Interference Alignment based Precoding for ISI Mitigation in OFDM with Insufficient Cyclic Prefix

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CANDIDATE's DECLARATION

I hereby declare that the work which is being presented in this dissertation report entitled, **"INTERFERENCE ALIGNMENT BASED PRECODING FOR ISI MITIGATION IN OFDM WITH INSUFFICIENT CYCLIC PREFIX"** towards the partial fulfillment of the requirements for the award of the degree of **MASTER OF TECHNOLOGY** with specialization in **Communication Systems**, submitted in the Department of Electronics and Communication Engineering, Indian Institute of Technology Roorkee, Roorkee (India) is an authentic record of my own work carried out during the period from May 2018 to September 2019, under the guidance of **Dr. ANSHUL TYAGI, Assistant Professor, Department of Electronics and Communications Engineering, Indian Institute of Technology Roorkee**.

I have not submitted the matter embodied in this dissertation for the award of any other Degree or Diploma.

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CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of my knowledge and belief.

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ABSTRACT

Interference Alignment (IA) based Channel Independent Precoding for SISO OFDM and MIMO OFDM systems with Insufficient Cyclic Prefix (CP) is proposed that can remove the Inter-Block Interference (IBI) experienced from the insufficient CP with bandwidth efficiency higher than the unprecoded (or conventional) OFDM system with sufficient CP. The BER performance of IA based channel independent precoding in SISO OFDM and MIMO OFDM has been analyzed in comparison to conventional OFDM. The proposed precoding technique is much more bandwidth efficient than the conventional OFDM using the insufficient CP because the IBI caused by the insufficient CP can be set to a subspace whose dimensions are no more than a half of the difference of the length of ISI channel and the length of insufficient CP, so that more information symbols can be sent by using the other half.

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Abbreviations

AM	**A**mplitude **M**odulation
AWGN	**A**dditive **W**hite **G**aussian **N**oise
BE	**B**andwidth **E**fficiency
BER	**B**it **E**rror **R**ate
BPSK	**B**inary **P**hase **S**hift **K**eying
CIR	**C**hannel **I**mpulse **R**esponse
CP	**C**yclic **P**refix
CSIT	**C**hannel **S**tate **I**nformation at the **T**ransmitter
DFT	**D**iscrete **F**ourier **T**ransform
FDM	**F**requency **D**ivision **M**ultiplexing
FFT	**F**ast **F**ourier **T**ransform
FM	**F**requency **M**odulation
IA	**I**nterference **A**lignment
IBI	**I**nter **B**lock **I**nterference
ICI	**I**nter **C**arrier **I**nterference
IDFT	**I**nverse **D**iscrete **F**ourier **T**ransform
IEEE	**I**nstitute of **E**lectrical and **E**lectronics **E**ngineers
IFFT	**I**nverse **F**ast **F**ourier **T**ransform
ISI	**I**nter **S**ymbol **I**nterference
LTE	**L**ong **T**erm **E**volution
MCM	**M**ulti **C**arrier **M**odulation
OFDM	**O**rthogonal **F**requency **D**ivision **M**ultiplexing
PM	**P**hase **M**odulation
PAPR	**P**eak to **A**verage **P**ower **R**atio

QAM	**Q**uadrature **A**mplitude **M**odulation
QPSK	**Q**uadrature **P**hase **S**hift **K**eying
SE	**S**pectral **E**fficiency
SISO	**S**ingle **I**nput **S**ingle **O**utput
SNR	**S**ignal to **N**oise **R**atio
WLAN	**W**ireless **L**ocal **A**rea **N**etwork
WIMAX	**W**orldwide **I**nteroperability for **M**icrowave **A**ccess

Symbols

E_b	Average energy per bit
d	Dimension of space spanned by IBI
u	Length of Cyclic Prefix
L	Order of Channel Impulse Response
M	Order of Modulation
N_f	Number of Sub-carriers
N_o	Noise Power
$\mathbf{W}_N$	IDFT Matrix
$\mathbf{W}_N^{-1}$	DFT Matrix
N_t	Number of Transmit Antennas
N_r	Number of Receive Antennas

Chapter 1

Introduction

1.1 Basic Introduction

In past few years, a massive increase in mobile data traffic has been encountered by wireless communication systems, due to a large growth in the number of users as well as to the development of new applications involving high bit rate. Thus, the need for high speed transmission without any interference is bound to increase. In such evolving environment, the development of interference mitigation methods using beamforming or IA based Precoding for OFDM systems to provide the communication reliability, serve as the most suitable topics. In most applications, it is necessary to separate out multiple sources and extract the source of importance while reducing the unwanted interfering signals and noise. The estimated signals are then directly heard or subjected to a particular process. In this situation, the utilisation of the spatial domain of the communication channel by using multiple antenna systems and precoding schemes can be a major advancement for improving the Spectral Efficiency (SE) / Bandwidth Efficiency (BE) which is the rate of information that can be sent over a specific bandwidth in a given communication system. The use of multiple antennas and precoding schemes enables the adoption of diversity and spatial multiplexing techniques for mitigating the interference effects. This results in increase of the signal to interference plus noise ratio at the receiver that may be used to produce suitable benefits in terms of communication reliability and capacity.

1.2 Motivation

There are scenarios in which insufficient Cyclic Prefix (CP) can occur e.g. in a relay network, where the relay nodes can have multipaths and thus, the order of Channel Impulse Response (CIR) becomes difficult to predict because it is hard to know the time delays of the multipaths from the relay nodes due to their different locations and their motions. In such situation, as the size of OFDM block is already known and cannot be too large, the length of CP is also known and may be limited too. Also, if low value of peak to average power ratio (PAPR) is required in an OFDM system to limit the IDFT size, the length of CP may not be too large which leads to insufficient CP. Insufficient CP results in increased ICI and ISI. The interference distorts the subcarriers used for estimation of channel and Ki the process of detection and if neglected, can result in large detection error. Hence, ICI and ISI need to be cancelled or supressed to boost the performance of the OFDM system.

If IBI is to be removed completely, CP length has to be more than or equal to the number of taps of the frequency selective wireless channel. This results in a large overhead, thereby, decreasing the bandwidth efficiency, especially in channels with a large delay spread. This problem becomes worse in macro-cellular environments which are full of reflections from distant structures, that give rise to a large number of low power channel taps. In practice, OFDM system designs may ignore such low power taps, and thus suffer from a bit-error rate (BER). The need to eliminate this overhead motivates the design and analysis of precoded OFDM systems with insufficient CP.

1.3 Introduction to OFDM

Most commonly used single carrier modulation schemes or techniques are AM, PM, FM, BPSK, QPSK, etc, in which the incoming or message signal is modulated by mapping the information of the message signal to variations in frequency or amplitude or phase or a combination of them of a single carrier signal. However, in wideband systems of single carrier modulation schemes, transmission bandwidth of a single channel is typically much higher than the coherence bandwidth of the channel (which is the range of frequencies over which the channel frequency response is considerably flat or channel is frequency flat fading channel). Thus, channel becomes frequency selective fading channel which in time domain results in ISI (Inter-symbol Interference). Hence, single carrier system experiences ISI which is a significant distortion because symbols are interfering with each other and thus reliability of detection of symbols is a problem at the receiver end. However, in Multi-Carrier Modulation (MCM) transmission schemes, wideband systems are divided into multiple parallel independent narrowband subcarriers and by transmitting them in parallel, overall data rate or symbol rate remains the same as in the case of single carrier transmission schemes. But, implementation wise, it provides a major improvement as narrowband subcarriers experience frequency flat fading channel and thus it is easy to implement reliable detection schemes at the receiver for narrowband subcarriers because they are ISI free. But the bottleneck in implementing MCM systems is that the implementation of bank of N_f modulators and N_f demodulators is difficult because N_f is large and scaling proportional to N_f subcarriers (i.e. N_f can be 64, 256, 512 etc.). So, in order to remove this bottleneck in implementing MCM systems , there was a key advancement proposed by Wienstein and Ebert in the form of OFDM Systems in 1971.

OFDM refers to Orthogonal Frequency Division Multiplexing, a technique used to send large quantities of digital data across a multipath channel. OFDM or the more general Multi-Carrier Modulation (MCM) technique is chosen mostly for new communication systems. OFDM is a orthogonal multicarrier modulation or transmission scheme which splits the available broadband channel or available frequency band into a number of multiple independent parallel narrowband sub-carriers or subchannels by using IFFT/FFT algorithm and each subchannel (signal) is modulated using a common modulation scheme (such as QAM or PSK i.e. BPSK or QPSK) at a low symbol rate. A cyclic prefix (CP) is generally added to each OFDM symbol prior to sending it to the channel. To remove ICI and ISI, the CP length must be equal to or more than the length of the channel impulse response (CIR), else, the system faces distortion from insufficient CP comprised of both ICI and ISI. When the channel impulse response consists of a sequence of pulses, the received symbol at a given time instant is not directly proportional to the transmitted symbol but is equal to the convolution of the channel impulse response with the input sequence. In a multi-user scheme, data streams have to be separated to provide the corresponding data stream to each user. A solution for suppressing ISI is the OFDM technique which changes a frequency selective channel into a set of parallel orthogonal flat fading channels.

1.3.1 Basic Concept of OFDM

Let $\{X_i\}_{i=0}^{N_f-1}$ be the complex symbols which are to be transmitted by OFDM, then the OFDM (modulated) signal can be shown as

$$S(t) = \sum_{i=0}^{N_f-1} X_i U_i(t) = \sum_{i=0}^{N_f-1} X_i e^{j2\pi f_i t} = \sum_{i=0}^{N_f-1} X_i e^{j2\pi i \Delta f t}; 0 \leq t \leq T_s \tag{1.1}$$

where

$$f_i = i\Delta f$$

and

$$U_i(t) = \begin{cases} e^{j2\pi f_i t} & 0 \leq t \leq T_s \\ 0 & otherwise \end{cases}$$

for i = 0,1,2,.....,N_f-1

X_i = Complex-valued Modulated Symbols (e.g., QAM)
N_f = Number of Sub-carriers
f_i = Centre frequency of i^{th} Sub-carrier
T_s = Symbol Duration,
Δf = Sub-channel or Sub-carrier Spacing; $\Delta f = 1/T_s$
In order to demodulate OFDM signal at receiver, the duration of symbol must be large so that $T_s\Delta f = 1$, which is also termed as orthogonality condition.

Condition of Orthogonality : In communication systems, orthogonality means two signals are uncorrelated or independent over a symbol interval.

$$\frac{1}{T_s}\int_0^{T_s} U_k(t)U_l^*(t)dt = \begin{cases} 1 & k = l \\ 0 & otherwise \end{cases}$$

1.3.2 SISO OFDM Transmitter and Receiver Model

The block diagram shown below is of SISO OFDM system which consists of single input (i.e. single transmit antenna) and single output (i.e. single receive antenna). In this diagram, at the transmitter end, the binary source generates digital input data sequence. This binary data is encoded by using common digital modulation schemes like QAM,BPSK,QPSK with several different single constellations. The serial to parallel block makes data symbols parallelized in N_f different data substreams. Each data substream will modulate a separate subcarrier using the IFFT modulation block. The IFFT block at the transmitter converts parallel N_f data sub-streams of frequency domain data symbol into a time domain OFDM symbol. After that, cyclic prefix is inserted to block inter block interference and inter symbol interference. The CP copies the last specific length of data bits from the end to the beginning of OFDM symbol. The data is then converted back from parallel to serial form. This serially converted data is transmitted from single transmit antenna to single receive antenna through the frequency selective fading channel and Additive White Gaussian Noise (AWGN) is added in received OFDM symbol through the channel. At receiver end, the received data are first changed from serial to parallel form and also cyclic prefix is removed from each symbol. The FFT block at the receiver tranforms this data from time domain symbol to frequency domain symbol and again covert it from parallel to serial form. The bits are retrieved through demodulation and the stream is serialized and estimated output is taken after decoding.

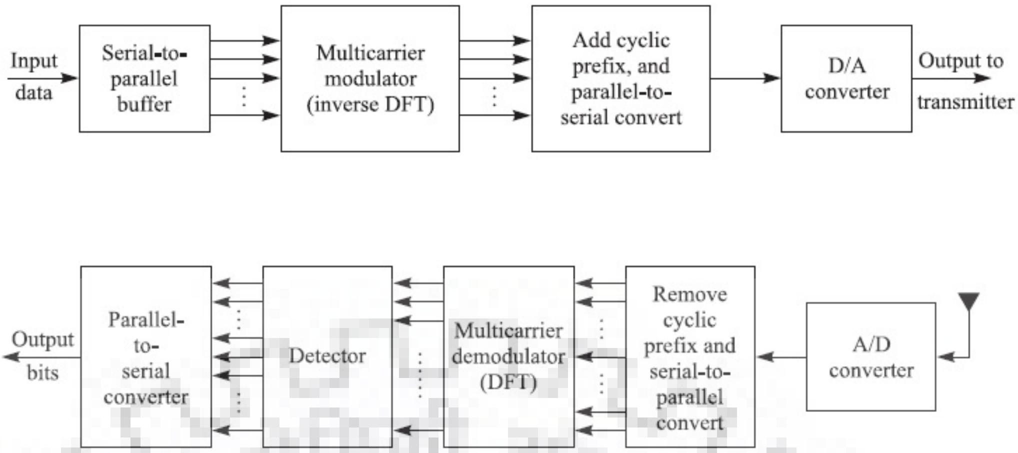

FIGURE 1.1: Block Diagram of SISO OFDM System

1.3.3 Cyclic Prefix for OFDM

Cyclic prefix is a process of prefixing the symbol or sample corresponding to it with a repetition of the end and then cycling the samples from the end towards the beginning. Cyclic prefix insertion is mainly used in OFDM systems as a method to diminish the effects of ICI and ISI. The Cyclic Prefix separates the different OFDM blocks from each other or restores the orthogonality of carriers that is lost when the wireless channel is having multiple paths i.e. channel is frequency selective. The CP blocks the ISI between adjacent OFDM symbols. The CP converts the linear convolution of a frequency selective multipath channel into a circular convolution using FFT. Normally, the CP length is longer than the dispersive channel length to completely block ISI.

OFDM uses large symbol duration T_s compared to the duration of the impulse response τ_{max} of the channel. To limit the ISI, guard interval larger than that of the estimated delay spread is added. If guard interval remains empty, the subcarriers loose their orthogonality i.e. ICI still persists. To block both the ICI and ISI, OFDM symbol is cyclically expanded into the guard interval.

Let T_{cp} denote the length or duration of CP in terms of samples and T_{sym} denote the length or duration of OFDM symbol without CP.

Figure 1.2 shows two consecutive OFDM symbols, each of which has the CP of length or duration T_{cp}, while illustrating the OFDM symbol of length $= T_{sym} + T_{cp}$.

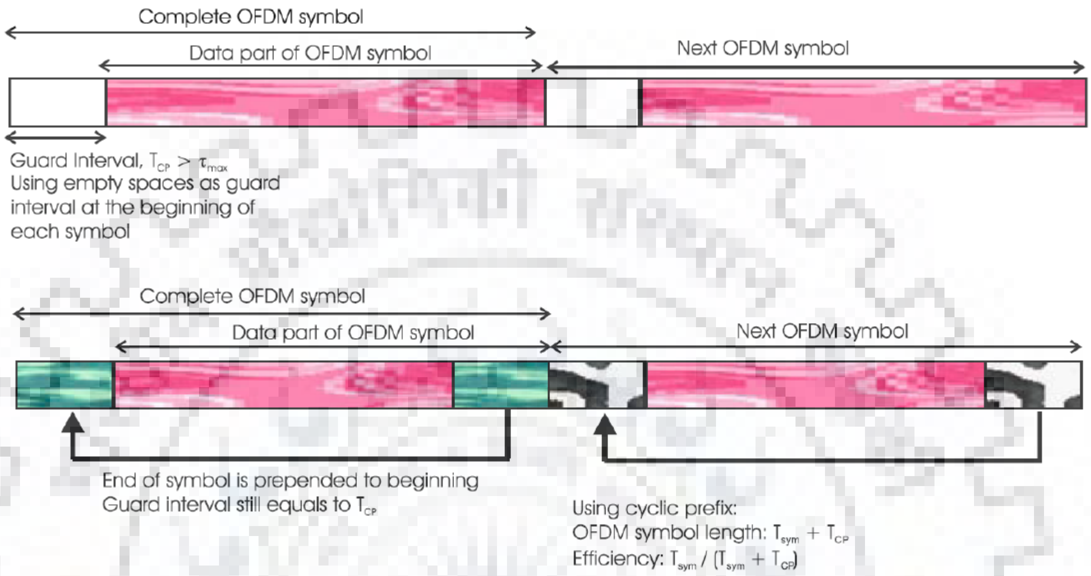

FIGURE 1.2: OFDM Signal with Cyclic Prefix

Guard Time	Cyclic Prefix
Eliminates ISI	Eliminates ISI
Suffers from ICI	Eliminates ICI
Causes a reduction in data rate as a result of the increased OFDM symbol time.	Causes a reduction in data rate as a result of the increased OFDM symbol time.
Does not consume additional power associated with OFDM symbol time expansion due to the Guard Time.	Necessitates additional power associated with OFDM symbol expansion due to the introduction of Cyclic Prefix.

TABLE 1.1: Guard Time Vs Cyclic Prefix

1.3.4 Advantages and Applications of OFDM

OFDM is the latest wireless technology which forms the foundation for 4G and 5G wireless communication systems. OFDM is a major broadband wireless technique which favours data rates beyond 100 Mbps which is of great importance in future broadband wireless communication systems. OFDM has high spectral efficiency in removing effects of ISI with very low complexity because it utilises orthogonality of subcarriers and employs IFFT operation at transmitter and FFT operation at receiver which are very fast and efficient algorithms. OFDM lowers the degree of crosstalk in signal transmissions and is extensively used in the radio communications in a specific IEEE standard technologies such as IEEE 802.11a/g/n, IEEE 802.15.3a, 802.16d/e and 4G standard technologies LTE, WiMAX. WLAN 802.11n supports data rates upto 200 Mbps. The applications of OFDM also cover wireless metropolitan networks, wireless local area networks and wireless personal area networks.

1.3.5 Limitations of OFDM

Occurrence of very high peak values leads to high PAPR (Peak to average power ratio). High PAPR limits the efficiency of circuits such as analog to digital converters and power amplifiers. OFDM is highly sensitive to synchronization errors and it results in interference and loss of orthogonality because non-linear effects generated by the power amplifier result in ICI and thus ruin the orthogonality.
To perfectly eliminate both ISI and ICI, the CP length has to be at least equal to the channel impulse response length . The limitation of the cyclic prefix insertion is that it reduces the spectral efficiency of OFDM systems since it contains no user data. As the cyclic prefix retransmits data that is already being sent, it takes up system capacity and lowers the overall data rate.

1.4 Introduction to Precoding

Precoding refers to transmission of multiple data streams from the transmit antenna with suitable and independent weightings in order to maximise the link throughput at the receiver output.
IA based Precoding is a technique used to block the interference by working with the information symbols at the transmitter and it needs the full channel state information at the transmitter (CSIT). This technique decreases the distorted effect of the communication channel. e.g if information sent is x and it is passed through channel g and gaussian noise n is added to it, then the signal received at the receiver is $y = x * g + n$. The receiver knows the information about n and g and lowers the effect of n by increasing SNR, but information about the channel g is required, and this increases the complexity. The receiver (mobile units) is kept simple to reduce cost or size of mobile unit so that the transmitter (the base station) does the hard work and predicts the channel.

1.5 Introduction to MIMO-OFDM

MIMO when combined with OFDM is suitable for applications that demand high data rate. MIMO helps to achieve higher data rate by transmitting data over multiple antennas. OFDM provides reliable communication over fading channels by dividing a broadband channel into narrow bandwidth flat fading channels. In this way, MIMO OFDM provides good quality and higher data rate communication which is the demand of the modern communication.

1.5.1 Performance Gain in MIMO System

The multiple antennas are deployed either at transmitter or receiver to get reliable transmission over multi-path fading channels and improves capacity of the system. In MIMO, multiple transmit and receive antennas are used to get the spatial multiplexing gain which increases the spectral efficiency.

1.5.1.1 Diversity Gain

To get the diversity gain, same data is sent over independent fading channels. When same data is sent multiple times over different fading channels, the fading suffered by same data over different channels is different. This assures that data through one of the channels will get less faded as compared to other copies of same data. Thus, there is more probability of receiving the correct data. Hence, reliability of the communication system improves. Also, co-channel interference reduces significantly. The gain introduced by above technique is called spatial diversity gain.

$$\text{Diversity Gain} = N_t \times N_r \tag{1.2}$$

where N_t is the number of transmitting antennas and N_r is the number of receiving antennas. In Figure 1.3 diversity gain for a 2X2 MIMO system is 4.

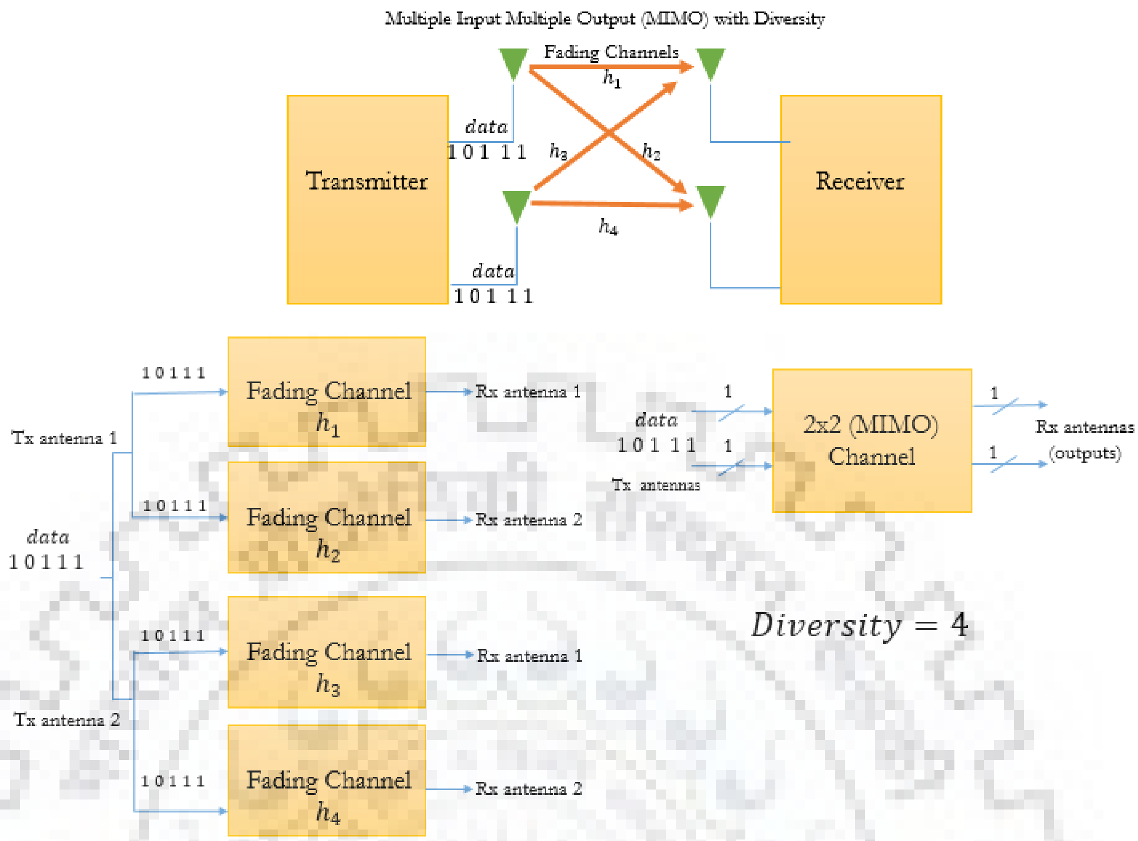

FIGURE 1.3: Diversity Gain in MIMO System

[13]

1.5.1.2 Spatial Multiplexing Gain

In comparison to single antenna system used at transmitting or receiving end, Spatial multiplexing results in linear capacity increase at no extra power or bandwidth requirement. In spatial multiplexing, every transmitting antenna transmits independent data. The multiplexing gain is obtained provided that the channel has abundant scattering and it allows the independent data to be transmitted simultaneously at the same frequency. As independent data transmitted through MIMO channel at same frequency experience different fading, thus the receiver is able to separate the different signals and capacity gain is obtained. This gain is also called as spatial multiplexing gain.

$$\text{Multiplexing Gain} = \min(N_t, N_r) \tag{1.3}$$

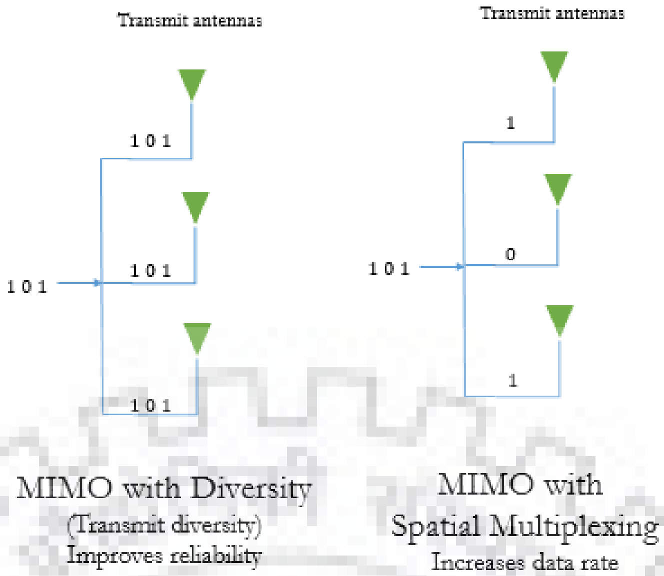

FIGURE 1.4: Diversity Gain vs Spatial Multiplexing Gain in MIMO System [13]

In figure 1.4 for a 3X3 MIMO with spatial multiplexing, multiplexing gain is 3. For the same 3X3 MIMO with diversity, diversity gain is 9.

1.5.1.3 Array gain

Array gain is achieved by using coherent combining at the transmitting and the receiving end. It increases the average SNR at the receiver and thus coverage is improved.

Multiple antennas at transmitter and receiver can eliminate co-channel interference that causes an increase in cellular system capacity.

Chapter 2

Literature Survey

2.1 SISO-OFDM Model

Taking a OFDM SISO system in which there are N_f subcarriers. Let the channel be a frequency selective fading channel. The channel impulse response (CIR) is expressed as:

$$\mathbf{g} = [g(0), g(1), ...g(L)]^T \tag{2.1}$$

where, L is the order of Channel Impulse Response and $L + 1$ is the Channel Impulse Response length. Consider elements of $\mathbf{g}$ as independent and identically distributed, complex gaussian with zero mean. Let $s_n = [s_n^0, s_n^1, s_n^2, ..., s_n^{N_f-1}]^T$ be the input signal for the nth OFDM block. Let $\mathbf{W}_N$ is the $N_f \times N_f$ IDFT matrix where $[\mathbf{W}_N]_{k,l} = (1/\sqrt{N_f})\exp(\frac{j2\pi kl}{N_f})$.

A CP of length u is added to each IDFT block in time domain. The CP length u is chosen to be less than or equal to the length of channel. As a result, the transmitted OFDM block is altered by IBI and ICI. At the receiver, the received nth OFDM block in time domain after the removal of insufficient CP is given as [7]:

$$\mathbf{v}_n = (\mathbf{G} - \mathbf{P})\mathbf{W}_N\mathbf{s}_n + \mathbf{Q}\mathbf{W}_N\mathbf{s}_{n-1} + \mathbf{z}_n \tag{2.2}$$

where $\mathbf{z}_n$ is the noise vector added to transmitted signal in time domain and $\mathbf{G}$ is a $N_f \times N_f$ channel matrix. Let $\mathbf{P}$ and $\mathbf{Q}$ denote the $N_f \times N_f$ ICI and IBI matrices respectively [2].

$$\mathbf{P} = \begin{bmatrix} \mathbf{0}_{(L-u)\times(N_f-L)} & \mathbf{S} & \mathbf{0}_{(L-u)\times(u)} \\ \mathbf{0}_{(N_f-L+u)\times(N_f-L)} & \mathbf{0}_{(N_f-L+u)\times(L-u)} & \mathbf{0}_{(N_f-L+u)\times(u)} \end{bmatrix} \tag{2.3}$$

$$\mathbf{Q} = \begin{bmatrix} \mathbf{0}_{(L-u)\times(N_f-L+u)} & \mathbf{S} \\ \mathbf{0}_{(N_f-L+u)\times(N_f-L+u)} & \mathbf{0}_{(N_f-L+u)\times(L-u)} \end{bmatrix} \tag{2.4}$$

where matrix $\mathbf{S}$ is a $(L-u)\times(L-u)$ order matrix and is given as:

$$\mathbf{S} = \begin{bmatrix} g(L) & g(L-1) & \dots & g(u+1) \\ 0 & g(L) & \dots & g(u+2) \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & g(L) \end{bmatrix} \tag{2.5}$$

The presence of matrices $\mathbf{P}$ and $\mathbf{Q}$ in the received signal indicate ICI and IBI which is due to the CP chosen less than the length of the CIR. Let $(\mathbf{G}-\mathbf{P}) = \mathbf{R}$ where matrix $\mathbf{R}$ is of size $N_f \times N_f$ and is given as [1]:

$$\mathbf{R} = \begin{bmatrix} g(0) & 0 & \dots & 0 & \dots & 0 & g(u) & \dots & g(1) \\ \vdots & \ddots & \ddots & & \ddots & \vdots & \vdots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & & 0 & g(L) & & g(u) \\ \vdots & & & \ddots & \ddots & & 0 & \ddots & \vdots \\ \vdots & & & & \ddots & \ddots & & \ddots & g(L) \\ g(L) & & & & & \ddots & \ddots & & 0 \\ 0 & \ddots & & & & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & & & & \ddots & 0 \\ 0 & \dots & 0 & g(L) & \dots & \dots & g(u-1) & \dots & g(0) \end{bmatrix} \tag{2.6}$$

The received signal in frequency domain obtained by the DFT matrix $\mathbf{W}_N^{-1}$ of size $N_f \times N_f$ is given as:

$$\mathbf{y}_n = \mathbf{W}_N^{-1}\mathbf{R}\mathbf{W}_N\mathbf{s}_n + \mathbf{W}_N^{-1}\mathbf{Q}\mathbf{W}_N\mathbf{s}_{n-1} + \widehat{\mathbf{z}}_n \tag{2.7}$$

where $\widehat{\mathbf{z}}_n = \mathbf{W}_N^{-1}\mathbf{z}_n$, is the noise vector in frequency domain.

2.2 Implementation of Precoding

To perform precoding on input signal of $N_f \times 1$ vector $\mathbf{t}_n$, it is multiplied by a precoding matrix $\mathbf{A}$ of size $N_f \times N_f$ as given below [3]:

$$\mathbf{s}_n = \mathbf{A}\mathbf{t}_n \tag{2.8}$$

So, the received signal in frequency domain for nth OFDM block is given by:

$$\mathbf{y}_n = \mathbf{W}_N^{-1}\mathbf{R}\mathbf{W}_N\mathbf{A}\mathbf{t}_n + \mathbf{W}_N^{-1}\mathbf{Q}\mathbf{W}_N\mathbf{A}\mathbf{t}_{n-1} + \widehat{\mathbf{z}}_n \tag{2.9}$$

or

$$\mathbf{y}_n = \mathbf{W}_N^{-1}\mathbf{R}\mathbf{B}\mathbf{t}_n + \mathbf{W}_N^{-1}\mathbf{Q}\mathbf{B}\mathbf{t}_{n-1} + \widehat{\mathbf{z}}_n \tag{2.10}$$

where $\mathbf{B} = \mathbf{W}_N\mathbf{A}$, $\mathbf{A}$ and $\mathbf{B}$ are called as precoding matrices in frequency domain and time domain respectively.

2.3 SISO OFDM Precoding Matrix Design

The precoding matrix $\mathbf{B}$ is designed on the concept of interference alignment technique such that the desired signal occupies a subspace other than the interference subspace. So, in our case, the received IBI is aligned in small dimension subspace that is separate from the subspace of the information symbols. The received signal in time domain is as below:

$$\mathbf{v}_n = \mathbf{R}\mathbf{B}\mathbf{t}_n + \mathbf{Q}\mathbf{B}\mathbf{t}_{n-1} + \mathbf{z}_n \tag{2.11}$$

The IBI term in the above equation is $\mathbf{Q}\mathbf{B}\mathbf{t}_{n-1}$. Assume noise $\mathbf{z}_n$ to be negligible for convenience. The signal subspace occupied by the matrix $\mathbf{RB}$ and the IBI subspace occupied by the matrix $\mathbf{QB}$ need to be separate so that variables in $\mathbf{t}_n$ can be solved freely . In other words, below conditions need to be met for precoding matrix design [1]:

1. $span\{\mathbf{RB}\} \cap span\{\mathbf{QB}\} = \{0\}$;
2. $dim(\mathbf{QB})$ should be minimized;
3. $dim(\mathbf{RB})$ should be maximized.

where dim is the dimension of the space that is linearly spanned by the column vectors of the matrix, which is also the column rank of the matrix.

Since the elements of matrix $\mathbf{R}$ are independent and identically distributed, it has non zero determinant and thus, its rank is N_f. The rank of matrix $\mathbf{Q}$ is $L-u$. Assuming rank of the precoding matrix $\mathbf{B}$ is $N_f - d$ and it is designed in such a way that the IBI matrix $\mathbf{QB}$ has rank as $L-u-d$. This can be done by keeping the last d columns of matrix $\mathbf{B}$ all zeros. By choosing this type of precoder $\mathbf{B}$, the rank of the matrix $\mathbf{RB}$ will be $N_f - d$. To separate the signal subspace from IBI subspace, the sum of ranks of the matrices $\mathbf{RB}$ and $\mathbf{QB}$ has to be less than or equal to N_f, i.e.,

$$N_f - d + L - u - d \leq N_f$$

Using the above condition, precoding aligns the IBI into a space of dimension $L-u-d$. As a result, we get value of d as:

$$d \geq \frac{L-u}{2}$$

and the minimum value of d is:

$$d = \frac{L-u}{2}$$

Also, the sub-space occupied by the IBI has dimension $(L-u)/2$. In the OFDM system without precoding, extra $(L-u)$ redundant symbols or zeroes are required to eliminate the IBI. But in OFDM system with IA based precoding, only half of $(L-u)$ redundant symbols or zeroes are required to segregate the signal subspace from IBI subspace so that the desired signal can be detected accurately and thus, the other half can be used for sending more information symbols.

For N_f dimensional vector space,$\{e_1, e_2, ...e_{N_f}\}$ is a set of $N_f \times 1$ orthonormal basis elements where $e_i = [\underbrace{0, ..., 0}_{i-1}, 1, 0, ..., 0]^T$

Let Precoding matrix $\mathbf{B}$ be given as:

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 \end{bmatrix} \tag{2.12}$$

Assume,

$$\begin{aligned} \mathbf{B}_1 &= \begin{bmatrix} e_{(N_f-L+u)+1}, & \cdots & , e_{(N_f-L+u)+n_1} \end{bmatrix} \\ &= \begin{bmatrix} e_{n_{\alpha+1}}, & \cdots & , e_{n_\beta} \end{bmatrix} \end{aligned} \tag{2.13}$$

$$\begin{aligned} \mathbf{B}_2 &= \begin{bmatrix} e_1, & \cdots & , e_{(N_f-L+u)+1}, e_1, & \cdots & , e_1 \end{bmatrix} \\ &= \begin{bmatrix} e_1, & \cdots & , e_{n_\alpha}, \underbrace{e_1, ..., e_1}_{N_f-n_1-n_\alpha} \end{bmatrix} \end{aligned} \tag{2.14}$$

where $n_\alpha = (N_f - L + u)$, $n_\beta = n_\alpha + n_1$, $n_1 = d = \frac{L-u}{2}$

For example, for the case:N_f=64,L=16,u=12, the time domain precoding matrix **B** is a 64×64 matrix that is given as:

$$\mathbf{B} = \begin{bmatrix} \mathbf{e}_{61} & \mathbf{e}_{62} & \mathbf{e}_1 & \dots & \mathbf{e}_{60} & \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{2.15}$$

where, $e_i = [\overbrace{0, ..., 0}^{i-1}, 1, 0, ..., 0]^T, 1 \leq i \leq 64$ is a set of 64×1 orthonormal vectors. Here, $rank(\mathbf{RB}) = 62$, $rank(\mathbf{QB}) = 2$ and the column vectors of **RB** are linearly independent of the only two non zero column vectors of **QB**. In this example, 62 independent information symbols can be detected easily. Whereas, if block size in conventional OFDM is $N_f = 64$, then 4 additional redundant symbols or zeroes are added in the OFDM block to remove IBI perfectly and hence, only 60 independent information symbols can be transmitted per block.

2.4 SISO OFDM Decoding

By using n_1 linearly independent columns $\{\mathbf{q}_1, ..., \mathbf{q}_{n_1}\}$ in $\mathbf{QB}_1$, a full column rank matrix is obtained $\mathbf{Q'} = [\mathbf{q}_1 ... \mathbf{q}_{n_1}]$ and then IBI term $\mathbf{QB}_1\mathbf{t}_{n-1}$ is eliminated by forming a zero forcing matrix as

$$\mathbf{F}_{zf} = [\mathbf{I} - \mathbf{Q}'(\mathbf{Q'^H}\mathbf{Q}')^{-1}\mathbf{Q'^H}] \tag{2.16}$$

Here, IBI exists in the first n_1 rows of the IBI vector and thus $\mathbf{F}_{zf}$ is expressed in the below diagonal form:

$$\mathbf{F}_{zf} = diag(\underbrace{0, ..., 0}_{n_1}, 1, ..., 1) \tag{2.17}$$

The received signal after applying the zero forcing is given as

$$\mathbf{F}_{zf}\mathbf{v}_n = \mathbf{F}_{zf}\mathbf{R}\mathbf{B}_1\bar{\mathbf{t}}_n + \mathbf{F}_{zf}\mathbf{z}_n \tag{2.18}$$

where $\bar{\mathbf{t}}_\mathbf{n} = [\mathbf{t}_\mathbf{n}(\mathbf{0}), ..., \mathbf{t}_\mathbf{n}(\mathbf{n}_\beta - \mathbf{1})]^T$ is the independent information symbol part. As $\mathbf{F}_{zf}$ maps the first n_1 rows of $\mathbf{v}_n$, $\mathbf{R}$ and $\mathbf{z}_n$ to be zero rows, the signal is expressed as

$$\mathbf{v}'_n = \mathbf{R}'\mathbf{B}_1\bar{\mathbf{t}}_n + \mathbf{z}'_n \tag{2.19}$$

where $\mathbf{v}'_n$, $\mathbf{R}'$ and $\mathbf{z}'_n$ are obtained by truncating the first n_1 rows from $\mathbf{v}_n$, $\mathbf{R}$ and $\mathbf{z}_n$. For the signal detection, the MMSE equalizer is given as:

$$\mathbf{G} = (\frac{1}{\sigma_s^2}\mathbf{I} + \mathbf{B}_1^H\mathbf{R}'^H\mathbf{R}'\mathbf{B}_1)^{-1}\mathbf{B}_1^H\mathbf{R}'^H \tag{2.20}$$

Hence, the equalized signal after applying the MMSE equalizer is given as:

$$\hat{\mathbf{t}}_n = \mathbf{G}\mathbf{R}'\mathbf{B}_1\bar{\mathbf{t}}_n + \mathbf{G}\mathbf{z}'_n \tag{2.21}$$

2.5 MIMO-OFDM Model

Let $\bar{\mathbf{s}}_n = [(\mathbf{s}_n^0)^T, (\mathbf{s}_n^1)^T, (\mathbf{s}_n^2)^T, ..., (\mathbf{s}_n^{N_f-1})^T]^T$ be the input to the MIMO OFDM system, where $\mathbf{s}_n^i$ is the $N_t \times 1$ vector in frequency domain for the N_t transmit antennas at the i^{th} subcarrier, $0 \leq i \leq N_f - 1$. To get the input $\bar{\mathbf{s}}_n$ in time domain at N_t transmit antennas, the overall IDFT operation is given as: $\overline{\mathbf{W}} \triangleq \mathbf{W}_N \otimes \mathbf{I}_{N_t}$. Assume the multipath channel between i^{th} receive antenna and j^{th} transmit antenna as: $\mathbf{g}_{ij} = [g_{ij}(0), g_{ij}(1), ...g_{ij}(L)]^T$ and all the entries in $\mathbf{g}_{ij}$ are i.i.d complex gaussian random variables having zero mean and all the channels have same length as L+1. A length of CP,u is added to signal input block at each transmit antenna. The multipath channel matrices $\mathbf{G}(l)$, $l = 0, 1, \ldots, L$ for time domain vectors serially transmitted at N_t transmit antennas are given as:

$$\mathbf{G}(l) = \begin{bmatrix} g_{11}(l) & \ldots & g_{1N_t}(l) \\ \vdots & \ddots & \vdots \\ g_{N_r1}(l) & \ldots & g_{N_rN_t}(l) \end{bmatrix} \tag{2.22}$$

As channel coefficients are random so the matrices $\mathbf{G}(l)$ are having full rank almost assuredly. After removing CP at the receiver, the received block in time domain is as below:

$$\bar{\mathbf{v}}_n = \mathbf{R}\overline{\mathbf{W}}\bar{\mathbf{s}}_n + \mathbf{Q}\overline{\mathbf{W}}\bar{\mathbf{s}}_{n-1} + \mathbf{z}_n \tag{2.23}$$

where, $\mathbf{z}_n$ is the $N_fN_r \times 1$ noise vector having complex gaussian distribution. $\mathbf{R}$ is the overall channel matrix of size $N_fN_r \times N_fN_t$ and $\mathbf{Q}$ is the IBI matrix of size $N_fN_r \times N_fN_t$ as given below:

$$
\mathbf{R} = \begin{bmatrix}
\mathbf{G}(0) & \mathbf{0} & \dots & \mathbf{0} & \dots & \mathbf{0} & \mathbf{G}(u) & \dots & \mathbf{G}(1) \\
\vdots & \ddots & \ddots & & \ddots & \vdots & \vdots & \ddots & \vdots \\
\vdots & & \ddots & \ddots & & \mathbf{0} & \mathbf{G}(L) & & \mathbf{G}(u) \\
\vdots & & & \ddots & \ddots & & \mathbf{0} & \ddots & \vdots \\
\vdots & & & & \ddots & \ddots & & \ddots & \mathbf{G}(L) \\
\mathbf{G}(L) & & & & & \ddots & \ddots & & \mathbf{0} \\
\mathbf{0} & \ddots & & & & & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & & & & & \ddots & \mathbf{0} \\
\mathbf{0} & \dots & \mathbf{0} & \mathbf{G}(L) & \dots & \dots & \mathbf{G}(u-1) & \dots & \mathbf{G}(0)
\end{bmatrix} \tag{2.24}
$$

$$
\mathbf{Q} = \begin{bmatrix}
\mathbf{0} & \dots & \mathbf{0} & \mathbf{G}(L) & \dots & \mathbf{G}(u+1) \\
\vdots & & & \ddots & \ddots & \vdots \\
\vdots & & & & \ddots & \mathbf{G}(L) \\
\vdots & & & & & \mathbf{0} \\
\vdots & & & & & \vdots \\
\mathbf{0} & \dots & \dots & \dots & \dots & \mathbf{0}
\end{bmatrix} \tag{2.25}
$$

$\bar{\mathbf{s}}_n$ is the precoded output obtained by multiplying the information symbol vector $\bar{\mathbf{t}}_n$ with the precoding matrix $\mathbf{A}$ of size $N_f N_t \times N_f N_t$ as below:

$$
\bar{\mathbf{s}}_n = \mathbf{A}\bar{\mathbf{t}}_n \tag{2.26}
$$

The precoding matrix in time domain is given as:

$$
\mathbf{B} \triangleq \overline{\mathbf{W}}\mathbf{A} \tag{2.27}
$$

Hence, the received signal in frequency domain is given as:

$$
\mathbf{y}_n = (\mathbf{W}_N^{-1} \otimes \mathbf{I}_{N_r})\mathbf{R}\mathbf{B}\bar{\mathbf{t}}_n + (\mathbf{W}_N^{-1} \otimes \mathbf{I}_{N_r})\mathbf{Q}\mathbf{B}\bar{\mathbf{t}}_{n-1} + \widehat{\mathbf{z}}_n \tag{2.28}
$$

2.6 MIMO-OFDM Precoding Matrix Design

The channel matrix $\mathbf{R}$ is full rank almost assuredly i.e. N_f. The IBI matrix $\mathbf{Q}$ has $N_r(L-u) \times N_t(L-u)$ non zero submatrix $\mathbf{Q}_s$.

$$\mathbf{Q}_s = \begin{bmatrix} \mathbf{G}(L) & \dots & \dots & \mathbf{G}(u+1) \\ \mathbf{0} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{G}(L) \end{bmatrix} \tag{2.29}$$

The precoding matrix is designed so as to cope with $\mathbf{Q}_s$. Consider a $N_tN_f \times N_tN_f$ precoding matrix as follows:

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix} \tag{2.30}$$

where $\mathbf{B}_{21}$ and $\mathbf{B}_{22}$ are of order $N_t(L-u) \times n_1$ and $N_t(L-u) \times n_2$ respectively. Number of rows of these two submatrices are same as the number of columns of $\mathbf{Q}_s$ and these submatrices are designed to block IBI. The size of $\mathbf{B}_{11}$ is $N_t(N_f - L+u) \times n_1$ and the size of $\mathbf{B}_{12}$ is $N_t(N_f - L+u) \times n_2$. Submatrices $\mathbf{B}_{11}$ and $\mathbf{B}_{12}$ are designed to have maximum possible signal space dimension and $n_1 + n_2 = N_tN_f$. The objective of design of Precoding matrix is to align the IBI matrix i.e $\mathbf{Q}$ in a subspace that is separate from the signal subspace taken by the information symbols vector $\bar{\mathbf{t}}_n$. The design criteria is as follows:

- $span\{\mathbf{RB}\} \cap span\{\mathbf{QB}\} = \{0\}$;
- $dim(\mathbf{QB})$ should be minimum;
- $dim(\mathbf{RB})$ should be maximum.

where *dim* is the dimension of the space taken by the column vectors of the matrix.

2.6.1 Precoding when $N_r \leq N_t$

For MIMO-OFDM system, total $N_r N_f$ symbols can be solved such that the symbols transmitted by N_t transmit antennas should not exceed $N_r N_f$.

2.6.1.1 When $N_t(N_f - L + u) < N_r N_f$

The precoding matrix will satisfy the properties as below:

1. $span\{\mathbf{B}_{22}\} \subset span\{\mathbf{B}_{21}\}$.
2. Take
$$n_1 \triangleq \lfloor \frac{N_r N_f - N_t(N_f - L + u)}{2} \rfloor$$
Both $\mathbf{B}_1$ and $\mathbf{B}_{21}$ are full column rank matrices.
3. Sub-matrix $\mathbf{B}_{12}$ has $N_t(N_f - L + u)$ linearly independent column vectors and matrix $\mathbf{B}_2$ has column rank $N_t(N_f - L + u)$.
4. Any non-all-zero linear combinations of the first $N_t(N_f - L + u)$ column vectors of $\mathbf{B}_2$ do not belong to $span\{\mathbf{B}_1\}$.

2.6.1.2 When $N_t(N_f - L + u) \geq N_r N_f$

The precoding matrix $\mathbf{B}$ is constructed such that $[\mathbf{B}_{11} \quad \mathbf{B}_{12}]$ has full row rank as $N_t(N_f - L + u)$ and by setting $\mathbf{B}_{21} = \mathbf{B}_{22} = \mathbf{0}$, we get

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{2.31}$$

The total number of independent information symbols which can be decoded in each OFDM block by the zero forcing operator when $N_r \leq N_t$ and CP used is insufficient is given as:

$$\begin{cases} N_t(N_f - L + u) + \lfloor \frac{N_r N_f - N_t(N_f - L + u)}{2} \rfloor, & \text{if } N_t(N_f - L + u) < N_r N_f \\ N_r N_f, & \text{if } N_t(N_f - L + u) \geq N_r N_f \end{cases} \tag{2.32}$$

For $N_t N_f$ dimensional vector space, $\{e_1, e_2, ...e_{N_t N_f}\}$ is a set of $N_t N_f \times 1$ orthonormal basis elements where $e_i = [\underbrace{0, ..., 0}_{i-1}, 1, 0, ..., 0]^T$

Assume,

$$\begin{aligned} \mathbf{B}_1 &= \left[e_{N_t(N_f-L+u)+1}, \quad \dots \quad , e_{N_t(N_f-L+u)+n_1} \right] \\ &= \left[e_{n_{\alpha+1}}, \quad \dots \quad , e_{n_\beta} \right] \end{aligned} \tag{2.33}$$

$$\begin{aligned} \mathbf{B}_2 &= \left[e_1, \quad \dots \quad , e_{N_t(N_f-L+u)+1}, e_1, \quad \dots \quad , e_1 \right] \\ &= \left[e_1, \quad \dots \quad , e_{n_\alpha}, \underbrace{e_1, ..., e_1}_{N_t N_f - n_1 - n_\alpha} \right] \end{aligned} \tag{2.34}$$

where $n_\alpha = N_t(N_f - L + u)$, $n_\beta = n_\alpha + n_1$

2.6.2 IBI Cancellation when $N_r > N_t$

For MIMO-OFDM system with insufficient CP and $N_r > N_t$, no zero padding or precoding is required to solve $N_t N_f$ independent information symbols by zero forcing operator, when

$$N_f \geq \frac{N_t}{N_r - N_t}(L - u)$$

In this case, IBI space span $\mathbf{Q}$ is isolated from the signal space span $\mathbf{R}$. In other words, the receive signal space has dimension greater than or equal to the sum of the dimensions of the IBI matrix $\mathbf{Q}$ and channel matrix $\mathbf{R}$.

2.7 MIMO-OFDM Decoding

The received signal in time domain is as below:

$$\bar{\mathbf{v}}_n = \mathbf{RB}\bar{\mathbf{t}}_n + \mathbf{QB}\bar{\mathbf{t}}_{n-1} + \mathbf{z}_n \tag{2.35}$$

The rank of the matrix $\mathbf{QB}$ is same as the rank of the matrix $\mathbf{Q}_s\mathbf{B}_{21}$ and it is equal to n_1. Let $\mathbf{Q}' = \mathbf{Q}_s\mathbf{B}_{21}$. Thus, the IBI term $\mathbf{Q}_s\mathbf{B}_{21}\bar{\mathbf{t}}_{n-1}$ is eliminated by forming a zero forcing matrix as

$$\mathbf{F}_{zf} = [\mathbf{I} - \mathbf{Q}'(\mathbf{Q}'^{\mathbf{H}}\mathbf{Q}')^{-1}\mathbf{Q}'^{\mathbf{H}}] \tag{2.36}$$

Here, IBI exists in the first n_1 rows of the IBI matrix and thus, $\mathbf{F}_{zf}$ being a $N_fN_r \times N_fN_r$ matrix, is expressed in the below diagonal form:

$$\mathbf{F}_{zf} = diag(\underbrace{0, ..., 0}_{n_1}, 1, ..., 1) \tag{2.37}$$

The received signal after applying the zero forcing is given as

$$\mathbf{F}_{zf}\bar{\mathbf{v}}_n = \mathbf{F}_{zf}\mathbf{RB}\bar{\mathbf{t}}_n + \mathbf{F}_{zf}\mathbf{z}_n \tag{2.38}$$

where $\bar{\mathbf{t}}_{\mathbf{n}} = [(\mathbf{t}_{\mathbf{n}}^{\mathbf{0}})^{\mathbf{T}}, ..., (\mathbf{t}_{\mathbf{n}}^{(\mathbf{n}_\beta-\mathbf{1})})^{\mathbf{T}}]^T$ is the independent information symbols part. As $\mathbf{F}_{zf}$ maps the first n_1 rows of $\bar{\mathbf{v}}_n$, $\mathbf{R}$ and $\mathbf{z}_n$ to be zero rows, the signal is expressed as

$$\bar{\mathbf{v}}'_n = \mathbf{R}'\mathbf{B}\bar{\mathbf{t}}_n + \mathbf{z}'_n \tag{2.39}$$

where $\mathbf{v}'_n$, $\mathbf{R}'$ and $\mathbf{z}'_n$ are obtained by truncating the first n_1 rows from $\bar{\mathbf{v}}_n$, $\mathbf{R}$ and $\mathbf{z}_n$. For the signal detection, the MMSE equalizer is given as:

$$\mathbf{G} = (\frac{1}{\sigma_s^2}\mathbf{I} + \mathbf{B}^H\mathbf{R}'^H\mathbf{R}'\mathbf{B})^{-1}\mathbf{B}^H\mathbf{R}'^H \tag{2.40}$$

Hence, the equalized signal after applying the MMSE equalizer is given as:

$$\hat{\bar{\mathbf{t}}}_n = \mathbf{GR}'\mathbf{B}\bar{\mathbf{t}}_n + \mathbf{Gz}'_n \tag{2.41}$$

Chapter 3

BER Performance of IA based Precoding in SISO OFDM

Bit Error Rate (BER) is the percentage of bits that have errors with respect to the total number of bits received in a transmission system as a result of noise, interference or other issues, normally expressed as ten to a negative power. It is used to determine the quality of a digital transmission system. Mathematically, BER can be expressed by a simple formula :

BER = Number of bits in error / Total number of bits received

Spectral efficiency or Bandwidth efficiency refers to the rate of information that can be sent over a specific bandwidth in a given communication system. Spectral Efficiency is measured in $b/s/Hz$. For SISO OFDM, it is mathematically given by:

$$\text{Spectral Efficiency}_{(\text{SISO-OFDM})} = \frac{N_f}{(N_f + u)} \times \log_2 M \tag{3.1}$$

where, N_f is no. of subcarriers, u is the length of cyclic prefix and M is the order of modulation scheme.

3.1 Simulation Results

3.1.1 Order of CIR, L=16 and No. of Sub-carriers, N_f=64

The parameters used to compute simulations are given below in table 3.1. BPSK has been used as a modulation scheme. Theoretical bit error rate for conventional OFDM has also been plotted for comparison. The theoretical BER for conventional OFDM with BPSK is given by the expression:

$$\text{BER} = 1/2 \times \left(1 - \sqrt{\frac{E_b/N_o}{E_b/N_o + 1}}\right) \tag{3.2}$$

where E_b/N_o = Signal to Noise Ratio in linear scale

Parameters	Values
No. of Sub-carriers (N_f)	64
Order of CIR (L)	16

TABLE 3.1: Simulation parameters

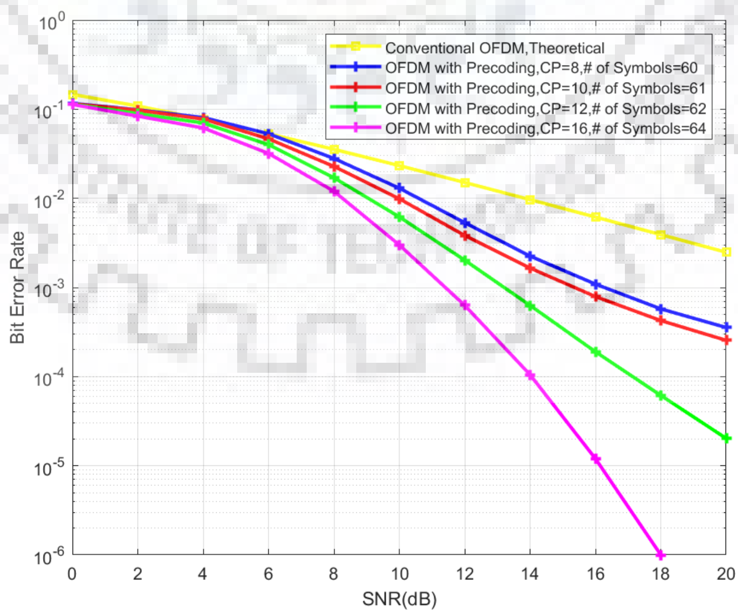

FIGURE 3.1: SISO : BER performance of Precoding based on IA for different values of CP when $L = 16$ and $N_f = 64$

Figure 3.1 shows the BER performance of IA based Precoding for different values of CP i.e. 8,10,12 and 16. For plots with insufficient CP i.e. where value of CP is less than the channel length L=16, BER increases for IA based Precoding as value of CP decreases.

For CP=8, Number of independent information symbols that can be detected for a IA based precoding system is 60 with bandwidth efficiency of 0.8888 bits/s/Hz. For CP=10, 61 information symbols can be detected successfully with bandwidth efficiency of 0.8648 bits/s/Hz. Similarly, for CP=12, 62 symbols can be detected with bandwidth efficiency of 0.8421 bits/s/Hz.

For CP=16, BER performance is same as that of conventional OFDM and all the 64 information symbols can be detected successfully with bandwidth efficiency of 0.8000 bits/s/Hz.

3.1.2 Order of CIR, L=16 and No. of Sub-carriers, N_f=256

The parameters used to compute simulations are given below in table 3.2. BPSK has been used as a modulation scheme. Theoretical bit error rate for conventional OFDM has also been plotted for comparison.

Parameters	Values
No. of Sub-carriers (N_f)	256
Order of CIR (L)	16

Table 3.2: Simulation parameters

Figure 3.2 shows the BER performance of IA based Precoding for different values of CP i.e. 8,10,12 and 16. For plots with insufficient CP i.e. where value of CP is less than the channel length L=16, BER increases for IA based Precoding as value of CP decreases.

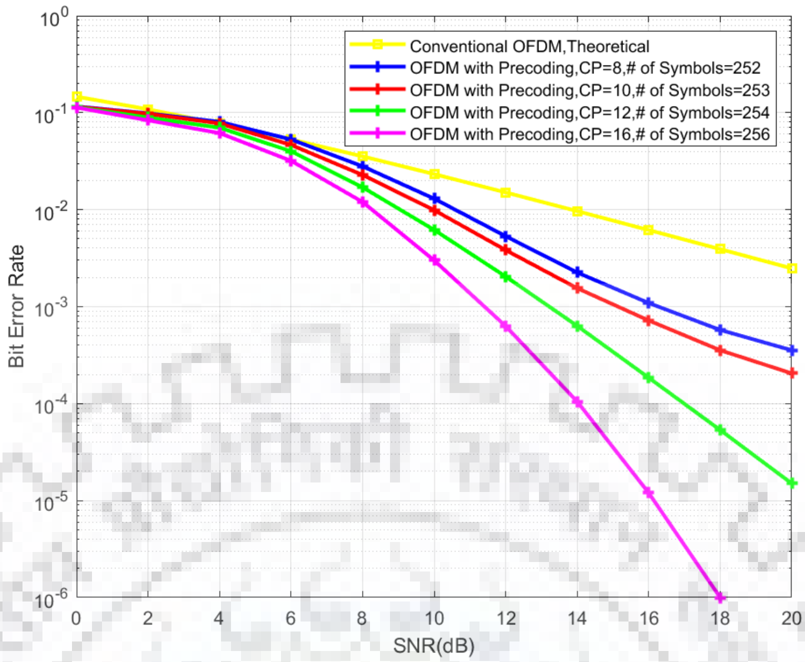

FIGURE 3.2: SISO : BER performance of Precoding based on IA for different values of CP when $L = 16$ and $N_f = 256$

For CP=8, Number of independent information symbols that can be detected for a IA based precoding system is 252 with bandwidth efficiency of 0.9696 bits/s/Hz. For CP=10, 253 information symbols can be detected successfully with bandwidth efficiency of 0.9624 bits/s/Hz. Similarly, for CP=12, 254 symbols can be detected with bandwidth efficiency of 0.9552 bits/s/Hz.

For CP=16, BER performance is same as that of conventional OFDM and all the 256 information symbols can be detected successfully with bandwidth efficiency of 0.9411 bits/s/Hz.

3.1.3 Order of CIR,L=19 and No. of Sub-carriers,N_f=256

The parameters used to compute simulations are given below in table 3.3. BPSK has been used as a modulation scheme. Theoretical bit error rate for conventional OFDM has also been plotted for comparison.

Parameters	Values
No. of Sub-carriers (N_f)	256
Order of CIR (L)	19

TABLE 3.3: Simulation parameters

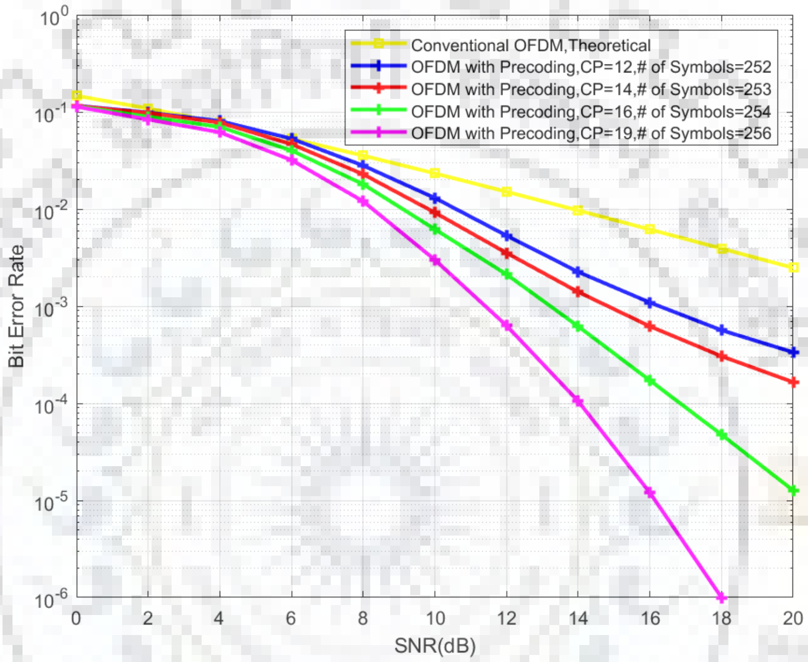

FIGURE 3.3: SISO : BER performance of Precoding based on IA for different values of CP when $L = 19$ and $N_f = 256$

Figure 3.3 shows the BER performance of IA based Precoding for different values of CP i.e. 12,14,16 and 19. For plots with insufficient CP i.e. where value of CP is less than the channel length L=19, BER increases for IA based Precoding as value of CP decreases.

For CP=12, Number of independent information symbols that can be detected for a IA based precoding system is 252 with bandwidth efficiency of 0.9552 bits/s/Hz. For CP=14, 253 information symbols can be detected successfully with bandwidth efficiency of 0.9481 bits/s/Hz. Similarly, for CP=16, 254 symbols can be detected with bandwidth efficiency of 0.9411 bits/s/Hz.

For CP=19, BER performance is same as that of conventional OFDM and all the

256 information symbols can be detected successfully with bandwidth efficiency of 0.9309 bits/s/Hz.

3.1.4 Order of CIR, L=19 and No. of Sub-carriers, N_f=512

The parameters used to compute simulations are given below in table 3.4. BPSK has been used as a modulation scheme. Theoretical bit error rate for conventional OFDM has also been plotted for comparison.

Parameters	Values
No. of Sub-carriers (N_f)	512
Order of CIR (L)	19

TABLE 3.4: Simulation parameters

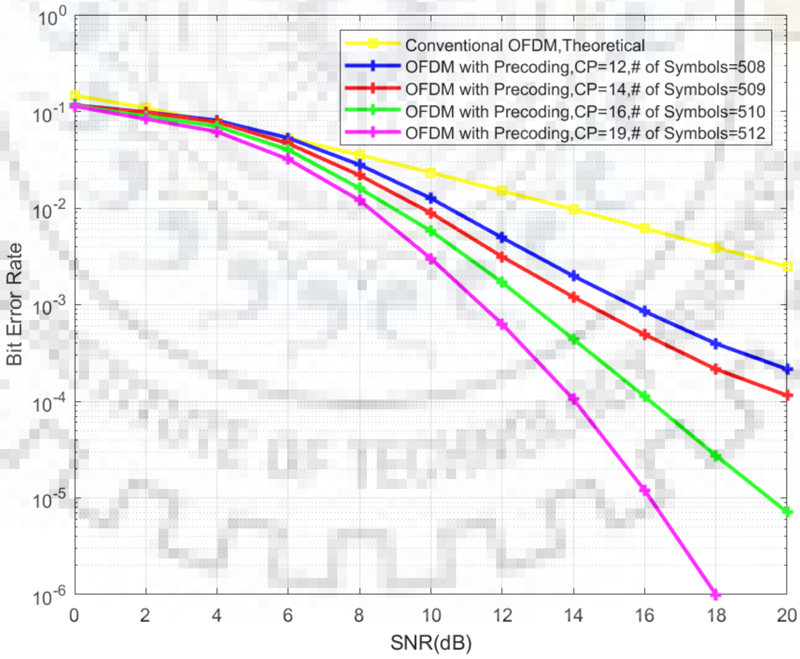

FIGURE 3.4: SISO : BER performance of Precoding based on IA for different values of CP when $L = 19$ and $N_f = 512$

Figure 3.4 shows the BER performance of IA based Precoding for different values of CP i.e. 12,14,16 and 19. For plots with insufficient CP i.e. where value of CP is less than the channel length L=19, BER increases for IA based Precoding as value of CP decreases.

For CP=12, Number of independent information symbols that can be detected for a IA based precoding system is 508 with bandwidth efficiency of 0.9770 bits/s/Hz. For CP=14, 509 information symbols can be detected successfully with bandwidth efficiency of 0.9733 bits/s/Hz. Similarly, for CP=16, 510 symbols can be detected with bandwidth efficiency of 0.9696 bits/s/Hz.

For CP=19, BER performance is same as that of conventional OFDM and all the 512 information symbols can be detected successfully with bandwidth efficiency of 0.9642 bits/s/Hz.

Chapter 4

BER Performance of IA based Precoding in MIMO OFDM

In MIMO-OFDM, different symbols are transmitted on multiple antennas at the same time to get the higher data rate. The spatial multiplexing gain depends on number of transmitters as well as number of receivers.
For MIMO OFDM, Spectral Efficiency is mathematically given by:

$$\text{Spectral Efficiency}_{(\text{MIMO-OFDM})} = \min(N_t, N_r) \times \frac{N_f}{(N_f + u)} \times \log_2 M \tag{4.1}$$

where,

N_t = Number of Transmit antennas
N_r = Number of Receive antennas
N_f = Number of Sub-carriers
u = Length of Cyclic Prefix
M = Order of Modulation Scheme.
Spectral efficiency is measured in b/s/Hz.
The increase in data rate is main advantage of MIMO-OFDM over SISO-OFDM. By using low value of CP and IA based precoding in MIMO-OFDM, much better data rates can be achieved in comparison to conventional MIMO-OFDM.

4.1 Simulation Results

4.1.1 Order of CIR, L=19, No. of Sub-carriers, N_f=32, No. of Transmit antennas, N_t=2 and No. of Receive antennas, N_r=1

The parameters used to compute simulations are given below in table 4.1. BPSK has been used as a modulation scheme. Theoretical bit error rate for conventional OFDM has also been plotted for comparison.

Parameters	Values
No. of Sub-carriers (N_f)	32
Order of CIR (L)	19
No. of Transmit antennas (N_t)	2
No. of Receive antennas (N_r)	1

TABLE 4.1: Simulation parameters

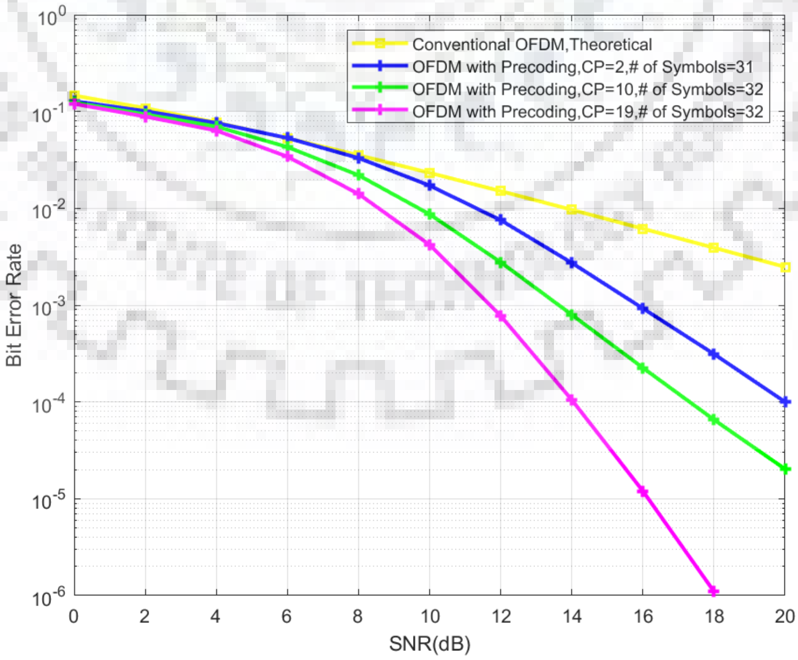

FIGURE 4.1: MISO : BER performance of Precoding based on IA for different values of CP when $L = 19$, $N_f = 32$, $N_t = 2$ and $N_r = 1$

Figure 4.1 shows the BER performance of IA based Precoding for different values of CP i.e. 2,10 and 19. For plots with insufficient CP i.e. where value of CP is less than the channel length L=19, BER increases for IA based Precoding as value of CP decreases.

For CP=2, Number of independent information symbols that can be detected for a IA based precoding system is 31 with bandwidth efficiency of 0.9411 bits/s/Hz. For CP=10, 32 information symbols can be detected successfully with bandwidth efficiency of 0.7619 bits/s/Hz. Similarly, for CP=19, 32 symbols can be detected with bandwidth efficiency of 0.6274 bits/s/Hz.

4.1.2 Order of CIR, L=19, No. of Sub-carriers, N_f=32, No. of Transmit antennas, N_t=4 and No. of Receive antennas, N_r=2

The parameters used to compute simulations are given below in table 4.2. BPSK has been used as a modulation scheme. Theoretical bit error rate for conventional OFDM has also been plotted for comparison.

Parameters	Values
No. of Sub-carriers (N_f)	32
Order of CIR (L)	19
No. of Transmit antennas (N_t)	4
No. of Receive antennas (N_r)	2

Table 4.2: Simulation parameters

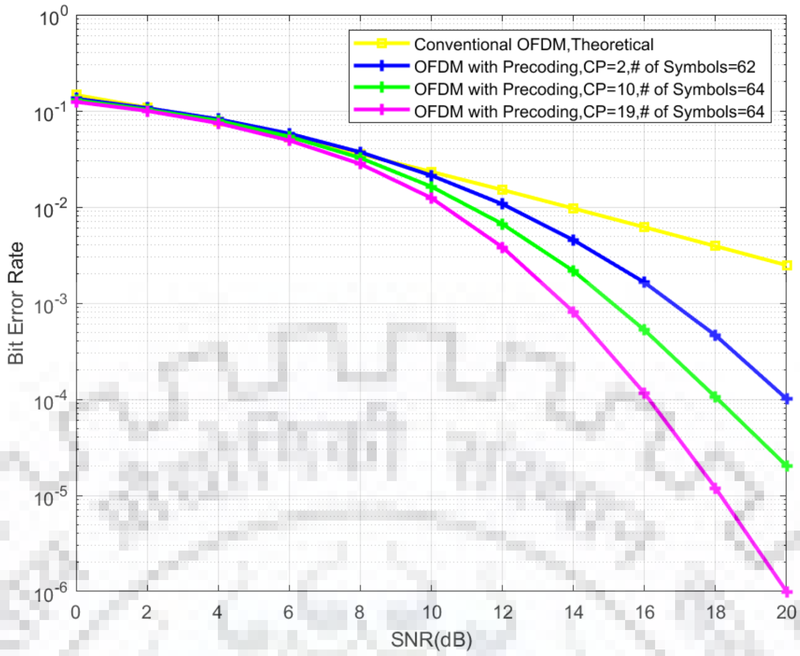

FIGURE 4.2: MIMO : BER performance of Precoding based on IA for different values of CP when $L = 19$, $N_f = 32$, $N_t = 4$ and $N_r = 2$

Figure 4.2 shows the BER performance of IA based Precoding for different values of CP i.e. 2,10 and 19. For plots with insufficient CP i.e. where value of CP is less than the channel length L=19, BER increases for IA based Precoding as value of CP decreases.

For CP=2, Number of independent information symbols that can be detected for a IA based precoding system is 62 with bandwidth efficiency of 1.8823 bits/s/Hz. For CP=10, 64 information symbols can be detected successfully with bandwidth efficiency of 1.5238 bits/s/Hz. For CP=19, BER performance is same as that of conventional OFDM and all the 64 information symbols can be detected successfully with bandwidth efficiency of 1.2549 bits/s/Hz.

4.1.3 Order of CIR, L=12, No. of Sub-carriers, N_f=32, No. of Transmit antennas, N_t=2 and No. of Receive antennas, N_r=2

The parameters used to compute simulations are given below in table 4.3. BPSK has been used as a modulation scheme. Theoretical bit error rate for conventional OFDM has also been plotted for comparison.

Parameters	Values
No. of Sub-carriers (N_f)	32
Order of CIR (L)	12
No. of Transmit antennas (N_t)	2
No. of Receive antennas (N_r)	2

TABLE 4.3: Simulation parameters

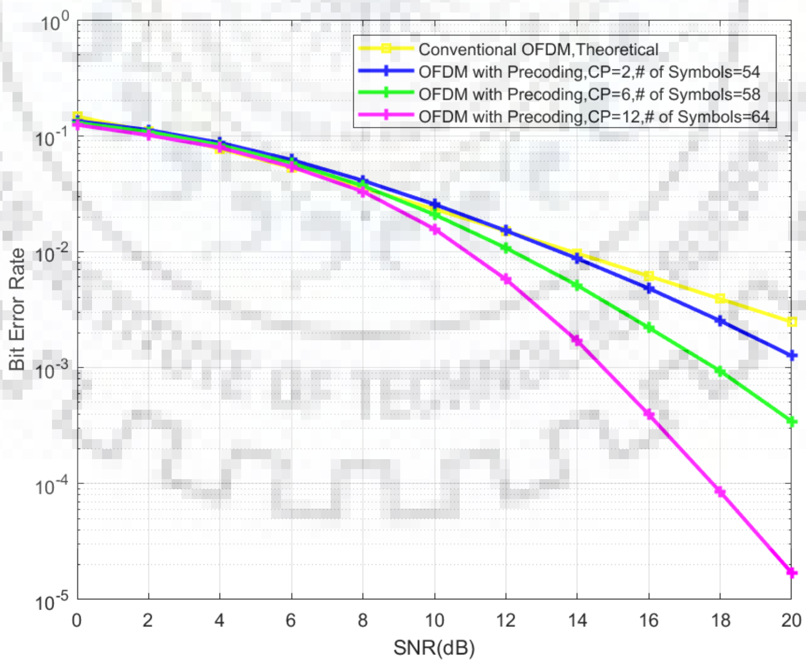

FIGURE 4.3: MIMO : BER performance of Precoding based on IA for different values of CP when $L = 12$, $N_f = 32$, $N_t = 2$ and $N_r = 2$

Figure 4.3 shows the BER performance of IA based Precoding for different values of CP i.e. 2,6 and 12. For plots with insufficient CP i.e. where value of CP is less than the channel length L=19, BER increases for IA based Precoding as value of CP decreases.

For CP=2, Number of independent information symbols that can be detected for a IA based precoding system is 54 with bandwidth efficiency of 1.882 bits/s/Hz. For CP=6, 58 information symbols can be detected successfully with bandwidth efficiency of 1.6842 bits/s/Hz. For CP=12, BER performance is same as that of conventional OFDM and all the 64 information symbols can be detected successfully with bandwidth efficiency of 1.4545 bits/s/Hz.

4.1.4 Order of CIR, L=12, No. of Sub-carriers, N_f=32, No. of Transmit antennas, N_t=4 and No. of Receive antennas, N_r=4

The parameters used to compute simulations are given below in table 4.4. BPSK has been used as a modulation scheme. Theoretical bit error rate for conventional OFDM has also been plotted for comparison.

Parameters	Values
No. of Sub-carriers (N_f)	32
Order of CIR (L)	12
No. of Transmit antennas (N_t)	4
No. of Receive antennas (N_r)	4

TABLE 4.4: Simulation parameters

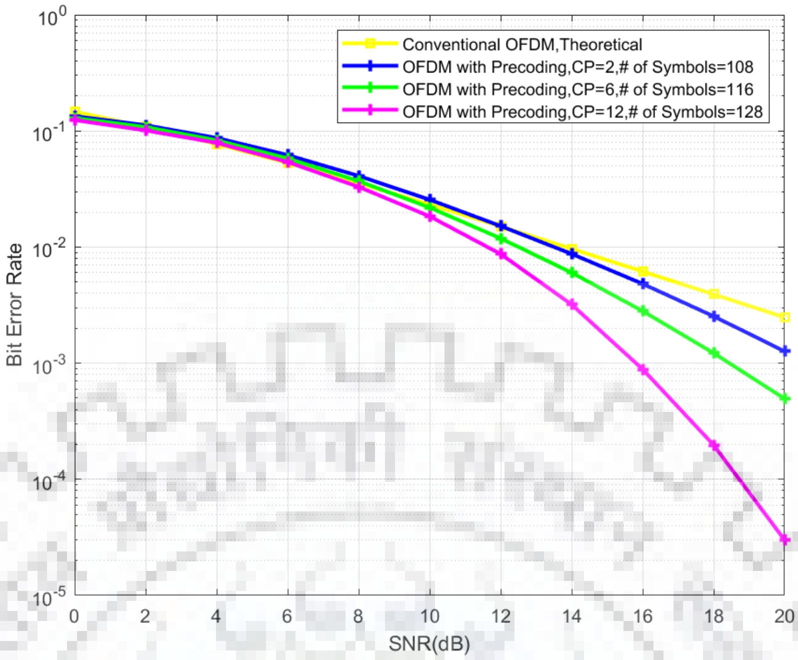

FIGURE 4.4: MIMO : BER performance of Precoding based on IA for different values of CP when $L = 12$, $N_f = 32$, $N_t = 4$ and $N_r = 4$

Figure 4.4 shows the BER performance of IA based Precoding for different values of CP i.e. 2,6 and 12. For plots with insufficient CP i.e. where value of CP is less than the channel length L=19, BER increases for IA based Precoding as value of CP decreases.

For CP=2, Number of independent information symbols that can be detected for a IA based precoding system is 108 with bandwidth efficiency of 3.7647 bits/s/Hz. For CP=6, 116 information symbols can be detected successfully with bandwidth efficiency of 3.3684 bits/s/Hz. For CP=12, BER performance is same as that of conventional OFDM and all the 128 information symbols can be detected successfully with bandwidth efficiency of 2.909 bits/s/Hz.

Chapter 5

Conclusion and Future Work

Simulation results show that IA based Precoding with insufficient CP in OFDM is superior over conventional OFDM in terms of bandwidth efficiency and power efficiency. The bandwidth efficiency improvement is important in terms of high data rate requirements of modern wireless system. By varying the different parameters, we get an increase in bandwidth efficiency. Also, bit error rate of IA based Precoding with insufficient CP in OFDM is comparable to conventional OFDM. The BER performance by varying length of cyclic prefix, number of sub-carriers and channel length has been analyzed.

Application of IA based Precoding to MIMO (Multiple Input Multiple Output) OFDM systems further increases the data rate. There is high demand for bandwith and power efficiency in 5G wireless networks. So, performance of IA based Precoding along with MIMO has been analyzed.

In the simulations, it has been assumed that the Channel State Information is already available at the transmitter. So, as a future work, IA based precoding for OFDM system with imperfect Channel State Information at the transmitter can be studied.

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