BINARY IMAGE RECONSTRUCTION WITH DIAGONAL AND ANTI-DIAGONAL PROJECTIONS

A THESIS

Submitted in partial fulfilment of the

requirements for the award of the degree

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in

MATHEMATICS

by

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis, entitled **"BINARY IMAGE RECONSTRUCTION WITH DIAGONAL AND ANTI-DIAGONAL PROJECTIONS"** in partial fulfilment of the requirements for the award of the Degree of Doctor of Philosophy and submitted in the Department of Mathematics of Indian Institute of Technology Roorkee, Roorkee is an authentic record of my own work carried out during a period from July, 2009 to November, 2013 under the supervision of Dr. Tanjua Srivastava, Professor, Department of Mathematics, Indian Institute of Technology Roorkee, Roorkee.

The matter presented in the thesis has not been submitted by me for the award of any other degree of this or any other Institute.

(SHIV KUMAR VERMA)

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

(Tanuja Srivastava)
Supervisor

Date: November, 2013

The Ph.D. Viva-Voce Examination of **Mr**. **Shiv Kumar Verma**, Research Scholar, has been held on……

Signature of Supervisor Chairman, SRC Signature of External Examiner

Head of the Department/ Chairman, ODC

ABSTRACT

Discrete tomography refers to reconstruction of discrete functions from its projections and the function has discrete finite range and the domain of discrete function is bounded which may be continuous or discrete. In particular, when the range of discrete function consists only two values 0 and 1 and the domain consists the finite discrete set, the problem of discrete tomography is changed to determination of discrete sets by their projections in few directions, here projection represents the number of lattice points on each parallel line in few directions. Thus problem of discrete tomography can be referred as reconstruction of binary images from its line sum, whereas in classical problem of Discrete Tomography the lines are rows and columns.

In present thesis, the projections are considered from two directions as diagonal (45^0) and anti-diagonal (135^0) only. Hence the problem of discrete tomography is to reconstruct the function f on finite lattice set X from its projections in diagonal and anti-diagonal direction. Mathematically, this problem is to get the binary matrix from its diagonal and anti-diagonal sums.

The compatibility and consistency of the projection data and the existence of solution of unique reconstruction is a challenging task of discrete tomography. In case of diagonal and anti-diagonal projection set it has not yet been reported. Thus in present thesis it has been achieved to get the consistency of the projection set then the algorithms proposed to get the unique reconstruction of two orthogonal projections without using any constraints. Mathematical formulation and characterization of projection set in diagonal and anti-diagonal direction has been determined for the reconstruction of binary images and to achieve the goal of research work.

The analysis of all proposed reconstruction algorithms has been performed on the basis of misclassification percentage of pixels between the reconstructed and original binary images. The reconstruction algorithms proposed by Chang, without using any constraints or a priori information about the object, thus algorithms in present thesis have been compared to verify the quality of proposed reconstruction algorithms.

Binary images have been Reconstructed and the error occurred in reconstruction process has been reported as of percentage of misclassification. The outcome of the proposed algorithms reveals that the reconstruction of binary images is possible from orthogonal projection (Diagonal & anti-diagonal) only without using any constraints. Maximum average misclassification percentage is achieved 7.3 % and minimum 2.43% which is much significant as compare to Chang's (max.32.4% & min 10.77%). In case of hv-convex binary images, it has been noted that no proposed algorithms has crossed this upper and lower limit of misclassification, although for non convex binary images this limit is 47% and 0%, where zero percent misclassification signifies the uniqueness of binary image.

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With profound gratitude, love and devotion, I dedicate this Thesis to my family.

Roorkee
November, 2013 (SHIV KUMAR VERMA)

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Acronyms

ART	Algebraic Reconstruction Techniques
CT	Computerized Tomography
CAT	Computerized Axial Tomography
dad-Projection	Diagonal and Anti-Diagonal- Projections
DART	Discrete Algebraic Reconstruction Technique
DT	Discrete Tomography
hv-convex	Horizontal and Vertical- Convex
MRI	Magnetic Resonance Imaging
MSART	Modified Simultaneous Algebraic Reconstruction Technique
NDT	Non Destructive Testing
NMR	Nuclear Magnetic Resonance
PET	Positron Emission Tomography
SART	Simultaneous Algebraic Reconstruction Technique
SIRT	Simultaneous Iterative Reconstruction Technique

Author's publications

1. Tanuja Srivastava, Shiv Kumar Verma and Divyesh Patel, "Reconstruction of Binary Images from Two Orthogonal Projections", International Journal of Tomography and Simulations, Vol.21 (2), pp. 105-114, 2012.
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