EARTH PRESSURE AND SEISMIC DESIGN OF RETAINING WALLS

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree

of

MASTER OF TECHNOLOGY

in

EARTHQUAKE ENGINEERING

(With specialization in Soil Dynamics)

By

SURESH KUMAR

DEPARTMENT OF EARTHQUAKE ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY ROORKEE

ROORKEE- 247 667 (INDIA)

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CANDIDATE'S DECLARATION

I hereby declare that the work which is being presented in this dissertation entitled, "**EARTH PRESSURE AND SEISMIC DESIGN OF RETAINING WALLS**" in partial fulfillment of the requirements for the award of the degree of **Master of Technology** in **Earthquake Engineering** with specialization in **Soil Dynamics** submitted to the Department of Earthquake Engineering, Indian Institute of Technology, Roorkee is the authentic record of my own work carried out under the supervision of **Dr. B.K. Maheshwari**, Professor, Department of Earthquake Engineering, Indian Institute of Technology Roorkee, India.

The matter presented in this dissertation report has not been submitted by me for the award of any other degree of this or any other institute.

Place: Roorkee

Date: Suresh Kumar

CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

Dr. B.K. Maheshwari
Professor
Department of Earthquake Engineering
Indian Institute of Technology, Roorkee
Roorkee-247667, Uttarakhand, India

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Place: Roorkee

Date: Suresh Kumar

ABSTRACT

This dissertation described study into earth retaining cantilever walls with relief shelves behaviour under seismic conditions, using numerical modelling technique. The effects of provision of relief shelves on lateral earth pressure and backfill soil settlement were studied with the help of numerical models using small strain finite element methods available in commercial software ABAQUS. The soil response was simulated using the Mohr-Coulomb's model. Two seismic events (i.e., 1994 Northridge earthquake and 1983 Trinidad earthquake) were considered for dynamic loading.

The results for static surcharge and soil self-weight loading showed that the lateral earth pressure and therefore total thrust on the wall reduces significantly when the cantilever retaining wall is provided with monolithic relief shelves. The backfill-soil-surface settlement also decreased substantially for the wall with one relief shelf, in comparison to the one with no relief shelf. When the number of relief shelves provided increased, the settlement further reduced. However, it was observed that the soil settlement remains almost similar for the cases of the wall with two and three relief shelves. The effect of relief shelf thickness on the lateral earth pressure and the backfill soil settlement were also investigated.

The response of the cantilever retaining wall with and without relief shelves were studied for the considered seismic events. The lateral earth pressure decreased for the wall with no relief shelves as the wall tilted towards opposite of the backfill soil. This led to a drastic increase (about 85%) in vertical settlement of the backfill soil. This shows that the cantilever retaining walls without any additional measure for settlement resistance can lead to failure of structures constructed on the backfill soil. However, as expected, the retaining wall with relief shelves resisted the tilting wall due to earthquake. Thus, the lateral earth pressure and the backfill soil settlement remained almost similar to those before the occurrence of earthquake.

The importance of the provision of relief shelves with the cantilever retaining wall is clear from this study. Such walls behave better in the time of seismic events.

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Chapter 1

INTRODUCTION

Retaining walls have been most important type of structure in the field of geotechnical engineering. These are constructed to resist the lateral earth pressure of the backfill and find a wide range of applications in situations such as slope stability, geotechnical excavations and temporary constructions etc. Various types of such retaining structures are in practice which include gravity retaining walls, cantilever retaining walls, sheet pile walls, anchored sheet pile walls, diaphragm walls, buttress retaining walls and basement walls. In certain circumstances, special types of retaining wall are also provided. These include retaining walls with counterforts, reinforced soil retaining walls and buttress retaining walls.

Retaining walls are susceptible to failure if their responses under static and dynamic conditions are not properly predicted and designed accordingly. Therefore, while designing retaining walls, stress and displacement criteria become important. For analysis and design of retaining walls magnitude and distribution of earth pressure and displacement of the wall under static and dynamic conditions is required.

The lateral earth pressure influences the dimension and indirectly the cost of construction of these walls heavily. Geotechnical engineers have been trying various techniques to reduce the lateral earth pressure on the retaining structures. Few widely used methods are the provision of EPS geofoam between wall and soil interface, reinforcing the backfill soils by geo-synthetic materials and many more. In recent times, a special type of retaining wall is being tried to implement in sustaining high earth pressures from steep and very high slopes. These walls are known as retaining walls with relief shelves. Relief shelves are horizontal reinforced concrete slabs, which are provided monolithically with the retaining wall stem, and extend into the soil at 90°, throughout the wall. The number and sectional dimensions of such shelves provided on the wall-back depend on the height of the wall, acting lateral earth pressure and the required decrease in earth pressure. High cantilever retaining walls with such relief shelves provided on backfill side of the wall, may be the most economical earth retaining solution. The relief shelves decrease the lateral earth pressure on the wall back and increases the overall stability of the retaining wall. The study of this type of retaining wall has not been an active topic of research in the investigations of retaining structures. Few studies have been carried out on the real behaviour of this type of wall. Therefore, studying the effectiveness of this type of retaining wall is required for its use in practical application.

In this report, a study on the behavior of the retaining walls with pressure relief shelves under static as well as seismic conditions with varying geometrical parameter has been done.

Chapter 2

LITERATURE REVIEW

2.1 General

The seismic behavior of retaining walls depends on the total lateral earth pressures that develop during earthquake shaking. These total pressures include both the static gravitational pressures that exist before an earthquake occurs and the transient dynamic pressures induced by the earthquake. Static earth pressures on retaining structures are strongly influenced by wall and soil movements. One common approach to the seismic design of retaining walls involves estimating the load imposed on the wall during earthquake shaking and then ensuring that the wall can resist those loads. Since the actual loading on retaining walls is extremely complicated, seismic pressures on retaining walls are usually estimated using simplified methods.

In the seismic zones, the retaining walls are subjected to dynamic earth pressure, the magnitude of which is more than the static earth pressure due to ground motion. For better approximation of the seismic response, evaluation of the dynamic earth pressures acting on the wall should be as accurate as realistically possible, both in terms of magnitude as well as point of application.

2.2 Types of Retaining Walls

Retaining walls are often classified in terms of their relative mass, flexibility and anchorage conditions.

2.2.1 Gravity walls use their mass or weight to resist the pressure exerted by the earth behind them. These walls usually have an average height of three to four feet. They are made from mortarless stone or masonry units. These walls comprise of a volume of materials which are stacked together in the making of the walls. The weight or force of friction that is created by these materials is greater than the force exerted by the soil. A process known as 'battering' helps the walls to improve stability by leaning back into the retained soil. In this process, as the walls get taller and they slant backwards. Battering is done to prolong the lifespan of gravity walls, which otherwise would tilt outward [Kramer (1996)].

2.2.2 Cantilever walls are among the taller retaining walls. Here, the walls have uniform thickness and are tied to a footing. Properly engineered cantilever walls hold back sufficient amount of soil. These walls bens and as well as translate and rotate and rely on their flexural strength to resist lateral earth pressure. The actual distribution of lateral earth pressure on a cantilever wall is influenced by the relative stiffness and deformation of both the wall and soil. The walls are built with steel-reinforcement in both the footing and wall structures [Kramer (1996)].

2.2.3 Sheet piling retaining walls are used in areas having soft soils and tight spaces. Materials such as steel, vinyl or wood planks are used for making these types of retaining walls. A cable or a rod is used as a tie-back anchor to the walls. The rods are placed at a distance and tied to the back of the walls. Hydrostatic pressure is one of the main causes of the instability of the wall and hence proper drainage has to be ensured during construction [Kramer (1996)].

2.2.4 Concrete retaining walls are common in gardens and other outdoor landscapes. They offer better support for vertically-slanting slopes. Concrete retaining walls are high-built and have deeper and heavier soil underneath them. These properties make them offer better resilience and solidity. These types of retaining walls require greater base depth so as to create a better foundation. Properly installed concrete retaining walls do not face problems like tilting, bowling or cracking [Kramer (1996)].

2.2.5 Braced walls are constrained against certain types of movement by the presence of external bracing elements. In case of basement walls and bridge abutment walls lateral movement of the top of the wall are restrained by the structures they support. The provision of lateral support at different locations along a braced wall keeps bending moment so low that relatively flexible structural sections can be used [Kramer (1996)].

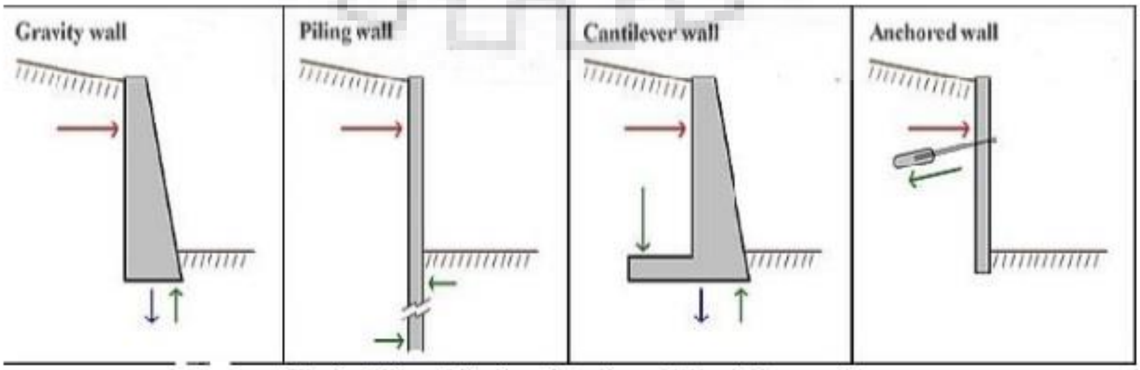


Figure 2.1 Simplified explanation of Retaining wall (www.wikipedia.com)

2.3 Types of Retaining Wall Failures

Under static conditions retaining walls are acted upon by body forces related to mass of the wall, by soil pressures and by external forces such as those transmitted by braces. A properly designed retaining wall will achieve equilibrium of these forces without inducing shear stress that approach the shear strength of the soil. During an earthquake inertial forces and changes in soil strength may violate equilibrium and cause permanent deformation of wall. Failure by sliding, tilting, bending or by other mechanism occurs when these permanent deformations become excessive [Kramer (1996)].

2.3.1 Overturning: This occurs when the turning moment due to lateral forces exceeds that due to the self-weight of the wall. Thus it occurs when moment equilibrium is not satisfied. The factor of safety against overturning should be at least two.

2.3.2 Sliding: The wall will slide if the lateral pressure on the back of the wall produces a thrust that exceeds the frictional resistance developed between the base of the wall and the soil. Thus it occurs when horizontal equilibrium is not satisfied The factor of safety against sliding should be about two.

2.3.3 Bearing on ground: The normal pressure between the base of the wall and the soil beneath can cause a bearing failure of the soil, if the ultimate bearing capacity is exceeded. The allowable bearing pressure is generally assumed as one-third of the ultimate value.

2.3.4 Flexural failure: Soil pressures and bending moment in cantilever walls depend upon geometry, stiffness and strength of the soil-wall system. If the bending moment required for equilibrium exceeds the flexural strength of the soil, flexural failure may occur.

2.3.5 Tilting: The wall and a large amount of the retained material rotate about some point if the shear resistance developed along a circular arc is exceeded. Tilting of braced walls involves rotation about the point at which the brace acts on the wall, often the top of the wall as in the cases of basement and bridge abutment walls.

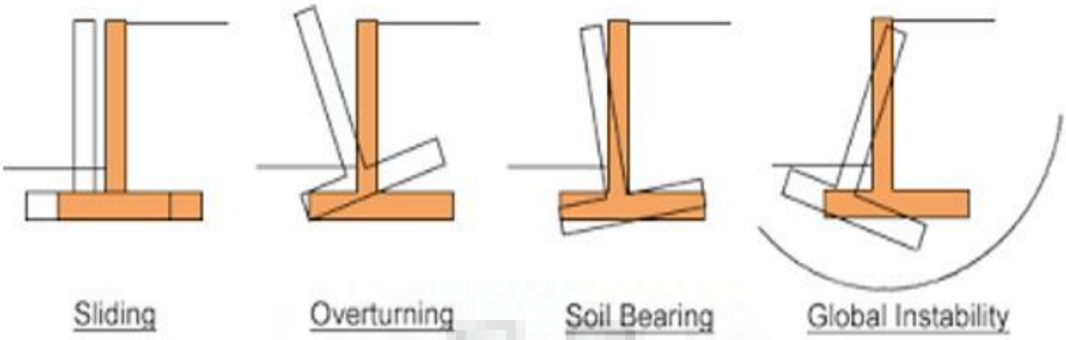


Figure 2.2 Failure mechanism of Retaining wall (www.wikipedia.com)

2.4 Classical Earth Pressure Theories

Coulomb's (1773) theory of earth pressure involves the consideration of a sliding wedge which tends to break away from rest of the backfill upon wall movement. A rupture surface is assumed and equations of equilibrium of the wedge are written taking care that the unknown stresses on the rupture surface do not enter the equations. These equations for pressures on the wall are then either maximized (active pressure) or minimized (passive pressure). Coulomb however realized, the rupture surface could be curved; he used only a plane surface [Kramer (1996)].

Rankine's (1857) theory is based on the theory of plasticity. He considered the state of limiting equilibrium at any point in a soil mass bounded by a plane surface. Using the principle of conjugate stresses between the major and minor stresses, it was assumed that the introduction of the wall does not affect the state of stresses, i.e. the wall is frictionless. The pressure on the wall at any point would be equal to the stress on the vertical plane. The earth pressure on the wall was thus determined in terms of the strength parameters of the soil c, the cohesion and ϕ the angle of internal friction of soil [Kramer (1996)].

Culmann (1866) presented a graphical method to determine the magnitude of the earth pressure and to locate Coulomb's most critical rupture surface which is based on Coulomb's theory.

However neither of the above discussed theory takes into account cohesion as a soil parameter in lateral earth pressure computations. The first ever attempt to introduce cohesion in earth pressure computations was made by Bell (1915). Bell's equations were directly obtained from Mohr's circle. Currently these equations are recommended for computing lateral earth pressure in cohesive soils. In the later stage a number of modifications of Coulomb's wedge theory have been reported by Satyanarayan (1966, 68), Prabhakaran (1965) and Prakash and Saran (1966) by considering different geometries of the problem.

2.5 Dynamic Earth Pressure Theories - Analytical Methods

The first theoretical study in this field dates back to 1916 when Sano introduced the seismic coefficient method for seismic design of structures. He suggested substitution of ϕ, the angle of internal friction of the soil with $\left[\emptyset - \frac{\tan^{-1}\alpha_h}{1-\tan^{-1}\alpha_v}\right]$ in either Rankine's or Coulomb's theory where α_h and α_v are the horizontal and vertical seismic coefficients. These coefficients stand for a horizontal acceleration of $\alpha_h g$ and a vertical acceleration of $\alpha_v g$ where, g is the acceleration due to gravity. Also the term $\left[1 - \frac{\tan^{-1}\alpha_h}{1-\tan^{-1}\alpha_v}\right]$ denotes the angle of repose of the soil under earthquake conditions [Kramer (1996)].

The analyses carried out by Okabe (1926) and Mononobe (1929) for dynamic lateral earth pressure is a straight forward extension of the Coulomb sliding wedge theory. This considers additional quasi-static inertial forces of the fill material. This method has been the conventional method of estimating the lateral soil pressures for seismic design of retaining walls [Kramer (1996)].

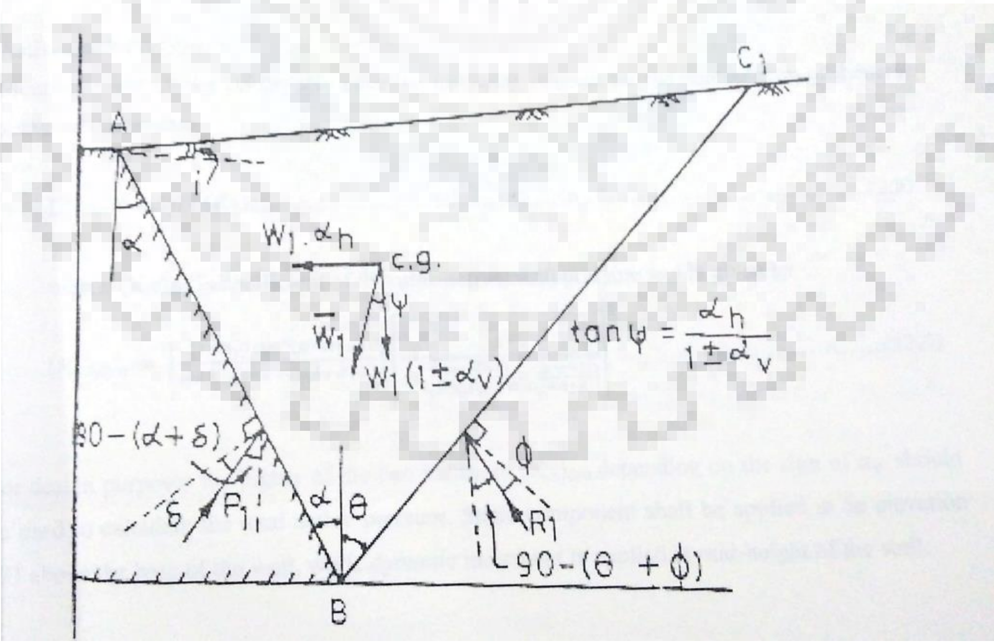


Figure 2.3 Forces acting on failure wedge in active state [Kramer (1996)]

where, H is the wall height, a is the vertical inclination of wall, y is the unit weight of soil, Ø is the angle of shearing resistance, BCI is the trial failure plane, 0 is the inclination of failure plane to vertical, t is the inclination of backfill with horizontal, ö is the angle of friction between wall and backfill,

$$\Psi = \tan^{-1}\left[\frac{\alpha_h}{1 \pm \alpha_v}\right] \qquad \textbf{(2.1)}$$

Where, α_h and α_v are the horizontal αh and αv are the horizontal are the horizontal and vertical seismic coefficients such that $\alpha_h = a_h/g$ t and$\alpha_v = a_v/g$, and a_h and a_v are the horizontal and vertical accelerations caused by the earthquake on wedge ABC_1.

Active Earth Pressure

Mononobe and Okabe (1929) also gave the following relation for the computation of dynamic Active Earth Pressure

$$(P_a)_{dyn} = 0.5\gamma H^2 (K_A)_{dyn} \qquad \textbf{(2.2)}$$

Where, $(K_A)_{dyn}$ is coefficient of dynamic active earth pressure and is given by

$$(K_A)_{dyn} = \left[\frac{(1 \pm \alpha_y) cos^2(\emptyset - \lambda - \alpha)}{cos\lambda cos^2\alpha \cos(\delta + \alpha + \lambda)}\right]\left[\frac{1}{1 + \sqrt{\left(\frac{\sin(\phi+\delta)\sin(\phi - i - \lambda)}{\cos(\alpha - i)\cos(\delta + \alpha + \lambda)}\right)}}\right]^2 \qquad \textbf{(2.3)}$$

For design purposes the higher of the two values of $(K_A)_{dyn}$ depending on the sign of α_v. should be used to calculate the total active pressure. Static component shall be applied at an elevation H/3 above the base of the wall, while dynamic increment is applied at mid-height of the wall.

Passive Earth Pressure

Mononobe and Okabe (1929) gave the expression for the computation of dynamic passive earth pressure as $(P_p)_{dyn}$ which is

$$(P_p)_{dyn} = 0.5\gamma H^2 (K_P)_{dyn} \qquad \textbf{(2.4)}$$

Where, $(K_P)_{dyn}$ is coefficient of dynamic passive earth pressure and is given by

$$(K_P)_{dyn} = \left[\frac{(1 \pm \alpha_y) cos^2(\emptyset + \lambda - \alpha)}{cos\lambda cos^2\alpha \cos(\delta - \alpha + \lambda)}\right]\left[\frac{1}{1 + \sqrt{\left(\frac{\sin(\phi+\delta)\sin(\phi - i - \lambda)}{\cos(\alpha - i)\cos(\delta - \alpha + \lambda)}\right)}}\right]^2 \qquad \textbf{(2.5)}$$

For design purposes the lesser value of the coefficient will be taken out of its two values corresponding to $\pm\alpha_v$ should be used to calculate the total passive pressure. Static component of total earth pressure will be applied at H/3 above the base of wall, while dynamic decrement at 0.66 H above the base of the wall.

Kapila (1962) gave approximate modifications for the well known graphical solution of the coulomb's theory by Culmann and Melbye to take into account the effect of dynamic forces. Prakash and Saran (1966) developed a general solution for determination of dynamic pressures exerted by c-φ soil on retaining walls. Gravity effects, surcharge effects and cohesive forces are considered one at a time and the principle of superposition has been utilized.

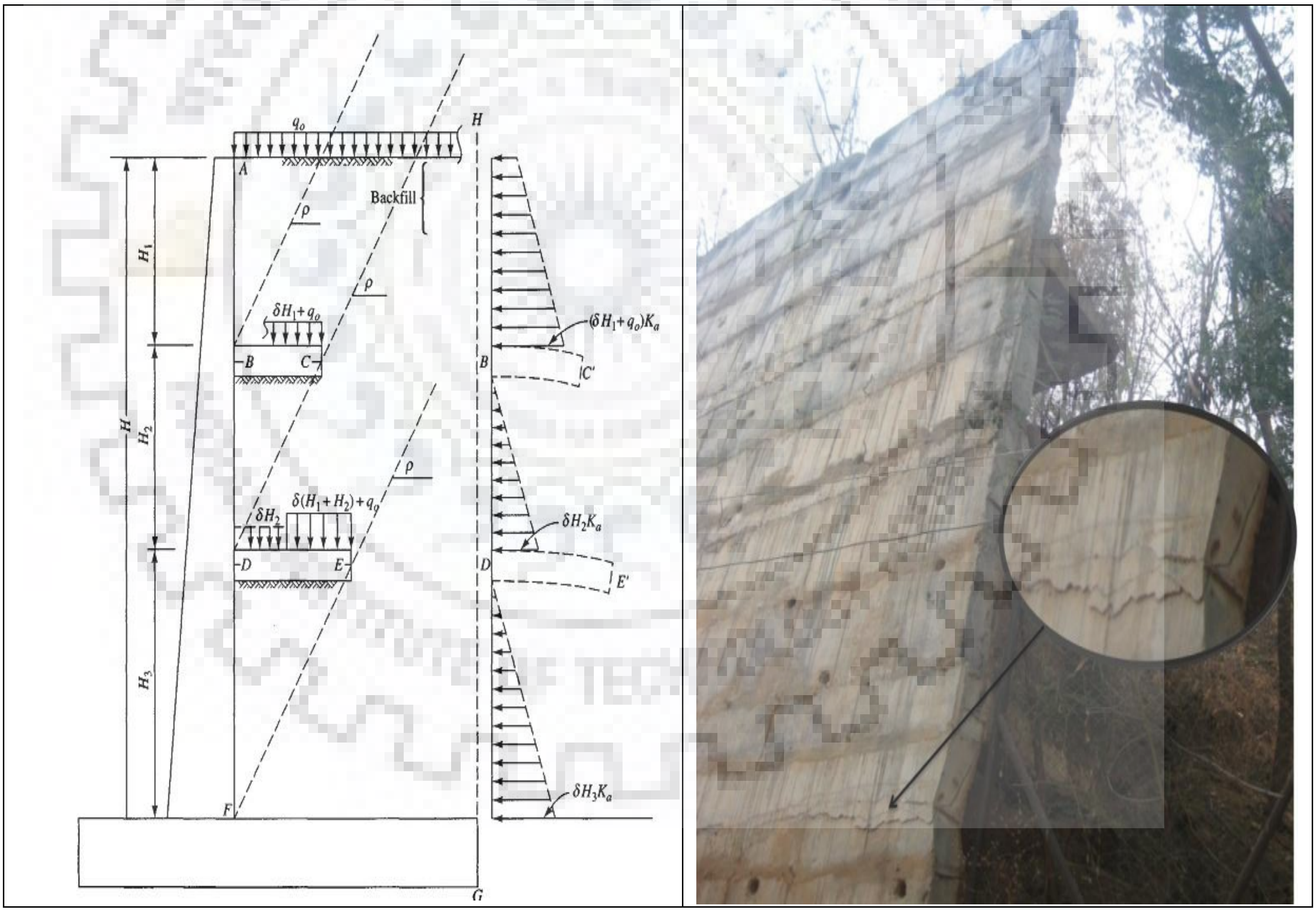
2.6 Retaining Walls with Pressure Relief Shelves

The stability of earth retaining structures have been widely studied in the literature. However, a majority of the previous reported studies do not consider the behaviour of retaining walls with relief shelves. As per Shehata H. F. (2016), the earliest investigation of such retaining structures started in 1927 at the University of Western Australia during an lecture on applied Geo-mechanics. In this lecture, such walls were considered as a flexible retaining wall. After a long time, Jumikis (1964) contemplated the impact of providing one or more relief shelves to a counterfort retaining wall in order to enhance the stability of the retaining wall. He found that the provision of relief shelves lead to decrease in the lateral earth pressure on the retaining wall and enhances it's stability. Chaudhuri et al. (1973) studied the response of wall with one relief shelf on the total thrust acting on the cantilever retaining wall. The total thrust was evaluated from the wedges stability analysis using Coulomb's theory the wall, and excluded the weight of soil above the relief shelf in calculations of weight of failure wedge. With the help of physical models of retaining walls, it was observed that the maximum height of retained sandy soil before the overturning was greater in case of walls with relief shelf than the retaining walls without the shelves. Later it was suggested by Bowles (1997) that these walls can be a possible solution for high cantilever retaining walls on ensuring the proper compaction of backfill soil. The concept of such walls is shown in the Figure 1(a). Figure 2.4(b) shows a cantilever retaining wall with relief shelves constructed in Hyderabad, India.

Recently, Chauhan et al. (2016) has performed a parametric study focussing on the retaining wall with pressure relief shelves using commercial software FLAC3. The reported that the lateral earth thrust on the wall with relief shelves decrease by 43-48%, but mentioned that the consideration of inappropriate lateral earth pressure magnitude and distribution in the design calculations might be cause of the failure of the wall shown in figure 2.4(b).

However, a thorough understanding of the change in earth pressure and backfill settlement profiles in the time of a seismic event is lacking. As these structures are widely used to stabilize the steep mountain slopes in India’s northeast region, which is highly prone to earthquakes, the investigation of the response of such wall is the need of hour.

A series of small-strain finite-element (FE) analyses have been carried out for cantilever retaining walls with and without relief shelves considering different backfill and foundation soils with uniform shear strength for one, two and three relief shelves. The analysis were performed for both static and earthquake loading conditions. The finite element model is initially validated with available results in the literature.



(a) **(b)**

Figure 2.4: (a) Cantilever retaining wall with relief shelves (Bowles, 1997), (b) Cantilever retaining wall with relief shelves in Hyderabad, India (after Chauhan et al., 2016)

Chapter 3

NUMERICAL METHODOLOGY

3.1 Details of finite element model

All analyses in the current study were carried out using the commercial finite element software Abaqus (Dassault Systèmes, 2016). Schematic representations of the retaining wall with relief shelves used in this study are shown in figures 1, 3, 5 and 7. Dimensions are not to the scale in the figures. A two-dimensional plane strain finite element model was made assuming the cantilever retaining wall and the backfill and foundation soil as solid deformable parts. The size of backfill and foundation soils were adopted, such that boundary effects do not influence behaviour of retaining wall. Vertical movement of side boundaries of the model were allowed but the horizontal movement was restrained (i.e., roller boundary condition). Along the bottom boundary, both vertical and horizontal displacements were not allowed (i.e., hinge boundary condition). For static equilibrium of the model under soil self-weight and surcharge loading, the wall right top node of the wall was provided with hinge boundary condition. During earthquake loading conditions, all horizontal boundary conditions were freed to allow the movement in horizontal direction as the acceleration record of Northridge earthquake and Trinidad earthquake were applied to all nodes at the bottom and vertical boundaries of the soil region in horizontal direction. For discretization of backfill and foundation soil, three-noded linear plane strain triangular elements (CPE3 of the Abaqus Standard Library) were considered. Good mesh quality was ensured by considering the mesh density both at near-field and far-field locations to minimize approximation and discretization errors. The minimum side length of triangular elements used was 0.3 m near the retaining wall back and base. A maximum size was used of 0.5m at the side and 1.0 m at bottom boundaries of the model. A typical undeformed FE mesh for a cantilever retaining wall with considered number of relief shelves are shown in figures 3.2, 3.4, 3.6 and 3.8. Zero wall-soil interface tensile properties were prescribed using "allow separation" option available under "Normal Behavior: Constraint enforcement method" in Abaqus. The surface tangential interaction between the retaining wall and the backfill soil was prescribed using Lagrange Multiplier (Standard only) with friction coefficient of 0.37. Damping behaviour between wall and backfill soil was defined using damping coefficient of 0.1.

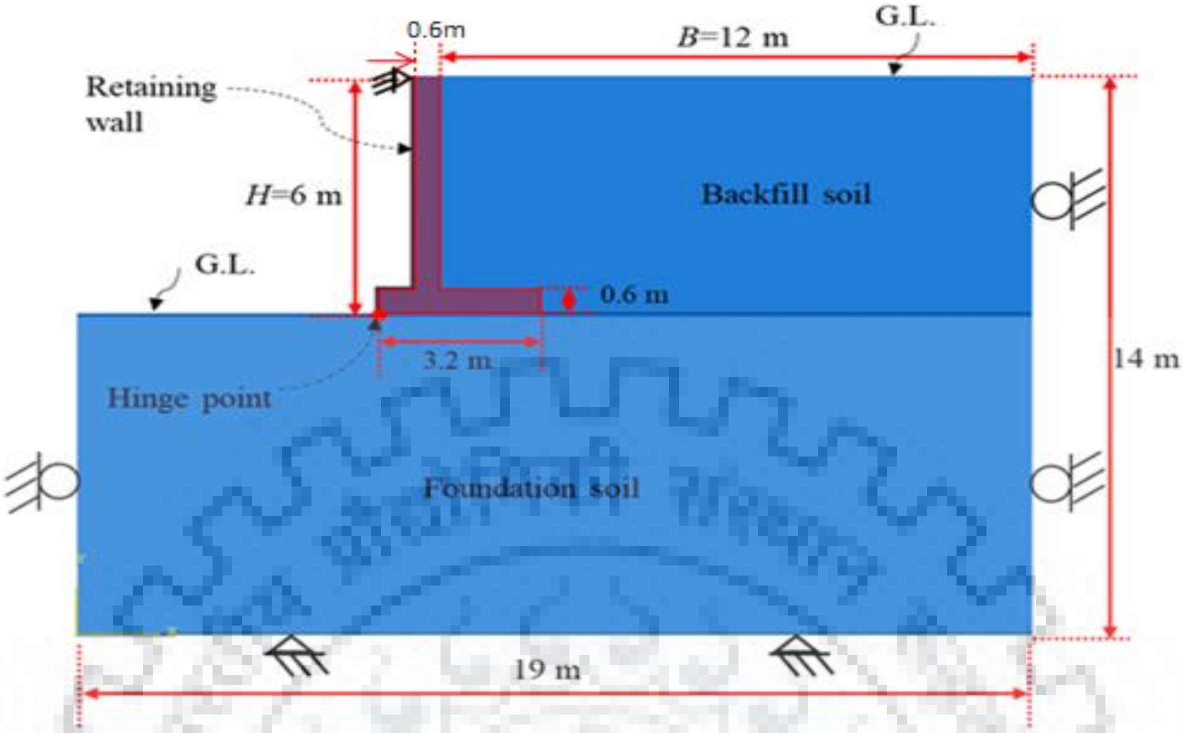


Figure 3.1: Schematic representation of the problem geometry and boundary conditions for retaining wall with no relief shelf

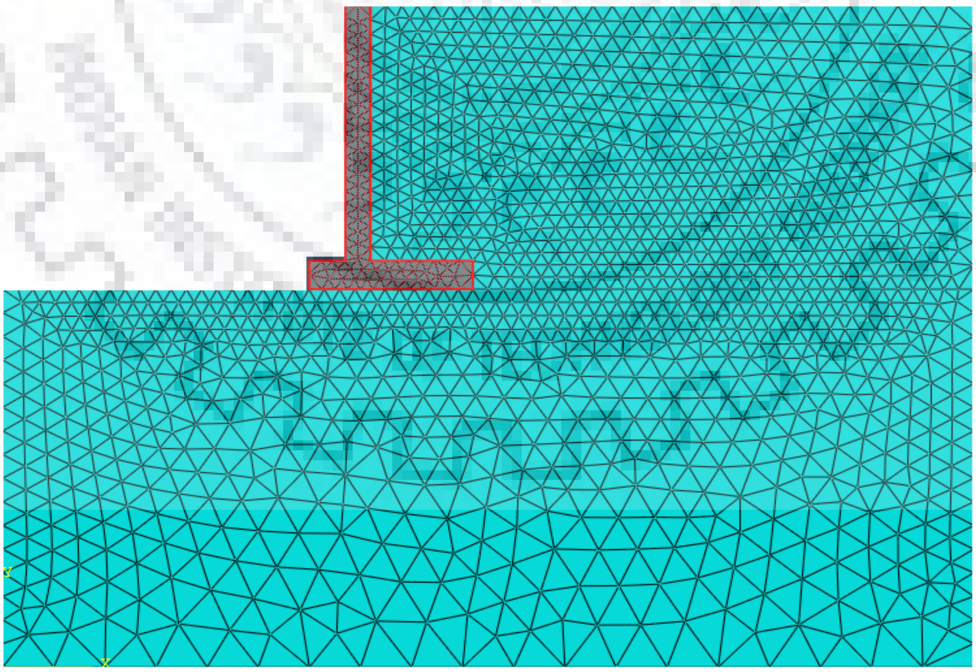


Figure 3.2: Typical finite element mesh for the case of wall with no relief shelves

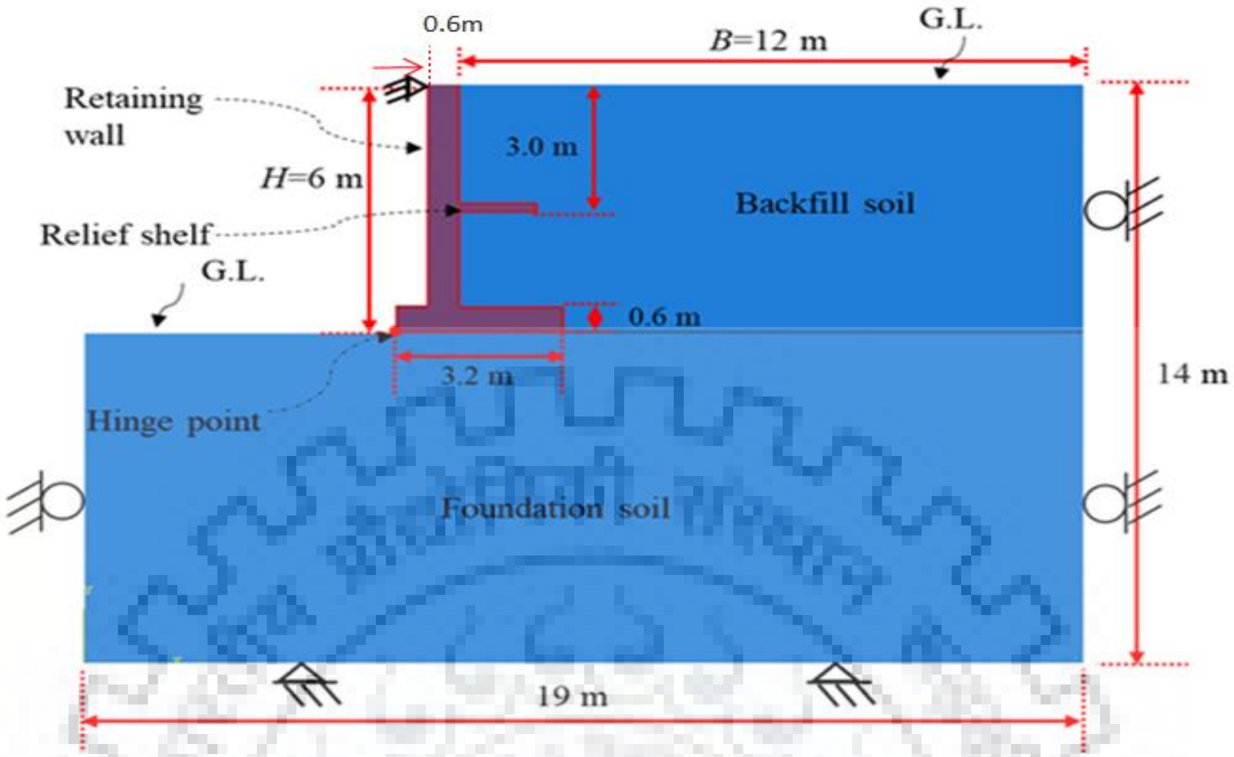


Figure 3.3: Schematic representation of the problem geometry and boundary conditions for retaining wall with one relief shelf

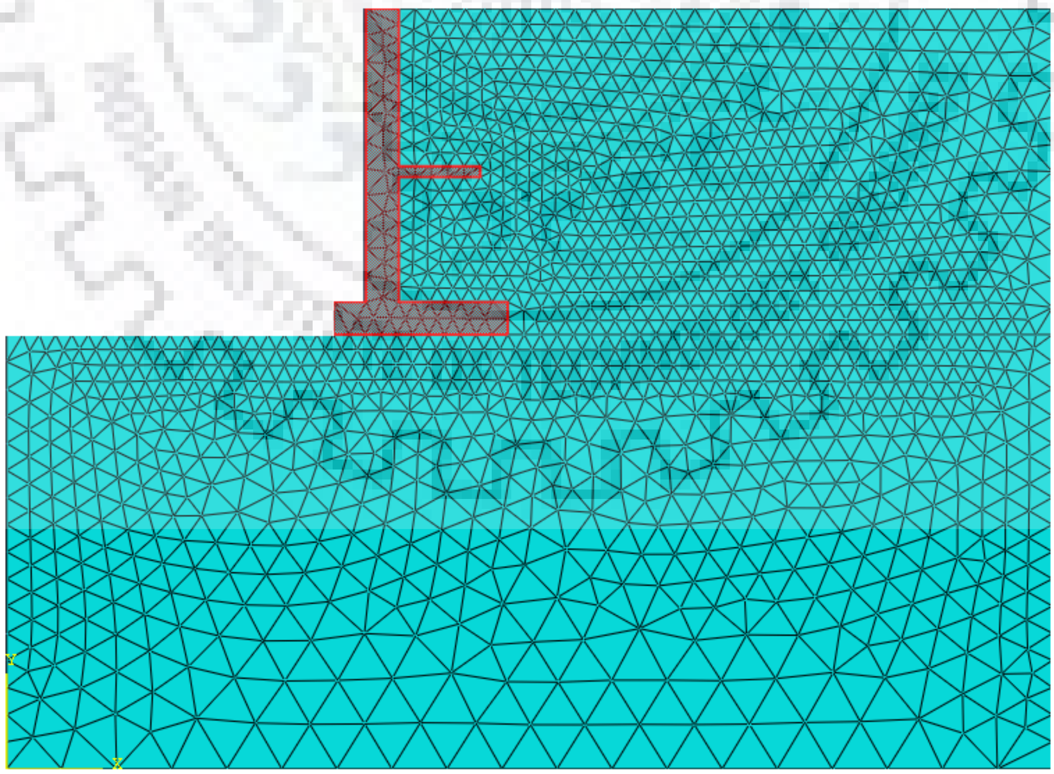


Figure 3.4: Typical finite element mesh for the case of wall with one-relief shelves

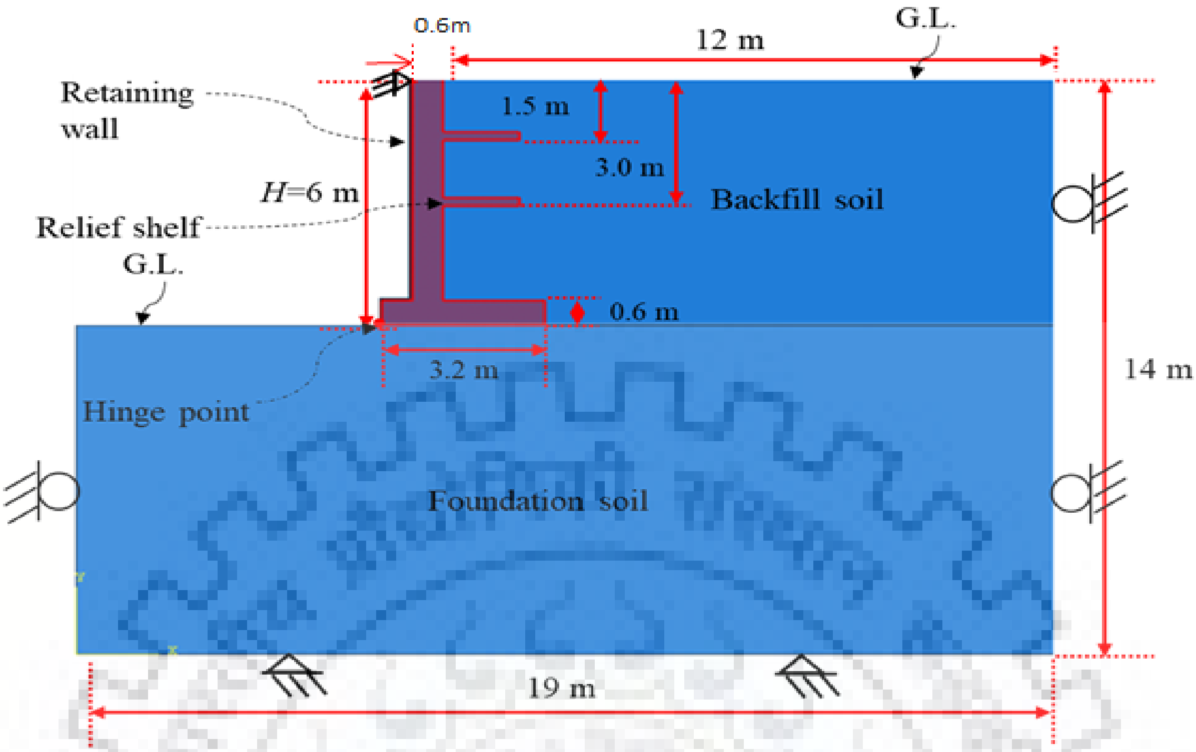


Figure 3.5: Schematic representation of the problem geometry and boundary conditions for retaining wall with two relief shelves

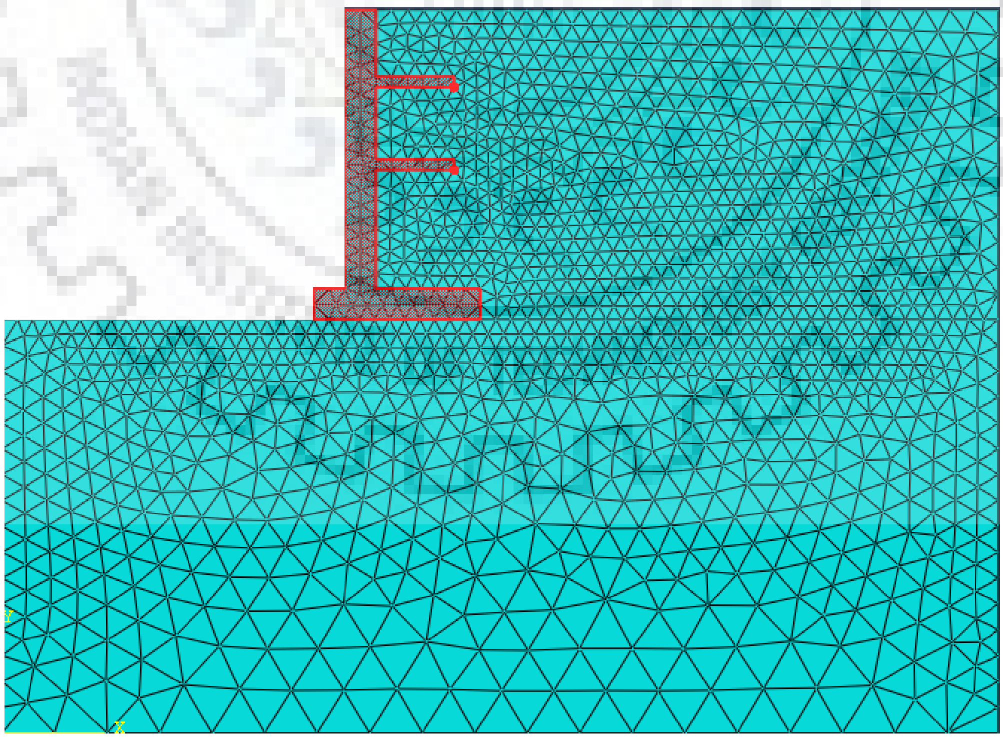


Figure 3.6: Typical finite element mesh for the case of wall with two relief shelves

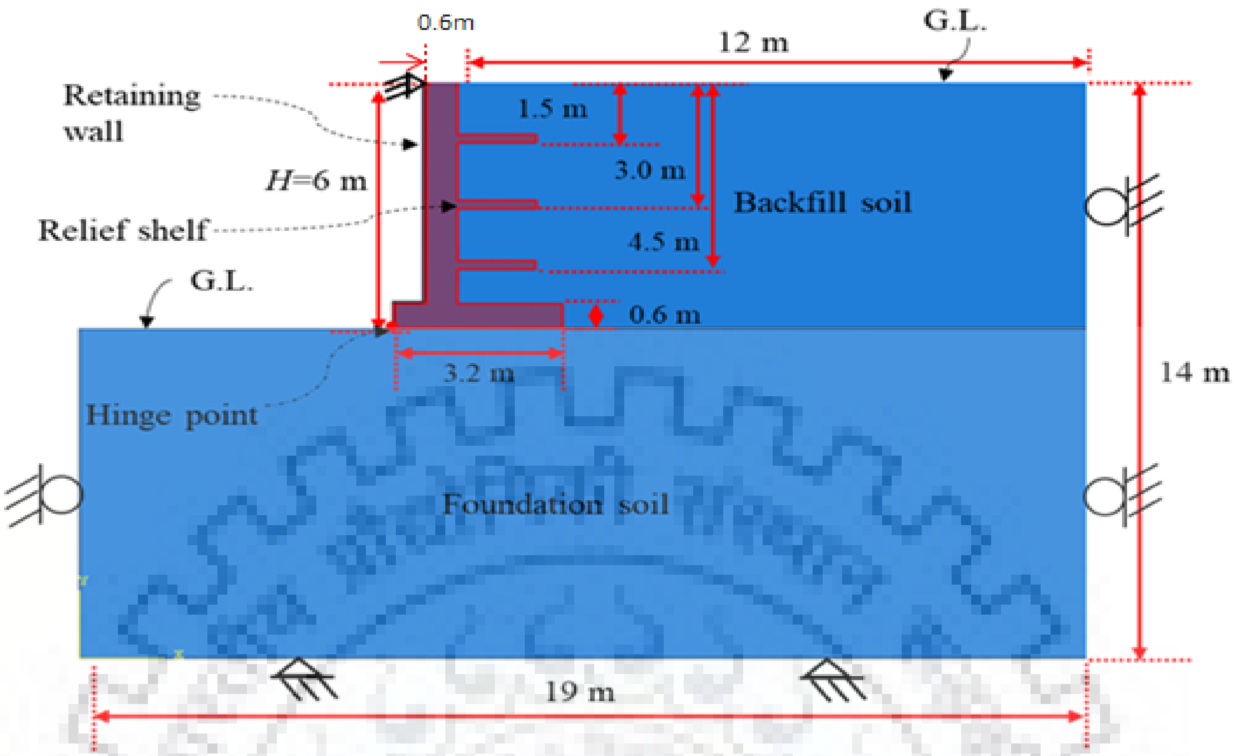


Figure 3.7: Schematic representation of the problem geometry and boundary conditions for retaining wall with three relief shelves

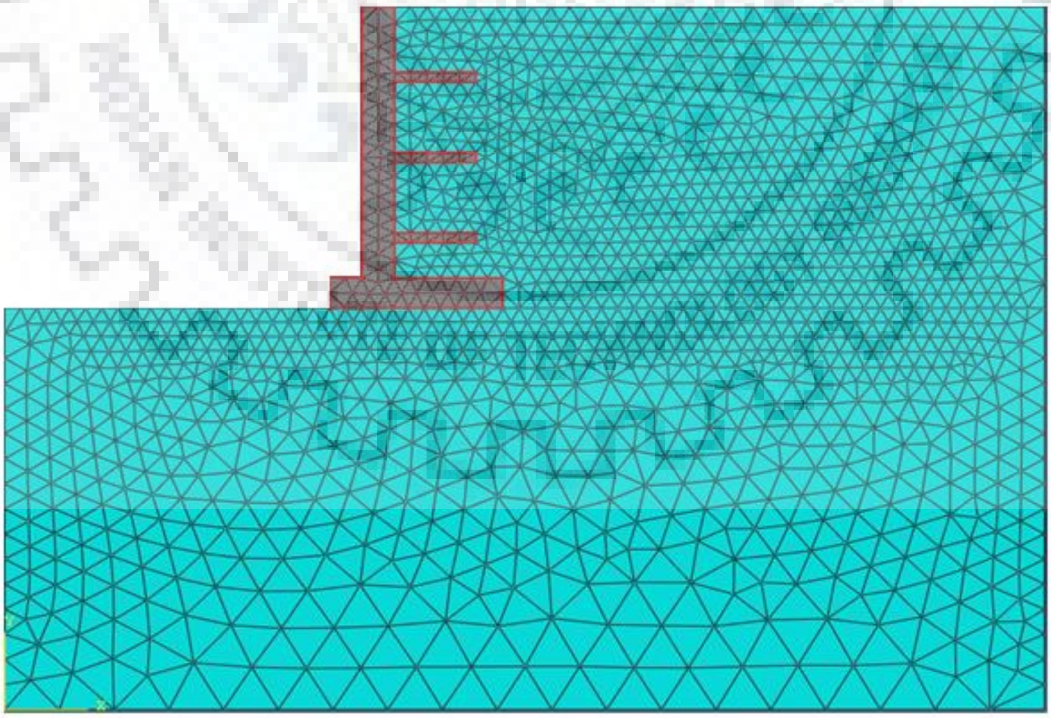


Figure 3.8: Typical finite element mesh for the case of wall with three relief shelves

3.2 Soil and wall parameters

The elastic-plastic soil constitutive model with Mohr-Coulomb failure (MC) was used to simulate backfill and foundation soil response. The backfill and foundation soil parameters of the MC model used for this work are shown in Table 1 and 2. The soil was considered with self-weight. The retaining wall was simulated as a concrete part with elastic properties shown in Table 3. The damping coefficient value of 0.1 was used.

In geo-mechanics problems, the Mohr-Coulomb failure or strength criterion is widely used and most of the solutions for routine geotechnical problems are designed using the Mohr-Coulomb criterion. This criterion considers that the failure is governed by the maximum shear stress. The maximum shear stress, which is also failure stress, depend on the normal stress. To represent failure graphically, Mohr's circle for states of stress and strains at failure are used. Maximum and minimum values of all principal stresses and strains are used for this plot. The line, which touches these Mohr's circles is known as MC failure line (see Figure 3.9). The MC failure criterion can be written as

$$\tau = c + \sigma \tan \phi ,$$

Where τ is the shear stress, σ is the normal stress, c is the cohesion of soil, and ϕ is internal friction angle of soil.

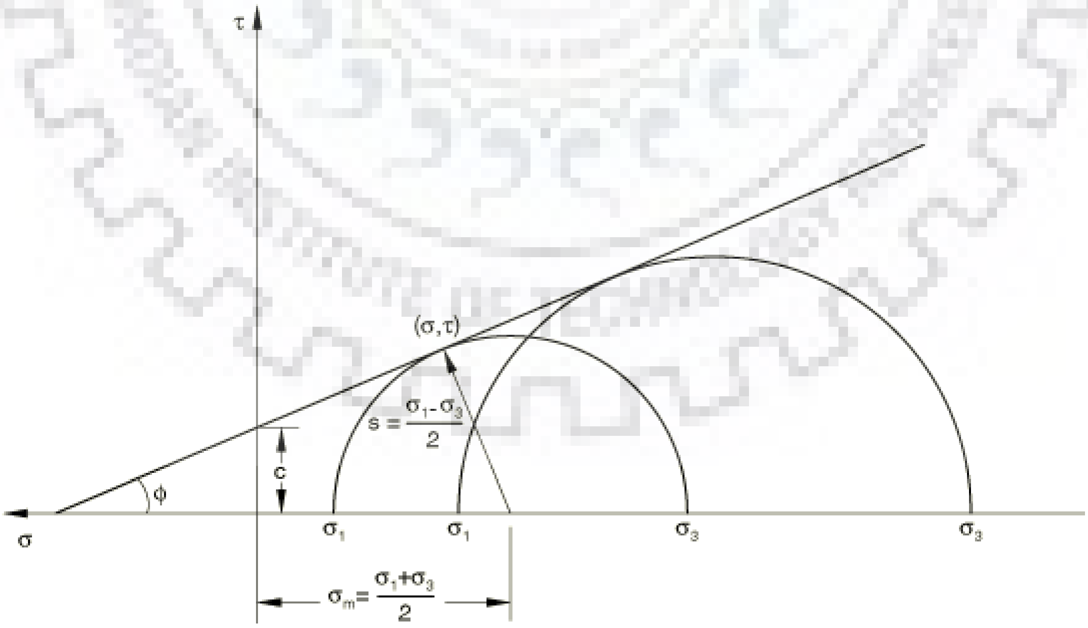


Figure 3.9: Failure criterion of Mohr-Coulomb model [Ranjan and Rao (2000)]

Table 3.1 Different foundation soil parameters for numerical analysis

Foundation Soil Parameters	Values
Unit weight, γ (kN/m^3)	16
Cohesion, c (kN/m^2)	10
Internal friction angle, ϕ	30.0°
Dilation angle, ψ	0.1°
Poisson's ratio, v	0.33
Young's modulus, E (MPa)	34.0

Table 3.2 Different backfill soil parameters for numerical analysis

Backfill Soil Parameters	Values
Unit weight, γ (kN/m^3)	16
Cohesion, c (kN/m^2)	1
Internal friction angle, ϕ	39.0°
Dilation angle, ψ	0.1°
Poisson's ratio, v	0.30
Young's modulus, E (MPa)	13.0

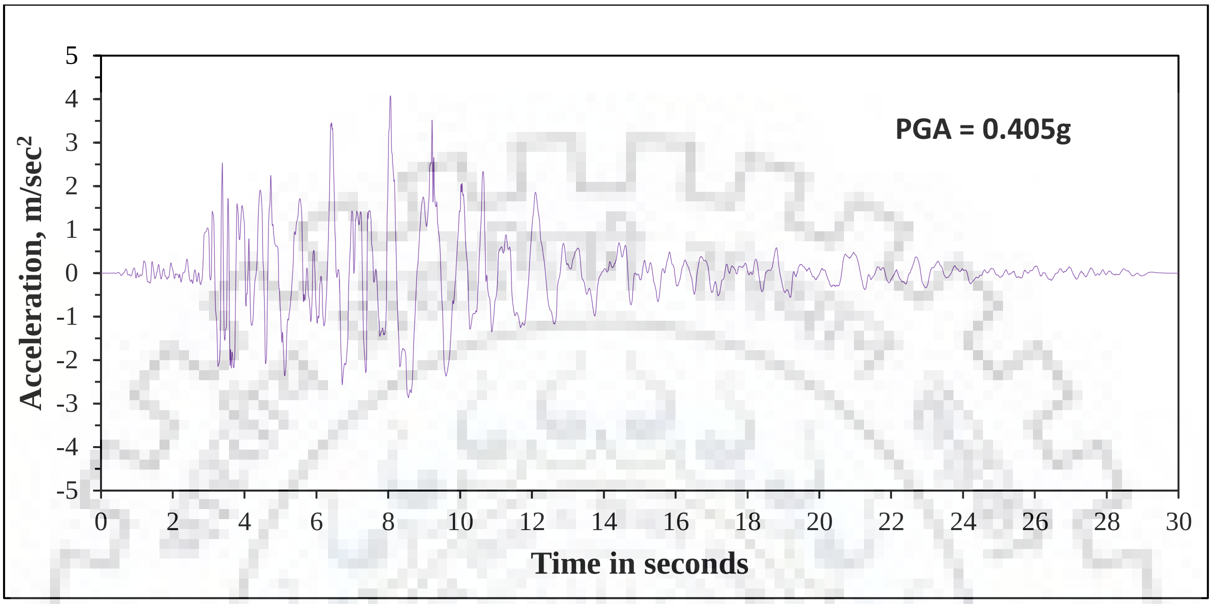
Table 3.3. Different retaining wall concrete parameters for numerical analysis

Concrete Wall Parameters	Values
Unit weight, γ (kN/m^3)	25
Poisson's ratio, v	0.15
Young's modulus, E (MPa)	2.75E4

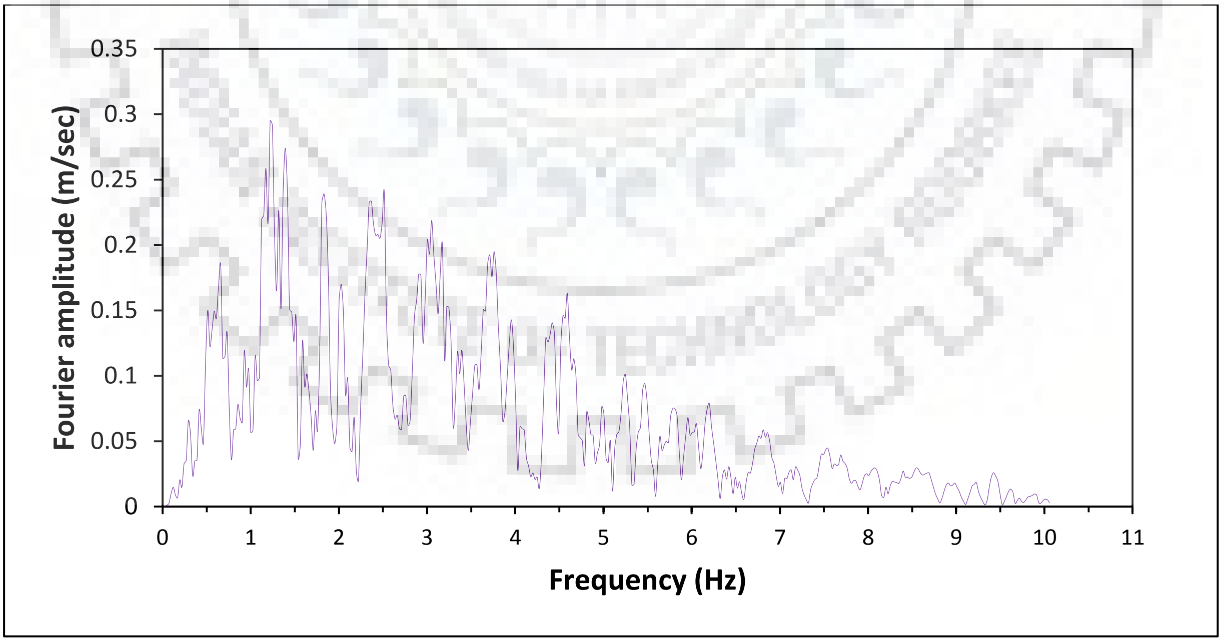
3.3 Earthquake loading data

Two seismic events e.g., 1994 Northridge earthquake (recorded at 090CDMG STATIO N 24278) and 1983 Trinidad earthquake (recorded at 090CDMG STATION 1498) were considered in the present study.

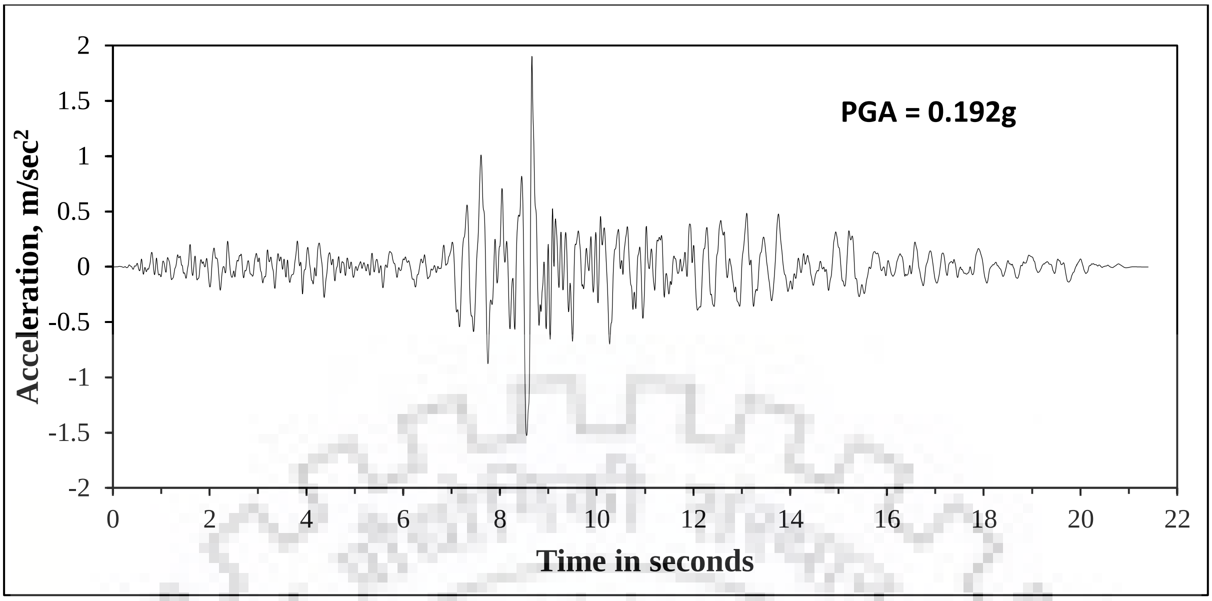
Ground acceleration-time history plots are given in figures 3.10 (a) and (c). Peak Ground Acceleration (PGA) for Northridge and Trinidad earthquakes are 0.405g and 0.192g respectively; Figures 3.10 (b) and (d) show the Fourier amplitude of seismic events considered in the current work.



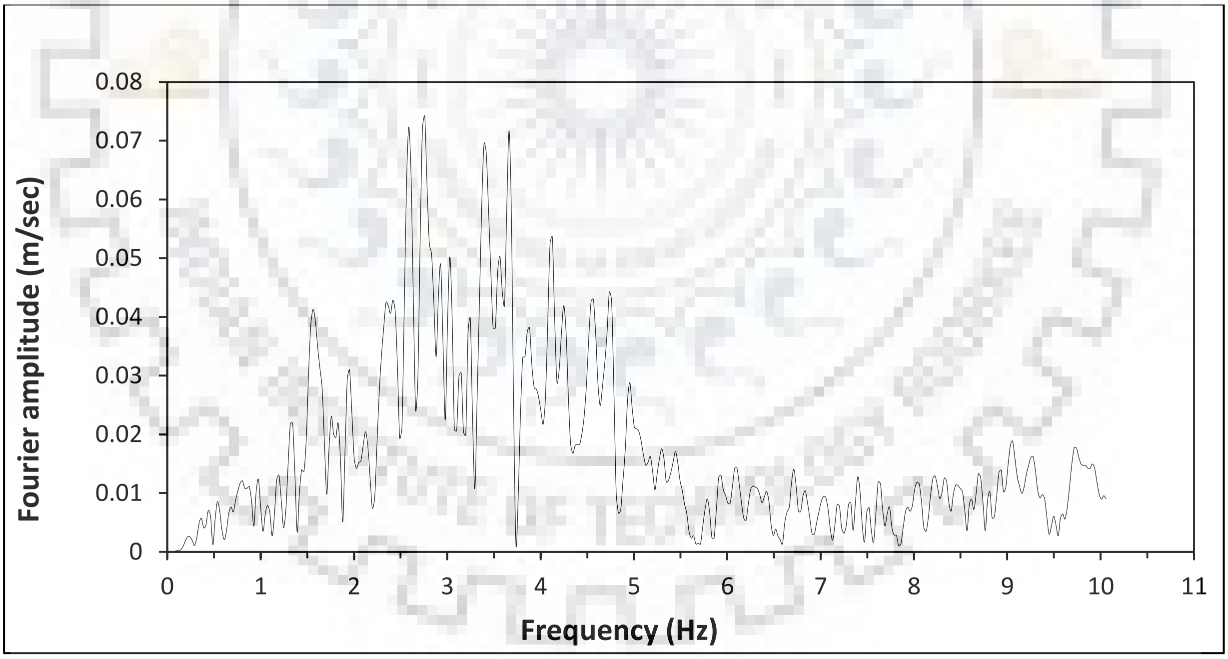
(a)



(b)



(c)



(d)

Figure 3.10: Ground acceleration for (a) 1994 Northridge earthquake, (c) 1997 Trinidad earthquake; and Fourier amplitude for (b) 1994 Northridge earthquake, (d) 1997 Trinidad earthquake

CHAPTER 4

VALIDATION OF NUMERICAL MODEL

4.1 INTRODUCTION

The validation of a numerical model is most important aspect of any numerical study. This can be achieved by performing few analyses of which the results are already available in literature and the comparison of obtained and published results can provide the basis of assuming the model simulation correct. Another method, which is widely used to establish model validity, is evaluating the field variables of interest (e.g., lateral earth pressure) for the simulated problem from available analytical methods. Then, the calculated values are compared with the obtained numerical results.

In present study, the plane strain 2D finite element model was validated by comparing the FE solutions for retaining wall in uniform soil with available theoretical and numerical results. The lateral earth pressure on the back of the retaining wall was obtained from various methods and compared with the obtained results. This investigation is first validated by comparing the results with previous studies available in the literature.

Various theories have been given to estimate the lateral pressure exerted by the soil on a retaining structure. This pressure is dependent on the soil structure and the interaction or movement with the retaining system. Due to many variables, earth-retaining problems can be highly indeterminate. Therefore, it is essential that good engineering judgment be used. For the cantilever retaining wall considered in the present work, the famous Rankine's theory and Jaky's relationship are used to compare the results.

Jaky (1948) proposed a formula to calculate at rest lateral earth pressure. It was basically given for normally consolidated soils and follows as [Ranjan and Rao (2000)].

$\sigma_h = K_0 \times \sigma_v$, where K_0 is earth pressure coefficient at rest.

$K_0 = 1 - \sin\phi'$, where ϕ' is internal friction angle of soil.

The Rankine theory assumes that there is no wall friction and the ground and failure surfaces are straight planes, and that the resultant force acts parallel to the backfill slope (i.e., no friction acting between the soil and the backfill). The coefficient of active earth pressure according to Rankine's theory for horizontal backfill is given by the following expression

$$K_a = \frac{1 - \sin\phi'}{1 + \sin\phi'}, \qquad (4.1)$$

The following model analyses (Figure 4.1 a, b and c) were carried out for validation:

(a) Cantilever retaining wall with horizontal backfill and no provision of relief shelf
(b) Cantilever retaining wall with horizontal backfill and one relief shelf
(c) Cantilever retaining wall with horizontal backfill and two relief shelves

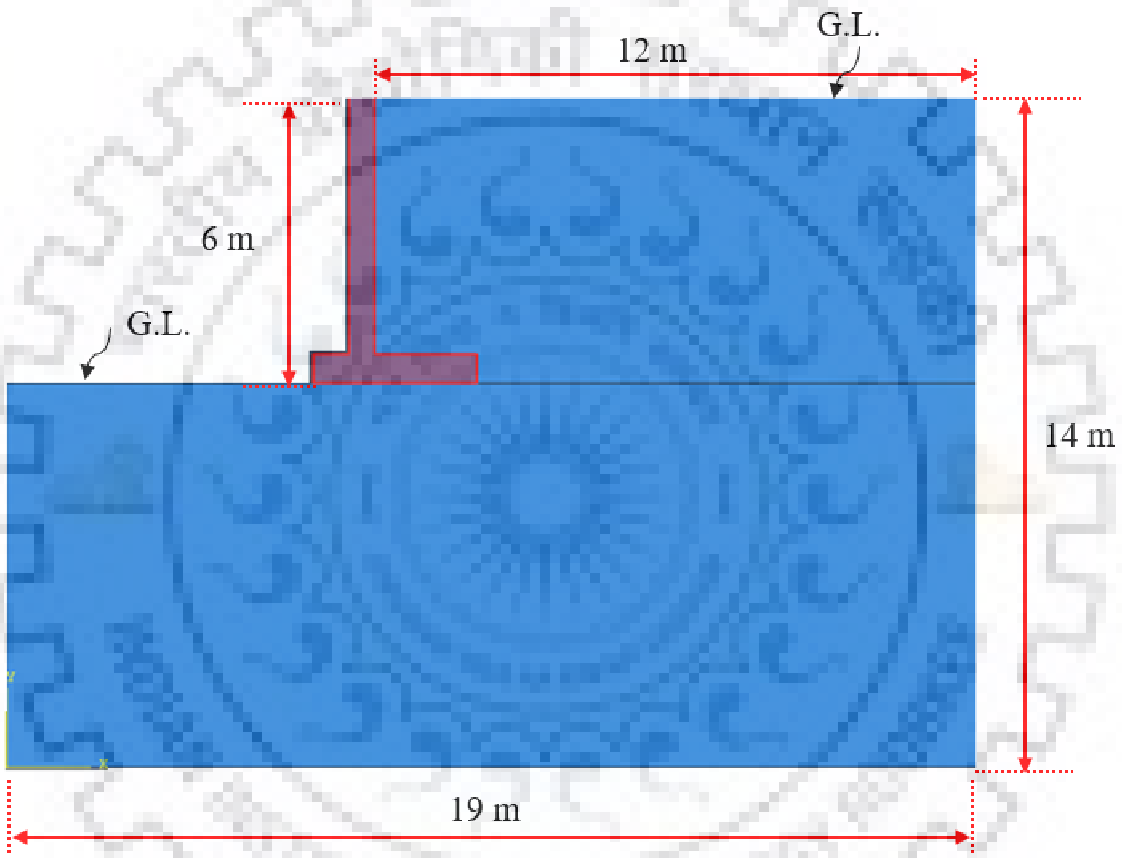


Figure: 4.1(a)

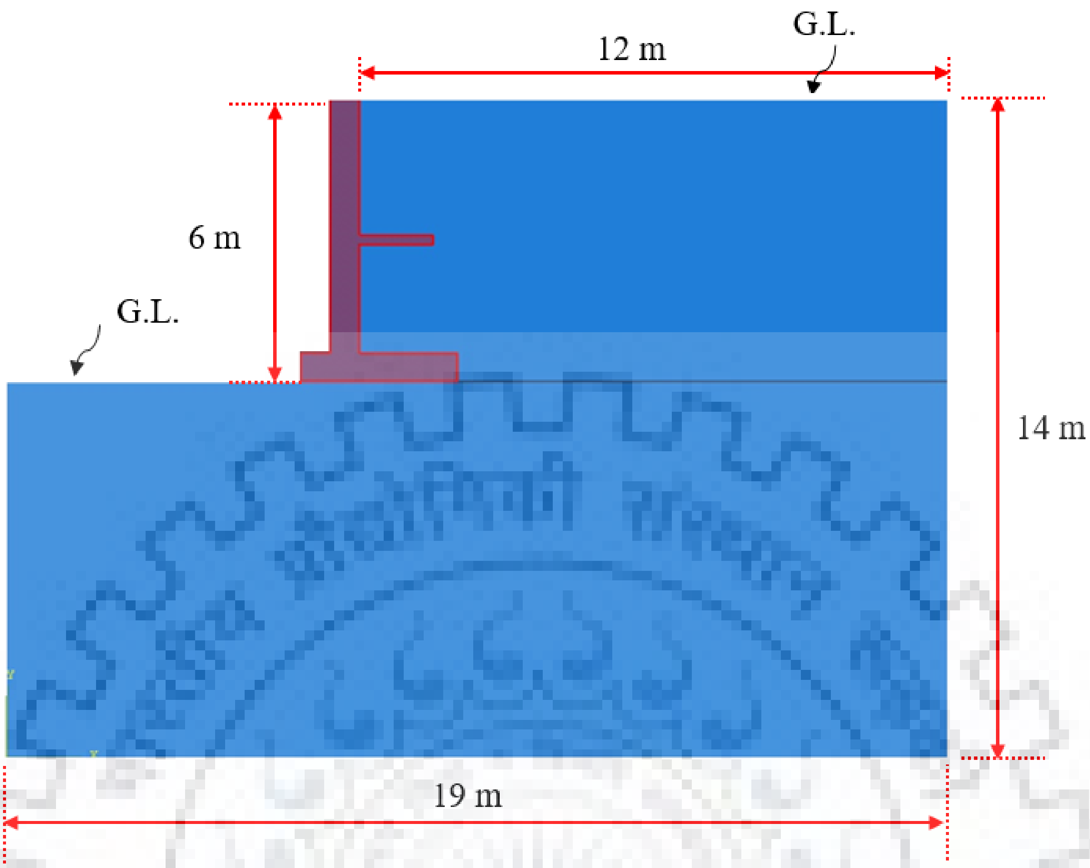


Figure: 4.1(b)

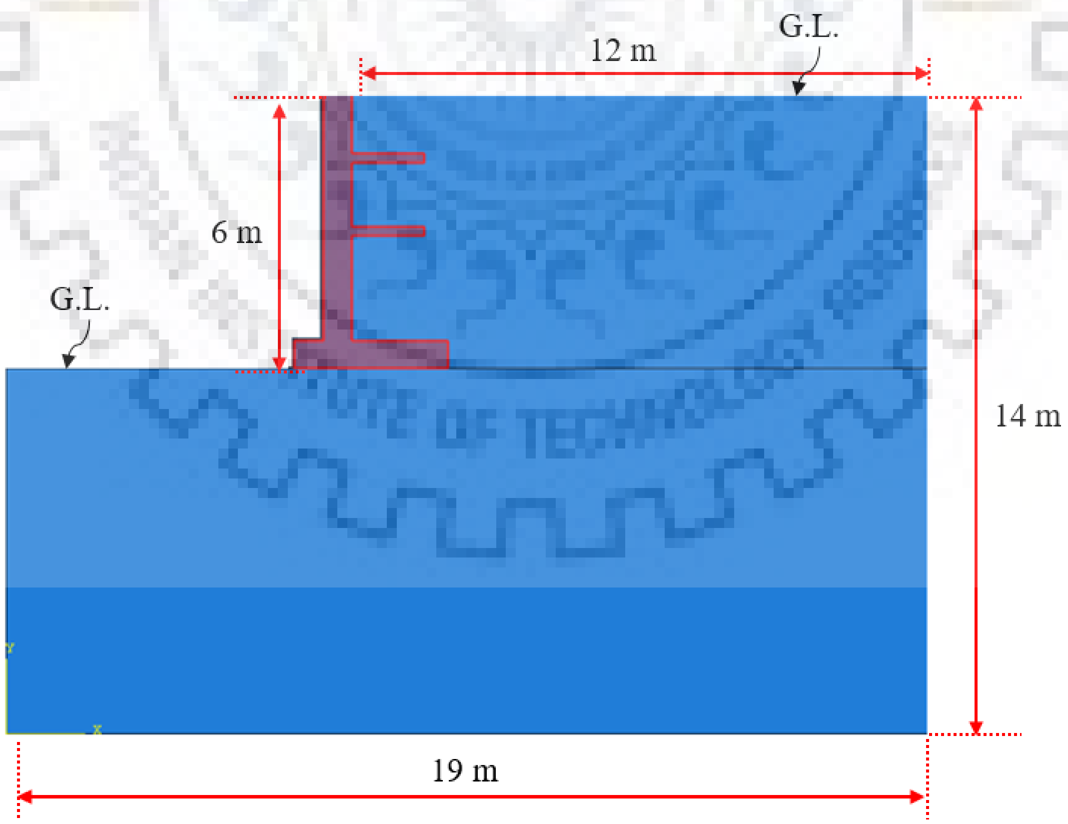


Figure: 4.1(c)

The dimensions of soil domain were considered such that the boundary effects have minimum influence of the retaining wall behaviour.

The results for retaining wall with no relief shelf, one-relief shelves and two relief shelves for static surcharge loading with wall height obtained from the present study are also compared with solutions given by Khan et al. (2016). Khan et al. (2016) considered similar soil and wall properties, which makes the comparison robust which are given in Table 4.1.

Table 4.1. Different backfill soil parameters for numerical analysis [Khan et al. (2016]

Parameters	Backfill Soil	Foundation Soil	Wall Concrete
Unit weight, γ (kN/m^3)	16	16	25
Cohesion, c (kN/m^2)	1	10	-
Internal friction angle, ϕ	39.0°	30.0°	-
Dilation angle, ψ	0.1°	0.1°	-
Poisson's ratio, ν	0.30	0.33	0.15
Young’s modulus, E (MPa)	13.0	34.0	2.75E4

Figure 4.2 and 4.3 shows a comparison between the lateral earth pressure results obtained from the present and previous works. The lateral earth pressure is plotted against the normalised wall height *z/H*. The figure shows that results from solutions of Khan et al. (2016) and present finite element analyses are in good agreement. It can be clearly seen that results for no relief shelf case of the present study and solutions by Khan et al. (2016) show some discrepancy with Rankine’s active earth pressure values at all depths. The discrepancy is lesser at smaller depths but becomes very high at the base of the wall. These differences can be due to inherent limitations of numerical modelling methods. However, these were consistent with the results given by Khan et al. (2016).

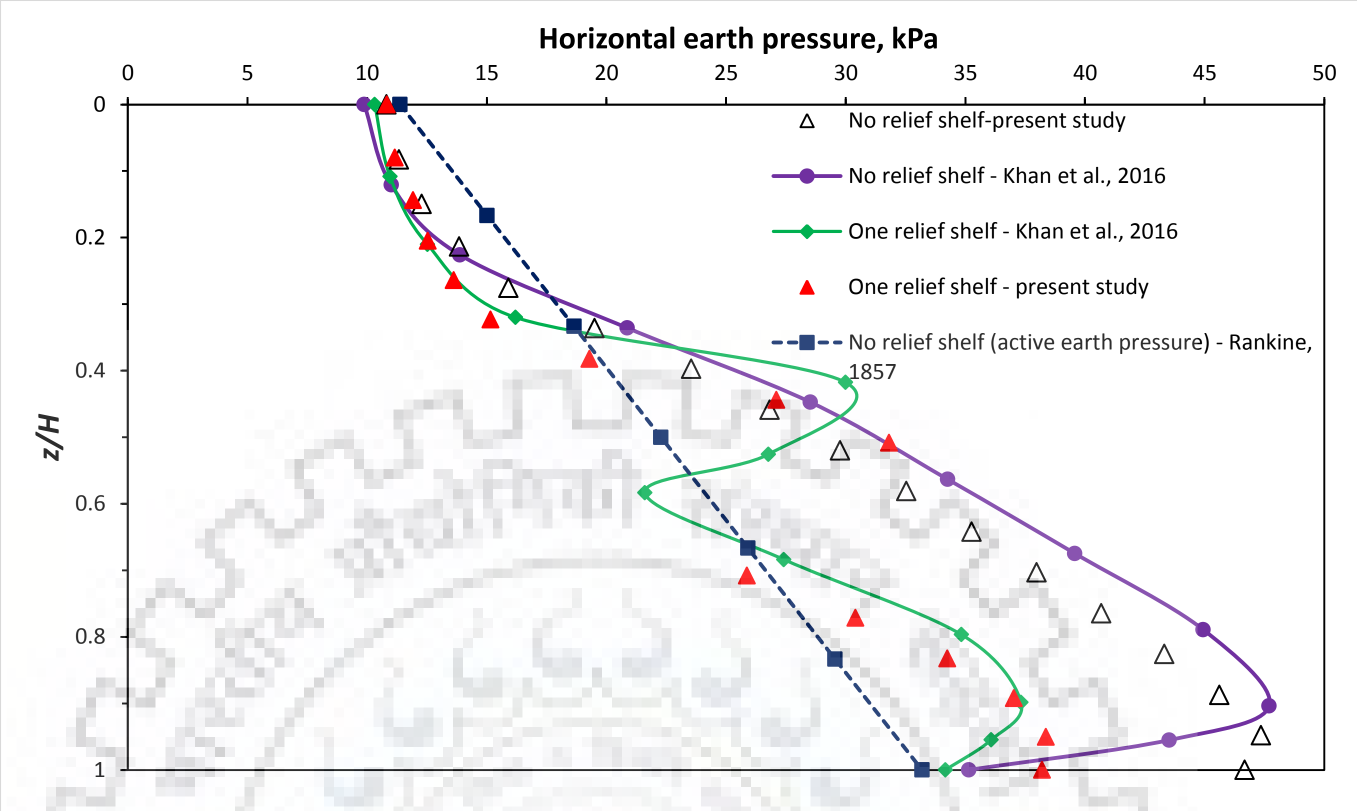


Figure 4.2: Comparison of lateral earth pressure on wall with single and without relief shelves

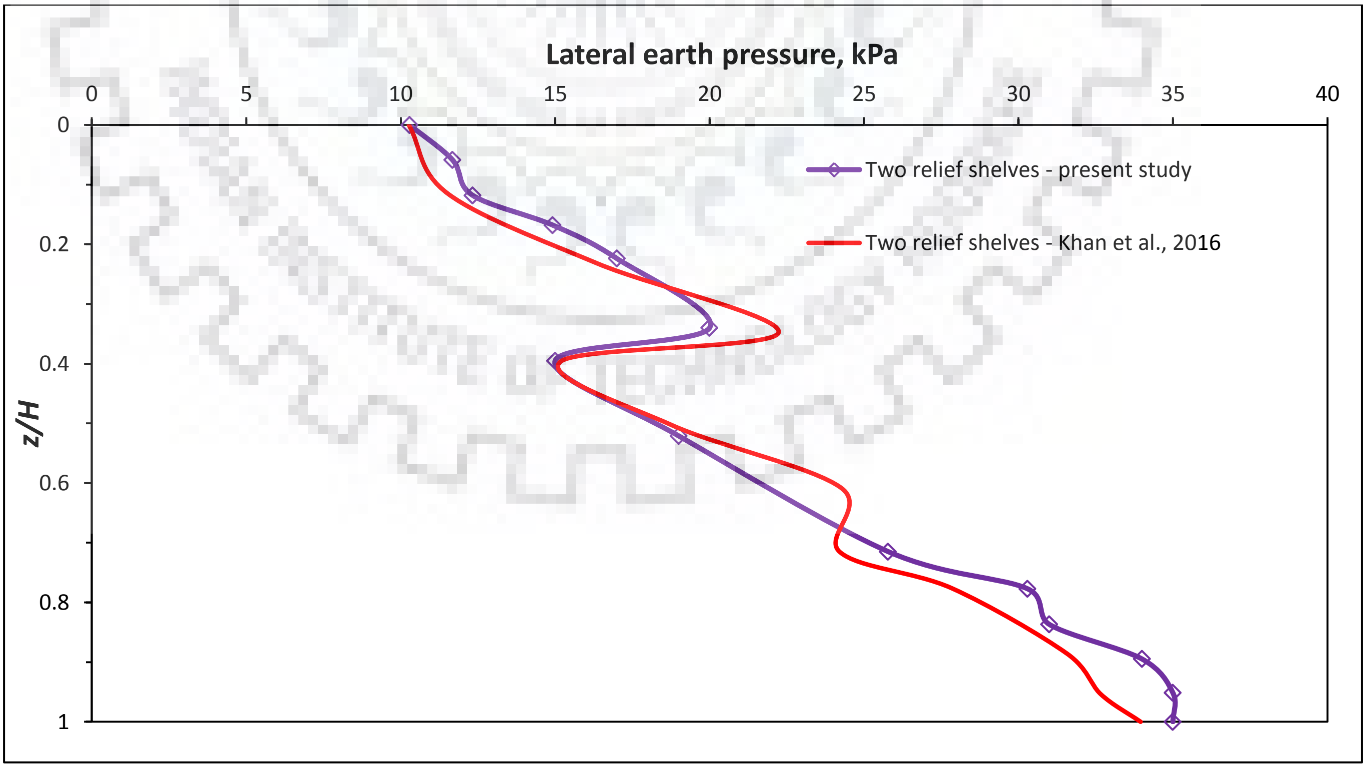


Figure 4.3: Comparison of lateral earth pressure on wall with two relief shelves

CHAPTER 5

RESPONSE OF RETAINING WALL UNDER STATIC LOAD

A series of analyses was undertaken to investigate the response of retaining walls with and without relief shelves under the influence of backfill surcharge under static conditions. The analyses were carried out in four steps to simulate the loading stages in the field conditions. The series of steps were set in following sequence.

1) Step 1 - Geostatic step: To establish in-situ stress conditions in the backfill and foundation soils, the geostatic stress field was introduced in Abaqus using the step 'geostatic'. The coefficient of lateral earth pressure can be prescribed using keyword condition
2) Step 2 – Static General step: A general static step was defined for surcharge loading while the wall movement was restrained both horizontally and vertically at the top of the wall to achieve static equilibrium. In this step, 10kPa pressure was applied as surcharge on backfill surface.
3) Step 3 – Static General Step: In this step, additional pressure applied is 20 kPa so that total pressure reached to 30 kPa.
4) Step 4 – Static General Step: The backfill surcharge is further increased by 20 kPa in this step which made total backfill surcharge loading of 50 kPa.

5.1 Effect of shelf providing on lateral earth pressure on the wall

The retaining walls with relief shelves have not been an active topic of research in past. Recently, it has attracted the attention of few researchers but there are many conflicting views about the effectiveness of such retaining structures. Therefore, the response of such walls under the influence of static loading is a matter of great significance and can help in investigating their response in seismic events as well. The results from the present work are discussed in following sections of this chapter.

5.1.1 Effect of total number of shelves provided

The effectiveness of the provided shelves on the retaining wall backfill side was studied by varying the number of shelves. Figure 5.1 and 5.2 show the resulting lateral earth pressures after the application of 50 kPa surcharge on the backfill surface for the cases of a cantilever retaining wall with none, one shelf, two shelves, and three shelves of 0.2 m and 0.4 m thickness respectively. The width of shelves in all the cases is 1.5m. Khan et al. (2016) showed through their numerical study that the reduction in lateral earth pressure increases with increase in width of relief shelves for the cases with shelf width less than 1.5 m. Thereafter, the width does not have influence of lateral pressure. It can be clearly seen that providing relief shelves to the retaining structure significantly reduces the lateral earth pressure. For the single shelf provided at mid height of the wall, the earth pressure distribution is similar to the distribution for the cantilever without a shelf until the shelf location, with a concentration of the later earth pressure directly above the relief shelf. This increased ‘‘concentration’’ of the pressure is attributed to the shelf rigidity. The lateral earth pressure decreases directly under the shelf and then again increases linearly. By providing two and three shelves, the lateral earth pressure directly above the shelves also shows a small increase in the earth pressure for the same reason. It can be clearly observed that the earth pressure in the part between the two shelves starts from a lesser value and then returns to the path of the cantilever wall without shelf case.

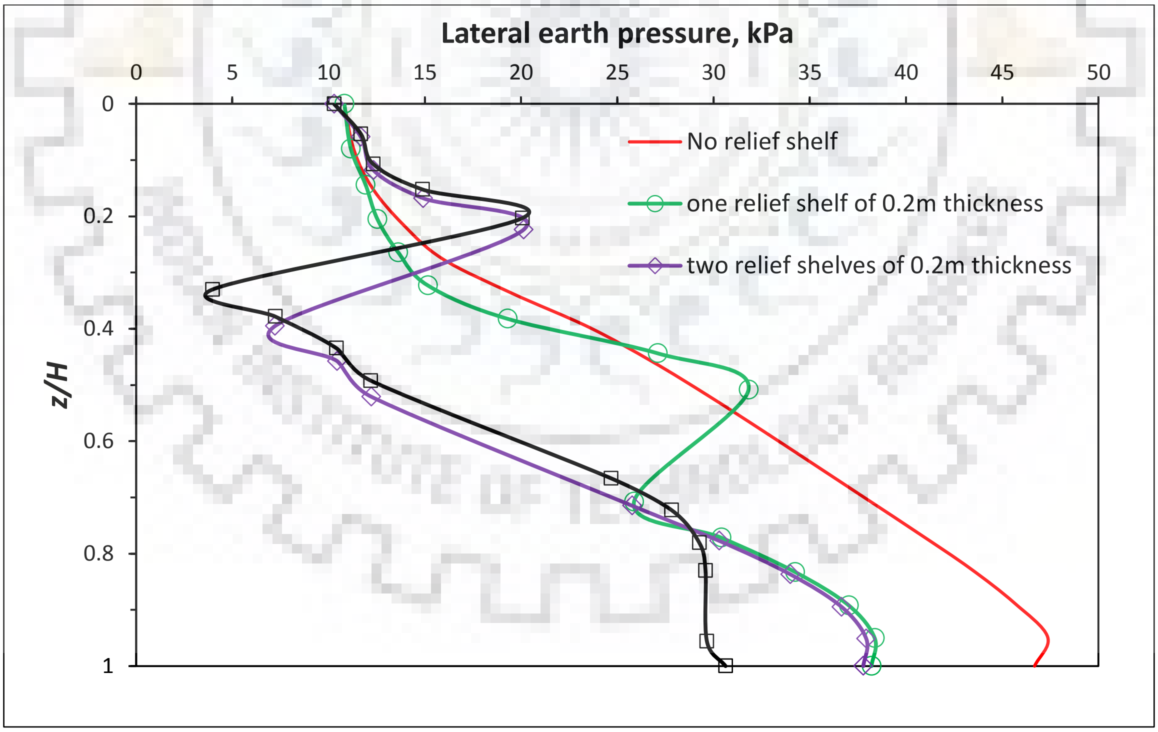


Figure 5.1: Effect of number of relief shelves 0.2 m thickness on earth pressure distribution under static loading

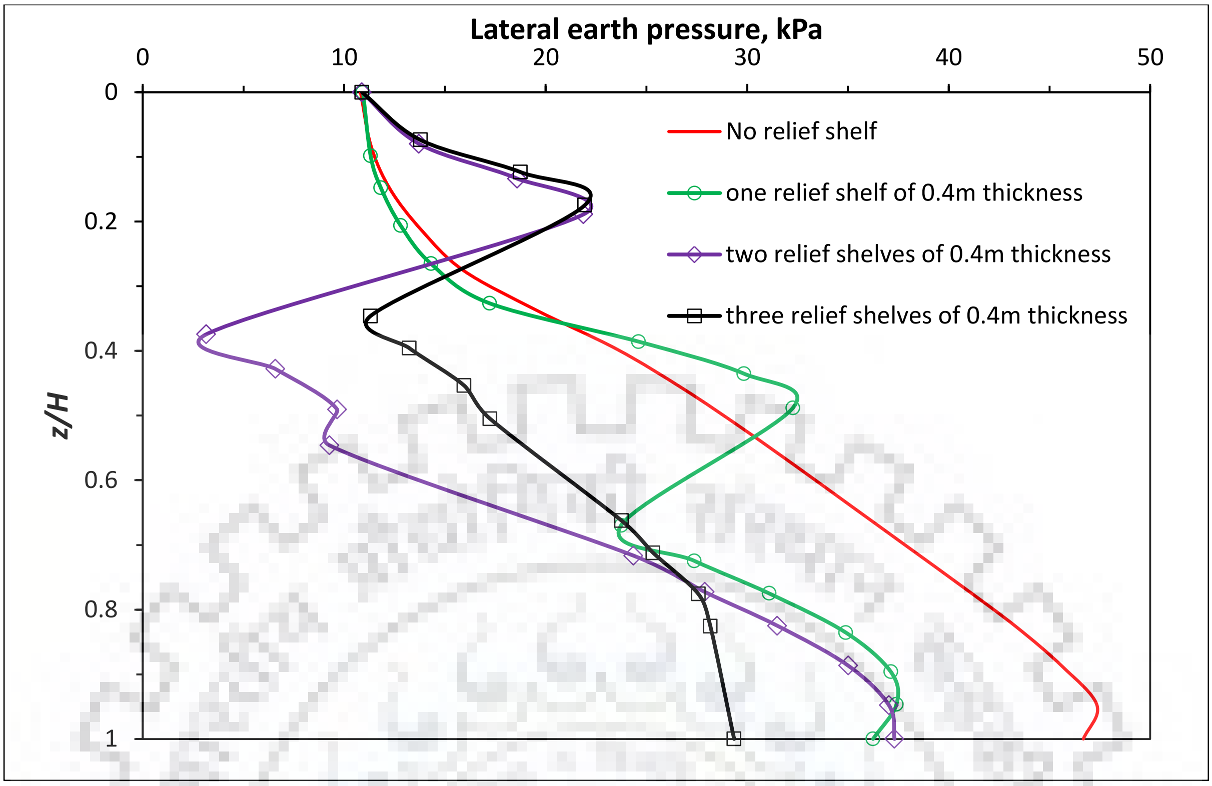


Figure 5.2: Effect of number of relief shelves of 0.4 m thickness on earth pressure distribution under static loading

5.1.2 Effect of thickness of shelves provided

The effect of relief shelf thickness is investigated for the case of the shelf width of 1.5 m. Two values of shelf thickness (e.g., 0.2m and 0.4 m) were considered. The case of providing a single relief shelf at a mid-depth *z/H* equal to 0.5, two relief shelves at *z/H* of 0.2 and 0.5 and three relief shelves at *z/H* of 0.18, 0.45 and 0.75 were investigated, as shown in figure 5.3. The small thickness of relief shelf (0.2 m) should deflect more than the one with large thickness (0.4 m). This deflection of relief shelf causes the relief shelf to rest on the lower soil, which releases the vertical loading directly above the relief shelf and increases the vertical loading below the relief shelf. The curves shown in figure 5.3 for the case of two relief shelves clearly demonstrate and support this hypothesis. Such release of the vertical pressure directly above the relief shelf enhances the vertical loading under the relief shelf and leads to the decrease of the overall stability of the retaining structure. In the figure, the pressure for the case of one-relief shelves slightly increases when the shelf thickness is increased from 0.2 m to 0.4m. However, this was not the case when two relief shelves are provided.

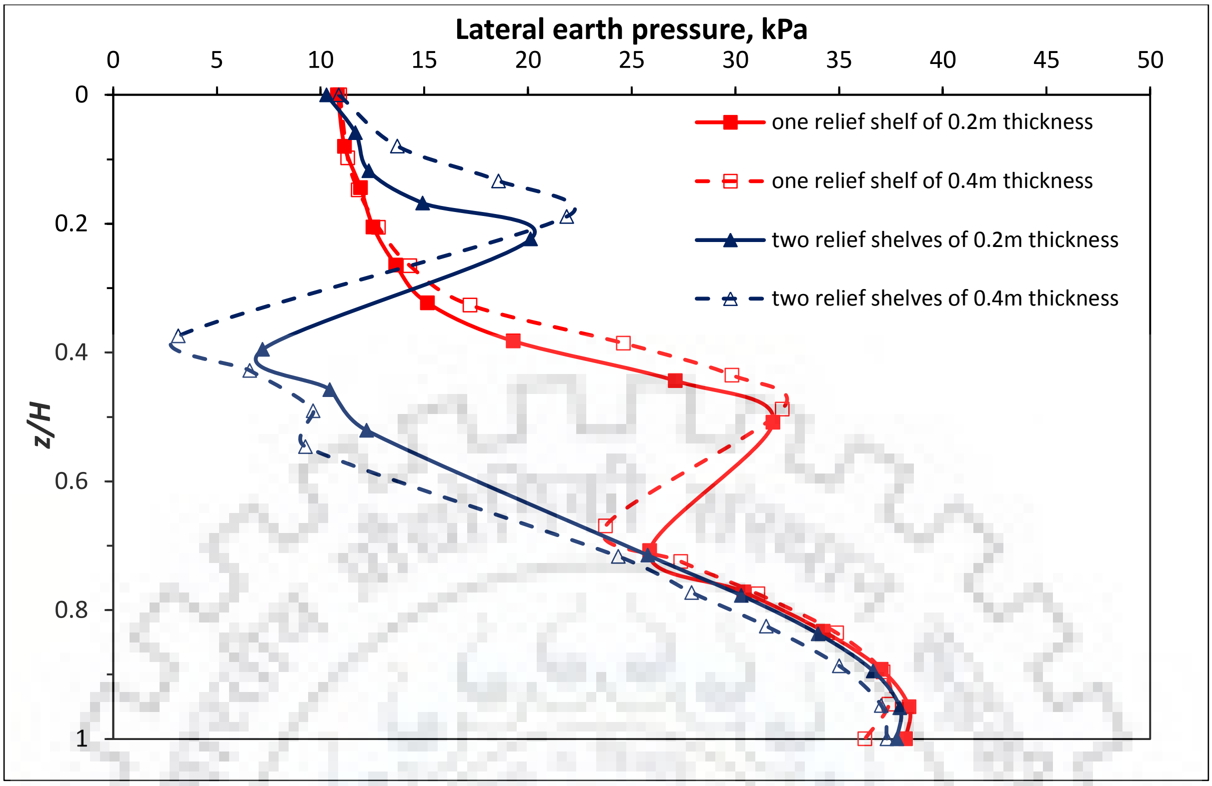


Figure 5.3: Effect of thickness of relief shelves of 1.5 m width on earth pressure distribution under static loading

5.2 Effect of shelf providing on settlement profile of backfill

The effectiveness of provision of the relief shelves for cantilever retaining wall was also investigated by studying the backfill settlement profile. The backfill settlement is a very important criterion of serviceability of cantilever retaining walls. Uncontrolled settlement at backfill can lead to the collapse of the structures directly resting on the backfill and those, which are in close proximity.

5.2.1 Effect of total number of shelves provided

Figure 5.4 shows the backfill surface settlement of all retaining walls considered in this study at the end of application of 50 kPa surcharge loading. The settlement is plotted with the normalised distance from the wall (*x/B*), where B is the total length of backfill top surface from the wall edge to farthest edge of backfill. It can be observed that the settlement is minimum close to the wall (at top) and gradually increases to a constant value of 25.37 mm far away from the wall, where these settlements are taken along the height of the wall. Among different cases considered, the settlement is maximum for a wall without relief shelf (15.27 mm). The increase in the number of provided shelves reduces the settlement by 3 mm for the single shelf case and by approximately 11 mm for the 2 and 3 shelves cases. These relief shelves resist the backfill surface settlement near the stem of cantilever retaining wall. Therefore, the effect of relief shelf provision can be clearly seen on surface settlement profile near the wall. Then, the effect continuously diminish at farther distances from wall and follows the settlement profile that of retaining wall without relief shelves beyond *x/B* of 0.55 from the wall. Interestingly, the increase in number of relief shelves from 2 to 3 hardly influence the settlement profile. Therefore, the two relief shelves at upper 2/3 height of wall is optimum from serviceability point of view.

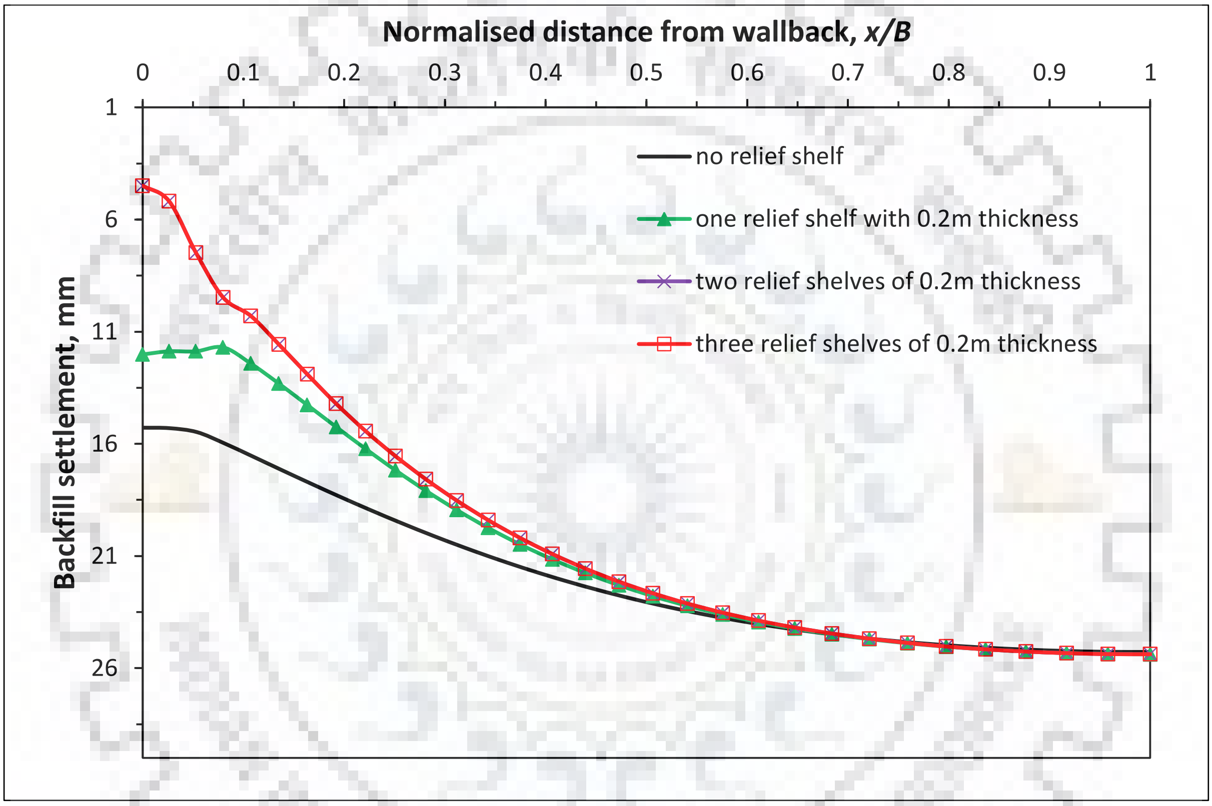


Figure 5.4: Effect of number of relief shelves of 0.2 m and 0.4 m thickness on backfill settlement profile under static loading

Vertical settlement mechanisms for different cases of retaining wall with relief shelves under the effect of static surcharge loading are represented in figures 5.5-5.8, in which contours of soil vertical displacement are plotted. For retaining wall without relief shelf, the vertical settlement near the wall back at the top is about 15 mm, which is maximum among all the cases considered in this study (compare figure 5.5 with 5.6, 5.7 and 5.8). The backfill settlement above the relief shelf slightly decreases when retaining wall is provided with one relief shelf. However, significant reduction in vertical settlement (nearly zero) can be observed for the soil below the relief shelf (see figure 5.6). Furthermore, the backfill settlement near the wall decreases with increase in number of relief shelves from one to two and three. The settlement is almost negligible near the wall for these two cases (see figure 5.7 and 5.8)

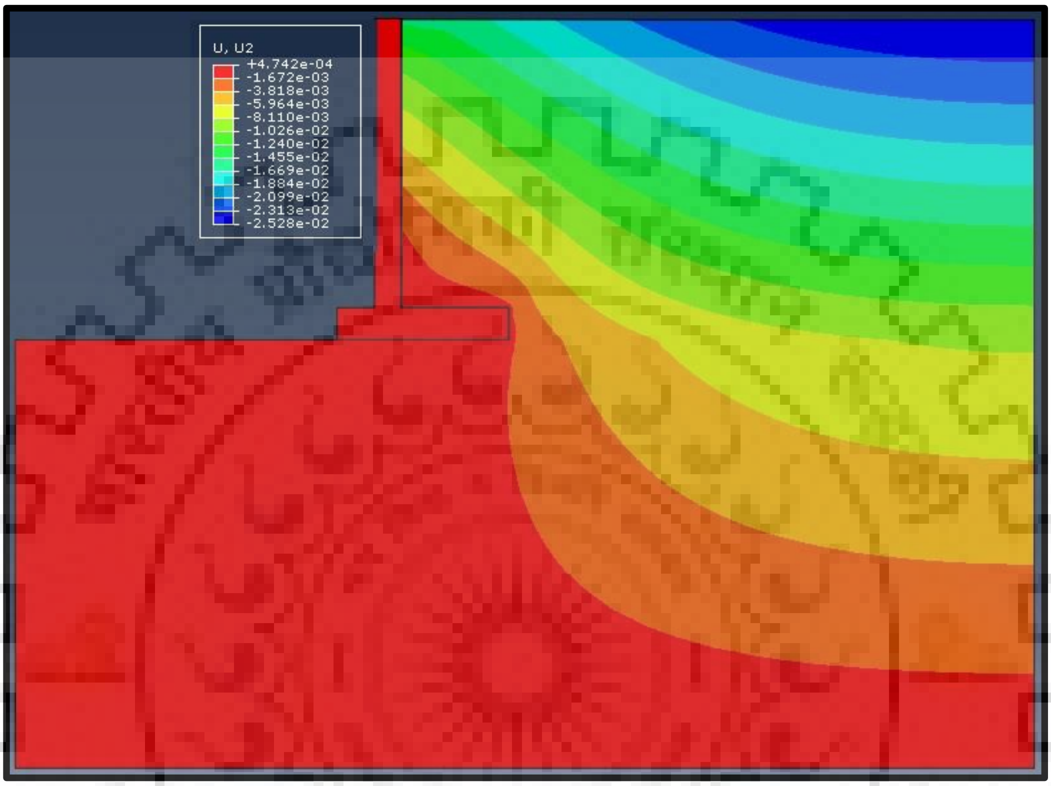


Figure 5.5: Vertical settlement distribution for the case of retaining wall without relief shelf under static loading

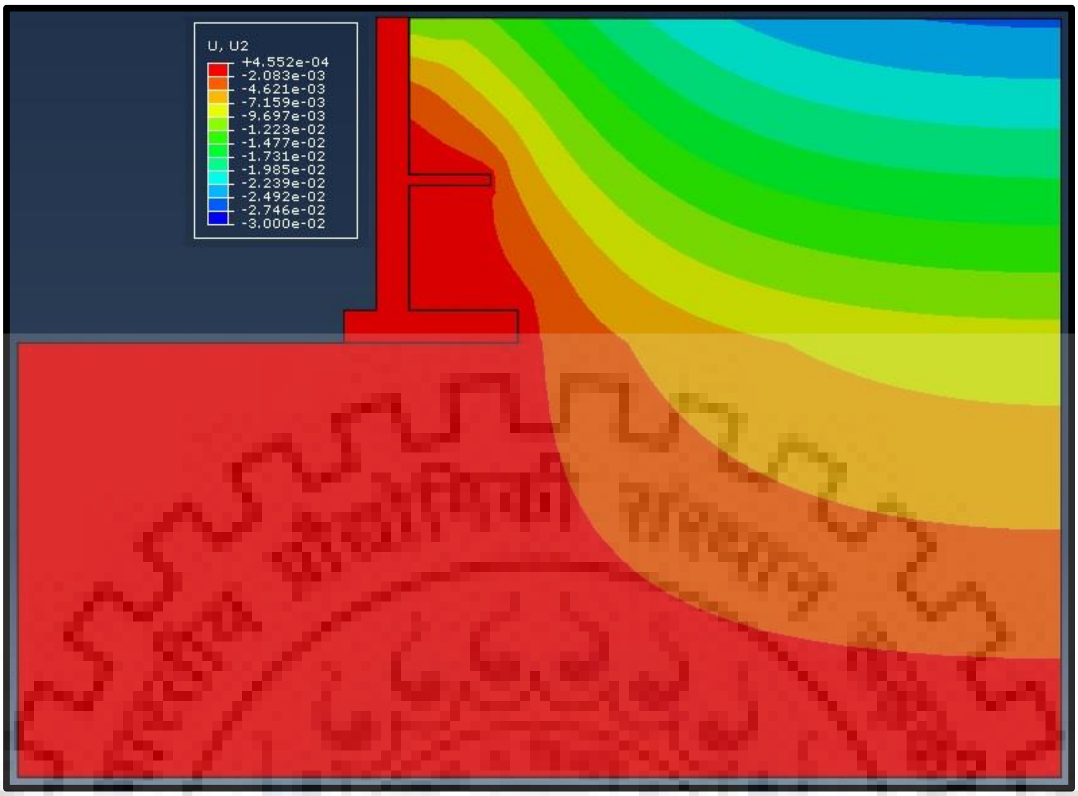


Figure 5.6: Vertical settlement distribution for the case of retaining wall with one relief shelf of 0.2 m thickness under static loading

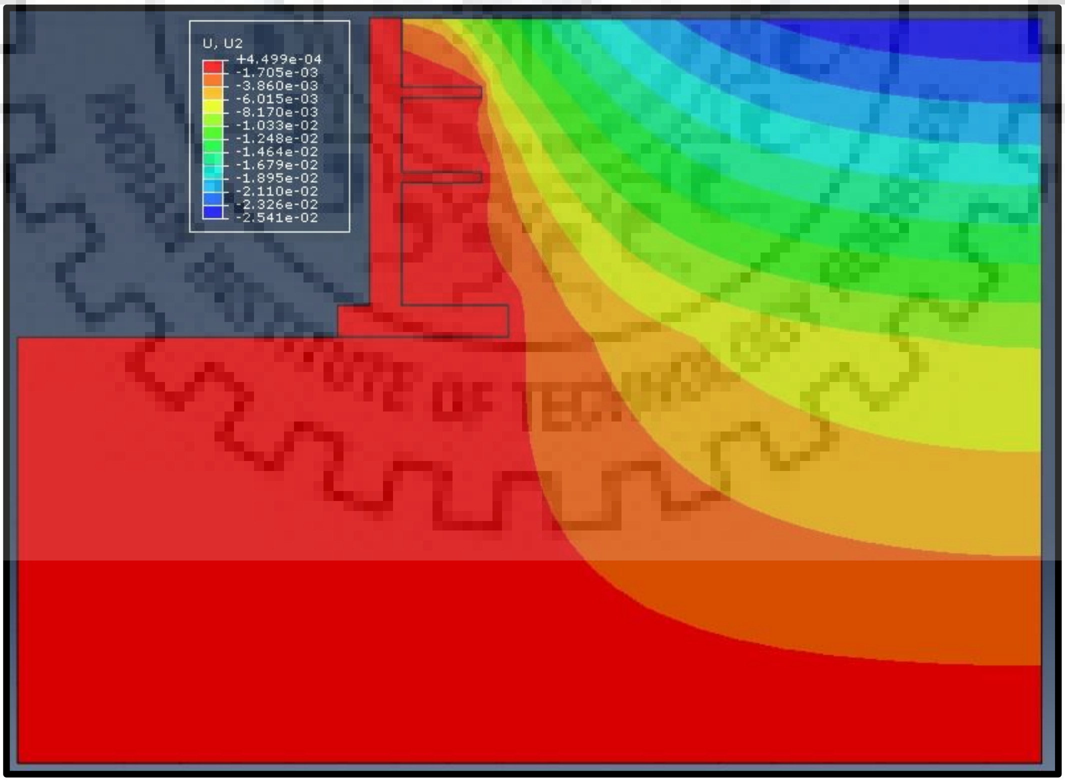


Figure 5.7: Vertical settlement distribution for the case of retaining wall with two relief shelves of 0.2 m thickness under static loading

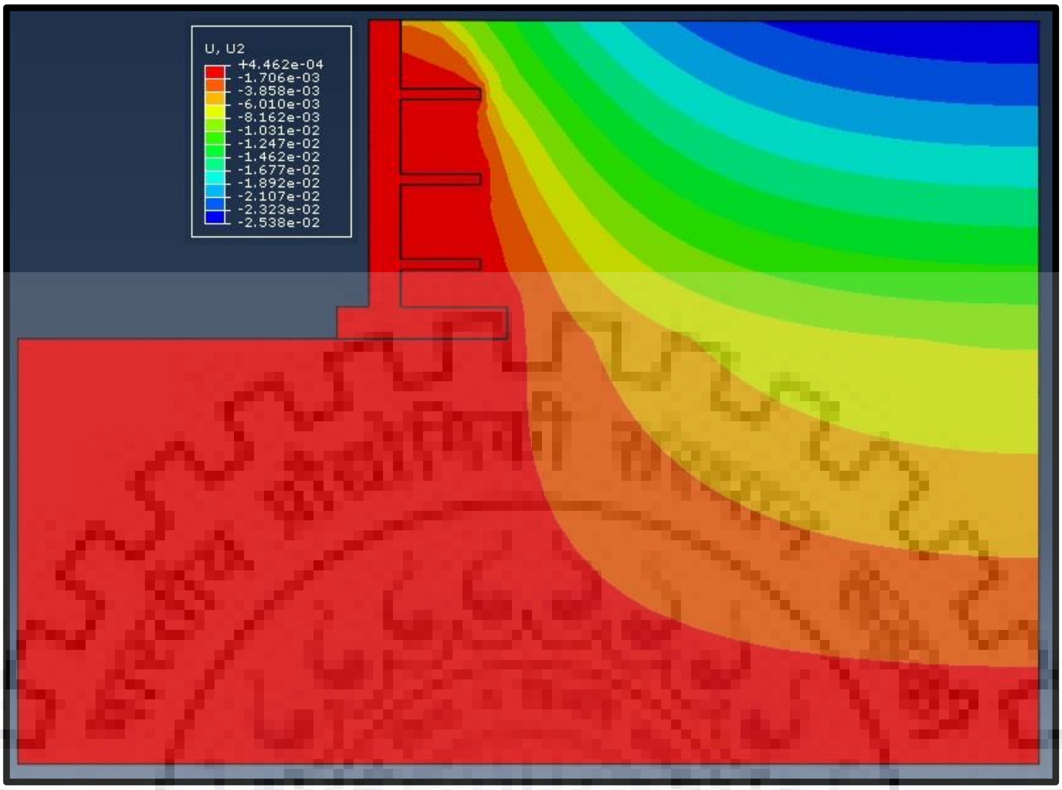


Figure 5.8: Vertical settlement distribution for the case of retaining wall with three relief shelves of 0.2 m thickness under static loading

5.2.2 Effect of thickness of shelves provided

Figure 5.9 shows the effect of shelf thickness on the backfill settlement profile for the case of wall with one-relief shelves of 1.5 m width each. Two values of shelf thickness (e.g., 0.2m and 0.4 m) were considered. The figure demonstrates that the thickness has negligible effect upon the surface settlement profile negligibly. Figure 5.10 shows the contour plot of vertical settlement of soil for these two cases. It can be clearly seen that the soil displacement profile is almost similar for both the cases of shelf thickness of 0.2 m and 0.4 m. Similar response was observed for the other cases.

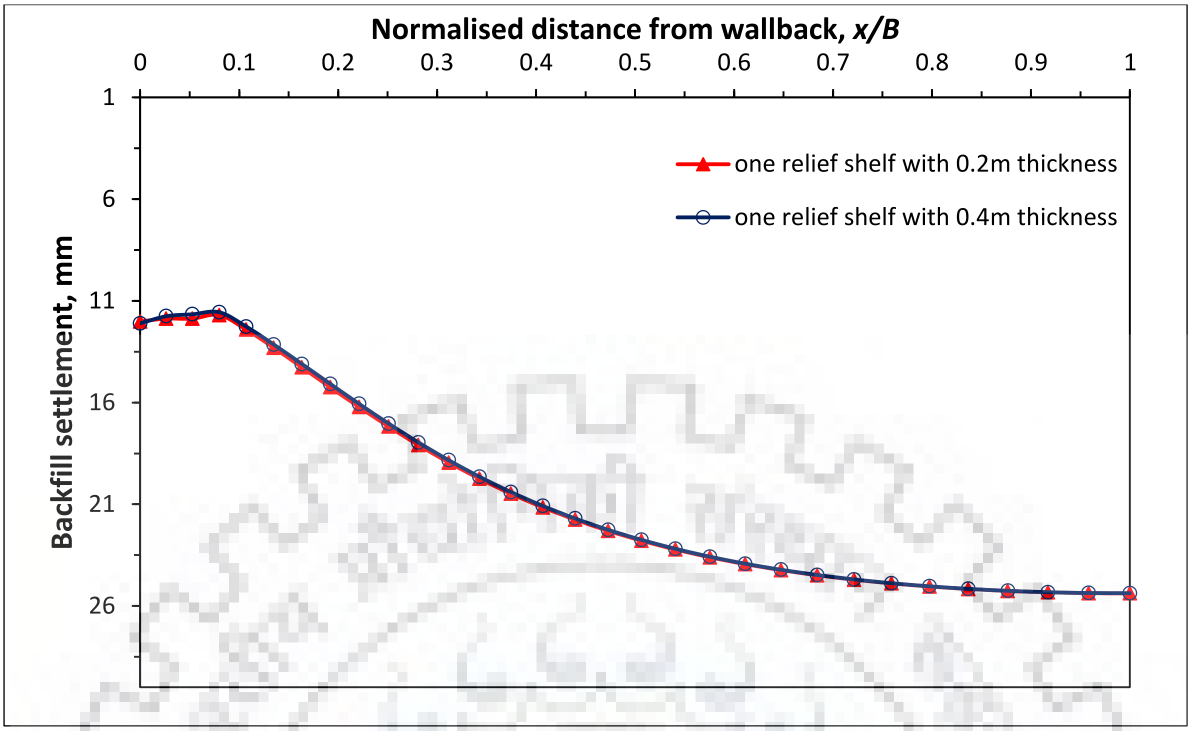
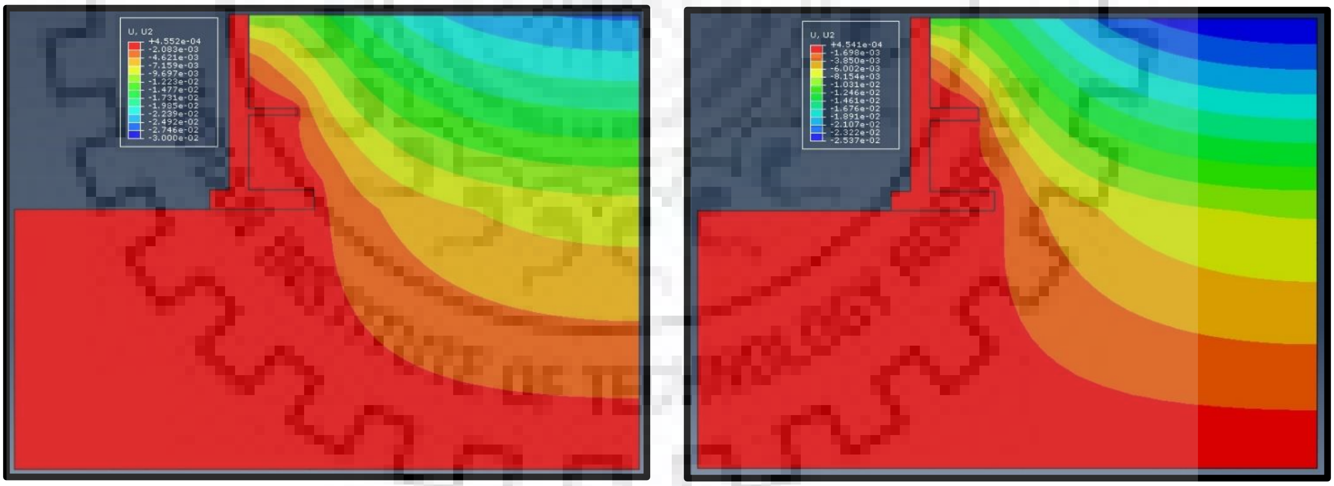


Figure 5.9: Effect of thickness of relief shelves of 1.5 m width on earth pressure distribution under static loading



(a) **(b)**

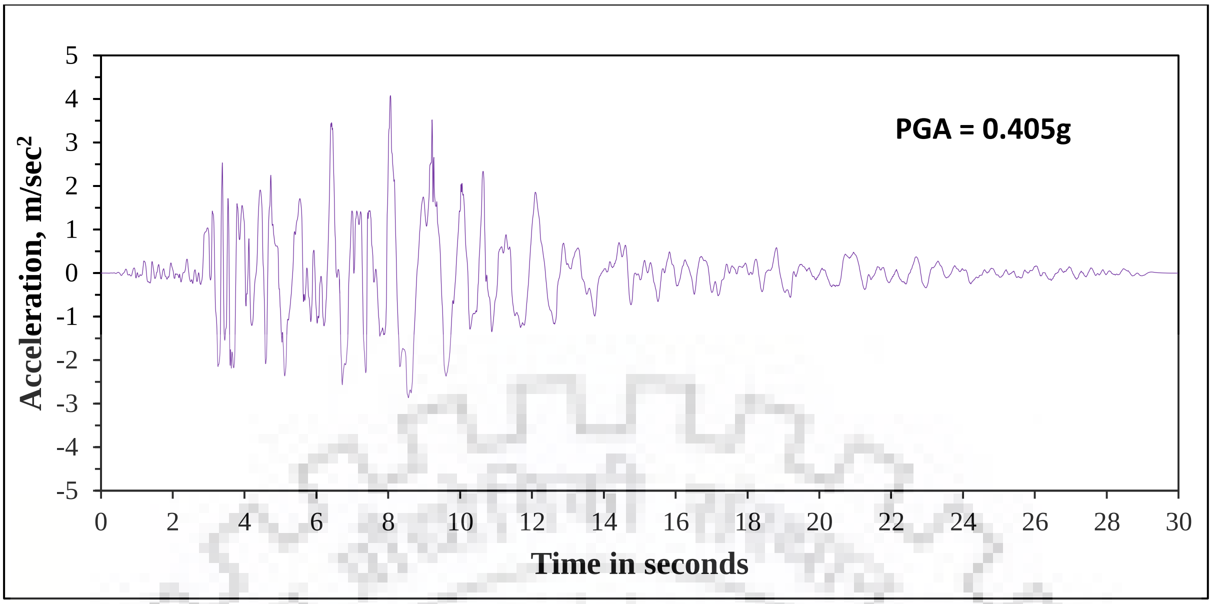
Figure 5.10: Comparison of vertical settlement distribution for the cases of retaining wall with one-relief shelf of (a) 0.2 m, and (b) 0.4 m thickness under static loading

CHAPTER 6

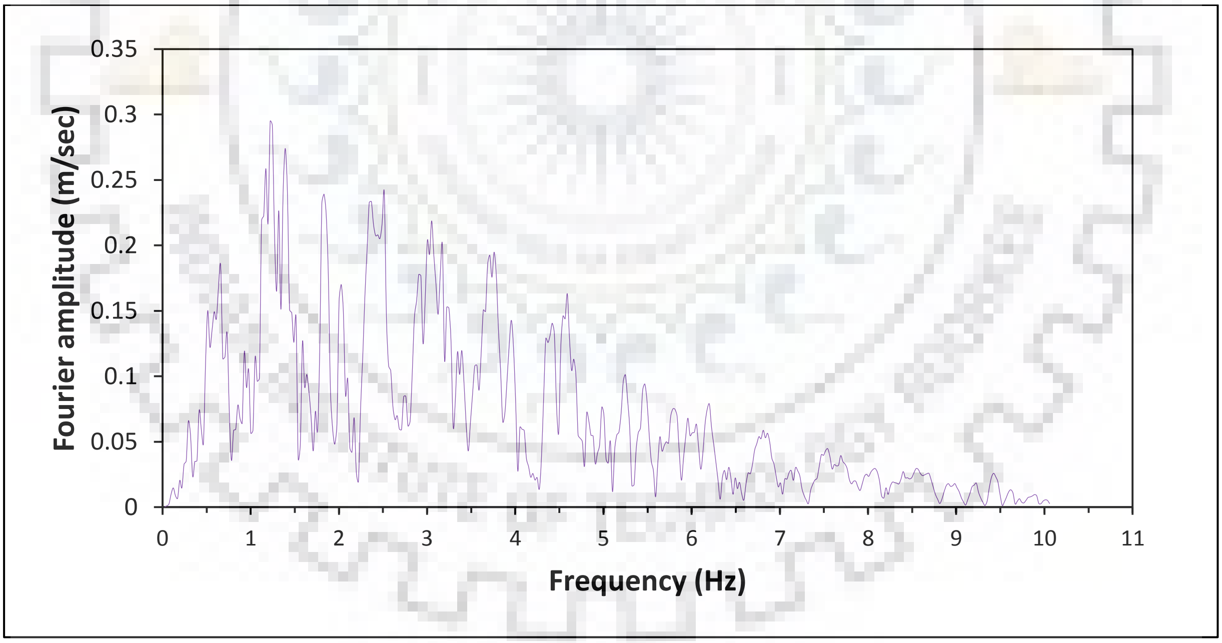
RESPONSE OF RETAINING WALL UNDER SEISMIC LOAD

The changes in lateral earth pressures on the retaining wall from the initial rest position, due to seismic event have been investigated in this chapter. Two seismic events e.g., 1994 Northridge earthquake (recorded at 090CDMG STATIO N 24278) and 1983 Trinidad earthquake (recorded at 090CDMG STATION 1498) were considered. A series of analyses was undertaken to study the response of retaining walls with and without relief shelves under the influence of dynamic loading after static loading conditions. The analyses were carried out in five steps to simulate a seismic event following the static loading stages in the field conditions. For static loading, step 1-4 were similar to those described in chapter 5. The series of steps were set in following sequence. The step 5 was simulated using Implicit Dynamic step available in Abaqus step module. In this step, the wall-soil domain was subjected to a seismic loading. In this step, the acceleration record of Northridge and Trinidad earthquakes were applied horizontally to all nodes at the bottom and side vertical boundaries of the foundation and backfill soil region. Following the study for the cantilever retaining walls by Upadhyay et al. (2011), it was assumed that horizontal acceleration over the depth of the foundation and backfill soils away from the wall is with uniform distribution.

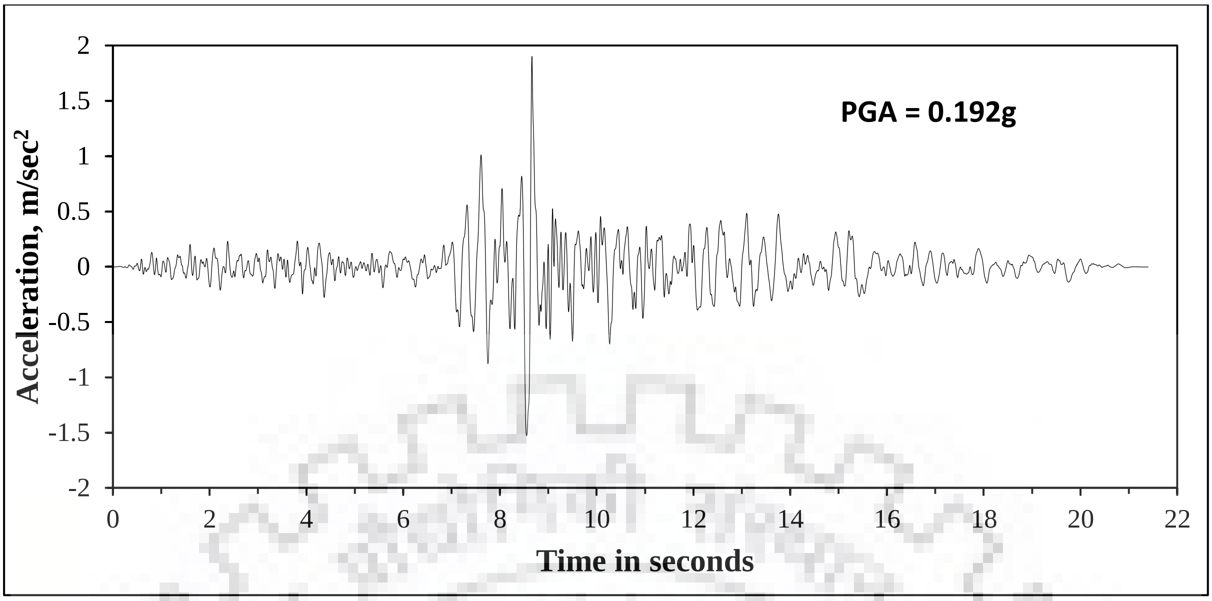
Ground acceleration-time history plots are given in figures 6.10 (a) and (c). Peak Ground Acceleration (PGA) for Northridge and Trinidad earthquakes are 0.405g and 0.192g respectively; Figures 6.10 (b) and (d) show the Fourier amplitude of seismic events considered in the current work. From these data, it can be seen that the Northridge earthquake lasted for longer duration as well as with a higher PGA.


PGA = 0.405g
Acceleration, m/sec^2
Time in seconds

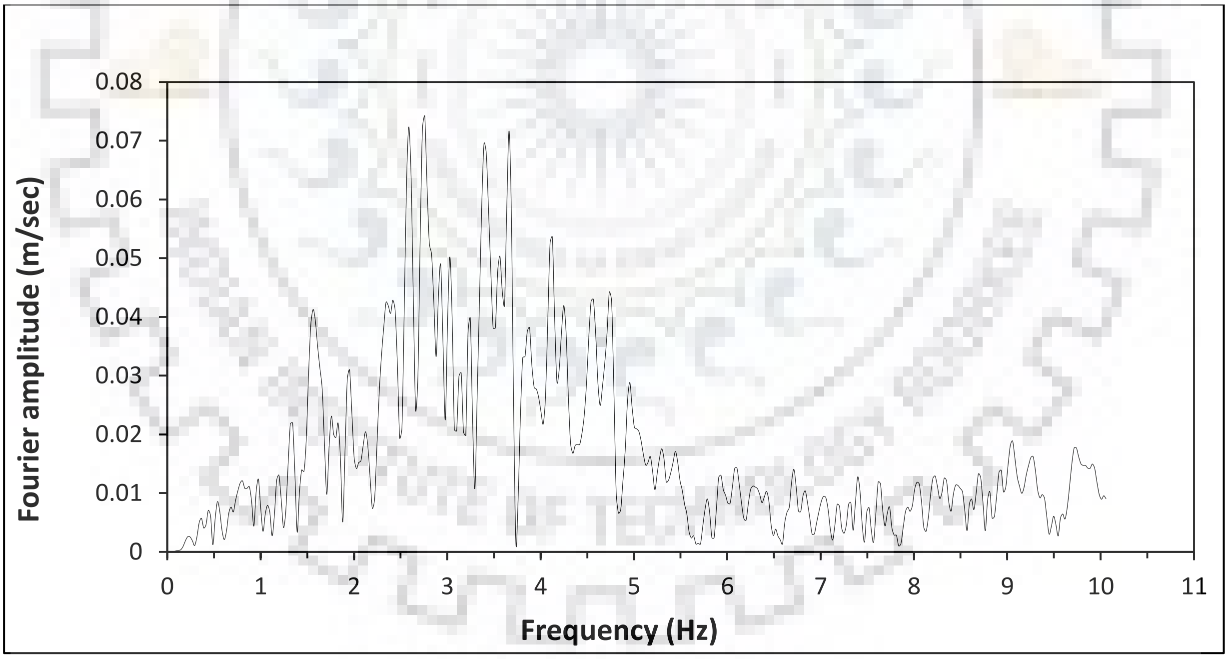
(a)


Fourier amplitude (m/sec)
Frequency (Hz)

(b)



(c)



(d)

Figure 6.1: Ground acceleration for (a) 1994 Northridge earthquake, (c) 1997 Trinidad earthquake; and Fourier amplitude for (b) 1994 Northridge earthquake, (d) 1997 Trinidad earthquake

6.1 Effect of shelf providing on lateral earth pressure on the wall

The lateral earth pressures on the wall were investigated to check the efficacy of the relief shelf provision in maintaining the stability of the wall after the considered seismic event in this study.

Figures 6.2, 6.3, 6.4 and 6.5 show the variation of lateral earth pressures with normalised wall height post the Northridge and Trinidad earthquakes for the all cases with 0.2 m shelf thickness. Figures 6.6 and 6.7 show the changes in lateral earth pressure due to these earthquakes from at rest pressure. The very interesting thing to see is the lateral earth pressure profile obtained along the normalised height of the retaining wall with no relief shelf is nonlinear. As compared to the 'at rest' earth pressure obtained at the end of 50 kPa surcharge loading, the post-earthquake lateral earth pressures have lower values. The reason behind this is the separation of wall and backfill soil during a seismic event. As the retaining wall was allowed to tilt during the earthquake, this excess lateral earth pressure was mobilised and resulted in earth pressure reduction and a permanent deformation of wall after earthquake.

It is noteworthy to see that the separation between wall and soil could not occur for the walls with relief shelves and hence the lateral earth pressure distribution was approximately similar before and post-earthquake (see figures 6.6 and 6.7). Therefore, the provision of relief shelves is proved to maintain the whole retaining wall system more stable and intact post a seismic event.

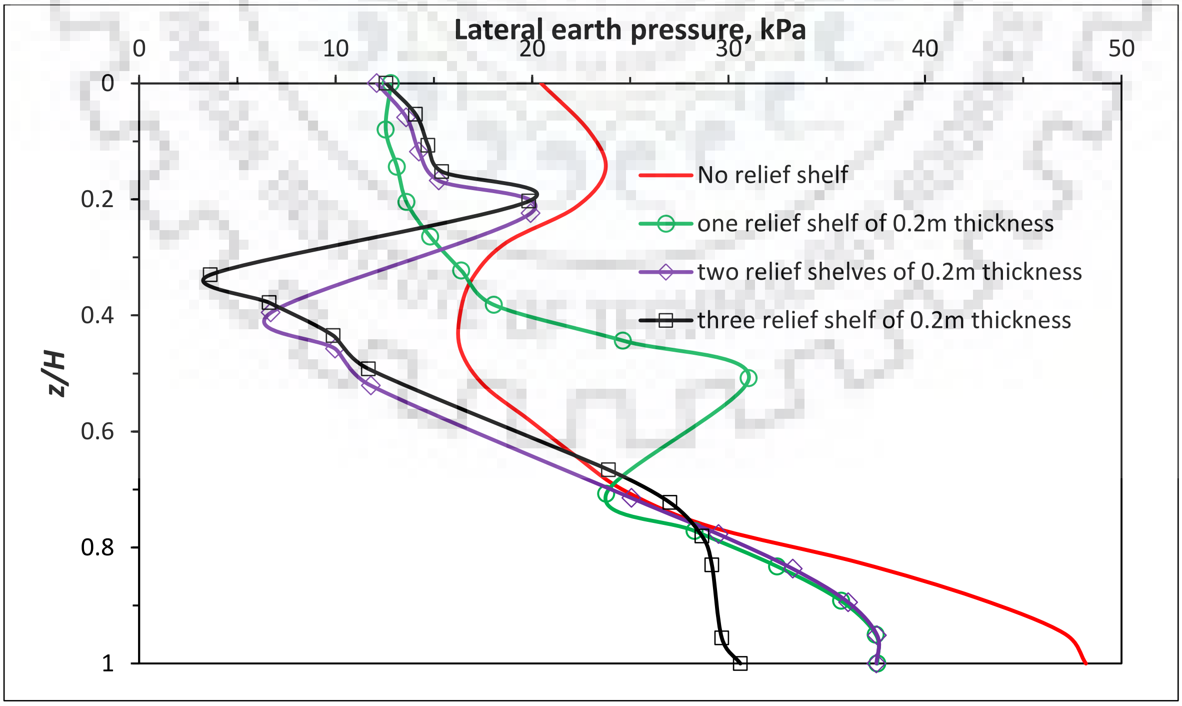


Figure 6.2: Effect of number of relief shelves of 0.2 m thickness on earth pressure distribution after Northridge earthquake

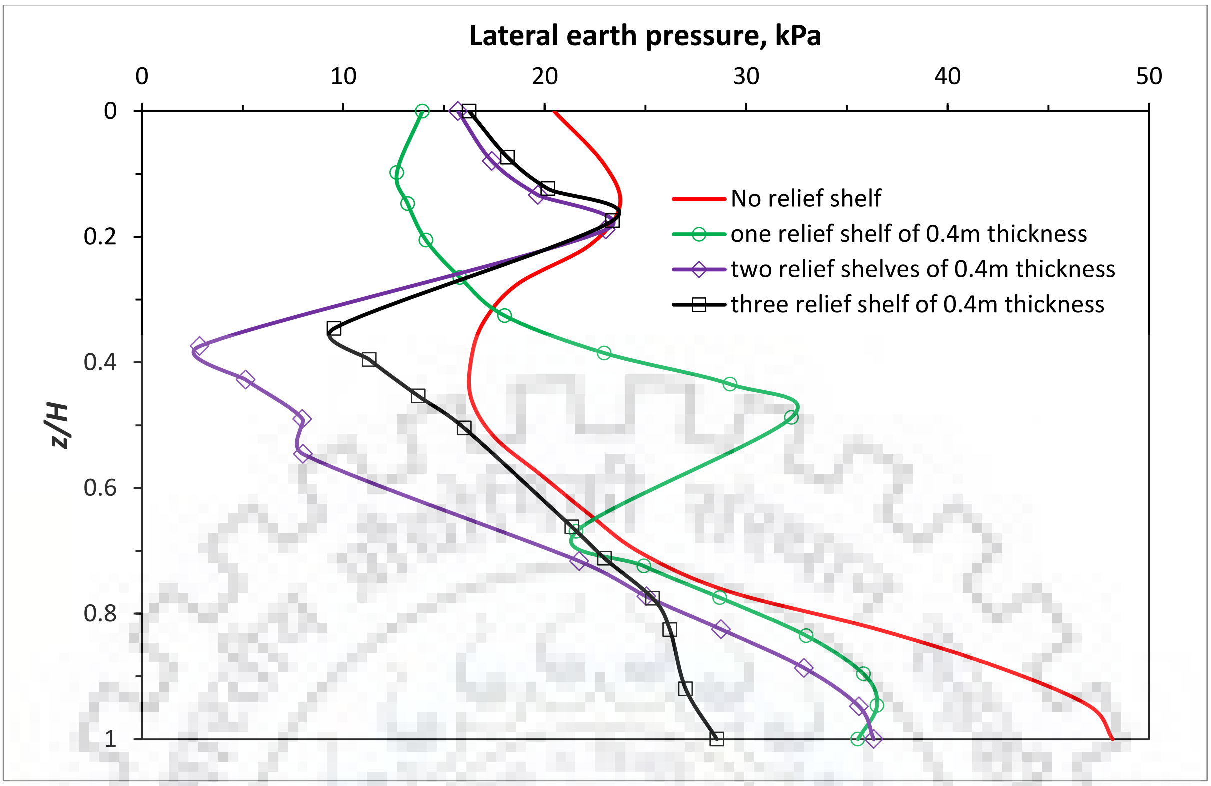


Figure 6.3: Effect of number of relief shelves of 0.4 m thickness on earth pressure distribution after Northridge earthquake

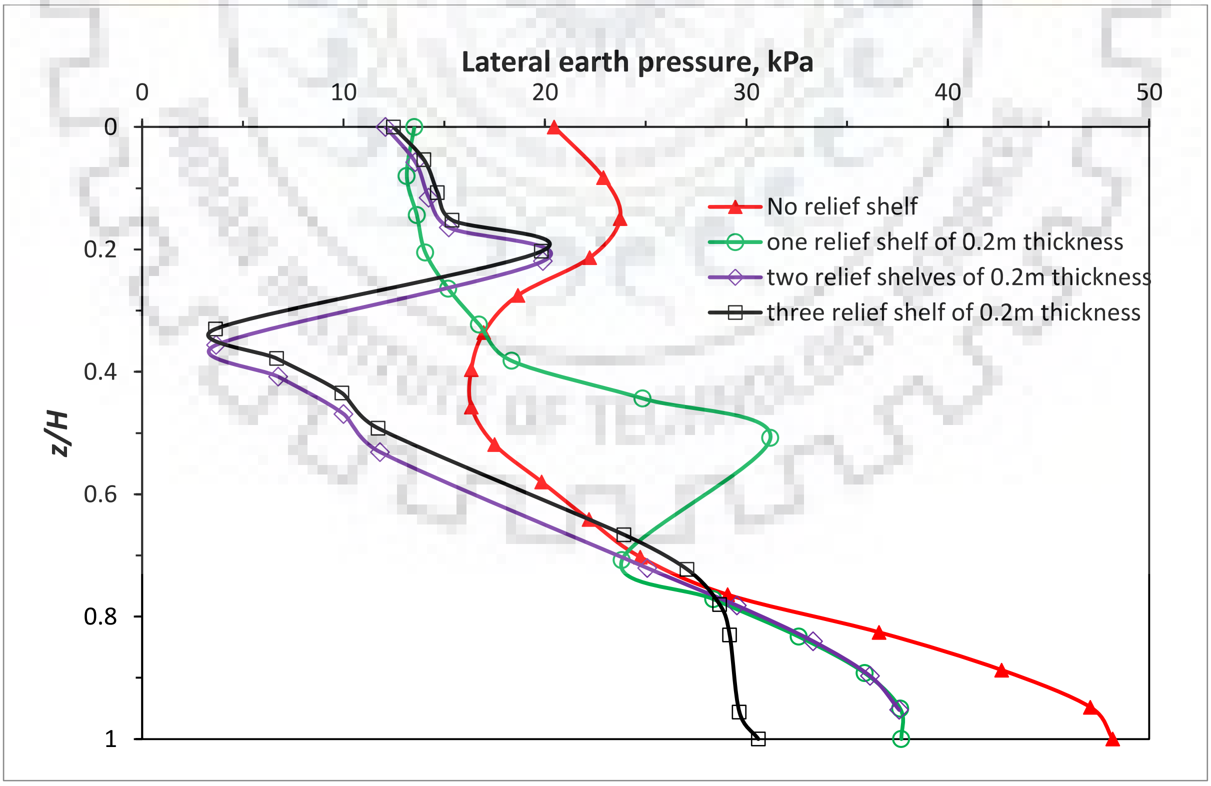


Figure 6.4: Effect of number of relief shelves of 0.2 m thickness on earth pressure distribution after Trinidad earthquake

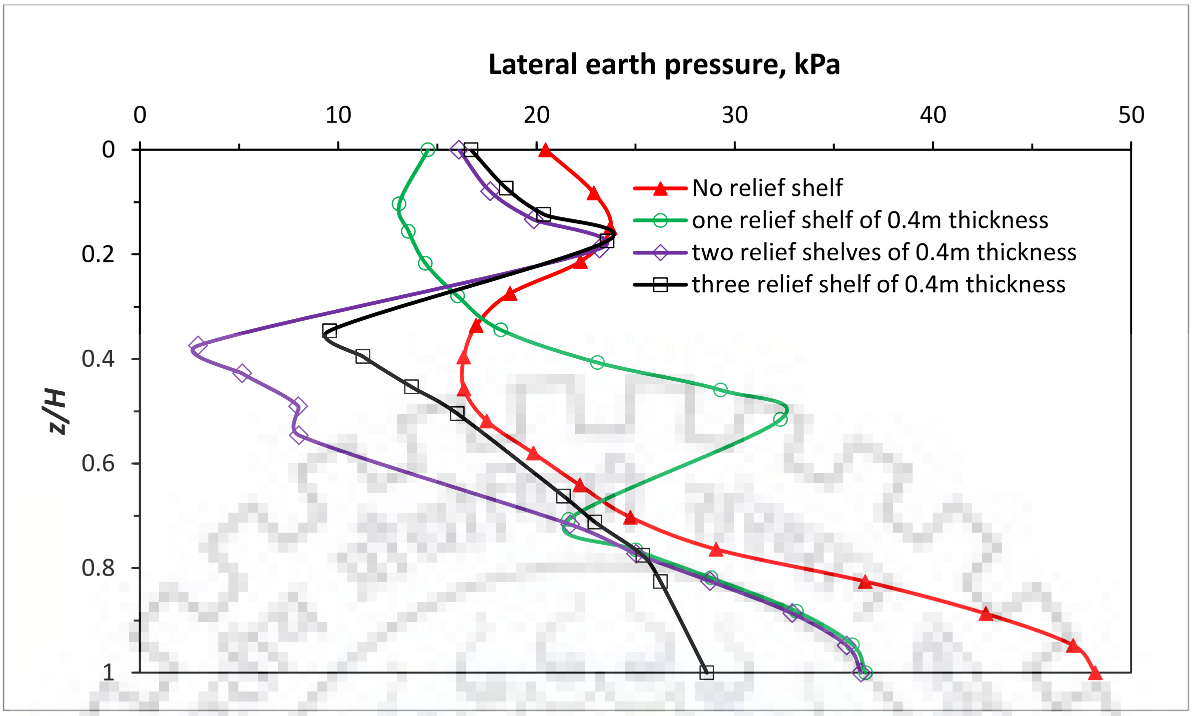


Figure 6.5: Effect of number of relief shelves of 0.4 m thickness on earth pressure distribution after Trinidad earthquake

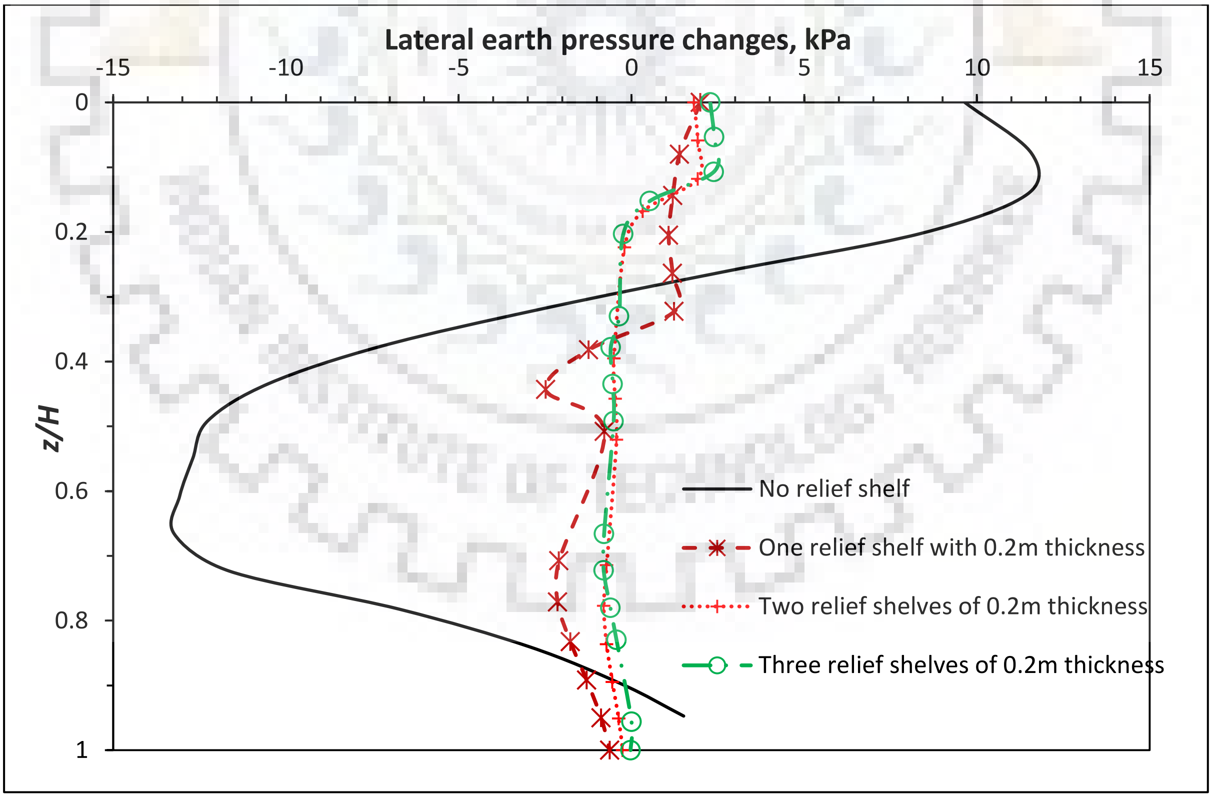


Figure 6.6: Changes in lateral earth pressure before and after Northridge earthquake

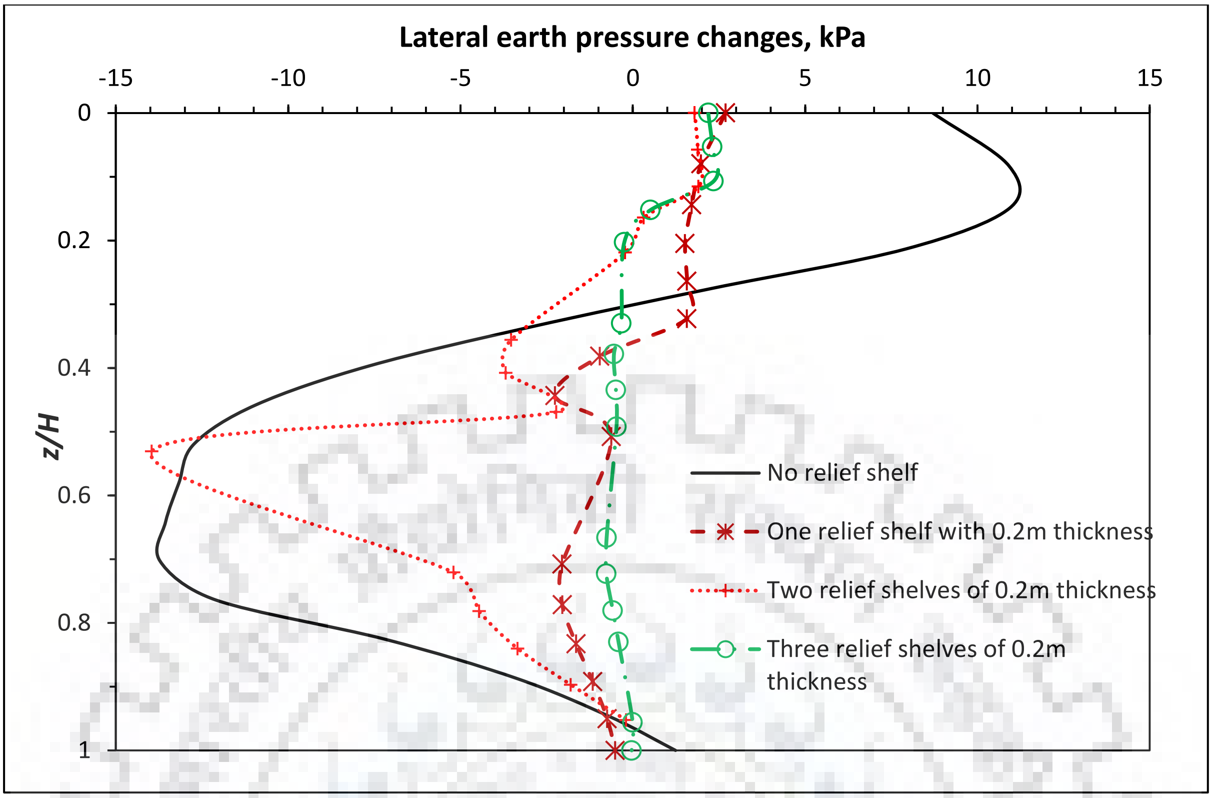


Figure 6.7: Changes in lateral earth pressure before and after Trinidad earthquake

6.2 Effect of shelf providing on settlement profile of backfill

Figures 6.8 and 6.10 show the backfill surface settlement of all cases with 0.2 m shelf thickness performed in this study post the Northridge and Trinidad earthquakes. The settlement is plotted with the normalised distance from the wall (*x/B*). The settlement profile is similar to the one before the seismic event. However, the values have increased simultaneously for no relief shelf. Figures 6.12 and 6.13 show the comparison of settlement before and after both earthquakes for all the cases with no relief shelf and two relief shelves with 0.2 m thickness. It can be observed that the settlement profile for the cases with relief shelves does not change post-earthquake, indicating their effectiveness in maintaining the ground level unaltered in the event of earthquake. On the other side, as expected, the settlement near the wall increased by approximately 80% for the wall without relief shelf. This was also seen in the previous section, where the reduction in lateral earth pressure post-earthquake for the wall without relief shelf was discussed. Both these observations are in good agreement and shows that walls without relief shelves are not much effective in sustaining the backfill in the event of an earthquake. Figures 6.9 and 6.11 show the effect of shelf thickness on the backfill settlement profile for the case of wall with one-relief shelves of 1.5 m width each. Two values of shelf thickness (e.g., 0.2m and 0.4 m) were considered. The figure demonstrates that the thickness has negligible effect upon the surface settlement profile negligibly. Similar response was observed for the other cases.

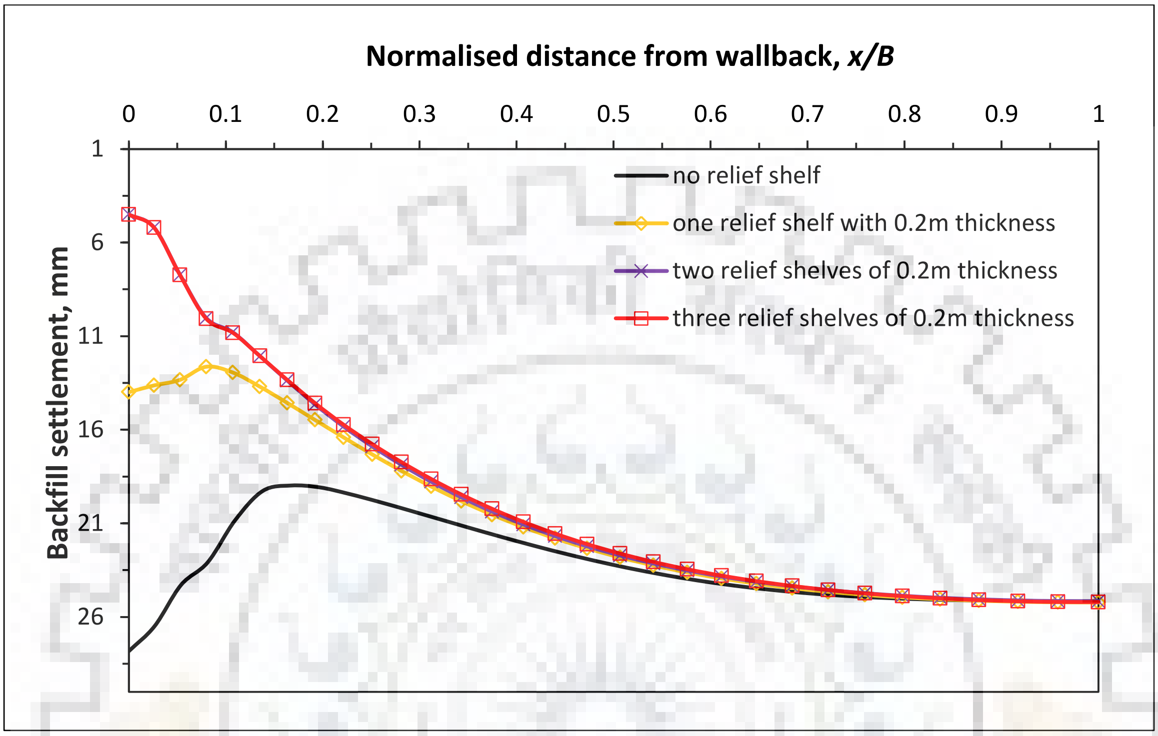


Figure 6.8: Effect of number of relief shelves of 0.2 m thickness on backfill settlement profile after Northridge earthquake

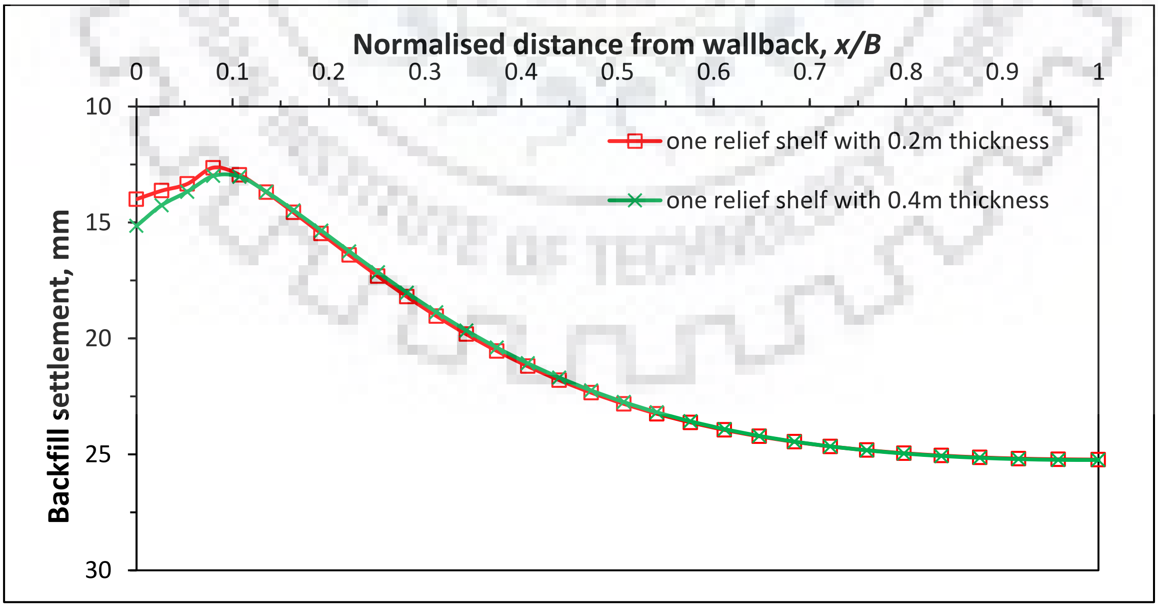


Figure 6.9: Effect of thickness of relief shelves of 1.5 m width on backfill settlement profile after Northridge earthquake

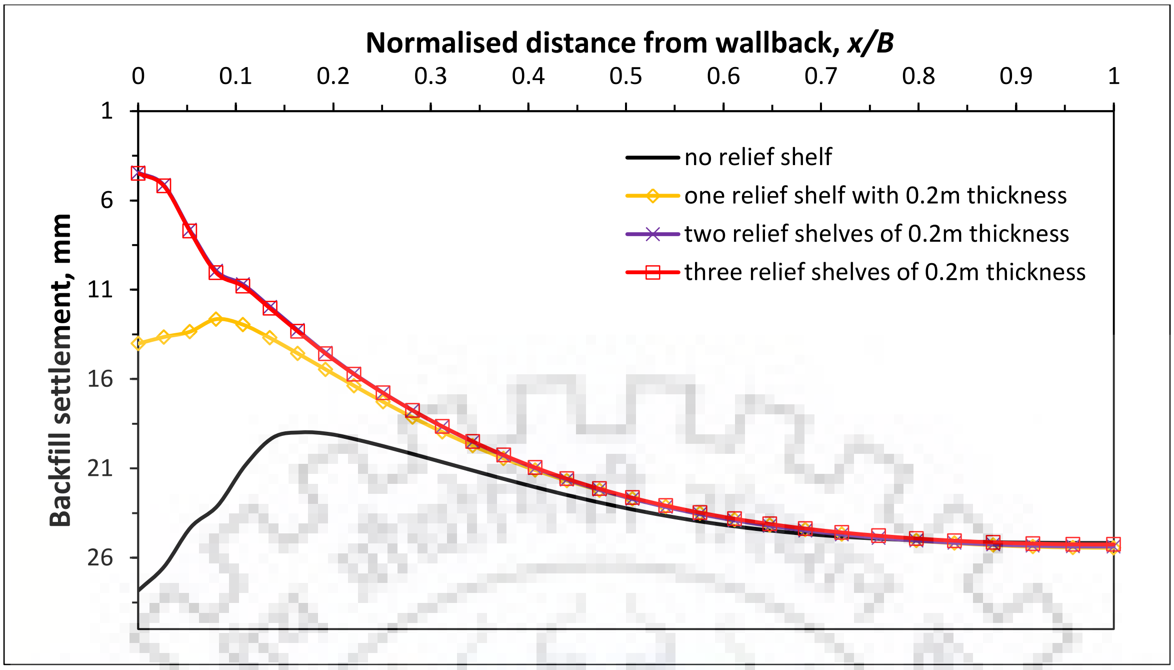


Figure 6.10: Effect of number of relief shelves of 0.2 m and 0.4 m thickness on backfill settlement profile after Trinidad earthquake

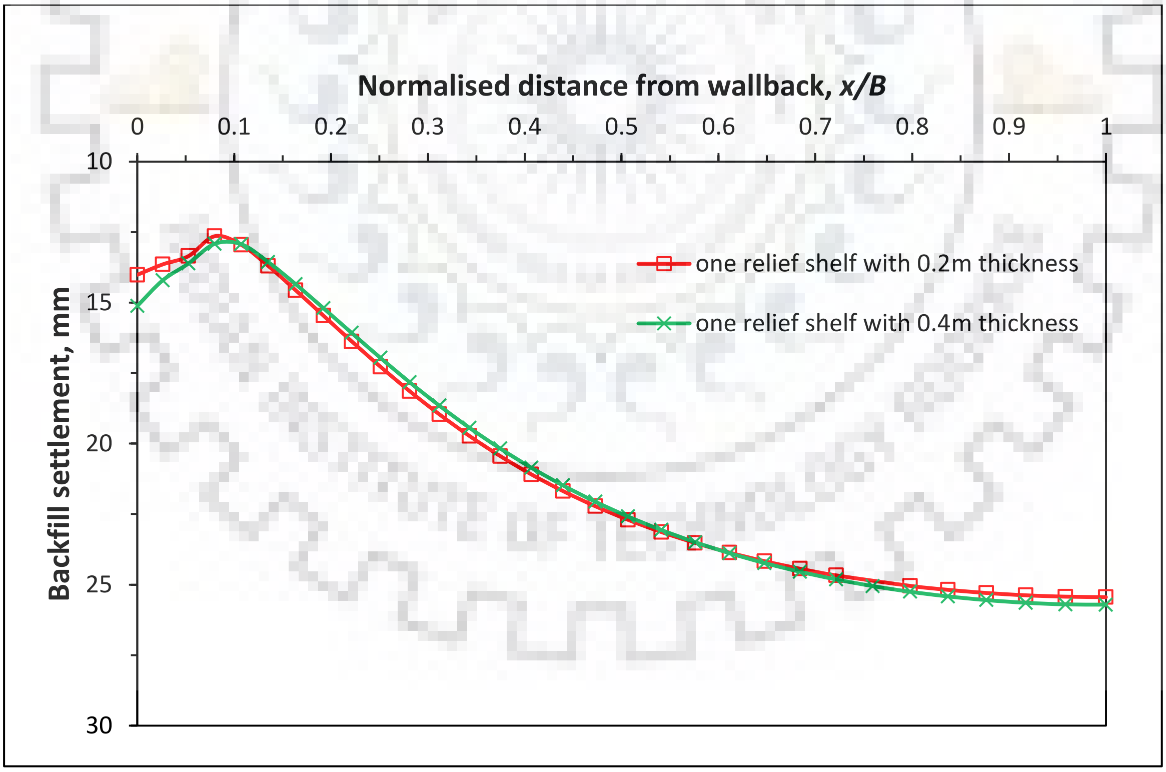


Figure 6.11: Effect of thickness of relief shelves of 1.5 m width on backfill settlement profile after Trinidad earthquake

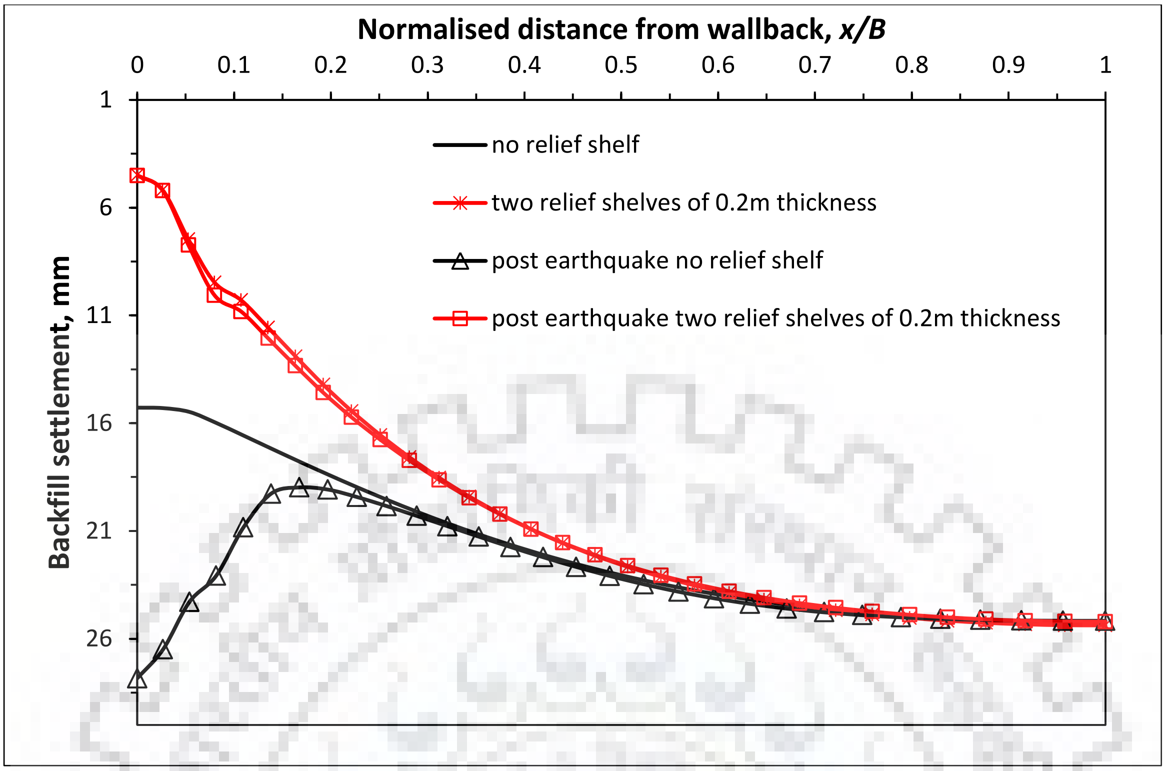


Figure 6.12: Comparison of backfill settlement profile of cases with 0 and 2 relief shelves before and after Northridge earthquake

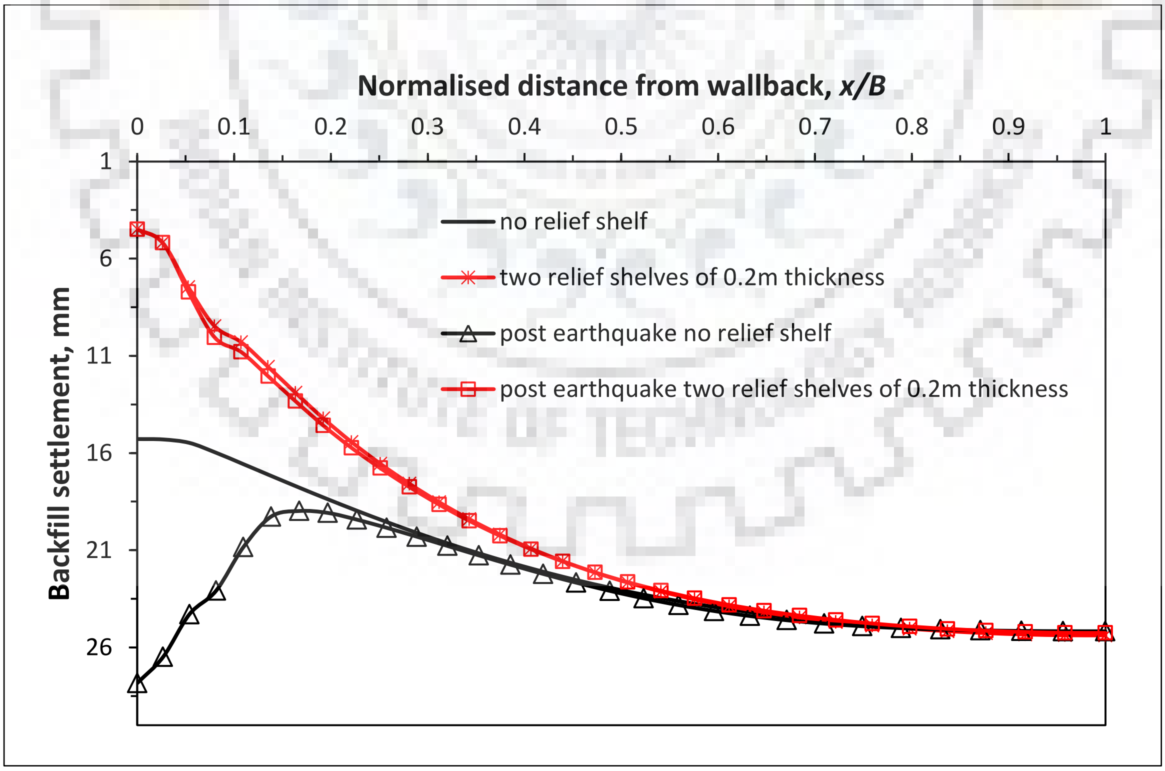
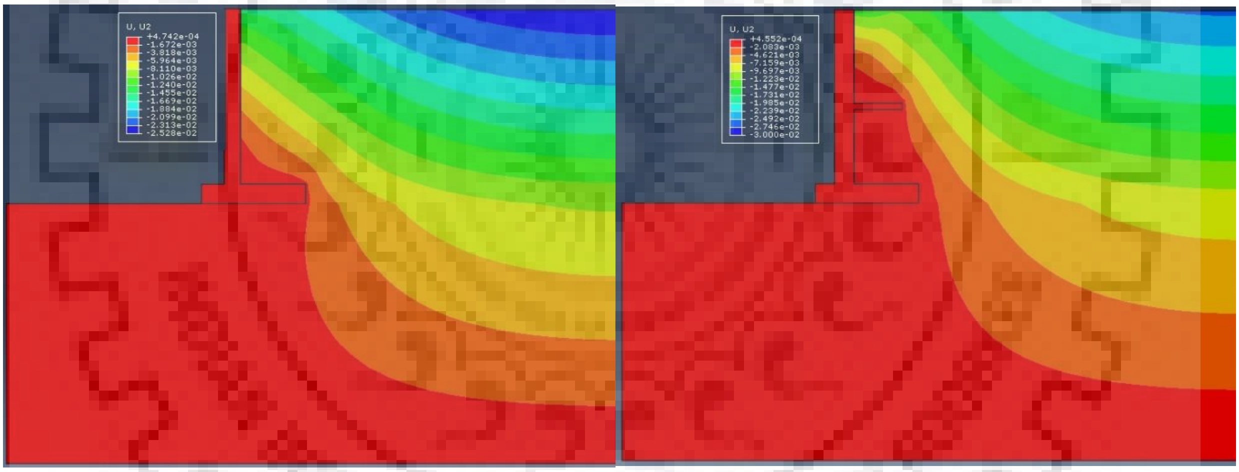


Figure 6.13: Comparison of backfill settlement profile of cases with 0 and 2 relief shelves before and after Trinidad earthquake

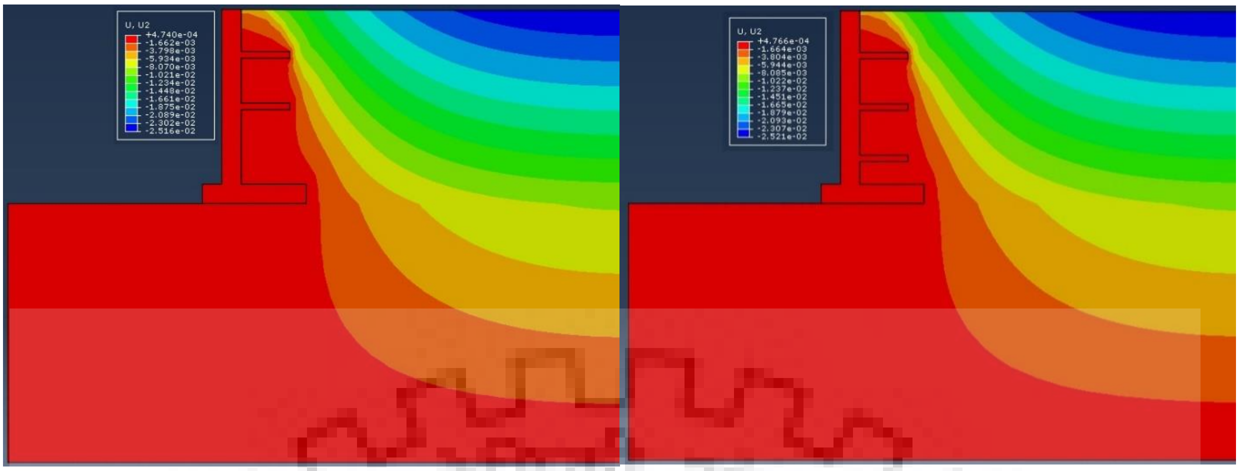
Vertical settlement mechanisms post-earthquakes for different cases of retaining wall with relief shelves under the effect of dynamic loading are represented in figures 6.13-6.16, in which contours of soil vertical displacement are plotted. For retaining wall without relief shelf, the vertical settlement near the wall back (at top) is about 27 mm, which is maximum among all the cases considered in this study (compare figure 6.13(a) with 6.13b, 6.14a and b, and 6.15(a) with 6.15b, 6.16a, and b. The backfill settlement above the relief shelf slightly decreases when retaining wall is provided with one relief shelf (see figures 6.13(b) and 6.15(b). However, significant reduction in vertical settlement (nearly zero) can be observed for the soil below the relief shelf. Furthermore, the backfill settlement near the wall decreases with increase in number of relief shelves from one to two and three. The settlement is almost negligible near the wall for these two cases (see figure 6.14a and b, and 6.16a and b).

In conclusion, the retaining walls with relief shelves behave well in the event of considered seismic events.



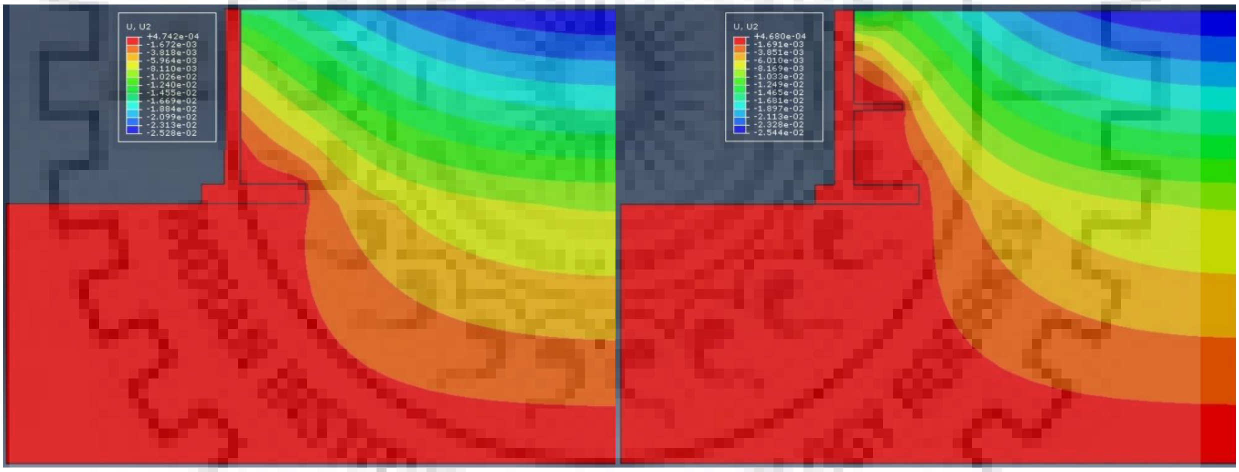
(a) (b)

Figure 6.14 Vertical settlement distribution for the case of retaining wall with (a) no relief shelf and (b) one relief shelf post Northridge earthquake



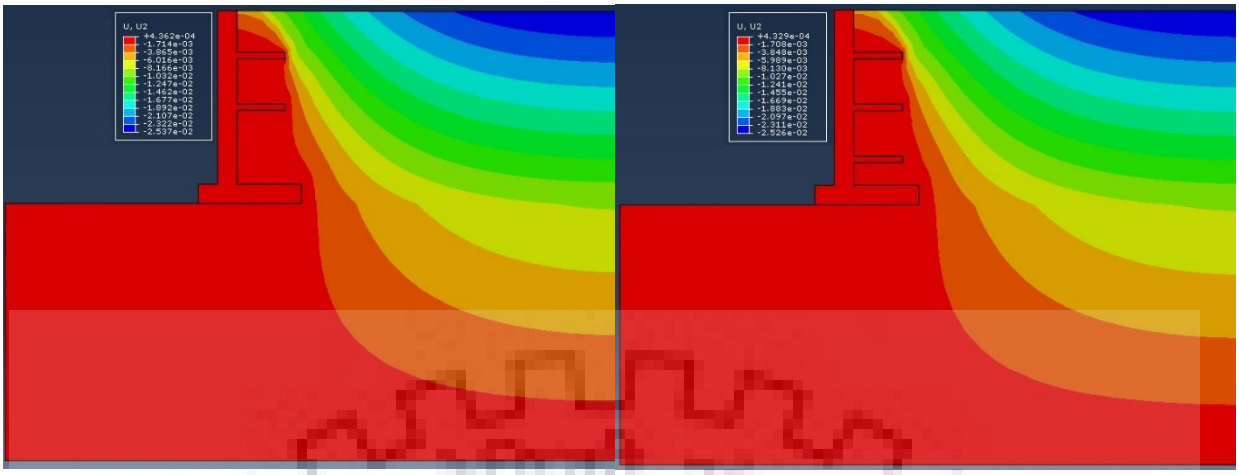
(a) **(b)**

Figure 6.15 Vertical settlement distribution for the case of retaining wall with (a) two relief shelves and (b) three relief shelves post Northridge earthquake



(a) (b)

Figure 6.16 Vertical settlement distribution for the case of retaining wall with (a) no relief shelf and (b) one relief shelf post Trinidad earthquake



(a) **(b)**

Figure 6.17 Vertical settlement distribution for the case of retaining wall with (a) two relief shelves and (b) three relief shelves post Trinidad earthquake

Chapter 7

CONCLUSIONS

7.1 Conclusions

The cantilever retaining wall-soil system was simulated into finite element model to study the response of the walls attached with relief shelves during a seismic event. It was observed that a little consideration has been given to this type of retaining walls. Providing monolithic relief shelves decreases the total lateral earth pressure acting on the wall and resists the backfill soil settlement efficiently. These very significant effects of such walls enable the structure to remain stable during extreme loading events and ensures the stability of natural or man-made structures resting on the backfill soil. This also can be utilised while deciding the dimensions of the retaining structure and can significantly save the material and construction costs. Recently, few researchers have studied the static response of these walls. However, as discussed previously, the performance of such walls in an event of earthquake is still an open question.

This dissertation present a brief investigation of the effect of providing monolithic relief shelves with cantilever retaining walls. Both static and dynamic responses of walls with varying number of shelves of different thickness were studied. The cantilever retaining wall with none, one, two and three relief shelves of 0.2 m and 0.4 m thickness were considered. Initially, the primary results produced by the finite element model were compared to the available results for static conditions in literature and the results were in good agreement. It was observed that both the lateral earth pressure and the backfill soil settlement decreased significantly, which increased the retaining wall stability. It was seen that provision of one relief shelf at the mid-height of the wall resulted in a decreased lateral earth pressure of approximately 15% of its value for the case with no relief shelf. Similarly, this value for the backfill settlement was approximately 20%. For the cases with two relief shelves, the settlement reduced by more than 70% of its value for the case with no shelf. The retaining wall remained stable during and after both the earthquakes. In contrast, the cantilever wall with no relief shelf tilted opposite to the backfill soil and showed sudden increase in the backfill surface settlement close to the wall. After analysing and discussing the obtained results, the following major conclusions were drawn from this study.

- Retaining walls with relief shelves can be considered as effective solutions for maintaining the stability of high retaining walls during both static and dynamic loading conditions.
- Retaining walls with relief shelves can considerably reduce the lateral earth pressure (and in result the total thrust) on the wall.
- Retaining walls with relief shelves showed to resist the settlement of backfill in both pre and post-earthquake conditions. In contrast, the walls with no relief shelf were seen to be unable in doing so and the backfill settlement increased by about 80% post-earthquake.

7.2 Limitations and Future Scope

This study has shown the capabilities of a cantilever retaining wall with relief shelves in maintaining the stability under static as well as dynamic loadings. Despite the good results and observations of the present study, there are many limitations and need to be extended in future.

- The optimum number and appropriate location of relief shelves for fulfilling both lateral thrust and serviceability criterion should be obtained from a detailed parametric study. In addition, the economic and feasibility aspects should be considered when designing the wall.
- A more detailed study extending the present work is required to study the various height of walls, number of shelves provided, width and thickness of the shelves and different seismic events.
- Experimental investigations are needed to understand the behaviour under seismic loadings, which will validate their efficacy in resisting and decreasing lateral thrust and backfill soil settlements.

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