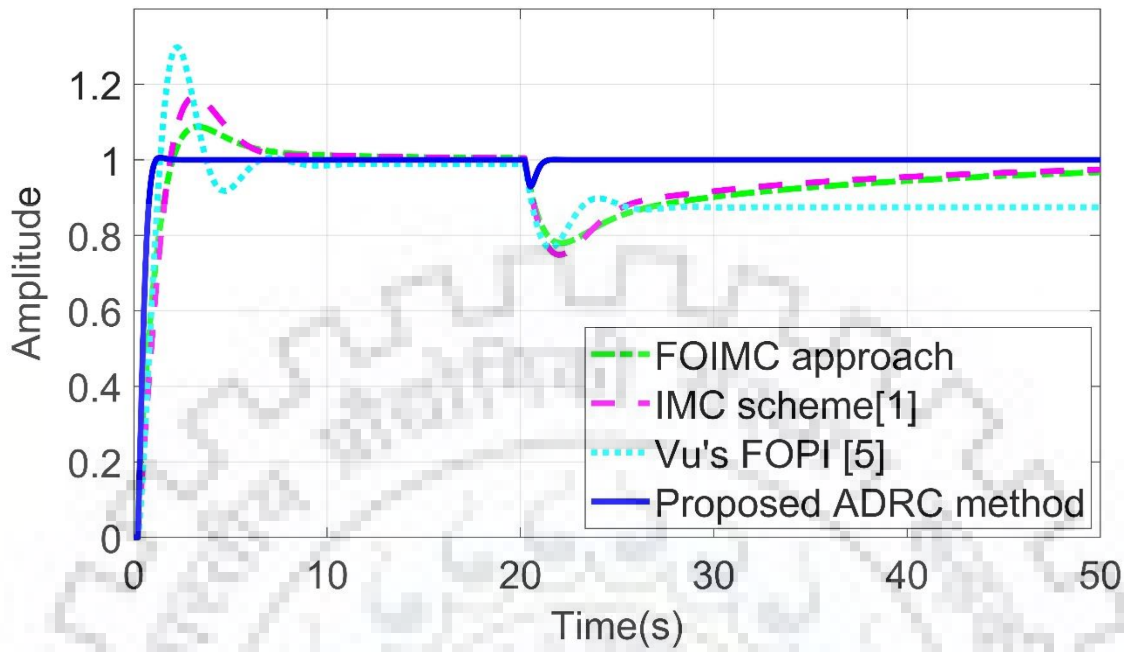


Figure 9.2.2 Robustness analysis for +20% for example 1

For example 2 -

$$G(s)=\frac{0.99932\,e^{-0.1922s}}{1.0842s^{1.0132}+1}$$

There k’ ∈ [0.799456, 1.199184]

θ' ∈ [0.15376, 0.23064]

α ∈ [0.881056, 1.321584]

τ' ∈ [0.86736, 1.30104]

constant term 1 also can vary from 0.8 to 1.2.

Methods \ Errors	ISE		IAE		ITAE	
	-20%	+20%	-20%	+20%	-20%	+20%
FOIMC approach	0.3784	0.4093	0.9109	0.9697	7.927	7.871
Monje approach[13]	1.479	1.542	7.086	6.945	147.7	147.9
Vu's FOPI[5]	0.7161	1.009	1.778	2.582	18.79	25.17
Das method[4]	0.9009	0.9718	5.06	4.958	116	115.3
Proposed ADRC method	0.3012	0.329	0.7436	0.7039	1.962	0.3068

Table 9.2.2 Errors for robustness by proposed FOADRC and existing methods in example 2

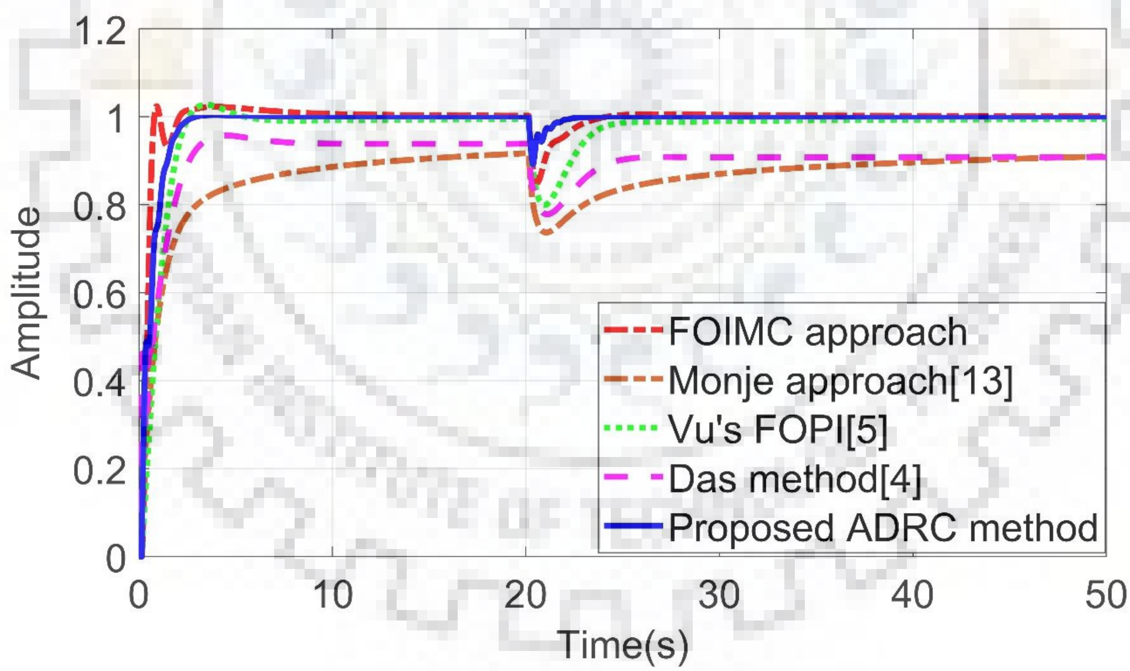


Figure 9.2.3 Robustness analysis for -20% for example 2

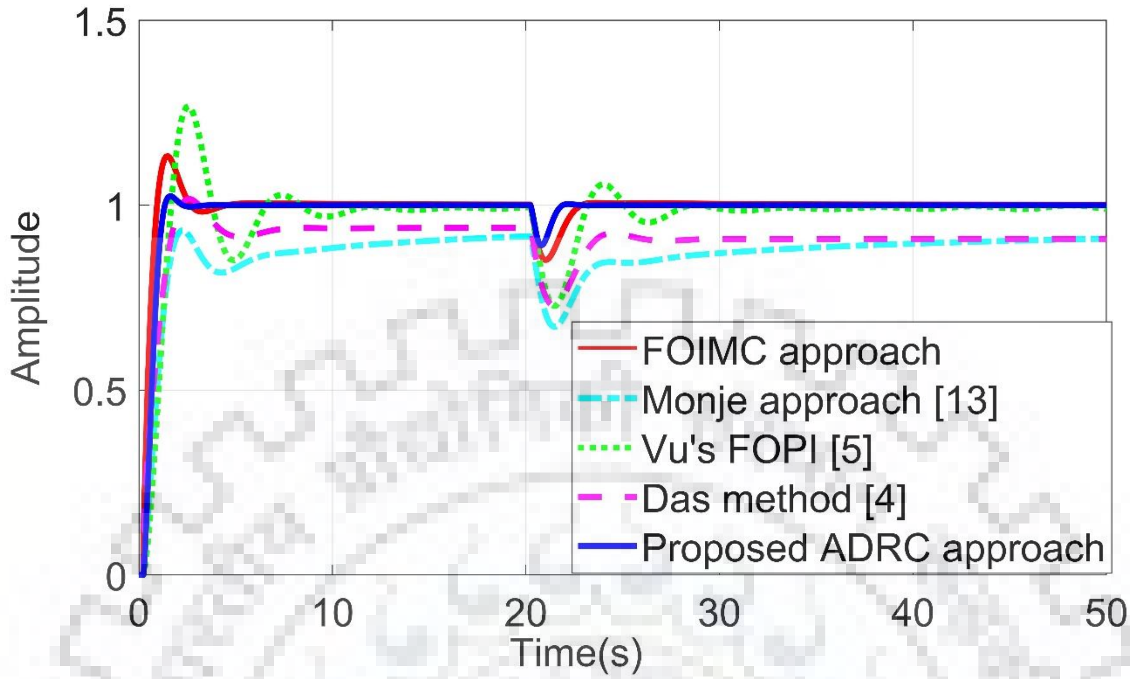


Figure 9.2.4 Robustness analysis for +20% for example 2

For example 3 -

$$G(s) = \frac{(8-s)e^{-0.05s}}{(10s+1)}$$ can be written in this form

$$G(s) = \frac{(T1-k1s)e^{-\theta s}}{(T2s+1)}$$

There T1’ ∈ [6.4, 9.6]

k1’ ∈ [0.8, 1.2]

θ' ∈ [0.04, 0.06]

T2’ ∈ [8, 12]

constant term 1 also can vary from 0.8 to 1.2

Errors / Methods	ISE		IAE		ITAE	
	-20%	+20%	-20%	+20%	-20%	+20%
FOIMC approach	0.5502	0.6696	1.461	1.482	6.296	6.313
Monje approach[13]	0.5516	0.7753	1.939	2.345	32.62	33.38
PSO scheme[12]	0.5841	0.9207	1.679	2.212	29.24	29.96
Cuckoosearch method[11]	0.6803	1.242	1.762	2.59	28.4	29.85
Proposed ADRC method	0.5719	0.5852	1.063	1.064	1.125	1.103

Table 9.2.3 Errors for robustness by proposed FOADRC and existing methods in example 3

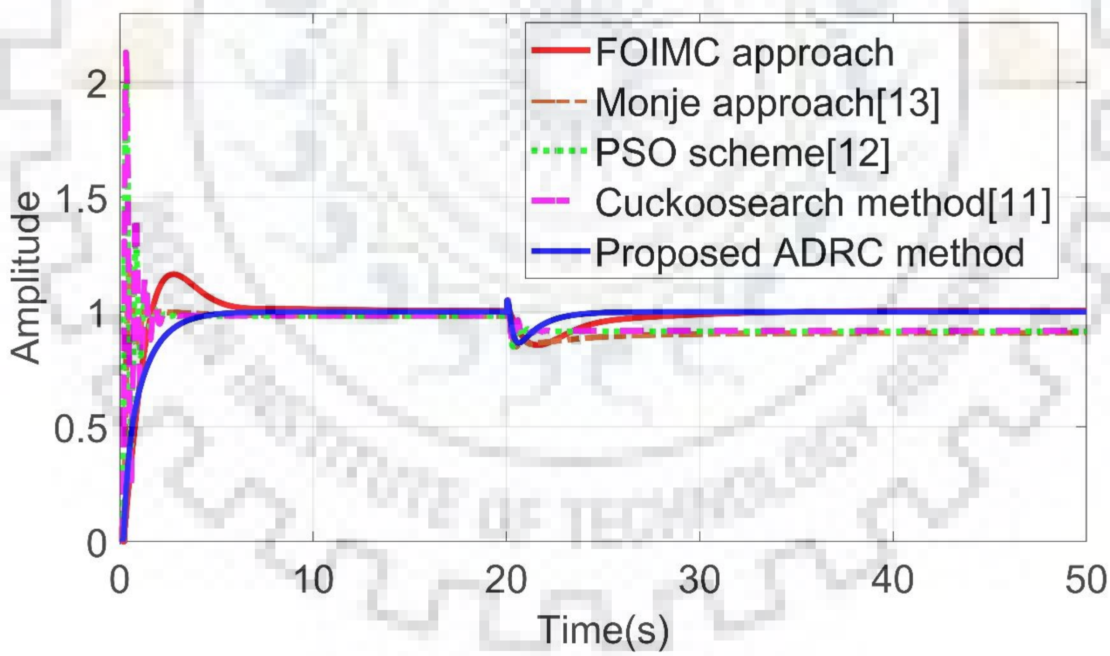


Figure 9.2.5 Robustness analysis for -20% for example 3

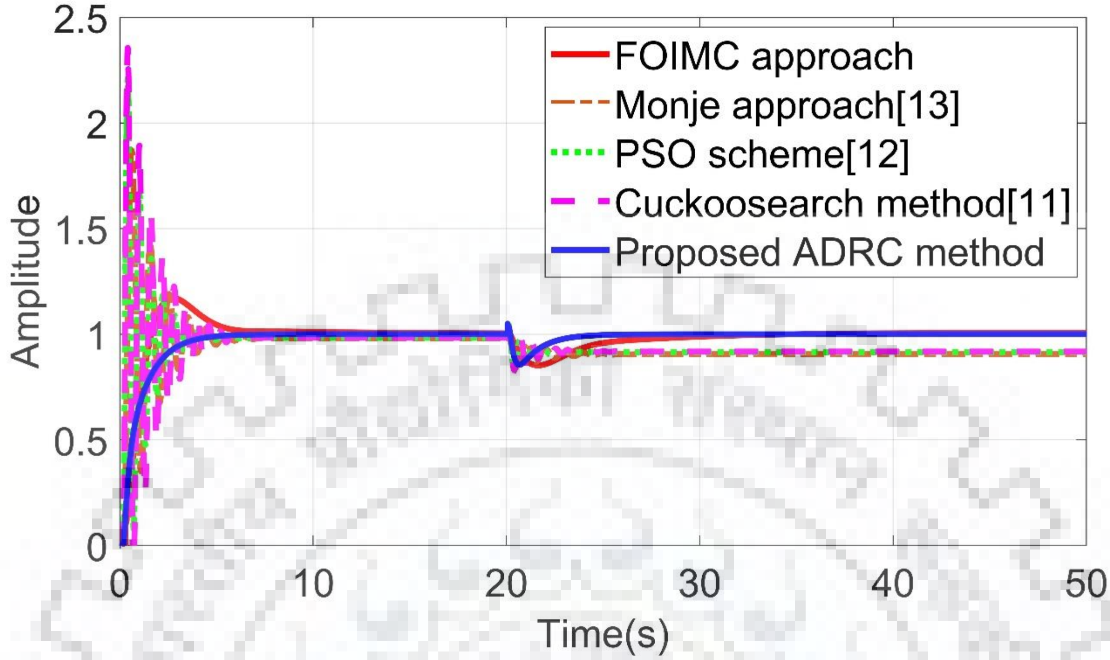


Figure 9.2.6 Robustness analysis for +20% for example 3

For example 4 -

$$G(s) = \frac{(1-0.02s^{0.1})e^{-0.12s}}{(1+15s^{0.2})}$$

can be written in this form

$$G(s) = \frac{(T1-k1s)e^{-\theta s}}{(T2s+1)}$$

There T1’ ∈ [0.8, 1.2]

k1’ ∈ [0.016, 0.024]

θ' ∈ [0.096, 0.144]

T2’ ∈ [12, 18]

constant term 1 also can vary from 0.8 to 1.2

Errors / Methods	ISE		IAE		ITAE	
	-20%	+20%	-20%	+20%	-20%	+20%
FOIMC approach	0.5502	0.6696	1.461	1.482	6.296	6.313
Monje scheme[13]	0.5516	0.7753	1.939	2.345	32.62	33.38
PSO scheme[12]	0.5841	0.9207	1.679	2.212	29.24	29.96
Cuckoosearch method[11]	0.6803	1.242	1.762	2.59	28.4	29.85
Proposed ADRC method	0.425	0.5605	0.8754	1.277	1.044	5.093

Table 9.2.4 Errors for robustness by proposed FOADRC and existing methods in example 4

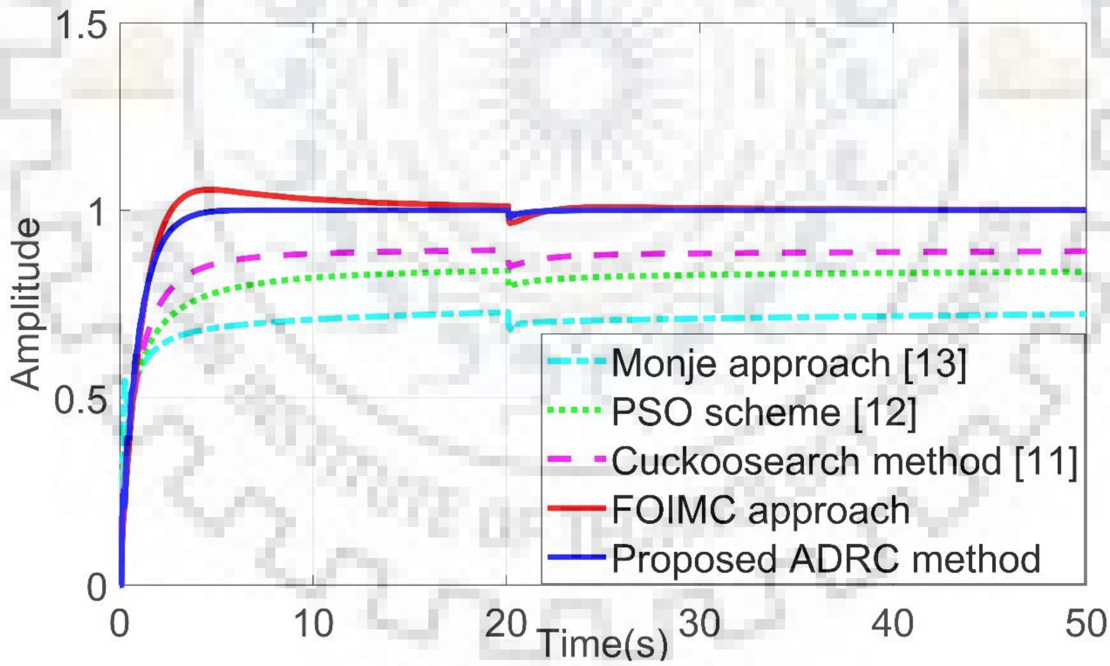


Figure 9.2.7 Robustness analysis for -20% for example 4

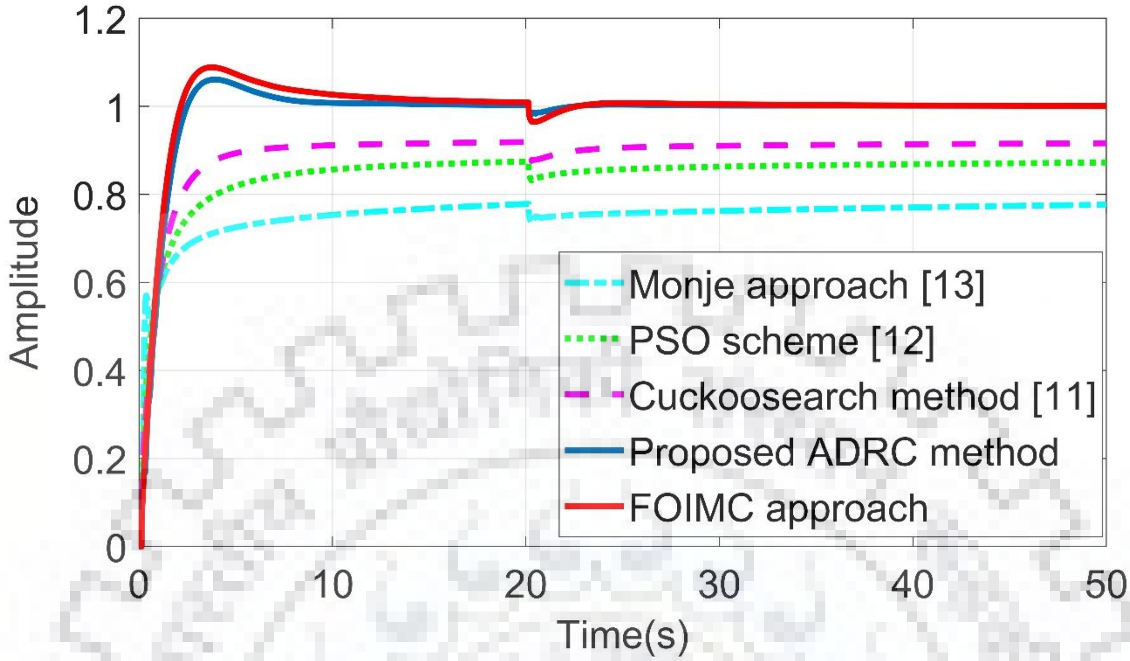


Figure 9.2.8 Robustness analysis for +20% for example 4

Above results shows the best results for all the systems after perturbation in the parameters of the system.

9.3 CONTROL ENERGY ANALYSIS

For any system analysis given energy to operate system should be minimum that is operating energy should be minimum for controller but for fraction it is also future research point to decrease it's operating energy but in this thesis energy analysis of controller is done.

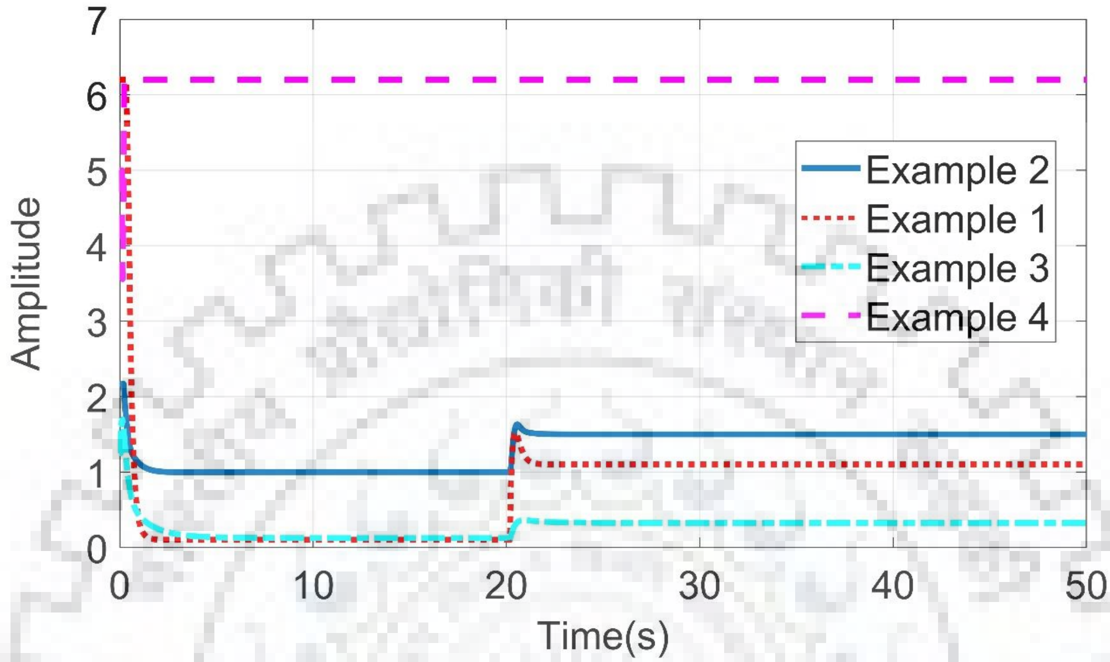


Figure 9.3.1 Control energy analysis for proposed FOADRC

Above plot shows the energy value of the controllers. Energy increases with good response ,for best result there it is need to provide more energy to controller. Sometimes more energy is given when system is complex like non minimum system or fractional sytem but FOADRC works on minimum enegy so it is easy to implement practically.

CHAPTER 10

CONCLUSION AND FUTURE SCOPE

In this thesis proposed method is useful for every type of systems minimum, non minimum, integer and non integer and their combination too. One method suitable for every system without affecting the parameters, stability and robustness. It gives a high level robustness by changing parameters like gain, delay, pole and zero coefficients. It shows minimum errors as compared to other existing method that are shown in this thesis. This comparision is shown for every example. Stability is shown by the input and output disturbance and noise. It also performs it's best as compared to other in all these conditions. This thesis is basically based on time delay first order system but it can be further implement for higher order systems too. No doubt fractional order pid is always better than simple pid controller because it gives flexible, stable and robust result. It has five variable parameters that can be changed according to our requirement and in any plant main aim is always that the system should give output according to our desire.

Fractional calculus is 300 years old technique but it became more popular when it comes tuse in control and system, it helps in designing of better pid controller. FOPID is using in real time systems and giving better result than pid. Real time system can be interfaced by MATLAB and FOMCON toolbox and mathematical fractional model of real time system can be obtained by FOMCON's fractional system time domain identification tool. Fractional order pid controller are experimented on real time systems and getting good results. They are robust and stable but using high input energy to design fractional order controller. This is the main isssue of this technique.

In every example errors, simulation result and robustness are dicussed to show FOPID is giving better result than existing tuning method. These tuning methods are different for every type of system so different approach is used for different system but proposed method gives result for every type of system so no need to calculate again for other system. So calculation becomes easy.

Proposed FOADRC approach is used to reject the nonlinear disturbance that can be internal or external for integer and fractional system. In this thesis, proposed FOADRC is applied for fractional order time delay system and differnet pole locations that are not done in literature work. Different pole locations give the flexibilty to the system without making it complex. ADRC can be controlled easily by using the only two tuning parameters. This approach is applied on all type of systems like integer, fractional, non minimum integer and non minimum fractional systems considering the delay. In all the cases this approach gives the best results and these results are comapred by other existing tuning methods. Error analysis in time domain are done for FOADRC and it is observed that errors are minimal. Robustness analysis also done by pertubing all the parameters by +20% and -20% and stable and best results are observed. Control energy for this approach is minimal so it is easy to implement practically.

In future work, this proposed FOADRC method will be analysed for unstable systems and Fractional order pid controller consume more energy as compared to pid controller so it can be future research in this direction that FOPID works best on low energy. Fopid design papers are increasingly steadily but this issue is unresolved still. FOPID uses more energy so it is still unwilling to experiment for industy aspects. Thus the future research should revolve around minimisation of energy required for implementation of FOPID controller. Fractional complex order pid controller is also interesting research direction that will give more flexible result.

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