Study of Thermal Conductance in Topological Josephson Junctions: Application as Thermal Sensor

A

DISSERTATION

Submitted in partial fulfilment of the

Requirement for the award of the degree

Of

MASTER OF TECHNOLOGY

In

SOLID STATE ELECTRONIC MATERIAL

By

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Under the supervision of

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DEPARTMENT OF PHYSICS

INDIAN INSTITUTE OF TECHNOLOGY, ROORKEE

CANDIDATE'S DECLARATION

I hereby declare the report which is being presented in this Dissertation, entitled, "**Study of Thermal Conductance in Topological Josephson Junction: Application as Thermal Sensor"** in partial fulfilment of the requirement for the award of the degree of **Master of Technology** with specialization in "**Solid State Electronic Materials**", submitted in **Department of Physics, Indian Institute of Technology Roorkee,** is an authentic record of my own work carried out during the period from July 2018 to May 2019 under the supervision and guidance of **Dr. Ajay,** Associate professor, Department of Physics, Indian Institute of Technology Roorkee.

I also declare that I have not submitted the matter embodied in this report for award of any other degree.

Date: MAY, 2019

Place: Roorkee. **(BHUPENDRA KUMAR)**

CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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ACKNOWLEDGEMENT

The satisfaction of successful completion of any task would be incomplete without mention of the people who made it possible and whose constant guidance and encouragement crowns all efforts with success.

I express of deep sense of gratitude to my supervisor **Dr. AJAY, Associate Professor**, **Department of Physics, Indian Institute of Technology Roorkee**, for his initiative and constant inspiration. I have to thank my parents for their endless love, support and encouragement. I would also thanks to my senior Tanuj Chamoli (PhD, Department of Physics) and Anand Omar (PhD, Department of Metallurgical and Material's engineering) and my friends for their cooperation. Lastly I would like to thank science and the giant called the internet which has let me stand on the shoulders of giants to help me reach the end of the project.

(Bhupendra Kumar)

ABSTRACT

The present work deals with thermal transport in Josephson junction. In Josephson junction, heat current is phase dependent just like Josephson super current. Using topological insulators in Josephson junctions provide edge states for transport of heat current from one superconducting lead to other. Bogoliubov-de gennes transformation is used to set the theoretical formulation for thermal conductance in topological Josephson junctions. Andreev Reflection mechanism is used to describe thermal transport in topological Josephson junctions. In one dimensional short Topological Josephson junction the thermal conductance is function of phase only and shows less oscillation in thermal conductance. But in two dimensional Topological Josephson junction thermal conductance is function of phase and interface barrier strength and oscillations are more pronounced here. In one dimensional long topological Josephson junction thermal conductance shows abrupt behavior as a function of junction length and phase difference. These results of thermal conductance in topological Josephson junction can be used in thermal sensing devices where switching behavior is controlled by junction length. For short junctions, the system shows a sharp switching behavior while for long junctions the switching is smooth, which indicates a credential to use these systems for thermal switching device.

Keywords: *Josephson junction (JJ), Andreev Reflection (AR), Andreev Bound State (ABD), Topological insulators (TI), Quantum spin Hall Effect (QSHE), superconductor-topological insulator-superconductor (S-TI-S), critical width (dc), Quantum well (QW), Two dimensional topological insulator (2DTI)*

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CHAPTER 1

Introduction

1.1 Background

Quantum tunneling is responsible for many physical phenomena that amazed scientists in the early 20th century. According to classical physics an electron can't tunnel through the barrier unless its kinetic energy is greater than the potential barrier. But as per quantum physics an electron can tunnel through the barrier as shown in figure 1.1, if its kinetic energy is less than the magnitude of barrier potential, therefore produces a tunneling current. In quantum tunneling the electron tunnels through the barrier without changing energy although the amplitude reduced.

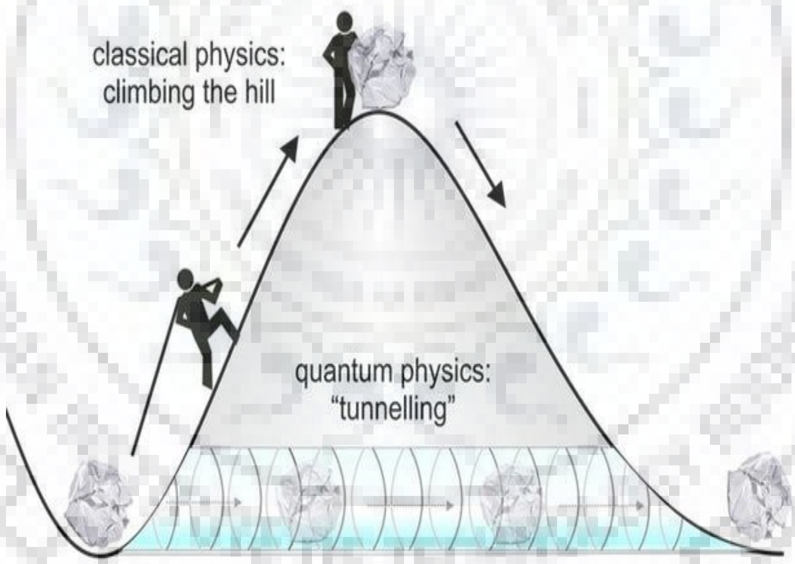

Fig 1. 1 Phenomenon of tunneling across a barrier: in classical and quantum physics [1]

The transmission probability of electron through a barrier is given by $T \approx e^{-2kL}$ where

$$k = \sqrt{\frac{2m}{\hbar^2}(V - E)} \tag{1.1}$$

1.2 Josephson Tunneling

In 1962, B. D. Josephson discovered a different kind of tunneling in superconductors, in which superconducting pairing of electrons is important [1]. Experimentally it was proved by P. W. Anderson and J.M. Rowell in 1963 [2]. Electrons can attract each other via distortion of lattice. It was first realized by Frolich in 1950. When an electron goes through a crystal, lattice distortion produced and sets the heavier ions into slow forced oscillations. Because electron moves with high speed so it cross the region before the oscillations stop. At the same time if another electron pass through this region, it will experience a force which is attractive. So this attractive force lowers the energy of second electron. Since coulomb's repulsion is instantaneous so repulsive force between the electrons is small while the attraction mediated by lattice distortion is highly retarded in time. So the attraction caused by weak lattice distortion can overcome a stronger Coulomb's repulsion. Thus the net effect is the attractive in nature of two electrons via a lattice distortion (or phonon) to form a pair of electrons known as the cooper pair. When two superconducting leads separated through a weak link, then a dissipation less current flows across the junction. This current is known as Josephson current and junction called Josephson junction. The weak link may be insulator (I), normal metal (N), or a topological insulator (TI). An S-I-S junction is shown in figure 1.2.

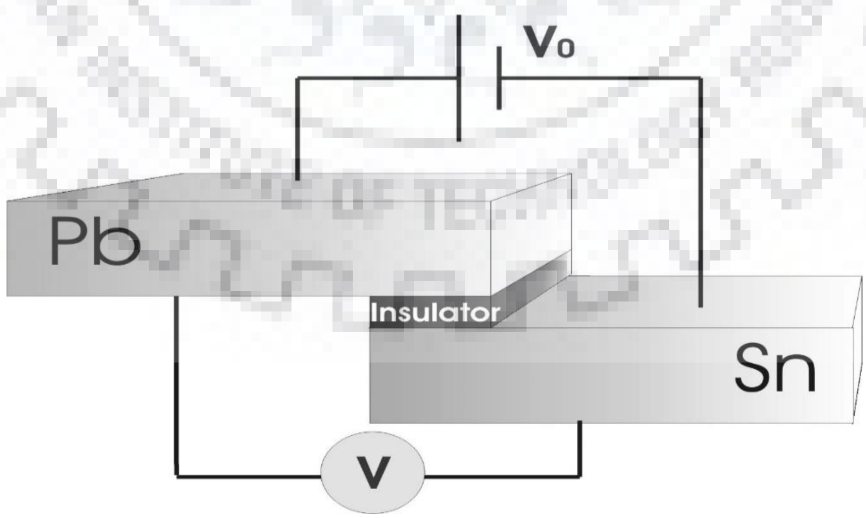

Fig 1. 2 The SIS Josephson junction used to detect the Josephson tunneling current. [1]

In figure 1.2 an insulator is connected to two superconductors. If the width of insulator is thin (about few micrometer) then the current will flow across the junction. But if the width of insulator is large then current will not flow.

1.3 DC and AC Josephson Effect

There are two types of Josephson Effect DC Josephson effect and AC Josephson effect. In DC Josephson Effect a DC current flows across junction without applying any voltage. The equation is given as

$$I = I_0 sin(\theta_2 - \theta_1) \tag{1.2}$$

A DC voltage applied across the junction causes RF current oscillations across the junction. This is called AC Josephson effect [3].

$$I = I_0 \sin[\delta(0) - \frac{2eVt}{\hbar}] \tag{1.3}$$

The current oscillates with frequency $\omega = \frac{2eV}{\hbar}$

The current – voltage relation of Josephson junction can describe with the help of diagram.

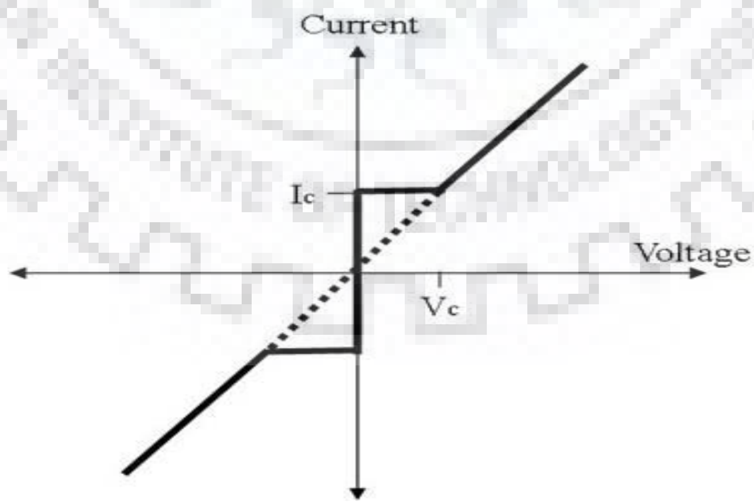

Fig 1. 3 current voltage characteristics of a Josephson junction [3]

DC current flows in the absence of applied voltage. At voltage above V_c junction has a finite resistance, but the current has an oscillatory component of frequency $\omega = \frac{2eV}{\hbar}$.

1.4 Topological Josephson junctions

Josephson junction is made by sandwiching a thin (30-40 angstrom) insulator between two layers of superconducting material. But the discovery of topological insulators and unique properties of topological insulators leads Physicists to use it in Josephson junctions because the bulk of topological insulators behaves as an insulator and its surface provides channels for conduction. The conduction from the channels in topological insulators is quite interesting and flows without backscattering.

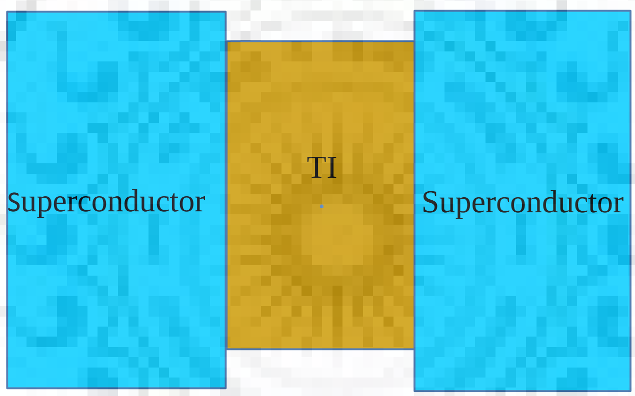

Fig 1. 4 Schematic of topological Josephson junction

In figure 1.4 two superconductors are connected through a topological insulator. In chapter 2 all basic information about topological insulators is discussed.

1.5 Motivation

In electric charge transport Joule heating is major disadvantage. For this proper thermal management and active cooling are required in electronic devices. Interestingly not only Josephson current but also heat current between two superconductors kept at different temperatures depends on phase difference across the junction. So theory what we use for electric part is not changed for thermal current and thermal current flows without scattering so we can say that thermal current is more prominent. Phase dependence of heat currents is recently discovered in superconducting quantum interferences devices (SQUIDs) and using topological insulator instead of insulator opens new era of Josephson junctions called topological Josephson junctions. The unique properties of topological insulators makes it so interesting.

1.6 Organization of Thesis

This thesis is organized as follows:

Chapter 1 gives all the information about Josephson effect, its types and briefly about the topological Josephson junction. Motivation part describes the reason for doing this work.

Section 2.1, 2.2 and 2.3 explains the origin of topological insulator and how topology related with Physics. We describe theoretical and experimental part of this discovery. Section 2.4 and 2.5 gives the idea about two dimensional and three dimensional topological insulators respectively, their electronic band structures, properties and benefits in Josephson junction.

Section 3.1 describes the concept of thermal current in Josephson junctions and in section 3.2 transport mechanism is discussed. Section 3.3 contains all the mathematical work and theories used to calculate thermal conductance in one dimension. In section 3.4 and 3.5 the behavior of thermal conductance in short and long one dimensional topological Josephson junction is discussed respectively. In section 3.6 we extend the earlier case for two dimension.

Chapter 4 is an application part of topological Josephson junction and we showed how topological junction can be used as thermal sensor in that chapter. It is discussed for short junction.

In chapter 5 we conclude our thesis and scope of work in future.

CHAPTER 2

Review of Relevant Literature

In chapter 1 we briefly describe the topological Josephson junctions. Now in chapter 2 we will give review about topological materials and we will describe how and why topology is used in condensed matter Physics. All the necessary information about topological insulators is given in this chapter which will be used in chapter 3 and onwards.

2.1 Electronic Band Structures of Solids

Metals, insulator and semiconductor are first three states of matter which describes electronic phase. After that magnet, superconductors are more exotic phase and in recent year topological insulator emerge as a new electronic phase which fascinates the research world. Electrical conduction in metals, insulators, and semiconductors is described by band theory of solids [4]. The band structure of these three states is shown in figure 2.1.

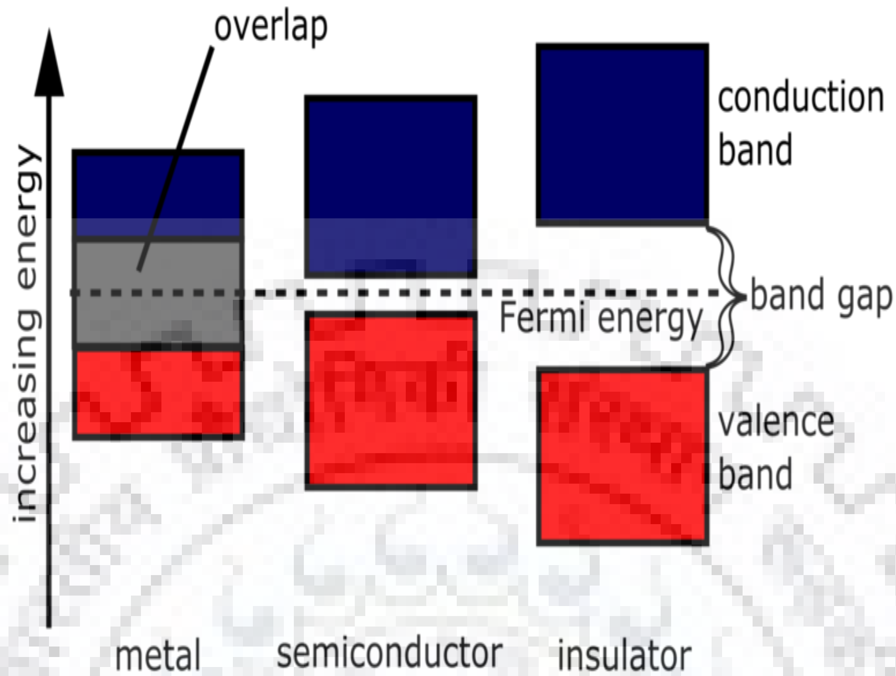

Fig 2. 1 Schematic of band structure of metal, semiconductor and insulator [3]

Conduction band is present above the Fermi level and valence band is below the Fermi level. Fermi level is highest energy state occupied by electrons in materials at absolute zero temperature. In metals conduction and valence band overlaps so there are free electrons to flow. This makes metals good conductors. Silver, copper, gold, aluminum are some examples of conductors. In insulator the energy gap between conduction and valence band is large (about 6eV).Because of this large gap electrons can not jump into conduction band from valence band. So conduction is not possible in insulators. Diamond (energy gap about 5.4eV) is good example of insulator. In case of semiconductor the energy gap between conduction and valence band is about 1ev. On increasing the temperatures the electrons can jump from valence band to conduction band. At zero temperature semiconductors act like insulators but with increasing temperature their conductivity increases. Germanium (0.7eV) and Silicon (1.1eV) are examples of semiconductors.

2.2 Use of Topology in Condensed Matter and Classification of Topological Matter

D. J. Thouless was the first who describes this new state of matter, quantum hall state and he shares a Noble prize in 2016 with Duncan, Haldane and Kosterlitz. They showed how to use topology in condensed matter Physics [5]. Topology is the branch of Mathematics which studies the quantities that does not change under continuous changes. To relate the topology in physics one can consider Hamiltonian of many-particle system separating the ground state and excited state through an energy gap. Here topology is in the line that on changing the Hamiltonian the bulk is not close. A unique point of view is to associate some integer numbers for each topological classes. These are called topological invariants and helpful in defining the band structure. A simple example to understand this is given in figure 2.3.

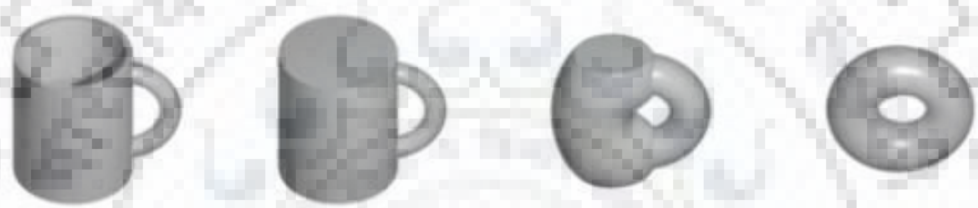

Fig 2. 2 Continuous deformation of a cup into doughnut. In terms of topology both are same because number of holes for objects is equal i.e. 1 adapted from [5]

A doughnut has one hole that is representing the topological invariant. We can see the continuous deformation from a doughnut to a cup. Both cup and doughnut belongs the same topological class because they have same number of holes i.e. 1. There are mainly three symmetries on which class of topological matters depends: The time-reversal symmetry Θ, the particle-hole symmetry and the Chiral symmetry [6, 7]. The quantum hall state does not follow any of the symmetries so edge states are not safe in this state. These are also called trivial topological insulators. On the other hand in quantum spin hall state the edge states are protected by time-reversal symmetry and called non-trivial topological insulators.

2.3 Topological State of Matter

Other than these three states there exist some material for which the bulk and edge shows different behavior. The bulk shows insulating behavior and edge shows conducting nature [8]. They are called topological insulator. The most common system which shows this type of behavior is two dimensional electron gas (2DEG). When a strong perpendicular magnetic field is applied on 2DEG its bulk behaves as insulator and edge as conductor. An electron travelling in these states can not backscatter as the counter propagating channel is on the other side. The quantum Hall (QH) state breaks time-reversal symmetry due to the presence of high magnetic field. In recent year one new state of matter is in light which does not breaks the time-reversal symmetry but breaks two other symmetries. This state is called Quantum spin Hall state (QSH) or normally topological insulator [9]. This state is driven by Spin-orbit coupling. In this state of matter spin up electron is carried by one mover and spin down electron is carried by another which can be shown in figure 2.2.

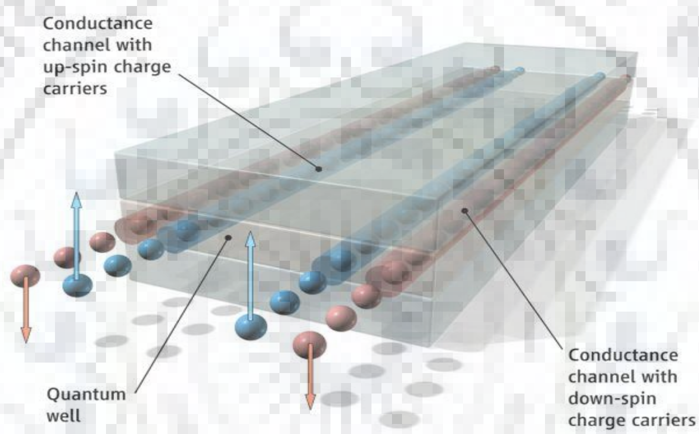

Fig 2. 3 Schematic of the spin polarized edge channels in a quantum spin hall insulator [26].

The quantum spin hall insulator state is invariant under time reversal symmetry and has a charge excitation gap in the 2D bulk, but has topologically protected 1D gapless edge states that lie inside the bulk insulating gap. The edge states have a distinct helical property: two states with opposite spin polarization counter propagate at a given edge. Due to this reason they are also called helical edge states i.e. the spin is correlated with direction of motion. In this precise sense the QSH insulator represents a new topologically distinct state of matter.

2.4 Two Dimensional Topological Insulator

As we see in figure 2.2 QSH state needs the counter-propagation of opposite spin states. This type of coupling between spin and the orbital motion is relativistic effect and we know the spin orbit coupling depends on Z^4 . Light elements also show spin-orbit coupling but did not turned into topological insulator, while heavy materials turned into topological insulator. Two groups

Kane and Mele group and Berniveg, Hughes and Zhang group (BHZ) started work on this topic independently. Graphene is the first predicted (According to Kane and Mele model) topological insulator but this is not used in experiments because it has small energy gap $10^{-3}eV$ [10]. The model which is used by BHZ is considered more general which is proposed in 2006. They predicted that HgTe/CdTe quantum wells with band inversion mechanism behave as two dimensional topological insulator. In the band inversion mechanism valence band and conduction band inverted at critical thickness. The quantum well behaves as conventional insulator when critical thickness is less than 6.5 nm and behaves as topological insulator beyond critical thickness [11, 12, 13]. Unlike the Graphene in which energy gap is too small for direct experiments, the energy gap in HgTe/CdTe quantum wells is enough large for performing direct experiments. The mechanism which is used in BHZ model is proved mathematically based on continuum models and also in first principle and tight binding methods, which all produced the Quantum spin hall state and topological phase transition. When HgTe - based quantum well structures are grown, the special properties of the well material can be utilized to tune the electronic structure. For wide QW layers, quantum confinement is weak and the band structure remains "inverted". However, the confinement energy increases when the wall width is reduced. Thus the energy levels will be shifted and finally the energy bands will be aligned in a "normal" way, if the QW thickness d_{QW} falls below a critical thickness dc.

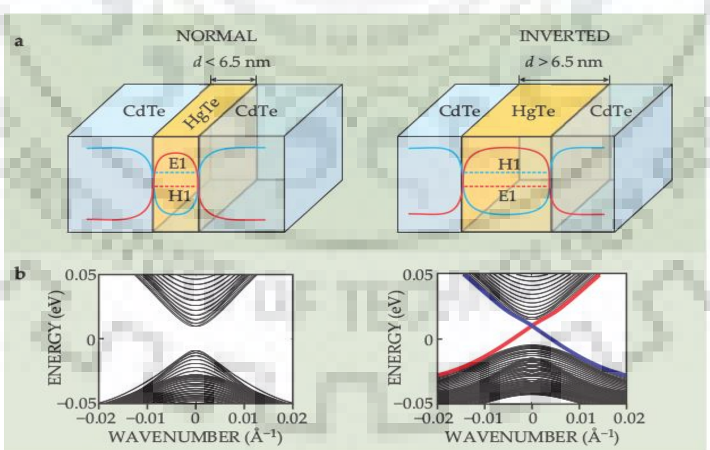

Fig 2. 4 (a) band inversion mechanism in HgTe/CDTE quantum wells, (b) energy spectra of quantum wells [11]

As in semiconductors the conduction band is filled with s-orbital electrons and valence band is filled with p-orbital electrons. Bands are inverted in Hg and Te elements because spin-orbit coupling is strong in heavy elements. The edge states of two dimensional topological insulators are protected by time- reversal symmetry. These edge states are also called helical edge states. They have one pair of 1D edge states crossing at zero momentum. In figure 2.4 (b) before the critical thickness there is an insulating energy gap between valence band and conduction band but after critical thickness there are edge states shown by blue and red lines.

From the explicit solution of BHZ model, there is a pair of helical states exponentially localized at the edge, and described by the effective helical edge theory. The concept of "helical" edge states refers to the fact that states with opposite spin counter-propagate at a given edge as we see in fig 2.4 (b). This is in sharp contrast to the "chiral" edge states in the QH state, where the edge states propagate in one direction only, as shown in fig 2.4 (a). In the QH effect the chiral edge states cannot be backscattered for sample widths larger than the decay length of the edge states. In the QSH effect, one may naturally ask whether backscattering of the helical edge states is possible. It turns out that TR symmetry prevents the helical edge states from backscattering. The absence of backscattering relies on the destructive interference between all possible backscattering paths taken by the edge electrons.

2.5 Three Dimensional Topological Insulators

Fu, Kane and Mele, in 2007 first discovered the three dimensional topological insulators. In two dimensional topological insulators conduction occur from edge states and it also has an insulating bulk but in three dimensional topological insulator bulk is insulating and its surfaces behaves as conductor. As shown in figure the bulk of a three dimensional topological insulator is gapped but the surface of 3DTI are gapless and this is the reason for developing the metallic two dimensional electron gas. The surface of three dimensional topological insulator supports electronic motion in any direction along surface, but the direction of electron's motion determines its spin direction. The dispersion relation describes a cone with a spin perpendicular to the momentum which rotates with it. This cone is called Dirac cone.

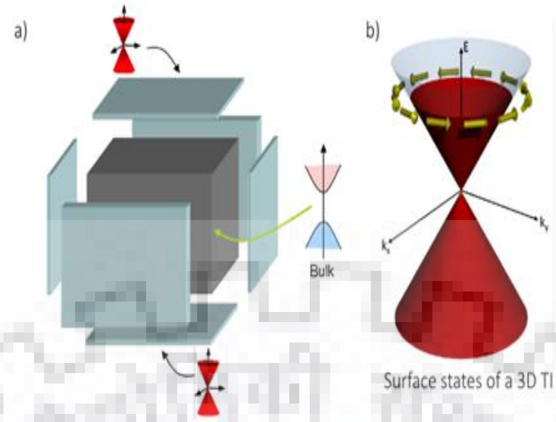

Fig 2. 5 (a) Schematic of three dimensional topological insulator, (b) band structures of surface states on a 3DTI. Arrows represent the spin of the electrons and always perpendicular to its momentum [27].

The first predicted three dimensional topological insulator is Bi_2Sb_{1-x} in 2007 [14]. After that in 2009 Bi_2Te_3, Sb_2Te_3 and Bi_2Se_3 are experimentally proved three dimension topological insulators. Soon after the theoretical prediction of the three dimensional insulator in Bi_2Te_3, Sb_2Te_3 and Bi_2Se_3 class of materials, the surface states with a single Dirac cone is observed using angle resolved photo Spectroscopy. Unlike Graphene, the Hamiltonian of topological insulators is the function of real spin rather than a sub-lattice pseudo-spin degree of freedom. This implies spin dynamics will be qualitatively different from the Graphene. Due to the dominant spin-orbit interaction it is also very different from ordinary spin-orbital coupled semiconductors.

Experimental studies have provided evidence of the existence of chiral surface states and of their protection by time reversal symmetry. Several efforts have focused on the role of magnetic impurities on the surface states of three dimensional topological insulators. The key to the eventual success of topological insulators in technological materials is linked to their transport properties which is described in section 3.2. Potential applications of topological surface states necessarily rely on the realization of an edge metal allowing continuous tuning of the Fermi energy through the Dirac point, the presence of a minimum conductivity at zero carrier density and bipolar transport.

CHAPTER 3

Study of Thermal Conductance in Topological Josephson Junctions

In chapter 2 we discussed how topology is used in condensed matter and all the necessary information about topological insulators which will be used in studying the thermal conductance in topological Josephson junctions. In the present chapter the origin of thermal current in Josephson junction is discussed and then the variation of thermal conductance in one dimensional and two dimensional Topological Josephson junction is discussed.

3.1 Thermal Current in Josephson Junctions

In superconductor-insulator-superconductor Josephson junction, the total electrical current is contribution of three parts: Quasi-particle current, an interference current, The Josephson current. [15] Quasi-particle current has dissipation nature because its response to temperature drop or voltage cross is non-equilibrium. An interference current represented as $I_c cos(\emptyset)$) and it's the result of coupling between quasi-particle and condensate. The Josephson current is represented as $I_c sin(\emptyset)$ and can flow without the voltage across the junction [16].

These results lead the Physicists to study the energy transfer through junction. In Josephson junction energy current is divided just as its electrical counterpart. The current carried by the quasi-particles are dissipative in nature. Interference current and pair current are non-dissipative.

The interference current flows only when there is a temperature or voltage drop across the junction but Josephson current can flow in absence of voltage too [17].

As we discussed in chapter 1 that heat current or energy current is also phase dependent like Josephson current. So the theory we are using for Josephson current may be applied for thermal current also. Here we only talk about the thermal current.

3.2 Transport Mechanism: Andreev Reflection

Here we are connecting two superconducting leads through a three dimensional topological insulator so we know that bulk of 3DTI is insulating and surfaces are conducting. To understand the transport mechanism of surfaces Andreev Reflection mechanism is used.

The surface of three dimensional topological insulators is conducting so there are electrons to move. It is like interface of N-S junction. From the conducting surface when one electron with energy less than superconducting gap (Δ) comes toward the superconducting lead, it cannot enter in the superconducting regime because of the zero density of states at this energy. Instead electron is back reflected as a process called Andreev reflection [18, 19].

In Andreev reflection electron reflects back as a hole from the interface to N-side and make a bound state called Andreev Bound State (ABS). At the same time one Cooper pair is transmitted in superconductor regime as shown in figure 3.1a. We can extend this mechanism for S-3DTI-S Josephson junction as we see in figure 3.2. This process happens in both interfaces. One Cooper pair is transmitting in right lead and same process happens in the left lead. So Cooper pair is transmitting through left lead to right lead and this process conserves charge and energy. In figure 3.2 in the right superconducting lead Andreev reflection proceeds as: one electron with energy $E_F+\varepsilon$ and wave vector k_F+q and back reflects as hole with opposite spin, energy $E_F-\varepsilon$ and wave vector $-k_F+q$.

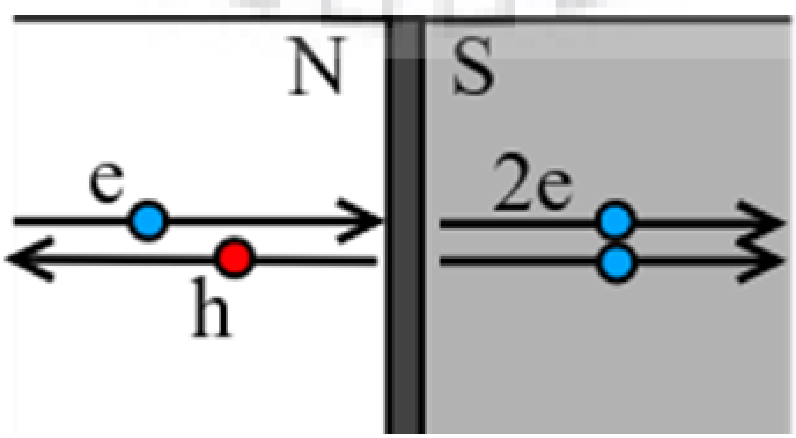

Fig 3. 1 N- S interface of a Josephson junction [18]

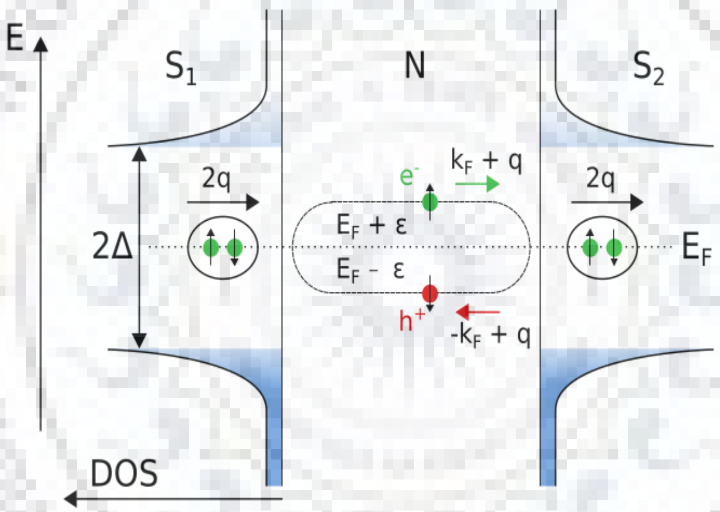

Fig 3. 2 Andreev reflection in S-TI-S Josephson junction. The two superconductor leads have a gap 2Δ [18]

3.2 Theoretical Formulation for Thermal Conductance in Topological Josephson Junctions

We had already discussed transport mechanism of cooper pairs in Superconductor-topological insulator-superconductor junction. Quasi-particles with energy above or below the superconducting gap carries the heat current in junction. Heat current does not suffer backscattering and it is more faster than electrical part. Here we are studying the thermal conductance in Josephson junctions based on surface states of three dimensional topological insulators. There is a phase difference ØR-ØL across the junction and the pair potential is given by $\Delta e^{i\emptyset_r}$. Here we take an assumption that change on pair potential is small than superconducting coherence length [20].

The mathematical formulation for the study of thermal conductance is set by the help of Bogoliubov-De Gennes equation. Josephson junctions are typical example of inhomogeneous system. To describe the inhomogeneous systems Ginzburg-Landu theory is used but disadvantage with this theory is that it is not applicable for quasi-particles. The microscopic description to discuss inhomogeneity and to make BCS mean field Hamiltonian spatially dependent, Bogoliubov-De gennes Hamailtonian is used.

The Bogoliubov-De Gennes Hamiltonian for electron-like and hole-like quasi-particles is given as

$$H= -h_k h_k^* + \sigma_y^2 \Delta^2 e^{-i\emptyset_r} e^{i\emptyset_r} \tag{3.1}$$

Where $h_k = \frac{h}{2\pi}\vartheta_F k.\sigma - \mu\sigma_0$ is the Dirac Hamiltonian for single-particle which defines the helical surface of topological insulators, k is the charge carrier wave vector, ϑ_F is the Fermi velocity.

The eigenfunctions for the electron-like quasi-particle which is moving right side and hole-like quasi-particle which is moving left are given by

$$\varphi_1(x,y) = \left(u, e^{ik\theta_e}u, -e^{-i\emptyset_r}e^{i\theta_e}v, e^{-i\emptyset_r}v\right)^T e^{ik_e.r} \tag{3.2}$$

$$\varphi_2(x,y) = \left(v, e^{i\theta_h}v, -e^{-i\emptyset_r}e^{i\theta_h}u, e^{-i\emptyset_r}u\right)^T e^{ik_h.r} \tag{3.3}$$

Where r = (x, y) and $k_{e,h}$= $k_{e,h}(cos\theta_{e,h}, sin\theta_{e,h})$

u, v are coherent factors and given as

$$u = \frac{1}{2}\sqrt{1 + \frac{\sqrt{\omega^2 - \Delta^2}}{w}}, v = \frac{1}{2}\sqrt{1 - \frac{\sqrt{\omega^2 - \Delta^2}}{w}} \tag{3.4}$$

In one dimension the barrier potential plays no role so we skip the barrier potential for one dimension. Although potential leads to the boundary condition.

The wave functions of electron like quasi-particles for the three regions(S, TI, S) may be written as

A) $\varphi_L = \varphi_S^{e\pm} + r^1\varphi_S^{e-} + r_A^1\varphi_S^{h-}$
B) $\varphi_M = f\varphi_N^{e+} + g\varphi_N^{h+} + m\varphi_N^{e-} + n\varphi_N^{h-}$ (3.5)
C) $\varphi_S = t^1\varphi_S^{e+} + t_A^1\varphi_S^{h+}$

Where, r^1 and r_A^1 amplitudes of normal and Andreev reflections. f, g, m, n are corresponding transmission and reflection amplitude in NM and t^1, t_A^1 are the amplitudes of electron-like and hole-like quasi-particles in the right superconducting lead.

Similarly wave functions for hole-like quasi-particles for three regions may be given as

A) $\varphi_L = \varphi_S^{h+} + r^2\varphi_S^{h-} + r_A^2\varphi_S^{e-}$
B) $\varphi_M = f'\varphi_N^{e+} + g'\varphi_N^{h+} + m'\varphi_N^{e-} + n'\varphi_N^{h-}$ (3.6)
C) $\varphi_S = t^2\varphi_S^{e+} + t_A^2\varphi_S^{h+}$

The thermal conductance of a one dimensional Josephson junction is given by

$$k(\emptyset) = \frac{1}{h}\int_{\Delta}^{\infty} d\omega\omega\big(\tau_e(\omega,\emptyset) + \tau_h(\omega,\emptyset)\big)\frac{df}{dt} \tag{3.7}$$

Where f $=\frac{1}{e^{\omega/kbT}+1}$ is the Fermi distribution function.

3.3 Result and Discussion for Thermal Conductance in Short One Dimensional Topological Josephson Junctions

The transmission probability for short one dimensional topological Josephson junction is given by

$$\tau_{e,h}(\omega, \emptyset) = \frac{\omega^2 - \Delta^2}{\omega^2 - \Delta^2 cos^2 \frac{\emptyset}{2}} \tag{3.8}$$

It is pointed out that transmission probability is independent of strength of barrier potential in one dimension. Here we calculate the thermal conductance as a function of phase with different superconducting gap.

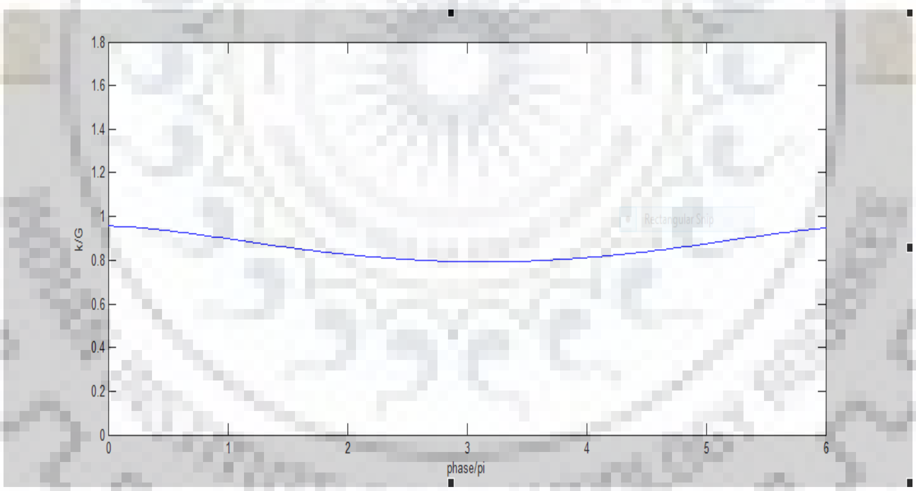

Fig 3. 3 Thermal conductance as a function of phase in units of thermal conductance quantum for $k_BT = \Delta$

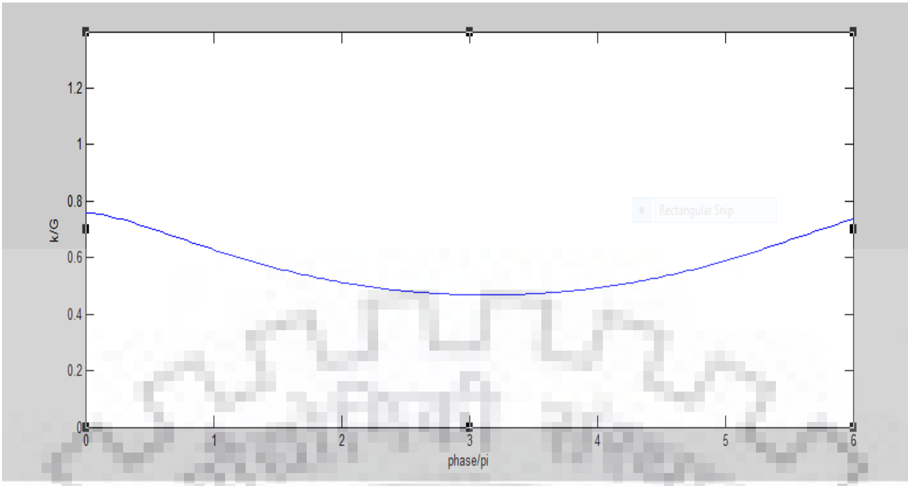

Fig 3. 4 Thermal conductance as function of phase in units of thermal conductance quantum for $\boldsymbol{k_B T} = \Delta/2$

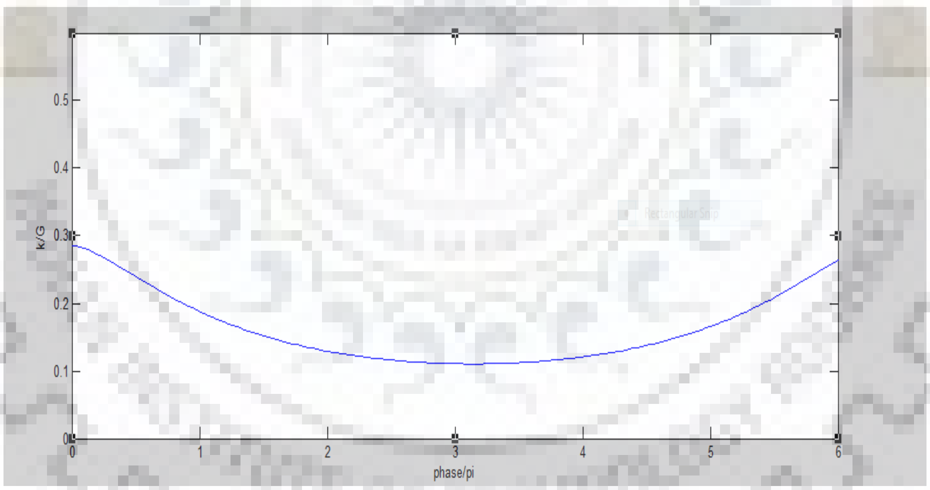

Fig 3. 5 Thermal conductance as function of phase in units of thermal conductance quantum for $\boldsymbol{k_B T} = \Delta/4$

From these plots of thermal conductance as a function of phase we observe that thermal conductance decreases with decreasing temperature. This is due to less number of thermally excited quasi-particles on decreasing temperature. The oscillations starts on lowering the temperature but temperature can't be lowered too much because thermal conductance will be decreased. As we see that in one dimension transmission probability is only the function of phase and superconducting gap. We neglect the barrier strength potential case but in two dimensional case we have to take in account the strength of barrier potential.

3.4 Result and Discussion for Long One Dimensional Topological Josephson Junction

In one dimensional short topological Josephson junctions we consider only phase dependence in transmission function because the intermediate region is very thin. For numerical simplicity L=0 is taken into account. But in long one dimensional S-TI-S junctions the intermediate region plays an important role because in intermediate region hole and electron wave vector depends on energy as we have shown in equation number 3.4. So the modified transmission function for long topological junction is given as…

$$\tau_{e,h}(\omega,\emptyset)=\frac{\omega^2-\Delta^2}{\omega^2-\Delta^2 cos^2(\frac{\emptyset}{2}\mp\frac{\omega L}{\hbar\vartheta_F})} \tag{3.9}$$

Here transmission function as well as thermal conductance is a function of junction length L and phase difference $\emptyset$. The three dimensional profile of thermal conductance as a function of phase and junction length is given in figure 3.6, 3.7 and 3.8.

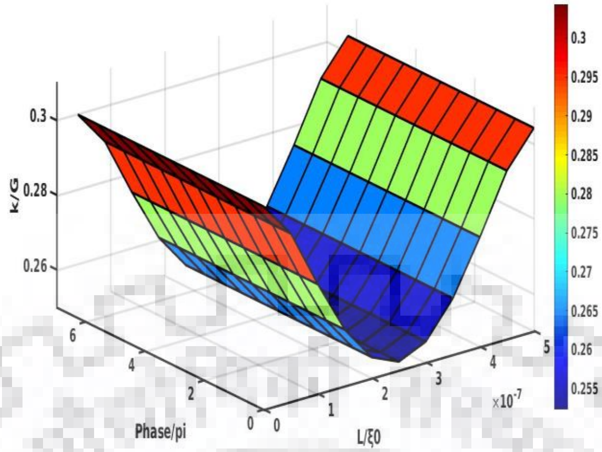

Fig 3. 6 Thermal conductance of a long one dimensional S-TI-S Josephson junction in units of GQ as a function of junction length and phase difference for $\boldsymbol{k_B T} = \Delta$

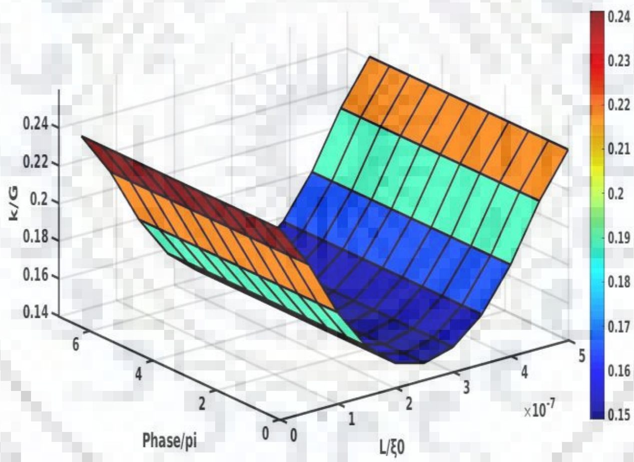

Fig 3. 7 Thermal conductance of a long one dimensional S-TI-S Josephson junction in units of GQ as a function of junction length and phase difference for $\boldsymbol{k_B T} = \Delta/\mathbf{2}$

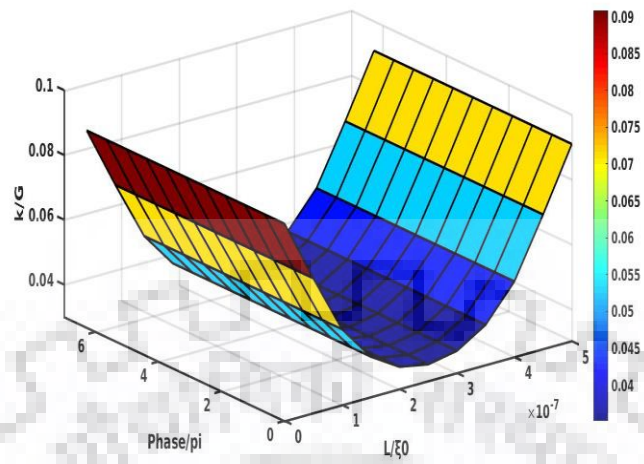

Fig 3. 8 Thermal conductance of a long one dimensional S-TI-S Josephson junction in units of GQ as a function of junction length and phase difference for $\boldsymbol{k_B T} = \Delta/4$

Thermal conductance in long one dimensional topological Josephson junctions as a function of phase difference and junction length shows abrupt behavior.

This type of behavior of thermal conductance in short and long topological Josephson junctions clearly indicates that topological Josephson junctions may be used as thermal switches because it can achieve a large temperature difference between the on and off state which is discussed in chapter 4.

3.5 Result and Discussion for Two Dimensional Topological Josephson Junction

As discussed in section 3.1 that, in two dimensional case we have to take into account the strength of barrier potential. It is modeled by a delta potential U δ(x), with barrier height U. The boundary condition with the help of this potential is given by $\varphi_L(0, y) = cosZ\tau_0\sigma_0 + isinZ\tau_z\sigma_x\varphi_R(0, y)$ where $Z = \frac{U}{\hbar v_F}$ and τ is the Pauli matrix. In two dimensional case there are more than one channels N>>1 to carry the quasi-particles. So here we are plotting the thermal conductance as a function of interface barrier strength with N>>1 for $k_B T = \Delta/2$.

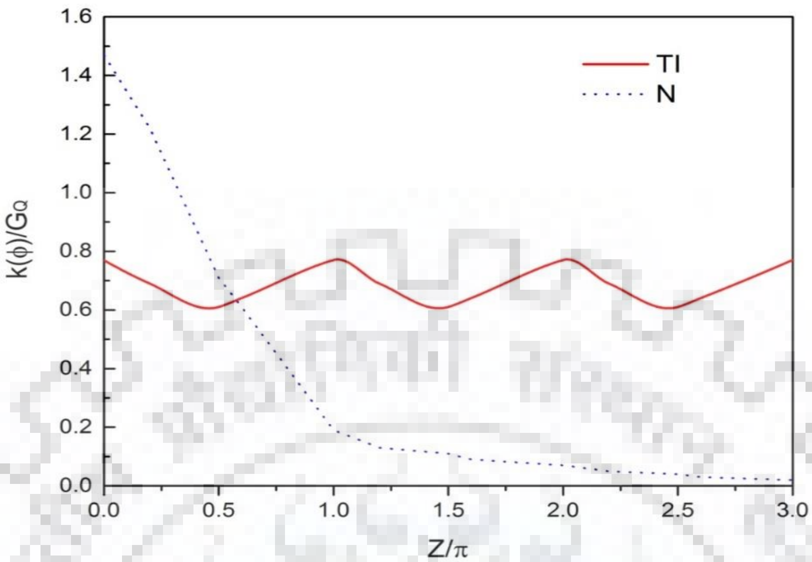

Fig 3. 9 Thermal conductance as a function of interface barrier strength Z for $\boldsymbol{k_B T = \Delta/2}$

In two dimensional topological Josephson junction the thermal conductance as a function of interface barrier strength shows oscillations with temperature. Incidence of quasi-particles play an important role here.

Oblique incidence of quasi-particles in topological insulator from superconductor does not show unit transmission because it can backscattered. This type of incoming of quasi-particles experiences a higher barrier potential at interface. Quasi-particles which have normal incidence show unit transmission. We got oscillations in thermal conductance for two dimensional topological Josephson junction due to resonance formation at interface when incidence of quasi-particles is normal. We see that in heat current the contribution of oblique incidence is less than the normal incidence of quasi-particles.

CHAPTER 4

Application of Topological Josephson Junctions as Thermal Switch

In the previous chapter we have discussed the thermal conductance behavior in one dimensional and two dimensional topological Josephson junction and find that it is showing oscillations more rapidly in two dimensional topological Josephson junction. In the present chapter we will discuss how topological Josephson junction based on two dimensional topological insulator can be used as thermal switch.

4.1 Two Dimensional Topological Insulator in Josephson Junctions

We make some changes here in junction three dimensional topological insulator is replaced by two dimensional topological insulator and we study the behavior of thermal conductance as a function of magnetic flux. The motive of using the two dimensional topological insulator is that it has helical edge states which are conducting and conductance increased from $\frac{2e^2}{h}\,to\,\frac{4e^2}{h}$ due to Andreev reflection [22]. Experimentally it has proven that heat current also depends on phase and it is found on superconducting quantum interference device (SQUID) [23]. Experimentally it is also proven that heat current also diffracted with magnetic flux [24, 25].

On the basis of above discussion, temperature biased topological Josephson junction can be used as thermal switch and it can be controlled by weak magnetic field. The switching behavior can be controlled by junction length i.e. junction is short or long. The two superconducting electrodes are separated by the edge channels of two dimensional topological insulators. Heat current is produced when we put these two superconducting leads at different temperatures i.e. $\Delta T = T_L - T_R$.

4.2 Thermal Conductance for Short Junction

Here we show the flux dependence of thermal conductance for short junction $L = \xi_0$, for different temperatures. The short junction and long junction is decided according to characteristic length.

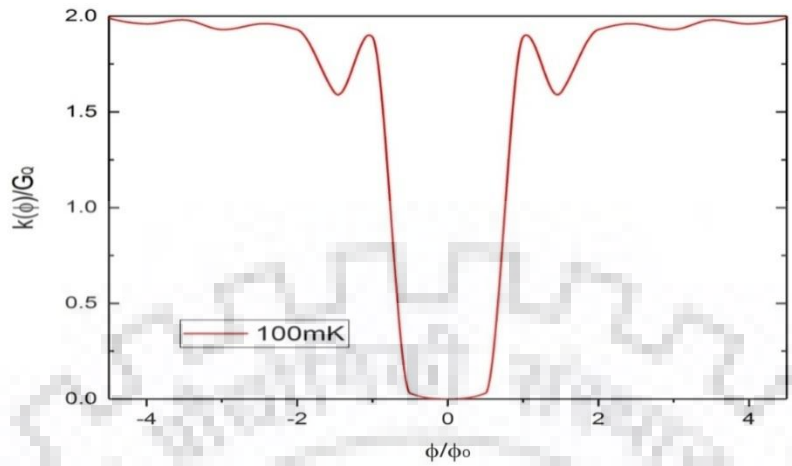

Fig 4. 1 Flux dependence of thermal conductance for short junction for 100mK [28]

Thermal conductance is suppressed exponentially as we see in figure 4.1. At low temperature, less number of thermally excited quasi-particles and also the superconducting gap is present. That's why thermal conductance is decreasing. When we apply some magnetic flux, the superconducting gap is closed so that we can get higher thermal conductance of k=2G_Q where $G_Q = \frac{\pi^2 k_B^2 T}{3h}$ is the thermal conductance quantum. The change in thermal conductance is rapid. The oscillations we got in thermal conductance can be related to the point that electron and hole wave vectors are energy dependent.

CHAPTER 5

Conclusion and Scope of Future Work

5.1 Conclusions

Heat current depends on phase just like the Josephson current. The thermal conductance in topological Josephson junction, (either one dimensional or two dimensional, based on three dimensional topological insulator) shows oscillations in possible range of superconducting gap and temperature. In one dimension the oscillations are not too pronounce and the transmission function has only phase dependence. In one dimensional short topological Josephson junction, the transmission function as well as thermal conductance is independent from the strength of interface barrier. With lowering the temperature the thermal conductance decreases rapidly. But in two dimensional topological Josephson junction we see that thermal conductance depends on barrier potential also. We got oscillations in thermal conductance as a function of Z (strength of interface barrier). Because of this potential, incoming quasi-particle experiences some force there. In two dimensional topological Josephson junctions, the normal incidence of quasi-particles gives the unit transmission because at interface resonance occurs. While in oblique incidence unit transmission not occurred. In long topological Josephson junctions the transmission function is function of phase and junction length. The study of thermal conductance for short and long one dimensional topological Josephson junction and two dimensional topological Josephson junction indicates that topological Josephson junctions can be used in thermal sensing devices. It can be controlled by small magnetic field and the switching behavior is controlled by junction length. For short junction the behavior is sharp and for long junction the thermal conductance behavior is smooth.

5.2 Scope of Future Work

1. Experimental set up for Thermal switch using topological Josephson junction

2. Phase tunable- thermal rectification in the topological SQUPIT.

3. Phase coherent heat circular based on multiple Josephson junctions.

4. Phase-dependent heat and charge transport through superconductor–quantum dot hybrids

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