TIME DEPENDENT SEISMIC HAZARD ANALYSIS

A DISSERTATION

Submitted in the partial fulfilment of the requirements for the award of the degree of

MASTER OF TECHNOLOGY

in

EARTHQUAKE ENGINEERING

(With specialization in Structural Dynamics)

by

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CANDIDATE'S DECLARATION

I hereby, declare that the work which is being presented in this dissertation entitled, **"TIME DEPENDENT SEISMIC HAZARD ANALYSIS"**, being submitted in partial fulfilment of the requirements for the award of degree of "Master of Technology" in "Earthquake Engineering" with specialization in Structural Dynamics, to the Department of Earthquake Engineering, Indian Institute of Technology Roorkee, under the supervision of Dr. M.L.Sharma, Professor, Department of Earthquake Engineering, Indian Institute of Technology Roorkee, is an authentic record of my own work carried out during the period of June 2017 to May 2018.

I declare that I have not submitted the material embodied in this dissertation for the award of any other degree or diploma.

Place: Roorkee
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CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of my knowledge and belief.

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ABSTRACT

The use of deterministic seismic approach is not so prevalent nowadays .in recent years we consider the probabilistic seismic hazard analysis. It takes into account the uncertainties regarding time, space and magnitude. For considering the temporal uncertainties we have considered two models lognormal and weibull distribution. We have calculated the parameters of both the models and worked on the hazard rate curve .we have further predicted the probability of occurrence of earthquake in future years. The region we have considered for study is Jammu and Kashmir region.

ACKNOWLEDGEMENT

Firstly I would like to express my heartfelt gratefulness to my guru, **Dr. M.L Sharma**, Professor, Dept. of Earthquake Engineering, for his dedication and due diligence. He has mentored me throughout this course work and his expertise and patience gave me the strength to move forward the hard times.

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Akanksha Agarwal

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CHAPTER 1

INTRODUCTION

1.1PREAMBLE

The Indian subcontinent is very seismically active regions of the word. It can be mainly divided into three basic units which are, the Himalaya's, the Indo- gangetic plains and the Peninsular India. Each part is geographically very different and exhibit a different seismic behaviour. The major seismicity of Indian subcontinent was of interrogate type and is confirmed into Himalayan range. This seismicity is mainly because to continental collision between Indian plates and Eurasian plates. The north most part of indo gangetic plains which forms the foot of Himalayas show the sign of moderate eve of seismicity. Peninsular India shows low event of seismicity but has occasionally experienced some damaging earthquake.

The Himalayas are the major newly formed mountain range in the word is seismic and tectonically very active. The epicentral distribution of the Indian subcontinent shows that most of earthquake are concerned in the plate boundary of Himalayas. The Himalayan range has witnessed for major earthquake (m>8) in the past of about 100 years and many moderate earthquakes. These are Shilong earthquake, Kangra earthquake, Uttarkashi earthquake and Chamoli earthquake.

The indo gangetic plains are "great plains" and are the rages food aids of the Indus and ganga-brahmaputra river system. The indo gangetic plains represent the Himalayan fore and basin system. It covers length of 700,000 km and varies in its width a through the way.

Earthquakes are natural disaster which if gone wrong may case heavy devastations sufficient to get attention. Most of the areas of the world are sitting on the brewing earthquake, and since it's a stochastic phenomenon this can never be predicted. Many studies on earthquake researchers have been undertaken without any satisfying results. Hence the most defensive one in earthquake risk mitigation is to construct facility which are earthquake resistant. It is righty said that earthquakes do not kill anybody but buildings do.

Many countries have responded to this peculiar situation by giving building codes for earthquake resistant designs. In order to ascertain appropriate design methodology, the first question which needs to be answered is the process of ground shaking which is expected at the site of interest. None of the structures can be made earthquake resistant, in terms of economics. Therefore objective of earthquake resistant design is to strengthen the structure to perform without much damage when it is subjected to any ground motion

Seismic hazard assessment involves the quantitative determination of ground shaking disaster at any specific site. Seismic hazard can be studied in determinate, in which uncertainties and probability in size and, magnitude is considered. Earthquake is defined as the rapid vibrations of earth masses which is mainly cased due to movement of the huge section of the rocks .It is one of major sudden phenomenon which disturbs daily life to a very wide extend and may turn it upside down. The science which deals with the earthquake studies is referred as seismology. The increasing threat and the loss thus created is sufficient to design the structures to be earthquake resistant. Hence or main aim is to create a structures and facilities which can withstand certain ground motion avoiding much damage Seismic hazard is the probability of an earthquake in a given region, within a specific time elapsed, and with ground motion parameters exceeding a given threshold. Risk hence is defined as:

Risk = Hazard x Vulnerability x Exposure

Two standard measures of anticipated ground motion are:

(1) Maximum considered earthquake which is probabilistic and simpler and is used in the standard building codes

(2) Maximum Credible Earthquake that is more detailed and deterministic .it is used in designing larger buildings

This earthquake shaking may be defined, in terms of design ground motion parameters. Thus defining and evaluating design ground motion parameters is major function of seismic hazard assessment.

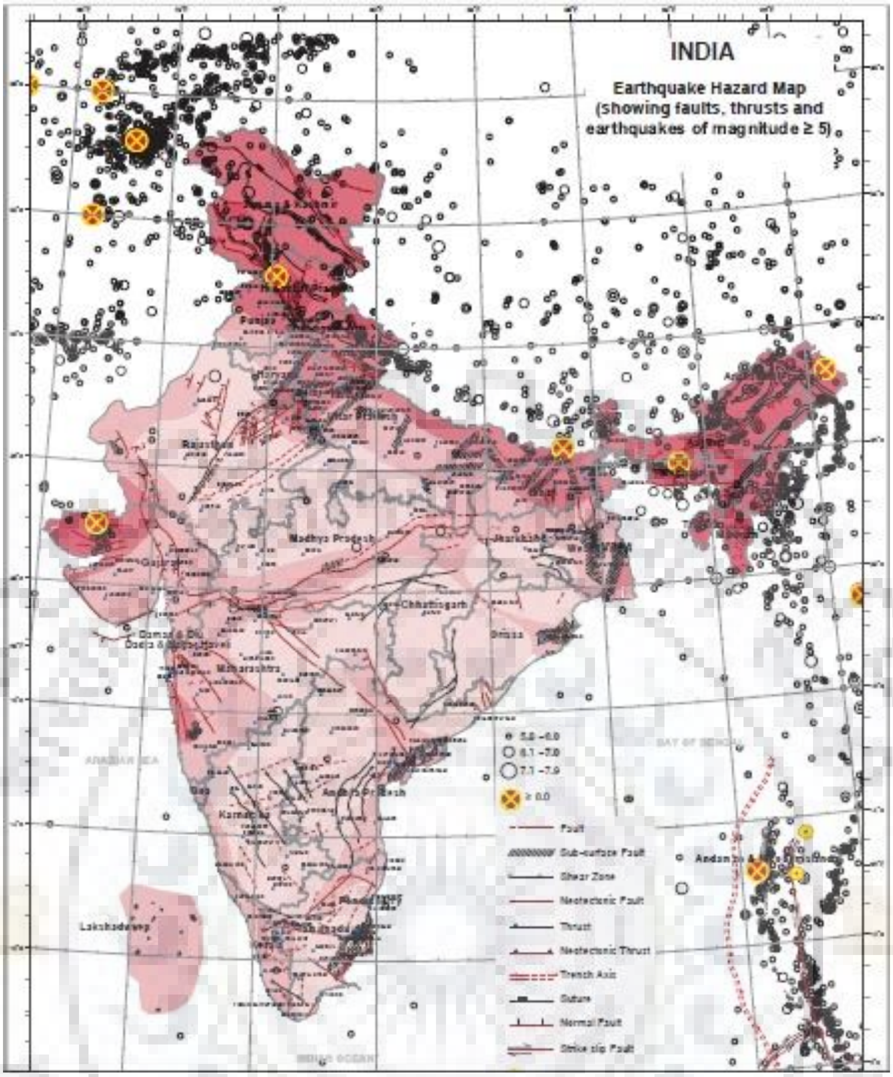

Fig 1 Earthquake hazard map of India (vulnerability atlas, second edition, peer group)

1.2 OBJECTIVE OF STUDY

The decade 1990-2000 was defined as the international decade for natural disaster reduction (idndr) by the united nation. This provides the institutions worldwide to evaluate current methods and predicting the new methods for mitigating risk.

The following study considers the time dependent seismic hazard analysis .in the earlier periods the used method was Poisson model or the time independent model. Although it was simpler to apply because of its shortcomings it is absolute nowadays .so in our following work we have done the case study of Jammu and Kashmir region, mainly focusing on majorly Kashmir valley .Jammu and Kashmir comes under the highest seismic zone .it has some part under zone 4 and some under zone 5 making it the major hotspot of seismic tectonic research. Thru recent 2005 earthquake on Muzaffarabad fault has caused for about more than 80,000 loss of lives thus causing destruction of property which was of worth millions. The studies done by seismologists suggest that in Kashmir region earthquake has occurred randomly with respect to time and doesn't follows a particular pattern. So the danger or threat of earthquake in Kashmir shouldn't be taken lightly. Hence seismologists works continuously on predicting the future major earthquake to reduce the risks which is created by them.

Hence in our following study we consider the lognormal and Weibull distribution to study the hazard occurrence. We worked on co probability for future earthquake for magnitude which are greater than 6.

1.3 METHODOLOGY AND SCOPE.

The following thesis work consists of the Poissons model lognormal model and the weibull distribution. In all the three methods we have taken the declustered catalogue for magnitude more than 6. The Kashmir area which we have chosen for our study is between 32 N to 37 N and longitude 72 E to 81 E. firstly in the poissons model we have worked on the map of hazard rate, proving that it is time independent. Further we have worked on the lognormal distribution. We have found the parameters by maximum likelihood method .we have worked upon the conditional probability of future earthquake by all three methods .Comparison is done on the instant failures curve on all three methods. Lastly we have depicted that our practical curves are coming similar to the theoretical curves.

1.4 ORGANISATION OF WORK

The fundamental objective of the thesis work is to evaluate the temporal uncertainty hence the work has been divided to study the poissons distribution and then it is compared with the renewal models. The chapter 1 and chapter 2 concerns with the basics of seismic hazard analysis .chapter 3 and chapter 4 deals with the theoretical knowledge of poisson distribution, weibull distribution and lognormal distribution. Chapter 5 concerns with the conditional probability and methods to give the probability of future occurrence. Chapter 6 dealt with the brief seismo-tectonics study of Kashmir region .the final chapter 7 contains results and conclusion.

CHAPTER 2

CLASSES OF SEISMIC HAZARD ANALYSIS

2.1 DETERMINISTIC SEISMIC HAZARD ANALYSIS

Deterministic seismic hazard assessment (DSHA) was used in the past years of seismic analysis. It is the classical model.it is absolute nowadays A DSHA involves the postulated occurrence of an earthquake of definite size occurring at particular location.

The four definite steps (Reiter, 1990) consisting of:

1. The first step includes Identification and characterization of all earthquake sources
2. The second step involves randomly selecting of a source-to-site distance and usually the least distance is selected.
3. The particular earthquake which produces the strongest shaking is than selected and the controlling earthquake is defined in terms of magnitude and distance from site from the site.
4. The final step involves the controlling earthquake .this may be defined in terms of peak acceleration. Sometimes peak velocity, and response spectrum ordinate are also selected.

2.2 PROBABILISTIC SEISMIC HAZARD ANALYSIS

In the recent years the use of probabilistic seismic hazard analysis is increasing. It has allowed variations in the size, location, and reoccurrence rate of earthquakes .these uncertainties can be considered in evaluating of seismic hazards. Usually it gives a complete framework for these uncertainties. (Reiter, 1990), gave four steps for PSHA which are little bit similar to D.S.H.AEA.

1. The initial step, involves identification and characterization of earthquake sources, that the uncertainties of potential rupture locations at a particular site must also be characterized. Usually we define a uniform probability distributions to each site zone
2. Next, we define a renewable model for the seismicity or temporal distribution
3. Next the ground motion prevalent at the site by earthquakes of any possible size occurring at a particular location are defined by the recurrence model.
4. The last step involves joining the probability of time and ground motion. The following steps can be depicted as follows

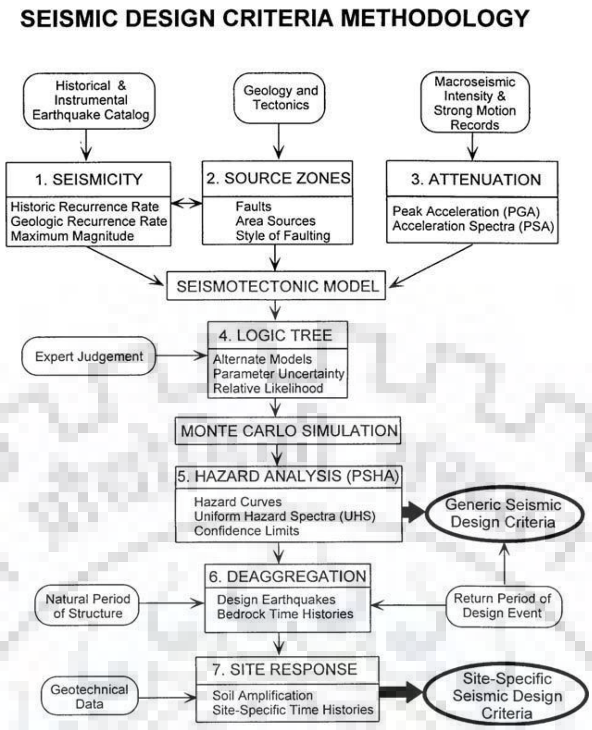

Fig 2 Methodology followed in probabilistic seismic analysis (Steven L. Kramer (1996), "Geotechnical Earthquake Engineering)

CHAPTER3

TEMPORAL UNCERTAINTY

To work on the probability of various hazards occurring in given time period we have to work on the distribution of earthquake occurrence with respect to time. In past times earthquakes were known to occur randomly with time. Moreover study of available records has revealed little evidence when aftershocks are removed, of temporal patterns in earthquake recurrence. This consideration of memory less occurrence allows the use of simple probability models yet they cannot satisfy with the results of elastic rebound theory .Time-independent probabilistic seismic-hazard study consider each source as being temporally and spatially independent. Hence foreshocks and aftershocks, which are both spatially and temporally dependent on the main shock can be removed from earthquake catalogues or data. Yet, these earthquakes should be considered part of the seismic hazard, capable of producing damaging ground motions.

We have assumed the random occurrence. This allows the use of simple probability models, but it is not satisfying with elastic rebound theory. As regarding this phenomenon of elastic rebound (Reid, 1911), earthquakes occurring because of successive build-up and release of strain energy. This strain energy is in the rock which ae close or on the two faults. Thus, the time for the second earthquake depends on the stress released in the previous earthquake. This depicts that earthquakes follow a memory pattern. However, in practical applications, the earthquake is commonly approximated as a Poisson process under the assumption of independence, that is they follow a memory-less pattern since the computations are easy.

3.1 TIME INDEPENDENT MODEL (POISSON MODEL)

The occurrence of earthquakes with respect to time, is mainly considered by a Poisson model. The Poisson model gives a simple yet useful method for studying probabilities of events that describes Poisson process it helps in giving values of a random variable describing the number of occurrences of particular event during a given time interval or in a specified spatial region.. Poisson processes deals with the following specifications:

1. The number of occurrences in particular time interval cannot depend on number that happens in any other time interval.

2. The occurrence probability in a very minute or small time interval is depending on the length of that time interval.

3. The occurrence probability of greater than one during a very short time interval is minute. For a Poisson process, the probability of a random variable N, which depicts the number of occurrences of specific event

$$P\,[N = n] = \mu^{n}\, e^{-\mu} / n! \qquad (3.1)$$

Where μ, is the average number of occurrences of the event in that time interval. The time happening between events in a Poisson process is exponentially distributed.

To characterize the temporal distribution of earthquake recurrence for PSHA purposes, the Poisson probability is described as

$$P\,[N= n] = (\lambda t)^{n}\, e^{-\lambda t} / n! \qquad (3.2)$$

Hence λ can be said as average rate of occurrence

Probability of no event may be determined by P(N=0)=e

Probability of occurrence of minimum one event in a period of time t is given by

$$P[N \geq 1\,] = P[N = 1] + P[N=2] + P[N=3] + \ldots + P\,[N = \infty] = 1 - P\,[N = 0] = 1 - e^{-\lambda t} \qquad (3.3)$$

The corresponding cumulative density function can be defined as:

$$F(t)= 1 - e\; {-\lambda t} \qquad (3.4)$$

we can define the density function as:

$$f(t)=Df(t)\backslash dt = \lambda\; e\; {-\lambda t} \qquad (3.5)$$

Thus we can see that the hazard rate function is having a constant value λ

Moreover,

$$P[N \geq 1] = l - e^{-\lambda mt} \qquad (3.6)$$

Drawbacks of the model

Although the poisson model is simpler to apply yet it is contradictory to elastic rebound theory by H.F REID .according to REID theory earthquake occur due to building and release of stress in the rock stratum near the faults .hence we can conclude that if a large earthquake has occurred at a particular site than it will take some time to build and accumulate the strain energy so that the next earthquake can occur .but the poisson distribution says that at time t=0 the probability of occurrence of earthquake is non zero since according to it ,the hazard rate is constant with value equal to λ according to poisson model .the occurrence of earthquake is random and do not depend on the previous history but as we discussed the earthquake occurring at some other time t depend on the previous time lapse .hence we can conclude that the poisson model is against the elastic rebound theory.so in the recent times poisson model is being succeeded by more efficient methods .these are often referred as renewal models.

CHAPTER 4

TIME-DEPENDENT MODELS

Since the poisons model is having a major drawback that it assumes a constant hazard rate .hence it can be physically interpolated that the strain energy which has been accumulated has never been released which is physically not possible.so seismologists further gave some renewal models.

A renewal prototypes are the time dependent models. These are usually those model that satisfies all the Poisson postulates except the constant hazard rate .hence these models are locally referred as "non-homogeneous Poisson process". The most versatile method tp depict that the large earthquake has some periodicity and they are indicating the memory of the last event thus occurs.

The hazard rate is assumed to have some temporal relations ,that is it is time dependent .hence they usually satisfy the elastic rebound theory that the strain energy once accumulated in the previous event would be released (e.g.; Thacher, 1984; Sykes and Nishenko, 1984). A number of studies (e.g.; Hagiwara, 1974; Rikitake, 1976; Utsu, 1984; Nishenko and Buland, 1987; Papazachos and Papaionnou, 1993; etc.) have used such distributions, most common among which are Lognormal, Brownian Time Passage, Weibull, and Gamma. Renewal models apply different distributions, allowing for the probability of occurrence, probability to increase with elapsed time since the previous event (Cornell & Winterstein, 1988).in the following models inter arrival time of the events with magnitude greater than the particular threshold magnitude is computed and is further used to apply these relations.

Both poisons and the renewal models follow the assumption of "characteristic earthquake model". According to this ,all the earthquakes occurring along or adjacent to a particular rupture fault are assumed to have same magnitude displacements, and rupture lengths.it makes the calculations simpler but the results thus coming may not be that much satisfactory. Sometimes we use more versatile relations like Gutenberg-Richter distributions. The time dependent models needs more variables and constants to be included and hence the results will be varied than those of time dependent models. Both poissons and renewal models calculations require moment-balanced models .these should be consistent with the global plate rate models as well as slip rates found on individual faults.

The main four types of time dependent seismic hazard model are

- Gamma Distribution
- Weibull Distribution
- Lognormal Distribution
- Brownian Passage Time (BPT) Model

4.1LOG NORMAL DISTRIBUTION

The lognormal distribution is useful in depicting any variables which are continuous and not less than zero for evaluating the continuous random variable whose logarithm is normally distributed us use the following distribution.

Hence we can say that if a given variant, which is x is log-normally distributed, then Y=ln(X) is having normal distribution. Similar to it if Y is having normal distribution, then the exponential function of X has lognormal distribution. It can be written in both two constants and three constants form. The three constant form is usually written as

$$f(\mathrm{X}|\theta,\mu,\sigma^2)=\frac{1}{\sqrt{(2\pi\sigma^2)}(\mathrm{X}-\theta)}\exp\left[\frac{-\left(\ln(\mathrm{X}-\theta)-\mu^2\right)}{2\sigma^2}\right], \forall\ \mathrm{X}>\theta,\ -\infty<\mu<\theta,\ \sigma>0 \qquad (4.1)$$

The two parametric form is

$$f(\mathrm{X}|\mu,\sigma^2)=\frac{1}{\sqrt{(2\pi\sigma^2)}\mathrm{X}}\exp\left[\frac{-\left(\ln(\mathrm{X})-\mu^2\right)}{2\sigma^2}\right], \forall\ \mathrm{X}>0,\ -\infty<\mu<0, \sigma>0 \qquad (4.2)$$

In the three parametric form the location parameter decides the movement of the density function along axis .since it has no involvement for the shape of density function, the three parametric form is usually avoided. Indeed we use the two parametric estimation.

The lognormal distribution was first explained by F. Galton in 1879.After Galton to the lognormal distribution remained left unnoticed until 1903, Kapteyn defined the lognormal distribution as a particular class of the transformed normal distribution. Note that the lognormal is sometimes called the anti-lognormal distribution, because it is not the distribution of the logarithm of a normal variable, but is instead the anti-log of a normal variable (Brezina 1963; Johnson and Kotz 1970).

Multiplicative property

Considering the additive property of normal distribution we can derive the multiplicative property according to it if two independent random variables, X1 and X2, are distributed respectively as Lognormal than we can state that their product is also log normally distributed. Also for very minute value the lognormal and normal plot gives the same plots. However it does not provides a moment generating function. Any random variable X can be considered log-normally distributed with parameters μ and σ if Y = ln(X) is normally distributed provided with mean μ and standard deviation σ. The parameter σ is the shape parameter of X while eμ is the scale parameter of X.

On a logarithmic scale, μ and σ are known as the location parameter and the scale parameter, respectively. The scale parameter provides the knowledge about how the graph shrinks or stretches. The location parameter, which tells you where on the x-axis the graph is located.

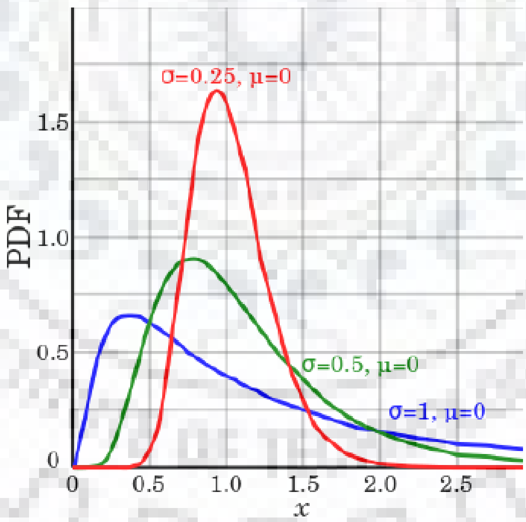

Fig. 3 Probability function of earthquake occurrence in case oflognormal distribution model (A. D. Telang and Mariappan V (2007), "Hazard Rate of Lognormal Distribution: An Investigation)

The hazard rate for lognormal distribution starts with a zero value. It than increases y to a finite maximum value near the mean recurrence time. After that it decreases asymptotically to zero value for very long recurrence times.

4.1.1 Parameter calculation

The most frequent methods of parameter estimation for the lognormal distribution is maximum likelihood method .Sometimes Method of Moments is also used. Although both these formulations gives both of the closed-form solutions, but in our following work we have concerned on the .maximum likelihood method.

Maximum Likelihood Estimators

Maximum Likelihood is the most common used method due to the fact that it chooses those parametric values which makes the continuous data more likely compared to any secondary parametric values. For this purpose the technique used is such that we maximise the likelihood function.

Likelihood function may be derived by taking product of the probabilistic density function for a series of X_is (i = 1, 2, ... n) .this product is than maximised .this implies we take the derivative of the likelihood function with respect to μ and σ^2Some appealing features of Maximum Likelihood estimators include that they are asymptotically unbiased, in that the bias tends to zero as the sample size n increases; they are asymptotically e client, in that they achieve the Cramer-Rao lower bound as n approaches 1; and they are asymptotically normal.

Mathematical formulations

The likelihood function may be written as following

$$\begin{aligned} L\left(\mu,\sigma^2 \middle| X\right) &= \prod_{i=1}^{n}\left[f\left(X_i \middle| \mu,\sigma^2\right)\right] \\ &= \prod_{i=1}^{n}\left(\left(2\pi\sigma^2\right)^{-1/2} X_i^{-1}\exp\left[\frac{-(\ln(X_i)-\mu)^2}{2\sigma^2}\right]\right) \\ &= \left(2\pi\sigma^2\right)^{-n/2}\prod_{i=1}^{n} X_i^{-1}\exp\left[\frac{-(\ln(X_i)-\mu)^2}{2\sigma^2}\right] \end{aligned} \quad (4.3)$$

We have to maximise the above relation to yield the following equations

$$\tilde{\mu} = \frac{\sum_{i=1}^{n} \ln(X_i)}{n}$$

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{n} \left\{ \ln(X_i) - \frac{\sum_{i=1}^{n} \ln(X_i)}{n} \right\}^2}{n} \tag{4.4}$$

4.2 WEIBULL DISTRIBUTION

Weibull distribution is a continuous probability distribution. It was first described by Fréchet (1927) and first applied by Rosin & Rammler (1933) to describe a particle size distribution but Later in 1951 a Swedish mathematician Waloddi Weibull, listed it in detail, this method is mainly in reliability engineering and in many other studies due to its vast use and relative simplicity.

The weibull distribution similar to lognormal distribution can be described in three, two as well as one parametric form.

- β is the shape parameter. It is also known as the Weibull slope
- η is known as scale parameter
- γ is the location parameter

Usually the location parameter value is taken equal to zero in order to reduce three parametric form to two parametric form. In case of single parametric, value of β is known initially thus we have to calculate the scale parameter. It is suggested that the specialist have a very good and remarkable estimate for β before using the single-parameter Weibull distribution for analysis. An important characterstic of the Weibull distribution is that the way in which the values of the shape parameter, β, and the scale parameter, η, disturb such distribution characteristics as the shape of the *pdf* curve, and the failure rate.

Weibull Shape Parameter, β

The Weibull shape parameter, β. It is often referred as the Weibull slope. It is due to theory that value of β gives the slope of probability graph. That is why the value of β is equal to the slope of the line of a probability plot. Different values of this parameter have different effect on the hazard rate curve, the feature which is knowledge full to study. The parameter is non dimensional.

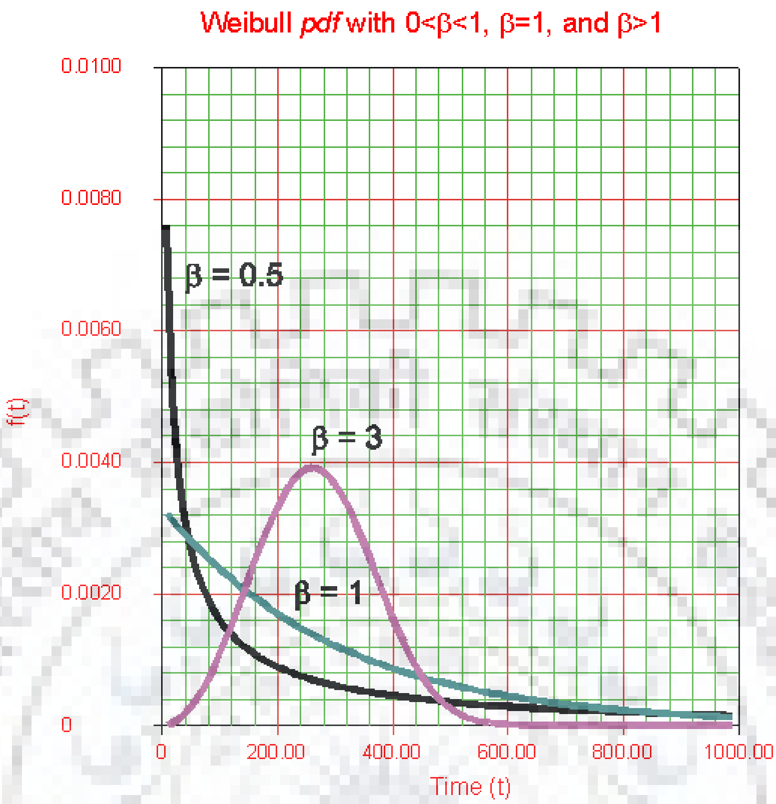

Fig 4 probablity graph for weibull distribution for constant η and variable β (Brenda F. Ginos (2009), "Parameter Estimation for the weibull Distribution'', pg 1-111)

Weibull Scale parameter, η

If we change the, η, than there would be variation on the abscissa scale. Changing the value of η, if we keep β constant results of stretching out the *pdf*. Since the area under a *pdf* curve is a constant value of peak will change.

- If η is increased, considering β and γ are kept uniform, the distribution gets stretched out to the right and its height decreases this is done while maintaining its shape and location.
- If η is decreased, while β and γ are kept uniform, the distribution gets pushed in towards the left (i.e., towards its beginning or towards 0 or γ) also its height increases.
- It is not a dimensionless quantity. Infact it has unit of time.

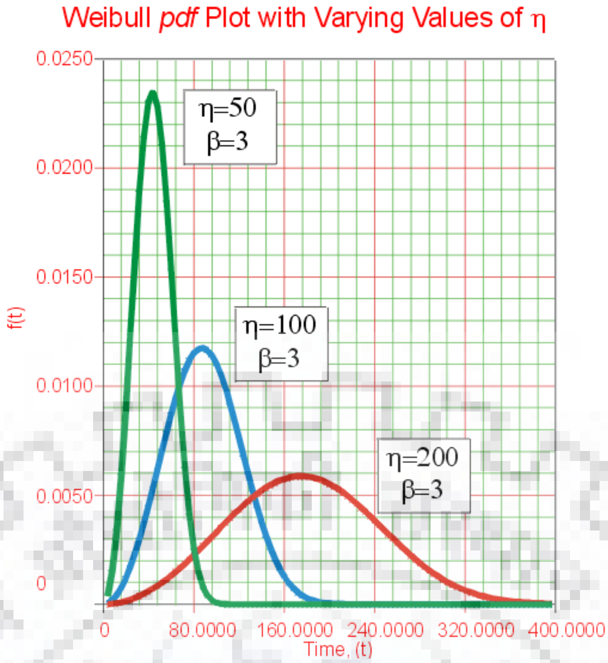

Fig 5 probability graph for weibull distribution for constant β and variable η (Brenda F. Ginos (2009), "Parameter Estimation for the weibull Distribution'', pg 1-111)

4.2.1 Parametric calculation

Usually the graphical method and analytical method are used for parametric calculation. But since there is high error in graphical method we use the later one. This is motivated by the availability of high-speed computers.

The analytical methods are maximum likelihood method, method of moment and maximum likelihood method. In our following we have discussed all the three methods, but we have worked on maximum likelihood method.

The method of maximum likelihood was developed by Harter and Moore in 1965 due to its desirable property and simplicity it is often used. If there are n random variables than the likelihood function us the product of probability density function .this joint function when maximized yields the value of the parameters.

In case of weibull transformation we differentiate the likelihood function with respect to β and η. The equations are than solved to evaluate the following equation.

$$f(x) = \frac{\beta}{\eta}\left(\frac{x-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{x-\gamma}{\eta}\right)^{\beta}}, \beta > 0, \eta > 0, x \geq 0$$

$$F(x) = 1 - e^{-\left(\frac{x-\gamma}{\eta}\right)^{\beta}}$$

$$\frac{\sum_{i=1}^{n} x_i^{\beta} \ln x_i}{\sum_{i=1}^{n} x_i^{\beta}} - \frac{1}{\beta} - \frac{1}{n}\sum_{i=1}^{n} \ln x_i = 0$$

$$\eta = \frac{\sum_{i=1}^{n} x_i^{\beta}}{n} \qquad (4.5)$$

CHAPTER 5

CONDITIONAL PROBABILITY

In probability theory, we usually define the definitions in term of some sample space conditional probability is a method to calculate the probability of an event Aby assuming that any other event B has already happened. The conditional probability is mainly written in the form as $P(A|B)$, or sometimes $P_B(A)$. In the form of equation we can write as:

$$P(A|B) = P(A\cap B)/P(B) \tag{5.1}$$

The method of conditional probability is very much fundamental and peculiar method in probability theory. But it may sometimes turn out to be very difficult to apply. Events are said to be mutually exclusive if P(A|B) = P(A).

Given two events A and B, from the sigma-field of a probability space, with P(B) > 0, the conditional probability of A given B is defined as the quotient of the probability of the joint of events A and B, and the probability in the context of earthquake occurrence A and B are events of same magnitude range and conditional probability p(c)of next event b within time (t,t+dt)/probability of preceding event a at t.

CHAPTER 6

REGION OF STUDY – JAMMU AND KASHMIR

The state of Jammu & Kashmir is the western most extension of the Himalayan mountain range in India. The Himalayan range were formed due to collision of the two continents. It is due to collision of Indian and Eurasian plate and the process is still undergoing today. The Himalayan range thus have many basins .one of the important basin is Kashmir basin .it is often referred as "thrust top" or "piggy back " basin.it is most often cited similar to upper siwalik group. Fig (6) and Fig (7) suggest longitudinal and transverse length of the bas8in is 100 km and 50 km.

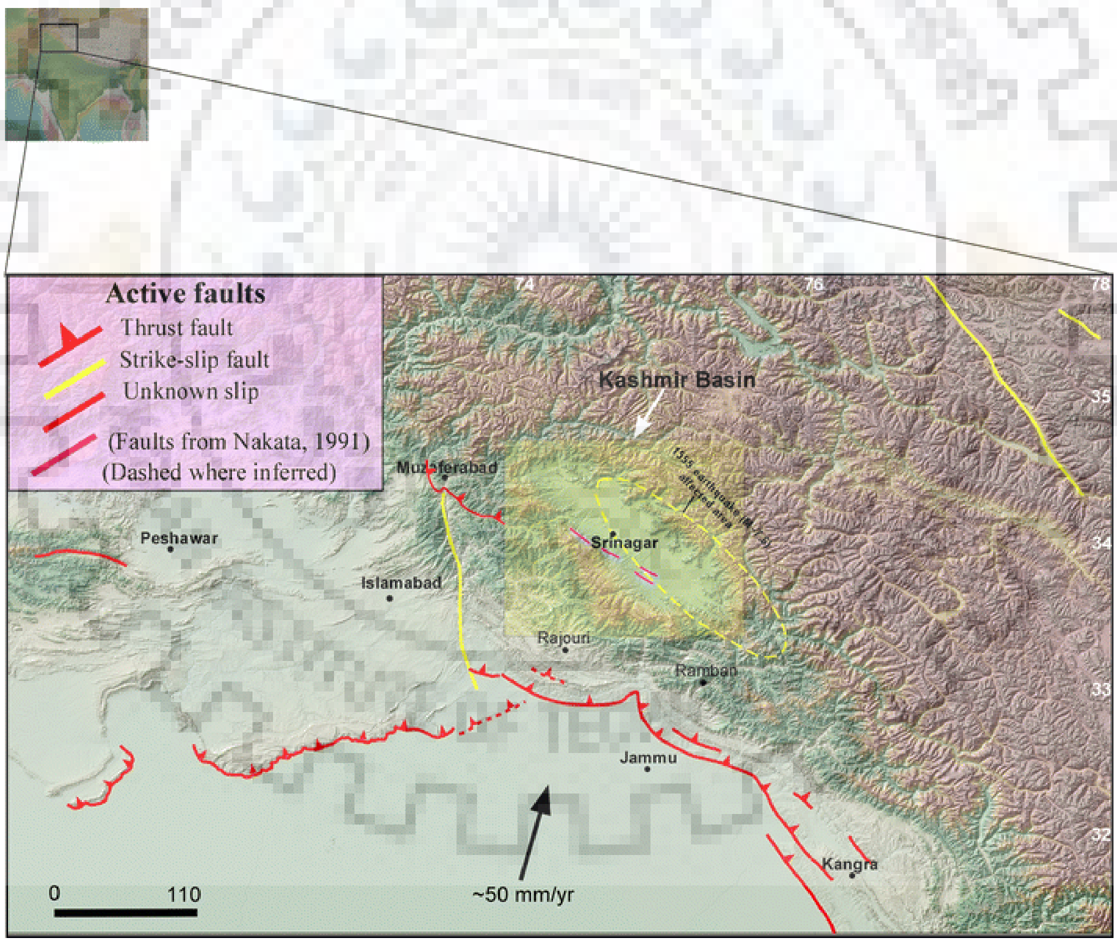

Fig 6 following,is Topographic map giving the major active faults in and around the Kashmir Basin.A few of these faults, are depicted in detail the arrow gives the average motion of Indian plate relative to Eurasia (. Nakata et al. 1991; Malik and Nakata 2003; Avouac et al. 2006; Kumar et al. 2010; Valli et al. 2007, 2008; Rajendran 2004 and references herein).

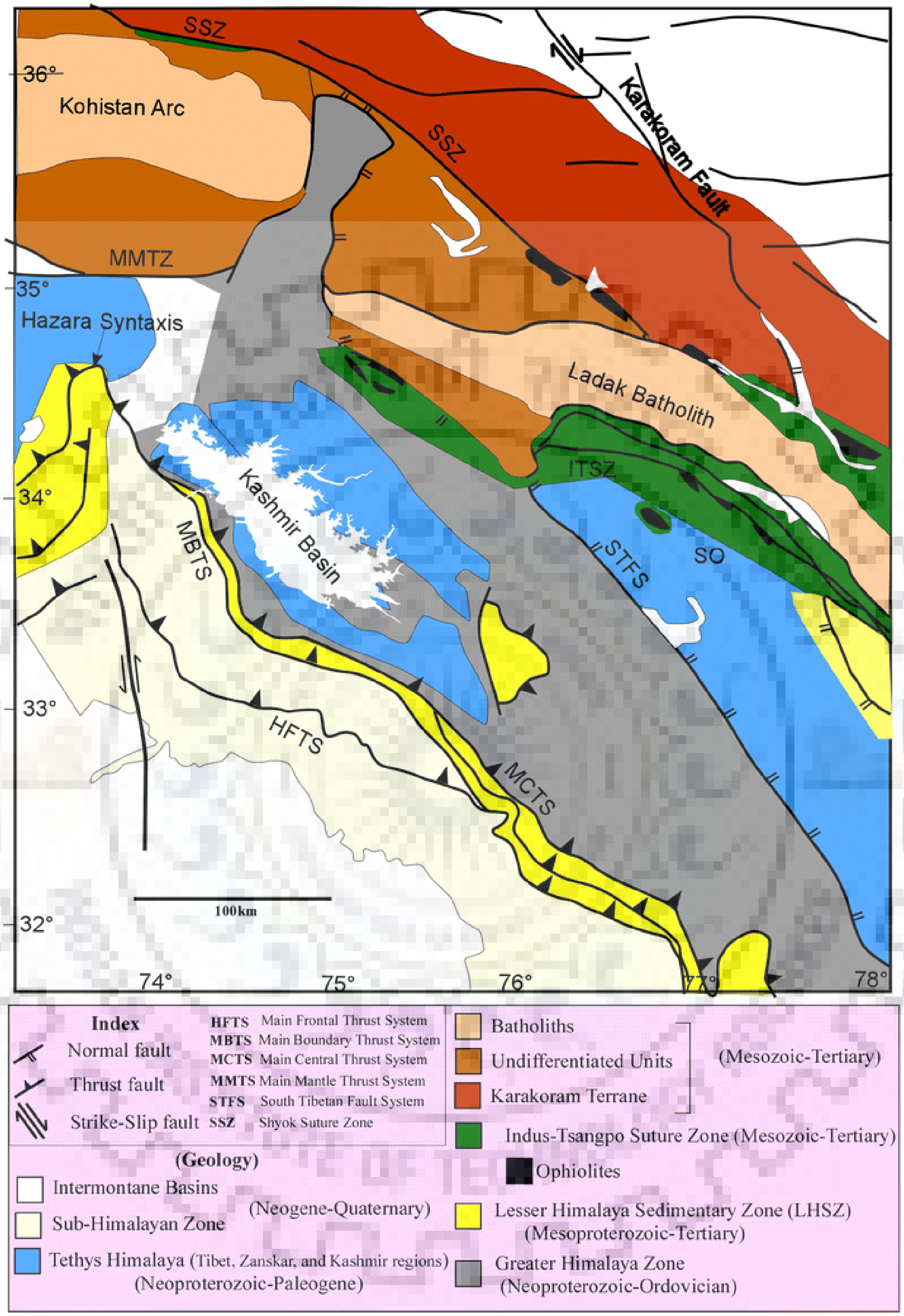

Fig. 7Tectonic map of of NW Himalayas (it is modified after Hodges 2000)

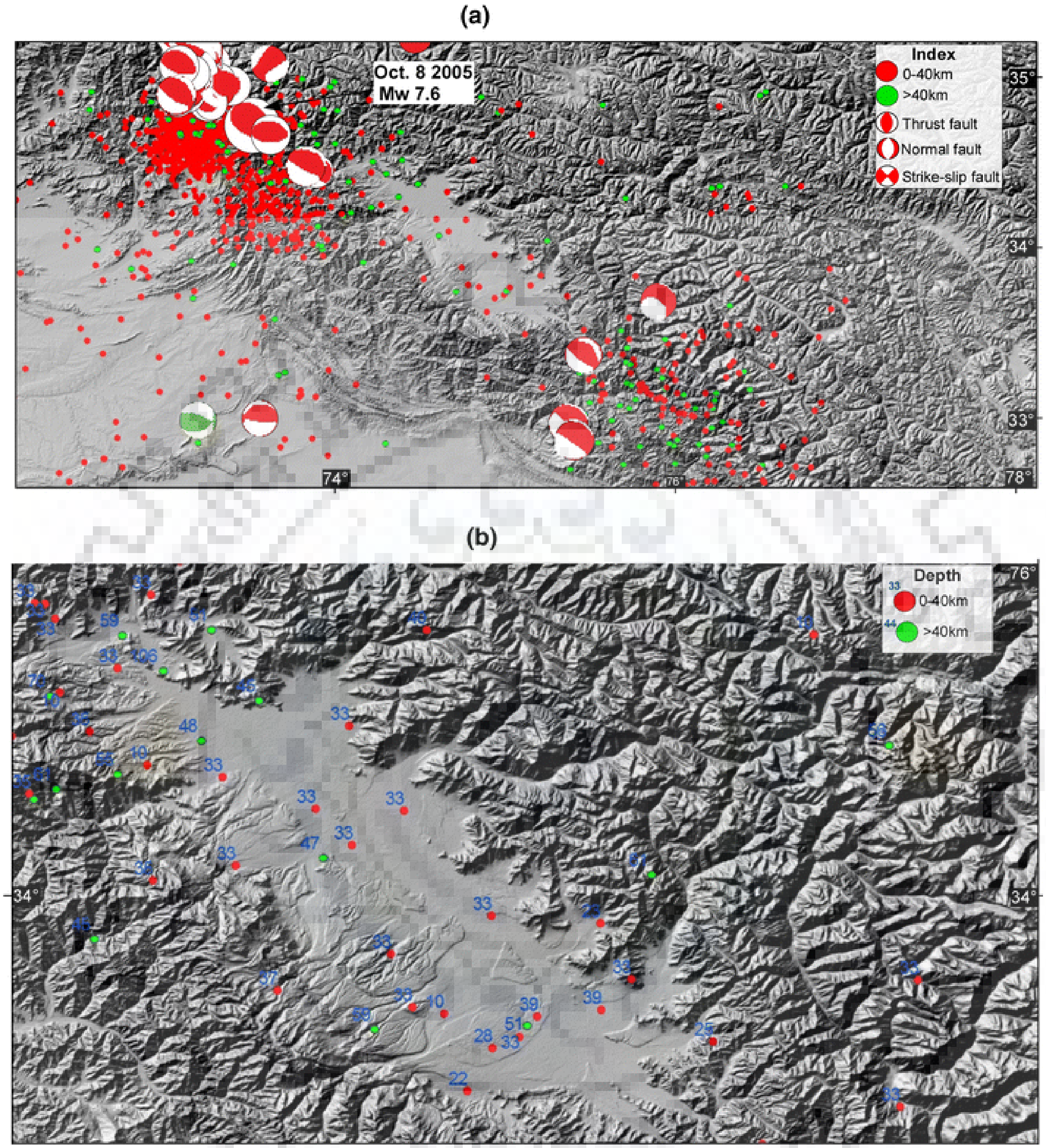

Fig. 8 a The seismicity data (NEIC, 1973–2012 and CMT catalogue, 1976–2012) it is being plotted on the SRTM topographic image of the Kashmir Basin (KB) and the adjacent regions. Figure b gives the view of the KB and the NEIC earthquakes ,with annotated depth in blue (Nakata et al. 1991; Malik and Nakata 2003; Avouac et al. 2006; Kumar et al. 2010; Valli et al. 2007, 2008;)

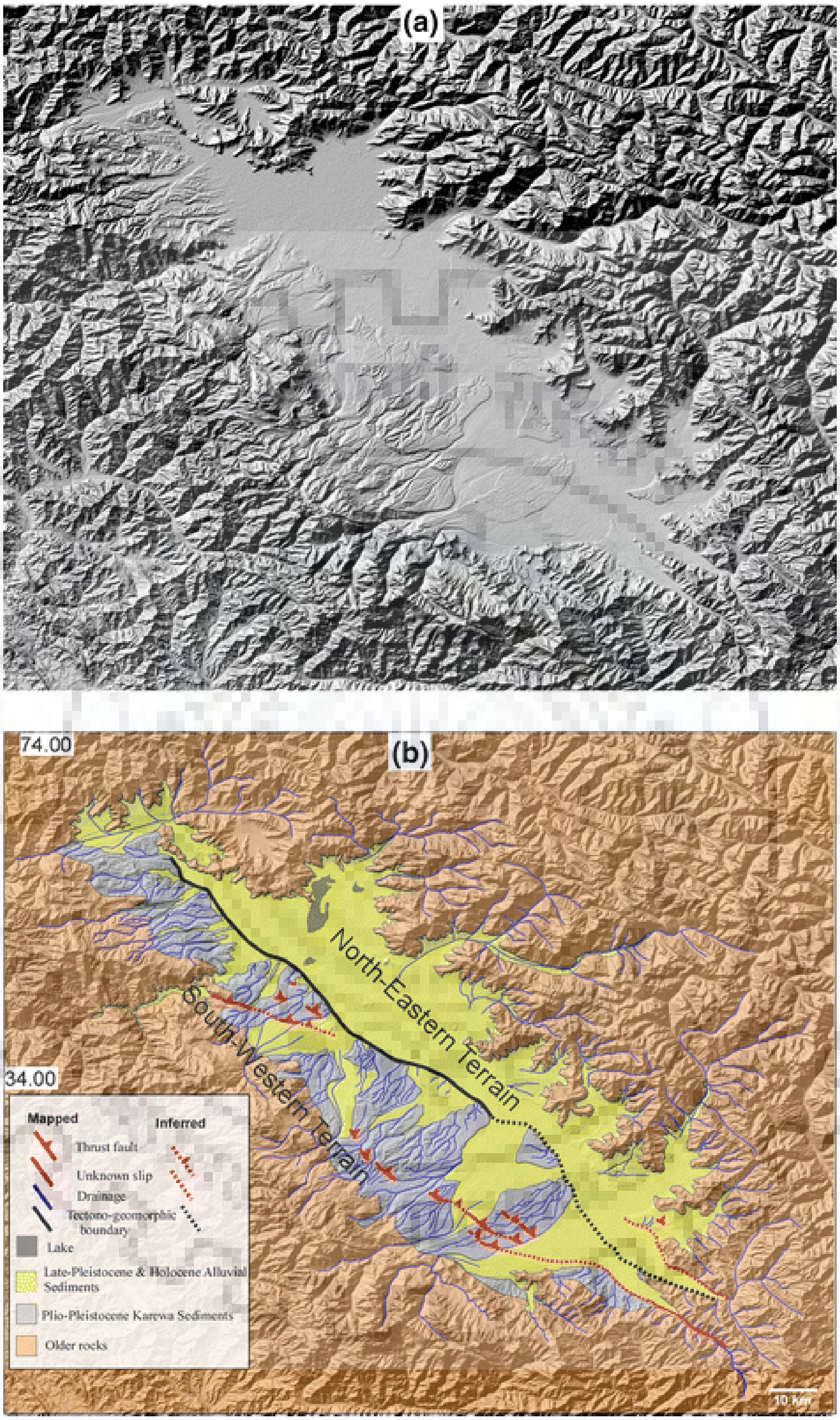

Fig. 9 Evidence of active thrust faults in the Kashmir Basin. Uninterrupted topography is lying on the top. Mapped active faults and bedrock are depicted at the bottom (freely available 90-m-resolution SRTM data used)

While collecting the data from national earthquake information centre it was seen that in the Kashmir basin the major earthquakes are of low depths only .these earthquake are distributed very un evenly. The Kashmir basin is mainly oval shaped. The north east region is drowned while the south west region is uplifted.

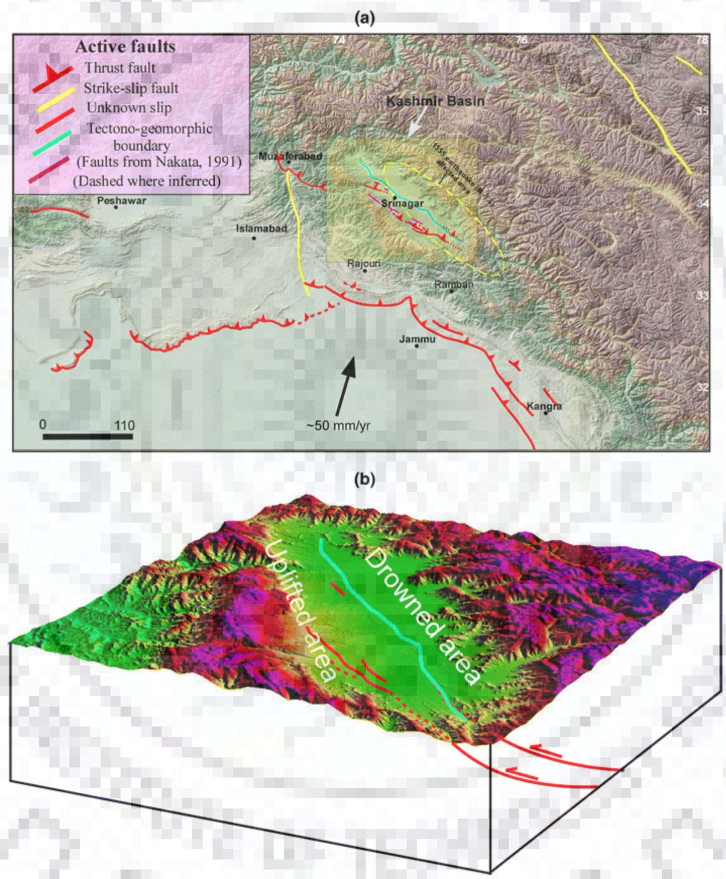

Fig. 10 the following is STRM image shows the Kashmir Basin and its tectonic geomorphology. b The active thrust faults have uplifted ,half of the valley and drowned the other half. (Nakata et al. 1991; Malik and Nakata 2003; Avouac et al. 2006; Kumar et al. 2010; Valli et al. 2007, 2008;)

Over the last hundred years more than 35 earthquakes have strike Kashmir basin which have magnitude greater than 7 suggest that the Kashmir sits on the top list of high seismicity.

Recently the Moment Magnitude (Mw) 7.8 earthquake ruptured part of this plate boundary fault. It occurred on 25th April 2015, in Nepal .during this earthquake we lost more than 15000 people. The fault is called as, Main Frontal Thrust (MFT) fault. It a megathrust fault that accumulates an average value of 2 cm/year the regional convergence between India plate and Eurasia plates. The accumulated length of this fault is considered more than 2000 kilometre .It marks present day active, plate boundary along which, accumulated stress is sometimes released through medium to large magnitude earthquakes. Thus, we can say that it is not surprising that the ongoing collision has caused in more than seven major earthquakes along the Himalayan region in the past 100 years.

Fig11. Region of study

Fig. 12 Latitudes and Longitudes - Region of study

These faults could pose a lot of danger and could potentially nucleate earthquake in the near future. Thus due to following reasons we choose the following region of Jammu and Kashmir for our study.

CHAPTER 7

MEHODOLOGY FOLLOWED

7.1POISSON DISTRIBUTION

In case of poisson distribution firstly we take the diclustered catalogue .than we find the inter arrival time for magnitude greater than 6 .we take the distinct value and arrange them in decreasing order. We than find the average rate of occurrence, λ. It may be found by dividing number of years to the difference of first and last year of the catalogue. We apply the specific formula of probability density function and cumulative density function. We than find reliability function. The hazard rate function is find out by following formula.

$$\text{Hazard rate}= \frac{p.d.f}{r(t)} = \frac{p.d.f}{1-c.d.f} \qquad (7.1)$$

Hazard rate curve is than plotted and matched with the theoretical value.

7.2LOGNORMAL DISTRIBUTION AND WEIBULL DISTRIBUTION

For the lognormal distribution the method followed is similar to poisson distribution. The catalogue is taken and is diclusterd .the inter arrival time for magnitude greater than 6 is found out .the 8unique value of the inter arrival time are taken .these values are arranged in descending order for these inter arrival time the lognormal distribution is than applied. For applying lognormal distribution the first step is to find the shape parameter and the scale parameter.
As discussed earlier the shape and scale parameter are calculated by maximum likelihood method

$$\mu= \frac{\sum ln\ x}{n} \qquad (7.2)$$

$$\sigma_X = \sum(\frac{\ln x - \mu}{n}) \qquad (7.3)$$

By finding the parameters of the distribution find the probability density function and cumulative density function .find the reliability function also which is numerically equal to 1-c.d.f .the hazard rate function is than calculated respective to each inter arrival time.

The last step is to find the conditional probability of earthquake occurrence in the future period. Calculating the conditional probability let us assume that last event of magnitude 6 occurred in 2015 from the catalogue .so if we ought to calculate the occurrence after 10 years after the present 2018

$$p(I\,\frac{10}{3}) = \frac{c.d.f(13) - c.d.f(3)}{1 - c.d.f(13)} \qquad (7.4)$$

Generalising the above method we can calculate the probability of occurrence after the number of years desired.

CHAPTER 8

OBSERVATION AND RESULTS

8.1 POISSONS DISTRIBUTION

For the poisons method the following mathematical calculations are done. Value of average rate of occurrence λ= 0.5377

λ= No. of events/Time period

No. of events = 114

Time period = 212

λ= 0.5377

The probability density function =p.d.f=f(t)= $\lambda e^{-\lambda t}$

The reliability function = r (t) =1- c.d.f = $e^{-\lambda t}$

where c.d.f =1- $e^{-\lambda t}$

t	PDF	1-CDF =RELIABLITY R(T)	HR
1	0.3141	0.4159	0.5377
2	0.1834	0.6589	0.5377
3	0.1071	0.8008	0.5377
4	0.0626	0.8836	0.5377
5	0.0366	0.9320	0.5377
6	0.0213	0.9603	0.5377
8	0.0073	0.9865	0.5377
10	0.0025	0.9954	0.5377
15	0.0002	0.9997	0.5377
20	0.0000	1.0000	0.5377
50	0.0000	1.0000	0.5377

Table1 showing calculation of hazard rate value

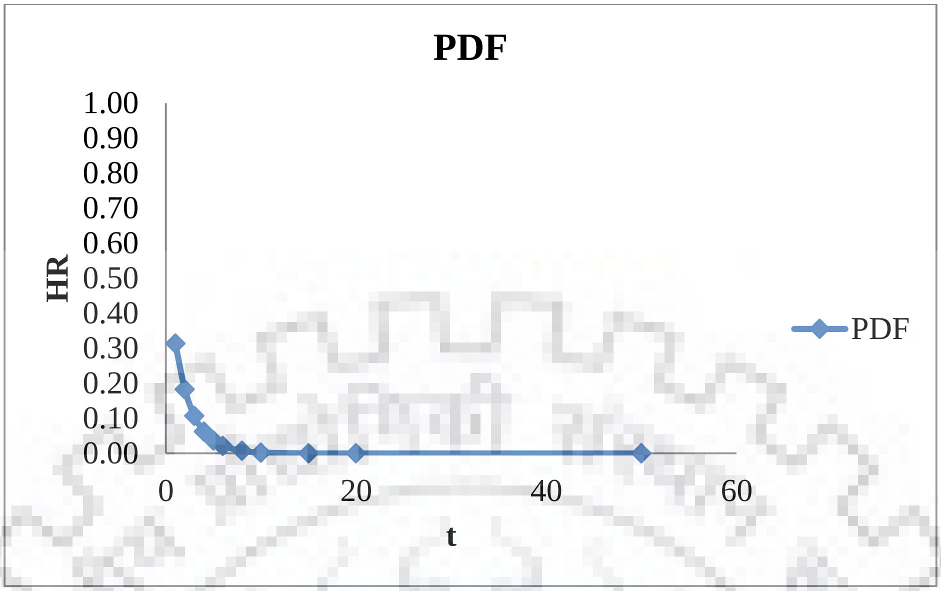

Fig 13 Graph showing probability graph for poisson distribution

Further the probability of occurrence of earthquake of magnitude greater than 6 in the upcoming years can be computes as

$$P(n\geq1)= 1- e^{-\lambda t}$$

t	probbality
1	0.4159
2	0.6589
3	0.8008
4	0.8836
5	0.9320
6	0.9603
8	0.9865
10	0.9954
15	0.9997
20	1.0000
50	1.0000

Table 2 Table depicting calculation of probability

The hazard rate curve thus can be plotted as given below

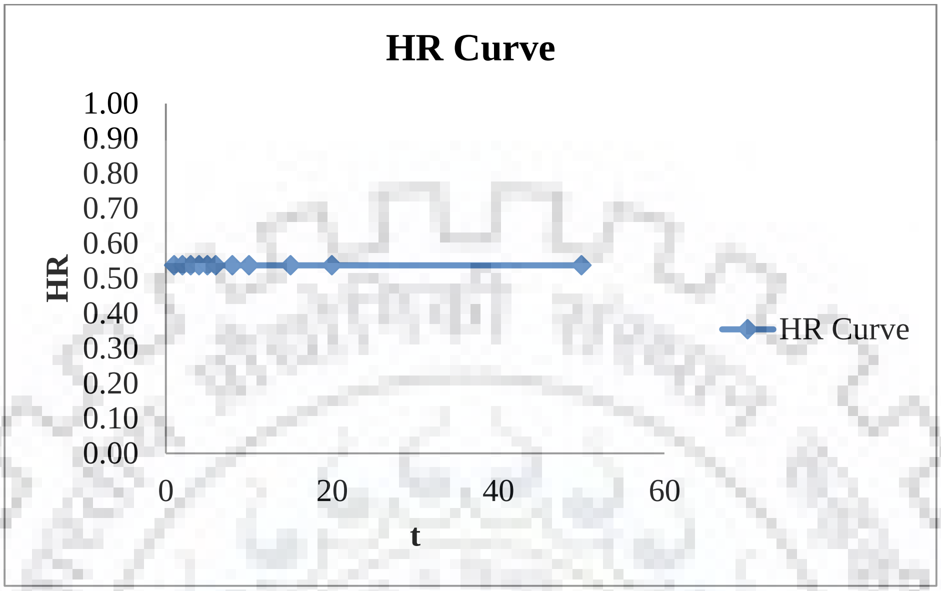

Fig 14 hazard rate curve for poisson distribution

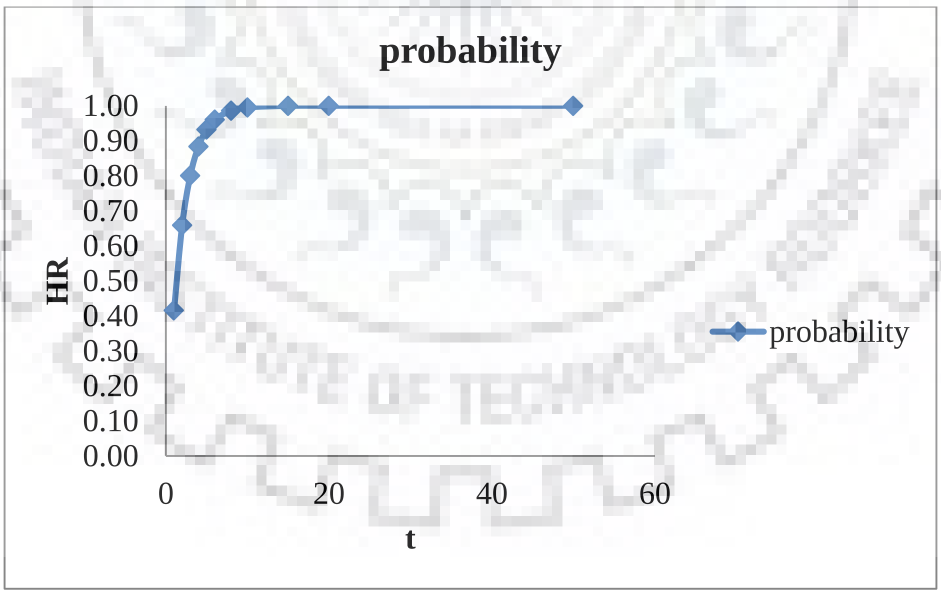

Fig 15 graph showing the trend of probability

8.2 LOGNORMAL DISTRIBUTION

For the lognormal distribution the parameters calculated are as follows; $\mu=0.4596$ and $\sigma=0.6537$

From these parameter calculated use the standard p.d.f and c.d.f equations to find the value and further the hazard rate is calculated.

Probability Density Function (PDF)	**Cumulative Density Function (CDF)**
0.0034	1
0.0095	0.918
0.0178	0.756
0.0256	0.729
0.0342	0.66
0.0424	0.618
0.0526	0.569
0.061	0.512
0.0706	0.447
0.081	0.371
0.0909	0.285
0.097	0.19
0.0904	0.095
0.0518	0.02

Table 3 calculated pdf and cdf

Inter arrival time	Hazard Rate
32	0.0695
25	0.0793
13	0.106
12	0.109
10	0.1155
9	0.1188
8	0.1221
7	0.1251
6	0.1276
5	0.1287
4	0.1271
3	0.1198
2	0.0999
1	0.0529

Table 4 table showing hazard rate function corresponding to inter arrival time

From the following tables the graph for the hazard rate and probability density function is plotted

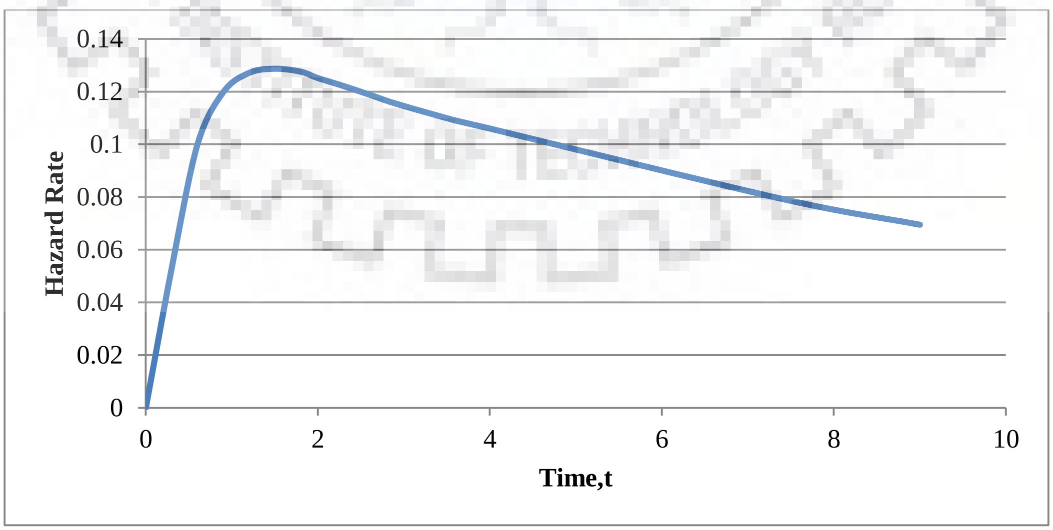

Fig.16. Hazard Rate versus time plot

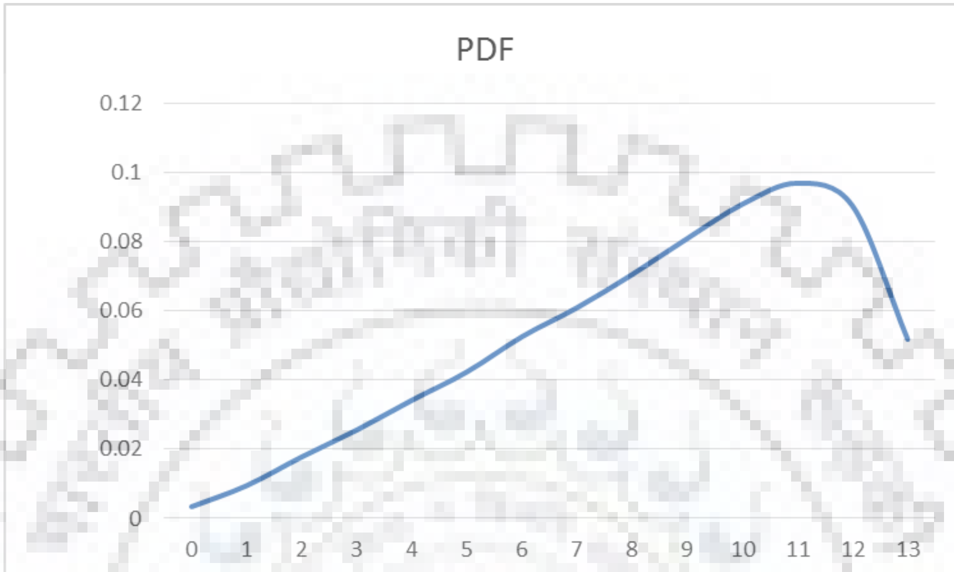

Fig 17 Figure showing pdf graph

No. of Years	**Probability of EQ with magnitude ≥ 6**
10	0.65
20	0.71
30	0.84
40	0.878

Table.5 Calculation of Probability

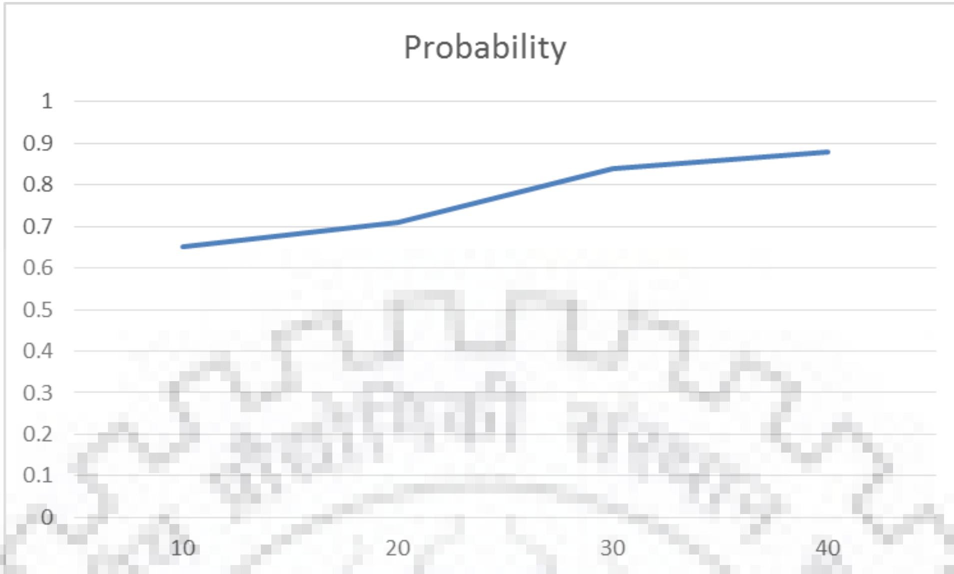

Fig 18 figure showing the trend of probability of occurrence of earthquake

8.3 WEIBULL DISTRIBUTION

From the given inter arrival time the parameters are calculated by applying the maximum likelihood method and newton - raphson method

η= scale parameter =4.648

β= shape parameter=0.9894

Inter arrival time	Probability Density Function (PDF)	Cumulative Density Function (CDF)
32	0.00025	1
25	0.00106	0.99493
13	0.01024	0.93712
12	0.01636	0.92238
10	0.02499	0.88164
9	0.0309	0.8538
8	0.03823	0.81937
7	0.04731	0.77676
6	0.05859	0.72401
5	0.0726	0.65867
4	0.09004	0.57766

3	0.11182	0.47714
2	0.13913	0.35219
1	0.17386	0.19642

Table 6 pdf and cdf of weibull distribution

Inter arrival time	Hazard Rate
32	0.958187
25	0.643475
13	0.450979
12	0.3668
10	0.306227
9	0.279253
8	0.25206
7	0.231613
6	0.203808
5	0.191496
4	0.181496
3	0.173808
2	0.17014
1	0.16854

Table 7 table showing calculated hazard rate

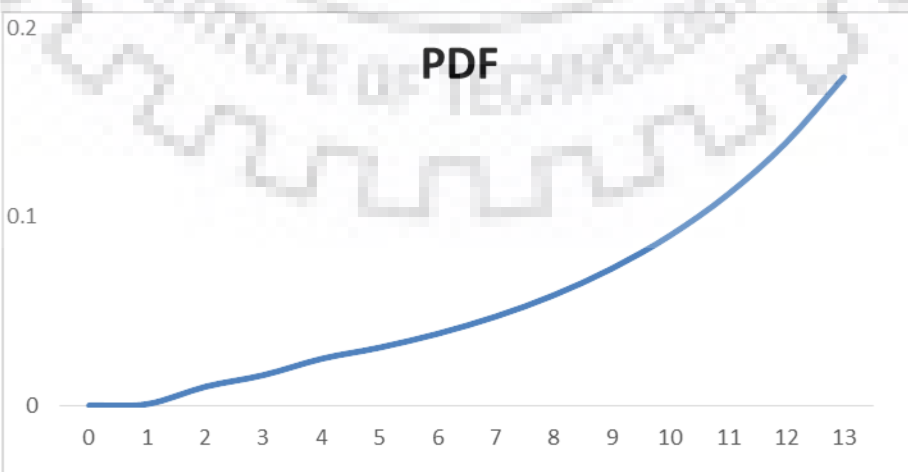

Fig 19 Figure showing the variation of pdf for weibull distribution

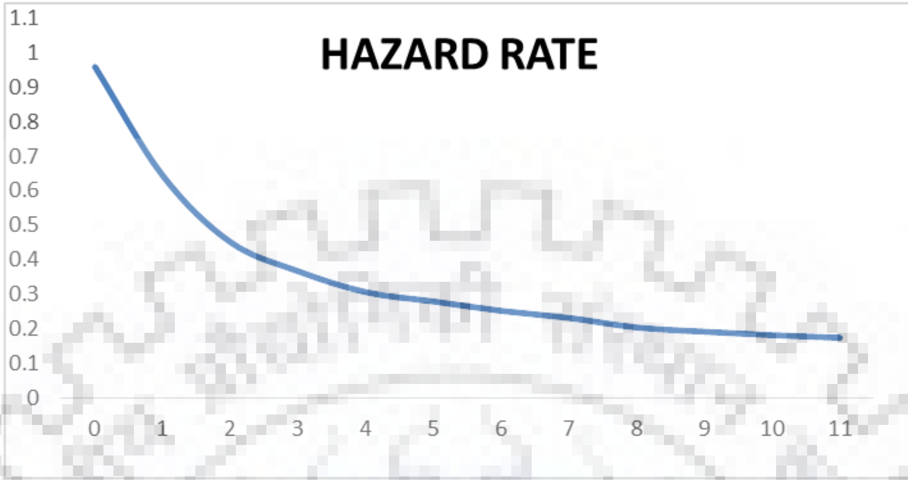

Fig 20 Figure showing the hazard rate curve

CHAPTER 8

CONCLUSION

From the above hazard rate curve or instantaneous failure rate curve it can be concluded that the strain energy is accumulated and then released out .from the previous calculation on the poisson model we have seen that hazard rate curve is linear with a constant value of .this implies that the strain energy if accumulated during the build-up of stress is never released .or in other words we can say that if the strain energy is released during an earthquake process than it is never accumulated back .

In seismological terms both these conditions are not physically possible. If once the strain energy is accumulated it will definitely get released till the further stress bold up. Both these conditions are also not justifying the elastic rebound theory by H.F.REID. Thus although the poisson model is simpler to use but due to its following drawbacks it is not in frequent seismological studies nowadays. On the contrary the hazard rate curve of the lognormal distribution is having the shape which is according to the seismological theory it is unimodal with convexity upwards. Hazard rate h(t) or sometimes may be called as the instantaneous failure rate is , is an important reliability measure, it is given as a ratio of f(t) to R(t). Where f(t) is the probability density function and r(t) is the reliability function . The reliability function is numerically equal to 1-c.d.f.

So the curve depicts that as the time progresses the strain energy is accumulated in the fault or adjacent to it in the region of Kashmir considered. This reaches to its peak value which can be correlated to the fact that a major earthquake is believed to occur .once this event has occurred the strain energy will start getting released .this can be studied in accordance to the elastic rebound theory .

We can also conclude that at time t=0 the hazard rate function is zero and at time approaches to infinity ,t=∞ the failure rate is also zero .thus clearly showing that it needs sometime for the energy to get accumulated and once all of this energy is released after the fore shocks no big earthquake can occur. For the next big event the energy should once again get accumulated thus in failure data analysis, the hazard rate behaviour plays a key role and therefore, it is necessary to know its correct form.

The probability of occurrence of earthquake greater than magnitude 6 is different in the lognormal distribution and the poisons distribution. It is due to the fact that the parametric calculation for poisons model includes many assumptions .it assumes to be earthquake as a memory less event, without considering any temporal uncertainties.

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