REDUCED ORDER MODELLING IN CONTROL SYSTEM

Ph.D. THESIS

by

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INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
ROORKEE – 247 667 (INDIA)
AUGUST, 2013

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A THESIS

Submitted in partial fulfilment of the requirements for the award of the degree of

DOCTOR OF PHILOSOPHY
in
ELECTRICAL ENGINEERING

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ROORKEE

CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in this thesis entitled **"REDUCED ORDER MODELLING IN CONTROL SYSTEM"** in partial fulfilment of the requirements for the award of *the Degree of Doctor of Philosophy* and submitted in the Department of Electrical Engineering of Indian Institute of Technology Roorkee, Roorkee is an authentic record of my own work carried out during a period from July, 2010 to August, 2013 under the supervision of Dr. Rajendra Prasad, Professor, Department of Electrical Engineering, Indian Institute of Technology Roorkee, Roorkee.

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other Institution.

(SANTOSH RANGARAO DESAI)

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

(Rajendra Prasad)
Supervisor

Date:

The Ph.D. Viva-Voce Examination of ***Mr. Santosh Rangarao Desai***, Research Scholar, has been held on …………….

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ABSTRACT

In the present digital world, the advancement of modern day technology towards miniaturization associated is with societal and environmental processes which are contributing to the soaring system complexities, thereby resulting in outsized systems. It's a known fact, that the behavioral study of any system starts with building up of a mathematical model based on theoretical considerations. Accordingly, a set of ordinary differential equations (ODE) or partial differential equations (PDE) are derived by applying physical laws, signifying a mathematical model. In nature and industry, most of these mathematical models turn out to be of higher order, hereafter called as "original model". The direct simulation or design of such models is neither computationally desirable nor physically convenient to handle. Additionally, such models pose difficulty during analysis, control, synthesis and identification as the said tasks are not so easy as they seem to be. It is really grueling, sometimes not feasible and also prove to be a costly affair because of what it may be called as "the curse of dimensionality". In engineering and science, it is often desirable to use the simplest mathematical model that “does the job”. Hence a systematic approximation of the original model is very much in need which results in a reduced order model. The systematic procedure that ends up in reduced order model is called Model Order Reduction (MOR). Hence MOR has born out of the necessity to provide simplified/reduced models, that address the ill effects of higher dimensional models.

The order reduction phase consists of reducing the number of ODE's appropriately, using model reduction technique, to form a reduced model. But, the derived reduced model should provide a good approximation for the original model by preserving some vital features viz. stability, realizibility, good time/frequency response matching etc. It is therefore desirable that the original model can be replaced by the reduced model enabling easy analysis, design, simulation, control and cost effective on line implementation, apart from ensuring the following qualities

(a) Simplify the understanding of the system.
(b) Reduction of computational and hardware complexity.
(c) Reduction of storage requirements.
(d) Ease of efficient controller design and implementation.
(e) Cost effectiveness.

Consequently, order reduction ends up as a necessary procedure for simulating large complex systems; the same is generally practiced in systems and control engineering in spite of having high speed processors and is active area of research.

In the existing literature, abundant order reduction methods have been developed by several authors and are mainly categorized as time and frequency domain reduction methods. Normally, the time domain methods start with a state space description whereas the latter rely on the transfer function model. However, the reduced models obtained from different reduction techniques are unique and the quality is ultimately judged by the way it is utilized. But, none can be judged as the universally best reduction method as these methods depend upon various reason. The best method is one, which shields the vital dynamics of the system under consideration; how well it satisfies the application specifications with reasonable error/computational efforts, in addition to storage. Consequently, the need for better approximation techniques persist. An effort is being made in the present research work to address this issue.

The initial objective of this thesis is to recapitulate most of the model reduction methods available in the research literature. This is succeeded by the purpose to promote some new model order reduction methods applicable to SISO/MIMO time-invariant continuous time systems. The work presented here is confined to linear systems/models and the examples therein. The task mentioned involves the use of both conventional and evolutionary strategies. The considered systems may be represented in frequency domain or time domain. In addition, the other objective is to ensure the superiority of the new reduction methods by comparing with other well known methods, beside checking its validity for LTI discrete system. Lastly, to solve the problem of designing a suitable PID controller for the higher ordered model, utilizing the newly developed method is being considered. In addition, direct and indirect approaches of controller design are dealt with apart from alternative approach to check its applicability for the original model.

At the outset, introduction followed by importance and applications of MOR is presented, subsequently followed by statement of MOR problem in both time and frequency domain for continuous time systems (SISO and MIMO). Besides brief overview about the developments that have taken place in the area of MOR, various existing reduction methods and their associated qualities/drawbacks are also reflected. Composite reduction methods are developed for reduction of higher ordered LTI continuous systems. Stability Equation (SE), Eigen Spectrum Analysis (ESA), Dominant Pole (DP), Modified Pole Clustering (MPC) are employed to propose composite methods. These methods are applicable for SISO/MIMO systems taken from the available literature and are comparable to the available reduced models. The same proposed methods are also extended for SISO/MIMO discrete systems. Comparison of responses to step input and their associated performance indices justifies the proposed methods.

Evolutionary schemes including the recently introduced Big Bang Big Crunch(BBBC) optimization technique are adopted to float new reduction methods. Mixed methods using BBBC in combination with Routh Approximation (RA) and Stability Equation (SE) method yield good results. In addition, BBBC also plays an important role in optimizing the linear shift point 'a' for order reduction in least square sense. Further, systems of higher order represented in both time and frequency domain are considered for reduction using BBBC and the same is extended for discrete systems as well. Original models having the order upto 200, are considered for reduction. The proposed methods are applied on SISO/MIMO systems and are justified by considering the available bench mark examples.

The TMS320C54X processor, grouped under a fixed point DSP is a low-cost, comprehensive development tool that allows new DSP designers to explore the TMS320C5000 DSP architecture and begin developing DSP based applications. It has functional adaptability to a great extent and processing speed. BBBC is roped in to do the required task. The order of Butterworth and Chebyshev filters are designed, and their order is reduced and implemented on TMS320C54x processor. Simulations are carried out in MATLAB and Code Composer Studio (CCS). The input/output waveforms obtained are compared and substantiated. In addition, the frequency response and FFT power spectrum of the input/ output signals are also plotted.

The design of controller for the original models representing practical systems are also dealt to ensure the suitability of developed MOR methods. Further, fractional order PID controller is discussed and shown to perform better than the integer order PID controller using an example. Both direct and indirect approach of controller design are employed in addition to an alternative approach for controller design. The design examples are confined to frequency domain. The unit step response of closed loop transfer functions obtained from the original and reduced plant transfer function are compared with the unit step response of the reference model.

Overall the viability/validity and use of the MOR techniques developed are conclusively established through several numerical examples.

ACKNOWLEDGEMENT

No word would suffice to express my deep gratitude towards my supervisor Dr. Rajendra Prasad, Professor, Department of Electrical Engineering, Indian Institute of Technology Roorkee, Roorkee, without whose sustained supervision, wide experience, sharp and incisive intellect deep insight of the subject this thesis would not have come to fruition. I humbly acknowledge my life time gratitude to him for managing to find time to have stimulated technical discussions and providing me proficient and expert guidance on the subject area that shaped this thesis.

My deepest gratitude and sincere thanks to Prof. M. K. Vasantha, Professor, Indian Institute of Technology Roorkee, for his inspiring guidance, constant encouragement and moral support.

I owe thanks to Dr. Pramod Agarwal, Professor and Head, Dr. Vinod Kumar, Professor, Department of Electrical Engineering, Indian Institute of Technology Roorkee for their invaluable direction, encouragement and support and for providing me all the necessary facilities for carrying out this work till its completion.

I am immensely thankful to Dr. Indra Gupta, Associate Professor Department of Electrical Engineering, Indian Institute of Technology Roorkee, for her moral support and providing lab resources during my work.

I am extremely grateful to all the faculty members of Electrical Engineering Department whose kind nature, enthusiasm, perfectionism and willingness to help, has always inspired me in carrying out this work.

I am thankful to Dr. Rama Bhargava, Co-ordinator, Quality Improvement Programme Centre (QIP), IITR and all the staff for providing support during my course of study.

I am also grateful to AICTE, the Ministry of Human Resources and Development, Government of India, and the staff at my parent institute B. M. S. College of Engg., Bengaluru for sponsoring me to pursue doctoral research work.

I extend my sincere thanks to all research scholars especially to Mr. K. P. Guruswamy, Mr. H. Nagendra, Mr. Nagashettappa Biradar, Mr. Chandrashekar, Mr. Govardhan Bhat, Mr. Praveen Kamat, Mr. Subramanya M. B., Mr. Ajay Shiv Sharma, Mr. Neeraj Gupta for sharing and supporting me directly or indirectly during my research work.

I express my sense of gratitude to Mr. Rajendra Singh, Mr. C. M. Joshi, the Lab staff of Microprocessor and Computer Lab for their cooperation and assistance.

I owe a debt of gratitude to my parents/inlaws for their eternal support and everlasting blessings. I gratefully acknowledge the inspiration and unflagging support of my family members and in laws. I have no words to express the appreciation for my wife Ms Leena, who stood by me at every moment and kept me encouraging in low moments. Words can hardly explain the cooperation and patience of my son Varad.

My sincere pranams to H. H. Sri. Sri. Ravishankar Guruji for his kind blessings to reach at this stage.

I am thankful to the Almighty who gave me the strength and health for completing the work.

Santosh Rangarao Desai

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LIST OF SYMBOLS

Symbol	**Explanation**
$G(s)$	Transfer function of original high order system
$R(s)$	Transfer function of reduced order model
$[G(s)]$	Transfer function matrix of original high order system
$[R(s)]$	Transfer function matrix of reduced order model
A	System matrix
B	Input matrix
C	Output matrix
D	Transmission matrix
a_i b_i	Numerator and denominator coefficients of original high order system
c_i d_i	Numerator and denominator coefficients of reduced order model.
e_i	Power series expansion coefficients
σ_p	Pole centroid
k	System stiffness
I	Integral square error
J	Impulse response energy
J_{org}	Impulse response energy of original system
$M(s)$	Open loop specification model transfer function
a	Linear Shift point
Δ	Incremental change
λ	Poles of system
ω	Angular velocity deviation of the machine rotor in radians per second.
δ , D	Synchronous machine torque angle, damping coefficient,
E_q	Excitation voltage or open circuit voltage of the machine
$G(z)$	Transfer function of discrete time high order model
$G_c(s)$	High order controller transfer function
$G_{cl}(s)$	Closed loop control system with unity feedback
$g_{ij}(s)$	Elements of transfer function matrix
$G_p(s)$	Original plant open loop transfer function
K, K_1, K_2, K_3	Parameters of controller
M_i	Markov parameters
$M(s)$	Specification/reference model transfer function
$R(z)$	Transfer function of discrete time reduced order model

$R_c(s)$	Reduced order controller transfer function
$R_{cl}(s)$	Reduced order closed loop control system with unity feedback
$x(t)$	State vector
$u(t)$	Input vector
$y(t)$	Output vector
n	Order of the original system
r	Order of the reduced model
A_r	Reduced model system matrix
B_r	Reduced model input matrix
C_r	Reduced model output matrix
$x_r(t)$	Reduced model state vector
$u_r(t)$	Reduced model input vector
$y_r(t)$	Reduced model output vector
$\dot{x}(t)$	Derivative state vector
$\dot{x}_r(t)$	Reduced model derivative state vector

LIST OF ABBREVIATIONS

A M	Arithmetic Mean
ALU	Arithmetic Logic Unit
BB	Big Bang
BBBC	Big Bang Big Crunch
BC	Big Crunch
BSP	Buffer Serial Port
CCS	Code Composer Studio
CFE	Continued Fraction Expansion
CPU	Central Processing Unit
CRM	Classical Reduction Methods
DMA	Direct Memory Access
DP	Dominant Pole
DRAM	Direct Routh Approximation Method
DSK	DSP Starter Kit
DSP	Digital Signal Processing
ESA	Eigen Spectrum Analysis
ESP	Eigen Spectrum Points
EVM	Evolution Module
FFT	Fast Fourier Transform
FOPID	Fractional Order Proportional Integral Derivative
GM	Geometric Mean
GA	Genetic Algorithm
H M	Harmonic Mean
HNA	Hankel Norm Approximation
HOM	Higher Order Model
HPI	Host Port Interface
IAE	Integral Absolute Error
IC	Integrated Circuit
IRE	Impulse Response Energy

ISE	Integral Square Error
ITAE	Integral Time Absolute Error
LTI	Linear Time Invariant
McBSP	Multichannel Buffer Serial Port
MEMS	Micro Electro Mechanical Systems
MIMO	Multiple Input Multiple Output
MOR	Model Order Reduction
MP	Markov Parameters
MPC	Modified Pole Clustering
ODE	Ordinary Differential Equations
PDE	Partial Differential Equations
PSO	Particle Swarm Optimization
RA	Routh Approximation
RISE	Relative Integral Square Error
ROM	Reduced Order Model
SCM	Stability Criterion Methods
SE	Stability Equation
SIM	Simulation
SISO	Single Input Single Output
SPM	Stability Preservation Methods
SSE	Summation Square Error
TM	Time Moments
VLSI	Very Large Scale Integration

CHAPTER - 1

INTRODUCTION

Presently, the world is witnessing great changes in almost all areas including system engineering arena, with steeply growing system complexities, thereby resulting in an outsized systems. Generally these systems are best described by large number of differential or difference equations, to form a mathematical model and ease the purpose of analysis, simulation, design and control. However, the said task is not so easy as it seems to be. It is really grueling, sometimes not feasible and also proves to be a costly affair because of what may be called as "the curse of dimensionality". Hence model order reduction (MOR) was born out of the necessity to provide simplified models, thereby addressing the effects of higher dimensional models and continue to rule, in spite of the emergence of present day technology.

1.1 THE IMPORTANCE OF MODEL ORDER REDUCTION

Today, variety of systems appear from diverse areas such as microgrid, mechanical, hydraulic, thermal, space exploration, space communication, global earth observation, robotics, refineries, transport, security, aerospace, manufacturing, molecular systems, Micro-Electro-Mechanical systems (MEMS), electrical power, environmental, urban traffic network, control systems including cruise control etc. Most of these systems, in turn comprises of many subsystems, sharing some common characteristics such as structure, behavior and interconnectivity. These interconnected subsystems themselves being complex, helps to escalate the size and complexity of the overall system, thereby posing difficulty in understanding the system behavior appropriately.

The outlook of this discussion is depicted in fig. 1. At the outset, a complex physical system or data of large dimension is considered. It's a known fact, that the behavioral study of any system, starts with building up of a mathematical model based on theoretical considerations. Accordingly, a set of ordinary differential equations (ODE) or partial differential equations (PDE) are derived by applying physical laws, signifying a mathematical model. In the latter case, the equations are further discretized to obtain large set of ODE's. The mathematical models obtained are quite simpler and understandable than the system it represents. In other words, a good model is a judicious tradeoff between realism and simplicity. Currently, as most of the systems existing in nature/industry turns out to be higher order models, also called as "original model", the direct simulation or design is neither computationally desirable nor physically convenient to be handled. Additionally, such models pose difficulties during its analysis, control, synthesis and identification. Thus, an increasing need exists for a systematic procedure to derive a lower order model, which may be called as "reduced model" from the original model. The subsequent phase consists of reducing the number of ODE's (order of the system) appropriately, using model reduction technique, to form a reduced model. But, the derived reduced model should provide a good approximation for the original model, by preserving some vital features viz. stability, realizibility, good time/frequency response matching. It is therefore desirable that the original model can be replaced by the reduced model enabling easier to analyze, design, simulate, control and cost effective for on line implementation apart from ensuring the following qualities

(a) Simplify the understanding of the system.
(b) Reduction of computational and hardware complexity.
(c) Reduction of storage requirements.
(d) Ease of efficient controller design and implementation.
(e) Cost effectiveness.

Realizing a common goal of reducing higher order models has become the focus of the following areas.

(a) Industrial applications viz structural mechanics, thermal modelling, diffusion, acoustics, MEMS etc [1].
(b) Parameter optimization of large scale dynamical systems [2].
(c) Transient electromagnetic phenomena, Power system, Electrical machines [3].
(d) Transient response sensitivity of large dimensional systems [4].
(e) VLSI design[5]
(f) Predicting the dynamic errors of large dimensional models using reduced order model [6].
(g) Design of suboptimal control by simplified models [7, 8].
(h) Design of control system [9-12].
(i) Adaptive control [13, 14].
(j) Design of reduced order estimators [15].
(k) Intelligent controller [10, 16].
(l) Aircraft maneuver- control dynamics [17].

(m) Controlling thin film growth in a High Pressure Chemical Vapor Deposition reactor [18].

(n) Circuit simulation [19].

(o) A flat-plate solar collector system [20].

(p) IC modelling [21].

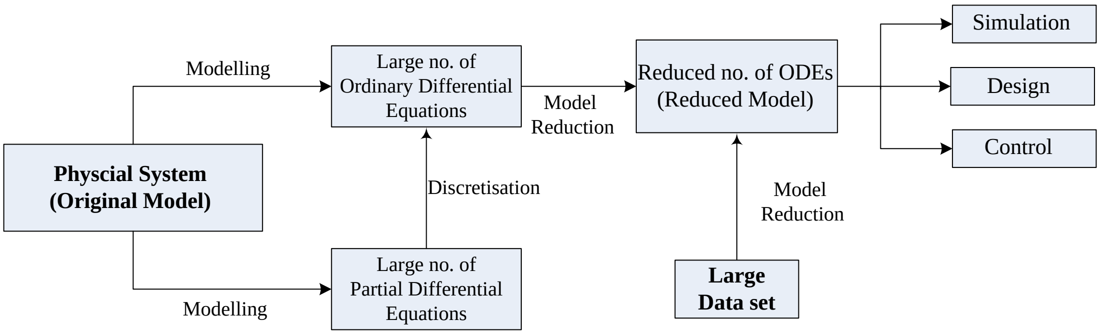

Fig. 1 The outlook

1.2 MODEL REDUCTION PROBLEM FORMULATION

There are enormous of MOR techniques capable of approximating original model in the form of differential or difference equations, resulting in lower order reduced model. Unfortunately, none of these techniques can be termed as universal best in an overall sense, since the system characteristics strongly influence the reliability, performance and adequacy of the reduced system. Although most of the available techniques succeed in preserving the adequate response characteristics and unsolved problems need to be addressed. Hence, an unsurpassed reduction technique has been sought, instead of the available model reduction technique, which will approximate the original model to a reduced model that will take a closer look efficiently. On the other hand, it is observed that in spite of having high computing power and advanced algorithms, there is a need for model order reduction to cope up with even more complex problems. This is due to the fact that, increase in computational power seems to go hand-in-hand with more complicated systems in this fast changing real world.

The present research work is devoted to the category of time-invariant systems operating in continuous time, discrete time domain and the examples therein. Time invariant models are very useful, especially if the time scale of the model is small compared to the life span of the modeled process. Further, many nonlinear models can be approximated by a linear one. The work presented here is confined to linear systems only. During modelling and analysis process of such linear systems, the mathematical model takes either of the following forms/representations

(a) State space form or Time domain representation.

(b) Transfer function form or Frequency domain representation.

The reduction method accept the original model in state space form are called as time domain reduction methods; in transfer function form are called as frequency domain reduction methods. As mentioned earlier/above, the main objective is to obtain a reduced order model from a given original higher order model. Nevertheless, the reduced model should pose all the imperative features of the given higher order model.

1.2.1Time Domain Representation

In this approach, mathematical model of the original system is represented by a set of first order differential equations called state space representation of the system.

Assume an n^{th} order linear time invariant system be expressed in state space form as

$$\left.\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned}\right\} \qquad (1.1)$$

$x(t) = n \times 1$ state vector, $u(t) = p \times 1$ input vector,

$y(t) = m \times 1$ output vector, $A = n \times n$ system matrix,

$B = n \times p$ input matrix and $C = m \times n$ output matrix

The problem of order reduction is to derive a reduced model of order ' r ' $(r < n)$ described by

$$\left.\begin{aligned} \dot{x}_r(t) &= A_r x_r(t) + B_r u(t) \\ y_r(t) &= C_r x_r(t) \end{aligned}\right\} \qquad (1.2)$$

where $x_r(t) = r \times 1$ state vector, $y_r(t) = m \times 1$ output vector,

$A_r = r \times r$ system matrix, $B_r = r \times p$ input matrix and

$C_r = m \times r$ output matrix

such that the reduced r^{th} order model retains the salient characteristics of the original n^{th} order system for a given set of inputs and y_r(t) is a close approximation of original output y(t).

1.2.2 Frequency Domain Representation

Here, a generalized n^{th} order SISO system is reprsented in the form of transfer function given by

$$G_n(s) = \frac{N_n(s)}{D_n(s)} = \frac{a_o + a_1 s + \ldots + a_m s^m}{b_o + b_1 s + b_2 s^2 + \ldots + b_n s^n}; \quad m < n \qquad (1.3)$$

The equation (1.3) is to be approximated to reduced order model of order ' r ' represented by

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{d_o + d_1 s + ... + d_p s^p}{e_o + e_1 s + e_2 s^2 + ... + e_r s^r}; \quad p \le r < n \tag{1.4}$$

It is required to determine the unknown coefficients $e_0, e_1, ... e_r$ and $d_0, d_1 ... d_p$ such that G_r(s) is a close approximation of G_n(s) and the response of G_n(s) and G_r(s) matches as closely as possible and behave identically for a given set of inputs.

The equivalent of (1.3) in terms of MIMO system is given by

$$N_n(s) = [G_n(s)] D_n(s) \tag{1.5}$$

where the n^{th} order transfer function matrix [G_n(s)] is given by

$$[G_n(s)] = C(sI - A)^{-1} B \tag{1.6}$$

$$= \frac{A_o + A_1 s + ... + A_{n-1} s^{n-1}}{b_o + b_1 s + ... + b_n s^n} \tag{1.7}$$

and a reduced transfer function matrix is given by

$$[R(s)] = \frac{D_o + D_1 s + ... + D_{r-1} s^{r-1}}{e_o + e_1 s + ... + e_r s^r} \tag{1.8}$$

where A_{n-1} and D_{r-1} are matrices of appropriate dimension and b_n and e_r are scalar constants. A reduced order model is sought in the form (1.8) such that the responses of system described by (1.7) and (1.8) closely match in some sense as much as possible for a given set of inputs.

1.3 OVERVIEW OF ORDER REDUCTION METHODS

Till date, various authors have introduced numerous MOR techniques and are broadly categorized as frequency domain and time domain reduction methods. A substantial coverage of these reduction techniques have been given in [3, 22-33] respectively. However, the reduced models obtained from different reduction techniques are unique in itself and the quality is ultimately judged by the way it is utilized.

1.3.1 Frequency Domain Order Reduction Methods

The order reduction methods falling under frequency domain are further classified into three groups namely [34-36]

(i) Classical Reduction Methods (CRM)

(ii) Stability Preservation Methods (SPM)

(iii) Stability Criterion Methods (SCM)

The CRM are algebraic in nature and based on the classical theories of mathematical approximation or mathematical concepts such as continued fraction expansion and truncation, Pade approximation etc. [16, 37-44]. In this group of reduction methods, situations may arise where stable original model results in unstable reduced model and vice versa. Further, non-minimum phase behavior, low accuracy in the mid and high frequency ranges are other disadvantages of this group.

The Stability Preservation Methods (SPM), retains the stability of the original model. However, the main disadvantage of SPM is lack of flexibility, especially when the reduced model doesn't provide good approximation. Methods such as Routh approximation, reduction using Mihailov criterion, Hurwitz polynomial approximation, differentiation method, Routh-Hurwitz array method, stability equation, dominant pole retention, factor division method etc fall under the category of SPM [35, 45-52].

SCM form the third group which includes mixed methods. These mixed methods are derived by the combination of CRM and SPM. SPM is used to find the reduced denominator while numerator terms are obtained by using one of the CRM. Resultantly this ensures less error during low frequency range but the stability of SCM is achieved at the price of loss of accuracy [35, 53-58].

Some notable order reduction methods under frequency domain are discussed as follows

1.3.1.1 Continued Fraction Expansion Method

This method was introduced by Chen and Shieh [59] for approximating linear time invariant SISO system. Later, the same technique was extended to MIMO systems [38, 60], mixed with other methods and modified in [40, 42, 53, 61, 62].The main highlight of this method is that, the reduced model obtained possess the vital features of the original model and avoids the need of calculating the eigen values or eigen vectors. Apart from this, steady state response matching, computational simplicity, fitting of time moments are some added features. Bosley *et. al.*[37] have shown that this method is a special case of Pade approximation, which is equivalent to the time moment matching method for asymptotically stable systems. However, this method, doesn't yield stable system even though the original model is stable and lacks in approximating the transient response. Later, Chuang [39] proposed a modified continued fraction expansion method which ensures the stability of the reduced order model and improves the initial transient response by combining expansions about $s = 0$ and $s = \infty$ alternately. Poor response under steady state is one of the disadvantages of this method. This method came to be known as modified Cauer continued fraction method. Khatwani *et. al.*[42] shown that this method is similar to the mixed complete Pade approximation about $s = 0$ and $s = \infty$. However this modified method still lacks in providing guaranteed stability and this is overcome by Lucas [63].The Cauer third form was later on proposed by Shieh and Goldman [64] combining the advantages of Cauer first and Cauer second form.

Further, it can be deduced that the first Cauer form gives good matching in the transient region only, Cauer second form provides excellent matching in steady state region and finally the third grasps the advantage of the former and the latter. CFE has also been useful in reducing the original model represented in discrete form [65, 66]. Likewise various authors have proposed modifications and extensions for better performance. Davidson and Lucas [40] showed system reduction using CFE about a general point. CFE was combined with stability equation by Chen *et. al.*[53]; Routh Hurwitz array by Pal [67]; Mihailov criterion and evolutionary technique based on PSO by Panda *et. al.*[68]; ESA by Parmar *et. al.*[69], Parmar and Bhandari [70]; dominant pole retention by Parmar *et. al.*[71] and so on.

1.3.1.2 Time Moment Matching Method

Paynter and Takahashi [72] initially introduced time moment matching method of model reduction, based on finding a initial set of time moments of original model and matching them with those for the reduced model. Initial time-moments are matched to obtain good approximations at low frequencies whereas initial Markov parameters matching ensures good approximation at higher frequencies. The remaining time/Markov parameters are not of considerable interest. However, it is observed that stability and the response matching during the transient period cannot be guaranteed and may not be up to the expected level. Additionally, the problem of computations creeps in as the number of constants to be evaluated in the reduced model are very large. In this regard, a new computer oriented algorithm for evaluating the time moments was proposed by Lal and Mitra [73] to overcome these disadvantages.

The reduction of SISO systems are implemented by Gibilaro and Lees [44] and Zakian [74]. The same method was further applied on multivariable systems by Shih and Shieh [75] successfully by matching the coefficients of power series expansion about $s = 0$ and $s = \infty$, where s is the Laplace transform variable. Later, the moment matching technique was shown to be applicable to discrete systems by Hwang and Shih [76], multirate linear systems by Williamson *et al.* [77]. Shamash [78] used the moments concept in composite systems. Parthasarathy and Singh [79] proposed minimal realization of symmetric transfer function matrices using time moments and Markov parameters. Jonckheere and Ma [80], Krajewski et al. [81] proposed reduction methods by using impulse response energies, matching both Markov parameters and time moments. Later, a novel method to find out the reduced order model in comparatively lesser time period with good accuracy was intorduced by Salimbahrami and Lohmann [82]. This technique uses Krylov subspace method based on moment matching. Another feature of this method is that, it can be extended to multivariable cases also by using Arnoldi or Lanczos algorithm [83, 84]. Recently Parmar [85] suggested a reduction method, where the concept of shifted time moment proportional's matched in least-square sense is applicable to LTI systems.

1.3.1.3 Pade Approximation Method

One of the powerful reduction method and computationally simpler in the category of frequency domain is Pade approximation method. Pade is the name of the person who introduced this method to the world in 1892 [86] and hence the name Pade approximation. The main features of this method are, good approximation during the steady state period between the responses of the original and reduced models; fitting of time moments. Hence, many researchers have used this method to devise new stable reduction methods [28, 63, 87-91]. Basically, the method is based on matching the $2r-1$ coefficients of the power series expansion, about $s = 0$ of the original model with that of the reduced model. But, the limitation is that reduced model may turn out to be unstable for a given stable original model and vice versa. This is due to the approximation of non-dominant poles. Shamash [91, 92] showed another way of approximating the original model by retaining the dominant poles of the original model in the reduced model thus preserving stability. Similarly, there are suggestions from several authors about devising mixed methods for reduction like Bistritz and Shaked [93], Pal [56, 57, 67], Shamash [94], Singh [95], Wan [35] and so on. A new way of formulating a multipoint Pade approximant of a linear system transfer function was suggested by Lucas [96] where, the expansion points can be a mixture of one or more real, complex and purely imaginary points. The work was modified to generalize the method by extending to expansion points at infinity. Further, Aguirre [97, 98] introduced a new method of model order reduction named Least square Pade method. Prasad *et.al* [99-101] successfully applied pade approximation for reducing multivariable systems. The reduced model so obtained is in the time domain irrespective of the domain of the original model. The same reduction method is also carried out on discrete systems by Hwang and Chow [102], Prasad and Devi [103] etc. Recently it is shown that mixed reduction methods are also possible using spectrum analysis by Parmar *et al.* [104, 105], differentiation and Pole clustering method by Vishwakarma and Prasad [106-108].

1.3.1.4 Routh Approximation Method

In 1975 Hutton and Friedland [45] devised a new reduction technique from a given higher order transfer function. To begin with, reciprocal transformation is applied on the given transfer function and then the denominator/numerator polynomials of the reduced model is obtained from the α-β table. The denominator coefficients of the reciprocated original transfer function are used in preparing the α table. Likewise, the β table is formed using the numerator coefficients in which β coefficients are determined by using the α table and successive elements of β table [22]. The resultant model is reciprocated back again to get the final reduced model. This method also turns out to be computer oriented, simple algebraic calculations, guaranteed stability along with good steady state matching. However, the task of reciprocating the transfer function twice; during the initial step and at the last step seems to be an disadvantage. Krishnamurthy and Seshadri [49] suggested regrouping of entries of α table thus avoiding reciprocal transformation, similar to direct Routh approximation method (DRAM). This method sometimes proves to be fruitless due to the approximation of non dominant poles of the original model [109].

Shamash [58] used this method for generating stable biased reduced order models. Further, it is shown that unstable models can also be simplified by modifying the said method according to Rao *et al.* [110]. Original models represented in z- domain can also be reduced as suggested by Therapos [111], Choo [112], Hwang and Hsieh [113] proved via bilinear transformation. Reduced models was obtained for interval systems by Bandyopadhyay *et al.* [114, 115], Sastry *et. al.*[116]; interval systems using mixed methods by Dolgin and Zeheb [117]. Hwang *et al.*[118] proposed multi-frequency Routh approximation for stable reduced models. Singh *et. al.*[119] derived a reduced SISO model by combining with improved Pade approximants- a computer based approach. Later, Panda *et al.* [120] optimized the reduced model by combining with PSO.

1.3.1.5 Routh Hurwitz Array Method

This simple method of deriving a lower order model was proposed by Krishnamurthy and Seshadri [48, 49]. This method is based on Routh Hurwitz stability criterion, where the numerator and denominator coefficients of the derived reduced transfer function is obtained from the numerator and denominator coefficients of the original transfer function. According to the Routh stability array, for a given N^{th} degree numerator polynomial, a polynomial of *(n-1)*th order can be constructed using the second and third rows of Routh array. Likewise, a polynomial of *(n-2)*th order can be constructed from the third and fourth row of the array respectively. The denominator coefficients of the reduced transfer function is obtained in a similar way. The main attraction of this method is that the reduced transfer function is stable for a given stable original transfer function. Singh [121] commented that a high degree of nonuniqueness exists in this method. Mixed methods using factor division method was proposed by Singh *et al.*[122], Pade approximation by Pal [57], balanced realizations and Pade approximation by Singh *et al.*[123], spectrum analysis by Parmar *et. al.*[124], error minimization method by Mittal *et. al.*[125], continued fraction expansion by Pal [67] etc.

1.3.1.6 Stability Equation Method

This method is one of the popular reduction method commonly used in frequency domain. Chen *et. al.*[50] initially proposed the concept of stability equations, by separating the numerator and denominator polynomials of the original model into their even and odd parts. The factors with large of z_i and p_i magnitudes are discarded successively in the reduction process. The roots which are closer to the origin after factorization, are considered in the formation of reduced model. The important feature of this particular method is that, stability of the original model is protected while deriving the reduced model. Moreover, the first two time moments are also retained, thus providing good matching during steady state response for a given step, impulse and ramp type of inputs. Lucas [126] simplified the method further, by suggesting the tabular approach for reducing the degree of the stability equations. With this, the problem of calculating the roots/factors became a procedure of the past. As time passed, various authors proposed different mixed reduction methods by combining this method with continued fraction expansion [53], Pade approximation [56, 127], complex curve fitting [128] etc. Prasad *et al.* [129] used this method in combination with Pade approximation, *Mittal et al.* [130] with error minimization for reducing MIMO systems. Therapos applied this method for deriving reduced models in discrete domain [131, 132] and from fast oscillating systems [133]. Later, Lucas [89] showed that this method is a two stage multipoint Pade approximation. Tsay and Han [134] concluded that the method can also be applied for analysis and design of models with several adjustable and variable parameters. In recent times, Parmar *et al.* [135] introduced a composite method by combining with Genetic Algorithm for devising the reduced model.

1.3.1.7 Differentiation Method

Gutman *et al.*[47] differentiated the reciprocated numerator and denominator polynomials of original model represented in transfer function form, for n times till the desired order of the reduced model is reached. Then the reduced model is normalized and reciprocated back to give the final lower order model. Gutman *et al.*[47] concluded that the method is simple, applicable to unstable non-minimum phase models. Apart from these advantages, the method suffers from a drawback of steady state matching. As this reduction method is based on differentiating the polynomials, hence the name. Later Lucas [136] showed that this method [47] is equal to forming successive ratios of multipoint Taylor polynomial approximations of numerator, denominator polynomials respectively. It is concluded that the method is computationally easy because of the Routh array structure being formulated. Prasad *et. al.* [137] extended the benefits of this method to MIMO systems also. Several authors have formed mixed methods by combining this method with continued fraction expansion by Pal and Prasad [138, 139], Pade approximations by Lepschy and Viaro [140], factor division method by Vishwakarma and Prasad [141]. using PSO by Tomar *et. al.*[142].

1.3.1.8 Truncation Method

Gustafson [143] showed that a new simple reduction method can be developed by gradually neglecting the terms of higher order, present in the numerator and denominator polynomial of the original transfer function. Later, Shamash [144] applied on MIMO models successfully and concluded that this method is also comparable. Prasad *et al.* [145] suggested a modification of the truncation method. The main advantage is that, the stability is guaranteed provided that, the poles of the higher order transfer function is well damped [146].

1.3.1.9 Dominant Pole Retention Method

This method is one among the stable reduction methods introduced by Davison [51], which always results in stable reduced model (provided original model is stable) while retaining the dominant behavior of the original model. Always, poles located far from origin are discarded, since they have lesser effect on the response of the model. The disadvantage of this method lies, when difficulty arises while deciding the most dominant pole among the multiple poles present near the imaginary axis. Moreover, when the original model is represented in state space form, the reduction process becomes computationally tedious. This is due to computation of eigen values and vectors of large dimensional space matrix, linear transformations and matrix diagonalization.

Till date, several authors have reaped the benefits of this method and have suggested their own mixed reduction methods both in continuous and discrete domain. The denominator polynomial is formed by retaining the dominant poles of the original model while, numerator polynomial is found out by some other technique. Shamash [92], Mukherjee and Mishra [147], Sinha and Pal [148], Pal [149], Parmar *et al.*[71, 150, 151] are some authors to name a few. Shieh and Wei [152], Lamba *et al.* [153], Prasad *et al.*[101] extended the application of this method to MIMO models.

1.3.1.10 Factor Division Method

This method of order reduction was introduced by Lucas [52]. This method is simple to compute, helps in retaining dominant modes and preserve initial time moments in the reduced model by itself. Thus, eliminating the need to calculate moments beforehand and solve Pade equations [92, 154]. Further, this method was extended to find biased reduced order models by Lucas [155]. This was possible by retaining the initial time moments and Markov parameters. Later, modification to the existing factor division method [52, 155] was suggested by Lucas [156]. The modified factor division method guaranteed stable reduced models (provided original model is stable). Also, this method has the ability to formulate the reduced models of varying orders by simply varying a single parameter in the denominator of the modified transfer function. Several authors have obtained reduced models for SISO/MIMO system, in combination with Mihailov criterion by Prasad *et al.* In recent times, composite reduction method was suggested by Parmar *et al.*[157] using ESA and Singh *et al.*[122] using Routh-Hurwitz array, Vishwakarma *et al.* using differentiation method [141] and pole clustering [158].

1.3.1.11 Mihailov Stability Criterion

Wan [35] suggested a new method of order reduction by using Mihailov criterion and Pade approximation method. The main feature of this method is stability preservation in the reduced model, simple to compute, spared from determining the initial Markov parameters and time moments. This method in combination with factor division method was shown to be applicable by Prasad *et al.*[159] for SISO systems. Vishwakarma and Prasad [160] used the combination of this method with GA whereas Panda *et al.* [68] mixed with PSO to generate the stable reduced order model recently.

1.3.1.12 Error Minimization Technique

The name itself indicates the concept involved in this type of order reduction method. The reduced model is formulated by reducing the difference occurring between the responses of the original and the reduced model. There are variety of error criteria to choose namely ISE, ITSE, IAE, ITAE. But, ISE is the one most commonly used, for reducing the error between the responses of the original and reduced model. These responses may be either in time domain or frequency domain. Consequently, error minimization can be carried out by

(a) Time response matching
(b) Frequency response matching

(a) Time response matching

Some authors like Mukherjee and Mishra [147, 161], Hwang [162], Puri and Lan [90], Lamba *et al.* [153], Howitt and Luus [163] have reduced the ISE between the step (impulse) responses of the original and reduced model for devising the reduced models. Hwang [162] made use of Routh approximation; matrix formula to calculate ISE from the coefficients of the error transfer function. Matrix formula was used to avoid actual evaluation of the time response. Mukherjee and Mishra reduced a linear SISO system [147] having distinct real poles using dominant pole retention and then extended to MIMO system [161]. Puri and Lan [90] introduced a another stable reduction method based on reducing impulse response error using stability and Pade approximation approach. Lamba *et al.* [153] combined retention of dominant pole/Routh approximation along with reducing step response error. Reduction of SISO models were carried out by Howitt and Luus [163], where zeros and poles are considered to be free parameters and are chosen to minimize ISE. Later, Singh *et al.*[164] introduced a computer aided order reduction approach using ISE minimization technique. Mittal *et al.* [125, 130, 165] proposed several order reduction methods, where the denominator are reduced using Routh-Hurwitz array, stability equation, dominant pole retention etc. The numerator polynomial are obtained by error minimization of step response.

Hwang *et al.* [166] suggested a new reduction method for discrete systems which employs methods belonging to both time and frequency domain. Later, higher order discrete models are also reduced by Puri and Lim [167]. Recently, Mukherjee *et al.*[168] introduced a new concept of using secant method for minimizing the ISE between the transient parts of the original and reduced models subjected to step/impulse input. Parmar *et al.* [135, 150, 169] proposed several methods for reducing a given higher order SISO/MIMO model using error minimization approach and GA, PSO, stability equation.

(b) Frequency response matching

As the name says it all, this technique is based on matching the frequency response of the original and the reduced model. Levy [170] minimized an error function spread over a interested frequency range in Least square sense. Reduction method based on complex curve fitting was also suggested by Rao and Lamba[171]. Discrete system reduction via frequency response matching was proposed by Sahani and Nagar [172], Nagar and Singh [173]. The denominator is obtained based on the concept of power decomposition by preserving the poles of large dispersion and numerator polynomial are found out by frequency response matching technique.

1.3.1.13 Least Square Method

Initially, Shoji *et al.* [174] used least square matching of time moments of the original model to obtain a reduced lower order model. This method was introduced to overcome the drawback of Pade approximation and provide the user an extra degree of freedom in the design of the simplified model. Modification to this method was proposed by Lucas and Beat [175]. These methods [174] [175] are referred as partial least squares method since only the denominator polynomials are formed in least square sense. Aguirre [97] extended it to include the use of Markov parameters; the method came to be known as full least squares method since both the numerator and denominator polynomials are formed in a single generalized inverse operation. Lalonde [176] generated reduced models in discrete domain using Markov parameters only. Lucas *et al.* [177] suggested a stability preserving least squares Pade method for discrete system and modified [178] it after few years.

Aguirre [179] proposed a method, retaining the exact poles and zeros in a reduced model. The remaining coefficients of the reduced model are calculated, by means of least squares matching of Pade coefficients and Markov parameters. Later, an extended least squares model reduction was proposed by Aguirre [180]. The convenience provided here is that, numerator polynomial of the reduced model are formed by means of least square method while the denominator polynomial is previously determined by any suitable method. In the recent times, Parmar [85] simplified a given higher order model using least squares moment matching about 'a' and generalized least squares method about 'a', where the value of 'a' may be arithmetic, geometric or harmonic mean.

1.3.2 Time Domain Order Reduction Methods

Time domain order reduction methods, deals with original/reduced model represented in state space form; require information regarding the eigen values and eigen vectors or overall characteristic of the original model. Consequently, this will help in ensuring close matching between the time responses of the original and reduced model. The below section briefly describes some vital time domain order reduction methods.

1.3.2.1 Aggregation Method

This method being the most general projective reduction method, was brought to light by Aoki [181] from his pioneering work. It is shown that, the simplified model can be derived by aggregating the original state vector into a lower dimensional vector. Further, the internal structural properties (dominant eigen values) of the original model are considered for retaining in the reduced model, so as to match the responses of both original and reduced model. This further results in analysis and deriving state feedback suboptimal controllers [181]. Later, Hickin and Sinha [182] compared this method with singular perturbation method [183] and showed that aggregation is a generalization of projection method.

1.3.2.2 Singular Perturbation Method

Kokotovic *et al.* [184] proposed this method as a tool to reduce the original model having two-scale property *i.e.* the eigen values can be divided into two modes viz. 'fast' and 'slow' modes. Initially the order of the original model is reduced by neglecting fast phenomena. Later on, approximation is improved by re-introducing their effect as a boundary layer. Fernando and Nicholson [185] concluded that this method is compatible with the balanced realization method. Further, the same author [186] tried singular perturbation reduction method for both continuous and discrete time models. Resultantly, the technique guaranteed the preservation of dominant eigen values of the original model but unable to handle large scale models. This is due to the non availability of slow and fast subsystems.

1.3.2.3 Modal Analysis

This approach is based on retaining the dominant eigen values of the original model initially and then computing the remaining parameters of the reduced model. The parameters computed are such that, the response of the original and reduced model closely matches as much as possible for a given set of inputs. Authors like Aoki [181], Davison [51], Marshall [187] proposed reduction methods belongs to this category. Aoki [181] method was based on aggregation having a more general approach. The Davidson method assumes that all the eigen values are distinct neglecting large eigen values and the input is of unit step. Hickin [188] showed that these three methods [51, 181, 187] can be regarded as special cases of aggregation method proposed by Aoki [181]. Gruca [189] introduced delay in the output vector of aggregated model thereby, leading to the improvement equality of simplified aggregated model of the system without increasing the order of the state differential equations.

1.3.2.4 Optimal Order Reduction

This category of reduction technique, generates a simplified model of specific order, so as to closely match the response of the original and reduced model in an optimum way. However there is no limitation on the location of eigen values. The reduced model is obtained based on minimizing some performance criterion such as ISE. An orthogonal projection based geometric approach is employed to obtain the reduced model by Anderson [190]. Another method of optimal order reduction was proposed by Sinha and Bereznai [191] using the pattern search method [192]. Wilson and Mishra [193] studied the approximations for step and impulse responses in his optimal order reduction method. Further, reduced models are devised in frequency domain using optimal order reduction methods by Langholz and Bistritz [194]. The advantage of this method is that it requires lesser computational time provided the gradient of the objective function is evaluated. Pseudo inverse of a matrix is used for least square fit by Sinha and Pille [195].

1.3.2.5 Minimal Realization Algorithm

This algorithm is utilized to obtain state variable model from a system represented in frequency domain. Several authors have used various forms of Hankel matrices for minimal realization. A linear state model was realized in its minimal form by an algorithm suggested by Ho and Kalman [196]. The same author also obtained a non-minimal realization in the form of a block companion matrix. Another method was introduced by Tether [197], which provides good approximation in transient response by retaining few initial Markov parameters of the original model. An internally balanced minimal realization of a stable SISO model was computed in the method suggested by Therapos [198]. An efficient algorithm for minimal order realization of a given system model was suggested by Rozsa and Sinha [199]. Later, Shamash [200] in his proposed method for multivariable system, showed that the reduced model obtained by minimal realization approach is equivalent to the one derived by time moments and continued fraction methods. Parthasarathy and Singh disclosed minimal realization of symmetric transfer function matrix by utilizing moments and Markov parameters in [79]. Minimal realization linear time varying systems was proposed by Lal and Singh [201]. A new method based on generation of successive partial realization of large dimensional system (MIMO) represented in state space is given by Hickin and Sinha [202]. A comparative study in terms of computational efficiency and suitability for practical implementation, for obtaining minimal-order realizations of MIMO systems was also carried out.

1.3.2.6 Balanced Realization Approach

The concept of balancing is first encountered in the work of Mullis and Roberts [203]. Later, Moore [204] used appropriate similarity transformations to introduce this reduction technique in systems and control literature. Basically, this approach is based on simultaneous diagonalization of observability grammian (W_o) and controllability grammian (W_c) matrices. Lyapunov equations are solved to obtain W_o and W_c either in semi positive definite or semi-definite matrix form. Further, these matrices are used to define the measures of controllability and observability. In this reduction procedure, the insignificant states are directly eliminated to form a lower dimensional model. These insignificant states are least controllable and least observable. Hence, they have less/no influence on the impulse response of the system and can be discarded.

Several authors have used this method to realize reduced models viz. Yang *et al.*[205] for unstable systems, Sandberg and Rontzer [206] for linear time varying systems, Nagar and Singh proposed a two step procedure for discrete system [173]; algorithmic approach for system decomposition [207] and controller reduction using balanced realization [208] , Meyer [209] applied to fractional balanced reduction, Perev and Shafai [210] for reduction of singular systems, Kenny and Hewer [211] for balancing unstable minimal MIMO systems, Therapos for unstable nonminimal linear systems [212] and discrete SISO system [213]. Pernebo and Silverman [214] extended this method to obtain lower dimensional models. Chin [215] contributed by reducing unstable higher order models using low frequency approximation balancing technique. Lastman and Sinha [216] compared with the aggregation method. Al-Saggaf and Franklin [217] uses a new frequency weighing technique for approximating large scale discrete and continuous time models. A reduction technique in frequency domain, based on the impulse response grammian applicable for linear continuous models was introduced by Agathoklis and Sreeram [218]. Gugercin and Antoulas [219] surveyed different model order reduction by balanced truncation.

1.3.2.7 Hankel Norm Approximation

Today, Hankel norm reductions are mostly sought order reduction technique in literature of system theory. This method constitutes a beautiful theory associated with the names of Arov-Adamjan-Krein [220]. Glover [221] introduced state space ideas and characterized all stable approximations of a linear time-variant stable system. Kung and Lin [222] extended for MIMO systems. A program for solving the L_2 reduced-order problem while the denominator is fixed was contributed by Krajewski *et al.* [223] while, Kemin [224] proposed a new approximating technique by using frequency-weighted balanced realization. Gao *et al.*[225] examined the problem of H_∞ model reduction for discrete time-delay systems while Wang *et al.*[226] obtained reduced model for fast subsystems. Ferrante *et al.* [227]proposed an algorithm using the bounds on the eigenvalues of the Jacobian of the associated transition function. This is an alternate to the one proposed recently [223]. HNA method belongs to the class absolute (or additive) error model order reduction methods and relies on a guarantee error bound.

1.4 OBJECTIVE OF THE THESIS

The initial objective of this thesis is to recapitulate most of the model reduction methods available in the research literature, succeeded by the purpose to promote some new model order reduction methods applicable to SISO/MIMO linear time-invariant continuous time systems. The task mentioned involves the use of both conventional and evolutionary strategies. The systems considered may be represented in frequency domain (preferable) or time domain. In addition, the superiority of the new reduction methods can be ensured, by comparing with other well known methods, besides checking its validity for LTI discrete systems. Moreover, reduction methods are also proposed for reducing the order of digital filters. Lastly, to solve the problem of designing a suitable PID controller for the higher ordered model, utilizing the newly developed method. Both, direct and indirect approaches of controller design, are planned to check its applicability for the original model.

1.5 ORGANISATION OF THE THESIS

The entire research work is structured through seven chapters in this thesis, with the introduction to model order reduction at the outset. Subsequently followed by importance and applications of model order reduction, statement of model order reduction problem in both time and frequency domain for continuous time systems (SISO and MIMO). Besides brief overview about the developments that have taken place in the area of model order reduction, various existing reduction methods and their associated qualities/drawbacks are also reflected.

The second chapter encompasses the reduction of higher order LTI continuous systems using the developed composite reduction methods. Stability Equation (SE), Eigen Spectrum Analysis (ESA), Dominant Pole (DP), Modified Pole Clustering (MPC) are employed to propose composite methods which are comparable to available reduced systems. Further, the application of these methods are extended to include linear multivariable systems. Comparison of reaction curves (unit step response) and their associated performance indices justifies the proposed methods.

The proposed reduction of higher ordered LTI discrete models are dealt in chapter three. The composite methods proposed in the previous chapter are applied on systems having higher order and discrete in nature. The qualities of the proposed methods are judged by comparing the results obtained and is seen to be comparable. Later, MIMO systems are also reduced to lower order successfully to confirm the worthiness of the proposed methods.

Evolutionary schemes including the recently introduced Big Bang Big Crunch optimization technique are adopted in the chapter four to float new reduction methods. Mixed methods using BBBC in combination with Routh Approximation (RA) and Stability Equation (SE) method yields good results. In addition, BBBC plays important role in optimizing the linear shift point 'a' for order reduction in least square sense. Further, systems of higher order represented in both time and frequency domain are considered for reduction using BBBC and the same is extended for discrete systems as well. The proposed methods are applied on SISO/MIMO systems and is justified by considering the available higher order systems.

In chapter five, approximation of filters having higher dimensions are reduced and implemented on TMS320C5402 processor. This chapter provides a concise view of TMS320C5402 - a fixed point DSP and its application to model reduction. Simulations are carried out in MATLAB, Code Composer Studio (CCS) and input/output waveforms obtained are compared. In addition, the frequency response and FFT power spectrum of the input/ output signals are also plotted for clarity.

The design of controller for the original higher order models are dealt in chapter six to ensure the suitability of developed model order reduction methods. Further, fractional order PID controller are shown to perform better than the integer order PID controller. Both direct and indirect approach of controller design are employed for controller design. Illustrative examples available in the literature are solved to substantiate the methods. The reaction curves of closed loop transfer functions, obtained from the original and reduced plant, are compared with the reaction curves of the reference model. This ensures the suitability of the method. It is seen that the responses are in close agreement with that of the reference model.

The conclusions of the thesis, suggestions for future scope on the research work are mentioned in the last chapter.

CHAPTER - 2

REDUCTION OF CONTINUOUS TIME SYSTEMS USING NEW COMPOSITE METHODS

In the previous chapter, various frequency and time domain reduction methods were discussed. The stability preservation in the reduced system was noticed as a major issue in some reduction techniques [94]. Apart from this, lower accuracy in the mid, high frequency ranges and exhibiting non minimum phase characteristics are additional. The popular Routh algorithm cannot be used for simplification in the continued fraction expansion or matching of time-moments, if the order of the given original system is very high. Pade approximation method yields good steady-state approximation to the system response. However, the only drawback is that unstable reduced model may arise from a stable full system [228]. As a remedy for this situation, several variants were suggested and one such suggestion is that a new order reduction method based on least square fitting of time moments of the given higher order system with singularity was proposed [174]. The highlight of this technique is that it facilitates an extra degree of freedom in the design of the stable reduced system. Later, Aguirre [180] suggested a procedure where, the poles are retained in a reduced model while the numerator terms are computed by means of least squares matching.

In this chapter, new composite methods are proposed for reduction of continuous LTI systems represented in frequency domain. The advantages of ESA [157, 229], stability equation [50], dominant pole [51, 165] and modified pole clustering techniques [230] are reaped in combination with least squares method [174, 180]. The mixed methods presented are devoid of instability issues (except for unstable system) and provides good accuracy while retaining most of the key characteristics of the original system. One common philosophy used in the suggested methods is that, the denominator polynomial is computed initially, using the stability preserving methods. Further, the numerator polynomial is found out using the least squares method. Numerical examples of LTI continuous system from the available literature are solved using the suggested technique and the results obtained are found to be comparable.

2.1 PROBLEM STATEMENT

Consider the n^{th} order original system represented in the transfer function form as

$$G_n(s) = \frac{N_n(s)}{D_n(s)} = \frac{a_o + a_1 s + \ldots + a_m s^m}{b_o + b_1 s + b_2 s^2 + \ldots + b_n s^n}; \quad m < n \tag{2.1}$$

The objective is to compute r^{th} (r < n) order reduced system $G_r(s)$ from (2.1) in the form of

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{d_o + d_1 s + \ldots + d_p s^p}{e_o + e_1 s + e_2 s^2 + \ldots + e_r s^r}; \quad p < r \tag{2.2}$$

a_m, b_n, d_p and e_r's are the scalar constants.

2.2 LEAST SQUARES MOMENT MATCHING METHOD OF ORDER REDUCTION

Consider the n^{th} order transfer function of the form (2.1), the time moment proportional's c_i are obtained by expanding $G_n(s)$ about s = 0 as

$$G_n(s) = \sum_{i=0}^{\infty} c_i s^i \tag{2.3}$$

Similarly the Markov parameters, m_j are obtained by expanding $G_n(s)$ about s = ∞ as

$$G_n(s) = \sum_{j=1}^{\infty} m_j s^{-j} \tag{2.4}$$

The r^{th} order reduced system derived by Pade approximation method [174] has a denominator polynomial $D_r(s)$

$D_r(s) = \sum_{i=0}^{r} e_i s^i ;(e_r = 1)$, given by the solution of the linear set

$$\begin{bmatrix} c_r & c_{r-1} & \cdots & c_1 \\ c_{r+1} & c_r & \cdots & c_2 \\ \vdots & \vdots & \cdots & \vdots \\ c_{2r-1} & c_{2r-2} & \cdots & c_r \end{bmatrix} \begin{bmatrix} e_0 \\ e_1 \\ \vdots \\ e_{r-1} \end{bmatrix} = \begin{bmatrix} -c_0 \\ -c_1 \\ \vdots \\ -c_{r-1} \end{bmatrix} \tag{2.5}$$

where e_i coefficients constitute the denominator of the reduced system. If the solution of (2.5) do not yield a stable denominator, then another equation is added to this set [174], such that the system assumes a fitting of the next time moment from the original system.

$$\begin{bmatrix} c_r & c_{r-1} & \cdots & c_1 \\ c_{r+1} & c_r & \cdots & c_2 \\ \vdots & \vdots & \cdots & \vdots \\ c_{2r-1} & c_{2r-2} & \cdots & c_r \\ c_{2r} & c_{2r-1} & \cdots & c_{r+1} \end{bmatrix} \begin{bmatrix} e_0 \\ e_1 \\ \\ \vdots \\ e_{r-1} \end{bmatrix} = \begin{bmatrix} -c_0 \\ -c_1 \\ \vdots \\ -c_{r-1} \\ -c_r \end{bmatrix} \tag{2.6}$$

In matrix vector form, (2.6) will be *[H][e] = [c]*.

The denominator vector estimate 'e' can be obtained by solving in the least squares sense by means of generalized inverse method given by

$$e = \left(H^T H\right)^{-1} H^T c \tag{2.7}$$

If the reduced denominator is not stable, then H and c in (2.7) are extended by another row. The elements of the next row will be the next time moments from the original system in a least squares match.

2.3 GENERALIZED LEAST SQUARES METHOD OF ORDER REDUCTION

According to [231] the generalized least-squares method is illustrated below.Let the r^{th} order reduced system of $G_r(s)$ be in the form of (2.2) obtained by retaining $(r + t)$ time moments and $(r - t)$ Markov parameters $(0 \le t \le r)$. The e_k , d_k coefficients are derived by solving the following set of equations

$$\left.\begin{aligned}
d_0 &= e_0 c_0 \\
d_1 &= e_1 c_0 + e_0 c_1 \\
\vdots \quad & \vdots \quad \vdots \\
d_{r-1} &= e_{r-1} c_0 + \ldots + e_0 c_{r-1} \\
0 &= e_{r-1} c_1 + \ldots + e_0 c_r \\
0 &= e_{r-1} c_2 + \ldots + e_0 c_{r+1} \\
\vdots \quad & \vdots \quad \vdots \\
0 &= e_{r-1} c_t + \ldots + e_0 c_{r+t-1}
\end{aligned}\right\} \tag{2.8}$$

and

$$\left.\begin{aligned}
d_{r-1} &= m_1 \\
d_{r-2} &= m_1 e_{r-1} + m_2 \\
\vdots \quad & \vdots \quad \vdots \\
d_t &= m_1 e_{t+1} + m_2 e_{t+2} + \ldots + m_{r-t}
\end{aligned}\right\} \tag{2.9}$$

where, c_i and m_j are the time moment proportionals and Markov parameters of the system. By substituting (2.9) in (2.8) and discarding the d_j $(j=t, t+1, ..., r-1)$in (2.9) results

$$\begin{bmatrix}
c_{r+t-1} & c_{r+t-2} & \cdots & \cdots & \cdots & \cdots & c_t \\
c_{r+t-2} & c_{r+t-3} & \cdots & \cdots & \cdots & c_t & c_{t-1} \\
\vdots & \vdots & \cdots & \cdots & \cdots & \vdots & \vdots \\
c_{r-1} & c_{r-2} & \cdots & \cdots & \cdots & c_1 & c_0 \\
c_{r-2} & c_{r-3} & \cdots & \cdots & \cdots & c_0 & -m_1 \\
c_{r-3} & c_{r-4} & \cdots & \cdots & c_0 & -m_1 & -m_2 \\
\vdots & \vdots & \cdots & \cdots & \vdots & \vdots & \vdots \\
c_t & c_{t-1} & \cdots & c_0 & -m_1 & \cdots & -m_{r-t-1}
\end{bmatrix}
\begin{bmatrix} e_0 \\ e_1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ e_{r-1} \end{bmatrix}
=
\begin{bmatrix} 0 \\ e_1 \\ \vdots \\ m_1 \\ m_2 \\ m_3 \\ \vdots \\ m_{r-t} \end{bmatrix} \tag{2.10}$$

In matrix vector form, (2.10) will be *[H][e] =[m]*.

If the denominator vector estimate '*e*' due to (2.10) is unstable, then the next Markov parameter $m_{(r-t+1)}$ can be assumed to be matched by extending (2.9) with

$$d_{t-1} = m_1 e_{t+2} + m_2 e_{t+1} + \ldots + m_{r-t+1} \tag{2.11}$$

This results in additional row to the *H* matrix and the *m* vector in (2.10) given by $[c_{t-1} \quad c_{t-2} \quad \cdots \quad c_0 \quad -m_1 \quad -m_2 \quad \cdots \quad -m_{r-t}]$ and $[-m_{r-t+1}]$. Similar to (2.6) using least-squares the value of '*e*' is calculated from this non-square matrix by

$$e = \left(H^T H\right)^{-1} H^T m \tag{2.12}$$

Further, the *H* matrix and the *m* vector can be extended by assuming a matching of the next Markov parameter if the denominator value is unstable. Later, (2.12) is solved to obtain another value of' '*e*'.

2.3.1 Eigen Spectrum Analysis and Least Squares Method

The composite reduction method presented here, is a combination of ESA [157] and least squares method [174]. The ESA is used for finding new poles, whereas least squares method is used for deriving the numerator polynomials. The concept of ESA is based on the following criteria [157].

1) Centroid (arithmetic mean of real parts of the poles) of both original and reduced system must be equal.

2) Stiffness (ratio of real part of nearest to farthest pole) of both original and reduced system must be equal.

3) The steady state response of both original and reduced system must match when subjected to step input.

The procedure for obtaining the reduced system using the proposed method is as follows.

Step 1: Considering D_n(s) of G_n(s) in (2.1) and rearranging in the form of

$$D_n(s) = (s + p_1)(s + p_2)\ldots(s + p_n) \tag{2.13}$$

such that $-p_1 < -p_2 < \ldots < -p_n$ are the poles of the higher order original system G_n(s).

Step 2: Locate the poles (Eigen Spectrum Points (ESP)) of G_n(s) in (2.13) on the negative real axis of the ‘s’ plane as shown in Fig. 2.1.

Step 3: Calculate the pole centroid σ_p given by

$$\sigma_p = \frac{\sum \textit{absolute value of real part of poles}}{\textit{number of poles}} = \frac{\sum_{j=1}^{n} |\mathrm{Re}\, p_j|}{n} \tag{2.14}$$

Step 4: Determine the system stiffness *k* using the formula

$$k = \frac{|\mathrm{Re}\, p_1|}{|\mathrm{Re}\, p_n|}\, ; |\mathrm{Re}\, p_1| < |\mathrm{Re}\, p_2| < \ldots < |\mathrm{Re}\, p_n| \quad (2.15)$$

Step 5: Computation of *Re*p'$_r$, M.

Let σ`$_p$ , k` be the pole centroid and system stiffness of G_r(s). If σ`$_p$ = σ$_p$ and k`= k, then

$$k = \frac{|\mathrm{Re}\, p_1^{'}|}{|\mathrm{Re}\, p_r^{'}|} = k^{'} \text{ and } \sigma_p = \frac{\sum_{j=1}^{r} |\mathrm{Re}\, p_j^{'}|}{r} = \sigma`_p \quad (2.16)$$

Rep`$_j$ are the real parts of poles of G_r(s). σ_p can rewritten as

$$\sigma_p = \frac{\mathrm{Re}\, p_1^{'} + \mathrm{Re}\, p_2^{'} + \mathrm{Re}\, p_3^{'} + \ \ldots \ + \mathrm{Re}\, p_{r-1}^{'} + \mathrm{Re}\, p_r^{'}}{r}$$

$$= \frac{\mathrm{Re}\, p_1^{'} + (\mathrm{Re}\, p_1^{'} + M) + (\mathrm{Re}\, p_1^{'} + 2M) + \ldots + (\mathrm{Re}\, p_1^{'} + (r-2)M) + \mathrm{Re}\, p_r^{'}}{r}$$

$$= \mathrm{Re}\, p_1^{'} + \mathrm{Re}\, p_1^{'}(r-2) + (M + 2M + \ldots + (r-2)M + \mathrm{Re}\, p_r^{'})$$

$$M = \frac{\mathrm{Re}\, p_r^{`} - \mathrm{Re}\, p_1^{`}}{(r-1)} \ ,$$

replacing Re $p`_1$ = $k$ Re $p`_r$,

$$M(r-1) = \mathrm{Re}\, p_r^{'} - k\, \mathrm{Re}\, p_r^{'} \quad (2.17)$$

We have,

$$N = \sigma_p r \quad (2.18)$$

$$QM = (M + 2M + \ldots + (r-2)M)$$

Substituting the values of σ_p, QM , (2.18) will be

$$N = k\, \mathrm{Re}\, p_r^{'}(r-1) + \mathrm{Re}\, p_r^{'} + QM \quad (2.19)$$

(2.17) and (2.19) in matrix form is given by

$$\begin{bmatrix} k(r-1)+1 & Q \\ (1-k) & (1-r) \end{bmatrix} \begin{bmatrix} \mathrm{Re}\, p_r^{'} \\ M \end{bmatrix} = \begin{bmatrix} N \\ 0 \end{bmatrix} \quad (2.20)$$

The value of Rep_r`, M is obtained by solving (2.20).

Step 6: Calculate ESP from Rep_r`, M and form the denominator polynomial D_r(s) of G_r(s). Let these denominator coefficients be 'e_i', $i = 0,1, 2 , \ldots \infty$.

Step 7: Determination of numerator polynomial N_r(s) of G_r(s).

Compute the time moment proportional's 'c_i' by expanding $G_n(s)$ about s=0 using (2.3). Substitute the value of 'e_i', $i = 0,1, 2, \ldots \infty$ and 'c_i' in (2.8) and (2.9) to obtain the coefficients 'd_i', $i = 0,1, 2, \ldots(r-1)$.

Step 8: The ratio of the numerator 'N_r(s)' and denominator polynomial 'D_r(s)' gives the reduced order stable LTI continuous system G_r(s).

2.3.1.1 Illustrative examples

The proposed method is applied on six numerical examples successfully chosen from the available literature. One example is solved in detail while the results obtained are mentioned directly for the other remaining examples. The performance of the reduced system is measured in terms of an error indices such as Integral Square Error (ISE) '*I*' and Impulse Response Energy (IRE) '*J*'. A smaller value of ISE is expected which is a measure of the closeness between G_n(s) and G_r(s). The ISE [119] and IRE [45, 94] values are calculated by using the formula

$$I = \int_0^\infty [y(t) - y_r(t)]^2 \, dt \qquad (2.21)$$

$$J = \int_0^\infty g^2(t) \, dt \qquad (2.22)$$

y(t), y_r(t) are the unit step responses of G_n(s) and G_r(s) respectively.

g(t) is the impulse response of the system under consideration.

Example 2.1: Consider a sixth order original system taken from Jamshidi [22]

$$G_n(s) = \frac{N_n(s)}{D_n(s)}$$

$$= \frac{2s^5 + 3s^4 + 16s^3 + 20s^2 + 8s + 1}{2s^6 + 33.6s^5 + 155.94s^4 + 209.46s^3 + 102.42s^2 + 18.3s + 1}$$

A second order reduced system is to be synthesized and the steps followed are as below.

Step 1: The D_n(s) of G_n(s) is rearranged in the form of (2.13) as

$$D_n(s) = (s+0.1)(s+0.2)(s+0.5)(s+1)(s+5)(s+10)$$

and the ESP/poles are Rep_1 = -0.1, Rep_2 = -0.2, Rep_3 = -0.5, Rep_4 = -1, Rep_5 = -5, Rep_6 = -10. These ESP's are located on the negative real axis of the 's' plane as shown in Fig. 2.1.

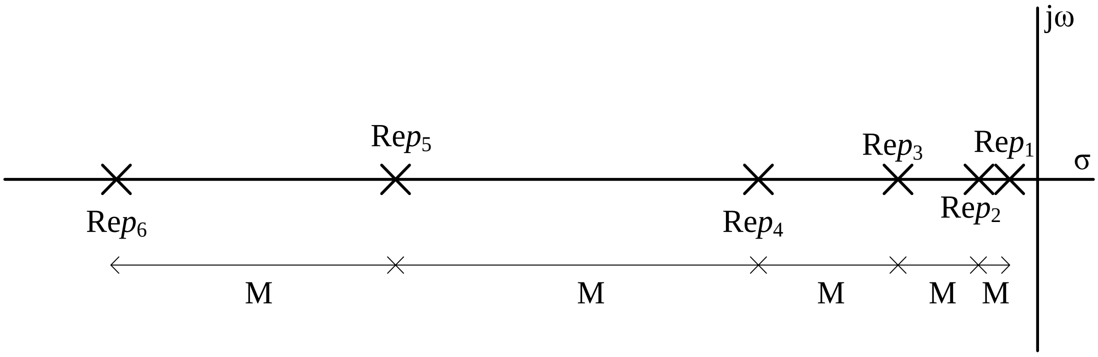

Fig. 2.1 ESP of original system $G_n(s)$ of example 2.1

Step 2: Pole centroid is calculated using (2.14) and is given by

$$\sigma_p = \frac{|-0.1|+|-0.2|+|-0.5|+|-1|+|-5|+|-10|}{6} = 2.8$$

Step 3: System stiffness using (2.15) will be

$$k = \frac{0.1}{10} = 0.01 = k'$$

Step 4: Substituting the values of k, r = 2 (order of the reduced system), $N = (\sigma_p * r) = (2.8*2) = 5.6$ in (2.20)

$$\begin{bmatrix} 0.01(2-1) & 0 \\ (1-0.01) & (1-2) \end{bmatrix} \begin{bmatrix} \operatorname{Re} p_r' \\ M \end{bmatrix} = \begin{bmatrix} 5.6 \\ 0 \end{bmatrix}$$

$$or \begin{bmatrix} \operatorname{Re} p_r' \\ M \end{bmatrix} = \begin{bmatrix} 5.5446 \\ 5.4891 \end{bmatrix}$$

$$or\ p_1' = -0.055446,\ p_2' = -5.5446$$

Step 5: Then, the denominator polynomial of $G_r(s)$ will be

$$\begin{aligned} D_r(s) &= (s + \operatorname{Re} p_1')(s + \operatorname{Re} p_2') \\ &= (s + 0.055446)(s + 5.5446) \\ &= s^2 + 5.6s + 0.3074 \end{aligned}$$

Step 6: The first five time moment proportional's c_i are obtained using (2.2) and is given in Table 2.1

Table 2.1 Time moment proportionals obtained for example 2.1

i	c_i
0	1.0000
1	-10.300
2	106.070
3	-1079.615
4	10897.7631

Step 7: The numerator coefficients of G_r(s) are computed by substituting coefficients of 'D_r(s)' and c_i's in (2.8) resulting in

$$N_r(s) = 2.433s + 0.3072$$

The reduced second order system 'G_r(s)' after matching the steady state errors will be

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{2.4333s + 0.3074}{s^2 + 5.6s + 0.3074}$$

The unit step responses of the original and reduced system are shown in Fig. 2.2. The table 2.2 compares the results for example 2.1 in terms of *'I'* and *'J'* with other available methods and are comparable. The *'J'* value of original system 'J_{org}' is 88.73.

Table 2.2 Comparison of reduced order systems for example 2.1

Order Reduction Method	Reduced System G_r(s)	ISE 'I'	IRE 'J'
Proposed Method	$\frac{2.4333s + 0.3074}{s^2 + 5.6s + 0.3074}$	0.3896	87.881
Jamshidi [22]	$\frac{13.06\ s + 1}{8.75s^2 + 18\ s + 1}$	1.230	68.695
Mahmoud and Singh [23]	$\frac{6.5\ s + 5}{s^2 + 4\ s + 5}$	4.329	824.52
Singh *et. al.* [232]	$\frac{1.987s + 154.044}{1.987s^2 + 33.58s + 154.0}$	2.882	223.18

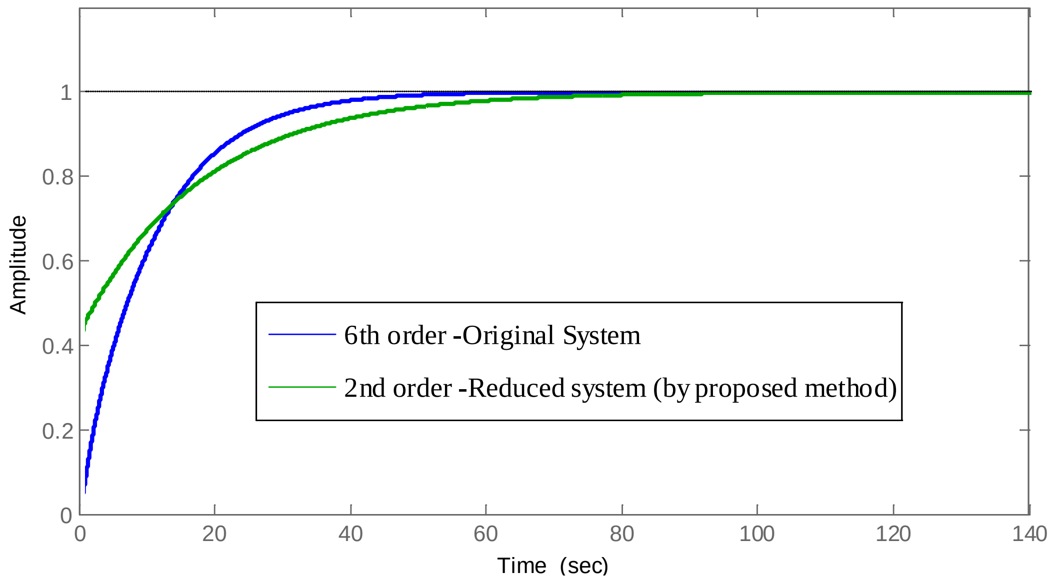

Fig. 2.2 Comparison of step responses for example 2.1

Example 2.2: Consider a fourth order system taken from Mittal *et. al.*[165] and Mukherjee and Mishra [147]

$$G_n(s) = \frac{s^3+7s^2+24s+24}{s^4+10s^3+35s^2+50s+24}$$

The ESP of D_n(s) are found out to be Rep_1 = -1, Rep_2 = -2, Rep_3 = -3, Rep_4 = -4 and all the poles lie on the negative real axis of *'s'* plane. A second order reduced system is desired and the steps described in 2.3.1 are followed to obtain

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{0.677s+4}{s^2+5s+4}$$

Table 2.3 Comparison of reduced order systems for example 2.2

Order Reduction Method	Reduced System G_r(s)	ISE *'I'*	IRE *'J'*
Proposed Method	$\frac{0.677s+4}{s^2+5s+4}$	0.265 x 10^{-3}	27.96
Chen *et. al.* [53]	$\frac{0.699\ s + 0.699}{s^2+1.45771\ s + 0.699}$	33.3 x 10^{-3}	33.4
Gutmen *et. al.*[47]	$\frac{2(48\ s + 144)}{70s^2+ 300s + 288}$	45.6 x 10^{-3}	79.7
Phillip and Pal [233]	$\frac{0.9315s + 1.609}{s^2+2.756s + 1.609}$	1.719 x 10^{-3}	29.65
Krishnamurthy and Seshadri [49]	$\frac{20.5714\ s + 24}{30s^2+42\ s + 24}$	8.9 x 10^{-3}	47.8
Lucas [52]	$\frac{0.833\ s + 2}{s^2+3\ s + 2}$	0.328 x 10^{-3}	48.4
Mittal *et. al.*[165]	$\frac{0.799\ s + 2}{s^2+3\ s + 2}$	0.267 x 10^{-3}	47.2
Moore [204]	$\frac{0.8217\ s + 0.4543}{s^2+1.268\ s + 0.4663}$	2.9 x 10^{-3}	50.0
Mukherjee and Mishra [147]	$\frac{0.800000033\ s + 2}{s^2+3\ s + 2}$	0.237 x 10^{-3}	47.2
Pal [57]	$\frac{16\ s + 24}{30s^2+42\ s + 24}$	11.1 x 10^{-3}	49.1
Prasad and Pal [234]	$\frac{s + 34.2465}{s^2+239.8082\ s + 34.2465}$	1331 x 10^{-3}	16.6
Safonov and Chang [235]	$\frac{0.8213\ s + 0.4545}{s^2+1.268\ s + 0.4664}$	2.855 x 10^{-3}	50.1
Safonov *et. al.*[236]	$\frac{0.7431\ s + 1.057}{s^2+1.879\ s + 1.084}$	0.622 x 10^{-3}	47.3

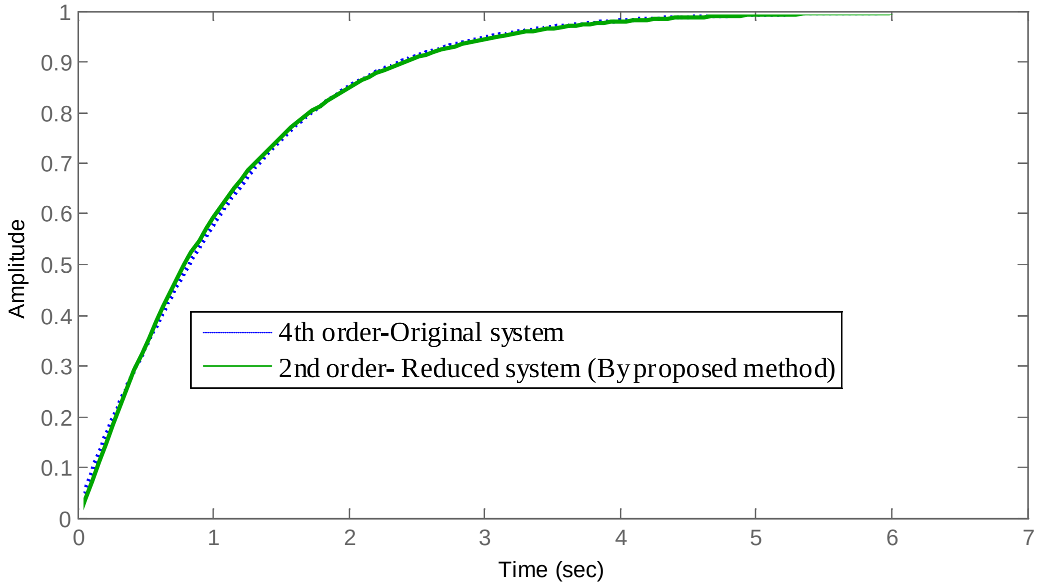

Fig. 2.3 Comparison of step responses for example 2.2

The unit step responses of the original and reduced system are compared in Fig. 2.3. The results are compared with other methods in terms of *'I'* and *'J'* in Table 2.3.

Example 2.3: Consider a sixth order system taken from Mukherjee *et. al.* [168] and Philip and Pal [233]

$$G_n(s) = \frac{s^5 + 1014s^4 + 14069s^3 + 69140s^2 + 140100s + 1000000}{2^6 + 222s^5 + 14541s^4 + 248420s^3 + 1.454\times10^6 s^2 + 2.22\times10^6 s + 1000000}$$

A second order reduced system is desired and the steps described in 2.3.1 are followed to obtain

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{0.03796s + 0.7913596}{s^2 + 1.6843s + 0.7913596}$$

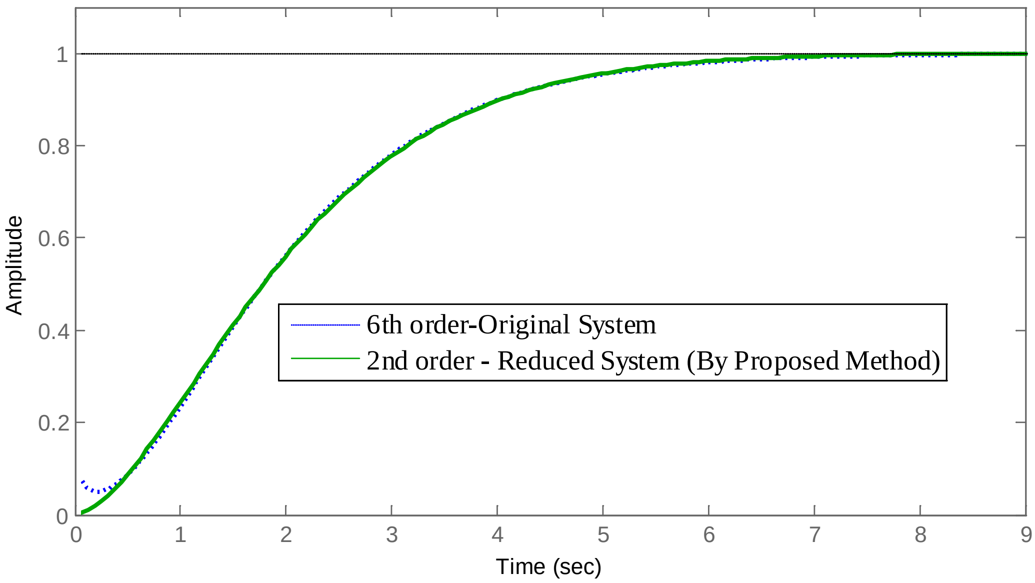

Fig. 2.4 Comparison of step responses for example 2.3

The original, second order reduced system are subjected to a unit step input and their responses obtained are as shown in Fig. 2.4. The results are compared with other methods in terms of *'I'* and *'J'* in Table 2.4. (J_{org} is 19.89).

Table 2.4 Comparison of reduced order systems for example 2.3

Order Reduction Method	Reduced System $G_r(s)$	ISE *'I'*	IRE *'J'*
Proposed Method	$\frac{0.03796s+0.7913596}{s^2+1.6843s+0.7913596}$	1.139	14.11
Mukherjee *et. al.* [168] (impulse response matching)	$\frac{9.71s^2+1.256\times10^4 s+9.189\times10^4}{s^3+252.8s^2+1.67\times10^4 s+9.189\times10^4}$	2.9423	569.38
Mukherjee *et. al.* [168] (step response matching)	$\frac{46.63s^2+271.48s+509.6}{s^3+55.35s^2+692.5s+509.6}$	1.9196	13107.1
Lee *et. al.*[237] (impulse response matching)	$\frac{13.09s^2+922s+4855}{s^3+205.9s^2+10681s+4855}$	2.6861	1028.7
Lee *et. al.*[237] (step response matching)	$\frac{34.09s^2+797.3s+683.5}{s^3+41.982s^2+1504s+683.5}$	1.995	7078.7
Shamash [91] (step response matching)	$\frac{53.67s^2+152.8s+196.8}{s^3+103.1s^2+314s+196.8}$	1.895	17289
Shamash [91] (second order)	$\frac{37.55s+77.25}{s^2+100.8s+77.25}$	1.8245	8468.8
Philip and Pal [233]	$\frac{43.64s^2+310.8s+490.8}{s^3+56.55s^2+736.8s+490.8}$	1.9158	11467.1

Example 2.4: An eighth order system investigated by Shamash [92] is considered for obtaining a second order reduced system. The original system is given by

$$G_n(s)=\frac{N_n(s)}{D_n(s)}$$

where,

$$N_n(s)=18s^7+514s^6+5982s^5+36380s^4+122664s^3+222088s^2+185760s+40320$$

$$D_n(s)=s^8+36s^7+546s^6+4536s^5+22449s^4+67284s^3+118124s^2+109584s+40320$$

Applying the proposed method, G_n(s) is reduced to second order and is given by

$$G_r(s)=\frac{N_r(s)}{D_r(s)}=\frac{24.22s+8}{s^2+9s+8}$$

Table 2.5 Comparison of reduced order systems for example 2.4

Order Reduction Method	Reduced System $G_r(s)$	ISE 'I'	IRE 'J'
Proposed Method	$\frac{24.22s+8}{s^2+9s+8}$	0.050	4228.5
Chen *et. al.*[238]	$\frac{0.72058\text{ s} + 0.3669}{s^2+0.02768\text{s} + 0.3669}$	7.2067	161.23
Gutmen *et. al.*[47]	$\frac{5.35\times10^8 \text{ s} + 8.129\times10^8}{8.505\times10^7 s^2+5.523\times10^8\text{s} + 8.129\times10^8}$	1.376	365.05
Hutton and Friedland [45]	$\frac{1.99\text{ s} + 0.4318}{s^2+1.174\text{s} + 0.4318}$	1.917	124.08
Krishnamurthy and Seshadri [49]	$\frac{1.557\times10^5 \text{ s} + 40320}{65520s^2+75600\text{s} + 40320}$	1.6532	180.05
Lucas [52]	$\frac{6.779\text{ s} + 2}{s^2+3\text{s} + 2}$	0.27973	629.72
Mittal *et. al.*[165]	$\frac{7.12\text{ s} + 2}{s^2+3\text{s} + 2}$	0.27205	693.82
Mukherjee *et. al.*[168]	$\frac{11.39\text{ s} +4.436}{s^2+4.212\text{s} + 4.436}$	0.0578	1394.2
Pal [57]	$\frac{1.518\times10^5 \text{ s} +40320}{65520s^2+75600\text{s} + 40320}$	1.6509	171.97
Prasad and Pal [234]	$\frac{17.99\text{ s} +500}{s^2+13.25\text{s} + 500}$	1.4585	2279.1
Safonov *et. al.* [236]	$\frac{16.96\text{ s} +5.011}{s^2+7.028\text{s} + 5.011}$	0.0173	2581.1
Shamash [92]	$\frac{6.779\text{ s} +2}{s^2+3\text{s} + 2}$	0.27923	629.72

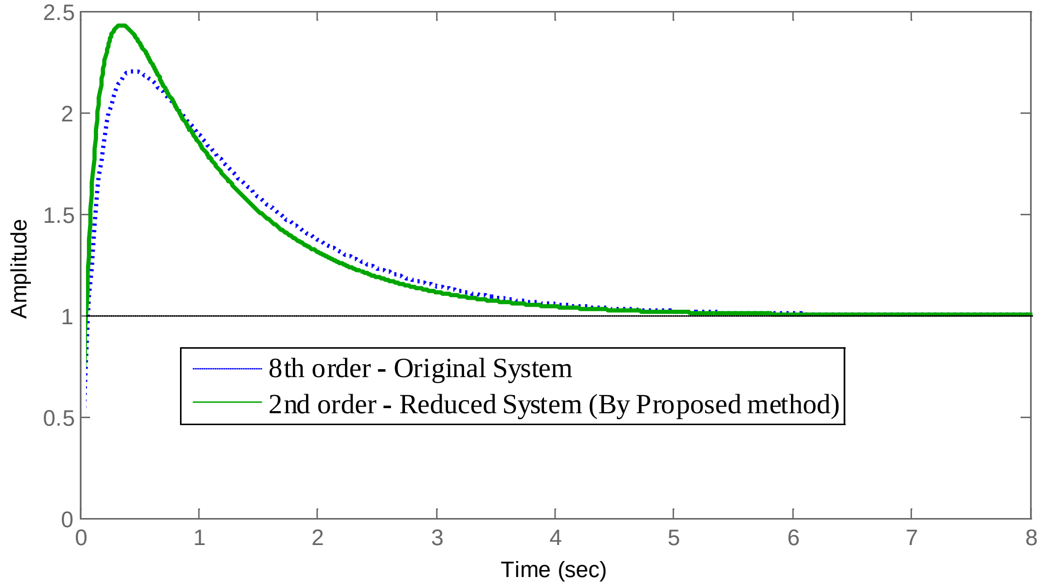

Fig. 2.5 Comparison of step responses for example 2.4

The results obtained by proposed method is compared with other methods in terms of *'I'* and *'J'*, are according to Table 2.5. The value of J_{org} is 2509.2. The unit step responses of G_n(s) and G_r(s) for example 2.4 are depicted in Fig. 2.5.

2.3.1.2 Extension to Multivariable Systems

The proposed method is extended to MIMO systems which is a direct application of the SISO method on the elements of the transfer function matrix of MIMO system as discussed below.

Let the n^{th} order MIMO system having *'p'* inputs and *'m'* outputs be described as

$$[G_n(s)] = \frac{1}{D_n(s)} \begin{bmatrix} A_{11}(s) & A_{12}(s) & A_{13}(s) & \cdots & A_{1p}(s) \\ A_{21}(s) & A_{22}(s) & A_{23}(s) & \cdots & A_{2p}(s) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ A_{m1}(s) & A_{m2}(s) & A_{m3}(s) & \cdots & A_{mp}(s) \end{bmatrix}$$

or, $[G_n(s)] = [g_{ij}(s)]$, $i=1,2,\ldots m;\ j=1,2,\ldots p$ (2.23)

is a $m \times p$ transfer matrix.

The general form of $g_{ij}(s)$ of $[G_n(s)]$ in (2.23) will be

$$g_{ij}(s) = \frac{A_{ij}(s)}{D_n(s)}$$

$$= \frac{A_0 + A_1 s + A_2 s^2 + \ldots + A_{n-1} s^{n-1}}{b_0 + b_1 s + b_2 s^2 + \ldots + b_{n-1} s^{n-1} + b_n s^n} \qquad (2.24)$$

where A_i , b_i (i = 0,1,2…n-1) are scalar constants.

The objective is to find the r^{th} (r<n) order reduced system $[R(s)]$ having *'p'* inputs and *'m'* outputs described by

$$[R(s)] = \frac{1}{D_r(s)} \begin{bmatrix} B_{11}(s) & B_{12}(s) & B_{13}(s) & \cdots & B_{1p}(s) \\ B_{21}(s) & B_{22}(s) & B_{23}(s) & \cdots & B_{2p}(s) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ B_{m1}(s) & B_{m2}(s) & B_{m3}(s) & \cdots & B_{mp}(s) \end{bmatrix} \qquad (2.25)$$

or, $[R(s)] = [r_{ij}(s)]$, $i=1,2,\ldots m;\ j=1,2,\ldots p$

is a m x p transfer matrix.

The general form of $r_{ij}(s)$ of $[R(s)]$ in (2.25) is taken as

$$r_{ij}(s) = \frac{B_{ij}(s)}{D_r(s)}$$

$$= \frac{B_0 + B_1 s + B_2 s^2 + \ldots + B_{r-1} s^{r-1}}{d_0 + d_1 s + d_2 s^2 + \ldots + d_{r-1} s^{r-1} + d_r s^r} \qquad (2.26)$$

where B_i , d_i (i = 0,1,2…r-1) are scalar constants.

The proposed method is applied to (2.26) by following the steps described in 2.3.1. Initially the denominator D_n(s) is reduced using ESA, followed by the determination of the coefficients of the numerator polynomials of each element of [*R*(s)] by least squares method. The method proposed is verified by solving two numerical examples as given below.

2.3.1.2.1 Illustrative Examples

Example 2.5: Consider an aircraft gas turbine [239] represented by linearised perturbation model (A,B,C) in state space form

$$A = \begin{bmatrix} -1.268 & -0.04528 & 1.498 & 951.5 \\ 1.002 & -1.957 & 8.52 & 1240 \\ 0.0 & 0.0 & -10 & 0.0 \\ 0.0 & 0.0 & 0.0 & -100 \end{bmatrix}; B = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \\ 10.0 & 0.0 \\ 0.0 & 100.0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \end{bmatrix}$$

where the state variables are

$x_1 = y_1$= high pressure spool speed

$x_2 = y_2$= low pressure spool speed

x_3 = jet pipe nozzle area

x_4 = fuel flow rate

y_1 and y_2 are the outputs, the control inputs are:

u_1= demanded jet pipe nozzle area

u_2= demanded fuel flow rate

The plant transfer function according to (2.23)

$$[G_n(s)] = \frac{1}{D_n(s)} \begin{bmatrix} A_{11}(s) & A_{12}(s) \\ A_{21}(s) & A_{22}(s) \end{bmatrix}$$

$$A_{11}(s) = 14.96s^2 + 1521.432s + 2543.2$$

$$A_{12}(s) = 95150s^2 + 1132094.7s + 1805947.0$$

$$A_{21}(s) = 85.2s^2 + 8642.688s + 12268.8$$

$$A_{22}(s) = 124000s^2 + 1492588s + 2525880.0$$

and

$$D_n(s) = s^4 + 113.225s^3 + 1357.275s^2 + 3502.75s + 2525$$

It is desired to reduce [G_n(s)] to a second order system represented in the form

$$[R(s)]=\frac{1}{D_r(s)}\begin{bmatrix} B_{11}(s) & B_{12}(s) \\ B_{21}(s) & B_{22}(s) \end{bmatrix}$$

The ESP/poles of D(s) are Rep_1 = -0.13384, Rep_2 = -0.1.8866, Rep_3 = -10, Rep_4 = -100. By following the description in 2.3.1.2.1,

$$D_r(s)=s^2+56.16s+41.73$$

and $B_{11}(s)=23.84s+42.03$, $B_{12}(s)=1.78\times10^4 s+2.985\times10^4$

$B_{21}(s)=136.6s+202.8$, $B_{22}(s)=2.339\times10^4 s+4.179\times10^4$

Table 2.6 Comparison of ISE and IRE for example 2.5

r_{ij} (i,j=1,2)	Proposed Method		Prasad [240]	
	ISE 'I'	IRE 'J'	ISE 'I'	IRE 'J'
r_{11}	0.112	57.43	0.1660	38.497
r_{12}	0.008	3.1744 x10^8	0.0068	3.18 x10^7
r_{21}	0.020	18.68 x10^4	0.0478	1216.3
r_{22}	0.007	5.48 x10^8	0.0119	1.26 x10^8

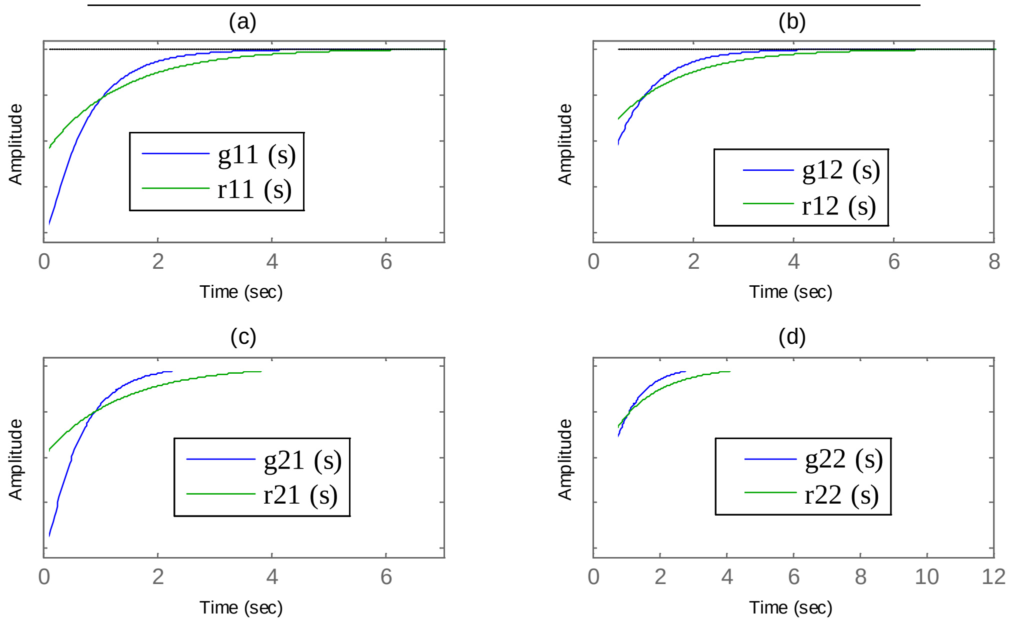

Fig. 2.6 (a)-(d) Comparison of step responses for example 2.5

The results obtained by the proposed method is compared with other method in terms of *'I'* and *'J'* for each element of transfer function matrix are tabulated in Table 2.6. The value J_{org} of each element of plant transfer function matrix are 64.526, 3.054 X 10^7,1735.7 and 5.7 X 10^7 respectively. The unit step responses of [G_n(s)] and [R(s)] are depicted in Fig. 2.6 (a)-(d).

Example 2.6: Consider a sixth-order two input two output system [33] described by the transfer function matrix

$$[G(s)] = \begin{bmatrix} \frac{2(s+5)}{(s+1)(s+10)} & \frac{(s+4)}{(s+2)(s+5)} \\ \frac{(s+10)}{(s+1)(s+20)} & \frac{(s+6)}{(s+2)(s+3)} \end{bmatrix}$$

$$= \frac{1}{D_n(s)} \begin{bmatrix} A_{11}(s) & A_{12}(s) \\ A_{21}(s) & A_{22}(s) \end{bmatrix}$$

The denominator $D_n(s)$ is given by

$$D_n(s) = (s+1)(s+2)(s+3)(s+5)(s+10)(s+20)$$
$$= s^6 + 41s^5 + 571s^4 + 3491s^3 + 10060s^2 + 13100s + 6000$$

and

$$A_{11}(s) = 2s^5 + 70s^4 + 762s^3 + 3610s^2 + 7700s + 6000$$
$$A_{12}(s) = s^5 + 38s^4 + 459s^3 + 2182s^2 + 4160s + 2400$$
$$A_{21}(s) = s^5 + 30s^4 + 331s^3 + 1650s^2 + 3700s + 3000$$
$$A_{22}(s) = s^5 + 42s^4 + 601s^3 + 3660s^2 + 9100s + 6000$$

By following the steps described in 2.3.1.2.1,

$$D_r(s) = s^2 + 13.66s + 8.457$$

and $B_{11}(s) = 6.0498s + 8.457$, $B_{12}(s) = 3.94s + 3.4$

$B_{21}(s) = 2.813s + 4.3$, $B_{22}(s) = 8.02s + 8.457$

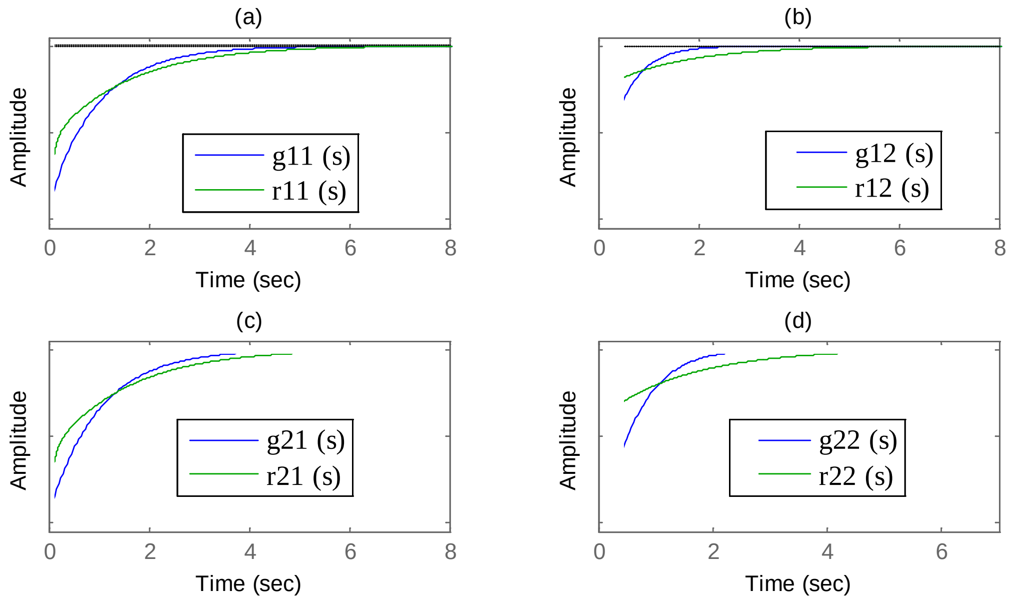

Fig. 2.7 (a)-(d) Comparison of step responses for example 2.6

Table 2.7 Comparison of reduced order systems for example 2.6

Order Reduction Method	ISE '*I*' for r_{ij} (i, j = 1,2)			
	r_{11}	r_{12}	r_{21}	r_{22}
Proposed Method	0.0227	0.00691	0.0024	0.066
Parmar *et. al.*[71]	0.0525	0.0020	0.0168	0.033
Nahid and Prasad [139]	0.0658	0.1645	0.0066	0.157
Parmar *et. al.*[135]	0.0145	0.00874	0.00254	0.0157
Prasad and Pal [241]	0.1365	0.00245	0.04029	0.0679
Safanov and Chiang [242]	0.5906	0.03713	0.00733	1.0661
Prasad *et. al.* [137]	0.0307	0.00026	0.26197	0.0217
Parmar *et. al.* [157]	0.0266	0.0069	0.0061	0.0683
Parmar *et. al.* [70]	0.0449	0.0344	0.0088	0.1577

Table 2.7 shows a comparison with the available existing reduced second order models in terms of ISE values. Further, the step responses are compared in Fig. 2.7(a)-(d). It is seen that the proposed method is comparable with other existing methods.

2.3.2 Stability Equation and Least Squares Method

In the proposed method, a stability criteria based reduction technique named SE method [50] is used in combination with least squares method [174, 175, 231], to obtain the reduced order system. The denominator and numerator polynomial of the higher order continuous time system are reduced using SE and least squares method.The results obtained are comparable as shown in the examples solved below. The same method is also extended for multivariable systems successfully. The method comprise of the following steps.

Step 1: Let an n^{th} order original system be $G_n(s)$ as in (2.1). Consider $D_n(s)$ and bifurcate into odd and even parts to obtain the following stability equations.

$$D_n(s) = D_e(s) + D_o(s) \tag{2.27}$$

$$\left.\begin{aligned} D_e(s) &= a_{11}\prod_{i=1}^{k_1}(1+s^2/z_i^2) \\ D_o(s) &= a_{12}s\prod_{i=1}^{k_2}(1+s^2/p_i^2) \end{aligned}\right\} \tag{2.28}$$

where k_1 and k_2 are the integer part of $n/2$ and $(n-1)/2$ respectively and $z_1^2 < p_1^2 < z_2^2 < p_2^2$ ...

Step 2: Discard the factors with larger magnitudes of z_i ,p_i, and find the reduced stability equations of the desired order 'r' become [56]

$$\left.\begin{aligned} D_{re}(s) &= a_{11}\prod_{i=1}^{r_1}(1+s^2/z_i^2) \\ D_{ro}(s) &= a_{12}s\prod_{i=1}^{r_2}(1+s^2/p_i^2) \end{aligned}\right\} \qquad (2.29)$$

where r_1 and r_2 are the integer parts of $r/2$ and $(r-1)/2$.

Step 3: Thus the reduced denominator is constructed as

$$D_{r_1}(s) = D_{re}(s) + D_{ro}(s)$$

Step 4: Now, apply the reciprocal transformation to D(s) resulting in

$$\tilde{D}(s) = s^n D(\frac{1}{s}) = \tilde{D}_e(s) + \tilde{D}_o(s)$$

Step 5: Reducing the denominator further

$$\tilde{D}_{r_2}(s) = \tilde{D}_{re}(s) + \tilde{D}_{ro}(s)$$

Step 6: Compute D_r(s) for various combinations of r_1 and r_2

$$\begin{aligned} D_r(s) &= D_{r_1}(s).D_{r_2}(s) \\ &= e_{11} + e_{12}s + e_{13}s^2 + \ldots + e_{1r+1}s^r \end{aligned}$$

with $((r = r_1 + r_2) < n)$, where $D_{r_2}(s)$ is reciprocal of $\widetilde{D}_{r_2}(s)$

The reduced denominator D_r(s) is taken as

$$\begin{aligned} D_r(s) &= D_{re}(s) + D_{ro}(s) \\ &= e_o + e_1 s + e_2 s^2 + \ldots + s^r \end{aligned} \qquad (2.30)$$

Step 7: Determine numerator polynomial N_r(s) of G_r(s):

Compute the time moment proportional's 'c_i' by expanding G(s) about s=0 using (2.2). Substitute the value of 'e_i', $i = 0,1, 2$, ... ∞ and 'c_i' in (2.8) and (2.9) to obtain the coefficients 'd_i', $i = 0,1, 2, \ldots(r-1)$.

Step 8: The ratio of the numerator 'N_r(s)' and denominator polynomial 'D_r(s)' gives the reduced order stable LTI continuous system G_r(s).

2.3.2.1 Illustrative examples

Numerical examples from the available literature, are chosen for applying the proposed reduction method and the results are compared with other methods. The comparison is done in terms of ISE, IRE values calculated using (2.21) and (2.22) respectively. The first example is solved in detail while the transfer function of the reduced systems are mentioned directly for the other examples.

Example 2.7: Consider a fourth order system taken from Mittal *et. al.*[165] and Mukherjee and Mishra [147]

$$G(s) = \frac{N_n(s)}{D_n(s)} \; \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24}$$

A second order reduced system is desired.

Step 1: The denominator polynomial of G_n(s) is bifurcated into odd and even terms as

$$D_n(s) = D_e(s) + D_o(s)$$

$$D_e(s) = 24 + 35s^2 + s^4$$

$$D_e(s) = 24\left(1 + \frac{s^2}{0.6858}\right)\left(1 + \frac{s^2}{34.71}\right)$$

$$D_o(s) = 50s + 10s^3 = 50s\left(1 + \frac{s^2}{5}\right)$$

Step 2: Neglecting the factors with larger magnitudes of z_i^2 and p_i^2 successively in D_e(s) and D_o(s) the reduced second order equation will be

$$D_{r_1}(s) = D_{re}(s) + D_{ro}(s)$$

$$D_{re}(s) = 24\left(1 + \frac{s^2}{0.6858}\right)$$

$$D_{ro}(s) = 50s \quad D_{ro}(s) = 50s$$

$$D_{r_1}(s) = 24(1 + \frac{s^2}{0.6858}) + 50s = s^2 + 1.48s + 0.699$$

Step 3: For r_1=0, r_2=2; the reciprocal transformed $\tilde{D}(s)$ is expressed into the following stability equations

$$\tilde{D}(s) = 24s^4 + 50s^3 + 35s^2 + 10s + 1$$

$$= \widetilde{D}_e(s) + \tilde{D}_o(s)$$

$$\tilde{D}_e(s) = 24s^4 + 35s^2 + 1$$

$$\tilde{D}_o(s) = 50s^3 + 10s$$

Step 4: Neglecting the factors with larger magnitudes of z_i^2 and p_i^2 in $\widetilde{D_e}(s)$ and $\widetilde{D_o}(s)$ the reduced second order equation will be

$$\tilde{D}_{r_2}(s) = \tilde{D}_{re}(s) + \tilde{D}_{ro}(s)$$

$$= 34.3s^2 + 10s + 1$$

$$D_{r_2}(s) = s^2 + 10s + 34.3$$

Step 5: Thus the three second order reduced denominators which are properly normalized are given as

$$D_r(s) = D_{r_1}(s).D_{r_2}(s)$$

$$D_r(s) = s^2 + 1.428s + 0.6858 \; ; r_1 = 2, \; r_2 = 0$$

$$D_r(s) = s^2 + 10s + 34.3 \; ; r_1 = 0, \; r_2 = 2$$

$$D_r(s) = s^2 + 3.913s + 1.6464 \; ; r_1 = 1, \; r_2 = 1$$

Step 6: The first five time moment proportional's c_i are obtained using (2.2) and is given in Table 2.8

Table 2.8 Time moment proportionals obtained for example 2.7

i	c_i
0	1.0000
1	-1.0833
2	1.0903
3	-1.0666
4	1.0417

Step 7: The numerator coefficients of $G_r(s)$ are computed by substituting coefficients of '$D_r(s)$' and c_i's in (2.8) resulting in

$$N_r(s) = 0.7008s + 0.69$$

The reduced second order system '$G_r(s)$' for r_1=2, r_2= 0, after matching the steady state errors will be

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{0.7008s + 0.69}{s^2 + 1.48s + 0.69}$$

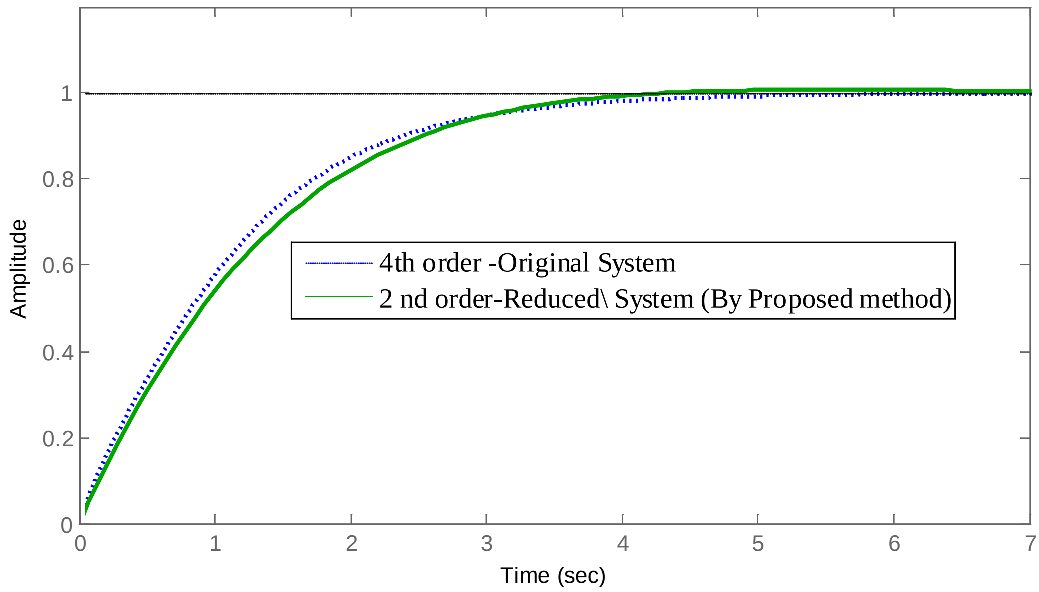

Fig. 2.8 Comparison of step responses for example 2.7

Table 2.9 Comparison of reduced order systems for example 2.7

Order Reduction Method	Reduced System $G_r(s)$	ISE '*I*'	IRE '*J*'
Proposed Method	$\frac{0.7008s+0.69}{s^2+1.48s+0.69}$	2.63×10^{-3}	25.9
Chen *et. al.* [53]	$\frac{0.699\ s+0.699}{s^2+1.45771\ s+0.699}$	33.3×10^{-3}	33.4
Gutmen *et. al.*[47]	$\frac{2(48\ s+144)}{70s^2+300s+288}$	45.6×10^{-3}	79.7
Krishnamurthy and Seshadri [49]	$\frac{20.5714\ s+24}{30s^2+42\ s+24}$	8.9×10^{-3}	47.8
Moore [204]	$\frac{0.8217\ s+0.4543}{s^2+1.268\ s+0.4663}$	2.9×10^{-3}	50.0
Pal [57]	$\frac{16\ s+24}{30s^2+42\ s+24}$	11.1×10^{-3}	49.1
Prasad and Pal [234]	$\frac{s+34.2465}{s^2+239.8082\ s+34.2465}$	1331×10^{-3}	16.6
Safonov and Chang [235]	$\frac{0.8213\ s+0.4545}{s^2+1.268\ s+0.4664}$	2.855×10^{-3}	50.1
Safonov and Chang [242]	$\frac{s+5.403}{s^2+8.431\ s+4.513}$	60.9×10^{-3}	34.7

The unit step responses of the original and reduced system are shown in Fig. 2.8. Table 2.9 compares the results in terms of '*I*' and '*J*' with other available methods (J_{org} is 29.86) and are comparable.

Example 2.8: Consider a sixth order original system mentioned in example 2.1 in 2.3.1.1. Following the steps described in 2.3.2, the second order reduced system for r_1=2, r_2= 0 as

$$G_r(s) = \frac{0.07933s+0.009916}{s^2+0.1815s+0.009916}$$

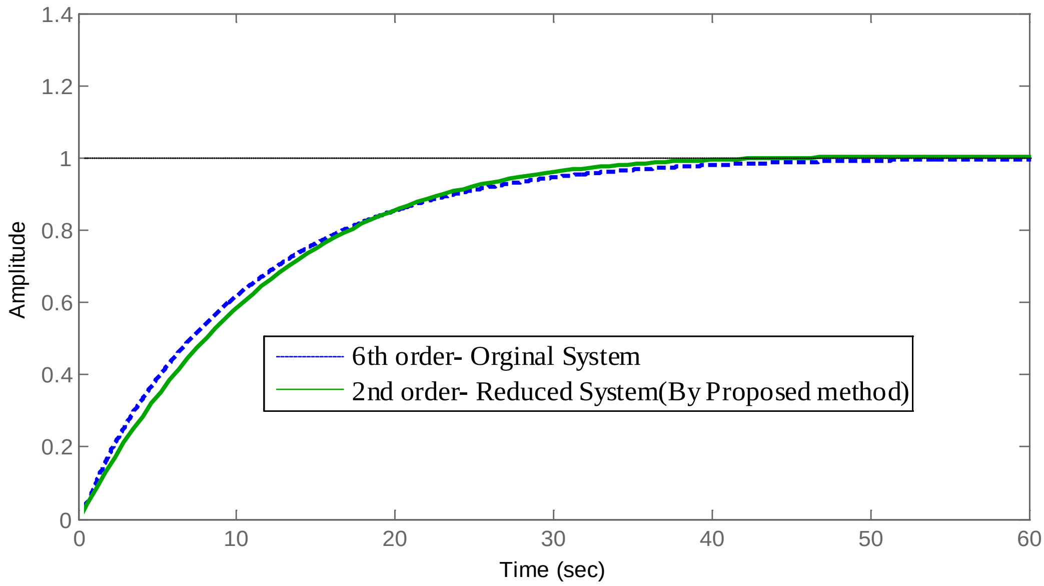

Fig. 2.9 Comparison of step responses for example 2.8

Table 2.10 Comparison of reduced order systems for example 2.8

Order Reduction Method	Reduced System $G_r(s)$	ISE '*I*'	IRE '*J*'
Proposed Method	$\frac{0.07933s+0.009916}{s^2+0.1815s+0.009916}$	0.00933	87.31
Jamshidi [22]	$\frac{13.06\, s+1}{8.75s^2+18\, s+1}$	1.230	68.695
Mahmoud and Singh [23]	$\frac{6.5\, s+5}{s^2+4\, s+5}$	4.329	824.52
Singh *et. al.* [232]	$\frac{1.987s+154.044}{1.987s^2+33.58s+154.044}$	2.882	223.18
Singh [243]	$\frac{5.99\, s+1}{87.97s^2+15.96\, s+1}$	0.02	1.4365

The unit step responses of the original and reduced system are shown in Fig. 2.9. Table 2.10 compares the results in terms of '*I*', '*J*' with other available methods and are comparable, J_{org} being 88.73.

Example 2.9: Revisiting a sixth order system mentioned in example 2.3 in 2.3.1.1, we have

$$G_n(s) = \frac{s^5+1014s^4+14069s^3+69140s^2+140100s+1000000}{2^6+222s^5+14541s^4+248420s^3+1.454\times10^6 s^2+2.22\times10^6 s+1000000}$$

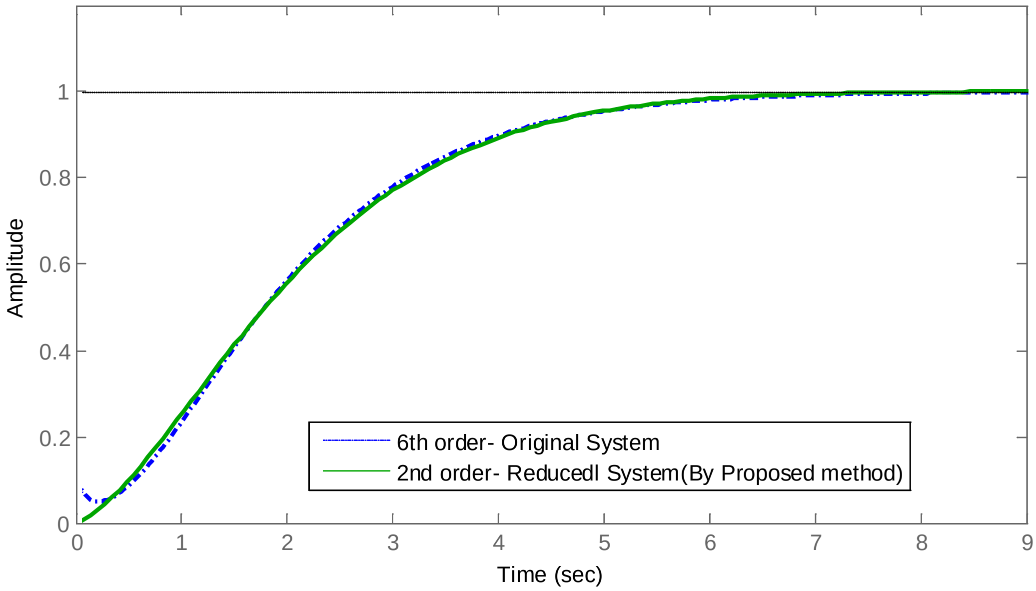

Fig. 2.10 Comparison of step responses for example 2.9

A second order reduced system is desired and the steps described in 2.3.2 are followed to obtain

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{0.09563s + 0.693}{s^2 + 1.537s + 0.693} \text{ for } r_1=2,\ r_2=0$$

Table 2.11 Comparison of reduced order systems for example 2.9

Order Reduction Method	Reduced System G_r(s)	ISE 'I'	IRE 'J'
Proposed Method	$\frac{0.09563s + 0.693}{s^2 + 1.537s + 0.693}$	1.15	13.718
Mukherjee *et. al.* [168] (impulse response matching)	$\frac{9.71s^2 + 1.256\times10^4 s + 9.189\times10^4}{s^3 + 252.8s^2 + 1.67\times10^4 s + 9.189\times10^4}$	2.9423	569.38
Mukherjee *et. al.* [168] (step response matching)	$\frac{46.63s^2 + 271.48s + 509.6}{s^3 + 55.35s^2 + 692.5s + 509.6}$	1.9196	13107.1
Lee *et. al.*[237] (impulse response matching)	$\frac{13.09s^2 + 922s + 4855}{s^3 + 205.9s^2 + 10681s + 4855}$	2.6861	1028.7
Lee *et. al.*[237] (step response matching)	$\frac{34.09s^2 + 797.3s + 683.5}{s^3 + 41.982s^2 + 1504s + 683.5}$	1.995	7078.7
Shamash [91] (step response matching)	$\frac{53.67s^2 + 152.8s + 196.8}{s^3 + 103.1s^2 + 314s + 196.8}$	1.895	17289
Shamash [91] (second order)	$\frac{37.55s + 77.25}{s^2 + 100.8s + 77.25}$	1.8245	8468.8
Philip and Pal [233]	$\frac{43.64s^2 + 310.8s + 490.8}{s^3 + 56.55s^2 + 736.8s + 490.8}$	1.9158	11467.1

The original, second order reduced system are subjected to a unit step input and their responses obtained are as shown in Fig. 2.10. The goodness/quality of the results are compared with other methods (J_{org} = 19.89) in terms of *'I'* and *'J'* in Table 2.11.

Example 2.10: An eighth order system investigated by Shamash [92] is considered for obtaining a second order reduced system. The original system is given by

$$G_n(s) = \frac{N_n(s)}{D_n(s)}$$

$$N_n(s) = 18s^7 + 514s^6 + 5982s^5 + 36380s^4 + 122664s^3 + 222088s^2 + 185760s + 40320$$

$$D_n(s) = s^8 + 36s^7 + 546s^6 + 4536s^5 + 22449s^4 + 67284s^3 + 118124s^2 + 109584s + 40320$$

Applying the proposed method, G_n(s) is reduced to second order and is given by

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{61.39s + 13.248}{s^2 + 36.36s + 13.248} \quad \text{for } r_1=1, r_2=1$$

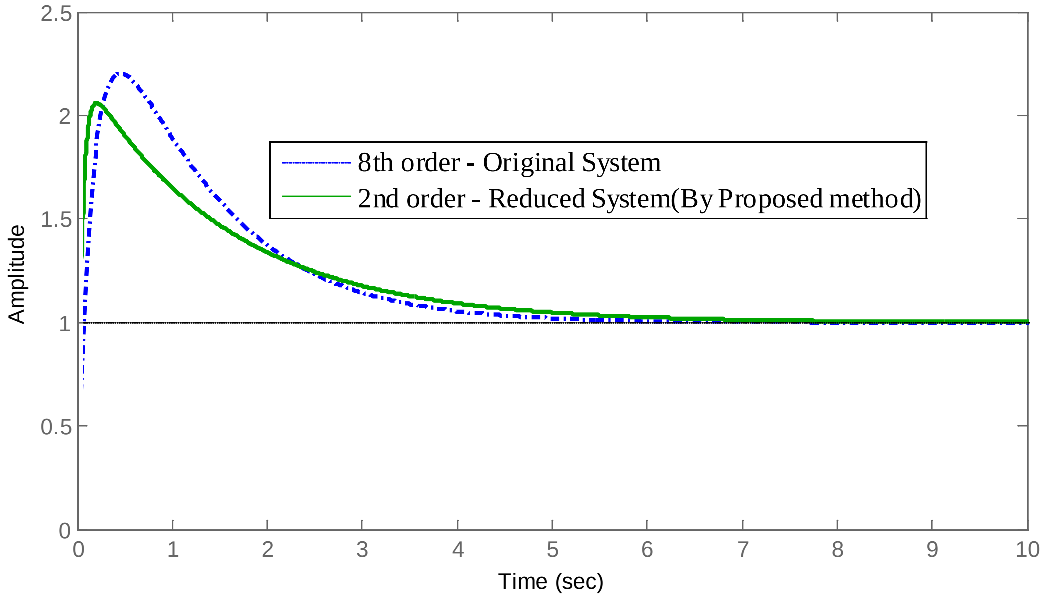

Fig. 2.11 Comparison of step responses for example 2.10

Table 2.12 Comparison of reduced order systems for example 2.10

Order Reduction Method	Reduced System G_r(s)	ISE 'I'	IRE 'J'
Proposed Method	$\frac{61.39s + 13.248}{s^2 + 36.36s + 13.248}$	0.3257	22629.0
Chen *et. al.*[238]	$\frac{0.72058\,s + 0.3669}{s^2 + 0.02768s + 0.3669}$	7.2067	161.23
Gutmen *et. al.*[47]	$\frac{5.35\times10^8\,s + 8.129\times10^8}{8.505\times10^7 s^2 + 5.523\times10^8 s + 8.129\times10^8}$	1.376	365.05
Hutton and Friedland [45]	$\frac{1.99\,s + 0.4318}{s^2 + 1.174s + 0.4318}$	1.917	124.08
Krishnamurthy and Seshadri [49]	$\frac{1.557\times10^5\,s + 40320}{65520s^2 + 75600s + 40320}$	1.6532	180.05
Lucas [52]	$\frac{6.779\,s + 2}{s^2 + 3s + 2}$	0.27973	629.72
Pal [57]	$\frac{1.518\times10^5\,s + 40320}{65520s^2 + 75600s + 40320}$	1.6509	171.97
Prasad and Pal [234]	$\frac{17.99\,s + 500}{s^2 + 13.25s + 500}$	1.4585	2279.1

The results obtained by proposed method is compared with other methods in terms of *'I'* and *'J'*, are according to Table 2.12 (J_{org} = 2509.2). The unit step responses of G_n(s) and G_r(s) are depicted in Fig. 2.11.

Example 2.11: An ninth order system investigated by Mukherjee *et. al.*[168] is considered for obtaining a second order reduced system. The original system is given by

$$G_n(s)=\frac{s^4+35s^3+291s^2+1093s+1700}{s^9+9s^8+66s^7+294s^6+1029s^5+2541s^4+4684s^3+5856s^2+4620s+1700}$$

Following the steps described in 2.3.2, for r_1=2, r_2= 0 a second and third order reduced system will be

$$G_r^2(s)=\frac{N_r(s)}{D_r(s)}=\frac{61.39s+13.248}{s^2+36.36s+13.248} \quad \text{and}$$

$$G_r^3(s)=\frac{N_r(s)}{D_r(s)}=\frac{-0.3309s^2+0.3252s+0.493}{s^3+1.3s^2+1.348s+0.493} \quad \text{for } r_1=3,\ r_2=0 \text{ respectively}$$

Table 2.13 Comparison of reduced order systems for example 2.11

Order Reduction Method	Reduced System G_r(s)	ISE 'I'	IRE 'J'
Proposed Method (second order)	$\frac{61.39s+13.248}{s^2+36.36s+13.248}$	0.0012 x10^{-2}	12.784
Proposed Method (third order)	$\frac{-0.3309s^2+0.3252s+0.493}{s^3+1.3s^2+1.348s+0.493}$	2.857x10^{-2}	29.103
Phillip and Pal [233]	$\frac{0.5058s^2-1.985s+3.534}{s^3+3s^2+5.534s+3.534}$	2.87x10^{-2}	29.42
Mukherjee *et. al.* [168] (impulse response matching)	$\frac{0.2945s^2-2.203s+2.32}{s^3+2.5008s^2+4.778s-2.32}$	8.77x10^{-2}	51.01
Mukherjee *et. al.* [168] (step response matching)	$\frac{-3.49s^2-4.14s+2.078}{s^3+3.828s^2+4.884s+2.078}$	72.6 x10^{-2}	364.36
Chen *et. al.*[50]	$\frac{285s^2+1093s+1700}{3408s^3+5031s^2+4620s+1700}$	29.6 x10^{-2}	25.43
George and Rein Method I [163]	$\frac{-0.29913s+0.73912}{s^2+0.95727s+0.73912}$	4.23x10^{-2}	26.03
George and Rein Method II [163]	$\frac{-0.57072s+0.98330}{s^2+1.42381s+0.98330}$	1.87x10^{-2}	28.636

The responses of G_n(s) and G_r(s) for a unit step input are shown in Fig. 2.12. Table 2.13 compares the results obtained by proposed method in terms of '*I*' and '*J*' with other methods (J_{org} = 28.23).

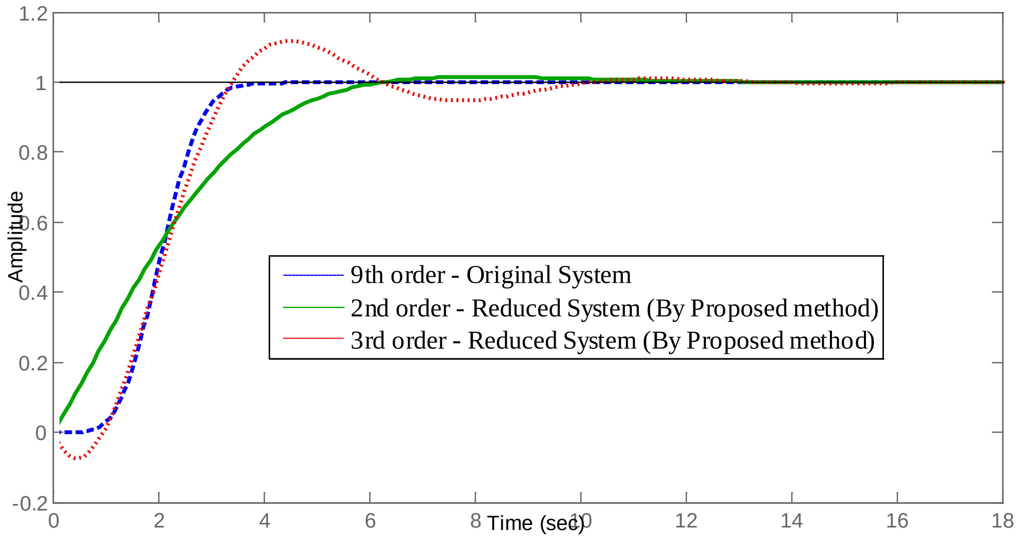

Fig. 2.12 Comparison of step responses for example 2.11

2.3.2.2 Extension to Multivariable systems

The proposed method can be applied on multivariable system to obtain the reduced system. The extension to MIMO systems is a direct application of the SISO method on the elements of the transfer function matrix of MIMO system as discussed below.

Consider a n^{th} order MIMO system having '*p*' inputs and '*m*' outputs as described in (2.23). The proposed method is applied to (2.24), by initially reducing the denominator polynomial using SE method (following the steps described in 2.3.2). The numerator coefficients of the numerator polynomials of each element of [*R*(s)] is then found out using least squares method. The method proposed is justified by solving two numerical examples as given below.

2.3.2.2.1 Illustrative Examples

Example 2.12: Consider an aircraft gas turbine [239] taken from example 2.5 in 2.3.1.2.1 is given by

$$[G_n(s)] = \frac{1}{D_n(s)}\begin{bmatrix} A_{11}(s) & A_{12}(s) \\ A_{21}(s) & A_{22}(s) \end{bmatrix}$$

$$A_{11}(s) = 14.96s^2 + 1521.432s + 2543.2$$

$$A_{12}(s) = 95150s^2 + 1132094.7s + 1805947.0$$

$$A_{21}(s) = 85.2s^2 + 8642.688s + 12268.8$$

$A_{22}(s) = 124000s^2 + 1492588s + 2525880.0$

and

$D_n(s) = s^4 + 113.225s^3 + 1357.275s^2 + 3502.75s + 2525$

It is desired to reduce $[G_n(s)]$ to a second order system represented in the form

$$[R(s)] = \frac{1}{D_r(s)}\begin{bmatrix} B_{11}(s) & B_{12}(s) \\ B_{21}(s) & B_{22}(s) \end{bmatrix}$$

Following the steps described in 2.3.2.2, for $r_1=2$, $r_2=0$,

$D_r(s) = s^2 + 2.58s + 1.863$

and $B_{11}(s) = 1.118s + 1.876 \quad B_{12}(s) = 832.4s + 1332$

$B_{21}(s) = 6.355s + 9.052 \quad B_{22}(s) = 1097s + 1864$

Table 2.14 Comparison of ISE and IRE for example 2.12

r_{ij} (i,j=1,2)	Proposed Method		Prasad [240]	
	ISE 'I'	IRE 'J'	ISE 'I'	IRE 'J'
r_{11}	0.00124	40.376	0.0028	46.426
r_{12}	256.74	21.29 x10^6	1064.8	3.47 x10^7
r_{21}	0.037	1106.9	0.086	1080
r_{22}	413.87	39.38 x10^8	27821	6.799 x10^7

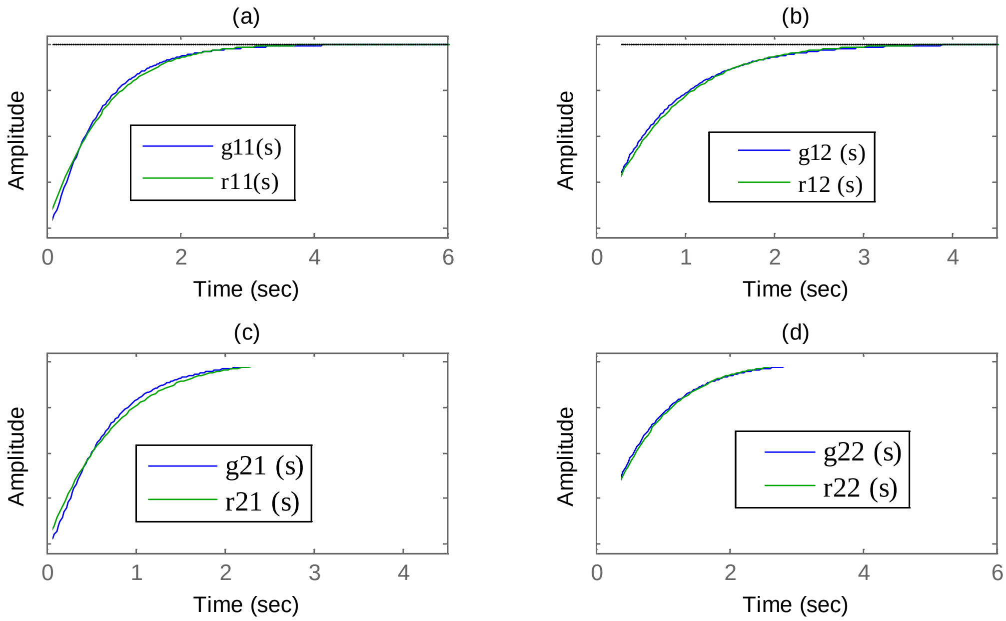

Fig. 2.13 (a)-(d) Comparison of step responses for example 2.12

The results obtained by proposed method, is compared with other method in terms of *'I'* and *'J'*, for each element of transfer function matrix are according to Table 2.14. The value J_{org} of each element of plant transfer function matrix are 64.526, 3.054 X 10^7,1735.7 and 5.7 X 10^7 respectively. The unit step responses of [G_n(s)] and [R(s)] are depicted in Fig. 2.13 (a)-(d).

Example 2.13: Consider a sixth-order two input two output system [33] described by the transfer function matrix and taken in example 2.6 in 2.3.1.2.1 given by

$$[G_n(s)] = \frac{1}{D_n(s)}\begin{bmatrix} A_{11}(s) & A_{12}(s) \\ A_{21}(s) & A_{22}(s) \end{bmatrix}$$

$$D_n(s) = s^6 + 41s^5 + 571s^4 + 3491s^3 + 10060s^2 + 13100s + 6000$$

and

$$A_{11}(s) = 2s^5 + 70s^4 + 762s^3 + 3610s^2 + 7700s + 6000$$
$$A_{12}(s) = s^5 + 38s^4 + 459s^3 + 2182s^2 + 4160s + 2400$$
$$A_{21}(s) = s^5 + 30s^4 + 331s^3 + 1650s^2 + 3700s + 3000$$
$$A_{22}(s) = s^5 + 42s^4 + 601s^3 + 3660s^2 + 9100s + 6000$$

By following the steps described in 2.3.2.2,

$D_r(s) = s^2 + 1.349s + 0.6181$ for r_1=2, r_2=0.

and $B_{11}(s) = 0.79328s + 0.6181$, $B_{12}(s) = 0.4288s + 0.242$

$B_{21}(s) = 0.3814s + 0.309$, $B_{22}(s) = 0.9379s + 0.6181$

Table 2.15 Comparison of ISE for example 2.13

Order Reduction Method	ISE '*I*' for r_{ij} (i, j = 1,2)			
	r_{11}	r_{12}	r_{21}	r_{22}
Proposed Method	0.0165	0.0095	0.003	0.0194
Parmar *et. al.* [71]	0.0525	0.0020	0.0168	0.033
Nahid and Prasad [139]	0.0658	0.1645	0.0066	0.157
Parmar *et. al.* [135]	0.0145	0.00874	0.00254	0.0157
Prasad and Pal [241]	0.1365	0.00245	0.04029	0.0679
Safanov and Chiang [242]	0.5906	0.03713	0.00733	1.0661
Prasad *et. al.* [137]	0.0307	0.00026	0.26197	0.0217
Parmar *et. al.* [157]	0.0266	0.0069	0.0061	0.0683
Parmar *et. al.* [70]	0.0449	0.0344	0.0088	0.1577
Saraswathi [244]	0.084	0.08	0.305	0.346

Table 2.15 shows a comparison with the available existing reduced second order models in terms of ISE values. Further, the waveforms are compared in Fig. 2.14(a)-(d). It is seen that the proposed method is comparable with other existing methods.

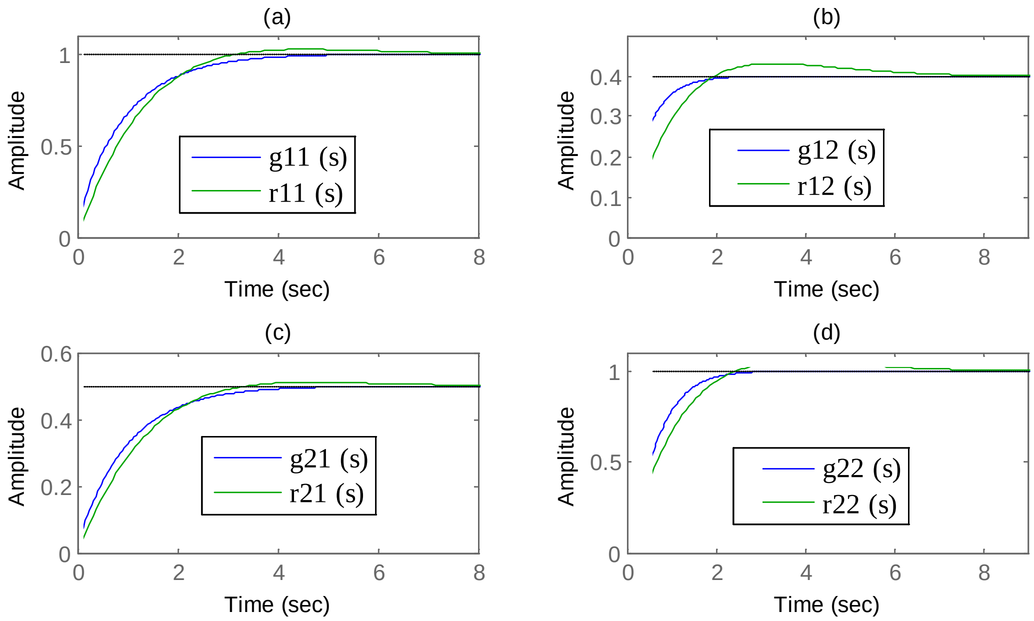

Fig. 2.14 (a)-(d) Comparison of step responses for example 2.13

2.3.3 Dominant Pole and Least Squares Method

In the proposed method, the concept of dominant pole retention [51, 147, 161, 165] is combined with least squares method [174, 175, 231], to generate the reduced order stable system. The process of obtaining the denominator coefficients of the reduced system, comprises of retaining the dominant poles of the original system. Whereas, the numerator coefficients are found out using the least square sense. Further the same method is also extended for multivariable systems. The proposed procedure for deriving the reduced system are as follows.

Step 1: Let an nth order original system be $G_n(s)$ as in (2.1). The denominator polynomial $D_n(s)$ is represented in the form

$$D_n(s) = (s + p_1)(s + p_2)\ldots(s + p_n) \tag{2.31}$$

where $-p_1 < -p_2 < \ldots < -p_n$ are the poles of the higher order original system $G_n(s)$.

Step 2: Selection and retention of dominant pole of $G_n(s)$:

The number of dominant pole to be retained, is directly based on the desired order of the reduced system. The poles nearest to the origin are selected and retained to maintain the overall behavior of the reduced system, similar to that of the original system. On the other hand, the non-dominant poles being responsible during the transient period are neglected. Now, the denominator D_n(s) is reduced to rth order and is given by

$$\begin{aligned} D_r(s) &= (s+p_1)(s+p_2)\ldots(s+p_r) \\ &= e_o + e_1 s + e_2 s^2 + \ldots + s^r \end{aligned} \tag{2.32}$$

Step 3: Determination of numerator polynomial N_r(s) of G_r(s):

Compute the time moment proportional's 'c_i' by expanding G(s) about s=0 using (2.2). Substitute the value of 'e_i', $i = 0,1,2,\ldots\infty$ and 'c_i' in (2.8) and (2.9) to obtain the coefficients 'd_i', $i = 0,1,2,\ldots(r-1)$.

Step 4: The ratio of the numerator 'N_r(s)' and denominator polynomial 'D_r(s)' gives the reduced order stable LTI continuous system G_r(s).

2.3.3.1 Illustrative examples

The above mentioned steps are applied on numerical examples and the results are compared with other methods available in the literature. The ISE, IRE values are calculated using (2.21) and (2.22) for each reduced system and are tabulated. The step by step procedure to obtain the reduced system is described in detail only for the following example. However, in case of the remaining examples, only the results are mentioned directly, though the procedure adopted remains the same.

Example 2.14: Consider a sixth order original system taken from Jamshidi [22] and mentioned in example 2.1 in 2.3.1.1.

$$G_n(s) = \frac{N_n(s)}{D_n(s)} = \frac{2s^5+3s^4+16s^3+20s^2+8s+1}{2s^6+33.6s^5+155.94s^4+209.46s^3+102.42s^2+18.3s+1}$$

A second order reduced system is desired and the procedure adopted are as follows.

Step 1: The denominator polynomial D_n(s) is represented in the form

$$D_n(s) = (s+0.1)(s+0.2)(s+0.5)(s+1)(s+5)(s+10)$$

The poles of D_n(s) are p_1 = -0.1, p_2 = -0.2, p_3 = -0.5, p_4 = -1, p_5 = -5, p_6 = -10 respectively

Step 2: Since the order of the reduced system is equal to two, only two poles that are nearest to the origin (dominant poles) are selected for retaining in the reduced system. Therefore the reduced denominator will be

$$D_r(s) = (s+0.1)(s+0.2) = s^2 + 0.3s + 0.02$$

Step 3: According to (2.2) the first few time moment proportionals obtained are given in Table 2.16

Table 2.16 Time moment proportionals obtained for example 2.14

i	c_i
0	1.0000
1	-10.300
2	106.070
3	-1079.615
4	10897.7631

Step 4: The coefficients of the numerator polynomial of G_r(s) are then computed by substituting coefficients of 'D_r(s)' and c_i's in (2.8) resulting in

$$N_r(s) = 0.094s + 0.02$$

Therefore the desired reduced system 'G_r(s)' will be

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{0.094s + 0.02}{s^2 + 0.3s + 0.02}$$

The original and reduced system are then subjected to unit step input and their responses are plotted in Fig. 2.15. Further, the ISE, IRE values (J_{org} being 88.73) are also calculated and compared with other available methods in Table 2.17.

Table 2.17 Comparison of reduced order systems for example 2.14

Order Reduction Method	Reduced System G_r(s)	ISE 'I'	IRE 'J'
Proposed Method	$\frac{0.094s + 0.02}{s^2 + 0.3s + 0.02}$	0.0010	81.99
Jamshidi [22]	$\frac{13.06\text{ s} + 1}{8.75s^2 + 18\text{ s} + 1}$	1.230	68.69
Mahmoud and Singh [23]	$\frac{6.5\text{ s} + 5}{s^2 + 4\text{ s} + 5}$	4.329	824.52
Singh *et. al.* [232]	$\frac{1.987\text{s} + 154.044}{1.987s^2 + 33.58\text{s} + 154.0}$	2.882	223.18
Singh [243]	$\frac{5.997\text{ s} + 1}{87.97s^2 + 15.96\text{ s} + 1}$	0.02	1.4365

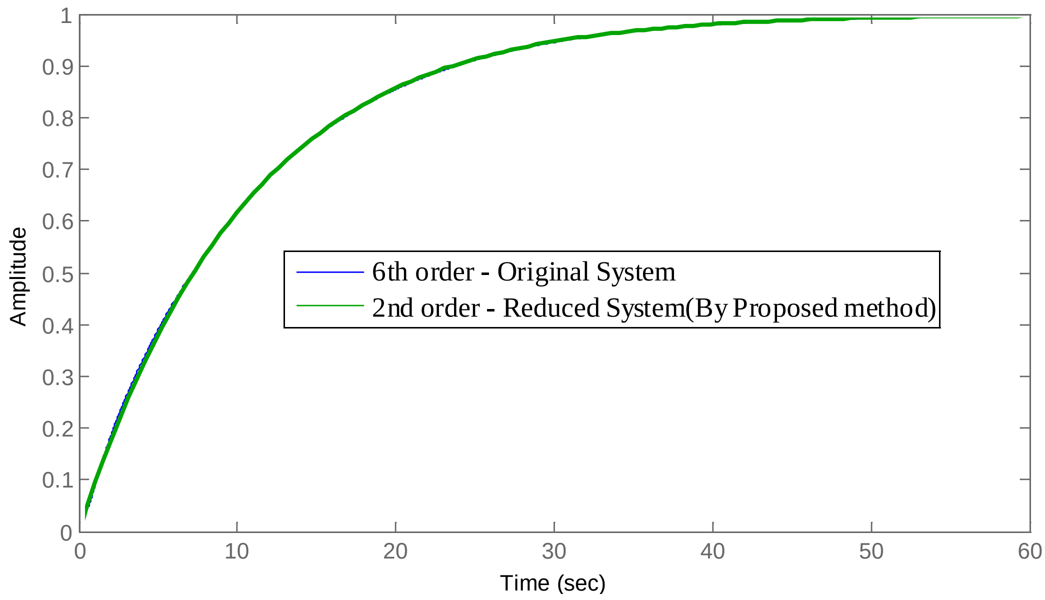

Fig. 2.15 Comparison of step responses for example 2.14

Example 2.15: Revisiting a sixth order system mentioned in example 2.3 in 2.3.1.1

The steps described in 2.3.3 are followed to obtain the desired system as

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{-0.0691s + 0.99}{s^2 + 1.99s + 0.99}$$

Table 2.18 Comparison of reduced order systems for example 2.15

Order Reduction Method	Reduced System $G_r(s)$	ISE 'I'	IRE 'J'
Proposed Method	$\frac{-0.0691s + 0.99}{s^2 + 1.99s + 0.99}$	1.229	15.01
Mukherjee *et. al.* [168] (impulse response matching)	$\frac{9.71s^2 + 1.256\times10^4 s + 9.189\times10^4}{s^3 + 252.8s^2 + 1.67\times10^4 s + 9.189\times10^4}$	2.9423	569.38
Mukherjee *et. al.* [168] (step response matching)	$\frac{46.63s^2 + 271.48s + 509.6}{s^3 + 55.35s^2 + 692.5s + 509.6}$	1.9196	13107.1
Lee *et. al.*[237] (impulse response matching)	$\frac{13.09s^2 + 922s + 4855}{s^3 + 205.9s^2 + 10681s + 4855}$	2.6861	1028.7
Lee *et. al.*[237] (step response matching)	$\frac{34.09s^2 + 797.3s + 683.5}{s^3 + 41.982s^2 + 1504s + 683.5}$	1.995	7078.7
Shamash [91] (step response matching)	$\frac{53.67s^2 + 152.8s + 196.8}{s^3 + 103.1s^2 + 314s + 196.8}$	1.895	17289
Shamash [91] (second order)	$\frac{37.55s + 77.25}{s^2 + 100.8s + 77.25}$	1.8245	8468.8
Philip and Pal [233]	$\frac{43.64s^2 + 310.8s + 490.8}{s^3 + 56.55s^2 + 736.8s + 490.8}$	1.9158	11467.1

The responses of original, second order reduced system when subjected to a unit step input are plotted in Fig. 2.16. The goodness/quality of the results are compared with other methods in terms of *'I'* and *'J'* (J_{org} is 19.89) in Table 2.18.

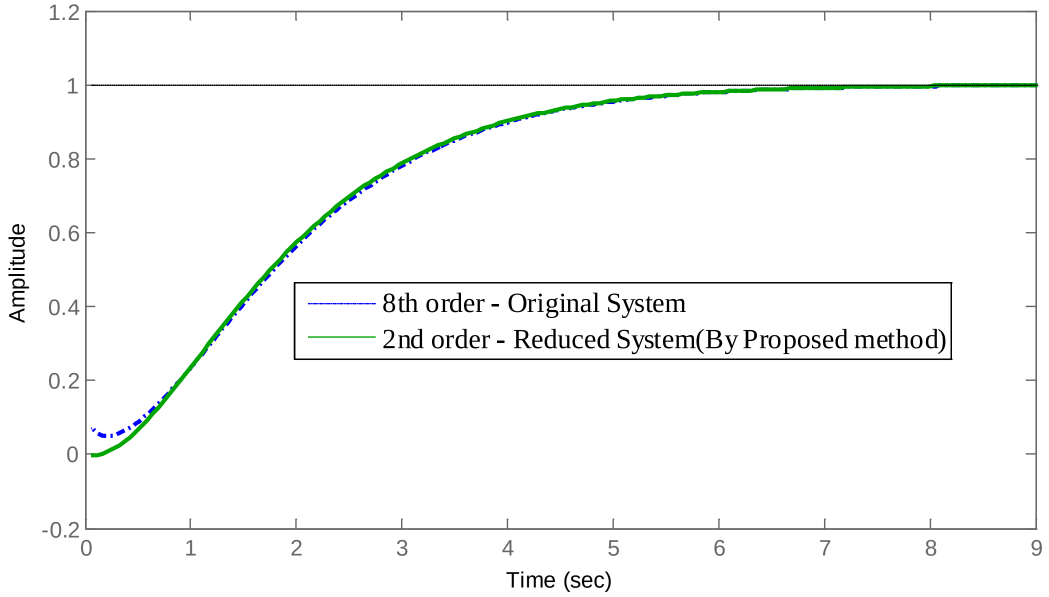

Fig. 2.16 Comparison of step responses for example 2.15

Example 2.16: An eighth order system investigated by Shamash [92] and mentioned in example 2.10 in 2.3.2.1 is considered for order reduction.

The poles of $G_n(s)$ are found to be -1, -2.99, -3.99, -5, -6, -7, -8 respectively. Since a second order system is desired two dominant poles located at -1 and -2.99 are considered. Following the steps mentioned in 2.3.3,

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{6.75s + 1.99}{s^2 + 2.99s + 1.99}$$

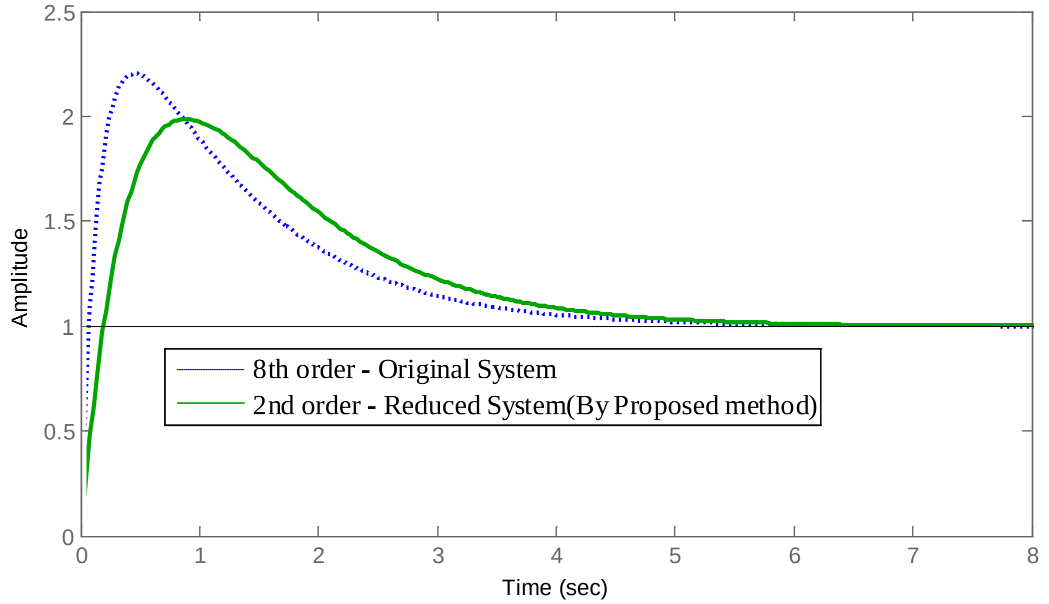

Fig. 2.17 Comparison of step responses for example 2.16

Table 2.19 Comparison of reduced order systems for example 2.16

Order Reduction Method	Reduced System G_r(s)	ISE 'I'	IRE 'J'
Proposed Method	$\frac{6.75s+1.99}{s^2+2.99s+1.99}$	0.2823	625.97
Chen *et. al.*[238]	$\frac{0.72058\text{ s} + 0.3669}{s^2+0.02768\text{s} + 0.3669}$	7.2067	161.23
Gutmen *et. al.*[47]	$\frac{5.35\times10^8\text{ s} + 8.129\times10^8}{8.505\times10^7 s^2+5.523\times10^8\text{s} + 8.129\times10^8}$	1.376	365.05
Hutton and Friedland [45]	$\frac{1.99\text{ s} + 0.4318}{s^2+1.174\text{s} + 0.4318}$	1.917	124.08
Krishnamurthy and Seshadri [49]	$\frac{1.557\times10^5\text{ s} + 40320}{65520s^2+75600\text{s} + 40320}$	1.6532	180.05
Lucas [52]	$\frac{6.779\text{ s} + 2}{s^2+3\text{s} + 2}$	0.27973	629.72
Pal [57]	$\frac{1.518\times10^5\text{ s} +40320}{65520s^2+75600\text{s} + 40320}$	1.6509	171.97
Prasad and Pal [234]	$\frac{17.99\text{ s} +500}{s^2+13.25\text{s} + 500}$	1.4585	2279.1

The results of other methods are compared with the proposed method in terms of *'I'* and *'J'* and are according to Table 2.19 (J_{org} = 2509.2). The unit step responses of G_n(s) and G_r(s) are also depicted in Fig. 2.17.

Example 2.17: Consider a fourth order system as mentioned in example 2.7 in 2.3.2.1
The reduced system 'G_r(s)' obtained is given by

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{0.833s+0.2}{s^2+3s+0.2}$$

The unit step responses of the original and reduced system are shown in Fig. 2.18. Table 2.20 compares the results in terms of *'I'*, *'J'* (J_{org} is 2.986 x 10^1) with other available methods and are found to be comparable.

Table 2.20 Comparison of reduced order systems for example 2.17

Order Reduction Method	Reduced System $G_r(s)$	ISE 'I'	IRE 'J'
Proposed Method	$\frac{0.833s+0.2}{s^2+3s+0.2}$	0.327×10^{-3}	29.87
Chen *et. al.* [53]	$\frac{0.699\,s + 0.699}{s^2+1.45771\,s + 0.699}$	33.3×10^{-3}	33.4
Gutmen *et. al.*[47]	$\frac{2(48\,s + 144)}{70s^2 + 300s + 288}$	45.6×10^{-3}	79.7
Phillip and Pal [233]	$\frac{0.9315s + 1.609}{s^2+2.756s + 1.609}$	1.719×10^{-3}	29.65
Krishnamurthy and Seshadri [49]	$\frac{20.5714\,s + 24}{30s^2+42\,s + 24}$	8.9×10^{-3}	47.8
Lucas [52]	$\frac{0.833\,s + 2}{s^2+3\,s + 2}$	0.328×10^{-3}	48.4
Mittal *et. al.*[165]	$\frac{0.799\,s + 2}{s^2+3\,s + 2}$	0.267×10^{-3}	47.2
Moore [204]	$\frac{0.8217\,s + 0.4543}{s^2+1.268\,s + 0.4663}$	2.9×10^{-3}	50.0
Mukherjee and Mishra [147]	$\frac{0.800000033\,s + 2}{s^2+3\,s + 2}$	0.237×10^{-3}	47.2
Pal [57]	$\frac{16\,s + 24}{30s^2+42\,s + 24}$	11.1×10^{-3}	49.1
Prasad and Pal [234]	$\frac{s + 34.2465}{s^2+239.8082\,s + 34.2465}$	1331×10^{-3}	16.6
Safonov and Chang [235]	$\frac{0.8213\,s + 0.4545}{s^2+1.268\,s + 0.4664}$	2.855×10^{-3}	50.1
Safonov *et. al.*[236]	$\frac{0.7431\,s + 1.057}{s^2+1.879\,s + 1.084}$	0.622×10^{-3}	47.3

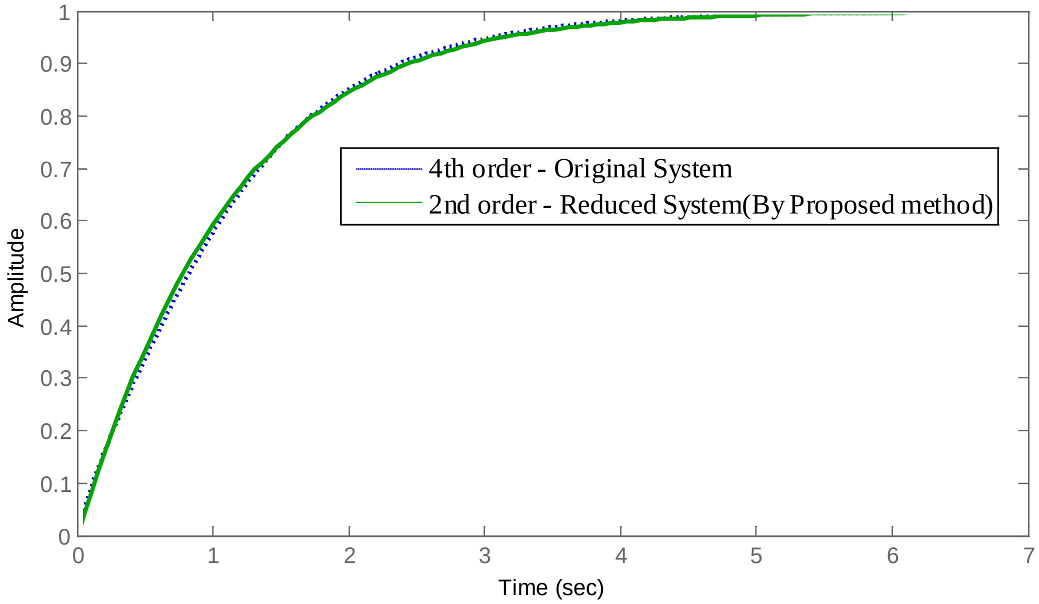

Fig. 2.18 Comparison of step responses for example 2.17

2.3.3.2 Extension to Multivariable systems

The proposed method described in 2.3.3.1is extended for multivariable systems. The procedure involves the direct application of the SISO method on the elements of the transfer function matrix of MIMO system. The same is mentioned as follows.

MIMO system of n^{th} order having *'p'* inputs and *'m'* outputs is assumed to be in the form (2.23). The proposed method is applied to (2.24) by initially reducing the denominator polynomial using DP method by following the steps described in 2.3.3. Later, the numerator coefficients of each element of [*R*(s)] is then found out using least squares method. The applicability of the proposed method is justified by solving two numerical examples.

2.3.3.2.1 Illustrative Examples

Example 2.18: Consider an aircraft gas turbine [239] taken from example 2.5 in 2.3.1.2.1 is given by

$$[G_n(s)] = \frac{1}{D_n(s)}\begin{bmatrix} A_{11}(s) & A_{12}(s) \\ A_{21}(s) & A_{22}(s) \end{bmatrix}$$

$$A_{11}(s) = 14.96s^2 + 1521.432s + 2543.2$$
$$A_{12}(s) = 95150s^2 + 1132094.7s + 1805947.0$$
$$A_{21}(s) = 85.2s^2 + 8642.688s + 12268.8$$
$$A_{22}(s) = 124000s^2 + 1492588s + 2525880.0$$

and

$$D_n(s) = s^4 + 113.225s^3 + 1357.275s^2 + 3502.75s + 2525$$

It is desired to reduce G_n(s) to a second order system represented in the form

$$[R(s)] = \frac{1}{D_r(s)}\begin{bmatrix} B_{11}(s) & B_{12}(s) \\ B_{21}(s) & B_{22}(s) \end{bmatrix}$$

By following the steps described in 2.3.3.2,

$$D_r(s) = s^2 + 3.225s + 2.525$$

and $B_{11}(s) = 1.242s + 2.543$, $B_{12}(s) = 933.4s + 1806$

$B_{21}(s) = 7.2933s + 12.27$, $B_{22}(s) = 1215s + 2526$

The results obtained by proposed are method is compared with other method in terms of *'I'* and *'J'* for each element of transfer function matrix are according to Table 2.21. The value J_{org} of each element of plant transfer function matrix are 64.526, 3.054 X 10^7,1735.7 and 5.7 X 10^7 respectively. The unit step responses of [G_n(s)] and [*R*(s)] are depicted in Fig. 2.19 (a)-(d).

Table 2.21 Comparison of ISE and IRE for example 2.18

r_{ij} (i,j=1,2)	Proposed Method		Prasad [240]	
	ISE 'I'	IRE 'J'	ISE 'I'	IRE 'J'
r_{11}	0.0017	42.977	0.0028	46.426
r_{12}	13.083	22.85×10^6	1064.8	3.47×10^7
r_{21}	0.0612	1217.1	0.086	1080
r_{22}	22.157	41.83×10^6	27821	6.799×10^7

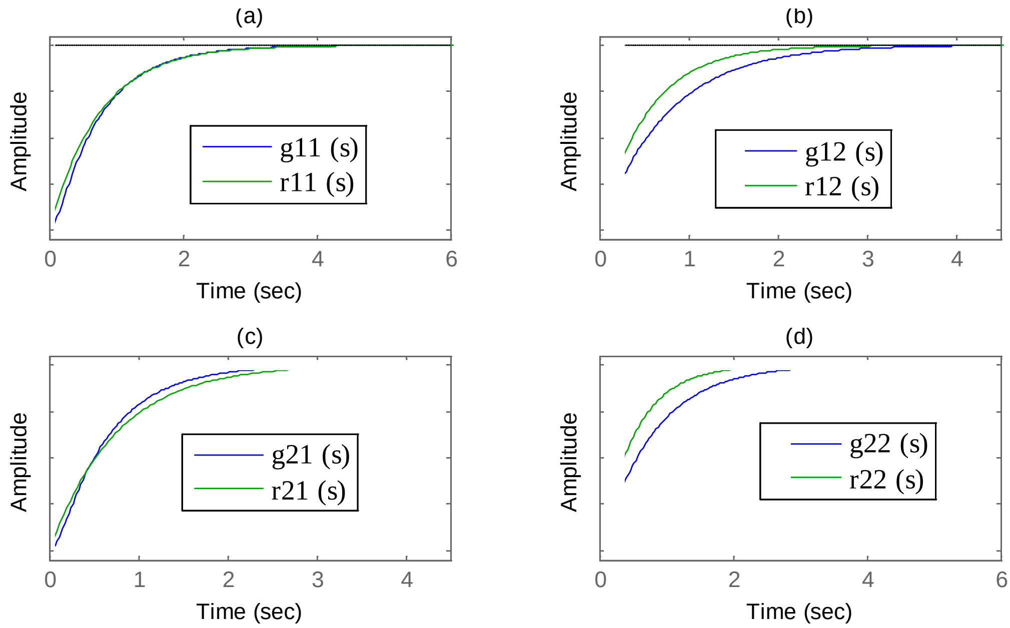

Fig. 2.19 (a)- (d) Comparison of step responses for example 2.18

Example 2.19: Consider a sixth-order two input two output system [33] described by the transfer function matrix and taken in example 6 in 2.3.1.2.1.

By following the steps described in 2.3.3.2 ,

$$D_r(s) = s^2 + 3s + 2$$

and $B_{11}(s) = 1.2s + 2,\ B_{12}(s) = 0.84s + 0.8$

$$B_{21}(s) = 0.5s + 1,\ B_{22}(s) = 1.667s + 2$$

Table 2.22 shows a comparison with the available existing reduced second order models in terms of ISE values. Further, the responses for a given step input are compared in Fig. 2.20(a)-(d). It is observed that the proposed method is comparable with other existing methods.

Table 2.22 Comparison of reduced order systems for example 2.19

Order Reduction Method	ISE '*I*' for r_{ij} (i, j = 1,2)			
	r_{11}	r_{12}	r_{21}	r_{22}
Proposed	0.0008	0.0001	0.000073	0.0037
Parmar *et. al.* [71]	0.0525	0.0020	0.0168	0.033
Nahid and Prasad [139]	0.0658	0.1645	0.0066	0.157
Parmar *et. al.* [135]	0.0145	0.00874	0.00254	0.0157
Prasad and Pal [241]	0.1365	0.00245	0.04029	0.0679
Safanov and Chiang [242]	0.5906	0.03713	0.00733	1.0661
Prasad *et. al.* [137]	0.0307	0.00026	0.26197	0.0217
Parmar *et. al.* [157]	0.0266	0.0069	0.0061	0.0683
Parmar *et. al.* [70]	0.0449	0.0344	0.0088	0.1577
Saraswathi [244]	0.084	0.08	0.305	0.346

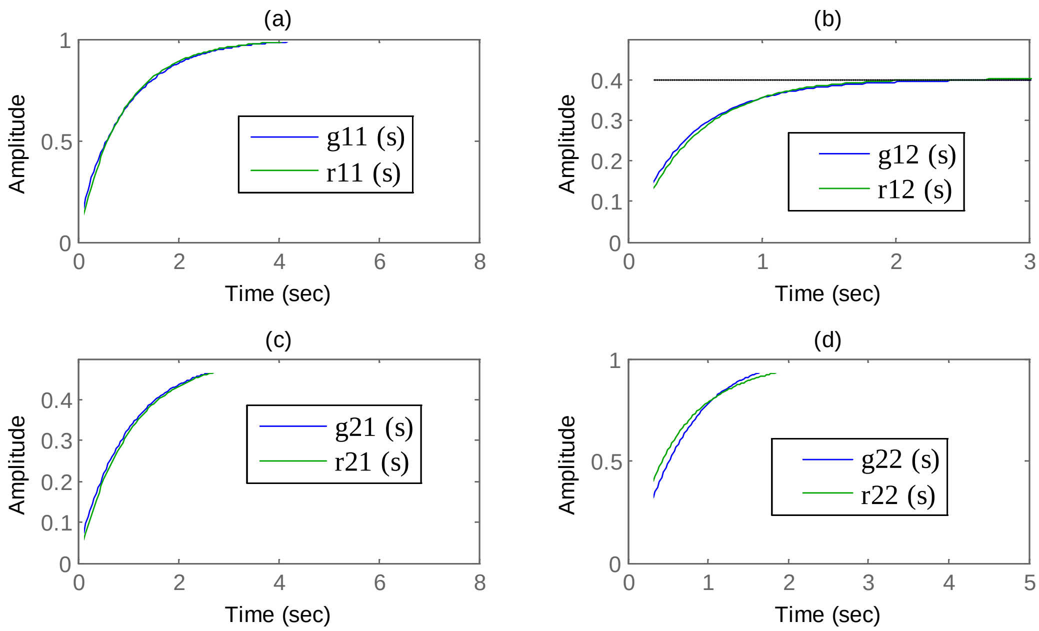

Fig. 2.20 (a)- (d) Comparison of step responses for example 2.19

2.3.4 Modified Pole Clustering and Least Squares Method

The proposed method combines the advantages of modified pole clustering and least squares methods. The modified pole clustering differs from the pole clustering method in the way the pole clusters are calculated [149]. Hence, the results obtained by modified pole cluster method are more dominant than that obtained by pole clustering method. Added to this, the proposed method is computer oriented. The steps to compute the coefficients of the reduced system are mentioned below.

Step 1: Let an n^{th} order original system be $G_n(s)$ as in (2.1).The denominator polynomial $D_n(s)$ is represented in the form

$$D_n(s) = (s+p_1)(s+p_2)...(s+p_n) \tag{2.33}$$

where $|-p_1| < |-p_2| < ... < |-p_n|$ are the poles of the higher order original system $G_n(s)$.

Step 2: Form the clusters, depending upon the order of the reduced system desired and then compute the modified pole cluster centre for each cluster.

Step 3: Computation of modified pole cluster centre P_{ei} :

An algorithm for finding the modified pole cluster centre is as follows [230].

(1) consider a cluster having 'x' no of poles with $|-p_1| < |-p_2| < ... < |-p_x|$

(2) set m =1,

(3) the pole cluster centre is found out by using

$$\sigma_m = \left[\frac{\sum_{k=1}^{x} \left(\frac{-1}{|p_k|} \right)}{x} \right]^{-1} \tag{2.34}$$

(4) increment m

(5) compute modified cluster centre using the formula

$$\sigma_m = \left[\frac{\left(\frac{-1}{|p_k|} \right) + \left(\frac{-1}{|\sigma_{m-1}|} \right)}{2} \right]^{-1} \tag{2.35}$$

(6) check if m $= x$?, if No, then go to step (4)

(7) the modified pole cluster centre of the i^{th} cluster is $p_{ei} = \sigma_m$.

Step 4: Determination of denominator polynomial $D_r(s)$

The denominator polynomial is constructed using the modified pole cluster centre using

$$D_r(s) = (s-p_{e1})(s-p_{e2})...(s-p_{er})$$

The following cases may occur during the formation of the reduced denominator polynomial.

Case 1: If the modified cluster centre's are real, then the r^th^ order reduced denominator will be given by

$$\begin{aligned} D_r(s) &= (s-p_{e1})(s-p_{e2})\dots(s-p_{er}) \\ &= e_o + e_1 s + e_2 s^2 + \dots + e_r s^r \end{aligned} \quad (2.36)$$

*Case 2:*If the modified cluster centre's are complex conjugate, then the r^th^ order reduced denominator will be given by

$$\begin{aligned} D_r(s) &= (s-\overset{*}{\phi}_{e1}|(s-\overset{\circ}{\phi}_{e2})\dots(s-\overset{*}{\phi}_{er/2})(s-\overset{\circ}{\phi}_{er/2}) \\ &= e_o + e_1 s + e_2 s^2 + \dots + e_r s^r \end{aligned} \quad (2.37)$$

$$\phi_{ei} = A_{ei} \pm jB_{ei}$$

*Case 3:*If some of the modified cluster centre's are real (assume (r-2)) and others are complex conjugate, then the r^th^ order reduced denominator will be given by

$$\begin{aligned} D_r(s) &= (s-p_{e1})(s-p_{e2}|\dots(s-p_{e(r-2)})(s-\overset{*}{\phi}_{e1})(s-\overset{\circ}{\phi}_{e2}) \\ &= e_o + e_1 s + e_2 s^2 + \dots + e_r s^r \end{aligned} \quad (2.38)$$

$$\phi_{ei} = A_{ei} \pm jB_{ei}$$

Step 5: Determination of numerator polynomial N_r(s) of G_r(s):

Compute the time moment proportional's 'c_i' by expanding *G*(s) about s=0 using (2.2). Substitute the value of 'e_i', $i = 0,1, 2, \dots \infty$ and 'c_i' in (2.8) and (2.9) to obtain the coefficients 'd_i', $i = 0,1, \dots(r-1)$.

Step 6: The ratio of the numerator 'N_r(s)' and denominator polynomial 'D_r(s)' gives the reduced order stable LTI continuous system G_r(s).

2.3.4.1 Illustrative examples

The above mentioned steps are followed for reducing higher order systems taken from the literature. The results obtained are then compared with other available methods. ISE and IRE values are calculated using (2.21), (2.22) for each reduced system and are tabulated. The following example is solved in depth while, only the results are mentioned directly for the remaining examples.

Example 2.20: Consider a sixth order original system taken from Jamshidi [22] and mentioned in example 2.1 in 2.3.1.1 with J_{org}= 8.09.

$$G(s) = \frac{N_n(s)}{D_n(s)} = \frac{2s^5+3s^4+16s^3+20s^2+8s+1}{2s^6+33.6s^5+155.94s^4+209.46s^3+102.42s^2+18.3s+1}$$

A second order reduced system is desired and the procedure described in 2.3.4 are as follows.

Step 1: The roots of the denominator polynomial D_n(s) are (-0.1,-0.2 -0.5 -1, -5, -10)

Step 2: Since a second order is desired, two clusters are formed with (-0.1, -0.2) and (-0.5 -1, -5, -10)

Step 3: The modified pole cluster centre P_{ei} are computed. Considering the cluster with (-0.1, -0.2) , the value of x=2.For m=1, using (2.34), the value of σ_m will be -0.1333

Step 4: Now, m is incremented to 2 and is equal to the value of x.

Step 5: Using (2.35) the value of σ_m=-0.1143 Therefore the pole centre of cluster (-0.1,-0.2) is p_{e1}= 0.1143.

Step 6: Similarly, for the next cluster with (-0.5 -1, -5, -10), m=1, σ_m=-1.212 and m is incremented to 2 .

Step 7: Follow step 4 - 6 as in 2.3.4 till the value of m and x are same. Then σ_m=-0.5395 or pole centre of cluster (-0.5 -1, -5, -10) is p_{e2}= -0.5395.

Step 8: As the both the pole centers are real using (2.36),

$$\begin{aligned} D_r(s) &= (s - p_{e1})(s - p_{e2}) \\ &= (s + 0.114)(s + 0.5395) \\ &= s^2 + 0.6539s + 0.0617 \end{aligned}$$

Step 9: Compute the time moment proportional's 'c_i' by expanding *G*(s) about s=0 using (2.2). Substitute the value of 'e_i', *i = 0,1, 2 , ..∞* and 'c_i' in (2.8) and (2.9) to obtain the coefficients 'd_i', *i = 0,1, 2 , ...(r-1).*

Step 10: The numerator polynomial of G_r(s) are then given as

$$N_r(s) = 0.01839s + 0.0617$$

Step 11: The ratio of the numerator 'N_r(s)' and denominator polynomial 'D_r(s)' gives the reduced order stable LTI continuous system G_r(s) as

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{0.01839s + 0.0617}{s^2 + 0.6539s + 0.0617}$$

The unit step responses(reaction curves) of the original and reduced system are shown in Fig. 2.21. Table 2.23 compares the proposed reduced order model with other available reduced models obtained by different methods in terms of *'I'* and *'J'*. It is seen that the values of of *'I'* and *'J'* for the proposed reduced model are comparable.

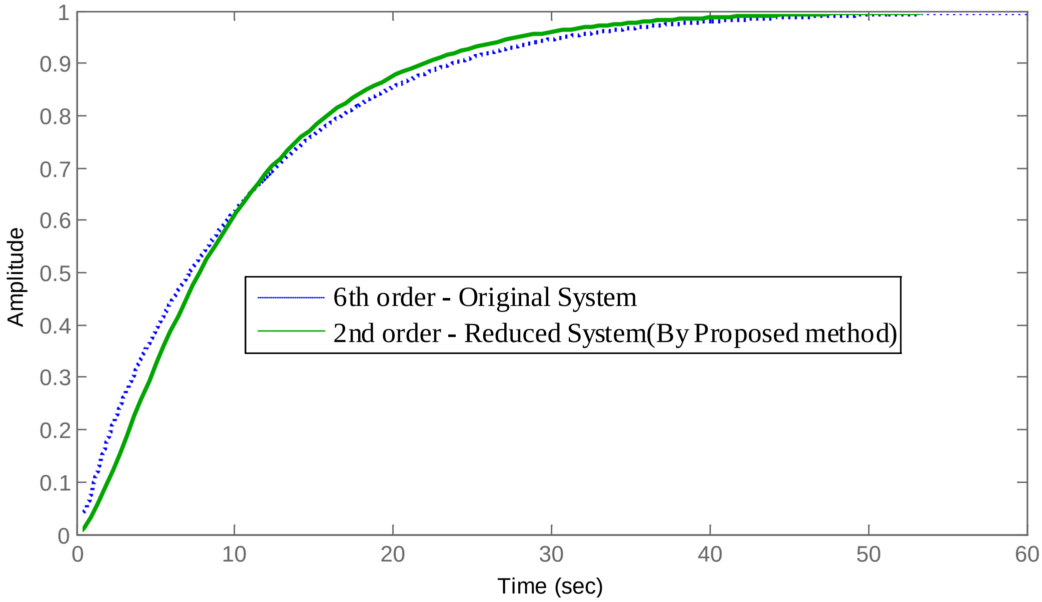

Fig. 2.21 Comparison of step responses for example 2.20

Table 2.23 Comparison of reduced order systems for example 2.20

Order Reduction Method	Reduced System $G_r(s)$	ISE 'I'	IRE 'J'
Proposed Method	$\frac{0.01839s+0.0617}{s^2+0.6539s+0.0617}$	0.0298	1.6141
Jamshidi [22]	$\frac{13.06\ s+1}{8.75s^2+18\ s+1}$	1.230	68.695
Mahmoud and Singh [23]	$\frac{6.5\ s+5}{s^2+4\ s+5}$	4.329	824.52
Singh *et. al.* [232]	$\frac{1.987s+154.044}{1.987s^2+33.58s+154.044}$	2.882	223.18
Singh [243]	$\frac{5.99\ s+1}{87.97s^2+15.96\ s+1}$	0.02	1.4365

Example 2.21: Revisiting a fourth order system in example 2.7 in 2.3.2.1 with J_{org}= 29.86 . The roots of D_r(s) are (-1,-2,-3,-4). Two clusters (-1,-2) and (-3,-4) are formed and following the steps described in 2.3.4 the modified pole cluster centers as p_{e1} = -1.33, p_{e2}=-3.4236

The reduced second order system 'G_r(s)' obtained will be

$$G_r(s)=\frac{N_r(s)}{D_r(s)}=\frac{0.3806s+3.658}{s^2+4.343s+3.658}$$

Table 2.24 Comparison of reduced order systems for example 2.21

Order Reduction Method	Reduced System $G_r(s)$	ISE 'I'	IRE 'J'
Proposed Method	$\frac{0.3806s+3.658}{s^2+4.343s+3.658}$	1.4×10^{-3}	26.6
Chen *et. al.* [53]	$\frac{0.699 s + 0.699}{s^2+1.45771 s + 0.699}$	33.3×10^{-3}	33.4
Gutmen *et. al.*[47]	$\frac{2(48 s + 144)}{70s^2 + 300s + 288}$	45.6×10^{-3}	79.7
Krishnamurthy and Seshadri [49]	$\frac{20.5714 s + 24}{30s^2 + 42 s + 24}$	8.9×10^{-3}	47.8
Moore [204]	$\frac{0.8217 s + 0.4543}{s^2 + 1.268 s + 0.4663}$	2.9×10^{-3}	50.0
Pal [57]	$\frac{16 s + 24}{30s^2 + 42 s + 24}$	11.1×10^{-3}	49.1
Phillip and Pal [233]	$\frac{0.9315s + 1.609}{s^2 + 2.756s + 1.609}$	1.71×10^{-3}	29.7
Prasad and Pal [234]	$\frac{s + 34.2465}{s^2 + 239.8082 s + 34.2465}$	1331×10^{-3}	16.6
Safonov and Chang [235]	$\frac{0.8213 s + 0.4545}{s^2 + 1.268 s + 0.4664}$	2.855×10^{-3}	50.1
Safonov and Chang [242]	$\frac{s + 5.403}{s^2 + 8.431 s + 4.513}$	60.9×10^{-3}	34.7

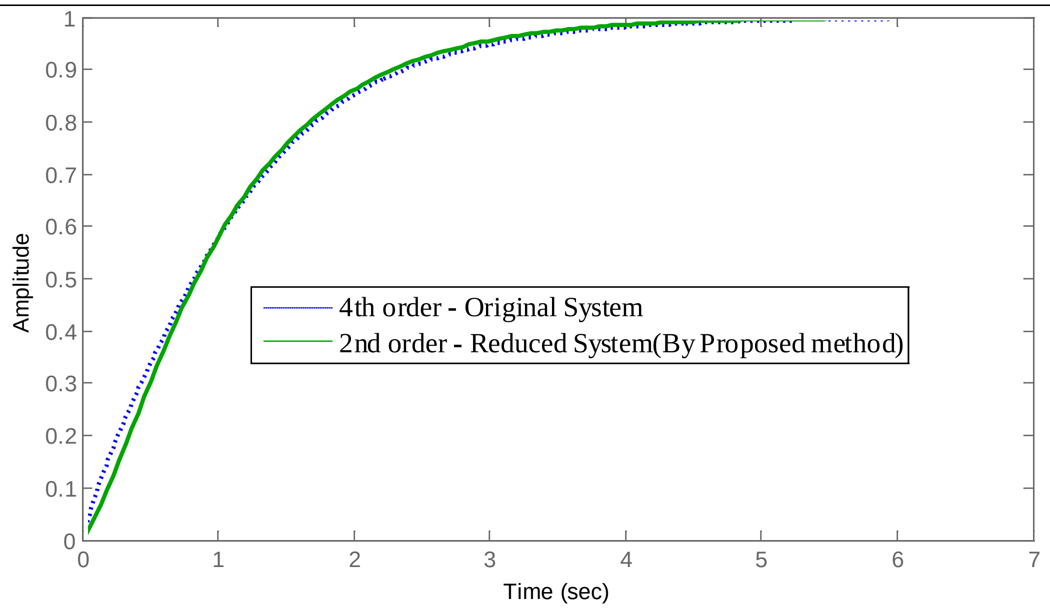

Fig. 2.22 Comparison of step responses for example 2.21

The unit step responses of the original and reduced system are shown in Fig. 2.22. Table 2.24 compares the results in terms of *'I'* and *'J'* with other available methods and are found to be comparable.

Example 2.22: Revisiting example 3 in 2.3.2.1, the roots of D_r(s) are (-1,-1,-10,-10,-100,-100). Two clusters containing the poles (-1) and (,-1,-10,-10,-100,-100) are formed. The steps described in 2.3.4 are followed to get the modified pole cluster centers as

p_{e1} = -1 , p_{e2} = -1.0496

The reduced second order system 'G_r(s)' is then obtained as

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{0.1335s + 1.05}{s^2 + 2.058s + 1.05}$$

Table 2.25 Comparison of reduced order systems for example 2.22

Order Reduction Method	Reduced System G_r(s)	ISE *'I'*	IRE *'J'*
Proposed Method	$\frac{0.1335s + 1.05}{s^2 + 2.058s + 1.05}$	1.1139	15.691
Mukherjee *et. al.* [168] (impulse response matching)	$\frac{9.71s^2 + 1.256\times10^4 s + 9.189\times10^4}{s^3 + 252.8s^2 + 1.67\times10^4 s + 9.189\times10^4}$	2.9423	569.38
Mukherjee *et. al.* [168] (step response matching)	$\frac{46.63s^2 + 271.48s + 509.6}{s^3 + 55.35s^2 + 692.5s + 509.6}$	1.9196	13107.1
Lee *et. al.*[237] (impulse response matching)	$\frac{13.09s^2 + 922s + 4855}{s^3 + 205.9s^2 + 10681s + 4855}$	2.6861	1028.7
Lee *et. al.*[237] (step response matching)	$\frac{34.09s^2 + 797.3s + 683.5}{s^3 + 41.982s^2 + 1504s + 683.5}$	1.995	7078.7
Shamash [91] (step response matching)	$\frac{53.67s^2 + 152.8s + 196.8}{s^3 + 103.1s^2 + 314s + 196.8}$	1.895	17289
Shamash [91] (second order)	$\frac{37.55s + 77.25}{s^2 + 100.8s + 77.25}$	1.8245	8468.8
Philip and Pal [233]	$\frac{43.64s^2 + 310.8s + 490.8}{s^3 + 56.55s^2 + 736.8s + 490.8}$	1.9158	11467.1

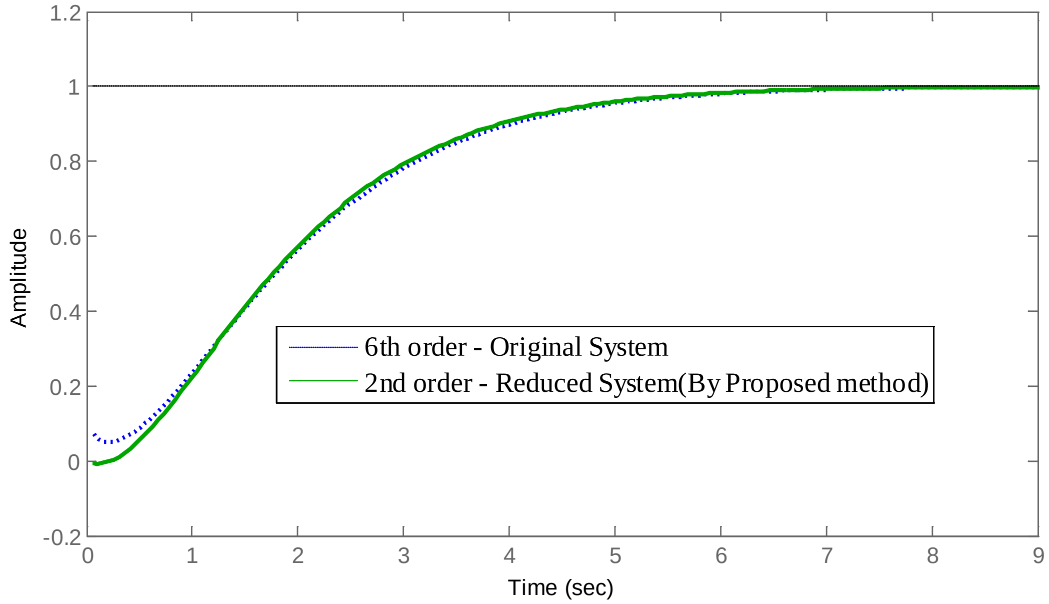

Fig. 2.23 Comparison of step responses for example 2.22

The original, second order reduced systems are subjected to a unit step input and their responses obtained are as shown in Fig. 2.23. The results are compared with other methods in terms of *'I'* and *'J'* (J_{org}= 19.89) in Table 2.25.

Example 2.23: An eighth order system investigated by Shamash [92] as mentioned in example 2.4 in 2.3.1.1 is considered.

The poles of G_n(s) are computed and are grouped into two clusters containing the poles (-1,-2,-3,-4) and (-5,-6,-7,-8). The steps described in 2.3.4 are followed to get the modified pole cluster centers as

p_{e1} = -1.0637 , p_{e2}= -5.1327

The reduced second order system 'G_r(s)' is then obtained as

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{16.51s + 5.46}{s^2 + 6.1964s + 5.4597}$$

The results obtained by proposed method is compared with other methods in terms of *'I'* and *'J'* (J_{org}= 2509.2) are according to Table 2.26. The unit step responses of G_n(s) and G_r(s) are depicted in Fig. 2.24.

Table 2.26 Comparison of reduced order systems for example 2.23

Order Reduction Method	Reduced System $G_r(s)$	ISE 'I'	IRE 'J'
Proposed Method	$\frac{16.51s+5.46}{s^2+6.1964s+5.4597}$	0.014	2321
Chen *et. al.*[238]	$\frac{0.72058\ s + 0.3669}{s^2+0.02768s + 0.3669}$	7.2067	161.23
Gutmen *et. al.*[47]	$\frac{5.35\times10^8\ s + 8.129\times10^8}{8.505\times10^7 s^2+5.523\times10^8 s + 8.129\times10^8}$	1.376	365.05
Hutton and Friedland [45]	$\frac{1.99\ s + 0.4318}{s^2+1.174s + 0.4318}$	1.917	124.08
Krishnamurthy and Seshadri [49]	$\frac{1.557\times10^5\ s + 40320}{65520s^2+75600s + 40320}$	1.6532	180.05
Lucas [52]	$\frac{6.779\ s + 2}{s^2+3s + 2}$	0.27973	629.72
Pal [57]	$\frac{1.518\times10^5\ s +40320}{65520s^2+75600s + 40320}$	1.6509	171.97
Prasad and Pal [234]	$\frac{17.99\ s +500}{s^2+13.25s + 500}$	1.4585	2279.1

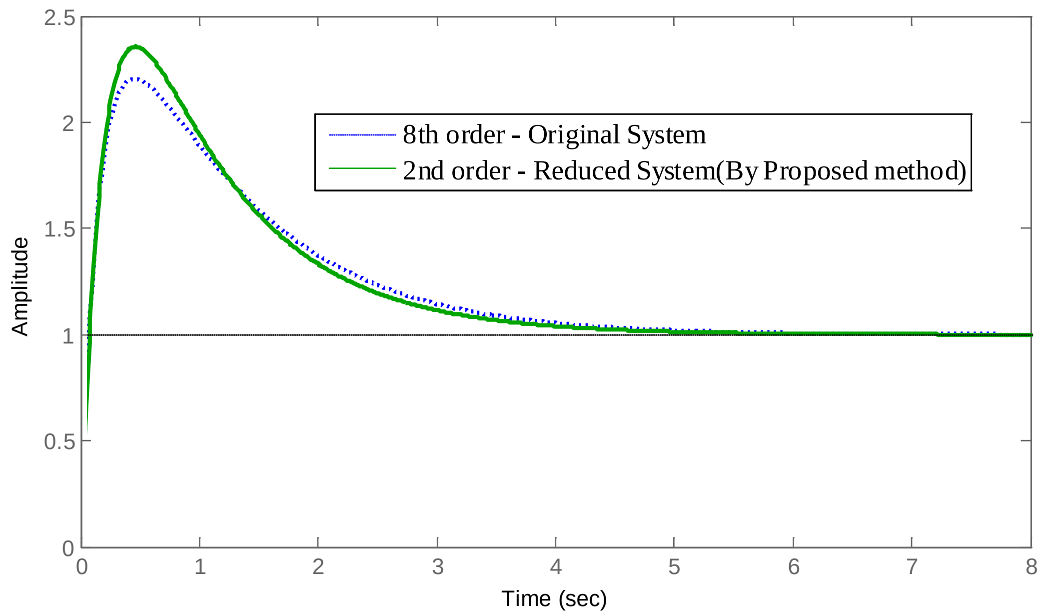

Fig. 2.24 Comparison of step responses for example 2.23

Example 2.24: An ninth order system investigated by Mukherjee *et. al.*[168] is considered for obtaining a second order reduced system. The original system with J_{org}= 28.23 is given by

$$G_n(s) = \frac{s^4 + 35s^3 + 291s^2 + 1093s + 1700}{s^9 + 9s^8 + 66s^7 + 294s^6 + 1029s^5 + 2541s^4 + 4684s^3 + 5856s^2 + 4620s + 1700}$$

The poles of G_n(s) are (-1,-1±j, -1±j2, -1±j3, -1±j4). These poles are grouped into two clusters containing the poles (-1) in the first and remaining poles in the second cluster.. The procedure as in 2.3.4 are followed to get the modified pole cluster centers as

p_{e1} = -1, $p_{e2,3}$ = -1±j1.0637

Since the modified pole cluster comprises of real and complex conjugates case 3 (2.38) is followed to obtain the reduced denominator. The reduced second order system 'G_r(s)' is then obtained as

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{-0.05311s^2 - 0.2907s + 2.132}{s^3 + 3s^2 + 4.132s + 2.132}$$

The responses of G_n(s) and G_r(s) for a unit step input are shown in Fig. 2.25. Table 2.27 compares the results obtained by proposed method in terms of *'I'* and *'J'* with other methods.

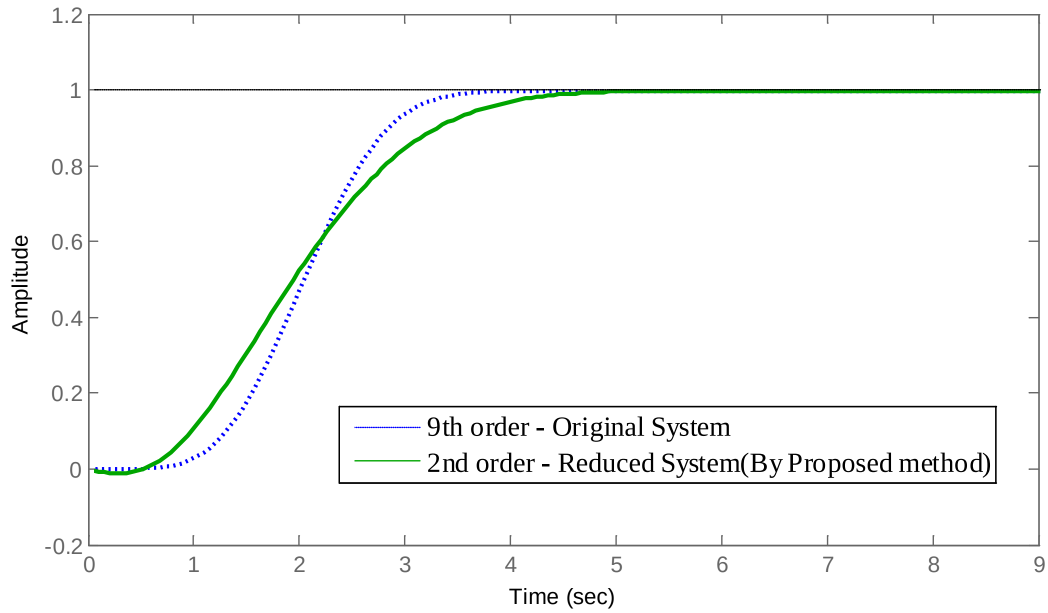

Fig. 2.25 Comparison of step responses for example 2.24

Table 2.27 Comparison of reduced order systems for example 2.24

Order Reduction Method	Reduced System $G_r(s)$	ISE 'I'	IRE 'J'
Proposed Method (second order)	$\frac{-0.05311s^2-0.2907s+2.132}{s^3+3s^2+4.132s+2.132}$	0.0151	19.645
Mukherjee *et. al.* [168] (impulse response matching)	$\frac{0.2945s^2-2.203s+2.32}{s^3+2.5008s^2+4.778s-2.32}$	0.0877	51.01
Mukherjee *et. al.* [168] (step response matching)	$\frac{-3.49s^2-4.14s+2.078}{s^3+3.828s^2+4.884s+2.078}$	0.726	364.36
Chen *et. al.*[50]	$\frac{285s^2+1093s+1700}{3408s^3+5031s^2+4620s+1700}$	0.296	25.43
Phillip and Pal [233]	$\frac{0.5058s^2-1.985s+3.534}{s^3+3s^2+5.534s+3.534}$	0.0282	29.42
George and Rein Method I [163]	$\frac{-0.29913s+0.73912}{s^2+0.95727s+0.73912}$	0.0423	26.03
George and Rein Method II [163]	$\frac{-0.57072s+0.98330}{s^2+1.42381s+0.98330}$	0.0187	28.636

2.3.4.2 Extension to Multivariable systems

The proposed method described in 2.3.4.1 can also be extended for multivariable system according to the procedure mentioned below. The procedure is rather simple and involves direct application of the SISO method on the elements of the transfer function matrix of MIMO system. Consider a system of n^{th} order represented in the form of (2.23) having *'p'* inputs and *'m'* outputs. The proposed reduction procedure is applied to (2.24) by following the steps in 2.3.4. The justification of the proposed method is provided by solving two numerical examples.

2.3.4.2.1 Illustrative Examples

Example 2.25: Consider an aircraft gas turbine [239] taken from example 2.5 in 2.3.1.2.1 is given by

$$[G_n(s)] = \frac{1}{D_n(s)}\begin{bmatrix} A_{11}(s) & A_{12}(s) \\ A_{21}(s) & A_{22}(s) \end{bmatrix}$$

$$A_{11}(s) = 14.96s^2 + 1521.432s + 2543.2$$
$$A_{12}(s) = 95150s^2 + 1132094.7s + 1805947.0$$
$$A_{21}(s) = 85.2s^2 + 8642.688s + 12268.8$$
$$A_{22}(s) = 124000s^2 + 1492588s + 2525880.0$$

and

$$D_n(s) = s^4 + 113.225s^3 + 1357.275s^2 + 3502.75s + 2525$$

It is desired to reduce $[G_n(s)]$ to a second order system represented in the form

$$[R(s)] = \frac{1}{D_r(s)} \begin{bmatrix} B_{11}(s) & B_{12}(s) \\ B_{21}(s) & B_{22}(s) \end{bmatrix}$$

The poles of the original system are found to be (-1.338 ,-1.887, -10, -100) respectively. Forming two clusters having poles (-1.338) and (-1.887, -10, -100) will give

p_{e1} = -1.338, p_{e2} = -2.218

by following the steps in 2.3.4, the reduced denominator will be,

$$D_r(s) = s^2 + 3.56s + 2.9748$$

and $B_{11}(s) = 1.222s + 2.996$, $B_{12}(s) = 928.4s + 2128$, $B_{21}(s) = 7.429s + 14.45$ $B_{22}(s) = 1192s + 2976$

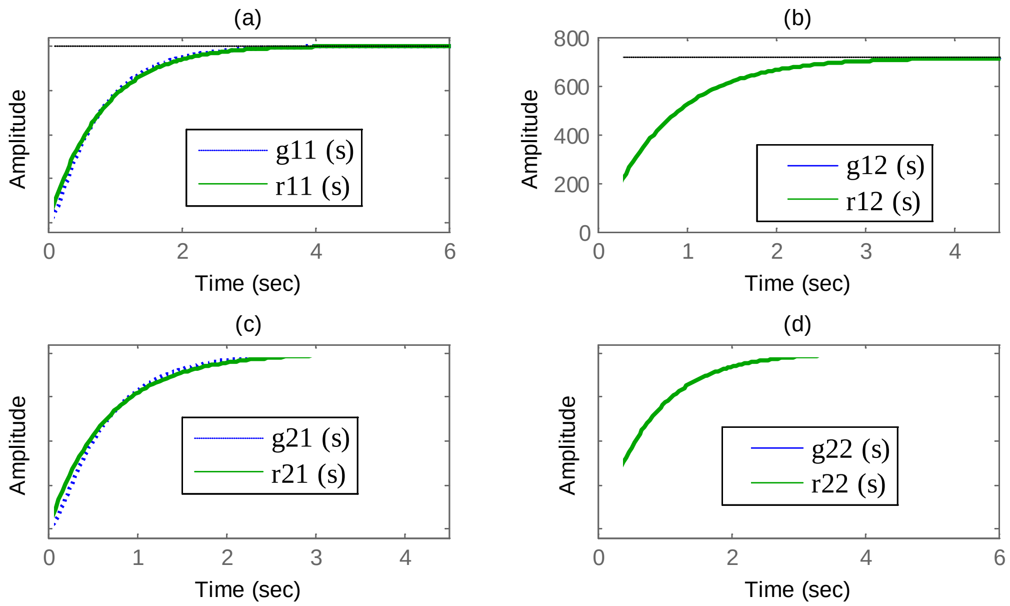

Fig. 2.26 (a)-(d) Comparison of step responses for example 2.25

The results are compared with other method in terms of '*I*' and '*J*' for each element of transfer function matrix are according to Table 2.28. The value J_{org} of each element of plant transfer function matrix are 64.526, 3.054 X 10^7,1735.7 and 5.7 X 10^7 respectively). The unit step responses of $[G_n(s)]$ and $[R(s)]$ are depicted in Fig. 2.26 (a)-(d).

Table 2.28 Comparison of ISE and IRE for example 2.25

r_{ij} (i,j=1,2)	Proposed Method		Prasad [240]	
	ISE *'I'*	IRE *'J'*	ISE *'I'*	IRE *'J'*
r_{11}	0.00163	42.65	0.0028	46.426
r_{12}	11.064	22.78×10^{6}	1064.8	3.47×10^{7}
r_{21}	0.0670	1231.1	0.086	1080
r_{22}	13.98	41.47×10^{6}	27821	6.799×10^{7}

2.4 CONCLUSION

In this chapter four mixed/composite methods of order reduction have been proposed for reducing higher order LTI continuous time systems. These methods are based on least squares method in combination of spectrum analysis, stability equation, dominant pole and modified pole clustering technique. The responses of the reduced systems indicate that the key qualities of the original system are retained and show overall better performance. The methods are simple, rugged and computer viable. Added to this, a stable original system results in stable reduced system. Further these methods are also extended for MIMO systems successfully. The results obtained are compared with other available methods in terms of ISE and IRE. It is observed that the proposed methods are comparable in quality with the other known existing methods.

CHAPTER - 3

REDUCTION OF DISCRETE TIME SYSTEMS USING NEW COMPOSITE METHODS

The world has seen tremendous development in the area of approximation algorithms and metaheuristics during the past decades. Today, metaheuristics is being recognized as an essential research field [245]. However, efforts are on to cope up with problems arising from the fast advancing world. Order reduction is one such field, which seeks for more efficient lower order approximation algorithm, so as to satisfy the need of the hour. Moreover, in this digital era, usage of digital processors in the design and implementation of control systems has become inevitable and can be termed of paramount importance. Also, there is a strong belief that there is an enormous potential and future for their use. Thus, resulting in the need for more model order reduction methods in discrete domain. In the preceeding chapter, systems in continues time were considered and some new order reduction methods were also discussed. In the present chapter, higher order systems represented in discrete domain are taken up for order reduction.

The order reduction methods proposed in chapter 2 are valid for continuous time systems represented in frequency domain. Here, the same methods are extended for order reduction of discrete time systems. In order to do so, initially the original system in z-domain has to be transformed to *w*-domain. Then the reduction technique is applied and finally converted back into z-domain. Many continuous time reduction methods have been successfully extended, to reduce discrete systems either by using bilinear transformation ($z=(1+w)/(1-w)$), where 'w' is a new variable or linear transformation ($z = p+1$), where 'p' is a new variable or homographic transformation ($z= p/(A+Bp)$), where A,B are constants [246-248]. In the proposed methods, both bilinear transformation as well as linear transformation are used to meet the objective. Numerical examples belonging to SISO, are taken up for reduction initially and are later extended for MIMO systems.

3.1 PROBLEM STATEMENT

Let the n^{th} order discrete-time system be represented in the frequency domain as

$$G_n(z) = \frac{N_n(z)}{D_n(z)} = \frac{a_o + a_1 z + \ldots + a_m z^m}{b_o + b_1 z + b_2 z^2 + \ldots + b_n z^n}; \quad m < n \tag{3.1}$$

where a_i's and b_i's are the scalar constants. The r^{th} order (r < n) reduced system is to be derived from (3.1) comprising of scalar constants d_j's and e_j's represented in the form of

$$G_r(z) = \frac{N_r(z)}{D_r(z)} = \frac{d_o + d_1 z + ... + d_p z^p}{e_o + e_1 z + e_2 z^2 + ... + e_r z^r}; \quad p < r \tag{3.2}$$

provided that the reduced system (3.2) also exhibit almost the same characteristics as that of (3.1) on the application of the same input.

As mentioned earlier, the proposed mixed methods for reducing discrete systems consists of converting a given system in z- domain (3.1) either to *w*-domain or *p*-domain and vice-versa.This issue is resolved by using bilinear or linear transformation.

Bilinear transformation method converts a given system in *z*-domain to *w*-domain. This is achieved by substituting $z=(1+w)/(1-w)$, both in the numerator and denominator polynomial separately using synthetic division [249] to obtain

$$\left.\begin{aligned} N_n(w) &= N_n(z)\big|_{z=(1+w)/(1-w)} = \frac{N_n(w)}{(1-w)^{n-1}} = 0 \\ D_n(w) &= D_n(z)\big|_{z=(1+w)/(1-w)} = \frac{D_n(w)}{(1-w)^{n}} = 0 \end{aligned}\right\} \tag{3.3}$$

thus

$$\begin{aligned} G_n(w) &= \frac{N_n(w)}{D_n(w)} \\ &= \frac{a_{10} + a_{11} w + ... + a_{1n-1} w^{n-1}}{b_{10} + b_{11} w + b_{12} w^2 + ... + b_{1n} w^n} \end{aligned} \tag{3.4}$$

The rank in (3.4) remains the same as that of (3.1), hence matching the response of the reduced system (3.2) with that of (3.1) for a given step input at t =0. Further, G_n(w) can be reduced using the proposed reduction methods and can be converted back to the form (3.2), using inverse bilinear transformation. This transformation comprises of substituting $w=(z-1)/(z+1)$, separately for numerator and denominator polynomials as given below.

$$\left.\begin{aligned} N_r(z) &= N_r(w)\big|_{w=(z-1)/(z+1)} = \frac{N_r(z)}{(z+1)^{r-1}} = 0 \\ D_r(z) &= D_r(w)\big|_{w=(z-1)/(z+1)} = \frac{D_r(z)}{(z+1)^{r}} = 0 \end{aligned}\right\} \tag{3.5}$$

$$G_r(z) = \frac{N_r(z)}{D_r(z)} \tag{3.6}$$

However, some continuous time reduction methods via bilinear transformation may sometimes yield poor approximations in case of discrete time systems [247, 250]. This kind of unwanted quality of approximation is due to the deformation taking place during bilinear transformations. This can be reduced by using linear transformation where the system (3.1) is converted to *p*-domain by substituting *z=p+1* resulting in

$$G_n(p) = \frac{N_n(p)}{D_n(p)} = \left.\frac{N_n(z)}{D_n(z)}\right|_{z=p+1} = \frac{a_{1o} + a_{11}p + ... + a_{1n-1}p^{n-1}}{b_{1o} + b_{11}p + b_{12}p^2 + ... + p^n} \tag{3.7}$$

The reduced system in the form (3.2) is obtained by reducing (3.7) using reduction techniques. The reduced system in *p*-domain is converted to *z*-domain by substituting *p=z-1*

$$G_r(z) = \frac{N_r(z)}{D_r(z)} = \left.\frac{N_r(p)}{D_r(p)}\right|_{p=z-1} \tag{3.8}$$

3.2 EIGEN SPECTRUM ANALYSIS AND LEAST SQUARES METHOD

The composite reduction method presented here, is a combination of ESA [157] and least squares method [174]. ESA is used for finding the new poles, whereas least squares method is used for deriving the numerator polynomials. The procedure for obtaining the reduced system using the proposed method is as follows.

Step 1: Consider G_n(z) (3.1) and convert using linear transformation using (3.7). Find roots of the denominator polynomial D_n(p) such that $-p'_1 < -p'_2 < ... < -p'_n$.

Step 2: Locate the Eigen Spectrum Points (ESP) of G_n(p).

Step 3: Follow the steps 3 to 5 of section 2.3.1.

Step 4: Calculate ESP from Rep_r', *M* and form the denominator polynomial D_r(p) of G_r(p). Let these denominator coefficients be 'e_i', $i = 0,1, 2 , ... \infty$.

Step 5: Determine of numerator polynomial N_r(p) of G_r(p)

Compute the time moment proportional's 'c_i' by expanding G_n(p) about p=0 using

$$G_n(p) = \sum_{i=0}^{\infty} c_i\, p^i \tag{3.9}$$

Substitute the value of 'e_i', $i = 0,1, 2 , ... \infty$ and 'c_i' in (2.8) and (2.9) to obtain the coefficients 'd_i', $i = 0,1, 2 , ...(r-1)$.

Step 6: Then numerator 'N_r(p)' and denominator polynomial 'D_r(p)' is substituted in (3.8) and G_r(z) is obtained by substituting *p=z-1*.

3.2.1 Illustrative examples

The proposed method is justified by solving the following numerical examples choosen from the literature. The first example is solved in detail according to the steps described in 3.2. In the remaining examples the reduced system is mentioned directly. The goodness/quality of the reduced system is judged by measuring Summation Square Error SSE [251]. The SSE value is a measure of the closeness between $G_n(z)$ and $G_r(z)$. The SSE value is calculated by using the formula

$$SSE = \sum_{k=0}^{N} [y(k) - y_r(k)]^2 \tag{3.10}$$

where $y(k)$, $y_r(k)$ are the outputs of $G_n(z)$ and $G_r(z)$ respectively at the kth sampling instant t_k. N is the number of sampling instances.

Example 3.1: A supersonic inlet model transfer function from Lalonde *et. al.*[176]

$$G_n(z) = \frac{N_n(z)}{D_n(z)} = \frac{2.0434z^6 - 4.982z^5 + 6.57z^4 - 5.819z^3 + 3.636z^2 - 1.41z + 0.2997}{z^7 - 2.46z^6 + 3.433z^5 - 3.33z^4 + 2.546z^3 - 1.584z^2 + 0.7478z - 0.252}$$

Step 1: Consider $G_n(z)$ and convert to p-domain using (3.7) resulting in

$$G_n(p) = \frac{N_n(p)}{D_n(p)}$$

$$= \frac{20430p^6 + 72760p^5 + 123055p^4 + 115011p^3 + 64240p^2 + 20330p + 3377}{10000p^7 + 45400p^6 + 96730p^5 + 119355p^4 + 93560p^3 + 45040p^2 + 13028p + 1008}$$

Step 2: The ESP of $G_n(p)$ are p'_1 = -0.1119, $p'_{2,3}$ = -0.3132 ±j0.5843, $p'_{4,5}$ = -0.7024±j0.7559 and $p'_{6,7}$ = -1.1984±j0.6989.

Step 3: The pole centroid is calculated using (2.14) as

$$\sigma_p = \frac{|-0.1119| + |-0.3132| + |-0.3132| + |-0.7024| + |-0.7024| + |-1.1984| + |-1.1984|}{7}$$

$$= 0.6486$$

Step 4: The system stiffness is

$$k = \frac{0.1119}{1.1984} = 0.0933 = k'$$

Step 5: For a second order reduced system, we have r =2, $Q=0$; $N = (\sigma_p\, r)$ = (0.6486×2) =1.2972.

Now, substitute the values of k, r, N, Q in (2.20)

$$\begin{bmatrix} 0.0933(2-1) & 0 \\ (1-0.0933) & (1-2) \end{bmatrix} \begin{bmatrix} \operatorname{Re} p'_r \\ M \end{bmatrix} = \begin{bmatrix} 1.2972 \\ 0 \end{bmatrix}$$

$$or \begin{bmatrix} \mathrm{Re}\, p_r^{'} \\ M \end{bmatrix} = \begin{bmatrix} 1.1864 \\ 1.2983 \end{bmatrix}$$

$$or \ p_1^{'} = -1.1864 \ and \ p_2^{'} = -0.1109$$

Step 6: Then, the denominator polynomial of G_r(p) will be

$$D_r(p) = (p + p_1^{'})(p + p_2^{'})$$
$$= p^2 + 1.297p + 0.1314$$

Step 7: The first five time moment proportional's c_i are obtained using (3.9) and is given in Table 3.1

Table 3.1 Time moment proportionals obtained for example 3.1

i	c_i
0	3.35019841269841
1	-23.1313342466616
2	212.99810124759
3	-1916.20786912305
4	17121.3108524937

Step 8: The numerator coefficients of G_r(p) are computed by substituting coefficients of 'D_r(p)' and c_i in (2.8) resulting in

$$N_r(p) = 1.306p + 0.4402$$

The second order reduced system in *p*-domain is

$$G_r^2(p) = \frac{N_r(p)}{D_r(p)} = \frac{1.306p + 0.4402}{p^2 + 1.297p + 0.1314}$$

Step 9: Convert the reduced system G_r(p) to G_r(z) by substituting $p=(z-1)$ to give

$$G_r^2(z) = \frac{N_r(z)}{D_r(z)} = \frac{6530z - 4329}{5000z^2 - 3515z - 828}$$
$$= \frac{1.306(z - 0.663)}{z^2 - 0.703z - 0.1656}$$

Similarly, third and fifth reduced order systems thus obtained are

$$G_r^3(z) = \frac{21850z^2 - 30840z + 11848}{10000z^3 - 10540z^2 + 808z + 585}$$
$$= \frac{2.1850(z^2 - 1.4114z + 0.5422)}{z^3 - 1.0540z^2 + 0.0808z + 0.0585}$$

$$G_r^5(z) = \frac{4227z^4 + 31602z^3 - 93338z^2 - 82951z - 24497}{10000z^5 - 17570z^4 + 8730z^3 - 520z^2 - 375z + 32}$$

$$= \frac{0.4227z^4 + 3.157z^3 - 9.33z^2 - 8.29z - 2.447}{z^5 - 1.76z^4 + 0.87z^3 - 0.05z^2 - 0.037z + 0.0032}$$

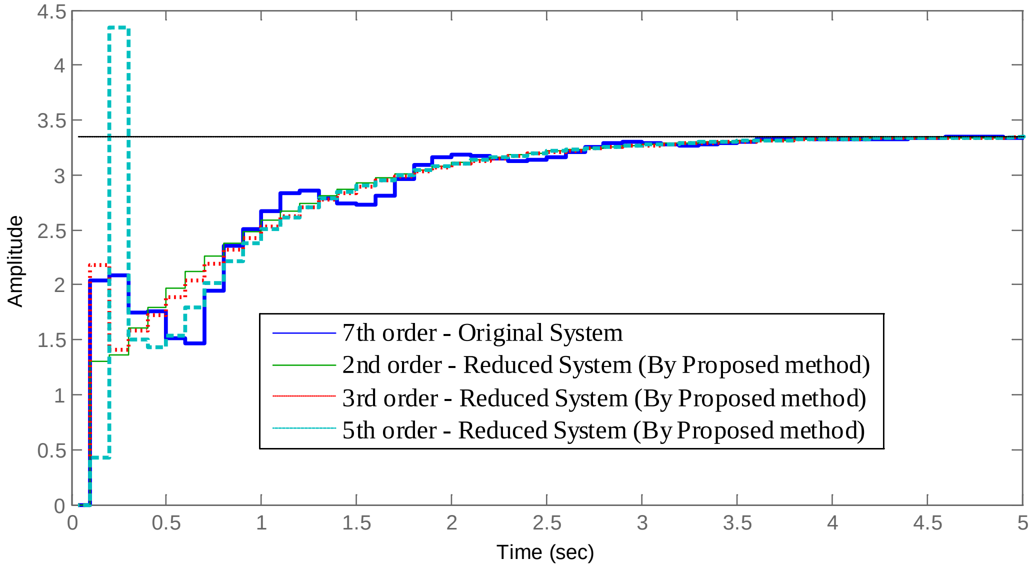

Fig. 3.1(a) Comparison of step responses for example 3.1

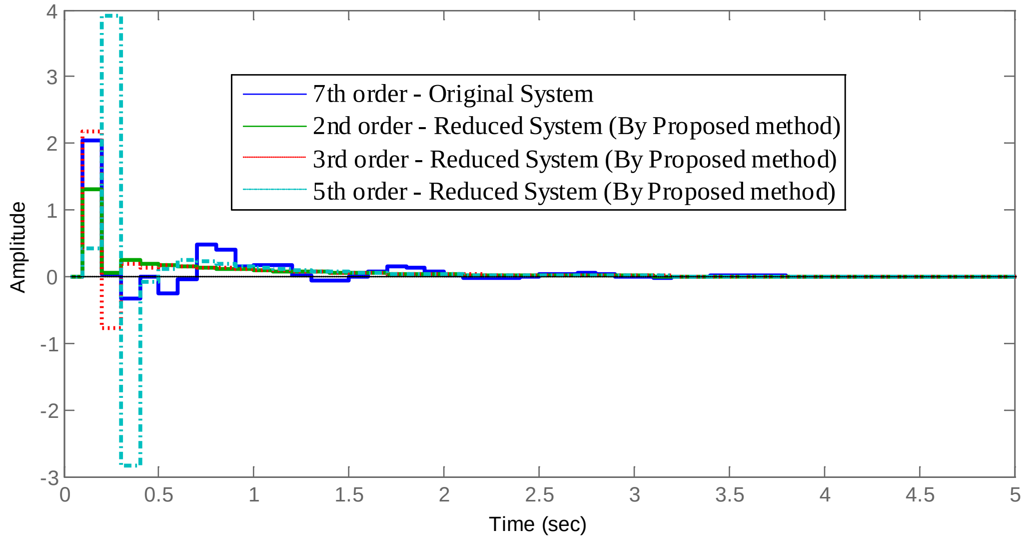

Fig. 3.1(b) Comparison of impulse responses for example 3.1

The original and reduced systems are subjected to step, impulse input and their corresponding responses are shown in Fig. 3.1(a)-(b). These responses are compared in Table 3.2 with respect to SSE given by (3.10) with other available methods.

Table 3.2 Comparison of reduced order systems for example 3.1

Order Reduction Method	Reduced System $G_r(z)$	SSE
Proposed Method (second order)	$\frac{1.306z - 0.866}{z^2 - 0.703z - 0.1656}$	0.2039
Proposed Method (third order)	$\frac{2.1850z^2 - 3.083z + 1.1847}{z^3 - 1.0540z^2 + 0.0808z + 0.0585}$	0.1594
Proposed Method (fifth order)	$\frac{0.4227z^4 + 3.157z^3 - 9.33z^2 - 8.29z - 2.447}{z^5 - 1.76z^4 + 0.87z^3 - 0.05z^2 - 0.037z + 0.0032}$	0.14662
Lalonde *et. al.*[176] (third order)	$\frac{0.0627z^3 - 2.106z^2 + 1.569z + 0.0371}{z^3 - 0.8204z^2 + 0.1697z - 0.1648}$	0.55879
Lalonde *et. al.*[176] (fifth order)	$\frac{1.72\times10^{-4}z^5 + 2.1z^4 - 2.9z^3 + 2.15z^2 - 1.5z - 0.66}{z^5 - 1.488z^4 + 1.231z^3 - 0.96z^2 - 0.6693z - 0.3247}$	0.17453

Example 3.2: An eighth order discrete time system is considered in this exapmle .[252].

$$G_n(z) = \frac{N_n(z)}{D_n(z)}$$

$$= \frac{280.3z^7 + 186z^6 - 35z^5 + 25.33z^4 - 86z^3 - 43.66z^2 + 7.33z - 1}{666.7z^8 - 280.3z^7 - 186z^6 + 35z^5 - 25.33z^4 + 86z^3 + 43.66z^2 - 7.33z + 1}$$

Converting G_n(z) to *p*-domain using (3.7)

$$G_n(p) = \frac{N_n(p)}{D_n(p)}$$

$$= \frac{28030p^7 + 214811p^6 + 696733p^5 + 1.245\times10^6 p^4 + 1.32\times10^6 p^3 + 817666p^2 + 266644p + 33330}{66670p^8 + 505333p^7 + 1.652\times10^6 p^6 + 3.037\times10^6 p^5 + 3.422\times10^6 p^4 + 2.414\times10^6 p^3 + 1.049\times10^6 p^2 + 266722p + 33340}$$

The ESP of G_n(p) are $p'_{1,2}$ = -0.2491±j0.3068, $p'_{3,4}$ = -0.9072±j 0.1248, $p'_{5,6}$ = -1.0577 ±j 0.6323 and $p'_{7,8}$ = -1.5758 ±j 0.1758. A second order reduced system is desired and the steps described in 3.2 are followed to obtain

$$G_r(p) = \frac{N_r(p)}{D_r(p)} = \frac{1.889p + 0.4199}{p^2 + 1.89p + 0.42}$$

or $$G_r^2(z) = \frac{N_r(z)}{D_r(z)} = \frac{1.889z - 1.469}{z^2 - 0.11z - 0.47}$$

Similarly, the third order reduced system obtained is

$$G_r^3(z) = \frac{N_r(z)}{D_r(z)}$$
$$= \frac{0.06217z^2 + 2.094z - 1.755}{z^3 - 0.158z^2 - 0.466z + 0.0249}$$

The step and impulse responses of the original and reduced system are shown in Fig. 3.2(a)-(b). These responses are compared in Table 3.3 in terms of SSE with other available methods.

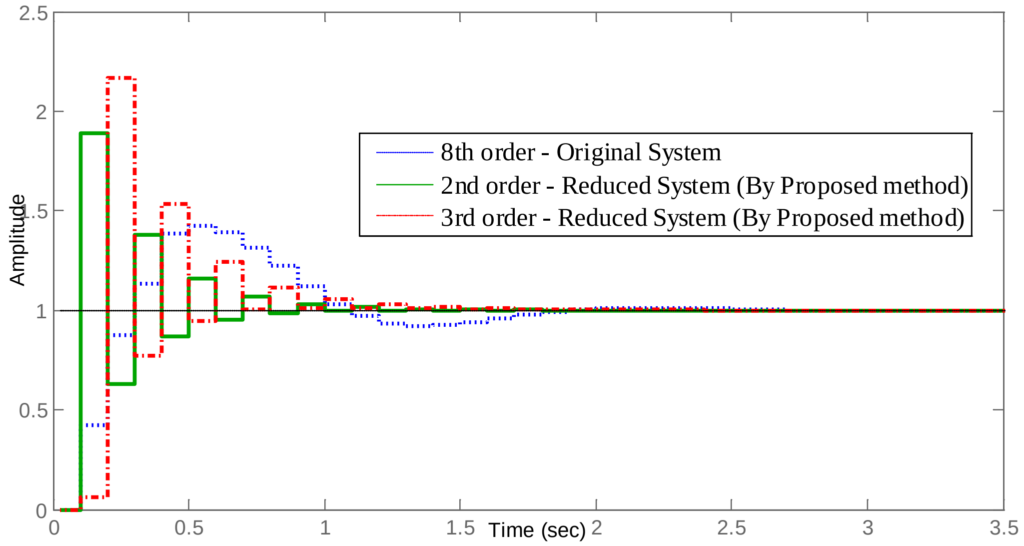

Fig. 3.2 (a) Comparison of step responses for example 3.2

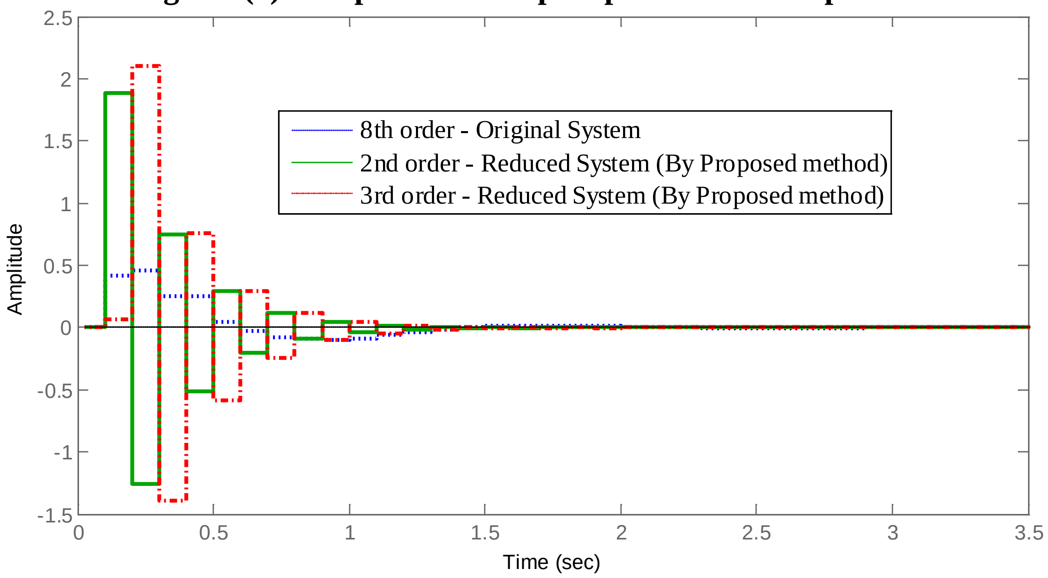

Fig. 3.2 (b) Comparison of impulse responses for example 3.2

Table 3.3 Comparison of reduced order systems for example 3.2

Order Reduction Method	Reduced System G_r(z)	SSE
Proposed Method (second order)	$\frac{1.889z-1.469}{z^2-0.11z-0.47}$	0.232
Proposed Method (third order)	$\frac{0.06217z^2+2.094z-1.755}{z^3-0.158z^2-0.466z+0.0249}$	0.195
Chung *et. al.*[252]	$\frac{0.3975z-0.318}{z^2-1.6025z+0.682}$	1.336
Satakshi *et. al.*[251]	$\frac{0.463z+0.30172}{z^2-1.5312z+0.6868}$	0.1466
Prasad [240] (using 2 TM)	$\frac{0.5282z-0.4225}{z^2-1.472z+0.577}$	1.176
Prasad [240] (using one TM and MP)	$\frac{0.420499z-0.31486}{z^2-1.472z+0.577}$	0.4238

Example 3.3: A discrete time system of fifth order [50] is considered

$$G_n\left(z\right) = \frac{3z^4-8.886z^3+10.0221z^2-5.091975z+0.9811125}{z^5-3.7z^4+5.47z^3-4.037z^2+1.4856z-0.2173}$$

Converting G_n(z) to p-domain using (3.7)

$$G_n\left(p\right) = \frac{N_n(p)}{D_n(p)}$$

$$= \frac{30000p^4+31140p^3+13620p^2+2900p+231}{10000p^5+13000p^4+6700p^3+1730p^2+220p+17}$$

It is desired to have a reduced system of second order. The steps described in 3.2 are followed resulting in

$$G_r(p) = \frac{N_r(p)}{D_r(p)} = \frac{6.909p+0.4076}{p^2+0.5201p+0.03}$$

or

$$G_r^2(z) = \frac{N_r(z)}{D_r(z)} = \frac{1.167\times10^5z-1.098\times10^5}{10000z^2-14799z+5099}$$

$$= \frac{11.67z-\ 10.98}{z^2-1.4799z+0.5099}$$

The step and impulse responses of the original and reduced system are shown in Fig. 3.3(a)-(b). These responses are compared in Table 3.4 in terms of SSE with other available methods.

Table 3.4 Comparison of reduced order systems for example 3.3

Order Reduction Method	Reduced System $G_r(z)$	SSE
Proposed Method	$\dfrac{11.67z - 10.98}{z^2 - 1.4799z - 0.5099}$	109.69
Chen *et. al.* [50]	$\dfrac{0.008634z - 0.003272}{0.6636z^2 - 1.265z + 0.6066}$	109.79
Prasad [240] (using stability equation and continued fraction)	$\dfrac{1.68157z - 1.587947}{z^2 - 1.8542z - 0.86384}$	178.11

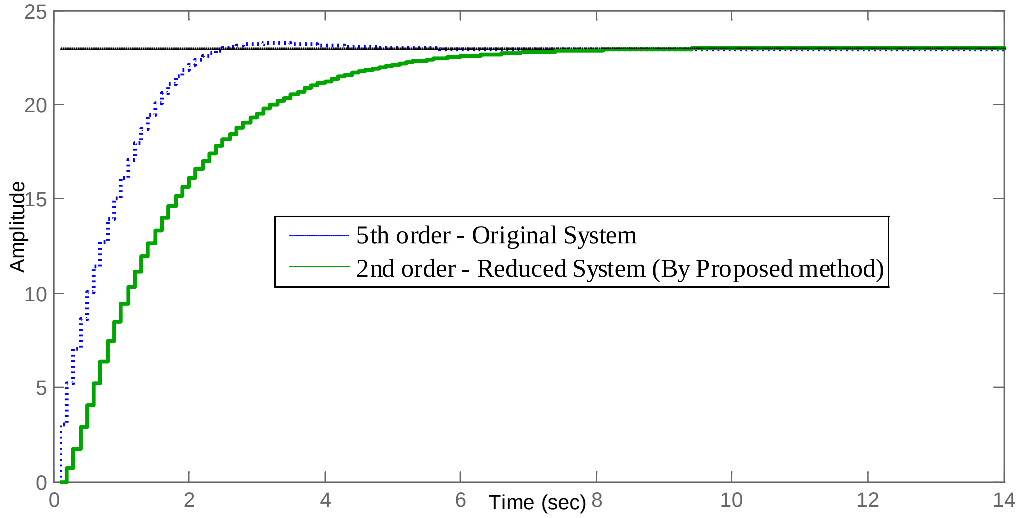

Fig. 3.3 (a) Comparison of step responses for example 3.3

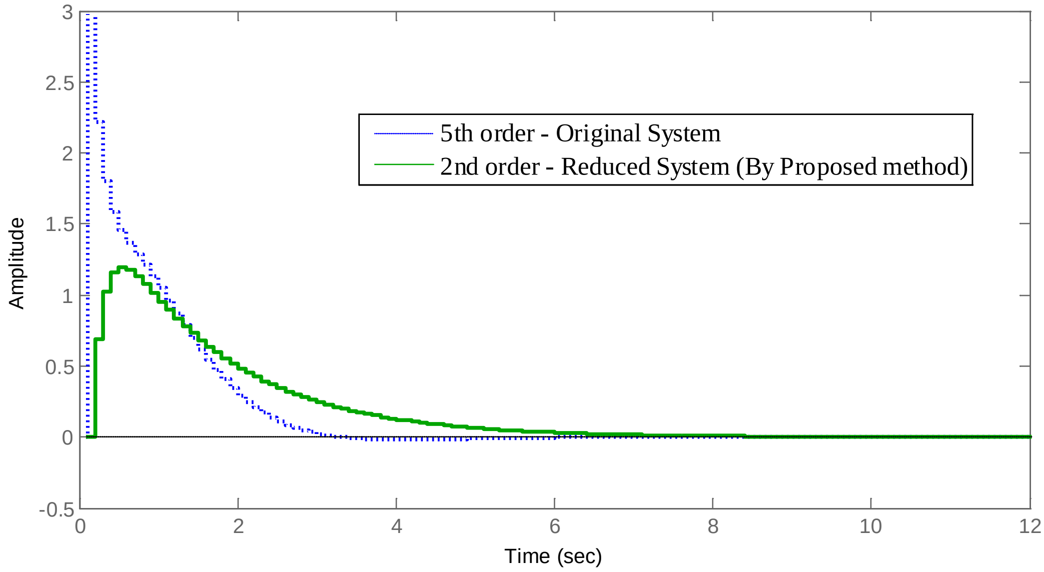

Fig. 3.3 (b) Comparison of impulse responses for example 3.3

Example 3.4: Consider eighth order discrete transfer function taken from Bistritz [253]

$$G_n(z) = \frac{N_n(z)}{D_n(z)}$$

$$= \frac{1.68z^7 + 1.116z^6 - 0.21z^5 + 0.152z^4 - 0.516z^3 - 0.262z^2 + 0.044z - 0.006}{8z^8 - 5.046z^7 - 3.348z^6 + 0.63z^5 - 0.456z^4 + 1.548z^3 + 0.786z^2 - 0.132z + 0.018}$$

using (3.7)

$$G_n(p) = \frac{N_n(p)}{D_n(p)}$$

$$= \frac{840p^7 + 6438p^6 + 20883p^5 + 37321p^4 + 39556p^3 + 24773p^2 + 8517p + 1261}{4000p^8 + 29477p^7 + 92665p^6 + 161299p^5 + 167933p^4 + 105233p^3 + 38404p^2 + 8000p + 1000}$$

following the steps in 3.1 are

$$G_r(p) = \frac{N_r(p)}{D_r(p)} = \frac{1.139p + 0.9483}{p^2 + 1.84p + 0.752}$$

or $$G_r^2(z) = \frac{N_r(z)}{D_r(z)} = \frac{0.3731\, z - 0.2985}{z^2 - 1.6268z - 0.702}$$

Table 3.5 Comparison of reduced order systems for example 3.4

Order Reduction Method	Reduced System $G_r(z)$	SSE
Proposed Method	$\frac{0.3731z - 0.2985}{z^2 - 1.6268z - 0.702}$	0.032
Hwang *et. al.* [254]	$\frac{0.316331z - 0.262395}{z^2 - 1.73034z - 0.784276}$	0.062
Bistritz [253]	$\frac{0.2696z - 0.2157}{z^2 - 1.73z - 0.7842}$	0.052
Bistritz [132]	$\frac{0.37131242z - 0.298}{z^2 - 1.626873z - 0.701497}$	0.057
Hwang *et. al.* [254]	$\frac{0.3664429z - 0.28918}{z^2 - 1.626873z - 0.701497}$	0.065
Hwang and Shih [248]	$\frac{0.2018z^2 + 0.04484z - 0.156}{1.2z^2 - 1.955z + 0.843}$	0.0791

The reaction curves and impulse responses of the original, reduced system are shown in Fig. 3.4(a)-(b). The results obtained are compared in terms of SSE with other available methods in Table 3.5 and is comparable.

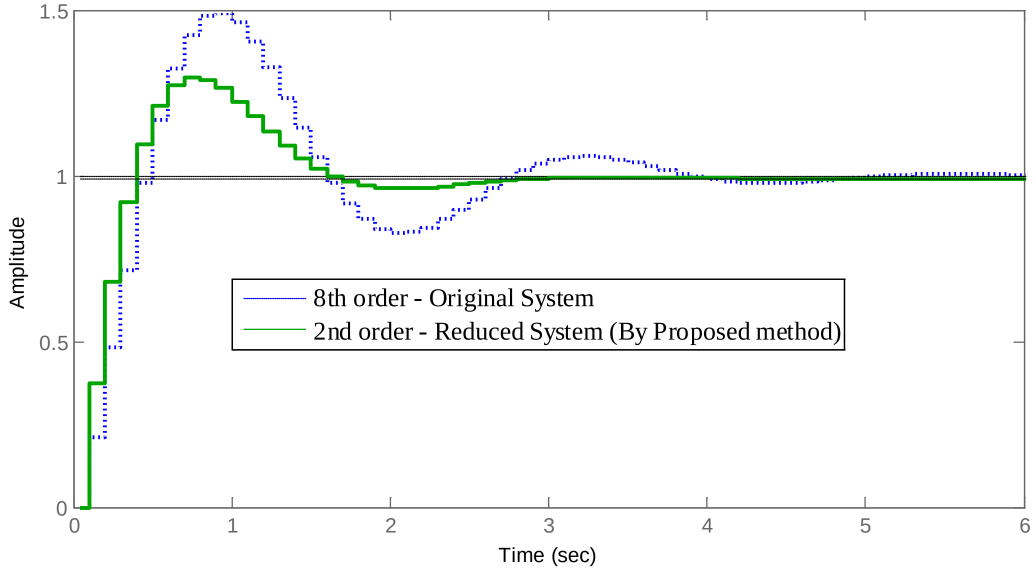

Fig. 3.4 (a) Comparison of step responses for example 3.4

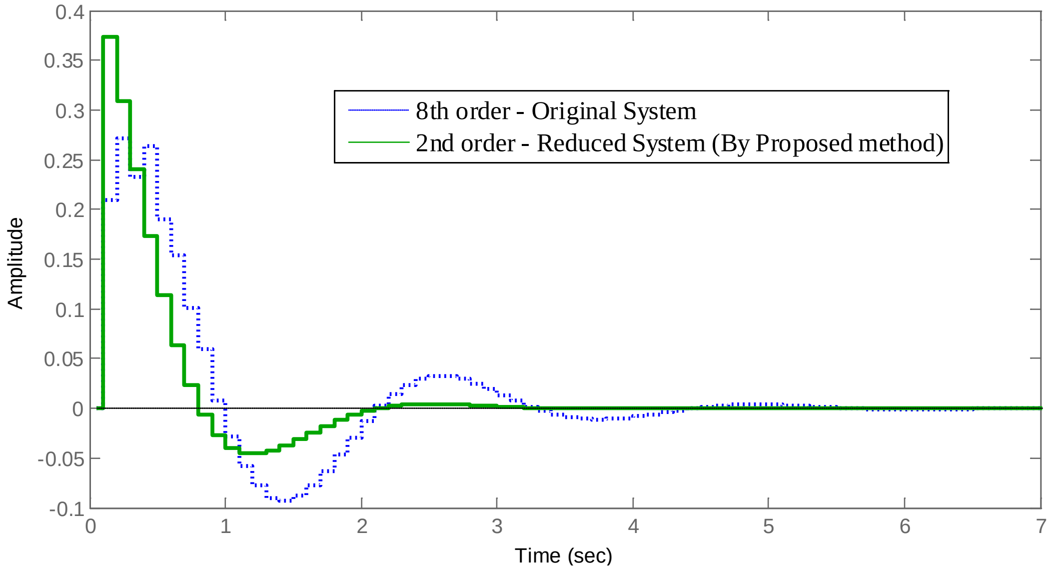

Fig. 3.4 (b) Comparison of impulse responses for example 3.4

In this proposed method of reducing discrete time systems, the overall performance of the reduced system is comparable with other methods of reduction. The comparison tables, step and impulse responses mentioned justifies the same. In example 3.1, the proposed fifth order reduced system performs better as compared to its counterparts.

3.3 STABILITY EQUATION AND LEAST SQUARES METHOD

The advantages of stability equation (SE) method [50] is reaped, in combination with least squares method [174, 175, 231], to obtain the reduced order discrete system. Initially, the denominator polynomial of the higher order system in *z*-domain, is converted to *p*-domain and is then reduced using SE. Similarly, the numerator terms are obtained using least squares method. The procedural steps to be followed are similar to (2.27) to (2.30). The results obtained using the proposed mixed method are as shown in the following solved examples. Further, the method is also extended for the order reduction of discrete time systems having multiple inputs and multiple outputs.

3.3.1 Illustrative examples

The detailed reduction procedure using the proposed mixed method is described in example 1, where as in the later consecutive examples the reduced system are mentioned directly. *SSE* [251] using (3.10) is calculated to measure the goodness/quality of the reduced system.

Example 3.5: Consider a discrete system taken from example 3 in 3.2.1

$$G_n(z) = \frac{3z^4 - 8.886z^3 + 10.0221z^2 - 5.091975z + 0.9811125}{z^5 - 3.7z^4 + 5.47z^3 - 4.037z^2 + 1.4856z - 0.2173}$$

Step 1: The fifth order discrete system G_n(z) is converted using bilinear transformation by substituting $z=(1+w)/(1-w)$ both in the numerator and denominator polynomial according to (3.3)

$$N_n(w) = N_n(z)|_{z=(1+w)/(1-w)}$$
$$= 3z^4 - 8.886z^3 + 10.0221z^2 - 5.091975z + 0.9811125\,|_{z=(1+w)/(1-w)}$$
$$= 3\frac{(1+w)^4}{(1-w)^4} - 8.886\frac{(1+w)^3}{(1-w)^3} + 10.0221\frac{(1+w)^2}{(1-w)^2} - 5.091975\frac{(1+w)}{(1-w)} + 0.9811125$$

$$N_n(w) = \frac{\begin{array}{c}3(1+w)^4 - 8.886(1+w)^3(1-w) + 10.0221(1+w)^2(1-w)^2 \\ -5.091975(1+w)(1-w)^3 + 0.9811125(1-w)^4\end{array}}{(1-w)^4} = 0$$

$$N_n(w) = 27.9791w^4 + 15.6636w^3 + 3.8466w^2 + 0.4876w + 0.0231$$

$$D_n(w) = D_n(z)|_{z=(1+w)/(1-w)}$$
$$= z^5 - 3.7z^4 + 5.47z^3 - 4.037z^2 + 1.4856z - 0.2173\Big|_{z=(1+w)/(1-w)}$$
$$= \frac{(1+w)^5}{(1-w)^5} - 3.7\frac{(1+w)^4}{(1-w)^4} + 5.47\frac{(1+w)^3}{(1-w)^3} - 4.037\frac{(1+w)^2}{(1-w)^2} + 1.4856\frac{(1+w)}{(1-w)} - 0.2173$$

$$= \frac{\begin{array}{c}(1+w)^5 - 3.7(1+w)^4(1-w) + 5.47(1+w)^3(1-w)^2 - 4.037(1+w)^2(1-w)^3 \\ +1.4856(1+w)(1-w)^4 - 0.2173(1-w)^5\end{array}}{(1-w)^5} = 0$$

$$D_n(w) = 15.9103w^5 + 11.9885w^4 + 3.531w^3 + 0.533w^2 + 0.0355w + 0.0017$$

Step 2:using (3.4)

$$G_n(w) = \frac{N_n(w)}{D_n(w)} = \frac{27.9791w^4 + 15.6636w^3 + 3.8466w^2 + 0.4876w + 0.0231}{15.9103w^5 + 11.9885w^4 + 3.531w^3 + 0.533w^2 + 0.0355w + 0.0017}$$

Step 3: Bifurcate D_n(w) into odd and even parts

$$D_n(w) = D_o(w) + D_e(w)$$
$$D_o(w) = 15.9103w^5 + 3.531w^3 + 0.0355w$$
$$D_e(w) = 11.9885w^4 + +0.533w^2 + +0.0017$$

Step 4: From D_e(w) and D_o(w), following the tabular approach [126], the D_{re}(w) and D_{ro}(w) are computed as shown in the Table 3.6 and 3.7. Let Q_m be the ratio of the successive elements in the first columns of the following Table 3.6 and 3.7 respectively.

Table 3.6 Even degree alternates

Q_m	*Even degree alternates*		
	11.9885	0.533	0.0017
0.044	0.533	0.0017	
0.041	0.4947	0.0017	
0.041	0.4918	0.0017	(2nd)

Table 3.7 Odd degree alternates

Q_m	*Odd degree alternates*		
	15.9103	3.531	0.0355
0.221	3.531	0.0355	
0.211	3.371	0.0355	
0.211	3.363	0.0355	(2nd)

Therefore D_r(w) are

$$D_{re}(w) = 0.4918w^2 + 0.0017$$
$$D_{ro}(w) = 3.363w^3 + 0.0355w$$
$$D_r(w) = D_{re}(w) + D_{ro}(w)$$
$$= 0.4918w^2 + 0.0355w + 0.0017$$
$$D_r(w) = w^2 + 0.072w + 0.00346$$

Step 5: In order to find the numerator terms of N_r(w), the time moment proportional's 'c_i' are computed by expanding G_n(w) about w=0 by using

$$G_n(w) = \sum_{i=0}^{\infty} c_i w^i \tag{3.11}$$

The first five time moment proportionals of G_n(w) are tabulated in Table 3.8

Table 3.8 Time moment proportionals obtained for example 3.5

i	c_i
0	13.5882352941176
1	3.06920415224912
2	-2061.69774068797
3	23081.1324457322
4	78672.8678106316

Substitute the value of ‘e_i’, $i = 0,1, 2, \ldots \infty$ and ‘c_i’ in (2.8) and (2.9) to obtain the coefficients ‘d_i’, $i = 0,1, 2, \ldots(r-1)$.

Step 6: N_r(w) is

$$N_r(w) = 0.989w + 0.04702$$

Step 7: The ratio of N_r(w), D_r(w) is G_r(w).

$$G_r(w) = \frac{N_r(w)}{D_r(w)} = \frac{0.989w + 0.04702}{w^2 + 0.072w + 0.00346}$$

Step 8: Applying inverse bilinear transformation using (3.3), the reduced order discrete time system G_r(z) is obtained as (3.4).

$$N_r(z) = N_r(w)|_{w=(z-1)/(z+1)}$$

$$= 0.989w + 0.04702 \,|_{w=(z-1)/(z+1)}$$

$$= 0.989\frac{(z-1)}{(z+1)} + 0.04702 = 0$$

$$N_r(z) = 0.989(z-1) + 0.04702(z+1)$$

$$D_r(z) = D_r(w)|_{w=(z-1)/(z+1)}$$

$$= w^2 + 0.072w + 0.00346\Big|_{w=(z-1)/(z+1)}$$

$$= \frac{(z-1)^2}{(z+1)^2} + 0.072\frac{(z-1)}{(z+1)} + 0.00346 \quad = 0$$

$$D_r(z) = (z-1)^2 + 0.072(z-1)(z+1) + 0.00346(z+1)^2$$

the second order reduced discrete system is

$$G_r^2(z) = \frac{N_r(z)}{D_r(z)}$$

$$= \frac{1.474\times10^5 z - 1.344\times10^5}{53773z^2 - 99654z + 46573}$$

$$= \frac{3.156z - 2.858}{z^2 - 1.853z + 0.866}$$

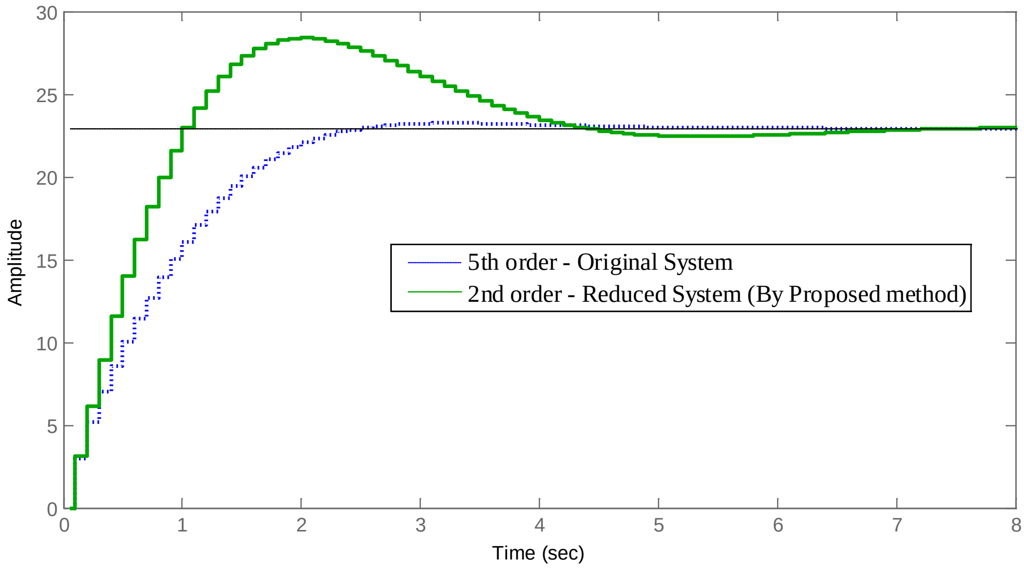

Fig. 3.5 (a) Comparison of step responses for example 3.5

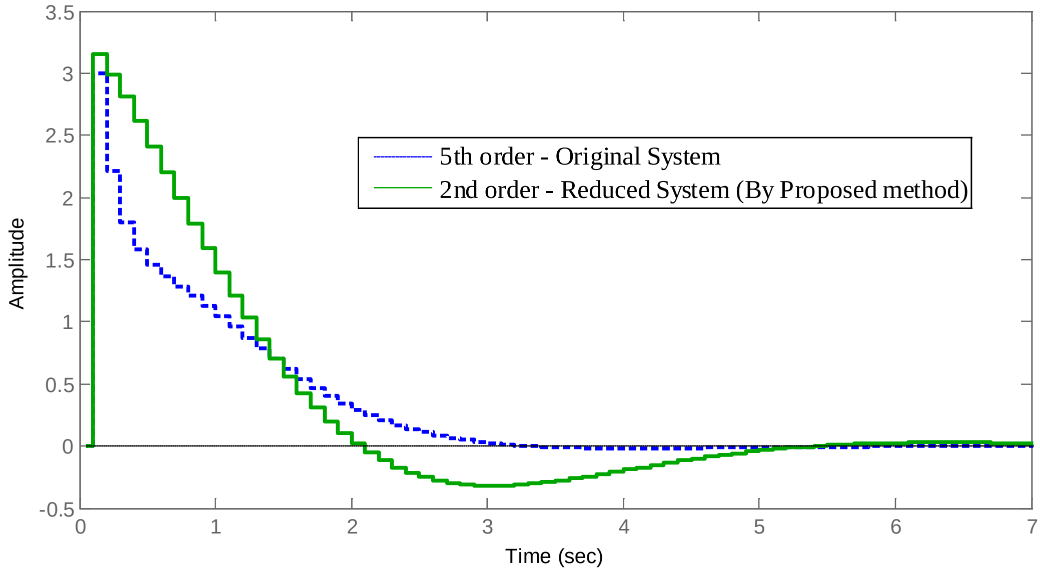

Fig. 3.5 (b) Comparison of impulse responses for example3.5

The step and impulse responses of the original and reduced system are shown in Fig. 3.5 (a)-(b). These responses are compared in Table 3.9 in terms of SSE with other available methods.

Table 3.9 Comparison of reduced order systems for example 3.5

Order Reduction Method	Reduced System $G_r(z)$	SSE
Proposed Method	$\frac{3.156z-2.858}{z^2-1.853z+0.866}$	93.12
Chen *et. al.* [50]	$\frac{0.008634z-0.003272}{0.6636z^2-1.265z+0.6066}$	109.79
Prasad [240] (using stability equation and continued fraction)	$\frac{1.68157z-1.587947}{z^2-1.8542z-0.86384}$	178.11

Example 3.6: Consider eightth order higher order discrete transfer function [253] taken from example 3.4 in 3.2.1. The equivalent G_n(w) is

$$G_n(w)=\frac{N_n(w)}{D_n(w)}=\frac{-0.002w^7+4.866w^6+27.814w^5+54.058w^4+62.106w^3+46.214w^2+17.986w+1.998}{8w^8+78.64w^7+292.928w^6+526.816w^5+584.144w^4+400.24w^3+139.232w^2+16w+2}$$

following steps 3-8 in 3.3.1 in example 3.5, the reduced system obtained is

$$G_r^2(z)=\frac{N_r(z)}{D_r(z)}=\frac{152.8\,z-122.9}{569z^2-985z+446}=\frac{0.269\,z-0.216}{z^2-1.731z+0.784}$$

Table 3.10 Comparison of reduced order systems for example 3.6

Order Reduction Method	Reduced System $G_r(z)$	SSE
Proposed Method	$\frac{0.269\,z-0.216}{z^2-1.731z+0.784}$	0.032
Hwang *et. al.* [254]	$\frac{0.316331z-0.262395}{z^2-1.73034z-0.784276}$	0.062
Bistritz [253]	$\frac{0.2696z-0.2157}{z^2-1.73z-0.7842}$	0.052
Bistritz [132]	$\frac{0.37131242z-0.298}{z^2-1.626873z-0.701497}$	0.057
Hwang *et. al.* [254]	$\frac{0.3664429z-0.28918}{z^2-1.626873z-0.701497}$	0.065
Hwang and Shih [248]	$\frac{0.2018z^2+0.04484z-0.156}{1.2z^2-1.955z+0.843}$	0.0791

The responses of the original and reduced system are shown in Fig. 3.6 (a)-(b). The results obtained are compared in terms of SSE with other available methods in Table 3.10

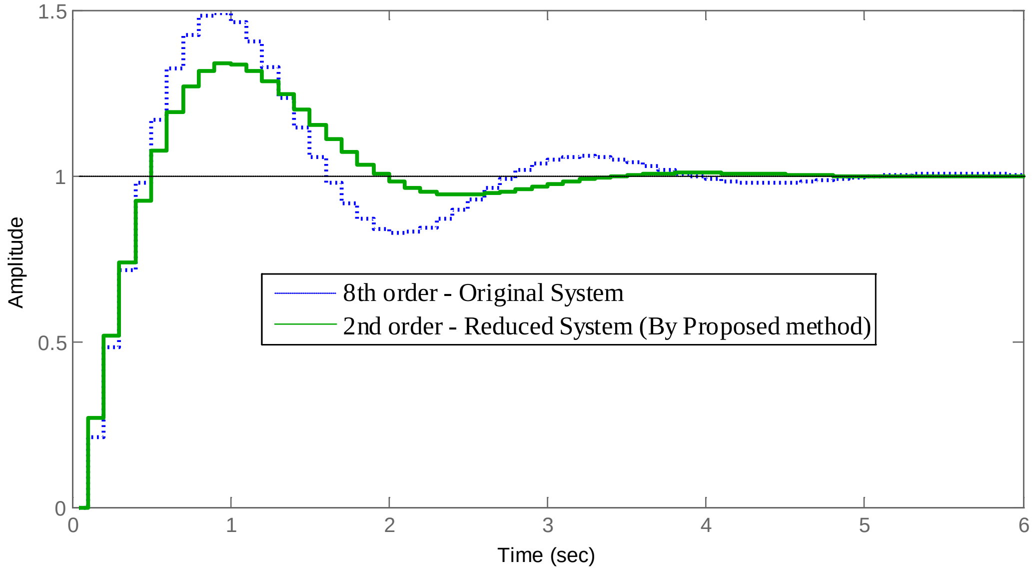

Fig. 3.6 (a) Comparison of step responses for example 3.6

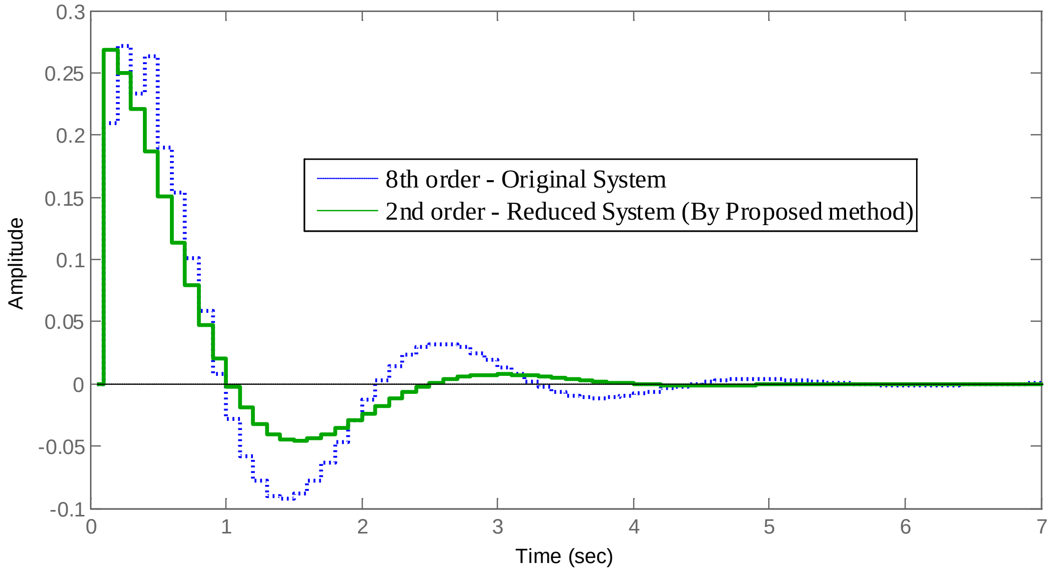

Fig. 3.6 (b) Comparison of impulse responses for example 3.6

Example 3.7: Consider eightth order discrete time system from example 3.2 in 3.2.1.

Using bilinear transformation, G_n(w)

$$G_n(w) = \frac{-0.04w^7 + 813.14w^6 + 4641.9w^5 + 9020.1w^4 + 10362w^3 + 7708.6w^2 + 2999.8w + 333.3}{666.66w^8 + 6146.8w^7 + 22496w^6 + 41713w^5 + 48010w^4 + 34682w^3 + 13959w^2 + 2667.1w + 333.4}$$

using the proposed mixed method as in example 3.5 in 3.3.1, the second order reduced system is obtained as

$$G_r^2(z) = \frac{N_r(z)}{D_r(z)} = \frac{131\,z - 105}{309z^2 - 487z + 204}$$
$$= \frac{0.424\,z - 0.339}{z^2 - 1.576z + 0.66}$$

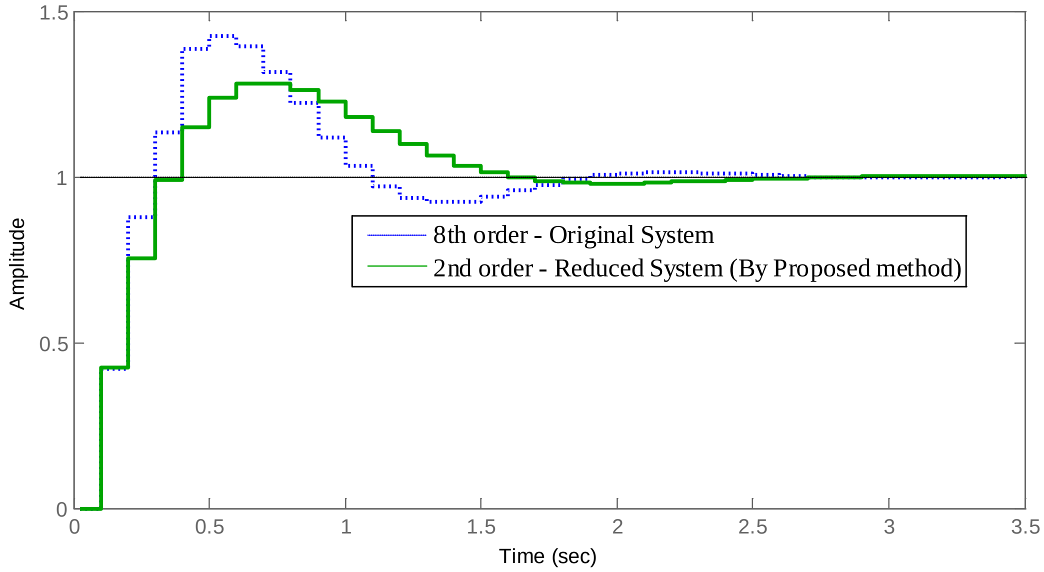

Fig. 3.7 (a) Comparison of step responses for example 3.7

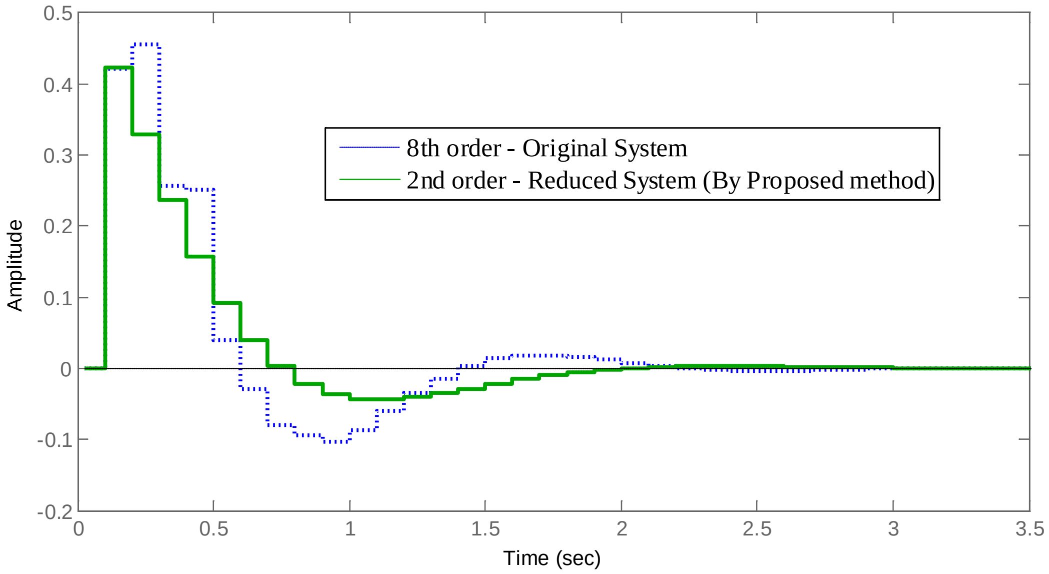

Fig. 3.7 (b) Comparison of impulse responses for example 3.7

The reaction curves (unit step responses) of the original system '$G_n(z)$' and reduced system '$G_r(z)$' are shown in Fig. 3.7 (a). Similarly, Fig. 3.7 (b) depicts the impulse responses of '$G_n(z)$' and '$G^2_r(z)$'. The results obtained by the proposed reduction method, are compared with other reduction methods proposed by several authors in Table 3.11. SSE is taken as the base for comparison.

Table 3.11 Comparison of reduced order systems for example 3.7

Order Reduction Method	Reduced System $G_r(z)$	SSE
Proposed Method (second order)	$\frac{0.424z-0.339}{z^2-1.576z+0.66}$	0.022
Satakshi *et. al.*[251]	$\frac{0.625z-0.5443}{z^2-1.4323z+0.5129}$	0.0245
Chung *et. al.*[252]	$\frac{0.3975z-0.318}{z^2-1.6025z+0.682}$	1.336
Satakshi *et. al.*[251]	$\frac{0.7769z-0.694}{z^2-1.4248z+0.5075}$	0.0486
Satakshi *et. al.*[251]	$\frac{0.463z+0.30172}{z^2-1.5312z+0.6868}$	0.1466
Prasad [240] (using two TM)	$\frac{0.5282z-0.4225}{z^2-1.472z+0.577}$	1.176
Prasad [240] (using one TM/ MP)	$\frac{0.420499z-0.31486}{z^2-1.472z+0.577}$	0.4238

Example 3.8: A supersonic inlet model transfer function in example 3.1 in 3.2.1.

G_n(w) is obtained by substituting z=(1+w)/(1-w) in G_n(z)

$$G_n(w) = \frac{24.76w^6+30.616w^5+39.438w^4+23.13w^3+10.432w^2+2.0398w+0.3377}{15.353w^7+20.128w^6+42.551w^5+23.144w^4+20.324w^3+4.4992w^2+1.9w+0.1008}$$

Table 3.12 Comparison of reduced order systems for example 3.8

Order Reduction Method	Reduced System $G_r(z)$	SSE
Proposed Method (second order)	$\frac{0.789z-0.559}{z^2-1.289z+0.358}$	0.3269
Proposed Method (third order)	$\frac{1.014z^2-1.515z+0.627}{z^3-2.245z^2+1.906z-0.622}$	0.2896
Lalonde *et. al.*[176] (third order)	$\frac{0.0627z^3-2.106z^2+1.569z+0.0371}{z^3-0.8204z^2+0.1697z-0.1648}$	0.55879
Lalonde *et. al.*[176] (fifth order)	$\frac{1.72\times10^{-4}z^5+2.1z^4-2.9z^3+2.15z^2-1.5z-0.66}{z^5-1.488z^4+1.231z^3-0.96z^2-0.6693z-0.3247}$	0.17453

The reduced system obtained using steps 3-8 of 3.3.1 obtained is

$$G_r^2(z) = \frac{1192\,z - 843.9}{1511z^2 - 1948z + 541} = \frac{0.789\,z - 0.559}{z^2 - 1.289z + 0.358}$$

$$G_r^3(z) = \frac{140500z^2\text{-}209900\,z + 86970}{138566z^3 - 3111332z^2 + 264077z - 86243} = \frac{1.014z^2\text{-}1.515\,z + 0.627}{z^3 - 2.245z^2 + 1.906z - 0.622}$$

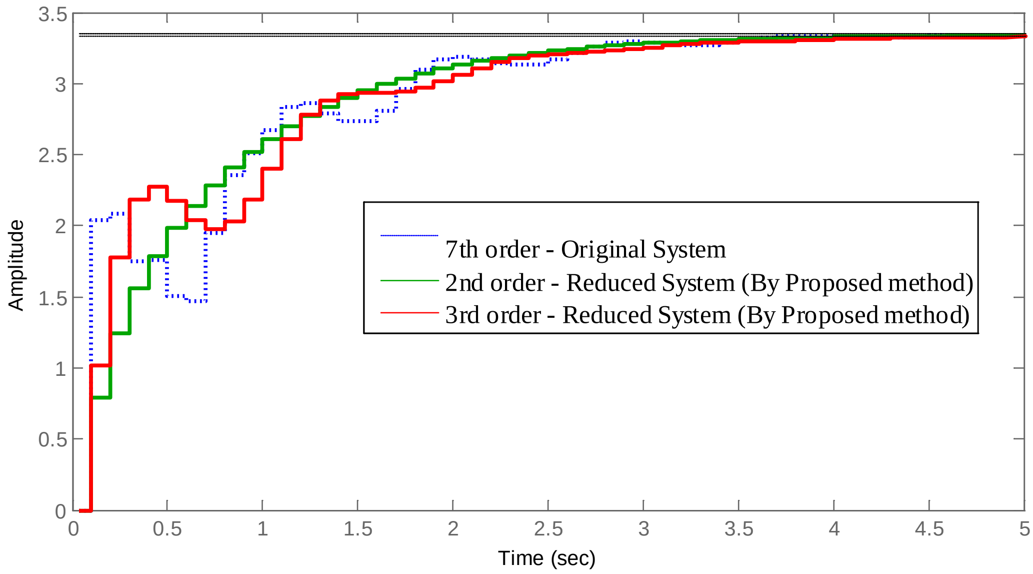

Fig. 3.8 (a) Comparison of step responses for example 3.8

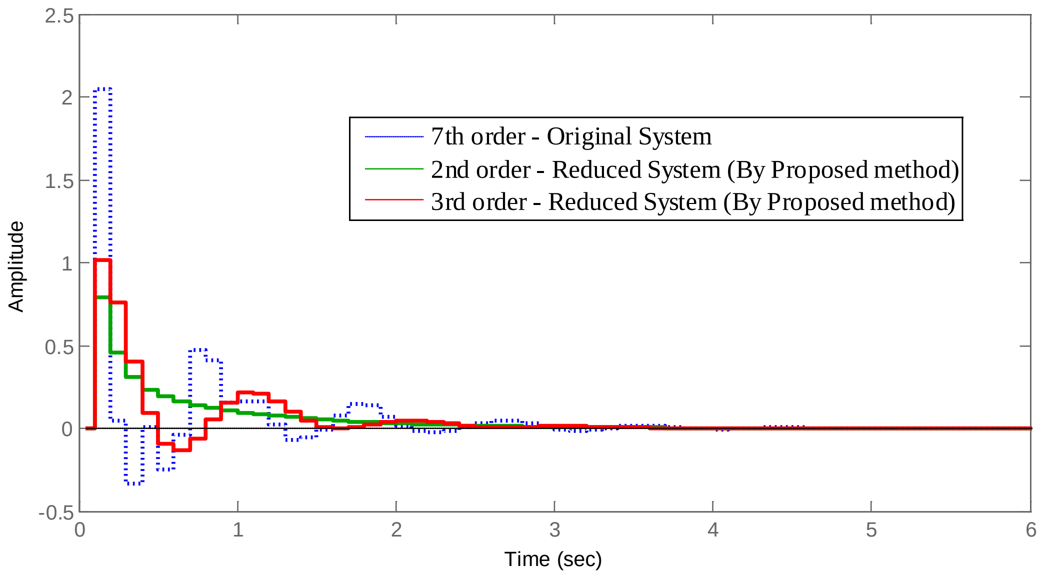

Fig. 3.8 (b) Comparison of impulse responses for example 3.8

The original, reduced system are subjected are to step and impulse input. Their corresponding responses are shown in Fig. 3.8 (a)-(b). These responses are compared in Table 3.12 with respect to SSE given by (3.10) with other available methods.

3.3.2 Extension to Multivariable systems

The proposed method described in 3.3 is extended for discrete systems having multiple input and multiple output. The procedure comprises of direct application of the SISO method on the elements of the transfer function matrix of MIMO system. The steps to be followed are simple and is clear in the following example 3.9.

3.3.2.1 Illustrative Examples

Example 3.9: A multivariable system of dimension (m×l) with m=3 , l=2 taken from [250] is given by

$$[G_n(z)] = \frac{1}{D_n(z)}\begin{bmatrix} A_{11}(z) & A_{12}(z) & A_{13}(z) \\ A_{21}(z) & A_{22}(z) & A_{23}(z) \end{bmatrix}$$

$$A_{11}(z) = 1.3z^7 + z^6 - 0.02z^5 + 0.042z^4 - 0.181z^3 - 0.007z^2 + 0.024z - 0.0033$$
$$A_{12}(z) = 0.082z^7 + 0.095z^6 - 0.14z^5 + 0.01z^4 - 0.13z^3 - 0.2z^2 + 0.017z - 0.0015$$
$$A_{13}(z) = 0.3z^7 + 0.021z^6 - 0.05z^5 - 0.1z^4 - 0.205z^3 - 0.055z^2 + 0.003z - 0.0012$$
$$A_{21}(z) = 1.081z^7 + 0.3z^6 - 0.286z^5 - 0.092z^4 + 0.113z^3 - 0.08z^2 - 0.0354z - 0.004$$
$$A_{22}(z) = 0.3z^7 + 0.621z^6 - 0.253z^5 - 0.116z^4 + 0.247z^3 - 0.26z^2 - 0.212z - 0.004$$
$$A_{23}(z) = 1.05z^7 + 0.13z^6 + 0.27z^5 - 0.043z^4 + 0.071z^3 - 0.17z^2 - 0.085z - 0.003$$
$$D_n(z) = 8z^8 - 5.046z^7 - 3.348z^6 + 0.63z^5 - 0.456z^4 + 1.548z^3 + 0.786z^2 - 0.132z + 0.018$$

It is desired to reduce [G_n(z)] to a reduced order system represented in the form

$$[R(z)] = \frac{1}{D_r(z)}\begin{bmatrix} B_{11}(z) & B_{12}(z) & B_{13}(z) \\ B_{21}(z) & B_{22}(z) & B_{23}(z) \end{bmatrix} \tag{3.12}$$

Step 1: G_n(z) is converted using bilinear transformation by substituting *z*=(*1*+*w*)/(*1*-*w*) both in the numerator and denominator of each A_{ij}(z) according to (3.3),

$$[G_n(w)] = \frac{1}{D_n(w)}\begin{bmatrix} A_{11}(w) & A_{12}(w) & A_{13}(w) \\ A_{21}(w) & A_{22}(w) & A_{23}(w) \end{bmatrix}$$

$$A_{11}(w) = 0.0913w^7 + 4.0149w^6 + 19.241w^5 + 39.982w^4 + 49.891w^3 + 36.837w^2 + 14.187w + 2.1547$$
$$A_{12}(w) = -0.0745w^7 - 0.8965w^6 + 1.5315w^5 + 3.5975w^4 + 2.5925w^3 + 2.7185w^2 + 1.2945w - 0.2675$$
$$A_{13}(w) = 0.1832w^7 + 1.9616w^6 + 6.4832w^5 + 9.948w^4 + 10.292w^3 + 7.3008w^2 + 2.3184w - 0.0872$$
$$A_{21}(w) = 0.7486w^7 + 5.097w^6 + 18.945w^5 + 38.265w^4 + 41.297w^3 + 24.569w^2 + 8.449w + 0.9966$$
$$A_{22}(w) = -0.159w^7 - 1.643w^6 - 2.195w^5 + 11.273w^4 + 15.859w^3 + 8.991w^2 + 5.951w + 0.323$$
$$A_{23}(w) = 1.392w^7 + 7.376w^6 + 20.276w^5 + 36.004w^4 + 36.072w^3 + 22.408w^2 + 9.652w + 1.22$$
$$D_n(w) = 8w^8 + 78.64w^7 + 292.93w^6 + 526.82w^5 + 584.14w^4 + 400.24w^3 + 139.23w^2 + 16w + 2$$

Step 2: Consider D_n(w) and following step 3,4 in example 1 in 3.3.1,

$$D_r(w) = w^2 + 0.123w + 0.0153$$

Step 3: Computation of the elements of transfer function matrix [R(w)]:

By following the steps 5-6 of example 3.5 in 3.3.1 and applying on each element of [G_n(w)], results in

$$B_{11}(w) = 0.1096w + 0.01616 \qquad B_{12}(w) = 0.009307w - 0.002006$$
$$B_{13}(w) = 0.01726w - 0.000654 \qquad B_{21}(w) = 0.06486w + 0.007475$$
$$B_{22}(w) = 0.04512w + 0.002423 \qquad B_{23}(w) = 0.07422w + 0.00915$$

substitute the values of B_{ij}, D_r (w-domain) in [R(w)]

$$[R(w)] = \frac{1}{D_r(w)} \begin{bmatrix} B_{11}(w) & B_{12}(w) & B_{13}(w) \\ B_{21}(w) & B_{22}(w) & B_{23}(w) \end{bmatrix}$$

Step 4: Convert B_{ij} in w-domain to z-domain using inverse bilinear transformation using (3.5). Similarly convert D_r(w) to D_r(z). Substituting the B_{ij} in z-domain and D_r(z) in (3.12) ,

$$[R(z)] = \frac{1}{D_r(z)} \begin{bmatrix} B_{11}(z) & B_{12}(z) \\ B_{21}(z) & B_{22}(z) \end{bmatrix}$$

$$B_{11}(z) = 0.221z - 0.164 \qquad B_{12}(z) = 0.0128z - 0.0198$$
$$B_{13}(z) = 0.0291z - 0.0314 \qquad B_{21}(z) = 0.127z - 0.1008$$
$$B_{22}(z) = 0.0835z - 0.075 \qquad B_{23}(z) = 0.146z + 0.1139$$
$$D_r(z) = z^2 - 1.731z + 0.784$$

Table 3.13 Comparison of SSE for example 3.9

r_{ij} (i,j=1-3)	Proposed Method	Prasad [240]
r_{11}	0.02610	0.04475
r_{12}	0.00175	0.00203
r_{13}	0.00189	3.5437
r_{21}	0.00631	0.01026
r_{22}	0.00516	0.006618
r_{23}	0.01160	0.01747

The results obtained by proposed method is compared with other method in terms of *SSE* for each element of transfer function matrix are according to Table 3.13. The unit step and impulse responses of [G_n(z)] and [R(z)] are depicted in Fig. 3.9 (a)-(b).

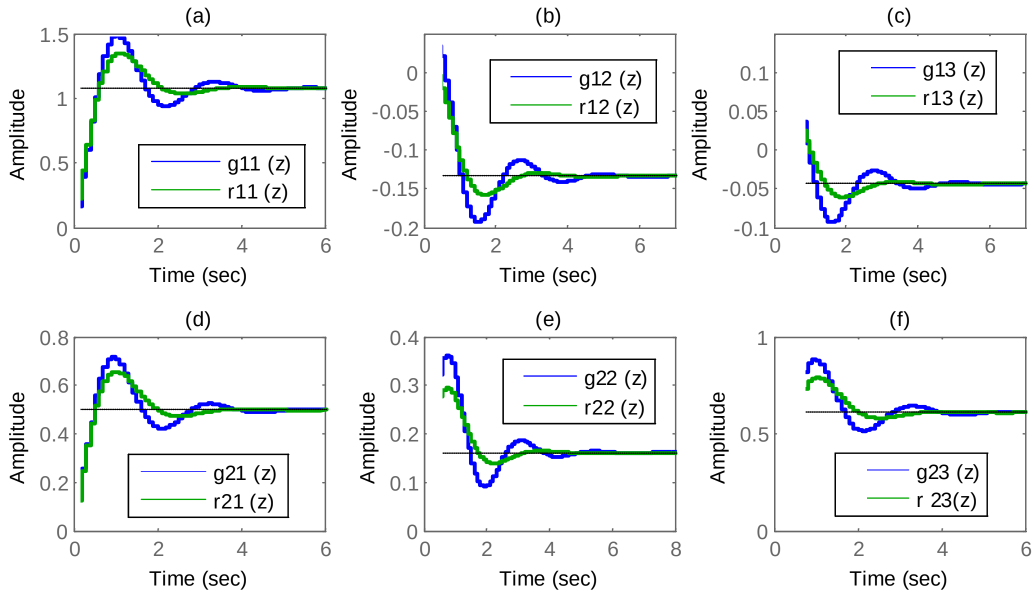

Fig. 3.9 (a) Comparison of step responses for example 3.9

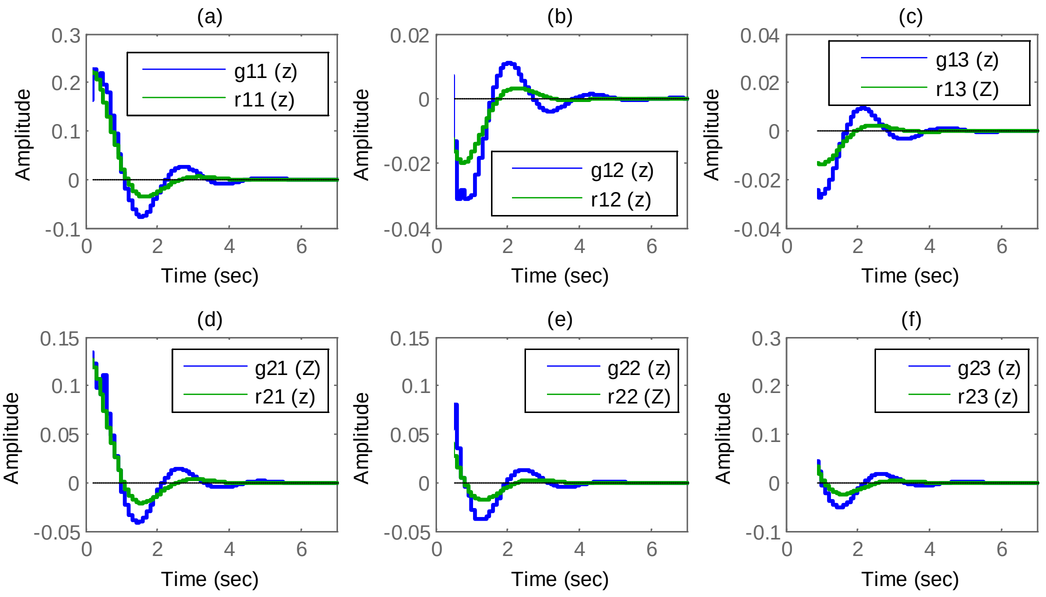

Fig. 3.9 (b) Comparison of impulse responses for example 3.9

Example 3.10: Given a z-transfer function matrix by

$$[G_n(z)] = \begin{bmatrix} \dfrac{2.25(z-0.75)}{(z-0.95)(z-0.5)} & \dfrac{1.5(z-0.8)}{(z-0.9)(z-0.75)} \\ \dfrac{1.04(z-0.65)}{(z-0.95)(z-0.5)} & \dfrac{(z-0.7)}{(z-0.9)(z-0.35)} \end{bmatrix}$$

Applying bilinear transformation according to (3.3),

$$[G_n(w)] = \frac{1}{D_n(w)} \begin{bmatrix} A_{11}(w) & A_{12}(w) \\ A_{21}(w) & A_{22}(w) \end{bmatrix}$$

$$A_{11}(w) = 22.976w^5 + 31.208w^4 + 14.7w^3 + 2.872w^2 + 0.238w + 0.0064$$
$$A_{12}(w) = 13.861w^5 + 20.652w^4 + 10.908w^3 + 2.388w^2 + 0.188w + 0.0032$$
$$A_{21}(w) = 13.861w^5 + 20.652w^4 + 10.908w^3 + 2.388w^2 + 0.188w + 0.0032$$
$$A_{22}(w) = 11.313w^5 + 13.765w^4 + 5.8088w^3 + 1.0368w^2 + 0.0748w + 0.0016$$
$$D_n(w) = 17.068w^6 + 26.872w^5 + 15.55w^4 + 4.03w^3 + 0.461w^2 + 0.0198w - 0.0008$$

following the steps 2-4 of example 5 in 3.3.2.1, the reduced discrete system obtained is

$$[R(z)] = \frac{1}{D_r(z)}\begin{bmatrix} B_{11}(z) & B_{12}(z) \\ B_{21}(z) & B_{22}(z) \end{bmatrix}$$

$$B_{11}(z) = 2.606z - 2.469 \qquad B_{12}(z) = 2.231z - 4.809$$
$$B_{21}(z) = 1.132z - 1.21 \qquad B_{22}(z) = 0.713z - 0.683$$
$$D_r(z) = w^2 - 1.733w + 0.7395$$

Table 3.14 gives the SSE values obtained for r_{ij}. Fig. 3.10 (a)-(b) depicts the step and impulse responses of G_n(z) and R(z) respectively.

Table 3.14 SSE values for example 3.10

r_{ij} (i,j=1-3)	Proposed Method
r_{11}	0.02610
r_{12}	0.00175
r_{13}	0.00189
r_{21}	0.00631
r_{22}	0.00516
r_{23}	0.01160

In this proposed method, the *z*-domain is transformed to *w*-domain unlike to the method discussed in section 3.2. The method is applied on SISO systems initially and then extended to MIMO systems. In example 3.8 the third order proposed system gives much lower SSE as compared to Lalonde *et. al.*[176] of the same order. Error analysis is carried out on all examples considered in this section, to establish the validity of the introduced method.

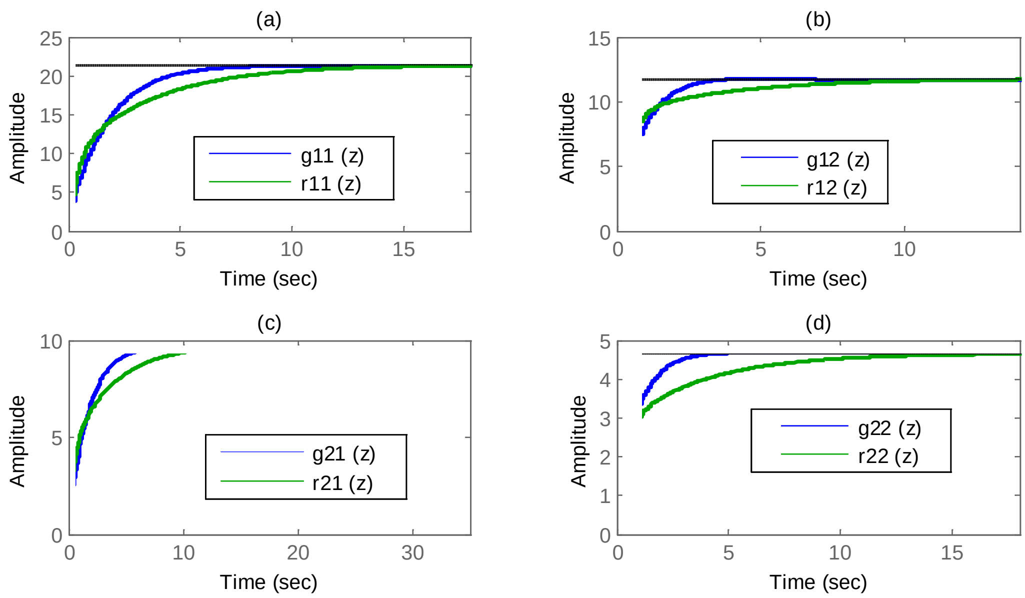

Fig. 3.10 (a) Comparison of step responses for example 3.10

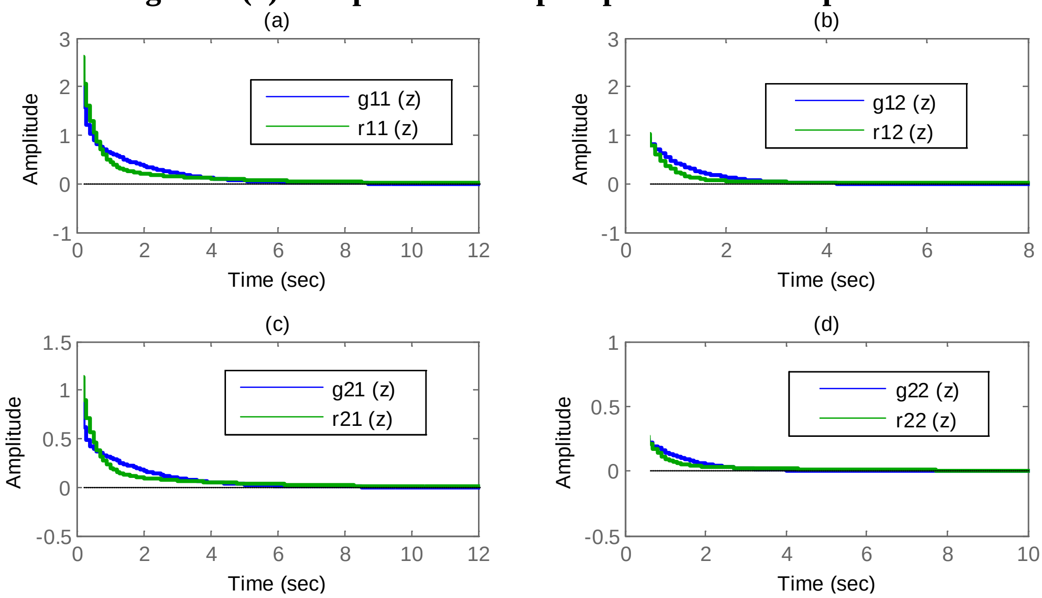

Fig. 3.10 (b) Comparison of impulse responses for example 3.10

3.4 DOMINANT POLE AND LEAST SQUARES METHOD

As discussed in 2.3.3 the proposed method combines the concept of dominant pole retention [51, 147, 161, 165] is with least squares method [174, 175, 231] to generate the reduced order stable system. Once again, the given higher order discrete system is converted to *w*-domain using bilinear transformation (3.3) and then the proposed method is applied. Then the reduced system is converted back to *z*-domain using (3.5).The same method is then extended for multivariable systems also. The procedure for generating the reduced discrete system is as follows.

Step 1: Consider $G_n(z)$ of the form (3.1) and convert using bilinear transformation (3.3) to give

$$G_n(w) = \frac{N_n(w)}{D_n(w)} = \frac{a_o + a_1 z + ... + a_m z^m}{b_o + b_1 z + b_2 z^2 + ... + b_n z^n};$$

Step 2: Find the roots of the denominator polynomial D_n(w), such that $-p_1 < -p_2 < ... < -p_n$ are the poles of the higher order system G_n(w).

Step 3: Retain of dominant pole of G_n(w)\

Let us assume that rth order reduced system is desired. Then, 'r' number of poles nearest to the origin are selected and retained whereas the other poles are neglected. Doing so, the overall behavior of the reduced system is maintained similar to that of the original system. Now, the reduced denominator D_r(w) is

$$D_r(w) = (w + p_1)(w + p_2)...(w + p_r)$$
$$= e_o + e_1 w + e_2 w^2 + ... + w^r$$

Step 4: The numerator terms of N_r(w) are found initially computing the time moment proportional's 'c_i' by expanding G_n(w) about w=0 using the formula (3.11).Substitute the value of 'e_i', $i = 0,1, 2, ... \infty$ and 'c_i' in (2.8) and (2.9) to obtain the coefficients 'd_i', $i = 0,1, 2, ...(r-1)$ of N_r(w).

Step 5: The ratio of N_r(w), D_r(w) is

$$G_r(w) = \frac{N_r(w)}{D_r(w)}$$

Step 6: Applying inverse bilinear transformation using (3.5), the reduced order system G_r(z) is obtained

$$G_r(z) = \frac{N_r(z)}{D_r(z)}$$

3.4.1 Illustrative examples

The steps 1-6 of the proposed method are applied on numerical examples taken from the literature. The first example is solved in detail while the rest of the examples are solved briefly. The results obtained are compared with other methods available in the literature in terms of *SSE* given by (3.10).Further, the step and impulse responses are also plotted for the original and reduced discrete system.

Example 3.11: Consider eighth order higher order discrete transfer function [253] taken from example 3.4 in 3.2.1.

Step 1: The given discrete system G_n(z) is converted using bilinear transformation by substituting $z=(1+w)/(1-w)$ both in the numerator and denominator polynomial according to (3.3)

$$G_n(w)=\frac{N_n(w)}{D_n(w)}=\frac{-0.002w^7+4.866w^6+27.814w^5+54.058w^4+62.106w^3+46.214w^2+17.986w+1.998}{8w^8+78.64w^7+292.928w^6+526.816w^5+584.144w^4+400.24w^3+139.232w^2+16w+2}$$

Step 2: The roots of D_n(w) are $p_{1,2}$ = -0.0463 ± j0.1362, $p_{3,4}$ = -0.4281 ± j0.9899, $p_{5,6}$ =-0.8380 ± j0.1839,$p_{7,8}$ = -3.6025 ± j1.0655.Since a second order reduced systems is desired, the complex poles near the origin $p_{1,2}$ are retained. The reduced denominator obtained is

$$D_r(w)=(w+p_1)(w+p_2)=(w+0.0463+\text{j}0.1362)(w+0.0463-\text{j}0.1362)$$
$$=w^2+0.0926\text{w}+0.0207$$

Step 3: In order to find the numerator terms of N_r(w), the time moment proportional's 'c_i' are computed by expanding G_n(w) about w=0 by using (3.11). The first five time moment proportionals of G_n(w) as in Table 3.15.

Table 3.15 Time moment proportionals obtained for example 3.11

i	c_i
0	0.9990
1	1.0010
2	-54.4474
3	197.0266
4	1749.1254

Substitute the value of 'e_i', i = 0,1, 2 , ... ∞ and 'c_i' in (2.8) and (2.9) to obtain the coefficients 'd_i', i = 0,1, 2 , ...(r-1).

Step 4:N_r(w) is $N_r(w)=0.1132w+0.02068$

Step 5: The ratio of N_r(w), D_r(w) is

$$G_r(w)=\frac{N_r(w)}{D_r(w)}=\frac{0.1132w+0.02068}{w^2+0.0926w+0.0207}$$

Step 6: Applying inverse bilinear transformation using (3.3), the reduced order discrete time system G_r(z) is obtained as (3.4).

$$N_r(z) = N_r(w)|_{w=(z-1)/(z+1)}$$
$$= 0.1132w + 0.02068\,|_{w=(z-1)/(z+1)}$$
$$= 0.1132\frac{(z-1)}{(z+1)} + 0.02068 = 0$$
$$N_r(z) \quad = 0.1132(z-1) + 0.02068(z+1)$$

$$D_r(z) = D_r(w)|_{w=(z-1)/(z+1)}$$
$$= w^2 + 0.0926w + 0.0207\Big|_{w=(z-1)/(z+1)}$$
$$= \frac{(z-1)^2}{(z+1)^2} + 0.0926\frac{(z-1)}{(z+1)} + 0.0207 \quad = 0$$
$$D_r(z) = (z-1)^2 + 0.0926(z-1)(z+1) + 0.0207(z+1)^2$$

The second order reduced discrete system is

$$G_r^2(z) = \frac{N_r(z)}{D_r(z)}$$
$$= \frac{2678z - 1850}{11133z^2 - 19586z + 9281}$$
$$= \frac{0.24z - 0.166}{z^2 - 1.759z + 0.8336}$$

Table 3.16 Comparison of reduced order systems for example 3.11

Order Reduction Method	Reduced System G_r(z)	SSE
Proposed Method	$\frac{0.24z-0.166}{z^2-1.759z+0.8336}$	0.00043
Hwang *et. al.* [254]	$\frac{0.316331z-0.262395}{z^2-1.73034z-0.784276}$	0.062
Bistritz [253]	$\frac{0.2696z-0.2157}{z^2-1.73z-0.7842}$	0.052
Bistritz [132]	$\frac{0.37131242z-0.298}{z^2-1.626873z-0.701497}$	0.057
Hwang *et. al.* [254]	$\frac{0.3664429z-0.28918}{z^2-1.626873z-0.701497}$	0.065
Hwang and Shih [248]	$\frac{0.2018z^2+0.04484z-0.156}{1.2z^2-1.955z+0.843}$	0.0791

The responses of the original and reduced system are shown in Fig. 3.11 (a)-(b). The results obtained are compared in terms of SSE with other available methods in Table 3.16.

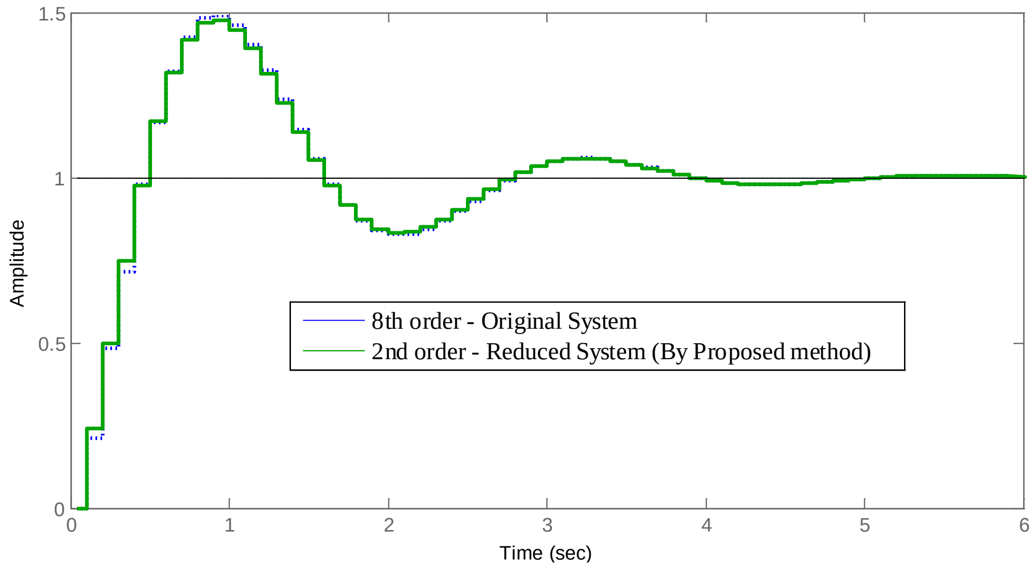

Fig. 3.11 (a) Comparison of step responses for example 3.11

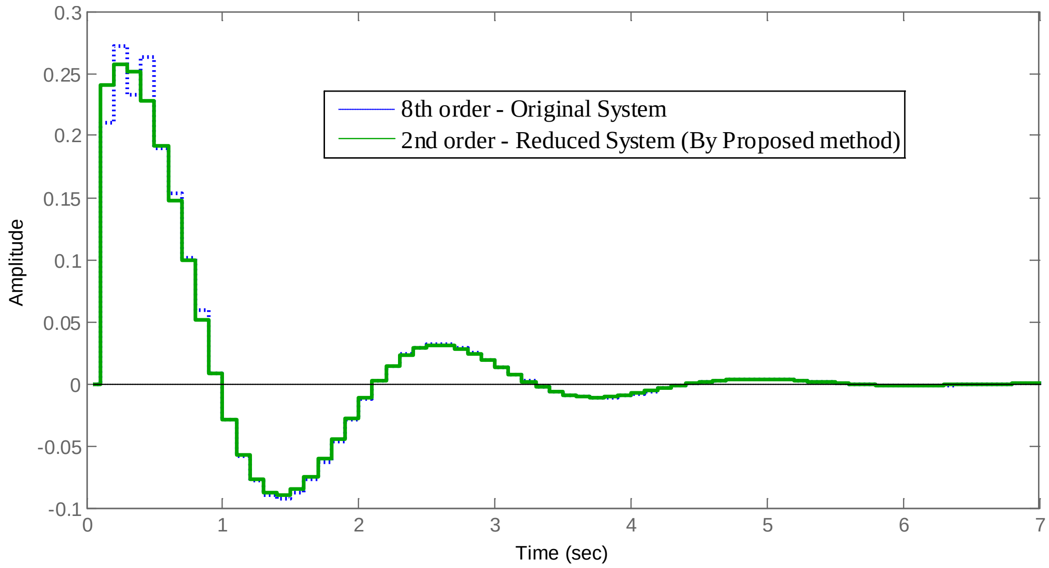

Fig. 3.11 (b) Comparison of impulse responses for example 3.11

Example 3.12: Consider a supersonic inlet model transfer function taken from Lalonde *et. al.*[176]

$$G_n(z) = \frac{N_n(z)}{D_n(z)} = \frac{2.0434z^6 - 4.982z^5 + 6.57z^4 - 5.819z^3 + 3.636z^2 - 1.41z + 0.2997}{z^7 - 2.46z^6 + 3.433z^5 - 3.33z^4 + 2.546z^3 - 1.584z^2 + 0.7478z - 0.252}$$

G_n(w) is given by

$$G_n(w) = \frac{24.76w^6 + 30.616w^5 + 39.438w^4 + 23.13w^3 + 10.432w^2 + 2.0398w + 0.3377}{15.353w^7 + 20.128w^6 + 42.551w^5 + 23.144w^4 + 20.324w^3 + 4.4992w^2 + 1.9w + 0.1008}$$

Then the reduced second and fifth order discrete system is obtained by following steps 2-6 in 3.4, are

$$G_r^2(z) = \frac{9303z - 56.46}{16277z^2 - 8620z - 4897}$$
$$= \frac{0.572z - 0.003}{z^2 - 0.5296z + 0.3}$$
$$G_r^3(z) = \frac{140500z^2 - 209900z + 86970}{138566z^3 - 311133z^2 + 264077z - 86243}$$
$$= \frac{1.013z^2 - 1.513z - 0.627}{z^3 - 2.245z^2 - 1.905z + 0.622}$$

The original and reduced system subjected to step and impulse input and their corresponding responses are shown in Fig. 3.12 (a)-(b). These responses are compared in Table 3.17 with respect to SSE given by (3.10) with other available methods.

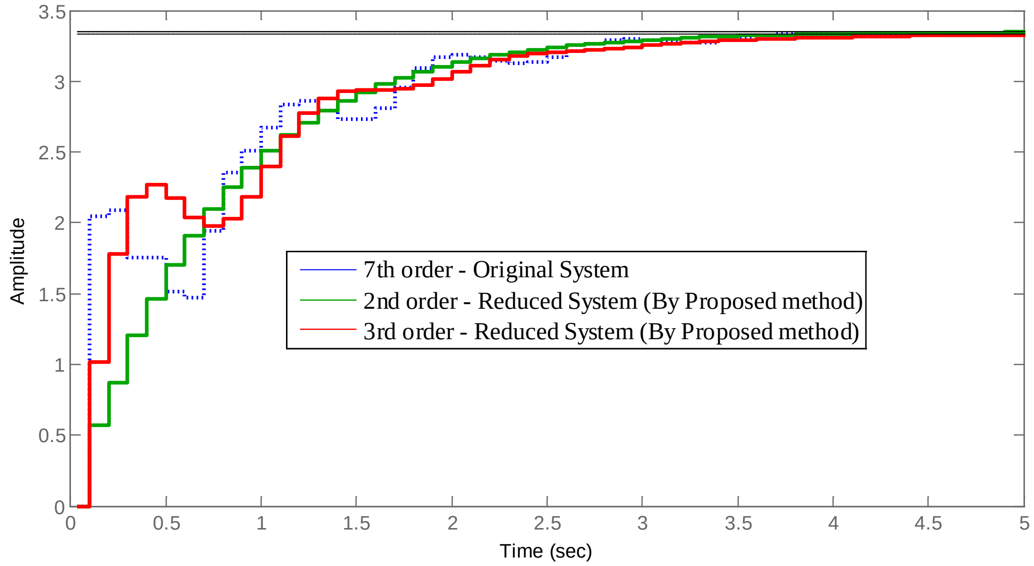

Fig. 3.12 (a) Comparison of step responses for example 3.12

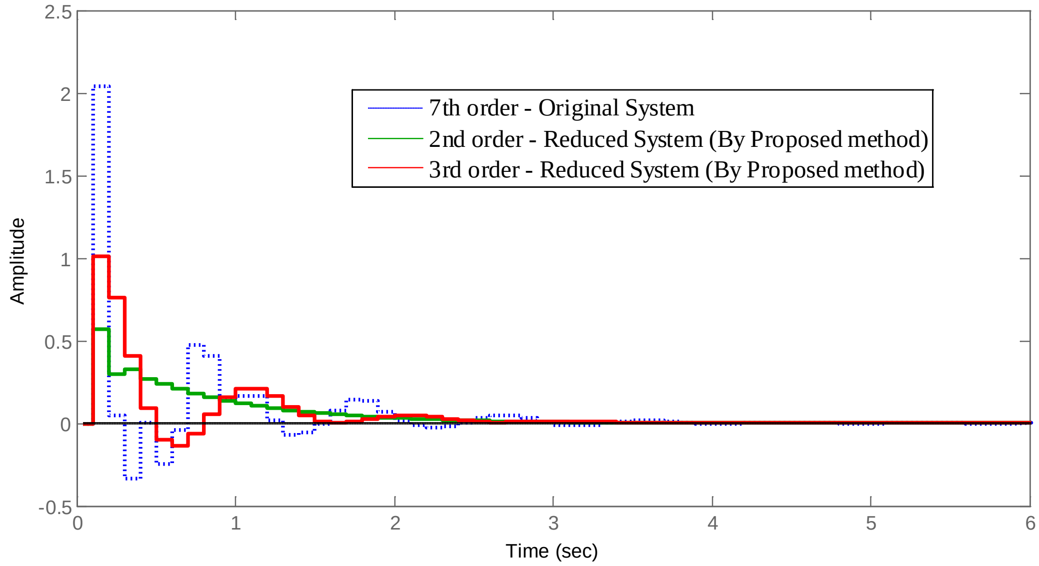

Fig. 3.12 (b) Comparison of impulse responses for example 3.12

Table 3.17 Comparison of reduced order systems for example 3.12

Order Reduction Method	Reduced System $G_r(z)$	SSE
Proposed Method (second order)	$\frac{0.572z-0.003}{z^2-0.5296z+0.3}$	0.4535
Proposed Method (third order)	$\frac{1.013z^2-1.513z-0.627}{z^3-2.245z^2-1.905z+0.622}$	0.2896
Lalonde *et. al.*[176] (third order)	$\frac{0.0627z^3-2.106z^2+1.569z+0.0371}{z^3-0.8204z^2+0.1697z-0.1648}$	0.55879

Example 3.13: Consider 8th order discrete time system taken in 3.2.1 in example 3.2. Using the procedure of the proposed method (3.4), the reduced second order discrete system is

$$G_r^2(z)=\frac{N_r(z)}{D_r(z)}=\frac{0.498z-0.342}{z^2-1.5z+0.658}$$

Table 3.18 Comparison of reduced order systems for example 3.13

Order Reduction Method	Reduced System $G_r(z)$	SSE
Proposed Method (second order)	$\frac{0.498z-0.342}{z^2-1.5z+0.658}$	0.00148
Chung *et. al.*[252]	$\frac{0.3975z-0.318}{z^2-1.6025z+0.682}$	1.336
Satakshi *et. al.*[251]	$\frac{0.463z+0.30172}{z^2-1.5312z+0.6868}$	0.1466
Prasad [240] (using 2 TM)	$\frac{0.5282z-0.4225}{z^2-1.472z+0.577}$	1.176
Prasad (using one TM and MP)	$\frac{0.420499z-0.31486}{z^2-1.472z+0.577}$	0.4238

The step and impulse responses of the original and reduced system are shown in Fig. 3.13 (a)-(b). These responses are compared in Table 3.18 in terms of SSE with other available methods.

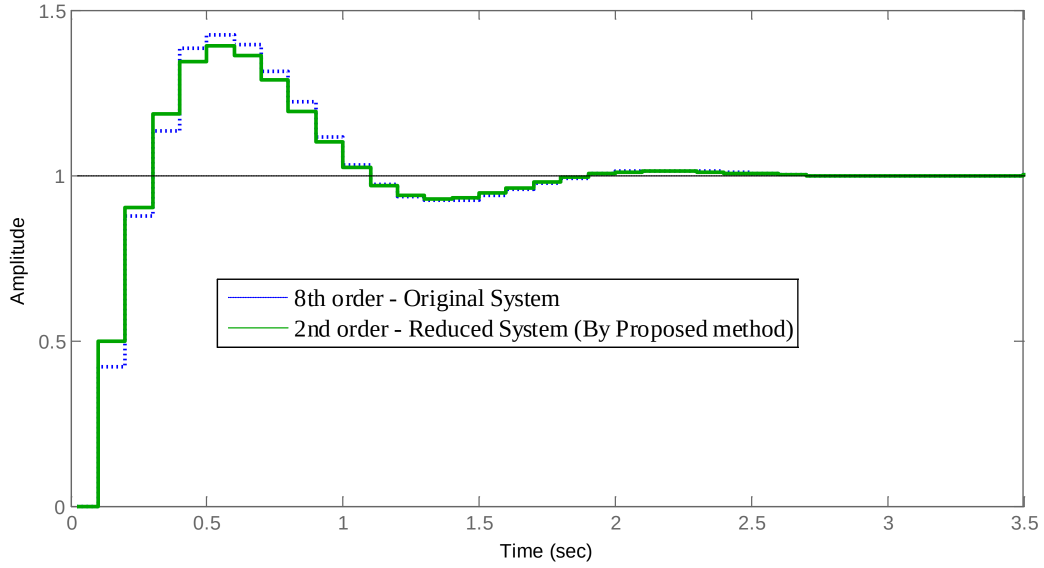

Fig. 3.13 (a) Comparison of step responses for example 3.13

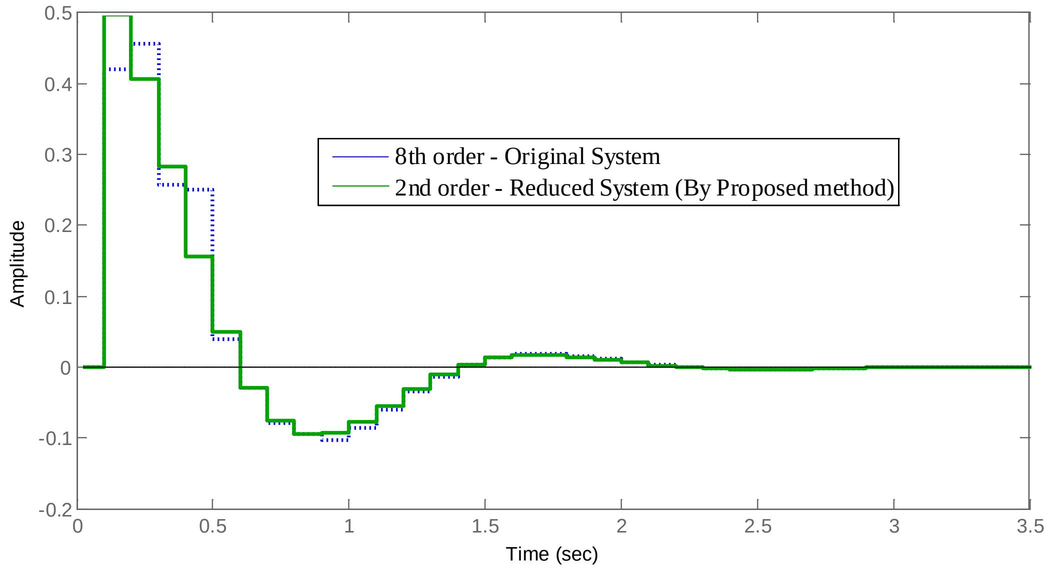

Fig. 3.13 (b) Comparison of impulse responses for example 3.13

Example 3.14: Consider fifth order discrete time system taken from Chen *et. al.* [50]

$$G_n(z) = \frac{3z^4 - 8.886z^3 + 10.0221z^2 - 5.091975z + 0.9811125}{z^5 - 3.7z^4 + 5.47z^3 - 4.037z^2 + 1.4856z - 0.2173}$$

The reduced system of second order is obtained by applying the proposed method (3.4) and is given by

$$G_r^2(z) = \frac{N_r(z)}{D_r(z)} = \frac{1.647z - 1.161}{z^2 - 1.757z - 0.7786}$$

The step and impulse responses of the original and reduced system are shown in Fig. 3.14 (a)-(b). These responses are compared in Table 3.19 in terms of SSE with other available methods.

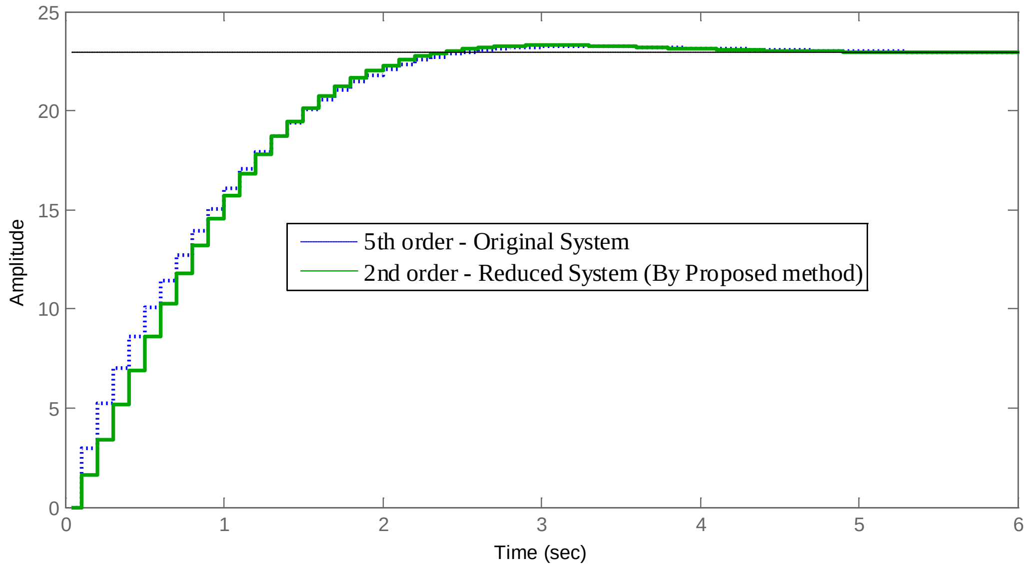

Fig. 3.14 (a) Comparison of step responses for example 3.14

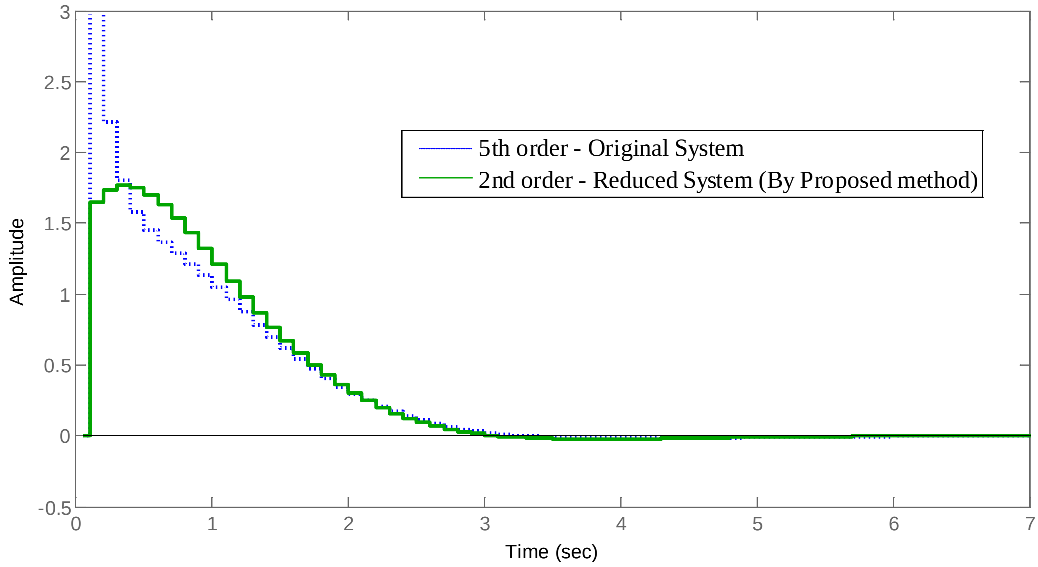

Fig. 3.14 (b) Comparison of impulse responses for example 3.14

Table 3.19 Comparison of reduced order systems for example 3.14

Order Reduction Method	Reduced System $G_r(z)$	SSE
Proposed Method	$\frac{1.647z - 1.161}{z^2 - 1.757z - 0.7786}$	1.737
Chen *et. al.* [50]	$\frac{0.008634z - 0.003272}{0.6636z^2 - 1.265z + 0.6066}$	109.79
Prasad [240] (using SE and continued fraction)	$\frac{1.68157z - 1.587947}{z^2 - 1.8542z - 0.86384}$	178.11

3.4.2 Extension to Multivariable systems

The mixed method using DP and least square is extended for multivariable systems. The procedure of 3.3 applies on each of the elements of the transfer function matrix of MIMO system. Example 5 is solved in depth to signify the proposed procedure.

3.4.2.1 Illustrative Examples

Example 3.15: Revisit the multivariable system of example 3.10 in 3.3.2

$$[G_n(z)] = \frac{1}{D_n(z)}\begin{bmatrix} A_{11}(z) & A_{12}(z) & A_{13}(z) \\ A_{21}(z) & A_{22}(z) & A_{23}(z) \end{bmatrix}$$

Step 1:[G_n(z)] is converted to **[**$G_n(w)$] following step 1- 2 of example 1 in 3.3.2.1.

Step 2: The roots of D_n(w), are $p_{1,2}$ = -0.0463 ± j0.1362, $p_{3,4}$ = -0.4281 ± j0.9899, $p_{5,6}$ = -0.8380 ± j0.1839, $p_{7,8}$ = -3.6025 ± j1.0655.Since a second order reduced systems is desired, the complex poles near the origin $p_{1,2}$ are retained.The reduced denominator obtained are

$$D_r(w) = (w + p_1)(w + p_2) = (w + 0.0463 + \text{j}0.1362)(w + 0.0463 - \text{j}0.1362)$$
$$= w^2 + 0.0926\text{w} + 0.0207$$

Step 3: Using each element of **[**G_n(z)] using steps 3-4 of example 3.11 in 3.4.1 resulting in

$$[R(w)] = \frac{1}{D_r(w)}\begin{bmatrix} B_{11}(w) & B_{12}(w) & B_{13}(w) \\ B_{21}(w) & B_{22}(w) & B_{23}(w) \end{bmatrix}$$

$$B_{11}(w) = 0.06819w + 0.0223 \qquad B_{12}(w) = 0.02316w - 0.002769$$
$$B_{13}(w) = 0.02718w - 0.0009025 \qquad B_{21}(w) = 0.05107w + 0.01031$$
$$B_{22}(w) = 0.0498w + 0.003343 \qquad B_{23}(w) = 0.05337w + 0.01263$$
$$D_r(w) = w^2 + 0.0926w + 0.0207$$

Step 4: Convert B_{ij} in *w*-domain to z-domain using inverse bilinear transformation using (3.5) and D_r(w) to D_r(z). Substituting B_{ij} in *z*-domain and D_r(z) in (3.12),

$$[R(z)] = \frac{1}{D_r(z)}\begin{bmatrix} B_{11}(z) & B_{12}(z) & B_{13}(z) \\ B_{21}(z) & B_{22}(z) & B_{23}(z) \end{bmatrix}$$

$$B_{11}(z) = 0.1625z - 0.0824 \qquad B_{12}(z) = 0.0366z - 0.0465$$
$$B_{13}(z) = 0.0472z - 0.05 \qquad B_{21}(z) = 0.11z - 0.0732$$
$$B_{22}(z) = 0.0954z - 0.0834 \qquad B_{23}(z) = 0.122z + 0.0767$$
$$D_r(z) = w^2 - 1.759w + 0.8334$$

The results are tabulated in Table 3.20 in terms of *SSE* for each element of transfer function matrix. The unit step and impulse responses of **[**G_n(z)] and [*R*(z)] are depicted in Fig. 3.15 (a)-(b).

Table 3.20 Comparison of SSE for example 3.15

r_{ij} (i,j=1-3)	Proposed Method	Prasad [240]
r_{11}	0.000192	0.04475
r_{12}	0.00202	0.00203
r_{13}	0.0000717	3.5437
r_{21}	0.01026	0.01026
r_{22}	0.006619	0.006618
r_{23}	0.00058	0.01747

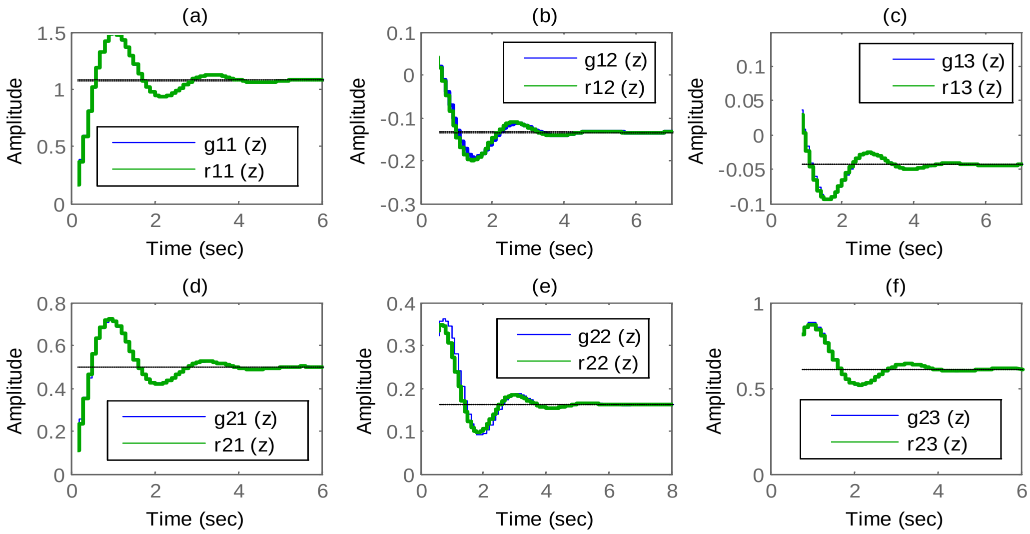

Fig. 3.15 (a) Comparison of step responses for example 3.15

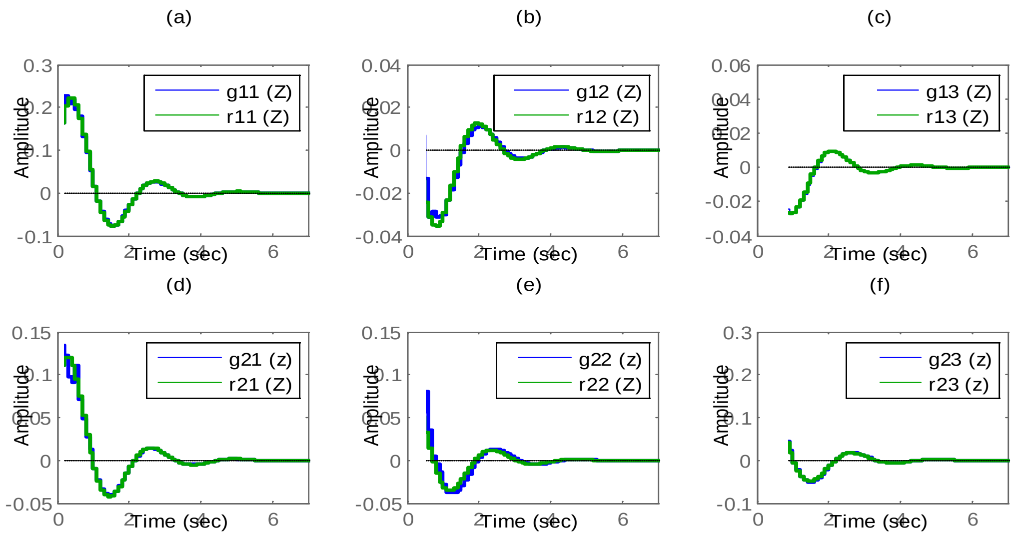

Fig. 3.15 (b) Comparison of impulse responses for example 3.15

Example 3.16: Given a z-transfer function matrix of example 3.9 in 3.3.2.1

The reduced system obtained by applying method 3 in 3.4.2 ,

$$[R(z)] = \frac{1}{D_r(z)} \begin{bmatrix} B_{11}(z) & B_{12}(z) \\ B_{21}(z) & B_{22}(z) \end{bmatrix}$$

$$B_{11}(z) = 4.9z - 4.142$$
$$B_{12}(z) = 2.81z - 2.39$$
$$B_{21}(z) = 1.395z - 1.049$$
$$B_{22}(z) = 0.63z - 4.63$$
$$D_r(z) = z^2 - 0.922z - 0.0425$$

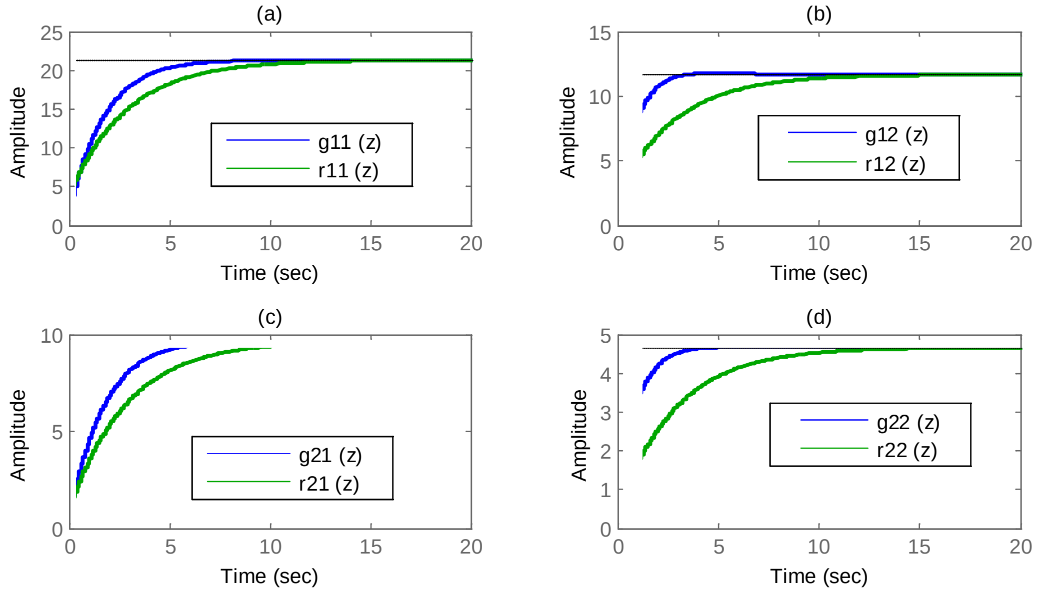

Fig. 3.16 (a) Comparison of step responses for example 3.16

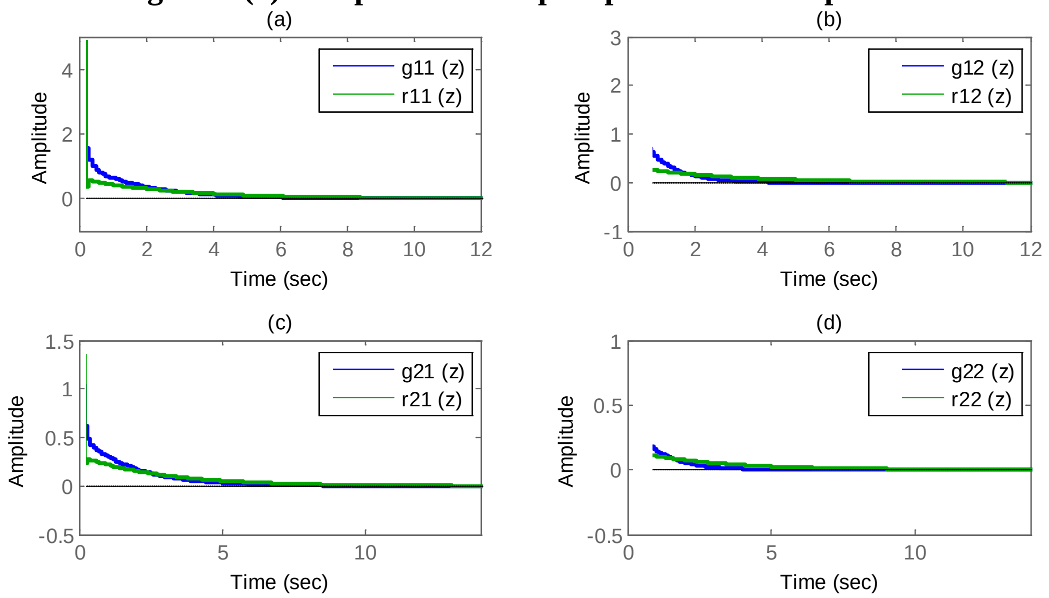

Fig. 3.16 (b) Comparison of impulse responses for example 3.16

Table 3.21 gives the SSE values obtained for r_{ij}. Fig. 3.16 (a)-(b) depicts the step and impulse responses of **[**G_n(z)] and [R(z)] respectively.

Table 3.21 SSE values for example 3.16

r_{ij} (i,j=1-2)	Proposed Method
r_{11}	27.909
r_{12}	43.84
r_{21}	157.2
r_{22}	9.2436

3.5 MODIFIED POLE CLUSTERING AND LEAST SQUARES METHOD

The advantages of modified pole clustering method and least squares method are utilized in proposed method. The results obtained by modified pole cluster method are better than that obtained by pole clustering method. The procedure to compute the coefficients of the reduced system are as follows.

Step 1: G_n(z) in the form of (3.1) and convert using bilinear transformation using (3.3) to give

$$G_n(w) = \frac{N_n(w)}{D_n(w)}$$

Step 2: Find the roots of the denominator polynomial D_n(w), such that $|-p_1| < |-p_2| < ... < |-p_n|$ are the poles of the higher order original system G_n(w).

Step 3: The step 2 and 3 of section 2.3.4 are followed

Step 4: Determine of denominator polynomial D_r(w)

The denominator polynomial is constructed using modified pole cluster centre

$$D_r(w) = (w - p_{e1})(w - p_{e2})...(w - p_{er})$$

The following cases may occur during the formation of the reduced denominator polynomial.

Case 1:If the modified cluster centre's are real, then the rth order reduced denominator is

$$\begin{aligned} D_r(w) &= (w - p_{e1})(w - p_{e2})...(w - p_{er}) \\ &= e_o + e_1 w + e_2 w^2 + ... + e_r w^r \end{aligned} \quad (3.13)$$

*Case 2:*If the modified cluster centre's are complex conjugate, then the rth order reduced denominator obtained is given by

$$D_r(w) = (w - \overset{*}{\phi}_{e1})(w - \overset{\circ}{\phi}_{e2})\ldots(w - \overset{*}{\phi}_{er/2})(w - \overset{\circ}{\phi}_{er/2}) \\ = e_o + e_1 w + e_2 w^2 + \ldots + e_r w^r \quad (3.14)$$

$$\phi_{ei} = A_{ei} \pm jB_{ei}$$

Case 3: If some of the modified cluster centre's are real (assume (r-2)) and others are complex conjugate, then the r[th] order reduced denominator obtained is given by

$$D_r(w) = (w - p_{e1})(w - p_{e2})\ldots(w - p_{e(r-2)})(w - \overset{*}{\phi}_{e1})(w - \overset{\circ}{\phi}_{e2}) \\ = e_o + e_1 w + e_2 w^2 + \ldots + e_r w^r \quad (3.15)$$

$$\phi_{ei} = A_{ei} \pm jB_{ei}$$

Step 5: The numerator terms of N_r(w) are found out by initially computing the time moment proportional's 'c_i' by expanding G_n(w) about w=0 using the formula (3.11). Substitute the value of 'e_i', $i = 0,1,2,\ldots\infty$ and 'c_i' in (2.8) and (2.9) to obtain the coefficients 'd_i', $i = 0,1,2,\ldots(r-1)$ of N_r(w).

Step 6: The ratio of $N_r(w)$, $D_r(w)$ gives $G_r(w)$

$$G_r(w) = \frac{N_r(w)}{D_r(w)}$$

Step 7: Now, applying inverse bilinear transformation using (3.5), the reduced order system G_r(z) is obtained as

$$G_r(z) = \frac{N_r(z)}{D_r(z)}$$

3.5.1 Illustrative examples

Example 3.17: Revisit higher order discrete transfer function [253] taken from example 4 in 3.2.1.

Step 1: The eighth order discrete system G_n(z) is converted using bilinear transformation by substituting $z=(1+w)/(1-w)$ both in the numerator and denominator polynomial according to (3.3) resulting in $N_n(w)$ and $D_n(w)$

Step 2: According to (3.4)

$$G_n(w) = \frac{N_n(w)}{D_n(w)}$$

$$= \frac{-0.002w^7 + 4.866w^6 + 27.814w^5 + 54.058w^4 + 62.106w^3 + 46.214w^2 + 17.986w + 1.998}{8w^8 + 78.64w^7 + 292.928w^6 + 526.816w^5 + 584.144w^4 + 400.24w^3 + 139.232w^2 + 16w + 2}$$

Step 3: The roots of D_n(w), are -0.0463± j0.1362, -0.4281± j0.9899, -0.8380± j0.1839, -3.6025± j1.0655.

Step 4:A second order reduced system is desired, two clusters are formed (-0.0463, -0.4281, -0.8380, -3.6025) and (0.1362, 0.9899, 0.1839, 1.0655).

Step 5: The modified pole cluster centre P_{ei} are computed. Considering the first cluster (-0.0463, -0.4281, -0.8380, -3.6025), the value of x=4. For m=1, according to (3.13), the value of σ_m is obtained as -21.5983.

Step 6: m is incremented to 2 and σ_m=-23.9342.

Step 7: Repeat until the value of m =4 and is equal to the value of x. Using (3.22) the value of σ_m= -0.0508. Therefore the pole centre of cluster (-0.0463, -0.4281, -0.8380, -3.6025) is p_{e1}= -0.0508.

Step 8: Similarly, for the next cluster (0.1362, 0.9899, 0.1839, 1.0655), m=1, σ_m=--7.3421 and m is incremented to 2 .

Step 9: Follow the rules in step 3 of 3.5 till the value of m and x are same. Finally, the value of σ_m will be -0.1453 or pole centre of cluster (0.1362, 0.9899, 0.1839, 1.0655), is p_{e2}= -0.1453.

Step 10: Now using (3.14),

$$D_r(w) = (w + 0.0508 + j0.1453)(w + 0.0508 - j0.1453) = w^2 + 0.1016w + 0.0237$$

Step 11: In order to find the numerator terms of N_r(w), the time moment proportional's 'c_i' are computed by expanding G_n(w) about w=0 by using

$$G_n(w) = \sum_{i=0}^{\infty} c_i\, w^i \tag{3.16}$$

The first five time moment proportionals of G_n(w) as in Table 3.22

Table 3.22 Time moment proportionals obtained for example 3.17

i	c_i
0	0.9990
1	1.0010
2	-54.4474
3	197.0266
4	1749.1254

Substitute the value of 'e_i', $i = 0,1, 2 , ... \infty$ and 'c_i' in (2.8) and (2.9) to obtain the coefficients 'd_i', $i = 0,1, 2 , ...(r-1)$.

Step 12: N_r(w) is then given by $N_r(w) = 0.1252w + 0.02368$

Step 13: The ratio of N_r(w), D_r(w) gives G_r(w).

$$G_r(w) = \frac{N_r(w)}{D_r(w)} = \frac{0.1252w + 0.02368}{w^2 + 0.1016w + 0.0237}$$

Step 14: Applying inverse bilinear transformation using (3.3), the reduced order discrete time system G_r(z) is obtained as (3.4).

$$N_r(z) = N_r(w)|_{w=(z-1)/(z+1)}$$
$$= 0.1252w + 0.02368\,|_{w=(z-1)/(z+1)}$$
$$= 0.1252\frac{(z-1)}{(z+1)} + 0.02368 = 0$$
$$N_r(z) \quad = 0.1252(z-1) + 0.02368(z+1)$$

$$D_r(z) = D_r(w)|_{w=(z-1)/(z+1)}$$
$$= w^2 + 0.1016w + 0.0237\Big|_{w=(z-1)/(z+1)}$$
$$= \frac{(z-1)^2}{(z+1)^2} + 0.1016\frac{(z-1)}{(z+1)} + 0.0237 \quad = 0$$
$$D_r(z) = (z-1)^2 + 0.1016(z-1)(z+1) + 0.0237(z+1)^2$$

Therefore the second order reduced discrete system is

$$G_r^2(z) = \frac{N_r(z)}{D_r(z)} \quad = \frac{2977z - 2030}{11253z^2 - 19526z + 9221} = \frac{0.2645z - 0.180}{z^2 - 1.735z + 0.819}$$

Table 3.23 Comparison of reduced order systems for example 3.17

Order Reduction Method	Reduced System G_r(z)	SSE
Proposed Method	$\frac{0.2645z - 0.180}{z^2 - 1.735z + 0.819}$	0.0101
Hwang *et. al.* [254]	$\frac{0.316331z - 0.262395}{z^2 - 1.73034z - 0.784276}$	0.062
Bistritz [253]	$\frac{0.2696z - 0.2157}{z^2 - 1.73z - 0.7842}$	0.052
Bistritz [132]	$\frac{0.37131242z - 0.298}{z^2 - 1.626873z - 0.701497}$	0.057
Hwang *et. al.* [254]	$\frac{0.3664429z - 0.28918}{z^2 - 1.626873z - 0.701497}$	0.065
Hwang and Shih [248]	$\frac{0.2018z^2 + 0.04484z - 0.156}{1.2z^2 - 1.955z + 0.843}$	0.0791
Prasad [240]	$\frac{0.2871\text{ z - }0.1436}{z^2 - 1.722z + 0.8564}$	0.1765

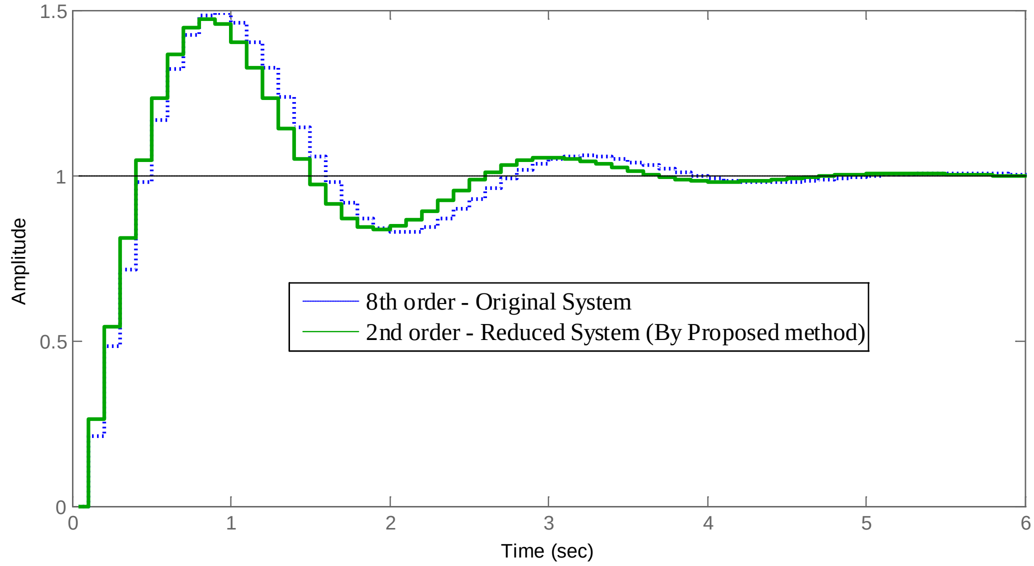

Fig. 3.17 (a) Comparison of step responses for example 3.17

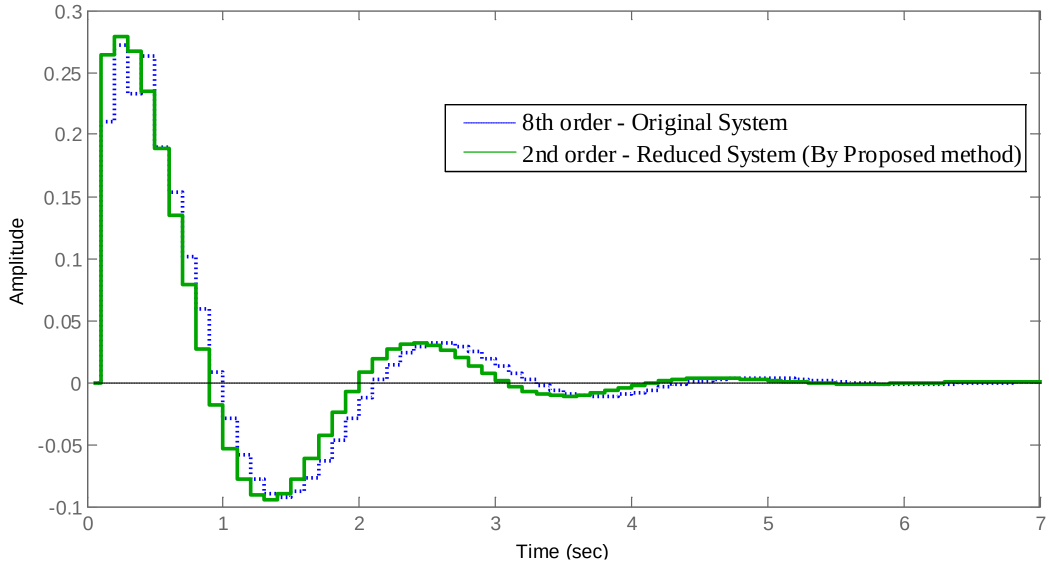

Fig. 3.17 (b) Comparison of impulse responses for example 3.17

The step and impulse responses of the original and reduced system are shown in Fig. 3.17 (a)-(b). These responses are compared in Table 3.23 in terms of SSE with other available methods.

Example 3.18: Consider a supersonic inlet model transfer function taken from Lalonde *et. al.*[176]

$$G_n(z) = \frac{N_n(z)}{D_n(z)} = \frac{2.0434z^6 - 4.982z^5 + 6.57z^4 - 5.819z^3 + 3.636z^2 - 1.41z + 0.2997}{z^7 - 2.46z^6 + 3.433z^5 - 3.33z^4 + 2.546z^3 - 1.584z^2 + 0.7478z - 0.252}$$

G_n(w) is given by

$$G_n(w) = \frac{24.76w^6 + 30.616w^5 + 39.438w^4 + 23.13w^3 + 10.432w^2 + 2.0398w + 0.3377}{15.353w^7 + 20.128w^6 + 42.551w^5 + 23.144w^4 + 20.324w^3 + 4.4992w^2 + 1.9w + 0.1008}$$

It is desired to obtain a second order reduced system. Hence two pole clusters are formed as (-0.0587, -0.0593, -0.1508, -0.4164) and (-0.3668,0.00001, -0.6696, -1.2371). Then the second order reduced system is obtained by following the steps mentioned in 3.5 as

$$G_r^2(z)=\frac{1302z-1175}{2817z^2-4981z+2202}=\frac{0.462z-0.193}{z^2-1.768z+0.782}$$

A third order reduced system can be obtained by forming three clusters (0.0593), (-0.0587± j0.3668) and (-0.1508± j0.6696, -0.4164± j1.2371). The reduced discrete system is obtained by following steps in 3.5, resulting in

$$G_r^3(z)=\frac{1.264z^2-1.855z+0.785}{z^3-2.16z^2+1.926z-0.705}$$

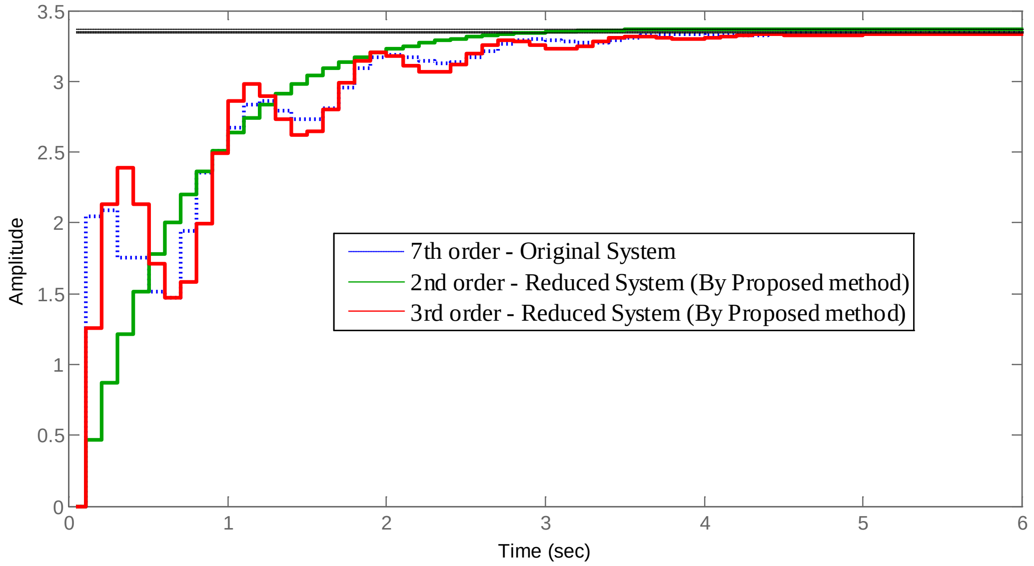

Fig. 3.18 (a) Comparison of step responses for example 3.18

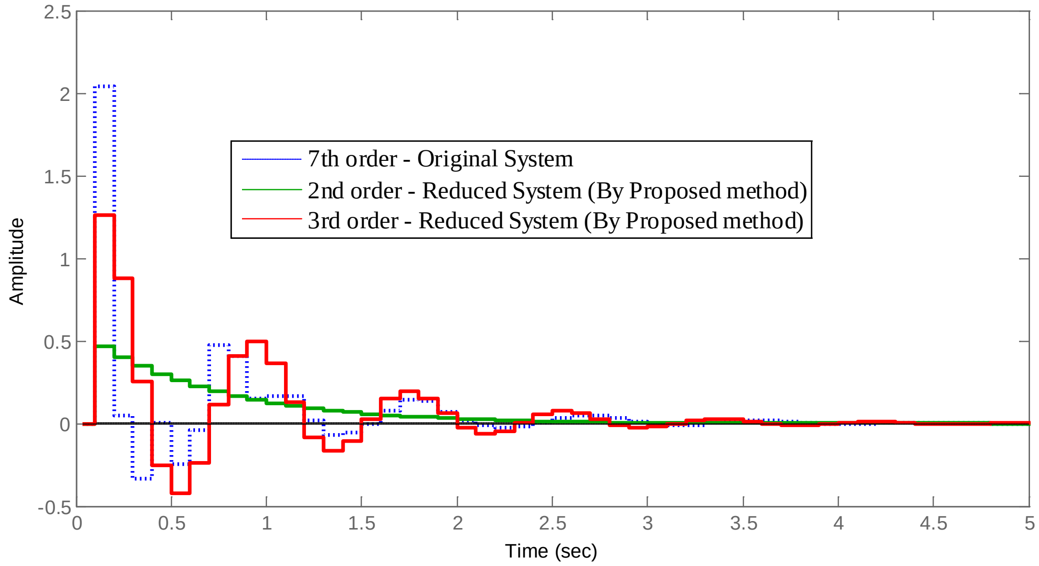

Fig. 3.18 (b) Comparison of impulse responses for example 3.18

Table 3.24 Comparison of reduced order systems for example 3.18

Order Reduction Method	Reduced System G_r(z)	SSE
Proposed Method (second order)	$\frac{0.462z-0.193}{z^2-1.768z+0.782}$	0.5113
Proposed Method (third order)	$\frac{1.264z^2-1.855z+0.785}{z^3-2.16z^2+1.926z-0.705}$	0.15714
Lalonde *et. al.*[176] (third order)	$\frac{0.0627z^3-2.106z^2+1.569z+0.0371}{z^3-0.8204z^2+0.1697z-0.1648}$	0.55879
Lalonde *et. al.*[176] (fifth order)	$\frac{1.72\times10^{-4}z^5+2.1z^4-2.9z^3+2.15z^2-1.5z-0.66}{z^5-1.488z^4+1.231z^3-0.96z^2-0.6693z-0.3247}$	0.17453

The original and reduced system is subjected to step and impulse input. Their corresponding responses are shown in Fig. 3.18 (a)-(b). These responses are compared in Table 3.24 with respect to SSE given by (3.10) with other available methods.

Example 3.19: Consider 8th order discrete time system taken in 3.2.1 in example 3.2.
By following the procedure of the proposed method (3.5), the reduced second order discrete system is

$$G_r^2(z)=\frac{N_r(z)}{D_r(z)}=\frac{0.537z-0.365}{z^2-1.462z+0.635}$$

Table 3.25 Comparison of reduced order systems for example 3.19

Order Reduction Method	Reduced System G_r(z)	SSE
Proposed Method (second order)	$\frac{0.537z-0.365}{z^2-1.462z+0.635}$	0.0053
Chung *et. al.*[252]	$\frac{0.3975z-0.318}{z^2-1.6025z+0.682}$	1.336
Satakshi *et. al.*[251]	$\frac{0.463z+0.30172}{z^2-1.5312z+0.6868}$	0.1466
Prasad [240] (using 2 TM)	$\frac{0.5282z-0.4225}{z^2-1.472z+0.577}$	1.176
Prasad [240] (using one TM and MP)	$\frac{0.420499z-0.31486}{z^2-1.472z+0.577}$	0.4238

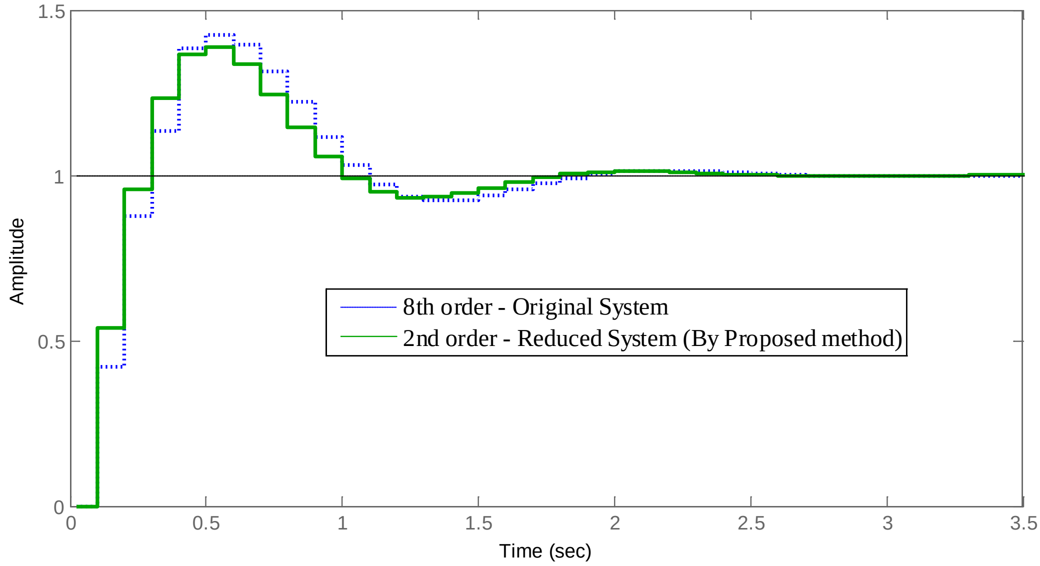

Fig. 3.19 (a) Comparison of step responses for example 3.19

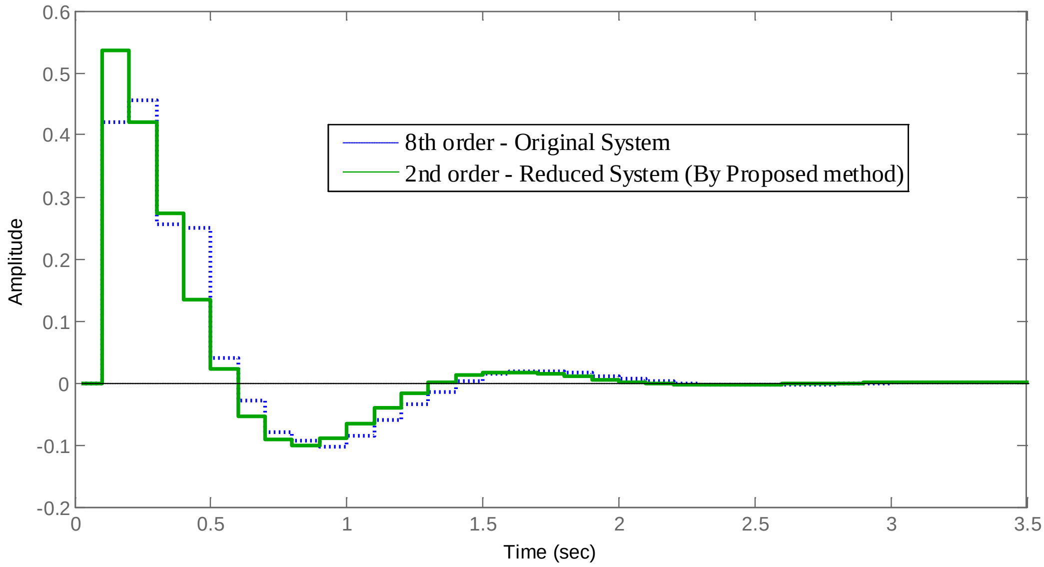

Fig. 3.19 (b) Comparison of impulse responses for example 3.19

The step and impulse responses of the original and reduced system are shown in Fig. 3.19 (a)-(b). These responses are compared in Table 3.25 in terms of SSE with other available methods.

3.5.2 Extension to Multivariable systems

The composite method using modified pole clustering and least square is extended for multivariable systems. The procedure in 3.5.1 is applied separately on each of the elements of the transfer function matrix of MIMO system. Example 3.20 is solved in depth to signify the proposed procedure.

3.5.2.1 Illustrative Examples

Example 3.20: A multivariable system of dimension (m×l) with m=3 , l=2 taken from [250] and also example 3.9 in 3.3.2 is given by

$$[G_n(z)] = \frac{1}{D_n(z)} \begin{bmatrix} A_{11}(z) & A_{12}(z) & A_{13}(z) \\ A_{21}(z) & A_{22}(z) & A_{23}(z) \end{bmatrix}$$

It is desired to reduce G(z) to a reduced order system represented in the form (3.12)

Step 1: Using bilinear transformation **[**G_n(z)] is converted to **[**$G_n(w)$] following step 1 and 2 as in 3.3.2.1.

Step 2: The roots of D_n(w) are (-0.0463 ± j0.1362, -0.4281 ± j0.9899, -0.8380 ± j0.1839, -3.6025 ± j1.0655).Since a second order reduced systems is desired, two clusters (-0.0463, -0.4281, -0.8380, -3.6025) and (0.1362, 0.9899, 0.1839, 1.0655) are formed. Then using step 4, the pole centre are obtained as $p_{e1,2}$=0.0508±j0.1452.

Step 3: Therefore the reduced denominator obtained is

$$D_r(w) = (w + 0.0508 + \text{j}0.1452)(w + 0.0508 - \text{j}0.1452)$$
$$= w^2 + 0.1016\text{w} + 0.0237$$

Step 4: Now, consider each element of **[**G_n(z)] and compute the values of B_{ij} (w-domain) from step 11-12 in 3.5.1 resulting in

$$[R(w)] = \frac{1}{D_r(w)} \begin{bmatrix} B_{11}(w) & B_{12}(w) \\ B_{21}(w) & B_{22}(w) \end{bmatrix}$$

$$B_{11}(w) = 0.07331w + 0.02553 \qquad B_{12}(w) = 0.02711w - 0.00317$$
$$B_{13}(w) = 0.03131w - 0.001033 \qquad B_{21}(w) = 0.05627w + 0.01181$$
$$B_{22}(w) = 0.05631w + 0.003828 \qquad B_{23}(w) = 0.0607w + 0.01446$$
$$D_r(w) = w^2 + 0.1016w + 0.0237$$

Step 5: Now**,** convert B_{ij} in w-domain to z-domain using inverse bilinear transformation using (3.5). Similarly convert D_r(w) to D_r(z). Substituting the B_{ij} (z-domain) and D_r(z) in (3.12) ,

$$[R(z)] = \frac{1}{D_r(z)} \begin{bmatrix} B_{11}(z) & B_{12}(z) \\ B_{21}(z) & B_{22}(z) \end{bmatrix}$$

$$B_{11}(z) = 0.175z - 0.0849 \qquad B_{12}(z) = 0.0425z - 0.0538$$
$$B_{13}(z) = 0.0538z - 0.0574 \qquad B_{21}(z) = 0.121z - 0.079$$
$$B_{22}(z) = 0.1068z - 0.0932 \qquad B_{23}(z) = 0.133z - 0.082$$
$$D_r(z) = w^2 - 1.735w + 0.819$$

Table 3.26 Comparison of SSE for example 3.20

r_{ij} (i,j=1-3)	Proposed Method	Prasad [240]
r_{11}	0.0058	0.04475
r_{12}	0.0011	0.00203
r_{13}	0.00050	3.5437
r_{21}	0.00151	0.01026
r_{22}	0.00576	0.006618
r_{23}	0.00432	0.01747

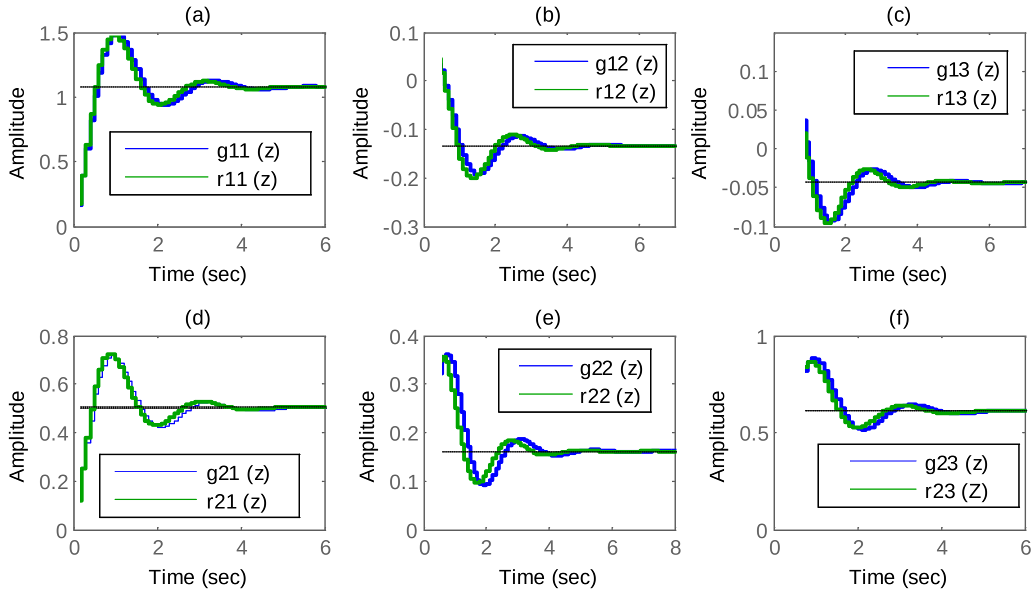

Fig. 3.20 (a) Comparison of step responses for example 3.20

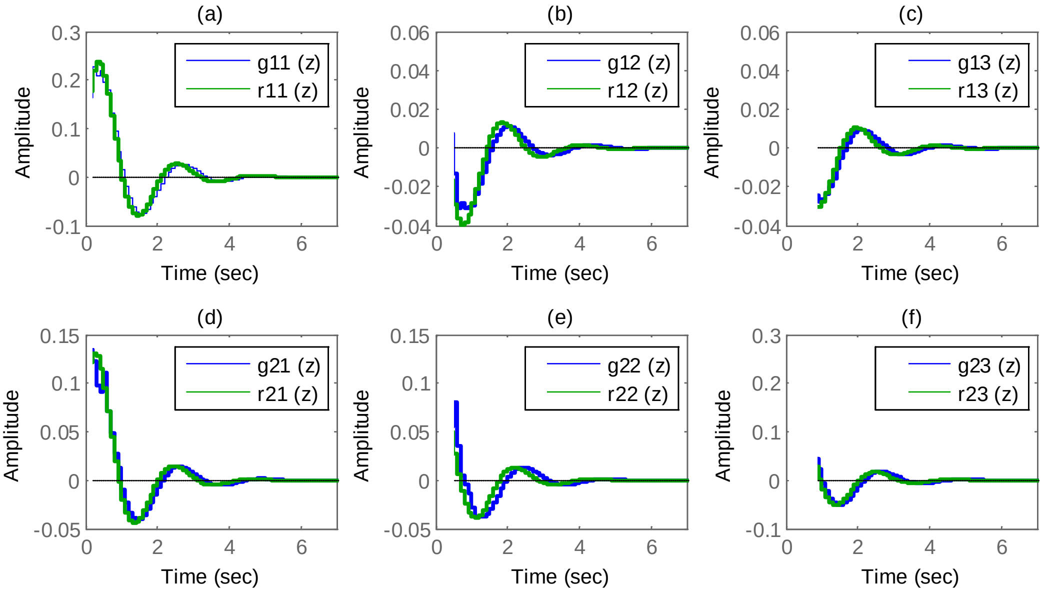

Fig. 3.20 (b) Comparison of impulse responses for example 3.20

The results are tabulated in Table 3.26 in terms of *SSE* for each element of transfer function matrix. The unit step and impulse responses of **[**G_n(z)] and [*R*(z)] are depicted in Fig. 3.20 (a)-(b).

Example 3.21: Given a z-transfer function matrix of example 3.10 in 3.3.2

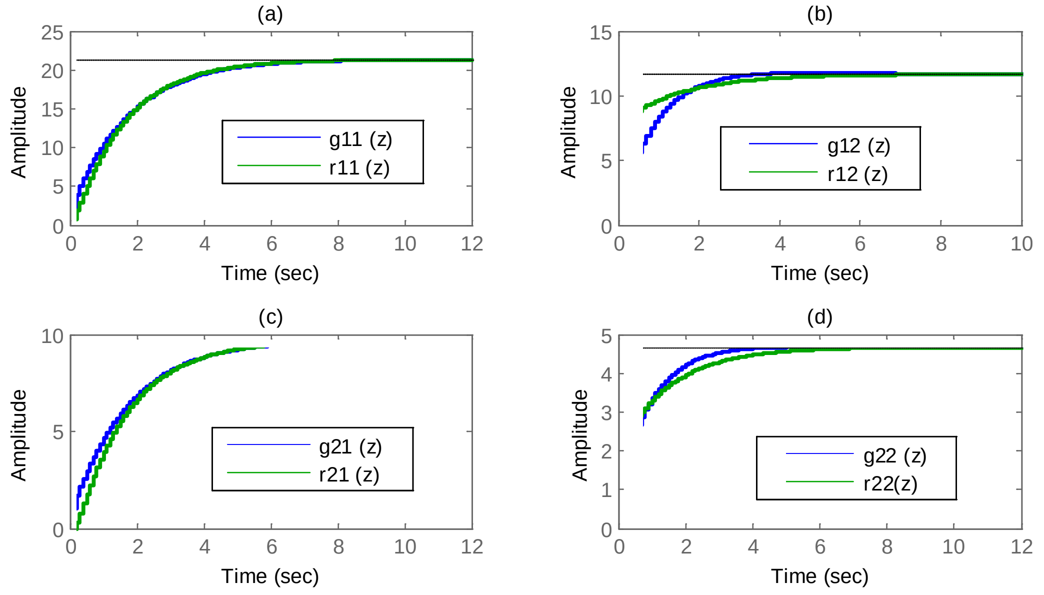

Fig. 3.21 (a) Comparison of step responses for example 3.21

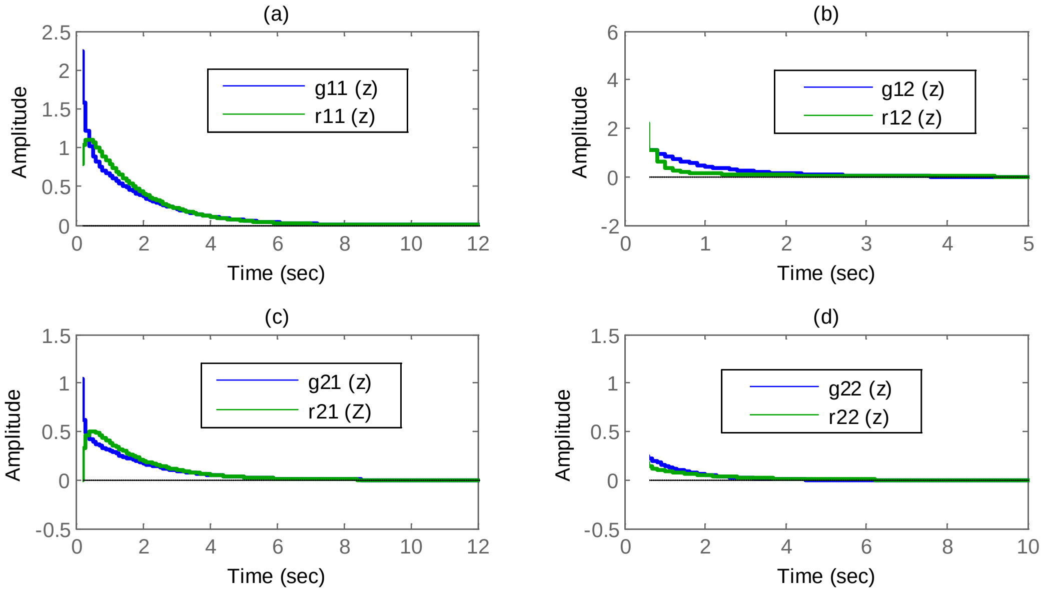

Fig. 3.21 (b) Comparison of impulse responses for example 3.21

The reduced system obtained by following steps 1-5 of example 3.20 in 3.5.2.1 obtained is

$$[R(z)] = \frac{1}{D_r(z)} \begin{bmatrix} B_{11}(z) & B_{12}(z) \\ B_{21}(z) & B_{22}(z) \end{bmatrix}$$

$$B_{11}(z) = 0.789z - 0.0723 \qquad B_{12}(z) = 4.472z - 4.08$$
$$B_{21}(z) = -0.0044z + 0.329 \quad B_{22}(z) = 1.26z - 1.116$$

and

$$D_r(z) = z^2 - 1.4z + 0.4354$$

Table 3.27 represents the SSE values obtained for r_{ij} by the proposed method. Fig. 3.21 (a)-(d) depicts the step, impulse responses of **[**G_n(z)] and [R(z)] respectively and are found to be comparable.

Table 3.27 SSE for example 3.21

r_{ij} (i,j=1-2)	Proposed Method
r_{11}	2.94
r_{12}	8.931
r_{21}	1.395
r_{22}	0.2344

The step and impulse responses of the examples (SISO and MIMO) solved in this section by the introduced method justifies its applicability. Further, the proposed reduced systems are also compared with the other available methods in terms of SSE and tabulated.

3.6 CONCLUSION

The proposed mixed methods for continuous time systems are extended the higher order discrete time systems. This is achieved by first transforming the given original discrete time system into either *p* and *w* domain and then the transformed system is reduced. Later on, the reduced system in *p/w* domain is reverted back to *z*- domain. The proposed methods are justified for both SISO and MIMO systems with the aid of numerical examples. The stable denominators are found by using ESA or stability equation or dominant pole retention or modified pole clustering method. Hence, the reduced systems are stable in nature for stable original system. The other advantages of proposed methods are:

1. Computationally simpler and flexible.

2. It is validated that, the methods can be applied for MIMO systems even though the number of inputs and outputs are unequal.
3. Algorithm for minimization process is not at all required.

Further, the results viz. step, impulse responses for original and reduced discrete time systems are plotted. The quality of these responses are compared with the responses obtained by other methods available in the literature in terms of SSE and are tabulated accordingly. Lower value of SSE is always desirable, as it signifies the closeness between the step response of the original and reduced discrete time systems. The same is justified in the examples (3.1 to 3.21) solved.

CHAPTER - 4

REDUCTION OF LTI SYSTEMS USING EVOLUTIONARY TECHNIQUES

The unending need for miniaturization, driven by the demand for increased system complexity, necessitates the speeding up of simulation process in the design and validation stage. This further ends up as a necessary procedure for simulating large complex systems. Abundant order reduction techniques are available in literature today [23, 30, 140] and the best technique is one which shields the vital dynamics of the system under consideration. Also, disentangle the best available model in light of the purpose for which the model is to be used- namely, to design a control system to meet certain specifications that helps to find reduced order approximated models, without incurring too much error. For such complex problems, nature inspired approaches are among the methods, that have proved to be useful.

Approximation algorithms were formally introduced about four decades ago [245] to generate near-optimal solutions to optimization problems that could not be solved efficiently by the computational techniques available at that time. Later, introduction of new methodologies referred to as metaheuristics kicked off a new wave with the tremendous growth of their usefulness for solving practical problems. Today, the evolutionary optimization methods namely Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) have fascinated both researchers as well as industry to solve a wide variety of problems including the reduction of large scale systems [255, 256]. A possibility of combining conventional method and optimization techniques has also proved to be fruitful in the order reduction domain [68, 135, 233]; further these techniques have influenced deeply in the controller design as well [257, 258]. In spite of these current optimization methods, there is a great emphasis for the advancement of the so called global optimization methods [135]. Researchers are still striving to attain a universal optimization method that can be applied to all multifaceted problems with equal efficiency. In the chapters discussed till now, conventional methods were used to develop new reduction methods for reducing higher order systems.But, in this chapter nature inspired approach called BBBC having proven track record is being introduced and explored for the purpose of order reduction [233].

In recent times, BBBC method of evolutionary computation has been praised for effectively solving issues related to areas such as target motion analysis, fuzzy model inversion, space trusses design, nonlinear controller design and airport gate assignment problem [259-261]. The significant features of this method is that it converges quicker, requires no rigid first guess algorithms and explores the majority of problem space; it is simple, intuitive in nature and easy to implement [262, 263]. Further, it is unfussy to code and understand its most basic form. Hence, it is found to be useful in solving mixed integer optimization problems that are typical of complex engineering systems [263]. In this chapter, a mixed method of order reduction technique combining the merits of BBBC with Routh Approximation (RA)[263], Stability Equation (SE) [261]method are utilized to meet the purpose. BBBC, being a numerical optimizer, assists in rationally searching the best values among the alternative ones, to suit the needs of the system. Stable reduction methods such as RA and SE maintains stability while matching the steady state. This new combinatorial method comes out to be comparable than the other conventional techniques available in the literature.

4.1 BIG BANG BIG CRUNCH ALGORITHM

The BBBC method, was developed and proposed as a novel optimization method by Erol and Eksin [264] in 2006; is inspired from one of the evolution of the universe theories in physics and astronomy; describes how the universe was created, evolved and would end namely the BBBC. The Big Bang(BB) phase is random energy dissipation over the entire search space or the transformation from an ordered state to a disordered or chaotic state. After the BB phase, a contraction occurs in the Big Crunch(BC) phase. Here, the particles that are randomly distributed are drawn into an order. This reduces the computational time and has quick convergence even in long, narrow parabolic shaped flat valleys.

The BB phase is similar to the creation of initial random population in GA. The designer should handle the impermissible candidates at this phase. Once the population is created, fitness values of the individuals are calculated [263, 265]. The crunching phase is a convergence operator that has many inputs but only one output, which can be named as the centre of 'mass'. The center of mass is the weighted average of the candidate solution positions. After a number of sequential banging and crunching phases, the algorithm converges to a solution. The term ‘mass’ refers to the inverse of the merit function value. The point representing the center of mass ‘X_c’ of the population is calculated according to the formula

$$X_c = \frac{\sum_{k=0}^{N-1} \frac{X_k}{E}}{\sum_{k=0}^{N-1} \frac{1}{E}} \tag{4.1}$$

where 'X_k' is a point within an n-dimensional search space generated. Here, it is related to the numerator polynomial coefficients. '*E*' is a fitness function or objective value of the candidate *k*, '*N*' is the population size in banging phase. The convergence operator in the crunching phase is different from wild selection, since the output term may contain additional information (new candidate or member having different parameters than others) than the participating ones. In the next cycle of the BB phase, new solutions are created by using the previous knowledge (center of mass). The fitness function '*E*' [264] is

$$E = \sum_{i=0}^{M-1} [y(i\Delta t) - y_r(i\Delta t)]^2 \tag{4.2}$$

where $M = \frac{T}{\Delta t}$

$y(i\Delta t)$ and $y_r(i\Delta t)$ are the unit step responses of the given original and reduced model at time $t=\Delta t$. Usually time 'T' is taken as 10 sec and $\Delta t = 0.1$ sec.

The basic BBBC algorithm is as follows.

Step 1:[Start] The big bang starts by generating the new population.

Step 2:[Evaluate Fitness value] For each iteration, the algorithm will act such that each candidates will move in a direction to improve its fitness function. The action involves movement updating of individuals and evaluating the fitness function for the new position.

Step 3:[Compare Fitness Function]Compare the fitness function of the new position with the specified fitness function. Repeat the above steps for the whole candidates.

Step 4:[Maximum iteration] Check if maximum iteration is reached or a specified termination criteria is satisfied. Stop and return the best solution, otherwise update and go to the next iteration.

Step 5:[Loop] Go to step2 for fitness evaluation.

Table 4.1 Typical parameters for BBBC

Population size	75
Number of iterations	50
Reduction rate	0.75
Termination method	Maximum generation

The basic flowchart and typical parameters of the BBBC algorithm[261] is depicted in Fig. 4.1 and Table 4.1 respectively.

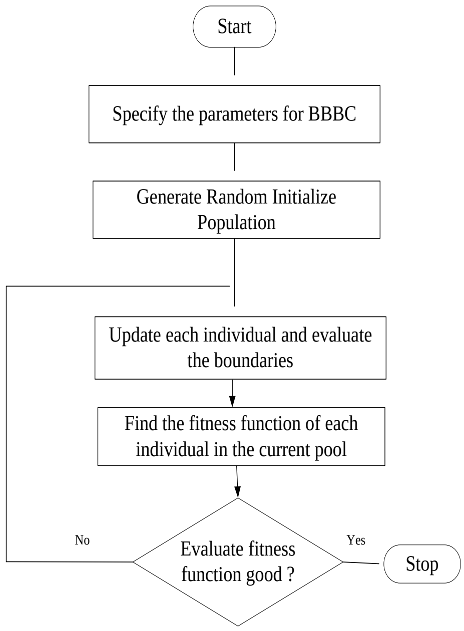

Fig. 4.1 Basic flowchart of BBBC

4.2 STATEMENT OF THE PROBLEM

The original nth order LTI system is taken into account having

Case a) Single Input and Single Output (SISO).

The LTI-SISO system is represented by the transfer function

$$G_n(s) = \frac{N_n(s)}{D_n(s)} = \frac{a_o + a_1 s + \ldots + a_m s^m}{b_o + b_1 s + b_2 s^2 + \ldots + b_n s^n} \; ; \quad m < n \tag{4.3}$$

The objective is to compute r^{th} (r < n) order reduced system $G_r(s)$ from (4.3) and represent in the form of

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{d_o + d_1 s + \ldots + d_q s^q}{e_o + e_1 s + e_2 s^2 + \ldots + e_r s^r} \; ; \quad q < r \tag{4.4}$$

where a_i, b_i, d_j and e_j's are the scalar constants.

Case b) Multiple Input Multiple Output (MIMO)

Let the n^{th} order MIMO system having *'p'* inputs and *'m'* outputs be described as

$$[G_n(s)] = \frac{1}{D_n(s)} \begin{bmatrix} A_{11}(s) & A_{12}(s) & A_{13}(s) & \cdots & A_{1p}(s) \\ A_{21}(s) & A_{22}(s) & A_{23}(s) & \cdots & A_{2p}(s) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ A_{m1}(s) & A_{m2}(s) & A_{m3}(s) & \cdots & A_{mp}(s) \end{bmatrix}$$

$$[G_n(s)] = [g_{ij}(s)]\ ,\ i=1,2,\ldots m;\ j=1,2,\ldots p \qquad (4.5)$$

The general form of $g_{ij}(s)$ of $[G_n(s)]$ in (4.5) will be

$$g_{ij}(s) = \frac{A_{ij}(s)}{D_n(s)}$$
$$= \frac{A_0 + A_1 s + A_2 s^2 + \ldots + A_{n-1} s^{n-1}}{b_0 + b_1 s + b_2 s^2 + \ldots + b_{n-1} s^{n-1} + b_n s^n} \qquad (4.6)$$

where A_i constants are matrices of appropriate dimension, b_i (i = 0,1,2…n-1) are scalar constants.

The objective is to find the r[th] (r<n) order reduced system $[R(s)]$ having *'p'* inputs and *'m'* outputs described by

$$[R(s)] = \frac{1}{D_r(s)} \begin{bmatrix} B_{11}(s) & B_{12}(s) & B_{13}(s) & \cdots & B_{1p}(s) \\ B_{21}(s) & B_{22}(s) & B_{23}(s) & \cdots & B_{2p}(s) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ B_{m1}(s) & B_{m2}(s) & B_{m3}(s) & \cdots & B_{mp}(s) \end{bmatrix} \qquad (4.7)$$

or, $[R(s)] = [r_{ij}(s)],\ i=1,2,\ldots m;\ j=1,2,\ldots p$

The general form of $r_{ij}(s)$ of $[R(s)]$ in (4.7) could be

$$r_{ij}(s) = \frac{B_{ij}(s)}{D_r(s)}$$
$$= \frac{B_0 + B_1 s + B_2 s^2 + \ldots + B_{r-1} s^{r-1}}{d_0 + d_1 s + d_2 s^2 + \ldots + d_{r-1} s^{r-1} + d_r s^r} \qquad (4.8)$$

where B_i are constant matrices, d_i (i = 0,1,2…r-1) are scalar constants.

4.3 ROUTH APPROXIMATION AND BBBC METHOD

In this novel method, the merits of BBBC and RA [45] method is reaped to meet the desired objective. RA is used to determine the stable reduced denominator polynomial while BBBC completes the reduced system by finding the numerator polynomials. The algorithm for finding the aimed reduced order system is as follows.

i) Determination of the denominator polynomial

Step 1: Given a stable original SISO system (4.3), the denominator $D_n(s)$ is reciprocated resulting in $\tilde{D}_n(s)$

$$\tilde{D}_n(s) = s^n D_n\left(\frac{1}{s}\right)$$
$$= b_0 s^n + b_1 s^{n-1} + b_2 s^{n-2} + \ldots + 1 \qquad (4.9)$$

Step 2: Form the alpha table as shown in Table 4.2 from the coefficients of $\widetilde{D_n}(s)$ and obtain the values of $\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_n$ parameters.

Table 4.2 Alpha Table

	$b_0^0 = b_0$	$b_2^0 = b_2$	$b_4^0 = b_4$	$b_6^0 = b_6$	$\ldots$
	$b_0^1 = b$	$b_2^1 = b_3$	$b_4^1 = b_5$	$\ldots$	
$\alpha_1 = \dfrac{b_0^0}{b_0^1}$	$b_0^2 = b_2^0 - \alpha_1 \alpha_2^1$	$b_2^2 = b_4^0 - \alpha_1 \alpha_4^1$	$b_4^2 = b_6^0 - \alpha_1 \alpha_6^1$	$\ldots$	
$\alpha_2 = \dfrac{b_0^1}{b_0^2}$	$b_0^3 = b_2^1 - \alpha_2 \alpha_2^2$	$b_2^3 = b_4^1 - \alpha_2 \alpha_4^2$	$\ldots$		
$\alpha_3 = \dfrac{b_0^2}{b_0^3}$	$b_0^4 = b_2^2 - \alpha_3 \alpha_2^3$	$b_2^4 = b_4^2 - \alpha_3 \alpha_4^3$	$\ldots$		
$\alpha_4 = \dfrac{b_0^3}{b_0^4}$	$b_0^5 = b_2^3 - \alpha_4 \alpha_2^4$	$\ldots$	$\ldots$		
$\alpha_5 = \dfrac{b_0^4}{b_0^5}$	$b_0^6 = b_2^4 - \alpha_5 \alpha_2^5$	$\ldots$	$\ldots$		
$\ldots$	$\ldots$	$\ldots$			

Step 3: Compute the r[th] order denominator polynomial using

$$\begin{aligned} &\tilde{D}_r(s) = \alpha_r s D_{r-1}(s) + D_{r-2}(s) \ \textit{for} \ r = 1,2,... \\ &\textit{and} \ D_{-1}(s) = D_0(s) = 1 \end{aligned} \tag{4.10}$$

Step 4: Apply the reciprocal transformation to $\widetilde{D_r}(s)$ to obtain the reduced denominator $D_r(s)$ of the reduced system

$$D_r(s) = s^r \tilde{D}_r\left(\frac{1}{s}\right) \tag{4.11}$$

ii) Determination of the numerator polynomial

In the present study, the numerator coefficients are obtained by using BBBC to minimize the fitness function '*E*' which is the error between the transient responses of the given original system and the reduced order system as given in (4.2). The IRE values represented by '*J*' is calculated by using the formula (2.22).

4.3.1 Illustrative examples

The proposed method is justified by solving the following numerical examples taken from the available literature. The step, frequency responses of the original, reduced systems are plotted and compared. Further, the results obtained are compared with other methods in terms of '*E*', '*J*' using (4.2) and (2.22) respectively.

Example 4.1: Consider a fourth-order system [135, 147] with J_{org}= 29.86 described by the transfer function as

$$G_n(s) = \frac{N_n(s)}{D_n(s)} = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24}$$

Step 1: Apply the reciprocal transformation to $D_n(s)$ resulting in

$$\tilde{D}_n(s) = s^n D_n\left(\frac{1}{s}\right)$$

$$= 1 + 10s + 35s^2 + 50s^3 + 24s^4$$

Step 2: Forming the alpha array as

	24	35	1
	50	10	
α_1 = 0.48	30.2	1	
α_2=1.655	8.345	0	

Step 3: The second order reduced denominator is then obtained by using

$$\tilde{D}_r(s) = 1 + \alpha_1 s + \alpha_1\alpha_2 s^2$$

$$= 1 + 1.65s + 0.7944s^2$$

Step 4: Reverting back by reciprocating once again

$$D_r(s) = s^2 + 1.65s + 0.7944$$

Step 5: Using BBBC, the numerator coefficients are generated according to (4.2) for initial population size as 75 feasible solutions, the number of iterations is limited to 50 and reduction rate is fixed at 0.75.

$$N_r(s) = 0.8058s + 0.7944$$

Step 6: Thus the reduced second order system is given as

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{0.8058s + 0.7944}{s^2 + 1.65s + 0.7944}$$

The reaction curves of the original system G_n(s), the proposed reduced system G_r(s) are as compared in Fig. 4.2 (a). Similarly, the frequency responses are compared and displayed in Fig. 4.2 (b). It is seen that the responses are matching both in steady and transient states. Table 4.3 exhibits the superiority of the proposed method by comparing with the alternative methods available in terms of fitness function and IRE.

Table 4.3 Comparison of reduced order systems for example 4.1

Order Reduction Method	Reduced System G_r(s)	'*E*'	'*J*'
Proposed Method [263]	$\frac{0.8058s+0.7944}{s^2+1.65s+0.7944}$	0.24×10^{-3}	28.2
Philip and Pal [233]	$\frac{0.9315s+1.609}{s^2+2.756s+1.609}$	1.72×10^{-3}	29.7
Chen *et. al.* [53]	$\frac{0.699\,s+0.699}{s^2+1.45771\,s+0.699}$	33.3×10^{-3}	33.4
Gutmen *et. al.*[47]	$\frac{2(48\,s+144)}{70s^2+300s+288}$	45.6×10^{-3}	79.7
Krishnamurthy and Seshadri [49]	$\frac{20.5714\,s+24}{30s^2+42\,s+24}$	8.9×10^{-3}	47.8
Lucas [52]	$\frac{0.833\,s+2}{s^2+3\,s+2}$	0.328×10^{-3}	48.4
Mittal *et. al.*[165]	$\frac{0.799\,s+2}{s^2+3\,s+2}$	0.267×10^{-3}	47.2
Moore [204]	$\frac{0.8217\,s+0.4543}{s^2+1.268\,s+0.4663}$	2.9×10^{-3}	50.0
Mukherjee and Mishra [147]	$\frac{0.800000033\,s+2}{s^2+3\,s+2}$	0.237×10^{-3}	47.2
Pal [57]	$\frac{16\,s+24}{30s^2+42\,s+24}$	11.1×10^{-3}	49.1
Prasad and Pal [234]	$\frac{s+34.2465}{s^2+239.8082\,s+34.2465}$	1331×10^{-3}	16.6
Safonov and Chang [235]	$\frac{0.8213\,s+0.4545}{s^2+1.268\,s+0.4664}$	2.855×10^{-3}	50.1
Safonov *et. al.*[236]	$\frac{0.7431\,s+1.057}{s^2+1.879\,s+1.084}$	0.622×10^{-3}	47.3

Example 4.2. Consider the system described by the transfer function [68] with J_{org}= 5.19 x 10^2

$$G_n(s) = \frac{N_n(s)}{D_n(s)} = \frac{10s^4 + 82s^3 + 264s^2 + 369s + 156}{2s^5 + 21s^4 + 84s^3 + 173s^2 + 148s + 40}$$

The reciprocal transformation for D_n(s) is applied and the steps 2-6 in 4.3.1 are followed resulting in the reduced system as

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{2.60001s + 1.03701}{s^2 + 1.01s + 0.26651}$$

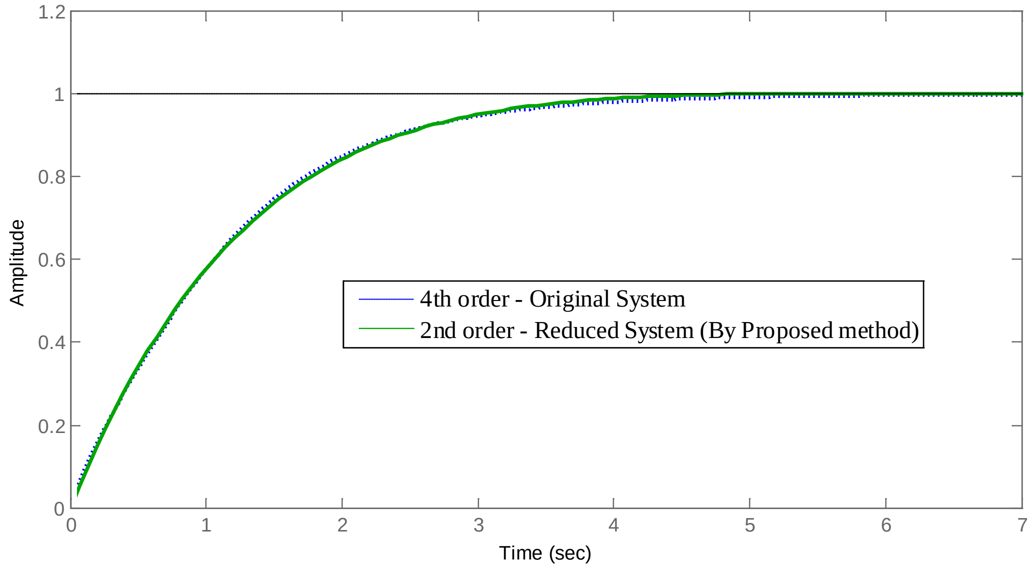

Fig. 4.2 (a) Comparison of step responses for example 4.1

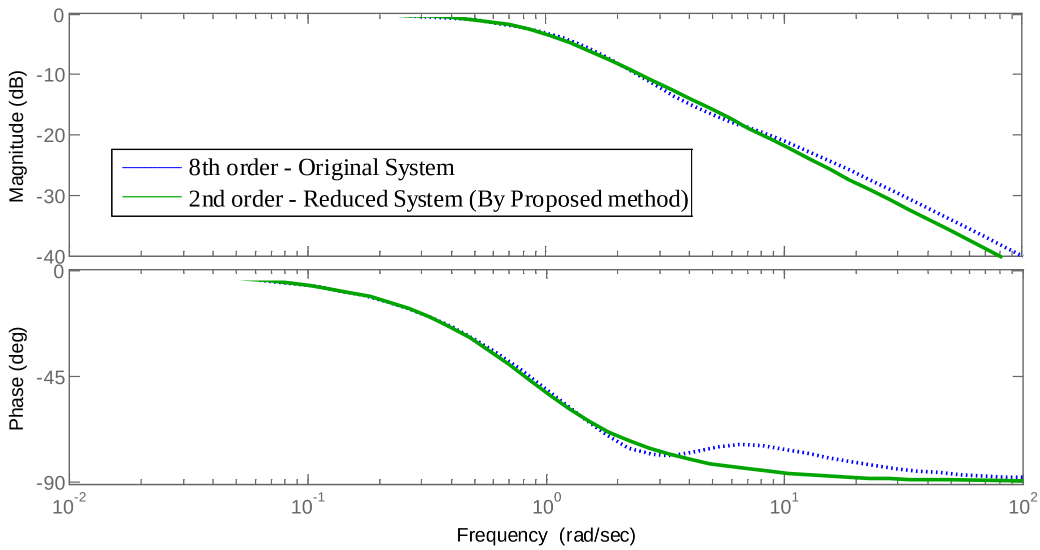

Fig. 4.2 (b) Comparison of frequency responses for example 4.1

The comparison of step, frequency responses are depicted in Fig. 4.3 (a) and Fig. 4.3 (b) respectively. The values of '*E*' and '*J*' are compared in Table 4.4 with other methods.

Table 4.4 Comparison of reduced order systems for example 4.2

Order Reduction Method	Reduced System $G_r(s)$	'*E*'	'*J*'
Proposed Method	$\frac{2.60001s+1.03701}{s^2+1.01s+0.26651}$	0.275	3.29×10^2
Panda et. al. [68]	$\frac{369\ s+156}{239.5s^2+148\ s+40}$	1.567	2.45×10^2
Krishnamurthy and Seshadri [266]	$\frac{5s^2+2.045s+2.4729}{s^3+2.709s^2+2.3176\ s+0.6264}$	5.057	0.406×10^2
Panda et. al. [68]	$\frac{347.025\ s+225.61}{135.68s^2+166.38\ s+57.85}$	0.283	3.39×10^2

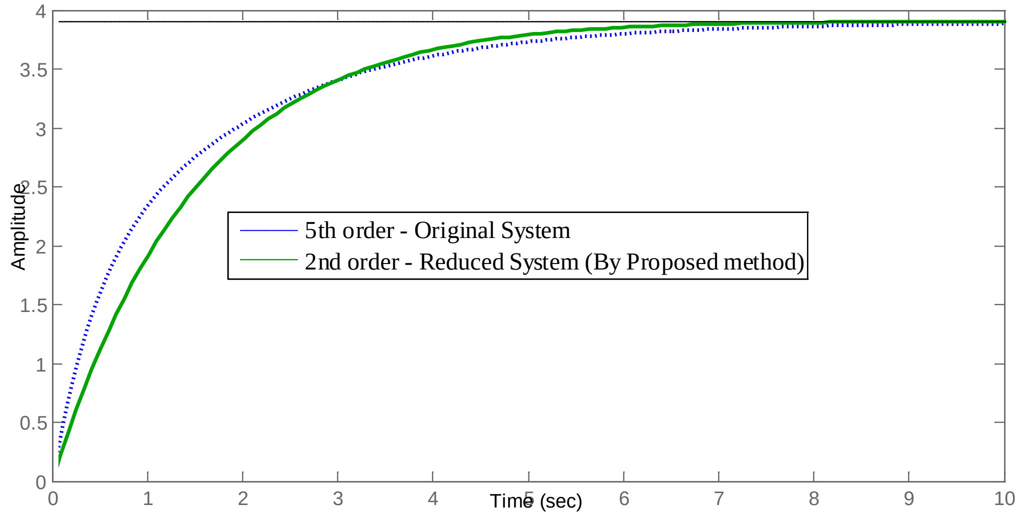

Fig. 4.3 (a) Comparison of step responses for example 4.2

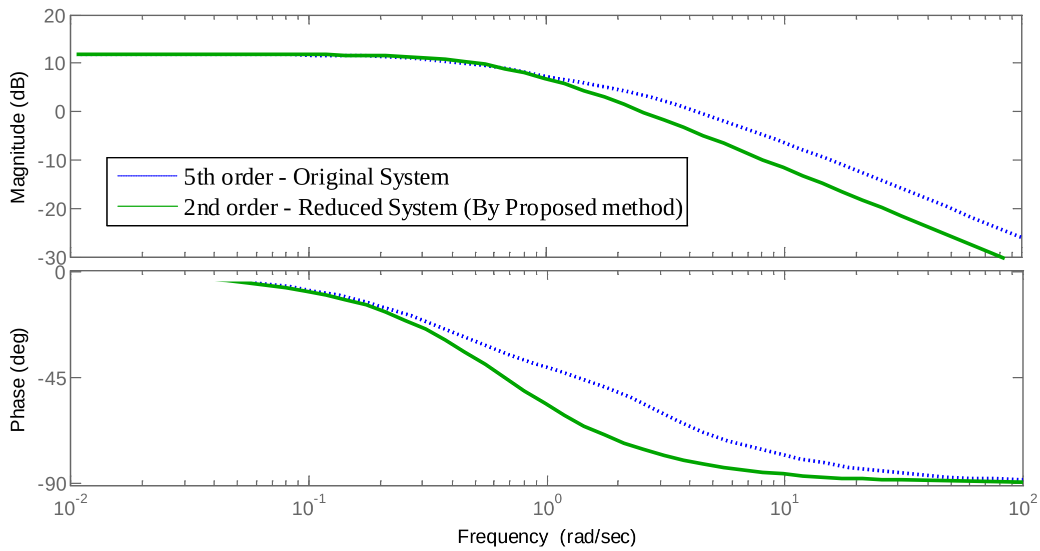

Fig. 4.3 (b) Comparison of frequency responses for example 4.2

4.3.2 Extension to Multivariable Systems

The proposed reduction procedure described in 4.3 is extended to systems having multiple input and multiple outputs. The technique involves direct application of the proposed method on each element of the transfer function matrix of MIMO system as discussed below.

An n^{th} order MIMO system having *'p'* inputs and *'m'* outputs of the form (2.23) is considered. The motive is to find the r^{th} (r<n) order reduced system [R(s)] having *'p'* inputs and *'m'* outputs described by (2.25). The proposed method is applied to (2.23) by following the steps described in 4.3. To start with, the denominator D_n(s) is reduced using RA. Then the coefficients of the numerator polynomials of each element of [G_n(s)] is found out by BBBC. The method proposed is verified by solving an example.

4.3.2.1 Illustrative Examples

Example 4.3: Consider a sixth order two input two output system [135] having transfer function matrix

$$[G(s)] = \begin{bmatrix} \dfrac{2(s+5)}{(s+1)(s+10)} & \dfrac{(s+4)}{(s+2)(s+5)} \\ \dfrac{(s+10)}{(s+1)(s+20)} & \dfrac{(s+6)}{(s+2)(s+3)} \end{bmatrix}$$

$$= \frac{1}{D_n(s)} \begin{bmatrix} A_{11}(s) & A_{12}(s) \\ A_{21}(s) & A_{22}(s) \end{bmatrix}$$

The denominator D_n(s) is given by

$$D_r(s) = (s+1)(s+2)(s+3)(s+5)(s+10)(s+20)$$
$$= s^6 + 41s^5 + 571s^4 + 3491s^3 + 10060s^2 + 13100s + 6000$$

and

$$A_{11}(s) = 2s^5 + 70s^4 + 762s^3 + 3610s^2 + 7700s + 6000$$
$$A_{12}(s) = s^5 + 38s^4 + 459s^3 + 2182s^2 + 4160s + 2400$$
$$A_{21}(s) = s^5 + 30s^4 + 331s^3 + 1650s^2 + 3700s + 3000$$
$$A_{22}(s) = s^5 + 42s^4 + 601s^3 + 3660s^2 + 9100s + 6000$$

A second order reduced system is desired of the form

$$[R\ (s)] = \frac{1}{D_r(s)} \begin{bmatrix} B_{11}(s) & B_{12}(s) \\ B_{21}(s) & B_{22}(s) \end{bmatrix}$$

Step 1: Consider the denominator polynomial D_n(s) and obtain the reciprocated denominator $\widetilde{D}_n(s)$ according to step 1 in 4.3.1.

Step 2: Apply the RA technique to $\widetilde{D}_n(s)$ by forming the alpha table as in step 2 of 4.3.1. Therefore the second order polynomial D_r(s) obtained, after reciprocating back will be

$$\widetilde{D}_r(s) = 1 + 1.548267s + 0.7091s^2 \quad \text{or} \quad D_r(s) = s^2 + 1.548267s + 0.7091$$

Step 3: Applying the BBBC algorithm for minimizing (4.2) for each element of the transfer function matrix [G(s)], the reduced order elements B_{ij} of the reduced system [R(s)] are obtained as [263]

$$B_{11}(s) = 0.9475s + 0.7091; \qquad B_{12}(s) = 0.4892s + 0.2837$$
$$B_{21}(s) = 0.455s + 0.3546\ ; \qquad B_{22}(s) = 1.126s + 0.7091$$

The step responses of the original, the proposed reduced system are depicted in Fig. 4.4 (a) for inputs $\begin{bmatrix}1\\0\end{bmatrix}$ and $\begin{bmatrix}0\\1\end{bmatrix}$. Similarly Fig. 4.4 (b) displays the comparison of frequency response of each element of the original and reduced transfer function matrix. The error analysis is also carried out and compared with other methods in Table 4.5.

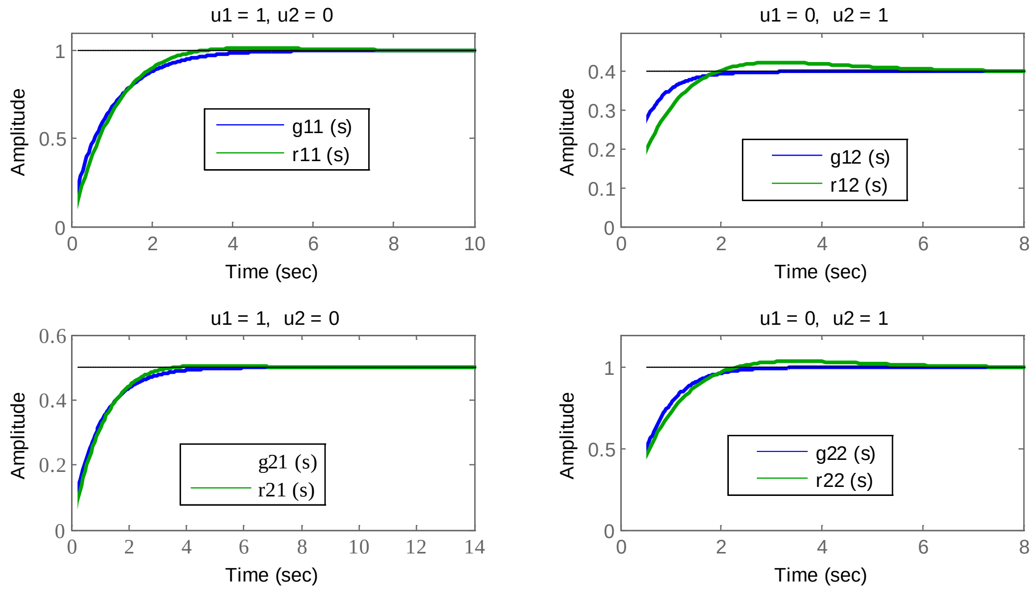

Fig. 4.4 (a) Comparison of step responses for example 4.3

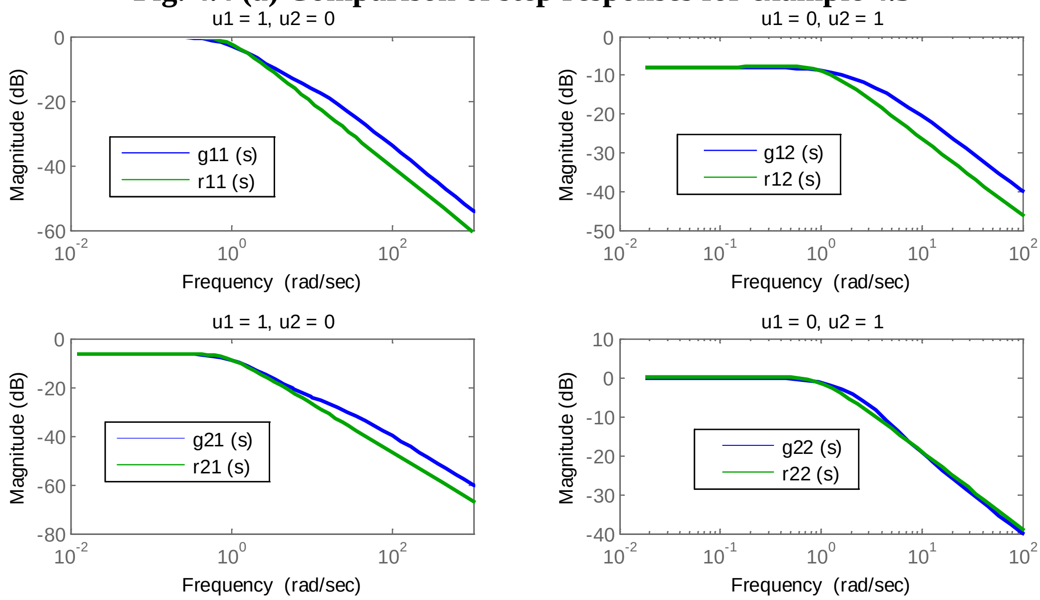

Fig. 4.4 (b) Comparison of frequency responses for example 4.3

Table 4.5 Comparison of error analysis for example 4.3

Order Reduction Method	'E' for r_{ij} (i, j = 1,2)			
	r_{11}	r_{12}	r_{21}	r_{22}
Proposed method [263]	0.0067	0.00591	0.00096	0.0059
Parmar *et. al.* [71]	0.0525	0.0020	0.0168	0.033
Nahid and Prasad [139]	0.0658	0.1645	0.0066	0.157
Parmar *et. al.* [135]	0.0145	0.00874	0.00254	0.0157
Prasad and Pal [241]	0.1365	0.00245	0.04029	0.0679
Safanov and Chiang [242]	0.5906	0.03713	0.00733	1.0661
Prasad *et. al.* [137]	0.0307	0.00026	0.26197	0.0217
Parmar *et. al.* [157]	0.0266	0.0069	0.0061	0.0683
Parmar *et. al.* [70]	0.0449	0.0344	0.0088	0.1577

It can be noticed that, the introduced method is successfully applied for the above examples and is justified from the step, frequency responses. Added to this, the tabulated error analysis confirms the same.

4.4 STABILITY EQUATION AND BBBC METHOD

SE method is essentially a stability criteria based reduction method and is one of the most popular frequency domain techniques available in the literature [26,27]. The SE method has the privilege of yielding stable reduced order system, provided the original system is stable. In other words, it retains the stability of the original system and also nullifies the steady state response matching issues. Here, the proposed method which is a combination of SE and BBBC is dealt with [261]. The procedure for obtaining the reduced denominator polynomial is same as mentioned in 2.3.2. However the numerator coefficients are optimized using BBBC. Numerical examples are solved to illustrate the proposed method.

4.4.1 Illustrative Examples

Example 4.4: Consider the transfer function of a system taken in example 4.1 in 4.3.1

Adopting the SE method of reduction, the D_r(s) of the original system is reduced according to the procedure mentioned in 2.3.2, resulting in

$$D_r(s) = D_{r_1}(s).D_{r_2}(s)$$

$$D_r(s) = s^2 + 1.47s + 0.686 \text{ ; } r_1 = 2,\ r_2 = 0$$

$$D_r(s) = s^2 + 10s + 34.3 \text{ ; } r_1 = 0,\ r_2 = 2$$

$$D_r(s) = s^2 + 3.913s + 1.6464 \text{ ; } r_1 = 1,\ r_2 = 1$$

Now, selecting $D_r(s)$ for $r_1 = 2$, $r_2 = 0$, the numerator coefficients are optimized using BBBC according to (4.2) resulting in

$$N_r(s) = 0.8s + 0.678$$

Therefore, the reduced second order system obtained is

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{0.8s + 0.678}{s^2 + 1.47s + 0.686}$$

Fig. 4.5 (a) and (b) shows the reaction curves, frequency responses of the original system and proposed reduced system. It is seen that the responses are matching both in steady and transient states. The error analysis is carried out, compared with other methods and tabulated in Table 4.6 (J_{org}=29.86).

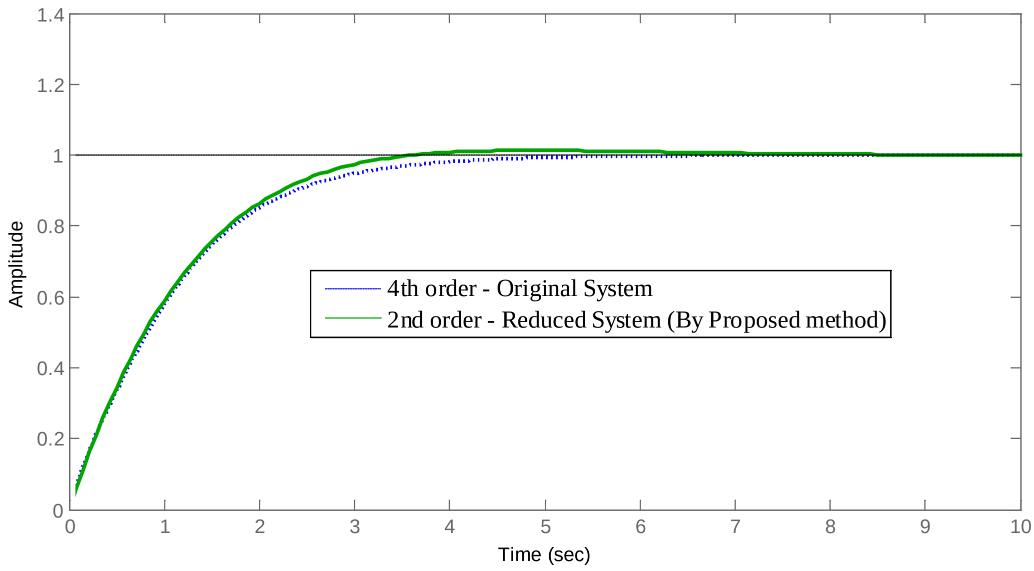

Fig. 4.5 (a) Comparison of step responses for example 4.4

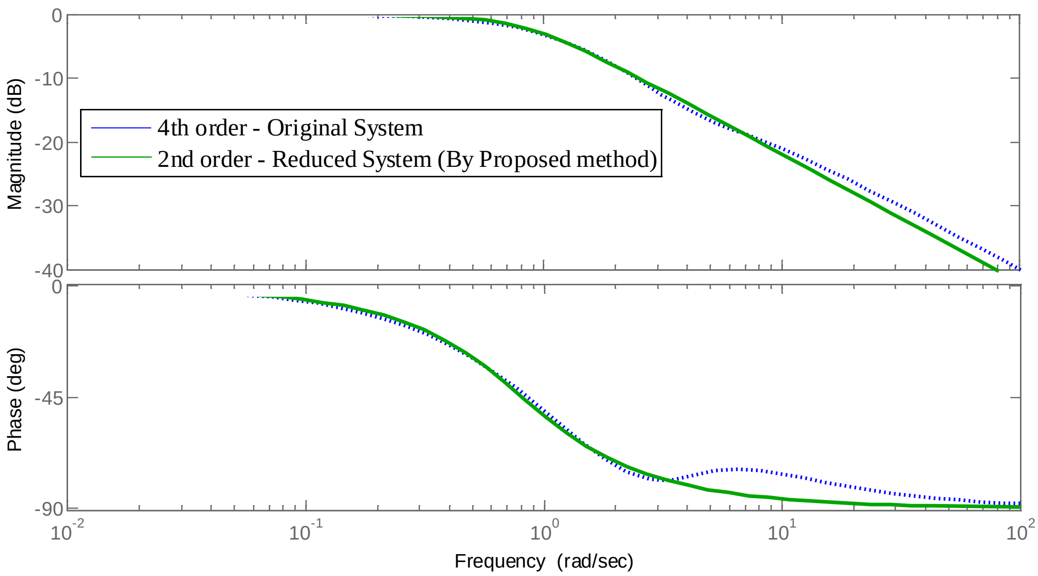

Fig. 4.5 (b) Comparison of frequency responses for example 4.4

Table 4.6 Comparison of reduced order systems for example 4.4

Order Reduction Method	Reduced System	'*E*'	'*J*'
Proposed Method [261]	$\frac{0.8s+0.678}{s^2+1.47s+0.686}$	2.2 x 10^{-3}	29.3
Chen *et. al.* [53]	$\frac{0.699\text{ s}+0.699}{s^2+1.45771\text{ s}+0.699}$	33.3 x 10^{-3}	33.4
Gutmen *et. al.*[47]	$\frac{2(48\text{ s}+144)}{70s^2+300\text{s}+288}$	45.6 x 10^{-3}	79.7
Krishnamurthy and Seshadri [49]	$\frac{20.5714\text{ s}+24}{30s^2+42\text{ s}+24}$	8.9 x 10^{-3}	47.8
Lucas [52]	$\frac{0.833\text{ s}+2}{s^2+3\text{ s}+2}$	0.328 x 10^{-3}	48.4
Mittal *et. al.*[165]	$\frac{0.799\text{ s}+2}{s^2+3\text{ s}+2}$	0.267 x 10^{-3}	47.2
Moore [204]	$\frac{0.8217\text{ s}+0.4543}{s^2+1.268\text{ s}+0.4663}$	2.9 x 10^{-3}	50.0
Mukherjee and Mishra [147]	$\frac{0.800000033\text{ s}+2}{s^2+3\text{ s}+2}$	0.237 x 10^{-3}	47.2
Pal [57]	$\frac{16\text{ s}+24}{30s^2+42\text{ s}+24}$	11.1 x 10^{-3}	49.1
Prasad and Pal [234]	$\frac{\text{s}+34.2465}{s^2+239.8082\text{ s}+34.2465}$	1331 x 10^{-3}	16.6
Safonov and Chang [235]	$\frac{0.8213\text{ s}+0.4545}{s^2+1.268\text{ s}+0.4664}$	2.855 x 10^{-3}	50.1
Safonov *et. al.*[236]	$\frac{0.7431\text{ s}+1.057}{s^2+1.879\text{ s}+1.084}$	0.622 x 10^{-3}	47.3

Example 4.5: Consider a ninth order system having transfer function [168] with J_{org} = 28.23

$$G_n(s)=\frac{s^4+35s^3+291s^2+1093s+1700}{s^9+9s^8+66s^7+294s^6+1029s^5+2541s^4+4684s^3+5856s^2+4620s+1700}$$

Using SE reduction method, the reduced denominators are found as

$$D_r(s)=s^3+1.3s^2+1.34s+0.493\ ;\ r_1=3,\ r_2=0$$
$$D_r(s)=s^3+9s^2+46.54s+187.43\ ;\ r_1=0,\ r_2=3$$
$$D_r(s)=s^3+9.96s^2+8.994s+3.1813\ ;\ r_1=2,\ r_2=1$$
$$D_r(s)=s^3+9.367s^2+49.84s+17.08\ ;\ r_1=1,\ r_2=2$$

$D_r(s)$ for $r_1=3$, $r_2=0$ is selected and numerator coefficients are then generated using BBBC.

$$N_r(s) = 0.08717s^2 + 0.3142s + 0.493$$

Then, the reduced third order model obtained is

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{0.08717s^2 + 0.3142s + 0.493}{s^3 + 1.3s^2 + 1.34s + 0.493}$$

Fig. 4.6 (a) shows the step responses of the original system, proposed reduced system. It is seen that the responses are comparable. Similarly Fig. 4.6 (b) shows the comparison of the frequency plots. Table 4.7 compares various reduced order systems in terms of '*E*' and '*J*' values.

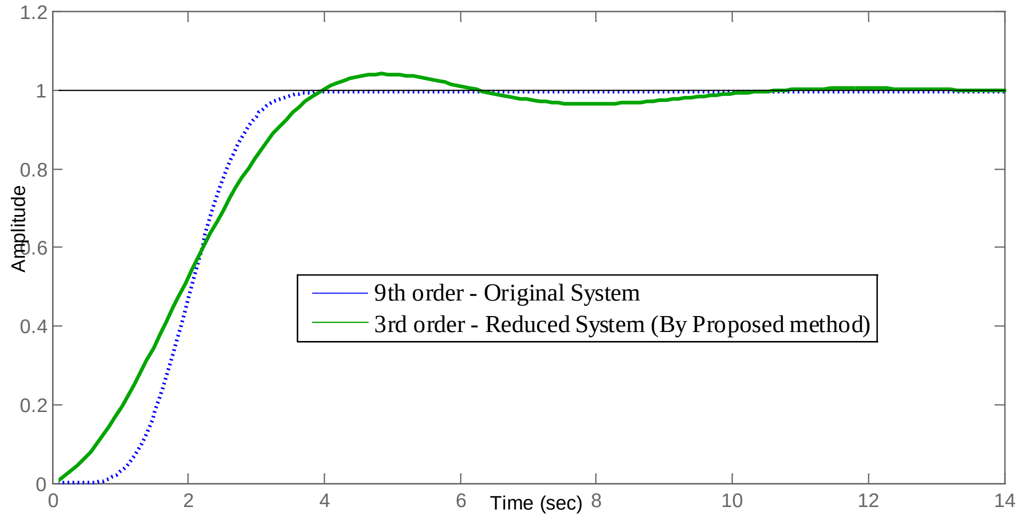

Fig. 4.6 (a) Comparison of step responses for example 4.5

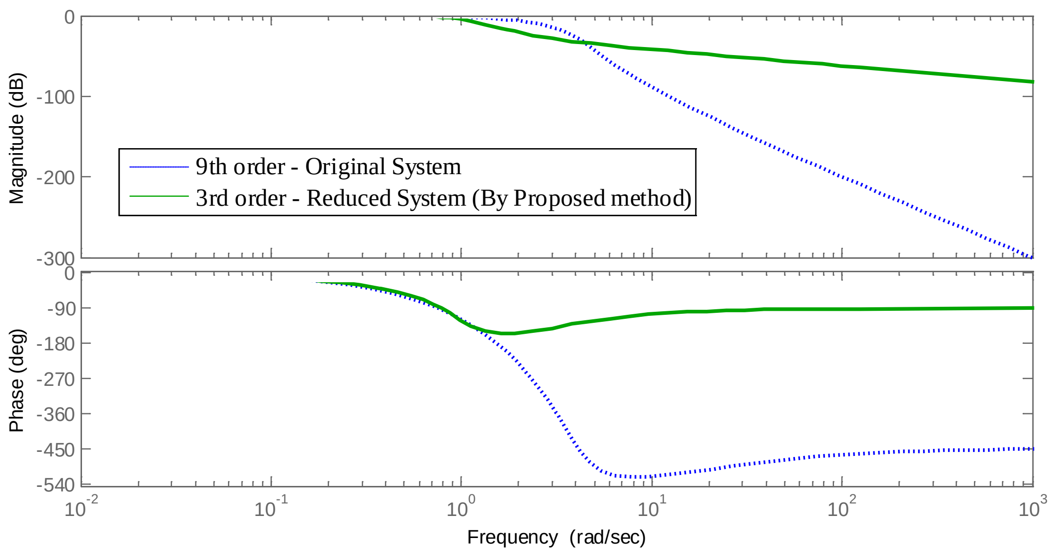

Fig. 4.6 (b) Comparison of frequency responses for example 4.5

Table 4.7 Comparison of reduced order systems for example 4.5

Order Reduction Method	Reduced System	'*E*'	'*J*'
Proposed Method [261]	$\frac{0.08717s^2+0.3142s+0.493}{s^3+1.3s^2+1.34s+0.493}$	2.857x10^{-2}	29.103
Boby and Pal [233]	$\frac{0.5058s^2-1.985s+3.534}{s^3+3s^2+5.534s+3.534}$	2.82x10^{-2}	29.42
Mukherjee *et. al.* [168] (impulse response matching)	$\frac{0.2945s^2-2.203s+2.32}{s^3+2.5008s^2+4.778s-2.32}$	8.77x10^{-2}	51.01
Mukherjee *et. al.* [168] (step response matching)	$\frac{-3.49s^2-4.14s+2.078}{s^3+3.828s^2+4.884s+2.078}$	7.26x10^{-1}	364.36
Chen *et. al.*[50]	$\frac{285s^2+1093s+1700}{3408s^3+5031s^2+4620s+1700}$	2.96x10^{-1}	25.43

Example 4.6: Consider a eighth order system [267] having J_{org}=2509.2.

$$G_n(s)=\frac{18s^7+514s^6+5982s^5+36380s^4+122664s^3+222088s^2+185760s+40320}{s^8+36s^7+546s^6+4536s^5+22449s^4+67284s^3+118124s^2+109584s+40320}$$

The reduced denominators are found using SE reduction method as

$$D_r(s)=s^2+6.867s+5.255\ ;\ r_1=2,\ r_2=0$$

$$D_r(s)=s^2+36s+501.94\ ;\ r_1=0,\ r_2=2\quad t$$

$$D_r(s)=s^2+36.36s+13.25\ ;\ r_1=1,\ r_2=1$$

Using BBBC, the numerator coefficients for r_1=2, r_2=0 is found to be

$$N_r(s)=16.91s+5.255$$

Table 4.8 Comparison of reduced order systems for example 4.6

Order Reduction Method	Reduced System G_r(s)	'*E*'	'*J*'
Proposed Method [261]	$\frac{16.91s+5.255}{s^2+6.87s+5.26}$	0.68x10^{-3}	23.17 x10^2
Dia Abu--Nadi *et. al.* [267]	$\frac{17.099s+5.074}{s^2+6.972s+5.151}$	3.01x10^{-3}	24.19 x10^2
Parmar *et. al.* [105]	$\frac{24.11s+8}{s^2+9s+8}$	48 x10^{-3}	41.92 x10^2
Parmar *et. al.* [135]	$\frac{22.8212s+8.01}{s^2+9s+8}$	0.37x10^{-3}	37.42 x10^2
Mittal *et. al.* [165]	$\frac{7.091s+1.9906}{s^2+3s+2}$	272 x10^{-3}	6.94 x10^2
Lucas [52]	$\frac{6.7786s+2}{s^2+3s+2}$	279 x10^{-3}	6.297 x10^2

The reduced second order model obtained is

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{16.91s + 5.255}{s^2 + 6.87s + 5.26}$$

Fig. 4.7 (a)- (b) shows the step, frequency responses of the original and reduced system. Error analysis is performed for reduced systems obtained by other methods also and are tabulated in Table 4.8.

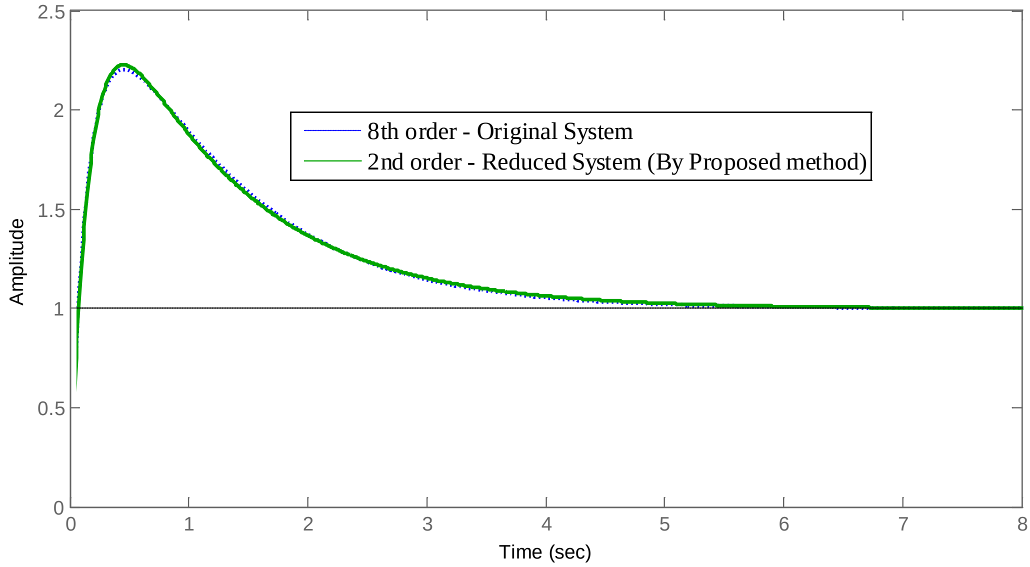

Fig. 4.7 (a) Comparison of step responses for example 4.6

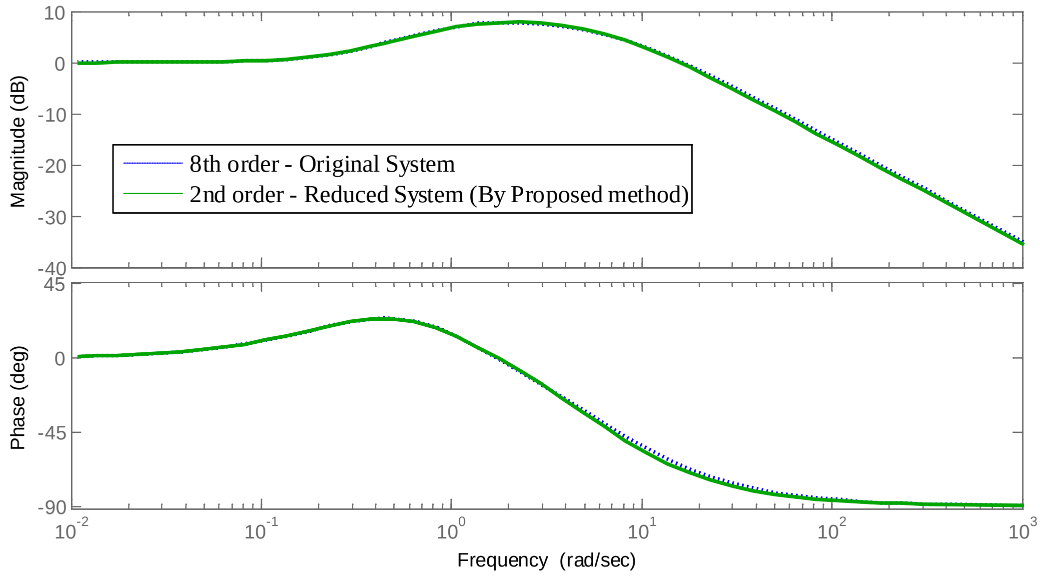

Fig. 4.7 (b) Comparison of frequency responses for example 4.6

4.5 BBBC OPTIMIZED LINEAR SHIFT POINT 'a' FOR ORDER REDUCTION BY LEAST SQUARES METHOD

Pade approximation method of deriving reduced models, is a popular approach which yields good steady-state approximation to the system response. The only drawback is that unstable reduced model may arise from a stable full system [228].To overcome this hitch, several variants were suggested and one such suggestion was to use least squares time moment fit, to obtain a reduced denominator and then the numerator terms using exact time moment matching [174]. In order to minimize the sensitivity towards pole distribution of the original system, Lucas *et. al.*[175] proposed a linear shift point about a general point ‘a’. The value of a ≈ (1- α) and – α is the real part of the smallest magnitude pole. Further, the method of model reduction by least squares moment matching, was generalized by including Markov parameters in the process along with the time moments to cope with a wider class of transfer functions [85].

This section deals with the concept of least squares time moment matching which has been extended about a general point ‘a’, to obtain lower order reduced system. The main contribution in this section, lies in selecting the critical value of ‘a’, as the behavior of the reduced system is dependent on it. Optimized value of ‘a’, will help to realize more accurate approximation of the original system while safeguarding the crucial characteristics of the same as far as possible. This is accomplished here, by availing recently erupted optimization technique called BBBC. The denominator polynomial is computed by using the shifted time moment proportionals. Then, the numerator coefficients are found by matching the appropriate number of time moments. The validity of this criterion is illustrated by solving numerical examples and comparing it with the existing techniques available in the literature. The procedure for deriving the reduced system is as follows.

Step 1: Given a nth order system $G_n(s)$ (4.3), find the time moment proportionals c_i by expanding $G_n(s)$ about s=0 to give

$$G_n(s) = \sum_{i=0}^{\infty} c_i\, s^i \tag{4.12}$$

Step 2: It is well known that a reduced rth order model derived by the Pade approximation method [174] has a denominator polynomial $D_r(s)$ given by

$$D_r(s) = \sum_{i=0}^{r} e_i\, s^i \;\; ;(e_r = 1) \tag{4.13}$$

Given by the solution of the linear set

$$\begin{bmatrix} c_r & c_{r-1} & \cdots & c_1 \\ c_{r+1} & c_r & \cdots & c_2 \\ \vdots & \vdots & \cdots & \vdots \\ c_{2r-1} & c_{2r-2} & \cdots & c_r \end{bmatrix} \begin{bmatrix} e_0 \\ e_1 \\ \vdots \\ e_{r-1} \end{bmatrix} = \begin{bmatrix} -c_0 \\ -c_1 \\ \vdots \\ -c_{r-1} \end{bmatrix} \tag{4.14}$$

Here the e_i coefficients constitute the denominator of the reduced system.

Step 3: The numerator polynomial can be constituted from the following set of equations

$$\begin{aligned} d_0 &= e_0c_0 \\ d_1 &= e_1c_0 + e_0c_1 \\ \vdots \quad &\vdots \quad \vdots \\ d_{r-1} &= e_{r-1}c_0 + \ldots + e_0c_{r-1} \\ 0 &= e_{r-1}c_1 + \ldots + e_0c_r \\ 0 &= e_{r-1}c_2 + \ldots + e_0c_{r+1} \\ \vdots \quad &\vdots \quad \vdots \\ 0 &= e_{r-1}c_t + \ldots + e_0c_{r+t-1} \end{aligned} \tag{4.15}$$

where ($r \geq t \geq 0$)

Step 4: Calculate the numerator and denominator of the reduced system G_r(s) by matching proper number of time moments of G_n(s) to the reduced model.

Step 5: Applying BBBC, optimize the value of ‘a’ obeying (4.2).

Step 6: Find the shifted time moments ($\hat{c_i}$) by replacing G_n(s) by G_n(s + a) and expanding it about s = 0 in (4.12). Then, repeat the above steps 2-3.

Step 7:Apply inverse shift s → (s - a) to the reduced denominator formed by ‘*e*’.

Step 8: Repeat step 4 to obtain reduced system G_r(s).

4.5.1 Illustrative Examples.

Example 4.7: Consider a ninth order system having transfer function [168]

$$G_n(s) = \frac{s^4 + 35s^3 + 291s^2 + 1093s + 1700}{s^9 + 9s^8 + 66s^7 + 294s^6 + 1029s^5 + 2541s^4 + 4684s^3 + 5856s^2 + 4620s + 1700}$$

Step 1: The first ten time moment proportionals c_i obtained by expanding G_n(s) about s=0 (4.12) and is given in Table 4.9

Table 4.9 Time Moment Proportionals

i	c_i
0	1.0000
1	-2.0747
2	2.3648
3	-2.0146
4	1.5513
5	-1.2961
6	1.2775
7	-1.3817
8	1.4924
9	-1.5509

Step 2: The third order model coefficients obtained by following steps 2-4 in 4.5 are shown in Table 4.10. 'J_r' and 'I_r' forms the relative Integral Square Error (RISE) for step and impulse input, calculated to measure the goodness of reduced model given by [85]

$$J_r = \frac{\int_0^\infty [y(t)^2 + y_r(t)^2 - 2y(t)y_r(t)]\, dt}{\int_0^\infty [y(t)^2 + y_r(\infty)^2 - 2y(t)y_r(\infty)]\, dt} \tag{4.16}$$

$$I_r = \frac{\int_0^\infty [g(t)^2 + g_r(t)^2 - 2g(t)g_r(t)]\, dt}{\int_0^\infty g^2(t)\, dt} \tag{4.17}$$

Table 4.10 Comparison of third order models

Moments used in least squares fit	d_2	d_1	d_0	e_2	e_1	e_0	J_r	I_r
6	0.0850	-0.9719	2.392	2.708	3.990	2.392	0.00220	0.05265
7	-0.0344	-0.7065	2.151	2.673	3.757	2.151	0.00336	0.07392
8	-0.1445	-0.4967	1.984	2.673	3.623	1.984	0.00467	0.10305
9	-0.2203	-0.3702	1.895	2.687	3.561	1.895	0.00568	0.12794

Step 3: Now, by employing BBBC the value of 'a' is being optimized to 0.6615 according to (4.2) so as to achieve considerable improvement in the values of 'J_r' and 'I_r'.

Step 4: According to step 6 in 4.5, the shifted time moments ($\hat{c}_i$) are found as given in Table 4.11.

Table 4.11 Shifted time moment proportionals

i	$\hat{c}_i$
0	0.2748
1	-0.5059
2	0.5097
3	-0.3695
4	0.2151
5	-0.1071
6	0.0484
7	-0.0214
8	0.0104
9	-0.0060

Step 5: It is noticed that the rate of increase in the magnitude of $\widehat{c_i}$ is quite small in Table 4.11. Using these values the reduced third order models are as shown in Table 4.12. It can be judged easily that the RISE values (4.16), (4.17) obtained here are far better compared to those given in Table 4.10. This increases the approximation of the reduced model.

Step 6: Since the value of J_r is least, when six shifted time moments are used, d_i , e_i for $i = 0, 1, 2$ are selected and substituted in (4.4) to give the reduced system as

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{0.1748s^2 - 1.156s + 2.504}{s^3 + 2.606s^2 + 4.052s + 2.504}$$

Table 4.12 Comparison of third order models (a = 0.6615)

Shifted Time Moments used in least squares fit	d_2	d_1	d_0	e_2	e_1	e_0	J_r	I_r
6	0.1748	-1.156	2.504	2.606	4.052	2.504	0.00116	0.04025
7	0.2064	-1.302	2.722	2.776	4.35	2.722	0.00128	0.04303
8	0.2214	-1.369	2.821	2.850	4.485	2.821	0.00127	0.04480
9	0.2255	-1.387	2.848	2.869	4.521	2.848	0.00126	0.04531

Table 4.13 Comparison of reduced order systems for example 4.7

Order Reduction Method	Reduced System	J_r	I_r
Proposed Method	$\frac{0.1748s^2 - 1.156s + 2.504}{s^3 + 2.606s^2 + 4.052s + 2.504}$	0.001160	0.04025
Lucas [268]	$\frac{0.217s^2 - 1.35s + 2.791}{s^3 + 2.814s^2 + 4.456s + 2.791}$	0.001239	0.04394
George and Rein Method I [163]	$\frac{-0.29913s + 0.73912}{s^2 + 0.95727s + 0.73912}$	0.026575	0.19418
George and Rein method II [163]	$\frac{-0.57072s + 0.98330}{s^2 + 1.42381s + 0.98330}$	0.01117	0.26409
Boby and Pal [233]	$\frac{0.5058s^2 - 1.985s + 3.534}{s^3 + 3s^2 + 5.534s + 3.534}$	0.00168	0.11
Mukherjee *et. al.* [168] (impulse response matching)	$\frac{0.2945s^2 - 2.203s + 2.32}{s^3 + 2.5008s^2 + 4.778s - 2.32}$	5.336	12.421
Mukherjee *et. al.* [168] (step response matching)	$\frac{-3.49s^2 - 4.14s + 2.078}{s^3 + 3.828s^2 + 4.884s + 2.078}$	0.726	5.97
Chen *et. al.*[50]	$\frac{285s^2 + 1093s + 1700}{3408s^3 + 5031s^2 + 4620s + 1700}$	0.0296	0.278

The results obtained are compared with that of the available existing methods and are listed in Table 4.13. It is noticed that the values of 'J_r' and 'I_r' obtained by proposed method is challenging for the existing methods. Fig. 4.8 (a) and (b) compares the step and impulse responses of original, proposed reduced system. It is seen that the responses of the proposed system are quite appealing.

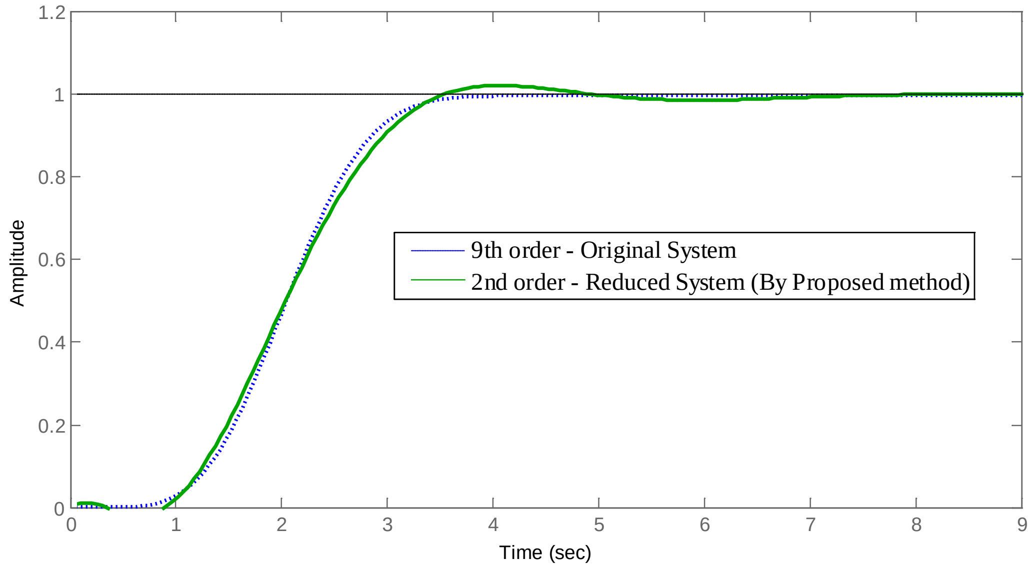

Fig. 4.8 (a) Comparison of step responses for example 4.7

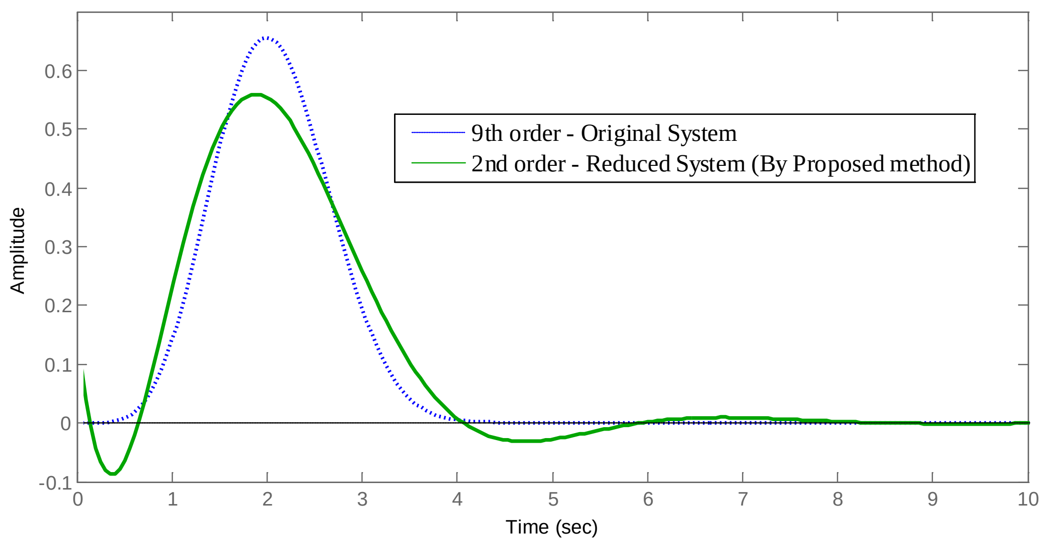

Fig. 4.8 (b) Comparison of impulse responses for example 4.7

Example 4.8: Consider a third order system characterized by transfer function [85]

$$G_n(s) = \frac{8s^2+6s+2}{s^3+4s^2+5s+2}$$

The first ten time moment proportionals c_i are obtained as in Table 4.14. The second order model coefficients obtained by following step 2-4 in 4.5 are shown in Table 4.15

Table 4.14 Time moment proportionals

i	c_i
0	1.0000
1	0.5000
2	0.7500
3	-3.3750
4	6.6875
5	-10.3438
6	14.1719
7	-18.0859
8	22.0430
9	-26.0215

Table 4.15 Comparison of second order models

Moments used in least squares fit	d_1	d_0	e_1	e_0	J_r	I_r
4	-1.7778	-0.2222	-1.6667	-0.2222	unstable	unstable
5	-0.1418	-0.1099	-0.0869	-0.1099	unstable	unstable
6	0.6641	0.1110	0.6086	0.1110	3.361240	0.862037
7	1.1202	0.2798	0.9803	0.2798	2.574628	0.750469
8	1.4076	0.4026	1.2063	0.4026	2.176359	0.680474
9	1.6025	0.4933	1.3558	0.4933	1.94334	0.643025

Table 4.16 Shifted time moment proportionals

i	$\hat{c}_i$
0	1.3233
1	0.1069
2	-0.2271
3	0.1203
4	-0.0318
5	-0.0113
6	0.0250
7	-0.0248
8	0.0198
9	-0.0142

It can be seen in Table 4.15, that the method dosen't produce good approximated reduced system; especially reduced system obtained by using 4 and 5 time moments will result in unstable model. Now, applying BBBC the optimized value of 'a' is found according to (4.2) as 0.8751. The shifted time moments ($\hat{c}_i$) are obtained as in Table 4.16. Then the second reduced order model coefficients being calculated are noted in Table 4.17. Finally, it can be concluded that the values of 'J_r' and 'I_r' shown in Table 4.17 have improved largely as compared to that in Table 4.15.

Table 4.17 Comparison of second order models (a = 0.8751)

Shifted Time Moments used in least squares fit	d_1	d_0	e_1	e_0	J_r	I_r
4	7.541	4.677	2.987	4.677	0.06781	0.01468
5	7.107	4.730	2.731	4.730	0.07453	0.02522
6	6.889	4.719	2.593	4.719	0.08382	0.03338
7	6.852	4.679	2.563	4.679	0.08760	0.03538
8	6.884	4.644	2.576	4.644	0.08792	0.03440
9	6.921	4.623	2.594	4.623	0.08750	0.03318

Table 4.18 Comparison of reduced order models for example 4.8

Order Reduction Method	Reduced System	J_r	I_r
Proposed Method	$\frac{7.541s+4.677}{s^2+2.987s+4.677}$	0.06781	0.01468
Parmar Method (a=AM) [85]	$\frac{5.5932s+4.5293}{s^2+3.3285s+4.5293}$	0.1905	0.06705
Parmar Method (a=HM) [85]	$\frac{5.4959s+4.5546}{s^2+3.2186s+4.5546}$	0.1812	0.06864
Parmar Method (a=GM) [85]	$\frac{5.5395s+4.4179}{s^2+3.3305s+4.4179}$	0.1999	0.07151
Lucas and Beat (a=0) [175]	$\frac{1.4076s+0.4206}{s^2+1.2063s+0.4206}$	2.17636	0.68047
Lucas and Mumro (a=0) [231]	$\frac{4.0135s+1.9248}{s^2+3.0511s+1.9248}$	0.66326	0.24050
Chuang [269]	$\frac{8s+7}{s^2+4.2s+7.6}$	0.16801	0.02236
Marshall [270]	$\frac{12.08696s+4.34783}{s^2+5.34783s+4.34783}$	0.29366	0.11030
Chen *et. al* [53]	$\frac{1.5s+0.5}{s^2+1.25s+0.5}$	2.05089	0.66030
Pal [57]	$\frac{1.375s+0.5}{s^2+1.125s+0.5}$	2.20010	0.69327
Lepschy and Viaro [88]	$\frac{0.906268s+0.350005}{s^2+0.731265s+0.350005}$	3.05349	0.82127

Since the value of J_r is least for four shifted time moments, d_i , e_i for $i = 0, 1$ are selected and substituted in (4.4) resulting in the reduced system

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{7.541s + 4.677}{s^2 + 2.987s + 4.677}$$

The proposed method is compared in terms of 'J_r' and 'I_r' with the other available methods for validation in Table 4.18. It is seen that the proposed method excel as compared to other method. The step, impulse responses of the original, proposed system are depicted in Fig. 4.9 (a) and (b) respectively.

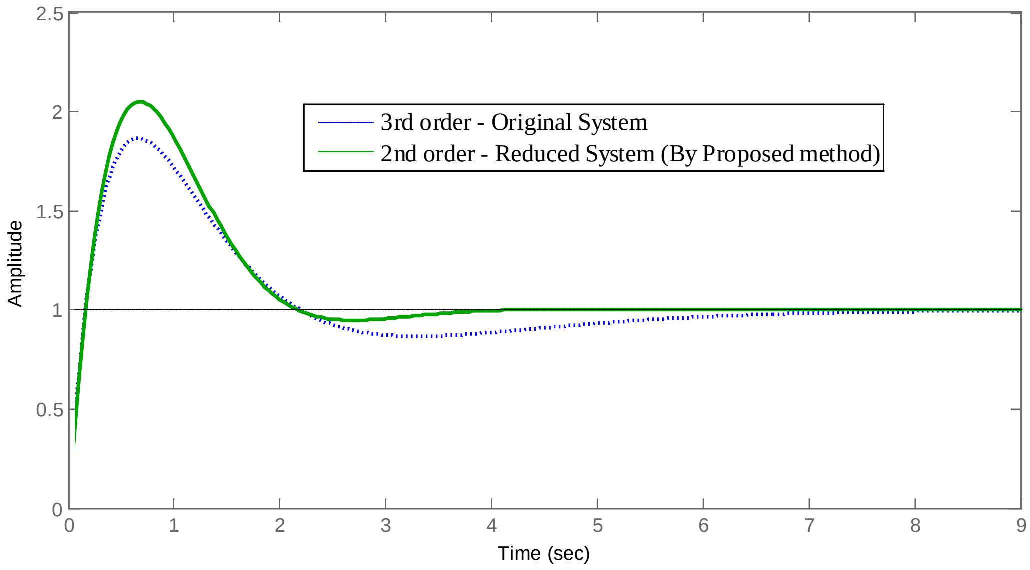

Fig. 4.9 (a) Comparison of step responses for example 4.8

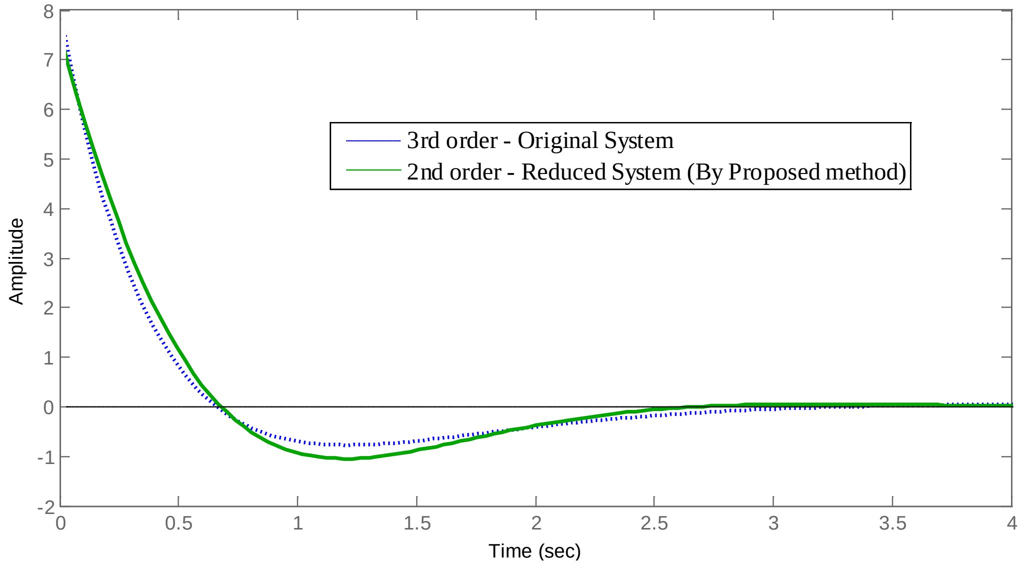

Fig. 4.9 (b) Comparison of impulse responses for example 4.8

Example 4.9: Revisiting an eighth order system considered in example 4.6 in 4.4.1
The possible combination of second order reduced system is shown in Table 4.19.

Table 4.19 Comparison of second order models

Moments used in least squares fit	d_1	d_0	e_1	e_0	J_r	I_r
4	15.1008	4.8206	5.9933	4.8206	0.002767	0.01475
5	14.5023	4.6325	5.7503	4.6325	0.004379	0.021948
6	14.0980	4.5040	5.5886	4.5040	0.005841	0.02777
7	13.8238	4.4164	5.4800	4.4164	0.007025	0.032167
8	13.6348	4.3557	5.4055	4.3557	0.007943	0.03542
9	13.5015	4.3129	5.3532	4.3129	0.00864	0.03783

Using BBBC the value of 'a' is optimized at 0.8972 satisfying (4.2). The shifted time moments are calculated and the possible combination of reduced second order models with 'J_r' and 'I_r' are in Table 4.20.

Table 4.20 Comparison of second order models (a = 0.8972)

Moments used in least squares fit	d_1	d_0	e_1	e_0	J_r	I_r
4	17.12	5.212	6.89	5.212	0.000940	0.00157
5	16.94	5.22	6.821	5.22	0.00069	0.002041
6	16.87	5.223	6.791	5.223	0.000626	0.002271
7	16.84	5.2240	6.779	5.2240	0.000611	0.002364
8	16.83	5.2244	6.775	5.2244	0.000607	0.002400
9	16.82	5.2243	6.774	5.2243	0.000605	0.002407

Table 4.21 compares the proposed system (for 4 time shifted time moments) with various reduced systems available in the literature. It is observed that the proposed system excels in both 'J_r' and 'I_r' terms comparatively. This further can be validated by the step, impulse responses shown in Fig. 4.10 (a) and (b) respectively.

Table 4.21 Comparison of reduced order models for example 4.9

Order Reduction Method	Reduced System $G_r(s)$	J_r	I_r
Proposed Method	$\frac{17.12s+5.212}{s^2+6.89s+5.212}$	0.000940	0.00157
Lucas Method [268]	$\frac{18s+5.6026}{s^2+7.4151s+5.6026}$	0.001340	0.0062
Shamash [92]	$\frac{6.7786s+2}{s^2+3s+2}$	0.18978	0.3068
Lucas Method [268]	$\frac{1.9896s+0.4318}{s^2+1.1737s+0.4318}$	1.31279	0.76358
Nidhi *et al*	$\frac{17.96s+5.012}{s^2+7.028s+5.012}$	0.01156	0.00198
Chen *et. al.*[238]	$\frac{0.72058\text{ s} + 0.3669}{s^2+0.02768\text{s} + 0.3669}$	4.669	0.991
Gutmen *et. al.*[47]	$\frac{5.35\times10^8\text{ s} + 8.129\times10^8}{8.505\times10^7s^2+5.523\times10^8\text{s} + 8.129\times10^8}$	0.89	0.3979
Hutton and Friedland [45]	$\frac{1.99\text{ s} + 0.4318}{s^2+1.174\text{s} + 0.4318}$	1.241	0.7831
Krishnamurthyy and Seshadri [49]	$\frac{1.557\times10^5\text{ s} + 40320}{65520s^2+75600\text{s} + 40320}$	1.069	0.7486
Lucas [52]	$\frac{6.779\text{ s} + 2}{s^2+3\text{s} + 2}$	0.1802	0.355
Pal [57]	$\frac{1.518\times10^5\text{ s} +40320}{65520s^2+75600\text{s} + 40320}$	1.068	0.753
Prasad and Pal [234]	$\frac{17.99\text{ s} +500}{s^2+13.25\text{s} + 500}$	0.9464	0.3418
Safonov *et. al.* [236]	$\frac{16.96\text{ s} +4.729}{s^2+7.028\text{s} + 5.011}$	0.00387	0.00268
Mukherjee *et. al.*[168]	$\frac{11.39\text{ s} +4.435}{s^2+4.2122\text{s} + 4.4357}$	0.0365	0.1207
Mittal *et. al.*[165]	$\frac{7.09\text{ s} +1.9907}{s^2+3\text{s} + 2}$	0.1734	0.337

It is observed from the above illustrative examples, that optimizing the value of point 'a' using BBBC has resulted in excellent results both in terms of 'J_r' and 'I_r' as compared with other reduced systems. Moreover the step and impulse responses are also pleasing.

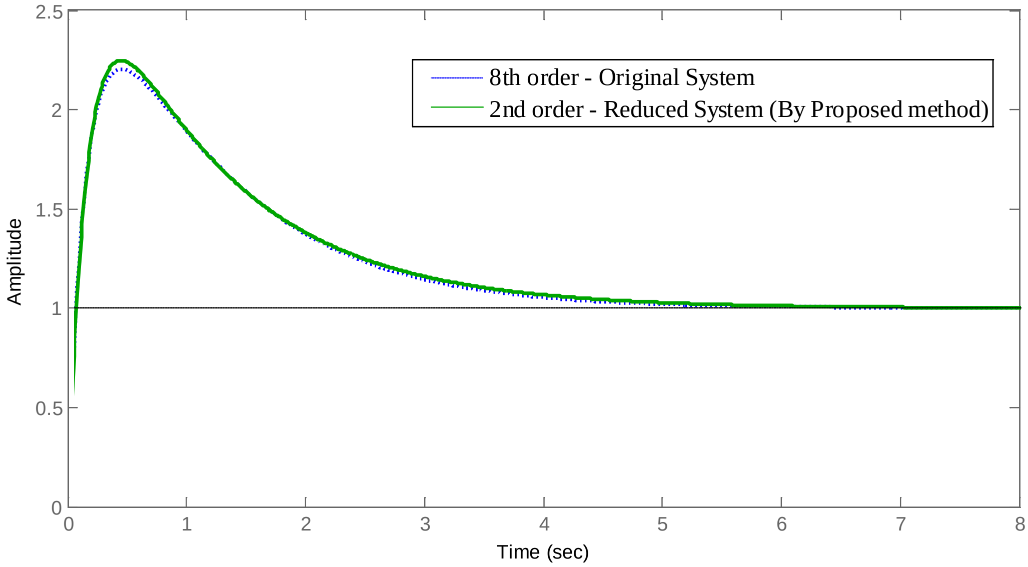

Fig. 4.10 (a) Comparison of step responses for example 4.9

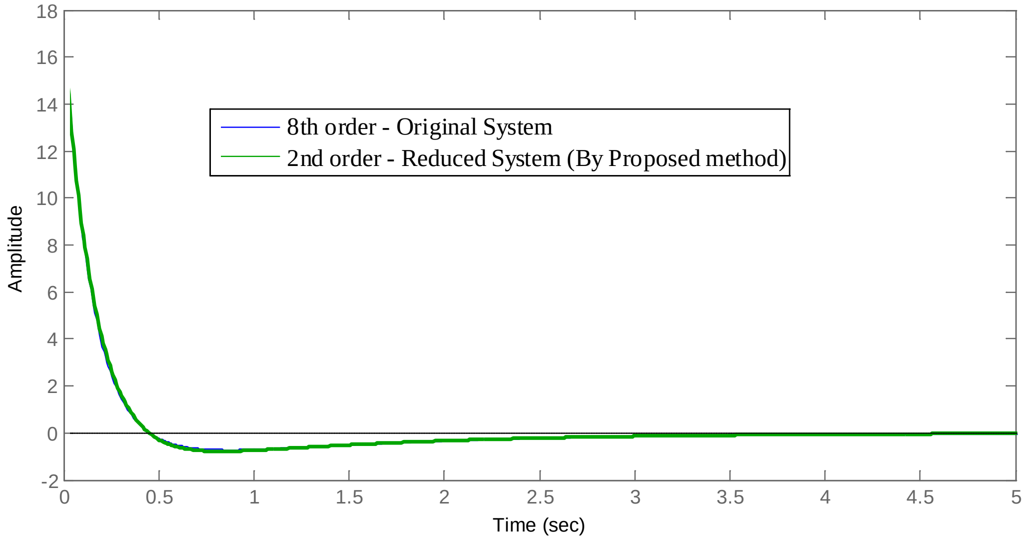

Fig. 4.10 (b) Comparison of impulse responses for example 4.9

4.6 BBBC METHOD FOR CONTINUOUS TIME SYSTEMS

In this proposal, the higher order continuous time system is accepted and reduced directly by using BBBC optimization technique while satisfying (4.2). The typical parameters while using BBBC are as mentioned in Table 4.1 in 4.1. The effectiveness of BBBC is justified by solving illustrative examples.

4.6.1 Time Domain

Let an n^{th} order stable large scale LTI continuous system be described in time domain by (1.1). The problem of order reduction is to derive a reduced model of order ' r ' $(r < n)$

described by (1.2), such that the reduced r^{th} order model retains the salient characteristics of the original n^{th} order system for a given set of inputs given by

$$\begin{aligned}[G_r(s)] &= C_r(sI - A_r)^{-1} B_r \\ &= \frac{N_r(s)}{D_r(s)} = \frac{\sum_{i=0}^{(r-1)} d_i s^i}{\sum_{i=0}^{r} e_i s^i} ; e_i = 1 \;\; for \; i = r\end{aligned} \tag{4.18}$$

4.6.1.1 Illustrative Examples

Example 4.10: Consider a benchmark example of 84th order [271] represented in time domain form (1.1)

$$[A] = \begin{bmatrix} A_1 & 0 & 0 & 0 & 0 & 0 & 0 & a_{78} & 0 & 0 \\ 0 & A_2 & 0 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & A_3 & 0 & 0 & 0 & 0 & 0 & 0 & a_{154} \\ 0 & 0 & 0 & A_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & A_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & A_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_7 & 0 & 0 & 0 \\ a_1 & 0 & 0 & 0 & 0 & 0 & 0 & A_8 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & a_{77} & 0 & 0 & 0 & 0 & 0 & 0 & A_{42} \end{bmatrix}$$

Table 4.22 Coefficients of [B(i)]

i	*B(i)*	*i*	*B(i)*	*i*	*B(i)*	*i*	*B(i)*
1.	7.624824	22	9.201152	43.	8.562598	64.	8.77164
2.	2.354606	23	4.575071	44.	5.995426	65.	7.548852
3.	7.570281	24	6.616429	45.	4.822466	66.	3.25237
4.	2.321555	25	5.214498	46.	0.120089	67.	2.581167
5.	4.608591	26	9.829012	47.	2.787891	68.	7.341292
6.	0.353796	27	8.996692	48.	5.741295	69.	4.374441
7.	5.242211	28	6.302044	49.	8.2792	70.	3.124567
8.	3.992345	29	3.468259	50.	7.045744	71.	9.036562
9.	8.994307	30	3.058394	51.	3.418584	72.	6.950758
10.	1.71435	31	9.01239	52.	4.020264	73.	7.567942
11.	0.247185	32	1.065961	53.	5.389039	74.	2.666435
12.	5.921826	33	2.963187	54.	1.897445	75.	6.112229
13.	5.702193	34	8.515574	55.	4.346914	76.	1.722831
14.	5.824618	35	7.294136	56.	4.086833	77.	1.518785
15.	3.413381	36	6.427816	57.	2.056365	78.	6.839423
16.	5.595597	37	2.681877	58.	2.93394	79.	2.179521
17.	3.976737	38	7.281299	59.	2.561231	80.	7.636844
18.	9.456895	39	8.92205	60.	3.126108	81.	0.669466
19.	8.76504	40	5.099291	61.	5.486768	82.	2.401817
20.	9.884494	41	5.530897	62.	8.308178	83.	3.542152
21.	3.913699	42	8.808669	63.	2.974988	84.	6.076599

where the diagonal elements are

$$\underset{i=1,\ldots 42}{A_i} = \begin{bmatrix} -734 & 171 \\ -9 & -734 \end{bmatrix}$$

and the off diagonal elements

$$\underset{j=1,\ldots 154}{a_j} = 196$$

$$B = [B_i]_{84\times 1} \; i = 1,2,3,\ldots 84$$

The values of $[B_i]$ are in Table 4.22

$$C = [C_{1,j}]_{1\times 84} \;\; j = 1,2,3,\ldots 84$$
$$= B^T$$

The associated transfer function matrix is then obtained given by (4.18)

$$[G(s)] = C(sI - A)^{-1} B$$

Using BBBC, obeying the fitness function (4.2) the reduced 13th order LTI system becomes

$$[G_r(s)] = \frac{N_r(s)}{D_r(s)}$$

where denominator polynomial is given by

$$D_r(s) = s^{13} + 534.4 s^{12} + 2.355\times 10^6 s^{11} + 1.109\times 10^9 s^{10} + 2.237\times 10^{12} s^9 + 9.204\times 10^{14} s^8 + 1.097\times 10^{18} s^7 + 3.896\times 10^{20} s^6 + 2.93\times 10^{23} s^5 + 8.841\times 10^{25} s^4 + 4.035\times 10^{28} s^3 + 1.016\times 10^{31} s^2 + 2.238\times 10^{33} s + 4.594\times 10^{35}$$

and numerator polynomial is

$$N_r(s) = 2780 s^{12} + 7.432\times 10^5 s^{11} + 6.326\times 10^9 s^{10} + 399\times 10^{12} s^9 + 5.805\times 10^{15} s^8 + 1.021\times 10^{18} s^7 + 2.747\times 10^{21} s^6 + 3.606\times 10^{23} s^5 + 7.063\times 10^{26} s^4 + 6.164\times 10^{28} s^3 + 9.4\times 10^{31} s^2 + 4.1\times 10^{33} s + 4.978\times 10^{36}$$

Fig. 4.11 (a) and (b) shows the comparison of step and impulse responses of the original, reduced system. It is seen that the responses are matching both in steady and transient states. The value of 'J_r', 'I_r' obtained are 4.87 x 10^{-09} and 1.10 x10^{-05} respectively.

Example 4.11: Consider a 200th order benchmark example represented in time domain form [271] as

$$[A] = \begin{bmatrix} P_1 & q_{200} & 0 & 0 \\ q_1 & P_2 & \ddots & 0 \\ 0 & \ddots & \ddots & q_{398} \\ 0 & 0 & q_{199} & P_{200} \end{bmatrix}$$

where the diagonal and off diagonal elements are

$$\underset{i=1,\ldots200}{P_i} = -808.02 \quad and \quad \underset{j=1,\ldots398}{q_j} = 404.01$$

$$B = [B_{i,1}]_{200\times1} = 0 \quad \forall \quad i = 1,2,3\ldots200$$
$$= 1 \quad for\, i = 67$$
$$C = [C_{1,j}]_{1\times200} = 0 \quad \forall \quad j = 1,2,3\ldots200$$
$$= 1 \quad for\; j = 133$$

The numerator and denominator coefficients are obtained by using BBBC while obeying (4.2) resulting in the fourth order reduced system in the form (4.18).

$$[G_r(s)] = \frac{N_r(s)}{D_r(s)} = \frac{0.0001169s^3 + 0.0005669s^2 - 0.008125s + 0.03507}{s^4 + 3.511s^3 + 16.24s^2 + 7.928s + 0.6248}$$

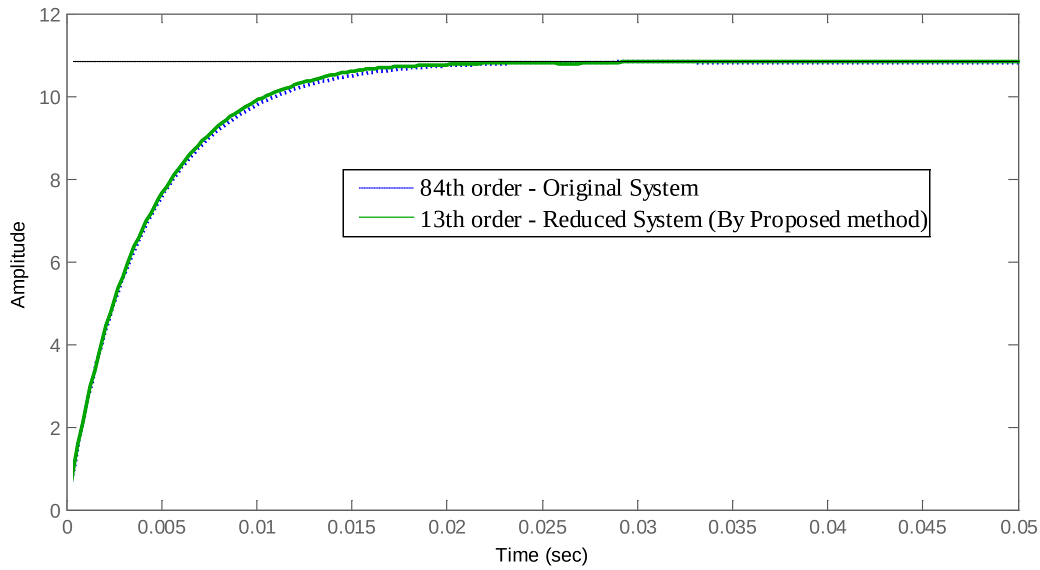

Fig. 4.11 (a) Comparison of step responses for example 4.10

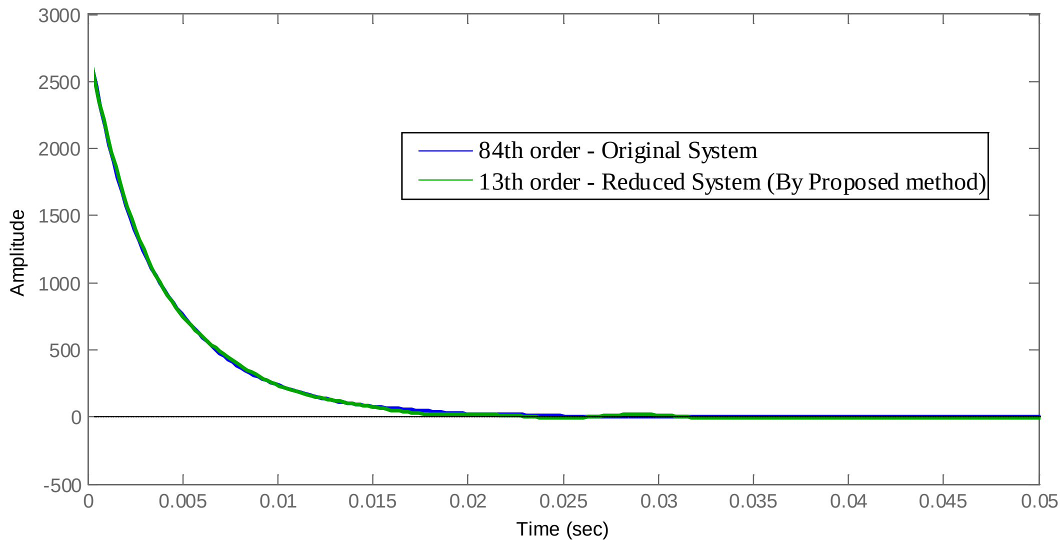

Fig. 4.11 (b) Comparison of impulse responses for example 4.10

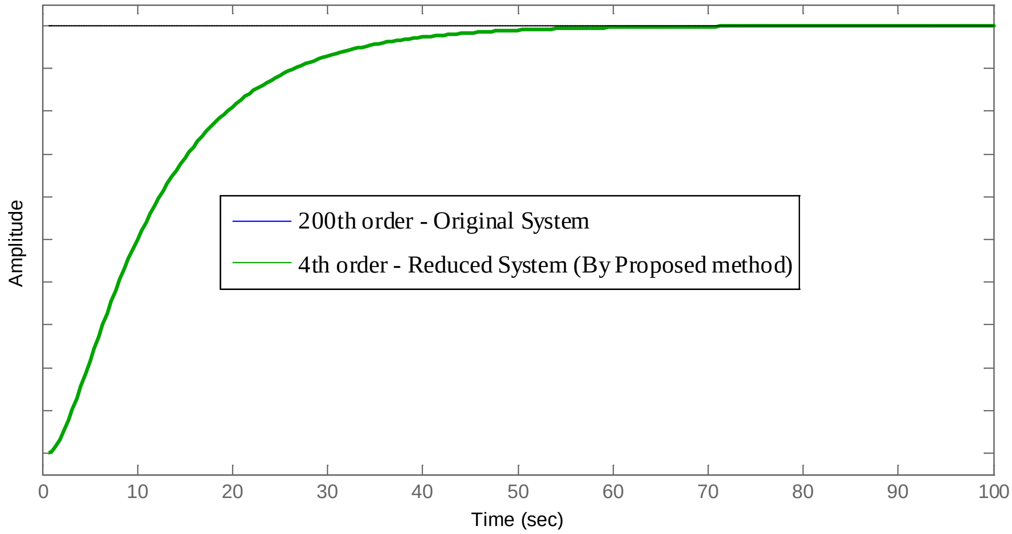

Fig. 4.12 (a) Comparison of step responses for example 4.11

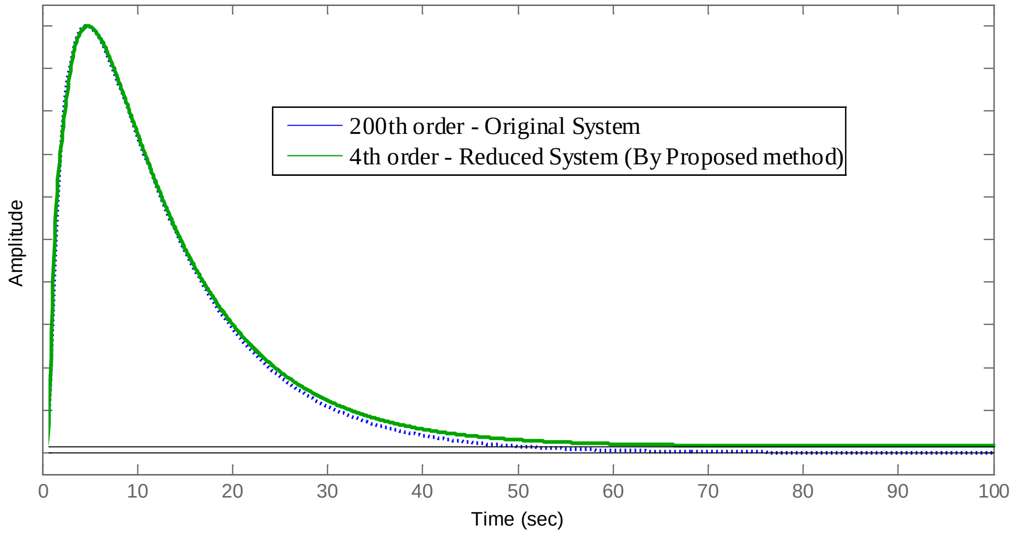

Fig. 4.12 (b) Comparison of impulse responses for example 4.11

Fig. 4.12 (a) and (b) depicts the responses of the original, reduced system for a given step and impulse input respectively. It is seen that the responses are overlapping one over the other, with values of J_r, I_r being 1.187x10^{-9} and 6.747x10^{-04} .

4.6.2 Frequency Domain

Here, the higher order continuous time system represented in the form of (4.3) or (4.5) is reduced to the form (4.4) or (4.6) respectively using BBBC obeying (4.2). Numerical examples are solved in the following for illustration.

4.6.2.1 Illustrative Examples

Example 4.12: Consider a ninth order system having transfer function [168] having J_{org}=28.23.

$$G_n(s)=\frac{s^4+35s^3+291s^2+1093s+1700}{s^9+9s^8+66s^7+294s^6+1029s^5+2541s^4+4684s^3+5856s^2+4620s+1700}$$

Using BBBC the reduced system obtained while minimizing (4.2) is

$$G_r(s)=\frac{N_r(s)}{D_r(s)}=\frac{0.376s^2-1.613s+3}{s^3+2.686s^2+4.65s+3}$$

The results obtained are compared with that of the available existing methods and are listed in Table 4.23. It is noticed that the values of '*E*' and '*J*' obtained by proposed method is challenging for the existing methods. Fig. 4.13 compares the reaction curves of original and proposed reduced system. It is seen that the responses of the proposed system are quite appealing.

Table 4.23 Comparison of reduced order systems for example 4.12

Order Reduction Method	Reduced System G_r(s)	'*E*'	'*J*'
Proposed Method	$\frac{0.376s^2-1.613s+3}{s^3+2.686s^2+4.65s+3}$	1.35x10^{-3}	29.154
Chen and Chang [50]	$\frac{285s^2+1093s+1700}{3408s^3+5031s^2+4620s+1700}$	296 x10^{-3}	25.43
Lucas [268]	$\frac{0.217s^2-1.35s+2.791}{s^3+2.814s^2+4.456s+2.791}$	2.15x10^{-3}	26.88
George and Rein Method I [163]	$\frac{-0.29913s+0.73912}{s^2+0.95727s+0.73912}$	42.3 x10^{-3}	26.03
George and Rein method II [163]	$\frac{-0.57072s+0.98330}{s^2+1.42381s+0.98330}$	18.7 x10^{-3}	28.636
Boby and Pal [233]	$\frac{0.5058s^2-1.985s+3.534}{s^3+3s^2+5.534s+3.534}$	28.7 x10^{-3}	29.42
Mukherjee *et. al.* [168] (impulse response matching)	$\frac{0.2945s^2-2.203s+2.32}{s^3+2.5008s^2+4.778s-2.32}$	87.7 x10^{-3}	51.01
Mukherjee *et. al.* [168] (step response matching)	$\frac{-3.49s^2-4.14s+2.078}{s^3+3.828s^2+4.884s+2.078}$	726 x10^{-3}	364.36
Proposed Method (MPC and Least Squares)	$\frac{-0.05311s^2-0.2907s+2.132}{s^3+3s^2+4.132s+2.132}$	15.1 x10^{-3}	19.645

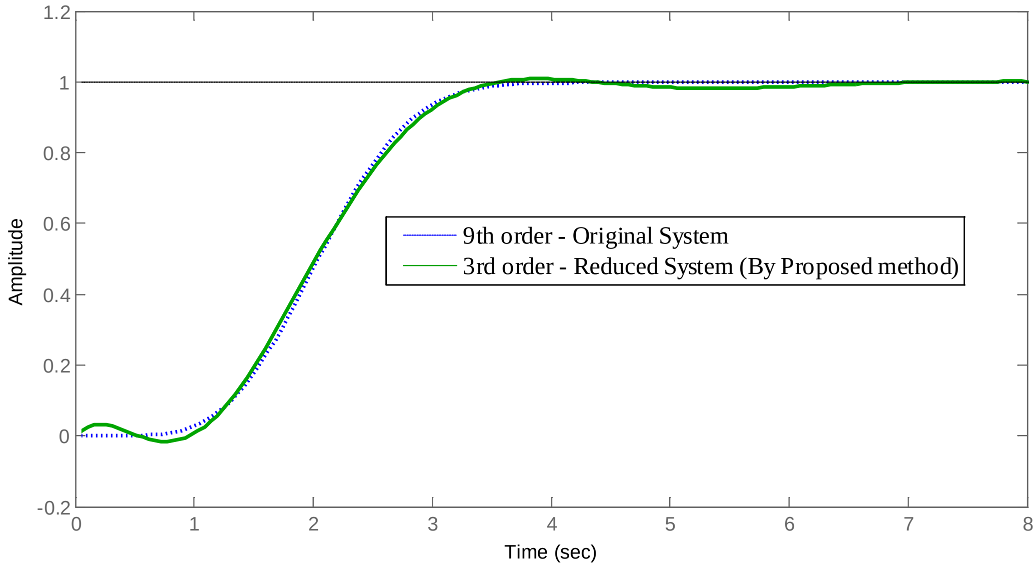

Fig. 4.13 Comparison of step responses for example 4.12

Example 4.13: Consider an eighth order transfer function taken from [49] having J_{org}=26286

$$G_n(s)=\frac{35s^7+1086s^6+13285s^5+82402s^4+278376s^3+511812s^2+482964s+194480}{s^8+33s^7+437s^6+3017s^5+11870s^4+27470s^3+37492s^2+28880s+9600}$$

The reduced system obtained by applying BBBC is given by

$$G_r(s)=\frac{N_r(s)}{D_r(s)}=\frac{31.98\ s+79.03}{s^2+3.541s+3.901}$$

Table 4.24 Comparison of reduced order systems for example 4.13

Order Reduction Method	Reduced System G_r(s)	'E'	'J'
Proposed Method	$\frac{31.98\ s+79.03}{s^2+3.541s+3.901}$	39.52	25408
Krishnamurthyy and Seshadri [49]	$\frac{334828\ s+194480}{2.012e004s^2+1.812e004s+9600}$	135.66	16522
Gutman *et. al.*[47]	$\frac{1.391e009\ s+3.921e009}{2.699e007s^2+1.456e008s+1.935e008}$	73.65	39788
Agathoklis and Sreeram [272]	$\frac{35\ s+181.9}{s^2+7.169s+8.029}$	67.856	21018
Chen *et. al.*[50]	$\frac{482964\ s+194480}{34194s^2+28880s+9600}$	172.1	11745
Lepschy and Viaro [140]	$\frac{4.601\ s+20.26}{0.1395s^2+0.7521s+1}$	40.238	25990
Sivanandam and Deepa [255]	$\frac{482964\ s+194480}{37492s^3+2s^2+28880s+9600}$	1570.3	134733.0
Sivanandam and Deepa [255]	$\frac{35\ s+66.27}{s^2+3.357s+3.271}$	42	26950
Sivanandam and Deepa [255]	$\frac{35\ s+61.95}{s^2+3.247s+3.058}$	41.225	26880

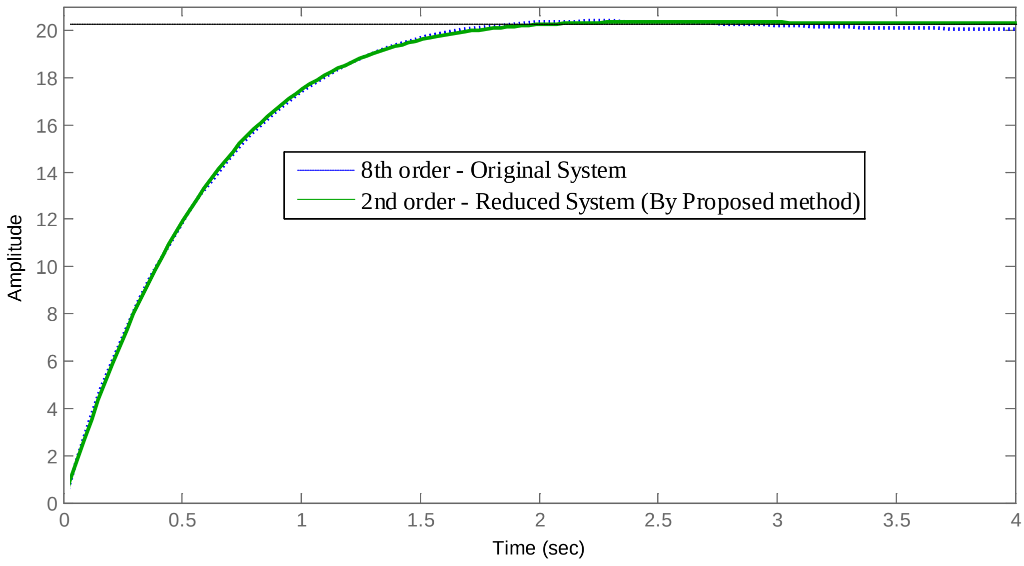

Fig. 4.14 (a) Comparison of step responses for example 4.13

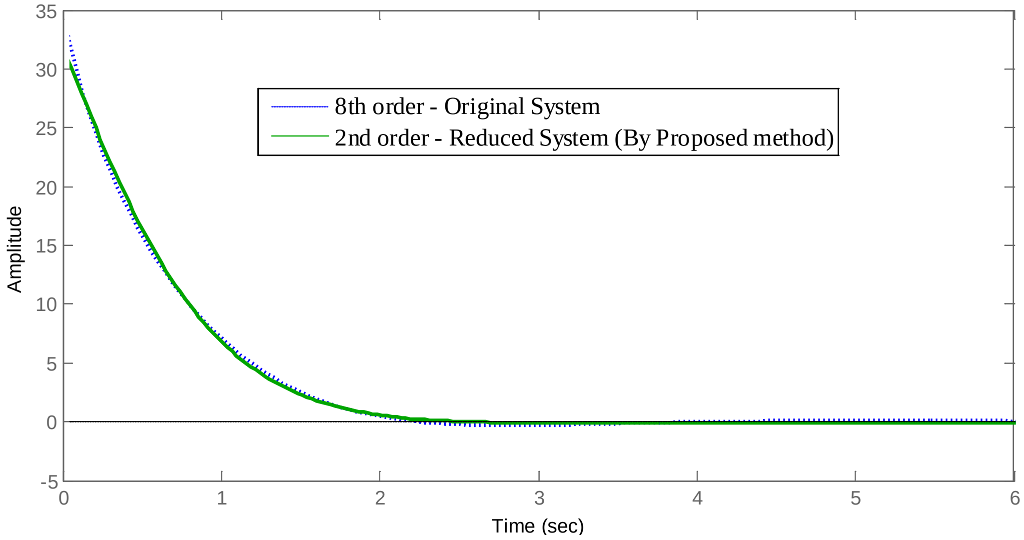

Fig. 4.14 (b) Comparison of impulse responses for example 4.13

Table 4.24 compares the reduced systems obtained by various authors in terms of '*E*' and '*J*'. Fig. 4.14 (a) and (b) compares the step, impulse responses of original and proposed reduced system. It is seen that the responses of the proposed system overlaps the original system indicating the goodness of the proposed method.

4.6.2.2 Extension to Multivariable Systems

In 4.6.1 and 4.6.2, the proposed method has been successfully applied on SISO system to obtain the reduced system. The same proposed method can also be extended to MIMO systems, which is a direct application of the SISO method, on the elements of the transfer function matrix of MIMO system as discussed below.

MIMO system of n^{th} order described in the form (2.24) is considered to obtain (2.26). The proposed method is applied to (2.24) by following the steps described in 4.1. The method proposed is verified by solving two illustrative examples as given below.

4.6.2.2.1 ***Illustrative Examples***

Example 4.14: Consider a practical power system of 10th order taken from [273]. It consists of a 3-ph synchronous generator with an automatic excitation control system, supplying power through a step-up transformer and a high voltage transmission line connected to an infinite grid. The outputs are the torque angle δ and terminal voltage v_t. Inputs being exciter voltage ΔV_{ref} and mechanical torque ΔT_m. The system is described by

$$x^T = \begin{bmatrix} E_q & \omega & \delta & v_1 & v_2 & v_3 & v_4 & v_5 & v_R & E_{FD} \end{bmatrix}$$

$$u^T = \begin{bmatrix} \Delta V_{ref} & \Delta T_m \end{bmatrix}$$

$$y^T = \begin{bmatrix} \delta & v_t \end{bmatrix}$$

The state space form will be

A=

-0.5517	0	-0.3091	0	0	0	0	0	0	0.1695
-0.0410	0	-0.035	0	0	0	0	0	0	0
0	314.15930	0	0	0	0	0	0	0	0
9.554	0	-0.866	-20	0	0	0	0	0	0
0	0	0	0	-1	0	0	0	0.04210	-0.0328
-0.1962	10.869600	-0.1672	0	0	-10.8696	0	0	0	0
-0.9386	51.984900	-0.7999	0	0	-41.1153	-10.8696	0	0	0
-0.9386	51.984900	-0.7999	0	0	-41.1153	-10.8696	-0.10	0	0
0	0	0	-100	-1000	0	0	1000	-20	0
0	0	0	0	0	0	0	0	1.0526	-0.8211

B^T =

0	0	0	0	0	0	0	0	10000	
0	0.0926	0	0	0	0.4428	2.1179	2.1179	0	0

C =

0	0	0	0	0	0	0	0	1000	0
0	0.09260		0	0	0.4428	2.1179	2.11790		0

The plant transfer function matrix is then given by

$$[G_n(s)] = \frac{1}{D_n(s)} \begin{bmatrix} A_{11}(s) & A_{12}(s) \\ A_{21}(s) & A_{22}(s) \end{bmatrix} \tag{4.19}$$

$$A_{11}(s) = -2298s^5 - 9.845\times10^4 s^4 - 1.376\times10^6 s^3 - 6.838\times10^6 s^2 - 6.1\times10^6 s - 5.43\times10^5$$

$$A_{12}(s) = 29.09s^8 + 1868s^7 + 4.61\times10^4 s^6 + 5.459\times10^5 s^5 + 3.14\times10^6 s^4 + 7.683\times10^6 s^3 + 5.719\times10^6 s^2 + 1.709\times10^6 s + 1.21\times10^5$$

$$A_{21}(s) = 85.1s^7 + 3646s^6 + 5.201\times10^4 s^5 + 2.976\times10^5 s^4 + 8.46\times10^5 s^3 + 3.1\times10^6 s^2 + 2.748\times10^6 s + 2.446\times10^5$$

$$A_{22}(s) = -1.26s^8 - 85.17s^7 - 2089s^6 - 2.568\times10^4 s^5 - 1.908\times10^5 s^4 - 7.117\times10^5 s^3 - 1.083\times10^6 s^2 - 2.969\times10^5 s - 1.94\times10^4$$

and

$$D_n(s) = s^{10} + 64.21s^9 + 1596s^8 + 1.947\times10^4 s^7 + 1.253\times10^5 s^6 + 4.686\times10^5 s^5 + 1.333\times10^6 s^4 + 2.611\times10^6 s^3 + 1.39\times10^6 s^2 + 4.39\times10^5 s + 3.261\times10^4$$

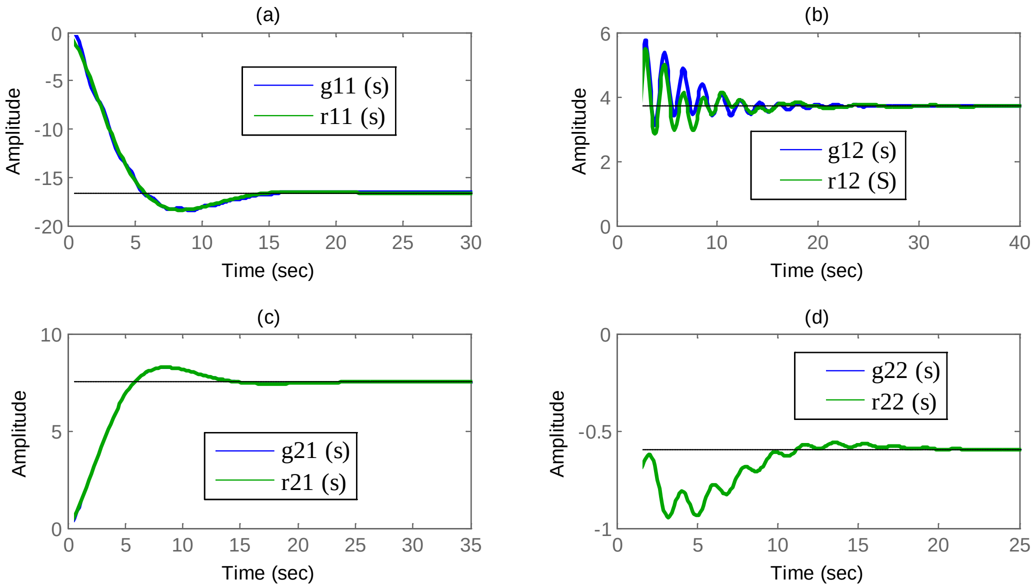

Fig. 4.15 (a) Comparison of step responses for example 4.14

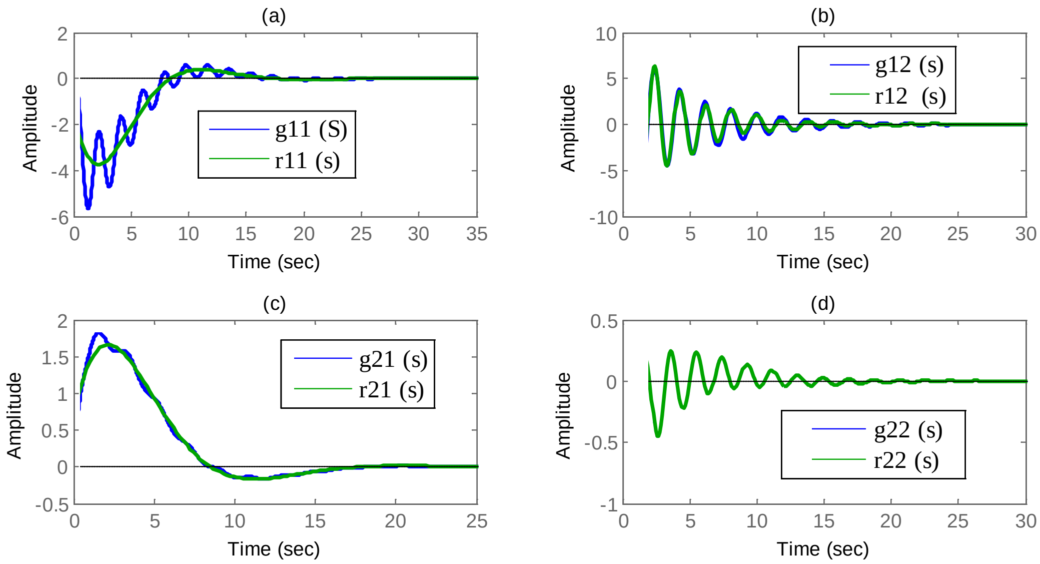

Fig. 4.15 (b) Comparison of impulse responses for example 4.14

It is desired to reduce [G_n(s)] to a second order system represented in the form

$$[R'(s)] = \begin{bmatrix} B'_{11}(s) & B'_{12}(s) \\ B'_{21}(s) & B'_{22}(s) \end{bmatrix} \tag{4.20}$$

Applying BBBC, the reduced order system [R'(s)] is obtained, where

$$B'_{11}(s) = \frac{-1.408\ s - 3.15}{s^2 + 0.5122s + 0.1892}$$

$$B'_{12}(s) = \frac{35.62s^2 + 4.815s + 34.33}{s^4 + 0.6925s^3 + 11.87s^2 + 2.6s + 9.249}$$

$$B'_{21}(s) = \frac{0.6895\ s + 1.38}{s^2 + 0.5078s + 0.1839}$$

$$B'_{22}(s) = \frac{-1.016s^2 - 5.491s - 1.127}{s^4 + 0.9044s^3 + 11.44s^2 + 5.815s + 1.895}$$

The results obtained by proposed method is compared with other method in terms of *'E'* and *'J'* for each element of transfer function matrix are according to Table 4.25. The value J_{org} of each element of plant transfer function matrix are 6090.9, 9516.2, 1073.2 and 65.05 respectively. The step, impulse responses of [G_n(s)] and [R'(s)] are depicted in Fig. 4.15 (a) and (b).

Table 4.25 Comparison of error for example 4.14

r_{ij} (i,j=1,2)	Proposed Method		Parmar et. al. [274]		Abu-Al-Nadi et. al.[273]	
	'E'	*'J'*	*'E'*	*'J'*	*'E'*	*'J'*
r_{11}	0.945	5163	1125.2	397266	217.4	3459.3
r_{12}	0.8625	10865	2.9236	10048	259.88	53163
r_{21}	0.031	1035.3	89.536	22712	30.339	794.66
r_{22}	1.45×10^{-5}	64.33	26.952	11601	0.8352	125.72

Example 4.15: Consider the state space representation of a longitudinal motion of a flexible bomber aircraft given by [275]

$$A = \begin{bmatrix} 0.14158 & 1.025 & -0.00267 & -0.0001106 & -0.08021 & 0 \\ -5.5 & -0.8302 & -0.06549 & -0.0039 & -5.115 & 0.809 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -1040 & -78.35 & -34.83 & -0.6214 & -865.6 & -631 \\ 0 & 0 & 0 & 0 & -75 & 0 \\ 0 & 0 & 0 & 0 & 0 & -100 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 75 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \end{bmatrix}$$

$$C = \begin{bmatrix} -1491 & -146.43 & -40.2 & -0.9412 & -1285 & -564.65 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

using BBBC the reduced order system [R'(s)] is obtained according to (4.20)

$$B'_{11}(s) = \frac{-1.501e006\ s + 4.334e007}{s^2 + 0.9708s + 38.87}$$

$$B'_{12}(s) = \frac{1554\ s - 0.09772}{s^2 + 0.9179s + 25.94}$$

$$B'_{21}(s) = \frac{-3.098\ s + 0.7651}{s^2 + 0.3279s + 3.265}$$

$$B'_{22}(s) = \frac{2.012\ s - 0.585}{s^2 + 0.3289s + 3.343}$$

Table 4.26 Comparison of error for example 4.15

r_{ij} (i,j=1,2)	Proposed Method	
	'*E*'	'*J*'
r_{11}	0.865	472.33×10^8
r_{12}	1.17	861.12×10^5
r_{21}	0.0647	795.1
r_{22}	0.02	335.4

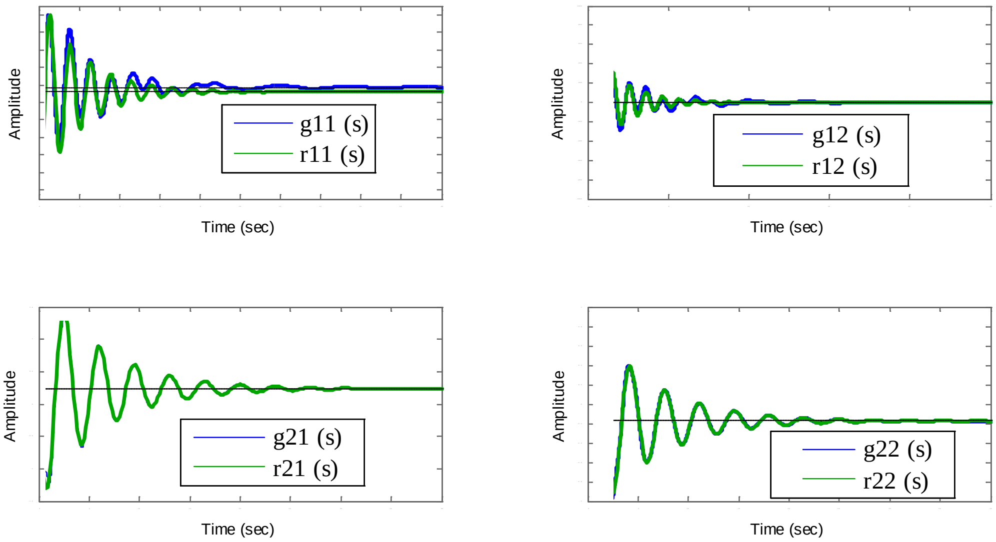

Fig. 4.16 Comparison of step responses for example 4.15

The proposed system is compared with other reduced systems for each element of transfer function matrix in terms of *'E'* and *'J'*. The value J_{org} of each element of plant transfer function matrix are 5.723 X 10^{10}, 1.9642 X 10^{10}, 831.087 and 367.43 respectively. The values obtained are tabulated in Table 4.26. The step responses of original, reduced models are depicted in Fig. 4.16.

Example 4.16: Consider the two input two output turbo-generator system given by [275] having the value J_{org} of each element of plant transfer function matrix are 80.845, 2535.99 X 10^3, 4.0086 and 2.9789 X 10^2 respectively.

$$A = \begin{bmatrix} -18.4456 & 4.2263 & -2.283 & 0.226 & 0.422 & -0.0951 \\ -4.0977 & -6.0706 & 5.6825 & -0.6966 & -1.2246 & 0.2873 \\ 1.4449 & 1.4336 & -2.6477 & 0.6092 & 0.8979 & -0.23 \\ -0.0093 & 0.2302 & -0.50022 & -0.1764 & -6.3152 & 0.135 \\ -0.0464 & -0.3489 & 0.7238 & 6.3117 & -0.6886 & 0.3645 \\ -0.06002 & -0.2361 & 0.23 & 0.0915 & -0.3214 & -0.2087 \end{bmatrix}$$

$$B^T = \begin{bmatrix} -0.2748 & -0.0501 & -0.155 & 0.0716 & -0.0814 & 0.0244 \\ 3.1463 & -9.3737 & 7.4296 & -4.9176 & -10.2648 & 13.7943 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.5971 & -0.7697 & 4.885 & 4.8608 & -9.8177 & -8.861 \\ 3.1013 & 9.3422 & -5.6 & -0.749 & 2.9974 & 10.5719 \end{bmatrix}$$

Applying BBBC, each element of transfer function matrix of [R'(s)] is obtained, where

$$B'_{11}(s) = \frac{-0.8983\ s - 0.4476}{s^2 + 1.522s + 0.3061} \qquad B'_{12}(s) = \frac{138.3\ s - 1217}{s^2 + 9.199s + 2.079}$$

$$B'_{21}(s) = \frac{-0.06748\ s + 0.084}{s^2 + 0.8489s + 0.151} \qquad B'_{22}(s) = \frac{-153.3\ s + 4528}{s^2 + 28.86s + 6.824}$$

Table 4.27 Error analysis for example 4.16

r_{ij} (i, j =1,2)	*'E'*	*'J'*
r_{11}	0.22	30.95
r_{12}	151.3	231.42×10^4
r_{21}	0.0016	1.43
r_{22}	30.01	310.26×10^4

Table 4.27 details the values of *'E'* and *'J'* for each element of transfer function matrix. The step responses of original, reduced models are plotted in Fig. 4.17 respectively. It is noticed that the response of the reduced system and the original system are comparable.

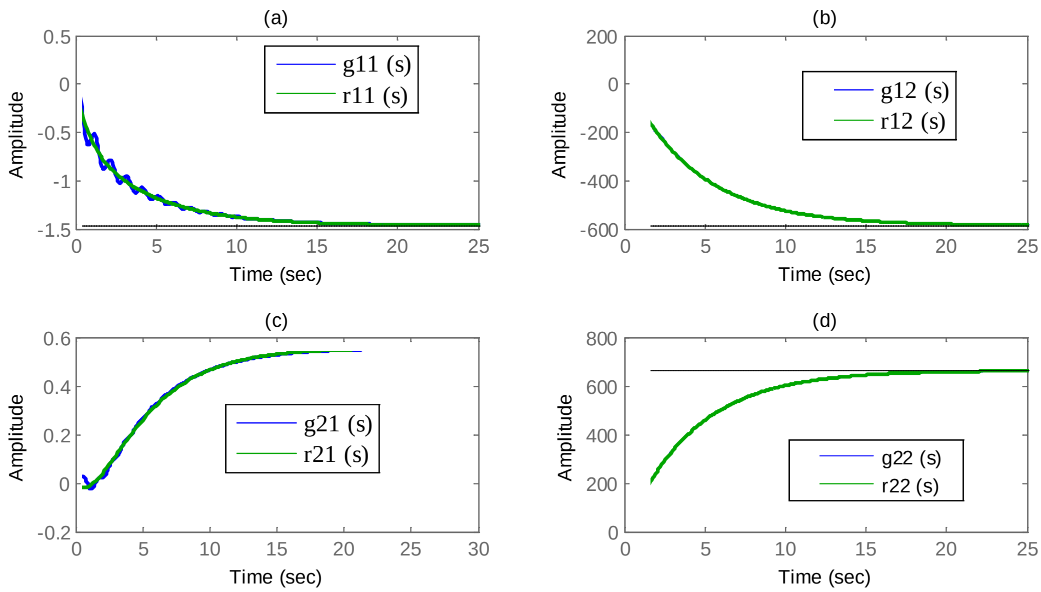

Fig. 4.17 Comparison of step responses for example 4.16

Example 4.17: A electric power system consisting of a salient pole synchronous generator connected to an infinite bus bar is considered. Taking into account the well known parameters, a very accurate mathematical model in state space form is given by [3]

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -52.08 & -23.54 & 9.415 & -23.54 & -99.2 & 27.88 \\ -541.2 & 0.6953 & -2.216 & -36.23 & 24.15 & 664.2 & -362.3 \\ -1136 & 1.46 & -0.664 & -76.08 & -12.08 & 1395 & -760.8 \\ 541.2 & 0.6953 & 1.328 & -36.23 & -38.65 & 664.2 & -362.3 \\ 398 & 3.403 & 897.1 & -1346 & 897.1 & -53.83 & -26.91 \\ 298.5 & 2.552 & 672.9 & -1009 & 672.9 & -40.37 & -35.89 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 0 & 0 & 2.121 & 0.6642 & -1.328 & 0 & 0 \\ 0 & 52.08 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -0.17 & 0 & 0 & 0.3018 & 0 & -0.0375 & 0 \end{bmatrix}$$

As per (4.20), the reduced order system [R'(s)] obtained by applying BBBC will be

$$B'_{11}(s) = \frac{0.009518\ s - 0.3161}{s^2 + 2.606s + 0.5733} \qquad B'_{12}(s) = \frac{0.9534\ s + 0.7437}{s^2 + 2.431s + 0.5298}$$

$$B'_{21}(s) = \frac{-0.2922\ s}{s^2 + 2.409s + 0.5301} \qquad B'_{22}(s) = \frac{80.17\ s}{s^2 + 114.7s + 74.17}$$

$$B'_{31}(s) = \frac{0.1436\ s + 0.2324}{s^2 + 2.311s + 0.5016} \qquad B'_{32}(s) = \frac{0.002814\ s - 0.4068}{s^2 + 2.611s + 0.5744}$$

The values of '*E*' and '*J*' for each element of transfer function matrix are according to Table 4.28. The value J_{org} of each element of plant transfer function matrix are 1.8591, 22.452, 1.011, 16310, 1.788 and 3.07 respectively. The step and impulse responses of original, reduced models are depicted in Fig. 4.18 (a) and (b) respectively.

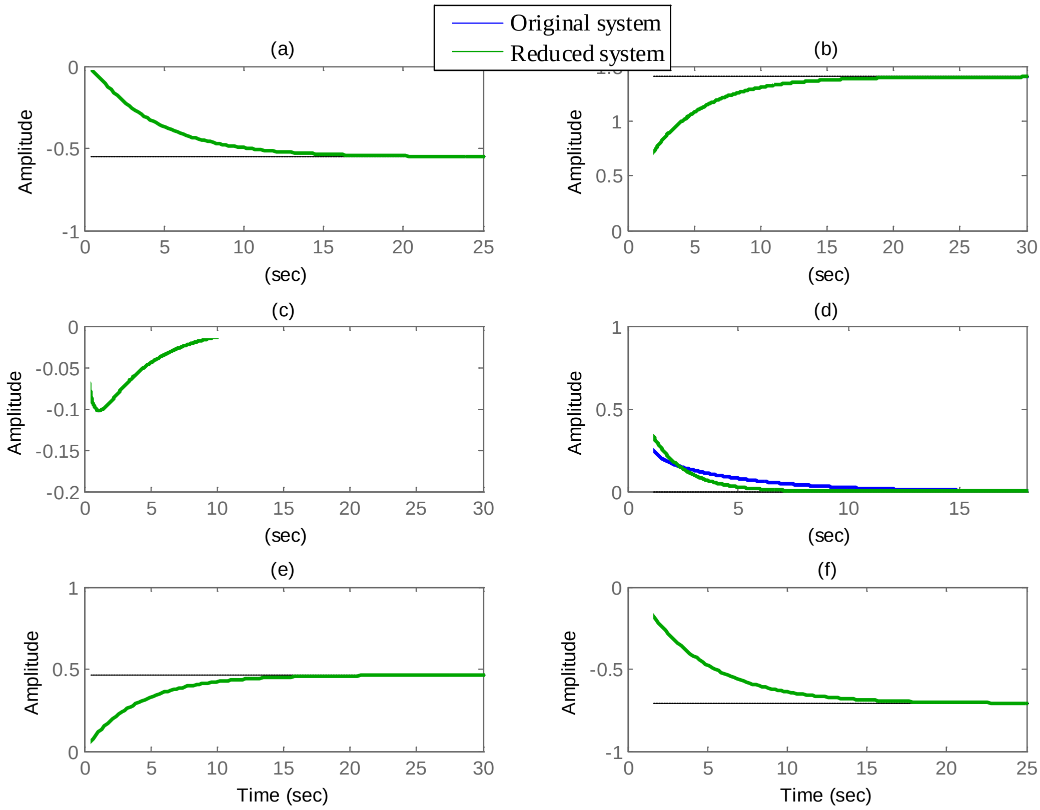

Fig. 4.18 (a) Comparison of step responses for example 4.17

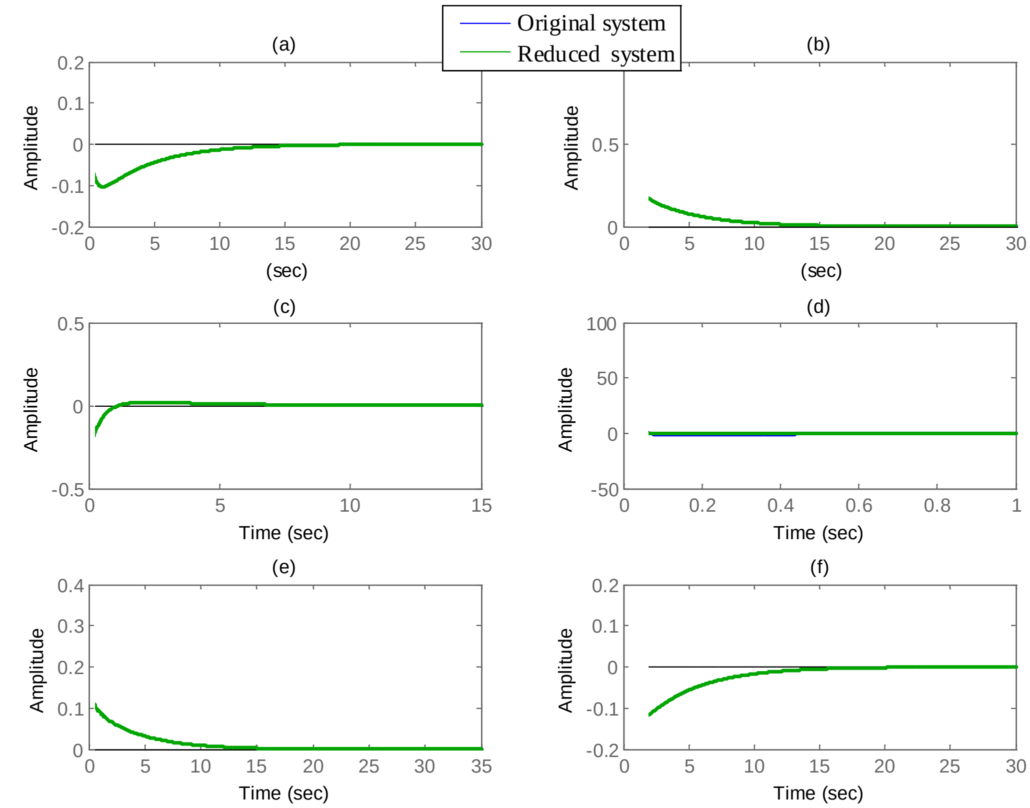

Fig. 4.18 (b) Comparison of impulse responses for example 4.17

Table 4.28 Comparison of error for example 4.17

r_{ij} (i=1,2,3; j=1,2)	'*E*'	'*J*'
r_{11}	0.54×10^{-8}	1.86
r_{12}	2900×10^{-8}	26.51
r_{21}	28.6×10^{-8}	1.51
r_{22}	1850×10^{-8}	15110
r_{31}	22.4×10^{-8}	1.652
r_{32}	1.49×10^{-8}	3.07

4.7 BBBC METHOD FOR REDUCTION OF DISCRETE TIME SYSTEMS

This section deals with the higher order systems represented in discrete domain. The typical parameters employed while using BBBC are according to Table 4.1 in 4.1.

Consider the n^{th} order discrete-time system be represented in (3.1). The objective is to find the r^{th} (r < n) order reduced system having the form (3.2) The effectiveness of BBBC is justified by solving illustrative examples as given below.

4.7.1 Illustrative Examples

Example 4.18: Consider eighth order higher order discrete transfer function [253]

$$G_n(z) = \frac{N_n(z)}{D_n(z)} = \frac{1.68z^7 + 1.116z^6 - 0.21z^5 + 0.152z^4 - 0.516z^3 - 0.262z^2 + 0.044z - 0.006}{8z^8 - 5.046z^7 - 3.348z^6 + 0.63z^5 - 0.456z^4 + 1.548z^3 + 0.786z^2 - 0.132z + 0.018}$$

Using BBBC the second order reduced system obtained is

$$G_r^2(z) = \frac{N_r(z)}{D_r(z)} = \frac{0.2339\text{ z - }0.1597}{z^2 - 1.765z + 0.839}$$

Table 4.29 Comparison of reduced order systems for example 4.18

Order Reduction Method	Reduced System G_r(z)	SSE
Proposed Method	$\frac{0.2339\text{ z - }0.1597}{z^2 - 1.765z + 0.839}$	0.00025
Hwang *et. al.* [254]	$\frac{0.316331z - 0.262395}{z^2 - 1.73034z - 0.784276}$	0.062
Bistritz [253]	$\frac{0.2696z - 0.2157}{z^2 - 1.73z - 0.7842}$	0.052
Bistritz [132]	$\frac{0.37131242z - 0.298}{z^2 - 1.626873z - 0.701497}$	0.057
Hwang *et. al.* [254]	$\frac{0.3664429z - 0.28918}{z^2 - 1.626873z - 0.701497}$	0.065
Hwang and Shih [248]	$\frac{0.2018z^2 + 0.04484z - 0.156}{1.2z^2 - 1.955z + 0.843}$	0.0791

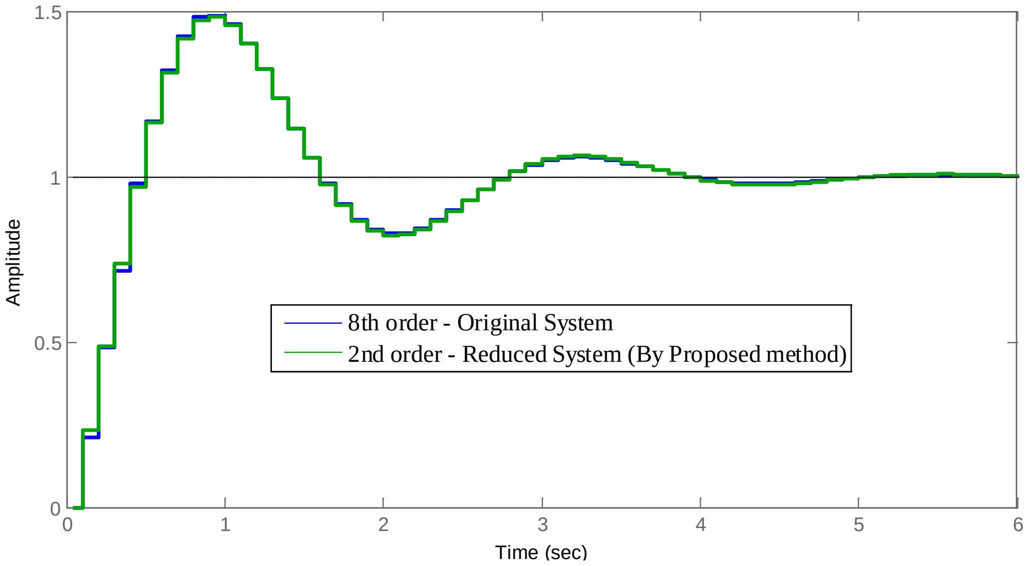

Fig. 4.19 (a) Comparison of step responses for example 4.18

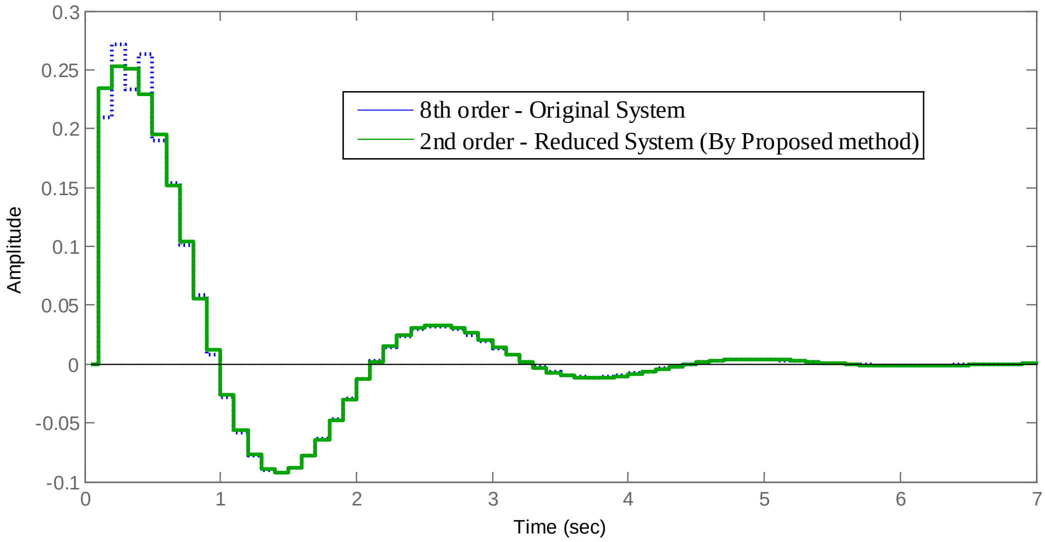

Fig. 4.19 (b) Comparison of impulse responses for example 4.18

The responses of the original and reduced system are shown in Fig. 4.19 (a)-(b). It is observed that the responses obtained are more or less exactly same. The results in terms of SSE (3.10) are compared with other available methods in Table 4.29.

Example 4.19: Revisiting a supersonic inlet model transfer function of example 2 in 3.5.1 and using BBBC, the second order reduced system obtained will be

$$G_r(z) = \frac{N_r(z)}{D_r(z)} = \frac{1.456\text{ z - }1.16}{z^2 - 0.9245z + 0.01274}$$

The original and reduced system is subjected to step and impulse input. Their corresponding responses are shown in Fig. 4.20 (a)-(b). These responses are compared in Table 4.30 with respect to SSE with other available methods.

Table 4.30 Comparison of reduced order systems for example 4.19

Order Reduction Method	Reduced System $G_r(z)$	SSE
Proposed Method (second order)	$\dfrac{1.456z - 1.16}{z^2 - 0.9245z + 0.01274}$	0.1713
Lalonde *et. al.*[176] (third order)	$\dfrac{0.0627z^3 - 2.106z^2 + 1.569z + 0.0371}{z^3 - 0.8204z^2 + 0.1697z - 0.1648}$	0.55879
Lalonde *et. al.*[176] (fifth order)	$\dfrac{1.72\times10^{-4}z^5 + 2.1z^4 - 2.9z^3 + 2.15z^2 - 1.5z - 0.66}{z^5 - 1.488z^4 + 1.231z^3 - 0.96z^2 - 0.6693z - 0.3247}$	0.17453

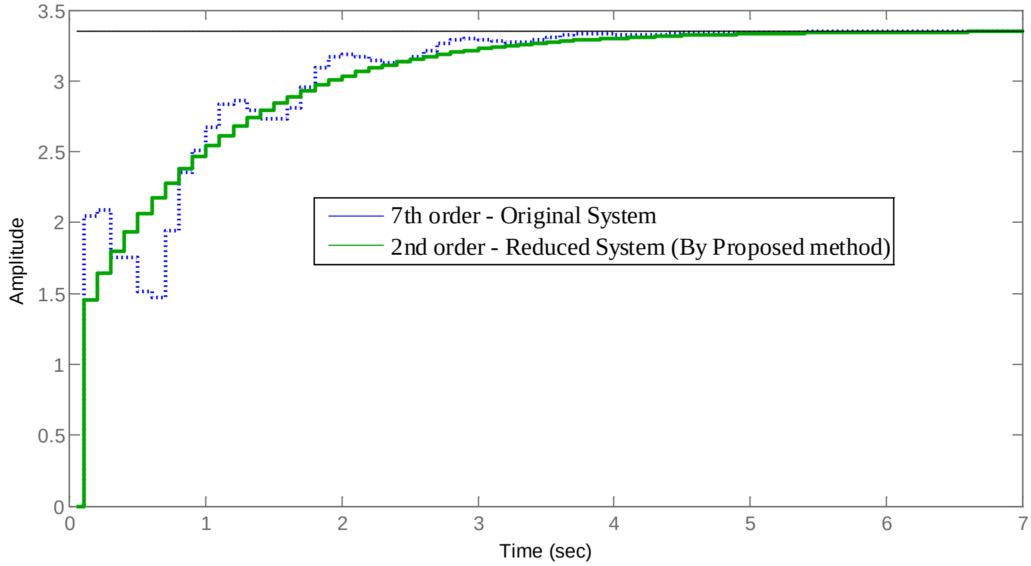

Fig. 4.20 (a) Comparison of step responses for example 4.19

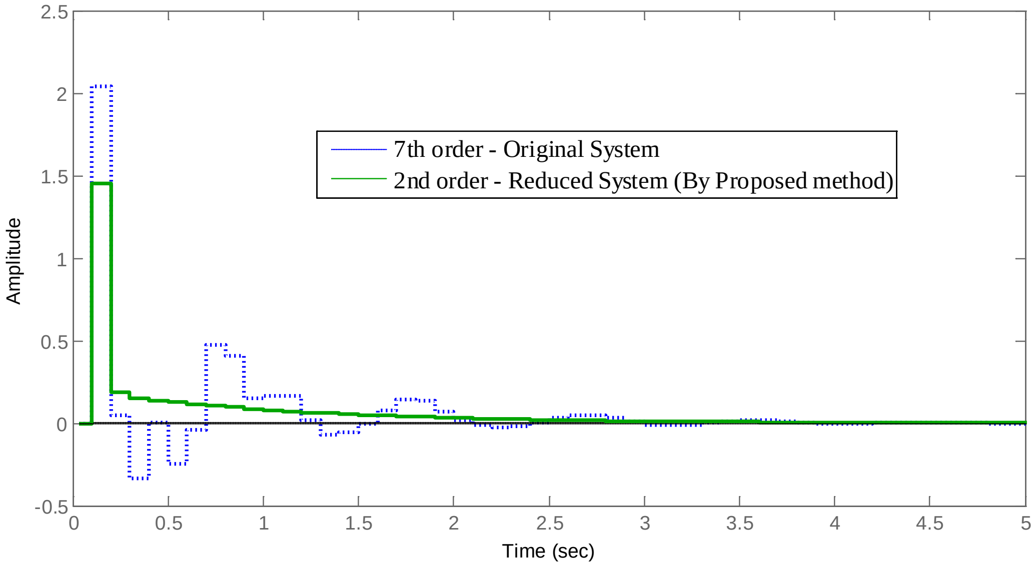

Fig. 4.20 (b) Comparison of impulse responses for example 4.19

Example 4.20: Consider the 8th order transfer function investigated by Alsmadi et al. [276] given as

$$G_n(z) = \frac{0.1625z^7 + 0.125z^6 - 0.0025z^5 + 0.00525z^4 - 0.02263z^3 - 0.00088z^2 + 0.003z + 0.000413}{z^8 - 0.6307z^7 - 0.4185z^6 + 0.078z^5 - 0.057z^4 + 0.1935z^3 + 0.09825z^2 - 0.0165z + 0.00225}$$

Using BBBC the second order reduced system obtained is

$$G_r(z) = \frac{N_r(z)}{D_r(z)} = \frac{0.1663\ z - 0.08625}{z^2 - 1.756z + 0.8301}$$

The corresponding responses of the original and reduced system, when subjected to step and impulse input are shown in Fig. 4.21 (a)-(b). It is worth to mention here that, the $G_n(z)$ and $G_r(z)$ behaves exactly similar for a given step/impulse input. These responses are compared in Table 4.31 in terms of SSE with other available methods.

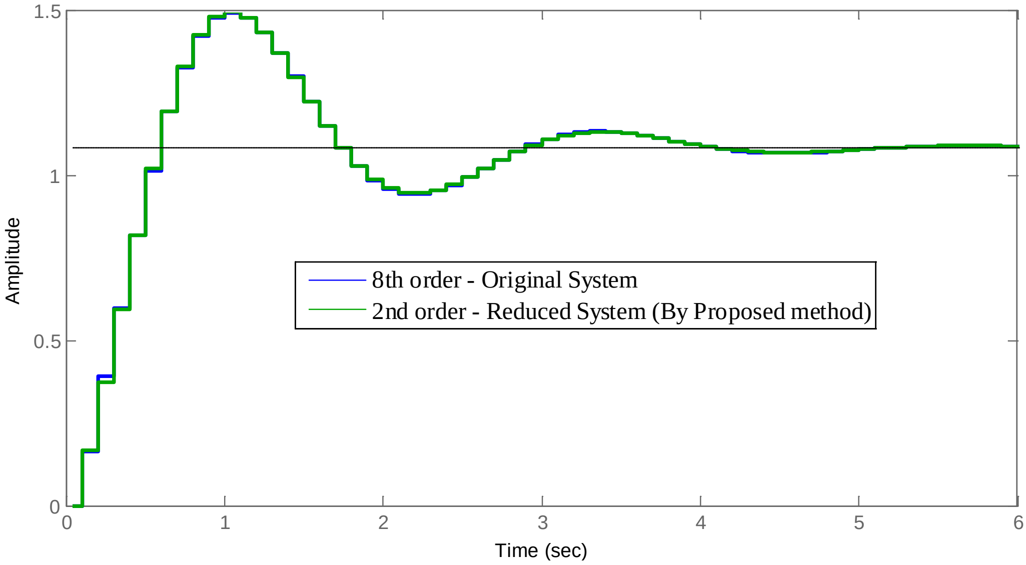

Fig. 4.21 (a) Comparison of step responses for example 4.20

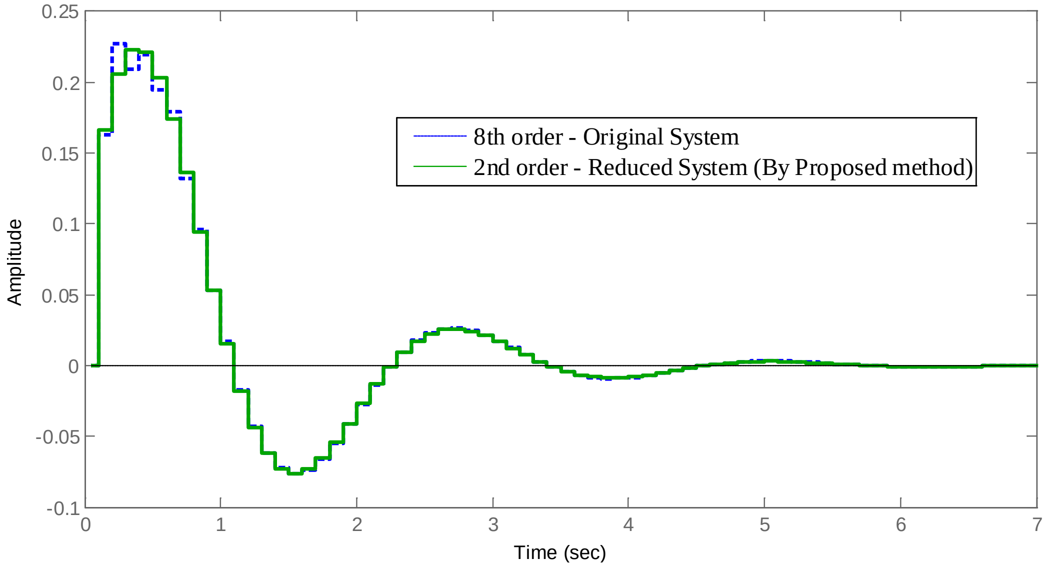

Fig. 4.21 (b) Comparison of impulse responses for example 4.20

Table 4.31 Comparison of reduced order systems for example 4.20

Order Reduction Method	Reduced System	SSE
Proposed Method (second order)	$\dfrac{0.1663z - 0.08625}{z^2 - 1.756z + 0.8301}$	0.18×10^{-3}
Alsmadi et al. [276]	$\dfrac{0.1623z + 0.08245}{z^2 - 1.759z + 0.8335}$	48.7×10^{-3}
Ramesh [277] (Method 1)	$\dfrac{0.004756z - 0.002325}{0.02927z^2 - 0.05133z + 0.0243}$	0.43×10^{-3}
Ramesh [277] (Method 2)	$\dfrac{0.02387z^2 + 0.0576z + 0.0337}{1.145z^2 - 1.946z + 0.9098}$	15.9×10^{-3}
Ramesh [277] (Method 3)	$\dfrac{0.4094z - 0.2947}{1.235z^2 - 1.948z + 0.8158}$	6.55×10^{-3}
Ramesh [277] (Method 4)	$\dfrac{0.1521z^2 + 0.0497z - 0.1024}{1.204z^2 - 1.956z + 0.84}$	4732×10^{-3}

4.8 CONCLUSION

The new reduction methods suggested in this chapter comprises of a recently introduced evolutionary technique named BBBC. Some of the suggested new methods is a combination of BBBC and stability preserving methods such as Routh approximation and stability equation. Another new concept of reduction method is suggested which optimizes the value of linear shift point 'a' using least square method. Some of the advantages of the proposed methods are

1. No knowledge of eigen values/ vectors are needed.
2. Problem of solving nonlinear equations does not exist in some method.
3. Knowledge of possible reduced system coefficients beforehand not necessary as in some search techniques.

CHAPTER - 5

ORDER REDUCTION OF DIGITAL FILTERS IMPLEMENTED ON TMS320C5402 DSP

According to the discussion in the previous chapters, it was noted that the key ingredients of any system viz. complexity and system order, are escalating exponentially in this fast ever-changing era. The analysis, design of such systems has assumed greater importance and generally this is being accomplished with the help of simulation. However, simulation becomes untenable with increasing, complexity and order of the systems under consideration. This is so, because they prove to be costly, time consuming, larger memory usage and inefficient sometimes. The remedy is to find an alternate competent reduced order model, which preserves the vital dynamic elements while eliminating the spurious ones.

Digital Signal Processing has been exciting and growing technology during the past few decades [278]. Its applications have also been expanded vigorously to encompass almost all fields resulting in tremendous progress in both the theoretical and practical aspects. While more DSP algorithms are being discovered, better tools are also being developed to implement these algorithms. In the recent years, the availability of cheap DSP chips (both general purpose and specific purpose), have created a great impact on different disciplines from electronic and mechanical engineering to economics and metallurgy [279]. This has motivated to pioneer alternate way of implementing reduction technique on TMS320C5402 DSP, to generate a reduced model. The reduced model is ought to inherit all the significant behavioral qualities of the original model.

In today's digital world, the design of digital filter has emerged as a vital player in the signal processing field. This is due to the fact, that digital filters have their applications soaring, day by day. It is penetrating in almost all areas including control systems, audio/video processing, communication systems and systems for medical applications to name just a few. Nowadays, online and offline data can be processed using digital filters, which are being realized both in hardware and software. Digital filters in hardware form can perform tasks, that were almost exclusively performed by analog systems in the past. On the other hand, software realization can be achieved using programming languages. In this work, both lowpass and highpass digital Infinite Impulse Response (IIR) filter is realized, reduced and is implemented on TMS320C5402 DSP for a given filter specifications. Since the order of the designed IIR filter is high, a recently erupted Big Bang Big Crunch (BBBC) based optimization technique is employed for reducing the order of the designed high order IIR filter. This further ensures easy, less complex, economical, realization of the filter. Moreover the reduced filter will be comparatively compact because of lesser number of adders and multipliers being used. The results obtained for reduced and original high order IIR filter, justifies the proposed approach.

The generation of TMS320C5402 DSPs integrate functions to improve performance, lower chip count and reduced power consumption to enable greater system cost savings. These devices combine high-performance, a large degree of parallelism and a specialized instruction set to effectively implement a variety of complex algorithms and applications. The features list include a Viterbi accelerator, four internal buses, dual-address generators, 40-bit adder, two 40-bit ALUs, eight auxiliary registers and a software stack [280]. Further, these DSP's are also highly programmable, which makes them very attractive for system upgrades and multitasking. Because of the above mentioned attributes and many more, DSP's are on the verge to be mushroomed in our day-to-day activities as well as in the near future.

5.1 TMS320C5402 DSP

Texas Instruments (TI) introduced its first general purpose fixed point DSP TMS32010 in late 1982. The TMS320 product line contains a family of DSP's designed to support a wide range of high-speed or numeric-intensive DSP applications [278]. It has extended into two major classes: the floating point and the fixed point processors. The TMS320C54X DSP, grouped under the latter category [281-283] is a low-cost, comprehensive development tool; allows new DSP designers to explore the TMS320C5000 DSP architecture and begin developing DSP based applications. It has functional adaptability to a great extent and processing speed. Advanced modified Harvard architecture comprising of dedicated buses for program memory, data memory and address is employed. CPU with application specific hardware logic, on-chip memory, on-chip peripherals, large read only memory spaces for integrating entire algorithms on chip, highly specialized instruction set are additional features. The C54x devices have modular architecture design for fast development of byproduct devices and advanced integrated circuit processing technology, for increased performance with low power consumption [284].The foremost characteristics [285] of the 'C54x family of DSP's are:

1. Performance ranging from 30-200 MIPS

2. Multiplier of 17 x 17-bit
3. ALU (40-bit) and dual accumulators (40-bit),
4. Variety of versions operating on 1.2 -5 voltage range are available,
5. Addressing mode for 8Mx16-bit max. (extended)
6. Addressable external space
7. On-chip RAM (up to 200 K words)
8. Direct memory access (DMA) controller (six-channel)
9. 8/16-bit host port interface (HPI)
10. Full-duplex type of serial ports are available
11. Serial port to support 8/16 bit transfers, time-division
12. Multiplexed (TDM) serial port, buffered serial port (BSP) and multi-channel BSP (McBSP)
13. Ultra-thin packaging (100, 128, and 144-pin TQFPs)

5.1.1 Fixed Point Representation

The TMS320C5402 DSP is basically a 16 bit processor with the dynamic number range varying from 32767 to -32768. These fixed point devices assume the binary point after the sign bit as shown in Fig. 5.1.This fractional number representation is called Q15 format since there are15 magnitude bits. The approximate allowable range of numbers in Q15 format representation is 0.999 to -1.

$$x = b_0 . b_1 b_2 ... b_{M-1} b_M$$

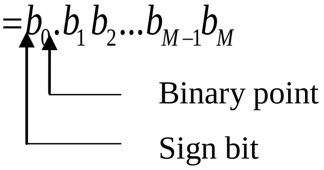

Fig. 5.1 Fixed point representation of binary fractional numbers.

5.1.2 Code Composer Studio

Code Composer Studio (CCS) is the integrated development environment used to implement various algorithms in TI owned DSPs and microcontrollers [286]. It comprises of tools required for developing and debugging embedded applications. TI's device specific compilers, code (source) editor, environment for building projects, debugger, profiler, simulators etc are some of the additional features. To conclude, it provides a single user interface, facilitating the users to go through every step of the application development. CCS helps in speeding up process development stage, for the users working on real time signal processing and embedded applications. Further, provision for coding in C/assembly language, development and debugging are also provided [282]. Similar kind of tools/interfaces used elsewhere are incorporated, which helps the new users feel comfortable and start working quickly, modify/add additional features to their applications [287].

The CCS supports four phases of the development cycle as shown in the Fig. 5.2. The initial phase is the design phase, where all the conceptual planning is done. The CCS creates project, writes the code and configures the file in the code and build phase. Debugging is done in debug phase and lastly, the real time debugging is done in the analysis phase. In other words, CCS provides the interface with the C54X simulator(SIM), DSP starter kit (DSK), evaluation module (EVM) or in circuit emulator (XDS).

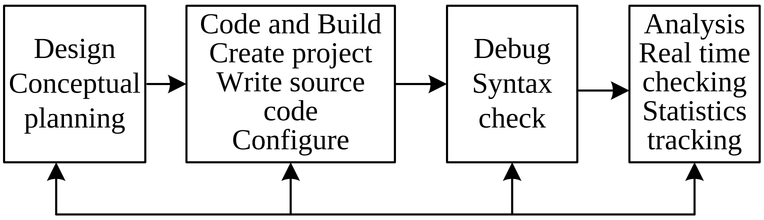

Fig. 5.2 Development cycle of CCS

In order to implement the filter design on TMS320C5402 processor the following steps are being followed

Step 1: Generate a test signal comprising of different frequency components.

Step 2: Design an IIR filter $H(z)$ of N^{th} order for desired specifications.

Step 3: Using the filter coefficients, obtain the filter output.

Step 4: Obtain the impulse and frequency response of the designed filter.

Step 5: Applying BBBC, obtain the reduced filter $H_{red}(z)$ coefficients.

Step 6: Compare the impulse and frequency response with the original after obtaining the filter output.

Step 7: Export the scaled data in Q15 format into TMS320C5402 processor using CCS and execute the program.

Step 8: Plot the responses in CCS for comparison and justification.

5.2 BBBC METHOD FOR ORDER REDUCTION

In this section, BBBC optimization technique described in 4.1, is utilized to optimize the coefficients of the reduced order filter.

5.2.1 Problem Statement

A finite-dimensional digital IIR filter under stable condition can be represented in the z-domain as

$$H(z)=\frac{\sum_{i=1}^{N} b_i z^{-i}}{\sum_{i=1}^{N} a_i z^{-i}} \quad ; a_1 = 1$$
$$= \frac{b_1 + b_2 z^{-1} + \ldots + b_{N+1} z^{-N}}{a_1 + a_2 z^{-1} + \ldots + a_{N+1} z^{-N}} \tag{5.1}$$

It is desired to find a reduced 'r' (r < N) order digital IIR filter $H_{red}(z)$ given by

$$H_{red}(z)=\frac{\sum_{i=1}^{r} d_i z^{-i}}{\sum_{i=1}^{r} c_i z^{-i}} \quad ; c_1 = 1$$
$$= \frac{d_1 + d_2 z^{-1} + \ldots + d_{r+1} z^{-r}}{c_1 + c_2 z^{-1} + \ldots + c_{r+1} z^{-r}} \tag{5.2}$$

5.2.2 Illustrative Examples

Example 5.1: Consider a fifteenth order highpass butterworth filter designed for 70Hz cutoff frequency, sampling frequency being 300Hz is represented by *H*(z). It is desired to find a approximate reduced order filter.

$$H(z)= \frac{\begin{array}{c}0.0003-0.0051z^{-1}+0.0354z^{-2}-0.1534z^{-3}+0.4602z^{-4}-1.0125z^{-5}+1.6876z^{-6}-2.1697z^{-7}+2.1697z^{-8}-\\ 1.6876z^{-9}+1.0125z^{-10}-0.4602z^{-11}+0.1534z^{-12}-0.0354z^{-13}+0.0051z^{-14}-0.0003z^{-15}\end{array}}{\begin{array}{c}1-0.9982z^{-1}+2.4783z^{-2}-1.8191z^{-3}+2.1660z^{-4}-1.1766z^{-5}+0.8526z^{-6}-0.3384z^{-7}+0.1587z^{-8}-0.0444z^{-9}+0.0133z^{-10}\\ -0.0024z^{-11}+0.0004z^{-12}-0.00004z^{-13}+0.0000034z^{-14}-0.0000001z^{-15}\end{array}}$$

Scaling appropriately the filter coefficients in Q15 format is given by

$$H_{Q15}(z)= \frac{\begin{array}{c}1h+0Chz^{-1}+04Ehz^{-2}+0150hz^{-3}+03EEhz^{-4}+08A4hz^{-5}+0E67hz^{-6}+01284hz^{-7}+01284hz^{-8}+0E67hz^{-9}\\ +08A4hz^{-10}+03EEhz^{-11}+0150h\,z^{-12}+04Ehz^{-13}+0Chz^{-14}+01h\,z^{-15}\end{array}}{\begin{array}{c}0889h+0885hz^{-1}+01526hz^{-2}+0F86hz^{-3}+0127Chz^{-4}+0A0Bhz^{-5}+0747h\,z^{-6}+02E4hz^{-7}+015Bhz^{-8}+061h\,z^{-9}+\\ 01Ehz^{-10}+06hz^{-11}+01hz^{-12}+01hz^{-13}+01hz^{-14}\end{array}}$$

Applying BBBC for minimizing the fitness function given by

$$err = \max_{\omega \le \omega_{pass}}\left(\left|W_f(\omega)[H(e^{j\omega})-H_{red}(e^{j\omega})]\right|-\delta^{pass}\right)+\max_{\omega_{stop} \le \omega}\left(\left|W_f(\omega)[H(e^{j\omega})-H_{red}(e^{j\omega})]\right|-\delta^{stop}\right) \tag{5.3}$$

where *err* is the error, W_f is the weighting function, $H(e^{j\omega})$, $H_{red}(e^{j\omega})$ is the frequency response of the original high order and reduced order filter, δ^{pass} and δ^{stop} are the pass band and stop band ripples, ω_{pass} and ω_{stop} are the normalized pass band and stop band cut off frequencies.

The reduced order filter in Q15 format is obtained as

$$H_{Q15red}(z) = \frac{1h + 0Ahz^{-1} + 03Bhz^{-2} + 0CFhz^{-3} + 01F1hz^{-4} + 0315hz^{-5} + 0385hz^{-6} + 026Bhz^{-7} + 0118hz^{-8}}{0889h + 070Dhz^{-1} + 0108Ahz^{-2} + 0C6Ahz^{-3} + 0BC3hz^{-4} + 06A4hz^{-5} + 0340hz^{-6} + 010Bhz^{-7} + 030hz^{-8}}$$

or

$$H_{red}(z) = \frac{0.00033 - 0.0043z^{-1} + 0.0268z^{-2} - 0.0944z^{-3} + 0.227z^{-4} - 0.361z^{-5} + 0.412z^{-6} - 0.283z^{-7} + 0.1281z^{-8}}{1 + 0.826z^{-1} + 1.937z^{-2} + 1.454z^{-3} + 1.378z^{-4} + 0.778z^{-5} + 0.38z^{-6} + 0.121z^{-7} + 0.0218z^{-8}}$$

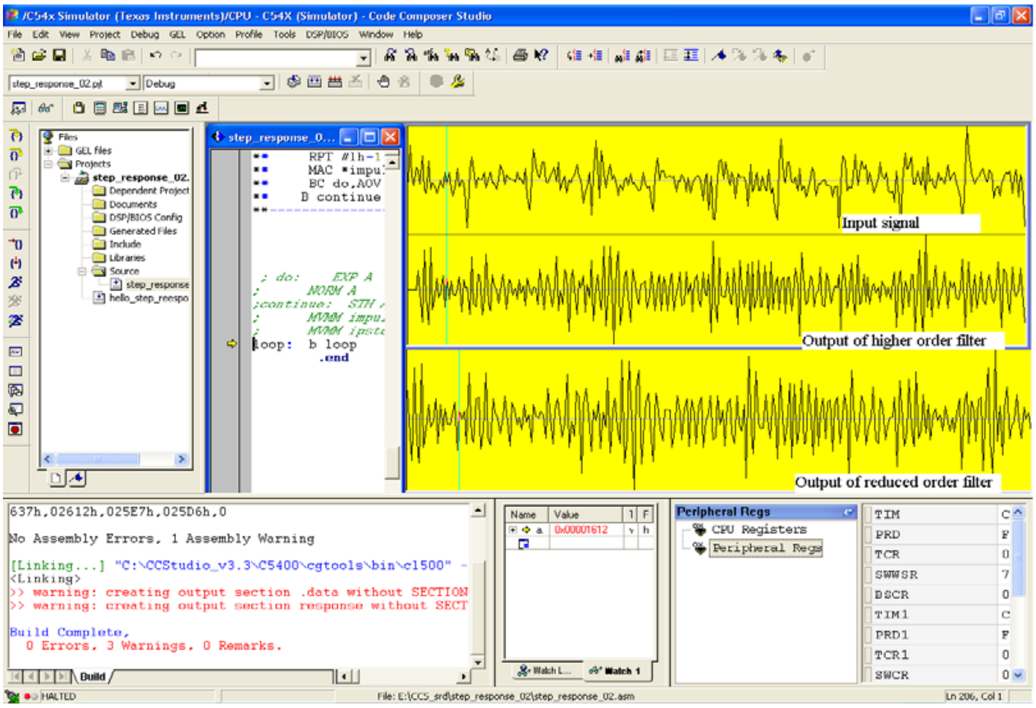

Fig. 5.3 Comparison of filter responses obtained in CCS for example 5.1

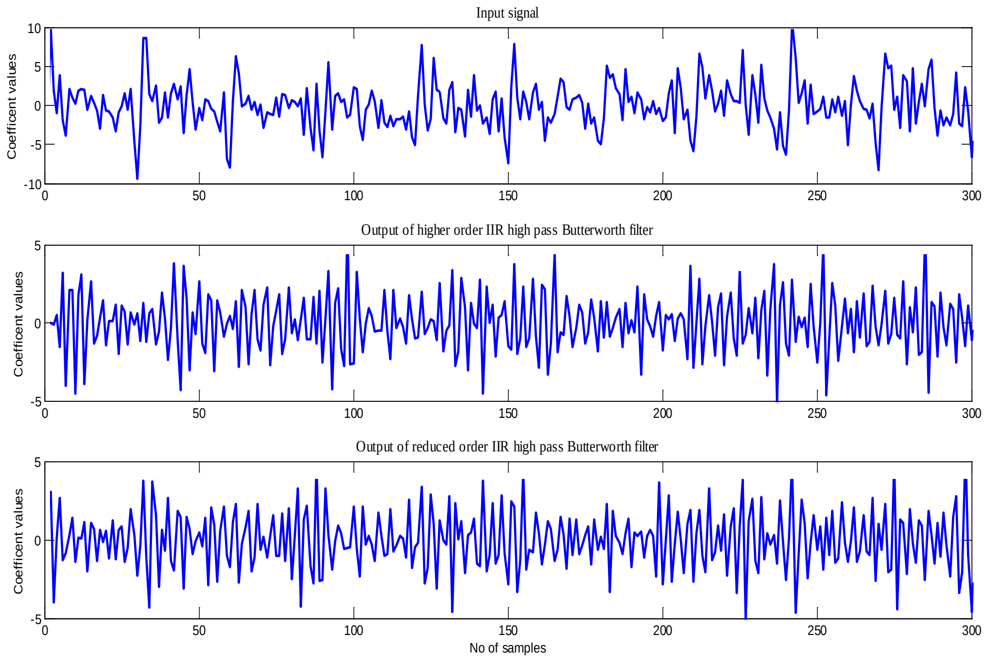

Fig. 5.4 Comparison of filter responses for example 5.1

Fig. 5.3 and 5.4 depicts the waveforms of the input (test) signal, filter response of the higher order (H(z)) and reduced order (H_{red}(z)) highpass butterworth filter obtained in CCS and MATLAB respectively. Fig. 5.5 compares the magnitude and phase responses of *H*(z) and H_{red}(z) obtained in MATLAB using fvtool. Similarly FFT power spectrum of input signal, *H*(z) and H_{red}(z) are compared in Fig. 5.6. It is seen that the frequency components above the cuttoff frequency (70 Hz) are passed through the filter as designed.

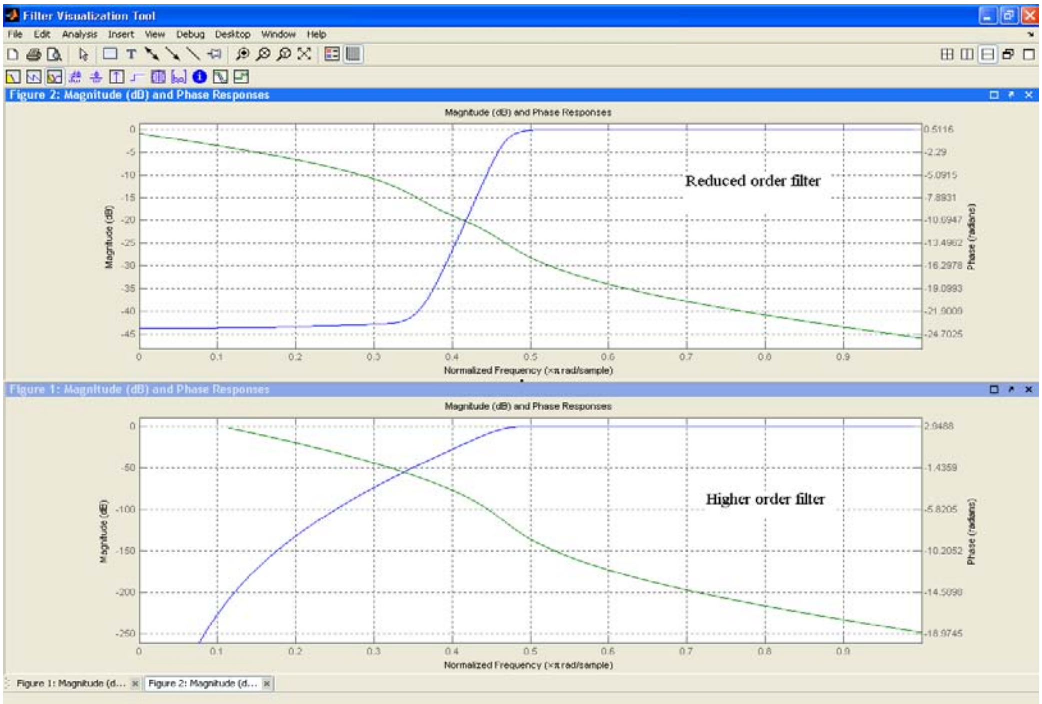

Fig. 5.5 Comparison of magnitude and phase responses for example 5.1

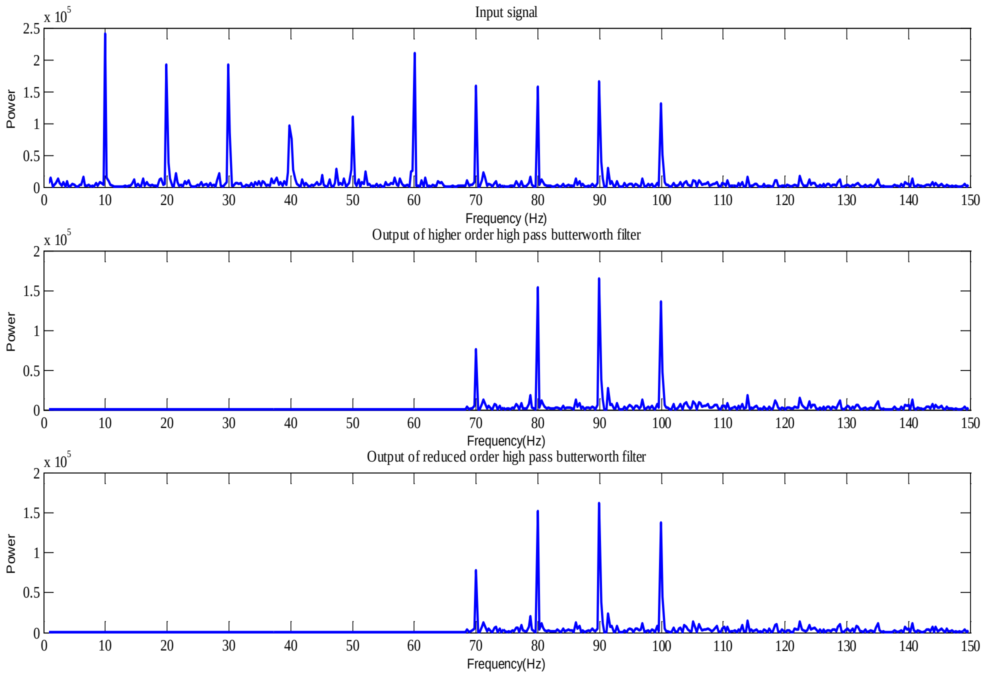

Fig. 5.6 Comparison of FFT power spectrum for example 5.1

Example 5.2: A thirteenth order highpass Chebyshev filter with cutoff frequency equal to 70Hz and sampling frequency is 300Hz having ripple factor of 0.1 is described by *H*(z).

$$H(z)=\frac{0.000088-0.00115z^{-1}+0.0069z^{-2}-0.025z^{-3}+0.0635z^{-4}-0.114z^{-5}+0.152z^{-6}-0.1527z^{-7}+0.114z^{-8}-0.0635z^{-9}+0.0254z^{-10}-0.0069z^{-11}+0.00115z^{-12}-0.000088z^{-13}}{1+2.766z^{-1}+6.749z^{-2}+11.398z^{-3}+16.258z^{-4}+19.056z^{-5}+19.0395z^{-6}+16.1408z^{-7}+11.618z^{-8}+7.0115z^{-9}+3.47z^{-10}+1.35142z^{-11}+0.3795z^{-12}+0.06296z^{-13}}$$

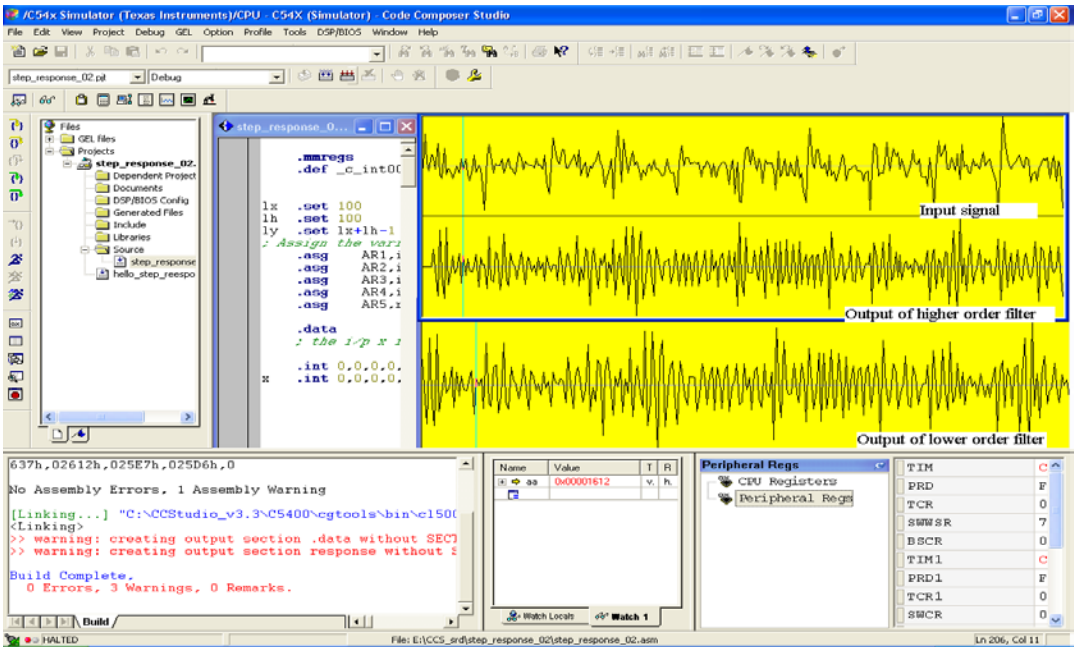

Fig. 5.7 Comparison of filter responses obtained in CCS for example 5.2

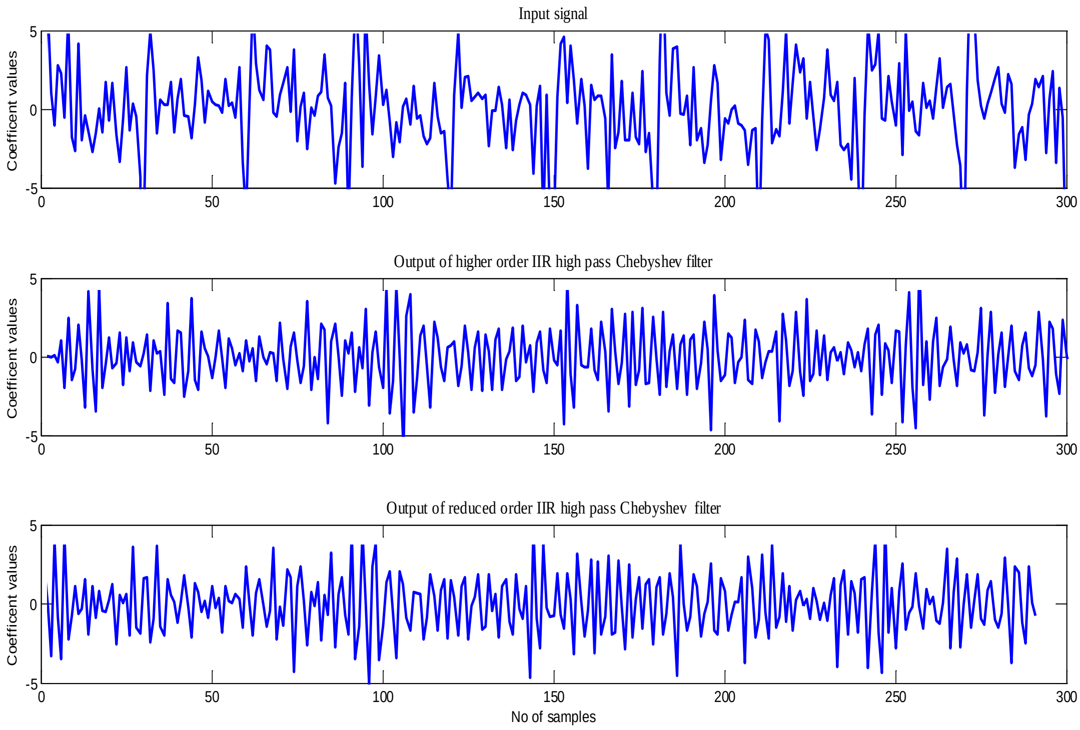

Fig. 5.8 Comparison of filter responses for example 5.2

Scaling appropriately the filter coefficients in Q15 format is given by

$$H_{Q15}(z)=\frac{1h+02hz^{-1}+0Bhz^{-2}+028hz^{-3}+064hz^{-4}+0B3hz^{-5}+0EEhz^{-6}+0EEhz^{-7}+0B3hz^{-8}+064hz^{-9}+028hz^{-10}+0Bhz^{-11}+02hz^{-12}+01hz^{-13}}{0619h+010DDhz^{-1}+02924hz^{-2}+0457Ahz^{-3}+0631Ahz^{-4}+07428hz^{-5}+0740Dhz^{-6}+06262hz^{-7}+046D1hz^{-8}+02ABDhz^{-9}+01528hhz^{-10}+083Dhz^{-11}+0251hz^{-12}+063hz^{-13}}$$

Using BBBC, satisfying (5.3) the eight order filter is obtained in Q15 format as

$$H_{Q15red}(z)=\frac{1h+010hz^{-1}+05Chz^{-2}+069hz^{-3}+0244hz^{-4}+038hz^{-5}+04D6hz^{-6}+08Bhz^{-7}+031Ehz^{-8}}{0925h+0C37hz^{-1}+018A6hz^{-2}+018E3hz^{-3}+017E4hz^{-4}+01039hz^{-5}+08ADhz^{-6}+0336hz^{-7}+0A0hz^{-8}}$$

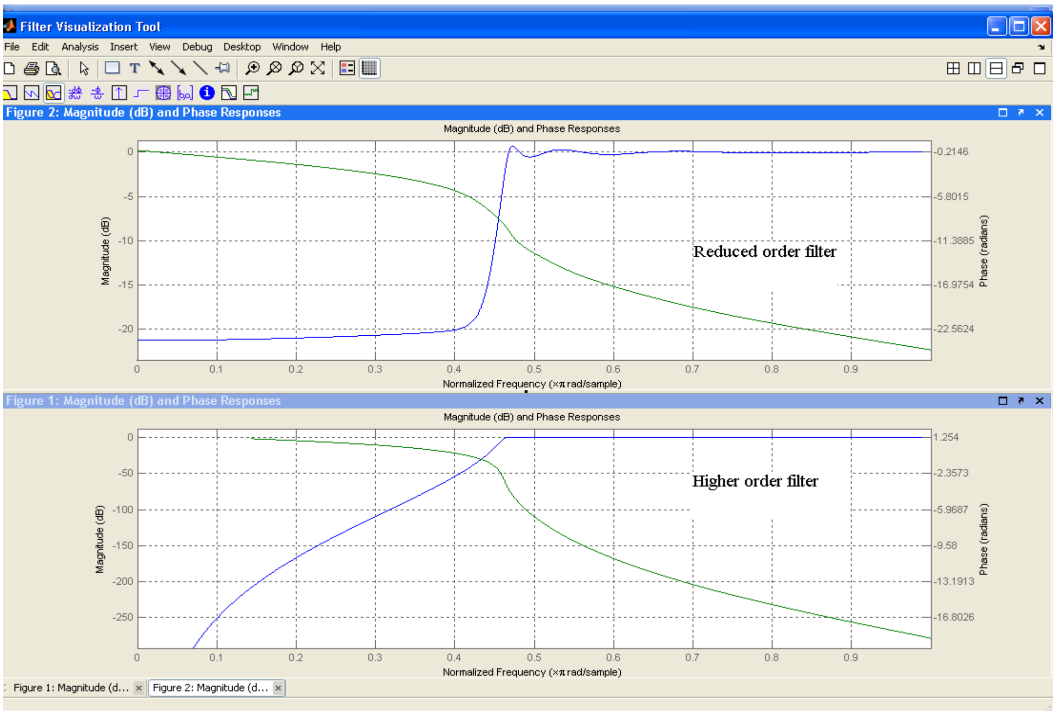

Fig. 5.9 Comparison of magnitude and phase responses for example 5.2

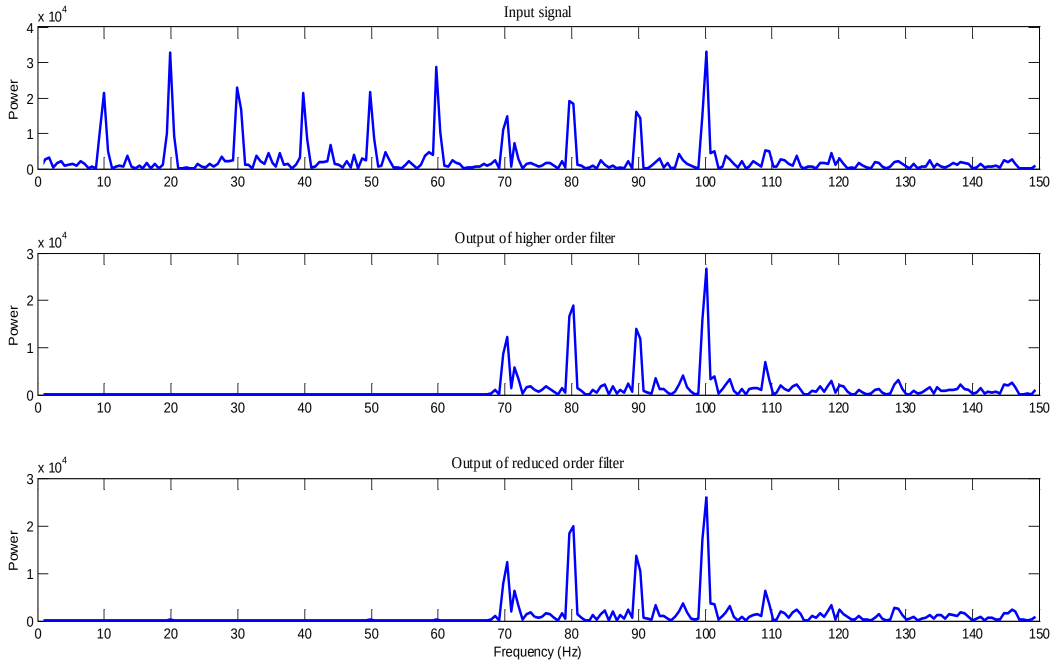

Fig. 5.10 Comparison of FFT power spectrum for example 5.2

The input signal, filtered output of higher order (H(z)) and reduced order chebyshev filter (H_{red}(z)) obtained in CCS are shown in Fig. 5.7. The same set of signals obtained in MATLAB are depicted in Fig. 5.8. It is observed that the signals obtained in CCS and MATLAB are alike. The response obtained from higher order filter has introduced some delay in the beginning. Hence the magnitude is zero initially. This delay is nullified in the response of the reduced order filter. The magnitude and phase responses of H(z) and H_{red}(z) are compared in Fig. 5.9 using fvtool in MATLAB. The phase responses in both cases are similar. However, the magnitude response of reduced order filter exhibits ripple in the pass band due to the characteristic of the chebyshev filter. The highpass filter under discussion is designed for 70Hz cutoff frequency. The same is clearly visible in Fig. 5.10, which depicts the FFT power spectrum of input signal, output of higher order and reduced order filter.

Example 5.3: A twelfth order lowpass Butterworth filter with cutoff frequency equal to 50Hz and sampling frequency 300Hz is described by H(z).

$$H(z)=\frac{0.000019+0.000229z^{-1}+0.00126z^{-2}+0.0042z^{-3}+0.00946z^{-4}+0.0151z^{-5}+0.0176z^{-6}+0.0151z^{-7}+0.00946z^{-8}+0.0042z^{-9}+0.00126z^{-10}+0.000229z^{-11}+0.000019z^{-12}}{1-3.99z^{-1}+8.569z^{-2}-12.15z^{-3}+12.473z^{-4}-9.6z^{-5}+5.638z^{-6}-2.528z^{-7}+0.855z^{-8}-0.211z^{-9}+0.036z^{-10}-0.0038z^{-11}+0.00019z^{-12}}$$

Scaling appropriately the filter coefficients in Q15 format is given by

$$H_{Q15}(z)=\frac{1h+01hz^{-1}+03hz^{-2}+0Ahz^{-3}+017hz^{-4}+024hz^{-5}+02Ahz^{-6}+024hz^{-7}+017hz^{-8}+0Ahz^{-9}+03hz^{-10}+01hz^{-11}+01hz^{-12}}{0925h+0247Dhz^{-1}+04E59hz^{-2}+06F2Bhz^{-3}+0720Chz^{-4}+057C8hz^{-5}+0338Ehz^{-6}+0171Fhz^{-7}+07D2hz^{-8}+01F1hz^{-9}+056hz^{-10}+0Ahz^{-11}+01hz^{-12}}$$

Using BBBC, the eight order filter is obtained in Q15 format as

$$H_{Q15red}(z)=\frac{1h+01hz^{-1}+04hz^{-2}+0Chz^{-3}+018hz^{-4}+029hz^{-5}+029hz^{-6}+021hz^{-7}+012hz^{-8}}{09D9h+025D5hz^{-1}+04BAAhz^{-2}+060B7hz^{-3}+05523hz^{-4}+03455hz^{-5}+015DDhz^{-6}+05ADhz^{-7}+0B5hz^{-8}}$$

The input signal comprising of 10Hz to 100Hz frequency components in steps of 10, is added with random noise and is shown in Fig. 5.11 and Fig. 5.12. In addition to this, the respective output of the higher order and reduced order butterworth lowpass filters are also obtained accordingly. It can be observed that the filtered output of H(z) and H_{red}(z) are identical. In Fig. 5.13, the phase response of H(z) and H_{red}(z) are similar but, the magnitude response is better in case of reduced order filter. Fig. 5.14 indicates the comparison of FFT power spectrums of input signal, output of higher order and reduced order filter, as obtained in MATLAB. It is clearly visible that the reduced order filter passes all the frequency components within 50Hz and attenuates thereafter. This is similar to that of higher order filter.

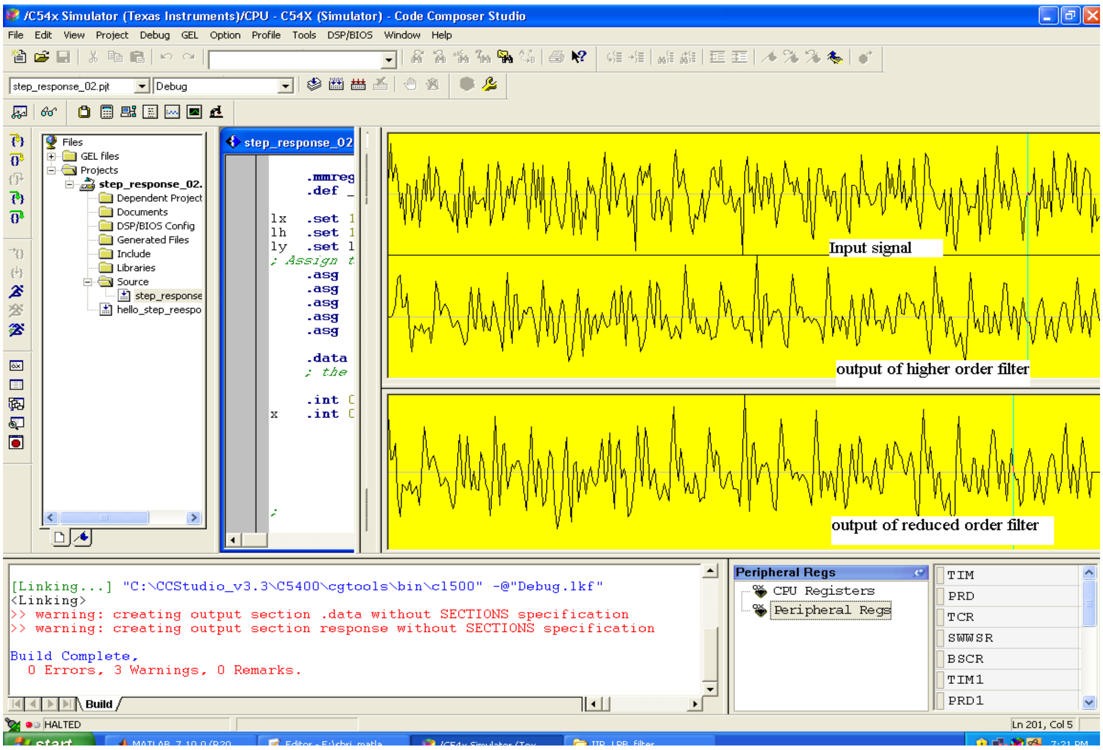

Fig. 5.11 Comparison of filter responses obtained in CCS for example 5.3

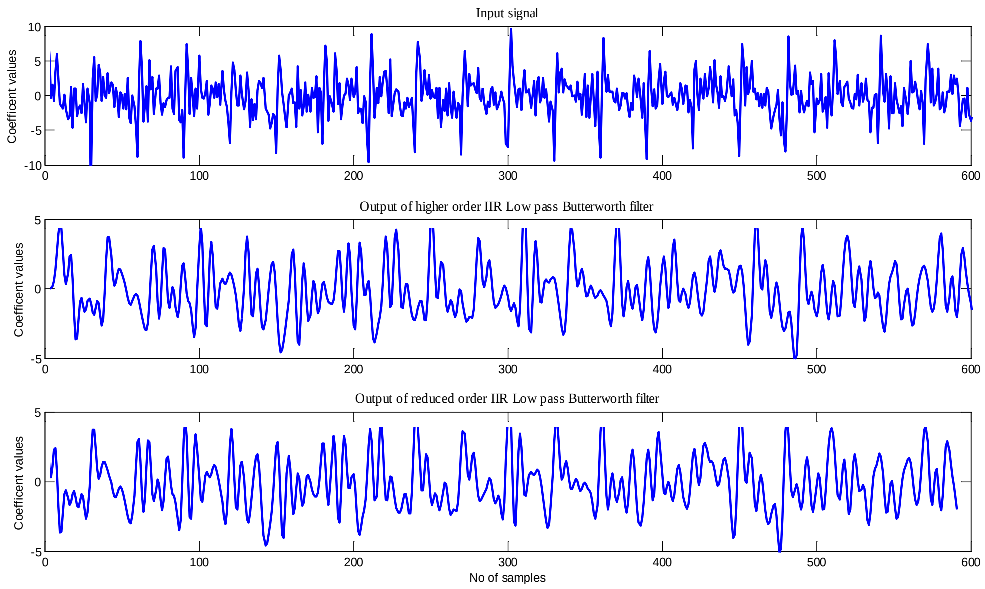

Fig. 5.12 Comparison of filter responses for example 5.3

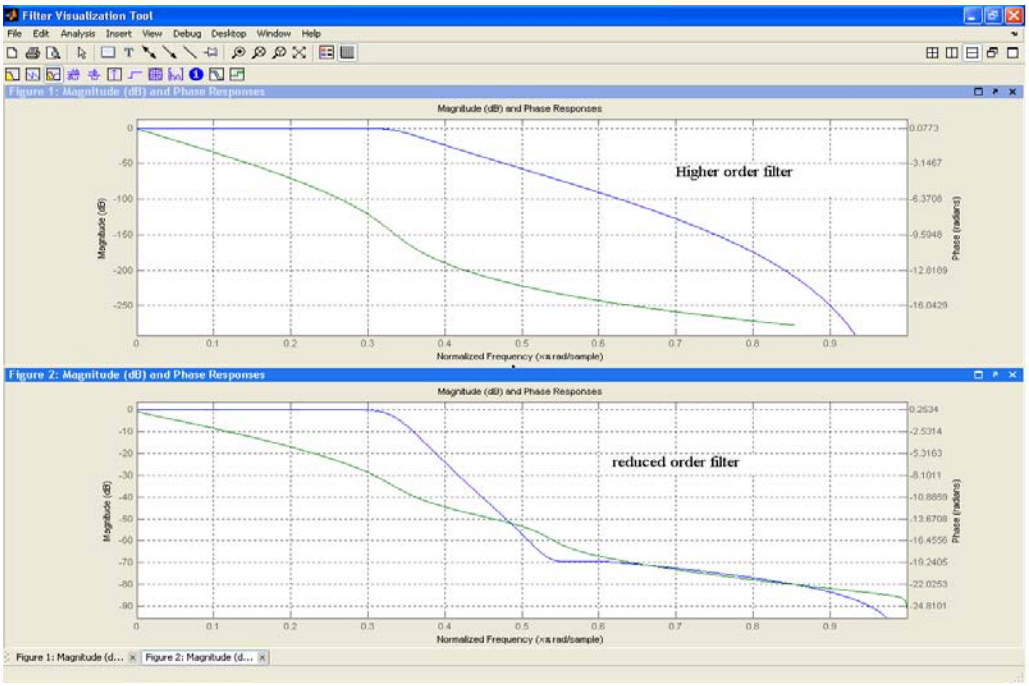

Fig. 5.13 Comparison of magnitude and phase responses for example 5.3

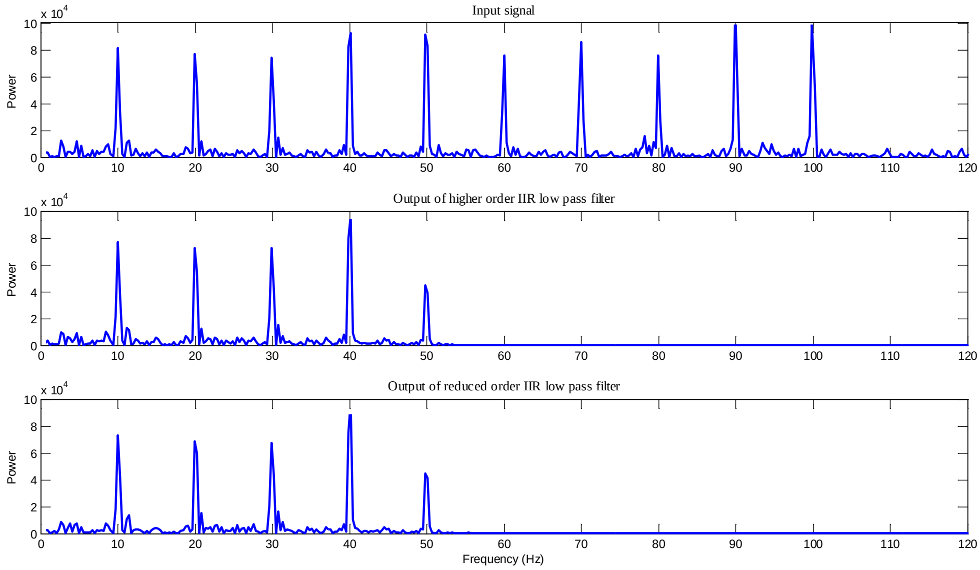

Fig. 5.14 Comparison of FFT power spectrum for example 5.3

5.3 GENETIC ALGORITHM FOR ORDER REDUCTION

5.3.1 Introduction

GA is now a days, has become a general-purpose search/optimization method. It is one among the various optimization techniques available, that can be employed to provide solutions for complex problems that are difficult to solve. It can also be applied for problems having discontinuous objective function, stochastic or highly nonlinear which cannot be solved easily by other standard optimization methods [288]. GA keeps and modifies a population of solutions and uses survival of the fittest strategy in search of better solutions. During each iteration, best solution of any population will reproduce and survive to the next iteration thus ameliorating successively. However, there is every chance that subordinate solutions can also remain alive and reproduce. Usually the solutions are represented as strings of fixed length, called chromosomes. The quality of each solution will be according to the fitness/objective function. To start with GA optimization, a random population is initialized and is executed in cycles called generations, as follows [289] :

1. Objective/fitness function is used for evaluating every individual of the current population.
2. Reproduction occurs during iterations. Random selection of single or multiple parents are choosen, however the strings with better fitness values have more tendency of providing an offspring.
3. Offspring are yielded by applying crossover and mutation to parents. These offspring's are injected into the next population and thus repeating the cycle.

Some fundamental issues have to be finalized before implementing GA. All these issues are described concisely in the below sections.

5.3.1.1 Chromosome Representation

This ascertains the structure of the problem and also the genetic operators used in GA. Every individual/ chromosome is formed by a series of genes. There are variety of individual/chromosome representations viz. binary digits, matrices, integers, real values, floating point numbers. In order to obtain efficient and better solutions one must go for natural representations. On the other hand, real-coded representation offers more efficiency as far as CPU time is considered. Additionally, it provides results of higher precision with good consistency.

5.3.1.2 Selection Function

The selection function plays a crucial role in deciding the individuals that survive and move forward to the next generation. This is performed based upon the individual's fitness. The chances of inferior individuals being selected is lesser than that of the superior ones. There are variety of schemes that can be followed for the selection viz. elitist models, roulette wheel selection and its extensions, normal geometric, ranking methods. However, the right type of selection plays an important role as it helps in yielding series of generations. In this communication, the geometric selection function (normalized) is found to be suitable for use.

The selection process comprises of assigning a possibility of selection, denoted by p_i to each individuals based on its fitness value. The possibility of individual selection p_i is given as

$$p_i = q'(1-q)^{r-1}$$
$$q' = \frac{q}{1-(1-q)^P} \quad (5.4)$$

where q = probability of selecting the best individual, r = rank of the individual (with best equals 1) and P = population size

5.3.1.3 Genetic Operators

Genetic operators assist in the basic search operation in GA. Crossover and mutation are the two basic types of operators available. New solutions are yielded depending on the existing solutions in the current population. In case of crossover, two individuals are considered to be parents. this results in two new individuals. Whereas, mutation yields a one solution (new) by altering single individual. The different types of crossover employed are simple, arithmetic and heuristic crossover. Similarly the various types of mutations are uniform, multi-non-uniform and non-uniform mutation etc. In this proposal, arithmetic crossover and non-uniform mutation are being used. A random number 'r_n', is generated by crossover from a uniform distribution starting 1 to m and generates two new individuals. This is possible by using

$$x_i' = \begin{cases} x_i, \ if \ \ i < r_n \\ y_i \ otherwise \end{cases}$$
$$y_i' = \begin{cases} y_i, \ if \ \ i < r_n \\ x_i \ otherwise \end{cases} \quad (5.5)$$

Two linear combinations (complimentary) of the parents are yielded by arithmetic crossover, where r_n = U (0, 1)

$$\overline{X'} = r_n \overline{X} + (1 - r_n)\overline{Y}$$
$$\overline{Y'} = r_n \overline{Y} + (1 - r_n)\overline{X} \quad (5.6)$$

the value of variable 'j' is randomly selected by non-uniform mutation and initializes to an non-uniform number randomly.

$$x_i' = \begin{cases} x_i + (b_i - x_i)\ f(G_c) \ \ if \ r_1 < 0.5, \\ x_i + (x_i + a_i)\ f(G_c) \ \ if \ r_1 \geq 0.5, \\ x_i, \quad otherwise \end{cases} \quad (5.7)$$

$$f(G_c)=\left(r_2\left(1-\frac{G_c}{G_{\max}}\right)\right)^b$$

where r_1, r_2 are uniform random numbers between 0 and 1, G_c is current generation, G_{max} is maximum number of generations and b is the shape parameter

5.3.1.4 Initialization, Termination and Evaluation Function

Optimization method such as Particle Swarm Optimization (PSO) starts by floating an initial population randomly. The same case applies to GA also. The execution takes place and continues further from iteration to iteration until some stopping criterion is satisfied. The stopping criterion can be the limit for number of generations, criteria for population convergence, unsatisfactory improvement in the best solution provided over a span generations or desired value for the fitness function.

There are different forms of evaluation/objective that can be made use of. Doing so helps in mapping the population into a partially ordered set. The optimizing process continues until the best solution is obtained according to the fitness function under the limitation of the number of generations. The same is noticeable from the illustrative examples being solved.

5.3.2 Problem Statement

Consider a n^{th} order linear time invariant single input single output (LTI-SISO) system, also called Higher Order Model (HOM), described by

$$G(s)=\frac{a_o+a_1 s+\ldots+a_m s^m}{b_o+b_1 s+b_2 s^2+\ldots+b_n s^n}; \quad m<n \tag{5.8}$$

The objective is to compute r^{th} (r < n) order Reduced Order Model (ROM) R(s) from (5.8) in the form

$$R(s)=\frac{d_o+d_1 s+\ldots+d_p s^p}{e_o+e_1 s+e_2 s^2+\ldots+e_r s^r}; \quad p<r \tag{5.9}$$

Further, it should also be noted that the ROM obtained must be stable and closely approximate the HOM, in such a way that the difference between the transient parts of the G(s) and R(s) is as minimum as possible.

5.3.3 Illustrative Examples

Example 5.4: Consider the system given by Phillip and Pal [233]

$$G(s)=\frac{s^3+7s^2+24s+24}{s^4+10s^3+35s^2+50s+24} \tag{5.10}$$

or in Q15 format, equation(5.8) is rewritten as

$$G_{Q15}(s) = \frac{08000\text{h}s^3 + 038000\text{h}s^2 + 0\text{C}0000\text{h}s + 0\text{C}0000\text{h}}{08000\text{h}s^4 + 050000\text{h}s^3 + 0118000\text{h}s^2 + 0190000\text{h}s + 0\text{C}0000\text{h}}$$

The reduced model obtained in TMS320C5402 DSP using GA is given by [290]

$$R_{Q15}(s) = \frac{061\text{DC}\text{h}s + 0\text{D}832\text{h}}{08000\text{h}s^2 + 014\text{BA}6\text{h}s + 0\text{D}832\text{h}}$$

or in decimal numbering system R(S) is

$$R(s) = \frac{0.7645s + 1.689}{s^2 + 2.591s + 1.689}$$

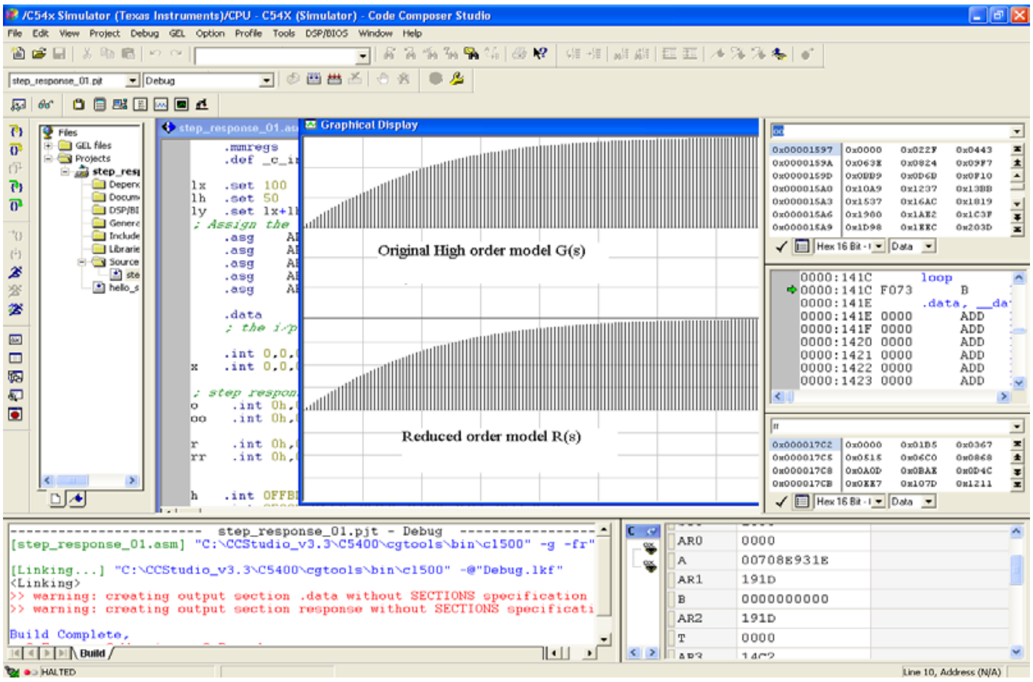

Fig. 5.15 Comparison of step responses obtained in CCS for example 5.4

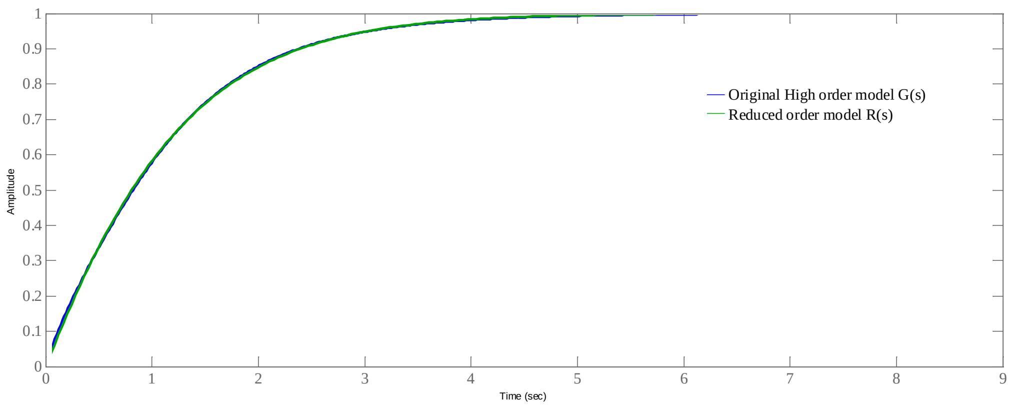

Fig. 5.16 Comparison of step responses for example 5.4

Fig. 5.15 and 5.16 shows the snap shot view of the results obtained in CCS and MATLAB respectively. It is seen that the step response of reduced model closely approximate with the original model. Table 5.1 compares the values of '*I*' using (2.21), obtained by various authors using different techniques.

Table 5.1 Comparison of reduced order systems for example 5.4

Order reduction Method	Reduced System *R*(s)	'*I* '
Proposed Method	$\frac{0.7645s+1.689}{s^2+2.591s+1.689}$	7.23334 x 10^{-5}
Phllip and Pal [233]	$\frac{0.9315s+1.609}{s^2+2.756s+1.609}$	1.75399 x 10^{-3}
Chen [53]	$\frac{0.699\text{ s} + 0.699}{s^2+1.45771\text{ s} + 0.699}$	2.780534 x 10^{-3}
Krishnamurthy and Sheshadri [49]	$\frac{20.5714\text{ s} + 24}{30s^2+42\text{ s} + 24}$	9.742003 x 10^{-3}
Pal [57]	$\frac{16\text{ s} + 24}{30s^2+42\text{ s} + 24}$	11.861676 x10^{-3}
Gutman et. al [47]	$\frac{2(48\text{ s} + 144)}{70s^2+300\text{s} + 288}$	45.5957 x 10^{-3}

Example 5.5: Consider the eight order transfer function [267]

$$G(s)=\frac{18s^7+514s^6+5982s^5+36380s^4+122664s^3+222088s^2+185760s+40320}{s^8+36s^7+546s^6+4536s^5+22449s^4+67284s^3+118124s^2+109584s+40320}$$

or in Q15 format the above equation is becomes

$$G_{Q15}(s)=\frac{90000\text{h}s^7+01010000\text{h}s^6+0\text{BAF}0000\text{h}s^5+0470\text{E}0000\text{h}s^4+0\text{EF}940000\text{h}s^3+01\text{B}1\text{C}40000\text{h}s^2+1016\text{AD}00000\text{h}s+04\text{EC}00000\text{h}}{8000\text{h}s^8+0120000\text{h}s^7+01110000\text{h}s^6+08\text{DC}0000\text{h}s^5+02\text{BD}88000\text{h}s^4+0836\text{A}0000\text{h}s^3+0\text{E}6\text{B}60000\text{h}s^2+0\text{D}6080000\text{h}s+04\text{EC}00000\text{h}}$$

The second order reduced model obtained in TMS320C5402 DSP using GA is given by [290]

$$R_{Q15}(s)=\frac{0875\text{C}3\text{hs}+02\text{A}148\text{h}}{08000\text{h}s^2+036\text{F}5\text{Dh}s+02\text{A}148\text{h}}$$

or in decimal numbering system *R*(S) is

$$R(s)=\frac{16.92s+5.26}{s^{2}+6.89s+5.26}$$

Fig. 5.17 shows the preview of the results obtained in CCS with the step response for the original and reduced model closely matching. Fig. 5.18 shows the step responses obtained in MATLAB. The comparison of 'I' (2.21) values is calculated for other reduced models and are tabulated in Table 5.2.

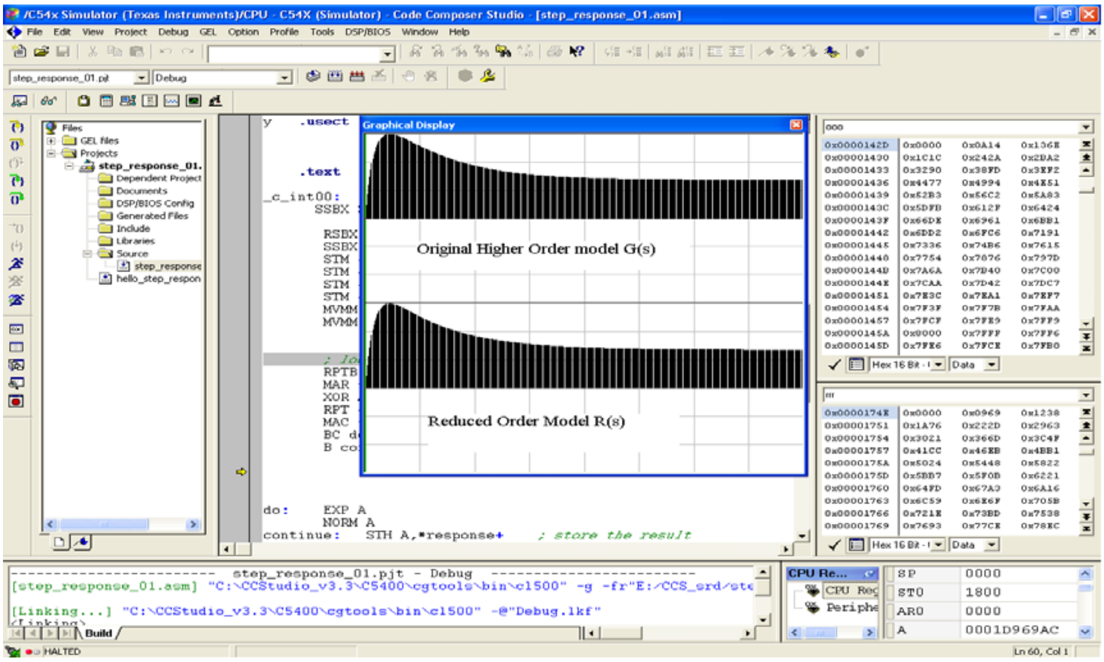

Fig. 5.17 Comparison of step responses obtained in CCS for example 5.5

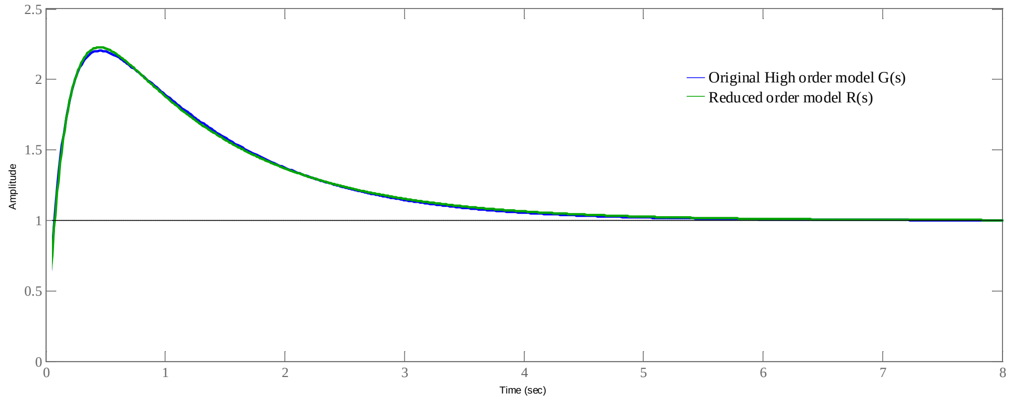

Fig. 5.18 Comparison of step responses for example 5.5

Table 5.2 Comparison of reduced order systems for example 5.5

Order reduction Method	Reduced System *R*(s)	'*I*'
Proposed Method	$\frac{16.92s+5.26}{s^2+6.89s+5.26}$	0.000610
Dia et.al [267]	$\frac{17.099s+5.074}{s^2+6.972s+5.151}$	0.00062

Example 5.6: The transfer function of a original system [53] is given by

$$G(s)=\frac{s^4+35s^3+291s^2+1093s+1700}{s^9+9s^8+66s^7+294s^6+1029s^5+2541s^4+4684s^3+5856s^2+4620s+1700}$$

In Q15 format the original model becomes,

$$G_{Q15}(s)=$$

$$\frac{8000\text{h}s^4+0118000\text{h}s^3+0918000\text{h}s^2+02228000\text{h}s+03520000\text{h}}{\begin{array}{c}8000\text{h}s^9+048000\text{h}s^8+0210000\text{h}s^7+0930000\text{h}s^6+02028000\text{h}s^5+04F68000\text{h}s^4\\+09260000\text{h}s^3+0B700000\text{h}s^2+09060000\text{h}s+03520000\text{h}\end{array}}$$

The numerator and denominator coefficients of reduced model obtained in TMS320C5402 DSP using GA is given by

$$R_{Q15}(s)=\frac{03155\text{h}s^2+02D50\text{h}s+018667\text{h}}{08000\text{h}s^3+015E98\text{h}s^2+025AA0\text{h}s+018667\text{h}}$$

or *R*(S) is

$$R(s)=\frac{0.3854s^2-1.646s+3.05}{s^3+2.739s^2+4.708s+3.05}$$

Table 5.3 Comparison of reduced order systems for example 5.6

Order reduction Method	Reduced System *R*(s)	'*I*'
Proposed Method	$\frac{0.3854s^2-1.646s+3.05}{s^3+2.739s^2+4.708s+3.05}$	1.17822 x 10^{-3}
Philip and Pal [233]	$\frac{0.5058s^2-1.985s+3.534}{s^3+3s^2+5.534s+3.534}$	3 x 10^{-3}
Mukherjee [168]	$\frac{0.2945s^2-2.203s+2.32}{s^3+2.5008s^2+4.778s-2.32}$	20.5 x 10^{-3}

The results obtained in CCS and MATALB are shown in Fig. 5.19 and 5.20. It is seen that the step response for the original model approximates with that of reduced model. Table 5.2 lists the comparison of '*I*' (2.21) values calculated for various reduced models.

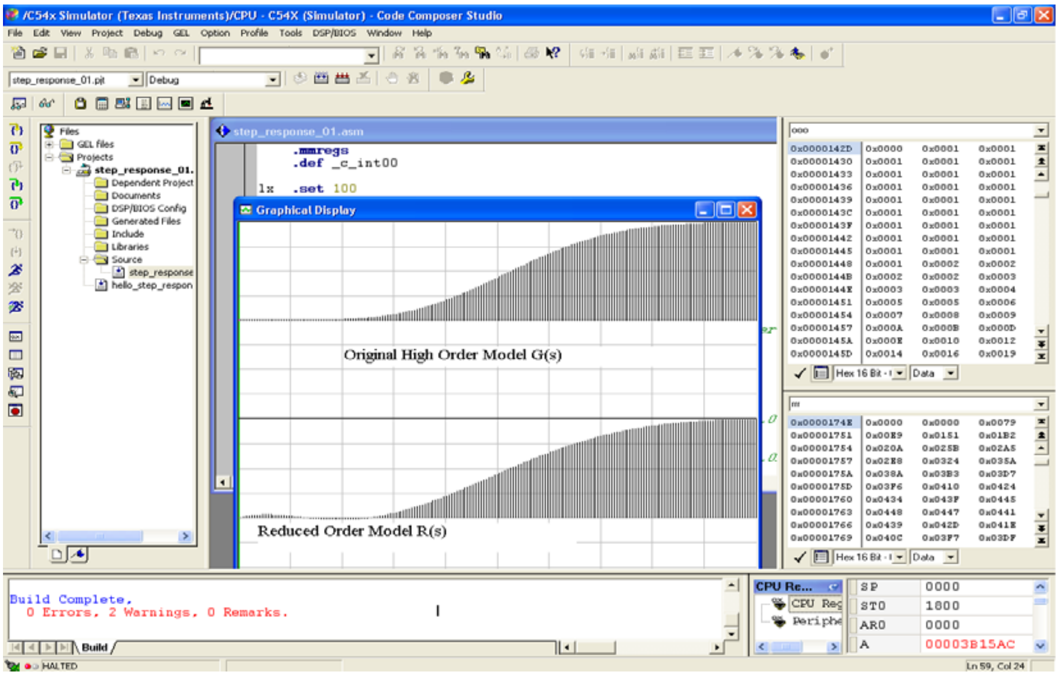

Fig. 5.19 Comparison of step responses obtained in CCS for example 5.6

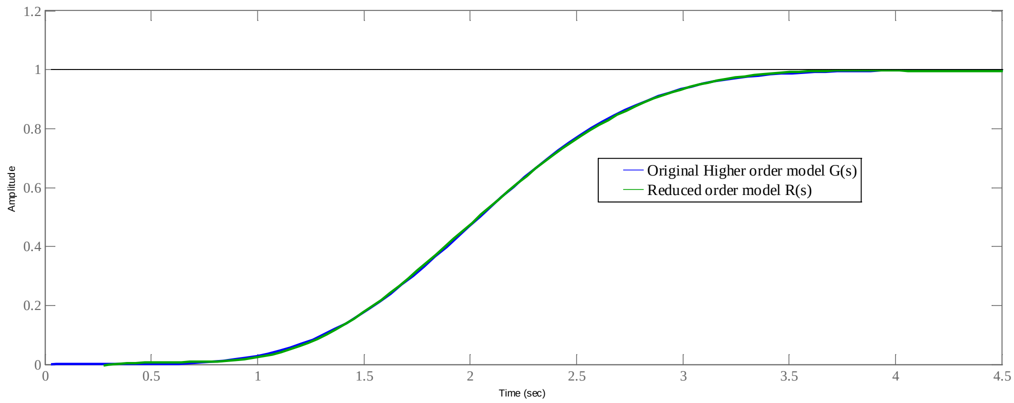

Fig. 5.20 Comparison of step responses for example 5.6

5.4 CONCLUSION

The nature inspired technique such as BBBC, GA has been roped in to reduce the order of the given transfer function. The same is implemented on TMS320C5402 DSP, MATLAB and the results obtained are comparable. In 5.2, order reduction process is applied on higher order chebyshev and butterworth filter. Cases of lowpass, highpass are considered and it can be concluded that order reduction results in elimination of delay in the output of the reduced order filter. Furthermore, the number of multipliers, adders required reduces drastically thereby reducing the implementation cost of the filter. Lastly, GA is used to optimize the ROM coefficients so as to follow the step response of the HOM on TMS320C5402 DSP successfully. The responses show the effectiveness of the technique.

CHAPTER - 6

CONTROLLER DESIGN

The chapters that were dealt till now projected the need for order reduction; various techniques available; means to combat/improve drawbacks of the prevailing methods, by developing/proposing new reduction techniques, thus satisfying the need of the hour. The methods proposed, is not only limited to continuous time systems, but can also be applied to discrete time systems. The same is justified by solving several numerical examples and comparing the results with other well known methods, when subjected to specified test inputs. However, these simulations were carried out to determine the open loop behavior of the system. But, in most of the practical cases, some sort of controller always exists to control the system behavior. The design of such controller becomes a crucial task, especially when the plant size is very large. In such cases, the size of the controller also increases thereby resulting in complicated and costly design. Apart from this, more computational time, difficulties posed during analysis, simulation and understanding of the system are additional hurdles. Hence, there is a need for suitable lower order controller, which can be derived by preserving the crucial dynamics of the higher order controller. Furthermore, the derived reduced controller should be in a position to control the original higher order system satisfactorily and hence results in application of order reduction methods to controller reduction problems. In this chapter, the design of PID controller is dealt with using evolutionary techniques such as PSO and BBBC. Later, these techniques are adopted, to present a computer aided mode of optimizing the Fractional Order Proportional Integral Derivative (FOPID) controller parameters, including the integral order 'λ' and derivative order 'μ'.

6.1 PID CONTROLLER

Today, abundant methods are proposed in [9, 258, 277, 291-293] are available. But, choosing the best technique is still at large because of various reasons. One of them is due to the fact that the designed system can only be accepted, if it satisfies the design constraints. Consequently this results in simple, low-order approximations without sacrificing accuracy. The heart of this chapter lies in the design of a PID controller $G_c(s)$ connected in series with an uncontrolled plant $G_p(s)$. The $G_c(s)$ designed should be in a position to drive the plant in stable mode, when the response of the closed loop plant with unity feedback is considered. In spite of a desired quicker response, the designed PID controller G_c(s), must also be able to closely match the time responses of the controlled system with those of the reference model. In order to carry out the above mentioned task, two different types of approaches [15, 294] for controller design are dealt with namely plant/process reduction and controller reduction. The same are being reflected in Fig. 6.1.

The process reduction approach comprises of reducing the original plant G_p(s) to R_p(s). Then a suitable controller R_c(s) is designed and placed in series with R_p(s) as shown in Fig. 6.2. Further, the closed loop response of R_{CL}(s) is obtained with unity feedback. The block diagram in Fig. 6.2 depicts the original, reduced controller configurations with that of the reference model M(s). This method is also referred as direct approach.

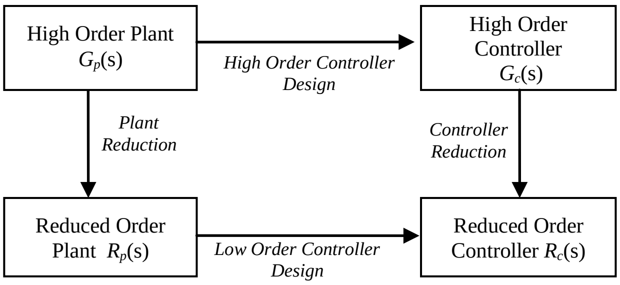

Fig. 6.1 Controller design approaches

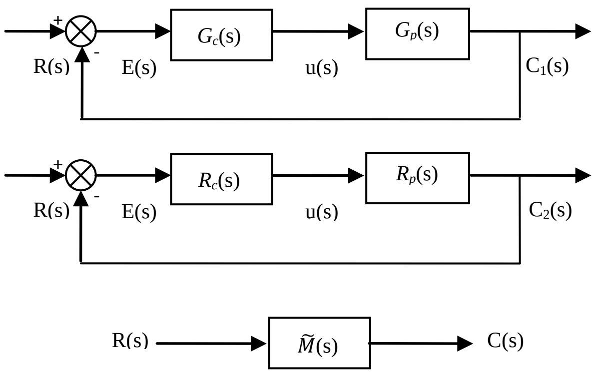

Fig. 6.2 Original and reduced controller configuration with reference model

In the controller reduction approach, the (higher order) controller G_c(s) is designed on the basis of higher order original uncontrolled plant G_p(s). Then, transfer function of the closed loop configuration G_{CL}(s) with unity feedback is reduced appropriately and compared with R_{CL}(s). This approach is also called as indirect approach. The general criticism in the process reduction approach, is that the error creeps in, as the reduction is carried out during the initial stage of design. Whereas, in the controller reduction approach, the issue of error propagation dosen't persist, as the reduction process is carried out in the final stage of the design.

In the present study, both direct and indirect design approaches have been considered. In order to carry out the design task, a recently erupted evolutionary technique [264] called as Big Bang Big Crunch (BBBC) optimization algorithm is sought for the purpose. In other words, BBBC being another type of evolutionary computation is being roped in to assist in the design of PID controller. This approach comes out to be better than the other conventional techniques like HNA. Apart from this, PSO being another evolutionary method is also used for optimizing the design parameters. Such numerical technique not only aids in rationally searching, but also in selecting an appropriate combination of the best parameters among the available collection, so as to satisfy the design requirements. Once the design task is completed, the closed loop responses are then compared with the reference model *M*(s). The reference model *M*(s) also called as specification model or standard model is the desired transfer function of the closed loop system. To conclude, *M*(s) meets all the desired performance specifications and act as the basis for comparison.

6.1.1 Design Procedure

The direct and indirect approaches of controller design [294] are shown in the Fig.6.1. Initially a controller is designed for high order system and is reduced to obtain a low order controller. Then the closed loop response of higher order controller with original plant and low order controller with reduced plant are compared with the reference model. The controller parameters are obtained using approximate model matching in the Pade sense. The performance of full order controller is then compared, with that of the reduced order controller as shown in Fig. 6.2.

6.1.1.1 Direct Approach: Plant Reduction and Controller Design

The design procedure is based on approximate model matching in Pade sense and consists of the following steps.

Step1: For the plant having a transfer function G_p(s), construct a reference model *M*(s) on the basis of time/frequency domain specifications. The closed loop response of the controlled system with unity feedback approximates the reference model response.

Let the transfer function of the plant G_p(s) and the reference model *M*(s) are given by

$$G_p(s) = \frac{a_o + a_1 s + \ldots + a_m s^m}{b_o + b_1 s + b_2 s^2 + \ldots + b_r s^n}\ ; \quad m < n \tag{6.1}$$

$$M(s) = \frac{g_o + g_1 s + \ldots + g_u s^u}{h_o + h_1 s + h_2 s^2 + \ldots + h_v s^v}\ ;\ u < v \tag{6.2}$$

Step 2: Determine an equivalent open loop specification model. If *M*(s) is the desired closed loop system (reference model) then the equivalent open loop specification model transfer function is obtained by

$$\widetilde{M}(s) = \frac{M(s)}{1 - M(s)} \tag{6.3}$$

Step 3: Structure of the controller:

Let the controller structure $G_c(s)$ is given by

$$G_c(s) = \frac{p_o + p_1 s + \ldots + p_k s^k}{q_o + q_1 s + q_2 s^2 + \ldots + q_j s^j}\ ;\ k < j \tag{6.4}$$

Step 4: For determining the unknown controller parameters, the response of the closed loop system is matched with that of the reference model as

$$G_c(s)G_p(s) = \widetilde{M}(s)$$

$$G_c(s) = \frac{\widetilde{M}(s)}{G_p(s)} = \sum_{i=0}^{\infty} e_i s^i \tag{6.5}$$

where e_i are the power series expansion coefficients about s = 0. Now the unknown control parameters p_k and q_j are obtained by equating (6.4) and (6.5) in Pade sense.

$$\left.\begin{aligned}
p_0 &= q_0 e_0 \\
p_1 &= q_0 e_1 + q_1 e_0 \\
p_2 &= q_0 e_2 + q_1 e_1 + q_2 e_0 \\
&\vdots \\
p_i &= q_0 e_i + q_1 e_{i-1} + \ldots + q_1 e_0 \\
0 &= q_0 e_{i+1} + q_1 e_1 + \ldots + q_{i+1} e_0 \\
&\vdots \\
0 &= q_0 e_{i+j} + q_1 e_{i+j-1} + \ldots + q_j e_i
\end{aligned}\right\} \tag{6.6}$$

The controller with the desired structure is obtained by solving (6.6).

Step 5: After obtaining the controller parameters, the closed loop transfer function can be obtained as

$$G_{CL}(s) = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)} \tag{6.7}$$

Step 6: Reduce the plant G_P(s) to R_P(s) using reduction method. Repeat steps 4 and 5. The closed loop transfer function for the reduced order model is

$$R_{CL}(s)=\frac{R_c(s)R_p(s)}{1+R_c(s)R_p(s)} \quad (6.8)$$

6.1.1.2 Indirect Approach: Controller Design and Reduction

In this approach a higher order controller G_c(s) is designed for G_p(s) and closed loop transfer function with unity feedback is obtained. Then, the closed loop transfer function G_{CL}(s) is reduced to obtain reduced closed loop transfer function R_{CL}(s). The procedure will become more transparent when refered to the illustrative examples in 6.1.2.1.2.

6.1.2 Particle Swarm Optimization

PSO being a global optimization algorithm, a subset of evolutionary computation has gained popularity in academia and industry; simplicity and intuitiveness, capable of handling both discrete and continuous variables, requires no rigid first guess algorithms, ease of implementation, exploring majority of the problem space are its advantages. As a result, it is found to be useful in solving mixed integer optimization problems, that are typical of complex engineering systems. Another main attraction of PSO is that, it works well for any dimensional problem. Hence, it is used at finding the optimum for single objective and multi-objective functions (nonlinear and linear). Although, the problem of being stuck in local minima exists, it is uncomplicated to code and understand its most basic form. Conceptually, PSO is similar to Genetic Algorithms due to the stochastic population based nature, but is easier to implement with the same. Further, the stochastic population based optimization technique comes with a simple memory component. To conclude, PSO has similar or better results than GA [295, 296].

The PSO algorithm is originally introduced in terms of social and cognitive behavior by Kennedy and Eberhart in 1995 [297]. Swarm can be formally defined as a group of mobile agents that communicate with each other directly or indirectly [298]. Since its inception, many problems in various engineering fields are benefitted because of its fairly simple computations; sharing of information within the algorithm as it derives its internal communications from the social behavior of individuals. The individuals, henceforth called particles, are flown through the multi-dimensional search space with each particle representing a possible solution to the multi-dimensional optimization problem [299]. Each solution's fitness is based on a performance function related to the optimization problem being solved.

The process of PSO, begins by initializing the population of particles; randomly positioned across the search range with an initial random velocity having values not greater than a certain percentage of the search space in each direction. Each particle (candidate solution), is expressed as a position within the search space of the problem; flies through search space by updating its individual velocity at regular intervals toward both the best position or location it personally has found called *pbest* [300] (i.e. the personal best), and toward the globally best position found by the entire swarm called *gbest* (i.e. the global best). The *pbest* and *gbest* are iteratively updated for each particle, till a better or more dominating solution (in terms of fitness) is found. This process continues, until the maximum iterations is reached or specified criteria is met. The particle swarm consists of a swarm of particles each moving or flying through the search space of the problem according to velocity update equation

$$v_i = v_i + c_1 r_1 (p_i - x_i) + c_2 r_2 (p_g - x_i) \tag{6.9}$$

where v_i is the velocity vector of i^{th} particle, x_i is the position vector of i^{th} particle, p_i is the n-dimensional personal best of i^{th} particle found from initialization, p_g is the n-dimensional global best of the swarm found from initialization, c_1 is the cognitive acceleration coefficient c_2 the social acceleration coefficient, r_1 , r_2 are the random numbers drawn from a uniform distribution and the position is updated using

$$x_i = x_i + v_i \tag{6.10}$$

The classical version of PSO algorithm defined by velocity update equations (6.9) and (6.10) inherits a weakness that can be fixed by the introduction of an inertia weight '*w*' or constriction coefficient 'χ'. The method of introducing inertia weight, was first introduced by Shi and Eberhart [301] and the modified velocity update equation is given by

$$v_i = w * v_i + c_1 r_1 (p_i - x_i) + c_2 r_2 (p_g - x_i) \tag{6.11}$$

According to Eberhart and Shi [302] the optimal strategy is to initially set *w* to 0.9 and reduce it linearly to 0.4, allowing initial exploration followed by acceleration toward an improved global optimum.

The problems in velocity update equations (6.9) and (6.10) was addressed by Clerc [296] by introducing constriction coefficient 'χ' so as to result in

$$v_i = \chi [v_i + c_1 r_1 (p_i - x_i) + c_2 r_2 (p_g - x_i)] \tag{6.12}$$

where χ is computed as

$$\chi = \frac{2}{\left|2 - \phi - \sqrt{\phi(\phi - 4)}\right|} \tag{6.13}$$

where $\phi = c_1 + c_2$, $\phi > 4$

The velocity equation, is the heart of the PSO algorithm; and expresses each particle's velocity as a balance between attraction to its own personal best position and the current global best position among all particles. This is the difference between local and global searching. This is one of the reasons the algorithm is so resistant to getting stuck in local minima [303]. The basic PSO algorithm is as follows:

Step 1: [start] The PSO starts by randomly initializing the position, velocity, and the personal best of each particle in the swarm.

Step 2: [Evaluate Fitness value] For each iteration, the particles will be moved into the solution space. The algorithm will act on each particle such that each particle will move in a direction to improve its fitness function. The action involves updating the particles velocity, movement updating of particles and evaluating the fitness function for the new position.

Step 3: [Compare Fitness Function] Compare the fitness function of the new position with the fitness function of *gbest.* Repeat the above steps for the whole particles.

Step 4: [Maximum iteration] Check if maximum iteration reached or a specified termination criteria is satisfied. Stop, and return the best solution *gbest* otherwise, update w and go to the next iteration.

Step 5: [Loop] Go to step2 for fitness evaluation.

The flowchart showing the process of PSO [304] is as shown in Fig. 6.3. Table 6.1 gives the typical parameters used for PSO in the present study.

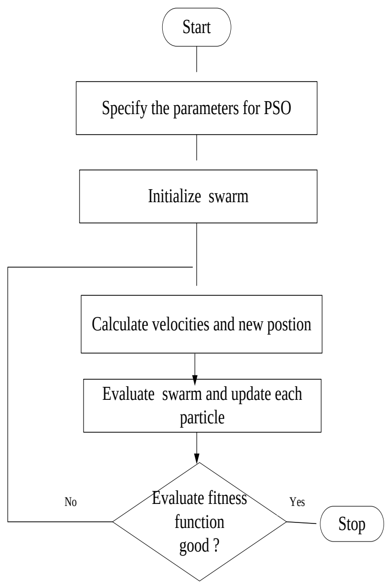

Fig. 6.3 PSO optimization process

Table 6.1 Typical Parameters

Parameters	Value
Swarm size	20
Maximum generations	100
c_1, c_2	2, 2
w_{start}, w_{end}	0.9, 0.4

6.1.2.1 Illustrative Examples

Numerical examples are presented to illustrate both direct and indirect methods of PID controller design. The design parameters are optimized using PSO [305]and solved in detail for the first example. Later, the results are compared with other methods.

6.1.2.1.1 *Direct Method*

Example 6.1: Consider the regulator problem, whose transfer function and the reference model are given as [22]

$$G_p(s) = \frac{s^5 + 8s^4 + 20s^3 + 16s^2 + 3s + 2}{2s^6 + 36.6s^5 + 204.8s^4 + 419s^3 + 311.8s^2 + 67.2s + 4}$$

$$M(s) = \frac{0.023s + 0.0121}{s^2 + 0.21s + 0.0121}$$

Step 1: Consider M(s) and determine the equivalent open loop transfer function using (6.3)

$$\widetilde{M}(s) = \frac{0.023s^3 + 0.01693s^2 + 0.002819s + 0.0001464}{s^4 + 0.397s^3 + 0.05137s^2 + 0.002263s}$$

Step 2: Let the desired controller be according to (6.5) and is given by

$$G_c(s) = \frac{\widetilde{M}(s)}{G_p(s)} = \frac{1}{s}(0.064707 + 0.767859s + 0.801795s^2 - 4.681159s^3 + ...)$$

Step 3: Taking the PID controller structure as

$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_1 s + K_2 + K_3 s^2}{s}$$

Step 4: Comparing the controller G_c(s) with the power series expansion, parameters K_1, K_2 and K_3 of the controller are obtained, which results in the PID controller as

$$G_c(s) = \frac{0.064707 + 0.767859s + 0.801795s^2}{s}$$

Step 5: The corresponding closed loop transfer function is found using (6.7)

$$G_{CL}(s) = \frac{0.8228s^7 + 7.349s^6 + 22.66s^5 + 29.02s^4 + 16.03s^3 + 4.981s^2 + 1.728s + 0.1294}{1.823s^7 + 25.65s^6 + 125.1s^5 + 238.5s^4 + 172s^3 + 38.58s^2 + 3.728s + 0.1294}$$

Step 6: The original system is reduced to second order model using proposed PSO technique by minimizing the ISE between the G_p(s) and R_p(s) using (2.21). Where y(t), y_r(t) are the unit step responses of G_p(s) and R_p(s) respectively.

Thus,

$$R_{ppso}(s)=\frac{0.02555s+0.01036}{s^2+0.4756s+0.01036}$$

Step 7: Similarly using GA and HNA, the reduced system obtained is

$$R_{pga}(s)=\frac{0.01414s+0.009369}{s^2+0.1436s+0.009369}$$

$$R_{phna}(s)=\frac{0.0113s+0.0736}{s^2+0.1436s+0.009369}$$

Step 8: Now, following step 4 in 6.1.1.1, the controller structure obtained is

$$R_{cpso}(s)=\frac{\widetilde{M}(s)}{R_{pga}(s)}$$
$$=\frac{1}{s}(0.06471+0.714036s+1.89926s^2+...)$$

Step 9: Considering the PID controller structure as

$$R_c(s)=K_1+\frac{K_2}{s}+K_3 s$$
$$=\frac{K_1 s+K_2+K_3 s^2}{s}$$

Step 10: Comparing the coefficients with the power series expansion the parameters, K_1, K_2 and K_3 of the controller are obtained, which gives the PID controller as

$$R_{cpso}(s)=\frac{0.06471+0.714036s+1.89926s^2}{s}$$

Step 11: The closed loop transfer function of the reduced second order model and the controller using PSO [305],GA and HNA [306] according to step 6 in 6.1.1.1, results in

$$R_{CLPSO}(s)=\frac{0.04853s^3+0.03861s^2+0.00933s+0.0006703}{1.049s^3+0.2142s^2+0.01969s+0.0006703}$$

$$R_{CLGA}(s)=\frac{0.01165s^3+0.07664s^2+0.005604s+0.004762}{1.012s^3+0.417s^2+0.0792s+0.004762}$$

$$R_{CLHNA}(s)=\frac{0.05177s^3+0.0445\ s^2+0.007102s+0.0006152}{1.052s^3+0.1885s^2+0.0167s+0.0006152}$$

Table 6.2 Comparison of original and reduced order plants for example 6.1

System	Rise Time t_r (sec)	Peak Overshoot % M_p	Settling Time t_s (sec)
M (s)	28.1	0.00383	46.2
G_{CL}(s)	22.3	0	45.1
R_{CLPSO}(s)	28.8	0	43.6
R_{CLGA}(s)	28	0	45.3
R_{CLHNA}(s)	30	0.95	42.1

Fig 6.4 shows the comparison of step response of closed loop transfer function of the original plant, the reduced model using PSO with that of the reference model. It is seen that all the three responses are matching in both steady state and transient regions. Table 6.2 gives the qualitative comparison of original and reduced order plants in terms of transient response parameters.

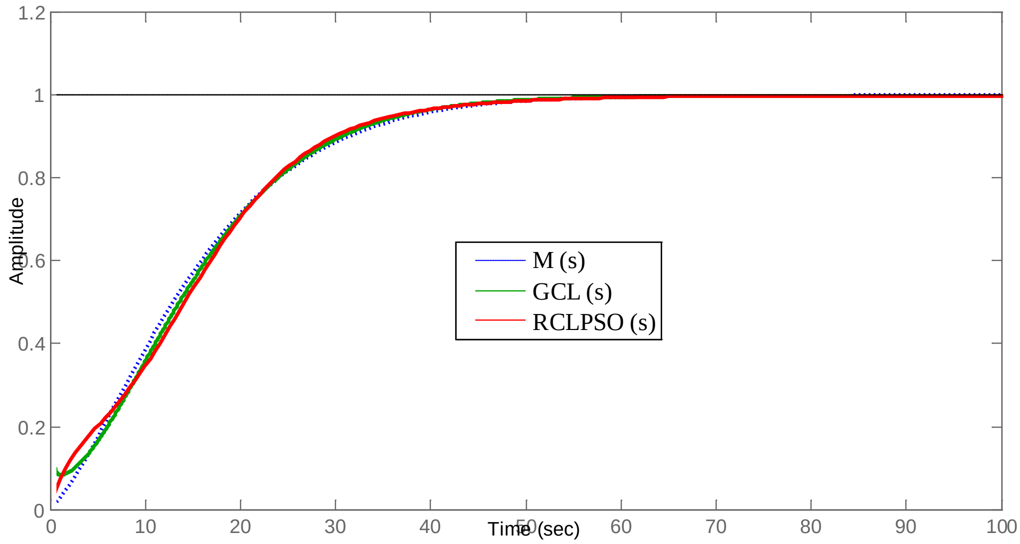

Fig. 6.4 Comparison of step responses for example 6.1

6.1.2.1.2 *Indirect Method*

Example 6.2: Consider a sixth order rational minimum phase stable practical system taken from Prasad [240] having transfer function and the reference model as

$$G_p(s) = \frac{248.05s^4 + 1483.3s^3 + 91931s^2 + 468730s + 634950}{s^6 + 26.24s^5 + 1363.1s^4 + 26803s^3 + 326900s^2 + 859170s + 528050}$$

$$M(s) = \frac{4}{s^2 + 4s + 4}$$

Step 1: Considering M(s), the equivalent open loop transfer function using (6.3) is given by

$$\widetilde{M}(s)=\frac{4s^2+16s+16}{s^4+8s^3+20s^2+16s}$$

Step 2: According to (6.5), the controller transfer function is given as

$$\widetilde{M}(s)=G_c(s)G_p(s)$$

or

$$G_c(s)=\frac{\widetilde{M}(s)}{G_p(s)}$$

$$=\frac{1}{s}(0.8316+0.5313s-0.2841s^2+0.1159s^3+..)$$

Step 3: Choose the controller structure as

$$G_c(s)=\frac{k(1+k_1s)}{s(1+k_2s)}$$

Step 4: Matching controller structure with power series expansion coefficients gives

$$K=0.8316,\ K_1=1.1735,\ K_2=0.5347$$

Hence the controller G_c(s) for the original plant is given as

$$G_c(s)=\frac{0.976s+0.8316}{0.5347s^2+s}$$

Step 5: Then the corresponding closed loop transfer function G_{CL}(s) is

$$G_{CL}(s)=\frac{242.5s^5+1656s^4+9.11\times10^4s^3+5.347\times10^5s^2+1.011\times10^6s+5.28\times10^5}{0.5363s^8+15.07s^7+757.3s^6+1.598\times10^4s^5+2.038\times10^5s^4+8.788\times10^5s^3+1.677\times10^6s^2+1.539\times10^6s+5.28\times10^5}$$

Step 6: The closed loop transfer function is reduced to third order using PSO[305] by following step 6 in example 1 in 6.1.2.1.1 and is given by

$$R_{CLPSO}(s)=\frac{0.8622s^2+2.05s+0.9609}{s^3+3.258\ s^2+\ 3.172\ s+0.9609}$$

$$R_{CLGA}(s)=\frac{0.4844s^2+2.393s+1.674}{s^3+3.233\ s^2+4.045s+1.674}$$

$$R_{CLHNA}(s)=\frac{0.9633s^2+3.88s+1014}{1.176s^3+1.404s^2+1190s+\ 1013}$$

The comparison of step responses of *M*(s), G_{CL}(s) and R_{CLPSO}(s) is depicted in Fig.6.5. These responses are compared in terms of transient response parameters with responses from other methods in Table 6.3. It can be concluded that the result obtained by PSO is comparable.

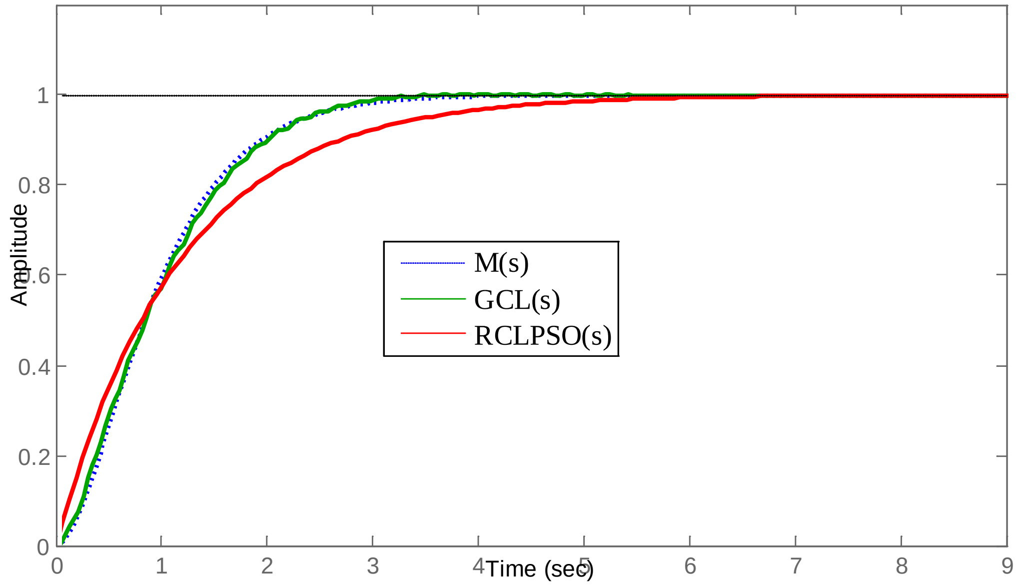

Fig. 6.5 Comparison of step responses example for 6.2

Table 6.3 Comparison of original and reduced order plants for example 6.2

System	Rise Time t_r (sec)	Peak Overshoot % M_p	Settling Time t_s (sec)
M (s)	1.68	0	2.92
G_{CL}(s)	1.76	0.239	2.81
R_{CLPSO}(s)	2.57	0	4.57
R_{CLGA}(s)	1.82	0.158	2.79
R_{CLHNA}(s)	2.57	0.02	4.6

6.1.3 BIG BANG BIG CRUNCH OPTIMIZATION

The BBBC method relies on one of the theories of the evolution of the universe; namely, the Big Bang and Big Crunch theory and then realized to be useful for optimization [264]. The details of this recently introduced technique is already dealt in detail in the previous chapter 4. In the present section, BBBC optimization technique is utilized to optimize the parameters of the PID controller. It is seen that, this approach comes out to be better than the other conventional techniques including HNA in the following solved examples.

6.1.3.1 Illustrative Examples

The numerical examples considered in 6.3.1 are taken up to illustrate that BBBC outperforms better as compared to PSO,GA and HNA method. This same is verified in both methods (indirect and direct) of PID controller design by comparing in terms of time domain specifications.

6.1.3.1.1 ***Direct Method***

Example 6.3: Consider the regulator problem taken in example 6.1 in 6.1.2.1.1

$$G_P(s) = \frac{s^5 + 8s^4 + 20s^3 + 16s^2 + 3s + 2}{2s^6 + 36.6s^5 + 204.8s^4 + 419s^3 + 311.8s^2 + 67.2s + 4}$$

$$M(s) = \frac{0.023s + 0.0121}{s^2 + 0.21s + 0.0121}$$

Step 1: Following the steps 1 to 5 of example 6.1 in 6.1.2.1.1,

$$G_{CL}(s) = \frac{0.8228s^7 + 7.349s^6 + 22.66s^5 + 29.02s^4 + 16.03s^3 + 4.981s^2 + 1.728s + 0.1294}{1.823s^7 + 25.65s^6 + 125.1s^5 + 238.5s^4 + 172s^3 + 38.58s^2 + 3.728s + 0.1294}$$

Step 2: The original system is reduced to second order model using BBBC resulting in

$$R_{PBBBCOA}(s) = \frac{0.0233s + 0.01176}{s^2 + 0.2035s + 0.01176}$$

Step 3: Similarly using PSO,GA and HNA, the reduced transfer function is

$$R_{ppso}(s) = \frac{0.02555s + 0.01036}{s^2 + 0.4756s + 0.01036}$$

$$R_{pga}(s) = \frac{0.01414s + 0.009369}{s^2 + 0.1436s + 0.009369}$$

$$R_{phna}(s) = \frac{0.0113s + 0.0736}{s^2 + 0.1436s + 0.009369}$$

Step 4: Now, the reduced controller obtained is

$$R_{CBBBC}(s) = \frac{\widetilde{M}(s)}{R_{PBBBC}(s)} \quad = \frac{1}{s}(0.06191 + 0.76252s + 0.5764s^2 - ...)$$

Step 5: Taking the PID controller structure as

$$R_c(s) = K_1 + \frac{K_2}{s} + K_3 s$$

$$= \frac{K_1 s + K_2 + K_3 s^2}{s}$$

Step 6: Comparing the coefficients with the power series expansion the parameters, K_1, K_2 and K_3 of the controller are obtained, which gives the PID controller as

$$R_{CBBBC}(s) = \frac{0.06191 + 0.7625s + 0.5764\, s^2}{s}$$

Step 7: The closed loop transfer function of the reduced second order model and the controller using BBBC, PSO, GA and HNA according to step 6 in 6.1.1.1, results in [307]

$$R_{CLBBBC}(s)=\frac{0.01343s^3+0.02455\ s^2+0.01041s+0.000728}{1.013s^3+0.2281s^2+0.02217\,s+0.000728}$$

$$R_{CLPSO}(s)=\frac{0.04853s^3+0.03861s^2+0.00933s+0.0006703}{1.049s^3+0.2142\,s^2+0.01969\,s+0.0006703}$$

$$R_{CLGA}(s)=\frac{0.01165s^3+0.07664\,s^2+0.005604s+0.004762}{1.012s^3+0.417\,s^2+0.0792\,s+0.004762}$$

$$R_{CLHNA}(s)=\frac{0.05177s^3+0.0445\ s^2+0.007102s+0.0006152}{1.052s^3+0.1885s^2+0.0167\,s+0.0006152}$$

Table 6.4 Comparison of original and reduced order plants for example 6.3

System	Rise Time t_r (sec)	Peak Overshoot % M_p	Settling Time t_s (sec)
M (s)	28.1	0.00383	46.2
G_{CL}(s)	22.3	0	45.1
R_{CLBBBC}(s)	28.2	0	47.5
R_{CLPSO}(s)	28.8	0	43.6
R_{CLGA}(s)	28	0	45.3
R_{CLHNA}(s)	30	0.95	42.1

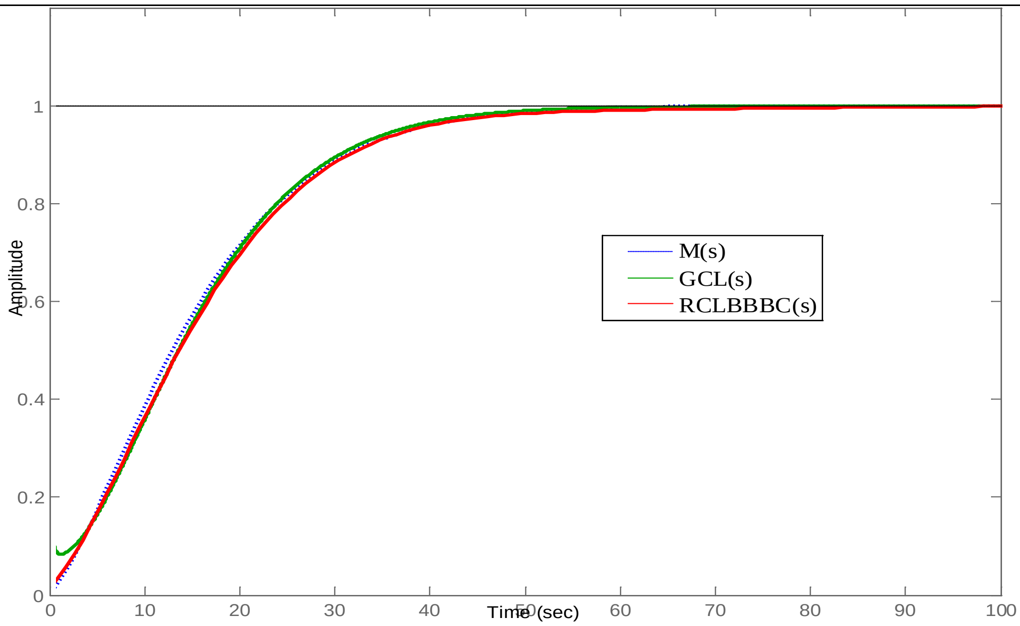

Fig. 6.6 Comparison of step responses example for 6.3

The response of M(s), G_{CL}(s) and R_{CLBBBC}(s) for a given step input is plotted in Fig 6.6. Further, the results are compared with other methods in terms of t_r, M_p, t_s and are tabulated in Table 6.4.

6.1.3.1.1 *Indirect Method*

Example 6.4: Consider a 6th order rational minimum phase stable practical system taken from example 6.2 in 6.1.2.1.2

$$G_p(s)=\frac{248.05s^4+1483.3s^3+91931s^2+468730s+634950}{s^6+26.24s^5+1363.1s^4+26803s^3+326900s^2+859170s+528050},\quad M(s)=\frac{4}{s^2+4s+4}$$

Step 1: Following the same steps 1 to 5 of example 6.2 in 6.1.2.1.2, the closed loop transfer function G_{CL}(s) as

$$G_{CL}(s)=\frac{242.5s^5+1656s^4+9.11\times10^4s^3+5.347\times10^5s^2+1.011\times10^6s+5.28\times10^5}{0.5363s^8+15.07s^7+757.3s^6+1.598\times10^4s^5+2.038\times10^5s^4+8.788\times10^5s^3+1.677\times10^6s^2+1.539\times10^6s+5.28\times10^5}$$

Step 2:G_{CL}(s) is reduced to third and second order using BBBC [307]

$$R_{CL3BBBC}(s)=\frac{-0.04996\ s^2+7.323s+25.83}{s^3+14.13\ s^2+\ 33.04\ s+25.83}$$

$$R_{CL2BBBC}(s)=\frac{0.2917s+2.797}{s^2+3.083s+2.797}$$

Step 3: The reduced third order system obtained [307] by PSO, GA, HNA are

$$R_{CL3PSO}(s)=\frac{0.8622s^2+2.05s+0.9609}{s^3+3.258\ s^2+\ 3.172\ s+0.9609}$$

$$R_{CL3GA}(s)=\frac{0.4844s^2+2.393s+1.674}{s^3+3.233\ s^2+4.045s+1.674}$$

$$R_{CL3HNA}(s)=\frac{0.9633s^2+3.88s+1014}{1.176s^3+1.404s^2+1190s+1013}$$

Table 6.5 Comparison of original and reduced order plants for example 6.4

System	Rise Time t_r (sec)	Peak Overshoot % M_p	Settling Time t_s (sec)
M (s)	1.68	0	2.92
G_{CL}(s)	1.76	0.239	2.81
$R_{CL3BBBC}$(s)	1.76	0.213	2.81
$R_{CL2BBBC}$(s)	1.76	0.0583	2.84
R_{CLPSO}(s)	2.57	0	4.57
R_{CLGA}(s)	1.82	0.158	2.79
R_{CLHNA}(s)	2.57	0.02	4.6

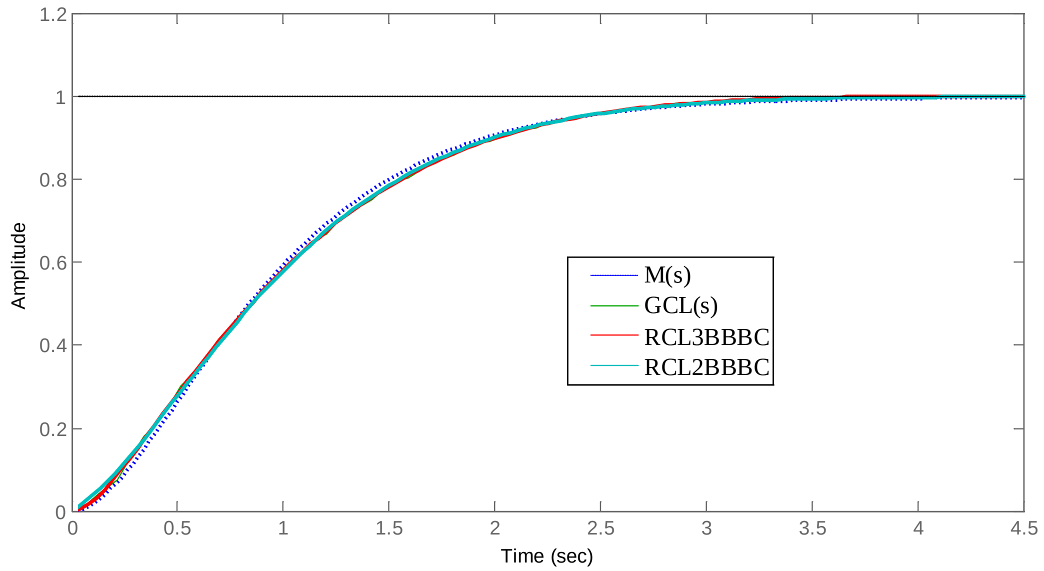

Fig. 6.7 Comparison of step responses for example for 6.4

The comparison of step responses of M(s), G_{CL}(s), $R_{CL3BBBC}$(s) and $R_{CL2BBBC}$(s) is depicted in Fig.6.6. It is seen that G_{CL}(s), $R_{CL3BBBC}$(s) and $R_{CL2BBBC}$(s) almost overlaps each other and are competitive. The same can be concluded by observing Table 6.5, which compares responses from other methods in terms of transient response parameters. It can be concluded that the result obtained by BBBC is comparable.

6.2 FOPID CONTROLLER

The Proportional Integral Derivative (PID) controllers have become popular during the past decades and its widespread use has attested it. This is because of its performance, robustness, simplicity, availability of many effective and simple tuning methods [308]. However, according to the reports of Van Overschee and De Moor (2000), 80% of the PID controllers are badly tuned [267]. Over the decades the PID control techniques have undergone many changes and the same will continue in the days to come. Today, various PID controller tuning methods are available but, the determination of the best PID controller parameter is still a challenging task and is under active research [309]. Also, the progress of design methods for classical PID control is approaching the point of diminishing returns. Therefore this study proposes to use a FOPID controller which provides relief to some of the problems. FOPID controllers are similar to usual PID controllers, except for the fractional derivative λ and integral μ orders, which helps to provide additional design flexibility.These two parameters ensure a robust performance of the controlled system in terms of gain variations and phase characteristics. Hence fractional order controller becomes a powerful tool in designing robust control system with less controller parameters to tune.

The concept of FOPID controller was initially proposed by Igor Podlubny [310] with a drawback, that its applicability is limited only to linear systems with constant coefficients [310]. However, a new tuning method for the problem of identifying of parameters of FOPID model was kept open. In this regard several analytical ways of tuning were proposed [311]. Since then, FOPID controllers are increasingly being used. Recently evolutionary techniques such as Particle Swarm Optimization (PSO) [312] and Genetic Algorithm (GA) [313, 314] based FOPID controller design was proposed and proved to be more effective.

The FOPID controller also known as $PI^{\lambda}D^{\mu}$ was studied in time domain in [310] and in frequency domain [311]. Generally the FOPID controller takes the form

$$G_{FOPID}(s) = \frac{K_D s^{\lambda+\mu} + K_P s^{\lambda} + K_I}{s^{\lambda}} \tag{6.14}$$

where λ and μ are positive real numbers, Kp is the proportional gain, K_I the integral gain and K_D the differentiation gain. Considering λ and μ as unity, we obtain a classical PID controller. The graphical representation highlighting the flexibility provided by FOPID controller in PID control design is as shown in Fig. 6.8. The FOPID controller extends the four control points (P, PI, PD, PID) of the classical PID to a variety of control points on the quarter-plane defined by selecting the values of λ and μ. Thus generalizes the conventional integer order PID controller and expands it from point to plane.

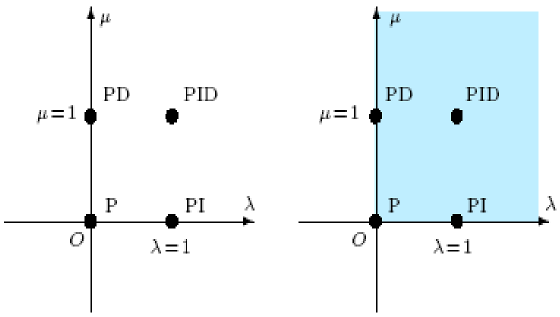

Fig. 6.8 FOPID v/s Classical PID : from points to plane a) integer order PID b) fractional order PID

6.2.1 Problem Statement

The original n^{th} order LTI-SISO system is taken into account represented by the transfer function

$$G_p(s) = \frac{\sum_{j=1}^{n} b_j s^{j-1}}{\sum_{i=1}^{m} a_i s^{i-1}} \tag{6.15}$$

where b_j's, a_i's are scalar constants and n, m are the order of the numerator, denominator polynomial respectively. The objective is to find the FOPID controller G_{FOPID}(s) given by (6.14). Further the unit step response of closed loop transfer function of G_p(s), G_{FOPID}(s) connected in series for a given unit step input meets the desired design specifications such as peak overshoot 'M_P', settling time 'T_S', rise time 'T_r', Integral Square Error (*ISE)* etc. The feedback gain of the closed loop system is assumed to be unity.

6.2.2 Optimizing FOPID Controller Parameters using BBBC

During the past decade, the usage of evolutionary technique has transformed the way of solving problems successfully in almost all fields and is drawing much attention. Big Bang Big Crunch (BBBC) as discussed in chapter 4, is one such optimization technique recently introduced and belongs to the family of evolutionary method [264]. Hence, BBBC [315] is used here to find the optimal regions of complex search spaces through the interaction of individuals and turns out to be successful. As a result, the main objective of optimizing the FOPID controller parameters for an uncontrolled plant G_P(s) is fulfilled; the closed loop response with unity feedback is stable and may have suitable fast response. This results in rationally searching the best design among the alternative designs to meet the desired.

6.2.2.1 Illustrative Examples

Example 6.5: Consider an original system characterized by transfer function G_p(s) [313] for which a FOPID controller has to be designed which will provide lesser M_P , T_S, *ISE* for a given unit step input.

$$G_p(s) = \frac{40}{2s^3 + 10s^2 + 82s + 10}$$

Step 1: Taking the FOPID controller structure as

$$G_{FOPID}(s) = \frac{K_D s^{\lambda+\mu} + K_P s^{\lambda} + K_I}{s^{\lambda}}$$

Step 2: Substitute the values of K_P, K_I, K_D, λ, μ generated using BBBC by minimizing the fitness function 'E' given by (4.2) and the peak overshoot given by

$$M_p = \exp^{\left(\frac{-\sigma_d \pi}{\omega_d}\right)} 100 \qquad (6.16)$$

($\sigma_d \pm j\omega_d$ is the location of the dominant pole)

Step 3: This results in

$$G_{FOPIDBBBC}(s) = \frac{5.3896 s^{0..9+1.1} + 27.0406 s^{0.9} + 19.6417}{s^{0.9}}$$

Step 4: The open loop transfer function is then obtained by

$$G_{OL}(s) = G_P(s)G_{FOPIDBBBC}(s)$$
$$= \left(\frac{40}{2s^3+10s^2+82s+10}\right)\left(\frac{5.3896s^{\lambda+\mu}+27.0406s^{\lambda}+19.6417}{s^{\lambda}}\right)$$

Step 5: The closed loop transfer function is then obtained by

$$G_{CLBBBC}(s) = \frac{G_p(s)G_{FOPIDBBBC}(s)}{1+G_p(s)G_{FOPIDBBBC}(s)}$$
$$= \frac{215.584s^2+1081.624s^{0.9}+785.668}{2s^{3.9}+10s^{2.9}+215.584s^2+82s^{1.9}+1091.624s^{0.9}+785.668}$$

Step 6: The FOPID controller designed using GA [313] is given by

$$G_{FOPIDGA}(s) = \frac{0.36015s^{\lambda+\mu}+6s^{\lambda}+12.24}{s^{\lambda}}$$

Step 7: The closed loop transfer function using $G_{FOPIDGA}(s)$ for $\lambda = 0.9$ and $\mu = 1.1$

$$G_{CLGA}(s) = \frac{14.40s^2+240s^{0.9}+489.6}{2s^{3.9}+10s^{2.9}+14.406s^2+82s^{1.9}+250s^{0.9}+489.6}$$

Step 8: The approximated closed loop transfer function $G_{CLBBBCRED}(s)$ obtained for $G_{CLBBBC}(s)$ using BBBC [315] minimizing (4.2)

$$G_{CLBBBCRED}(s) = \frac{4.858s^2+3.25\times10^{-5}s+0.07416}{s^3+4.859s^2+0.0153s+0.07416}$$

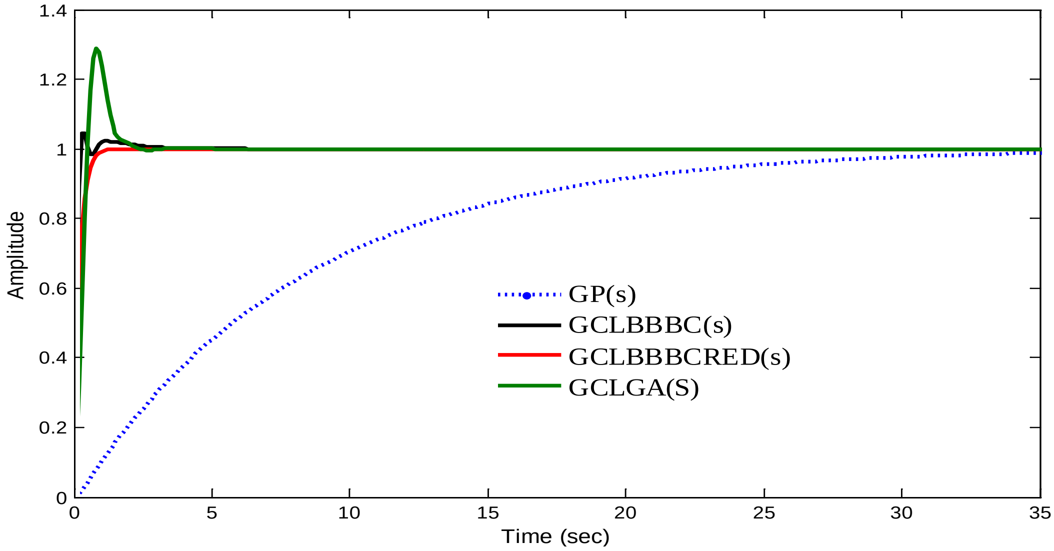

Fig. 6.9 Comparison of unit step response of closed loop system using FOPIDBBBC and FOPIDGA controller for λ=0.9 and μ=1.1

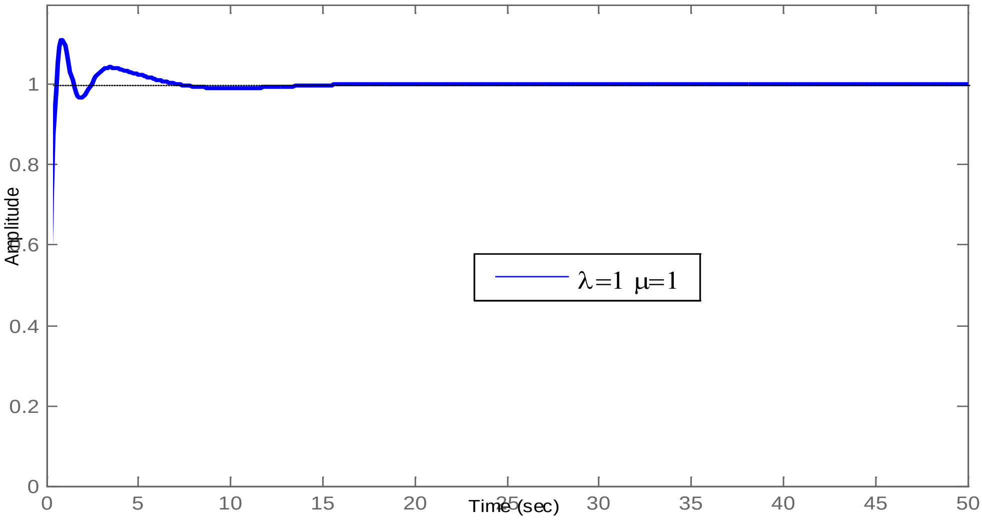

Fig. 6.10 Unit step response of system using FOPID controller for unity λ and μ

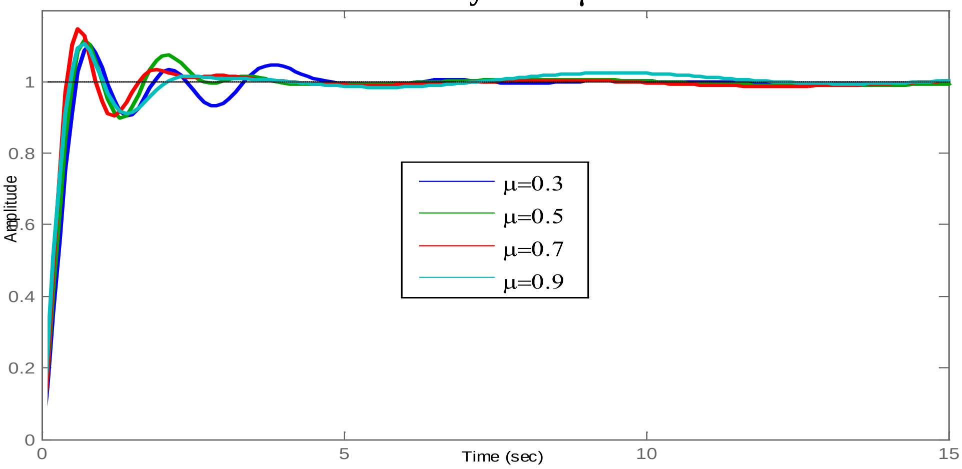

Fig. 6.11 Unit step response of system using FOPID controller for varying μ (μ< 1)

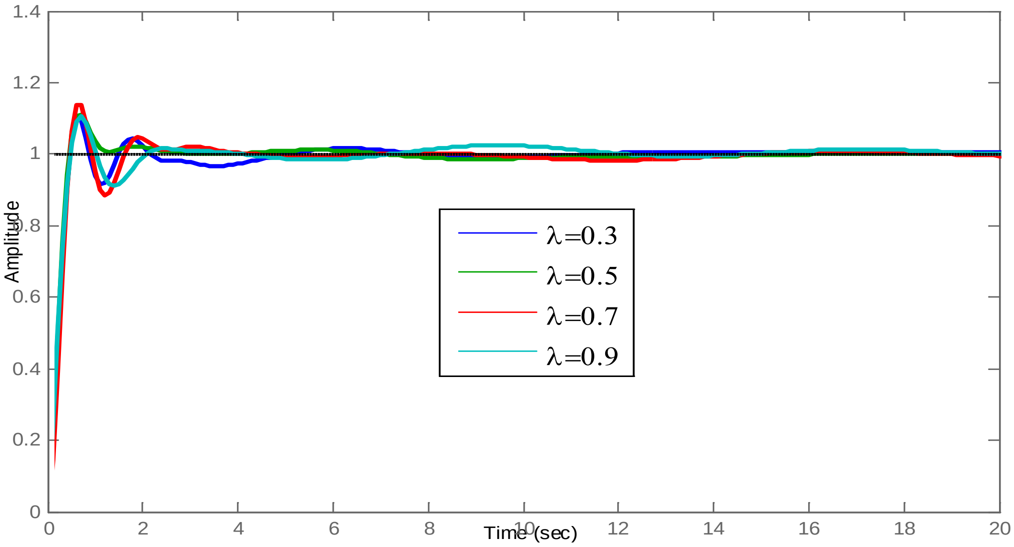

Fig. 6.12 Unit step response of system using FOPID controller for varying λ (λ< 1)

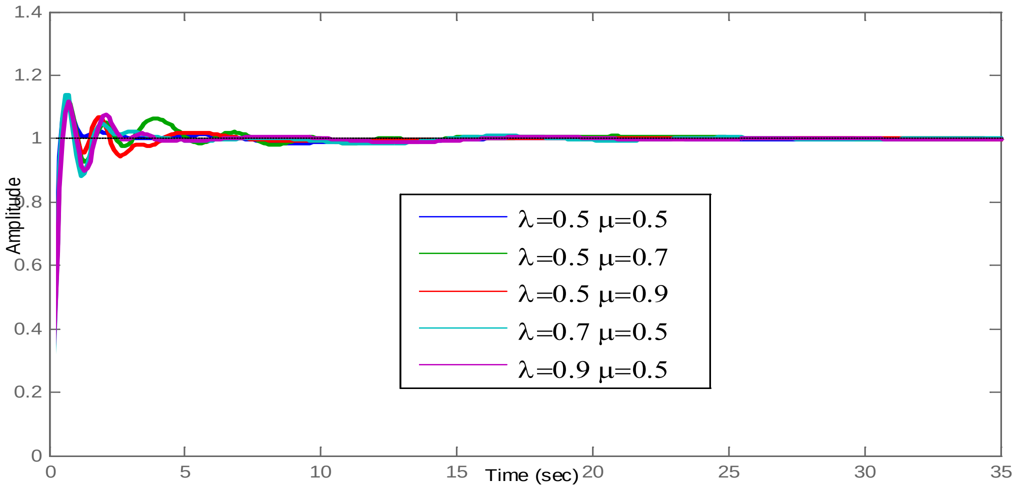

Fig. 6.13 Unit step response of system using FOPID controller for varying λ and μ (λ< 1,μ< 1)

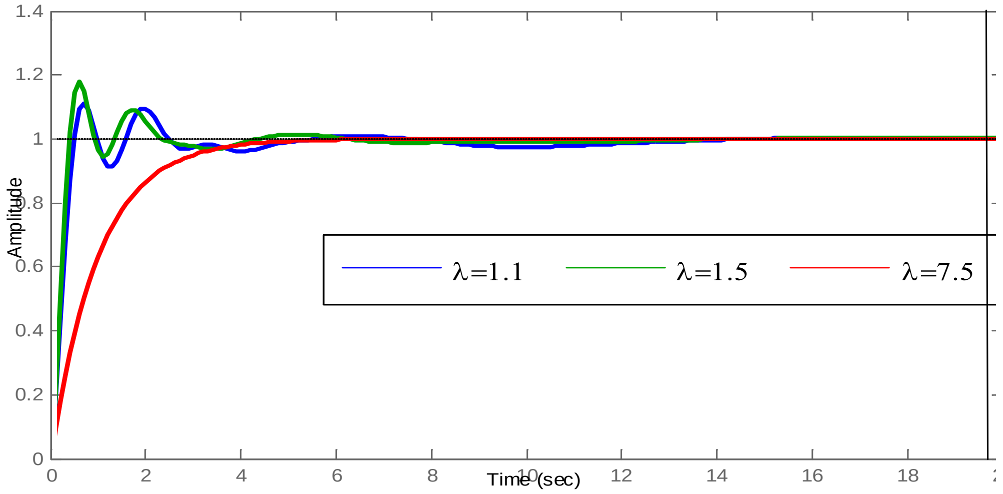

Fig. 6.14 Unit step response of system using FOPID controller for varying λ (λ> 1)

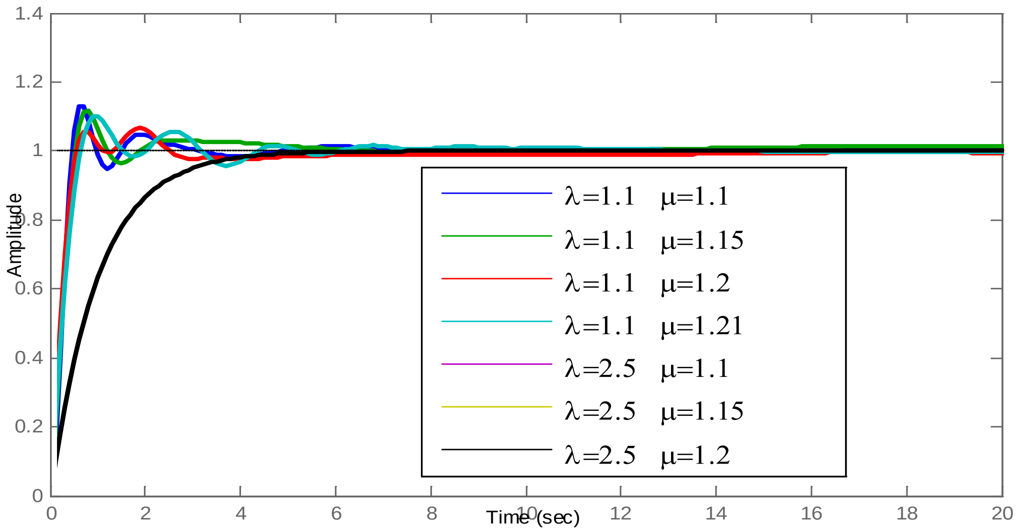

Fig. 6.15 Unit step response of system using FOPID controller for varying λ and μ (λ> 1,μ>1)

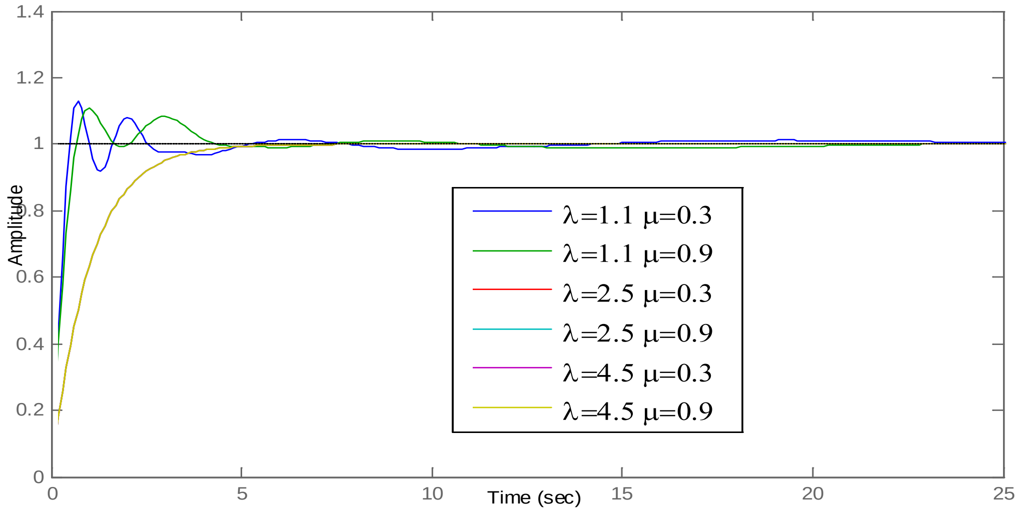

Fig. 6.16 Unit step response of system using FOPID controller for varying λ and μ (λ> 1,μ< 1)

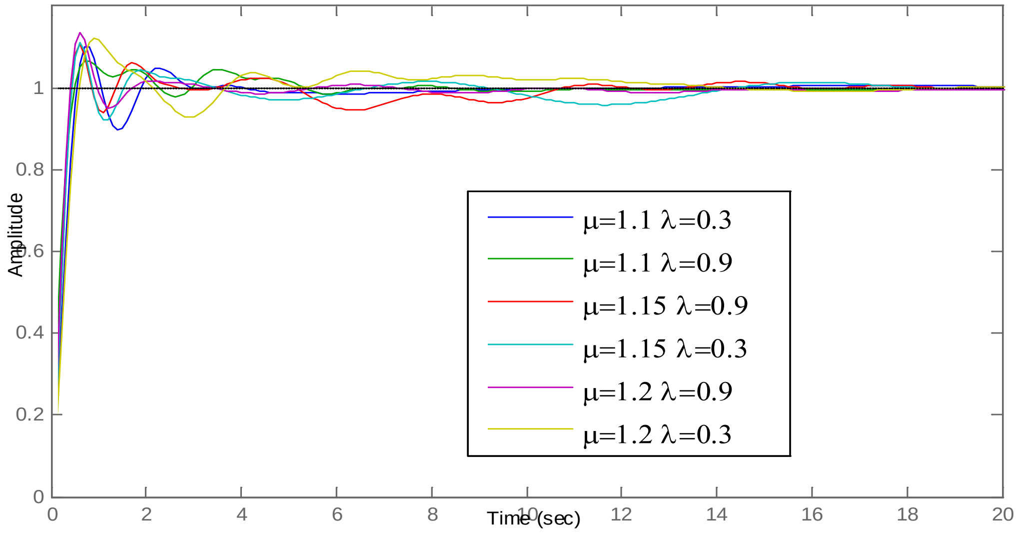

Fig. 6.17 Unit step response of system using FOPID controller for varying λ and μ (λ< 1,μ>1)

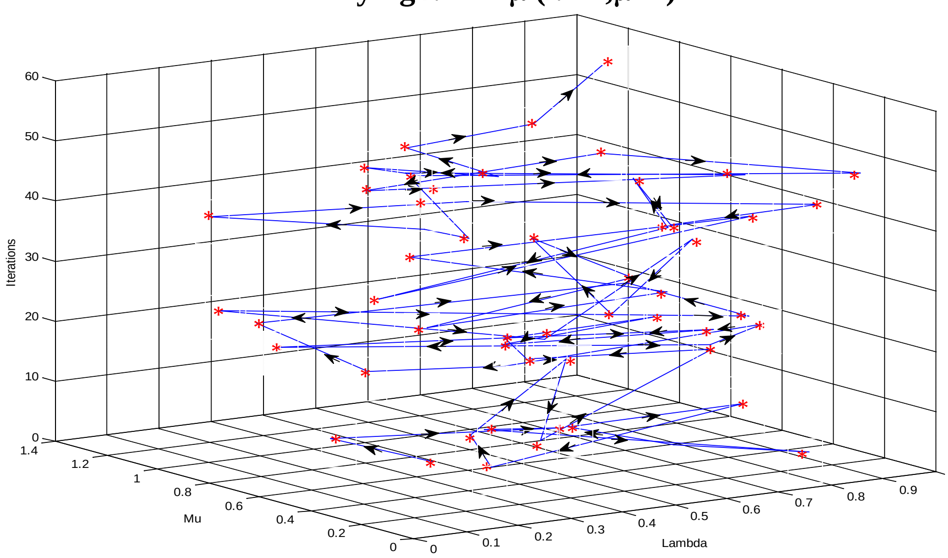

Fig. 6.18 Path traced to by λ and μ during the optimization process

Table 6.6 Comparison of parameters for different combinations of λ and μ for example 6.5

λ	μ	Values obtained by BBBC (proposed)				Values obtained by GA [313]			
		Settling Time 'T_s'	Peak Overshoot 'M_p'	ISE	IATE	Settling Time 'T_s'	Peak Overshoot 'M_p'	ISE	IATE
1	1	5.4627	11.3171	1.9246	36.3443	23.65	47.15	2.350	53.70
1	0.3	4.3278	10.0349	2.4982	33.1984	41.742	46.0245	2.889	123.7
1	0.5	2.5002	11.6848	2.2635	23.6048	34.903	46.9016	2.659	89.85
1	0.7	2.2034	14.3641	2.0940	18.2592	25.293	47.2256	2.505	67.85
1	0.9	10.4159	10.6855	1.9705	35.1073	24.148	47.3067	2.405	56.97
0.3	1	4.2231	10.8044	2.1812	29.7341	27.228	37.3379	2.243	68.56
0.5	1	1.9948	11.3069	2.0830	23.9988	24.263	40.0857	2.253	56.97
0.7	1	3.2528	14.6011	2.3424	28.4420	24.031	42.9715	2.291	54.11
0.9	1	10.4159	10.6855	1.9705	35.1073	23.632	45.8391	2.335	53.58
0.5	0.5	1.9548	11.4000	1.9830	24.1121	35.404	38.7205	2.543	95.84
0.5	0.7	6.8865	12.2687	2.2163	38.9573	26.266	39.7392	2.397	72.19
0.5	0.9	3.7937	11.5378	2.1081	23.6011	24.703	40.1304	2.300	60.52
0.7	0.5	3.3538	14.8011	2.3436	28.4520	28.661	42.1173	2.589	92.45
1.1	0.5	11.1732	11.2030	2.2012	24.1625	23.4050	48.5314	2.381	54.04
1.5	0.5	3.8488	0.9096	2.0339	23.2997	27.4584	53.5798	2.475	56.14
7.5	0.5	3.9123	0.1060	5.5167	11.0425	37.8834	141.1292	5.643	121.3
1.1	1.1	2.4884	13.5817	2.2104	18.5385	22.925	45.8945	2.142	49.08
1.1	1.15	4.4859	11.8160	2.2241	502059	22.096	36.124	1.438	38.42
1.1	1.2	4.5773	6.7630	1.9964	70.8083	13.319	44.7987	0.2839	13.54
1.1	1.21	4.1228	10.1151	2.2047	19.7048	20.628	93.2290	0.8556	30.17
2.5	1.1	3.9123	0.0	5.5167	11.0425	25.475	60.6300	2.441	57.84
2.5	1.15	3.9120	0.0	5.5180	11.0200	24.189	46.7059	1.658	44.59
2.5	1.2	3.9200	0.0	5.5200	11.0100	17.147	44.2505	0.4092	21.84
1.1	0.3	4.5326	12.9902	2.2375	36.9991	41.357	47.7478	2.918	123.5
1.1	0.9	3.9314	10.7146	2.3835	28.9681	23.860	48.6390	2.428	57.27
2.5	0.9	3.9123	0.0	5.5167	11.0425	30.375	64.9865	2.764	67.17
4.5	0.3	3.9122	0.0	5.5169	11.0430	40.946	102.1544	4.574	135.4
0.3	1.1	2.6811	10.5548	2.2649	22.5673	26.564	36.0403	2.021	63.14
0.9	1.1	4.9125	6.7860	1.6417	23.2709	23.366	43.4405	2.098	48.6
0.3	1.15	13.3011	11.1522	2.0382	47.0520	25.260	30.5292	1.344	48.47
0.9	1.15	10.1522	10.8518	2.0804	27.5545	22.705	34.3791	1.403	38.11
0.3	1.2	1.5322	13.6581	0.1837	2.98015	4.8701	47.5611	0.1939	5.328
0.9	1.2	1.4958	12.2436	0.2507	5.36410	13.042	45.1063	0.2591	11.7

6.3 CONCLUSION

In this chapter, the task of designing both PID and FOPID controllers have been accomplished successfully. In case of PID controller, both direct and indirect methods of controller design is considered. The reduction of the closed loop system is then performed using PSO and BBBC by reducing the error between the reference model and the reduced model. Later, the unknown controller parameters have been found. The suitability of the proposed methods have been justified by solving illustrative examples from the literature available. The step responses illustrates the goodness of the proposed method.

In the case FOPID controller, the values of λ and μ are tuned by using BBBC and then the results are compared with the existing technique. Qualitative comparison in terms of settling time, peak overshoot, ISE and IATE are tabulated apart from step responses to justify the proposed method.

CHAPTER - 7

CONCLUSIONS AND FUTURE SCOPE

The area of devising new reduced order methods has been on the move from past few decades. The work presented here pertains to the development of new techniques for order reduction and design of controllers. However, most of the proposed methods are confined to frequency domain, where a given system is primarily described by transfer function. This concluding chapter is primarily devoted for summarizing main results of each chapter. Further, the scope for future work is also discussed.

7.1 RESULTS AND DISCUSSION

The chapters deliberated in this thesis until now, deals with the order diminution methods developed for linear systems, including nature guided techniques. These methods are applicable for both SISO and MIMO time invariant systems delineated in frequency domain. The same is also extended for discrete systems as well and is exemplified. Subsequently, effort has been made to circumvent some of the drawbacks related with frequency domain reduction methods. Later, their applicability in the field of controller design is also proposed. The whole work presented here, is carried out in MATLAB 7.10 (R2010a) and Spectrum Digital Code Compose Studio (CCStudio) Version 3.1 with the view of using TMS320C5402 DSP processor.

The introductory chapter gives an insight regarding the importance and applications of model order reduction, various available methods of order reduction along with its merits/demerits. However, an emphasis has given for the methods being employed in developing new reduction methods. Apart from this, problem statement of model order reduction for SISO/MIMO systems in both time and frequency domain are also included. Further, it has been noted that order reduction field has received widespread interest among research community.

In chapter 2, four composite reduction methods, are proposed based on mixed reduction method leading to superior solutions. These methods were applied to reduce the order of the original stable system (continuous time) represented in frequency domain. The first and second method reaps the benefits of ESA and stability equation for reducing the denominator terms, whereas the third and fourth method employs dominant pole and modified pole clustering respectively. All these methods were proposed in combination with least squares method to preserve the stability/vital features of the original system, apart from improving ISE values as exemplified. Further, the same algorithms were also extended for MIMO systems and results are comparable.

In chapter 3 the developed algorithms were discussed for reducing the order of discrete time (stable) systems. In order to accomplish the task a, continuous time reduction methods such as ESA, stability equation, dominant pole and modified pole clustering techniques were used in combination with least squares method. The developed combinative methods were applied on original higher order stable system (discrete time). This was possible only, by using discrete to continuous time transformation and vice versa during the initial and final stages. The same methods were further extended for MIMO systems and proved to be comparable in quality. The comparison of step/impulse responses along with tables indicating SSE values confirm it.

In chapter 4, four new approximation methods were introduced based on the recently erupted evolutionary technique called BBBC. The first and second method used the merits of Routh approximation and stability equation method in combination with BBBC to obtain the reduced order system. The said method well well suited, for both linear dynamic stable SISO and MIMO systems. The third method comprise of optimizing the linear shift point about a general point 'a' by employing BBBC optimization technique, to obtain the resultant system comparable to reduced systems derived from other methods. This has been apparent from the examples solved by considering SISO systems and comparing the results with other available reduced order systems. In case the fourth approximation method, both numerator and denominator terms were optimized using BBBC, while satisfying the fitness function. Benchmark examples belonging to SISO/MIMO linear stable systems available, were considered and then operated upon. The responses obtained for step and impulse input were compared along with performance indices values. It has been observed that the results obtained were encouraging. The same approximation technique, was then extended for generating reduced system in discrete time. The suitability of the proposed method was shown by solving illustrative examples.

In chapter 5, TMS320C5402 processor were introduced to reduce the order of the digital filters and original systems. BBBC was used to perform the task. The order of Butterworth and Chebyshev filters designed were reduced and implemented. The input/output waveforms obtained in MATLAB and CCS were compared. In addition, the frequency response and FFT power spectrum of the input/ output signals were also plotted for clarity. In the second case, GA was employed to optimize the system coefficients and implemented on CCS. The resultant waveforms show the validity of the said method.

The various controllers were designed and discussed in chapter 6. The PID values of the controller were optimized by using PSO in the first method and BBBC in the second. Both direct and indirect approach of designing the controller were adopted and exemplified. The flexibility offered by FOPID controller was discussed and compared to PID controller. An original system was picked up from the available literature for consideration and FOPID controller parameters were then optimized using BBBC. The results obtained were comparable to its counterpart.

7.2 FUTURE SCOPE

Research is a continuous process and hence, there will be opportunities for further amelioration all the time. The same is true in the present work and some suggestions are given below which may motivate the researchers to take up research work in this direction.

The developed reduction methods works well for SISO systems. Apart from this, MIMO systems are also benefitted from these methods, provided the common denominator polynomial is available. If this condition is not met, then direct extension of the methods applicable to SISO cannot be extended to MIMO systems. This is the research gap that one can think to fill it up. In this regard, introducing a new reduction algorithm can be thought of which considers all the elements of the MIMO system together and delivers a reduced MIMO system respectively.

Scope for devising some norms which provides an idea about the least order an original system can be reduced, without altering its vital characteristics is another research area to be explored upon. Predicting beforehand, reduces the time consumed to arrive at the appropriate order of the reduced system. Further, one can also extend the proposed methods in the reduction of interval systems as well.

The proposed techniques applicable for continuous time systems discussed so far are extended for discrete time systems. This has been achieved by incorporating *z-p* transformations and vice versa during the initial and final stages of reduction process respectively. Approximation methods dealing with discrete time systems without the need of *z-p* transformations can be further explored and developed.

The evolutionary techniques viz. BBBC, GA are roped in to generate reduced systems by using ISE as fitness function. However, performance indices such as IAE, ITAE and ITSE can also be minimized to generate reduced order systems.

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LIST OF PUBLICATIONS

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