A STUDY ON PROPAGATION OF FLOOD WAVE IN A STREAM CONSIDERING STREAM-AQUIFER INTERACTION USING VARIABLE PARAMETER MUSKINGUM METHODS

Ph.D. THESIS

by

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled "**A STUDY ON PROPAGATION OF FLOOD WAVE IN A STREAM CONSIDERING STREAM-AQUIFER INTERACTION USING VARIABLE PARAMETER MUSKINGUM METHODS**" in partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy and submitted in the Department of Mathematics of the Indian Institute of Technology Roorkee, Roorkee, is an authentic record of my own work carried out during a period from July, 2012 to March, 2019 under the supervision of Dr. T. R. Gulati, Professor, Department of Mathematics, and Dr. M. Perumal, Professor, Department of Hydrology, Indian Institute of Technology Roorkee, Roorkee.

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other institution.

(MAMTA SAXENA)

This is to certify that the above statement made by the candidate is correct to the best of our knowledge.

(T. R. GULATI)
Supervisor

(M. PERUMAL)
Supervisor

Dated:

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ABSTRACT

Stream aquifer interaction is one of the important components of the stream flow processes, especially in deserts and water deficient regions. Apart from the natural recharge which contributes to the groundwater storage due to rainfall-runoff process over a catchment, the stream-aquifer interaction process also contributes to this storage over the catchment and also sustains the flow in a river as baseflow. However, many a times this process is not given its due importance and consequently not taken into account in the water availability assessment of the catchment. The major objective of this study is to present different approaches of accounting this process during the passage of a flood wave in a stream, when stream-aquifer is enabled by the prevailing stream bed geological conditions of a stream, under the scenarios of stream fully or partially penetrating the adjoining aquifer. It is assumed that the flow in the stream is one-dimensional, but the flow in the aquifer can be one-dimensional or two-dimensional, depending on the existing flow scenarios in the aquifer and the data availability. With this background, the following related problems are investigated in this study.

Stream-aquifer interaction process play a dominant role in the form of lateral flow which affects the flood wave transformation process in the considered study reach in the absence of runoff causing precipitation in the intervening catchment of the study reach. Tracking a flood wave along a stream requires the use of appropriate tools, like the flood routing model. The available literature in this regard reveal the use of a variant of the well-known Muskingum method, known as the non-linear Muskingum method which routes the flood hydrograph considering non-linearity in the routing process by duly accounting for the river-aquifer interaction process in the study reach. However, this study uses the rating curve relevant to the study reach of the stream to convert the discharge to stage or flow depth at mid-section of the considered sub-reach of the study reach for using the governing equation of the stream-aquifer interaction process during the flood propagation study. Therefore, the use of rating curve for converting discharge of the nonlinear Muskingum method into the corresponding stage hydrograph required for considering stream-aquifer interaction process in the study reach restricts the application of the nonlinear rating method only to steep slope river reaches.

To overcome such a limitation in the use of such simplified routing methods in streams of moderate bed slopes, this study proposes a better alternative simplified routing method which has the capability to route a flood hydrograph characterized by discharge variable, but capable of estimating the associated stage hydrograph required for accounting stream-aquifer interaction process in the study reach without the use of established rating curve of the study reach. This method known as the Variable Parameter Muskingum Method (VPMM) was proposed by Perumal and Price (2013). Alternatively, one can directly employ a stage hydrograph variable based simplified routing method for accounting stream-aquifer interaction process during the passage of a flood wave in a stream reach duly accounting for stream-aquifer interaction process in the study reach. This method known as the Variable Parameter Muskingum stage-hydrograph (VPMS) routing method was proposed by Perumal and Ranga Raju (1998) which is capable of estimating associated discharge hydrograph at the location of the study reach without using the established rating curve of the stream-reach. This study employs both these two routing methods for accounting stream-aquifer interaction process during the passage of a flood wave propagation in a stream reach, where this interaction process is conducive to take place. Different possible stream scenarios with reference to the surrounding aquifer environment are explored for accounting stream-aquifer interaction in a study reach of a stream, like the stream fully or partially penetrating the surrounding aquifer. Also, the flow scenarios of flow in aquifer being one-dimensional or two-dimensional can be considered in the study. A brief summary of the outcome of the investigations carried out in the study are presented herein.

The present study is conducted with the following objectives: 1) Streamflow routing using the VPMM method and assuming one-dimensional flow in the aquifer perpendicular to the stream axis for the case of fully penetrating stream; 2) Streamflow routing using the VPMS method and assuming one-dimensional flow in the aquifer perpendicular to the stream axis for fully penetrating stream case. 3) Use of the VPMM and VPMS methods for streamflow routing and assuming two-dimensional flow in the aquifer perpendicular to the stream axis as well as parallel to it for fully penetrating stream case. 4) Use of the VPMS and VPMM methods for streamflow routing and assuming two-dimensional flow in the aquifer for partially penetrating stream case. The stream-aquifer model for fully penetrating stream has been developed for solving the one-dimensional aquifer flow, where the stream flow is also considered as one-dimensional. The VPMM and the VPMS methods considering stream-aquifer interaction have been verified using hypothetical data and their field applicability have been demonstrated in this study. The Nash-Sutcliffe Efficiency

(NSE) estimate of the VPMM and VPMS methods for the hypothetical case studies are estimated to be 0.9979 and 0.9994 respectively and the respective Root-Mean Square (RMSE) estimates are found to be 0.84 and 0.43. The field application of the VPMM and VPMS show the NSE estimates with respect to the monitored data are found to be 0.9557 and 0.9304 respectively and the respective RMSE estimates of 0.0217 and 0.027, respectively.

The routing model considering stream-aquifer interaction process for fully penetrating stream has been developed for solving the two-dimensional aquifer flow, where the stream flow is considered as one-dimensional. The validity of the proposed methods has been checked by applying these methods for the hypothetical case of step-rise input and by comparing the obtained bank storage with the same obtained using an analytical method available in the literature for the same step-rise input. The methods have also been applied on a hypothetical pulse input and the results show very close reproductions with the corresponding hypothetical analytical results.

The stream-aquifer model for partially penetrating stream has been developed for solving the two-dimensional aquifer flow. The lateral flow estimation of this model has been obtained using conformal mapping approach, which is based on the method developed by Aravin and Numerov (1965). The conformal mapping approach has been applied on the field data of the Platte River of Nebraska, USA using the VPMM method for streamflow routing. Results reveal that the VPMM method can closely reproduce the hydrograph recorded at the downstream of the considered routing reach. The NSE and RMSE estimates of the VPMM method are found to be 0.9922 and 10.26, respectively with reference to the available monitored data. The results reveal that with the moderate data requirements, the VPMM method can be chosen to evaluate the bank storage for a river segment. In the present study, two verification approaches have been employed for producing discharge hydrographs considering bank storage on both the sides of the channel cross-section. In the first verification case of the considered approach of modeling stream-aquifer interaction, the explicit finite difference method was used for the solution of the full Saint-Venant equations and the second using the VPMM method (Perumal and Price, 2013). Also, the hydrographs reproduced by the above-mentioned procedures have been compared with the explicit solution. Moreover, the VPMM method simultaneously computes the stage hydrograph corresponding to a given inflow or routed discharge hydrograph. Therefore, for the evaluation of bank storage, this method provides the values of hydraulic heads which are equal to the river stages at each river-section. Overall, the study shows the appropriateness of using the VPMM and VPMS methods accounting for stream-aquifer interaction can be very useful field applications while routing flood in streams where stream aquifer interaction process is dominant.

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CHAPTER 1

INTRODUCTION

1.1 GENERAL

In field practices, the surface water and groundwater are frequently treated as isolated components for water resources assessment in spite of their close linkage and interaction in many ways (Sophocleous, 2002). Streams and aquifers are hydraulically connected with water passing between the adjoining aquifer and the stream channel, and vice versa during the passage of flood wave in the stream reach (Castro and Hornberger, 1991; Bencala, 1993). Hence, degradation or exhaustion of one will distress the other in terms of both quality and quantity of water. The stream-aquifer interaction proceeds on various temporal and spatial scales, and are complex in nature (Kalbus, 2009). In other words, this process can be explained by two terms; 1) The gaining or effluent streams defined as the flow of groundwater into the stream through streambed, and 2) The losing or influent streams, where seepage from stream is lost through the streambed into the groundwater storage. Most often, a combination of these two processes may prevail over the same stretch of a river reach (Winter, 1998).

The hydraulic gradient is the main factor which decides the nature of flow exchange, i.e., losing or gaining stream. In gaining stretches of a river, the groundwater table elevation is generally higher, in comparison with the prevailing stream stage, and vice versa for the losing stream reaches. A saturated zone or an unsaturated zone can act as a medium for connection or disconnection to the stream, when the water table is below the streambed. The level of groundwater table and stream stage can alter or cause temporal changes in magnitude and direction of flows due to an unusual precipitation event or due to the seasonal variations in the precipitation. The gaining or losing condition of a stream can be found out by observing the elevation of the groundwater table and stream water level in a nearby monitoring well. The complexities in stream-aquifer interaction process arise mainly due to the complex porosity of aquifers, catchment physiographical characteristics, surface water positioning and also due to the difference in the opinions of hydrologists and hydrogeologists regarding the selection of methods or models to investigate the interaction between them (Madlala, 2015). Surplus water after a high precipitation and flood events may contribute to aquifer storage by percolation process.

The storage capacity of the aquifer alon with the magnitude and duration of flood hydrograph are the main factors which decide the volume of water stored within the bank (Brunke and Gonser, 1997). Several investigators provided the solution for stream-aquifer interaction problem by assuming a fully penetrating rectangular channel and one-dimensional aquifer flow (Cooper and Rorabaugh, 1963; Hornberger et al., 1970; Zitta and Wiggert, 1971).

Many models have been developed to simulate flood processes due to spatially variable and non-uniform precipitation event. Every so often, these developed models have been divided into two components; 1) surface and 2) subsurface components. These two components are commonly linked with each other. Due to their artificial separation, most of the problems were eradicated, but it resulted in growth of the number of discrete models. It also failed to describe the system accurately (McDonald and Harbaugh, 1998; Aral, 1990; Fread, 1993). Although, good and acceptable simulation results are being provided by these discrete models related to the hydrological processes of their particular domains, and when the interactions amid the domains became more dominant, the model results show deviation from the actual or observed events. Hence to minimize the deviation between the simulated and observed runoff processes, it is necessary to consider the interaction between the stream and aquifer, even using a simple coupling process. Streamflow routing models considering stream-aquifer interaction have been proposed in the literature (Bear, 1972). Either these models investigate streamflow routing using the full one-dimensional flow equations (Zitta and Wiggert, 1971), like the Saint Venant equations (SVE), which may not be suitable for application under the scenario of insufficient cross-sections information of the river, or too simplified methods, like the nonlinear Muskingum routing method (Birkhead and James, 1998) which may be suitable for application only in steep slope rivers. This shortcoming in the stream-aquifer interaction models present the scope for improving the simplified routing models with the capability to vary the routing parameters at every routing time level which may be applicable for steep as well as intermediate slope rivers. Such models can be employed for considering stream-aquifer modeling purposes.

Based on the identified scope, a study on the passage of flood wave in a channel has been carried out by Perumal and Ranga Raju (1998) and Perumal and Price (2013). During the passage of a flood wave, the stream-aquifer interaction takes place, i.e., during the rising stage, the aquifers are recharged and bank storage is increased and when the flood recedes, there is a return flow from the aquifer to the river. River flow to some extent is controlled by stream-aquifer interaction process. The study has been carried out considering flow in the aquifer perpendicular to the stream-axis. However, in nature, the flow in the aquifer will be two-dimensional. Earlier, wave propagation studies have been carried out assuming the stream banks to be vertical, the stream to be fully penetrating. In reality, as vertical stream banks do not exist always, the stream banks can also be curvilinear. It is also very usual that stream does not penetrate the aquifer completely and it penetrates partially. In the present thesis, flood propagation analysis has also been carried out considering fully as well as partially penetrating rivers.

1.2 SCOPE OF THE STUDY

In the light of above discussion, the scope of investigating the variable parameter flood routing methods for stream-aquifer interaction study exists and they can be briefly summarized as below:

a) Use of a Variable Parameter stage-hydrograph routing method incorporating stream-aquifer interaction process, assuming the aquifer flow to be one-dimensional perpendicular to the stream and assuming fully penetrating stream.

b) Use of a Variable Parameter discharge-hydrograph routing method incorporating stream-aquifer interaction process, assuming the aquifer flow to be one-dimensional perpendicular to the stream and assuming fully penetrating stream.

c) Development of a stream-aquifer interaction model using a simplified stage hydrograph routing method in the study reach and considering the simultaneous solution of the surface water and groundwater flow equations assuming i) fully penetrating and ii) partially penetrating stream.

d) Development of a stream-aquifer interaction model using a simplified discharge hydrograph routing method in the study reach and considering the simultaneous solution of the surface water and groundwater flow equations assuming i) fully penetrating and ii) partially penetrating stream.

1.3 OBJECTIVES

Taking into consideration the above-stated scope of the study, the objectives of the stream-aquifer interaction study can be stated as follows:

1) Streamflow routing using the Variable Parameter Muskingum Stage-hydrograph (VPMS) method and assuming one-dimensional flow between the aquifer and the stream for fully penetrating stream case.

2) Streamflow routing using the Variable Parameter McCarthy-Muskingum discharge hydrograph (VPMM) method and assuming one-dimensional flow between the aquifer and the stream for fully penetrating stream case.

3) Use of the VPMS and VPMM methods for streamflow routing and assuming two-dimensional flow in the aquifer for fully penetrating stream case.

4) Use of the VPMS and VPMM methods for streamflow routing and assuming two-dimensional flow in the aquifer for partially penetrating stream case.

1.4 ORGANIZATION OF THE THESIS

The thesis has been organized as follows:

In Chapter 1, a general introduction to the stream-aquifer interaction and flood routing approaches has been presented with an emphasis on the application of VPMS and VPMM methods for routing floods. The objectives of the study have been identified.

In Chapter 2, a general review of the literature about flood routing and stream-aquifer interaction processes has been presented. It includes, the subject matter on the description of various flood routing methods, including the VPMS and VPMM methods applied in the study. It also presents the explicit finite difference solution and analytical solution of the Boussinesq equation.

In Chapter 3, the extension of VPMM method has been presented considering stream-aquifer interaction. Also, the solution procedures have been described for the stream-aquifer interaction process. Also, the Simpson's (3/8)- rule has been explained for evaluating the lateral flow where the hydraulic heads have been obtained using the explicit finite difference solution of the Boussinesq equation and the aquifer flow has been considered as one-dimensional, assuming the channel is fully penetrating the aquifer. The solutions obtained from the routing procedure have been verified using the existing solution. The methodology has been verified for the field data of Sabie River, South Africa.

In Chapter 4, the extension of the VPMS method has been presented considering stream-aquifer interaction. The methodolgy has been applied on the hypothetical stage hydrogrpah used by Zitta and Wiggert (1971) and the solutions obtained have been verified using the existing solution. Also, the methodology has been verified for the field data of Sabie River, South Africa.

In Chapter 5, dealing with channel discharge and stage routing, a methodology for evaluating the lateral flow using Darcy's law has been presented where the hydraulic heads have been obtained using the mass balance approach on each grid in the direction of aquifer flow. The channel has been considered as fully penetrating and aquifer flow has been considered as two-dimensional. The volume storage obtained using presented methodology has been compared with the one obtained using the analytical solution by Morel-Seytoux (Mishra, unpublished book) for the case of step-rise input. Also, the methodology has been verified for the field application using the discharge data of Sabie River, South Africa.

In Chapter 6, the reach transmissivity has been estimated for the rectangular channel using the Aravin and Numerov approach (1965) for the case of partially penetrating streams. The obtained reach-transmissivity per hydraulic conductivity has been compared with the reach transmissivity obtained from the Morel-Seyotux approach (Morel-Seytoux et al., 1979) and the Aravin and Numerov approach (1965). Using this reach transmissivity, the lateral flow has been obtained for the partially penetrating channels. The methodology has been applied on the hypothetical stage hydrograph. Also, this methodology has been applied by studying this problem of Platte River, Nebraska (USA) and compared with the monitored data.

CHAPTER 2

REVIEW OF LITERATURE

2.1 GENERAL

Floods can be sometimes menacing which can result in loss of human lives and property and several methods have been developed for predicting the flood wave movement along the river and improve the water management of natural rivers. The changes in a flood wave as it propagates through a river channel downstream can be predicted with the help of a technique known as flood routing. Applications of flood routing methods include problems such as flood forecasting, rating curve development, peak flow estimation, dam-breach analysis, watershed dynamics modeling, flood insurance, and floodplain zoning studies (USACE, 1994; Henderson, 1996). The salient features of a flood wave are its shape, magnitude, and celerity. The magnitudes of the salient features change gradually as the flood wave moves downstream through a river channel. Flood wave attenuation and translation are two main primary concerns while predicting the changes during flood movement along the river. The relative decrease in the magnitude of the peak discharge is termed as attenuation, whereas translation is the delay in time of peak discharge, determined based on the travel time of flood wave downstream.

The investigations of the mathematical techniques to predict the wave propagation in the river channel have been continually developed since the early of the seventeenth century. The fundamental theory for one-dimensional analysis of flood wave propagation was originally developed by Barre-de Saint-Venant in 1871. The theoretical equation proposed by Saint-Venant for the analysis of flood wave propagation was based on the equations of continuity and momentum balance. However, due to the mathematical complexities in solving those theoretical equations proposed by Saint-Venant, simplifications were necessary in order to obtain the feasible solutions of the salient properties of the flood wave. With the advent of computers in the middle of the twentieth century, it was possible to obtain the complete solution of SVE.

The dynamics of the unsteady flow in rivers or channels is adequately described by the SVE based full-scale dynamic wave models. But the methods demand more input information and also require the data to represent correctly the main channel's resistance and geometry characteristics of the main channel in shorter spatial intervals along the routing reach (Rutschmann and Hager, 1966). Obtaining the cross-sectional information at a closer spatial interval is highly expensive. This high cost is sometimes prohibitive to carry out such exercises.

Thus, its application in engineering practice was hampered. In such cases, the use of simplified momentum equation was inevitable (Ferrick, 1985). Because of the successful application to the simplified flood routing methods in the British rivers, the British Flood Studies Report (NERC, 1975) emphasized the need to focus the future research on the advancement of simplified methods for flood routing. Hence, the revisit of the flood routing which are more simplified has been discussed in this Chapter.

Fread (1981) broadly classified the flood routing method as; 1) purely emotional techniques, 2) linearized version of the SVE, 3) hydrologic (storage) routing technique based on mass conservation and an approximation of the relationship between flow and storage, and 4) hydraulic routing based on the concept of mass conservation and simplified form of the conservative momentum equation. A brief discussion of the flood routing methods is given in the subsequent sections.

2.2 CLASSIFICATION OF FLOOD ROUTING METHODS

2.2.1 Empirical Methods

Empirical models are basically the interpolation formulae based on the past observations of a flood wave. The lag model can be categorized as one of the empirical models where the term 'lag' is defined as the time difference between the inflow and outflow for a routing reach. In practice, this method has been applied to long reaches for routing. Tatum (1940) has developed the successive average-lag model, assuming that at some point downstream, the average flow is 0.5 (I_1+I_2) at times t_1 and t_2 respectively. In this process, Tatum (1940) approximated that the number of successive averages occurring within the reach is equal to the wave travel time by the reach length. Thus, the outflow computed at the end of reach is given by

$$O_{n+1} = C_1 I_1 + C_2 I_2 + \ldots + C_1 I_{n+1} \quad (2.1)$$

where, n denotes the number of sub-reaches and the coefficients used in the method can be calculated using a trial and error approach for the observed inflow and outflow hydrographs.

Power law equations: Leopold et al. (1953) advocated the power law equation, which computes the depths of water in a stream using the following equations for a stream section, that relates stream width and depth as

$$y = c_1 Q^m \quad (2.2)$$

$$W = aQ^b \quad (2.3)$$

where, y is the stream depth, W is the stream width, c_1 and a are the empirical coefficients and m and b are the empirical exponents which can be obtained using the regression methods.

Gage relation: Linsely et al. (1949) developed the relationship between the flows of the upstream station to that at a downstream point. Gage relation is based either on stream depth, or flow or on both. The advantage of establishing this empirical relationship between the gages is that the lateral flow (in / out) the reach is automatically inherited in that relation.

2.2.2 Linearized Models

The complexity involved in solving the SVE has encouraged the scientists and engineers to simplify the equations required for obtaining the amenable solutions. Since different discharges travel at different celerity, flood waves are inherently nonlinear in nature. Linear flood routing methods can model the nonlinear behavior of flood either totally ignoring the least important nonlinear terms or linearizing the nonlinear terms of the Saint-Venant's momentum equation. Linear system theory has also been used to develop routing techniques (Dooge, 1973). Linear model is generally applicable to the higher flows of interest (Kulundaiswamy and Subramanian, 1967; Bates and Piligrim, 1983). Moreover, the linear models perform better than the nonlinear models due to the reason that they do not allow magnifying the input errors (Singh and Woolhiser, 1976). Therefore, the linearized models may be sub-divided as; 1) simple impulse response model, 2) complete linearized model and 3) multiple linearized model.

Simple impulse response model: The impulse response approach is based on the composition of linear reservoirs and linear channels arranged in series or parallel or their combinations. The linear system is uniquely characterized by its unit impulse response. By knowing the unit impulse response, all the system outputs may be determined for all the inputs. The input-output relationship is shown with the help of the convolution integral

$$O(t) = \int_0^t I(t)H(t-\tau')d\tau' \qquad (2.4)$$

where, $O(t)$ is the outflow, $I(t)$ is the inflow and $H(t-\tau')$ is the unit impulse response. It is also related to the 'Lag method' and the approach is analogous to the unit hydrograph used by the hydrologists to model the rainfall-runoff transformation process.

Complete linearized model: Lighthill and Whitham (1955) developed the first linearized model was and was followed by Harley (1967). This model is similar to the diffusion analogy model developed by Hayami (Chow, 1959). However, the only difference is that it does not overattenuate the flood wave similar to the diffusion analogy model. The accuracy of the model is dependent on the reference flow considered for the routing.

2.3 HYDROLOGICAL ROUTING METHODS

The exact description of one-dimensional flow in the channel was proposed by Saint-Venant (de Saint-Venant, 1871), but their solution procedure requires a high degree of computational facilities. The engineers and hydrologists could not attempt to solve these equations till the mid-fifties of the twentieth century due to non-availability of computational facilities. But, flood routing is an essential component in the planning of hydrological projects. Therefore, a class of methods which is simple for field applications with minimal data requirement was introduced. Later, these methods came to be known as the hydrological methods, wherein the channel reach can be considered as a lumped system which involves continuity equation and storage equation. In hydrological routing, the flow can be computed only as a function of time along the channel. The data requirements for these methods are the flood events from the past at both the channel sections. These data can also be applicable even when the roughness and topographical characteristics of the channel sub-sections are not known.

Based on the storage equation used in the model framework, the hydrologic routing methods can be classified as linear and nonlinear. The flood wave propagation dynamics in a natural channel is characterized by, namely, uniform translation and reservoir action (Langbien, 1942). These two characteristics of flood wave propagation are modeled by the SVE in a distributed manner, wherein the hydrological methods do it in a lumped manner. The classical Muskingum method (in linear form) (McCarthy, 1938) has been extensively applied for river flood routing that employs two routing parameters. These are the travel time, which accounts for the convection flood wave dynamics, and the weighting parameter, which accounts for flood wave diffusion wave characteristics.

However, in most of the natural river systems, a nonlinear relationship between the discharge and storage exists, i.e., all the discharges of different magnitudes in a flood wave travel at different wave speeds and, therefore, the use of the linear models from a theoretical perspective is inappropriate. To overcome this deficiency, methods which use a nonlinear storage equation were proposed by Rockwood (1958), Laurenson (1962), and Mein et al. (1974). It is pertinent to mention here that the parameter estimation for the storage equations using only the inflow, outflow and the corresponding channel storage information pertaining to the particular flood event. However, these do not directly involve the channel characteristics such as geometry and roughness. Thus, hydrological routing methods are only applicable those flood events which are in the range of the events used in the parameters calibration. Therefore, simplified wave models can provide a moderately good characterization for the flood wave propagation under a number of circumstances (Ponce et al., 1978; Tsai, 2003), provided the physical parameters are valid under particular ranges. This led to the inference that the storage equation may be considered as a substitute for the Saint-Venant momentum equation (Apollov et al., 1964; Koussis and Osborne, 1986). From hydrodynamics, a lumped nonlinear state model similar to that of the linearized Muskingum model was derived by Napiorkowski, et al. (1981). The physical-based approach used by them returns the functional relationships between the hydrodynamic characteristics and model parameters of the system.

Therefore, the hydrological routing methods may be linked to the hydrodynamics-based methods (Zoppou and O'Neill, 1982; Weinmann and Laurenson, 1979). Similar conclusions have been drawn by Dooge et al. (1982), Dooge (1973), Cunge (1969) and Apollov et al. (1964). The above statements reveal that some of the storage routing methods are derivable as a simplification of the physically based hydrodynamics methods. Thus, the distinction between the hydraulic and hydrologic routing methods should be on the basis of the estimation of routing parameters of the method. In this context, Perumal (1995) has suggested that if the parameters of the storage routing method are estimated using the storage, and the corresponding inflow and outflow information, then it may be categorized as the hydrologic method. Else, if they are estimated using the established relationships based on the channel and flow characteristics, then it may be categorized as the hydraulic routing method or the physically based hydrologic routing method (Kundzewicz, 1986). Some of the hydrologic routing methods are discussed below.

2.3.1 Storage Routing Method

The equation for storage routing is based on the conservation of mass. Specifically, the storage $\left(S(t)\right)$, inflow ($I(t)$), outflow ($O(t)$), are related by

$$I(t) - O(t) = \frac{\Delta S(t)}{\Delta t} \tag{2.5}$$

where, ΔS is the change in storage in Δt time interval. Both I and O are time-varying functions with I and O being the inflow and outflow, respectively. Eq. (2.5) can also be written as

$$I(t)\Delta t - O(t)\Delta t = \Delta S(t) \tag{2.6}$$

If the subscripts '1' and '2' are used to indicate the values at time t and (t+Δt), respectively, then Eq. (2.6) becomes

$$\frac{1}{2}\left(I_1(t)+I_2(t)\right)\Delta t-\frac{1}{2}\left(O_1(t)+O_2(t)\right)\Delta t=S_2(t)-S_1(t) \tag{2.7}$$

While, $I_1(t)$, $I_2(t)$, $O_1(t)$ and $S_1(t)$ are known at any time t, the values for $O_2(t)$ and $S_2(t)$ are unknown. Eq. (2.7) can, therefore, be rearranged such that the on the left side of equal sign known values are placed and the unknown values are on the right side:

$$\frac{1}{2}\left(I_1(t)+I_2(t)\right)\Delta t+\left(S_1(t)-\frac{1}{2}O_1(t)\Delta t\right)=\left(S_2(t)+\frac{1}{2}O_2(t)\Delta t\right) \tag{2.8}$$

Eq. (2.8) represents one equation with two unknowns. To find a solution to the problem, it is necessary to have a second equation. In storage routing, the outflow-storage relationship is the second equation. The outflow is usually assumed only to be a function of the storage. Although this may not be exactly true for the small reservoirs, it is the assumption that usually made. In practice, the outflow-storage relationship is expressed as a stage-storage-discharge relationship. In conclusion, Eq. (2.8) represents the routing equation for routing a hydrograph through a storage facility. Given the stage-storage-discharge relationship for a site and the facility, the outflow hydrograph can be computed using Eq. (2.8).

2.4 HYDRAULIC FLOOD ROUTING METHOD

The motion of a long wave in a river or estuary such as flood wave, tide or storm surge is usually considered as unsteady flow and because of the flow rate, this is described as a distributed process, where the depth (elevation) and velocity differ in space. The flood wave is usually considered as one-dimensional, i.e., the acceleration and velocity components of the wave in the transverse and vertical directions are not considered. Hence, the motion of the wave is described solely in the direction of the longitudinal axis of the river by the one-dimensional differential equation. The estimates of the properties such as depth, velocity, and flow rate in a channel can be achieved by applying the distributed flow routing. Alternatively, the hydraulic flood routing is derived from the complete differential equations of one-dimensional unsteady flow, popularly known as the SVE (Saint-Venant, 1871). These equations consist of; 1) the continuity equations which conserve the mass of the wave

$$\frac{\partial Q}{\partial x}+\frac{\partial A}{\partial t}=0 \tag{2.9}$$

and 2) the equation of motion or dynamic equilibrium which conserves the momentum of the wave

$$S_f=S_o-\frac{\partial y}{\partial x}-\frac{v}{g}\frac{\partial v}{\partial x}-\frac{1}{g}\frac{\partial v}{\partial t} \tag{2.10}$$

where, Q = discharge, A = cross-sectional area, S_f = energy slope, S_o = bed slope, y = flow depth, v = average velocity over cross-section, x = space variable, t = time variable, $\partial y/\partial x$ = longitudinal water surface gradient, $(1/g)$ $(\partial v/\partial t)$ = local acceleration gradient and (v/g) $(\partial v/\partial x)$ = convective acceleration gradient.

The above equations permit the water level and flow rate to be estimated as a function of time and space rather than time only as given in the lumped flow routing method described in Section 2.2.3. The basic assumptions adopted to develop the system (Abott, 1979; Henderson, 1963; Chow, 1959) are given below:

- The flow rate is one-dimensional
- The pressure has a hydrostatic distribution
- The distributed friction losses can be evaluated with the usual uniform flow formula and most account for the concentrated head losses, and
- There are no lateral inflows or outflows.

Within the basic model, the categorization of the river waves may be categorized as kinematic, gravity, diffusion or dynamic waves, which corresponds to momentum equation in various forms (Daluz Vieira, 1983; Bocquillon, 1978; Weinmann and Laurenson, 1974; Dooge and Harley, 1967).

To simplify the Saint-Venant momentum equation, there have been various approaches so that the channel routing becomes more flexible to the solution (Dooge, 1986). The acceleration terms are usually of two orders of magnitude smaller than S_0 and S_f and one or two orders of magnitude less than $\frac{\partial y}{\partial x}$ (Cunge et al., 1980; Kutchment, 1972; Henderson, 1966). This advocates that by dropping these acceleration terms, a good approximation to the solution of the simplified equation may be produced on the basis of the full SVE. For the above reasons, the hydraulic methods may be classified into two major groups; 1) the dynamic wave method on the basis of the solution of the full SVE and 2) the simplified hydraulic methods or physically based hydrologic methods (Sahoo, 2007).

A simple model was developed by Moramarco et al. (2005) which reconstructed the discharge hydrographs at a stage monitoring site in a river where the discharge is recorded at the upstream section. The model was also designed to incorporate the lateral inflow without applying the flood routing method where the rating curve at local site was also not required. A simple methodology was gain developed by Moramarco et al. (2008) for reconstruction of the discharge hydrograph at a river section where stage data is monitored and at another upstream river section the discharge data is monitored with the assumption of negligible lateral inflows. The method was calibrated and validated by comparing the results of a hydrodynamic model (MIKE 11) with the proposed method results.

In the dynamic wave method, the Saint-Venant momentum equation are used in its totality including all the terms. The simplified hydraulic methods are based on the continuity equation, which is either distributed or lumped and a simplified momentum equation arrived at by truncating or approximating or linearizing the pressure and acceleration terms in the full Saint-Venant momentum equation. These simplified methods are useful when there is a high computational demand for solving the full SVE as in case of the hydrological land-surface schemes of the climate change models (Sahoo, 2007). For hydraulic models (Singh, 1996; Li and Singh, 1993), the complete SVE or their simplified representations are usually adopted and the discharge and the river stage are simultaneously forecasted. Jaiswal et al. (2010) addressed two hydrodynamic dispersion problems for n =2 power of the velocity. The Laplace transform technique was used to obtain the solutions for 2D advection-diffusion equations for pulse type point source. The point sources considered were both varying and uniform in nature.

The applicability of the diffusive wave (DW) and the kinematic wave (KW) for steady flow in prismatic channels was investigated by Moramarco et al. (2008). A Lax-Wendorff numerical scheme which is of second-order coupled with characteristic method was used for evaluation at the boundaries. For the solution of the full SVE, the numerical techniques have accomplished great progress. In the numerical techniques, a comprehensive mathematical depiction of the physical model is not only comparatively complex and expensive but is also demanding with respect to its data requirement, e.g., the information about river cross-section, its roughness (Rutschmann and Hager, 1996) and initial as well as boundary conditions. Therefore, its application in the field of engineering is not very convenient. In such case, the use of simplified momentum equation is inevitable (Ferrick, 1985).

Sahoo (2013) developed a time distribution based multilinear discharge-hydrograph routing method. A linear-sub model known as the variable parameter Muskingum-type routing approach was used and chosen as the framework for the proposed methodology.

Swain and Sahoo (2015) revised the previously developed VPMM method to exclusively include the lateral flow which non-uniformly distributed in the routing scheme accounting for stream channel flows.

2.5 VARIABLE PARAMETER MUSKINGUM STAGE-HYDROGRAPH (VPMS) ROUTING METHOD

The VPMS method proposed by Perumal and Ranga Raju (1998a) directly derived from the full governing SVE for unsteady in the channel without the consideration of the lateral flow. As determined for the physically based Muskingum method, the routing parameters change at every time interval concerning to the flow and channel characteristics by the similar form (Perumal, 1994a, 1994b; Dooge et al., 1982; Cunge, 1969; Apollov et al., 1964, Singh et al., 1980). The developed routing equation differs from the routing equation of classical Muskingum method as the stage variable has been used here in place of the discharge variable.

2.5.1 The VPMS method - Concept

The development of VPMS routing method was based on the concepts; 1) Between the stage and the discharge, a unique relationship exists at a prismatic section in a river reach throughout steady flow condition which forms a steady flow rating curve and 2) The similar stage and discharge relationship does not exist at the same location throughout unsteady flow, but the corresponding discharge occurs somewhere downstream from that section in the river reach as hypothesized in the definition sketch (Fig. 2.1). At any given instant of time, a unique stage and corresponding discharge relationship is maintained, recorded at the downstream section (Section 3) in Fig. 2.1) which precedes the corresponding steady stage section.

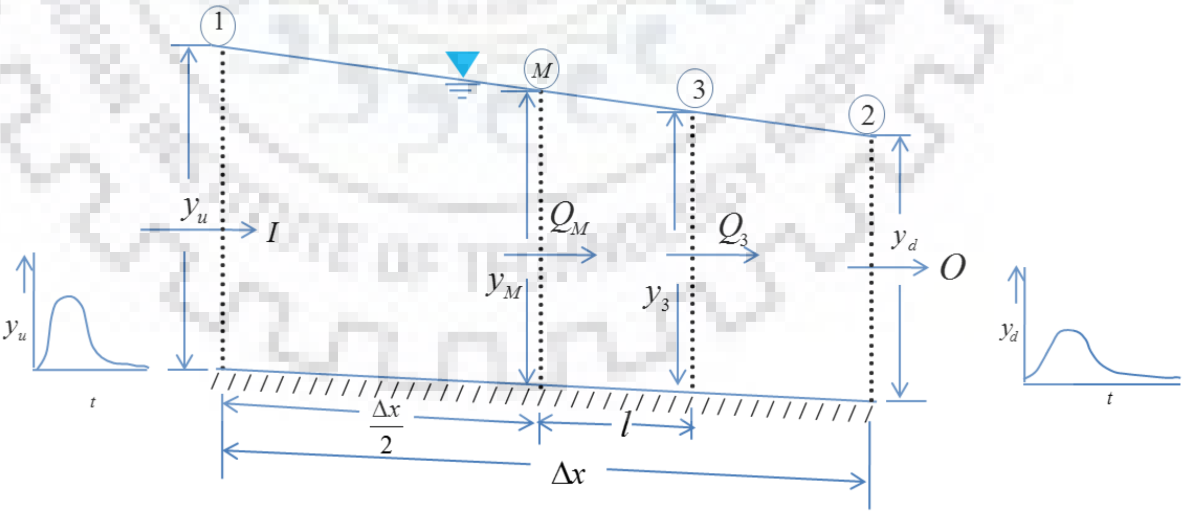

Fig. 2. 1 Definition sketch of the variable parameter Muskingum stage routing (VPMS) method computational reach

2.5.2 Theoretical background of the VPMS method

The expression for one-dimensional full Saint-Venant unsteady flow equation in a river channel without lateral flow consideration are described as

$$\frac{\partial Q}{\partial x}+\frac{\partial A}{\partial t}=0 \qquad (2.9)$$

$$S_f = S_o - \frac{\partial y}{\partial x} - \frac{v}{g}\frac{\partial v}{\partial x} - \frac{1}{g}\frac{\partial v}{\partial t} \qquad (2.10)$$

where, Q = discharge, A = area of cross-section, S_o = the bed slope, S_f = energy slope, g = acceleration due to gravity, v = average velocity, y = depth of flow at the cross-section, and t and x are the time and space variables, respectively.

The assumptions made in the derivation of the method in simplifying the unsteady flow are (i) the channel reach is prismatic and (ii) the longitudinal water surface gradient $\frac{\partial y}{\partial x}$, the local acceleration gradient and the convective acceleration gradient $\frac{v}{g}\frac{\partial v}{\partial x}$ and remain nearly constant at any instant of time for a given reach. Further, this assumption indicates that at any instant of time, the friction slope S_f does not get altered over the computational reach length which in result depicts that the flow depth fluctuates approximately linearly.

Based on the Manning's friction law and above assumptions, the expression for simplified momentum equation can be developed as

$$\frac{\partial Q}{\partial x} = v\left[\frac{\partial A}{\partial y} + mP\frac{\partial R}{\partial y}\right]\left[\frac{\partial y}{\partial x}\right] \qquad (2.11)$$

where, m = 2/3 for the Manning's friction law, P = wetted perimeter and R = hydraulic radius.

Using Eq. (2.11), the flood wave celerity can be expressed as

$$c = \frac{\partial Q}{\partial A} = \left[1 + \left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)\right]v \qquad (2.12)$$

Using Eqs. (2.9) - (2.11) and the discharge expression using the Manning's friction law, S_f can be formulated as

$$S_f = S_o - \left[1 - \frac{1}{S_o}\left\{1 - \left(mF\left(\frac{P\,\partial R/\partial y}{\partial A/\partial y}\right)\right)^2\right\}\right] \tag{2.13}$$

where, $F = \left(\frac{V^2\,\partial A/\partial y}{gA}\right)^{1/2}$ is termed as Froude number. (2.14)

As shown in Fig. 2.2, after applying the continuity equation (Eq. (2.9)) at the center of the numerical grid network, the numerical approximation of the derivatives yields the routing equation as

$$y_{d,j+1} = C_1\, y_{u,j+1} + C_2\, y_{u,j} + C_3\, y_{d,j} \tag{2.15}$$

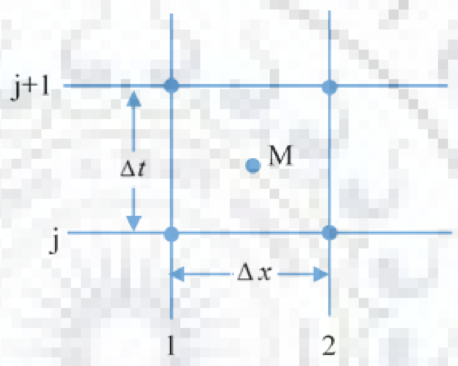

Fig. 2. 2 A computational grid of the VPMS method

The coefficients C_1, C_2 and C_3 can be shown as

$$C_1 = \frac{-K\theta + \Delta t/2}{K(1-\theta) + \Delta t/2} \tag{2.16a}$$

$$C_2 = \frac{K\theta + \Delta t/2}{K(1-\theta) + \Delta t/2} \tag{2.16b}$$

$$C_3 = \frac{K(1-\theta) - \Delta t/2}{K(1-\theta) + \Delta t/2} \tag{2.16c}$$

where,

$$K = \frac{\Delta x}{\left[1 + m\left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)_3\right] v_3} \tag{2.17a}$$

$$\theta = \frac{1}{2} - \frac{Q_3\left[1 - m^2 F_M^2 \left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)_M^2\right]}{2\,S_0 \left.\frac{\partial A}{\partial y}\right|_3 \left[1 + m\left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)_3\right] v_3\ \Delta x} \tag{2.17b}$$

2.6 VARIABLE PARAMETER MCCARTHY-MUSKINGUM (VPMM) METHOD

A physically based Variable Parameter McCarthy-Muskingum method (VPMM) has been proposed by Perumal and Price (2013), under the objectives of the full mass conservation for the routed hydrograph and non-linearity in routing procedure.

2.6.1 Concept of the VPMM method

The approximation of the momentum equation of the SVE was used for development for the VPMM method. The VPMM method has the same form of the routing equation as the classical Muskingum method as proposed by McCarthy in 1938. In this method, it is assumed that for a steady flow having prismatic cross-section shape, the discharge at one point is uniquely related to cross-sectional area of flow at that point. Though in case of the unsteady flow, this stage-discharge relationship at the downstream section at any given instant of time preceding to the corresponding midsection of the routing reach.

2.6.2 The VPMM method- The Theoretical Background

The following assumptions were made in the development of VPMM method: i) prismatic channel reach having consistent cross-section shapes with that of a natural; ii) no lateral inflow or outflow along the reach; and iii) relative to the bottom slope, the slope due to local acceleration, the slope due to convective acceleration and the slope of water surface are small but not negligible. Definition sketch of VPMM method has been shown in Fig. 2.3.

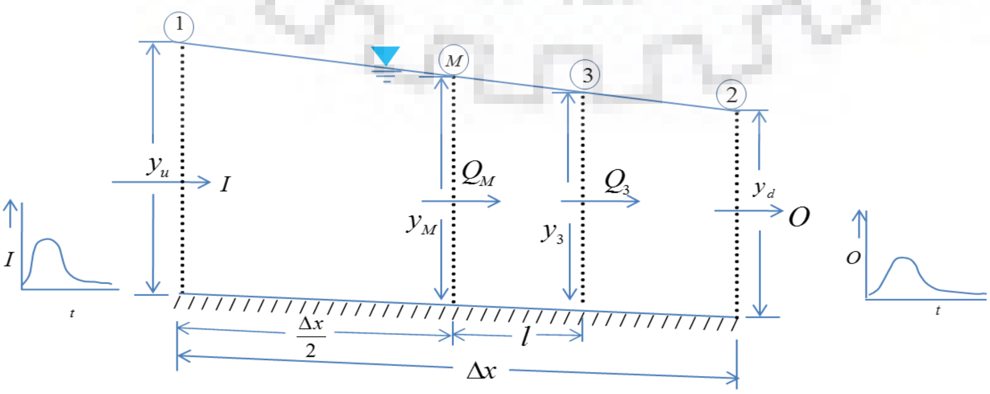

Fig. 2. 3 Definition sketch of the variable parameter McCarthy-Muskingum (VPMM) method

The VPMM method has been derived from the full SVE (Eqs. (2.9) and (2.10)) considering that the flood wave propagates in one dimension in the channels where the stream-aquifer interaction has not been considered. This method, developed on the basis of principles of hydraulics, characterizes the channel storage as prism and wedge storages, as envisioned by McCarthy (1938) in his classical Muskingum method. In the VPMM method, the routing equation is in the same form as in the classical Muskingum method proposed by McCarthy in 1938. Therefore, Perumal and Price (2013) acknowledged the contribution of McCarthy in the development of the renowned Muskingum method by naming their newly developed method as the Variable Parameter McCarthy-Muskingum (VPMM) method.

In the VPMM method, the discharge at the mid-section of any reach using the momentum equation of SVE in approximate form, can be expressed as

$$Q_M = Q_{o,M}\left\{1-\frac{1}{S_o}\frac{\partial y}{\partial x}\left[1-\frac{4}{9}F_M^2\left(\frac{P}{B}\frac{dR}{dy}\right)_M^2\right]\right\}^{\frac{1}{2}} \tag{2.15}$$

where, Q_M = average discharge, $Q_{o,M}$ = normal discharge, P_M = wetted perimeter, B_M = top width and R_M = hydraulic radius corresponding to flow depth Y_M at the mid-section of any reach at any time, S_o = bed slope of the channel and $(\partial y/\partial x)$ = longitudinal water surface gradient.

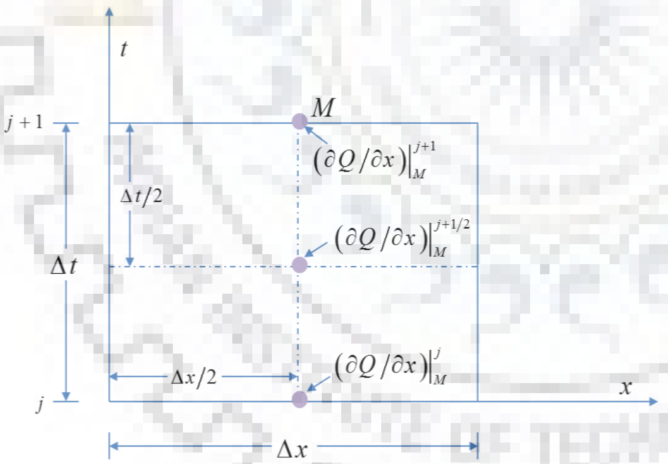

Fig. 2. 4(a) Spatial approximation of

$$\left(\frac{\partial Q}{\partial x}\right)\Bigg|_M^{j+1/2} = 0.5\left[\frac{\partial Q}{\partial x}\Bigg|_M^{j} + \frac{\partial Q}{\partial x}\Bigg|_M^{j+1}\right]$$

Source: Perumal and Price (2013)

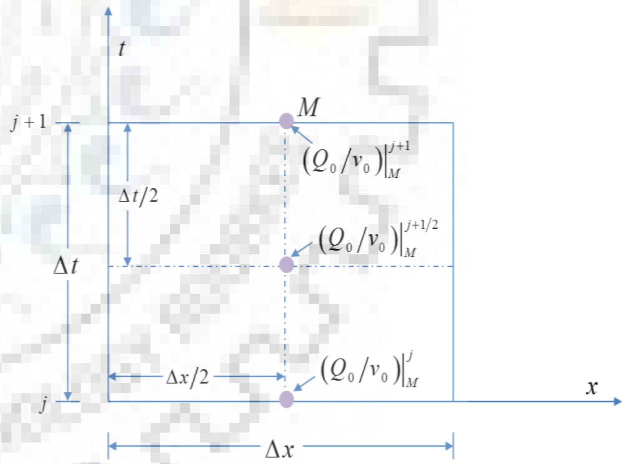

Fig. 2.4 (b). Temporal approximation of

$$\left(\frac{\partial A}{\partial t}\right)\Bigg|_M^{j+1/2} = \left(\frac{\partial\left(Q_0/v_0\right)}{\partial t}\right)\Bigg|_M^{j+1/2} = \left[\frac{Q_0}{v_0}\Bigg|_M^{j+1} - \frac{Q_0}{v_0}\Bigg|_M^{j}\right]\Bigg/\Delta t$$

Source: Perumal and Price (2013)

On applying the continuity equation at the center point 'M' shown in Figs. 2.4(a) and 2.4(b), the routing equation can be written as

$$O_{j+1}=\frac{\Delta t-2.K_{j+1}.\theta_{j+1}}{\Delta t+2.K_{j+1}.\left(1-\theta_{j+1}\right)}.I_{j+1}+\frac{\Delta t+2.K_{j}.\theta_{j}}{\Delta t+2.K_{j+1}.\left(1-\theta_{j+1}\right)}.I_{j}+\frac{-\Delta t+2.K_{j}.\left(1-\theta_{j}\right)}{\Delta t+2.K_{j+1}.\left(1-\theta_{j+1}\right)}.O_{j} \tag{2.16}$$

where, j in subscript indicates the time at t and $j+1$ indicates the time at $t+\Delta t$ (Δt is the routing time interval).

At time level$\left(j+1\right)$, the travel time and weighting coefficient represented by K and θ respectively can be estimated by the following equations

$$K_{j+1}=\frac{\Delta x}{V_{0M,j+1}} \tag{2.17}$$

$$\theta_{j+1}=0.5-\frac{Q_{0M,j+1}}{2.S_0.B_{M,j+1}.c_{0M,j+1}.\Delta x} \tag{2.18}$$

where, the subscript $0M$ indicates that the variables calculated at the mid-section of the sub-reach corresponds to the normal discharge and V is the flow velocity.

The discharge $Q_{0M,j+1}$ can be estimated by the following equation

$$Q_{0M,j+1}=\theta_{j+1}I_{j+1}+(1-\theta_{j+1})O_{j+1} \tag{2.19}$$

According to Perumal et al. (2017) the VPMM method can be considered as a quasi-linear method.

2.7 EXPLICIT FINITE DIFFERENCE METHOD FOR SOLVING THE TRANSIENT FLOW (WANG AND ANDERSON, 1995)

Consider the two-dimensional transient flow equation

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}=\frac{S}{T}\frac{\partial h}{\partial t}-\frac{R\left(x,y,t\right)}{T} \tag{2.20}$$

where, $h=$ hydraulic head $\left(m\right)$, $S=$ storage coefficient of the aquifer (dimensionless), $T=KD=$ transmissivity of the aquifer $\left(m^2/\mathrm{s}\right)$, $D=$ thickness of the aquifer $\left(m\right)$, $K=$ hydraulic conductivity of the aquifer $\left(m/\mathrm{s}\right)$, $t=$ time parameter, and x and y are the spatial parameters.

Using explicit finite difference approximation, Eq. (2.20) can be written as:

$$\frac{h^n_{i+1,j}-2h^n_{i,j}+h^n_{i-1,j}}{\left(\Delta x^2\right)}+\frac{h^n_{i,j+1}-2h^n_{i,j}+h^n_{i,j-1}}{\left(\Delta y^2\right)}=\frac{S}{T}\frac{h^{n+1}_{i,j}-h^n_{i,j}}{\left(\Delta t\right)}-\frac{R^n_{i,j}}{T} \tag{2.21}$$

where, the sub-scripts i and j are the indices in the direction of the spatial variables x and y, respectively. The previous time step at which the hydraulic heads are denoted by superscript n the previous time step at which the hydraulic heads are known for the space derivatives and $n+1$ represents the current time-step for which the value of hydraulic head is to be evaluated.

2.8 THE SOLUTION FOR A FULLY PENETRATING STREAM WITH VERTICAL BANKS AND AN INFINITE AQUIFER

The governing partial differential equation for one-dimensional flow is given by,

$$\frac{\partial^2 h}{\partial x^2} = \frac{\phi}{T}\frac{\partial h}{\partial t} \tag{2.22}$$

and the initial and boundary conditions are:

$$h(x,0) = 0 \tag{2.23a}$$

$$h(0,t) = \sigma \tag{2.23b}$$

$$h(\infty,t) = 0 \tag{2.23c}$$

where various notations are as defined for Eq. (2.20).

The solution to the Eq. (2.21), satisfying the prescribed initial and boundary conditions has been given by Carslaw and Jaeger (1965):

$$h(x,t) = \sigma\left\{1 - erf\left(\frac{x}{\sqrt{4\beta\, t}}\right)\right\} \tag{2.24}$$

where, $\beta = \frac{T}{\phi} =$ hydraulic diffusivity$\left(m^2/s\right)$,

$erf(X) = \frac{2}{\sqrt{\pi}}\int_0^X e^{-v^2}\,dv$, v is a dummy variable

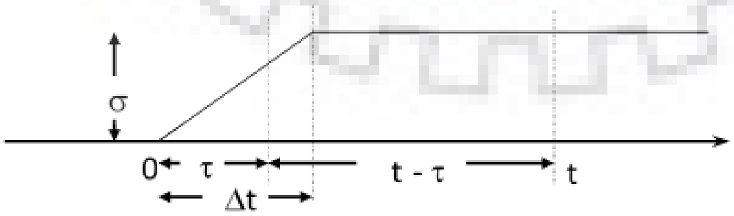

Fig. 2. 5 One ramp rise

A stream and a confined aquifer establish a linear system. Its response to a ramp perturbation in stream stage can be derived from the response of the system to a unit step perturbation in stream stage using convolution technique. A description of a ramp perturbation is shown in Fig.2.5. The stream stage rises linearly from zero at $t = 0$ and attains a unit height at $t = \Delta t$ after which it remains unchanged.

The volume of recharge R_{vr} up to time *t* corresponding to a unit ramp rise in Δt time, has been derived by Morel –Seytoux (Mishra, unpublished book) using response to unit step rise

$$R_{vr}\left(t,\Delta t\right) = \int_{0}^{\Delta t} \frac{1}{\Delta t} 2\sqrt{\frac{T\varphi}{\pi}}\sqrt{t-\tau}\,d\tau = \frac{2}{\Delta t}\sqrt{\frac{T\varphi}{\pi}}\frac{2}{3}\left\{-\left(t-\tau\right)^{3/2}\right\}_{0}^{\Delta t} = \frac{4}{3\Delta t}\sqrt{\frac{T\varphi}{\pi}}\left\{t^{3/2}-\left(t-\Delta t\right)^{3/2}\right\} \quad (2.25)$$

Using $t = m\Delta t$, where m is an integer, in Eq. (2.25), we get

$$R_{vr}\left(m,\Delta t\right) = \frac{4}{3}\sqrt{\frac{T\Delta t\phi}{\pi}}\left\{m^{3/2}-\left(m-1\right)^{3/2}\right\} \quad (2.26)$$

The volume of aquifer recharge from the unit length of the stream to the aquifer is given as

$$V_r\left(t\right) = 2\sigma\sqrt{\frac{T\varphi}{\pi}}\sqrt{t} \quad (2.27)$$

The volume of flow that has entered the aquifer by the end of $n\Delta t$ can be computed using a convolution technique that can be described as

$$V\left(n\Delta t\right) = \int_{0}^{n\Delta t} \frac{d\sigma}{d\tau} 2\sqrt{\frac{T\varphi}{\pi}}\sqrt{t-\tau}\,d\tau \quad (2.28)$$

On substituting the dummy variable τ by $\zeta + \left(\gamma - 1\right)\Delta t$ and after some simplifications, the expression for volume is obtained as

$$V\left(n\Delta t\right) = \sum_{\gamma=1}^{n}\left[\left\{\sigma\left(\gamma\right)-\sigma\left(\gamma-1\right)\right\}R_{vr}\left\{\left(n-\gamma+1\right),\Delta t\right\}\right] \quad (2.29)$$

2.9 STREAM-AQUIFER INTERACTION PROCESS

The most elementary understanding of stream-aquifer interaction can be interpreted by the flux direction between a surface water body and the underlying aquifer. Stream-Aquifer Interaction is an interesting field, which has not been given importance by most of the Hydrogeologists. It has been found to be a matter of great concern in India and other parts of the World as an estimation of the water budget is essential for planning. Ray (2008) in his article discussed the potential of riverbank filtration (RBF) to supply water to a number of cities worldwide. He iterated that a number of cities supply poor quality water and inclusion of riverbank filtration gradually into the water supply system can improve the overall quality of the supplied water. Anuraga et al. (2006) proposed a methodology incorporating soil–water–atmosphere–plant (SWAP) model for some data provided by government agencies and integrated their method with geographical information system (GIS) to evaluate the influence of land use and soil on groundwater budgets at the sub-watershed scale. The study revealed that for Bethamangala sub-watershed, soil type affected the groundwater recharge more compared to land use. Gaur et al. (2011) developed a groundwater evaluation methodology by combining both numerical model and spatial model using GIS. They applied this newly developed approach on the Banganga River sub-basin in India. At first, the groundwater potential zones were delineated using spatial modelling. Further, groundwater flow model for the study area was developed using the numerical model MODFLOW and then the results were superimposed with the spatial modelling ones and were validated. The location of new rainwater harvesting structures were decided based on these results.

Hantush et al. (2002) addressed the transport and fate of agricultural pollutants with the help of proposed mass fraction models. The developed models were based on semi-infinite domain describing various process like leaching, adsorption, vapour loss through soil and soil degradation. The results showed that for immobile-water-regions the diffusive mass transfer (lateral) significantly affects processes like volatilization, leeching and degradation losses and in case of mobile-water regions the advection process impacts the process.

Hantush et al. (2011) presented fundamental equations and their numerical and analytical solutions for the stream-aquifer interaction processes in riparian aquifer environments and hillslope. They presented the derivation of physically-based models for sub-surface flow from the renowned Boussinesq equations and their connection with the conversant hyperbolic and exponential empirical recession curves.

Bharati et al. (2017) derived an analytical solution of 1D advection-dispersion equation (ADE) for solute transport for any permissible n values. In case of groundwater, the dispersion coefficient for solute transport is considered as proportional to the nth power of groundwater velocity. The analytical solution for a non-homogenous medium were obtained at n=1,1.5 and 2.0 considering the groundwater velocity as a linear function of space.

Sanskrityayn et al. (2016) solved analytically 1D advection diffusion equation (ADE) with variable coefficients using the Green's Function Method (GFM). The mass pollutant transport in a heterogeneous medium is described by ADE and originates from the instantaneous source. The variable coefficients of the ADE were reduced to constant ones using a newly developed moving coordinate transformation approach.

Sanskrityayn et al. (2017) in their technical note addressed the two most commonly used to obtain the solution of ADE with time-dependence coefficients which describes the transportation of solute due to a continuous source in semi-infinite and infinite porous media.

Morbidelli et al. (2018) summarized the salient features and also provided credible reasons for discrepancies from the previous studies by various authors regarding the role of slope on infiltration. They also offered creative suggestions for future studies for enhancing the knowledge of surface slope and its effect on sloping surfaces.

Any groundwater development should be undertaken only after a careful study, which includes an analysis of the probable influence of the aquifer surrounding any potential zones. This analysis should be based on a thorough understanding of groundwater movement, surface water flow and the relationship between them. Carefully planned groundwater development can result in a more efficient and beneficial utilization of available water resources (*Rovey*, 1975). He adopted an integrated systematic hydrological approach to model the domain, which in turn can be used in several other stream-aquifer systems. In the subsequent sub-sections, a brief theoretical background of the work done on the stream-aquifer interaction process for the cases of fully and partially penetrating stream is given.

2.9.1 The interaction between fully penetrating stream and aquifer

Stream and aquifer interaction (fully penetrating) received the attention of the investigators in early fifties. Todd (1955) highlighted the groundwater flow in relation to a flooding stream. He studied the stream-aquifer interaction with the help of experiments on a Hale-Shaw model and analyzed the bank storage volume and magnitude and time distribution of groundwater flow. Todd (1955) observed that the large changes in the magnitude and direction of groundwater movement are brought about during the occurrence of the flood wave in a river.

A generalized flood wave in a stream was solved by Cooper and Rorabaugh (1963), which fully penetrates an isotropic and homogeneous aquifer. They proposed a generalized form of flood waves in the stream and derived expressions for the piezometric head in finite as well as infinite confined aquifers, discharge from the stream at any section and bank storage volumes when a generalized flood wave passes in the stream.

Hornberger et al. (1970) have given a numerical solution of the Boussinesq equation and analyzed the groundwater flow due to stream stage changes. Verma and Brutsaert (1970) developed a numerical scheme to analyze the flow in a two-dimensional unconfined aquifer considering water table fall and quantified the rate of outflow into an adjoining fully penetrating stream.

Pinder and Sauer (1971) studied the flood wave modifications as a result of bank storage using a numerical solution of coupled dynamic equations describing the 1D open channel flow and the 2D transient groundwater flow simultaneously. They indicated that the flood wave may be modified considerably by bank storage. The degree of modification is influenced by the hydraulic conductivity of the aquifer.

Swamee et al. (2000) acquired an algebraic approximation to the integral present in Theis solution (1941).

Choudhary and Chahar (2007) obtained an exact analytical solution for the seepage/recharge from rectangular channel array using the Schwartz-Christoffel transformation and an inverse hydrograph. The obtained solution was useful in quantifying seepage loss and/or artificial recharge from rectangular channels array.

Kim et al. (2008) performed experiments to evaluate head and flow variations along perforated screens using sand tanks with varying filter parameters. A mathematical model was also developed for axial flow velocity distribution predictions. Results showed that the rate of production increased with increase in well diameter, drawdown and lateral length.

Majumdar et al. (2013) determined both the recharge rates and rising heads in the confined aquifer which were not possible in case of slug test solutions by developing semi-analytical equations for confined aquifers. The newly developed equations include well storage as a function of aquifer diffusiveness for a fully penetration aquifer. The friction parameter k was more suitable for head loss, which is a function of the well-known Reynolds number.

2.9.2 Interaction of semi-pervious/partially penetrating stream and aquifer

Hantush (1965) proposed a conceptual model for the study of interaction among a partially penetrating stream, an aquifer, and a well. He introduced the concept of retardation coefficient and gave the analytical solution for drawdowns due to pumping a well in the vicinity of a semi-pervious stream. He defined the retardation coefficient as the aquifer's effective length essential to cause the equivalent head loss similar to the semi-pervious stream bank. The derivation assumed that there is no storage in the semi-pervious layer.

Hall and Moench (1972) studied the stream-aquifer interaction with and without semi-pervious stream banks in finite and semi-infinite aquifers. Equations for the instantaneous unit response function and unit step response function are given for each case. The authors used the approach of retardation coefficient to model the semi-pervious stream.

Marino (1973) developed expressions which were analytical in nature describing the piezometric head fluctuations in a system containing semi-pervious stream and unconfined aquifer. The storage capacity of the stream-bed has been neglected. He solved the governing differential equation using the Laplace transformation method. The solution gives the profits of the piezometric head when the level in the semi-pervious stream is abruptly lowered or raised to a new height and maintained persistent afterwards. He has obtained the expressions for the water table fluctuations when the stream bank has same hydraulic conductivity as that of the aquifer. The expressions are valid when the decline or rise of the water table is lesser than 50% of the initial depth of saturation and they have been obtained in terms of averaged head over the saturation depth.

A feature, indirectly relevant to the stream-aquifer interaction is the concept of additional resistance which may be needed to represent the deformation of the stream-lines in the vicinity of the stream. Streltsova (1974) described the principle of the additional seepage resistance method. However, the method was not exclusively used to represent stream-aquifer interaction.

A discrete kernel approach was proposed by Morel-Seytoux and Daly (1975) to study stream-aquifer interactions for the time-varying stage in a partially penetrating stream. They assumed that the recharge from a reach at any time is directly proportional to the difference between the stream stage and head in the aquifer in the vicinity of the stream. The stream was idealized as a reach, each reaches acting either as a recharge well or a discharge well depending upon whether the reach is influent or effluent. Morel-Seytoux (1975) presented an efficient and accurate hydraulic model of the interaction between a river and an alluvial aquifer as presented by Morel-Seytoux and Daly (1975) centered on the discrete kernel approach.

Halek and Svec (1979) have shown that the effect of the resistance induced by the curvature of the streamlines as well as by the semi-perviousness of the stream-bed can be accounted for by means of ‘substitute length’. In the range of this length, a pressure flow without storage of water is expected. A partially penetrating semi-pervious stream poses the boundary condition of the third kind (Fourier’s condition). The linearity of the governing differential equation and the nature of the boundary condition of the third kind suggest the possibility of superposition of suitably formulated solutions upon one another for time-varying boundary perturbation. If a flood wave passes in the stream, the time distribution of piezometric head in the aquifer can be obtained using superposition of discretized step inputs.

Mishra and Seth (1988) studied the recharge to a shallow water table aquifer from a river of large width. Under steady state condition, the unconfined flow problem was solved using

Zhukovsky's function and Schwarz-Christoffel conformal mapping (Harr, 1962), where the distance between the observation well and the river bank is more than half the saturated thickness of the aquifer below the river bed. The reach transmissivity constant is independent of the drawdown at the observation well. Knowing the recharge for a given position of the water table at a point near the river, the corresponding reach transmissivity can be obtained. The reach transmissivity values are also found to be dependent on the river stage.

Shen and Sun (1987) used the R-C network to estimate the vertical resistance of the soil layer on the bed of the partially penetrating stream overlying the aquifer. They also emphasized the development aspects of R-C networks.

Chin (1991) studied the clogged channels leakage partially penetrating surficial aquifers. In this study, he proposed a general formulation to describe leakage from finite width channels with symmetric drawdowns on either side where the perimeter of the channel is logged.

Sekhar et al. (1992) estimated parameter using the weighted least squares approach in aquifer-aquitard system. Six parameters namely: leakage coefficient, storage coefficient of the aquifer, specific yield of the aquitard, specific storage, equivalent transmissivity and the degree of anisotropy were evaluated by them. An iterative numerical procedure was used for solving the equations and the optimisation was done using sensitivity analysis.

Sekhar et al. (1994) estimated parameters for a leaky aquifer system which is anisotropic in nature and also its principal axes are not known and have a declining water table. Seven parameters governing the principal axes direction were identified and a more computationally efficient modified parameter perturbation technique was used for sensitivity coefficients determination.

Genereux and Guardiaro (1998) conducted a canal drawdown experiment to estimate aquifer parameters. They also compared reach transmissivity using several approaches. The reach transmissivity was found location dependent. The authors also observed the deposition of fine sediments in the vicinity of the recharge boundary and emphasized to consider their role in studying groundwater exchange with the recharge boundary.

Hantush et al., (2000) developed analytical solutions describing stream-aquifer interactions considering the bank-storage impact on channel discharges. The results indicated that an increase in hydraulic conductivity of the river banks reduces the stream impulse response functions. Whereas, increase in aquifer conductivity reduces the unit impulse response functions.

The routed stream flows showed attenuated peaks and an extended tail with higher values of hydraulic conductivity.

2.10 MODEL EFFICIENCY CRITERIA

2.10.1 Nash-Sutcliffe efficiency (NSE)

The model efficiency in the reproduction of the runoff hydrograph was assessed by Nash-Sutcliffe efficiency (NSE) (Nash and Sutcliffe, 1970), which can be expressed as:

$$NSE = \left[1 - \left(\frac{\sum_{i=1}^{N} \left(Q_{obs} - Q_{sim} \right)^2}{\sum_{i=1}^{N} \left(Q_{obs} - \overline{Q}_{obs} \right)^2} \right) \right] \times 100 \qquad (2.30)$$

where, Q_{obs} = observed values of streamflow

Q_{sim} = simulated values of streamflow

$\overline{Q}_{obs}$ = average of Q_{obs} values over N number of data point

2.10.2 Root-mean-square error (RMSE)

The accuracy of the method has also been assessed by Root-mean-square error (RMSE), which can be expressed as (Reusser et al., 2009):

$$RMSE = \sqrt{\frac{1}{N} \left(\sum_{i=1}^{N} \left(Q_{obs} - Q_{sim} \right)^2 \right)} \qquad (2.31)$$

2.11 CONCLUSIONS

From the review of literature of the subjects, it is observed that though the stream-aquifer interaction problem has been studied by many investigators, but not many (e.g., Hantush 1975, Halek and Svec 1979, Hall and Moench 1972, Marino 1973, Mishra and Seth 1988, Morel-Seytoux and Daly 1975, Sekhar et al. 1994, Shen and Sun 1987) have analyzed the interaction of a partially penetrating stream and an aquifer. In other studies (e.g., Hornberger et al. 1970, Pinder and Sauer 1971, Verma and Brutsaert 1970, Kim et al. 2008) stream has been assumed to completely penetrate the entire depth of aquifer. The VPMM and VPMS methods among the number of available approaches which physically represents the Muskingum method, was found to be most suitable because of their ability to explain more features of the Muskingum method compared to all the other available approaches. The VPMM (Perumal and Price 2013) and VPMS

(Perumal and Ranga Raju 1998a) methods have been developed using the numerical solution of St.Venant equations. In the light of the above mentioned studies, the scope of investigating the variable parameter flood routing methods for stream-aquifer interaction study exists and the same has been explored herein. In the present thesis, flood propagation analysis has been carried out considering fully penetrating as well as partially penetrating streams.

CHAPTER 3

VARIABLE PARAMETER McCARTHY-MUSKINGUM METHOD CONSIDERING STREAM-AQUIFER INTERACTION

3.1 GENERAL

Stream-aquifer interaction process plays an important role in modulating flood wave propagation in a channel. The use of a realistic flood propagation model would enable the appropriate modelling of both the stream-aquifer interaction and flood propagation processes. Propagation of flood wave in a stream considering stream-aquifer interaction has been investigated using linear and nonlinear Muskingum methods (Gill, 1978; Tung, 1985; Yoon and Padmanabhan, 1993; Kim et al., 2001). Recently improved Muskingum methods, known as the Variable Parameter Muskingum methods have been studied (Todini (2007), Price (2009), and Perumal and Price (2013)) for flood routing in channels, using discharge as the operating variable, and by Perumal and Ranga Raju (1998) using stage or flow depth as the operating variable. These improved methods have been demonstrated to serve their purpose much better than the linear and nonlinear Muskingum methods. However, no research has been undertaken to study the stream-aquifer interaction process that arises during the passage of a flood wave using the Variable parameter Muskingum method.

In this chapter, the stream-aquifer interaction process has been integrated with channel routing process based on the VPMM method proposed by Perumal and Price (2013) with the enhanced capability of accounting lateral flow in the reach. The routed discharge hydrograph accounting for lateral flow is compared first, for the hypothetical cases of flood wave movement in hypothetical channels, and subsequently verified using recorded flood hydrograph in rivers. The routing algorithm developed herein has been applied for routing the input discharge hydrograph in a reach of Sabie River, South Africa wherein the stream-aquifer interaction process exists. Besides, the VPMM estimated bank storage hydrograph is compared with the corresponding monitored bank-storage hydrograph, at the end of the routing reach.

3.2 GOVERNING EQUATIONS

Though a realistic flood propagation study in a river reach requires the use of equations governing the flood wave propagation in two-dimensions, but considering the model simplicity and data availability limitations for two-dimensional modeling, it is proposed in this study to use one-dimensional governing equations for stream flow routing, but two-dimensional flow modelling in the aquifer. However, the equations governing flood wave propagation in channels should consider the presence of lateral outflow and inflow in the channel or river reach to consider river-aquifer interaction process in the study reach. It is assumed that this interaction process takes place perpendicular to the river banks at the interface of the stream and aquifer, and this assumption enables to modify only the continuity equation governing the flood propagation process, and not the governing momentum equation. Accordingly, continuity equation considering lateral inflow and outflow in the river reach is expressed as:

$$\frac{\partial Q}{\partial x}+\frac{\partial A}{\partial t}=-2q_L \qquad (3.1)$$

where, Q = channel flow $\left(m^3/s\right)$; A = channel cross-sectional area $\left(m^2\right)$; B =channel width (m); x = distance in the direction of channel flow (m); t = time (s); q_L = lateral flow in the study reach per unit length on one side of the river bank $\left(m^3/s/m\right)$;

The momentum equation governing the one-dimensional flood propagation is expressed as:

$$\frac{1}{A}\frac{\partial Q}{\partial t}+\frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+g\frac{\partial y}{\partial x}-g\left(S_0-S_f\right)=0 \qquad (3.2)$$

The non-linear Boussinesq equation which governs unsteady, one-dimensional lateral flow in the unconfined aquifer adjacent to the study river reach is expressed as (Aravin and Numerov, 1965; Jacob, 1950; Polubarinova-Kochina, 1962):

$$\frac{\partial^2 h^2}{\partial z^2}=\frac{2S_y}{K}\frac{\partial h}{\partial t} \qquad (3.3)$$

where, S_0 = slope of the channel bottom(dimensionless); S_f = friction slope(dimensionless); R = hydraulic radius (m); h = height from the datum to the phreatic surface (m); z = distance perpendicular to the path of the channel (m); S_y = specific yield (dimensionless); K = hydraulic conductivity (m/s).

3.3 VARIABLE PARAMETER McCARTHY-MUSKINGUM METHOD CONSIDERING STREAM-AQUIFER INTERACTION

In the present study, the VPMM method (Perumal and Price, 2013) is extended for accounting the seepage through the stream banks in the direction perpendicular to the stream along the study reach. It is assumed that the aquifers adjacent to both the river banks are having the same uniform aquifer characteristics. The lateral flow, say $2q_L$ has been equally distributed towards both aquifers adjacent to the river, though its contribution to the river reach or vice-versa from either side of the aquifer can vary, depending on the prevailing aquifer characteristics.

The VPMM method considering bank seepage has been diagrammatically presented in Fig. 3.1.

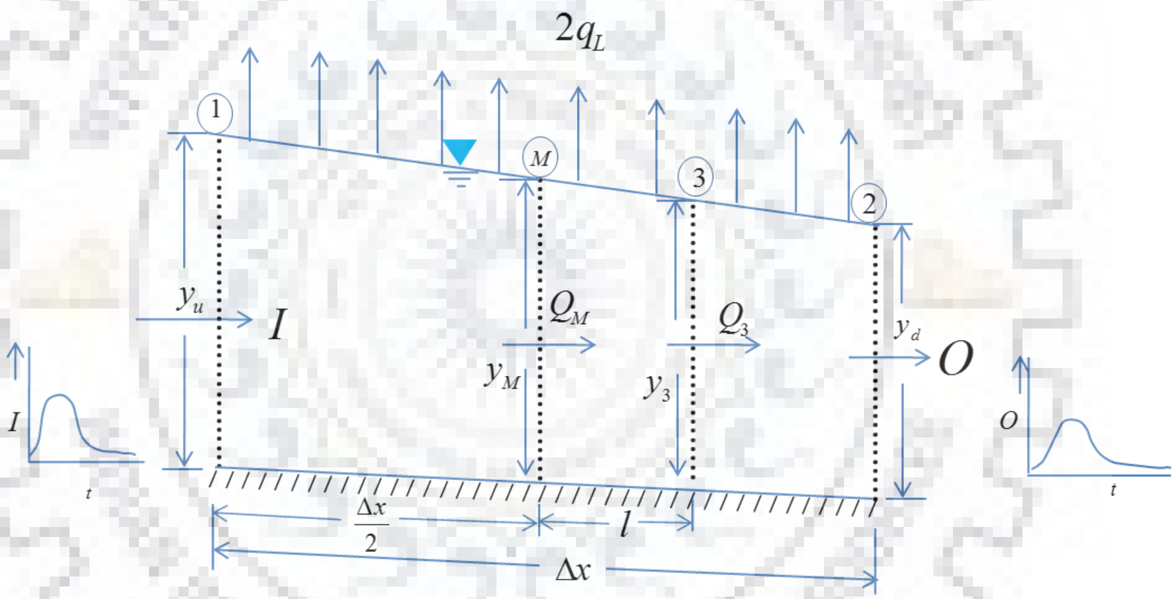

Fig.3. 1 Definition outline of VPMM method considering bank seepage

Using the continuity equation given by Eq. (3.1), which includes the sub-surface interactive flow induced by the flood propagation process in the study reach, and is distributed on both sides of the channel as shown in Fig. 3.1, and the momentum equation given by Eq. (3.2), the following routing equation using VPMM method is arrived at (Yadav at al., 2015):

$$O^{j+1} = C_1 I^{j+1} + C_2 I^j + C_3 O^j + C_4 \left(2 \times \left(-q_L^{j+1}\right) \times \Delta x\right) \quad (3.4)$$

where, the coefficients C_1, C_2, C_3 and C_4 are expressed as

$$C_1 = \frac{-K^{j+1}\theta^{j+1} + 0.5\Delta t}{K^{j+1}\left(1-\theta^{j+1}\right) + 0.5\Delta t} \quad (3.5a)$$

$$C_2 = \frac{K^{j}\theta^{j} + 0.5\Delta t}{K^{j+1}\left(1-\theta^{j+1}\right) + 0.5\Delta t} \quad (3.5b)$$

$$C_3 = \frac{K^{j}\left(1-\theta^{j}\right) - 0.5\Delta t}{K^{j+1}\left(1-\theta^{j+1}\right) + 0.5\Delta t} \quad (3.5c)$$

$$C_4 = \frac{0.5\Delta t}{K^{j+1}\left(1-\theta^{j+1}\right) + 0.5\Delta t} \quad (3.5d)$$

The flood wave travel time in the study reach or sub-reach is given as

$$K^{j+1} = \frac{\Delta x}{v_{0,M}^{j+1}} \quad (3.6a)$$

The weighting parameter θ is expressed as

$$\theta^{j+1} = \left(\frac{1}{2} - \frac{a_0}{c_0}\bigg|_M^{j+1} \frac{1}{\Delta x} \right) \quad (3.6b)$$

where,

$$\frac{a_0}{c_0}\bigg|_M^{j+1} = \frac{Q_{0,M}^{j+1}}{2S_0 B_M^{j+1} c_{0,M}^{j+1}} \left[1 - \frac{4F_M^2}{9}\left(\frac{P}{B}\frac{dR}{dy} \right)_M^2 \right]^{j+1} \quad (3.6c)$$

$$F_M = \left[\frac{\left(Q_M^{j+1}\right)^2 B_M^{j+1}}{g\left(A_M^{j+1}\right)^3} \right]^{\frac{1}{2}} \quad (3.6d)$$

In Eq. (3.4), the sub-surface flow term q_L can be obtained from the solution of the groundwater flow equations for each interface section between river and aquifer, and for every time interval. The negative sign associated with q_L implies the lateral outflow from the considered routing reach when it receives the rising inflow hydrograph.

The subscript $(0, M)$ indicates that the estimated or computed variable at the mid-section of the routing sub- reach or reach correspond to the normal discharge; $v_{0,M}$ = normal velocity at the mid-section of sub-reach (m/s); $c_{0,M}$ = celerity corresponding to normal discharge at the mid-section of sub-reach (m/s).

3.3.1 Routing process

For routing the discharge-hydrograph using the VPMM routing method, the following procedure is used.

1) To follow the assumption of approximately linearly varying flow depth along the longitudinal length of the given routing channel reach, the total length of the routing reach is divided into N equal sub-reaches of length Δx.

2) Corresponding to the initial steady-state discharge Q_0, the unrefined values of the travel time K, and the weighting parameter θ can be obtained at j^{th} time-step using Eq. (3.6) where celerity ($c_{0,M}$) and Froude number (F_M) can be estimated from the following equations (Perumal and Price, 2013):

$$c_0 = \frac{dQ}{dA} = \left[1 + \frac{2}{3}\frac{P}{B}\frac{dR}{dy}\right]v_0 \tag{3.7}$$

$$F_M = \left(\frac{v^2 B}{gA}\right)^{\frac{1}{2}} \tag{3.8}$$

3) Employing the unrefined values of K and θ for the j^{th} time step (previous time) in step 2, the values of C_1, C_2, C_3 and C_4 can be obtained for $(j+1)^{th}$ time step (current time) using Eq. 3.5.

4) The outflow discharge at $(j+1)^{th}$ time step is estimated using the equation

$$O^{j+1} = C_1 I^{j+1} + C_2 I^j + C_3 O^j + C_4\left(2\times\left(-q_L^{j+1}\right)\times\Delta x\right) \tag{3.9}$$

5) The discharge at section-3 can be assessed as

$$Q_3^{j+1} = \theta^{j+1} I^{j+1} + \left(1-\theta^{j+1}\right)O^{j+1} \tag{3.10}$$

which is corresponding to $Q_{0,M}$, the normal discharge at the midpoint of the reach.

6) The normal flow depth occurring at the midpoint of the Muskingum reach, denoted by y_M^{j+1}, with respect to the discharge at section-3 (also corresponding to $Q_{0,M}$), can be assessed by applying the Newton-Raphson iteration method on the equation given below:

$$Q_3 = Q_{0,M} = \frac{1}{n} A_M \, R_M^{\frac{2}{3}} \, S_0^{\frac{1}{2}} \tag{3.11}$$

7) Corresponding to the normal depth y_M^{j+1}, the estimated normal velocity is $v_{0,M}^{j+1}$ for the evaluation of the refined value of K which can be calculated by Eq. (3.6a).

8) Estimate $Q_M^{j+1} = \frac{1}{2}\left(I^{j+1} + O^{j+1}\right)$ at the center of the Muskingum reach.

9) Estimate the Froude number $F_M = \left(\frac{Q_M^2 B_M}{g A_M^3}\right)^{\frac{1}{2}}$ at the center of the Muskingum reach.

10) Estimate celerity $c_{0,M}^{j+1}$ using

$$c = \frac{dQ}{dA} = \left[1 + \frac{2}{3} \frac{P}{B} \frac{dR}{dy}\right] v \tag{3.12}$$

11) The refined value of the weighting parameter can be estimated by the Eq. (3.6b).

11) Employing the refined values of K and θ at the $(j+1)^{th}$ time step, find the refined values of C_1, C_2, C_3 and C_4 using Eq. (3.5).

12) Find the refined values of outflow discharge O^{j+1} , Q_3^{j+1}, y_M^{j+1}, and Q_M^{j+1} using steps (4),(5), (6), and (8), respectively.

13) Estimate the flow depth y_{i+1}^{j+1} with respect to the outflow O^{j+1} using the following equation:

$$y_{i+1}^{j+1} = y_M^{j+1} + \frac{\left(O^{j+1} - Q_M^{j+1}\right)}{B_M c_M^{j+1}} \tag{3.13}$$

14) Repeat the process from steps (2)-(13) for all sub-sections of the reach considering the fact that the outflow discharge for one sub-reach will be the inflow discharge for the adjacent downstream sub-reach.

15) Now, solve for the lateral flow for the same time level using the groundwater flow equations which has been discussed in section 3.5 for one-dimensional flow in the aquifer. This method will be modified for the two-dimensional flow in the aquifer in the later chapters.

16) Again, calculate the values of C_1, C_2, C_3, C_4 and then the outflow discharge using Eq. (3.9).

17) Now, repeat the procedure using steps (5) to (13) to find out the depth at the outlet.

18) Proceed further in the similar manner for the subsequent time steps till the end of the simulation time.

3.4 ZITTA AND WIGGERT SOLUTION (1971)

Consider a rectangular channel which fully penetrates the adjoining unconfined bank aquifer, i.e., channel banks have been taken as vertical (Pinder and Sauer, 1971; Zitta and Wiggert, 1971; Perkins and Koussis, 1996; Hantush et al., 2002; Miracapillo and Morel-Seytoux, 2014). For the solution of stream-aquifer system provided by Zitta and Wiggert (1971), they used the explicit finite difference solution of the full SVE for channel routing and the solution of Boussinesq equation for flow transfer into and from aquifer. They used Simpson's (3/8)- rule to evaluate stream-aquifer interaction flow from river to adjacent aquifer, and vice-versa. Here, the VPMM method has been used for channel routing and the stream-aquifer interaction flow has been estimated using Simpson's (3/8)- rule. The cross-section and plan view of a stream channel reach under investigation is shown in Fig. 3.2.

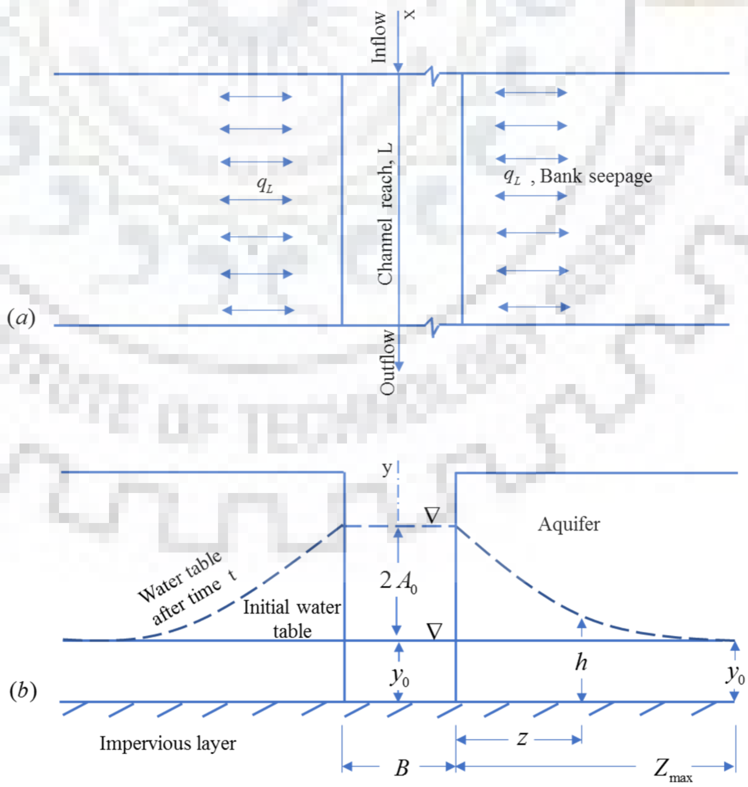

Fig.3. 2 Plan (a) and section (b) views of the considered stream and aquifer interaction system (from Zitta and Wiggert, 1971)

The lateral flow per side per unit length can be estimated using the following equation (Zitta and Wiggert, 1971),

$$q_L = -S_y \int \frac{\partial h}{\partial t} dz \tag{3.14}$$

where, h = height from the datum to the phreatic surface(m); z = distance perpendicular to the path of the channel (m); S_y = specific yield(dimensionless); q_L = lateral flow in the study reach per unit length on one side of the river bank$\left(m^3/s/m\right)$; $t = \text{time}(s)$.

For a rectangular channel which fully penetrates the aquifer, the input details as used by Zitta and Wiggert (1971) is described in Fig.3.3 and the loop rating curve at the inlet section is shown in the inset of the Fig. 3.3.

The hypothetical input stage hydrograph used in the study is expressed mathematically as

$$y(t) = y_0 + A_0\left[1 - \cos\left(\pi t / T_c\right)\right] \tag{3.15}$$

where, $x = 0$ and $0 \le t \le 2T_c$

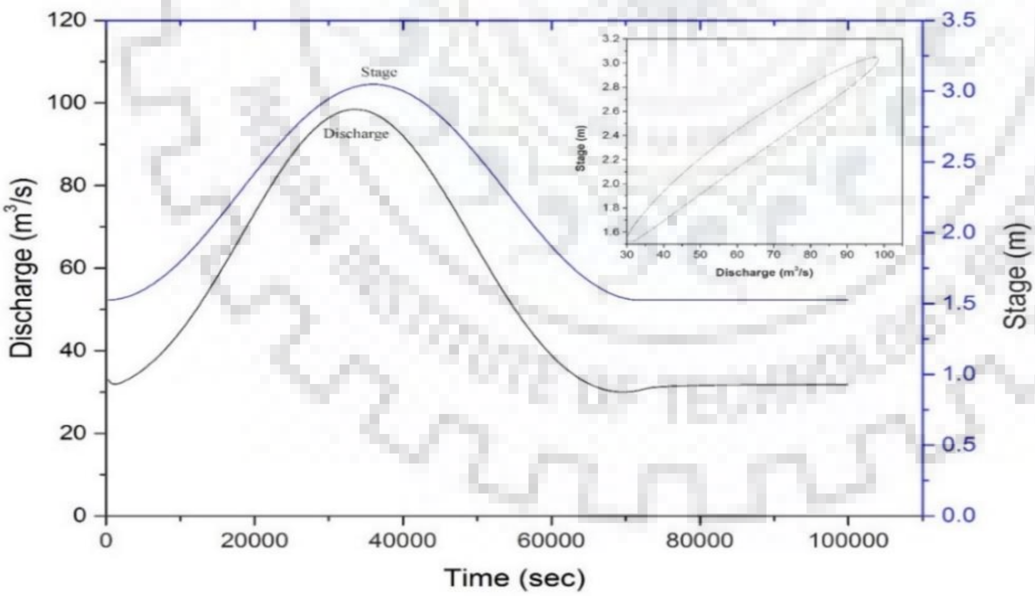

Fig.3. 3 Input stage and corresponding discharge hydrograph

The initial and boundary conditions used for this routing problem, respectively, are given as

i) $h(t) = y_0$ (3.16)

at $0 \le z \le Z_{\max}$, $t = 0$

ii) $h(t) = y_0$ (3.17)

at $z = Z_{\max}$, $t > 0$

iii) $h(t) = y(t)$ (3.18)

at $z = 0$, $t > 0$

The channel and aquifer characteristics details are given in Table 3.1.

Table 3. 1 Numerical data

Channel Characteristics	***Aquifer characteristics***
$y_0 = 1.524\,m$	Hydraulic conductivity (K) = 0.000945 m/s
Channel width = 30.48 m	Specific yield (S_y) = 0.16
Length = 16 Km	$Z_{\max}$ = 300 m
Bed slope = 0.000189	*Finite difference routing parameters*
Manning's n = 0.025	Δx = 2000 m
T_c = 36000s	Δt = 300 s

3.4.1 Solution Procedure using the Simpson's (3/8)- rule

To evaluate the lateral flow using Simpson's (3/8)-rule, the following procedure has been adopted. The estimation of lateral flow has been shown for a sub-reach of the channel.

1) Solve Eq. (3.1) and (3.2) using any of the three routing procedures; i.e., explicit finite difference solution of the full SVE or VPMS method or VPMM method considering $q_L = 0$ and subsequently find out the river stage and discharge for all sub-sections.
2) Discretize the width of the aquifer z into n equal (a multiple of 3) sub-reaches as $\Delta z = \frac{z_{\max}}{n}$.

3) Initially, the hydraulic head and the river stage are at the same level.

4) For the present time $\left(t+\Delta t\right)$, consider the hydraulic depth to be equal to the average flow depth at the interface of the river.

5) Using Eq. (3.3), obtain the values of hydraulic heads at all the nodal points of the aquifer using an explicit finite difference scheme.

6) From Eq. (3.14), consider $f\left(z\right)=\frac{\partial h}{\partial t}=\frac{h(t+\Delta t,z)-h\left(t,z\right)}{\Delta t}$ where z has been divided into 'n' equal sub-intervals from i = 0, 1, 2,…, n. So, it can be said that

$$f\left(z_i\right)=\frac{h(t+\Delta t,z_i)-h\left(t,z_i\right)}{\Delta t},\quad \text{i = 0, 1, 2,…, n.} \tag{3.19}$$

Evaluate the values of $f\left(z_i\right)$ for all the values from 0, 1, 2,…,n.

7) Apply the Simpson's (3/8)- rule to find out the value of the integral in Eq. (3.14) which can be defined as (Matthews, 2004),

$$\int_{z_0}^{z_{max}} f\left(z\right)dz=\frac{3\Delta z}{8}\left[f\left(z_0\right)+3\sum_{i=1,4,7,...}^{n-2} f\left(z_i\right)+3\sum_{i=2,5,8,...}^{n-1} f\left(z_i\right)+3\sum_{i=3,6,9,...}^{n-3} f\left(z_i\right)+f\left(z_n\right)\right] \tag{3.20}$$

8) Multiply the value of the integral obtained in step (7) with the specific yield $\left(S_y\right)$ to find the lateral flow as given in Eq. (3.14).

9) Again solve Eqs. (3.1) and (3.2) using the value of q_L obtained from step (8), and find out the river stage and discharge for all sub-sections.

10) Repeat the steps (1) to (9) for each time interval while routing the complete inflow hydrograph in the channel.

3.5 PROPOSED APPROACH FOR STUDYING FLOW MOVEMENT IN THE RIVER BANK AQUIFER SYSTEM

In this chapter, the sub-surface flow in the riverbank aquifer is considered as one-dimensional perpendicular to the stream-section. The assumptions considered in the development of river routing using VPMM and VPMS methods considering stream-aquifer interaction are as follows:

1) The study river reach is characterized by a prismatic section.
2) The considered unconfined bank river aquifer located on either side adjacent to the stream is symmetrical in form and assumed to be characterized by the same aquifer properties.
3) The river channel fully penetrates the aquifer.

4) The flow in the stream is one-dimensional and one-dimensional flow perpendicular to the river face prevails in the river bank aquifer.
5) The initial water level in the aquifer is same as the water level in the stream prior to the arrival of flood wave in the stream channel.
6) During the progress of stream-aquifer interaction process, no rain is recorded and this assumption avoids the aquifer storage variation due to recharging.

The bisymmetrical stream-aquifer system studied herein is shown in Fig. 3.4 with sectional detail of the stream-aquifer system and its representation in plan form with the stream reach discretized in n sub-reaches for enabling the application of stream routing using the VPMM and VPMS methods.

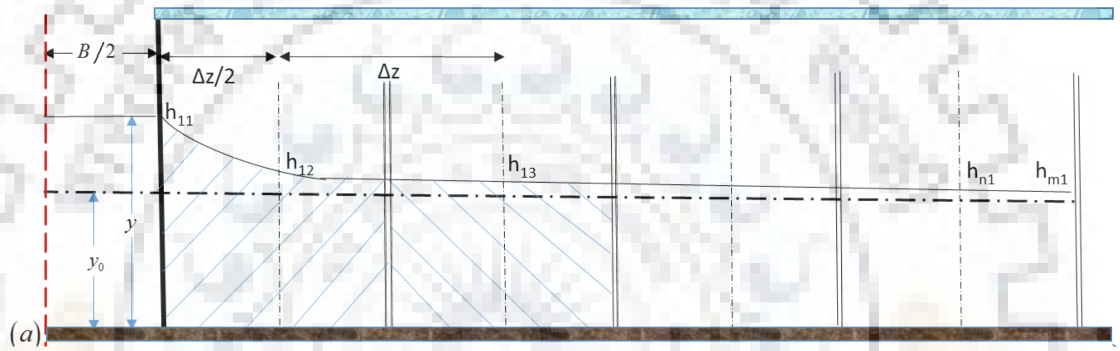

Fig.3. 4 (a) Cross-section of the stream-aquifer system

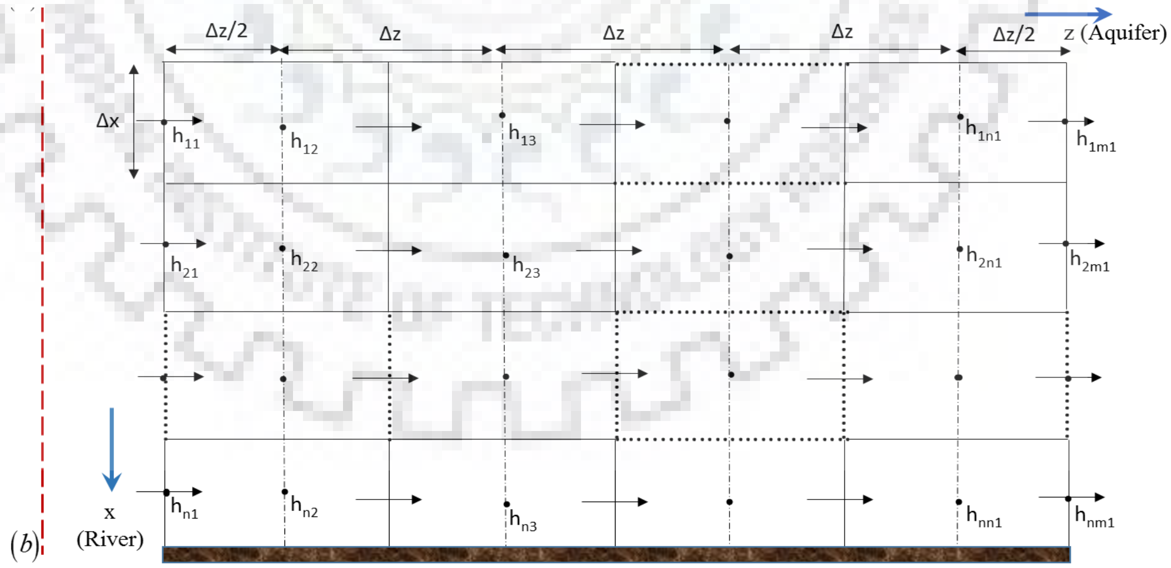

Fig.3. 4 (b) Plan view of stream-aquifer system

3.5.1 Determination of the first grid size

For the development of stream-aquifer interaction model, the aquifer is discretized into seamlessly connected rectangular grids to form an equal size grid network except for two narrow strips, one adjacent to the river face and the other at the far end of the aquifer boundary parallel to the river face. The size of the grid is decided in such a manner that the end section of the first grid located immediately adjacent to the stream is at a distance of greater than 0.75 times of the maximum possible saturated thickness perpendicular to the stream face formed due to the passage of flood hydrograph. The restriction on the size of the narrow strips is imposed to satisfy the applicability of one of the Dupuit's assumption, as advocated by Bear (1972).

Therefore, in the present case, the aquifer has been discretized in the direction perpendicular to the channel flow with a grid size of $\Delta z = 6m$ and the first observation point has been taken at the distance half of the selected grid size (i.e. $\Delta z/2$) following the Dupuit's assumptions (Bear, 1972). The locations of all observation points are marked as shown in Fig. 3.4(b). For each of the considered grid, the mass balance equation in the respective flow direction is expressed as

$$I - O = \Delta S/\Delta t \tag{3.21}$$

where, $I =$ rate of inflow volume$\left(m^3/s\right)$; $O =$ rate of outflow volume$\left(m^3/s\right)$; $\Delta S/\Delta t =$ rate of change of storage$\left(m^3/s\right)$; $S =$ aquifer storage volume$\left(m^3\right)$; $t =$ time$\left(s\right)$.

In the present study, the explicit finite difference equations for the mass balance approach has been solved for each grid in the flow direction of the aquifer to find out the value of water table level for each grid.

Application details of the mass balance equation for a typical grid of the considered narrow strips located on the boundaries of the main aquifer grid network along with the flow directions in the considered grids are shown in Figs.3.5-3.7.

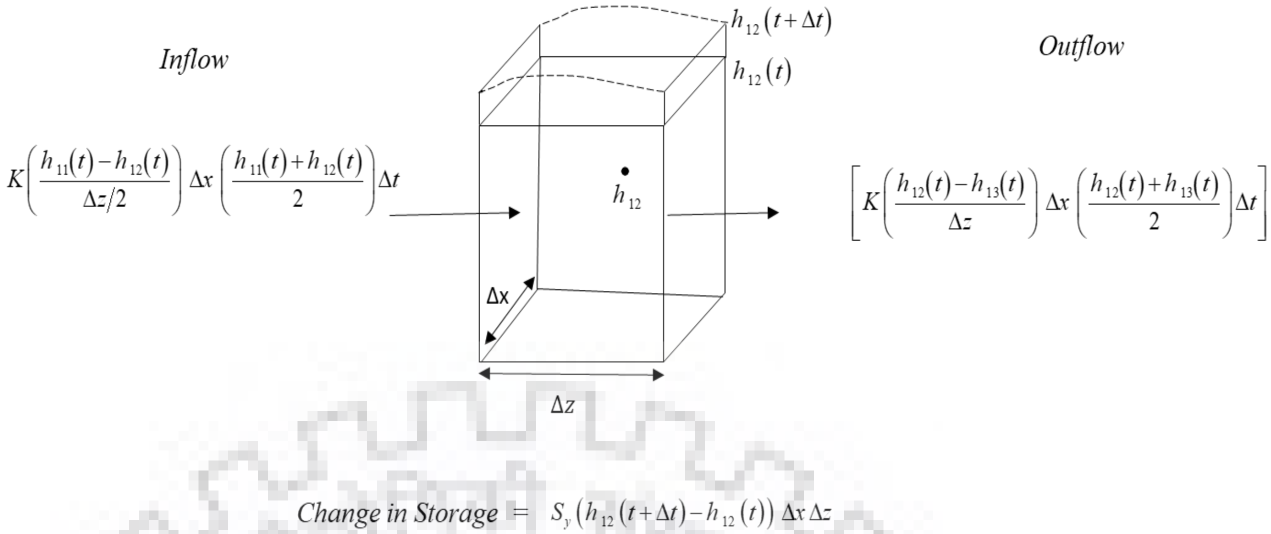

Fig. 3.5 Mass balance in h_{12} grid

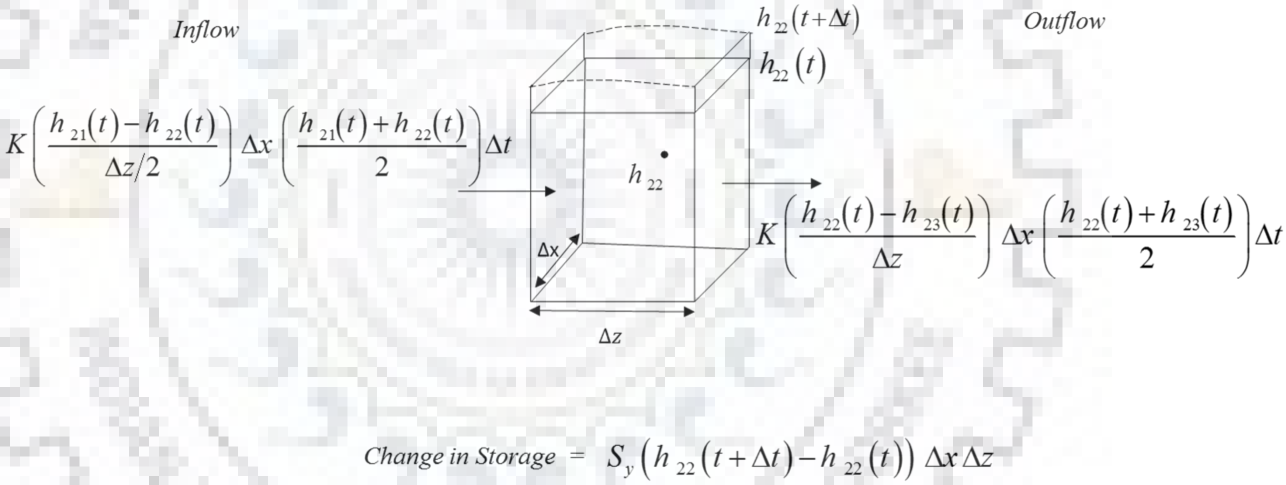

Fig. 3. *6* Mass balance in h_{22} grid

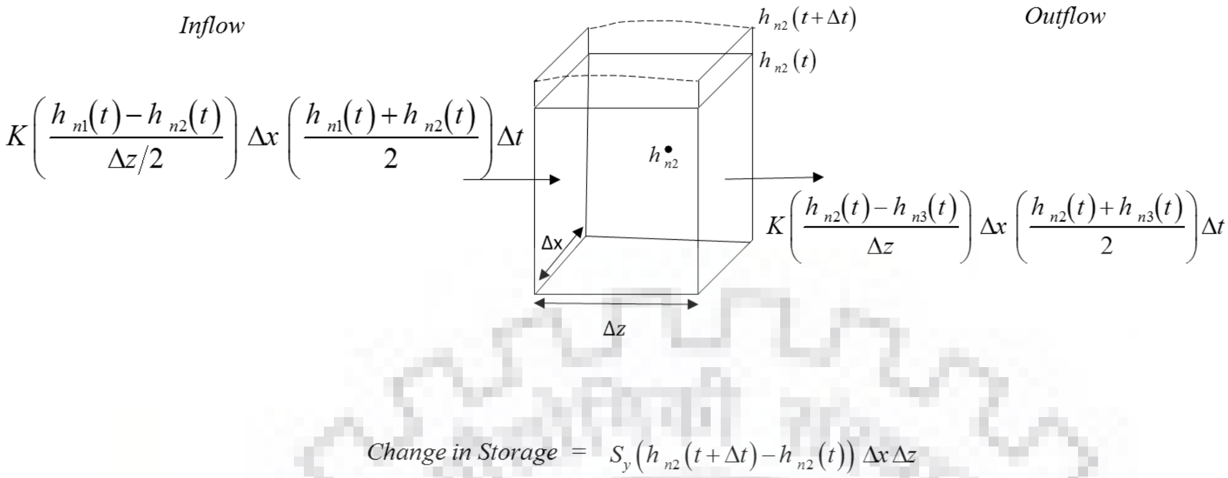

Fig. 3. 7 Mass balance in h_{n2} grid

Using Eq. (3.21), the value of $h_{12}(t+\Delta t)$ can be obtained for the time step $t+\Delta t$ considering the values at time t are known.

The flow in/out to the aquifer computed for the grid h_{12} at the time $t+\Delta t$ in the finite difference form has been written as:

$$q_{L,1}(t+\Delta t) = K \times \frac{\left(y_{M,1}(t+\Delta t) - h_{12}(t+\Delta t)\right)}{(\Delta z/2)} \times \frac{\left(y_{M,1}(t+\Delta t) + h_{12}(t+\Delta t)\right)}{2} \qquad (3.22)$$

where $y_{M,1}(t+\Delta t)$, is the depth at mid-section for the first sub-reach along the channel length for the time step $t+\Delta t$.

The positive value of $q_{L,1}(t+\Delta t)$ in Eq. (3.22) represents the influent stream (losing stream), whereas the negative value represents the effluent stream (gaining stream) (Sophocleous, 2002).

Following the same procedure, the values of water table levels in the direction of the aquifer and the flow in/out to the aquifer for all the channel sub-sections can be obtained.

In this study, the explicit finite difference method has been used for the solution of the groundwater flow equations. The procedure of solving this system of equations follows two steps; in the first step, the open channel flow equations are solved for steady flow conditions. Then in the second step, the groundwater equations are solved for steady flow by using the calculated stream elevations. Further, the open channel flow equations must now be solved once again because the groundwater inflow term q_L has been modified by the new head distribution in the aquifer.

To proceed with the solution of the transient problem the procedure indicated above is repeated for each time interval Δt .

The volume of bank storage can be calculated from the equation given in Eq. (3.23) (Chen and Chen, 2003);

$$V = \int_0^t q_L \, dt \tag{3.23}$$

3.6 FIELD APPLICATION

The study area details as extracted from Birkhead and James (1998, 2002), Heritage et al. (2001) are as follows:

The catchment area of the Sabie river (Fig. 3.8) is about 7100 km^2 (709,600 ha). The river originates from the Mauch Berg's eastern slopes at an altitude of 2200m and flows eastward into the various geomorphic zones, the low relief of the Lowveld and Lebombo for around 201 km before converging into the Incomati River in Mozambique. It is a perennial river, however, like any other semi-arid systems it is also subjected to various discharge extremes. The river system is immensely affected by the seasonal rainfall in summer which brings high flows and floods in the catchment and the winter results in the low flows. The land use in the upper part of the Sabie river catchment is mainly human settlements and agricultural, whereas the Kruger National Park (KNP) covers most of the lower part of the catchment which is dominated by the grasslands also it covers the majority of the Sabie river and hence, the less abstractions can be observed in this region (Saraiva Okello et al., 2015). The study site (shown in Fig. 3.8) selected for the analysis is same as chosen by Birkhead and James (1998) located at 4.6 km downstream of the gauging site where the continuous discharge record is available.

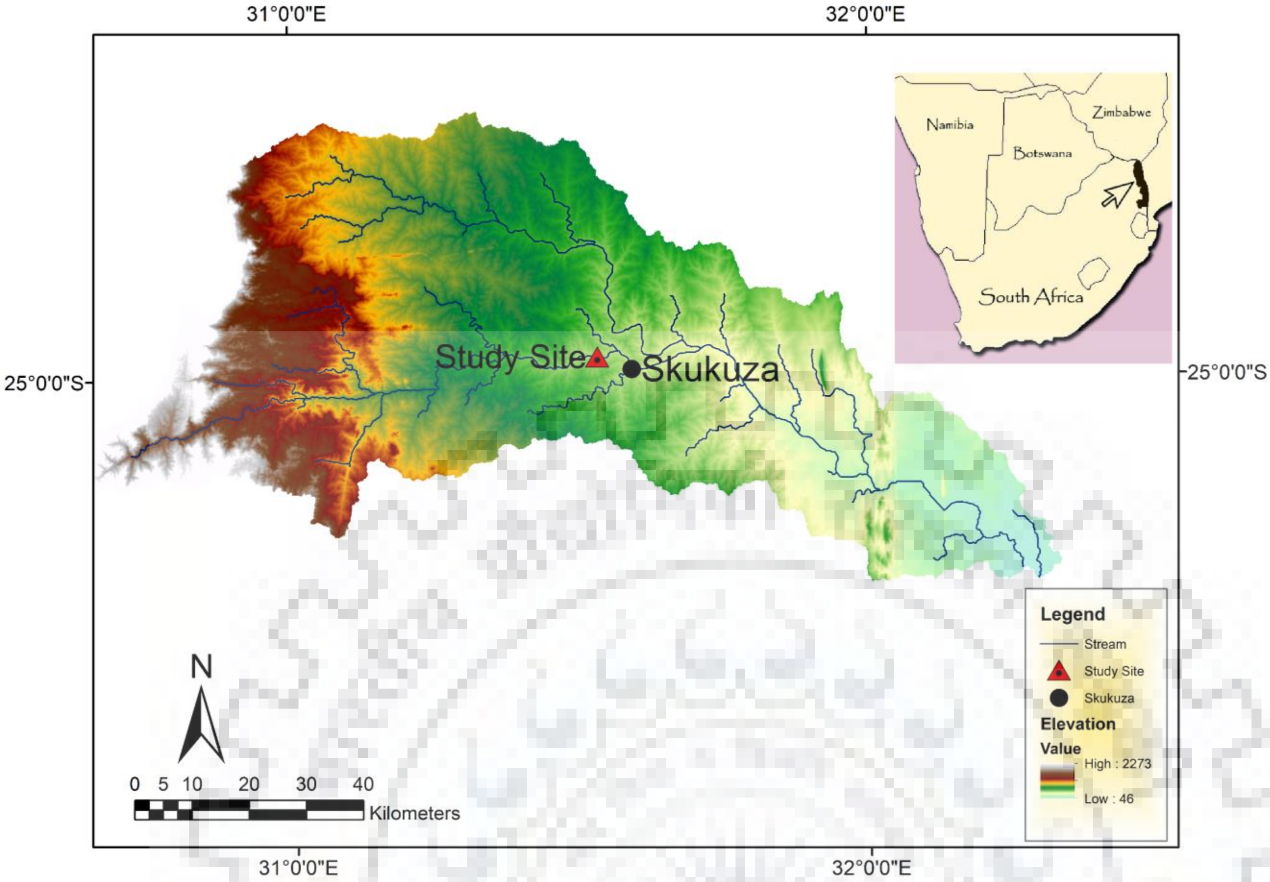

Fig.3. 8 Sabie River catchment and location of the study site

The flood event data with upstream discharge hydrograph and the corresponding event stage hydrograph recorded downstream of the considered 4.6 km reach as extracted from the study of Birkhead and James (1998) is shown in Fig. 3.9. Also, the related hydraulic data, channel characteristics, and routing parameters for the unsteady flow computations used by Birkhead and James (2002) are given in Table 3.2.

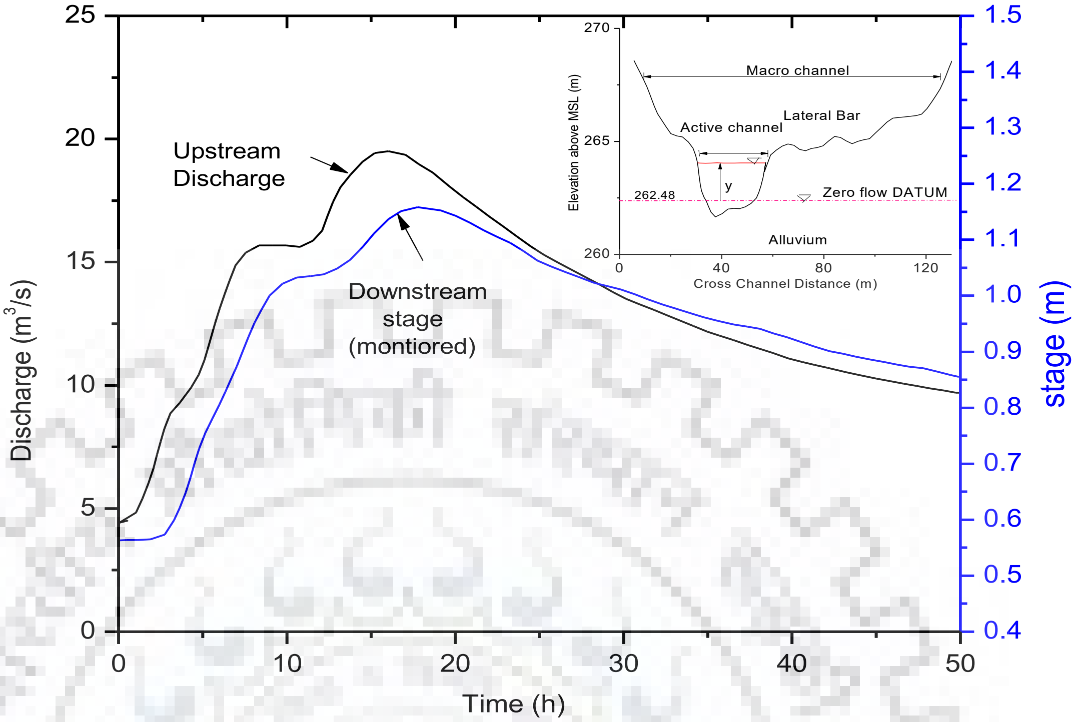

Fig.3. 9 Monitored discharge and stage hydrographs at the upstream and downstream, respectively. (Birkhead and James, 1998)

Table 3. 2 River characteristics

Reach length (m)	4600
Discharge range (m^3/s)	4.39-19.51
Bank characteristics:	
Hydraulic conductivity $K\ (m/s)$	0.014
The extent of bank/side (m)	62
Effective porosity (dimensionless)	0.34
Saturated depth (m)	1.56

The Sabie River is a mixed bedrock/alluvial influenced semi-arid river system, with fluvial deposits within a bedrock macro-channel. The width of the macro-channel has been considered as 146 m from the analysis of an aerial photograph of scale 1:10000. The average active channel length over the reach length is 23 *m*. The bed slope of the reach is 0.00278 which has been calculated using the Manning's roughness coefficient given as 0.0617 (Heritage et. al., 2001). The river bed is located approximately 1 *m* above the bedrock (Birkhead and James, 2002) and due to this reason one can assume that the river channel fully penetrates the aquifer. For the present study, inflow discharge has been taken as the digitized values from the figure given by Birkhead and James (1998) (Fig. 3.9, herein). The spatial and temporal grids are $\Delta x = 575\ m$ and $\Delta t = 90s.$ The VPMM method has been applied to find out the bank seepage using the given river and bank characteristics.

3.7 RESULTS AND DISCUSSION

The simulations have been performed in this chapter using FORTRAN-77 code that solves the river-aquifer interaction flow problem formulated herein using the related governing flow equations discussed. For analyzing the modification of flood wave due to interactive bank flow, the given stage hydrograph has been routed through the channel reach with and without the consideration of interactive flow. The channel has been considered as impervious in the first cycle of routing and the interactive flow then has been considered in the second cycle of routing. In the first cycle of routing which consists of two iterative routings with the first iterative routing used for the determination of unrefined discharge and the second one used for estimating the refined discharge using the refined routing parameters estimated using the unrefined routed discharge of the first iteration. Prior to the second cycle of routing, the river-aquifer interaction flow is taken into account for determining outflow.

For the application of VPMM method, the discharge hydrograph corresponding to stage hydrograph given by Eq. (3.15) is estimated. This discharge was estimated by using the explicit finite difference solution of the full Saint-Venant equations for the stage hydrograph given in Eq. (3.15) without considering stream-aquifer interaction. Then using that discharge input, routing has been performed using the VPMM method to find the river-aquifer interaction flow. The comparison between the stream discharges for these simulations at the upstream and downstream with and without considering stream-aquifer interaction indicates the effect of stream-aquifer interaction flow on the flood wave. The diagrammatic representation of results for all the proposed methods has been shown in Fig. 3.10.

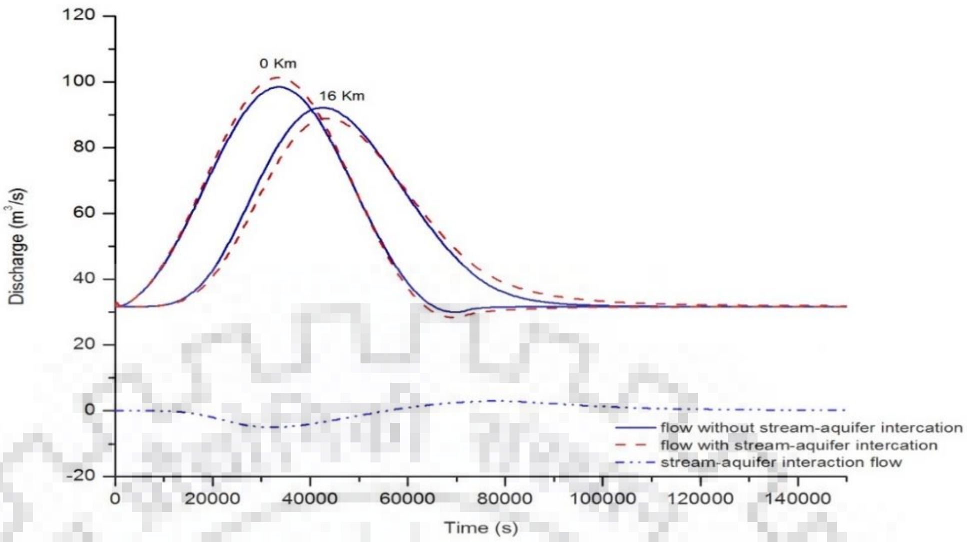

Fig.3. 10 Effect of bank seepage in a 16 Km reach using explicit method

In Fig. 3.10, the upstream discharge considering stream-aquifer interaction is more than without considering interaction. It is due to the steeping of the energy grade line (Zitta and Wiggert, 1971).

The comparison between the downstream discharge hydrograph obtained by the VPMM and by the explicit routing method considering the stream-aquifer interaction flow in the routing process (Zitta and Wiggert, 1971) are shown in Fig. 3.11.

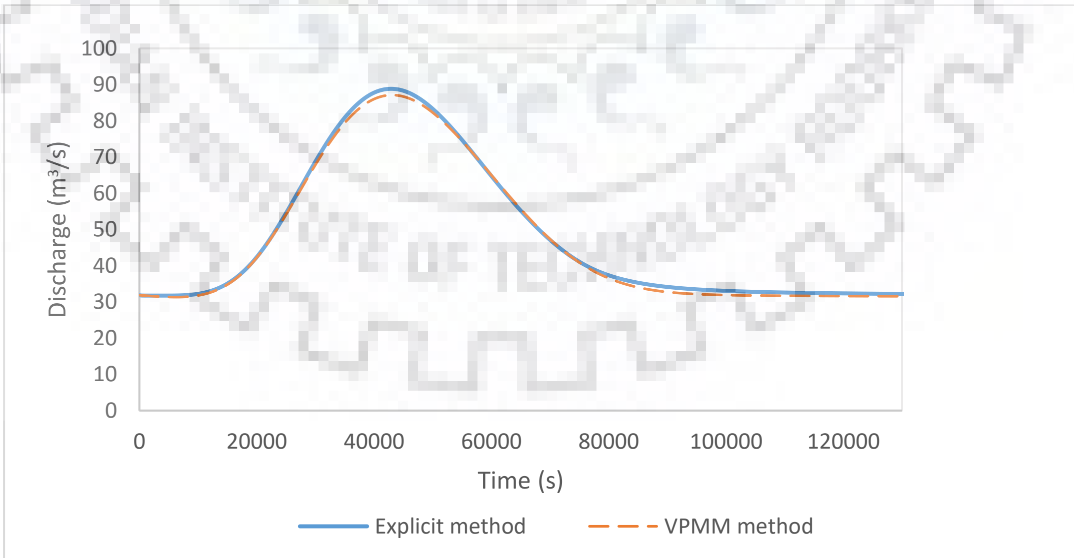

Fig.3. 11 Comparison of downstream discharge obtained by the VPMM method with the results of Zitta and Wiggert (1971)

The NSE and RMSE estimates assessed for the simulated hydrographs of the VPMM method with respect to the corresponding explicit finite difference method considering as the benchmark solution are 0.9979 and 0.84, respectively. Therefore, the VPMM method can be adopted for the simulation of hydrographs considering the stream-aquifer interaction.

For the Sabie River analysis, the comparison between the monitored and simulated flow-depth hydrographs are shown in Fig.3.12.

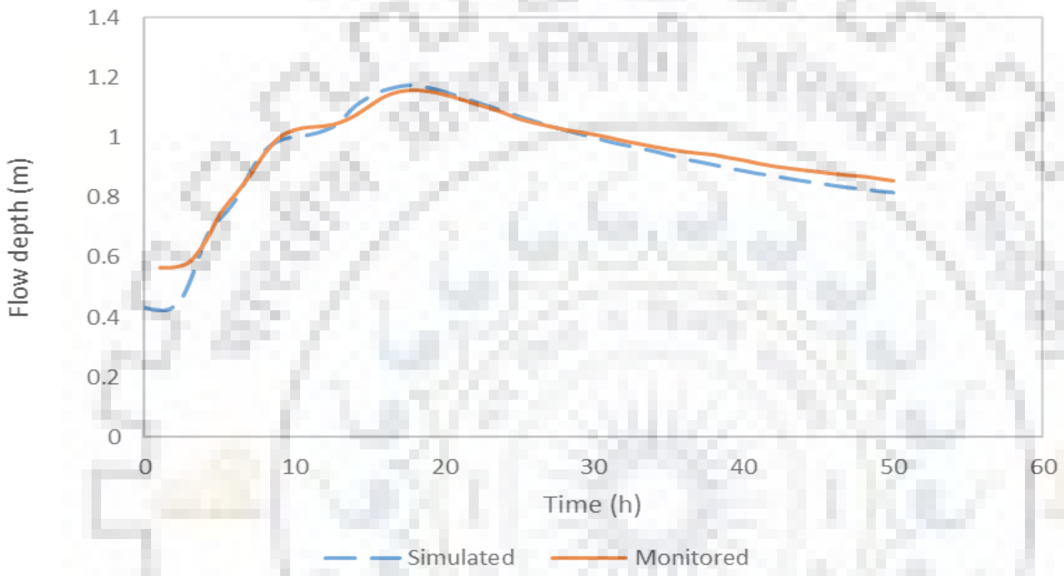

Fig.3. 12 Comparison of monitored downstream depth with simulated using the VPMM method

It is obvious from Fig.3.12 that the VPMM method reproduces the monitored hydrograph closely. The NSE estimate for the solution of the VPMM method is 0.9557 and that of the RMSE is 0.0217. The first few points of the monitored depth are omitted for the comparison because of the differences in the values of the monitored upstream discharge hydrograph (Birkhead and James, 1998), and the initial values given by Birkhead and James (2002).

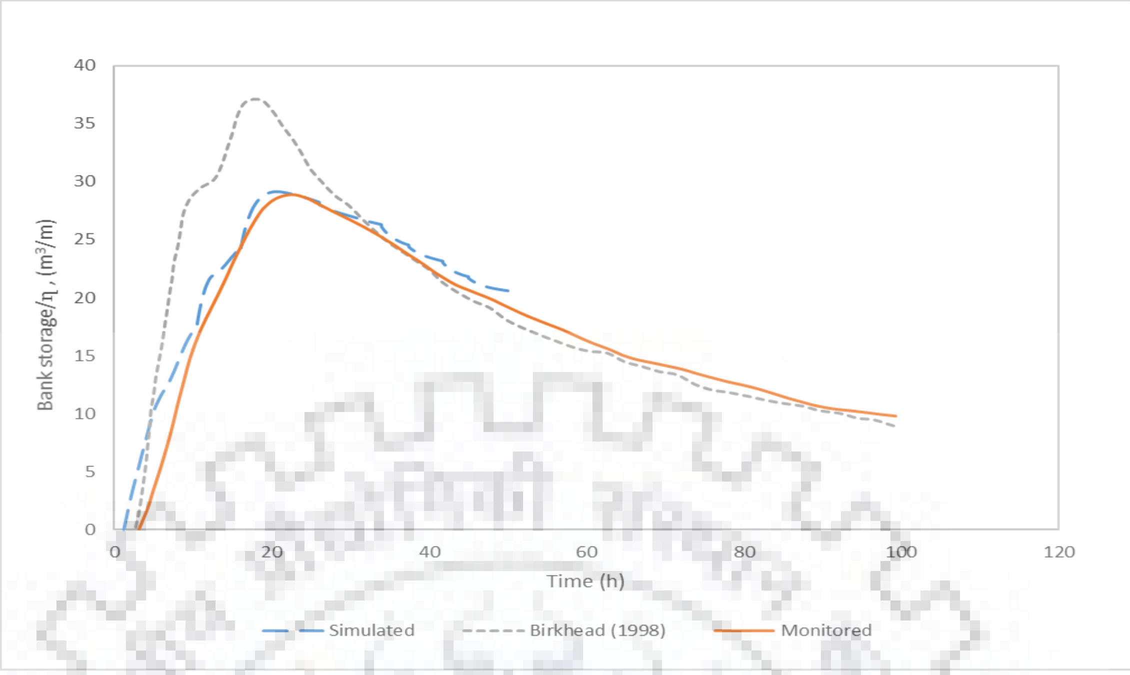

Fig.3. 13 Comparison between monitored and simulated bank storage per effective bank porosity

Fig.3.13 shows the comparison between the monitored and simulated bank-storage per effective bank porosity, as used in the study of Birkhead and James (1998). The bank-storage has been divided by the average of the sediment porosity at the upstream and the downstream section. The simulated bank storage per porosity has been evaluated for 50 hours, as in the present study the upstream discharge hydrograph was available for 50 hours only.

3.8 CONCLUSIONS

For the Simpson's (3/8) - rule the number of subdivisions should always be a multiple of 3. The values of phreatic surfaces at each grid point in the direction of the aquifer are required for the evaluation of bank seepage using this method. Therefore, the sensitivity of the method got increased because of the involvement of hydraulic heads at each grid points. In the evaluation of the bank seepage, only two values of phreatic surfaces play a major role to evaluate the lateral flow, one at the interface of the stream and aquifer which is actually the stream stage and other at just adjacent to the stream in the direction of the aquifer at the first observation point. Therefore, the use of Simpson's (3/8)-rule is not suggestible due to its complicated calculation and its sensitivity.

With the moderate data requirements, the VPMM method can be chosen to evaluate the bank storage for a river segment. The required data are the channel cross-sectional details and the discharge hydrograph at the section. In the present study, two procedures have been adopted for producing discharge hydrographs considering bank storage on both the sides of the channel cross-section. In the first verification of the considered approach of modelling river-aquifer interaction, the explicit finite difference method was used for the solution of the full Saint-Venant equations and the second using the VPMM method (Perumal and Price, 2013). Also, the hydrographs reproduced by the above-mentioned procedures have been compared with the explicit solution. Moreover, the VPMM method simultaneously computes the stage hydrograph corresponding to a given inflow or routed discharge hydrograph. Therefore, for the evaluation of bank storage, this method provides the values of hydraulic heads which are equal to the river stages at each river-section. The VPMM method is a simplified routing procedure and provides the river stage at each section along with the discharge hydrographs. The application of the VPMM method is relatively simple and will give better results in comparison to the procedures following the rating curve conversion approach as adopted by Birkhead and James (1998).

CHAPTER 4

VARIABLE PARAMETER STAGE- HYDROGRAPH METHOD CONSIDERING STREAM-AQUIFER INTERACTION

4.1 GENERAL

Stream discharge information is often considered as an important quantification variable useful for the analysis and decision-making process of various hydrological problems. However, information about stream stage is considered more relevant during flood forecasting, the design of flood protection embankments, automated operation of canal network systems and in environmental flow assessment issues, particularly for aquatic-ecosystem needs. As of now, direct discharge measurement in river engineering practices is possible only in day time. However, stage measurements along rivers can be made all the time using automatic recordings. In order to estimate the inflow hydrograph required for flood routing, quite often a measured stage hydrograph is converted to discharge hydrograph using the rating-curve developed for that section where the corresponding stage measurements are recorded. However, such transformation introduces errors in the estimated inflow hydrograph, especially when the flood wave is non-kinematic. To overcome this problem, it would be desirable to use the measured stage itself as the operating variable for routing, rather than using discharge variable.

Several investigators (Cooper and Rorabaugh, 1963; Hornberger et al., 1970; Pinder and Sauer, 1971; Swamee et al., 2000) provided solutions for stream-aquifer interaction problem assuming the channel section of the interacting reach with bank aquifer fully penetrates the aquifer. In addition, it is assumed that the river-aquifer interaction takes place perpendicular to the river bank. This assumption implies that the flow is one-dimensional in the river-bank aquifer (Cooper and Rorabaugh, 1963; Hornberger et al., 1970; Zitta and Wiggert, 1971). Many models have been developed to simulate flood processes due to non-uniform and spatially varying precipitation event. Often, these models are separated into surface and subsurface components. Although this artificial separation of an otherwise linked system helped to reduce most of the problems, but failed to describe the system accurately and resulted in numerous discrete models of limited applicability (McDonald and Harbaugh, 1998; Fread, 1993; Aral, 1990). Even though these models provide good results in simulating the related hydrological processes of their particular domains, they show a deviation from the observed events when the interactions between these domains become dominant. Hence to minimize the deviation between the simulated and observed runoff processes, it is necessary to consider the interaction between the stream and aquifer using a simple coupling process.

Most of the available stage or flow-depth routing methods are based on the solution of the governing equations (Saint-Venant's equations) using explicit or implicit numerical schemes, and only a few simplified routing methods using stage as the operating variable are available in the literature (Hayami, 1951; Franchini and Lamberti, 1994). However, these simplified methods employ routing schemes based on linear theory which in turn employ constant velocity for the entire routing process of a flood event which is contradictory to the nonlinear characteristics of flood-wave movement in channels and streams. Further, to estimate discharge hydrographs corresponding to the routed stage hydrographs, these methods employ the established rating-curve at a section which again estimates the erroneous discharge hydrographs due to non-kinematic behavior of the flood wave propagation. In this chapter, the stream-aquifer interaction process has been integrated with the channel routing processes proposed by Perumal and Ranga Raju (1998), known as the VPMS method. The solution for the propagation of flood wave in a channel reach subject to one-dimensional river-aquifer interaction in the study reach using the VPMS method has been proposed in this chapter. The lateral flow due to stream-aquifer interaction has been estimated based on the application of mass conservation equation applied to cascade of sub-reaches in the bank-aquifer strip perpendicular to the stream, as explained in Section3.5. This solution is compared with the lateral flow which has been obtained using the approach employed by Zitta and Wiggert (1971), as explained in Section 3.4. The appropriateness of the proposed routing method considering river-aquifer interaction is first verified with the hypothetical stage-hydrograph routing solution obtained by Zitta and Wiggert (1971) and its practical application is demonstrated using the case study of Birkhead and James (1998) for the Sabie River in South Africa.

4.2 VARIABLE PARAMETER STAGE-HYDROGRAPH ROUTING METHOD CONSIDERING STREAM-AQUIFER INTERACTION

Similar to the application of the VPMM method for studying the stream-aquifer interaction, the VPMS method has also been extended for accounting the river-aquifer interaction along the routing reach. It is considered that this interaction with the bank aquifer is uniformly distributed on both sides of the routing channel reach. The lateral flow, say $2q_L$ has been uniformly distributed on both sides of the river.

The VPMS method routing system considering stream-aquifer interaction is depicted in Fig.4.1.

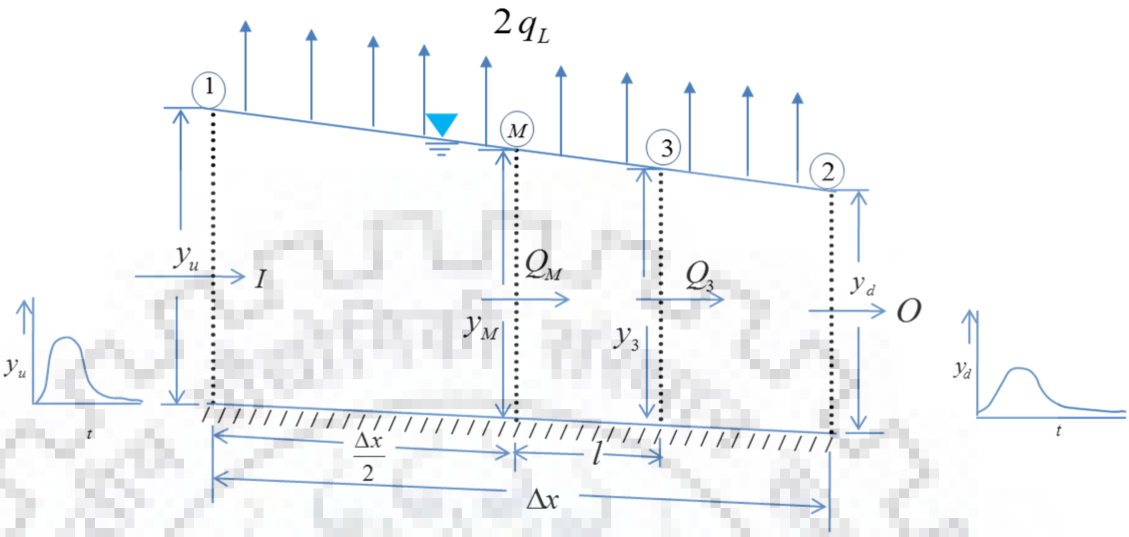

Fig.4. 1 Definition sketch of the VPMS method routing scheme considering river-aquifer interaction

Applying the modified continuity equation given by Eq. (3.1) with the river-aquifer interaction process taking place with uniformly distributed lateral flow q_L m^2/s along the study reach, as described in Fig. 4.1, the routing equation in terms of flow depth variable is expressed as (Perumal and Ranga Raju, 1998):

$$y_{d,j+1} = C_1\, y_{u,j+1} + C_2\, y_{u,j} + C_3\, y_{d,j} - C_4 \left(\frac{q_L}{B}\right) \tag{4.1}$$

where,

$$C_1 = \frac{-K\theta + 0.5\Delta t}{K(1-\theta) + 0.5\Delta t} \tag{4.2a}$$

$$C_2 = \frac{K\theta + 0.5\Delta t}{K(1-\theta) + 0.5\Delta t} \tag{4.2b}$$

$$C_3 = \frac{K(1-\theta) - 0.5\Delta t}{K(1-\theta) + 0.5\Delta t} \tag{4.2c}$$

$$C_4 = \frac{0.5K\,\Delta t}{K(1-\theta)\; + \; 0.5\Delta t} \tag{4.2d}$$

$$K = \frac{\Delta x}{\left[1 + m\left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)_3\right] v_3} \tag{4.3a}$$

$$\theta = \frac{1}{2} - \frac{Q_3\left[1 - m^2 F_M^2\left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)_M^2\right]}{2\,S_0 \left.\frac{\partial A}{\partial y}\right|_3 \left[1 + m\left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)_3\right] v_3\;\Delta x} \tag{4.3b}$$

The q_L term in Eq. (4.1) can be obtained from the groundwater flow equations which will be discussed later in this chapter for each river sub-reach.

4.2.1 Routing procedure

The step-wise procedure for stage-hydrograph routing using the VPMS method in the presence of stream-aquifer interaction in the reach is described herein:

1) Following the assumption of approximately linearly varying flow depth along the longitudinal length of the given routing channel reach, the total length of the routing reach is divided into N equal sub-reaches of length Δx.

2) Corresponding to the initial steady-state depth y_0, the unrefined values of the travel time K, and the weighting parameter θ can be obtained at j^{th} time-step using Eq. (4.3).

3) Employing the unrefined values of K and θ for the j^{th} time step, i.e., at the previous time step in step (2), the values of C_1, C_2, C_3 and C_4 can be obtained for the $(j+1)^{th}$ time step using Eq. (4.2).

4) Estimate the unrefined stage $y_{d,j+1}$ at the reach outlet using the following equation:

$$y_{d,j+1} = C_1\, y_{u,j+1} + C_2\, y_{u,j} + C_3\, y_{d,j} \tag{4.4}$$

5) Using this unrefined stage $y_{d,j+1}$, the stage at Section-3, as shown in Fig. 4.1, can be estimated as

$$y_{3,j+1} = \theta\, y_{u,j+1} + (1-\theta)\, y_{d,j+1} \tag{4.5}$$

6) Estimate $A_{3,j+1}$ corresponding to $y_{3,j+1}$.

7) Using the unrefined stage $y_{d,j+1}$ obtained in step (4), the stage at the mid-reach is estimated as

$$y_{M,j+1} = \frac{\left(y_{u,j+1} + y_{d,j+1}\right)}{2} \tag{4.6}$$

8) Estimate $A_{M,j+1}$ and $\left(\partial A/\partial y\right)_M$ with respect to $y_{M,j+1}$.

9) Estimate $Q_{3,j+1} = A_{M,j+1} C_f \, R_{M,j+1}^m \, S_0^{1/2}$.

10) Using $A_{3,j+1}$ estimated in step (6), estimate $v_{3,j+1} = \dfrac{Q_{3,j+1}}{A_{3,j+1}}$.

11) Using the estimated geometric and flow variables at the mid-section, the discharge at the mid-reach can be calculated as

$$Q_{M,j+1} = A_{M,j+1} C_f \, R_{M,j+1}^m \left[S_0 - \frac{\left(y_{d,j+1} - y_{u,j+1}\right)}{\Delta x} \right]^{\frac{1}{2}} \tag{4.7}$$

where, C_f = friction coefficient

12) Estimate the Froude number at the mid-section $F_{M,j+1} = \left(\dfrac{Q_M^2 \left(\partial A/\partial y\right)_M}{g \, A_M^3} \right)^{1/2}$.

13) Estimate the refined values of K and θ for the $(j+1)^{th}$ Δt time level using Eq. (4.3).

14) Repeat the process from steps (3) to (13) to estimate the refined values of flow and stage at the outlet of channel sub-reach.

15) Now find out the discharge at the sub-reach outlet using the following equation (Perumal and Ranga Raju, 1998)

$$O_{j+1} = Q_{M,j+1} + \left(\partial A/\partial y\right)_{M,j+1} \left[1 + \left(\frac{P \, \partial R/\partial y}{\partial A/\partial y} \right)_{M,j+1} \right]^{\frac{1}{2}} v_{M,j+1} \left(y_{d,j+1} - y_{M,j+1}\right) \tag{4.8}$$

16) Repeat the process from steps 3-15 for all the channel sub-reaches of the stream considering the fact that the downstream depth for one sub-reach will be the upstream depth of the subsequent adjacent sub-reach.

17) Now solve for the lateral flow for the same time level using groundwater flow equations which has been discussed in the Sections 3.4.1 and 3.5 for the one-dimensional flow in the aquifer. This procedure of obtaining lateral flow will be modified for the two-dimensional flow in the aquifer in the later chapter (Chapter-5).

18) Again, estimate the values of C_1, C_2, C_3, C_4, and then calculate the downstream depth as

$$y_{d,j+1} = C_1\, y_{u,j+1} + C_2\, y_{u,j} + C_3\, y_{d,j} - C_4 \left(q_{L\,j+1} / B \right) \qquad (4.9)$$

19) Now, repeat the steps from (5)-(15) to find out the discharge at the outlet of each sub-reach corresponding to a given stage at the inlet of the study reach.

20) In this way, one can move to the next simulation time level $(t+\Delta t)$ till the end of the simulation time. The duration of the simulation time corresponds to the duration till the steady flow condition is reduced at the outlet of the considered routing reach.

4.3 VERIFICATION USING HYPOTHETICAL DATA

The VPMS method considering stream-aquifer interaction process has been verified using the hypothetical data used by Zitta and Wiggert (1971) as discussed in Section3.4 in Chapter 3. For the use of VPMS method, the stage input hydrograph given in Eq. (3.15) has been used along with the initial and boundary conditions given in Eqs. (3.16)- (3.18), and the channel and aquifer characteristics are the same as given in Table 3.1. For the estimation of lateral flow, the procedure as given in Section3.5 has been used.

4.4 APPLICATION TO SABIE RIVER REACH

The stream flow routing considering stream-aquifer interaction using the VPMS method has been applied for the same reach of the Sabie River as described in Section3.6 in Chapter 3. The input information for the Sabie River has been given in terms of discharge and the channel section is very steep. Therefore, for the application of VPMS method for the same river reach, it is required to obtain a stage hydrograph corresponding to the given discharge hydrograph using the explicit finite difference method of solving the full Saint-Venant equations assuming no stream-aquifer interaction in the first instant. After obtaining the input stage hydrograph, the routing process given in Section3.5 was applied to find the downstream discharge considering stream-aquifer interaction.

4.5 RESULTS AND DISCUSSION

The comparison between the downstream discharge hydrograph obtained by the VPMS method and the discharge obtained using the explicit method (Zitta and Wiggert, 1971) is shown in Fig. 4.2. In both the approaches, the lateral flow in the routing reach due to stream-aquifer interaction is considered.

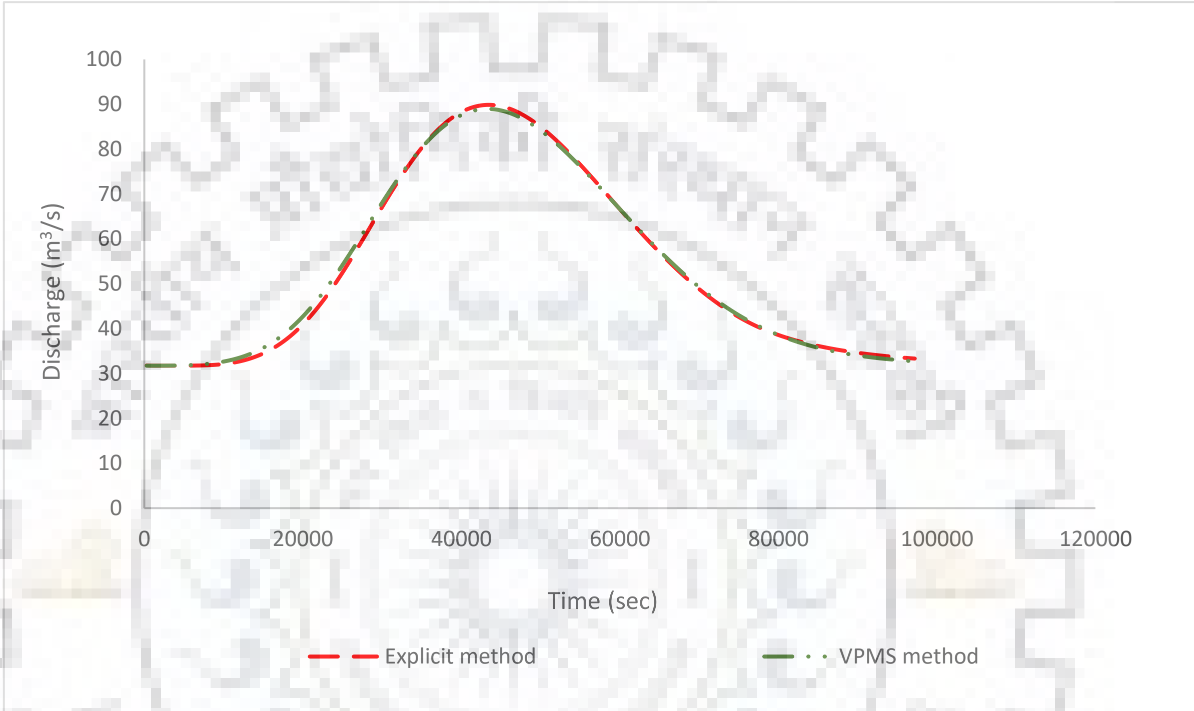

Fig.4. 2 Comparison of downstream discharge obtained by the VPMS method with the results of

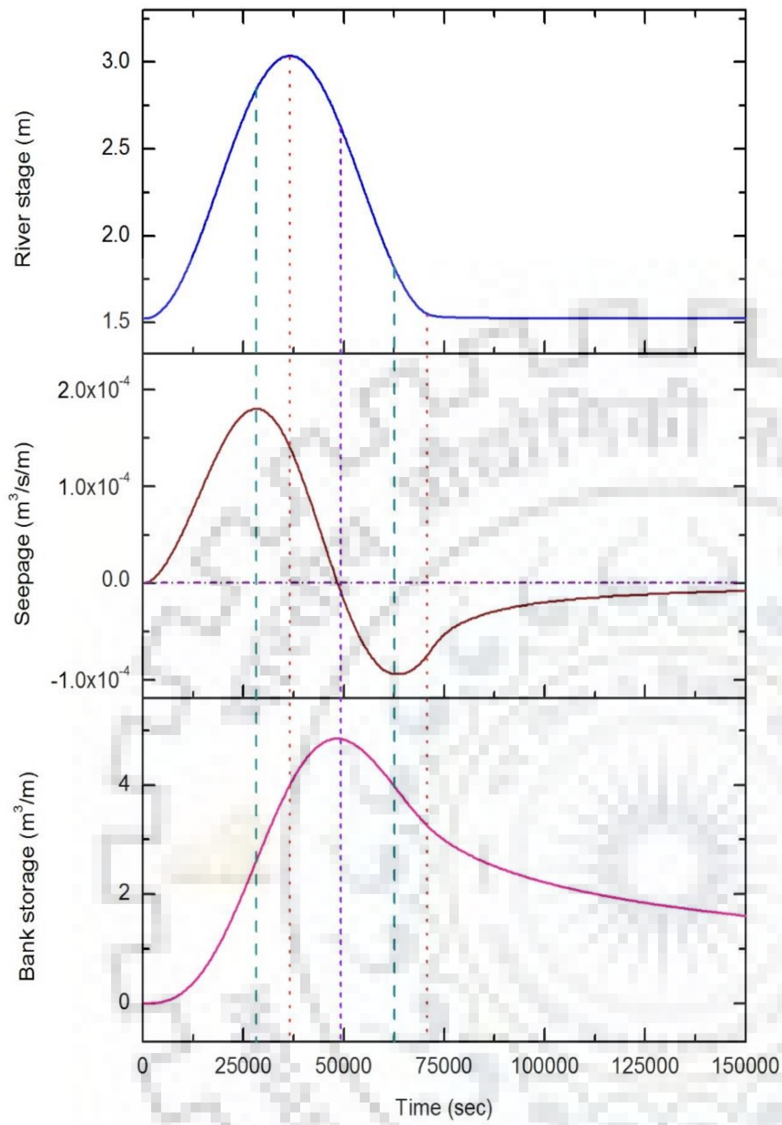

Fig.4. 3 Bank storage and bank seepage hydrographs simulated at the outlet of the first sub-reach using the VPMS method for the case study of Zitta and Wiggert (1971)

In Fig. 4.3, the lateral flow for the first sub-section of the considered channel reach along with the corresponding average flow depth and the bank storage hydrographs are shown. The positive value of the bank seepage (q_L) represents the flow towards the aquifer during the flood period and the negative value of the bank seepage represents the return flow towards the river. The bank seepage becomes maximum at the time prior to the maximum river stage during the flood period. At the later stage of flood hydrograph, the return flow to the river starts at a lower rate, but it remains continuous for a long time after the flood period. The bank storage becomes maximum at the time where the bank seepage becomes zero. After that, the bank storage will reduce at a lower rate.

The obtained downstream discharge hydrograph for the case study of Sabie River (Birkhead and James, 1998) given in Section 4.4 using the VPMS method has been compared with the monitored flow depth at the end of the considered study reach as shown in Fig.4.4.

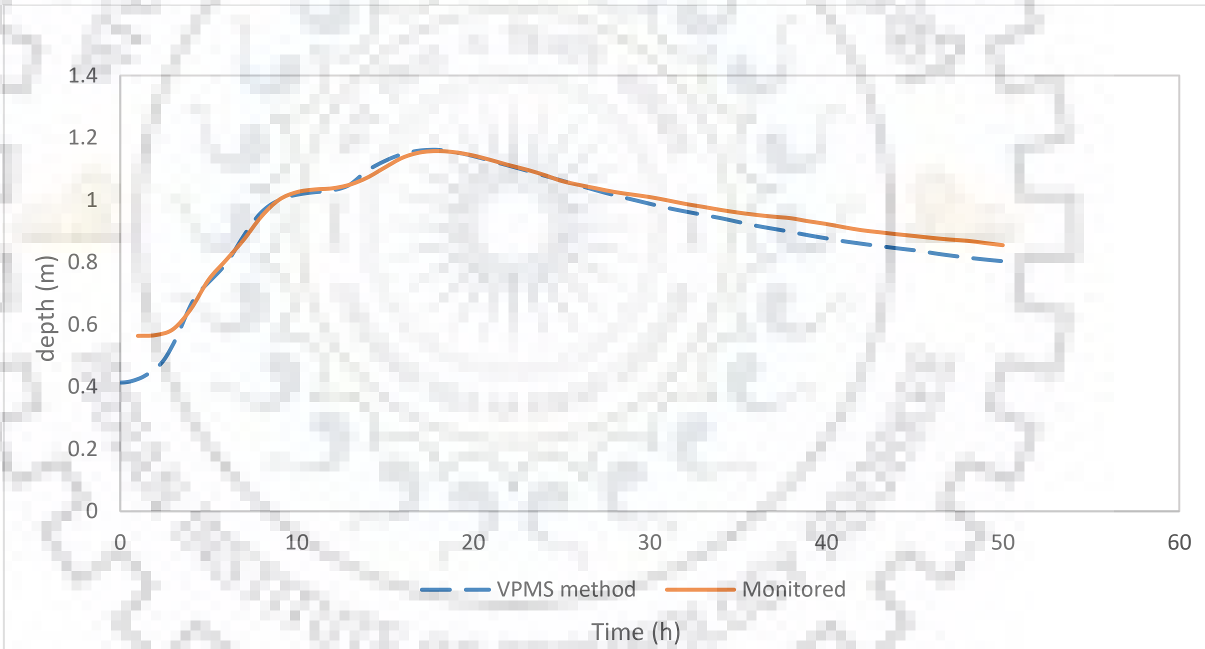

Fig.4. 4 Comparison of monitored downstream depth with the corresponding simulated hydrograph using the VPMS method

The NSE and RMSE estimates for the VPMS method with respect to the explicit finite difference method for the hypothetical stage hydrograph are 0.9994 and 0.43, where the explicit finite difference solution is considered as the benchmark solution. The NSE and RMSE estimates for the VPMS method for the Sabie River analysis with respect to the monitored one are 0.9304 and 0.027.Therefore, the VPMS method described herein considering stream-aquifer interaction can be applied for flood propagation studies in natural rivers and in man-made canals. Perumal and Ranga Raju (1998b) demonstrated that the VPMS method without considering lateral flow can be applied for routing the input-stage hydrographs characterized by the criterion $\left|\frac{1}{S_0}\frac{\partial y}{\partial x}\right| << 1$.

4.6 CONCLUSION

The VPMS routing method considering stream-aquifer interaction is more straightforward than the VPMM method considering stream-aquifer interaction. Accordingly, the VPMS method for flood propagation studies considering stream-aquifer interaction has been developed herein. The procedure for the application of the VPMS method considering stream-aquifer interaction has been described in this chapter. The developed method has been verified for the hypothetical data by reproducing the benchmark solution. Subsequently, the proposed method was applied for a field problem of the Sabie River as investigated by Birkhead and James (1998) who used the non-linear Muskingum method for routing in the channel reach.

CHAPTER 5

ONE-DIMENSIONAL CHANNEL ROUTING CONSIDERING TWO-DIMENSIONAL FULLY PENETRATING STREAM-AQUIFER INTERACTION

5.1 GENERAL

Ground water and surface water of the hydrological cycle are not isolated components, instead the storages and flow processes within them are hydraulically connected with each other. Interaction (stream-aquifer) among these components affect both their quality and quantity. A realistic accounting of the stream-aquifer interaction is the required for effective water resources management of a basin in order to cater for the increased water demand due to ever growing population around the world (Courbis et al., 2008; Ma et al., 2008). A stream-aquifer interaction can take place under: (a) fully-penetrating or (b) partially-penetrating conditions i.e., when the level of stream channel bed corresponds to the bed level of the unconfined aquifer surrounding the stream channel. Both scenarios may prevail in nature. In this chapter, the former scenario is considered. The interaction of a fully penetrating stream and an aquifer has been analyzed by several investigators in the past (Ferris 1952, Cooper and Rorabaugh 1963, Hornberger et al., 1970, Morel-Seytoux 1988). In real scenarios, however, the aquifer flow is generally two-dimensional. Therefore, in the present chapter, the numerical models have been developed for solving the stream-aquifer system when the interface of a fully penetrating stream channel is surrounded by the aquifer wherein the flow is considered to be two-dimensional.

5.2 PROPOSED METHOD FOR SOLVING THE AQUIFER SYSTEM

In this chapter, the sub-surface flow has been considered as two-dimensional which implies that the flow in aquifer is perpendicular to the stream-section as well as parallel to the river-reach. Such a generalized flow movement assumption in the aquifer is valid when the sub-soil is anisotropic in nature and the hydraulic conductivities along both the directions varies significantly in magnitude, but not individually negligible. However, this model can be applied for the one-dimensional flow as well, if it is considered that the hydraulic conductivity along the direction to the stream axis is very low in comparison to that along the direction normal to the stream axis.

The assumptions considered in the development of the VPMM and VPMS methods considering river-aquifer interaction, are as follows:

1) The channel is a prismatic.

2) The considered unconfined stream-bank aquifer located on either side adjacent to the stream is symmetrical in form and assumed to be characterized by the same aquifer properties.

3) The channel fully penetrates the aquifer.

4) The flow in the stream is one-dimensional and two-dimensional flow prevails in the adjacent aquifer.

5) The initial water level in the aquifer is same as the water level in the stream prior to the arrival of flood wave in the stream channel.

6) During the progress of river-aquifer interaction process, no rain is recorded. This assumption avoids the aquifer storage variation due to percolation process.

However, the use of this assumption (4) does not restrict the application of the proposed method when one- dimensional flow prevails in the aquifer. Therefore, when the hydraulic conductivity of the bank soil strata along the direction of stream axis is very low or negligible in comparison to that along the direction normal to the stream axis, one may consider that the flow in the aquifer is predominately one-dimensional. The definition sketch of 2D flow in one-half of the symmetrical part the river-bank aquifer system is shown in Figs. 5.1(a) and 5.1 (b)

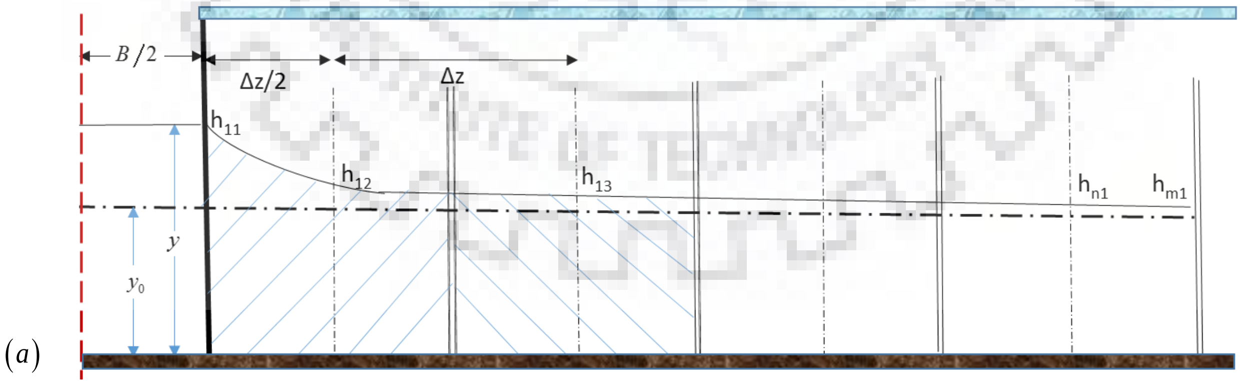

Fig. 5. 1 (a) Cross-section of the stream-aquifer system

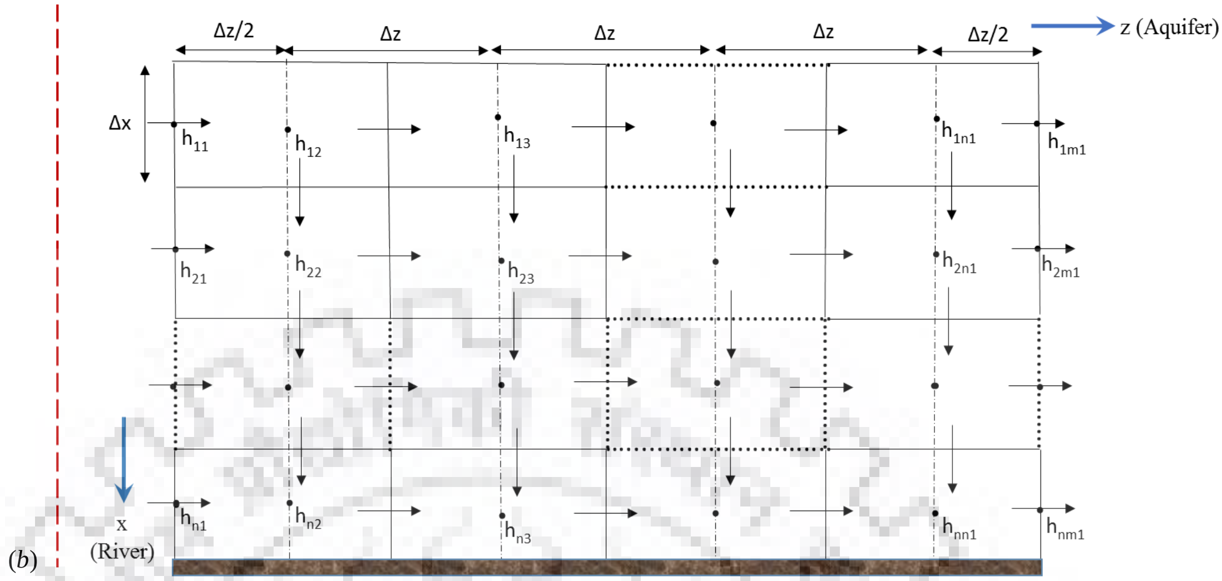

Fig. 5. 1 (b) Plan view of the stream-aquifer system

Consider a fully penetrating rectangular channel reach having length L, width B which is affected by the stream-aquifer interaction process. Assume the initial flow depth in the reach prior to the passage of flood wave in the reach as y_0 and after time *t*, it becomes y which is dependent on time (*t*). Let us denote the flow in the channel or river reach takes place along the x-direction and that in the stream-aquifer takes place along the z-direction. Let us consider that the aquifer adjacent to the river is characterized by the specific yield S_y, the hydraulic conductivity K_z along the direction perpendicular to the channel direction and K_x is the hydraulic conductivity along the direction of channel flow.

5.2.1 Determination of the first observation location in the bank-aquifer

For the purpose of developing the stream-aquifer interaction model, the aquifer is discretized into a seamlessly connected rectangular grid system to form an equal size grid-network, except for the two narrow strips, one immediately adjacent to the stream-aquifer interaction face and the other at the end of the aquifer grid- network far away from the stream. The size of the grid is decided in such a manner that the end section of the first grid located immediately adjacent to the stream is at a distance of 0.75 times greater than the maximum possible saturated thickness perpendicular to the stream-aquifer interaction face. The restriction on the size of the narrow strips is imposed in order to satisfy the applicability of one of the Dupuit's assumptions as advocated by Jacob Bear (1972). The grid size of the considered grid network is taken as twice that of the narrow strip, i.e., twice that of the distance of the first observation section; whereas, the last grid size is equal to that of the first grid (Fig.3 (b)). For each of the considered grids, the mass balance equation in the respective flow direction is expressed as

$$I - O = \Delta S/\Delta t \tag{5.1}$$

where, $I =$ rate of inflow volume $\left(m^3/s\right)$; $O =$ rate of outflow volume $\left(m^3/s\right)$; $\Delta S/\Delta t =$ rate of change of storage $\left(m^3/s\right)$; $S =$ aquifer storage volume $\left(m^3\right)$; $t =$ time $\left(s\right)$.

Application details of the mass balance equation for a typical grid of the considered grid-network of the aquifer, shown in Fig.5.1, are shown in Figs. 5.2-5.4.

For solving the aquifer system, the explicit finite difference method has been used in which the hydraulic head at the current time step can be evaluated using the information of head values of the previous time steps at the corresponding considered grid (Wang and Anderson, 1995).

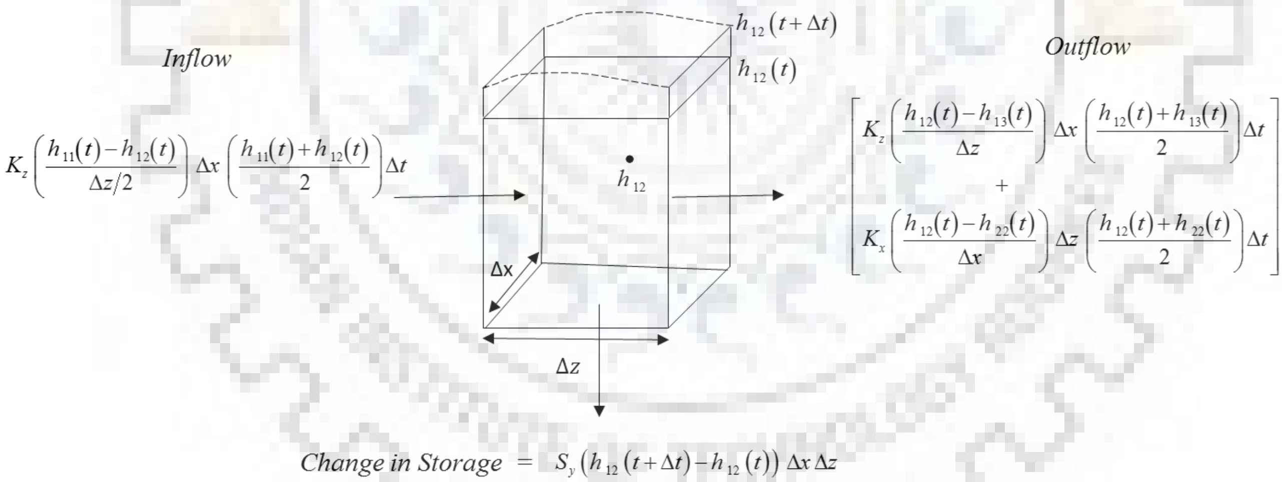

Fig. 5. 2 Mass conservation in h12 grid

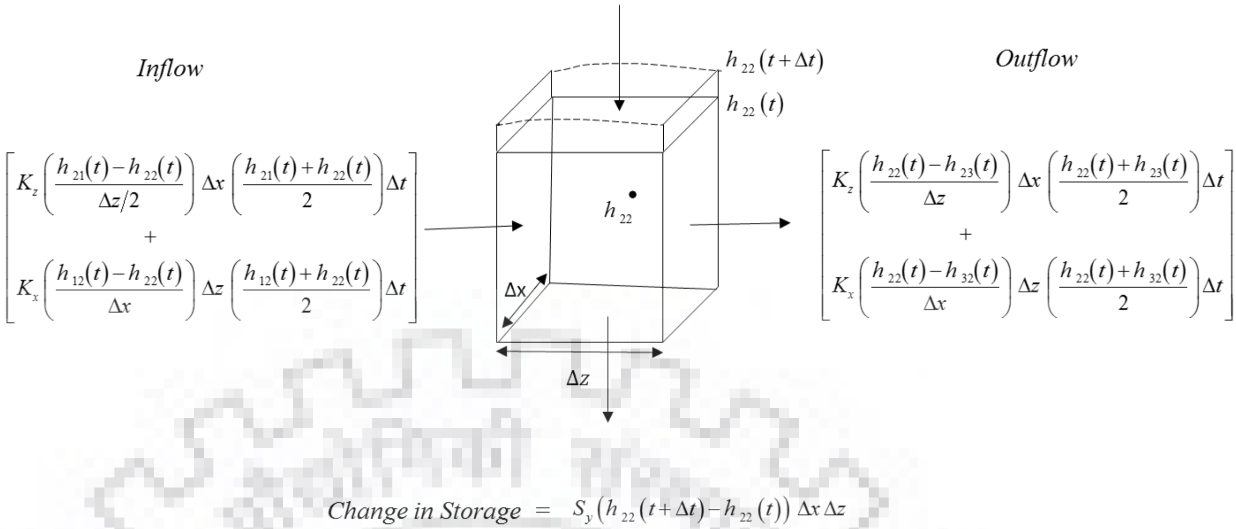

Fig. 5. 3 Mass conservation in h22 grid

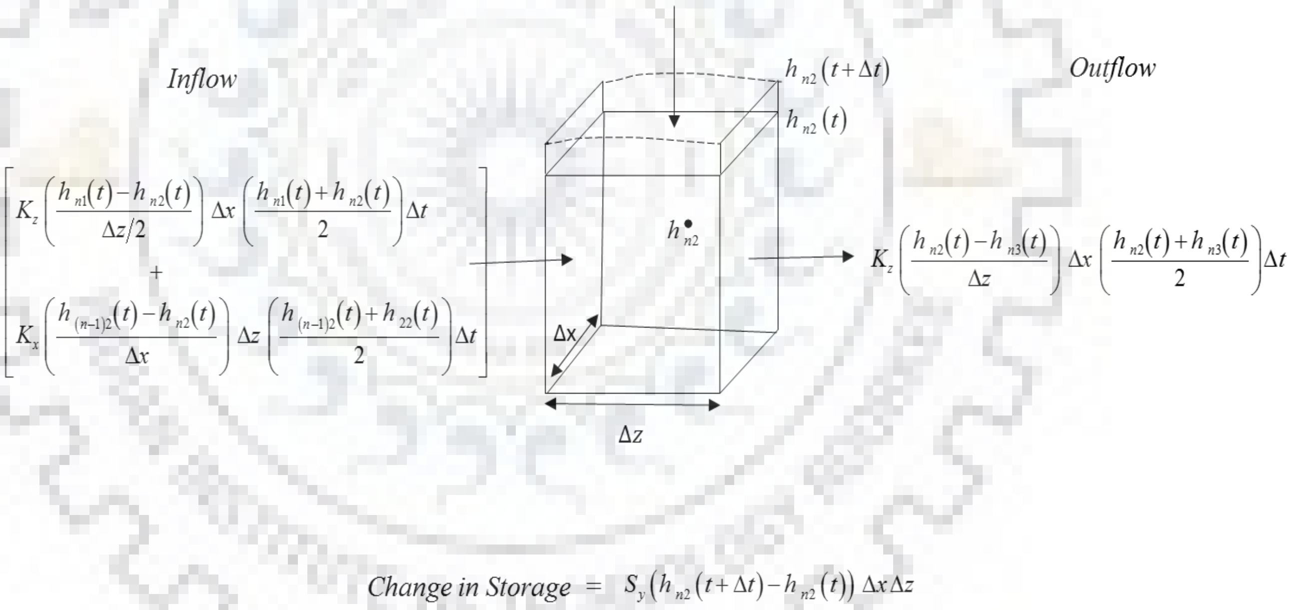

Fig. 5. 4 Mass conservation in hn2 grid

The flow between stream and aquifer per unit length per side can be defined by Darcy's law. It can be expressed as

$$q_L = K_z i A_a \tag{5.2}$$

where, q_L = rate of water flow between stream and aquifer $\left(m^3/s/m\right)$; K_z = hydraulic conductivity of the aquifer normal to the stream-axis (m/s); K_x = hydraulic conductivity of the aquifer parallel to the stream-axis (m/s); i = hydraulic gradient (dimensionless); A_a = aquifer cross-sectional grid area of flow $\left(m^2\right)$;

The lateral flow/unit reach length in/out to the aquifer per side, following the Darcy's law, for the grid h_{12} at the time $t+\Delta t$ in the finite difference form is written as:

$$q_{L,1}(t+\Delta t) = K \times \frac{\left(y_{M,1}(t+\Delta t) - h_{12}(t+\Delta t)\right)}{(\Delta z/2)} \times \frac{\left(y_{M,1}(t+\Delta t) + h_{12}(t+\Delta t)\right)}{2} \tag{5.3}$$

where, $y_{M,1}(t+\Delta t)$ is the flow depth at the mid-section for the first sub-reach along the channel length for the time step $t+\Delta t$.

The positive value of in Eq. (5.3) represents the influent stream (losing stream), whereas the negative value represents the effluent stream (gaining stream) (Sophocleous, 2002). Following the same procedure as used in Eq. (5.3) for the other points of the grid-network of the aquifer, the values of phreatic surfaces at any location along the flow direction of the aquifer and the flow in/out to the aquifer can be obtained at any instant of time.

5.3 VALIDATION OF THE METHOD

5.3.1 Step-rise input

Consider a rectangular channel which fully penetrates the aquifer. The channel and aquifer characteristics are given in Table. 4.1.

The stream stage at the upstream is given by

$$y(x,0) = y_0 \quad \text{where} \quad x=0 \ \text{ and } \ t=0 \tag{5.4a}$$

$$y(x,t) = 2y_0 \quad \text{where} \quad x=0 \ \text{and} \ t>0 \tag{5.4b}$$

The initial and boundary conditions for the solution of the Boussinesq equation can be described as

$$h(z,0) = y_0 \quad \text{where} \quad 0 \le z \le Z_{max} \quad , t = 0 \tag{5.5a}$$

$$h(Z_{max}, t) = y_0 \quad \text{where} \quad z = Z_{max} \quad , \quad t > 0 \tag{5.5b}$$

$$h(0,t) = y(0,t) \quad \text{where} \quad z = 0, \; t > 0, \; x = 0 \tag{5.5c}$$

For the given stage hydrograph, the amount of bank storage has been calculated here using the VPMS method and the explicit finite difference method for the flow generated by the full SVE. In this procedure, the temporal step-size has been taking as $\Delta t = 30s$, and the spatial step size is $\Delta x = 2000m$. Again, the bank storage has been evaluated analytically using the Eqs. (2.25), (2.26) and (2.28) where the first observation point at $\Delta z = 5m$. The obtained stage hydrographs at the upstream and downstream sections of the stream reach with and without the consideration of stream-aquifer interaction has been shown in Fig. 5.5.

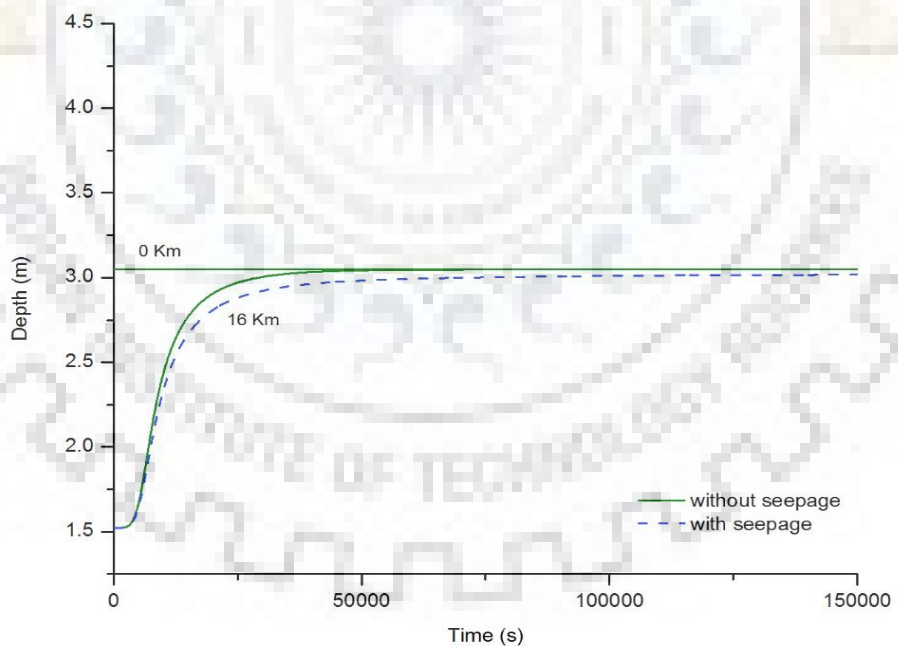

Fig. 5. 5 Stage hydrograph with and without considering seepage at the downstream

The bank storage and seepage corresponding to the stream stage at the middle of the first sub-section of the stream reach are shown in Fig. 5.6.

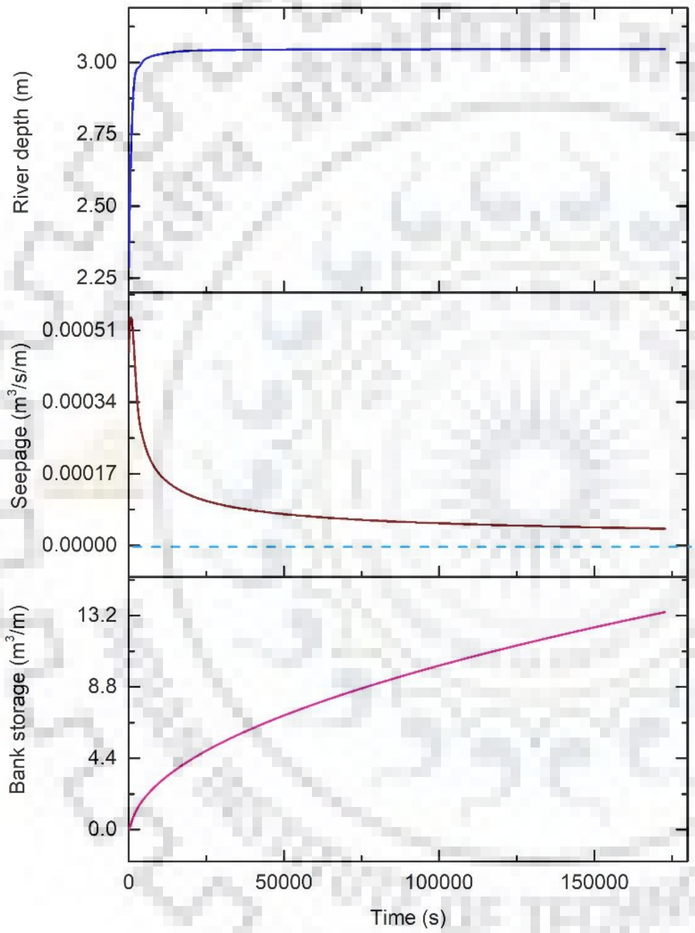

Fig. 5. 6 Stream stage, seepage and bank storage for the first sub-section

From the Fig. 5.6., it is clear that for the first sub-section in the direction of channel flow, the stream stage rises rapidly from the initial stage height to maximum stage height as given in Eqs. (5.4a) and (5.4b) and maintained the same thereafter. The corresponding flow depth for channel flow continuously contributed to the bank seepage per unit length per side towards aquifer in the direction normal to the streamflow. Therefore, the value of seepage is always positive for the simulation time. As a result of this, the bank storage volume per unit length per side is continuously increasing.

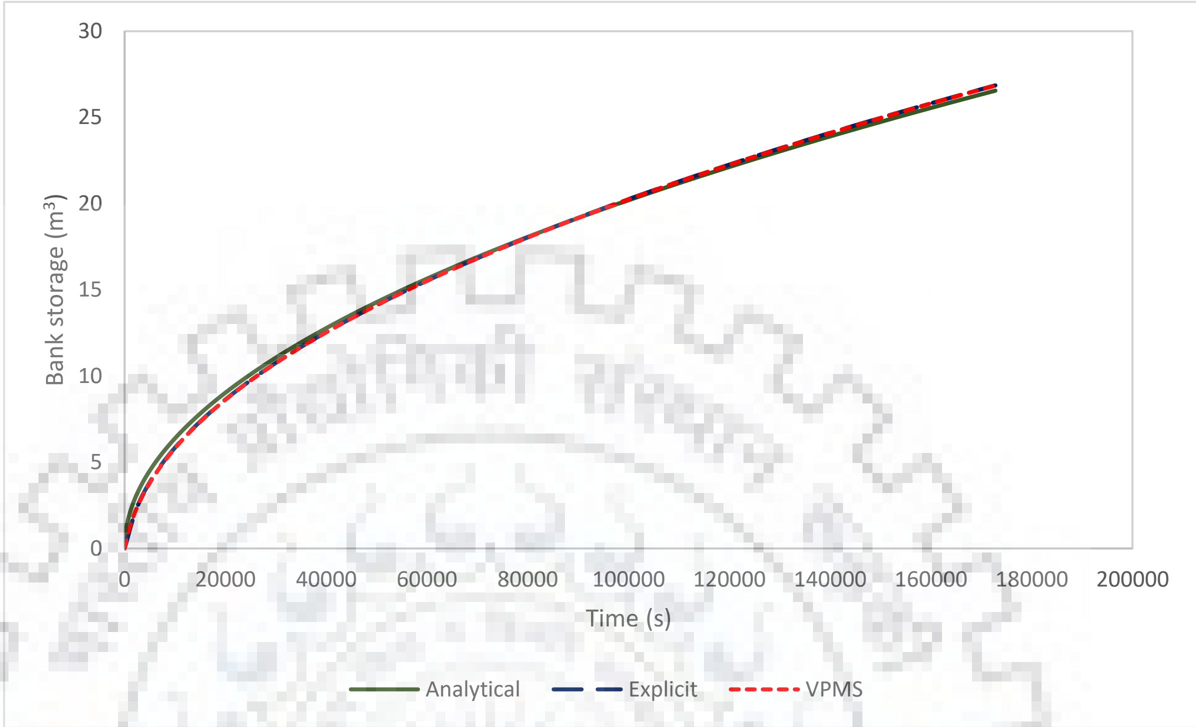

Fig. 5. 7 Comparison of bank storage using numerical methods with analytical solution for step-rise input

The bank storage obtained from the analytical method given in Eq. (2.29) in Chapter 2, has been compared with the explicit method and the VPMS method. The comparison is also shown in Table. (5.1.)

Table 5. 1 Comparison of the estimated bank storage using the Explicit and VPMS methods with the analytical solution of Morel-Seytoux (Mishra, unpublished book)

Time(sec)	Analytical	Explicit	VPMS
3600	3.831457	3.11586	3.07968
7200	5.419339	4.82082	4.77888
10800	6.637648	6.11058	6.08598
14400	7.664692	7.197	7.18008
18000	8.569518	8.15136	8.13702
21600	9.387532	9.0102	8.99742
25200	10.13977	9.7974	9.78588
28800	10.83993	10.5285	10.51782
32400	11.49753	11.21418	11.20416
36000	12.1195	11.86194	11.85246
39600	12.71107	12.47766	12.46866
43200	13.27631	13.0653	13.05678
46800	13.81845	13.629	13.62084
50400	14.34011	14.1711	14.1633
54000	14.84345	14.6943	14.68686
57600	15.33027	15.19986	15.19266
61200	15.80209	15.68994	15.68298
64800	16.26024	16.16586	16.15914

5.3.2 Zitta and Wiggert (1971) solution

The methodology for evaluating the flow considering stream-aquifer interaction, where the flow in the aquifer is two-dimensional has also been verified for Zitta and Wiggert (1971) solution from chapter 3, wherein the flow in the aquifer was considered as one-dimensional. In the present methodology, if it is considered that the hydraulic conductivity along the direction of channel is very less than that of the direction perpendicular to the channel, then the flow in the aquifer can be considered as one-dimensional. The numerical grid size of the computational mesh of the aquifer along the direction of the channel is taken as the same as that of the sub-reach length of the channel and in the direction perpendicular to the channel path, which is $\Delta z = 6m$ with a time step as $\Delta t = 60s$. The first observation point in the aquifer is considered at the distance of half of the selected grid size (i.e. $\Delta z/2$) following the Dupuit's assumptions (Bear, 1972), as discussed earlier.

The comparison of the hydrographs obtained by the proposed method with the benchmark solution is shown in Fig.5.8.

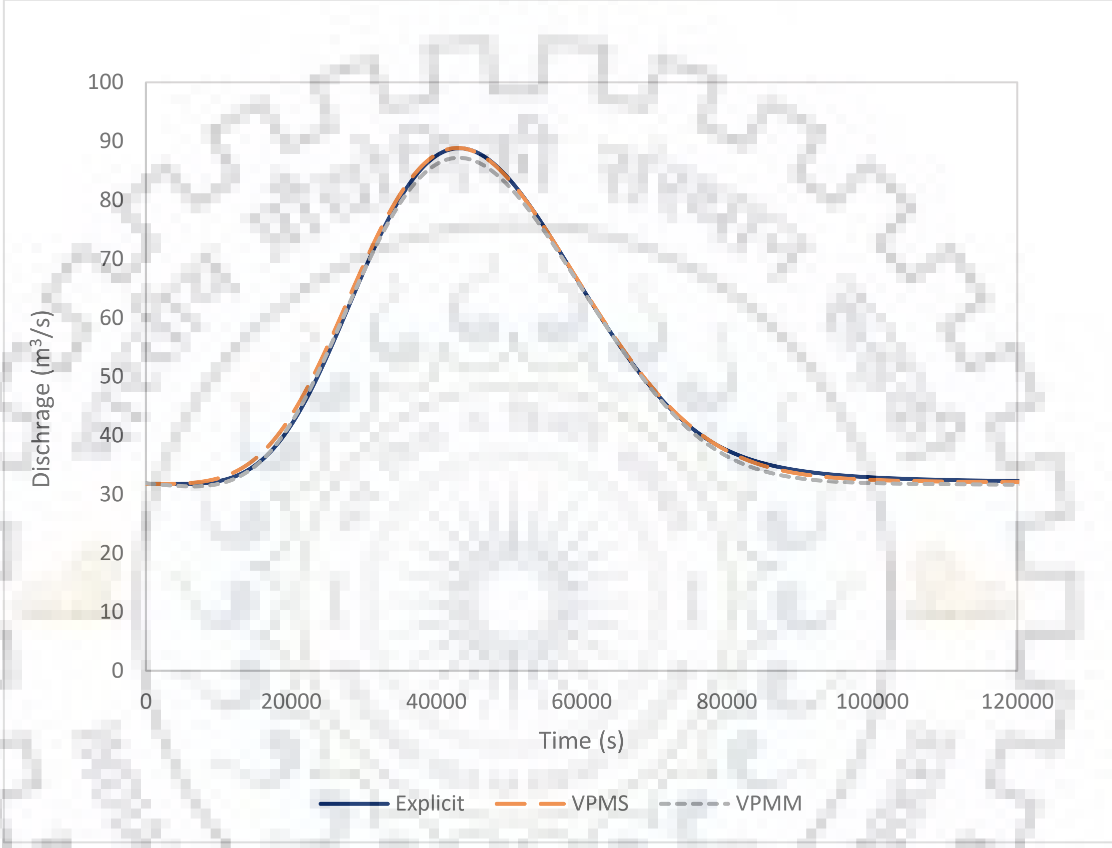

Fig. 5. 8 Comparison of discharge hydrographs obtained by the VPMM and VPMS methods with the explicit solution

Table 5. 2 Efficiency of adopted Methods for Zitta and Wiggert (1971) solution

Procedure adopted	**NSE**	**RMSE**
VPMS method	0.9992	0.52
VPMM method	0.9983	0.76

The conservation of mass has been proved using the Simpson's 1/3 rule for averaged flow. The detailed computation process is explained in Appedix A.

5.3.3 Validation using Birkhead (2002) solution

Birkhead and James (1998, 2002) used a hypothetical hydrograph for routing in a prismatic rectangular channel subjected to stream-aquifer interaction. Birkhead and James (1998) modified the traditional nonlinear Muskingum routing equations to synthesize the rating relationship and Birkhead and James (2002) modified the nonlinear Muskingum procedure for the stream-aquifer interaction considering permeable river banks of varying hydraulic conductivity. The form of the hydrograph is expressed as:

$$Q = Q_p + \left(\frac{Q_p - Q_0}{2}\right)\left(1 - \cos\frac{\pi t}{t_p}\right) \tag{5.6}$$

In this Chapter, using the given inflow hydrograph, the downstream discharge has been obtained using the explicit, VPMM and VPMS methods considering stream-aquifer interaction as explained earlier in the Chapters-3 and 4. The obtained hydrograph has been compared with the solution obtained by Birkhead and James (2002). The channel characteristics and other routing parameters are shown in Table 5.3., as follows:

Table 5. 3 Data for unsteady flow computations in a rectangular channel (Birkhead and James, 2002)

Characteristics of channel	
Reach length	1250 m
Channel width	10 m
Bed slope	0.001
Manning's roughness	0.030
Hydraulic data	
Q_0	5 m^3/s
Q_p	50 m^3/s
t_p	1.5 h
t_b	3.0 h
Routing parameters	
Δx	250 m
Δt	20 s

The analysis of stream-aquifer interaction for the given channel characteristics and flood hydrograph can be done along with the riverbank characteristics (Birkhead and James, 2002) given in Table.5.4.

Table 5. 4 Riverbank characteristics

Hydraulic conductivity K_z (m/s)	0.01
Hydraulic conductivity K_x (m/s)	0.0001
The extent of river bank per side (m)	100
Effective porosity	0.3

5.3.3.1 Determination of the first grid size in the direction of aquifer flow

For the considered channel sections, the grid size in the direction of the aquifer has been taken as $\Delta z = 31\, m$. The first observation point in the direction of the aquifer-flow has been taken at the distance $\Delta z/2$, in order to satisfy one of the Dupuit's assumptions (Bear, 1972).

The upstream hydrographs and the obtained downstream hydrographs are shown in Fig. 5.9.

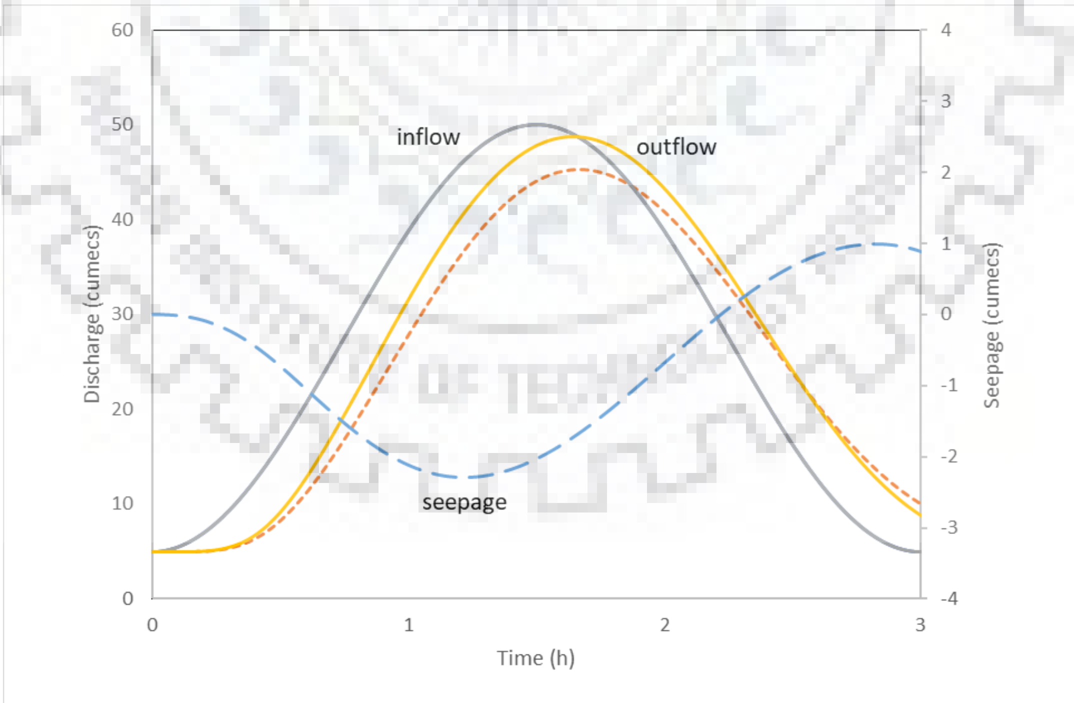

Fig. 5. 9 Discharge hydrographs at upstream and downstream with seepage

The hydrographs at the downstream section considering seepage are also compared with the previously available solution (Birkhead and James, 2002), which is shown in Fig. 5.10.

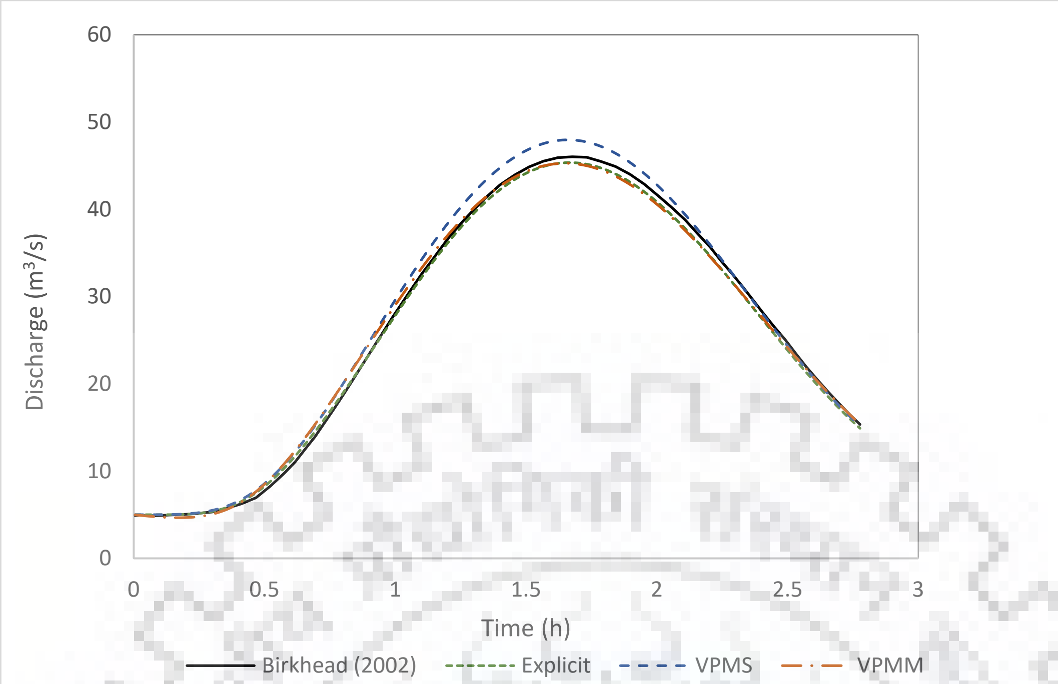

Fig. 5. 10 Downstream discharge hydrographs compared with the corresponding digitized hydrograph of Birkhead (2002) solution

The NSE and RMSE estimates with respect to the Birkhead and James solution (2002) for the adopted methods are shown in Table 5.5.

Table 5. 5 Efficiency of adopted Methods for Birkhead and James (2002) solution

Procedure adopted	NSE	RMSE
Explicit method	0.9983	0.59
VPMM method	0.997	0.76
VPMS method	0.9927	1.23

For the application of VPMS method for the given data in Tables 5.3 and 5.4 and Eq. (5.6), firstly the channel routing has been performed using the solution of full SVE using explicit finite difference method without considering the stream-aquifer interaction so that the flow depth at the upstream can be obtained for the application of VPMS method. Then the VPMS method has been applied for the given data.

It can be inferred from Fig. 5.10, that the explicit solution and the VPMM method have reproduced approximately the same hydrograph as compared to the Birkhead (2002) because the discharge at the upstream has been used as the input for these methods. In the case of the VPMS method, it is clear that the discharge slightly deviates at the peak of the hydrograph. The reason may be attributed to the conversion of discharge to stage and then the use of stage as the input information. Therefore, it can also be said that due to the conversion of discharge into the stage, the final output may slightly deviate from the monitored data. Therefore, for the close reproduction, it is suggested to use the original data.

5.4 CONCLUSIONS

With the moderate data requirements, the VPMM and VPMS methods can be chosen to evaluate the bank storage for a river segment. The required data is the channel cross-sectional details and the discharge hydrograph at the section. In this chapter, three procedures have been adopted for producing discharge hydrographs considering bank storage on both the sides of the channel cross-section. The first procedure is the explicit solution of the full Saint-Venant equations, the second is the VPMS method (Perumal and Ranga Raju, 1998a) and the third is the VPMM method (Perumal and Price, 2013). Also, the hydrographs reproduced by the above-mentioned procedures have been compared with the existing solutions. Moreover, these methods simultaneously compute the stage hydrograph corresponding to a given inflow or routed discharge hydrograph. Therefore, for the evaluation of bank storage, these method provides the value of the hydraulic head which in terms equal to the river stage at each river-section. The VPMS and VPMM methods are relatively simple to understand and apply and would yield more accurate results in comparison to the procedures following the rating curve approach.

CHAPTER 6

ONE-DIMENSIONAL CHANNEL ROUTING CONSIDERING TWO-DIMENSIONAL PARTIALLY PENETRATING STREAM-AQUIFER INTERACTION

6.1 GENERAL

The study of stream-aquifer interaction under the condition of stream partially penetrating the surrounding aquifer is a very complex problem and, in general, many rivers are in partially penetrating form. In order to find the analytical solutions for this complex problem, studies were carried out by Hunt (1999) and Hunt et al. (2001) who provided an approximate analytical solution for the case of stream which penetrates the top of the aquifer and having a semi-permeable bottom with width zero. Following the Hunt's analytical solution, Darama (2001) and Fox et al., (2002) developed a stream-aquifer model to predict the aquifer drawdown. Butler et al. (2007) provided a new semi-analytical solution to describe the impact of pumping of groundwater on streams nearby.

An analytical model was developed by Li and Wang (2007) for an unconfined aquifer recharging a nearby lake. Considering stream-aquifer interaction process, an analytical model which considers the effect of stream stage on the hydraulic head of the aquifer was proposed by Zlotnik and Huang (1999) and Szilagyi et al., (2006). Srivastava et al. (2006) provided a new analytical solution to examine the stream stage fluctuations on the adjacent alluvial valley aquifer. Kim et al. (2007) presented a 2-D semi-analytical solution to analyze the stream-aquifer interaction in a coastal aquifer, where the groundwater level responds to the tides. Intaraprasong and Zhan (2009) improved the analytical solutions for this complex problem by considering pumping wells near the streams with low-permeability streambeds, and most importantly by simultaneously considering temporally and spatially varying stream stages. Assumptions like negligible drawdown in the source bed of a leaky aquifer and horizontal flow in an aquifer of infinite extent has been considered for the development of analytical solutions (Swamee et al., 2000; Zlotnik and Tartakovsky, 2008). Furthermore, in order to identify the range of applicability and validity of these analytical solutions, a series of two- and three-dimensional solutions were developed by Christensen et al. (2009, 2010).

In Chapters 3, 4 and 5, wave propagation studies have been carried out assuming the stream banks to be vertical and the stream to be fully penetrating the bank-aquifers. However, in some cases curvilinear stream banks also exist in nature. Under this scenario, it is also very probable that a stream partially penetrates the aquifer. In fact, most of the streams only partially penetrate the aquifers. Therefore, in this chapter, the flood propagation analysis using the VPMM and VPMS methods have been carried out for the case of stream-aquifer interaction taking place under partially penetrating conditions. In this study the streams are considered to be characterized by prismatic rectangular sections.

6.2 PROPOSED METHOD FOR CONSIDERING STREAM-AQUIFER INTERACTION DURING FLOOD WAVE PROPAGATION

In this chapter, it has been considered that the stream partially penetrates the adjoining aquifer and the sub-surface flow has been considered as two-dimensional which implies that the flow in the aquifer takes place both in perpendicular to the stream-axis as well as parallel to the stream-reach.

The assumptions made in the application of the VPMM and VPMS methods considering stream-aquifer interaction are as follows:

1) The channel is assumed to be characterized by a rectangular prismatic section.

2) The considered unconfined river bank aquifer located on either side adjacent to the stream is symmetrical in form and assumed to be characterized by the same aquifer properties.

3) The channel partially penetrates the aquifer.

4) The flow in the stream is one-dimensional and two-dimensional flow prevails in the adjacent aquifer.

5) The initial water level in the aquifer is same as the water level in the stream prior to the arrival of flood wave in the stream channel.

6) During the progress of river-aquifer interaction process, no rain is recorded. This assumption avoids the aquifer storage variation in the stream-bank aquifer due to percolation process.

However, the use of assumption (4) does not restrict the application of the proposed method when one- dimensional flow prevails in the aquifer, i.e., when the hydraulic conductivity of the bank soil strata along the direction of stream-axis is very low or negligible in comparison to that along the direction normal to the stream-axis. The definition sketch of 2D flow in one-half of the symmetrical part the river-bank aquifer system is shown in Figs. 6.1(a) and 6.1 (b)

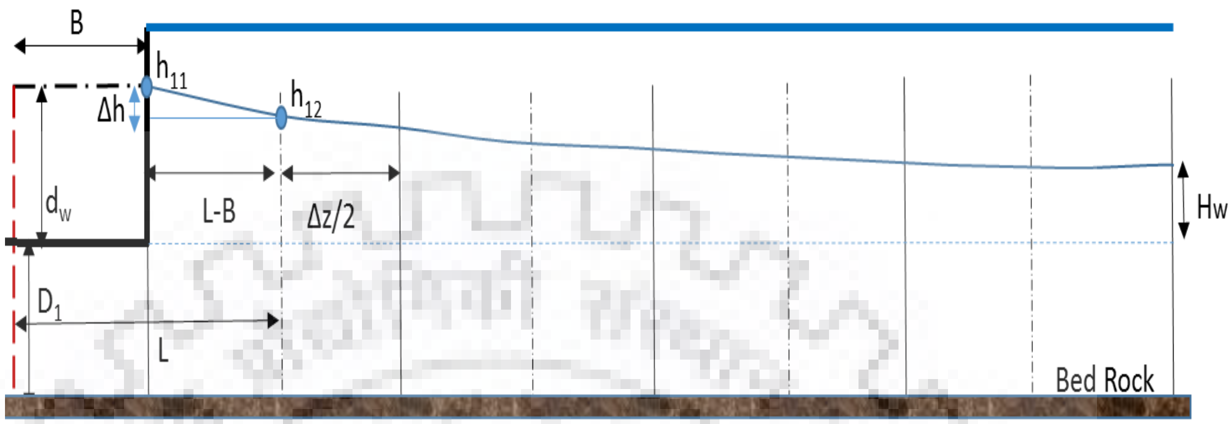

Fig.6. 1(a) Cross-section of the stream-aquifer system

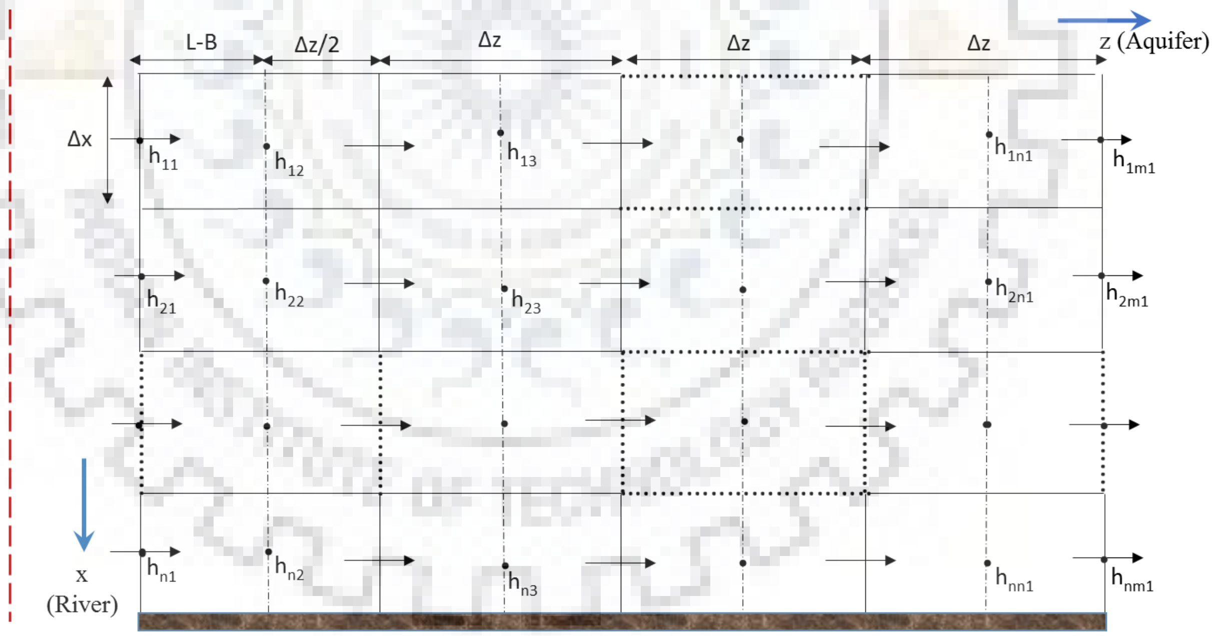

Fig.6. 1(b) Plan view of the stream-aquifer system

6.3 REACH TRANSMISSIVITY CONSTANT BASED ON THE SOLUTION OF MOREL-SEYTOUX et al. (1979)

Morel-Seytoux et al. (1979) derived the following approximate expression for seepage from a partially penetrating stream in an unconfined aquifer

$$Q = K_z \left[\frac{\Delta h}{L + 0.5D_1 - 0.5B} \right] \left[0.5w_p + D_1 + d_w - \Delta h \right] \Delta x \cong \Delta x \frac{T}{D_1} \frac{0.5w_p + D_1 + d_w}{L + 0.5D_1 - 0.5B} \Delta h \qquad (6.1)$$

where, $Q =$ seepage through a stream reach of length Δx, K_z = hydraulic conductivity of the aquifer perpendicular to the stream-axis, $2B =$ the width of the stream at the water surface and $T = K_z D_1 =$ transmissivity of the aquifer. The term in the first square bracket is an average hydraulic gradient. The term in the second square bracket is an average of the inflow and outflow areas. The inflow area is estimated as wetted perimeter, w_p of the stream multiplied by reach length Δx. The solution provided by Morel-Seytoux et al., (1979) is applicable for any type of cross-section. The total outflow area on both sides of the stream is equal to $2(D_1 + d_w - \Delta h)\Delta x$, $D_1 =$ the saturated thickness of the aquifer below the stream bed, $d_w =$ depth of water in the stream and $\Delta h =$ the difference in hydraulic heads in the stream reach and at an observation well located at a distance $L(= 5w_p)$ from the stream axis where Dupuit's assumptions are valid. The constant of proportionality Γ_r between seepage and the potential difference has been designated as reach transmissivity (Morel-Seytoux and Daly, 1975). The reach transmissivity constant is specific for the piezometer where the measurement of potential difference obtained (Morel-Seytoux et al., 1979).Therefore, the reach transmissivity Γ_r for unit length of stream reach is given by

$$\Gamma_r \cong K_z \frac{0.5w_p + D_1 + d_w}{L + 0.5D_1 - 0.5B} \qquad (6.2)$$

It is implied that the reach transmissivity would vary with L, the distance of the observation well from the stream axis. It is also a function of the wetted perimeter, thickness of aquifer below stream bed, width of the stream and the hydraulic conductivity of the aquifer.

6.4 REACH TRANSMISSIVITY CONSTANT DERIVED BY ARAVIN AND NUMEROV (1965)

Aravin and Numerov (1965) proposed an approximate solution for the evaluation of effluent seepage for a stream having rectangular cross-section which partially penetrates an unconfined aquifer. The flow domain has been decomposed into two sections: the unconfined flow section above the bed level of the stream and the confined flow section below the stream bed level. Conformal mapping technique has been applied for the computation of the effluent seepage through the stream bed, and the Dupuit's theory has been applied to quantify the component of effluent seepage through stream banks.

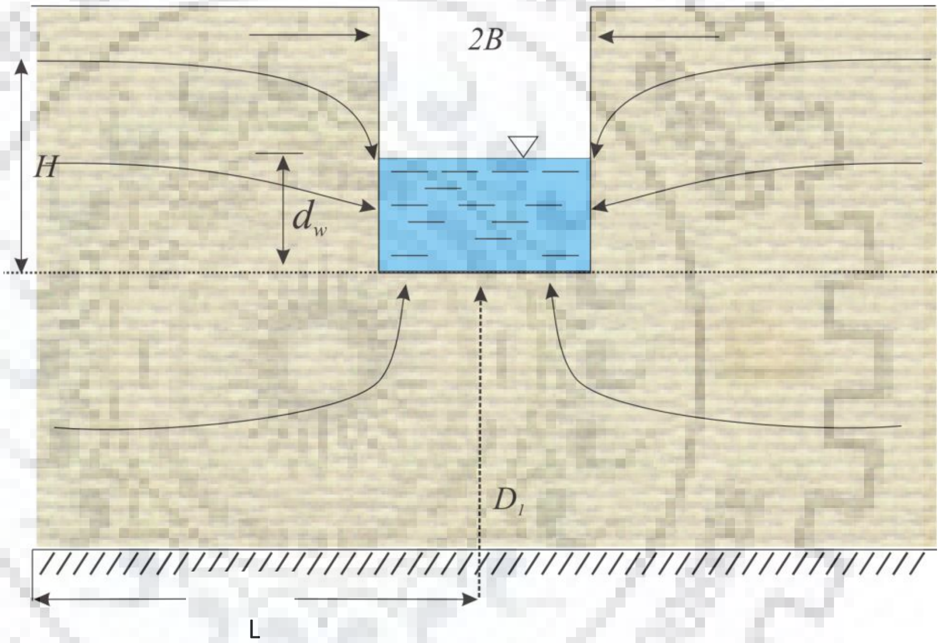

Fig.6. 2 A partially penetrating effluent stream with rectangular cross section

Using the solution of Aravin and Numerov (1965), the reach transmissivity constant for unit length of the stream having rectangular cross-section of top-width $2B$ and depth of water d_w is expressed as:

$$\Gamma_r = \frac{K_z\left(H+d_w\right)}{L-B} + \frac{K_z\pi}{0.5\pi L/D_1 - \ln\left\{\sinh\left(0.5\pi B/D_1\right)\right\}} \tag{6.3}$$

where, $D_1 =$ aquifer thickness under the stream bed, $H =$ the observed height of water table above the stream bed in an observation well preferably located at a distance L more than $\left(B+D_1+d_w\right)$ from the stream axis. The reach-transmissivity constant derived from Aravin and

Numerov's solution is also applicable to influent stream, if the phreatic lines are above the bed level of the partially penetrating stream.

6.5 REACH TRANSMISSIVITY CONSTANT BASED ON ARAVIN AND NUMEROV'S SOLUTION (Mishra, Unpublished work)

In a steady or quasi-steady state flow condition, the flow exchange between a hydraulically connected stream and an aquifer, where water table lies at shallow depth, is often assumed to be linearly proportional to the boundary potential difference (Ernst, 1962; Aravin and Numerov, 1965; Herbert, 1970; Morel-Seytoux, 1964; Morel-Seytoux and Daly, 1975; Besbes et al., 1978; Flug et al., 1980). Hydraulic connection of a stream with an aquifer infers that when there is a rise in water level in the aquifer below the stream bed, the seepage from the stream gets reduced and *vice versa*. Bouwer (1969) has stated that the seepage from a stream to an aquifer is directly proportional to the difference of the existing water levels in the stream and the aquifer, in the vicinity of the stream, where water table lies at a shallow depth. The constant of proportionality is known as reach transmissivity (Morel-Seytoux and Daly, 1975), which is related to the hydraulic conductivity of the aquifer medium, the stream cross section, and distance of the observation well location from the stream boundary at which the head difference is measured (Morel-Seytoux, 1964; Bouwer, 1969). The reach transmissivity constant is specific for the piezometer at which the potential difference is measured (Morel-Seytoux et al., 1979).

The seepage from a stream under steady as well as quasi-steady state flow, is estimated by multiplying the difference in hydraulic heads at the stream boundary and at a piezometer or at an observation well in the vicinity of the stream with the corresponding reach transmissivity constant. Aravin and Numerov (1965) and Bouwer (1969) have investigated the steady seepage from a stream relating to various stream geometry and boundary conditions. These solutions for steady state flow yield reach transmissivity constants, which are applicable to streams with similar sections.

6.5.1 A Rigorous Derivation of Reach Transmissivity Γ_r

In the present chapter, a rederived expression for reach transmissivity has been presented and the stream conductance of a partially penetrating stream with rectangular section has been comprehended in an unconfined aquifer assuming the phreatic lines lying above stream bed (Fig. 6.3). Following the Aravin and Numerov's approach (1965), the flow domain is decomposed into two sections. The seepage through stream banks is quantified using Dupuit's theory. The seepage through the stream bed is quantified using conformal mapping. For the application of the conformal mapping, the symmetry in the flow domain has been considered and, therefore, one-half of the flow domain has been shown in Fig.6.3. The steps of mapping are also given in Fig.6.3.

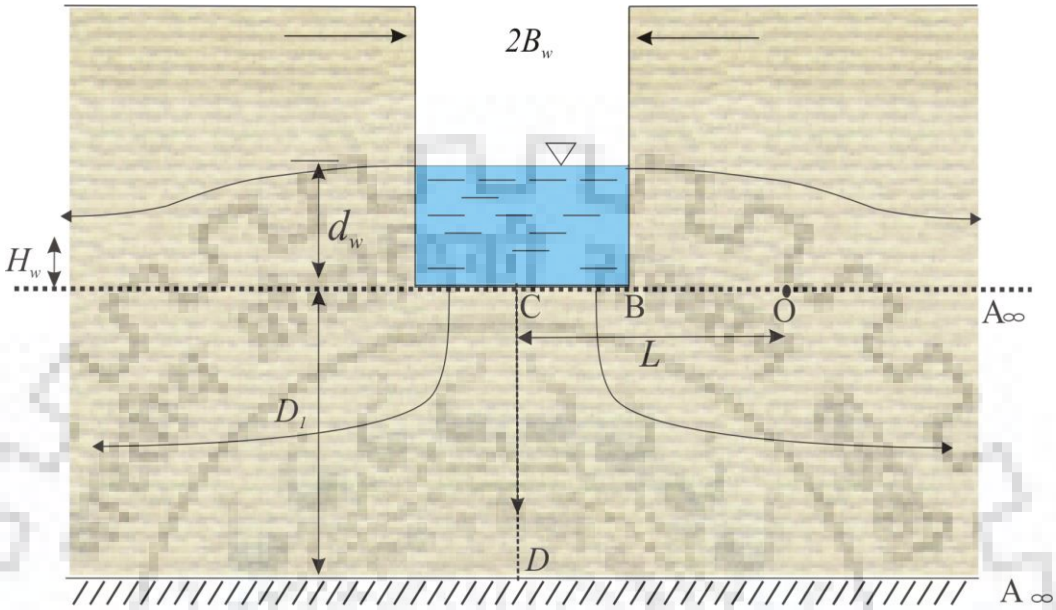

Fig.6. 3 A Partially Penetrating Influent Stream with Rectangular Cross Section in z=(x+iy) Plane

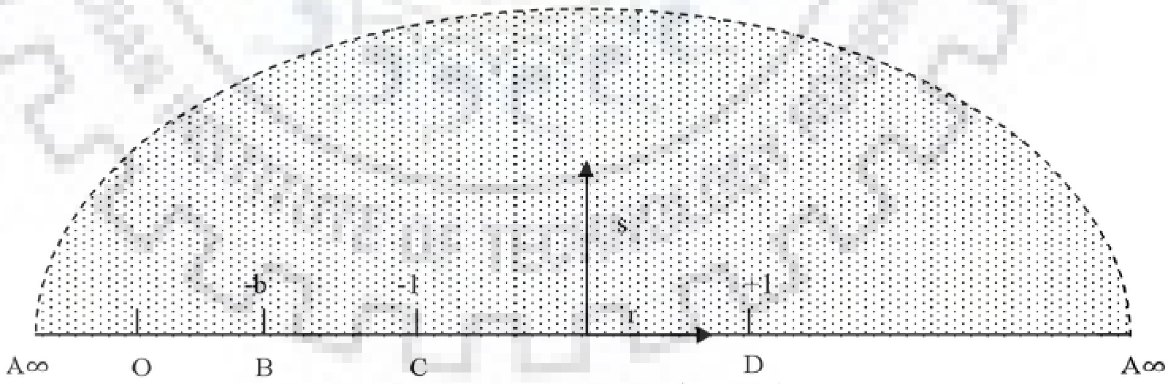

Fig.6. 4 Auxiliary t (=r+is) plane

The vertices A_∞, C, D, A_∞ in z - plane have been mapped onto points $-\infty, -1, 1, \infty$ in auxiliary t plane respectively.

According to Schwarz-Christoffel transformation (Harr, 1962), the conformal mapping of the flow domain in $z-$plane onto the auxiliary $t-$plane is given by (Harr, 1962):

$$z = M\int \frac{dt}{\sqrt{(1+t)(1-t)}} + N = M\sin^{-1}t + N \tag{6.4a}$$

For the vertex D, z = 0, and t =1; hence, the constant $N = -M\pi/2$. For vertex C, $z = iD_1$ and $t = -1$; therefore, the constant $M = -iD_1/\pi$. Substituting the constants M and N in (6.4a), we obtain the mapping function as:

$$z = \frac{-iD_1}{\pi}\sin^{-1}t + \frac{iD_1}{2} \tag{6.4b}$$

Conversely,

$$t = \cosh\frac{\pi z}{D_1} \tag{6.5}$$

Let the point $z = B + iD_1$ be mapped onto $t = -b$. Therefore, substituting $z = B + iD_1$ and $t = -b$ in Eq. (6.5), we find

$$b = \cosh\frac{\pi B}{D_1} \tag{6.6}$$

The complex potential $w(=\phi + i\psi)$ pertaining to the flow domain is shown in Fig.6.4, where $\psi =$ stream function, and $\phi =$ velocity potential function defined as:

$$\phi = -K_z\{p/\gamma_w + y\} + C^* \tag{6.7}$$

where, $K_z =$ hydraulic conductivity of the aquifer medium perpendicular to the stream-axis; $\gamma_w =$ unit weight of water ; $p =$ water pressure at location $z(=x+iy)$. The constant $C^* = K_z(D_1 + d_w)$ has been chosen and accordingly $\phi = 0$ has been assigned for stream boundary BC. $\psi = 0$ to the stream line CDA_∞ and $\psi = q_1$ to the to the stream line BOA_∞ have been assigned for the influent stream. For satisfying the Cauchy Riemann condition, q_1 needs to have a positive value.

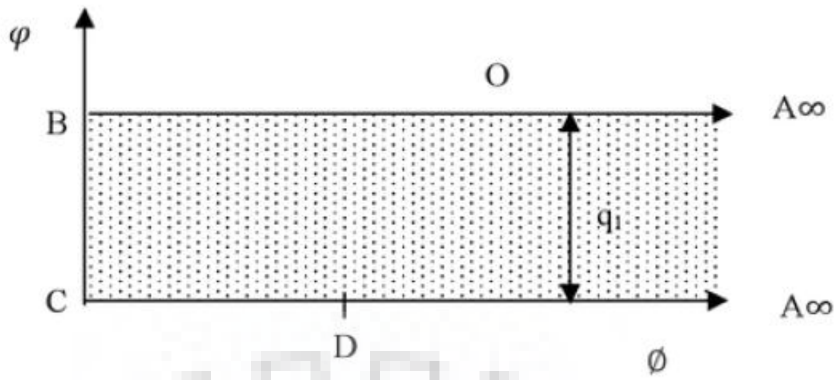

Fig.6. 5 Complex potential $w(=\phi+i\psi)$ Plane

The conformal mapping of the complex potential plane onto *t*- plane is given by:

$$w=M_1\int\frac{dt}{\sqrt{(1+t)^{1/2}(b+t)^{1/2}}}+N_1 \tag{6.8}$$

Substituting $t=(b-1)\tau-1, dt=(b-1)d\tau, \tau=\sinh^2 v, d\tau=2\sinh v\cosh v\,dv$ in succession, we get

$$w=2M_1\int dv+N_1=2M_1\sinh^{-1}\sqrt{\frac{1+t}{b-1}}+N_1 \tag{6.9}$$

For vertex C, $w=0$, and $t=-1, N_1=0$. For the vertex B in $w-$plane, $w=iq_1$, and $t=-b$. Therefore, $M=q_1/\pi$. The relation between w and t plane is

$$w=2\frac{q_1}{\pi}\sinh^{-1}\sqrt{\frac{1+t}{b-1}} \tag{6.10}$$

Conversely,

$$t=(b-1)\sinh^2\left(\frac{\pi w}{2q_1}\right)-1 \tag{6.11}$$

Incorporating Eq. (6.5), i.e. $t=\cosh\frac{\pi z}{D_1}$ in Eq. (6.11) $1+\cosh\frac{\pi z}{D_1}=2\cosh^2\frac{\pi z}{2D_1}$ and simplifying it,

$$\cosh\left(\frac{\pi z}{2D_1}\right)=\sqrt{\frac{(b-1)}{2}}\sinh\left(\frac{\pi w}{2q_1}\right) \qquad (6.12a)$$

For $\psi = 0$, i.e., along CDA, $w=\phi$, therefore,

$$\cosh\left(\frac{\pi z}{2D_1}\right)=\sqrt{\frac{(b-1)}{2}}\sinh\left(\frac{\pi \phi}{2q_1}\right) \qquad (6.12b)$$

At $z=iD_1/2$, the potential ϕ_a is given by

$$\phi_a=\frac{2q_1}{\pi}\sinh^{-1}\left\{\frac{1}{\sqrt{b-1}}\right\}=\frac{q_1}{\pi}\ln\frac{\sqrt{\cosh(\pi B/D_1)+1}}{\sqrt{\cosh(\pi B/D_1)-1}} \qquad (6.13)$$

Along BOA, the complex potential $w=\phi+iq$, therefore, from Eq. (6.12a)

$$\cosh\left(\frac{\pi z}{2D_1}\right)=\sqrt{\frac{(b-1)}{2}}\sinh\left(i\frac{\pi}{2}+\frac{\pi\phi}{2q_1}\right)=i\sqrt{\frac{(b-1)}{2}}\cosh\left(\frac{\pi\phi}{2q_1}\right) \qquad (6.14)$$

Substituting $z=x+iD_1$, $\cosh\left(\frac{\pi z}{2D_1}\right)=\cosh\left(i\frac{\pi}{2}+\frac{\pi x}{2D_1}\right)=i\sinh\left(\frac{\pi x}{2D_1}\right)$ in Eq. (6.14) and simplifying

$$\phi=\frac{2q_1}{\pi}\cosh^{-1}\left[\sqrt{\frac{2}{(b-1)}}\sinh\left(\frac{\pi x}{2D_1}\right)\right]=\frac{2q_1}{\pi}\cosh^{-1}\left[\operatorname{csc}h\left(\frac{\pi B}{2D_1}\right)\sinh\left(\frac{\pi x}{2D_1}\right)\right] \qquad (6.15)$$

The potential ϕ being positive, as shown in Fig.6.4, the relation $\cosh^{-1}(X)=\ln\left(X+\sqrt{X^2-1}\right)$ is applicable for an influent stream, whereas $\cosh^{-1}(X)=\ln\left(X-\sqrt{X^2-1}\right)$ is applicable for an effluent stream. Mathematically, $\left|\ln\left(X+\sqrt{X^2-1}\right)\right|=\left|\ln\left(X-\sqrt{X^2-1}\right)\right|$.

6.5.2 Reach transmissivity

Consider a piezometer be located at a distance $L(>B)$ from the stream axis on the stream line $\psi=q_1$.

The potential ϕ_0 at the piezometer from Eq. (6.15) is

$$\phi_0 = \frac{2q_1}{\pi}\cosh^{-1}\left[\csc h\left(\frac{\pi B}{2D_1}\right)\sinh\left(\frac{\pi L}{2D_1}\right)\right] \quad ;L > B \tag{6.16}$$

Let the piezometric head at $z = L + iD_1$ be p_0/γ_w. The potential at this location is given by

$\phi_0 = -K_z\left(D_1 + p_0/\gamma_w\right) + C^* = K_z\left(h_r - h_0\right)$, where h_r is the hydraulic head at the stream boundary equal to $D_1 + d_w$, and h_0 is the hydraulic head at the piezometer equal to $D_1 + p_0/\gamma_w$.

Incorporating ϕ_0 in Eq. (6.16) , and solving for q_1

$$q_1 = \frac{K_z\pi}{2\cosh^{-1}\left[\csc h\left(\frac{\pi B}{2D_1}\right)\sinh\left(\frac{\pi L}{2D_1}\right)\right]}\left(h_r - h_0\right) \tag{6.17}$$

Let H_w be the water table height above stream bed level in the conceptualized unconfined flow domain at distance $L - B$ from the stream bank. Applying Dupuit's theory, the unconfined seepage through the stream bank is

$$q_2 = k\frac{\left(d_w + H_w\right)}{2}\frac{\left(d_w - H_w\right)}{L - B} \tag{6.18}$$

The total seepage from the stream is given by

$$Q = 2\left(q_1 + q_2\right) = \frac{K_z\pi}{\cosh^{-1}\left[\csc h\left(\frac{\pi B}{2D_1}\right)\sinh\left(\frac{\pi L}{2D_1}\right)\right]}\left(h_r - h_0\right) + \frac{K_z\left(d_w + H_w\right)}{L - B}\left(d_w - H_w\right) \tag{6.19}$$

As seen from Eq. (6.19), the component of seepage through the conceptual confined flow domain is linearly related to $\left(h_r - h_0\right)$; whereas, the component of seepage through the conceptual unconfined flow domain is non-linearly related to H_w. Eq. (6.19) suggests that a piezometer at bed level of the stream in the conceptual confined aquifer and an observation well in the unconfined flow domain above the piezometer need to be constructed in the vicinity of the stream to assess the exchange of flow between the stream and the aquifer.

Alternatively, we can express Q as

$$Q = \Gamma_{rc}\left(h_r - h_0\right) + \Gamma_{ru}\left(d_w - H_w\right) \tag{6.20}$$

where, $\Gamma_{rc} = \dfrac{K_z \pi}{2\cosh^{-1}\left[\csc h\left(\dfrac{\pi B}{2D_1}\right)\sinh\left(\dfrac{\pi L}{2D_1}\right)\right]}$, and $\Gamma_{ru} = \dfrac{K_z\left(d_w + H_w\right)}{L - B}$ (6.21)

The constant Γ_{rc} pertaining to confined flow depends only on the stream section and distance of the piezometer from stream-axis. It is independent of the depth of water in the stream and observed head at the piezometer. The constant Γ_{ru} pertaining to the unconfined aquifer flow depends on the depth of water in the stream, head at the observation well and its distance from stream bank, but not on the stream cross section. The constant Γ_{rc} remains same, whether the stream is influent or effluent, as $\left|\ln\left(X + \sqrt{X^2 - 1}\right)\right| = \left|\ln\left(X - \sqrt{X^2 - 1}\right)\right|$.

For $L \geq \left(B + D_1 + d_w\right)$, as implied in Aravin and Numerov's analysis (1969), the stream lines would be nearly horizontal and the equipotential lines would be vertical. Therefore, for $L \geq \left(B + D_1 + d_w\right)$, at $z = L + iD_1$, the piezometric head, p_0/γ_w, in the conceptual confined flow domain would be nearly equal to the water table height, H_w, above the stream bed level in the conceptual unconfined flow domain. Accordingly, the difference $\left(d_w - H_w\right)$ could be expressed as

$$\left(d_w - H_w\right) = \left(d_w + D_1 - D_1 - H_w\right) = \left(d_w + D_1 - D_1 - p_0/\gamma_w\right) = \left(h_r - h_0\right) \quad (6.22)$$

Making use of the observed head $L \geq \left(B + D_1 + d_w\right)$, seepage from the stream can be expressed as:

$$Q = \left[\frac{K_z \pi}{\cosh^{-1}\left[\csc h\left(\dfrac{\pi B}{2D_1}\right)\sinh\left(\dfrac{\pi L}{2D_1}\right)\right]} + \frac{K_z\left(d_w + H_w\right)}{L - B}\right]\left(h_r - h_0\right) = \Gamma_r\left(h_r - h_0\right) \quad (6.23)$$

The reach transmissivity constant, Γ_r, is given by

$$\Gamma_r = \frac{K_z \pi}{\cosh^{-1}\left[\csc h\left(\dfrac{\pi B}{2D_1}\right)\sinh\left(\dfrac{\pi L}{2D_1}\right)\right]} + \frac{K_z\left(d_w + H_w\right)}{L - B}, \; L \geq \left(B + D_1 + d_w\right) \quad (6.24)$$

The total seepage from the stream can be computed by observing the water table position in an observation well which should be located preferably at a distance more than $D_1 + d_w$ from the stream bank. In Table 1, we make a comparison of the rederived reach transmissivity with those derived by Aravin and Numerov (1965) and Morel-Seytoux (1979) corresponding to the location of an observation well at $D_1 + d_w$ from stream bank. As seen from Table 6.1, the simple method adopted by Morel-Seytoux is accurate enough to compute seepage from stream having $B/D_1 \geq 2.5$. The rederived reach transmissivity is same as that derived by Aravin and Numerov for $L \geq (B + D_1 + d_w)$.

The piezometric head at any location can be predicted by numerical modeling of groundwater flow in a stream-aquifer system. For simulation of the piezometric surface, the flow domain is discretized by a grid pattern, and at each grid node, the mass balance equation is written. The set of simultaneous linear equations in terms of unknown hydraulic heads are solved in discrete time domain satisfying the initial and boundary conditions. The stream is treated as a head dependent type hydrologic boundary. In the mass balance equation near the stream nodes, the exchange between the stream reach and the aquifer is incorporated assuming the exchange rate to be linearly proportional to the difference in stream stage and unknown piezometric head at the node under the stream axis.

Table 6. 1 Variation of reach-transmissivity constant Γ_r/K_z with B/D_1 , for $d_w/D_1=0.1$, $H_w/D_1=0.06$ distance of observation well from stream bank $(L-B)/D_1=1.1$

B/D_1	Γ_r/K_z (Morel-Seytoux)	Γ_r/K_z (Aravin and Numerov)	Γ_r/K_z (Rederived)
0.1	.788	.987	.992
0.2	.824	1.132	1.137
0.3	.857	1.223	1.227
0.4	.889	1.285	1.287
0.5	.919	1.329	1.330
0.6	.947	1.360	1.360
0.7	.974	1.383	1.382
0.8	1.000	1.399	1.397
0.9	1.024	1.411	1.409
1.0	1.048	1.420	1.417
1.5	1.149	1.438	1.434
2.0	1.231	1.442	1.438
2.5	1.298	1.443	1.439
3.0	1.355	1.443	1.439
3.5	1.403	1.443	1.439
4.0	1.444	1.443	1.439
4.5	1.481	1.443	1.439
5.0	1.512	1.443	1.439
6.0	1.565	1.443	1.439
7.0	1.608	1.443	1.439
8.0	1.643	1.443	1.439
9.0	1.672	1.443	1.439
10.0	1.697	1.443	1.439

6.6 APPLICATION OF THE METHOD

6.6.1 Channel Routing on the basis of Morel-Seytoux et al. (1979) solution based on reach transmissivity constant

Consider a rectangular channel which partially penetrates the adjoining aquifer. As shown in the Fig. 6.1(a) and 6.1(b), consider the stream flows in the x-direction and perpendicular to the stream direction, the aquifer lies i.e., in z-direction considering the remaining assumptions given in Section 6.2. The stage hydrograph at the inlet of the study reach has been described in Eq. (6.25).

$$y(t)=y_0+A_0\left[1-\cos\left(\pi t/T_c\right)\right] \tag{6.25}$$

where, $x = 0$ and $0 \le t \le 2T_c$, y = flow depth*(m)*, y_0 = initial flow depth*(m)*, t = time*(s)*, T_c = time of concentration*(m)*,

The initial and boundary conditions for ground water flow are given as:

i) $$h(t) = y_0 \tag{6.26}$$

where $0 \le z \le Z_{\max}$, $t = 0$

ii) $$h(t) = y_0 \tag{6.27}$$

where $z = Z_{\max}$, $t > 0$

iii) $$h(t) = y(t) \tag{6.28}$$

where $z = 0$, $t > 0$

where, h = height from the datum to the phreatic surface(m); z = distance perpendicular to the path of the channel (m); $Z_{\max}$ = maximum distance perpendicular to the channel path(m);

Table 6. 2 Numerical data

Channel Characteristics
y_0 =1.524m
Channel width=30.48 m
Length=16 Km
Bed slope=0.000189
Manning's n =0.025
T_c = 36000s
Aquifer characteristics
Hydraulic conductivity (K) =0.000945 m/s
Specific yield (S_y) =0.16
Z_{max} = 670 m
$D_1 = 2m$
Finite difference routing parameters
Δx =2000 m
Δt =60 s

6.6.1.1 Discretization of the distance of observation well

For the development of the model, the aquifer is discretized into seamlessly connected rectangular grids to form an equal size grid network except for two narrow strips, one is just adjacent to the river face and the other at the end of the aquifer boundary parallel to the stream-aquifer interaction face. The location of the observation well is considered at the first grid of the grid network which can be considered at the distance $L(=5w_p)$, where w_p is the wetted perimeter of the channel cross-section following Morel-Seytoux (1979) approach. Other grids will follow the pattern as given in Fig. 6.1(b). Using the developed methodology on the above described hydrograph, the outflow hydrograph obtained at the downstream section has been shown in Fig. 6.5 as follows. Therefore, for the given hypothetical case, consider $L \approx 167.5\,m$. Therefore $\Delta z = 335m$. The locations of all the observation points in the direction perpendicular to the channel are marked as shown in Fig. 6.1(b). For each of the considered grid using the mass balance equation in the respective flow direction, estimate the hydraulics heads.

Lateral flow estimation for the partial penetration case study using the VPMM and VPMS methods have been made following the same procedures as described in Sections 3.3.1 and 4.2.1 respectively. Applying Eq. (6.2), the value of reach transmissivity can be obtained and by using Eq.(6.1), the value of lateral flow can be estimated. This is used in the channel routing process described in the Sections 3.3.1 and 4.2.1, respectively to find the discharge at the downstream section.

6.6.2 Application of reach transmissivity approach using conformal mapping based on Aravin and Nemerov approach (1965) on the Platte River, Nebraska (U.S.A.)

The Platte River Basin (PRB) located in the heartlands of the United States of America. It encompasses an area of 230,362 km^2 having a wide variation of the climatic patterns. The basin stretches across three states: Colorado, Wyoming and Nebraska. The North and South Platte originates in Colorado, flows through Wyoming and both meet in Central Nebraska to form central and lower Platte River. A series of federal reservoirs have been built on the river throughout its course before reaching Nebraska. In Nebraska, the river flows into one of the largest reservoirs (14,447 hectares) known as Lake McConaughey. The mean annual temperature of the Platte River Basin ranges between 9 and 11^o C, and the precipitation patterns shows an increment from west (437.13 mm/year) to east (722.41 mm/year).

The study area for the present study lies between the cities of Grand Island and Kearney having a total reach length of 66 km. This river reach section has wide channel width of several hundred meters and a shallow depth of less than 1 m. Sands and gravels are the main deposited materials found in the study river area. The hydraulic conductivity of the river bed is found to be on higher side (average of 40.2 m/d) as found through numerous permeameter tests (Chen, 2004; Chen et al., 2008; Cheng et al., 2011; Landon et al., 2001) which means that the river is hydrologically well connected to the adjacent aquifers. The adjacent aquifers also known as the High Plains aquifers, also referred as “Ogallala Aquifer” which is an unconfined aquifer formed on alluvium. This aquifer is considered as the largest groundwater reserve in the United States of America with an estimated storage of about 3700.45 m^3 spread over eight states. An aquitard which consists of clay and silt unit beneath the alluvium separates it from the underlying Ogallala aquifer. The alluvium which consists mainly of sand and gravel has a high permeability with a hydraulic conductivity of about 100 m/day (Chen et al., 2003). But the aquitard beneath the alluvium has a very low hydraulic conductivity 0.001 m/day as discovered by Chen et al., (2005) through pumping tests. Whereas, the Ogallala aquifer which lies underneath the aquitard has a high permeability and hydraulic conductivity (5 m/d) than the aquitard, but less than the aquitard.

Table 6. 3 Channel and aquifer characteristics (Huang et. al., 2015)

Channel width (m)	250
Specific yield, S_y (dimensionless)	0.15
Transmissivity, $T\ \left(m^2/d\right)$	2000
Hydraulic conductivity, $K_h\ \left(m/d\right)$	100
Saturated thickness, $M\ \left(m\right)$	20

The discharge and stream stage were recorded at the Kearney gauge (USGS Station #06770200) and the Grand Island gauge (USGS Station #06770500). A groundwater monitoring well with a depth of 7.5 m is situated 42 m from the right river bank near the Kearney gauge. A groundwater monitoring well (USGS Well #405227098165601) with a depth of 9 m is situated 17.7 m from the left river bank near the Grand Island gauge.

6.7 RESULTS AND DISCUSSIONS

A comparative analysis among the three approaches Morel-Seytoux approach (1979), Aravin and Numerov approach (1965) and the approach using conformal mapping (Mishra, unpublished work) has been done for the partially penetrating stream with rectangular cross-section which is shown in the Table 6.1. The comparison shows that these approaches give nearly similar values.

The stage hydrograph given in Eq. (6.25) is simulated using explicit method, VPMM method and VPMS methods to find the discharge at the outlet of the study reach considering stream-aquifer interaction where stream is partially penetrating the adjacent aquifer. The discharge hydrographs obtained from explicit solution of full SVE have been shown in the Fig. 6.6 considering with and without lateral flow.

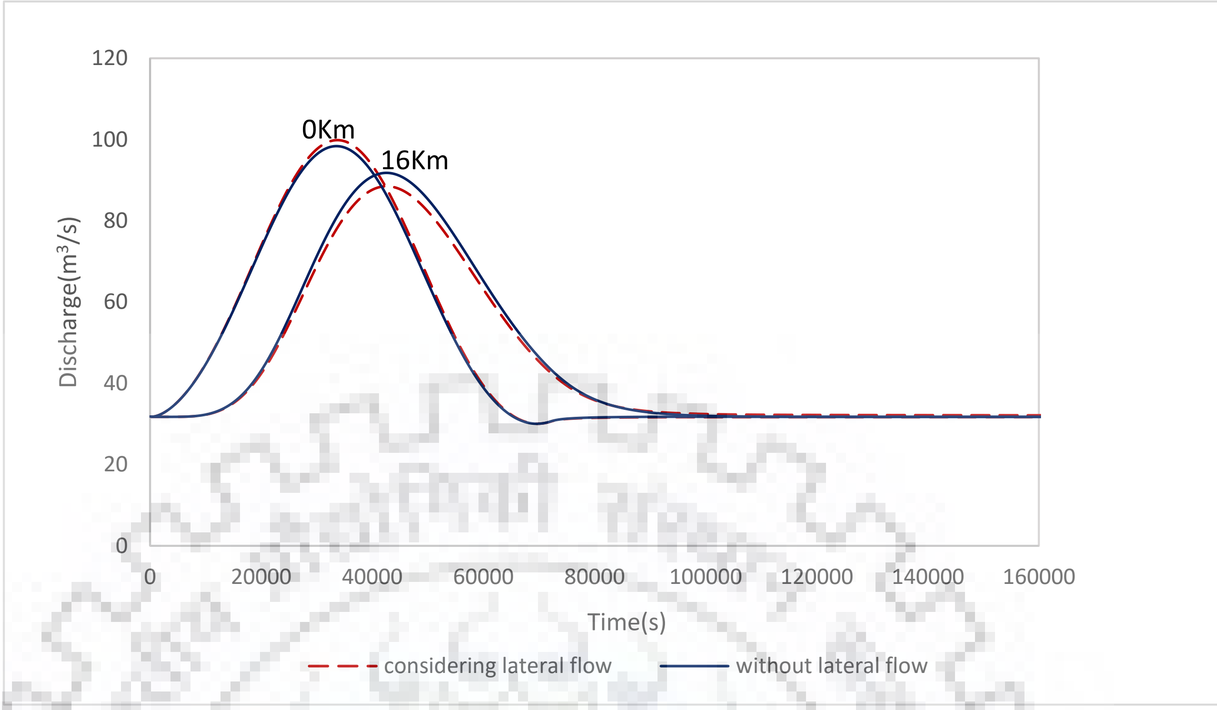

Fig.6. 6 Discharge hydrographs obtained using explicit solution of full SVE

The downstream discharge hydrographs in the Fig.6.6 shows very insignificant return flow in this case. If the discharges obtained from Fig. 6.6 and Fig.3.10 in Chapter 3, then it can be observed that the return flow in the fully penetrating stream is greater than the partially penetrating stream. Discharge hydrographs obtained at the downstream sections using VPMM and VPMS method for the input stage hydrograph given in Section6.6.1 are given in Fig. 6.7. Here explicit solution of full SVE is considered as benchmark solution.

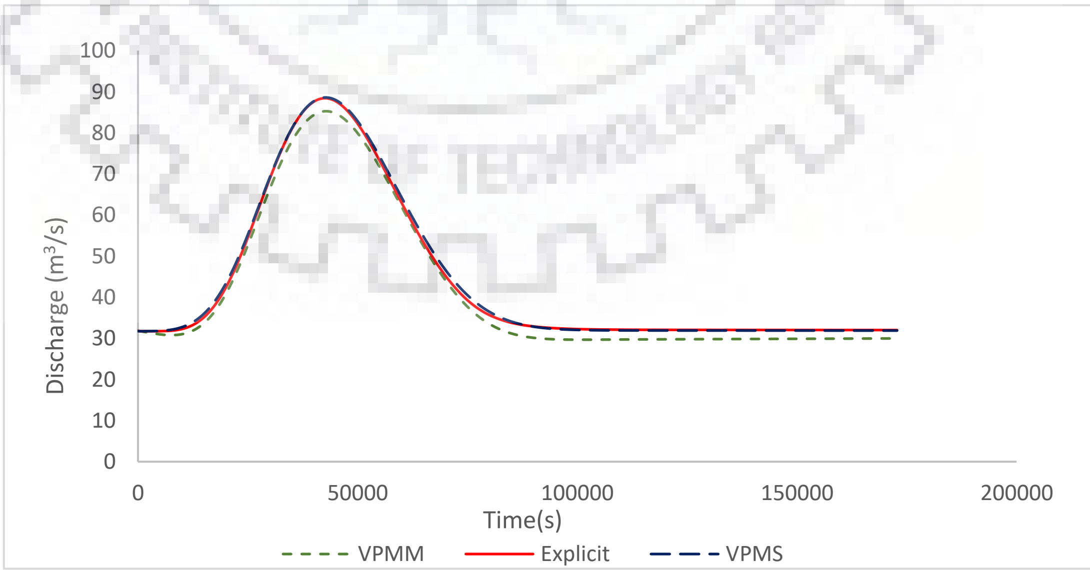

Fig.6. 7 Comparison of discharge obtained at the downstream of the study reach

The NSE estimate assessed for the simulated hydrographs of the VPMM and VPMS methods with respect to the corresponding explicit finite difference method considering as the benchmark solution is 0.9863 and 0.9989, respectively and the RMSE estimate for VPMM and VPMS methods is 2.15 and 0.59 respectively.

A flow event between 20th September and 1st October, 2013 recorded in the study reach to 1/10/2013 has been analyzed herein. The study reach has been divided into 20 each sub-reaches, i.e., the length of each sub-reach is equal to 3300 m and the time-step is equal to 15 min. Using the discharge recorded at the Kearney gauge (USGS Station #06770200), the VPMM method has been applied to simulate the discharge at the Grand Island gauge (USGS Station #06770500). The simulated discharge hydrograph at the downstream section of the study reach has been compared with the available monitored data. The discharge hydrographs obtained from the analysis has been shown in the Fig.6.8 as follows,

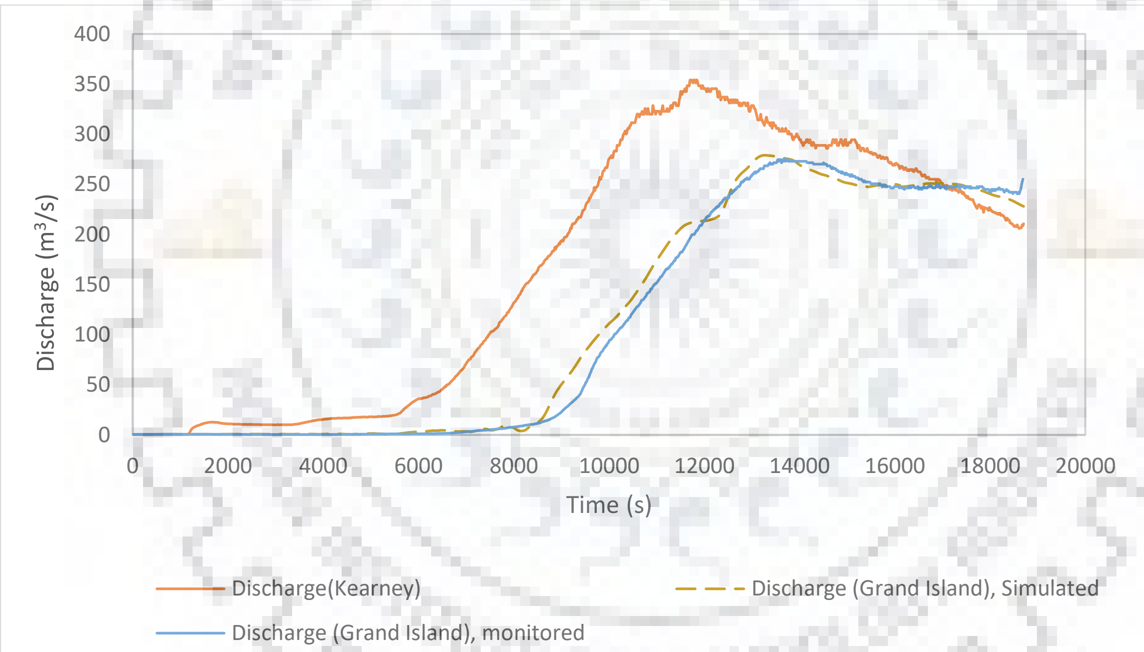

Fig.6. 8 Simulated and Monitored discharge hydrographs at the Grand Island using VPMM method

The NSE and RMSE estimates for the VPMM methods corresponding to the monitored data are 0.9922 and 10.26 respectively.

6.8 CONCLUSION

For a partially penetrating stream having rectangular cross-section, the lateral flow estimation using reach transmissivity has been explained using three approaches. Morel-Seytoux approach (1979) is relatively simple that has been derived using Darcy's law, average area of flow and average gradient for which the distance of the observation well should be at the distance of 5 time the wetted perimeter of the channel cross-section. Therefore the conformal mapping approach is better that does not required such condition. In this approach the seepage through the stream bank and the bed identifies separately. The application of the conformal mapping approach on the study reach of Platte River, Nebraska shows that it can give a good approximation of the lateral flow estimation for the channel routing processes considering stream-aquifer interaction.

CHAPTER 7

CONCLUSIONS

GENERAL

The major objectives of the study are concerned with the extension of the available one-dimensional streamflow routing methods, viz., the VPMM and the VPMS routing methods by incorporating the stream-aquifer interaction process. After incorporating with this process these methods the extended capabilities of these methods were verified using hypothetical inflow and the corresponding benchmark outflow hydrographs simulated with stream-aquifer interaction process, and subsequently applying these verified methods for field flood routing applications wherein the stream-aquifer interaction process is dominant under the scenarios of stream fully and partially penetrating the surrounding aquifers wherein, one-dimensional and two-dimensional flow scenarios may be present. Based on the study carried out under the above given background, the following conclusions are arrived at from the study:

CONCLUSIONS

Based on the considered objectives of the study, it is found that both the extended VPMM and VPMS methods as developed in this study can be used for streamflow routing assuming all the above-mentioned scenarios involving stream-aquifer interaction. These two routing methods were used for both one-dimensional and two-dimensional flow scenarios in the aquifer and the stream fully or partially penetrating the surrounding aquifer.

On the basis of the study carried out in this thesis, the following conclusions can be drawn:

1) With moderate data requirements, the VPMM and VPMS methods can be chosen to evaluate the lateral flow and bank storage for a river segment. The required data are the channel cross-sectional details and the discharge and stage hydrographs of the study reach at the section.
2) The proposed methodology for lateral flow estimation using the Darcy's law is more simple and less prone to errors in the comparison to the Simpson's (3/8) - rule used in the earlier studies for assessing aquifer flow study in the groundwater storage, as it requires only two values of hydraulic heads for the lateral flow estimation; one at the stream-

aquifer interface section and the other at just adjacent to that for each sub-section of the reach at a time.

3) VPMS and VPMM methods are relatively simple and will give more accurate results in comparison to the procedures based on the rating curve conversion approach.

4) Morel-Seytoux approach (1979) is not simple in application for stream-aquifer interaction process study in field under the scenario of partially penetrating stream condition, which require, the distance of the observation well should be at the distance of 5 time the wetted perimeter of the channel cross-section. However, the conformal mapping approach does not require such condition as required in the Morel-Seytoux approach. Therefore, conformal mapping approach is more suitable for the field applications when stream-aquifer interaction is dominant under the scenario of stream partially penetrating the aquifer.

APPENDIX I

The validity of conservation of mass using the Simpson's 1/3 rule

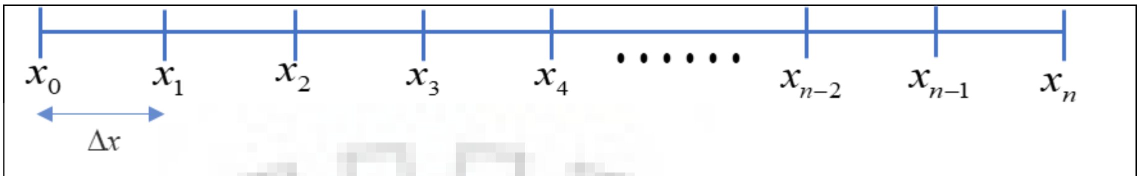

Fig. Division of a line into *n* equal segments of width Δx

Simpson's rule for the numerical approximation of the definite integral has been defined as:

$$\int_{x_0}^{x_n} f(x)\,dx \approx \frac{\Delta x}{3}\Big[\big\{f(x_0)+4f(x_1)+f(x_2)\big\}+\big\{f(x_2)+4f(x_3)+f(x_4)\big\}+\cdots+\big\{f(x_{n-2})+4f(x_{n-1})+f(x_n)\big\}\Big]$$

where, $\Delta x = \dfrac{x_n - x_0}{n}$, where n is an even number

Calculation of average flow to the control section $(x, x+2\Delta x)$ **using Simpson's 1/3 rule**

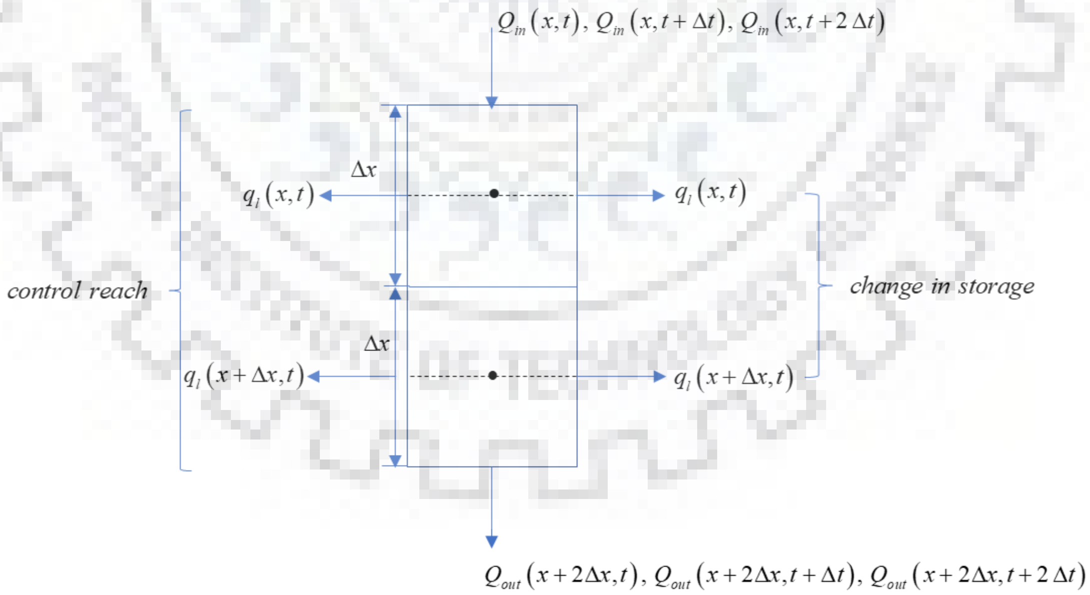

Fig. Definition sketch of a control reach

$$\overline{Q_{in}}(x,t)=\frac{1}{6}\big\{Q_0(x,t)+4Q_0(x,t+\Delta t)+Q_0(x,t+2\Delta t)\big\}$$

Total inflow= $\overline{Q_{in}}(x,t)\times 2\Delta t$

(i)

$$\overline{Q_{out}}(x+2\Delta x,t)=\frac{1}{6}\left\{Q_2(x+2\Delta x,t)+4Q_2(x+2\Delta x,t+\Delta t)+Q_2(x+2\Delta x,t+2\Delta t)\right\}$$

Total outflow= $\overline{Q_{out}}(x+2\Delta x,t)\times 2\Delta t$

(ii)

i) Calculation of average depth to the control section $(t,t+2\Delta t)$ using Simpson's 1/3 rule:

$$\bar{y}(x,t)=\frac{1}{6}\left\{y(x,t)+4y(x+\Delta x,t)+y(x+2\Delta x,t)\right\}$$

$$\bar{y}(x,t+2\Delta t)=\frac{1}{6}\left\{y(x,t+2\Delta t)+4y(x+\Delta x,t+2\Delta t)+y(x+2\Delta x,t+2\Delta t)\right\}$$

Change in storage = $width\,of\;the\,channel\times 2\Delta x\times\left\{\bar{y}(x,t+2\Delta t)-\bar{y}(x,t)\right\}$

(iii)

Bank seepage = $q_l(x,t)\times 2\Delta x\times 2\Delta t$

(iv)

The values of total inflow, total outflow, change in storage and bank seepage has been calculated using Eqs. (i)-(iv) respectively. For the cases, where seepage has not been considered, the term $q_l(x,t)$ will be taken as zero.

a) **Mass Balance at x=0,500,1000 m and t=35900 to 36000s.**

Time (sec)	$Q(0,t)$ (m^3/s)	$Q(0,t+\Delta t)$ (m^3/s)	$Q(0,t+2\Delta t)$ (m^3/s)	Total inflow (m^3)
35900	101.9212			
35910	101.913	101.913		
35920	101.9048	101.9048	101.9048	2038.26
35930	101.8965	101.8965	101.8965	2038.095667
35940	101.8882	101.8882	101.8882	2037.93
35950	101.8798	101.8798	101.8798	2037.763667
35960	101.8714	101.8714	101.8714	2037.596
35970	101.8629	101.8629	101.8629	2037.427667
35980	101.8544	101.8544	101.8544	2037.258
35990		101.8459	101.8459	2037.088
36000			101.8373	2036.917667

Time (sec)	$Q(1000,t)$ (m^3/s)	$Q(1000,t+\Delta t)$ (m^3/s)	$Q(1000,t+2\Delta t)$ (m^3/s)	Outflow (m^3)	Bank seepage (m^3)	Total outflow (m^3)
35900	100.9372					
35910	100.9313	100.9313				
35920	100.9254	100.9254	100.9254	2018.626	18.2788	2036.9048
35930	100.9193	100.9193	100.9193	2018.507333	18.2678	2036.775133
35940	100.9133	100.9133	100.9133	2018.386333	18.2568	2036.643133
35950	100.9073	100.9073	100.9073	2018.266	18.246	2036.512
35960	100.9012	100.9012	100.9012	2018.145667	18.235	2036.380667
35970	100.895	100.895	100.895	2018.023667	18.224	2036.247667
35980	100.8889	100.8889	100.8889	2017.900333	18.213	2036.113333
35990	100.8828	100.8828	100.8828	2017.778	18.202	2035.98
36000	100.8766	100.8766	100.8766	2017.655667	18.1908	2035.846467

Time (sec)	$\bar{y}(x,t)$ (m)	$\bar{y}(x,t+2\Delta t)$ (m)	Change in storage (m^3)	Total inflow-Total outflow (m^3)	Error (in %)
35900	3.034017				
35910	3.034042				
35920	3.034065	3.034065	1.44272	1.3552	-6.458087367
35930	3.034087	3.034087	1.39192	1.320533333	-5.405896607
35940	3.03411	3.03411	1.37668	1.286866667	-6.979226027
35950	3.034132	3.034132	1.36144	1.251666667	-8.770173104
35960	3.034153	3.034153	1.31064	1.215333333	-7.84201865
35970	3.034173	3.034173	1.25984	1.18	-6.766101696
35980	3.034194	3.034194	1.24968	1.144666667	-9.174140942
35990		3.034213	1.2192	1.108	-10.03610108
36000		3.034232	1.16332	1.0712	-8.599701271

b) Mass Balance at x=7,7.5,8 Km and t=35900 to 36000 sec.

Time (sec)	$Q(7000,t)$ (m^3)	$Q(7000,t+\Delta t)$ (m^3)	$Q(7000,t+2\Delta t)$ (m^3)	Total inflow (m^3)
35900	95.29854			
35910	95.30222	95.30222		
35920	95.30595	95.30595	95.30595	1906.044567
35930	95.30956	95.30956	95.30956	1906.1186
35940	95.31314	95.31314	95.31314	1906.1911
35950	95.31676	95.31676	95.31676	1906.262933
35960	95.32032	95.32032	95.32032	1906.335
35970	95.32389	95.32389	95.32389	1906.406433
35980	95.32742	95.32742	95.32742	1906.477667
35990	95.33089	95.33089	95.33089	1906.5482
36000	95.33434	95.33434	95.33434	1906.617733

Time (sec)	$Q(8000,t)$ (m^3)	$Q(8000,t+\Delta t)$ (m^3)	$Q(8000,t+2\Delta t)$ (m^3)	Total outflow (m^3)
35900	94.64009			
35910	94.64539	94.64539		
35920	94.65067	94.65067	94.65067	1892.907733
35930	94.65595	94.65595	94.65595	1893.0134
35940	94.66119	94.66119	94.66119	1893.118867
35950	94.66636	94.66636	94.66636	1893.223567
35960	94.67152	94.67152	94.67152	1893.327167
35970	94.67666	94.67666	94.67666	1893.430333
35980	94.6818	94.6818	94.6818	1893.5332
35990	94.68684	94.68684	94.68684	1893.635667
36000	94.69193	94.69193	94.69193	1893.736967

Time (sec)	$\overline{y}(x,t)$ (m)	$\overline{y}(x,t+2\Delta t)$ (m)	Change in storage (m^3)	Total inflow-Total outflow (m^3)	Error (in %)
35900	2.9478255				
35910	2.9480418				
35920	2.948257	2.948257	13.15212	13.13683333	-0.116364928
35930	2.948472	2.948472	13.11148	13.1052	-0.04791991
35940	2.9486863	2.948686	13.08608	13.07223333	-0.105924262
35950	2.9489012	2.948901	13.081	13.03936667	-0.319289536
35960	2.9491143	2.949114	13.04544	13.00783333	-0.289107845
35970	2.9493272	2.949327	12.98448	12.9761	-0.064580267
35980	2.9495395	2.94954	12.95908	12.94446667	-0.11289251
35990		2.949751	12.9286	12.91253333	-0.124426913
36000		2.949963	12.89812	12.88076667	-0.13472283

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