TRAFFIC FLOW MODELLING:
A CONTINUUM APPROACH

A DISSERTATION

Submitted in partial fulfillment of the requirements
for the award of the degree
of

MASTER OF TECHNOLOGY
in
INFRASTRUCTURE SYSTEMS

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CANDIDATE'S DECLARATION

I hereby declare that the work carried out in this seminar report entitled, "**Traffic Flow Modelling: A Continuum Approach",** is presented in partial fulfilment of the requirements for the award of degree of "**Master of Technology**" In Centre for Excellence in Transportation Systems with specialization in Infrastructure Systems, submitted to the Centre for Excellence in Transpiration Systems, Indian Institute of Technology Roorkee, under the guidance of, Dr. Ameeya Kumar Nayak, Associate Faculty, Centre for Transportation Systems.

Date: 22.05.2018 **(P. N. Aravind)**

Place: Roorkee

This is to certify that the above statement made by the candidate is correct to the best of my knowledge and belief.

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1. INTRODUCTION

Traffic networks which includes many kinds of networks like highways, streets etc., provide convenient and economical conveyance of passengers and goods. The basic activity in transportation being a trip, which is defined by destination and origin, arrival and departure times and the route taken for travel. Different trips interact with each other on the network to produce complex pattern of traffic flows. Since traffic conditions in major cities and urban conglomerations are becoming increasingly congested, affecting the overall operational efficiency of networks as well as the cost of travel of each individual trip, modelling of traffic flow is being seen as essential rather than secondary process in traffic engineering and the policy making process in transport sector.

Traffic flow is one such phenomenon which is highly difficult to model mathematically due to its extreme complexity. The most basic traffic scenario is a one dimensional road with one-way traffic. This is a simple scenario to model as we have eliminated various factors.

There are three main approaches taken to model traffic flow; microscopic approaches, macroscopic approaches and mesoscopic approach. Microscopic models map traffic flow as a set of individual vehicles, while macroscopic models map traffic flow as fluid flow where each vehicle is analogous to a molecule of fluid. Mesoscopic models describe vehicle behaviour in aggregate terms such as in probability distributions. They essentially cover the ground in between a macroscopic and a microscopic model.

In this report, the main emphasis is on macroscopic models. Macroscopic models place more emphasis on traffic flow as a continuum versus a collection of individual vehicles. Continuum traffic flow modelling generally uses a macroscopic perspective, although microscopic principles can be incorporated into continuum models.

1.1 Traffic Modelling

In modelling traffic, it is necessary to visualize a coupled system consisting the car and the driver. The driver is responsible for operating the car and making it become a part of the traffic flow. Thus the traffic is not just a mechanical process but one in which human decisions are involved. But on a whole, this individual vehicle is not as important as the overall flow of traffic. Macroscopic models of traffic flow exploit these conditions and use them to come up with a set of assumptions. When these cars are viewed as a moving gas or liquid, it is called a continuum model of traffic flow. When the traffic theory is based on individual drivers responding to surrounding traffic, it is called car-following theory. Here we will look at the background and evolution of traffic models (van Wageningen-Kessels et al. 2014).

1.1.1. Fundamental Diagram

Traffic flow models are based on the assumption that there is some relation between the distance between vehicles and their velocity. This relation between distance and velocity was first studied by Greenshields (van Wageningen-Kessels et al. 2014) and called the fundamental relation. Originally, Greenshields studies the relation between spacing (s) and velocity (v). However, the fundamental relation can be expressed in other variables such as density (q, average number of vehicles per unit length of road) and flow (q, average number of vehicles per time unit).

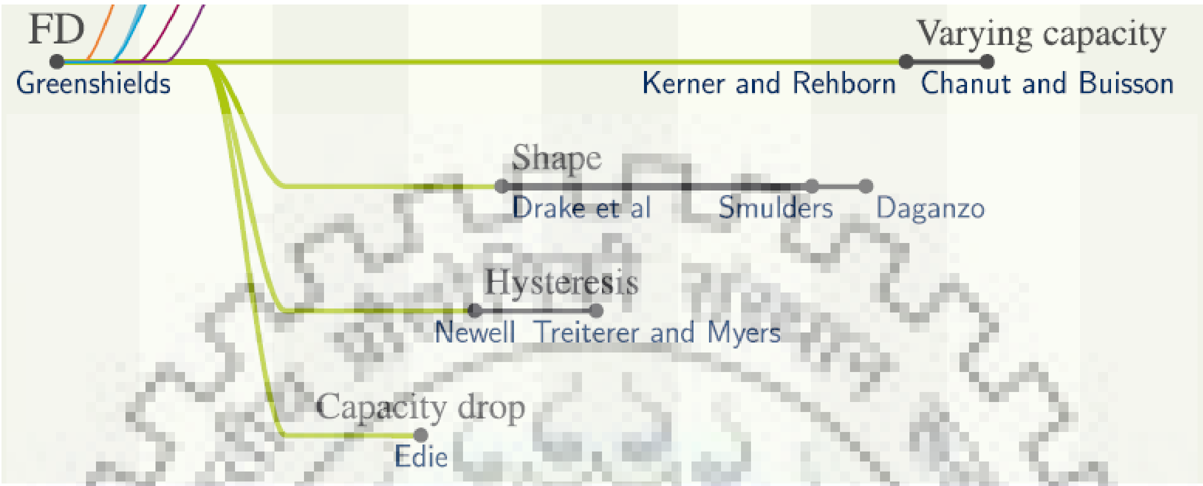


Figure 1-1 Branch of Traffic Flow models showing fundamental diagram family

Greenshields proposed a linear relationship in the density-velocity plane and parabolic in density-flow plane, whereas Daganzo (Daganzo 1995) relation is triangular in the density-flow plane. Smulders is a combination of both. It is parabolic for low densities and linear for higher densities. Drake proposes a characteristic curve for the density-flow plot. However, observed density flow plots show a wide scatter. These are being explained by a range of phenomenon like capacity drop, hysteresis and the 3-phase fundamental relation proposed by Kerner (Kerner 2004)

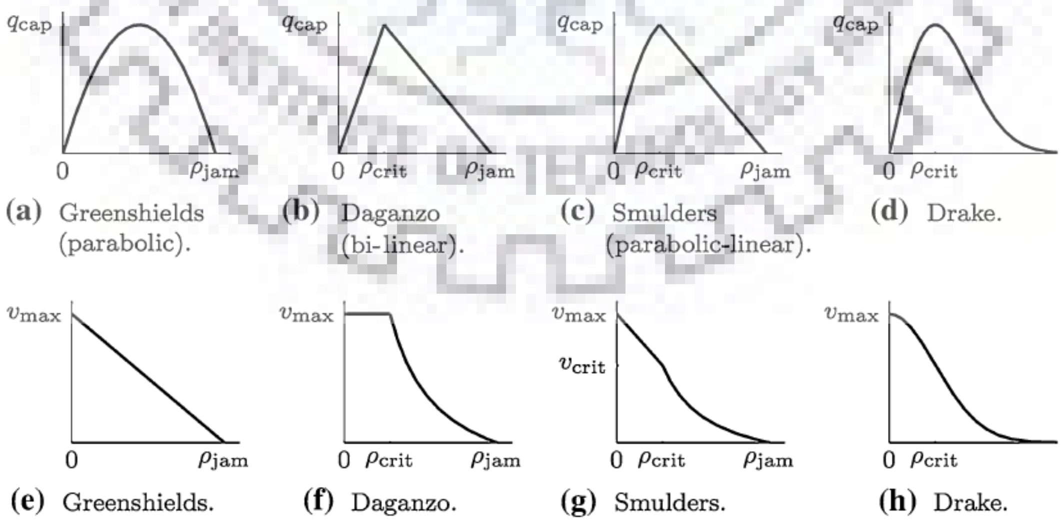
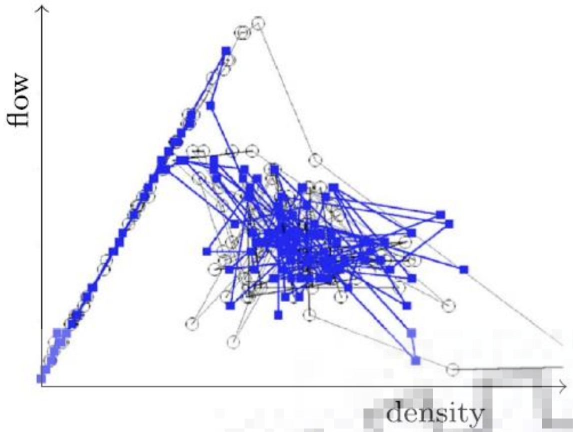
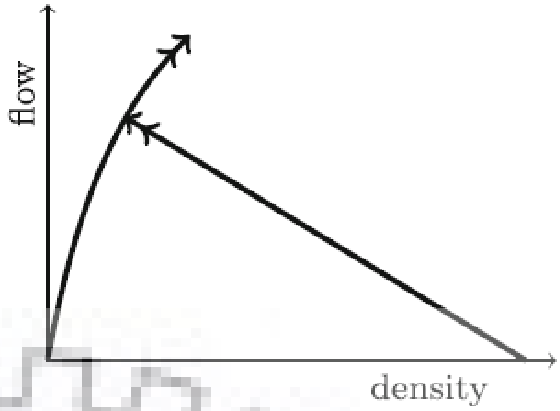


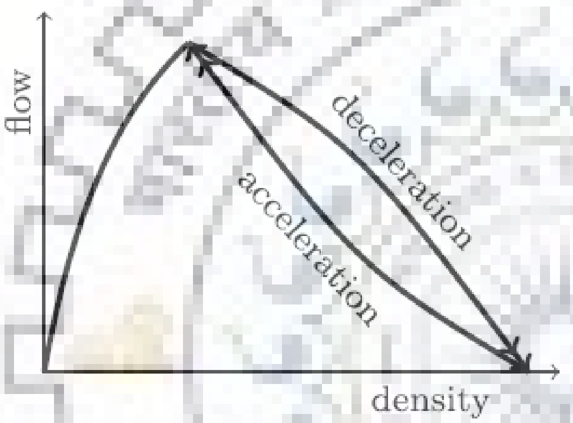
Figure 1-2 Different shapes of Fundamental Relations



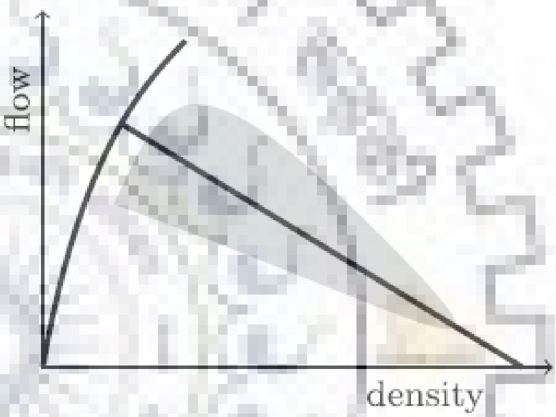
a. Scatter observed in a density-flow plot



b. Fundamental relation with capacity drop



c. Fundamental relation with hysteresis



d. 3-phase fundamental relation line

Figure 1-3 Fundamental 'relations' based on scatter in observations

Fundamental relations are important in all families of traffic models since in any model, traffic is assumed to be in a state of fundamental relation or proceeds toward it. Macroscopic and mesoscopic models explicitly include it but microscopic models have some assumptions on the fundamental relations which differ from model to model.

1.1.2. Microscopic Models

Microscopic models, one of the earliest models, are based on the assumption that drivers adjust their behaviour based on the vehicle they are following. They describe both the longitudinal (car-following) and lateral (lane-changing) behaviour of vehicles.

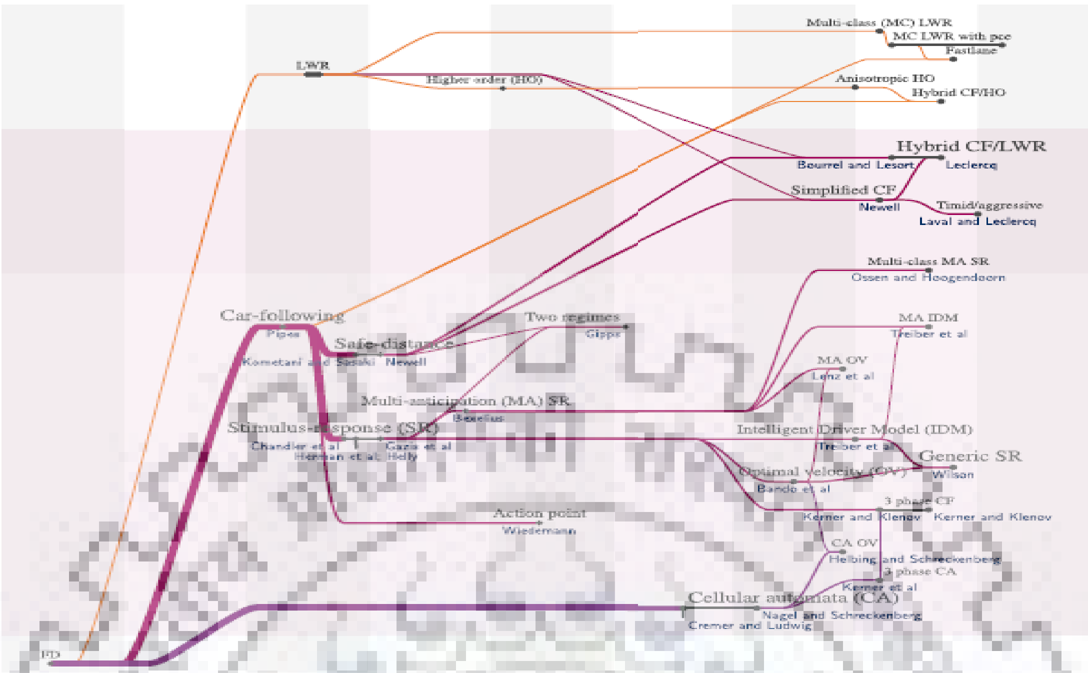


Figure 1-4 Details of traffic models showing microscopic model family

1.1.2.1. Safe-Distance Model

The earliest car following models were based on a safe following distance. Pipes [7] proposes a model where the position of the leader is expressed in terms of the position of the follower.

$$x_{n-1} = x_n + d + Tv_n + l_{n-1}^{veh}$$

Where d is the distance between vehicles at standstill and Tv_n is the legal distance assumed by Pipes. Kometani and Sasaki [8] derive an improved version of this model with the help of Newtonion equations of motion. They replace distance at standstill "d" with a velocity dependant term and also include a time delay, τ.

1.1.2.2 Stimulus-Response Model

These are based on the assumption that drivers accelerate or decelerate based on three stimuli; own velocity (v_n), spacing with leader (s_n) and relative velocity with leader (s_n'). It is called the GHR Model named after Gazis et al [9].

$$a_n(t) = \gamma \frac{\left(v_{n-1}(t)\right)^{c1}}{\left(s_n(t-\tau)\right)^{c2}} \dot{s_n}'(t-\tau)$$

Where ϒ is the sensitivity parameter. $S_n(t-\tau)$ is the stimulus and $a_n(t)$ is the response, hence it is called the stimulus response model.

Recent improvements over this model include the optimal velocity model which states that the driver accelerates or decelerates until their optimal velocity, a function of headway.

$$a_n(t) = \gamma(v^*(s_n(t)) - v_n(t))$$

$$v^*(s) = v_{max}(\tanh(s - c_1) + c_2)$$

1.1.2.3 Action Point Model

This branch of car-following model is based on the fact that a driver reacts to a situation only when he/she approaches a vehicle. There is a proposed perception threshold before a driver reacts. The assumptions incorporated in this model are that at large headways, driving behaviour is least affected by other vehicles. Whereas at small headways, it is only influenced by other vehicles.

1.1.2.4 Conclusions

Microscopic traffic models are often criticized for having too many parameters. Models like the one proposed by Gazis et al [9] have parameters c1 and c2 which don't have a physical interpretation. Other models have parameters which are too difficult to observe or tabulate which makes the whole exercise futile. That is why continuum models having lesser parameters are preferred.

1.1.3 Macroscopic Models

These models treat traffic as a continuum where individual vehicles are not modelled. However aggregates variables such as flow and density are used. Some of the models are explained below.

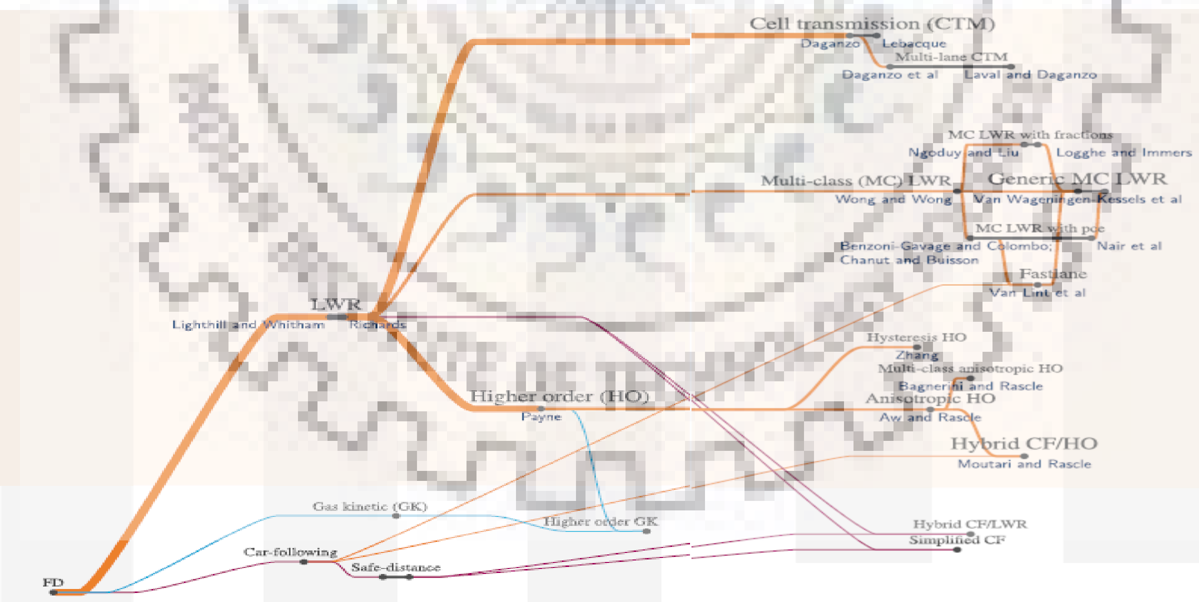


Figure 1-5 Details of traffic models showing macroscopic model family

1.1.3.1 Kinematic Wave Models

Macroscopic traffic flow models were first introduced by Lighthill and Whitham [2] in 1955 and independently by Richards in 1956. This is commonly known as the LWR Model. It is based around the assumption that the number of vehicles is conserved between any two points if there are no entrances (sources) or exits (sinks).The dynamics of traffic are given by a partial differential equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\big(q(\rho)\big) = 0$$

Where flow (q) is a function of density (ρ). This model has been used to analyse a number of traffic flow problems. Notably, both Lighthill & Whitham and Richards used the model to demonstrate the existence of shockwaves in transport systems. The main drawback of this model is that the vehicles are assumed to attain the equilibrium velocity almost instantly after the change of state, which implies infinite acceleration. Another disadvantage is that the transition from a free flow regime to a congested regime always occurs at the same densities. Further, the model does not contain any inertial effects, which implies that the vehicles adjust their speeds instantaneously, nor does it contain any diffusive terms, which would model the ability of drivers to look ahead and adjust to changes in traffic conditions, such as shocks, before they arrive at the vehicle itself. Some variants of the LWR model have proposed bounded-acceleration while a stochastic kinematic wave model uses breakdown probabilities to predict that a breakdown may occur at different densities.

In order to address some of the limitations, Lighthill & Whitham propose a second-order model of the form

$$\frac{\partial \rho}{\partial t} + c\frac{\partial \rho}{\partial x} + T\frac{\partial^2 \rho}{\partial t^2} + D\frac{\partial^2 \rho}{\partial x^2} = 0$$

where T is the inertial time constant for speed variation, c is the wave speed (obtained from the relationship between q and ρ, and D is a diffusion coefficient-representing how vehicles respond to nonlocal changes in traffic conditions.

1.1.3.2 Multi class Kinematic Wave Models

A multi class multi-lane model was proposed by Daganzo [4] based on the LWR model which distinguishes between two types of drivers: slugs who drive slow and don't overtake whereas rabbits who drive fast and overtake more often.

Wong and Wong [6] first introduced a class specific version of the conservation equation.

$$\frac{\partial \rho_u}{\partial t} + \frac{\partial q_u}{\partial x} = 0$$

Here ρ_u represents class specific density. Effectively, the vertical axes of the density–velocity fundamental relations are scaled differently for each class. It has been found that multiclass models are able to reproduce phenomena related to scatter in the fundamental diagram better than mixed-class models.

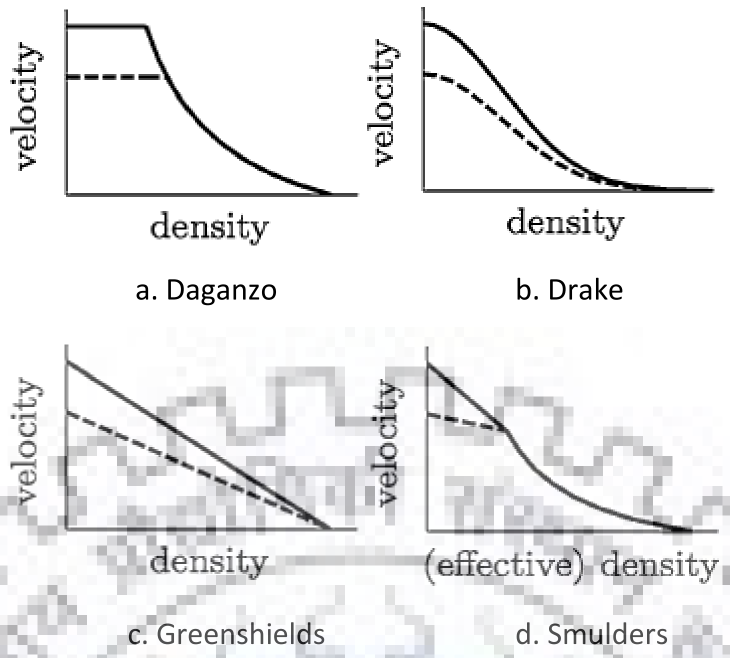


Figure 1-6 Different shapes of two-class fundamental relations.

The model provided by Nair et al is a recent multi class kinematic wave model. It is known as the porous flow model. It is different in the sense that it considers heterogeneous traffic instead of homogenous. It assumes that small vehicles can drive through pores which in this case are the gaps between two vehicles. This model tries to explain traffic which is discontinuous and disordered. It also has different types of vehicle classes like bikes, cars etc as is generally the case with Indian conditions.

1.1.3.3 Higher Order Models

Higher order models give an equation which describes the acceleration towards the equilibrium velocity that has been discussed in the fundamental relations. Payne [10] derived a macroscopic traffic flow model from a simple stimulus–response car-following model. It yielded a model consisting of the fundamental relation and two-coupled partial differential equations, hence the name higher-order model.

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \frac{v*(\rho) - v}{t_{relax}} - \frac{c^2\partial\rho}{\rho\partial x}$$

Here $v^*(\rho)$ is the equilibrium velocity described in the fundamental relations. But several authors have argued that higher-order models are flawed because they are not anisotropic.
Anisotropy generally means that the speed of the traffic wave cannot be faster than the speed of individual vehicles inside the flow. Hence improvements were made to the model and instead of Payne's velocity,

$$\frac{\partial}{\partial t}\big(v + p(\rho)\big) + v\frac{\partial}{\partial x}\big(v + p(\rho)\big) = 0$$

The $p(\rho)$ is the pressure term. This implies that when parameters have been correctly chosen, the characteristic speed cannot be faster than vehicles.

1.1.3.4 Conclusions

Although an analogy is assumed between a traffic flow and fluid flow, the number of particles in a traffic flow is extremely small as compared to a fluid. This means that the descriptive accuracy achieved by these models will never be the same as that in fluids. Moreover, the so called particles in the flow, i.e. the drivers, all behave differently and change behaviour over time and distance unlike fluid particles which obey simple physical laws. However if the level of descriptive detail is overlooked or compromised a little, then the continuum assumption can prove reasonable.

1.1.4 Mesoscopic Models

This class of models were originally developed to fill voids or gaps left by macro models which consider traffic flow as that of a fluid and find analogues between the two and micro models which see the individual vehicles in a traffic flow. They describe vehicle behaviour in aggregate terms. It can be probability distributions.

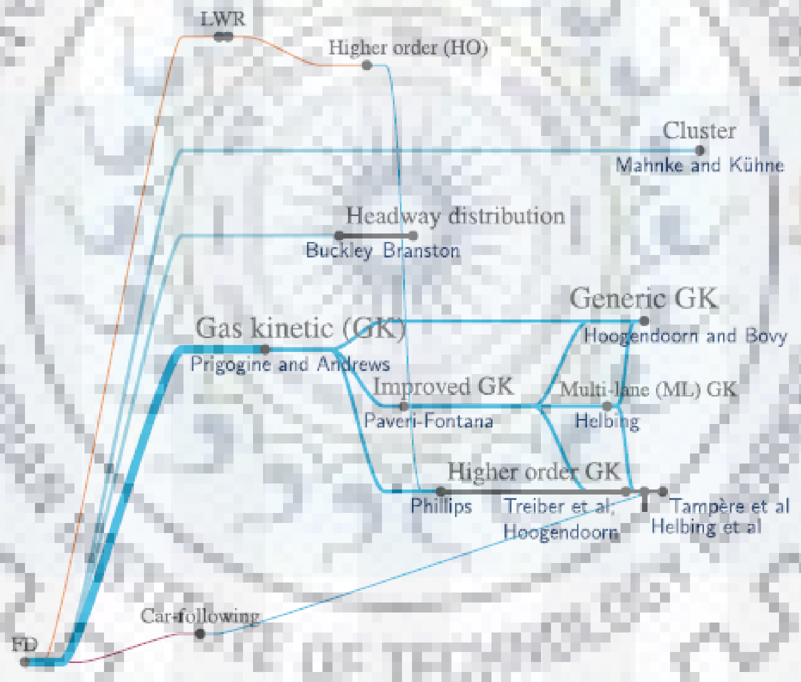


Figure 1-7 Details of traffic models showing mesoscopic model family

1.1.4.1 Gas-Kinetic Models

The continuum models of traffic flow presents an analogy between traffic and fluid flow. Similarly gas kinetic models describes these motions in terms of gas particles. When applied to traffic flow, these models describe the dynamics of velocity distribution functions of vehicles.

$$\frac{\partial\tilde{\rho}}{\partial t} + v\frac{\partial\tilde{\rho}}{\partial x} = \left(\frac{\partial\tilde{\rho}}{\partial t}\right)_{acc} + \left(\frac{\partial\tilde{\rho}}{\partial t}\right)_{int}$$

With reduces phase space density $\tilde{\rho}$. At any time t, the expected number of vehicles between location x and x & dx that drive with a velocity between v and v & dv is the integral of the reduced phase-space density over this two-dimensional area [1].

$$\int_{x}^{x+dv}\int_{v}^{v+dv} \tilde{\rho}(x,v,t)dxdv \approx \tilde{\rho}(x,v,t)dxdv$$

1.2. Need for Study

This report aims to study the traffic flow modelling incorporating driver's forecast effect. These type of driving behaviour models try to capture a drivers' decisions when manoeuvring in various traffic conditions. They are essential for traffic simulation and for several other fields of transportation science such as studies for safety and capacity, in which aggregate traffic flow characteristics might be needed.

It is highly required since the ability to map traffic conditions can save a lot of time and effort as the future predictions can be done accordingly and steps taken to ensure that those scenarios do not play out the way they did in the simulations.

This can be used in traffic reduction which means accident prevention, better travel times since flow is managed efficiently. All these can be done theoretically to get a first glance of the outside situation and then if required further predictions or models can be developed.

1.3. Objectives of the Study

The main objectives of the study are:

1. The macroscopic model or equation is solved and basic parameters are measured to check whether it can be used any further before calibration.

2. The model to be used to simulate some default conditions to authenticate its seriousness.

3. Compare it with a previous model and look for signs of improvement

1.4. Composition of Dissertation

The entire thesis is divided into 6 chapters.

Chapter 1 gives a general idea about traffic modelling, a brief history about various models and the current scenario along with the merits and demerits of the various methods.

Chapter 2 is the background study required to proceed with the modelling. Various derivations of the basic formulas and ideas is given to be used in the following chapters.

Chapter 3 is the literature review of the dissertation. It gives a brief about the main sources of literature helpful in the thesis.

Chapter 4 is the research methodology where the traffic model is analysed using numerical methods.

Chapter 5 is Results and Discussions where graphs are given and its occurrence is discussed.

Chapter 6 is the limitations and future scope of the study

2. MATHEMATICAL BACKGROUND STUDY

2.1. The Conservation Law

The fluid flow analogy is where macroscopic mathematical modelling rests, where the traffic stream is treated as a 1-D compressible fluid. Any conservation law states that the change in some physical quantity in a region of space is equal to the net influx, provided there are no sinks.

If in a one lane road, $\rho(x,t)$ denotes the density at some interval (x1,x2), then the integral of the conservation law can be written as

$$\frac{d}{dt}\int_{x1}^{x2} \rho(x,t)dx = f_{x=x1} - f_{x=x2}$$

The left hand side can be written as $\int_{x1}^{x2} \frac{\partial \rho}{\partial t} dx$ and right hand side as $-\int_{x1}^{x2} \frac{\partial f}{\partial x} dx$, we can rewrite (1) as

$$\int_{x1}^{x2} \frac{\partial \rho}{\partial t} + \frac{\partial f}{\partial x}\, dx =$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial f}{\partial x} = 0 \qquad (2)$$

This is the conservation law or also known the continuity equation in its partial differential form.

Since f takes the form $f = f(\rho(x,t))$, it is safe to assume that $v(\rho) = v_0(1-\frac{\rho}{\rho max})$.

$$f(\rho) = v(\rho) * \rho$$
$$= v_0\left(1 - \frac{\rho}{\rho_{max}}\right) * \rho \quad for\ 0 < \rho < \rho_{max}$$

Let us set σ as the value for ρ at which f is maximised.

$$\text{Then } \sigma = \frac{\rho max}{2} \qquad (4)$$

a. flow with respect to ρ greater than σ is referred as heavy traffic
b. flow with respect to ρ lower than σ is referred to as light traffic

2.2 Characteristics

If the equation has to become solvable, the initial density distribution must be given,

$$\rho_t + f(\rho)_x = 0 \ , \qquad x \in R\,, t > 0$$

$$\rho(x,0) = \rho_0(x) \quad , \quad x \in R$$

Level curve of x = x(t) on the x-t plane is given by

$$\rho(x(t),(t) = constant = P_0$$

Differentiating w.r.t t and using $\rho_t = -f'(\rho) * \rho_x$

$$0 = \rho_x x'(t) + \rho_t = \rho_x(x'(t) - f'(P_0))$$

It can be seen that $x'(t) = f'(P_0)$ which comes down to a straight curve as a characteristic in the x-t plane. Here $f'(P_0)$ is the signal speed at which wavefront will propagate. Since it is a derivative term, it is the slope of the equation also.

2.3. Discontinuities and the Jump

Despite having initial data, when the characteristics are drawn, we find that a continuous solution is not possible after a certain point of time, since different concentration characteristics interact with each other.

To let us calculate the solution after the discontinuity, generalisation of the solution concept needs to be done. The below equation

$$x = f'(\rho_0(x_0))t + x_0$$

$$\rho = \rho_0(x_0)$$

the above eqn is multiplied with a test function ϕ and integrated by parts

$$\int_0^\infty \int_{-\infty}^\infty (\rho\varphi_t + f(\rho)\varphi_t)dxdt + \int_{-\infty}^\infty \rho(x,0)\varphi(x,0)dx = 0, \forall\, \varphi \in C_0^1$$

Now let $\rho+ = \rho(x(t) + 0,t)$ and $\rho- = \rho(x(t) – 0,t)$ be the values of ρ on the left and the right.
The conservation law gives that

$$f(\rho(a,t)) - f(\rho(b,t)) = \frac{d}{dt}\int_a^b \rho dx = \frac{d}{dt}\left(\int_a^{x(t)} \rho dx + \int_{x(t)}^b \rho dx\right) =$$

$$= \int_{a}^{x(t)} \rho_t dx + \rho^- x'(t) + \int_{x(t)}^{b} \rho_t dx - \rho^+ x'(t) = [\rho_t = -f_x]$$

$$= f(\rho(a,t)) - f(\rho(b,t)) + f(\rho^+) - f(\rho^-) - (\rho^+ - \rho^-)x'(t)$$

Solving for x', we can find that

$$x'(t) = \frac{f(\rho^+) - f(\rho^-)}{\rho^+ - \rho^-} = s$$

That last expression is known as the jump condition and it further proves the above mentioned case that the slope of the graph between two points gives the speed of the shock waves.

2.4. The Riemann Problem

The Riemann problem is a conservation law combined with a piecewise constant with just one single discontinuity. Let us consider the example below:

$$\rho(x,0) = \begin{cases} \rho_l\,, & x < 0 \\ \rho_r\,, & x > 0 \end{cases}$$

Where ρl and ρr corresponds to an arbitrary point to the left and right side of the data respectively. The points are chosen in such a way that they are not too far from the surface to give appropriate values closest to the actual value. And they are not too close to the surface that they start to interact with the surface itself.

Case 1: when $\rho_l < \rho_r$

The shock acts as a barrier between the two sides. The characteristics from either side go into the shock.

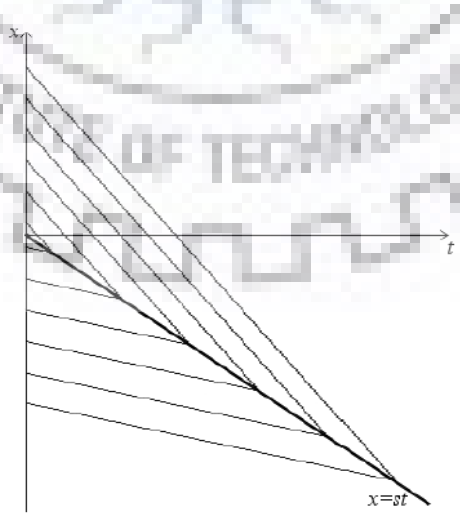


Figure 2-1 Shock Waves

Case 2: when $\rho_l > \rho_r$

Theoretically, it results in the jump being taken from infinitely many places or suggests infinitely many solutions. The characteristics travel away from shock.

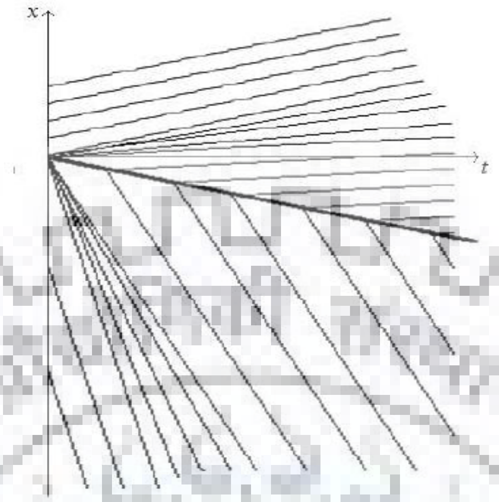


Figure 2-2 Rarefaction waves

3. LITERATURE REVIEW

3.1. Formation of a Continuum Model

One of the first continuum models was proposed by Lighthill and Whitham (1955) and Richards (1956) [2]. It provided an analogous between fluid flow and traffic flow. It was assumed that there is a conservation of the number of vehicles in a road section given that there are no entry or exit ramps.

Since fluctuation of speed is not permitted around equilibrium speed, the model is unable to explain non equilibrium conditions which are what real life problems are based on. Hence the model was improvised by Lighthill et al and a new model containing inertial time constant, T and diffusion coefficient, D, were introduced. The inertial time constant accounted for adjustments in speed implying that the decision to accelerate or brake is not instantaneous. The diffusion coefficient implies for the dependency of flow on concentration gradient.

3.2. Three Phase traffic Theory

(Kerner 2004) introduces us to the concept of three phase traffic flow and their applications in traffic flow modelling. Understanding traffic congestion is the key to effective management and control of transportation. According to this theory, there are two types of flow: free flow and congested flow. Further subdivisions of congested flow are synchronized flow and wide-moving jams.

The difference between a synchronized flow and a wide moving jam is that in wide moving jam, the velocity of the jam front remains the same even after the passing of the bottleneck or other complex traffic states. But in synchronized flows, the jam front velocity is fixed at the bottleneck, meaning after the bottleneck the flow is converted to a free flow.

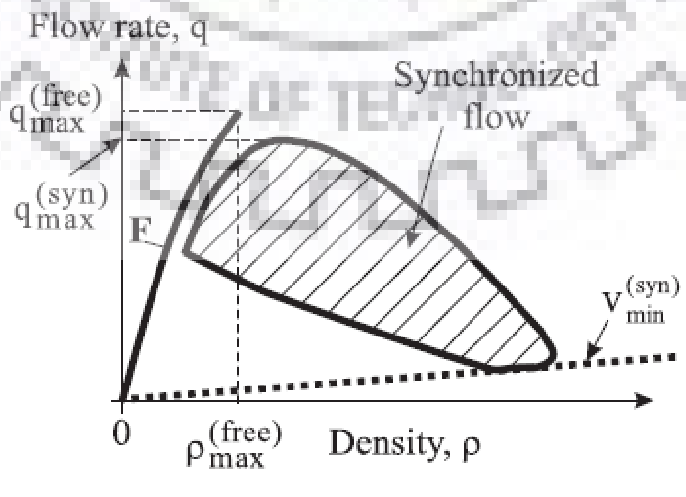


Figure 0-1 Density-flow plot for 3-phase traffic flow.

Curve F shows free flow while the dashes region is synchronized flow for a multi-lane homogenous road. The dotted line is the minimum possible speeds in steady states of synchronized flow.

This graph shows that for a given density, there a multitude of flows or speeds and vice-versa. This means that there is no single fundamental diagram for the steady state speed for a synchronized flow unlike other models. This also explains the scattering of flow in a better way.

The results of this work are important since previously all mathematical models had to provide steady state solutions which belong to a curve going through the origin and has at least one maximum.

3.3. A new macro model for traffic flow with consideration of DFE

In this paper, (Tang et al. 2010a) discusses about various developments in macroscopic traffic flow models. Acceleration equations of improved optimal velocity model, multi velocities difference model are given. Even though the models explain a magnitude of complex phenomenon, they cannot be used to study Driver's Forecast Effect since they do not consider it in the acceleration equation. With the advent of ITS, the forecast information will be crucial and driver's will adjust acceleration based on this information.

First the acceleration equation with DFE coefficient is given:

$$\frac{dv_n(t)}{dt} = \kappa\left(V\left(\Delta x_n(t)\right) - v_n(t)\right) + \beta\kappa(V(\Delta x_n(t+\tau) - v_n(t+\tau))$$

Here β is the coefficient for DFE, k is the reactive coefficient in the Optimal Velocity model and τ is the time step of the driver's forecast. The first term is the acceleration from normal conditions while the second term is the acceleration from the forecast information at time t+τ.

For the new macro model, the micro variables are converted into macro variables and the non-linear terms are neglected.

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial(\rho v)}{\partial x} = 0\,, \\ \dfrac{\partial v}{\partial t} + (v - \beta\tau u'_e(h)c_0)\dfrac{\partial v}{\partial x} = \dfrac{(1+\beta)(v_e(\rho) - v)}{T + \beta\tau} \end{cases}$$

3.4. Macro Modelling and Analysis of Traffic Flow using road width

(Tang et al. 2011)have proposed a traffic model which takes into consideration the road width. The changes in road width is a major reason for the decrease or increase in speeds of the flow in general and for drivers in particular. The changes can be in the form of disturbances due to activities like construction work or in the event of an accident. This paper in particular focuses on a speed gradient model since density gradient models were found to have characteristic speeds greater that the vehicular speed which resulted in a backward movement of traffic under certain situations.

Hence they proposed an improvised model which is anisotropic, heterogeneous and which accounts for the change of road width.

$$\begin{cases} \rho_t + (\rho v)_x = 0\,, \\ v_t + v v_x = \dfrac{v_e - v}{\tau} + c_0 v_x + \dfrac{v dA}{\sigma dx} \end{cases}$$

Here c_0 is the speed of propagation for a disturbance whereas the term $\frac{1}{\sigma}\frac{dA}{dx}v$ accounts for the gradient of road width where A is the road width and σ is the reaction time for the driver.

This model portrays a directly proportional relationship of road width and equilibrium speed and flow. It investigates the effects of small disturbances in width to the speed and finds that the under moderate to high densities the effect of disturbances is heavy while under low densities, it is negligible.

3.5. A new continuum model for traffic flow and numerical tests

The model proposed in the paper (Jiang et al. 2002) is given below

$$\begin{cases} \dfrac{\partial k}{\partial t} + \dfrac{\partial (ku)}{\partial x} = g(x,t), \\ \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} = \dfrac{u_e - u}{T} + c_0 \dfrac{\partial u}{\partial x} \end{cases}$$

The model successfully removed various discrepancies occurring in previous models like isotropic behaviour etc. it successfully proved that the characteristics equation is not greater than flow speed at any point in the interval. Further numerical tests showed that linear stability analysis and local cluster effect is also successfully reproduced.

But the drawback is that at higher densities, the phenomenon of stop and go traffic takes frequent occurrences and the model is incapable of reducing these type of conditions.

4. RESEARCH METHODOLOGY

4.1. Overview

The focus of this study is to test out a macroscopic traffic flow model to find out how well it can map traffic conditions and find its true potential. Since these models are purely deterministic, it is considered that drivers always behave according to the same laws and are predictable as well. The model predicts a uniform model at low densities whereas after a certain threshold density, flow becomes unstable as small perturbations are amplified.

The model considered here (Tang et al. 2010b) is the one developed with consideration for driver's forecast effect

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial(\rho v)}{\partial x} = 0 \,, \\ \dfrac{\partial v}{\partial t} + (v - \beta\tau u'_e(h)c_0)\dfrac{\partial v}{\partial x} = \dfrac{(1+\beta)(v_e(\rho) - v)}{T + \beta\tau} \end{cases} \qquad 1$$

Here, β is the driver's forecast effect.

The above equation is similar to the advection equation. The term advection means transport of a substance by bulk motion. Here it might denote the movement of a disturbance in the traffic flow and the speed with which it is transferred along the stream.

It is convenient to write this system in vector form, i.e.,

$$U + F(U) = S(U) \qquad 2$$

Where

$$U = \begin{pmatrix} \rho \\ v \end{pmatrix}, \quad F(U) = \begin{pmatrix} \rho v \\ v - \beta\tau u_e(h)c_0 \end{pmatrix}, \quad S(U) = \begin{pmatrix} 0 \\ (v_e(\rho) - v)\frac{(1+\beta)}{(T+\beta\tau)} \end{pmatrix}$$

Here U is the vector of conserved variables, F(U) is the flux vector and S(U) is the vector of source terms.

The homogenous form of this equation is

$$U_t + F(U)_x = 0$$

We can linearize the equation by writing in the form Ut + J*Ux = 0 where J is the Jacobian Matrix defined by

$$J = \frac{\partial F(U)}{\partial U} = \begin{pmatrix} \frac{\partial F1}{\partial U1} & \frac{\partial F1}{\partial U2} \\ \frac{\partial F2}{\partial U1} & \frac{\partial F2}{\partial U2} \end{pmatrix} \quad 3$$

Where, $U = \binom{U1}{U2} = \binom{\rho}{v}$ and $F(U) = \binom{F1}{F2} = \binom{\rho v}{v - \beta\tau u_e(h)c_0}$

since we have to write F in terms of U, we convert the F matrix to the following,

$F(U) = \binom{U1 * U2}{U1 - \beta\tau u_{e(h)}c_0}$

Where the term $\beta\tau u_e(h)c_0$ is a constant.

it is easily shown that the Jacobian matrix J is

$$J = \begin{pmatrix} U2 & U1 \\ 0 & 1 - \beta\tau u_e(h)c_0 \end{pmatrix}$$

Therefore, Ut + J*Ux = 0 becomes

$$\binom{\rho}{v} + \begin{pmatrix} U2 & U1 \\ 0 & 1 - \beta\tau u_e(h)c_0 \end{pmatrix} * \binom{\rho v}{v - \beta\tau u_e(h)c_0} = 0$$

The Jacobian matrix J has two eigenvalues $\lambda_1 = v - \beta\tau u_e(h)c_0$ and $\lambda_2 = v$

These are the characteristic speeds of the model. Here we can see that at no time interval, these characteristic speeds will be greater than the flow velocity. Since the characteristic speeds represents the speed of information, it can be safe to assume that it cannot travel from upstream to downstream. This signifies is that the driver will not be affected by vehicles from behind.

As we can see that the eigenvalues are real and distinct which classifies the above equations as hyperbolic PDEs. The solutions of hyperbolic equations are distinctive, in the sense that they are wave-like. Disturbances generally have a finite propagation or perturbation speed. In the case of the above equation, c_0 is the speed of small perturbations and it is equal to $\varepsilon / \beta\tau + T > 0$.

4.2. Numerical Simulations

There are different types of numerical methods:

1. Method of characteristics

2. Finite Element Method

3. Finite Difference Method

4. Finite Volume Method

Generally, method of characteristics is not used for hyperbolic PDEs since they don't necessarily contain exact solutions. Some earlier models like LWR model can be solved using this method. Hence FEM, FDM or FVM are used. In this case, we go with FDM to provide numerical solutions.

Before we proceed to carry out the solution, we have to look at the numerical scheme of the above mentioned equation, since in spite of being a hyperbolic system, it is not possible to write it in a conservative scheme. Hence we use upwind scheme to carry out the discretization.

In this, all continuous functions, models, variables etc. are transferred into their discrete counterparts. This enables the user to carry out suitable numerical evaluation, albeit with some approximations.

$$\rho_k^{m+1} = \rho_k^m + \frac{\Delta t}{\Delta x} v_k^m (\rho_{k-1}^m - \rho_k^m) + \frac{\Delta t}{\Delta x} \rho_k^m (v_k^m - v_{k+1}^m) \quad (4)$$

if $v_k^m < \beta \tau u'_e(h_k^m) c_0$

$$v_k^{m+1} = v_k^m + \frac{\Delta t}{\Delta x} (\beta \tau u'_e(h_k^m) c_0 - v_k^m)(v_{k+1}^m - v_k^m) + \frac{\Delta t(1+\beta)}{T+\beta\tau} (v_e(\rho_k^m) - v_k^m) \quad (5)$$

Else

$$v_k^{m+1} = v_k^m + \frac{\Delta t}{\Delta x} (\beta \tau u'_e(h_k^m) c_0 - v_k^m)(v_k^m - v_{k-1}^m) + \frac{\Delta t(1+\beta)}{T+\beta\tau} (v_e(\rho_k^m) - v_k^m) \quad (6)$$

Here a first-order upwind scheme has been used to convert the continuous variables in the hyperbolic PDE to discrete variables. Note that the notations k, m, Δt, Δx denote space index, time index, time step and special step respectively.

ρ_k^m and v_k^m are density and speed at the corresponding point (k,m).

4.3. Shock and Rarefaction Waves

By definition, shock waves occur when a particular stream of traffic with certain characteristics meets another stream with different characteristics.

As pointed out by Daganzo, the realistic description of shock fronts in traffic is a particularly difficult problem. We will investigate how the traffic flow fronts between a congested and a nearly free traffic evolve under two Riemann initial conditions. These two initial conditions are:

$$\rho_u^1 = 0.04, \quad \rho_d^1 = 0.18, \tag{7}$$

$$\rho_u^2 = 0.18, \quad \rho_d^2 = 0.04, \tag{8}$$

Where ρ_u and ρ_d are upstream and downstream densities for cases 1 and 2 respectively. The plot below shows a theoretical representation of shockwaves in traffic flow.

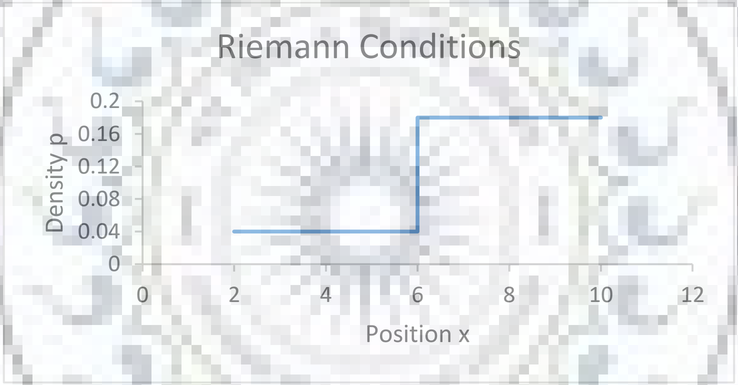


Figure 0-1Riemann Conditions as step wise function

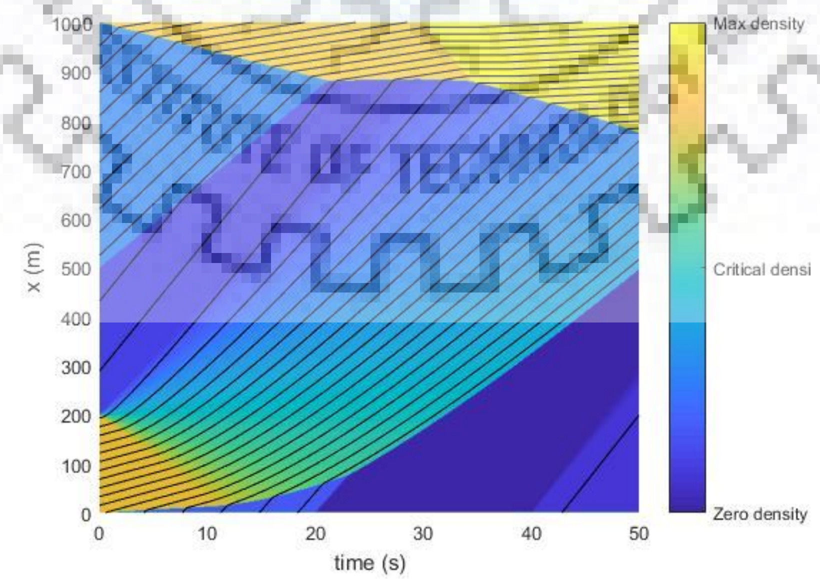


Figure 0-2 Shock Wave Fronts

The above plot shows shock waves. The changing colour gradients re the shock fronts and the empty area has almost zero densities or free flow speeds.

The initial speeds are

$$v_u^{1,2} = v_e\left(\rho_u^{1,2}\right), \qquad v_d^{1,2} = v_e\left(\rho_d^{1,2}\right) \tag{9}$$

We use the following equilibrium speed:

$$v_e(\rho) = v_f\left(1 - exp\left(1 - exp\left(\frac{c_m}{v_f}\left(\frac{\rho_j}{\rho} - 1\right)\right)\right)\right) \tag{10}$$

Where v_f is the free speed and c_m is the kinematic wave speed at jam density. The above equation can also be written as

$$u_e(h) = v_f\left(1 - exp\left(1 - exp\left(\frac{c_m}{v_f}\left(h\rho_j - 1\right)\right)\right)\right) \tag{11}$$

It is essentially the same equation with mean headway, h=1/ρ being substituted in place. Other parameters are:

v_f = 30 m/s, ρ_j = 0.2 veh/m, T = 10 s, τ = 5 s,

C_0 = C_m = 11 m/s, β = 0.3, Δx = 100 m, Δt = 1 s

4.4. Evolution of Small Perturbations

Whenever something unexpected happens in a traffic flow like vehicles changing lanes or vehicles entering or exiting the flow, its continuity is disturbed. This disturbance can travel like a ripple inside the flow and might cause a major problem if left unchecked. Hence it is essential to map such disturbances and to know the conditions which cause them and also increase the chances and frequency of its occurrence.

To describe this effect, the initial condition (Herrmann and Kerner 1998)

$$\rho(x,0) = \rho_0 + \Delta\rho\left\{cosh^{-2}\left(\frac{160}{L}\left(x - \frac{5L}{16}\right)\right) - \frac{1}{4}cosh^{-2}\left(\frac{40}{L}\left(x - \frac{11L}{32}\right)\right)\right\} \tag{12}$$

We will also use speed; (B. Kerner 1993)

$$v_e(\rho) = v_f \left(\left(1 + exp \left\{ \frac{\frac{\rho}{\rho_{jam}} - 0.25}{0.06} \right\} \right) - 3.72 * 10^{-6} \right)^{-1} \quad 13$$

And also the equilibrium speed;

$$u_e(h) = v_f \left(\left(1 + exp \left\{ \frac{\frac{1}{h\rho_j} - 0.25}{0.06} \right\} \right) - 3.72 * 10^{-6} \right)^{-1} \quad 14$$

4.5. Traffic Data

Traffic data obtained from RITES LTD on the National Highway 44 (previously NH-7) on the Nagpur-Hyderabad section near Multimodal International Hub at Nagpur is used for some real world correlation with the traffic model to see if it can be adapted to the existing conditions.

A sample of flow (q), densities (ρ) and observed velocity (v) data for 1-day are tabulated below:

Density (veh/km)	Velocity (km/hr)	Flow (veh/hr)
15	59	852
17	57	948
25	47	1176
14	52	720
17	53	912
15	50	744
15	60	876
19	54	1044
20	55	1104
15	62	912
29	51	1476
11	62	684
15	59	852
17	57	948
25	47	1176
14	52	720
17	53	912
15	50	744
11	108	1224
9	90	780

16	65	1032
9	62	540
2	48	120
5	65	312

The flow obtained for 15-min intervals were converted to 1-hr data to match the units of vehicular speed in km/hr. Then densities were obtained by the fundamental equation $\mathbf{q = \rho * \mathit{v}}$.

The values from these data, i.e., jam densities, free flow velocities and flow, have been used alongside the plots from the model and inferences are drawn from the comparison.

The inferences are shown alongside Figure 5-4 and Figure 5-5.

5. RESULTS AND DISCUSSION

Simulations have been carried out

The graphs below depict the shock wave and rarefaction wave from the model.

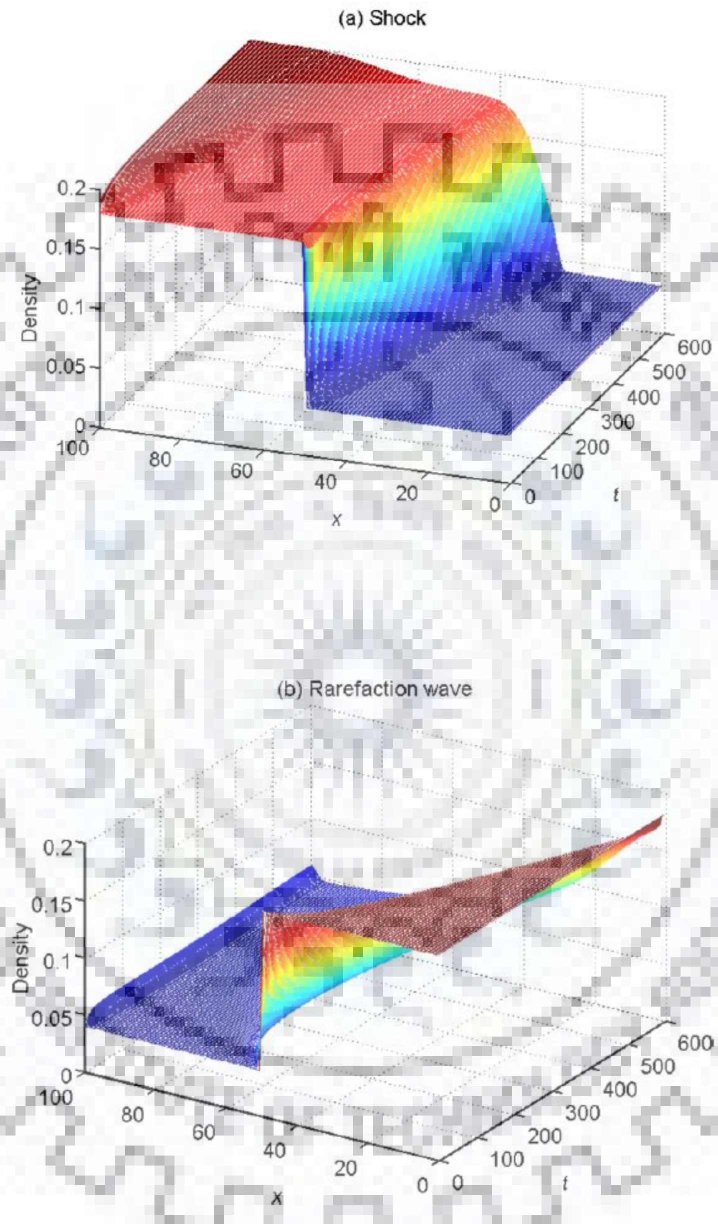
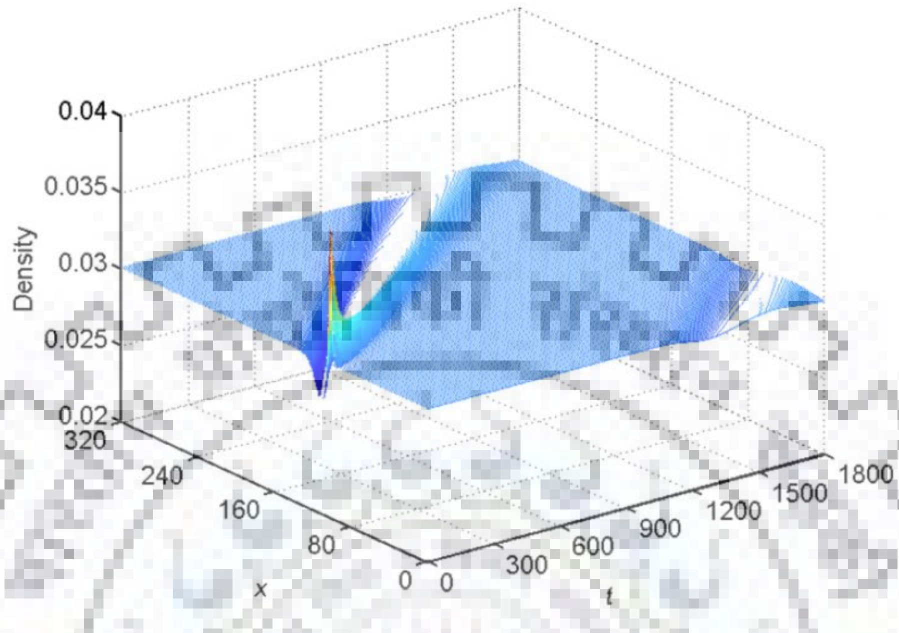


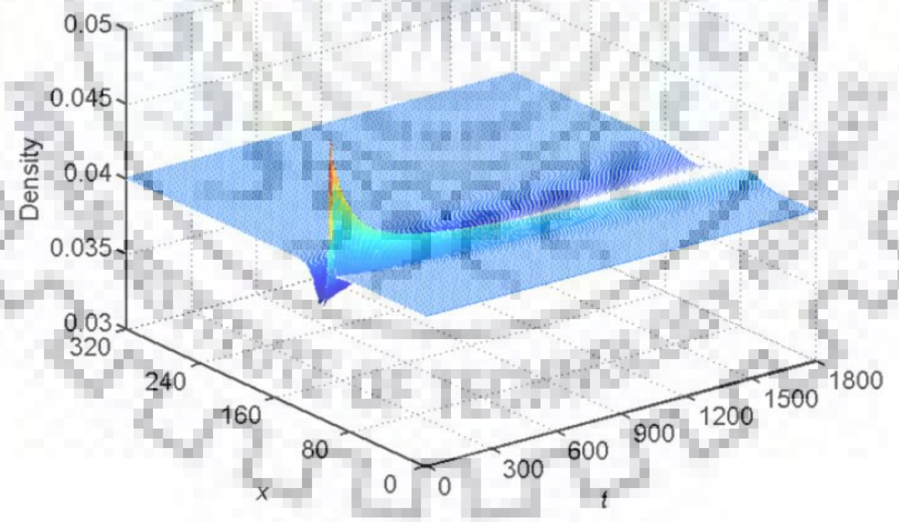
Figure 0-1 3-D Shock and Rarefaction Waves

The Cauchy-Riemann initial conditions were applied and it can be seen that the model is successfully able to produce shock waves and rarefaction waves. The shock wave is essentially when a high density stream meets a low density stream or a low flow stream meets a high flow stream. Rarefaction waves are the opposite of this and happens when the traffic is cleared.

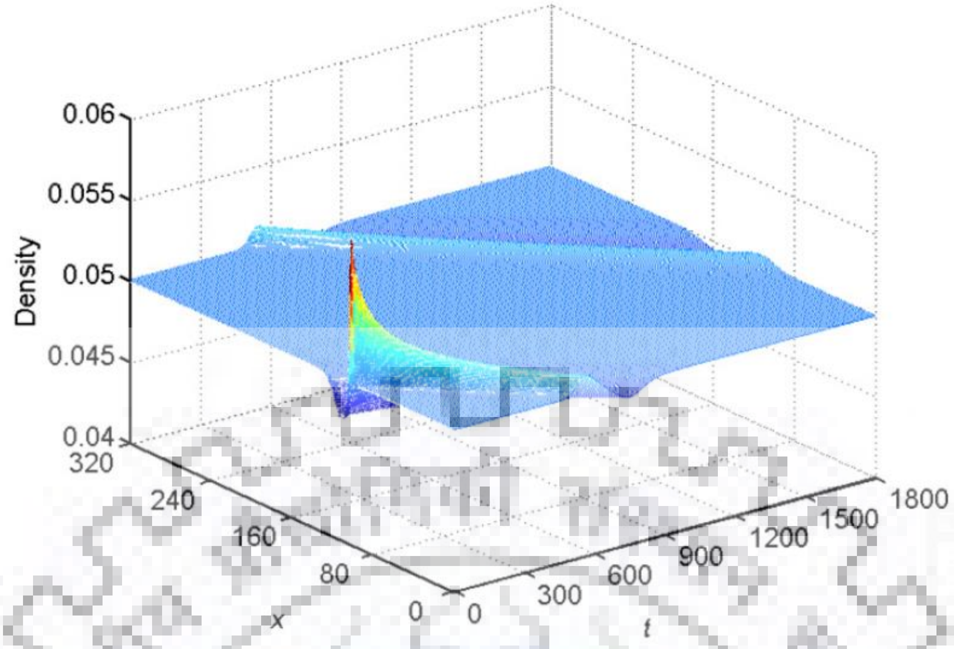
Note that in 2-D, i.e., when only the density vs position axis are considered, the graph resembles a plot of Riemann conditions with upstream and downstream densities of 0.04 and 0.18 respectively for shock waves and vice-versa for rarefaction waves.



(a)



(b)

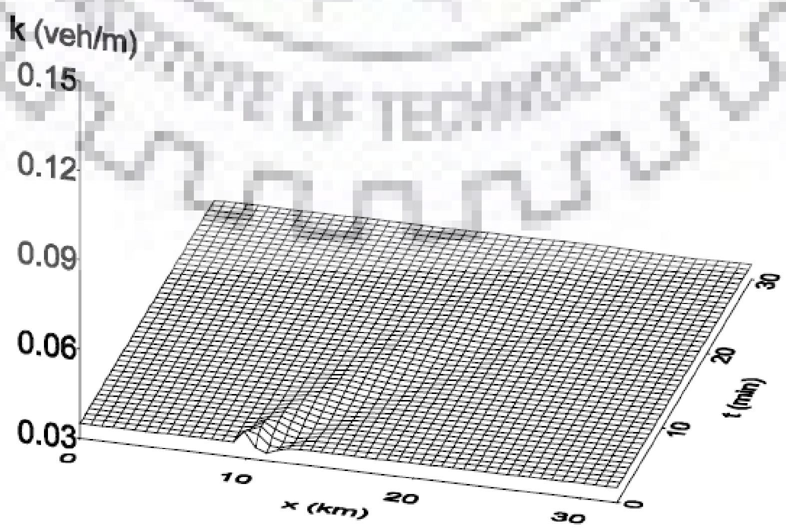


(c)

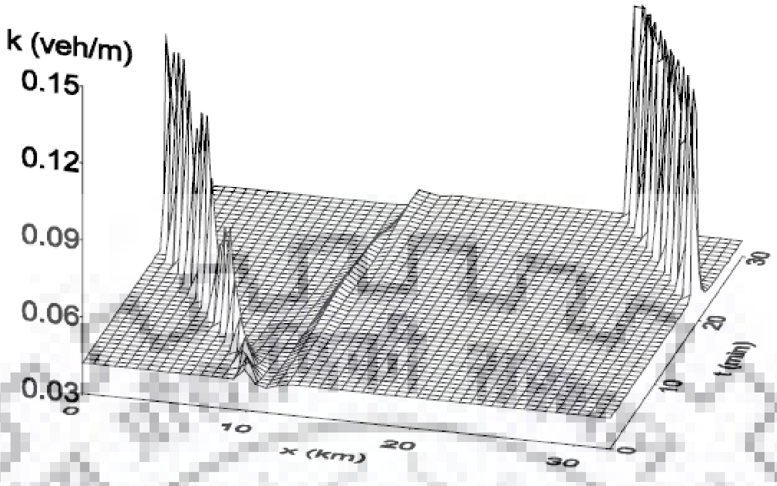
Figure 0-2 Evolution of perturbations
When the initial velocities are (a) 0.03, (b) 0.04 and (c) 0.05

The above figures show evolution of perturbations when the initial densities are 0.03, 0.04 and 0.05 respectively. The model shows the propagation direction of the disturbances along with their amplitudes. At the point of origin, the perturbation shows a peak which gradually smoothens out as it fans outwards.

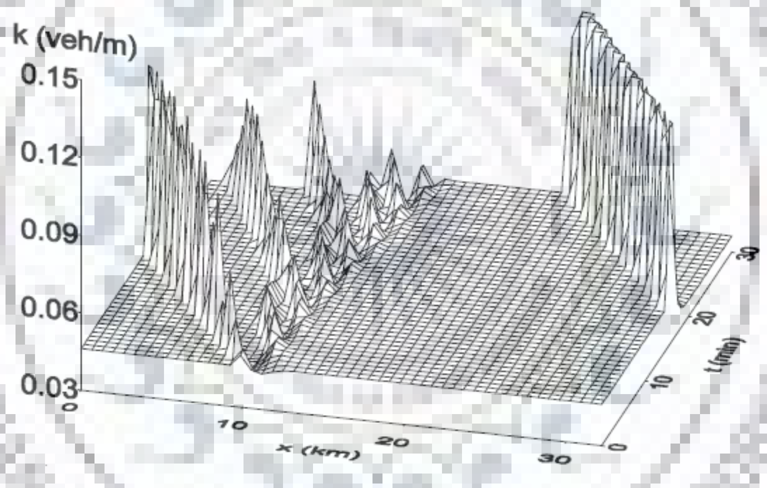
Also at initial density of 0.04, which was the initial condition for a shock wave, the perturbation starts at the same place for all times t, which can be correlated with the earlier plot for shock wave condition for comparison and verification.



(a)



(b)



(c)

Figure 0-3 Local Cluster Effect in Jiang Model

The graphs show a similar plot of perturbations from a different model proposed (Jiang et al. 2002)
The proposed model is presented below:

$$\begin{cases} \dfrac{\partial k}{\partial t} + \dfrac{\partial (ku)}{\partial x} = g(x,t), \\ \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} = \dfrac{u_e - u}{T} + c_0\dfrac{\partial u}{\partial x} \end{cases}$$

The drawback of this model compared to the present one is that small perturbations lead to huge aberrations and causing stop-and-go traffic conditions at different times. As we can see the wave fronts presented in this paper are smoother. They also are eliminated or die-out as it moves forward.

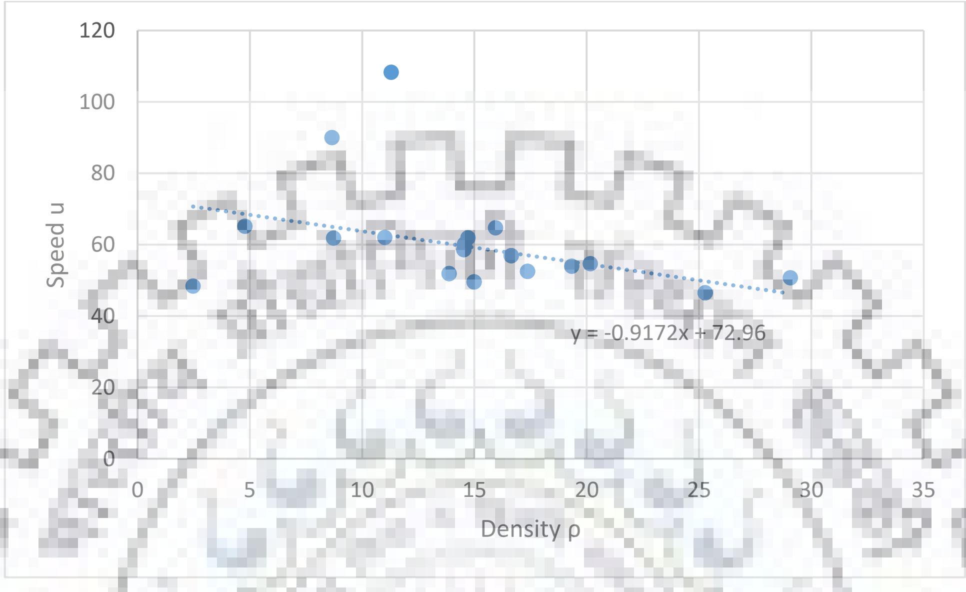


Figure 0-4 Speed Density Curve for local data

From the sample of traffic values obtained, speed vs density graph is plotted. A trend line is formed which conforms to the Greenshields speed-density relation.

The equation of the trend line is y = -0.9172x + 72.96. Using this equation, we can find the free flow velocity v_f by substituting x=0 and jam density ρ_{jam} by substituting y=0.

the values of v_f = 72.96 km/hr or **20.26 m/s**.

ρ_{jam} = 79.55 veh/km or **0.079 veh/km**

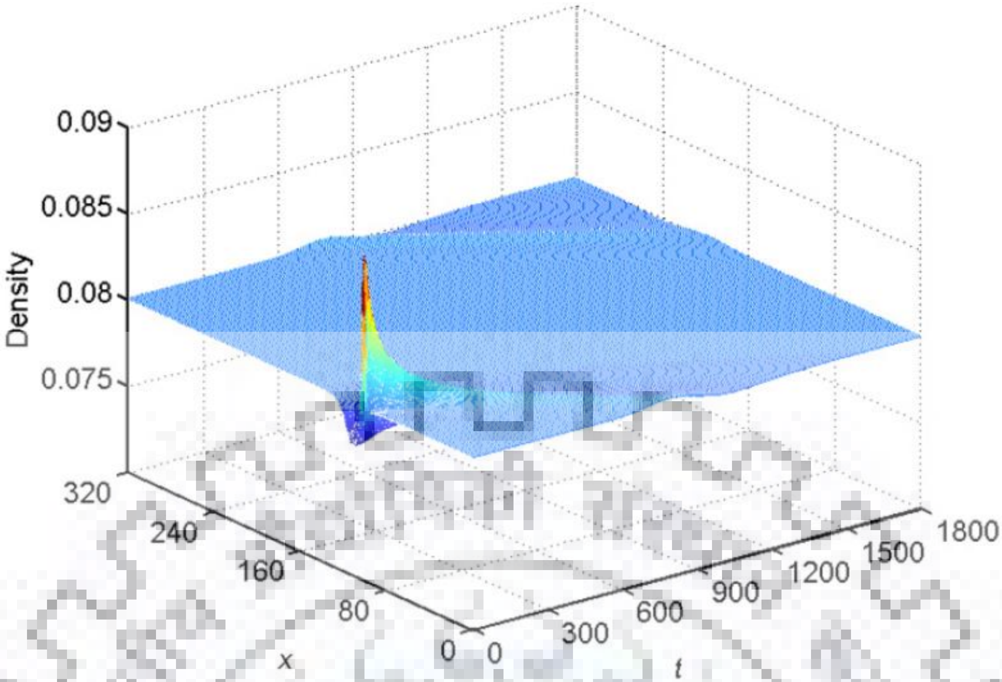


Figure 0-5 Evolution of perturbations for local data

In this plot the initial density is taken as 0.079 veh/m ≈ 0.08 veh/m. we can see that the results are in accordance with other plots. Since 0.08 veh/m is the calculated jam density of our data, it is shown that even at these high densities, the model behaves properly without any significant alterations.

Even though the data is fairly small and the values obtained from it can be classified as approximate in value, it is the closest to the real world data feasible.

6. Conclusions and Recommendations

From the results we can conclude the following:

1. The macroscopic model is able to prove its anisotropic nature, i.e., information does not reach the driver from behind. This means that anything happening on the upstream cannot affect the drivers in the downstream which is physically right.

2. The model successfully reproduces shock and rarefaction waves. This is essential as these phenomenon has been associated with macroscopic models since the very first model by Lighthill and Witham, Payne (LWR) Model. This is one of the most basic criteria's for a model in this domain.

3. The effect of small perturbations or disturbances are created. It is also compared with similar plots from another model. It can be concluded that this model smoothens the effect greatly and there is also no occurrence of any stop and go traffic in the temporal distribution.

4. The sample traffic data obtained is used to calculate densities and speeds of the actual road conditions. These data is used to create a perturbation plot. It can be concluded that even at jam densities for the road section, this model is able to give encouraging results. It must be noted that the data values are highly limited and further huge amounts of data is required to model perspectives more clearly.

7. Limitations and Future Scope

- One of the major limitations of macroscopic traffic modelling in general is that they are highly theoretical in nature. Real world data is seldom used to validate these models as computer simulated graphics show sufficiently good results. Hence their real world simulation capabilities are limited in nature.

- Having said that, they are still used among researchers because of their relative ease during developmental stages and during simulations than their microscopic counterparts.

- Another drawback of these models are that they take only homogenous traffic conditions into account while model creation and validation. Since no traffic in the world can be classified as perfectly homogenous, it is difficult to obtain the exact results we are hoping for.

- Continuing from the previous point, since their noon-heterogeneous nature has been established, their ability to model Indian conditions comes under question. Not only is Indian conditions highly heterogeneous, the driver behaviour is also very erratic.

- Hence a heterogeneous model taking into account factors such as irrational driving or road conditions etc must be created which hopefully can more successfully model real world conditions.

8. REFERENCES

1. Wageningen-Kessels, F., Lint, H., Vuik, K. & Hoogendoorn, S., 2015. Genealogy of traffic flow models, *EURO Journal on Transportation and Logistics*, pp.445–473

2. Lighthill, MJ., Whitham, GB., 1955. On kinematic waves II: a theory of traffic flow on long crowded roads. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Science*, 229(1178), pp.317–345.

3. Greenshields, BD., 1935. A study of traffic capacity. In: *Proceedings of the 14th Annual Meeting of the Highway Research Board*, pp 448–477.

4. Daganzo, CF., 1994. The cell transmission model: a dynamic representation of highway traffic consistent with the hydrodynamic theory. *Transportation Research Part B:Methodological* 28(4), pp.269–287

5. Kerner, BS., Klenov, SL., 2006. Deterministic microscopic three-phase traffic flow models. *Journal of Physics A: Mathematical and General* 39(8)

6. Wong, GCK., Wong, SC., 2002. A multi-class traffic flow model: an extension of LWR model with heterogeneous drivers. *Transport Research Part A: Policy & Practice* 36(9), pp.827–841

7. Pipes, LA., 1953. An operational analysis of traffic dynamics. *Journal of Applied Physics* 24(3), pp.274–281

8. Kometani, E., Sasaki, T., 1961. Dynamic behaviour of traffic with a nonlinear spacing–speed relationship In: Herman R (ed) *Theory of traffic flow*, proceedings. Elsevier, Amsterdam, pp. 105–119

9. Gazis DC, Herman R, Rothery RW 1961) Nonlinear follow-the-leader models of traffic flow. *Operational Research* 9(4), pp.545–567

10. Payne HJ (1971) Models of freeway traffic and control. In: Mathematical models of public systems. Simulation Council Proceedings, pp 51–61

11. Tang TQ, Li CY, Huang HJ, Shang HY (2011) Macro modeling and analysis of traffic flow with road width, Journal of Central South University of Technology18:1757–1764

12.Herrmann, M., and Kerner, B. S. (1998). "Local cluster effect in different traffic flow models." 255(September 1997), 163–188.

13. Jiang, R., Wu, Q., and Zhu, Z. (2002). "A new continuum model for traffic flow and numerical tests." 36, 405–419.

14. Bando, M., Hasebe, K., Nakayama, A., Shibata, A., & Sugiyama, Y. (1995). Dynamical model of traffic congestion and numerical simulation. *Physical review E, 51*(2), 1035.

15. Del Castillo, J. M., Pintado, P., & Benitez, F. G. (1994). The reaction time of drivers and the stability of traffic flow. *Transportation Research Part B: Methodological*, *28*(1)