SIMPLIFIED SEP APPROXIMATION OF HQAM IN COMBINED TIME SHARED NAKAGAMI-LOGNORMAL AND RICIAN FADING CHANNEL

A Dissertation

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CANDIDATE'S DECLARATION

I declare that the work presented in this dissertation with title " **Simplified SEP Approximation Of HQAM In Combined Time Shared Nakagami-Lognormal And Rician Fading Channel**" towards the fulfillment of the requirement for the award of the degree of **Master of Technology** submitted in the **Department of Electronics and Communication Engineering**, **Indian Institute of Technology Roorkee**, India. It is an authentic record of my own work carried out under the supervision of **Dr. P.M. Pradhan**, Assistant Professor, Department of Electronics and Communication Engineering, IIT Roorkee.

The content of this dissertation has not been submitted by me for the award of any other degree of this or any other institute.

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CERTIFICATE

This is to certify that the statement made by the candidate is correct to the best of my knowledge and belief.

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Date : ....

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Abstract

Due to different atmospheric conditions, signals transmitted through wireless medium experience several distortions. As a result, the received signal randomly gets attenuated in amplitude and distortion in phase. Therefore, irregularity is observed in received signal strength. The performance of the wireless communication system gets degraded in fading environment, relative to the additive white Gaussian noise (AWGN) environment. The mobile user experiences different fading scenarios, which can be characterized as multi-path fading, shadow fading and mixture of both (composite fading). There are some known fading models such as Rayleigh, Rician, Nakagami-m, Weibull and composite fading channel (Rayleigh/Log-Normal (R/L), Rician/Log-Normal (Rice/L), Weibull/Log-Normal (W/L)or Nakagami/Log Normal (N/L)).

In this dissertation, different fading scenarios are discussed and analyzed, and the corresponding probability distribution function (PDF) has been analysed. The Gaussian Q function has been approximated using the trapezoidal rule.

The computational complexity involved in the calculation of the double integral in the expression of SEP for composite N/L channel is very high. Therefore, two closed form approximations for the PDF of the composite N/L channel are obtained using the approximations proposed by Holtzman and Gauss Hermite. Further, these two approximations are incorporated in the expression of SEP to reduce the computational complexity. In order to evaluate the performance of these two approximations, they are compared with the exact SEP expression. It is found that SEP result from the approximations proposed by Holtzman and Gauss Hermite are very close to the exact SEP obtained for low and high SNR, respectively.

Further, the performance of the two approximations are evaluated in a combined (time shared) N/L and Rician fading channel. Afterwards, the sensitivity analysis is carried out to study the effect of Rician k factor, Nakagami-m factor and time shared factor in the combined channel.

Contents

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Abbreviations

PDF	Probability Distribution Function
SEP	Symbol Error Probability
N/L	Nakagami-Lognormal
SNR	Signal to Noise Ratio
AWGN	Additive White Gausian Noise
HA	Holtzman Approximation
GHA	Gauss Hermite Approximation
LOS	Line of Sight
BER	Bit Error Ratio

Symbols and Notations

γ	Avergae SNR
σ	Standard Deviation
μ	Mean
ω	Slow Varying Power for Log-normal Fading Channel
Q	Gaussian Q Function
m	Nakagami-m Fading Parameter
K	Rician K Factor
K_c	Average Number of Couples of Adjacent Nearest Neighbours
ζ	System Parameter for HQAM Constellations
k	Average Number of Nearest Neighbour

Chapter 1

Introduction

1.1 What is fading?

Fading is defined as weakening of a signal due to different effects such as multi-path, diffraction, reflection and shadowing etc. These effects are caused by geographical positioning, different radio frequencies and time variance.

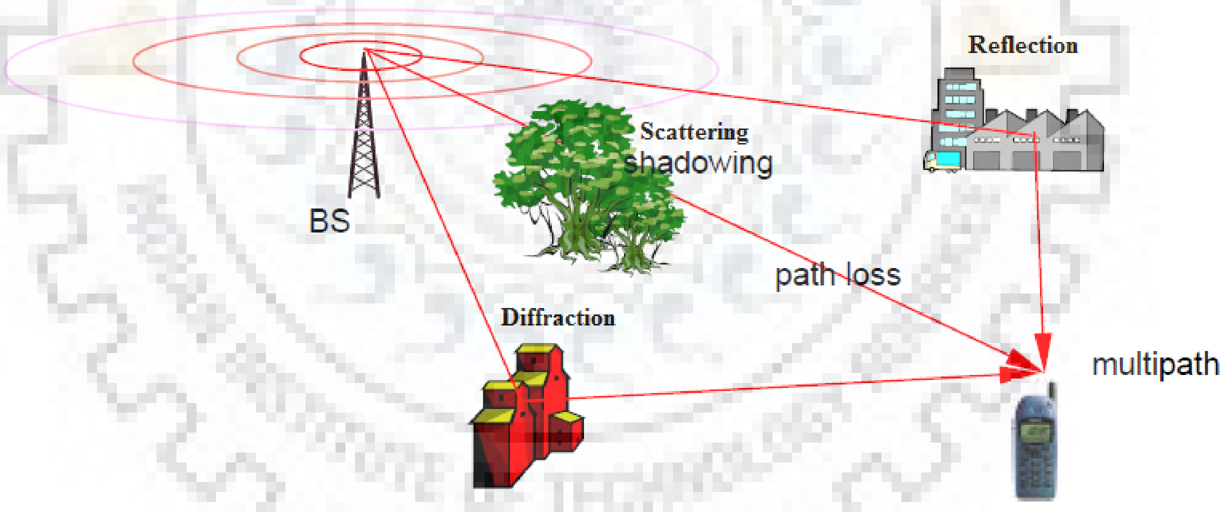


Fig. 1.1. Basic Fading Model [1]

The transmitted signal follows different paths to reach at the receiver. As a result, various replicas of the transmitted signal are received at the receiver. These replicas have different path lengths, i.e. different phases. The shortest path is known as Line of Sight (LOS) path [2]. The signals which are in-phase with the LOS component increases the received signal strength. The signals which are out of phase with the LOS component reduces the signal strength. Because of this the received signal strength gets fluctuations.

This phenomenon is called channel multi-path fading.

1.2 Phenomena which define the fading

1.2.1 Coherence bandwidth

The range of frequencies over which frequency correlation is greater than 0.9 is defined as coherence bandwidth expressed below

$$\mathrm{B}_c = \frac{1}{50\sigma_c} \tag{1.1}$$

when this correlation is greater than 0.5, coherence bandwidth becomes

$$B_c = \frac{1}{5\sigma_c} \tag{1.2}$$

where σ_c = RMS delay spread

1.2.2 Coherence time

Coherence time (Tc) is defined as the duration over which impulse response of the channel is constant [1]. If symbol period greater than coherence time, then the channel changes during symbol period. Thus the received symbols gets distorted. The coherence time is defined as [1]

$$T_c = \frac{1}{f_m} \tag{1.3}$$

where f_m = RMS delay spread.

1.3 Types of fading

1.3.1 Flat fading

If all components of the transmitted signal experiences the same change in amplitude and in phase, the channel is known as flat fading channel [1].

$$B_s << B_c \tag{1.4}$$

Also,

$$T_s >> \sigma_\tau \tag{1.5}$$

where B_s is the signal bandwidth , B_c is the coherence bandwidth, T_s is the symbol period and σ_τ is the rms delay spread.

1.3.2 Frequency selective fading

If all components of the transmitted signal experiences variable change in amplitude and in phase, the channel is known as frequency selective fading channel [1].

$$B_s >> B_c \tag{1.6}$$

Also,

$$T_s << \sigma_\tau \tag{1.7}$$

1.3.3 Fast fading

If the impulse response of the fading channel experience rapid change with in symbol period, than the channel is known as fast fading channel [1].

$$T_s >> T_c \tag{1.8}$$

where T_c is the coherence time and T_s is the symbol period. The signal is dispersed and distorted due to the doppler spreading [1].

$$B_s << B_d \tag{1.9}$$

where B_d is the Doppler spread and B_s is the signal bandwidth.

1.3.4 Slow fading

If the impulse response of the fading channel experience less changes with in symbol period, then the channel is known as slow fading channel [1]. These types of channels are constant during one symbol duration [1].

$$T_s << T_c \tag{1.10}$$

Also,

$$B_s >> B_d \tag{1.11}$$

Chapter 2

Fading channel

2.1 Multi-path fading

In wireless communication, transmitted signal is scattered due to environmental effects [3]. As the signal is passes through different paths it under goes channel impairments such as interference, reflection and scattering. The received signal is obtained from these multi-path component.

2.1.1 Models for multi-path propagation

Rayleigh fading model

In this case the channel is modelled as Rayleigh distribution, where the distribution is obtained by the summation of two uncorrelated Gaussian random variables. The rayleigh distribution is defined as

$$f_R r = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) \tag{2.1}$$

Rician fading model

This channel model is same as Rayleigh fading model, except in this case LOS component is also present along with the multi-path component [4]. The rician distribution is defined as

$$f_R r = \frac{r}{\sigma^2} \exp\left(-\frac{(r^2 + A^2)}{2\sigma^2}\right) \mathrm{I}_o \left(\frac{A_r}{\sigma^2}\right) \tag{2.2}$$

where factor υ is defined as

$$\upsilon_{dB} = 10 \exp\left(-\frac{A^2}{2\sigma^2}\right) \tag{2.3}$$

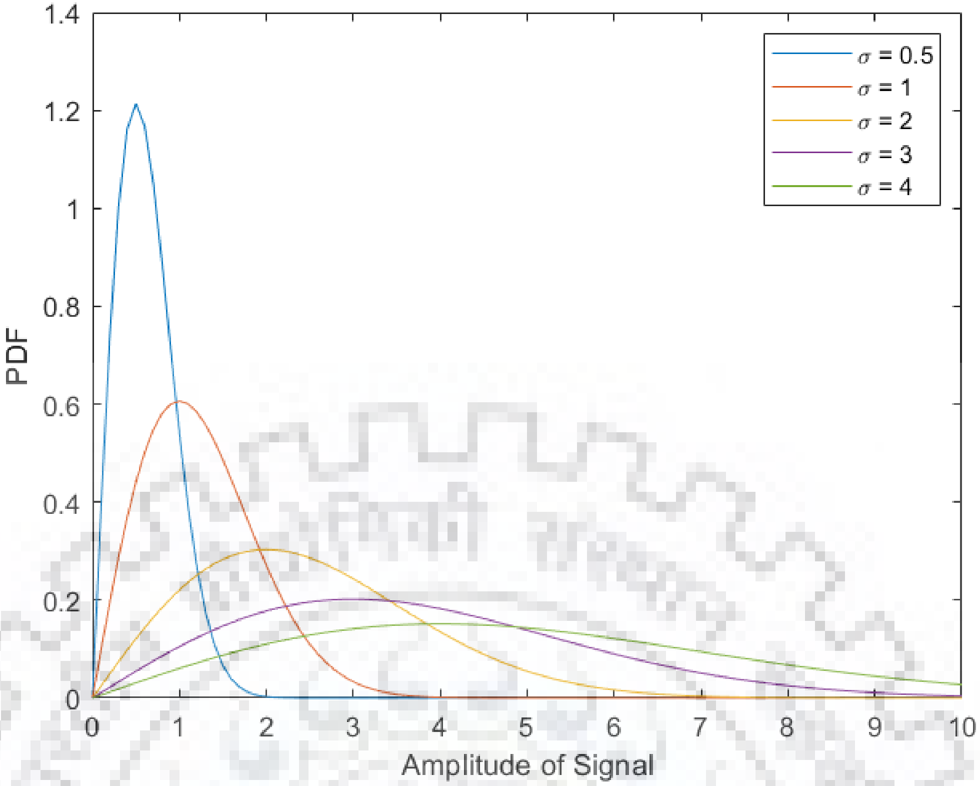


Fig. 2.1. PDF vs Amplitude of Signal (Rayleigh distribution)

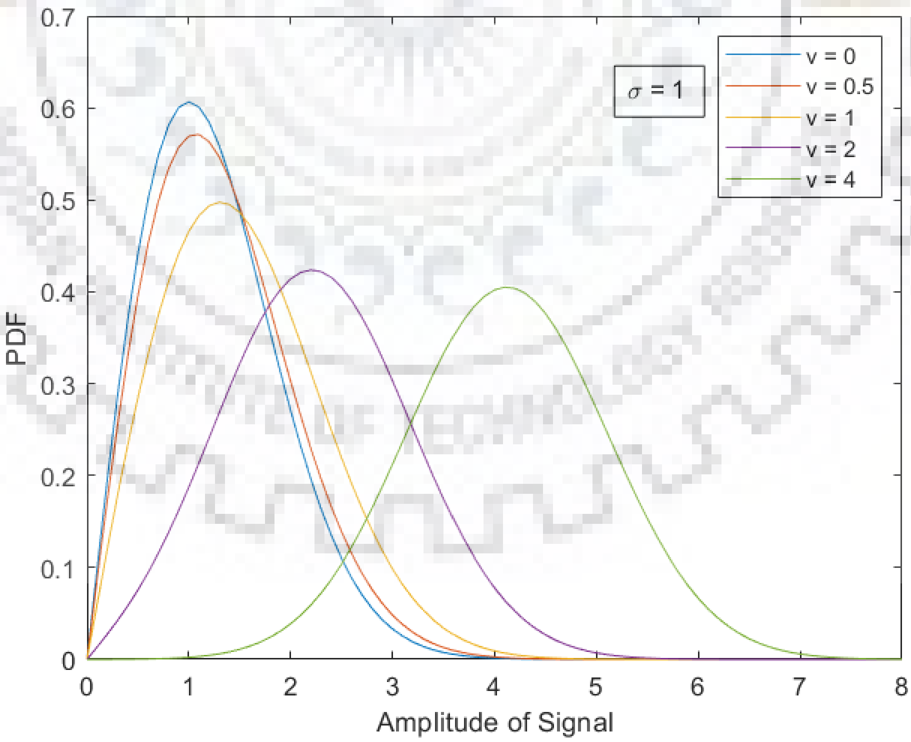


Fig. 2.2. PDF vs Amplitude of Signal (Rician distribution)

v is the shape parameter

Nakagami-m fading model

The sum of multiple independent and identically distributed (i.i.d.) Rayleigh-fading signals have a Nakagami distributed signal amplitude [5]. The Nakagami-m distribution is defined as

$$f_R(r) = \frac{2r^{\omega-1}}{\Gamma(\omega)}\left(\frac{\omega^{\omega}}{\Omega^{\omega}}\right)\exp\left(-\frac{\omega r^2}{\Omega}\right) \tag{2.4}$$

ω=shape parameter

$$\omega = \frac{(M+1)^2}{2M+1} \tag{2.5}$$

when $\omega = 1$, Nakagami model is the Rayleigh fading model.

where,

$$M = \frac{A}{2\sigma} \tag{2.6}$$

Ω represents average signal power and $\Gamma(m)$ represents gamma function.

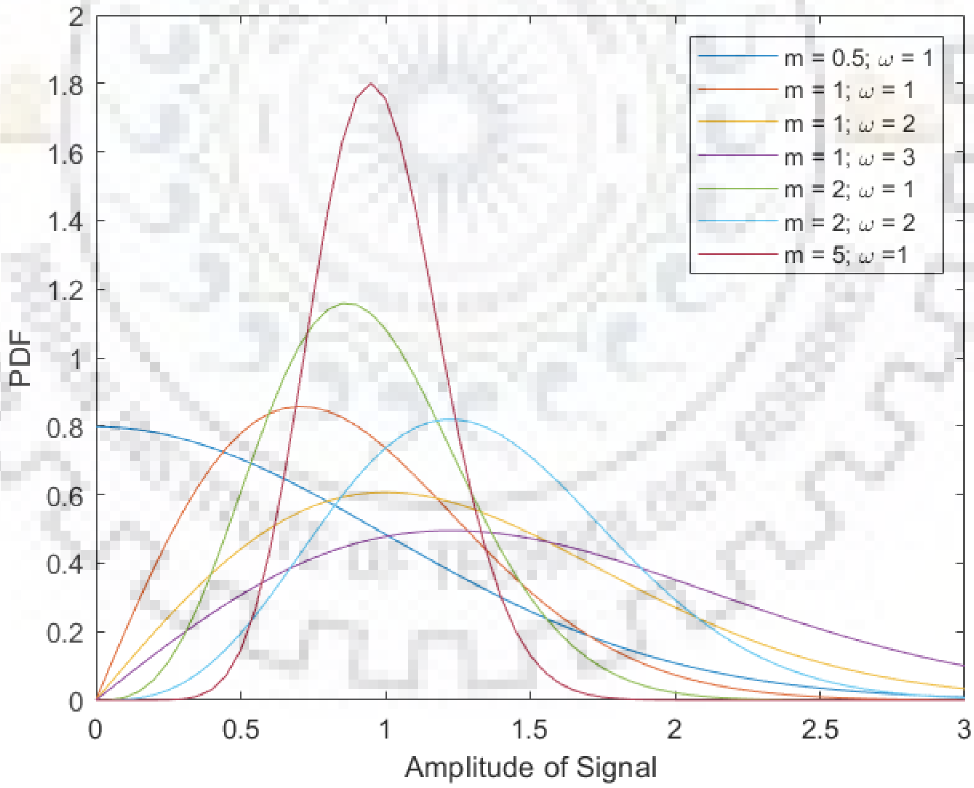


Fig. 2.3. PDF vs Amplitude of Signal (Nakagami distribution)

2.2 Shadowing fading model

Shadowing is the long distance effect which is caused by trees, building and mountains. The shadowing reduces the received signal power and remove the fluctuations due to multi-path fading [3].The signal effected by shadowing can be modelled using different mathematical distribution such as log-normal distribution and gamma distribution. The fluctuation in the received signal SNR is observed because of intermediate interfering objects between the transmitter and receiver. These fluctuations are experienced on local-mean powers, that is to eliminate fluctuation due to multi-path fading.

2.2.1 Models for shadow fading propagation

2.2.2 Log-Normal fading distribution

Log-normal shadowing incorporates the effects of surrounding environment clutter on the received signal strength for the same transmitter and receiver separation. There are many factors contributing to the overall path loss such as free space loss, diffraction, reflection, transmissions, etc [6]. All these losses are a random in nature thus can be modelled as RV. The total loss can be estimated by adding all of these losses (each in decibels). Due to central limit theorem the expression of the overall loss (in decibels) will tend to have a gaussian distribution with zero mean. The PDF of log-normal distribution is defined as [7]

$$f_x(x) = \frac{1}{\sigma x\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right) \tag{2.7}$$

2.2.3 Gamma fading distribution

The gamma distribution is a continuous distribution of two parameters. By choosing suitable values for these parameters it can achieve different distributions such as Erlang distribution, chi-square distribution [8]. The PDF is defined as

$$f(x;\alpha;\beta) = \frac{\beta^\alpha x^{\alpha-1}\exp(-\beta x)}{\Gamma(\alpha)} \tag{2.8}$$

$\alpha = K$; $\beta = \frac{1}{\theta}$

where α is a shape parameter and β is a rate parameter.

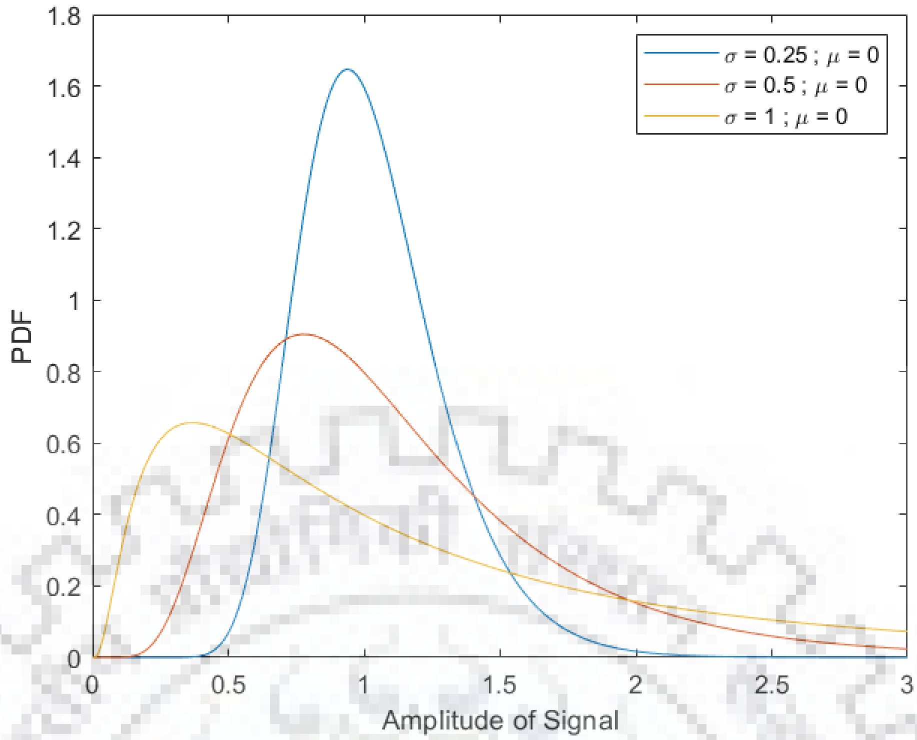


Fig. 2.4. PDF vs Amplitude of Signal (Log-Normal)

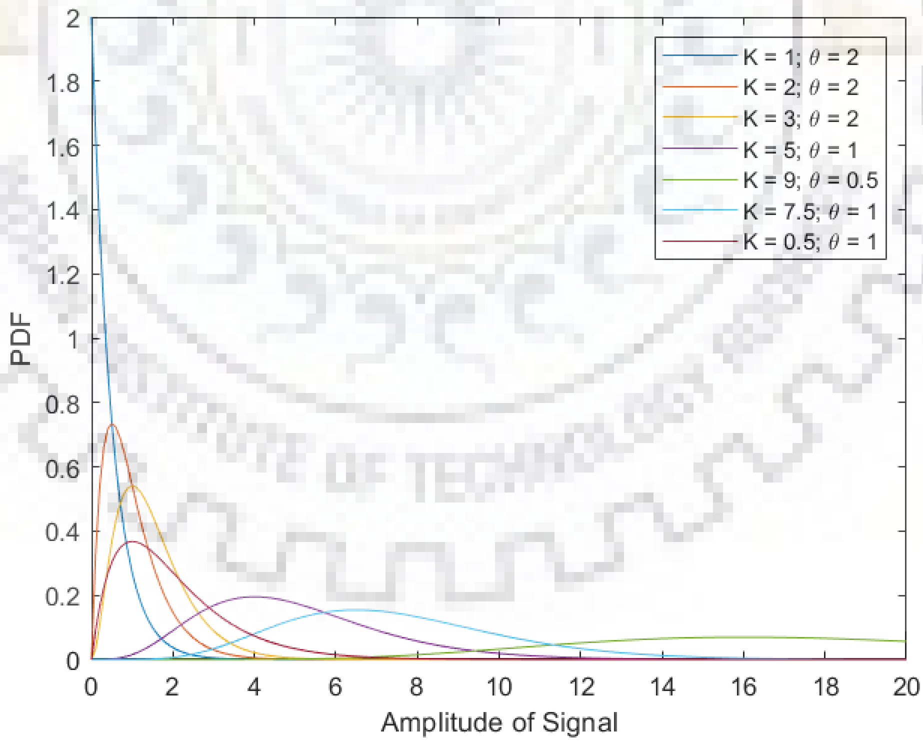


Fig. 2.5. PDF vs Amplitude of Signal (Gamma)

2.3 Literature review and discussion

The Q function plays an important role in estimating the SEP for various digital modulation schemes in AWGN and as well as in fading channel.

In [9], authors has used the Trapezoidal rule of numerical integration to approximate the Q function and compared the results while calculating the SEP for the Nakagami-m fading channel. The value of SEP for Nakagami-m channel using the Q function approximation already been carried out by various other authors. The table provided by [9] , gives the precise detail that the approximations provided for the Q function by authors in [9] is more accurate as compared to other authors but only for the low SNR's, for high SNR's other authors approximation is working better.

In [10], the authors have used α- μ, η- μ, κ- μ fading channel for calculating the SEP using the Q function approximations provided in [9]. Problem with the solution provided by [9] is that after multiplying two approximations, the number of terms become very large. To reduce these elements, authors has used Trapezoidal rule for the SEP calculation. The estimated SEP from this method has been compared from the one obtained from the Q funtion approximations [11–13].

In [14] the PDF for N/L fading and Rician fading has been used for the estimation of SEP. The variation in the PDF for N/L and Rician fading channel using different parameters had been studied.

In [15], author has proposed a method to find the mean of an equation involving gaussian PDF. In this paper author has improved the accuracy of approximation by using simplified method. In this, author has expanded the function with the Taylor series which it further has started with an expansion in differences (Stirling formula). This expansion is further used to estimate the mean up to two terms. To increase the accuracy of the proposed calculation, author has estimated the most optimized parameter value from [16]. In this author has also presented the graph for the same which shows the accuracy of the proposed method of approximation. Authors in [17] have used the proposed approximation presented in [15] for the calculation of channel capacity in gaussian fading channel. The same reference has been taken of [15, 17] and used in estimation of closed form analysis for the Nakagami-Lognormal fading channel.

In [18], a systematic method is proposed for transforming the variable of integration, so that the integrand is sampled in an appropriate area. This method is often used in statistics as a numerical integration because of the relation with the gaussian densities. In this method, the term associated with the gaussian density is being taken as a main function and approximated into a sum of weights and abscissas (k-th root of an N-th order Hermite Polynomial) which can be found from [19]. The sum of all the terms including the weights, comprise of whole approximated numerical integration.

In [20], authors has provided the closed form expressions for the PDF of instantaneous SNR using the method provided in [15]. The same has been used to calculate amount of fading (AOF), outage probability (OP), and average channel capacity for the channel model based on combined (time shared) N/L shadowing and unshadowing (Rice) fading.

In [21] various mathematical integral solution is provided which could be used for various closed form analysis. For every numerical integration there is more than one solution which can be used to get the closed form solution. These solutions can be used to reduce the computational complexity.

In [22], author has presented the SEP calculation for Hexagonal-QAM (HQAM) in AWGN channel. This paper brought out advantages of using HQAM constellation over Square-QAM constellation and showed various constellation patters for HQAM.

A lot of work has been proposed for unshadow fading channel but actual practical scenario which is combination of multi-path and shadowing fading also known as composite fading channel is not been calculated.

Chapter 3

Approximations for Q function

The Q function plays an important role in finding the SEP for various digital modulation schemes, in AWGN as well as in fading channels.
In this, using the Trapzoidal rule for numerical integration, authors in [9, 10] have proposed unique, simple and tighter approximations for the Q function.
The complementary error function is defined as

$$erfc(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} \exp\left(-t^2\right) dt \tag{3.1}$$

The Q function is defined in term of error function as

$$Q(t) = \frac{1}{2} erfc\left(-\frac{t}{\sqrt{2}}\right) \tag{3.2}$$

Further error function can be expanded into approximated form defined as [9]
For $n = 3$,

$$erfc(x) = \frac{1}{6}\exp\left(-x^2\right) + \frac{1}{3}\exp\left(-4x^2\right) + \frac{1}{3}\exp\left(-\frac{4x^2}{3}\right) \tag{3.3}$$

Similarly, Q function in expanded approximated form can be expressed as

$$Q(x) = \frac{1}{12}\exp\left(-\frac{x^2}{2}\right) + \frac{1}{6}\exp\left(-\frac{2x^2}{1}\right) + \frac{1}{6}\exp\left(-\frac{2x^2}{3}\right) \tag{3.4}$$

For $n = 4$,

$$erfc(x) = \frac{1}{8}\exp\left(-x^2\right) + \frac{1}{4}\exp\left(-2x^2\right) + \frac{1}{4}\exp\left(-\frac{20x^2}{3}\right) + \frac{1}{4}\exp\left(-\frac{20x^2}{17}\right) \tag{3.5}$$

Similarly, Q function in expanded approximated form can be expressed as

$$Q(x) = \frac{1}{16}\exp\left(-\frac{x^2}{2}\right) + \frac{1}{8}\exp\left(-\frac{x^2}{1}\right) + \frac{1}{8}\exp\left(-\frac{10x^2}{3}\right) + \frac{1}{8}\exp\left(-\frac{10x^2}{17}\right) \tag{3.6}$$

Eq. (3.6) is working better then the existing approximation in different papers [11–13, 23–25].

Table 3.1: Comparision of Existing Approximation for erfc(x)

x	**0.8**	**1.2**	**1.6**	**2.0**	**2.4**
erfc(x)	0.25899	0.08686	0.02361	0.004677	0.000688
chiani [11]	0.300878	0.112791	0.029349	0.005466	0.000756
karagianndis [12]	0.260420	0.089024	0.023006	0.004465	0.000647
prony p=2 [13]	0.270179	0.090204	0.022881	0.004584	0.000700
Gauss Hermite [25]	0.255456	0.086565	0.021769	0.004018	0.000541
Invertible Approx [24]	0.268294	0.087692	0.023873	0.004662	0.000679
Trapezoidal n=3 [9]	0.255647	0.089407	0.023873	0.004662	0.000679
Trapezoidal n=4 [9]	0.256697	0.089928	0.023613	0.004678	0.000689

The relative error (RE) is defind as

$$RE = \frac{|erfc(x) - F(x)|}{erfc(x)} \times 100 \tag{3.7}$$

Eq. (3.7) of relative error is used as a tool to find the proposed method theory Table 3.1 confirms the accuracy of the proposed approximation by the authors [9].

The SEP calculation for the constellation of HQAM in AWGN channel has been a main topic of interest The exact SEP for a general class of HQAM signals for the AWGN channel presented in [22]

$$P_{AWGN}^{exact} = KQ\left(\sqrt{\zeta\gamma}\right) - K_c C \tag{3.8}$$

where

$$C = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{\infty} \left[2Q\left(\sqrt{\zeta\gamma}\right) - 2Q(x) + Q^2(x)\right] e^{-\left(\left(x-\sqrt{2\zeta\gamma}\right)^2/2\right)} dx \tag{3.9}$$

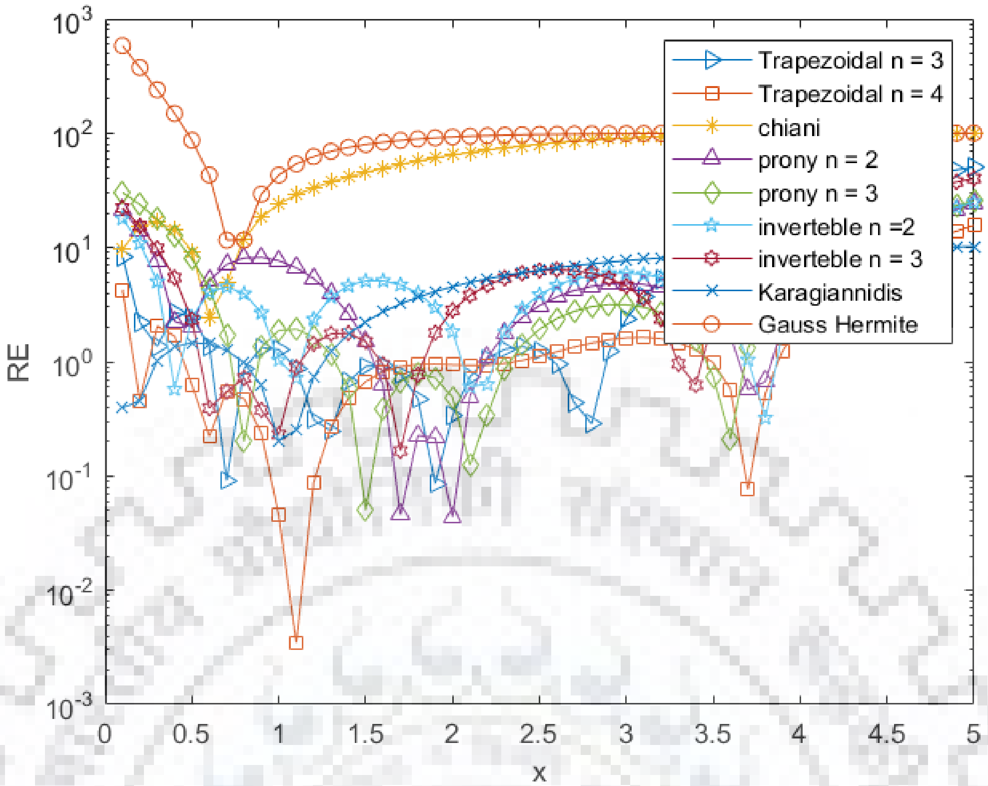


Fig. 3.1. Relative Error

Further, using [22, (6)], the authors in [9] approximated (3.9) as

$$C \simeq 2Q\left(\sqrt{\zeta\gamma}\right)Q\left(\sqrt{\frac{\zeta\gamma}{3}}\right) - \frac{2}{3}Q^2\left(\sqrt{\frac{2\zeta\gamma}{3}}\right) \tag{3.10}$$

It yields an approximation of SEP accuratly of (3.8) as

$$P_{AWGN} \simeq KQ\left(\sqrt{\zeta\gamma}\right) + \frac{2}{3}K_cQ\left(\sqrt{\frac{2\zeta\gamma}{3}}\right) - 2K_cQ(\sqrt{\zeta\gamma})Q\left(\sqrt{\frac{\zeta\gamma}{3}}\right) \tag{3.11}$$

A new approximation of the Q function proposed in [10] and therefore used for the estimation of SEP.

$$Q\left(\sqrt{\zeta\gamma}\right) \simeq \frac{1}{12}\exp\left(-\frac{\zeta\gamma}{2}\right) + \frac{1}{6}\exp\left(-\frac{2\zeta\gamma}{1}\right) + \frac{1}{6}\exp\left(-\frac{2\zeta\gamma}{3}\right) \tag{3.12}$$

$$Q\left(\sqrt{\zeta\gamma}\right).Q\left(\sqrt{\frac{\zeta\gamma}{3}}\right) \simeq \frac{1}{12}\exp\left(-\frac{2\zeta\gamma}{3}\right) + \frac{1}{12}\exp\left(-\frac{2\zeta\gamma}{1}\right) \tag{3.13}$$

$$Q^2\left(\sqrt{\zeta\gamma}\right) \simeq \frac{1}{8}\exp\left(-\frac{4\zeta\gamma}{3}\right) + \frac{1}{12}\exp\left(-\frac{2\zeta\gamma}{1}\right) \tag{3.14}$$

For fading channel the SEP is defined as

$$P_{fading} = \int_0^\infty P_{AWGN}.p_\gamma(\gamma)d\gamma \tag{3.15}$$

where $p_\gamma(\gamma)$ is the probability distribution function for the fading channel.

The SEP is defind for different constellation, and its coefficient for the AWGN is defind in the Table 3.2

Table 3.2: SEP parameters for HQAM constellation

Constellation,M	ζ	**K**	K_c
optimum, 4	1	5/2	3/2
double diamond, 8	4/9	13/4	9/4
HEX,16	2/9	33/8	27/4
sub-optimum TQAM, 32	8/71	75/16	33/8
codex, 64	3/567	81/16	297/64
irragular TQAM, 256	8192/577595	711/128	171/32
TQAM, 64	2/9	33/8	27/8
regular TQAM, 32	8/71	75/16	33/8
TQAM, 64	2/37	161/32	147/32
optimum, 32	512/4503	75/16	33/8
optimum, 64	8/141	163/32	75/16
TQAM, 256	2/149	705/128	675/128

Chapter 4

Simulation results and discussion

4.1 Nakagami-Lognormal fading channel

The channel taken into consideration is Nakagami-Lognormal. The PDF for Nakagami-Lognormal fading channel is defined as

$$p_\gamma(\gamma) = \int_0^\infty \left\{ \frac{m^m}{\omega^m \Gamma(m)} \gamma^{m-1} \exp\left(-\frac{m\gamma}{\omega}\right) \right\} \times \left\{ \frac{1}{\sigma\omega\sqrt{2\pi}} \exp\left(-\frac{(\log_e \omega - \mu)^2}{2\sigma^2} \right) \right\} d\omega \quad (4.1)$$

where $\sigma_{db} = \xi\sigma$, $\mu_{db} = \xi\mu$, parameters μ and σ are mean and standard deviation of the random variable (RV), ω is slowly varying power, γ is the average SNR and m is the Nakagami-m fading paramter.

4.1.1 Holtzman approximation based approach

It is computationally intensive to find the solution for the (4.1). Using the solution provided by [15, 17].

$$P(\theta) = P(\mu) + (\theta - \mu)P^{'}(\mu) + \frac{1}{2}(\theta - \mu)^2 P^{''}(\mu) + ... \quad (4.2)$$

Eq. (4.2) is defined as the taylor series of a function.

$$E[P(\theta)] \approx P(\mu) + \frac{1}{2}\sigma^2 P^{''}(\mu) \quad (4.3)$$

Taking the mean of (4.2) and reducing the infinite series only up to two terms. Rather than going for the derivative expansion, it can also be proceeded with expansion in differences i.e. using the Stirling formula. Eq. (4.2) becomes

$$P(\theta) = P(\mu) + (\theta - \mu)\frac{P(\mu + h) - P(\mu - h)}{2h} + \frac{1}{2}(\theta - \mu)^2\frac{P(\mu + h) - 2P(\mu) + P(\mu - h)}{h^2} \tag{4.4}$$

Taking expectation of (4.4)

$$E[P(\theta)] = P(\mu) + \frac{1}{2}\sigma^2\frac{P(\mu + h) - 2P(\mu) + P(\mu - h)}{h^2} \tag{4.5}$$

where $h = \sqrt{3}\sigma$ is the most optimized value for the accuracy of (4.5) [16]. So (4.5) becomes

$$E[P(\theta)] \approx \frac{2}{3}P(\mu) + \frac{1}{6}P\left(\mu + \sqrt{3}\sigma\right) + \frac{1}{6}P\left(\mu - \sqrt{3}\sigma\right) \tag{4.6}$$

Authors in [17] has used the above method to find the channel capacity for gaussian distribution

$$E[C^n] = E[(\log(1 + \gamma))^n] = \int_{\gamma=0}^{\infty} (\log(1 + \gamma))^n \frac{1}{\sqrt{2\pi}\sigma\gamma} e^{-\frac{(\ln\gamma - \mu)^2}{2\sigma^2}} d\gamma \tag{4.7}$$

taking $\log_e \omega = x$

$$E[C^n] = \int_{-\infty}^{+\infty} g(x)\phi(x)dx = \int_{x=-\infty}^{\infty} (\log(1 + e^x))^n \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \tag{4.8}$$

$$E[C] \approx \frac{2}{3}g(\mu) + \frac{1}{6}g\left(\mu + \sqrt{3}\sigma\right) + \frac{1}{6}g\left(\mu - \sqrt{3}\sigma\right) \tag{4.9}$$

$$E[C] \approx \frac{2}{3}log(1 + e^{\mu}) + \frac{1}{6}log\left(1 + e^{\mu + \sqrt{3}\sigma}\right) + \frac{1}{6}log\left(1 + e^{\mu - \sqrt{3}\sigma}\right) \tag{4.10}$$

PDF of N/L fading channel presented in (4.1) is similar with the (4.8). Therefore (4.1) can be simplified as

$$p_\gamma(\gamma) = \int_{\theta}^{\infty} \psi(\gamma; x) \times \left\{\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right)\right\} dx \tag{4.11}$$

$$p_\gamma(\gamma) = \frac{2}{3}\psi(\gamma; \mu) + \frac{1}{6}\psi\left(\gamma; \mu + \sigma\sqrt{3}\right) + \frac{1}{6}\psi\left(\gamma; \mu - \sigma\sqrt{3}\right) \tag{4.12}$$

where $\psi(\gamma; x) = \frac{m^m}{e^{x^m}\Gamma(m)}\gamma^{m-1}\exp(-\frac{m\gamma}{e^x})$

SEP for the fading channel is defined from (3.15) as

$$P_{fading} = \int_0^\infty P_{AWGN}.p_\gamma(\gamma)d\gamma \tag{4.13}$$

where

$$P_{AWGN} \simeq KQ\left(\sqrt{\zeta\gamma}\right) + \frac{2}{3}K_cQ\left(\sqrt{\frac{2\zeta\gamma}{3}}\right) - 2K_cQ(\sqrt{\zeta\gamma})Q\left(\sqrt{\frac{\zeta\gamma}{3}}\right) \tag{4.14}$$

After multiplying the closed form of N/L fading channel from (4.12) with the approximated form of Q function (4.14). The integral function, consist of the γ variable to be solved as

$$\mathrm{I} = \int_\theta^\infty \gamma^{m-1}\exp(-\gamma a)exp(\gamma b)d\gamma \tag{4.15}$$

$$\mathrm{I} = \int_\theta^\infty \gamma^{m-1}\exp(-\gamma(a+b))d\gamma \tag{4.16}$$

Using the solution provided in [26]

$$\int_\theta^\infty x^n e^{-\mu x} = n!\mu^{-n-1} \tag{4.17}$$

Equation to be integrated is given by

$$\int_\theta^\infty \gamma^{m-1}\exp\left(-\frac{m\gamma}{e^x}\right)\exp(-b\xi\gamma)d\gamma = (m-1)!\left(\frac{m}{e^x}+b\xi\right)^{-m} \tag{4.18}$$

Defining

$$\phi(x; b) = \frac{m^m}{e^{x^m}\Gamma(m)}(m-1)!\left(\frac{m}{e^x}+b\xi\right)^{-m} \tag{4.19}$$

Eq. (4.19) is defined as to compensate the long terms into standard form. The complete solution of SEP for the N/L fading channel using the approximation method provided by the Holtzman is given by

$$
\begin{aligned}
P_{fading}(Proposed\ HA) = {} & \frac{K}{6}\Bigg[\ \frac{1}{12}\phi\left(\mu+\sigma\sqrt{3};\frac{1}{2}\right)+\frac{1}{6}\phi\left(\mu+\sigma\sqrt{3};2\right) \\
& +\frac{1}{6}\phi\left(\mu+\sigma\sqrt{3};\frac{2}{3}\right)\Bigg]\ +\frac{K}{6}\Bigg[\ \frac{1}{12}\phi\left(\mu-\sigma\sqrt{3};\frac{1}{2}\right) \\
& +\frac{1}{6}\phi\left(\mu-\sigma\sqrt{3};2\right)+\frac{1}{6}\phi\left(\mu-\sigma\sqrt{3};\frac{2}{3}\right)\Bigg] \\
& +\frac{2K}{3}\left[\frac{1}{12}\phi\left(\mu;\frac{1}{2}\right)+\frac{1}{6}\phi\left(\mu;2\right)+\frac{1}{6}\phi\left(\mu;\frac{2}{3}\right)\right] \\
& -\frac{4K_c}{3}\left[\frac{1}{12}\phi\left(\mu;\frac{2}{3}\right)+\frac{1}{12}\phi\left(\mu;2\right)\right] \\
& -\frac{K_c}{3}\left[\frac{1}{12}\phi\left(\mu+\sigma\sqrt{3};\frac{2}{3}\right)+\frac{1}{12}\phi\left(\mu+\sigma\sqrt{3};2\right)\right] \\
& -\frac{K_c}{3}\left[\frac{1}{12}\phi\left(\mu-\sigma\sqrt{3};\frac{2}{3}\right)+\frac{1}{12}\phi\left(\mu-\sigma\sqrt{3};2\right)\right] \\
& +\frac{4K_c}{9}\left[\frac{1}{8}\phi\left(\mu;\frac{4}{3}\right)+\frac{1}{24}\phi\left(\mu;\frac{2}{3}\right)\right] \\
& +\frac{K_c}{9}\left[\frac{1}{8}\phi\left(\mu+\sigma\sqrt{3};\frac{4}{3}\right)+\frac{1}{24}\phi\left(\mu+\sigma\sqrt{3};\frac{2}{3}\right)\right] \\
& +\frac{K_c}{9}\left[\frac{1}{8}\phi\left(\mu-\sigma\sqrt{3};\frac{4}{3}\right)+\frac{1}{24}\phi\left(\mu-\sigma\sqrt{3};\frac{2}{3}\right)\right]
\end{aligned}
\tag{4.20}
$$

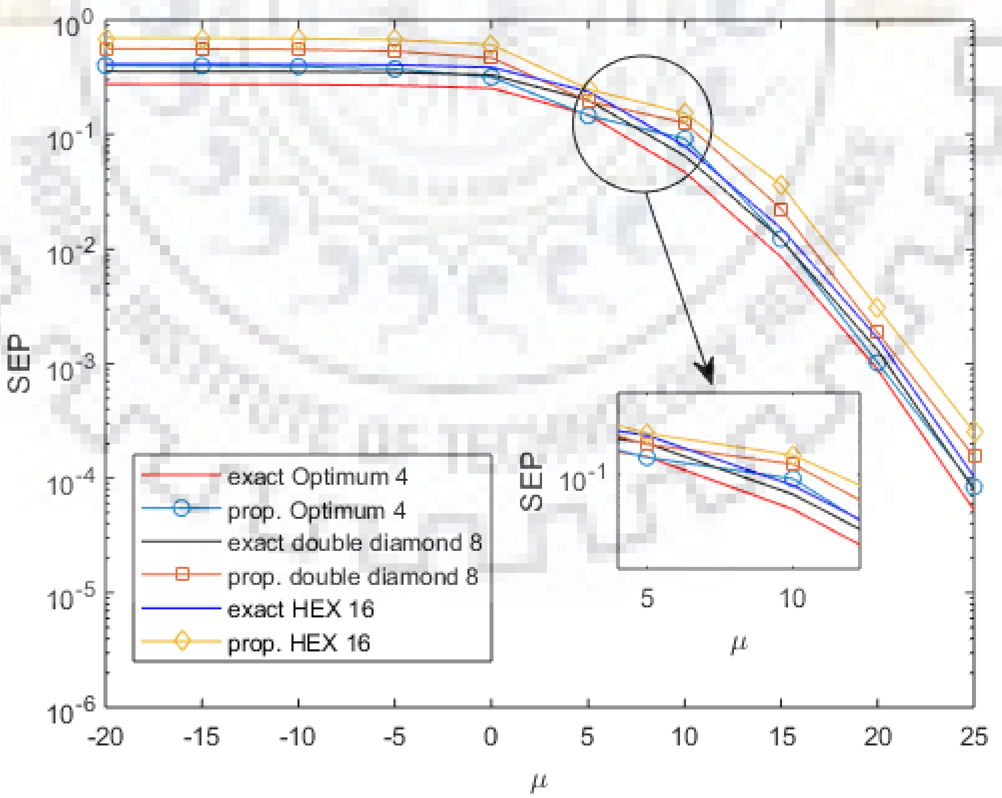


Fig. 4.1. Comparison of exact SEP with that of proposed HA-based approach (m= 0.5; σ = 8 dB)

Fig. 4.1 clearly shows that there is a decrease in the SEP for higher Constellation as compared to lower constellation i.e. the error performance improves. This observation is done on the fixed values of m and σ. The proposed approximation by the method of Holtzman based approach matches with the exact form.

4.1.2 Gauss hermite approximation based approach

$$p_\gamma(\gamma) = \int_0^\infty \left\{ \frac{m^m}{\omega^m \Gamma(m)} \gamma^{m-1} \exp\left(-\frac{m\gamma}{\omega}\right) \right\} \times \left\{ \frac{1}{\sigma\omega\sqrt{2\pi}} \exp\left(-\frac{(\log_e \omega - \mu)^2}{2\sigma^2}\right) \right\} d\omega \tag{4.21}$$

Taking the reference from [27],
substituting $t = \frac{(\log_e \omega - \mu)}{\sqrt{2}\sigma}$

$$\omega = \exp\left(\sqrt{2}\sigma t + \mu\right) \tag{4.22}$$

using (4.22), (4.21) converterted into the form of $\int\limits_{-\infty}^{+\infty} \exp(-t^2)\psi(t)dt$. This is similar to the problem statement for the Gauss Hermite integration [27].

$$y = c \int_{-\infty}^{+\infty} f(x) exp(-b(x-a)^2) dx \tag{4.23}$$

$$y = \sum_{1}^{N} w(i) \times f(x_i) \tag{4.24}$$

where $c = 1$, $w(i)$ are weights , x_i are abscissas and N = 6
Using (4.23-4.24), (4.21) approximated into the closed form as

$$p_\gamma(\gamma) = \frac{m^m \gamma^{m-1}}{\Gamma(m)\sqrt{\pi}} \sum_{i}^{N} W(i)\psi(t_i) \tag{4.25}$$

where $\psi(t) = exp\left(-\exp\left(-\sqrt{2}\sigma t - \mu\right) m\gamma\right) \times \left(\exp\left(\sqrt{2}\sigma t + \mu\right)\right)^{-m}$
After multiplying the closed form of N/L fading channel from (4.25) with the approximated form of Q function (4.14), The integral function, consist of the γ variable to be solved is

$$\mathrm{I} = \int_{\theta}^{\infty} \gamma^{m-1} exp\left(-\exp\left(-\sqrt{2}\sigma t - \mu\right) m\gamma\right) \times \exp\left(-\alpha\gamma\right) d\gamma \tag{4.26}$$

$$\mathrm{I} = \int_{\theta}^{\infty} \gamma^{m-1} \exp\left(-\beta\gamma\right) \exp\left(-\alpha\gamma\right) d\gamma \tag{4.27}$$

where $\beta = \exp\left(-\sqrt{2}\sigma t - \mu\right) m$

Using the solution provided in [26]

$$\int_{\theta}^{\infty} x^{n} e^{-\mu x} = n! \mu^{-n-1} \tag{4.28}$$

Equation to be integrated is given by

$$\int_{\theta}^{\infty} \gamma^{m-1} \exp\left(-\frac{\beta\gamma}{1}\right) \exp(-\alpha\gamma) = (m-1)!(\beta+\alpha)^{-m} \tag{4.29}$$

Defining

$$\phi\left(\beta_i; \alpha\right) = W(i) \times \frac{m^m}{\Gamma(m)\sqrt{\pi}} (m-1)!(\beta_i+\alpha)^{-m} \tag{4.30}$$

Eq. (4.30) is defined as to compensate the long terms into standard form. The complete solution of SEP for the N/L fading channel using the approximation method provided by the Gauss Hermite is given by

$$\begin{aligned} P_{fading}(Proposed\ GHA) = K &\left[\frac{1}{12}\sum_{i}^{N}\phi\left(\beta_i; \frac{1}{2}\right) + \frac{1}{6}\sum_{i}^{N}\phi\left(\beta_i; 2\right) + \frac{1}{6}\sum_{i}^{N}\phi\left(\beta_i; \frac{2}{3}\right)\right] \\ &- 2K_c\left[\frac{1}{12}\sum_{i}^{N}\phi\left(\beta_i; \frac{2}{3}\right) + \frac{1}{12}\sum_{i}^{N}\phi\left(\beta_i; 2\right)\right] \\ &+ \frac{2K_c}{3}\left[\frac{1}{8}\sum_{i}^{N}\phi\left(\beta_i; \frac{4}{3}\right) + \frac{1}{24}\sum_{i}^{N}\phi\left(\beta_i; \frac{2}{3}\right)\right] \end{aligned} \tag{4.31}$$

Fig. 4.2 clearly shows that there is a decrease in the SEP for higher Constellation as compared to lower constellation. The proposed approximation by the method of Gauss Hermite based approach matches with the exact form.

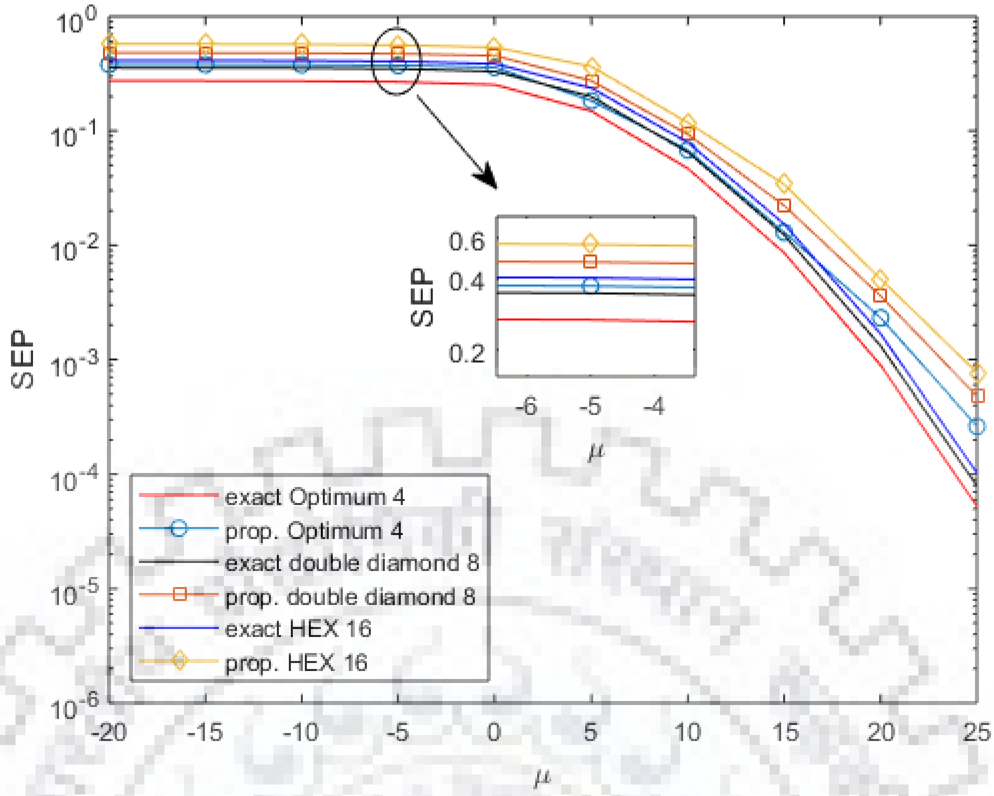
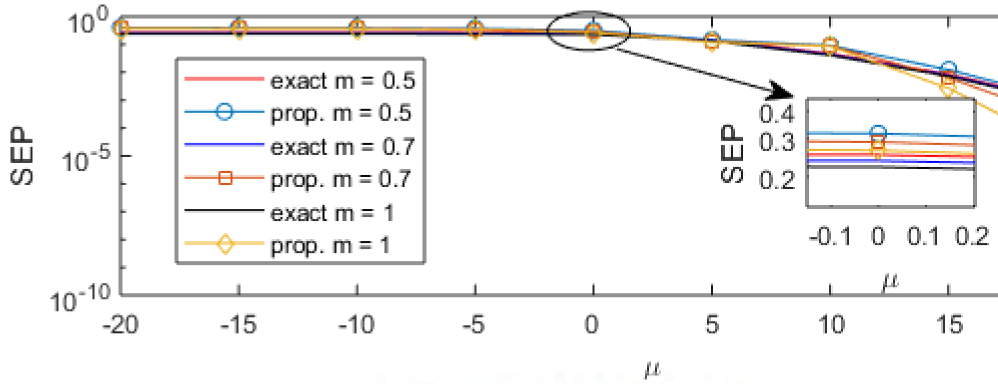


Fig. 4.2. Comparison of exact SEP with that of proposed GHA-based approach (m= 0.5; σ = 8 dB)

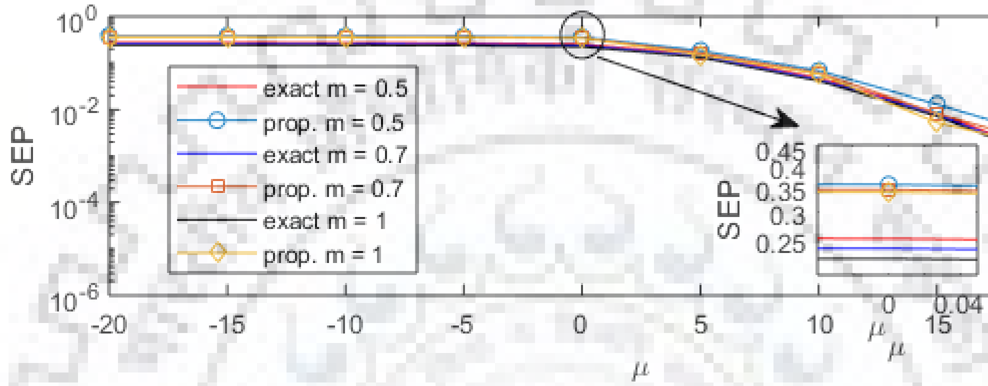
Table 4.1: Comparision of Proposed and Existing SEP Approximation with μ = 10 dB

m	**0.5**	**0.7**	**1**
Exact	0.046840992	0.043550551	0.040532553
Proposed HA-based Approach	0.091569571	0.087685435	0.0857428
Proposed GHA-based Approach	0.0678377	0.0603981	0.05733571

From Fig. 4.3, it is observed that, on increase the value of m of Nakagami-paramter, the SEP decreases. This observation is done for the Optimum 4 constellation of HQAM. Also Table 4.1 depicts that the Holtzman based approximation approach is more acurate at lower SNR as compared to Gauss Hermite based approximation approach which is favourable for higher SNR.



(a)



(b)

Fig. 4.3. Comparison of exact SEP with that of proposed (a) HA-based approach; (b) GHA-based approach (σ=8 dB; Optimum 4)

4.2 Rician fading channel

Assuming the scenario of mobile user, where user is receiving signal from both direct (LOS) and shadowing-multi-path. The channel is taken into consideration is Rician fading channel. The PDF for the Rician Distribution is defined as

$$P_\gamma(\gamma) = \frac{(1+k)\,e^{-k}}{\bar{\gamma}} e^{-\frac{(1+k)\gamma}{\bar{\gamma}}} \mathrm{I}_o\left[2\sqrt{\frac{k\,(1+k)\,\gamma}{\bar{\gamma}}}\right] \tag{4.32}$$

where $\bar{\gamma} = averageSNR$ and I_o = modified bessel function of first kind.
SEP of fading channel from (3.15)

$$P_{fading} = \int_0^\infty P_{AWGN}.p_\gamma(\gamma)d\gamma \tag{4.33}$$

After multiplying the Rician fading channel from (4.32) with the approximated form of Q function (4.14), the integral function which consist of the γ variable becomes

$$\mathrm{I} = \int_{\theta}^{\infty} e^{-\frac{(1+k)\gamma}{\bar{\gamma}}} \mathrm{I}_o \left[2\sqrt{\frac{k(1+k)\gamma}{\bar{\gamma}}} \right] e^{-\alpha\xi\gamma} d\gamma \tag{4.34}$$

Using the solution provided in [26]

$$\int_{\theta}^{\infty} e^{-\alpha x} \mathrm{I}_{2\nu} \left[2\sqrt{\beta x} \right] = \frac{e^{\frac{\beta}{2\alpha}} \Gamma(\nu+1)}{\sqrt{\alpha\beta}\,\Gamma(2\nu+1)} \times \mathrm{M}_{-\frac{\nu}{2}} \left(\frac{\beta}{\alpha} \right) \tag{4.35}$$

where $\mathrm{M}_a(b)$ is the WhittakerM function

$$\int_{\theta}^{\infty} e^{-\frac{(1+k)\gamma}{\bar{\gamma}}} \mathrm{I}_o \left[2\sqrt{\frac{k(1+k)\gamma}{\bar{\gamma}}} \right] e^{-\alpha\xi\gamma} d\gamma = \exp\left(\frac{-\frac{(1+k)k}{\bar{\gamma}}}{2\left(\frac{(1+k)}{\bar{\gamma}} + \alpha\xi\right)} \right) \times \frac{\Gamma(1)}{\Gamma(1)\sqrt{\frac{(1+k)k}{\bar{\gamma}} \times \left(\frac{(1+k)}{\bar{\gamma}} + \alpha\xi\right)}} \times \mathrm{M}_0 \left[\frac{\frac{(1+k)k}{\bar{\gamma}}}{\left(\frac{(1+k)}{\bar{\gamma}} + \alpha\xi\right)} \right] \tag{4.36}$$

Defining

$$\phi(\alpha) = \exp\left(\frac{-\frac{(1+k)k}{\bar{\gamma}}}{2\left(\frac{(1+k)}{\bar{\gamma}} + \alpha\xi\right)} \right) \times \frac{\Gamma(1)}{\Gamma(1)\sqrt{\frac{(1+k)k}{\bar{\gamma}} \times \left(\frac{(1+k)}{\bar{\gamma}} + \alpha\xi\right)}} \times \mathrm{M}_0 \left[\frac{\frac{(1+k)k}{\bar{\gamma}}}{\left(\frac{(1+k)}{\bar{\gamma}} + \alpha\xi\right)} \right] \times \frac{(1+k)e^{-k}}{\bar{\gamma}} \tag{4.37}$$

Eq. (4.37) is defined as to compensate the long terms into standard form. The complete solution of SEP for the Rician fading channel is given by

$$P_{fading}(Rician) = K\left[\frac{1}{12}\phi\left(\frac{1}{2}\right) + \frac{1}{6}\phi(2) + \frac{1}{6}\phi\left(\frac{2}{3}\right) \right] + \frac{2K_c}{3}\left[\frac{1}{8}\phi\left(\frac{4}{3}\right) + \frac{1}{24}\phi\left(\frac{2}{3}\right) \right] - 2K_c\left[\frac{1}{12}\phi\left(\frac{2}{3}\right) + \frac{1}{12}\phi(2) \right] \tag{4.38}$$

Fig. 4.4 depicts that for higher constellation the proposed closed form for SEP is less acurate. It is also observed that the SEP for lower constellation decrease significantly as compared to higher constellation.

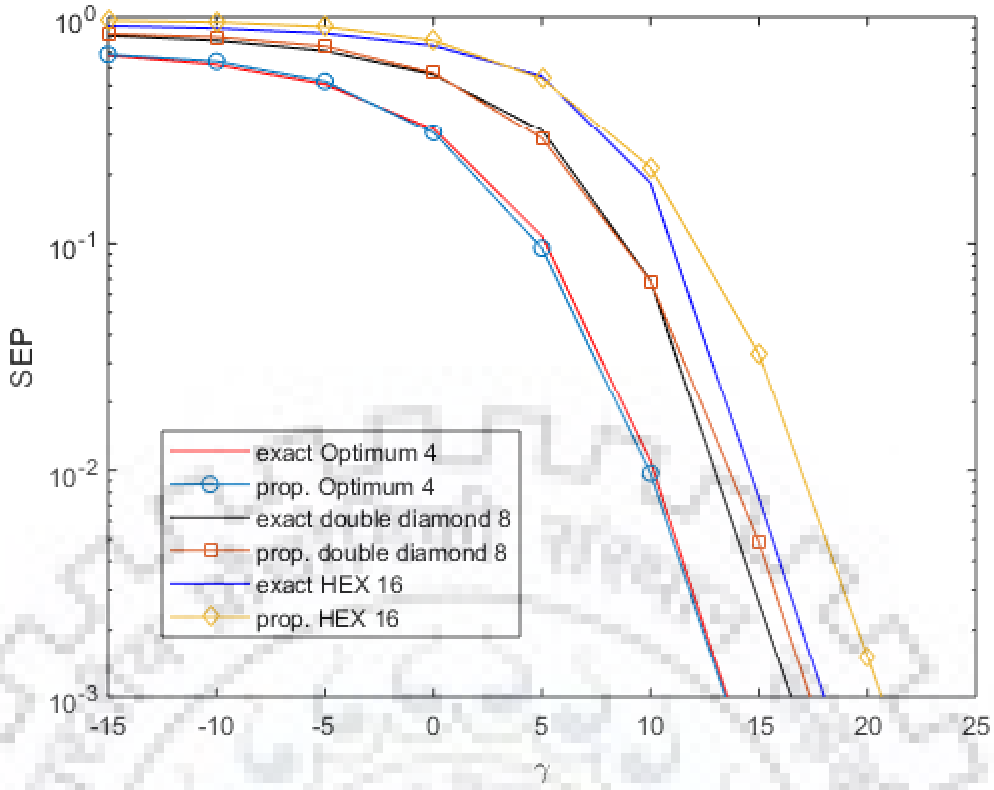


Fig. 4.4. Comparison of exact SEP with that of proposed for rician channel (K = 9.9 dB)

4.3 Combined time shared shadowed/unshadowed fading

Land–mobile satellite systems can be seen as combination of composite multi-path/shadowed fading and unshadowed multi-path fading [14]. Combining the shadowed and unshadowed SEP with the probability as presented in Table 4.2

Table 4.2: Parameter Probability(A) and Rician K factor for various scenario

Envirnnment	**A**	K **[dB]**
Urban Area	0.67	3.1
Suburban Area	0.55	9.8
Highways	0.30	11.7

Table 4.2 shows the variation of combined time shared shadowed/unshadowed. For urban areas the pribability of N/L fading channel is high due to conjustion, therefore the Rician

K factor is less. Further on highways where Rician fading channel dominates due to open field, chances of direct LOS is more, therefore share of N/L is less and value of Rician K is high. Combined (time shared) shadowed/unshadowed is defined as

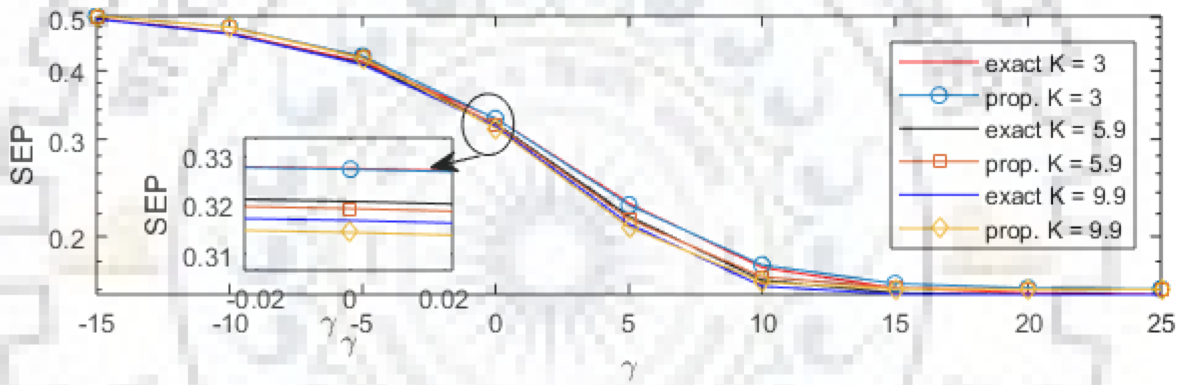
$P(\gamma)$ = (1-A) $\times$ SEP of Rician fading channel + A $\times$ SEP of N/L shadowing fading channel.

The solution of the proposed approximation used by the Holtzman based approach using (4.20) & (4.38) becomes

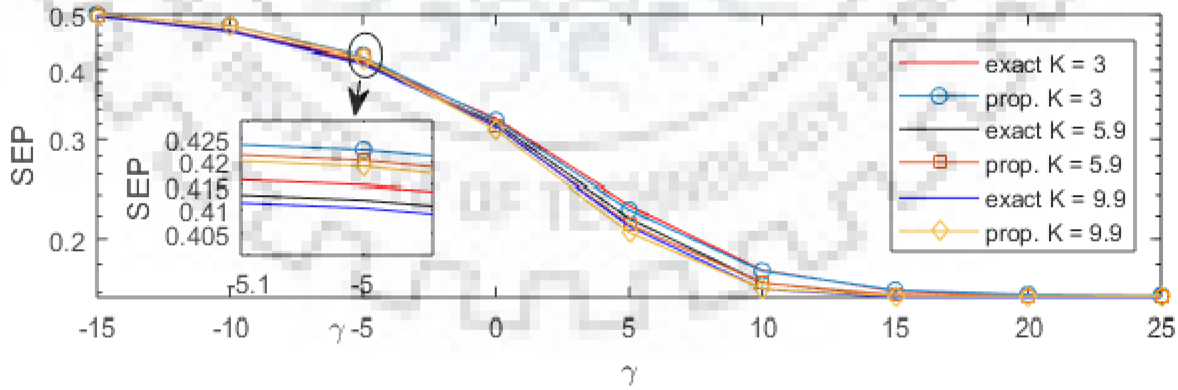
$$\begin{aligned}
P_{fading}(HA+Rice) = A\Bigg[& \frac{K}{6}\left[\frac{1}{12}\phi\left(\mu+\sigma\sqrt{3};\frac{1}{2}\right)+\frac{1}{6}\phi\left(\mu+\sigma\sqrt{3};2\right)+\frac{1}{6}\phi\left(\mu+\sigma\sqrt{3};\frac{2}{3}\right)\right] \\
& +\frac{K}{6}\left[\frac{1}{12}\phi\left(\mu-\sigma\sqrt{3};\frac{1}{2}\right)+\frac{1}{6}\phi\left(\mu-\sigma\sqrt{3};2\right)+\frac{1}{6}\phi\left(\mu-\sigma\sqrt{3};\frac{2}{3}\right)\right] \\
& +\frac{2K}{3}\left[\frac{1}{12}\phi\left(\mu;\frac{1}{2}\right)+\frac{1}{6}\phi\left(\mu;2\right)+\frac{1}{6}\phi\left(\mu;\frac{2}{3}\right)\right] \\
& -\frac{4K_c}{3}\left[\frac{1}{12}\phi\left(\mu;\frac{2}{3}\right)+\frac{1}{12}\phi\left(\mu;2\right)\right] \\
& -\frac{K_c}{3}\left[\frac{1}{12}\phi\left(\mu+\sigma\sqrt{3};\frac{2}{3}\right)+\frac{1}{12}\phi\left(\mu+\sigma\sqrt{3};2\right)\right] \\
& -\frac{K_c}{3}\left[\frac{1}{12}\phi\left(\mu-\sigma\sqrt{3};\frac{2}{3}\right)+\frac{1}{12}\phi\left(\mu-\sigma\sqrt{3};2\right)\right] \\
& +\frac{4K_c}{9}\left[\frac{1}{8}\phi\left(\mu;\frac{4}{3}\right)+\frac{1}{24}\phi\left(\mu;\frac{2}{3}\right)\right] \\
& +\frac{K_c}{9}\left[\frac{1}{8}\phi\left(\mu+\sigma\sqrt{3};\frac{4}{3}\right)+\frac{1}{24}\phi\left(\mu+\sigma\sqrt{3};\frac{2}{3}\right)\right] \\
& +\frac{K_c}{9}\left[\frac{1}{8}\phi\left(\mu-\sigma\sqrt{3};\frac{4}{3}\right)+\frac{1}{24}\phi\left(\mu-\sigma\sqrt{3};\frac{2}{3}\right)\right]\Bigg] \\
& +(1-A)\Bigg[K\left[\frac{1}{12}\phi\left(\frac{1}{2}\right)+\frac{1}{6}\phi\left(2\right)+\frac{1}{6}\phi\left(\frac{2}{3}\right)\right] \\
& +\frac{2K_c}{3}\left[\frac{1}{8}\phi\left(\frac{4}{3}\right)+\frac{1}{24}\phi\left(\frac{2}{3}\right)\right] \\
& -2K_c\left[\frac{1}{12}\phi\left(\frac{2}{3}\right)+\frac{1}{12}\phi\left(2\right)\right]\Bigg]
\end{aligned} \tag{4.39}$$

The solution of the proposed approximation used by the Gauss Hermite based approach using (4.30) & (4.38) becomes

$$
\begin{aligned}
P_{fading}(GHA + Rice) = A\Bigg[\, & K\left[\frac{1}{12}\sum_{i}^{N}\phi\left(\beta_i;\frac{1}{2}\right) + \frac{1}{6}\sum_{i}^{N}\phi\left(\beta_i;2\right) + \frac{1}{6}\sum_{i}^{N}\phi\left(\beta_i;\frac{2}{3}\right)\right] \\
& - 2K_c\left[\frac{1}{12}\sum_{i}^{N}\phi\left(\beta_i;\frac{2}{3}\right) + \frac{1}{12}\sum_{i}^{N}\phi\left(\beta_i;2\right)\right] \\
& + \frac{2K_c}{3}\left[\frac{1}{8}\sum_{i}^{N}\phi\left(\beta_i;\frac{4}{3}\right) + \frac{1}{24}\sum_{i}^{N}\phi\left(\beta_i;\frac{2}{3}\right)\right]\Bigg] \\
& + (1-A)\Bigg[\, K\left[\frac{1}{12}\phi\left(\frac{1}{2}\right) + \frac{1}{6}\phi\left(2\right) + \frac{1}{6}\phi\left(\frac{2}{3}\right)\right] \\
& + \frac{2K_c}{3}\left[\frac{1}{8}\phi\left(\frac{4}{3}\right) + \frac{1}{24}\phi\left(\frac{2}{3}\right)\right] \\
& - 2K_c\left[\frac{1}{12}\phi\left(\frac{2}{3}\right) + \frac{1}{12}\phi\left(2\right)\right]\Bigg]
\end{aligned}
\tag{4.40}
$$



(a)



(b)

Fig. 4.5. Comparison of exact SEP with that of proposed (a) HA-based approach; (b) GHA-based approach (A = 0.5; μ = -3.914 dB; σ = 0.8 dB; Optimum 4; m=1)

K factor of Rician fading channel denotes the ratio of the power of the direct LOS component to the average power of the scattered component. From Fig. 4.5, it is clearly seen that on increasing K of Rician fading channel, the SEP decrease. Increasing K, increases the chance of direct LOS at the reciever end which leads to lower SEP. This observation is done for equal sharing between N/L and Rician channel i.e. for suburban areas. Table 4.3-4.4 validates the accuracy of the proposed SEP estimation with the the exact form.

Table 4.3: Comparison of Exact and Proposed SEP Approximation with γ = 5 dB

K	**3**	**5.9**	**9.9**
Exact	0.22980187	0.2177677	0.2108202
Proposed HA-based Approach	0.220349	0.2152312	0.20797517
Proposed GHA-based Approach	0.22551423	0.2127105	0.20545441

Table 4.4: Comparison of Exact and Proposed SEP Approximation with γ = 20 dB

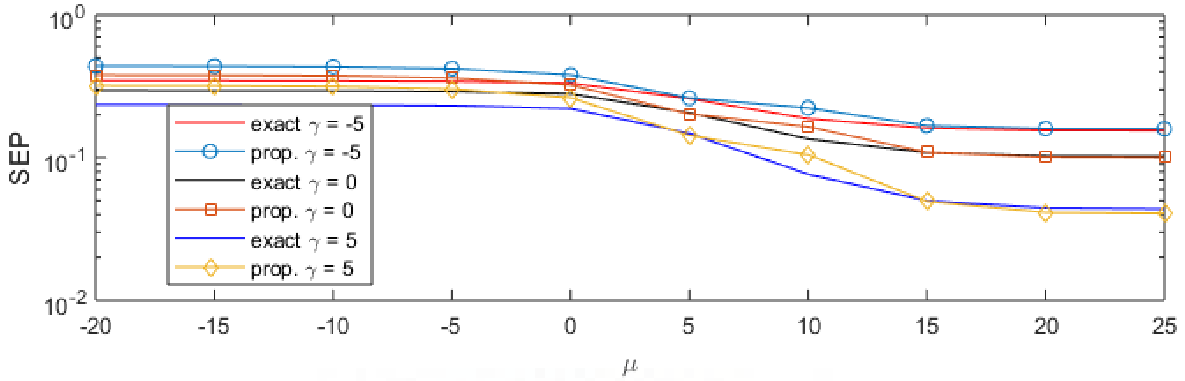
K	**3**	**5.9**	**9.9**
Exact	0.1578946	0.156965	0.1568271
Proposed HA-based Approach	0.1612487	0.160373	0.160245
Proposed GHA-based Approach	0.158727	0.157852	0.157724

Table 4.5: Comparision of Proposed and Existing SEP Approximation with μ = -5 dB

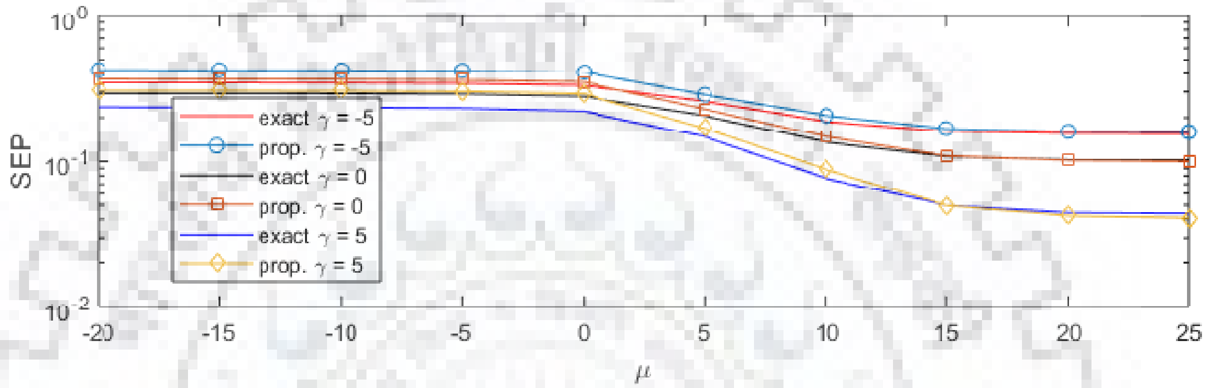
γ	**-5**	**0**	**5**
Exact	0.342434016	0.2899545	0.2313410
Proposed HA-based Approach	0.41041766	0.3517908	0.29221649
Proposed GHA-based Approach	0.42029207	0.361665210	0.30209089

Table 4.6: Comparision of Proposed and Existing SEP Approximation with μ = 0 dB

γ	**-5**	**0**	**5**
Exact	0.3321032	0.27962379	0.2210102
Proposed HA-based Approach	0.38075737	0.3221305	0.2625561
Proposed GHA-based Approach	0.4098058	0.35117901	0.29160469



(a)



(b)

Fig. 4.6. Comparison of exact SEP with that of proposed (a) HA-based approach; (b) GHA-based approach (A = 0.7; σ = 8 dB; Optimum 4; m=0.5)

Table 4.7: Comparision of Proposed and Existing SEP Approximation with $\mu = 5$ dB

γ	**-5**	**0**	**5**
Exact	0.2588255	0.20634611	0.1477325
Proposed HA-based Approach	0.261182384	0.202555	0.1429812
Proposed GHA-based Approach	0.2871766	0.228549	0.16897545

Table 4.8: Comparision of Proposed and Existing SEP Approximation with $\mu = 10$ dB

γ	**-5**	**0**	**5**
Exact	0.1876724	0.1351929	0.0765794
Proposed HA-based Approach	0.2229793	0.164352473	0.1047781
Proposed GHA-based Approach	0.2063670	0.1477402	0.0881659

On increasing the average SNR γ of Rician fading channel, the SEP decreases as shown in Fig. 4.6. Table 4.5-4.8 compares the accuracy of the SEP approximation (4.39) & (4.40) that appear for the fixed digital modulation schemes of HQAM i.e. optimum 4 with existing integrals, for different values of the SNR γ.

From Fig. 4.7 , it clearly shows that SEP increases for lower SNR even after increasing the share of dominant shadowing as compared with higher SNR. The reason being, the value of SEP of the Rician channel at lower SNR is very high and therefore it superimposed its SEP on the N/L which contribute to lower SEP. For higher SNR there is sudden drop in the SEP for the Rician channel, due to which N/L fading take dominancy over the Rician channel. Therefore SEP increases on increasing the share of N/L fading for higher SNR.

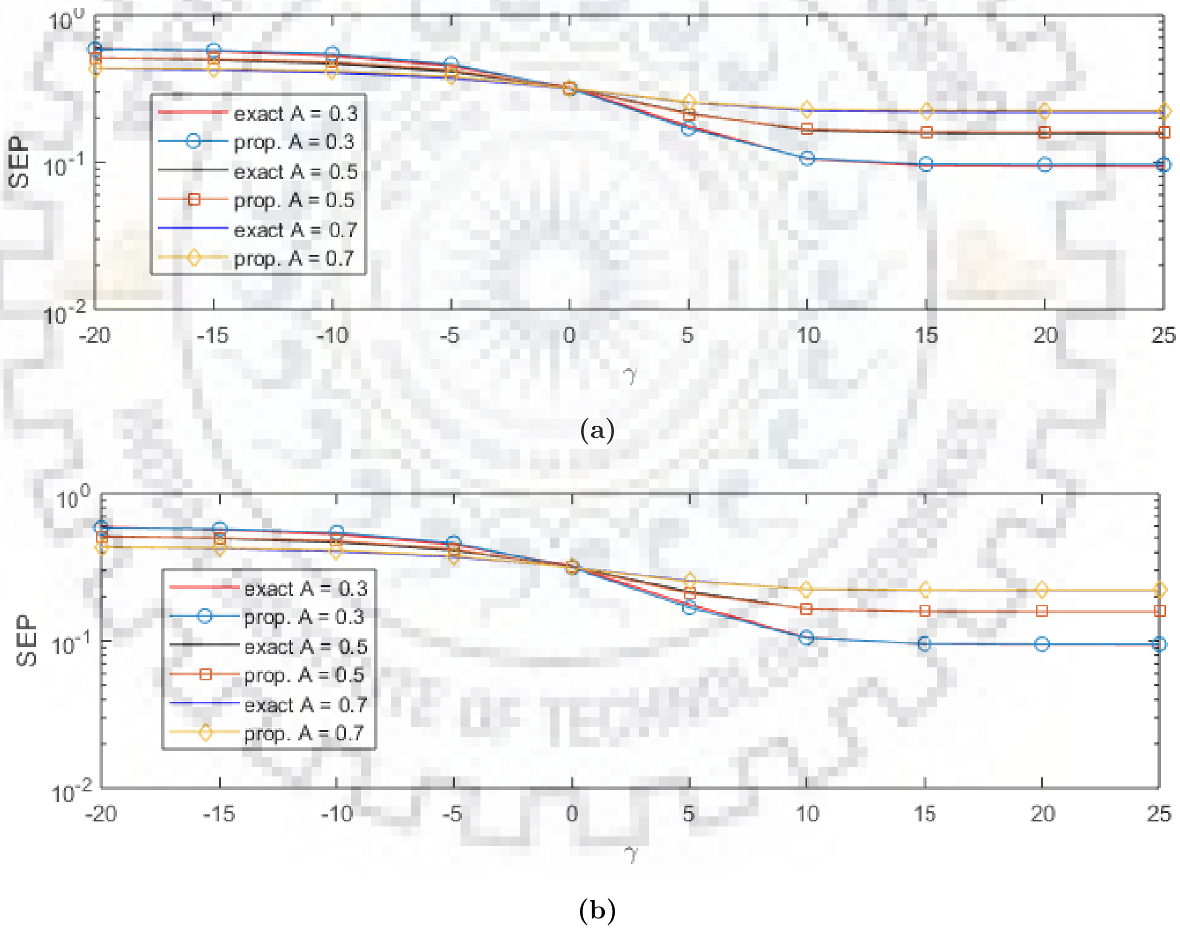


Fig. 4.7. Comparison of exact SEP with that of proposed (a) HA-based approach; (b) GHA-based approach (μ = -3.914 dB; σ = 0.8 dB; Optimum 4; m=1; K = 9.9 dB)

To prove the accuracy of proposed approximations, Fig. 4.8 presents the SEP plot for different constellation of HQAM over combined (time shared) N/L and Rician fading channel. It shows that for higher constellation SEP increases as compared with lower constellation of HQAM. This observation is done for equal sharing between N/L and Rician channel i.e. for suburban areas. Table 4.11-4.12 confirms the accuray of proposed SEP estimation.

Table 4.9: Comparision of Proposed and Existing SEP Approximation with γ = 0 dB

Probability A	**0.3**	**0.5**	**0.7**
Exact	0.321682365	0.3193829	0.3170836
Proposed HA-based Approach	0.3164447	0.3175953	0.3187459
Proposed GHA-based Approach	0.3119322	0.3140745	0.31421684

Table 4.10: Comparision of Proposed and Existing SEP Approximation with γ = -10 dB

Probability A	**0.3**	**0.5**	**0.7**
Exact	0.5281634	0.466869	0.405575
Proposed HA-based Approach	0.54410	0.480207	0.4163129
Proposed GHA-based Approach	0.554588	0.487686	0.41978387

Table 4.11: Comparision of Proposed and Existing SEP Approximation with γ = -5 dB

HQAM Constellation	**Optimum 4**	**Double Diamond 8**	**HEX 16**
Exact	0.4505327	0.6046527	0.713761
Proposed HA-based Approach	0.4630611	0.6308350	0.761511
Proposed GHA-based Approach	0.466548	0.638244	0.76294385

Table 4.12: Comparision of Proposed and Existing SEP Approximation with γ = 10 dB

HQAM Constellation	**Optimum 4**	**Double Diamond 8**	**HEX 16**
Exact	0.105549	0.2896112	0.519369
Proposed HA-based Approach	0.106323	0.298283	0.55827
Proposed GHA-based Approach	0.104811	0.294693	0.55770509

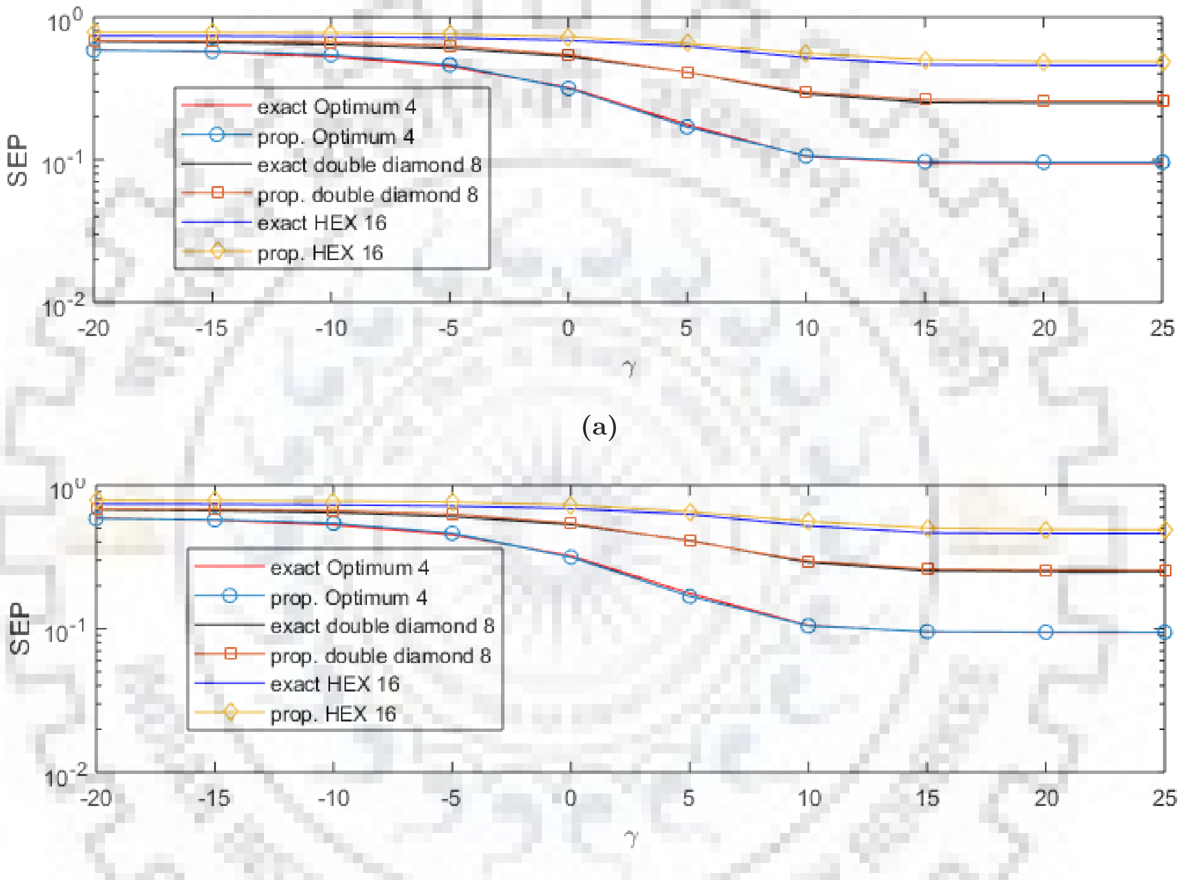


Fig. 4.8. Comparison of exact SEP with that of proposed (a) HA-based approach; (b) GHA-based approach (A = 0.5; μ = -3.914 dB; σ = 0.8 dB; m=1; K = 6.9 dB)

Chapter 5

Conclusion

In this report, results are calculated for SEP is estimated by evaluating the approximation of Q function using trapezoidal rule. It is found that SEP obtained from the approximation proposed by Holtzman and Gauss Hermite are very close to the exact SEP for low and high SNR, respectively. Mathematical closed forms show that SEP calculated through the method proposed by Holtzman is simple as compared to that proposed by Gauss Hermite at the cost of computational time complexity. Increasing the parameter m of Nakagami channel and K of Rician channel, which leads to non fading scenario, the SEP should decrease in nature, which can also be observed in the results. It is seen that for higher constellation, the SEP increases as compared to lower constellation. From the analysis of SEP for Rician channel with different HQAM constellation, it is observed that proposed calculation obtained is less acurate for higher constellation. Results from the combined (time shared) N/L and Rician fading channel depict that, SEP increases for lower SNR with less dominant shadowing as compared to higher SNR with dominant shadowing.

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