RELIABILITY ANALYSIS OF SOME INDUSTRIAL SYSTEMS IN FUZZY ENVIRONMENT

Ph. D. THESIS

by

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A THESIS

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled "**RELIABILITY ANALYSIS OF SOME INDUSTRIAL SYSTEMS IN FUZZY ENVIRONMENT**" in partial fulfilment of the requirements for the award of the Degree of Doctor of Philosophy and submitted in the Department of Mathematics of the Indian Institute of Technology Roorkee, Roorkee is an authentic record of my own work carried out during a period from August, 2013 to July, 2018 under the supervision of Dr. S. P. Sharma, Professor, Department of Mathematics, Indian Institute of Technology Roorkee, Roorkee.

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other Institution.

(NEHA SINGHAL)

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

(S. P. Sharma)
Supervisor

Date: **July , 2018**

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Neha Singhal

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Abstract

An industrial system is consists of numerous components/subsystems and the probability that the system survives, depends directly on each of its constituent components/subsystems. These components/subsystems are expected to be operational and accessible for the most possible time to maximize profit and overall production. But failure is nearly unavoidable phenomenon with technological products and systems. Further, age and undesirable operating conditions of production/manufacturing processes affect each part of the system differently. Thus, there is a need to develop a suitable approach for analyzing the performance of these complex systems so that timely actions may be taken for achieving the goal of high production and hence more profit. The performance analysis includes the study of main reliability attributes such as system reliability, availability, maintainability and risk and safety analysis of the system as well as of its components/units. Generally, system analysts model and analyze the system behavior through various qualitative and quantitative tools/techniques. These techniques require precise knowledge of numerical probabilities and systems'/components' functional dependencies which may be difficult to be obtained in any large-scale system as the data collected or available from the historical records are mostly uncertain, limited and imprecise in nature. In order to predict the behavior of a system, it is necessary to develop mathematical model that deals with the uncertain behavior of the system. With the growing complexity of system, advancement in technology and demand of product quality, the significance of reliability and availability becomes very important. Most

of the systems in industry are repairable and it is expected that one should attain maximum profit from them.
Systems always exhibit some kind of uncertainty in their behavior because of the impreciseness of the data associated with these systems. The objective of this thesis is to develop methodology for analyzing performance and behavior of various repairable industrial systems under uncertain environment in different forms. The validation of the methodology is also a part of the objective. For that performance and behavior of Butter-Oil Processing Plant (BOPP), Condensate System, Piston manufacturing Plant and Cattle feed plant have been analyzed by using the available information about the systems' primary data. Herein the methodology is based on the amalgamation of techniques: namely, fuzzy set theory (and generalized fuzzy set theory), Runge-Kutta fourth order method and Particle Swarm Optimization. Reliability/Availability has also been studied through the solution of fuzzy differential equations. System availability in steady state has also been studied in this thesis. The main advantage of the proposed approach is that it provides system analyst a valid range of prediction for all reliability measures by elaborating uncertain data. Through these approaches, system analyst may also optimize the reliability of system.
Apart from this analysis, system reliability also has been studied through Intuitionistic fuzzy set theory. Sensitivity analysis has also been carried out for the reliability indices and effects on system are addressed which will be helpful for the system analyst/plant maintenance personnel to decide the best suited action and to assign the repair priorities as per the system requirements.

The whole work of the thesis is divided into eight chapters and chapter-wise summary of the thesis is as follows:
Chapter 1 covers the literature related to evaluation of system reliability/availability, behavior analysis using conventional methods, fuzzy approach based reliability analysis, reliability optimization etc.

Chapter 2 describes preliminaries and terminologies needed for the understanding of overall research work, presented in the subsequent chapters. The concepts of reliability, availability and their measures are discussed. Concepts related to Markov process, Particle Swarm Optimization, Fuzzy Set Theory, Generalized fuzzy and Intuitionistic fuzzy set theory have been described.

Chapter 3 formulates a new methodology for behavior analysis of systems through fuzzy Kolmogorov's differential equations and Particle Swarm Optimization. For handling the uncertainty in data, differential equations have been formulated by Markov modeling of system in fuzzy environment. Firstly solution of these derived fuzzy Kolmogorov's differential equations has been found by Runge-Kutta fourth order method and thereafter the solution has been improved by Particle Swarm Optimization. Fuzzy availability is estimated in its transient as well as steady states. Sensitivity analysis has also been performed to find the relative importance of a particular component of the system. Butter oil processing plant as an industrial system has been studied as a case for application of the proposed approach. Obtained results by the proposed technique have been compared with the results obtained by existed techniques.

Chapter 4 is an extension of chapter 3 in the sense that here a technique for solving first order linear differential equations with fuzzy constant coefficients and fuzzy initial values is given. It is based again on α-cut of a fuzzy set by formulation of optimization model. The approach, named as RKPSO, for solution of fuzzy differential equation is an amalgamation of Runge-Kutta (RK) fourth order method and Particle Swarm Optimization (PSO) technique. Some examples are discussed to illustrate the suggested approach. Furthermore, a concrete example of system of fuzzy differential equations in more than one dependent variable is taken. The whole process is presented by evaluating the availability of a Piston manufacturing plant, which is a repairable industrial system. Sensitivity analysis of Piston manufacturing plant has also been studied in this chapter, which shows the simultaneous effects of failure and repair rates on the system's steady state availability.

Chapter 5 deals with performance analysis of an industrial system having uncertain behavior. In this chapter, reliability/availability has been computed through Markov process. Uncertainty in data has been dealt with generalized fuzzy numbers. Availability of system in transient as well as in steady state has been examined in this chapter. Results have been computed and then compared by performing different arithmetic operations' approaches. For application perspective of proposed approach, butter-oil processing plant has been considered. Impacts of different arithmetic approaches in the methodology are reflected by numerical calculations and are depicted through the graphs.

Chapter 6 discusses the behavior analysis of a cattle feed plant, which has been investigated by using the approach, proposed through Particle Swarm Optimization and generalized fuzzy methodology. Uncertainties in the data are handled with the help of generalized fuzzy numbers and then behavior of the system has been analyzed in the form of various reliability parameters. In this methodology, availability analysis has been discussed through Markov process having uncertainties in the form of generalized fuzzy numbers in data. Obtained optimization problem, from the proposed approach, has been solved through particle swarm optimization. Application of the method has been shown by the evaluation of the availability of an industrial system.

Chapter 7 studies a technique to examine the performance analysis of an industrial system in a more steady and logical manner. In this chapter, we have proposed a structured and methodological framework, to analyze a complex industrial system. In quantitative framework, a set of differential equations is formulated through Markov modeling of industrial system in intuitionistic fuzzy environment. Intuitionistic fuzzy system availability is estimated in its transient as well as steady states. Effects of variations in failure and repair rates' have been studied for the purpose of sensitivity analysis and to determine the system's most crucial component. To study the behavior of the system, availability of the system for different (α, β)-cuts has been evaluated. The suggested approach is explained through the study of condensate system of Thermal power plant.

Chapter 8 deals with overall summary of this study and brief discussion on the scope for future work.

List of Publications

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Abbreviations

AGREE	Advisory Group on Reliability of Electronic Equipment
Av	Availability
DM	Decision Maker
EA	Evolutionary algorithm
FDEs	Fuzzy Differential Equations
FMEA	Failure Mode and Effect Analysis
FMECA	Failure Mode Effect and Critical Analysis
FTA	Fault Tree Analysis
GA	Genetic Algorithm
GFN	Generalized Fuzzy Number
IFDEs	Intuitionistic Fuzzy Differential Equations
IFS	Intuitionistic Fuzzy Set
MCS	Monte Carlo Simultaion
MM	Markov Modeling
MTBF	Mean Time Between Failures
MTTR	Mean Time to Repair
PN	Petri Nets
PSO	Particle Swarm Optimization
RAM	Reliability, Availability and Maintainability
RAP	Reliability Allocation Problem
RBD	Reliability Block Diagram
RCM	Reliability Centred Maintenance
RK-IV	Runge-Kutta Fourth Order
TFN	Triangular Fuzzy Number

Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

1.1 Background

With modern technology and higher reliability/availability requirements, systems are getting complicated day by day and hence job of system analyst/plant personnel is becoming more and more difficult to run the system under failure free pattern. However, failure is an inevitable fact related with technological products and systems. In recent years, the importance of reliability theory has been increasing rapidly with the innovation of recent technology for the purpose of making good products and highly reliable systems.

During World War II, mathematical theory of reliability took a shape of an aspect and grew as a result of the demands of developing technology. In World War II, the need for reliability was felt because of failure of many military operations in spite of the best efforts from the users. It was reported that during World War II electronic equipments used by Navy were operative for only 30% of the total available time because of frequent failures. At the onset of the war, it was discovered that over 50% of the airborne electronic equipments in storage were unable to meet the requirements of the Air Core and Navy. According to the study, army equipments were either under breakdown/repair for almost $60 - 75\%$ of its total time, which created problems for a successful mission. These facts may be for the reason that during war period the availability of equipments is of prime importance besides its cost. Since then efforts are continuing in this area to achieve the desired goals in industrial organizations. In view of these difficulties, reliability engineering emerged as a separate discipline in USA in the early 1950s. A study group “Advisory Group on the reliability of Electronic Equipment” (AGREE) was formulated in 1950. In 1952, an initial report by this group recommended for the creation of reliable systems which stated need to develop better components with more consistency and establishment of reliability and quality requirements from suppliers. In 1957, a final report was published by AGREE and provided the classic definition of reliability: reliability is the probability of a product performing a specified function without failure under given conditions for a specified period of time.

In recent years, due to the involvement of advancement in technologies, industrial systems are becoming complex in structural design and operational states therefore it has become hard for system analyst to achieve overall system performance with maximum profit, minimum system cost and less utilization resources. Therefore, now a days system reliability has become an important issue in analyzing the performance of an engineering system and for reducing their likelihood failures. For this elaborated knowledge of failure behavior of the system as well as its components are needed for analyzing system performance as well as for the maintenance strategies in order to enhance the system performance. Regarding this, number of techniques have been used which are based on probabilistic information about the systems and their constituents' failures. It is well thought of, that uncertainties in the failure data are insufficient to be handled by probability theory. This is because of the requirement of the large data for analysis, which is difficult to be obtained from various resources. Further, age and undesirable operating conditions of a manufacturing process affect each part of the system differently. Therefore, it is not simple to achieve a satisfactory and reliable system behavior. Thus, in the present scenario of global competition and faster delivery time, it is an important topic for system analyst to consider business with quality requirements together. This is the reason why there is a growing interest in investigation and implementation of reliability principles for industrial systems.

The objective of this work is to predict/analyze the performance and behavior analysis of industrial systems more closely by utilizing uncertain and imprecise data. The present work deals with analysis and optimization of reliability of some industrial systems. A brief literature on various issues related to reliability evaluation, analysis and optimization of a system have been discussed and are written section-wise hereafter.

1.2 Literature Review

In this section, a brief review of literature on reliability/availability analysis evaluation is given.

1.2.1 Reliability and Availability analysis using Conventional Methods

Reliability is a popular concept, defined according to the Oxford English Directory as 'the quality of being reliable, that may be relied upon; in which confidence may be put; trustworthy, safe, sure'. Today, reliability has grown into an omnipresent attribute with qualitative and quantitative connotations that prevades every aspects of our present day technologically intensive world. As reliability deals with the duration of breakdowns. The usefulness of the reliability analysis for the systems was discussed almost half century back by the researchers [77, 231]. It has been always considered as a useful tool for production availability, risk analysis and design of systems. Various techniques/methods exist in the literature for evaluating the reliability analysis of systems. These methods/techiques include Reliability Block Diagram (RBD), Fault Tree Analysis (FTA), Failure Mode and Effect Analysis (FMEA), Petri Nets (PN), Monte Carlo Simulation and Markov Modeling [26, 31, 39, 55, 115, 121, 159, 161, 199, 254, 273] etc.

In 1970, Buzacott [40] inspected the computation of reliability measures, based on successive reduction of complex models and incorporating minimal cut and path sets and also discussed the effect of redundancy by making use of exponential distribution to model system failure and repair distribution. In 1970, Vesely and Narum [276] developed a computer code, KITTO systems to estimate system availability and to analyze the repairable reliability parameters with an assumption that the failure and repair events of considered system components must be independent. In 1972, Kim [145] recommended a three phase approach for complex system reliability analysis in which at first phase, all series-parallel subsystems are reduced to non series-parallel subsystems. In the second phase, all the possible paths are traced from source to sink and based on these paths system reliability is calculated in third phase. Cherry et al. [53] performed reliability analysis of a system by calculating long term availability of system with the assumption of constant failure and repair rates.

Dhillon and Singh [66] studied new techniques and applications of engineering reliability. In 1986, Cafaro et al. [41] discussed the use of Markov chains in evaluating the reliability and availability of a system with time-dependent transition rates using analytical matrix- based methods. In 1990, Kumar et al. [161] studied design and cost analysis of a refining system in the sugar industry. In 1991, Kumar et al. [157] evaluated availability of equipment used for the decomposition process in the urea production system. In 1992, McCluer and Whittle [193] reviewed three petroleum refineries with reliability block diagrams (RBDs) to identify potential effects of single failures. Kumar et al. [158–160] discussed the performance of paper and sugar industry. Kumar and Pandey [156] used the Markov model in order to analyze the reliability of fertilizer plants. Mitchel and Murry [202], in 1996, simulated system structure using RBDs and forecasted the availability.

In 1997, Du and Nicholson [70] described sensitivity analysis for a degradable transportation system. In 1998, Atkinson and Nevil [21] discussed statistical methods for assessing measurement error in variables relevant to sports medicine. In 1999, Iida [128] studied basic concepts and future directions of road network reliability analysis. Tu et al. [267] studied reliability-based design optimization in 1999. Kumar et al. [166] analyzed the concept of maintenance free operating period (MFOP) and the reliability requirement driven by the Ministry of Defence (UK) for the next generation of future aircraft to be included in the fleet. Liu and Yang [187] developed an expert system named as EASYDFQR for quality and reliability design. This expert base system contains essential information about reliability and quality, such as design approaches, reliability models, fault tree analysis (FTA), failure modes, effects, and criticality analysis (FMECA) etc. In 2000, Haldar and Mahadevan [117] discussed probability, reliability and statistical methods in engineering design. Sarhan [233] studied the reliability equivalence factors of n independent and non-identical components' series system. Crocker and Kumar [60] proposed a new approach to Reliability Centred Maintenance (RCM) using the concepts of soft life and hard life to optimise the total maintenance cost. In 2001, Sarhan [234] computed the maximum likelihood and Bayes estimates of component reliabilities when the system components have constant failure rates. In 2002, Adamyan and He [3] represented a methodology that can be used for identifying the failure sequences and assessing the probability of their occurrence in a manufacturing system. Sarhan [235] extended the concept of reliability equivalence from simple series and parallel systems to some complex systems. Sarhan [236] studied the problem of estimating parameters included in the life time distributions of the individual components in a series system. Sarhan and Bassiouny [237] estimated the reliability of the individual components that belong to a parallel system using masked-system life test data. Further in 2004, Adamyan and He [4] discussed method that allows the system failures to be modeled using general Petri nets with inhibitor arcs and loops, which employs fewer variables than existing marking-based methods and substantially accelerates the computations. Hauptmanns [118] studied semi-quantitative fault tree analysis for process plant safety using frequency and probability ranges. Gupta et al. [113, 114] evaluated availability of butter-oil and plastic pipe manufacturing plants using Markov model. Kumar et al. [168] developed new allocation models based on total cost of ownership that allocate both reliability and maintainability for a series-parallel system subject to meeting a system-level availability target. Kumar and Knezevic [165] developed mathematical models for spare components with exponential, gamma, normal and Weibull time to failure distribution using a renewal process. Kumar et al. [167] developed strategies by both public and private sectors to focus on the reliability, maintainability and supportability characteristics inherent to the design of a system. Verma et al. [274] discussed systemic approach to integrated E-maintenance of large engineering plants. Misra et al. [201] discussed standby redundancy allocations in series and parallel systems. Gupta et al. [112] studied various aspects of aging and statistical dependence in frailty models. Misra et al. [200] discussed active redundancy allocations in series systems. Kumar et al. [164] discussed reliability, maintenance and logistic support.

1.2.2 Reliability analysis using Markov Process

In order to estimate the system behavior, several techniques are available in the literature. Some of them are fault tree [221, 253], Petri nets [4, 186, 203] and Monte Carlo Simulation [131] whereas some statistical analysis techniques include Bayesian method [127, 143], Redundancy allocations [101] and Markov analysis [116, 248] etc. All these techniques are independent to each other. In different situations for the study of availability, different methods are used. Markov analysis is most prominently used technique. A.A. Markov [23] explained one such class of processes in which the probability of the process, being in a given state at a particular time is related to the immediately preceding state of that process.

In literature, many authors have analyzed reliability/availabaility through Markov model (MM). In 1978, Beaudry [25] studied performance related reliability measures for computing systems through Markov models. In 1989, Smotherman and Zemoudeh [256] discussed non-homogeneous Markov model for phased-mission reliability analysis. In 1997, Anderson et al. [12] studied improved reliability model for redundant protective systems-Markov models. In 1998, Pukite and Pukite [219] discussed Markov modeling for reliability analysis. In 2004, Tanrioven et al. [261] discussed a new approach to real-time reliability analysis of transmission system using fuzzy Markov model. Mohanta et al. [204] presented a fuzzy Markov model to incorporate the influences of maintenance scheduling as well as aging of generating units on failure-repair cycle for computation of state probabilities. Shun Chun [252] studied Markov model for reliability analysis of dual-redundant relays. Gupta et al. [113, 114] studied analysis of reliability and availability of serial processes of butter-oil processing plant and plastic-pipe manufacturing plant. Guo and Yang [111] in 2008, discussed automatic creation of Markov models for reliability assessment of safety instrumented systems. Garg et al. [85, 86] in 2009, discussed reliability analysis of Cattle feed and Pharmaceutical Plant through Markov modeling.

In recent years, many researchers [140, 241] have studied reliability analysis through Markov model. In 2011, Shakuntla et al. [240, 242] discussed availability analysis of Polytube Industry by Markov modeling. Baazi et al. [24] studied Markov reliability modeling for induction motor drives under field-oriented control in 2012. Tewari et al. [265] in 2012, studied availability analysis of steam generating system in a thermal power plant. In 2013, Yu et al. [288] studied the cost-effectiveness of pharmacist care for diabetes in prevention of cardiovascular diseases through Markov model. In 2013, D'Amico et al. [61] studied the use of three semi-Markov models for wind speed modeling. Jiang et al. [132] developed Markov chain model to derive performance metrics for evaluating the proposed hybrid strategy that combines overlay and underlay dynamic spectrum access. Murthy et al. [210] presented a methodology using a Markov model in conjunction with event tree analysis to embark upon a set of health indices for phasor measurement units. In 2013, Zhang et al. [290] studied model and analyzed the reliability of such a modular converter through Markov model. Lal et al. [170] in 2013, explored performance analysis of piston manufacturing plant through stochastic models. In 2014, Guilani et al. [110] evaluated reliability of non-reparable three-state systems using Markov model. In 2014, Gowid et al. [108] discussed maintenance and optimization of reliability of liquefaction system on FLNG terminals using Markov modelling. In 2014, Ram and Manglik [224] discussed stochastic behaviour analysis of a Markov model under multi-state failures. In 2015, Minion et al. [198] evaluated cost-effectiveness of antiangiogenesis therapy using bevacizumab in advanced cervical cancer through Markov model. Yang et al. [284] studied support vector machine enhanced Markov model for short-term wind power forecast. Vrignat et al. [281] studied failure event prediction using hidden markov model approaches. In 2016, Lisnianski et al.[183] studied multi-state Markov model for reliability analysis of a combined cycle gas turbine power plant. Jun et al. [133] in 2016, discussed application of neural network and Markov model in the water reliability of river ecological environment. Hong et al. [124] in 2016, discussed multi-scenario passive filter planning in factory distribution system by using Markov model and probabilistic Sugeno fuzzy reasoning. In 2017, Roy and Chatterjee [227] studied reliability analysis of a multi-state wind farm using Markov process. In 2018, Bolvashenkov et al. [34] studied Markov reward model for decision making in the choice of optimal type of traction electric motor for icebreaking ship. Tan et al. [260] designed method to evaluate the reliability, availability, maintainability, and safety (RAMS) of the system by Markov modeling. In 2018, Manesh et al. [192] discussed a new procedure for determination of availability and reliability of complex cogeneration systems by improving the approximated Markov method. Singh and Tiwari [255] in 2018, studied review of reliability and availability evaluation of MPPGCL Sirmour Hydropower Station using Markov modeling.

1.2.3 Reliability analysis using Fuzzy approach

Reliability analysis is always one of the most important task for performing and analyzing the uncertain behavior of system using various techniques which require the knowledge of precise numerical probabilities and component functional dependencies, the information which is rather difficult to obtain. Even if data is available, it is based on past actions of the system so it is often inaccurate and incapable of forecasting the upcoming behavior of the system. As such the system reliability is affected by many factors such as design, installation and manufacturing and hence it may be extremely difficult to construct complete and accurate mathematical model for the system in order to assess the reliability because of inadequate knowledge about the basic failure events. This leads to problems of uncertainty in reliability assessment. Impact of the uncertainties on the system has always been the interest of engineers and scientists. Numerically, modeling and input uncertainties come in two flavors. Some of them are truly random in nature. Uncertainty is generally categorized into two ways: Aleatory and Epistemic. Aleatory uncertainty is truly random in nature and represents unknowns that differ each time when same experiment runs, whereas epistemic uncertainty arises due to lack of information about the process which could be reduced with more time and resources. Uncertainty quantification intends to work towards reducing epistemic uncertainties. Both probabilistic and non-probabilistic approaches are used to treat the element of uncertainty in reliability analysis. Conventionally, reliability theory is based on the probabilistic and binary state assumptions. Although, the probability approach has been applied successfully for many real world engineering reliability problems but still there are some limitations to the probabilistic method [42, 82, 134, 139]. Due to these limitations, the results based on probability theory do not always provide useful information to the practitioners and hence probabilistic approach to the conventional reliability analysis is inadequate to account for such built-in uncertainties in the data. To overcome these difficulties, methodologies based on fuzzy set theory [289] are being used in the risk analysis for propagating the basic event uncertainty.

Many researchers have discussed reliability/availability using the concept of fuzzy set theory. In 1990, Singer [254] discussed fuzzy set approach to fault tree and reliability analysis. Yuan et al. [135] discussed reliability where, binary-state assumption is reserved and the possibility assumption is taken in place of the probability assumption. Yuan et al. [134] introduced the concept of fuzzy success/failure (state) to represent the system structure, and performance. Chowdhury and Misra [54] introduced the concept of "fuzzy probability" in the evaluation of reliability of a general non-series parallel network. Cheng and Mon [52] evaluated fuzzy system reliability by interval arithmetic and α-cuts. Mon and Cheng [208] studied fuzzy system reliability analysis for components with different membership functions. Chen [48] presented a new method to analyze fuzzy system reliability using fuzzy number arithmetic operations. Bowles and Pelaez [35] discussed application of fuzzy logic to reliability engineering. Verma and Knezevic [275] presented the development of a weighted wedge to facilitate compliance analysis between fuzzy required and predicted system reliability values. Cai [42] studied failure-oriented view to system failure engineering using fuzzy methodology. Hong and Do [123] simplified fuzzy arithmetic and obtained the solutions for L-R type fuzzy system reliability. Chanda and Bhattacharjee [44] presented a fuzzy fault-tree based reliability analysis of an optimally planned transmission system. Dodagoudar and Venkatachalam [67] presented a methodology to process the fuzzy uncertainties in a slope reliability analysis. Knezevic and Odoom [147] developed a methodology which uses Petri nets instead of the fault tree methodology and solves for reliability indices utilising fuzzy lambdatau method. Savoia [238] proposed an approach to perform reliability analysis using extended fuzzy operations. Chen [49] discussed a method for analyzing fuzzy system reliability using vague set theory. Biondini et al. [32] presented a methodological approach of wide generality for assessing the reliability of reinforced and prestressed concrete structures.Liu et al. [184] discussed fuzzy rule based evidential reasoning approach for engineering system safety analysis. Sharma et al. [244] studied fuzzy modeling of system behavior for risk and reliability analysis. Sharma et al. [246] studied performance analysis of a complex robotic system by using fuzzy methodology and fault tree analysis. Sharma et al. [247] analyzed reliability of complex robotic system using Petri nets and fuzzy lambda-tau methodology. Donighi and Khanmohammadi [68] discussed fuzzy reliability model for series-parallel systems. Sharma et al. [245] studied reliability of complex multi-robotic system using GA and fuzzy methodology. Garg [88] proposed an approach using Petri nets and Vague Lambda-Tau methodology for reliability analysis of repairable systems. Garg et al. [90, 98] presented a novel technique for analyzing the behavior of an industrial system by utilizing vague, imprecise, and uncertain data. Komal et al. [149] studied the fuzzy reliability analysis of dual-fel steam turbine propulsion system in LNG carriers. Lin et al. [181] discussed dependence between degradation processes within piecewise-deterministic Markov process (PDMP) modeling framework. He and Zhang [119] focused on the performance evaluation of networks, whose arc failure rates are imprecise numbers. Komal [148] presented an integrated approach for fuzzy reliability analysis and resource allocation for a paper machine of a paper mill.

1.2.4 Reliability analysis using Generalized and Intuitionistic fuzzy set theory

Fuzzy set theory proposed by Zadeh [289] has achieved a great success in various fields to handle the uncertainties in the data by defining the fuzzy set which accommodates the various degree of membership on the real interval $[0, 1]$ by the membership function $\mu_{\tilde{A}} \in [0, 1]$. After the introduction of the concept of fuzzy set theory, several researchers worked on the extensions of the notion of fuzzy set theory and several theories have been given as the extension of fuzzy set theory. Two of them namely, Generalized fuzzy set theory and Intuitionistic fuzzy set theory have drawn the attention of many researchers during the last decades [11, 19, 195, 196, 215]. Several authors [47, 50, 96] have worked in the application of generalized fuzzy set theory . In order to generalize the concept of arithmetic operations, Chen [45] has discussed the arithmetic operations on generalized fuzzy numbers in 1985. In 2013, Dat et al. [62] have discussed the improved arithmetic operations on generalized fuzzy numbers. In 2016, Dutta [73] has studied normalized approach to avoid computational difficulties.

In 2010, Mahapatra et al. [189] discussed intuitionistic fuzzy multi-objective mathematical programming on reliability optimization model. Kumar et al. [162] analyzed fuzzy system availability using intuitionistic fuzzy number. Lata and Kumar [172] discussed method for solving intuitionistic fuzzy differential equations along with the evaluation of intuitionistic fuzzy reliability of industrial system. Garg et al. [87, 97] studied reliability analysis of the engineering systems using intuitionistic fuzzy set theory. Garg et al. [100] have discussed intuitionistic fuzzy optimization technique for solving multi-objective reliability optimization problems. In 2016, Vishwakarma and Sharma [278] studied uncertainty analysis of industrial system using intuitionistic fuzzy set theory.

1.2.5 Reliability optimization

One of the most important factors for industrial products in face of the competitiveness of international market is reliability and quality problem. Improvement of system reliability by applying various reasonable techniques is always a challenging task for engineers/system analysts. Thus, reliability is a key index to enhance the performance of any system. As more and more complex systems are growing, the interest in assessing system reliability and the need for improving the reliability of systems have become very important. System reliability can be improved by using more reliable components, increasing redundant components in parallel and enabling repeatedly the allocation of entire system framework. For each manufacturing system, the subsystems have specific failure time and repair time distributions. In order to optimize the system performance, optimum failure and repair pattern is needed so that system can run failure free for maximum time period and perform its intended work at desired satisfaction level.

Many researchers [80, 109, 153, 175, 177, 209] have drawn attention in both problem characteristics and solution methodologies. In 1990, Fu and Frangopool [81] suggested three step reliability based vector optimization searching strategy. Gen et al. [104] proposed an efficient and specific algorithm for solving large-scale 0-1 GP problems in particular structures and introduced two numerical examples from among the problems of system reliability. Belli and Jedrzejowicz [28] proposed approach for optimization of software reliability, where emphasis was put on the software redundancy to achieve fault tolerance. Liu and Der Kiureghian[185] determined the suitability of the algorithms for application to linear and nonlinear finite element reliability problems. Hikita et al. [122] presented models for reliability optimization problems for general systems. Atiqullah and Rao [20] presented an algorithm which selects the optimal set of links that maximizes the overall reliability of the network subject to a cost restriction, given the allowable node-link incidences, the link costs and the link reliabilities. Coit and Smith [58] developed problem-specific genetic algorithm to analyze series-parallel systems and to determine the optimal design configuration when there were multiple component choices available for each of several k-out-of-n:G subsystems. Utkin et al. [268] discussed a method to solve the fuzzy reliability optimization problem for systems with unloaded reserve. Zhang and Der Kiureghian [292] discussed two improved optimization algorithms for reliability analysis. Finn and Kayande [78] proposed a general method to optimize the design of marketing measurement in applied studies. Chen et al. [51] presented a new method for reliability based optimization which requires only a modest increase in computational cost over that of deterministic design algorithm. Altiparmak et al. [10] presented a meta-heuristic approach using a genetic algorithm (GA) to optimize reliability of computer communication networks considering cost. Kumar [163] presented two optimization models for the independent recovery blocks with exponential execution time and conditions for the optimal arrangement of versions within a recovery block. Berman and Kumar [29] presented optimization models to maximize the reliability of the software satisfying a budget limitation.

In last decade, a lot of work has been done to optimize the performance of system by various approaches. In 2000, Coit et al. [57] developed a methodology to accommodate the redundancy allocation problem for systems designed with multiple k-out-of-n subsystems in series. Roco et al. [226] proposed an approach using Cellular evolutionary Strategies (CES) to solve various types of reliability optimization problems. Levitin and Lisnianski [178] presented a technique for solving a family of multi state systems reliability optimization problems, such as optimal expansion, structure optimization, maintenance optimization and optimal multistage modernization. Levitin and Lisnianski [179] suggested a method which allows the reliability of weighted voting system to be exactly evaluated without imposing constraints on unit weights or threshold value. Papadrakakis and Lagaros [213] explored the application of neural networks to reliability-based structural optimization of large-scale structural systems. Goel et al. [106] proposed optimization framework for the synthesis of the hydrodealkylation process (HDA) process. Frangopol and Maute [79] discussed a review of the life-cycle reliability-based optimization field with emphasis on civil and aerospace structures. Du et al. [69] proposed an integrated framework for optimization under uncertainty that brings both the probabilistic design constraints and the design objective robustness into account. By Youn and Choi [287] the hybrid mean value (HMV) method had been proposed for highly efficient and stable RBDO by evaluating the probabilistic constraint effectively. Nahas and Nourelfath [211] presented an application of ant system in a reliability optimization problem for a series system with multiple-choice constraints incorporated at each subsystem, to optimize the system reliability subject to the system budget. Shao et al. [243] studied the reliability optimization of distributed access networks subject to a constraint on the total cost. Gen and Yun [105] introduced the hybrid approaches for combining GA with fuzzy logic, neural network and other conventional search techniques for various reliability optimization problems. Mahapatra and Roy [190] studied multi-objective reliability optimization problem for reliability of system where reliability enhancement is involved with several mutually conflicting objectives. Saranga and Kumar [232] developed a mathematical model for repair analysis and proposed a methodology based on genetic algorithms. Konak et al. [152] presented genetic algorithms (GA), developed specifically for problems with multiple objectives. Zhao et al. [293] developed multiobjective ant colony system (ACS) technique for the reliability optimization problem of series-parallel systems. Mohanta et al. [205] presented a comparison of results for optimization of captive power plant maintenance scheduling using genetic algorithm as well as hybrid techniques. Tavakkoli et al. [263] proposed genetic algorithm for a redundancy allocation problem for the series-parallel system when the redundancy strategy can be chosen for individual subsystems. Taboada et al. [259] proposed and implemented custom genetic algorithm to solve multiple objective multi-state reliability optimization design problems. Azaron et al. [22] proposed an approach which is used to solve a multi-objective discrete reliability optimization problem in a k dissimilar-unit nonrepairable cold-standby redundant system. Li et al. [180] proposed a two-stage approach for solving multi-objective system reliability optimization problems. Komal et al. [150, 151] proposed an hybridized approach genetic algorithms-based Lambda-Tau (GABLT) technique to analyze the behavior of complex repairable industrial systems stochastically up to a desired degree of accuracy.

In recent years, researchers [269] have proposed many algorithms for reliability optimization. In 2010, Aoues and Chateauneuf [13] presented an overview of various reliability-based design optimization approaches which are tested on a various benchmark problems. Mahapatra et al. [189] proposed an intuitionistic fuzzy optimization approach to solve a multi-objective nonlinear programming problem in the context of a reliability application. Spence and Gioffrè [257] proposed an efficient reliability-based design optimization based on decoupling the traditionally nested optimization loop. Lin and Yeh [182] discussed reliability optimization of component assignment problem for a multistate network. Arya et al. [16] proposed a methodology for reliability enhancement of radial distribution system by determining optimal values of repair and failure rates. Sahoo [230] solved the constrained multi-objective reliability optimization problem of a system with interval valued reliability of each component. Safari [229] proposed a variant of the Non-dominated Sorting Genetic Algorithm (NSGA-II) for solving a mathematical model for multi-objective redundancy allocation problems (MORAP). Valian et al. [272] proposed an improved cuckoo search algorithm, enhancing the accuracy and convergence rate of the cuckoo search algorithm for reliability optimization. Garg and Sharma [99, 101] considered the multi-objective reliability redundancy allocation problem of a series system where designing cost and reliability of the system are considered as two different objectives. Garg [92] discussed the solution of reliability redundancy allocation problems of seriesparallel system under the various nonlinear resource constraints using biogeography based optimization.

Meng et al. [197] applied a modified chaos control to the advanced mean value iterative procedure through modifying the iterative step of the chaotic dynamics analysis. Mellal and Zio [194] developed penalty guided stochastic fractal search approach for solving redundancy allocation, reliability allocation and reliabilityredundancy allocation problems. Kumar et al. [155] solved some complex reliability optimization problems by using nature-inspired metaheuristic called gray wolf optimizer algorithm. Heidari et al. [120] presented a mixed-integer nonlinear programming to model the optimal placement of manual and protective devices and automatic sectionalizing switches in distribution networks. Jain et al. [130] incorporate the efficient function of a multi-objective evolutionary algorithm. Ge et al.[103] developed an optimization model to deduce the reliability design of critical components in a serial system.

1.3 Thesis Objectives

Systems always exhibit some kind of uncertainty in their behavior because of the impreciseness of the data associated with these systems. The objective of this thesis is to develop methodology for analyzing performance and behavior of various repairable industrial systems under uncertain environment in different forms. The validation of the methodology is also a part of the objective. For that performance and behavior of Butter-Oil Processing Plant (BOPP), Condensate System, Piston manufacturing Plant and Cattle feed plant have been analyzed by using the available information about the systems' primary data. Herein the methodology is based on the amalgamation of techniques: namely, fuzzy set theory (and generalized fuzzy set theory), Runge-Kutta method and Particle Swarm Optimization. Reliability/Availability has also been studied through the solution of fuzzy differential equations. System availability in steady state has also been studied in this thesis. The main advantage of the proposed approach is that it provides system analyst a valid range of prediction for all reliability measures by elaborating uncertain data. Through these approaches, system analyst may also optimize the reliability of the system.

Apart from this analysis, system reliability also has been studied through Intuitionistic fuzzy set theory. Sensitivity analysis has also been carried out for the reliability indices and effects on system are addressed which will be helpful for the system analyst/plant maintenance personnel to decide the best suited action and to assign the repair priorities as per the system requirements.

The whole work of the thesis is divided into eight chapters and chapter-wise summary of the thesis is as follows:

Chapter 1 covers the literature related to evaluation of system reliability/availability, behavior analysis using conventional methods, fuzzy approach based reliability analysis, reliability optimization etc.

Chapter 2 describes preliminaries and terminologies needed for the understanding of overall research work, presented in the subsequent chapters. The concepts of reliability, availability and their measures are discussed. Concepts related to Markov process, Particle Swarm Optimization, Fuzzy Set Theory, Generalized fuzzy and Intuitionistic fuzzy set theory have been described.

Chapter 3 formulates a new methodology for behavior analysis of systems through fuzzy Kolmogorov's differential equations and Particle Swarm Optimization. For handling the uncertainty in data, differential equations have been formulated by Markov modeling of system in fuzzy environment. Firstly solution of these derived fuzzy Kolmogorov's differential equations has been found by Runge-Kutta fourth order method and thereafter the solution has been improved by Particle Swarm Optimization. Fuzzy availability is estimated in its transient as well as steady states. Sensitivity analysis has also been performed to find the relative importance of a particular component of the system. Butter-oil processing plant as an industrial system has been studied as a case for application of the proposed approach. Obtained results by the proposed technique have been compared with the results obtained by existed techniques.

Chapter 4 is an extension of chapter 3 in the sense that here a technique for solving first order linear differential equations with fuzzy constant coefficients and fuzzy initial values is given. It is based again on α-cut of a fuzzy set and formulation of optimization model. The approach, named as RKPSO, for solution of fuzzy differential equation is an amalgamation of Runge-Kutta (RK) fourth order method and Particle Swarm Optimization (PSO) technique. Some examples are discussed to illustrate the suggested approach. Furthermore, a concrete example of system of fuzzy differential equations in more than one dependent variable is taken. The whole process is presented by evaluating the availability of a Piston manufacturing plant, which is a repairable industrial system. Sensitivity analysis of Piston manufacturing plant has also been studied in this chapter, which shows the simultaneous effects of failure and repair rates on the system's steady state availability.

Chapter 5 deals with performance analysis of an industrial system having uncertain behavior. In this chapter, reliability/availability has been computed through Markov process. Uncertainty in data has been dealt with generalized fuzzy numbers. Availability of system in transient as well as in steady state has been examined in this chapter. Results have been computed and then compared by performing different arithmetic operations' approaches. For application perspective of proposed approach, butter-oil processing plant has been considered. Impacts of different arithmetic approaches in the methodology are reflected by numerical calculations and are depicted through the graphs.

Chapter 6 discusses the behavior analysis of a cattle feed plant, which has been investigated by using the approach, proposed through Particle Swarm Optimization and generalized fuzzy methodology. Uncertainties in the data are handled with the help of generalized fuzzy numbers and then behavior of the system has been analyzed in the form of various reliability parameters. In this methodology, availability analysis has been discussed through Markov process having uncertainties in the form of generalized fuzzy numbers in data. Obtained optimization problem, from the proposed approach, has been solved through particle swarm optimization. Application of the method has been shown by the evaluation of the availability of an industrial system.

Chapter 7 studies a technique to examine the performance analysis of an industrial system in a more steady and logical manner. In this chapter, we have proposed a structured and methodological framework, to analyze a complex industrial system. In quantitative framework, a set of differential equations is formulated through Markov modeling of industrial system in intuitionistic fuzzy environment. Intuitionistic fuzzy system availability is estimated in its transient as well as steady states. Effects of variations in failure and repair rates' have been studied for the purpose of sensitivity analysis and to determine the system's most crucial component. To study the behavior of the system, availability of the system for different (α, β)-cuts has been evaluated. The suggested approach is explained through the study of condensate system of Thermal power plant.

Chapter 8 deals with overall summary of this study and brief discussion on the scope for future work.

Chapter 2

Preliminaries

The basic motive of studying reliability engineering is to search and generate methods and tools for evaluating reliability/availability, safety of systems and their components/equipments, and side by side that help support engineers in building in these characteristics. Subsequent sections of this chapter cover the main and basic aspects of reliability engineering of a system i.e. reliability and availability along with some soft computing techniques, which have been used in this thesis.

2.1 Reliability

Reliability of a system or its components is defined by the probability that the system (component) will perform its required function under the given conditions for a stated time interval [33, 74]. Qualitatively, it can also be viewed as the ability of a system to remain functional. Quantitatively, reliability specifies the probability that no operational interruptions will occur during stated time interval. To express it mathematically: one can define a continuous random variable T as the time to failure of the system; $T \geq 0$. Then reliability as a function of time 't' can be expressed as

$$R(t) = Pr\{T \geq t\} = \int_t^\infty f(u)du \tag{2.1.1}$$

where, $f(t)$ is failure probability density function. Clearly, $R(t) \geq 0, R(0) = 1$, and $\lim_{t\to\infty} R(t) = 0$ and the function $R(t)$ is a non-increasing function of t. For a given value of t, $R(t)$ is the probability that the time to failure is greater than or equal to t. In literature, reliability function is also called survivor function. The cumulative distribution function (CDF) of the failure distribution is defined as

$$F(t) = Pr\{T \leq t\} = \int_{-\infty}^{t} f(u)du \tag{2.1.2}$$

The expressions in Eq. (2.1.1) and Eq. (2.1.2) manifests that both the reliability function and the CDF represent areas under the curve defined by $f(t)$.

In addition to the probability function, there is another function, called the failure rate or hazard rate function which is often used in reliability. It provides an instantaneous (at time t) rate of failure. If the conditional probability of a failure in the time interval from t to $t + \Delta t$ given that system has survived to the time t, is

$$Pr\{t \leq T \leq t + \Delta t | T \geq t\} = \frac{R(t) - R(t + \Delta t)}{R(t)} \tag{2.1.3}$$

then $\frac{R(t)-R(t+\Delta t)}{R(t)\Delta t}$ is the conditional probability of failure per unit of time (failure rate).

If a particular hazard rate function is uniquely determined by

$$\lambda(t) = \frac{-dR(t)}{dt}.\frac{1}{R(t)} = \frac{f(t)}{R(t)} \tag{2.1.4}$$

then, $\lambda(t)$ is known as the instantaneous hazard rate or failure rate function. Based on these hazard function, the reliability function can be derived as

$$R(t) = exp\Big[-\int_{0}^{t} \lambda(u)du\Big] \tag{2.1.5}$$

The mean time to failure (MTTF) of the system is defined as

$$MTTF = \int_{0}^{\infty} R(t)dt \tag{2.1.6}$$

2.2 Availability

Availability is one of the most important measures in the reliability theory. **Availability** is defined as the probability of a product or system working satisfactorily at any given point of time when used under the given conditions of use [33, 74]. Thus availability signifies the probability that the system is available and is working satisfactorily at a given point of time. Availability is a more meaningful parameter of performance of a maintained system than reliability. Similar to the reliability function, it also gives a probability that a system will be available to function at the given time t. For defining the availabilities of the system, let

$$X(t) = \begin{cases} 1, & \text{if system is up at time t;} \\ 0, & \text{if the system is down at time t.} \end{cases} \tag{2.2.1}$$

Availability can be divided into the following categories.

(a) *Pointwise Availability*: It is the probability that the system will be up at a given instant of time. This availability is given by

$$A(t) = Pr\{X(t) = 1\} = E\{X(t)\}. \tag{2.2.2}$$

(b) *Average Availability*: Average availability over the interval $[0, T]$ is defined as

$$A(T) = \frac{1}{T}\int_0^T A(t)dt. \tag{2.2.3}$$

(c) *Interval Availability*: Interval or mission availability represents average availability over the mission time t_1 to t_2 is defined as

$$A_{t_2-t_1} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} A(t)dt. \tag{2.2.4}$$

(d) *Steady State Availability*: The steady state availability of the system is the limit of the instantaneous availability function as time approaches infinity and is given as

$$A = \lim_{T\to\infty} A(T) = \frac{MTBF}{MTBF + MTTR} \tag{2.2.5}$$

where, $MTBF$ and $MTTR$ are the mean time between failure and mean time to repair of the system/component respectively.

With the introduction of a repair capability that will restore a system to an operative state, to predict system availability as an alternative measure of system performance, both the failure and repair probability distributions must be considered.
To quantify the repair time, let T be the continuous random variable representing the time to repair a failed unit, having a probability density function of $m(t)$. Then cumulative distribution function is

$$Pr\{T \leq t\} = M(t) = \int_0^t m(t')dt' \tag{2.2.6}$$

Equation (2.2.6) gives the probability that a repair will be accomplished within time t.
The mean time to repair may be found from

$$MTTR = \int_0^\infty (1 - M(t))dt \tag{2.2.7}$$

Instantaneous repair rate (or repair hazard rate) is a conditional probability $\mu(t)$, defined as

$$\mu(t) = lim_{\Delta t \to 0} \frac{1}{\Delta t} \text{ Probability [unit will be repaired in the interval } (t, t + \Delta t)$$
$$\text{given that it has not been repaired in the interval } (0, t)] \tag{2.2.8}$$

or

$$\mu(t) = lim_{\Delta t \to 0} \frac{dM(t)/dt}{1 - M(t)} \tag{2.2.9}$$

2.3 Markov Process

A **Markov Chain** is a stochastic model describing a sequence of possible events in which the probability of each event depends only on the state attained in the previous event.

In probability theory and related fields, **Markov Process** is a random/stochastic process indexed by time, and with the property that future is independent of the past, given the present. Markov processes, named for Andrei Markov, are among the most important of all random processes. The calculation of the reliability of a system with elements exhibiting dependent failures and involving repair or standby operations is, in general, complicated and several approaches have been suggested to carry out the computations. A technique that has appeal and works well when failure and repair hazards are constant requires the use of Markov models.
Consider a system with components $x_1, x_2, x_3, ...$ such that the system state is a function of the states of the components. The system state is denoted by X. This system state changes with time t. The system state X and time t are two random variables. Each of these variables can be either continuous or discrete. Consequently, there are four possible combinations, namely,

1. Continuous-state, continuous-time;

2. Discrete-state, continuous-time;

3. Continuous-state, discrete-time;

4. Discrete-state, discrete-time.

If the state of the system is probability based, then the model is a Markov probability model. In reliability analysis, we deal with discrete-state continuous-time model, also called Markov process.
In a Markov model, state of the system is associated with probability P_{ij}, indicating the probability of the system moving from one state i to state j. This probability P_{ij} is called the transition probability. In Markov process, fundamental assumption is that the transition probability from i to j depends entirely on states i and j, and is independent of all previous states except the last one, i.e., state i.
To investigate the reliability/availability of complex repairable systems various tools

such as: Markov process, Semi-Markov process, Petri nets and Fault tree analysis etc., are developed. It is found from the bath-tub curve that the most favorable region is useful life (or normal life) which requires failure rates to be constant. Markov process, a stochastic process exhibiting memoryless property is a powerful technique in the analysis of reliability and availability of complex repairable industrial systems where the stay time in the system states follows an exponential distribution i.e. failure rate (λ) and repair rate (μ) are constant for all units during this process. System governing Markov process has the property that the transition probability of the system from one state to another state depends only on the current state and not on the previous states, the system may have experienced. The diagram indicating the states and transitions is known as Transition diagram.

In any given system, Markov process consists of all of its possible states, their transitions from one state to other, and corresponding transition rates. These transitions occur on account of failures and repairs. In system reliability and availability analysis, the respective transition probabilities must satisfy the following conditions [33, 258]:

- The transition probability from one state i to another state $(i+1)$ in time Δt is given by $\lambda \Delta t$, where λ is failure rate associated with states.

- The transition probability from state $(i+1)$ to the state i in time Δt is given by $\mu \Delta t$, where μ is repair rate associated with states.

- The probability of more than one transition in Δt is negligible and ignored.

Consider a case of repairable system that consists of single element, the element can be in one of two states: s_0- functioning state or s_1- the non-functioning state (shown in Figure 2.1). According to the above probabilistic considerations of Markov process, the probability that the system is in state s_0 at time $(t+\Delta t)$ is expressed as:

$$P_{s_0}(t+\Delta t) = (\text{Probability that it will not fail (non functioning) during } \Delta t).P_{s_0}(t)$$
$$+(\text{Probability that it will become good (functioning) during } \Delta t).P_{s_1}(t) \tag{2.3.1}$$

$$P_{s_0}(t+\Delta t) = P_{00}(\Delta t)P_{s_0}(t) + P_{10}(\Delta t)P_{s_1}(t). \tag{2.3.2}$$

Similarly,

$$P_{s_1}(t+\Delta t) = P_{11}(\Delta t).P_{s_1}(t) + P_{01}(\Delta t).P_{s_0}(t). \tag{2.3.3}$$

For non-repairable systems with constant hazard rate λ:

$$P_{00}(\Delta t) = (1-\lambda\Delta t), \qquad P_{10}(\Delta t) = 0,$$
$$P_{11}(\Delta t) = 1, \qquad P_{01}(\Delta t) = \lambda\Delta t. \tag{2.3.4}$$

For repairable systems with constant rates:

$$P_{00}(\Delta t) = (1-\lambda\Delta t), \qquad P_{10}(\Delta t) = \mu\Delta t,$$
$$P_{11}(\Delta t) = (1-\mu\Delta t), \qquad P_{01}(\Delta t) = \lambda\Delta t. \tag{2.3.5}$$

where, λ and μ are respectively the failure and repair rates of the system. The diagram representing the process is shown in Figure 2.1.

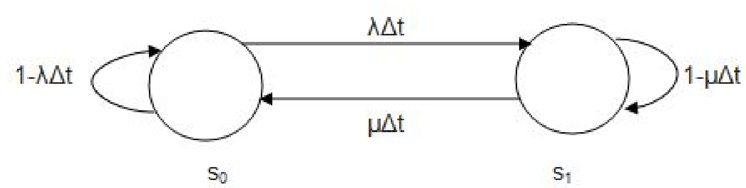


Figure 2.1: Transition diagram for a system having two states

2.4 Fuzzy Set Theory

In the last century there have been many paradigmatic changes in science and mathematics, one such change concerns the concept of uncertainty. Of course, this change in science has been manifested by a gradual transition from the traditional view, which insists that uncertainty is undesirable in science and should be avoided by all possible means, but as per the modern view, uncertainty is considered essential to science and has, in fact, a great utility.

In general, systems are constructed as models of either some aspects of reality or some desirable man-made objects. Uncertainty is thus an important commodity in the modeling business, which can be traded for gains in the other essential characteristics of models. Nowadays, complexity arises from uncertainty in the form of ambiguity. One should closely look into the real world complex problems to find an accurate solution, amidst the existing uncertainties using certain methodologies. Hence for handling ambiguity and uncertainty that exist in complex problems, the concept of fuzzy set theory was introduced by Lofti A. Zadeh [289] in 1965. Fuzzy set theory, compared to other mathematical theories, is perhaps the most easily adoptable theory to practice. The main reason is that fuzzy set theory has the property of relatively, variability and inexactness in the definitions of its elements. Many researchers [138, 206, 207] have worked in the field of application of fuzzy set theory.

Basically, a set is defined as a well defined collection of distinct objects, which shares a certain characteristic. The classical set (crisp set) is defined in such a way that the universe of discourse is divided into two groups: members and non-members. Consider an object x and a crisp set A. This object x is either a member or a non-member of the given set A. In case of crisp sets, no partial membership exists. This binary issue of membership can be represented mathematically by the characteristic function,

$$\chi_A(x) = \begin{cases} 1, & \text{if } x \in A; \\ 0, & \text{if } x \notin A \end{cases} \tag{2.4.1}$$

where, χ_A is the membership in set A for element x in the universe. The membership concept represents mapping from an element x in universe X to one of the two elements in universe $Y = \{0, 1\}$. The whole set X is assigned a membership value 1, and the null set ϕ is assigned a membership value 0.

2.4.1 Fuzzy Sets

An extension and generalization of crisp set theory was introduced by L.A. Zadeh in 1965 as fuzzy set theory [72, 294] which allows partial membership from 0 to 1 and named this partial membership of an element as its degree of membership. Members of a crisp set would not be members unless their membership is full or complete (i.e. having degree 1), but in contrast, elements in a fuzzy set, because their membership need not to be complete, can also be members of other fuzzy sets on the same universe.

A **fuzzy set** $\tilde{A}$ is completely characterized by the set $\{(u, \mu_{\tilde{A}}(u)) \mid u \in U\}$ where $\mu : U \to [0, 1]$. $\mu_{\tilde{A}}(u)$ determines the degree of belonging of element u in set A.

2.4.2 Membership Functions

Membership function defines the fuzziness in a fuzzy set irrespective of the elements in the set, which are discrete or continuous. The membership functions are generally represented graphically. There exist certain limitations for the shapes used to represent graphical form of membership function. The membership function defines all the information contained in a fuzzy set; hence it is important to discuss the various features of the membership functions. For a fuzzy set $\tilde{A}$, a membership function, denoted by $\mu_{\tilde{A}}$ maps U to the interval $[0, 1]$, i.e. $\mu_{\tilde{A}} : U \to [0, 1]$. Some basic features involved in characterizing membership function are the following [72, 146, 294].

(i) Core: The core of a membership function for some fuzzy set $\tilde{A}$ is defined as that region of universe that is characterized by complete membership in the set $\tilde{A}$. The core has elements u of the universe such that

$$\mu_{\tilde{A}}(u) = 1. \tag{2.4.2}$$

(ii) Support: The support of a membership function for a fuzzy set $\tilde{A}$ is defined as that region of universe that is characterized by a non-zero membership in the set $\tilde{A}$. The support comprises elements u of the universe U such that

$$\mu_{\tilde{A}}(u) > 0. \tag{2.4.3}$$

(iii) Height: The height, $h(\tilde{A})$, of a fuzzy set $\tilde{A}$ is the largest membership grade obtained by any element in that set $\tilde{A}$. Formally,

$$h(\tilde{A}) = \sup_{u \in U} \mu_{\tilde{A}}(u). \tag{2.4.4}$$

(iv) Boundary: The boundary of a membership function for a fuzzy set $\tilde{A}$ is defined as the region of universe that contains a nonzero but not a complete membership. Those elements u of the universe whose boundary comprises such that

$$0 < \mu_{\tilde{A}}(u) < 1. \tag{2.4.5}$$

In other words, the boundary elements are those which possess partial membership in the fuzzy set $\tilde{A}$.

2.4.3 Alpha-cut of Fuzzy Sets

The concept of α- cut of a fuzzy set is an important concept in fuzzy set theory, denoted by $\tilde{A}^{\alpha}$. The **$\boldsymbol{\alpha}$- cut** $\tilde{A}$ of a fuzzy set $\tilde{A}$ is defined as the crisp set of elements of universe of discourse U which have their membership grades in A , greater than or equal to the specified value α where $\alpha \in [0, 1]$ i.e.

$$\tilde{A}^{\alpha} = \{u \in U | \mu_{\tilde{A}}(u) \geq \alpha\}; \quad \alpha \in [0, 1]. \tag{2.4.6}$$

On the other hand, if the inequality in $\tilde{A}^{\alpha}$ is strict inequality then the α- cut is called a **strong $\boldsymbol{\alpha}$- cut**, denoted as $\tilde{A}^{\alpha+}$, i.e.

$$\tilde{A}^{\alpha+} = \{u \in U | \mu_{\tilde{A}}(u) > \alpha\}; \quad \alpha \in [0,1]. \tag{2.4.7}$$

All the α- cut sets form a family of crisp sets.
In this thesis, α**- cut** of a fuzzy number $\tilde{A}$, $0 \leq \alpha \leq 1$, denoted by $\tilde{A}^{\alpha}$, is defined as

$$\begin{cases} \{u \in R | \mu_{\tilde{A}}(u) \geq \alpha\}, & \text{if } \ 0 < \alpha \leq 1; \\ cl(supp\tilde{A}), & \alpha = 0. \end{cases} \tag{2.4.8}$$

where, cl denotes the closure and $supp$ denotes the support of a set.

2.4.4 Fuzzy Numbers and Arithmetic Operations

A fuzzy set for which there exists an element u in the universe with membership value unity (i.e. 1) or for which height $h(\tilde{A}) = 1$ is called **normal fuzzy set**. A fuzzy set for which no element u of the universe has its membership value equal to 1 (i.e. $h(\tilde{A}) < 1$) is called **subnormal fuzzy set**. Another important property of fuzzy sets is their convexity. Mathematically, a fuzzy set $\tilde{A}$ is called **convex fuzzy set** if for any elements $u_1, u_2, u_3 \in R$ such that $u_1 < u_2 < u_3$, $\mu_{\tilde{A}}(u_2) \geq \min[\mu_{\tilde{A}}(u_1), \mu_{\tilde{A}}(u_3)]$ is satisfied. It can also be stated as:

$$\mu_{\tilde{A}}(\lambda u_1 + (1-\lambda)u_2) \geq \min[\mu_{\tilde{A}}(u_1), \mu_{\tilde{A}}(u_2)] \tag{2.4.9}$$

for all $u_1, u_2 \in R$ and all $\lambda \in [0,1]$.
A fuzzy set $\tilde{A} = \{(u, \mu_{\tilde{A}}(u)) \mid u \in R\}$ defined on the real line R is said to be a **fuzzy number** if

(i) $\tilde{A}$ is normal.

(ii) All α-cuts of $\tilde{A}$ must be closed intervals for each $\alpha \in (0,1]$.

(iii) Support of $\tilde{A}$ i.e. $\{u \in R | \mu_{\tilde{A}}(u) > 0\}$ is bounded.

As all the α-cuts are assumed to be closed intervals therefore a fuzzy number governs the property of convexity. Another property followed by membership functions of a fuzzy number is that the membership functions can be represented as piecewise continuous functions.

A **Triangular Fuzzy Number** is defined by an ordered triplet (a_1, a_2, a_3) with $a_1 < a_2 < a_3$ in R representing, respectively, the lower value, the modal value, and the upper value of a triangular fuzzy membership function and it is called a Triangular Fuzzy Number, denoted as $\tilde{A} = (a_1, a_2, a_3)$, if its membership function $\mu_{\tilde{A}}(u)$ is defined by

$$\mu_{\tilde{A}}(u) = \begin{cases} \frac{u-a_1}{a_2-a_1}, & a_1 \leq u < a_2 \\ \frac{a_3-u}{a_3-a_2}, & a_2 \leq u < a_3 \\ 0, & \text{otherwise.} \end{cases} \tag{2.4.10}$$

An $\alpha-$ cut of a triangular fuzzy number $\tilde{A} = (a, b, c)$ is defined below and shown in Figure 2.2.

$$\tilde{A}^{\alpha} = [a^{\alpha}, c^{\alpha}] = [(b-a)\alpha + a, c - (c-b)\alpha] \tag{2.4.11}$$

Similarly, a **Trapezoidal fuzzy number** is defined with four parameters $a_1,\ a_2\ , a_3,\ a_4$ with $a_1 < a_2 < a_3 < a_4$ in R and is denoted by $\tilde{A} = (a_1, a_2, a_3, a_4)$ with membership function $\mu_{\tilde{A}}(u)$, defined as

$$\mu_{\tilde{A}}(u) = \begin{cases} \frac{u-a_1}{a_2-a_1}, & a_1 \leq u < a_2 \\ 1, & a_2 \leq u < a_3 \\ \frac{a_4-u}{a_4-a_3}, & a_3 \leq u < a_4 \\ 0, & \text{otherwise.} \end{cases} \tag{2.4.12}$$

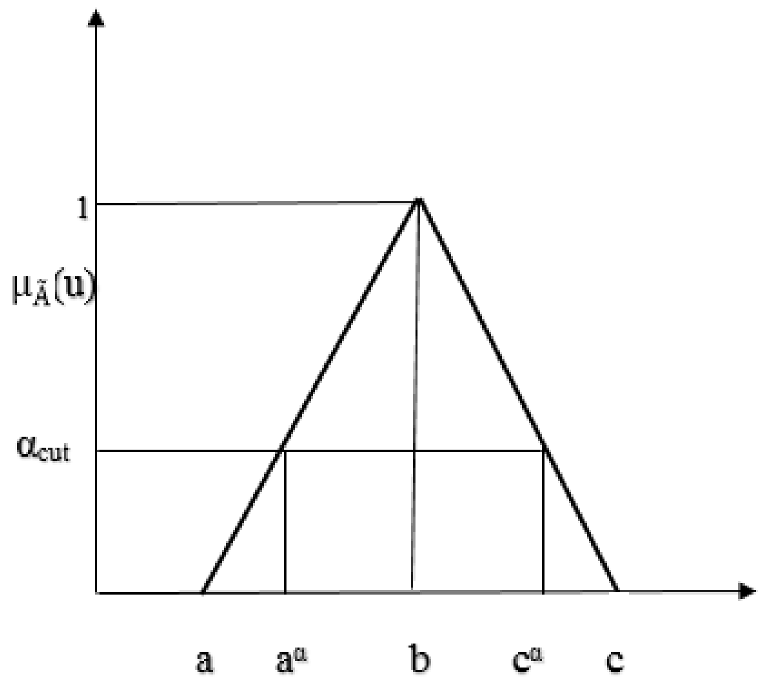


Figure 2.2: $\alpha-$ cut of a triangular fuzzy number $\tilde{A} = (a, b, c)$

Arithmetic approach on fuzzy numbers is based on interval arithmetic, let $*$ denotes any of the four basic arithmetic operations; addition, subtraction, multiplication and division then the fuzzy set $\tilde{A}_1 * \tilde{A}_2$ defined on R for two fuzzy numbers $\tilde{A}_1$ and $\tilde{A}_2$, is defined by defining its α-cut, $(\tilde{A}_1 * \tilde{A}_2)^\alpha$, as

$$(\tilde{A}_1 * \tilde{A}_2)^\alpha = \tilde{A}_1^\alpha * \tilde{A}_2^\alpha \tag{2.4.13}$$

for any $\alpha \in (0, 1]$. (When $* = /$, it is required that $0 \notin \tilde{A}_2^\alpha$, $\forall \alpha \in (0, 1]$) Since α-cut of any fuzzy number is a closed interval, so is $(\tilde{A}_1 * \tilde{A}_2)^\alpha$ for each $\alpha \in (0, 1]$ and consequently the fuzzy set $\tilde{A}_1 * \tilde{A}_2$ is also a fuzzy number. Let $\tilde{A}_1{}^\alpha = [a_1^\alpha,\ c_1^\alpha]$ and $\tilde{A}_2{}^\alpha = [a_2^\alpha,\ c_2^\alpha]$ be the two α-cuts of two fuzzy numbers $\tilde{A}_1$ and $\tilde{A}_2$ respectively. Then four arithmetic operations on these, in terms of their α-cuts, are defined as below:

1. **Addition:**

$$\tilde{A}_1^\alpha + \tilde{A}_2^\alpha = [a_1^\alpha + a_2^\alpha, c_1^\alpha + c_2^\alpha] \tag{2.4.14}$$

2. **Subtraction:**

$$\tilde{A}_1^\alpha - \tilde{A}_2^\alpha = [a_1^\alpha - c_2^\alpha, c_1^\alpha - a_2^\alpha] \tag{2.4.15}$$

3. **Multiplication:**

$$\tilde{A}_1^\alpha . \tilde{A}_2^\alpha = [\min(a_1^\alpha.a_2^\alpha, a_1^\alpha.c_2^\alpha, a_2^\alpha.c_1^\alpha, c_1^\alpha.c_2^\alpha), \max(a_1^\alpha.a_2^\alpha, a_1^\alpha.c_2^\alpha, a_2^\alpha.c_1^\alpha, c_1^\alpha.c_2^\alpha)] \tag{2.4.16}$$

4. **Divison:**

$$\tilde{A}_1^\alpha / \tilde{A}_2^\alpha = \left[\min\left(\frac{a_1^\alpha}{a_2^\alpha}, \frac{a_1^\alpha}{c_2^\alpha}, \frac{a_2^\alpha}{c_1^\alpha}, \frac{c_1^\alpha}{c_2^\alpha}\right), \ \max\left(\frac{a_1^\alpha}{a_2^\alpha}, \frac{a_1^\alpha}{c_2^\alpha}, \frac{a_2^\alpha}{c_1^\alpha}, \frac{c_1^\alpha}{c_2^\alpha}\right)\right] \tag{2.4.17}$$

2.5 Generalized Fuzzy Set Theory

After the introduction of fuzzy set theory, researchers have made further developments in this theory and extended it further to the one developed with generalized fuzzy set theory [46, 125] which has been applied to many fields such as risk analysis, reliability analysis, pattern recognition etc. [47, 50]. Many researchers have worked in the field of generalized fuzzy numbers. In definition of fuzzy number, the membership function $\mu_{\tilde{A}}(u)$ was restricted to the normal form, that is there exists at least one support point u_0 with value $\mu_{\tilde{A}}(u_0) = 1$. But in many cases, membership function is not restricted to the normal form, so fuzzy numbers have been further generalized and named as generalized fuzzy numbers.

A **Generalized fuzzy number** $\tilde{A}$, a fuzzy subset of the real line R, is said to be a generalized fuzzy number, if its membership function has following characteristics:

1. $\mu_{\tilde{A}} : R \to [0, w]$ is upper semi-continuous.
2. $\mu_{\tilde{A}}(u) = 0 \ \forall u \in (-\infty, a] \cup [d, \infty)$.
3. $\mu_{\tilde{A}}(u)$ is strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$.

4. $\mu_{\tilde{A}}(u) = w \ \forall u \in [b, c]$, where $0 \leq w \leq 1$, $a < b \leq c < d$.

where, a, b, c, d and w are real numbers. If $w = 1$, generalized fuzzy number is called fuzzy number.

A generalized fuzzy number $\tilde{A} = (a, b, c, d; w_{\tilde{A}})$ is called generalized trapezoidal fuzzy number, if it is described as any fuzzy subset of the real line R with membership function $\mu_{\tilde{A}}(x)$ is expressed as :

$$\mu_{\tilde{A}}(u) = \begin{cases} \frac{w_{\tilde{A}}(u-a)}{b-a}, & a \leq u < b \\ w_{\tilde{A}}, & b \leq u \leq c \\ \frac{w_{\tilde{A}}(u-d)}{c-d}, & c \leq u < d \\ 0, & \text{otherwise} \end{cases} \tag{2.5.1}$$

when $b = c$, the generalized trapezoidal fuzzy number is reduced to a generalized triangular fuzzy number and can be denoted by $\tilde{A} = (a, b, d; w_{\tilde{A}})$.

2.6 Intuitionistic Fuzzy Set Theory

Fuzzy set theory is unable in giving analytical information about the membership values of an element based on the evidences in favour and against of that element. To develop more precise knowledge and relevant information, Intuitionistic Fuzzy Set (IFS) theory was introduced by Atanassov [17, 18] in 1983. He pointed out that the degree of membership of an element should be measured in an interval form rather than the point value as in fuzzy set theory. Fuzzy set theory, based on the notion of membership functions gives an estimate of how likely an element belongs to specific set whereas Intuitionistic fuzzy set theory provides more information of that particular element by giving lower and upper bounds of its likelihood in that set. Intuitionistic fuzzy set theory deals uncertainty with hesitation.

Intuitionistic Fuzzy Sets: In IFS theory, the element u in the universe U is associated with its membership (called acceptance) and non membership (called rejection) value such that sum of these values always belongs to unit interval $[0,1]$. Mathematically, Let U be a universe of discourse. Then the IFS $\tilde{A}$ in U is stated as $\tilde{A} = \{< u, \mu_{\tilde{A}}(u), \nu_{\tilde{A}}(u) >| \ u \in U\}$ where the functions $\mu_{\tilde{A}} : U \rightarrow [0,1]$ and $\nu_{\tilde{A}} : U \rightarrow [0,1]$ are subjected to the condition $0 \leq \mu_{\tilde{A}}(u) + \nu_{\tilde{A}}(u) \leq 1 \ \forall u \in U$. The values $\mu_{\tilde{A}}(u)$ and $\nu_{\tilde{A}}(u)$ symbolize respectively the degree of acceptance and rejection of element u in set $\tilde{A}$.

Here, $\pi_{\tilde{A}}(u) = 1 - (\mu_{\tilde{A}}(u) + \nu_{\tilde{A}}(u))$ is the degree of hesitation or uncertainty of the element u in the set $\tilde{A}$. If $\mu_{\tilde{A}}(u)$ and $1 - \nu_{\tilde{A}}(u)$ are equal then there is zero degree of hesitation i.e. the degrees of membership and non-membership are exact and the theory reverts back to that of fuzzy sets. If $\mu_{\tilde{A}}(u)$ and $1 - \nu_{\tilde{A}}(u)$ are both 1 or 0, depending on the fact that whether $u \in \tilde{A}$ or $u \notin \tilde{A}$, then the knowledge about u is very exact and the theory reverts back to that of ordinary crisp sets.

(α, β)- Cut: An (α, β)- cut of IFS $\tilde{A}$, denoted as $\tilde{A}[\alpha, \beta]$, or here as $\langle \tilde{A}[\alpha]; \tilde{A}[\beta] \rangle$, is defined by $\tilde{A}[\alpha, \beta] = \tilde{A}[\alpha] \cap \tilde{A}[\beta]$, where $\tilde{A}[\alpha] = \{u \in U \mid \mu_{\tilde{A}}(u) \geq \alpha\}$ and $\tilde{A}[\beta] = \{u \in U \mid \nu_{\tilde{A}}(u) \leq \beta\}$ for $\alpha \in [0,1]$ and $\beta \in [0,1]$ such that $\alpha + \beta \leq 1$.
In this thesis, an (α, β)- cut of IFS $\tilde{A}$ is defined by $\tilde{A}[\alpha, \beta] = \tilde{A}[\alpha] \cap \tilde{A}[\beta]$, where $\tilde{A}[\alpha] = \{x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha\}$ and $\tilde{A}[\beta] = \{x \in X \mid \nu_{\tilde{A}}(x) \leq \beta\}$ for $\alpha \in (0,1]$ and $\beta \in [0,1)$ such that $\alpha + \beta \leq 1$.
Here, we separately define $\tilde{A}[\alpha]$, for $\alpha = 0$ as the closure of the union of all $\tilde{A}[\alpha]$'s for $\alpha \in (0,1]$. Similarly, $\tilde{A}[\beta]$, for $\beta = 1$ as the closure of the union of all $\tilde{A}[\beta]$'s for $\beta \in [0,1)$.

Intuitionistic Fuzzy Number (IFN): An intuitionistic fuzzy subset $\tilde{A} = \{< u, \mu_{\tilde{A}}(u), \nu_{\tilde{A}}(u) >| \ u \in R\}$ of the real line R is called Intuitionistic Fuzzy Number (IFN) if

1. $\tilde{A}$ is normal i.e. $\exists u_0 \in R : \mu_{\tilde{A}}(u_0) = 1$.

2. $\tilde{A}$ is convex IFS i.e. $\mu_{\tilde{A}}(\lambda u_1 + (1-\lambda)u_2) \geq \min(\mu_{\tilde{A}}(u_1), \mu_{\tilde{A}}(u_2))$ and $\nu_{\tilde{A}}(\lambda u_1 + (1-\lambda)u_2) \leq \max(\nu_{\tilde{A}}(u_1), \nu_{\tilde{A}}(u_2))$, $\forall u_1, u_2 \in R, \ 0 \leq \lambda \leq 1$.

3. $\mu_{\tilde{A}}(u)$ is upper semi-continuous and $\nu_{\tilde{A}}(u)$ is lower semi-continuous.

4. Set $\{u \in R | \nu_{\tilde{A}}(u) < 1\}$ is bounded.

Triangular Intuitionistic Fuzzy Number (TIFN): A TIFN $\tilde{A}$ with parameters $a_1' \leq a_1 < a_2 < a_3 \leq a_3'$ is a subset of IFS in R, denoted as $\tilde{A} = \langle (a_1, a_2, a_3); (a_1', a_2, a_3') \rangle$ with membership and non-membership functions defined respectively by

$$\mu_{\tilde{A}}(u) = \begin{cases} \frac{u-a_1}{a_2-a_1}, & a_1 \leq u < a_2 \\ \frac{a_3-u}{a_3-a_2}, & a_2 \leq u < a_3 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad \nu_{\tilde{A}}(u) = \begin{cases} \frac{a_2-u}{a_2-a_1'}, & a_1' \leq u < a_2 \\ \frac{u-a_2}{a_3'-a_2}, & a_2 \leq u < a_3' \\ 1, & \text{otherwise} \end{cases} \tag{2.6.1}$$

Trapezoidal Intuitionistic Fuzzy Number (TrIFN): A TrIFN $\tilde{A}$ with parameters $a_1' \leq a_1 < a_2 \leq a_3 < a_4 \leq a_4'$ is a subset of IFS in R and is denoted by $\tilde{A} = \langle (a_1, a_2, a_3, a_4); (a_1', a_2, a_3, a_4') \rangle$ with membership and non-membership functions defined as:

$$\mu_{\tilde{A}}(u) = \begin{cases} \frac{u-a_1}{a_2-a_1}, & a_1 \leq u < a_2 \\ 1, & a_2 \leq u < a_3 \\ \frac{a_4-u}{a_4-a_3}, & a_3 \leq u < a_4 \\ 0, & \text{otherwise} \end{cases} \qquad \nu_{\tilde{A}}(u) = \begin{cases} \frac{a_2-u}{a_2-a_1'}, & a_1' \leq u < a_2 \\ 0, & a_2 \leq u < a_3 \\ \frac{u-a_3}{a_4'-a_3}, & a_3 \leq u < a_4' \\ 1, & \text{otherwise} \end{cases} \tag{2.6.2}$$

2.7 Particle Swarm Optimization (PSO)

The PSO technique [75, 217] simulates the behavior of individuals in a group to maximize the species survival. Particle Swarm Optimization (PSO) is a population based stochastic optimization technique inspired by social behavior of bird flocks or fish schooling, developed by Eberhart and Kennedy in 1995 [75, 142]. Each particle flies in a direction that is based on its experience and that of the whole group. Individual particles move stochastically toward the position affected by the present velocity, previous best performance, and the best previous performance of the group. PSO uses a population of particles, wherein each particle represents a solution to the problem. These particles “fly” through a multidimensional search space, where the position of each particle is adjusted according to your own experience and that of its neighbours. The PSO approach is simple in concept and easily implemented with few coding lines, meaning that many can take advantage of it. Particle Swarm Optimization (PSO) has become a candidate for many optimization applications due to its high-performance and flexibility. Compared with other evolutionary algorithms, the main advantages of PSO are its robustness in controlling parameters and its high computational efficiency. As in GA, PSO exploits a population of potential solutions to explore the search space. Different from GA, in PSO, no operators motivated by natural evolution are applied to extract a new generation of solutions. PSO relies on the exchange of information between individuals called particles of population. Starting from a randomly distributed set of particles, algorithm tries to improve the solutions according to fitness function. In PSO, global sharing of information takes place and previous experience of all other companions during the search for promising regions of environment is taken into account. The improvisation is performed through moving the particles around the search space by means of a set of simple mathematical expressions. These mathematical expressions, in the most basic form, suggest the movement of each particle towards its own best experienced position and swarm’s best position so far along with some random disturbance.

PSO has had many proposed improvements and applications [220, 270]. Most of the modifications to PSO are to improve convergence and to increase the diversity of the swarm [141, 266]. PSO has been proved useful on diverse engineering design applications such as control systems [83], manufacturing [176, 262], robotics [218, 223], reliability [144, 286], communication networks [174], reliability-redundancy optimization [56, 101] and many others [65, 191].

2.7.1 PSO Algorithm

Particle swarm optimization (PSO) is a stochastic optimization technique which maintains a swarm of candidate solutions, referred to as particles. This algorithm starts by initializing a flock randomly over a search space where every bird is called a particle. These particles have certain position and fly with a certain velocity. Particles are flown through hyper-dimensional search space, with each particle being attracted towards the best solution found by the particle as neighborhood and the best solution found by the particle. At each iteration, each particle adjusts its velocity based on its momentum and impact of its best position (Pbest) as well as the best position of its neighbors (Gbest) and then evaluate new position that the particle is fly to.

Suppose the searching space dimension is D, total number of particles are N. Let x_k^i and v_k^i respectively be the position and velocity of i^{th} particle in the search space at k^{th} iteration then the position of this particle at $(k+1)^{th}$ iteration is updated through the following equation:

$$x_{k+1}^i = x_k^i + v_{k+1}^i. \tag{2.7.1}$$

Initial position x_0^i of i^{th} particle taken randomly as $x_0^i \sim U(x_{\min}^i, x_{\max}^i)$ from uniform distribution in the range $[x_{\min}^i, x_{\max}^i]$, where $x_{\min}^i$ and $x_{\max}^i$ are lower and upper bounds of the i^{th} variable respectively. Position of the updated velocity v_{k+1}^i at $(k+1)^{th}$ iteration is calculated through the following equation:

$$v_{k+1}^i = w.v_k^i + c_1.r_1.(p_k^i - x_k^i) + c_2.r_2.(p_k^g - x_k^i). \tag{2.7.2}$$

where, v_k^i is the velocity vector at k^{th} iteration, c_1 and c_2 are constants and have influence in the movement of particles, r_1 and r_2 are random variables with uniform distribution between 0 and 1. p_k^i represents the best position of the i^{th} particle, and

p_k^g corresponds the global best position of the swarm upto k^{th} iteration and w is inertia weight which shows the effect of previous velocity vector on the new velocity vector.

The parameter c_1, named as cognitive factor, characterizes the level of importance given by the particle to its previous positions whereas parameter c_2, called the social factor, signifies the level of importance that particle gives to the overall position. The pseudo code for PSO is given in Table 2.1 and diagram has been shown in Figure 2.3.

```
1:  For each particle
       Initialize particle position and velocity
    end
2:  Do
3:    Update the best known position (p_k^i) for each particle as:
        (a) Evaluate fitness value.
        (b) If the fitness value is better than the best fitness value (Pbest) in history.
        (c) Set current value as the new Pbest.
      end
4:    Select the particle with the best fitness value of all the particles as the Gbest (p_k^g).
5:    For each particle:
        (a) Evaluate particle velocity according equation (2.7.2).
        (b) Update particle position according to equation (2.7.1).
      end
6:  While minimum error criteria or maximum iterations is not attained.
```

Table 2.1: Pseudo Code of Particle Swarm Optimization

2.7.2 Parametric aspects of PSO

The basic PSO is governed by a number of control parameters, namely the dimension of the problem, number of particles, acceleration coefficient, inertia weight, number of iterations, and random value which scale the contribution of the cognitive and social components. In addition, if velocity clamping or constriction is used, the maximum velocity and constriction coefficient also influence the performance of the

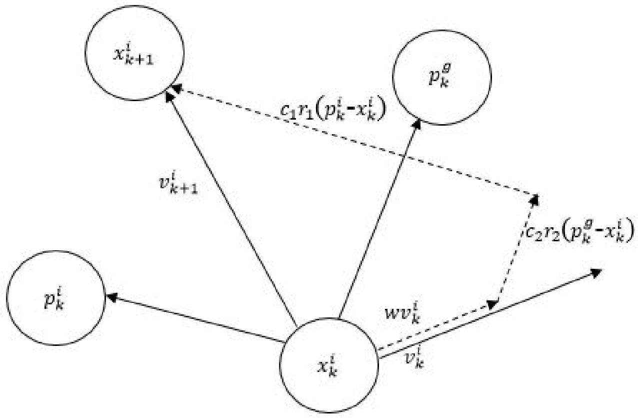


Figure 2.3: Movement of a particle in search space

PSO. Many researchers [212, 250, 271] worked for parameters' selection for many benchmark problems. This section discusses these parameters as below:

- **Swarm Size:** The swarm size is a critical parameter in the original PSO algorithm - too few particles will cause the algorithm to become stuck in local minima, while too many particles will slow down the algorithm. However, sometimes it is also the case that more particles may lead to fewer iterations to reach a good solution, compared to smaller swarms.

- **Number of Iterations:** The number of iterations is also very important factor. To reach a good solution, number of iterations is problem dependent. Too few iterations may terminate the search prematurely. A too large number of iterations have the consequence of unnecessary added computational complexity (provided that the number of iterations is the only stopping condition).

- **Inertia Weight:** The variable w, called the inertia weight plays an important role in the PSO convergence behavior since it is employed to control the exploration abilities of the swarm. It directly impacts the current velocity, which in turn is based on the previous history of velocities. Large inertia weights allow for wide velocity updates allowing to globally explore the design space, while small inertia values concentrate the velocity updates to nearby regions of the design space. Due to the importance of the inertia weight in controlling the global/local search behavior of the PSO algorithm, a dynamic improvement has proven useful by forcing an initial global search with a high inertia weight ($w \approx 1$) and subsequently narrowing down the algorithm exploration to feasible areas of the design space by decreasing its value toward local search values ($w < 0.5$). Larger values for w result in smoother, more gradual changes in direction through search space. Toward the end of the training run smaller inertia coefficients allow particles to settle into the minimum. A dynamic variation of inertia weight is proposed by Shi and Eberhart [250, 251] in which w is linearly decreasing with each algorithmic iteration as shown in Eq. (2.7.3).

$$w_{k+1} = w_{\max} - \left(\frac{w_{\max} - w_{\min}}{k_{\max}} k\right) \tag{2.7.3}$$

- **Acceleration Weight:** The acceleration coefficient, c_1 and c_2 called cognitive and social parameters respectively, together with the random vectors r_1 and r_2, control the stochastic influence of the cognitive and social components on the overall velocity of a particle. Acceleration coefficients c_1 and c_2 also control how far a particle will move in a single iteration. The constant c_1 expresses how much confidence a particle has in itself, while c_2 expresses how much confidence particle has in its neighbours. Typically these are both set to a value of 2, although assigning different values to c_1 and c_2 sometimes leads to improved performance [249, 291].

- **Velocity Clamping:** Velocity clamping [75, 76] was introduced by Eberhart and Kennedy it helps particles to stay within the boundary and to take reasonably step size in order to comb through the search space. Without this velocity clamping in the searching space the process will be prone to explode and particles' positions change rapidly. Maximum velocity controls the granularity of the search space by clamping velocities and creates a better balance between global exploration and local exploitation.

Chapter 3

Availability analysis through fuzzy differential equations and particle swarm optimization

This chapter formulates a new technique for behavior analysis of systems through fuzzy Kolmogorov's differential equations and Particle Swarm Optimization. For handling the uncertainty in data, differential equations have been formulated by Markov modeling of system in fuzzy environment. Firstly solution of these derived fuzzy Kolmogorov's differential equations has been found by Runge-Kutta fourth order method and thereafter the solution has been improved by Particle Swarm Optimization. Fuzzy availability is estimated in its transient as well as steady states. Sensitivity analysis has also been done to find the relative importance of a particular component of the system. Butter-oil processing plant as an industrial system has been studied as an application of the proposed approach.

3.1 Introduction

Reliability of a system is ability to execute a required function under operational and environmental conditions in stipulated period of time. Reliability is considered as one of the most important quality features of technical products and used to improve the productivity of system. Availability is also considered as a critical measure of behavior of a system, as mostly the systems are repairable ones. The main objective of reliability/avilability study is to provide information as a basis for making decisions. A system normally comprises a number of subsystems that are interconnected in such a manner that the system is able to execute a set of required functions. One of the importance of system reliability/availability is to discover the weakness in the system and quantify the influence of subsystems' failures. Reliability measures are used to estimate and order the impact of a particular subsystem within a system design. In realistic situations, the analyst has to derive stochastic models of the system. A mathematical model is essential in order to handle data and use statistical and mathematical methods to evaluate reliability/availability or risk parameters. For this, number of techniques are available in literature. Some of them are fault tree, petri nets, bayesian and Markovian approach [94, 171, 224, 239, 282] etc.

To estimate and improve reliability/availability of a system from its mathematical model, we need input data. Conventional reliability theory is based on probabilistic approach, but results obtained from probabilistic approach do not always provide helpful information. Data, we collect, are either from past history or as observed by experts. Usually these data are incomplete, vague or uncertain. These types of data usually do not provide certain information. Thus probabilistic approach to the traditional reliability is insufficient to tackle uncertainties in data. To handle these types of difficulties, methodologies based on fuzzy set theory, proposed by Zadeh [289] came into existence. A lot of work has been done in fuzzy set theory [72, 294]. Applications of fuzzy set theory in different fields have been found by many researchers [5, 216, 264].

Importance of differential equations is well known. Fuzzy set theory has been successfully implemented in differential equations also. In 1982, Dubois and Parade [71] have discussed differentiation in fuzzy environment. Kaleva [136, 137] has discussed existence and uniqueness of a solution of fuzzy differential equations. Buckley and

Feuring [36, 37] have discussed approaches for the solution of fuzzy initial value problem for n^{th} order differential equations. Bede et al. [27] have discussed first order linear fuzzy differential equations under generalized differentiability. The topic of fuzzy differential equations has been growing rapidly in recent years and a lot of work has been done by several authors [9, 14, 64, 93, 228].
The concept of fuzzy set theory has been implemented in the estimation of reliability/availability of the system in different approaches by several researchers [90–92, 147, 254]. For instance, Sharma et al.[244] have discussed fuzzy modeling of system behavior for risk and reliability analysis in 2008. Park et al. [214] have discussed the probability of failure in rock slopes through fuzzy set theory in 2012. Garg et al. [98] have presented a technique for examining the reliability using soft computing based techniques. Yazdi et al. [285] have discussed failure probability analysis by employing fuzzy fault tree analysis. In 2018, Garg [95] has discussed analysis of industrial systems using different fuzzy membership functions.
Out of the many discussed mathematical techniques for evaluating reliability, Markov model is a commonly used and widespread technique. In 2005, Gupta et al. [113] has studied the availability of butter-oil processing plant through crisp approach. Kumar and Lata [154] in 2012 have discussed reliability evaluation of condensate system using fuzzy Markov model. In 2013, Lata and Kumar [173] have discussed the reliability through solving the fuzzy differential equations. In 2015, Garg [91] has discussed the approach for reliability analysis by solving the fuzzy differential equations through numerical techniques. Results provided by existing approaches [91, 154] deal with uncertainties but do not optimize availability through Markov process. In this chapter we extend the idea of finding availability by solving fuzzy differential equations through the optimization model. Numerical solution of availability has been optimized. Out of many meta-heuristic techniques available in literature, authors have used particle swarm optimization. Among all, PSO is powerful and faster for many benchmark functions[126, 277]. Also PSO is simple and

easy to implement since there are not many parameters to be adjusted. A novel approach has been discussed here for reliability/availability evaluation containing data uncertainty through Markov model and particle swarm optimization.

In this approach, first a system has been mathematically modelled through Markov process. A set of differential equations with fuzzy parameters and fuzzy initial conditions has been composed to handle uncertainty. Here solution of thus obtained fuzzy differential equations has been found by α-cut method with the help of Runge-Kutta fourth order method and thereafter solution of the equations has been improved with the help of particle swarm optimization. Obtained solution has been compared with the existing methods. Apart from that, impacts of components on the system have been analyzed by varying their repair and failure rates individually and simultaneously. Based on this analysis, system analyst may analyze system performance and can plan suitable maintenance.

This chapter is divided into following sections. Proposed approach for reliability/availability evaluation has been described in Section 3.2. In Section 3.3, a case study of an industrial system as an application of the proposed approach has been taken. Results for performance analysis of the industrial system have been given in Section 3.4. Conclusion of the chapter is given in Section 3.5.

3.2 Proposed Approach

Availability Av is the probability that the system is operating satisfactorily at time t. The following basic assumptions are made in the proposed approach.

(i) Repair and failure rates are independent to each other.

(ii) Probability of two or more components failed or repaired at the same time is zero.

(iii) Repaired unit is assumed as good as new and repair is done according to first in, first out strategy.

(iv) At any time, system is either in working or in failed state.

In order to evaluate the availability of the system by Markov process, having uncertainty in parameters, following steps have been taken:

Step 1: [***Derivation of Kolmogorov differential equations through Markov process*** [84, 225]]: Consider the markov process $\{Y(t); t \geq 0\}$ with state space $\Psi = \{0, 1, 2, ...r\}$ and transition probabilities $P_{ij}(t)$. The transition probabilities of Markov process are:

$$P_{ij}(t) = Pr\{Y(t) = j|Y(0) = i\} \text{ for all } i, j \in \Psi.$$

Then by first considering a transition from state i to k in $(0, t)$ and then a transition from k to j in $(t, t + \Delta t)$, Chapman-Kolmogorov equations give $P_{ij}(t + \Delta t)$ as

$$P_{ij}(t+\Delta t) = \sum_{k=0}^{r} P_{ik}(t).P_{kj}(\Delta t) = \sum_{k=0, k\neq j}^{r} P_{ik}(t).P_{kj}(\Delta t) + P_{jj}(\Delta t)P_{ij}(t).$$

or

$$P_{ij}(t+\Delta t) - P_{ij}(t) = \sum_{k=0, k\neq j}^{r} P_{ik}(t).P_{kj}(\Delta t) - [1 - P_{jj}(\Delta t)]P_{ij}(t).$$

Dividing by Δt and taking limit as $\Delta t \to 0$, one gets

$$\lim_{\Delta t\to 0} \frac{P_{ij}(t+\Delta t) - P_{ij}(t)}{\Delta t} = \lim_{\Delta t\to 0} \sum_{k=0, k\neq j}^{r} P_{ik}(t).\frac{P_{kj}(\Delta t)}{\Delta t} - \frac{[1 - P_{jj}(\Delta t)]}{\Delta t}P_{ij}(t).$$

As the summation is finite, it leads to

$$\frac{dP_{ij}(t)}{dt} = \sum_{k=0, k\neq j}^{r} q_{kj}P_{ik}(t) - v_j P_{ij}(t), \tag{3.2.1}$$

where v_j is the rate at which process leaves state j and q_{kj} is the transition rate from state k to state j.

This gives rise to a set of first order linear differential equations through Markov process. Here, availability Av of a system is sum of the probabilities of working states i.e. $Av = \sum_s P_{is}(t)$, where s represents working state.

Step 2: [***Formulation of Fuzzy differential equations (FDEs)***]: In practical situations, when equation represents a physical situation, the values of coefficients may depend on the various sources, and cannot be obtained accurately. In most of the cases, the information collected from various sources are based on the past behavior of the system and consequently do not necessarily identify the performance of the system. To deal with such type of uncertainties in the coefficients and initial values, the corresponding differential equation becomes fuzzy differential equation. For solution of such differential equations, consider a general set of linear first order fuzzy differential equation in fuzzy function

$$\tilde{Z}(t) = (\tilde{z}_1(t), \tilde{z}_2(t), ..., \tilde{z}_n(t))^T,$$

as

$$\frac{d\tilde{Z}(t)}{dt} = \tilde{C}\tilde{Z}(t)+h(t), \text{ with initial conditions } \tilde{Z}(0) = (\tilde{\phi}_1, \tilde{\phi}_2, ..., \tilde{\phi}_n)^T, \quad (3.2.2)$$

where

(i) $\tilde{\phi}_i$'s are fuzzy numbers.

(ii) $\tilde{C} = [\tilde{c}_{ij}]$ is an $n \times n$ matrix of fuzzy numbers.

(iii) $(h(t))^T = (h_1(t), h_2(t), ..., h_n(t))$, with all the $h_i(t)$'s for $i = 1, 2..., n$ as continuous functions on the interval I.

(iv) all the $\tilde{z}_i(t)$'s for $i = 1, 2, ..., n$ are fuzzy subsets of real numbers for $t \in I$.

Step 3: [***Computation of $\boldsymbol{\alpha}$-cuts***]: Let $(\tilde{z}_i(t))^\alpha$ be closed and bounded intervals for all t and i, defined as

$$(\tilde{z}_i(t))^\alpha = [(\tilde{z}_i(t))^\alpha_{(L)}, (\tilde{z}_i(t)^\alpha_{(R)}], \quad (3.2.3)$$

where $(\tilde{z}_i(t))^\alpha_{(L)}$ and $(\tilde{z}_i(t)^\alpha_{(R)}$ are functions of t and α. Assume that all $(\tilde{z}_i)^\alpha_{(L)}$ and $(\tilde{z}_i)^\alpha_{(R)}$ are continuously differentiable functions of t for all $\alpha \in (0, 1)$, $1 \leq i \leq n$.

Step 4: [***Substitution of $\boldsymbol{\alpha}$-cuts in FDEs***]: Now substitute the α-cuts of $\tilde{Z}(t)$ into Eq. (3.2.2). Then using the concepts of interval arithmetic, system of differential equations (3.2.2) reduces to following system of differential equations:

$$(\tilde{z}_i'(t))^{\alpha}_{(L)} = \sum_{j=1}^{n} b_{ij}x_j + h_i(t), \tag{3.2.4}$$

$$(\tilde{z}_i'(t))^{\alpha}_{(R)} = \sum_{j=1}^{n} d_{ij}x_j + h_i(t), \tag{3.2.5}$$

where,

$$b_{ij}x_j = \min\{(\tilde{c}_{ij})^{\alpha}_{(L)}(\tilde{z}_j)^{\alpha}_{(L)}, (\tilde{c}_{ij})^{\alpha}_{(L)}(\tilde{z}_j)^{\alpha}_{(R)}, (\tilde{c}_{ij})^{\alpha}_{(R)}(\tilde{z}_j)^{\alpha}_{(L)}, (\tilde{c}_{ij})^{\alpha}_{(R)}(\tilde{z}_j)^{\alpha}_{(R)}\}.$$

and

$$d_{ij}x_j = \max\{(\tilde{c}_{ij})^{\alpha}_{(L)}(\tilde{z}_j)^{\alpha}_{(L)}, (\tilde{c}_{ij})^{\alpha}_{(L)}(\tilde{z}_j)^{\alpha}_{(R)}, (\tilde{c}_{ij})^{\alpha}_{(R)}(\tilde{z}_j)^{\alpha}_{(L)}, (\tilde{c}_{ij})^{\alpha}_{(R)}(\tilde{z}_j)^{\alpha}_{(R)}\}.$$

with the initial conditions

$$(\tilde{z}_i(0))^{\alpha}_{(L)} = (\tilde{\phi})^{\alpha}_{(L)} \quad \text{and} \quad (\tilde{z}_i(0))^{\alpha}_{(R)} = (\tilde{\phi})^{\alpha}_{(R)},$$

for $1 \leq i \leq n$.

Step 5: [***Formulation of Optimization Problem***]: With the help of above equations, an optimization problem is developed by using $\tilde{c}_{ij}$'s and $\tilde{z}_j$'s for α-cut level. In the form of bounded interval, input data at α-cut level is substituted in the expression. Lower and upper boundary values of these equations are obtained at α-cut level by solving the following optimization problem.

$$\min / \max \; Av(\tilde{c}_{ij}, \tilde{z}_j),$$

$$\text{subject to} \quad \mu_{\tilde{c}_{ij}} \geq \alpha,$$
$$\mu_{\tilde{z}_j} \geq \alpha,$$
$$0 \leq \alpha \leq 1, \tag{3.2.6}$$

where, Av is the fitness function, obtained by solving the differential equations (3.2.4) and (3.2.5) for availability by using RK-IV method. The obtained maximum and minimum value of Av denoted by $Av_{\max}$ and $Av_{\min}$ respectively corresponding to α-cut level satisfy

$$\mu_{\tilde{Av}}(Av_{min}) = \mu_{\tilde{Av}}(Av_{max}) = \alpha. \tag{3.2.7}$$

There are many efficient techniques available for finding the global or near global solution. Out which PSO is one of the most popular evolutionary algorithm, briefly described in the Section 2.7 of Chapter 2. Optimization problem 3.2 has been solved by Particle swarm optimization technique.

Step 6: [***Solution of Availability***]: We say $\tilde{Av}(t)$ is a required fuzzy solution for all t if the obtained values of $(\tilde{Av}(t))^{\alpha}_{(L)}$ and $(\tilde{Av}(t))^{\alpha}_{(R)}$ define the α-cuts $[(\tilde{Av}(t))^{\alpha}_{(L)}, (\tilde{Av}(t))^{\alpha}_{(R)}]$ of fuzzy numbers. Thus we can say that $\tilde{Av}(t)$ is a fuzzy solution of Eq. (3.2.1) if following conditions are met out.

1. $\frac{\partial(\tilde{Av})^{\alpha}_{(L)}}{\partial\alpha} \geq 0$ and $\frac{\partial(\tilde{Av})^{\alpha}_{(R)}}{\partial\alpha} \leq 0$,
   i.e. $(\tilde{Av})^{\alpha}_{(L)}$ increases while $(\tilde{Av})^{\alpha}_{(R)}$ decreases as α increases.
2. $(\tilde{Av})^{\alpha}_{(L)} \leq (\tilde{Av})^{\alpha}_{(R)}$ for $\alpha = 1$,

for all $\alpha \in [0, 1]$ and $t \in I$.

3.3 Case Study

To illustrate the suggested approach, butter-oil manufacturing plant [113], as a repairable industrial system has been considered. Figure 3.1 gives the flow diagram

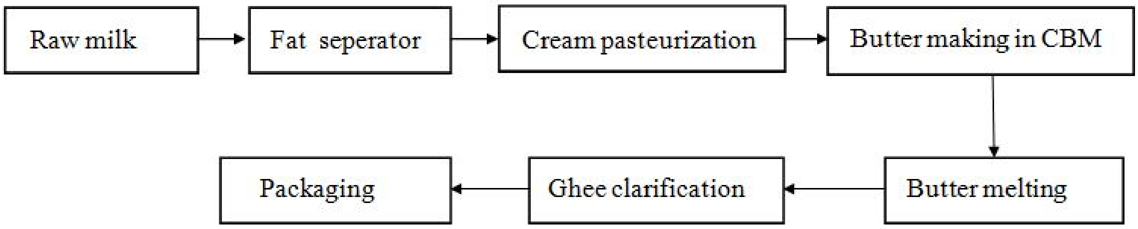


Figure 3.1: Flow diagram of Butter-oil processing plant

of the process of butter-oil processing plant. Concise description of the system is given here.

3.3.1 System Description

Butter-oil processing plant consists of eight sub-systems out of which pump and chiller are supported by standby units with perfect switch over devices. It has been considered that these two sub-systems never fail. The mathematical modelling has been taken out by the consideration of remaining six sub-systems.

1. **Subsystem A (Separator):** This subsystem of plant consists of motors, bearings and high speed gearbox in series. It works on the principle of centrifugal force. Fats from milk are taken away in the form of cream and the retained skimmed milk is used for preparation of milk powder.

2. **Subsystem B (Pasteuriser):** Pasteuriser includes a motor and bearings connected in series. This subsystem is used to destroy pathogenic organisms, to desirable organisms and to inactivate the enzymes present, and to remove volatile flavours by heating the cream up to 80^oC. Tanning substance present in cream is also removed by this subsystem. It fails through reduced state B^1 only.

3. **Subsystem C (Continuous Butter Making):** The subsystem includes gearbox, motor and bearings in series. First butter granules are obtained with

the help of this machine. The homogeneous butter is taken out from machine into butter trolleys and shifted to melting vats.

4. **Subsystem D (Melting Vats):**This subsystem consists of monoblock pumps, motors and bearing in series. In this subsystem, butter is melted at about 107^oC very gently so that water evaporates from butter.

5. **Subsystem E (Butter-Oil Clarifier):** This subsystem consists of gearbox and motor in series. In this subsystem, fine particles of butter-oil are separated from butter-oil by settling it for few hours. For storage of butter-oil, it is cooled to a temperature of $28^o - 30^oC$.

6. **Subsystem F (Packaging):** In this subsystem F, packets of processed butter-oil are produced using pouch filling machine. It is a fill, flow and seal automatic machine. This subsystem is composed of circuit board and pneumatic cylinder.

Input data in terms of failure and repair rates is given in Table 3.1.

3.3.2 Notations

In this section, notations that are used for performance analysis of the system are given below.

○	Represents working state of the system.
⬭	Represents reduced state of the system.
▭	Represents failed state of the system.
A, B, C, D, E, F	Working states of the subsystem.
a, b, c, d, e, f	Failed states of the subsystem.
B^1	Represents the reduced state of subsystem B.
$P_1(t)$	Indicates the probability of the system working in full capacity at time t.
$P_2(t)$	Indicates the probability of the system in reduced state at time t.
$P_3(t)$ to $P_{13}(t)$	Indicate the probabilities of the system in failed state at time t.
$\lambda_i, i = 1, 2..., 7$	Represent failure rates of the subsystems A, C, D, E, F, B and B^1 respectively.
$\mu_i, i = 1, 2..., 6$	Represent repair rates of the subsystems A, C, D, E, F and B respectively.
Av	System availability

The transition diagram of butter-oil processing plant is given here in Figure 3.2.

Component	Separator (A)	Pasteuriser (B)	Continuous Butter Making (C)	Melting Vats (D)	Butter-oil clarifier (E)	Packaging (F)
Failure rate	0.008	0.01111*, 0.0055**	0.0054	0.0027	0.0009	0.0027
Repair rate	0.41	6.00	0.40	0.70	0.30	0.65

*corresponding to failure rate of Pasteuriser (B) and **corresponding to failure rate of Pasteuriser (B) in reduced state.

Table 3.1: Input data for the system

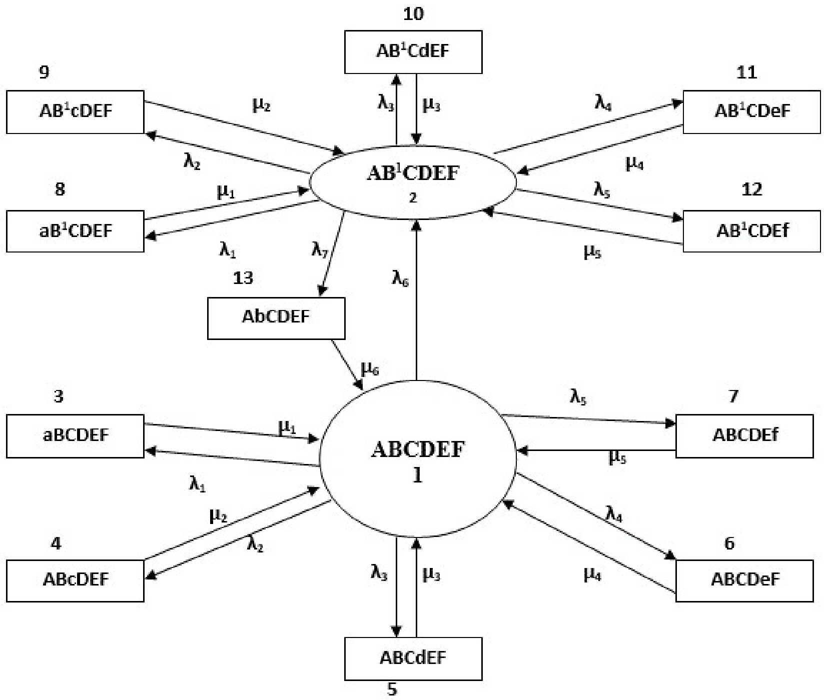


Figure 3.2: Transition diagram of Butter-oil processing plant

3.3.3 Mathematical Formulation

Applying the concepts of Markov modeling and probability theory as described in Step-1 and Step-2 of proposed approach, the transition diagram (Figure 3.2) of this system leads to the formulation of following fuzzy differential equations:

$$
\begin{aligned}
\frac{d\tilde{P}_1(t)}{dt} \oplus \tilde{\delta}_1 \tilde{P}_1(t) &= \sum_{j=1}^{5} \tilde{\mu}_j \tilde{P}_{j+2}(t) \oplus \tilde{\mu}_6 \tilde{P}_{13}(t) \\
\frac{d\tilde{P}_2(t)}{dt} \oplus \tilde{\delta}_2 \tilde{P}_2(t) &= \sum_{j=1}^{5} \tilde{\mu}_j \tilde{P}_{j+7}(t) \oplus \tilde{\lambda}_6 \tilde{P}_1(t) \\
\frac{d\tilde{P}_{i+2}(t)}{dt} \oplus \tilde{\mu}_i \tilde{P}_{i+2}(t) &= \tilde{\lambda}_i \tilde{P}_1(t), \qquad i = 1, 2, ..., 5 \\
\frac{d\tilde{P}_{i+7}(t)}{dt} \oplus \tilde{\mu}_i \tilde{P}_{i+7}(t) &= \tilde{\lambda}_i \tilde{P}_2(t), \qquad i = 1, 2, ..., 5 \\
\frac{d\tilde{P}_{13}(t)}{dt} \oplus \tilde{\mu}_6 \tilde{P}_{13}(t) &= \tilde{\lambda}_7 \tilde{P}_2(t)
\end{aligned} \tag{3.3.1}
$$

with $\tilde{\delta}_1 = \sum_{j=1}^{6} \tilde{\lambda}_j$ and $\tilde{\delta}_2 = \sum_{j=1}^{5} \tilde{\lambda}_j \oplus \tilde{\lambda}_7$

and the given initial conditions as:

$\tilde{P}_1(0) = 1$ and $\tilde{P}_j(0) = 0$ for j=2 to 13.

Availability function $\tilde{A}v(t)$ of the system in terms of $\tilde{P}_1(t)$ and $\tilde{P}_2(t)$ can be obtained by

$$\tilde{A}v(t) = \tilde{P}_1(t) \oplus \tilde{P}_2(t). \tag{3.3.2}$$

3.3.4 Steady State Analysis

For long term availability of the system, steady state probabilities of the system are obtained by applying following limitations on probabilities:

$$\frac{d}{dt} \to 0 \text{ as } t \to \infty$$

In this case study, following system of equations are obtained by imposing the above restrictions.

$$P_2 = \frac{\lambda_6}{\lambda_7} P_1; \quad P_3 = \frac{\lambda_1}{\mu_1} P_1; \quad P_4 = \frac{\lambda_2}{\mu_2} P_1; \quad P_5 = \frac{\lambda_3}{\mu_3} P_1; \quad P_6 = \frac{\lambda_4}{\mu_4} P_1; \quad P_7 = \frac{\lambda_5}{\mu_5} P_1;$$

$$P_8 = \left(\frac{\lambda_1}{\mu_1}\right)\left(\frac{\lambda_6}{\lambda_7}\right) P_1; \quad P_9 = \left(\frac{\lambda_2}{\mu_2}\right)\left(\frac{\lambda_6}{\lambda_7}\right) P_1; \quad P_{10} = \left(\frac{\lambda_3}{\mu_3}\right)\left(\frac{\lambda_6}{\lambda_7}\right) P_1;$$

$$P_{11} = \left(\frac{\lambda_4}{\mu_4}\right)\left(\frac{\lambda_6}{\lambda_7}\right)P_1; \quad P_{12} = \left(\frac{\lambda_5}{\mu_5}\right)\left(\frac{\lambda_6}{\lambda_7}\right)P_1; \quad P_{13} = \left(\frac{\lambda_6}{\mu_6}\right)P_1.$$

Substituting these values of P_1 to P_{13} in the normalizing condition $\sum_{i=1}^{13} P_i = 1$, Steady state availability becomes:

$$Av = P_1 + P_2 = \left[\left(1 + \frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} + \frac{\lambda_3}{\mu_3} + \frac{\lambda_4}{\mu_4} + \frac{\lambda_5}{\mu_5}\right) + \left\{\mu_6\left(\frac{1}{\lambda_6} + \frac{1}{\lambda_7}\right)\right\}^{-1}\right]^{-1}$$

3.4 Results and Discussion

System availability for Butter-oil processing plant obtained through proposed approach in terms of transient and steady state has been discussed in this section.

3.4.1 Transient State

The proposed approach has been performed in MATLAB (Mathworks). In the present analysis, 20 independent runs have been made that imply 20 different random initial solutions with swarm size equal to 25×(no. of variables). In this case, acceleration coefficient parameters c_1 and c_2 are taken as c_1=c_2=2 with inertia weight w is explained as $w = w_{max} - (w_{max} - w_{min}) * iter/iter_{max}$, here $w_{max} = 0.9$ and $w_{min} = 0.4$ are maximum and minimum values of inertial weight respectively and $iter_{max}$ indicates the maximum generation number (=100). The termination criterion has been set either to relative error equal to 10^{-6} or maximum number of generations, whichever is obtained first.

Fuzzy system availability has been calculated by solving the set of fuzzy differential equations (3.3.1) for mission time t=100 days for $\alpha = 0, 0.1, 0.2, ..., 1$ with $\pm 15\%$, $\pm 35\%$ and $\pm 50\%$ uncertainties. It has been observed that for $\pm 15\%$ uncertainty, fuzzy system availability lies in the intervals $[0.9573611, 0.9574162]$ and $[0.9431705, 0.9681699]$ by existing and proposed approaches respectively. Results for system availability for different α-cuts have been summarized in tabular form

(Table 3.2 and 3.3) and in graphs (Figure 3.3), obtained by proposed and existing approaches. Comparison has also been shown through graphs. Following results are concluded by using the proposed method.

	±15% uncertainty					
	Gupta et al. approach [113]		Garg approach [91]		Proposed approach	
$\alpha \downarrow$	$(\tilde{Av}(t))^{\alpha}_{(L)}$	$(\tilde{Av}(t))^{\alpha}_{(R)}$	$(\tilde{Av}(t))^{\alpha}_{(L)}$	$(\tilde{Av}(t))^{\alpha}_{(R)}$	$(\tilde{Av}(t))^{\alpha}_{(L)}$	$(\tilde{Av}(t))^{\alpha}_{(R)}$
0	0.9573854	0.9573854	0.9573611	0.9574162	0.9431705	0.9681699
0.1	0.9573854	0.9573854	0.9573633	0.9574128	0.9447922	0.9672100
0.2	0.9573854	0.9573854	0.9573655	0.9574095	0.9463640	0.9662264
0.3	0.9573854	0.9573854	0.9573679	0.9574062	0.9478880	0.9651708
0.4	0.9573854	0.9573854	0.9573702	0.9574030	0.9493665	0.9641844
0.5	0.9573854	0.9573854	0.9573725	0.9573999	0.9508474	0.9631240
0.6	0.9573854	0.9573854	0.9573750	0.9573969	0.9521947	0.9620068
0.7	0.9573854	0.9573854	0.9573775	0.9573939	0.9535481	0.9609193
0.8	0.9573854	0.9573854	0.9573801	0.9573910	0.9548803	0.9597728
0.9	0.9573854	0.9573854	0.9573827	0.9573881	0.9561419	0.9585872
1.0	0.9573854	0.9573854	0.9573854	0.9573854	0.9573854	0.9573854

Table 3.2: Availability of the butter-oil processing plant at t = 100 days with ±15% uncertainty

(i) Results provided by the existing approach of Gupta et al. [113] do not deal with uncertainty and imprecise information. Proposed approach deals with uncertainty and provides more realistic results. System availability for mission time $t = 100$ days is 0.9573854 by existing approach while keeping ±15% uncertainty in view in the proposed approach it is $(0.9431705, 0.9573854, 0.9681699)$. For instance, it can be seen (from Table 3.2) for level of uncertainty $\alpha = 0.5$, availability lies in the interval $[0.9508474,\ 0.9631240]$ which shows the possibility to optimize the availability from 0.9573854 to 0.9631240.

(ii) For different uncertainty levels ±35% and ±50%, results have been computed and depicted in Table 3.3. For instance, for ±50% uncertainty in data, system availability lies in $[0.8820623, 0.9854269]$ and for ±35% uncertainty in data, system availability lies in $[0.9152954, 0.9790589]$.

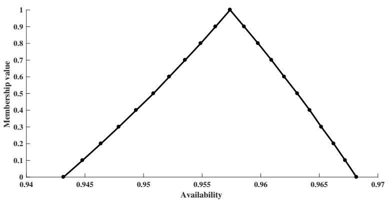
	±35% uncertainty				±50% uncertainty			
	Garg approach [91]		Proposed approach		Garg approach [91]		Proposed approach	
$\alpha \downarrow$	$(\tilde{Av}(t))^{\alpha}_{(L)}$	$(\tilde{Av}(t))^{\alpha}_{(R)}$	$(\tilde{Av}(t))^{\alpha}_{(L)}$	$(\tilde{Av}(t))^{\alpha}_{(R)}$	$(\tilde{Av}(t))^{\alpha}_{(L)}$	$(\tilde{Av}(t))^{\alpha}_{(R)}$	$(\tilde{Av}(t))^{\alpha}_{(L)}$	$(\tilde{Av}(t))^{\alpha}_{(R)}$
0	0.9573366	0.9574706	0.9152954	0.9790589	0.9573227	0.9575243	0.8820623	0.9854269
0.1	0.9573403	0.9574598	0.9212073	0.9773771	0.9573269	0.957505	0.8948612	0.9834425
0.2	0.9573443	0.9574496	0.9266106	0.9756092	0.9573316	0.9574871	0.9058148	0.9813244
0.3	0.9573485	0.9574399	0.9315681	0.9736623	0.9573366	0.9574706	0.9152954	0.9790589
0.4	0.9573529	0.9574308	0.9361329	0.9717881	0.957342	0.9574553	0.9235813	0.9766303
0.5	0.9573576	0.9574221	0.9403499	0.9697191	0.9573479	0.9574412	0.9308851	0.9740205
0.6	0.9573626	0.9574139	0.9442573	0.9675326	0.9573542	0.9574282	0.9373715	0.9712084
0.7	0.9573678	0.9574062	0.9478879	0.9652182	0.9573611	0.9574162	0.9431705	0.9681699
0.8	0.9573734	0.9573989	0.9512703	0.9627644	0.9573686	0.9574051	0.9483858	0.9648765
0.9	0.9573792	0.957392	0.954429	0.9601584	0.9573767	0.9573949	0.95310124	0.9612948
1	0.9573854	0.9573854	0.9573854	0.9573854	0.9573854	0.9573854	0.9573854	0.9573854

Table 3.3: Availability of the butter-oil processing plant at t = 100 days with different uncertainties

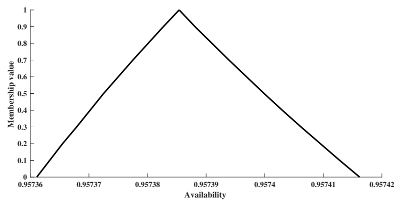
(iii) For ±15% uncertainty in data, results obtained by existing approach of Garg [91] deal with uncertainty and are obtained by solving fuzzy differential equations by RK-IV method. Fuzzy system availability is (0.9574162,0.9573854, 0.9573611) by existing approach and $(0.9431705, 0.9573854, 0.9681699)$ by proposed approach. It can be seen (from Table 3.2) that proposed approach has an improvement on the availability by improving the solutions through particle swarm optimization. For instance, for $\alpha = 0.7$ availability lies in the interval [0.9573775, 0.9573939] and [0.9535481, 0.9609193] by existing and proposed approaches, whose spread increases by 0.40% and 0.37% in left and right cut respectively, which shows the possibility to optimize the availability. For $\alpha = 0.5$, it is observed that right cut of the availability increases 0.60% from 0.9573999 to 0.9631240 by using proposed approach instead of existing one.

Based on these obtained results, system analyst may improve his target goals, rather from traditional analysis. In case, if system analyst wants to optimize availability of system with ±15% uncertainty in data, then new target would be greater than 0.9574162, rather it will be 0.9681699 which comes from proposed approach.

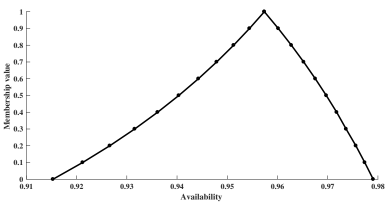
(iv) For $\pm35\%$ uncertainty in data, fuzzy system availability is $(0.9573366, 0.9573854, 0.9574706)$ by existing approach and $(0.9152954, 0.9573854, 0.9790589)$ by proposed approach. For $\pm50\%$ uncertainty in data, fuzzy system availability is $(0.9573227, 0.9573854, 0.9575243)$ by existing approach and $(0.8820623, 0.957\text{-}3854, 0.9854269)$ by proposed approach. It may be observed from Table 3.3 that proposed approach provides an improvement in the availability by improving the solutions through PSO.



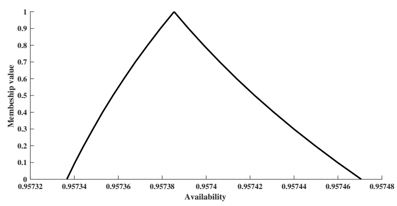
(a) $\pm15\%$ uncertainty by proposed approach



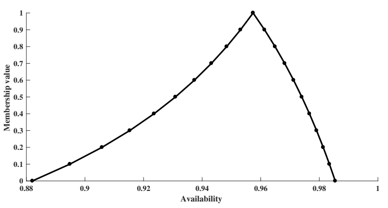
(b) $\pm15\%$ uncertainty by existed approach



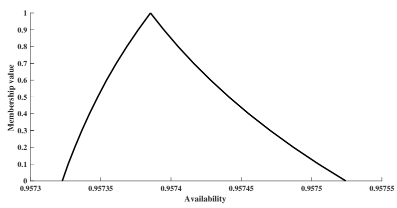
(c) $\pm35\%$ uncertainty by proposed approach



(d) $\pm35\%$ uncertainty by existed approach



(e) $\pm50\%$ uncertainty by proposed approach



(f) $\pm50\%$ uncertainty by proposed approach

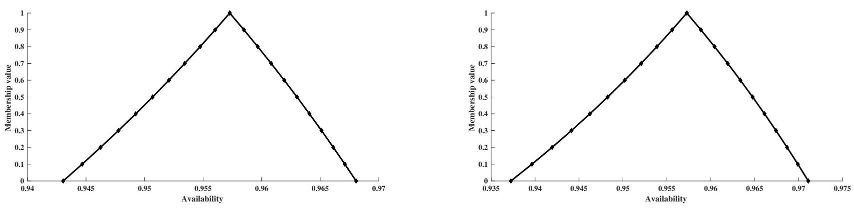
Figure 3.3: Transient state availability for $t=$ 100 days with different uncertainties in data

3.4.2 Steady state

To find the long term availability of the system, fuzzy steady state availability has been found at uncertainty levels $\pm15\%$ and $\pm20\%$. Corresponding to different α-cuts ($\alpha = 0, 0.1, 0.2, ..., 1$), fuzzy steady state availability has been discussed in Table 3.4 and graphs (Figure 3.4) at uncertainty levels $\pm15\%$ and $\pm20\%$. From these results, it has been seen that steady state availability lies in the intervals $[0.9430492, 0.9680618]$ and $[0.9372473, 0.9711025]$ for $\pm15\%$ and $\pm20\%$ uncertainty levels respectively.

	Lal et al. approach [113]		Proposed approach ($\pm15\%$ uncertainty)		Proposed approach ($\pm20\%$ uncertainty)	
$\alpha \downarrow$	$(\tilde{Av})^{\alpha}_{(L)}$	$(\tilde{Av})^{\alpha}_{(R)}$	$(\tilde{Av})^{\alpha}_{(L)}$	$(\tilde{Av})^{\alpha}_{(R)}$	$(\tilde{Av})^{\alpha}_{(L)}$	$(\tilde{Av})^{\alpha}_{(R)}$
0	0.9572711	0.9572711	0.9430492	0.9680618	0.9372473	0.9711025
0.1	0.9572711	0.9572711	0.9446717	0.9671013	0.9396443	0.9699149
0.2	0.9572711	0.9572711	0.9462442	0.9661171	0.9419386	0.9686893
0.3	0.9572711	0.9572711	0.9477690	0.9651082	0.9441365	0.9674241
0.4	0.9572711	0.9572711	0.9492482	0.9640738	0.9462442	0.9661171
0.5	0.9572711	0.9572711	0.9506838	0.9630128	0.9482670	0.9647663
0.6	0.9572711	0.9572711	0.9520778	0.9619242	0.9502100	0.9633694
0.7	0.9572711	0.9572711	0.9534318	0.9608069	0.9520778	0.9619242
0.8	0.9572711	0.9572711	0.9547477	0.9596598	0.9538746	0.9604279
0.9	0.9572711	0.9572711	0.9560270	0.9584816	0.9556045	0.9588779
1.0	0.9572711	0.9572711	0.9572711	0.9572711	0.9572711	0.9572711

Table 3.4: Steady state availability of the butter-oil processing plant



(a) By 15% uncertainty (b) By 20% uncertainty

Figure 3.4: Steady state availability with different uncertainties in data

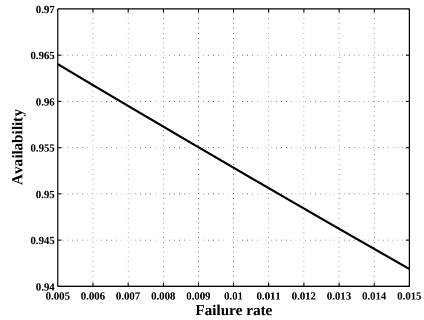
As the goal of system analyst is to maximize profit and failure free performance of system, it is important to find the impact of individual parameters of the components on the system. For this analysis, individual effects of failure and repair rates have been done to find the impact of that particular subsystem on the system performance. Effect of each parameter on the system availability has been shown through subplots given in Figure 3.5. For instance, failure rate of subsystem Separator varies from 0.005 to 0.015, keeping other parameters fixed, shows that availability varies from 0.9418774 to 0.9640235, which gives rise an increment of 2.35%. Minimum and maximum value of the availability by variation in each parameter in each component is outlined in the Table 3.5.

Component	Failure rate (λ)	Availability (Min,Max)	Repair rate (μ)	Availability (Min,Max)
Separator (A)	0.005-0.015	(0.9418774,0.9640235)	0.30-0.50	(0.9507596, 0.9605004)
Pasteuriser (B)	0.0025-0.025* 0.0045-0.0065**	(0.9571443,0.9575704) (0.4900633,0.4900992)	5.00-7.00	(0.4900507,0.4901012)
Continuous Butter Making (C)	0.002-0.008	(0.9513516,0.9651241)	0.30-0.50	(0.9531651, 0.9597518)
Melting Vats (D)	0.001-0.005	(0.9542696,0.9595018)	0.60-0.70	(0.9566824, 0.9572711)
Butter-Oil Clarifier (E)	0.0005-0.0012	(0.9563556,0.9584945)	0.20-0.40	(0.9558985, 0.9579589)
Packaging (F)	0.001-0.005	(0.3457415,0.7087912)	0.55-0.75	(0.9565795,0.95777891)

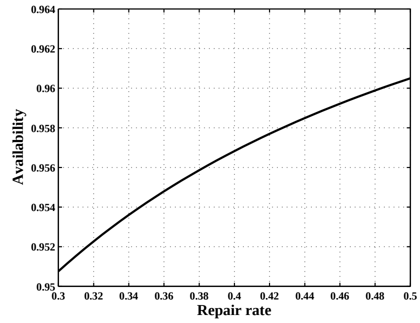
*corresponding to failure rate of Pasteuriser (B) and **corresponding to failure rate of Pasteuriser (B) in reduced state.

Table 3.5: Individual effects of failure and repair rates on availability of butter-oil processing plant

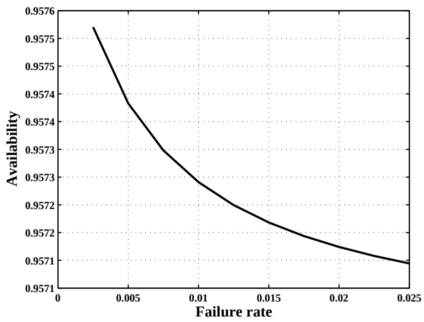
In order to find the crucial components in the system, in preferential order, so that suitable maintenance may be taken by system analyst, the effects on availability have been studied by varying failure and repair rates of a component simultaneously. Effect of each component on the system behavior has been illustrated through the subplots given in Figure 3.6. From this investigation, it may be observed that, variation in the failure and repair rates of Separator (Subsystem A) from 0.005 to 0.015 and 0.30 to 0.50 respectively shows an increment of 3.86% on system availability. The complete ranges of impacts on availability by varying failure and repair rates are depicted in Table 3.6. From this analysis, it has been seen that for long term availability, more attention should be paid as per the preferential order ; Pasteuriser,



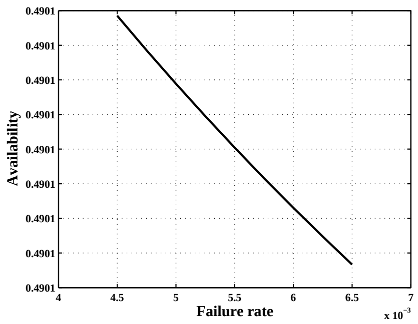
(a) Seperator



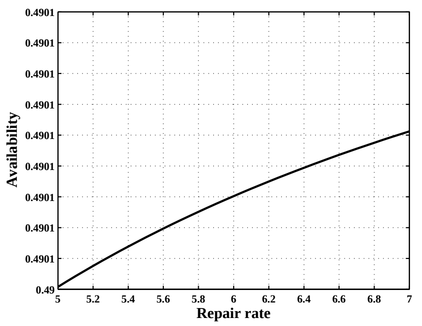
(b) Separator



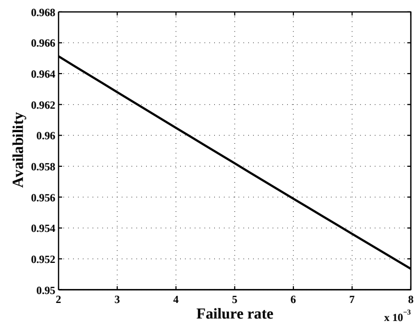
(c) Pasteuriser



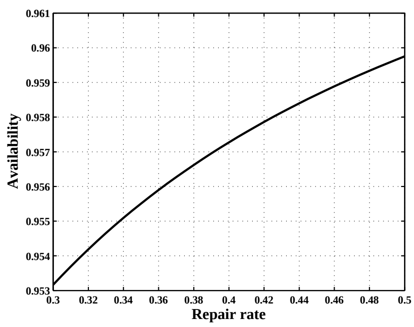
(d) Pasteuriser (reduced state)



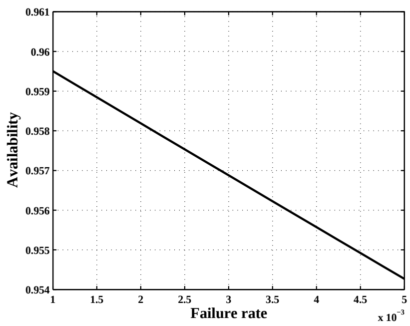
(e) Pasteuriser



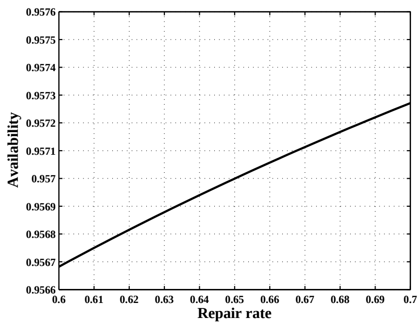
(f) Continuous Butter Making



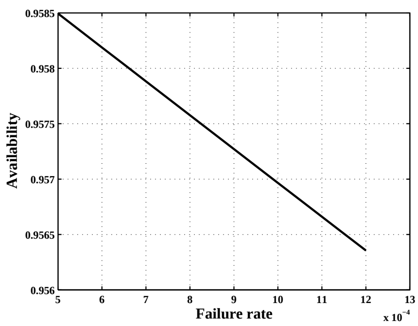
(g) Continuous Butter Making



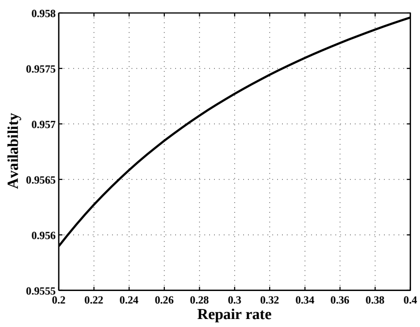
(h) Melting Vats



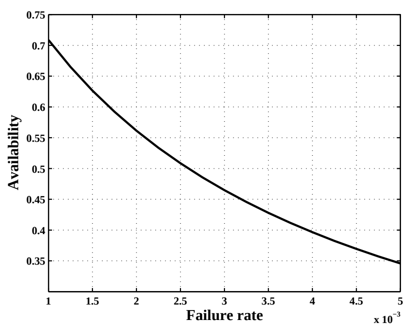
(i) Melting Vats



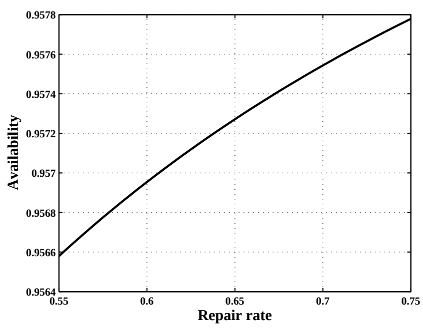
(j) Butter-Oil Clarifier



(k) Butter-Oil Clarifier



(l) Packaging



(m) Packaging

Figure 3.5: Individual effects of repair and failure rates on availability of butter-oil processing plant

Separator, Continuous Butter Making, Packaging, Melting Vats, Butter-Oil Clarifier. Thus system analyst may plan suitable maintenance policy to attain target goals.

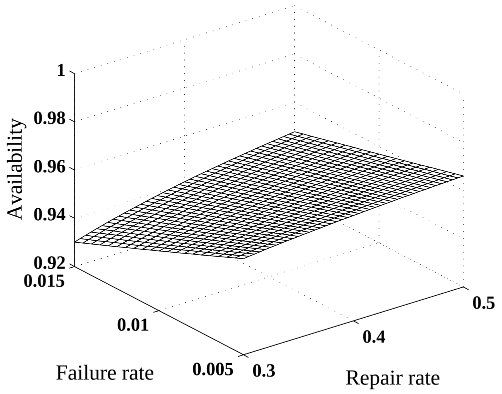
Component	Failure rate (λ)	Repair rate (μ)	Availability (Min,Max)
Separator (A)	0.005-0.015	0.30-0.50	(0.9301253, 0.9660679)
Pasteuriser (B)	0.0025-0.025* 0.0045-0.0065**	5.00-7.00	(0.4900109,0.4901685) (0.4900305, 0.4901176)
Continuous Butter Making (C)	0.002-0.008	0.30-0.50	(0.9453558, 0.9660565)
Melting Vats (D)	0.001-0.005	0.60-0.70	(0.9531868, 0.9595018)
Butter-Oil Clarifier (E)	0.0005-0.0012	0.20-0.40	(0.9545299, 0.9588775)
Packaging (F)	0.001-0.005	0.55-0.75	(0.9527682,0.9598627)

*corresponding to failure rate of Pasteuriser (B) and **corresponding to failure rate of Pasteuriser (B) in reduced state.

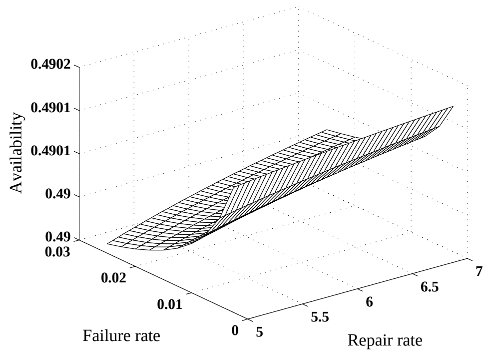
Table 3.6: Simultaneous effects of failure and repair rates on availability of butter-oil processing plant

3.5 Conclusion

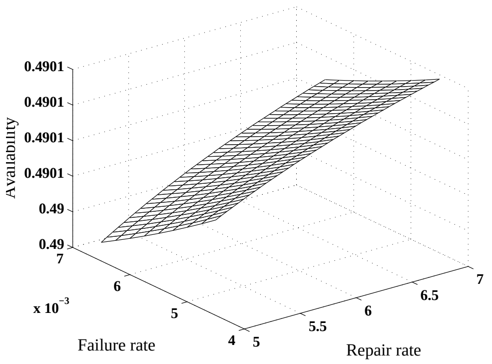
In this chapter, we have discussed an efficient approach based on RK-IV method and Particle swarm optimization for examining the availability indices of an industrial system. Performance analysis of Butter-oil processing plant as a case study of repairable industrial system has also been discussed. An organized framework has been developed to handle uncertain, vague information related to system behavior. This approach has been discussed for analyzing the availability through Markov process and handles uncertainty through fuzzy set theory. It has been noticed that the solution found by RK-IV method is improved by Particle swarm optimization. This methodology optimizes the spread of quantitative availability which may be useful to system analyst to draw more relevant conclusions. It has been observed that improvement of solution also provides the possibility to increase the availability, which may be useful for the system analyst. Results provided by existing approach [91]



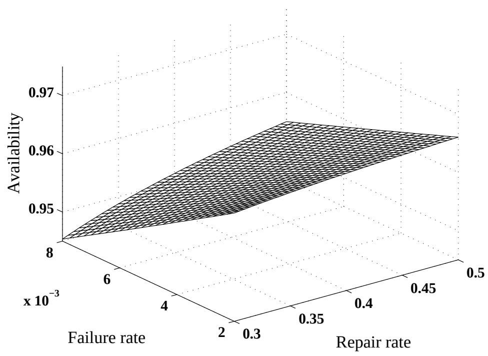
(a) Seperator



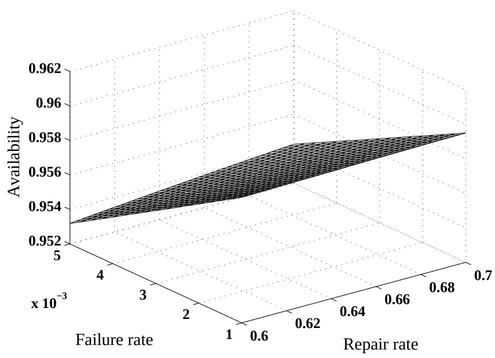
(b) Pasteuriser



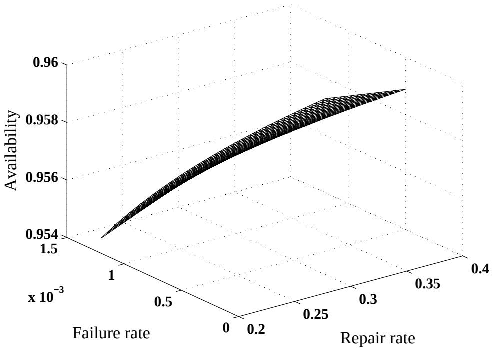
(c) Pasteuriser (reduced state)



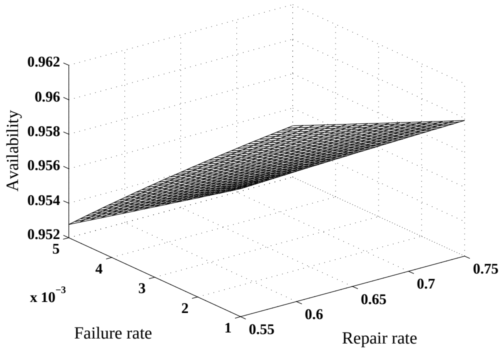
(d) Continuous Butter Making



(e) Melting Vats



(f) Butter-Oil Clarifier



(g) Packaging

Figure 3.6: Simultaneous effects of repair and failure rates on availability

deal with uncertainties but do not optimize availability through Markov process. Sensitivity analysis as well as individual performance analysis in order to study the behavior effect has been carried out for various combinations of availability parameters. On the basis of these results, system analyst may analyze the behavior of system and plan the suitable maintenance to enhance the performance of system and therefore reduce maintenance and operational cost.

Chapter 4

A numerical approach of solving fuzzy differential equations through optimization modelling

In this chapter, solution of first order linear differential equations with fuzzy constant coefficients and fuzzy initial values has been studied. An algorithm, named as Runge-Kutta Particle Swarm Optimization (RKPSO) has been proposed. It is based on α-cut of a fuzzy set by formulation of optimization model. An alternative approach RKPSO for solution of fuzzy differential equation is an amalgamation of Runge-Kutta fourth order (RK-IV) method and Particle Swarm Optimization (PSO) technique. Some examples are discussed to illustrate the suggested approach. Furthermore, a concrete example of system system of fuzzy differential equations in more than one dependent variable is taken. The whole process is presented by evaluating the availability of a Piston manufacturing plant, which is a repairable industrial system.

4.1 Introduction

Differential equations have a remarkable ability to model the physical world problems/phenomenon around us. They are used in a wide variety of disciplines as in economics, physics, biology, chemistry and engineering. Differential equations are among the linchpins of modern mathematics which are essential for solving and analyzing complex problems in engineering, natural sciences and even in business. In most of the real life problems, differential equations are not directly solvable i.e. don't have closed form solutions. In such cases one seeks for an approximate solution. Many numerical methods such as Euler, finite element and Runge-Kutta etc. already exist in literature for finding the approximate solutions for such type of differential equations.

Further in some of real life situations, parameters which are incorporated with the governing differential equations may not be certain. Parameter values are subjected to inaccuracies caused by some variations to estimate the experimental data. The vagueness and impreciseness in variables and parameters may be due to errors in measurement, observation. Impact of the uncertainties on the system has always been the interest of engineers and scientists. To overcome uncertainties or lack of precision, fuzzy set theory [289] is a good tool. One can use a fuzzy environment in variables, parameters and initial conditions in place of exact ones, and thereby turning general differential equations into Fuzzy Differential Equations (FDEs).

Fuzzy differential equations play an important role in an increasing number of system models in engineering, physics and other sciences [43, 63]. Initially, according to Vorobiev and Seikkala [280], term fuzzy differential equation was originated in 1978. Since then it became the subject of interest for many researchers and scientists. In 1982, Dubois and Parade [71] have discussed differentiation of ordinary functions and differentiation of fuzzy valued functions at non-fuzzy point. In 1983, Puri and Ralescu [222] have studied the concept of differential of a fuzzy function. Elementary fuzzy calculus has been discussed by Goetschel and Voxman [107] in 1986. In 1987, Kaleva [136] has studied the existence and uniqueness theorem for solution of fuzzy differential equations. Solution of fuzzy initial value problem with two approaches has been studied by Seikkala in 1987. A lot of work has been done in the theoretical as well as in applied field of fuzzy differential equations [30, 43, 59, 169, 283]. In order to find the solution of fuzzy differential equations, Ma et al. [188] have discussed the numerical solution of fuzzy differential equations in 1999. Buckley and Feuring [36–38] have studied the different approaches for the solutions of fuzzy differential equations. Abbasbandy and Viranloo [1, 2] have also studied the numerical solution of fuzzy differential equations using Runge-Kutta method. Recently, in 2013, Lata and Kumar [173] have discussed an analytical method for solving fuzzy differential equations. Ahmad et al. [7] have studied analytical and numerical solutions of fuzzy differential equations. In 2014, Gasilov et al.[102] have discussed solution of linear differential equations with fuzzy boundary values. In 2016, Arqub et al. [15] have studied numerical solutions of fuzzy differential equations using reproducing kernel Hilbert space method. Ahmadian et al. [8] have discussed numerical solutions of fuzzy differential equations by Runge-Kutta method with generalized differentiability. Jafari and Razvarz [129] have studied the solution of fuzzy differential equations using fuzzy Sumudu transforms. Different approaches have been studied by many researchers [63, 91, 279].

This chapter extends the idea for solving fuzzy differential equations through formation of multi-objective optimization problem. It can be observed from the literature that different methods for solving same fuzzy differential equations provide different solutions. This motivates authors' interest, set out in this chapter, to solve fuzzy differential equations using multi-objective optimization problem. In this chapter, solution of first order fuzzy differential equations has been found by formulating an optimization problem and thereby obtaining its solution.

First order linear fuzzy differential equations are one of the simplest fuzzy differential equations which may appear in many applications. However the form of such an equation is very simple, it raises many problems since under different fuzzy differential equation concepts, the behavior of the solutions is different. In real applications it can be complicated to obtain exact solution of fuzzy differential equations due to complexities in fuzzy arithmetic, creating the need for use of reliable and efficient numerical techniques in the solution of fuzzy differential equations.

The aim of this chapter is to solve fuzzy differential equations through optimization model and has been solved using α-cuts and Runge-Kutta method. Some examples are solved through this approach and their solutions have been illustrated with tables and graphs. Application of this solution obtaining approach has been shown by evaluation of availability of Piston manufacturing plant, an industrial system. Its availability has been discussed through tables and graphs.

This chapter is organized as follows: Section 4.2 comprises a solution approach for solving fuzzy differential equations by constructing optimization model of a system. In order to demonstrate the validation and efficiency of the solution approach, some examples have been discussed in Section 4.2.1. In Section 4.3, a case study as an application of fuzzy differential equations has been presented. Availability analysis of Piston manufacturing plant along with the results and discussion has been presented along with sensitivity analysis. In Section 4.4, conclusions have been drawn.

4.2 The Solution Approach

A system of first order linear differential equations with constant coefficients is represented as

$$\frac{dX}{dt} = AX + G(t) \tag{4.2.1}$$

where, $(X(t))^T = (x_1, x_2, ..., x_n)$, $A = [a_{ij}]$ a $n \times n$ matrix of constants and $(G(t))^T = (g_1(t), g_2(t), ..., g_n(t))$. The independent variable t belongs to an interval (closed and bounded). Let $(X(0))^T = (\zeta_1, \zeta_2, ..., \zeta_n)$. It is assumed that the functions $g_i(t)$ are all continuous on that interval.

But in practical life situations, values of parameters and initial conditions depend on the information available from various sources and hence do not tell the exact behavior of system. Due to imprecise nature of the parameters, above system of differential equations (4.2.1) has been converted into system of fuzzy differential equations by representing each of the parameters as fuzzy number. Thus system of first order linear fuzzy differential equations is represented as

$$\frac{d\tilde{X}}{dt} = \tilde{A}\tilde{X} + G(t) \tag{4.2.2}$$

where, $\tilde{A} = [\tilde{a}_{ij}]$ with every $\tilde{a}_{ij}$ a fuzzy number and $(\tilde{X}(0))^T = (\tilde{\zeta}_1, \tilde{\zeta}_2, ..., \tilde{\zeta}_n)$ with all $\tilde{\zeta}_i$ as fuzzy numbers.

In order to find a solution of the form of differential equations Eq. (4.2.2), firstly α-cuts of the parameters have been computed. The α-cuts of the parameters are

$$\tilde{a}_{ij}^{\alpha} = [\tilde{a}_{ij(L)}^{\alpha}, \tilde{a}_{ij(R)}^{\alpha}],$$

for every $1 \leq i, j \leq n$

$$\tilde{x}_i^{\alpha} = [\tilde{x}_{i(L)}^{\alpha}, \tilde{x}_{i(R)}^{\alpha}], \tag{4.2.3}$$

for every $1 \leq i \leq n$ and

$$\tilde{\zeta}_i^{\alpha} = [\tilde{\zeta}_{i(L)}^{\alpha}, \tilde{\zeta}_{i(R)}^{\alpha}]$$

for every $1 \leq i, j \leq n$.

Based on these α-cuts, set of first order linear fuzzy differential equations (4.2.2) can be written as:

$$\frac{d\tilde{x}_i^{\alpha}}{dt} = \sum_{j=1}^{n} \tilde{a}_{ij}^{\alpha}\ \tilde{x}_j^{\alpha} + G(t),$$

$$\Big[\frac{d\tilde{x}_{i(L)}^{\alpha}}{dt}, \frac{d\tilde{x}_{i(R)}^{\alpha}}{dt}\Big] = \sum_{j=1}^{n} [\tilde{a}_{ij(L)}^{\alpha}, \tilde{a}_{ij(R)}^{\alpha}][\tilde{x}_{j(L)}^{\alpha}, \tilde{x}_{j(R)}^{\alpha}] + [G(t),\ G(t)], \tag{4.2.4}$$

for $i = 1, 2, ..., n$ with initial conditions

$$[\tilde{x}_{i(L)}^{\alpha}(0), \tilde{x}_{i(R)}^{\alpha}(0)] = [\tilde{\zeta}_{i(L)}^{\alpha}, \tilde{\zeta}_{i(R)}^{\alpha}].$$

In next step, interval arithmetic operations, based on the $\alpha-$ cut of fuzzy numbers, are performed on the obtained system of equations (4.2.4). Then the system of equations (4.2.4) reduces to

$$\begin{aligned}
\frac{d\tilde{x}^{\alpha}_{i(L)}}{dt} &= \sum_{j=1}^{n} \min\{\tilde{a}^{\alpha}_{ij(L)}\tilde{x}^{\alpha}_{j(L)},\ \tilde{a}^{\alpha}_{ij(L)}\tilde{x}^{\alpha}_{j(R)},\ \tilde{a}^{\alpha}_{ij(R)}\tilde{x}^{\alpha}_{j(L)},\ \tilde{a}^{\alpha}_{ij(R)}\tilde{x}^{\alpha}_{j(R)}\} + G(t),\\
\frac{d\tilde{x}^{\alpha}_{i(R)}}{dt} &= \sum_{j=1}^{n} \max\{\tilde{a}^{\alpha}_{ij(L)}\tilde{x}^{\alpha}_{j(L)},\ \tilde{a}^{\alpha}_{ij(L)}\tilde{x}^{\alpha}_{j(R)},\ \tilde{a}^{\alpha}_{ij(R)}\tilde{x}^{\alpha}_{j(L)},\ \tilde{a}^{\alpha}_{ij(R)}\tilde{x}^{\alpha}_{j(R)}\} + G(t),
\end{aligned} \tag{4.2.5}$$

with initial conditions as

$$\tilde{x}^{\alpha}_{i(L)}(0) = \tilde{\zeta}^{\alpha}_{i(L)} \quad \text{and} \quad \tilde{x}^{\alpha}_{i(R)}(0) = \tilde{\zeta}^{\alpha}_{i(R)}, \quad \text{for } i = 1, 2, ..., n. \tag{4.2.6}$$

Now according to the mathematical model, an optimization problem is developed for α-cut level. In the form of bounded interval, input data at α-cut level is substituted in the expression. Lower and upper boundary values of these equations are obtained at α-cut level by solving the following optimization problem.

$$\begin{aligned}
&\min / \max \ y(\tilde{x}_i, \tilde{a}_{ij}),\\
&\text{subject to} \quad \mu_{\tilde{a}_{ij}} \geq \alpha,\\
&\qquad\qquad\quad\ \mu_{\tilde{x}_i} \geq \alpha,\\
&\qquad\qquad\quad\ 0 \leq \alpha \leq 1,
\end{aligned} \tag{4.2.7}$$

where, y is the fitness function of $\tilde{x}_i$ according to mathematical model of the problem and obtained from Eq. (4.2.5) by using Runge-Kutta fourth order method. The obtained maximum and minimum values of y, denoted by $y_{\max}$ and $y_{\min}$ respectively corresponding to α-cut level satisfy

$$\mu_{\tilde{y}}(y_{min}) = \mu_{\tilde{y}}(y_{max}) = \alpha.$$

Solution of optimization problem so obtained is modified by Particle swarm optimization. We say the obtained solution is fuzzy solution if the solution gives intervals defining α-cut of a fuzzy number. Mathematically, one can say that $\tilde{y}(t)$ is a fuzzy solution for all t if the obtained values of $(\tilde{y}(t))^{\alpha}_{(L)}$ and $(\tilde{y}(t))^{\alpha}_{(R)}$ define the α-cuts $[(\tilde{y}(t))^{\alpha}_{(L)}, (\tilde{y}(t))^{\alpha}_{(R)}]$ of fuzzy numbers. Thus we can say that $\tilde{y}(t)$ is a fuzzy solution if following conditions are met out.

1. $\frac{\partial(\tilde{y})^{\alpha}_{(L)}}{\partial\alpha} \geq 0$ and $\frac{\partial(\tilde{y})^{\alpha}_{(R)}}{\partial\alpha} \leq 0$,
   i.e. $(\tilde{y})^{\alpha}_{(L)}$ increases while $(\tilde{y})^{\alpha}_{(R)}$ decreases as α increases.

2. $(\tilde{y})^{\alpha}_{(L)} \leq (\tilde{y})^{\alpha}_{(R)}$ for $\alpha = 1$.

for all $\alpha \in [0, 1]$ and $t \in I$.

4.2.1 Examples

In this section, some examples taken from [7] have been solved by proposed approach and compared with analytical approach.

Example 1: Consider the following fuzzy initial value problem.

$$\begin{cases} \frac{d\tilde{X}}{dt} = -\frac{1}{2}\tilde{X} + 2sin(3t), t \in [0, 4], \\ \tilde{X}(0) = (-1, 0, 1). \end{cases} \tag{4.2.8}$$

Analytical solution of Eq. (4.2.8) is given as:

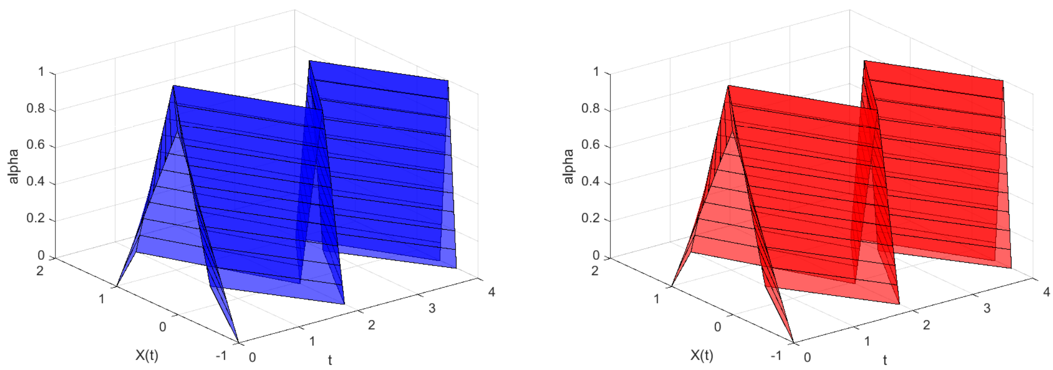
$$\tilde{X}^{\alpha}_{1}(t) = -\frac{24}{37}cos(3t) + \frac{4}{37}sin(3t) + \big(-(1-\alpha) + \frac{24}{37}\big)\exp^{-\frac{1}{2}t},$$
$$\tilde{X}^{\alpha}_{2}(t) = -\frac{24}{37}cos(3t) + \frac{4}{37}sin(3t) + \big((1-\alpha) + \frac{24}{37}\big)\exp^{-\frac{1}{2}t}.$$

Results obtained using proposed approach have been depicted in Table 4.1. For different values of t, solutions of Eq. (4.2.8) through analytical and proposed approach have also been shown through graphs in Fig. 4.1.

For different times $t = 1, 2, 3, 4$, results for Eq. (4.2.8) have been computed. It can be observed that for every given time, obtained solution is a fuzzy solution. For instance, left cut solution values for each value of t increases as α increases and right cut solution values for each value of t decreases as α increases. Obtained solutions for different values of t are shown in Figure 4.1.

	$t=1$		$t=2$		$t=3$		$t=4$	
α	$x^{\alpha}_{(L)}$	$x^{\alpha}_{(R)}$	$x^{\alpha}_{(L)}$	$x^{\alpha}_{(R)}$	$x^{\alpha}_{(L)}$	$x^{\alpha}_{(R)}$	$x^{\alpha}_{(L)}$	$x^{\alpha}_{(R)}$
0	0.4443081	1.6573695	-0.7822752	-0.0465163	0.5571597	1.0034200	-0.6529228	-0.3822523
0.1	0.5049612	1.5967164	-0.7454872	-0.0833042	0.5794727	0.9811070	-0.6393893	-0.3957858
0.2	0.5656143	1.5360633	-0.7086993	-0.1200922	0.6017857	0.9587940	-0.6258558	-0.4093193
0.3	0.6262673	1.4754103	-0.6719113	-0.1568801	0.6240987	0.9364810	-0.6123223	-0.4228529
0.4	0.6869204	1.4147572	-0.6351234	-0.1936681	0.6464117	0.9141679	-0.5987887	-0.4363864
0.5	0.7475735	1.3541041	-0.5983355	-0.2304560	0.6687248	0.8918549	-0.5852552	-0.4499199
0.6	0.8082265	1.2934511	-0.5615475	-0.2672440	0.6910378	0.8695419	-0.5717217	-0.4634535
0.7	0.8688796	1.2327980	-0.5247596	-0.3040319	0.7133508	0.8472289	-0.5581881	-0.4769870
0.8	0.9295327	1.1721449	-0.4879716	-0.3408199	0.7356638	0.8249159	-0.5446546	-0.4905205
0.9	0.9901857	1.1114919	-0.4511837	-0.3776078	0.7579768	0.8026029	-0.5311211	-0.5040540
1	1.0508388	1.0508388	-0.4143957	-0.4143957	0.7802898	0.7802898	-0.5175876	-0.5175876

Table 4.1: Solutions of Example 1 for different times t



(a) By analytical method

(b) By given approach

Figure 4.1: Solution of Example 1.

Example 2: Consider the following fuzzy initial value problem.

$$\begin{cases} \frac{d\tilde{X}}{dt} = \frac{1}{2}\tilde{X} + 2sin(3t), t \in [0,4], \\ \tilde{X}(0) = (-1,0,1). \end{cases} \tag{4.2.9}$$

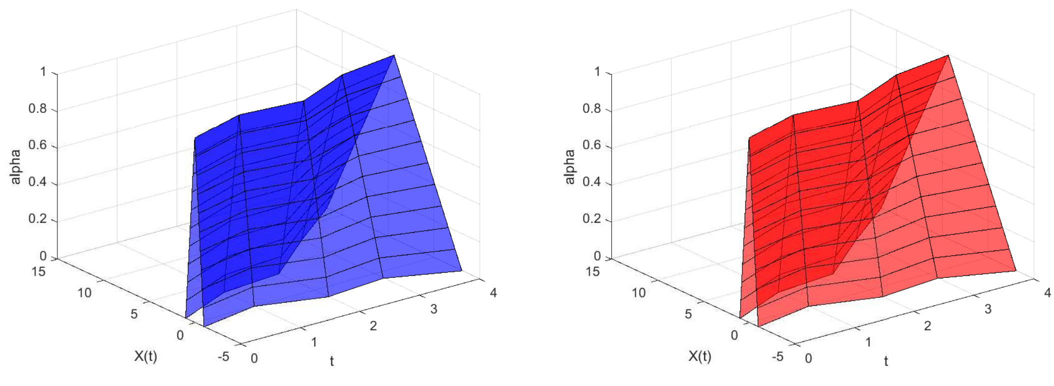
Analytical solution of Eq. (4.2.9) is given as:

$$\tilde{X}^{\alpha}_1(t) = -\frac{24}{37}cos(3t) - \frac{4}{37}sin(3t) + \big(-(1-\alpha) + \frac{24}{37}\big)\exp^{\frac{1}{2}t},$$
$$\tilde{X}^{\alpha}_2(t) = -\frac{24}{37}cos(3t) - \frac{4}{37}sin(3t) + \big((1-\alpha) + \frac{24}{37}\big)\exp^{\frac{1}{2}t}.$$

Results obtained using proposed approach have been depicted in Table 4.2. For different values of t, Solutions of Eq. 4.2.9 through analytical and proposed approach have also been shown through graphs in Figure 4.2. For different times $t = 1, 2, 3, 4$, results for Eq. (4.2.9) have been computed. It can be observed that for every given time, obtained solution is a fuzzy solution. For instance, left cut solution values for each value of t increases as α increases and right cut solution values for each value of t decreases as α increases. Obtained solutions for different values of t are shown in Figure 4.2.

	$t=1$		$t=2$		$t=3$		$t=4$	
α	$x^{\alpha}_{(L)}$	$x^{\alpha}_{(R)}$	$x^{\alpha}_{(L)}$	$x^{\alpha}_{(R)}$	$x^{\alpha}_{(L)}$	$x^{\alpha}_{(R)}$	$x^{\alpha}_{(L)}$	$x^{\alpha}_{(R)}$
0	0.047621	3.345063	-1.547678	3.888886	-1.028197	7.935181	-3.085512	11.692601
0.1	0.212493	3.180191	-1.275850	3.617057	0.580029	7.487012	-2.346606	10.953695
0.2	0.377365	3.015319	-0.800000	3.345229	0.131859	7.038843	-1.607700	10.214789
0.3	0.542237	2.850447	-0.732194	3.073401	0.316309	6.590674	-0.8687948	9.475884
0.4	0.707109	2.685575	-0.460365	2.801573	0.764478	6.142505	-0.129889	8.736978
0.5	0.871981	2.520702	-0.188537	2.529745	1.212647	5.694336	0.609016	7.998072
0.6	1.036853	2.355830	0.083291	2.257916	1.660816	5.246167	1.347922	7.259167
0.7	1.201726	2.190958	0.355119	1.986088	2.108985	4.797998	2.086827	6.520261
0.8	1.366598	2.026086	0.626947	1.714260	2.557154	4.349829	2.825733	5.781356
0.9	1.531470	1.861214	0.898776	1.442432	3.005323	3.901661	3.564639	5.042450
1	1.696342	1.696342	1.170604	1.170604	3.453492	3.453492	4.303544	4.303544

Table 4.2: Solutions of Example 2 for different times t



(a) By analytical method

(b) By given approach

Figure 4.2: Solution of Example 2.

In the next example, fuzzy coefficients have been taken into account. Example 3 has been extended from Example 2 in view of fuzzy coefficients in place of crisp coefficients. Solution has been found through suggested approach.

Example 3: Consider the following fuzzy initial value problem.

$$\begin{cases} \frac{d\tilde{X}}{dt} = \tilde{a}\tilde{X} + \tilde{b}sin(3t), t \in [0,4], \\ \tilde{X}(0) = (-1,0,1), \\ \text{where } \tilde{a} = (\frac{1}{4}, \frac{1}{2}, \frac{3}{4}) \text{ and } \tilde{b} = (1.75, 2, 2.25). \end{cases} \quad (4.2.10)$$

Results obtained using proposed approach have been depicted in Table 4.3. For different values of t, Solutions of Eq. (4.2.10) through proposed approach have also been shown through graph in Figure 4.3.

In Example 3, parameters are considered as fuzzy numbers. Solution obtained by proposed approach follows to be a fuzzy solution. For different values of t, left cut solution values increases as α increases and right cut solution values decreases as α increases.

	$t=1$		$t=2$		$t=3$		$t=4$	
α	$x^{\alpha}_{(L)}$	$x^{\alpha}_{(R)}$	$x^{\alpha}_{(L)}$	$x^{\alpha}_{(R)}$	$x^{\alpha}_{(L)}$	$x^{\alpha}_{(R)}$	$x^{\alpha}_{(L)}$	$x^{\alpha}_{(R)}$
0	-0.430569	4.285268	-2.509955	7.016775	-3.835119	16.755385	-9.447828	33.762591
0.1	-0.169675	3.974931	-1.952648	6.19863	-2.547719	14.658698	-6.596289	28.59165
0.2	0.079452	3.677113	-1.445927	5.441999	-1.422487	12.779039	-4.207312	24.092657
0.3	0.317232	3.391383	-0.986225	4.742876	-0.442682	11.096042	-2.218847	20.184797
0.4	0.54407	3.117324	-0.570201	4.097502	0.406869	9.591146	-0.576456	16.796535
0.5	0.760355	2.854532	-0.325111	3.502344	0.884403	8.247439	0.039098	13.864581
0.6	0.935923	2.602617	-0.07963	2.954086	1.270741	7.04952	0.620328	11.332968
0.7	1.11347	2.361202	0.190754	2.449615	1.713957	5.983363	1.318156	9.152238
0.8	1.299187	2.129918	0.487907	1.986011	2.220573	5.036205	2.150541	7.278705
0.9	1.493373	1.90841	0.813816	1.560531	2.797778	4.196434	3.137924	5.673811
1	1.696341	1.696342	1.170603	1.56053	3.453491	3.453491	4.303544	4.303544

Table 4.3: Solution of Example 3 for different times t

4.3 Case Study

For an application point of view, availability of piston manufacturing plant [170, 173], an industrial system has been evaluated by solving the corresponding fuzzy differential equations. In this section, system availability has been discussed.

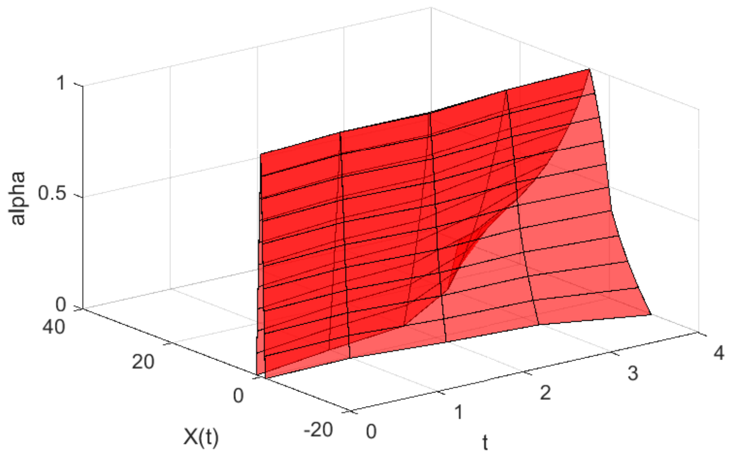


Figure 4.3: Solution of Example 3 by given approach

4.3.1 System Description

In order to reduce the complexity for the effective performance analysis, the piston manufacturing plant is categorized into two subsystems namely, R_1 and R_2. Brief description of these subsystems has been given here. For detailed description of the system, one may refer [170]. The flow chart of the piston manufacturing plant is given in Figure 4.4.

System R_1 is constituted by six subsystems, namely, A, B, C, D, E and F, briefly described given below.

(i) Subsystem A denotes a fixture seat machining operation which is carried out for clamping of pistons.

(ii) Subsystem B is again a machining operation of rough grooving and turning operation.

(iii) Subsystem C denotes the rough pin hole boring machine which is used to make pin holes on the piston for the pin, used to connect it to the crankshaft by a connecting rod.

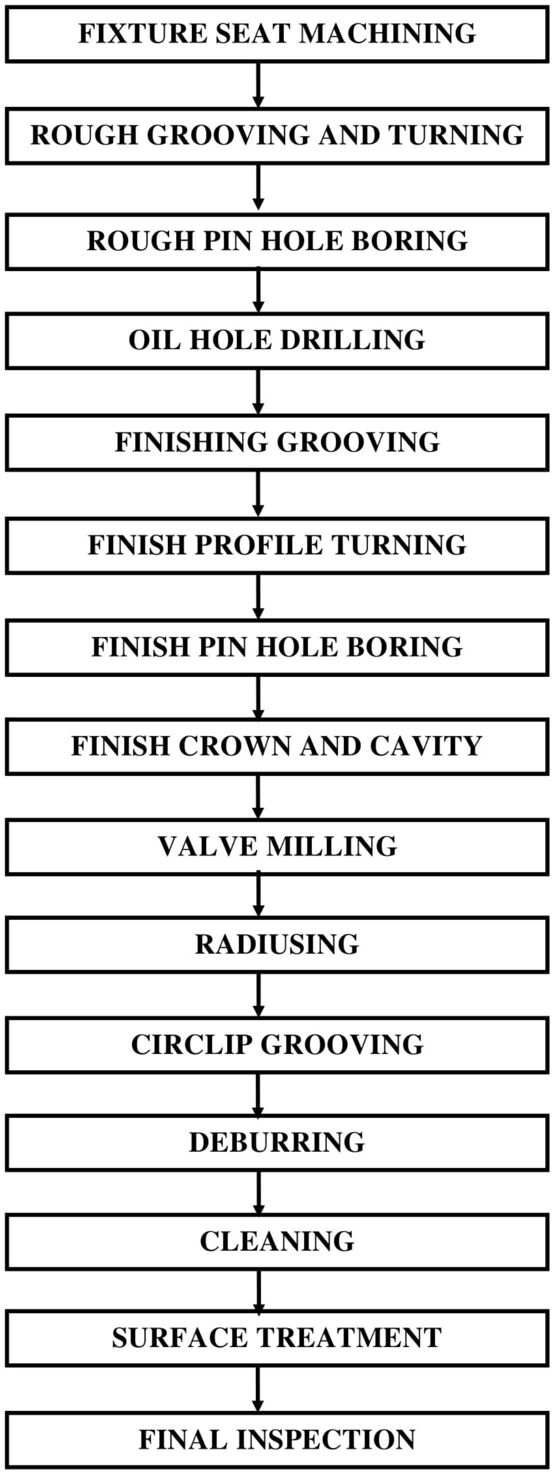


Figure 4.4: Flow chart of Piston manufacturing plant

(iv) Subsystem D consists of an oil hole drilling machine which is used to drill oil hole on the piston. Through this hole oil passes to lubricate the operating machine.

(v) Subsystem E denotes the finishing grooving machine which is used to give finishing to the rough grooves.

(vi) Subsystem F denotes the finish profile turning operation to form the piston in oval shape for overcoming the expansion problems at high temperature in working condition.

System R_2 is composed of many subsystems out of which major subsystems whose failure show complete breakdown of the system, are of importance. These are described below.

(i) Subsystem G that denotes the finish pin hole boring machine.

(ii) Subsystem H that denotes the finish crown and cavity which is used to give finishing of the upper part of the piston i.e. crown.

(iii) Subsystem I that denotes the valve milling machine to create the valve recession of the piston.

(iv) Subsystem J which is a chamfering machine used to round off the corners of the piston for its smooth running.

(v) Subsystem K that denotes the circlip grooving machine for making the circlip grooves on the piston.

(vi) Subsystem L which is the deburring machine that is used to neaten and smoothen the rough edges of the piston.

cleaning machine that helps to clean the piston.

In addition to these subsystems in R_2, there are subsystems described below which are considered to have no failure.

(vii) Subsystem M which cleans the inner and outer surfaces of the piston is the cleaning machine.

(viii) Subsystem N is the equipment that is used to coat the piston with some mixture.

(ix) Subsystem O represents the process by which the manufactured product is finally inspected before packaging.

4.3.2 Notations

In this section, notations that are used for examining the availability of the system are given.

○	Represents that system is in full working state.
⬭	Represents reduced state of the system.
▭	Represents that system is in failed state.
A,B,C,D,E,F	Represent full working states of the subsystem for R_1
G,H,I,J,K,L	Represent full working states of the subsystem for R_2
a,b,c,d,e,f	Represent failed states of the subsystem for R_1
g,h,i,j,k,l	Represent failed states of the subsystem for R_2
$\overline{C}$, $\overline{E}$ and $\overline{G}$	Represent reduced states of the subsystems C, E and G.
$\lambda_i,\ i = 1, 2..., 7$	Represent failure rates of subsystems, when the transition is from H to h, I to i, J to j, K to k, L to l, G to $\overline{G}$ and $\overline{G}$ to g respectively.
$l_j,\ j = 1, 2, ..., 8$	Represent failure rates of subsystems, when the transition is from A to a, B to b, D to d, F to f, C to $\overline{C}$, E to $\overline{E}$, $\overline{C}$ to c and $\overline{E}$ to e respectively.
$\mu_i,\ i = 1, 2..., 7$	Represent repair rates of subsystems, when the transition is from h to H, i to I, j to J, k to K, l to L, $\overline{G}$ to G and g to G respectively.
$m_j,\ j = 1, 2, ..., 8$	Represent repair rates of subsystems, when the transition is from a to A, b to B, d to D, f to F, $\overline{C}$ to C, $\overline{E}$ to E, c to C and e to E respectively.

4.3.3 Input Parameters

Piston manufacturing plant has been discussed by many researchers [170, 173]. Input parameters as given below (in Table 4.4 and 4.5) have been taken in the form of trapezoidal fuzzy numbers from Lata and Kumar [173].

Failure rate	Repair rate
$\tilde{l}_1$ = (0.00105,0.00126,0.00154,0.00175)	$\tilde{m}_1$ = (1.026,1.0584,1.1016,1.134)
$\tilde{l}_2$ = (0.00045,0.00054,0.00066,0.00075)	$\tilde{m}_2$ = (0.04085,0.04214,0.04386,0.04515)
$\tilde{l}_3$ = (0.000675,0.00081,0.00099,0.001125)	$\tilde{m}_3$ = (0.475,0.49,0.51,0.525)
$\tilde{l}_4$ = (0.000675,0.00081,0.00099,0.001125)	$\tilde{m}_4$ = (0.2717,0.28028,0.29172,0.3003)
$\tilde{l}_5$ = (0.0156,0.01872,0.02288,0.026)	$\tilde{m}_5$ = (0.1463,0.15092,0.15702,0.1617)
$\tilde{l}_6$ = (0.0156,0.01872,0.02288,0.026)	$\tilde{m}_6$ = (0.2375,0.245,0.255,0.2625)
$\tilde{l}_7$ = (0.000675,0.00081,0.00099,0.001125)	$\tilde{m}_7$ = (0.05605,0.05782,0.06018,0.06195)
$\tilde{l}_8$ = (0.002925,0.00351,0.00429,0.004875)	$\tilde{m}_8$ = (0.08265,0.08526,0.08874,0.09135)

Table 4.4: Input data for the system R_1

Failure rate	Repair rate
$\tilde{\lambda}_1$ = (0.00105,0.00126,0.00154,0.00175)	$\tilde{\mu}_1$ = (0.3135,0.3234,0.3366,0.3465)
$\tilde{\lambda}_2$ = (0.00023,0.00027,0.00033,0.00038)	$\tilde{\mu}_2$ = (0.475,0.49,0.51,0.525)
$\tilde{\lambda}_3$ = (0.00008,0.00009,0.00011,0.00013)	$\tilde{\mu}_3$ =(0.6365,0.6566,0.6834,0.7035)
$\tilde{\lambda}_4$ = (0.00023,0.00027,0.00033,0.00038)	$\tilde{\mu}_4$ =(0.03325,0.0343,0.0357,0.03675)
$\tilde{\lambda}_5$ = (0.00008,0.00009,0.00011,0.00013)	$\tilde{\mu}_5$ =(2.8785,2.9694,3.0906,3.1815)
$\tilde{\lambda}_6$ = (0.0156,0.01872,0.02288,0.026)	$\tilde{\mu}_6$ =0.2109,0.21756,0.22644,0.2331)
$\tilde{\lambda}_7$ = (0.003,0.0036,0.0044,0.005)	$\tilde{\mu}_7$ =(0.11875,0.1225,0.1275,0.13125)

Table 4.5: Input data for the system R_2

Now first we discuss the mathematical formulation and their solutions for the subsystems R_1 and R_2 separately.

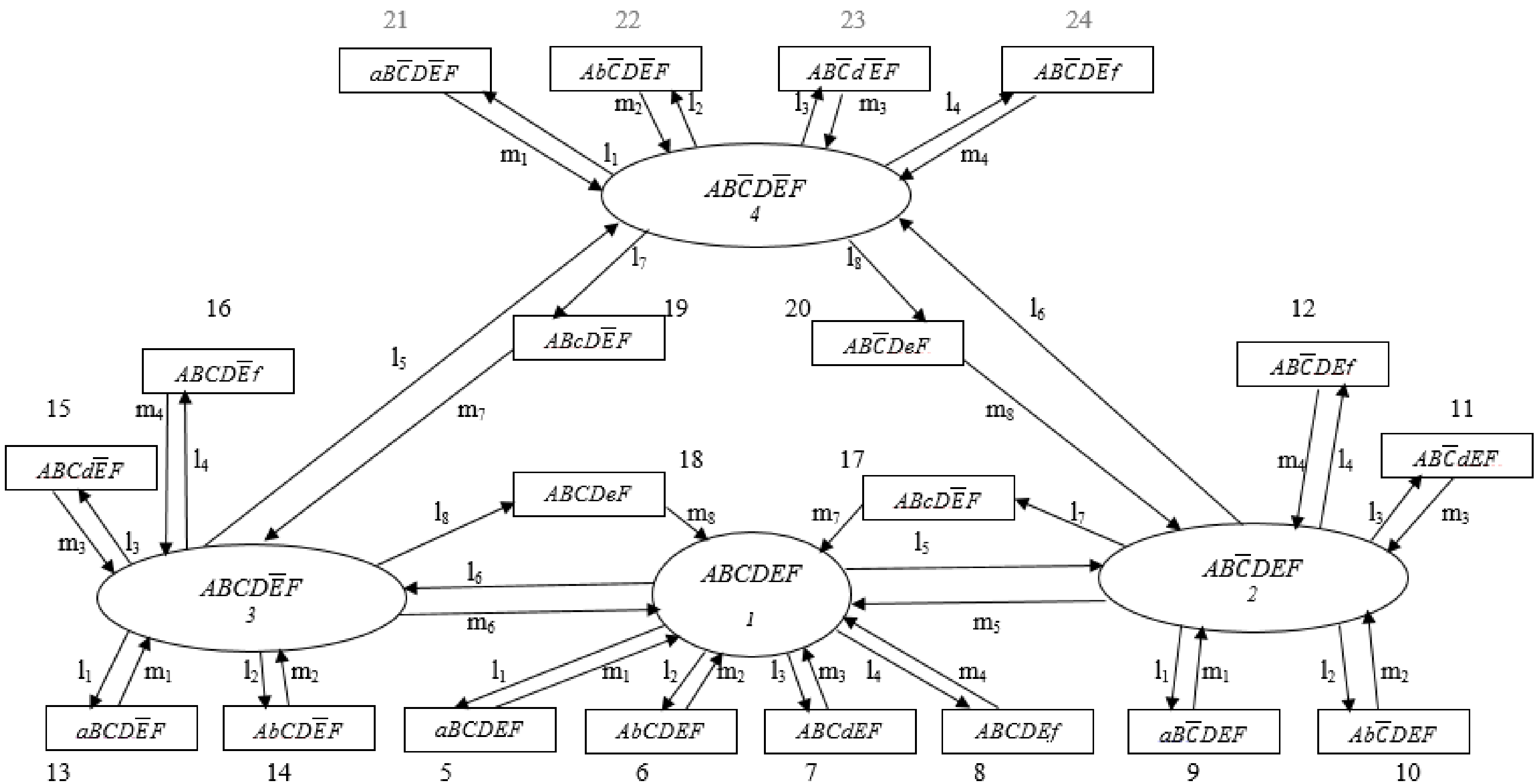


Figure 4.5: Transition diagram of subsystem R_1 of piston manufacturing plant

4.3.4 Mathematical Formulation for R_1

Applying the concepts of Markov modeling and probability theory, the transition diagram (Figure 4.5) of this system leads to the formulation of following differential equations for subsystem R_1.

$$
\begin{aligned}
\frac{d\tilde{P}_1(t)}{dt} &= \sum_{i=1}^{4} \tilde{m}_i \tilde{P}_{i+4}(t) \oplus \sum_{i=2}^{3} \tilde{m}_{i+3} \tilde{P}_i(t) \oplus \sum_{i=7}^{8} \tilde{m}_i \tilde{P}_{i+10}(t) \ominus \tilde{\phi}_1 \tilde{P}_1(t) \\
\frac{d\tilde{P}_2(t)}{dt} &= \sum_{i=1}^{4} \tilde{m}_i \tilde{P}_{i+8}(t) \oplus \tilde{m}_8 \tilde{P}_{20}(t) \oplus \tilde{l}_5 \tilde{P}_1(t) \ominus \tilde{\phi}_2 \tilde{P}_2(t) \\
\frac{d\tilde{P}_3(t)}{dt} &= \sum_{i=1}^{4} \tilde{m}_i \tilde{P}_{i+12}(t) \oplus \tilde{m}_7 \tilde{P}_{19}(t) \oplus \tilde{l}_6 \tilde{P}_1(t) \ominus \tilde{\phi}_3 \tilde{P}_3(t) \\
\frac{d\tilde{P}_4(t)}{dt} &= \sum_{i=1}^{4} \tilde{m}_i \tilde{P}_{i+20}(t) \oplus \tilde{l}_5 \tilde{P}_3(t) \oplus \tilde{l}_6 \tilde{P}_2(t) \ominus \tilde{\phi}_4 \tilde{P}_4(t) \\
\frac{d\tilde{P}_{i+4}(t)}{dt} &= \tilde{l}_i \tilde{P}_1(t) \ominus \tilde{m}_i \tilde{P}_{i+4}(t), \quad i = 1,2,3,4 \\
\frac{d\tilde{P}_{i+8}(t)}{dt} &= \tilde{l}_i \tilde{P}_2(t) \ominus \tilde{m}_i \tilde{P}_{i+8}(t), \quad i = 1,2,3,4 \\
\frac{d\tilde{P}_{i+12}(t)}{dt} &= \tilde{l}_i \tilde{P}_3(t) \ominus \tilde{m}_i \tilde{P}_{i+12}(t), \quad i = 1,2,3,4 \\
\frac{d\tilde{P}_{17}(t)}{dt} &= \tilde{l}_7 \tilde{P}_2(t) \ominus \tilde{m}_7 \tilde{P}_{17}(t), \\
\frac{d\tilde{P}_{18}(t)}{dt} &= \tilde{l}_8 \tilde{P}_3(t) \ominus \tilde{m}_8 \tilde{P}_{18}(t), \\
\frac{d\tilde{P}_{19}(t)}{dt} &= \tilde{l}_7 \tilde{P}_4(t) \ominus \tilde{m}_7 \tilde{P}_{19}(t), \\
\frac{d\tilde{P}_{20}(t)}{dt} &= \tilde{l}_8 \tilde{P}_4(t) \ominus \tilde{m}_8 \tilde{P}_{20}(t), \\
\frac{d\tilde{P}_{i+20}(t)}{dt} &= \tilde{l}_i \tilde{P}_4(t) \ominus \tilde{m}_i \tilde{P}_{i+20}(t), \quad i = 1,2,3,4.
\end{aligned}
\tag{4.3.1}
$$

where,

$$
\begin{aligned}
\tilde{\phi}_1 &= \tilde{l}_1 \oplus \tilde{l}_2 \oplus \tilde{l}_3 \oplus \tilde{l}_4 \oplus \tilde{l}_5 \oplus \tilde{l}_6 \\
\tilde{\phi}_2 &= \tilde{l}_1 \oplus \tilde{l}_2 \oplus \tilde{l}_3 \oplus \tilde{l}_4 \oplus \tilde{l}_6 \oplus \tilde{l}_7 \oplus \tilde{m}_5
\end{aligned}
$$

$$\tilde{\phi}_3 = \tilde{l}_1 \oplus \tilde{l}_2 \oplus \tilde{l}_3 \oplus \tilde{l}_4 \oplus \tilde{l}_5 \oplus \tilde{l}_8 \oplus \tilde{m}_6$$
$$\tilde{\phi}_4 = \tilde{l}_1 \oplus \tilde{l}_2 \oplus \tilde{l}_3 \oplus \tilde{l}_4 \oplus \tilde{l}_7 \oplus +\tilde{l}_8$$

with initial conditions

$$\tilde{P}_1(0) = (0.94, 0.945, 0.955, 0.96)$$
$$\tilde{P}_2(0) = (0.006, 0.0065, 0.0075, 0.008)$$
$$\tilde{P}_3(0) = (0.004, 0.0045, 0.0055, 0.006)$$
$$\tilde{P}_4(0) = (0.002, 0.0025, 0.0035, 0.004)$$
$$\tilde{P}_j(0) = 0,\ j = 5, 6, ..., 24.$$

Thus availability of the subsystem R_1 is

$$\tilde{A}v_{R_1} = \tilde{P}_1(t) \oplus \tilde{P}_2(t) + \tilde{P}_3(t) + \tilde{P}_4(t), \tag{4.3.2}$$

as the states having the probabilities $\tilde{P}_1$, $\tilde{P}_2$, $\tilde{P}_3$ and $\tilde{P}_4$ are the only working states of R_1.

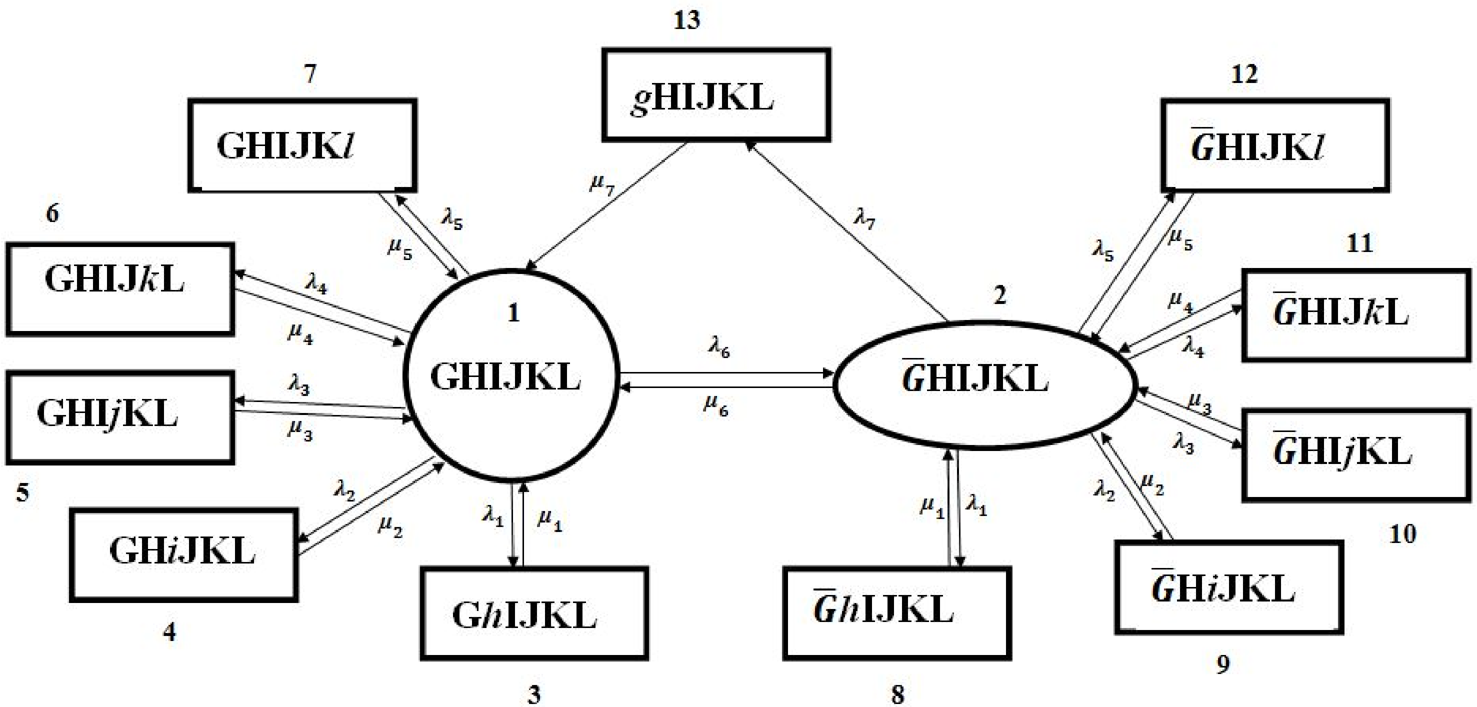


Figure 4.6: Transition diagram of subsystem R_2 of piston manufacturing plant

4.3.5 Mathematical Formulation for R_2

Using the concepts of probability and Markov modeling, differential equations corresponding to the transition diagram for system R_2 (Figure 4.6) are formulated as:

$$
\begin{aligned}
\frac{d\tilde{P}_1(t)}{dt} &= \sum_{j=1}^{5} \tilde{\mu}_j \tilde{P}_{j+2}(t) \oplus \tilde{\mu}_6 \tilde{P}_2(t) \oplus \tilde{\mu}_7 \tilde{P}_{13}(t) \ominus \tilde{\delta}_1 \tilde{P}_1(t) \\
\frac{d\tilde{P}_2(t)}{dt} &= \sum_{j=1}^{5} \tilde{\mu}_j \tilde{P}_{j+7}(t) \oplus \tilde{\lambda}_6 \tilde{P}_1(t) \ominus \tilde{\delta}_2 \tilde{P}_2(t) \\
\frac{d\tilde{P}_{i+2}(t)}{dt} &= \tilde{\lambda}_i \tilde{P}_1(t) \ominus \tilde{\mu}_i \tilde{P}_{i+2}(t), \quad i = 1,2,3,4,5 \\
\frac{d\tilde{P}_{i+7}(t)}{dt} &= \tilde{\lambda}_i \tilde{P}_2(t) \ominus \tilde{\mu}_i \tilde{P}_{i+7}(t), \quad i = 1,2,3,4,5 \\
\frac{d\tilde{P}_{13}(t)}{dt} &= \tilde{\lambda}_7 \tilde{P}_2(t) \ominus \tilde{\mu}_7 \tilde{P}_{13}(t)
\end{aligned}
\tag{4.3.3}
$$

where

$$\tilde{\delta}_1 = \tilde{\lambda}_1 \oplus \tilde{\lambda}_2 \oplus \tilde{\lambda}_3 \oplus \tilde{\lambda}_4 \oplus \tilde{\lambda}_5 \oplus \tilde{\lambda}_6$$

$$\tilde{\delta}_2 = \tilde{\lambda}_1 \oplus \tilde{\lambda}_2 \oplus \tilde{\lambda}_3 \oplus \tilde{\lambda}_4 \oplus \tilde{\lambda}_5 \oplus \tilde{\lambda}_7 \oplus \tilde{\mu}_6$$

with initial conditions in R_2 as

$$\tilde{P}_1(0) = (0.95, 0.955, 0.965, 0.97)$$

$$\tilde{P}_2(0) = (0.004, 0.0045, 0.0055, 0.006)$$

$$\tilde{P}_j(0) = 0,\ j = 3, 6, ..., 13.$$

Thus availability of the subsystem R_2 is

$$\tilde{Av}_{R_2} = \tilde{P}_1(t) \oplus \tilde{P}_2(t), \tag{4.3.4}$$

as the states having the probabilities $\tilde{P}_1$ and $\tilde{P}_2$ are the only working states of R_2. Hence, the availability of the whole system (i.e. piston manufacturing plant) is the product of the availabilities of R_1 and R_2 as given by Eq. (4.3.2) and (4.3.4) i.e.

$$\tilde{Av} = \tilde{Av}_{R_1} \otimes \tilde{Av}_{R_2}. \tag{4.3.5}$$

4.3.6 Steady State Analysis

For long term availability of the system R_1, steady state probabilities of system R_1 are obtained by applying following limitations on probabilities.

$$\frac{d}{dt} \to 0, \quad \text{as } t \to \infty.$$

In this case study, following system of equations are obtained by imposing the above restrictions.
$P_5 = \frac{l_1}{m_1}P_1,\ P_6 = \frac{l_2}{m_2}P_1,\ P_7 = \frac{l_3}{m_3}P_1,\ P_8 = \frac{l_4}{m_4}P_1,\ P_9 = \frac{l_1}{m_1}P_2,\ P_{10} = \frac{l_2}{m_2}P_2,$
$P_{11} = \frac{l_3}{m_3}P_2,\ P_{12} = \frac{l_4}{m_4}P_2,\ P_{13} = \frac{l_1}{m_1}P_3,\ P_{14} = \frac{l_2}{m_2}P_3,\ P_{15} = \frac{l_3}{m_3}P_3,\ P_{16} = \frac{l_4}{m_4}P_3,$
$P_{17} = \frac{l_7}{m_7}P_2,\ P_{18} = \frac{l_8}{m_8}P_3,\ P_{19} = \frac{l_7}{m_7}P_4,\ P_{20} = \frac{l_8}{m_8}P_4,\ P_{21} = \frac{l_1}{m_1}P_4,\ P_{22} = \frac{l_2}{m_2}P_4,$
$P_{23} = \frac{l_3}{m_3}P_4,\ P_{24} = \frac{l_4}{m_4}P_4.$
Substituting these values of the probabilities in the normalizing condition $\sum_{i=1}^{24} P_i = 1$, steady state availability for system R_1 becomes:

$$Av = P_1 + P_2 + P_3 + P_4. \tag{4.3.6}$$

For long term availability of the system R_2, steady state probabilities of system R_2 are obtained by applying following limitations on probabilities.

$$\frac{d}{dt} \to 0, \quad \text{as } t \to \infty.$$

In this case study, following system of equations are obtained by imposing the above restrictions.
$P_2 = \frac{\lambda_6}{(\mu_6 + \lambda_7)}P_1,\ P_3 = \frac{\lambda_1}{\mu_1}P_1,\ P_4 = \frac{\lambda_2}{\mu_2}P_1,\ P_5 = \frac{\lambda_3}{\mu_3}P_1,\ P_6 = \frac{\lambda_4}{\mu_4}P_1,\ P_7 = \frac{\lambda_5}{\mu_5}P_1,$
$P_8 = \frac{\lambda_1}{\mu_1}P_2,\ P_9 = \frac{\lambda_2}{\mu_2}P_2,\ P_{10} = \frac{\lambda_3}{\mu_3}P_2,\ P_{11} = \frac{\lambda_4}{\mu_4}P_2,\ P_{12} = \frac{\lambda_5}{\mu_5}P_2,\ P_{13} = \frac{\lambda_7}{\mu_7}P_2.$
Substituting these values of P_1 to P_{13} in the normalizing condition $\sum_{i=1}^{13} P_i = 1$,

Steady state availability becomes:

$$Av = P_1 + P_2 = \frac{1}{\left[1 + \frac{\lambda_6}{(\mu_6 + \mu_7)} + \frac{\lambda_7\lambda_6}{\mu_7(\mu_6 + \mu_7)}\right] + \left[\frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} + \ldots + \frac{\lambda_5}{\mu_5}\right]\left(1 + \frac{\lambda_6}{\mu_6 + \mu_7}\right)}. \tag{4.3.7}$$

4.3.7 Results and Discussion

In this section, availability of piston manufacturing plant has been evaluated by solving the set of differential equations (4.3.2) and (4.3.4) through proposed approach. For different α- cut values, availability of piston manufacturing plant has been evaluated for mission time $t = 360$ hours. Results have been computed by formulating optimization problem. Availability of any system is sum of probabilities of its working states. In order to evaluate availability, fuzzy differential equations have been formulated first, in multi-objective optimization problem through proposed algorithm and then subsequent multi-objective optimization problem has been solved by linearisation. Weights are considered equal and non-zero corresponding to working state probabilities and zero otherwise.

This solution approach has been performed in MATLAB (Mathworks). In the present analysis, population size are set to be randomly as $25 \times D$, where D is the dimension of the problem. In order to eliminate stochastic discrepancy, 15 independent runs have been made that involves 15 different initial trial solutions. In this case, acceleration coefficient parameters c_1 and c_2 are taken as c_1=c_2=2 with inertia weight w, explained as $w = w_{max} - (w_{max} - w_{min}) * iter/iter_{max}$. Here $w_{max} = 0.9$ and $w_{min} = 0.4$ are taken as maximum and minimum values of inertial weight respectively and $iter_{max}$ indicates the maximum generation number(=100). The termination criterion has been set either to relative error equal to 10^{-6} or maximum number of generations, whichever is obtained first.

Availability has been computed through Garg approach [91] and by given approach and results have been summarized in Table 4.6 and Table 4.7. It may be noted that results obtained from Garg approach, shown in Table 4.6 do not follow to be a fuzzy number as such. However, since we have taken intervals the left and right values of the intervals are to be interchanged to see that the results obtained by Garg approach also follow to be fuzzy number.

	Availability of subsystem R_1		Availability of subsystem R_2		Availability of the system	
$\alpha\downarrow$	$(\tilde{A}v_{R_1}(t))^{\alpha}_{(L)}$	$(\tilde{A}v_{R_1}(t))^{\alpha}_{(R)}$	$(\tilde{A}v_{R_2}(t))^{\alpha}_{(L)}$	$(\tilde{A}v_{R_2}(t))^{\alpha}_{(R)}$	$(\tilde{A}v(t))^{\alpha}_{(L)}$	$(\tilde{A}v(t))^{\alpha}_{(L)}$
0	0.921100	0.922437	0.942109	0.956778	0.867776	0.882567
0.1	0.920975	0.922467	0.942449	0.956452	0.867972	0.882296
0.2	0.920851	0.922501	0.942789	0.956128	0.868168	0.882028
0.3	0.920728	0.922538	0.943130	0.955803	0.868366	0.881765
0.4	0.920607	0.922579	0.943471	0.955480	0.868566	0.881505
0.5	0.920488	0.922623	0.943812	0.955157	0.868768	0.881250
0.6	0.920371	0.922672	0.944154	0.954834	0.868972	0.880999
0.7	0.920256	0.922724	0.944496	0.954513	0.869178	0.880751
0.8	0.920143	0.922779	0.944839	0.954192	0.869387	0.880508
0.9	0.920032	0.922839	0.945182	0.953871	0.869598	0.880269
1	0.919925	0.922902	0.945526	0.953551	0.869812	0.880034

Table 4.6: Availability of the system at $t = 360$ hours by Garg approach [91]

	Availability of subsystem R_1		Availability of subsystem R_2		Availability of the system	
$\alpha\downarrow$	$(\tilde{A}v_{R_1}(t))^{\alpha}_{(L)}$	$(\tilde{A}v_{R_1}(t))^{\alpha}_{(R)}$	$(\tilde{A}v_{R_2}(t))^{\alpha}_{(L)}$	$(\tilde{A}v_{R_2}(t))^{\alpha}_{(R)}$	$(\tilde{A}v(t))^{\alpha}_{(L)}$	$(\tilde{A}v(t))^{\alpha}_{(L)}$
0	0.891303	0.949526	0.933012	0.964967	0.831596	0.916261
0.1	0.893620	0.946656	0.933917	0.964298	0.834567	0.912858
0.2	0.894563	0.946395	0.934774	0.963429	0.836214	0.911784
0.3	0.896429	0.945401	0.935740	0.962696	0.838824	0.910134
0.4	0.897993	0.943839	0.936579	0.961889	0.841041	0.907868
0.5	0.899701	0.942103	0.937483	0.961078	0.843454	0.905434
0.6	0.901600	0.940689	0.938639	0.960264	0.846277	0.903310
0.7	0.903280	0.938646	0.939477	0.959448	0.848611	0.900582
0.8	0.904865	0.937457	0.940364	0.958594	0.850902	0.898641
0.9	0.906547	0.935546	0.941074	0.957770	0.853128	0.896038
1	0.908705	0.934300	0.941963	0.956986	0.855966	0.894112

Table 4.7: Availability of the system at $t = 360$ hours by proposed approach

(i) Results for system availability for different α-cuts have been summarized in tabular form (Table 4.6 and Table 4.7) as obtained by Garg approach and our approach.

We note that the whole system availability lies in the intervals $[0.831596,\ 0.916261]$ and $[0.867776,\ 0.882567]$ by the proposed and Garg approach respectively. It can be observed from the Table 4.6 that Garg approach does not always generate fuzzy solution as such whereas results obtained through proposed solution approach provides fuzzy solution. For instance by Garg appraoch, left cuts for availability of system R_1 are 0.921100, 0.920975 and 0.920851 corresponding to $\alpha = 0, 0.1$ and 0.2 respectively which shows that as α increases, left cuts of availability are not increasing. However, by proposed approach, left cuts of availability of system R_1 are 0.891303, 0.893620 and 0.894563 corresponding to $\alpha = 0,\ 0.1$ and 0.2 respectively which clearly shows that left cut of availability are increasing as α increases. It reflects that Garg approach is not suitable for this study.

(ii) Fuzzy availability analysis of the piston manufacturing plant by JMD approach has also been studied by Lata and Kumar [173]. System availability of the system lies in the intervals $[0.831596,\ 0.916261]$ and $[0.867774,\ 0.908151]$ for different presumption level by proposed approach and JMD approach respectively. While comparing the results, it can be observed that proposed solution approach optimizes the spread in comparison to other existing methods. For instance, for $\alpha = 0.4$, availability of system are $[0.871850,\ 0.903996]$ and $[0.841041,\ 0.907868]$ by JMD and proposed approach respectively which shows an increment of 107% in the interval. Proposed approach provides increment in the interval for every corresponding presumption level which shows the optimized interval solution. Comparison has been shown in Figure 4.7.

(iii) Proposed solution approach optimizes results, by which system analyst can predict the behavior of industrial systems in a more consistent manner. Based on the above analysis, maintenance schedule can be prepared which might help the maintenance managers to improve the system effectiveness by adopting suitable preventive maintenance actions.

(iv) Steady state sensitivity analysis has also been discussed (described in Section 4.3.6). Table 4.8 shows the simultaneous effect of failure and repair rates on availability on each subsystem of system R_1. Sensitivity analysis on each components on R_1 has been shown in Figure 4.8. The performance of system can be improved by this analysis and appropriate maintenance strategies. From the results as shown in Figure 4.8, it has been obtained that, to save time and money, necessary actions should be taken as per preferential order so that the system analyst can obtain high production goals along with maintaining its performance. It may be observed from Table 4.8 that 40% decrement in failure rates and 10.53% increment in repair rates produce different variations in overall availability of R_1. For instance, it result 0.07% gain in availability by these variations in Subsystem A while 0.80% gain in availability by these variations in Subsystem B. These variations in Subsystem C result 1.09% gain in availability, Subsystem $\overline{C}$ result 0.31% gain in availability, Subsystem D result 0.10% gain in availability, Subsystem E result 0.33% gain in availability and Subsystem $\overline{C}$ result 0.69% gain in availability of R_1.

(v) Steady state analysis on R_2 has been studied and depicted in Table 4.9. Figure 4.9 shows the simultaneous effects of failure and repair rates of each component on availability as per preferential order for long term availability. It may be observed from Table 4.9 that 40% decrement in failure rate and 10.53% increment in repair rate of Subsystem H result 0.25% gain in availability of R_2, 39.47% decrement in failure rate and 10.53% increment in repair rate of Subsystem I result 0.025% gain in availability of R_2, 38% decrement in failure rate and 10.53% increment in repair rate of Subsystem J result 0.0063% gain in availability, 30.30% decrement in failure rate and 9.52% increment in repair of Subsystem K rate produce 6.48% gain in availability, 20% decrement in failure rate and 10.53% increment in repair rate show 0.00129% gain in availability, 40% decrement in failure rate and 10.53% increment in repair rate of Subsystem G produce 1.98% gain in availability, 40% decrement in failure rate and 10.53% increment in repair rate produce .11% gain in availability of R_2.

It may be observed in this study that variations in rates result variation in availability, which may be important for system analyst to obtain high production goals along with maintaining its performance.

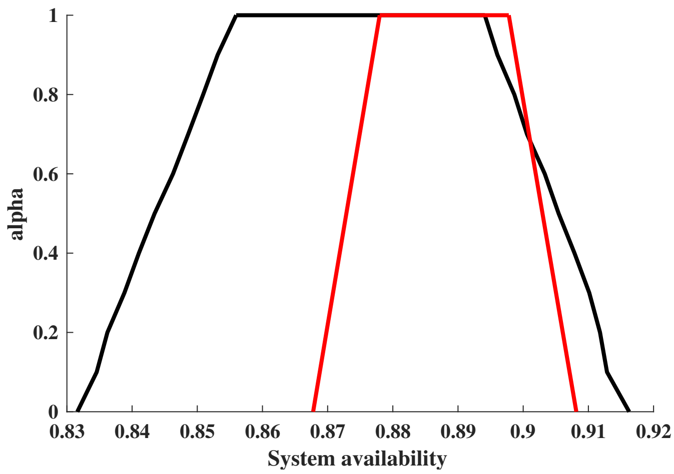
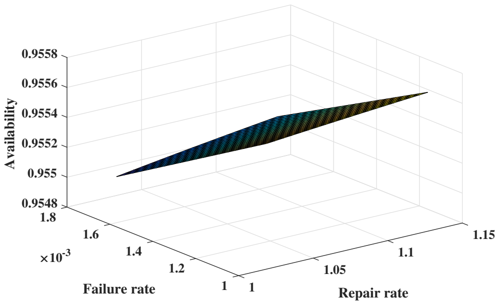


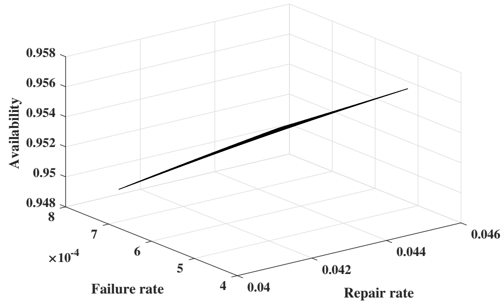
Figure 4.7: Availability of Piston manufacturing plant by JMD (in red) and given solution approach (in black)

4.4 Conclusions

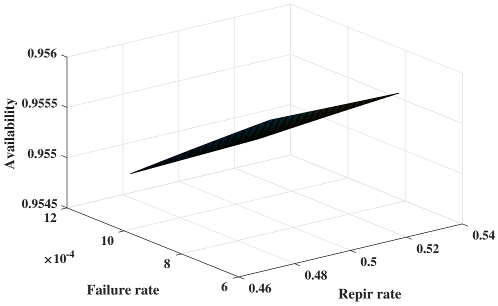
In this chapter, an approach for numerical solution of differential equations has been given by formulating objective optimization problem. Additionally, some examples



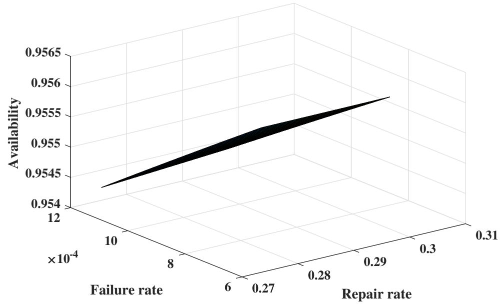
(a) Subsystem A



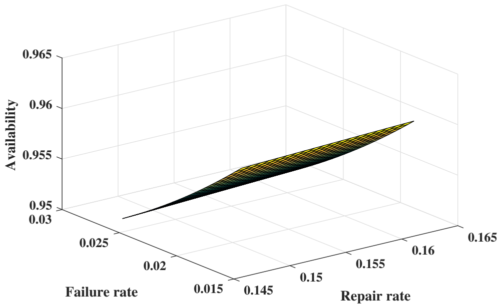
(b) Subsystem B



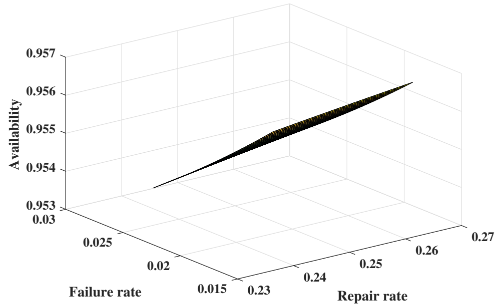
(c) Subsystem D



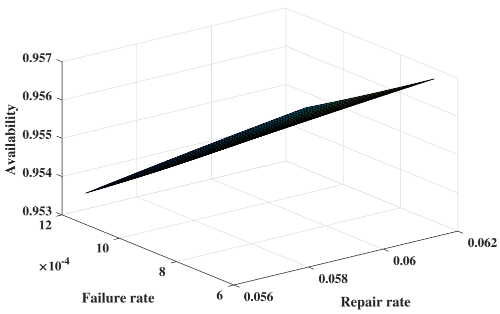
(d) Subsystem F



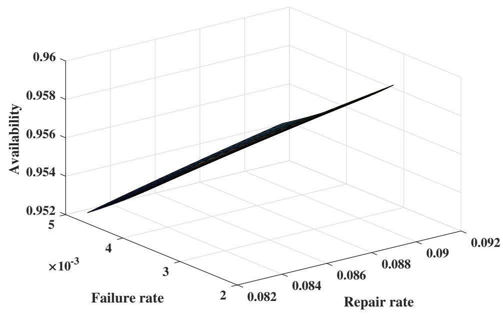
(e) Subsystem C



(f) Subsystem E

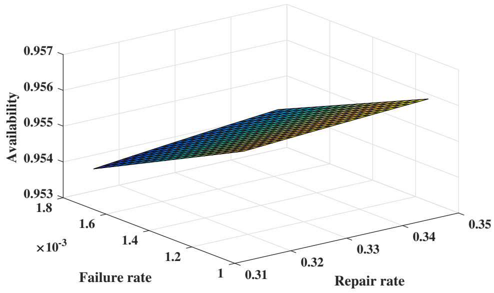


(g) Subsystem $\overline{C}$

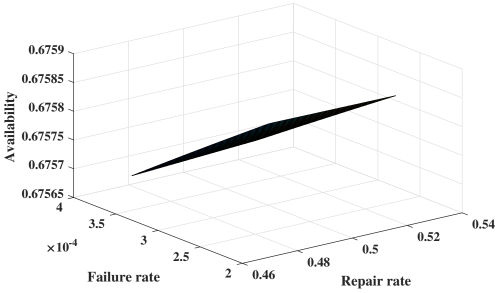


(h) Subsystem $\overline{E}$

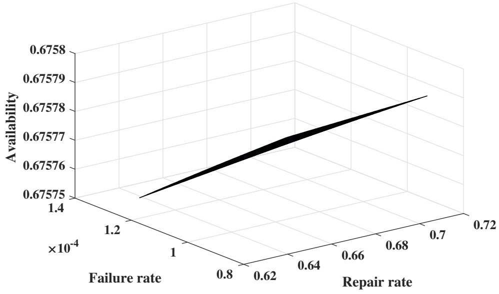
Figure 4.8: Simultaneous effects of repair and failure rates on availability of system R_1



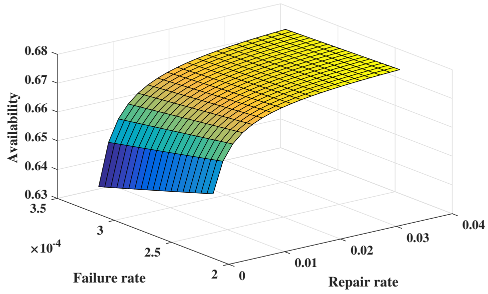
(a) Subsystem H



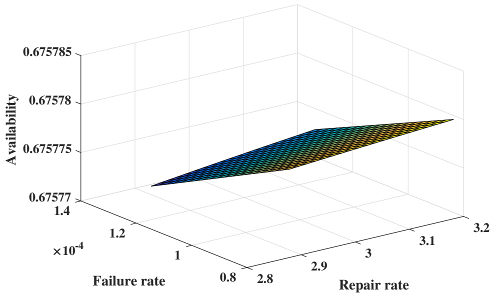
(b) Subsystem I



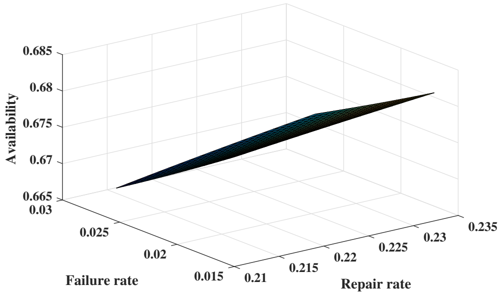
(c) Subsystem J



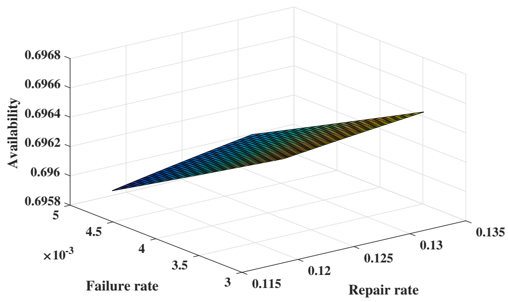
(d) Subsystem K



(e) Subsystem L



(f) Subsystem G



(g) Subsystem $\overline{G}$

Figure 4.9: Simultaneous effects of repair and failure rates on availability of system R_2

Component	Failure rate (λ)	Repair rate (μ)	Availability (Min,Max)
Subsystem A	0.00105-0.00175	1.026-1.134	(0.954971, 0.955682)
Subsystem B	0.00045-0.00075	0.04085-0.04515	(0.949239, 0.956863)
Subsystem D	0.000675-0.001125	0.475-0.525	(0.954826, 0.955814)
Subsystem F	.000675-0.001125	0.2717-0.3003	(0.954438, 0.956165)
Subsystem C	.0156-0.026	0.1463-0.1617	(0.950590,0.960956)
Subsystem E	0.0156-0.026	0.2375-0.2625	(0.953788,0.956957)
Subsystem $\overline{C}$	0.000675-0.001125	.05605,0.06195	(0.953781,0.956764)
Subsystem $\overline{E}$	.002925-0.004875	0.08265,0.09135	(0.952065,0.9586529)

Table 4.8: Simultaneous effects of failure and repair rates on availability of system R_1

Component	Failure rate (λ)	Repair rate (μ)	Availability (Min,Max)
Subsystem H	0.00105-0.00175	0.3135-0.3465	(0.982675, 0.985145)
Subsystem I	0.00023-0.00038	0.475-0.525	(0.696141, 0.696316)
Subsystem J	0.00008-0.00013	0.6365-0.7035	(0.696211, 0.696255)
Subsystem K	0.00023-0.00033	0.03325-0.03675	(0.654893, 0.697361)
Subsystem L	0.00008-0.0001	2.8785-3.1815	(0.696232,0.696241)
Subsystem G	0.0156-0.026	0.2109-0.2331	(0.668817,0.682057)
Subsystem $\overline{G}$	0.003-0.005	0.11875-0.13125	(0.695832,0.696608)

Table 4.9: Simultaneous effects of failure and repair rates on availability of system R_2

have been solved through proposed algorithm. It has been observed that proposed solution approach not only finds a solution of fuzzy differential equations but also optimize the solution. Optimization increases the uncertainty but it also provides the wide range for the solution. Along with algorithm, availability of piston manufacturing plant has been computed as an application of proposed approach. This chapter replies to an interesting question, how to evaluate the solution of fuzzy differential equations for practical applications through multi-objective optimization problem? In this chapter, it has been shown through the evaluation of availability of the system, by formulating the multi-objective optimization problem. In this case, multi-objective optimization has been dealt by linearisation and suitable weights have been chosen for the problem. It has been found that solution through the approach predicts the behavior of industrial systems in more realistic and consistent manner. The prediction of system behavior through this approach is useful for system analyst in order to improve the system performance.

Chapter 5

Availability analysis of Industrial systems using Markov process and generalized fuzzy numbers

In this chapter, reliability/availability has been discussed using Markov process and generalized fuzzy numbers. An approach has been discussed to evaluate reliability/availability through different arithmetic operations. Results have been computed and then compared by performing different arithmetic operations' approaches. For application perspective of proposed approach, a butter-oil processing plant has been considered. Impacts of degree of confidence through different arithmetic approaches in the methodology are reflected by numerical calculations and are depicted through the graphs.

5.1 Introduction

Today with growing complexity of the repairable industrial systems along with advances in technology, it is difficult, for the system analyst to analyze and predict the behavior of the industrial system in a more realistic and proper manner. Thus, system reliability analysis is an important issue for academic research and practice. Realising this, various researchers [6, 90, 264, 278] have paid more attentions to the system behavior by using conventional and non-conventional techniques. In order to predict their behavior, data related to system parameters are generally evaluated from historical records or existing databases, which are generally imprecise in nature. Thus, if data are used as in the analysis then computed results contain a high amount of uncertainties. Therefore, there is a need for developing such type of methodology which will reduce the uncertainties, for each reliability index, up to a desired degree of accuracy so that plant personnel may use these indices to analyze the system behavior more closely and take more sound decisions to improve the performance of the plant. In this study, reliability parameters like failure rates and repair rates are considered as generalized fuzzy numbers as a generalization of fuzzy numbers and using different arithmetic operations, reliability has been computed.

In the present chapter, a methodology has been explored for availability analysis in more generalized way. To handle uncertainties in data, generalized fuzzy numbers have been used in both failure and repair rates. In this chapter, constant failure and repair rates model has been taken during analysis. Using different arithmetic operations, system availability has been discussed in its transient as well as in steady state. For application point of view, butter-oil processing plant as a repairable industrial system has been studied.
In the following section, we describe some different types of arithmetic operations on generalized fuzzy numbers.

5.2 Arithmetic Operations

The scientific literature on fuzzy arithmetic operations is rich in terms of several approaches to define fuzzy operations having many desired properties. In order to generalize the concepts of arithmetic operations for generalized fuzzy numbers, following approaches have been given with their own advantages/disadvantages. Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2; w_2)$ be two generalized trapezoidal fuzzy numbers, where $a_1, b_1, c_1, d_1, w_1, a_2, b_2, c_2, d_2$ and w_2 are all real numbers, with the assumption that both w_1 and w_2 ($w_1 \leq w_2$) belong to the closed interval $[0, 1]$. If $b_1 = c_1$ and $b_2 = c_2$, then above generalized trapezoidal fuzzy numbers can be reduced to generalized triangular fuzzy numbers.

5.2.1 Chen's arithmetic operations

Based on extension principle, Chen [45] adopted the four basic arithmetic operations between two generalized fuzzy numbers $\tilde{A}_1$ and $\tilde{A}_2$ as follows:

- **Addition:** $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; w)$
- **Subtraction:** $\tilde{A}_1 \ominus \tilde{A}_2 = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2; w)$
- **Multiplication:** $\tilde{A}_1 \otimes \tilde{A}_2 = (a_1.a_2, b_1.b_2, c_1.c_2, d_1.d_2; w)$
- **Division:** $\tilde{A}_1 \oslash \tilde{A}_2 = (a_1/d_2, b_1/c_2, c_1/b_2, d_1/a_2; w)$

where, $w = min(w_1, w_2)$.

5.2.2 Normalized arithmetic operations

Computational complications are avoided in normalized approach [63] as following steps are taken in performing the arithmetic operations.

(a) Step 1: The given generalized fuzzy numbers are normalized to get the corresponding normal fuzzy numbers

(b) Step 2: Fuzzy arithmetic operations are performed in the obtained normalized fuzzy numbers. The resulting fuzzy number will also be a normalized fuzzy number.

(c) Step 3: The resulted generalized fuzzy number is obtained by truncating the resulting normalized fuzzy number to the minimum height of the given generalized fuzzy numbers.

For two generalized trapezoidal fuzzy numbers, $\tilde{A}_1$ and $\tilde{A}_2$, the membership function of resulting generalized fuzzy numbers after performing arithmetic operations according to the these three steps are given below.

- **Addition:**

  (i) When $x \in [a_1 + a_2, a_1 + a_2 + w((b_1 + b_2) - (a_1 + a_2))]$,
  $$\mu_{\tilde{A}_1 \oplus \tilde{A}_2}(x) = \frac{x - (a_1 + a_2)}{(b_1 + b_2) - (a_1 + a_2)}.$$

  (ii) When $x \in [a_1+a_2+w((b_1+b_2)-(a_1+a_2)), d_1+d_2-w((d_1+d_2)-(c_1+c_2))]$,
  $$\mu_{\tilde{A}_1 \oplus \tilde{A}_2}(x) = w.$$

  (iii) When $x \in [d_1 + d_2 - w((d_1 + d_2) - (c_1 + c_2)), d_1 + d_2]$,
  $$\mu_{\tilde{A}_1 \oplus \tilde{A}_2}(x) = \frac{(d_1 + d_2) - x}{(d_1 + d_2) - (c_1 + c_2)}.$$

- **Subtraction:**

  (i) When $x \in [a_1 - d_2, (a_1 - d_2) + w((b_1 - c_2) - (a_1 - d_2))]$,
  $$\mu_{\tilde{A}_1 \ominus \tilde{A}_2}(x) = \frac{x - (a_1 - d_2)}{(b_1 - c_2) - (a_1 - d_2)}.$$

  (ii) When $x \in [a_1-d_2+w((b_1-c_2)-(a_1-d_2)), (d_1-a_2)-w((d_1-a_2)-(c_1-b_2))]$,
  $$\mu_{\tilde{A}_1 \ominus \tilde{A}_2}(x) = w.$$

  (iii) When $x \in [(d_1 - a_2) - w((d_1 - a_2) - (c_1 - b_2)), d_1 - a_2]$,
  $$\mu_{\tilde{A}_1 \ominus \tilde{A}_2}(x) = \frac{(d_1 - a_2) - x}{(d_1 - a_2) - (c_1 - b_2)}.$$

- **Multiplication:**

(i) When $x \in [a_1a_2, a_1a_2 + w\{(b_1 - a_1)a_2 + (b_2 - a_2)a_1\} + w^2(b_1 - a_1)(b_2 - a_2)]$,

$$\mu_{\tilde{A}_1 \otimes \tilde{A}_2}(x) = \frac{-\{(b_1 - a_1)a_2 + (b_2 - a_2)a_1\}}{2(b_1 - a_1)(b_2 - a_2)} + \frac{1}{2(b_1 - a_1)(b_2 - a_2)}\Big\{\{(b_1 - a_1)a_2 + (b_2 - a_2)a_1\}^2 - 4(b_1 - a_1)(b_2 - a_2).(a_1a_2 - x)\Big\}^{1/2}.$$

(ii) When $x \in [a_1a_2 + w\{(b_1 - a_1)a_2 + (b_2 - a_2)a_1\} + w^2(b_1 - a_1)(b_2 - a_2), d_1d_2 - w\{(d_1 - c_1)d_2 + (d_2 - c_2)d_1\} + w^2(d_1 - c_1)(d_2 - c_2)]$,

$\mu_{\tilde{A}_1 \otimes \tilde{A}_2}(x) = w$.

(iii) When $x \in [a_1a_2 + w\{(b_1 - a_1)a_2 + (b_2 - a_2)a_1\} + w^2(b_1 - a_1)(b_2 - a_2), d_1d_2 - w\{(d_1 - c_1)d_2 + (d_2 - c_2)d_1\} + w^2(d_1 - c_1)(d_2 - c_2)]$,

$$\mu_{\tilde{A}_1 \otimes \tilde{A}_2}(x) = \frac{\{(d_1 - c_1)d_2 + (d_2 - c_2)d_1\}}{2(d_1 - c_1)(d_2 - c_2)} - \frac{1}{2(d_1 - c_1)(d_2 - c_2)}\Big\{\{(d_1 - c_1)d_2 + (d_2 - c_2)d_1\}^2 - 4(d_1 - c_1)(d_2 - c_2).(d_1d_2 - x)\Big\}^{1/2}.$$

- **Division:**

  (i) When $x \in \left[\frac{a_1}{d_2}, \frac{a_1 + w(b_1 - a_1)}{d_2 - w(d_2 - c_2)}\right]$, $\mu_{\tilde{A}_1 \oslash \tilde{A}_2}(x) = \frac{d_2x - a_1}{(b_1 - a_1) + x(d_2 - c_2)}$.

  (ii) When $x \in \left[\frac{a_1 + w(b_1 - a_1)}{d_2 - w(d_2 - c_2)}, \frac{d_1 - w(d_1 - c_1)}{a_2 + w(b_2 - a_2)}\right]$, $\mu_{\tilde{A}_1 \oslash \tilde{A}_2}(x) = w$.

  (iii) When $x \in \left[\frac{d_1 - w(d_1 - c_1)}{a_2 + w(b_2 - a_2)}, \frac{d_1}{d_2}\right]$, $\mu_{\tilde{A}_1 \oslash \tilde{A}_2}(x) = \frac{d_1 - a_2x}{x(b_2 - a_2)(d_1 - c_1)}$.

5.2.3 Improved Arithmetic Operations

Again based on extension principle, Dat et al. [62] derived in 2013 the arithmetic operations on generalized fuzzy numbers. To obtain the arithmetic operations between two generalized trapezoidal fuzzy numbers $\tilde{A}_1$ and $\tilde{A}_2$, firstly, take w-cut where

$w = w_1(< w_2)$ of fuzzy number $\tilde{A}_2 = (a_2, b_2, c_2, d_2; w_2)$, and transform the generalized fuzzy number $\tilde{A}_2$ into new generalized fuzzy number $\tilde{A}'_2 = (a_2, b'_2, c'_2, d_2; w'_2)$, where $w'_2 = w$ and $b'_2 = a_2 + w(b_2 - a_2)/w_2$ and $c'_2 = d_2 + w(d_2 - c_2)/w_2$.
Improved arithmetic approach is based on interval arithmetic, let $*$ denote any of the four basic arithmetic operations. Fuzzy set $\tilde{A}_1 * \tilde{A}_2$ defined on R, the set of real numbers, is defined in terms of its α-cut, $(\tilde{A}_1 * \tilde{A}_2)^\alpha$, as

$$(\tilde{A}_1 * \tilde{A}_2)^\alpha = (\tilde{A}_1 * \tilde{A}'_2)^\alpha = \tilde{A}_1^\alpha * \tilde{A}_2^{\prime\alpha} \text{ for any } \alpha \in (0, w]$$

The membership function of resulting generalized trapezoidal fuzzy numbers after performing improved arithmetic operations on two generalized trapezoidal fuzzy numbers will be,

- **Addition:**

  (i) When $x \in \left[a_1 + a_2, b_1 + a_2 + \dfrac{w}{w_2}(b_2 - a_2)\right]$, $\mu_{\tilde{A}_1 \oplus \tilde{A}_2}(x) = \dfrac{x - (a_1 + a_2)}{\left(\frac{b_1 - a_1}{w}\right) + \left(\frac{b_2 - a_2}{w_2}\right)}$.

  (ii) When $x \in \left[b_1 + a_2 + \dfrac{w}{w_2}(b_2 - a_2), d_1 + d_2 - \dfrac{w}{w_2}(d_2 - c_2)\right]$, $\mu_{\tilde{A}_1 \oplus \tilde{A}_2}(x) = w$.

  (iii) When $x \in \left[d_1 + d_2 - \dfrac{w}{w_2}(d_2 - c_2), d_1 + d_2\right]$, $\mu_{\tilde{A}_1 \oplus \tilde{A}_2}(x) = \dfrac{(d_1 + d_2) - x}{\left(\frac{d_1 - c_1}{w}\right) + \left(\frac{d_2 - c_2}{w_2}\right)}$.

- **Subtraction**

  (i) When $x \in \left[a_1 - d_2, b_1 - d_2 + \dfrac{w}{w_2}(d_2 - c_2)\right]$, $\mu_{\tilde{A}_1 \ominus \tilde{A}_2}(x) = \dfrac{x - (a_1 - d_2)}{\left(\frac{b_1 - a_1}{w}\right) - \left(\frac{d_2 - c_2}{w_2}\right)}$.

  (ii) When $x \in \left[b_1 - d_2 + \dfrac{w}{w_2}(d_2 - c_2), c_1 - a_2 - \dfrac{w}{w_2}(b_2 - a_2)\right]$, $\mu_{\tilde{A}_1 \ominus \tilde{A}_2}(x) = w$.

(iii) When $x \in \left[c_1 - a_2 - \dfrac{w}{w_2}(b_2 - a_2), d_1 - a_2\right]$, $\mu_{\tilde{A}_1 \ominus \tilde{A}_2}(x) = \dfrac{(d_1 - d_2) - x}{\left(\frac{d_1 - c_1}{w}\right) + \left(\frac{b_2 - a_2}{w_2}\right)}$.

- **Multiplication**

(i) When $x \in \left[a_1 a_2, \dfrac{w}{w_2}(b_1 b_2 - b_1 a_2) + b_1 a_2\right]$,

$$\mu_{\tilde{A}_1 \otimes \tilde{A}_2}(x) = \frac{-N_1 + \sqrt{N_1^2 + 4\ M_1(x - P_1)}}{2\ M_1}.$$

(ii) When $x \in \left[\dfrac{w}{w_2}(b_1 b_2 - b_1 a_2) + b_1 a_2, \dfrac{w}{w_2}(c_1 c_2 - c_1 d_2) + c_1 d_2\right]$,

$$\mu_{\tilde{A}_1 \otimes \tilde{A}_2}(x) = w.$$

(iii) When $x \in \left[\dfrac{w}{w_2}(c_1 c_2 - c_1 d_2) + c_1 d_2, d_1 d_2\right]$,

$$\mu_{\tilde{A}_1 \otimes \tilde{A}_2}(x) = \frac{-N_2 - \sqrt{N_2^2 + 4\ M_2(x - P_2)}}{2\ M_2},$$

where

$$\begin{aligned} M_1 &= \frac{a_1(b_2 - a_2)}{w_2} + \frac{a_2(b_1 - a_1)}{w}, \quad M_2 = \frac{d_1(d_2 - c_2)}{w_2} + \frac{d_2(d_1 - c_1)}{w}, \\ N_1 &= \frac{(b_1 - a_1)(b_2 - a_2)}{w\ w_2}, \quad N_2 = \frac{(d_1 - c_1)(d_2 - c_2)}{w\ w_2}, \\ P_1 &= a_1 a_2, \quad P_2 = d_1 d_2. \end{aligned}$$

- **Division**

(i) When $x \in \left[\dfrac{a_1}{d_2}, \dfrac{b_1}{d_2 - \frac{w}{w_2}(d_2 - c_2)}\right]$, $\mu_{\tilde{A}_1 \oslash \tilde{A}_2}(x) = \dfrac{x\ d_2 - a_1}{\left[\left(\frac{b_1 - a_1}{w}\right) + w\left(\frac{d_2 - c_2}{w_2}\right)\right]}$.

(ii) When $x \in \left[\dfrac{b_1}{d_2 - \frac{w}{w_2}(d_2 - c_2)}, \dfrac{c_1}{a_2 + \frac{w}{w_2}(b_2 - a_2)}\right]$, $\mu_{\tilde{A}_1 \oslash \tilde{A}_2}(x) = w$.

(iii) When $x \in \left[\frac{c_1}{a_2 + \frac{w}{w_2}(b_2 - a_2)}, \frac{d_1}{d_2}\right]$, $\mu_{\tilde{A}_1 \oslash \tilde{A}_2}(x) = \frac{d - x\ a_2}{\left[\left(\frac{d_1 - c_1}{w}\right) + \frac{x}{w_2}(b_2 - a_2)\right]}$.

To demonstrate all the three types of above discussed arithmetic operations on generalized trapezoidal fuzzy numbers, arithmetic operations on $\tilde{A}_1 = (20, 22, 25, 30; 0.8)$ and $\tilde{A}_2 = (4, 6, 8, 10; 0.6)$ are computed and listed in Table 5.1.

Arithmetic operation	**Chen's Approach**	**Normalized Approach**	**Improved Arithmetic operations**
Addition	$\begin{cases} 0.6\left(\frac{x-24}{4}\right) & 24 \le x \le 28 \\ 0.6 & 28 \le x \le 33 \\ 0.6\left(\frac{40-x}{7}\right) & 33 \le x \le 40 \end{cases}$	$\begin{cases} \frac{x-24}{4} & 20 \le x \le 26.4 \\ 0.6 & 26.4 \le x \le 35.8 \\ \frac{40-x}{7} & 35.8 \le x \le 40 \end{cases}$	$\begin{cases} \frac{6}{35}(24-x) & 24 \le x \le 27.5 \\ 0.6 & 27.5 \le x \le 34.25 \\ \frac{12}{115}(40-x) & 34.25 \le x \le 40 \end{cases}$
Subtraction	$\begin{cases} 0.6\left(\frac{x-10}{4}\right) & 10 \le x \le 14 \\ 0.6 & 14 \le x \le 19 \\ 0.6\left(\frac{26-x}{7}\right) & 19 \le x \le 26 \end{cases}$	$\begin{cases} \frac{x-10}{4} & 10 \le x \le 12.4 \\ 0.6 & 12.4 \le x \le 21.8 \\ \frac{26-x}{7} & 21.8 \le x \le 26 \end{cases}$	$\begin{cases} \frac{6}{35} & 10 \le x \le 13.5 \\ 0.6 & 13.5 \le x \le 20.25 \\ 12\left(\frac{26-x}{115}\right) & 20.25 \le x \le 26 \end{cases}$
Multiplication	$\begin{cases} 0.6\left(\frac{x-80}{52}\right) & 80 \le x \le 132 \\ 0.6 & 132 \le x \le 200 \\ 0.6\left(\frac{300-x}{100}\right) & 200 \le x \le 300 \end{cases}$	$\begin{cases} \frac{-24+\sqrt{416+2x}}{256} & 80 \le x \le 110.24 \\ 0.6 & 110.24 \le x \le 237.6 \\ \frac{55-\sqrt{25+10x}}{10} & 237.6 \le x \le 300 \end{cases}$	$\begin{cases} \frac{-230}{3} + \sqrt{\left(\frac{230}{3}\right)^2 - \frac{200}{6}(80-x)} & 80 \le x \le 129 \\ 0.6 & 129 \le x \le 243.75 \\ \frac{425}{4} - \sqrt{\left(\frac{425}{4}\right)^2 - \frac{250}{3}(300-x)} & 243.75 \le x \le 300 \end{cases}$
Division	$\begin{cases} 0.6\left(\frac{x-2}{0.75}\right) & 2 \le x \le \frac{22}{8} \\ 0.6 & \frac{22}{8} \le x \le \frac{25}{6} \\ 0.6\left(\frac{7.5-x}{3.33}\right) & \frac{25}{6} \le x \le \frac{30}{4} \end{cases}$	$\begin{cases} \frac{50x-10}{1+x} & 2 \le x \le \frac{53}{22} \\ 0.6 & 2.4 \le x \le \frac{135}{26} \\ \frac{30-4x}{2x+5} & \frac{135}{26} \le x \le 7.5 \end{cases}$	$\begin{cases} \frac{10x-20}{\frac{5}{2}+\frac{10}{3}x} & 2 \le x \le \frac{43}{16} \\ 0.6 & \frac{43}{16} \le x \le \frac{105}{24} \\ \frac{30-4x}{\left(\frac{25}{4}+\frac{10}{3}x\right)} & \frac{105}{24} \le x \le \frac{30}{4} \end{cases}$

Table 5.1: Arithmetic operations on A_1 and A_2 by different approaches

5.3 Suggested Approach

In this section, a methodology for behavior analysis of large-scale system having uncertainties in data has been suggested using Markov process. In this technique, two important tools, namely Markov modeling and Generalized fuzzy set theory are hybridized. The following basic assumptions are made in the proposed methodology.

1. Uncertainties in failure and repair rates have been handled by generalized fuzzy numbers and are independent to each other.
2. Repaired unit is assumed as good as new and probability that two or more failed components could be repaired at the same time is zero.
3. At any given time, system is either in working or in failed state.
4. System structure is precisely known.

Details of the strategy through this approach are given herein.

Step 1: [***Evaluation of availability through Markov process*** [84, 225]] For the Markov process $\{Y(t); t \geq 0\}$ with state space $\Psi = \{0, 1, 2, ...r\}$ and transition probabilities $P_{ij}(t)$, expressed as

$$P_{ij}(t) = Pr\{Y(t) = j|Y(0) = i\} \text{ for all } i, j \in \Psi,$$

the Kolmogorov differential equations, as described in Step 1 of section 3.2 are

$$\frac{dP_{ij}(t)}{dt} + P_{ij}(t)v_j = \sum_{k \neq j} P_{ik}(t)q_{kj}, \tag{5.3.1}$$

where v_j is the rate at which process leaves state j and q_{kj} is the transition rate from state k to state j. **Availability** Av is the probability that the system is operating satisfactorily at time t. Here, availability of a system is sum of the probabilities of working states and Av is a function of $P_{ij}(t)$ i.e. $Av = \sum_s P_{is}(t)$ where s represents working state.

Step 2: [***Data uncertainties in generalized form of fuzzy numbers***] By this step we handle data uncertainties in availability evaluation, parameters like failure and repair rates, we take them in form of generalized trapezoidal fuzzy numbers i.e. we take here v_j and q_{kj} as
$\tilde{v}_j = (v_{j(1)}, v_{j(2)}, v_{j(3)}, v_{j(4)}; w_{v_j})$ and $\tilde{q}_{kj} = (q_{kj(1)}, q_{kj(2)}, q_{kj(3)}, q_{kj(4)}; w_{q_{kj}})$.

For handling uncertainty, system of differential eq. (5.3.1) is considered in the form:

$$\frac{d\tilde{P}_{ij}(t)}{dt} + \tilde{P}_{ij}(t)\tilde{v}_j = \sum_{k\neq j} (\tilde{P}_{ik}(t)\tilde{q}_{kj}) \tag{5.3.2}$$

where $\tilde{P}_{ij}(t)$ are fuzzy subsets of R for real number t.
Let $\tilde{P}_{ij}(t) = (P_{ij(1)}(t), P_{ij(2)}(t), P_{ij(3)}(t), P_{ij(4)}(t); w_{P_{ij}})$ where, $w_{P_{ij}}$ are the weights associated with corresponding probabilities P_{ij} and are taken as $min(w_{q_{kj}}, w_{v_j})$. Availability in form of uncertainty is obtained by summation of probabilities of its working states. i.e. $\tilde{Av} = \sum_s \tilde{P}_{is}(t)$ where s represents working states.

Step 3: [***Determination of α-cut of generalized fuzzy numbers***] In this step we take the α-cuts of the Eq. (5.3.2). Then the corresponding system of equations become

$$\Big(\frac{d\tilde{P}_{ij}(t)}{dt}\Big)^{\alpha} + \big(\tilde{P}_{ij}(t)\tilde{v}_j\big)^{\alpha} = \big(\sum_{k\neq j} (\tilde{P}_{ik}(t)\tilde{q}_{kj})\big)^{\alpha},$$

Here,

$$(\tilde{P}_{ij}(t))^{\alpha} = \big[\tilde{P}_{ij}(t)^{\alpha}_{(L)}, \tilde{P}_{ij}(t)^{\alpha}_{(R)}\big],$$

$$\tilde{q}^{\alpha}_{kj} = \big[\tilde{q}^{\alpha}_{kj(L)}, \tilde{q}^{\alpha}_{kj(R)}\big] \text{ and } \tilde{v}^{\alpha}_{j} = \big[\tilde{v}^{\alpha}_{j(L)}, \tilde{v}^{\alpha}_{j(R)}\big].$$

Step 4: [***Implementation of arithmetic operations on generalized fuzzy numbers***] After the deduction of α-cuts, in this step we perform arithmetic operations by using different operations as discussed in previous section, i.e. the system of differential equations:

$$\Big(\frac{d\tilde{P}_{ij}(t)}{dt}\Big)^{\alpha} + \big(\tilde{P}_{ij}(t)\tilde{v}_j\big)^{\alpha} = \big(\sum_{k\neq j}(\tilde{P}_{ik}(t)\tilde{q}_{kj})\big)^{\alpha}$$

is bifurcated into two systems:

$$\begin{gathered}\Big(\frac{d\tilde{P}_{ij}(t)}{dt}\Big)^{\alpha}_{(L)} + \big(\tilde{P}_{ij}(t)\tilde{v}_j\big)^{\alpha}_{(L)} = \big(\sum_{k\neq j}(\tilde{P}_{ik}(t)\tilde{q}_{kj})\big)^{\alpha}_{(L)} \\ \text{and} \\ \Big(\frac{d\tilde{P}_{ij}(t)}{dt}\Big)^{\alpha}_{(R)} + \big(\tilde{P}_{ij}(t)\tilde{v}_j\big)^{\alpha}_{(R)} = \big(\sum_{k\neq j}(\tilde{P}_{ik}(t)\tilde{q}_{kj})\big)^{\alpha}_{(R)}\end{gathered} \tag{5.3.3}$$

Now $P_{ij}(t)^{\alpha}_{(L)}$ and $P_{ij}(t)^{\alpha}_{(R)}$ can be obtained after solving the system of equations 5.3.3 by using different arithmetic operations. After getting the values of $P_{ij}(t)$, availability $Av(t)^{\alpha}_{(L)}$ and $Av(t)^{\alpha}_{(R)}$ can be obtained by the following sums

$$\tilde{Av}(t)^{\alpha}_{(L)} = \sum_s \tilde{P}_{is}(t)^{\alpha}_{(L)} \text{ and } \tilde{Av}(t)^{\alpha}_{(R)} = \sum_s \tilde{P}_{is}(t)^{\alpha}_{(R)}$$

where, $\sum$ is taken over s for the working states of the system.

Step 5: [***Evaluation of Availability in terms of generalized fuzzy numbers***] Finally after solving the above framed set of differential equations in each case by using Runge-Kutta fourth order method, the values of P_{ij} are obtained. Substitute these values in the expression of Availability ($\tilde{Av}$). These obtained values of Availability ($\tilde{Av}$) corresponding to real number $t = t_0$ represent α-cut for the solution $\tilde{Av}$, provided the following conditions are satisfied.

1. $\frac{d\tilde{Av}^{\alpha}_{(L)}(t_0)}{d\alpha} \geq 0$, $\forall \alpha \in [0, w)$
2. $\frac{d\tilde{Av}^{\alpha}_{(R)}(t_0)}{d\alpha} \leq 0$, $\forall \alpha \in [0, w)$
3. $\tilde{Av}^{\alpha}_{L}(t_0) \leq \tilde{Av}^{\alpha}_{R}(t_0)$, when $\alpha = w$

5.4 Case Study

For illustration of the suggested approach with different arithmetic approaches, availability of butter-oil processing plant, as an industrial system has been studied. System description of butter-oil processing plant is already given in Chapter 3. Same notations have been used here in this chapter. Input data in terms of generalized fuzzy numbers, for system availability has been depicted in Table 5.2.

Component	Failure rate	Repair rate
Separator	(0.0068,0.008,0.0092;0.70)	(0.3485,0.41,0.4715;0.70)
Continuous Butter Making	(0.00459,0.0054,0.00621;0.80)	(0.34,0.40,0.46;0.80)
Melting Vats	(0.002295,0.0027,0.003105;0.85)	(0.595,0.70,0.805;0.85)
Butter oil clarifier	(0.000765,0.0009,0.001035;0.75)	(0.255,0.30,0.345;0.75)
Packaging	(0.002295,0.0027,0.003105;0.85)	(0.5525,0.65,0.7475;0.85)
Pasteuriser	(0.0094435,0.01111,0.0127765;0.80)	(5.10,6.00,6.90;80)
Pasteuriser* reduced state	(0.004675,0.0055,0.006325;0.80)	

Table 5.2: Input data for the system

5.4.1 Mathematical Formulation

Applying the concepts of Markov modeling and probability theory as described in Step-1 and Step-2 of proposed approach, the transition diagram (Figure 3.2) of this system leads to the formulation of following fuzzy differential equations:

$$
\begin{aligned}
\frac{d\tilde{P}_1(t)}{dt} \oplus \tilde{\delta}_1 \tilde{P}_1(t) &= \sum_{j=1}^{5} \tilde{\mu}_j \tilde{P}_{j+2}(t) \oplus \tilde{\mu}_6 \tilde{P}_{13}(t), \\
\frac{d\tilde{P}_2(t)}{dt} \oplus \tilde{\delta}_2 \tilde{P}_2(t) &= \sum_{j=1}^{5} \tilde{\mu}_j \tilde{P}_{j+7}(t) \oplus \tilde{\lambda}_6 \tilde{P}_1(t), \\
\frac{d\tilde{P}_{i+2}(t)}{dt} \oplus \tilde{\mu}_i \tilde{P}_{i+2}(t) &= \tilde{\lambda}_i \tilde{P}_1(t), \qquad i = 1, 2, ...5 \\
\frac{d\tilde{P}_{i+7}(t)}{dt} \oplus \tilde{\mu}_i \tilde{P}_{i+7}(t) &= \tilde{\lambda}_i \tilde{P}_2(t), \qquad i = 1, 2, ...5 \\
\frac{d\tilde{P}_{13}(t)}{dt} \oplus \tilde{\mu}_6 \tilde{P}_{13}(t) &= \tilde{\lambda}_7 \tilde{P}_2(t),
\end{aligned}
\tag{5.4.1}
$$

with $\tilde{\delta}_1 = \sum_{j=1}^{6} \tilde{\lambda}_j$ and $\tilde{\delta}_2 = \sum_{j=1}^{5} \tilde{\lambda}_j \oplus \tilde{\lambda}_7$,
and the given initial conditions as:

$$y(1) = (0.94, 0.96, 0.98; 0.9),$$

$$y(2) = (0.004, 0.005, 0.006; 0.9),$$

$$y(j) = 0 \quad \text{for} \quad j = 3, ..., 13.$$

Availability function $\tilde{Av}(t)$ of the system in terms of $\tilde{P}_1(t)$ and $\tilde{P}_2(t)$ from system of equations (5.4.1) can be obtained by

$$\tilde{Av}(t) = \tilde{P}_1(t) \oplus \tilde{P}_2(t). \tag{5.4.2}$$

5.4.2 Steady State Analysis

For long term availability of the system, steady state probabilities of the system are obtained by applying following limitations on probabilities:

$$\frac{d}{dt} \to 0 \text{ as } t \to \infty. \tag{5.4.3}$$

In this case study, following system of equations are obtained by imposing the above restrictions.

$$P_2 = \frac{\lambda_6}{\lambda_7} P_1; \quad P_3 = \frac{\lambda_1}{\mu_1} P_1; \quad P_4 = \frac{\lambda_2}{\mu_2} P_1; \quad P_5 = \frac{\lambda_3}{\mu_3} P_1; \quad P_6 = \frac{\lambda_4}{\mu_4} P_1; \quad P_7 = \frac{\lambda_5}{\mu_5} P_1; \tag{5.4.4}$$

$$P_8 = \Big(\frac{\lambda_1}{\mu_1}\Big)\Big(\frac{\lambda_6}{\lambda_7}\Big) P_1; \quad P_9 = \Big(\frac{\lambda_2}{\mu_2}\Big)\Big(\frac{\lambda_6}{\lambda_7}\Big) P_1; \quad P_{10} = \Big(\frac{\lambda_3}{\mu_3}\Big)\Big(\frac{\lambda_6}{\lambda_7}\Big) P_1; \tag{5.4.5}$$

$$P_{11} = \Big(\frac{\lambda_4}{\mu_4}\Big)\Big(\frac{\lambda_6}{\lambda_7}\Big) P_1; \quad P_{12} = \Big(\frac{\lambda_5}{\mu_5}\Big)\Big(\frac{\lambda_6}{\lambda_7}\Big) P_1; \quad P_{13} = \Big(\frac{\lambda_6}{\mu_6}\Big) P_1; \tag{5.4.6}$$

Substituting these values of P_1 to P_{13} in the normalizing condition $\sum_{i=1}^{13} P_i = 1$, steady state availability becomes:

$$Av = P_1 + P_2 = \Big[\Big(1 + \frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} + \frac{\lambda_3}{\mu_3} + \frac{\lambda_4}{\mu_4} + \frac{\lambda_5}{\mu_5}\Big) + \Big\{\mu_6\Big(\frac{1}{\lambda_6} + \frac{1}{\lambda_7}\Big)\Big\}^{-1}\Big]^{-1}. \tag{5.4.7}$$

5.5 Results and Discussion

System availability in transient state with uncertain parameters (as in Table 5.2) for different time periods have been discussed in this section.
Availability of the system in terms of generalized fuzzy numbers by proposed approach for different time periods $t = 50, 100, 150, 200$ days with different presumption levels are depicted in Table 5.3.

(i) Results calculated by the traditional approach **(crisp)** [113] do not always provide the exact idea about the behavior of the system. Uncertainties in data cannot be dealt with crisp approach.

(ii) Differential equations having uncertainties in form of fuzzy numbers deal with uncertainties in the data. It can be observed that while ignoring the additional information (i.e. degree of confidence in data is one) they provide the availability in the form of triangular fuzzy number. While dealing with generalized fuzzy numbers not only generalizes the results but also reduces uncertainties in the results (in Table 5.3). Availability of Butter-Oil processing plant for $t = 100$ days is (0.903799, 0.923875, 0.943956) by existed Garg approach [91] while it is (0.903799, 0.919411, 0.928340, 0.943956; 0.7) by proposed approach with improved arithmetic operations. In order to see the reduction in uncertainties, it can be observed (from Table 5.3) that with $t = 100$ days, for $\alpha = 0.5$ availability of system is [0.913836, 0.933915] and [0.914950, 0.932801], which shows 11.10% decrease in uncertainty. It has been observed that how preservation of degree of confidence w in data provides reduction in uncertainties.

(iii) Results obtained by proposed methodology using Chen's arithmetic operations [45] have been depicted in Table 5.3. Following observations have been made from the results obtained through Chen's arithmetic operations.

- System availability obtained from suggested approach using Chen's arithmetic operations is in the form of generalized triangular fuzzy number as shown in Figure 5.1.
- It reduces troubleness and tediousness of arithmetical operations during the solution of differential equations.
- Results obtained from this approach show that it reduces uncertainty as compared to other approach which can be observed from Fig 5.1. For instance, corresponding to α= 0.5 and t= 50 days, System availability is $[0.913968, 0.934047]$, $[0.915082, 0.932933]$ and $[0.918271, 0.929744]$ through normalized, improved operations and Chen's opeations which show reduction of 42.79 % and 35.75 % in uncertainty respectively.
- Major drawback by suggested approach using Chen's arithmetic operations is that it loses the importance of extra information related to fuzzy numbers. Arithmetic operations between generalized fuzzy numbers are the same when we change the degree of confidence w of generalized fuzzy numbers which causes the loss of information.

(iv) Results computed by suggested approach using normalized arithmetic operations are outlined in Table 5.3. Following observations have been made in normalized approach.

- Solution obtained by suggested approach using normalized arithmetic operations shows increase in uncertainty compared to the solutions obtained by other approaches. For example, corresponding to α=0.4 and t=100 days, System availability is $[0.911829, 0.935923]$ and $[0.912720, 0.935032]$ by normalized and improved operations respectively which shows an increment of 8.07%. Corresponding to α=0.4 and t=100 days, system availability is $[0.911829, 0.935923]$ and $[0.915270, 0.932481]$ by normalized and Chen's arithmetic operations respectively which shows an increment of

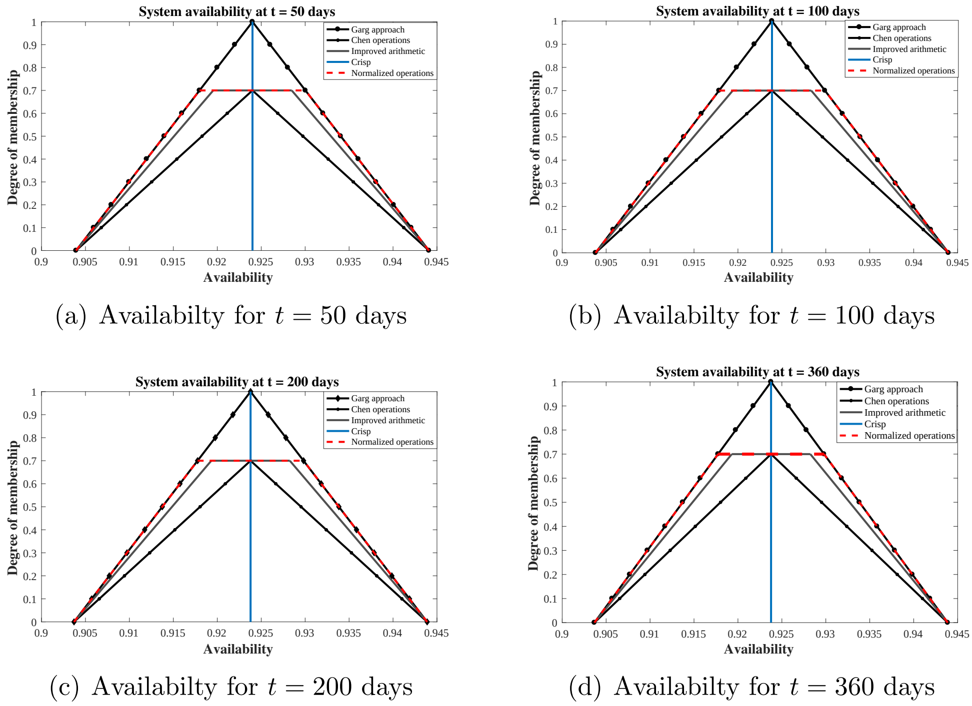
40.12%.

- Normalized arithmetic operations in suggested approach overcomes computational arithmetic difficulties.

(v) Results evaluated by suggested approach using arithmetic operations through improved arithmetic operations are given in Table 5.3. Using improved arithmetic operations, following observations have been made in the evaluation of availability.

- Solutions obtained through the improved arithmetic operations use all given data and does not lose information.
- System availability corresponding to different times have been evaluated in form of generalized trapezoidal fuzzy numbers as shown in Figure 5.1.
- In the evaluation of availability, there is less amount of uncertainty in comparison to normalized operations. For instance, corresponding to α=0.3 and t= 200 days, system availability is $[0.909725, 0.937854]$ and $[0.910394, 0.937185]$ by normalized and improved arithmetic operations respectively which shows reduction of 4.85% in uncertainty, although in same observations, system availability is $[0.912308, 0.935271]$ and $[0.910394, 0.937185]$ by Chen's and improved arithmetic operations which shows an increment of 16.52% in uncertainty.

(vi) In order to find the long term availability of the system, generalized fuzzy availability has been computed for different $\alpha = 0, 0.1...w$ using proposed appraoch with different arithmetic operations. It has been observed that availability of the system lies in $[0.943049, 0.968062]$ by suggested approach using different arithmetic operations. For different presumption level, results are computed for different approaches and have been outlined in Table 5.4. On the basis of these computed results, system analyst may analyze the behavior of system and plan the suitable maintenance to enhance the performance of system and therefore reduce maintenance and operational cost.



(a) Availabilty for $t = 50$ days

(b) Availabilty for $t = 100$ days

(c) Availabilty for $t = 200$ days

(d) Availabilty for $t = 360$ days

Figure 5.1: System availability of butter-oil processing plant at different times

System availability for t = 50days									
	Crisp approach[113]	Garg Approach[91]		Result computed by suggested approach through					
				Normalized operations		Chen's operations		Improved arithmetic operations	
$\alpha \downarrow$	Av	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$
0	0.924007	0.903930	0.944087	0.903930	0.944087	0.903930	0.944087	0.903930	0.944087
0.1	0.924007	0.905937	0.942079	0.905937	0.942079	0.906798	0.941218	0.906160	0.941856
0.2	0.924007	0.907945	0.940071	0.907945	0.940071	0.909666	0.938350	0.908390	0.939625
0.3	0.924007	0.909953	0.938063	0.909953	0.938063	0.912534	0.935481	0.910621	0.937394
0.4	0.924007	0.911960	0.936055	0.911960	0.936055	0.915402	0.932612	0.912851	0.935164
0.5	0.924007	0.913968	0.934047	0.913968	0.934047	0.918271	0.929744	0.915082	0.932933
0.6	0.924007	0.915976	0.932039	0.915976	0.932039	0.921139	0.926876	0.917312	0.930702
0.7	0.924007	0.917984	0.930031	0.917984	0.930031	0.924007	0.924007	0.919542	0.928472
0.8	0.924007	0.919991	0.928023	-	-	-	-	-	-
0.9	0.924007	0.921999	0.926015	-	-	-	-	-	-
1.0	0.924007	0.924007	0.924007	-	-	-	-	-	-

System availability for t = 100days									
	Crisp approach[113]	Garg Approach[91]		Result computed by suggested approach through					
				Normalized operations		Chen's operations		Improved arithmetic operations	
$\alpha \downarrow$	Av	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$
0	0.923875	0.903799	0.943956	0.903799	0.943956	0.903799	0.943956	0.903799	0.943956
0.1	0.923875	0.905806	0.941948	0.905806	0.941948	0.906667	0.941087	0.906029	0.941725
0.2	0.923875	0.907814	0.939940	0.907814	0.939940	0.909535	0.938218	0.908259	0.939494
0.3	0.923875	0.909821	0.937932	0.909821	0.937932	0.912402	0.935350	0.910489	0.937263
0.4	0.923875	0.911829	0.935923	0.911829	0.935923	0.915270	0.932481	0.912720	0.935032
0.5	0.923875	0.913836	0.933915	0.913836	0.933915	0.918139	0.929612	0.914950	0.932801
0.6	0.923875	0.915844	0.931907	0.915844	0.931907	0.921007	0.926744	0.917180	0.930570
0.7	0.923875	0.917852	0.923875	0.917852	0.929899	0.923875	0.923875	0.919411	0.928340
0.8	0.923875	0.919860	0.927891	-	-	-	-	-	-
0.9	0.923875	0.921867	0.925883	-	-	-	-	-	-
1.0	0.923875	0.923875	0.923875	-	-	-	-	-	-

System availability for t = 200days									
	Crisp approach[113]	Garg Approach[91]		Result computed by suggested approach through					
				Normalized operations		Chen's operations		Improved arithmetic operations	
$\alpha \downarrow$	Av	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$
0	0.923789	0.903699	0.943883	0.903699	0.943883	0.903699	0.943883	0.903699	0.943883
0.1	0.923789	0.905708	0.941874	0.905708	0.941874	0.906568	0.941012	0.905931	0.941650
0.2	0.923789	0.907716	0.939864	0.907716	0.939864	0.909438	0.938142	0.908162	0.939418
0.3	0.923789	0.909725	0.937854	0.909725	0.937854	0.912308	0.935271	0.910394	0.937185
0.4	0.923789	0.911734	0.935845	0.911734	0.935845	0.915178	0.932400	0.912626	0.934952
0.5	0.923789	0.913743	0.933835	0.913743	0.933835	0.918048	0.929530	0.914858	0.932720
0.6	0.923789	0.915752	0.931826	0.915752	0.931826	0.920918	0.926659	0.917090	0.930487
0.7	0.923789	0.917761	0.929817	0.917761	0.929817	0.923789	0.923789	0.919323	0.928255
0.8	0.923789	0.919770	0.927807	-	-	-	-	-	-
0.9	0.923789	0.921779	0.925798	-	-	-	-	-	-
1.0	0.923789	0.923789	0.923789	-	-	-	-	-	-

System availability for t = 360days									
	Crisp approach[113]	Garg Approach[91]		Result computed by suggested approach through					
				Normalized operations		Chen's operations		Improved arithmetic operations	
$\alpha \downarrow$	Av	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$
0	0.923768	0.903668	0.943870	0.903668	0.943870	0.903668	0.943870	0.903668	0.943870
0.1	0.923768	0.905678	0.941860	0.905678	0.941860	0.906539	0.940998	0.905901	0.941636
0.2	0.923768	0.907688	0.939850	0.907688	0.939850	0.909411	0.938126	0.908134	0.939403
0.3	0.923768	0.909698	0.937839	0.909698	0.937839	0.912282	0.935255	0.910368	0.937169
0.4	0.923768	0.911708	0.935829	0.911708	0.935829	0.915154	0.932383	0.912601	0.934936
0.5	0.923768	0.913718	0.933819	0.913718	0.933819	0.918025	0.929511	0.914834	0.932702
0.6	0.923768	0.915728	0.931809	0.915728	0.931809	0.920897	0.926640	0.917068	0.930469
0.7	0.923768	0.917738	0.929799	0.917738	0.929799	0.923768	0.923768	0.919301	0.928235
0.8	0.923768	0.919748	0.927788	-	-	-	-	-	-
0.9	0.923768	0.921758	0.925778	-	-	-	-	-	-
1.0	0.923768	0.923768	0.923768	-	-	-	-	-	-

Table 5.3: System availability of Butter-oil processing plant

	Crisp approach	Steady state availability computed by					
		Chen arithmetic operations		Normalized arithmetic operations		Improved arithmetic operations	
$\alpha\downarrow$	Av	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$
0	0.957271	0.943049	0.968062	0.943049	0.968062	0.943049	0.968062
0.1	0.957271	0.945351	0.966682	0.944671	0.967101	0.94518	0.966788
0.2	0.957271	0.947554	0.965254	0.946244	0.966117	0.947225	0.965473
0.3	0.957271	0.949663	0.963773	0.947769	0.965108	0.949189	0.964113
0.4	0.957271	0.951683	0.962238	0.949248	0.964074	0.951077	0.962706
0.5	0.957271	0.953622	0.960645	0.950684	0.963013	0.952892	0.961250
0.6	0.957271	0.955483	0.958990	0.952078	0.961924	0.954640	0.959743
0.7	0.957271	0.957271	0.957271	0.953432	0.960807	0.956323	0.958181

Table 5.4: Steady state availability of the butter-oil processing plant

5.6 Conclusion

In this chapter, a methodology has been discussed for analyzing availability in more generalized way through Markov process. Uncertainty in data has been dealt in form of generalized fuzzy numbers. Methodology for the evaluation of availability analysis has been done through different arithmetic operations. Obtained results through different arithmetic operations have been compared in tabular form. Comparison of results obtained by different arithmetic operations has shown the reduction in epistemic uncertainties. Availability analysis has been done in its transient as well as steady state. Impact of additional information (i.e. w) in uncertainty in form of fuzzy numbers has been seen as it reduces uncertainty in the solution of the differential equations.

For application point of view, availability analysis of butter oil processing plant, as a repairable industrial system has been studied. For improving reliability/availability, system analyst can observe the impact of failure and repair rates on the system by the proposed approach. Obtained results can help decision makers in deciding the characteristics of each components and also can provide repair policies for more reliable and efficient system.

Chapter 6

Availability analysis of Industrial systems using generalized fuzzy numbers and particle swarm optimization

In this chapter, an approach has been presented to optimize availability of the system having uncertain parameters in the form of generalized fuzzy numbers. In this approach, solution has been obtained from α- cut and RK-IV method and then optimized by formulating an optimization problem. Improvement in the obtained solution optimizes the spread in uncertainty. Approach has been implemented for the behavior study of cattle feed plant. Sensitivity analysis of cattle feed plant has also been done in this chapter.

6.1 Introduction

In recent years industrial systems are becoming more complex and getting more complicated due to modern technology, innovation and higher reliability requirements. In addition to complexity of the system, most of the real-world industrial systems are repairable in nature and hence getting failed, they are to be repaired based on different distributions and with additional constraints such as spare parts

availability, repair crew response time, etc. The effectiveness of production processes and the equipments that are part of them are generally measured according to the results of reliability and availability indicators, as well as through the economic analysis of its life cycle. It is necessary to have both a high reliability and maintainability in order to achieve a high availability. In classical reliability model, system performance is evaluated by using the probability theory. In practical cases, the probability function of the elements' lifetimes may be unknown or imprecise.
In fact, from a practical viewpoint one may consider ambiguous situations like uncertain parameters. In such situations, the traditional reliability theory, based on binary state assumptions do not always provide useful information to the practitioners due to the limitation of being able to handle only quantitative information. Then consideration of subjective information along with qualitative databases to deduce useful results become very important. Again the use of fuzzy set theory [289] is a suggestive approach to handle the subjective information or uncertainties. Due to incomplete and uncertain input information, mathematical models of such problems are developed in fuzzy environment. As has already been pointed out in previous chapters, generalized fuzzy set theory [45, 46] becomes another important tool to handle uncertainties. Various approaches for arithmetic operations between generalized fuzzy numbers have been discussed in the literature.

In this chapter, behavior analysis of a system is analyzed in the form of generalized fuzzy functions. An attempt has been made to optimize the performance of the system. In this chapter, performance analysis of cattle feed plant with limited, vague and imprecise data has been analyzed through the proposed approach in fuzzy environment. Results have been computed by various arithmetic approaches on generalized fuzzy numbers. An attempt has been made to optimize the behavior and performance analysis of cattle feed plant with limited, vague and imprecise data through the proposed interactive approach using gerenalized fuzzy numbers.

6.2 Solution Technique

The suggested technique is a hybridized technique in which the important tool, Particle swarm optimization is hybridized with the Markov process and the concept of generalized fuzzy numbers. In order to find the availability, the uncertainties are handled through generalized fuzzy numbers. The following basic assumptions have been made in the proposed technique.

(i) Uncertainties in failure and repair rates have been handled by generalized fuzzy numbers and are independent to each other.

(ii) Repaired unit is assumed as good as new and the probability that two or more failed components could be repaired at the same time is zero.

(iii) At any given time, system is either in working or in failed state.

(iv) System structure is precisely known.

Following steps are to be taken in order to evaluate the availability.

Step 1: First step in this technique is the information extraction phase. In this, information is extracted from the available historical data in the form of failure and repair rates of each component of the system which is imprecise in nature.

Step 2: Since collected data are generally vague, imprecise or limited in nature, so to account the uncertainties in the analysis, the obtained data are fuzzified into generalized fuzzy numbers. Usually, crisp numbers in the extracted data are converted into triangular/trapezoidal fuzzy numbers with known spread suggested by decision maker/system analyst. For the consideration of both cases, parameters have been taken in trapezoidal form as it can be reduced to triangular form by imposing the restriction.

Step 3: In this step, system is mathematically modeled with the help of Markov Process. For the Markov process $\{Y(t); t \geq 0\}$ with state space $\Psi = \{0, 1, 2, ...r\}$

and transition probabilities $P_{ij}(t)$, expressed as

$$P_{ij}(t) = Pr\{Y(t) = j|Y(0) = i\} \text{ for all } i, j \in \Psi, \tag{6.2.1}$$

the Kolmogorov differential equations, as described in Step 1 of Section 3.2 are

$$\frac{dP_{ij}(t)}{dt} + P_{ij}(t)v_j = \sum_{k \neq j} P_{ik}(t)q_{kj}, \tag{6.2.2}$$

where v_j is the rate at which process leaves state j and q_{kj} is the transition rate from state k to state j. **Availability** Av is the probability that the system is operating satisfactorily at time t so the availability of the system, here is the sum of the probabilities of working states at time $t$and Av is a function of $P_{ij}(t)$ i.e. $Av = \sum_s P_{is}(t)$, where s represents working state.

Step 4: In this step the uncertain parameters are introduced as generalized trapezoidal fuzzy numbers as
$\tilde{v}_j = (v_{j(1)}, v_{j(2)}, v_{j(3)}, v_{j(4)}; w_{v_j})$ and $\tilde{q}_{kj} = (q_{kj(1)}, q_{kj(2)}, q_{kj(3)}, q_{kj(4)}; w_{q_{kj}})$. The special case $v_{j(2)} = v_{j(3)}$ and $q_{kj(2)} = q_{kj(3)}$ leads to generalized triangular fuzzy numbers. To handle uncertainty, system of differential equations (6.2.2) is considered in the form:

$$\frac{d\tilde{P}_{ij}(t)}{dt} + \tilde{P}_{ij}(t)\tilde{v}_j = \sum_{k \neq j} (\tilde{P}_{ik}(t)\tilde{q}_{kj}) \tag{6.2.3}$$

where $\tilde{P}_{ij}(t)$ are fuzzy subsets of R for real number t.
Let $\tilde{P}_{ij}(t) = (P_{ij(1)}(t), P_{ij(2)}(t), P_{ij(3)}(t), P_{ij(4)}(t); w_{P_{ij}})$ where, $w_{P_{ij}}$ is the weight associated with corresponding probabilities P_{ij} and is taken as $min(w_{q_{kj}}, w_{v_j})$. Availability in form of uncertainty is obtained by summation of probabilities of its working states. i.e. $\tilde{Av} = \sum_s \tilde{P}_{is}(t)$, where s represents working states.

Step 5: Now, α- cuts of all the parameters involved in the obtained differential equations are evaluated as

$$(\tilde{P}_{ij}(t))^\alpha = \left[\tilde{P}_{ij}(t)^\alpha_{(L)}, \tilde{P}_{ij}(t)^\alpha_{(R)}\right],$$

$$\tilde{q}_{kj}^{\alpha} = \left[\tilde{q}_{kj(L)}^{\alpha}, \tilde{q}_{kj(R)}^{\alpha}\right] \text{ and } \tilde{v}_j^{\alpha} = \left[\tilde{v}_{j(L)}^{\alpha}, \tilde{v}_{j(R)}^{\alpha}\right].$$

Step 6: In this step different arithmetic operations as discussed in section 5.2 of chapter 5 have been implemented on the following equation.

$$\Big(\frac{d\tilde{P}_{ij}(t)}{dt}\Big)^{\alpha} + \big(\tilde{P}_{ij}(t)\tilde{v}_j\big)^{\alpha} = \big(\sum_{k\neq j}(\tilde{P}_{ik}(t)\tilde{q}_{kj})\big)^{\alpha}. \tag{6.2.4}$$

Step 7: Following optimization model for the optimization of availability $\tilde{Av}$ with the conditions on α-cut level of $\tilde{v}_j$ and $\tilde{q}_{kj}$ is formed.

$$\min / \max \;\; Av(\tilde{q}_{ij}, \tilde{v}_j),$$

$$\begin{aligned} \text{subject to} \quad & \mu_{\tilde{q}_{ij}} \geq \alpha, \\ & \mu_{\tilde{v}_j} \geq \alpha, \\ & 0 \leq \alpha \leq 1, \end{aligned} \tag{6.2.5}$$

where, Av is the fitness function, obtained by solving the differential equations (6.2.4) for availability by using RK-IV method. The obtained maximum and minimum values of Av denoted by $Av_{\max}$ and $Av_{\min}$ respectively corresponding to α-cut level satisfy

$$\mu_{\tilde{Av}}(Av_{min}) = \mu_{\tilde{Av}}(Av_{max}) = \alpha. \tag{6.2.6}$$

Step 8: The solution of the above framed set of differential equations, having generalized fuzzy parameters, by using RK-IV method is then optimized through Particle Swarm Optimization approach. The obtained values of availability $(\tilde{Av})$ corresponding to real number $t = t_0$ represent α- cut for the solution $\tilde{Av}$, provided the following conditions are satisfied through different arithmetic operations.

1. $\frac{d\tilde{Av}_{(L)}^{\alpha}(t_0)}{d\alpha} \geq 0$, $\forall \alpha \in [0, w)$
2. $\frac{d\tilde{Av}_{(R)}^{\alpha}(t_0)}{d\alpha} \leq 0$, $\forall \alpha \in [0, w)$
3. $\tilde{Av}_L^{\alpha}(t_0) \leq \tilde{Av}_R^{\alpha}(t_0)$, when $\alpha = w$ where $w = \min(w_{P_{ij}}) \;\; \forall i, j$.

6.3 Case Study

Now we illustrate the above described approach through a concrete example of an industrial system, namely a cattle feed plant. A short description of of this plant is given below.

6.3.1 System Description

The cattle feed plant [86, 89] consists of seven subsystems namely, Elevator, Grinder, Hooper, Mixer, Winch, Palletiser and Screw Conveyor. Initially, elevator lifts the material and put it into the grinder. Grinder grinds the raw material and then the material is put into the hopper. Hopper is used for the storage and cooling of material. Cooling is done by the fans present in the hopper. Then the material is put into the mixer for proper mixing of certain additives in specified ratio. This mixture is lifted by winch which put this mixture into the palletiser. Palletiser allows the mixture to move forward and passes through holes which give them a proper shape. Finally screw conveyor carries the final product to the store where it is packed for final delivery. The transition diagram of the cattle feed plant is given in Figure 6.1. This plant consists mainly of the following seven subsystems:

(i) Subsystem A (Elevator): Elevator consists of one unit. System fails when this unit fails.

(ii) Subsystem B (Grinder): It consists of one unit whose failure causes major failure.

(iii) Subsystem C (Hopper): It consists of one unit whose failure causes failure of the system.

(iv) Subsystem D (Mixer): It consists of two units, in which one is working and one is at cold standby. Complete failure of the system occurs when both of them fail.

(v) Subsystem E (Winch): This subsystem consists of one unit whose failure causes the failure of the system.

(vi) Subsystem F (Palletiser): It consists of two units, one working and one in cold standby. Complete failure occurs when both unit fails.

(vii) Subsystem G (Screw Conveyor): This subsystem consists of one unit. System fails if this subsystem fails.

6.3.2 Notations

Notations that are used for behavior analysis of the system are given below. Input

○	Represents working state of the system.
▭	Represents failed state of the system.
⬭	Represents reduced state of the system.
A, B, C, D, E, F, G	Working states of the subsystem.
a, b, c, d, e, f, g	Failed states of the subsystem.
$\overline{D}$ and $\overline{F}$	Represent the reduced states of subsystem D and F.
$P_1(t)$	Indicates the probability of the system working in full capacity at time 't'.
$P_2(t)$, $P_3(t)$ and $P_4(t)$	Indicate the probabilities of the system in reduced states at time 't'.
$P_5(t)$ to $P_{28}(t)$	Indicate the probabilities of the system in failed state at time 't'.
$\lambda_i, i = 1, 2..., 9$	Represent failure rates of the subsystems, when the transition is from A to a B to b, C to c, D to $\overline{D}$, $\overline{D}$ to d, E to e, F to $\overline{F}$, $\overline{F}$ to f and G to g respectively.
$\mu_i, i = 1, 2..., 9$	Represent failure rates of the subsystems, when the transition is from a to A b to B, c to C, $\overline{D}$ to D, d to $\overline{D}$, e to E, $\overline{F}$ to F, f to $\overline{F}$ and g to G respectively.

parameters in the form of failure and repair rates have been given in next section.

6.3.3 Input parameters

Failure and repair rates corresponding to each subsystem of cattle feed plant, in terms of generalized fuzzy numbers are depicted in Table 6.1 with ±15% uncertainty.

Subsystem	Failure rate	Repair rate
Subsystem A (Elevator)	(0.0017, 0.0020, 0.0023; 0.8)	(0.0170, 0.0200, 0.0230; 0.8)
Subsystem B (Grinder)	(0.0008, 0.0010,0.0012; 0.7)	(0.0085, 0.0100, 0.0115; 0.7)
Subsystem C (Hopper)	(0.0034, 0.0040, 0.0046; 0.8)	(0.0340 ,0.0400,0.0460; 0.8)
Subsystem D (Mixer)	(0.0021, 0.0025, 0.0029; 0.8)	(0.0170, 0.0200,0.0230; 0.8)
Subsystem D (Mixer*)	(0.0021, 0.0025 , 0.0029; 0.9)	(0.0170, 0.0200 ,0.0230; 0.9)
Subsystem E (Winch)	(0.0043 , 0.0050 ,0.0057; 0.7)	(0.0425, 0.0500, 0.0575; 0.7)
Subsystem F (Palletiser)	(0.0026, 0.0030, 0.0034; 0.9)	(0.0255, 0.0300, 0.0345; 0.9)
Subsystem F (Palletiser*)	(0.0026, 0.0030, 0.0034; 0.8)	(0.0255, 0.0300, 0.0345; 0.8)
Subsystem G (Screw Conveyor)	(0.0017, 0.0020, 0.0023; 0.8)	(0.0170, 0.020,0.0230; 0.8)

*corresponding to reduced state.

Table 6.1: Input data for cattle feed plant

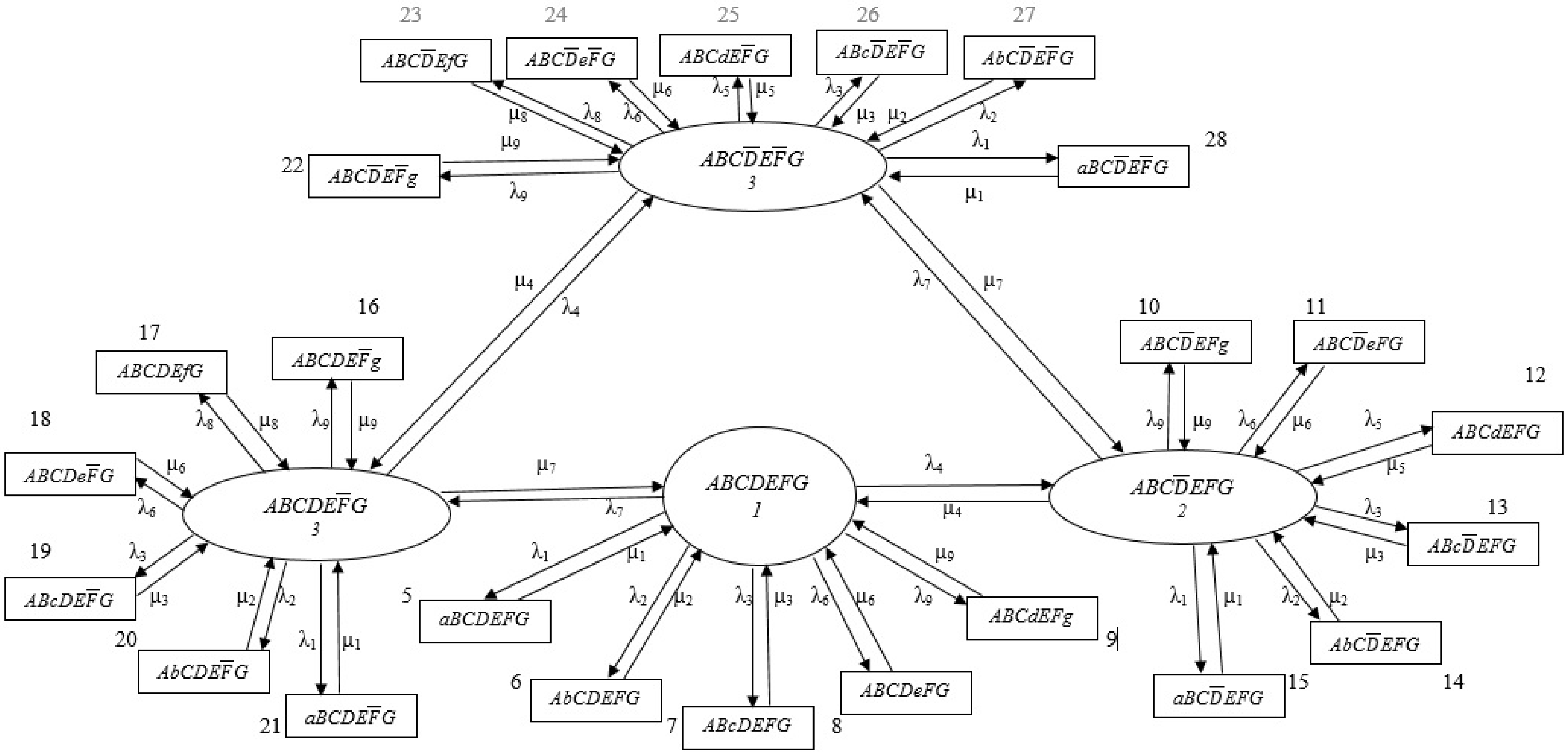


Figure 6.1: Transition diagram of cattle feed plant

6.3.4 Mathematical Formulation

Applying the concepts of Markov modeling and probability theory, the transition diagram (Figure 6.1) of this system leads to the formulation of following set of differential equations.

$$\frac{d\tilde{P}_1(t)}{dt} = \tilde{\mu_1}\tilde{P}_5(t) \oplus \tilde{\mu_2}\tilde{P}_6(t) \oplus \tilde{\mu_3}\tilde{P}_7(t) \oplus \tilde{\mu_6}\tilde{P}_8(t) \oplus \tilde{\mu_9}\tilde{P}_9(t) \oplus \tilde{\mu_4}\tilde{P}_4(t) \oplus$$
$$\tilde{\mu_7}\tilde{P}_2(t) \ominus \tilde{\chi}_1\tilde{P}_1(t)$$
$$\frac{d\tilde{P}_2(t)}{dt} = \tilde{\mu_1}\tilde{P}_{21}(t) \oplus \tilde{\mu_2}\tilde{P}_{20}(t) \oplus \tilde{\mu_3}\tilde{P}_{19}(t) \oplus \tilde{\mu_4}\tilde{P}_3(t) \oplus \tilde{\mu_6}\tilde{P}_{18}(t) \oplus \tilde{\mu_8}\tilde{P}_{17}(t) \oplus$$
$$\tilde{\mu_9}\tilde{P}_{16}(t) \oplus \tilde{\lambda}_7\tilde{P}_1(t)\tilde{\chi}_2\tilde{P}_2(t)$$
$$\frac{d\tilde{P}_3(t)}{dt} = \tilde{\mu_1}\tilde{P}_{28}(t) \oplus \tilde{\mu_2}\tilde{P}_{27}(t) \oplus \tilde{\mu_3}\tilde{P}_{26}(t) \oplus \tilde{\mu_5}\tilde{P}_{25}(t) \oplus \tilde{\mu_6}\tilde{P}_{24}(t) \oplus \tilde{\mu_8}\tilde{P}_{23}(t) \oplus$$
$$\tilde{\mu_9}\tilde{P}_{22}(t) \oplus \tilde{\lambda}_4\tilde{P}_2(t) \oplus \tilde{\lambda}_7\tilde{P}_4(t) \ominus \tilde{\chi}_3\tilde{P}_3(t)$$
$$\frac{d\tilde{P}_4(t)}{dt} = \tilde{\mu_1}\tilde{P}_{15}(t) \oplus \tilde{\mu_2}\tilde{P}_{14}(t) \oplus \tilde{\mu_3}\tilde{P}_{13}(t) \oplus \tilde{\mu_5}\tilde{P}_{12}(t) \oplus \tilde{\mu_6}\tilde{P}_{11}(t) \oplus \tilde{\mu_9}\tilde{P}_{10}(t) \oplus$$
$$\tilde{\mu_7}\tilde{P}_3(t) \oplus \tilde{\lambda}_4\tilde{P}_1(t) \ominus \tilde{\chi}_4\tilde{P}_4(t)$$

$$\frac{d\tilde{P}_i(t)}{dt} = \tilde{\lambda}_j\tilde{P}_1(t) \ominus \tilde{\mu_j}\tilde{P}_i(t), \quad i = 5, 6, 7, 8, 9; j = 1, 2, 3, 6, 9 \tag{6.3.1}$$

$$\frac{d\tilde{P}_i(t)}{dt} = \tilde{\lambda}_j\tilde{P}_4(t) \ominus \tilde{\mu_j}\tilde{P}_i(t), \quad i = 10, 11, 12, 13, 14, 15; j = 9, 6, 5, 3, 2, 1$$

$$\frac{d\tilde{P}_i(t)}{dt} = \tilde{\lambda}_j\tilde{P}_2(t) \ominus \tilde{\mu_j}\tilde{P}_i(t), \quad i = 16, 17, 18, 19, 20, 21; j = 9, 8, 6, 3, 2, 1$$

$$\frac{d\tilde{P}_i(t)}{dt} = \tilde{\lambda}_j\tilde{P}_3(t) \ominus \tilde{\mu_j}\tilde{P}_i(t), \quad i = 22, 23, 24, 25, 26, 27, 28; j = 9, 8, 6, 5, 3, 2, 1$$

where,

$$\tilde{\chi}_1 = \tilde{\lambda}_1 \oplus \tilde{\lambda}_2 \oplus \tilde{\lambda}_3 \oplus \tilde{\lambda}_6 \oplus \tilde{\lambda}_9 \oplus \tilde{\lambda}_4 \oplus \tilde{\lambda}_7,$$

$$\tilde{\chi}_2 = \tilde{\lambda}_1 \oplus \tilde{\lambda}_2 \oplus \tilde{\lambda}_3 \oplus \tilde{\lambda}_4 \oplus \tilde{\lambda}_6 \oplus \tilde{\lambda}_8 \oplus \tilde{\lambda}_9 \oplus \tilde{\mu}_7,$$

$$\tilde{\chi}_3 = \tilde{\lambda}_1 \oplus \tilde{\lambda}_2 \oplus \tilde{\lambda}_3 \oplus \tilde{\lambda}_5 \oplus \tilde{\lambda}_6 \oplus \tilde{\lambda}_8 \oplus \tilde{\lambda}_9 \oplus m_4 \oplus \tilde{\mu}_7,$$

$$\tilde{\chi}_4 = \tilde{\lambda}_1 \oplus \tilde{\lambda}_2 \oplus \tilde{\lambda}_3 \oplus \tilde{\lambda}_5 \oplus \tilde{\lambda}_6 \oplus \tilde{\lambda}_7 \oplus \tilde{\lambda}_9 \oplus \tilde{\mu}_4,$$

with initial conditions,
$\tilde{P}_1(0) = (0.965, 0.97, 0.975; 0.9)$
$\tilde{P}_2(0) = (0.005, 0.0055, 0.006; 0.8)$
$\tilde{P}_3(0) = (0.004, 0.0045, 0.005; 0.8)$
$\tilde{P}_4(0) = (0.003, 0.0035, 0.004; 0.8)$
$\tilde{P}_j(0) = 0$ otherwise.
Thus availability of the cattle feed plant is

$$\tilde{Av} = \tilde{P}_1(t) \oplus \tilde{P}_2(t) \oplus \tilde{P}_3(t) \oplus \tilde{P}_4(t), \tag{6.3.2}$$

as the states having the probabilities $\tilde{P}_1$, $\tilde{P}_2$, $\tilde{P}_3$ and $\tilde{P}_4$ are the only working states.

6.4 Results and Discussion

Availability of cattle feed plant has been evaluated in its transient state. Results are obtained by solving mathematical formulation of cattle feed plant as a set of differential equations Eq. (6.3.1) using proposed technique. MATLAB program has been developed in order to solve the differential equations by all the three arithmetic operations. For each arithmetic operations, population size are set randomly as $25 \times D$, where D is the dimension of the problem. In order to eliminate stochastic discrepancy, 15 independent runs have been made that involve 15 different initial trial solutions. In this case, acceleration coefficient parameters c_1 and c_2 are taken as 2 i.e. as c_1=c_2=2 with inertia weight w, explained as $w = w_{max} - (w_{max} - w_{min}) * iter/iter_{max}$. Here $w_{max} = 0.9$ and $w_{min} = 0.4$ are taken as maximum and minimum values of inertial weight respectively and $iter_{max}$ indicates the maximum generation number(=100). The termination criterion has been set either to relative error equal to 10^{-6} or maximum number of generations, whichever is obtained first.
Obtained results have been depicted in Table 6.2 and comparison is also shown in Figure 6.2. Following observations have been made through this analysis.

(i) Availability of cattle feed plant at time $t = 100$ hrs and 200 hrs has been evaluated. Availability through crisp data [86] is 0.668940 and 0.6510207 for mission time $t = 100$ hrs and $t = 200$ hrs respectively. Results obtained from traditional method do not deal with uncertainties in the data. It can be observed that the result obtained from Chen's approach lies in $[0.602008, 0.732328]$ and $[0.578687, 0.7182922]$ for $t = 100$ hrs and $t = 200$ hrs respectively.

(ii) Results obtained from data in the form of generalized fuzzy numbers provide reduction in uncertainty as compared to the results from the data in fuzzy numbers. Here, results have been computed through different arithmetic operations. It shows that including the degree of confidence w reduces the uncertainty.

(iii) Results obtained using Chen's arithmetic operations tackle the computational difficulties. It reduces the uncertainty in the obtained results but it loses the importance of degree of confidence i.e w in the data. Availability function obtained from the proposed approach using Chen's arithmetic operations is in the form of generalized triangular fuzzy number, as shown in Figure 6.2, and the availability lies in the interval $[0.602008, 0.732328]$ for $t = 100$ hrs.

(iv) Availability at time $t = 100$ hrs from the proposed approach using normalized arithmetic operations lies in the interval $[0.602691, 0.732391]$. For $t = 200$ hrs, availability obtained through proposed approach using normalized arithmetic operations lies in the interval $[0.5794196, 0.718787]$. Obtained availability functions for $t = 100$ hrs and 200 hrs using normalized arithmetic operations are in the form of generalized trapezoidal fuzzy numbers as shown in Figure 6.2 and Figure 6.3.

(v) Results for $t = 100$ hrs obtained using improved arithmetic operations lies in the interval $[0.601056, 0.733057]$. It is reflected form Table 6.3 that results for $t = 200$ hrs obtained using improved arithmetic operations lies in the interval $[0.5792986, 0.733057]$. Obtained availability function using improved arithmetic operations is in the form of generalized trapezoidal fuzzy number as shown in Figure 6.2. Results obtained using improved arithmetic operations involve computational complexity but it does emphasis the importance of degree of confidence.

(vi) It may be seen from the obtained results (Table 6.2 and 6.3) that availability of the system decreases by 2.68% from 100 hrs to 200 hrs.

System availability for t=100 hrs

	Crisp approach[86]	Result computed by suggested approach through					
		Normalized operations		Chen's operations		Improved arithmetic operations	
$\alpha\downarrow$	Av	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$
0	0.668940	0.602691	0.732391	0.602008	0.732328	0.601056	0.733057
0.1	0.668940	0.611886	0.723632	0.613422	0.726849	0.610539	0.724229
0.2	0.668940	0.622006	0.714815	0.616179	0.719165	0.619686	0.715807
0.3	0.668940	0.634649	0.705603	0.624919	0.713162	0.628369	0.708089
0.4	0.668940	0.641517	0.696986	0.630935	0.707116	0.636695	0.700379
0.5	0.668940	0.651820	0.687211	0.636577	0.701291	0.646449	0.691336
0.6	0.668940	0.660747	0.678251	0.642567	0.693738	0.654551	0.683198
0.7	0.668940	0.668940	0.668940	0.651221	0.688037	0.662829	0.674949

Table 6.2: Availability of cattle feed plant at t = 100 hrs

System availability for t=200 hrs

	Crisp approach[86]	Result computed by suggested approach through					
		Normalized operations		Chen's operations		Improved arithmetic operations	
$\alpha\downarrow$	Av	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$	$\tilde{Av}^{\alpha}_{(L)}$	$\tilde{Av}^{\alpha}_{(R)}$
0	0.651021	0.579419	0.718787	0.578687	0.718292	0.579299	0.733057
0.1	0.651021	0.585179	0.711841	0.591512	0.708869	0.595342	0.723770
0.2	0.651021	0.595226	0.705664	0.599599	0.699772	0.600818	0.716873
0.3	0.651021	0.600905	0.699023	0.611839	0.690804	0.608512	0.707771
0.4	0.651021	0.611384	0.692190	0.622976	0.680316	0.610056	0.699631
0.5	0.651021	0.616859	0.685453	0.631703	0.670523	0.618704	0.690992
0.6	0.651021	0.622806	0.678621	0.642283	0.660825	0.634380	0.682256
0.7	0.651021	0.631339	0.671719	0.651021	0.651021	0.64616	0.675033

Table 6.3: Availability of cattle feed plant at t = 200 hrs

Simultaneous effects of failure and repair rates on steady state availability have been observed as given in Figure 6.4. Impacts of various parameters on steady state availability have also been observed and depicted in Table 6.4. It may be observed that 26.09% decrement in failure rate and 35.29% increment in repair

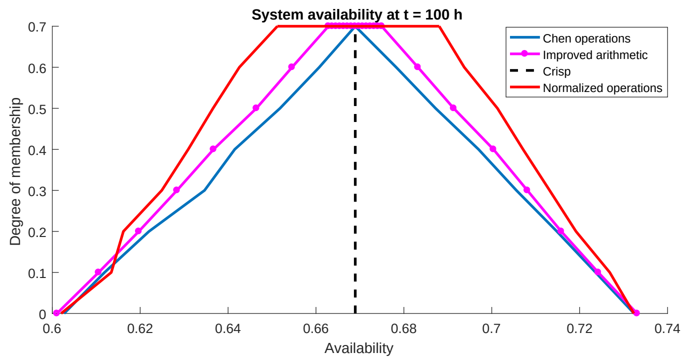


Figure 6.2: Availability of cattle feed plant at t = 100 hrs

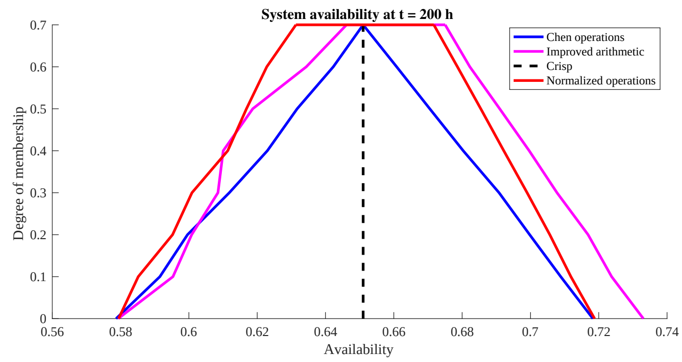


Figure 6.3: Availability of cattle feed plant at t = 200 hrs

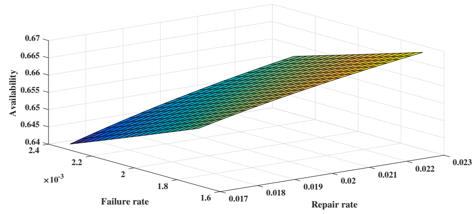
rate of Elevator, keeping other parameters fixed, results 4.1% increase in overall availability. 33.33% decrement in failure rate and 35.29% increment in repair rate of Grinder leads to the 4.1% gain in overall availability. 26.09% decrement in failure rate and 35.29% increment in repair rate of Hopper leads to the 4.1% gain in overall availability. 27.59% decrement in failure rate and 35.29% increment in repair rate of Mixer leads to the 0.49% gain in overall availability. 27.59% decrement in failure rate and 35.29% increment in repair rate of Mixer in reduced state affects availability by 0.56% gain. 24.56% decrement in failure rate and 35.29% increment in repair rate of Winch leads to the 4.1% gain in overall availability. Similary, 23.53% decrement in failure rate and 35.29% increment in repair rate of Palletiser leads to the 0.33% gain in overall availability and 23.53% decrement in failure rate and 35.29% increment in repair rate of reduced state of Palletiser leads to the 0.37% gain in overall availability. 26.09% decrement in failure rate and 35.29% increment in repair rate of Screw Conveyor leads to the 4.1% gain in overall availability of Cattle feed plant. Variations in failure and repair rates reflect variation in availability. These results may be useful for system analysts in order to take the decisions about system performance.

Component	Failure rate (λ)	Repair rate (μ)	Availability (Min,Max)
Elevator	0.0017- 0.0023	0.017- 0.023	(0.6417357,0.6680505)
Grinder	0.0008-0.0012	0.0085-0.0115	(0.6417357, 0.6680505)
Hopper	0.0034- 0.0046	0.0340 -0.0460	(0.6417357,0.6680505)
Mixer	0.0021- 0.0029	0.0170-0.0230	(0.6548048,0.6580406)
Mixer (Reduced state)	0.0021-0.0029	0.0170-0.0230	(0.6545009, 0.6581733)
Winch	0.0043-0.0057	0.0425- 0.0575	(0.6417356,0.6680504)
Palletiser	0.0026-0.0034	0.0255-0.0345	(0.6553913,0.6575609)
Palletiser (Reduced state)	0.0026-0.0034	0.0255-0.0345	(0.6552271,0.6576315)
Screw Conveyor	0.0017- 0.0023	0.0170-0.0230	(0.6417356,0.6680504)

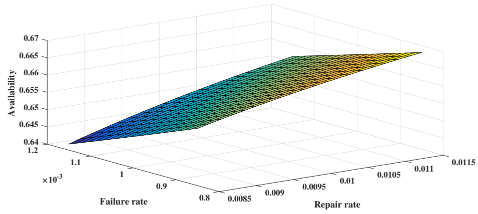
Table 6.4: Simultaneous effect of failure and repair rates on steady state availability in a cattle feed plant

6.5 Conclusion

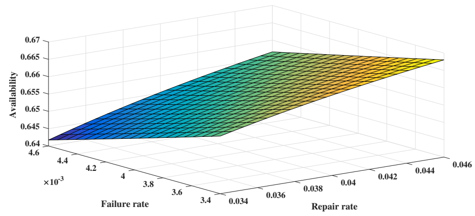
In this chapter, an approach has been introduced for behavior analysis and reliability optimization of a repairable industrial system. An attempt has been made to analyze and optimize the performance of a cattle feed plant with limited and uncertain data. In the proposed approach, uncertainty has been dealt with generalized fuzzy numbers. This approach is divided into two folds: First is finding solution through α-cut and RK-IV method and second fold is to optimize the obtained solution through Particle swarm optimization. In this chapter, this approach has been applied on a cattle feed plant. This approach optimizes the availability of cattle feed plant. Availability of the system has been computed by using different arithmetic



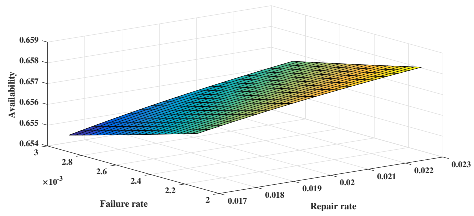
(a) Elevator



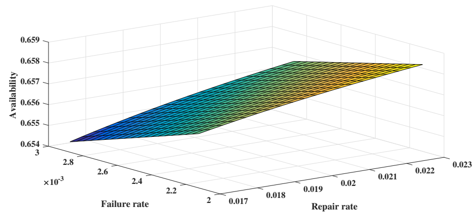
(b) Grinder



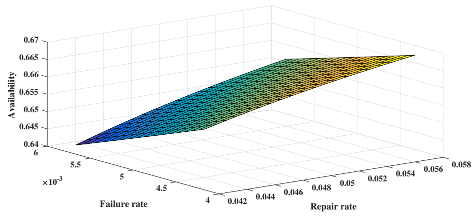
(c) Hopper



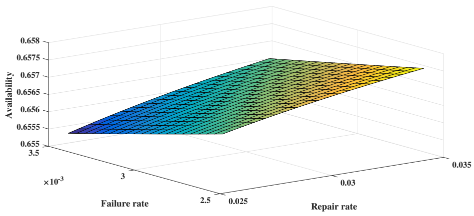
(d) Mixer



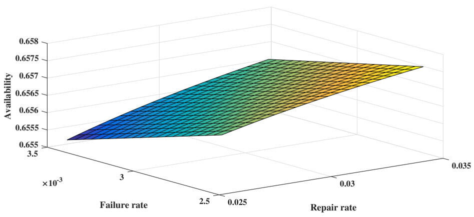
(e) Mixer (Reduced state)



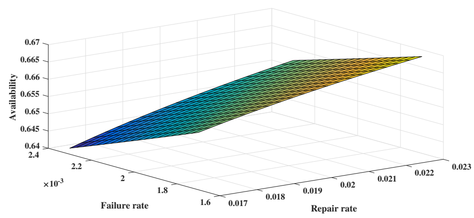
(f) Winch



(g) Palletiser



(h) Palletiser (Reduced state)



(i) Screw Conveyor

Figure 6.4: Simultaneous effect of repair and failure rates on steady availability in a cattle feed plant

operations. Results have been computed which will help the plant maintenance personnel in deciding his or her future strategy to gain optimum performance of the system.

Chapter 7

Availability analysis of Industrial systems through Intuitionistic fuzzy differential equations

The objective of this chapter is again to discuss a technique for analyzing and predicting the behavior of a complex repairable industrial system by utilizing uncertain data, consequently of forming intuitionistic fuzzy differential equations through Markov process and (α, β)- cuts. For better understanding of the technique, this chapter comprises the study of two systems: namely, Condensate system of power plant and Butter-Oil Processing plant. The effects of variations in failure and repair rates have been studied for the purpose of sensitivity analysis and to determine the system's most crucial component. The obtained results will be useful to the system manager/analyst to plan and execute the future course of action in the industry.

7.1 Introduction

In previous chapters, we have discussed uncertainty in terms of degree of membership i.e. in terms of fuzzy sets/fuzzy numbers. But there is always a scope of hesitation in degree of membership. It is taken into account through Intuitionistic fuzzy set theory which was introduced by Atanassov in 1983 [17, 18].

This chapter takes care of these types of uncertainties and the technique for availability analysis has been extended in terms of Intuitionistic fuzzy differential equations (IFDEs). Firstly, differential equations have been generated through mathematical modeling of the system. There differential equations are then transformed into intuitionistic fuzzy differential equations with the help of intuitionistic fuzzy numbers for tackling the uncertainties. Suggested approach/technique is explained through the study of condensate system of Thermal power plant located in northern part of India. System availability in transient and steady state has been evaluated. Corresponding to different presumption level, probability value of each state has been evaluated with the help of RK-IV method. System performance by varying its failure and repair rates on the availability of the system has also been analyzed. The performance of system can be improved by this analysis and appropriate maintenance strategies may be implemented.

7.2 Proposed Approach

An approach of availability analysis of any system having uncertainties, expressed in the form of intuitionistic fuzzy numbers has been presented here. In this approach, Markov process has been used and the assumptions used in the process are the same as discussed in Chapter 2. Along with all the assumptions, the parameters are in the form of intuitionistic fuzzy numbers, instead of fuzzy numbers. Following steps have been taken in order to evaluate availability.

Step 1: [***Derivation of differential equations through Markov process***]**:**

First step is to derive a system of differential equations from Markov model of a system, as discussed in Section 3.2 of Chapter 3.

Step 2 :[***Formation of Intuitionistic Fuzzy differential equation (IFDE)***]

We come across many physical situations that are described by an n^{th} order linear differential equation with constant coefficients for some integral values of n i.e. a differential equation of the form:

$$a_n x^{(n)} + a_{n-1} x^{(n-1)} + ... + a_1 x^{(1)} + a_0 x = g(t), \quad x^{(p)} = \frac{d^p x}{dt^p}, \ p = 1, 2, ..., n$$

with initial conditions

$$x(0) = \gamma_0, x^{(1)}(0) = \gamma_1, ..., x^{(n-1)}(0) = \gamma_{n-1}, \tag{7.2.1}$$

and all its coefficients a_i having crisp values. However in real life situations, when the differential equation represents a physical situation, the values of coefficients may depend on the various sources, and cannot be obtained precisely. In most of the cases, the information collected from various sources are based on the history or the past behaviour of the system and consequently do not identify the precise behavior of the system. To deal with such type of uncertainties in the coefficients and initial values of the derivatives the differential equation is changed into intuitionistic fuzzy differential equation of the form:

$$\tilde{a}_n \tilde{x}^{(n)} \oplus \tilde{a}_{n-1} \tilde{x}^{(n-1)} \oplus ... \oplus \tilde{a}_1 \tilde{x}^{(1)} \oplus \tilde{a}_0 \tilde{x} = g(t),$$

with

$$\tilde{x}(0) = \tilde{\gamma}_0, \ \tilde{x}^{(1)}(0) = \tilde{\gamma}_1, \ ..., \tilde{x}^{(n-1)}(0) = \tilde{\gamma}_{n-1}. \tag{7.2.2}$$

Here, $\tilde{a}_n$ is non-zero intuitionistic fuzzy number, $\tilde{x}^{(p)} = \frac{d^p \tilde{x}}{dt^p}$ for $p = 1, 2, ..., n$, and $\tilde{a}_i, \ \tilde{\gamma}_i$ for $i = 0, 1, 2, ..., n-1$ are intuitionistic fuzzy numbers. In the proposed approach for the solution of this type of intuitionistic fuzzy differential equations, following steps are performed.

Step 3: [***Determination of*** (α, β)**- *cuts***]

Herein we obtain the (α,β)-cuts corresponding to all intuitionistic fuzzy parameters $\tilde{a}_n,\ \tilde{a}_{n-1},...,\tilde{a}_0,\ \tilde{x}^{(n)},\ \tilde{x}^{(n-1)},...,\tilde{x}^{(1)},\tilde{x}$ and $\tilde{\gamma}_0,\tilde{\gamma}_1,...,\tilde{\gamma}_{n-1}$ as

$$\tilde{a}_n(t)[\alpha,\beta]=\langle\tilde{a}_n(t)[\alpha];\tilde{a}_n(t)[\beta]\rangle$$
$$=\langle[\tilde{a}_{n(L)}(t)[\alpha],\tilde{a}_{n(R)}(t)[\alpha]];[\tilde{a}_{n(L)}(t)[\beta],\tilde{a}_{n(R)}(t)[\beta]]\rangle;$$

$$\tilde{a}_{n-1}(t)[\alpha,\beta]=\langle[\tilde{a}_{n-1(L)}(t)[\alpha],\tilde{a}_{n-1(R)}(t)[\alpha]];[\tilde{a}_{n-1(L)}(t)[\beta],\tilde{a}_{n-1(R)}(t)[\beta]]\rangle,$$

$$...,\tilde{a}_0(t)[\alpha,\beta]=\langle[\tilde{a}_{0(L)}(t)[\alpha],\tilde{a}_{0(R)}(t)[\alpha]];[\tilde{a}_{0(L)}(t)[\beta],\tilde{a}_{0(R)}(t)[\beta]]\rangle,$$

$$\tilde{x}^n(t)[\alpha,\beta]=\langle\tilde{x}^n(t)[\alpha];\tilde{x}^n(t)[\beta]\rangle$$
$$=\langle[\tilde{x}^n_{(L)}(t)[\alpha],\tilde{x}^{(n)}_{(R)}(t)[\alpha]];[\tilde{x}^{(n)}_{(L)}(t)[\beta],\tilde{x}^{(n)}_{(R)}(t)[\beta]]\rangle,$$

$$\tilde{x}^{(n-1)}(t,)[\alpha,\beta]=\langle[\tilde{x}^{(n-1)}_{(L)}(t)[\alpha],\tilde{x}^{(n-1)}_{(R)}(t)[\alpha]];[\tilde{x}^{(n-1)}_{(L)}(t)[\beta],\tilde{x}^{(n-1)}_{(R)}(t)[\beta]]\rangle,$$

$$...,\tilde{x}(t)[\alpha,\beta]=\langle[\tilde{x}_{(L)}(t)[\alpha],\tilde{x}_{(R)}(t)[\alpha]];[\tilde{x}_{(L)}(t)[\beta],\tilde{x}_{(R)}(t)[\beta]]\rangle,$$

and

$$\tilde{\gamma}_i(0)[\alpha,\beta]=\langle[\tilde{\gamma}_{i(L)}(0)[\alpha],\tilde{\gamma}_{i(R)}(0)[\alpha]];[\tilde{\gamma}_{i(L)}(0)[\beta],\tilde{\gamma}_{i(R)}(0)[\beta]]\rangle,$$

for $i=0,1,...,n-1$.

Step 4: [***Substitution of*** (α,β)***-cut in IFDE***]

On the basis of these (α,β)-cuts, the intuitionistic fuzzy differential equation (7.2.2) becomes:

$$[\tilde{a}_{n(L)}(t)[\alpha],\tilde{a}_{n(R)}(t)[\alpha]][\tilde{x}^{(n)}_{(L)}(t)[\alpha],\tilde{x}^{(n)}_{(R)}(t)[\alpha]]+[\tilde{a}_{n-1(L)}(t)[\alpha],\tilde{a}_{n-1(R)}(t)[\alpha]]$$
$$[\tilde{x}^{(n-1)}_{(L)}(t)[(\alpha),\tilde{x}^{(n-1)}_{(R)}(t)[\alpha]]+...+[\tilde{a}_{0(L)}(t)[\alpha],\tilde{a}_{0(R)}(t)[\alpha]][\tilde{x}_{(L)}(t)[\alpha],\tilde{x}_{(R)}(t)[\alpha]]=$$
$$[g(t),g(t)], \tag{7.2.3}$$

and

$$[\tilde{a}_{n(L)}(t)[\beta], \tilde{a}_{n(R)}(t)[\beta]][\tilde{x}^{(n)}_{(L)}(t)[\beta], \tilde{x}^{(n)}_{(R)}(t)[\beta]] + [\tilde{a}_{n-1(L)}(t)[\beta], \tilde{a}_{n-1(R)}(t)[\beta]]$$
$$[\tilde{x}^{(n-1)}_{(L)}(t)[\beta], \tilde{x}^{(n-1)}_{(R)}(t)[\beta]] + ... + [\tilde{a}_{0(L)}(t)[\beta], \tilde{a}_{0(R)}(t)[\beta]][\tilde{x}_{(L)}(t)[\beta], \tilde{x}_{(R)}(t)[\beta]] =$$
$$[g(t), g(t)] \tag{7.2.4}$$

with the following initial conditions:

$$[\tilde{x}_{(L)}(0)[\alpha], \tilde{x}_{(R)}(0)[\alpha]] = [\tilde{\gamma}_{0(L)}(0)[\alpha], \tilde{\gamma}_{0(R)}(0)[\alpha]],$$

$$[\tilde{x}_{(L)}(0)[\beta], \tilde{x}_{(R)}(0)[\beta]] = [\tilde{\gamma}_{0(L)}(0)[\beta], \tilde{\gamma}_{0(R)}(0)[\beta]], ...,$$

$$[\tilde{x}^{(n-1)}_{(L)}(0)[\alpha], \tilde{x}^{(n-1)}_{(R)}(0)[\alpha]] = [\tilde{\gamma}_{n-1(L)}(0)[\alpha], \tilde{\gamma}_{n-1(R)}(0)[\alpha]],$$

$$[\tilde{x}^{(n-1)}_{(L)}(0)[\beta], \tilde{x}^{(n-1)}_{(R)}(0)[\beta]] = [\tilde{\gamma}_{n-1(L)}(0)[\beta], \tilde{\gamma}_{n-1(R)}(0)[\beta]].$$

Step 5: [***Formulation of differential equations by*** (α, β)***-cut***]

Using the concepts of arithmetic operations through (α, β)- cuts, above system of intuitionistic fuzzy differential equations is transformed into the following ordinary differential equations.

$$\sum_{j=0}^{n} b_j x^{(j)} = g(t) \text{ and } \sum_{j=0}^{n} b'_j x'^{(j)} = g(t)$$
$$\tilde{x}^{(j)}_{(L)}(0)[\alpha] = \tilde{\gamma}_{j(L)}(0)[\alpha] \text{ and } \tilde{x}^{(j)}_{(L)}(0)[\beta] = \tilde{\gamma}_{j(L)}(0)[\beta] \tag{7.2.5}$$

$$\sum_{j=0}^{n} c_j x^{(j)} = g(t) \text{ and } \sum_{j=0}^{n} c'_j x'^{(j)} = g(t)$$
$$\tilde{x}^{(j)}_{(R)}(0)[\alpha] = \tilde{\gamma}_{j(R)}(0)[\alpha] \text{ and } \tilde{x}^{(j)}_{(R)}(0)[\beta] = \tilde{\gamma}_{j(R)}(0)[\beta]$$

where

$$b_j x^{(j)} = min(\tilde{a}_{j(L)}(t)[\alpha]\tilde{x}^{(j)}_{(L)}(t)[\alpha],\ \tilde{a}_{j(L)}(t)[\alpha]\tilde{x}^{(j)}_{(R)}(t)[\alpha],$$
$$\tilde{a}_{j(R)}(t)[\alpha]\tilde{x}^{(j)}_{(L)}(t)[\alpha],\ \tilde{a}_{j(R)}(t)[\alpha]\tilde{x}^{(j)}_{(R)}(t)[\alpha])$$

$$c_j x^{(j)} = max(\tilde{a}_{j(L)}(t)[\alpha]\tilde{x}^{(j)}_{(L)}(t)[\alpha],\ \tilde{a}_{j(L)}(t)[\alpha]\tilde{x}^{(j)}_{(R)}(t)[\alpha],$$
$$\tilde{a}_{j(R)}(t)[\alpha]\tilde{x}^{(j)}_{(L)}(t)[\alpha],\ \tilde{a}_{j(R)}(t)[\alpha]\tilde{x}^{(j)}_{(R)}(t)[\alpha])$$

and

$$b'_j x'^{(j)} = min(\tilde{a}_{j(L)}(t)[\beta]\tilde{x}^{(j)}_{(L)}(t)[\beta],\ \tilde{a}_{j(L)}(t)[\beta]\tilde{x}^{(j)}_{(R)}(t)[\beta]),$$
$$\tilde{a}_{j(R)}(t)[\beta]\tilde{x}^{(j)}_{(L)}(t)[\beta],\ \tilde{a}_{j(R)}(t)[\beta]\tilde{x}^{(j)}_{(R)}(t)[\beta])$$

$$c'_j x'^{(j)} = max(\tilde{a}_{j(L)}(t)[\beta]\tilde{x}^{(j)}_{(L)}(t)[\beta],\ \tilde{a}_{j(L)}(t)[\beta]\tilde{x}^{(j)}_{(R)}(t)[\beta]),$$
$$\tilde{a}_{j(R)}(t)[\beta]\tilde{x}^{(j)}_{(L)}(t)[\beta],\ \tilde{a}_{j(R)}(t)[\beta]\tilde{x}^{(j)}_{(R)}(t)[\beta])$$

Step 6: [***Solution of IFDE through*** (α, β)***- cut***]

Now the above framed set of differential equations is solved by using Runge-Kutta fourth order method to obtain the values of $\tilde{x}_{(L)}(t_0)[\alpha]$, $\tilde{x}_{(R)}(t_0)[\alpha]$, $\tilde{x}_{(L)}(t_0)[\beta]$ and $\tilde{x}_{(L)}(t_0)[\beta]$ corresponding to real number $t = t_0$. These obtained values represent the $(\alpha, \beta)-$ cuts for the solution $\tilde{x}(t)$, provided following conditions are satisfied.

(i) $\frac{d\tilde{x}_{(L)}(t_0)[\alpha]}{d\alpha} \geq 0, \frac{d\tilde{x}_{(R)}(t_0)[\alpha]}{d\alpha} \leq 0\ \forall \alpha \in [0, 1]$

(ii) $\frac{d\tilde{x}_{(L)}(t_0)[\beta]}{d\beta} \leq 0, \frac{d\tilde{x}_{(R)}(t_0)[\beta]}{d\beta} \geq 0\ \forall \beta \in [0, 1]$

(iii) $\tilde{x}_L(t_0)[\alpha] \leq \tilde{x}_R(t_0)[\alpha]$, when $\alpha = 1$
and $\tilde{x}_L(t_0)[\beta] \leq \tilde{x}_R(t_0)[\beta]$, when $\beta = 0$.

7.3 Case Study

The approach described above has been illustrated by studying and analyzing system reliability of Condensate system [116, 172] and Butter-oil processing plant [113] through intuitionistic fuzzy differential equations.

7.3.1 Condensate System

Operating power plants efficiency is very important in the economics of power generation. This requires that all the systems function at their peak performance over long term operation. Condensate system helps the power plants to function efficiently and keeps them in continuous operation for optimal performance.
A repairable industrial system, namely Condensate system of Thermal power plant located in the Panipat, northern part of India has been taken. This system is concisely described as below.

System Description:

This system consists of the following subsystems.

- **Subsystem (A): Condenser** having only one unit connected in series whose failure causes the complete failure of the system.
- **Subsystem (B): Gland steam condenser** is connected in series with Condenser, having single unit whose failure causes the failure of the system.
- **Subsystem (C): Drain Cooler** having single unit connected in series. The failure of this unit causes total failure of the system.
- **Subsystem (D): Heaters** consists of three units of low pressure heaters connected in series. Entire system will fail, if any one of them fails.
- **Subsystem (E): Deaerator** having one unit arranged in series whose failure will cause the entire failure of the system.
- **Subsystem (F): Extraction pumps** having two units connected in parallel with one operative and other in cold standby. Entire failure of the pumps occur when both of them will fail.

Corresponding to each main component, failure and repair rates in the form of trapezoidal intuitionistic fuzzy numbers are outlined in Table 7.1.

Component	Failure rate (λ_i)	Repair rate (μ_i)
Condenser	$\langle(0.00615, 0.00684, 0.00836, 0.00919);$ $(0.00553, 0.00684, 0.00836, 0.01011)\rangle$	$\langle(0.243, 0.27, 0.33, 0.363);$ $(0.218, 0.27, 0.33, 0.399)\rangle$
Gland steam condenser	$\langle(0.00818, 0.00909, 0.01111, 0.01222);$ $(0.00736, 0.00909, 0.01111, 0.01344)\rangle$	$\langle(0.122, 0.135, 0.165, 0.182);$ $(0.109, 0.135, 0.165, 0.2)\rangle$
Drain cooler	$\langle(0.00332, 0.00369, 0.00451, 0.00496);$ $(0.003, 0.00369, 0.00451, 0.00545)\rangle$	$\langle(0.284, 0.315, 0.385, 0.424);$ $(0.256, 0.315, 0.385, 0.466)\rangle$
Heaters	$\langle(0.00616, 0.00684, 0.00836, 0.00919);$ $(0.00554, 0.00684, 0.00836, 0.01011)\rangle$	$\langle(0.203, 0.225, 0.275, 0.303);$ $(0.183, 0.225, 0.275, 0.333)\rangle$
Deaerator	$\langle(0.00267, 0.00297, 0.00363, 0.00399);$ $(0.00241, 0.00297, 0.00363, 0.00439)\rangle$	$\langle(0.151, 0.168, 0.206, 0.226);$ $(0.136, 0.168, 0.206, 0.241)\rangle$
Extraction pumps	$\langle(0.0243, 0.027, 0.033, 0.0363);$ $(0.02187, 0.027, 0.033, 0.0399)\rangle$	$\langle(0.223, 0.248, 0.303, 0.333);$ $(0.2, 0.248, 0.303, 0.366)\rangle$

Table 7.1: Input data for Condensate system

Notations:

In this section, notations that are used for examining the availability of the system are given.

○	Represents that system is in full working state.
⬭	Represents reduced state of the system.
▭	Represent that system is in failed state.
A,B,C,D,E,F	Represent full working states of the subsystem.
F_1	Represent that the subsystem F is working on standby unit.
a,b,c,d,e,f	Represents failed states of the subsystem
$P_0(t)$	Probability of working of the system in full capacity at time 't'.
$P_1(t)$	Probability in standby state of the system at time 't'.
$P_2(t)$ to $P_{12}(t)$	Probability of the system in failed state.
$\lambda_i, i = 1, 2..., 6$	Failure rates of A, B, C, D, E and F subsystems respectively.
$\mu_i, i = 1, 2..., 6$	Repair rates of A, B, C, D, E and F subsystems respectively.

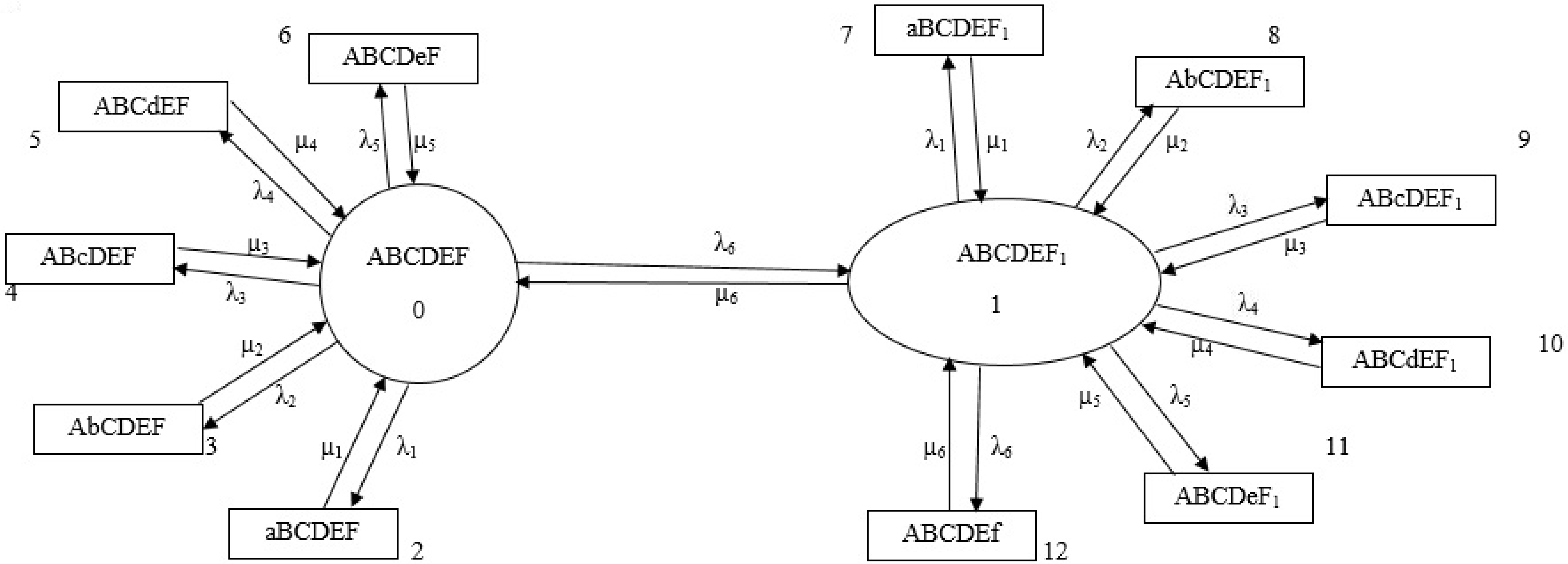


Figure 7.1: Transition diagram of the Condensate System

Transient Analysis:

In this section, availability analysis of Condensate system of Thermal power plant [116, 172] is examined through Markov state transition diagram. All the possible states for the Condensate system under consideration have been illustrated in the transition diagram (Figure 7.1).

Mathematical formulation

Applying the concepts of Markov modeling and probability theory, following intuitionistic fuzzy differential equations (IFDEs) are obtained from the transition diagram (Figure 7.1):

$$\begin{aligned}
\frac{d\tilde{P}_0(t)}{dt} \oplus \tilde{\delta}_1\tilde{P}_0(t) &= \sum_{j=1}^{5}\tilde{\mu}_j\tilde{P}_{j+1}(t) \oplus \tilde{\mu}_6\tilde{P}_1(t)\\
\frac{d\tilde{P}_1(t)}{dt} \oplus \tilde{\delta}_2\tilde{P}_1(t) &= \sum_{j=1}^{6}\tilde{\mu}_j\tilde{P}_{j+6}(t) \oplus \tilde{\lambda}_6\tilde{P}_0(t)\\
\frac{d\tilde{P}_{i+1}(t)}{dt} \oplus \tilde{\mu}_i\tilde{P}_{i+1}(t) &= \tilde{\lambda}_i\tilde{P}_0(t), \qquad i = 1,2,3,4,5\\
\frac{d\tilde{P}_{i+6}(t)}{dt} \oplus \tilde{\mu}_i\tilde{P}_{i+6}(t) &= \tilde{\lambda}_i\tilde{P}_1(t) \qquad i = 1,2,3,4,5,6
\end{aligned} \tag{7.3.1}$$

with $\tilde{\delta}_1 = \sum_{j=1}^{6}\tilde{\lambda}_j$, $\tilde{\delta}_2 = \sum_{j=1}^{6}\tilde{\lambda}_j \oplus \tilde{\mu}_6$ and initial conditions
$\tilde{P}_0(0) = \langle(0.95, 0.955, 0.965, 0.97); (0.945, 0.955, 0.965, 0.975)\rangle$
$\tilde{P}_1(0) = \langle(0.004, 0.0045, 0.0055, 0.006); (0.0035, 0.0045, 0.0055, 0.0065)\rangle$ and
$\tilde{P}_j(0) = 0$ for j=2 to 13.
Corresponding to assigned $(\alpha, \beta)-$ cuts as described in Step 2 to 3 of Section 7.2, the system of intuitionistic fuzzy differential equations obtained by the system are solved by Runge-Kutta fourth order method since the system is quite involved and it is not easy to obtain its analytic solution. The availability function $\tilde{A}v(t)$ so obtained is

$$\tilde{A}v(t) = \tilde{P}_0(t) \oplus \tilde{P}_1(t) \tag{7.3.2}$$

where $\tilde{P}_0(t)$ and $\tilde{P}_1(t)$ are the probabilities of system working in full and standby state respectively.

Steady State Analysis:

In order to obtain the availability in long run, we substitute derivatives of all probabilities individually equal to zero. i.e.,
$\frac{dP_i(t)}{dt} = 0$ and $P_i(t) \to P_i$ as $t \to \infty$.
Then following probability values in terms of P_0 are obtained by solving the system equations recursively.

$P_1 = \frac{\lambda_6}{\mu_6}P_0;\qquad P_2 = \frac{\lambda_1}{\mu_1}P_0;\qquad P_3 = \frac{\lambda_2}{\mu_2}P_0;$

$P_4 = \frac{\lambda_3}{\mu_3}P_0;\qquad P_5 = \frac{\lambda_4}{\mu_4}P_0;\qquad P_6 = \frac{\lambda_5}{\mu_5}P_0;$

$P_7 = (\frac{\lambda_1}{\mu_1})(\frac{\lambda_6}{\mu_6})P_0;\qquad P_8 = (\frac{\lambda_2}{\mu_2})(\frac{\lambda_6}{\mu_6})P_0;\qquad P_9 = (\frac{\lambda_3}{\mu_3})(\frac{\lambda_6}{\mu_6})P_0;$

$P_{10} = (\frac{\lambda_4}{\mu_4})(\frac{\lambda_6}{\mu_6})P_0;\qquad P_8 = (\frac{\lambda_5}{\mu_5})(\frac{\lambda_6}{\mu_6})P_0;\qquad P_9 = (\frac{\lambda_6}{\mu_6})(\frac{\lambda_6}{\mu_6})P_0;$

Based on these probabilities, after substituting the values of (P_0 to P_{12}) in the equation $\sum_{i=0}^{12} P_i = 1$, we get

$$\tilde{P}_0 = \frac{1}{(1 + \frac{\tilde{\lambda}_1}{\tilde{\mu}_1} \oplus \frac{\tilde{\lambda}_2}{\tilde{\mu}_2} \oplus \frac{\tilde{\lambda}_3}{\tilde{\mu}_3} \oplus \frac{\tilde{\lambda}_4}{\tilde{\mu}_4} \oplus \frac{\tilde{\lambda}_5}{\tilde{\mu}_5} \oplus \frac{\tilde{\lambda}_6}{\tilde{\mu}_6}) \oplus (\frac{\tilde{\lambda}_1}{\tilde{\mu}_1} \oplus \frac{\tilde{\lambda}_2}{\tilde{\mu}_2} \oplus \frac{\tilde{\lambda}_3}{\tilde{\mu}_3} \oplus \frac{\tilde{\lambda}_4}{\tilde{\mu}_4} \oplus \frac{\tilde{\lambda}_5}{\tilde{\mu}_5} \oplus \frac{\tilde{\lambda}_6}{\tilde{\mu}_6})\frac{\tilde{\lambda}_6}{\tilde{\mu}_6}}$$

Hence the system availability in the steady state is

$$\tilde{Av} = \left(1 + \frac{\tilde{\lambda}_6}{\tilde{\mu}_6}\right)\tilde{P}_0 \tag{7.3.3}$$

It may be noted that, perhaps there is a typographical error in the expression of $\tilde{P}_0$ as given in [91, 116].

Results and Discussion:

Availability for Condensate system in terms of transient and steady state has been discussed in this section.

(a) Transient State:

System availability in terms of Intuitionistic fuzzy number (IFN) is computed by the proposed approach for mission time $t = 48$ hrs. Solution so obtained from set of differential equations (7.3.1) is summarized in tabular form (Table 7.2) for $\alpha, \beta = 0, 0.2, 0.4, 0.8, 1.0$. This analysis reveals that the results evaluated by suggested method are better than the existing results. For instance corresponding to $\alpha = 0, \beta = 1$, the probabilities of the working of the states P_0 and P_1 are $[0.7399581, 0.757268]$ and $[0.0805787, 0.0825391]$ respectively by suggested method while these are $[0.739958, 0.756819]$ and $[0.080578, 0.082496]$ by existing approach. Based on the obtained values of probabilities, the corresponding $((\alpha, \beta)-$ cuts of the overall system availability, by existing and proposed approach for the time $t = 48$ hrs is $[0.820536, 0.839315]$ and $[0.8205368, 0.8397459]$ respectively. There is 2.29% increment in the availability corresponding to $\alpha = 0,\ \beta = 1$. Effects on the system availability for different assumption level of uncertainties are evaluated and outlined in Table 7.3. Following conclusions are drawn from this analysis and basic concepts.

(i) The results evaluated by the traditional approach (crisp) [116] do not always provide the exact idea about the behavior of the system. As these methods deal with the precise data, cannot deal with the data containing uncertainties.

(ii) The results computed by the fuzzy method [91, 172] do not deal with the degree of hesitation. Results obtained by the suggested approach deal with the various degree of membership and non-membership function. For instance, the availability of the system corresponding to $\alpha = 0.7$ and $\beta = 0.1$ is $[0.8240099, 0.8353281]$

(iii) Here the results are calculated by the suggested method by handling the data uncertainties in the form of intuitionistic trapezoidal fuzzy numbers. From this, corresponding to different assumption level, the availability of the system has been computed for $t = 48$ hrs. It has been seen that suggested approach gives a better range than the one, given by Lata and Kumar [172], at any $(\alpha, \beta)-$ cut as the method of $(\alpha, \beta)-$ cuts deals accurately with the intuitionistic fuzzy differential equations.

The complete results of system availability are outlined in Table 7.3. With the help of (α, β)-cut, membership and non-membership function in approximated form of intuitionistic fuzzy availability at $t = 48$ hrs are explained here as:

$$\mu_{\tilde{A}v}(x) = \begin{cases} \frac{x-0.8205368}{0.0044256}, & 0.8205368 \leq x \leq 0.8249624 \\ 1, & 0.8249624 \leq x \leq 0.8344188 \\ \frac{0.8397459-x}{0.0053271}, & 0.8344188 \leq x \leq 0.8397459 \\ 0, & \text{otherwise} \end{cases}$$

$$\nu_{\tilde{A}v}(x) = \begin{cases} \frac{0.8249624-x}{0.0094872}, & 0.8154752 \leq x \leq 0.8249624 \\ 0, & 0.8249624 \leq x \leq 0.8344188 \\ \frac{x-0.8344188}{0.0091394}, & 0.8344188 \leq x \leq 0.8435582 \\ 1. & \text{otherwise} \end{cases}$$

Intuitionistic fuzzy availability of the system at $t = 48$ hrs is shown in Figure 7.2.

This model provides the system analyst an effective tool to decide the related features of the components and will assist him in design modifications to minimize the failures and to help in maintenance decision making.

j	$\tilde{P}_j(t)[\alpha]$ for $\alpha = 0$		$\tilde{P}_j(t)[\alpha]$ for $\alpha = 0.2$		$\tilde{P}_j(t)[\alpha]$ for $\alpha = 0.4$		$\tilde{P}_j(t)[\alpha]$ for $\alpha = 0.6$		$\tilde{P}_j(t)[\alpha]$ for $\alpha = 0.8$		$\tilde{P}_j(t)[\alpha]$ for $\alpha = 1$	
	$\tilde{P}_{j(L)}(t)[\alpha]$	$\tilde{P}_{j(R)}(t)[\alpha]$	$\tilde{P}_{j(L)}(t)[\alpha]$	$\tilde{P}_{j(R)}(t)[\alpha]$	$\tilde{P}_{j(L)}(t)[\alpha]$	$\tilde{P}_{j(R)}(t)[\alpha]$	$\tilde{P}_{j(L)}(t)[\alpha]$	$\tilde{P}_{j(R)}(t)[\alpha]$	$\tilde{P}_{j(L)}(t)[\alpha]$	$\tilde{P}_{j(R)}(t)[\alpha]$	$\tilde{P}_{j(L)}(t)[\alpha]$	$\tilde{P}_{j(R)}(t)[\alpha]$
0	0.7399581	0.7572068	0.7407585	0.7562609	0.7415624	0.7553146	0.7423694	0.7543679	0.7431791	0.7534208	0.7439914	0.7524733
1	0.0805787	0.0825391	0.0806567	0.0824217	0.0807349	0.0823037	0.0808134	0.0821850	0.0808920	0.0820657	0.0809710	0.0819456
2	0.0187312	0.0191702	0.0187549	0.0191486	0.0187787	0.0191271	0.0188024	0.0191057	0.0188260	0.0190843	0.0188497	0.0190631
3	0.0495506	0.0508388	0.0496581	0.0508014	0.0497630	0.0507648	0.0498654	0.0507291	0.0499658	0.0506944	0.0500641	0.0506605
4	0.0086515	0.0088580	0.0086646	0.0088492	0.0086776	0.0088404	0.0086905	0.008831	0.0087033	0.0088233	0.0087160	0.0088148
5	0.0224602	0.0229664	0.0224926	0.0229476	0.0225248	0.0229291	0.0225569	0.0229110	0.0225889	0.0228932	0.0226207	0.0228760
6	0.0130856	0.0133687	0.0130992	0.0133473	0.0131129	0.0133258	0.0131266	0.0133041	0.0131404	0.0132822	0.0131542	0.0132602
7	0.0020382	0.0020896	0.0020408	0.0020868	0.0020433	0.0020841	0.0020458	0.0020814	0.0020483	0.0020786	0.0020507	0.0020758
8	0.0053656	0.0055394	0.0053799	0.0055340	0.0053936	0.0055286	0.0054067	0.0055232	0.0054192	0.0055177	0.0054314	0.0055123
9	0.0009416	0.0009655	0.0009430	0.0009644	0.0009444	0.0009633	0.0009457	0.0009621	0.0009470	0.0009610	0.0009483	0.0009599
10	0.0024430	0.0025033	0.0024466	0.0025008	0.0024502	0.0024983	0.0024537	0.0024958	0.0024571	0.0024934	0.0024605	0.0024909
11	0.0014209	0.0014570	0.0014227	0.0014544	0.0014245	0.0014518	0.0014262	0.0014491	0.0014279	0.0014463	0.0014295	0.0014436
12	0.0087747	0.0089972	0.0087823	0.0089829	0.0087899	0.0089684	0.0087974	0.0089538	0.0088050	0.0089390	0.0088125	0.0089241

j	$\tilde{P}_j(t)[\beta]$ for $\beta = 0$		$\tilde{P}_j(t)[\beta]$ for $\beta = 0.2$		$\tilde{P}_j(t)[\beta]$ for $\beta = 0.4$		$\tilde{P}_j(t)[\beta]$ for $\beta = 0.6$		$\tilde{P}_j(t)[\beta]$ for $\beta = 0.8$		$\tilde{P}_j(t)[\beta]$ for $\beta = 1$	
	$\tilde{P}_{j(L)}(t)[\beta]$	$\tilde{P}_{j(R)}(t)[\beta]$	$\tilde{P}_{j(L)}(t)[\beta]$	$\tilde{P}_{j(R)}(t)[\beta]$	$\tilde{P}_{j(L)}(t)[\beta]$	$\tilde{P}_{j(R)}(t)[\beta]$	$\tilde{P}_{j(L)}(t)[\beta]$	$\tilde{P}_{j(R)}(t)[\beta]$	$\tilde{P}_{j(L)}(t)[\beta]$	$\tilde{P}_{j(R)}(t)[\beta]$	$\tilde{P}_{j(L)}(t)[\beta]$	$\tilde{P}_{j(R)}(t)[\beta]$
0	0.7439914	0.7524733	0.7422271	0.7540959	0.7404626	0.7557245	0.738699	0.7573580	0.7369382	0.7589959	0.7351821	0.7606377
1	0.0809710	0.0819456	0.0808306	0.0821426	0.0806927	0.0823384	0.0805572	0.0825333	0.0804240	0.0827272	0.0802931	0.0829205
2	0.0188497	0.0190631	0.0188098	0.0191049	0.0187704	0.0191469	0.0187317	0.0191890	0.0186937	0.0192312	0.0186568	0.0192734
3	0.0500641	0.0506605	0.0499587	0.0507484	0.0498518	0.0508377	0.0497426	0.0509285	0.0496302	0.0510206	0.0495131	0.0511141
4	0.0087160	0.0088148	0.0086961	0.0088305	0.0086763	0.0088464	0.0086566	0.0088627	0.0086372	0.0088792	0.0086180	0.0088959
5	0.0226207	0.0228759	0.0225521	0.0229181	0.0224825	0.0229610	0.0224120	0.0230046	0.0223404	0.0230487	0.0222677	0.0230934
6	0.0131542	0.0132602	0.0131284	0.0133901	0.0131029	0.0135141	0.0130778	0.0136327	0.0130531	0.0137464	0.0130286	0.0138557
7	0.0020507	0.0020758	0.0020475	0.0020810	0.0020442	0.0020860	0.0020411	0.0020911	0.0020379	0.0020961	0.0020347	0.0021010
8	0.0054314	0.0055123	0.0054190	0.0055244	0.0054057	0.0055363	0.0053909	0.0055480	0.0053743	0.0055596	0.0053553	0.0055711
9	0.0009483	0.0009599	0.0009467	0.0009618	0.0009451	0.0009638	0.0009435	0.0009658	0.0009419	0.0009678	0.0009402	0.0009698
10	0.0024605	0.0024909	0.0024541	0.002496	0.0024476	0.0025015	0.0024409	0.0025068	0.0024339	0.0025121	0.0024265	0.0025174
11	0.0014295	0.0014436	0.0014270	0.0014582	0.0014244	0.0014721	0.0014217	0.0014854	0.0014187	0.0014982	0.0014155	0.0015104
12	0.0088125	0.0089241	0.0088029	0.0089478	0.0087937	0.0089711	0.0087849	0.0089941	0.0087766	0.0090169	0.0087684	0.0090396

Table 7.2: Solution of set of Intuitionistic fuzzy differential equations (7.3.1) at t=48 hrs

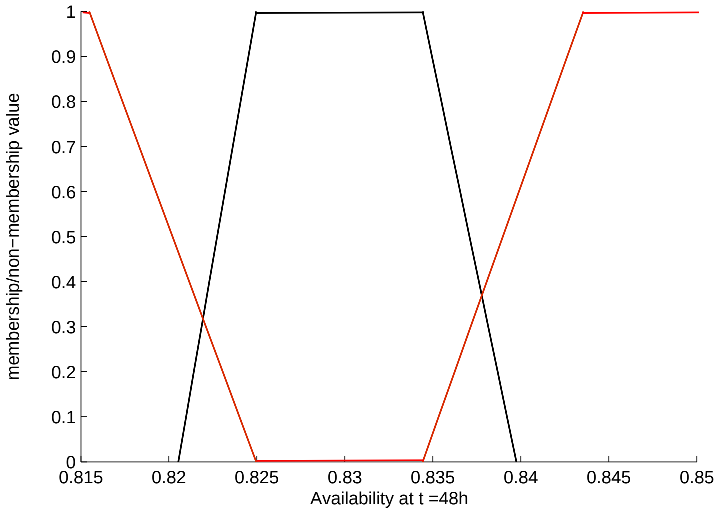


Figure 7.2: Intuitionistic fuzzy availability of Condensate system (membership and non-membership function are shown by black and red lines respectively)

	Proposed Approach		Existed Approach [172]		Proposed Approach		Existed Approach [172]	
$\alpha, \beta \downarrow$	$\tilde{A}v_{(L)}(t)[\alpha]$	$\tilde{A}v_{(R)}(t)[\alpha]$	$\tilde{A}v_{(L)}(t)[\alpha]$	$\tilde{A}v_{(R)}(t)[\alpha]$	$\tilde{A}v_{(L)}(t)[\beta]$	$\tilde{A}v_{(R)}(t)[\beta]$	$\tilde{A}v_{(L)}(t)[\beta]$	$\tilde{A}v_{(R)}(t)[\beta]$
0	0.8205368	0.8397459	0.8205360	0.8393150	0.8249624	0.8344188	0.8249610	0.8344180
0.1	0.8209756	0.8392144	0.8209785	0.8388253	0.8240099	0.8353281	0.8240124	0.8353744
0.2	0.8214152	0.8386826	0.8214210	0.8383356	0.8230577	0.8362386	0.8230638	0.8363308
0.3	0.8218559	0.8381506	0.8218635	0.8378459	0.8221062	0.8371502	0.8221152	0.8372872
0.4	0.8222973	0.8376183	0.8223060	0.8373562	0.8211553	0.8380629	0.8211666	0.8382436
0.5	0.8227396	0.8227485	0.8368665	0.8373562	0.8202053	0.8389766	0.8202180	0.8392000
0.6	0.8231827	0.8365529	0.8231910	0.8363768	0.8192563	0.8398913	0.8192694	0.8401564
0.7	0.8236266	0.8360198	0.8236335	0.8358871	0.8183085	0.8408068	0.8183208	0.8411128
0.8	0.8240712	0.8354864	0.8240760	0.8353974	0.8173622	0.8417232	0.8173722	0.8420692
0.9	0.8245164	0.8349528	0.8245185	0.8349077	0.8164176	0.8426403	0.8164236	0.8430256
1.0	0.8249624	0.8344188	0.8249610	0.8344180	0.8154752	0.8435582	0.8154750	0.8439820

Table 7.3: Availability of the Condensate system at t=48 hrs

(b) Steady State:

As the goal of system analyst is to maximize the profit and production of the industrial system. So it is required that the system should be operated for maximum possible time. But failure is an inevitable event, allied with the industries. To tackle this and for long term availability, the analysis has been done by steady state availability and by sensitivity analysis. The corresponding results of availability of the system in steady state are summarized in Table 7.4. It has been noticed that by the suggested method, we find the maximum possible span of steady state availability of the system corresponding to each presumption levels of failure and repair rates. From these results, it has been concluded that the whole system availability lies between 0.8000445 and 0.9034856. In order to save manpower, time and money,

$\alpha, \beta \downarrow$	$\tilde{Av}_{(L)}[\alpha]$	$\tilde{Av}_{(R)}[\alpha]$	$\tilde{Av}_{(L)}[\beta]$	$\tilde{Av}_{(R)}[\beta]$
0	0.8000445	0.9034856	0.8319701	0.8834051
0.1	0.8034636	0.9016214	0.8259024	0.8876222
0.2	0.8068294	0.8997272	0.8196578	0.8916957
0.3	0.8101430	0.8978023	0.8132291	0.8956326
0.4	0.8134058	0.8958460	0.8066086	0.8994397
0.5	0.8166186	0.8938576	0.7997885	0.9031230
0.6	0.8197825	0.8918362	0.7927604	0.9066884
0.7	0.8228986	0.8897811	0.7855154	0.9101414
0.8	0.8259680	0.8876914	0.7780443	0.9134872
0.9	0.8289915	0.8855664	0.7703374	0.9167304
1.0	0.8319701	0.8834051	0.7623846	0.9198758

Table 7.4: Steady state system availability of Condensate system

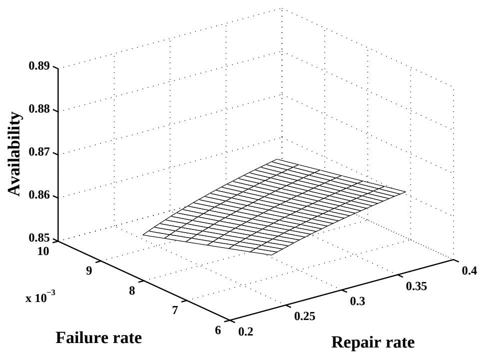
conditions of the system should be modified according to their effectiveness on the availability. As the repair and failure rates of each component of the system influence the system availability directly. Effect of variations in failure and repair rates' has been studied for the purpose of sensitivity analysis (in Table 7.5). The performance of system can be improved by this analysis and appropriate maintenance strategies may be implemented.

It may be observed from Table 7.5 that 33.08% decrement in failure rate and 49.38% increment in repair rate of Condenser, keeping other parameters fixed, lead to 1.8% increase in overall availability. 33.06% decrement in failure rate and 49.18% increment in repair rate of Gland steam condenser affect the overall availability by 4.81% gain. Variation of 33.06% decrement in failure rate and 49.29% increment in repair rate in Drain cooler leads to 0.82% gain in overall availability. 32.97% decrement in failure rate and 49.26% increment in repair rate of Heaters lead to 2.15% in overall availability of the system. Similarly, 33.08% decrement in failure rate and 49.67% increment in repair rate of Deaerator lead to 1.25% in overall availability and 33.06% decrement in failure rate, 49.33% increment in repair rate of Extraction pumps lead to 1.54% gain in overall availability of Condensate system. This is also consistent with the practical point of view: whenever there is a decrease in failure rate and increase in repair rate, the availability should increase. This validates our methodology.

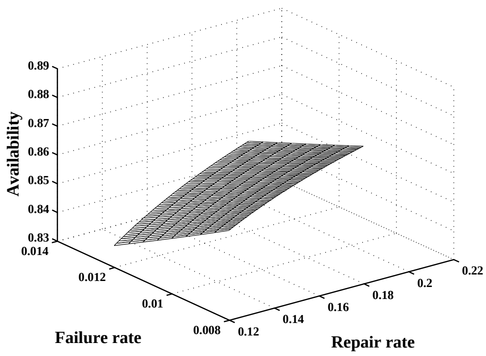
From the results shown in Figure 7.3, it has been obtained that, to save time and money, necessary actions should be taken in the components: Gland steam condenser, Heaters, Extraction pumps, Condenser, Deaerator and Drain Cooler; as per preferential order so that the system analyst can obtain high production goals along with maintaining its performance.

Component	Failure rate (λ)	Repair rate (μ)	Availability (Min,Max)
Condenser (A)	0.00615-0.00919	0.243-0.363	(0.852319, 0.867635)
Gland steam condenser (B)	0.00818-0.01222	0.122-0.182	(0.836383, 0.876629)
Drain cooler (C)	0.00332-0.00496	0.284-0.424	(0.857222, 0.864256)
Heaters (D)	0.00616-0.00919	0.203-0.303	(0.850577, 0.868901)
Deaerator (E)	0.00267-0.00399	0.151-0.226	(0.854998, 0.865714)
Extraction pumps (F)	0.0243-0.0363	0.223-0.333	(0.850907,0.864012)

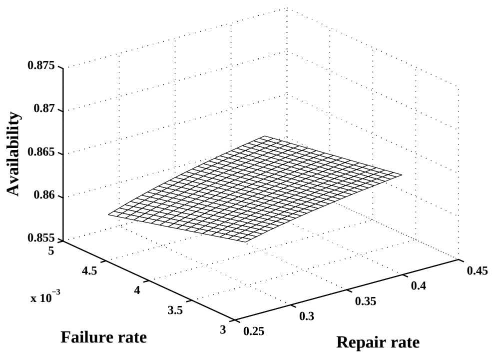
Table 7.5: Simultaneous effects of failure and repair rates on availability of Condensate system



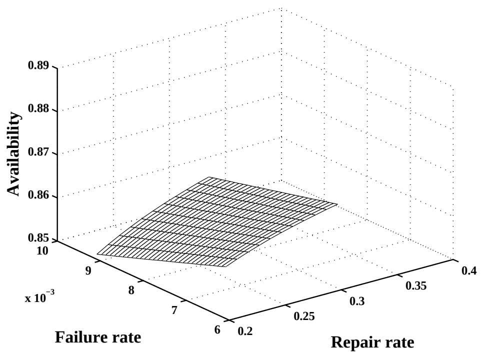
(a) Condenser



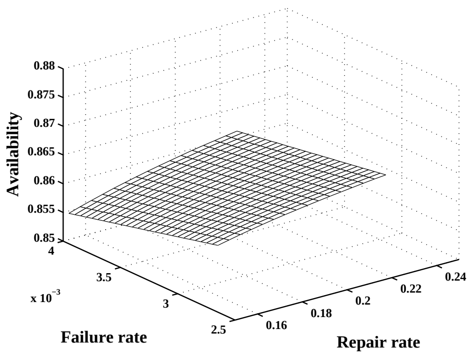
(b) Gland Steam Condenser



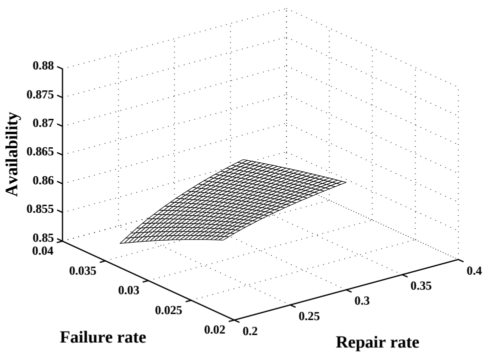
(c) Drain Cooler



(d) Heaters



(e) Deaerator



(f) Extraction Pumps

Figure 7.3: Simultaneous effects of repair and failure rates on steady state availability of Condensate system

7.3.2 Butter-Oil Processing Plant

Firstly, to analyze the suggested approach, butter oil processing plant, as a repairable industrial system, has been taken. Gupta et al. [113] used Markov model with crisp parameters to calculate crisp reliability. Instead of crisp parameters we use intuitionistic fuzzy parameters and the methodology described in section 7.2 is applied to Butter Oil processing plant. System description along with notations are same as used in Chapter 3. The data of failure and repair rates corresponding to each subsystem of the system are taken from [113] as:

Failure rate (λ) = [0.008 0.0054 0.0027 0.0009 0.0027 0.0055 0.01111]

Repair rate (μ) = [0.41 0.40 0.70 0.30 0.65 6.00]

As the data collected for evaluation of reliability contains uncertainty. So, to account for uncertainties and vagueness in data, the obtained crisp data are converted into intuitionistic fuzzy numbers as suggested by decision makers/system analyst. An input data for intuitionistic fuzzy failure rate (λ_i) and intuitionistic fuzzy repair rate (μ_i) for ith component of the system is in the form of triangular intuitionistic fuzzy numbers with 15% span in both the directions with membership and 20% in both the directions with non-membership functions.

Mathematical Formulation:

Using the concepts of probability and Markov modeling, intuitionistic fuzzy differential equations corresponding to the transition diagram (Figure 3.2) are formulated as follows:

$$\frac{d\tilde{P}_1(t)}{dt} \oplus \tilde{\delta}_1 \tilde{P}_1(t) = \sum_{j=1}^{5} \tilde{\mu}_j \tilde{P}_{j+2}(t) \oplus \tilde{\mu}_6 \tilde{P}_{13}(t)$$

$$\frac{d\tilde{P}_2(t)}{dt} \oplus \tilde{\delta}_2 \tilde{P}_2(t) = \sum_{j=1}^{5} \tilde{\mu}_j \tilde{P}_{j+7}(t) \oplus \tilde{\lambda}_6 \tilde{P}_1(t)$$

$$\frac{d\tilde{P}_{i+2}(t)}{dt} \oplus \tilde{\mu}_i \tilde{P}_{i+2}(t) = \tilde{\lambda}_i \tilde{P}_1(t), \qquad i = 1, 2, ...5$$

$$\begin{aligned} \frac{d\tilde{P}_{i+7}(t)}{dt} \oplus \tilde{\mu}_i \tilde{P}_{i+7}(t) =& \tilde{\lambda}_i \tilde{P}_2(t), \qquad i = 1, 2, ...5 \\ \frac{d\tilde{P}_{13}(t)}{dt} \oplus \tilde{\mu}_6 \tilde{P}_{13}(t) =& \tilde{\lambda}_7 \tilde{P}_2(t) \end{aligned} \tag{7.3.4}$$

with $\tilde{\delta}_1 = \sum_{j=1}^{6} \tilde{\lambda}_j$ and $\tilde{\delta}_2 = \sum_{j=1}^{5} \tilde{\lambda}_j \oplus \tilde{\lambda}_7$

with the initial conditions:

$\tilde{P}_1(0) = \langle(0.94, 0.96, 0.98); (0.935, 0.96, 0.985)\rangle$

$\tilde{P}_2(0) = \langle(0.004, 0.005, 0.006); (0.0035, 0.005, 0.0065)\rangle$ and

$\tilde{P}_j(0) = 0$ for j=3 to 13.

The availability function $\tilde{A}v(t)$ of the butter-oil processing plant in terms of $\tilde{P}_1(t)$ and $\tilde{P}_2(t)$ can be obtained by

$$\tilde{A}v(t) = \tilde{P}_1(t) \oplus \tilde{P}_2(t) \tag{7.3.5}$$

Results and Discussion:

Intuitionistic fuzzy system availability is evaluated by the set of first order intuitionistic fuzzy differential equations at different (α, β)- cuts and mission time $t = 365$ days. Solution obtained from the set of differential equations (7.3.4) is summarized in Table 7.6 for $\alpha, \beta = 0, 0.2, 0.4, 0.8, 1.0$. From the analysis, it has been observed that results computed by proposed approach are better than the existing results. Based on these probabilities, the corresponding $(\alpha, \beta)-$ cut of the overall system availability, for the mission time $t = 365$ days by proposed approach lies in the interval $[0.9036677, 0.9438700]$. Similar effect on the overall system availability at different levels of uncertainty is computed and summarized in Table 7.7. From these results it is concluded that

(i) Results provided by the proposed method deal with the various degrees of membership and non-membership functions. For instance, the system availability corresponding to $\alpha = 0.7$ and $\beta = 0.1$ is intersection of intervals $[0.9177379, 0.9297987]$ and $[0.9212317, 0.9263048]$ i.e. availability of system at $\alpha = 0.7$ and $\beta = 0.1$ is $[0.9212317, 0.9263048]$. Corresponding to $\alpha = 0.7$ and $\beta = 0.3$, availability of the system is the intersection of intervals $[0.9177379, 0.9297987]$ and $[0.9161586, 0.9313781]$ i.e. $[[0.9177379, 0.9297987]$ which corresponds to the result obtained from fuzzy numbers.

(ii) On the other hand, the results are computed by the proposed approach by handling the uncertainties in the data in the form of intuitionistic triangular fuzzy numbers. From this, corresponding to different presumption levels, the system availability has been computed for $t = 365$ days.

The complete results of system availability are summarized in Table 7.7. With the help of (α, β)-cut, approximated values of membership and non-membership function of intuitionistic fuzzy availability at $t = 365$ days are defined here.

$$\mu_{\tilde{A}v}(x) = \begin{cases} \frac{x-0.9036677}{0.0201005}, & 0.9036677 \le x \le 0.9237682 \\ 1, & x = 0.9237682 \\ \frac{0.9438700-x}{0.0201018}, & 0.9237682 \le x \le 0.9438700 \\ 0, & \text{otherwise} \end{cases}$$

$$\nu_{\tilde{A}v}(x) = \begin{cases} \frac{0.9237682-x}{0.0253642}, & 0.8984040 \le x \le 0.9237682 \\ 0, & x = 0.9237682 \\ \frac{x-0.9237682}{0.0253666}, & 0.9237682 \le x \le 0.9491348 \\ 1. & \text{otherwise} \end{cases}$$

System availability of butter oil processing plant at $t = 365$ days in term of intuitionistic fuzzy set is shown in Figure 7.4.

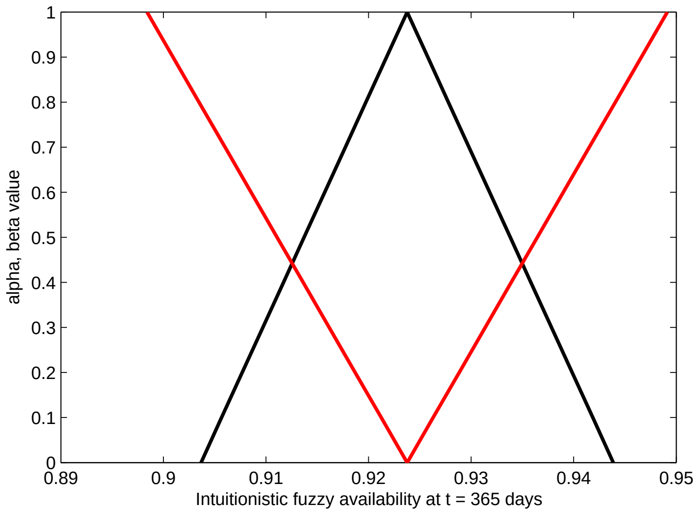


Figure 7.4: Intuitionistic fuzzy availability of Butter Oil Processing Plant(membership and non-membership functions are shown by black and red lines respectively)

j	$\tilde{P}_j(t)[\alpha]$ for $\alpha = 0$		$\tilde{P}_j(t)[\alpha]$ for $\alpha = 0.2$		$\tilde{P}_j(t)[\alpha]$ for $\alpha = 0.4$		$\tilde{P}_j(t)[\alpha]$ for $\alpha = 0.6$		$\tilde{P}_j(t)[\alpha]$ for $\alpha = 0.8$		$\tilde{P}_j(t)[\alpha]$ for $\alpha = 1$	
	$\tilde{P}_{j(L)}(t)[\alpha]$	$\tilde{P}_{j(R)}(t)[\alpha]$	$\tilde{P}_{j(L)}(t)[\alpha]$	$\tilde{P}_{j(R)}(t)[\alpha]$	$\tilde{P}_{j(L)}(t)[\alpha]$	$\tilde{P}_{j(R)}(t)[\alpha]$	$\tilde{P}_{j(L)}(t)[\alpha]$	$\tilde{P}_{j(R)}(t)[\alpha]$	$\tilde{P}_{j(L)}(t)[\alpha]$	$\tilde{P}_{j(R)}(t)[\alpha]$	$\tilde{P}_{j(L)}(t)[\alpha]$	$\tilde{P}_{j(R)}(t)[\alpha]$
1	0.6063945	0.6315314	0.6087487	0.6289154	0.6111551	0.6263127	0.6136056	0.6237259	0.6160933	0.6211579	0.6186124	0.6186124
2	0.2972732	0.3123386	0.2989390	0.3109342	0.3005526	0.3095165	0.3021222	0.3080829	0.3036547	0.3066305	0.3051558	0.3051558
3	0.0118338	0.01232287	0.0118794	0.0122719	0.0119262	0.0122212	0.0119738	0.0121708	0.0120222	0.01212076	0.0120712	0.0120712
4	0.0081875	0.0085259	0.0082191	0.0084906	0.0082514	0.0084555	0.0082844	0.0084207	0.0083179	0.0083861	0.0083518	0.0083518
5	0.0023391	0.0024359	0.0023482	0.0024259	0.0023574	0.0024158	0.0023669	0.0024059	0.0023765	0.0023959	0.0023862	0.0023862
6	0.0018195	0.0018947	0.0018265	0.0018868	0.0018337	0.0018790	0.0018410	0.0018713	0.0018485	0.0018636	0.0018559	0.0018559
7	0.0025191	0.0026233	0.0025288	0.0026125	0.0025388	0.0026016	0.0025489	0.0025909	0.0025593	0.0025803	0.0025697	0.0025697
8	0.0057988	0.0060941	0.0058315	0.0060666	0.0058632	0.0060389	0.0058941	0.0060109	0.0059241	0.0059824	0.0059535	0.0059535
9	0.0040119	0.0042164	0.0040347	0.0041974	0.0040566	0.0041782	0.004077	0.0041588	0.0040987	0.0041391	0.0041191	0.0041191
10	0.0011464	0.0012047	0.0011529	0.0011993	0.0011591	0.0011938	0.0011652	0.0011883	0.0011711	0.0011826	0.0011769	0.0011769
11	0.0008914	0.0009369	0.0008965	0.0009327	0.0009014	0.0009284	0.0009061	0.0009241	0.0009107	0.00091978	0.0009153	0.0009153
12	0.0012346	0.0012974	0.0012415	0.0012915	0.0012483	0.0012856	0.0012548	0.0012796	0.0012612	0.0012736	0.0012675	0.0012675
13	0.0005499	0.0005778	0.0005530	0.0005752	0.0005560	0.0005726	0.0005589	0.0005699	0.0005618	0.0005673	0.0005645	0.0005645

j	$\tilde{P}_j(t)[\beta]$ for $\beta = 0$		$\tilde{P}_j(t)[\beta]$ for $\beta = 0.2$		$\tilde{P}_j(t)[\beta]$ for $\beta = 0.4$		$\tilde{P}_j(t)[\beta]$ for $\beta = 0.6$		$\tilde{P}_j(t)[\beta]$ for $\beta = 0.8$		$\tilde{P}_j(t)[\beta]$ for $\beta = 1$	
	$\tilde{P}_{j(L)}(t)[\beta]$	$\tilde{P}_{j(R)}(t)[\beta]$	$\tilde{P}_{j(L)}(t)[\beta]$	$\tilde{P}_{j(R)}(t)[\beta]$	$\tilde{P}_{j(L)}(t)[\beta]$	$\tilde{P}_{j(R)}(t)[\beta]$	$\tilde{P}_{j(L)}(t)[\beta]$	$\tilde{P}_{j(R)}(t)[\beta]$	$\tilde{P}_{j(L)}(t)[\beta]$	$\tilde{P}_{j(R)}(t)[\beta]$	$\tilde{P}_{j(L)}(t)[\beta]$	$\tilde{P}_{j(R)}(t)[\beta]$
1	0.6186124	0.6186124	0.6154528	0.6218193	0.6123527	0.6250639	0.6093271	0.6283387	0.6063950	0.6316373	0.6035804	0.6349551
2	0.3051558	0.3051558	0.3032423	0.3070221	0.3012694	0.3088507	0.2992221	0.3106494	0.2970815	0.3124241	0.2948236	0.3141797
3	0.0120712	0.0120712	0.0120097	0.0121336	0.0119495	0.0121968	0.0118907	0.0122606	0.0118339	0.0123249	0.0117794	0.0123896
4	0.0083518	0.0083518	0.0083092	0.0083950	0.0082676	0.0084387	0.0082269	0.0084828	0.0081876	0.0085273	0.0081499	0.0085721
5	0.0023862	0.0023862	0.0023739	0.0023985	0.0023621	0.0024110	0.0023504	0.0024236	0.0023392	0.0024363	0.0023284	0.0024491
6	0.0018559	0.0018559	0.0018466	0.0018656	0.0018729	0.0018753	0.0018283	0.0018851	0.0018196	0.0018949	0.0018112	0.0019049
7	0.0025697	0.0025697	0.0025566	0.0025830	0.0025438	0.0025965	0.0025312	0.0026101	0.0025191	0.0026238	0.0025075	0.0026375
8	0.0059535	0.0059535	0.005916	0.0059901	0.0058773	0.0060259	0.0058371	0.0060611	0.0057949	0.0060958	0.0057504	0.0061301
9	0.0041191	0.0041191	0.0040931	0.0041444	0.0040663	0.0041692	0.0040385	0.0041935	0.004009	0.0042175	0.0039785	0.0042413
10	0.0011769	0.0011769	0.0011695	0.0011842	0.0011619	0.0011912	0.0011539	0.0011982	0.0011457	0.0012050	0.0011369	0.0012118
11	0.0009153	0.0009153	0.0009095	0.0009209	0.0009036	0.0009264	0.0008974	0.0009391	0.0008908	0.0009372	0.0008839	0.00094249
12	0.0012675	0.0012675	0.0012595	0.0012752	0.0012513	0.0012829	0.0012427	0.0012903	0.0012338	0.0012977	0.0012244	0.0013050
13	0.0005645	0.0005645	0.0005609	0.0005679	0.0005573	0.0005714	0.0005560	0.0005726	0.0005496	0.0005779	0.0005454	0.0005812

Table 7.6: Solution of set of Intuitionistic fuzzy Kolmogorov's differential equations (7.3.4) at t=365 days

$\alpha, \beta \downarrow$	$\tilde{Av}_{(L)}(t)[\alpha]$	$\tilde{Av}_{(R)}(t)[\alpha]$	$\tilde{Av}_{(L)}(t)[\beta]$	$\tilde{Av}_{(R)}(t)[\beta]$
0	0.9036677	0.9438700	0.9237682	0.9237682
0.1	0.9056777	0.9418598	0.9212317	0.9263048
0.2	0.9076877	0.9398496	0.9186951	0.9288415
0.3	0.9096977	0.9378394	0.9161586	0.9313781
0.4	0.9117077	0.9358292	0.9136221	0.9339147
0.5	0.9137178	0.9338190	0.9110856	0.9364514
0.6	0.9157278	0.9318089	0.9085492	0.9389881
0.7	0.9177379	0.9297987	0.9060128	0.9415247
0.8	0.9197480	0.9277885	0.9034765	0.9440614
0.9	0.9217581	0.9257784	0.9009402	0.9465981
1.0	0.9237682	0.9237682	0.8984040	0.9491348

Table 7.7: System availability of Butter-Oil Processing Plant at t=365 days

7.4 Conclusion

In this chapter, availability analysis has been discussed through Markov model with intuitionistic fuzzy parameters. Solution of generated system of IFDEs has been evaluated using (α, β)- cut arithmetic operations on Intuitionistic fuzzy parameters. From application point of view, Condensate system of Thermal Power Plant and Butter-Oil Processing Plant have been considered. The data uncertainties in failure and repair rates have been tackled with the help of intuitionistic fuzzy numbers. Based on the summary given in the tabular form, system analyst can predict the behavior of the system in more consistent manner. This methodology will assist the plant managers in design modifications to reduce the failures and to help in maintenance decision making. System availability in transient and steady state have been discussed corresponding to different presumption levels which are summarized in the tabular form. Results are computed for different (α, β)- cuts so that analyst can deal with different presumption levels. System analyst/Engineers may consider different (α, β)- cuts so that to obtain the required availability of the system at different uncertainty levels. System performance by varying its failure and repair rates on the availability of the system has also been analyzed. The performance of system can be improved by this analysis and appropriate maintenance strategies may be implemented accordingly.

Chapter 8

Summary and Future Scope

This chapter highlights the major research contribution and presents a comprehensive summary of the research work discussed in this thesis. It also outlines the recommendations to system analysts for improving the systems' performance. The scope for future work to extend the frontiers of the research reported in this thesis have also been outlined.

8.1 Summary of the work

The research work discussed in this thesis is an attempt to facilitate the system analysts/engineers for studying, analyzing, characterizing and predicting the behavior of the system more closely with uncertain data. An overview of the available literature on reliability analysis in different scenario using conventional methods and fuzzy methodology has been given. From the reviewed literature, it has been concluded that the job of the system analyst is quite challenging to maintain the performance of the system for maximum possible duration of time by using vague, limited and uncertain data. Proper attention has been given to this effect in the present study by making use of fuzzy set theory, which is an effective tool to solve the problems related to quantify the uncertainty in the analysis. To help system analyst, Markov model has been used in order to understand the interactions among different components of the system.

The problem of performance analysis for a real complex system has been solved by presenting soft computing techniques involving fuzzy set theory and particle swarm optimization. As the primary data corresponding to system failure and repair rates involves various uncertainties the proposed approaches deal not only with fuzzy numbers but with generalization of fuzzy numbers also. Behavior analysis in terms of fuzzy reliability/availability is done individually for some industrial systems. Major advantage of the proposed techniques is that they provide more realistic results for reliability analysis of complex industrial systems to system analysts/engineers. The analysis will be useful for the plant maintenance personnel to decide the best suited action and to assign the repair priorities as per system requirements. The conclusion made from the work presented in this thesis are summarized below:

1. Fuzzy reliability methodology has important implications in the managerial perspective with respect to plant maintenance and operation.

2. In our study, a technique has been discussed through fuzzy Kolmogorov's differential equations and Particle Swarm Optimization to handle uncertainty. Fuzzy availability is estimated in its transient as well as steady states. As an application of the proposed approach, butter-oil processing plant as an industrial system has been studied. Based on the behavioral and sensitivity analysis results, the system analyst may analyze the critical behavior of the system and plan for suitable strategy.

3. In the suggested approach, formation of optimization model deals with these kinds of situations. In this proposed methodology, solution of fuzzy differential equations is obtained by forming optimization model. Some examples are discussed to illustrate the suggested approach. Furthermore, an application of the approach is presented by evaluating the availability of a repairable system. Piston manufacturing plant, as a repairable industrial system has been taken for the application. Sensitivity analysis has also been studied in order to

discuss the behavior analysis of Piston manufacturing plant. These analysis will help the system analyst for finding the most critical component of the system on which more attention should be given for saving money, manpower and time by adopting a suitable maintenance strategy.

4. In this thesis, a strategy for the assessment of dependability investigation of industrial systems has been contemplated in more summed up way. In this methodology, reliability/availability has been computed through Markov process. Uncertainty in data has been dealt with generalized fuzzy numbers. Availability of system in transient as well as in steady state has been examined in this article. Results have been computed and then compared by performing different arithmetic operations' approaches. For application perspective of proposed approach, a butter oil processing plant has been considered. Impact of different arithmetic approaches in the methodology are reflected by numerical calculations and are depicted through the graphs.

5. In this thesis, solution of differential equations having uncertainties in the form of generalized fuzzy numbers has been discussed through optimization model. Optimization model has been solved through PSO. Performance analysis of Cattle feed plant has been studied. Results have been obtained through different arithmetic operations. Based on their analysis, the system analyst may plan the suitable maintenance strategies for improving the performance of the system.

6. In this thesis, structured and methodological framework has been proposed to analyze a complex industrial system. In quantitative framework, a set of differential equations are formulated through Markov modeling of industrial system in intuitionistic fuzzy environment. Intuitionistic fuzzy system availability is estimated in its transient and steady states. Effects of variations in failure and repair rates' have been studied for the purpose of sensitivity

analysis and to determine the systems' most crucial component. To study the behavior of the system, availability of the system for different (α, β)-cuts has been evaluated. The suggested approach is explained through the study of condensate system of Thermal power plant.

7. Computed results will facilitate the concerned plant managers to plan and adopt suitable maintenance strategies for improving system performance and thereby reducing operational and maintenance costs.

8.2 Future Scope

The methods of design, reliability analysis and optimization aspects in production and manufacturing system can be extended in the following directions.

1. To illustrate the proposed approaches, failure and repair rates are considered as linear membership functions. Uncertainty handled by non linear membership functions in failure and repair rates for examining system availability may be studied in future.

2. Several other factors like maintainability, risk analysis etc. which are not evaluated in uncertain environment are the factors for the future study and develop the methods for their evaluation and analysis.

3. In our study, we have considered the failure and repair rates of different subsystems as constant. In future, we may try to extend the proposed approach to analyze reliability/availability with arbitrary rates instead of constant rates.

4. Different fuzzy arithmetic operations are available in literature. In future, proposed approaches may be extended for different fuzzy operations as well and the results obtained thus may be compared with the existing results.

5. Presented study can be performed equally well to evaluate the system behavior of other process industries such as sugar industry, power plant, cement

industry, petroleum, food processing etc. as the considered methodology can overcome various kind of problems in the area of quality, reliability and maintainability, which strongly needs the management attention.

6. For future research, one may implement this technique for solving models of fuzzy differential equations in other application areas of science and engineering disciplines. This approach may work well for any system of differential equations by formulation of optimization model.

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