Hybrid Pre-Coding Technology in mm-wave MIMO Systems

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree

of

MASTER OF TECHNOLOGY

in

ELECTRONICS & COMMUNICATION ENGINEERING

(With Specialization in Communication Systems)

By

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CANDIDATE'S DECLARATION

I hereby declare that the work presented in this dissertation report entitled **"HYBRID PRE-CODING TECHNOLOGY FOR mm-WAVE MIMO SYSTEMS"** towards the partial fulfilment of the requirements for the award of the degree of **MASTER OF TECHNOLOGY** with specialization in **Communication Systems**, submitted in the Department of Electronics and Communication Engineering, Indian Institute of Technology Roorkee, Roorkee (India) is an authentic record of my own work carried out during the period from July 2018 to June 2019, under the guidance and supervision of **Dr. Meenakshi Rawat**, Assistant Professor, Department of Electronics and Communication Engineering, Indian Institute of Technology Roorkee, Roorkee (India). The matter presented in this dissertation has not been submitted by me for the award of any other degree of this or any other institution.

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CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of my knowledge and belief.

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Abstract

Millimeter wave (mm-wave) MIMO system using hybrid pre-coding is one of the promising technologies for improvement in the channel capacity. The ever growing traffic exploration in cellular communication has recently drawn increased attention to a large amount of underutilized spectrum in the millimeter-wave frequency bands as a potentially visible solution for achieving tens of thousand times more capacity compared to current 4G cellular bands.

There are several alternating minimization techniques which can be used to explore the underutilized band. Hybrid pre-coding is one of the promising technique which can best utilize the underutilized band. Hybrid pre-coding has advantages such as lower complexity and lower cost over its completely analog or digital counter parts. However, there is a need to ascertain an optimum number of RF branches to allow for highest channel efficiency with minimum implementation complexity.

This works represent the theoretical guarantee for alternate minimization using hybrid pre-coding for any variety of phase retrieval problems. The use of the well known least square algorithm to the PE-AltMin and extract the phase of the analog pre-coder and alternatively optimize the digital and analog pre-coder, which significantly improves the performance of the system. Finally, the recent from the proposed algorithm attesting the practicability of the mm-wave bands for cellular usage with an acceptable amount of complexity and improvement in the performance.

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Symbol	Stands For
N_{RF}	Number of RF chain
N_{s}	Number of data stream
N_{R}	Number of receive antenna
N_{T}	Number of Transmit antenna
N_{RF}^{T}	Number of RF chain at transmitter
N_{RF}^{R}	Number of RF chain at receiver
N_{PS}	Number of phase shifter
R	Spectral efficiency
P_{common}	Common power
P_{T}	Transmitter power
P_{R}	Receiver power
P_{PA}	Transmit power of power amplifier
P_{PS}	Transmit power of phase shifter
P_{RF}	Power of the RF chain
$\mathbf{F}_{\mathrm{AA}}$	Updated analog pre-coder
$\mathbf{F}_{\mathrm{BB}}$	Digital pre-coder
$\mathbf{F}_{\mathrm{DD}}$	Orthogonal digital pre-coder
$\mathbf{F}_{\mathrm{opt}}$	Optimal digital pre-coder
$\mathbf{F}_{\mathrm{RF}}$	Analog pre-coder
$\mathbf{W}_{\mathrm{BB}}$	Digital decoder
$\mathbf{W}_{\mathrm{RF}}$	Analog decoder
$\mathbf{W}_{\mathrm{opt}}$	Optimal digital decoder
$\mathbf{H}$	Channel Matrix
$\mathbf{n}$	Noise vector
$\mathbf{x}$	Input signal at transmitter
$\mathbf{y}$	Output signal at receiver decoder
$\mathbf{Tr}(.)$	Trace of
$\Re(.)$	Real part of
$\Im(.)$	Imaginari part of
$\Vert\cdot\Vert_{\mathrm{F}}$	Frobeniuos norm
α	Constant
A_f	Feasible set of pre-coder
η	Energy efficiency
σ_{n}^2	Average noise power
ρ	Average signal power
ϵ,$\forall$	Belong to and For all respectively
$\dagger$	Psuedo inverse of
θ, ϕ	Elevation and Azimuth angle of departure and arrival

List of Abbreviations

Acronyms	**Stands for**
ADC	Analog to Digital Converter
Altmin	Alternating Minimization
BB	Base Band
CSI	Channel State Information
DAC	Digital to Analog Converter
DD	Subscript used for Orthogonal
D2D	Device to Device
eNB	Evolved Node B, that handles radio communications
IFFT	Inverse Fast Fourier Transform
i.i.d	Independent and Identically Distributed
LTE	Long Term Evolution, a 4G mobile communications standard
LNA	Low Noise Amplifier
MIMO	Multiple Input Multiple Output
mm-wave	Millimeter Wave
MO-AltMin	Manifold Optimization AltMin
OFDM	Orthogonal Frequency Division Multiplexing
OMP-AltMin	Orthogonal Matching Pursuit AltMin
P2P	Point to Point or Peer to Peer communication
P2M	Point to Multipoint
PA	Power Amplifier
PE-Altmin	Phase Extraction Alternating Minimization
PPE-AltMin	Proposed Phase Extraction Alternating Minimization
QCQP	Quadratic Constraints Quadratic Programming
RAN	Radio Access Network
RF	Subscript used for analog
RF chain	Radio Frequency Chain
Rx	Receiver
SDR-AltMin	Semidefinite Relaxation Alternating Minimization
SIC-AltMin	Successive Interference Cancellation AltMin
SNR	Signal to Noise Ratio
SVD	Singular Value Decomposition
Tx	Transmitter
UDN	Ultra-Dense Network
USPA	Uniform Square Planner Array

Chapter 1

Introduction

Millimeter wave (mm-wave) in wireless communication is a key technology for cellular communication systems. The upcoming 5G networks aim at carrying out the projected 1000X increase in capacity by 2020 [5]. There are three key approaches to achieve a dramatic increase in system capacity. One way to boost the capacity is to improve the spectral efficiency through the physical layer technique, e.g. massive MIMO [5]. Actually, this is by assuming minimizing objective function is approximately lead to maximizing the spectral efficiency[8]. The second approach is by channel coding result in large bandwidth for the wireless communication systems[9]. For example Error correcting codes have two opposite effects on the efficiency of cellular mobile radio systems. Although they increase the bandwidth per channel, the codes also make signals more robust and thereby reduce the required distance between users of the same frequency band. The third one is the densification of the network by locating small cell for D2D communication[7]. Evolved Node B (eNB) is the only mandatory node in the radio access network (RAN) of Long Term Evolution (LTE). The D2D communication enables new service opportunities and reduces the eNB load for short range data intensive peer-to-peer communication. The eNB is a complex base station that handles radio communications with multiple devices in the cell and carries out radio resource management and handover decisions.

1.1 Motivation

The range of mm-wave is 1 mm – 10 mm i.e., 30 GHz -300 GHz, have now been put forward as a prime candidate for new spectrum in the 5G cellular system, which previously considered for outdoor p2p communication or for carrying high-resolution multimedia, gaming streams. In past days problem associated with the mm-wave were huge path loss due to large obstacle such as large building, mountain and rain attenuation, etc. All these have been overcome by MIMO transceiver by deploying spatial multiplexing [4]. However, this results in ten-time increment in the carrier frequency. High directional beamforming increases the reliability of the mm-wave MIMO system, which significantly minimizes the path loss.

For the conventional MIMO systems, pre-coding is typically accomplished at baseband through digital pre-coder, which takes care of amplitude and phase of the transmitted signal. For fully digital pre-coder needs a large number of low noise amplifier (LNA), analog to digital converter (ADC), signal mixer, up-converter which significantly increase the cost of the system design. These limits the use of the fully digital pre-coder. However, the analog pre-coder gives poor privilege than that of the digital pre-coder. High cost and power consumption of RF chains make the digital pre-coder design impractical [8]. However, the Hybrid pre-coder gives satisfactory performance in the design of the pre-coder. Since hybrid pre-coding is a combination of low dimension digital pre-coder and high dimension analog pre-coder make it simple to use. The high dimension of analog pre-coder further makes a problem in the designs of pre-coder due to power consumer variable voltage amplifiers. These things lead to fixation of the analog pre-coder with costless phase shifter with constant magnitude. Thus hybrid pre-coder becomes practical anyhow in between pure digital pre-coder and analog pre-coder.

1.2 Chapter Description

Chapter 1 deals with an introduction to the work done during the specified period. Various types of pre-coding techniques available are shown in chapter 1. Chapter 2 inform about pre-coder design, as-well chapter 2 deals with system model. Chapter 3 tells about Manifold Optimization Alternating Minimization (MO-AltMin) based hybrid pre-coding for fully connected structure, as-well chapter 3 gives a clear description about Phase Extraction Alternating Minimization (PE-AltMin) and modified PE-AltMin based hybrid pre-coding for the same structure. Actually chapter 3 deals with fully connected hybrid pre-coders.

In chapter 4 we see the behavior of the parts connected structure on Semidefinite Relaxation based Alternating Minimization (SDR-AltMin) technique. In chapter 4 we extend the proposed AltMin algorithms for mm-wave MIMO OFDM systems. We discuss the simulation result in chapter 5 with complexity analysis. In chapter 6 we conclude the thesis and mention the future prospection.

1.3 Article and Inspiration

Our aim is to design an algorithm which can optimize the system with improved performance at significant low complexity. Here orthogonal matching pursuit provides low spectral efficiency with high computational complexity and hardware complexity as well. This inspires us to design a system with significantly lower complexity in both computational and hardware point of views. Then, we think about SDR AltMin algorithm, which significantly improves spectral efficiency.

However MO-AltMin gives the best result as compared to all technique aforementioned, but the problem here is the highest computation complexity with the lowest hardware complexity. This makes us search an intermediate complexity and good spectral efficient algorithm which will take care of both hardware complexity and computational complexity. However we got satisfactory performance here, but still, some improvement is needed. Because by nature human is curious about possibility available. Now we left the rest for such curious person.

Chapter 2

Pre-Coding Techniques

Pre-coding is a generalization of beam-forming to support multi-stream transmission in MIMO communication systems. The same signal is transmitted from each transmit antenna with appropriate weighting such that the signal power is maximized at the receiver. There are basically three types of pre-coding techniques available. These are listed below:

- Analog Pre-coder
- Digital Pre-coder
- Hybrid Pre-coder

2.1 Analog Pre-coding

In the case of analog pre-coding, there is only one data stream for one user. It is restricted to power consumption and phase control, however, the hardware requirement is least, also it gives the least performance since we are using only phase shifter to realize analog pre-coder with fixed magnitude and low energy consumption at a lower cost.

2.2 Digital Pre-coding

A digital pre-coder can control both the amplitude and phase of the signal. Here the number of RF chain is equal to a number of the transmit antennas. The number of data stream increases from one to many number of the users. Since energy consumption and cost are more with the optimal solution it is still impractical. Because of the power constraints, we can't use such a system which consumes more power along with complex design.

2.3 Hybrid Pre-coding

To design system with cost-efficient, hybrid pre-coder uses low dimension digital pre-coder and transmit directional beam to the high dimension analog pre-coder [16]. This pre-coding technique yields the benefit of both analog and digital pre-coding, which is a practical and convincing solution to the available pre-coding method.

Another thing we want to reveal here is that analog beam-forming [3] and pre-coding enable us to change its phase which carries information through the defined channel via antenna array in a specific direction. This provides greater flexibility since one can assign different phases to the signal. Whereas in the beam-forming the phase is fixed for all data stream desired for spatial multiplexing, where we want to transmit the superposition of the signal. Here phase and power can be jointly designed to achieve higher capacity using spatial multiplexing. This can be used for both line of sight and non-line of site mm-wave MIMO communication systems. Comparison among Analog, Digital and Hybrid Pre-coding technique is shown in[17].

Table 2.1: Comparison among Analog, Digital and Hybrid Pre-coding

Parameters	**Analog**	**Digital**	**Hybrid**
No. of User	Single	Multiple	Multiple
No. of stream	Single	Multiple	Multiple
Signal control	Phase	Amplitude, Phase	Amplitude, Phase
Energy consumption	Less	Maximum	Intermediate
Hardware Complexity	Simple	Complex	Intermediate

According to link from RF chain to antennas, which determine the number of phase shifter in use, the hybrid pre-coding transceiver architecture can be categorized into the fully-connected and partially-connected structure as shown in Fig. (2.1), (2.2) and (2.3). Based on the number of RF chains at transmitter and receiver, the hybrid pre-coder is classified into two types. The transmitter and receiver architecture can be classified into two sorts as shown in the Fig.(2.2) and (2.3). Fully and partially connected structures are the basic structure discussed in various literature. We will discuss about these two structures separately in details. These are listed here.

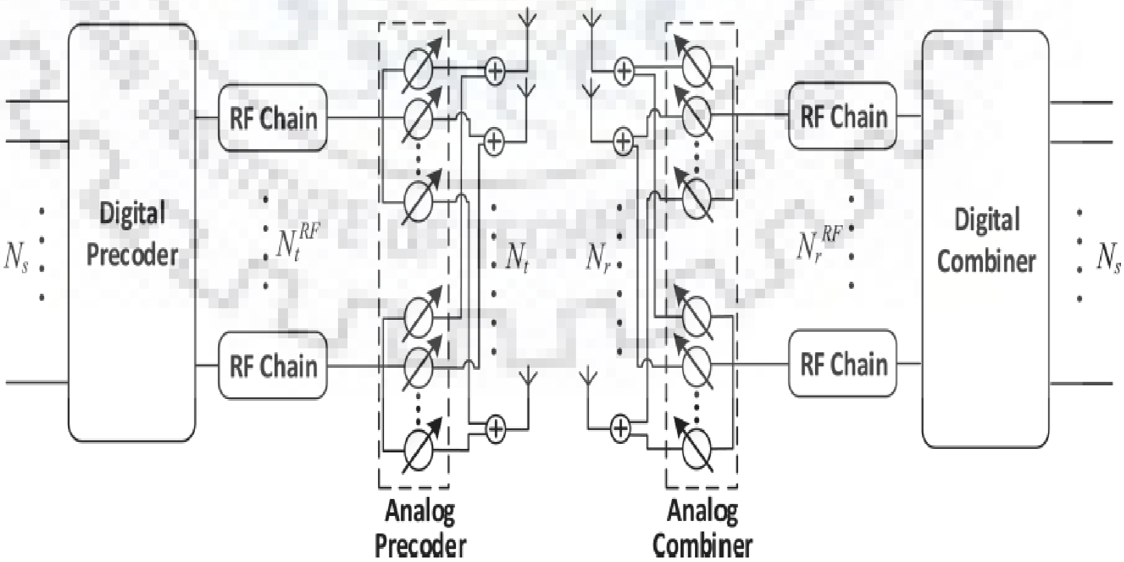

Figure 2.1: Hybrid pre-coder Design

- Fully connected structure
- Partially connected structure

2.3.1 Fully connected structure

The fully connected structure access full beam forming gain for each and every RF chains with a natural combination between RF chains and antenna elements, as we know in this case each antenna is connected to all RF chain. In other words, we can say that all RF chains are connected to all antennas [13].

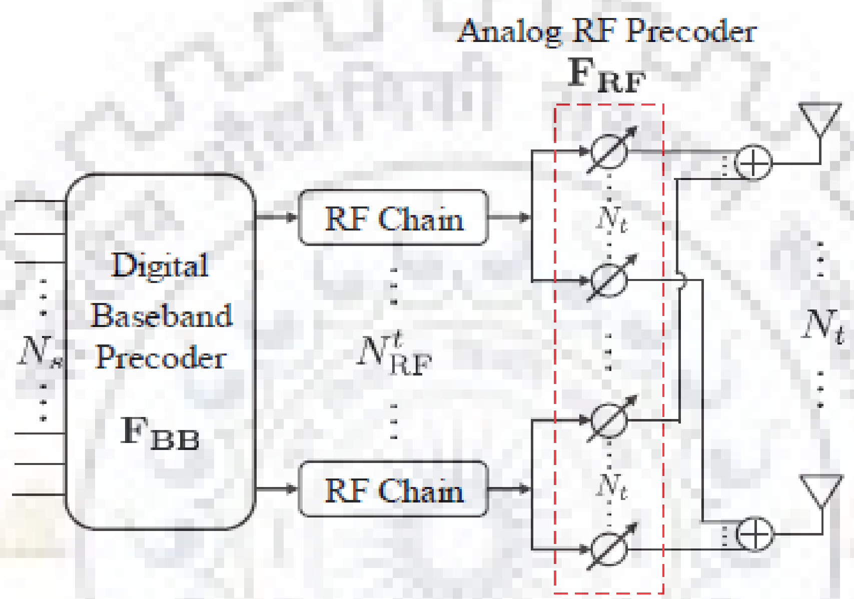

Figure 2.2: Fully connected structure of MIMO transmitter system

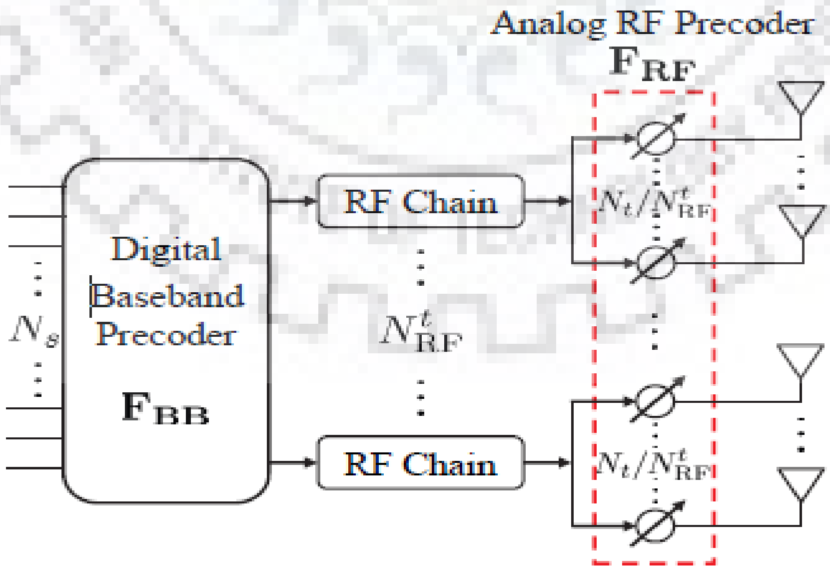

Figure 2.3: Partially connected structure of MIMO transmitter system

Due to this complex structure, it consumes more power than the partially connected structure. Consequently, it becomes energy inefficient as the number of RF chain increases. In fully connected structure unit modulus constraints is needed for analog pre-coder. On overall its performance is good. However, for the lower number of RF chain, it gives a superior result than the partially connected one.

2.3.2 Partially connected structure

In this structure, each RF chains is connected with Nt/Nrf part of the antennas, which notably reduces the hardware design complexity. It is energy efficient but at the expense of loss in performance [20]. However, in this structure, the energy becomes almost constant with respect to the number of RF chain. On the other hand, scarifying some beamforming gain partially connected structure provides the least complexity.

No study tells the direct optimization of the hybrid pre-coder without extra restriction in the partially connected structure. That is why where boost performance is needed; research pays less attention to it. However, we believe any researcher will further renovate it in the coming days to achieve greater efficiency at low complexity.

Table 2.2: Comparison between fully and partially connected structure

Algorithm vs Parameter	**Fully Connected**	**Partially Connected**
No. of phase shifters	$N_t.N_{\text{RF}}$	N_t
Max. no. of RF chain	N_t	N_t/N_{RF}
Analog precoder dimension	Large	Intermediate
Energy Efficiency($N_{\mathbf{RF}}$ = 18 **and** $N_s = 2$)	≤ 40	≥ 60

2.4 System Modeling

2.4.1 Mathematical model of transmitted and received signal

Consider the single-user mm-wave MIMO system as shown in Fig. (2.1) where input size to digital pre-coder is $N_s \times 1$, transmitter and receiver antenna size of $N_t \times 1$ and $N_t \times 1$. Finally, decoded output is of size $N_s \times 1$. Where N_s, N_t, N_r are number of data stream, number of transmitter and number of receiver antenna respectively. The transceiver is subjected to following restriction i.e., $N_s \leq N_{\text{RF}}^t \leq N_t$ and $N_s \leq N_{\text{RF}}^r \leq N_r$. Transmitted signal by the transmitter is given by

$$\mathbf{x} = \mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\mathbf{s} \tag{2.1}$$

Where, s is the $N_s \times 1$ system vector following constraints $\mathbb{E}\left[\mathbf{s}\mathbf{s}^H\right] = \frac{1}{N_s}\mathbf{I}_{N_s}$. Size of digital pre-coder $\mathbf{F}_{\text{BB}}$ is $N_{RF}^t \times N_s$ and size of analog pre-coder $\mathbf{F}_{\text{RF}}$ is $N_t \times N_{RF}^t$ .The normalized transmit power restriction is given by $\|\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\|_F^2 = N_s$. Received signal after the signal processing at receiver for the narrowband block fading channel is given by

$$\mathbf{y} = \sqrt{\rho}\mathbf{W}_{\text{BB}}^H\mathbf{W}_{\text{RF}}^H\mathbf{H}\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\mathbf{s} + \mathbf{W}_{\text{BB}}^H\mathbf{W}_{\text{RF}}^H\mathbf{n} \tag{2.2}$$

Where, ρ is average received power, $\mathbf{H}$ is channel matrix, $\mathbf{W}_{\text{BB}}$ is digital base-band decoder of size $N_{\text{RF}}^r \times N_s$ and $\mathbf{W}_{\text{RF}}$ is analog RF decoder of size $N_r \times N_{\text{RF}}^r$ and $\mathbf{n}$ is the noise vector of independent and identically distributed (i.i.d.) Gaussian noise having zero mean and variance. Spectral efficiency for the Gaussian distribution is given by following equation.

$$\begin{aligned} R = \log\det\Bigg(\mathbf{I}_{N_s} + \frac{\rho}{\sigma_n^2 N_s}(\mathbf{W}_{\text{RF}}\mathbf{W}_{\text{BB}})^{\dagger}\mathbf{H}\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}} \\ \times \mathbf{F}_{\text{BB}}^H\mathbf{F}_{\text{RF}}^H\mathbf{H}^H(\mathbf{W}_{\text{RF}}\mathbf{W}_{\text{BB}})\Bigg). \end{aligned} \tag{2.3}$$

Our assumption is that perfect channel state information (CSI) is available at Tx and Rx. To achieve this, we need to effective feedback technique which is suggested in [15]. Analog pre-coder is limited to the phase adjustment at transmitter. Thus analog pre-coder and decoder $\mathbf{F}_{\text{RF}}$ and $\mathbf{W}_{\text{RF}}$ ought to satisfy the unit modulus constraints. i.e. $|(\mathbf{F}_{\text{RF}})_{i,j}| = |(\mathbf{W}_{\text{RF}})_{i,j}| = 1$, for all non-zero elements.

2.4.2 Channel Matrix Modeling

By keeping free space path loss in the mind, the environment of mm-wave propagation is defined by clustered channel model which follow the Saleh-Valenzuela model [15]. This model defines the channel matrix for the mm-wave MIMO as shown below.

$$\mathbf{H} = \sqrt{\frac{N_t N_r}{N_{cl} N_{ray}}} \sum_{i=1}^{N_{cl}} \sum_{l=1}^{N_{ray}} \alpha_{il} \mathbf{a}_r(\phi_{il}^r, \theta_{il}^r) \mathbf{a}_t(\phi_{il}^t, \theta_{il}^t)^H \tag{2.4}$$

Where, N_{cl} and N_{ray} are number of cluster and the number of the rays respectively and α_{il} denote the path gain of lth ray in the ith propagation cluster. For the complex Gaussian channel it should satisfy $\mathbb{E}\left[\|\mathbf{H}\|_F^2\right] = N_t N_r$. Array response vector at the transmitter and receiver for specified rays and cluster respectively, is given by

$$\begin{aligned} \mathbf{a}(\phi_{il}, \theta_{il}) = \frac{1}{\sqrt{N}}\Big[1, \ldots, e^{j\frac{2\pi}{\lambda}d(p\sin\phi_{il}\sin\theta_{il} + q\cos\theta_{il})}, \\ \ldots, e^{j\frac{2\pi}{\lambda}d\left((\sqrt{N}-1)\sin\phi_{il}\sin\theta_{il} + (\sqrt{N}-1)\cos\theta_{il}\right)}\Big]^T \end{aligned} \tag{2.5}$$

where, d is the antenna spacing and λ is the signal wavelength with constraints $0 \le p < \sqrt{N}$ and $0 \le q < \sqrt{N}$. Here N is N_t and N_r at transmitter and receiver respectively. Here, we assume that antenna is for half-wave dipole i.e., antenna spacing $d = \lambda/2$.

2.4.3 Problem Formulation

In [8] and [13] the pre-coder and decoder design is subdivided into two subproblems, which makes the design simpler and easier. However, there is an extra power constraint in the pre-coder design. The problem can be formulated as shown below.

$$\begin{aligned} \underset{\mathbf{F}_{\text{RF}}, \mathbf{F}_{\text{BB}}}{\text{minimize}} \quad & \|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\|_F \\ \text{subject to} \quad & \begin{cases} \mathbf{F}_{\text{RF}} \in \mathcal{A} \\ \|\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\|_F^2 = N_s \end{cases} \end{aligned} \tag{2.6}$$

Where $\mathbf{F}_{\text{opt}}$ is optimal digital pre-coder, $\mathbf{F}_{\text{RF}}$ and $\mathbf{F}_{\text{BB}}$ is analog and digital pre-coder equation (4.6) shows difference between optimal digital pre-coder and hybrid pre-coder. In [8] it has been shown that minimizing objective function is equivalent to the maximizing the spectral efficiency. Actually it acts like Euclidean distance between the optimal digital pre-coder and the hybrid pre-coder. Here A_f is feasible set of fully connected and partially connected analog pre-coder. Optimal digital and optimal analog pre-coder can be achieved by taking singular value decomposition of the channel matrix $\mathbf{H}$ i.e., $\mathbf{H} = \mathbf{USV}^{\text{H}}$. Here $\mathbf{U}$ and $\mathbf{V}$ are the unitary matrix, which satisfy condition $\mathbf{U}^* = \mathbf{U}^{-1}$ or $\mathbf{UU}^* = \mathbf{I}$, where $\mathbf{I}$ is identity matrix. We have choose first N_s column of $\mathbf{U}$ and $\mathbf{V}$ to get the optimal digital decoder and optimal digital pre-coder respectively.

(2.6) is a main matrix factorization problem, for which alternating minimization algorithm is applied as main tool [10]. Many papers show the proof of it as a successful tool for the solving problem [14] such as phase retrieval, image reconstruction, and blind de-convolution. In alternating minimization, we work on analog and digital pre-coder. Alternatively we solve $\mathbf{F}_{\text{RF}}$ and $\mathbf{F}_{\text{BB}}$. First fix $\mathbf{F}_{\text{RF}}$ and solve for $\mathbf{F}_{\text{BB}}$ and then fix $\mathbf{F}_{\text{BB}}$ solve for $\mathbf{F}_{\text{RF}}$, while maintaining unit modulus constraint on the analog pre-coder $\mathbf{F}_{\text{RF}}$.

Chapter 3

Fully Connected Hybrid Pre-Coding

3.1 Manifold Optimization based Hybrid Pre-coding

The fully connected structure is most widely used in many places, where each RF chain is connected to all the antenna arrays, as shown in Fig.2.1. Due to unit modulus constraint the design of the pre-coder become intractable. In this manifold technique, the unit modulus constraint is defined in the Riemannian manifold region, that's why named as manifold optimization [2]. Here for the MO AltMin, we are dealing for the fully connected structure only.

In paper [20], author motivated by [22] has proved that for fully connected structure objective function in eq.(2.6) can be made strictly zero under condition that number of RF chain must be greater or equal to twice of the number of data stream used. Therefore the author of [19] focus in the range of N_{RF} where $N_s \leq N_{\mathrm{RF}}^t < 2N_s$. However author has got performance of fully digital pre-coder for $N_{\mathrm{RF}}^t \geq 2N_s$.

3.1.1 Digital Baseband Pre-coder Design

First we design the digital pre-coder by fixing analog pre-coder $\mathbf{F}_{\mathrm{RF}}$ and optimizing the digital pre-coder by using globally optimizer accepted least square. From (2.6) we can write the objective function as follow.

$$\underset{\mathbf{F}_{\mathrm{BB}}}{\text{minimize}} \quad \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F \,, \tag{3.1}$$

Whose, solution for the digital pre-coder is given by least square, given by

$$\mathbf{F}_{\mathrm{BB}} = \mathbf{F}_{\mathrm{RF}}^{\dagger}\mathbf{F}_{\mathrm{opt}} \tag{3.2}$$

No doubt (3.2) provides globally optimized solution for the given objective function in (2.6).

3.1.2 Analog RF Pre-coder Design

Analog pre-coder can be specified by unit modulus constraints i.e., $|(\mathbf{F}_{\mathrm{RF}})_{i,j}| = 1$, no need to say that each RF chain is connected to all the antennas. In analog pre-coder $\mathbf{F}_{\mathrm{RF}}$ design we fix digital pre-coder $\mathbf{F}_{\mathrm{BB}}$, and optimize analog pre-coder $\mathbf{F}_{\mathrm{RF}}$ while minimizing the Euclidean distance between the optimal digital pre-coder and hybrid pre-coder as shown below.

$$\begin{aligned} \underset{\mathbf{F}_{\mathrm{RF}}}{\text{minimize}} \quad & \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \\ \text{subject to} \quad & |(\mathbf{F}_{\mathrm{RF}})_{i,j}| = 1, \forall i, j. \end{aligned} \tag{3.3}$$

The main obstacles here are the unit modulus constraints, which are intrinsically non-convex. One thing is to be noted that (3.3) is square of the Frobenius norm, which will make the error function smooth and quadratic. A Riemannian manifold is the complex circle region where we can find the required gradient of the cost function. It is said in [19] that Riemannian manifold is analogous to Euclidean space with smooth constraints. To optimize the analog pre-coder and minimize the cost function we use tree searching algorithm namely conjugate gradient, steepest descent, and the trust region. In former we define the gradient followed by the Armijo backtracking line search and by Polak-ribiere parameter. Then define Riemannian gradient, now we find the direction of the conjugate line search. Armijo and polak-Ribiere assure the convergence of the Euclidean distance and optimize the analog pre-coder $\mathbf{F}_{\mathrm{RF}}$. They decrease the cost function in each iteration towards zero and non-increasing which guarantee the converging cost function.

3.1.3 Hybrid Pre-coder Design

In the step of hybrid pre-coder design after optimizing the analog and digital pre-coder we just take the norm of the digital pre-coder by multiplying the square root of the data stream and dividing the Frobenius norm of the product of analog and digital pre-coder i.e., we normalize $\mathbf{F}_{\mathrm{BB}}$ by a factor $\frac{\sqrt{N_s}}{\|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F}$ at the step (3.2)) to achieve $\widehat{\mathbf{F}}_{\mathrm{BB}}$. The following lemma reveals the effects of this normalization.

Lemma in [19] says, if the Euclidean distance before normalization is given by inequality $\|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F \leq \delta$ then, after normalization we have relation as $\left\|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\widehat{\mathbf{F}}_{\mathrm{BB}}\right\|_F \leq 2\delta$. Since error factor delta is very small therefore twice of it is not very big quantity, it should be almost zero. **Proof:** Define the normalize factor $\frac{\sqrt{N_s}}{\|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F} = \frac{1}{\lambda}$ and thus $\|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F = \lambda\sqrt{N_s} = \lambda \|\mathbf{F}_{\mathrm{opt}}\|_F$ by norm inequality, we have

$$\begin{aligned} \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F &\geq \left| \|\mathbf{F}_{\mathrm{opt}}\|_F - \|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F \right| \\ &= |1-\lambda| \, \|\mathbf{F}_{\mathrm{opt}}\|_F , \end{aligned} \tag{3.4}$$

Which is equivalent to $\|\mathbf{F}_{\mathrm{opt}}\|_F \leq \frac{1}{|\lambda-1|}\delta$. When $\lambda \neq 1$, which indicates that the objective function $\|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F \neq 0$. Normalized digital pre-coder for MO-AltMin is given by

$$\widehat{\mathbf{F}}_{\mathrm{BB}} = \frac{\sqrt{N_s}}{\|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F}\mathbf{F}_{\mathrm{BB}} \tag{3.5}$$

Now,

$$
\begin{aligned}
&\left\|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}}\widehat{\mathbf{F}}_{\text{BB}}\right\|_F \\
&= \left\|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}} + \left(1 - \frac{1}{\lambda}\right)\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\right\|_F \\
&\leq \left\|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\right\|_F + \left|1 - \frac{1}{\lambda}\right| \left\|\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\right\|_F \\
&\leq \delta + |\lambda - 1| \left\|\mathbf{F}_{\text{opt}}\right\|_F \leq \delta + \frac{|\lambda - 1|}{|\lambda - 1|}\delta = 2\delta.
\end{aligned}
\tag{3.6}
$$

Lemma guarantee the convergence of the cost function at cost of high complexity. Because of the high dimension of analog pre-coder, it will take too much time to achieve the final result. Hence limit the practical implementation of the design. Only the Euclidean Gradient of objective function will have the complexity of $N_{\text{RF}}^t N_t \times N_s N_t$ which is very large. Despite the high complexity it directly solves the unit modulus constraint of analog pre-coder using alternating minimization which will dramatically improve the spectral efficiency.

3.2 Hybrid Pre-coder design for PE-AltMin Algorithm

The high complexity of MO-AltMin prevents practical implementation. It inspires the researcher to design a hybrid pre-coding [6] algorithm with little lower complexity both in hardware and practical implementation point of view. Hybrid pre-coding is the best to substitute for it. It not only reduces complexity but makes the design practical with slightly lower performance.

3.2.1 Digital Baseband Structure

Here we utilize the orthogonal property on digital pre-coder matrix as defined in (3.7) where $\mathbf{F}_{\text{DD}}$ is a unitary matrix known as orthogonal digital pre-coder with same dimension as of digital pre-coder $\mathbf{F}_{\text{BB}}$.

$$\mathbf{F}_{\text{BB}}^H \mathbf{F}_{\text{BB}} = \alpha \mathbf{F}_{\text{DD}}^H \alpha \mathbf{F}_{\text{DD}} = \alpha^2 \mathbf{I}_{N_s} \tag{3.7}$$

Orthogonal digital pre-coder is obtained from the product of unitary matrix obtained from the singular value decomposition (SVD). This orthogonal digital pre-coder significant simplify the pre-coder structure. Here we extract the phase of the product of unconstrained optimal digital pre-coder and orthogonal digital pre-coder which gives the analog pre-coder.

$$\textbf{SVD} : \mathbf{F}_{\text{opt}}^H \mathbf{F}_{\text{RF}}^{(k)} = \mathbf{U}^{(k)} \mathbf{S}^{(k)} \mathbf{V}_1^{(k)H} \quad \text{and} \quad \mathbf{F}_{\text{DD}}^{(k)} = \mathbf{V}_1^{(k)} \mathbf{U}^{(k)H} \tag{3.8}$$

3.2.2 Hybrid Pre-coder Design

By using the orthogonal properties from (3.7) and replacing $\mathbf{F}_{\text{BB}}$ to $\alpha \mathbf{F}_{\text{DD}}$, cost function in (2.6) can be rewritten as follow.

$$
\begin{aligned}
&\left\|\mathbf{F}_{\text{opt}}-\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\right\|_F^2\\
=&\operatorname{Tr}\left(\mathbf{F}_{\text{opt}}^H\mathbf{F}_{\text{opt}}\right)-\operatorname{Tr}\left(\mathbf{F}_{\text{opt}}^H\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\right)\\
&-\operatorname{Tr}\left(\mathbf{F}_{\text{BB}}^H\mathbf{F}_{\text{RF}}^H\mathbf{F}_{\text{opt}}\right)+\operatorname{Tr}\left(\mathbf{F}_{\text{BB}}^H\mathbf{F}_{\text{RF}}^H\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\right)\\
=&\left\|\mathbf{F}_{\text{opt}}\right\|_F^2-2\alpha\Re\operatorname{Tr}\left(\mathbf{F}_{\text{DD}}\mathbf{F}_{\text{opt}}^H\mathbf{F}_{\text{RF}}\right)\\
&+\alpha^2\left\|\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{DD}}\right\|_F^2.
\end{aligned}
\tag{3.9}
$$

We note that the square of the Frobenius norm $\left\|\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{DD}}\right\|_F^2$ has the following upper bound.

$$
\begin{aligned}
\left\|\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{DD}}\right\|_F^2&=\operatorname{Tr}\left(\mathbf{F}_{\text{DD}}^H\mathbf{F}_{\text{RF}}^H\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{DD}}\right)\\
&=\operatorname{Tr}\left\{\begin{pmatrix}\mathbf{I}_{N_s}&\\&\mathbf{0}\end{pmatrix}\mathbf{K}^H\mathbf{F}_{\text{RF}}^H\mathbf{F}_{\text{RF}}\mathbf{K}\right\}\\
&\leq\operatorname{Tr}\left\{\mathbf{K}^H\mathbf{F}_{\text{RF}}^H\mathbf{F}_{\text{RF}}\mathbf{K}\right\}\\
&=\left\|\mathbf{F}_{\text{RF}}\right\|_F^2,
\end{aligned}
\tag{3.10}
$$

where $\mathbf{F}_{\text{DD}}\mathbf{F}_{\text{DD}}^H=\mathbf{K}\begin{pmatrix}\mathbf{I}_{N_s}&\\&\mathbf{0}\end{pmatrix}\mathbf{K}^H$ is SVD of the $\mathbf{F}_{\text{DD}}\mathbf{F}_{\text{DD}}^H$ and the inequality holds when $N_{\text{RF}}^t=N_s$, i.e., $\mathbf{F}_{\text{DD}}$ is a square matrix. Hence, the objective function in the (4.6) is upper-bounded by $\left\|\mathbf{F}_{\text{opt}}\right\|_F^2-\frac{\left\{\Re\operatorname{Tr}\left(\mathbf{F}_{\text{DD}}\mathbf{F}_{\text{opt}}^H\mathbf{F}_{\text{RF}}\right)\right\}^2}{\left\|\mathbf{F}_{\text{RF}}\right\|_F^2}$. In order to get $\mathbf{F}_{\text{RF}}$ rid of the product with $\mathbf{F}_{\text{BB}}$, we choose to add the constant term $\left(\frac{1}{2\left\|\mathbf{F}_{\text{RF}}\right\|_F^2}-1\right)\left\|\mathbf{F}_{\text{opt}}\right\|_F^2+\frac{1}{2}$ to the bound and multiply it by the positive constant term $2\left\|\mathbf{F}_{\text{RF}}\right\|_F^2$. Then we have

$$
\begin{aligned}
&\left\|\mathbf{F}_{\text{opt}}\right\|_F^2-2\Re\operatorname{Tr}\left(\mathbf{F}_{\text{DD}}\mathbf{F}_{\text{opt}}^H\mathbf{F}_{\text{RF}}\right)+\left\|\mathbf{F}_{\text{RF}}\right\|_F^2\\
=&\operatorname{Tr}\left(\mathbf{F}_{\text{RF}}^H\mathbf{F}_{\text{RF}}\right)-2\Re\operatorname{Tr}\left(\mathbf{F}_{\text{DD}}\mathbf{F}_{\text{opt}}^H\mathbf{F}_{\text{RF}}\right)\\
&+\operatorname{Tr}\left(\mathbf{F}_{\text{DD}}\mathbf{F}_{\text{opt}}^H\mathbf{F}_{\text{opt}}\mathbf{F}_{\text{DD}}^H\right)\\
=&\left\|\mathbf{F}_{\text{opt}}\mathbf{F}_{\text{DD}}^H-\mathbf{F}_{\text{RF}}\right\|_F^2.
\end{aligned}
\tag{3.11}
$$

The objective function for the hybrid pre-coder is given by the following equation which is modified in terms of the orthogonal digital pre-coder.

$$
\begin{aligned}
\underset{\mathbf{F}_{\text{RF}},\mathbf{F}_{\text{DD}}}{\text{minimize}}\quad&\left\|\mathbf{F}_{\text{opt}}\mathbf{F}_{\text{DD}}^H-\mathbf{F}_{\text{RF}}\right\|_F^2\\
\text{subject to}\quad&\begin{cases}\left|(\mathbf{F}_{\text{RF}})_{i,j}\right|=1,\forall i,j\\ \mathbf{F}_{\text{DD}}^H\mathbf{F}_{\text{DD}}=\mathbf{I}_{N_s}.\end{cases}
\end{aligned}
\tag{3.12}
$$

For fully connected hybrid pre-coding analog pre-coder $\mathbf{F}_{\text{RF}}$ is given by following equation. This solution is obtained by alternating minimization by fixing orthogonal digital pre-coder and extracting the phase of the product of optimal and orthogonal digital pre-coder.

$$
\arg\left(\mathbf{F}_{\text{RF}}\right)=\arg\left(\mathbf{F}_{\text{opt}}\mathbf{F}_{\text{DD}}^H\right)
\tag{3.13}
$$

3.3 Proposed Phase Extraction (PPE) Alternating Minimization Algorithm

We work on PE-Altmin in the same way as proposing in the [19], the slight modification in the algorithm works fine to reach the performance of the fully optimal digital pre-coder, without constraints on the number of data stream or number of the RF chain. We are happy to apply least square to update the digital pre-coder $\mathbf{F}_{\mathrm{DD}}$ as well analog pre-coder $\mathbf{F}_{\mathrm{AA}}$ after last iteration. The formula can be written below.

$$\mathbf{F}_{\mathrm{DD}} = (\mathbf{F}_{\mathrm{RF}}^{H}\mathbf{F}_{\mathrm{RF}})^{\dagger}\mathbf{F}_{\mathrm{RF}}^{H}\mathbf{F}_{\mathrm{opt}} \tag{3.14}$$

$$\mathbf{F}_{\mathrm{AA}} = \mathbf{F}_{\mathrm{opt}}(\mathbf{F}_{\mathrm{BB}}^{H}\mathbf{F}_{\mathrm{BB}})^{\dagger}\mathbf{F}_{\mathrm{BB}}^{H} \tag{3.15}$$

Proposed Phase Extraction (PPE) Alternating Minimization

---

1. **Take input as:** $\mathbf{F}_{\mathrm{opt}}, \mathbf{N}_{\mathrm{RF}}$
2. Generate $\mathbf{F}_{\mathrm{opt}}$with uniformaly distributed phase and set k = 0
3. **loop start**
4. Now fix $\mathbf{F}_{\mathrm{RF}}^{(k)}$ and get SVD of $\mathbf{F}_{\mathrm{opt}}^{H}\mathbf{F}_{\mathrm{RF}} = \mathbf{U}^{(k)}\mathbf{S}^{(k)}\mathbf{V}_{1}^{(k)^{H}}$
5. Get orthogonal digital pre-coder $\mathbf{F}_{\mathrm{DD}} = \mathbf{V}^{(k))}\mathbf{U}^{(k)H}$
6. Fix $\mathbf{F}_{\mathrm{DD}}$, update $arg(\mathbf{F}_{\mathrm{RF}}^{(k+1)}) = arg(\mathbf{F}_{\mathrm{opt}}\mathbf{F}_{\mathrm{DD}}{}^{H})$
7. $k+1 \leftarrow k$
8. **until** condition terminate itself
9. Optimize orthogonal digital pre-coder $\mathbf{F}_{\mathrm{DD}}$ and $\mathbf{F}_{\mathrm{AA}}$ by least square
10. Normalize the digital pre-coder $\widehat{\mathbf{F}}_{\mathrm{BB}} = \frac{\sqrt{N_s}}{\|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F}\mathbf{F}_{\mathrm{BB}}$
11. **End**

Digital and analog pre-coder is regularly optimized after each iteration, which results in significant improvement in the spectral efficiency of the PE- Alternating minimization.

3.4 Comparison between MO-Alt Min and PE-AltMin

3.4.1 Complexity

Both in MO-AltMin and PE-AltMin we have seen that digital pre-coder design has almost the same complexity but the analog pre-coder design leaves significant complexity in the MO-AltMin algorithm. No doubt MO-AltMin takes the complexity of $N_{\mathrm{RF}}^{t}N_t \times N_s N_t$ due to the involvement of the Kronecker product in the conjugate gradient algorithm. Whereas in the case of PE-AltMin analog pre-coder is simply obtained by extracting the phase of the product of unconstrained optimal digital pre-coder and orthogonal digital pre-coder.

3.4.2 Accuracy

The gap between optimal digital pre-coder and hybrid pre-coder defines the accuracy of the algorithm. Larger will be the gap, lower will be the performance and vice versa. However, in the case of MO-AltMin, we got a better result as compared to the PE-AltMin at the cost of computational complexity. The simulation result is shown in chapter 5.

3.5 Comparison between PE-Alt Min and Proposed PE-AltMin

3.5.1 Complexity

Both PE-AltMin and Proposed AltMin, the digital pre-coder design gives almost the same amount of complexity, but analog design imparts more complexity in the proposed AltMin algorithm as compared to the PE-AltMin. There is one more reason behind the increase in complexity of the Proposed AltMin, that is due to the use of the least square which involves pseudo-inverse of the complex matrices. In former we extract just the phase of the analog pre-coder whereas in later one we extract the phase of same after the update it by least square.

3.5.2 Accuracy

As we showed in the plot section, the accuracy of the proposed AltMin algorithm is obviously better than that of not only other fully connected structure but also all partially connected structure. However, a slight increment in complexity without losing error performance gives the performance of the optimal digital pre-coder.

Chapter 4

Partially Connected Hybrid Pre-coding

In [1],[18] and [21] many authors admired about partially connected structure because of its simple structure and energy efficient technique. As we already said that partially connected structure only N_t/N_{RF}^t antenna are connected to each RF chain. This results in a significant reduction in the hardware complexity as well as computational complexity. This structure is also known as an array of sub-array structure. Its effect is mostly in RF domain design. Therefore A_p now becomes the feasible set of the partially connected analog pre-coder, where each block consist of N_t/N_{RF}^t size. Here analog pre-coder also follows the unit modulus constraint. Its structure is shown in bellow.

$$\mathbf{F}_{\mathrm{RF}} = \begin{bmatrix} \mathbf{p}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_2 & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{p}_{N_{\mathrm{RF}}^t} \end{bmatrix} \tag{4.1}$$

where, $\mathbf{p}_i = \left[\exp\left(j\theta_{(i-1)\frac{N_t}{N_{\mathrm{RF}}^t}+1} \right), \ldots, \exp\left(j\theta_{i\frac{N_t}{N_{\mathrm{RF}}^t}} \right) \right]^T$ and θ_i is phase of ith phase shifter.

4.1 Analog RF Design

Due to special structure of the analog pre-coder $\mathbf{F}_{\mathrm{RF}}$, power constraints can be reformulated at transmitter side from (4.6) as bellow.

$$\|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 = \frac{N_t}{N_{\mathrm{RF}}^t} \|\mathbf{F}_{\mathrm{BB}}\|_F^2 = N_s \tag{4.2}$$

Therefore analog design of the objective function can be formulated as

$$\begin{aligned} \underset{\mathbf{F}_{\mathrm{RF}}}{\text{minimize}} \quad & \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \\ \text{subject to} \quad & \mathbf{F}_{\mathrm{RF}} \in \mathcal{A}_p. \end{aligned} \tag{4.3}$$

Using the property of analog pre-coder $\mathbf{F}_{\mathrm{RF}}$, problem (8.3) can be recast as

$$\underset{\{\theta_i\}_{i=1}^{N_t}}{\text{minimize}} \quad \left\| (\mathbf{F}_{\mathrm{opt}})_{i,:} - e^{j\theta_i} (\mathbf{F}_{\mathrm{BB}})_{l,:} \right\|_2^2, \tag{4.4}$$

Where, $l = \left\lceil i\frac{N^t_{\mathrm{RF}}}{N_t}\right\rceil$. There exists a closed-form for all non-zero elements of analog pre-coder $\mathbf{F}_{\mathrm{RF}}$. Here analog pre-coder can be extracted from the digital pre-coder as shown below for the specified range.

$$\arg\{(\mathbf{F}_{\mathrm{RF}})_{i,l}\} = \arg\left\{(\mathbf{F}_{\mathrm{opt}})_{i,:}(\mathbf{F}_{\mathrm{BB}})_{l,:}{}^{H}\right\}, \\ 1 \leq i \leq N_t, l = \left\lceil i\frac{N^t_{\mathrm{RF}}}{N_t}\right\rceil. \tag{4.5}$$

The design of the analog pre-coder becomes simple due to the special structure of the analog pre-coder. Hence the unit modulus constraint gives no more problems.

4.2 Digital Base band pre-coder Design

From (4.2), the pre-coder design at the transmitter side can be rewritten like the following.

$$\begin{aligned} \underset{\mathbf{F}_{\mathrm{BB}}}{\text{minimize}} \quad & \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \\ \text{subject to} \quad & \|\mathbf{F}_{\mathrm{BB}}\|_F^2 = \frac{N^t_{\mathrm{RF}}N_s}{N_t}. \end{aligned} \tag{4.6}$$

Here both objective function and constraint are quadratic in nature. If all the coefficient of the quadratic function is positive semidefinite, then only we can say the problem is convex and we can apply the convex optimization technique. On the other hand, if the coefficient is a combination of positive and negative the problem is non-convex. Here we use semidefinite relaxation based alternating minimization algorithm using quadratic constraint quadratic program (QCQP).

4.3 Comparison between two Hybrid Pre-coding Structure

Energy efficiency for hybrid pre-coding is given by (4.6). In terms of design partially connected structure gives more simplicity than that of fully connected structure. As the mapping of RF chain to antennas defines the structure of the pre-coder. Energy efficiency is in [19] is defined as the ratio of spectral efficiency to the of total power consumption.

$$\eta = \frac{R}{P_{\mathrm{common}} + N^t_{\mathrm{RF}}P_{\mathrm{RF}} + N_tP_{\mathrm{PA}} + N_{\mathrm{PS}}P_{\mathrm{PS}}} \tag{4.7}$$

where, η is energy efficiency in bits/Hz/J and P_{common} is the common power of the transmitter. N_t, N_r and N_{PS} are number of transmitter antenna, receiver antenna, and RF chain respectively. $P_{\mathrm{RF}}, P_{\mathrm{PS}}$ and P_{PA} are the power of each RF chain, phase shifter and power amplifier respectively. Here N_{PS} is defined as:

$$N_{\mathrm{PS}} = \begin{cases} N_tN^t_{\mathrm{RF}} & \text{fully-connected} \\ N_t & \text{partially-connected} \end{cases} \tag{4.8}$$

It can be noted that the number of the phase shifter is independent of the number of RF chain for the partially connected structure. That is the reason it acts as a power efficient technique.

4.4 Hybrid Precoding in MmWave MIMO-OFDM Systems

In the past section, all the techniques shown are for the narrowband mm-wave system. For large bandwidth, we can use mm-wave OFDM. This is one of the best techniques to overcome the multipath fading. In traditional mm-wave MIMO-OFDM system digital pre-coding is done in the frequency domain for each subcarrier. In addition to this inverse fast Fourier transform is post-digital pre-coding, which combines all the subcarrier together. Analog pre-coding is the post-inverse fast Fourier transform (IFFT) signal processing. After all these mathematical manupulation, the received signal at the receiver is given by

$$\mathbf{y}[k] = \sqrt{\rho}\mathbf{W}_{\mathrm{BB}}^{H}[k]\mathbf{W}_{\mathrm{RF}}^{H}\mathbf{H}[k]\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}[k]\mathbf{s}+\mathbf{W}_{\mathrm{BB}}^{H}[k]\mathbf{W}_{\mathrm{RF}}^{H}\mathbf{n} \tag{4.9}$$

Where, $k \in [0, K-1]$ is the sub carrier index. $\mathbf{H}[k]$ is channel matrix given by [11] and [12] in frequency domain for kth subcarrier as:

$$\mathbf{H}[k] = \gamma \sum_{i=0}^{N_{cl}-1} \sum_{l=1}^{N_{ray}} \alpha_{il}\mathbf{a}_r(\phi_{il}^r, \theta_{il}^r)\mathbf{a}_t(\phi_{il}^t, \theta_{il}^t)^H e^{-j2\pi ik/K} \tag{4.10}$$

where $\gamma = \sqrt{\frac{N_t N_r}{N_{cl}N_{ray}}}$ and K represents the total number of subcarrier. Hybrid pre-coder design for MIMO-OFDM system can be recast as:

$$\begin{aligned} &\underset{\mathbf{F}_{\mathrm{RF}},\mathbf{F}_{\mathrm{BB}}[k]}{\text{minimize}} \quad \sum_{k=0}^{K-1} \|\mathbf{F}_{\mathrm{opt}}[k] - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}[k]\|_F^2 \\ &\text{subject to} \quad \begin{cases} \mathbf{F}_{\mathrm{RF}} \in \mathcal{A} \\ \|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}[k]\|_F^2 = N_s, \end{cases} \end{aligned} \tag{4.11}$$

Formulation point of view there is a very small difference in mm-wave MIMO-OFDM system and narrowband MIMO system. Digital pre-coder for PE-AltMin can be given as

$$\begin{aligned} &\underset{\mathbf{F}_{\mathrm{RF}}}{\text{minimize}} \quad \sum_{k=0}^{K-1} \left\|\mathbf{F}_{\mathrm{opt}}[k]\mathbf{F}_{\mathrm{DD}}^{H}[k] - \mathbf{F}_{\mathrm{RF}}\right\|_F^2 \\ &\text{subject to} \quad |(\mathbf{F}_{\mathrm{RF}})_{i,j}| = 1, \forall i, j, \end{aligned} \tag{4.12}$$

For analog pre-coder design, analog pre-coder can be extracted from the optimal and orthogonal digital pre-coder for mm-wave MIMO-OFDM system is given by

$$\arg(\mathbf{F}_{\mathrm{RF}}) = \arg\left(\sum_{k=0}^{K-1} \mathbf{F}_{\mathrm{opt}}[k]\mathbf{F}_{\mathrm{DD}}^{H}[k]\right) \tag{4.13}$$

We can get closed form solution for lth rays in ith cluster for analog pre-coder for the given constraint as:

$$\begin{aligned} \arg\{(\mathbf{F}_{\mathrm{RF}})_{i,l}\} &= \arg\left\{\sum_{k=0}^{K-1} (\mathbf{F}_{\mathrm{opt}}[k])_{i,:}(\mathbf{F}_{\mathrm{BB}}[k])_{l,:}{}^{H}\right\}, \\ 1 \le i \le N_t, l &= \left\lceil i\frac{N_{\mathrm{RF}}^t}{N_t}\right\rceil, \end{aligned} \tag{4.14}$$

These are the extension of the previous technique explained before. MO-AltMin, PE-AltMin, SDR-AltMin can be directly extended from the narrowband mm-wave MIMO system.

Chapter 5

Result Evaluation

In this section we are going to discuss the various results obtained by different scheme. The transmitter which has $N_t = 144$ send the data stream to receiver with $N_r = 36$ antennas using USPA. Assume number of cluster $N_{cl} = 5$ with each cluster can have maximum number of $N_{ray} = 10$ rays. Here AoDs and AoAs follow the Laplacian distribution with uniform mean over $[0, 2\pi)$. Average power of each cluster is assumed as $\sigma^2_{\alpha,i} = 1$. Angular spread of 10 degree and for 1000 realization.

5.1 Spectral efficiency evaluation

Fig.5.1 shows the spectral efficiency versus SNR plot for the different pre-coding scheme.Here we assume $N_s = N_{\text{RF}}$. This is the worst case scenario since we know that the number of RF chain is always greater or equal to twice the number of the data stream.

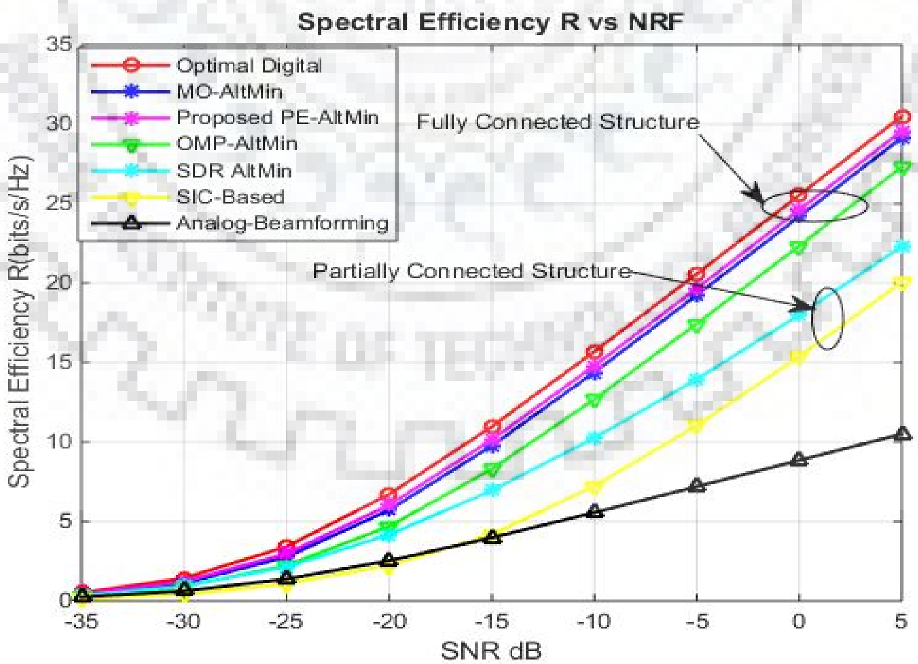

Figure 5.1: Spectral Efficiency R for SNR = -35 to 5 dB, $N_s = N_{\text{RF}} = 3$

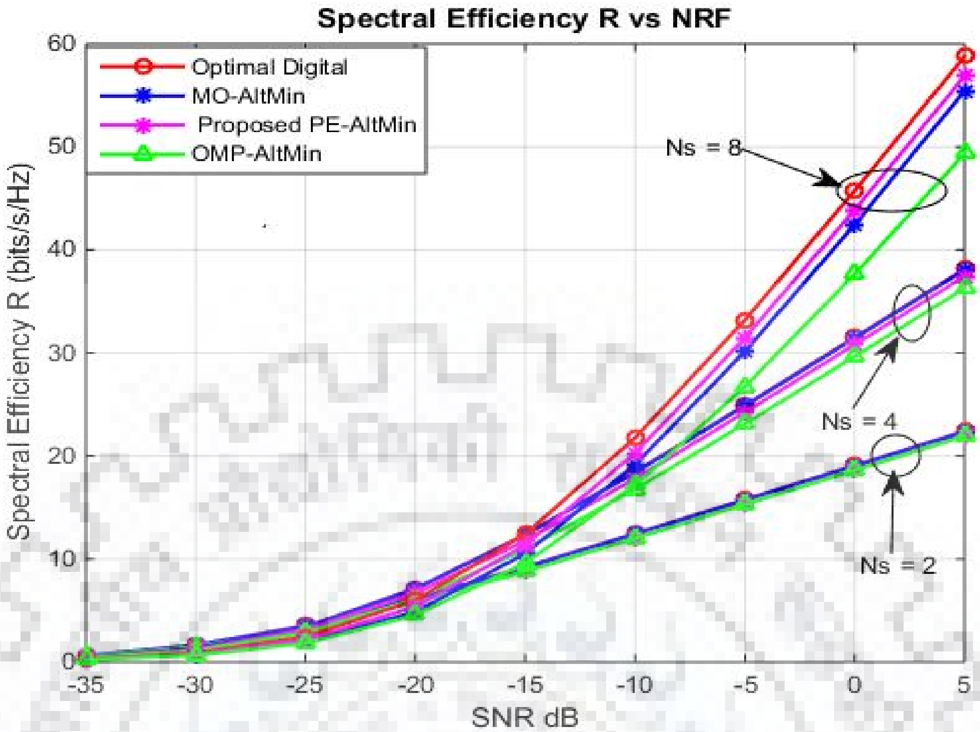

Figure 5.2: Spectral efficiency R for SNR = -35 to 5 dB, $N_s = N_{\text{RF}} = 2, 4$ and 8

Here we are seeing that optimal digital pre-coder gain highest spectral efficiency whereas the SIC-based method satisfies with least spectral efficiency. However, proposed PE-AltMin gain region in between the MO-AltMin and optimal digital pre-coder. Fig.5.2 represent the effect of a number of RF chain on the spectral efficiency for the specified range of SNR from -35 to 5 dB. We observed for N_s equal to 2, 4 and 8 with constraint that NRF at Tx and Rx equal to the number of data stream i.e., $N_s = N_{\text{RF}}^t = N_{\text{RF}}^r$.

5.2 Energy efficiency evaluation

Here we are going to observe the energy efficiency for the different scheme i.e., partially and fully connected structure. For the given value of $N_s = 2$, $N_{\text{RF}}^t = N_{\text{RF}}^r = N_{RF}$ and SNR = 0 dB with power P_{common} = 10 W, P_{PS} = 10 mW and $P_{\text{PA}} = P_{\text{RF}}$ = 100 mW.

Here we can see the effect of the number of RF chain for the optimal digital case and normal case. In case of fully connected structure, it loses spectral efficiency as the N_{RF} increases, whereas partially connected structure enjoys with almost constant energy efficiency.

Fig.4.3. shows the effect of number of RF chain on the spectral efficiency. In case of partially connected structure we see that energy as well with respect to the N_{RF} while power becomes almost constant hence there is no chance of energy down. We also see that for optimal case fully connected structure show better energy efficiency than without the optimal case. However, for the partially connected case, we see that without optimal and optimal both cases are almost the same.

We can also see the effect of the increase in the data stream on the spectral

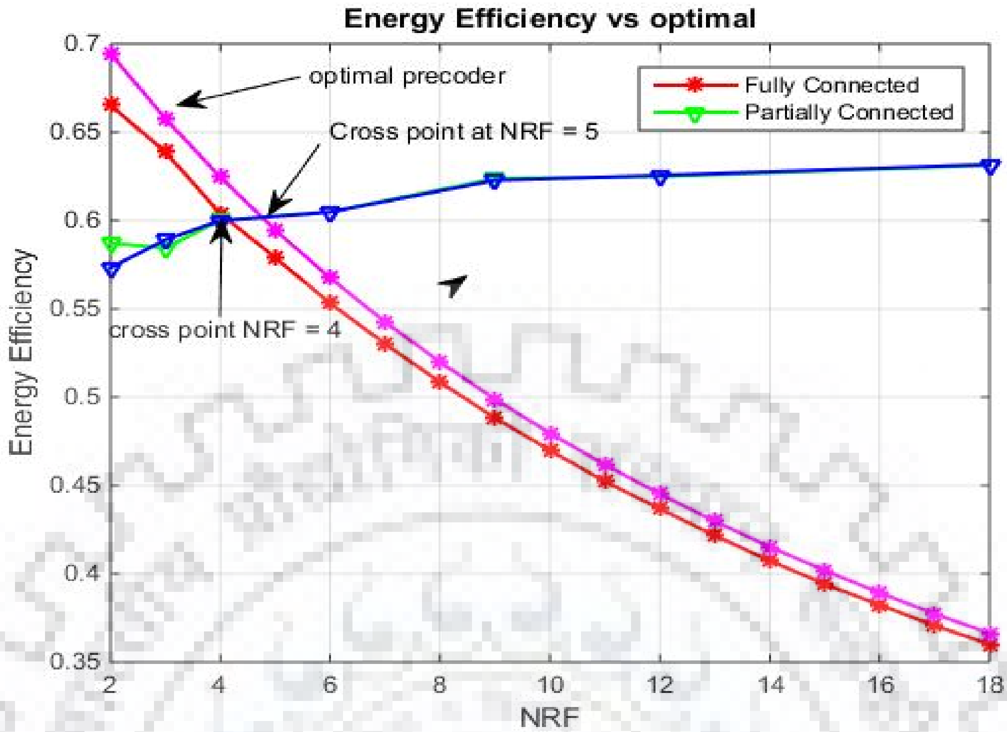

Figure 5.3: Energy efficiency R versus N_{RF} plot

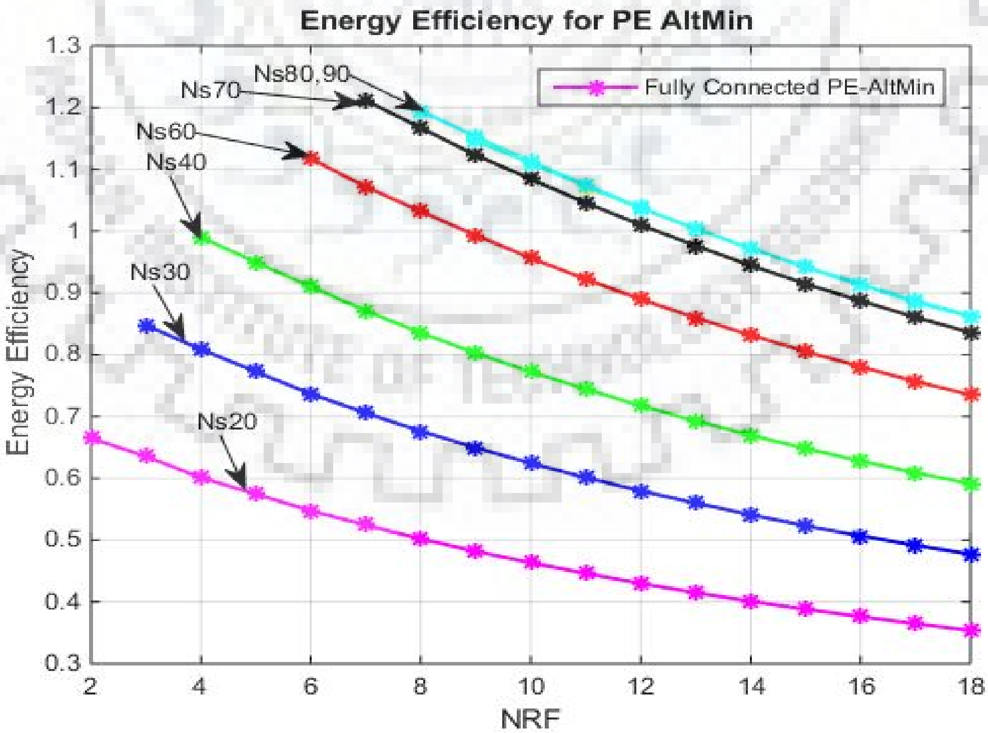

Figure 5.4: Effect of data stream N_s on Energy efficiency R plot

efficiency as shown in fig.3.4. Here as we increase the number of the data stream the energy efficiency increases proportionally up to $N_s = 2$ to 7, after then there is no significant improvement in the spectral efficiency. However, the computational complexity increases as the number of data stream N_s increases.

5.3 Spectral efficiency versus $\mathbf{N_{RF}}$ plot

Author of [19] tried to get the better spectral efficiency for the range of RF chain $N_s \leq N^t_{\text{RF}} < 2N_s$. Since in cases of MO-AltMin got optimal case response for the $N^t_{\text{RF}} \geq 2N_s$. Therefore our main problem is to achieve the optimal case response at acceptable complexity. Fig.10.5 and 10.6 show the plot for the different pre-coding scheme.

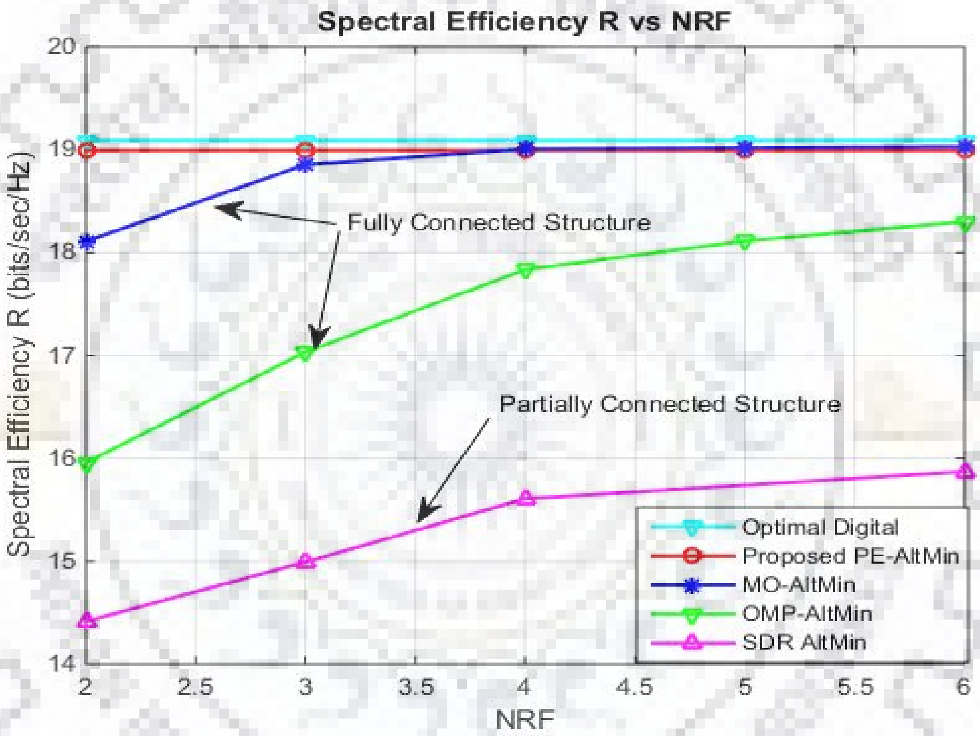

Figure 5.5: Spectral efficiency R for $N_s = 2$, and $N_{\text{RF}} = 2$ to 6

Here we compare the effect of RF chain on the performance of the different available algorithms. Assuming 6 data stream is being transmitted. Since the optimal solution for the $N^t_{\text{RF}} \geq 2N_s$ region has fully developed in the [20] so, here we focus on the remaining region i.e., $N^t_{\text{RF}} = N^r_{\text{RF}} = N_{\text{RF}} \in [6, 11]$. We found that PE-AltMin has small gap compared with the MO- AltMin algorithm. This is because the algorithm tries to minimize the upper bound instead of the original objective function.

As we had seen that the upper bound is tight when $N_{RF} = N_s$ and get looser when N_{RF} increases, which determine the gap between the MO-AltMin and the PE-AltMin algorithm. However, the spectral efficiency provided by the proposed PE-AltMin algorithm is far higher than that of the existing OMP algorithm.

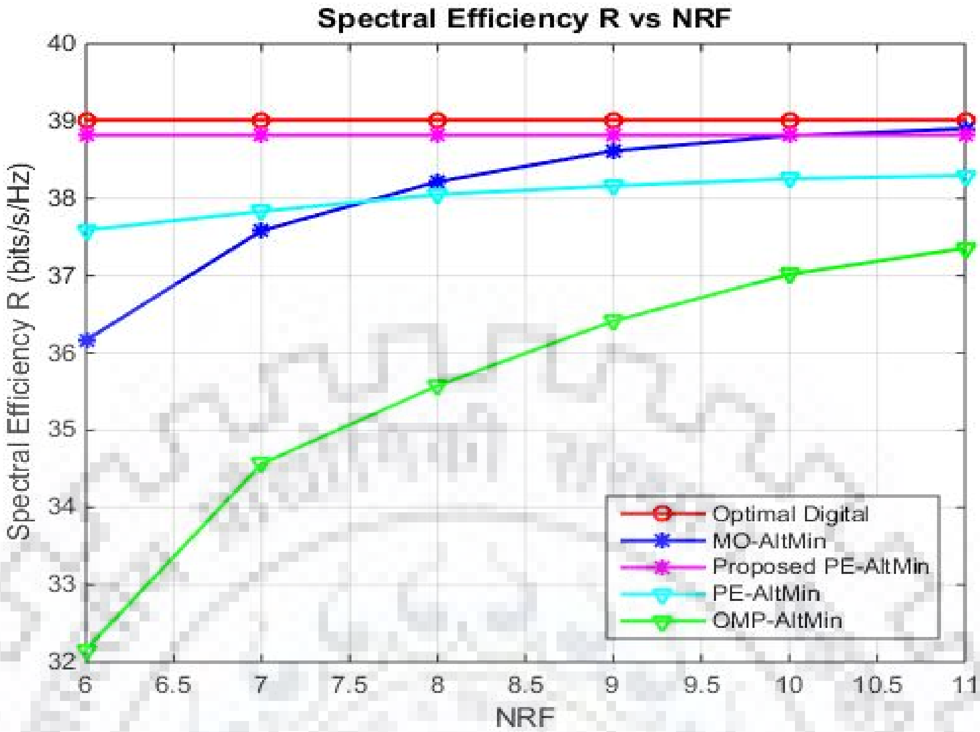

Figure 5.6: Spectral efficiency R for $N_s = 6$, and $N_{\mathrm{RF}} = 6$ to 11

5.4 Hybrid Pre-coding in mm-wave MIMO-OFDM Systems

In this section, we will know the performance of the AltMin algorithm when applied to the mm-wave MIMO-OFDM systems. We assume that here the number of the subscriber is K = 128. No doubt the MO-AltMin algorithm gets the close performance of the optimal digital pre-coder with a large number of the RF chains(see Fig.10.7).

Here we interestingly observe that low complexity PE-AltMin algorithm can achieve almost the same spectral efficiency as that of the MO-altMin algorithm when the number of RF chains and data stream is equal. This phenomenon is the same as that in the narrowband system. It is also found that in mm-wave system PE-AltMin outperforms the MO-AltMin algorithm. This indicates that the PE-AltMin algorithm can serve as an outstanding candidate for the low complexity hybrid pre-coding, both in a narrowband and broadband OFDM system, when the transceiver has limited RF chains available.

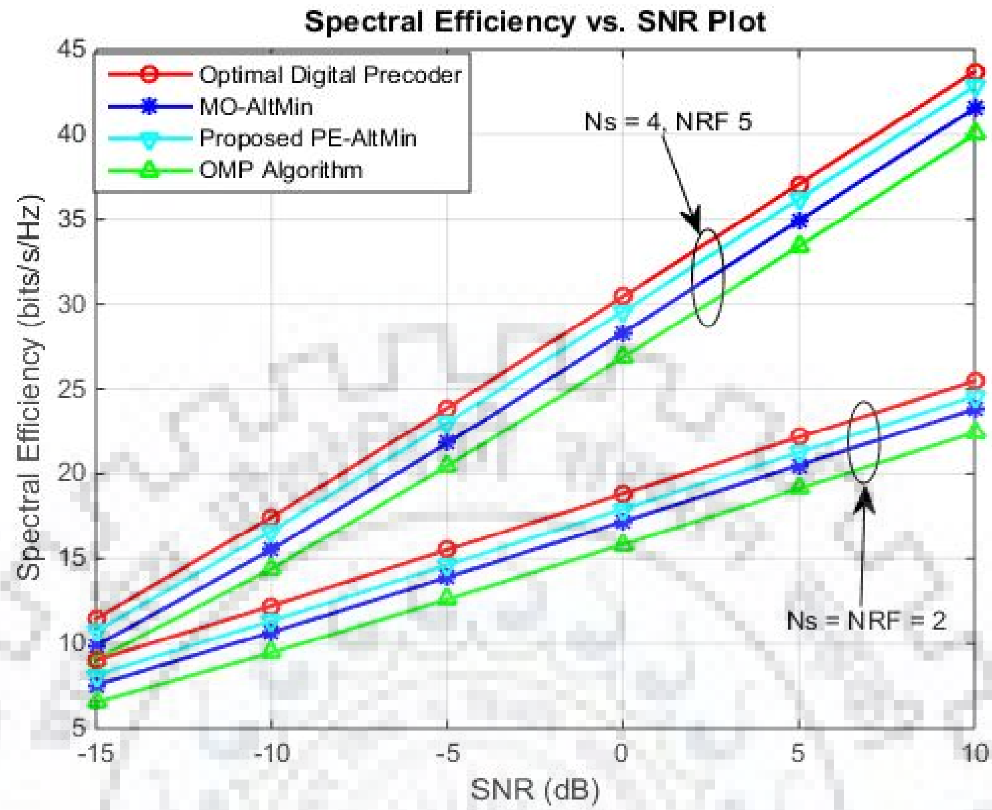

Figure 5.7: Spectral efficiency achieved for different pre-coding algorithm

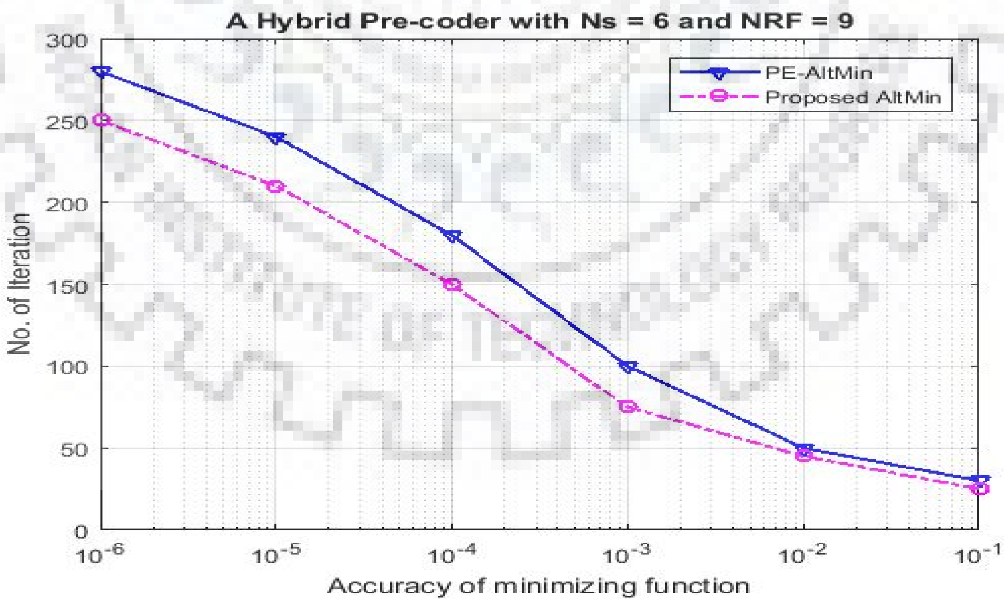

Figure 5.8: Accuracy versus iteration plot for proposed and PE-AltMin

5.5 Proposed Algorithm Complexity Analysis

In this section, we can see the accuracy and error plot for the proposed and PE-AltMIn algorithm. In Fig. 10.8 we can see the accuracy of the optimizing function with respect to the number of the iteration. Here we found that the proposed Altmin take less number of iteration to reach the same result than that of PE-AltMin algorithm with slight increment in the computation complexity.

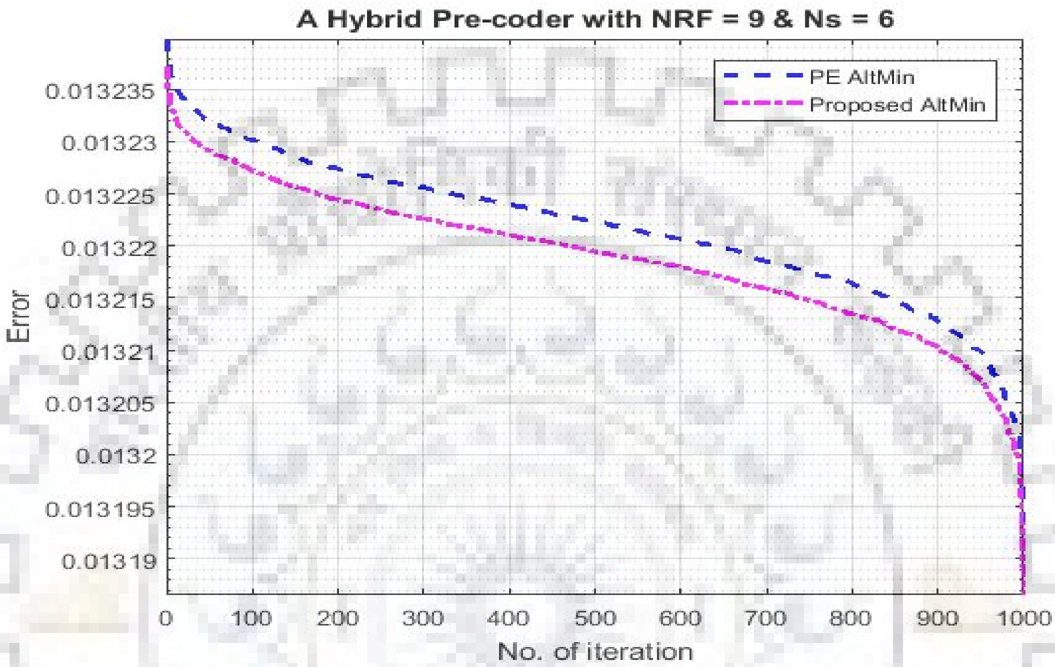

Figure 5.9: Error versus iteration plot for Proposed and PE-AltMin

In figure, we have shown the error versus iteration plot. Here also we can see proposed AltMin is more error efficient than the PE-AltMin. In the above two plots data stream as $N_s = 6$ and number of RF chain as N_{RF} varies from 9 to 18. Moreover, we observe that both Fig. 10.8 and 10.9 gives better performance in all respect, without compromising the loss in spectral efficiency.

Chapter 6

Conclusion and Future Scope

6.1 Conclusion

From the previous section, we have already seen that hybrid pre-coder become one of the best technique for hardware simplification as well as cost-efficient than either analog or digital pre-coder. Among fully connected structure: OMP, MO-AltMin, PE-AltMin, our Proposed PE-AltMin gives better performance. However, a partially connected structure such as an SDR-AltMin is far behind from a fully connected structure except that it is an energy efficient technique. In brief, we can say

- OMP, however, gives the low computation complexity, but its performance is poor with respect to all other fully connected structure used here(See simulation result). For each iteration it goes through three steps viz. matching orthogonal projection and residue update. This thing reduces its hardware complexity.
- MO-AltMin, however, gives better performance but it restricts the practical implementation because of its higher complexity.
- PE-AltMin inset the new idea of designing hybrid pre-coder with slight performance less than MO-AltMIn. Moreover, it promises to practical design.
- Proposed AltMin also serves itself as a practical solution but with slight increment in the complexity of the design. However, the accuracy and performance of the proposed algorithm are better than all fully connected structure.

Table 6.1: Comparison among various AltMin algorithms

Algorithm vs Parameter	SDR	MO	OMP	PE	PPE-AltMin
Performance	V	II	IV	III	I
Computational Complexity	V	I	IV	II	II
Hardware complexity	IV	I	III	II	II
Energy Consumption	I	IV	II	III	III
Compatibility	Y	N	Y	Y	Y

- SDR AltMin proves itself as energy efficient technique than all fully connected structure with an increased number of RF chain.

6.2 Future Scope

Our proposed PE-AltMin algorithm is superior to MO-AltMin and reaches to optimal pre-coder, still, we believe that there are many possibilities which need further remark to achieve most sufficient, least complex from both hardware and computation point of view.

- Partially connected structure is one of the best substitutes when energy efficient design is given priority.
- Simple hybrid pre-coder design can be extended to another hybrid pre-coder with channel training and feedback.
- To solve no-convex problem other effective techniques can be found. The optimal design needs further investigation.

This research aimed to identify the best possible solution to optimize the spectral efficiency at reduced complexity. Based on formulation and graphical result the effectiveness of the proposed algorithm has been shown here. To reach the peak of the mountain there isn't a single way but may have many ways. Similarly, we say this is one of the ways to get an optimal solution. There may have other best ways, which further needs deep investigation.

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