TUNING OF IMC CONTROLLER USING LQR

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by

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CANDIDATE'S DECLARATION

I hereby certify that this dissertation work titled **TUNING OF IMC CONTROLLER USING LQR** in partial fulfillment of the requirement of award of Degree of **Master of Technology** in **Electrical Engineering** with specialization in **Systems And Control**, submitted to the Department of Electrical Engineering, Indian Institute of Technology Roorkee, is an authentic record of the work carried out during a period from July 2018 to May 2019 under the supervision of **Dr. Yogesh Vijay Hote**, **Department of Electrical Engineering, Indian Institute of Technology, Roorkee**. The matter presented in this dissertation has not been submitted by me for the award of any other degree of this institute or any other institute.

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ABSTRACT

The controller design has been a prime focus since the evolution of control theory. Since then, various methodologies for controller design have come up. In classical controller design domain, one such technique is the Internal Model Control. This dissertation work presents a deep insight in the Internal Model Control (IMC) theory. The IMC theory is built around the internal model principle. The overall IMC strategy is analyzed thoroughly. Although there are many other advantages of IMC, one exciting fact about this technique is that the control is achieved via only one tuning parameter (λ). In spite of having some fine advantages, there is a particular drawback that there isn't any systematic tuning rules for finding the value of λ. In literature, some techniques have been developed so far for the evaluation of λ but most of them have failed in some or the other way. Because of this, usually researchers opt for the hit and trial method. To overcome this difficulty, this dissertation presents a new approach in the quantitative computation of the tuning parameter λ. This has been carried out by incorporating the advanced control techniques in the classical control, i.e. the Linear Quadratic Regulator (LQR) approach in IMC. A systematic algorithm for determining the tuning parameters is developed and elucidated in an orderly manner. The proposed approach was successfully applied on the different types of plant models and for Load Frequency Control (LFC) problem for power systems. The system responses are simulated and the results are neatly depicted. The simulation results are testimony to the efficacy of the proposed technique.

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CHAPTER 1

Introduction

1.1 Motivation

Control and automation has a huge potential in a world where everything is progressing so rapidly. Thus the study of control system plays a very important role for the development of the world. The role of a control engineer revolves mainly around meeting the following four objectives system modeling, stability analysis, performance analysis and controller design. The designing of a controller plays an important role since it has to be done keeping in mind the several uncertainties such as the effect of disturbances on the model, etc.

1.2 Problem Statement and Author's Contribution

Since the evolution of control theory, various methodologies for controller design have come up. One such technique is the Internal Model Control (IMC), in which the controller is based mainly on the exact model of the plant. The highlight of this technique is that the controller has only one tuning parameter i.e. λ. And because of only one parameter, the tuning can be easily achieved, as compared to other popular methods which have more tuning parameters, and finding these controller parameters can be more complex. However, there isn't any systematic method for finding the value of λ. Some techniques have been developed so far for the evaluation of λ but most of them are not as effective. In fact, the selection of the tuning parameter has been often done by trial and error method.

Thus, keeping this in mind, a method has been proposed to find the value of the tuning parameter λ. In this method, we incorporate the advanced control techniques with the classical control technique i.e., the Linear Quadratic Regulator approach in IMC Scheme. The aim is to construct a systematic algorithm for determining the tuning parameters with the help of LQR.

1.3 Dissertation Outline

This dissertation is divided into the sections as follows. In the second chapter, the Internal Model Control theory is studied thoroughly. The IMC principle is defined around which the whole IMC theory is built. The properties and the designing procedure of IMC is explained in depth. The drawbacks of one-degree of freedom IMC are studied. The discussion is then moved over to the two-degree of freedom IMC. Its structural analysis is done and its advantages over one-degree of freedom IMC are discussed. In the third chapter, the Linear Quadratic Regulator approach is discussed. The basic LQR technique is studied comprehensively. The significance of the Q and R matrices are discussed. In the fourth chapter, the proposed theory is explained exhaustively. The design algorithm that is suggested is jotted down in step by step methodology. To check the effectiveness of the proposed approach, it is applied on different systems and models, which is discussed in the fifth chapter. Also the simulation results and the performance analysis that has been carried out is shown neatly. In chapter sixth, the dissertation has been concluded and the future work that can be carried out is discussed. Finally, the publication summary has been included in the end, which is based on the work carried out in the dissertation.

CHAPTER 2

Internal Model Control

2.1 Introduction

In this chapter the Internal Model Control method is studied thoroughly. The studies on IMC structure came into light during the 1980s. A theory of inferential control was put forth by Joseph and Brosilow [1-2]. And in 1982, researchers named Garcia and Morari put down a firm theoretical methodology in this area [3]. Since then, Manfred Morari and his colleagues worked meticulously in this research area and greatly expanded it [4]. Basically, the IMC controller is designed to achieve control over these two basic objectives [5]. Firstly, the system's response to set point changes should be in a particularly desired manner. Secondly, the system should oppose the consequences of disturbance in the process. Thus the Internal Model Control technique was developed keeping in mind these main objectives. To attain these objectives, the IMC is designed accordingly and the design methodology and various other features of IMC are discussed below.

2.2 Internal Model Principle

The Internal Model Control theory is developed around the internal model principle. This principle states that, "if any control system involves, implicitly or explicitly, some representation of the process to be controlled then a perfect control is easily achieved." In fact, if the controller has been designed based on the exact model of the process then perfect control is theoretically possible [3].

2.3 IMC Strategy

To demonstrate the effectiveness of the IMC principle, an open loop structure is considered below. Fig.1 shows the structure of an open loop model. $C(s)$ represents the controller and $G(s)$ represents the plant model [5].

Figure 1: Open loop model

The controller $C(s)$, sets control on the plant $G(s)$ and the output $Y(s)$ can be found as:

$$Y(s) = R(s)C(s)G(s) \tag{1}$$

Consider a plant model $\tilde{G}(s)$, such that it is an exact representation of the process $G(s)$**,** i.e.

$$\tilde{G}(s) = G(s) \tag{2}$$

then set point tracking is achieved by taking a controller so that:

$$C(s) = \tilde{G}(s)^{-1} \tag{3}$$

Substituting (2) and (3) in (1), the output will be,

$$Y(s) = R(s) \tag{4}$$

Thus, it can be seen that the output follows the input and a perfect control is achieved. Open loop control systems are not as commonly used as closed loop control systems because of the issue of accuracy. To get a deeper insight, a closed loop control system is considered.

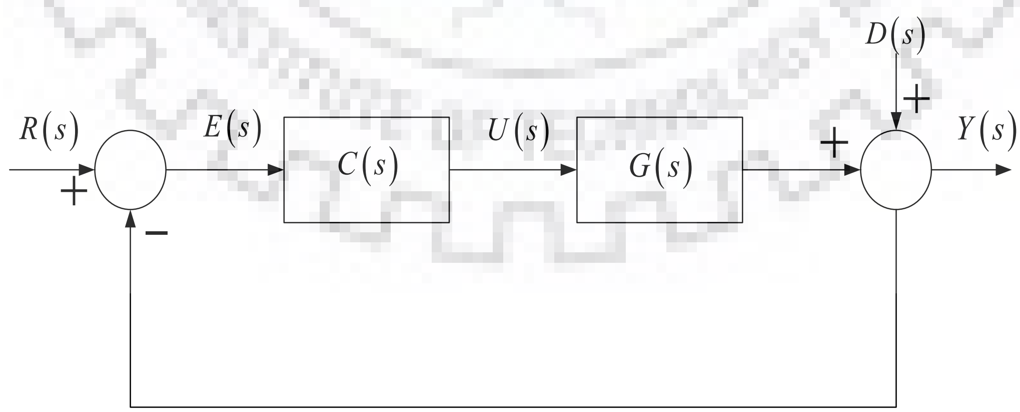

Figure 2: Structure of a classical feedback control.

A block diagram of a classical feedback control structure is shown in the Fig.2. IMC can be regarded as an exceptional case of classical feedback structure. For demonstrating this, we add and subtract the plant model $\tilde{G}(s)$, as shown in Fig. 3.

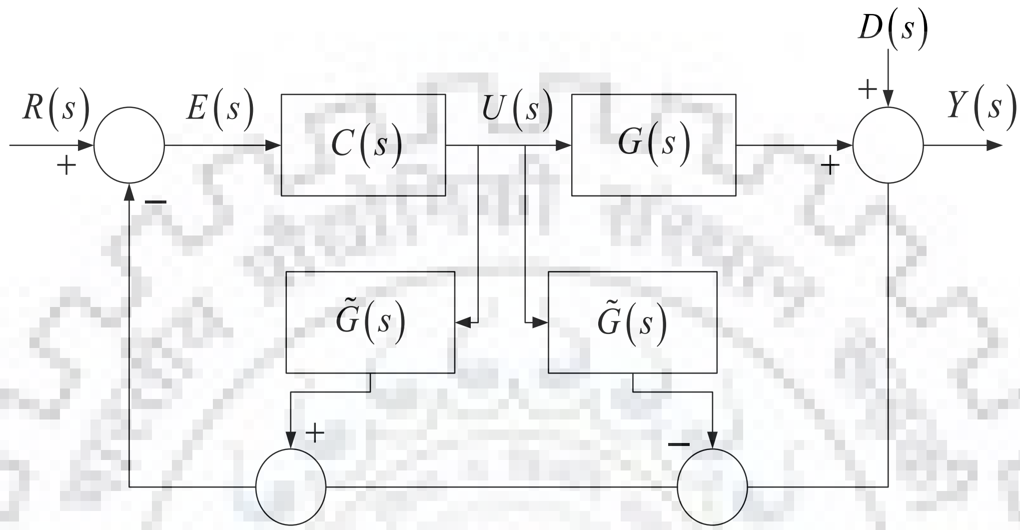

Figure 3: Plant model inserted to feedback path [3]

The structure then obtained, can be further adjusted to get a structure as shown in Fig. 4. The model $\tilde{G}(s)$ and $C(s)$ together forms a new controller $Q(s)$, as shown in Fig. 5, which is a basic IMC structure.

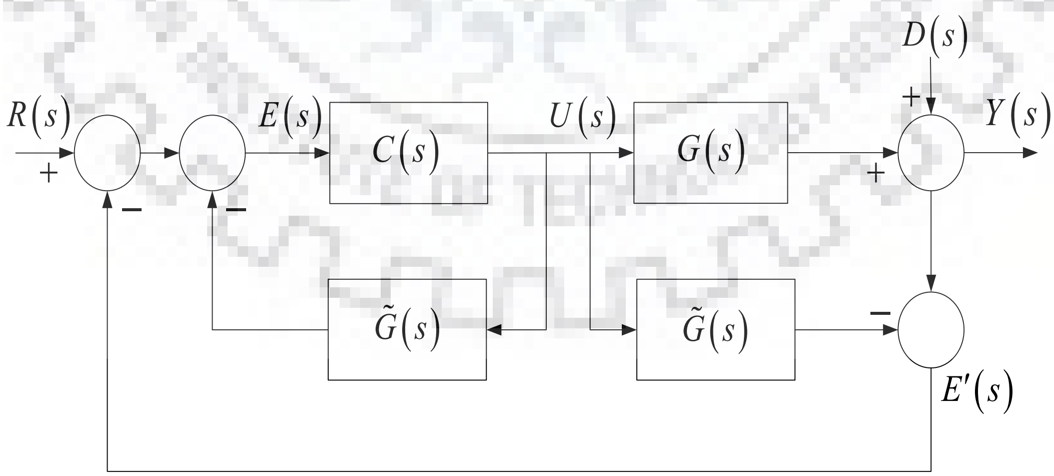

Figure 4: Basic feedback controller with plant model [3]

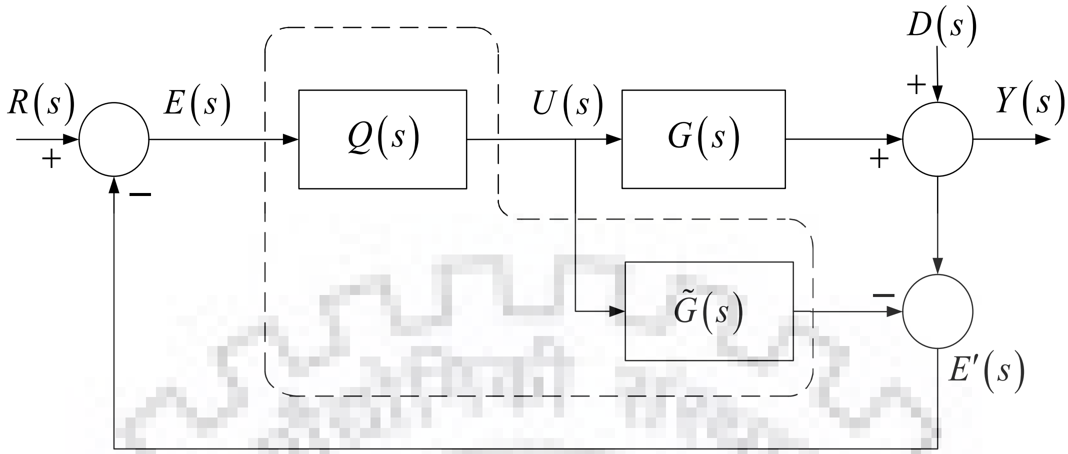

Figure 5: Basic IMC structure [3]

Now the relation between IMC controller $Q(s)$ and the classical controller can be given by the following two equations [5].

$$Q(s)=\frac{C(s)}{1+\tilde{G}(s)C(s)} \tag{5}$$

$$C(s)=\frac{Q(s)}{1-\tilde{G}(s)Q(s)} \tag{6}$$

The controller $Q(s)$ and the internal model $\tilde{G}_p(s)$ together characterize the IMC system.

2.4 Analysis of the IMC Structure

Consider the block diagram of IMC given in Fig.5. From the block diagram, it is seen that the error signal $E(s)$ is given by,

$$E(s)=R(s)-E'(s) \tag{7}$$

The control signal $U(s)$ is given as,

$$U(s)=E(s)Q(s) \tag{8}$$

Substituting (7) in (8), $U(s)$ can thus be written as,

$$U(s)=\left[R(s)-E'(s)\right]Q(s) \tag{9}$$

Now, $E'(s)$ can be written as,

$$E'(s)=\left[G(s)-\tilde{G}(s)\right]U(s)+D(s) \tag{10}$$

In the above expression, $D(s)$ is the disturbance affecting the system. Using (10), (9) can be modified as,

$$U(s)=\left[R(s)-\left\{\left[G(s)-\tilde{G}(s)\right]U(s)+D(s)\right\}\right]Q(s) \tag{11}$$

$$U(s)=\frac{\left[R(s)-D(s)\right]Q(s)}{1+\left[G(s)-\tilde{G}(s)\right]Q(s)} \tag{12}$$

From the block diagram, the output $Y(s)$ can be written by

$$Y(s)=G(s)U(s)+D(s) \tag{13}$$

Substituting $U(s)$ in (13),

$$Y(s)=G(s)\frac{\left[R(s)-D(s)\right]Q(s)}{1+\left[G(s)-\tilde{G}(s)\right]Q(s)}+D(s) \tag{14}$$

Further simplifying,

$$Y(s)=\frac{G(s)R(s)Q(s)-G(s)D(s)Q(s)+D(s)+D(s)G(s)Q(s)-D(s)\tilde{G}(s)Q(s)}{1+\left[G(s)-\tilde{G}(s)\right]Q(s)} \tag{15}$$

$$Y(s)=\frac{G(s)R(s)Q(s)+\left[1-\tilde{G}(s)Q(s)\right]D(s)}{1+\left[G(s)-\tilde{G}(s)\right]Q(s)} \tag{16}$$

Thus the output equation of an IMC structure [5] is obtained as,

$$Y(s)=\frac{G(s)R(s)Q(s)}{1+\left[G(s)-\tilde{G}(s)\right]Q(s)}+\frac{\left[1-\tilde{G}(s)Q(s)\right]D(s)}{1+\left[G(s)-\tilde{G}(s)\right]Q(s)} \tag{17}$$

The above expression can be modified as,

$$Y(s) = \eta R(s) + \varepsilon D(s) \tag{18}$$

The complimentary sensitivity function is represented by η and the sensitivity function is given as ε, where,

$$\eta = \frac{G(s)Q(s)}{1+\left[G(s)-\tilde{G}(s)\right]Q(s)} \;\&\; \varepsilon = \frac{\left[1-\tilde{G}(s)Q(s)\right]}{1+\left[G(s)-\tilde{G}(s)\right]Q(s)} \tag{19}$$

The complimentary sensitivity function η accounts for the tracking performance of the system, whereas, the sensitivity function ε tells about the robustness of the system.

From the above expressions it can be seen that for a perfect control, i.e., for the system's output to follow the set reference point, the plant model $\tilde{G}(s)$ must be an exact approximation of the process $G(s)$. In other words,

$$\tilde{G}(s) = G(s) \tag{20}$$

Also, to bring the effects of disturbance to zero, the controller $Q(s)$ must be designed such that it is inverse of the plant model $\tilde{G}(s)$. That is,

$$Q(s) = \tilde{G}(s)^{-1} \tag{21}$$

The above strategy is thus proved to deliver perfect set point tracking and disturbance rejection.

2.5 Properties of IMC

As suggested by Garcia and Morari [3], the advantages of IMC structure can be explained with the following three properties.

1. **Dual Stability**: This property of IMC states that if there exists a model that is exact representation of a the system, i.e., if $\tilde{G}(s) = G(s)$, then one can easily say that if the

open loop system is stable, the closed loop stability is certainly ensured. In other words, if model is exact, for overall stability the stability condition of the plant and the controller is sufficient.

2. **Perfect Control**: If the plant model is exact, i.e., if $\tilde{G}(s) = G(s)$ and if the controller is designed such that it is inverse of the plant model, i.e., $Q(s) = \tilde{G}(s)^{-1}$ then the system will be perfectly controlled, provided the closed loop system is stable. The output will follow the set point perfectly.

3. **Zero Offset**: If we select the IMC controller $Q(s)$ as the inverse of plant model $\tilde{G}(s)$ then the equations (19) the denominator terms will be equal to $G(s)Q(s)$ and the gain between the output $Y(s)$ and $R(s)$ will be unity. Also the gain between output $Y(s)$ and $D(s)$ will be zero. This validates that the steady state deviation between the output and $R(s)$ will not exist and there will be zero offset.

2.6 Internal Model Control Design Algorithm

In this section, the process of designing the controller by IMC technique [6] is discussed. Represent the plant as $G(s)$ and the process model as $G(s)$, such that

$$\tilde{G}(s) = G(s) \tag{22}$$

Since the inverse of the process model is to be taken, it is important to make sure that it is invertible. For this, we factorise the process model into minimum phase (invertible) and non-minimum phase (non-invertible) parts,

$$\tilde{G}(s) = \tilde{G}^{+}(s)\tilde{G}^{-}(s) \tag{23}$$

$\tilde{G}^{+}(s)$ represents the Non-minimum phase elements (noninvertible), which means the right half plane (RHP) zeros andλtime delays. $\tilde{G}^{-}(s)$ represents Minimum phase elements that are invertible.

Thus we take the inverse of the Minimum phase element as,

$$Q_1(s) = \tilde{G}^{-}(s)^{-1} \tag{24}$$

If it is not possible to factorize the elements of the process model, then we check if it is stable and that all the poles are on the left half side of the s-plane, then we can say that the model is invertible.

If there are only non-invertible elements in the process model, then we have to use other factorization techniques because the non-invert ability of the components may cause instability when they are inverted.

If the controller $Q_1(s)$ is improper, we need to augment a filter with the controller to make it proper. To improve the robustness of the system, it is necessary that the effect of model mismatch is reduced.

Since, at high frequency end, the occurrence of model mismatch between the plant and the process is highly possible, a low pass filter $F(s)$ is normally included to suppress these effects .Thus the IMC controller $Q(s)$ is designed as the low pass filter augmented with the inverted model $Q_1(s)$.

$$Q(s) = Q_1(s)F(s) \tag{25}$$

$$Q(s) = \left[\tilde{G}^{-}(s)\right]^{-1} F(s) \tag{26}$$

The low pass filter that is to be adjoined is usually of the form,

$$F(s) = \frac{1}{(\lambda s + 1)^n} \quad \text{or} \tag{27}$$

$$F(s) = \frac{n\lambda + 1}{(\lambda s + 1)^n} \tag{28}$$

where, n is used to make $Q_1(s)$ proper or semi-proper and λ is the filter tuning parameter. Filters other than the form given above can also be used to improve performance but the robustness of the system is compromised.

The parameter λ is used to vary the speed of response of the closed loop system. Thus the practical IMC structure with a single adjustable parameter λ is depicted in the Fig.6

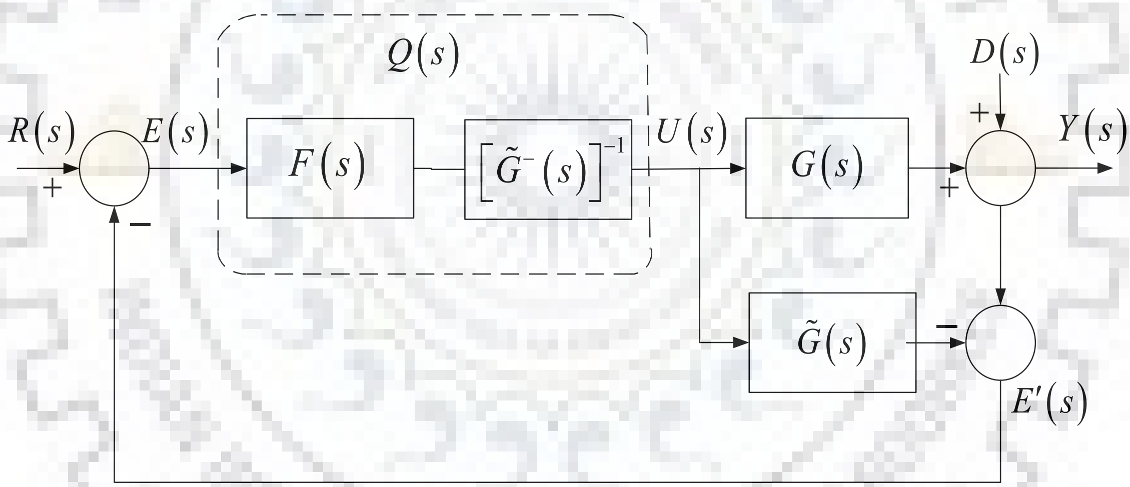

Figure 6: Schematic of practical IMC structure [5]

When the value of λ is increased, higher robustness performance is assured, but the time constant increases and thus slows the speed of response and of the system. On the other hand if the value of λ is decreased beyond a certain point, the reference tracking performance is perfect but the disturbance rejection is compromised [6].

2.7 Modified IMC structure

In control systems, there are some applications in which the problem of set-point tracking is not as important as that of dealing with problem of disturbance rejection. The problem of disturbance rejection becomes severe with systems having a time delay. Some systems have a set-point that barely changes, but time-delay occurs. So, there is a need for a control theory that enables us to design systems that can independently control the disturbance rejection and the set-point tracking. Due to this concern, modified forms of the IMC structure were developed. One of the most effectively used structures is discussed henceforth.

2.8 Two Degree of Freedom IMC

Various forms of 2-DOF IMC structures have been put forth for different kinds of systems. For example, the integrating processes, unstable processes and systems in need of optimal set-point rejection and disturbance tracking require a little distinct internal model control structure.

Here we are mainly focusing on the optimization of eliminating the effects of disturbance and better tracking of the set-point. As discussed previously, the IMC structure is capable of giving excellent reference tracking but is unsuccessful in achieving satisfactory disturbance rejection. In the original IMC structure, there is a trade-off between the desired set-point tracking performance and the disturbance rejection performance of the system. Because of this, we can't achieve good performance for both elimination of disturbance and the reference tracking simultaneously, we have to make a compromise between either of them.

So, to eliminate the aforementioned shortcomings a two degree of freedom IMC structure [5] is established, in which two controllers $Q_d(s)$ and $Q(s)$ are included in the original IMC structure. $Q_d(s)$ is responsible for disturbance rejection performance and $Q(s)$ is responsible for the set point tracking.

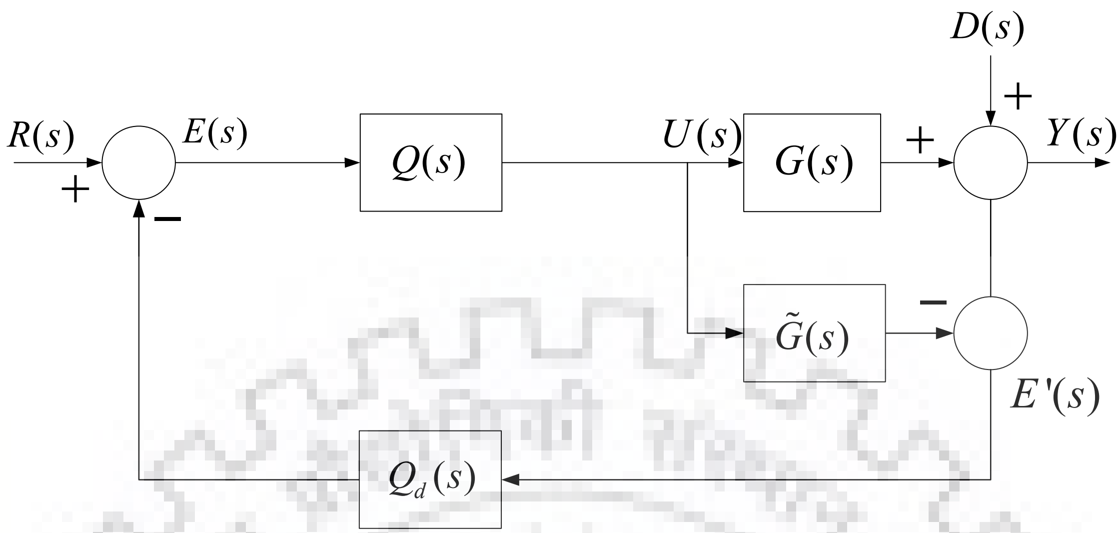

Figure 7: Two Degree of Freedom IMC structure [7]

To apply this design scheme developed by Liu and Gao [7], we consider the IMC structure shown in the Fig.7.

From the block diagram of two degree of freedom IMC the error signal $E(s)$ is given by,

$$E(s) = R(s) - Q_d(s)E'(s) \tag{29}$$

The control signal $U(s)$ is given as,

$$U(s) = E(s)Q(s) \tag{30}$$

Substituting (29) in (8), $U(s)$ can thus be written as,

$$U(s) = \left[R(s) - Q_d(s)E'(s)\right]Q(s) \tag{31}$$

Now, $E'(s)$ can be written as,

$$E'(s) = \left[G(s) - \tilde{G}(s)\right]U(s) + D(s) \tag{32}$$

In the above expression, $D(s)$ is the disturbance affecting the system. Using (32), (31) can be modified as,

$$U(s)=\left[R(s)-Q_d(s)\left\{\left[G(s)-\tilde{G}(s)\right]U(s)+D(s)\right\}\right]Q(s) \tag{33}$$

$$U(s)=\frac{R(s)Q(s)-Q(s)Q_d(s)D(s)}{1+\left[G(s)-\tilde{G}(s)\right]Q(s)Q_d(s)} \tag{34}$$

From the block diagram, the output $Y(s)$ can be written by

$$Y(s)=G(s)U(s)+D(s) \tag{35}$$

Substituting $U(s)$ in (35),

$$Y(s)=G(s)\frac{R(s)Q(s)-Q(s)Q_d(s)D(s)}{1+\left[G(s)-\tilde{G}(s)\right]Q(s)Q_d(s)}+D(s) \tag{36}$$

Further simplifying,

$$Y(s)=\frac{G(s)Q(s)R(s)-G(s)Q(s)Q_d(s)D(s)+D(s)+G(s)Q(s)Q_d(s)D(s)-\tilde{G}(s)Q(s)Q_d(s)D(s)}{1+\left[G(s)-\tilde{G}(s)\right]Q(s)Q_d(s)} \tag{37}$$

$$Y(s)=\frac{G(s)Q(s)R(s)+\left[1-\tilde{G}(s)Q(s)Q_d(s)\right]D(s)}{1+\left[G(s)-\tilde{G}(s)\right]Q(s)Q_d(s)} \tag{38}$$

Thus the output equation of a two degree of freedom IMC structure is obtained as,

$$Y(s)=\frac{G(s)Q(s)R(s)}{1+\left[G(s)-\tilde{G}(s)\right]Q(s)Q_d(s)}+\frac{\left[1-\tilde{G}(s)Q(s)Q_d(s)\right]D(s)}{1+\left[G(s)-\tilde{G}(s)\right]Q(s)Q_d(s)} \tag{39}$$

From the above expressions it can be seen that for a perfect control, i.e., for the system's output to follow the set reference point, the plant model $\tilde{G}(s)$ must be an exact approximation of the process $G(s)$. In other words,

$$\tilde{G}(s) = G(s) \tag{40}$$

Also, the controller $Q(s)$ must be designed such that it is inverse of the plant model $\tilde{G}(s)$. That is,

$$Q(s) = \tilde{G}(s)^{-1} \tag{41}$$

Now, in two degree of freedom IMC, to bring the effects of disturbance to zero, the equation (42) must also be satisfied.

$$1 - \tilde{G}(s)Q(s)Q_d(s) = 0 \tag{42}$$

Thus the disturbance rejection controller $Q_d(s)$ is designed such that the above expression is satisfied. This is achieved by using a filter of the form (43) in designing of $Q_d(s)$ which is different from the one used in basic IMC.

$$f(s) = \frac{\beta_m + \cdots + \beta_1 + 1}{(\lambda_d s + 1)^m} \tag{43}$$

Where, m is the no. of poles of $\tilde{G}(s)$ and λ_d is the filter tuning parameter of disturbance rejection controller. $\beta_1, \beta_2 \cdots \beta_m$ are selected such that they should satisfy the condition (44) for each pole $p_1, p_2, \cdots p_m$ of the plant.

$$\left(1 - \tilde{G}(s)Q(s)Q_d(s)\right)\Big|_{s=p_1, p_2, \cdots p_m} = 0 \tag{44}$$

The designing procedure for $Q(s)$ is similar to that used in basic IMC (26). But, $Q_d(s)$ is designed by using filter of the form given by (43). Thus $Q_d(s)$ will be of the form,

$$Q_d(s) = \left[\tilde{G}(s)\right]^{-1} f(s) \tag{45}$$

$$Q_d(s) = \frac{\left[\tilde{G}(s)\right]^{-1}\left(\beta_m + \cdots + \beta_1 + 1\right)}{\left(\lambda_d s + 1\right)^m} \tag{46}$$

By employing the above strategy in the new modified two degree of freedom IMC, one can achieve independent control over the reference tracking and disturbance rejection performance of the system easily.

CHAPTER 3

Linear Quadratic Regulator

3.1 LQR Approach

The quadratic regulator technique lays a theory that allows determining an optimal solution of a particular problem. This is accomplished by defining a certain performance index and then working out through a procedure to obtain the optimum solution. The LQR method gives an organised way of evaluating the control gain matrix. The prime focus of this method is to find out a control signal that usually minimizes a certain cost function; this cost function is also called as performance index [8].

Let us consider a system,

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned} \tag{47}$$

The state feedback control is given by,

$$u = -Kx \tag{48}$$

Here, K is the gain matrix. We have to find the optimum solution of K so that it minimizes the cost function given as,

$$J = \int_0^\infty \left(x^T Q x + u^T R u \right) dt \tag{49}$$

In the above performance index, Q is a symmetric positive semi-definite matrix and R is a symmetric positive definite matrix. The value of R defines the amount of control energy applied; large value of R stabilizes the system with less control effort. The matrix Q governs the effect of changes in the states; large value of Q implies, the system is stabilized with least possible changes in states.

From (47) and (48), we get the system equation as,

$$\dot{x} = \left(A - BK\right)x \tag{50}$$

We can prove that $\left(A - BK\right)$ is stable, i.e. the eigenvalues will be on the left side of the imaginary axis.

From (49) and (50) the performance index becomes,

$$J = \int_0^\infty x^T \left(Q + K^T R\, K\right) x\, dt \tag{51}$$

Considering,

$$\frac{d}{dt}\left(x^T P x\right) = -\left(x^T \left(Q + K^T R\, K\right) x\right) \tag{52}$$

Where is P symmetric positive definite matrix, we get,

$$x^T P\dot{x} + \dot{x}^T P x = -\left(x^T \left(Q + K^T R\, K\right) x\right) \tag{53}$$

From (50) and (53), we get,

$$x^T \left(\left(A - BK\right)^T P + P\left(A - BK\right)\right) x = -x^T \left(Q + K^T R\, K\right) x \tag{54}$$

$$\left(A - BK\right)^T P + P\left(A - BK\right) = -Q + K^T R\, K \tag{55}$$

Since $\left(A - BK\right)$ matrix is stable, we can say that there exists a matrix P that satisfies the equation (55).

The gain matrix *K* is found to be as,

$$K = R^{-1} B^T P \tag{56}$$

The positive definite matrix P in the equation must satisfy the Algebraic Riccati Equation (ARE) given by (57) which is the reduced form of (55).

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \tag{57}$$

Hence, the gain matrix K gives the optimum state feedback control.

The matrices Q and R are to be assumed for solving the Riccati Equation. There are different ways of selecting the matrices Q and R which depend on the particular application that it has been used in [9]. Some of them are mentioned below.

- One of the most basic way is to use trial and error method. However, it doesn't ensure accuracy.
- Choose Q as an identity matrix and R as some factor σ multiplied with an identity matrix. This is an elementary choice of Q and R

$$Q = I \;\&\; R = \sigma I \tag{58}$$

- Another way is by choosing diagonal weights for Q such that it is of the form (59) where, $q_{11} > q_{22} > \cdots q_{nn}$ and R is selected such that $R = \sigma I$.

$$Q = diag\left(q_{11}, q_{22}, \cdots, q_{nn}\right) \tag{59}$$

- In one particular way of selecting matrices Q and R , the output matrix C from (47) is used. In this Q is taken as,

$$Q = C^T C \;\&\; Q = C^T C \tag{60}$$

One can select any of these methods according to their desired performance and suitability.

CHAPTER 4

Internal Model Control based on LQR

4.1 Proposed Approach

In the IMC controller, the tuning parameter has been usually determined by trial and error method until now. Hence a new method of determining the tuning parameters of IMC controller is proposed. The aim is to determine the tuning parameter by a step by step algorithm. A brief overview of the method is explained below.

Initially, the controller is determined by applying IMC scheme. It is seen that the controller equation is in terms of the tuning parameters. This controller and the plant model is used to get the closed loop characteristic equation, that will also come in terms of tuning parameter. Further, through LQR, we find out the feedback gain matrix. With this feedback gain matrix we find out the eigenvalues. A closed loop characteristic equation with the help of the eigenvalues is found out.

Now, both the closed loop equations are compared and the values of tuning parameters are determined. Hence, the proposed approach provides a systematic method for determining the values of tuning parameters.

4.2 Proposed IMC Design using LQR

The designing of the IMC controller based on LQR approach is described as follows. This method can be applied to a system of any order. Suppose the plant model is considered to be a third order system as,

$$\widehat{G}(s) = \frac{B_0}{s^3 + A_1 s^2 + A_2 s + A_3} \tag{61}$$

Step 1: Model order reduction by using Routh Approximation method [10]:

First, reciprocate the plant model [4, 11]. Using (61), we can write,

$$\breve{G}(s) = \frac{B_0}{A_3 s^3 + A_2 s^2 + A_1 s + 1} \tag{62}$$

Using [10], reduced order model can be determined as follows. The parameters of reduced model are expressed in terms of the α and β. In [10], they are expressed in a tabular form as shown in Table 1. Thus, α_i and β_i terms are used to find the reduced i^{th} order numerator P_i and denominator Q_i. Using this approach, 2nd order reduced model can be expressed as

$$G(s) = \frac{P_2(s)}{Q_2(s)} \tag{63}$$

where,

$$\begin{aligned} P_2(s) &= \beta_2 + \alpha_2 \beta_1 s \\ Q_2(s) &= 1 + \alpha_2 s + \alpha_1 \alpha_2 s^2 \end{aligned} \tag{64}$$

Then, reciprocate the numerator and denominator to get the final reduced order model. Thus, the reduced model comes out to be in the form,

$$G(s) = \frac{b_0}{s^2 + a_1 s + a_2} \tag{65}$$

where, $b_0 = \beta_1, a_1 = \alpha_2, a_2 = \alpha_1 \alpha_2$.

Table 1: Alpha and Beta table for Routh Approximation.

α - Table		
	A_3	A_1
$\alpha_1 = \frac{A_3}{A_2}$	A_2	1
$\alpha_2 = \frac{A_2^2}{A_2 A_1 - A_3}$	$\frac{A_2 A_1 - A_3}{A_2}$	
β - Table		
	B_0	0
$\beta_1 = \frac{B_0}{A_2}$	0	
$\beta_2 = 0$	0	

Step 2: Design of IMC controller for the given reduced model [11]:

Represent the plant as $G(s)$, and the process model as $\tilde{G}(s)$, such that,

$$\tilde{G}(s) = G(s) \tag{66}$$

Factorise the process model into minimum phase (invertible) and non-minimum phase (non-invertible) parts,

$$\tilde{G}(s) = \tilde{G}^{+}(s)\tilde{G}^{-}(s) \tag{67}$$

$\tilde{G}^{+}(s)$ represents the non-minimum phase elements (noninvertible) and $\tilde{G}^{-}(s)$ represents minimum phase elements that are invertible. The IMC controller is defined by,

$$Q(s) = \left[\tilde{G}^{-}(s)\right]^{-1} f(s) \tag{68}$$

The proposed filter is of the form ,

$$f(s) = \frac{p}{(\lambda s+1)(s+p)} \tag{69}$$

where, λ is the filter tuning parameter and p is the pole that is augmented to make $Q(s)$ proper. Assume $G(s)$ is invertible. Thus, from (65), (68) and (69), we write,

$$Q(s) = \frac{\left(s^2 + a_1 s + a_2\right)p}{b_0(\lambda s+1)(s+p)} \tag{70}$$

The relation between IMC and the classical controller can be given by the following equation.

$$C(s) = \frac{Q(s)}{1 - G(s)Q(s)} \tag{71}$$

Substituting (65) and (70) into (71), the controller equation comes out to be,

$$C(s) = \frac{\left(s^2 + a_1 s + a_2\right)p}{b_0\left[(\lambda s+1)(s+p) - p\right]} \tag{72}$$

Step 3: Get the closed loop characteristic equation from the controller (72) designed by IMC approach. The above controller $C(s)$ and system $G(s)$ can be written in terms of conventional closed-loop characteristic equation. We know the characteristic equation is given by,

$$1+G(s)C(s)=0 \tag{73}$$

$$1+\left(\frac{b_0}{\left(s^2+a_1s+a_2\right)}\right)\left(\frac{\left(s^2+a_1s+a_2\right)p}{b_0\left[(\lambda s+1)(s+p)-p\right]}\right)=0 \tag{74}$$

Simplifying (74), we get,

$$(\lambda s+1)(s+p)-p+p=0 \tag{75}$$

$$(\lambda s+1)(s+p)=0 \tag{76}$$

$$\lambda s^2+(\lambda p+1)s+p=0 \tag{77}$$

Simplifying further, the closed loop characteristic equation in terms of λ and p is obtained as,

$$s^2+\frac{(\lambda p+1)}{\lambda}s+\frac{p}{\lambda}=0 \tag{78}$$

Step 4: Design of LQR approach for state space model: Consider the state space model of (65) as,

$$\begin{aligned}\dot{x}&=Ax+Bu\\ y&=Cx\end{aligned} \tag{79}$$

The above system is expressed in terms of controllable form. This form can be written as

$$\begin{aligned}\dot{x}&=A_1x+B_1u\\ y&=C_1x\end{aligned} \tag{80}$$

The above system matrices A_1, B_1 and C_1 can be written as

$$A_1=\begin{bmatrix}0 & 1\\ -a_2 & -a_1\end{bmatrix},\quad B_1=\begin{bmatrix}0\\ b_0\end{bmatrix},\quad C_1=\begin{bmatrix}1 & 0\end{bmatrix} \tag{81}$$

The performance index to be minimized [12] is taken as,

$$J = \int_0^{\infty} \left(x^T Q x + u^T R u \right) dt \tag{82}$$

State feedback control matrix u is given as,

$$u = -Kx \tag{83}$$

Hence, system equation becomes,

$$\dot{x} = \left(A_1 - B_1 K \right) x \tag{84}$$

The feedback gain matrix K is,

$$K = R^{-1} B_1^T P \tag{85}$$

The positive definite matrix P is found by solving the Algebraic Riccati Equation (ARE) [13] given as,

$$PA_1 + A_1^T P - PB_1 R^{-1} B_1^T P + Q = 0 \tag{86}$$

The positive semi definite matrix Q is taken as, $Q = diag\left(q_{11}, q_{22} \right)$, where, $q_{11} > q_{22}$, and the positive definite matrix R is taken as, $R > 0$. The matrix P is of the form given below,

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \tag{87}$$

Substituting (81) and (87) in (85), gives,

$$\begin{aligned} K &= R^{-1} B_1^T P \\ &= \left[R \right]^{-1} \left[0 \quad b_0 \right] \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \end{aligned} \tag{88}$$

Thus the gain matrix K is,

$$K = \begin{bmatrix} \dfrac{b_0 p_{12}}{R} & \dfrac{b_0 p_{22}}{R} \end{bmatrix} \tag{89}$$

Step 5: From the gain matrix K , computed in (89), get the matrix $[A_1 - B_1K]$ as,

$$A_1 - B_1K = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix} - \begin{bmatrix} 0 \\ b_0 \end{bmatrix} \begin{bmatrix} \frac{b_0 . p_{12}}{R} & \frac{b_0 . p_{22}}{R} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ \frac{b_0{}^2 . p_{12}}{R} & \frac{b_0{}^2 . p_{22}}{R} \end{bmatrix} \tag{90}$$

$$[A_1 - B_1K] = \begin{bmatrix} 0 & 1 \\ -a_2 - \frac{b_0{}^2 p_{12}}{R} & -a_1 - \frac{b_0{}^2 p_{22}}{R} \end{bmatrix} \tag{91}$$

From (91), the eigenvalues can be found by the following characteristic equation,

$$|sI - (A_1 - B_1K)| = 0 \tag{92}$$

$$[sI - (A_1 - B_1K)] = \begin{bmatrix} s & -1 \\ \left(a_2 + \frac{b_0{}^2 . p_{12}}{R}\right) & s + \left(a_1 + \frac{b_0{}^2 . p_{22}}{R}\right) \end{bmatrix} \tag{93}$$

$$|sI - (A_1 - B_1K)| = \begin{vmatrix} s & -1 \\ \left(a_2 + \frac{b_0{}^2 . p_{12}}{R}\right) & s + \left(a_1 + \frac{b_0{}^2 . p_{22}}{R}\right) \end{vmatrix}$$

$$= s^2 + \left(a_1 + \frac{b_0{}^2 . p_{22}}{R}\right)s + \left(a_2 + \frac{b_0{}^2 . p_{12}}{R}\right) \tag{94}$$

Hence, (95) gives the closed loop characteristic equation as (LQR approach)

$$s^2 + \left(a_1 + \frac{b_0{}^2 p_{22}}{R}\right)s + \left(a_2 + \frac{b_0{}^2 p_{12}}{R}\right) = 0 \tag{95}$$

Step 6: By comparing the characteristic equations from (78) and (95), the values of λ and p can be evaluated.

Comparing (78) and (95), the following two equations are obtained,

$$\frac{p}{\lambda} = \left(a_2 + \frac{b_0{}^2 p_{12}}{R}\right) \tag{96}$$

$$\frac{(\lambda p+1)}{\lambda}=\left(a_1+\frac{b_0{}^2 p_{22}}{R}\right) \tag{97}$$

Now, (96) can be written as,

$$p=\left(a_2+\frac{b_0{}^2 p_{12}}{R}\right)\lambda \tag{98}$$

Substituting (98) in (97), we get,

$$\frac{\left(a_2+\frac{b_0{}^2 . p_{12}}{R}\right)\lambda^2+1}{\lambda}=a_1+\frac{b_0{}^2 . p_{22}}{R} \tag{99}$$

$$\left(a_2+\frac{b_0{}^2 . p_{12}}{R}\right)\lambda^2+1=\left(a_1+\frac{b_0{}^2 . p_{22}}{R}\right)\lambda \tag{100}$$

Further simplifying, the following polynomial equation is obtained,

$$\lambda^2-\frac{\left(Ra_1+b_0{}^2 p_{22}\right)}{\left(Ra_2+b_0{}^2 p_{12}\right)}\lambda+\frac{R}{\left(Ra_2+b_0{}^2 p_{12}\right)}=0 \tag{101}$$

The solution of the polynomial equation (101) gives the optimum value of λ. From this, λ calculate p from (98).

Step 7: The IMC controller $C(s)$ which was formulated in (72), is reconfigured into an ideal PID form [5]. From (72), $C(s)$ is given as,

$$C(s)=\frac{\left(s^2+a_1 s+a_2\right)p}{b_0\left[(\lambda s+1)(s+p)-p\right]} \tag{102}$$

The ideal PID form is given as,

$$C(s)=\frac{C_0+C_1\, s+C_2\, s^2}{s}$$
$$=\frac{K_i}{s}+K_p+K_d\, s \tag{103}$$

Therefore,

$$C_0=K_i,\quad C_1=K_p,\quad C_2=K_d \tag{104}$$

Rearrange (102), such that it is of the form (105),

$$C(s)=\frac{1}{s}F(s) \tag{105}$$

Using (102) and (105), we get,

$$C(s)=\frac{1}{s}\frac{\left(\frac{p}{b_0(\lambda p+1)}\ s^2+\frac{pa_1}{b_0(\lambda p+1)}\ s+\frac{pa_2}{b_0(\lambda p+1)}\right)}{\left[\left(\frac{\lambda}{\lambda p+1}\right)\ s+1\right]} \tag{106}$$

From (105) and (106), we can write,

$$F(s)=\frac{\left(\frac{p}{b_0(\lambda p+1)}\ s^2+\frac{pa_1}{b_0(\lambda p+1)}\ s+\frac{pa_2}{b_0(\lambda p+1)}\right)}{\left[\left(\frac{\lambda}{\lambda p+1}\right)\ s+1\right]} \tag{107}$$

Applying Taylor series to the function $F(s)$, we get,

$$F(s)=C_0+C_1\ s+C_2\ s^2 \tag{108}$$

where,

$$C_0=F(0),\quad C_1=F'(0),\quad C_2=\frac{F''(0)}{2!} \tag{109}$$

By comparing (104), (109) and using (107), the PID controller parameters can be obtained as

$$K_i=\frac{p}{b_0(\lambda p+1)}\ a_2 \tag{110}$$

$$K_p=\frac{p}{b_0(\lambda p+1)}\left[a_1-a_2\left(\frac{\lambda}{\lambda p+1}\right)\right] \tag{111}$$

$$K_d=\frac{p}{2b_0(\lambda p+1)}\times\left[1-a_1\left(\frac{\lambda}{\lambda p+1}\right)-a_2\left(\frac{\lambda}{\lambda p+1}\right)^2\right] \tag{112}$$

The formulae for PID parameters in terms of IMC tuning parameter λ and p are constructed. From step 7, the optimum value of λ and p are obtained by (101) and (98). By substituting these values in the formulae, constructed by (110), (111) and (112), PID controller parameters are obtained.

Thus LQR approach has been successfully applied to design the IMC controller, which gives a proper method for determining the tuning parameters of IMC.

This controller design technique is applied on some plant models in the next segment.

CHAPTER 5

Numerical Studies

5.1 Illustrative Examples

In this section the application of proposed approach is shown. The examples that are considered are discussed thoroughly so that one can get a better understanding of this technique. The plant models considered are of different types, so that the usefulness of the proposed technique is highlighted.

Example 1: Consider a second order system with time delay. This plant model is taken from [14] by S. Skogestad. The controller is designed by our proposed method and the results are compared with that of [14].

$$G(s) = \frac{e^{-s}}{(s+1)^2}$$

The plant model can be approximated as,

$$G(s) = \frac{(1-s)}{s^2+2s+1} \tag{113}$$

The state space model, as given by (81) is,

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & -1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \end{bmatrix} \tag{114}$$

The matrices Q and R are selected in a way such that $Q = diag(q_{11}, q_{22})$ and $R > 0$ as,

$$Q = \begin{bmatrix} 20 & 0 \\ 0 & 17 \end{bmatrix}, \quad R = [1] \tag{115}$$

Using (115), and solving equations (86) and (88). The feedback gain matrix is evaluated as,

$$K = \begin{bmatrix} 3.5826 & 3.3071 \end{bmatrix} \tag{116}$$

From (89), p_{12} and p_{22} are found out to be,

$$p_{12} = 3.5826, \quad p_{22} = 3.3071 \tag{117}$$

Using (117) and solving the polynomial (101). The value of λ is taken as the minimum of the two values, that are obtained by solving (101), for faster speed of response. Then by substituting the value of λ into (98), the value of p can be determined. These values of λ and p are obtained as

$$\lambda = 0.2368, \quad p = 1.0855 \tag{118}$$

Substituting the values of λ and p in (107), and applying Taylor series, the values of PID parameters are obtained as,

$$K_p = 0.8799, K_i = 0.4634, K_d = 0.3744 \tag{119}$$

Thus, the controller designed by the proposed method is,

$$C(s) = 0.8799 + \frac{0.4634}{s} + 0.3744\, s \tag{120}$$

Simulation Results: For a set point of unity, the response is simulated as shown by Fig. 8.

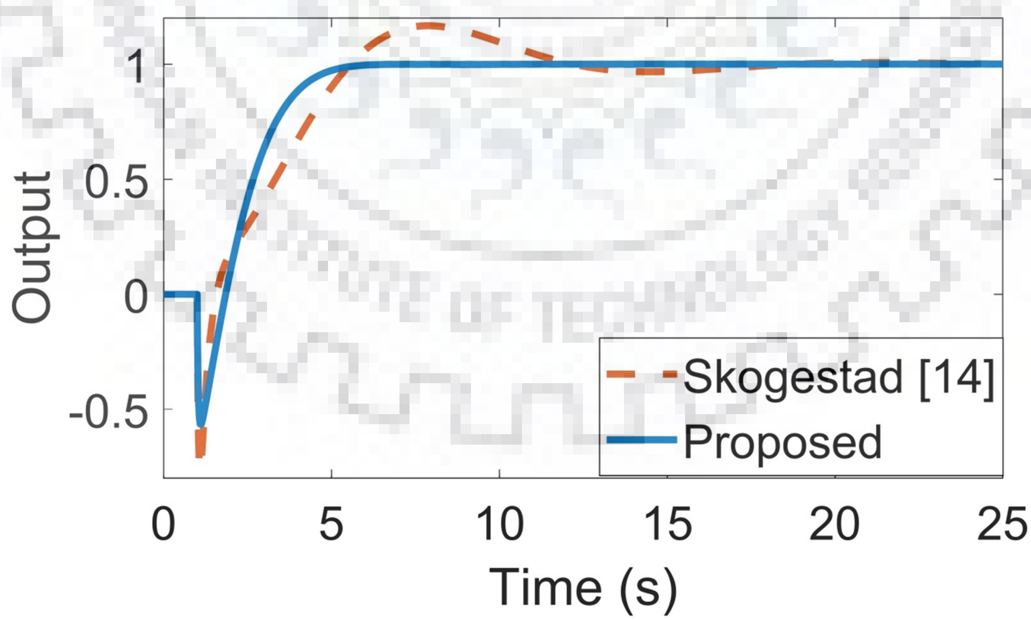

Figure 8: Response of a second order plus time delay system of Example 1

Example 2: This plant model is taken from [15]. The plant is such that it has integral action in the plant itself. The proposed method is applied as follows.

$$G(s)=\frac{e^{-s}}{s(s+1)}$$

The plant model of this process can be approximated as,

$$G(s)=\frac{1-s}{s^2+s} \tag{121}$$

The state space model, as given by (81) is,

$$A=\begin{bmatrix}0 & 1\\ 0 & -1\end{bmatrix},\quad B=\begin{bmatrix}0\\ 1\end{bmatrix}$$
$$C=\begin{bmatrix}1 & -1\end{bmatrix},\quad D=\begin{bmatrix}0\end{bmatrix} \tag{122}$$

The matrices Q and R are selected in a way such that $Q=diag(q_{11},q_{22})$ and $R>0$ as,

$$Q=\begin{bmatrix}10 & 0\\ 0 & 4\end{bmatrix},\quad R=[0.01] \tag{123}$$

Using (123) solve the equation (86) and (88). Thus, feedback gain matrix is evaluated as,

$$K=\begin{bmatrix}31.6228 & 20.5464\end{bmatrix} \tag{124}$$

From (89), p_{12} and p_{22} are found out to be,

$$p_{12}=0.3162,\quad p_{22}=0.2055 \tag{125}$$

Using (125) and solving the polynomial (101). The value of λ is taken as the minimum of the two values, that are obtained by solving (101), for faster speed of response. Then by substituting the value of λ into (98), the value of p can be determined. These values of λ and p are obtained as

$$\lambda=0.05,\quad p=1.58 \tag{126}$$

Substituting the values of λ and p in (107), and applying Taylor series, the values of PID parameters are obtained as,

$$K_p = 0.5948, K_i = 0, K_d = 0.5836 \tag{127}$$

Thus, the controller designed by the proposed method is,

$$C(s) = 0.5948 + 0.5836\, s \tag{128}$$

Simulation Results:

For example 2 the reference is set at unity and the output response is simulated as shown by Fig. 9.

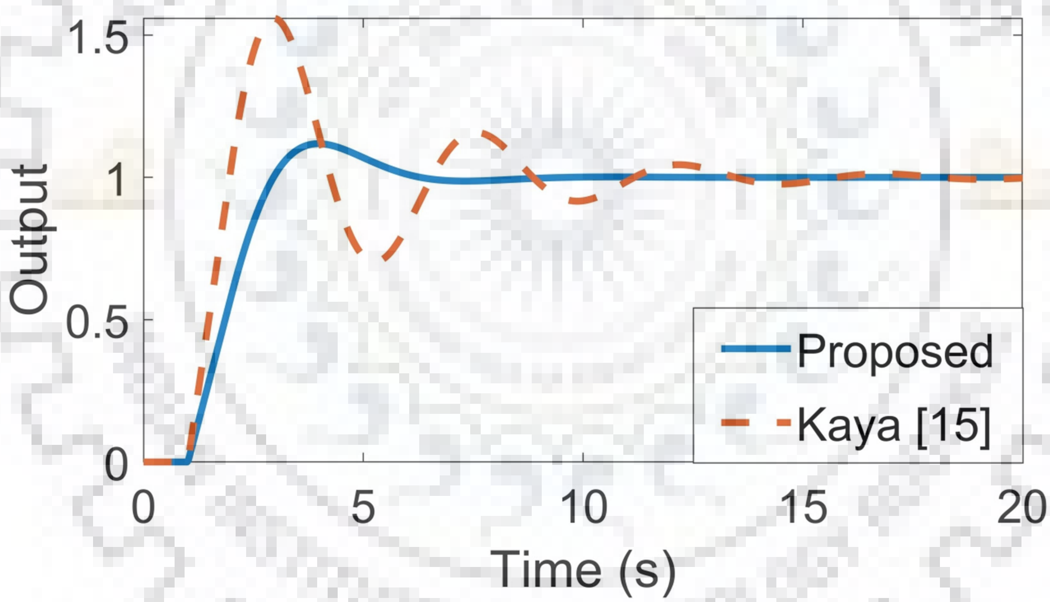

Figure 9: System response for Example 2

Example 3: Consider a plant model given in [16]. The example taken in [16] is of a nonlinear so called separable system. The linear part is defined by its transfer function. Thus controller is designed taking in consideration the transfer function, i.e.,

$$G(s) = \frac{4}{s(0.5s+1)}$$

The plant model of this process can be approximated as,

$$G(s) = \frac{8}{s^2 + 2s} \tag{129}$$

The state space model, as given by (81) is,

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 8 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \end{bmatrix} \tag{130}$$

The matrices Q and R are selected in a way such that $Q = diag(q_{11}, q_{22})$ and $R > 0$ as,

$$Q = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}, \quad R = [0.01] \tag{131}$$

Using (131) solve the equation (86) and (88). Thus, feedback gain matrix is evaluated as,

$$K = \begin{bmatrix} 31.6228 & 10.9323 \end{bmatrix} \tag{132}$$

From (89), p_{12} and p_{22} are found out to be,

$$p_{12} = 0.3162, \quad p_{22} = 0.1093 \tag{133}$$

Using (133) solve the polynomial (101). The value of λ is taken as the minimum of the two values, that are obtained by solving (101), for faster speed of response. Then by substituting the value of λ into (98), the value of p can be determined. These values of λ and p are obtained as

$$\lambda = 0.1035, \quad p = 3.2742 \tag{134}$$

Substituting the values of λ and p in (107), and applying Taylor series, the values of PID parameters are obtained as,

$$K_p = 4.8905, K_i = 0, K_d = 2.0671 \quad (135)$$

Thus, the controller designed by the proposed method is,

$$C(s) = 4.8905 + 2.0671\, s \quad (136)$$

Simulation Results:

The simulation is done for set point of unity. Fig. 10 shows the output response obtained for example 3.

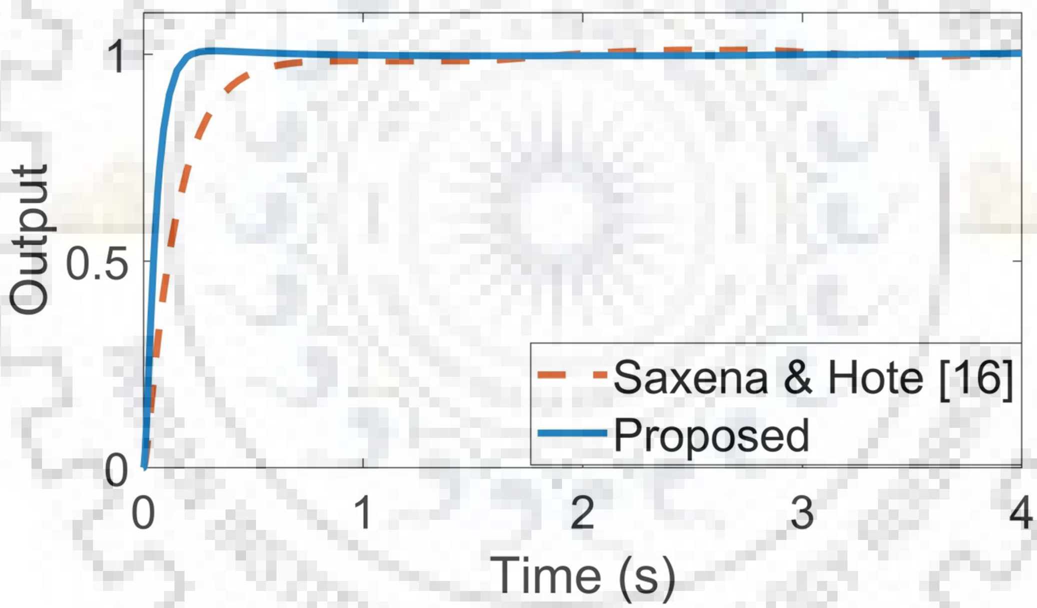

Figure 10: Response of the system of Example 3.

Performance Analysis:

Integral error criterion is used to evaluate the optimal performance of the system. They are ISE stands for integral of squared error, IAE stands for integral of absolute error and ITAE stands for integral of time weighted absolute error. The error analysis has been carried out for the examples worked out in this section. Table 2 shows the results obtained for each of the example and is compared with some existing approaches.

Table 2: Performance analysis of proposed approach with other techniques for the examples considered.

Examples	Method	Reference Tracking		
		ISE	IAE	ITAE
1	Proposed	1.826	2.402	3.681
	Skogestad [14]	2.1	3.551	12.5
2	Proposed	1.599	2.135	3.098
	Kaya [15]	1.82	3.201	10.62
3	Proposed	0.03423	0.07823	0.08876
	Saxena & Hote [16]	0.08481	0.228	0.3301

5.2 Tuning of Load Frequency Controller for Power Systems

For application purposes the plant model of load frequency control of power systems is considered. The load frequency control of power systems with non-reheated, reheated and hydro turbines is discussed in [5]. The plant model of the non-reheated and reheated turbines is taken to apply the proposed technique for designing of controller.

LFC System Description:

A power system is usually characterized by complex nonlinear large scale systems [17]. However, it is possible to linearize the system about its operating point for the problem of load frequency control. The single area power system consists of a governor, turbine and machine represented by the Fig. 11.

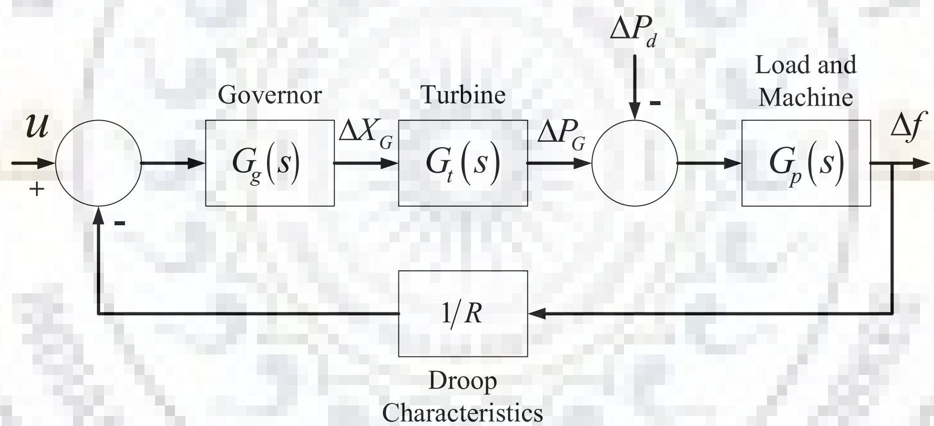

Figure 11: Linear model of a single-area power system

In Fig. 2, u is the control signal, ΔX_G is the Incremental change in governor valve position, ΔP_G Incremental change in generator output, ΔP_d represents the Load disturbance and Δf is the Incremental frequency deviation.

The dynamics of these components [18] can be given by the following,

- Governor:

$$G_g(s) = \frac{1}{T_G s + 1} \tag{137}$$

where, T_G is the governor time constant.

- Turbine: The turbine is classified into non-reheated and reheated turbine. The transfer function for non-reheated turbine is given by,

$$G_t(s) = \frac{1}{T_T s + 1} \tag{138}$$

where, T_T is the turbine time constant.

The transfer function of the reheated turbine is given by,

$$G_t(s) = \frac{cT_r s + 1}{(T_r s + 1)(T_T s + 1)} \tag{139}$$

where, T_r is the reheated turbine constant and c is the percentage of the power generated in the reheated turbine.

- Load and Machine:

$$G_p(s) = \frac{K_P}{T_P s + 1} \tag{140}$$

where, K_P is the electric system gain and T_P is the electric system time constant

Without considering the droop characteristics, the overall transfer function, $G_{wd}(s)$, is written as,

$$G_{wd}(s) = G_g(s) G_p(s) G_t(s) \tag{141}$$

Whereas, when the droop characteristics are considered, the overall transfer function becomes,

$$G_d(s) = \frac{G_g(s) G_p(s) G_t(s)}{1 + G_g(s) G_p(s) G_t(s)/R} \tag{142}$$

where, R is speed regulation due to governor action.

- **LFC system having Non-Reheated turbine:**

The plant model of non-reheated turbine with droop characteristics [18] is given by,

$$G(s) = \frac{250}{s^3 + 15.88s^2 + 42.46s + 106.2} \tag{143}$$

This model is reduced to the form (65) by using Routh approximation Method. Thus the reduced model is

$$G(s) = \frac{18.68}{s^2 + 3.173s + 7.94} \tag{144}$$

The state space model, as given by (81) is,

$$\begin{aligned} A &= \begin{bmatrix} 0 & 1 \\ -7.94 & -3.173 \end{bmatrix} \\ B &= \begin{bmatrix} 0 \\ 18.68 \end{bmatrix} \\ C &= \begin{bmatrix} 1 & 0 \end{bmatrix} \\ D &= \begin{bmatrix} 0 \end{bmatrix} \end{aligned} \tag{145}$$

The matrices Q and R are selected in a way such that $Q = diag(q_{11}, q_{22})$ and $R > 0$ as,

$$\begin{aligned} Q &= \begin{bmatrix} 5000 & 0 \\ 0 & 4 \end{bmatrix} \\ R &= [0.1] \end{aligned} \tag{146}$$

Using (146) solve the equation (86) and (88). Thus, feedback gain matrix is evaluated as,

$$K = \begin{bmatrix} 223.1821 & 7.8254 \end{bmatrix} \tag{147}$$

From (89), p_{12} and p_{22} are found out to be,

$$p_{12} = 119.4765, \quad p_{22} = 4.1892 \tag{148}$$

Using (148) solve the polynomial (101). The value of λ is taken as the minimum of the two values, that are obtained by solving (101), for faster speed of response. Then by substituting the value of λ into (98), the value of p can be determined. These values of λ and p are obtained as

$$\lambda = 6.852\times10^{-5}, \quad p = 28.5704 \tag{149}$$

Substituting the values of λ and p in (110), (111), and (112), the values of PID parameters are obtained as,

$$K_p = 4.8427, K_i = 12.1202, K_d = 0.7631 \tag{150}$$

Thus the controller designed by the proposed method is,

$$C(s) = 4.8427 + \frac{12.1202}{s} + 0.7631\, s \tag{151}$$

Simulation Results

The frequency deviation for the LFC system having non-reheated turbine is shown in Fig. 12. The simulations are done for a load disturbance of 0.01 units.

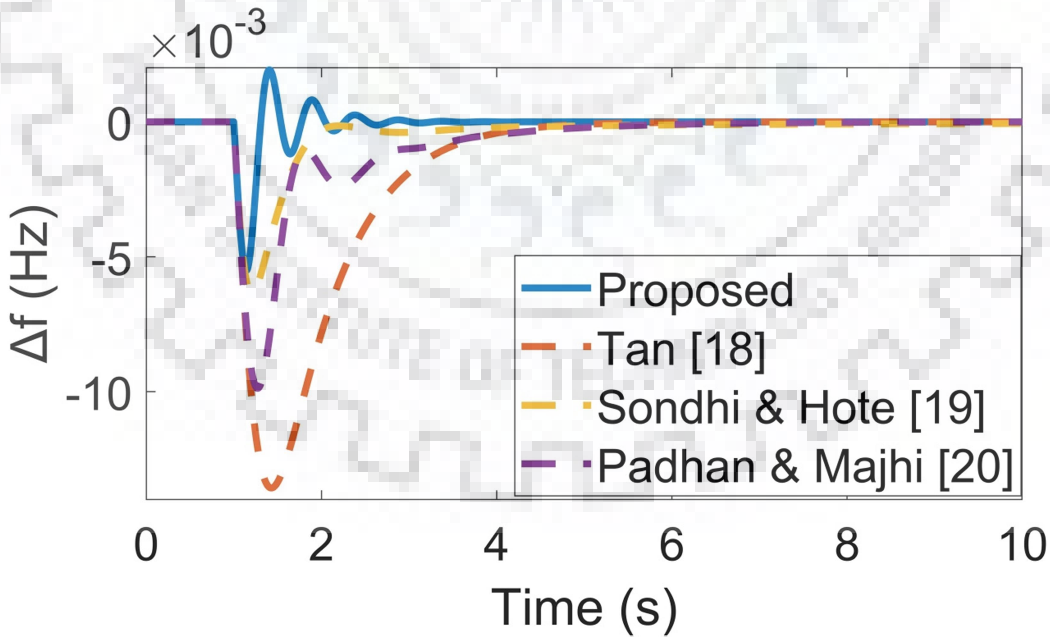

Figure 12: Response of power system with Non-Reheated turbine

- **LFC system having Reheated turbine:**

The plant model of reheated turbine with droop characteristics [18] is given by,

$$G(s)=\frac{87.5\,s+59.52}{\left(s^4+16.12\,s^3+46.24\,s^2+48.65\,s+25.3\right)} \tag{152}$$

This model is reduced to the form (5) by using Routh approximation Method. Thus the reduced model is

$$G(s)=\frac{1.572}{s^2+1.285s+0.6682} \tag{153}$$

The state space model, as given by (81) is,

$$A=\begin{bmatrix}0 & 1\\ -0.6683 & -1.285\end{bmatrix}$$
$$B=\begin{bmatrix}0\\ 1.572\end{bmatrix}$$
$$C=\begin{bmatrix}1 & 0\end{bmatrix}$$
$$D=\begin{bmatrix}0\end{bmatrix} \tag{154}$$

The matrices Q and R are selected in a way such that $Q=diag\left(q_{11},q_{22}\right)$ and $R>0$ as,

$$Q=\begin{bmatrix}1911 & 0\\ 0 & 0.01\end{bmatrix}$$
$$R=[0.1] \tag{155}$$

Using (155) solve the equation (86) and (88). Thus, feedback gain matrix is evaluated as,

$$K=\begin{bmatrix}137.8145 & 12.453\end{bmatrix} \tag{156}$$

From (89), p_{12} and p_{22} are found out to be,

$$p_{12}=2876.68,\quad p_{22}=79.2176 \tag{157}$$

Using (157) solve the polynomial (101). The value of λ is taken as the minimum of the two values, that are obtained by solving (101), for faster speed of response. Then by substituting the value of λ into (98), the value of p can be determined. These values of λ and p are obtained as

$$\lambda = 5.134 \times 10^{-4}, \quad p = 11.122 \tag{158}$$

Substituting the values of λ and p in (110), (111), and (112), the values of PID parameters are obtained as,

$$K_p = 9.0382, K_i = 4.7018, K_d = 3.5155 \tag{159}$$

Thus the controller designed by the proposed method is,

$$C(s) = 9.0382 + \frac{4.7018}{s} + 3.5155\, s \tag{160}$$

Simulation Results

The frequency deviation for the LFC system having reheated turbine is shown in Fig. 13. The simulations are done for a load disturbance of 0.01 units. It is clearly seen that the frequency deviation response of proposed controller is the better than the existing controllers.

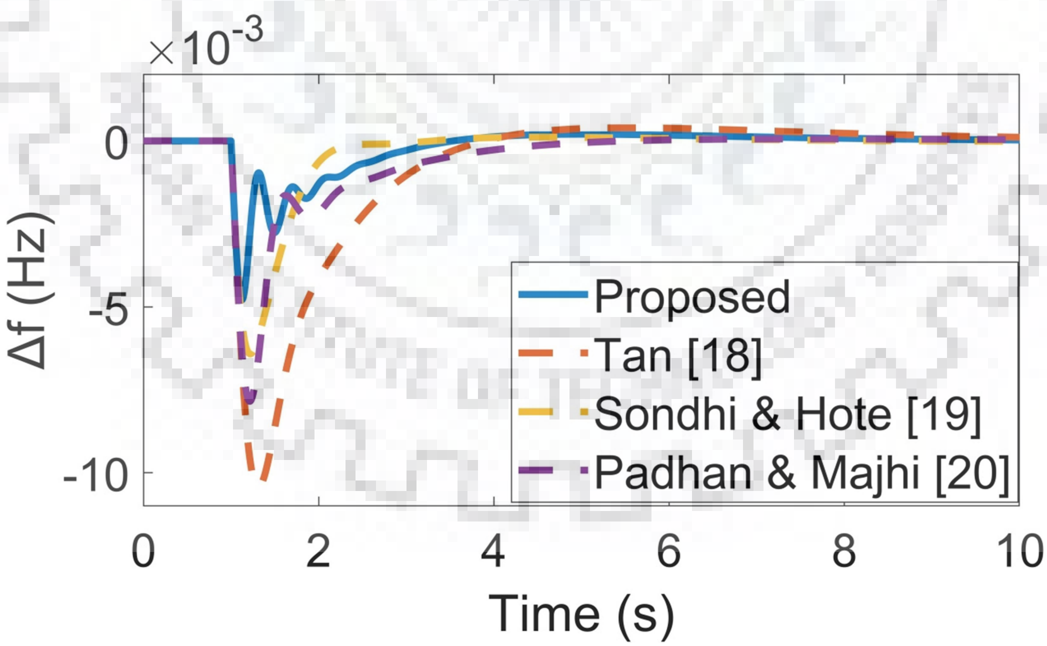

Figure 13: Response of power system with Reheated turbine

Performance Analysis:

Integral error criterion is used to evaluate the optimal performance of the system. ISE stands for integral of squared error, IAE stands for integral of absolute error and ITAE stands for integral of time weighted absolute error. The comparison between the proposed technique and some existing methods, on the basis of performance indices, is given in Table 3 and Table 4.

Table 3: Performance analysis of Power system for Non-Reheated Turbine.

	Nominal		
	ISE	**IAE**	**ITAE**
Proposed	5.166×10^{-6}	1.772×10^{-3}	2.482×10^{-3}
Tan [18]	137.4×10^{-6}	15.73×10^{-3}	30.26×10^{-3}
FO-PID Sondhi& Hote [19]	14.7×10^{-6}	4.719×10^{-3}	16.04×10^{-3}
Padhan&Majhi [20]	36.26×10^{-6}	7.665×10^{-3}	16.05×10^{-3}

Table 4: Performance analysis of Power system for Reheated Turbine.

	Nominal		
	ISE	**IAE**	**ITAE**
Proposed	6.336×10^{-6}	3.739×10^{-3}	1.022×10^{-2}
Tan [18]	61.85×10^{-6}	11.59×10^{-3}	3.098×10^{-2}
FO-PID Sondhi & Hote [19]	16.04×10^{-6}	4.23×10^{-3}	1.039×10^{-2}
Padhan&Majhi [20]	20.07×10^{-6}	5.912×10^{-3}	1.406×10^{-2}

CHAPTER 6

Conclusion and Future Prospects

This dissertation presents a new and straightforward approach for controller design. The IMC theory was discussed briefly, and its advantages were studied. Also an elementary study of the LQR technique was carried out. The proposed approach has been discussed thoroughly. The designing of IMC controller is done by incorporating the advanced control techniques in the classical control, i.e. the Linear Quadratic Regulator approach in IMC. A mathematical algorithm for determining the tuning parameters is developed. The proposed approach was successfully applied on various plant models and two models of the Load Frequency controller for power systems. The system responses were simulated and a comparison is made with some existing approaches. And after comparing them with the results of the other techniques, it is clearly seen that the proposed technique exhibits much better system response. The error analysis was carried out. Compared to the results of other methods, the errors were much smaller in the proposed approach. It is quite evident that the performance of the proposed technique is much superior to the existing techniques.

In future, further study will be carried out in this direction. We will extend this technique for controller design for a practical hardware system. In this approach, 1DOF IMC structure was used. In future, designing of the controller will be carried out by 2DOF IMC structure.

List of Publication

1. Nikita S. Raut, Yogesh V. Hote and Jitendra Sharma, "PID Design based on Internal Model Control for Load Frequency Control using Linear Quadratic Regulator", Manuscript 1791 submitted to 2019 IEEE Conference on Decision and Control (CDC).Received March 17, 2019.

The enlisted paper is under review.

REFERENCES

[1] B. Joseph, and C.B. Brosilow, "Inferential control of processes, Part I, Steady-state analysis and design," *AIChE J.*, Vol. 24, no. 3, pp. 485-92, 1978 a.

[2] B. Joseph, and C.B. Brosilow, "Inferential control of processes, Part III, Construction of optimal and suboptimal dynamic simulators," *AIChE J.*, Vol. 24, no. 3, pp. 500-509, 1978b.

[3] Garcia & Morari, "Internal Model Control. 1. A Unifying Review and Some New Results." *Industrial & Engineering Chemistry Process Design and Development*. Vol 21, 1982, pp. 308-323.

[4] M. Morari, B. R. Holt, and S. Lehmann, "A frequency domain approach to control structure synthesis," *22nd IEEE Conf. Decision and Contrl.*, Vol. 20, part 1, pp. 478-82, 1981.

[5] C. Brosilow and B. Joseph, *Techniques of Model-Based Control*. Englewood Cliffs, NJ: Prentice-Hall, 2002.

[6] Saxena & Hote, "Advances in Internal Model Control Technique: A Review and Future Prospects", *IETE Technical Review*, Vol. 29 no. 6, 2012, pp. 461-472

[7] T. Liu and F. Gao, "New insight into internal model control filter design for load disturbance rejection," in *IET Control Theory & Applications*, vol. 4, no. 3, pp. 448-460, March 2010.

[8] Ogata, K, *Modern Control Engineering*, 5th Edition, Pearson, Upper Saddle River, 2010.

[9] Murray RM, Optimization-based control. California Institute of Technology, CA. 2009 Jan:111-28.

[10] M. F. Hutton and B. Friedland, "Routh approximations for reducing order of linear, time-invariant systems," IEEE Trans. Autom. Control, vol. AC-20, pp. 329-337. Jun. 1975.

[11] M. Morari and E. Zafiriou, Robust Process Control. Englewood Cliffs, NJ: Prentice-Hall, 1998

[12] D. S. Naidu, *Optimal Control Systems.* Richard C. Dorf (Ed.). CRC Press, Inc., Boca Raton, FL, USA. 2002

[13] D.E. Kirk, *Optimal Control Theory: An Introduction*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1970.

[14] S. Skogestad, “Simple analytic rules for model reduction and PID controller tuning”, *Journal of Process Control*, vol. 13, no. 4, 2003, pp 291-309.

[15] I. Kaya, "Two-degree-of-freedom IMC structure and controller design for integrating processes based on gain and phase-margin specifications," in *IEE Proceedings - Control Theory and Applications*, vol. 151, no. 4, pp. 481-487, 24 July 2004

[16] Saxena & Hote, “Internal model control based PID tuning using first-order filter.” *International Journal of Control Automation and Systems*, vol 15, no. 1, 2017.

[17] Kundur, P., *Power system stability and control.* McGraw-Hill,1994

[18] W. Tan, “Unified tuning of PID load frequency controller for power systems via IMC,” *IEEE Trans. Power Syst.*, vol. 25, no. 1, Feb. 2010, pp. 341–350

[19] Swati Sondhi, Yogesh V. Hote, “Fractional order PID controller for load frequency control”, *Energy Conversion and Management,* Vol. 85,2014, pp. 343-353.

[20] W.D. G. Padhan and S. Majhi, “A new control scheme for PID load frequency controller of single area and multi area power systems,” *ISA Trans.*, vol. 52, issue 2, March. 2013, pp. 242–251

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