ANALYSIS OF CORRELATION, NETWORK & QUANTUM PRINCIPLES IN FINANCE AND DECISION MAKING

Ph.D. THESIS

by

SHAKTIKANTA NAYAK

DEPARTMENT OF MANAGEMENT STUDIES
INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
ROORKEE-247 667 (INDIA)
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ROORKEE

CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled "**ANALYSIS OF CORRELATION, NETWORK & QUANTUM PRINCIPLES IN FINANCE AND DECISION MAKING**" in partial fulfilment of the requirements for the award of the Degree of Doctor of Philosophy and submitted in the Department of Management Studies of the Indian Institute of Technology Roorkee, Roorkee, is an authentic record of my own work carried out during the period from July, 2010 to February, 2015 under the supervision of Dr. J.P.Singh, Professor , Department of Management Studies, Indian Institute of Technology Roorkee, Roorkee.

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other Institute.

(**SHAKTIKANTA NAYAK**)

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

(J.P.Singh)
Supervisor

Date:_____

The Ph.D. Viva-Voce examination of **Mr. Shaktikanta Nayak,** Research Scholar, has been held on_____

Chairman, SRC

Signature of External Examiner

This is to certify that the student has made all the corrections in the thesis.

Signature of Supervisor

Head of the Department

ABSTRACT

The purpose of this thesis is to study the correlation and network behavior of several small cap, mid cap and large cap Indian companies indexed in Bombay Stock Exchange separately. Random matrix theory has been employed to compare the statistical properties of the correlation matrix obtained from the empirical market data. The spectral properties of market data were tested against random matrix predictions and find out some agreement between the distributions of eigenvalues, eigenvector components and the inverse participation ratios for eigenvectors. A bulk of eigenvalues falls within the bounds expected for a random matrix constructed from mutually uncorrelated time series. The eigenvalues below and above the bulk have been investigated. It is observed that few largest and smallest eigenvalues deviate significantly from the bulk, the largest is identified with market mode while the smallest with entanglement characteristics of companies. Entanglement physically means non-separability of two different systems or entities. In the context of companies, it will reveal correlated companies, whose dynamics are intertwined. It is observed that the largest eigenvalue is independent of the dimension of the correlation matrix and dependent upon the capitalization of the stocks. The intermediate eigenvalues that fall between the large and the bulk have been associated with specific business sectors with strong intra-group interactions. The intermediate eigenvalues that fall between the large and the bulk are few in number and lie close to the bulk are associated with specific business sectors with strong intra-group interactions. We propose that this is due to the lack of distinct sector identity in the market to influence the overall market trend. The correlation analysis is carried out to investigate the behavior of stocks during the market crash and it is observed that during crisis stocks belonging to similar or related business sector to move together. Further the internal dynamics of stocks and visualization of financial networks have been investigated, first by constructing Minimum Spanning Tree from the unfiltered correlation matrix. The minimum spanning tree method facilitated to find closest stocks to create a tree. Later market mode and random noise have been removed from the data to show the clustering of stocks into economic sectors based on their proximity. Both methods show absence of clustering of co-moving stocks that belong to same business sector. The analysis of the Indian market crash has been investigated using the minimum spanning tree method and it is observed that during crisis period the stocks are becoming closer than before and after the crisis.

Further we have also investigated some financial and decision problems using principles of quantum theory. The principal goal of the quantum approach to business and artificial intelligence is to develop a unified theory for functional aspects of the brain that, from one side, could formalize cognitive decision making process in terms of quantum language and, from another side, would suggest a scheme of thinking quantum systems that could be engaged for creating computational intelligence. Quantum computation based on the quantum mechanical nature of physics, which is inherently a parallel distributed processor having the exponential memory capacity and easily trainable, but it has severe hardware limitations. On the other hand the power of artificial neural network is due to its massive parallel, distributed processing of information and nonlinear transformation. Hence attempt is made to explain present understanding and applicability of quantum approach to neural information processing. Quantum computing techniques have successfully explained many computationally hard problems which were impossible in classical computing. Quantum Deutsch's algorithm has been introduced to predict the financial market price. The Grover's algorithm for database search has been explained in the case of four qubit system. In some experiments of psychology the classical decision theory is unable to explain the paradoxical departure from logical statements of principle of a sure thing. Such type of paradoxical departure of human behavior in decision making can be explained using the probabilistic mathematical framework of quantum mechanics. An attempt is made to introduce a quantum decision model to explain two stage gambling experiment which violates the principle of sure thing and other decision related problems like merger acquisition problem.

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LIST OF ABBREVIATIONS

BSE Bombay Stock Exchange

RMT Random Matrix Theory

IPR Inverse Participation Ratio

QDT Quantum Decision Theory

QC Quantum Computing

ANN Artificial Neural Network

NYSE New York stock exchange

MST Minimum spanning tree

ANN Artificial Neural Network

CNN Classical Neural Network

XOR Exclusive OR

QANN Quantum Artificial Neural Network

LIST OF PUBLICATIONS

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CHAPTER-1

INTRODUCTION

1.1 Background

In the early twentieth century physicists have faced difficulty to explain certain physical phenomena using classical logic, probability and dynamics. Quantum theory was discovered to explain the experimental findings which were not explainable by classical physics [103, 113, 41]. During 1950 existing models were unable to explain the energy spectrum of complex nuclei. Random Matrix Theory was introduced by Wigner to explain the structural properties of energy levels of nuclei [47, 101]. The application of the above mentioned theories was restricted only in science and engineering community to study the behavior of atomic and subatomic particles for example the motion of electrons. Scientists have tried to generalize the mathematical basis of these theories and opened it up to all branches starting from science to business and management. The dynamics of financial stock market are one of the complex systems where millions of transactions take place. The nature of interaction among the stocks in a stock market is not known much similar to that of a quantum dynamical system. In a stock market every agent is concerned for his/her own profit looking at the current transaction. If the transactions occur randomly then the prices will be random. Hence knowing the current status of a stock will not help to predict its future. The randomly fluctuating stock prices can be modelled using correlation matrix. Such random matrices are extensively studied in nuclear physics where random matrix models have been employed to study the spectral behavior of complex many body systems; stock market being one of them [89, 127, 131]. In this thesis Random matrix theory is used to extract the noise from correlated stocks. The objective of this thesis is to examine the correlation and network behavior of several small cap, mid cap and large cap Indian companies separately indexed in Bombay Stock Exchange. A bulk of eigenvalues falls within the bounds expected for a random matrix constructed from mutually uncorrelated time series. The eigenvalues below and above the bulk have been investigated. It is observed that few largest and smallest eigenvalues deviate significantly from the bulk, the largest is identified with market mode while the smallest with entanglement characteristics of companies. Entanglement physically means non-separability of two different systems or entities. In the context of companies, it will reveal correlated companies, whose dynamics are intertwined. It is observed that the largest eigenvalue is independent of the dimension of the correlation matrix and dependent upon the capitalization of the stocks. The intermediate

eigenvalues that fall between the large and the bulk have been associated with specific business sectors with strong intra-group interactions. The intermediate eigenvalues that fall between the large and the bulk are few in number and lie close to the bulk are associated with specific business sectors with strong intra-group interactions. We propose that this is due to the lack of distinct sector identity in the market to influence the overall market trend. The correlation analysis is carried out to investigate the behavior of stocks during the market crash and it is observed that during crisis stocks belonging to similar or related business sector to move together.

The discovery of the complex network structure of developed countries like USA and Japan insists us to investigate correlation based network behavior of developing markets like India. The complex network technique of graph theory has been successfully applied in the field of computer science, statistical physics, biological sciences, social sciences and financial market [18, 20, and 44,172]. The complex network is a tool to represent the spatial structure of complex real world system in the form of a graph. Hence the question arises how to represent the temporal dynamics of the time series using complex network techniques. The advances in statistical physics make it possible to apply the complex network tool to analyze the financial time series. In this thesis a correlation based network method is used to construct the network of financial time series and a threshold method is used to construct the group correlation matrix. The group structure of the correlation matrix is obtained by filtering market mode and the noise predicted by Random matrix theory of the actual market. The more common algorithms used to extract hierarchical structure present in the data are given by minimum spanning tree method [99]. Very few studies have been carried out to analyze the spectral characteristics of correlation matrix. But no study in our knowledge has been performed to analyze small capitalized companies, mid capitalized companies and large capitalized companies separately of a particular stock exchange. The correlation and networking of financial time series have been studied and these methods are applied to study the behavior of stocks during economic crisis. The analysis of the Indian market crash has been investigated using the minimum spanning tree method and it is observed that during crisis period the stocks are becoming closer than before and after the crisis.

Further quantum computation uses quantum mechanical effects to perform computational tasks and in some cases its results are exponentially faster than their classical counterparts. In recent years quantum techniques, applied in the field of computing, have successfully explained many computationally hard problems which were impossible in classical computing. The study of financial market is highly sensitive, uncertain and

unpredictable and depends upon many external factors. There is no well defined analytical procedure to predict the financial market. But Quantum Mechanics, based upon uncertainty and probability theory, have explained many physical phenomena most accurately which were impossible in classical computing. Quantum Deutsch's algorithm has been introduced to predict the financial market price. The Grover's algorithm for database search has been explained in the case of four qubit system. Quantum computation based on the quantum mechanical nature of physics is inherently a parallel distributed processor having exponential memory capacity and easily trainable. But it is linear in nature and suffers from hardware implementation. On the other hand the power of artificial neural network is due to its massive parallel, distributed processing of information and nonlinear transformation. Hence an attempt is made to explain present understanding and applicability of quantum approach to neural information processing [82]. The quantum probability theory is the generalization of classical probability theory which is able to explain the effects that were impossible in classical domain. Apart from information processing some of the important application domains, where quantum theory has proven to be successful, are decision science, cognitive science and game theory [130]. In many cases the business and management science deal with various ill-structured problems for example the complex process of understanding the human decision making. Hence we need sophisticated mathematical tools to deal with such type of problems. Quantum mechanical tools based upon probability and uncertainty are suitable candidates to explain the decision making process. In some experiments of psychology the classical decision theory is unable to explain the paradoxical departure from logical statements of principle of sure thing. Such type of paradoxical departure of human behavior in decision making can be explained using the probabilistic mathematical framework of quantum mechanics. Quantum decision model has been analysed to explain the two stage gambling experiment which violates the principle of sure thing. This model has also been used to explain other decision related problems like merger and acquisition of business firms.

1.2 Objectives

The two major objectives of the present work are identified as follows:

1. To study the financial time series comprising of daily closing price indices of several Indian companies, belonging to Bombay Stock exchange. In particular, large cap, mid-cap and small cap companies have been studied separately to investigate their correlation and network behavior.
2. The quantum approach to business and artificial intelligence has been studied to develop a unified theory for functional aspects of the brain that, from one side, could formalize cognitive decision making processing in terms of quantum language and, in other side, would suggest a scheme of thinking quantum systems that could be engaged for creating computational intelligence.

1.3 Organization of Thesis

The thesis is divided in to seven chapters. The main body of the thesis is as follows.

Chapter 1: Introduction

Chapter 1 describes the basic research background of the thesis along with important objectives. Other chapters are made according to the theme outlined within the objective of the thesis.

Chapter 2: Review of Literature

A recent development and application of fundamental theories of physics in the non physics domains like decision science, computer science, business and management sciences are discussed in chapter 2. A brief review of basic theory on the topics of random matrix theory, correlation based network, quantum computing, quantum logic gates, quantum artificial neural networks and quantum decision theory have been discussed.

Chapter 3: Correlation Analysis of Indian Companies

The correlation analysis using Random matrix theory has been explained in chapter 3. First, the correlation analysis technique is used to examine the interrelationship of several Indian companies belonging to BSE. Then Random matrix theory is applied to remove noise from the signal. Then higher eigenvalues above the noise threshold and lower eigenvalues below noise threshold are investigated. Finally the behavior of stocks during market crash has been investigated.

Chapter 4: Network Behavior of Indian Companies

In chapter 4, the dynamics of the stock market are captured by using graph theoretical analysis. Minimum spanning tree method of Krushakal has been used to establish the relationship between various stocks. The visualization and analysis of the complex network are made by using Pajek software. Then group correlation structure is extracted by removing market mode and noise from the correlation matrix. Finally the Minimum spanning tree method is used to analyze the market crash.

Chapter 5: Quantum Computing

The motivation of this chapter is to discuss possible applications of quantum algorithms in the financial market price prediction and searching an unstructured database. The application of quantum computing principles to the field of artificial neural computing has been discussed in a hope to enhance the computational capabilities not available in classical Neurocomputing.

Chapter 6: Quantum Model for Decision Making

In chapter 6, the basic concepts of quantum mechanics, interference of probability amplitudes, is introduced to explain the violation of "the principle of sure thing" and an attempt is made to introduce a quantum decision model to explain two stage gambling experiment. The quantum model has been discussed to study the mental state of the acquiring firm in case of Merger and acquisition of companies.

Chapter 7: Conclusions

It contains the major contributions made out of the present research work.

CHAPTER 2

REVIEW OF LITERATURE

2.1 Motivation of Random Matrix Theory

Over the year's physicists, economist and computer professionals have given their effort to understand the financial markets in a better way. Many theories of statistics, physics and computational sciences have been applied successfully to get relevant information from huge data base. Network theory (originated from graph theory) is one of them which have drawn attention to explain and understand financial markets. The main area of study is the correlation based network analysis of financial market. So when we construct a financial network based on the correlation of stocks then the network is referred as correlation network .The prime objective of the study of networks is to reduce complexity of financial dependencies and to understand the internal dynamics of financial market. Financial correlation based network models allow us to quantify and model the information present in a correlation matrix. It also helps for better visualization of stock interactions to make good decisions and forecast the dynamics in financial markets. In general the construction of reliable correlation matrix is difficult as it contains noise (random values) if the length of financial time series is not large compared with respect to the number of stocks taking part in the correlation analysis of stocks. Various methods have been employed to remove noise from the correlation matrix in order to get reliable correlation among the stocks. Random Matrix Theory (RMT) is one of the methods used by Econo-physicists to remove noise to get better information related to financial markets.

The study of matrix theory with random elements is known as random matrix theory has a rich historical background in the nuclear physics, probability theory and statistics. In order to understand the statistical properties of energy levels RMT was invented by Wigner, Dyson and Mehta etc. The above mentioned researchers postulated that the Hamiltonian describing a heavy nucleus can be described by a matrix H where all the elements of the matrix are independent and random [101]. Based on this postulate a large number of the remarkable predictions were made that agrees well with the experimental data. Similarly for complex quantum systems RMT is used for predictions to represent an average over all possible interactions. The deviations from RMT universal prediction identify system specific non random properties of the system under consideration which provides clues about the underlying interactions. The theory successfully explained experimental observations.

Nowadays, RMT is a well established theory and is applied in almost all scientific fields. For example in the statistics multivariate analysis is carried out by the application of RMT. Image denoising, wireless communications are few areas where RMT is used extensively. The successful application of RMT is possible only due to a computing facility available to store and process large dimensional data sets. Recent studies [128,89] applying RMT to study the properties of correlation matrix show that almost 98% of the eigenvalues of correlation matrix agree well with RMT predictions which suggest a considerable degree of randomness in the measured cross correlation matrix. It is also observed 2 % of the largest eigenvalues deviates significantly from RMT predictions. These results prompt the following questions: i) what is the possible interpretation for the deviation from RMT? ii) What can be inferred about the structure of correlation matrix from the above mentioned results? iii) What can be the practical implications of these results? The theory of random matrices is potentially of great interest to understand the statistical structure of the empirical correlation matrices appearing in the study of multivariate financial time series. A remarkable agreement between the theoretical prediction (based on the assumption that the correlation matrix is random) and empirical data concerning the density of eigenvalues associated with the time series of the different stocks of the S&P500 (or other major markets).

2.2 Networks in Finance

The complex nature of linkage among financial companies/institutions can be captured by use of network representation of financial systems. The network describes the collection of nodes and links between them. In general the nodes may represent individual stocks or firms or countries or even a collection of firms. The linkage between nodes represents the direct relationship between them. In general for social network point of view the linkage could be friendship tie, while in case of countries the linkage may be free trade agreement or a mutual defense pact. In the perspective of financial systems the nodes of the network represent financial institutions or financing companies and the linkage are created out of mutual exposures between banks. A network approach to the financial system is important for accessing financial stability and can be involved to confine the extremities that the risk associated with a single firm may create for the entire system. The financial companies are interdependent so using network technique one can prevent a local crisis from the global crisis. More generally network analysis helps in handling two types of questions: the effect of network structure and the process of network formation. The first question captures the features related to the social efficiency and the second question emphasizes on the tension between socially desirable outcomes and the outcomes that arise as a result of self interested action of

individuals. The first type of question, on network effects, studies processes that take place on fixed networks. For example one can apply network methods to study financial network structure on the way the banking system responds to contagion. Not only can we show the network structure responding differently to the propagation of stocks but the weakness of the system that depends upon the location in the network of the institution that was initially affected. The second type of question on network formation studies how stocks form connections. Network theory has been successfully applied to a wide range of situations. We will consider a few examples from economics to illustrate the scope of theories of networks: labor markets, buyer and seller networks, risk sharing networks and product adoption. For example studies show the involvement of networks in labor market examines the imperfections in the matching process between workers and their employers. The relevant information in a labor market is the available number of job vacancies and the number of capable workers. Networks can make establish relationship to transmit information across individual workers and among employers and workers. In fact, evidence shows on an average 50% of workers get their job through their personal contact and 40% to 50% employers use social networking sites to find employees (i.e. They hire recommended applicants to fill their vacancies) [137,106]. This motivated a thorough study of the effect of social networks on employment and wage inequality .Beside labor market researchers have investigated the role of networks in markets in general. The Arrow-Debreu model of the economy suggests agents interact anonymously in centralized markets and the prices are formed following their independent decisions. Corominas-Bosch [34] developed a model of bargaining between buyers and sellers who are connected by an externally given network. Transactions occur only between buyer and sellers that are connected by a link, but if someone has multiple links, then several possibilities transact become possible. Hence, the network structure enhances the bargaining power of various buyers and sellers. Recently, Gale and Kariv [54] experimentally investigated the effect of intermediation between buyers and sellers on networks. In this model traders are organized in an incomplete network.

2.2.1 Correlation Network

Recent years have witnessed the development of network theory. So the natural focus is on how network theory can be used to explain for better understanding financial market. The major area of the study is correlation based networks. The networks can be utilized to understand and forecast the dynamical nature of financial market and at the same time it can be used to trim down the complex nature of financial markets. We will make a short review and discuss recent developments and application of network theory in the field of finance. Some of

the papers develop methods used to study correlation based networks and discuss applications of the methods to various financial markets.

In 1999 Rosario Mantegna [99] in his paper hierarchical structure in financial markets tried to find a topological arrangement of USA stocks that reproduce economic classification. The paper introduced a technique how to extract a subgraph of the most significant links from a complex network to produce an asset tree. By constructing the minimum spanning tree Mantegna found that USA stocks are grouped according to their business sectors. The paper Asset tree and asset graphs in financial markets [117] by A. Chakrobati and J. P. Onnela et al in 2003 established an innovative network known as a dynamic asset graph. This is same as that of the asset tree introduced by Mantegna using a minimum spanning tree method but the difference between the approaches are in the function for mapping correlations into link lengths i.e. the distance function is non-linear. For an asset graph the distances are placed in ascending order as with the minimum spanning tree construction and each distance is added as link until there are $N-1$ links in total ,where N is the number of nodes. As they have taken asset graphs so the links may form loops and also unconnected nodes are allowed in this approach. Further studies by these authors are discussed in [116]. In 2004 G. Bonanno and G. Caldarelli et al published a paper Networks of equities in financial markets [18]. They have considered how returns of stocks traded in the market are affected by changing the time horizons used to compute the correlation coefficients. They observed as the time horizon decreases the graph structure progressively changes from a complex organization to a simple form where clusters are sparser. The paper also discusses volatility time series of stock prices and global financial markets. M. Tumminello, T. Ast, T.Di Matteo and R. N. Mantegna in 2005 [162] published a paper “A tool for filtering information in complex systems” , where they have extracted the subgraph of the most relevant links from a complex network by using the correlation coefficient matrix but instead of using Euclidean distance matrix formula they have used Gower’s metric distance . The paper outlined the method for constructing a minimum spanning tree and then directly constructs from this a planar maximally filtered graph. The basic constraint imposed to construct the MST is that each new link added into the tree must not form a cycle where as the new method “planar maximally filtered graph” is a little bit more complex graph as it allows loops and cliques of up to four elements. This can be drawn on a planar surface without link crossings. The paper develops minimum spanning trees and planar maximally filtered graph of USA stock correlations and also considers the economic properties of the four clique structures in planar maximally filtered graph. In the same year T. Mizuno, H. Takayasu and M. Takayasu [112] constructed minimum spanning trees of global currencies

using Kruskal's minimum spanning tree algorithm. The paper "correlation networks among currencies" finds that the currencies cluster geographically. They have taken forty four currencies for a time period of six years to construct a minimum spanning tree and a hierarchical tree i.e. a rooted tree graph showing the node labels on the base and measuring the correlations or distances on the side, which clusters the nodes based on correlation. In this paper topology of foreign exchange markets using heretical structure methods the distance matrices were calculated using Gower's metric distance function for two networks- one using the New Zealand dollar as the numeraire and other using USA dollar. The New Zealand dollar based minimum spanning tree has a star structure with the USA dollar node as the central hub whereas the USA dollar based minimum spanning tree has much sparser clustering. In 2008 again Rosario Mantegna, Michele Tumminello and Fabrizio Lillo in the paper [163] correlation, hierarchies and networks in financial market have taken correlation coefficient matrix and have given detail methods for constructing hierarchical structures. They have given emphasis on single linkage cluster analysis and average linkage cluster analysis. The two methods give slightly different structures as each one focuses on different aspects of the correlation matrix. The paper compares the structures of minimum spanning tree and planar maximally filtered graph. They have given details of the construction algorithm for both. Dror Y. Kenett, Yoash Shapiral in 2010 published a paper dynamics of stock market correlations. They have analyzed stationary correlations between stocks and introduced a new tool to study the structure and dynamics of the market. They have taken data from the New York stock exchange (NYSE) and Tel Aviv stock exchange. They have constructed correlation matrix and studied it using stock market holography, a method introduced by Shapira et al in 2009. The stock market holography method works by presenting stocks in a correlation based three dimensional space using data reduction techniques. In 2010 Dror Y. Kenett, Michele Tumminello, Rosario N. Mantegna [79] published a paper dominating clasp of the financial sector revealed by partial correlation analysis of the stock market. They used a concept of partial correlations: the correlation between two variables adjusted for their relationship with other variables. It is a tool that can be used to filter the most relevant links. The planner maximally filtered graph, discovered by the same author, has been adapted so that it can be constructed from partial correlations to form a partial correlation planar maximally filtered graph. The paper also gives a detail construction of the partial correlation threshold network- a type of dependency network where the inclusion of links depends on means and standard deviation of nodes. All the methods are demonstrated using data from NYSE.

2.2.2 Correlation Based Network Methods

Due to the successful applications of RMT and network theory it is penetrated in the field of financial research for data analysis and predictions. A large group of Physicists, Computer scientists in collaboration with Economists are actively involved to study the correlation based network analysis of financial stocks. The history of financial networks research is exciting and can be considered in different ways. In developed countries like USA the networks based on company ownership have been studied. This study shows how a small number of company owners control over a large number of companies. Another study demonstrated how two companies are connected or linked if a single person sits on the board of two companies. Many researchers have tried to figure out the hierarchical structure of the financial market by approaching the problem from different point of views. The financial market is one of these complex and dynamic systems which has so many technical hitches to model. In order to model financial systems, correlation is a fundamental variable to define the system and interactions of multivariate time series. A correlation network can also be defined as a weighted network. In network terminology the financial assets are nodes and the correlation between the assets are the weights of the links. The weighted network is constructed by studying different methods. The basic purpose of using different methods during formation of networks is to observe the relationship of clusters and find the most informative illustration of the network. Correlation network was first introduced by Mantegna in 1999. He wanted to demonstrate the whole market and the interaction of the assets. The correlation network is constructed by using cross-correlations of the changing stock prices on the same interval. There are several financial networks which are constructed with the help of correlation. Some of the recent methods which are used to illustrate the financial networks are the minimum spanning tree (MST), maximal spanning tree, the asset graph with threshold and planar maximally filtered graph]. The minimum spanning tree can be obtained using two most famous algorithms: i) Kruskal's minimum spanning tree algorithm ii) Prim's minimum spanning tree algorithm. In our thesis Kruskal's minimum spanning tree algorithm is used to construct MST to visualize the correlation network. Mantegna has introduced "distance" a new parameter to construct the MST, which represents high correlation between the stocks with a short distance in the network. A financial network branches into two main parts called tree network and graph. The prime difference between them is that the graphs are allowed to include cycles while the trees are the acyclic graphs.

2.3 Basic Concepts of Quantum Computing

The mathematical basis of quantum mechanics discovered new type of logic, probability and dynamics called quantum logic, quantum probability and quantum dynamics respectively and scientists tried to generalize the mathematical theory of quantum mechanics and opened it up to all branches of science, engineering, business and management sciences. Quantum computation (QC) [109, 114] uses quantum mechanical effects to perform computational tasks and in some cases its results are exponentially faster than their classical counterparts. The purpose of this section is to give a brief outline of the postulates of quantum mechanics and quantum logic gates in order to understand quantum algorithms, quantum neural networks and quantum decision making. It provides the general introduction of the basic ideas of quantum logic gates. The main concepts of quantum mechanics are wave function, superposition, and coherence and decoherence, operators, measurement, entanglement, and unitary transformations. Quantum mechanics provides the basis to understand quantum computation and information processing. The theory of quantum mechanics is based upon some fundamental postulates which are discussed in the section. This section reviews necessary principles, terminologies and standard notation which we will use throughout the thesis.

Let us first discuss the fundamental necessity of quantum computing. As we all know the development of classical computers is in enormous progress to provide better computing facilities. Then what is the necessity to think of a new paradigm of computing. There are few good reasons to think beyond the conventional computing.

a) Efficient Simulation of Other Quantum Systems

The physical world is fundamentally quantum mechanical. All the computers are physical devices. In early 1980 Feynman tried to simulate quantum systems using classical computers. He failed and came to the conclusion that classical computers have certain limitation to simulate quantum features properly. The fundamental limitation of classical computers and models of classical computation are based on classical physics. There is nothing wrong with these models but they are not enough to explore fully the potential of the physical world of information processing. Alternatively Feynman thought of a quantum computer which can simulate all the quantum features. Hence quantum computers could be used as a link between theoretical models formulated using fundamental concepts and experimental observations.

b) Statistical Reduction

In 1965 Intel cofounder Gordon Moore predicted that the number of transistors per square inch on integrated circuits has doubled approximately every eighteen months. If this trend of minimization of computing devices continues, it will approach the microscopic level, where the laws of classical physics fail. We thus have to cope up with the quantum effects and we can either try to overcome from these effects or try to make use of them and start doing quantum computing .If we adapt one electron will be enough to store one bit. Hence technological progress is required to explore the potentials of quantum computing.

c) Reduced Complexity

The computational problems which can be solved in polynomial time are called tractable or easy problems and the problems which require super polynomial time are called intractable or hard problems. Hence there is a sharp borderline between easy and hard problems based upon the time complexity for solving the problem. Examples of some common hard problems are travelling salesman problem and problem of factoring a number into primes, which is commonly used in public key encryption. In 1994 Peter Shore showed that the problem of factoring a number into primes can be solved in polynomial time using a quantum computer .Hence the problem which was previously a hard problem for a classical computer is no more a hard problem for the quantum computer. So the exponential speedup gained by quantum algorithms may reduce the complexity of other computational hard problems.

2.3.1 Elements of Quantum Computation

Quantum mechanics is the most accurate and complete science for the description of the world. Now we will discuss some mathematical definitions and standard notations needed for the discussion of quantum computing [138, 62, 83].

a) Superposition

Superposition means a system remains in two or more of its basis states simultaneously. Linear superposition is the physical representation of mathematical principle of linear combination of vectors. The wave function Ψ , which describes the quantum system, exists in a complex Hilbert space .The Hilbert space has a set of states Φ_i that form a basis .Hence the quantum system can be described mathematically as follows: $|\Psi\rangle = \sum_i \ C_i \quad |\Phi_i\rangle$ Where, $|\Psi\rangle$ is the superposition of basis states $|\Phi_i\rangle$ and C_i is the complex coefficients.

b) Coherence and Decoherence

A quantum system is said to be coherent if it is a linear superposition of its basis states. If a quantum system is in a linear superposition of states and interacts with its environment, the superposition is destroyed. This loss of coherence is called decoherence.

c) Entanglement

It is a quantum mechanical phenomenon to establish correlation between two or more quantum states. Entanglement seems to be intuitive from a computational point of view but it is little understood from a physical stand point of view. The correlation between states comes into picture when states exist as superposition .When superposition is destroyed the proper correlation is somehow communicated between the qubits and this communication is achieved due to entanglement. The power of quantum computation derives from the exponential state spaces of multiple quantum bits: just as a single qubit can be in a superposition of 0 and 1, a register of n qubits can be in a superposition of all 2^n possible values. The "extra" states that have no classical analog and lead to the exponential size of the quantum state space are the entangled states. Mathematically entanglement can be described using density matrix formalism.

d) Tensor Product

If a system is composed of two subsystems A and B respectively, them Hilbert space of the entire system can be represented by the tensor product of the individual subsystems, denoted by $A \otimes B$. If $|\Psi_1\rangle$ is a ket in A and $|\Psi_2\rangle$ in B then the tensor product is a ket in $A \otimes B$.This is denoted as $|\Psi_1\rangle|\Psi_2\rangle$, $|\Psi_1\rangle \otimes |\Psi_2\rangle$,$|\Psi_1\Psi_2\rangle$, $|\Psi_1, \Psi_2\rangle$. Suppose A is $m \times n$ a matrix and B is $p \times q$ .Then tensor product or Kronecker product, is defined as $mp \times nq$ a matrix.

Let $|\Phi_1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|\Phi_2\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Then we can have the various tensor products as follows:

$$|\Phi_1\rangle \otimes |\Phi_1\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, |\Phi_1\rangle \otimes |\Phi_2\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$|\Phi_2\rangle \otimes |\Phi_1\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, |\Phi_2\rangle \otimes |\Phi_2\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

In general

$$\left(\alpha\begin{bmatrix}a\\b\end{bmatrix}\right)\otimes\begin{bmatrix}c\\d\end{bmatrix}=\begin{bmatrix}\alpha ac\\ \alpha ad\\ \alpha bc\\ \alpha bd\end{bmatrix}=\begin{bmatrix}a\\b\end{bmatrix}\otimes\left(\alpha\begin{bmatrix}c\\d\end{bmatrix}\right)$$

$$=\left(\alpha\begin{bmatrix}a\\b\end{bmatrix}\otimes\begin{bmatrix}c\\d\end{bmatrix}\right)$$

Example:

Let $|\Phi\rangle=\begin{bmatrix}2\\6i\end{bmatrix}$ and $|\Psi\rangle=\begin{bmatrix}3\\4\end{bmatrix}$

Then the inner product can be written as

$$\langle\Phi|\Psi\rangle=[2\quad 6i]\begin{bmatrix}3\\4\end{bmatrix}=6+24\text{i}$$

The outer product can be written as

$$|\Phi\rangle\langle\Psi|=\begin{bmatrix}2\\6i\end{bmatrix}[3\quad 4]=\begin{bmatrix}6 & 8\\ 18i & 24i\end{bmatrix}$$

The tensor product can be written as

$$|\Phi\rangle\otimes|\Psi\rangle=\begin{bmatrix}2\\6i\end{bmatrix}\otimes\begin{bmatrix}3\\4\end{bmatrix}=\begin{bmatrix}6\\8\\18i\\24i\end{bmatrix}$$

e) Quantum Bit (Qubit)

The bit is the smallest unit of information in classical computation, information and communication .Similarly a quantum bit or qubit is the smallest unit for quantum computing, quantum information processing and quantum communication. Just as the classical bit has two possible states 0 and 1, the qubit has possible states $|0\rangle$ and $|1\rangle$ .These two states are also known as computational basis states. The main difference between classical bit and quantum bit is that, a qubit can stay in the superposition of basis states. The superposition is nothing but the linear combination of two states $|0\rangle$ and $|1\rangle$ as$|\Psi\rangle=\alpha|0\rangle+\beta|1\rangle$. Where α and β are two complex numbers. In general a qubit is a vector in the two dimensional complex vector spaces. When we try to measure a qubit, we get either the result 0 with probability $|\alpha|^2$ or 1 with probability$|\beta|^2$.As a sum of probabilities is equal to 1, $|\alpha|^2+|\beta|^2=1$.Hence we can interpret this as the condition that the qubit's state can be normalized to length 1.

Example:

$$|\Psi\rangle=\frac{1}{\sqrt{2}}|0\rangle+\frac{1}{\sqrt{2}}|1\rangle$$

Here $\alpha=\beta=\frac{1}{\sqrt{2}}$. When we measure, it gives a result of 0 for $\left|\frac{1}{\sqrt{2}}\right|^2=\frac{1}{2}=50\%$ of the time the chance of getting 0, similarly for 1 we can find 50% chance of getting 1. Hence we come to

the conclusion that a qubit can exist in a coherent superposition of states $|0\rangle$ and $|1\rangle$ before it is measured. Upon measurement, it only gives one of the states, 0 or 1.

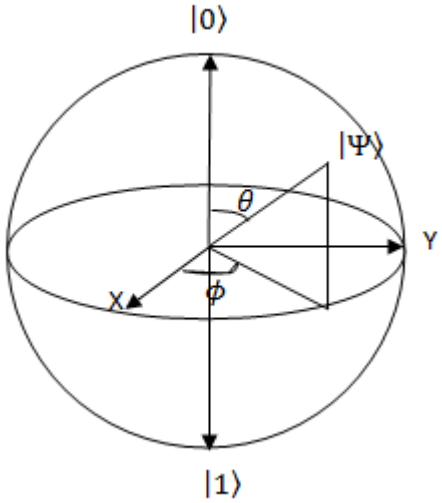


Figure 2.1: Bloch sphere depiction of a quantum bit

A complex number $Z = X + iY$, can be written in polar form as:

$$X = r\cos\theta$$

$$Y = r\sin\theta$$

$$Z = r\cos\theta + r\sin\theta = re^{i\theta}$$

Let $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$ and $|\alpha|^2 + |\beta|^2 = 1$

Let $\alpha = e^{i\gamma}\cos\frac{\theta}{2}$ and $\beta = e^{i(\gamma+\phi)}\sin\frac{\theta}{2}$ then

$$|\Psi\rangle = e^{i\gamma}\cos\frac{\theta}{2}|0\rangle + e^{i(\gamma+\phi)}\sin\frac{\theta}{2}|1\rangle$$

$$\Rightarrow |\Psi\rangle = e^{i\gamma}\left(\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle\right)$$

Under measurement $e^{i\gamma}|\Psi\rangle \cong |\Psi\rangle$, the global phase is not observable. Hence we can write the qubit $|\Psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$ .So any single qubit state is a point on the surface of Bloch sphere and any unitary transformation is a rotation in some arbitrary angle. Since there are an infinite number of points on the unit sphere, can we store infinite number of information in a single qubit? The answer is no because of a single measurement we can get a single bit of information from the state of a qubit.

f) Multiple Qubit

In classical two bits we have four possible states 00, 11, 01, 10.Correspondingly, for two qubits we have four computational basis states$|00\rangle, |10\rangle \ |01\rangle \ |11\rangle$ .So the pair of qubits can also exist in superposition of these four states ,so that

$$|\Psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

Similarly the probability has to sum up to 1

$$|\alpha_{00}|^2 + |\alpha_{01}|^2 \ + |\alpha_{10}|^2 + |\alpha_{11}|^2 \ = 1$$

Let us suppose we measure the first qubit of the two-pair quantum system, the chances of getting 0 is $|\alpha_{00}|^2 + |\alpha_{01}|^2$. So after measurement we have to remove the coefficient α_{10} and α_{11} and the state of the qubit becomes:

$$|\Psi'\rangle = \frac{\alpha_{00}|00\rangle + \alpha_{01}|01\rangle}{\sqrt{|\alpha_{00}|^2 \ + |\alpha_{01}|^2}}$$

g) Bell State

An important example of two qubit pair is the bell state :

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

If we measure the first qubit, then the probability of getting 0 is 50%.The resulting post measurement state will be $|00\rangle$, so if we will measure the second qubit it will give 0.On the other hand if we measure the first qubit as 1 with probability 50%, then the resulting post measurement state will be $|11\rangle$.So measuring the second qubit will now give 1. Hence, measurement of the second qubit always gives the same result as the measurement of first qubit. So in other words the measurable outcomes are correlated.

h) Postulates of Quantum Mechanics

Two types of operations undergo in a quantum system: measurement and quantum state transformations. In the classical computing set of universal gates are used for computational purposes, whereas in QC most algorithms follow a sequence of quantum state transformations followed by measurement. Actual quantum computation processes are very different from that of classical counterpart. In classical computer we give input data through the input devices .The input signal is stored in computer memory ,then fed into the microprocessor and the result is stored in memory before it is displayed in the screen .Thus the information travels around the circuit .In contrast , information in quantum computation is first stored in a register and then external fields , such as oscillating magnetic fields , electric fields or laser beams are

applied to produce gate operations on the register. These external fields are designed so that they produce desired gate operation, i.e. unitary matrix acting on a particular set of qubits .Hence information sits in the register and they are updated each time the gate operation acts on the register. In order to understand the manipulation of quantum registers we must know how quantum systems evolve. For better understanding we will discuss the basic postulates of quantum mechanics.

Postulate-1

This postulate states the association of mathematical concept with any isolated physical system called Hilbert space or state space. The instantaneous configuration of the system is represented by a state vector, which is a member of state space. The state vectors must be unit vectors in state space.

Example: - A qubit (simplest quantum mechanical system) has two dimensional state space. Suppose $|0\rangle$ and $|1\rangle$ form an orthonormal basis for that state space .Then an arbitrary state vector in the state space can be written as:

$$|\Psi\rangle = \text{a}\,|0\rangle + \text{b}\,|1\rangle$$

Where a and b are complex numbers and $|a|2 + |b|^2 = 1$ (due to the condition that $|\Psi\rangle$ must be unit vector).

Postulate-2

The evolution of a closed quantum system is described by a unitary transformation**.** That is, the state $|\Psi\rangle$ of the system at time t_1 is related to the state $|\Psi'\rangle$ at time t_2 by a unitary operator U, which only depends upon times t_1 and t_2 .That is:

$$|\Psi'\rangle = U(t_1, t_2)|\Psi\rangle$$

The unitary operator is a matrix with the property $UU^* = I$, where the U^*denotes the conjugate transpose of the matrix U and I is the identity matrix. As we know in classical computation *logic gates* transform input state to the final state. This postulate is important in quantum computing point of view because the input state can be transformed to output state by unitary operators. Therefore in quantum computing the analogues of classical logic gates are unitary operators.

Example: - One interesting unitary operator is the Hadamard operator, denoted by H, where:

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

If we take $|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$ and $|1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$ then

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}\ and\ H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

The beauty of Hadamard operator is that, it transforms the pure basis states to equal superposition states. This operator is very useful in many quantum algorithms.

Postulate-3

When we observe the closed system to find out what is going on inside the system, an interaction which makes the system no longer closed. Postulate-3 provides a means for describing the effects of measurements (i.e. Its state is observed) on quantum systems. If the quantum system is measured, then it immediately undergoes a discontinuous and unpredictable jump into one of its basis states. That is, superposition is destroyed.

Example: - A single qubit is represented by the state vector $|\Psi\rangle = a|0\rangle + |b1\rangle$, the measurement of the state gives either the result $|0\rangle$, with probability $|a|^2$ or the result $|1\rangle$, with probability $|b|^2$.

Postulate-4

The state space of a composite physical system is the tensor product of the state spaces of the component physical systems. The tensor product is a way of representing vector spaces together. The tensor product between two state vectors $|\Psi_1\rangle$ and $|\Psi_2\rangle$ can be written as $|\Psi_1\rangle \otimes |\Psi_2\rangle$ or $|\Psi_1\Psi_2\rangle$ or $|\Psi_1\rangle|\Psi_2\rangle$ or $|\Psi_1, \Psi_2\rangle$. We can generalize tensor product to tensor exponentiation as follows:

$$|\Psi\rangle^{\otimes 3} = |\Psi\rangle \otimes |\Psi\rangle \otimes |\Psi\rangle$$

Example:

Suppose

$$|\Psi_1\rangle = a|0\rangle + b|1\rangle \; and \; |\Psi_2\rangle = c|0\rangle + d|1\rangle$$

Then

$$|\Psi_1\rangle \otimes |\Psi_2\rangle = a.c|0\rangle|0\rangle + a.d|0\rangle|1\rangle + b.c|1\rangle|0\rangle + b.d|1\rangle|1\rangle$$

$$= ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle$$

2.3.2 Logic Gates in Quantum Computing

In quantum mechanics the state of a physical system is represented by its wave function which contains all information to describe the system completely. A quantum bit or qubit is the smallest unit of information used for quantum information processing. Just as the classical bit has two possible states 0 and 1, the qubit has two possible states ket $|0\rangle$ and ket $|1\rangle$ .These two states are also known as computational basis states. In general a qubit is a vector in the two dimensional complex vector spaces. The main difference between a classical bit and quantum bit is that, a qubit can stay in the superposition of basis states. The superposition is the linear combination of the two basis states $|0\rangle$ and $|1\rangle$ to form the state $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where α and β are two complex scalar numbers called amplitudes. The amplitudes can be thought of as

the quantum probabilities. The classical probability amplitudes are represented by real numbers, whereas the quantum probability amplitudes are represented by complex numbers. The probabilities of a classical system must sum to 1 to form a complete probability distribution. Similarly the squares of the absolute values of the amplitudes of states in a quantum system must add up to 1. The logic that can be implemented with qubits is quite distinct from Boolean logic, and that is why quantum computing is exciting by opening new possibilities.

Mathematically quantum logic gates are represented by transformation matrices or linear operators. The quantum logic gate in the form of the transformation matrix interacts with the quantum register through tensor operations. All linear operators that correspond to the quantum logic gates must be unitary i.e. the inverse of that matrix must be equal to its complex conjugate transpose. Unitary operators preserve the inner product of the two vectors. Geometrically the lengths of vectors and the angle between them are preserved i.e. $\langle u|\mathrm{U}^{\dagger}\mathrm{U}\,|v\rangle = \langle u|I|v\rangle = \langle u|v\rangle$. The unitary transformation on a single qubit may be visualized as rotation and reflection about the x, y and z axes of the Bloch sphere. If U and V are two unitary operators then the composition is also unitary:

$$(UV)^{\dagger} = V^{\dagger}U^{\dagger} = V^{-1}U^{-1} = (UV)^{-1}$$

Classical gates like NAND and XOR are irreversible but the quantum gates, represented in the form of unitary matrices, are always reversible. The matrix representation of single as well as multiple qubit gates is a convenient mathematical way of expressing their input output relationship. If the i^{th} truth-table combinations of a gate are denoted as $|input_i\rangle$ and $|output_i\rangle$, then the matrix representation of the gate can be computed as $\sum_i |input_i\rangle\langle output_i|$.

Single Qubit Gates

As the NOT gate is a single bit gate in classical computers, in the similar way there exists single qubit quantum gate in quantum computers. Hence single qubit gates are those that act on only a single quantum bit.

Quantum NOT-Gate

The quantum NOT gate for qubits can be defined as a process that takes $|0\rangle$ and produces $|1\rangle$ and vice-versa. However in case of superposition, the NOT gate acts linearly in the state $\alpha|0\rangle + \beta|1\rangle$ to $\alpha|1\rangle + \beta|0\rangle$

The matrix representation of quantum NOT gate is $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

The matrix representation of $\alpha|0\rangle + \beta|1\rangle$ is $\begin{bmatrix} \alpha \\ \beta \end{bmatrix}$

$$\left(\because \quad \alpha|0\rangle + \beta|1\rangle \;=\; \alpha \begin{bmatrix}1\\0\end{bmatrix} + \beta \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}\alpha\\0\end{bmatrix} + \begin{bmatrix}0\\\beta\end{bmatrix} = \begin{bmatrix}\alpha\\\beta\end{bmatrix}\right)$$

If we apply quantum NOT gate to the state $\alpha|0\rangle + \beta|1\rangle$ then

$$X\begin{bmatrix}\alpha\\\beta\end{bmatrix} = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}\alpha\\\beta\end{bmatrix} = \begin{bmatrix}\beta\\\alpha\end{bmatrix} = \alpha|1\rangle + \beta|0\rangle$$

The matrix representation of NOT gate $X = \sum_i |input_i\rangle\langle output_i|$

$$= |0\rangle\langle 1| + |1\rangle\langle 0| = \begin{bmatrix}1\\0\end{bmatrix}[0 \quad 1] + \begin{bmatrix}0\\1\end{bmatrix}[1 \quad 0] = \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} + \begin{bmatrix}0 & 0\\1 & 0\end{bmatrix} = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$$

Unitary Constraint on Quantum Gate

A single quantum bit gate can be represented by 2×2 matrices. The normalization condition requirement for any α and β in the form $\alpha|0\rangle + \beta|1\rangle$ is$|\alpha|^2 + |\beta|^2 = 1$, due to the requirement of the conservation of probability to be 1. Hence in general it can be said that if a single quantum bit gate is defined by a unitary matrix U, then $adj(U).U = I$.

Verification of Unitary Constraint of Quantum NOT Gate:

Let $X = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$

Then $adj(X) = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ and $adj(X).X = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}.\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} = I$

Hence we can conclude from the above discussion that there exists a single non-trivial single qubit bit gate i.e. NOT gate. But in case of single qubit NOT gates, several possible cases are possible as long as the unitary constraint is satisfied. Some of these are Hadamard gates and Z gates.

Z-Gate

The Matrix Representation of Z-Gate is defined as:

$$Z = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$$

It inverts sign of $|1\rangle$ to give $-|1\rangle$ and leaves $|0\rangle$ unaltered.

For $|0\rangle$ the output is $|0\rangle$:

$$Z|0\rangle = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} = |0\rangle$$

For $|1\rangle$ the output is $-|1\rangle$:

$$Z|1\rangle = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\-1\end{bmatrix} = -|1\rangle$$

For the state $\alpha|0\rangle + \beta|1\rangle : Z(\alpha|0\rangle + \beta|1\rangle) = \alpha Z|0\rangle + \beta Z|1\rangle = \alpha|0\rangle - \beta|1\rangle$

The matrix representation of Z- gate $Z = \sum_i |input_i\rangle\langle output_i|$

$$= |0\rangle\langle 0| - |1\rangle\langle 1| = \begin{bmatrix}1\\0\end{bmatrix}[1 \quad 0] - \begin{bmatrix}0\\1\end{bmatrix}[0 \quad 1] = \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} - \begin{bmatrix}0 & 0\\1 & 0\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$$

Verification of Unitary Constraint for Z-Gate:

We have to show: $adj(Z).Z = I$

$$\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = I$$

Hadamard gate

The Matrix Representation of Hadamard gate is defined as:

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$$

For $|0\rangle$ the output is $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$:

$$H|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} = \frac{1}{\sqrt{2}}\left[\begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}\right] = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

Similarly, for $|1\rangle$ the output is $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$:

$$H|1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix} = \frac{1}{\sqrt{2}}\left[\begin{bmatrix}1\\0\end{bmatrix} - \begin{bmatrix}0\\1\end{bmatrix}\right] = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Hence, the Hadamard gate is also known as "square- root" of a NOT gate as it transforms $|0\rangle$ into $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|1\rangle$ into $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$, which may be considered as the midway between $|0\rangle$ and $|1\rangle$.

For the state $\alpha|0\rangle + \beta|1\rangle$:

$$H(\alpha|0\rangle + \beta|1\rangle)\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}\alpha\\\beta\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}\alpha+\beta\\\alpha-\beta\end{bmatrix}$$

Or $H(\alpha|0\rangle + \beta|1\rangle) = \alpha H|0\rangle + \beta H|1\rangle = \alpha\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right) + \beta\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) = \frac{\alpha+\beta}{2}|0\rangle + \frac{\alpha-\beta}{2}|1\rangle$

If we apply $\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)$ to H:

$$\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right).\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\frac{1}{2}\begin{bmatrix}2\\0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} = |0\rangle$$

Similarly, if we apply $\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$to H: we will get$|1\rangle$.

Verification of Unitary Constraint for Hadamard-Gate:

We have to show: $adj(H).H = I$

$$\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = I$$

Phase Gate (S-Gate)

The S-Gate is defined as: $S = \begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$

For $|0\rangle$ the output is

$$\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = \begin{bmatrix}1\\ 0\end{bmatrix} = |0\rangle$$

For $|1\rangle$ the output is

$$\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix} = \begin{bmatrix}0\\ i\end{bmatrix} = i|1\rangle$$

For $\alpha|0\rangle + \beta|1\rangle$ the output is

$$\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \begin{bmatrix}\alpha\\ i\beta\end{bmatrix} = \alpha|0\rangle + i\beta|1\rangle$$

It can be verified that S gate satisfies unitary constraint.

T-Gate

The T-Gate is defined as follows:

$$T = \begin{bmatrix}1 & 0\\ 0 & exp\left(\frac{i\pi}{4}\right)\end{bmatrix}$$

This is also known as $\frac{\pi}{8}$ gate since it can also be expressed as:

$$T = exp\left(\frac{i\pi}{4}\right)\begin{bmatrix}exp\left(-\frac{i\pi}{8}\right) & 0\\ 0 & exp\left(\frac{i\pi}{8}\right)\end{bmatrix}$$

For $|0\rangle$ the output is

$$\begin{bmatrix}1 & 0\\ 0 & exp\left(\frac{i\pi}{4}\right)\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = \begin{bmatrix}1\\ 0\end{bmatrix} = |0\rangle$$

For $|1\rangle$ the output is

$$\begin{bmatrix}1 & 0\\ 0 & exp\left(\frac{i\pi}{4}\right)\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix} = \begin{bmatrix}0\\ exp\left(\frac{i\pi}{4}\right)\end{bmatrix} = exp\left(\frac{i\pi}{4}\right)|1\rangle$$

For $\alpha|0\rangle + \beta|1\rangle$ the output is

$$\begin{bmatrix} 1 & 0 \\ 0 & exp\left(\frac{i\pi}{4}\right) \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta exp\left(\frac{i\pi}{4}\right) \end{bmatrix} = \alpha|0\rangle + \beta exp\left(\frac{i\pi}{4}\right)|1\rangle$$

Relation between S and T Gates:

The algebraic relation between S and T is as follows:

$S = T^2$ as

$$T^2 = \begin{bmatrix} 1 & 0 \\ 0 & exp\left(\frac{i\pi}{4}\right) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & exp\left(\frac{i\pi}{4}\right) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} = S$$

2.4 Quantum Artificial Neural Networks

This literature review aims to narrate the summary of research into quantum artificial neural networks (QANN). An outline of foundational material in quantum computation in particular quantum information theory and neural networks is given. The incentives behind QANN are discussed and different models of QANN and their properties are examined indepth with covering a series of physical interpretation. Richard P. Feynman, in an important speech to a conference of physicists, proposed about quantum computer in the year 1982. He anticipated that in order to simulate quantum mechanical systems efficiently, computers that used quantum phenomena to perform calculations might be needed. After some years, the field of quantum computation emerged through the work of David Deutsch, Peter Shor and many others. It indicates that not only such computers were physically constructible, but in many cases was exponentially faster than their classical counterparts. Today this field of research is one of the fastest growing among others. Exactly in the same year 1982, John Hopfield published his pioneer research on associative memory network. This was a breakthrough of decades of research worldwide in the field of neural networks. He demonstrated layers of interconnected processing units whose behavior was modeled on the basis of biological neurons of the human brain, and which surpass over traditional computers in many tasks such as pattern recognition. Such parallel processors were used in a different applications ranging from modeling financial stocks to basic artificial intelligence problems, and through periods of varying expectations are entering more practical domains. During the last fifteen years there has been interest in the models which combine the quantum computation and neural computation in the form of quantum artificial neural network (QANN): Artificial neural networks (ANN) which use quantum formalism in their operations. The expectation behind this model is that such a QANN may exploit the advantages of quantum computer such as inherent

quantum parallelism and allow a physical implementation which is easier to construct than a quantum computer.

Many researchers use their own analogy in establishing a connection between quantum mechanics and neural networks by M. Perus [125], D. Ventura and T. Martinez [169], T.Menneer [111], E.Behrman [13], S. Kak [75]. These models focus on different aspects of quantum computations and neural processing. The main concepts of quantum mechanics are: wave function, superposition, measurement, entanglement, unitary transformation, with the similarity of neural networks are: neuron, interconnections, evolution to attractor, learning rule, gain function. Indeed the establishment of the correspondence between the fields is a major challenge in the development of models of quantum neural networks. In order to establish a correlation between two models of computation one must understand physical interpretations of quantum mechanics provided by different school of thought. The choice of interpretation is vital for establishing different analogies between quantum and neurocomputing. M.Perus has outlined the similarities existing between quantum and neural formalisms. ANN demands nonlinearity but quantum computing is a linear theory then the question comes how to introduce nonlinearity for QNN is first explained by E. Behrman [13] and clarified the role of nonlinearity using Feynman path integral approach and proposed possible physical implementation of this model. D. Ventura showed how quantum superposition can be used to develop associative memory with exponential capacity. R. Chrisley described how quantum neural networks with continuous weights able to process gradual signals can be implemented. It is suggested that quantum neural network can have many advantages such as exponential acceleration of computations, smaller number of hidden units and, as the consequence, better generalization.

2.5 Quantum Decision Making

Information processing in Quantum decision theory (QDT) is based on mathematical theory of separable Hilbert spaces is a novel variant of decision making. This mathematical model interprets the superposition effect of composite prospects of many integrated intended actions. The theory is illustrated intension interfering, entangled decision making and non commutatively of subsequent decisions. This quantum approach to decision making overcomes the paradoxes of classical decision theory. The conditional maximization of entropy makes it possible to connect classical and quantum decision theory and shows former is the limit of later under vanishing interference terms.

Now we will introduce the historical perspective, the basic ideas and related studies of QDT. First we will focus the objective of the approach. The basic objective of QDT is to extend the process of decision making by using quantum mechanical tools and techniques. Decision theory is concerned how to arrive at the best possible decisions. The prime hypotheses of decision theory are that people are fully informed and rational. These hypotheses have moved up many inquires on with the evidence provided by the many behavioral sciences like the Allais paradox [3]. It is revealed that the humans frequently seem to deviate from rational decision theory due to cognitive and emotional biases. The hypothesis of bounded rationality of behavioral finance and behavioral economics [155] has tried to make clear these deviations. In literature different models have been developed to explanation for the deviations observed in human agents [135, 56,145]. However, many paradoxes hang about unsolved on an ad hoc basis.

A new approach to decision theory can be projected using mathematical theory of Hilbert spaces which is chiefly used in quantum theory as a mathematical means [53]. In recent times a broad framework of QDT is introduced to give an explanation of the many paradoxes of traditional decision theory in a unifying and predictive approach [176]. This formalism permits to explain different anomalies (ex-the conjunction and disjunction effects. When human being does not follow the principle of sure thing of conventional probability theory is called disjunction effect and the logical fallacy that takes place when people assume specific conditions are more probable than a single general one is called the conjunction effect) in a quantitative way. QDT provides a relationship between disjunction and conjunction effect.

QDT employs the conventional mathematical formalism of quantum mechanics based on the theory of separable Hilbert spaces. Quantum mechanics have proven wave particle duality i.e. waves originated from discrete energy packets, called quanta, which behave like to that of particles. Likewise in QDT framework quantum is a qualifier which gives importance on the fact that a decision is a discrete choice from a large set of entangled options. The basic idea of QDT is to generalize the classical probability theory a basis for decision making. The decision making is a complex dynamical process of the human brain with many hidden non local variables such as emotions, unknown states of nature and subconscious processes. The mathematical theory of complex Hilbert space provides a direct way to avoid dealing with unknown hidden variables and reflects the complexity of nature .Let us summarize previous studies related to decision making associated with the cognitive process concerned with functional aspects of human brain i.e. focus is made towards the mathematical modeling.

One class of approach of decision making is based upon the theory of neural networks and dynamical systems [68, 69, 4]. This approach suffers from modeling upper mental faculties from a microscopic neuron based picture. In literature researchers invoke quantum as qualifier to represent brain as a quantum like object [95, 142]. Several mechanisms have been suggested in the literature [158, 12]. However the existence of a genuine quantum effect in the brain is controversial and has been criticized by many researchers [159].

The second class of theories does not assume the quantum nature of the brain. Rather this approach uses quantum techniques as a tool to explain classical probability theory. The quantum technique has successfully explained non quantum phenomenon such as quantum game theory [46] and quantum algorithms [153]. Hence there is no contradiction of applying quantum tools and techniques to describe classical problems. Thus quantum technique is the most efficient tool to explain classically complicated problems. In case of decision making the underlying hidden variables (example- emotion) performed by a human being is difficult quantify can be possible to capture using quantum theory. The QDT is a convenient mathematical model which can characterize the complicated process of mind for decision making. This approach comes into the picture in a natural way due to its several features such as: its probabilistic nature, existence of entangled decisions, the interference of several decisions and the non commutativity of decisions. The classical decision theory is based on utility theory [171]. Decision making in the presence of uncertainty is discussed in various literature using statistical decision theory [92]. Quantum decision theory is different from that of classical decision theory which is primarily based upon utility theory.

2.6 Summary of Literature Review

In this chapter we have discussed the application of basic principles and recent development of quantum principles in non quantum domains like computing, information processing and decision science are discussed. The RMT can be used as a tool to remove noise from the signal of the financial time series. This method has been applied successfully in the field of image processing, biology and financial time series analysis.

CHAPTER-3

CORRELATION ANALYSIS OF INDIAN COMPANIES

3.1 Introduction

Financial time series is closely linked to the evolution of a large number of interacting systems. The price change of stocks is affected by the information arriving in the market (for example: the news break, political instability, economic crisis, outbreak of war etc.) and increase uncertainty in the stock market leads to the high fluctuation in prices. The correlation analysis of price movements for different kind of stocks is a cutting edge research for those who are actively involved in the financial markets [55, 32, 36, 74]. Due to the complex nature of financial market, correlation analysis has been employed to gather information on combined modes underlying the dynamic nature of stock prices [98, 65]. It is expected that the correlation of price movements will come into picture when the stocks interact strongly among themselves. The assumption behind such interactions among the stocks, occur due to the identical or related business sectors i.e. those stocks face similar challenges for the market condition and struggle for the same set of consumers. For example the automobile and transportation sectors will be affected equally due to the rise in oil price. In addition, the response of all stocks shows similar behavior when an event affects the whole market and this induces the market wide correlations. From another point of view, the event which is only concerned with individual stock will have a tendency to decorrelate the price movement from others. It has been found that during financial crisis the correlation increases and stocks become closer i.e. the linkage between stocks becomes tightened. It is believed that during crisis high transaction stocks fluctuate more to create uncertainty in the market. Hence the effects that govern the correlation behavior of stock prices can be classified as follows: (a) Market Mode: this is universal to all the stocks (b) Sector Mode: associated with particular business group (c) Individual: related to an individual stock.

The physicists have investigated financial market structure to analyze the spectral characteristics of the correlation matrix. A number of researchers have successfully applied random matrix technique in different areas such as quantum field theory, quantum chaos, biology, image processing and disordered systems [136, 17]. Recently random matrix theory (RMT) has been applied to investigate the correlation structure of financial market [90,127]. It is observed that a large volume of experimental eigenvalue distribution obey the theoretical prediction of RMT and few eigenvalues deviate significantly from RMT. The basic objective

of this chapter is to investigate the interrelationship of several Indian companies, in particular small cap, mid cap and large cap companies indexed in Bombay Stock Exchange separately. Spectral properties of the data correlation has been examined. Random matrix theory has been applied to remove noise from the structured correlated stocks. The eigenvalues below and above noise threshold have been investigated. Finally, the correlation analysis is conducted to analyze the Indian financial market crash.

3.2 Over View of Data

There are twenty three stock markets in India. The Bombay Stock Exchange (BSE)[67] is the oldest stock market compared to all other stocks in India. It is thus an excellent source of data for studying correlation structure of Indian companies. We have considered daily closing price data of the stocks traded on the BSE. The data is downloaded from Yahoo finance [68]. For correlation analysis we have focused on daily closing price data on BSE large cap, mid cap and small cap stocks. From April 5, 2007 to March 30, 2012, which corresponds to $T = 1293$ the data points of fifty five large cap companies, April 3, 2007 to March 30, 2012, which corresponds to $T = 1295$ the data points of forty six mid cap companies and May 18, 2007 to March 30, 2012, which corresponds to $T = 1260$ the data points of eighty seven small cap companies are taken according to their business sectors. Refer Appendix-I for the name of companies.

3.3 Proposed Methodology

In this section we will discuss the methodology applied to analyze the data. The procedure adopted to analyze the data is as follows: we have taken data of large-cap, mid-cap and small-cap companies indexed in BSE with different sectors separately. The data are then grouped according to sector wise. Interpolation method is used for missing data. The noise from the correlation matrix is removed using RMT. The data are analyzed using MATLAB software. The basic methodology applied to analyze the time series data are given in the following subsections.

3.3.1 Eigenvalue Decomposition

Eigenvalue decomposition is a factorization technique for complex matrix, used for the transformation of the eigenvalues and associated eigenvectors in a number of applications in statistics. It is engaged to decompose the correlation matrix into eigenvalues and eigenvectors. The fundamental idea is that most of the information is surrounded in the greater eigenvectors. As a result using a determined boundary condition we can remove eigenvectors that can be explained as noise.The eigenvalue equation is given below as:

$$A\vec{x} = \lambda\,\vec{x}$$

Where λ is a scalar. A is a square matrix and $\vec{x}$ is the eigenvector matrix corresponding to the eigenvalues. The eigenvalues and eigenvectors of the above equation are known as characteristic roots and characteristic vectors respectively. We can find characteristic roots and characteristic vectors by treating matrix A as a system of linear equations and solving for the values of the variables that make up the components of the eigenvector. Eigenvalue decomposition is used by many authors to measure the correlations of stock price change of assets [119, 89]. Stanley and Rosenow [128] applied the eigenvalue decomposition method in RMT to demonstrate the validity of the universal predictions of RMT. In the next section RMT has been discussed.

Example:

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Applying the formula in the above equation,

$$A\vec{x} = \lambda\,\vec{x} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

After simplification and rearranging,

$$(2-\lambda)x_1 + x_2 = 0$$
$$x_1 + (2-\lambda)\,x_2 = 0$$

The necessary and sufficient condition to have non zero vectors $[x_1, x_2]$, the determinant of the coefficient matrix must be equal to zero.

Hence

$$\begin{vmatrix} (2-\lambda) & 1 \\ 1 & (2-\lambda) \end{vmatrix} = 0$$
$$(2-\lambda)(2-\lambda) - 1.1 = 0$$
$$\lambda^2 - 4\lambda + 3 = 0$$
$$(\lambda - 3)(\lambda - 1) = 0$$

The last equation has two solutions for λ .So there exist two eigenvalues of the matrix A:

$$\lambda_1 = 3, \lambda_2 = 1$$

The eigenvectors of the eigenvalues 3 and 1 can be calculated as follows:

For $\lambda = 3$

$$(2-\lambda)x_1 + x_2 = 0$$
$$(2-3)x_1 + x_2 = 0$$
$$\Rightarrow\ x_1 = x_2$$

We can find an infinite number of values for x_1 that can satisfy the above equation. The only restriction imposed is that not all components in an eigenvector can equal to zero.

Therefore, if $x_1 = 1$, then $x_2 = 1$ and eigenvector corresponding to $\lambda = 3$ is [1,1].

For $\lambda = 1$

$$(2 - \lambda)x_1 + x_2 = 0$$

$$\Rightarrow\ x_1 = -\,x_2$$

So eigenvector for $\lambda = 1$ is [1, -1].

3.3.2 Noise Cleaning Using RMT

In order to measure the correlation among different stocks, the price fluctuation is to be measured first such that the outcome of the result does not depend upon the scale of measurement. Therefore, logarithmic return of the stock price indices is calculated. Defining daily closing price return of the stock $i\ (= 1, \ldots..,\ N)$

$$R_i\ (\mathrm{t})\ \equiv\ log_e\ S_i(\mathrm{t}+1)\ -log_e\ S_i(\mathrm{t}) \qquad (3.1)$$

Where $S_i(\mathrm{t})$ is the stock price at the time $t\ (= 1, \ldots..,\ T)$. Then normalize the return as

$$m_i(t) = \frac{R_i(t) - \langle R_i \rangle}{\sigma_i} \qquad (3.2)$$

Where $\sigma_i \equiv \sqrt{\langle {R_i}^2 \rangle - \langle R_i \rangle^2}$, the standard deviation of R_i and $\langle \ldots.. \rangle$, corresponds to time average over the period T.

The correlation matrix is constructed from the time series of price change $m_i(t)$ (where i labels the stock and t the time) through the following equation.

$$C_{ij}\ = \frac{1}{T}\sum_{t=1}^{T} m_i(t)\ m_j(t)$$

The above equation symbolically can be written as

$$C \equiv \tfrac{1}{T} M M^t \qquad (3.3\ (a))$$

Where M is a $N \times T$ rectangular matrix with elements $M_{it} \equiv m_i(t)$ and M^t, its transpose. So the component of the cross-correlation matrix C is given as

$$C_{ij}\ = \langle m_i(t)\ m_j(t) \rangle \qquad (3.3\ (b))$$

Where C_{ij} , represents the correlation between returns for stocks i and j. The construction of reliable correlation matrix is difficult. For N number of stocks the correlation matrix contains $N(N-1)/2$ entires. This can be determined from N number of time series with time period T days. The covariance will contain noise if T is not very large compared to the number of

stocks (N). Hence the empirical correlation matrix will contain random values. In that case the correlation matrix structure will be dominated by noise and therefore one must be careful while using it for applications. In particular the smallest eigenvalues of the correlation matrix are very sensitive to the noise. We have applied RMT to remove noise from trend.

According to RMT, in the limit $T \to \infty \; and \; N \to \infty$ such that $\mathrm{Q} = \frac{\mathrm{T}}{\mathrm{N}} \geq 1$ is fixed. The probability density $\mathrm{P}(\lambda)$, for the eigenvalue λ of the corresponding random matrices is given [45,146,101,21].

$$\mathrm{P}(\lambda) = \frac{\mathrm{Q}}{2\pi} \frac{\sqrt{(\lambda - \lambda_{\min})\ (\lambda_{\max} - \lambda)}}{\lambda} \tag{3.4}$$

Where $\mathrm{Q} = \frac{\mathrm{T}}{\mathrm{N}} \geq 1$ with $T \; and \; N$ tend to ∞. The range of the distribution is represented by

$$\lambda_{\substack{\max \\ \min}} = [\,1 \pm \left(1/\sqrt{\mathrm{Q}}\right)]^2 \tag{3.5}$$

In case of $N \times T$ random matrix M the accurate distribution of eigenvalues λ falls in the interval $[\lambda_{max}\,, \lambda_{min}\,]$ and $Q = T/N$. But in our case the matrix $N \times T$ is finite. Hence there is a small probability of finding λ above the noise threshold. The eigenvector distribution of a random matrix is given by Peter-Thomas distribution as follows:

$$P(u) = \frac{1}{2\pi} exp\left(-\frac{u^2}{2}\right) \tag{3.6}$$

We will apply above mentioned equations to the time series data to study the correlation behavior.

3.4 Analysis of Eigenvalues above the Noise Threshold

In this section the correlation of small cap, mid cap and large cap Indian companies belonging to BSE has been investigated separately. For this purpose we have taken daily closing price indices of large, mid and small cap companies belonging to BSE. In each cap we have taken different companies belonging to different business sectors. The stocks are arranged according to their business sector. After obtaining the correlation matrix it is decomposed into eigenvalues and eigenvectors by eigenvalue decomposition method. In order to isolate noisy components from structured correlated behavior, RMT has been employed. This is possible since noisy fluctuations are known to be present within a specified eigenvalue range of the correlation matrix. It also has a well defined distribution making it easy to identify.

Small Cap Companies

We have considered eighty seven small cap companies in our study. The number of data points: $T = 1260$, the number of companies $N = 87$ and $Q = T/N = 14.4$, this implies the distribution of eigenvalues to be bounded within the noise threshold $\lambda_{max} = 1.59$ $(using\ Equation\ (3.5))$, in the absence of any correlations. In fact a bulk of eigenvalues of experimental data falls below the noise threshold as shown in figure (3.1).

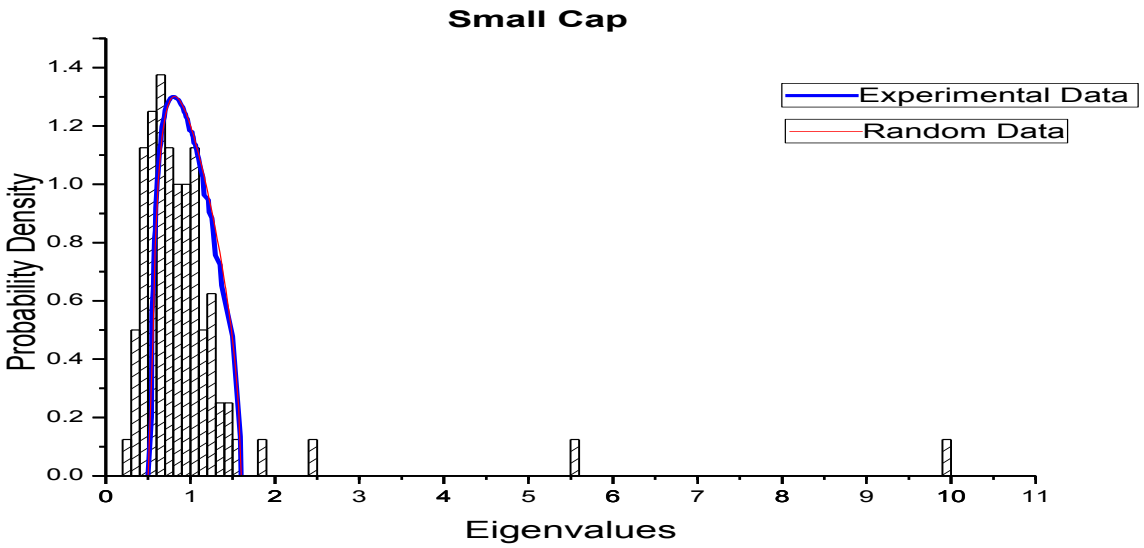


Figure 3.1: The Probability Density Function of eigenvalues of the correlation matrix C for eighty seven small cap companies in the BSE of India for the period May 2007-March-2012.

In figure (3.1) the probability density vs. eigenvalue plot of the correlation matrix C for eighty seven small cap stocks for the period 18-05-2007 to 30-03-2012 is shown. The theoretical distribution predicted by RMT is shown in the red line. The experimental distribution of the correlation matrix is shown in blue line. The theoretical curve overlaps with the experimental curve.

A small portion of eigenvalues departs from random matrix behavior. The analysis of this deviation above the noise threshold λ_{max}, will provide better understanding of the internal structure of financial market behavior of small cap companies. The largest (highest), second largest and third largest eigenvalues of the experimental data are $\lambda_0 = 9.93,\ \lambda_1 = 5.52,\ \lambda_2 = 2.47$ respectively, as shown in the table (3.1(a)), deviates significantly from random matrix theory indicating their non-random nature. On the other hand the largest, second largest and third largest eigenvalues of the random data are $\lambda_0^R = 1.55$,$\lambda_1^R = 1.48$ and $\lambda_2^R = 1.42$ respectively, as shown in the table (3.1 (b)).

Table 3.1: Represents the three largest eigenvalues (a) experimental and (b) random data of eighty seven BSE Small Cap Companies

(a)

Eigenvalues	Experimental Data
λ_0	9.93
λ_1	5.52
λ_2	2.47

(b)

Eigenvalues	Random Data
λ_0^R	1.55
λ_1^R	1.48
λ_2^R	1.4

Table 3.1 has been prepared to show how much the largest eigen value of random matrix and experimental matrix are deviating from the noise threshold predicted by random matrix theory in case of BSE small cap companies.

Many factors that are affecting the correlated price movements of stocks are discussed in the section 3.1. The common assumption behind the correlated price movement of stocks is due to the strong interaction among the companies. This interaction may arise due to the similar or interdependent nature of companies. Hence, the effects that govern the correlated price movement of stocks can be classified into (a) Market Mode: this is universal to all the stocks (b) Sector Mode: associated with particular business group (c) Individual: related to an individual stock. In figure (3.1) the bulk of eigenvalue distribution of experimental data matches fare well with the random matrix. But few large eigenvalues deviate significantly from random matrix predictions. The highest eigenvalue represents the market mode, i.e. the influence of entire market common to all the stocks, while the remaining eigenvalues above the bulk of eigenvalues and below the highest eigenvalue represent different business sectors [58], as indicated by the composition of the corresponding eigenvectors [127].

Hence, we will examine the behavior of stocks considering three consecutive largest eigenvalues deviating significantly as shown in figure (3.1). For this we have considered three consecutive largest eigenvalues and plotted the absolute values of eigenvector components on the y-axis and the number of stocks on the x-axis as shown in the figure (3.2).

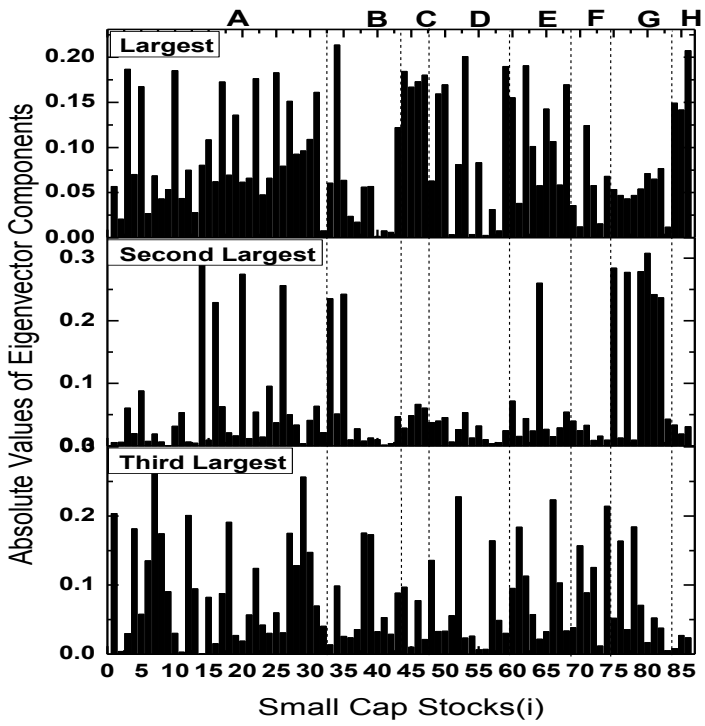


Figure 3.2: Experimentally obtained absolute values of Eigenvector components of three largest eigenvalues of the correlation matrix C of eighty seven BSE Small Cap Companies

In figure (3.2) the absolute values of the eigenvector components i.e. $u_i(\lambda)$ corresponding to three consecutive highest eigenvalues of the correlation matrix C of small cap companies. The companies i are arranged according to their business sectors from left to right are as follows: - A: Industrial, B: Textile, C: Automobile, D: Food, E: Construction, F: Finance, G: Pharmacy, H: Chemical and I: Energy.

Each eigen value has a corresponding eigen vector that can be represented by its components. The i^{th}component of a given eigenvector is related to the contribution of i^{th}company. In figure (3.2), the absolute values of the eigen vector components, for a specific eigenvalue, are taken in y-axis to quantify the contribution of each small cap companies. Hence, the values of components contain information about the companies contributing to a specific eigenvector. Further, in order to measure the components in a same scale of measurement we have chosen the absolute value of each component. In the figure (3.2) we have merged three plots of eigenvector components vs. number of stocks for three different eigenvalues i.e. largest, second largest and third largest, in order to compare the contributions of the group of stocks. In the x-axis we have taken number of stocks/ companies and in the y-axis we have taken the contribution of each company which is the absolute values of

eigenvector components. In the plot the dotted lines separate the group of stocks and those are indicated in the upper part of the plot with capital letters

Mid Cap Companies

We have considered forty six mid cap companies in our study. The number of data points: $T = 1295$, the number of companies: 46 and $Q = 28.15$, this implies the distribution of eigenvalues to be bounded with the limit, $\lambda_{max} = 1.41$ and $\lambda_{min} = 0.65$, in the absence of any correlations. In fact a bulk of eigenvalues of experimental data falls below the noise threshold as shown in figure (3.3).

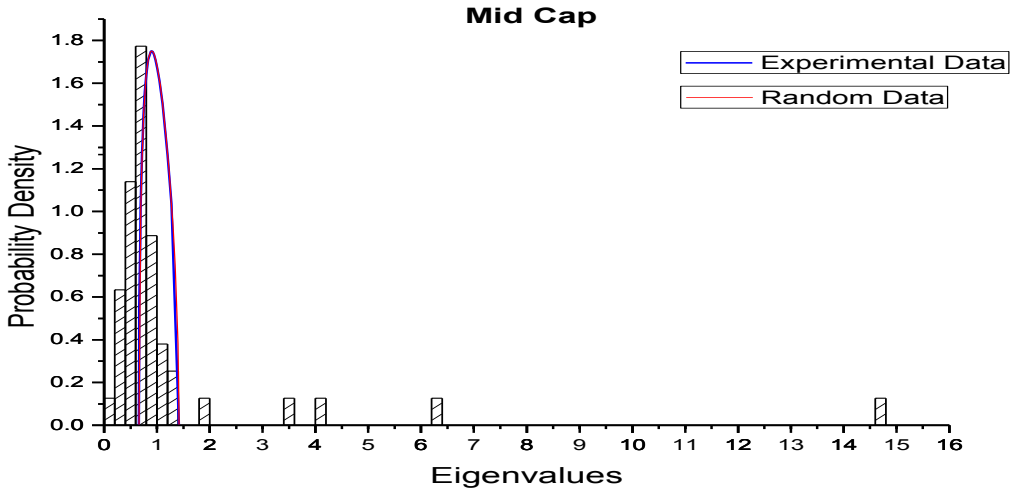


Figure 3.3: The Probability Density Function of eigenvalues of the correlation matrix C for forty six mid cap companies in the BSE of India for the period April 2007-March-2012

In figure (3.3) the probability density vs. eigenvalue plot of the correlation matrix C for forty six mid cap stocks for the period 3-04-2007 to 30-03-2012 is shown. The theoretical distribution predicted by RMT is shown in the red line. The experimental distribution of the correlation matrix is shown in blue line. The theoretical curve overlaps with the experimental curve.

A small portion of eigenvalues departs from random matrix behavior. The analysis of this deviation above the threshold λ_{max} , will provide better understanding of the internal structure of financial market behavior of mid cap companies. The largest, second largest and third largest eigenvalues of the experimental data are found to be $\lambda_0 = 14.77$, $\lambda_1 = 6.2$, $\lambda_2 = 4.1$ respectively as shown in the table (3.2(a)).On the other hand the largest, second largest and third largest eigenvalues of the random data are found to be $\lambda_0^R = 1.44$,$\lambda_1^R = 1.40$ and $\lambda_2^R = 1.39$ respectively, as shown in the table (3.2 (b)).

Table 3.2: Represents the three largest eigenvalues (a) experimental and (b) random data of forty six BSE Mid Cap Companies

(a)

Eigenvalues	Experimental Data
λ_0	14.7
λ_1	6.2
λ_2	4.1

(b)

Eigenvalues	Random Data
λ_0^R	1.44
λ_1^R	1.40
λ_2^R	1.39

Let us examine the behavior of stocks considering three largest eigenvalues deviating significantly as shown in figure (3.3). For this we have taken three consecutive largest eigenvalues and plotted the absolute values of eigenvector components on the y-axis and the number of stocks on the x-axis as shown in the figure (3.4).

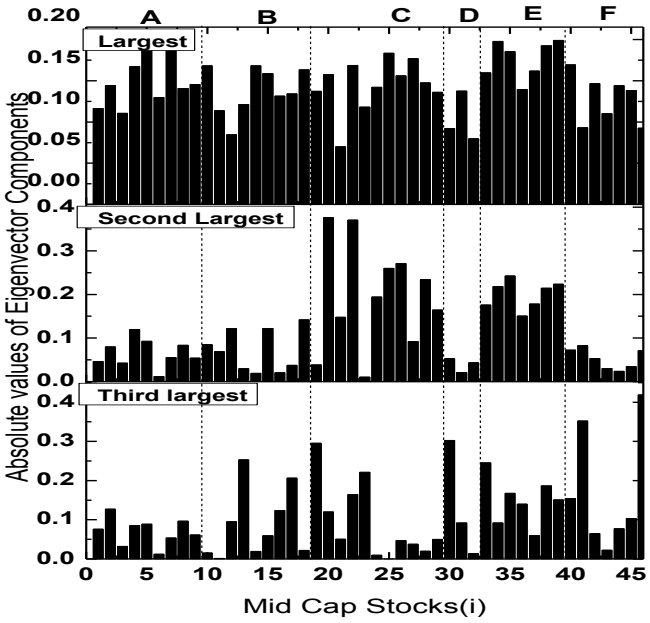


Figure 3.4: Experimentally obtained absolute values of eigenvector components of three largest eigenvalues of the correlation matrix C of forty six BSE Mid Cap Companies

In figure (3.4) the absolute values of the eigenvector components i.e. $u_i(\lambda)$ corresponding to three highest eigenvalues of the correlation matrix C of mid cap companies. The companies i are arranged according to their business sectors from left to right are as follows: - A: Textile, B: Industrial, C: Chemical, D: Pharmacy, E: Bank, F: Computer.

Large Cap Companies

We have considered fifty five BSE large cap companies in our study. The number of data points: $T = 1295$, the number of companies: 55 and $Q = 23.54$, which means that the eigenvalue distribution should be restricted within noise threshold $\lambda_{max} = 1.45$ and $\lambda_{min} = 0.63$ in the absence of any correlations. In fact a bulk of eigenvalues of experimental data falls below the noise threshold as shown in figure (3.5).

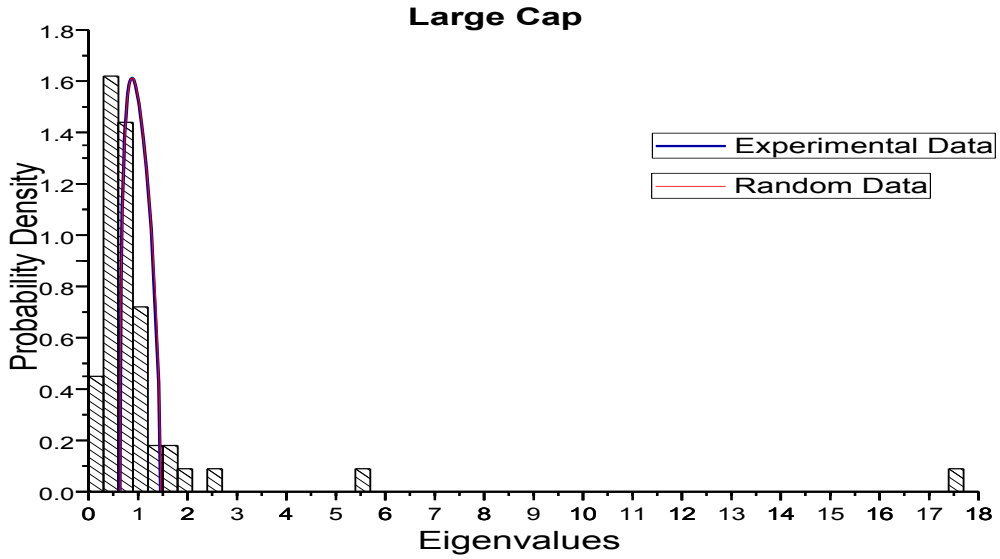


Figure 3.5: The Probability Density Function of eigenvalues of the correlation matrix C for fifty five large cap companies in the BSE of India for the period April 2007-March-2012

The probability density function of BSE large cap companies is shown in figure (3.5).The eigenvalues of the correlation matrix C for fifty five large cap stocks for the period 3-04-2007 to 30-03-2012 is taken. The theoretical distribution predicted by RMT is shown in the red line. The experimental distribution of the correlation matrix is shown in blue line. The theoretical curve overlaps with the experimental curve.

A small portion of eigenvalues departs from random matrix behavior. The analysis of this deviation above the threshold λ_{max} , will provide better understanding of the internal organization of the market behavior of large cap companies. The largest, second largest and third largest eigenvalues of the experimental data are found to be $\lambda_0 = 17.5$, $\lambda_1 = 5.5$, $\lambda_2 = 2.5$ respectively as shown in the table (3.3) , which deviates significantly from random matrix theory indicating their non-random nature. On the other hand the largest, second largest and third largest eigenvalues of the random data are found to be $\lambda_0^R = 1.53$,$\lambda_1^R = 1.44$ and $\lambda_2^R = 1.40$ respectively.

Table 3.3: Represents the three largest eigenvalues (a) experimental and (b) random data of fifty five BSE Large Cap Companies.

(a)

Eigenvalues	Experimental Data
λ_0	17.5
λ_1	5.5
λ_2	2.5

(b)

Eigenvalues	Random Data
λ_0^R	1.53
λ_1^R	1.44
λ_2^R	1.40

Let us examine the behavior of stocks considering three consecutive largest eigenvalues of fifty five BSE large cap companies deviating significantly as shown in figure (3.5). For this we have considered three consecutive largest eigenvalues and plotted the absolute values of eigenvector components on the y-axis and the number of stocks on the x-axis as shown in the figure (3.6).

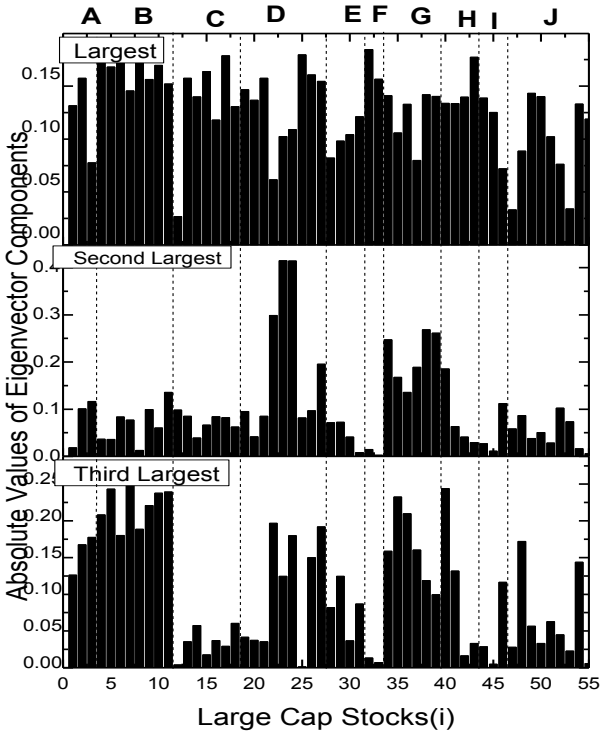


Figure 3.6: Experimentally obtained absolute values of eigenvector components of three largest eigenvalues of the correlation matrix C of fifty five BSE Large Cap Companies

In figure (3.6) the absolute values of the eigenvector components i.e. $u_i(\lambda)$ corresponding to three highest eigenvalues of the correlation matrix C of large cap companies. The companies i are arranged according to their business sectors from left to right are as follows: - A: Automobile, B: Bank, C: Metal, D: Energy, E: Pharma, F: Power, G: Software, H: Telecom, I: Trading, J: Miscellaneous.

3.4.1 Results and Discussions-1

1. The largest eigenvalues for BSE small cap, mid cap and large cap companies are almost ten times larger than that of random matrix prediction as shown in table (3.1), (3.2) and (3.3) .
2. The highest eigenvalue is independent of the number of companies participating to calculate the eigenvalue. For eighty seven small cap companies the highest eigenvalue is 9.93, for forty six mid cap companies the highest eigenvalue is 14.7 and for fifty five

large cap companies the highest eigenvalue is 17.5 as shown in table (3.1), (3.2) and (3.3) respectively. From the experimental observation some indication of the linear dependency of the highest eigenvalue upon the capitalization of a company is obtained.

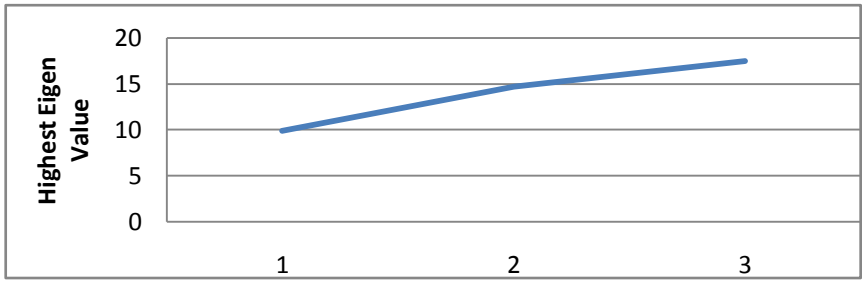


Figure 3.7: Highest eigenvalue vs. numbers of caps

In figure (3.7) the x-axis represents the number of caps and the y-axis represents the highest eigenvalues of small cap, mid cap and large cap companies.

3. Observing top part of the figures (3.2) , (3.4) and (3.6) we can clearly visualize that there exists a uniform composition of eigenvector components of leading (largest) or dominant eigenvalue. This indicates equal contribution of all stocks to the leading eigenvalue. Hence the largest eigenvalue is common for all stocks. So it is associated with market mode. So the higher eigenvalues above the noise threshold reveal the so called market behavior and other correlated dynamics among subgroups.
4. Considering the eigenvalues which lie between the bulk of eigenvalues expected by RMT and the largest eigenvalue. From figures (3.2), (3.4) and (3.6) we can clearly visualize the localized effect of eigenvector components corresponding to second and third largest eigenvalue. The behavior of the same stocks for different eigenvectors varies unequally. For example in figure (3.2) the chemical stocks have significant contribution to the eigenvector that corresponds to the second highest eigenvalue. From figure (3.2) it is apparent that the chemical stocks are not contributing significantly for largest and third largest eigenvalues. From figure (3.4) it is apparent that the chemical stocks are not contributing significantly for third highest eigenvalues.
5. The localized effects show that a small number of stocks have significant contribution to the second and third eigenvalue mode. It can be observed for a particular eigenvector the dominant contribution of stocks belong to same or related business sectors. But there is an exception for second largest eigenvalue where the stocks having large market capitalization contribute significantly (here large market capitalization means the stocks having a good financial condition within the specified list of small cap, mid cap and large cap companies). From figure (3.2), (3.4) and (3.6) the list of companies

for dominant localized eigenvector components for second largest eigenvalue are shown in table (3.2), (3.4) and (3.6).

Table 3.4: Small Cap stocks contributing to the second largest eigenvalue

Sector	Name of Company
Industry	Alok Industries Ltd, Binani Industries Ltd, Finolex Industries Ltd, Relience Industrial Infrastructure
Textile	Nahar Spinning Mills Ltd, Syaram Silk Mills Ltd
Finance	Gruh Finance
Chemical	Aditya Birla Chemicals, Bodal Chemicals Ltd, Empee Sugars & Chemicals Ltd, Gujrat Alkalies & Chemicals Ltd, India Glycols Ltd

Table 3.5: Mid Cap stocks contributing to the second largest eigenvalue

Sector	Name of Company
Chemical	National Fertilizers, Rastriya Chemicals
Bank	Dena Bank , Syndicate Bank

Table 3.6: Large Cap stocks contributing to the second largest eigenvalue

Sector	Name of Company
Energy	Oil India Ltd., Indian Oil Corporation Ltd.
Software	Tata Consultancy Services Ltd., Wipro Ltd.

3.5 Inverse Participation Ratio (IPR)

The considerable contribution of a group of stocks that deviate significantly from RMT cannot be obtained by direct examination of the eigenvector compositions. The reason behind for this assumption is due to the large eigenmode which corresponds to market may be dominating the intra-group correlations. Hence we need a more comprehensive investigation of the eigenvector compositions; for this we employ IPR, that can be defined for k-th eigenvector as,

$$I_k = \sum_{i=1}^{N} [u_{ki}]^4 \qquad (3.7)$$

Where, u_{ki} is the component of k-the eigenvector. The understanding of IPR can be demonstrated by two limiting conditions: (a) a vector with identical components $u_{ki} \equiv 1/\sqrt{N}$, has $I_k = 1/N$, whereas (b) a vector with one component $u_{k1} \equiv 1$ and $u_{ki} = 0$ for $i \neq 1$, we have $I_k = 1$. Hence, IPR measures the reciprocal number of elements that make a significant contribution to each eigenvector. The IPR is the expectation value of a matrix whose elements are random. The random matrix is generated from N time series that are

mutually uncorrelated by using the command rand ( ) in Matlab. Form the eigenvectors corresponding to the eigenvalues of the random correlation matrix, the value of the IPR would be $\langle I \rangle \cong 3/N$. The dotted lines in figures (3.7), (3.8) and (3.9) shows the eigenvalues belong to the bulk that are predicted by RMT. The extreme ends of the eigenvalues deviate significantly from the value$\langle I \rangle \cong 3/N$, which suggests the presence of localized eigenvectors.

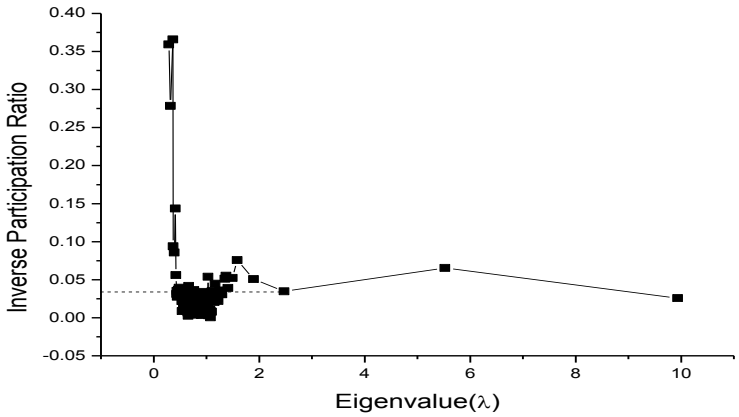


Figure 3.8: The IPR for different eigenvalues of BSE small cap stocks. The broken line shows IPR $= 3/N$ (where$N = 87$)

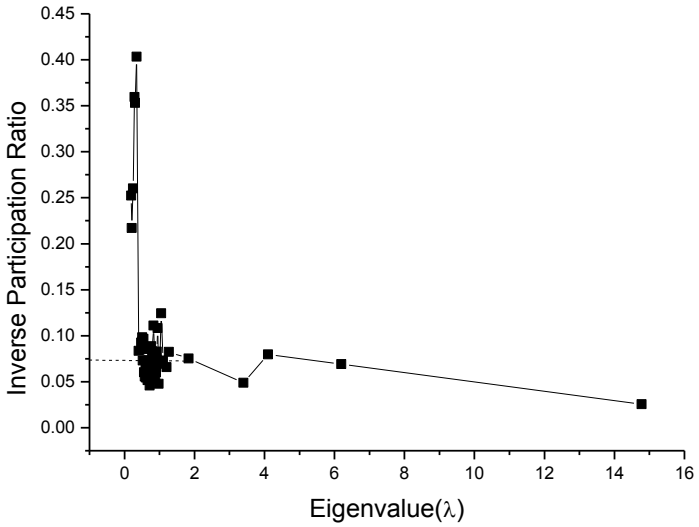


Figure 3.9: The IPR for different eigenvalues of BSE mid cap stocks. The broken line shows IPR $= 3/N$ (where$N = 46$)

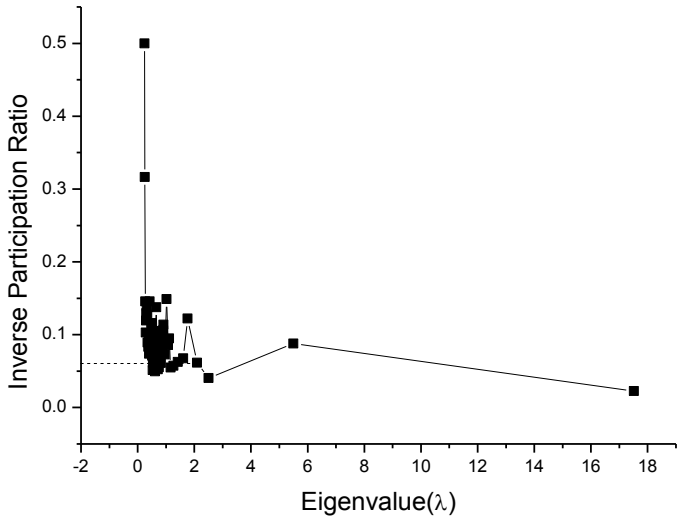


Figure 3.10: The IPR for different eigenvalues of BSE large cap stocks. The broken line shows IPR $= 3/N$ (Where $N = 55$)

Using IPR we found that there exists bulk of noisy eigenvalues in the correlation matrix and localized eigenvalues at the higher and lower end for small cap, mid cap and large cap stocks.

3.6 Analysis of Eigenvalues below the Noise Threshold

In this section we will examine the deviations for smallest eigenvalues lying below the noise threshold predicted by RMT. From figures (3.8), (3.9) and (3.10) it is clear that the market deviates significantly at the extreme ends of eigenvalues. This suggests the existence of localized eigenvectors i.e. only a small number of stocks have significant contribution.

Small Cap Companies

Let us examine the behavior of small cap stocks considering three consecutive smallest eigenvalues deviating significantly as shown in figure (3.1).In table (3.7) the smallest eigenvalues of large cap companies below the noise threshold is shown. The lowest, second lowest and third lowest eigenvalues are denoted by λ_{55} , λ_{54} and λ_{53} , with values 0.24, 0.25 and 0.26 respectively.

Table 3.7: Represents the three smallest eigenvalues of the experimental data of eighty seven BSE Small Cap Companies

Eigenvalues	Experimental Data
λ_{87}	0.28
λ_{86}	0.31
λ_{85}	0.36

In order to examine the behavior of small cap stocks we have considered three consecutive smallest eigenvalues and plotted the absolute values of eigenvector components on the y-axis and the number of stocks on the x-axis as shown in the figure (3.10).

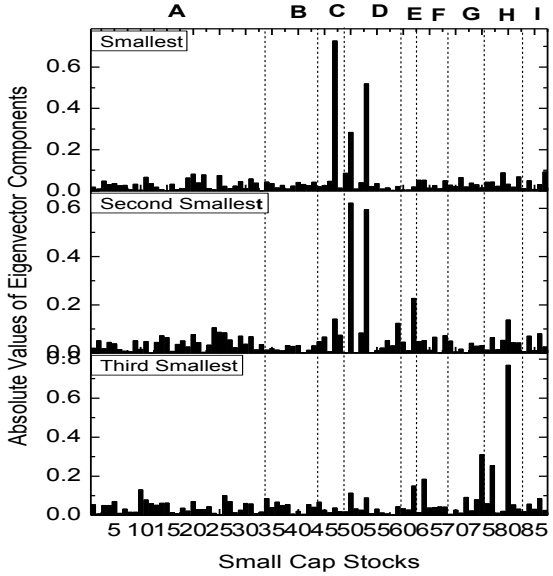


Figure 3.11: Experimentally obtained absolute values of eigenvector components of three smallest eigenvalues of the correlation matrix C of eighty seven BSE Small Cap Companies

Figure (3.11) shows the absolute values of eigenvector components i.e. $u_i(\lambda)$ corresponding to three smallest eigenvalues of the correlation matrix C. The companies i are arranged according to their business sectors from left to right are as follows: - A: Industry, B: Textile, C: Automobile, D: Food, E: Construction, F: Finance, G: Pharmacy, H: Chemical and I: Energy.

Mid Cap Companies

Let us examine the behavior of three consecutive smallest eigenvalues of mid cap stocks that are deviating significantly as shown in figure (3.3).In table (3.8) the smallest eigenvalues of mid cap companies below the threshold is shown. The lowest second lowest and third lowest eigenvalues are denoted by $\lambda_{46}, \lambda_{45}$ and λ_{44} , with values 0.19, 0.21 and 0.24respectively.

Table 3.8: Represents the three smallest eigenvalues of the experimental data of forty six BSE Mid Cap Companies

Eigenvalues	Experimental Data
λ_{46}	0.19
λ_{45}	0.21
λ_{44}	0.24

In order to examine the behavior of mid cap stocks we have considered three consecutive smallest eigenvalues and plotted the absolute values of eigenvector components on the y-axis and the number of stocks on the x-axis as shown in the figure (3.11).

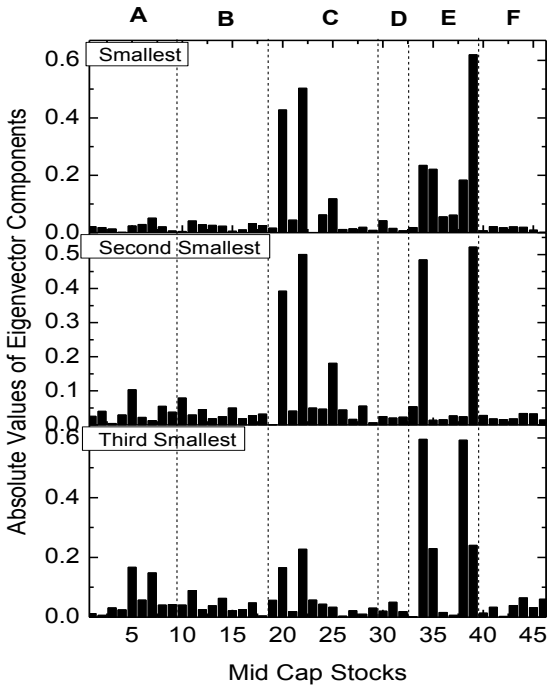


Figure 3.12: Experimentally obtained absolute values of eigenvector components of three smallest eigenvalues of the correlation matrix C of forty six BSE mid cap companies

Figure (3.12) shows the absolute values of eigenvector components i.e. $u_i(\lambda)$ corresponding to three smallest eigenvalues of the correlation matrix C of mid cap companies. The arrangement of the stocks i is made according to their business sectors from left to right - A: Textile, B: Industries, C: Chemical, D: Pharmacy, E: Bank, F: Computer

Large Cap Companies

Let us examine the behavior of three consecutive smallest eigenvalues of large cap stocks which are deviating significantly as shown in figure (3.5). In table (3.9) the smallest eigenvalues of large cap companies below the noise threshold is shown. The lowest, second lowest and third lowest eigenvalues are denoted by λ_{55} , λ_{54} and λ_{53} , with values 0.24, 0.25 and 0.26 respectively.

Table 3.9: Represents the three smallest eigenvalues of the experimental data of fifty five BSE Large Cap Companies

Eigenvalues	Experimental Data
λ_{55}	0.24
λ_{54}	0.25
λ_{53}	0.26

In order to examine the behavior of large cap stocks we have considered three consecutive smallest eigenvalues and plotted the absolute values of eigenvector components on the y-axis and the number of stocks on the x-axis as shown in the figure (3.12).

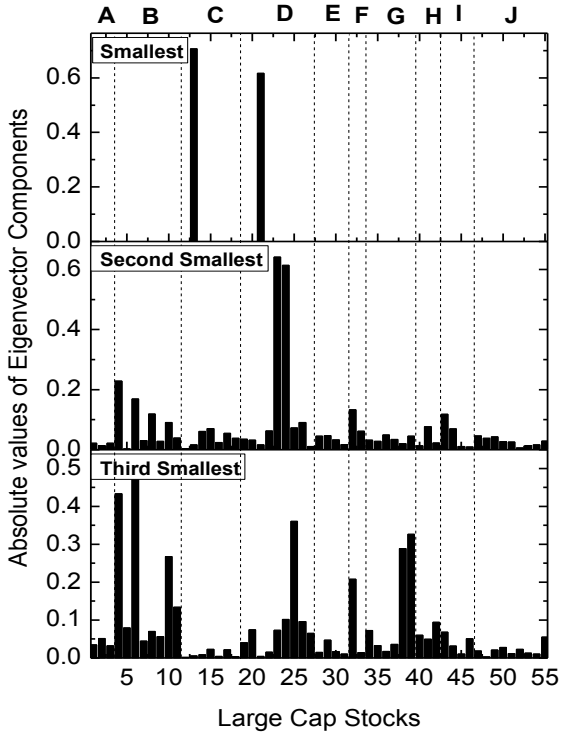


Figure 3.13: Experimentally obtained absolute values of eigenvector components of three smallest eigenvalues of the correlation matrix C of fifty five BSE large cap companies

Figure (3.13**)** shows the absolute values of eigenvector components i.e. $u_i(\lambda)$ corresponding to three smallest eigenvalues of the correlation matrix C of fifty five large cap companies. We have arranged the stocks i according to their business sectors from left to right are as follows: - A: Automobile, B: Bank, C: Metal, D: Energy, E: Pharma, F: Power, G: Software, H: Telecom, I: Trading, J: Miscellaneous

3.6.1 Results and Discussions-2

1. From figure (3.11), (3.12) and (3.13) it is clear that the number of deviating eigenvalues is small in number which can be interpreted in a qualitative manner as the entangled charactaristics of different companies. Entanglement physically means non-separability of two different systems or entities.
2. In the context of companies, it will reveal correlated companies, whose dynamics are intertwined.

3. From figure (3.11), the dominant localized eigenvector components of forty six mid cap companies are shown in table (3.10). This deviation shows strong correlation between few stocks. In table (3.8), for λ_{87}, we have two dominant stocks "Munjal Showa" belongs to automobile sector and "Ram Sugars" belongs to the food sector. But for the second lowest eigenvalue we have two dominant stocks are "Tata Coffee" and "Rana Sugars" where both the companies belong to the food sector and are highly correlated. Similarly for the third smallest eigenvalue the dominant stock is from a Chemical sector and the name of the company is "Gujrat Alkalis and chemical" which is not entangled strongly with any of the companies. This fact shows there must be some kind of interdependency or correlation such that market condition of one influences the prices of the other company .This strong interdependency or correlation can be interpreted as the "financial entanglement" of stocks.

Table 3.10: Small cap stocks with dominant contribution to the three smallest eigenvalues

Sectors	$\boldsymbol{\lambda_{87}}$	$\boldsymbol{\lambda_{86}}$	$\boldsymbol{\lambda_{85}}$
Automobile	Munjal Showa Ltd		
Food	Rana Sugars Ltd.	Tata Coffee Ltd. Rana Sugars Ltd.	
Chemical			Gujarat Alkalis & Chem. Ltd.

4. From figure (3.12), the dominant localized eigenvector components of forty six small cap companies are shown in table (3.11). This deviation shows strong correlation between few stocks. In table (3.10), for λ_{46}, we have three dominant stocks out of three two stocks belong to Chemical sector and the name of the stocks are "National Fertilizers" and "Rashtriya Chemicals and Fertilizers". The third stock "Vijaya Bank" belongs to Banking sector. But for the second lowest eigenvalue λ_{45}, we have four dominant stocks two are from Chemicals and the rest two are from banking Sector. The Chemical stocks are same as that found for lowest eigenvalue λ_{46} , and in banking stock we have a common stock as found in lowest eigenvalue λ_{46}, and the rest one is "Dena Bank". Similarly for the third smallest eigenvalue the dominant stocks are from Bank sector the "Dena Bank" and the "UCO bank". This fact shows there is some kind of strong interdependency or correlation among the lowest eigenvalue stocks "National Fertilizers", "Rashtriya Chemicals and Fertilizers" and "Vijaya Bank" of mid cap

companies such that market condition of one influences the prices of the other company.

Table 3.11: Large cap stocks with dominant contribution to the three smallest eigenvalues

Sectors	$\boldsymbol{\lambda_{46}}$	$\boldsymbol{\lambda_{45}}$	$\boldsymbol{\lambda_{44}}$
Chemical	National Fertilizers Ltd. Rastriya Chemicals & Fertilizers Ltd.		
Bank	Vijaya Bank Ltd.	Vijaya Bank Ltd. Dena Bank Ltd.	Dena Bank Ltd. UCO Bank

5. From figure (3.13), the dominant localized eigenvector components of fifty five large cap companies are shown in table (3.12). This deviation shows strong correlation between few stocks. In table (3.12), for λ_{55}, we have two dominant stocks which are "NMDC Ltd." and "Oil and Natural Gas" and they belong to Metal and Energy sectors respectively. But for the second lowest eigenvalue λ_{54} , we have two dominant stocks they are "Oil India Ltd." and "Indian Oil" and both belong to the same sector. Similarly for the third smallest eigenvalue the dominant stocks are from Bank sector the "AXIS Bank" and the "ICICI Bank". This fact shows there is some kind of strong interdependency or correlation among the lowest eigenvalue stocks "NMDC Ltd." and "Oil and Natural Gas" of large cap companies such that market condition of one influences the prices of the other company.

   Table 3.12: Mid cap stocks with dominant contribution to the three smallest eigenvalues

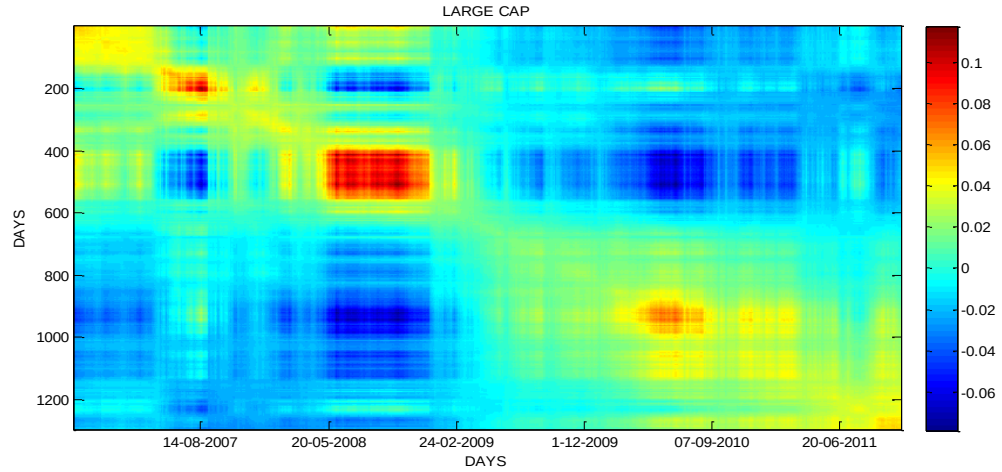
Sectors	$\boldsymbol{\lambda_{55}}$	$\boldsymbol{\lambda_{54}}$	$\boldsymbol{\lambda_{53}}$
Metal	NMDC Ltd.		
Energy	ONGC Ltd.	Oil India Ltd. Indian Oil Corporation Ltd.	
Bank			AXIS Bank Ltd. ICICI Bank Ltd.

3.7 Analysis of Market Crash

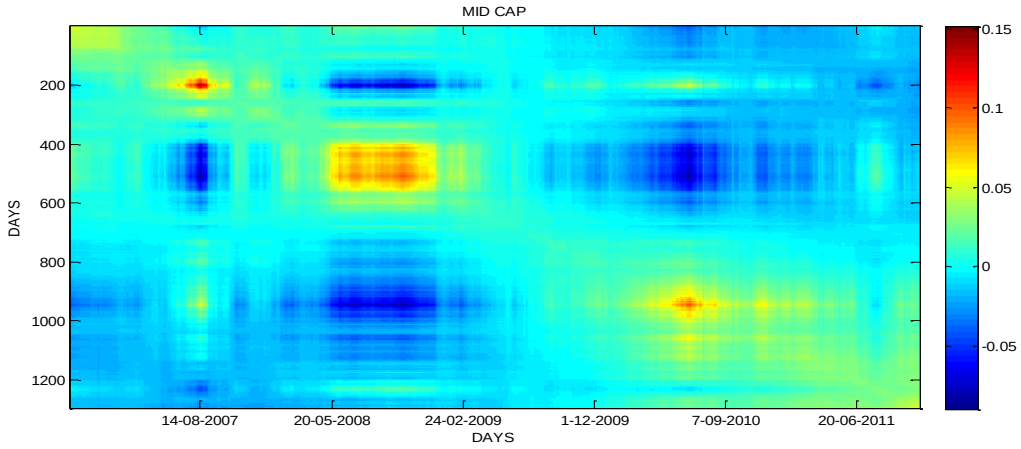
In this section market crash is examined considering the correlation matrix obtained in equation (3.3 (a)). The basic steps applied to analyze the time series data are as follows:

1. Calculate correlation matrix using equation (3.3 (a)) for large-cap, mid-cap and small-cap time series data as shown in figure (3.14 (a), (b) and (c)).

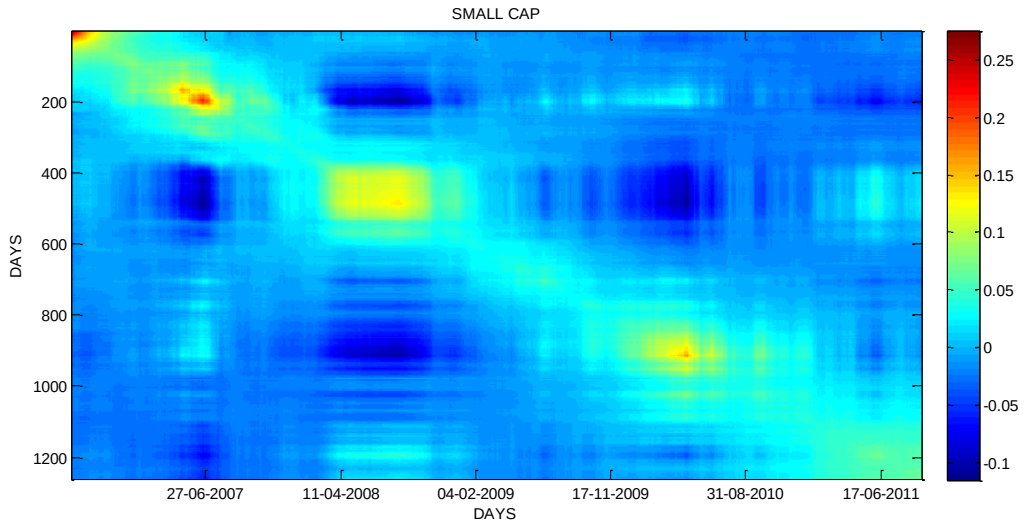
2. From figure (3.14), we found three clear time periods- before, during and after crisis for all three set of caps.
3. Then the time series data are spliced into three segments according to three time periods. The time 1-5-2008 to 30-7-2009 is taken as crisis period.
4. Again using equation (3.3 (a)), we constructed correlation matrix during the crisis period and plotted stocks verses stock graph as shown in figure (3.15 (a), 3.15 (b) and 3.15 (c)).
5. For further investigation we picked three correlated stocks from each set of stocks to show their behavior during the crisis.



(a)



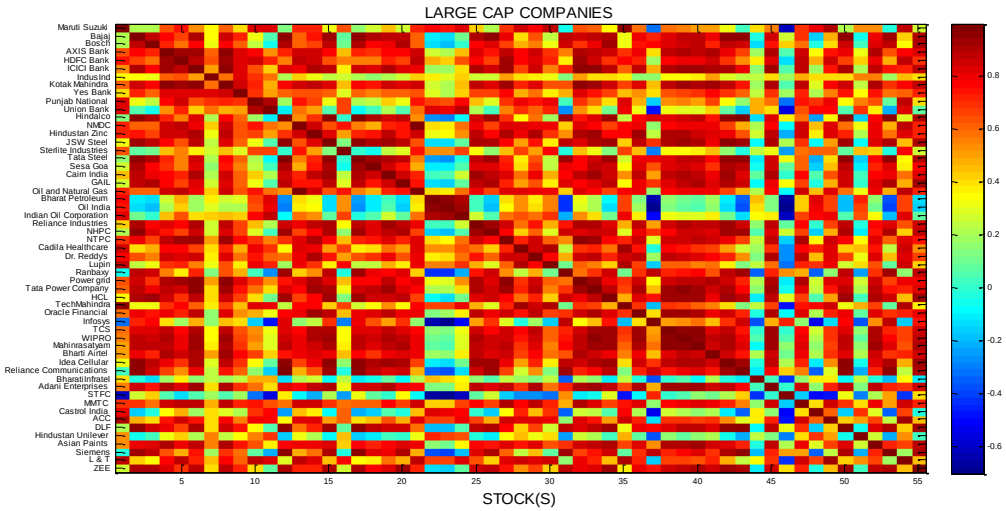
(b)



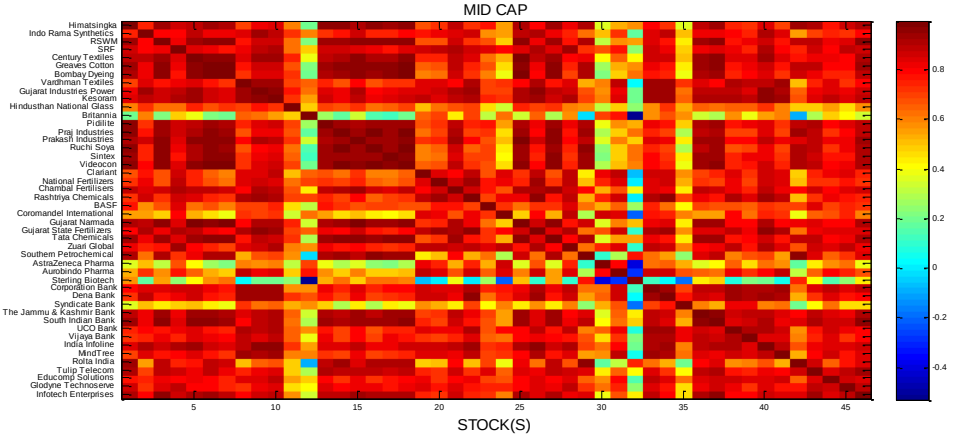
(c)

Figure 3.14 : Represents, the image diagram of the correlation matrix for (a) large cap companies, (b) mid cap companies and (c) small cap companies

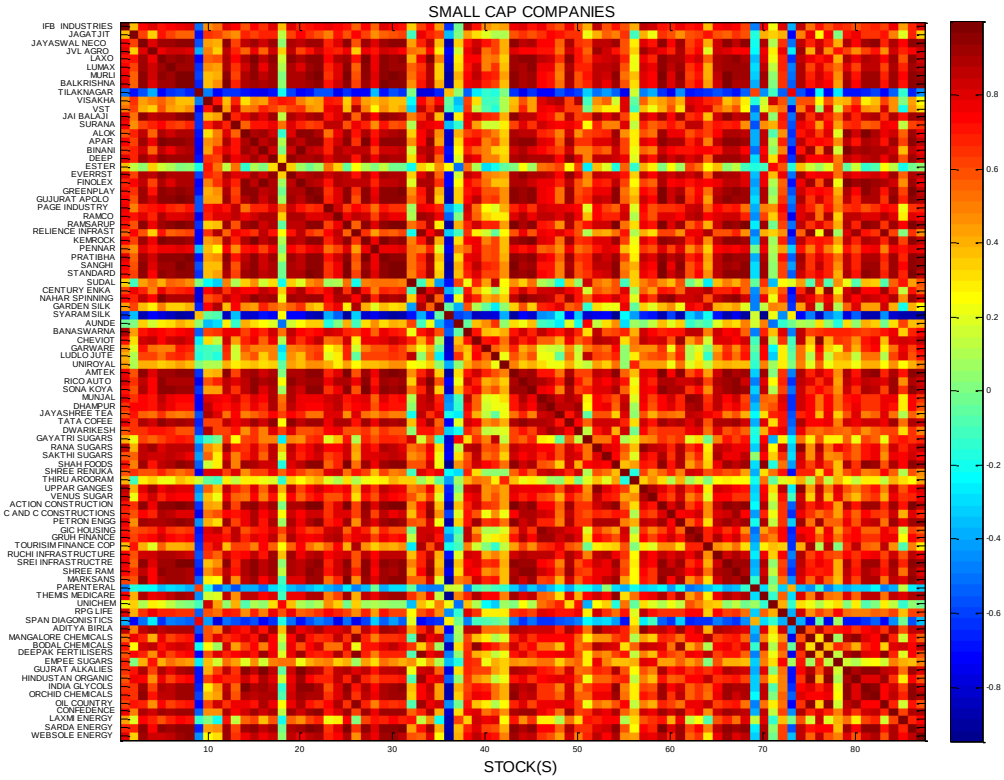
In figure (3.14 (a), (b) and (c)) the correlation matrices are represented in image form for large-cap,mid-cap and small-cap companies. Red and yellow patches correspond to high correlation region, while dark blue represents uncorrelated region. The figures show clearly three distinct time periods , which are before , during and after the crisis. From the figure , 1-5-2008 to 30-7-2009, is taken as the time period during the crisis. The color intensity represents the degree of correlation before, during and after a crisis.



(a)



(b)

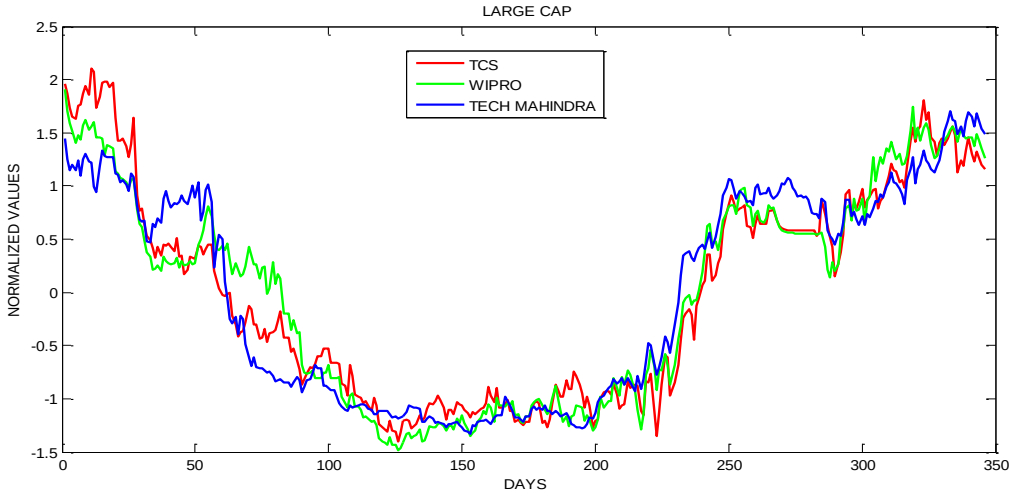


(c)

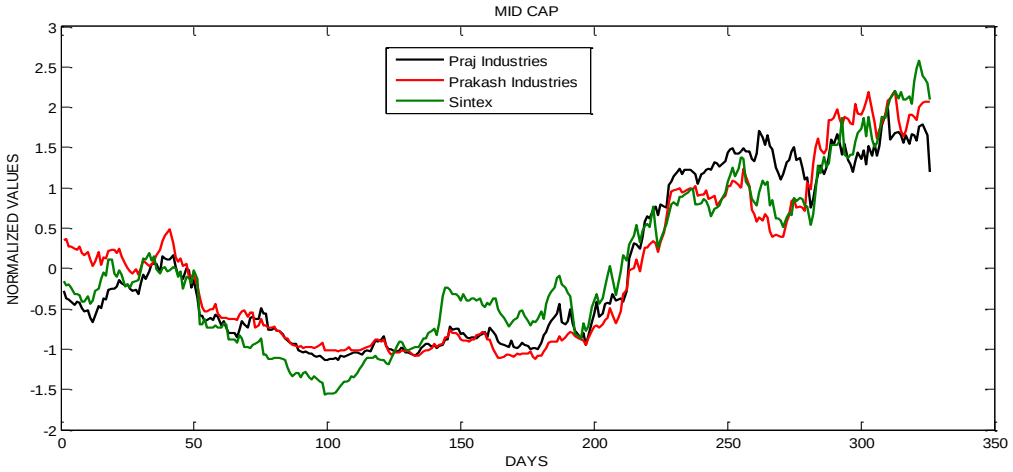
Figure 3.15: Represents the correlation of stocks during the crisis for (a) large cap companies, (b) mid cap companies and (c) small cap companies

In figure (3.15) we have constructed the correlation matrix among the companies using equation (3.4). The red and yellow blocks to show high correlation. In the y-axis the name of companies is mentioned for direct inspection of the correlation among other companies. The x-axis represents a number of stocks. The color bar shows the intensity. The color intensity level

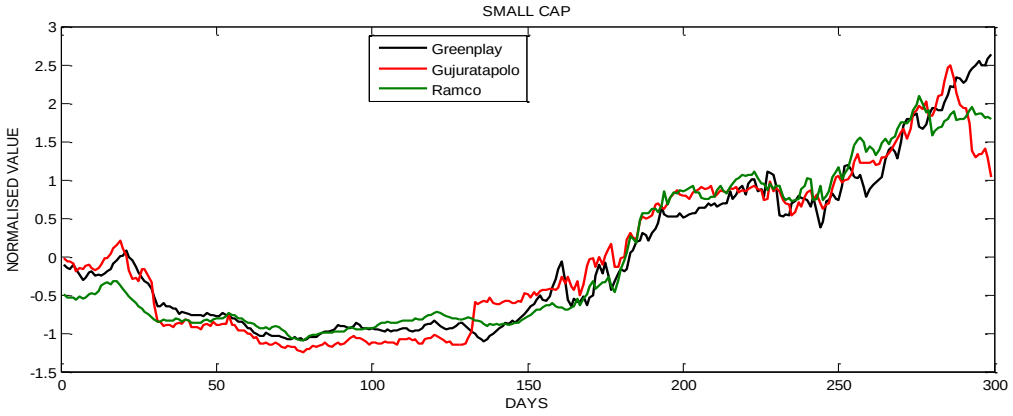
increases from lower intensity (blue) to higher intensity (red). The color intensity represents the degree of correlation among the companies.



(a)



(b)



(c)

Figure 3.16: Normalized closing price vs time during crisis graph for (a) large cap, (b) mid cap and (c) small cap companies

In figure (3.16 (a), (b) and (c)) we have taken the plot of normalized closing price values vs time during crisis graph for better visualization of stock movements. In (c) Ramco, Greenplay and Gujurat Apolo belong to Industrial are chosen, (b) Praj industries,Prakash industries and Sintex industries belonging to industrial sector are chosen and (a) Wipro, Tech mahinra and TCS belong to IT sector of large cap companies are chosen.

3.7.1 Results and Discussion-3

1. During the crisis the stocks belonging to similar or related business sector moves together is shown in figure (3.16 (a) (b) and (c)) respectively.
2. For small cap companies Ramco , Greenplay and Gujurat Apolo belong to Industrial sector move together as shown in figure (3.16 (c)).
3. For mid cap companies Praj industries , Prakash industries and Sintex industries belong to Industrial sector move together as shown in figure (3.16 (b)).
4. Wipro, Tech Mahindra and TCS belong to IT sector of large cap companies move together as shown in figure (3.16 (a)).
5. From the figure it is clear that large-cap "IT Sector" companies drop down during the crisis. And the small-cap industry sector and mid-cap industry sector behavior are almost similar.
6. During the crisis the comparison of small-cap and large-cap stocks behavior shows that the large-cap stocks go down. And the small-cap and mid-cap stock's behavior is almost similar.
7. From this we can conclude that the emerging market are more correlated during crisis.

3.8 Summary and Conclusions

In this chapter the correlation analysis technique was introduced to examine the interrelationship of several Indian companies belonging to BSE. Different size of companies were taken to study the correlation behavior. RMT is applied to remove noise from the signal. The higher eigenvalues above random noise shows market mode and group or sector behavior of companies. The small eigenvalues below the bulk of eigenvalues show strong correlation which can be interpreted in the language of quantum physics as the entanglement characteristics of financial companies. We observed small cap, large cap and mid cap companies correlation behavior is almost same. Finally behavior of stocks during market crash was studied for different capitalization of companies. During the crisis the result shows that the correlation of stocks belonging to similar or related business sector moves together. We observed that the large-cap stocks go down during the crisis with respect to small cap

companies. And the behavior of small-cap and mid-cap stocks is almost similar. From this we conclude that the emerging market is more correlated during crisis.

CHAPTER-4

NETWORK BEHAVIOR OF INDIAN COMPANIES

4.1 Introduction

Networks help to visualize and understand the relationships among various interacting entities. Complex network is a real world phenomenon which exists in many different fields of studies. The most common networks are social networks ecological networks and in the human brain the neural networks. Recently the network theory has been successfully introduced in the field of financial market and has gained attention of researchers. The newly introduced field is popularly known as financial networks [110, 140, 7]. The most studied network theory in finance originates from the application of laws of graph theory [162,172,117]. It provides a way to visualize the relationship between the stocks, helps for better prediction and accessing market dynamics [67]. The stock market is a real world complex network, where the price fluctuations among various stocks have complicated relationships. The objective of the chapter is to investigate the dynamics of stock market by using graph theoretical method. Krushakal's Minimum spanning tree (MST) is one of the methods used to establish the relationship between various stocks [84]. The visualization and analysis of the complex network are made by using Pajek software [11], which is used for analysis and visualization of large social networks. In this chapter the network behavior of the BSE large - cap, mid-cap and small-cap companies are investigated separately. For this purpose we have taken correlation matrix of stocks (discussed in chapter-3) and transformed it into a distance matrix to produce a connected network. The values of the correlation coefficient lie between -1 to $+1$. According to the definition of distance matrix the value of the distance between stocks lie between 0 to 2. From this we can conclude that the closer the relationship between two vertices the shorter the distance is. Therefore the distance characterizes how closely stocks go jointly based on their correlation. This fact resembles with the bond length between the two atoms i.e. if the bond length decreases then the atoms become closer to each other and if bond length increases the atoms go away from each other. In this chapter we have explored the distance characteristics of MST to determine market crash. During the crisis the stocks come closer to each other and go away after crisis. Using this fact we can analyze the network behavior of the stock market during crisis period. The important result we obtained from the analysis is that the total minimum distance of the MST would become smaller during crises for three sets of capitalization

of companies and the edge sum of the MST is far more before and after crises. The result verifies the theoretical prediction. From this we can conclude that the edge sum of MST is a basic parameter to decide economic crises, i.e. if edge sum of the MST is more, then the stock market is far from economic crises and if edge sum of MST is small, then the stock market is under the threat of economic crises.

4.2 Proposed Methodology

In this section we will discuss the technical methodology used for the treatment of the data. The primary objective is to establish relationship between the stocks. A clear and concise relationship can be achieved by introducing the correlation matrix. Each element of the correlation matrix C represents correlation coefficient. The correlation coefficient measures the degree of correlation i.e. the strength and direction of a linear relationship between two stocks. Correlation matrix provides the information about the degree of linear dependence between the variables. The value of correlation lies in the range of $[-1, +1]$. If the value of the correlation is $+1$, there exists a positive or increasing linear relationship and when it is -1 there exists a negative or a decreasing linear relationship. When the value of the correlation is 0 then there is no correlation between the variables.

4.2.1 Distance Matrix

In order to represent the interaction among the stocks in a graphical manner the method proposed by Mantegna is used [99]. A distance matrix is constructed from the correlation matrix using equation (4.1(a)). Then connected network is formed taking the coefficients of the distance matrix. It is observed that the co-moving stocks form cluster in developed countries. The equation to calculate the distance (d_{ij}, between stocks i and j) is obtained by the formula:

$$d_{ij} = \sqrt{2(1 - C_{ij})} \qquad (4.1(a))$$

Using equation (4.1 (a)) the MST can be generated using Kruskal's algorithm for minimum spanning tree. In an MST N node of the tree is connected with $N - 1$ edges such that the sum total of edge length is minimum i.e. $\sum_{i,j} d_{ij}$, is minimized. Where nodes correspond to the stock (firm) and the edge between nodes correspond the price fluctuation relationships of the stocks. The value of the correlation coefficient lies between -1 to $+1$. Hence, the range of the distance matrix lies between 0 to 2. From the definition of the distance matrix we can infer that the closer the

relationship between two vertices the shorter the distance is. Hence, the distance characterizes how closely stocks go jointly based on their correlation. As the distance matrix is derived from the correlation matrix it should obey the properties of the correlation matrix. Hence, the distance matrix has similar properties as that of correlation matrix. Considering a $N \times N$ distance matrix, we will get non negative real symmetrical elements, where all the diagonal elements are zero. But in case of a correlation matrix, we will get symmetric non-negative elements with diagonal elements 1.

The derivation of the distance matrix formula

Let, Δt be the time period of the $i\ th$ stock return. Where Δt is the daily return.

$$x_i(t, \Delta t) = \frac{S_i(t)}{S_i(t - \Delta t)} - 1$$

A stock return vector can be generated by taking all the stock returns for a time period T as follows:

$$x_i = \begin{bmatrix} x_i(t) - \mu_i \\ x_i(t-1) - \mu_i \\ \vdots \\ \vdots \\ x_i(t - T + 1) - \mu_i \\ x_i(t - T) - \mu_i \end{bmatrix}$$

The normalized stock return for the $i\ th$ stock will be

$$x_i = \begin{bmatrix} (x_i(t) - \mu_i)/\sigma_i \\ (x_i(t-1) - \mu_i)/\sigma_i \\ \vdots \\ \vdots \\ (x_i(t - T + 1) - \mu_i)/\sigma_i \\ (x_i(t - T) - \mu_i)/\sigma_i \end{bmatrix}$$

Where μ_i and σ_i are the mean and standard deviation of $i\ th$ stock.

According to Mantega the metric distance between the $i\ th$ stock and $j\ th$ stock is defined as

$$d_{ij} = \sqrt{\sum_{k=0}^{T} \left(\frac{x_{ik}}{|x_i|} - \frac{x_{jk}}{|x_j|} \right)^2}$$

$$d_{ij} = \sqrt{\sum_{k=0}^{T}\left(\frac{{x_{ik}}^2}{|x_i|^2} + \frac{{x_{jk}}^2}{|x_j|^2} - \frac{2x_{ik}x_{jk}}{|x_i||x_j|}\right)} \qquad (4.1\ (b))$$

The equation (4.1 (b)) can further be simplified by taking $\sum_{k=0}^{T} {x_{ik}}^2 = |x_i|^2$ and $\sum_{k=0}^{T} {x_{jk}}^2 = xj2$

Hence equation(4.1 (b)) can be rewritten as

$$d_{ij} = \sqrt{2\left(1 - \sum_{k=0}^{T}\frac{x_{ik}x_{jk}}{|x_i||x_j|}\right)} \qquad (4.1\ (c))$$

Using the definition of correlation matrix C_{ij} :

$$C_{ij} = \sum_{k=0}^{T}\frac{x_{ik}x_{jk}}{|x_i||x_j|}$$

Equation (4.1(c)) can be rewritten as

$$d_{ij} = \sqrt{2\left(1 - C_{ij}\right)}$$

4.2.2 Minimum Spanning Tree Method

Minimum spanning tree (MST) of a weighted graph defines the lowest cost subset of edges that keeps the graph in one connected component. There are many applications of MST for example the most practical application of MST is in the telephone companies for designing a method of laying cables to a new neighboring city with the lowest total cost. Route finding for airlines is another important application of MST. Where vertices represent the cities and the edges represent the routes between cities. MST is used in airlines to find the optimized route cost path without forming any cycle. So MST is the mother of all network design problems. MST is important for several reasons as given below:

1. MST can be computed easily and quickly. They create a sparse subgraph to where all the vertices of the original graph are available.

2. It provides a natural way to identify clusters in sets of points. Delete the long edges from an MST. The leaves are connected components that define natural clusters in the data set, as shown in the figure (4.1).
3. It can be used to give approximate solution to computationally hard problems such as travelling salesman problem.
4. MST algorithm is a tool that provides graphic evidence of greedy algorithms that can give provably optimal solutions.

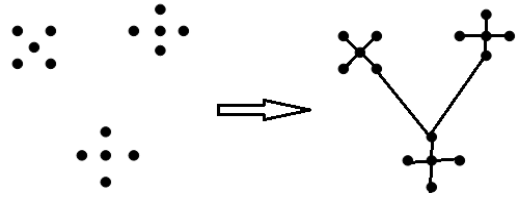


Figure 4.1: The natural cluster and MST

If we are given a graph $G = (V,\ E)$, we may have more than one MST. This can be verified from the graphs shown in the figures.

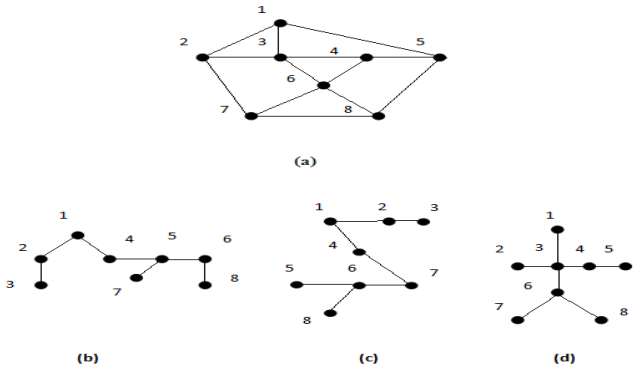


Figure 4.2: Represents the original graph (a) and three MST of the original graph (a) are (b), (c) and (d)

Looking at figure (4.2 (a), (b) and (c)) we can notice the structural difference of these trees from each other significantly. The basic differences are as follows:

1. The number of vertices is same in all the trees as that of the graph (a).
2. The edge set is a subset of the graph (a).
3. There is no cycle.

Such a structure is called spanning tree of the graph. Now let us define the formal definition of the spanning tree. A tree T is a spanning tree of a connected graph $G = (V,\ E)$ such that:

1. Every vertex of G belonging to an edge in T
2. The edges in T form a tree.

Let us see how we are able to construct a spanning tree for a given graph. Take any vertex V as an initial partial tree and add edges one by one so that each edge joins a new vertex in the partial tree. In general if there are n vertices in a graph we shall construct a spanning tree $(n-1)$ steps i.e. $(n-1)$ edges are needed to be added. Frequently we need weighted graphs and we need to build a subgraph that must be connected and must include every vertex in the graph. To construct a subgraph with least weight or least cost, we must not have cycles in it. Actually we need to construct a spanning tree with minimum cost or a minimum spanning tree for this we need a weighted graph. In a weighted graph each edge or path has assigned a cost or weight, which is a number representing the relationship between two vertices. Figure (4.3) shows a weighted graph.

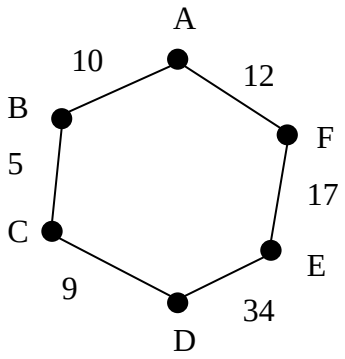


Figure 4.3 Weighted graph

Hence MST is an acyclic connected graph. The sum of edge lengths of MST is minimized. Let us define a graph $G = (V, E)$ and $(u, v) \in E$, where V is the vertices and E is the edges. Define, the weight associated with each edge (u, v) as $w(u, v)$. Hence, a tree T is a subset of E and said to be a minimum spanning tree if the weight (T) , is minimized.

Where

$$w(T) = \sum_{(u,v)\in T} w(u, v)$$

In order to create the MST from stock data first the correlation matrix C_{ij} , is generated then a complete graph is created with a metric distance d_{ij} .Where, d_{ij}, is assigned to the edge connecting stock i with stock j.

Minimum Spanning Tree (MST)

There are a number of algorithms employed to compute the minimum spanning tree. The two famous algorithms used for computing minimum spanning tree are Kruskal's algorithm and the Prim's algorithm. Both of the algorithms follow Greedy choice property: local optimal is part of global optimal i.e. the decision is taken for each step based on what seems best at the current step. We have chosen Kruskal's algorithm in our research for its easy understanding and best one to solve the problem by hand. Hence, Kruskal's algorithm have been employed to analyze the financial network. The Kruskal's algorithm is applied to find the MST from a connected weighted graph. So in our case, from the distance matrix by applying Kruskal's minimum spanning tree algorithm we will be able to find the minimum spanning tree. The working principle of Kruskal's algorithm is as follows:

1. Arrange all the edges in increasing order of weight.
2. Pick the smallest weighted edge (in case of more than one, pick one randomly). Mark it with a given color say green.
3. Next find the edge with minimum weight (uncolored) from the graph add that edge to the tree if it does not form a cycle.
4. Continue step-2 until all the edges are visited. The green edges form MST.
5. Add the cost of all edges to get the minimum cost of the spanning tree.

The pseudo code of Kruskal's algorithm which is used in a financial correlation network:

MST-KRUSKAL$(\boldsymbol{G}, \boldsymbol{w})$

$A \leftarrow \varphi$

For each vertex $v \in V[G]$

do MAKE-SET (v)

Sort the edges of graph G in non decreasing order by weight w

For each edge$(u, v) \in G(E)$, taken in non decreasing order by weight

do If FIND-SET (u) ≠ FIND-SET (v)

$then\ A \leftarrow A \cup \{(u, v)\}$

UNION (u, v)

Return A

Explanation of the Algorithm

Figure (4.4) describes the complete process of drawing MST with seven vertices. The original graph is shown in figure (4.4 (a)) with the corresponding edge cost. The sub graph (b) has two cost edges CE and AD but we have chosen AD arbitrarily from the original graph and marked it green. Then CE edge is chosen in the next step as it does not form any cycle shown in the sub graph (c). The edge DF, AB and BE are chosen following the similar steps. Finally we get MST having six green colored edges shown in the sub figure (g).

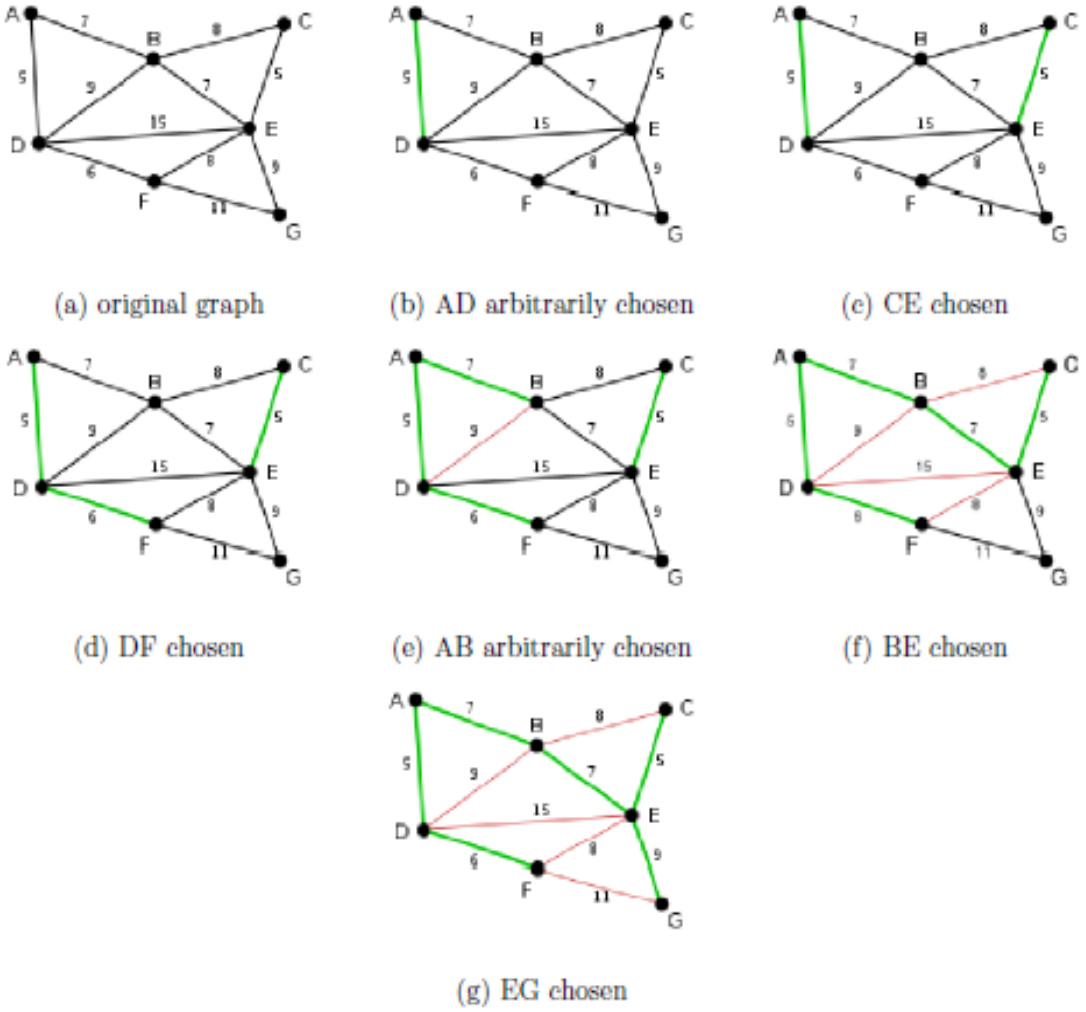


Figure 4.4: A Complete process of drawing MST with seven vertices

Mantegna first introduced a method to represent the interaction of stocks. According to this method the correlation matrix among stocks is changed into a distance matrix to form a connected network of co-moving stocks. If i and j are two stocks then d_{ij} is the distance between stocks. The

distance between stocks is evaluated using the correlation matrix C, according to the equation (4.1). Applying Kruskal's MST method the least weight edge is extracted from the list of edges again the next smallest weighted edge is extracted and added into the tree. Finally the minimum spanning tree is obtained from the distance matrix. After getting the list of edges and their weights of the MST the tree is constructed by the use of Pajek software.

4.3 Graphical Representation of Stocks

In this section the behavior of financial market is represented in tree structure where stocks (companies) are represented as nodes and the correlation coefficients by edge weights of the tree. The MST is an efficient tool for discovering the hierarchical structure of the network. In a fully network all the information cannot be seen easily. Many links among nodes makes it harder to obtain the desired results. Using MST algorithms, we can figure out the basic structure of the data. Hence it can be used as a filtering tool.

The study starts by considering three matrices consisting of all the correlation coefficients for the three sets of BSE small cap, mid cap and large cap companies. The set of data for small cap companies are formed by: eighty seven small cap stocks which include daily closing prices between (May 18, 2007) and (March 30, 2012). The stocks are separated into 9 main sectors. Fifty five large cap stocks which include daily closing prices between (April 5, 2007) and (March 30, 2012). The stocks are separated into 10 main sectors. Forty six mid cap stocks which include daily closing prices between (April 5, 2007) and (March 30, 2012). The stocks are separated into 6 main sectors. The sectors and number of companies corresponding to each sector is given in the appendix-I.

MST was used for financial implementation for the first time by Mantegna in 1999. MST has been applied to BSE market sample data. After constructing correlation matrix, we used the distance metric of Mantegna to construct the distance matrix given in equation (4.1(a)). Next we used the Kruskal algorithm to construct the MST from the fully connected graph. The principle of Kruskal algorithm is to add an edge with minimum weight as long as it does not create a cycle. We use the Kruskal algorithm to construct a financial network, which is a connected undirected graph, into MST and find out the statistical properties of the network.

The MST is obtained by implementing the Kruskal algorithm in Matlab software and displayed using Pajek software. The color of a node shows sector that node belongs to and the number assigned to a node corresponds to its name. The input to the algorithm is the distance

metric which is obtained by correlation matrix according to the Mantegna distance metric. Then the output of MST is used as input in Pajek software for visualization.

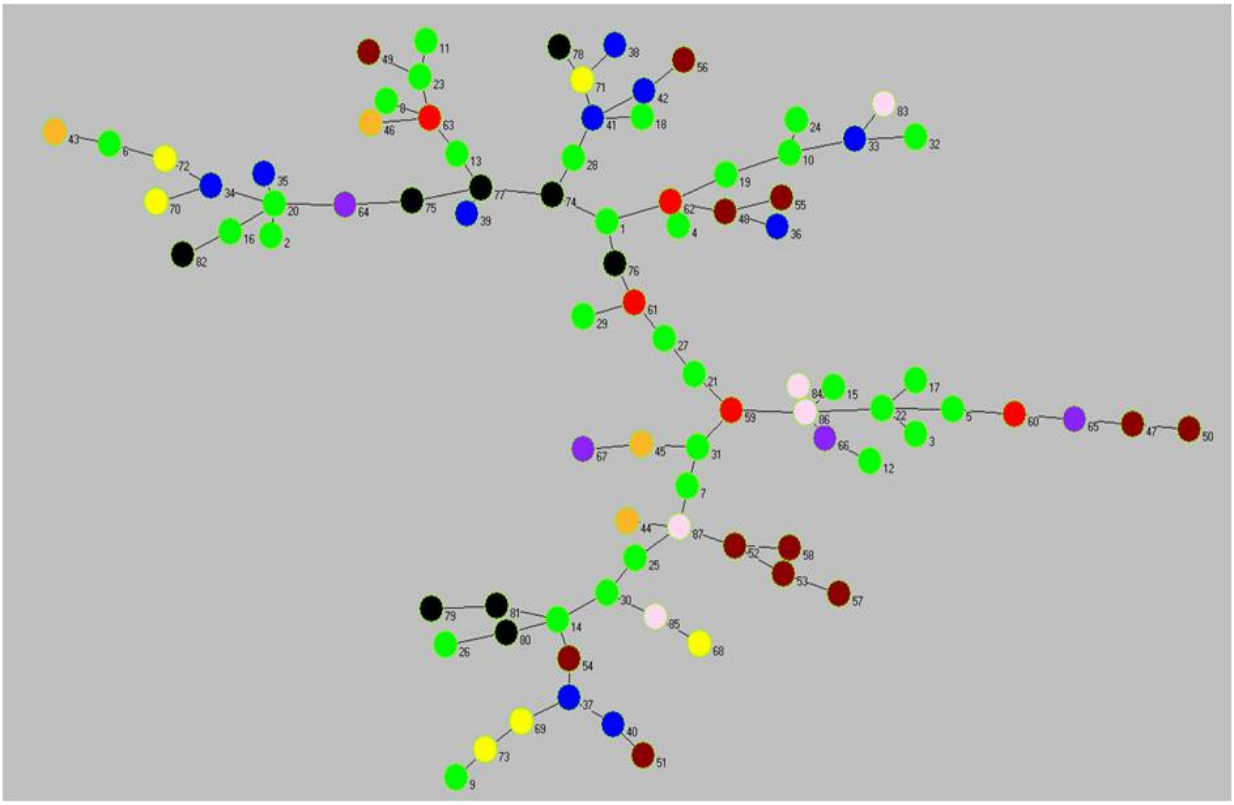


Figure 4.5: Represents Minimum Spanning Tree of Bombay Stock Exchange Small Cap Companies

Figure (4.5) shows the minimum spanning tree of eighty seven small cap companies. Each company is represented by a number. Stocks belonging to a particular sector are assigned with a unique color. There are some sector specific groups which indicate the strong relation. Referring to Appendix-I the name of the stocks can be figured out. In the MST there exist eighty six edges. Each edge carries some weight or cost. But in the tree structure we have considered the cost to be unity. The color codes and corresponding sectors of the stocks are as follows: Green-Industry, Blue-Textile, Dandelion-Automobile, Maroon-Food, Red-Construction, Purple-Finance, Yellow-Pharma, Black-Chemical, Pink-Energy. The stocks are represented by numbers.

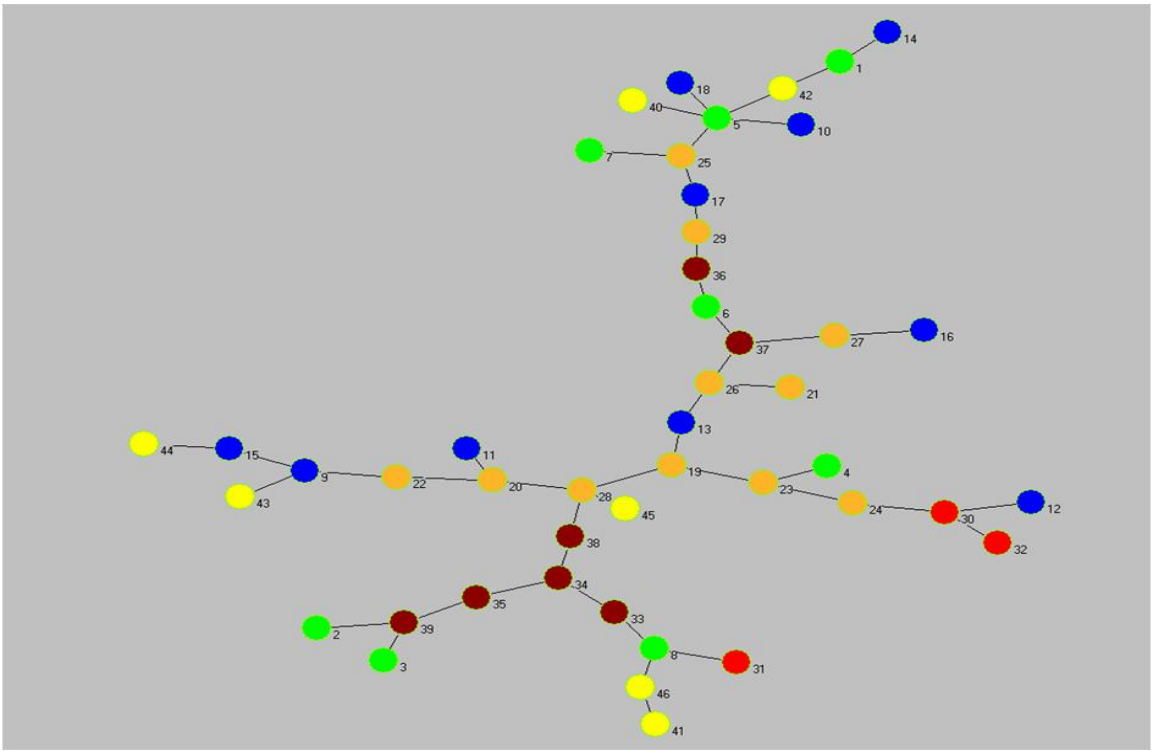


Figure 4.6: Represents Minimum Spanning Tree of Bombay Stock Exchange mid Cap Companies

Figure (4.6) shows the minimum spanning tree of forty six mid cap companies. Each company is represented by a number. Stocks belonging to a particular sector are assigned with a unique color. There are some sector specific groups which indicate the strong relation. Referring to Appendix-I the name of the stocks can be figured out. In the MST there exist forty five edges. Each edge carries some weight or cost. But in the tree structure we have considered the cost to be unity. The color codes and corresponding sectors of the stocks are as follows: Text-Green, Industrial-Blue, Chemical- Dandelion, Pharma- Red, Bank- Maroon and Computer- Yellow.

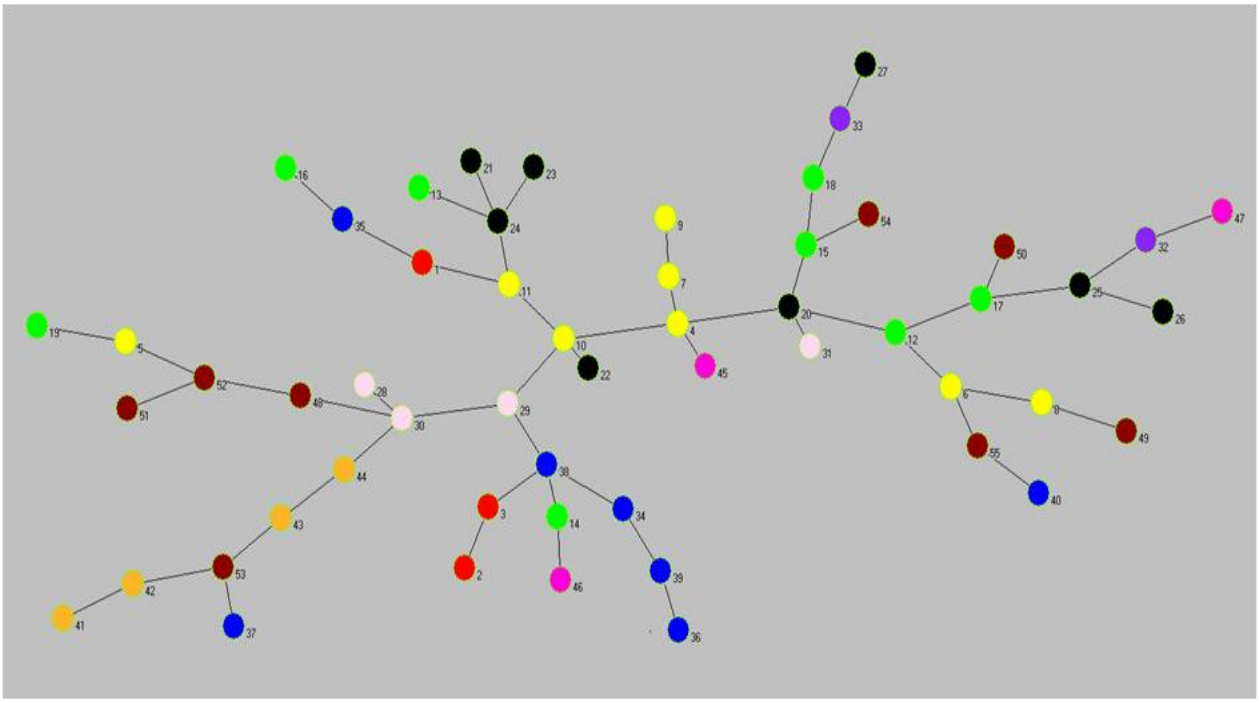


Figure 4.7: Represents Minimum Spanning Tree of Bombay Stock Exchange large Cap Companies

Figure (4.7) shows the minimum spanning tree of fifty five large cap companies. Each company is represented by a number. Stocks belonging to a particular sector are assigned with a unique color. There are some sector specific groups which indicate the strong relation. Referring to Appendix-I the name of the stocks can be figured out. In the MST there exist fifty four edges. Each edge carries some weight or cost. But in the tree structure we have considered the cost to be unity. The color codes and corresponding sectors of the stocks are as follows: Automobile-Red, Bank-yellow, Metal-Green, Energy-Black, Pharma-Pink, Power-Purple, Software-Blue, Telecom- Dandelion, Trading- Rubin red, Mislanious-Maroon.

A minimal spanning tree figures out the dynamic system of the market. For NYSE (The New York Stock Exchange), the construction of such type of MST forms the cluster of stocks belonging to the same business sector [116]. But in case of our experimental data very few sectors form clusters and the rest of the stock does not form any cluster. For example: the Banking sectors of large cap stocks form cluster which are yellow in color as shown in figure 4.8. Such type of deviation may happen due to the dominating nature of market mode over the intra-sector interaction of the stocks. And the spectral analysis of one day data did not give clear clustering information.

4.4 Group Correlation Structure Analysis

From figure (4.5), (4.6), and (4.7) the group behavior of stocks are not prominent. For this we will make a group correlation analysis to find the group characteristics of stocks or clustering of stocks. The set of highly inter correlated stocks in their price change are called group stocks. In the experimental correlation matrix there exists several types of noise which affects to identify the group of stocks or sectors as shown in figure (4.5), (4.6), and (4.7). To remove the noise, the intergroup correlation method proposed by Mantegna is used [81]. The expanded correlation matrix will be:

$$C = \sum_i \lambda_i u_i u^T{}_i \tag{4.2}$$

Where, λ_i , is the eigenvalue and u_i , is the corresponding eigenvector. The eigenvalues λ_i , are sorted in increasing order (the largest eigenvalue is labelled as the size of the correlation matrix) and u_i , is the corresponding eigenvector. Equation (4.2) allows decomposing the correlation matrix into three parts:

1. The market mode
2. The sector mode
3. The random component

As we know (1) and (3) do not contribute anything for group identification. Hence we may assume few large eigenvalues lying above the bulk of eigenvalues and below the largest eigenvalue contribute significantly for group identification. From the experimental data it is not possible to figure out the exact number of eigenvalues contributing for group identification. From figures (3.1), (3.3) and (3.5), of chapter-3 three intermediate large eigenvalues are identified to produce filtered correlation matrix for sector or stock group identification by taking partial sum of $\sum_{i=1}^{N_g} \lambda_i u_i u^T{}_i$, where N_g is the number of eigenvalues contributing to find group correlation matrix and the total summation produces the group correlation or the sector correlation matrix. Considering previous results of Plerous et al. [127] we put forward the correlation matrix into three segments by the market wide part of the largest eigenmode, the group correlation part of the intermediate eigenvalues and the random part of the bulk of eigenvalues. Let us expand the equation (4.2) into three separate correlation matrices as follows:

$$C = C_m + C_g + C_r$$

$$= \lambda_0 u_0 u^T{}_0 + \sum_{i=1}^{N_g} \lambda_i u_i u^T{}_i + \sum_{i=N_g+1}^{N-1} \lambda_i u_i u^T{}_i \qquad (4.3)$$

Where, C_m, C_g and C_r are the market mode, group mode and random mode respectively. Determination of C_m , is simple and straightforward. The stock prices which have no correlation are called bulk eigenvalues. In the experimental correlation matrix the eigenvalue spectrum deviates significantly from the equation (4.1) due to the combination with basic structured correlations, such as the group correlation surrounded by C_g [73,77]. For that reason we can use a graphical approach to determine N_g. In the eigenvalue spectrum as shown in the figure (3.1), (3.3) and (3.5) we choose the cut $N_g = 3$.

Next, we will construct a network structure to visualize the interactions among the stocks by using the group correlation matrix. A threshold method is used to construct the group behavior of stocks: The basic idea of threshold method is to specify a threshold value c^{th} ,$-1 \leq c^{th} \leq +1$. If the value of correlation coefficient is greater or equal to the value c^{th} an edge is added between vertices i and j.

Steps to construct networks of interactions:

1. Generate an adjacency matrix A from the group correlation matrix C_g by using a threshold value.
2. Choose the threshold value c^{th} in such a way that $A_{ij} = 1$ if $(C_g)_{ij} > c^{th}$ and $A_{ij} = 0$ otherwise.

4.4.1 Results and Discussions-1

In this section the group correlation or clustering behavior of stocks for small cap, large cap and mid cap companies are shown graphically. Pajek software has been used for graphical visualization of clustering of stocks.

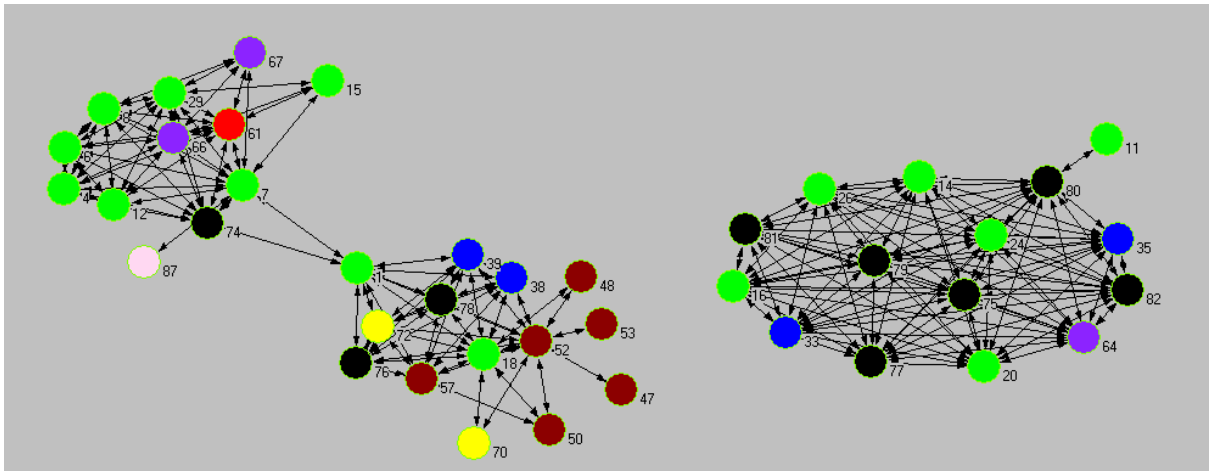


Figure 4.8: Represents Clustering of BSE Small cap Stocks generated from the group correlation matrix C_g

By taking the best choice of threshold $c^{th} = 0.09$ we can construct the largest clusters for BSE small cap companies. Figure (4.8) shows with best choices of threshold industrial sectors (green), Food sectors (Maroon) and chemical sectors (black) are forming clusters. But other sectors fail to form a cluster signifies the intra-group correlations are fragile in contrast to the market-wide correlation in the case of small cap companies.

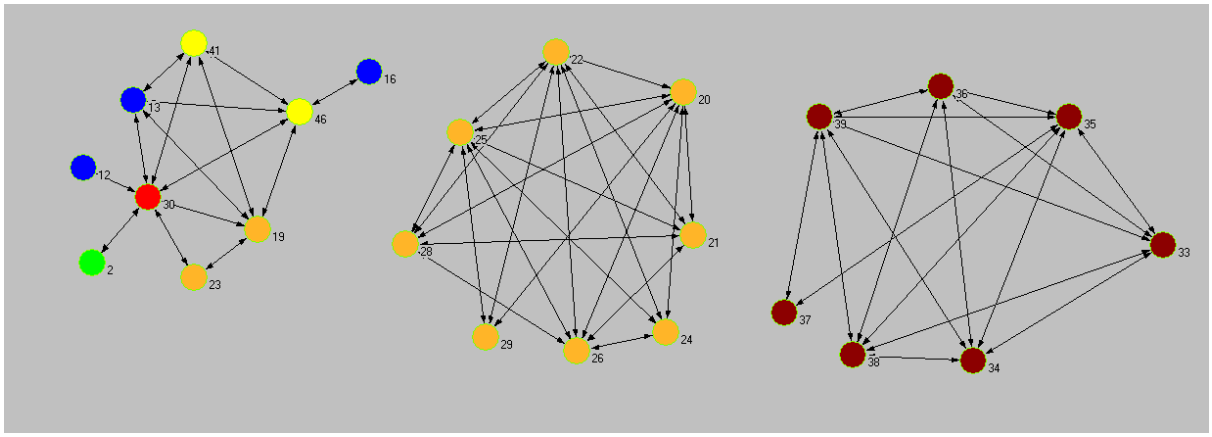


Figure 4.9: Represents Clustering of BSE Mid Cap Stocks generated from the group correlation matrix C_g

By taking the best choice of threshold $c^{th} = 0.09$ we constructed the largest clusters for BSE mid cap companies. Figure (4.9) shows with best choices of threshold the chemical and banking sector companies form groups but other companies do not cluster signifies the intra-group correlations fragile in contrast to the market-wide correlation.

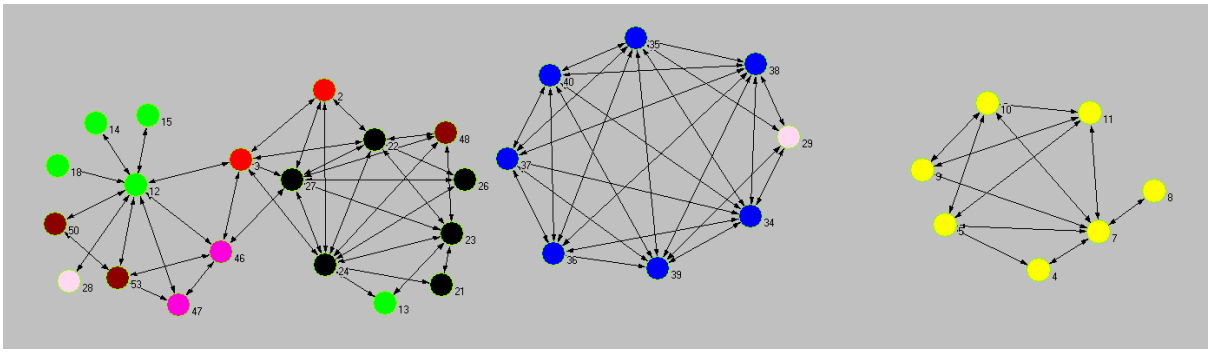


Figure 4.10: Represents Clustering of BSE Large Cap Stocks generated from the group correlation matrix C_g

By taking the best choice of threshold $c^{th} = 0.09$ we can construct the largest clusters for BSE large cap companies. Figure (4.10) shows with best choices of threshold sectors are not forming any cluster. We are getting three clusters banking sector (yellow), software (Blue), Energy (Black). But the rest of sectors do not form cluster shows the intra-group correlations are fragile in contrast to the market-wide correlation in BSE large cap companies.

4.5 Network Analysis of Market Crash

In this section network method has been employed to analyze 2008 Indian market crash. We have taken data of large-cap, mid-cap and small-cap companies according to their business sectors. Interpolation is used for missing data. The data are analyzed using MATLAB software. The network is drawn using Pajek software.

Here the time series data have been analyzed for before, during and after crisis for large cap, mid cap and small cap companies separately. For this the time series data has been splited into three time periods such as before the crisis, during the crisis and after the crisis respectively. From figure (3.14 (a), 3.14 (b) and 3.14 (c)) we can clearly find three time periods. We have taken some time gap between before the crisis and during the crisis in order to make to avoid overlapping effect between the time periods. Similarly some time gap is taken between during crisis and after the crisis.

The data points for large cap companies are divided into three parts as follows: The total data length, $T = 1293$, is divided into three parts as,

a) From April 5, 2007 to March 20, 2008, corresponds to $T = 251$ data points before crisis.

b) From May 20, 2008 to May 05, 2009, corresponds to $T = 251$ data points during the crisis.

c) From February 09, 2010 to Jan 25, 2011, corresponds to $T = 251$ data points after crisis.

The data points for mid cap companies are divided into three parts as follows: The total data length, $T = 1295$, of mid cap companies is divided into three parts as,

a) From April 3, 2007 to March 18, 2008, corresponds to $T = 251$ data points before crisis.
b) From May 20, 2008 to May 05, 2009, corresponds to $T = 251$ data points during the crisis.
c) From February 09, 2010 to Jan 25, 2011, corresponds to $T = 251$ data points after crisis.

The data points for small cap companies are divided into three parts as follows: The total data length, of small cap companies are divided into three parts as,

a) From May 18, 2007 to May 5, 2008, corresponds to $T = 251$ data points before crisis.
b) From May 20, 2008 to May 05, 2009, corresponds to $T = 251$ data points during the crisis.
c) From February 09, 2010 to Jan 25, 2011, corresponds to $T = 251$ data points after crisis.

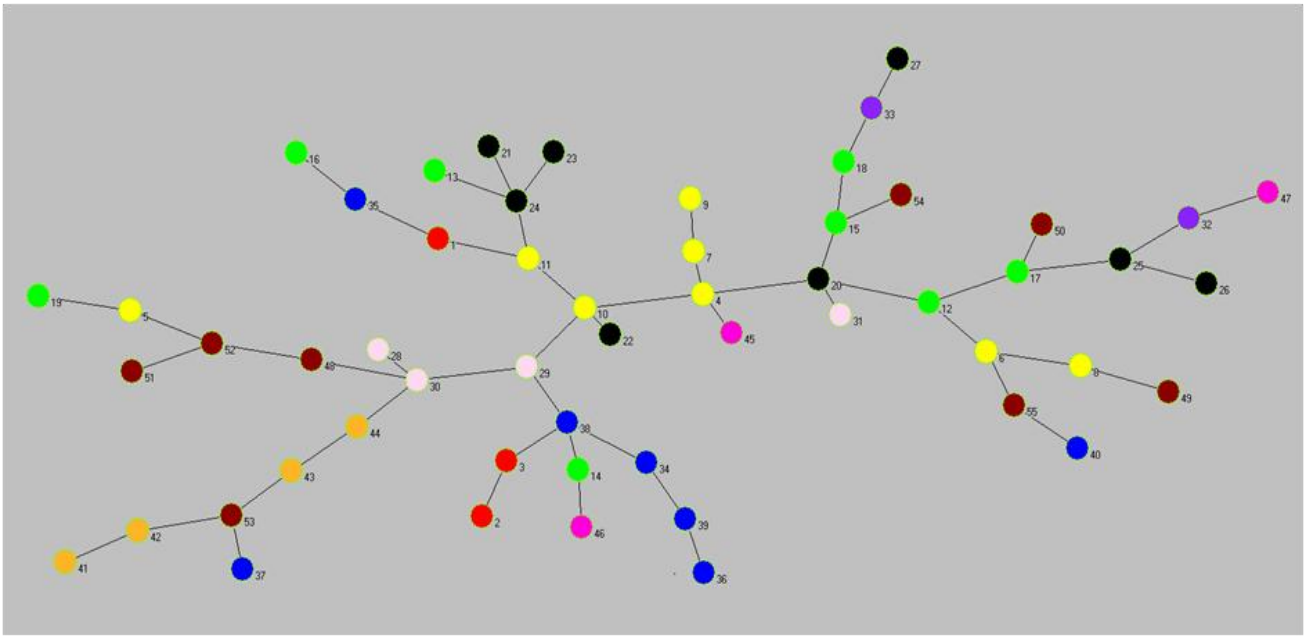
The basic steps applied to analyze the time series data are as follows:

1. Construct the correlation matrix
2. Find the distance matrix from the correlation matrix using equation (4.1(a))
3. Extract the MST applying the Kruskal's algorithm (in case of before, during and after the crisis)
4. Draw the MST graph using Pajek software (in case of before, during and after the crisis)
5. Add the edge cost to compare.

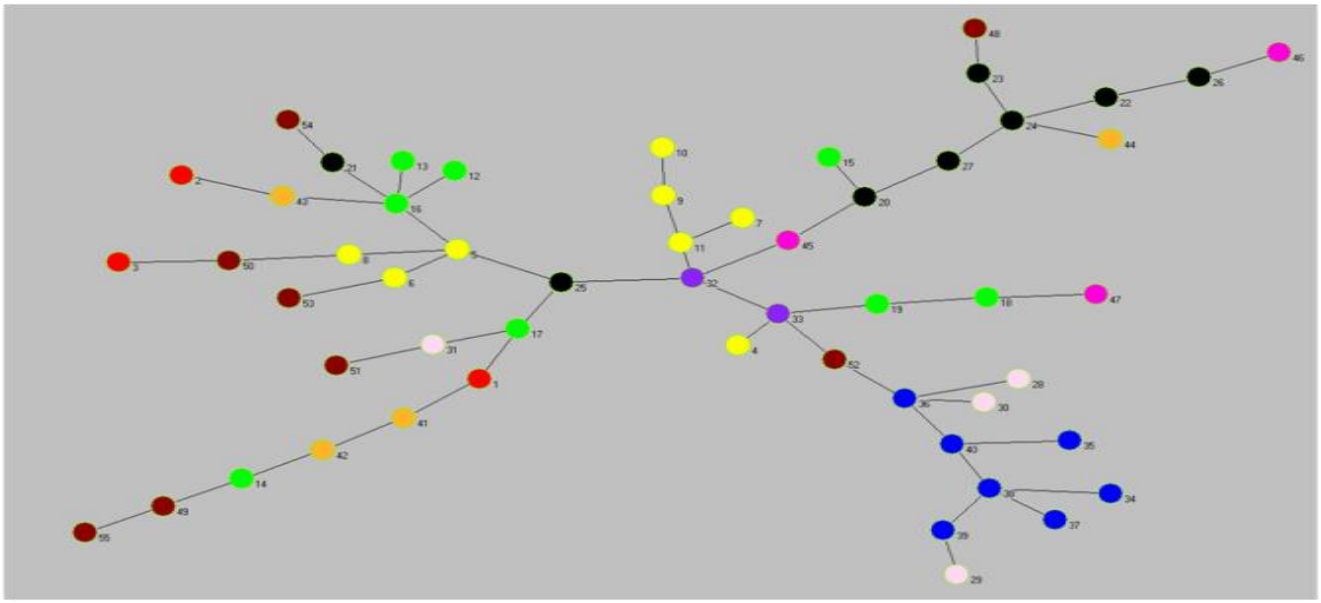
4.5.1 Results and Discussions-2

Now we will discuss the results obtained constructing the MST for three time periods (before, during and after the crisis) and three sets of capitalization companies.

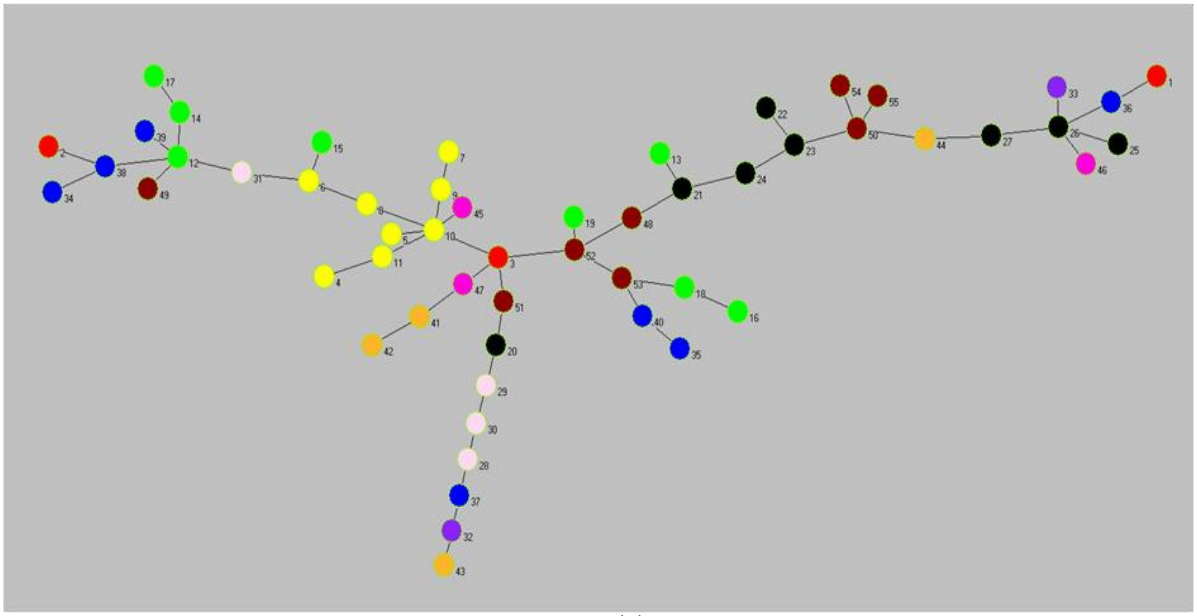
Large Cap Companies



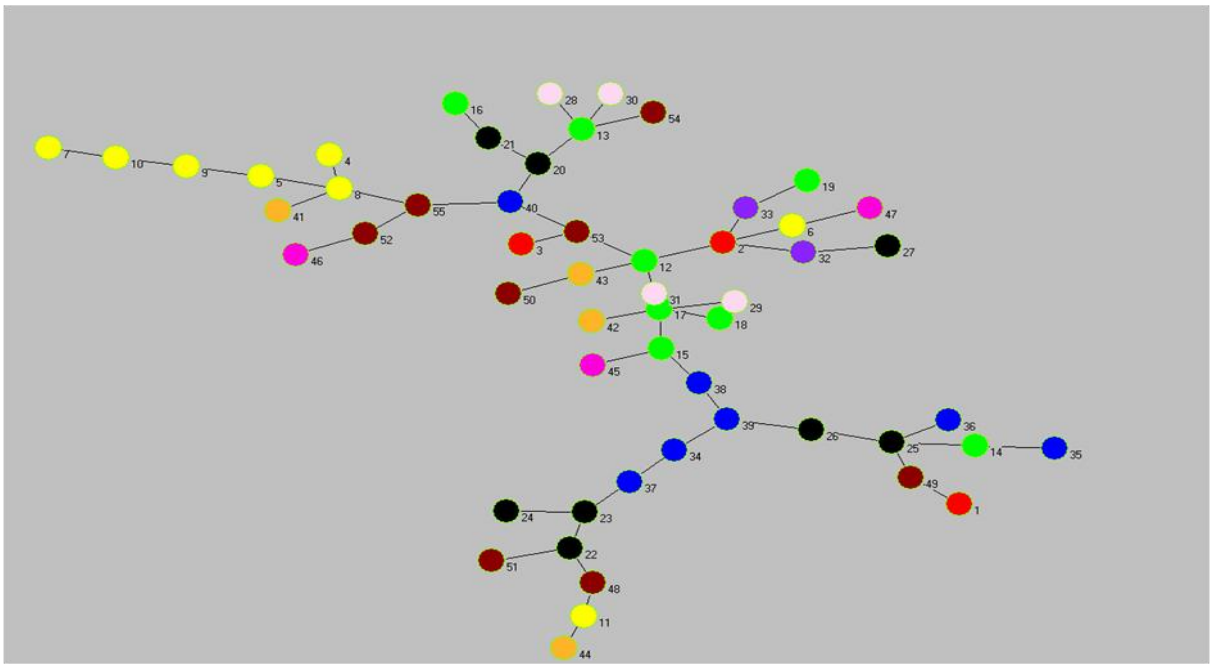
(a)



(b)



(c)



(d)

Figure 4.11: Represents MST of Large cap stocks for (a) total time period, (d) During crisis, (b) before crisis and (c) after crisis

Figure (4.11 (a)) shows the graphical representation of the MST of overall data points $T = 1293$. The sub figure (b) shows the MST before the crisis. Similarly sub figure (c) and (d) shows the MST after and during crisis respectively. The color code and corresponding sectors of large cap stocks are as follows: Automobile-Red, Bank-yellow, Metal-Green, Energy-Black, Pharma- Pink, Power-Purple, Software-Blue, Telecom-Dandelion, Trading-Rubin red, Mislanious-Maroon. Each node is represented by a number. Referring to appendix-I we can find the name of the company corresponding to each number. From figure (4.11 (c) and (d)) i.e. after and during crisis period the banking sector stocks form strong group but the group behavior is less prominent in case of before crisis as shown in figure (4.11 (b)). Before crisis and during the crisis the software companies form clusters as shown in figure (4.11 (d) & (b)) but they are not forming strong cluster after crisis as shown in the figure (4.11. (c)). Similarly energy sector stocks form a cluster before the crisis and they do not form clusters during crisis.

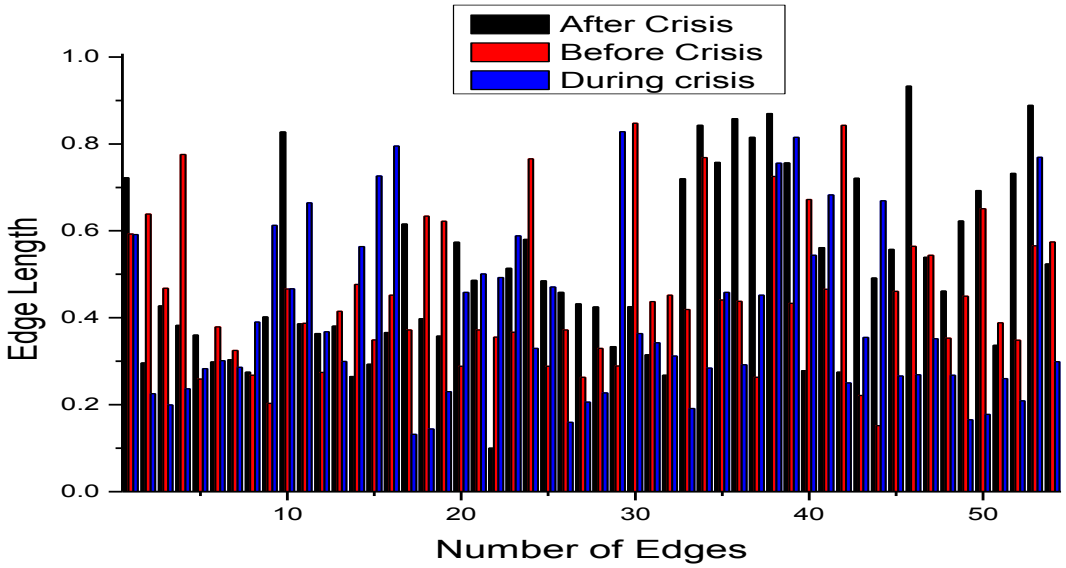


Figure 4.12 : Represents Edge length vs. Number of Edges of Large Cap Stocks

In figure (4.12), the plot is drawn taking the number of edges of the MST of large cap companies before, during and after a crisis in the x-axis and edge length or edge cost of the MST during, before and after the crisis in the y-axis. As the MST has 55 nodes so in the x-axis 54 number of edges have been considered for large cap companies. In order to analyze the effect of closeness of stocks we have added the edge cost of the MST for the three sets of time periods i.e. before the crisis, during the crisis and after a crisis.

In order to analyze the effect of closeness of large cap stocks we have added the edge cost of the MST for the three sets of time periods i.e. before the crisis, during the crisis and after the crisis. Edge sum of the MST of large cap stocks before, during and after the crisis have been taken in the y-axis and the three time periods before, during and after crisis is taken in the x-axis.

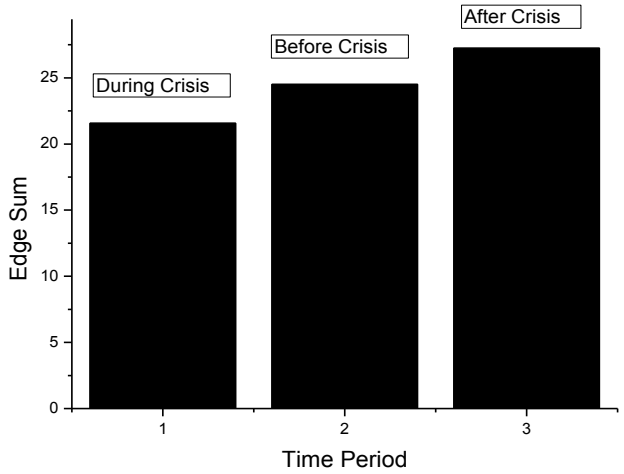
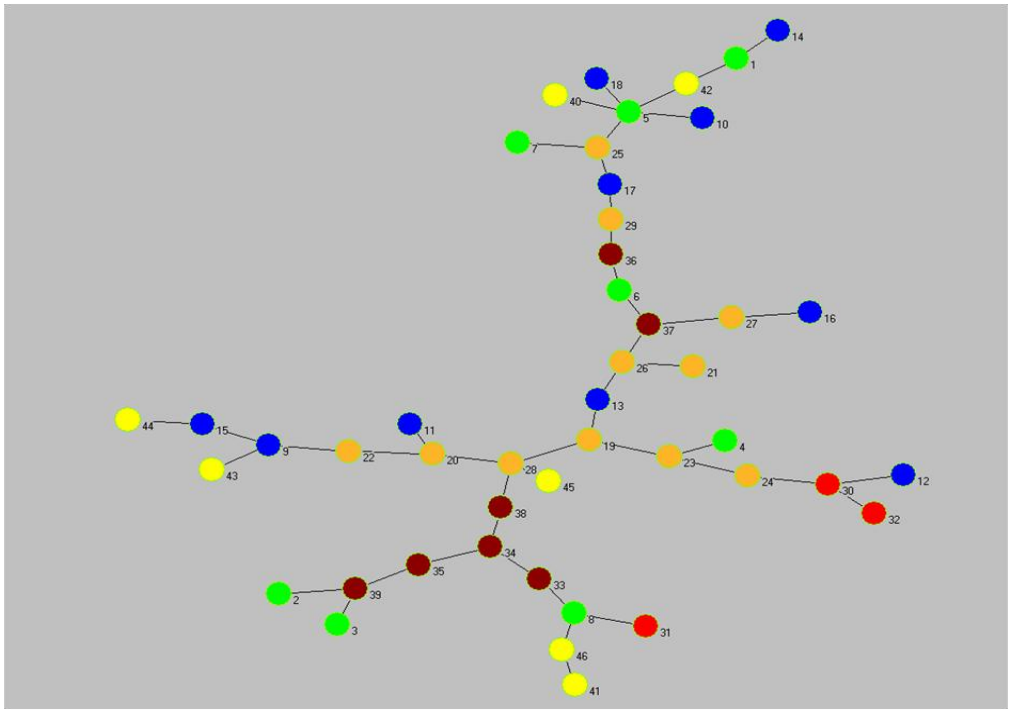


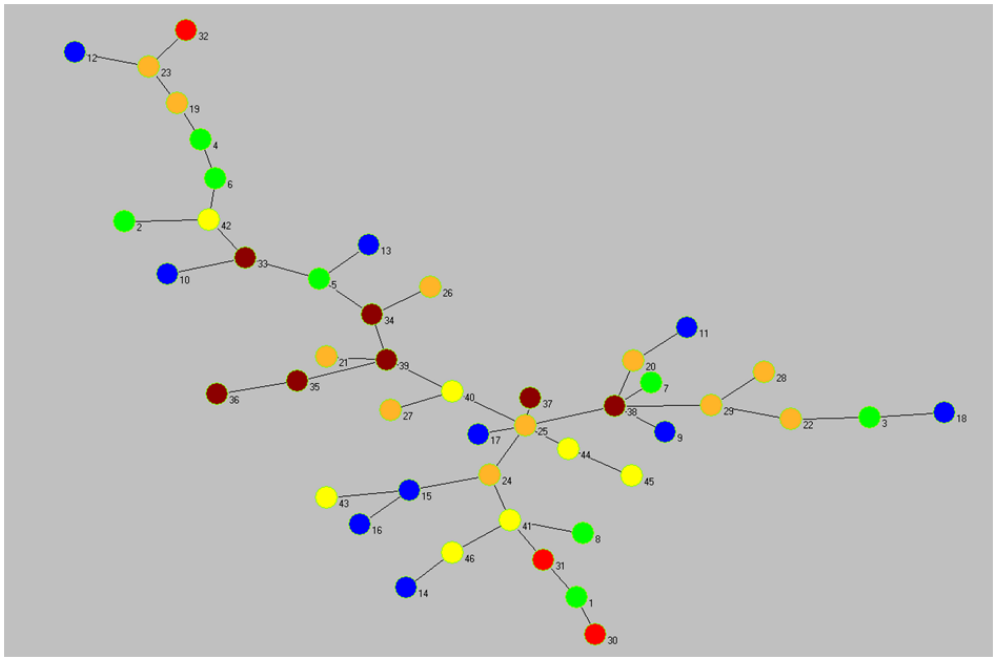
Figure 4.13: Edge sum of large cap stocks vs. time period

In figure (4.13), the plot is drawn taking edge sum of the MST of large cap stocks before, during and after the crisis in the y-axis and the three time periods before, during and after crisis is taken in the x-axis.The cost of edge sum, before the crisis, during the crisis and after the crisis is found to be 24.5211, 21.5774 and 27.2498 respectively.

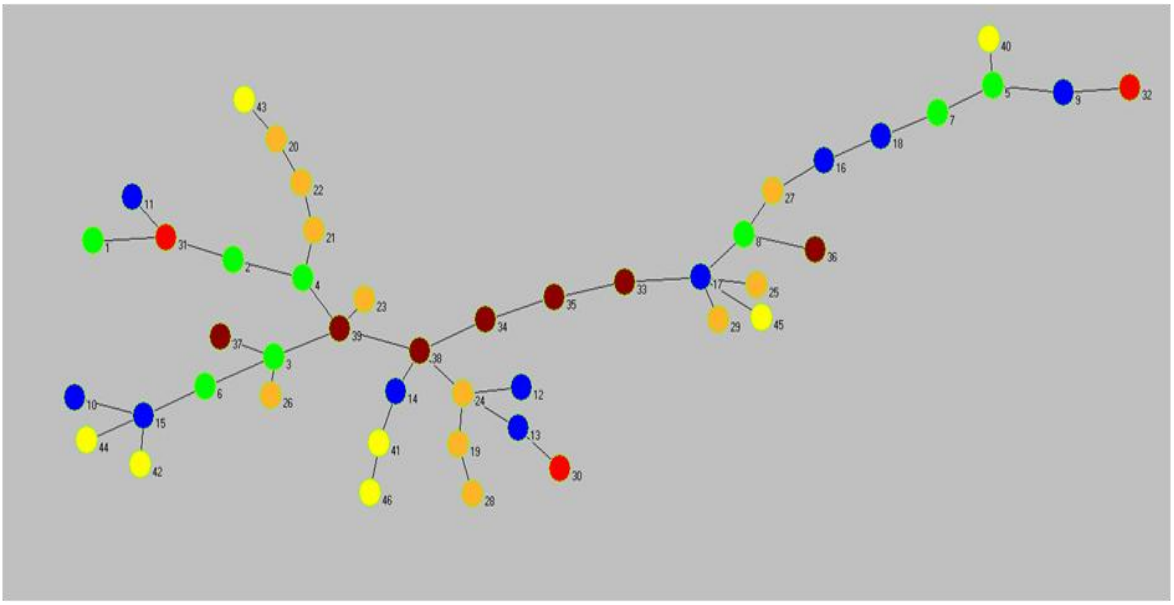
Mid Cap Companies



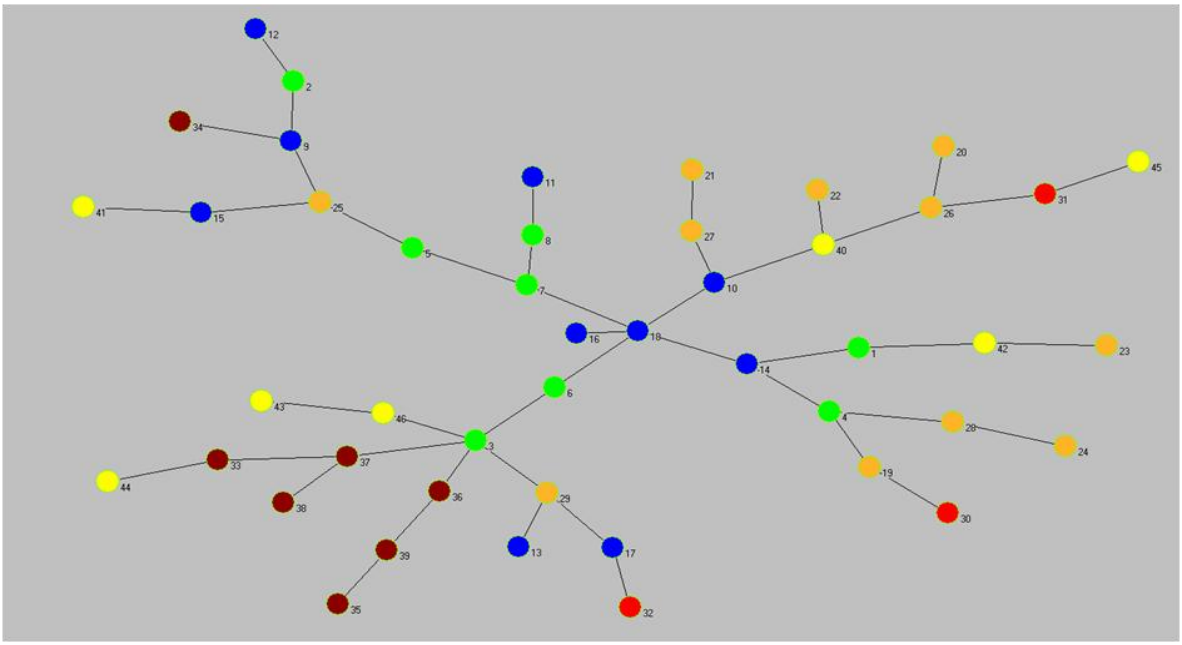
(a)



(b)



(c)



(d)

Figure 4.14: Represents MST of Mid cap stocks for (a) total time period, (d) During crisis, (b) before crisis and (c) after crisis

Figure (4.14 (a)) shows the graphical representation of the MST of overall data points $= 1295$. The sub figures (b), (c) and (d) represent the MST before, after and during crisis respectively. The color code of each node is given as per the industry sector they belong to are as follows: Text-

Green, Industrial-Blue, Chemical- Dandelion, Pharma- Red, Bank- Maroon, and Computer- Yellow. Each node is represented by a number. Referring to appendix-I we can find the name of the company corresponding to each number. From figure (4.13 (c) & (d)) the banking sector stocks form strong group during and after the crisis but the group behavior is less prominent in case of before crisis as shown in figure (4.13 (b)).

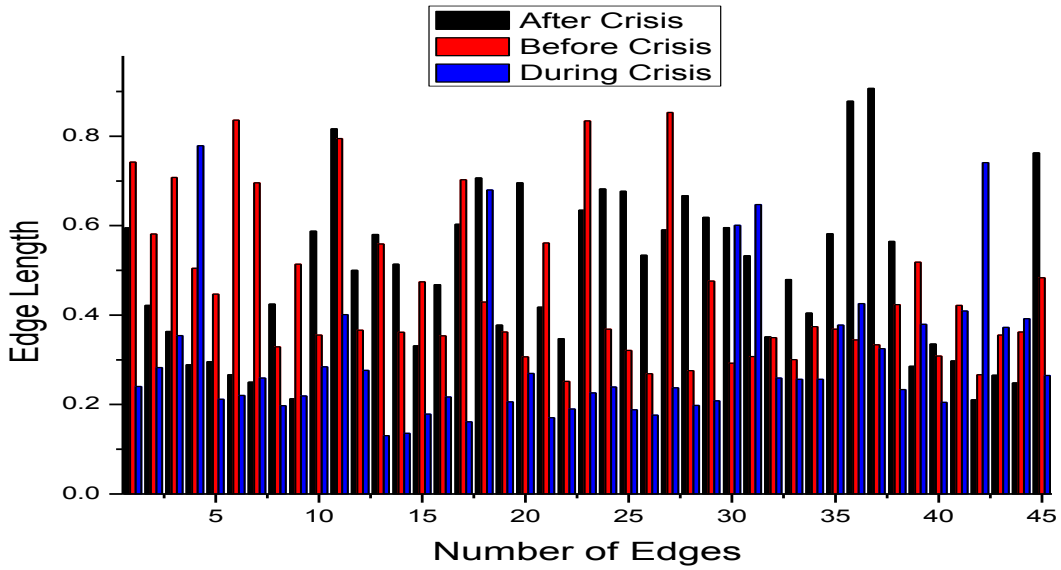


Figure 4.15 : Represents Edge length vs. Number of Edges of Mid Cap Stocks

In figure (4.15), the plot is drawn taking the number of edges of the MST of mid cap companies before, during and after a crisis in the x-axis and edge length or edge cost of the MST during, before and after the crisis in the y-axis. As the MST has 46 nodes so in the x-axis 45 numbers of edges have been considered for mid cap companies.

In order to analyze the effect of closeness of mid cap stocks we have added the edge cost of the MST for the three sets of time periods i.e. before the crisis, during the crisis and after a crisis. Edge sum of the MST of mid cap stocks before, during and after a crisis have been taken in the y-axis and the three time periods before, during and after a crisis is taken in the x-axis.

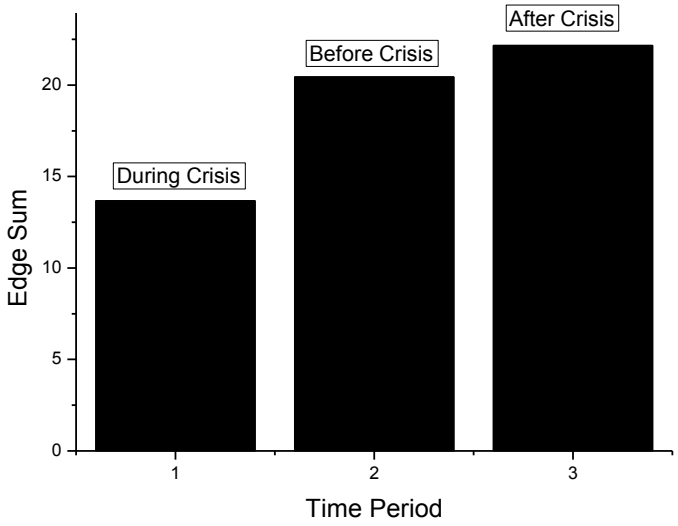
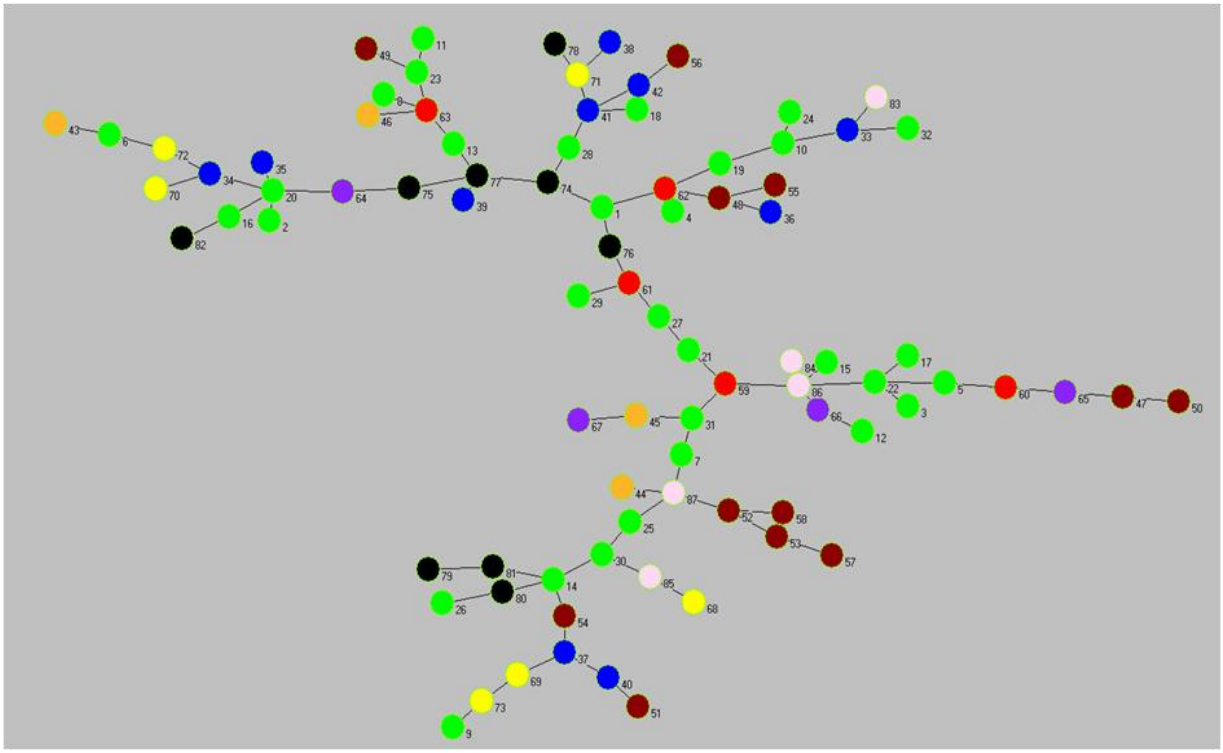


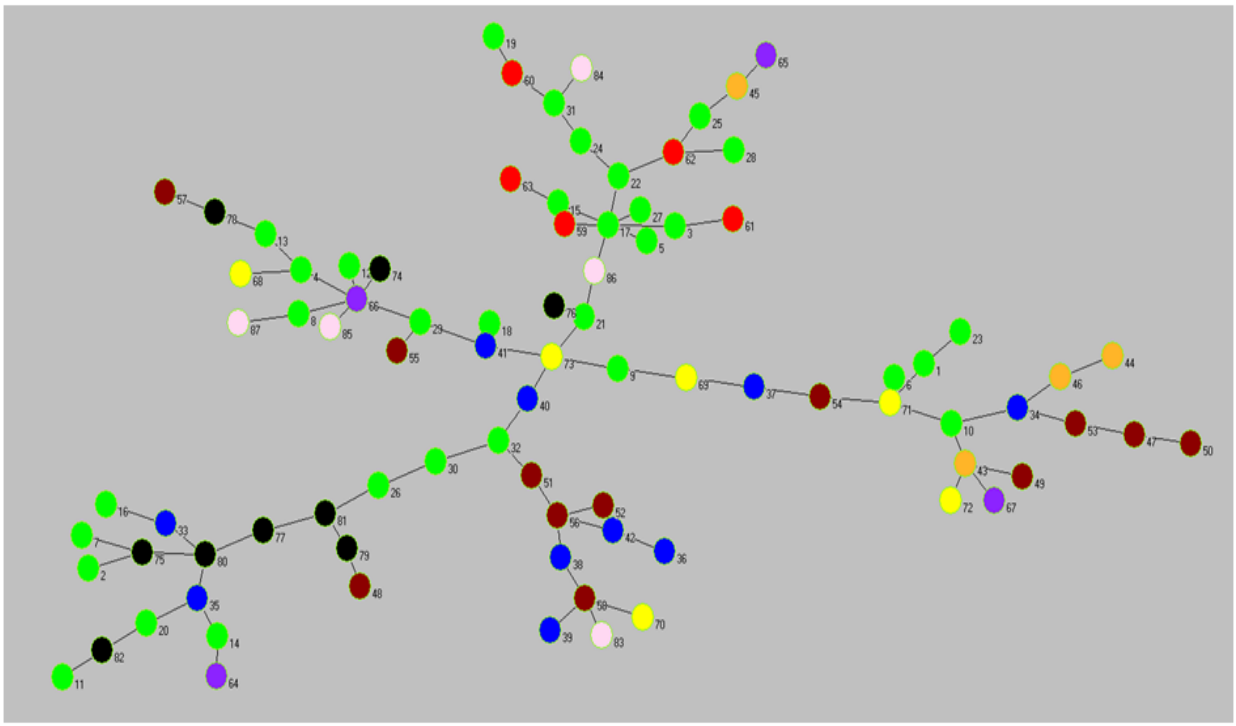
Figure 4.16: Edge sum of mid cap stocks vs. time period

In figure (4.16), the plot is drawn taking edge sum of the MST of mid cap stocks before, during and after a crisis in the y-axis and the three time periods before, during and after a crisis is taken in the x-axis. The cost of edge sum, before the crisis, during the crisis and after the crisis is found to be 20.4441, 13.6838 and 22.1702 respectively.

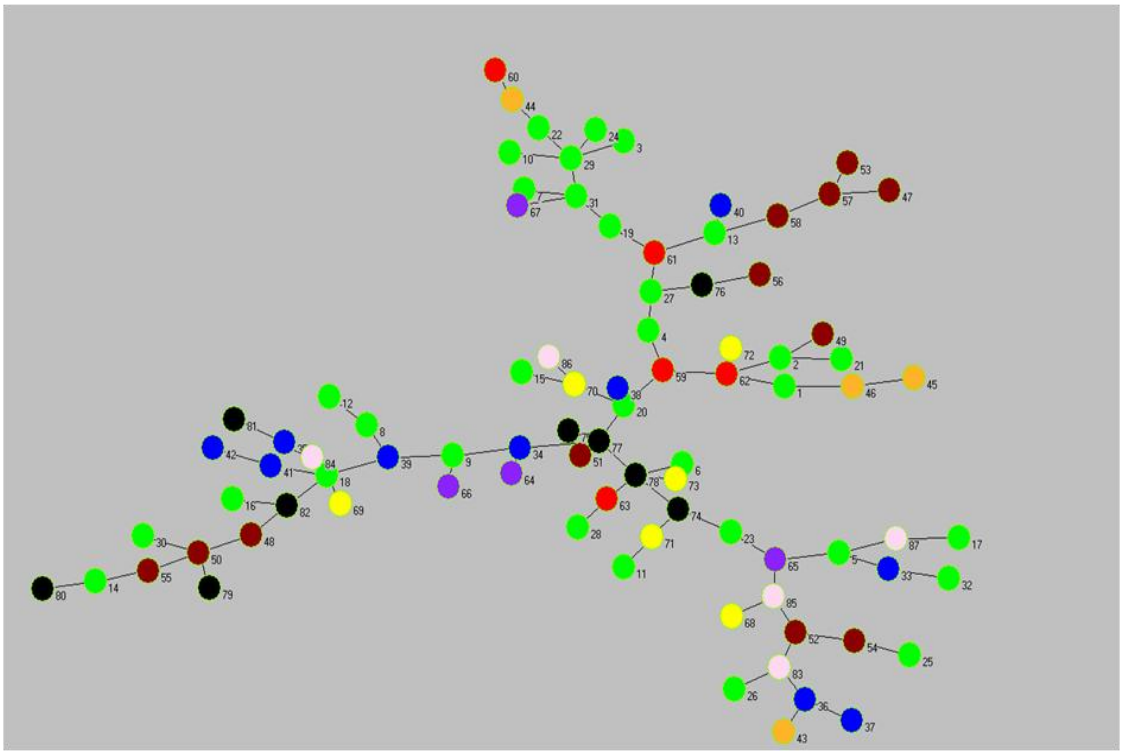
Small Cap Companies



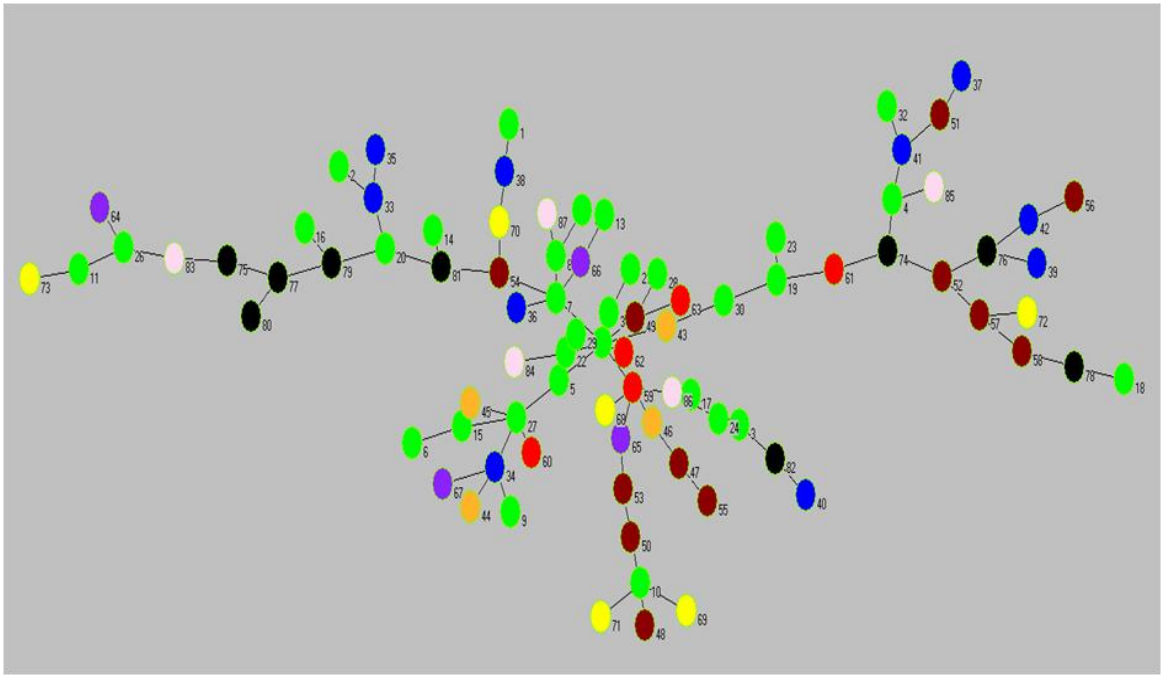
(a)



(b)



(c)



(d)

Figure 4.17: Represents MST of Small cap stocks for (a) total time period, (d) During crisis, (b) before crisis and (c) after crisis

Figure (4.17 (a)) shows the graphical representation of MST small cap companies and the overall data points $T = 1260$. The sub figures (b), (c) and (d) represent the MST before, during and after crisis respectively. The color of each node is given as per the industry sector they belong to are as follows: The color code and corresponding sectors of the stocks are as follows: Green-Industry, Blue-Textile, Dandelion-Automobile, Maroon/ Raw Sienna-Food, Red-Construction, Purple-Finance, Yellow-Pharma Black-Chemical, Pink-Energy. The stocks are represented by numbers. Referring to appendix- I we can find the name of the company corresponding to each number. For small cap companies there is no clear group structure among the stocks.

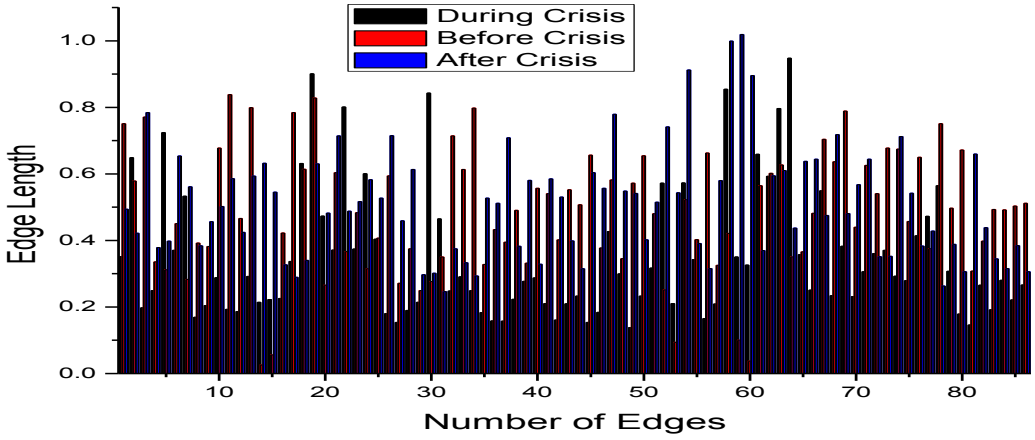


Figure 4.18: Represents Edge length vs. Number of Edges of Small Cap Stocks

In figure (4.18), the plot is drawn taking the number of edges of MST of small cap companies before, during and after the crisis in the x-axis and edge length or edge cost of the MST during, before and after the crisis in the y-axis. As the MST has 87 nodes so in the x-axis 86 number of edges have been considered for small cap companies. In order to analyze the effect of closeness of stocks we have added the edge cost of the MST for the three sets of time periods i.e. before the crisis, during the crisis and after the crisis.

In order to analyze the effect of closeness of small cap stocks we have added the edge cost of the MST for the three sets of time periods i.e. before the crisis, during the crisis and after a crisis. Edge sum of the MST of small cap stocks before, during and after a crisis have been taken in the y-axis and the three time periods before, during and after a crisis is taken in the x-axis.

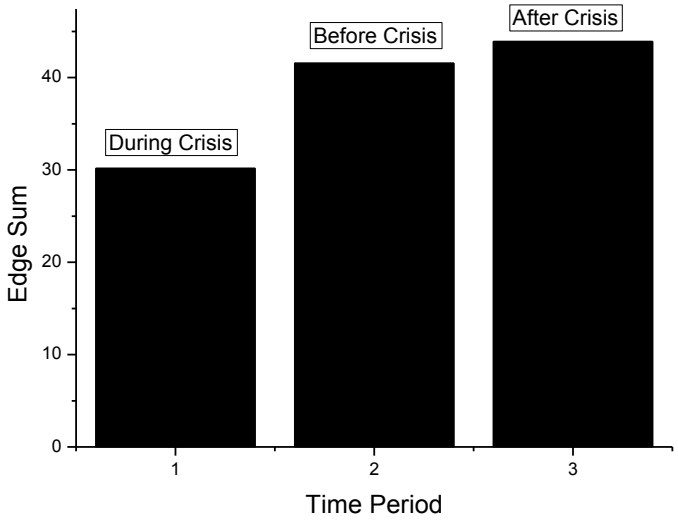


Figure 4.19: Edge sum of small cap stocks vs. time period

In figure (4.19), the plot is drawn taking edge sum of the MST of small cap stocks before, during and after a crisis have been taken in the y-axis and the three time periods before, during and after a crisis is taken in the x-axis. The cost of edge sum, before the crisis, during the crisis and after the crisis is found to be 41.5615, 30.1652 and 43.9121 respectively.

Summary of observations

1. From figure (4.13), (4.16) and (4.19) it is clear that during the crisis the sum of edge cost of the MST is smallest.
2. From figure (4.13), (4.16) and (4.19) it is clear that the sum of edge cost of the MST before crisis is also smaller than that found after crisis. So we can conclude that before the crisis the stocks are becoming closer to each other and during the crisis they become closer and after crisis they move away or they try to become independent from each other.
3. The behavior of stocks (small cap, large cap and mid cap) during the crisis, before the crisis and after the crisis is shown in the figure (4.20).

Table 4.1: Edge Sum of MST

	Large	Mid	Small
Before	24.5211	20.4441	41.5615
During	21.5774	13.6838	30.1652
After	27.2498	22.1702	43.9121

In table (4.1) we have taken the values of edge sum of large cap, mid cap and small cap companies before, during and after the crisis.

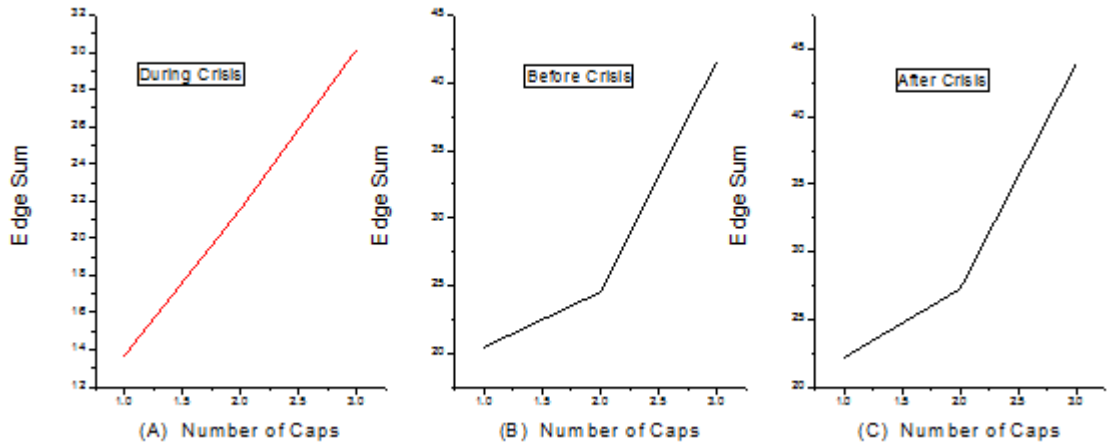


Figure 4.20: Edge sum vs. Number of caps

4. In figure (4.20) three plots are drawn for during, before and after the crisis. In the x-axis number of caps (here it is small cap, mid cap and large cap) and in the y-axis edge sum of MST of small cap, large and mid cap companies are taken during the crisis, before the crisis and after the crisis. During the crisis the edge sum of MST for small cap, large cap and mid cap companies show a linear trend as shown in figure (4.17 (a)) i.e. the stocks are coming close to each other in an equal proposition. But the edge sum of MST is almost same in case of before and after crisis for small cap, large cap and mid cap companies.
5. From figure (4.11 (c) and (d)) after and during crisis period the banking sector stocks of large cap companies form strong group but the group behavior is less prominent in case of before crisis as shown in figure (4.11 (b)). Similar patterns of behavior are obtained in case of mid cap banking sector companies. From figure (4.14 (c) & (d)) the banking sector

stocks of mid cap companies form strong group during and after the crisis but the group behavior is less prominent in case of before crisis as shown in figure (4.14 (b)).

6. By taking the best choice of threshold $c^{th} = 0.09$ the largest cluster is constructed for small cap, large cap and mid cap companies. Very few stocks form cluster but other sectors fail to form the cluster signifies the intra-group correlations are fragile in contrast to the market-wide correlation in the case of small cap , large and mid cap companies.

4.6 Summary and Conclusions

The minimum spanning tree method is used to represent the stock interactions of the BSE large cap, mid cap and small cap companies. From the tree structure very few stocks form clusters. In order to show clustering of stocks we have taken a threshold $c^{th} = 0.09$. From the experiment we find chemical and banking sectors of mid cap form clusters, industrial and chemical sectors of small cap companies form clusters and banking, software and energy sectors of large cap form clusters. The rest of the companies do not form clusters evidences intra-group correlations are fragile in contrast to the market-wide correlation in BSE companies. The Minimum spanning tree method is applied to analyze market crash. Before the crisis the stocks are becoming closer to each other and during the crisis they become closer and after the crisis they move away or they try to become independent from each other. This indicates during crises the bond length between stocks decrease i.e. the stocks become closer to each other. During crisis small cap, large cap and mid cap companies are equally affected. And the nature of stocks is similar before and after the crisis.

CHAPTER-5

QUANTUM COMPUTING

5.1 Introduction

Computation for, say, the cost of searching the database, market price prediction, financial forecasting, etc. strain the capabilities of the fastest and most powerful supercomputers. The difficulty is not so much that the microprocessors are too slow; it is that computers are inherently inefficient. Modern computers divide the task into elementary operations and then process it serially, one operation at a time. Computer designers have tried to devise parallel computing architecture, but the progress of designing such concurrent processing unit is slow and fitful. The main cause behind the failure is due to the inherently serial processing capability of microprocessors. In nature truly parallel computer is possible because of its inherent parallel processing capability. In principle such computing facilities do exist. They are called quantum computers. The fundamental unit of information in a quantum computer is a qubit. A qubit can exist in any superposition of states of $|0\rangle$ and $|1\rangle$. The inherent parallelism of quantum computer arises due to the superposition principles of quantum mechanics; this parallelism is the fundamental features of many quantum algorithms. Quantum computing is becoming popular due to its massive parallel information processing ability though it is facing problem to implement physical hardware. But the application of quantum computing in the field of computational intelligence can enrich the performance of solving a large number of complex problems. The concept of classical computing based on principles of quantum physics is first introduced by Benioff [14], Feynman [50] and formalized by Deutsch [40] in 1985. Quantum computing became popular after shore's factorization algorithm [152]. The motivation for applying quantum computing in the field of artificial neural computing is the natural step to make the generalization of classical Neurocomputing to quantum Neurocomputing in a hope to enhance the computational capabilities not available using classical Neurocomputing. Quantum computing has some truly surprising applications, which range from searching to artificial neural information processing. The objective of this chapter is to discuss possible applications of quantum computational algorithms in the financial market price prediction and searching an unsorted database. The application of quantum computing principles in the field of artificial neural network has been discussed in a hope to enhance the computational capabilities not available in classical neurocomputing

5.2 Working principle of Quantum Computation

Consider a classical register with 3 bits, then possible outcome will be 2^3=8 (it would be possible to use this register to represent any one of the numbers from 0 to 7 at any instance of time). If we will consider a register of 3 qubits, then the register can represent all the numbers from 0 to 7 simultaneously. A single processor having qubit registers will be able to perform calculations using all possible values of the input registers simultaneously. This phenomenon is called quantum parallelism. Unlike classical bits qubits can exist simultaneously as $|0\rangle$ and $|1\rangle$, with probability for each state given by numerical coefficients. A quantum computation [109] is a collection of the following three elements:

1. A register or a set of registers.
2. A unitary matrix, which is used to execute the quantum algorithms.
3. Measurements to extract information.

Hence quantum computation is the set $\{H, U, \{M_m\}\}$, where H is the Hilbert space of n-qubit register. $U \in U(2^n)$ is the quantum algorithm. $\{M_m\}$ is the set of measurement operators .Actual quantum computation processes are very different from that of classical counterpart. In classical computer we give input data through the input devices .The input signal is stored in computer memory, then fed into the microprocessor and the result is stored in memory before it is displayed on the screen. Thus the information travels around the circuit. In contrast information in quantum computation is first stored in a register and then external fields such as oscillating magnetic fields, electric fields or laser beams are applied to produce gate operations on the register. These external fields are designed so that they produce desired gate operation, i.e. unitary matrix acting on a particular set of qubits. Hence information sits in the register and they are updated each time the gate operation acts on the register.

5.2.1 Quantum Algorithm

Just like classical algorithms, quantum algorithms work with the probability distribution over various states. The idea to implement quantum mechanics for algorithmic tasks was introduced by Feynman [50, 51]. The most important quantum algorithms discovered to date all perform tasks for which there are no classical equivalents. Deutsch's algorithm [40] is designed to solve the problem of identifying whether a binary function is constant or balanced. Its running time is $O(n)$ while the classical method requires $O(2^n)$. Simon's algorithm [154] is designed for finding the periodicity in a 2-1 binary function that is guaranteed to possess a periodic element. Here exponential speed up is also achieved. Another famous algorithm called Grover's algorithm is meant for searching an unsorted database in time $O(\sqrt{N})$, where classical search algorithm has running time $O(N)$. This is an example of a

real world problem for which quantum algorithm provides the performance that is classically impossible. Finally the most important quantum algorithm is Shor's algorithm [152,153] for prime factorization. This algorithm finds the prime factors of very large numbers in polynomial time for which the best classical algorithm requires exponential time. The basic steps used in the quantum algorithms are as follows:

1. Initialize the quantum registers.
2. Put the registers in superposition states.
3. Evolve the registers using unitary operators.
4. Measure the states to get the result.

5.2.2 Quantum Oracle

Before going to discuss Deutsch algorithm let us first discuss the notion of quantum black box function. A black box function is modelled to calculate a subroutine of a problem. We are not interested to know details of the calculation in the subroutine. We simply model the function that is computed as the black-box. For certain problems quantum algorithm makes less call or queries to the black-box than any classical algorithm. Classically a black-box function is a box that evaluates an unknown function f .The input is n-bit string x and the output is given by m-bit string $f(x)$ .According to quantum mechanics such a box can only exist if it is reversible. To create a reversible box, the input x is output together with $f(x)$ and the black box looks like as shown in the figure. In order to make the box reversible, an additional m-bit input y is added and the output of the result is $f(x) \oplus y$, where $\oplus$ denotes bit wise addition modulo 2 operator .If $y = 0$ then the output is simply $f(x)$.

Figure-5.1: Quantum Black Box

In a quantum circuit (model of computation), first the qubits are initialized to known states, in the second step unitary transformation is performed on the qubits, finally qubits are measured. In the second step the dynamics that govern the evolution of the states with respect to time are unitary (i.e. unitary matrix). The unitary matrix is invertible, so the quantum computing is reversible. Hence the black box or oracle must be reversible. To create a reversible box, the input x is output together with $f(x)$ and the black box looks like as shown in the figure (5.1).

5.3 Deutsch's Algorithm

The study of financial market is highly sensitive, uncertain and unpredictable and depends upon many external factors. There is no well defined analytical procedure to predict the financial market. But Quantum Mechanics, based upon uncertainty and probability theory, have explained many physical phenomena most accurately. So introducing quantum concepts in financial domain may lead better approximation to make a good decision. Since there is no well-defined mathematical or analytical solution for financial market predictions, people generally apply numerical methods and computer simulation to get better results. The introduction of computer science to solve the financial problems is known as computational finance, where computer simulation tries to find out the most approximate result of a given problem. But the traditional classical computers have limited computational capacity with respect to the complexity of the problem. There are two major limitations of a classical computer which are space and time. The space complexity is the physical memory space required for computation. The time complexity is the time required by the CPU (central processing unit) to perform the basic operation. The space and time complexity can be resolved by using quantum computing depending upon the complexity of the problem. Quantum computer processes the information parallely (due to principle of super position) while the classical computer does it sequentially. So quantum computer posses inherent parallel computing capacity, which speed up the computational task. Similarly, the quantum computer provides exponential memory space with respect to classical computer. But the time complexity is the dominating feature to determine the complexity of an algorithm. Hence, we are silent about physical space and talked about time only. Quantum computing overcomes the drawback of classical computing and in some cases it provides faster solution. Hence it is worth to introduce quantum algorithms in the context of computational finance to solve financial problem in a faster manner

Statement of Deutsch's Algorithm

It is one of the earliest quantum algorithms which determine whether a function is constant or balanced in a single evaluation. Deutsch's algorithm is concerned with functions from the set $\{0,1\}$ to the set $\{0,1\}$. There will be four possible ways of mapping:

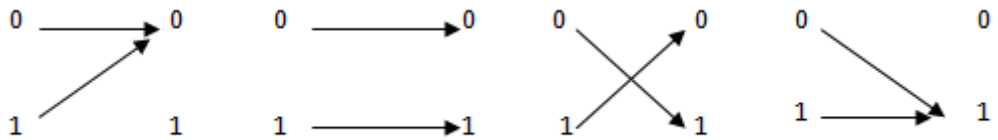


Figure 5.2: Balanced and Constant

A function $f\{0,1\} \rightarrow \{0,1\}$ is balanced if $f(0) \neq f(1)$ i.e. one to one and constant if $f(0) = f(1)$ .The job is to determine whether f is constant or balanced [89]. Classically it would take two evaluations of the function to tell whether it is one or the other. Quantum mechanically, we can answer this question in a single evaluation only. The reason for this is that quantum mechanically we can pack 0 and 1 into *x* at the same time. Note that for a classical algorithm to solve this problem with a success probability more than one half one has to query the black box twice, for both $f(0)$ and $f(1)$. Obviously, the worst-case time complexity of the best classical algorithm will take $2^n + 1$ time with different inputs.

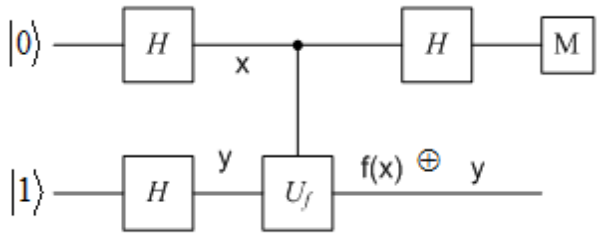


Figure 5.3: A circuit representation of quantum network that realizes Deutsch's algorithm

In figure (5.3) the circuit takes two inputs, $|1\rangle$ and $|0\rangle$, respectively. Each input independently subjected to the Hadamard gate. The operator U_f carries out the XOR (exclusive-OR) transformation.

$$U_f|xy\rangle = |x\rangle \otimes |y \oplus f(x)\rangle \qquad (5.1)$$

The symbol $\otimes$ represents the tensor product of two vectors. Function f maps $\{0,1\}$ to $\{0,1\}$. For a classical computer two measurements are required to determine f is balanced or constant.

H is the Hadamard operator defines as:

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \; and \;\; H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

The Hadamard gate in the upper qubit converts $|0\rangle$ to $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$.

Thus super position of states $|0\rangle$ and $|1\rangle$ act simultaneously to the input x.

5.3.1 Explanation of the Algorithm

The basic steps of execution of Deutsch's algorithm are given below.

1. The first pair of Hadamard gates produce:

$$|01\rangle \rightarrow \frac{1}{2}[(|0\rangle + |1\rangle)\otimes(|0\rangle - |1\rangle)] \tag{5.2}$$

2. The controlled-U_f gate is applied.

$$\begin{aligned} & U_f \frac{1}{2}[(|0\rangle + |1\rangle)\otimes(|0\rangle - |1\rangle)] \\ &= \frac{1}{2}\ U_f[(|0\rangle + |1\rangle)\otimes(|0\rangle - |1\rangle)] \\ &= \tfrac{1}{2}\ U_f(|0\rangle\otimes(|0\rangle - |1\rangle) + |1\rangle\otimes(|0\rangle - |1\rangle)) \end{aligned} \tag{5.3}$$

Now we will do little bit mathematics to simplify above equation:

$$U_f|x\rangle\otimes(|0\rangle - |1\rangle) = |x\rangle\otimes\big((|0\rangle - |1\rangle) \oplus f(x)\big) \tag{5.4}$$

If $f(x) = 0$ then

$$\begin{aligned} & \big((|0\rangle - |1\rangle) \otimes f(x)\big) = |0\rangle - |1\rangle \\ &= (-1)^0(|0\rangle - |1\rangle) = (-1)^{f(x)}(|0\rangle - |1\rangle) \end{aligned} \tag{.55}$$

Similarly, if $f(x) = 1$ then

$$[(|0\rangle - |1\rangle) \otimes f(x)] = [|1\rangle - |0\rangle] = (-1)^1[|0\rangle - |1\rangle]$$

$$= (-1)^{f(x)}\ [|0\rangle - |1\rangle] \tag{5.6}$$

From equations (5.5) and (5.6) it is clear that the same formula is valid for all values of $f(x)$.
So equation (5.4) will be

$$U_f|x\rangle\otimes(|0\rangle - |1\rangle) = |x\rangle\otimes(-1)^{f(x)}(|0\rangle - |1\rangle)$$

$$= (-1)^{f(x)}|x\rangle\otimes(|0\rangle - |1\rangle) \tag{5.7}$$

Applying equation (5.7) to equation (5.3)

$$U_f \frac{1}{2}[(|0\rangle + |1\rangle)\otimes(|0\rangle - |1\rangle)] = \frac{1}{2}\ U_f[(|0\rangle + |1\rangle)\otimes(|0\rangle - |1\rangle)]$$

$$= \frac{1}{2}\ U_f(|0\rangle\otimes(|0\rangle - |1\rangle) + |1\rangle\otimes(|0\rangle - |1\rangle))$$

$$= \frac{1}{2}\left[U_f|0\rangle\otimes(|0\rangle - |1\rangle) + U_f|1\rangle\otimes(|0\rangle - |1\rangle)\right]$$

$$= \frac{1}{2}\left[(-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle\right]\otimes(|0\rangle - |1\rangle) \tag{5.8}$$

3. Finally the Hadamard gate is applied to the 1st vector of equation (5. 8)

$$\frac{1}{2}\left(\left((-1)^{f(0)}H|0\rangle + (-1)^{f(1)}H|1\rangle\right)\otimes(|0\rangle - |1\rangle)\right)$$

$$= \frac{1}{2}\left((-1)^{f(0)}\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) + (-1)^{f(1)}\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)\right)\otimes(|0\rangle - |1\rangle)$$

$$= \frac{1}{2}\left(|0\rangle\left((-1)^{f(0)} + (-1)^{f(1)}\right) + |1\rangle\left((-1)^{f(0)} - (-1)^{f(1)}\right)\right)\otimes\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \quad (5.9)$$

When $f(x)$ is balanced then $(-1)^{f(0)} + (-1)^{f(1)} = 0$ this reduces the upper qubit to equation (5.10).

$$\frac{1}{2}\left(|1\rangle\left((-1)^{f(0)} - (-1)^{f(1)}\right)\right) = \pm|1\rangle \quad (5.10)$$

4. Hence to determine the function $f(x)$ is constant or balanced we have to measure the upper qubit vector. If it is $|0\rangle$ then $f(x)$ is a constant, if it is $|1\rangle$ then $f(x)$ is a balanced function.

5.3.2 Mathematical Analysis of Market Price Prediction

Let us suppose we have developed a function $f(x, M)$ that can predict whether the market price of a given financial instrument will increase or decrease after certain time period (let us take after one day). In principle there are many factors which affect the market price of a given financial instrument. Here x is the most significant variable (factor) which is responsible to predict future market price and for mathematical simplification we have taken *x* as a binary variable (either 0 or 1). The output of the function is binary i.e. 0 and 1. The variable M includes relevant market data (which is taken constant for a given market day). So we can write $f(x, M) = f(x)$. The objective is to show how Deutsch's algorithm can reduce the number of calculation steps.

The function works as follows:

1. If $f(0) = f(1)$ then market price of the instrument will increase tomorrow.
2. If $f(0) \neq f(1)$ then market price of the instrument will decrease tomorrow.

If we solve this problem in a traditional way it works as follows:

1. Calculate $f(x)$ for $x = 0$ and $x = 1$
2. Compare, if $f(0) = f(1)$ then market price of the instrument will increase tomorrow.
3. If $f(0) \neq f(1)$ then market price of the instrument will decrease tomorrow.

The problem in this method is that the function $f(x)$ is very complex in nature and the calculation of $f(x)$ for the value 0 and 1 will take a major time. Suppose we engage a single super-computer to calculate first $f(0)$ then $f(1)$ and compare the results to get a conclusion it takes more than one day to do this task. So we are not able to forecast the market price of the instrument for tomorrow. Hence our financial problem reduced to a computational problem and

its solution depends upon the computing power of the processor and the efficiency of the algorithm.

We are facing problem while calculating both $f(0)$ and $f(1)$ simultaniously in order to compare their values. The problem will be solved if we can calculate the joint property of $f(0)$ and $f(1)$. Quantum computer is able to answer about the joint properties. Define an operator U that transforms a 2-qubit register as follows:

$$\mathrm{U}: |x\rangle|y\rangle \ \rightarrow |x\rangle|y \oplus f(x)\rangle \tag{5.11}$$

The symbol $\oplus$ is the *XOR* operator.

As our objective is to find the value of both functions simultaneously at both inputs $|0\rangle$ and $|1\rangle$ .So we need to work with the super positions of both the states. So initialize the second qubit with the equal superposition of states $|0\rangle$ and $|1\rangle$ with $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$.

$$U: |x\rangle \oplus \frac{|0\rangle - |1\rangle}{\sqrt{2}} \rightarrow |x\rangle \oplus \frac{|0 \oplus f(x)\rangle - |1 \oplus f(x)\rangle}{\sqrt{2}} \tag{5.12}$$

Now put the possible values of $f(x)$ in the right side of equation (5.12)

$$\left.\begin{array}{l} \text{If } f(x) = 0 : \text{then } |x\rangle \oplus \frac{|0\oplus 0\rangle - |1\oplus 0\rangle}{\sqrt{2}} = |x\rangle \oplus \frac{(-1)^0(|o\rangle - |1\rangle)}{\sqrt{2}} \\ \text{If } f(x) = 1\text{: then } |x\rangle \oplus \frac{|0\oplus 1\rangle - |1\oplus 1\rangle}{\sqrt{2}} = |x\rangle \oplus \frac{(-1)^1(|o\rangle - |1\rangle)}{\sqrt{2}} \end{array}\right\} \tag{5.13}$$

Equation (5.13) can be written as follows:

$$|x\rangle \oplus \frac{|0 \oplus f(x)\rangle - |1 \oplus f(x)\rangle}{\sqrt{2}} = |x\rangle \oplus \frac{(-1)^{f(x)}(|o\rangle - |1\rangle)}{\sqrt{2}} \tag{5.14}$$

Equation (5.14) is the result of Deutsch's algorithm. The operator U allows us to change the sign of the second qubit. Now if we allow the 1st qubit to be in equal super position of $|0\rangle$ and $|1\rangle$, we will be able to use the fact that U only needs to evaluate the function once to give properties of $f(0) and\ f(1)$.

Apply Hadamard operator to the state $|0\rangle$ and apply U operator, we get:

$$U: \frac{|0\rangle + |1\rangle}{\sqrt{2}} \oplus \frac{|0\rangle - |1\rangle}{\sqrt{2}} \rightarrow \frac{|0\rangle}{\sqrt{2}} \otimes \frac{(-1)^{f(0)}(|o\rangle - |1\rangle)}{\sqrt{2}} + \frac{|1\rangle}{\sqrt{2}} \otimes \frac{(-1)^{f(1)}(|o\rangle - |1\rangle)}{\sqrt{2}}$$

$$= \left[\frac{(-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle}{\sqrt{2}}\right] \otimes \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

If we apply Hadamard operator to the 1st qubit again, we get

$$\left[\frac{\left((-1)^{f(0)} + (-1)^{f(1)}\right)|0\rangle + \left((-1)^{f(0)} - (-1)^{f(1)}\right)|1\rangle}{2}\right] \otimes \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

If we measure the 1st qubit according to postulate-3, we get:

$|0\rangle$ with probability $\left|\frac{(-1)^{f(0)}+(-1)^{f(1)}}{2}\right|^2$ and $|1\rangle$ with probability $\left|\frac{(-1)^{f(0)}-(-1)^{f(1)}}{2}\right|^2$

Hence if $f(0) = f(1)$ we get the state $|0\rangle$ with probability 1 and if $f(0) \neq f(1)$ we get state $|1\rangle$ with probability1. Therefore, using Deutsch algorithm, we only need to evaluate the function $f(x)$ once and get the result that allows us to determine with certainty if the value of $f(x)$ is equal for the two inputs or if $f(x)$ is not equal.

Therefore, using Deutsch's algorithm, we only need to evaluate the function $f(x)$ once and get the result that allows us to determine with certainty if the value of $f(x)$ is equal for the two inputs or if $f(x)$ is not equal. In this section we have discussed working principle of quantum computer, quantum algorithm and a possible application of Deutsch's algorithm to financial market price prediction. Here we have discussed an example that will predict the market price of a financial instrument evaluating the function only once using Deutsch's algorithm. Thus it is computationally much faster than the classical approach. This algorithm solves our problem efficiently with less time.

5.4 Quantum Search Algorithm

Efficient data base search is highly required for solving different business problems. In recent years quantum algorithms, like Grover's algorithm for unstructured database search [138, 59,60], has successfully explained the problem exponentially faster than their classical counterparts .Which is based upon the principles of quantum computing [114, 62, 134, 109], is meant for searching an unsorted database in O ($\sqrt{N}$) times, where as a classical search algorithm takes O (N) times. Searching is a real world problem for which quantum algorithm provides the performance which was classically impossible. The quantum search technique takes the advantage of quantum parallelism to construct superposition of all possible values of the states and then increase the probability amplitude of the solution state. This parallelism provides the distinction between classical and quantum algorithmic strategies. The objective of a classical algorithm is to reduce the amplitude of non target states, whereas a quantum search algorithm tries to amplify the amplitude of the target states. Here the term amplification indicates to increase the probability of the target state.

Information storage and retrieval are the most important applications of database system. In a brute force search technique the worst case time complexity to locate an item is $O(N)$ where, N is the size of the data base. Binary search technique is an improved algorithm for searching which is based upon sorting. The worst case time complexity to locate

an item using this technique is $O(\log N)$. Suppose the telephone number of a person is the *key* instead of his *name* and we want to find the name of the person. In this case binary search is not possible to search this unsorted database. The major drawback of binary search is the sorting then searching. In the real world we have certain problems which have to be searched in an efficient way without sorting. Is there any technique which can search more efficiently than the best known classical search techniques with less computing time? The answer is a new search algorithm which is based upon principles of quantum computing, the computing inspired by quantum mechanics [41]. The main idea of quantum search technique is to rotate the state vector from an initial state to a final state (target) in a two-dimensional Hilbert space. The algorithm is iterative and each iteration causes the same amount of rotation. The speedup is achieved due to the quantum parallelism and the probability, which is the square of the amplitude. The Grover's algorithm is based upon two transformations: Inversion about the mean and Phase inversion. Repeated applications of these two transformations increase the probability amplitude of the desired state with high accuracy.

5.4.1 Phase Inversion and Inversion about the Mean

Suppose we have a set of integers $X = \{x_i\}$ where i $= 1, 2 \ldots .$ N, with mean

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad (5.15)$$

The classical inversion about the mean is denoted by Y, transforms x_i to $x_i{}'$:

$$Y: x_i \rightarrow x_i{}' \; with$$
$$x_i{}' = \bar{x} + (\bar{x} - x_i) = 2\bar{x} - x_i \qquad (5.16)$$

Example: 1 (Inversion about the Mean)

Suppose we have a set of eight integers

$$\mathrm{X} = \{34, 66, 47, 63, 54, 28, 42, 48\}$$

The average is

$$\bar{x} = \frac{1}{8}\sum_{i=1}^{8} x_i = 49$$

Graphically representation of eight integers with average 49:

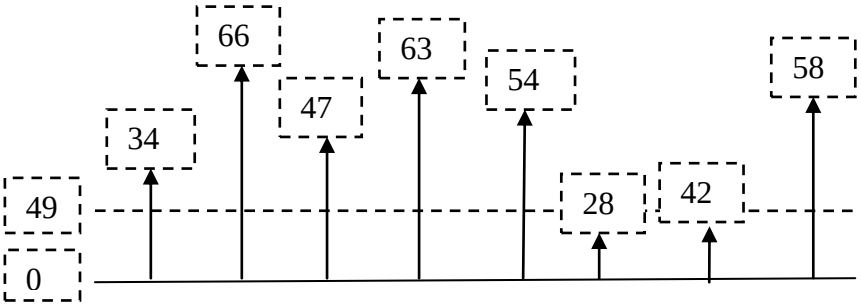


Figure 5.4: Set of eight integers with average 49

In figure (5.4) the amplitude of vertical bars is proportional to the modulus of the numbers. The transformation of individual integers about the mean is $\bar{x} = 49$

The transformation of individual integers is given below:

$34 \rightarrow 98 - 34 = 64$,$66 \rightarrow 98 - 66 = 32$, $47 \rightarrow 98 - 47 = 51$, $63 \rightarrow 98 - 63 = 35$

$54 \rightarrow 98 - 54 = 44$,$28 \rightarrow 98 - 28 = 70$, $42 \rightarrow 98 - 42 = 56$, $58 \rightarrow 98 - 58 = 40$

Graphical representation of inversion about the mean:

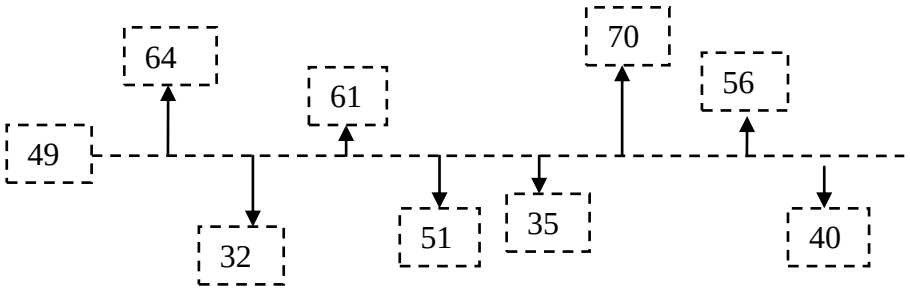


Figure 5.5: Graphical Representation of Inversion about Mean

Example: 2 (Inversion about the Mean and Phase Inversion)

Next, let us level the N items with complex numbers of equal amplitude $x_i = \frac{1}{\sqrt{N}}$ and impose the condition that after each transformation the sum of the squares of the modulus of the complex numbers to be one. Suppose there are eight numbers and we want to search sixth number. Assume that there is an oracle (a black box) which is capable to invert the phase of the complex number and identify the item we are searching for. Gradually we will notice that the repeated application of phase inversion and inversion about the mean will amplify the amplitude of the desired state and will decrease the amplitude of other states.

Step-0

Initially all the 8 numbers are of equal amplitude shown in the figure below.

Graphical representation of eight complex numbers with equal amplitude $\frac{1}{\sqrt{8}}$:

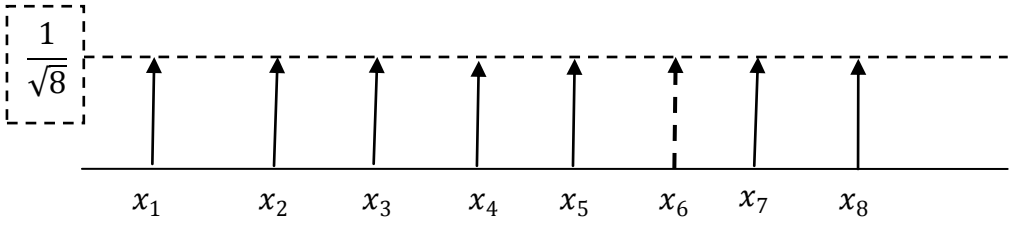


Figure 5.6: Graphical representation of eight complex numbers with equal amplitude $\frac{1}{\sqrt{8}}$

Graphical representation of Phase inversion of the sixth element:

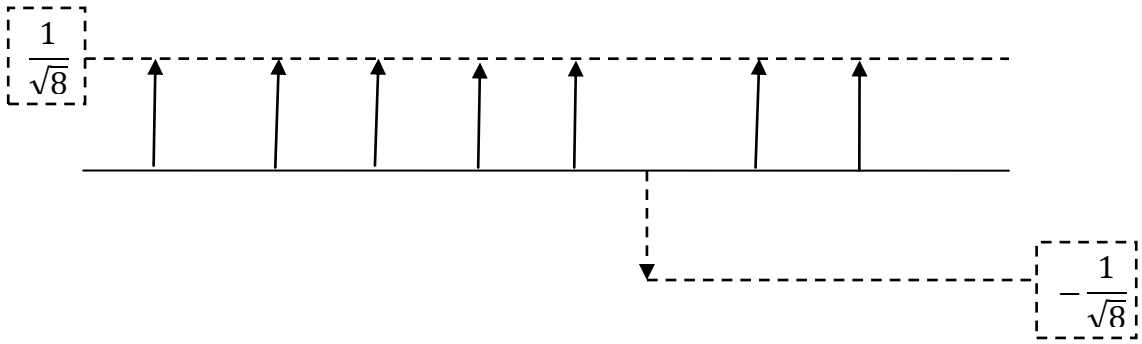


Figure 5.7: Graphical representation of Phase inversion of the sixth element

Step-1

Calculate the new mean from the phase inversion figure (5.7) of Step-0

$$\bar{x} = \frac{1}{8}\sum_{i=1}^{8} x_i = \frac{1}{8}\left[7 \times \frac{1}{\sqrt{8}} - \frac{1}{\sqrt{8}}\right] = \frac{3}{4\sqrt{8}} \tag{5.17}$$

Calculate the new inversion about the mean using the formula using equation (5.16)

$$x_1{}' = x_2{}' = x_3{}' = x_4{}' = x_5{}' = x_7{}' = x_8{}'$$
$$= 2 \times \frac{2}{4\sqrt{8}} - \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{8}} \tag{5.18}$$

And

$$x_6{}' = 2 \times \frac{3}{4\sqrt{8}} + \frac{1}{\sqrt{8}} = \frac{5}{2\sqrt{8}} \tag{5.19}$$

The inversion about the mean amplifies the amplitude of the sixth item we are searching for and reduce the amplitude of the rest seven items.

This can be visualized graphically as follows:

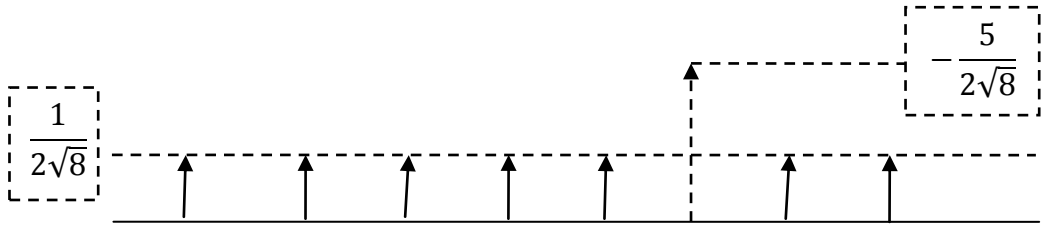


Figure 5.8: The inversion about the mean amplifies the amplitude of the sixth item

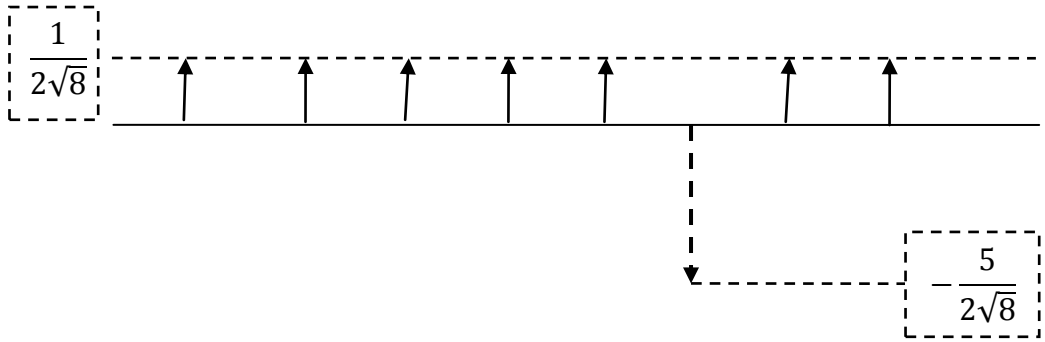


Figure 5.9: The phase inversion of the sixth item

Again make a second time phase inversion of the sixth item which is shown graphically in figure (5.9).

Step-2

When we are going for a second phase inversion of the sixth item we have to find a new inversion about the mean. Hence the new mean will be:

$$\bar{x}' = \frac{1}{8}\sum_{i=1}^{8} x_i = \frac{1}{8}\left[7 \times \frac{1}{2\sqrt{8}} - \frac{5}{2\sqrt{8}}\right] = \frac{1}{8\sqrt{8}} \tag{5.20}$$

The inversion about the mean using the formula will be

$$x_i'' = x_2'' = x_3'' = x_4'' = x_5'' = x_7'' = x_8''$$

$$= 2 \times \frac{1}{8\sqrt{8}} - \frac{1}{2\sqrt{8}} = -\frac{1}{4\sqrt{8}} \tag{5.21}$$

$$x_6'' = 2 \times \frac{1}{8\sqrt{8}} + \frac{5}{2\sqrt{8}} = \frac{11}{4\sqrt{8}} \tag{5.22}$$

The second time inversion about the mean amplifies the amplitude of the sixth item more than previous and reduces more the amplitude of the rest of the seven items than before. This can be visualized graphically as below:

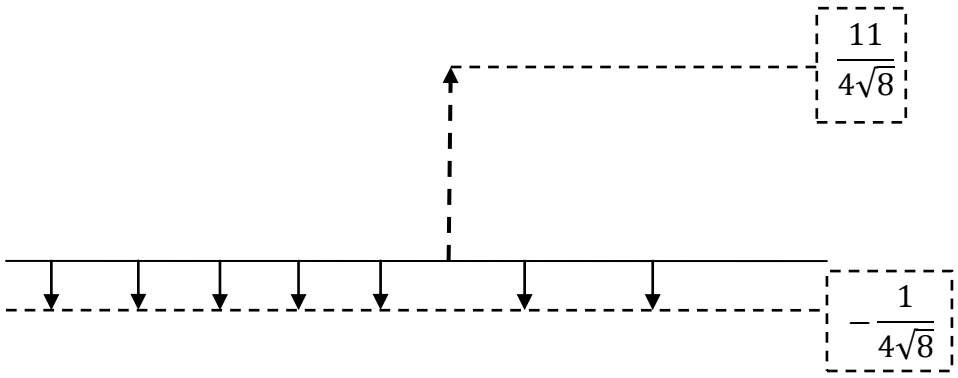


Figure 5.10: Second time inversion about the mean of the sixth item

From the above discussion we can analyze, without going into the detail of the Grover search algorithm, the effect of inversion about the mean with phase inversion considering a set of eight complex numbers x_i to be the probability amplitude of the basis states of the Hilbert space. The initial state of a system with eight elements:

$$\begin{aligned} |\Psi\rangle &= x_1|000\rangle \\ &+ x_2|001\rangle + x_3|010\rangle + x_4|011\rangle \\ &+ x_5|100\rangle + x_6|101\rangle + x_7|110\rangle \\ &+ x_8|111\rangle \end{aligned} \tag{5.23}$$

The probability amplitude of the sixth element $|101\rangle$ is $1/8 = 0.125$

After two iterations of phase inversion followed by inversion about the mean the final state will be:

$$\begin{aligned} |\Psi\rangle'' &= x_1''|000\rangle + x_2''|001\rangle + x_3''|010\rangle + x_4''|011\rangle \\ &+ x_5''|100\rangle + x_6''|101\rangle + x_7''|110\rangle \\ &+ x_8''|111\rangle \end{aligned} \tag{5.24}$$

If we measure the probability amplitude of the sixth element $|101\rangle$ will be $\left(\frac{11}{4\sqrt{8}}\right)^2 = \frac{121}{128} = 0.97227$ .Hence in conclusion the concept of inversion about mean with phase inversion increases the probability amplitude of the desired state with decreasing amplitude of other

states. In other words the probability of finding the desired state is tending towards one compared with other states.

5.4.2 Grover's Search Method

Let us consider an unstructured database with $N = 2^n$ elements which are numbered 0 to $N - 1$. Grover's algorithm uses two registers where the first register contains n-qubits to hold the address of the *label* in binary form. The second register holds the oracle qubit. Apply n-dimensional Hadamard gate to create $|\Psi\rangle$, an equal superposition of the indices of all items in the database. Apply Grover operator G for $O(\sqrt{N})$ times on $|\Psi\rangle$ .Where G consists of oracle U_f followed by an inversion about the mean D_Ψ.Hence $G = D_\Psi U_f = (2|\Psi\rangle\langle\Psi| - I\ )U_f$.Finally measure the state $|\Psi\rangle$.

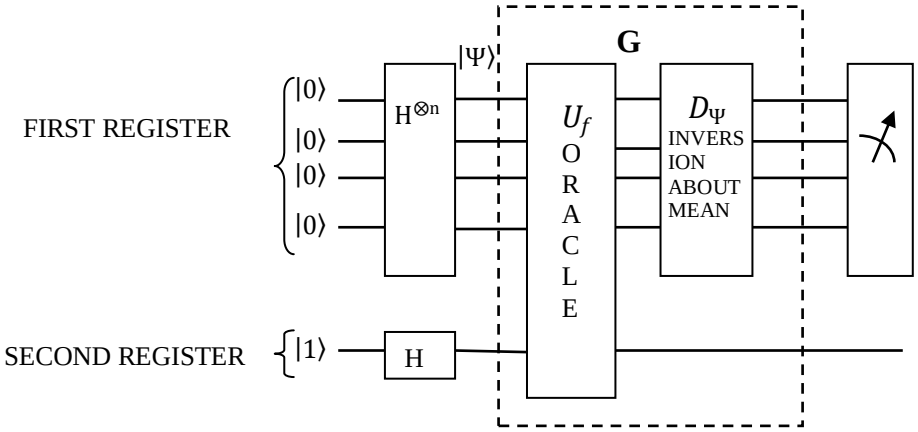


Figure 5.11: The schematic representation of Grover's search algorithm

In figure (5.11) an n-dimensional Hadamard transform produces, $|\Psi\rangle$, an equal superposition of the indices of all items in the data base. Then we carry out $\sqrt{N}$ Grover iterations, G. The Grover iteration G consists of the transformations performed by the oracle, U_f, followed by an inversion about the mean, D_Ψ.Finally we measure the result.

Step-0

Initialize first register and second register in the state $|0\rangle^{\otimes n}$ and $|1\rangle$ respectively.

Step-1

Apply n-dimensianl Hadamard gate to first and 1-dimensianl Hadamard gate to second register respectively.

Step-2

Apply Grover operator G for $O(\sqrt{N})$ times on step-1

(a) Apply oracle U_f (Phase inversion)

(b) Apply $D_\Psi = 2|\Psi\rangle\langle\Psi| - I$ (Amplitude amplification)

Step-3

Measure the result

5.4.3 Application in four Qubit System

In this section we will discuss the working of Grover's search algorithm with an example. We have taken a data base of 16 elements and we are searching the eleventh record from the given database. Let $N = 2^4 = 16$, so we need a register of length $n = 4$ bits. The number of iterations $k = \text{round}\left(\frac{\pi}{4}\sqrt{N}\right) = round\left(\frac{\pi}{4}\sqrt{16}\right) = 3$ .Refer Grover circuit in figure (5.11) and Grover algorithm steps for execution of the problem.

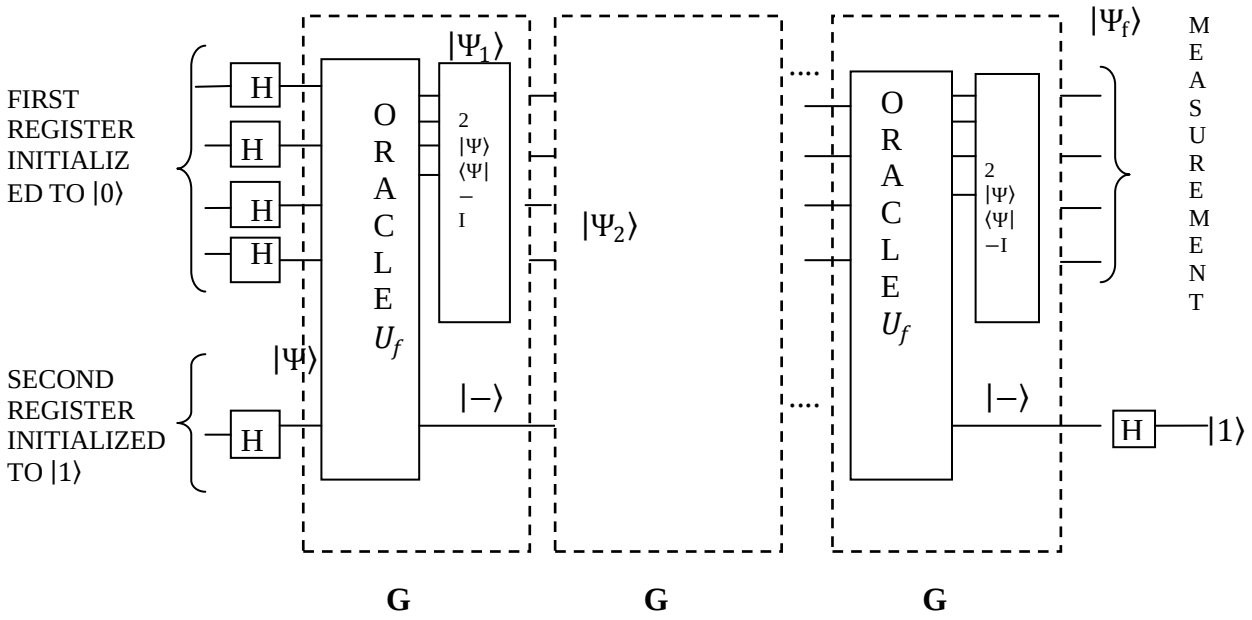


Figure 5.12: Grover Circuit for $n = 4$

We will start by putting four qubits in the first register and one qubit in the second register respectively.

Step-0

Then initialize the first register to be in the state $|0000\rangle$ and second register to be $|1\rangle$

Step-1

Apply the Hadamard operator to the first register:

$$|\Psi\rangle = H^{\otimes 4}\,|0000\rangle = (H|0\rangle)^{\otimes 4}$$

$$= \left[\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right]^{\otimes 4} = \frac{1}{\sqrt{2^4}} \sum_{x=0}^{2^4-1} |x\rangle = \frac{1}{\sqrt{16}} \sum_{x=0}^{15} |x\rangle$$

$|\Psi\rangle = \frac{1}{\sqrt{16}}\{|0000\rangle + |0001\rangle + |0010\rangle + |0011\rangle + |0100\rangle + |0101\rangle + |0110\rangle + |0111\rangle +$

$1000+1001+1010+1011+1100+1101+1110+1111$

Apply the Hadamard operator to the second register:

$$|-\rangle = H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

Step-2

Apply Grover operator G for three times and stop when $|1011\rangle = 11$ is reached:

We want to find the 11^{th} item that is $|1011\rangle = 11$

So we will modify the state $|\Psi\rangle$ as follows:

$$|\Psi\rangle = \frac{1}{\sqrt{16}}\left[\sum_{\substack{x=0 \\ x\neq 11}}^{15} |x\rangle + |1011\rangle\right] = \frac{1}{\sqrt{16}}\left[\sqrt{15}|\Phi\rangle + |1011\rangle\right]$$

Where,

$$|\Phi\rangle = \frac{1}{\sqrt{15}} \sum_{\substack{x=0 \\ x\neq 11}}^{15} |x\rangle$$

$$= \frac{1}{\sqrt{15}} [\,|0000\rangle + |0001\rangle + |0010\rangle + |0011\rangle + |0100\rangle + |0101\rangle + |0110\rangle + |0111\rangle$$
$$+ |1000\rangle + |1001\rangle + |1010\rangle + |1100\rangle + |1101\rangle + |1110\rangle + |1111\rangle]$$

Therefore

$$|\Psi\rangle = \frac{\sqrt{15}}{\sqrt{16}}|\Phi\rangle + \frac{1}{\sqrt{16}}|1011\rangle$$

<u>First iteration</u>

Apply the oracle operator U_f :

$$|\Psi_1\rangle|-\rangle = U_f(|\Psi\rangle|-\rangle) = \frac{\sqrt{15}}{\sqrt{16}}U_f[|\Phi\rangle|-\rangle] + \frac{1}{\sqrt{16}}U_f[|1011\rangle|-\rangle]$$

$$= \frac{\sqrt{15}}{\sqrt{16}}|\Phi\rangle|-\rangle - \frac{1}{\sqrt{16}}|1011\rangle|-\rangle$$

Hence

$$|\Psi_1\rangle = \frac{\sqrt{15}}{\sqrt{16}}|\Phi\rangle - \frac{1}{\sqrt{16}}|1011\rangle$$

$$|\Psi_1\rangle = \frac{1}{\sqrt{16}}\{\,|0000\rangle + |0001\rangle + |0010\rangle + |0011\rangle + |0100\rangle + |0101\rangle + |0110\rangle + |0111\rangle$$
$$+ |1000\rangle + |1001\rangle + |1010\rangle - |1011\rangle + |1100\rangle + |1101\rangle + |1110\rangle$$
$$+ |1111\rangle\,\}$$

Using state $|\Phi\rangle$ we can write $|\Psi_1\rangle$ as:

$$|\Psi_1\rangle = \frac{1}{4}\sum_{\substack{x=0 \\ x\neq 11}}^{15}|x\rangle - \frac{1}{4}|1011\rangle$$

Apply the inversion operator $(2|\Psi\rangle\langle\Psi| - I)$:

$$|\Psi_2\rangle = [2|\Psi\rangle\langle\Psi| - I]|\Psi_1\rangle = \frac{3}{4}|\Psi\rangle + \frac{1}{2}|1011\rangle$$

$$\Rightarrow |\Psi_2\rangle = \frac{3}{16}\sum_{\substack{x=0 \\ x\neq 11}}^{15}|x\rangle + \frac{11}{16}|1011\rangle$$

Similarly the second and third iterations will be carried out as follows:

Second iteration

Apply the oracle operator U_f :

$$|\Psi_3\rangle|-\rangle = U_f(|\Psi_2\rangle|-\rangle) = \frac{3}{4}|\Psi\rangle - \frac{7}{8}|1011\rangle$$

$$= \frac{3}{16}\sum_{\substack{x=0 \\ x\neq 11}}^{15}|x\rangle - \frac{11}{16}|1011\rangle$$

Apply the inversion operator $(2|\Psi\rangle\langle\Psi| - I)$:

$$|\Psi_4\rangle = [2|\Psi\rangle\langle\Psi| - I]|\Psi_3\rangle = \frac{5}{64}\sum_{\substack{x=0 \\ x\neq 11}}^{15}|x\rangle + \frac{61}{64}|1011\rangle$$

Third iteration

Apply the oracle operator U_f :

$$|\Psi_5\rangle|-\rangle = U_f(|\Psi_4\rangle|-\rangle) = \frac{5}{16}|\Psi\rangle - \frac{33}{32}|1011\rangle$$

Apply the inversion operator $(2|\Psi\rangle\langle\Psi| - I)$:

$$|\Psi_f\rangle = [2|\Psi\rangle\langle\Psi| - I]|\Psi_5\rangle = -\frac{13}{256}|x\rangle + \frac{251}{256}|1011\rangle$$

Step-4

Measure the state $|\Psi_f\rangle$

Finally calculate the probability of the target state $|1011\rangle$

$$P = \left|\frac{251}{256}\right|^2 \cong 0.96 \tag{5.25}$$

Hence finding the desired state $|1011\rangle$ with probability is around 96%.From this we can conclude that the oracle requires three iterations to execute the algorithm. In order to execute the same problem classically we need eight queries in an average.

Table 5.1: Comparison of quantum and classical search algorithm

Algorithm	**Quantum**	**Classical**
Executes on	Quantum computer	Classical computer
Uses	Qubits	Bits
Number of operations	$O(\sqrt{N})$	$O(N)$
Nature	Probabilistic	Generally Non probabilistic
Implementation	Quantum Gates	Classical Gates

In this section, a simple explanation of inversion about the mean and phase inversion is carried out considering a register of four qubits. The limitation of the algorithm is that it terminates earlier before reaching the target state due to some error factor. A comparison between quantum and classical search algorithm is shown in table (5.1).

5.5 Quantum Artificial Neural Network

The new field which integrates classical neurocomputing with quantum computing is quantum neural computing. Quantum computation is a linear theory but ANN depends upon non linear approach. The field of ANN contains several important ideas, which includes the concept of a processing element (neuron), the transformation performed by this element (in general, input summation and nonlinear mapping of the result in to an output value), the interconnection structure between neurons, the network dynamics and the learning rule which governs the modification of interaction strengths. The main concepts of quantum mechanics are wave function, superposition (coherence), measurement (Decoherence), entanglement, and unitary transformations. In order to establish such correspondence is a major challenge in the development of a model of QANN. Lewestein [91] proposed quantum perceptron, where classical weights in the perceptron are replaced as unitary operator to map input to output. Chrisly proposed a design for feed forward artificial neural network using double-slit

experiment. Menneer and Narayanan [111] have extended Chrisly proposal for single pattern quantum neural network .Where the concept of multiple universes or superposition principle of quantum theory is applied to neural computing .The architecture of double-slit experiment provides the basis for quantum artificial neural network .Hence a quantum neural network can be modeled on the basis of double-slit experiment, where the photon gun is equivalent to the input pattern, the slit is equivalent to input neurons, the waves between the slits and the screen is equivalent to the connections between the input and output neurons and the screen is equivalent to the output neurons.

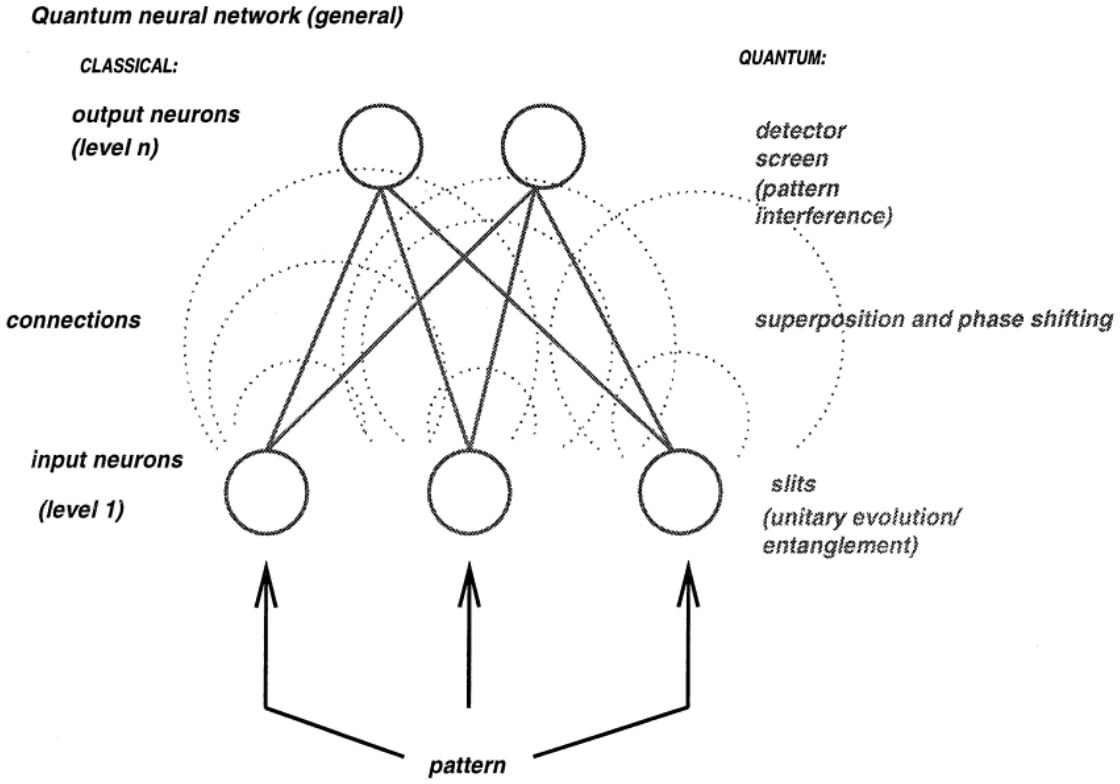


Figure 5.13: A quantum artificial neural network utilizing a many slit experimental setup

Table 5.2: Quantum Analogy of ANN

ANN	**Double Slit Experiment**	**QANN**
Pattern	Photon gun	Quantum Register holds input Pattern
Input Neuron	Slits	Entanglement/unitary evolution of input pattern
Connection	Waves	Superposition of waves created by input pattern
Output neuron	Detector screen	Pattern Interference
Weight	Link	Phase shift

5.5.1 Mathematical Representation of a Quantum Neuron

Artificial Neuron

A classical artificial neuron is an adjustable unit performing, in general, a non linear mapping of the set of many, N, input (perceptron) values x_1 ..... x_N to a single output value Y. The output value of a classical perceptron is [104]

$$Y = f\left(\sum_{j=1}^{N} w_j x_j\right) \tag{5.26}$$

Where $f(\,)$ is the perceptron activation function and w_j are the weights tuning during learning process.

The perceptron learning algorithm works as follows:

1. The weights w_j are initialized to small random numbers.
2. A *pattern* vector ($x_{1.....}x_N$) is presented to the perceptron and the output Y generated according to the equation (5.26).
3. The weights are updated according to the rule

$$w_j(t+1) = w_j(t) + \eta(d - Y) \tag{5.27}$$

Here t is discrete time (iteration number) , d is the desired output provided for training and $0 < \eta < 1$ is the step size.

4. Repeat steps-1 and 2 until the iteration error is less than user specified error threshold or a predetermined number of iterations have been completed.

Necessary Functionality Required for a Quantum Neuron

While the classical artificial neuron has its natural analog, the nervous cell, there is no natural analog of quantum artificial neuron .Therefore in order to understand the effect of quantum concepts on neural computation, it is necessary to imagine the functionalities of a quantum neuron.

A quantum neuron should have the ability to perform quantum computation and neural computation. Neural computing can process both binary signals (bit strings) and analog signals (patterns having continuous components).Qubit is the quantum analog of a bit .Hence the quantum neuron should process not only qubits but analog signals too.

The development in classical neural computing has been partly connected with the generalization of the form of input and output (real, complex, vector, binary etc.). Hence it can be expected that a quantum neuron should also have arbitrary input and output and even new types of input and output. As in the quantum domain a superposition of states can be created, it

is not obvious in advance if quantum neurons must be connected by networks. So the nature of inputs and outputs may be rather different.

Quantum Neuron

Many researchers use their own analogies in establishing a connection between quantum mechanics and neural networks [49]. The power of ANN is due to their massive parallel, distributed processing of information and due to the nonlinearity of the transformation performed by the network nodes (neurons) .On the other hand; quantum mechanics offers the possibility of an even more powerful quantum parallelism which is expressed in the principle of superposition .This principle provides the advantage in processing large data sets. Consider a neuron with N inputs$|x_1\rangle \dots \dots |x_N\rangle$ as shown in the figure as shown in the figure (5.14).

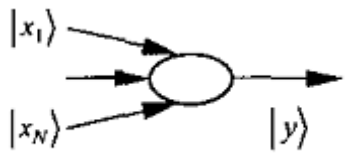


Figure-5.14: Quantum Neuron

$|x_j\rangle$ is a quantum bit of the form

$$|x_j\rangle = a_j|0\rangle + b_j|1\rangle = (a_j, b_j)^{\mathrm{T}} \tag{5.28}$$

Where $|a_j|^2 + |b_j|^2 = 1$.The output $|y\rangle$ can be derived by the rule [100]

$$|y\rangle = \hat{F}\sum_{j=1}^{N} \widehat{w_j}\,|x_j\rangle \tag{5.29}$$

Where $\widehat{w_j}$ is a 2×2 matrices acting on the basis $\{|0\rangle, |1\rangle\}$ and $\hat{F}$ is an unknown operator that can be implemented by the network of quantum gates. Let us consider $\hat{F} = \hat{I}$ be the identity operator .The output of the perceptron at the time t will be

$$|y\,(t)\rangle = \hat{I}\sum_{j=1}^{N} \widehat{w_j}(\mathrm{t})\,|x_j\rangle \tag{5.30}$$

In analogy with classical case equation (5.27) we can update the weights as follows

$$\widehat{w_j}(\mathrm{t+1}) = \widehat{w_j}(\mathrm{t}) + \eta(|d\rangle - |y\,(t)\rangle)\langle x_j| \tag{5.31}$$

Where $|d\rangle$the desired output .It is is can be shown equation (5.32) derives the quantum neuron in to desire state$|d\rangle$. Using equation (5.31) and taking modulo-square difference of real and desired outputs, we can get

$$\||d\rangle - |y\,(t+1)\rangle\|^2 = \left\||d\rangle - \textstyle\sum_{j=1}^{N} \widehat{w_j}(\mathrm{t}+1)\,|x_j\rangle\right\|^2$$

$$= \left\| |d\rangle - \sum_{j=1}^{N} \widehat{w_j}(\mathrm{t}) \, |x_j\rangle \; + \eta \left(|d\rangle - |y\,(t)\rangle \, \langle x_j | x_j \rangle \right) \right\|^2$$

$$= \left\| |d\rangle - |y\,(t)\rangle \; - \sum_{\mathrm{j}=1}^{\mathrm{N}} \eta(\, |d\rangle - \; |y\,(t)\rangle) \right\|^2$$

$$= (1 - N\eta)^2 \, \| |d\rangle - |y\,(t)\rangle \|^2 \qquad (5.32)$$

For small $\eta (0 < \eta < \frac{1}{N})$ and normalized input states the result of iteration converges to the desired state $|d\rangle$.The whole network can be then composed from the primitives elements using the standard rules of ANN architecture.

Learning Rule

The gradient-descent-based algorithm can be used for training the QANN.

Step-1

Choose $\eta > 0$and lower limit of error E_{min}

Step-2

Initialize $\widehat{w_j} = \widehat{w_j}(0)$, as N random matrices, set error $E = 0$ and step counter $t = 1$

Step-3

Calculate output of QNN

$$|y\,(t)\rangle = \hat{F} \sum_{j=1}^{N} \widehat{w_j}(\mathrm{t}) \, |x_j\rangle$$

Step-4

Update the weight according to learning rule

$$\widehat{w_j}\,(t+1) = \widehat{w_j}\,(\mathrm{t}) + \eta(|d\rangle - |y\,(t)\rangle)\langle x_j|$$

Step-5

Calculate the error

$$E = \| |d\rangle - |y\,(t)\rangle \|^2$$

Step-6

If $E < E_{min}$ go to step-7 otherwise set $t = t + 1$

Step-7

Save the weight matrix $\widehat{w_j}$ and stop.

5.5.2 Computational Power of QANN

In quantum information processing the Hadamard transformation or Hadamard gate is a one qubit rotation that maps the qubit basis states $|0\rangle$and $|1\rangle$ into two superposition of states with equal weight of the computational basis states $|0\rangle$and $|1\rangle$.Quantum algorithms use Hadamard gate as an initial step to map n-qubits with $|0\rangle$ to a superposition of all 2^northogonal

states in the $|0\rangle$, $|1\rangle$ basis with equal weight. When the Hadamard gate is applied twice in succession, then the final state is always the same as the initial state. Here we will see how the Hadamard transformation can be implemented using quantum artificial neural network. Considering a quantum neuron with input $|x\rangle$ and output $Y = f\left(\sum_{j=1}^{N} w_j x_j\right)$

Taking weight $\hat{w}$ and operator $\hat{F}$ as

$$\hat{w} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } \hat{F} = \hat{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{5.33}$$

Here $\hat{H}$ is the Hadamard transformation.

When input $|x\rangle = |0\rangle$

$$|y\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

When input $|x\rangle = |1\rangle$

$$|y\rangle = |0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

When $\hat{H}$ is applied to n qubits in the state $|0\rangle$ individually, $\hat{H}$ generates a superposition of all 2^n possible states .i.e. $0\ to\ 2^n - 1$.

$$(\hat{H}\otimes\hat{H}\otimes\hat{H}\otimes \dots \dots \otimes\hat{H})|0000 \dots .0\rangle$$

$$= \frac{1}{\sqrt{2}^n}(|0\rangle + |1\rangle)\otimes(|0\rangle + |1\rangle)\otimes \dots \dots \otimes(|0\rangle + |1\rangle)|0000 \dots .0\rangle$$

$$= \frac{1}{\sqrt{2^n}} \sum_{j=0}^{2^n-1} |x_j\rangle \tag{5.34}$$

Comparing equation (5.34) with equation (5.29) we can conclude that Hardamard transformation can be implemented by a quantum neuron with 2^n inputs and one output choosing $\hat{w} = \frac{1}{\sqrt{2^n}}\hat{I}$ and $\hat{F} = \hat{I}$.

5.5.3 Logic Gates via QANN

Here we will discuss the computational power of QANN

NOT Function

Consider a QANN with one input $|x\rangle$ and one output $|y\rangle$.According to equation (5.29), the output is $|y\rangle = \hat{F}\,\hat{w}\,|x_j\rangle$

Let

$$\hat{w} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } \hat{F} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Then we can get

$$|y\rangle = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle \text{ , When } |x\rangle = |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|y\rangle = \hat{w}|x\rangle = |0\rangle \text{ , When } |x\rangle = |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

From this we can conclude that the QANN has same computational power as NOT gate

XOR Function

Consider a QNN with two inputs $|x_1\rangle$and $|x_2\rangle$According to equation (5.29), the output is

$$|y\rangle = \hat{F}\sum_{j=1}^{2} \widehat{w_j}\ |x_j\rangle = \hat{F}(\widehat{w_1}\ |x_1\rangle + \widehat{w_2}\ |x_2\rangle)$$

Choosing

$$\widehat{w_1} = \widehat{w_2} = \hat{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \; and \; \hat{F} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} sgn(.) & 0 \\ 0 & sgn(.) \end{pmatrix}$$

Where $sgn(.)$ is signum function, $(sgn(x) = +1, for\ x > 0\ and\ sgn(x) = -1, for\ x < 0)$ we can find that the output of the QNN will produce the value of the XOR function as follows:

When

$$|x_1\rangle = |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \; and \; |x_2\rangle = |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Then the output becomes

$$|y\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} sgn(.) & 0 \\ 0 & sgn(.) \end{pmatrix}\left(\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right)$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} sgn(.) & 0 \\ 0 & sgn(.) \end{pmatrix}\sqrt{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

When

$$|x_1\rangle = |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \; and \; |x_2\rangle = |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Then the output becomes

$$|y\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} sgn(.) & 0 \\ 0 & sgn(.) \end{pmatrix}\left(\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right)$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}sgn(.) & 0\\ 0 & sgn(.)\end{pmatrix}\sqrt{2}\begin{pmatrix}1\\ 0\end{pmatrix}$$

$$= \begin{pmatrix}0 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix} = \begin{pmatrix}0\\ 1\end{pmatrix} = |1\rangle$$

Similarly we can find the other two outputs of XOR function.

Advantage over ANN

1. N-set of training patterns forms a set of N-homogenous components in QANN where each training pattern is channeled to one component part and the set of weights connected to this component is changed to learn this training pattern .Interference does not take place in the leaning patterns due to the independent components for each training pattern .But in CNN there is most possibility of pattern interference where the network unlearns i.e. a form of catastrophic forgetting.
2. Patterns are separated by using different components in QANN .So there is no necessity of decision plane to separate patterns in to classes which is required in CNN.
3. The components can be trained as a superposition of networks.
4. Also we can train classical networks and combine in to superposition later.
5. Less training time is required as compared to classical neural network.

5.6 Summary and Conclusions

In this chapter we have discussed fundamental concepts of quantum computing, quantum algorithm and its application to a financial market price prediction. Here we have proposed a method that predicts the market price of a financial instrument using Deutsch's algorithm and it is computationally much faster than the classical approach. Using Deutsch's algorithm we have evaluated the function $f(x)$ once and the result allowed us to determine with certainty if the value of $f(x)$ is equal for the two inputs or if $f(x)$ is not equal. Hence Deutsch's algorithm allows us to predict the moment of a specified financial instrument. This algorithm solves our problem efficiently but the drawback is in the implementation of the algorithm using quantum computer. Then we have discussed searching an unsorted database in quantum domain using Grover's data base search algorithm. We have discussed the working of Grover's search algorithm in a four qubit system. Finally mathematical representation of quantum neuron is discussed with necessary functionalities with learning rule. The computational power of QANN and the advantage of QANN over ANN are discussed.

CHAPTER-6

QUANTUM MODEL FOR DECISION MAKING

6.1 Introduction

In the early twentieth century physicists faced problem to explain certain physical phenomena using classical logic, probability and dynamics. Quantum mechanics was discovered to explain the paradoxical departure from classical physics [41, 103]. The mathematical basis of quantum mechanics discovered new type of logic, probability and dynamics called quantum logic, quantum probability and quantum dynamics respectively to study the behavior of atomic and subatomic particles for example the motion of electrons, the attraction and repulsion among proton and electron etc [113]. Similarly, the decision theory approach to social science is facing paradoxes of human decision making that are quite difficult to explain using classical decision theory. For example the principle of sure thing plays a great role in explaining classical decision theory [92]. But for some experiments of psychology the principle of sure thing contradicts with experimental results. The two famous experimental results of psychology, the two stage gambling and the prisoner's dilemma show that people violate the principle of sure thing of classical decision theory. The classical decision theory fails to explain the experimental findings for over a decade. In recent years, quantum theory applied in the field of computing and have successfully explained many computationally hard problems which were impossible in classical computing for example quantum search algorithm, quantum factorization algorithm and quantum cryptography etc. Hence we have enough evidence of the successful application of quantum techniques [6,8,67] in non-quantum mechanical problems. The basic objective of this chapter is to study the recent progress of quantum principles and it's applications in human decision making process. In this chapter we have reviewed the explanation of violation of principle of sure thing in case of two stage gambling experiment performed by Amos Tversky and Eldar Shafir [165] and the scope of this model in a large range of business and management related problem domains by P. Agrawal and R.Sharda [1].

6.2 Experimental Observations

In recent years there is growing interest in the application of quantum theory in non quantum frameworks such as quantum cryptography [15,31], quantum information processing and quantum computation [153,94,64]. The legend physicist Nils Bohr tried to apply quantum mechanics outside the preview of physical sciences and taken endeavor to find similarity between quantum processes and mental process of human being. An attempt has been made to describe the mind, brain and consciousness using quantum mechanical tools [26, 95,150,126,142]. The work of Tversky, Shafir and Kahenman has contributed a lot in the area of human judgments under uncertainty and biases [148,164,165]. For this remarkable contribution Kahenman was awarded the Nobel Prize in the year 2002. Classical statistics could not explain the experimental results in the field of human psychology carried out by Tversky and Shafir [165]. For example classical decision theory failed to explain the disjunction effect of the human mind.

The experimental observations conducted by Amos Tversky and Eldar Shafir are as follows: The strategy of the game is to toss a coin having 50% the possibility of winning $\$200$ and 50% possibility of losing $\$100$. The participants are allowed to play the same game again for the second time with or without knowing the outcome of the 1^{st} game. As an observation of the experiment it is found that a maximum number of participants are prepared to play the second game after knowing the success or failure of the first game. But a very few are ready to go for a second game without knowing the result of the first game. In view of the principle of sure thing [139] all the participants should go for second game even if they know or don't know the outcome of the first game. Hence the contradiction arises in the principle of sure thing [If X prefers Y, with the knowledge that the event Z occurred and if X prefers Y, knowing that the event Z did not happen. Then we can conclude that the happening/not happening of event Z does not affect the preference of X to Y. Another way we can say X prefers Y in spite of knowing the result of the occurrence of the event Z. This principle becomes a basic axiom of classical decision theory].

6.3 Quantum Decision Model

The classical decision theory is unable to explain the contradiction of the experiment from the logical statement of sure thing principle. Amos Tversky and Eldar Shafir observed the contradiction of the principle of sure thing related to two stage gambling experiment. Such type of paradoxical departure of human behavior in decision making can be modeled using the probabilistic mathematical framework of quantum mechanics. Some state of the human mind can be abstracted similar with quantum states in Hilbert space [113, 104] and the process of decision making formulated based on the postulates of quantum mechanics.

6.3.1 Mathematical Notations

In order to explain the paradoxes of psychology Khrennikov [80] used quantum interference in the form of an adjustable parameter called coefficient of interference. Sornette and Yukalov [176-178] provided detailed theory known as QDT (quantum decision theory). They provide a postulate analogous to postulate of quantum theory. In this section, an attempt is made to explain the discrepancy of this experiment with the sure-thing principle of savage [143] using a quantum decision model.

In quantum mechanics the state of a system under consideration is described by a quantum state. The process of changing a state function from one form to another form is described by an operator. For example in quantum mechanics we have energy, momentum and position operators similarly in quantum decision theory we will interpret price, buy, win etc. in the form of price operator, "buy" operator, "win" operator etc. Now we will illustrate the close correspondence of quantum postulates and quantum decision model. The dynamical state corresponding to a quantum system is represented by a state $|\Psi\rangle$, similarly state $|\Psi\rangle$ represents the state of system in quantum decision model. In a quantum system for each dynamical variable O there exists a corresponding operator $\hat{O}$ associated with each other.

Similarly in quantum decision model each variable O is associated with an operator $\hat{O}$. The expectation value $\langle O\rangle$ of the dynamical variable O in quantum state $|\Psi\rangle$ will be represented as

$$\langle O\rangle = \frac{\langle\Psi|\hat{O}|\Psi\rangle}{\langle\Psi|\Psi\rangle} = \langle\Psi|\hat{O}|\Psi\rangle\,, \quad \text{if } \langle\Psi|\Psi\rangle = 1, i.e\ normailzed \tag{6.1}$$

Similarly the quantum decision model measurement $\langle O \rangle$ of dynamical variable O in state $|\Psi\rangle$ can be represented as equation (6.1).

Description of Probabilities

Suppose $X_1 \rightarrow$ Success in the first game and $A_1 \rightarrow$ Agree to play the game for the second time. $X_2 \rightarrow$ Failure in the first game and $A_2 \rightarrow$ Do not agree to play the game for the second time. Let $P(X_1)$ be the probability of success of the participants in the first game and $P(X_2)$ be the probability of failure of the participants in the first game.

Similarly $P(A_1 |X_1) \rightarrow$ is the conditional probability of participants those agree to play the game for 2nd time with the knowledge of success in the 1st game.

$P(A_2|X_1) \rightarrow$ Is the conditional probability of participants those do not agree to play the game for 2nd time with the knowledge of success in the 1st game.

$P(A_1\, X_1) \rightarrow$ Is the joint probability of success in 1st game and agrees to play the game for 2nd time.

$$P(A_1\, X_1) = P(X_1)P(A_1\, |X_1) \qquad (6.1\ (a))$$

$P(A_2X_1) \rightarrow$ Is the joint probability of failure in 1st game and agrees to play the game for 2nd time.

$$P(A_2X_1) = P(X_1)P(A_2|X_1) \qquad (6.1\ (b))$$

$P(A_1\, |X_2) \rightarrow$ Is the conditional probability of participants agreed to play the game for 2nd time with knowledge of failure in the 1st game.

$P(A_2|X_2) \rightarrow$ Is the conditional probability of participants those do not agree to play the game for 2nd time with knowledge of failure in the 1st game.

$P(A_1\, X_2) \rightarrow$ Is the joint probability of failure in 1st game and agrees to play the game for 2nd time.

$$P(A_1\, X_2) = P(X_2)P(A_1\, |X_2) \qquad (6.1\ (c))$$

$P(A_2X_2) \rightarrow$ Is the probability of failure in 1st game and do not agree to play the game for 2nd time.

$$P(A_2X_2) = P(X_2)P(A_2|X_2) \qquad (6.1\ (d))$$

$P(A_1\ ) \rightarrow$ Is the probability of participants those agree to play the game for the 2nd time without the knowledge of success or failure in the 1st game.

$P(A_2)$ → Is the probability of participants those do not agree to play the game for the 2nd time without knowledge of success or failure in the 1st game.

From this we can conclude that the sum of participants agrees to play the game for 2nd time and those who do not agree to play the game for the 2nd time without the knowledge of success or failure in the 1st game can be written as:

$$P(A_1) + P(A_2) = 1 \qquad \text{(6.1 (e))}$$

Quantum States and Operators

The quantum states and operators are as follows:

Let $|A_1\rangle$ → be the state that corresponds to agree to play the game for the second time with 100% probability. Similarly, $|X_1\rangle$ → be the state that corresponds success in the first game with 100% probability, $|A_2\rangle$ → be the state that corresponds not agree to play the game for the second time with 100% probability, $|X_2\rangle$ → be the state that corresponds success in first gamble with 0% probability, $|A_1X_1\rangle$ → be the state in which Probability of success in the first game is 100% as well as agreeing to play the game for the second time is 100%, i.e.

$$|A_1X_1\rangle = |A_1\rangle|X_1\rangle$$

And $|A_2X_1\rangle$ → be the state in which Probability of success in the first game is 100% as well as does not agree to play the game for the second time 100%, i.e.

$$|A_2X_1\rangle = |A_2\rangle|X_1\rangle$$

Similarly let $|A_1X_2\rangle$ → be the state in which Probability of not success in the first game is 100% as well as agrees to play the game for the second time is 100%, i.e.

$$|A_1X_2\rangle = |A_1\rangle|X_2\rangle$$

And $|A_2X_2\rangle$ → be the state in which Probability of not success in the first game is 100% as well as not agree to play second gamble is 100%, i.e.

$$|A_2X_2\rangle = |A_2\rangle|X_2\rangle$$

O_{SUC} → $Success$ operator that corresponds to the probability of success in the first game.

O_{A_1} → Agree to play operator which corresponds to agree to play a second game.

Experimental Data to be analyzed

The violation of sure thing principle is empirically verified by Tversky and Shafir [161]. In this section we will discuss some examples of violation of sure thing principle using quantum theory. The experimental observation is shown below:

$P(A_1|X_1)$ =Probability of success in the first game and agrees to play second game=0.69

$P(A_2|X_1)$ =The Probability of success in the first game is 100% as well as not agree to play second game is 100%= 0.31

$P(A_1|X_2)$= The Probability of not success in the first game is 100% as well as agreeing to play second game is 100%= 0.59

$P(A_2|X_2)$ = The Probability of not Success in the first game is 100% as well as not agree to play second game is 100%= 0.41

$P(A_1)$= Probability of participants agrees to play the game for a second time without knowledge of success or failure in the first game = 0.36

$P(A_2)$ = Probability of not agree to play second game without knowledge of success or failure in first game = 0.64

From the above data 69% participants are agreeing to play a second game after knowing that they have succeeded the first game. 59% are ready for the second game after knowing that they have failed first game and only 36% are ready for the second game in the absence of knowing the result of the first game. Hence the majority of the participants prefers for second game knowing the result of the first game but very few are not ready for the second game without knowing the result of the first game. In literature [165] this effect is called disjunction effect of choice under uncertainty. This effect contradicts the principle of the sure-thing given by [143].

Eigenkets and Eigenvalues

Let us define the operators corresponding to the eigenstates related to the experiment. Suppose $|A_1\rangle$ be the eigenstate of the operator O_{A_1} with eigenvalue 1. In the same way$|A_2\rangle$ is an eigenstate of the operator O_{A_1} with eigenvalue 0. Similarly $|X_1\rangle$ is an eigenstate of the operator O_{SUC} whose eigenvalue is 1 and $|X_2\rangle$ is the eigenstate of the operator O_{SUC} with eigenvalue 0. As we know eigenkets belong to different eigenvalues of a given operator are orthogonal to each other, so let us try to utilize these concepts in quantum decision model as follows:

$$O_{A_1}|A_1\rangle = |A_1\rangle, O_{A_1}|A_2\rangle = 0 \qquad (6.2\ (a))$$

$$O_{A_1}\,|A_1X_1\rangle = |A_1X_1\rangle\,, O_{SUC}|A_1X_1\rangle = |A_1X_1\rangle \quad \text{(6.2 (b))}$$

$$O_{SUC}|X_1\rangle = |X_1\rangle\,, O_{SUC}|X_2\rangle = 0 \quad \text{(6.2 (c))}$$

$$O_{A_1}\,|A_2X_1\rangle = 0\,, O_{SUC}|A_2X_1\rangle = |A_2X_1\rangle \quad \text{(6.2 (d))}$$

$$O_{A_1}\,|A_1X_2\rangle = |A_1X_2\rangle, O_{SUC}|A_1X_2\rangle = 0 \quad \text{(6.2 (e))}$$

$$O_{A_1}\,|A_2X_2\rangle = 0\,, O_{SUC}|A_2X_2\rangle = 0 \quad \text{(6.2 (f))}$$

$$\langle X_1|X_2\rangle = \langle X_2|X_1\rangle = 0\,, \langle A_1X_1|A_1X_2\rangle = \langle A_1X_2|A_1X_1\rangle = 0$$
$$\langle A_1|A_2\rangle = \langle A_2|A_1\rangle = 0\,, \langle A_2X_1|A_2X_2\rangle = \langle A_2X_2|A_2X_1\rangle = 0$$

Representation of States in Success-Failure Space

Already the basic operators and eigenstates related to our experiment are discussed. Let us construct the ket representation of the state present in success-failure space. Let the state $|\Phi\rangle$ is representing the success/failure of the system as a linear combination.

$$|\Phi\rangle = \alpha_1|X_1\rangle + \alpha_2|X_2\rangle \quad (6.3)$$

The normalization condition holds $|\alpha_1|^2 + |\alpha_2|^2 = 1$. The success/failure state $|\Phi\rangle$ can be represented as a linear combination of success state $|X_1\rangle$ and of failure state $|X_2\rangle$ with probability amplitude α_1 and α_2 respectively shown in the figure (6.1). As O_{SUC} is the success operator so the probability of success in first gamble can be calculated:

$$P(X_1) = \langle\Phi|O_{SUC}|\Phi\rangle \quad (6.4)$$

From Equation (6.3) and Equation (6.2 (c))

$$O_{SUC}|\Phi\rangle = O_{SUC}(\alpha_1|X_1\rangle + \alpha_2|X_2\rangle) = \alpha_1|X_1\rangle \quad (6.5)$$

The bra vector corresponding of Equation (6.3) will be:

$$\langle\Phi| = \alpha^*{}_1\langle X_1| + \alpha^*{}_2\langle X_2| \quad (6.6)$$

By combining Equations (6.4), (6.5), (6. 6)

$$P(X_1) = \langle\Phi|O_{SUC}|\Phi\rangle = [\alpha^*{}_1\langle X_1| + \alpha^*{}_2\langle X_2|\,][\alpha_1|X_1\rangle]$$
$$= \alpha^*{}_1\alpha_1\langle X_1||X_1\rangle + \alpha^*{}_2\alpha_1\langle X_2||X_1\rangle = \alpha^*{}_1\alpha_1 = |\alpha_1|^2 \quad (6.7)$$

Hence

$$P(X_2) = 1 - P(X_1) = 1 - |\alpha_1|^2 = |\alpha_2|^2 \quad (6.8)$$

From Equations (6.7), (6. 8), we can conclude that success/failure is not an issue of a decision by the players in this experiment.

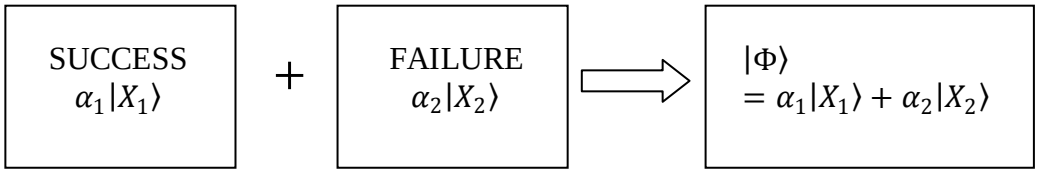


Figure 6.1 : The success-failure state $|\Phi\rangle$ in the case of the first game is a linear combination of success state $|X_1\rangle$ with probability amplitude α_1 and the failure state $|X_2\rangle$ with probability amplitude α_2.

CASE-1

From the experimental findings it is known some agree to play the second game while getting success in the first game and some do not agree to play the second game: As we have already denoted $|A_1X_1\rangle \rightarrow$ The Probability of success in the first game and agree to play a second game (success-agree to play) and $|A_2X_1\rangle \rightarrow$ The Probability of success in the first game and not agree to play the game for the second time (success-not agree to play). Therefore the state of mind $|\Psi_1\rangle$ associated with success-agree to play and success-not agree to play can be represented as a linear combination of $|A_1X_1\rangle$ and $|A_2X_1\rangle$ is shown in the figure (6.2).

$$|\Psi_1\rangle = a_1|A_1X_1\rangle + b_1|A_2X_1\rangle \qquad (6.9)$$

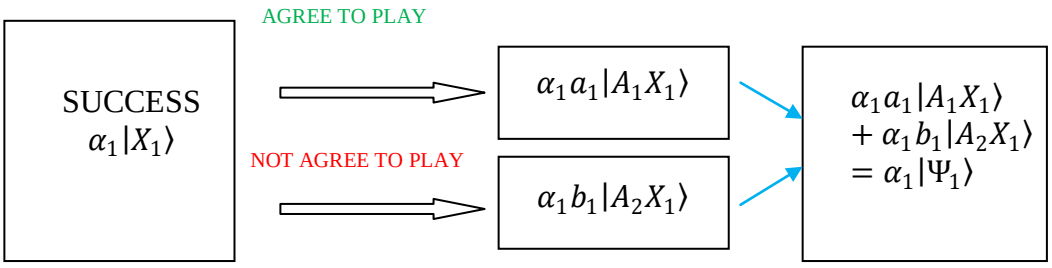


Figure 6.2: Success state leads to success and agree to play state with probability amplitude a_1 and success-not agree to play state with probability amplitude b_1. The linear combination of these two states has been characterized by $|\Psi_1\rangle$. The factor α_1 in each block indicates the probability amplitude of success state (see figure (6 .1)).

The normalization condition leads to $|a_1|^2 + |b_1|^2 = 1$. For computing success-agree to play we can use the state $|\Psi_1\rangle$ and operator O_{A_1} . Hence Probability of success in the first game and agrees to play second game will be:

$$P(A_1|X_1) = \langle\Psi_1|O_{A_1}|\Psi_1\rangle \tag{6.10}$$

$$O_{A_1}|\Psi_1\rangle = a_1 O_{A_1}|A_1X_1\rangle + b_1 O_{A_1}|A_2X_1\rangle = a_1|A_1X_1\rangle \; (\because Equn\ 6.2 - b, d)$$

$$\langle\Psi_1| = a^*{}_1\langle A_1X_1| + b^*{}_1\langle A_2X_1|$$

$$P(A_1|X_1) = \langle\Psi_1|O_{A_1}|\Psi_1\rangle = [a^*{}_1\langle A_1X_1| + b^*{}_1\langle A_2X_1|]a_1|A_1X_1\rangle$$
$$= a^*{}_1a_1\langle A_1X_1||A_1X_1\rangle + b^*{}_1a_1\langle A_2X_1||A_1X_1\rangle = |a_1|^2 \tag{6.11}$$

As sum of $P(A_1|X_1)$ and $P(A_2|X_1)$ equals to one.

$$P(A_2|X_1) = 1 - P(A_1|X_1) = |b_1|^2$$

Thus the success state leads to success and agree to play a second game with probability amplitude a_1 and the success state not agree to play a second game with probability amplitude b_1 respectively.

CASE-2

Again from the experiment we know $|A_1X_2\rangle \rightarrow$ the probability of not success in the first game (failure) and agree to play a second game (fail-agree) and $|A_2X_2\rangle \rightarrow$ The probability of not success in the first game (failure) and not agree to play a second game (fail-not agree). Therefore the state of mind $|\Psi_2\rangle$ associated with fail-agree and fail-not agree can be represented as a linear combination of $|A_1X_2\rangle$ and $|A_2X_2\rangle$ is shown in the figure (6.3).

$$|\Psi_2\rangle = a_2|A_1X_2\rangle + b_2|A_2X_2\rangle \tag{6.12}$$

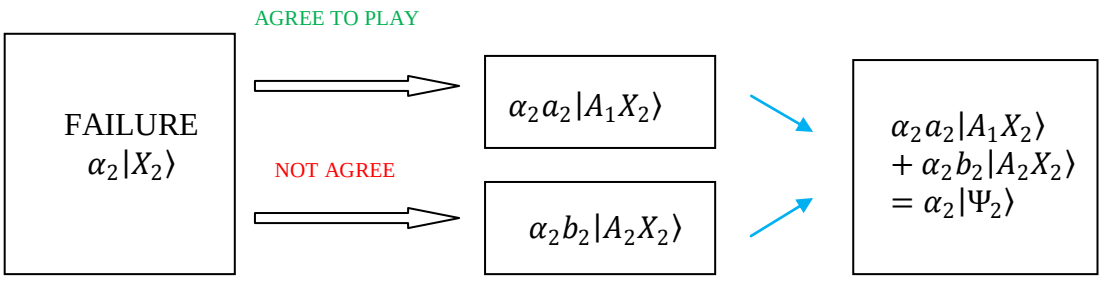


Figure 6.3: Failure state leads to agree to play state with probability amplitude a_2 and failure-not agree to play state with probability amplitude b_2. The linear combination of these two states has been characterized by $|\Psi_2\rangle$. The factor a_2 in each block indicates the probability amplitude of success state (see figure 6.1).

The normalization condition leads to $|a_2|^2 + |b_2|^2 = 1$. Again for computing fail-agree we can use the state $|\Psi_2\rangle$ and operator O_{A_1} . Hence the probability of getting success in the first game and agree to play second game will be:

$$P(A_1|X_2) = \langle\Psi_2|O_{A_1}|\Psi_2\rangle \tag{6.13}$$

$$O_{A_1}|\Psi_2\rangle = a_2 O_{A_1}|A_1X_2\rangle + b_2 O_{A_1}|A_2X_2\rangle = a_2|A_1X_2\rangle \; (\because Equn\ (6.2\ (e),(f)))$$

$$\langle\Psi_2| = a^*{}_2\langle A_1X_2| + b^*{}_2\langle A_2X_2|$$

$$P(A_1|X_2) = \langle\Psi_2|O_{A_1}|\Psi_2\rangle = [a^*{}_2\langle A_1X_2| + b^*{}_2\langle A_2X_2|]a_2|A_1X_2\rangle$$

$$= a^*{}_2a_2\langle A_1X_2||A_1X_2\rangle + b^*{}_2a_2\langle A_2X_2||A_1X_2\rangle = |a_2|^2 \tag{6.14}$$

As sum of $P(A_1|X_2)$ and $P(A_2|X_2)$ equals to one.

$$P(A_2|X_2) = 1 - P(A_1|X_2) = |b_2|^2$$

Thus fail state leads to fail and agree to play a second game with probability amplitude a_2 and the fail state not agree to play a second game with probability amplitude b_2 respectively. Hence the linear combination of these states can be represented as $|\Psi_2\rangle$.

CASE-3

Let us consider the state of mind $|\Psi\rangle$, which is representing the sum of those who fails and those who are getting success in the first game i.e. the linear combination of states $|\Psi_1\rangle$ and $|\Psi_2\rangle$:

$$|\Psi\rangle = \alpha_1|\Psi_1\rangle + \alpha_2|\Psi_2\rangle \tag{6.15}$$

$$|\Psi\rangle = \alpha_1a_1|A_1X_1\rangle + \alpha_1b_1|A_2X_1\rangle + \alpha_2a_2|A_1X_2\rangle + \alpha_2b_2|A_2X_2\rangle \tag{6.16(a))}$$

Where

$$d_1 = \alpha_1a_1, d_2 = \alpha_2a_2, d_3 = \alpha_1b_1, d_4 = \alpha_2b_2 \tag{6.16(b))}$$

Equation (6.16 (a)) is represented in figure (6.4).

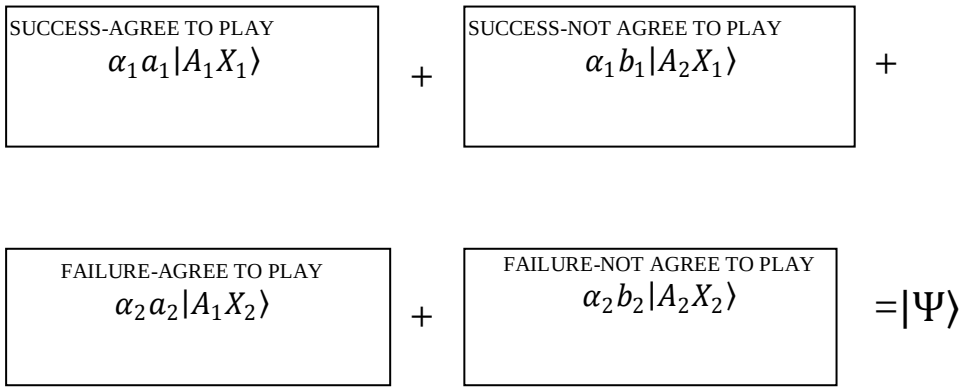


Figure 6.4: The state of the system $|\Psi\rangle$ is a linear combination of four states success-agrees to play, success- not agree to play, failure-agree to play and failure- not agree to play

Equation (6.3) represents the states $|X_1\rangle$ and $|X_2\rangle$ in the success - failure state become the state $|\Psi_1\rangle$ and $|\Psi_2\rangle$ in success-failure-agree to play-not agree to play state of equation (6.15) respectively. Similarly, the state of equations (6.3) in the success - failure state becomes the state $|\Psi\rangle$ in success-failure-agree to play-not agree to play state as shown in the figure (6.4).

Analysis of the Probabilities

Let us calculate the joint probability of getting success in the first game and agree to play the second game of the state $|\Psi\rangle$ as given in equation (6.16 (a)) using success operator (O_{SUC}) and agree to play (O_{A_1}) we get:

$$P(A_1X_1) = \langle\Psi|O_{A_1}O_{SUC}|\Psi\rangle$$

$$O_{SUC}|\Psi\rangle = O_{SUC}[\alpha_1 a_1|A_1X_1\rangle + \alpha_1 b_1|A_2X_1\rangle + \alpha_2 a_2|A_1X_2\rangle + \alpha_2 b_2|A_2X_2\rangle]$$

$$(\because Equation\ (6.16-a))$$

$$= \alpha_1 a_1|A_1X_1\rangle + \alpha_1 b_1|A_2X_1\rangle \quad (6.17)$$

$$(\because\ Equation\ (6.2(e),(f),(b),(d)))$$

$$O_{A_1}O_{SUC}|\Psi\rangle = O_{A_1}[\alpha_1 a_1|A_1X_1\rangle + \alpha_1 b_1|A_2X_1\rangle] \quad (6.18)$$
$$= \alpha_1 a_1|A_1X_1\rangle = d_1|A_1X_1\rangle$$

$$(\because Equation\ (2\ (b)) and\ \alpha_1 a_1 = d_1)$$

$$P(A_1X_1) = \langle\Psi|O_{A_1}O_{SUC}|\Psi\rangle = |d_1|^2 \quad (6.19)$$

Similarly we can find:

$$P(A_1X_2) = |d_2|^2, P(A_2X_1) = |d_3|^2, P(A_2X_2) = |d_4|^2 \quad (6.20)$$

Using (6.19) and (6.20)

$$P(A_1X_1) + P(A_1X_2) + P(A_2X_1) + P(A_2X_2)$$

$$= |d_1|^2 + |d_2|^2 + |d_3|^2 + |d_4|^2 = 1 \qquad (6.21)$$

Equation (6.19) in combination with equation (6.16 (b)) gives:

$$P(A_1X_1) = |\alpha_1|^2|a_1|^2$$

This equation with the help of equations (6.11) and (6.7) gives:

$$P(A_1X_1) = P(X_1)P(A_1|X_1) \qquad (6.22)$$

As expected in equation (6.1 (a)) .Similarly, equations (6.20) can be used to obtain equations (6.1 (b)), (6.1 (c)), (6.1 (d)) respectively. Such verifications show the reliability of various probabilities computed in this section.

6.3.2 When the result of first Gamble is not known

In equation (6.15) the state $|\Psi\rangle$ contains information regarding success and failure. In order to determine the probability of agree to play a second game without any knowledge of the result of first game, $P(A_1)$, Shafir and Tversky performed the experiment after some days. Tversky and Shafir have taken some time between the first game and second game to clean the information of success and failure from the mind of participants. One can delete the information regarding success or failure included in the mathematical formulation by removing levels X_1 and X_2 from equation (6.16 (a)) producing a new set of equation as follows:

$$|\Psi'\rangle = d_1'|A_1\rangle + d_2'|A_1\rangle + d_3'|A_2\rangle + d_4'|A_2\rangle \qquad (6.22\ (a))$$

$$|\Psi'\rangle = (d_1' + d_2')|A_1\rangle + (d_3' + d_4')|A_2\rangle \qquad (6.22\ (b))$$

The constant factors d_i' of equation (6.22 (a)), where $i = 1\ to\ 4$ are same as that of d_i of equation (6.18) .So we can write

$$|d_i'| = |d_i|, \text{where } i = 1\ to\ 4 \qquad (6.22\ (c))$$

In order to find the probability of participants agreed to play the game for a second time without knowing the result of success or failure in the first game, the average value of the operator O_{A_1} has to be evaluated using equation (6.16).

$$P(A_1) = \langle O_{A_1} \rangle = \langle \Psi | O_{A_1} | \Psi \rangle = d_1 |A_1X_1\rangle + d_2 |A_1X_2\rangle$$
$$= |d_1|^2 + |d_2|^2 = P(A_1X_1) + P(A_1X_2) \quad (6.23)$$

Similarly, if we calculate the average value of the operator O_{A_1} using equation (6.22 (a))we will get the probability of participants agreed to play a second game without knowing the result of the first game as the sum of $P(A_1X_1), P(A_1X_2)$ and an interference term.

$$P(A_1) = \langle O_{A_1} \rangle = \langle \Psi' | O_{A_1} | \Psi' \rangle = |d_1{}' + d_2{}'|^2$$
$$= |d_1{}'|^2 + |d_2{}'|^2 + (d_1{}' * d_2{}') + (d_2{}' * d_1{}')$$
$$= P(A_1X_1) + P(A_1X_2) + (d_1{}' * d_2{}') + (d_2{}' * d_1{}') \quad (6.24)$$

Using equations (6.15) and (6.22 (a)) the average value of the operator O_{A_1} is computed to find the probability of participants agree to play the game for a second time without any knowledge of success and failure. If we will compare the results obtained in equation (6.23) and (6.24) we can clearly visualize the distinction: that one finds $P(A_1)$ which is equal to sum of $P(A_1X_1)$, $P(A_1X_2)$ and an extra interference term and in equation (6.23) there is no such interference terms d_1 and d_2 i.e. we find the probability as the sum of $P(A_1X_1)$ and $P(A_1X_2)$. This extra term was not present in the classical framework, which is the exceptional characteristic of quantum framework.

We can write equation (6.24) as:

$$P(A_1) = P(A_1X_1) + P(A_1X_2) + Q_{int}(A_1) \quad (6.25)$$

Where

$$Q_{int}(A_1) = (d_1{}' * d_2{}') + (d_2{}' * d_1{}') \quad (6.26)$$

As $d_1{}'$ and $d_2{}'$ are complex numbers we can represent them as follows:

$$d_1{}' = |d_1{}'| exp(i\theta_1) = \big(P(A_1X_1)\big)^{\frac{1}{2}} exp(i\theta_1) \quad (6.27)$$

$$d_2{}' = |d_2{}'| exp(i\theta_2) = \big(P(A_1X_2)\big)^{\frac{1}{2}} exp(i\theta_2) \quad (6.28)$$

Where θ_1 and θ_2 are the phase angles.

Hence equation (6.26) will be:

$$Q_{int}(A_1) = (d_1{}' * d_2{}') + (d_2{}' * d_1{}')$$
$$= 2[P(A_1X_1)P(A_1X_2)]^{\frac{1}{2}} \cos(\theta_2 - \theta_1) \quad (6.29)$$

$$\left(\because we\ have\ taken, exp(i\theta) = \cos(\theta) + isin(\theta) and\ i = \sqrt{-1}\right)$$

As the extreme values of cosine function are ± 1 hence equation (6.29) can be written as:

$$-2[P(A_1X_1)P(A_1X_2)]^{\frac{1}{2}} \leq Q_{int}(A_1)$$
$$\leq 2[P(A_1X_1)P(A_1X_2)]^{\frac{1}{2}} \quad (6.30)$$

Similarly, the operator for not agree to play second gamble is $I - O_{A_1}$, where $I =$ unit operator. Hence the probability of not agree to play second gamble without knowledge of success or failure in the first game is similar to equation (6.25) will be:

$$P(A_2) = P(A_2X_1) + P(A_2X_2) + Q_{int}(A_2) \quad (6.31)$$

Where $$Q_{int}(A_2) = (d_3{}' * d_4{}') + (d_4{}' * d_3{}') \quad (6.32)$$

For $d_3{}' and\ d_4{}'$ we can get an equivalent equation similar to equation (6.29) as follows:

$$Q_{int}(A_2) = (d_3{}' * d_4{}') + (d_4{}' * d_3{}')$$

$$= 2[P(A_2X_1)P(A_2X_2)]^{\frac{1}{2}} \cos(\theta_4 - \theta_3) \quad (6.33)$$

Combining equation (6.25) and (6.31) we get

$$P(A_1) + P(A_2)$$
$$= P(A_1X_1) + P(A_1X_2) + P(A_2X_1)$$
$$+ P(A_2X_2) + Q_{int}(A_1) + Q_{int}(A_2) \quad (6.34)$$

$$\Rightarrow Q_{int}(A_1) + Q_{int}(A_2) = 0 \quad (6.35)$$

$$(\because Equation\ (6.21)\ and (\ 6.1(e))$$

This equality shows at least one term is non zero. So one of the terms necessarily is positive and another one will be negative. Hence when the probability of one term will increase the

probability of another term will decrease. This interference term is the pivotal feature to explain the violation of sure thing principle.

6.4 Two Stage Gambling Experiment

$P(X_1) =$ The probability of wining first gamble = 0.5

$P(X_2)$= Probability of not wining first gamble = 0.5

$$P(A_1|X_1) = 0.69$$

$$P(A_1|X_2) = 0.59$$

$$P(A_1X_1) = P(X_1)P(A_1|X_1) = 0.5 * 0.69 = 0.345$$

$$P(A_1X_2) = P(X_2)P(A_1X_2) = 0.5 * 0.59 = 0.295$$

Classically one expects:

$$P(A_1) = P(A_1X_1) + P(A_1X_2) = 0.345 + 0.295 = 0.64$$

But in the experiment we get:

$$P(A_1) = 0.36$$

This anomaly can be explained using equation (6.25)

$$P(A_1) = P(A_1X_1) + P(A_1X_2) + Q_{int}(A_1)$$

$$\Longrightarrow Q_{int}(A_1) = 0.36 - 0.64 = -0.28$$

Hence

$$Q_{int}(A_1) = -0.28$$

The factor $\mathrm{Q_{int}}(\mathrm{A_1})$ is compatible with the equation (6.30) that suggests:

$$+0.6380 \geq \mathrm{Q_{int}}(\mathrm{A_1}) \geq -0.6380 \qquad (6.36)$$

This compatibility proposes to facilitate the analysis based upon interference happening in the quantum model of decision, may not be capable to predict the outcome of this gambling experiment in advance, however can clearly describe the result. The experimental findings violate sure-thing principle (1954). On the other hand considering $\cos(\theta_2 - \theta_1)$ of the equation (6.29) as an adjustable parameter the data can be described by assigning

$$\cos(\theta_2 - \theta_1) = -0.439 \qquad (6.37)$$

Hence the value of $\cos(\theta_2 - \theta_1)$ with equation (6.29) leads to $Q_{int}(A_1) = -0.28$ that can describe the experimental outcome for agreeing to play a second game.

Similarly, the probability of not accepting second game, we can find the following results:

$$P(A_2|X_1) = 0.155, P(A_2|X_2) = 0.205\ , P(A_2) = 0.64 \tag{6.38}$$

$$Q_{int}\,(A_2) = 0.28\ and\ \cos(\theta_2 - \theta_1) = 0.785 \tag{6.39}$$

Hence under uncertainty- without knowledge of success or failure in the first game the decision in favor of playing the game for a second time is indeed more difficult than in favor of not to play the game for the second time. From equation (6.35) , as the sum $Q_{int}\,(A_1)\ and\ Q_{int}\,(A_2)$ is zero, we can draw a conclusion in advance that $Q_{int}\,(A_1)$ will be negative and $Q_{int}\,(A_2)$ will be positive. Hence we can conclude that using a quantum model of decision making we are capable to describe the outcomes of two-stage gambling which was not possible in classical formalism such as the principle of sure thing [143] or classical Markov model [130].

6.5 Buy or Not to Buy Experiment

Again we will consider a classical experiment of Amos Tversky and Eldar Shafir. Three groups of undergraduate students were taken in this experiment. They were asked to imagine suppose you have just taken one taught qualifying examination. It is the end of fall and you do not sure you passed the examination. You got an opportunity to buy a Christmas vacation package to Hawaii. The offer expires before the announcement of exam result. The first group consists of sixty seven students who were instructed to assume that they have cleared the examination. It is observed that only 54% students agreed to purchase a vacation package. The second group consists of sixty seven students were instructed to assume that they have not cleared the examination. It is observed that only 57% students agreed to purchase a vacation package. The third group consists of sixty six students were instructed to assume that they do not know the result of the examination. It is observed that only 32% students agreed to purchase a vacation package. The classical decision theory cannot explain the experiment so we will try quantum framework to explain the experiment. Let us try to generalize the conclusions discussed in the case of two-stage-gambling experiment for buy or not to buy as follows:

$Y_1 \rightarrow$ Pass the examination,$B_1 \rightarrow$ Buy the vacation

$Y_2 \rightarrow$ Not to pass in the examination, $B_2 \rightarrow$ Not to buy the vacation

$O_{PAS} \rightarrow\ Pass$ Operator, $O_{A_2} \rightarrow$ Buy Operator

Outline of experimental data to be analyzed:

$P(B_1|Y_1) \rightarrow$ Conditional Probability of buying a vacation if pass in the examination= 0.54

$P(B_1|Y_2) \rightarrow$ The conditional probability of buying a vacation if fail in the examination= 0.57

$P(B_1)$ →Probability of buying a vacation without knowledge of pass or fail in the examination = 0. 32

From the above data we can calculate:

$P(B_2|Y_1)$ → Conditional Probability of not buying a vacation if pass in the examination

$$P(B_2|Y_1) = 1 - P(B_1|Y_1) = 1 - 0.54 = 0.46$$

$P(B_2|Y_1)$ → Conditional Probability of not buying a vacation if fail in the examination

$$P(B_2|Y_2) = 1 - P(B_1|Y_2) = 1 - 0.57 = 0.43$$

$P(B_2)$ →Probabilities of not buying a vacation without knowledge of pass or fail in the examination.

$$P(B_2) = 1 - P(B_1) = 1 - 0.32 = 0.43$$

Considering

$$P(Y_1) = P(Y_2) = 0.5$$

We get

$$P(B_1Y_1) = P(Y_1)P(B_1|Y_1) = 0.5 * 0.54 = 0.270$$
$$P(B_1Y_2) = P(Y_2)P(B_1|Y_2) = 0.5 * 0.57 = 0.285$$

From quantum interference of probability we know:

$$P(B_1) = P(B_1Y_1) + P(B_1Y_2) + Q_{int}(B_1)$$
$$\Longrightarrow 0.32 - 0.270 - 0.285 = -0.235 = Q_{int}(B_1)$$

Hence

$$Q_{int}(B_1) = -0.235$$

This value is consistent with the equation given below:

$$-2[P(B_1Y_1)P(B_1Y_2)]^{\frac{1}{2}} \leq Q_{int}(B_1) \leq 2[P(B_1Y_1)P(B_1Y_2)]^{\frac{1}{2}}$$

So,

$$-0.555 \leq Q_{int}(B_1) \leq 0.555$$

Following equation (6.29) we can get

$$\cos(\theta_2 - \theta_1) = -0.424$$

Hence we can conclude that using a quantum model of decision the experimental outcome of buy or not to buy which was impossible in classical formalism such as the principle of sure thing [139].

6.6 Explanation of Merger and Acquisition Problem

From the discussion it can be concluded that there are some situations in which the classical decision theory is unable to explain but by using quantum formarmalisim we can explain the same situation in a better way. Hence the successful explanation of quantum models of human related decision making suggests that it can be applied to other fields where human decision making plays an important role. Therefore it is natural to adopt quantum theory in management science which promises for other decision making problems for example: supply chain management [63], group decision making [39] and Merger and acquisition [38]. We will investigate the merger and acquisition problem in the perspective of quantum decision model.

Let us explain merger and acquisition of real world companies in the notion of quantum decision model. In case of merger and acquisition of companies various issues play significant role such as the financial interests of both firms, the psychology of managers and board members of both firms, the ego factor in acquiring a business firm becoming a big firm etc. to reach at the successful merger deal. The share price of both merger and actuation firm is being affected after the announcement of the merger deal due to several reasons. The perception of the public about the companies is affected by the opinion and critical analysis of experts.

Subsequently after the broadcast of the final merger deal, the opinion of public and experts comment regarding the physiological factors linked to the merger influences the market value of both companies. In fact merger and acquisition is a complex process. We will apply the quantum model to study the mental state of the company who is buying another company considering psychology and financial factors of both firms in case of Merger and acquisition of companies. In literature merger and acquisition [97,108, 151, 160, 5] problem is studied in the context of business strategy.

Consider a simple view of merger and acquisition problem. Suppose the acquiring firm and merger firm be denoted as A_1 and A_2 respectively.

Let $|\Psi_{A_1}\rangle$ represents the state of mind of the firm A_1

The state can be written as a linear combination of price operator as follows:

$$|\Psi_{A_1}\rangle = a_1|E_{A_1}\rangle + a_2|F_{A_1}\rangle \tag{6.40}$$

Where a_1 and a_2 are complex coefficients. The normalization condition gives:

$$|a_1|^2 + |a_2|^2 = 1 \tag{6.41}$$

Where $|E_{A_1}\rangle$ corresponds the state of mentality of the acquiring firm A_1 dealing with financial factors after the declaration of the merger deal. If p_a be the current market price of stock A_1 then after the declaration of merger deal the market value of stock A_1 will be E_{A_1} times the present market value p_a . After the merger the total cost of all shares of firms A_1 and A_2 will be S^{A_1} times of the latest market price. Where S^{A_1} is the synergy (the technical definition of synergy is the ability of the merged company to generate higher shareholders than the stand alone entities) dependent factor absorbed in E_{A_1} .

The state $|F_{A_1}\rangle$ denotes the part of the state of mind of the firm A_1 associated with psychological factors of the acquiring firm A_1. The psychological factors such as: the fear of competitors may acquire A_2 . The psychological factors of the firm A_1 are not apparent to the public so it will not add any value to the existing market price after the merger and acquisition deal. The coefficient a_2 contains the outcome of such synergy factors.

The states $|\mathrm{E}_{A_1}\rangle$ and $|\mathrm{F}_{A_1}\rangle$ are eigenstates of price operator P_{A_1}:

1. Ortho normality condition of states $|\mathrm{E}_{A_1}\rangle$ and $|\mathrm{F}_{A_1}\rangle$ are given in equations (6.42) and (6.43)

$$\langle \mathrm{E}_{A_1}|\mathrm{E}_{A_1}\rangle = \langle \mathrm{F}_{A_1}|\mathrm{F}_{A_1}\rangle = 1 \tag{6.42}$$

$$\langle \mathrm{E}_{A_1}|\mathrm{F}_{A_1}\rangle = \langle \mathrm{F}_{A_1}|\mathrm{E}_{A_1}\rangle = 1 \tag{6.43}$$

2. The states $|\mathrm{E}_{A_1}\rangle$ and $|\mathrm{F}_{A_1}\rangle$ are eigenstates of price operator P_{A_1} with eigenvalues $\mathrm{p}_a \mathrm{E}_{A_1}$ and zero respectively.

$$\mathrm{P}_{A_1}|\mathrm{E}_{A_1}\rangle = \mathrm{p}_a \mathrm{E}_{A_1}|\mathrm{E}_{A_1}\rangle \tag{6.44}$$

$$\mathrm{P}_{A_1}|\mathrm{F}_{A_1}\rangle = 0|\mathrm{F}_{A_1}\rangle = 0 \tag{6.45}$$

Let the expected market value of stock A_1 in state $|\Psi_{A_1}\rangle$ of the firm A_1 be P^{A_1} .

So, the market value of A_1 in the state $|\Psi_{A_1}\rangle$ will be

$$P^{A_1} = \langle \Psi_{A_1}|\mathrm{P}_{A_1}|\Psi_{A_1}\rangle$$

Calculating

$$\mathrm{P}_{A_1}\left|\Psi_{A_1}\right\rangle = \mathrm{P}_{A_1}\left[a_1\left|E_{A_1}\right\rangle + a_2\left|F_{A_1}\right\rangle\right] = \mathrm{p}_a \mathrm{E}_{A_1}\left|\mathrm{E}_{A_1}\right\rangle$$

$$and$$

$$\langle\Psi_{A_1}| = {a_1}^*\langle E_{A_1}| + {a_2}^*\langle F_{A_1}|$$

We can find

$$P^{A_1} = \left\langle\Psi_{A_1}\left|\mathrm{P}_{A_1}\right|\Psi_{A_1}\right\rangle = |a_1|^2 \mathrm{p}_a \mathrm{E}_{A_1} \tag{6.46}$$

Equation (6.46) can be written in view of equation (6.41) as follows:

$$P^{A_1} = (1 - |a_2|^2)\mathrm{p}_a \mathrm{E}_{A_1} \tag{6.47}$$

From equation (6.47) if $a_2 = 0$ then the firm A_1 will expect the cost of their stock to be $\mathrm{p}_a \mathrm{E}_A$ after the declaration of merger. If a_2 is non zero then the stock price of firm A_1 will be $(1 - |a_2|^2)$ times of $\mathrm{p}_a \mathrm{E}_A$. Analyzing equation (6.47) we can conclude that the firm A_1 is losing an amount equal to $|a_2|^2 \mathrm{p}_a \mathrm{E}_{A_1}$ immediately after the deal in favor of the firm A_2 due to the fear factor of other competitors. The firm A_2 gains due to the factor a_2 present in the term. If a firm A_1 has N_1 number of shares and the firm A_2 has N_2 number of shares then firm A_1 scarifies $|a_2|^2 \mathrm{p}_a \mathrm{E}_{A_1}$ per share and firm A_2 gains an additional share equal to $|a_2|^2 \mathrm{p}_a \mathrm{E}_{A_1}(\frac{N_1}{N_2})$ to one share of A_2 . As the value of $(\frac{N_1}{N_2})$ being very big with respect to 1 hence the profit per share to A_2 possibly be very large even for a small change a_2. This fact is experimentally verified that the market price of the acquiring business firm generally decreases on the announcement of the merger. Study on merger and actuation by Andrade et al. [32] shows the acquiring firm bears a loss of 0.7% and the merged firm gets a 16% gain .The simple model explains the merger and acquisition problem.

6.7 Comparison of Classical vs Quantum Model

1. If we will put equation (6.29) in (6.24) we will get

$$P(A_1) = P(A_1X_1) + P(A_1X_2) + [P(A_1X_1)P(A_1X_2)]^{\frac{1}{2}}\cos(\theta_2 - \theta_1)$$

.

Hence in general the quantum probability can be written

$$P = P_1 + P_2 + 2\,cos\theta\sqrt{P_1P_2}$$

This way of representation is called quantum interference of probabilities and it is coming into the picture due to the transformation of vectors in complex Hilbert space.

Whereas the classical probability can be written as simple addition of two probabilities i.e. $P = P_1 + P_2$ .Which shows the major difference between classical and quantum probability.

2. From equations (6.19) and (6.20) we can conclude that the probabilities represented as squares of probability amplitudes $d_1, d_2\ etc$ in the case of quantum decision model. But in case of the classical decision framework there does not exist any probability amplitude terms.
3. The probability amplitudes are added in case of the quantum model to get $P(A_1)$. Using equation (6.24) we get the interference term $Q_{int}\,(A_1)$ naturally while calculating $\left|d_1{}' + d2'2\right.$. But in the classical decision model the respective probabilities are added.
4. In general the probability amplitudes are complex numbers. The phase factors influence the interference term $Q_{int}\,(A_1)$. In the classical decision making process we have not encountered the phase factors. In a limiting condition the value of the term $Q_{int}\,(A_1)$ found in quantum decision framework may lead to a zero. Under this circumstances quantum result merges with the classical outcome. This is possible when the cosine term of the equations (6.29) or (6.33) becomes zero.

6.8 Summary and Conclusions

In this chapter we explored the potential application of the quantum decision theory to explain some experimental findings of psychology which could not be described by the traditional classical theories. We have considered more general operators such as success-operator and failure-operator etc. to derive the results. Also we have analyzed the scope of this model to apply in a large range of business and management related problem domains such as merger and acquisition of business firms.

CHAPTER-7

CONCLUSIONS

7.1 Summary

The empirical correlation analysis has been introduced to examine the interrelationship of several Indian companies belonging to BSE. Different size of companies have been considered to study the correlation behavior. RMT has been applied to remove noise from the correlated stocks. The higher eigenvalues above random noise shows market mode and group or sector behavior of companies. The highest eigenvalue is independent of the number of companies participating to calculate the eigenvalue. The small eigenvalues below the bulk of eigenvalues show the strong correlationship which can be interpreted in the language of quantum physics as the entanglement characteristics of financial companies. We observed that the correlation behavior of small cap, large cap and mid cap companies is almost same. Finally behavior of stocks during financial market crash has been studied for different capitalization of companies. During the crisis the result shows that the correlation of stocks belonging to similar or related business sector moves together. We observed that the large-cap IT stocks go down during the financial crisis with respect to small cap companies. And the behavior of small-cap and mid-cap stocks is almost similar. From this we conclude that the emerging market is more correlated during crisis.

The network behavior of Indian companies was analyzed using Minimum spanning tree method. It is observed from the tree structure that a very few stocks form clusters. In order to show clustering of stocks we have taken a threshold $c^{th} = 0.09$. From the experiment we find chemical and banking sectors of mid cap stocks, industrial and chemical sectors of small cap companies and banking, software and energy sectors of large cap stocks form clusters. The rest of the companies do not form clusters evidences intra-group correlations are fragile in contrast to the market-wide correlation in BSE companies. Minimum spanning tree method has been applied to analyze market crash. It is observed that before the financial crisis the stocks are becoming closer to each other and during the crisis they become more closer and after the crisis they move away or they try to become independent from each other. This indicates that during crises the bond length between stocks decreases i.e. the stocks become closer to each other. During crisis small cap, large cap and mid cap companies are equally affected. And the nature of stocks is similar before and after the crisis.

Further we have discussed the fundamental concepts of quantum computing, quantum algorithm and its application to a financial market price prediction. Here we have analysed a method that predicts the market price of a financial instrument using the Deutsch's algorithm and it is computationally much faster than the classical approach. By evaluating the function $f(x)$ once using the Deutsch's algorithm we can predict the moment of a specified financial instrument much faster than the classical computational methods. This algorithm solves our problem efficiently but the drawback is in its implementation using quantum computers. Searching an unsorted database in quantum domain has been discussed using Grover's search algorithm for four qubit system. The mathematical representation and its necessary functionalities of quantum neuron have been shown along with the learning rule. The computational power of QANN and its advantage over ANN have been discussed. Finally we have explored the potential application of quantum decision model to explain some experimental findings of psychology which could not be described by the traditional classical probability theories. We have considered more general operators such as success-operator and failure-operator etc. to derive the results. Also we have analyzed the scope of this model to apply in a large range of business and management related problem domains such as merger and acquisition of business firms.

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APENDIX-I

LARGE CAP COMPANIES

sl.	Company	Sector
1	Maruti Suzuki India Limited	Auto
2	Bajaj Auto Ltd	Auto
3	Bosch Ltd	Auto
4	AXIS Bank Limited	Bank
5	HDFC Bank Ltd.	Bank
6	ICICI Bank Ltd.	Bank
7	IndusInd Bank Ltd	Bank
8	Kotak Mahindra Bank Limited	Bank
9	Yes Bank Ltd.	Bank
10	Punjab National Bank	Bank
11	Union Bank of India	Bank
12	Hindalco Industries Ltd.	Metal
13	NMDC Ltd.	Metal
14	Hindustan Zinc Ltd.	Metal
15	JSW Steel Ltd.	Metal
16	Sterlite Industries (India) Ltd.	Metal
17	Tata Steel Limited	Metal
18	Sesa Goa Limited	Metal
19	Cairn India Limited	Energy
20	GAIL (India) Limited	Energy
21	Oil and Natural Gas Corp. Ltd.	Energy
22	Bharat Petroleum Corp. Ltd.	Energy
23	Oil India Ltd.	Energy
24	Indian Oil Corporation Limited	Energy
25	Reliance Industries Limited	Energy
26	Nhpc Ltd.	Energy
27	Ntpc Ltd.	Energy
28	Cadila Healthcare Limited	Pharma
29	Dr. Reddy's Laboratories Ltd.	Pharma
30	Lupin Limited	Pharma

31	Ranbaxy Laboratories Ltd.	Pharma
32	Power grid Corporation Ltd.	Power
33	Tata Power Company Limited	Power
34	HCL Technologies Ltd.	Software
35	Tech Mahindra Limited	Software
36	Oracle Financial Services Software Limited	Software
37	Infosys Ltd.	Software
38	Tata Consultancy Services Limited	Software
39	WIPRO LTD.	Software
40	Mahindra Satyam	Software
41	Bharti Airtel Limited	Telecom
42	Idea Cellular Limited	Telecom
43	Reliance Communications Ltd.	Telecom
44	Bharati Infratel Limited	Telecom
45	Adani Enterprises Ltd.	Trading
46	Shriram Transport Finance Company Ltd	Trading
47	MMTC Ltd.	Trading
48	Castrol India Ltd.	Lubricant
49	ACC Limited	Cement
50	DLF Ltd.	Real estate
51	Hindustan Unilever Ltd. (HUL)	Personal care
52	Asian Paints Limited	Paints
53	Siemens Ltd.	Electric Equipment
54	Larsen & Toubro Ltd.	Engineering
55	Zee Entertainment Enterprises Ltd.	Entertainment

MID CAP COMPANIES

sl.	Company	Sector
1	Himatsingka Seide Ltd.	Textile
2	Indo Rama Synthetics India Ltd.	Textile
3	RSWM Limited	Textile
4	SRF Limited	Textile
5	Century Textiles and Industries Ltd	Textile
6	Greaves Cotton Limited	Textile
7	The Bombay Dyeing Limited	Textile

8	Vardhman Textiles Limited	Textile
9	Gujarat Industries Power Co. Ltd.	Industrial
10	Kesoram Industries Ltd.	Industrial
11	Hindusthan National Glass Ind Ltd.	Industrial
12	Britannia Industries Limited	Industrial
13	Pidilite Industries Limited	Industrial
14	Praj Industries Ltd.	Industrial
15	Prakash Industries Limited	Industrial
16	Ruchi Soya Industries Limited	Industrial
17	Sintex Industries	Industrial
18	Videocon Industries Ltd.	Industrial
19	Clariant Chemicals (India) Ltd	Chemical
20	National Fertilizers Ltd.	Chemical
21	Chambal Fertilisers & Chemicals Ltd.	Chemical
22	Rashtriya Chemicals & Fertilizers Ltd	Chemical
23	BASF India Limited	Chemical
24	Coromandel International Limited	Chemical
25	Gujarat Narmada Valley Limited	Chemical
26	Gujarat State Fertilizers Ltd.	Chemical
27	Tata Chemicals Limited	Chemical
28	Zuari Global Limited	Chemical
29	Southern Petrochemical Limited	Chemical
30	AstraZeneca Pharma India Limited	Pharma
31	Aurobindo Pharma Ltd.	Pharma
32	Sterling Biotech Limited	Pharma
33	Corporation Bank	Bank
34	Dena Bank Ltd.	Bank
35	Syndicate Bank Limited	Bank
36	Jammu & Kashmir Bank Limited	Bank
37	South Indian Bank Ltd.	Bank
38	UCO Bank	Bank

39	Vijaya Bank Ltd.	Bank
40	India Infoline Limited	Comp
41	MindTree Limited	Comp
42	Rolta India Limited	Comp
43	Tulip Telecom Limited	Comp
44	Educomp Solutions Limited	Comp
45	Glodyne Technoserve Limited	Comp
46	Infotech Enterprises Limited	Comp

SMALL CAP COMPANIES

sl.	Company	Sector
1	Ifb Industries Ltd.	Industrial
2	Jagatjit Industries Ltd.	Industrial
3	Jayaswal Neco Industries Ltd.	Industrial
4	Jvl Agro Industries Ltd.	Industrial
5	Laxo Industries Ltd.	Industrial
6	Lumax Industries Ltd.	Industrial
7	Murli Industries Ltd.	Industrial
8	Balkrishna Industries Ltd.	Industrial
9	Tilaknagar Industries Ltd.	Industrial
10	Vishakha Industries Ltd.	Industrial
11	Vst Industries Ltd.	Industrial
12	Jaibalaji Industries Ltd.	Industrial
13	Surana Industries Ltd.	Industrial
14	Alok Industries Ltd.	Industrial
15	Apar Industries Ltd.	Industrial
16	Binani Industries Ltd.	Industrial
17	Deep Industries Ltd.	Industrial
18	Ester Industries Ltd.	Industrial
19	Everest Industries Ltd.	Industrial
20	Finolex Industries Ltd.	Industrial

21	Greenplay Industries Ltd.	Industrial
22	Gujurat Apolo Industries Ltd.	Industrial
23	Page Industry Industries Ltd.	Industrial
24	Ramco Industries Ltd.	Industrial
25	Ramsarup Industries Ltd.	Industrial
26	Relience Industrial Infra.	Industrial
27	Kemrock Industries Ltd.	Industrial
28	Pennar Industries Ltd.	Industrial
29	Pratibha Industries Ltd.	Industrial
30	Sanghi Industries Ltd.	Industrial
31	Standard Industries Ltd.	Industrial
32	Sudal Industries Ltd.	Industrial
33	Century Enka Ltd.	Textile
34	Nahar Spinning Mills Ltd.	Textile
35	Garden Silk Mills Ltd.	Textile
36	Siyaram Silk Mills Ltd.	Textile
37	Aunde India Ltd.	Textile
38	Banaswara Syntex Ltd.	Textile
39	Cheviot Co. Ltd.	Textile
40	Garware Marine Ltd.	Textile
41	Ludlow Jute & Specialties Ltd.	Textile
42	Uniroyal industries ltd	Textile
43	Amtek India Ltd.	Automobile
44	Rico Auto Industries Ltd.	Automobile
45	Sona koyo steering systems	Automobile
46	Munjal Showa Ltd.	Automobile
47	Dhampur Sugar Mills Ltd.	Food
48	Jay Shree Tea & Industries Ltd.	Food
49	Tata Coffee Ltd.	Food
50	Dwarikesh Sugar India Ltd.	Food
51	Gayatri Sugars Ltd.	Food
52	Rana Sugars Ltd.	Food
53	Sakthi Sugars Ltd.	Food
54	Shah Food Ltd.	Food
55	Shree Renuka Sugars Ltd.	Food
56	Thiru Arooran Sugar ltd	Food
57	Upper Ganges sugar Industries Ltd.	Food

58	Venus Sugars Ltd.	Food
59	Action Construction Equipment	Construction
60	C and C Cnstructions	Construction
61	Petron Engg. Construction	Construction
62	Gic Housing	Finance
63	Gruh Finance	Finance
64	Tourism Finance Crop of India	Finance
65	Ruchi Infrastructure	Finance
66	Srei infrastructure finance	Finance
67	Shree Ram Urban Infrastructure Ltd.	Finance
68	Marksans Pharma	Pharma
69	Parenteral Drug (India)	Pharma
70	Themis Medicare	Pharma
71	Unichem laboratories Ltd.	Pharma
72	Rpg Life Science Ltd.	Pharma
73	Span diagnostics ltd.	Pharma
74	Aditya Birla Chemicals	Chemical
75	Mangalore Chemicals & Fert.	Chemical
76	Bodal Chemicals Ltd.	Chemical
77	Deepak Fertilisers & Petrochemicals Corporation. Ltd.	Chemical
78	Empee Sugars & Chemicals Ltd.	Chemical
79	Gujrat Alkalies & Chem. Ltd.	Chemical
80	Hindustan Organic Chem. Ltd.	Chemical
81	Indian Glycols Ltd.	Chemical
82	Orchid Chemicals & Pharma. Ltd.	Chemical
83	Oil Country Tubular Ltd.	Energy
84	Confidence Petroleum.india Ltd.	Energy
85	Laxmi Energy Ltd.	Energy
86	Sarda Energy Ltd.	Energy
87	Websol energy system ltd.	Energy