REDUCED ORDER MODELLING FOR CONTROL SYSTEM DESIGN

Ph. D. Thesis

by

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REDUCED ORDER MODELLING FOR CONTROL SYSTEM DESIGN

A THESIS

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled "**REDUCED ORDER MODELLING FOR CONTROL SYSTEM DESIGN**" in partial fulfilment of the requirements for the award of the Degree of Doctor of Philosophy and submitted in the Department of Electrical Engineering of the Indian Institute of Technology Roorkee, Roorkee, is an authentic record of my own work carried out during a period from December, 2014 to June, 2019 under the supervision of Dr. Rajendra Prasad, Professor, Department of Electrical Engineering, Indian Institute of Technology Roorkee, Roorkee.

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other Institution.

(SUDHARSANA RAO POTTURU)

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

(Rajendra Prasad)
Supervisor

The Ph. D. Viva-Voce Examination of Sudharsana Rao Potturu, Research Scholar, has been held on **September 9, 2019**

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This is to certify that the student has made all the corrections in the thesis.

Supervisor **Head of the Department**

Dated: ____________

Abstract

The physical system can be represented in mathematical models. The mathematical procedure of system modelling often leads to a comprehensive description of a process in the form of higher order ordinary differential equations or partial differential equations which are difficult to use and sometimes necessary to find the possibility of some equations of the same type but of lower order that may adequately reflect all essential characteristics of the original system. Hence a systematic approximation of the original model is required which results in a reduced order model. The systematic procedure that leads to reduced order model is termed as model order reduction (MOR), which tries to quickly capture the essential features of an original system.

A large number of order reduction techniques have been suggested by several authors in the literature. These are broadly categorized as time and frequency domain reduction techniques. The frequency domain reduction methods also utilized to reduce the order of interval systems based on Kharitonov's theorem and interval arithmetic operation (IAO). Furthermore, combined methods have been developed by several authors in which denominator polynomials are determined by one method and numerator terms are determined by another method. In spite of many existing reduction techniques, there is always a scope of developing new techniques. Therefore, the model order reduction of original higher order systems is in demand in the field of system and control due to the various issues like good time/frequency response matching, stability and realizability etc. So, it is of great interest to investigate the efficacy of new algorithms.

The initial aim of this thesis is to highlight the frequency domain and interval domain order reduction methods available in the literature. This lead to motivate to develop some new algorithm for order reduction of linear time invariant single input single output (SISO) and multi input multi output (MIMO) systems. The work represented in this thesis involves the use of both conventional and interval approach for order reduction of continuous and discrete time systems. In addition, the other objective is to ensure the superiority of the new reduction methods by comparing

with other well-known reduction methods available in the literature. Lastly, to solve the problem of designing the controller both in direct and indirect approaches by using proposed reduced order methods.

The introduction followed by importance and application of model order reduction is presented, subsequently followed by the mathematical preliminaries, then the concept of interval systems is introduced. Besides a brief overview of the development that have taken place in the area of model order reduction, various existing reduction methods and their associated qualities/ drawbacks are also reflected. New composite reduction methods are developed for reduction of higher order linear time invariant systems. Time moment matching method, factor division algorithm, Pade approximation method and differentiation method are employed to propose composite MOR methods. These methods are applicable to SISO/MIMO systems taken from the literature and the results are compared with the some available reduction models. The comparative analysis has been done on the basis of their performance indices which justify the proposed methods.

New composite reduction methods are developed for reduction of higher-order linear-time invariant (LTI) interval systems using differentiation method, stability equation method and time moment matching method based on Kharitonov's theorem. Further, based on interval arithmetic operations new mixed methods have also been proposed by using Pade approximation method, factor division algorithm and differentiation method. To show the efficacy and powerfulness of the proposed reduction methods the popular numerical examples available in the literature are considered. Some of these methods are also extended to model reduction of discrete time systems.

The controller is designed on the basis of approximate model matching, with both the direct and indirect approaches, using the proposed reduction methods. The desired performance specifications of the plant are translated into a specification/reference model transfer function. In direct approach the original higher order plant is reduced and the controller designed for reduced order model. In indirect approach of controller design, a controller is designed for original plant transfer function and the higher order closed loop transfer function is obtained with unity feedback. Then this higher order closed loop transfer function is reduced to lower order model and performance is compared with that of the reference model.

The performance comparison of various models has been carried out using MATLAB software package.

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(Sudharsana Rao Potturu)

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LIST OF ACRONYMS

ACO	Ant Colony Optimization
BFO	Bacterial Foraging Optimization
CFE	Continued Fraction Expansion
DM	Differentiation Method
FDA	Factor Division Algorithm
GA	Genetic Algorithm
HOIS	Higher Order Interval System
HOS	Higher Order System
IAE	Integral Absolute Error
IAO	Interval Arithmetic Operation
IDM	Inverse Distance Measure
ISE	Integral Square Error
ITAE	Integral Time Absolute Error
ITSE	Integral Time Square Error
IWO	Invasive Weed Optimization
LOM	Lower Order Model
LTI	Linear Time Invariant
MOR	Model Order Reduction
MIMO	Multi Input Multi Output
PA	Pade Approximation
PID	Proportional-Integral-Derivative
PSO	Particle Swarm Optimization
ROM	Reduced Order Model
ROIM	Reduced Order Interval Model
SAE	Summation Absolute Error
SISO	Single Input Single Output
SRAM	Simplified Routh Approximation Method
SSE	Summation Square Error

STAE	Summation Time Absolute Error
TMM	Time Moment Matching

LIST OF SYMBOLS

$\left[a_i^-, a_i^+\right], \left[b_j^-, b_j^+\right]$	Numerator and denominator coefficients of lower and upper bounds of original higher order system
$\left[c_i^-, c_i^+\right], \left[d_j^-, d_j^+\right]$	Numerator and denominator coefficients of lower and upper bounds of reduced order model
$[G(s)]$	n^{th} order transfer matrix of original system
$[G(z)]$	n^{th} order pulse transfer matrix of original system
$[G(s, e_{ij}, D_n)]$	n^{th} order interval transfer matrix of original model
$[R(s)]$	k^{th} order transfer matrix of reduced order model
$[R(z)]$	k^{th} order pulse transfer matrix of reduced model
$[R(s, h_{ij}, D_k)]$	k^{th} order interval transfer matrix of reduced model
$\tilde{M}(s)$	Open loop specification model
$g(h), r(h)$	Original and reduced order models unit step response at the h^{th} sampling instant
$G(s)$	n^{th} order transfer function of original model
$G(z)$	n^{th} order pulse transfer function of original system
$G(s, a, b)$	n^{th} order interval transfer function of original system
$G(z, p, q)$	n^{th} order pulse interval transfer function of original system
$M(s)$	Reference model
n, k	Order of higher order system and reduced order model
$N(s), D(s)$	Numerator and denominator polynomials of original higher order system

$n(s), d(s)$	Numerator and denominator polynomials of reduced order model
$N(s,a), D(s,b)$	Numerator and denominator interval polynomials of original higher order system
$n(s,c), d(s,d)$	Numerator and denominator interval polynomials of reduced order model
$R(s)$	k^{th} order transfer function of reduced order model
$R(z)$	k^{th} order pulse transfer function of reduced order model
$R(s,c,d)$	k^{th} order interval transfer function of reduced model
$R(z,u,v)$	k^{th} order discrete-time interval transfer function of reduced model
$y(t), y_k(t)$	Original and reduced order models unit step responses
a_i, b_j	Numerator and denominator coefficients of original system
c_i, d_j	Numerator and denominator coefficients of reduced order model
$G_C(s)$	High order controller transfer function
$G_p(s)$	Original plant transfer function
$G_{CL}(s)$	Higher order closed loop transfer function with unity feedback
K_1, K_2, K_3	PID controller parameters
M_i	i^{th} Markov parameter
M_p	Peak overshoot
p_i, q_i	Numerator and denominator coefficients of controller
$R_C(s)$	Reduced order controller transfer function

$R_p(s)$	Reduced plant transfer function
$R_{CL}(s)$	Reduced order closed loop transfer function with unity feedback
T_i	i^{th} time moment
t_r	Rise time
t_{ss}	Steady state time

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

The large scale systems exists everywhere in different diverse fields such as aeronautics, biomedical systems, complex chemical process, ecological systems, economic systems, electric power systems, mechanical environment systems, hydraulic pneumatic and thermal systems etc. A system is said to be large scale when its order and dimensions are so high, such that classical techniques of controller design, modelling, analysis, and computation fail to give accurate solutions with reasonable computational efforts.

The model order reduction (MOR) is defined in several ways depends on the context which one is preferred. Initially, the reduction methods were developed in the area of control systems, which studies the characteristics and properties of the dynamical systems to reduce their computational effort and complexity, while preserving their input-output behavior as much as possible. Later, the mathematicians has been taken up the field of MOR. Nowadays, MOR is a flourishing and demanding field of research in many different fields such as numerical analysis, systems and control theory etc. This has an encouraging and healthy effort towards model order reduction as a whole, bringing different view points and different techniques together, to push the MOR field forward rapidly.

The work presented in this thesis is focused on model order reduction of linear time invariant conventional and interval systems. The second part of this comprises of the application of reduced order modelling in control system design.

1.2 MODEL ORDER REDUCTION

In general, the available physical systems are complex and which contain very high order in their transfer function. The analysis, simulation and controller design of such systems becomes tedious and difficult. To deal with such systems, the order reduction methods plays an important role to reduce order of the original higher order systems to the lower order models (LOM) by retaining dominant properties of the original system.

Most of the real world processes are non-linear in nature, as a result the mathematical models which are used for the modelling, analysis and controller design of such systems become more cumbersome and rigid. Therefore, an appropriate linear model is considered to represent the system, which provides an easier way to analyze the complex processes. Further, the design of suitable controller and observer to gain the knowledge about the real world system also becomes difficult task even for linear model. Hence further simplification is required to reduce the computational complexity involved in the analysis and design of the system. This is achieved by mathematical procedures, known as model reduction techniques.

A reduced order model (ROM) means a system which has fewer state variables than the linear time invariant original higher order system. In recent development the accuracy and computational speed has increased in a large extent in processor design. The computational speed can be improved by providing high hardware configurations. But still providing accurate results is a challenging task for real time situations arising in nuclear reactors, control of chemical plants, process industries, estimation and filtering. The implementation difficulties involved in the design of controller and observer for higher order linear time invariant systems become more simple with the help of reduced order models. A reduced order model is useful in the design, analysis, and simulation of controller or compensator for stabilization of the output response of the given system.

1.2.1 Need for Model Order Reduction

Every physical system can be converted into a respective mathematical model. These mathematical model give a complete information about a physical system in the form of higher order differential equations. It is important and sometimes necessary to find a reduced order models which retain the dominant characteristics of the comprehensive model. The reasons for model order reduction are as follows [1]:

- **Quick and easy understanding of the system:** A complex dynamic systems possesses difficulties in its modelling, analysis and identification. An alternate method to deal with such systems is MOR. The MOR methods tries to quickly capture the essential properties of the original higher order system such as damping ratio, time constant and natural frequency.
- **Reduced computational burden:** The higher order systems are computationally heavy and time consuming. The model reduction methods are simple and avoids the computational effort in simulation.
- **Reduced hardware complexity:** The controller design for reduced order model is less costly, more reliable and easy to implement due to less hardware complexity.
- **Making feasible designs:** The effectiveness of the MOR in controller design is given bellow
  - Model reference adaptive and parameterized control methods [2]
  - Hierarchical control programme
  - Suboptimal control systems
  - Decentralized controllers
  - Power system stability [3, 4]
- **Generalization:** The reduced order model results are easily generalized to the other comparable models.

1.3 MATHEMATICAL PRELIMINARIES

1.3.1 Model Order Reduction Problem Statement Representation

The mathematical models of higher order dynamic systems described in the state space form is known as time domain representation and those in the transfer function form is known as frequency domain representation. The methods which reduce a higher order state space model are called time domain order reduction methods whereas those which reduce a transfer function or a transfer function matrix are called frequency domain order reduction methods. The aim of the model order reduction is to find a reduced system, which approximates the higher order system in some sense and gives nearly same response for the same type of inputs.

In the time domain, let an n^{th} order linear time invariant (LTI) system is expressed in state space form as

$$\left.\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned}\right\} \tag{1.1}$$

where, $x \in R^n, u \in R^p, y \in R^q$ are state, input and output variable vectors; A, B, C and D are constant matrices with dimensions $n \times n,\ n \times q,\ q \times n,\ q \times q$ respectively.

The model order reduction problem is to find the appropriate $k^{th}\,(k < n)$ order reduced model which reflects the dominant properties of the original high order system eq. (1.1) be expressed as

$$\left.\begin{aligned} \dot{x}_k(t) &= A_k x_k(t) + B_k u(t) \\ y_k(t) &= C_k x_k(t) + D_k u(t) \end{aligned}\right\} \tag{1.2}$$

such that original n^{th} order system and the reduced k^{th} order model are similar in the important aspects of their characteristics where $x_k \in R^r, u \in R^p, y_k \in R^q$ and $A_k,\ B_k,\ C_k,$ and D_k are constant matrices of reduced dimensions and y_k should be close approximation of y for given set of inputs.

The transfer function corresponding to eq. (1.1) may be written as:

$$G(s) = C(sI - A)^{-1}B + D \tag{1.3}$$

In the frequency domain, for single input single output (SISO) case, $G(s)$ is the n^{th} order higher order system and $R(s)$ is the reduced model of order $k\ (k < n)$. While, for multi input multi output (MIMO) case, $[G(s)]$ is the n^{th} order transfer function matrix with p inputs and q outputs and $[R(s)]$ is the k^{th} order reduced transfer function matrix with p inputs and q outputs.

In the interval domain, for SISO case, $G(s,a,b)$ represents the n^{th} order interval system and $R(s,c,d)$ represents the reduced interval model of order k. While for MIMO case, $[G(s,e_{ij},D_n)]$ is the n^{th} order transfer function matrix with p inputs and q outputs and $[R(s,h_{ij},d_k)]$ is the k^{th} order reduced transfer function matrix with p inputs and q outputs.

1.3.2 Time Moments and Markov Parameters

Let the impulse response of high order asymptotically stable system be $g(t)$ then

$$\begin{aligned} G(s) &= \int_0^\infty g(t)\, e^{-st} dt \\ &= \int_0^\infty g(t)\left[1 - \tfrac{st}{1!} + \tfrac{s^2t^2}{2!} - \tfrac{s^3t^3}{3!} + \cdots\right] dt \\ &= \int_0^\infty g(t)\, dt - s\int_0^\infty tg(t)\, dt + s^2\int_0^\infty \tfrac{t^2}{2!} g(t)\, dt \cdots \end{aligned}$$

or

$$G(s) = c_0 + c_1 s + c_2 s^2 + \cdots \tag{1.4}$$

Where,

$c_i = \frac{(-1)^i}{i!}\int_0^\infty t^i g(t)\, dt = \frac{(-1)^i}{i!} T_i$, and T_i is defined to be the i^{th} time moment of $g(t)$, it can be shown that

$$T_i = (-1)^i \left.\frac{d^i G(s)}{ds^i}\right|_{s=0}$$

Thus, the time moments [5] of the system are proportional to the coefficients of the power series expansion of $G(s)$ about $s = 0$. Alternatively $G(s)$ may be expanded about $s = \infty$, i.e.,

$$G(s) = M_1 s^{-1} + M_2 s^{-2} + M_3 s^{-3} + \cdots \tag{1.5}$$

The coefficients M_i are called the Markov parameters of $G(s)$

1.3.3 The Interval Arithmetic Operations

The interval arithmetic operations (IAO) summarize as follows [6, 7].

A real closed interval $[m]$, for computing on subsets of $\mathbb{R}$. The closed intervals of $\mathbb{R}$ are denoted by $\mathbb{IR}$. It is defined by its lower bound m^- and its upper bound m^+. For simplicity, we say that $[m] = [m^-, m^+]$, we have $m^- \le m \le m^+$. Let the two intervals $[m] = [m^-, m^+]$ and $[n] = [n^-, n^+]$ the arithmetic rules are:

$$[m] + [n] = \left[m^- + n^-, m^+ + n^+\right] \tag{1.6}$$

$$[m] - [n] = \left[m^- - n^+, m^+ - n^-\right] \tag{1.7}$$

$$\begin{aligned}[m] \times [n] = [&\mathrm{Min}\left\{m^-n^-, m^-n^+, m^+n^-, m^+n^+\right\}, \\ &\mathrm{Max}\left\{m^-n^-, m^-n^+, m^+n^-, m^+n^+\right\}]\end{aligned} \tag{1.8}$$

If we define,

$$\begin{aligned}1/[n] &= \phi \mathrm{if}\, [n] = [0,0]\,, \\ &= [1/n^+, 1/n^-]\, \mathrm{if} 0 \notin [n]\,, \\ &= [1/n^+, \infty]\, \mathrm{if} n^- = 0 \mathrm{and} n^+ > 0, \\ &= [-\infty, 1/n^-]\, \mathrm{if} n^- < 0 \mathrm{and} n^+ = 0, \\ &= [-\infty, \infty]\, \mathrm{if} n^- < 0 \mathrm{and} n^+ > 0.\end{aligned}$$

then

$$[m]/[n] = [m] \times (1/[n]) \tag{1.9}$$

A non-empty interval $[m]$ and a real number δ, then the interval is

$$\begin{aligned}\delta\,[m] &= [\delta m^-, \delta m^+]\, \mathrm{if} \delta \geq 0 \\ &= [\delta m^+, \delta m^-]\, \mathrm{if} \delta < 0\end{aligned} \tag{1.10}$$

The properties of the basic operators for intervals differs from their properties in $\mathbb{R}$. For instance, $[m] - [m]$ is generally not equal to $[0, 0]$. This is because, $[m] - [m] = \{m - n\, |m \in [m]\,, n \in [m]\}$, rather then $\{m - m | m \in [m]\}$. If $m^- = m^+ = m$, i.e. if $[m]$ consists only of the element m , then we identify the real number m with the degenerate interval $[m, m]$ keeping the real notation, i.e., $m \equiv [m, m]$. We have $[m] > [n]$ if $m^- > n^+$. The real interval $[m]$ is positive if $m^- > 0$. The interval numbers $[0, 0] = 0$ and $[1, 1] = 1$ perform as additive and multiplicative identities, respectively.

The addition and multiplication of arithmetic interval is remain associative and commutative, but multiplication is no longer distributive with respect to addition. Instead a property known as *subdistributivity* holds for:

$$[m] \times ([n] + [z]) \subset [m] \times [n] + [m] \times [z]$$

Further, the cancelation holds for both addition and multiplication.

If $[m] + [n] = [m] + [z]\,\mathrm{then}\,[n] = [z]$

If $[m] \times [n] = [m] \times [z]\,\mathrm{then}\,[n] = [z]\,;0 \notin [m]$

We have the property:

$$[m] \subseteq [n] \Leftrightarrow \delta\,[m] \subseteq \delta\,[n] \tag{1.11}$$

where δ is real point number.

1.3.4 Kharitonov's Theorem

Let us consider an interval polynomial of the form,

$$p\,(s,x) = \sum_{i=0}^{n} \left[x_i^-, x_i^+\right] s^i \tag{1.12}$$

Where $\left[x_i^-, x_i^+\right]$ represents the lower and upper bound interval for the i^{th} component of uncertainty x_i, In order to describe Kharitonov's theorem [8, 9] for robust stability, we first define four fixed Kharitonv polynomials associated with an interval polynomial family eq. (1.12).

$$\begin{aligned}
K^{--}\,(s) &= x_0^- + x_1^- s + x_2^+ s^2 + x_3^+ s^3 + x_4^- s^4 + ... \\
K^{-+}\,(s) &= x_0^- + x_1^+ s + x_2^+ s^2 + x_3^- s^3 + x_4^- s^4 + ... \\
K^{+-}\,(s) &= x_0^+ + x_1^- s + x_2^- s^2 + x_3^+ s^3 + x_4^+ s^4 + ... \\
K^{++}\,(s) &= x_0^+ + x_1^+ s + x_2^- s^2 + x_3^- s^3 + x_4^+ s^4 + ..
\end{aligned} \tag{1.13}$$

According to Kharitonov's theorem, it is sufficient to test the above four Kharitonov polynomials to guarantee the robust stability of the given interval polynomial in order to guarantee robust stability of the given interval polynomial eq. (1.12), This can be achieved by using an algebraic stability criterion, e.g., the Routh-Hurwitz criterion.

The stability test for interval polynomials of degree:

$$\begin{aligned}
&K^{+-}\,(s)\,, K^{++}\,(s)\,, K^{-+}\,(s)\,, K^{--}\,(s) && \mathrm{For} \quad n > 5 \\
&K^{+-}\,(s)\,, K^{++}\,(s)\,, K^{-+}\,(s) && \mathrm{For} \quad n = 5 \\
&K^{+-}\,(s)\,, K^{++}\,(s) && \mathrm{For} \quad n = 4 \\
&K^{+-}\,(s) && \mathrm{For} \quad n = 3
\end{aligned} \tag{1.14}$$

For $n = 1$ and 2, the necessary and sufficient condition for stability is positive lower bounds of the interval coefficients.

1.3.5 Sixteen Plant Theorem

Let us consider an Interval plant transfer function as given below,

$$G(s,a,b) = \frac{N(s,a)}{D(s,b)} = \frac{\sum_{i=0}^{m} \left[a_i^-, a_i^+\right] s^i}{s^n + \sum_{j=0}^{n-1} \left[b_j^-, b_j^+\right] s^j}, \quad m < n \tag{1.15}$$

Where $\left[a_i^-, a_i^+\right] (0 \le i \le m)$ and $\left[b_j^-, b_j^+\right] (0 \le j \le n-1)$ are interval coefficients of numerator and denominator polynomials respectively.

According to Kharitonov's theorem, the interval transfer function eq. (1.15) may represents into four numerator and denominator fixed coefficient Kharitonov polynomials as follows,

For Numerator $N(s,a)$,

$$\left.\begin{aligned} N^1(s) &= a_0^- + a_1^- s + a_2^+ s^2 + a_3^+ s^3 + a_4^- s^4 + \cdots \\ N^2(s) &= a_0^- + a_1^+ s + a_2^+ s^2 + a_3^- s^3 + a_4^- s^4 + \cdots \\ N^3(s) &= a_0^+ + a_1^- s + a_2^- s^2 + a_3^+ s^3 + a_4^+ s^4 + \cdots \\ N^4(s) &= a_0^+ + a_1^+ s + a_2^- s^2 + a_3^- s^3 + a_4^+ s^4 + \cdots \end{aligned}\right\} \tag{1.16}$$

and for Denominator $D(s,b)$,

$$\left.\begin{aligned} D^1(s) &= b_0^- + b_1^- s + b_2^+ s^2 + b_3^+ s^3 + b_4^- s^4 + \cdots \\ D^2(s) &= b_0^- + b_1^+ s + b_2^+ s^2 + b_3^- s^3 + b_4^- s^4 + \cdots \\ D^3(s) &= b_0^+ + b_1^- s + b_2^- s^2 + b_3^+ s^3 + b_4^+ s^4 + \cdots \\ D^4(s) &= b_0^+ + b_1^+ s + b_2^- s^2 + b_3^- s^3 + b_4^+ s^4 + \cdots \end{aligned}\right\} \tag{1.17}$$

By taking all the combinations of the $N^i(s), i = 1,2,3,4$ and $D^k(s), k = 1,2,3,4$, we obtain the 16 Kharitonov plants [9, 10] as,

$$G^{ik}(s) = \frac{N^i(s)}{D^k(s)} \tag{1.18}$$

for $i.k = 1,2,3,4.$

1.4 OBJECTIVE OF THE THESIS

The objective of this thesis is to first critically examine some of the existing model order reduction methods and to develop some new methods in conventional and interval approach of model order reduction which are applicable to linear time invariant continuous and discrete time systems. It has been observed that there are some methods of order reduction in the literature having some drawbacks. These drawbacks have been rectified and the modified version of these methods has been presented. Secondly, to design a controller for the reduced system, obtained from the proposed methods. The controller has been designed using both direct and indirect approaches to check its suitability for the original system.

1.5 ORGANIZATION OF THE THESIS

The present thesis is organized into seven chapters and the work included in each chapter is presented in the following sequence:

Chapter-1, the current chapter, gives an overview on modelling of large scale systems and role of model order reduction. Further, it presents mathematical preliminaries related to model order reduction and concepts of interval systems. Finally, outlines the organization of the present thesis.

Chapter-2, presents, the brief literature review on various existing reduction techniques in frequency domain, time domain and interval domain and their associated qualities/ drawbacks.

Chapter-3 deals with the reduction of higher order linear time invariant systems using proposed composite model reduction techniques. Some new combined order reduction methods have been proposed by using frequency domain reduction methods for linear dynamical systems. In these methods the reduced order denominator coefficients are determined by one method and the reduced order numerator coefficients are obtained by another method. This approach guarantees the stability of ROM if the original higher-order system is stable. In the proposed methods, the time moment matching method and differentiation methods have been used to obtain reduced order denominator polynomial while the Pade approximation and factor division algorithm have been used to determine the reduced order numerator polynomials. The results obtained from the proposed reduction approaches are compared with well-known reduction methods. Further, the application of these approaches are also extended to linear multivariable systems also. The efficacy of the presented techniques are justified by comparison of time and frequency responses and their associated performance indices.

Chapter -4 presents the MOR of continuous time interval systems. The new mixed methods are proposed by using differentiation method, factor division algorithm, and Pade approximation method based on interval arithmetic operations. Further, the reduced order models are obtained by using frequency domain reduction methods differentiation method, stability equation method and time moment matching method these are being utilized along with Kharitonov's theorem. The obtained proposed model results are validated by comparing with some other well-known reduction methods and recently published work in terms of performance indices, step and bode responses.

Chapter -5 deals the MOR of the discrete time systems by extending the methods proposed in the previous chapters. The reduced order models are obtained by using linear transformation. The results are compared with recently published work in terms of performance indices, step and impulse responses.

Chapter -6, the controller has been designed to ensure the suitability of the proposed model order reduction methods. For controller design both the direct and indirect approaches have been considered. Different examples are given to illustrate the methods. The unit step response of closed loop transfer function obtained from the original and reduced plant transfer function are compared with the unit step response of the reference model.

Chapter -7 concludes the work presented in this thesis along with the scope for the future work.

CHAPTER 2

LITERATURE SURVEY

2.1 INTRODUCTION

An ample variety of order reduction methods have been presented by several authors since four decades. An extensive bibliography on the research area of model order reduction can be seen in papers [11–14] and also some text books [15–19] have been written on this area. The reduction methods are broadly classified as time domain and frequency domain reduction methods. The reduced order models obtained by individual techniques which are different from others; however, the quality of reduced model is ultimately judged by the way it is utilized.

2.2 FREQUENCY DOMAIN ORDER REDUCTION METHODS

The frequency domain reduction techniques are categorized in the following three groups:

2.2.1 Classical Approach

In this approach, reduction methods are based on the classical theories of pure mathematics and algebraic in nature such as Pade approximation method [20], time moment matching method [21] and continued fraction expansion method [22] etc. The major drawback of reduction methods of this group is that the stable original model may results in unstable reduced order model and vice versa [23–31]. Another problem is that sometimes reduced order model, provides low accuracy in the mid and high frequency range and exhibits non minimum phase behavior. Although there are number of methods suggested by the authors which are available in the literature but few vital methods are discussed as follows:

2.2.1.1 Pade Approximation Method

This method was originally put forward by Pade [20]. This method is simple from the computational point of view, it fits steady state value and initial time moments of the ROM and the original model matches. To obtain r^{th} order reduction method, $2r-1$ coefficients of the power series expansion about $s=0$ of ROM are matched with the corresponding coefficients of the higher order system. The drawback of this approximation technique is that, sometimes the reduced models may be unstable (stable) even though the original higher order system is stable (unstable). Also, this method may approximate non-dominant poles sometimes, thus giving bad approximation. To overcome these disadvantages, several substitute reduction methods have been implemented. An improvement in the Routh-Pade method is suggested by Wilson et al. [32] where the stability issue of the Pade approximation is dealt by Bandyopadhyay et al. [33, 34]. Furthermore, the method has also been extended to multivariable systems by Bandyopadhyay et al. [35]. A method has been further introduced by Shamash [36] based on retention of poles of high order system in ROM and concept of Pade approximation about more than one point. Bistritz [37] presented a mixed method for closed loop design and put forward the concept of minimal Pade model reduction using second Cauer form of continued fraction expansion. Bandyopadhyay [33, 38] gave method of stability Pade approximation. Chen and Chang [39] proposed a mixed method by combining the Pade approximation method and stability equation method for obtaining stable ROM. Also, the Pade approximation method has been improved by Wan [40] with the help of Mihailov stability criteria. In [41, 42], the original single point Pade approximation about $s=0$ has been extended to multi point Pade approximation. Xiheng [42] presented a method and solved the Pade equations for expansion about $s=0$ and also about the points along the imaginary axis $s=jw$. Lepschy and Viaro [43] presented a reduction method to guarantee the stability of reduced order models using Routh-Pade approximation. Krajewski et al. [44] proposed a reduction method to deal with continuous time MIMO systems by matching first and second order information. Later Krajewski et al. [45] proposed approximation method based on Markov and energy indices method. Lepschy and Viaro [46] presented a simplification of transfer functions using modified Pade-type method. Lucas [47] modified it to multipoint Pade approximants, where the expansion points can be mixture of multiple real, complex and purely imaginary points. In [48], further the work was extended to expansion points at infinity and thus method became generalized in nature. Aguirre [49, 50] introduced a least square Pade method as a novel method of model order reduction and also the Pade approximation method has been used by Lam [51] for time delay systems. Next, the Pade approximation method has also used by Daly and Colebourn [52] for linear systems in state space form. Prasad [53] and Prasad et al. [54] presented the methods of model order reduction of LTI multivariable systems, which provided reduced order systems in state space form irrespective of whether the original system is available in state space form or in transfer function matrix form. The Pade approximation method has also been extended to the model order reduction of the discrete time systems by Hwang and Chow [55], Prasad and Devi [56].

2.2.1.2 Continued Fraction Expansion (CFE) Method

This method has been introduced by Chen and Shieh [22] for getting reduced order model of linear SISO system. Initially, their is no requirement to get the knowledge about Eigen vectors or Eigen values and dominant properties of the original HOS. This method contains lots of helpful properties like computational ease, preservation of steady state responses and fitting of time moments in reduced order models. The convergence is also fast in this method. This method has proved so far to be a special case of Pade approximation, for asymptotically stable state system which is identical to the time moment matching method [23]. The disadvantage of this technique is that the stability is not assured even though the given higher order system is stable and it may not give good transient response matching. Further, to avoid these disadvantages, a modified CFE method has been proposed by Chuang [25] combined the expansions about $s = 0$ and $s = \infty$ alternatively to improve the initial transient response of the ROMs at later times. This modified reduction method has been named to as the modified Cauer continued fraction. Continued fraction expansion method has further extended as first Cauer form, second Cauer form and modified Cauer form. Later by combining the first and second Cauer methods the third Cauer form has been proposed by Hwang [27]. In methods based on CFE, the given transfer function is expanded into a particular kind of continued fraction and truncated after few terms. The transfer function of ROM is determined by inverting the truncated CFE.

Various extensions and modifications have been carried out by many authors. CFE about a general point has been presented by Davidson and Lucas [26], while extension of CFE to MIMO systems has been proposed by Chen [24]. Khatwani et al. [30] suggested an algorithm for obtaining the ROM of LTI systems from its state space model directly, without calculating corresponding transfer function. Later this method has been mixed with other methods for model order reduction by authors like John and Parthasarathy [57] combined it along with Routh approximation and Chen et al. [58] combined it with stability equation method. Recently, Sambariya and Gupta [59] proposed new modified Cauer form technique which overcome the drawbacks of Cauer form by guaranteeing the stable reduced order models.

2.2.1.3 Time Moment Matching Method

This reduction approach of moment matching was first put forward by Paynter and Takahashi [21]. In this technique, the reduced order model is achieved by matching few lower order moments of the given HOS. The matching of initial time moments gives better approximation at low frequencies while matching the initial Markov parameters leads to good approximation at high frequencies. This method preserves the low frequency response of the original system. The main drawback of this method is that there is no guarantee of obtaining stable ROM even though the higher order model is stable and also transient performance of ROM may not always be satisfactory. Further, there exists a computational difficulty in this technique if large numbers of constants are to be evaluated in ROM. To overcome these computational difficulties, Lal and Mitra [60] proposed a computer oriented algorithm for evaluation of the time moments.

This method has been used by Gibilaro and lees [61] for SISO system. Further, this technique extended to MIMO system by shih and Shieh [62]. The application of moment matching for the reduction of multi-variable systems has been accomplished by matching the coefficients of power series expansion about $s = 0$ and $s = \infty$, where s is the Laplace transform variable. Taiwo and Krebs [63] proves the moment matching technique is also suited to non-square continuous and discrete systems. The moment matching techniques has been used for model order reduction of multi rate linear system by Williamson et. al. [64]. Hwang and Shih [65] modified this method for discrete systems. Hwang and Shih [65] modifies this method for discrete systems. Hickin and Sinha [66] proposed a combination of aggregation and moment matching for multivariable systems. Feng et al. [67] proposed a model order reduction scheme in adaptive sense based on moment matching. Scarciotti and Astolfi [68] presented a reduction technique by estimating the moments of linear and nonlinear unknown parameters of the reduced model using time moments. Vasu et al. [69] presented a reduction method to preserve the stability of the ROMs by matching Markov parameters and time moments by minimizing the error. Sinha et al. [70] has proposed a mixed method by combining Routh approximation and moment matching method. Kumar et al., [71] proposed optimal multilevel Krylov model order reduction technique to improve the finite element band width by using moment matching criterion. Krajewski et. al. [72] obtained reduced order model by matching time moments and impulse response energies.

2.2.1.4 Error Minimization Method

In this method, the time response comparison of original system and the ROM gives the error function. Different error minimization criteria are ISE, integral absolute error (IAE), integral time absolute error (ITAE) and integral time square error (ITSE) are most frequently used criteria. The basic approach of most of the methods is to minimize error between the unit responses of the original HOS and ROM using ISE.

Mishra and Wilson [73] used this method for model order reduction. Hwang and Wang [74] combined Routh method with error minimization technique and applied it to SISO system. In this approach they determined the denominator by Routh method and numerator by error minimization technique. Mukherjee and Mishra [75, 76] combined the dominant pole retention method and error minimization technique and applied it to both SISO and MIMO system. Lamba et al. [77] minimized time domain error function. In this approach the numerator is obtained by minimizing the step response error with a steady state constraint and then converting the error function into the frequency domain for minima operation. Howitt and Luus [78] introduced model order reduction for SISO LTI systems, in which both poles and zeros are taken as free parameters to minimize the integral square error in impulse and step responses. Puri and Lan [79] introduced a stable MOR technique which was based on minimization of the impulse response error and stability equation approach along with Pade approximation. Hwang et al. [80] proposed a combined time and frequency domain method to reduce the order of discrete time systems in z-domain. Puri and Lim [81] also introduced a method for discrete time systems. Reddy [82] obtained the coefficients of the reduced order model by minimizing the integral squares of the error between the corresponding real and imaginary part of original and reduced order model. Ouyang et al. [83] presented a combined method for linear system order reduction, in which the denominator was obtained by retaining the poles of large dispersion based on the concept of power decomposition and numerator's parameters were obtained by using frequency matching technique, Method of model order reduction for discrete time systems via frequency response matching was proposed by Sahani and Nagar [84].

2.2.1.5 Truncation Method

This technique was proposed by Gustafson [85]. In this technique, the higher order numerator and denominator terms are truncated to produce the ROM. This method is very simple from computational point of view. Shamash [86] extended this method for multivariable systems and by comparing this method with Pade and Routh approximation method, and concluded that truncation method is as reliable as these methods. Yeung [87] proved that the reduced system is stable if the poles of the original system are well damped. Prasad et al. [88] gave the modified form of truncation method.

2.2.2 Stability Preserving Approach

Stability is one of the most important parameter of any system which would never be sacrificed therefore the most imperative group of reduced order modelling is based on stability preserving approach. In this approach the obtained reduced order model is always stable. However, the main drawback of the methods under this group is flexibility when the approximation produced by the reduced model is not good [89–92]. The number of methods available in the literature based on stability preserving approach; some of them are as follows:

2.2.2.1 Differentiation Method

This technique was presented by Gutman et al. [93]. The original system numerator and denominator polynomial are first reciprocated and then differentiated many times to yield the reduced order model coefficients. These reduced order model is reciprocated back and normalized to obtain required ROM. This technique is quite simple and is also applicable to both non minimum phase and unstable systems. The main disadvantage of this technique is that, the original and ROM steady state response may not match. Lucas [94] proved that this method [93] gives better approximation in reduced order numerator and denominator polynomials by giving equivalent successive ratios of multipoints. This helps the method to get easy computation of the ROM by using formulation of the Routh array structure. Lepschy and Viaro [95] mixed this method with Pade method to combine the advantages of simple calculation and stability preservation. Manohar and Sambariya [96] proposed a reduction technique for multivariable systems using differentiation method. Further, Pal and Prasad [97] combined this method with continued fraction expansion approach.

2.2.2.2 Dominant Pole Retention Method

This technique was introduced by Davison [98]. This method provides stable reduced order models if the original system, matrix diagonalization, linear transformation, computation of Eigen values and Eigen vectors are involved in the process of reduction, which are computationally difficult and does not work when the system has widely separated Eigen values. Using this method, various authors have developed their own model order reduction techniques for continuous and discrete time systems in frequency domain. The dominant poles, nearer to imaginary axis, are retained in ROM and poles which are far away from imaginary axis or insignificant poles are neglected because their effect is comparatively less on the overall performance of the system. The disadvantage of this technique is that it is difficult to distinguish which pole is more dominant if many poles are very near to the imaginary axis. This method was further extended to clustering technique [99]. Argoun [100] presented model order reduction technique which is applicable to diagonally dominant systems.

2.2.2.3 Routh Approximation Method

Hutton and Friedland [90] have introduced this technique for reducing the order of HOS to the ROMs. The reduced order denominator and numerator coefficients are obtained by reciprocating the original system and by making $\alpha - \beta$ canonical forms. α table is constructed by using denominator coefficients of original system and β table is constructed by using numerator coefficients of original system in this process elements of α table and successive elements of β table are also used. There is no need of determining Eigen values in this method. Further, this technique preserves system stability and the steady state value matches. It involves simple algebraic calculations of finite number of steps. The drawback of this method is that it has to proceed through reciprocal transformation twice before ROM is obtained. To overcome this drawback, Krishnamurthy and Seshadri [91] proposed a method in which the reciprocal transformations are avoided and named as direct Routh approximation method. But, another drawback of this method is that it may sometimes approximate non dominant poles of the system [101].

This method was extended by Rao et al. [102] to simplify unstable systems. Stable biased ROM was given by Shamash [103]. Sastry and Krishnamurthy [104] modified this method to determine ROM directly in state space form. For stable reduced order models, multi frequency Routh approximation procedure was given by Hwang et al. [105]. This technique was used by Bandyopadhyay et al. [106, 107] and, Sastry and Mallikarjuna Rao [108] for reducing interval systems. Dolgin and Zeheb [109] combined Routh approximation method with Pade method for order reduction of linear discrete stable system in z-transfer function while Choo [110], Hwang and Hsieh [111] extended this method to discrete time system via bilinear transformation. Recently, Narwal and Prasad [112] has proposed a method in which this method has been combined with evolutionary algorithm named Cuckoo Search.

2.2.2.4 Routh Hurwitz Array Method

The Routh Hurwitz array method [92] is proposed to obtain reduced order models by constructing Routh array using original system numerator and denominator polynomials. a reduced polynomial of $(n-1)^{th}$ order can be composed by considering second and third rows of the Routh array for n^{th} order denominator polynomial. Likely for $(n-2)^{th}$ order system, the third and fourth row of the array are utilized.The procedure is repeated in the same way for reducing the numerator polynomial. A stable ROM is obtained from a stable full order model, but drawback is that it is a non-unique procedure in which many other high order models may have the same reduced model. This shortcoming was noticed by Shamash [101, 103]. Rao et al. [102] mixed this method with Routh approximation for simplification of multivariable systems and also for suboptimal control [113].This method has also extended to the reduction of discrete time systems [114–116]. Further, a simple proof of the Routh test was proposed by Ferrante et al. [117].

2.2.2.5 Factor Division Method

Lucas [118] first introduced tis method . This method preserves the initial time moments and also it retains the dominant properties of the original system in the reduced order model. This method does not involve in the calculation of the time moments [36, 119]. Lucas [120] extended this method to produce a biased reduced order models by preserving the stability of ROMs, and retaining initial time moments and the Markov parameters of the HOS. The ideas of Lucas [118, 120] were further extended to obtain modified factor division approach [121]. Using this modified approach, stable reduced order model is obtained for a stable system. Modified factor division approach gives families of ROMs of different orders by varying a different parameter in the denominator.

2.2.2.6 Stability Equation Method

Chen et.al. [89] proposed this technique, the ROM is determined by separating the both numerator and denominator into their even and odd parts. Then factored to find the roots, from this the large magnitude terms are neglected to achieve required ROM. This technique preserves the stability of the ROM. Further, this method ensures the steady state response matching time response.

Pal [122] combined this method with Pade approximation technique to overcome the disadvantage of Pade method. Later, Bistritz [123] proposed discrete stability equation method. This technique also used by Therapos [124] for fast oscillating systems. This method was combined with continued fraction method by Chen et al. [58] to prove that this approach may be applied to non-minimum phase systems. A tabular approach to this method was proposed by Lucas [125], which avoids the problem of computing roots of stability equations. Further, Desai and Prasad [126] presented a reduction technique by combining the stability equation method and big bang big crunch optimization technique.

2.2.2.7 Mihailov Stability Criterion

This method was proposed by Wan [40]. In this method reduced order denominator is obtained by using Mihailov criterion and reduced order numerator is determined by using Pade approximation method. This technique does not include the calculation of the Markov parameters and the initial time moments. This technique is computationally easy and guarantees the stability if the ROM if original system is stable. Rawat and Jha [127] presented application oriented reduction method which automatically regulate the voltage using Mihailov criterion. Prasad et al. [128, 129] proposed a method by combining the advantages of factor division method and Mihailov method and also extended it for multivariable systems.

2.2.2.8 Least Square Method

This technique was first proposed by Shoji et al. [130]. The reduced order model is obtained by matching the least squares of the time moments of the HOS. The advantage of this technique is that, it gives an extra degree of freedom in the reduced order model design. Further, Lucas and Beat [131] proposed a reduction method by modifying of this method. Later, Aguirre [49] extended this method to include the use of Markov parameters. Smith and Lucas [132] obtained the numerator coefficients by exact moment matching and denominator coefficients by least square method. In [133], Aguirre developed an algorithm in which numerator coefficients of ROM were determined by least square method while the denominator coefficients were method while the denominator coefficients were method while the denominator coefficients were method while the denominator coefficients were method while the denominator coefficients were already determined by some method.

2.2.2.9 Eigen Spectrum Analysis

Initially this method was introduced by Mukherjee [134]. In this method, the original system and the ROM system stiffness and the pole centroid are remain same. Also, the response matching of the proposed reduced models with original system are quite good but sometimes due to same stiffness the ROM may turn out to be non minimum phase. Further, this method was used in combination of Cauer second form [135] and factor division algorithm [136].

2.2.2.10 Clustering Technique

This method proposed by Sinha and Pal [99]. This method is an extension of the dominant pole retention method [98]. In this method the cluster centers are obtained by forming the clusters of the poles and zeros of the original system by using inverse distance measure (IDM) criterion. This method is computationally simple and preserves the stability of the reduced order model if the original HOS is stable. Further, this technique is equally applicable for unstable systems. Later, Chen et al. [22] has mixed this technique with the time moment matching method. In which the reduced order denominator is determined by the pole clustering method and the reduced order numerator coefficients are determined by time moment matching method.

2.2.2.11 Modified Pole Clustering Technique

Vishwakarma [137] and Komarasamy et al. [138] proposed a modified pole clustering technique. These techniques generate much effective cluster centers than the [99, 139]. In this technique only reduced order denominators are determined by clustering technique and the numerator of ROM is determined by using any other existing technique. These modified pole clustering techniques result more dominant cluster centers and therefore, produce better ROM. Recently, Narwal and Prasad [140] have proposed a method by improving the clustering technique, which produce the better results than the methods [99, 137–139] and also, the proposed method has been extended to order reduction of discrete time systems [140].

2.2.3 Composite Reduction Approach

The above discussed classical methods and stability preservation methods have some drawbacks. To avoid these drawbacks composite or mixed methods were implemented. This approach considers the advantages of both classical approach and the stability preservation approach to obtain stable reduced order models. The reduced order denominator coefficients are achieved by any method under stability preservation approach and the reduced order numerator polynomials are obtained by any method from classical approach. Some of the mixed methods are [40, 57, 58, 103, 122, 128, 141–145].

2.3 TIME DOMAIN ORDER REDUCTION TECHNIQUES

Some well-known time domain order reduction techniques are briefly reviewed below:

2.3.1 Modal Analysis

This method retains the dominant Eigen values of the original system in the reduced order models. The response of ROM and the high order response to a given input are approximately close. The methods developed by Davison [98], Marshall [146] and

Aoki [147] all fall in this category. The first three methods considered as particular case of aggregation method given by Aoki. The Method proposed by Davison involves diagonalization of system matrix and neglecting non-dominant Eigen values. Here, all Eigen values are assumed to be distinct and the step function is taken as the input. Aoki gave a more generalized approach based on aggregation. Gruca and Bertrand [148] minimized the quality index function of the output vector by introducing a delay in the output vector of the aggregated model. This approach led to the improvement in the equality of simplified aggregated model of the system with no increase in the order of state differential equation. Inooka and Obinata [149] developed a technique based on combining the aggregation method and the integral square criterion.

2.3.2 Aggregation Method

This method was introduced by Aoki [147], in this the ROM is obtained by aggregating the HOS state vector into the lower order vector. This technique is the most general projective technique for MOR. Aoki also designed suboptimal controller by using aggregation matrices. Hickin and Sinha [150] verified this method by proving the aggregation method is a generalized method and also this method is compared with singular perturbation method [151]. Hwang [152] invented a direct method to obtain the aggregation matrix for ROM. In this, the modified CFE method is used. This method retains the important properties (Eigen values) of the HOS in its ROM, which are useful in deriving the state feedback suboptimal control and analyzing system.

2.3.3 Hankel Norm Approximation

Adamjan et al. [153] proposed this method. Later this method extended by Kung and Lin [154] to multivariable system. Keith [155] derived the frequency response error bounds by characterizing all the optional solutions of Hankel norm approximation of multivariable system. A program to solve L_2 reduced order problem with fixed denominator degree was given by Krajewski et al. [31]. Convergent algorithm for L_2 model order reduction was proposed by Ferrante et al. [156]. Zhou [157] introduced a new frequency weighted L_∞ norm and optimal Hankel norm model order reduction technique. The problem of computing an H_2 optimal lower order model was dealt by Yan and Lam [158]. Gao et al. [159] analyzed the problem of L_∞ model reduction for linear discrete time systems with delay. Model reduction problem for singular system was investigated by Wang et al. [160]. The main advantage of this method is that, by using full order system's Hankel singular values [157] the priori additive approximation error can be obtained.

This method comes under the class of additive error MOR methods and this method guarantees the stability of the error bounds of the ROMs. Hankel norm of a stable SISO system lies between the more conventional L_2 and L_∞ norms [154].This method was further extended to include some classes of frequency weights [161] for scalar system. The presence of weights usually reflects the desire that the approximation be more accurate at particular frequencies. This approach was further extended to MIMO system [162].

2.3.4 Singular Perturbation Method

Kokotovic et al. [163] proposed this technique. This method is useful for separating the time scales in controller design of a reduced order model. This method is used for reducing the HOS having two scale property. Two scale properties are those in which the Eigen values are separated into two groups, called 'fast' and slow modes. The reduced model is obtained by neglecting the rapid phenomena and then improving the approximation by re-introducing their effect as boundary layer. Fernando and Nicholson [164] further examined this approach and concluded that this method is capable of being used with the balanced realization. In [165] Fernando et al. introduced singular perturbation approximation for discrete and continuous time system in the vicinity of negative real axis in complex plane. This method retains the dominant Eigen values of original system, but non availability of slow and fast subsystems, limits its use on a general large scale system. Later, Hote and Jha [166] presented new reduction method using Gerschgorin theorem and singular perturbation technique, this method identify the position of dominant and non-dominant eigen values.

2.3.5 Balanced Realization Approach

This method was proposed by Moore [167], this method is based on similar diagonalization of observability diagonalization of observability grammian (W_o) and controllability grammian (W_c) matrices using the appropriate similarity transformations. W_o and W_c are symmetric semi definite matrices which are derived by solving the Lyapunov equations and are used to define measures of controllability and observability. In this method, a ROM is determined by ignoring the insignificant states whose contribution is negligible and by considering the dominant dominant states of the controllable and observable parts of the system, is an way of obtaining reduced order model.

Pernebo and Silverman [168] extended this work. Samar et. al. [169] explained the retention of DC gain of balanced truncated model for minimal systems. Lam [170] used this method for realization of the Pade approximants. Yang et al. [171] used balanced realization for unstable systems while Sandberg and Rontzer [172] used balanced realization for linear time varying systems. Lastman and Sinha [173] compared aggregation method with balanced truncation method. Agathoklis and Sreeram [174] proposed a frequency domain model order reduction method which was based on impulse response grammian for reducing the linear continuous system. Al-Saggaf and Franklin [175] introduced a method based on frequency weighing technique for discrete and continuous time systems. Al-Saggaf [176] proposed a method for model order reduction of unstable systems based on generalized normal representations. Prakash and Rao [177] proposed an improved balancing model order reduction method where lower order model is obtained by the approximating the states of weak subsystem around zero frequency. M. Ha et al. [178] presented a reduction method by comparing balanced truncation and modal truncation techniques for linear state-space symmetric systems. Therapos [179] gave an approach for obtaining an internally balanced state space representation of discrete SISO system while Gugercin and Antoulas [180] gave a survey of model order reduction by balanced truncation. Yousefi and Lohmann [181] presented a method for model order reduction of nonlinear time invariant high order systems. Krajewski et al. [182] developed a method to match the properties of reduced order model with original system through retention of first and second order information. Ghosh and Senroy [183] extended the balanced truncation technique to reduce the dynamics of the power system. Meyer [184] further extended this approach to fractional balanced reduction. Perev and Shafai [185] used the method of balanced realization for lower order reduction of singular systems. An algorithmic approach for controller reduction using balanced realization has been given in [186] and for system decomposition and balanced realized model has been given in [187]. Nagar and Singh [188] gave a twostep method for reducing discrete time system. Krajewski et al. [189] proposed a reduction method to reproduce the asymptotic response. Kenny and Hewer [190] proposed a reduction method for balancing unstable multivariable systems by developing necessary and sufficient condition. Therapos [191] proposed an algorithm for unstable non-minimal linear systems using Low frequency approximation of balancing approach for unstable systems [192].

2.3.6 Minimal Realization Algorithm

This method deals with the construction of state variable model from a given transfer functions. Various authors have represented the construction of state variable model from a given transfer functions. Various authors have represented the minimal realization method using Hankel matrices. Ho and Kalman [193] gave an algorithm for the effective construction of minimal realization of a linear system. In this algorithm, non-minimal realization is obtained in the form of block companion matrix using Hankel matrix and then reduce it to make both observable and controllable. Tether [194] proposed a method in which few initial Markov parameters of the original system are retained. Only the initial transient response of the system is approximated through this model. Lal and Singh [195] proposed method for minimal realization of linear time varying systems. Kumar et al. [196] proposed a generalized reduction method using Gramian technique. Therapos [197] presented a technique for the computation of an internally balanced minimal realization of given stable SISO transfer function. Rozsa and Sinha [198] proposed an efficient algorithm for obtaining the minimal order realization of a given rational transfer function matrix. The algorithm was further extended in [199] for obtaining minimal realization of transfer function matrix in a canonical form. Shamash [200] extended this method for multivariable systems. Hickin and Sinha [201] introduced a new approach which was based on the generation of successive partial realization of high order multivariable system in state space form. A comparative study of different minimal realization techniques is available in [202].

2.3.7 Optimal Order Reduction

This methods is based on determining a reduced model of specified order such that its response matches with the original HOS response in an optimum manner and also this method has no restriction on the location of the Eigen values. In this method, the selected performance criterion (which is the error difference between the original and reduced order model response) is minimized. Some numerical algorithms or the necessary conditions of optimality are used to obtain the parameters of ROM. Anderson [203] introduced a geometric method, which is obtained based on orthogonal projection, from this, the ROM is achieved by minimizing the error in time domain. Sinha and Pille [204] introduced a method in which matrix pseudo inverse is used for a least square fit with the sample of response. Sinha and Bereznai [205] suggested a method of model order reduction using the pattern search method given by Hooke and Jeeves [206]. Bandler et al. [207] gave the method of optimal order reduction using gradient method, which take less computational time but evaluation of gradient of objective function is involved. Optional order reduction was developed by Wilson and Mishra [208], in which the approximations have been studied for unit step and impulse responses. Krajewski et al. [209] proposed a L_2-otimal pole retention method to obtain stable reduced order models. Fortuna and Muscato [210] presented a model reduction technique by using balanced gains and an optimal weighted L_2- norm criterion. Langholz and Bistrltz [211], Elliott and

Wolovich [212] proposed methods for obtaining the ROM in frequency domain. This method has been explained in [213]. Further, the applications of the reduced order models are proposed by Nabi [214–217] in the area of nonlinear systems, SVD and Krylov-Subspace.

2.4 INTERVAL DOMAIN ORDER REDUCTION TECHNIQUES

The above discussed reduction methods are developed for fixed coefficient transfer functions or state space models. It is fact that designing a controller based on fixed coefficient transfer function or state space model is often unrealistic because the practical system parameters vary within certain interval. The model reduction techniques and design of interval systems given in [218]- [255] which have received a great deal of attention. Bandyopadhyay et. al. [218] extended the fixed parameter reduction methods to deal with interval systems. In this, the reduction method is found by using Routh-Pade approximation technique to deal with interval systems. The reduced order denominator polynomial is achieved by Routh approximation, and the lower-order numerator coefficients are found by the power series expansions of the interval systems. Later the concept of γ-δ Routh approximation has been extended to continuous interval systems by Bandyopadhyay et al. [219]. The following are the limitations of above two Routh based approximations claimed by Hwang and Yang [220]: (1) Interval Routh extension formula may not provide the successes in obtaining a full interval Routh array. (2) Sometimes the interval Routh approximation method may give unstable ROMs, even if the higher order interval system is stable. To reduce the computational effort, γ table formulation [221] has been introduced, instead of γ-δ table formulation [219]. However, the limitation of this method is that, the obtained ROM may be unstable for the stable higher order interval system. Later, Dolgin and Zehab [222] have proved that generalized Routh approximation for interval systems may give unstable ROM. To overcome this problem, Dolgin and Zehab [222] modified the generalized Routh array and claimed that this method could provide stable ROMs. Later, Yang [223] proved that Dolgin and Zehab [222] method does not guarantee the stability of the reduced order interval system. To overcome this problem, Dolgin [224] has proposed a modified method of Routh algorithm for obtaining stable reduced order models. It is noted that there is a limitation in this method, that the interval arithmetic subtraction rule has been changed to obtain stable reduced order models. To overcome the limitation of the existing methods [218, 219, 222, 224, 225] Bandyopadhyay et al. [226] introduced a new method based on stable gama-delta Routh approximation of interval systems using Kharitonov polynomials, which guarantee the stability of the reduced order systems. However, this method does not require any interval arithmetic rules. Another alternative method has been proposed to overcome the limitation of the gama-delta Routh approximation, which is based on stable Routh Pade approximation [227]. Bandyopadhyay et al. [218] and Shingare [228] extended some fixed model reduction techniques to interval systems. The above methods give us motivation to propose new techniques for reduction of interval systems.

In recent years many researchers are focusing on mixed method techniques of interval systems. Saraswathi et al. [229] proposed a method based on Eigen spectrum analysis for the reduction of denominator coefficients and for numerator reduction Pade approximation is used. The Eigen spectrum method provides the Eigen values of the reduced order interval systems by preserving some of the characteristics such as stiffness and centroid of the higher order interval systems. While the reduced order numerator coefficients are obtained by using Pade approximation, this method preserves some of the time moments and Markov parameters. In this method the Eigen values are achieved by using [230]. Later, Selvaganesan [231] introduced a mixed method. In which the reduced order denominator is obtained by generalized Routh table and reduced order numerator polynomial is obtained by factor division algorithm method and gain factor is used for minimizing the steady state error. The main drawback of this method is that, the generalized Routh table for obtaining reduced order denominator may fails to produce stable ROMs. Recently, Saini and Prasad [232] applied genetic algorithm technique to interval systems but the denominator reduction polynomial is reduced by generalized Routh array, which fails to obtain stable reduced order models, proved by Dolgin abd Zeheb [222]. Later, Yan Zhe et al. [233] extended genetic algorithm for reduction of MIMO interval systems. Further, Potturu and Prasad [234] proposed stable mixed reduction methods by using differentiation method, factor division algorithm and Pade approximation method based on interval arithmetic operations. Recently , many stable reduction methods were developed for linear interval systems using Kharitonov's theorem are presented in [233, 236–239] to obtain stable ROMs.

A number of order reduction methods are developed in the area of continuous interval systems, but very limited number of reduction techniques are extended to the discrete-time interval systems [240–246, 248]. O. Ismail et al. [241] proposed a discrete interval reduction method using Pade approximation and dominant poles. The reduced order denominator is obtained by using retention of dominant poles of the original system, while the numerator is obtained by using Pade approximation method by matching the time-moments. Later Choo et al. [243] proposed a model reduction method for discrete-time interval systems. This method preserves desired real dominant poles by overcoming the stability problems in [241]. Recently, Singh and Chandra [244, 245] proposed a order reduction technique for discrete-time interval systems, in which the reduced order denominator is determined by retaining dominant poles while the numerator coefficients are determined by matching time moments of the original HOS. Sastry et al. [221] proposed a simplified Routh approximation method (SRAM) for order reduction of interval models by preserving the initial time moments of the higher-order interval systems. Papa and Babu [246], proposed model reduction of discrete interval systems by differentiation technique. Choudhary and Nagar [247], proposed gamma-delta approximation for reduction of discrete-time interval systems. Kiran and Sastry [248] applied least square method to deal with discrete interval systems.

Classical control system design techniques are used for fixed plant transfer functions which are well known for engineers. Since last few decades much attention has been devoted to interval systems. The Kharitonov proposed a celebrated theorem which deal with interval systems. This famous Kharitonov theorem [8, 10] provided lot of scope to deal with interval systems and its robust stability. Later these ideas have been extended to frequency response representations such as Bode and Nyquist plots for interval plant transfer functions. Nowadays, much attention given for formulation of P, PI and PID controllers to stabilize an interval plant [249]. Tan and Atherton [250] discussed the robust stability and controller design of uncertain systems with various forms of uncertain polynomial structures. Later, Smagina and Brewer [251] proposed a technique to deal with multivariabe dynamic systems. In which a P, PI regulator has been designed for the uncertain parameters in the state space model. Huang and wang [252] developed a controller to stabilize the four Kharitonov polynomials simultaneously. Here, the controller is designed by searching the non-conservative Kharitonov regions in the controller parameters plane through graphically and systematically. Later, Pujara and Roy [253] proposed technique based on the 'Polytype Algorithm' to compute first order and higher order stabilizing controller for SISO interval systems. Tan et al., [254] proposed a method based on plotting stability boundary locus in the $(K_p,\ K_i)$ plane and then computing the stabilizing values of the parameters of PI controller. The advantage of this technique is that, it does not require to swap over the parameters and also does not need linear programing to solve set of inequalities. Irrespective of stabilization, this method shift all poles to the left half plane, this guarantees the stability of the ROM. Babu and pappa [255] presented a hybrid algorithm using Particle Swarm Bacterial Foraging Optimization (PSO-BFO)to find optimum PID controller parameters K_p, K_i and K_d. The best possible optimum PID controller values are obtained by ISE criterion. In this, the controller parameters are obtained for ROM, after that, HOS has been tested.

The analysis, stability and controller design for parametric uncertain systems largely ignored till 1980's. Due to fact that, there were no general theories which could give information to analyze or design a control systems with uncertain parameters. After that, Kharitonov [8] proposed a famous Kharitonov's theorem to deal with uncertain systems, this theorem filled huge gap in the area of uncertain systems. This Kharitonov's theorem gives robust stability information of the uncertain systems by checking four Kharitonov polynomials. Later, Barmish [257] simplified this method by giving simple proofs of the Kharitonov's theorem for both continuous and discrete interval systems [257]- [274]. An alternative method developed by Anderson et al. [256], shown the simplified way to find the robust stability for interval systems. If the order of the system is more then or equal to five then the four Kharitonov polynomials must be checked other wise not required to check all four polynomials. Later, Hote et al. [270] proved that Anderson et al. [256] method cannot be applicable to relative stability analysis. This method gives the information about the relative stability of gain margin and phase margin without using graphical approach. Finally, graphical approach for investigation of robust stability for discrete interval systems has been developed [271–274].

Different tool box is available for the analysis of interval systems [275–277]. Tan and Atherton [275] developed a software package AISTK for analysis of the interval systems. The INTLAB has been developed to deal with the interval arithmetic operations. This software powerful tool to deal with uncertain systems, this package also developed for MATLAB environment.

CHAPTER 3

NEW COMPOSITE TECHNIQUES FOR REDUCED ORDER MODELLING: CONVENTIONAL SYSTEMS APPROACH

3.1 INTRODUCTION

In preceding chapter, various model order reduction techniques have been discussed in frequency domain. It is observed that, all the reduction methods have their own merits and demerits and can be best applied in a specific situation. A common limitation of some of the model order reduction methods is that, even though the original higher order system is stable the reduced order model turn out to be unstable [19, 20, 25, 26]. The other drawback of the reduction methods is that they have low accuracy in the mid and high frequency ranges and may exhibit non minimum phase characteristics [36, 41]. Therefore, obtaining an approximate ROMs from the original higher order systems is a major challenge in the field of control systems due to various issues such as stability, large in system dimensionality, good time and frequency response matching etc. In this chapter some new mixed reduction methods have been proposed in the frequency domain for the linear continuous-time invariant systems.

3.2 PROBLEM STATEMENT

3.2.1 Single Input Single Output (SISO) Systems

Let us consider an n^{th} order higher-order system transfer function is represented as,

$$G(s) = \frac{N(s)}{D(s)} = \frac{a_0 + a_1 s + a_2 s^2 + \cdots + a_{n-1} s^{n-1}}{b_0 + b_1 s + b_2 s^2 + \cdots + b_n s^n} \tag{3.1}$$

Where $a_0, a_1, \cdots, a_{n-1}$ and $b_0, b_1, \cdots, b_n$ are the known numerator and denominator coefficients of high order system (HOS). Our objective is to compute an k^{th} $(k < n)$ order reduced model transfer function as given below,

$$R(s) = \frac{n(s)}{d(s)} = \frac{e_0 + e_1 s + e_2 s^2 + \cdots + e_{k-1} s^{k-1}}{d_0 + d_1 s + d_2 s^2 + \cdots + d_k s^k} \tag{3.2}$$

Where $e_0, e_1, \cdots, e_{k-1}$ and $d_0, d_1, \cdots, d_k$ are the unknown numerator and denominator coefficients of reduced order model.

3.2.2 Multiple Input Multiple Output (MIMO) System

Consider an n^{th} order transfer matrix represented as

$$[G(s)] = \frac{1}{D_n(s)} \begin{bmatrix} A_{11}(s) & A_{12}(s) & \cdots & A_{1n}(s) \\ A_{21}(s) & A_{22}(s) & \cdots & A_{2n}(s) \\ \vdots & \vdots & \vdots & \vdots \\ A_{m1}(s) & A_{m2}(s) & \cdots & A_{mn}(s) \end{bmatrix} \tag{3.3}$$

Or $[G(s)] = [g_{ij}(s)]$,

where $g_{ij}(s)$ can be written as $[g_{ij}(s)] = [A_{ij}(s)]/D_n(s)$, where $i = 1, 2, \cdots, m\,;\ j = 1, 2, \cdots, n.$

The goal is to obtain lower k^{th} order transfer matrix

$$[R(s)] = \frac{1}{D_k(s)} \begin{bmatrix} E_{11}(s) & E_{12}(s) & \cdots & E_{1p}(s) \\ E_{21}(s) & E_{22}(s) & \cdots & E_{2p}(s) \\ \vdots & \vdots & \vdots & \vdots \\ E_{q1}(s) & E_{q2}(s) & \cdots & E_{qp}(s) \end{bmatrix} \tag{3.4}$$

Or $[R(s)] = [r_{ij}(s)]$

where $r_{ij}(s)$ can be written as $[r_{ij}(s)] = [E_{ij}(s)]/D_k(s)$, where $i = 1, 2, \cdots, q\,;\ j = 1, 2, \cdots, p.$

3.3 MODEL REDUCTION USING MODIFIED TIME MOMENT MATCHING METHOD

A new reduction technique is presented to obtain lower order models from higher-order systems. The presented technique is based on combining the time moment matching (TMM) [21, 60] method and straightforward mathematical technique as explained in the proposed methodology. The lower order denominator coefficients are achieved by TMM technique and the lower order numerator terms are found by straightforward mathematical technique. This technique is computationally simple and provides stable reduced order models. The proposed technique is examined with popular numerical examples and compared with the help of error indices such as ISE, IAE, ITSE and ITAE. The performance indices [75, 278] are defined as follows,

$$\text{ISE} = \int_{0}^{\infty} [y(t) - y_k(t)]^2 dt \tag{3.5}$$

$$\text{IAE} = \int_{0}^{\infty} |y(t) - y_k(t)| \, dt \tag{3.6}$$

$$\text{ITSE} = \int_{0}^{\infty} t[y(t) - y_k(t)]^2 dt \tag{3.7}$$

$$\text{ITAE} = \int_{0}^{\infty} t \, |y(t) - y_k(t)| \, dt \tag{3.8}$$

Where $y(t)$ and $y_k(t)$ are the original and ROM responses respectively.

The k^{th} order ROM is achieved from the n^{th} order original system by following the below procedural steps:

Step 1: Procedure for obtaining lower order denominator,

Step 1.1: Equation (3.9) is obtained by dividing coefficients of numerator and denominators of eq. (3.1) with b_0

$$G(s) = \frac{\frac{a_0}{b_0} + \frac{a_1}{b_0}s + \frac{a_2}{b_0}s^2 + \cdots + \frac{a_{n-1}}{b_0}s^{n-1}}{1 + \frac{b_1}{b_0}s + \frac{b_2}{b_0}s^2 + \cdots + \frac{b_n}{b_0}s^n} \tag{3.9}$$

Equation (3.9) is rewritten as follows,

$$G(s) = \frac{p_{21} + p_{22}s + p_{23}s^2 + \cdots + p_{2n-1}s^{n-1}}{1 + p_{12}s + p_{13}s^2 + \cdots + p_{1n}s^n} \tag{3.10}$$

Step 1.2: Equation (3.10) is arranged in the array form as following

$$\begin{array}{r|cccc} & 1 & p_{12} & p_{13} & \cdots \\ c_0 = & p_{21} & p_{22} & p_{23} & \cdots \\ c_1 = & p_{31} & p_{32} & p_{33} & \cdots \\ c_2 = & p_{41} & p_{42} & p_{43} & \cdots \\ & \vdots & \vdots & \vdots & \end{array} \tag{3.11}$$

The first and second rows of eq. (3.11) are formed from the denominator and numerator coefficients of eq. (3.10) and the remaining rows are constructed by using eq. (3.12),

$$p_{x,y} = p_{x-1,1}p_{1,y+1} - p_{x-1,y+1} \tag{3.12}$$

where $x = 3, 4, 5, \cdots$ and $y = 1, 2, 3, \cdots$

Step 1.3: The time-moments are obtained by expanding $G(s)$ as given below,

$$G(s) = \sum_{i=0}^{\infty} c_i s^i \tag{3.13}$$

where $c_i = (-1)^i p_{j,1} \quad \text{for } j = 2, 3, 4, 5 \cdots$

From the below coefficient matrix eq. (3.14) the initial time moments are achieved for reduced order models

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_m \\ \hline c_{m+1} \\ c_{m+2} \\ \vdots \\ \\ c_{m+k} \end{bmatrix} = \left[\begin{array}{cccc|cccc} 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ -c_0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ -c_1 & -c_0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -c_{m-1} & -c_{m-2} & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \hline -c_m & -c_{m-1} & \cdots & c_0 & \cdots & & & 0 \\ -c_{m+1} & -c_m & \cdots & -c_1 & -c_0 & & \cdots & 0 \\ \vdots & & \ddots & & \vdots & & & \vdots \\ & & & & & -c_0 & 0 & 0 \\ -c_{m+k-1} & -c_{m+k-2} & & \cdots & & -c_1 & -c_0 & 0 \end{array}\right] \times \begin{bmatrix} q_{12} \\ q_{13} \\ q_{14} \\ \vdots \\ q_{1k} \\ \hline 0 \\ 0 \\ \vdots \\ \\ 0 \end{bmatrix} + \begin{bmatrix} q_{21} \\ q_{22} \\ q_{23} \\ \vdots \\ q_{2,k-1} \\ \hline 0 \\ 0 \\ \vdots \\ \\ 0 \end{bmatrix} \tag{3.14}$$

The coefficient matrix eq. (3.14) is partitioned into submatrices and its dimensions are, $c_{11} = (m+1) \times k$; $c_{12} = (m+1) \times (m+1)$; $c_{21} = k \times k$ and $c_{22} = k \times (m+1)$

$$\left[\begin{array}{c} \hat{c}_1 \\ \hline \hat{c}_2 \end{array}\right] = \left[\begin{array}{c|c} c_{11} & c_{12} \\ \hline c_{21} & c_{22} \end{array}\right] \times \left[\begin{array}{c} \hat{q}_1 \\ \hline 0 \end{array}\right] + \left[\begin{array}{c} \hat{q}_2 \\ \hline 0 \end{array}\right] \tag{3.15}$$

Step 1.4: The reduced order denominator polynomial is obtained as follows,

$$[\hat{q}_1] = [c_{21}]^{-1} [\hat{c}_2] = 1 + q_{12}s + q_{13}s^2 + \cdots + q_{1k}s^k \tag{3.16}$$

The required reduced order denominator is written as

$$d(s) = d_0 + d_1 s + d_2 s^2 + \cdots + d_k s^k \tag{3.17}$$

Step 2: The procedure for obtaining lower order numerator is,

Step 2.1: Equate both original system eq. (3.1) and the reduced order model eq. (3.2) as follows,

$$\frac{a_0 + a_1 s + a_2 s^2 + \cdots + a_{n-1}s^{n-1}}{b_0 + b_1 s + b_2 s^2 \cdots + b_n s^n} = \frac{e_0 + e_1 s + e_2 s^2 + \cdots + e_{k-1}s^{k-1}}{d_0 + d_1 s + d_2 s^2 \cdots + d_k s^k} \tag{3.18}$$

Step 2.2: The k number of unknown numerator parameters are obtained by cross multiplying eq. (3.18) and comparing like powers of s as given below

$$\left.\begin{array}{c} a_0 d_0 = b_0 e_0 \\ a_0 d_1 + a_1 d_0 = b_0 e_1 + b_1 e_0 \\ a_0 d_2 + a_1 d_1 + a_2 d_2 = b_0 e_2 + b_1 e_1 + b_2 e_0 \\ \vdots \end{array}\right\} \tag{3.19}$$

By solving eq. (3.19) the required reduced order numerator coefficients are obtained, the reduced numerator polynomial is,

$$n(s) = e_0 + e_1 s + e_2 s^2 + \cdots + e_{k-1}s^{k-1} \tag{3.20}$$

3.3.1 Numerical Examples and results

To show the effectiveness and powerfulness of the proposed reduction method we considered standard SISO/MIMO systems which are available in the literature. The first example is solved in detail, whereas in the remaining examples the reduced order models are mentioned directly. The results are compared in terms of system response and performance indices.

3.3.1.1 Single Input Single Output Systems

Example 3.1: The 4^{th} order original system represented in transfer function [279]

$$G(s) = \frac{28s^3 + 496s^2 + 1800s + 2400}{2s^4 + 36s^3 + 204s^2 + 360s + 240} \tag{3.21}$$

Step 1: By dividing numerator and denominators of eq. (3.21) with 240 we can get the following transfer function,

$$G(s) = \frac{0.1167s^3 + 2.067s^2 + 7.5s + 10}{0.008333s^4 + 0.15s^3 + 0.85s^2 + 1.5s + 1} \tag{3.22}$$

Step 1.2: The time-moments are obtained by using eq. (3.10)-(3.13) as follows,

$c_0 = (-1)^0 a_{21} = 10;\ c_1 = (-1)^1 a_{31} = -7.5;\ c_2 = (-1)^2 a_{41} = 4.817;\ c_3 = (-1)^3 a_{51} = -2.2338$

Step 1.3: The desired reduced order denominator is achieved by following eq. (3.14)-(3.17),

$$\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} -c_1 & -c_0 \\ -c_2 & -c_1 \end{bmatrix}^{-1} \begin{bmatrix} c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1.70618 \\ 0.79797 \end{bmatrix}$$

Therefore

$$D_r(s) = 1 + 1.70618s + 0.79797s^2 = s^2 + 2.13815s + 1.25317$$

Step 2: The required reduced order numerator polynomial is obtained by following eq. (3.18)-(3.20),

$$N_r(s) = 11.9827s + 12.5317$$

Finally, the desired reduced order model is obtained as,

$$R(s) = \frac{11.9827s + 12.5317}{s^2 + 2.13815s + 1.25317}$$

Figures 3.1 and 3.2 show the comparison of step and bode diagram responses of original and reduced order models for Example 3.1. From this it is observed that, the proposed reduced model gave close approximation with the original system response. Further, the performance of the proposed method validated in terms of

ISE, IAE, ITSE and ITAE values by comparing with other reduction methods which are tabulated in Table 3.1. From this, it is noticed that, the proposed method exhibits less error compared to other reduction techniques.

Example 3.2: The 8^{th} order original system represented in transfer function as [280]

$$G_8(s) = \frac{40320 + 185760s + 222088s^2 + 122664s^3 + 36380s^4 + 5982s^5 + 514s^6 + 18s^7}{40320 + 109584s + 118124s^2 + 67284s^3 + 22449s^4 + 4536s^5 + 546s^6 + 36s^7 + s^8}$$

The desired ROM is obtained by using proposed method,

$$R_2(s) = \frac{15.08999s + 4.81695}{s^2 + 5.9894s + 4.81695}$$

The time and frequency responses of proposed model is compared with original model and other reduced order models Afzal [280] and Amit [281] which are shown in Figures 3.3 and 3.4 for Example 3.2. From this it is clear that, the proposed method provided close approximation with the original system response. Further, to show the effectiveness of the proposed technique, error indices are measured and compared with some other reduction methods and are displayed in Table 3.2. It is observed that, the presented technique gave least error value.

Example 3.3: the 8^{th} order original system represented in transfer function as [92]

$$G_8(s) = \frac{35s^7 + 1086s^6 + 13285s^5 + 82402s^4 + 278376s^3 + 511812s^2 + 482964s + 194480}{s^8 + 33s^7 + 437s^6 + 3017s^5 + 11870s^4 + 27470s^3 + 37492s^2 + 28880s + 9600}$$

The reduced order model obtained by proposed technique,

$$R(s) = \frac{39.13413s + 9.54126}{s^2 + 2.179005s + 0.47098}$$

The reduced order model obtained by Krishnamurthy [92]

$$R(s) = \frac{334828.5s + 194480}{20123.7s^2 + 18116.2s + 9600}$$

The time and frequency responses of proposed model is compared with original model and Krishnamurthy method [92] for Example 3.3 are shown in Figure 3.5 and 3.6 respectively. From this it is observed that, the responses of the ROM obtained by the proposed method gave much closer approximation with HOS compared to [92]. Further, the error indices values are also depicted in Table 3.3. It is observed that, the presented technique exhibits the less error compared to [92].

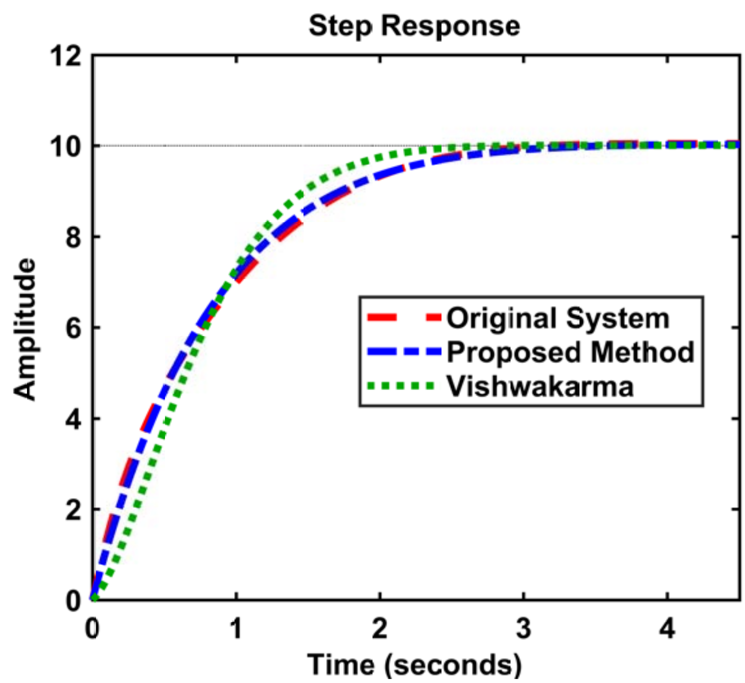


Fig. 3.1: Step response comparisons for Example 3.1.

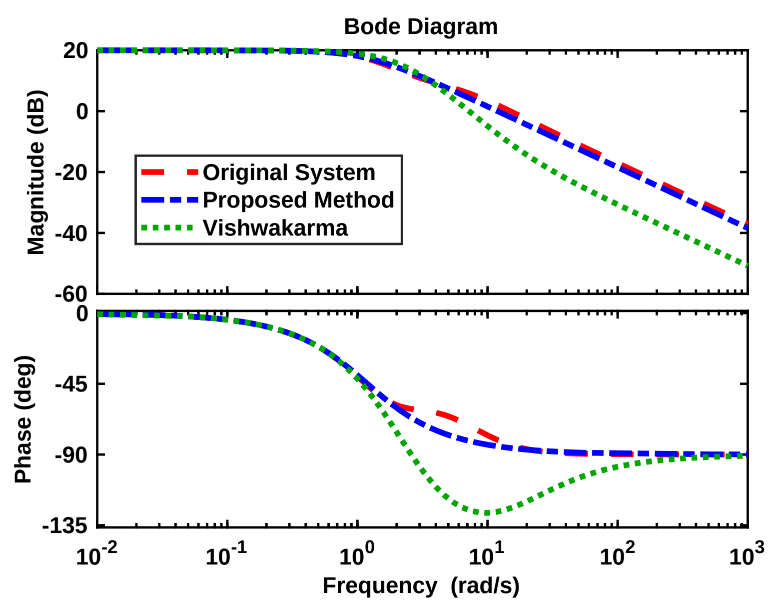


Fig. 3.2: Bode response comparisons for Example 3.1.

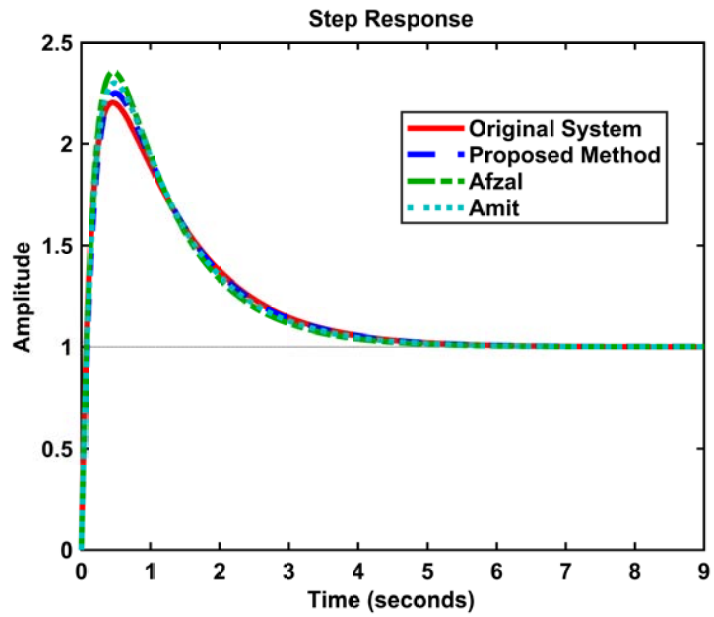


Fig. 3.3: Time response comparisons for Example 3.2.

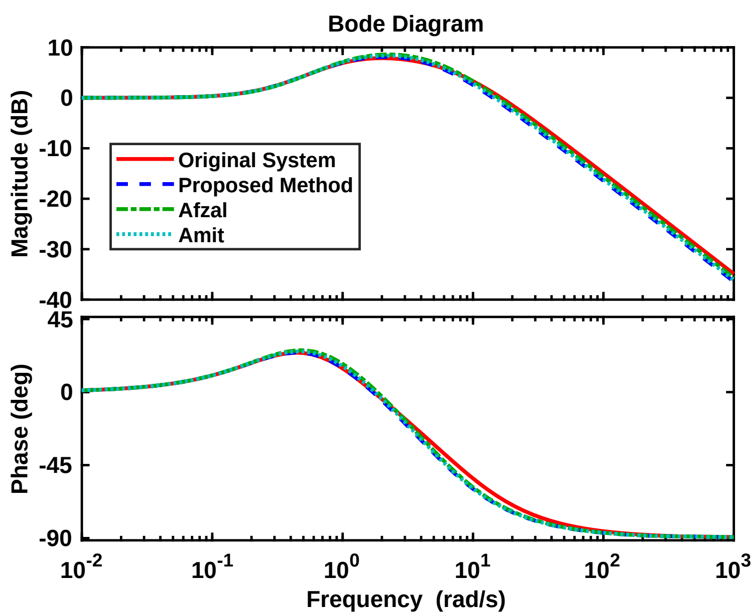


Fig. 3.4: Frequency response comparisons for Example 3.2.

Table 3.1: ISE, IAE, ITSE and ITAE comparison of ROMs for Example 3.1.

Reduction Methods	Transfer Function	ISE	IAE	ITSE	ITAE
Proposed method	$R(s) = \frac{11.9827s+12.5317}{s^2+2.13815s+1.25317}$	0.00137	0.0636	0.00506	0.511
B. Viswakarma [279]	$R(s) = \frac{2.8863s+51.4892}{s^2+4.15032s+5.14892}$	0.3379	0.8598	1.016	1.724
T.N.Lucas [118]	$R(s) = \frac{30s+40}{3s^2+6s+4}$	0.034	0.3147	0.1114	1.184
Narwal [140]	$R(s) = \frac{9.6686s+28.9503}{s^2+3.1144s+2.8953}$	0.02001	0.2012	0.1012	0.832

Table 3.2: ISE, IAE and ITAE comparison of ROMs for Example 3.2.

Reduction Methods	Transfer Function	ISE	IAE	ITAE
Proposed model	$R(s) = \frac{15.08999s+4.81695}{s^2+5.9894s+4.81695}$	1.967×10^{-4}	0.0223	0.0761
Afzal et al. [280]	$R(s) = \frac{16.504s+5.462}{s^2+6.197s+5.462}$	1.390×10^{-2}	0.1971	0.384
S. Afzal [282]	$R(s) = \frac{16.92s+5.263}{s^2+6.893s+5.262}$	7.2610×10^{-4}	0.0397	0.1842
S. Biradar et al. [283]	$R(s) = \frac{3.1084s+1.0005}{0.2075s^2+1.2434s+1}$	3.8124×10^{-4}	0.0743	0.1643
Amit [281]	$R(s) = \frac{15.6184s+5.0748}{s^2+6.0306s+5.0748}$	5.509×10^{-4}	0.0663	0.2038
Vishwakarma [284]	$R(s) = \frac{16.51145s+5.45971}{s^2+6.19642s+5.45971}$	1.406×10^{-2}	0.2366	1.3471
Abu Al Nadi [285]	$R(s) = \frac{17.0989s+5.0742}{s^2+6.9722s+5.1514}$	3.01×10^{-3}	0.0982	0.7192
Bansal et al. [286]	$R(s) = \frac{17.387s+5.3743}{s^2+7.091s+5.3743}$	8.50×10^{-4}	0.0507	0.2057
Parmar et al. [136]	$R(s) = \frac{24.11429s+8}{s^2+9s+8}$	4.8090×10^{-2}	0.1523	0.3891
Mukherjee et al. [287]	$R(s) = \frac{11.3909s+4.4357}{s^2+4.2122s+4.4357}$	5.6897×10^{-2}	0.3359	0.9475
Mittal et al. [288]	$R(s) = \frac{7.0908s+1.9906}{s^2+3s+2}$	0.2689	0.4743	1.21

Table 3.3: Performance indices comparison for Example 3.3.

Reduction method	ISE	IAE	ITSE	ITAE
Proposed method	0.587	1.23	1.81	2.658
Krishnamurthy [92]	16.92	5.65	73.9	40.1

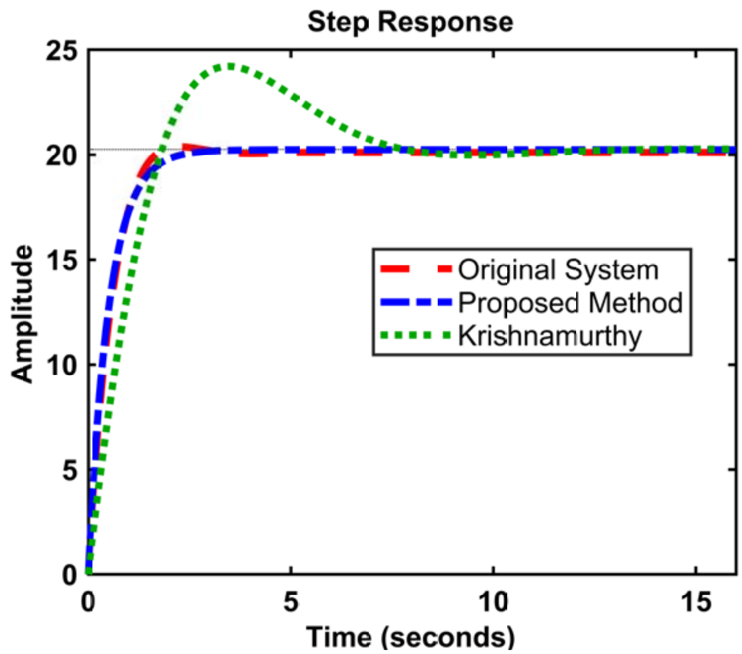


Fig. 3.5: Time response comparisons for Example 3.3.

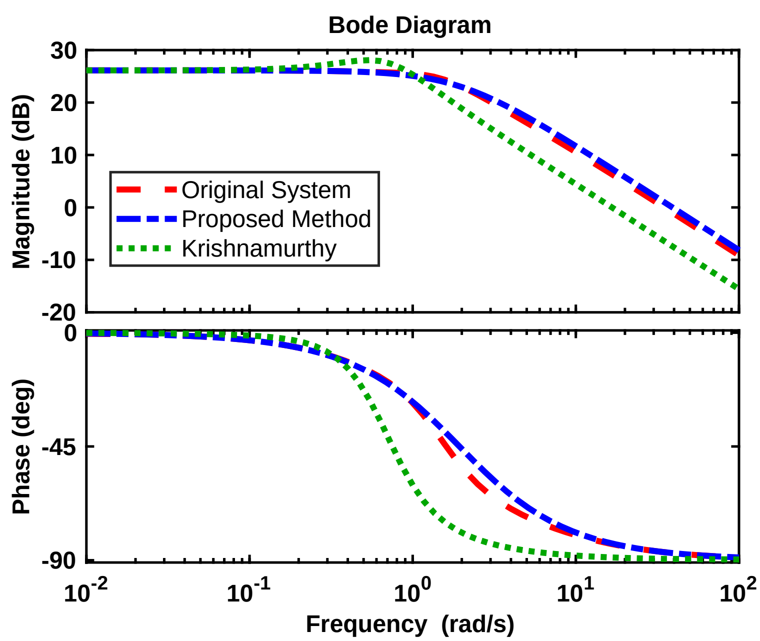


Fig. 3.6: Frequency response comparisons for Example 3.3.

3.3.1.2 Multiple Input Multiple Output System

Example 3.4: Further, the proposed method is extended to MIMO systems by direct application of SISO method on various elements of transfer function matrix of MIMO system. The 6^{th} order MIMO higher-order model [280] represented in transfer matrix form is given as,

$$[G(s)] = \begin{bmatrix} \frac{2(s+5)}{(s+1)(s+10)} & \frac{(s+4)}{(s+2)(s+5)} \\ \frac{(s+10)}{(s+1)(s+20)} & \frac{(s+6)}{(s+2)(s+3)} \end{bmatrix}$$
$$= \frac{1}{D_6(s)} \begin{bmatrix} h_{11}(s) & h_{12}(s) \\ h_{21}(s) & h_{22}(s) \end{bmatrix}$$

Where

$$D_6(s) = (s+20)(s+10)(s+5)(s+3)(s+2)(s+1)$$
$$= 6000 + 13100s + 10060s^2 + 3491s^3 + 571s^4 + 41s^5 + s^6$$

and

$$h_{11}(s) = 6000 + 7700s + 3610s^2 + 762s^3 + 70s^4 + 2s^5$$
$$h_{12}(s) = 2400 + 4160s + 2182s^2 + 459s^3 + 38s^4 + s^5$$
$$h_{21}(s) = 3000 + 3700s + 1650s^2 + 331s^3 + 30s^4 + s^5$$
$$h_{22}(s) = 6000 + 9100s + 3660s^2 + 601s^3 + 42s^4 + s^5$$

The reduced second order models obtained by the proposed technique and other methods are given in Table 3.4. The ISE values of second order model have been compared with other reduction methods [112,140,280,282,283] which are displayed in Table 3.4 for Example 3.4. From this it is observed that, the proposed technique gave lowest ISE value compared to other well-known reduction methods. Further, the accurate approximation of this method also shown in Figures 3.7 and 3.8 through step and bode diagram responses.

Example 3.5: Consider a MIMO model with real and complex poles which is Phillips-Heffron model of Single-Machine Infinite Bus Power System [289]. The 10^{th} order transfer matrix of practical power system is given as follows

$$[G(s)] = \frac{1}{D_{10}(s)} \begin{bmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{bmatrix}$$

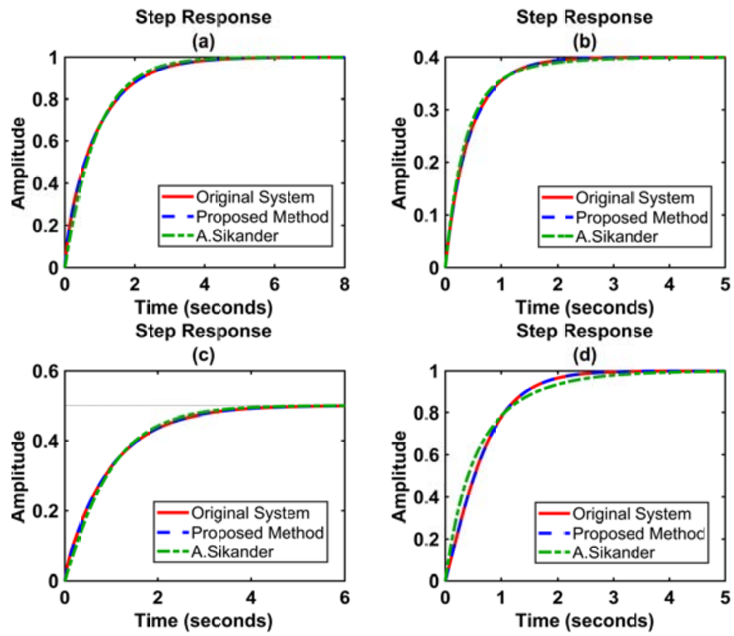


Fig. 3.7: Step response comparison of MIMO system (a) $r_{11}(s)$ (b) $r_{12}(s)$ (c) $r_{21}(s)$ (d) $r_{22}(s)$ for Example 3.4.

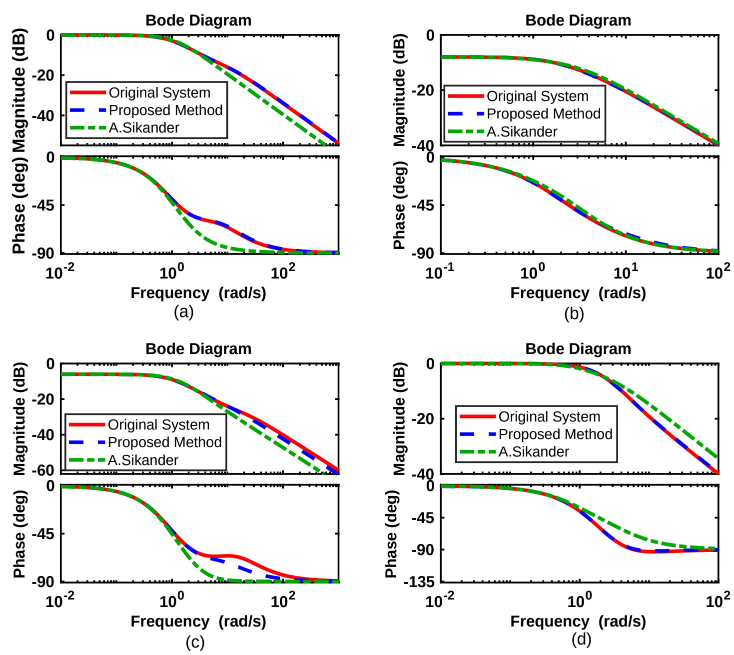


Fig. 3.8: Bode diagram response comparison of MIMO system (a) $r_{11}(s)$ (b) $r_{12}(s)$ (c) $r_{21}(s)$ (d) $r_{22}(s)$ for Example 3.4.

Where

$$D_{10}(s) = s^{10} + 64.21s^9 + 1596s^8 + 1.947 \times 10^4 s^7 + 1.268 \times 10^5 s^6 + 5.036 \times 10^5 s^5 + 1.569 \times 10^6 s^4 + 3.24 \times 10^6 s^3 + 4.061 \times 10^6 s^2 + 2.905 \times 10^6 s + 2.531 \times 10^5$$

and

$$H_{11}(s) = -2298s^5 - 9.845 \times 10^4 s^4 - 1.376 \times 10^6 s^3 - 6.838 \times 10^6 s^2 - 6.101 \times 10^6 s - 5.43 \times 10^5$$

$$H_{12}(s) = 29.09s^8 + 1868s^7 + 4.61 \times 10^4 s^6 + 5.459 \times 10^5 s^5 + 3.185 \times 10^6 s^4 + 8.703 \times 10^6 s^3 + 1.206 \times 10^7 s^2 + 7.606 \times 10^6 s + 6.483 \times 10^5$$

$$H_{21}(s) = 85.23s^7 + 3651s^6 + 5.208 \times 10^4 s^5 + 2.98 \times 10^5 s^4 + 8.472 \times 10^5 s^3 + 3.105 \times 10^6 s^2 + 2.752 \times 10^6 s + 2.45 \times 10^5$$

$$H_{22}(s) = -1.26s^8 - 85.18s^7 - 2089s^6 - 2.568 \times 10^4 s^5 - 1.909 \times 10^5 s^4 - 7.123 \times 10^5 s^3 - 1.084 \times 10^6 s^2 - 2.972 \times 10^5 s - 1.942 \times 10^4$$

The reduced order models of the presented technique and other reduction methods are presented in Table 3.5. Also to demonstration the competitiveness of the proposed reduced model the ISE values are measured and compared with other reduction methods GA [289] and IWO [290] which are displayed in Table 3.5 for Example 3.5. It is observed that, the presented method gave least ISE values compared to some other reduction methods. Further, the accurate approximation of this method also shown in Figure 3.9 by comparison through time responses.

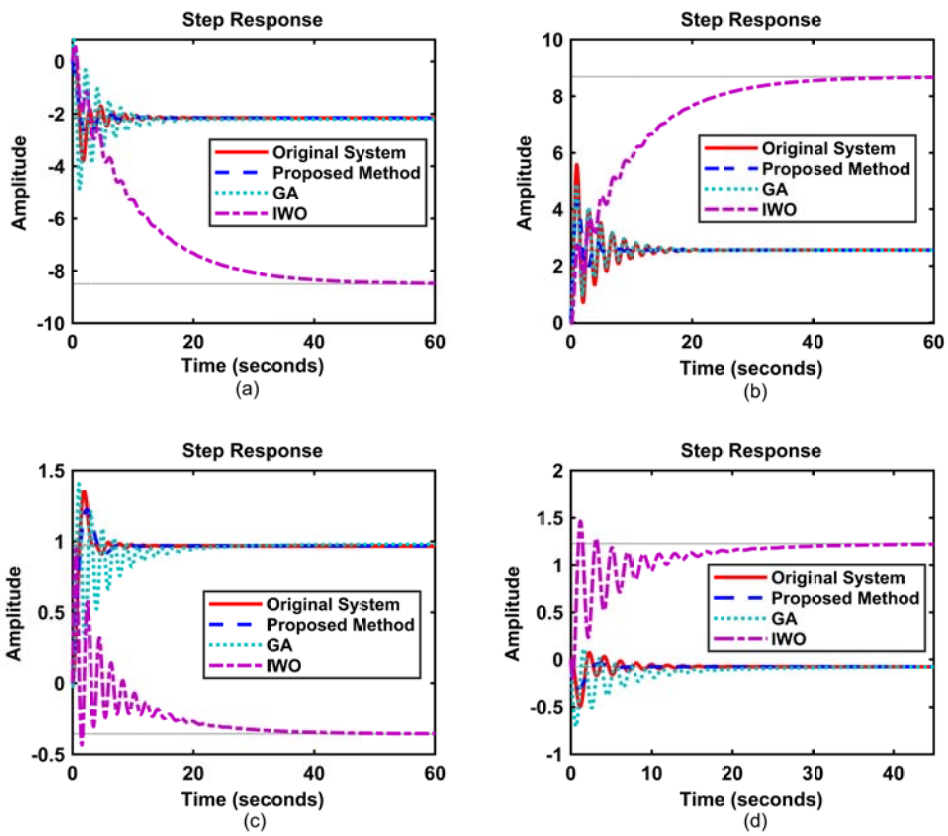


Fig. 3.9: Time response comparison of MIMO system (a) $r_{11}(s)$ (b) $r_{12}(s)$ (c) $r_{21}(s)$ (d) $r_{22}(s)$ for Example 3.5.

Table 3.4: ISE comparison of ROMs for Example 3.4.

Reduction methods	$R_{11}(s)$		$R_{12}(s)$	
	Transfer function	ISE	Transfer function	ISE
Proposed method	$\frac{2.055201s+10.61571}{s^2+11.60934s+10.61571}$	1.863×10^{-9}	$\frac{1.0492s+6.1538}{s^2+9.54615s+15.3846}$	1.003×10^{-8}
A.Sikander et al. [280]	$\frac{1.0546s+3.65079}{s^2+4.3374s+3.65079}$	2.02×10^{-3}	$\frac{1.0778s+1.4603}{s^2+4.3374s+3.65079}$	1.097×10^{-3}
A.Narwal et al. [140]	$\frac{1.3276s+3.0962}{s^2+4.0965s+3.0965}$	3.802×10^{-5}	$\frac{1.0447s+1.2444}{s^2+4.0965s+3.0965}$	8.779×10^{-6}
A.Sikander et al. [282]	$\frac{0.7938s+0.6181}{s^2+1.34952s+0.6181}$	0.00167	$\frac{0.4272s+0.2471}{s^2+1.34952s+0.6181}$	9.5814×10^{-3}
Amit et al. [112]	$\frac{0.8930s+0.6181}{s^2+1.3495s+0.6181}$	0.00112	$\frac{0.4517s+0.2472}{s^2+1.3495s+0.6181}$	7.312×10^{-3}
S. Biradar et al. [283]	$\frac{0.2023s+1}{0.10224s^2+1.10237s+1}$	7.9115×10^{-7}	$\frac{0.10084s+0.4}{0.10097s^2+0.7021s+1}$	1.2766×10^{-8}
	$R_{21}(s)$		$R_{22}(s)$	
Proposed method	$\frac{0.7622s+5.3078}{s^2+11.6093s+10.6157}$	9.331×10^{-9}	$\frac{1.0282s+5.5096}{s^2+4.7013s+5.5096}$	5.23×10^{-8}
A.Sikander et al. [280]	$\frac{0.43458s+1.8253}{s^2+4.3374s+3.65079}$	4.27×10^{-4}	$\frac{1.9035s+3.6507}{s^2+4.3374s+3.65079}$	6.107×10^{-3}
A.Narwal et al. [140]	$\frac{0.6116s+1.5480}{s^2+4.0965s+3.0965}$	7.585×10^{-6}	$\frac{1.7815s+3.0960}{s^2+4.0965s+3.0965}$	5.969×10^{-4}
A.Sikander et al. [282]	$\frac{0.3795s+0.309}{s^2+1.34952s+0.6181}$	3.122×10^{-3}	$\frac{0.9338s+0.6181}{s^2+1.34952s+0.6181}$	2.0033×10^{-2}
Amit et al. [112]	$\frac{0.4314s+0.3091}{s^2+1.3495s+0.6181}$	1.942×10^{-3}	$\frac{1.0579s+0.6181}{s^2+1.3495s+0.6181}$	1.117×10^{-2}
S. Biradar et al. [283]	$\frac{0.05025s+0.5}{0.05052s^2+1.0505s+1}$	5.6205×10^{-9}	$\frac{0.1676s+1}{0.1673s^2+0.8343s+1}$	8.4009×10^{-8}

Table 3.5: ISE comparison of reduced order models for Example 3.5.

Reduction method	$R_{11}(s)$		$R_{12}(s)$	
	Transfer function	ISE	Transfer function	ISE
Proposed method	$\frac{-7.556s^2-21.48s-2.145}{7.175s^3+6.678s^2+10.25s+1}$	1.395	$\frac{12.89s^2+26.94s+2.561}{2.31s^3+2.785s^2+10.26s+1}$	2.53
GA [289]	$\frac{7.4s^2-24s-2.3}{s^3+0.5785s^2+10.5690s+1.0532}$	21.52	$\frac{0.6250s^2+28.9013s+2.6745}{s^3+0.5785s^2+10.5690s+1.0532}$	0.6402
IWO [290]	$\frac{2.671s^2+0.232s-8.939}{s^3+0.5789s^2+10.57s+1.053}$	1065	$\frac{-0.8361s^2+11.45+9.147}{s^3+0.5789s^2+10.57s+1.053}$	1032
	$R_{21}(s)$		$R_{22}(s)$	
Proposed method	$\frac{4.486s^2+9.737s+0.968}{6.647s^3+7.718s^2+10.3s+1}$	0.0572	$\frac{-3.403s^2-1.112s-0.07673}{4.882s^3+7.657s^2+10.67s+1}$	0.0802
GA [289]	$\frac{-0.6161s^2+7.95482s+1.03278}{s^3+0.5785s^2+10.5690s+1.0532}$	2.93	$\frac{-1.5073s^2-2.9999s-0.0808}{s^3+0.5785s^2+10.5690s+1.0532}$	0.2697
IWO [290]	$\frac{2.643s^2+2.773s-0.3753}{s^3+0.5789s^2+10.57s+1.053}$	63.59	$\frac{-1.883s^2+7.123s+1.292}{s^3+0.5789s^2+10.57s+1.053}$	56.48

3.4 MODEL REDUCTION USING FACTOR DIVISION ALGORITHM AND DIFFERENTIATION METHOD

In this section, new reduction technique is proposed for order reduction of large scale systems. This method is based on combination of factor division algorithm (FDA) [118] and differentiation method (DM) [93]. The denominator polynomial of the ROM is obtained by using differentiation method while the reduced order numerator polynomial is achieved by using factor division algorithm. The proposed technique has been compared in terms of error indices (ISE, IAE, ITSE, and ITAE). The proposed technique explained in two steps as follows.

Step 1: The denominator coefficients of reduced order model eq. (3.2) is achieved by using DM

$$D_{n-r}\left(s\right) = D_n\left(s\right) - \frac{s}{n} D'_n\left(s\right) \tag{3.23}$$

Where n is order of the denominator polynomial

Differentiate eq. (3.23) into $(n-r)$ times to obtain the required reduced order denominator as

$$d\left(s\right) = d_0 + d_1 s + d_2 s^2 + ... + d_r s^r \tag{3.24}$$

Step 2: The reduced order numerator polynomial eq. (3.2) is determined by using FDA

The $G\left(s\right)$ may be taken as,

$$G_n\left(s\right) = \frac{N\left(s\right) d\left(s\right) / D\left(s\right)}{D\left(s\right)} \tag{3.25}$$

The numerator polynomial may be written as follows,

$$N\left(s\right) = \frac{N\left(s\right) d\left(s\right)}{D\left(s\right)} = \frac{f_0 + f_1 s + f_2 s^2 + f_3 s^3 + ...}{b_0 + b_1 s + b_2 s^2 + b_3 s^3 + ...} \tag{3.26}$$

From eq. (3.26) the reduced order numerator is obtained by following Routh recurrence formula as given below,

$$
\begin{aligned}
&e_0 = \frac{f_0}{b_0} \left\langle \begin{matrix} f_0 & f_1 & f_2 & f_3 \cdots \\ \\ b_0 & b_1 & b_2 & b_3 \cdots \end{matrix} \right. \\
&e_1 = \frac{q_0}{b_0} \left\langle \begin{matrix} q_0 & q_1 & q_2 & q_3 \cdots \\ \\ b_0 & b_1 & b_2 & b_3 \cdots \end{matrix} \right. \\
&\vdots \\
&e_{r-2} = \frac{u_0}{b_0} \left\langle \begin{matrix} u_0 & u_1 \\ \\ b_0 & b_1 \end{matrix} \right. \\
&e_{r-1} = \frac{v_0}{b_0} \left\langle \begin{matrix} v_0 \\ \\ b_0 \end{matrix} \right.
\end{aligned}
\tag{3.27}
$$

where

$$
\begin{aligned}
q_i &= f_{i+1} - e_0 b_{i+1}; i = 0, 1, 2, \cdots \\
u_i &= q_{i+1} - e_1 b_{i+1};\ i = 0, 1, 2, \cdots \\
&\cdots \\
v_0 &= u_1 - e_{r-2} b_1
\end{aligned}
$$

The required reduced order numerator polynomial is obtained as,

$$n(s) = e_0 + e_1 s + e_2 s^2 + \cdots + e_{r-1} s^{r-1} \tag{3.28}$$

3.4.1 Numerical Examples and Results

To show the efficacy and powerfulness of the proposed reduction method we considered some popular SISO/MIMO systems. The first example solved in detail, whereas in the remaining examples the reduced order models are mentioned directly. The results are compared in terms of system responses, and performance indices.

3.4.1.1 Single Input Single Output Systems

Example 3.6: Consider the 3^{rd} order system described by the transfer function [145]

$$G(s) = \frac{N(s)}{D(s)} = \frac{8s^2 + 6s + 2}{s^3 + 4s^2 + 5s + 2}$$

Step 1: The reduced order denominator is determined by following eq. (3.23)-(3.24)

$$d(s) = 4s^2 + 10s + 6$$

Step 2: The numerator polynomial of reduced order model obtained by following eq. (3.25)-(3.28)

$$n\left(s\right) = e_0 + e_1 s = 6 + 13s$$

Finally, the required reduced order model is obtained as,

$$R_2(s) = \frac{13s + 6}{4s^2 + 10s + 6}$$

The step and Nyquist responses of original system and reduced order models of proposed method and other reduction methods for Example 3.6 are shown in Figures 3.10 and 3.11. Further, The ISE, IAE, ITSE and ITAE for different reduction method comparisons are tabulated in Table 3.6. In Table 3.6, the proposed method exhibits less error compared to other methods.

Table 3.6: Performance indices comparison of ROMs for Example 3.6

Reduction methods	Transfer Function	ISE	IAE	ITSE	ITAE
Proposed Method	$R_2(s) = \frac{13s+6}{4s^2+10s+6}$	0.124	0.7998	0.4998	3.165
Chen [58]	$R_2(s) = \frac{6s+2}{4s^2+5s+2}$	0.264	1.199	1.25	5.676
Pal [145]	$R_2(s) = \frac{1.375s+0.5}{s^2+1.125s+0.5}$	0.3	1.248	1.407	5.779
Y.Shamash [86]	$R_2(s) = \frac{6s+2}{4s^2+5s+2}$	0.264	1.199	1.25	5.676

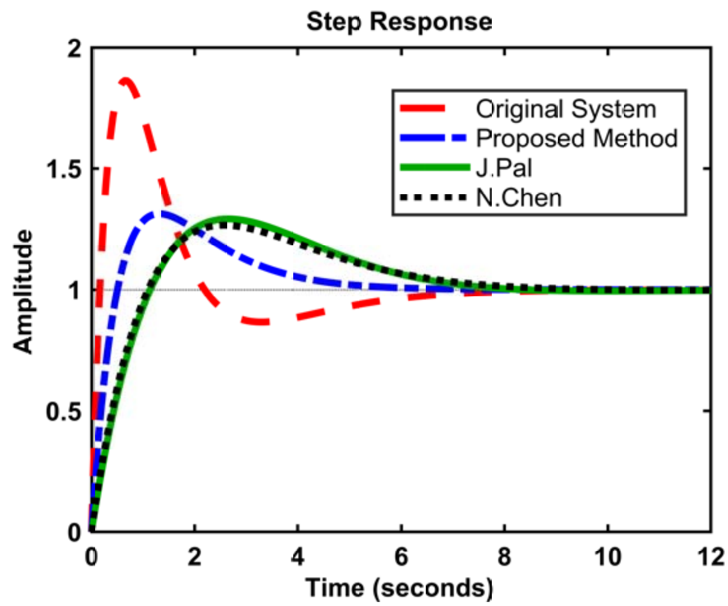


Fig. 3.10: Step response comparison for Example 3.6.

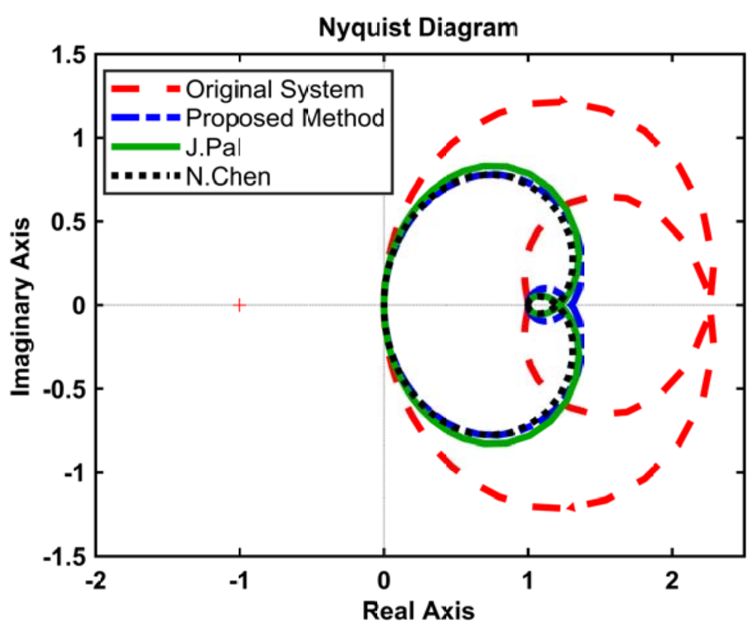


Fig. 3.11: Nyquist response comparison for Example 3.6.

Example 3.7: The 5^{th} order original system described by transfer function [291]

$$G_5(s) = \frac{s^4 + 7s^3 + 42s^2 + 142s + 156}{s^5 + 25s^4 + 258s^3 + 930s^2 + 1441s + 745}$$

The desired ROM is achieved by using FDA and DM is

$$R_2(s) = \frac{1501.2357s + 9359.7315}{5580s^2 + 49284s + 44700}$$

The comparison of step and bode Nyquist responses of original system and reduced order models proposed technique and other reduction methods are shown in Figures 3.12 and 3.13 for Example 3.7. The accuracy of the proposed method measured in terms of ISE, IAE, ITSE and ITAE, and are depicted in Table 3.7. From this, it is clearly observed that, the proposed method results in more accurate approximation with the original system compared to other methods.

Table 3.7: Performance indices comparison of proposed and other reduction methods for Example 3.7.

Reduction Methods	Transfer function	ISE	IAE	ITSE	ITAE
Proposed method	$R(s) = \frac{1501.2357s+9359.7315}{5580s^2+34584s+44700}$	0.0009023	0.04952	0.002753	0.136
Sikander [282]	$R(s) = \frac{0.007s+156}{909.5238s^2+1441s+745}$	0.007363	0.1416	0.02254	0.3485
Gutman et al. [93]	$R(s) = \frac{2130.041s+9360.18}{5580s^2+34584s+44700}$	0.001001	0.05189	0.003049	0.147
Krishnamurthy [92]	$R(s) = \frac{133.0285s+156}{770.2174s^2+1197.6291s+745}$	0.00091	0.05591	0.002971	0.1753

3.4.1.2 Multiple Input Multiple Output System

Example 3.8: Consider a 6^{th} order MIMO system as mentioned in Example 3.4 in section 3.3.1.2 The reduced order transfer matrix is obtained by proposed method given as,

$$R_2(s) = \frac{\begin{bmatrix} 12000s + 2160000 & 240000s + 864000 \\ 11345s + 1080000 & 132000s + 2160000 \end{bmatrix}}{241440s^2 + 1572000s + 2160000}$$

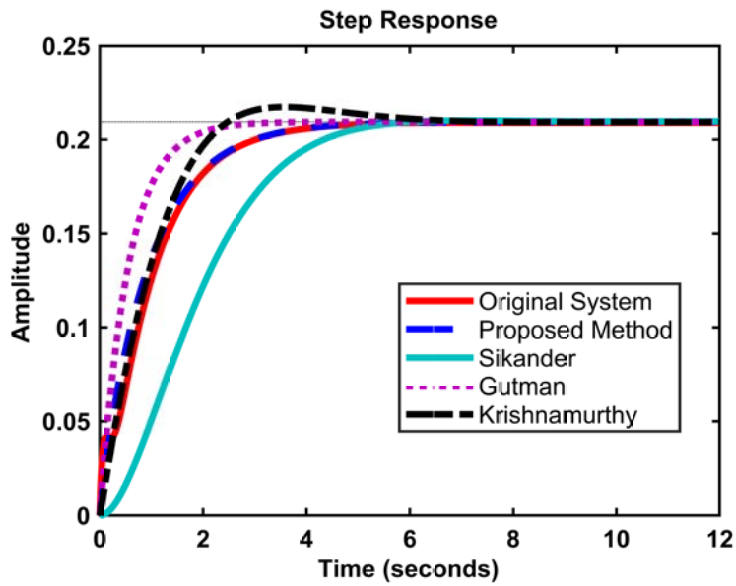


Fig. 3.12: Step response comparison for Example 3.7.

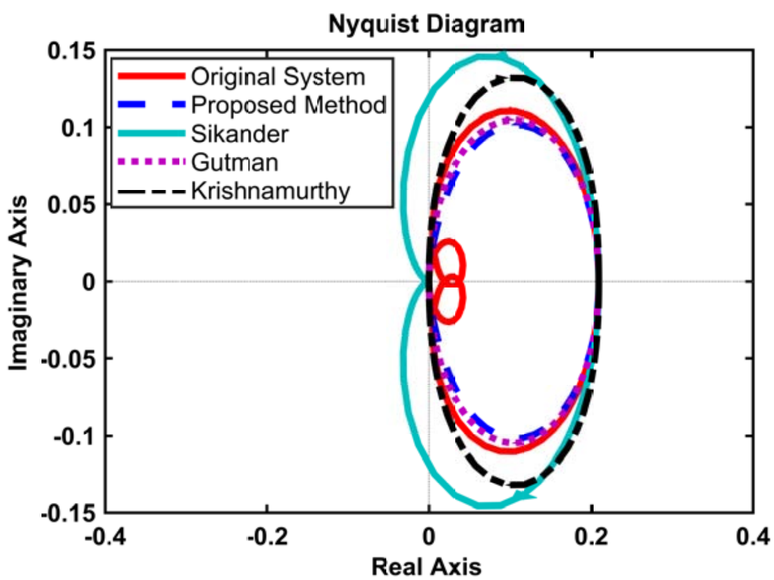


Fig. 3.13: Nyquist response comparison for Example 3.7.

The reduced order model is obtained by Parmar et al. [136] is

$$R_2(s) = \frac{\begin{bmatrix} 0.8503s + 0.6171 & 0.4617s + 0.2466 \\ 0.4093s + 0.3086 & 0.9977s + 0.6171 \end{bmatrix}}{s^2 + 1.34952s + 0.6181}$$

The reduced order model is obtained by Parmar et al. [292] is

$$R_2(s) = \frac{\begin{bmatrix} 6.0429s + 8.4707 & 3.9419s + 3.3883 \\ 2.8097s + 4.2354 & 8.0195s + 8.4707 \end{bmatrix}}{s^2 + 13.6666s + 8.4707}$$

Figures 3.14 and 3.15 show the comparison of time and frequency responses of original and reduced order models for Example 3.8. From this it is clear that the proposed method gave better approximation with original system response. Further, the proposed method is validated with ISE, IAE, ITSE and ITAE values by comparing with other well-known reduction methods shown in Table 3.8. It is observed that, the presented method gave quit comparable results with other reduction methods.

Table 3.8: Performance indices comparison of different reduction methods for Example 3.8.

Reduction Method	ISE			
	$R_{11}(s)$	$R_{12}(s)$	$R_{21}(s)$	$R_{22}(s)$
Proposed Method	0.01518	2.837×10^{-8}	0.003094	3.221×10^{-5}
Sikander [282]	0.01672	9.5814×10^{-3}	0.003122	2.1683×10^{-2}
Parmar et al. [136]	0.1471	0.0884	0.0258	0.1598
Parmar et al. [292]	0.225	0.0682	0.0613	0.678

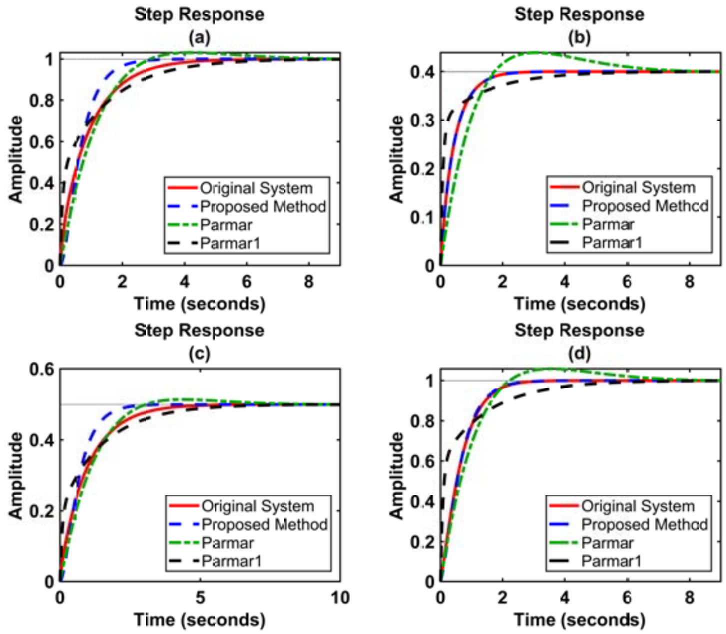


Fig. 3.14: Time response comparison of MIMO system (a) $r_{11}(s)$ (b) $r_{12}(s)$ (c) $r_{21}(s)$ (d) $r_{22}(s)$ for Example 3.8.

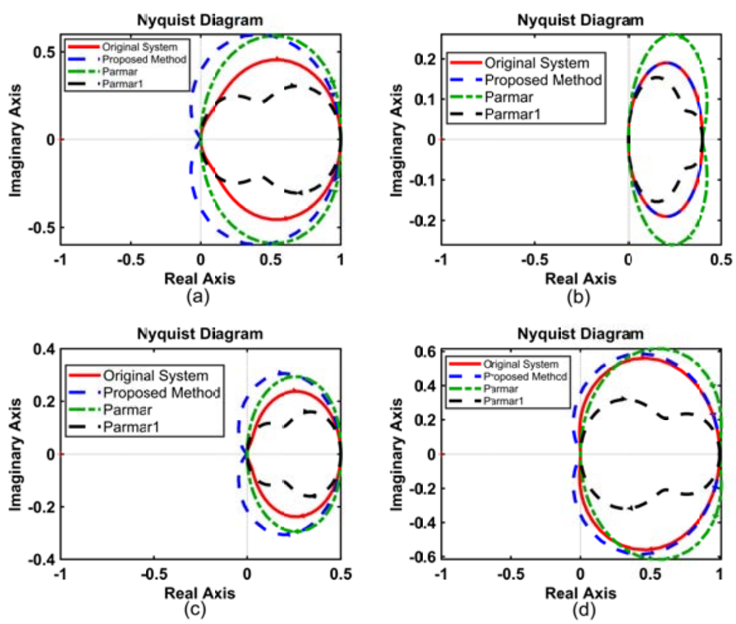


Fig. 3.15: Frequency response comparison of MIMO system (a) $r_{11}(s)$ (b) $r_{12}(s)$ (c) $r_{21}(s)$ (d) $r_{22}(s)$ for Example 3.8.

3.5 MODEL REDUCTION USING PADE APPROXIMATION AND DIFFERENTIATION METHOD

In this section, new model order reduction method is presented for order reduction of original systems. This technique is based on combination of Pade approximation (PA) [36, 40] and differentiation method (DM) [93]. The denominator coefficients of the ROM is obtained by using differentiation method while the reduced order numerator polynomial is determined by using Pade approximation. The presented technique has been compared with error indices (ISE, IAE, ITSE, and ITAE). The proposed technique explained in two steps as follows.

Step 1: The reduced order denominator polynomial in eq. (3.2) is obtained by using DM

$$D_{n-r}\left(s\right) = D_n\left(s\right) - \frac{s}{n} D'_n\left(s\right) \tag{3.29}$$

Where n is order of the denominator polynomial

Differentiate eq. (3.29) into $(n-r)$ times to obtain the required reduced order denominator as

$$d\left(s\right) = d_0 + d_1 s + d_2 s^2 + ... + d_r s^r \tag{3.30}$$

Step 2: The reduced order numerator polynomial in eq. (3.2) is determined by using PA

Equation eq. (3.1) is written as,

$$\frac{N(s)}{D(s)} = \frac{a_0 + a_1 s + a_2 s^2 + ... + a_{n-1} s^{n-1}}{b_0 + b_1 s + b_2 s^2 + ... + b_n s^n} = c_0 + c_1 s + c_2 s^2 + ... \tag{3.31}$$

We have the following set of linear simultaneous equations,

$$\left.\begin{aligned} a_0 &= b_0 c_0 \\ a_1 &= b_0 c_1 + b_1 c_0 \\ &\cdots \\ a_{n-1} &= b_0 c_{n-1} + b_1 c_{n-2} + \cdots + b_{n-1} c_0 \end{aligned}\right\} \tag{3.32}$$

The general form is as follows

$$c_i = \frac{1}{b_0}(a_i - \sum_{j=1}^{i} b_j c_{i-1}) i > 0 (\therefore a_i = 0 \text{if} i > n-1) \tag{3.33}$$

The required reduced order numerator polynomial is obtained as,

$$n(s) = e_0 + e_1 s + e_2 s^2 + \cdots + e_{r-1} s^{r-1} \tag{3.34}$$

where

$$e_0 = b_0 c_0$$
$$e_1 = b_0 c_1 + b_1 c_0$$
$$\ldots$$

3.5.1 Numerical Examples and Results

To show the efficacy and powerfulness of the proposed reduction method we considered some popular SISO/MIMO systems. The first example solved in detail, whereas in the remaining examples the reduced systems are mentioned directly. The results are compared in terms of system response, and performance indices .

3.5.1.1 Single Input Single Output Systems

Example 3.9: Consider the 4^{th} order system described by the transfer function [279].

$$G(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24} \tag{3.35}$$

Step 1: The reduced order denominator is obtained by using eq. (3.29) and (3.30)

$$d(s) = 70s^2 + 310s + 288$$

Step 2: The reduced order numerator is obtained by using eq. (3.31)-(3.34)

$$n(s) = 1.904s + 288$$

Finally, the required reduced order model is obtained as,

$$R_2(s) = \frac{1.904s + 288}{70s^2 + 310s + 288}$$

The comparison of step and Nyquist plot responses of original system and reduced order models proposed technique and other reduction methods are shown in Figures 3.16 and 3.17 for Example 3.9. The accuracy of the proposed method measured in terms of ISE, IAE, ITSE and ITAE, and are depicted in Table 3.9. From this, it is clearly observed that, the proposed method results in more accurate approximation with the original system compared to other methods.

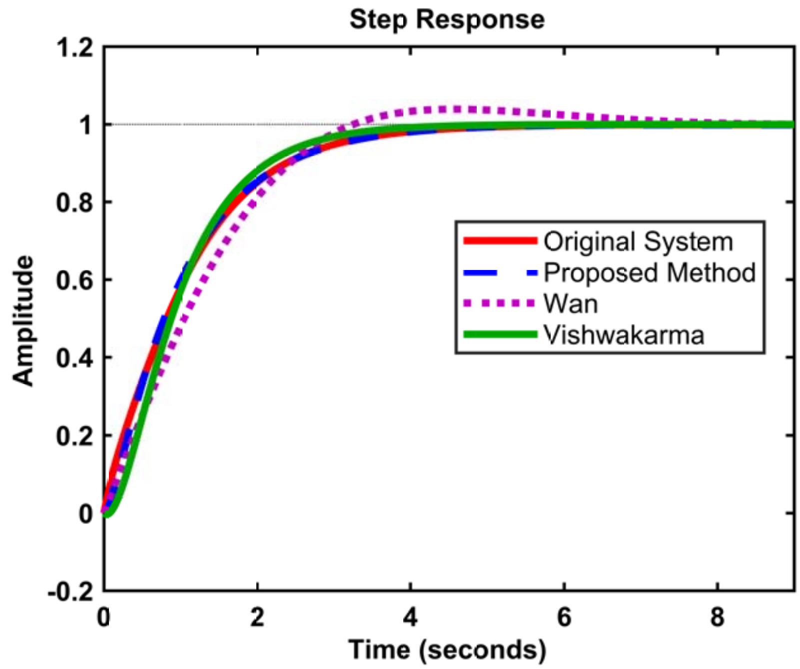


Fig. 3.16: Step response comparison for Example 3.9.

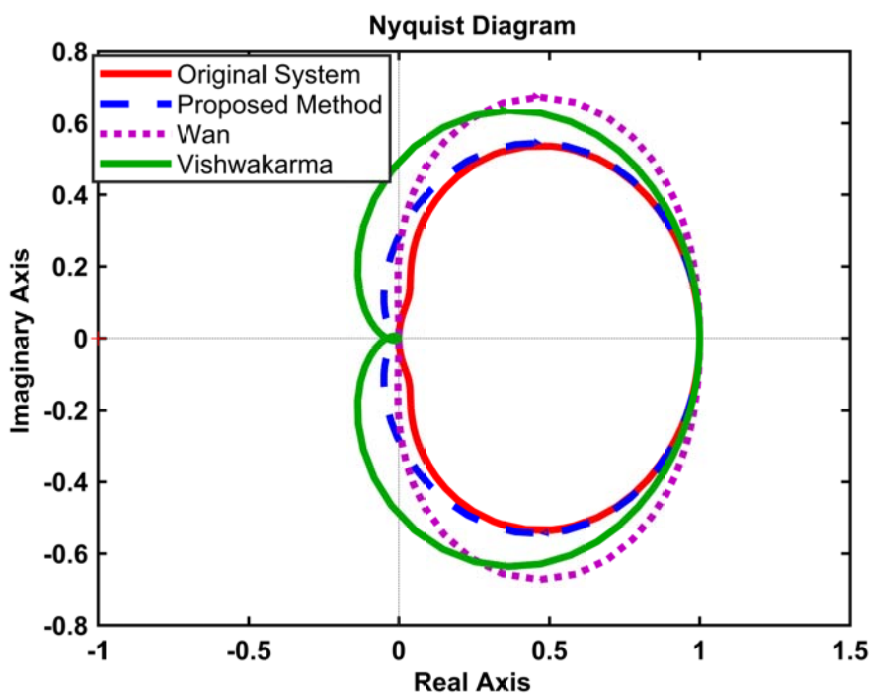


Fig. 3.17: Nyquist response comparison for Example 3.9.

Table 3.9: Performance indices comparison of different reduction methods for Example 3.9.

Reduction methods	Transfer function	ISE	IAE	ITSE	ITAE
Proposed method	$\frac{1.904s+288}{70s^2+310s+288}$	0.000192	0.01764	0.00252	0.0124
Mukherjee [75]	$\frac{0.8000003s+2}{s^2+3s+2}$	0.000914	0.02942	0.00524	0.1184
Lucas [118]	$\frac{0.833s+2}{s^2+3s+2}$	0.001142	0.07421	0.01024	0.8741
B.W. Wan [40]	$\frac{17.003s+24}{34.3004s^2+43.003s+24}$	0.000432	0.04191	0.00712	0.1347
Vishwakarma [279]	$\frac{-0.18976s+4.5713}{s^2+4.76187s+4.5713}$	0.000286	0.03128	0.00432	0.0627

Example 3.10: Consider the 3^{rd} order system as mentioned in Example 3.6 in section 3.4.1.1. The following reduced order model is obtained by proposed technique.

$$R_2(s) = \frac{14.5s + 6}{4s^2 + 10s + 6}$$

The step and Nyquist responses of original system and reduced order models of proposed method and other reduction methods for Example 3.10 are shown in Figures 3.18 and 3.19. Further, The ISE, IAE, ITSE and ITAE values for different reduction method comparisons are tabulated in Table 3.10. In Table 3.10, the proposed method exhibits less error compared to other methods

Table 3.10: Performance indices comparison of ROMs for Example 3.10

Reduction methods	Transfer Function	ISE	IAE	ITSE	ITAE
Proposed method	$R_2(s) = \frac{14.5s+6}{4s^2+10s+6}$	0.124	0.7998	0.4998	3.165
Chen [58]	$R_2(s) = \frac{6s+2}{4s^2+5s+2}$	0.264	1.199	1.25	5.676
Pal [145]	$R_2(s) = \frac{1.375s+0.5}{s^2+1.125s+0.5}$	0.3	1.248	1.407	5.779
Y.Shamash [86]	$R_2(s) = \frac{6s+2}{4s^2+5s+2}$	0.264	1.199	1.25	5.676

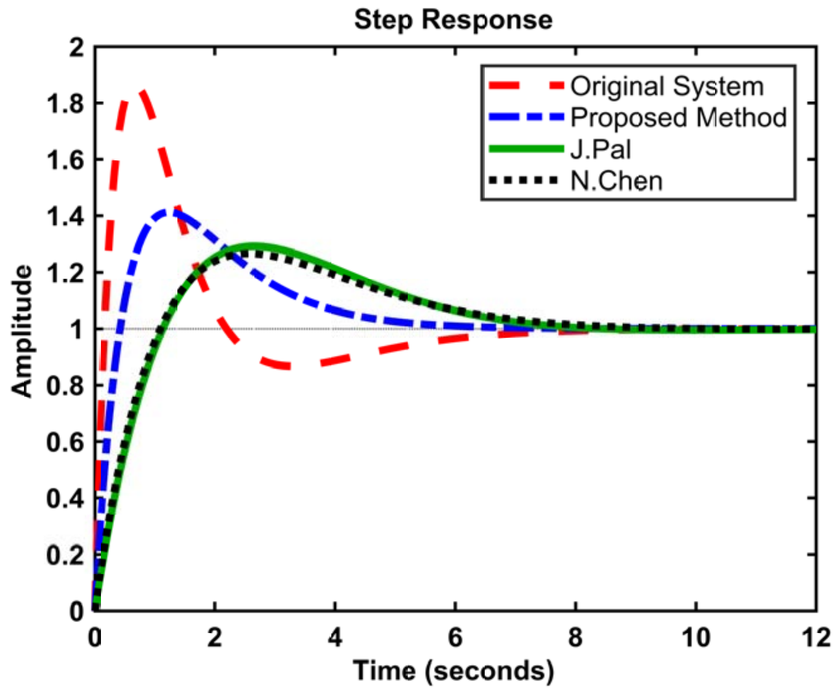


Fig. 3.18: Step response comparison for Example 3.10.

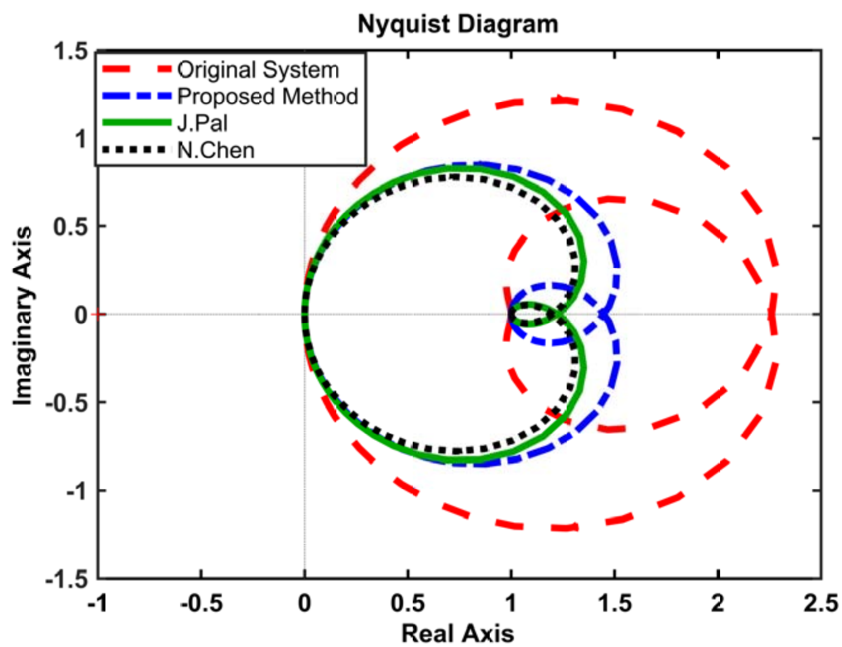


Fig. 3.19: Nyquist response comparison for Example 3.10.

3.5.1.2 Multiple Input Multiple Output System

Example 3.11: Consider a 6^{th} order MIMO system as mentioned in Example 3.4 in section 3.3.1.2. The reduced order transfer matrix of MIMO systems obtained by proposed technique is as follows,

$$R_2\left(s\right)=\frac{\begin{bmatrix}86000s+2160000 & 240000s+864000\\ 19871s+1080000 & 133440s+2160000\end{bmatrix}}{241440s^2+1572000s+2160000}$$

The reduced order model is obtained by Parmar et al. [136] is

$$R_2\left(s\right)=\frac{\begin{bmatrix}0.9098s+0.7091 & 0.4916s+0.2836\\ 0.4373s+0.3545 & 1.0753s+0.7091\end{bmatrix}}{s^2+1.548s+0.709}$$

The ROM is obtained by Parmar et al. [292] is

$$R_2\left(s\right)=\frac{\begin{bmatrix}6.0429s+8.4707 & 3.9419s+3.3883\\ 2.8097s+4.2354 & 8.0195s+8.4707\end{bmatrix}}{s^2+13.6666s+8.4707}$$

Figures 3.20 and 3.21 show the comparison of time and frequency responses of original and reduced order models for Example 3.11. From this it is clear that the proposed method gave better approximation with original system response. Further, the proposed method is validated with ISE, IAE, ITSE and ITAE values by comparing with other well-known reduction methods shown in Table 3.11. It is observed that, the presented method gave quit comparable results with other reduction methods.

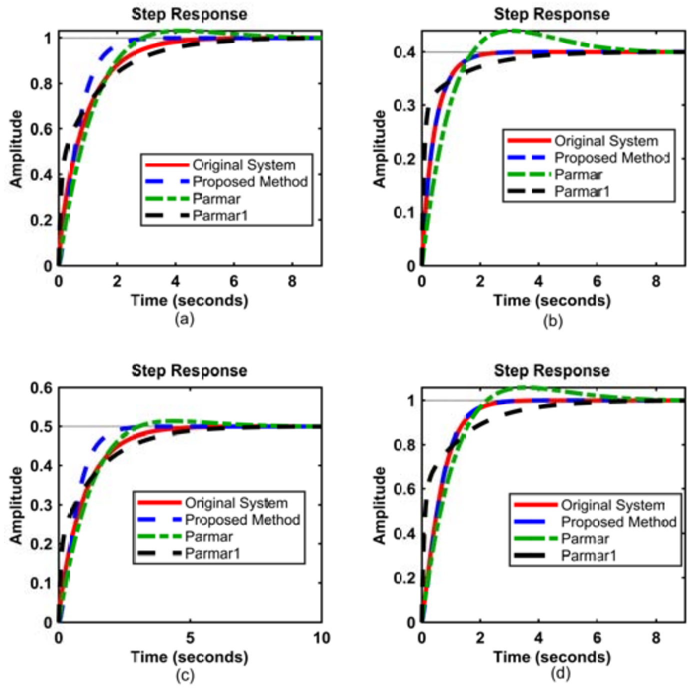


Fig. 3.20: Time response comparison of MIMO system (a) $r_{11}(s)$ (b) $r_{12}(s)$ (c) $r_{21}(s)$ (d) $r_{22}(s)$ for Example 3.11.

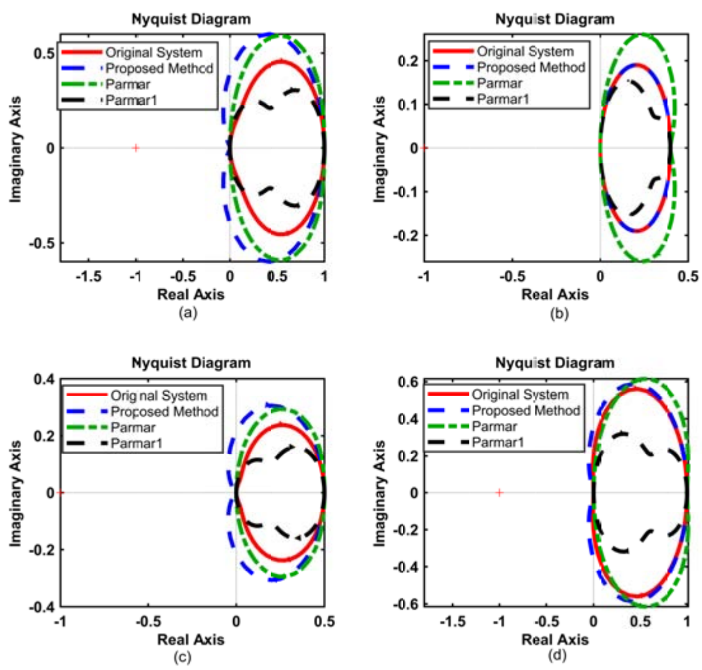


Fig. 3.21: Frequency response comparison of MIMO system (a) $r_{11}(s)$ (b) $r_{12}(s)$ (c) $r_{21}(s)$ (d) $r_{22}(s)$ for Example 3.11.

Table 3.11: Performance indices comparison of ROMs for Example 3.11

Reduction method	ISE			
	R_{11}	R_{12}	R_{11}	R_{11}
Proposed method	0.01439	2.837×10^{-8}	0.003041	3.294×10^{-5}
Parmar [136]	0.1471	0.0884	0.0258	0.1598
Parmar [292]	0.225	0.0682	0.0613	0.678
Sikander [282]	0.01672	9.581×10^{-3}	0.003122	2.1683×10^{-2}

3.6 CONCLUSION

In this chapter, three new mixed reduction techniques are presented for reducing the LTI higher order systems. First technique is based on modified time-moment matching method while the second and third techniques are based on differentiation method in combination with factor division algorithm and pade approximation. The time and frequency responses of original and reduced order models are plotted. It is observed that the ROMs obtained by these proposed techniques provided quit close approximation with the original system. Further, the proposed techniques are also extended for MIMO systems. Furthermore, to show the effectiveness and efficacy of the proposed methods, the obtained results are compared with different reduction methods in terms of ISE, IAE, ITSE and ITAE.

CHAPTER 4

NEW COMPOSITE TECHNIQUES FOR REDUCED ORDER MODELLING: INTERVAL SYSTEMS APPROACH

4.1 INTRODUCTION

The Several reduction methods are developed for fixed coefficient transfer functions or state space models [5, 60, 89, 90, 93, 280–283]. It is fact that designing a controller based on fixed coefficient transfer function or state space model is often unrealistic because the practical system parameters vary within certain interval. Bandyopadhyay et al. [218] first extended the conventional reduction methods to deal with interval systems. In this method, the reduction method is obtained by using Routh-Pade approximation technique to deal with interval systems. Later Hwang and Yang [220] said that, the Routh approximation method may loss its stability preservation property due to irreversibility of interval arithmetic operation. Later, Dolgin and Zeheb [222] and Yang [223] proposed modified Routh-Pade approximation method to avoid the limitations of [220]. Later Sastry et al. [108] presented reduced interval method by using modified Routh approximation method to avoid the complexity of [218]. This method requires only γ table formation to obtain stable reduced order interval model (ROIM). But still, this method has some limitations i.e. this method always provides some steady state error. The above discussed reduced order interval methods are obtained by using different frequency domain reduction methods and interval arithmetic operations. The interval arithmetic operations are complex and sometimes it may give unstable reduced order models even though the original higher order system is stable. To avoid these disadvantages, many interval reduction methods were developed for finding stable and better approximation of ROIMs [235, 240, 293–295].

4.2 PROBLEM STATEMENT OF INTERVAL SYSTEM

4.2.1 Single Input Single Output Systems

Let us consider a higher order interval transfer function is as follows,

$$G\left(s,a,b\right)=\frac{N\left(s,a\right)}{D\left(s,b\right)}=\frac{[a_0^-,a_0^+]+[a_1^-,a_1^+]s+\cdots+[a_{n-1}^-,a_{n-1}^+]s^{n-1}}{[b_0^-,b_0^+]+[b_1^-,b_1^+]s+\cdots+[b_n^-,b_n^+]s^n} \tag{4.1}$$

Where $\left[a_i^-,a_i^+\right]\left(i=0,1,2,\cdots,n-1\right)$ and $\left[b_j^-,b_j^+\right]\left(j=0,1,2,\cdots,n\right)$ are higher order interval coefficients of numerator and denominator polynomials.

Let us consider a reduced order interval transfer function is,

$$R\left(s,c,d\right)=\frac{n\left(s,c\right)}{d\left(s,d\right)}=\frac{[c_0^-,c_0^+]+[c_1^-,c_1^+]s+\cdots+[c_{k-1}^-,c_{k-1}^+]s^{k-1}}{[d_0^-,d_0^+]+[d_1^-,d_1^+]s+\cdots+[d_k^-,d_k^+]s^k} \tag{4.2}$$

Where $\left[c_i^-,c_i^+\right]\left(i=0,1,2,\cdots,k-1\right)$ and $\left[d_j^-,d_j^+\right]\left(j=0,1,2,\cdots,k\right)$ are unknown reduced order interval coefficients of numerator and denominator polynomials.

4.2.2 Multiple Input Multiple Output Systems

Let us consider a higher order multiple input multiple output interval transfer matrix is as follows,

$$\left[G\left(s,e_{ij},D_n\right)\right]=\frac{1}{D_n\left(s\right)}\begin{bmatrix} g_{11}\left(s\right) & g_{12}\left(s\right) & \cdots & g_{1p}\left(s\right)\\ g_{21}\left(s\right) & g_{22}\left(s\right) & \cdots & g_{2p}\left(s\right)\\ \vdots & \vdots & \ddots & \vdots\\ g_{q1}\left(s\right) & g_{q2}\left(s\right) & \cdots & g_{qp}\left(s\right)\end{bmatrix} \tag{4.3}$$

Where, $D_n\left(s\right)$ is the higher-order denominator polynomial, represented as,

$$\begin{aligned} D_n\left(s\right) &= \left[Q_0^-,Q_0^+\right]+\left[Q_1^-,Q_1^+\right]s+...+\left[Q_n^-,Q_n^+\right]s^n\\ &=\sum_{L=0}^{n}\left[Q_L^-,Q_L^+\right]s^L \end{aligned}$$

and

$$\begin{aligned} g_{qp}\left(s\right) &= \left[E_{qp}^-,E_{qp}^+\right]+\left[E_{qp}^-,E_{qp}^+\right]s+...+\left[E_{qp}^-,E_{qp}^+\right]s^{n-1}\\ &=\sum_{L=0}^{n-1}\left[E_{qp}^-,E_{qp}^+\right]s^L \end{aligned}$$

hence

$$G_{qp}\left(s\right)=\frac{\sum\limits_{L=0}^{n-1}\left[E_{qp}^-,E_{qp}^+\right]s^L}{\sum\limits_{L=0}^{n}\left[Q_L^-,Q_L^+\right]s^L} \tag{4.4}$$

where $q = 1, 2, 3...$ and $p = 1, 2, 3...$

The objective is to calculate the reduced k^{th} order multivariable interval transfer matrix expressed as follows

$$[R(s, h_{ij}, d_k)] = \frac{1}{d_k(s)} \begin{bmatrix} h_{11}(s) & h_{12}(s) & \cdots & h_{1p}(s) \\ h_{21}(s) & h_{22}(s) & \cdots & h_{2p}(s) \\ \vdots & \vdots & \ddots & \vdots \\ h_{q1}(s) & h_{q2}(s) & \cdots & h_{qp}(s) \end{bmatrix} \tag{4.5}$$
$$= \frac{h_{qp}(s)}{d_k(s)}$$

where

$$d_k(s) = \left[q_0^-, q_0^+\right] + \left[q_1^-, q_1^+\right] s + ...+ \left[q_k^-, q_k^+\right] s^k$$

and

$$h_{qp}(s) = \left[e_{qp}^-, e_{qp}^+\right] + \left[e_{qp}^-, e_{qp}^+\right] s + ... + \left[e_{qp}^-, e_{qp}^+\right] s^{k-1}$$

$$R_{qp}(s) = \frac{\sum_{L=0}^{k-1} \left[e_{qp}^-, e_{qp}^+\right] s^L}{\sum_{L=0}^{k} \left[q_L^-, q_L^+\right] s^L} \tag{4.6}$$

where $q = 1, 2, 3...$ and $p = 1, 2, 3...$

4.3 MODEL REDUCTION USING MIXED METHODS AND INTERVAL ARTHMETIC OPERATION

The three mixed methods are considered for reducing the linear dynamic interval systems. These three mixed methods are obtained based on interval arithmetic operations discussed in section 1.3.3, the denominator coefficients of the reduced order models are obtained by differentiation method while the numerator coefficients of the reduced order models are obtained by differentiation, factor division and Pade approximation methods. In addition, the mixed methods are compared qualitatively in terms ISE and IAE to know about the better reduction method among proposed three mixed methods.

In these three proposed mixed methods each method explained in two steps as follows

Method 1: Differentiation method

Step 1: Determination of the k^{th} order reduced interval denominator polynomial as given in eq. (4.2) is obtained by differentiation method [93].

$$D_{n-k}(s,b) = D_n(s,b) - \frac{s}{n} D_n'(s,b) \tag{4.7}$$

Where n is the order of original interval system denominator and k is the order of required reduced model.

Differentiate the reciprocal polynomial of Eq. (4.7) into $(n-k)$ times then reciprocate back and normalize to obtain the required reduced order interval coefficients of denominator polynomial.

$$d(s,d) = \left[d_0^-, d_0^+\right] + \left[d_1^-, d_1^+\right] s + \left[d_2^-, d_2^+\right] s^2 + ... + \left[d_k^-, d_k^+\right] s^k \tag{4.8}$$

This k^{th} order reduced interval denominator polynomial is common for remaining mixed methods.

Step 2: Determination of the $(k-1)^{th}$ order reduced interval numerator polynomial as given in Eq. (4.2) is obtained by using differentiation method [93]:

$$N_{n-k}(s,a) = N_n(s,a) - \frac{s}{n} N_n'(s,a) \tag{4.9}$$

Where n is the order of original interval system numerator and k is the order of required reduced model.

Differentiate the reciprocal polynomial Eq. (4.9) into $(n-k)$ times then reciprocate back and normalized to obtain the required reduced order interval numerator polynomial.

$$n(s,c) = \left[c_0^-, c_0^+\right] + \left[c_1^-, c_1^+\right] s + \left[c_2^-, c_2^+\right] s^2 + ... + \left[c_k^-, c_k^+\right] s^{k-1} \tag{4.10}$$

Method 2: Factor division algorithm and differentiation method

Step 1: The k^{th} order reduced interval coefficients of denominator polynomial $d(s)$ is already obtained in eq. (4.8).

Step 2: Determination of the k^{th} order reduced interval numerator polynomial as given in Eq. (4.2) is obtained by using factor division algorithm [118]

The eq. (4.1) may written as,

$$G_n(s) = \frac{N(s)\,d(s)}{D(s)\,d(s)} = \frac{N(s)\,d(s)/D(s)}{d(s)} \tag{4.11}$$

Therefore, the interval coefficients of numerator polynomial $n(s)$ of the reduced order model $R_k(s)$ is obtained by using the series expansion

$$N(s) = \frac{N(s)\,d(s)}{D(s)} = \frac{\left[f_0^-, f_0^+\right] + \left[f_1^-, f_1^+\right]s + \left[f_2^-, f_2^+\right]s^2 + ...}{\left[b_0^-, b_0^+\right] + \left[b_1^-, b_1^+\right]s + \left[b_2^-, b_2^+\right]s^2 + ...} \tag{4.12}$$

This is done by using the recurrence formula given as follows,

$$\left[c_0^-, c_0^+\right] = \frac{\left[f_0^-, f_0^+\right]}{\left[b_0^-, b_0^+\right]} \left\langle \begin{array}{llll} \left[f_0^-, f_0^+\right] & \left[f_1^-, f_1^+\right] & \left[f_2^-, f_2^+\right] ... \\ & & \\ \left[b_0^-, b_0^+\right] & \left[b_1^-, b_1^+\right] & \left[b_2^-, b_2^+\right] ... \end{array} \right.$$

$$\left[c_1^-, c_1^+\right] = \frac{\left[q_0^-, q_0^+\right]}{\left[b_0^-, b_0^+\right]} \left\langle \begin{array}{llll} \left[q_0^-, q_0^+\right] & \left[q_1^-, q_1^+\right] & \left[q_2^-, q_2^+\right] ... \\ & & \\ \left[b_0^-, b_0^+\right] & \left[b_1^-, b_1^+\right] & \left[b_2^-, b_2^+\right] ... \end{array} \right.$$

$$\cdots \tag{4.13}$$

$$\left[c_{r-2}^-, c_{r-2}^+\right] = \frac{\left[u_0^-, u_0^+\right]}{\left[b_0^-, b_0^+\right]} \left\langle \begin{array}{llll} \left[u_0^-, u_0^+\right] & \left[u_1^-, u_1^+\right] & \left[u_2^-, u_2^+\right] ... \\ & & \\ \left[b_0^-, b_0^+\right] & \left[b_1^-, b_1^+\right] & \left[b_2^-, b_2^+\right] ... \end{array} \right.$$

$$\left[c_{r-1}^-, c_{r-1}^+\right] = \frac{\left[v_0^-, v_0^+\right]}{\left[b_0^-, b_0^+\right]} \left\langle \begin{array}{l} \left[v_0^-, v_0^+\right] \\ \\ \left[b_0^-, b_0^+\right] \end{array} \right.$$

Where,

$$\left[q_i^-, q_i^+\right] = \left[f_{i+1}^-, f_{i+1}^+\right] - \left[c_0^-, c_0^+\right]\left[b_{i+1}^-, b_{i+1}^+\right] \text{ i} = 0, 1, 2, 3, ...$$

$$...$$

$$\left[v_0^-, v_0^+\right] = \left[u_1^-, u_1^+\right] - \left[c_{r-2}^-, c_{r-2}^+\right]\left[b_1^-, b_1^+\right]$$

The reduced order interval coefficients of numerator polynomial is

$$n\left(s,c\right)=\left[c_0^-,c_0^+\right]+\left[c_1^-,c_1^+\right]s+\left[c_2^-,c_2^+\right]s^2+...+\left[c_{r-1}^-,c_{r-1}^+\right]s^{r-1} \quad (4.14)$$

Method 3: Pade approximation and differentiation method.

Step 1: The k^{th} order reduced interval denominator polynomial $d\left(s\right)$ is already obtained in eq. (4.8).

Step 2: Determination of the k^{th} order reduced interval numerator polynomial as given in Eq. (4.2)is obtained by using pade approximation method [40]

The eq. (4.1) is written as,

$$\frac{N(s)}{D(s)}=\frac{\left[a_0^-,a_0^+\right]+\left[a_1^-,a_1^+\right]s+...+\left[a_{n-1}^-,a_{n-1}^+\right]s^{n-1}}{\left[b_0^-,b_0^+\right]+\left[b_1^-,b_1^+\right]s+...+\left[b_n^-,b_n^+\right]s^n}=\left[e_0^-,e_0^+\right]+\left[e_1^-,e_1^+\right]s+\left[e_2^-,e_2^+\right]s^2+... \quad (4.15)$$

We have the following set of linear simultaneous equations,

$$\begin{aligned}
&\left[a_0^-,a_0^+\right]=\left[b_0^-,b_0^+\right]\left[e_0^-,e_0^+\right]\\
&\left[a_1^-,a_1^+\right]=\left[b_0^-,b_0^+\right]\left[e_1^-,e_1^+\right]+\left[b_1^-,b_1^+\right]\left[e_0^-,e_0^+\right]\\
&\qquad\cdots\\
&\left[a_{n-1}^-,a_{n-1}^+\right]=\left[b_0^-,b_0^+\right]\left[e_{n-1}^-,e_{n-1}^+\right]+\left[b_1^-,b_1^+\right]\left[e_{n-2}^-,e_{n-2}^+\right]+\cdots+\left[b_{n-1}^-,b_{n-1}^+\right]\left[e_0^-,e_0^+\right]
\end{aligned} \quad (4.16)$$

The reduced order interval coefficients of numerator polynomial is obtained by using eq. 4.16

$$n\left(s,c\right)=\left[c_0^-,c_0^+\right]+\left[c_1^-,c_1^+\right]s+\left[c_2^-,c_2^+\right]s^2+...+\left[c_{k-1}^-,c_{k-1}^+\right]s^{k-1} \quad (4.17)$$

where

$$\begin{aligned}
&\left[c_0^-,c_0^+\right]=\left[d_0^-,d_0^+\right]\left[e_0^-,e_0^+\right]\\
&\left[c_1^-,c_1^+\right]=\left[d_0^-,d_0^+\right]\left[e_1^-,e_1^+\right]+\left[d_1^-,d_1^+\right]\left[e_0^-,e_0^+\right]\\
&\qquad\cdots
\end{aligned}$$

4.3.1 Numerical Examples and Results

To show the efficacy of the proposed reduction methods we considered popular SISO interval systems. The first example solved in detail whereas in the remaining examples the reduced systems are mentioned directly. The results are compare in terms of system response and performance indices.

4.3.1.1 Single Input Single Output Systems

Example 4.1: Consider a 3^{rd} order interval transfer function [106] given as

$$G_3(s) = \frac{[2,3]\,s^2 + [17.5,18.5]\,s + [15,16]}{[2,3]\,s^3 + [17,18]\,s^2 + [35,36]\,s + [20.5,21.5]}$$

Method 1: Differentiation method

The reduced order interval denominator polynomial is obtained by following eq. (4.7) and (4.8)

$$d(s) = \left(\frac{1}{3}\right)\left([15,18]\,s^2 + [69,73]\,s + [61.5,64.5]\right) \tag{4.18}$$

The reduced order interval numerator polynomial is achieved by following eq. (4.9) and (4.10)

$$n(s) = \left(\frac{1}{2}\right)\left([16.5,19.5]\,s + [30,32]\right)$$

Finally, The required reduced order interval transfer function is obtained as

$$R_2(s) = \frac{[24.75,29.25]\,s + [45,48]}{[15,18]\,s^2 + [69,73]\,s + [61.5,64.5]}$$

Method 2: Factor division algorithm and differentiation method.

The reduced order interval denominator polynomial is already obtained by using differentiation method in eq. (4.18) as

$$d(s) = [15,18]\,s^2 + [69,73]\,s + [61.5,64.5]$$

The reduced order interval numerator polynomial is obtained by following eq. (4.11) and (4.14)

$$n(s) = [13.9051,41.9273]\,s + [42.9069,50.3414]$$

Finally, The reduced order interval model of a factor division algorithm and differentiation method is as follows,

$$R_2(s) = \frac{[13.9051, 41.9273]\, s + [42.9069, 50.3414]}{[15, 18]\, s^2 + [69, 73]\, s + [61.5, 64.5]}$$

Method 3: Pade approximation and differentiation method

The reduced order interval denominator polynomial is already obtained by using differentiation method in eq. (4.18) as

$$d(s) = [15, 18]\, s^2 + [69, 73]\, s + [61.5, 64.5]$$

The reduced order interval numerator polynomial is obtained by using Pade approximation method eq. (4.11) and (4.14)

$$n(s) = [17.8334, 38.361]\, s + [42.9024, 50.3358]$$

Finally, The reduced order interval model of a Pade approximation and differentiation method is as follows,

$$R_2(s) = \frac{[17.8334, 38.361]\, s + [42.9024, 50.3358]}{[15, 18]\, s^2 + [69, 73]\, s + [61.5, 64.5]}$$

The comparison of step and bode responses of original interval system and reduced order interval systems of three mixed methods for Example 4.1 are shown in Figure 4.1 and 4.2. Further, ISE and IAE values for the three mixed methods are tabulated in Table 4.1. From these comparisons, it is clear that, among three mixed methods, the method 1 i.e. the differentiation method is giving better result.

Example 4.2: Consider a 3^{rd} order system [218] described by the interval Transfer function.

$$G_3(s) = \frac{[3, 4]\, s^2 + [25, 26]\, s + [14, 15]}{[7, 8]\, s^3 + [54, 55]\, s^2 + [90, 91]\, s + [35, 36]}$$

Method 1: The reduced order interval model is obtained by using differentiation method

$$R_2(s) = \frac{[36, 37.5]\, s + [42, 45]}{[52, 57]\, s^2 + [179, 183]\, s + [105, 108]}$$

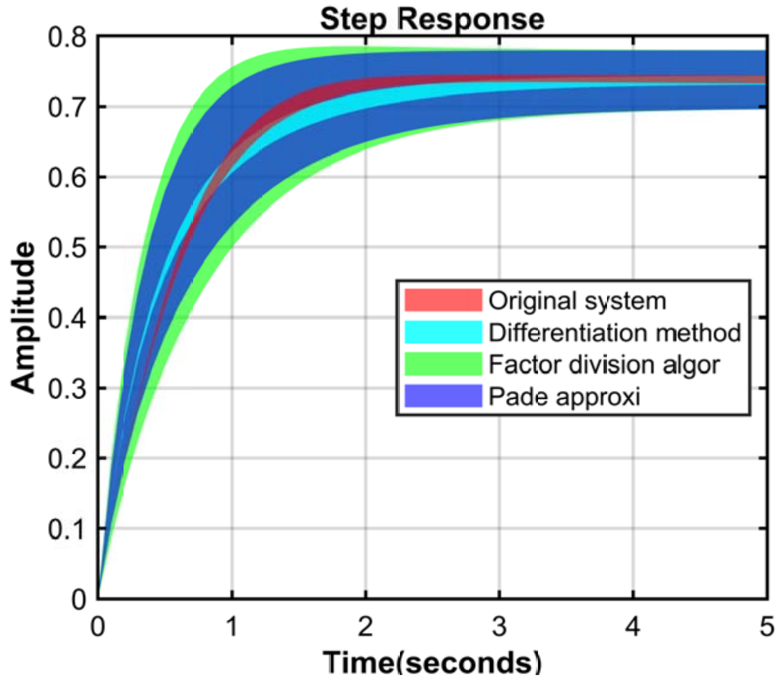


Fig. 4.1: Step response comparison of interval models for Example 4.1.

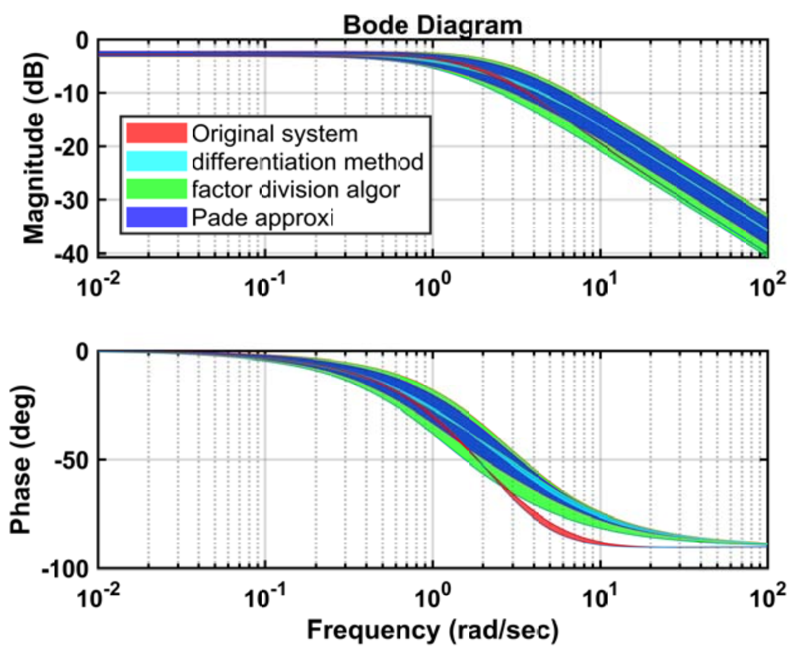


Fig. 4.2: Bode diagram response comparison of interval models for Example 4.1.

Table 4.1: Qualitative comparison of reduced order interval models with ISE and IAE for Example 4.1.

Reduction Methods	Lower Bound		Upper Bound	
	ISE	IAE	ISE	IAE
Method 1	0.002585	0.08396	0.002378	0.07216
Method 2	0.02418	0.2904	0.03248	0.2974
Method 3	0.01515	0.2295	0.02093	0.2424

Method 2: The reduced order interval model is obtained by using factor division algorithm and differentiation method

$$R_2\left(s\right)=\frac{\left[21.3673,29.6822\right]s+\left[13.611,15.428\right]}{\left[52,57\right]s^2+\left[179,183\right]s+\left[105,108\right]}$$

Method 3: The reduced order interval model is obtained by using pade approximation and differentiation method

$$R_2\left(s\right)=\frac{\left[21.3155,29.8207\right]s+\left[13.58,15.426\right]}{\left[52,57\right]s^2+\left[179,183\right]s+\left[105,108\right]}$$

The comparison of step and bode plot responses of original interval system and reduced order interval systems of three mixed methods are shown in Figure 4.3 and 4.4 for Example 4.2. The accuracy of the mixed methods measured in terms of ISE and IAE, and are depicted in Table 4.2. From this, it is clearly observed that, the differentiation method results in more accurate approximation with the original interval system among three mixed methods.

Example 4.3: Consider the 2^{nd} order system [296] described by the interval Transfer function

$$R_2\left(s\right)=\frac{\left[2,3\right]s+\left[15,16\right]}{\left[2,3\right]s^2+\left[12,13\right]s+\left[10,11\right]}$$

Method 1: The reduced order interval model is obtained by using differentiation method

$$R_1\left(s\right)=\frac{\left[30,32\right]}{\left[11,14\right]s+\left[20,22\right]}$$

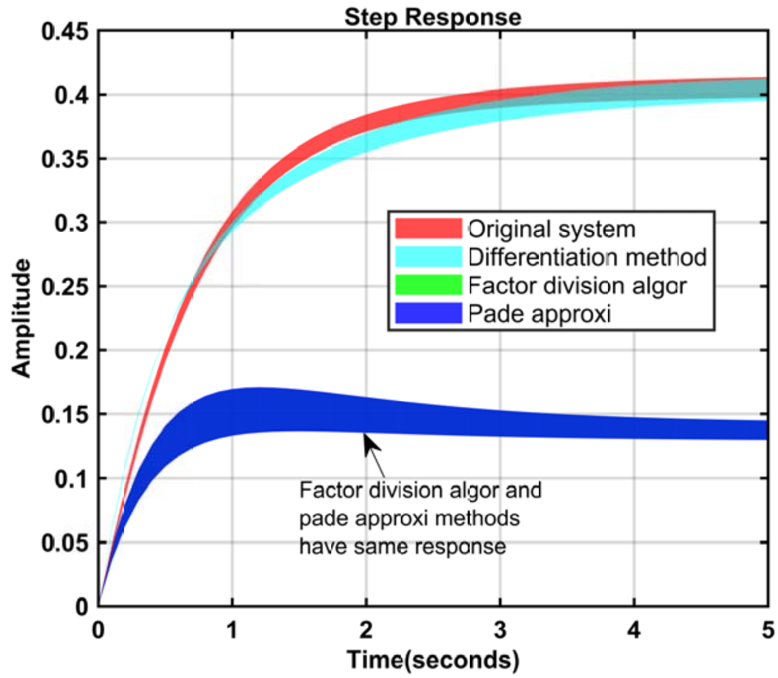


Fig. 4.3: Step response comparison of interval models for Example 4.2.

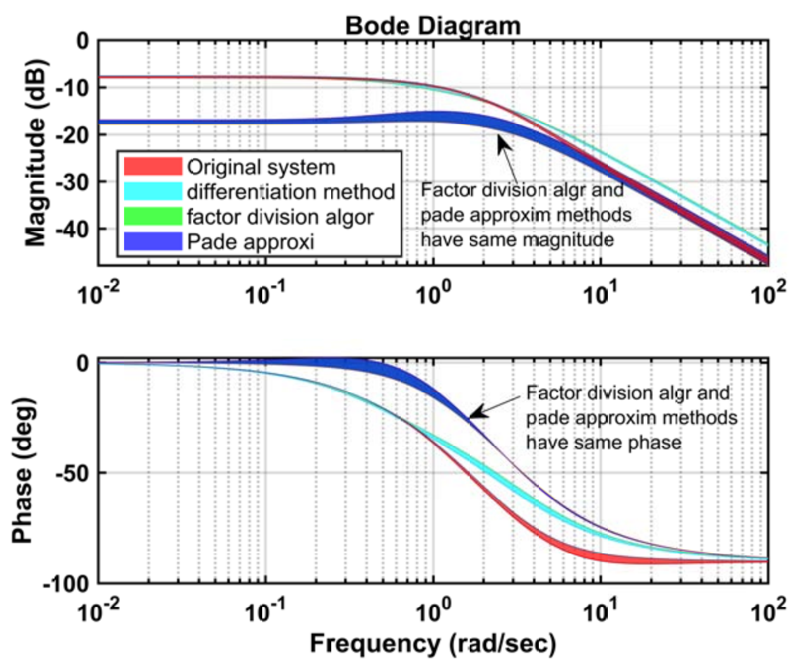


Fig. 4.4: Bode diagram response comparison of interval models for Example 4.2.

Table 4.2: Qualitative comparison of reduced order interval models with ISE and IAE for Example 4.2.

Reduction Methods	Lower Bound		Upper Bound	
	ISE	IAE	ISE	IAE
Method 1	0.000737	0.04905	0.000451	0.03899
Method 2	0.1813	0.7941	0.1652	0.7472
Method 3	0.1817	0.7952	0.1649	0.746

Method 2: The reduced order interval model is obtained by using factor division algorithm and differentiation method

$$R_1(s) = \frac{[27.27, 35.2]}{[11, 14]\, s + [20, 22]}$$

Method 3: The reduced order interval model is obtained by using pade approximation and differentiation method

$$R_1(s) = \frac{[27.2, 35.2]}{[11, 14]\, s + [20, 22]}$$

The step and bode plot responses of original interval system and reduced order interval systems of three mixed methods for Example 4.3 are shown in Figures 4.5 and 4.6. Further, The ISE and IAE for three mixed method comparisons are tabulated in Table 4.3. In Table 4.3, the differentiation method exhibits less error among three mixed methods.

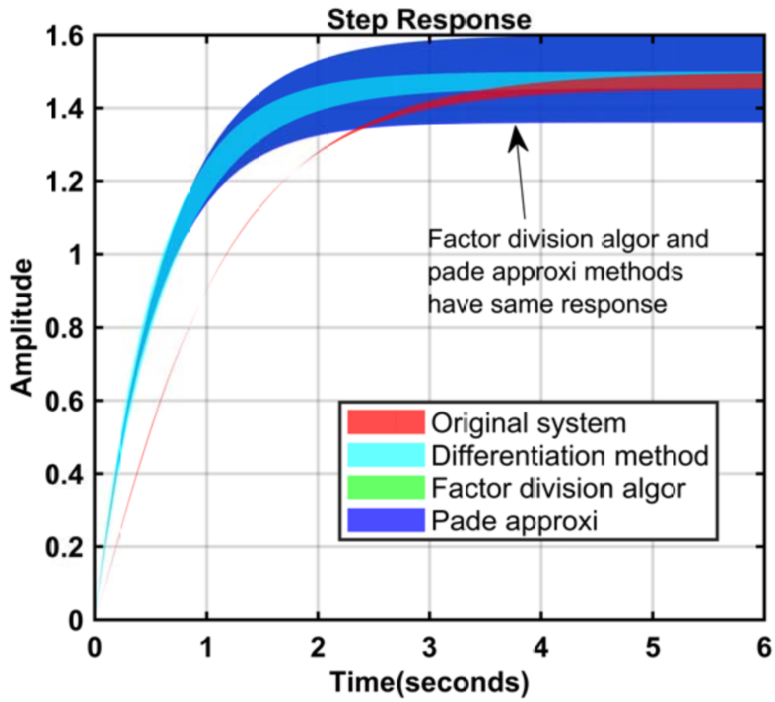


Fig. 4.5: Step response comparison of interval models for Example 4.3.

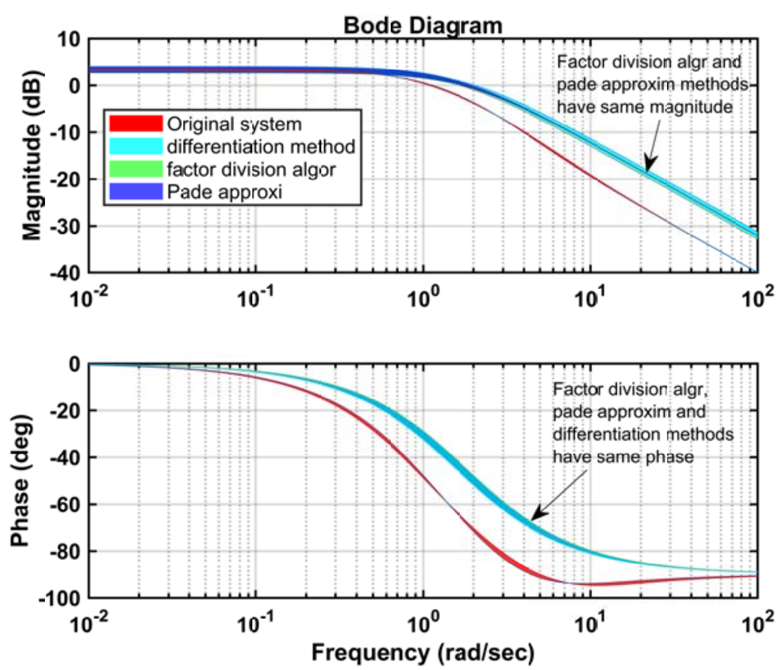


Fig. 4.6: Bode diagram response comparison of interval models for Example 4.3.

Table 4.3: Qualitative comparison of reduced order interval models with ISE and IAE for Example 4.3.

Reduction Methods	Lower Bound		Upper Bound	
	ISE	IAE	ISE	IAE
Method 1	0.198	0.764	0.09693	0.516
Method 2	0.107	0.613	0.2968	1.151
Method 3	0.1067	0.616	0.2968	1.151

4.4 MODEL REDUCTION USING DIFFERENTIATION METHOD AND KHARITONOV'S THEOREM

Kharitonov [8] proposed a pioneering technique to deal with interval systems. The Kharitonov's theorem is useful for finding the interval systems stability criterion by separating the interval polynomial into its four Kharitonov fixed coefficient polynomials, if these four Kharitonov polynomials meet the requirement of the stability criterion then the interval system is said to be stable. Further, Kharitonov's theorem minimize the computational burden of interval operations by avoiding the use of interval arithmetic operations.

In this section, a new reduction method is proposed to reduce the order of higher-order interval system (HOIS). The proposed ROIM is achieved based on differentiation method using Kharitonov's theorem. This method is significant for both SISO interval systems and MIMO interval systems. This technique is simple in mathematical calculation and preserves the dominant properties of the original interval system in its ROIM. To show the effectiveness of this technique, it is illustrated with some benchmark problems. The results are compared in terms of step and bode diagram responses along with their performance indices.

The procedural steps are given to describe the proposed technique as follows,

Step 1: According to Kharitonov's theorem, the HOIS of eq. (4.1) are represented into four fixed coefficient Kharitonov polynomials, Numerator,

$$\left.\begin{aligned}N^1(s) &= a_0^- + a_1^- s + a_2^+ s^2 + a_3^+ s^3 + a_4^- s^4 + \cdots \\ N^2(s) &= a_0^- + a_1^+ s + a_2^+ s^2 + a_3^- s^3 + a_4^- s^4 + \cdots \\ N^3(s) &= a_0^+ + a_1^- s + a_2^- s^2 + a_3^+ s^3 + a_4^+ s^4 + \cdots \\ N^4(s) &= a_0^+ + a_1^+ s + a_2^- s^2 + a_3^- s^3 + a_4^+ s^4 + \cdots\end{aligned}\right\} \tag{4.19}$$

Denominator,

$$\left.\begin{aligned}D^1(s) &= b_0^- + b_1^- s + b_2^+ s^2 + b_3^+ s^3 + b_4^- s^4 + \cdots \\ D^2(s) &= b_0^- + b_1^+ s + b_2^+ s^2 + b_3^- s^3 + b_4^- s^4 + \cdots \\ D^3(s) &= b_0^+ + b_1^- s + b_2^- s^2 + b_3^+ s^3 + b_4^+ s^4 + \cdots \\ D^4(s) &= b_0^+ + b_1^+ s + b_2^- s^2 + b_3^- s^3 + b_4^+ s^4 + \cdots\end{aligned}\right\} \tag{4.20}$$

Step 2: By using numerator and denominator Kharitonov polynomials of eq. (4.19) and (4.20), the Kharitonov transfer functions are obtained as given below

$$\left.\begin{aligned}G^1(s) &= \tfrac{N^1(s)}{D^1(s)} = \tfrac{a_0^- + a_1^- s + a_2^+ s^2 + a_3^+ s^3 + \cdots}{b_0^- + b_1^- s + b_2^+ s^2 + b_3^+ s^3 + \cdots} \\ G^2(s) &= \tfrac{N^2(s)}{D^2(s)} = \tfrac{a_0^- + a_1^+ s + a_2^+ s^2 + a_3^- s^3 + \cdots}{b_0^- + b_1^+ s + b_2^+ s^2 + b_3^- s^3 + \cdots} \\ G^3(s) &= \tfrac{N^3(s)}{D^3(s)} = \tfrac{a_0^+ + a_1^- s + a_2^- s^2 + a_3^+ s^3 + \cdots}{b_0^+ + b_1^- s + b_2^- s^2 + b_3^+ s^3 + \cdots} \\ G^4(s) &= \tfrac{N^4(s)}{D^4(s)} = \tfrac{a_0^+ + a_1^+ s + a_2^- s^2 + a_3^- s^3 + \cdots}{b_0^+ + b_1^+ s + b_2^- s^2 + b_3^- s^3 + \cdots}\end{aligned}\right\} \tag{4.21}$$

Step 3: By using differentiation method, the higher-order Kharitonov transfer functions given in eq. (4.21) are reduced to the lower order models as given below

1. The reciprocal of high-order transfer function $G^1(s)$ is,

$$\bar{G}^1(s) = \frac{1}{s} G^1\left(\frac{1}{s}\right) = \frac{a_0^- s^{n-1} + a_1^- s^{n-2} + a_2^+ s^{n-3} + \cdots}{b_0^- s^n + b_1^- s^{n-1} + b_2^+ s^{n-2} + \cdots} \tag{4.22}$$

2. The ROM is obtained by differentiating eq. (4.22) $(n-k)$ times, where n is the order of HOS and k is the order of ROM.

$$\bar{R}^1(s) = \frac{c_0^- s^{k-1} + c_1^- s^{k-2} + \cdots}{d_0^- s^k + d_1^- s^{k-1} + d_2^+ s^{k-2} + \cdots} \tag{4.23}$$

3. Reciprocate eq. (4.23) once again to bring reduced order model into its earlier form

$$R^1(s) = \frac{c_0^- + c_1^- s + \cdots}{d_0^- + d_1^- s + d_2^+ s^2 + \cdots} \tag{4.24}$$

4. Finally, the ROM is achieved by applying steady state correction k

$$R^1(s) = \frac{n^1(s)}{d^1(s)} = k\frac{c_0^- + c_1^- s + \cdots}{d_0^- + d_1^- s + d_2^+ s^2 + \cdots} \tag{4.25}$$

Similarly, the reduced order models for $G^2(s), G^3(s)$ and $G^4(s)$ are obtained by following Equations eq. (4.21) and (4.25).

$$\left.\begin{array}{l} R^2(s) = \frac{n^2(s)}{d^2(s)} = k\frac{c_0^- + c_1^+ s + \cdots}{d_0^- + d_1^+ s + d_2^+ s^2 + \cdots} \\ R^3(s) = \frac{n^3(s)}{d^3(s)} = k\frac{c_0^+ + c_1^- s + \cdots}{d_0^+ + d_1^- s + d_2^- s^2 + \cdots} \\ R^4(s) = \frac{n^4(s)}{d^4(s)} = k\frac{c_0^+ + c_1^+ s + \cdots}{d_0^+ + d_1^+ s + d_2^- s^2 + \cdots} \end{array}\right\} \tag{4.26}$$

Step 5: The four fixed parameter ROMs given in eq. (4.25) and (4.26) can be written into sixteen combinations of ROIMs (one to each), by following sixteen plant theorem [9, 10], the general form is,

$$R^{i,j}(s) = \frac{[n^i(s), n^j(s)]}{[d^i(s), d^j(s)]} \tag{4.27}$$

For $i, j = 1, 2, 3, 4$.

Step 6: The required ROIM eq. (4.2) is obtained by choosing least error ROIM by using ISE [75] comparison between step response of HOIS eq. (4.1) and the sixteen combinations of ROIMs of eq. (4.27).

$$\text{ISE} = \int_0^\infty \left[g_n(t) - r^{i,j}(t)\right]^2 dt \tag{4.28}$$

Where g_n is the HOIS and $r^{i,j}(t)$ is the ROIM.

4.4.1 Numerical Examples and Results

To show the efficacy and powerfulness of the presented reduction technique we considered well-known SISO/MIMO interval systems. The first example solved in detail whereas in the remaining examples the reduced systems are mentioned directly. The results are compared in terms of system response and performance indices.

4.4.1.1 Single Input Single Output Systems

Example 4.4: The 7^{th} order interval transfer function [293] is described as follows

$$G_7(s) = \frac{\begin{array}{c}[1.9, 2.1]s^6 + [24.7, 27.3]s^5 + [157.7, 174.3]s^4 + [542, 599]s^3 \\ +[930, 1028]s^2 + [721.8, 797.8]s + [187.1, 206.7]\end{array}}{\begin{array}{c}[0.95, 1.05]s^7 + [8.779, 9.703]s^6 + [52.23, 57.73]s^5 + [182.9, 202.1]s^4 \\ +[429, 474.2]s^3 + [572.5, 632.7]s^2 + [325.3, 359.5]s + [57.35, 63.39]\end{array}}$$

The interval transfer function is converted to fixed parameter Kharitonov transfer functions by following the eq. (4.19) and (4.21),

$$\begin{aligned}
G^1(s) &= \frac{2.1s^6+24.7s^5+157.7s^4+599s^3+1028s^2+721.8s+187.1}{1.05s^7+9.703s^6+52.23s^5+182.9s^4+474.2s^3+632.7s^2+325.3s+57.35} \\
G^2(s) &= \frac{2.1s^6+27.3s^5+157.7s^4+542s^3+1028s^2+797.8s+187.1}{0.95s^7+9.703s^6+57.73s^5+182.9s^4+429s^3+632.7s^2+359.5s+57.35} \\
G^3(s) &= \frac{1.9s^6+24.7s^5+174.3s^4+599s^3+930s^2+721.8s+206.7}{1.05s^7+8.779s^6+52.23s^5+202.1s^4+474.2s^3+572.5s^2+325.3s+63.39} \\
G^4(s) &= \frac{1.9s^6+27.3s^5+174.3s^4+542s^3+930s^2+797.8s+206.7}{0.95s^7+8.779s^6+57.73s^5+202.1s^4+429s^3+572.5s^2+359.5s+63.39}
\end{aligned} \tag{4.29}$$

The higher-order systems given in eq. (4.29) are reduced to the LOMs by following eq. (4.22)-(4.26).

$$\begin{aligned}
R^1(s) &= \frac{3.9928s+6.21}{s^2+3.0848s+1.9035} \\
R^2(s) &= \frac{0.4413s+0.6210}{s^2+0.3409s+0.1903} \\
R^3(s) &= \frac{4.4127s+7.582}{s^2+3.4092s+2.3252} \\
R^4(s) &= \frac{4.8773s+7.5820}{s^2+3.7676s+2.3252}
\end{aligned}$$

Finally, the required ROIM is obtained by using eq. (4.27) and (4.28), the second order interval model is,

$$R_2(s) = \frac{[3.9928, 4.8773]\, s + [6.21, 7.5820]}{[1, 1]\, s^2 + [3.0848, 3.7676]\, s + [1.9035, 2.3252]}$$

The ROIM obtained by using Kranthi (FDA & DM) et at [297] is,

$$R(s) = \frac{[128248.353, 735431.927]\, s + [426566.748, 575742.576]}{[68700, 75924]\, s^2 + [234216, 258840]\, s + [144522, 159742.8]}$$

The ROIM obtained by using Kranthi (PA&DM) et at [297] is,

$$R(s) = \frac{[124571.873, 735533.921]\, s + [419496.962, 569447.975]}{[68700, 75924]\, s^2 + [234216, 258840]\, s + [144522, 159742.8]}$$

The step and bode responses of original lower and upper bound interval systems of proposed method and some other reduction methods [231,236,240,255,297,298] of lower and upper bounds for Example 4.4 are compared and are shown in Figures 4.7 and 4.8. It is clear that proposed reduction method is closely matching with original higher order response. Moreover, the ISE and IAE values of this method and other reduction methods are shown in Table 4.4, which shows that the proposed method gives much lower ISE and IAE values are compared with given different reduction methods.

Table 4.4: Comparison of the ISE and IAE with proposed and other different reduction techniques for Example 4.4.

Reduction Methods	ISE		IAE	
	Lower Bound	Upper Bound	Lower Bound	Upper Bound
Proposed Method	1.0058	0.9574	2.912	2.798
Bandyopadhyay [298]	2.259	5.954	4.758	7.735
Kranthi(FDA&DM) [297]	2.434	6.346	6.125	11.231
Kranthi(PA&DM) [297]	3.181	7.379	7.132	11.982
Selvaganesan [231]	4.305	7.301	8.804	12.372
N. V. Anand [236]	0.991	3.2979	3.425	4.536
T. Babu [255]	2.428	4.323	5.734	9.325
Siva Kumar et al [240]	0.9357	1.0915	2.972	3.043

Example 4.5: Consider the 3^{rd} order interval system from Example 4.1 in section 4.3.1.1. Using proposed method, the required reduced order model is obtained as,

$$R_2(s) = \frac{[1.4583, 1.6323]\, s + [2.5, 2.8235]}{[1, 1]\, s^2 + [3.88, 4.2352]\, s + [3.416, 3.7941]}$$

The step and bode responses of original lower and upper bound interval systems and the proposed and other reduction methods [106, 221, 297] of lower and upper bounds for Example 4.5 are compared and are shown in Figures 4.9 and 4.10. Clearly, proposed method gave close approximation with original system in both

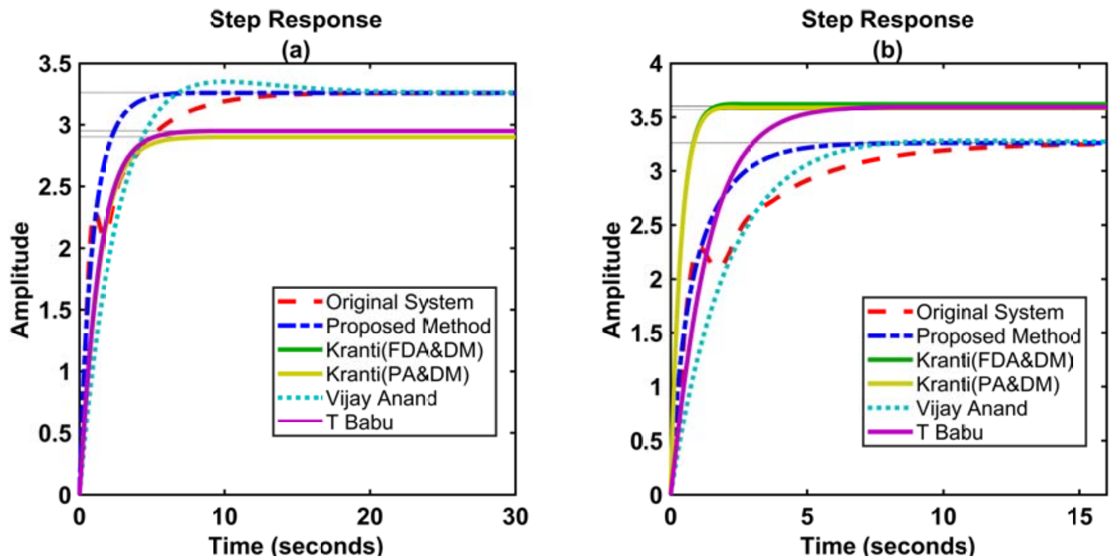


Fig. 4.7: Step response comparison of (a) Lower bound (b) Upper bounds for Example 4.4.

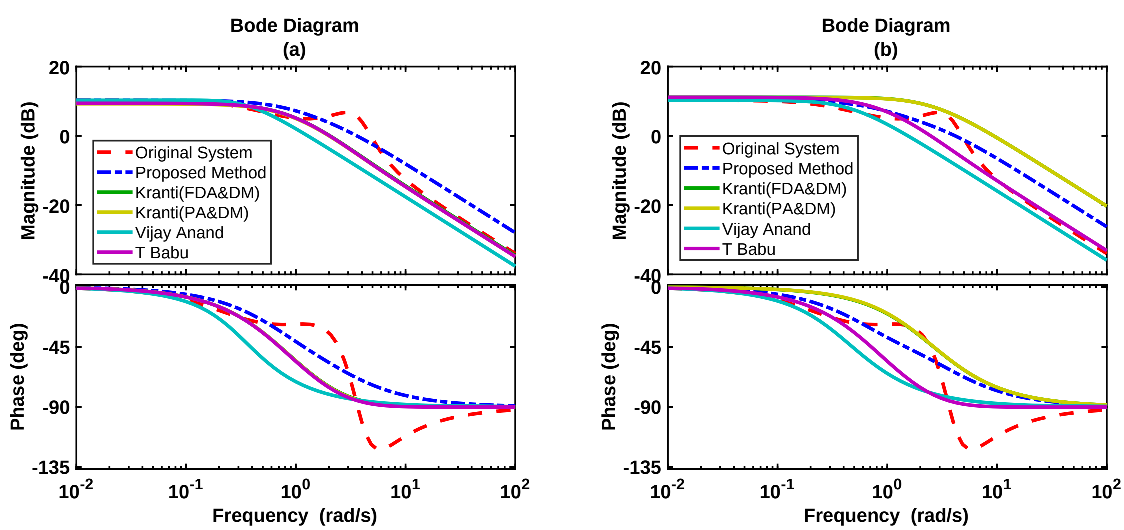


Fig. 4.8: Bode diagram response comparisons of (a) Lower bound (b) Upper bounds for Example 4.4.

lower bound and upper bound. Moreover, the ISE values of reduction methods are compared and are shown in Table 4.5, which shows that the proposed method obtained comparable low ISE values.

Table 4.5: The ISE comparison with proposed and other reduction methods for Example 4.5.

Reduced Methods	ISE	
	Lower Bound	Upper Bound
Proposed Method	3.6511×10^{-4}	7.6452×10^{-4}
Bandyopadhyay [106]	0.0015	2.9590×10^{-3}
Kumar D Kranthi et al. [297]	0.0196	0.0248
G.V.K R. Sastry et al. [221]	0.117	0.0084

4.4.1.2 Multiple Input Multiple Output System

Example 4.6: Further, to show the powerfulness of the proposed method it is also extended to the MIMO interval systems. Consider original multivariable interval transfer function matrix [108] given as,

$$[G_2(s)] = \frac{1}{D_2(s)} \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix}$$

where

$$D_2(s) = [0.537464, 1.537464]\, s^2 + [1.379131, 2.379313]\, s + [1, 2]$$

$$g_{11}(s) = [0.622, 1.622]\, s + [1.00721, 2.00721]$$

$$g_{12}(s) = [462.6, 463.6]\, s + [715.2653, 716.62653]$$

$$g_{21}(s) = [3.563, 4.563]\, s + [4.8589, 5.8589]$$

$$g_{22}(s) = [610.435, 611.435]\, s + [1000.5485, 1001.5485]$$

The desired ROM is obtained by proposed technique

$$[R_1(s)] = \frac{1}{D_1(s)} \begin{bmatrix} [2.01442, 4.01442] & [1430.5306, 1433.25306] \\ [9.7178, 11.7178] & [2001.097, 2003.097] \end{bmatrix}$$

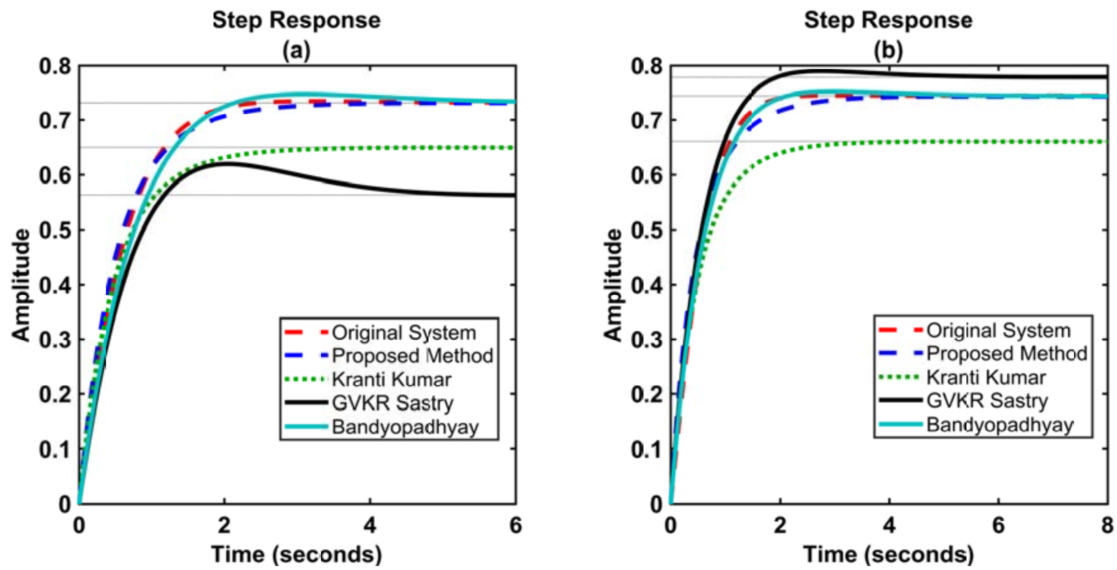


Fig. 4.9: Step response comparison of (a) Lower bound (b) Upper bounds for Example 4.5.

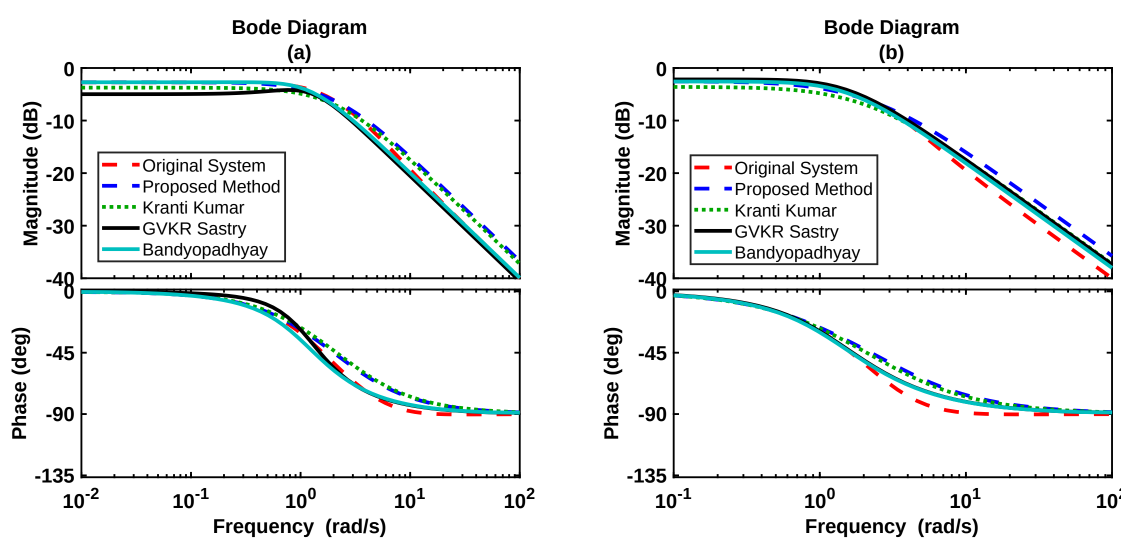


Fig. 4.10: Bode diagram response comparisons of (a) Lower bound (b) Upper bounds for Example 4.5.

where $D_1(s) = [1.379131, 2.379313]\, s + [2, 4]$

The ROM obtained by Sastry and Mallikarjuna Rao [108] is,

$$[R_1(s)] = \frac{1}{D_1(s)} \begin{bmatrix} [0.2115, 2.9] & [150.206, 1038.58] \\ [1.0204, 8.49] & [210.12, 1452.24] \end{bmatrix}$$

where $D_1(s) = [1, 1]\, s + [0.2, 2.9]$

The ROIM obtained by using Kranthi (FDA&DM) et at. [297] is,

$$[R_{FDA}\,(s)] = \frac{1}{D_{FDA}\,(s)} \begin{bmatrix} [1.00721, 8.0288] & [715.2653, 2866.5061] \\ [4.8589, 23.435] & [1000.5485, 4006.194] \end{bmatrix}$$

where $D_{FDA}\,(s) = [1.379131, 2.379313]\, s + [2, 4]$

The ROIM obtained by using Kranthi (PA&DM) et at. [297] is,

$$[R_{PA}\,(s)] = \frac{1}{D_{PA}\,(s)} \begin{bmatrix} [1.00721, 8.0288] & [715.2653, 2865.7021] \\ [4.8589, 23.4356] & [1000.6192, 4006.194] \end{bmatrix}$$

where $D_{PA}\,(s) = [1.379131, 2.379313]\, s + [2, 4]$

The step and bode response of original lower and upper bound interval systems and the proposed and other reduction methods [108,297] of lower and upper bounds for Example 4.6 are compared and are shown in Figures 4.11 - 4.14. Clearly, proposed method gave close approximation with original system in both lower bound and upper bounds. Moreover, the ISE values of reduction methods are compared and are shown in Table 4.6, which shows that the proposed method provided least ISE values.

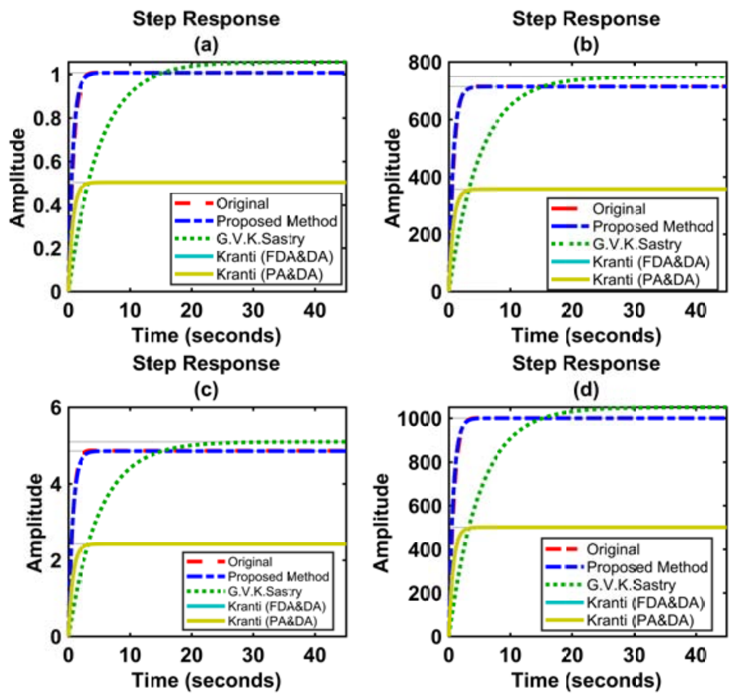


Fig. 4.11: Step response comparisons of lower bounds of (a)G_{r11} (b) G_{r12} (c) G_{r21} (d) G_{r22} for Example 4.6.

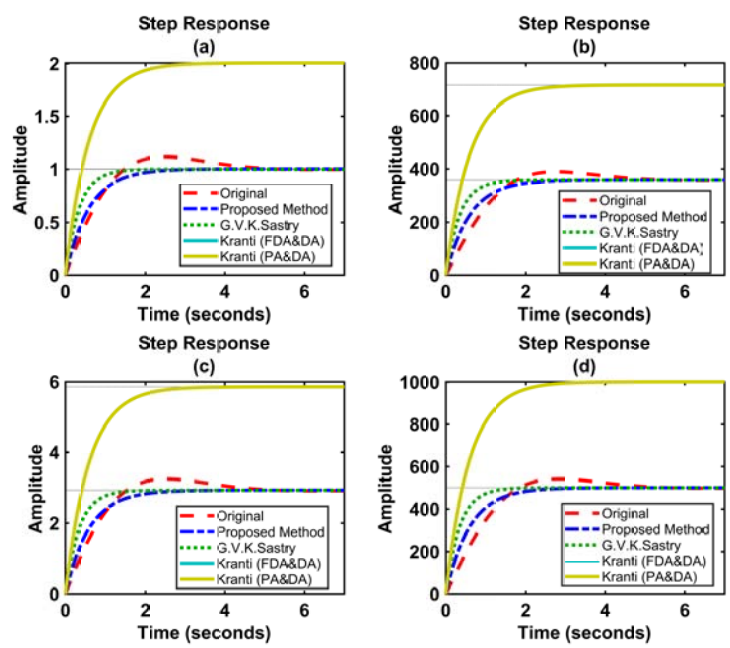


Fig. 4.12: Step response comparisons of upper bounds of (a)G_{r11} (b) G_{r12} (c) G_{r21} (d) G_{r22} for Example 4.6.

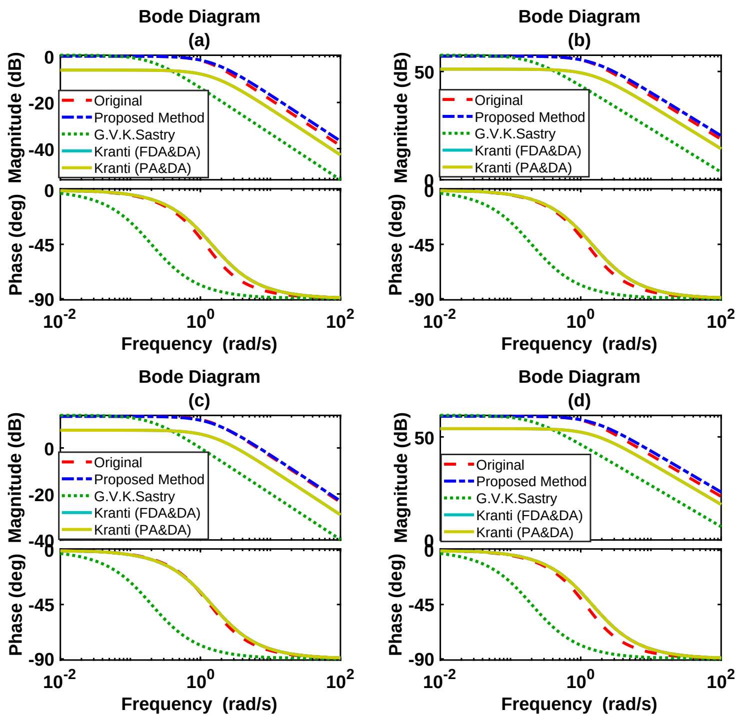


Fig. 4.13: Bode response comparisons of lower bounds of (a)G_{r11} (b) G_{r12} (c) G_{r21} (d) G_{r22} for Example 4.6.

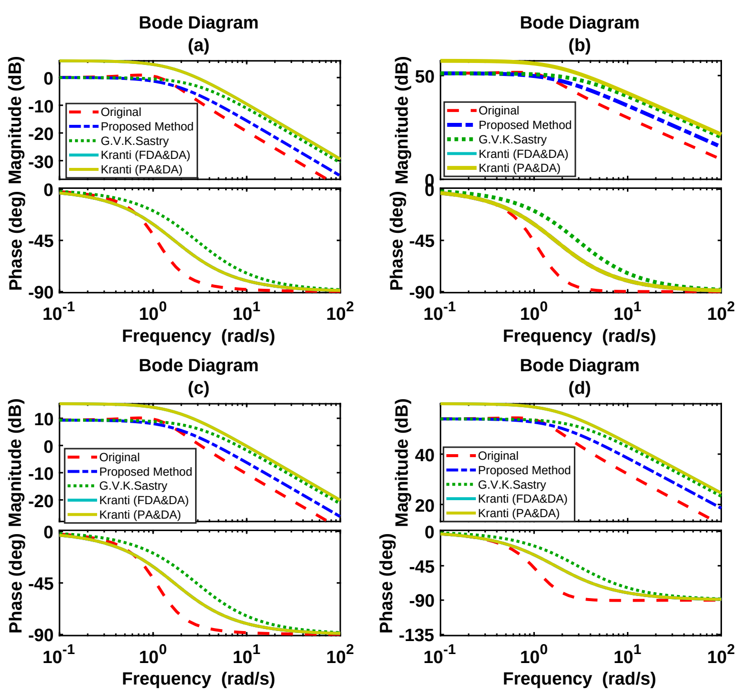


Fig. 4.14: Bode response comparisons of upper bounds of (a)G_{r11} (b) G_{r12} (c) G_{r21} (d) G_{r22} for Example 4.6.

Table 4.6: The ISE comparison with proposed and other reduction methods for Example 4.6.

Reduction Method	ISE							
	Lower Bound				Upper Bound			
	r_{11}	r_{12}	r_{21}	r_{22}	r_{11}	r_{12}	r_{21}	r_{22}
Proposed Method	0.0026	643.9382	0.0154	2.9457×10^3	0.0293	3.1809×10^3	0.2299	6.6898×10^3
Kranthi (FDA&DM) [297]	1.2	6.13×10^5	29.35	1.181×10^6	4.278	5.67×10^5	36.7	1.118×10^6
Kranthi (PA&DM) [297]	1.2	6.13×10^5	29.35	1.181×10^6	4.278	5.67×10^5	36.7	1.118×10^6
Sastry [108]	1.4323	7.3625×10^5	35.9562	1.4065×10^6	0.0413	7.9431×10^3	0.3666	1.7477×10^4

4.5 MODEL REDUCTION USING STABILITY EQUATION METHOD AND KHARITONOV'S THEOREM

A new hybrid order reduction technique is presented for reducing higher-order continuous linear time invariant interval systems. The reduced order interval model is obtained by using Kharitonov's theorem, stability equation method and the sixteen plant theorem [9, 10].

The procedural steps are given to describe the proposed technique as follows,

Step 1: The Kharitonov fixed parameter transfer functions are obtained by following eq. (4.19)-(4.21) in section 4.4

$$\left.\begin{array}{l}
G^1(s) = \frac{N^1(s)}{D^1(s)} = \frac{a_0^- + a_1^- s + a_2^+ s^2 + a_3^+ s^3 + \cdots}{b_0^- + b_1^- s + b_2^+ s^2 + b_3^+ s^3 + \cdots} \\
G^2(s) = \frac{N^2(s)}{D^2(s)} = \frac{a_0^- + a_1^+ s + a_2^+ s^2 + a_3^- s^3 + \cdots}{b_0^- + b_1^+ s + b_2^+ s^2 + b_3^- s^3 + \cdots} \\
G^3(s) = \frac{N^3(s)}{D^3(s)} = \frac{a_0^+ + a_1^- s + a_2^- s^2 + a_3^+ s^3 + \cdots}{b_0^+ + b_1^- s + b_2^- s^2 + b_3^+ s^3 + \cdots} \\
G^4(s) = \frac{N^4(s)}{D^4(s)} = \frac{a_0^+ + a_1^+ s + a_2^- s^2 + a_3^- s^3 + \cdots}{b_0^+ + b_1^+ s + b_2^- s^2 + b_3^- s^3 + \cdots}
\end{array}\right\} \tag{4.30}$$

Step 2: The numerator $N^1(s)$ and denominator $D^1(s)$ of Kharitonov plant $G^1(s)$ of eq. (4.30) are separating into their even and odd parts

$$G^1(s) = \frac{N^1{}_{even}(s) + N^1{}_{odd}(s)}{D^1{}_{even}(s) + D^1{}_{odd}(s)} \tag{4.31}$$

where

$$\begin{array}{l}
N^1{}_{even}(s) = \sum\limits_{i=0,2,4}^{m} a_i s^i = a_0 \prod\limits_{i=2}^{m} \left(1 + \frac{s^2}{\alpha_i^2}\right) \\
N^1{}_{odd}(s) = \sum\limits_{i=1,3,5}^{m} a_i s^i = a_1 s \prod\limits_{i=3}^{m} \left(1 + \frac{s^2}{\beta_i^2}\right)
\end{array} \tag{4.32}$$

and

$$D^1{}_{even}(s) = \sum_{i=0,2,4}^{n} b_i s^i = b_0 \prod_{i=2}^{n} \left(1 + \frac{s^2}{\alpha_i^2}\right)$$
$$D^1{}_{odd}(s) = \sum_{i=1,3,5}^{n} b_i s^i = b_1 s \prod_{i=3}^{n} \left(1 + \frac{s^2}{\beta_i^2}\right) \tag{4.33}$$

where

$$m = \begin{cases} m/2 & \text{if } m \text{ is even} \\ (m-1)/2 & \text{if } m \text{ is odd} \end{cases}$$
$$n = \begin{cases} n/2 & \text{if } m \text{ is even} \\ (n-1)/2 & \text{if } m \text{ is odd} \end{cases}$$

Eq. (4.32) and (4.33) are called stability equations [89]. The α_i and β_i are the roots of the even and odd parts of the numerator and denominators of the stability equations.

$$\alpha_1^2 < \alpha_2^2 < \alpha_3^2 < \cdots$$
$$\beta_1^2 < \beta_2^2 < \beta_3^2 < \cdots \tag{4.34}$$

Since smaller magnitude even or odd terms are more dominant than those of larger magnitude even or odd terms. Then, by neglecting the even or odd terms of numerator and denominators with larger magnitudes, the reduced order stability equations are obtained.

$$R^1(s) = \frac{n^1{}_{even}(s) + n^1{}_{odd}(s)}{d^1{}_{even}(s) + d^1{}_{odd}(s)} \tag{4.35}$$

where

$$n^1{}_{even}(s) = \sum_{i=0,2,4}^{r-1} c_i s^i = c_0 \prod_{i=1}^{m'} \left(1 + \frac{s^2}{\alpha_i^2}\right)$$
$$n^1{}_{odd}(s) = \sum_{i=1,3,5}^{r-1} c_i s^i = c_1 s \prod_{i=1}^{m'} \left(1 + \frac{s^2}{\beta_i^2}\right) \tag{4.36}$$

and

$$d^1{}_{even}(s) = \sum_{i=0,2,4}^{r} d_i s^i = d_0 \prod_{i=1}^{n'} \left(1 + \frac{s^2}{\alpha_i^2}\right)$$
$$d^1{}_{odd}(s) = \sum_{i=1,3,5}^{r} d_i s^i = d_1 s \prod_{i=1}^{n'} \left(1 + \frac{s^2}{\beta_i^2}\right) \tag{4.37}$$

Step 3: The reduced order model is obtained by combining the both numerator and denominators of stability equations Eq. (4.36) and (4.37), and, normalized to obtain the reduced order model as

$$R^1(s) = \frac{n^1(s)}{d^1(s)} = \frac{c_0^- + c_1^- s + \cdots}{d_0^- + d_1^- s + d_2^+ s^2 + \cdots} \tag{4.38}$$

Similarly, the lower order models of $G^2(s), G^3(s)$ and $G^4(s)$ are obtained by following above steps 1 – 3.

$$\left.\begin{array}{l} R^2(s) = \frac{n^2(s)}{d^2(s)} = \frac{c_0^- + c_1^+ s + \cdots}{d_0^- + d_1^+ s + d_2^+ s^2 + \cdots} \\ R^3(s) = \frac{n^3(s)}{d^3(s)} = \frac{c_0^+ + c_1^- s + \cdots}{d_0^+ + d_1^- s + d_2^- s^2 + \cdots} \\ R^4(s) = \frac{n^4(s)}{d^4(s)} = \frac{c_0^+ + c_1^+ s + \cdots}{d_0^+ + d_1^+ s + d_2^- s^2 + \cdots} \end{array}\right\} \tag{4.39}$$

Step 4: From the above reduced order fixed parameter transfer functions Eq. (4.38) and (4.39), the sixteen combinations of reduced order interval models can be constructed by using sixteen plant theorem. The general form is,

$$R^{i,j}(s) = \frac{[n^i(s), n^j(s)]}{[d^i(s), d^j(s)]} \tag{4.40}$$

with $i, j \in \{1, 2, 3, 4\}$

Step 5: The required lower order interval model eq. (4.2) is obtained by comparing ISE [75] between the transient parts of the unit step response of higher-order interval system eq. (4.1) with the sixteen combinations of reduced order interval models of eq. (4.40)

$$\mathrm{ISE} = \int_0^\infty \left[g(t) - r^{i,j}(t)\right]^2 dt \tag{4.41}$$

Where $g(t) = g_{\mathrm{Lor}U}(t)$ and $r^{i,j}(t) = {r^{i,j}}_{\mathrm{Lor}U}(t)$ are the original interval and reduced order interval models of lower and upper bounds respectively.

4.5.1 Numerical Examples and Results

In this section, the superiority of the proposed technique is justified by solving popular SISO/MIMO systems. The first example solved in detail whereas in the remaining examples the reduced systems are mentioned directly. The results are compared in terms of step and bode responses and performance indices viz. ISE, IAE, ITSE and ITAE.

4.5.1.1 Single Input Single Output Systems

Example 4.7: Consider a 6^{th} order interval system transfer function [108] described as follows.

$$G_6(s) = \frac{[1,2]\,s^5 + [30,34]\,s^4 + [330,360]\,s^3 + [1650,1800]\,s^2 + [3700,4200]\,s + [3000,3300]}{[1,2.5]\,s^6 + [40,46]\,s^5 + [570,620]\,s^4 + [3500,3800]\,s^3 + [10060,12000]\,s^2 + [13100,15080]\,s + [6000,6600]}$$

The higher-order fixed parameter transfer functions are obtained by using eq. (4.19) and (4.21),

$$G^1(s) = \frac{s^5+30s^4+360s^3+1800s^2+3700s+3000}{2.5s^6+40s^5+570s^4+3800s^3+12000s^2+13100s+6000}$$
$$G^2(s) = \frac{2s^5+30s^4+330s^3+1800s^2+4200s+3000}{2.5s^6+46s^5+570s^4+3500s^3+12000s^2+15080s+6000}$$
$$G^3(s) = \frac{s^5+34s^4+360s^3+1650s^2+3700s+3300}{s^6+40s^5+620s^4+3800s^3+10060s^2+13100s+6600}$$
$$G^4(s) = \frac{2s^5+34s^4+330s^3+1650s^2+4200s+3300}{s^6+46s^5+620s^4+3500s^3+10060s^2+15080s+6600}$$

The higher-order fixed parameter numerator $N^1(s)$ and denominator $D^1(s)$ of transfer function $G^1(s)$ are separated into their even and odd parts,

The stability equations are,

Numerator,

$$\left.\begin{aligned} N^1{}_{odd}(s) &= s^5 + 360s^3 + 3700s \\ &= 3700s\left(1+\frac{s^2}{10.5892}\right)\left(1+\frac{s^2}{349.4107}\right) \\ N^1{}_{even}(s) &= 30s^4 + 1800s^2 + 3000 \\ &= 3000\left(1+\frac{s^2}{1.7157}\right)\left(1+\frac{s^2}{58.2842}\right) \end{aligned}\right\} \quad (4.42)$$

Denominator,

$$\left.\begin{aligned} D^1{}_{odd}(s) &= 40s^5 + 3800s^3 + 13100s \\ &= 13100s\left(1+\frac{s^2}{3.5824}\right)\left(1+\frac{s^2}{91.4175}\right) \\ D^1{}_{even}(s) &= 2.5s^6 + 570s^4 + 12000s^2 + 6000 \\ &= 6000\left(1+\frac{s^2}{204.5965}\right)\left(1+\frac{s^2}{0.1524}\right)\left(1+\frac{s^2}{22.8910}\right) \end{aligned}\right\} \quad (4.43)$$

By neglecting the large magnitude factors of even and odd parts of eq. (4.42) and (4.43) the reduced order models are obtained as follows,

$$R^1(s) = \frac{3700s + 3000}{11709.6018s^2 + 13100s + 6000}$$

Similarly, we can obtain the reduced order models for $G^2(s), G^3(s)$ and $G^4(s)$ as follows,

$$R^2(s) = \frac{4200s+3000}{11709.6018s^2+15080s+6000}$$
$$R^3(s) = \frac{3700s+3300}{9636.4432s^2+13100s+6600}$$
$$R^4(s) = \frac{4200s+3300}{9636.4432s^2+15080s+6600}$$

The required lower order interval model is achieved by using eq. (4.40) and (4.41), the second order interval model is,

$$R_2(s) = \frac{[4200, 4200]\, s + [3000, 3300]}{[11709.6018, 9636.4432]\, s^2 + [15080, 15080]\, s + [6000, 6600]}$$

The step and bode responses of original lower and upper bound interval systems and the proposed and Sastry [108] reduction methods of lower and upper bounds for Example 4.7 are compared and are shown in Figures 4.15 and 4.16. Clearly, proposed method is quite comparable with original system in both lower bound and upper bound. Further, the error indices (ISE, IAE, ITSE and ITAE) values of reduction methods are compared and are shown in Table 4.7, which shows that the proposed method obtained comparable low error values.

Table 4.7: The performance indices comparisons of proposed and other reduction methods for Example 4.7.

Reduction Methods	Lower Bound				Upper Bound			
	ISE	IAE	ITSE	ITAE	ISE	IAE	ITSE	ITAE
Proposed Method	1.847×10^{-3}	0.0893	6.12×10^{-3}	0.3157	1.041×10^{-4}	0.02578	4.881×10^{-4}	0.118
Sastry [108]	0.01142	0.2841	0.0559	1.257	4.812×10^{-4}	0.0558	2.536×10^{-3}	0.3033

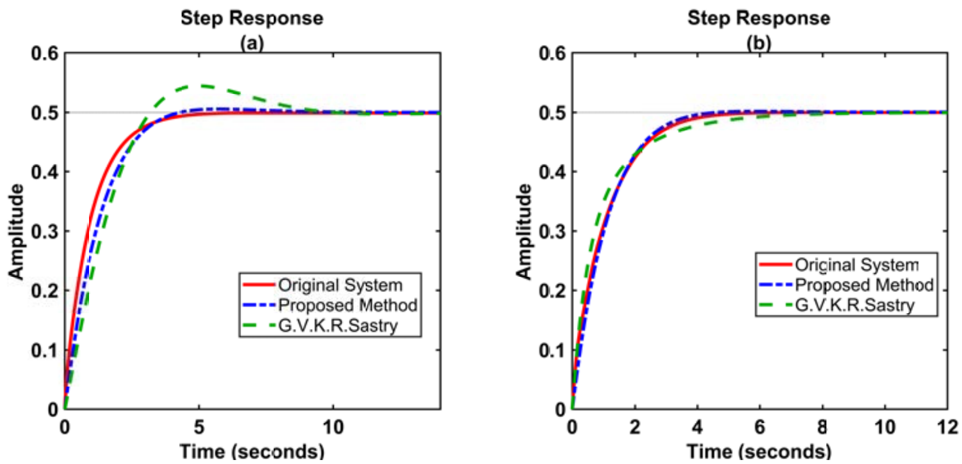


Fig. 4.15: Step response comparisons of (a) Lower bound (b) Upper bounds for Example 4.7.

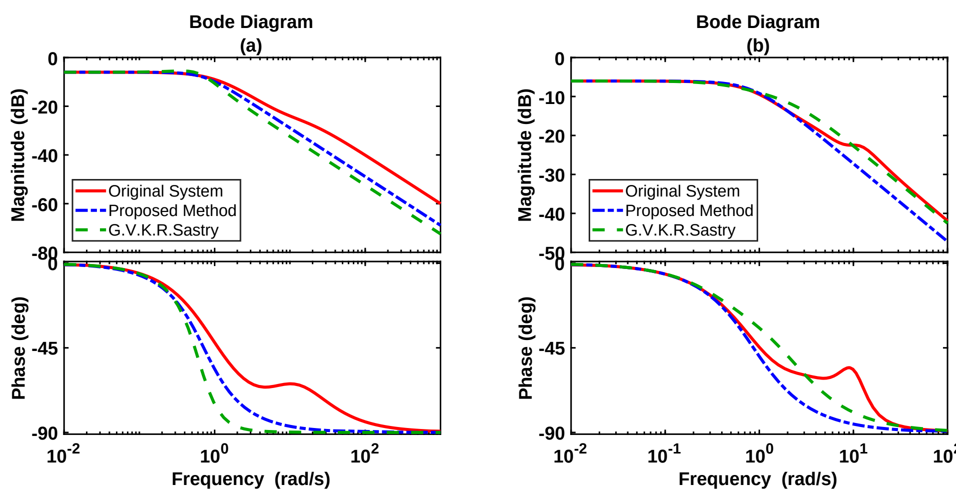


Fig. 4.16: Bode diagram response comparisons of (a) Lower bound (b) Upper bounds for Example 4.7.

Example 4.8: Consider a 15^{th} order interval system transfer function [299] described as follows,

$$G(s) = \frac{\begin{array}{l} [1.1, 2.925]\, s^{14} + [71.5, 149.6]\, s^{13} + [1987, 3583]\, s^{12} + [31788, 52140]\, s^{11} \\ + \left[3.3 \times 10^5, 5.084 \times 10^5\right] s^{10} + \left[2.368 \times 10^6, 3.49 \times 10^6\right] s^9 \\ + \left[1.217 \times 10^7, 1.738 \times 10^7\right] s^8 + \left[4.562 \times 10^7, 6.368 \times 10^7\right] s^7 \\ + \left[1.253 \times 10^8, 1.722 \times 10^8\right] s^6 + \left[2.504 \times 10^8, 3.406 \times 10^8\right] s^5 \\ + \left[3.576 \times 10^8, 4.83 \times 10^8\right] s^4 + \left[3.532 \times 10^8, 4.747 \times 10^8\right] s^3 \\ + \left[2.28 \times 10^8, 3.051 \times 10^8\right] s^2 + \left[8.616 \times 10^7, 11.48 \times 10^7\right] s \\ + \left[1.44 \times 10^7, 1.908 \times 10^7\right] \end{array}}{\begin{array}{l} [1, 2.88]\, s^{15} + [70, 142.1]\, s^{14} + [2100, 3482]\, s^{13} + [36132, 54070]\, s^{12} \\ + [401310, 572200]\, s^{11} + \left[3.064 \times 10^6, 4.265 \times 10^6\right] s^{10} \\ + \left[1.668 \times 10^7, 2.29 \times 10^7\right] s^9 + \left[6.616 \times 10^7, 8.991 \times 10^7\right] s^8 \\ + \left[1.934 \times 10^8, 2.605 \times 10^8\right] s^7 + \left[4.174 \times 10^8, 5.582 \times 10^8\right] s^6 \\ + \left[6.614 \times 10^8, 8.793 \times 10^8\right] s^5 + \left[7.571 \times 10^8, 10.02 \times 10^8\right] s^4 \\ + \left[6.071 \times 10^8, 8.012 \times 10^8\right] s^3 + \left[3.226 \times 10^8, 4.253 \times 10^8\right] s^2 \\ + \left[1.018 \times 10^8, 1.343 \times 10^8\right] s + \left[1.44 \times 10^7, 1.908 \times 10^7\right] \end{array}}$$

The required reduced order interval model is obtained by using proposed method,

$$R_2(s) = \frac{\left[8.616 \times 10^7, 11.48 \times 10^7\right] s + \left[1.908 \times 10^7, 1.908 \times 10^7\right]}{\left[2.385 \times 10^8, 2.385 \times 10^8\right] s^2 + \left[1.018 \times 10^8, 1.343 \times 10^8\right] s + \left[1.908 \times 10^7, 1.908 \times 10^7\right]}$$

The required reduced order interval model is obtained by using the method [299],

$$R(s) = \frac{[0.309346, 35.637894]\, s + [0.018173, 14.006786]}{[1, 1]\, s^2 + [0.333955, 50.850819]\, s + [0.018173, 14.006786]}$$

Figures 4.17 and 4.18 show the step and bode response comparisons of original and reduced order interval models of lower and upper bounds for Example 4.8. From these responses, it is observed that the presented technique is closely matching with original interval system. Moreover, the error indices of proposed and other reduction techniques are depicted in Table 4.8. It is clear that the proposed technique gives much better result than the other reduction method.

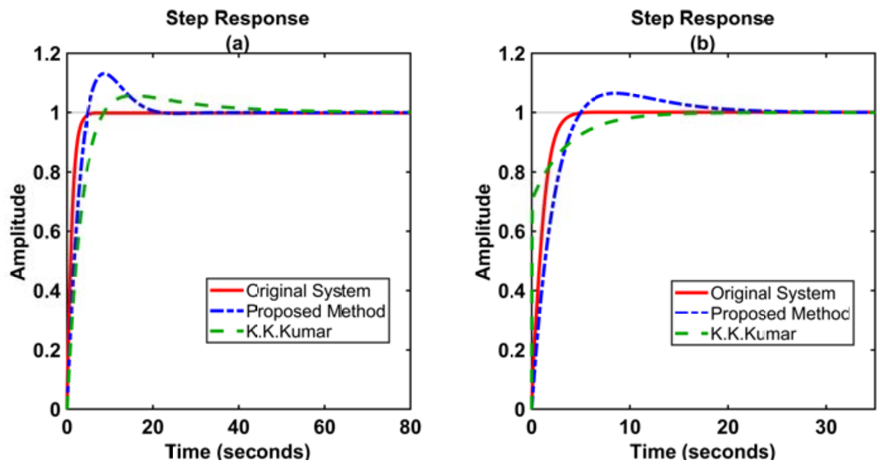


Fig. 4.17: Step response comparisons of (a) Lower bound (b) Upper bounds for Example 4.8.

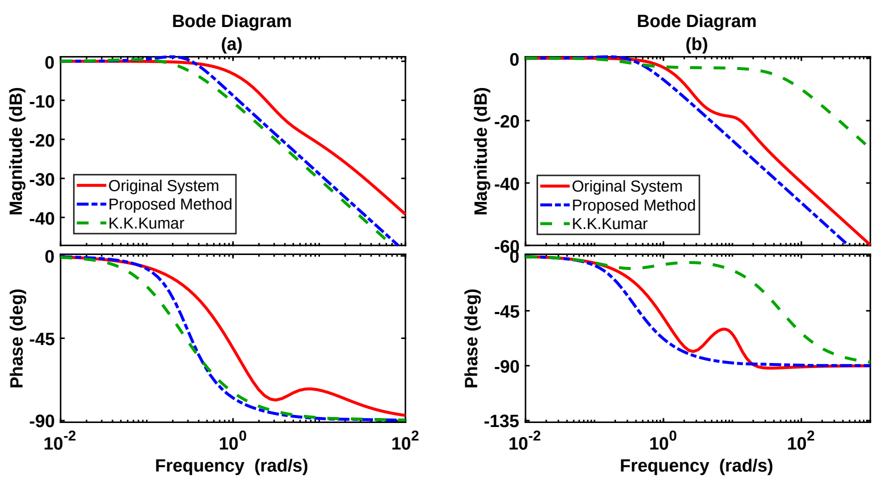


Fig. 4.18: Bode diagram response comparisons of (a) Lower bound (b) Upper bounds for Example 4.8.

4.5.1.2 Multiple Input Multiple Output System

Example 4.9: The proposed method further extended to the MIMO interval systems. Considered a 6^{th} order fixed coefficient MIMO system [283] and converted it to the interval system by incorporating $\pm 50\%$ uncertainty is as given below

$$G_6(s) = \frac{1}{D_6(s)} \begin{bmatrix} a_{11}(s) & a_{12}(s) \\ a_{21}(s) & a_{22}(s) \end{bmatrix}$$

Table 4.8: Performance indices comparison of reduction methods for Example 4.8.

Reduction Methods	Lower Bound				Upper Bound			
	ISE	IAE	ITSE	ITAE	ISE	IAE	ITSE	ITAE
Proposed Method	0.2617	1.805	1.63	13.98	0.1205	0.621	0.2375	3.657
K.K.Kumar [299]	0.4903	2.724	2.561	32	0.0783	0.7628	0.2586	4.792

where

$$d_6(s) = [0.5, 1.5]\, s^6 + [20.5, 61.5]\, s^5 + [285.5, 856.5]\, s^4 + [1745.5, 5236.5]\, s^3 + [5030, 15090]\, s^2 + [6550, 19650]\, s + [3000, 9000]$$

and

$$a_{11}(s) = [1, 3]\, s^5 + [35, 105]\, s^4 + [381, 1143]\, s^3 + [1805, 5415]\, s^2 + [3850, 11550]\, s + [3000, 9000]$$

$$a_{12}(s) = [0.5, 1.5]\, s^5 + [19, 57]\, s^4 + [229.5, 688.5]\, s^3 + [1091, 3273]\, s^2 + [2080, 6240]\, s + [1200, 3600]$$

$$a_{21}(s) = [0.5, 1.5]\, s^5 + [15, 45]\, s^4 + [165.5, 496.5]\, s^3 + [825, 2475]\, s^2 + [1850, 5550]\, s + [1500, 4500]$$

$$a_{22}(s) = [0.5, 1.5]\, s^5 + [21, 63]\, s^4 + [300.5, 901.5]\, s^3 + [1830, 5490]\, s^2 + [4550, 13650]\, s + [3000, 9000]$$

The required reduced order interval model is obtained by using proposed technique,

$$R_2(s) = \frac{1}{d_2(s)} \begin{bmatrix} c_{11}(s) & c_{12}(s) \\ c_{21}(s) & c_{22}(s) \end{bmatrix}$$

where

$$d_2(s) = [15037.5037, 3057.6756]\, s^2 + [19649.9812, 19649.9882]\, s + [2999.9821, 8999.9821]$$

and

$$c_{11}(s) = [11550, 11550]\, s + [2999.9185, 8999.9752]$$

$$c_{12}(s) = [6239.9905, 6239.9905]\, s + [1199.9351, 3599.9959]$$

$$c_{21}(s) = [5549.8291, 5549.8845]\, s + [1499.9211, 4499.5367]$$

$$c_{22}(s) = [13649.9958, 13649.9958]\, s + [2999.4781, 8999.9793]$$

The step and bode diagram responses of original and reduced order interval models of lower and upper bounds are compared and shown in Figures 4.19 - 4.22 for Example 4.9. From this it is observed that the proposed method gave close approximation with original interval system both lower and upper bounds, which show the effectiveness of the proposed method.

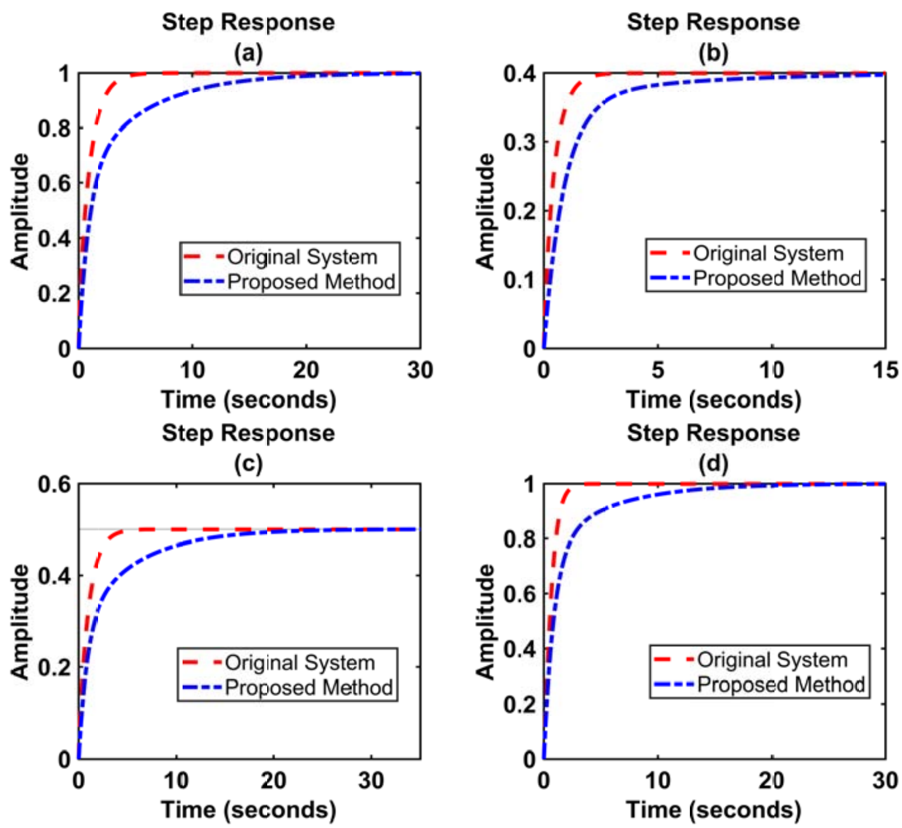


Fig. 4.19: Step response comparisons of lower bounds of (a)G_{r11} (b) G_{r12} (c) G_{r21} and (d) G_{r22} for Example 4.9.

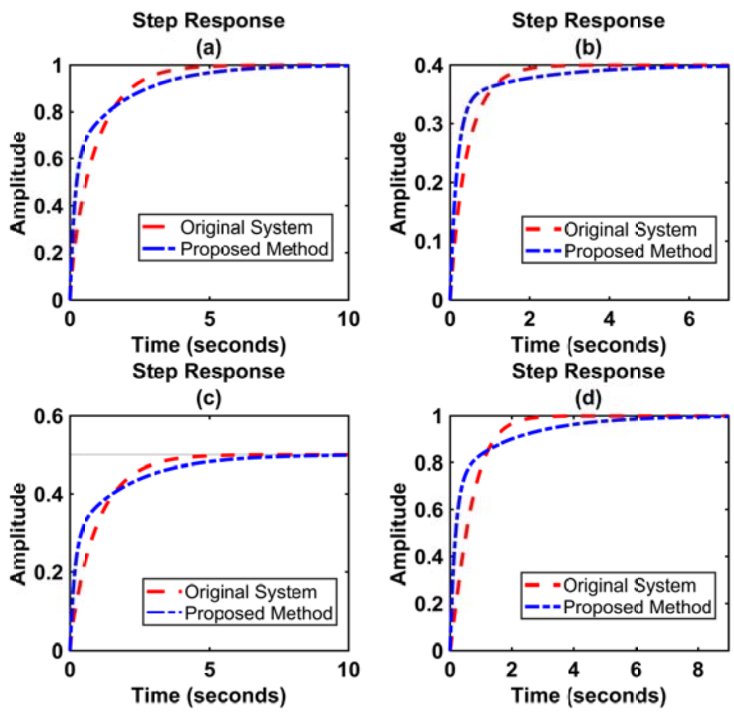


Fig. 4.20: Step response comparisons of upper bounds of (a)G_{r11} (b) G_{r12} (c) G_{r21} and (d) G_{r22} for Example 4.9.

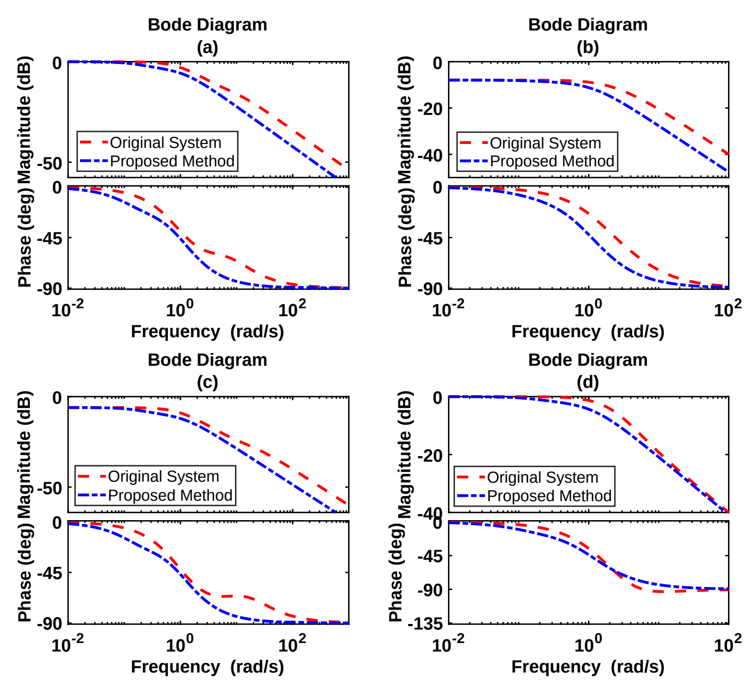


Fig. 4.21: Bode response comparisons of lower bounds of (a)G_{r11} (b) G_{r12} (c) G_{r21} and (d) G_{r22} for Example 4.9.

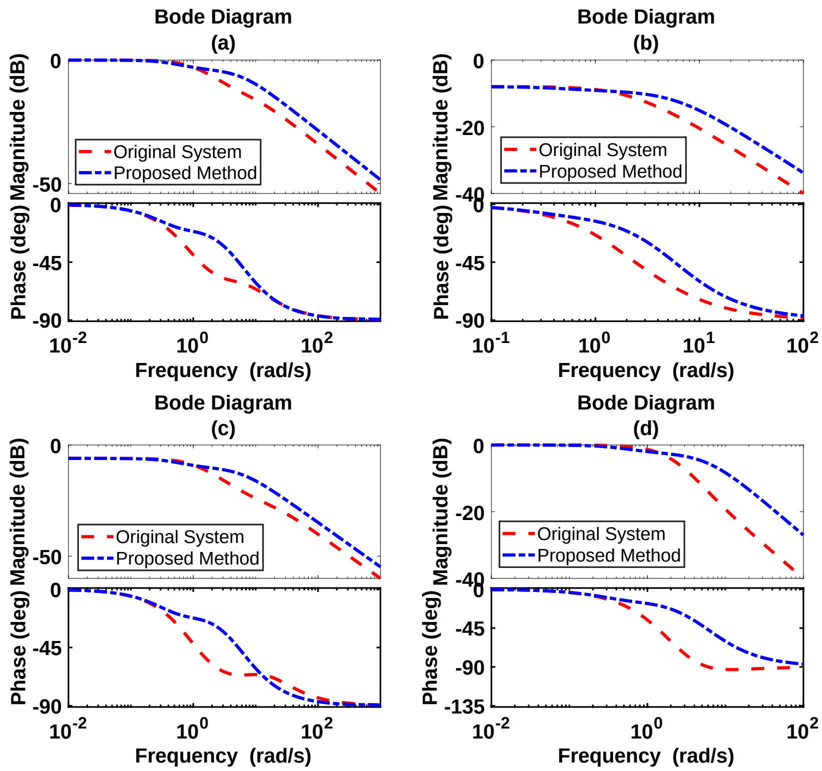


Fig. 4.22: Bode response comparisons of upper bounds of (a)G_{r11} (b) G_{r12} (c) G_{r21} and (d) G_{r22} for Example 4.9.

4.6 MODEL REDUCTION USING MODIFIED TIME MOMENT MATCHING AND KHARITONOV'S THEOREM

A new reduction method is proposed to reduce the order of higher-order interval system. The proposed reduced order interval model is achieved by using modified time-moment matching method [283], simple mathematical technique and Kharitonov's theorem. The reduced order denominator is obtained by using time moment matching method and the reduced order numerator is obtained by equating both higher order and reduced order transfer functions. The procedural steps are given to describe the proposed technique as follows,

Step 1: By using numerator and denominator Kharitonov polynomials of eq.

(4.19) and (4.21), the Kharitonov transfer functions are obtained as given below

$$\left.\begin{array}{l} G^1(s) = \frac{N^1(s)}{D^1(s)} = \frac{a_0^- + a_1^- s + a_2^+ s^2 + a_3^+ s^3 + \cdots}{b_0^- + b_1^- s + b_2^+ s^2 + b_3^+ s^3 + \cdots} \\ G^2(s) = \frac{N^2(s)}{D^2(s)} = \frac{a_0^- + a_1^+ s + a_2^+ s^2 + a_3^- s^3 + \cdots}{b_0^- + b_1^+ s + b_2^+ s^2 + b_3^- s^3 + \cdots} \\ G^3(s) = \frac{N^3(s)}{D^3(s)} = \frac{a_0^+ + a_1^- s + a_2^- s^2 + a_3^+ s^3 + \cdots}{b_0^+ + b_1^- s + b_2^- s^2 + b_3^+ s^3 + \cdots} \\ G^4(s) = \frac{N^4(s)}{D^4(s)} = \frac{a_0^+ + a_1^+ s + a_2^- s^2 + a_3^- s^3 + \cdots}{b_0^+ + b_1^+ s + b_2^- s^2 + b_3^- s^3 + \cdots} \end{array}\right\} \quad (4.44)$$

Step 2: The reduced order denominator and numerator polynomials of $G^1(s)$ are obtained by following step 1 and step 2 discussed in section 3.3.

Finally, the reduced order model is obtained as

$$R^1(s) = \frac{n^1(s)}{d^1(s)} = \frac{c_0^- + c_1^- s + \cdots}{d_0^- + d_1^- s + d_2^+ s^2 + \cdots} \quad (4.45)$$

Similarly, we can obtain the reduced order models of $G^2(s), G^3(s)$ and $G^4(s)$ as given below

$$\left.\begin{array}{l} R^2(s) = \frac{n^2(s)}{d^2(s)} = \frac{c_0^- + c_1^+ s + \cdots}{d_0^- + d_1^+ s + d_2^+ s^2 + \cdots} \\ R^3(s) = \frac{n^3(s)}{d^3(s)} = \frac{c_0^+ + c_1^- s + \cdots}{d_0^+ + d_1^- s + d_2^- s^2 + \cdots} \\ R^4(s) = \frac{n^4(s)}{d^4(s)} = \frac{c_0^+ + c_1^+ s + \cdots}{d_0^+ + d_1^+ s + d_2^- s^2 + \cdots} \end{array}\right\} \quad (4.46)$$

Step 3: The four fixed parameter ROMs given in eq. (4.45) and (4.46) can be written into sixteen combinations of ROIMs (one to each), by following sixteen plant theorem [9], the general form is,

$$R^{i,j}(s) = \frac{[n^i(s), n^j(s)]}{[d^i(s), d^j(s)]} \quad (4.47)$$

for $i, j = 1, 2, 3, 4.$

Step 4: The required ROIM is obtained by choosing least error ROIM by using ISE comparison between step response of HOIS eq. (4.1) and the sixteen combinations of ROIMs of eq. (4.47).

$$\text{ISE} = \int_0^\infty \left[g_n(t) - r^{i,j}(t)\right]^2 dt \quad (4.48)$$

Where $g_n(t)$ is the HOIS and $r^{i,j}(t)$ is the ROIM.

4.6.1 Numerical Examples and Results

To show the efficacy and powerfulness of the proposed reduction method we considered popular SISO/MIMO systems. The first example solved in detail whereas in the remaining examples the reduced systems are mentioned directly. The results are compared in terms of system response and performance indices.

4.6.1.1 Single Input Single Output Systems

Example 4.10: Consider the 7^{th} order interval system from Example 4.4 in section 4.4.1.1.

$$G_7(s) = \frac{[1.9, 2.1]s^6 + [24.7, 27.3]s^5 + [157.7, 174.3]s^4 + [542, 599]s^3 + [930, 1028]s^2 + [721.8, 797.8]s + [187.1, 206.7]}{[0.95, 1.05]s^7 + [8.779, 9.703]s^6 + [52.23, 57.73]s^5 + [182.9, 202.1]s^4 + [429, 474.2]s^3 + [572.5, 632.7]s^2 + [325.3, 359.5]s + [57.35, 63.39]}$$

The interval transfer function is converted to fixed parameter Kharitonov transfer functions by following eq. (4.19) - (4.21),

$$\begin{aligned}
G^1(s) &= \frac{2.1s^6+24.7s^5+157.7s^4+599s^3+1028s^2+721.8s+187.1}{1.05s^7+9.703s^6+52.23s^5+182.9s^4+474.2s^3+632.7s^2+325.3s+57.35} \\
G^2(s) &= \frac{2.1s^6+27.3s^5+157.7s^4+542s^3+1028s^2+797.8s+187.1}{0.95s^7+9.703s^6+57.73s^5+182.9s^4+429s^3+632.7s^2+359.5s+57.35} \\
G^3(s) &= \frac{1.9s^6+24.7s^5+174.3s^4+599s^3+930s^2+721.8s+206.7}{1.05s^7+8.779s^6+52.23s^5+202.1s^4+474.2s^3+572.5s^2+325.3s+63.39} \\
G^4(s) &= \frac{1.9s^6+27.3s^5+174.3s^4+542s^3+930s^2+797.8s+206.7}{0.95s^7+8.779s^6+57.73s^5+202.1s^4+429s^3+572.5s^2+359.5s+63.39}
\end{aligned} \tag{4.49}$$

The higher-order systems given in eq. (4.49) are reduced to the LOMs by following step 1 and step 2 discussed in section 4.6.

$$\begin{aligned}
R^1(s) &= \frac{-3.1282s-6.1097}{s^2-4.3536s-1.8730} \\
R^2(s) &= \frac{5.5395s+2.6011}{s^2+3.2967s+0.7974} \\
R^3(s) &= \frac{5.39025s+4.67258}{s^2+4.00169s+1.43287} \\
R^4(s) &= \frac{4.59042s+2.05531}{s^2+2.5486s+0.63027}
\end{aligned}$$

Finally, the required ROIM is obtained by using eq. (4.47) and (4.48), the second order interval model is,

$$R_2(s) = \frac{[5.5395, 5.39025]\,s + [2.6011, 4.67258]}{[1, 1]\,s^2 + [3.2967, 4.00169]\,s + [0.7974, 1.43287]}$$

The unit step and bode response comparisons of original system and reduced order interval models of lower and upper bounds are shown in Figures 4.23 and 4.24 for Example 4.10. Moreover, the ISE and IAE of proposed and other existing methods [231, 236, 240, 255, 297, 298] are tabulated in Table 4.9. From which it is clear that the proposed technique gives much closer approximation to the original system then the other methods.

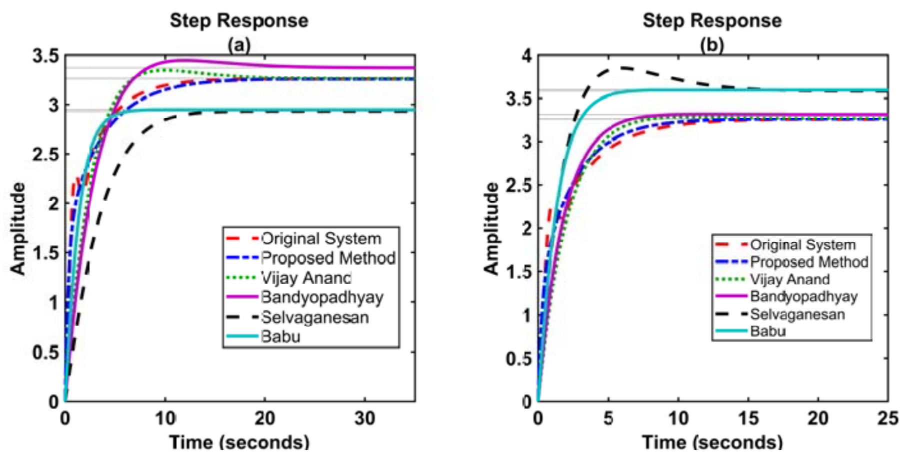


Fig. 4.23: Step response comparison of (a) lower bounds (b) upper bounds for Example 4.10

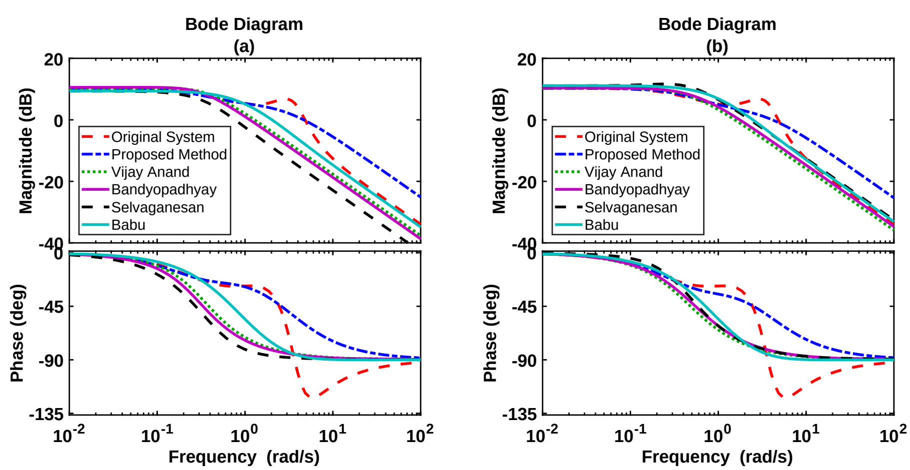


Fig. 4.24: Bode diagram response comparison of (a) lower bounds (b) upper bounds for Example 4.10

Example 4.11: Consider the 3^{rd} order interval system from Example 4.1 in section 4.3.1.1. The required reduced order model is obtained by using proposed method,

$$R_2(s) = \frac{[1.08902, 1.30976]\, s + [3.0272, 0.9869]}{[1,1]\, s^2 + [3.7252, 2.4464]\, s + [4.1373, 1.32622]}$$

Figures 4.25 and 4.26 shows the step and bode response comparison of original and reduced order interval models of lower and upper bounds for Example 4.11. From these responses, it is observed that the presented technique is closely matched with original interval system response. Moreover, the ISE of proposed and other reduction techniques are depicted in Table 4.10. It is clear that the proposed technique gives much better result than the other reduction method.

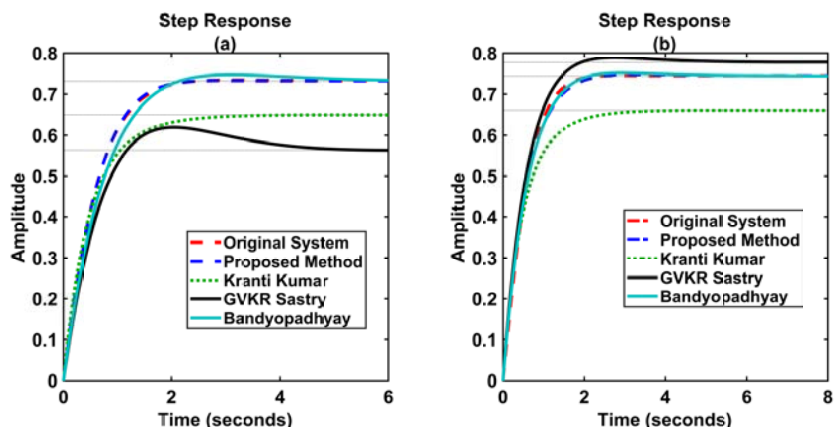


Fig. 4.25: Step response comparisons of (a) lower bounds (b) upper bounds of Example 4.11

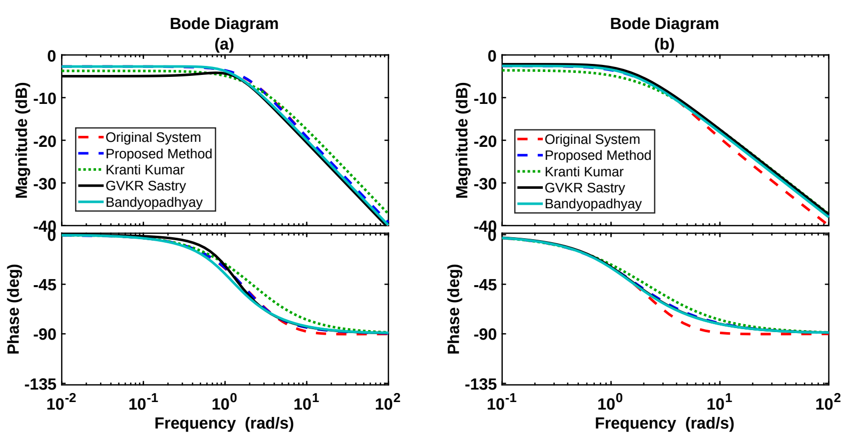


Fig. 4.26: Bode diagram response comparisons of (a) lower bounds (b) upper bounds of Example 4.11

Table 4.9: The ISE and IAE comparisons for original and other reduction methods for Example 4.10

Reduction Methods	ISE		IAE	
	Lower Bound	Upper Bound	Lower Bound	Upper Bound
Proposed Method	0.0739	0.1053	0.6911	0.8016
N. V. Anand [236]	0.991	3.2979	3.425	4.536
T. Babu [255]	2.428	4.323	5.734	9.325
Siva Kumar et al [240]	0.9357	1.0915	2.972	3.043
Bandyopadhyay [298]	2.259	5.954	4.758	7.735
Kranthi(FDA&DM) [297]	2.434	6.346	6.125	11.231
Kranthi(PA&DM) [297]	3.181	7.379	7.132	11.982
Selvaganesan [231]	4.305	7.301	8.804	12.372

Table 4.10: Comparison of the ISE with proposed and different reduction methods for Example 4.11.

Reduced Methods	ISE	
	Lower Bound	Upper Bound
Proposed Method	1.552×10^{-5}	9.321×10^{-5}
Bandyopadhyay et al. [106]	0.0015	2.9590×10^{-4}
Kranthi et al [297]	0.0196	0.0248
G.V.K R. Sastry et al. [221]	0.117	0.0084

Example 4.12: Consider an 8^{th} order interval system [294] having the transfer function

$$G_8(s) = \frac{\begin{array}{l}[2.67e^2, 2.89e^2]\, s^7 + [4.66e^5, 7.66e^5]\, s^6 + [4.03e^8, 7.31e^8]\, s^5 \\ + [2.4e^{11}, 3.3e^{11}]\, s^4 + [6.6e^{13}, 8.1e^{13}]\, s^3 + [5.5e^{15}, 9.0e^{15}]\, s^2 \\ + [1.28e^{17}, 1.46e^{17}]\, s + [6.42e^{17}, 7.85e^{17}]\end{array}}{\begin{array}{l}[1, 1]\, s^8 + [2.45e^3, 2.70e^3]\, s^7 + [2.16e^6, 2.30e^6]\, s^6 + [8.1e^8, 8.32e^8]\, s^5 \\ + [1.3e^{11}, 1.39e^{11}]\, s^4 + [8.7e^{12}, 9.41e^{12}]\, s^3 + [2.0e^{14}, 2.32e^{14}]\, s^2 \\ + [1.89e^{15}, 2.33e^{15}]\, s + [6.48e^{15}, 7.88e^{15}]\end{array}}$$

The required reduced order model is obtained by proposed method, as follows

$$R_2(s) = \frac{[899.7623, 965.7280]\, s + [4739.2229, 3115.7994]}{[1, 1]\, s^2 + [13.4965, 13.1252]\, s + [47.8351, 31.2770]}$$

Figures 4.27 and 4.28 shows the step and bode response comparisons of original and reduced order interval models of lower and upper bounds for Example 4.12. From these responses, it is observed that the presented technique is closely matched with original interval system response. Moreover, the ISE, IAE and ITAE of proposed and other reduction techniques are depicted in Table 4.11. It is clear that the proposed technique gives much better result than the other reduction method.

Table 4.11: The performance indices comparison of different reduction methods for Example 4.12.

Reduction methods	Lower bound			Upper bound		
	ISE	IAE	ITAE	ISE	IAE	ITAE
Proposed method	5.047×10^{-8}	3.177×10^{-4}	0.3422	1.392×10^{-4}	0.01668	0.3341
Bandopadhyay [106]	9.865×10^{-4}	0.0444	7.453	4.378×10^{-4}	0.03541	0.6769
Vijay Anand [294]	78.6	16.65	51.6	0.02354	1.157	3.4521

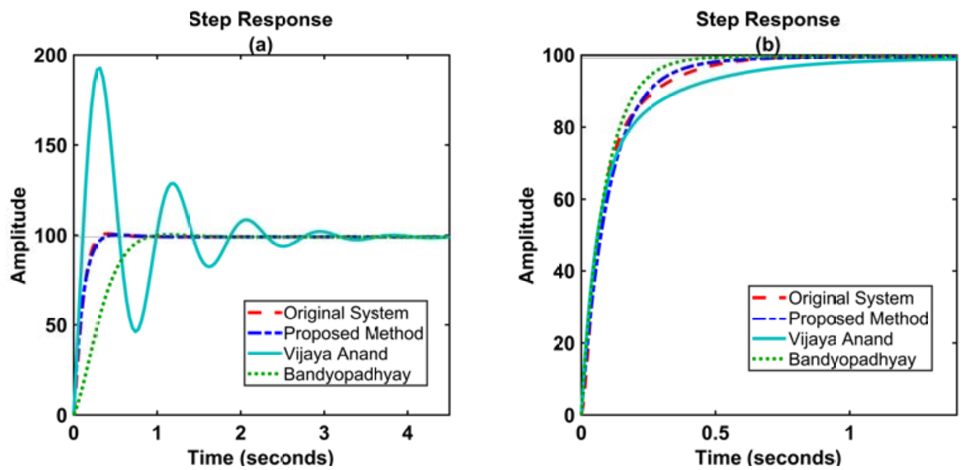


Fig. 4.27: Step response comparison of (a) lower bounds and (b) upper bounds of Example 4.12

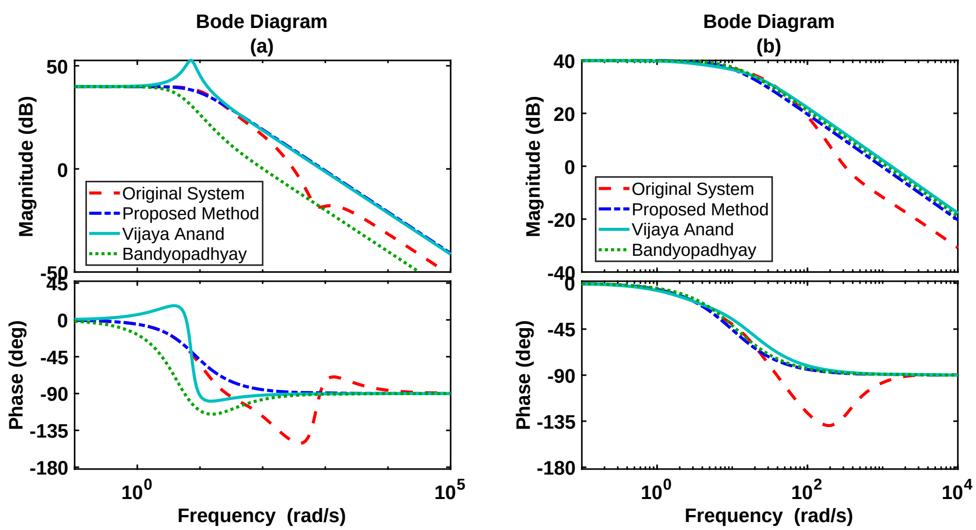


Fig. 4.28: Bode diagram response comparisons of (a) lower bounds and (b) upper bounds of Example 4.12

4.6.1.2 Multiple Input Multiple Output System

Example 4.13: The application of the proposed method further extended to MIMO interval systems. Consider a 5^{th} order MIMO interval transfer matrix [299]

$$G_5(s) = \frac{1}{D_5(s)} \begin{bmatrix} N_{11} \\ N_{21} \end{bmatrix}$$

where

$$D_5(s) = [1, 1.03]\, s^5 + [24.6, 25.34]\, s^4 + [136.14, 140.23]\, s^3 + [282.72, 291.2]\, s^2 + [236.51, 243.61]\, s + [66.34, 70.32]$$

and

$$N_{11}(s) = [3.83, 4.06]\, s^4 + [118.0, 125.0]\, s^3 + [339.6, 360.1]\, s^2 + [275.50, 280.10]\, s + [66.34, 70.32]$$

$$N_{21}(s) = [3.78, 4.00]\, s^4 + [95.8, 101.6]\, s^3 + [267.86, 283.9]\, s^2 + [233.53, 238.54]\, s + [66.34, 70.32]$$

The required reduced order interval model is obtained by using proposed technique,

$$R_2(s) = \frac{1}{d_2(s)} \begin{bmatrix} c_{11} \\ c_{21} \end{bmatrix}$$

where

$$d_2(s) = [1, 1]\, s^2 + [3.3694, 3.8267]\, s + [2.7162, 3.01959]$$

and

$$c_{11}(s) = [4.8754, 5.3936]\, s + [2.7162, 3.0195]$$

$$c_{21}(s) = [3.4478, 3.9089]\, s + [2.7161, 3.01959]$$

The reduced order interval model is obtained by using K. K. Kumar technique [299],

$$R_2(s) = \frac{1}{d_2(s)} \begin{bmatrix} c_{11} \\ c_{21} \end{bmatrix}$$

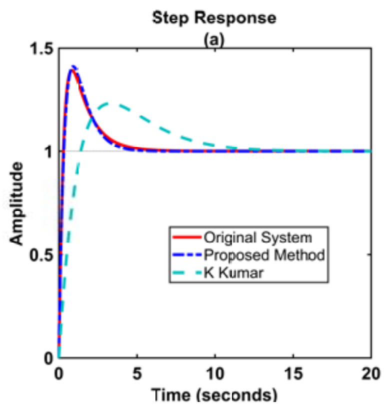
where

$$d_2(s) = [1, 1]\, s^2 + [1.0428, 1.1584]\, s + [0.2839, 0.3444]$$

and

$$c_{11}(s) = [1.2147, 1.3319]\, s + [0.2839, 0.3444]$$
$$c_{21}(s) = [1.03, 1.1343]\, s + [0.2839, 0.3444]$$

The step and bode response of original lower and upper bound interval systems of proposed and K.K. Kumar [299] reduction methods of lower and upper bounds for Example 4.13 are compared and are shown in Figures 4.29 - 4.32. Clearly, proposed method is closely matching with original system in both lower bound and upper bounds. Moreover, the ISE values of reduction methods are shown in Table 4.12, which shows that the proposed method obtained comparable lowest ISE values.



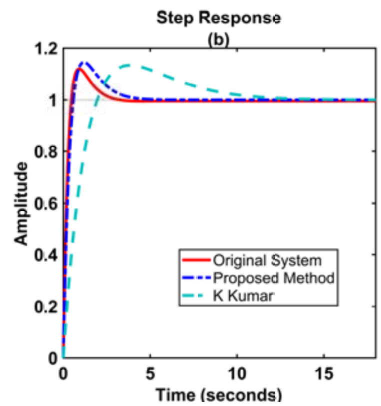


Fig. 4.29: Step response comparison of multivariable systems (a) $G_{r11}(s)$ (b) $G_{r21}(s)$ of lower bounds for Example 4.13

Table 4.12: ISE comparison of reduced order models for Example 4.13.

Reduction methods	ISE			
	Lower Bound		Upper Bound	
	r_{11}	r_{21}	r_{11}	r_{21}
Proposed method	6.091×10^{-4}	5.585×10^{-3}	2.569×10^{-4}	5.179×10^{-3}
K. Kumar [299]	0.1184	0.06108	0.1003	0.05419

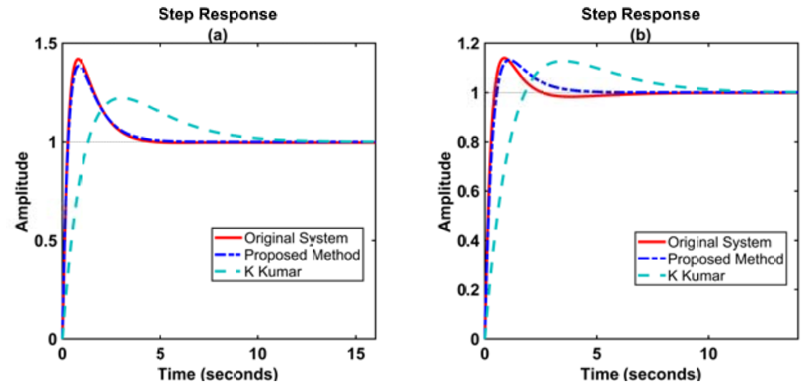


Fig. 4.30: Step response comparison of multivariable systems (a) $G_{r11}(s)$ (b) $G_{r21}(s)$ of upper bounds for Example 4.13

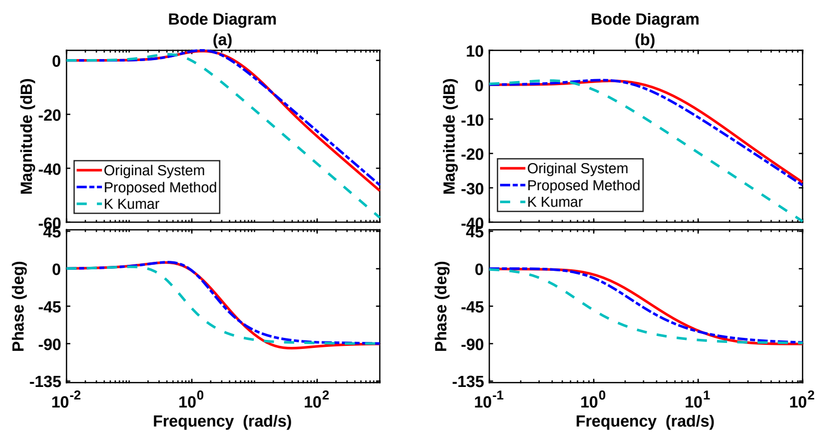


Fig. 4.31: Bode diagram response comparison of multivariable systems (a) $G_{r11}(s)$ (b) $G_{r21}(s)$ of lower bounds for Example 4.13

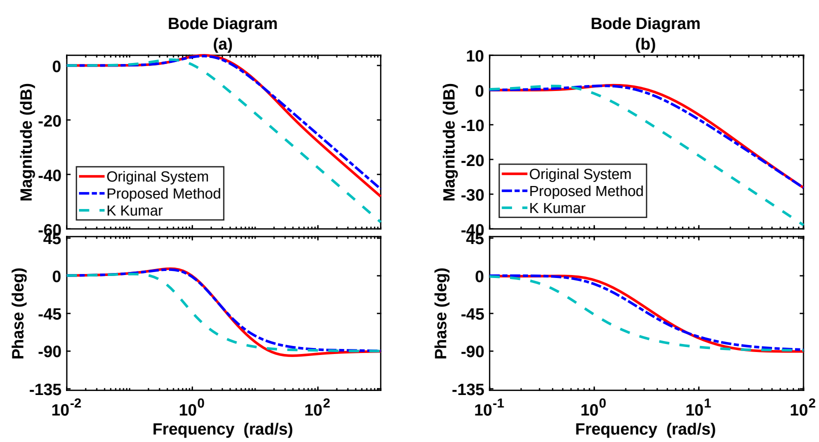


Fig. 4.32: Bode diagram response comparison of multivariable systems (a) $G_{r11}(s)$ (b) $G_{r21}(s)$ of upper bounds for Example 4.13

4.7 CONCLUSION

The new reduction methods are proposed for reducing the order of linear interval systems. In this chapter, four new combined order reduction methods are proposed, the first reduction technique is obtained by differentiation method, factor division algorithm, and Pade approximation method based on interval arithmetic operations. In this, the reduced order denominators are obtained by differentiation method and, the reduced order numerators are obtained by either of differentiation method, factor division algorithm, and Pade approximation method. The second, third and fourth reduction methods are developed by using differentiation method, stability equation method and modified time-moment matching method using Kharitonov's theorem. The proposed methods are justified by solving some benchmark numerical examples of both SISO and MIMO interval systems. It is observed that the proposed methods are computationally simple and applicable for MIMO systems also. Further, the time and frequency responses plotted to show the close response matching of reduced order models obtained by proposed methods with original systems. Furthermore, the results obtained by the proposed methods are also compared with recently available literature in terms of performance indices.

CHAPTER 5

NEW COMPOSITE TECHNIQUES FOR REDUCED ORDER MODELLING OF DISCRETE-TIME SYSTEMS

5.1 INTRODUCTION

Model reduction is an active area of research in control system engineering since 1960. The number of reduction methods were proposed for continuous time systems [40, 53, 92, 135, 145, 284, 300–304], but very limited number of reduction techniques are extended to the discrete-time systems [140, 281, 305–309]. Due to fast development of microprocessor and micro-controller based design of control systems, the importance of MOR for discrete-time systems is increasing day by day. The reduced order denominator models are obtained by first converting z-domain system to the w-domain by applying linear transformation $z = (w + 1)$, after that, this w-domain system is reduced by using proposed reduction technique. Finally, required reduced order model is obtained by converting the w-domain ROM to the z-domain ROM by applying inverse linear transformation $w = (z - 1)$.

5.2 PROBLEM STATEMENT

5.2.1 Single Input Single Output Systems

Consider a linear dynamic high-order discrete-time system represented as,

$$G(z) = \frac{N(z)}{D(z)} = \frac{b_0 + b_1 z + \cdots + b_{n-1} z^{n-1}}{a_0 + a_1 z + \cdots + a_n z^n} \tag{5.1}$$

Where $b_0, b_1, \cdots b_{n-1}$ and $a_0, a_1, \cdots a_n$ are the numerator and denominator coefficients. Our objective is to compute an $k^{th}\,(k < n)$ order LOS transfer function as given below,

$$R(z) = \frac{n(z)}{d(z)} = \frac{d_0 + d_1 z + \cdots + d_{k-1} z^{k-1}}{e_0 + e_1 z + \cdots + e_k z^k} \tag{5.2}$$

Where $d_0, d_1, \cdots d_{k-1}$ and $e_0, e_1, \cdots e_k$ are the unknown numerator and denominator coefficients.

The reduced order model achieved by using linear transformation, where the system given in z-domain is converted to w-domain by substituting $z = (w+1)$, resulting

$$G(z) = \frac{N(w)}{D(w)} = \left.\frac{N(z)}{D(z)}\right|_{z=(w+1)} = \frac{b_{20} + b_{21}w + \cdots + b_{2n-2}w^{n-1}}{a_{20} + a_{21}w + \cdots + a_{2n}w^{n}} \tag{5.3}$$

The above converted system in w-domain is then reduced by using the proposed methods and then converted back to z-domain by substituting $w = (z-1)$ to achieve the reduced order model in the form as given in eq. (5.2) and resulting

$$R(z) = \frac{n(z)}{d(z)} = \left.\frac{n(w)}{d(w)}\right|_{w=(z-1)} = \frac{d_0 + d_1 z + \cdots + d_{k-1}z^{k-1}}{e_0 + e_1 z + \cdots + e_k z^{k}} \tag{5.4}$$

5.2.2 Multiple Input Multiple Output System

Consider an n^{th} order transfer function matrix represented as

$$[G(z)] = \frac{1}{D_n(z)} \begin{bmatrix} A_{11}(z) & A_{12}(z) & \cdots & A_{1n}(z) \\ A_{21}(z) & A_{22}(z) & \cdots & A_{2n}(z) \\ \vdots & \vdots & \vdots & \vdots \\ A_{m1}(z) & A_{m2}(z) & \cdots & A_{mn}(z) \end{bmatrix} \tag{5.5}$$

Or $[G(z)] = [g_{ij}(z)]$, where $g_{ij}(z)$ can be written as $[g_{ij}(z)] = \frac{[A_{ij}(z)]}{D_n(z)}$, where $i = 1, 2, \cdots, m;\ j = 1, 2, \cdots, n$.

The goal is to obtain lower k^{th} order transfer function matrix

$$[R(z)] = \frac{1}{D_k(z)} \begin{bmatrix} E_{11}(z) & E_{12}(z) & \cdots & E_{1p}(z) \\ E_{21}(z) & E_{22}(z) & \cdots & E_{2p}(z) \\ \vdots & \vdots & \vdots & \vdots \\ E_{q1}(z) & E_{q2}(z) & \cdots & E_{qp}(z) \end{bmatrix} \tag{5.6}$$

Or $[R(z)] = [r_{ij}(z)]$, where $r_{ij}(z)$ can be written as $[r_{ij}(z)] = \frac{[E_{ij}(z)]}{D_k(z)}$, where $i = 1, 2, \cdots, q;\ j = 1, 2, \cdots, p$.

5.3 MODEL REDUCTION USING MODIFIED TIME MOMENT MATCHING METHOD

The order reduction methods proposed in chapter 3 are valid for continuous time systems represented in frequency domain. In this chapter, the method which is discussed in section 3.3 is extended to discrete-time systems by using linear transformation at initial and final stages to obtain ROMs. To demonstrate the superiority and efficacy of the proposed techniques, various benchmark numerical examples are considered from the literature. The results are compared in terms of system responses and performance indices with other well-known methods.

Further, the performance of the proposed method is also evaluated by using summation square error (SSE), summation absolute error (SAE) and summation time absolute error (STAE) is as follows,

$$\mathrm{SAE} = \sum_{h=0}^{n} \left| g\left(h\right) - r\left(h\right) \right| \tag{5.7}$$

$$\mathrm{SSE} = \sum_{h=0}^{n} \left[g\left(h\right) - r\left(h\right) \right]^2 \tag{5.8}$$

$$\mathrm{STAE} = \sum_{h=0}^{n} t \left| g\left(h\right) - r\left(h\right) \right| \tag{5.9}$$

n = number of sampling instances. Where $g\left(h\right)$ and $r\left(h\right)$ are the unit step responses of original HOS and the reduced order model respectively.

5.3.1 Numerical Examples and Results

To show the powerfulness and efficacy of the proposed reduction method we considered popular SISO/MIMO systems. The results are compared in terms of step response, impulse response and performance indices.

5.3.1.1 Single Input Single Output Systems

Example 5.1: The 4^{th} order original discrete-time model [310] represented in transfer function

$$G_4\left(z\right) = \frac{-0.216608 + 0.31926z - 0.40473z^2 + 0.0547377z^3}{0.282145 - 0.551205z + 0.875599z^2 - 1.36078z^3 + z^4}$$

The desired lower order discrete model is achieved by using proposed method is as follows

$$R_2(z) = \frac{-0.0066z - 0.1539}{z^2 - 1.478z + 0.6377}$$

The step and impulse responses of higher-order model is compared with proposed and recent reduced models [140, 281] which are shown in Figures 5.1 and 5.2 for Example 5.1. From this, it is understood that, the proposed model gave close response with the higher-order system than the other models. Further, the performance of the proposed model is also evaluated in terms of SSE, SAE, and STAE values with other existing methods which are shown in Table 5.1. It is noticed that, the suggested technique produced less error than other different reduction methods.

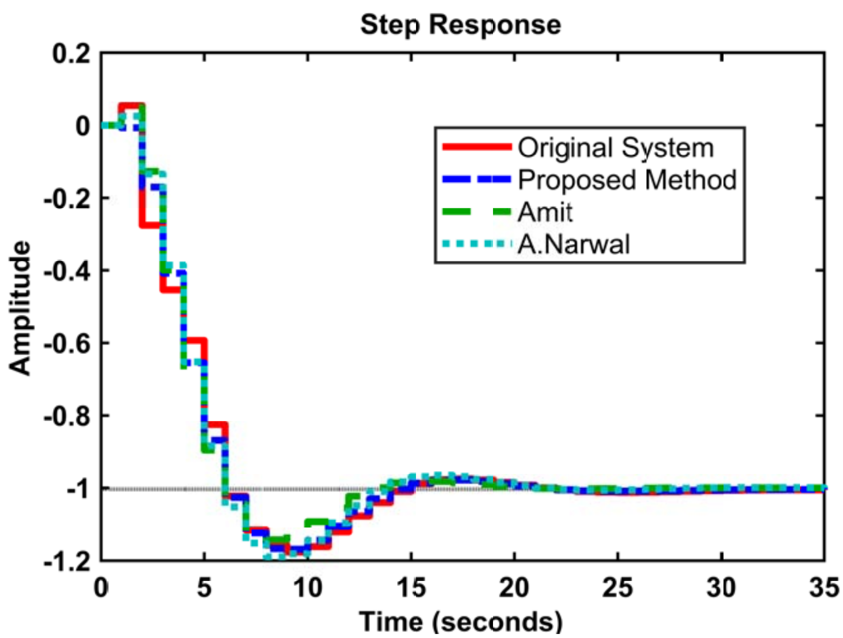


Fig. 5.1: Step response comparisons for Example 5.1.

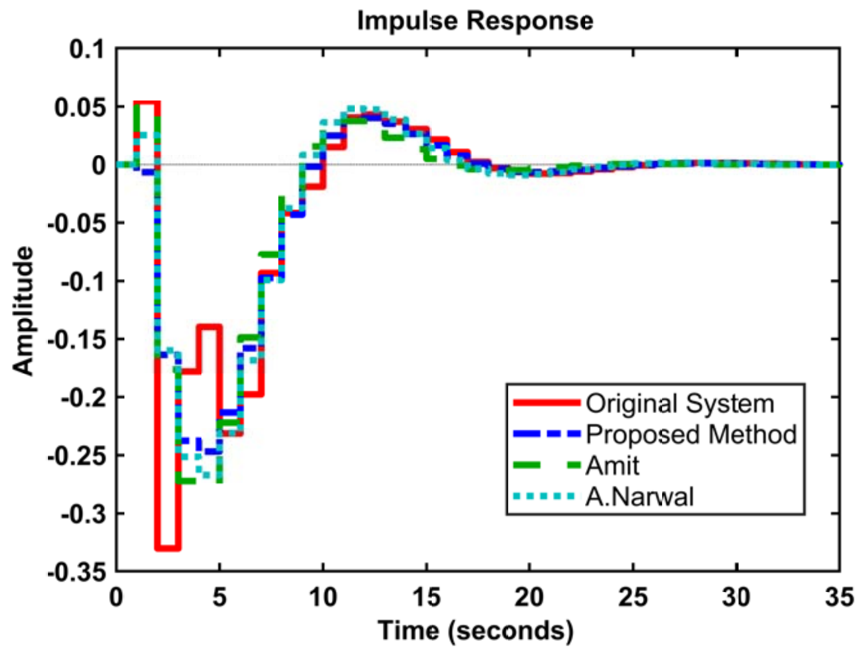


Fig. 5.2: Impulse response comparisons for Example 5.1.

Example 5.2: The 8^{th} order transfer function of discrete-time system [140] represented as

$$G_8(z) = \frac{280.3z^7 + 186z^6 - 35z^5 + 25.33z^4 - 86z^3 - 43.66z^2 + 7.33z - 1}{666.7z^8 - 280.3z^7 - 186z^6 + 35z^5 - 25.33z^4 + 86z^3 + 43.66z^2 - 7.33z + 1}$$

The required discrete lower order model is achieved by means of proposed method.

$$R_2(z) = \frac{0.4714z - 0.3275}{z^2 - 1.528z + 0.6724}$$

The step and impulse response of original model is compared with proposed and other reduced order models [140, 310] which are shown in Figure 5.3 and 5.4 for Example 5.2. From these responses it is observed that, the proposed model response is closely matching in both time and frequency domains compared to other reduced order models. Further, the obtained model also verified with SSE, SAE, and STAE by comparing with other lower order models which are displayed in Table 5.2. From this, it may be seen that, the presented technique produced very much improved results as compared to the other famous reduction methods.

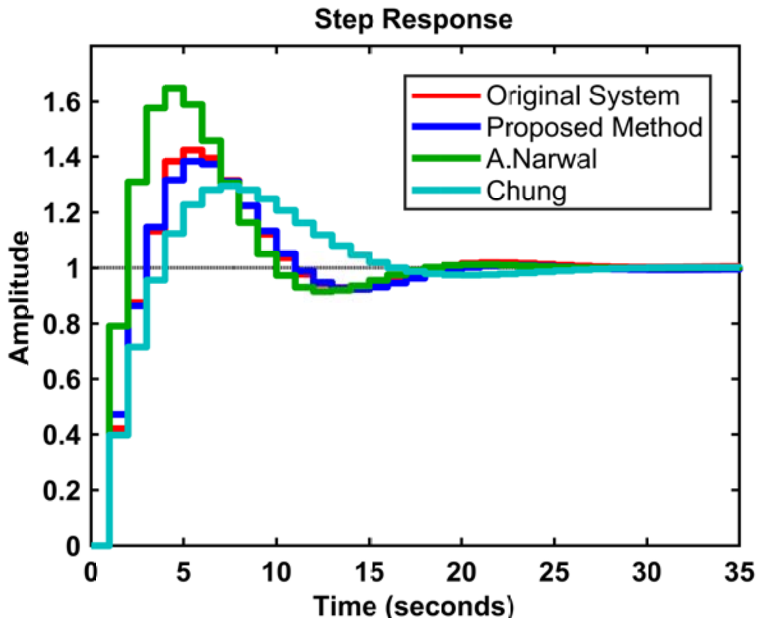


Fig. 5.3: Step response comparisons for Example 5.2

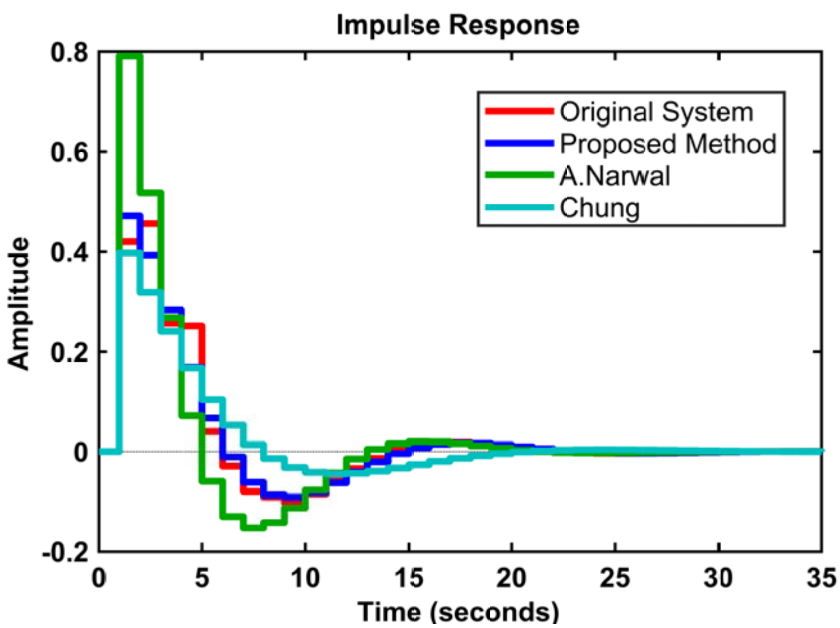


Fig. 5.4: Impulse response comparisons for Example 5.2

Table 5.1: The SSE, SAE, and STAE comparison of different reduced order models for Example 5.1.

Reduction method	SSE	SAE	STAE
Proposed method	1.13×10^{-6}	1.063×10^{-3}	3.614×10^{-2}
A.Narwal [140]	1.961×10^{-5}	5.560×10^{-3}	0.187
Satakshi et al. [305]	3.7812×10^{-3}	0.5462	2.436
Hsieh [306]	1.650×10^{-2}	0.9741	3.234
Amit Narwal [281]	2.584×10^{-4}	6.043×10^{-3}	0.6055

Table 5.2: Different reduced models SSE, SAE, and STAE comparison for Example 5.2.

Reduction methods	SSE	SAE	STAE
Proposed method	1.537×10^{-5}	3.92×10^{-3}	0.1333
A.Narwal et al. [140]	4.9582×10^{-4}	0.0183	1.237
Chung et al. [310]	0.2511	1.843	3.341
Satakshi et al. [305]	5.881×10^{-3}	0.3145	1.0547

5.3.1.2 Multiple Input Multiple Output System

Example 5.3: The application of the presented technique is also extended for discrete time MIMO systems. The proposed SISO method is applied successfully on each element of transfer matrix of MIMO system. The procedure is same as discussed earlier in section 3.3. Consider an 8^{th} order multivariable transfer function matrix [311] given as,

$$[G_n(z)] = \frac{1}{D_n(z)} \begin{bmatrix} A_{11}(z) & A_{12}(z) & A_{13}(z) \\ A_{21}(z) & A_{22}(z) & A_{23}(z) \end{bmatrix}$$

where

$$D_8(z) = 8z^8 - 5.046z^7 - 3.348z^6 + 0.63z^5 - 0.456z^4 + 1.548z^3 + 0.786z^2 - 0.132z + 0.018$$

and

$$A_{11}(z) = 1.3z^7 + z^6 - 0.02z^5 + 0.042z^4 - 0.181z^3 - 0.007z^2 + 0.024z - 0.0033$$

$$A_{12}(z) = 0.082z^7 + 0.095z^6 - 0.14z^5 + 0.01z^4 - 0.13z^3 - 0.2z^2 + 0.017z - 0.0015$$

$$A_{13}(z) = 0.3z^7 + 0.021z^6 - 0.05z^5 - 0.1z^4 - 0.205z^3 - 0.055z^2 + 0.003z - 0.0012$$

$$A_{21}(z) = 1.081z^7 + 0.3z^6 - 0.286z^5 - 0.092z^4 + 0.113z^3 - 0.08z^2 - 0.0354z - 0.004$$

$$A_{22}(z) = 0.3z^7 + 0.621z^6 - 0.253z^5 - 0.116z^4 + 0.247z^3 - 0.26z^2 - 0.212z - 0.004$$

$$A_{23}(z) = 1.05z^7 + 0.13z^6 + 0.27z^5 - 0.043z^4 + 0.071z^3 - 0.17z^2 + 0.085z - 0.003$$

The corresponding $[G_n(w)]$ is obtained using linear transformation

$$[G_n(w)] = \frac{1}{D_n(w)} \begin{bmatrix} A_{11}(w) & A_{12}(w) & A_{13}(w) \\ A_{21}(w) & A_{22}(w) & A_{23}(w) \end{bmatrix}$$

where

$$D_n(w) = 8w^8 + 58.95w^7 + 185.3w^6 + 322.6w^5 + 335.9w^4 + 210.5w^3 + 76.81w^2 + 16w + 2$$

and

$$E_{11}(w) = 1.3w^7 + 10.1w^6 + 33.28w^5 + 60.44w^4 + 65.29w^3 + 41.8w^2 + 14.64w + 2.155$$

$$E_{12}(w) = 0.082w^7 + 0.669w^6 + 2.152w^5 + 3.605w^4 + 3.28w^3 + 1.217w^2 - 0.289w - 0.2675$$

$$E_{13}(w) = 0.3w^7 + 2.121w^6 + 6.376w^5 + 10.47w^4 + 9.815w^3 + 4.845w^2 + 0.854w - 0.0872$$

$$E_{21}(w) = 1.081w^7 + 7.867w^6 + 24.22w^5 + 40.81w^4 + 40.72w^3 + 24.05w^2 + 7.713w + 0.9966$$

$$E_{22}(w) = 0.3w^7 + 2.721w^6 + 9.773w^5 + 18.43w^4 + 20.17w^3 + 12.87w^2 + 4.106w + 0.323$$

$$E_{23}(w) = 1.05w^7 + 7.48w^6 + 23.1w^5 + 40.01w^4 + 41.95w^3 + 26.48w^2 + 9.096w + 1.22$$

The desired reduced order model is obtained by using proposed method,

$$[R_2(z)] = \begin{bmatrix} E_{11}(z) & E_{12}(z) & E_{13}(z) \\ E_{21}(z) & E_{22}(z) & E_{23}(z) \end{bmatrix}$$

where

$$E_{11}(z) = \frac{0.1646z - 0.0833}{z^2 - 1.7561z + 0.8316}$$

$$E_{12}(z) = \frac{0.03002z - 0.0396}{z^2 - 1.7264z + 0.7984}$$

$$E_{13}(z) = \frac{0.0447z - 0.0478}{z^2 - 1.7486z + 0.8204}$$

$$E_{21}(z) = \frac{0.1146z - 0.0764}{z^2 - 1.7500z + 0.8267}$$

$$E_{22}(z) = \frac{0.0826z - 0.0731}{z^2 - 1.7634z + 0.8217}$$

$$E_{23}(z) = \frac{0.1199z - 0.0768}{z^2 - 1.7648z + 0.8354}$$

The reduced order model is obtained by Desai [311]

$$[R_2(z)] = \begin{bmatrix} E_{11}(z) & E_{12}(z) & E_{13}(z) \\ E_{21}(z) & E_{22}(z) & E_{23}(z) \end{bmatrix}$$

$$E_{11}(z) = \tfrac{0.221z-0.164}{z^2-1.731z+0.784}$$
$$E_{12}(z) = \tfrac{0.0128z-0.0198}{z^2-1.731z+0.784}$$
$$E_{12}(z) = \tfrac{0.0291z-0.0314}{z^2-1.731z+0.784}$$
$$E_{21}(z) = \tfrac{0.127z-0.1008}{z^2-1.731z+0.784}$$
$$E_{22}(z) = \tfrac{0.0835z-0.075}{z^2-1.731z+0.784}$$
$$E_{23}(z) = \tfrac{0.146z-0.1139}{z^2-1.731z+0.784}$$

The step and impulse responses of original system is compared with reduced order models obtained by proposed technique and Desai [311] for Example 5.3, which are shown in Figures 5.5 and 5.6. From this it is observed that, the ROM obtained by the proposed technique gave close approximation with the original system compared to [311]. Further, the SSE, SAE and STAE values are also calculated to show the accurate approximation of the proposed method by comparing with [311], which are displayed in Table 5.3 for Example 5.3. It is noticed that, the ROM obtained by proposed technique provided least SSE, SAE and STAE values compared to other reduction method.

Table 5.3: Different reduced models SSE, SAE and STAE comparison for Example 5.3.

Reduction Methods	PI	r_{11}	r_{12}	r_{13}	r_{21}	r_{22}	r_{23}
Proposed method	SSE	1.082×10^{-6}	3.064×10^{-7}	1.126×10^{-7}	9.44×10^{-7}	1.08×10^{-6}	9.693×10^{-7}
	SAE	0.00104	0.000553	0.000335	0.000971	0.001039	9.845×10^{-4}
	STAE	0.06137	0.03819	0.02316	0.05732	0.08211	0.05809
Desai [311]	SSE	5.238×10^{-5}	2.485×10^{-6}	5.728×10^{-8}	3.882×10^{-5}	2.073×10^{-6}	5.55×10^{-5}
	SAE	0.00723	0.001576	0.000239	0.00623	0.00144	0.00745
	STAE	0.427	0.1088	0.01651	0.3676	0.1138	0.4395

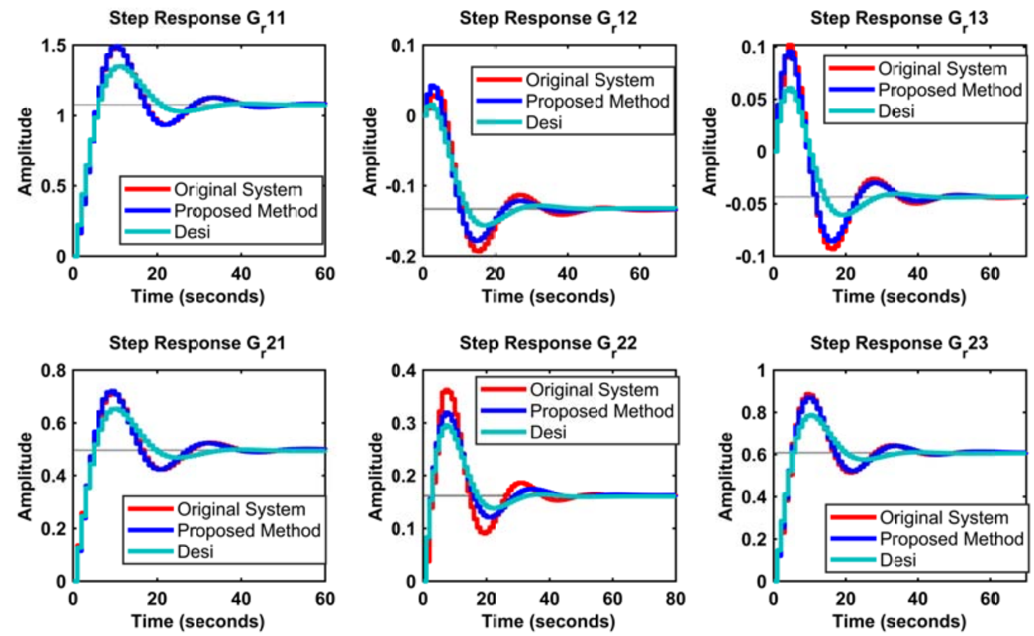


Fig. 5.5: Step response comparison of MIMO system for Example 5.3

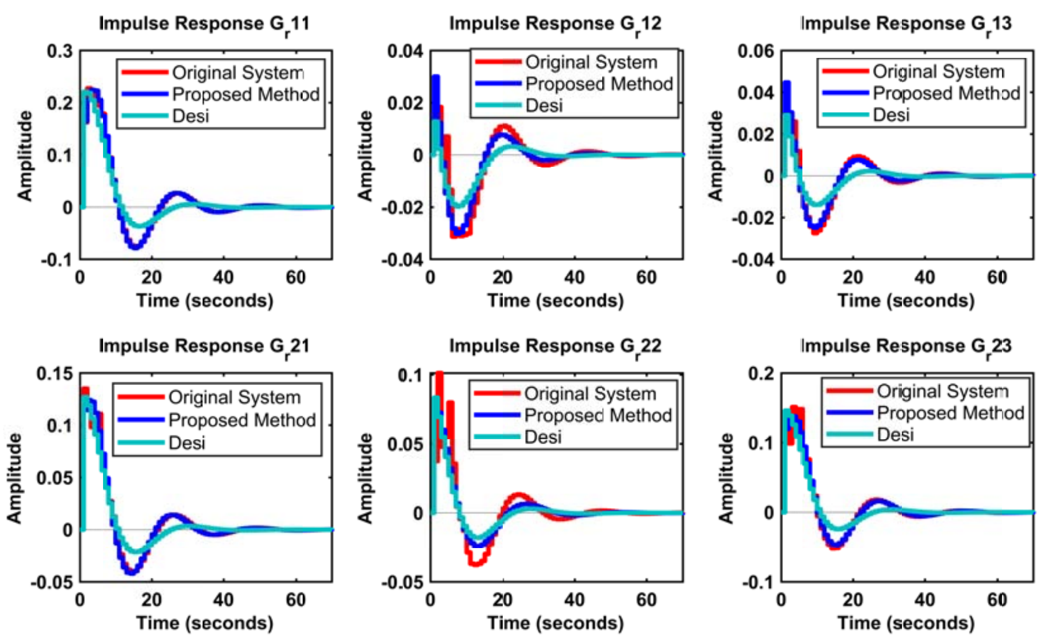


Fig. 5.6: Impulse response comparison of MIMO system for Example 5.3

5.4 INTERVAL SYSTEMS APPROACH IN DISCRETE DOMAIN

A number of reduction methods are developed in the area of continuous-time interval systems [108, 218, 231, 297, 298, 312–315], but very few reduction methods are extended to discrete-time interval systems. Ismail et al. [241] proposed a discrete interval reduction technique using Pade approximation and dominant poles. The lower order denominator is achieved by using retention of dominant poles of the higher order system, while the numerator is achieved by using Pade approximation method by matching the time-moments. Choo [243], proposed a model reduction method for discrete-time interval systems. This method preserves desired real dominant poles by overcoming the stability problems in [231]. Sastry and Mallikarjuna Rao [108] proposed simplified Routh approximation method (SRAM) for order reduction of interval models by preserving the initial time moments of higher-order interval systems. Pappa and Babu [316], proposed model reduction of discrete interval systems by differentiation method. Choudhary and Nagar [247], proposed γ-δ approximation for reduction of discrete-time interval systems.

The order reduction methods proposed in chapter 4 are developed for continuous time systems represented in frequency domain. In this chapter, some of the methods from chapter 4 are extended for order reduction of discrete time systems using linear transformation technique. To show the superiority, efficacy of the proposed technique various benchmark numerical examples have been considered. The results are compared in terms of step response, impulse response and performance indices with other well-known methods and recently published work.

5.4.1 PROBLEM STATEMENT

5.4.1.1 Single Input Single Output System

Let us consider a discrete-time original interval system described as follows,

$$G(z,p,q) = \frac{[p_0^-, p_0^+] + [p_1^-, p_1^+]z + \cdots + [p_{n-1}^-, p_{n-1}^+]z^{n-1}}{[q_0^-, q_0^+] + [q_1^-, q_1^+]z + \cdots + [q_n^-, q_n^+]z^n} \tag{5.10}$$

Our essential task is to determine the $k^{th}(k < n)$ order discrete-time reduced interval model as follows

$$R\left(z,u,v\right)=\frac{[u_0^-,u_0^+]+[u_1^-,u_1^+]z+\cdots+[u_{k-1}^-,u_{k-1}^+]z^{k-1}}{[v_0^-,v_0^+]+[v_1^-,v_1^+]z+\cdots+[v_k^-,v_k^+]z^k} \tag{5.11}$$

5.5 MODEL REDUCTION USING DIFFERENTIATION METHOD AND KHARITONOV'S THEOREM

The proposed technique is obtained by using differentiation method and Kharitonov's theorem. By using Kharitonov's theorem the higher order discrete-time interval systems are written into higher-order fixed parameter discrete-time systems. These z-domain fixed parameter HOS are converted to w-domain systems by applying linear transformation $z = (w + 1)$. Then these w-domain HOS are reduced by using differentiation method. These fixed parameter w-domain ROMs are converted back to z-domain ROMs by applying inverse linear transformation $w = (z - 1)$. Then, these fixed parameter reduced order models are rearranged to form sixteen combinations of reduced order interval models by using sixteen plant theorem. Finally, the required reduced interval model is obtained by comparing sixteen combinations of reduced order interval models with original interval system using SSE. The procedural steps to be followed are similar as discussed in section 4.4.

5.5.1 Numerical Examples and Results

To show the powerfulness and efficacy of the presented technique we considered popular SISO systems. The results are compared in terms of system response and performance indices.

Example 5.4: The 3^{rd} order original system [317] represented in discrete interval transfer function

$$G\left(z\right)=\frac{[1,2]\,z^2+[3,4]\,z+[8,10]}{[6,6]\,z^3+[9,9.5]\,z^2+[4.9,5]\,z+[0.8,0.85]}$$

The desired ROIM is obtained by using proposed technique

$$R_2\left(z\right)=\frac{[10.5,12]\,z+[28.5,30]}{[27.5,27.5]\,z^2+[28.8,29]\,z+[7.3,7.4]}$$

The step and impulse responses of original lower and upper bound interval systems and the proposed method and some other reduction methods [317–322] of lower and upper bounds for Example 5.4 are compared and are shown in Figures 5.7 and 5.8. It is clear that proposed reduction method is closely matching with original higher order response. Moreover, the SSE and SAE values of this method and other reduction methods are shown in Table 5.4, which shows that the proposed method gives much lower SSE and SAE value compared to other different reduction methods.

Table 5.4: The SSE and SAE comparison of different reduction methods for Example 5.4.

Reduction Methods	SSE		SAE	
	Lower Bound	Upper Bound	Lower Bound	Upper Bound
Proposed Method	0.001089	2.294×10^{-6}	0.03301	0.00151
Aseem et al. [318]	0.0672	0.0043	0.236	0.0932
Neeraj et al. [319]	0.188	0.2678	1.892	1.937
AK.Choudhary(Algor1) [320]	0.009332	2.89×10^{-4}	0.0966	0.017
AK.Choudhary(Algor2) [320]	0.1703	0.3794	0.4127	0.6159
Ruchira [321]	0.0105	0.025	0.6645	0.1388
Manish [322]	0.0852	0.0377	0.2893	0.38
AK.Choudhary(case1) [317]	0.02984	0.01938	0.1728	0.1392
AK.Choudhary(case3) [317]	0.002174	0.256	0.04663	0.506

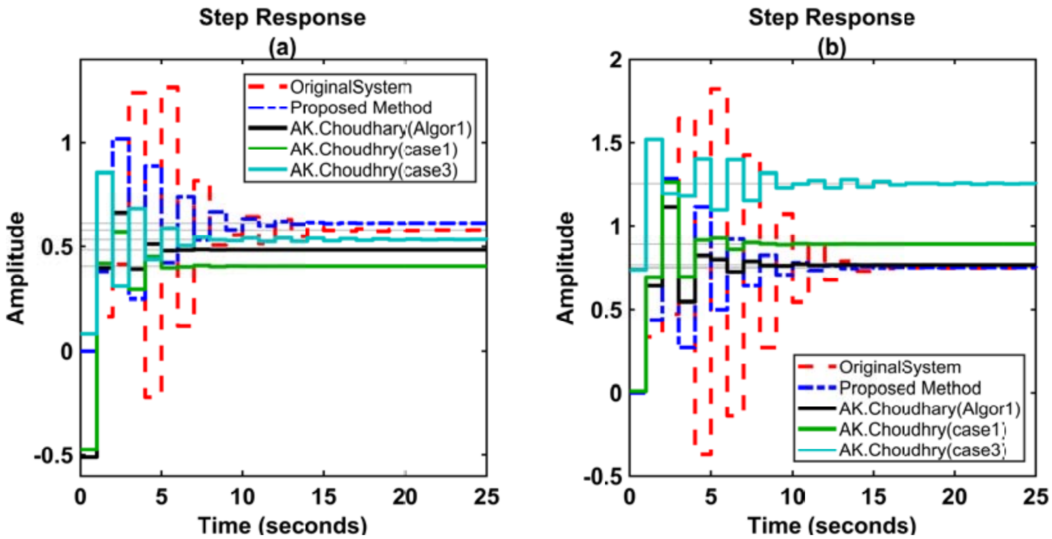


Fig. 5.7: Step response comparison of (a) lower bound (b) upper bounds for Example 5.4

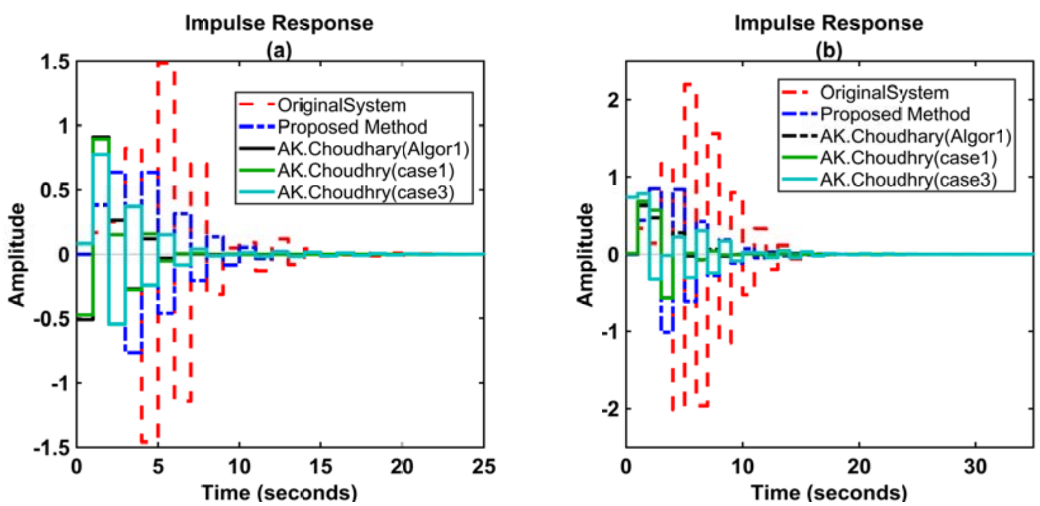


Fig. 5.8: Impulse response comparison of (a) lower bound (b) upper bounds for Example 5.4

Example 5.5: The 5^{th} order original system [323] described in discrete-time interval transfer function

$$G(s) = \frac{[2.3, 2.55]\,z^4 + [2.45, 2.65]\,z^3 + [3.25, 3.35]\,z^2 + [2.5, 2.65]\,z + [1.8, 2.2]}{[8.3, 8.35]\,z^5 + [4.6, 4.8]\,z^4 + [2.4, 2.5]\,z^3 + [2, 2.2]\,z^2 + [1.5, 1.8]\,z + [2.1, 2.15]}$$

The desired discrete-time ROIM is obtained by proposed technique is,

$$R(z) = \frac{[390, 400.5]\,z + [366, 385.5]}{[721.8, 729]\,z^2 + [315.2, 316.8]\,z + [199.8, 244.2]}$$

Figures 5.9 and 5.10 shows the step and impulse response comparisons of original and reduced order interval models of lower and upper bounds for Example 5.5. From these responses, it is observed that the presented technique is closely matched with original interval system. Moreover, the SSE and SAE of proposed and other reduction techniques are depicted in Table 5.5. It is clear that the proposed technique gives much better result than the other reduction method.

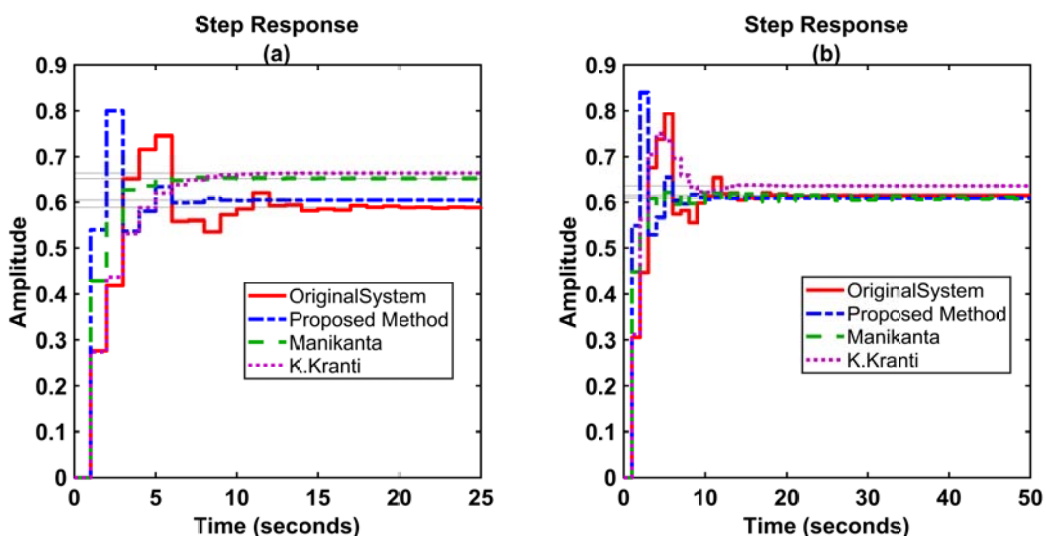


Fig. 5.9: Step response comparison of (a) lower bound (b) upper bounds for Example 5.5

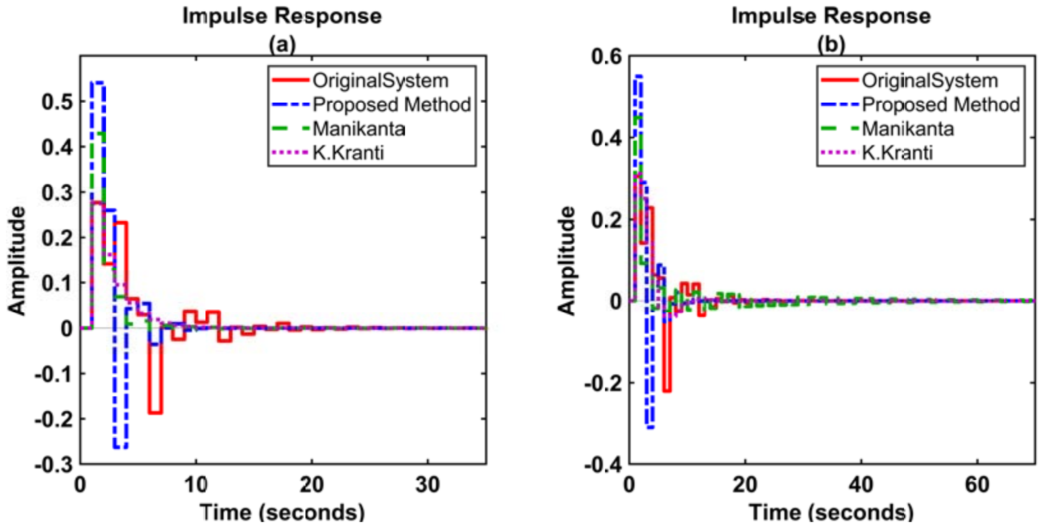


Fig. 5.10: Impulse response comparison of (a) lower bound (b) upper bounds for Example 5.5

Table 5.5: The SSE and SAE comparison of different reduction methods for Example 5.5.

Reduction Method	SSE		SAE	
	Lower Bound	Upper Bound	Lower Bound	Upper Bound
Proposed Method	2.782×10^{-4}	2.561×10^{-5}	1.668×10^{-2}	5.06×10^{-3}
Manikanta [323]	4.126×10^{-3}	1.125×10^{-4}	6.424×10^{-2}	1.061×10^{-2}
K. Kranthi [324]	5.644×10^{-3}	4.697×10^{-3}	7.513×10^{-2}	2.167×10^{-2}

5.6 MODEL REDUCTION USING STABILITY EQUATION METHOD AND KHARITONOV'S THEOREM

A new order reduction technique is proposed for reducing order of the higher-order discrete-time interval systems. The reduced order interval model is obtained by using Kharitonov's theorem, stability equation method and sixteen plant theorem. The Kharitonov's theorem converts the higher order discrete-time interval system into higher-order fixed parameter discrete-time systems. By applying linear transformation the z-domain systems are converted into w-domain systems. These

w-domain HOS are reduced by using stability equation method. Then, by applying inverse linear transformation the w-domain reduced order models are converted back to the z-domain fixed parameter reduced order models. Then, by using sixteen plant theorem [9] these reduced order fixed parameter models are rearranged to form sixteen combinations of reduced order discrete-time interval systems. The required reduced order interval model is achieved by comparing SSE between the transient parts of the HOIS and the ROIMs. The procedural steps to be followed are similar as discussed in section 4.5.

5.6.1 Numerical Examples and Results

To show the powerfulness and efficacy of the proposed technique we considered popular numerical examples. The results are compared in terms of step response, impulse response and performance indices.

Example 5.6: Consider the 5^{th} order discrete-time interval system from example 5.5 in section 5.5.1. The second order system is obtained by using proposed technique is as following

$$R_2(z) = \frac{[26.35, 26.7]\,z + [-13.75, -13.6]}{[111.5, 112.6]\,z^2 + [-149.8, -151.3]\,z + [59.44, 60.17]}$$

Figures 5.11 and 5.12 shows the step and impulse response comparisons of original and reduced order interval models of lower and upper bounds for Example 5.6. From these responses, it is observed that the presented technique is closely matched with original interval system. Moreover, the SSE and SAE of proposed and other reduction techniques are depicted in Table 5.6. It is clear that the proposed technique gives much better result than the other reduction method.

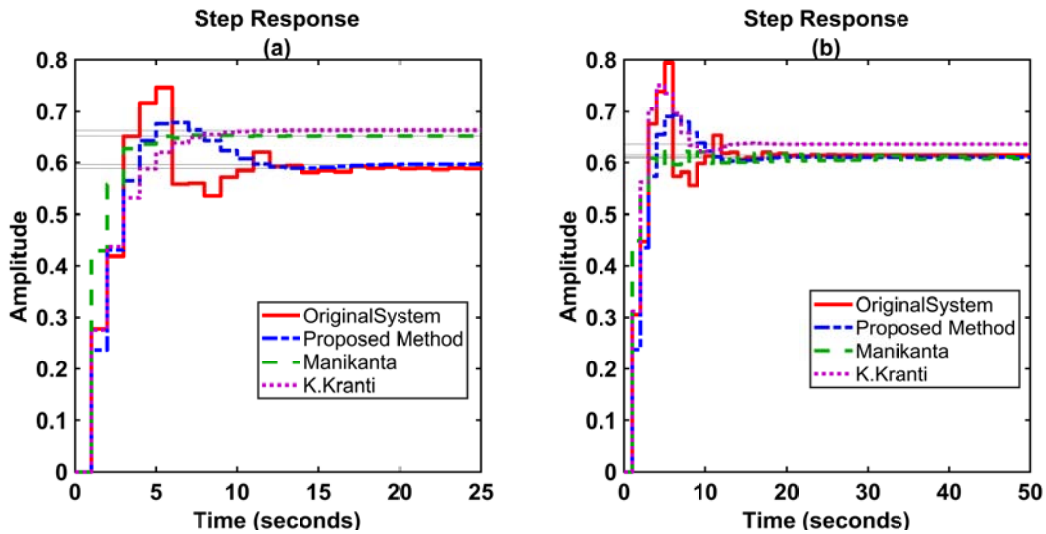


Fig. 5.11: Step response comparison of (a) lower bound (b) upper bounds for Example 5.6.

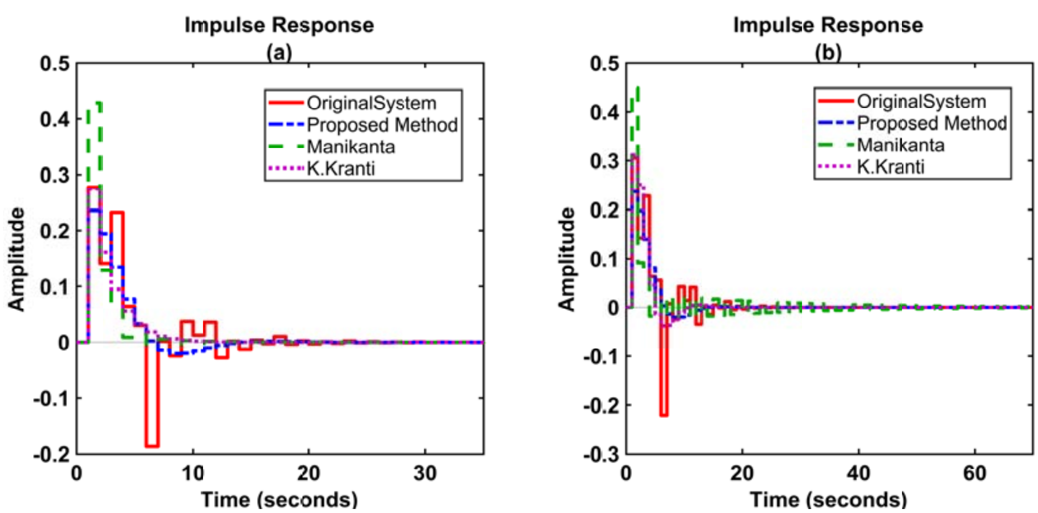


Fig. 5.12: Impulse response comparison of (a) lower bound (b) upper bounds for Example 5.6.

Table 5.6: The SSE and SAE comparisons of different reduction methods for Example 5.6.

Reduction Method	SSE		SAE	
	Lower Bound	Upper Bound	Lower Bound	Upper Bound
Proposed Method	9.611×10^{-5}	2.561×10^{-5}	9.804×10^{-3}	4.52×10^{-3}
Manikanta [323]	4.126×10^{-3}	1.125×10^{-4}	6.424×10^{-2}	1.061×10^{-2}
K. Kranthi [324]	5.644×10^{-3}	4.697×10^{-3}	7.513×10^{-2}	2.167×10^{-2}

Example 5.7: The 8^{th} order discrete-time interval system [322] described as follows,

$$G_8(z) = \frac{\begin{array}{l}[1.6484, 1.7156]\,z^7 + [1.0937, 1.1383]\,z^6 + [-0.2142, -0.2058]\,z^5 \\ + [0.1490, 0.1550]\,z^4 + [-0.5263, -0.5057]\,z^3 + [-0.2672, -0.2568]\,z^2 \\ + [0.0431, 0.0449]\,z + [-0.0061, -0.0059]\end{array}}{\begin{array}{l}[23.52, 24.48]\,z^8 + [-1.7156, -1.6484]\,z^7 + [-1.1383, -1.0937]\,z^6 \\ + [0.2058, 0.2142]\,z^5 + [-0.1550, -0.1490]\,z^4 + [0.5057, 0.5263]\,z^3 \\ + [0.2568, 0.2672]\,z^2 + [-0.0449, -0.0431]\,z + [0.0059, 0.0061]\end{array}}$$

The required reduced order interval model is obtained by using proposed technique,

$$R_2(z) = \frac{[16.11, 15.89]\,z + [-14.1, -13.9]}{[572.3, 548.7]\,z^2 + [-964.5, -925.4]\,z + [414.8, 398.2]}$$

The reduced order model obtained by using Choudhary and Nagar [317] method is,

$$R_2(z) = \frac{[2.27, 29.91]\,z^2 + [41.4, 42.7]\,z + [12.14, 39.78]}{[47.32, 62.95]\,z^2 + [-30, -2]\,z + [9.05, 24.68]}$$

The step and impulse response of original lower and upper bound interval systems and the proposed and Choudhary [317] reduction methods of lower and upper bounds for Example 5.7 are compared and are shown in Figures 5.13 and 5.14. Clearly, proposed method is quite comparable with original system in both lower bound and upper bound. Moreover, the SSE and SAE values of reduction methods are compared and are shown in Table 5.7, which shows that the proposed method obtained comparable low SSE and SAE values.

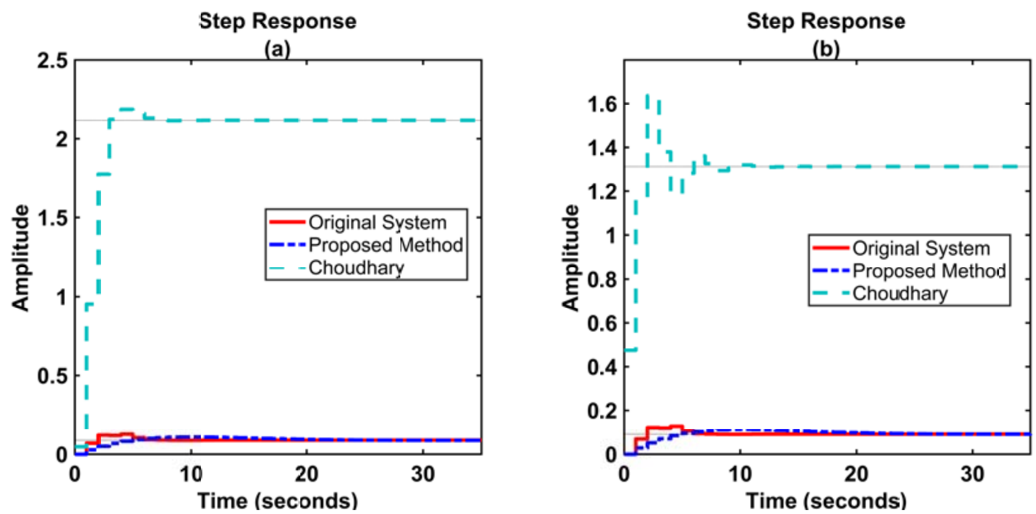


Fig. 5.13: Step response comparison of (a) lower bound (b) upper bounds for Example 5.7.

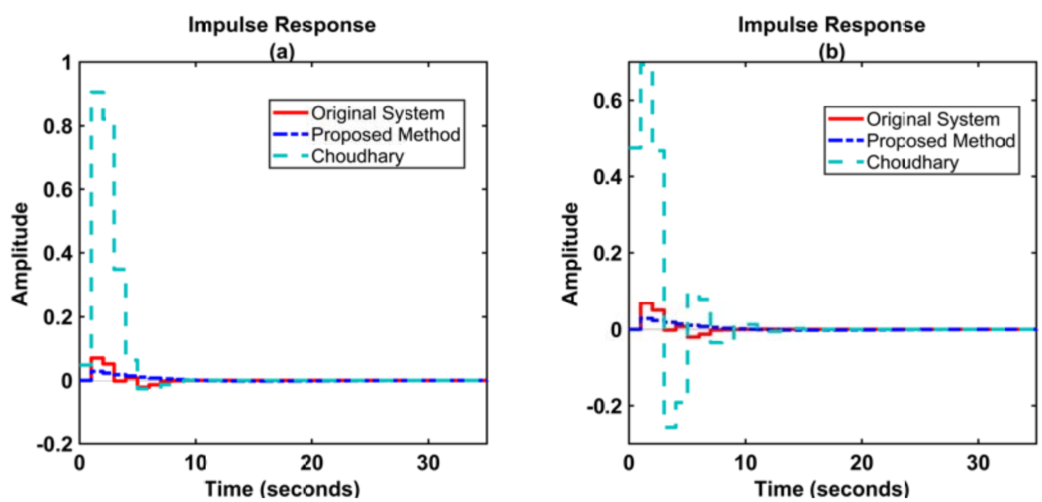


Fig. 5.14: Impulse response comparison of (a) lower bound (b) upper bounds for Example 5.7.

Table 5.7: The performance indices comparison of reduced order models for Example 5.7.

Reduction Method	Lower Bound			Upper Bound		
	SSE	SAE	STAE	SSE	SAE	STAE
Proposed Method	1.374×10^{-6}	1.172×10^{-3}	3.986×10^{-2}	3.417×10^{-8}	1.848×10^{-4}	6.285×10^{-3}
AK Choudhary [317]	4.108	2.027	68.91	1.489	1.22	41.49

5.7 MODEL REDUCTION USING MODIFIED TIME MOMENT MATCHING METHOD AND KHARITONOV'S THEOREM

A new order reduction technique is proposed for reducing order of the higher-order discrete-time interval systems. The proposed technique is obtained by using modified time-moment matching method and Kharitonov's theorem. The Kharitonov's theorem converts the higher order discrete-time interval system into higher-order fixed parameter discrete-time systems. These z-domain fixed parameter HOS are converted to w-domain systems by applying linear transformation $z = (w+1)$. Then these w-domain HOS are reduced by using modified time-moment matching method. After that, these w-domain ROMs are converted back to z-domain ROMs by applying inverse linear transformation $w = (z-1)$. Then, these fixed parameter reduced order models are rearranged to form sixteen combinations of reduced order interval models by using sixteen plant theorem. Finally, the required reduced interval model is obtained by comparing sixteen combinations of reduced interval models with original interval system using summation square error. The procedural steps to be followed are similar as discussed in section 4.6.

5.7.1 Numerical Examples and Results

To show the powerfulness and efficacy of the presented technique we considered popular SISO systems. The results are compared in terms of step response, impulse responses and performance indices.

Example 5.8: Consider the 3^{rd} order discrete-time interval system from example 5.4 in section 5.5.1. Using proposed method, the following second order system is obtained

$$R_2(z) = \frac{[0.252, 0.09128]\, z + [0.4276, 0.8665]}{[1,1]\, z^2 + [0.0043, 0.2117]\, z + [0.1041, 0.1197]}$$

The step and impulse responses of original lower and upper bound interval systems and the proposed method and some other reduction methods [317–322] of lower and upper bounds for Example 5.8 are compared and are shown in Figures 5.15 and 5.16. It is clear that proposed reduction method is closely matching with original higher order response. Moreover, the SSE and SAE values of this method and other reduction methods are shown in Table 5.8, which shows that the proposed method gives much lower SSE and SAE values compared to other different reduction methods.

Table 5.8: The SSE and SAE comparison of different reduction methods for Example 5.8.

Reduction Methods	SSE		SAE	
	Lower Bound	Upper Bound	Lower Bound	Upper Bound
Proposed Method	0.001031	1.214×10^{-6}	0.02301	0.00115
Manish [322]	0.0852	0.0377	0.2893	0.38
Ruchira [321]	0.0105	0.025	0.6645	0.1388
AK.Choudhary(Algor1) [320]	0.009332	2.89×10^{-4}	0.0966	0.017
AK.Choudhary(Algor2) [320]	0.1703	0.3794	0.4127	0.6159
Neeraj et al. [319]	0.188	0.2678	1.892	1.937
Aseem et al. [318]	0.0672	0.0043	0.236	0.0932
AK.Choudhary(case1) [317]	0.02984	0.01938	0.1728	0.1392
AK.Choudhary(case3) [317]	0.002174	0.256	0.04663	0.506

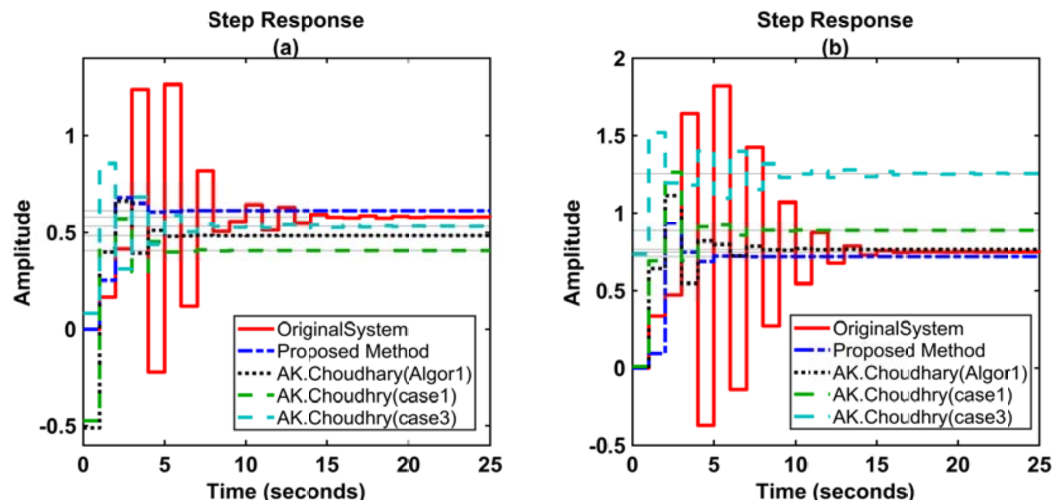


Fig. 5.15: Step response comparison of (a) lower bound (b) upper bounds for Example 5.8.

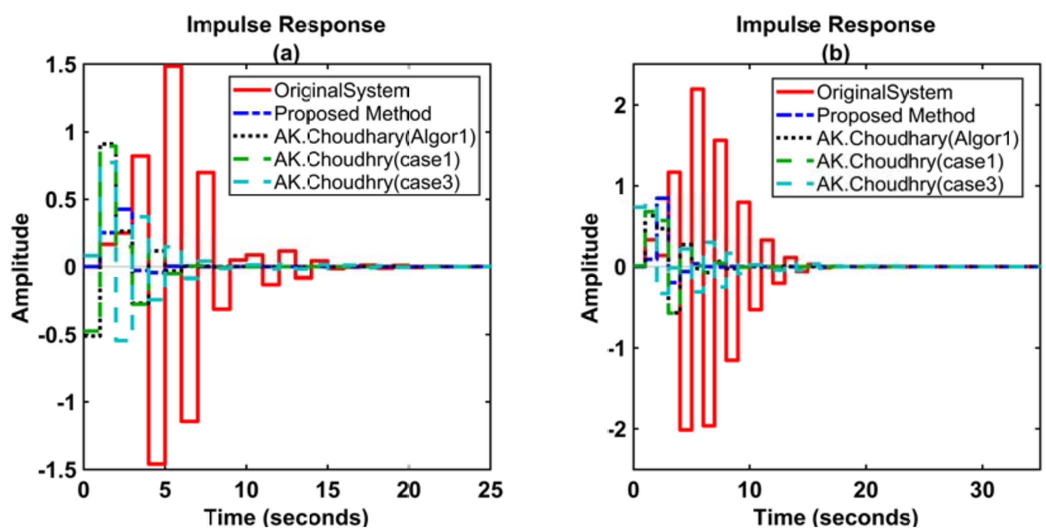


Fig. 5.16: Impulse response comparison of (a) lower bound (b) upper bounds for Example 5.8.

Example 5.9: Consider the 5^{th} order discrete-time interval system from example 5.5 in section 5.5.1. The required reduced order model is obtained by using proposed technique.

$$R_2(z) = \frac{[0.204582, 0.23630848]\,z + [0.1559464, 0.12482784]}{[1,1]\,z^2 + [-0.8091218, -0.7990144]\,z + [0.4253237, 0.38259972]}$$

Figures 5.17 and 5.18 shows the step and impulse response comparisons of original and reduced order interval models of lower and upper bounds for Example 5.9. From these responses, it is observed that the presented technique is closely matched with original interval system. Moreover, the SSE and SAE of proposed and other reduction techniques [323, 324] are depicted in Table 5.9. It is clear that the proposed technique gives much better result than the other reduction method.

Table 5.9: The SSE and SAE comparison of different reduction methods for Example 5.9.

Reduction Method	SSE		SAE	
	Lower Bound	Upper Bound	Lower Bound	Upper Bound
Proposed Method	1.15×10^{-5}	1.741×10^{-5}	3.377×10^{-3}	4.172×10^{-3}
Manikanta [323]	4.126×10^{-3}	1.125×10^{-4}	6.424×10^{-2}	1.061×10^{-2}
Kranthi [324]	5.644×10^{-3}	4.697×10^{-3}	7.513×10^{-2}	2.167×10^{-2}

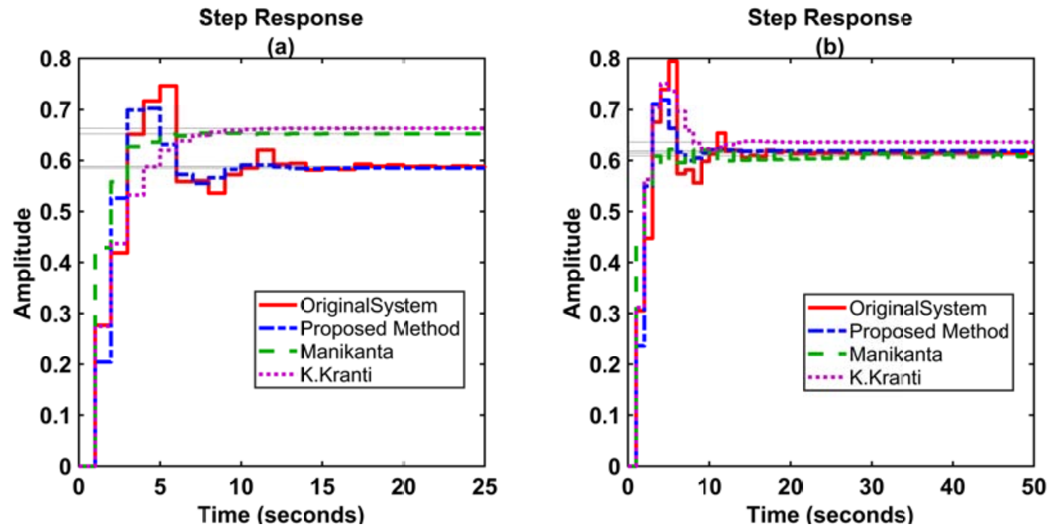


Fig. 5.17: Step response comparison of (a) lower bound (b) upper bounds for Example 5.9.

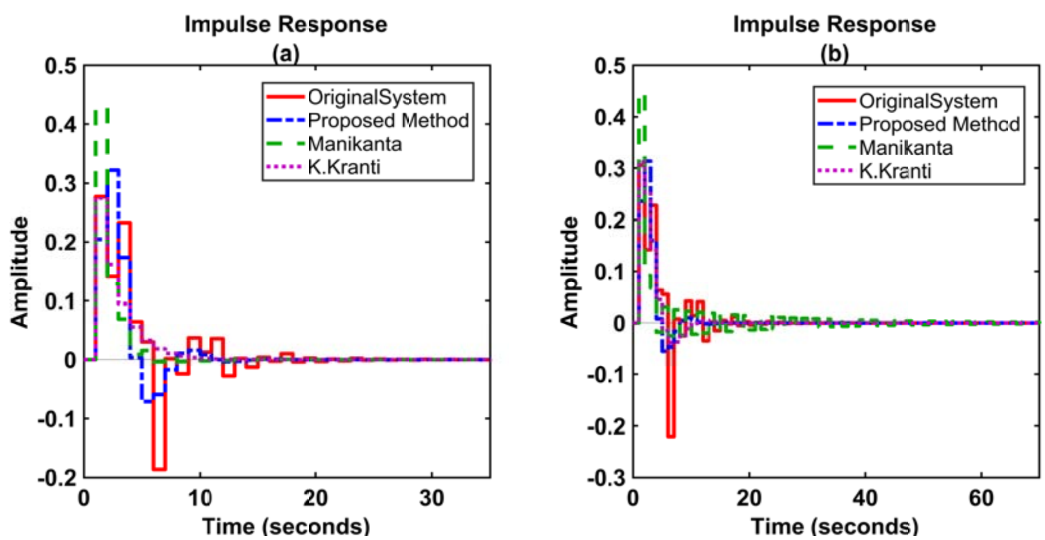


Fig. 5.18: Impulse response comparison of (a) lower bound (b) upper bounds for Example 5.9.

5.8 CONCLUSION

The new reduction methods proposed for reduced order modelling of continuous time systems which are discussed in chapter 3 and 4. These continuous time systems are extended for order reduction of discrete time systems. The first reduction technique in discrete domain is obtained by extending the continuous time reduction technique discussed in section 3.3. Remaining discrete-time reduction methods are obtained by extending the continuous time reduction methods discussed in section 4.4, 4.5 and 4.6. The reduced order models are achieved by first converting the given z-domain HOS to the w-domain systems using linear transformation and then the transformed system is reduced. After that, the reduced system is again converted back to the z-domain using inverse linear transformation. The step and impulse responses of the original and reduced order interval models are plotted. Further, the performance of the proposed methods evaluated in terms of performance indices. From these results it is observed that, the proposed techniques provided quite comparable results which are justified by solving benchmark numerical examples from the literature.

CHAPTER 6

CONTROLLER DESIGN

6.1 INTRODUCTION

In previous chapters several MOR techniques have been developed in continuous-time domain which approximate certain characteristics or properties of the original HOS. The quality of ROM is judged by the degree of its success in representing the desired characteristics of the system [325] and the way it is utilized. One of the main objective of MOR is to obtain low order controller to control the high order systems effectively so that, the overall system is of low order, which is easy to understand and analyze. It is thus important that the MOR methods should reduce the high order controller to a low order controller without incurring too much error. Model reduction is based on open loop considerations while closed loop stability performance is main concern in controller reduction.

In this chapter, suitability of the proposed reduction methods are examined for controller design. The problem is to design a controller $G_C(s)$ for an uncontrolled plant $G_p(s)$ such that the closed loop response with unity feedback is stable and has suitable fast response. The design problem may be stated as: It is required to find a controller $G_C(s)$ such that the time and frequency responses of the controlled system closely match with those of the reference model even for poor dynamic characteristics of the higher order plant $G_p(s)$.

To design a controller there are two common approaches. First approach is to design a controller for reduced order plant is called plant reduction [325] and second approach is to design a controller for high order system then reduce the closed loop higher order system is called controller reduction [326–329]. The common problem with the plant reduction approach is that, due to early stage reduction the error propagates in the design. While in controller reduction approach, error does not propagate as process reduction is carried out in the final stage of the design [330, 331]. It is shown that the proposed mixed methods based on Pade approximation and differentiation method, and Modified time-moment matching methods are suitable for both the approaches of controller design. By using Pade sense of approximate model matching the controller parameters are obtained. Figure 6.1 show the block diagram representation of direct and indirect controller design approaches [332]. Figure 6.2 show the comparison of full order controller and reduced order controller with reference model performance.

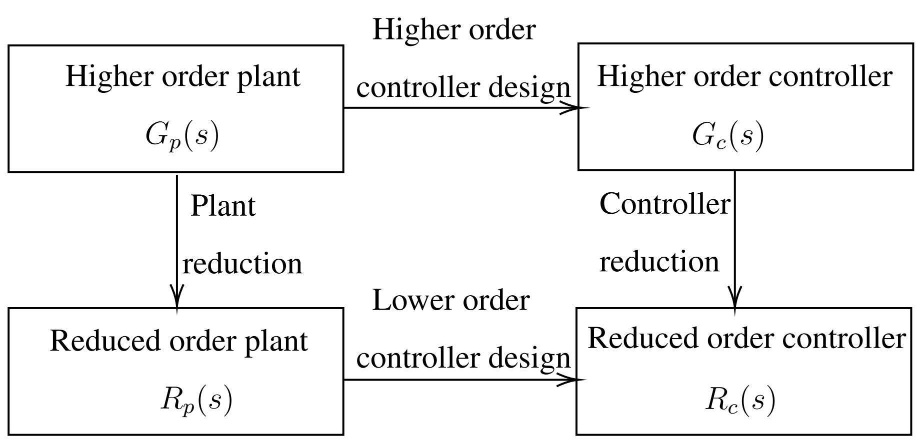


Fig. 6.1: Block diagram of direct and indirect controller design approaches

6.2 CHOICE OF REFERENCE MODEL

The choice of controller design and its complexity and structure depends on the kind of reference model which is chosen as desired closed loop system. The stability and the acceptable performance of the closed loop system must be ensured by the reference model. The design specifications [333, 334] of the reference model may chosen to meet as follows,

- The time domain specifications such as, settling time, rise time, steady state error, Max. overshoot.
- The complex domain specifications such as, undamped natural frequency, damping factor, damping ratio, and location of closed loop poles.

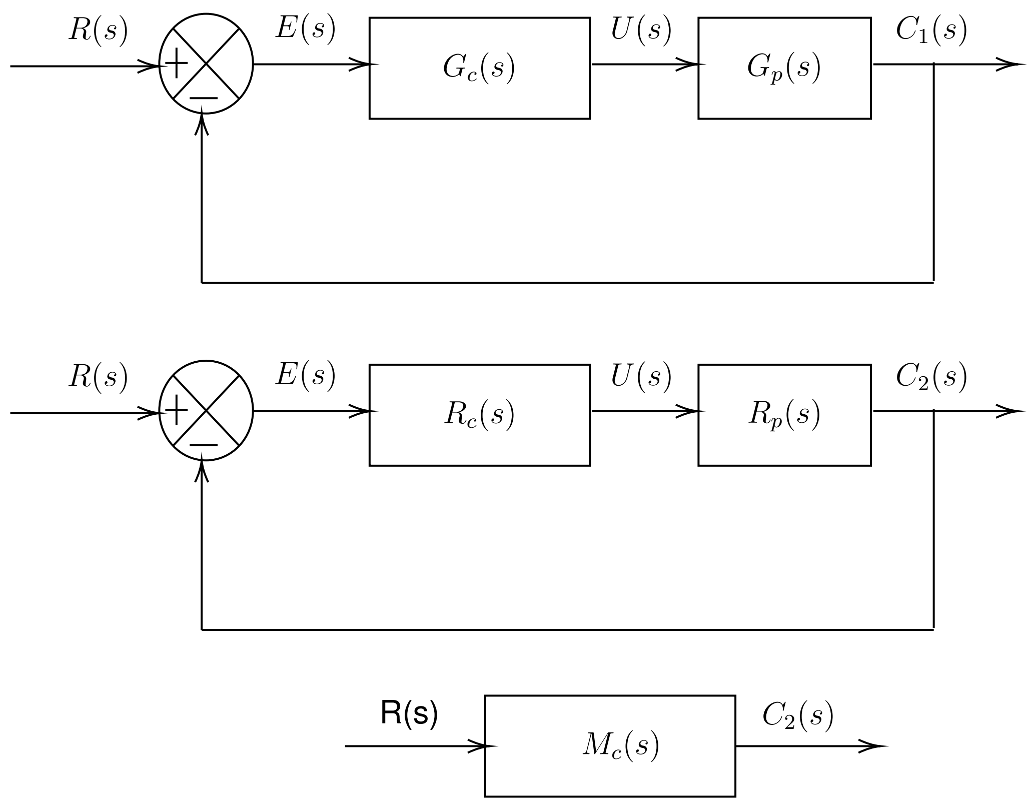


Fig. 6.2: Original and reduced order closed loop configuration with reference model.

- The frequency domain specifications such as, gain margin, phase margin, bandwidth and cut off rate.

The reference model can be constructed using the methods [335–337]. The reference model is specified such that the closed loop response of the controlled system should approximate the reference model.

6.3 PLANT REDUCTION AND CONTROLLER DESIGN: DIRECT APPROACH

The procedural steps to design a controller is based on approximate model matching in Pade sense as follows,

Step 1: Construct a reference model $M(s)$ for the plant having a transfer function $G_p(s)$ on the basis of specification given in section 6.2, such that the closed loop control system response approximates with reference model.

Let the plant transfer function $G_p(s)$ and the reference model $M(s)$ are given by

$$G_p(s) = \frac{\sum\limits_{i=0}^{m} a_i s^i}{\sum\limits_{i=0}^{n} b_i s^i} \; m \leq n \tag{6.1}$$

$$M(s) = \frac{\sum\limits_{i=0}^{u} g_i s^i}{\sum\limits_{i=0}^{v} h_i s^i} \tag{6.2}$$

Step 2: Determination of a corresponding open loop specification model $\tilde{M}(s)$. The corresponding transfer function of open loop specification model for reference model $M(s)$ is determined as follows.

$$\tilde{M}(s) = \frac{M(s)}{1 - M(s)} \tag{6.3}$$

Step 3: Controller structure specification

Let the controller structure $G_C(s)$ is given by

$$G_C(s) = \frac{\sum\limits_{i=0}^{k} p_i s^i}{\sum\limits_{i=0}^{l} q_i s^i} \quad k \leq l \tag{6.4}$$

Step 4: For obtaining the unknown parameters of the controller, the response of closed loop system is matched with the reference model as

$$G_C(s)\, G_p(s) = \tilde{M}(s) \tag{6.5}$$

which leads to

$$G_C(s) = \frac{\tilde{M}(s)}{G_p(s)} = \sum_{i=0}^{\infty} e_i s^i \tag{6.6}$$

Where $e_i \, (o \leq i \leq \infty)$ are the power series expansion coefficients about $s = 0$.

The controller may be taken as

$$G_C(s) = \frac{k\,(1 + T_1 s)}{s\,(1 + T_2 s)} \quad T_1 > T_2 \tag{6.7}$$

Now the unknown controller parameters $p_i\left(o\leq i\leq k\right)\ \ \&\ \ q_i\left(0\leq i\leq l\right)$ of the controller are calculated by equating the eq. (6.6) and (6.4) in Pade sense.

$$\begin{aligned}
&p_0 = q_0e_0\\
&p_1 = q_0e_1 + q_1e_0\\
&p_2 = q_0e_2 + q_1e_1 + q_2e_0\\
&\qquad\quad\cdots\\
&p_i = q_0e_i + q_1e_{i-1} + q_2e_{i-2} + \cdots + q_{i-1}e_i + q_ie_0\\
&0 = q_0e_{i+1} + q_1e_i + q_2e_{i-1} + \cdots + q_ie_1 + q_{i+1}e_0\\
&\qquad\quad\cdots\\
&0 = q_0e_{i+j} + q_1e_{i+j-1} + q_2e_{i+j-2} + \cdots + q_{j-1}e_{i+1} + q_je_i
\end{aligned} \tag{6.8}$$

By solving the above linear equations the desired structure of the controller is obtained.

Step 5: The closed loop transfer function is obtained by using the plant and controller transfer functions as follows,

$$G_{CL}\left(s\right) = \frac{G_C\left(s\right)G_p\left(s\right)}{1 + G_C\left(s\right)G_p\left(s\right)} \tag{6.9}$$

Step 6: The closed loop transfer function for the ROM is obtained by Reducing the plant $G_p\left(s\right)$ to $R_p\left(s\right)$ using one of the reduction methods discussed earlier in the chapters 3, and repeat the procedural steps of step 4 and 5 of section 6.3,

$$R_{CL}\left(s\right) = \frac{R_C\left(s\right)R_p\left(s\right)}{1 + R_C\left(s\right)R_p\left(s\right)} \tag{6.10}$$

6.3.1 Illustrative Examples and Results

Example 6.1: Consider a fuel control system taken from Aguirre [133] having transfer function and reference model as

$$G_p\left(s\right) = \frac{0.4299s^2 + 0.6010s + 0.1069}{s^3 + 0.7026s^2 + 0.8746s + 0.1107} \tag{6.11}$$

Step 1: Specification of the reference model $M\left(s\right)$ For this example, we specify the model as follows [133]

$$M\left(s\right) = \frac{0.16}{s^2 + s + 0.16} \tag{6.12}$$

Step 2: Determination of an equivalent open loop specification $\tilde{M}(s)$ is obtained by using eq. (6.3)

$$\tilde{M}(s) = \frac{0.16}{s(1+s)} \tag{6.13}$$

Step 3: Specification of the structure of controller $G_C(s)$ is

$$G_C(s) = \frac{k(1+T_1 s)}{s(1+T_2 s)} \quad T_1 > T_2 \tag{6.14}$$

Step 4: Determination of unknown controller parameters

Also the required controller is given by

$$\begin{aligned} G_C(s) = \frac{\tilde{M}(s)}{G_p(s)} &= \frac{0.017712+0.139936s+0.112416s^2+0.16s^3}{s(0.1069+0.7079s+1.0309s^2+0.4299s^3)} \\ &= \frac{1}{s}\begin{pmatrix} 0.165688+0.211841s-1.949049s^2+11.694259s^3 \\ -59.496367s^4+\cdots \end{pmatrix} \end{aligned} \tag{6.15}$$

Now by matching eq. (6.14) with the power series expansion eq. (6.15) in the Pade sense, we get

$$\begin{aligned} k &= 0.1657 \\ kT_1 &= 0.2118 + 0.1657T_2 \\ 0 &= -1.9490 + 0.2118T_2 \end{aligned}$$

Whose solution leads to,

$$\begin{aligned} k &= 0.1657 \\ T_1 &= 10.48028 \\ T_2 &= 9.20207 \end{aligned}$$

Consequently the controller $G_C(s)$ is given by

$$G_c(s) = \frac{0.165688(1+10.479096s)}{s(1+9.200540s)} \tag{6.16}$$

Step 5: Determination of closed loop transfer function

The closed loop system $G_{CL}(s)$ is given by

$$\begin{aligned} G_{CL}(s) &= \frac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)} \\ &= \frac{0.017712+0.285184s+1.114719s^2+0.746416s^3}{0.017712+0.395884s+3.007819s^2+9.495809s^3+7.464299s^4+9.200540s^5} \end{aligned} \tag{6.17}$$

Now we reduce the original plant transfer function $G_p(s)$ to its second order reduced model $R_p(s)$ by using the proposed method i.e., Pade approximation and differentiation method, discussed in section 3.5 of chapter 3.

$$R_p(s) = \frac{0.958439s + 0.32069}{0.7026s^2 + 1.7492s + 0.3321} \tag{6.18}$$

Then, the reduced order controller is given by

$$\begin{aligned} R_C(s) &= \tfrac{\tilde{M}(s)}{R_p(s)} = \tfrac{0.1124s^2+0.2799s+0.05314}{0.9584s^3+1.279s^2+0.3207s} \\ &= \tfrac{1}{s}\left(0.1657 + 0.2119415s - 0.98996s^2 + \cdots\right) \end{aligned} \tag{6.19}$$

Now $R_C(s)$ is of the form

$$R_C(s) = \frac{k(1+T_1 s)}{s(1+T_2 s)} \quad T_1 > T_2 \tag{6.20}$$

By matching eq. (6.20) with the power series expansion eq. (6.19) in the Pade sense, to get

$$k = 0.1657,\ T_1 = 5.949868,\ T_2 = 4.6708013$$

Therefore,

$$R_C(s) = \frac{0.985893s + 0.1657}{4.6708013s^2 + s} \tag{6.21}$$

Now the reduced order closed loop transfer function $R_{CL}(s)$ is obtained by using eq. (6.10)

$$R_{CL}(s) = \frac{0.945s^2 + 0.475s + 0.05314}{3.282s^4 + 8.873s^3 + 4.2449s^2 + 0.8071s + 0.05314} \tag{6.22}$$

The reduced order closed loop system obtained by using Narwal method [338] is

$$R_{CL}(s) = \frac{0.08835s^2 + 0.31401s + 0.143488}{2.51706s^4 + 2.514766s^3 + 2.947449s^2 + 1.21081s + 0.143488} \tag{6.23}$$

The step response comparisons of open loop original plant transfer function $G_p(s)$ and the reduced order plant models obtained by proposed technique and A. Narwal [338] are shown in Figure 6.3. It may be seen from the figure that the responses of the proposed reduced model gave close steady state matching with the $G_p(s)$ compared to other method. Also the step responses of closed loop transfer function of original system $G_{CL}(s)$ and the reduced order closed loop systems ($R_{CL}(s)$) of proposed model and A. Narwal model are compared with the step response of the reference model $M(s)$ which are shown in Figure 6.4. Further, the qualitative comparison of original and reduced order closed loop systems are compared with reference model in Table 6.1.

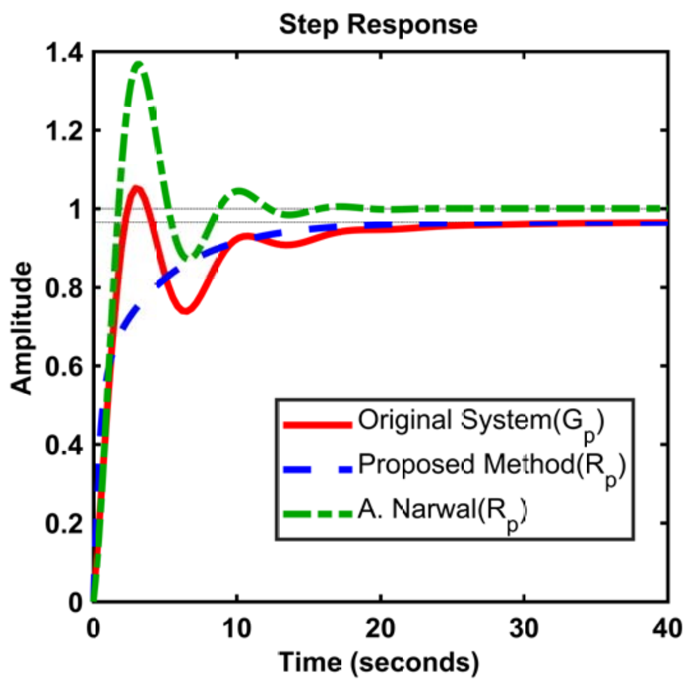


Fig. 6.3: Step response comparisons of original and reduced order plant for Example 6.1

Table 6.1: Qualitative comparison of original and reduced order models with reference model for Example 6.1

Systems	Rise time	Peak Overshoot	Steady state
$G_{CL}(s)$	12.2	0%	1
Reference model $M(s)$	11.6	0%	1
Proposed $R_{CL}(s)$	11.8	0%	1
A. Narwal $R_{CL}(s)$ [338]	11.1	0%	1

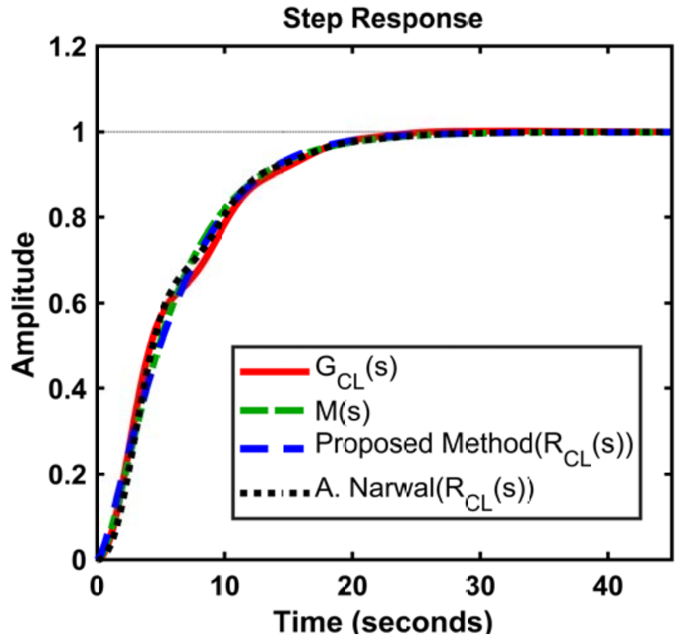


Fig. 6.4: Step response comparisons of original and reduced order closed loop models with reference model for Example 6.1

PID controller:

Generally, it is considered that the plant which is to be controlled is completely known to us, but in actual practice, it is not possible in each and every case. The performance of the closed loop system can be improved by introducing a PID (Proportional, integral and derivative) controller. The block diagram of the system with PID controller is given in Figure 6.5.

The input to the plant consists of three components: (i) K_1E, (ii) K_2E/s and (iii) K_3sE . The first component K_1E is proportional to the error, the second component K_2E/s is proportional to the integral of the error and the third component K_3sE is proportional to the derivative of the error. The first component K_1E increases the loop gain of the system which results in reduction of its sensitivity to plant parameter variations whereas the second component K_2E/s increases the order of the system and reduces the steady state error and the third component K_3sE helps in stabilizing the system by introducing the derivative term [339].

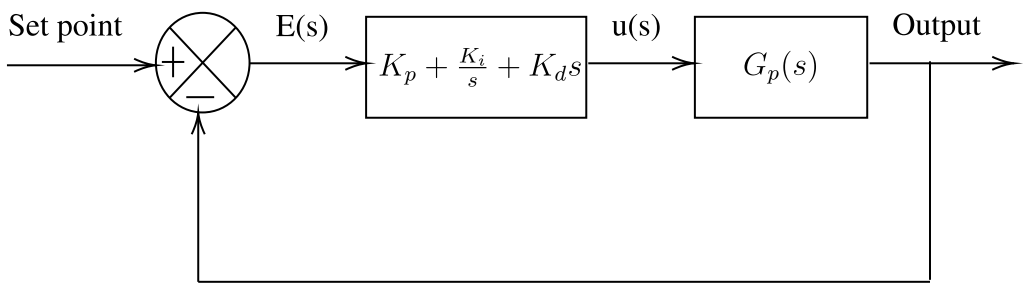


Fig. 6.5: Block diagram of PID Controller

Example 6.2: Consider the regulator problem [340] having the plant transfer function

$$G_p(s) = \frac{s^5 + 8s^4 + 20s^3 + 16s^2 + 3s + 2}{s^6 + 18.3s^5 + 102.42s^4 + 209.46s^3 + 155.94s^2 + 33.6s + 2} \tag{6.24}$$

The reference model

$$M(s) = \frac{0.0121 + 0.023s}{0.0121 + 0.21s + s^2} \tag{6.25}$$

From eq. (6.3)

$$\tilde{M}(s) = \frac{0.0121 + 0.023s}{s(0.187 + s)} \tag{6.26}$$

and

$$\begin{aligned}\frac{\tilde{M}(s)}{G_p(s)} &= \frac{\begin{array}{r}0.0242 + 0.452560s + 2.659674s^2 + 6.121086s^3 + 6.056862s^4 \\ +2.577090s^5 + 0.433s^6 + 0.023s^7\end{array}}{s(0.374+2.561s+5.992s^2+19.74s^3+21.496s^4+8.187s^5+s^6)} \\ &= \frac{1}{s}\left(0.064706 + 0.766974s + 0.822824s^2 - 4.971038s^3 - 7.148807s^4 + \cdots\right)\end{aligned} \tag{6.27}$$

Let the PID controller $G_C(s)$ be

$$G_C(s) = K_1 + \frac{K_2}{s} + K_3 s \tag{6.28}$$

Comparing the coefficients of equation eq. (6.27) and (6.28), the parameters $K_1, K_2\ \&\ K_3$ of the controller $G_C(s)$ are obtained and we get the PID controller as

$$G_C(s) = 0.766974 + \frac{0.064706}{s} + 0.822824s \tag{6.29}$$

The closed loop transfer function $G_{CL}(s)$ is given by

$$G_{CL}(s) = \frac{\begin{array}{l}0.129412 + 1.728065s + 4.981864s^2 + 16.034172s^3 + 29.022311s^4 \\ +22.656979s^5 + 7.349567s^6 + 0.822824s^7\end{array}}{\begin{array}{l}0.129412 + 3.728065s + 38.581864s^2 + 171.974172s^3 + 238.482311s^4 \\ +125.076979s^5 + 25.649567s^6 + 1.822824s^7\end{array}} \tag{6.30}$$

Now the given plant transfer function $G_p(s)$ is reduced to its second order reduced model $R_p(s)$ by using the proposed method i.e., time moment matching method, which is discussed in section 3.3 of chapter 3.

$$R_p(s) = \frac{0.026543s + 0.01266}{s^2 + 0.220241s + 0.01266} \tag{6.31}$$

The reduced order controller is obtained by using eq. (6.26) and (6.31)

$$\begin{aligned} R_C(s) = \frac{\tilde{M}(s)}{R_p(s)} &= \frac{0.023s^3+0.01717s^2+0.002956s+0.0001532}{0.02654s^3+0.01762s^2+0.002367s} \\ &= \frac{1}{s}\left(0.064723 + 0.76703s + 0.818359s^2 - \cdots\right) \end{aligned} \tag{6.32}$$

The reduced order controller is obtained as

$$R_C(s)\,\frac{0.76703s + 0.064723 + 0.818359s^2}{s} \tag{6.33}$$

Therefore, the closed loop reduced order model is

$$R_{CL}(s) = \frac{R_C R_p}{1 + R_C R_p} = \frac{0.02172s^3 + 0.03072s^2 + 0.01143s + 0.0008194}{1.02172s^3 + 0.25092s^2 + 0.02409s + 0.0008194}$$

The reduced order closed loop system obtained by using Narwal method [338] is

$$R_{CL}(s) = \frac{0.05187s^3 + 0.04385s^2 + 0.007202s + 0.0006062}{1.052s^3 + 0.1875s^2 + 0.01657s + 0.0006062} \tag{6.34}$$

The step responses of closed loop original and reduced order models of proposed and Narwal [338] are compared with the step responses of the reference model $M(s)$ in Figure 6.6. It is observed that the response of proposed model gave close response with $M(s)$ than the other reduction model. Further, the qualitative comparison of original and reduced order closed loop systems are compared with reference model in Table 6.2

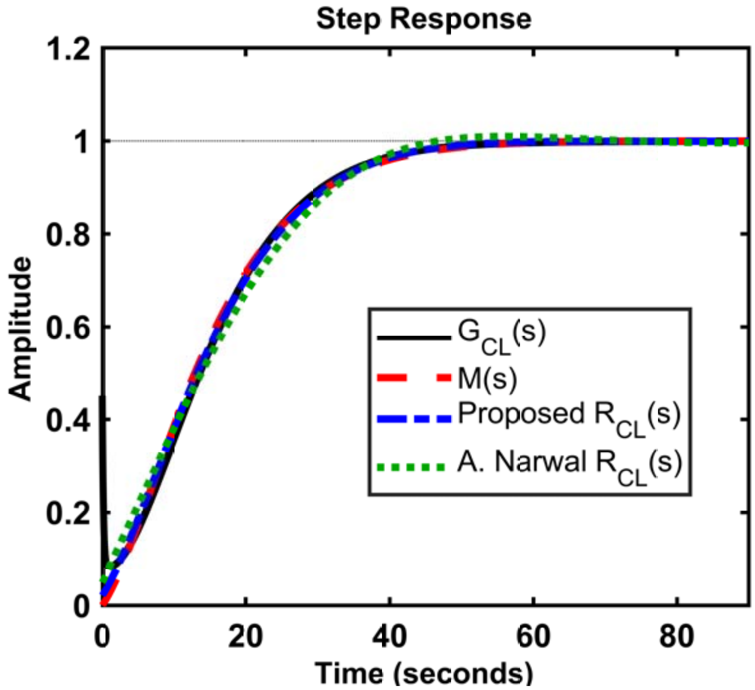


Fig. 6.6: Step response comparisons of original and reduced order closed loop models with reference model for Example 6.2

Table 6.2: Qualitative comparison of original and reduced order models with reference model for Example 6.2

Systems	Rise time	Peak Overshoot	Steady state
$G_{CL}(s)$	22.2	0%	1
Reference model $M(s)$	28.1	0%	1
Proposed $R_{CL}(s)$	28.3	0%	1
A. Narwal $R_{CL}(s)$ [338]	29.7	0.955%	1.019

6.4 INDIRECT APPROACH: PLANT REDUCTION AND CONTROLLER DESIGN

In this approach, first a controller is designed for the higher order plant, from this the high order closed loop transfer function with unity feedback is obtained by using high order plant transfer function and controller transfer function. Then the high order closed loop transfer function is reduced to lower order closed loop transfer function. The performance of the higher order closed loop transfer function $G_{CL}(s)$ and the lower order closed loop transfer function $R_{CL}(s)$ are compared with that of the reference model.

6.4.1 Illustrative Examples and Results

Example 6.3: Consider a plant transfer function taken from Prasad et. al. [341]

$$G(s) = \frac{s^3 + 12s^2 + 54s + 72}{s^4 + 18s^3 + 97s^2 + 180s + 100}$$

The design specifications are $w_n = 5.0,\ \zeta = 0.707$, using the method of Towil [337], the reference model comes out to be

$$M(s) = \frac{4.242s + 25}{s^2 + 7.07s + 25}$$

The equivalent open loop transfer function is obtained using eq. (6.3)

$$\tilde{M}(s) = \frac{4.242s^3 + 54.99s^2 + 282.8s + 625}{s^4 + 9.898s^3 + 44.99s^2 + 70.7s}$$

The controller structure is given by

$$G_C(s) = \frac{k(1 + k_1 s)}{s(1 + k_2 s)} \tag{6.35}$$

In order to match the response of closed loop system $G_{CL}(s)$ exactly with that of the reference model $M(s)$, the required controller is given by $G_C(s)\,G_p(s) = \tilde{M}(s)$

$$\begin{aligned} G_C(s) = \frac{\tilde{M}(s)}{G_p(s)} &= \frac{4.242s^7 + 131.3s^6 + 1684s^5 + 11810s^4 + 4.9e04s^3 + 1.17e05s^2 + 140780s + 62500}{s^7 + 21.9s^6 + 217.8s^5 + 1217s^4 + 3991s^3 + 7057s^2 + 5090s} \\ &= \frac{1}{s}\left(12.278 + 10.6337s - 1.3782s^2 + 0.2654s^3 + \cdots\right) \end{aligned} \tag{6.36}$$

By matching controller structure eq. (6.35) with power series expansion eq. (6.36) the coefficients of controller parameters are obtained as

$$\begin{aligned} k &= 12.278 \\ kk_1 &= 10.6337 + 12.278k_2 \\ 0 &= -1.3782 + 10.6337k_2 \end{aligned}$$

therefore,

$$k = 12.278,\ k_1 = 0.9957,\ k_2 = 0.1296$$

Hence the controller is

$$G_C(s) = \frac{12.278\,(1 + 0.9957s)}{s\,(1 + 0.1296s)}$$

The corresponding closed loop transfer function $G_{CL}(s)$ is

$$G_{CL}(s) = \frac{12.23s^4 + 159s^3 + 807.5s^2 + 1543s + 884}{0.1296s^6 + 3.333s^5 + 42.8s^4 + 279.3s^3 + 1000s^2 + 1643s + 884}$$

This high order closed loop transfer function is reduced to third order model using proposed Factor division algorithm and differentiation method discussed in section 3.4 of chapter 3.

$$R_{CL}(s) = \frac{12551.58371s^2 + 85770s + 106080}{2614.8s^3 + 23000s^2 + 98480s + 106080}$$

The reduced order closed loop system obtained by using Narwal method [338] is

$$R_{CL}(s) = \frac{4.7528s^2 + 31.2135s + 26.8022}{s^3 + 8.8282s^2 + 34.2448s + 26.8022}$$

The step responses of the high order closed loop transfer function $G_{CL}(s)$ and reduced order closed loop transfer functions of proposed and Narwal [338] are compared with that of the reference model which are shown in Figure 6.7. It is observed that the reduced order model responses are in close agreement with that of the reference model. Further, the qualitative comparison of original and reduced order closed loop systems are compared with reference model in Table 6.3

Table 6.3: Qualitative comparison of original and reduced order models with reference model for Example 6.3

Systems	Rise time	Peak Overshoot	Steady state
$G_{CL}(s)$	0.206	9.05%	1
Reference model $M(s)$	0.282	8.33%	1
Proposed $R_{CL}(s)$	0.263	7.12%	1
A. Narwal $R_{CL}(s)$ [338]	0.279	6.41%	1

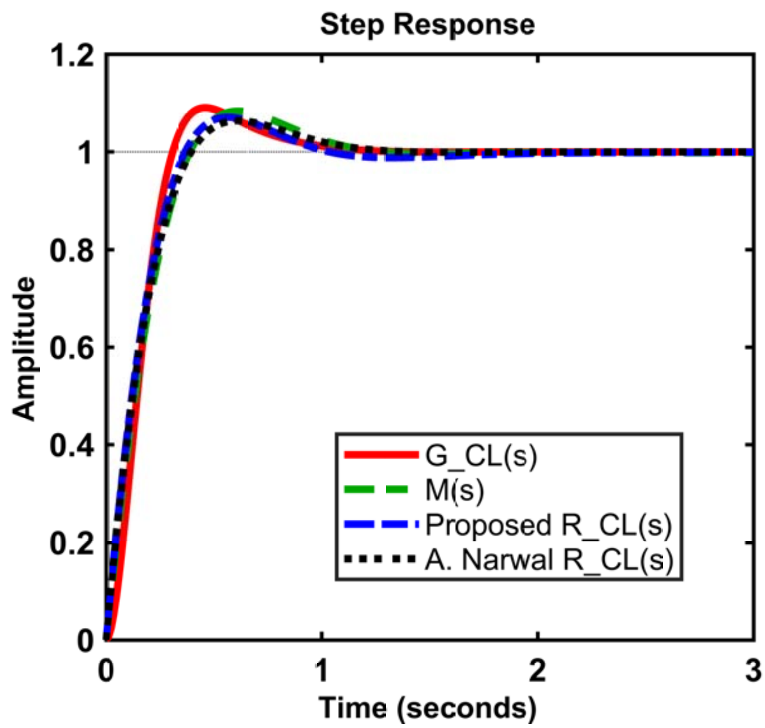


Fig. 6.7: Step response comparisons of original and reduced order closed loop models with reference model for Example 6.3

Example 6.4: Consider a 6^{th} order rational minimum phase practical system $G_p(s)$ taken by Prasad [333] which represents a typical open loop helicopter engine including a fuel controller. The input and output variables of the system are speed demand and propeller speed respectively. The step response of the system has undesirable oscillations because of elasticity of the propeller shaft. A controller is required to be designed so that the closed loop response of the system must follow the response of the reference model $M(s)$ which is given as

$$G_P(s) = \frac{248.05s^4 + 1483.3s^3 + 91931s^2 + 468730s + 634950}{s^6 + 26.24s^5 + 1363.1s^4 + 26803s^3 + 326900s^2 + 859170s + 528050}$$

The reference model is,

$$M(s) = \frac{4}{s^2 + 4s + 4}$$

then

$$G_C(s) = \frac{\tilde{M}(s)}{G_p(s)} = \frac{\begin{array}{l}4s^8 + 121s^7 + 5888s^6 + 1.2943e05s^5 + 1.7579e06s^4 + 9.0956e06s^3 \\ +2.1091e07s^2 + 2.22e07s + 8.449e06\end{array}}{\begin{array}{l}248.1s^8 + 3468s^7 + 1.088e05s^6 + 1.2082e06s^5 + 6.2473e06s^4 \\ +1.5933e07s^3 + 2.02e07s^2 + 1.016e07s\end{array}}$$

$$= \frac{1}{s}\left(0.8316 + 0.5313s - 0.2841s^2 + 0.1159s^3 + \cdots\right)$$

The structure of the controller is taken as

$$G_C(s) = \frac{k(1 + k_1 s)}{s(1 + k_2 s)}$$

The controller parameters are obtained by matching controller structure with power series expansion coefficients,

$$k = 0.8316,\ k_1 = 1.1735,\ k_2 = 0.5347$$

Hence, the required controller is finally given as

$$G_C(s) = \frac{0.9758s + 0.8316}{0.5347s^2 + s}$$

Therefore, the closed loop transfer function with the controller is given as

$$G_{CL}(s) = \frac{G_c(s)\,G_p(s)}{1 + G_c(s)\,G_p(s)} = \frac{\begin{array}{l}242.1s^5 + 1654s^4 + 9.095e04s^3 + 5.338e05s^2 \\ +1.011e06s + 5.281e05\end{array}}{\begin{array}{l}0.5347s^8 + 15.031s^7 + 755.1s^6 + 1.594e04s^5 + 2.032e05s^4 \\ +8.772e05s^3 + 1.6754e06s^2 + 1.538e06s + 5.281e05\end{array}}$$

The closed loop transfer function $G_{CL}(s)$ is reduced to third order closed loop transfer function $R_{CL}(s)$ by using the time moment matching method discussed in section 3.3 in chapter 3 is given as

$$R_{CL}(s) = \frac{0.0145267s + 3.921791}{s^2 + 3.928149s + 3.921791}$$

The reduced order closed loop system obtained by using Vishwakarma method [342] is

$$R_{CL}(s) = \frac{0.5339s^2 + 1.00996s + 0.52808}{0.84305s^3 + 1.6084s^2 + 1.5285s + 0.52808}$$

The step response of the original closed loop transfer function $G_{CL}(s)$ and the reduced closed loop transfer functions of proposed and Vishwakarma [342] are compared with the response of the reference model which is shown in Figure 6.8. It is observed that, the reduced proposed controller is closely matching with the reference model compared to other model. Further, the qualitative comparison of original and reduced order closed loop systems are compared with reference model in Table 6.4

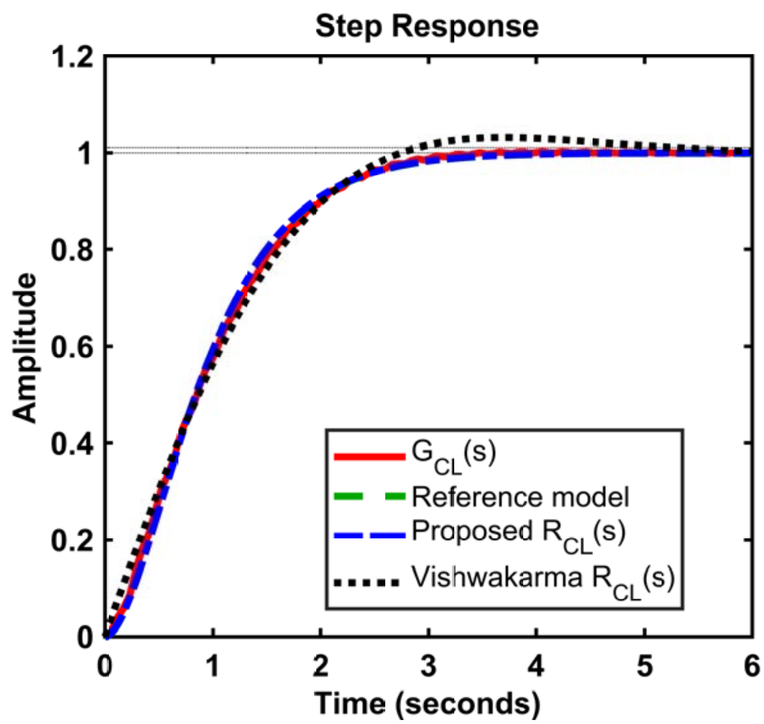


Fig. 6.8: Step response comparisons of original and reduced order closed loop models with reference model for Example 6.4

Example 6.5: Phase lead compensator design, let the plant model given by [334]

$$G_p(s) = \frac{20\,[1 + s/1.5]}{s\,[1 + s/4.5]\,[1 + s/10]\,[1 + s/30]}$$

The desired performance specifications are: Crossover frequency $w_c = 4.5;$ Damping ratio $\zeta = 0.785;$ Velocity error constant $k_v = 20$

The reference model is obtained by following the method of Chen and Shieh

Table 6.4: Qualitative comparison of original and reduced order models with reference model for Example 6.4

Systems	Rise time	Peak Overshoot	Steady state
$G_{CL}(s)$	1.76	0.289%	1
Reference model $M(s)$	1.68	0%	1
Proposed $R_{CL}(s)$	1.68	0%	1
Vishwakarma $R_{CL}(s)$ [342]	1.91	1.97%	1.03

[343],

$$M(s) = \frac{4.04265s + 8.009}{s^2 + 4.4431s + 8.009}$$

The compensator to be designed is assumed as phase lead, given by

$$G_C(s) = \frac{K[s+b]}{[s+a]}$$

By applying the proposed direct design method, the unknown parameters of the compensator are obtained as,

$$K = 0.106345;\ b = 2.261666;\ a = 0.262352$$

Therefore, the closed loop transfer function of the plant with the above designed compensator is given by

$$G_{CL}(s) = \frac{1914.21s^2 + 7200.618674s + 6493.955511}{s^5 + 44.762352s^4 + 491.674664s^3 + 3390.13896s^2 + 7554.79387s + 6493.955511}$$

Further, this $G_{CL}(s)$ is reduced by using proposed Pade approximation and differentiation method discussed in section 3.5 in chapter 3, and the reduced order closed loop control system is obtained as:

$$R_{CL}(s) = \frac{160064.541s + 389637.3307}{30340.83376s^2 + 181315.053s + 389637.3307}$$

The reduced order closed loop system obtained by using Narwal method [338] is

$$R_{CL}(s) = \frac{0.486536s + 37.731322}{s^2 + 6.538768s + 37.731322}$$

The step response of $G_{CL}(s)$, $M(s)$ and $R_{CL}(s)$ of proposed and Narwal [338] are compared in the below figure 6.9, from which it is clear that the response of $G_{CL}(s)$ and $R_{CL}(s)$ are found to close approximate the desired one $M(s)$. Further, the qualitative comparison of original and reduced order closed loop systems are compared with reference model in Table 6.5.

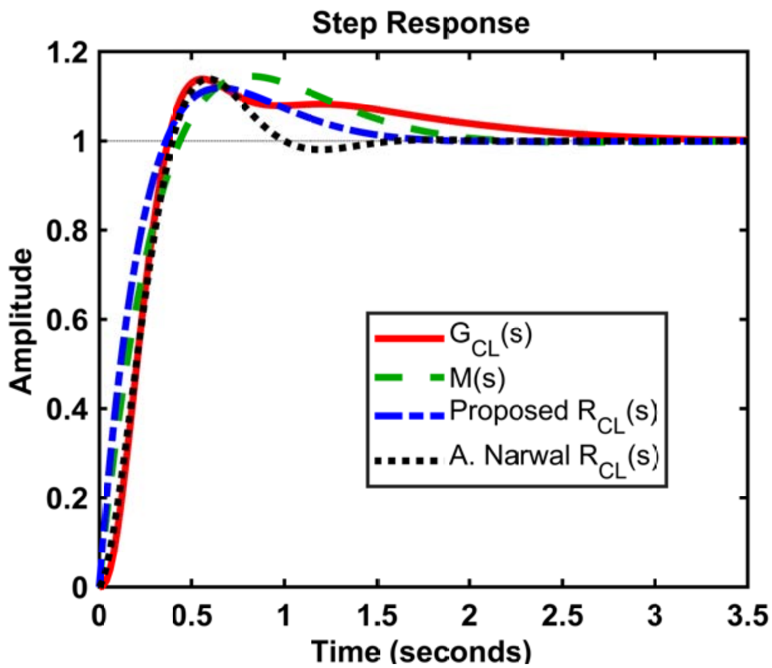


Fig. 6.9: Step response comparisons of original and reduced order closed loop models with reference model for Example 6.5

Table 6.5: Qualitative comparison of original and reduced order models with reference model for Example 6.5

Systems	Rise time	Peak Overshoot	Steady state
$G_{CL}(s)$	0.237	13.9%	1
Reference model $M(s)$	0.325	14.5%	1
Proposed $R_{CL}(s)$	0.206	11.9%	1
A. Narwal $R_{CL}(s)$ [338]	0.276	13.9%	1

Example 6.6: Phase Lead-Lag compensator design

Let the plant model be given by [334]:

$$G_p(s) = \frac{20}{s\left[1+s/10\right]\left[1+s/30\right]}$$

The desired performance specifications are: Crossover frequency $w_c = 5;$ Damping ratio $\zeta = 0.7;$ Velocity error constant $k_v = 20$

The reference model is obtained by following the method of Chen and Shieh [293],

$$M(s) = \frac{4.35s + 12.674}{s^2 + 4.984s + 12.674}$$

The compensator to be designed is assumed as phase lead-lag, given by

$$G_C(s) = \frac{s^2 + cs + d}{s^2 + as + b}$$

By applying the proposed direct design method, the unknown parameters of the compensator are obtained as

$$a = 22.420965;\ b = 13.552926;\ c = 6.180847;\ d = 13.951391$$

Therefore, the closed loop transfer function of the plant with the above designed compensator is given by

$$G_{CL}(s) = \frac{6000s^2 + 37085.082s + 83708.346}{s^5 + 62.420965s^4 + 1210.391526s^3 + 13268.40654s^2 + 41150.9598s + 83708.346}$$

Further, this $G_{CL}(s)$ is reduced by Pade approximation and differentiation method discussed in section 3.5 of chapter 3, and the reduced order closed loop control system is obtained as

$$R_{CL}(s) = \frac{455489.3672s + 5022500.76}{89610.44424s^2 + 847423.0352s + 5022500.76}$$

The reduced order closed loop system obtained by using Narwal method [338] is

$$R_{CL}(s) = \frac{1.181756s + 68.028254}{s^2 + 9.466647s + 68.028254}$$

The step response of $G_{CL}(s)$, $M(s)$ and $R_{CL}(s)$ of proposed and Narwal [338] are compared in the below Figure 6.10, from which it is clear that the response of the proposed model is quite comparable with the desired model $M(s)$. Further, the qualitative comparison of original and reduced order closed loop systems are compared with reference model in Table 6.6.

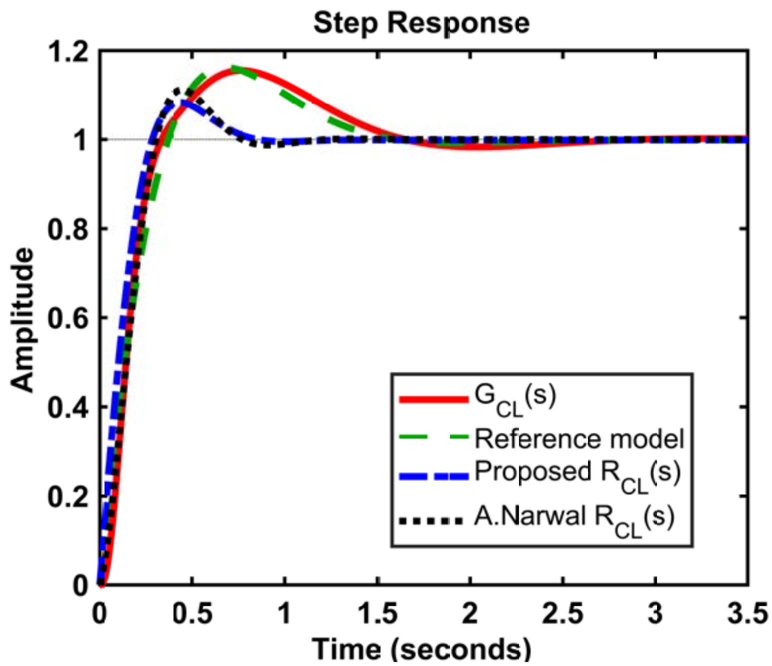


Fig. 6.10: Step response comparisons of original and reduced order closed loop models with reference model for Example 6.6

Table 6.6: Qualitative comparison of original and reduced order models with reference model for Example 6.6

Systems	Rise time	Peak Overshoot	Steady state
$G_{CL}(s)$	0.199	15.5%	1
Reference model $M(s)$	0.278	16.1%	1
Proposed $R_{CL}(s)$	0.207	8.35%	1
A. Narwal $R_{CL}(s)$ [338]	0.215	11.2%	1

6.5 CONCLUSION

In this chapter, the control systems are designed to ascertain the suitability of some of the MOR methods developed in previous chapters. The proposed reduction techniques discussed in chapter 3 have been used to design these control systems. Both, direct and indirect approaches have been considered in the present work in which the desired performance specifications of the plant are translated in to a specification/reference/model transfer function and then the linear algebraic equations are solved to obtain the unknown parameters of the controller. The methods are computer oriented, rugged and simple. The methods assure the reasonably well unit step response matching. It can be seen in the illustrative examples that the unit step responses of the overall control systems designed for the original and reduced order plants are in close agreement with that of the reference/specification model. Also, a qualitative comparison in terms of the transient response parameters for original and reduced order plants of these closed loop control systems is also shown in some examples, from which it is clear that the proposed design methods, i.e. direct and indirect, give good and acceptable closed loop performance.

CHAPTER 7

CONCLUSIONS

7.1 GENERAL

In this thesis the new techniques are developed for reduced order modelling of continuous and discrete time systems in both conventional and interval domains. The applicability of some of the proposed methods in the field of controller design for both direct and indirect approach is also explored. This chapter concludes the main contributions and results of each chapter. Furthermore, some important and useful suggestions for future work in this area of research are discussed.

7.2 SUMMARY OF IMPORTANT FINDINGS

In this thesis, the reduced order modelling of linear systems are presented in which the proposed techniques are developed for both conventional and interval LTI systems. Several new composite techniques are developed to circumvent the drawbacks of existing methods. Some of the model reduction techniques developed herein are extended for order reduction of multivariable and discrete time systems. Some benchmark numerical examples are chosen to demonstrate the efficacy of the proposed methods. The results obtained by the proposed methods are compared with other well-known reduction methods and recently published work in terms of ISE, IAE, ITSE and ITAE, Nyquist plot, step, impulse and bode diagram responses. Later, the applicability of the proposed methods in the field of controller design for both direct and indirect approach is also proposed.

An overview on modelling of large scale systems and role of model order reduction along with the mathematical preliminaries related to model order reduction and concepts of interval systems has been presented in chapter 1. Further, it included objectives and organization of the thesis also.

The brief literature review on various existing model reduction techniques in frequency domain, time domain and interval domain and their associated qualities/

drawbacks available in the literature has been included in chapter 2. It is observed that model order reduction is in demand in various fields, such as biomedical systems, transportation systems, complex chemical processes and systems and control.

In chapter 3, new composite mixed method techniques are suggested for reduced order modelling of linear dynamic continuous-time systems based on time moment matching method, factor division algorithm, Pade approximation and differentiation method. The first proposed technique utilized the benefits of time moment matching method to determine the reduced order denominator polynomial whereas in the second and third techniques the differentiation method employed to determine the reduced order denominator. The reduced order numerator polynomial is determined by equating higher-order and lower order models and by comparing like s terms after cross multiplication for first method whereas the second and third proposed methods employed factor division algorithm and Pade approximation method. Apart from preserving the stability and other essential characteristics of the original system, these methods also minimized the values of the error between original and reduced order systems. Further, these techniques are extended for order reduction of multi input multi output systems. The results obtained from proposed methods are found quite comparable with some other existing well-known reduction methods.

In chapter 4, the new reduction methods are proposed for reducing the order of continuous-time linear dynamic interval systems. In this chapter, four new order reduction techniques are proposed, the first reduction technique is obtained by differentiation method, factor division algorithm, and Pade approximation method based on interval arithmetic operations. In this, the reduced order denominators are obtained by differentiation method and, the reduced order numerators are obtained by either of differentiation method, factor division algorithm, and Pade approximation method. The second, third and fourth reduction methods are obtained by using differentiation method, stability equation method and modified time-moment matching method based on Kharitonov's theorem. The proposed methods are justified by solving some benchmark numerical examples of both SISO and MIMO interval systems. It is observed that the proposed methods are computationally simple and applicable for MIMO systems also. The results are compared with some other existing reduction techniques in terms of bode plot, step response and error indices such as ISE, IAE and ITAE. It is observed that, the results are encouraging and a vast improvement in the values of performance indices is achieved.

In chapter 5, some of the techniques developed in the previous chapters have been extended for reduced order modelling of discrete time systems. The concept of linear transformation is used during the initial and final stages, to convert the discrete domain systems to continuous domain systems and vice versa to obtain the reduced order models of proposed techniques. In this, the first technique has been further extended for reduced order modeling of multivariable discrete time systems. Here also, the proposed and some other order reduction techniques are compared in terms of performance indices such as SSE, SAE and STAE between original and reduced order systems by reducing the benchmark systems available in the literature. The original and reduced order system step and impulse responses are compared to show the close approximation of proposed reduced model response with the original system response. It is observed that the results obtained by the proposed techniques are comparable.

In chapter 6, the controllers are designed to ascertain the suitability of some of the proposed order reduction methods. The proposed conventional techniques which are time moment matching method, Pade approximation, factor division algorithm and differentiation methods have been tested while designing the low order controllers. Both the direct and indirect approaches have been considered in the present work. The desired performance specifications of the plant are translated into a specification/reference/model transfer function $M(s)$ and then the linear algebraic equations are solved to obtain the unknown parameters of the controller. These design methods have been implemented in MATLAB environment. Illustrative examples are given for both the approaches. It can be seen in the illustrative examples that, the unit step responses of the overall control systems designed for the original and reduced order plants are in close agreement with that of the reference/specification model and the proposed design direct and indirect methods. The obtained results are quite encouraging in terms of closed loop performance.

7.3 SCOPE FOR FUTURE WORK

There is always a scope for further improvement in any research work. Therefore, some suggestions are given for further research work in this area.

- In third chapter, some of the proposed mixed methods are based on factor division algorithm & differentiation methods, and Pade approximation & differentiation method these have been developed for linear continuous systems only. These techniques may be extended to order reduction of discrete time systems.

- The order reduction techniques in chapter three are proposed for the system given in frequency domain only. However, the same techniques may be explored for the system given in time domain directly.

- In fourth chapter, the proposed mixed interval technique based on interval arithmetic operation have been used for SISO interval systems only. This technique may be extended for continuous multivariable systems and discrete time interval systems.

- In fifth chapter, the discrete time interval techniques are extended for SISO systems only. These techniques may be extended for discrete time multivariable interval systems.

- The $z - w$ transformation and vice versa are used during the initial and final stages of reduced order modelling of discrete time systems. However, the reduced order discrete time system may be explored without using any transformation.

- In sixth chapter, the proposed techniques discussed in third chapter are used for controller design. In this chapter, the controller parameters can be found using any other optimization technique like genetic algorithm (GA), particle swarm optimization (PSO), ant colony optimization (ACO), etc. or using soft computing techniques like fuzzy controller design.

- The whole work presented in the thesis is for linear systems only. Therefore, the proposed work may be extended for non-linear systems also.

LIST OF PUBLICATIONS

JOURNALS

1. **Sudharsana Rao Potturu** and Rajendra Prasad, "Qualitative analysis of stable reduced order models for interval systems using mixed methods," *IETE Journal of Research*, Oct. 2018. doi: 10.1080/03772063.2018.1528185 (**SCI Indexed**)

2. **Sudharsana Rao Potturu** and Rajendra Prasad, "A new order reduction approach for continuous-time and discrete-time uncertain systems," *Applied Mathematical Modelling*, (Under Review).

3. **Sudharsana Rao Potturu** and Rajendra Prasad, "Model order reduction of LTI interval systems based on differentiation method using Kharitonov's theorem," *IETE Journal of Research*, (Under Review).

4. **Sudharsana Rao Potturu** and Rajendra Prasad, "A new model reduction technique for LTI systems using modified time-moment matching method," *IET Control Theory and Applications*. (Communicated).

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1. **Sudharsan Rao Potturu**, and Rajendra Prasad, "Model reduction of LTI discrete-time multivariable systems using Pade approximation method and stability equation method," *6th International Conference on Control, Decision and Information Technologies (CODIT'19)*, April 23-26, 2019, Paris, France.

2. **Sudharsan Rao Potturu**, and Rajendra Prasad, "Order reduction of interval Systems using Kharitonov's theorem and stability equation method," *in proc. on American Control Conference (ACC)*, Milwaukee WI, USA, June 27–29, 2018, pp. 6224-6229.

3. **Sudharsan Rao Potturu**, and Rajendra Prasad, "Differentiation method based order reduction of linear dynamic interval systems using Kharitonov's theorem," *in proc. on Third IFAC International Conference on Advances in Control and Optimization of Dynamical Systems (ACODS)*, Hyderabad, India, Feb 18-22, 2018, pp. 154-155. (Poster Presentation)

4. **Sudharsan Rao Potturu**, and Rajendra Prasad, "Reduction of interval systems using Kharitonov's polynomials and their derivatives," *6th International Conference on Computer Applications in Electrical Engineering-Recent Advances (CERA)*, Roorkee, India, Oct 5-7, 2017, 445-449.

5. **Sudharsan Rao Potturu**, and Rajendra Prasad, "Stable mixed reduced order models for linear dynamic systems and their qualitative comparison," *IEEE International Conference on Power Electronics, Intelligent Control and Energy Systems (ICPEICES)*, Delhi, India, July 4-6, 2016, pp. 1-4.

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