Synopsis for the Ph.D. Thesis

on

EFFICIENCY MEASURMENT OF PUBLIC SECTOR HOSPITALS IN UTTARAKHAND THROUGH *DEA* TECHNIQUE

by

Sandeep Kumar Mogha

under the supervision of

Dr. Shiv Prasad Yadav
Professor,
Department of Mathematics
IIT Roorkee, Roorkee

Dr. S.P. Singh
Professor,
Department of Humanities
and Social Sciences
IIT Roorkee, Roorkee

Department of Mathematics
Indian Institute of Technology Roorkee
Roorkee -247667 (India)
April-2014

Uttarakhand, the 27th state of India, attained its statehood on November 9, 2000. It consists of thirteen districts and is culturally divided into two regions: Garhwal and Kumaon. Out of thirteen districts, two districts namely Haridwar and Udham Singh Nagar are in the plain region and two districts namely Dehradun and Nanital are partially covered by plain areas while the rest area of the state is fully hilly. A large part of it is hilly and thinly populated where public hospitals are the main source of healthcare services; while in densely populated plain region, people have better access to private healthcare services. On an average, per capita public expenditure on health in Uttarakhand is higher than the national average, while reverse is true in case of per capita private expenditure on health. Since public resources at the disposal of the state government are limited and have competitive uses, it becomes imperative to make their efficient use so that maximum social welfare may be achieved.

Uttarakhand has an extensive network of public health institutions. It has 2 women & child welfare centers, 1765 women & child welfare sub-centers, 84 main centers, 250 additional primary health centers (PHCs), 55 community health centers (CHCs), 322 allopathic dispensaries, 39 rural female hospitals, 107 homeopathic dispensaries, 540 ayurvedic hospitals, 5 unani hospitals, 18 tuberculosis hospitals, 24 tehsil/district level post-mortem centers and 36 district / base / combined hospitals [15]. To cater to specific diseases, the State owns 14 T.B. hospitals, 23 blood banks, 3 leprosy hospitals, 9 urban leprosy centers and 7 urban family welfare centers. Only one private medical college and 2 government ayurvedic medical colleges are positioned in the State. However, there exists a wide disparity in the public healthcare infrastructure across districts and regions. For instant, 60% of government hospital beds are situated only in four districts, namely, Pauri, Almora, Dehradun and Nanital. The State also faces shortage of training institutions and public health management experts. It also suffers from insufficiency of medical and paramedical staff and their willingness to work in the inaccessible areas. Out of total 6143 sanctioned positions of doctors, 1799 are vacant.

Public sector hospitals play a significient role in the overall development of a nation’s economy. In the developing countries like India, public hospitals are the backbone of the healthcare system. Hospital’s efficiency is often difficult to evaluate because it is difficult to choice of input-output variables for efficiency evaluation and producing more and more outputs. Most sophisticated analytical tools are required to evaluate the efficiency of service sectors such as public healthcare sector. Data Envelopment Analysis (DEA) happens to be appropriate for such evaluation. It identifies the best performing decision making units (DMUs) without requiring a prior information of input and output prices or specification of the technologies. It can handle multiple inputs and outputs and does not require any assumption of a functional form relating inputs to outputs. Thus, it is well suited for comparative performance analysis of public sector hospitals.

Public sector hospitals, which provide un-priced services outside the market mechanism, bristle with the conceptual difficulty of such a precise delimitation of inputs and outputs. The problem becomes more acute in the absence of relevant data pertaining to the inputs and outputs. As such, this difficulty is partially by-passed by some performance measurement techniques which do provide scope for testing alternative input and output definitions using different combinations of inputs and outputs. One approach towards this end has been to examine the performance status of public hospitals on the basis of which policy decisions on the future course of action could be taken. It is in this context that this study applies DEA to measure the technical and scale efficiencies of public hospitals of Uttarakhand with a view to identify inefficient hospitals and input reduction (output augmentation) required to make them efficient [5, 6].

By critical examination of the available literature, it is found that DEA based studies dealing with the relative efficiency of public sector hospitals in India are extremely limited. The present study attempts to assess the performance of the public sector hospitals of Uttarakhand State through DEA technique. The CCR and BCC models are applied to determine the efficiencies of public sector hospitals of Uttarakhand. The study also measures the impact of various available background variables on the efficiency using Tobit regression model. Sensitivity analysis is conducted to identify the outliers on the frontier, verify the robustness of the efficiency scores and estimate the super efficiency scores by ranking the efficient hospitals. A non-oriented slack based model (SBM) is also applied to measure the efficiencies of the hospitals.

The basic DEA models (CCR and BCC models) cannot assess the impact of slacks on efficiency scores. Also the results obtained by these models show that many multipliers have zero value which indicates that the corresponding variables (inputs/outputs) have not been fully utilised in the assessment of the efficiency scores. To overcome this problem, new slack model (NSM) is applied to the data of 27 public sector hospitals of Uttarakhand.

Time series data for the period 2001-2011 have been used to assess the growth trends and efficiency patterns of the hospitals. The cross-sectional and panel data required for efficiency and productivity analysis of public sector hospitals of Uttarakhand are collected from the Directorate of Medical Health and Family Welfare, Government of Uttarakhand, Dehradun. We estimate the production correspondence between three inputs and four outputs , as per the availiblity of the data. Number of beds, number of doctors and number of paramedical staff (PMS) are considered as input variables and number of out-door patients (OPD), number of in-door patients (IPD), number of major surgery and number of minor surgery are considered as output variables. An attempt is also made to incorporate a comprehensive list of inputs and outputs reflects the general informative and robust results.

The objectives of the study are:

i. To measures the relative efficiencies of the public hospitals of Uttarakhand,
ii. To measures the effect of several background variables on the performance of the hospitals,
iii. To identify use of excess input and deficient output for the inefficient hospitals,
iv. To verify the robustness of the efficiencies of the hospitals,
v. To assess the impact of slacks on efficiencies and Super Efficiency scores of efficient hospitals,
vi. To analyse the growth trends and efficiency pattern of the public hospitals,
vii. To assess the total factor productivity changes, technical efficiency and technical changes in the hospitals.

The chapter-wise summary of the thesis is as follows:

Chapter 1 is introductory in nature. In brief, it deals with the aspects of performance measurement, history, infrastructure, statement of the problem, objectives and scope and limitations of the study. This chapter also discusses briefly about the DEA techniques used to assess the efficiency and productivity.

Chapter 2 is devoted to the review of relevant DEA and Malmquist Productivity Index (MPI) based studies on health sector and other sectors. Studies on health sector in Indian context as well as foreign context are reviewed separately. Studies related to other applications with DEA are also reviewed.

Chapter 3 shows the advancement in DEA techniques from the basic models to the new DEA based models. This chapter presents the basic (CCR and BCC) models, AR model, DEA models based on slacks such as Additive model, Two-stage model, slack based model (SBM), slack adjusted (SA) model, new slack model (NSM) and DEA models with non-discretionary variables and categorical DMUs.

Chapter 4 presents, the Cross-sectional analysis of the public sector hospitals of the state for the calendar year 2011. The efficiencies of the hospitals are assessed by using the basic DEA based CCR model [4]. For estimating that whether inefficiency of any hospital is due to inefficient production operation or due to unfavorable conditions displayed by the size of hospital, DEA based BCC model is also applied [2]. Region-wise, category-wise and area-wise analysis of efficiencies of the hospitals are also calculated. To identify the effect of background variables, Tobit regression [13] model and to check the robustness of the efficiency scores, sensitivity analysis [10] have also been conducted.

In **Chapter 5** we use a non-oriented model slack based model (SBM) [14] of DEA to evaluate the efficiency of hospitals. This model also deals with slacks. This model allows managers to work on both inputs and outputs to achieve efficiency. Generally, in case of public hospitals it is difficult to choose the orientation (input or output) for the evaluation of efficiencies. It is not admirable to reduce input levels or increase output levels regarding public sector hospitals. So, in this chapter, a non-oriented and non-radial model known as SBM-DEA model has been used.

In **Chapter 6** we measure the technical efficiencies of 27 public sector hospitals of the state with the actual impact of slacks, by using new slack model (NSM) [1] of DEA. In this model, all the inputs and outputs are utilized in the performance assessment. This chapter also discusses the relationship of NSM model with the basic CCR model. This model also identify the slacks and set the benchmarks for the inefficient hospitals. The region-wise performance of the hospitals has also been made. Sensitivity analysis is also applied to know the robustness of the efficiency scores. In this chapter we also identifies the **super efficiency** scores of the hospitals and ranks the efficient hospitals [11].

Chapter 7 deals with theoretical aspects of Malmquist Productivity Index (MPI) [12] and assesses the total factor productiviy (TFP) change [3, 7, 8, 9], technical efficiency change and technical changes that have taken place in public hospitals of Uttarakhand. It also examine the important reasons of low and high productivity growth in the public hospitals of the state. DEA based MPI approach is used to evaluate the TFP change in the public hospitals for the period 2001 to 2011. The technical efficiency change, technical change and TFP change in the public hospitals are calculated across region-wise and area-wise.

The last **Chapter 8** presents the summary of findings, conclusions and recommendations for the policy considerations along with some suggestions for the future research.

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4. **Sandeep Kumar Mogha**, Shiv Prasad Yadav and S.P. Singh, (2013), "Total Factor Productivity Growth in Private Sector Hospitals of India: An Assessment through MPI Approach", *Global Business and Economic Review*, **(Accepted)**.
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6. **Sandeep Kumar Mogha**, Shiv Prasad Yadav and S.P. Singh, (20**), "Technical Efficiency and Productivity Growth in Public Sector Hospitals of Uttarakhand (India)", *Health Systems*, **(Revision Submitted)**.
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Papers in Conference Proceedings

1. **Sandeep Kumar Mogha**, Shiv Prasad Yadav and S.P. Singh, "Technical and Relative Efficiency Assessment of Some Private Sector Hospitals in India*", Int. Conf. on Soft Computing for Problem Solving* (Soc.Pros.) held in Dec. 20-22, 2011 at IIT Roorkee, AISC 130, pp 657-666.
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3. **Sandeep Kumar Mogha**, Shiv Prasad Yadav and S.P. Singh, (2014), "SBM-DEA Model Based Efficiency Assessment of Public Sector Hospitals in Uttarakhand, India", *Int. Conf. on Soft Computing for Problem Solving* (Soc.Pros.-2013) held in Dec. 26-28, 2013 at Noida Extension center IIT Roorkee, Vol. 258, pp 467-480.
4. **Sandeep Kumar Mogha**, Shiv Prasad Yadav and S.P. Singh, (2014), "Efficiency Assessment of Secondary Level Public Hospitals of Uttarakhand (India)", *12th International Conference on Data Envelopment Analysis*", April, 14-17, 2014, Malaya Unversity, Kula Lumpur, Malaysia, pp. **-**.

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CANDIDATE’S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled **“EFFICIENCY MEASURMENT OF PUBLIC SECTOR HOSPITALS IN UTTARAKHAND THROUGH *DEA* TECHNIQUE”** in partial fulfilment of the requirements for the award of the Degree of Doctor of Philosophy and submitted in the Department of Mathematics of the Indian Institute of Technology Roorkee is an authentic record of my own work carried out during a period from July, 2009 to June, 2014 under the supervision of Dr. Shiv Prasad Yadav, Professor, Department of Mathematics and Dr. S.P. Singh, Professor, Department of Humanities and Social Sciences, Indian Institute of Technology Roorkee, Roorkee.

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other Institute.

(SANDEEP KUMAR MOGHA)

This is to certify that the above statement made by the candidate is correct to the best of our knowledge.

(Shiv Prasad Yadav)
(Supervisor)

(S.P. Singh)
(Supervisor)

Date: June , 2014

The Ph.D. Viva-Voce Examination of **Mr. Sandeep Kumar Mogha,** Research Scholar, has been held on……………………………………………………………………………..

Signature of Supervisor Signature of Chairman, SRC Signature of External Examiner

Head of the Department/Chairman, ODC

Abstract

Uttarakhand, the 27th state of India, attained its statehood on November 9, 2000. It consists of thirteen districts and is culturally divided into two regions: Garhwal and Kumaon. Out of thirteen districts, two districts namely Haridwar and Udham Singh Nagar are in the plain region and two districts namely Dehradun and Nanital are partially covered by plain areas, while the rest area of the state is fully hilly. A large part of it is hilly and thinly populated where public hospitals are the main source of healthcare services; while in densely populated plain region, people have better access to private healthcare services. On an average, per capita public expenditure on health in Uttarakhand has been higher than the national average, while the reverse is true in the case of per capita private expenditure on health. Since public resources at the disposal of the state government are limited and have competitive uses, it becomes imperative to make their efficient use so that maximum social welfare may be achieved.

Uttarakhand has an extensive network of public health institutions. It has 2 women & child welfare centers, 1765 women & child welfare sub-centers, 84 main centers, 250 additional primary health centers (PHCs), 55 community health centers (CHCs), 322 allopathic dispensaries, 39 rural female hospitals, 107 homeopathic dispensaries, 540 ayurvedic hospitals, 5 unani hospitals, 18 tuberculosis hospitals, 24 tehsil/district level post-mortem centers and 36 district / base / combined hospitals [27]. To cater to specific diseases, the State owns 14 T.B. hospitals, 23 blood banks, 3 leprosy hospitals, 9 urban leprosy centers and 7 urban family welfare centers. Only one private medical college and 2 government ayurvedic medical colleges are positioned in the State. However, there exists a wide disparity in the public healthcare infrastructure across districts and regions. For instant, 60% of government hospital beds are situated only in four districts, namely, Pauri, Almora, Dehradun and Nanital. The State also faces a shortage of training institutions and public health management experts. It also suffers from insufficiency of medical and paramedical staff and their willingness to work in the inaccessible areas. Out of total 6143 sanctioned positions of doctors, 1799 are vacant.

Public sector hospitals play a significant role in the overall development of a nation's economy. In the developing countries like India, public hospitals are the backbone of the

healthcare system. Hospitals efficiency is more difficult to evaluate than manufacturing business efficiency because it is difficult to choose the input-output variables for efficiency evaluation. More sophisticated analytical tools are required to evaluate the efficiency of service sectors, such as, public healthcare sector. Data Envelopment Analysis (DEA) happens to be appropriate for such evaluation. It identifies the best performing decision making units (DMUs) without requiring prior information of input and output prices or specification of the technologies. It can handle multiple inputs and outputs and does not require any assumption of a functional form relating inputs to outputs. Thus, it is well suited for comparative performance analysis of public sector hospitals.

Public sector hospitals, which provide un-priced services outside the market mechanism, bristle with the conceptual difficulty of such a precise delimitation of inputs and outputs. The problem becomes more acute in the absence of relevant data pertaining to the inputs and outputs. As such, this difficulty is partially bypassed by some performance measurement techniques which do provide scope for testing alternative input and output definitions using different combinations of inputs and outputs. One approach towards this end has been to examine the performance status of public hospitals on the basis of which policy decisions on the future course of action could be taken. It is in this context that this study applies DEA to measure the technical and scale efficiencies of public hospitals of Uttarakhand with a view to identify inefficient hospitals and input reduction (output augmentation) required to make them efficient.

By critical examination of the available literature, it is found that DEA based studies dealing with the relative efficiency of public sector hospitals in India are extremely limited. The present study attempts to assess the performance of the public sector hospitals of Uttarakhand State through this technique. The CCR and BCC models are applied to determine the efficiencies of public sector hospitals. The study also measures the impact of various available background variables on the efficiency using a Tobit regression model. Sensitivity analysis is conducted to identify the outliers on the frontier and verify the robustness of the efficiency scores. Super efficiency models are also applied to rank the efficient hospitals. A non-oriented slack based model (SBM) is also applied to measure the efficiencies of the hospitals.

The basic DEA models cannot assess the impact of slacks on efficiency scores. Also the results obtained from these models show that many multipliers have zero value which indicates that the corresponding variables (inputs/outputs) have not been fully utilized in the assessment of the efficiency scores. To overcome this problem, new slack model (NSM) is applied to the data of 27 public sector hospitals of Uttarakhand.

Time series data for the period 2001 to 2011 have been used to assess the growth trends and efficiency patterns of the hospitals. The data are collected from the Directorate of Medical Health and Family Welfare, Government of Uttarakhand, Dehradun for the period from 2001 to 2011. We estimate the production correspondence between three inputs and four outputs, as per the availability of the data. We consider number of beds, number of doctors and number of paramedical staff (PMS) as input variables and number of out-door patients (OPD), number of in-door patients (IPD), number of major surgery and number of minor surgery as output variables for the assessment of productivity and efficiency in the hospitals.

The main objectives of the study are to:

i. analyse the growth trends and efficiency pattern of the public hospitals;
ii. measure the relative efficiencies of the public hospitals of Uttarakhand;
iii. measure the effect of several background variables on the performance of the hospitals;
iv. identify use of excess input and deficient output for the inefficient hospitals;
v. verify the robustness of the efficiencies of the hospitals;
vi. measure the super efficiency scores of efficient hospitals; and
vii. assess the total factor productivity changes, technical efficiency and technical changes in the hospitals.

The chapter-wise summary of the thesis is as follows:

Chapter 1 is introductory in nature. It deals with the aspects of performance measurement, statement of the problem, objectives, scope, and limitations of the study. This chapter also discusses briefly about the DEA techniques used to assess the efficiency and productivity.

Chapter 2 is devoted to the review of literature on theme. DEA and MPI based studies on the health sectors in India and abroad along with some other relevant studies have been reviewed.

Chapter 3 shows the advancement in DEA techniques from the basic models to the new DEA based models. This chapter presents the basic (CCR and BCC) models, AR model, and DEA slack-based models, such as, Additive model, Two-stage model, SBM model, SA model, NSM model, and DEA models with non-discretionary and categorical variables.

Chapter 4 presents the efficiency evaluation of the public sector hospitals of Uttarakhand. This chapter presents a cross-sectional analysis of the public hospitals of the state for the calendar year 2011. Tobit regression and sensitivity analysis have also been carried out to identify the effect of background variables and robustness of the efficiency scores.

Chapter 5 deals with a non-oriented “slack based model” (SBM) of DEA to evaluate the efficiency of hospitals. This model allows managers to work on both inputs and outputs to achieve efficiency. Generally, in case of public hospitals it is difficult to choose the orientation (input or output) for the evaluation of efficiencies. It is not admirable to reduce input levels or increase output levels regarding public sector hospitals. So, in this chapter, a non-oriented and non-radial model known as SBM-DEA model has been used.

Chapter 6 deals with a new slack model (NSM) of DEA to evaluate the efficiency with the actual impact of slacks on the efficiency scores. This chapter also presents the super efficiencies of the hospitals and ranks of the efficient hospitals.

Chapter 7 deals with theoretical aspects of the Malmquist Productivity Index (MPI) and assesses the total factor productivity (TFP) change, technical efficiency change and technical changes that have taken place in public hospitals for the period 2001 to 2011.. It also examines the reasons of low productivity in the public hospitals.

The last chapter presents a summary of findings, conclusions and recommendations for the policy considerations along with some suggestions for the future research.

Acknowledgements

This work would never have been accomplished without **God Almighty** with his blessings and his power that work within me.

I wish to express my deep sense of gratitude towards my Supervisors **Dr. Shiv Prasad Yadav**, Professor, Department of Mathematics and **Dr. S.P. Singh**, Professor, Department of Humanities and Social Sciences, Indian Institute of Technology Roorkee, Roorkee, for their invaluable guidance, sincere advice and encouragement throughout the completion of this research work.

I find myself fortunate to have had the opportunity to work under their supervision during the course of this work. Their encouraging support and thorough involvement in the formulation of the research work and preparation of the manuscript are gratefully acknowledged. I sincerely appreciate their pronounced individualities, humanistic and warm personal approach, which has given me strength to carry out this research work smoothly. I humbly acknowledge a lifetime's gratitude to them.

I am very thankful to my SRC members **Dr. T.R. Gulati**, Professor, Department of Mathematics, Indian Institute of Technology Roorkee, **Dr. J.P. Singh**, Professor, Department of Management Studies, Indian Institute of Technology Roorkee, for their invaluable suggestions and encouragement throughout my work.

I would like to express my heartfelt gratitude to **Prof. R.C. Mittal**, Head, Department of Mathematics, Indian Institute of Technology Roorkee and **Prof. P.N. Agarwal**, DRC Chairman, Department of Mathematics, Indian Institute of Technology Roorkee, for their warm personal approach, providing a healthy environment for research and organizational support. I am also indebted to all the faculty members of the department for their support and help to carry out this work. I would like to thank administrative and technical staff members of the Department and the Institution who have been kind enough to advise and help in their respective roles.

For the financial support I would like to thank the University Grant Commission (UGC) New Delhi Government of India.

I express my appreciation to my seniors **Dr. Shivi Agarwal,** Assistant Professor, BITS Pilani Rajasthan and **Dr. Preeti Tyagi,** Assistant Professor, Delhi University and **Dr. R.N. Joshi**,

Associate Professor, Department of Textile Technology, SGGSIE&T Nanded, for generously sharing their time and knowledge during my research endeavor.

I would like to extend my appreciation and thanks to my colleague Sujeet Kumar Singh for his valuable suggestions and helping me during the work. I would also like to thank to my other colleagues Dr. Mohit Kumar, Dr. Sanoj Kumar, Dr. Amit Tomar, Dr. Pravesh Kumar and my other contemporaries for their association which has been instrumental in the smooth completion of this work. I wish to express my sincere thanks to my friends Vikash Tyagi, Sumit Malik, Fateh Singh, Sanjay Yadav, Varun Mahajan, Akarsh Arora, Ankur Doshi, Amol Garg, Arpit Baheti, Hitesh Kansal, Kartik MS and Sumita for their help and support throughout my research work. I would like to thank to all my friends for making life fun while working. Your entertaining and enjoyable news thereby always made me smile. Also, your social support was really fantastic.

Now, I wish to pay my utmost gratitude to my mother Smt. Satyavati Devi and my father Shri Bhagmal Singh for their invaluable blessings and delightful love. Your understanding, the faith and indescribable support to me throughout the whole life are invaluable

I would like to acknowledge unforgettable silent and unconditional support of my brother Rajeev and my sister Indu, brother-in-law Aashu and my lovely nephews Ankit (Harsh) and Ankur (Lalu) during this work. I also want to express my sincere thanks to all those who directly or indirectly helped me at various stages of the work, but I could not mention their names due to the shortage of the space.

It is a pleasant duty to acknowledge my gratitude to all my relatives, friends and only a few among many without help, encouragement and time; this thesis would never have been possible.

In the end, I am thankful to Almighty for bringing this day in my life.

(Sandeep Kumar Mogha)

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List of Abbreviations

AR	Assurance Region
BCC	Banker, Charnes and Cooper
CCR	Charnes, Cooper and Rhodes
CRS	Constant Returns to Scale
DCR	DEA Cross Reference
DCS	DEA Cross Sensitivity
DEA	Data Envelopment Analysis
DMUs	Decision Making Units
DRS	Decreasing Returns to Scale
GDP	Gross Domestic Product
IRS	Increasing Returns to Scale
LPP	Linear Programming Problem
MPI	Malmquist Productivity Index
MPPS	Most Productive Scale Size
NSM	New Slack Model
OTE	Overall Technical Efficiency
PFP	Partial Factor Productivity
PTE	Pure Technical Efficiency
PTECh	Pure Technical Efficiency Change
RTS	Returns to Scale
SA Model	Slack Adjusted Model
SBM	Slack Based Measure
SE	Scale Efficiency
SECh	Scale Efficiency Change
TECh	Technical Efficiency Change
TECHCh	Technical Change
TFP	Total Factor Productivity
VRS	Variable Returns to Scale

Chapter 1

Introduction

1.1 General

This study is inspired by two observations: (1) a greater need to pay attention towards measuring the efficiency of the healthcare sector in Uttarakhand, as health is a backbone of the overall development of a nation's economy, (2) the dearth of studies in healthcare sector in India as well as in Uttarakhand, a State of India, by DEA and the need to study in DEA literature to give special attention on empirical sensitivity analysis of DEA scores.

Since after Indian independence in 1947, there is a big pressure on the government to make healthcare sector more and more efficient. For this purpose, the government of India introduces time to time several health policies which have emphasis on 100 percent treatment in hospitals as well as healthcare of satisfactory quality with special focus on health for all. Keeping this in view, it becomes important for the government to know how efficiently the available resources are used to serve the population.

Delivery of public healthcare services in India is largely responsibility of the state governments. The central government frames health related policies and programs and finances medical education. District hospitals are controlled by the respective state governments and these hospitals provide medical care free of charge, the cost of which is covered by the funds the hospitals receive from the government.

The current scenario of the healthcare sector in Uttarakhand is disappointing. Public healthcare services are overburdened and collapsing. Hilly geographical areas, increase in population density, lack of transport facilities, inaccessibility, illiteracy, poverty, poor nutritional status, diversity in food habit and lifestyle are various impediments. Government priorities for providing healthcare services to rural areas are yet to be fulfilled. At this stage, low budget from state for health, lack of funds and coordination have triggered a downturn in health services in rural and hilly areas in the state.

Healthcare sector plays a significant role in the overall development of a nation's economy. In a developing country like India, public hospitals are the backbone of the healthcare system. It is the prime powered mode of health linking the remote and hilly areas with the rest of the country. India as a developing country seems to be riding on the information economy with the potential to become a developed nation in decades to come. However, much growth remains to be achieved in increasing literacy, public knowledge, and providing quality healthcare to the general masses. Education and health being vital components of human development play a significant role not only in the well-being of the people, but also contribute substantially to the economic development of the country.

Like other developing countries, India too is under increasing pressure to improve efficiency of the public healthcare system, as resource crunch and inefficient delivery system of public services have put the public sector in a position of comparative disadvantage. It is observed that public health investment over the years has been comparatively low. Its percentage in terms of gross domestic product (GDP) has declined from 1.30% in 1990 to 0.90% in 1999. However, it has increased to 1.25% in 2007 and further to 1.30% in 2011. The aggregate expenditure on the health sector in India is 5.2 % of the GDP of which public sector constitutes only 17% [49].

On an average, per capita public expenditure on health in Uttarakhand has been higher than the national average, while the reverse is true in the case of per capita private expenditure on health [49]. Since public resources at the disposal of the state government are limited and have competitive uses, it becomes imperative to make their efficient use so that maximum social welfare may be achieved. The public healthcare infrastructure and the health status of the state are given below.

1.1.1 Healthcare Infrastructure of the State

Uttarakhand, the 27th state of India, attained its statehood on November 9, 2000. It consists of thirteen districts and is culturally divided into two regions: Garhwal and Kumaon. Out of thirteen districts, two districts namely, Haridwar and Udham Singh Nagar are in the plain region and two districts, namely Dehradun and Nainital are partially covered by plain areas, while the rest area of the state is fully hilly. More than 80% of the land in the state is mountainous. Uttarakhand has a population of 10.12 million and is the most sparsely populated with an average population density of 189 per sq. km [58]. About 50% of its rural population live in villages of population less than 200 people and about 30% of the population lives in villages of population between 201 to 500. About 17% population lives in villages of population between 501 to 2000 people and about

3% population lives in villages of population between 2001 to 5000 people [142, 143]. Further, 62% of the villages are not connected by Pucca road. So, extending public healthcare facilities to such scattered population is a big challenge.

Uttarakhand has an extensive network of public health institutions. It has 2 women & child welfare centers, 1765 women & child welfare sub-centers, 84 main centers, 250 additional primary health centers (PHCs), 55 community health centers (CHCs), 322 allopathic dispensaries, 39 rural female hospitals, 107 homeopathic dispensaries, 540 Ayurvedic hospitals, 5 Unani hospitals, 18 tuberculosis hospitals, 24 Tesla/district level post-mortem centers and 36 district/base/combined hospitals [143]. To cater to specific diseases, the State owns 14 T.B. hospitals, 23 blood banks, 3 leprosy hospitals, 9 urban leprosy centers and 7 urban family welfare centers. Only one private medical college and 2 governments Ayurvedic medical colleges are positioned in the State. However, there exists a wide disparity in the public healthcare infrastructure across districts and regions. For instant, 60% of government hospital beds are situated only in four districts, namely, Pauri, Almora, Dehradun and Nainital. The State also faces a shortage of training institutions and public health management experts. It also suffers from insufficiency of medical and paramedical staff and their willingness to work in the inaccessible areas. Out of total 6143 sanctioned positions of doctors, 1799 are vacant [143].

1.1.2 Health Status of the State

The health status of Uttarakhand in terms of many indicators is better than its parent State (Uttar Pradesh) and the national average, but still the status is far away from satisfactory [49, 103, 104] as is evident from the figures given in Table 1.1.

Table 1.1: Comparative Statement of Uttarakhand State's Health Indicators with UP and India

Indicator	Uttarakhand	U.P.	India
Decadal Growth of Population (%)	19.17	20.09	17.64
Literacy Rate (%)	79.6	69.72	74.04
(a) Male	88.3	79.24	82.14
(b) Female	70.7	59.26	65.46
Crude Birth Rate (%)	19.7	22.5	24.99
Crude Death Rate (%)	6.5	7.3	8.04
Infant Mortality Rate (%)	41	50	46.07
Total Fertility Rate (%)	2.6	3.5	2.5
Female Ratio (in number)	963	908	940
Density per Square Km. (in number)	189	828	382

1.2 Statement of the Problem

Public sector hospitals, which provide un-priced services outside the market mechanism, bristle with the conceptual difficulty of such a precise delimitation of inputs and outputs. The problem becomes more acute in the absence of relevant data pertaining to the inputs and outputs. As such, this difficulty is partially bypassed by some performance measurement techniques which do provide scope for testing alternative input and output definitions using different combinations of inputs and outputs. One approach towards this end has been to assess the performance of public hospitals, on the basis of which policy decisions on the future course of action could be taken. It is in this context that this study applies DEA to measure the technical and scale efficiencies of public hospitals of Uttarakhand with a view to identify inefficient hospitals, and input reduction (output augmentation) required to make them efficient.

Public sector hospital's efficiency is more difficult to evaluate than the manufacturing firm's efficiency, because it is difficult to determine the efficient amount of resources required to produce various service outputs. The manufacturing efficient cost, which is the optimal cost used to produce the given level of output, can be used to identify operating inefficiencies through classical cost accounting variance analysis. However, in service organizations, like public healthcare system, it is difficult to identify the specific resources required to provide a specific service output. Another reason may be that the system being evaluated against a standard cost might not accept or agree on a standard because of the professional judgment involved in providing each type of health services. The professionals could convincingly argue that no two health services are alike, hence no standard or efficient input level can be identified as a basis for evaluating the efficiency of producing such services. Therefore, the productivity discussion in public sector hospitals should have an awareness of the difficulty of measuring inputs and determining the efficient levels of output produced.

It is possible that a less profitable organization may be more efficient in using its manpower and other inputs than the more profitable organizations. Since public sector hospitals have social obligation and the need for other types of performance measures would be even more warranted than it is in the profit service business.

More sophisticated analytical tools are required to evaluate the technical efficiency and productivity of public sector hospitals. A linear programming technique based DEA happens to be appropriate for such study. It identifies the best performing decision-making units (DMUs) without requiring a prior identification of the technologies or information on input and output prices. It is applied to identify efficient DMUs, evaluate managerial performance, allocate resources among sister concerns and diagnose the determinants of unsuccessful DMUs. Thus, it is well suited for comparative performance analysis of individual hospitals.

Farrell [52] was the first to measure the productive efficiency, drawing inspiration from Koopmans [73] and Debreu [43]. Farrell showed how to define economic efficiency and how to decompose it into technical and allocative efficiency components. The use of linear programming techniques of Farrell [52], eventually influenced Charnes, Cooper and Rhodes [27] to develop DEA. This methodology requires no assumption on a functional form for the efficient frontier and therefore, no parameter estimation, making it useful in a wide variety of applications. At present, it is a well-established non-parametric efficiency measurement technique widely used to assess the performance of DMUs.

The basic DEA models are used to distinguish between efficient and inefficient DMUs. A DMU is fully efficient if its efficiency score is one and all slacks are zero. Some DMUs are found to be radially efficient, but they are not fully efficient because of the slack in input and output variables. It clearly indicates that the slacks presented in the input and out variables influence the efficiency scores. The impact of slacks on efficiency scores cannot be assessed by basic DEA models. Also the results obtained from these models show that many multipliers have the zero values which indicate that the corresponding variable (input/output) has not been fully utilized in the efficiency assessment. To overcome this problem, extended DEA models have also been applied. In this study, we use a non-oriented, slack-based model (SBM) to deal with slacks. New slack model (NSM) is also applied to overcome this problem. This model directly deals with slacks and also estimates the impact of slacks on the efficiency scores and solves the problem of zero multipliers.

The main weakness with the DEA technique is that extreme observations highly influence the frontier. Therefore, some kind of sensitivity analysis is required to detect outliers and assess the robustness of the frontier. In this regards, we use “DEA Cross Reference (DCR) efficiency measure” given by Hibiki and Sueyoshi [56]. Also, we use a new model of sensitivity analysis given by Agarwal et al. [7], which detects the outliers and determines the robustness of the efficiency scores obtained by the NSM model. This model is also useful for ranking of the efficient DMUs and assessing the super efficiency scores of the DMUs.

The present study attempts to assess the performance of the public sector hospitals of Uttarakhand through DEA technique. The NSM model is applied to determine the impact of slacks on the efficiency of the hospitals. This study also measures the impact of several background variables on efficiency using a Tobit regression model. Sensitivity analysis is used to identify the outliers on the frontier, to verify the robustness of the efficiency scores and for ranking the efficient hospitals.

DEA based Malmquist Productivity Index (MPI) approach is used to evaluate the total factor productivity (TFP) change of the public sector hospitals. MPI provides important complementary information for productivity measurement. It also provides a natural way to measure the phenomenon of catching up. The technical change component of TFP captures shifts in the frontier of technology. This decomposition of productivity change into technical efficiency change and technical change is useful in distinguishing diffusion of technology and innovation, respectively.

1.3 Importance of the Study

The objective of any DMU is to increase the output (goods or services), reduce the cost, and obtain some profit (a reasonable return on investment). This objective can be achieved by improving performance. Performance broadly relates to the efficient and effective use of the existing resources within the constraints inherent in the business of producing goods and services. It, thus, is implied in every economic activity and primarily stands for producing more and more outputs from less and less resources [74]. It is essential to assess and monitor the efficiency and productivity of DMUs to check the degree to which inputs are utilized in the process of obtaining desired output. An assessment of efficiency and productivity of a service organization, like public healthcare sector, is quite relevant for improving the organizational performance and delivering the cost-effective services.

As Uttarakhand is a newly established state. A large part of it is hilly and thinly populated where public hospitals are the main source of healthcare services; while in the densely populated plain region, people have better access to private healthcare services. On an average, per capita public expenditure on health in Uttarakhand has been higher than the national average, while the reverse is true in the case of per capita private expenditure on health [49]. Since public resources at the disposal of the state government are limited and have competitive uses, it becomes imperative to make their efficient use so that maximum social welfare may be achieved.

Uttarakhand State healthcare system is in a peculiar situation. Its resources are shrinking while the demands for the services are expanding. Management of the health services needs to take appropriate steps for the efficient and effective utilization of available resources for maximizing output. Resources are needed to be used efficiently and judiciously in order to enhance productivity. In this context, it is necessary to study inter-regional comparison to know the best practices of efficient hospitals of Uttarakhand and use them to improve the performance of the inefficient hospitals.

A comparative analysis is fundamentally very useful for the purpose of monitoring the performance of the hospitals at an aggregate level. In fact, it would help in identifying the criterion for taking corrective action based on certain priority consideration. Indeed, apart from giving allowance for certain social consideration, the allocation of funds may be linked with their performance level so as to induce a spirit of competitiveness.

It is important to highlight the achievements of efficient hospitals, as it is to identify those hospitals having an unsatisfactory performance. It is indeed the later where there is much larger scope for improvement to ultimately lead to a higher utilization of the investible resources. This would obviously require an adequate monitoring of their performance. In order to execute such an exercise systematically, it is an important pre-requisite to have the knowledge of performance level together with an idea about the capacity and constraints of the hospital. This would surely help in visualizing the parameters of action plan to optimize the utilization of large input resources of hospitals of Uttarakhand.

Performance of a DMU (public hospital of Uttarakhand in our case) can be measured by using conventional methods such as ratio analysis and least square estimation methods. The ratio analysis is a simple two-dimensional measure and does not adequately reflect the complex nature of the DMUs operations. The ratio cannot capture the effect of factors, which affect the performance of the DMU [131]. Ordinary least square (OLS) methods are useful in identifying central tendencies, but fail to identify efficient DMUs in comparison to less efficient DMUs.

Furthermore, these methods cannot describe the changes needed for the relatively inefficient hospitals to improve their efficiency.

Evaluating the performance with respect to theoretical limits requires an assumption about the functional form of a frontier and estimation of its parameters. In the absence of qualifications for a performance frontier function or limitations in defining parameters one must resort to "non-parametric" methods. A non-parametric model to measure efficiency performance of a collection of functionally similar "decision units" to transform inputs into outputs has been well developed and widely used since Charnes et al. introduce DEA in 1978. Frequently, however, a measure of relative performance is required for "units" where a full complement of inputs and outputs are difficult to define or when the units do not behave as traditional DMUs. DEA is capable of setting targets for output augmentation and to estimate input-output slacks for inefficient DMUs to become efficient. From the policy point of view, it is interesting to distinguish an inefficient DMU from an efficient one.

The purpose of this study is to measure the efficiency and productivity of public sector hospitals of Uttarakhand. By developing a single index for efficiency, DEA will be able to assess the performance of a hospital in comparison to peer hospitals. This study also examines productivity change along with technical efficiency change and technical change in the hospitals. The findings of the study would not only be useful for individual hospitals for formulating appropriate strategy for enhancing productivity and efficiency, but would equally be relevant for the state government for framing and implementing suitable policies for the development of the public healthcare services.

1.4 Objectives of the Study

The objectives of the study are to

i. measure the relative efficiencies of public sector hospitals of Uttarakhand;
ii. verify the robustness of the efficiencies of the hospitals;
iii. measure the effect of several background variables on the performance of the hospitals;
iv. measure the impact of slacks on efficiencies;
v. identify use of excess inputs and deficient outputs for inefficient hospitals;
vi. measure the super efficiency of efficient hospitals; and

vii. assess the total factor productivity growth, technical change and technical efficiency change in public sector hospitals.

1.5 Data

The study examines the efficiency and productivity of public sector hospitals of Uttarakhand using both cross-sectional and time series data. The data for the study are collected from the Directorate of Medical Health and Family Welfare, Government of Uttarakhand, Dehradun, India. Estimation of the relative efficiencies of the public hospitals of Uttarakhand is based on cross-sectional data for the year 2011. Panel data for the period 2001 to 2011 are also collected from the Directorate of Medical Health and Family Welfare, Government of Uttarakhand, Dehradun, India [45] to measure TFP growth in the hospitals.

1.5.1 Input and Output Variables

As per the availability of data, we estimate the efficiencies of the hospitals using three inputs and four outputs. The three operational input variables are considered as the number of beds, number of doctors and the number of paramedical staff. We have selected four output variables as number of out-door patients, number of in-door patients including two case-mix variables number of major surgeries and number of minor surgeries. All the selected inputs and outputs are measured in numbers. Firstly, we discuss some input-output variables that were used in earlier studies. Some studies related to efficiency estimation of public sector healthcare services are listed in Table 1.2. It can be observed from the Table that all the researchers have selected the operational input-output variables to evaluate the efficiency and productivity. Number of beds and number of doctors are the major inputs and number of out-door patients and number of in-door patients are the major outputs used by the researchers in the efficiency evaluation. We have also selected the same input and output variables for this study.

Table 1.2: Input-Output Variables used in the Previous DEA studies on Healthcare Services

Study	Research Goal	Models Used	Inputs	Outputs
Osei et al. (2005) [99]	Efficiency evaluation	DEA based CCR and BCC	number of doctors, beds, other technical staff and subordinate staff	number of maternal and child care (MCH), number of patients discharge and number of child deliveries
Zere et al. (2006) [148]	Efficiency evaluation	DEA based CCR and BCC	total recurrent expenditure, beds and nursing staff	total outpatient visits and inpatient days
Agarwal et al. (2007) [3]	Efficiency evaluation	DEA based CCR and BCC	number of beds, doctors and paramedical staff	number of outdoor patients, indoor patients major surgery and minor surgery
Barros et al. (2007) [22]	Efficiency and productivity	Luenberger indicator and DEA based MPI	number of beds, total variable costs and number of full-time equivalent personal	number of patients, length of stay of the patient in the hospital, number of emergency cases and number of consultations
Gannon B. (2008) [53]	Total factor productivity	DEA based MPI	number of beds and number of full-time equivalent employee	outpatient attendance, total discharges and deaths and day cases
Dash U. (2009) [39]	Total factor productivity	DEA based MPI	number of beds, nursing staff, and physicians	number of inpatients, outpatients, and surgeries undertaken, emergency cases handled, medico legal cases, and deliveries
Dash et al. (2010) [40]	Efficiency evaluation	DEA based CCR and BCC	number of beds, nursing staff, assistant surgeons employed and number of civil surgeon employed	number of inpatients, outpatients, surgeries undertaken, emergency cases undertaken and deliveries
Dimas et al. (2010) [44]	Total factor productivity	DEA based MPI	number of beds, total personnel salary and total expenditure on medicines, supplies and other materials	number of patient-days, number of emergency cases and number of patients in OPD

Karagiannis and Velentzas (2010) [67]	Total factor productivity	DEA based MPI and quality adjusted MPI	number of beds, doctors and number of nursing and other personnel	number of inpatient days
Tlotlego et al. (2010) [137]	Efficiency and productivity	DEA based CCR, BCC and MPI	number of hospital beds and number of clinical staff	number of outpatient visit and inpatient days
Pham, L.T. (2011) [101]	Total factor productivity	DEA based MPI	total number of beds and total number of hospital's personnel (including physicians and non-physicians)	number of outpatient visits, surgical operations performed and inpatient days
Nedelea and Fannin (2012) [97]	Efficiency evaluation	DEA and SFA	total staffed and licensed hospital beds and Full time equivalent employee	total births, total hospital admissions, post admission days, total outpatient visits, emergency room visits and outpatient surgerie
Sheikhzadeh et al. (2012) [117]	Efficiency evaluation	DEA based CCR and BCC	number of physicians, nurses, medical team having a bachelor degree or above, active beds and medical team + nonmedical staff	number of emergency patients, number of outpatients, number of inpatients × average daily inpatients' residing
Kirigia and Asbu (2013) [72]	Efficiency evaluation	DEA based CCR and Tobit model	number of doctors, number of nurses and midwives, and number of operational beds and number of laboratory technicians	number of outpatients visit and number of inpatient discharges
Jat and Sebastian (2013) [62]	Efficiency evaluation	DEA based CCR and BCC	number of specialists, number of nurses, number of allied health and number of beds	number of outpatient, patients admissions to hospital, number of laboratory tests, and number of beneficiaries of radiological imaging

In reference to the above cited studies and as per the availability of the data, the three variables, number of beds, number of doctors, and number of paramedical staff (PMS) are taken as the input variables as they are the major indicators for determining the output of a hospital. The input and output variables used in the present study are described in Table 1.3 below

Table 1.3: Definitions of Input-Output Variables

Variables	Definitions
Inputs	
Number of beds	Total number of beds which is actually used by the hospital within a year
Number of doctors	Total number of doctors and nurses employed (full-time) in the hospital in a year
Number of paramedical staff	The total number of non-medical employees employed (full-time) by the hospital in a year
Outputs	
Number of out-door patients	Total number of outpatients who visits to the hospitals within a year without any stay in the hospital
Number of in-door patients	Total number of inpatients stayed in hospital beds and received inpatient services within a year
Number of major surgeries	Total number of major ambulatory surgical operations of inpatients in a hospital within a year
Number of minor surgeries	Total outpatient surgeries in a hospital within a year

1.6 Methodology

This study uses the basic CCR and BCC output-oriented DEA models to evaluate the efficiency scores. The study also uses a non-oriented, slack based model, for the efficiency evaluation. An output-oriented new slack model, developed by Agarwal et al. [6], which is the extension of the slack adjusted model [132] and slack based model [140], is also applied to assess the efficiency scores with slack inefficiency. The new slack model (NSM) also explores some of the underplaying hospitals for inefficiency, i.e., divergence from the MPPS, RTS and impact of slacks on efficiency. The NSM with VRS assumption separates OTE from SE. The study uses output-oriented model, i.e., how much outputs can be increased for the given level of inputs to make the hospital efficient [3, 93]. The study also examines the effect of various background variables on the efficiency scores by Tobit regression analysis. It identifies the outliers on the frontier, authenticates the robustness of the efficiency scores and determines the ranking of the efficient hospitals by using the DEA Cross Reference (DCR) given by Hibiki and Sueyoshi [56] and also using the extended method of sensitivity analysis used by Hibiki and Sueyoshi [56] and Jahanshahloo [60]. This sensitivity analysis is based on change of the reference set of the inefficient hospitals.

The framework of analysis adopted compares the inter-hospital performance. The cross-sectional study is based on data of 36 hospitals for the year 2011. The cross-sectional analysis is also carried out by using the new slack model. Time series analysis of the hospitals covers a period from the establishment of the state 2001 to 2011 in which we assess the total factor productivity change, technical efficiency change and technical change for the hospitals using 11-years' panel data. The data are available for 36 hospitals in the year 2011 which is used for cross-sectional analysis, but in the year 2001 these hospitals were only 27 in number. After 2001 these hospitals were extended up to 36 in numbers. Thus, for the time series analysis only 27 hospitals are taken for the analysis.

1.7 Data Envelopment Analysis (DEA)

DEA is a linear programming based technique that measures the relative efficiency of a homogenous set of DMUs. It optimizes on each individual observations with an objective of calculating a discrete piecewise frontier determined by the set of efficient DMUs. It compares entities that transform multiple resources into multiple outputs. In particular, the method is applicable when the DMUs are assumed to be homogenous in terms of the industry, their outputs and inputs, and when the DMUs can be assumed to be maximizing output and minimizing inputs. It does not require a priori judgment by the researcher as to the relative importance of the various outputs or knowledge of input prices. It does not require any specific assumption about the functional form. It calculates a maximal performance measure for each DMU relative to all other DMUs in the observed population with the sole requirement that each DMU lies on or below the frontier [30]. The DEA calculations define a production frontier on which all Pareto optimal entities are located. Each DMU not on the frontier is dominated by DMU or a combination of DMUs that define a facet of the frontier and form a reference set. DEA is regarded internationally as one of the most successful techniques of efficiency assessment proposed by researchers in management science and operations research. It is an alternative and a complement to the traditional central tendency analysis. In parametric analysis, the single optimized regression equation is assumed to apply to each DMU. DEA, in contrast optimizes the performance measure to each DMU. This result is an understanding of each individual DMU instead of a depiction of a hypothetical average DMU. The parametric approach requires imposition of a specific functional form (e.g. a regression equation, a production function etc.) relating the independent variables to the dependent variables. The functional form selected also requires specific assumptions about the distribution of the error terms and many other restrictions. In contrast, DEA does not require any assumption about the functional form.

DEA methodology has several advantages over traditional regression based production function approach. A few of them are as follows:

i. It can handle multiple inputs and outputs.
ii. It doesn’t require any assumption of a functional form relating to outputs and inputs.
iii. DMUs are directly compared against the combination of peers or peer.
iv. Outputs and inputs can have different units.
v. It sets targets for inefficient DMUs to make them efficient.
vi. It also identifies slacks in inputs and outputs.
vii. It estimates a single efficiency score for each DMU.

However, DEA has several limitations such as:

i. DEA is non-parametric technique so statistical hypothesis testing is difficult.
ii. It is an extreme point technique, so the measurement error cannot be measured.
iii. Efficiency obtained by DEA technique is relative, not an absolute measure.
iv. It is sensitive to the selection of input and output variables. Its results may be influenced by the number of observations.
v. Zero and negative values of input and outputs affect the results of DEA techniques.
vi. It is computationally intensive method.

1.8 Scope of the Study

The main thrust of this study is on the measurement of the performance of public hospitals of Uttarakhand, on using a new slack model to capture the impact of slacks on the efficiency scores, and on examining the effect of background variables on the efficiency scores using Tobit regression analysis. This study also identifies the outliers and determines the robustness of the efficiency scores through sensitivity analysis. The new slack and sensitivity analysis models are used in the public sector hospitals of Uttarakhand. However, these models can also be applied to other sectors. An attempt has also been made to evaluate total factor productivity change, technical change, and technical efficiency change in the hospitals.

1.9 Limitations of the Study

Limitations of the study are as follows:

i. Economic performance has not been analyzed in terms of price efficiency.
ii. Comparison of public and private sectors has not been done due to the unorganized management of the private sector.
iii. The performance of hospitals based on the quality of the service provided by them has not been evaluated due to the unavailability of the data.
iv. Limitations of DEA methodology can also be ruled out in the study.

Even so, these limitations will not seriously affect the conclusions and findings of the study. It is hoped that despite the above-mentioned limitations, the study will benefit to the DEA researchers for the theoretical aspects and the decision makers in revamping the structure and improving the performance of the public sector hospitals of the state.

1.10 Concepts and Definitions

1. **Decision Making Units (DMUs):** The term DMU is first used by Charnes, Cooper and Rhodes in 1978 in their seminal paper. A DMU is to be regarded as an entity responsible for converting inputs into outputs. It comprises banks, hospitals, educational institutes, government organizations, transport firms or any other decision making entities. The characterizations of the unit of assessment as “decision making” implies that it has control over the process as it employs to convert its resources into outcomes.
2. **Efficiency:** It refers to the degree to which the observed use of resources to produce outputs of a given quality matches the optimal use of resources to produce outputs of a given quality. A DMU is said to be fully efficient if and only if its outputs or inputs cannot be improved without worsening some of its other outputs or inputs.

3. **Inefficiency:** The amount by which a firm lies below its production and profit frontiers and the amount by which it lies above its cost frontier can be regarded as a measure of inefficiency.
4. **Input Orientated Measure:** The input-oriented technical efficiency measures the input quantities which can be proportionally reduced without changing the output quantities produced.
5. **Output Orientated Measure:** The output-oriented technical efficiency measures the output quantities that can be proportionally expanded without altering the input quantities used.
6. **Peer:** A peer is an efficient DMU which acts as a reference point for inefficient DMU.
7. **Peer Weight:** The multiplier value (λ-value) of an efficient DMU, which makes a reference set for an inefficient DMU, is called peer weight.
8. **Reference Set:** A set of efficient DMUs, whose combination makes an inefficient DMU efficient, is called a Reference Set.
9. **Production Frontier:** The production frontier measures either maximal outputs from the given level of inputs or minimal input used to produce the given level of output. DMUs operating on the production frontier are identified as technically efficient.
10. **Production Function:** Production function determines the maximum possible output, which can be produced from the given set of inputs.
11. **Overall Technical Efficiency:** The efficiency score evaluated from CCR model is defined as OTE. It reflects the ability of a DMU to obtain the maximum outputs from the given set of inputs.
12. **Pure Technical Efficiency:** The efficiency score evaluated from BCC model is defined as pure technical efficiency. It refers to the proportion of overall technical efficiency, which is attributed to the efficient conversion of inputs into outputs, given the scale size. A pure technical efficient DMU may still be over or under producing, if it is feasible for the DMU to alter its size towards the optimal size.
13. **Scale Efficiency:** The scale efficiency is defined as the ratio of OTE to PTE. It measures the impact of scale size on the efficiency of a DMU. It measures the divergence between VRS and CRS efficiency ratings lower the value of scale efficiency and the more adverse the impact of scale size on efficiency.
14. **Returns to Scale:** RTS refer to the magnitude of the change in the rate of output relative to the change in scale. In general, the output may change in proportion to the change in inputs

(Constant Returns to Scale). Output may increase more than in proportion (Increasing Returns to Scale) or less than in proportion (Decreasing Returns to Scale).

15. **Slacks:** The quantity of excess resources used and deficient outputs produced by an inefficient hospital to become efficient after radial change to reach the efficiency frontier are known as input slacks and output slacks, respectively.
16. **Productivity:** Productivity is the relationship between the output generated by a production or a service system and the input provided to create this output. It is determined dividing the output by the input. When productivity of two firms are compared, the more productive firms produces more output with the same input, or it produces the same output with lesser input.
17. **Partial Factor Productivity:** Partial Factor Productivity is a ratio of total output to the single input. It cannot provide the true performance of a resource. Labor productivity and capital productivity are the main example of the partial factor productivity.
18. **Total Factor Productivity (TFP):** TFP measures the overall productivity of a DMU by calculating the ratio of weighted sum of the outputs of the weighted sum of inputs. The TFP index having a value greater or lesser than one indicates the positive or negative growth in the productivity, respectively. The value of index of one implies no growth in the productivity.

1.11 Organization of the Thesis

The study has been divided into eight chapters. The introductory chapter deals with the aspects of the performance measurement, the history, infrastructure, statement of the problem, objectives, scope, and limitations of the study. This chapter also discusses briefly about the DEA techniques used to assess the efficiency and productivity.

Chapter 2 is devoted to the review of literature on theme. DEA and MPI based studies on the health sectors in India and abroad along with some other relevant studies have been reviewed.

Chapter 3 shows the advancement in DEA techniques from the basic models to the new DEA models. This chapter presents the basic (CCR and BCC) models, AR model, and DEA slack-based models, such as Additive model, Two-stage model, SBM model, SA model, NSM model, and DEA models with non-discretionary and categorical variables.

Chapter 4 presents the efficiency evaluation of the public sector hospitals of Uttarakhand. This chapter presents a cross-sectional analysis of the public hospitals of the state for the calendar year 2011. Tobit regression and sensitivity analysis have also been carried out to identify the effect of background variables and robustness of the efficiency scores.

Chapter 5 deals with a non-oriented “slack based model” (SBM) of DEA to evaluate the efficiency of hospitals. This model allows managers to work on both inputs and outputs to achieve efficiency. Generally, in case of public hospitals it is difficult to choose the orientation (input or output) for the evaluation of efficiencies. It is not admirable to reduce input levels or increase output levels regarding public sector hospitals. So, in this chapter, a non-oriented and non-radial model known as SBM-DEA model has been used.

Chapter 6 deals with a new slack model (NSM) of DEA to evaluate the efficiency with the actual impact of slacks on the efficiency scores. This also identifies the super efficiencies of the hospitals and ranks the efficient hospitals.

Chapter 7 deals with theoretical aspects of the Malmquist Productivity Index (MPI) and assesses TFP change, technical efficiency change and technical changes that have taken place in public hospitals of Uttarakhand. It also examines the sources of the TFP growth in the public hospitals of the state. DEA based MPI approach is used to evaluate the TFP change in the public hospitals for the period 2001 to 2011.

The last chapter presents a summary of findings, conclusions and recommendations for the policy considerations along with some suggestions for the future research.

Chapter 2

Review of Literature

2.1 Introduction

This Chapter reviews some relevant studies on the performance assessment of the healthcare sector. The review of literature covers the most reverent and cited findings, which contribute to the understanding of this study. The review has been divided into two categories: (i) DEA and MPI based studies on the health sector, (ii) DEA and MPI based studies on other sectors.

2.2 DEA and MPI Based Studies on Healthcare Sector

DEA and MPI are frequently used to measure relative efficiencies and TFP growth of organizations, such as, Banks, Hospitals, Schools, Universities, Transports, Airports, Manufacturing firms, etc. Among these, some relevant studies related to healthcare sector in Indian and foreign context are reviewed as follows:

2.2.1 Indian Studies

Bhatt et al. [23] attempt to provide an overview of the general status of the healthcare services provided by district and grant-in-aid hospitals in the state of Gujarat in terms of their technical and allocative efficiency. The study finds that the variation in efficiencies is significant within district hospitals and within the grant-in-aid hospitals. The overall efficiency levels of grant-in-aid hospitals are observed higher than that of the district level hospital. Further, the efficiency variations for district hospitals are found to be higher than that for grant-in-aid hospitals.

Agarwal et al. [1] measure the relative efficiency of 21 private sector hospitals of India to distinguish between efficient and inefficient hospitals. The paper is based on cross sectional data collected from “Prowess Database” supplied by the CMIE, Mumbai for the year 2001-02. The study finds that 33 percent of hospitals present the maximum degree of efficiency. The DEA (CCR-AR and BCC-AR) input-oriented models are used. The study concludes that on average, non-frontier hospitals may be able to reduce net fixed assets (NFA) by 15.53 percent, current assets (CA) by 17.46 percent, energy expenses (EE) by 13 percent and salary & wages (S&W) by

16.7 percent relative to the best performing hospital. Sensitivity analysis results suggest that the efficiency results are quite robust.

Agarwal et al. [3] measure and compare region-wise the technical and relative efficiency of 29 government hospitals of Uttaranchal State in India, using DEA-CCR and BCC models. The panel data used in this study were collected from the Directorate of Medical Health and Family Welfare, Government of Uttaranchal, for the years 2001 to 2004. The detailed information on input and output data reveals that there exist disparities in the distribution of healthcare facilities across hospitals of hill and plain regions and also between hospitals of Garhwal and Kumaon regions. The paper concludes that the performance of hospitals has improved in 2004 over 2001. Hospitals in Garhwal region achieved a higher average efficiency score compared to the hospitals in Kumaon region. The target setting results show that both outputs (miner and major surgeries) have significant scope for expansion.

Dash et al. [40] apply input-oriented DEA model to measure the technical and scale efficiencies of 29 district headquarter hospitals in Tamil Nadu for the year 2004-2005. Number of staff members and bed strength were two inputs and outpatient visits, number of inpatients, number of surgeries undertaken, the number of deliveries and the number of emergency case were five outputs used in the study. The results show that 52% of the hospitals were technically efficient and 81% were scale efficient, which implies that the scale inefficient hospitals could reduce their size by 19% without reducing the current level of outputs. The study also suggests that the inefficient hospitals (48%) have 151 excess beds, and the excess number of staff nurses, assistant surgeons and civil surgeons are 220, 43 and 33, respectively.

Lakshmana [78] investigates the district-wise healthcare infrastructure for children’s, based on the information collected from the children's outpatient departments of the district hospitals in Karnataka. The data of the study were collected through a primary survey during the period February and March of the year 2006. The study takes stock on the working status of doctors and the healthcare existing facilities for children in Children Outpatient Department (COPD). The paper is an analysis of public healthcare facilities available in COPDs in district hospitals of Karnataka. This study also attempts to understand the existing special services for children, such as, Neonatal Intensive Care Unit (NICU) and baby clinic in COPDs.

Mogha et al. [93] investigate the overall performance of 55 private sector hospitals in India for the year 2009-2010. DEA based CCR and BCC models are used to assess the efficiencies of the selected hospitals with Net Fixed Assets, Energy Expenses and wages & Salaries as input variables, and Operating Income as an output variable. The data of the study were collected from PROWESS database of CMIE. The study concludes that on average 23.70% of the technical potential of hospitals was not in use. The VRS results show that inefficient hospitals are able to augment their output by 21.20% relative to the best performing hospitals. The study shows that on average, inefficient hospitals have to augment their operating income by 44.27% with 18.12% reduction in net fixed asset and 0.85% reduction in energy expenses.

Mogha et al. [94] assess the technical and scale efficiencies of 50 private sector hospitals in India for the period 2004-05 to 2009-10 using CRS and VRS DEA models. The observed inputs used in the study were NFA, P&F, S&W and CA and the output was OI. The mean OTE, PTE and SE of the hospitals were found to be 79.40%, 87.70% and 90.70% during the whole study period. The RTS-wise results show that average OTE, PTE and SE scores of hospitals operating on IRS were 67.50%, 81.60% and 81.70%, respectively, while the same results for the hospitals operating on DRS were 73.40%, 81.90% and 89.90%, respectively. The analysis of target results show that inefficient hospitals could improve their output level following the best practices of efficient hospitals.

2.2.2 Foreign Studies

Zere et al [148] estimate the technical efficiency of 30 district hospitals over 13 regions of Namibia using DEA input oriented models for the four years period from 1997-98 to 2000-01. Jackknife analysis was also used to test the robustness of the efficiency scores. The study was based on three inputs, namely, total recurrent expenditure, number of beds, and nursing staff, and two outputs, namely, total outpatient visits and inpatient days. The results reveal that average technical efficiency scores of the hospitals range from 62.70% to 74.30%. The paper further shows that less than half of the district hospitals have remained on the efficiency frontier throughout the study period.

Miemieani et al. [87] estimate the health care efficiency and rank the DMUs in transition economies, using DEA based CCR output-oriented model. The data for the study are collected from World Development Indicators, UNICEF, and World Bank database. The input variables used in the study are: per capita healthcare expenditure in U.S. dollars, number of beds per thousand population, number of physicians per thousand population and percentage of children with measles inclusion; whereas the output variables are average life expectancy and infant mortality rate (male and Female). The resulted analysis shows that, during the entire study period OECD countries, Albania, and Armenia were top performer. Russia and Belarus were least efficient, followed by Latvia and Romania. The highest efficiency in Albania and Armenia could be attributable to their policies to control the consumption of alcohol and tobacco.

Ng [98] applies DEA and MPI techniques to measure efficiency and TFP growth in 12 Coastal and 17 Inland hospitals in China for the period from 2002-2003 to 2004-2005. The four inputs used in the study were number of beds, number of doctors, number of nurses and number of other medical staff. The two outputs taken in the study were numbered of inpatient stays and number of outpatient visits. The average OTE, PTE and SE efficiency scores for Coastal hospitals were 0.823, 0.852 and 0.966, respectively; whereas these scores for Inland hospitals were 0.832, 0.877 and 0.950, respectively. The MPI results show that mean values of TFP, technical efficiency change and technical change indices for coastal hospitals were 1.131, 0.972 and 1.147, respectively; whereas these indices for Inland hospitals were 0.985, 0.942 and 1.085, respectively. In Coastal hospitals, TFP growth was due to higher positive technical change as efficiency change was negative; whereas there was TFP regress in Inland hospitals, which was mainly due to regress in technical efficiency change.

Kirigia et al. [71] assess the TFP growth and its components in the public municipal hospitals of Angola. The analysis was based on panel data collected from 28 public municipal hospitals in the three year period from 2000 to 2002. The study applies output-oriented DEA based MPI approach to measure the technical efficiency and TFP growth. The input variables used in the study were doctors and nurses, drugs supply and beds, and the output variables were out-door patients and patient admissions. The results showed that on average the overall technical efficiency scores were 66.2%, 65.8% and 67.5% in the years 2000, 2001 and 2002, respectively. These efficiency scores show that these hospitals have to produce 33.8%, 34.2% and 32.5% more outputs with their existing input levels, if they have to operate efficiently. Out of the total 28 hospitals, about 39%, 43% and 36% hospitals were pure technical efficient in the years 2000, 2001 and 2002, respectively. Average scale efficiencies of the hospitals were 83%, 81% and 89% in the years 2000, 2001 and 2002, respectively. The MPI results show that on average, TFP in the hospitals grew by 4.5% per annum, which was contributed only by technical efficiency change, as technological change observed regress.

Tlotlego et al. [137] apply DEA based CCR, BCC and MPI approach on a sample of 21 non-teaching hospitals in Botswana over a three year period from 2006 to 2008. The study was undertaken with two inputs, namely, number of clinical staff and the number of hospital beds, and two outputs, viz., number of outpatient visits and number of inpatient days. The study concludes that out of 21 hospitals, 16 (76.2%), 16 (76.2%) and 13 (61.9%) hospitals were run inefficiently in 2006, 2007 and 2008, respectively. The average VRS efficiency scores of the hospitals were 70.4%, 74.2% and 76.3% in the years 2006, 2007 and 2008 respectively. The study also reveals that on average, TFP in the hospitals decreased by 1.5% per annum. The technical efficiency change has increased by 3.1%; while technical change has declined by 4.5%. Thus, TFP regress in hospitals was mainly due to technical regress as the hospitals achieved positive growth in technical efficiency during the same period.

Ismail [59] used output-oriented CCR and BCC models for the measurement of the technical efficiencies of state health institutions (hospitals and health centers) in Sudan for the year 2007. The data for the study were collected from the Annual Health Statistics Report 2007 published by the Federal Ministry of Health (FMoH) of Sudan. The efficiency analysis was carried out using five inputs and three output variables. The inputs were: number of hospitals, the number of health centers (primary health care units, dressing stations, dispensaries and health centers), number of beds, number of physicians and ancillary medical staff (pharmacists, midwives, nurses, medical assistants, technicians and others). The output variables considered in the study were number of inpatients, number of outpatients and number of surgical operations. The efficiency analysis concludes that 9 out of 15 hospitals were CRS efficient with an average efficiency score of 0.896, while the remaining 6 were inefficient with efficiency scores ranging between 0.618 and 0.961. The VRS efficiency results show that 10 out of 15 hospitals were efficient with an average efficiency score 0.935, while the remaining 5 were inefficient. These results suggest that each inefficient health institution has to upgrade its outputs by 10.4% with same input levels. The average scale efficiency 0.957 shows that these institutions have to scale down their scale size by

4.3%. The target setting results show that the outputs have to be increased by 4%, 9% and 8.2%, respectively, while inputs have to be decreased by 12.4%, 16.8%, 7.3%, 0.8% and 0.8%, respectively.

Jandaghi et al. [61] investigate and compare the efficiencies of 8 Qom (a city in Iran) hospitals using the DEA methodology. The five inputs used in the study were number of physicians, number of nurses, number of medical staff, number of official staff, and the amount of annual cost. The outputs of the study were clinical visits, emergency visits, and occupied cost-days. Input and output sets are taken in four different cases in the study. The efficiency results show that mean CRS and VRS efficiency scores were 0.950 and 0.968, respectively. Out of total 8 hospitals, 3 were on the efficiency frontier in all the four cases. The average scale efficiency was 0.981, suggesting that hospitals can increase their efficiency by 0.019 by optimizing their scale size.

Maredza [84] attempts to examine the efficiency of hospitals in Zimbabwe using DEA input-oriented models. The study examines whether for-profit hospitals were more efficient than non-profit hospitals. The input variables used in the study were number of beds, number of doctors, and number of nurses. The output variables used were inpatient days and total discharges. The study concludes that there was a marked deviation of efficiency scores from the best practice frontier with for-profit hospitals having the highest mean OTE score 61.4%. The mean OTE scores of mission and public hospitals were 35% and 50.3%, respectively. In the second stage, the paper applies Tobit regression model, which suggests that both, for-profit hospitals and government hospitals were important in influencing efficiency scores; however, for-profit hospitals had a higher marginal mean efficiency score than the non-profit hospitals.

Araujo et al. [15] use different DEA approaches in the assessment of efficiency of a sample of 20 Brazilian for-profit hospitals. The study uses the secondary data collected from the statistical database provided by the largest private healthcare provider corporation in Brazil, with a market share of 10.1% in terms of the number of enrollees, 6.3 million members and total net revenue from the services of $5.3 billion. The study assesses the input slacks in the hospitals as well as their output-increasing potentials. The nine input variables used in the study were hospital area (m^2), number of intensive care beds (ICU beds), number of emergency beds, total number of hospital beds, total number of staff, number of doctors, number of nurses, number of doctor's office in the hospital and number of surgical rooms. The study use 5 outputs which were number of ordinary inpatients (per year), number of ICU inpatients (per year), number of emergency inpatients (per year), total number of outpatient treatment (per year), and number of surgeries (per year). The nine input variables were extracted from Principal Component Analysis of data collected from 20 hospitals. The inputs hospital area, number of ICU beds, number of emergency beds, total number of hospital beds, total number of staff, number of nurses, and number of surgical rooms make up Factor 1, termed as the Hospital Infrastructure and Supporting Staff Index. The variable number of doctors and number of doctor's offices makes Factor 2, named Hospital Doctor and Office Index. With respect to the outputs, five production related variables were reduced into two factors: Longer-term Medical Procedures Index and Shorter Term Medical Procedures Index. The two most representative outputs for these two factors are, respectively, the number of ICU inpatients and the total number of outpatient treatments. The results of the study reveal that the average OTE and PTE of the for-profit hospitals in Brazil were 0.458, and 0.618, respectively.

Ramanathan and Ramanathan [109] propose balanced score card (BSC), advantageously integrated with DEA, to overcome some of the shortcomings of the DEA, such as the discriminating ability and the problem of a firm with extraordinary performance in terms of only one measure achieving high performance scores. The proposed integration of BSC with DEA is called the balanced efficiency assessment method. They applied this technique on balanced performance evaluation of health authorities in UK. Six BSC perspectives are employed to evaluate the performance of health authorities. Different sets of inputs and outputs are used in a DEA model for each perspective, and the DEA performance scores are aggregated across all the perspectives using arithmetic mean. The study finds that there is no single health authority that performs consistently well in terms of all the six perspectives and that a health authority that performs well in terms of one perspective does not seem to be doing well in terms of others.

2.3 DEA and MPI Based Studies on other Sectors

In recent years, DEA and MPI have become very popular among researchers for measuring the performance of DMUs because of their several merits over the traditional approach. DEA is extensively used in several areas and industries across the world to measure productive efficiencies and growth trends.

2.3.1 Indian Studies

Indian researchers have also applied DEA in several sectors, such as, Banks [115, 119, 130, 76, 77], Textile Industries [63, 64, 65], Sugar Industries [126, 128], Agro Food Industry [125], Manufacturing Industries [114], Transport sector [2, 4, 5, 6], Telecommunication industry [116], Thermal power plant [118], Water utilities [120], Rural development [129], Cricket [122, 123] and Education sector [141] to assess the performances and total factor productivity [4, 64, 66, 127]. Some of the Indian studies which use DEA are as follows:

Singh [125] applies DEA to measure performance of Agra-food industry of India, using time series data for the period 1981-1982 to 1997-1998. The study concludes that an increase in the share of the food industry in terms of number of units, fixed capital, employment and net value added of the entire organized industry and its relative higher growth in fixed capital, employment and net value added may provide misleading interpretation about its performance, if its relative efficiency is not studied. Further, the paper finds presence of high slacks in fixed and working capital, suggesting their rightsizing to improve the efficiency of the food-industry.

Singh [124], studies the performance of Indian industries in terms of employment, productivity and efficiency before and after economic reforms. Besides estimating the growth rate in public and private sector employment for the last 25 years and separately for pre and post reforms periods, the study estimate technical and scale efficiency trend in industrial sector during 1981-1982 to 1997-1998 and measures the relative efficiency of 22 industry groups for the year 1997-1998. The results of the DEA models reveal that the industrial sector has shown inefficiency in utilizing its resources during more than half of the whole study period. Identified the main reason for inefficiency is under utilization of labour, fuel and working capital. However, during the reform period, efficiency of Indian industry has increased, which appear to be due to improvement in labor efficiency. Analysis of cross-sectional data of the 22 industry groups for the year 1997-1998 demonstrates serious inefficiency in resource utilization, particularly in fixed capital, working capital and fuel consumption. Nevertheless, inefficient industry-groups have used their labour input efficiency.

Dash and Shanmugam [41] measure the technical, cost, revenue maximizing and profit efficiency of Indian banks for the period 1997- 2003. The inputs used in the study are borrowed funds, number of employees, fixed assets and equity. The results of the paper reveal that the Indian banks are still not much differentiated in terms of input or output-oriented technical or cost efficiency. However, they differ sharply in respect of revenue and profit efficiencies.

Singh [126] use DEA to assess the performance of 36 sugar mills of Uttar Pradesh for the period 1996-1997 to 2002-2003. The selected outputs in the study were sugar and molasses production. The inputs selected in the study were installed capacity, number of employees, raw material and energy & fuel. The results of the study show that the average mill can make radial reductions in all its inputs by 7 percent without detriment to its output level. The highest efficiency was achieved by the private mills, followed by the cooperative mills. The mills with bigger plant size attain relatively higher efficiency score. Also the mills located in the western region are found better performer. Labour and energy inputs are found highly underutilized in almost all the inefficient mills.

Mazumdar and Rajeev [86] examine the efficiency and productivity of the Indian pharmaceutical firms using Malmquist-Meta-Frontier approach. They measure the technical efficiency, technological gap ratio (TGR), and productivity change in the Indian pharmaceutical firms across different groups. The groups were formed based on their size, strategies and product varieties. The study indicates that vertically integrated firms that produce both bulk drugs and formulations exhibit higher technological innovation and efficiency. The analysis also reveals that increased export earnings do not necessarily lead to higher efficiency. They also find that installing capital-intensive techniques or imported technology propels the technological growth of firms.

Joshi and Singh [63] estimate the production efficiency of ready-made garment firms using the number of shirts produced as output and the number of stitching machines & operators used as inputs. The overall production efficiency, pure production efficiency and the scale efficiency of eight garment firms located in Bangalore, India are estimated using the DEA technique. The study finds that seven out of eight firms have not produced the maximum attainable output using the available inputs. The results show that on average, the firms have to increase the actual production of garments by 25 percent to achieve the targeted output. The technical inefficiency has been found due to both the inefficient scale size and inefficient utilization of resources.

Joshi and Singh [64] estimate the total factor productivity (TFP) growth of Indian garment industry for the period 2002 to 2007. The firm level panel data for the study was collected from a PROWESS database of the Center for Monitoring Indian Economy (CMIE). The input variables used in the study were net fixed assets, wages & salaries, raw material, and energy and fuel; the only output taken in the study was gross sale. The results of the study show that the Indian garment industry has achieved a moderate average TFP growth rate of 1.7% per annum during the study period. The small scale firms are found to be more productive than the medium and the large scale firms. The decomposition of TFP growth into technical efficiency change and technological change reveals that the productivity growth was contributed largely by technical efficiency change rather than the technological change.

Pannu et al. [100] used DEA models to analyze the relative efficiency and productivity change in Indian pharmaceutical industry (IPI) between 1998 and 2007 which covers the post-TRIPS (1995) and post Indian Patent Act Amendment (2005) period. BCC-DEA model and Malmquist productivity index were used to estimate the relative efficiency and productivity change of Indian pharmaceutical companies over the 10 year period. They tested several hypotheses on the average efficiency and productivity change of IPI to check if the indigenous and multinational companies differed in their efficiencies and productivity changes. They also analyzed the effect of firm size on several performance measures. Exploring the relationship between DEA efficiency and innovation, they find that innovative firms with R&D and patents have higher efficiency than non-innovative firms.

Joshi and Singh [65] use a two-stage DEA analysis to estimate the technical efficiency and its determinants in the Indian garment industry using cross-sectional data from 275 Indian garment firms for the year 2004-2005. The study uses five variables, namely; investment in plant and machinery (PM), raw martial consumed (MAT), fuels consumed (FUEL), number of workers (WORK) and the number of managerial staff (STAFF) and a single output variable, namely, the value of output. The results of the study show that on average, a garment firm can increase its output by 32.10% with the existing level of outputs. Further, the overall technical efficiency score were found to be more sensitive to the variation in the pure technical efficiency scores than that the scale efficiency scores. The results of the study also show that the micro-sized firms were more efficient in utilizing resources than the small and medium ones. The Tobit regression results show that investment in plant and machinery per employee was negatively influenced by the efficiency of the firms. Also, efficiency was found to be positively associated with labour productivity, wages per employee and labour-staff ratio.

Kumar and Kumar [75] used DEA to estimate the efficiency of 27 public sector banks in India. Out of these 27 public sector banks, 20 banks are nationalized and rest 7 are State banks along with its associates. The data for the study were collected from Reserve Bank of India website for the year 2008-2009. The study was undertaken with two inputs and two outputs. The inputs used in the study were interest expended and operating expenses and outputs were net interest and non-interest income. The results show that out of 27 banks only 6 are technically efficient with an average TE of 0.957; and 10 are pure technical efficient with an average PTE of 0.975. The scale efficiency of the banks is 0.982. The average TE 0.957 of the banks suggests that these banks can reduce on an average their inputs by at least 4.3% with the best practice efficiency frontier. These banks have the scope of producing 1.045 (1/0.957) times as much output with the same input level. Out of 21 inefficient banks 12 banks have IRS and remaining 9 banks are on DRS.

Tyagi et al. [141] use DEA based CCR and BCC models to evaluate the performance of 19 academic departments of IIT Roorkee (India). They assess the technical, pure technical and scale efficiencies using the CCR and BCC models. The inputs taken in this study were academic staff, non-academic staff and departmental operating cost, while total enrolled students, progress (teaching and research) and research index were the outputs used in the study. The paper has four assessments, namely, overall performance assessment, research performance assessment, teaching performance assessment and assessment of engineering departments by using 10 models. Sensitivity analysis has also been used to study the robustness of the results. The paper concludes that, for overall performance assessment, four departments, namely Chemistry, HSS, Management Studies and Mathematics are good example to follow the inefficient department to improve their performance. ECE and Management departments are efficient in the field of placement as well as teaching activities, while Mathematics is performing well only for enrolled students and students taught in other departments. ECE and Chemical Engineering are the best performing departments for both UG and PG programs among all engineering departments. Finally the overall performance assessment is good for all science departments and other departments need improvements in their activities. In the area of research only Biotechnology, Chemistry, Civil and Hydrology departments are found efficient.

Chatterjee and Sinha [33] apply DEA to compare the performance of commercial banks of India in the reform period. Output maximizing and cost minimizing approach with VRS technology is used for the study. Number of branches and borrowed capital are taken as input variables and loan as output variable. The results of the cost minimizing DEA reveal that the banks have diverged from the best practice cost frontier. The public sector banks lag behind the private sector banks in respect of technical and allocative efficiencies.

Puri and Yadav [105] measure the OTE, PTE and SE of 27 public and 22 private sector banks for the year 2009 to 2010 using DEA based CCR and BCC models. In this study, number of employees, fixed assets and loanable funds were used as input variables; while interest income and other income were used as output variables. The data on input and output variables have been taken from the RBI publication. The findings of the study show that: 1) public sector banks outperformed the private sector banks in all categories of efficiencies; 2) the contribution of scale inefficiency in overall technical inefficiency has been observed to be smaller than the contribution of pure technical inefficiency; 3) in public sector banks, State Bank of India & its Associates outperformed the other nationalized banks, while in private sector banks, new banks outperformed the old ones; 4) the highest and lowest levels of average overall technical inefficiency have been seen for old private banks (48.8%) and SBI & its Associates (2.2%), respectively; and 5) sensitivity analysis results of the study are quite robust to discriminate between efficient and inefficient banks.

Mahajan et al. [82] estimate technical efficiencies, slacks and input/output targets of 50 large Indian pharmaceutical firms for the year 2010-11. The study applied DEA approach, taking raw material, salaries and wages, advertisement and marketing and capital usage cost as input variables and net sales revenue as output variable. The BCC model identified that the inefficiency was either due to inefficient managerial performance or scale utilization. The study also analyzed the slacks which were found to be significant in regard of inputs, especially advertisement and marketing. The targets setting results have shown that all the inputs have significant scope for reduction. These results have practical implications, such as; managers and owners can take corrective actions to reduce the cost of operations by optimizing the input usage so as to improve their efficiency.

2.3.2 Foreign Studies

DEA is extensively used in several areas and industries in the whole world to measure their productive efficiencies and growth trends. Some studies related to DEA models and MPI approaches across foreign countries are discussed below.

Ali and Gstach [11] use MPI approach to investigate the relative performance of Austrian banks during 1990–1999 and test the hypothesis of increased competition. The study reveals the significant efficiency losses across all types of banks and size categories indicate that the gap in the performance of best practice and average practice Austrian banks grew between 1990 and 1996. The average technological improvement is found to be 0.6% per year for all banks in the 1990–1997 period. Commercial banks appear to have better adapted their year-wise improvements, of 1.4%, dominate the other three banks categories for which the corresponding figure was at 0.4%. The study reveals evidence of product diversification rather than increased price competition; a decrease in the spread of prices paid for inputs indicates increased competitiveness over the period, which can be attributed to deregulation brought about by EU-membership.

Ramanathan [107] uses DEA technique to synthesize the diverse characteristics of eight different systems in terms of their relative efficiency. Land was taken input variable. Three variables (loss of life expectancy, gain of life expectancy and CO_2 estimation per year) were three output variables used in the study. The results show those nuclear and solar photovoltaics are the most efficient technologies, followed by natural gas and oil. Sensitivity analysis shows that the main issue associated with the large-scale deployment of renewable technologies is the largest land area required by these technologies.

Alexander and Jaforullah [2] estimate pure technical and scale efficiencies of 324 secondary schools in respect of the capacity to produce academic qualifications for their students in New Zealand using DEA. Administrative expenses, expenditure of learning resources, depreciation expenses, expenditure for raising local funds, property management expenses, teaching staff and students are taken as input variables. The results show that very few schools could achieve optimal scale. Integrated schools are more efficient than the state schools. Single sex schools outperform co-educational ones. The study concludes that higher socioeconomic status of a school community confers both scale and pure technical efficiency advantages.

Ho Bruce [57] separates the efficiency and effectiveness of 59 listed electric companies in Taiwan, using a two-stage DEA model, and constructs a conceptual framework based on Financial Statement Analysis (FSA). In the first stage, a basic CCR output-oriented model has been applied with 3 inputs and one output. In the second stage, effective model, only one input and two outputs are taken in the analysis. The results of the analysis show that the companies whose efficiency scores were high, may not always have better effectiveness. In the first stage, CCR model results that only 9 corporations are efficient. In the second stage, effectiveness model indicates that only 5 companies are CCR efficient.

Lee [79] uses the DEA to evaluate the OTE, PTE and SE of 173 medium-sized audit firms for the year 2005. The data were obtained from the survey report of public audit firms in Taiwan, published by the Financial Supervisory Commission, Executive Yuan, Taiwan, ROC. To facilitate evaluating the efficiency, these 173 audit firms are divided equally into five groups (Group 1 to 5) in terms of total business revenues of audit firms in increasing order. Group 1 is the sample group with the lowest total business revenues; Group 5 is the sample group with the highest total business revenues. The four input variables of the study were number of branches, number of total employees, number of partners, and total expenditure. The four output variables taken in the study were attestation revenues, tax business revenues, management consultant revenues, and corporate registration & other business services. The results of the study show that in all the five groups and 173 firms only 28 are overall technical efficient, while 55 firms are pure technical efficient. The mean OTE, PTE and SE are found to be 0.778, 0.863 and 0.902, respectively. Out of these 55 PTE firms, 31 firms are scale inefficient which indicates that overall technical inefficiency in these firms is mainly due to their disadvantageous scale size.

Nassiri and Singh [96] apply DEA based CCR and BCC models in the category-wise and zone-wise evaluation of OTE, PTE and SE of paddy producers in Punjab state (India). The data for the study were collected from “All India Co-ordinate Research Project on Energy Requirement in Agriculture Sector” for the period 1997 to 2000. The study concludes that an increasing trend of total energy input was observed. It was also observed that smaller farmers had high energy-ratio and low specific energy as compared to larger ones. The OTE of marginal, small, semi-medium, medium and large firms were 0.681, 0.743, 0.666, 0.651 and 0.617, respectively; whereas PTE for the classified categories were 0.760, 0.819, 0.770, 0.770 and 0.791, respectively. Also high correlation was observed between energy-ratio and technical efficiency in both farm categories and zones.

Liu and Chen [80] proposed a worst-practice frontier (WPF) DEA model to identifying the bed performer firms, such as, bankrupt firms in the most unfavorable scenario in which the "worst efficient" DMUs constructs a worst practice frontier. To identify bad performers with the slack values, they formulate another model called worst-practice frontier with the slack based model. And at the end they develop a HypoSBM model to differentiate the worst performers from the bad ones. In the first WPF-DEA model, they made a change that reverses the constraints inequality, while in worst-practice frontier with slack based model; they made a change in the objective function minimization into maximization and the reverse of the numerator to denominator and vice-versa. Then, they form a HypoSBM model that ranks the worst performers using worst-practice frontier with the slack based model.

Amirteimoori et al. [13] proposed a modification in the standard DEA model to incorporate the recyclable outputs. In a system, there is some portion of produced outputs which are possibly used as inputs. Such types of outputs are called recyclable outputs, and they enter into the system again and treated as inputs. The study used these types of outputs on a group of 25 DMUs with two inputs x_1 and x_2 and four outputs y_1, y_2, z_1 and z_2. Here y_1 and y_2 were main outputs and z_1 and z_2 were recyclable outputs. When they use y_1, y_2, z_1 and z_2 as outputs, the general CCR model results only 7 DMUs being efficient, while when recyclable outputs are used, the number of efficient DMUs increased to 16. Such types of outputs are used in many real life problems, where some of the produced outputs are considered as inputs.

2.4 Other Applications with DEA

DEA is also used with Neural Networks [92], multi-objective programming [145, 146], semi-oriented radial measure [48], ranking with fuzzy [55, 88, 89, 90], multiparaetric sensitivity analysis of Additive models [121], super efficiencies or super inefficiencies [34], fuzzy input mix-efficiency [106] and hybrid mini-max reference point–DEA approach [147].

2.5 Summing Up

The review of above studies clearly shows that the researchers frequently use DEA to measure the efficiencies of different organizations including public healthcare. Broadly, one of the main objectives of these studies is an efficiency evaluation using CCR, BCC and modified DEA based models, and the second one is productivity measurement using MPI. Further, some of the studies have applied regression analysis to study the impact of background variables on the performance of the DMUs. These studies used the cross-sectional, time series and panel data and evaluated the performance at DMU level as well as organizational level. Their results show the efficiency scores of the individual DMUs, ranking of DMUs, excess use of inputs and deficit in outputs, benchmarks and provide the significant feedback for the policy considerations. It is observed that these studies use multiple inputs and multiple outputs for efficiency and productivity measurement and the background variables are used as explanatory variables in regression analysis to identify their impact on efficiency and productivity.

Also the review of the studies on the subject clearly indicates that there has been a very few studies on Indian public healthcare sector, based on DEA and MPI. However, the review of literature on the subject clearly indicates that there has hardly been any study conducted so far on Uttarakhand’s public healthcare sector, which has used DEA and MPI to measure the technical efficiency and productivity growth of public sector hospitals. The present study is an attempt to fill this gap.

Chapter 3

Advancement in DEA Techniques

3.1 Introduction

This Chapter presents an overview of Data Envelopment Analysis (DEA) technique. It describes the working of the basic (CCR and BCC) DEA models and also briefs the various modifications proposed in the basic models.

3.2 Data Envelopment Analysis (DEA): A Brief Review

Data envelopment analysis was developed by Charnes, Cooper and Rhodes in 1978 in their seminal paper [27]. Their paper was an extension of the Farrell [52] work. DEA was initially developed as a method for assessing the relative efficiencies of organizational units known as decision making units (DMUs). It constructs a non-parametric peace-wise frontier over the data using the mathematical programming method which is able to calculate the efficiencies relative to this frontier. It calculates a maximum performance measure for each DMU relative to all other DMUs in the dataset with the condition that each DMU lies on or below the frontier. The DMUs, not on the frontier, are scaled down against a convex combination of the DMUs on the frontier facet closest to it [30].

According to Charnes et al. [27], efficiency of any DMU is obtained as the maximum of the ratio of weighted output (virtual output) to weighted input (virtual input) subject to the condition that the similar ratio for all DMUs should be less than or equal to unity. This fractional programming problem was named after Charnes, Cooper and Rhodes as CCR ratio model. Inputs and output weights in DEA models provide a measure of the relative contribution of that input/output to the overall value of the efficiency. The weights are directly derived from the data in a manner that assign the best set of weights to each DMU. The term best is used here to mean that the resulting output-to-input ratio for each DMU is maximized relative to all other DMUs when these weights are assigned to the corresponding outputs and inputs for every DMU.

The fractional programming problem is transformed into a linear programming problem (LLP) by normalizing the denominator of the fractional programming objective function. Thus, the objective of the transformed LPP is to maximize virtual output subject to the unit virtual input while maintaining the condition that the virtual output cannot exceed the virtual input for each DMU. This is known as CCR multiplier model. The optimal weights generally vary from one to another DMU. The efficiency frontier is determined by the DMUs in the data set which cannot be dominated by any combination of the data of the other DMUs for whatever weights are applied to their inputs and outputs. There is no other common set of weights that gives a more favorable rating relative to the reference set. This means that an inefficient DMU with a set of weights would also be inefficient with any other set of weights. These weights differ from the customary weights, so they are also known as multipliers and the model is known as multiplier model.

In solving the above LPP, many computational difficulties arise. To overcome these problems, generally the dual of the LPP (CCR multiplier model), which is called CCR envelopment model, is used. The efficiency scores obtained by the envelopment model reflect the radial distance from the estimated production frontier to the DMU under consideration. The non-zero slacks and (or) radial efficiency scores lesser than unity identify the sources and amount of any inefficiency that may exist in the DMU. So, a DMU is called fully efficient if and only if it is not possible to reduce any input or increase any output without increasing some other input or reducing other output. When a DMU is CCR inefficient, then there must be at least one DMU, for which the virtual output is strictly equal to the virtual input. The set composed by these types of CCR efficient DMUs is known as the reference set or the peer group to that inefficient DMU. One version of a CCR model aims to minimize inputs while satisfying at least the given output levels. This is called an input-oriented model. There is another type of model called the output-oriented model that attempts to maximize output without requiring more of any of the observed input values.

Another basic model of DEA is BCC model, which is given by Banker, Charnes and Cooper [19]. The primary difference between BCC and CCR models is convexity constraint, which represents the returns to scale. Returns to scale (RTS) reflect the extent to which a proportional increase in all inputs increases outputs. The CCR model is based on the assumption that constant returns to scale (CRS) exist at the efficiency frontier, whereas BCC assumes variable returns to scale (VRS) frontier. The CCR efficiency is known as the overall technical efficiency (OTE), whereas the BCC efficiency is known as pure technical efficiency (PTE) [38]. The efficiency assessed by the BCC model is pure technical efficiency, net of any scale effect. If a DMU score value of both CCR-efficiency and BCC-efficiency unity, it is operating at the most productive scale size (MPSS). The impact of scale-size on the efficiency of a DMU is measured by scale efficiency (SE). Inefficiency in any DMU may be caused by the inefficient operation of the DMU itself (BCC-inefficiency) or by the disadvantageous conditions under which the DMU is operating (scale-inefficiency). SE is estimated dividing the CCR-efficiency (OTE) by the BCC-efficiency (PTE) for a DMU. As OTE of a DMU can never exceed its PTE, therefore SE is always less than or equal to 1. It suggests that a DMU is less productive when we control scale size, which means that scale of operation does impact the productivity of the DMU. SE measures the divergence the efficiency rating of a DMU under CCR BCC technical efficiency [134].

3.3 Growth of DEA

DEA is a non-parametric method for measuring the efficiencies of DMUs, such as schools, banks, transport services, hospitals etc.. In the subsequent years, a large number of researchers accumulated DEA into a significant volume of literature. After the seminal paper of Charnes, Cooper and Rhodes in 1978, there was an exponential growth in the number of publications on DEA. The rapid pace of dissemination of DEA as an acceptable method of efficiency analysis can be inferred from the fact that Seiford [112] introduced the bibliography of over 800 publications even as early as 1995. In a more recent bibliography, Tavares [133] introduced 3203 DEA references in 325 distinct journals from 2152 different authors of 213 different universities. After these, Emrouznejad et al. [47] introduced a more recent bibliography of 4015 DEA references by 2500 distinct authors in 2008. After this, a number of research papers have been added to the DEA bibliography up to 2013. One of the factors behind this explosive piece of DEA literature is that DEA brings theory and practice in a mutually reinforcing and beneficial dynamics. Practical applications of DEA follow theoretical developments in the field, while at the same time the applications highlight aspects of practical importance which research must address. On theoretical aspects, weight restriction, categorical DMUs, non-discretionary inputs and outputs, recyclable outputs, negative outputs, super efficiency, X-efficiency, mix-efficiency, input congestion, sensitivity analysis, slack based model and new slack model are the major extensions of DEA. Some of the major studies on the advancement of DEA approach are: Banker et al. [19], Charnes et al. [28], Banker and Morey [20, 21], Dyson and Thanassoulis [46], Charnes et al. [29], Thomption et al. [136], Wong and Beasley [144], Roll et al. [110], Ali and Seiford [10], Andersen and Petersen [14], Desai et al. [42], Thanassoulis et al. [135], Zhu [149], Seiford and Zhu [113], Pondinovski and Athanssopoulas [102], Hibiki and Sueyoshi [56], Sueyoshi et al. [132], Tone [140], Rouse and Lovell [111] Jahanshahloo et al. [60], Amin and Toloo [12], Toloo and Nalchigar [139], Agarwal et al. [6], Khezrimotlagh et al. [68, 69, 70]. In parallel with the theoretical development, a number of empirical studies have also been published, which evince the inexhaustible potential of DEA for innovative applications. DEA was applied to estimate relative efficiencies of non-profit as well as profit organizations, such as, Educational Institutions [141], Hospitals [1, 3, 39, 78, 93, 94], Banks [75, 105], Transport services [2, 4, 5], Sugar Mills [127, 128], Textile Industries [62, 63, 64], etc.. Moreover, development of DEA based Malmquist productivity index (MPI) for measuring total factor productivity (TFP) growth and its decomposition into technical efficiency change and technical change is the significant achievement in the field of productivity analysis [4, 64, 127].

3.4 DEA Models

To describe DEA efficiency evaluation, first assume that the performance of n DMUs (DMU$_j$: $j=1, 2,\ldots n$) be measured by DEA. The performance of DMU 'j' is characterized by a production process of m inputs (x_{ij}: $i= 1, 2,3,\ldots m$) to yield s outputs (y_{rj}: $j= 1, 2,3,\ldots s$). The essential characteristic of the given ratio construction is the reduction of the multiple-output and multiple-input situation to that of a single "virtual output" and a single "virtual input". Virtual output and virtual input can be calculated by the weighted sum of all outputs and weighted sum of all inputs respectively. In DEA, DMU$_j$ can select its own weights subject to three conditions:

- ❖ After DMU$_j$ picks its weights, these weights will be applied to all the rest of the DMUs.
- ❖ The weights must be given such that the highest score of the ratio is unity.
- ❖ No DMU is allowed to pick weights of zero to avoid the situation of weak efficiency. Weak efficiency occurs when two DMUs have the same score by choosing zero weight for any input or/and output. Charnes et al. [27] address the problem of weak efficiency by restricting the multiplier to be strictly positive. However, LPP formulation does not allow this restriction, so the non-Archimedean constant is used to resolve this problem.

The assumption in DEA is that $0 \le x^j \ne 0,\ 0 \le y^j \ne 0$. This assumption excludes the arbitrary of a DMU which transforms nothing into something or the uninteresting case of a unity with no output.

3.4.1 The CCR Model

According to Charnes et al. [27], the ratio of the single virtual output to single virtual input, which is to be maximized, forms the objective function for the k^{th} DMU (DMU_k) with the condition that the ratio of virtual output to virtual input of every DMU should be less than or equal to unity.

Mathematically, it can be written as:

$$\left.\begin{array}{lll} Max\, E_k^i = \dfrac{\sum_{r=1}^{s} v_{rk} y_{rk}}{\sum_{i=1}^{m} u_{ik} x_{ik}} & & \\ s.t. \quad \dfrac{\sum_{r=1}^{s} v_{rk} y_{rj}}{\sum_{i=1}^{m} u_{ik} x_{ij}} \le 1 & \forall j = 1, 2, ..., n & \\ \quad u_{ik} \ge \varepsilon & \forall i = 1, 2, ..., m & \\ \quad v_{rk} \ge \varepsilon, & \forall r = 1, 2, ..., s & \end{array}\right\} \qquad ....(3.1)$$

where E_k^i is input oriented efficiency of the k^{th} DMU; ($k=1, 2, 3, ...n$), y_{rk} is the amount of the r^{th} output produced by the k^{th} DMU, x_{ik} is the amount of the i^{th} input used by the k^{th} DMU, n is the number of DMUs, m is the number of inputs, s is the number of outputs, ε is non-Archimedean (infinitesimal) constant. v_{rk} is the weight given to the r^{th} output of the k^{th} DMU. The optimal value v_{rk} provides a measure of the relative contribution of y_{rk} to the overall value of E_k^i. Similarly u_{ik} is the weight given to the i^{th} input of the k^{th} DMU and the optimal value of u_{ik} expresses the relative contribution of x_{ik} to the overall value of E_k^i.

The model (3.1) is properly known as ***CCR ratio model,*** named after Charnes, Cooper and Rhodes. The problematic non-convex nonlinear fractional formulation of the model (3.1) is replaced by an equivalent LPP by imposing the constraint that the denominator of the objective function is equal to 1 [25] as follows:

$$\sum_{i=1}^{m} u_{ik} x_{ik} = 1 \qquad ....(3.2)$$

which provides:

$$\left.\begin{array}{ll} Max\, E_k^i = \sum_{r=1}^{s} v_{rk} y_{rk} & \\ s.t. \quad \sum_{i=1}^{m} u_{ik} x_{ik} = 1 & \\ \sum_{r=1}^{s} v_{rk} y_{rj} - \sum_{i=1}^{m} u_{ik} x_{ij} \le 0 & \forall j = 1,2,3,...,n \\ u_{ik} \ge \varepsilon & \forall r = 1,2,3,...,s \\ v_{rk} \ge \varepsilon & \forall i = 1,2,3,...,m \\ k = 1,2,3,...n & \end{array}\right\} \quad ....(3.3)$$

Model (3.3) is interpreted that the objective is to maximize the virtual output subject to unit virtual input, while maintaining the condition that the virtual output cannot exceed virtual input for any DMU. This is known as ***CCR Multiplier model***. If the optimal value of the objective function is 1 and all multipliers are strictly positive, then the DMU$_k$ is fully efficient. If efficiency score is 1 and at least one multiplier is zero, then the DMU$_k$ is weak efficient, so it may or may not be efficient. The dual of the model (3.3) is given by:

$$\left.\begin{array}{ll} Min\, Z_k^i = \theta_k - \varepsilon\left(\sum_{r=1}^{s} s_{rk}^{+} + \sum_{i=1}^{m} s_{ik}^{-}\right) & \\ s.t. \quad \sum_{j=1}^{n} \lambda_{jk} y_{rj} - s_{rk}^{+} = y_{rk} & \forall r = 1,2,3,...,s \\ \sum_{j=1}^{n} \lambda_{jk} x_{ij} + s_{ik}^{-} = \theta_k x_{ik} & \forall i = 1,2,3,...,m \\ \lambda_{jk} \ge 0 & \forall j = 1,2,3,...,n \\ s_{ik}^{-}, s_{rk}^{+} \ge 0 \quad \forall i = 1,2,3,...,m;\ r = 1,2,3,...,s & \\ \theta_k \ is\, unrestricted\ in\, sign & \end{array}\right\} \quad ....(3.4)$$

where, s_{rk}^{+} is the slack in the r^{th} output of the k^{th} DMU, s_{ik}^{-} is the slack in i^{th} input of the k^{th} DMU and λ_{jk}'s are the dual variables, known as intensity variables.

θ_k (Scalar) is the (proportional) reduction applied simultaneously to all inputs and results in a radial movement towards the envelopment surface. This is properly known as ***CCR envelopment model***. The interpretation of the results of the envelopment model (3.4) can be summarized as:

The DMU$_k$ is Pareto efficient if

(a) $\theta_k^* = 1$

(b) All slacks are zero

The DMU$_k$ is weakly efficient if

(a) $\theta_k^* = 1$

(b) $s_{rk}^{+*} \neq 0 \, and \, / \, or \, s_{ik}^{-*} \neq 0$ for some *r* and *i* in some alternate optima.

The non-zero slacks and (or) $\theta_k^* \leq 1$ identify the sources and amount of any inefficiency that may exist in the DMU. If the optimal value λ_{jk}^* of λ_{jk} is non zero, then the *j*th DMU represents the reference set (peer) of the *k*th DMU and the value for these reference set elements are the coefficients used to construct the benchmark. The reference set can be defined as the collection of DMUs used to construct the virtual DMU as benchmark which shows how inputs can be decreased and output increased to make the *k*th DMU efficient. It is to be noted that any uniform scaling of a DMU's input and output components does not change its relative efficiency status. This is why this model is also known as the constant returns to scale (CRS) model.

This version of CCR model aims to minimize inputs while satisfying at least the given output levels. This is called the input-oriented model. There is another type of model called the output-oriented model that attempts to maximize outputs without requiring more of any of the observed input values. In this model, we consider the ratio of virtual input to output in the ratio model. This would reorient the objective from maximization to minimization. The output-oriented multiplier and envelopment CCR models are shown in Table 3.1.

Let (θ^*, λ^*) be an optimal solution for the input-oriented model. Then $(1/\theta^*, \, \lambda^* / \theta^*) = (\phi^*, \tilde{\lambda}^*)$ is optimal solution for the corresponding output-oriented model. Similarly if $(\phi^*, \tilde{\lambda}^*)$ is an optimal solution for the output-oriented model, then $(1/\phi^*, \, \tilde{\lambda}^* / \phi^*) = (\theta^*, \lambda^*)$ is optimal solution for the input-oriented model. The correspondence needs not be 1-1, because of the possible presence of alternate optima. Table 3.1 presents the four different LPPs that correspond to the CCR DEA model.

Table 3.1: LPPs of CCR DEA Model

<table>
<tr><th colspan="2">Input-oriented CCR Model</th></tr>
<tr><th>Multiplier Model</th><th>Envelopment Model</th></tr>
<tr><td>$Max\,E_k^i = \sum_{r=1}^{s} v_{rk} y_{rk}$
$subject\ to$
$\sum_{i=1}^{m} u_{ik} x_{ik} = 1$
$\sum_{r=1}^{s} v_{rk} y_{rj} - \sum_{i=1}^{m} u_{ik} x_{ij} \le 0 \quad \forall j = 1,2,3,\dots,n$
$u_{ik} \ge \varepsilon \quad \forall i = 1,2,3,\dots,m$
$v_{rk} \ge \varepsilon \quad \forall r = 1,2,3,\dots,s$</td><td>$Min\,Z_k^i = \theta_k - \varepsilon\left(\sum_{r=1}^{s} s_{rk}^{+} + \sum_{i=1}^{m} s_{ik}^{-}\right)$
$subject\ to$
$\sum_{j=1}^{n} \lambda_{jk} y_{rj} - s_{rk}^{+} = y_{rk} \quad \forall r = 1,2,3,\dots,s$
$\sum_{j=1}^{n} \lambda_{jk} x_{ij} + s_{ik}^{-} = \theta_k x_{ik} \quad \forall i = 1,2,3,\dots,m$
$\lambda_{jk} \ge 0 \quad \forall j = 1,2,3,\dots,n$
$s_{ik}^{-}, s_{rk}^{+} \ge 0 \quad \forall i = 1,2,3,\dots,m;\ r = 1,2,3,\dots,s$
θ_k $is\ unrestricted\ in\ sign$</td></tr>
<tr><td colspan="2">$Max\,E_k^i = Min\,Z_k^i$ =Input-oriented efficiency of k^{th} DMU</td></tr>
</table>

Output-oriented CCR Model

<table>
<tr><th>Multiplier Model</th><th>Envelopment Model</th></tr>
<tr><td>$Min\,E_k^o = \sum_{i=1}^{m} u_{ik} x_{ik}$
$subject\ to$
$\sum_{r=1}^{s} v_{rk} y_{rk} = 1$
$\sum_{i=1}^{m} u_{ik} x_{ij} - \sum_{r=1}^{s} v_{rk} y_{rj} \ge 0 \quad \forall j = 1,2,3,\dots,n$
$u_{ik} \ge \varepsilon \quad \forall i = 1,2,3,\dots,m$
$v_{rk} \ge \varepsilon \quad \forall r = 1,2,3,\dots,s$</td><td>$Max\,Z_k^o = \phi_k + \varepsilon\left(\sum_{r=1}^{s} s_{rk}^{+} + \sum_{i=1}^{m} s_{ik}^{-}\right)$
$subject\ to$
$\sum_{j=1}^{n} \lambda_{jk} y_{rj} - s_{rk}^{+} = \phi_k y_{rk} \quad \forall r = 1,2,3,\dots,s$
$\sum_{j=1}^{n} \lambda_{jk} x_{ij} + s_{ik}^{-} = x_{ik} \quad \forall i = 1,2,3,\dots,m$
$\lambda_{jk} \ge 0 \quad \forall j = 1,2,3,\dots,n$
$s_{ik}^{-}, s_{rk}^{+} \ge 0 \quad \forall i = 1,2,3,\dots,m;\ r = 1,2,3,\dots,s$
ϕ_k $is\ unrestricted\ in\ sign$</td></tr>
<tr><td colspan="2">$Min\,E_k^o = Z_k^o$ =output-oriented efficiency of k^{th} DMU</td></tr>
</table>

The following example illustrates how these models are applied. We evaluate the performance of nine hospitals of Dehradun and Haridwar districts of Uttarakhand with two inputs and two outputs. The inputs and outputs are shown in Table 3.2. We begin with **multiplier** form of CCR-DEA model, with **output orientation.** For first DMU$_1$:

$$\left.\begin{array}{ll}
Min\, E_1^o = 715221v_{11} + 20006v_{21} & \\
subject\, to & \\
 & 402u11+55u21=1 \\
Hospital\,1: & 715221v_{11} + 20006v_{21} - 402u_{11} - 55u_{21} \le 0 \\
Hospital\,2: & 106535v_{11} + 22111v_{21} - 111u_{11} - 23u_{21} \le 0 \\
Hospital\,3: & 121075v_{11} + 3206v_{21} - 120u_{11} - 15u_{21} \le 0 \\
Hospital\,4: & 206844v_{11} + 15583v_{21} - 150u_{11} - 20u_{21} \le 0 \\
Hospital\,5: & 57283v_{11} + 1991v_{21} - 53u_{11} - 14u_{21} \le 0 \\
Hospital\,6: & 108594v_{11} + 2987v_{21} - 70u_{11} - 18u_{21} \le 0 \\
Hospital\,7: & 30426v_{11} + 2065v_{21} - 30u_{11} - 7u_{21} \le 0 \\
Hospital\,8: & 64963v_{11} + 1601v_{21} - 100u_{11} - 16u_{21} \le 0 \\
Hospital\,9: & 271514v_{11} + 5103v_{21} - 106u_{11} - 18u_{21} \le 0
\end{array}\right\} \qquad (3.5)$$

A full DEA analysis requires the solution of as many LPs as there are hospitals (i.e., nine in this example). A hospital (DMU) is relatively inefficient if its objective function value is strictly less than unity. Note that, out of all DMU inequalities, at least one must be binding, i.e., there must be at least one constraint (or DMU) for which the optimal weights produce equality between the left and right hand side. So, if a DMU gets a score less than “1”, then another DMU getting a score of “1” and therefore, is better. If the objective function value is “1” and all the multipliers are strictly positive, it is sufficient to conclude that the DMU evaluated is efficient. If the score is “1” and at least one multiplier is zero, then the DMU may or may not be efficient. This case may be of a week efficient DMU. Let the set of efficient DMU be

$$E_1^e = \{ j : \sum_{r=1}^{s} v_{r1}^* y_{rj} = \sum_{i=1}^{m} u_{i1}^* x_{ij} \} \qquad (3.6)$$

The subset E_1 of E_1^e composed of efficient DMUs, is called the reference set or the peer group to the DMU$_1$. These efficient DMUs force the DMU$_1$ to be inefficient. The set spanned by E_k is called the efficient frontier of DMU$_1$. v_{r1}^* and u_{i1}^* are the optimal weights for the r^{th} output and i^{th} input for DMU$_1$ respectively. The value of v_{rk}^* provides a measure of the relative contribution of y_{r1} to all overall efficiency score of DMU$_1$. Similarly the value of u_{i1}^* provides a measure of the relative contribution of x_{i1} to all overall efficiency score of DMU$_1$. The choice of weights is determined from observational data subject to the constraints established in the model. In DEA, these weights are known as multipliers, and therefore models (3.3 and 3.5) are known as multiplier models.

Table 3.2: Data for the Nine Hospitals (all values are measured in numbers)

			Input 1	*Input 2*	*Output 1*	*Output 2*
Code	Distt. Name	Hospital Name	Beds	Doctors	OPD	IPD
H1	Dehradun	Doon Male Hospital	402	55	715221	20006
H2	Dehradun	Female Hospital Dehradun	111	23	106535	22111
H3	Dehradun	Coronation Hosptial	120	15	121075	3206
H4	Dehradun	SPS Hospital Rishikesh	150	20	206844	15583
H5	Dehradun	St. Merry. Hospital Mussoorie	53	14	57283	1991
H6	Haridwar	HMG Hospital Haridwar	70	18	108594	2987
H7	Haridwar	CR Female Hospital	30	7	30426	2065
H8	Haridwar	Mela Hospital Haridwar	100	16	64963	1601
H9	Haridwar	Combined Hospital Roorkee	106	18	271514	5103

In our example, output-oriented CCR DEA multiplier model identifies hospitals H1, H3, H5, H6, H7 and H8 to be inefficient with respective objective function values of 0.926, 0.568, 0.482, 0.652, 0.568 and 0.282. When estimating the efficiency of hospital H1, the constraints corresponding to hospitals H2, H4 and H9 are binding. These hospitals are known as the reference set for hospital H1.

The envelopment form of the output-oriented model for hospital H1 is expressed as follows:

$$\left.\begin{aligned}
& Max\, Z_1^o = \theta_1 + \varepsilon \sum_{r=1}^{2} s_{r1}^{+} + \varepsilon \sum_{i=1}^{2} s_{i1}^{-} \\
& Subject\, to \\
& Output\,1: -715221\theta_1 + 715221\lambda_{11} + 106535\lambda_{21} + 121075\lambda_{31} + 206844\lambda_{41} + \\
& \qquad\qquad 57283\lambda_{51} + 108594\lambda_{61} + 30426\lambda_{71} + 64963\lambda_{81} + 271514\lambda_{91} - s_{11}^{+} = 0 \\
& Output\,1: -20006\theta_1 + 20006\lambda_{11} + 22111\lambda_{21} + 3206\lambda_{31} + 15583\lambda_{41} + 1991\lambda_{51} + \\
& \qquad\qquad 2987\lambda_{61} + 2065\lambda_{71} + 1601\lambda_{81} + 5103\lambda_{91} - s_{21}^{+} = 0 \\
& Input\,1: 402\lambda_{11} + 111\lambda_{21} + 120\lambda_{31} + 150\lambda_{41} + 53\lambda_{51} + 70\lambda_{61} \\
& \qquad\qquad + 30\lambda_{71} + 100\lambda_{81} + 106\lambda_{91} + s_{11}^{-} = 402 \\
& Input\,2: 55\lambda_{11} + 23\lambda_{21} + 15\lambda_{31} + 20\lambda_{41} + 14\lambda_{51} + \\
& \qquad\qquad 18\lambda_{61} + 7\lambda_{71} + 16\lambda_{81} + 18\lambda_{91} + s_{21}^{-} = 55 \\
& \qquad\qquad \lambda_{j1} \ge 0 \quad \forall\, j = 1, 2, \dots, 9 \\
& \qquad\qquad \theta_1 \text{ is unrestrected in sign,} \quad s_{r1}^{+}, s_{i1}^{-} \ge 0; r = 1, 2; i = 1, 2
\end{aligned}\right\} \qquad (3.7)$$

The optimal value of θ_1 less than unity for the envelopment form implies that the hospital is inefficient. If a DMU is inefficient, then all $\lambda's > 0$ corresponds to the reference set are used to construct the benchmark. The reference set can be defined as the collection of hospitals used to construct the virtual hospital. The virtual hospital is the benchmark for the inefficient hospital. For the output-oriented case, the improvement is achieved by uniformly augmenting the outputs while holding the inputs constant.

3.4.1.1 Input/output Targets for Inefficient DMUs

An inefficient DMU can be made efficient by projection onto the frontier. In an output-orientation, one improves efficiency through proportional augmentation of outputs, whereas an input-orientation requires proportional reduction of inputs.

For an output-orientation, the projection $(X_k, Y_k) \rightarrow (X_k, \theta^* Y_k)$ always yields a boundary point. However, technical efficiency is achieved only if all slacks are zero in all alternate optima so that $X_k = X\lambda^*$ and $\theta^* Y_k = Y\lambda^*$ for all optimal λ^*. Similarly, the input-oriented projection $(X_k, Y_k) \rightarrow (\phi^* X_k, Y_k)$ yields a boundary point which is technically efficient only if all slacks are zero in all alternate optima and $\phi^* X_k = X\lambda^*$ and $Y_k = Y\lambda^*$ for all optimal λ^*.

When a DMU is inefficient, the output-oriented level in equation (3.8) can be used as the basis for setting its targets so that it may improve its performance according to output-oriented CCR DEA model.

$$\left.\begin{aligned} \overline{x_k} &= x_k - s_k^- = X\lambda \\ \overline{y_k} &= \phi_k^* y_k + s_k^+ = Y\lambda \end{aligned}\right\} \tag{3.8}$$

Similarly, according to the input-oriented CCR DEA model

$$\left.\begin{aligned} \overline{x_k} &= \theta_k^* x_k - s_k^- = X\lambda \\ \overline{y_k} &= y_k + s_k^+ = Y\lambda \end{aligned}\right\} \tag{3.9}$$

The DEA Results for Output-oriented CCR Model are Summarized in Table 3.3.

Table 3.3: Results of Output-oriented CCR DEA Model

DMU	***Efficiency Score***	***u_{11}***	***u_{21}***	***v_{11}***	***v_{21}***	***Reference Set***	***Peer Weight***
H1	0.926	0	1.96E-02	1.10E-06	1.06E-05	H4, H9	0.61, 2.38
H2	1.00	9.01E-03	0	2.93E-06	3.11E-05	H2	1.00
H3	0.568	0	0.117277	6.59E-06	6.30E-05	H4, H9	0.14, 0.68
H4	1.00	0	0.050000	2.81E-06	2.69E-05	H4	1.00
H5	0.482	3.92E-02	0	1.28E-05	1.35E-04	H2, H9	0.09, 0.40
H6	0.652	2.19E-02	0	7.13E-06	7.56E-05	H2, H9	0.07, 0.59
H7	0.568	5.87E-02	0	1.91E-05	2.03E-04	H2, H9	0.13, 0.15
H8	0.282	0	0.221679	1.25E-05	1.19E-04	H4, H9	0.12, 0.76
H9	1.00	9.43E-03	0	3.68E-06	0	H9	1.00

DMU	s_{11}^{+}	s_{21}^{+}	s_{11}^{-}	s_{21}^{+}	***Projections or Target Values***			
	Beds	***Doctors***	***OPD***	***IPD***	***Beds***	***Doctors***	***OPD***	***IPD***
H1	58.59	0	0	0	343.41	55	772163.63	21598.79
H2	0	0	0	0	111	23	106535.00	22111.00
H3	27.16	0	0	0	92.84	15	212988.98	5639.83
H4	0	0	0	0	150	20	206844.00	15583.00
H5	0	4.61	0	0	53	9.39	118967.18	4134.97
H6	0	5.81	0	0	70	12.19	166479.34	4579.20
H7	0	1.36	0	0	30	5.64	53591.20	3637.21
H8	2.06	0	0	0	97.94	16	230415.09	5678.53
H9	0	0	0	0	106	18	271514.00	5103.00

In the envelopment form of our example, hospitals H1, H3, H5, H6, H7 and H8 are inefficient. For hospital H1, λ_2, λ_4 and λ_9 constitute the reference set. By calculating the projection, we obtain the virtual hospital (Beds = 343.41, Doctors = 55, OPD = 772163.63, and IPD = 21598.79) as required this projection obtain a score of “1” in the appropriate LPP. We can say similar about hospitals H3, H5, H6, H7 and H8 from Table 3.3.

For the same example, the envelopment form of the input-oriented CCR DEA model can be expressed as follows:

$$\left.\begin{aligned}
&Min\, Z_1^i = \phi_1 - \varepsilon\sum_{r=1}^{2} s_{r1}^{+} - \varepsilon\sum_{i=1}^{2} s_{i1}^{-}\\
&Subject\ to\\
&Output1:\ 715221\lambda_{11} + 106535\lambda_{21} + 121075\lambda_{31} + 206844\lambda_{41} + 57283\lambda_{51} +\\
&\qquad 108594\lambda_{61} + 30426\lambda_{71} + 64963\lambda_{81} + 271514\lambda_{91} - s_{11}^{+} = 715221\\
&Output1:\ 20006\lambda_{11} + 22111\lambda_{21} + 3206\lambda_{31} + 15583\lambda_{41} + 1991\lambda_{51} +\\
&\qquad 2987\lambda_{61} + 2065\lambda_{71} + 1601\lambda_{81} + 5103\lambda_{91} - s_{21}^{+} = 20006\\
&Input1:\ -402\phi_1 + 402\lambda_{11} + 111\lambda_{21} + 120\lambda_{31} + 150\lambda_{41} + 53\lambda_{51}\\
&\qquad + 70\lambda_{61} + 30\lambda_{71} + 100\lambda_{81} + 106\lambda_{91} + s_{11}^{-} = 0\\
&Input\,2:\ -55\phi_1 + 55\lambda_{11} + 23\lambda_{21} + 15\lambda_{31} + 20\lambda_{41} + 14\lambda_{51} +\\
&\qquad 18\lambda_{61} + 7\lambda_{71} + 16\lambda_{81} + 18\lambda_{91} + s_{21}^{-} = 0\\
&\qquad \lambda_{j1} \geq 0 \quad \forall\ j = 1, 2, \ldots, 9\\
&\qquad \phi_1 \text{ is unrestrected in sign,} \qquad s_{r1}^{+}, s_{i1}^{-} \geq 0;\ r = 1, 2;\ i = 1, 2
\end{aligned}\right\} \tag{3.10}$$

The DEA results for the input-oriented CCR DEA model are summarized in Table 3.4.

Table 3.4: Results of Input-oriented CCR DEA Model

DMU	***Efficiency Score***	***u11***	***u21***	***v11***	***v21***	***Reference Set***	***Peer Weight***
H1	0.926	0	1.82E-02	1.02E-06	9.77E-06	H4, H9	0.56, 2.21
H2	1	3.00E-03	0.029018	2.47E-06	3.33E-05	H2	1.00
H3	0.568	0	6.67E-02	3.75E-06	3.58E-05	H4, H9	0.08, 0.39
H4	1	2.91E-03	2.82E-02	2.40E-06	3.24E-05	H4	1.00
H5	0.482	1.89E-02	0	6.14E-06	6.51E-05	H2, H9	0.05, 0.19
H6	0.652	1.43E-02	0	4.65E-06	4.93E-05	H2, H9	0.05, 0.38
H7	0.568	3.33E-02	0	1.09E-05	1.15E-04	H2, H9	0.07, 0.08
H8	0.282	0	0.0625	3.51E-06	3.36E-05	H4, H9	0.03, 0.21
H9	1	0.00701	1.43E-02	3.02E-06	3.55E-05	H9	1.00

DMU	s_{11}^{+}	s_{21}^{+}	s_{11}^{-}	s_{21}^{+}	***Projections or Target Values***			
	Beds	***Doctors***	***OPD***	***IPD***	***Beds***	***Doctors***	***OPD***	***IPD***
H1	54.27	0	0	0	318.09	50.94	715221	20006
H2	0	0	0	0	111	23	106535	22111
H3	15.44	0	0	0	52.78	8.53	121075	3206
H4	0	0	0	0	150	20	206844	15583
H5	0	2.22	0	0	25.52	4.52	57283	1991
H6	0	3.79	0	0	45.66	7.95	108594	2987
H7	0	0.77	0	0	17.03	3.2	30426	2065
H8	0.58	0	0	0	27.61	4.51	64963	1601
H9	0	0	0	0	106	18	271514	5103

In the envelopment form of input-oriented model our example, hospitals H2, H3, H5, H6, H7 and H8 are inefficient. For hospital H1, λ_2, λ_4 and λ_9 constitute the reference set. By calculating the projection, we obtain the virtual hospital (Beds = 318.09, Doctors = 50.94, OPD = 715221, and IPD = 20006) as required this projection obtain a score of "1" in the appropriate LPP. We can say similar about hospitals H3, H5, H6, H7 and H8 from Table 3.4.

3.4.2 BCC Model

Another version of DEA is BCC model which is given by Banker, Charnes and Cooper [19]. Banker et al. [19] extended the idea of constant returns to scale (CRS) in CCR model for the variable returns to scale (VRS). They added a separate variable u_{01} to the CCR multiplier model to make it possible to determine whether operations were conducted in regions of increasing, constant, or decreasing returns to scale. The model is given in Table 3.5. They examined returns to scale locally at a point on the frontier, related to the value of the term u_{01} as:

- $u_{01} < 0$ implies increasing returns to scale;
- $u_{01} = 0$ implies constant returns to scale; and
- $u_{01} > 0$ implies decreasing returns to scale.

In the envelopment model, the primary difference between BCC model and CCR model is the convexity constraint, which represents the returns to scale. In the BCC model λ_{jk}'s are restricted to summing to one (i.e., $\sum_{j=1}^{n} \lambda_{jk} = 1$) which is known as convexity constraint.

$$\text{Scale Efficiency of } k^{th} \text{ DMU} = \frac{\text{Technical Efficiency of } k^{th} \text{ DMU}}{\text{Pure Technical Efficiency of } k^{th} \text{ DMU}}$$

$$= \frac{CCR \text{ efficiency } score \text{ of } k^{th} \text{ DMU}}{BCC \text{ efficiency score of } k^{th} \text{ DMU}}$$

Technical efficiency obtained from a CCR model is decomposed into two components, one is pure technical efficiency and another is scale efficiency [37]. If there is a difference in the two (CCR technical efficiency and BCC technical efficiency) technical efficiencies for a particular DMU, then this indicates that the DMU has scale efficiency, i.e., scale efficiency measures the divergence between the efficiency rating of a DMU under CCR and BCC technical efficiencies.

The overall technical efficiency (OTE) of a DMU can never exceed its pure technical efficiency (PTE). Table 3.5 presents the four different LPPs that correspond to the BCC DEA model.

Table 3.5: LPPs of BCC DEA Model

Input-oriented BCC Model	
Multiplier Model	Envelopment Model
$Max\, E_k^i = \sum_{r=1}^{s} v_{rk} y_{rk} + u_{0k}$ *subject to* $\sum_{i=1}^{m} u_{ik} x_{ik} = 1$ $\sum_{r=1}^{s} v_{rk} y_{rj} - \sum_{i=1}^{m} u_{ik} x_{ij} + u_{0k} \le 0 \quad \forall j = 1,2,3,...,n$ $u_{ik} \ge \varepsilon \quad \forall i = 1,2,3,...,m$ $v_{rk} \ge \varepsilon \quad \forall r = 1,2,3,...,s$	$Min\, Z_k^i = \theta_k - \varepsilon\left(\sum_{r=1}^{s} s_{rk}^{+} + \sum_{i=1}^{m} s_{ik}^{-}\right)$ *subject to* $\sum_{j=1}^{n} \lambda_{jk} y_{rj} - s_{rk}^{+} = y_{rk} \quad \forall\, r = 1,2,3...,s$ $\sum_{j=1}^{n} \lambda_{jk} x_{ij} + s_{ik}^{-} = \theta_k x_{ik} \quad \forall\, i = 1,2,3...,m$ $\sum_{j=1}^{n} \lambda_{jk} = 1$ $\lambda_{jk} \ge 0 \quad \forall j = 1,2,3...,n$ $s_{ik}^{-}, s_{rk}^{+} \ge 0 \quad \forall\, i = 1,2,3...,m;\ r = 1,2,3...,s$ θ_k *is unrestricted in sign*
$Max\, E_k^i = Min\, Z_k^i$ = Input-oriented efficiency of k^{th} DMU of BCC Model	

Output-oriented BCC Model

Multiplier Model	Envelopment Model
$Min\, E_k^o = \sum_{i=1}^{m} u_{ik} x_{ik} + u_{0k}$ *subject to* $\sum_{r=1}^{s} v_{rk} y_{rk} = 1$ $\sum_{i=1}^{m} u_{ik} x_{ij} - \sum_{r=1}^{s} v_{rk} y_{rj} + u_{0k} \ge 0, \forall j = 1,2,3,...,n$ $u_{ik} \ge \varepsilon \quad \forall i = 1,2,3,...,m$ $v_{rk} \ge \varepsilon \quad \forall r = 1,2,3,...,s$	$Max\, Z_k^o = \phi_k + \varepsilon\left(\sum_{r=1}^{s} s_{rk}^{+} + \sum_{i=1}^{m} s_{ik}^{-}\right)$ *subject to* $\sum_{j=1}^{n} \lambda_{jk} y_{rj} - s_{rk}^{+} = \phi_k y_{rk} \quad \forall\, r = 1,2,3,...,s$ $\sum_{j=1}^{n} \lambda_{jk} x_{ij} + s_{ik}^{-} = x_{ik} \quad \forall\, i = 1,2,3,...,m$ $\sum_{j=1}^{n} \lambda_{jk} = 1$ $\lambda_{jk} \ge 0 \quad \forall j = 1,2,3,...,n$ $s_{ik}^{-}, s_{rk}^{+} \ge 0 \quad \forall\, i = 1,2,3,...,m;\ r = 1,2,3,...,s$ ϕ_k *is unrestricted in sign*
$Min\, E_k^o = Z_k^o$ = output-oriented efficiency of k^{th} DMU of BCC Model	

All the three efficiencies (overall technical, pure technical and scale) are bounded by zero and one. Figure 3.1 presents the production frontier of single input (X) and single output (Y) data operated under constant returns to scale (CRS) and variable returns to scale (VRS) assumptions.

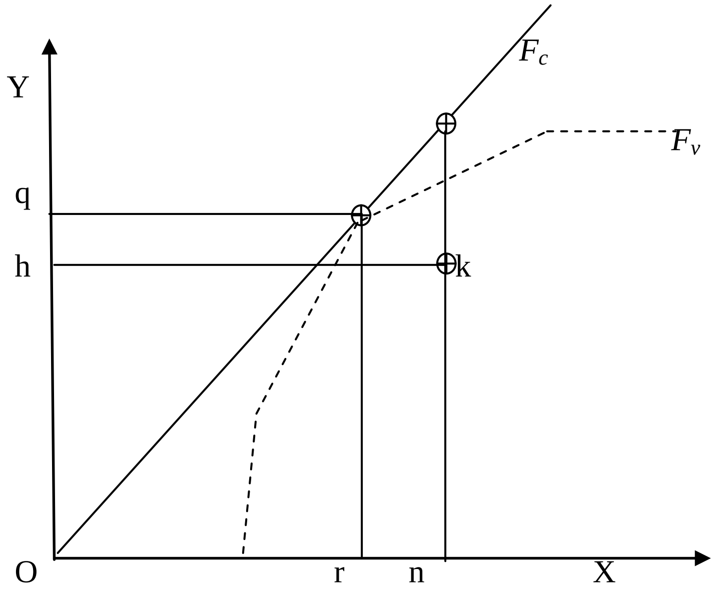


Figure 3.1: Production Frontier under CRS and VRS assumptions

Applying the BCC model to our example, we get the following LPP to measure the efficiency of DMU 1 (Hospital H1). The envelopment form of the output-oriented, BCC DEA model is as follows:

$$\left.\begin{aligned}
& Max\, Z_1^o = \theta_1 + \varepsilon \sum_{r=1}^{2} s_{r1}^{+} + \varepsilon \sum_{i=1}^{2} s_{i1}^{-} \\
& Subject\, to \\
& Output\,1: -715221\theta_1 + 715221\lambda_{11} + 106535\lambda_{21} + 121075\,\lambda_{31} + 206844\,\lambda_{41} + \\
& \qquad\qquad 57283\lambda_{51} + 108594\lambda_{61} + 30426\lambda_{71} + 64963\,\lambda_{81} + 271514\lambda_{91} - s_{11}^{+} = 0 \\
& Output\,1: -20006\theta_1 + 20006\lambda_{11} + 22111\lambda_{21} + 3206\lambda_{31} + 15583\,\lambda_{41} + 1991\lambda_{51} + \\
& \qquad\qquad 2987\lambda_{61} + 2065\lambda_{71} + 1601\lambda_{81} + 5103\lambda_{91} - s_{21}^{+} = 0 \\
& Input\,1: 402\lambda_{11} + 111\lambda_{21} + 120\lambda_{31} + 150\lambda_{41} + 53\lambda_{51} + 70\lambda_{61} \\
& \qquad + 30\lambda_{71} + 100\lambda_{81} + 106\lambda_{91} + s_{11}^{-} = 402 \\
& Input\,2: 55\lambda_{11} + 23\lambda_{21} + 15\lambda_{31} + 20\lambda_{41} + 14\lambda_{51} + \\
& \qquad 18\lambda_{61} + 7\lambda_{71} + 16\lambda_{81} + 18\lambda_{91} + s_{21}^{-} = 55 \\
& \qquad \lambda_{11} + \lambda_{21} + \lambda_{31} + \lambda_{41} + \lambda_{51} + \lambda_{61} + \lambda_{71} + \lambda_{81} + \lambda_{91} = 1 \\
& \qquad \lambda_{j1} \geq 0 \quad \forall\, j = 1, 2, \ldots, 9 \\
& \qquad \theta_1 \text{ is unrestrected in sign} \\
& \qquad s_{r1}^{+}, s_{i1}^{-} \geq 0; r = 1, 2; i = 1, 2
\end{aligned}\right\} \qquad (3.11)$$

Table 3.6 summarizes the output-oriented BCC DEA results.

In our example, H3, H5 and H6 are found to be BCC inefficient with respective objective function values of 0.618, 0.581 and 0.715. Hospitals H2, H4, H7 and H9 constitute the reference set of the inefficient hospitals. H1, H3, H5, H6, H7 and H8 hospitals are found to be scale inefficient, i.e., these hospitals are not operating on the optimal scale-size.

Table 3.6: Results of Output-oriented BCC DEA Model

DMU	***Efficiency Score***	u_{11}	u_{21}	v_{11}	v_{21}	u_{01}
H1	1.000	0	1.75E-02	1.11E-06	1.02E-05	4.00E-02
H2	1.000	9.01E-03	0	2.93E-06	3.11E-05	0
H3	0.618	0	0.159726	6.41E-06	7.00E-05	-0.7786
H4	1.000	0	0.050000	2.81E-06	2.69E-05	0
H5	0.581	4.58E-02	0	1.27E-05	1.37E-04	-0.7042
H6	0.715	2.56E-02	0	7.11E-06	7.65E-05	-0.3938
H7	1.000	6.84E-02	0	1.90E-05	2.04E-04	-1.0528
H8	0.297	0	0.302392	1.21E-05	1.33E-04	-1.4741
H9	1.000	9.43E-03	0	3.68E-06	0	0

DMU	***Reference Set***	***Peer Weights***	***Scale Efficiency***	***RTS***
H1	H1	1.000	0.926	DRS
H2	H2	1.000	1.000	CRS
H3	H4, H7, H9	0.09, 0.29, 0.62	0.919	IRS
H4	H4	1.000	1.000	CRS
H5	H2, H7, H9	0.03, 0.69, 0.27	0.829	IRS
H6	H2, H7, H9	0.03, 0.48, 0.49	0.912	IRS
H7	H7	1.000	0.568	IRS
H8	H4, H7, H9	0.08, 0.19, 0.72	0.949	IRS
H9	H9	1.000	1.000	CRS

In the envelopment form of our example, hospitals H3, H5, H6 and H8 are inefficient. For hospital H3, λ_4, λ_7 and λ_9 constitute the reference set. We can say similar about hospitals H5, H6 and H8 from Table 3.6.

For the same example, the envelopment form of the input-oriented CCR DEA model can be expressed as follows:

$$\left.\begin{array}{l}
Min\, Z_1^i = \phi_1 - \varepsilon \sum_{r=1}^{2} s_{r1}^{+} - \varepsilon \sum_{i=1}^{2} s_{i1}^{-} \\
Subject\, to \\
Output\,1:\ 715221\lambda_{11} + 106535\lambda_{21} + 121075\,\lambda_{31} + 206844\,\lambda_{41} + 57283\lambda_{51} + \\
\qquad\qquad 108594\lambda_{61} + 30426\lambda_{71} + 64963\,\lambda_{81} + 271514\lambda_{91} - s_{11}^{+} = 715221 \\
Output\,1:\ 20006\lambda_{11} + 22111\lambda_{21} + 3206\lambda_{31} + 15583\,\lambda_{41} + 1991\lambda_{51} + \\
\qquad\qquad 2987\,\lambda_{61} + 2065\lambda_{71} + 1601\lambda_{81} + 5103\lambda_{91} - s_{21}^{+} = 20006 \\
Input\,1:\ -402\phi_1 + 402\lambda_{11} + 111\lambda_{21} + 120\lambda_{31} + 150\lambda_{41} + 53\lambda_{51} \\
\qquad\qquad + 70\lambda_{61} + 30\lambda_{71} + 100\lambda_{81} + 106\lambda_{91} + s_{11}^{-} = 0 \\
Input\,2:\ -55\phi_1 + 55\lambda_{11} + 23\lambda_{21} + 15\lambda_{31} + 20\lambda_{41} + 14\lambda_{51} + \\
\qquad 18\lambda_{61} + 7\lambda_{71} + 16\lambda_{81} + 18\lambda_{91} + s_{21}^{-} = 0 \\
\qquad \lambda_{11} + \lambda_{21} + \lambda_{31} + \lambda_{41} + \lambda_{51} + \lambda_{61} + \lambda_{71} + \lambda_{81} + \lambda_{91} = 1 \\
\qquad \lambda_{j1} \geq 0 \quad \forall\ j = 1, 2, \ldots, 9 \\
\qquad \phi_1 \text{ is unrestricted in sign} \\
\qquad s_{r1}^{+}, s_{i1}^{-} \geq 0; r = 1, 2; i = 1, 2
\end{array}\right\} \qquad (3.12)$$

The findings of the input-oriented CCR DEA model are summarized in Table 3.4.

Table 3.7: Results of Input-oriented BCC DEA Model

DMU	***Efficiency Score***	u_{11}	u_{21}	v_{11}	v_{21}	u_{01}
H1	1	0	1.82E-02	1.16E-06	1.06E-05	-4.17E-02
H2	1	3.00E-03	0.029017	2.47E-06	3.33E-05	0
H3	0.742	0	6.67E-02	3.04E-06	0	0.3741178
H4	1	2.91E-03	2.82E-02	2.40E-06	3.24E-05	0
H5	0.726	1.89E-02	0	5.95E-06	0	0.3850675
H6	0.781	1.43E-02	0	4.50E-06	0	0.2915511
H7	1	3.33E-02	0	9.25E-06	9.96E-05	0.5128701
H8	0.536	0	0.0625	2.85E-06	0	0.3507355
H9	1	9.43E-03	0	3.68E-06	0	0

DMU	s_{11}^{+}	s_{21}^{+}	s_{11}^{-}	s_{21}^{+}	***Reference Set***	***Peer Weights***	***RTS***
H1	0	0	0	0	H1	1	DRS
H2	0	0	0	0	H2	1	CRS
H3	30.51	0	0	1.29	H7, H9	0.62, 0.37	IRS
H4	0	0	0	0	H4	1	CRS
H5	0	1.94	0	412.43	H7, H9	0.88, 0.11	IRS
H6	0	3.48	0	63.01	H7, H9	0.68, 0.32	IRS
H7	0	0	0	0	H7	1	IRS
H8	12.71	0	0	899.21	H7, H9	0.86, 0.14	IRS
H9	0	0	0	0	H9	1	CRS

3.4.3 Assurance Region Model (AR Model)

The common difficulty found in DEA applications is that multipliers often become zero, implying that each variable is not fully utilized in the DEA evaluation. This is equivalent to neglecting some inputs or outputs that is often not acceptable from practical point of view. To deal with this type of problem, weight restriction in different forms is often added to DEA models [29, 46, 110, 144], for example, Cone Ratio method [29] and Assurance Region technique [144]. These approaches can reduce the number of zero multipliers and as a result, number of efficient DMUs can also be reduced.

For AR Model, we add the weight restriction constraint into CCR multiplier model (3.3) in relative form as:

$$l_{1,i} \le \frac{v_i}{v_1} \le u_{1,i} \quad and \quad L_{1,r} \le \frac{u_r}{u_1} \le U_{1,r} \tag{3.13}$$

where the choice of numerator (v_1 and u_1) are arbitrary. Thus, the CCR/AR model is given by

$$\left.\begin{array}{ll}
Max\, E_k^i = \sum_{r=1}^{s} y_{rk} y_{rk} & \\
subject\, to & \\
\sum_{i=1}^{m} u_{ik} x_{ik} = 1 & \\
\sum_{r=1}^{s} y_{rk} y_{rj} - \sum_{i=1}^{m} u_{ik} x_{ij} \le 0 & \forall j = 1,2,3,....,n \\
l_{1,i} u_{1,k} - u_{ik} \le 0 & \forall i = 1,2,3,....,m \\
u_{ik} - u_{1,i} v_{1k} \le 0 & \forall i = 1,2,3,....,m \\
L_{1,r} v_{1k} - v_{rk} \le 0 & \forall r = 1,2,3,....,s \\
v_{rk} - U_{1,r} v_{1k} \le 0 & \forall r = 1,2,3,....,s \\
v_{rk} \ \ \ge 0 & \forall r = 1,2,3,....,s \\
u_{ik} \ \ \ge 0 & \forall i = 1,2,3,....,m
\end{array}\right\} \tag{3.14}$$

The envelopment form of the CCR/AR model in matrix notations is given by

$$\left.\begin{array}{l} Min\ Z_k^i = \theta_k \\ subject\ to \\ \theta_k x_k - X\lambda_k + P\pi \geq 0 \\ Y\lambda_k + Q\tau \qquad\quad \geq 0 \\ \lambda_k \geq 0 \\ \pi \geq 0; \tau \geq 0 \\ \theta_k\ is\ unrestricted\ in\ sign \end{array}\right\} \tag{3.15}$$

where $x_k = [x_{1k}, x_{2k}, ..., x_{mk}]^T$ = inputs of the k^{th} DMU; $y_k = [\mathrm{y}_{1k}, \mathrm{y}_{2k}, ..., \mathrm{y}_{sk}]^T$ = outputs of the k^{th} DMU; $\lambda_k = [\lambda_{1k}, \lambda_{2k}, ..., \lambda_{nk}]^T$ is the intensity variable. $X = (x_{ij})_{m\times n}$ is input matrix and $Y = (y_{rj})_{s\times n}$ is output matrix, π^* and τ^* are the column vectors of (2m-2) and (2s-2) dimensions, respectively corresponding to the weight restricted constraint.

$$P = \begin{bmatrix} l_{12} & -u_{12} & l_{13} & -u_{13} & .... \\ -1 & 1 & 0 & 0 & .... \\ 0 & 0 & -1 & 1 & .... \\ .... \end{bmatrix}$$, the m*(2m-2) matrix corresponding to the lower and upper bounds on input weights and

$$Q = \begin{bmatrix} L_{12} & -U_{12} & L_{13} & -U_{13} & .... \\ -1 & 1 & 0 & 0 & .... \\ 0 & 0 & -1 & 1 & .... \\ .... \end{bmatrix}$$, the m*(2s-2) matrix corresponding to the lower and upper bounds on input weights.

The DMU associated with (x_k, y_k) is AR efficient if $\theta_k^* = 1$ and all slacks are zero. The BCC/AR model differs from CCR/AR model only in the adjunction of the convexity constraint (i.e., $\sum_{j=1}^{n} \lambda_{jk} = 1$).

In our example, only three hospitals (H2, H4 and S9) are found to be efficient. Table 3.8 presents the result of CCR/AR input-oriented model.

Table 3.8: Results of Input-Oriented CCR/AR Model

DMU	***Efficiency Score***	u_{11}	u_{21}	v_{11}	v_{21}	***Reference Set***	***Peer Weights***
H1	0.926	0	0.018182	0.000001	0.000010	H4, H9	0.56, 2.21
H2	1	0.002996	0.029018	0.000002	0.000033	H2	1.00
H3	0.568	0.000000	0.066667	0.000004	0.000036	H4, H9	0.08, 0.39
H4	1	0.002910	0.028178	0.000002	0.000032	H4	1.00
H5	0.482	0.018868	0	0.000006	0.000065	H2, H9	0.05, 0.19
H6	0.652	0.014286	0	0.000005	0.000049	H2, H9	0.05, 0.38
H7	0.568	0.033333	0	0.000011	0.000115	H2, H9	0.07, 0.08
H8	0.282	0	0.062500	0.000004	0.000034	H4, H9	0.03, 0.21
H9	1	0.009434	0	0.000004	0	H9	1.00

By comparing the result of CCR/AR model to CCR model, we find that the most of the multiplier are at non zero level. It implies that all the variables are fully utilized in the performance assessment. The comparison also shows that the efficiency scores obtained by CCR/AR model are lesser and equal to the efficiency scores obtained by CCR model.

3.4.3.1 Input/output Targets for inefficient DMUs

According to the CCR/AR model, the input-output level in equation (3.16) can be used as the basis for setting the targets of any inefficient DMU$_k$, so that it may improves its performance.

$$\left.\begin{aligned} \overline{x_k} &= \theta_k^* x_k - S_k^- + P\pi^* = X\lambda \\ \overline{y_k} &= y_k + S_k^+ - Q\tau^* \quad\; = Y\lambda \end{aligned}\right\} \qquad (3.16)$$

where $\overline{x_k} = [\overline{x_{1k}}, \overline{x_{2k}}, ..., \overline{x_{mk}}]^T$ and $\overline{y_k} = [\overline{y_{1k}}, \overline{y_{2k}}, ..., \overline{y_{sk}}]^T$ are the target inputs and outputs respectively for k^{th} hospital; θ_k^* = optimal efficiency score of the k^{th} hospital; $S_k^- = [S_{1k}^-, S_{2k}^-, ...S_{mk}^-]^T$ = input slacks of the k^{th} hospital; and $S_k^+ = [S_{1k}^+, S_{2k}^+, ...S_{sk}^+]^T$ = output slack of the k^{th} hospital. The input-output levels $(\overline{x_k}, \overline{y_k})$ defined in (3.16) are the coordinate of the efficient frontier used as a benchmark for evaluating k^{th} hospital.

3.4.4 Returns to Scale (RTS)

RTS reflects the extent to which a proportional increase in all inputs increases outputs. Increasing returns to scale (IRS) occurs when a proportional increase in all inputs results in a more than proportional increase in outputs, Constant returns to scale (CRS) occurs when a proportional increase in the value of all inputs results in the same proportional increase in outputs of the DMUs, whereas decreasing returns to scale (DRS) occurs when proportional increase in all inputs results in a lesser than proportional increase in outputs.

If, in the CCR envelopment model, we impose $\sum_{j=1}^{n}\lambda_{jk} \leq 1$ instead of $\sum_{j=1}^{n}\lambda_{jk} = 1$, then the model is converted into Non-Increasing Returns to Scale (NIRS)**[1]**. Similarly if we impose $\sum_{j=1}^{n}\lambda_{jk} \geq 1$ instead of $\sum_{j=1}^{n}\lambda_{jk} = 1$, then the model is known as Non-Decreasing Returns to Scale (NDRS) **[2]**.

The constant returns to scale model is used when the uniform positive scaling of inputs and outputs of any of the DMUs does not affect its efficiency status. The variable returns to scale model is used when the uniform scaling above and below maximum observed values for any inputs and outputs cannot be assumed to be possible. Increasing returns to scale is used when the assumption of uniform downward scaling is always possible. Decreasing returns to scale is used when uniform upward scaling is possible.

If the DMU is pure technical efficient in terms of BCC input model, then the RTS are defined by the resulting peer weights of that DMU in terms of CCR input model as follows:

1. If $\sum_{j=1}^{n}\lambda_{j} = 1$in any alternate optima, then CRS prevails.
2. If $\sum_{j=1}^{n}\lambda_{j} > 1$ for all alternative optima, then DRS prevail.
3. If $\sum_{j=1}^{n}\lambda_{j} < 1$ for all alternate optima, then IRS prevails.

If DMU is found to be BCC inefficient, then we can identify RTS characteristics of its projected DMU ($\overline{x_0}, \overline{y_0}$) from those of its reference set as follows:

1. The projected DMU ($\overline{x_0}, \overline{y_0}$) displays CRS if the reference set of (x_0, y_0) consists of DMUs all belonging to CRS.
2. The projected DMU ($\overline{x_0}, \overline{y_0}$) displays IRS if the reference set of (x_0, y_0) consists of either IRS DMUs or a mixture of IRS and CRS DMUs.

3. The projected DMU ($\overline{x_0}, \overline{y_0}$) displays DRS if the reference set of (x_0, y_0) consists of either DRS DMUs or a mixture of DRS and CRS DMUs.

In our example, three hospitals (H2, H4, and H9) are operating under CRS, as μ_{01} is equal to zero. Hospital H1 operating at DRS, since corresponding μ_{01} is negative in multiplier model and $\sum_{j=1}^{n} \lambda_j > 1$in corresponding CCR envelopment model. Remaining five hospitals (H3, H5, H6, H7 and H8) are operating at IRS, since corresponding μ_{01} is positive in multiplier model and $\sum_{j=1}^{n} \lambda_j < 1$ in corresponding CCR envelopment model.

3.4.5 Non-Discretionary DEA Model

The DEA models described above implicitly assume that all inputs and outputs are discretionary, i.e., they can be controlled by the management of each DMU and varied at its discretion. Thus, failure of a DMU to produce maximal output levels with minimal input consumption results in a worsened efficiency score. However, there may be exogenously fixed (or non-discretionary) inputs or outputs that are beyond the control of a DMU's management. Banker and Morey [20] extended the basic DEA model to handle this type of problem.

Supposed that the input and output variables may each be partitioned into subsets of discretionary (D) and nondiscretionary (N) variable. Thus,

$I = \{1, 2, ...m\} = I_D \cup I_N$ with $I_D \cap I_N = \Phi$

And $O = \{1, 2, ...\text{s}\} = O_D \cup O_N$ with $\text{O}_D \cap O_N = \Phi$

where I_D, O_D and I_N, O_N refer to discretionary (D) and non-discretionary (N) input, I, and output, O, variables, respectively. The modified version of **input-oriented** CCR model is given by:

$$\left.\begin{aligned}
&Min\ Z_k^i = \theta_k - \varepsilon\left(\sum_{i\in I_D} S_{ik}^- + \sum_{r=1}^{s} S_{rk}^+\right)\\
&subject\ to\\
&\sum_{j=1}^{n}\lambda_{jk}y_{rj} - S_{rk}^+ = y_{rk} \qquad \forall r = 1,2,3,....,s\\
&\sum_{j=1}^{n}\lambda_{jk}x_{ij} + S_{ik}^- = \theta_k x_{ik} \qquad \forall i \in I_D\\
&\sum_{j=1}^{n}\lambda_{jk}x_{ij} + S_{ik}^- = \sum_{j=1}^{n}\lambda_{jk}x_{ik} \qquad \forall i \in I_N\\
&\lambda_{jk} \geq 0 \qquad \forall j = 1,2,3,...,n\\
&\theta_k\ is\ unrestricted\ in\ sign\\
&S_{rk}^+, S_{ik}^- \geq 0; r = 1,2,3,...,s, \quad i = 1,2,3,...,m
\end{aligned}\right\} \tag{3.17}$$

It is to be noted that θ_k to be minimized appears only in the constraints for which $i \in I_D$, whereas the constraints for which $i \in I_N$ operate only indirectly because the input levels are not subject to managerial control. It is also to be noted that the slack variables associated with I_N, the nondiscretionary inputs, are not included in the objective function and hence the non-zero slacks for these inputs do not enter directly into the efficiency scores to which the objective is oriented. The modified version of **input-oriented** BCC model is given by:

$$\left.\begin{aligned}
&Min\ Z_{kV}^i = \theta_k - \varepsilon\left(\sum_{i\in I_D} S_{ik}^- + \sum_{r=1}^{s} S_{rk}^+\right)\\
&subject\ to\\
&\sum_{j=1}^{n}\lambda_{jk}y_{rj} - S_{rk}^+ = y_{rk} \qquad \forall r = 1,2,3,....,s\\
&\sum_{j=1}^{n}\lambda_{jk}x_{ij} + S_{ik}^- = \theta_k x_{ik} \qquad \forall i \in I_D\\
&\sum_{j=1}^{n}\lambda_{jk}x_{ij} + S_{ik}^- = x_{ik} \qquad \forall i \in I_N\\
&\sum_{j=1}^{n}\lambda_{jk} = 1\\
&\lambda_{jk} \geq 0 \qquad \forall j = 1,2,3,...,n\\
&S_{rk}^+, S_{ik}^- \geq 0; r = 1,2,3,...,s, \quad i = 1,2,3,...,m
\end{aligned}\right\} \tag{3.18}$$

$$\theta_k\ is\ unrestricted\ in\ sign$$

3.4.6 DEA with Categorical DMUs

The previous development of DEA models assumed that all inputs and outputs are in the same category. However, this needs not be the case as when some DMUs have extra facilities and some do not. Banker and Morey [20] proposed a DEA model to handle this type of problem.

Suppose an input variable can assume one level out of L levels (1, 2… L). These L values effectively partition the set of DMUs into categories. Specifically, the set of DMUs $K=\{1,2,...,n\}=K_1\cup K_2\cup ....\cup K_L, where\ K_f=\{j: j\in K\ and\ input\ value\ is\ f\}$ and $K_i\cap K_j=\phi$ for $i\neq j$. We wish to evaluate a DMU with respect to the envelopment surface determined for the units contained in it and all preceding categories. The following model specification allows $DMU_k\in K_f$.

$$\left.\begin{array}{ll}
Min\ Z_k=\theta_k & \\
subject\ to & \\
\sum\limits_{j\in\cup_{f=1}^{K}K_f}^{n}\lambda_{jk}y_{rj}-s_{rk}^{+}=y_{rk} & \forall r=1,2,3,...,s\\
\sum\limits_{j\in\cup_{f=1}^{K}K_f}^{n}\lambda_{jk}x_{ij}+s_{ik}^{-}=\theta_k x_{ik} & \forall i=1,2,3,...,m\\
\lambda_{jk}\geq 0 & \forall j=1,2,3,...,n\\
\theta_k\ is\ unrestricted\ in\ sign & \\
s_{rk}^{+},s_{ik}^{-}\geq 0; r=1,2,3,....,s,\quad i=1,2,3,...,m &
\end{array}\right\} \tag{3.19}$$

Thus, the above specification allows one to evaluate all DMUs $1\in K_1$ with respect to the units in K_1, all DMUs $1\in K_2$ with respect to the units in K_2, etc. Although our presentation is for the input-oriented CCR model, it should be obvious that categorical DMUs can also be incorporated in this manner for any DEA model.

3.4.7 DEA Models with Slacks

A DMU with unit efficiency score and with no slacks in any optimal solution is called efficient. Otherwise, the DMU has a disadvantage against the DMUs in its reference set. Thus, in discussing total efficiency, it is important to observe both the efficiency score and the slacks. Slacks are also considered in the CCR and BCC models with the coefficient of ε(non-Archimedean) constant in their objective function. However, the numeric value for ε in computations should be chosen to be much smaller than input and output values so that they may not affect optimization. After it, many researchers [28, 10, 132, 140, 6] gave the different models to accumulate the impact of slacks on the efficiency scores. Some important models are given in this section.

3.4.7.1 Additive Model

In the objective function of the Additive model, given by Charnes et al. [28], only slacks are present.

$$\left.\begin{array}{ll}
Max\,E_k = \sum_{r=1}^{s} s_{rk}^{+} + \varepsilon\sum_{i=1}^{m} s_{ik}^{-} & \\
subject\,to & \\
\sum_{j=1}^{n}\lambda_{jk}y_{rj} - s_{rk}^{+} = y_{rk} & \forall r = 1,2,3,...,s \\
\sum_{j=1}^{n}\lambda_{jk}x_{ij} + s_{ik}^{-} = x_{ik} & \forall i = 1,2,3,...,m \\
\sum_{j=1}^{n}\lambda_{jk} = 1 & \forall j = 1,2,3,...,n \\
\lambda_{jk} \geq 0;\ j = 1,2,...,n & \\
s_{rk}^{+}, s_{ik}^{-} \geq 0;\ \ r = 1,2,3,...,s, i = 1,2,3,...,m &
\end{array}\right\} \qquad (3.20)$$

This model is non-radial**[3]** and non-oriented**[4]**. In this model, the efficiency score θ_k is not measured explicitly. A DMU$_k$ is efficient if and only if all slacks are zero, i.e., $s_{rk}^{+}\ and\ s_{ik}^{-} = 0$ for every *r* and *i*.

3.4.7.2 Two-Stage Optimization Model

Ali and Seiford [10] suggest a two-stage optimization procedure to avoid the use of ε in computations and also consider the slacks in the efficiency scores. The optimization models for stage 1 and 2 are given below.

Stage 1:

$$\left.\begin{aligned}
&Min\;\; Z_k^i = \theta_k \\
&subject\,to \\
&\sum_{j=1}^{n} \lambda_{jk} y_{rj} - s_{rk}^{+} = y_{rk} \qquad \forall r = 1,2,3,...,s \\
&\sum_{j=1}^{n} \lambda_{jk} x_{ij} + s_{ik}^{-} = \theta_k x_{ik} \qquad \forall i = 1,2,3,...,m \\
&\lambda_{jk} \geq 0 \qquad \forall j = 1,2,3,...,n \\
&s_{rk}^{+}, s_{ik}^{-} \geq 0; r = 1,2,3,...,s, i = 1,2,3,...,m
\end{aligned}\right\} \tag{3.21}$$

$$\theta_k\; is\, unrestricted\, in\, sign$$

Now this Stage 1 model is the same as the CCR DEA model with no ε constraints.

Stage 2:

$$\left.\begin{aligned}
&Max\, E_k = \sum_{r=1}^{s} s_{rk}^{+} + \varepsilon \sum_{i=1}^{m} s_{ik}^{-} \\
&subject\,to \\
&\sum_{j=1}^{n} \lambda_{jk} y_{rj} - s_{rk}^{+} = y_{rk} \qquad \forall r = 1,2,3,...,s \\
&\sum_{j=1}^{n} \lambda_{jk} x_{ij} + s_{ik}^{-} = \theta_k^{*} x_{ik} \qquad \forall i = 1,2,3,...,m \\
&\lambda_{jk} \geq 0 \qquad \forall j = 1,2,3,...,n \\
&s_{rk}^{+}, s_{ik}^{-} \geq 0; r = 1,2,3,...,s, i = 1,2,3,...,m
\end{aligned}\right\} \tag{3.22}$$

Where θ_k^{*} is the optimal value of θ_k in Stage 1. There are two major problem associated with the two stage LPP. The first problem is that the sum of slacks is maximized rather than minimized. Hence it identifies the furthest**5** efficient point, not the nearest point. The second problem is that it is not a unit invariant**6**.

3.4.7.3 Slack Based Model (SBM)

Tone [140] proposed a new measure of efficiency which deals directly with the slacks. This model is known as slack based measure (SBM) model. In order to estimate the efficiency of a DMU (x_0, y_0), SBM model is given as

$$\left.\begin{array}{ll}
Min\,\rho_k = \dfrac{1-\dfrac{1}{m}\sum\limits_{i=1}^{m}\dfrac{s_{ik}^{-}}{x_{ik}}}{1+\dfrac{1}{s}\sum\limits_{r=1}^{s}\dfrac{s_{rk}^{+}}{y_{rk}}} & \\
subject\,to & \\
\sum\limits_{j=1}^{n}\lambda_{jk}y_{rj} - s_{rk}^{+} = y_{rk} & \forall r = 1,2,3,...,s \\
\sum\limits_{j=1}^{n}\lambda_{jk}x_{ij} + s_{ik}^{-} = x_{ik} & \forall i = 1,2,3,...,m \\
\lambda_{jk} \geq 0 & \forall j = 1,2,3,...,n \\
s_{rk}^{+}, s_{ik}^{-} \geq 0; \quad r = 1,2,3,...,s, i = 1,2,3,...,m &
\end{array}\right\} \tag{3.23}$$

In the objective function of model (3.23), the numerator evaluates the mean reduction rate of inputs or input inefficiency. Similarly, the reciprocal of denominator evaluates the mean expansion rate of outputs or output inefficiency. Thus, ρ_k can be interpreted as the product of the input and output inefficiencies. This model is known as SBM-CRS model.

The interpretation of the results of model (3.23) can be summarized as follows:

- The DMU under reference is said to be Pareto efficient if all slacks are zero, i.e., s_{rk}^{+*} and s_{ik}^{-*} = 0 for every *r* and *i* which is equivalent to $\rho_k^* = 1$.
- The non-zero slacks and (or) $\rho_k^* \leq 1$ identify the sources and amount of any inefficiency that may exist in the DMU$_k$.

The detailed description of the model is described in Chapter 5.

3.4.7.4 Slack Adjusted (SA) Model

Sueyoshi et al. [132] suggested a new radial model in which the influence of slacks can directly be incorporated into the measurement of a DEA efficiency score. This model is known as stack adjusted (SA) model. The model is as follows:

$$\left.\begin{aligned}
&Min\ Z_k^i = \theta_k - \frac{1}{m+s}\left[\sum_{i=1}^{m}\frac{S_{ik}^-}{MR_{ik}^x} + \sum_{i=1}^{m}\frac{S_{rk}^+}{MR_{rk}^y}\right]\\
&subject\ to\\
&\sum_{j=1}^{n}\lambda_{jk}y_{rj} - S_{rk}^+ = y_{rk} \qquad \forall r = 1,2,3,...,s\\
&\sum_{j=1}^{n}\lambda_{jk}x_{ij} + S_{ik}^- = \theta_k x_{ik} \qquad \forall i = 1,2,3,...,m\\
&\lambda_{jk} \geq 0 \qquad \forall j = 1,2,3,...,n\\
&S_{rk}^+, S_{ik}^- \geq 0; r = 1,2,3,...,s, \quad i = 1,2,3,...,m
\end{aligned}\right\} \qquad (3.27)$$

$$\theta_k \ is\ unrestricted\ in\ sign$$

The influence of slack is adjusted by the following data range.

$$MR_{ik}^x = \max_j x_{ij}\,(i = 1,2,3,...,m)\ and\ MR_{rk}^y = \max_j y_{rj}\,(\mathrm{r} = 1,2,3,...,\mathrm{s})$$

3.4.7.5 New Slack Model (NSM)

Since the CCR model neglects the slacks in the evaluation of efficiencies. To overcome this shortcoming, Agarwal et al. [6] suggested a new slack model (NSM) which does not neglect the slacks in the evaluation of efficiencies. The NSM is given as follows:

$$\left.\begin{aligned}
&Max\,\overline{\theta}_k = \ \theta_k + \frac{1}{m+s}\left(\sum_{r=1}^{s}\frac{s_{rk}^+}{y_{rk}} + \sum_{i=1}^{m}\frac{s_{ik}^-}{x_{ik}}\right)\\
&subject\,to\\
&\sum_{j=1}^{n}\lambda_{jk}y_{rj} - s_{rk}^+ = \theta_k y_{rk} \qquad \forall\, r = 1,2,3,...,s,\\
&\sum_{j=1}^{n}\lambda_{jk}x_{ij} + s_{ik}^- = x_{ik} \qquad \forall\, i = 1,2,3,...,m,\\
&\lambda_{jk}, s_{ik}^-, s_{rk}^+ \geq 0,\ \forall\, i = 1,2,3,...,m;\ j = 1,2,3,...,n;\ r = 1,2,3,...,s,
\end{aligned}\right\} \qquad (3.28)$$

The interpretation of the results of Model 2 can be given as follows:

1. *The k^{th} hospital is total potential efficient or NSM efficient if and only if* $\overline{\theta}_k^* = 1$.

The above condition is equivalent to simultaneously $\theta_k^* = 1$ and all slacks are zero, i.e., radially efficient and no input access and no output shortfall exist in any optimal solution.

2. *The set of indices corresponding to positive* λ_{jk}*'s is called the reference set of the* k^{th} *inefficient hospital. The reference set denoted by* R_k *is defined as:*

   $R_k = \{DMU_j : \lambda_{jk} > 0, j = 1, 2, 3, ..., n\}$

3.5 Summing Up

In this chapter we have discussed the advancements in DEA techniques. There are two basic models (CCR and BCC models) with different types, such as, input or output orientation, multiplier or envelopment form constant or variable returns to scale. So, some considerations may be useful to select the appropriate DEA model. Whenever the inputs are controlled by the management, input-output model may be more appropriate. Output-oriented model is used when inputs are fixed by the management and performance of DMUs is assessed on the basis of outputs. Multiplier version is used when inputs and outputs are emphasized in an application, while envelopment version is used when the relations among the DMUs are emphasized. When the data are not normalized, i.e. data include large as well as small DMUs, variable returns to scale model may be used. Sometimes zero multiplier problem become in basic DEA models.

To solve zero multiplier problem in basic DEA models, DEA models with weight restriction may be used. DEA-AR model is one of the most commonly used models for the weight restriction. In DEA-AR model, weights can be restricted with the prior knowledge of that restriction and mathematically, these restrictions should satisfy the condition of positive multipliers. Non-discretionary DEA model can handle the real situation of the exogenously fixed variables. In this model, uncontrolled variables are not included in the objective function and the corresponding constraints do not have the radial change. Categorical DEA model deals with the variables that are in advantageous stage for some DMUs and are in worse for other DMUs. In this model, each DMU should be compared only with DMUs in its own and more disadvantaged categories.

The basic models do not emphasize the effect of slacks on the efficiency. To identify the role of slacks in the assessment of the efficiency, many DEA models are suggested, such as, additive model, two-stage model, SBM model, slack adjusted model and new slack model. All these models have their own advantages and limitations. The researchers have to choose the DEA models as per the requirement of their study as well as the situations of the selected DMUs. In this study, we use new slack model with output-orientation that measures the efficiency scores of DMUs with the actual impact of slacks on efficiency score with output-orientation.

NOTES

1. In this constraint, we cannot increase the scale of DMU but it is possible to reduce the scale.
2. In this constraint, scaling down of DMU is interdicted, while scaling up is permitted.
3. The radial models suggest a proportional change in all inputs an (or) outputs to improve the efficiency; while in non-radical model, proportional change in all inputs and outputs is not considered.
4. In an input orientation, one improves efficiency through the proportional reduction of inputs and an output orientation requires the proportional augmentation of outputs, whereas in non-orientation one can improve its efficiency through reduction in both inputs and augmentation of inputs.
5. Furthest point can be measured by the maximizing the objective function, which evaluates the maximum distance of the point from the frontier.
6. Unit invariance means the performance assessment is not based on the units of the variables.

Chapter 4

Technical Efficiency and its Determinants: A Cross Sectional Analysis

4.1 Introduction

This Chapter examines the technical and scale efficiencies of public sector hospitals of Uttarakhand through a two-stage analytical process using cross sectional data of 36 hospitals for the calendar year 2011. In the first stage, DEA is applied to measure technical efficiency scores, RTS, benchmarks for relatively inefficient hospitals, slacks in inputs and outputs. It also determines the region-wise, category-wise, and area-wise variations in the efficiency scores. Further, sensitivity analysis is done to test the robustness of the efficiency results. In the second stage, Tobit regression analysis is used to identify the impact of environmental factors on the efficiency scores.

4.2 Methodology

The application procedure of DEA analysis includes three stages: first stage is to select the hospitals. Then the inputs and output variables are to be selected for the analysis of efficiency of hospitals. Finally, DEA model is selected to analyze the data. This section discusses these three stages in brief.

4.2.1 Selection of Hospitals

We have selected government hospitals, having bed strength at least 24. As per the availability of data, a total of 36, district/base/combined hospitals are selected. Data have been collected from the Directorate of Medical Health and Family Welfare, Government of Uttarakhand. Since all the hospitals (district male/female/combined or base) are similar in structure, funded by the government and are of secondary level, they could be taken in the DEA study.

4.2.2 Selection of input/output Variables

Firstly, we discuss some input-output variables that were used in earlier studies. Some studies related to efficiency estimation of public sector healthcare services are discussed below.

Table 4.1 Input-output Variables used in the Previous Studies

S. N.	Author's	Year	Inputs	Outputs
1	Osei et al. [99]	2005	number of doctors, beds, other technical staff and subordinate staff	number of maternal and child care (MCH), number of child deliveries and number of patients discharged
2	Zere et al. [148]	2006	total recurrent expenditure, beds and nursing staff	total outpatient visits and inpatient days
3	Agarwal et al. [3]	2007	number of beds, number of doctors and number of paramedical staff	number of outdoor patients, number of indoor patients number of major surgery and number of minor surgery
4	Gannon B. [53]	2008	number of beds and full-time equivalent people employed	number of discharges and deaths, outpatient attendance, and day cases
5	Dash U. [39]	2009	number of beds, number of nursing staff, and number of physicians	number of inpatients, number of outpatients, and number of surgeries undertaken, emergency cases handled, medico legal cases, and deliveries
6	Dash et al. [40]	2010	number of beds, number of nursing staff, assistant surgeons employed and number of civil surgeon employed	number of inpatients, number of outpatients, number of surgeries undertaken, emergency cases undertaken and deliveries
7	Pham, L.T. [101]	2011	total number of beds, total number of hospital's personnel including physicians and non-physicians	outpatient visits, inpatient days and surgical operations performed
8	Nedelea and Fannin [97]	2012	total staffed and licensed hospital beds and Full time equivalent employee	total hospital admissions, post admission days, total outpatient visits, emergency room visits, outpatient surgeries and total births
9	Jat and Sebastian [62]	2013	number of specialists, number of nurses, number of allied health and number of beds	number of outpatient, number of patients admissions to hospital, number of laboratory tests, and number of beneficiaries of radiological imaging

In reference to the above mentioned studies and as per the availability of the data, three variables, namely, number of beds, number of doctors and the number of paramedical staff (PMS) are taken. These three variables are the main indicators to describing the status of a hospital. Also, as is indicated by Table 4.1, the major services provided by a hospital are treating of out-door and in-door patients, so, these variables are considered as the output variables for the study. Here we also added two cases-mix outputs, i.e., number of major and minor surgeries. Although the sample hospitals are homogeneous in the sense that they are in the same line of business i.e., providing almost free healthcare services to the patients; there are some minor differences in these hospitals. For example, some district male hospitals do not have maternity department and some district female hospitals do not have dental, orthopedic or eye departments. Therefore, to maintain the homogeneity of output measure (variables), only number of major surgeries and minor surgery are taken as the case-mix outputs, because the surgical department is common to all the hospitals. Thus, for estimating the technical efficiency of Uttarakhand's public hospitals, three variables, viz., number of beds, number of doctors and number of paramedical staff (PMS) are taken as input variables, and four variables namely, number of outdoor patients (OPD), number of indoor patients (IPD), number of major surgery, and number of minor surgeries are taken as output variables. All the input and output variables are measured in numbers. The thumb rule [141] "The number of DMUs is expected to be larger than twice the sum of inputs and outputs" is applied for the selection of number of hospitals, inputs and outputs. All the input and output variables are described in Chapter 1.

Table 4.2 presents mean and standard deviation for input and output variables for the year 2011. An average hospital in the state had 18.97 doctors, 39.83 paramedical staff and a capacity of 93.66 beds and served an average population of 126792.80. The variability in utilization of resources and in the production of outputs indicates that these hospitals have expanded their activities to different extents. For example, hospitals serving similar-sized population were found to differ even nine to ten times with respect to the output they produce. The average number of outpatients treated was 126792.80, ranging from 5491 to 715221.

Descriptive statistics of input and output variables are given in Table 4.2. It is clear from the maximum and minimum values of input and output variables and standard deviation that there is a perceptible variation in the selected inputs and outputs across the hospitals. In some cases, the input used by some hospitals is seventeen times more than that used by the other hospital. Variations in the level of outputs are also observed very high across the hospitals.

Table-4.2: Descriptive Statistics of Input and Output Variables

Inputs	Max	Min	Average	SD
Number of Beds	402	24	93.667	66.967
Number of Doctors	55	6	18.972	9.284
Number of PMS	140	11	39.833	25.403
Outputs	**Max**	**Min**	**Average**	**SD**
Number of Out-door Patients	715221	5491	126793	122709
Number of In-door Patients	22111	485	7221.42	5634.79
Number of Major Surgery	4128	76	562.944	738.654
Number of Minor Surgery	2834	231	692.361	522.973

Correlation analysis has also been worked out to know the relation between input and output variables (Table 4.3). It is observed that the output variables have statistically significant positive correlations with the input variables.

Table-4.3: Correlation Matrix between Inputs and Outputs

	No. of Beds	No. of Doctors	No. of PMS	No. of Out-door Patients	No. of In-door Patients	No. of Major Surgery	No. of Minor Surgery
No. of Beds	1						
No. of Doctors	0.876 0.000*	1					
No. of PMS	0.900 0.000*	0.906 0.000*	1				
No. of Out-door Patients	0.873 0.000*	0.776 0.000*	0.796 0.000*	1			
No. of In-door Patients	0.595 0.000*	0.559 0.000*	0.583 0.000*	0.612 0.000*	1		
No. of Major Surgery	0.473 0.004*	0.492 0.002*	0.619 0.000*	0.391 0.018**	0.477 0.003*	1	
No. of Minor Surgery	0.508 0.002*	0.527 0.001*	0.629 0.000*	0.443 0.007*	0.576 0.000*	0.959 0.000*	1

* Significant at 1% level of significance, ** Significant at 5% level of significance

4.2.3 Selection of the Models

The purpose of this chapter is twofold: first to assess the OTE, PTE and SE of 36 public sector hospitals of Uttarakhand using DEA and secondly to give special attention on empirical sensitivity analysis of DEA efficiency scores.

Efficiency scores can be estimated using DEA models with input-orientation or output-orientation. In case of CCR model, both orientations provide the same efficiency scores. However, when we use the BCC model to estimate the PTE, the input and output orientation may provide the different efficiency scores. Now, the next choice is an orientation of the model. In the input-orientation, the inputs are minimized while maintaining at least current levels of output; and in the output-orientation, the outputs are maximized while consuming at most the current level of inputs (Cooper 2004). The choice between the output and input orientation depends upon the type of application (Coelli et al., 1998) [36]. When our focus is to increase the output as much as possible without decreasing the resources (inputs) used, output orientation would be good. In contrast, when the focus is to reduce the inputs as much as possible without dropping the output levels, input-oriented model would be applied.

Generally, hospital management and healthcare providers anticipate demand for healthcare services and invest in inputs necessary to support the expected level of demand. It is not admirable to reduce input levels in public hospitals. In most of the public hospitals, there is an inadequacy of beds, medical and paramedical staff. More recruitment is required to meet the requirement of healthcare services. Hence, in such a situation, it is not desirable to assume that labor inputs (doctors and paramedical staff) have to be reduced. So, it is more sensible, practical view to consider output-based efficiency measure. Therefore, firstly, we use an envelopment form of CCR output-oriented model, given in Model 3.4, i.e., the relative efficiency of the hospitals is measured on their potential to increase output with their given level of inputs relative to the best practice hospital. To decompose OTE into PTE and SE, the envelopment form of output-oriented BCC model given in Table 3.5, is also applied. Secondly, we are to examine that how sensitive the DEA efficiency measurements are?

4.3 Empirical Findings and Discussions

This section reports the results obtained using the DEA models and Tobit regression. First, the results of efficiencies (OTE, PTE and SE) are explained in 4.3.1, 4.3.2 and 4.3.3 respectively. Slack and target analysis for inefficient hospitals have been done in section 4.3.4. Region-wise, category-wise and area-wise findings of the efficiency analyses are discussed in 4.3.5, 4.3.6 and 4.3.7, respectively. Finally, the results of sensitivity analysis and Tobit regression are discussed in Section 4.4.

4.3.1 Overall Technical Efficiency (OTE)

DEA evaluates the set of hospitals that construct a production frontier. The OTE scores indicate that hospitals having efficiency score equal to 1 are on the efficiency frontier under CRS assumption and those having efficiency score less than 1 are inefficient relative to the hospitals on the efficiency frontier. Table 4.4 evinces that out of 36 hospitals, only 10 (27.78%) are overall technical efficient and form the efficiency frontier. The remaining 26 (72.22%) hospitals are inefficient. The efficient hospitals H3, H5, H7, H12, H14, H15, H17, H18, H26 and H27 form the "reference sets" for inefficient hospitals. Base Hospital Almora (H25) is found to be most inefficient hospital as its efficiency score is only 25.30%. Among the 26 inefficient hospitals, 17 hospitals have the efficiency score below the average efficiency score and only 9 hospitals scored above the average efficiency score. The average OTE score 0.772 reveals that on average hospitals have to increase their output by 22.80% by maintaining the existing level of inputs.

We have also used the peer counts to discriminate among efficient hospitals. The higher peer count represents the extent of robustness of that hospital compared with other efficient hospitals. In other words, a hospital with higher peer count is likely to be a hospital, which is efficient with respect to a large number of factors and is probably a good example of a "global leader" or a hospital with high robustness.

Efficient hospitals that appear seldom in the reference set are likely to possess a very uncommon input/output mix. So, when the peer count is low, we can safely conclude that the hospital is somewhat of an odd unit and cannot be treated as a good example to be followed. Based on peer counts, the efficient hospitals are classified as follows

4.3.1.1 High Robustness ($13 \leq$ number of peer counts ≤ 17):

Female Hospital Dehradun (H7, number of peer counts =17) and Combined Hospital Roorkee (H14, number of peer counts =17) are considered high robust hospitals as they have maximum peer count. So these hospitals classified in the high robust group can be considered as global leaders in terms of OTE.

4.3.1.2 Middle Robustness ($7 \leq$ number of peer counts ≤ 12):

Combines Hospital Kotdwar Pauri (H5, number of peer counts =12), CR Female Hospital Haridwar (H12, number of peer counts =11) and L.D. Bhatt hospital Kashipur, U.S. Nagar (H26, number of peer counts =7) are classified in the middle robust group terms of OTE.

4.3.1.3 Low Robustness ($1 \leq$ number of peer counts ≤ 6):

Base Hospital Srinagar Pauri (H3, number of peer counts =4), B.D Pandey Male Hospital Nainital (H15, number of peer counts =5), Base hospital Haldwani Nainital (H17, number of peer counts =3), Female Hospital Haldwani Nainital (H18, number of peer counts =1) and District Hospital Rudrapur U.S. Nagar (H27, number of peer counts =1) are classified in the low robust group in terms of OTE.

4.3.2 Pure Technical Efficiency (PTE)

As CCR model is based on CRS assumption which does not assume the scale size of the hospital. So, it is relevant to assessing the OTE. Therefore, in order to know whether inefficiency in any hospital is due to the inefficient production operation or due to unfavorable conditions displayed by the size of the hospital, BCC model [19] is also applied.

Table 4.5 shows DEA results calculated by CCR and BCC models. It is evident from Table 4.5 that out of 36 hospitals, only 10 are overall technical efficient (OTE=1), and 16 hospitals are pure technical efficient (PTE = 1), i.e., none of these have scope for further improvement in outputs (maintaining the same input level) while remaining 20 hospitals are inefficient (score < 1). PTE measures how efficiently inputs are converted into outputs irrespective of the size of the hospital. The average PTE comes out to be 81%. This means that given the scale of operation, on average a hospital can increase its outputs by 19% of its observed level without increasing its input level. PTE is concerned with the efficiency converting inputs into outputs for the given scale size of the hospitals. We observe that H2, H6, H21, H28, H31 and H36 are overall technically inefficient while they are pure technically efficient. Inefficiency of these hospitals is due to scale size, which clearly evinces that these hospitals are able to convert their inputs into outputs with 100% efficiency, but they are overall inefficient due to their disadvantageous scale size. If we check their returns to scale, hospitals H2, H21, H28, H31 and H36 are found to operate at IRS and H6 at DRS. However, the inefficiency is much higher in H6 compared to all other hospitals. Hospital-wise OTE, PTE and SE scores are shown in Figure 4.1.

Table-4.4: Efficiency Scores, Reference Set, Peer Weights and Peer Count by CCR Model

Code	OTE	Reference Set	Peer Weights	Peer Count
H1	0.601	H5, H14, H17	0.427, 0.190, 0.256	0
H2	0.940	H7, H12	0.037, 0.688	0
H3	1.00	H3	1	4
H4	0.856	H7, H12, H14, H15	0.01, 0.193, 0.234, 0.354	0
H5	1.00	H5	1	12
H6	0.909	H5, H14	0.795, 2.349	0
H7	1.00	H7	1	17
H8	0.697	H7, H14	0.218, 0.554	0
H9	0.953	H3, H5, H17	0.251, 0.818, 0.063	0
H10	0.592	H12, H14, H15	0.797, 0.181, 0.158	0
H11	0.882	H12, H14	1.211, 0.318	0
H12	1.00	H12	1	11
H13	0.466	H5, H12, H14	0.434, 1.046, 0.096	0
H14	1.00	H14	1	17
H15	1.00	H15	1	5
H16	0.723	H3, H7, H18	0.284, 0.233, 0.207	0
H17	1.00	H17	1	3
H18	1.00	H18	1	1
H19	0.600	H7	0.304	0
H20	0.773	H5, H7, H14, H26	0.454, 0.075, 0.318, 0.161	0
H21	0.378	H12	0.8	0
H22	0.675	H12, H14, H15	0.438, 0.353, 0.372	0
H23	0.474	H7, H14	0.521, 0.001	0
H24	0.622	H5, H7, H14, H26	0.268, 0.220, 0.128, 0.366	0
H25	0.253	H5, H7, H12, H17	0.389, 0.328, 1.339, 0.286	0
H26	1.00	H26	1	7
H27	1.00	H27	1	1
H28	0.888	H7, H14, H15, H26	0.041, 0.125, 0.202, 0.143	0
H29	0.532	H12, H27	0.863, 0.172	0
H30	0.553	H5, H7, H14, H26	0.322, 0.155, 0.073, 0.934	0
H31	0.917	H3, H5, H7	0.037, 0.406, 0.080	0
H32	0.535	H3, H5, H26	0.009, 0.209, 0.817	0
H33	0.623	H5, H7, H14, H26	0.109, 0.224, 0.002, 1.083	0
H34	0.706	H7, H12, H14, H15	0.150, 0.046, 0.307, 0.149	0
H35	0.653	H5, H7, H12, H14	0.092, 0.055, 0.134, 0.397	0
H36	0.974	H5, H7, H14, H26	0.055, 0.098, 0.265, 0.231	0

Source: Author’s Calculation

Figure 4.1: Efficiency Pattern of the Hospitals

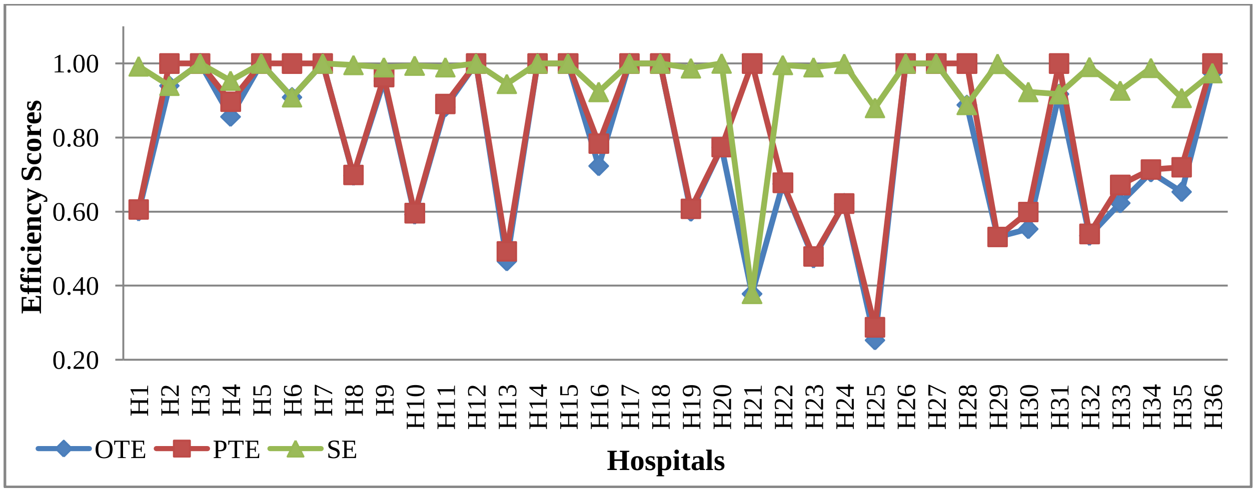


Table-4.5: OTE, PTE SE and RTS of Hospitals in Uttarakhand

Code	OTE	PTE	SE	RTS
H1	0.601	0.606	0.992	IRS
H2	0.940	1	0.940	IRS
H3	1	1	1	CRS
H4	0.856	0.897	0.954	IRS
H5	1	1	1	CRS
H6	0.909	1	0.909	DRS
H7	1	1	1	CRS
H8	0.697	0.699	0.996	IRS
H9	0.953	0.963	0.990	DRS
H10	0.592	0.596	0.994	DRS
H11	0.882	0.891	0.990	DRS
H12	1	1	1	CRS
H13	0.466	0.493	0.945	DRS
H14	1	1	1	CRS
H15	1	1	1	CRS
H16	0.723	0.784	0.923	IRS
H17	1	1	1	CRS
H18	1	1	1	CRS
H19	0.600	0.608	0.987	IRS
H20	0.774	0.774	1	CRS
H21	0.378	1	0.378	IRS
H22	0.675	0.678	0.996	DRS
H23	0.474	0.479	0.990	IRS
H24	0.622	0.622	0.999	IRS
H25	0.253	0.288	0.880	DRS
H26	1	1	1	CRS
H27	1	1	1	CRS
H28	0.888	1	0.888	IRS
H29	0.532	0.532	0.999	DRS
H30	0.553	0.599	0.923	DRS
H31	0.917	1	0.917	IRS
H32	0.535	0.54	0.991	DRS
H33	0.623	0.672	0.927	DRS
H34	0.706	0.714	0.988	IRS
H35	0.653	0.720	0.907	IRS
H36	0.974	1	0.974	IRS
Mean	0.772	0.810	0.955	

4.3.3 Scale Efficiency (SE)

Comparison of CCR and BCC results gives an assessment of whether the size of a hospital has an influence on its OTE or not. Scale efficiency (SE) is the ratio of OTE to PTE scores. If the value of the SE score is one, then the hospital is apparently operating at optimum scale size. If the value is less than one, then it appears either too small or too big relative to its optimum scale size [38]. From Table 4.5, we see that out of 36 hospitals, 11 hospitals are scale efficient, while the remaining 25 hospitals are scale inefficient. The hospital H20 is neither overall technical efficient nor pure technical efficient but it is scale efficient due to its scale size. The average scale efficiency is 0.955 (i.e., 95.50%) which indicates that on average a hospital may have to increase its scale efficiency by 4.50% beyond its best practice average targets under VRS, if it were to operate at CRS. Out of 25 scale-inefficient hospitals, 14 had IRS, while 11 revealed DRS. In order to operate at the most productive scale size (MPPS), a hospital exhibiting DRS should scale down both its outputs and inputs. Similarly, if a hospital is displaying IRS, it should expand both its outputs and inputs.

4.3.4 Input/ Output Slacks in the Inefficient Hospitals

It is important to study the slacks in inputs and outputs of the individual hospitals because the slack analysis provides additional insights about the magnitude of inefficiency for the inefficient hospitals. The magnitude of inefficiency is given by the quantity of excess inputs used (input slack) and/or deficient output produced (output slack) by the inefficient hospitals. Excess input utilization and/or deficient output production must be eliminated before a given hospital is said to be relatively efficient, compared to its reference set of the hospitals. The magnitude of estimated input and output slacks under CRS assumption are given in Table 4.6. The optimal input and output slacks calculated by CCR model for inefficient hospitals are given in Table 4.6.

The slacks in input variables show the underutilization of inputs. For instance H1 has underutilized Beds by 11.06% and PMS by 26.81%. On average, inefficient hospitals are not utilizing their Beds by 8.33 (8.99%), Doctors by 2.36 (12.63%) and PMS by 4.69 (11.99%). The slacks in all the efficient hospitals are zero and therefore, Table 4.6 provides details only for inefficient hospitals.

Table-4.6: Slacks in Inputs and Outputs for Inefficient Hospitals by CCR Model.

	Inputs			Outputs			
Code	Number of Beds	Number of Doctors	Number of Paramedical Staff	Number of out-door Patients	Number of in-door Patients	Number of Major Surgery	Number of Minor Surgery
H1	14.60	0.00	16.08	0.00	0.00	0.00	47.32
H2	5.21	1.32	0.00	5632.91	0.00	269.96	0.00
H4	0.00	0.00	8.29	0.00	0.00	205.56	0.00
H6	70.33	0.00	28.56	0.00	0.00	102.61	682.06
H8	37.01	0.00	3.82	0.00	3057.76	211.29	0.00
H9	26.86	0.00	19.09	0.00	0.00	0.00	26.84
H10	0.00	1.07	5.16	0.00	0.00	579.49	0.00
H11	0.00	3.81	23.55	0.00	735.76	0.00	178.83
H13	13.27	0.00	0.00	0.00	4686.40	304.87	0.00
H16	0.00	8.02	0.00	25834.29	0.00	313.40	0.00
H19	4.22	0.00	2.26	4119.83	5921.09	422.72	0.00
H20	0.00	3.01	0.00	0.00	0.00	48.24	0.00
H21	0.00	0.40	0.60	9831.25	235.66	560.78	0.00
H22	0.00	4.91	5.08	0.00	0.00	158.32	0.00
H23	11.06	0.00	4.31	0.00	4031.45	569.82	0.00
H24	0.00	3.23	0.00	0.00	0.00	38.78	0.00
H25	24.09	0.00	0.00	0.00	0.00	0.00	107.28
H28	0.00	2.54	0.00	0.00	0.00	128.85	0.00
H29	2.64	12.52	0.00	0.00	1174.20	595.56	0.00
H30	0.00	1.91	0.00	0.00	0.00	96.06	0.00
H31	7.21	0.00	0.00	53436.48	0.00	171.74	0.00
H32	0.00	1.60	5.07	0.00	0.00	269.96	0.00
H33	0.00	7.89	0.00	0.00	0.00	215.77	0.00
H34	0.00	1.82	0.00	0.00	0.00	190.49	0.00
H35	0.00	3.48	0.00	0.00	0.00	255.16	0.00
H36	0.00	3.85	0.00	0.00	0.00	201.26	0.00
Mean	8.33	2.36	4.69	3802.11	763.17	227.33	40.09

4.3.5 Input/Output Targets for Inefficient Hospitals

DEA allows setting targets to inputs and outputs for inefficient hospitals so that they may improve their performance and become efficient. We set input-output targets for inefficient hospitals by using the formulae given in Equations 4.3.4.1 and 4.3.4.2.

For outputs:

$$\overline{y_{rk}} = \phi_k^* y_{rk} + S_{rk}^{+*} = \sum_{j=1}^{n} \lambda_{jk}^* y_{rj} \tag{4.3.4.1}$$

For inputs:

$$\overline{x_{rk}} = x_{ik} - S_{ik}^{-*} = \sum_{j=1}^{n} \lambda_{jk}^* x_{ij} \tag{4.3.4.2}$$

where $\overline{y_{rk}}$ (r = 1,2,3,4) and $\overline{x_{ik}}$ (i = 1,2,3) are the output and input targets respectively for the k^{th} hospital; y_{rk} and x_{ik} are the actual r^{th} output and i^{th} input respectively of the k^{th} hospital; ϕ_k^* is the optimal efficiency score of the k^{th} hospital; s_{ik}^{-*} and s_{rk}^{+*} are the optimal input and output slacks of the k^{th} hospital.

Table 4.7 presents the target values of inputs and outputs for inefficient hospitals along with percentage reduction in inputs and augmentation in outputs. It reveals that an average hospital has significant scope to reduce inputs and augment outputs relative to the best performing hospitals. On average, 44.45% OPD, 61.11% IPD, 119.60% Major Surgery and 64.41% Minor Surgery should be increased with 9.98% reduction in Beds, 12.63% in Doctors and 11.99% reduction for PMS if an average hospital was to operate at the efficiency level.

Table-4.7: Targets for Inefficient Hospitals under CCR Output Oriented Model

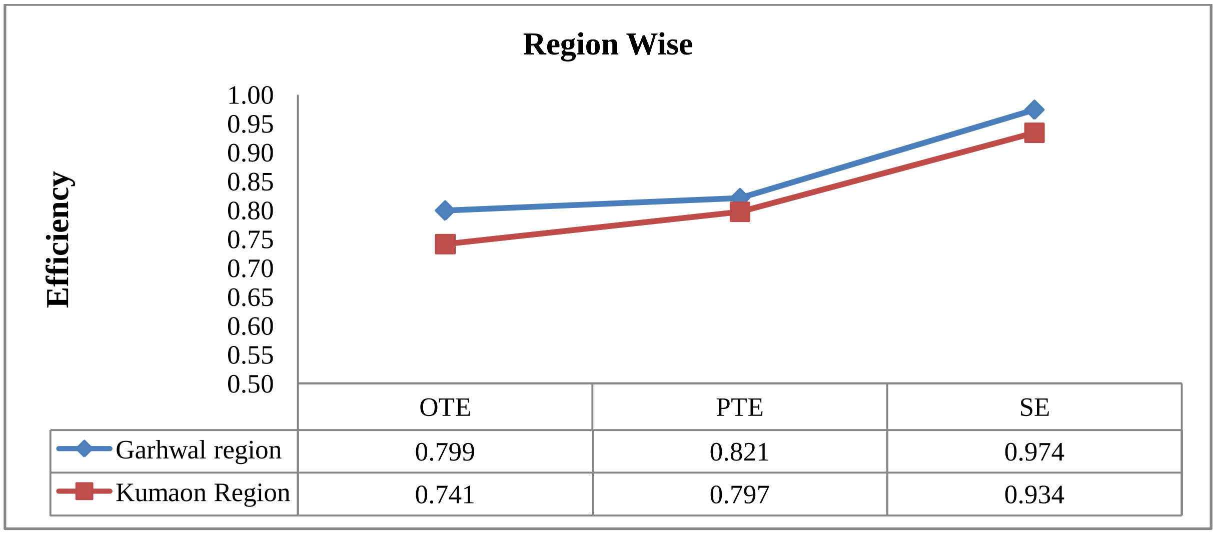
	Inputs			Outputs			
Code	No. of Beds	No. of Doctors	No. of PMS	No. of Out-door Patients	No. of In-door Patients	No. of Major Surgery	No. of Minor Surgery
H1	117.40(11.06)	20(0.00)	43.92(26.81)	216676.8(66.28)	10129.65(66.28)	1318.58(66.28)	1139.77(73.48)
H2	24.79(17.36)	5.68(18.91)	11(0.00)	24917.63(37.41)	2246.02(6.35)	745.33(66.74)	619.99(6.35)
H4	54(0.00)	15(0.00)	28.71(22.41)	122972.3(16.89)	3597.75(16.89)	418.29(129.83)	552.87(16.89)
H6	331.67(17.50)	55(0.00)	111.44(20.40)	786926.58(10.03)	22011.73(10.03)	1425.12(18.56)	2247.73(57.96)
H8	82.99(30.84)	15(0.00)	34.18(10.04)	173743.91(43.50)	7658.41(138.88)	788.17(96.06)	965.76(43.50)
H9	123.14(17.90)	20(0.00)	35.91((34.70)	216964.37(4.89)	16345.44(4.89)	694.39(4.89)	836.62(8.37)
H10	53(0.00)	12.93(7.65)	24.84(17.20)	96717.34(68.84)	3361.63(68.84)	888.47(385.50)	834.08(68.84)
H11	70(0.00)	14.19(21.14)	28.45(45.29)	123093.67(13.35)	4121.59(37.98)	1302.42(13.35)	1159.32(34.03)
H13	86.73(13.27)	16(0.00)	27(0.00)	139445.18(114.65)	8122.99(407.37)	1214.99(186.56)	1146.25(114.65)
H16	65(0.00)	13.99(36.43)	32(0.00)	87585.56(96.15)	13114.47(38.29)	747.65(138.10)	851.89(38.29)
H19	33.78(11.10)	7(0.00)	16.74(11.90)	32423.69(90.93)	6729.44(1287.51)	734.39(292.72)	773.35(66.67)
H20	100(0.00)	16.99(15.05)	33(0.00)	205267.52(29.29)	10612.14(29.29)	547.30(41.79)	769.28(29.29)
H21	24(0.00)	5.6(6.67)	10.40(5.54)	2434.80(343.29)	1652(208.21)	716.60(902.11)	610.40(164.24)
H22	74(0.00)	19.09(20.46)	36.94(12.09)	164392.24(48.08)	4578.75(48.08)	688.46(92.31)	812.98(48.08)
H23	57.94(16.03)	12(0.00)	28.69(13.05)	55813.17(110.86)	11521.28(224.36)	1257.23(285.65)	1324.22(110.86)
H24	90(0.00)	16.77(16.15)	37(0.00)	166458.28(60.86)	12506.65(60.86)	773.89(69.34)	942.62(60.86)
H25	175.91(12.05)	34(0.00)	74(0.00)	243295.77(294.58)	19125.43(294.58)	3401.30(294.58)	2912.76(309.67)
H28	40(0.00)	10.46(19.55)	22(0.00)	91024.87(12.57)	3975.81(12.57)	244.79(137.66)	361.34(12.57)
H29	47.36(5.28)	9.48(56.92)	15(0.00)	66394.58(88.13)	3621.76(178.38)	862.70(507.54)	731.82(88.13)
H30	120(0.00)	23.09(7.62)	54(0.00)	244589.20(80.86)	17080.77(80.86)	730.89(108.23)	954.96(80.86)
H31	54.79(11.64)	9(0.00)	15(0.00)	90960.85(164.47)	7640.38(9.10)	371.39(102.95)	482.23(9.10)
H32	78(0.00)	16.40(8.88)	42.93(10.55)	153173.07(86.92)	12876.89(86.92)	677.44(210.75)	800.02(86.92)
H33	108(0.00)	22.12(26.28)	56(0.00)	216588.29(60.58)	17058.76(60.58)	811.54(118.74)	992.41(60.58)
H34	60(0.00)	13.18(12.13)	28(0.00)	122922.19(41.73)	5732.84(41.73)	592.98(108.80)	731.31(41.73)
H35	60(0.00)	10.52(24.85)	22(0.00)	130339.49(53.07)	4602.78(53.07)	477.12(229.04)	583.19(53.07)
H36	60(0.00)	11.15(25.67)	26(0.00)	129407.37(2.62)	6504.18(2.62)	432.16(92.07)	556.21(2.62)
Mean	**84.33(9.98)**	**16.33(12.63)**	**34.43(11.99)**	**158709.03(44.45)**	**9097.29(61.11)**	**881.09(119.60)**	**949.74(64.41)**

Source: Author’s calculation

4.3.6 Region-wise Performance of Hospitals

Uttarakhand State is divided into thirteen districts and these districts are grouped into two regions- Kumaon and Garhwal. The six districts- Almora, Bageshwar, Champawat, Nainital, Pithoragarh and Udham Singh Nagar are in Kumaon region while rest seven districts- Dehradun, Haridwar, Tehri-Garhwal, Uttarkashi, Chamoli, Pauri and Rudraprayag are in Garhwal region. Figure 4.2 shows that the hospitals in Garhwal region are performing better than their counterparts in Kumaon region. It is because of the fact that out of 19 hospitals in Garhwal Region, 4 hospitals are in plain region; 5 hospitals are in partially plain region; and rest 10 hospitals are in hilly region; whereas in Kumaon Region, out of 17 hospitals, only 2 hospitals are in plain region; 7 hospitals are in partially plain region; and rest 8 hospitals are in hilly region. Utilization of hospitals in plain region is easier than that in hilly region due to relatively large size of population and better availability of sources to travel from villages to the hospitals. Nineteen hospitals, which are in Garhwal region have scored 79.90% OTE, 82.10% PTE and 97.40% SE, whereas the corresponding OTE, PTE and SE scores in the Kumaon region are 74.10%, 79.70% and 93.40% respectively. On average 8.94 (8.95%) beds, 1.71 (9.02%) doctors and 5.77 (13.81%) PMS are not utilized by the hospitals in the Garhwal region. On the other hand, hospitals in Kumaon region, on average, have not utilized their 2.74 (3.15%) beds, 1.69 (8.89%) doctors and 0.72 (1.91%) paramedical staff.

Fig. 4.2: Average OTE, PTE and SE scores of Hospitals in Garhwal and Kumaon regions



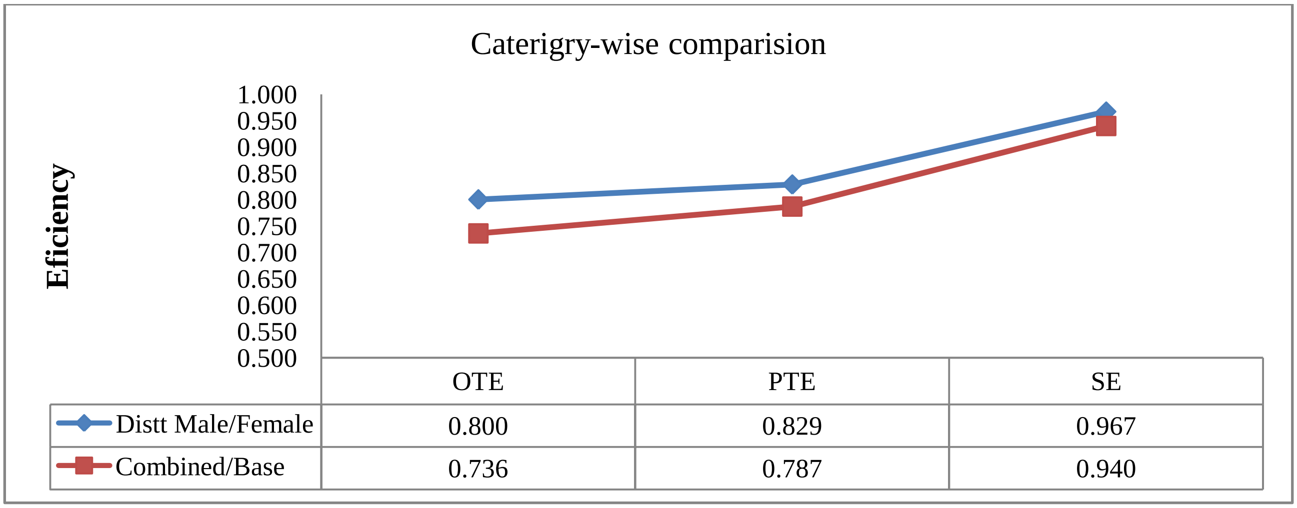
4.3.7 Category-wise Performance of Hospitals

To explain the source of inefficiency of hospitals, category-wise study of the performance of hospitals has also been conducted. Total 36 hospitals are divided into two groups, namely district male/female hospitals and combined/base hospitals. The results shown in Table 4.8 and Figure 4.2 show that there is a systematic pattern of OTE, PTE and SE with respect to both categories of hospitals. The results show that district male/female hospitals are performing better than combined/base hospitals as they have average OTE score (0.80) higher than the average efficiency score (0.736) of combined and base hospitals. The mean efficiency scores of both the category hospitals are given in Table 4.8.

Table 4.8: Category-wise Efficiencies of Public Hospitals

District Male/Female Hospitals						
	Min	Mean	Max	SD	No. of Efficient Hospitals (%)	Total No. of Hospitals
OTE	0.474	0.800	1	0.187	5 (25%)	20
PTE	0.479	0.829	1	0.194	10 (50%)	20
SE	0.888	0.967	1	0.038	5 (25%)	20
Combined and Base Hospitals						
	Min	Mean	Max	SD	No. of Efficient Hospitals (%)	Total No. of Hospitals
OTE	0.253	0.736	1	0.249	5 (31.25%)	16
PTE	0.288	0.787	1	0.229	6 (37.50%)	16
SE	0.378	0.940	1	0.154	5 (31.25%	16

Fig. 4.3: Category-wise Average OTE, PTE and SE in the Public Hospitals



District male/female hospitals are performing slightly better than the combined and base hospitals as they are utilizing their inputs: beds by 93.44%, doctors by 92.11% and PMS by 94.12%; whereas combined/base hospitals are utilizing their beds by 93.71%, doctors by 90.18% and PMS by 89.44%. In district male/female hospitals, an average doctor attains 7078.51, 379.30, 31.84 and 36.39 out-door patients, in-door patients, major surgeries and minor surgeries respectively, while the corresponding figures in combined and base hospitals are 6373.32, 381.67, 27.98 and 36.57 respectively. It is observed that district male/female hospitals take less in-door patients and minor surgeries relative to combined and base hospitals and attain better performance mainly because of higher per doctor out-door patients, and major surgeries.

4.3.8 Area-wise Performance of Hospitals

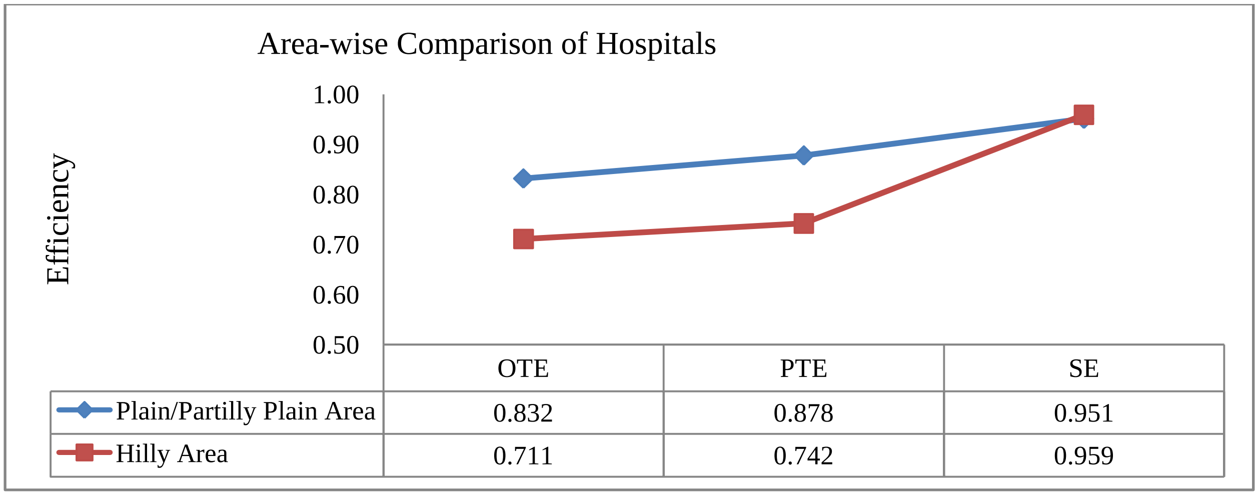
The area-wise efficiencies of hospitals are also estimated**.** Table 4.9 shows that the efficiency of hospitals in plain/partially-plain area is better than the hospitals situated in hilly areas.

Table 4.9: Area-wise Efficiencies of Public Hospitals

Plain/Partially Plain Area						
	Min.	Mean	Max.	SD	No. of Efficient Hospitals (%)	No. of Hospitals
OTE	0.378	0.832	1	0.208	8 (44.44%)	18
PTE	0.493	0.878	1	0.173	10 (55.56%)	18
SE	0.378	0.951	1	0.146	8 (44.44%)	18
Hilly Area						
	Min.	Mean	Max.	SD	No. of Efficient Hospitals (%)	No. of Hospitals
OTE	0.253	0.711	1	0.212	2 (11.11%)	18
PTE	0.288	0.742	1	0.223	6 (33.33%)	18
SE	0.88	0.959	1	0.042	2 (11.11%)	18

Area-wise performance of hospitals in Figure 4.4 shows that hospitals in plain/partially plain region perform better than the hospitals situated in hilly areas. There are many factors affecting the performance of these hospitals. Since private hospitals/nursing homes in the hilly areas are quite rare, most of the patients in the hill areas depend on the government hospitals. For this these hospitals must be located at appropriate places where most of the populations of that region be connected. Competent doctors and all the facilities must be available in the hospital so that people may have no need to go far from these hospitals, and ambulance facility should also be available in the hospital. The performance can be improved by recruiting motivated and trained health workers and capacity building of existing staff through training programs. Some motivational policies such as promotions, and performance based reward system should be introduced. Another point requiring attention is the lack of awareness of available facilities and services offered in the public hospitals, the large proportion of vacant healthcare positions, lack of people's awareness, the absence of advocacy groups, lack of adequate equipments, lack of training institutions and poor transport facility are the major factors affecting the efficiency of hospitals in hilly areas.

Fig. 4.4: Average Efficiency Scores of Hospitals in Plain/Partially Plain and Hilly Areas



	OTE	PTE	SE
Plain/Partilly Plain Area	0.832	0.878	0.951
Hilly Area	0.711	0.742	0.959

4.4 Post DEA Analysis

The drawback of DEA approach is that it is non-parametric technique and therefore hypothesis testing is very difficult. So, to determine the robustness of efficiency scores obtained by DEA, sensitivity analysis is used. Sensitivity analysis is applied to know how sensitive the solution value and efficiency scores of the hospitals are due to the inclusion or exclusion of input-output variables and DMUs. Tobit regression is used to examine the effect of various background/ environmental variables on the efficiency scores. In this chapter, we have applied both the methods.

4.4.1 Sensitivity Analysis

To investigate robustness of the efficiency scores, sensitivity analysis has been conducted. For this, we use "DEA Cross Reference (DCR) efficiency measure" given by Hibiki and Sueyoshi

[56]. DCR is used for evaluating sensitivity analysis between two hospitals. The DCR output-oriented model is given as follows:

$$\left.\begin{array}{l} Max\,\delta_{a,b} = \phi_{a} + \varepsilon \sum_{r=1}^{s} s_{rk}^{+} + \varepsilon \sum_{i=1}^{m} s_{ik}^{-} \\ s.t. \sum_{j \in J-\{b\}} \lambda_{ja} y_{rj} - s_{ra}^{+} = \phi_{k} y_{ra} \quad i = 1, 2, 3, \ldots, m \\ \quad \sum_{j \in J-\{b\}} \lambda_{ja} x_{ij} + s_{ia}^{-} = x_{ia} \qquad r = 1, 2, 3, \ldots, s \\ \quad \sum_{j \in J-\{b\}} \lambda_{ja} = 1 \\ \lambda_{ja} \geq 0,\ s_{ia}^{-}, s_{ra}^{+} \geq 0,\ \phi_{a} \geq 0, \end{array}\right\} \qquad \ldots Model 4$$

where J is the set of all hospitals, i.e., J = {H1, H2,…,H36}.

Here $\delta_{a,b}$ indicates the optimal level of an efficiency measure. It is defined as the DCR efficiency score of the a^{th} hospital under the condition that $j \in J - \{b\}$ [i.e., b^{th} hospital is excluded from the set of all hospitals]. s_{ia}^{-} *and* s_{ra}^{+} are the slacks related to inputs and outputs respectively.

The difference between optimal DCR efficiency score ($\delta_{a,b}^{*}$) and DEA efficiency score [CCR and BCC] refers to as "DEA Cross Sensitivity (DCS) efficiency score". The degree of DCS efficiency measure is formally defined as: $D_{a,b}^{*} = \delta_{a,b}^{*} - \phi_{a}^{*}$,

where

$\delta_{a,b}^{*}$ = Optimal DCR efficiency score of the a^{th} hospital when the b^{th} hospital is omitted from the set of hospitals.

ϕ_{a}^{*} = Optimal DEA [CCR and BCC] efficiency scores of the a^{th} hospital.

Table 4.10: DCR efficiency scores under CCR Model

Inefficient hospital (a)	Actual OTE Score	Efficient hospital No. (b) to be removed one by one											
		H3	H5	H7	H12	H14	H15	H17	H18	H26	H27	Mean	SD
H1	0.601	0.601	0.602	0.601	0.601	0.616	0.601	0.619	0.601	0.601	0.601	0.604	0.006
H2	0.940	0.940	0.940	0.953	1.00	0.940	0.940	0.940	0.940	0.940	0.940	0.947	0.017
H4	0.856	0.856	0.856	0.857	0.864	0.904	0.86	0.856	0.856	0.856	0.856	0.862	0.014
H6	0.909	0.909	0.909	0.909	0.909	1.00	0.909	0.909	0.909	0.909	0.909	0.918	0.027
H8	0.697	0.697	0.697	0.706	0.697	0.787	0.697	0.697	0.697	0.697	0.697	0.707	0.027
H9	0.953	0.960	0.956	0.953	0.953	0.953	0.953	0.955	0.953	0.953	0.953	0.954	0.002
H10	0.592	0.592	0.592	0.592	0.617	0.614	0.594	0.592	0.592	0.592	0.592	0.597	0.009
H11	0.882	0.882	0.882	0.882	0.916	0.911	0.882	0.882	0.882	0.882	0.882	0.888	0.009
H13	0.466	0.466	0.477	0.466	0.498	0.474	0.466	0.466	0.466	0.466	0.466	0.471	0.009
H16	0.723	0.736	0.723	0.841	0.723	0.723	0.723	0.723	0.724	0.723	0.723	0.736	0.035
H19	0.600	0.600	0.600	0.608	0.600	0.600	0.600	0.600	0.600	0.600	0.600	0.601	0.002
H20	0.773	0.773	0.798	0.783	0.773	0.815	0.773	0.773	0.773	0.776	0.773	0.781	0.013
H21	0.378	0.378	0.378	0.378	0.440	0.378	0.378	0.378	0.378	0.378	0.378	0.384	0.018
H22	0.675	0.675	0.675	0.675	0.686	0.708	0.678	0.675	0.675	0.675	0.675	0.679	0.009
H23	0.474	0.474	0.474	0.587	0.474	0.474	0.474	0.474	0.474	0.474	0.474	0.485	0.033
H24	0.622	0.622	0.630	0.648	0.622	0.629	0.622	0.622	0.622	0.626	0.622	0.627	0.007
H25	0.253	0.253	0.258	0.263	0.289	0.253	0.253	0.259	0.253	0.253	0.253	0.259	0.011
H28	0.888	0.888	0.888	0.895	0.888	0.913	0.893	0.888	0.888	0.909	0.888	0.894	0.009
H29	0.532	0.532	0.532	0.532	0.587	0.532	0.532	0.532	0.532	0.532	0.532	0.538	0.016
H30	0.553	0.553	0.558	0.565	0.553	0.555	0.553	0.553	0.553	0.566	0.553	0.556	0.004
H31	0.917	0.929	1.00	1.00	0.917	0.917	0.917	0.917	0.917	0.917	0.917	0.935	0.033
H32	0.535	0.535	0.535	0.572	0.535	0.535	0.535	0.535	0.535	0.574	0.535	0.543	0.015
H33	0.623	0.623	0.625	0.647	0.623	0.623	0.623	0.623	0.623	0.657	0.623	0.629	0.012
H34	0.706	0.706	0.706	0.720	0.707	0.742	0.707	0.706	0.706	0.706	0.706	0.711	0.011
H35	0.653	0.653	0.659	0.663	0.663	0.721	0.653	0.653	0.653	0.653	0.653	0.662	0.019
H36	0.974	0.974	0.977	1.00	0.974	1.00	0.974	0.974	0.974	0.99	0.974	0.981	0.011

Table 4.11: DCS scores under CCR model

Inefficient Hospital (a)	Efficient Hospital No. (b)									
	H3	H5	H7	H12	H14	H15	H17	H18	H26	H27
H1	0.000	0.001	0.000	0.000	0.015	0.000	0.018	0.000	0.000	0.000
H2	0.000	0.000	0.013	0.060	0.000	0.000	0.000	0.000	0.000	0.000
H4	0.000	0.000	0.001	0.008	0.048	0.004	0.000	0.000	0.000	0.000
H6	0.000	0.000	0.000	0.000	0.091	0.000	0.000	0.000	0.000	0.000
H8	0.000	0.000	0.009	0.000	0.090	0.000	0.000	0.000	0.000	0.000
H9	0.007	0.003	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.000
H10	0.000	0.000	0.000	0.025	0.022	0.002	0.000	0.000	0.000	0.000
H11	0.000	0.000	0.000	0.034	0.029	0.000	0.000	0.000	0.000	0.000
H13	0.000	0.011	0.000	0.032	0.008	0.000	0.000	0.000	0.000	0.000
H16	0.013	0.000	0.118	0.000	0.000	0.000	0.000	0.001	0.000	0.000
H19	0.000	0.000	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H20	0.000	0.025	0.010	0.000	0.042	0.000	0.000	0.000	0.003	0.000
H21	0.000	0.000	0.000	0.062	0.000	0.000	0.000	0.000	0.000	0.000
H22	0.000	0.000	0.000	0.011	0.033	0.003	0.000	0.000	0.000	0.000
H23	0.000	0.000	0.113	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H24	0.000	0.008	0.026	0.000	0.007	0.000	0.000	0.000	0.004	0.000
H25	0.000	0.005	0.010	0.036	0.000	0.000	0.006	0.000	0.000	0.000
H28	0.000	0.000	0.007	0.000	0.025	0.005	0.000	0.000	0.021	0.000
H29	0.000	0.000	0.000	0.055	0.000	0.000	0.000	0.000	0.000	0.000
H30	0.000	0.005	0.012	0.000	0.002	0.000	0.000	0.000	0.013	0.000
H31	0.012	0.083	0.083	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H32	0.000	0.000	0.037	0.000	0.000	0.000	0.000	0.000	0.039	0.000
H33	0.000	0.002	0.024	0.000	0.000	0.000	0.000	0.000	0.034	0.000
H34	0.000	0.000	0.014	0.001	0.036	0.001	0.000	0.000	0.000	0.000
H35	0.000	0.006	0.010	0.010	0.068	0.000	0.000	0.000	0.000	0.000
H36	0.000	0.003	0.026	0.000	0.026	0.000	0.000	0.000	0.016	0.000

Table 4.12: DCR Efficiency Scores under BCC Model

Inefficient Hospital No. (a)	Actual PTE Score	Efficient Hospitals No. (b)																Mean	SD
		H2	H3	H5	H6	H7	H12	H14	H15	H17	H18	H21	H26	H27	H28	H31	H36		
H1	0.606	0.606	0.606	0.608	0.606	0.606	0.616	0.634	0.606	0.635	0.606	0.606	0.606	0.606	0.606	0.606	0.606	0.610	0.009
H4	0.897	0.897	0.897	0.897	0.897	0.897	0.927	0.906	0.903	0.897	0.897	0.897	0.897	0.897	0.908	0.897	0.897	0.901	0.007
H8	0.699	0.699	0.699	0.699	0.699	0.711	0.729	0.798	0.699	0.699	0.699	0.699	0.699	0.699	0.699	0.699	0.699	0.708	0.025
H9	0.963	0.963	1.00	0.963	0.978	0.970	0.963	0.968	0.963	0.967	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.967	0.009
H10	0.596	0.596	0.596	0.596	0.596	0.602	0.663	0.622	0.596	0.596	0.596	0.596	0.596	0.596	0.596	0.596	0.596	0.602	0.017
H11	0.891	0.891	0.891	0.891	0.891	0.891	1.00	0.913	0.891	0.966	0.891	0.891	0.891	0.891	0.891	0.891	0.891	0.904	0.032
H13	0.493	0.493	0.493	0.501	0.493	0.55	0.503	0.493	0.493	0.493	0.493	0.493	0.493	0.493	0.493	0.493	0.493	0.498	0.014
H16	0.784	0.787	0.793	0.784	0.784	0.881	0.784	0.784	0.784	0.784	0.849	0.784	0.784	0.784	0.784	0.784	0.784	0.795	0.028
H19	0.608	0.608	0.608	0.608	0.608	0.608	0.867	0.608	0.608	0.608	0.608	0.608	0.608	0.608	0.608	0.608	0.608	0.624	0.064
H20	0.774	0.774	0.776	0.802	0.774	0.794	0.774	0.837	0.774	0.774	0.774	0.774	0.777	0.774	0.774	0.774	0.774	0.781	0.016
H22	0.678	0.678	0.678	0.678	0.678	0.686	0.692	0.721	0.679	0.678	0.678	0.678	0.678	0.678	0.678	0.678	0.678	0.682	0.011
H23	0.479	0.479	0.479	0.479	0.479	0.633	0.536	0.479	0.479	0.479	0.479	0.479	0.479	0.479	0.479	0.479	0.479	0.492	0.04
H24	0.622	0.622	0.622	0.635	0.622	0.672	0.622	0.639	0.622	0.622	0.622	0.622	0.630	0.622	0.622	0.622	0.622	0.628	0.013
H25	0.288	0.288	0.288	0.288	0.288	0.352	0.289	0.288	0.288	0.375	0.288	0.288	0.288	0.288	0.288	0.288	0.288	0.298	0.026
H29	0.532	0.532	0.532	0.536	0.532	0.532	0.588	0.532	0.532	0.532	0.532	0.532	0.532	0.534	0.532	0.532	0.532	0.536	0.013
H30	0.599	0.599	0.643	0.599	0.606	0.599	0.599	0.617	0.599	0.601	0.599	0.599	0.599	0.599	0.599	0.599	0.599	0.603	0.011
H32	0.540	0.540	0.545	0.540	0.540	0.594	0.540	0.541	0.540	0.540	0.540	0.540	0.577	0.540	0.540	0.540	0.540	0.546	0.015
H33	0.672	0.672	0.723	0.672	0.672	0.686	0.672	0.686	0.672	0.673	0.672	0.672	0.672	0.672	0.672	0.672	0.672	0.677	0.013
H34	0.714	0.714	0.714	0.714	0.714	0.731	0.752	0.748	0.715	0.714	0.714	0.714	0.724	0.714	0.714	0.714	0.714	0.720	0.012
H35	0.720	0.720	0.720	0.720	0.720	0.720	0.731	0.743	0.720	0.720	0.720	0.720	0.720	0.722	0.723	0.720	0.720	0.722	0.006

Source: Author’s Calculation

Table 4.13: DCS Scores under BCC Model

Inefficient	Efficient Hospital No. (b)															
Hospital No. (a)	H2	H3	H5	H6	H7	H12	H14	H15	H17	H18	H21	H26	H27	H28	H31	H36
H1	0.000	0.000	0.002	0.000	0.000	0.010	0.028	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H4	0.000	0.000	0.000	0.000	0.000	0.030	0.009	0.006	0.000	0.000	0.000	0.000	0.000	0.011	0.000	0.000
H8	0.000	0.000	0.000	0.000	0.012	0.030	0.099	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H9	0.000	0.037	0.000	0.015	0.007	0.000	0.005	0.000	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H10	0.000	0.000	0.000	0.000	0.006	0.067	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H11	0.000	0.000	0.000	0.000	0.000	0.109	0.022	0.000	0.075	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H13	0.000	0.000	0.008	0.000	0.057	0.010	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H16	0.003	0.009	0.000	0.000	0.097	0.000	0.000	0.000	0.000	0.065	0.000	0.000	0.000	0.000	0.000	0.000
H19	0.000	0.000	0.000	0.000	0.000	0.259	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H20	0.000	0.002	0.028	0.000	0.020	0.000	0.063	0.000	0.000	0.000	0.000	0.003	0.000	0.000	0.000	0.000
H22	0.000	0.000	0.000	0.000	0.008	0.014	0.043	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H23	0.000	0.000	0.000	0.000	0.154	0.057	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H24	0.000	0.000	0.013	0.000	0.050	0.000	0.017	0.000	0.000	0.000	0.000	0.008	0.000	0.000	0.000	0.000
H25	0.000	0.000	0.000	0.000	0.064	0.001	0.000	0.000	0.087	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H29	0.000	0.000	0.004	0.000	0.000	0.056	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.000
H30	0.000	0.044	0.000	0.007	0.000	0.000	0.018	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H32	0.000	0.005	0.000	0.000	0.054	0.000	0.001	0.000	0.000	0.000	0.000	0.037	0.000	0.000	0.000	0.000
H33	0.000	0.051	0.000	0.000	0.014	0.000	0.014	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H34	0.000	0.000	0.000	0.000	0.017	0.038	0.034	0.001	0.000	0.000	0.000	0.010	0.000	0.000	0.000	0.000
H35	0.000	0.000	0.000	0.000	0.000	0.011	0.023	0.000	0.000	0.000	0.000	0.000	0.002	0.003	0.000	0.000

Source: Author’s Calculation

The sensitivity analysis is done by removing each of the efficient hospitals one by one by using DCR model. Tables 4.10 and 4.12 present the result of DCR under CCR and BCC Output-oriented models respectively. The average and standard deviation of DCR scores of each hospital are listed in the last two columns of Tables 4.10 and 4.12. These results indicate that our DEA efficiency scores are robust and stable in the sense that removal of any efficient hospital from the set does not have any high influence on the mean OTE.

Table 4.10 gives the sensitivity in the CCR efficiency scores of the inefficient hospitals. The inefficient hospitals are classified as:

4.4.1.1 Low Sensitive Hospitals

The hospitals H1, H9, H19, H22 and H30 are low sensitive inefficient hospitals as they have difference between the actual OTE score and average of DCR score (i.e., excluding efficient hospital one by one) below 0.50 percent.

4.4.1.2 Middle Sensitive Hospitals

The hospitals H2, H4, H6, H10, H11, H13, H20, H21, H24, H25, H28, H29, H32, H33, H34, H35 and H36 are classified as middle sensitive inefficient hospitals as the difference between actual OTE score and average DCR score lies between 0.51 percent and 1 percent.

4.4.1.3 High Sensitive Hospitals

The hospitals H8, H16, H23 and H31 are graded as high sensitive inefficient hospitals as they have difference between the actual OTE score and average DCR score greater than 1 percent.

Tables 4.11 and 4.13 present the resulting DCS scores of CCR and BCC models respectively. From Table 4.11, we can conclude that H5 and H7 have a major impact on one inefficient hospital. For instance, efficiency score of H31 increases by 8.30% with the exclusion of H5 and H7. Similarly, efficiency score of H2 increases by 6% with the exclusion of H12, efficiency score of H6 increases by 9.10% with the exclusion of H14 and efficiency score of H36 increases by 2.60% with the exclusion of H7 and H14.

Similarly, Table 4.12 also indicates that H3 and H12 make a group of efficient hospitals that have the considerable influence in the magnitude of DCR results of other inefficient hospitals in terms of BCC output-oriented model.

4.4.2 Regression Analysis

In order to determine the effect of several background variables, which cannot be included in the efficiency assessment of the hospitals, regression analysis is done. There are two regression models commonly used: (i) Ordinary Least Squares (OLS) regression; (ii) Tobit regression [138]. However, because of efficient DMUs having a DEA efficiency score of 1 and a relatively large number of fully efficient DMU being estimated, the distribution of efficiency is truncated above from unity. As a result, the dependent variable (inefficiency scores) in the regression model becomes a limited dependent variable. In such a case, applying OLS regression is inappropriate [54], so, a Tobit regression model is used in this study to examine the impact of environmental factors, such as, category of hospitals, location of hospitals, size of hospitals, etc., on the (in) efficiency scores. In this study, area location of hospital (AREA), category of hospitals (CAT), size of hospital (SIZE), and hospital in the district headquarter (LOC), bed occupancy rate (BOR) are taken as independent variables in the Tobit regression. In order to normalize the DEA distribution and convenience for computation, the DEA efficiency scores obtained from the Model 4.2 are transformed into inefficiency scores and left a censoring point concentrated at zero by taking one minus efficiency score. The variables (dependent and independent) selected for the Tobit regression of the cross sectional data are described as follows:

INEF: It is calculated by subtracting the efficiency scores by one.

AREA: Equal to 1 if a hospital is located in plain/partially plain area; otherwise 0.

CAT: Equal to 1 if a hospital is district male/female; otherwise 0.

SIZE: Equal to 1 if bed strength of the hospital is above 100; otherwise 0.

LOC: Equal to 1 if a hospital is located in district headquarters; otherwise 0.

BOR: Bed occupancy rate of the hospitals.

Table 4.14: Results of the Tobit Regression Model.

INEF	Coefficient	Std. Err.	t – Value	p –Value
AREA	-0.169	0.100	-1.690	0.101
CAT	-0.180	0.126	-1.430	0.164
SIZE	-0.076	0.110	-0.690	0.493
LOC	0.159	0.120	1.330	0.193
BOR	-0.001	0.002	-0.340	0.737
Constant term	0.365	0.126	2.910	0.007
Sigma	0.259	0.038		

Source: *author's calculation*

Tobit regression is applied using the econometric software, STATA version 12. Table 4.14 reports the results obtained by the Tobit regression. The coefficients are interpreted to analyze the relationship between efficiency changes. In order to identify the effect of environmental variables, different parameters are taken as dummy variables. The results given in Table 4.14 show that estimated coefficient for the location of the hospitals is almost statistically significant at the 10 % level of significance. This indicates that hospitals located in the plain/semi-plain region of the state performed better than their counterparts located in the hilly region. All other variables are not found to have any statistically significant effect on the performance of the hospitals of the state. Some other factors, such as lack of awareness of available facilities and services offered in public hospitals, a large proportion of vacant healthcare positions, the absence of advocacy groups, lack of adequate equipments, lack of training facilities, and poor transport facility may also affect the efficiency scores of hospitals. However, data on these variables are difficult to get. The performance of the inefficient hospitals may be improved by recruiting motivated and trained health workers and capacity building of existing staff through training programs.

4.5 Summing Up

This chapter estimates OTE, PTE and SE through a two-stage analytical process using cross-sectional data of 36 government hospitals in Uttarakhand for the year 2011. In the first stage, DEA is employed to estimate the OTE, PTE, SE and RTS. The required improvements in inputs and outputs are also suggested. Sensitivity analysis is also used to test the robustness of the results. The variations in the efficiency scores across regions, areas and the location of hospitals are also studied using CCR and BCC models. In the second stage, Tobit regression model is applied to study the factors affecting the efficiency scores of the hospitals. The key findings of the Chapter are summarized as follows: The results of the chapter reveals that out of 36 hospitals only 10 (27.78%) are efficient.

1. The average OTE of public hospitals in Uttarakhand is 77.20 percent. This indicates that 22.80% of the technical potential of hospitals is not in use, implying that these hospitals have the scope of producing 22.80% more outputs with the same level of inputs.
2. The PTE score is found to be 81 percent and SE score is 95.50 percent. This shows that the hospitals are 19 percent inefficient in managerial performance and 4.5 percent in SE.
3. The RTS analysis shows that 38.88 percent of the hospitals operate at IRS, and 30.56 percent at DRS. This indicates that most of the hospitals operating at DRS are larger sized, while those at IRS are relatively smaller in size.
4. Slack analysis shows that on average, number of beds, number of doctors, and number of paramedical staff observe slacks of 8.33, 2.36 and 4.69, respectively.
5. The target setting results show that number of major surgeries has significant scope to expand. This can be expanded by using the proper referral system for secondary and tertiary hospitals so that these hospitals may not have to spend time and resources on providing primary healthcare or treating minors ailments.
6. The region-wise comparison shows that the hospitals of Garhwal region perform better than the hospitals of Kumaon region. On an average, hospitals in Garhwal and Kumaon regions have to increase their output by 20.10% and 25.90% respectively.
7. Category-wise comparison of hospitals shows that District male/female hospitals are performing better (OTE =0.80) than the combined and base hospitals (OTE =0. 736).
8. Area-wise comparison of efficiencies of the hospitals show that the hospitals of plain/ partially plain area are found to perform better than the hospitals located in hilly areas.
9. Sensitivity analysis results show that the efficiency scores of the hospitals are stable even after the exclusion of the most efficient hospitals.
10. Lastly, the regression analysis results show that the estimated coefficient for the location of the hospitals is almost statistically significant at 10% level of significance. This indicates that hospitals located in the plain/semi-plain region of the state performed better than their counterparts located in the hilly region.

Appendix 4A: Observed Input-Output Data of the Hospitals for the Year 2011.

	Inputs			**Outputs**			
Code	Beds	Doctors	PMS	OPD	IPD	Major	Minor
H1	132	20	60	130310	6092	793	657
H2	30	7	11	18134	2112	447	583
H3	100	18	45	169823	20348	422	542
H4	54	15	37	105207	3078	182	473
H5	104	16	22	187659	12607	401	638
H6	402	55	140	715221	20006	1202	1423
H7	111	23	55	106535	22111	2413	2541
H8	120	15	38	121075	3206	402	673
H9	150	20	55	206844	15583	662	772
H10	53	14	30	57283	1991	183	494
H11	70	18	52	108594	2987	1149	865
H12	30	7	13	30426	2065	952	763
H13	100	16	27	64963	1601	424	534
H14	106	18	40	271514	5103	471	741
H15	63	26	46	148385	5035	283	584
H16	65	22	32	44652	9483	314	616
H17	206	38	105	330964	14714	4128	2834
H18	52	17	31	70406	10576	318	513
H19	38	7	19	16982	485	187	464
H20	100	20	33	158765	8208	386	595
H21	24	6	11	5491	536	76	231
H22	74	24	42	111013	3092	358	549
H23	69	12	33	26469	3552	326	628
H24	90	20	37	103482	7775	457	586
H25	200	34	74	61659	4847	862	711
H26	66	14	38	158385	9885	208	324
H27	125	20	22	233711	10711	238	426
H28	40	13	22	80864	3532	103	321
H29	50	22	15	35292	1301	142	389
H30	120	25	54	135234	9444	351	528
H31	62	9	15	34394	7003	183	442
H32	78	18	48	81946	6889	218	428
H33	108	30	56	134876	10623	371	618
H34	60	15	28	86732	4045	284	516
H35	60	14	22	85151	3007	145	381
H36	60	15	26	126101	6338	225	542

Appendix 4B: Hospital Name and District Location.

Code	Distt. Name	Hospital Name
H1	Pauri	District Male Hospital
H2	Pauri	District Female Hospital
H3	Pauri	Base Hospital Srinagar
H4	Pauri	Combines Hospital Srinagar
H5	Pauri	Combines Hospital Kotdwar
H6	Dehradun	Doon Male Hospital
H7	Dehradun	Female Hospital Dehradun
H8	Dehradun	Coronation Hospital
H9	Dehradun	SPS Hospital Rishikesh
H10	Dehradun	St. Merry. Hospital Mussoorie
H11	Haridwar	HMG Hospital Haridwar
H12	Haridwar	CR Female Hospital
H13	Haridwar	Mela Hospital Haridwar
H14	Haridwar	Combined Hospital Roorkee
H15	Nainital	B.D Pandey Male Hospital
H16	Nainital	B.D Pandey Female Hospital
H17	Nainital	Base Hospitla Haldwani
H18	Nainital	Female Hospital Haldwani
H19	Nainital	G.B Pant Hospital Nainital
H20	Nainital	Combined Hospital Ramnagar
H21	Nainital	Combined Hospital Padampuri
H22	Almora	District Hospital Male
H23	Almora	District Hospital Female
H24	Almora	Combined Hospital Ranikhet
H25	Almora	Base Hospital Almora
H26	US Nagar	L.D Bhatt hospital Kashipur
H27	US Nagar	District Hospital Rudrapur
H28	Tehri	District Hospital Baurari Tehri
H29	Tehri	Combined Hospital Narendnagar
H30	Pithoragarh	DH Male Pithoragarh
H31	Pithoragarh	DH Female Pithoragarh
H32	Chamoli	District Hospital Gopeshwar
H33	Uttarkashi	District Hospital Uttarkashi
H34	Rudraprayag	District Hospital Rudraprayag
H35	Champawat	Combined Hospital Tanakpur
H36	Bageshwar	District Hospital Bageshwar

Chapter 5

Non-Oriented Measure of Efficiencies: A Slack Based Model

5.1 Introduction

In this chapter, we apply a slack based model (SBM) to determine the efficiency scores of a set of 36 public sector hospitals of Uttarakhand, State of India. We measure the relative efficiencies and slacks corresponding to selected inputs and outputs of the hospitals, set benchmarks for relative inefficient hospitals to improve their efficiencies. Stability of efficiency scores is also estimated using Jackknifing Analysis.

5.2 Methodology

In Chapter 4, we use DEA based radial models, CCR and BCC models, to measure the efficiencies of the public hospitals of Uttarakhand State. However, the radial CCR model [27] and BCC model [19] suffer from one shortcoming that they neglect the slacks in the evaluation of efficiencies. To overcome this shortcoming, efficiency scores can be computed using a non-radial and non-oriented model known as “slack-based model” given by Tone [140].

Since, in the basic CCR and BCC models the efficiency is measured either by changing inputs or by changing outputs, i.e., either input-oriented or output-oriented model is used for the efficiency measurement. When both inputs and outputs can be changed simultaneously, i.e., the DMU is able to reduce inputs and augment outputs simultaneously, a non-oriented SBM model is used. It is known as the Additive Model (AM) or a slack-based model (SBM) and this is based on input and output slacks. This model allows managers to work on both inputs and outputs to achieve efficiency. Generally, in case of public hospitals it is difficult to choose the orientation (input or output) for the evaluation of efficiencies. It is not admirable to reduce input levels or increase output levels regarding public sector hospitals. So, in this study, a non-oriented and non-radial SBM-DEA model has been used [38, 140].

In order to illustrate the model, let us assume that there are n DMUs (DMU$_j$, j=1, 2… n) with m inputs (x_{ij}, i=1, 2… m) and s outputs (y_{rj}, r=1, 2… s) for each DMU. Let u_i and v_j are the weights corresponding to the i^{th} input and r^{th} output. Then the SBM-DEA model can be described as follows:

$$\left.\begin{aligned}
& Min\rho_k = \frac{1-(1/m)\sum_{i=1}^{m} s_{ik}^{-}/x_{ik}}{1+(1/s)\sum_{r=1}^{s} s_{rk}^{+}/y_{rk}} \\
& Subject\ to \\
& \qquad \sum_{j=1}^{n} \lambda_{jk} x_{ij} + s_{ik}^{-} = x_{ik}, \forall i \\
& \qquad \sum_{j=1}^{n} \lambda_{jk} y_{rj} - s_{rk}^{+} = y_{rk}, \forall r
\end{aligned}\right\} \qquad Model\, 5.1$$

$$\lambda_{jk} \geq 0, s_{ik}^{-} \geq 0, s_{rk}^{+}, \forall i, r, j, k$$

The used notations in the model are described in Table 5.1.

Table 5.1: Description of Notations used in Model 5.1

Symbol	**Description**
n	Total number of DMUs (Hospitals)
m	Total number of inputs
s	Total number of outputs
i	Index of input
r	Index of output
j	Index for DMU
k	Index of specific DMU whose efficiency is being assessed
x_{ik}	Observed amount of the i^{th} input of the k^{th} hospital
y_{rk}	Observed amount of the r^{th} output of the k^{th} hospital
λ_{jk}	Multipliers used for computing linear combinations of inputs and outputs in the assessment of k^{th} hospital
ρ	The efficiency score of a hospital by SBM model
$\rho*$	The optimal efficiency score of a hospital by SBM model
s_{ik}^{-}	Non-negative slack or potential reduction of the i^{th} input for the k^{th} hospital
s_{rk}^{+}	Non-negative slack or potential increase of the r^{th} output for the k^{th} hospital
s_{ik}^{-*}	Optimal slack to identify an excess utilization of the i^{th} input for the k^{th} hospital
s_{ik}^{-*}	Optimal slack to identify a shortage utilization of the r^{th} output for the k^{th} hospital
$\overline{x_{ik}}$	Target for the i^{th} input of the k^{th} hospital after evaluation
$\overline{y_{rk}}$	Target for the r^{th} output of the k^{th} hospital after evaluation
λ_{jk}^{*}	Optimal value of λ_{jk}

In the objective function of Model 5.1, the numerator value evaluates the mean reduction rate of inputs or input inefficiency of k^{th} hospital. Similarly, the reciprocal of denominator evaluates the mean expansion rate of outputs or output inefficiency of k^{th} hospital. Thus, the value of ρ_k can be interpreted as the product of input and output inefficiencies. This model is known as SBM-CRS model [140].

Model 5.1 is a fractional programming problem. The theory of fractional linear programming [38] makes it possible to replace Model 5.1 with an equivalent linear programming problem. For this, let us multiply a scalar variable $t > 0$ to both the numerator and denominator of Model 5.1. This causes no change in ρ_k. We adjust t so that the denominator becomes 1. This gives the new constraint as:

$$t + (1/s)\sum_{r=1}^{s} ts_{rk}^{+} / y_{rk} = 1 \tag{5.1}$$

So, the objective is to minimize the numerator. Thus we have the following model:

$$\left.\begin{array}{l} Min\,\tau_k = t - (1/m)\sum_{i=1}^{m} ts_{ik}^{-} / x_{ik} \\ Subject\,to \\ \qquad t + (1/s)\sum_{r=1}^{s} ts_{rk}^{+} / y_{rk} = 1 \\ \qquad \sum_{j=1}^{n} \lambda_{jk} x_{ij} + s_{ik}^{-} = x_{ik}, \forall\, i \\ \qquad \sum_{j=1}^{n} \lambda_{jk} y_{rj} - s_{rk}^{+} = y_{rk}, \forall\, r \end{array}\right\} \qquad Model\,5.2$$

$$\lambda_{jk} \geq 0, s_{ik}^{-} \geq 0, s_{rk}^{+}, \forall i, r, j, k \;\&\, t > 0$$

Model 5.2 is a non-linear programming problem since it contains the non-linear terms ts_{ik}^{-} and ts_{rk}^{+}. Let us transform Model 5.2 into a linear programming problem. Let $S_{ik}^{-} = ts_{ik}^{-}, S_{ik}^{+} = ts_{ik}^{+}$ and $\delta = t\lambda$ then Model 5.2 becomes the following linear programming problem in $t,\; S_{ik}^{-}, S_{ik}^{+}$ *and* δ :

$$\left.\begin{array}{l} Min\,\tau_k = t - (1/m)\sum_{i=1}^{m} S_{ik}^{-} / x_{ik} \\ Subject\,to \\ \qquad t + (1/s)\sum_{r=1}^{s} S_{rk}^{+} / y_{rk} = 1 \\ \qquad \sum_{j=1}^{n} \delta_{jk} x_{ij} + S_{ik}^{-} = t x_{ik}, \forall\, i \\ \qquad \sum_{j=1}^{n} \delta_{jk} y_{rj} - S_{rk}^{+} = t y_{rk}, \forall\, r \end{array}\right\} \qquad Model\,5.3$$

$$\delta_{jk} \geq 0, S_{ik}^{-} \geq 0, S_{rk}^{+}, \forall i, r, j, k \ \& \ t > 0$$

Let an optimal solution of Model 5.3 be $(\rho^{*}, t^{*}, \delta^{*}, S_{ij}^{-*}, S_{ij}^{+*})$. Then the optimal solution of Model 5.1 is given by $\rho^{*} = \tau^{*}$, $\lambda^{*} = \delta^{*} / t^{*}$, $s_{ij}^{-*} = S_{ij}^{-*} / t^{*}$, $s_{ij}^{+*} = S_{ij}^{+*} / t^{*}$.

The interpretation of results of the Model 5.1 can be given as follows:

The k^{th} hospital is said to be Pareto efficient if all slacks are zero, i.e., $s_{ik}^{-*} = s_{rk}^{+*} = 0$ for all *i* and *r*, which is equivalent to $\rho_k^{*} = 1$.

The non-zero slacks and (or) $\rho_k^{*} \leq 1$ identify the sources and amount of any inefficiency that may exist in the k^{th} hospital. The reference set shows how input can be decreased and output can be increased to make the k^{th} hospital efficient.

The OTE for every sample hospital is calculated by using Model 5.1. The detailed information of SBM-CRS-DEA results is given in Table 5.1.

Pure technical efficiency (PTE) for every sample hospital is estimated by using Model 5.3 through adjoining the convexity constraint $\sum_{j=1}^{n} \delta_{jk} = 1$. And then scale efficiency (SE) is calculated for every hospital using SE = OTE / PTE.

If the optimal value λ_{jk}^{*} of λ_{jk} is non-zero, then the j^{th} hospital represents the reference set (peers) for the k^{th} hospital and the corresponding optimal value is known as the peer weight of the j^{th} hospital.

However some results are proven for the use of SBM-DEA model.

1. A DMU is said to be SBM-efficient if and only if $\rho^*=1$, i.e., when there is no input excess and no output shortfall in an optimal solution.
2. A DMU can become efficient and improve its performance by deleting excess inputs and augmenting the output shortfalls.
3. The optimal SBM efficiency score ρ^* for any DMU is not greater than the optimal CCR efficiency score θ^* [38].

The results of SBM-CRS and SBM-VRS models are calculated using MATLAB.

5.3 Data and Variables

In this Chapter, we use the same data and variables as have been used in Chapter 4.

5.4 Results and Discussions

The efficiency scores (OTE, PTE and SE) of 36 public hospitals have been estimated for the year 2011. Table 5.2 presents the efficiency scores obtained from SBM-CRS model and Table 5.3 from SBM-VRS model along with reference sets and peer weights of the sample hospitals. The DEA analysis evaluates the set of hospitals, which construct the efficiency frontier. The hospitals achieving the efficiency score equal to 1.00 constitute the efficiency frontier and those having the value less than 1.00 are inefficient and lie under the efficiency frontier.

5.4.1 Overall Technical Efficiency (OTE)

Table 5.2 evinces that out of 36 hospitals 10 hospitals (H3, H5, H7, H12, H14, H15, H17, H18, H26 and H27) are relatively overall technical efficient as their efficiency score are equal to one (OTE = 1), and thus they form the efficiency frontier. The remaining 26 hospitals are inefficient as they have efficiency scores less than 1 (OTE < 1). These (efficient) hospitals are on the best practice frontier and thus form the "reference sets", i.e., these hospitals can set an example of good operating practice for the remaining 26 inefficient hospitals to emulate. The average OTE score works out to be 54.10%, which reveals that on average a hospital can reduce its resources or increase output by 45.90% to become efficient.

Table 5.2: Efficiency Scores of Hospitals by SBM-DEA CRS based Model

Code	OTE	Reference Set	Peer Weights	RTS	Peer Count
H1	0.375	H5, H7	0.500, 0.750	DRS	0
H2	0.657	H5, H7	1.00, 2.00	DRS	0
H3	1	H3	1	CRS	0
H4	0.374	H7, H12, H14	0.002, 0.704, 0.308	DRS	0
H5	1	H5	1	CRS	10
H6	0.574	H7, H14	0.500, 2.667	DRS	0
H7	1	H7	1	CRS	22
H8	0.353	H5, H7	0.500, 0.250	IRS	0
H9	0.669	H5, H7, H14	0.820, 0.211, 0.112	DRS	0
H10	0.246	H12, H14	1.690, 0.021	DRS	0
H11	0.487	H7, H12, H14	0.333, 0.167, 0.167	IRS	0
H12	1	H12	1	CRS	14
H13	0.241	H7	0.333	IRS	0
H14	1	H14	1	CRS	15
H15	1	H15	1	CRS	0
H16	0.336	H7, H12	0.500, 0.750	DRS	0
H17	1	H17	1	CRS	0
H18	1	H18	1	CRS	0
H19	0.173	H7	0.500	IRS	0
H20	0.460	H5, H12, H14	0.411, 1.401, 0.144	DRS	0
H21	0.174	H5, H7	1.00, 0.667	DRS	0
H22	0.314	H7, H14	0.500, 0.250	IRS	0
H23	0.319	H7	0.400	IRS	0
H24	0.373	H5, H7, H12, H14	0.102, 0.075, 2.278, 0.026	DRS	0
H25	0.208	H7	0.600	IRS	0
H26	1	H26	1	CRS	1
H27	1	H27	1	CRS	0
H28	0.391	H7, H12, H14	0.092, 0.115, 0.249	IRS	0
H29	0.214	H5, H12	0.001, 1.151	DRS	0
H30	0.245	H7, H12, H14	0.137, 2.780, 0.133	DRS	0
H31	0.495	H7, H12	2.00, 1.00	DRS	0
H32	0.212	H7, H12, H14	0.108, 2.124, 0.021	DRS	0
H33	0.269	H7, H12, H14	0.245, 2.115, 0.164	DRS	0
H34	0.393	H7, H12, H14	0.018, 1.373, 0.159	DRS	0
H35	0.273	H5, H12, H14	0.155, 1.213, 0.070	DRS	0
H36	0.669	H5, H7, H14, H26	0.022, 0.137, 0.309, 0.148	IRS	0
Mean	0.541				

Source: Authors' Calculation

The hospital H19 is the most technical inefficient hospital as its efficiency is found to be 17.30%. Among the inefficient hospitals only 4 hospitals H2, H6, H9 and H36 have the efficiency score above the average efficiency score.

3.4.2 Pure Technical Efficiency (PTE)

SBM-CRS model is based on the assumption of constant returns to scale (CRS) which does not consider the scale size of hospital to be relevant in assessing OTE. Therefore, in order to know whether inefficiency in any hospital is due to the inefficient production operation or due to unfavorable conditions displayed by the size of the hospital, SBM-VRS model is also applied. SBM-VRS efficiency (PTE) is always greater than or equal to SBM-CRS efficiency (OTE). Hence number of hospitals on the frontier under SBM-VRS model is always greater than or equal to the number of hospitals on the frontier under SBM-CRS model.

The information about the results drawn from SBM-VRS model is also shown in Table 5.3. It is evident from the Table that out of 36 hospitals 18 (50%) are pure technical efficient (VRS score =1), while remaining 18 hospitals are inefficient as they scored efficiency score less than 1. The efficiency score obtained by this model measures how efficiently inputs are converted into output(s) irrespective of the size of the hospitals. The average PTE is worked out to be 73.80%. This means that given the scale of operation, on average, hospitals can reduce their inputs or increase outputs by 26.20% of their observed level to become pure technical efficient.

We observe that H2, H6, H19, H21, H28, H29, H31 and H36 are overall technical inefficient but pure technical efficient. This clearly evinces that these hospitals are able to convert its inputs into outputs with 100 percent efficiency, but their OTE is low due to their scale-size.

5.4.3 Scale Efficiency

A comparison of the results for SBM-CRS and SBM-VRS gives an assessment of whether the size of a hospital has an influence on its OTE or not. Scale efficiency (SE) is the ratio of OTE to PTE scores. If SE is less than one, then the hospital appears either small or big relative to its optimum scale-size. The SE score of the hospitals is shown in the last column of Table 5.4.

Table 5.3: Efficiency Scores of Hospitals by SBM-DEA VRS based Model

Code	PTE	Reference Set	Peer Weight	RTS	Peer Count
H1	0.392	H5, H7, H12	0.0002, 0.0001, 0.0007	IRS	0
H2	1.00	H2	1	CRS	6
H3	1.00	H3	1	CRS	2
H4	0.724	H12, H14, H21, H28	0.125, 0.093, 0.107, 0.677	DRS	0
H5	1.00	H5	1	CRS	7
H6	1.00	H6	1	CRS	0
H7	1.00	H7	1	CRS	2
H8	0.476	H12, H14, H21	0.608, 0.194, 0.199	DRS	0
H9	0.673	H5, H7, H12, H14,	0.737, 0.153, 0.048, 0.064	DRS	0
H10	0.537	H12, H14, H21	0.264, 0.088, 0.649	DRS	0
H11	0.530	H3, H12, H26	0.0001, 0.0007, 0.0002	IRS	0
H12	1.00	H12	1	CRS	17
H13	0.414	H2, H12, H21	0.0002, 0.0004, 0.0003	IRS	0
H14	1.00	H14	1	CRS	7
H15	1.00	H15	1	CRS	0
H16	0.586	H5, H7	0.0002, 0.0001	IRS	0
H17	1.00	H17	1	CRS	0
H18	1.00	H18	1	CRS	0
H19	1.00	H19	1	CRS	0
H20	0.505	H5, H12, H14, H21, H27	0.169, 0.615, 0.087, 0.099, 0.05	DRS	0
H21	1.00	H21	1	CRS	9
H22	0.418	H12, H28	0.0006, 0.0004	IRS	0
H23	0.453	H2, H12	0.0008, 0.0002	IRS	0
H24	0.385	H2, H3, H5, H12, H26	0.038, 0.03, 0.034, 0.896, 0.005	DRS	0
H25	0.223	H12	0.0001	IRS	0
H26	1.00	H26	1	CRS	5
H27	1.00	H27	1	CRS	2
H28	1.00	H28	1	CRS	3
H29	1.00	H29	1	CRS	0
H30	0.289	H2, H5, H12, H21	0.087, 0.103, 0.687, 0.127	DRS	0
H31	1.00	H31	1	CRS	0
H32	0.394	H2, H12, H21, H26	0.781, 0.029, 0.060, 0.133	DRS	0
H33	0.297	H2, H3, H12, H26	0.114, 0.051, 0.765, 0.069	IRS	0
H34	0.592	H12, H14, H21, H26, H28	0.463, 0.049, 0.229, 0.026, 0.237	DRS	0
H35	0.696	H5, H12, H14, H21, H27	0.0221, 0.093, 0.135, 0.688, 0.064	IRS	0
H36	1.00	H36	1	CRS	0
Mean	0.738				

Source: Author’s Calculation

Results also show that out of 36 hospitals, 10 hospitals are scale efficient while remaining 26 hospitals are scale inefficient. The average SE is found to be 74.20%, which indicates that on average a hospital may be able to decrease its inputs or increase its outputs by 25.80% beyond its best practice targets under variable returns to scale (VRS), if it were to operate at constant returns to scale (CRS).

5.4.4 OTE, PTE and SE of Hospitals

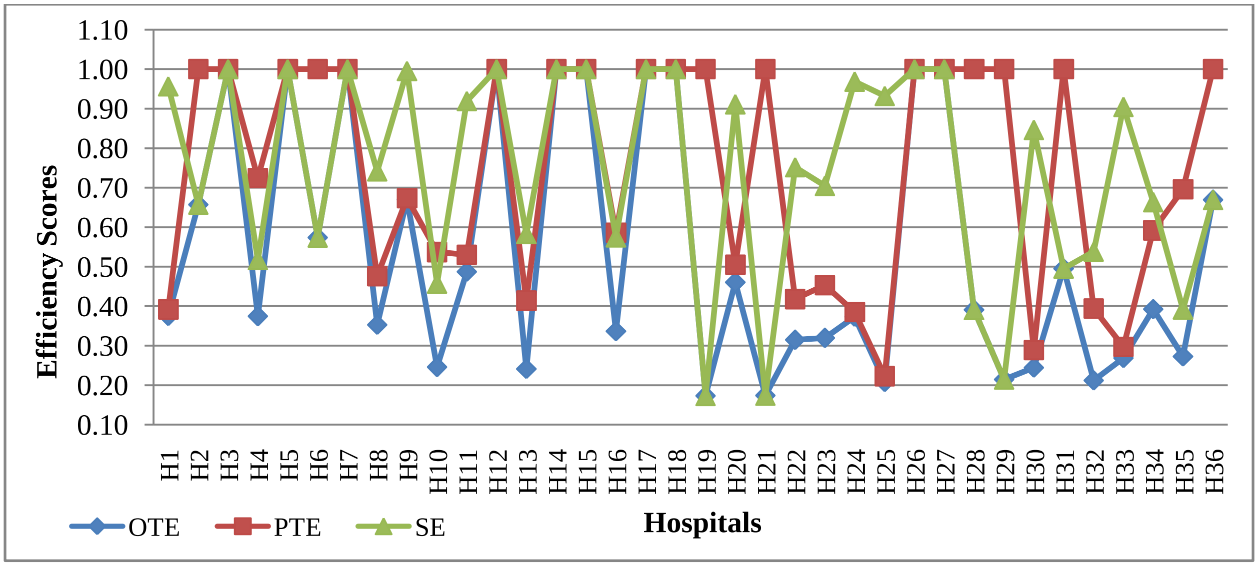
The OTE, PTE and SE of the hospitals obtained by SBM-DEA models are summarized in Table 5.4. A comparison between OTE, PTE and SE of the hospitals is given in Figure 5.1. The results show that there is no clear pattern of the efficiencies across the hospitals. The results also evince that the hospitals are overall technically inefficient due to their scale-size. The PTE results clearly evince that these hospitals are able to convert their inputs into outputs with 100% efficiency, but their OTE is low due to their scale-size.

Table 5.4: OTE, PTE and SE of the Hospitals by SBM Model

Code	OTE	PTE	SE	Code	OTE	PTE	SE
H1	0.375	0.392	0.957	H19	0.173	1.000	0.173
H2	0.657	1.000	0.657	H20	0.460	0.505	0.911
H3	1.000	1.000	1.000	H21	0.174	1.000	0.174
H4	0.374	0.724	0.517	H22	0.314	0.418	0.752
H5	1.000	1.000	1.000	H23	0.319	0.453	0.704
H6	0.574	1.000	0.574	H24	0.373	0.385	0.968
H7	1.000	1.000	1.000	H25	0.208	0.223	0.932
H8	0.353	0.476	0.741	H26	1.000	1.000	1.000
H9	0.670	0.673	0.995	H27	1.000	1.000	1.000
H10	0.246	0.537	0.457	H28	0.391	1.000	0.391
H11	0.487	0.530	0.919	H29	0.214	1.000	0.214
H12	1.000	1.000	1.000	H30	0.245	0.289	0.846
H13	0.241	0.414	0.582	H31	0.495	1.000	0.495
H14	1.000	1.000	1.000	H32	0.212	0.394	0.538
H15	1.000	1.000	1.000	H33	0.269	0.297	0.904
H16	0.336	0.586	0.574	H34	0.393	0.592	0.663
H17	1.000	1.000	1.000	H35	0.273	0.696	0.392
H18	1.000	1.000	1.000	H36	0.669	1.000	0.669
				Mean	0.541	0.738	0.733

Source: Author's Calculation

Figure 5.1: Comparison between OTE, PTE and SE of the Hospitals under SBM



5.4.5 Input and Output Slacks for Inefficient Hospitals

The study of input-output slacks is important because it provides additional insights about the extent of inefficiency for the inefficient hospitals. The extent of inefficiency is given by the quantity of excess inputs used (input slack) and/or deficient output produced (output slack) by inefficient hospitals. Excess input consumption and/or deficient output production must be eliminated before a given hospital is said to be relatively efficient, compared to its reference set of the hospital. The magnitude of estimated input and output slacks under CRS assumption are given in Table 5.5. The optimal input and output slacks for inefficient hospitals, calculated by SBM-CRS model, are given in Table 5.5.

The slacks in input variables show the underutilization of inputs. For instance H1 has underutilized Beds by 11.06% and PMS by 26.81%. On average inefficient hospitals are not utilizing their 12.67 beds (8.99%), 2.74 doctors (12.63%) and 6.21 paramedical staff (11.99%).

Table 5.5: Slacks in Inputs and Outputs under SBM-CRS Model

Code	Input 1	Input 2	Input 3	Output 1	Output 2	Output 3	Output 4
H1	29.75	0.00	19.75	0.00	13710.00	989.25	1322.00
H2	0.00	1.00	0.00	13321.00	364.00	164.00	0.00
H3	0.00	0.00	0.00	0.00	0.00	0.00	0.00
H4	0.00	4.48	15.41	0.00	0.00	638.97	298.31
H5	0.00	0.00	0.00	0.00	0.00	0.00	0.00
H6	96.67	0.00	17.17	0.00	3238.00	1184.00	1723.33
H7	0.00	0.00	0.00	0.00	0.00	0.00	0.00
H8	38.25	0.00	10.25	0.00	10731.75	637.50	507.75
H9	29.37	0.00	20.85	0.00	0.00	229.74	371.40
H10	0.00	1.78	7.16	0.00	1609.80	1436.46	811.81
H11	0.00	4.33	21.33	0.00	6360.00	0.00	358.67
H12	0.00	0.00	0.00	0.00	0.00	0.00	0.00
H13	41.67	4.33	0.00	0.00	9875.67	733.00	701.33
H14	0.00	0.00	0.00	0.00	0.00	0.00	0.00
H15	0.00	0.00	0.00	0.00	0.00	0.00	0.00
H16	0.00	7.50	1.50	18619.75	0.00	1324.75	912.00
H17	0.00	0.00	0.00	0.00	0.00	0.00	0.00
H18	0.00	0.00	0.00	0.00	0.00	0.00	0.00
H19	16.00	2.00	8.00	9137.00	5690.00	453.50	144.50
H20	0.00	1.03	0.00	0.00	599.46	1179.91	842.40
H21	13.33	4.00	5.67	7442.33	2236.67	224.00	63.00
H22	0.00	9.50	8.00	0.00	7724.75	803.00	733.50
H23	33.20	4.40	15.20	10881.20	4341.40	540.00	263.00
H24	0.00	0.24	0.00	0.00	0.00	1944.82	1426.06
H25	126.40	18.80	38.00	6695.80	9204.60	677.20	906.40
H26	0.00	0.00	0.00	0.00	0.00	0.00	0.00
H27	0.00	0.00	0.00	0.00	0.00	0.00	0.00
H28	0.00	5.61	5.51	0.00	0.00	344.28	183.60
H29	15.31	13.92	0.00	0.00	1093.97	954.72	490.45
H30	7.32	0.00	5.02	0.00	0.00	2688.51	2039.40
H31	9.00	1.00	0.00	18167.00	0.00	186.00	87.00
H32	0.00	0.26	13.58	0.00	0.00	2075.26	1483.46
H33	0.00	6.61	8.48	0.00	0.00	2311.38	1740.11
H34	0.00	2.12	2.81	0.00	0.00	1141.37	695.03
H35	0.00	1.76	0.00	0.00	1814.69	1105.07	695.66
H36	0.00	3.87	0.00	0.00	0.00	290.12	96.47
Mean	12.67	2.74	6.21	2340.67	2183.19	673.79	524.91

Source: Authors' Calculation

5.4.6 Targets for Inefficient Hospitals

When a hospital is inefficient, DEA allows setting targets of its inputs and outputs so that it can improve its performance. Thus, each of the inefficient hospitals can become overall technical efficient by adjusting its operation to the associated target point determined by the efficient hospitals that define its reference frontier.

Input and output targets, according to the SBM-DEA model, can be set by using the relations given in Equations (5.2) and (5.3) respectively.

$$\overline{x_{ik}} = x_{ik} - s_{ik}^{-*} \tag{5.2}$$

$$\overline{y_{rk}} = y_{rk} + s_{rk}^{+*} \tag{5.3}$$

where the used notations in Equations (5.2) and (5.3) are given in Table 5.1. The optimal input and output slacks for inefficient hospitals are given in Table 5.5. So, we can measure the targets for inputs and outputs for all inefficient hospitals. Table 5.6 presents the target values of all inputs and outputs for inefficient hospitals along with percentage reduction in inputs and percentage augmentation in outputs. It can be observed from Table 5.6 that on average a hospital has significant scope to reduce the inputs and expand the outputs relative to the best practice hospital.

Table 5.6: Percentage Reduction in Inputs and Augmentation in Outputs for Inefficient Hospitals under SBM-CRS Model

	Inputs			Outputs			
Code	Input 1	Input 2	Input 3	Output 1	Output 2	Output 3	Output 4
H1	22.54	0.00	32.92	0.00	225.05	124.75	201.22
H2	0.00	14.29	0.00	73.46	17.23	36.69	0.00
H4	0.00	29.86	41.65	0.00	0.00	351.08	63.07
H6	24.05	0.00	12.26	0.00	16.19	98.50	121.11
H8	31.88	0.00	26.97	0.00	334.74	158.58	75.45
H9	19.58	0.00	37.90	0.00	0.00	34.70	48.11
H10	0.00	12.70	23.87	0.00	80.85	784.95	164.33
H11	0.00	24.07	41.03	0.00	212.92	0.00	41.46
H13	41.67	27.08	0.00	0.00	616.84	172.88	131.34
H16	0.00	34.09	4.69	41.70	0.00	421.89	148.05
H19	42.11	28.57	42.11	53.80	1173.20	242.51	31.14
H20	0.00	5.16	0.00	0.00	7.30	305.68	141.58
H21	55.56	66.67	51.52	135.54	417.29	294.74	27.27
H22	0.00	39.58	19.05	0.00	249.83	224.30	133.61

H23	48.12	36.67	46.06	41.11	122.22	165.64	41.88
H24	0.00	1.18	0.00	0.00	0.00	425.56	243.35
H25	63.20	55.29	51.35	10.86	189.90	78.56	127.48
H28	0.00	43.15	25.06	0.00	0.00	334.25	57.20
H29	30.62	63.25	0.00	0.00	84.09	672.34	126.08
H30	6.10	0.00	9.29	0.00	0.00	765.96	386.25
H31	14.52	11.11	0.00	52.82	0.00	101.64	19.68
H32	0.00	1.43	28.29	0.00	0.00	951.95	346.60
H33	0.00	22.04	15.14	0.00	0.00	623.01	281.57
H34	0.00	14.13	10.05	0.00	0.00	401.89	134.70
H35	0.00	12.55	0.00	0.00	60.35	762.11	182.59
H36	0.00	25.78	0.00	0.00	0.00	128.94	17.80
Mean	12.57	13.16	14.04	17.53	66.55	208.23	110.73

Source: Authors' Calculation

5.5 Stability of Efficiency Scores (Jackknifing Analysis)

DEA is run, after dropping out the most efficient firm with the highest peer count one at a time, in order to test whether there are extreme outliers which may have affected the frontier and efficiency scores. The procedure, known as Jackknifing Analysis, tests the robustness of DEA results in regard to outliers [91]. In this analysis, five hospitals viz., H7, H14, H12, H5 and H26 which have peer counts 22, 15, 14, 10 and 1 respectively are dropped, one at a time. In the case of CRS assumption, to measure the change in efficiency scores and ranking of hospitals, we calculate Karl Pearson and Spearman rank correlation coefficient of OTE scores under five analyses, such as, JA1, JA2, JA3, JA4 and JA5. In JA1 analysis, we have included all 36 hospitals and calculated OTE scores. In further analysis like JA2, JA3, JA4, JA5 and JA6 the excluded hospitals are H7, H14, H12, H5 and H26 respectively. Karl Pearson and Spearman Rank correlation coefficients are given in Table 5.7 and Table 5.8 respectively.

It is observed that Karl Pearson coefficient of correlation ranges from 0.918 to 1.00. It suggests that the efficiency scores are stable even after the exclusion of the most efficient hospitals. In addition, the high and positive values of rank correlation coefficients (0.957 to 1.00) show that the rankings of hospitals are stable.

Table 5.7: Karl Pearson's Coefficients of Correlation

	JA1	JA2	JA3	JA4	JA5	JA6
JA1	1					
JA2	0.951*	1				
JA3	0.962*	0.933*	1			
JA4	0.982*	0.924*	0.973*	1		
JA5	0.964*	0.983*	0.918*	0.940*	1	
JA6	1.00*	0.951*	0.962*	0.981*	0.961*	1

*Significant at 1% level of significance

Table 5.8: Spearman Rank Correlation of Coefficients

	JA1	JA2	JA3	JA4	JA5	JA6
JA1	1					
JA2	0.974*	1				
JA3	0.987*	0.966*	1			
JA4	0.989*	0.957*	0.974*	1		
JA5	0.988*	0.981*	0.973*	0.973*	1	
JA6	1.00*	0.974*	0.988*	0.988*	0.988*	1

*Significant at 1% level of significance

5.6 Summing Up

This chapter estimates OTE, PTE and SE through a non-oriented model using cross-sectional data of 36 government hospitals in Uttarakhand for the year 2011. CCR and BCC model gives the results either towards input reduction or towards output augmentation. On contrast SBM model give the results in both input reduction and output augmentation. Thus, this Chapter suggests the required improvements in inputs and outputs. Jackknifing Analysis is also conducted to check the stability of efficiency scores.

- The study finds that 10 (27.78%) hospitals have the maximum degree of OTE.
- The average OTE (54.10%) of the hospitals indicates that on average 45.90% of the technical potential of hospitals is not in use, i.e., these hospitals have the scope of producing the more outputs with lesser inputs than their existing level.
- The results of SBM-VRS model show that out of 36 hospitals, 18 (50%) are pure technical efficient as they efficiently convert their inputs into outputs.

- However, out of 18 pure technical efficient hospitals, 8 hospitals are technically inefficient due to scale-size effect.
- The hospital H19 has the least SE (17.30%), implying that H19 has the maximum effect of scale-size of its efficiency score. It indicates that this hospital can improve its OTE by enhancing its scale of operation.
- The target setting results show that all the inputs have the significant scope of reduction and outputs have significant scope of augmentation.
- On average, inefficient hospitals may be able to reduce 12.57% of beds, 13.16% of doctors, 14.04% of paramedical staff, and to expand 17.53% of out-door patients, 66.55% of in-door patients, 208.23% of major surgeries and 110.73% of minor surgeries if they operate at the level of efficient hospitals.
- Karl Pearson coefficient of correlation ranges from 0.918 to 1.00 at 1% level of significance. It suggests that the efficiency scores are stable even after the exclusion of the most efficient hospitals.
- In addition, high and positive values of Spearman rank correlation coefficients, (0.957 to 1.00) show that the rankings of hospitals are stable.

Chapter 6

Estimating the Impact of Slacks on Efficiencies: A New Slack Model

6.1 Introduction

In this Chapter we apply a new slack model (NSM) to determine the impact of slacks on efficiency scores. We measure the relative efficiencies and slacks corresponding to selected inputs and outputs of the hospitals, set benchmarks for relative inefficient hospitals to improve their efficiencies, and also determine region-wise efficiencies of the hospitals. In this Chapter, efficiency of the same 36 hospitals, as in previous chapters, is assessed by using new slack model (NSM) and also identifies the slacks and sets the benchmarks for the inefficient hospitals. The region-wise performance of the hospitals has also been assessed.

6.2 Impact of Slacks on Efficiencies: An Illustration

A DMU with unit efficiency score and with no slack in any optimal solution is called efficient. Otherwise, it has a disadvantage against the other DMUs in the reference set. Thus, in discussing the total efficiency of a DMU, it is important to observe both the efficiency score and the slacks. The impact of slacks on the efficiency is illustrated for the two input case in Figure 6.1, where we use a simple example involving five DMUs which use two inputs x_1 and x_2 to produce a single output y_1, under the assumption of constant returns to scale. Consider case with two efficient DMUs that form the frontier, and three inefficient DMUs A, B and C. The vertical and horizontal boundaries DI and EI` beyond D and E, respectively, are not fully efficient. The three inefficient DMUs can radially reduce their inputs and move on the frontier if they adjust to their projected points A`, B` and C`, respectively. The technical efficiencies of A, B and C are given as OA`/OA, OB`/OB and OC`/OC, respectively. For DMU B, this movement is sufficient for it to become technical efficient. However, the point A` is not an efficient point since one could reduce the amount of input x2 used (by the amount CA`) and still produce the same output. This is known as input slack or input excess. Similarly, the output slack or output shortfall may also be identified.

Slacks are also considered in the CCR and BCC models with the coefficient of ε (non-Archimeden constant) in their objective functions. However, the numeric values for ε in computations are chosen to be much smaller than input and output values so that they may not affect optimization. Later on, many researchers [10, 132, 140] gave the different models to accumulate the impact of slacks on the efficiency scores. These models have been discussed in Chapter 3.

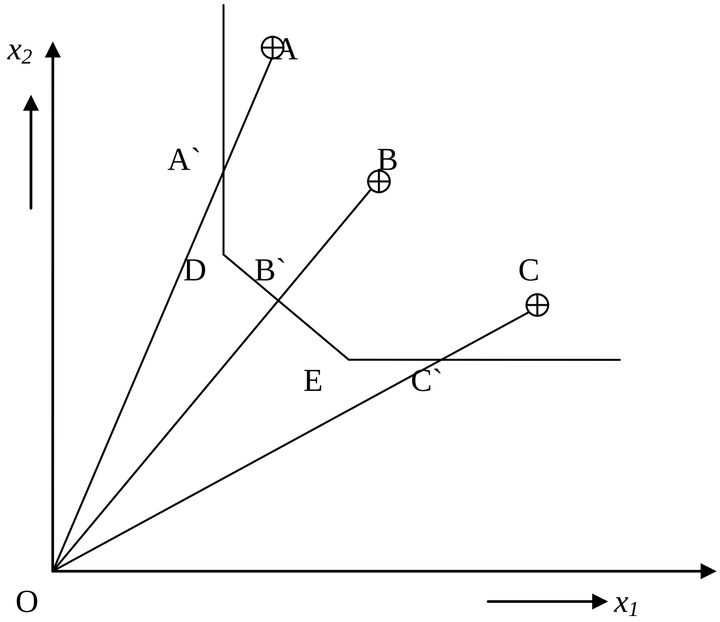


Figure 6.1: Concept of Slacks

There are two major problems associated with the two-stage LPP [9, 10]. The first problem is that the sum of slacks is maximized rather than minimized. Hence, it identifies the farthest efficient point not the nearest point. The second problem is that it is not a unit invariant. The additive and SBM models are non-radial models which deal only with the input and output slacks. They have no means to measure the depth of inefficiency. They do not deal with the proportional reduction of inputs and proportional augmentation of outputs which is the basic property of DEA models. SBM input-oriented model does not deal with the output slacks[1]. Contrary to it, Slack Adjusted Model deals with both radial inefficiency and slack inefficiency, but it measures the minimum impact of slacks on efficiency scores, not the actual impact of slacks, as it takes the maximum values of inputs and outputs in the denominator of the slack term in the objective function. To overcome these problems in the above stated models, we use New Slack Model with output-orientation, which directly deals with the radial efficiency and the slack term. It also handles the actual impact of slacks on efficiency scores instead of minimum impact.

6.3 New Slack Model (NSM)

The new slack model is developed by Agarwal et al. [6]. They use this model as an input-orientation in case of Uttar Pradesh State Road Transport (UPSRT). This model measures the efficiencies of the DMUs with the actual impact of slacks on these efficiency scores and in which all the inputs and outputs are utilized in the performance assessment. In this Chapter, we use the output-orientated new slack model. The detailed description of the model is given below:

Let us consider n hospitals with m inputs (x_{ij}: $i=1,\ldots,m$) and s outputs (y_{rj} : $j= 1,\ldots,s$) for measuring their technical efficiency. As shown in Chapter 3, the total output augmentation of the k^{th} hospital can be measured by

$$\overline{y}_{rk} = \sum_{j=1}^{n} \lambda_{jk} y_{rj} = \theta_k^* y_{rk} + s_{rk}^+ \tag{6.1}$$

where $\overline{y}_{rk}$ is the total potential of the r^{th} output; s_{rk}^+ is the value of slack in the optimal solution of the output-oriented CCR-DEA model. The total output produced efficiency measure $\overline{\theta}_{rk}$ is defined as the ratio of the potential produced by an output with the observed output.

$$\overline{\theta}_{rk} = \frac{\theta_k^* y_{rk} + s_{rk}^+}{y_{rk}} = \frac{\overline{y}_{rk}}{y_{rk}} \tag{6.2}$$

From (6.1) and (6.2), we get that

$$\overline{\theta}_{rk} = \frac{\overline{y}_{rk}}{y_{rk}} = \frac{\theta_k^* y_{rk} + s_{rk}^+}{y_{rk}} = \theta_k^* + \frac{s_{rk}^+}{y_{rk}} \tag{6.3}$$

Thus, the total output produced efficiency of the k^{th} hospital can be assessed by adding its inefficiency due to output slack in radial efficiency.

Thus the term s_{rk}^{+} / y_{rk} corresponds to the inefficiency in the k^{th} hospital due to the output shortfall in the r^{th} output. Similarly, we can define s_{ik}^{-} / x_{ik} as the inefficiency due to the existence of the slack in the i^{th} input of the k^{th} hospital. So, the term $\frac{1}{m+s}\left(\sum_{r=1}^{s}\frac{s_{rk}^{+}}{y_{rk}}+\sum_{i=1}^{m}\frac{s_{ik}^{-}}{x_{ik}}\right)$ measures the mean efficiency due to slack in all inputs and outputs. It can also calculate the mean reduction rate of all inputs and augmentation rate of all outputs. Thus the total output produced efficiency due to radial part and the slack part of all the inputs and outputs is defined by

$$\bar{\theta}_k = \theta_k + \frac{1}{m+s}\left(\sum_{r=1}^{s}\frac{s_{rk}^{+}}{y_{rk}}+\sum_{i=1}^{m}\frac{s_{ik}^{-}}{x_{ik}}\right) \tag{6.4}$$

Since the CCR model neglects the slacks in the evaluation of efficiencies, to overcome this shortcoming, efficiency score of the k^{th} hospital is computed using the output-oriented New Slack Model which is given as follows:

$$\left.\begin{aligned}
&Max\,\bar{\theta}_k = \theta_k + \frac{1}{m+s}\left(\sum_{r=1}^{s}\frac{s_{rk}^{+}}{y_{rk}}+\sum_{i=1}^{m}\frac{s_{ik}^{-}}{x_{ik}}\right)\\
&\quad s.t. \quad \sum_{j=1}^{n}\lambda_{jk}y_{rj} - s_{rk}^{+} = \theta_k y_{rk} \qquad \forall\, r = 1,2,3,\ldots,s,\\
&\qquad\qquad \sum_{j=1}^{n}\lambda_{jk}x_{ij} + s_{ik}^{-} = x_{ik} \qquad \forall\, i = 1,2,3,\ldots,m,\\
&\lambda_{jk}, s_{ik}^{-}, s_{rk}^{+} \geq 0,\ \forall\, i = 1,2,3,\ldots,m;\ j = 1,2,3,\ldots,n;\ r = 1,2,3,\ldots,s,
\end{aligned}\right\} \quad Model\,6.1$$

The interpretation of the results of Model 6.1 can be summarized as follows:

3. *The k^{th} hospital is total potential efficient or NSM efficient if and only if* $\bar{\theta}_k^{*} = 1$.

The above condition is equivalent to simultaneously $\theta_k^{*} = 1$ and all slacks are zero, i.e., radially efficient and no input access and no output shortfall exists in any optimal solution.

4. *The set of indices corresponding to positive* λ_{jk}*'s is called the reference set of the k^{th} inefficient hospital. The reference set denoted by R_k is defined as:*

   $R_k = \{DMU_j : \lambda_{jk} > 0,\ j = 1,2,\ldots,n\}$

6.4 Relationship between CCR Model and NSM Model

Output-oriented CCR model is given as follows:

$$\left.\begin{aligned}
&Max\, E_k = \phi_k + \varepsilon\left(\sum_{r=1}^{s} t_{rk}^{+} + \sum_{i=1}^{m} t_{ik}^{-}\right)\\
&\quad s.t. \quad \sum_{j=1}^{n} \mu_{jk} y_{rj} - t_{rk}^{+} = \phi_k y_{rk} \qquad \forall\, r = 1,\dots,s,\\
&\qquad\qquad \sum_{j=1}^{n} \mu_{jk} x_{ij} + t_{ik}^{-} = x_{ik} \qquad \forall\, i = 1,\dots,m,\\
&\mu_{jk}, t_{ik}^{-}, t_{rk}^{+} \geq 0,\ \forall\, i = 1,\dots,m;\ j = 1,\dots,n;\ r = 1,\dots,s,
\end{aligned}\right\} \qquad Model\, 6.2$$

The optimal value ϕ_k is fully CCR efficient if the optimal objective value $\phi_k^{*} = 1$ and all optimal slacks are zero ($t_{ik}^{-}, t_{rk}^{+} = 0$) for the optimal solution of Model 6.2.

Let the optimal solution of CCR model be ($\phi_k^{*}, \mu_k^{*}, t_k^{-}, t_k^{+}$). From the constraints of this model, the optimal solution holds

$$\phi_k y_{rk} = \sum_{j=1}^{n} \mu_{jk} y_{rj} - t_{rk}^{+} \tag{6.5}$$

$$x_{ik} = \sum_{j=1}^{n} \mu_{jk} x_{ij} + t_{ik}^{-} \tag{6.6}$$

Let us define $\theta_k = \phi_k^{*}, \lambda_k = \mu_k^{*}, s_{ik}^{-} = t_{ik}^{-*}, s_{rk}^{+} = t_{rk}^{+*}$

Then we can say that the feasibility criterion for both the models (NSM and CCR) is the same which implies that $\theta_k, \lambda_k, s_{ik}^{-}, s_{rk}^{+}$ are feasible for both models. The objective function of NSM model is given as

$$Max\,\bar{\theta}_k = \theta_k + \frac{1}{m+s}\left(\sum_{r=1}^{s}\frac{s_{rk}^{+}}{y_{rk}} + \sum_{i=1}^{m}\frac{s_{ik}^{-}}{x_{ik}}\right) \tag{6.7}$$

The following theorems [6], regarding the optimality and feasibility hold for both the models.

Theorem 6.4.1: *The optimal value* $\bar{\theta}_k^*$ *of NSM model is not greater than the optimal value* θ_k^* *of CCR model i.e.,* $\bar{\theta}_k^* \le \theta_k^*$.

Theorem 6.4.2: If the k^{th} *hospital is NSM efficient, then it is CCR efficient.*

6.5 Dual of NSM Model

In the direct use of basic CCR DEA model, there is possibility of assigning relatively higher weights to a particular input or output and lower weights to other inputs or outputs. The CCR model may also assign zero values to its multipliers which indicate that its corresponding input or output is not used for the DEA efficiency evaluation. This is equivalent to ignoring that input or output which is often not acceptable from practical point of view. Such types of problems are usually handled by imposing the multiplier restriction [29] on the CR model. However, it is often observed that these approaches cannot completely solve the problem of zero multiplier. These multiplier restrictions are based on the prior knowledge of the weights of inputs and outputs which is not possible in some cases. Contrary to it, the NSM model does not depend upon these multiplier restrictions, rather it directly imposes the lower bound on the multiplier in the dual of the Model 6.3, so that all the multipliers may always be positive in the optimal solution. To explain it more clearly, the dual of Model 6.1 is as follows:

$$\left.\begin{array}{l} Min\, E_k = \sum_{i=1}^{m} u_{ik} x_{ik} \\ \quad s.t. \\ \qquad \sum_{r=1}^{s} v_{rk} y_{rk} = 1 \\ \qquad \sum_{i=1}^{m} u_{ik} x_{ik} - \sum_{r=1}^{s} v_{rk} y_{rk} \ge 0 \quad \forall j = 1,2,...n \\ \qquad u_{ik} \ge 1/(m+s) x_{ik} \qquad \forall i = 1,2,...m \\ \qquad v_{rk} \ge 1/(m+s) y_{rk} \qquad \forall r = 1,2,...s \end{array}\right\} \qquad Model\, 6.3$$

The problem of zero multiplier is solved by the last two constraints in the above model. This dual model links the efficiency evaluation with the economic interpretation. The dual variables u_{ik} and v_{rk} $(i = 1,2,...m; r = 1,2,...s; k = 1,2,...n)$ can be interpreted as the virtual costs and prices of inputs and outputs, respectively. Thus the dual model finds the virtual costs u_{ik} and prices v_{rk} so that the ratio $\frac{\sum_{r=1}^{s} v_{rk} y_{rk}}{\sum_{i=1}^{m} u_{ik} x_{ik}}$ is minimized.

6.6 Empirical Findings and Discussions

The results of CCR-DEA model is obtained by using DEAP Software developed by Coelli [35] and results of NSM model are calculated by using mathematical software MATLAB.

Table 6.1 presents the results of CCR and NSM models respectively. The results reveal that the CCR efficiency scores are greater than or equal to the NSM efficiency scores of all the hospitals (Theorem 6.3.1). It is also observed that the hospitals, which are NSM efficient are also CCR efficient, but the converse is not true (Theorem 6.3.2). The hospitals H3, H5, H14, H15, H17, H18, H26 and H27 are found CCR efficient but NSM inefficient.

Table 6.1 presents the OTE score obtained from the output-oriented NSM model under the assumption of CRS along with reference set, peer weights and reference count (peer count).

Table 6.1: NSM-OTE, CCR-OTE, Reference Set, Peer Weight, Peer Count and Returns to Scale

Code	NSM OTE	CCR OTE	Reference Set	Peer Weight	Peer Count	RTS
H1	0.561	0.601	H7	0.0001	0	IRS
H2	0.779	0.940	H12	0.0001	0	IRS
H3	0.598	1.000	H7, H12	0.0003, 0.0017	0	IRS
H4	0.434	0.856	H12	0.0018	0	IRS
H5	0.855	1.000	H12	0.0017	0	IRS
H6	0.517	0.909	H7	0.0002	0	IRS
H7	1.000	1.000	H7	1	18	CRS
H8	0.514	0.697	H7	0.0001	0	IRS
H9	0.719	0.953	H12	0.00277	0	IRS
H10	0.390	0.592	H7	0.0005	0	IRS
H11	0.645	0.882	H7	0.0001	0	IRS
H12	1.000	1.000	H12	1	24	CRS
H13	0.347	0.466	H7, H12	0.0001, 0.0006	0	IRS
H14	0.630	1.000	H7, H12	0.0001, 0.0001	0	IRS
H15	0.529	1.000	H12	0.0021	0	IRS

H16	0.496	0.723	H7, H12	0.0006, 0.0001	0	IRS
H17	0.917	1.000	H7	0.0001	0	IRS
H18	0.648	1.000	H7, H12	0.0003, 0.0007	0	IRS
H19	0.263	0.600	H7	0.0003	0	IRS
H20	0.562	0.773	H12	0.0025	0	IRS
H21	0.243	0.378	H7, H12	0.0001, 0.0005	0	IRS
H22	0.455	0.675	H7	0.0001	0	IRS
H23	0.390	0.474	H7	0.0005	0	IRS
H24	0.507	0.622	H12	0.0025	0	IRS
H25	0.247	0.253	H7	0.0001	0	IRS
H26	0.431	1.000	H12	0.002	0	IRS
H27	0.583	1.000	H12	0.0017	0	IRS
H28	0.373	0.888	H12	0.0013	0	IRS
H29	0.363	0.532	H12	0.0008	0	IRS
H30	0.367	0.553	H12	0.0035	0	IRS
H31	0.641	0.917	H12	0.0001	0	IRS
H32	0.340	0.535	H7, H12	0.0001, 0.0023	0	IRS
H33	0.394	0.623	H7, H12	0.0001, 0.033	0	IRS
H34	0.514	0.706	H12	0.002	0	IRS
H35	0.381	0.653	H12	0.0017	0	IRS
H36	0.503	0.974	H12	0.002	0	IRS
Mean	0.532	0.772				

Source: Authors’ Calculation,

Note: Where, RTS stands for returns to scale.

6.6.1 Overall Technical Efficiency (OTE)

The OTE scores are calculated through NSM output-oriented model under CRS assumption. Table 6.1 shows that out of 36 hospitals, only 2 hospitals (H7 and H12) are overall technical efficient. The remaining 34 hospitals are inefficient as they have efficiency score less than one. The two efficient hospitals (H7 and H12) are on the best practice frontier and thus form the “reference set”, i.e., these hospitals can set an example of best operating practice for the remaining 34 inefficient hospitals. For every inefficient hospital, the NSM model identifies a set of corresponding efficient hospitals which constitute a reference set. These hospitals can be used as a benchmark for improving the performance of the inefficient hospitals. If the OTE score for an inefficient hospital is lower, there is a higher scope for it to increase outputs (maintaining input level) relative to the best practice hospital(s) in the reference set.

The average OTE scores 0.532, reveals that on average hospitals can increase their output by 46.80%. Twenty two hospitals (H4, H6, H8, H10, H13, H15, H16, H19, H21, H22, H23, H24, H25, H26, H28, H29, H30, H32, H33, H34, H35 and H36) attain an efficiency score lower than the average efficiency score. The results reveal that only two hospitals (H7 and H12) are fully efficient. The OTE score of hospitals H17 is in the range of 91.7%. This hospital may be able to increase its output by 8.3%, while maintaining the same level of inputs. The hospital H21 turns out to be the least efficient hospital as its OTE score is only 24.30%.

6.6.2 Input-output Slacks

The slack analysis provides additional insights about the magnitude of inefficiency for the under-performed hospitals. The magnitude of inefficiency is given by the quantity of deficient output produced (output slacks) and/or excess resources used (input slacks) by inefficient hospitals. If a hospital does not have slacks in inputs, then it implies that the hospital has utilized its inputs efficiently. The non-zero slacks in inputs show the over-utilization and non-zero slacks in outputs show under-production. From Table 6.2, we conclude that on average inefficient hospitals have to increase their outputs: number of outdoor patients, number of indoor patients, number of major surgery and number of minor surgery by 0.645, 1.678, 1.129 and 0.801 respectively with reduction of the inputs: number of beds, number of doctors and number of paramedical staff by 0.009, 0.002 and 0.004 respectively.

Table 6.2: Input and Output Slacks for Inefficient Hospitals by NSM Model

	Input Slack			Output Slack			
Code	IS1	IS2	IS3	OS1	OS2	OS3	OS4
H1	0.004	0.000	0.001	0.000	1.490	0.154	0.174
H2	0.000	0.000	0.000	1.923	0.040	0.025	0.000
H3	0.019	0.000	0.008	0.000	0.000	2.049	1.716
H4	0.000	0.002	0.014	0.000	2.115	1.619	1.127
H5	0.053	0.004	0.000	0.000	0.036	1.501	1.116
H6	0.014	0.000	0.001	0.000	4.575	0.534	0.557
H8	0.005	0.000	0.000	0.000	1.258	0.134	0.127
H9	0.064	0.000	0.017	0.000	0.000	2.410	1.856
H10	0.000	0.003	0.004	0.000	8.790	0.990	0.775
H11	0.000	0.000	0.002	0.000	1.210	0.081	0.107
H13	0.010	0.000	0.000	0.000	7.821	0.188	0.000
H14	0.002	0.000	0.000	0.000	1.197	0.192	0.176

H15	0.000	0.011	0.019	0.000	2.168	1.877	1.351
H16	0.000	0.008	0.000	3.646	0.000	1.032	0.684
H17	0.001	0.000	0.003	0.000	2.350	0.000	0.129
H18	0.000	0.006	0.007	0.000	0.000	1.106	0.871
H19	0.004	0.000	0.002	4.120	5.921	0.423	0.000
H20	0.024	0.002	0.000	0.000	1.249	2.229	1.647
H21	0.000	0.001	0.000	9.889	1.551	0.472	0.000
H22	0.000	0.001	0.001	0.000	1.276	0.138	0.134
H23	0.011	0.000	0.004	0.000	4.031	0.570	0.000
H24	0.000	0.000	0.000	0.000	0.000	2.078	1.516
H25	0.002	0.000	0.000	0.000	1.450	0.000	0.078
H26	0.006	0.000	0.012	0.000	0.332	1.824	1.402
H27	0.074	0.008	0.000	0.000	1.135	1.559	1.197
H28	0.000	0.004	0.005	0.000	0.981	1.218	0.856
H29	0.003	0.013	0.000	0.000	2.026	0.555	0.000
H30	0.013	0.000	0.007	0.000	0.000	3.107	2.301
H31	0.003	0.000	0.000	2.341	0.000	0.104	0.073
H32	0.000	0.000	0.014	0.000	0.000	2.171	1.549
H33	0.000	0.005	0.009	0.000	0.000	3.039	2.225
H34	0.000	0.001	0.002	0.000	1.292	1.705	1.164
H35	0.009	0.002	0.000	0.000	1.676	1.523	1.061
H36	0.000	0.001	0.000	0.000	1.072	1.795	1.264
Mean	0.009	0.002	0.004	0.645	1.678	1.129	0.801

Source: Authors’ Calculation

Table 6.2 also shows that the highest output slack observed in outdoor patients is 9.889 for H21, in indoor patients it is 8.790 for H10, in major surgery, it is 3.107 for H30 and in minor surgery, it is 2.301 for H30, whereas highest slacks in inputs: number of beds, number of doctors and the number of paramedical staff are 0.074 for H27, 0.013 for H29 and 0.019 for H15 respectively.

6.6.3 Input-output Targets for Inefficient Hospitals

DEA allow setting targets for the inputs and outputs for the inefficient hospitals to improve their performance and to make them efficient. The targets using the NSM model are calculated by the same relations as in CCR model. Thus, the input-output targets for inefficient hospitals are calculated by using the relations given in Equation 3.8.

Table 6.3: Percentage Reduction and Augmentation in the Corresponding Inputs and Outputs for Inefficient Hospitals under NSM-DEA Model

Code	Number of Beds	Number of Doctors	Number of PMS	Number of Out-door Patients	Number Indoor Patients	Number of Major Surgeries	Number of Minor Surgeries
H1	0.00	0.00	0.00	78.29	78.31	78.31	78.32
H2	0.00	0.00	0.00	28.34	28.33	28.34	28.33
H3	0.02	0.00	0.02	67.10	67.10	67.59	67.42
H4	0.00	0.02	0.04	130.52	130.59	131.41	130.76
H5	0.05	0.03	0.00	16.97	16.97	17.34	17.14
H6	0.00	0.00	0.00	93.41	93.43	93.45	93.45
H8	0.00	0.00	0.00	94.62	94.66	94.65	94.64
H9	0.04	0.00	0.03	39.08	39.08	39.44	39.32
H10	0.00	0.02	0.01	156.38	156.82	156.92	156.54
H11	0.00	0.00	0.00	54.96	55.00	54.97	54.97
H13	0.01	0.00	0.00	188.30	188.79	188.34	188.30
H14	0.00	0.00	0.00	58.84	58.86	58.88	58.86
H15	0.00	0.04	0.04	89.04	89.08	89.70	89.27
H16	0.00	0.04	0.00	101.42	101.41	101.74	101.52
H17	0.00	0.00	0.00	9.06	9.08	9.06	9.06
H18	0.00	0.03	0.02	54.22	54.22	54.57	54.39
H19	0.01	0.00	0.01	280.14	281.34	280.35	280.12
H20	0.02	0.01	0.00	77.86	77.88	78.44	78.14
H21	0.00	0.01	0.00	311.21	311.32	311.65	311.03
H22	0.00	0.00	0.00	119.91	119.95	119.95	119.93
H23	0.02	0.00	0.01	156.20	156.31	156.37	156.20
H24	0.00	0.00	0.00	97.30	97.30	97.75	97.56
H25	0.00	0.00	0.00	304.23	304.26	304.23	304.24
H26	0.01	0.00	0.03	131.79	131.79	132.67	132.22
H27	0.06	0.04	0.00	71.56	71.57	72.21	71.84
H28	0.00	0.03	0.02	168.19	168.22	169.37	168.46
H29	0.01	0.06	0.00	175.22	175.38	175.61	175.22
H30	0.01	0.00	0.01	172.61	172.61	173.50	173.05
H31	0.00	0.00	0.00	56.01	56.00	56.06	56.02
H32	0.00	0.00	0.03	193.98	193.98	194.98	194.34
H33	0.00	0.02	0.02	153.89	153.89	154.71	154.25
H34	0.00	0.01	0.01	94.67	94.70	95.27	94.90
H35	0.02	0.02	0.00	162.69	162.75	163.74	162.97
H36	0.00	0.01	0.00	98.95	98.97	99.75	99.18
Mean	0.01	0.01	0.01	98.41	98.44	98.64	98.54

Source: Authors’ Calculation

6.6.4 Region-wise Performance of Hospitals

Region-wise analysis of results shows that the performance of hospitals in Garhwal region is better than that of Kumaon region. The location of hospitals in hilly/partially hilly and plain areas is the main contributing factor of the higher efficiency of the Garhwal region hospitals. For instance, out of 19 hospitals in Garhwal Region, 4 hospitals are positioned in plain region; 5 in partially plain region; and rest 10 are in hilly region; whereas in Kumaon Region, out of 17 hospitals, only 2 hospitals are positioned in plain region; 7 hospitals are in partially plain region; and rest 8 hospitals are in hilly region. Larger size of population and better availability of travel facilities from villages to the hospitals is the main contributing factor of higher efficiency in the plain region hospitals and so, in Garhwal region hospitals. Out of 36 hospitals, 19 hospitals, which are in Garhwal region have scored 57.80% OTE, whereas the corresponding OTE score in the Kumaon region is 48.00%. The input resources beds, doctors and paramedical staff which are not utilized in the efficiency evaluation are 0.90%, 0.20% and 0.40% of the hospitals in Garhwal region, whereas the hospitals in Kumaon region are not utilizing their beds, doctors and paramedical staff by 0.90%, 0.20% and 0.30% respectively.

6.7 Sensitivity Analysis

Sensitivity analysis is an important topic in DEA analysis. It is important not only because the data set can be erroneous, and we need to justify the obtained efficiency at least for some change in data set, but also because some inefficient DMUs may turn out to be efficient after the changes in the data set. Charnes et al. [26] have introduced sensitivity analysis by changing in the output values in CCR model. Charnes and Neralic [32] determine the sufficient condition for a simultaneous change in all outputs and (or) all inputs of an efficient DMU, which preserves efficiency. Another method of sensitivity analysis given by Charnes et al. [31] is based on modification of DEA models in which the test DMU is excluded from the reference set. Continuing in this direction many modifications have been made in the DEA literature. `

Anderson and Peterson [14] proposed a new approach of sensitivity analysis by changing the reference set and named it as an "extended DEA measure" (EDM). In this approach, only a single decision making unit (DMU), whose efficiency is to be evaluated, is omitted. Hibiki and Sueyoshi [56] extended the EDM approach into a more general perspective for examining the stability of DEA efficiency. Jahanshahloo et al. [60] and Muller [95] use sensitivity approach for the ranking system of the efficient DMUs.

Agarwal et al. [7] proposed an approach for sensitivity analysis which satisfies both the requirements given in [60] and [95]. This approach determines the robustness of the efficiency scores by changing the reference set of the inefficient DMUs; ranks the efficient DMUs, identifies the outliers; and estimates the super efficiency of the DMUs. The super efficiency model excludes each observation from its own reference set so that it is possible to obtain efficiency scores that exceed one. We use the following model for sensitivity analysis as given in [7].

$$\left.\begin{aligned} & Max\, \eta_{a,b} = \theta_a + \frac{1}{m+s}\left(\sum_{r=1}^{s} \frac{s_{ra}^{+}}{y_{ra}} + \sum_{i=1}^{m} \frac{s_{ia}^{-}}{x_{ia}} \right) \\ & s.t. \quad \sum_{j \in J-\{b\}} \lambda_{ja} y_{rj} - s_{ra}^{+} = \theta_a y_{ra} \qquad \forall\, r = 1,\ldots,s, \\ & \qquad \sum_{j \in J-\{b\}} \lambda_{ja} x_{ij} + s_{ia}^{-} = x_{ia} \qquad \forall\, i = 1,\ldots,m, \\ & \lambda_{ja}, s_{ia}^{-}, s_{ra}^{+} \geq 0,\ \forall\, i = 1,2,\ldots,m;\ j = 1,2,\ldots,n;\ r = 1,2,\ldots,s, \end{aligned}\right\} \quad Model\ 6.4$$

where J is the set of all DMUs; a ∈ J_n, the set of inefficient DMUs; b ∈ J_e, the set of efficient DMUs. Efficiency of the a^th^ hospital to be evaluated under the condition that $j \in J - \{b\}$, i.e., the b^th^ hospital is excluded from the whole set. Table 6.4 shows the results of the sensitivity analysis obtained by changing the reference set of the inefficient hospital. So, we exclude these hospitals one by one from the whole set of hospitals. These results show that the NSM efficiency scores are robust and stable in the sense that removal of any efficient hospital H7 or H12 from the set does not have any high influence on the mean OTE. Removal of H7 gives no high influence on the mean OTE, whereas removal of H12 has slightly higher influence on its own efficiency.

The results also show that hospital H5 becomes efficient after removal of H12. It means that H5 has the structure similar to H12; it becomes inefficient due to the existence of H12. The hospital H12 has considerable influence on the efficiency of many inefficient hospitals, while H7 does not have any perceptible influence on the efficiency of inefficient hospitals. The results of sensitivity analysis after removing the efficient hospital one at a time have no significant change in the mean efficiency score of hospitals. This indicates that the resulted efficiency scores are robust.

Table 6.4: Results of Sensitivity Analysis and Super Efficiency

		Efficient Hospital		Increment in OTE		
Code	NSM OTE	H7	H12	H7	H12	Mean
H1	0.561	0.579	0.561	0.018	0.000	0.009
H2	0.779	0.779	0.793	0.000	0.014	0.007
H3	0.598	0.613	0.629	0.015	0.030	0.023
H4	0.434	0.434	0.465	0.000	0.031	0.016
H5	0.855	0.855	1.000	0.000	0.145	0.073
H6	0.517	0.528	0.517	0.011	0.000	0.005
H7	1.000	**1.217**	1.000	0.217	0.000	0.109
H8	0.514	0.581	0.514	0.067	0.000	0.033
H9	0.719	0.728	0.733	0.009	0.014	0.012
H10	0.390	0.392	0.390	0.002	0.000	0.001
H11	0.645	0.713	0.645	0.067	0.000	0.034
H12	1.000	1.000	**1.009**	0.000	0.009	0.004
H13	0.347	0.364	0.347	0.017	0.000	0.009
H14	0.630	0.668	0.644	0.039	0.015	0.027
H15	0.529	0.529	0.571	0.000	0.042	0.021
H16	0.496	0.537	0.497	0.040	0.000	0.020
H17	0.917	0.999	0.917	0.082	0.000	0.041
H18	0.648	0.671	0.683	0.023	0.034	0.028
H19	0.263	0.311	0.263	0.048	0.000	0.024
H20	0.562	0.562	0.649	0.000	0.087	0.043
H21	0.243	0.248	0.249	0.005	0.006	0.005
H22	0.455	0.471	0.455	0.016	0.000	0.008
H23	0.390	0.451	0.390	0.061	0.000	0.031
H24	0.507	0.515	0.566	0.009	0.059	0.034
H25	0.247	0.262	0.247	0.014	0.000	0.007
H26	0.431	0.431	0.478	0.000	0.046	0.023
H27	0.583	0.583	0.762	0.000	0.180	0.090
H28	0.373	0.373	0.440	0.000	0.067	0.034
H29	0.363	0.363	0.391	0.000	0.028	0.014
H30	0.367	0.368	0.403	0.001	0.037	0.019
H31	0.641	0.641	0.663	0.000	0.022	0.011
H32	0.340	0.347	0.386	0.007	0.045	0.026
H33	0.394	0.397	0.445	0.003	0.051	0.027
H34	0.514	0.514	0.556	0.000	0.042	0.021
H35	0.381	0.381	0.457	0.000	0.076	0.038
H36	0.503	0.503	0.636	0.000	0.134	0.067
Mean	0.532	0.553	0.565	0.021	0.033	0.027

Source: Authors' Calculation

From Table 6.4, we can observe the sensitivity in the efficiency scores of the inefficient hospitals. According to these results, the inefficient hospitals are classified into three categories [1, 93].

1. ***Low Sensitive Hospitals:*** A hospital is classified as the low sensitive hospital if the difference between the actual OTE score and average of the results of sensitivity model range from 0.001 to 0.020. Consequently, H1, H2, H4, H6, H9, H10, H12, h13, H16, H21, H22, H25, H29, H30 and H31 are classified as the low sensitive hospitals.

2 ***Middle Sensitive Hospitals:*** A hospital is classified as the middle sensitive hospital if the difference between the actual OTE score and average of the results of sensitivity model range from 0.021 to 0.040. Consequently, H3, H8, H11, H14, H15, H18, H19, H23, H24, H26, H28, H32, H33, H34, H35 and H36 are classified as the middle sensitive hospitals.

3 ***High Sensitive Hospitals:*** A hospital is classified as the high sensitive hospital if the difference between the actual OTE score and average of the results of sensitivity model range from 0.041 to 0.109. Consequently, H5, H7, H17, H20 and H27 are classified as the high sensitive hospitals.

Table 6.4 also shows the **Super Efficiency** scores of both the efficient hospitals H7 and H12. The super efficiency scores of hospitals H7 and H12 are 1.217 and 1.009 respectively.

6.8 Comparison of Efficiencies of Different DEA Models

The public sector hospitals are compared based on their efficiencies which are calculated by different radial and non-radial DEA models. The basic output-oriented CCR model, non-oriented SBM model and output-oriented NSM models are applied separately to measure their technical efficiency. The resulted efficiency scores show that the CCR efficiency scores are higher than efficiency scores calculated by both SBM and NSM models. It is because the SBM model deals only with slacks and measures the minimum impact of slacks on efficiency scores. NSM model is radial model which directly deals with the radial efficiency and the slack term. It also handles the actual impact of slacks on efficiency scores instead of minimum impact. The NSM model directly deals with slacks and measures the actual impact of slacks on efficiencies.

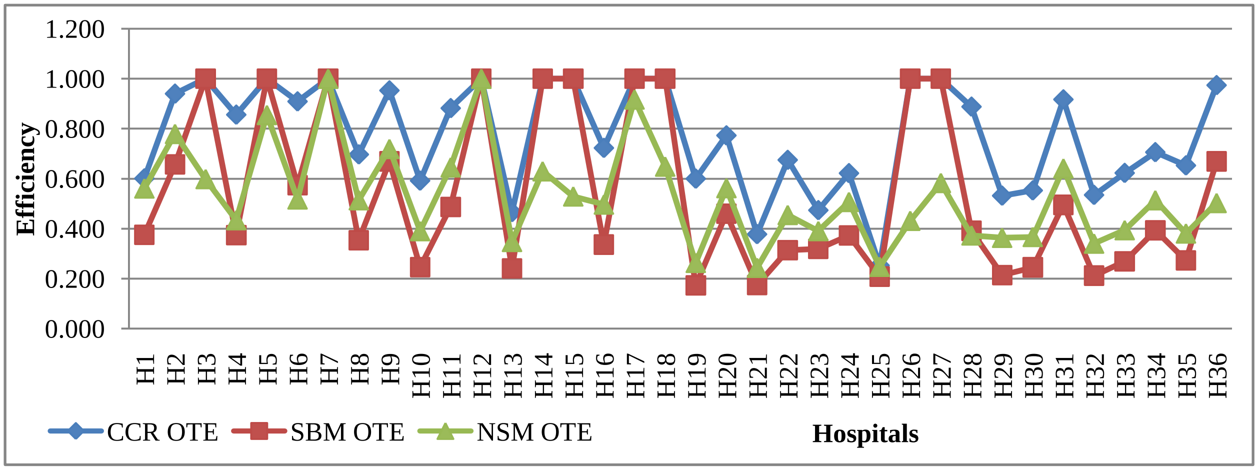
Table 6.5: Comparison of CCR, SBM and NSM OTE Scores

Code	CCR OTE	SBM OTE	NSM OTE
H1	0.601	0.375	0.561
H2	0.940	0.657	0.779
H3	1.000	1.000	0.598
H4	0.856	0.374	0.434
H5	1.000	1.000	0.855
H6	0.909	0.574	0.517
H7	1.000	1.000	1.000
H8	0.697	0.353	0.514
H9	0.953	0.669	0.719
H10	0.592	0.246	0.390
H11	0.882	0.487	0.645
H12	1.000	1.000	1.000
H13	0.466	0.241	0.347
H14	1.000	1.000	0.630
H15	1.000	1.000	0.529
H16	0.723	0.336	0.496
H17	1.000	1.000	0.917
H18	1.000	1.000	0.648
H19	0.600	0.173	0.263
H20	0.773	0.460	0.562
H21	0.378	0.174	0.243
H22	0.675	0.314	0.455
H23	0.474	0.319	0.390
H24	0.622	0.373	0.507
H25	0.253	0.208	0.247
H26	1.000	1.000	0.431
H27	1.000	1.000	0.583
H28	0.888	0.391	0.373
H29	0.532	0.214	0.363
H30	0.553	0.245	0.367
H31	0.917	0.495	0.641
H32	0.535	0.212	0.340
H33	0.623	0.269	0.394
H34	0.706	0.393	0.514
H35	0.653	0.273	0.381
H36	0.974	0.669	0.503
Mean	0.772	0.541	0.532

Source: Author's Calculation

The efficiency scores calculated by all the three models are given in Table 6.5. Figure 6.2 reveals that CCR efficiency is always higher than both the SBM and NSM efficiency.

Figure: 6.2: CCR, SBM and NSM Efficiency Scores



6.9 Summing Up

In this Chapter, we use NSM-DEA model to evaluate the OTE of 36 public sector hospitals in Uttarakhand, India. This model deals directly with input-output slacks. The model satisfies the monotone decreasing property with respect to slacks. It also satisfies the other properties of radial DEA model, such as unit invariance and translation invariance in inputs for the output-oriented model. Contrary to it, the CCR model does not account for slacks, whereas additive and SBM models do not have the radial properties of DEA model. Contrary to it, Slack Adjusted model deals with both radial inefficiency and slack inefficiency, but it measures the minimum impact of slacks on efficiency scores. Thus the NSM model shows a sharp contrast to other models. The dual of this model reveals that all multipliers become positive, i.e., all input and output variables are fully utilized in the performance assessment of the hospitals.

The key findings of the Chapter are summarized as follows:

- The results reveal that out of 36 hospitals only 2 (5.56 %) are efficient.
- The average OTE 53.20% indicates that 46.80% of the technical potential of hospitals is not in use, implying that these hospitals have the scope of producing 46.80% more outputs with same level of inputs.

- The hospitals H7 and H12 form efficiency frontier as they scored OTE equal to one.
- Combined Hospital Padampuri (H21) is found to be the most inefficient hospital as it has the least OTE score 24.30%. This hospital has to increase its number of out-door patients, number of in-door patients, number of major surgery and number of minor surgeries by 311.21%, 311.31%, 311.65% and 311.03% respectively with the same level of inputs.
- The hospitals of Garhwal region perform better than the hospitals in Kumaon region. On average, hospitals in Garhwal and Kumaon regions have to increase their output by 42.20% and 52% respectively with the given level of inputs.
- The target setting results show that number of indoor patients has significant scope to expand. This can be expanded by providing better facilities in the hospital, including medical and non-medical staff.
- Sensitivity analysis results show that the NSM efficiency scores are robust. Removal of efficient hospitals H7 and H12 gives no high influence on the mean OTE, however removal of H12 has somewhat high influence on it its own efficiency. So, it may be treated as an outlier.
- Hospital H5 becomes efficient after removal of H12. It means that H5 has the structure similar to H12; it becomes inefficient due to the existence of H12.
- Super Efficiency scores of the efficient hospitals H7 and H12 are 1.217 and 1.009 respectively.

Chapter 7

Total Factor Productivity Growth and its Sources in the Public Sector Hospitals

7.1 Introduction

In this chapter, DEA based MPI approach is used on the panel data of 27 public sector hospitals of Uttarakhand for the period of 11 years to measure the TFP growth and its sources (technical efficiency change and technical change). The decomposition of the technical efficiency change into pure technical efficiency change and scale efficiency change has been analyzed.

7.2 Concept of Productivity

Productivity is the relationship between the output generated by a production or a service system and the input provided to create this output. It is implied in every economic activity and primarily stands for producing more and more outputs from less and less input resources. It is determined by dividing the output from the inputs. When productivity of two firms are compared, the more productive firm produces more output with the same input, or it produce the same output with lesser input.

7.3 Productivity Measurement Approaches

It is important to understand different productivity measurement approaches. There are mainly two approaches of productivity measurement, namely, Partial Factor Productivity (PFP) and Total Factor Productivity (TFP). PFP is the value of output produced per unit of a single input and TFP measures the value of output when all input factors are used. The PFP approach cannot consider the input bundles together. Therefore, the TFP approach has advantages over the PFP approach since it considers multiple inputs and outputs. A TFP index measures change in the total output relative to the change in the use of all inputs. TFP is measured by Growth Accounting Approach (GAA), Stochastic Frontier Approach (SFA) and MPI approach.

Over the last two decades, researchers have used GAA to estimate TFP growth. This method represents output per unit of joint inputs and its interpretation is much less straightforward than of the PFP. GAA requires the prior specification of production function and is based on unrealistic assumptions of perfect competition and CRS. It assumes that a firm operates on its production frontier, implying that it has 100% technical efficiency. Thus, TFP growth measured through this approach is due to technical change (TECHCh), not due to technical efficiency change (TECh) [85]. It means one can interpret that the TFPCh is completely due to TECHCh and thus identical with TECHCh. However, the distinction between TECHCh and TECh can be made by conceiving the firm as operating in an exogenously determined environment, called the technology, which is the set of all feasible combinations of input and output quantities at a given period. A firm which operates on the boundary of this set is called technical efficient, whereas a firm which operates in the interior of this set is called technically inefficient. TECHCh then means that the set of feasible combinations expands or contracts while TECh means that the firm moves closer to or further away from the boundary. These two kinds of movement are clearly independent of each other; there can be TECHCh without TECh and TECh without TECHCh [16, 17, 18].

In recent years, parametric approach, SFA and non-parametric approach, DEA based MPI have become popular to estimate TFP. According to MPI approach, TFP can increase not only due to technological progress, but also due to increase in technical efficiency. The SFA and MPI do not consider that all firms are technically efficient. MPI has two advantages over SFA. First, it does not require any functional form for the production function. Second, it does not make a priori distinction between the relative importance of outputs and inputs [81]. The MPI has become a good tool for measurement of TFP for different profit and non-profit organizations. Looking at its advantages over other methods; we apply it to estimate the total factor productivity change (TFPCh), technical efficiency change (TECh), technology change (TECHCh), pure technical efficiency change (PTECh) and scale efficiency change (SECh) in Uttarakhand’s public sector hospitals. Here, we use output-oriented MPI between period’s *t* and *t+1* that is discussed in the next section.

7.4 Malmquist Productivity Index (MPI)

DEA based MPI approach was developed by Fare et al. [50, 51]. It was first suggested by Malmquist [83] as a quantity index for use in the analysis of consumption of inputs. Fare et al. [50]

combined the ideas from Farrell [52] on the measurement of efficiency and the measurement of productivity from Caves et al. [24], for developing the MPI approach that directly deals with input-output data using DEA. This approach has proven itself to be a good tool for measuring the TFP growth of DMUs.

In order to define the MPI of productivity change we consider, for each time period (t = 1, 2,..., T), a production technology S^t is given by

$$S^t = \{(x^t, y^t) : x^t \text{ can produce } y^t\} \tag{7.1}$$

Given the production technology at time *t*, the output distance function at *t* is defined as:

$$D_0^t(x^t, y^t) = \inf\{\theta : (x^t, y^t / \theta) \in S^t\} \tag{7.2}$$

This distance function is defined as the inverse of Farrell's [52] technical efficiency measure:

$$D_0^t(x^t, y^t) = \inf\{\theta : (x^t, y^t / \theta) \in S^t\} = (\sup\{\theta : (x^t, y^t / \theta) \in S^t\})^{-1} \tag{7.3}$$

This function returns the minimum value of θ by which the output may be divided and still be in the production set whose frontier is defined by technology S^t. Since $\theta \leq 1$, scaling back the output by the least possible factor gives the maximum proportional expansion of the output vector y^t, given input x^t and technology S^t.

Analogously, the following distance functions may be defined:

$$D_0^t(x^{t+1}, y^{t+1}) = \inf\{\theta : (x^{t+1}, y^{t+1} / \theta) \in S^t\} \tag{7.4}$$

$$D_0^{t+1}(x^t, y^t) = \inf\{\theta : (x^t, y^t / \theta) \in S^{t+1}\} \tag{7.5}$$

$$D_0^{t+1}(x^{t+1}, y^{t+1}) = \inf\{\theta : (x^{t+1}, y^{t+1} / \theta) \in S^{t+1}\} \tag{7.6}$$

These expressions are all distance functions with straightforward interpretations. For example, the first expression measures the maximal proportional expansion of the output vector y^{t+1} required to make (x^{t+1}, y^{t+1}) feasible when the technology of the time period *t* is used.

Similarly, we can define the input distance functions at time period t under the production technology S^t as

$$D_0^t(x^t, y^t) = \inf\{\theta : (x^t / \theta, y^t) \in S^t\} \tag{7.7}$$

This distance function gives the largest factor by which the input levels can be divided and still be in the production set whose frontier is defined by technology S^t.

Caves et al. [24] define the MPI, based on the technology of period *t*, as

$$M_{CCD}^t = D^t(x^{t+1}, y^{t+1}) / (D^t(x^t, y^t) \tag{7.8}$$

The distance functions used to define the MPI may be either input-oriented or output-oriented. It depends upon the statement of the problem of the DMUs of which productivity is to be estimated. An alternative specification uses the technology of period *t+1* as a basis to define the index:

$$M_{CCD}^{t+1} = D^{t+1}(x^t, y^t) / (D^{t+1}(x^t, y^t) \tag{7.9}$$

According to Fare et al [50], DEA based MPI approach is the geometric mean of two MPIs, given by Cave et al. [24]. Fare et al [50] decompose their MPI into two components, one change in technical efficiency (catching up effect) and the other change in technology (frontier shift). The change in technical efficiency determined by the efficient frontier is estimated using DEA for the set of DMUs (hospitals in this case). However, the changes in technology for a particular hospital under evaluation are only represented by a section of the DEA frontier.

The output-oriented MPI, which measures the productivity change of a particular DMU in time *t* and *t+1* is given as

$$MPI(X^{t+1}, Y^{t+1}, X^t, Y^t) = \left\{ \frac{D^t(X^{t+1}, Y^{t+1})}{D^t(X^t, Y^t)} \times \frac{D^{t+1}(X^{t+1}, Y^{t+1})}{D^{t+1}(X^t, Y^t)} \right\}^{\frac{1}{2}} \tag{7.10}$$

This represents the productivity of the production point (X^{t+1}, Y^{t+1}) relative to the production point (X^t, Y^t). The terms $D^t(X^t, Y^t)$, $D^{t+1}(X^{t+1}, Y^{t+1})$, $D^t(X^{t+1}, Y^{t+1})$ *and* $D^{t+1}(X^t, Y^t)$ are the distance functions in the time period *t* and *t+1*. A value greater than one $(MPI > 1)$ will indicate positive growth in TFP) (i.e., productivity gain from period *t* to period *t+1*; $(MPI < 1)$ indicates negative growth in TFP (i.e., productivity loss); and $(MPI = 1)$ means no change in the productivity from period *t* to period *t+1*.

This index is, in fact, the geometric mean of two output based Malmquist TFP indices [24]. One index uses period *t* technology and the other uses the period *t+1* technology. Fare et al. [50] decompose their MPI (given in equation 7.10) into two components:

$$MPI(X^{t+1},Y^{t+1},X^{t},Y^{t})=\left\{\frac{D^{t+1}(X^{t+1},Y^{t+1})}{D^{t}(X^{t},Y^{t})}\right\}\times\left\{\frac{D^{t}(X^{t+1},Y^{t+1})}{D^{t+1}(X^{t+1},Y^{t+1})}\times\frac{D^{t}(X^{t},Y^{t})}{D^{t+1}(X^{t},Y^{t})}\right\}^{\frac{1}{2}} \tag{7.11}$$

i.e.,

$$MPI = TECh \times TECHCh \tag{7.12}$$

$$MPI = \text{Technical Efficienc Change} \times \text{Technology Frontier Change}$$

The first component $TECh = \left\{\frac{D^{t+1}(X^{t+1},Y^{t+1})}{D^{t}(X^{t},Y^{t})}\right\}$ measures the changes in technical efficiency, between period *t* and the period *t+1*, which compares the closeness of hospitals in each period to that period’s efficient boundary. A value greater than one $(TECh>1)$ means DMU has become more efficient in period *t+1* compared to period *t*; $(TECh<1)$ means DMU has become lesser efficient in period *t+1* compared to period *t* and $(TECh=1)$ means DMU has the same distance from the respective boundaries in period *t+1* and period *t*.

The second component $TECHCh=\left\{\frac{D^{t}(X^{t+1},Y^{t+1})}{D^{t+1}(X^{t+1},Y^{t+1})}\times\frac{D^{t}(X^{t},Y^{t})}{D^{t+1}(X^{t},Y^{t})}\right\}^{\frac{1}{2}}$ measures the technology frontier shift, between time period *t* and period *t+1*. A value greater than one $(TECHCh>1)$ indicates a positive shift or technical progress; $(TECHCh<1)$ indicates a negative shift or technical regress and $(TECHCh=1)$ means no shift in technology frontier in period *t+1* and period *t*.

To evaluate the impact of any scale size changes on productivity changes, Fare et al. [10] decomposed the technical efficiency changes $(TECh)$ into two components as pure technical efficiency changes $(PTECh)$ and scale efficiency change $(SECh)$ that reflects the use of sub optimal scale of operation by the DMUs. Thus

$$TECh = PTECh \times SECh \tag{7.13}$$

Thus from Equation (7.12) and (7.13) we can write

$$MPI = PTECh \times SECh \times TECHCh \qquad (7.14)$$

The pure technical efficiency change $(PTECh)$ compares the closeness of DMU in each period to that period's efficient boundary corresponding to variable returns to scale (VRS) technology. $(PTECh > 1)$ indicates that DMU has become more pure technical efficient in period *t+1* compared to period *t* while $(PTECh < 1)$ indicates that DMU has become less pure technical efficient in period *t+1* compared to period *t.*

The scale efficiency change term $(SECh)$ reflects the impact of any change in scale size of the DMU on its productivity. The value $(SECh > 1)$ means DMU is more scale efficient in period *t+1* compared to period *t.* This represents a positive gain to its productivity attributable solely to changes to its scale size between period *t* and *t+1*. The value $(SECh < 1)$ means DMU is less efficient in period *t+1* compared to period *t.* This represents a negative impact on its productivity attributable to changes to its scale size. The value $(SECh = 1)$ means scale efficiency is same in period *t* and period *t+1* and so DMU has no impact to its productivity attributable to changes in its scale size. This does not necessarily mean that DMU has the same scale size in period *t* and *t+1*. Rather the impact of its scale size on its productivity is the same both in period *t* and period *t+1*.

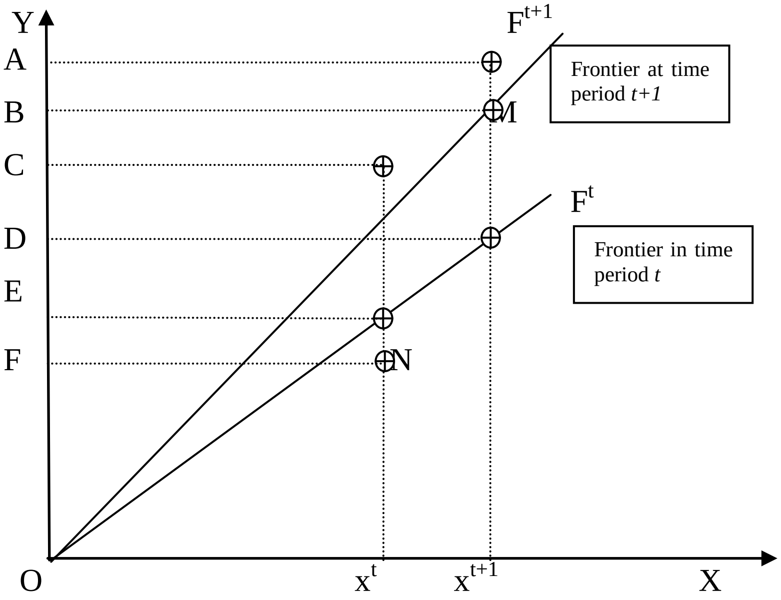


Figure 7.1: MPI under CRS technology

Figure 7.1 explains the main concept of MPI where we have depicted a CRS technology involving a single input ‘x’ and single output ‘y’. MPI under CRS technology indicates a rise in potential productivity as a technology frontier shifts from period *t* to period *t+1*. F^t and F^{t+1} in the Figure 7.1, represent the frontiers under CRS technology assumption at time period *t* and *t+1*, respectively. N represents the input-output combination (x^t, y^t) at time period *t*, while M represents the input-output combination (x^{t+1}, y^{t+1}) at time *t+1*. MPI and its components are represented by the distance functions. We consider the output distances along the y-axes such as A, B, C, D, E and F as shown in Figure 7.1. Hence, for technical efficiency change (TECh),

$$D^t(x^t, y^t) = F / E \text{ and } D^{t+1}(x^{t+1}, y^{t+1}) = B / A \tag{7.15}$$

Thus,

$$TECh = \left\{ \frac{D^{t+1}(X^{t+1}, Y^{t+1})}{D^t(X^t, Y^t)} \right\} = [(B/A)/(F/E)] \tag{7.16}$$

Where $TECh$ is the catching-up effect. It shows the direction of change in the input saving potential as measured by the relative distance to “own” frontiers. If, $TECh > 1$ than there is an increase in the technical efficiency of converting inputs into outputs from time period *t* to *t+1*.

For technology change (TECHCh),

$$D^t(x^{t+1}, y^{t+1}) = B / D \text{ and } D^{t+1}(x^{t+1}, y^{t+1}) = B / A \tag{7.17}$$

$$D^t(x^t, y^t) = F / E \text{ and } D^{t+1}(x^t, y^t) = F / C \tag{7.18}$$

$$D^t(x^{t+1}, y^{t+1}) / D^{t+1}(x^{t+1}, y^{t+1}) = (B/D)/(B/A) = (A/D) \tag{7.19}$$

$$D^t(x^t, y^t) / D^{t+1}(x^t, y^t) = (F/E)/(F/C) = (C/E) \tag{7.20}$$

Thus,

$$TECHCh = \left\{ \frac{D^t(X^{t+1}, Y^{t+1})}{D^{t+1}(X^{t+1}, Y^{t+1})} \times \frac{D^t(X^t, Y^t)}{D^{t+1}(X^t, Y^t)} \right\}^{\frac{1}{2}} = [(A/D) \times (C/E)]^{1/2} \tag{7.21}$$

The frontier function shift measures the relative distance between technologies of time period *t* and *t+1*, in terms of the relative efficiency for the same observation measured against two different frontiers. If we apply the technology of time *t* as the base technology, the relative distance between frontiers is measured at output level of observation of time period *t+1*.

The decomposition of technical efficiency change (TECh) into pure technical efficiency (PTECh) change and scale efficiency (SECh) change is illustrated in Figure 7.2, where we consider single input 'x' and single output 'y' case under both CRS and VRS technology assumption. The production frontiers showing in doted lines reflect the VRS technology in t and t+1 periods.

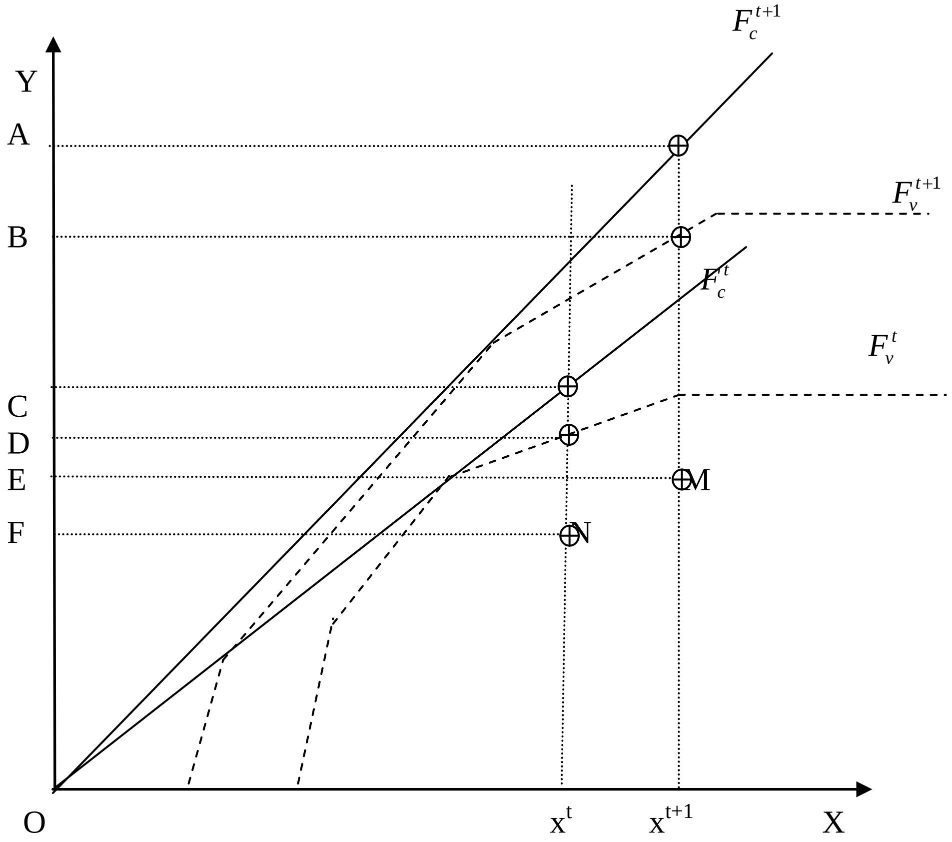


Figure 7.2: MPI under VRS technology

Consider a firm that operates at a point M in time period *t* and moved to point N in period *t+1*. In Figure 2, F_c^t *and* F_c^{t+1} represent the frontiers under CRS technology assumption at time period t and $t+1$ respectively; while F_v^t *and* F_v^{t+1} represent the frontiers under VRS technology assumption at time period t and $t+1$ respectively.

To explain MPI based on VRS technology, we consider the output distance along the Y-axes. The TECh under CRS technology can be written as

$$D_c^t(x^t, y^t) = F/C \text{ and } D_c^{t+1}(x^{t+1}, y^{t+1}) = E/A \tag{7.22}$$

Thus,

$$TECh = \left\{ \frac{D_c^{t+1}(X^{t+1}, Y^{t+1})}{D_c^t(X^t, Y^t)} \right\} = [(E/A)/(F/C)] \tag{7.23}$$

where, TECh is the catching-up effect. It shows the direction of change in the input saving potential as measured by the relative distance to “own” frontiers. For PTECh,

$$D_v^t(x^t, y^t) = F/D \text{ and } D_v^{t+1}(x^{t+1}, y^{t+1}) = E/B \tag{7.24}$$

Hence PTECh is given by,

$$PTECh = \frac{D_v^{t+1}(x^{t+1}, y^{t+1})}{D_v^t(x^t, y^t)} = (E/B)/(F/D) \tag{7.25}$$

Thus, scale efficiency change is given by,

$$\begin{aligned} &SECh = TECh / PTECh \\ &= [[(E/A)/(F/C)]/[(E/B)/(F/D)]] = [(C/D)/(A/B)] \end{aligned} \tag{7.26}$$

In DEA literature, there are two types of orientations (input and output). In input-oriented DEA measure, output(s) remains constant, but inputs are proportionally reduced. Similarly, in the output-oriented model, the outputs are maximized without altering the input quantities used [36]. In the healthcare sector, inputs, such as, number of beds, number of doctors and number of paramedical staff are fixed, at least, in short run and therefore they are not subject to easy change while outputs, such as, number of outpatients, number of inpatients, number of major surgeries and number of minor surgeries somewhat may be increased by providing better facilities to the patients. Therefore, in this study we use MPI index with output-orientation in the estimation of TFP of the healthcare sector, using DEAP software (version 2.1) developed by Coelli [35]. The output-oriented linear programming problems of MPI are given in Appendix 7A.

7.5 Data and Variables

Selection of input-output variables for the study is based on the variables used by the earlier studies and availability of data. In time-series analysis, the number of DMUs should be same during the entire study period, so, in this chapter, a total of 27 hospitals are selected for the entire analysis. In the starting year (2001) of the study period, there were only 27 hospitals situated in the state, therefore, the same hospitals are selected for the entire study period. Table 7.1 shows the input-output variables used in the earlier studies

Table 7.1: Inputs and Outputs Used in the Previous Efficiency and MPI Studies

S. N.	Author’s	Year	Inputs	Outputs
1	Agarwal et al. [3]	2007	number of beds, number of doctors and number of paramedical staff	number of outdoor patients, indoor patients, major surgery and minor surgery
2	Barros et al. [22]	2007	number of beds, number of full-time equivalent personal and total variable costs	number of patients, length of stay of the patient in the hospital, number of consultations and number of emergency cases
3	Gannon B. [53]	2008	number of beds and full-time equivalent people employed	number of discharges and deaths, outpatient , and day cases
4	Dash U. [39]	2009	number of beds, number of nursing staff, and number of physicians	number of inpatients, number of outpatients, and number of surgeries undertaken, emergency cases handled, medico legal cases, and deliveries
5	Dimas et al. [44]	2010	number of beds, total personnel salary and total expenditure on medicines, supplies and other materials	number of patient-days, number of patients in the outpatient department, and number of emergency cases
6	Karagiannis and Velentzas [67]	2010	number of beds, number of doctors and number of nursing and other personnel	number of inpatient days
7	Tlotlego et al. [137]	2010	number of clinical staff and number of hospitals beds	number of outpatient visit and number of inpatient days
8	Pham, L.T. [23]	2011	total number of beds, total number of hospital’s personnel including physicians and non-physicians	outpatient visits, inpatient days and surgical operations performed
9	Sheikhzadeh et al.	2012	number of physicians, nurses, medical team having a	number of emergency patients, number of outpatients, number of

	[117]		bachelor degree or above, active beds and medical team having 14 years diploma or lower + nonmedical and support staff	inpatients × average daily inpatients' residing
10	Kirigia and Asbu [72]	2013	number of doctors, number of nurses and midwives, number of lab technicians and number of operational beds and cots	number of outpatient department visits and number of inpatient department discharges

Keeping in view the variables used in the previous studies and as per the availability of data, we have taken number of beds, number of doctors and the number of paramedical staff (PMS) as input variables. Also, as indicated by Table 7.1, the major services provided by a hospital are out-door and in-door patients. These variables are considered as output variables for the study. We have also considered two cases-mix outputs, i.e., number of major and minor surgeries. Although all the sample hospitals are owned by the state government and are similar in nature and structure, there are some minor differences in these hospitals. For example, some district male hospitals do not have maternity department and some district female hospitals do not have dental, orthopedic or eye departments. Therefore, to maintain homogeneity of output measure (variables), only number of major and minor surgeries are taken as the case-mix outputs, as the surgical department is common to all the hospitals. Thus, for estimating TFP growth, three inputs, viz., number of beds, number of doctors and number of paramedical staff (PMS) and four outputs, namely, number of outdoor patients (OPD), number of indoor patients (IPD), number of major surgeries and number of minor surgeries are considered for the study. The input and output variables are defined in Table 1.3.

All the input and output variables are measured in numbers. The descriptive statistics of these input and output variables for the entire period are shown in Appendix 7C and the list of select hospitals in Appendix 7B. The descriptive statistics show that the sample hospitals vary significantly in terms of their inputs and output variables. Values of SD indicate that variation in output variables across hospitals is higher than that in input variables.

7.6 Results and Discussion

MPI is applied to construct the best-practice frontier, and compare the individual hospital to this frontier. Since the basic components of MPI are related to measures of technical efficiency and to the production frontiers, we first present the composition of the production frontier for each year under CRS and VRS technology assumptions and then estimate the TFP change.

7.6.1 Production Frontier

The best practice hospitals of the respective years construct the production frontier for different years. This part of the analysis is static in nature, as the performance of the hospitals of any given year is measured against the best practice hospitals in that year, and any movement of the production frontier from year to year is not taken into account. The descriptive statistics of technical efficiency (TE), pure technical efficiency (PTE), scale efficiency (SE), returns to scale (RTS) and composition of production frontiers under CRS and VRS assumptions over the period 2001 to 2011 are shown in Table 7.2.

7.6.1.1 Technical Efficiency (TE)

Table 7.2 shows that average TE of the sample hospitals does not evince any trend. Initially, it increased from 64.10% in 2001 to 70.50% in 2007, and thereafter it declined to 65.50% in 2008 and again increased to 80.30% in 2011. The average efficiency estimates across hospitals reveal that over the eleven year period, average TE is estimated to be 71.40%. This implies that on average, hospitals are by 29.60% off the best practice frontier under CRS assumption, and they can produce their output by using 29.60% lesser inputs if they operate on the best practice production frontier under the CRS assumption. Magnitude of SD in TE score also show variation across the years. It was highest in 2002 (0.254), followed by 2010 (0.249) and 2001 (0.241). As far as the number of hospitals on the frontier is concerned, Table 7.2 demonstrates that it was highest in 2010 (10 hospitals on the frontier) and lowest in 2006 and 2008 (4 hospitals on the frontier). It is quite evident from the estimated average TE scores that the hospitals in the state, on average, show consistent improvement in their performance during the first five years and the last three years.

7.6.1.2 Pure Technical Efficiency (PTE)

OTE is decomposed into PTE and SE. Decomposition is done to know sources of efficiency in the hospitals. As is indicated by Table 7.2, PTE scores show the pattern similar to that of OTE scores. Average PTE score increased from 76.00% in 2001 to 81.20% in 2005, and then it declined to 76.90% in 2006 and again increased to 82.10% in 2011 (Table 7.2). The value of SD in the PTE scores shows that PTE vary significantly across hospitals. The magnitude of SD was as high as 0.247 in 2001 and as low as 0.205 in 2005. The hospitals, nevertheless, have considerable scope to improve their ability to produce the maximum possible output from the inputs they employ. The average PTE score of the hospitals for the entire period is 77.50%, implying that the hospitals are on average 22.50% pure technical inefficient.

7.6.1.3 Scale Efficiency (SE)

SE is estimated by dividing OTE score from the PTE score. As PTE score is more than or equal to OTE score, value of SE score lies between zero and one. It is interesting to examine whether hospitals can improve their efficiency by changing their size or not. The mean SE score (91.30%) shows that hospitals are not operating at an optimal scale-size. They are able to make 8.70% improvement in their efficiency by adjusting their scale-size to the optimal level. This implies that the scale of production is also an important factor that affects the performance of hospitals.

It is observed that average SE score is much higher than the average PTE score during all the years under study. Moreover, value of SD is higher in PTE scores than the SE scores. This implies that overall technical inefficiency in the hospitals is largely due to inefficiency in the conversion of inputs into outputs. Therefore, variation in TE scores is mostly determined by the variation in PTE scores rather than the SE scores.

We also assess whether a hospital lies in the range of increasing, constant or decreasing returns to scale. If a hospital is operating with increasing/decreasing returns to scale, its efficiency can be increased if it attains CRS, because fewer resources are wasted due to hospital being either too small or too large. The measurement of RTS also helps to improve efficiency. Table 7.2 shows that during the study period, on average 13 (48.15%) out of 27 hospitals are found to operate at DRS and 7 (25.93%) at IRS. It is also observed that hospitals operated at DRS are larger in size, while those at IRS are relatively small.

Table 7.2: Descriptive Statistics of TE, PTE, SE and RTS for the Entire Period

		Max	Min	Mean	SD	No. EH	RTS	No. H	MPPS
2001	TE	1	0.211	0.641	0.241	5	*IRS*	11	5 (18.52%)
	PTE	1	0.304	0.76	0.247	8	*CRS*	7	
	SE	1	0.537	0.841	0.148	5	*DRS*	9	
2002	TE	1	0.232	0.721	0.254	8	*IRS*	14	8 (29.63%)
	PTE	1	0.320	0.774	0.246	12	*CRS*	8	
	SE	1	0.519	0.925	0.120	8	*DRS*	5	
2003	TE	1	0.213	0.747	0.239	8	*IRS*	14	8 (29.63%)
	PTE	1	0.253	0.785	0.231	11	*CRS*	8	
	SE	1	0.729	0.944	0.074	8	*DRS*	5	
2004	TE	1	0.308	0.762	0.235	9	*IRS*	9	9 (33.33%)
	PTE	1	0.346	0.805	0.222	13	*CRS*	9	
	SE	1	0.671	0.937	0.086	9	*DRS*	9	
2005	TE	1	0.274	0.767	0.217	7	*IRS*	6	7 (25.93%)
	PTE	1	0.304	0.812	0.205	11	*CRS*	7	
	SE	1	0.751	0.937	0.076	7	*DRS*	14	
2006	TE	1	0.266	0.705	0.212	4	*IRS*	6	4 (14.81%)
	PTE	1	0.354	0.769	0.214	9	*CRS*	4	
	SE	1	0.720	0.913	0.081	4	*DRS*	17	
2007	TE	1	0.266	0.705	0.212	4	*IRS*	6	4 (14.81%)
	PTE	1	0.354	0.769	0.214	9	*CRS*	4	
	SE	1	0.720	0.913	0.081	4	*DRS*	17	
2008	TE	1	0.211	0.654	0.235	4	*IRS*	2	4 (14.81%)
	PTE	1	0.324	0.737	0.238	9	*CRS*	5	
	SE	1	0.589	0.884	0.120	5	*DRS*	20	
2009	TE	1	0.194	0.661	0.236	4	*IRS*	1	4 (14.81%)
	PTE	1	0.329	0.743	0.233	8	*CRS*	4	
	SE	1	0.589	0.883	0.119	4	*DRS*	22	
2010	TE	1	0.178	0.683	0.249	5	*IRS*	5	5 (18.52%)
	PTE	1	0.29	0.754	0.242	9	*CRS*	5	
	SE	1	0.599	0.896	0.118	5	*DRS*	17	
2011	TE	1	0.253	0.803	0.220	10	*IRS*	8	10 (37.04%)
	PTE	1	0.288	0.821	0.218	13	*CRS*	10	
	SE	1	0.880	0.975	0.035	10	*DRS*	9	

Sources: *Author's calculation*

Where: -
No. EH stands for number of efficient hospitals;
No. H stands for number of hospitals and MPPS stands for most productive scale size.
Note: Figures in the parentheses are percentage of number of hospitals operating at MPSS.

Table 7.3 compares the mean TE, PTE and SE scores of individual hospitals for the entire period. It is evident from the table that out of 27 sample hospitals, 18 hospitals are on the frontier (under CRS assumption) at least once for the sample time period and H5 and H10 are on the frontier under the CRS assumption for every year. The hospitals H2, H5, H7, H10, H14, and H22 are found to be pure technical efficient in every year and remaining 21 hospitals are found on the frontier for at least once. On average, H20 observes lowest TE (25.60%) and PTE (34.10%), while the SE is found lowest for H2 (72.40%). Table 7.3 shows that two hospitals, namely H5 and H10 have been consistently on the CRS frontier throughout the period. They are found to operate at the most productive scale size (MPSS). An inefficient hospital may follow their best practices to improve its performance. Another important point that emerges from the analysis of Table 7.3 is that efficiency scores of individual hospitals vary significantly across years and consequently some hospitals are found to operate at IRS in some years and at DRS in other years. It may be due to fluctuations in the level of inputs and outputs across the years. For instance, number of average beds in the hospitals has increased from 82.19 in 2001 to 99.26 in 2011. Similarly, the average number of out-door patients have increased from 64874.44 in 2001 to 129910.60 in 2011. Values of SD in input-output variables are also found varying across years. It is also observed that some hospitals have achieved 100 percent PTE in almost all the years (for example H2, H6, H7, H14, and H22); however they are inefficient mainly due to their disadvantageous scale size. Their OTE may be increased by adjusting their scale-size to the optimum level.

Table 7.3: Average TE, PTE, SE, Frequency of CRS & VRS and RTS

Code	TE	PTE	SE	Freq. CRS	Freq. VRS	RTS (number of years)		
						IRS	CRS	DRS
H1	0.646	0.699	0.930	1	1	2	1	8
H2	0.724	1	0.724	0	11	11	0	0
H3	0.710	0.742	0.962	2	4	2	3	6
H4	0.731	0.831	0.888	0	2	9	0	2
H5	1	1	1	11	11	0	11	0
H6	0.917	0.933	0.921	4	10	0	4	7
H7	0.895	1	0.895	2	11	0	2	9
H8	0.941	0.963	0.976	5	6	1	5	5
H9	0.775	0.912	0.950	2	2	4	2	5
H10	1	1	1	11	11	0	11	0
H11	0.741	0.766	0.965	2	2	2	3	6
H12	0.758	0.805	0.931	3	4	3	3	5

H13	0.839	0.898	0.928	5	7	1	5	5
H14	0.881	1	0.881	5	11	0	5	6
H15	0.672	0.717	0.924	1	1	5	1	5
H16	0.346	0.437	0.791	0	0	8	0	3
H17	0.704	0.754	0.934	0	0	4	0	7
H18	0.401	0.464	0.867	0	0	1	0	10
H19	0.659	0.709	0.934	0	2	5	0	6
H20	0.256	0.341	0.748	0	0	0	0	11
H21	0.685	0.715	0.952	1	1	8	1	2
H22	0.977	1	0.977	10	11	1	10	0
H23	0.476	0.547	0.900	0	0	8	1	2
H24	0.656	0.673	0.957	1	1	1	1	9
H25	0.843	0.885	0.952	2	3	2	2	7
H26	0.525	0.553	0.944	0	0	3	0	8
H27	0.518	0.634	0.827	0	0	1	0	10
Mean	0.714	0.775	0.913					

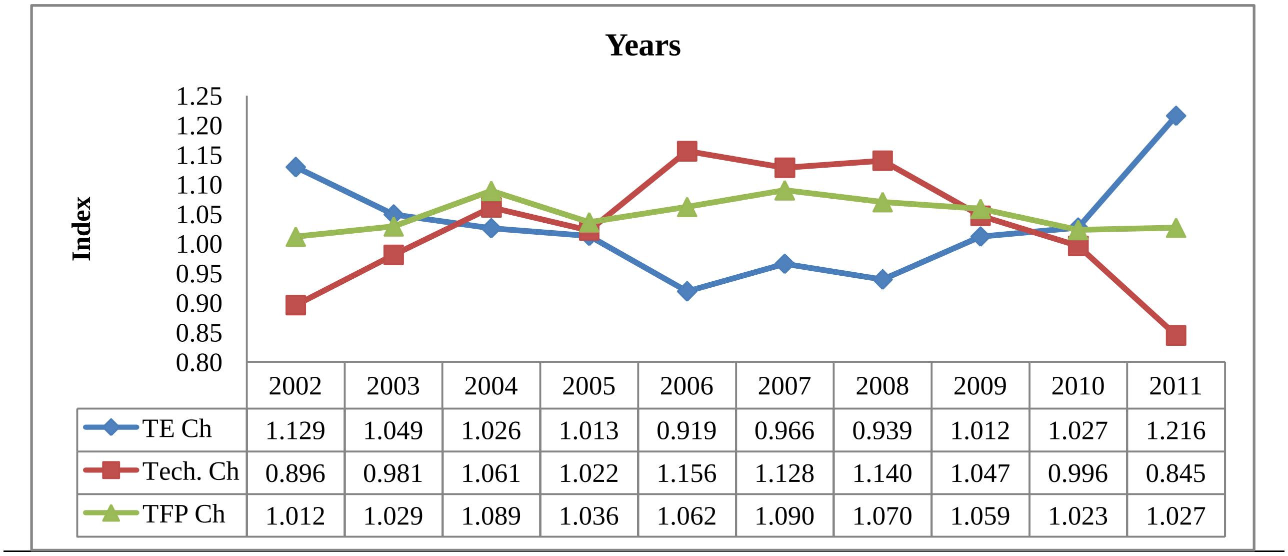
Sources: *authors' calculation*

7.6.2 Productivity Change

This section presents the dynamic aspects of performance of hospitals by incorporating shift in the production frontier over time. Figure 7.3 and Table 7.4 shows the TFP growth and its decomposition into technical efficiency change (catching up effect) and technical change (frontier shift) year-wise and hospital-wise, respectively. Unlike the efficiency scores that are based on the frontier of an indicated year, the MPI compares changes in TFP and its sources across two years. Since MPIs are multiplicative, the geometric mean is used to calculate the averages. The index is constructed in such a way that its value above one implies TFP growth, while its value below one indicates productivity regress.

Figure 7.3 shows trends in average TFP change indices along with technical efficiency change and technological change indices. It is evident from the Figure that hospitals have experienced positive growth in the TFP change indices throughout the study period. However, the average TFP growth shows variation across the years. The growth is observed highest in 2007 (9.0%), closely followed by 2004 (8.9%) and 2008 (7.0%). It is found lowest in 2002 (1.2%). Overall, TFP grew at an average rate of 4.9% per annum during the entire period. Both technical efficiency change (catch up) and technical change (frontier shift) have contributed to the TFP growth. However, contribution of technical efficiency change is slightly higher (2.6%) than that of technical change (2.2%). Technical change index shows improvement in six out of 11 years with the most pronounced growth of 15.6% observed in 2006; whereas technical efficiency change index shows progress in seven out of 11 years, with most pronounced growth of 21.6% found in 2011. The magnitudes of SD in the estimated indices of technical efficiency change and technical change indicate that the value of these indices varies significantly across years; whereas the value of SD in TFP indices shows that variation in the TFP indices across years is relatively much lower. It is because of the fact that sources of TFP growth, technical efficiency change and technical change do not appear to move in the same direction in some years. For instance, during 2006 to 2008 (three year period), technical efficiency change indices observed significant regress, while during the same period, technical change indices recorded the remarkable growth. Figure 7.3 shows that out of 11 years, only in three years, both the sources of TFP achieved progress in their indices. In the rest years, TFP progress is either due to technical change or due to technical efficiency change.

Figure 7.3: Change in TFP and its Components for the Entire Period



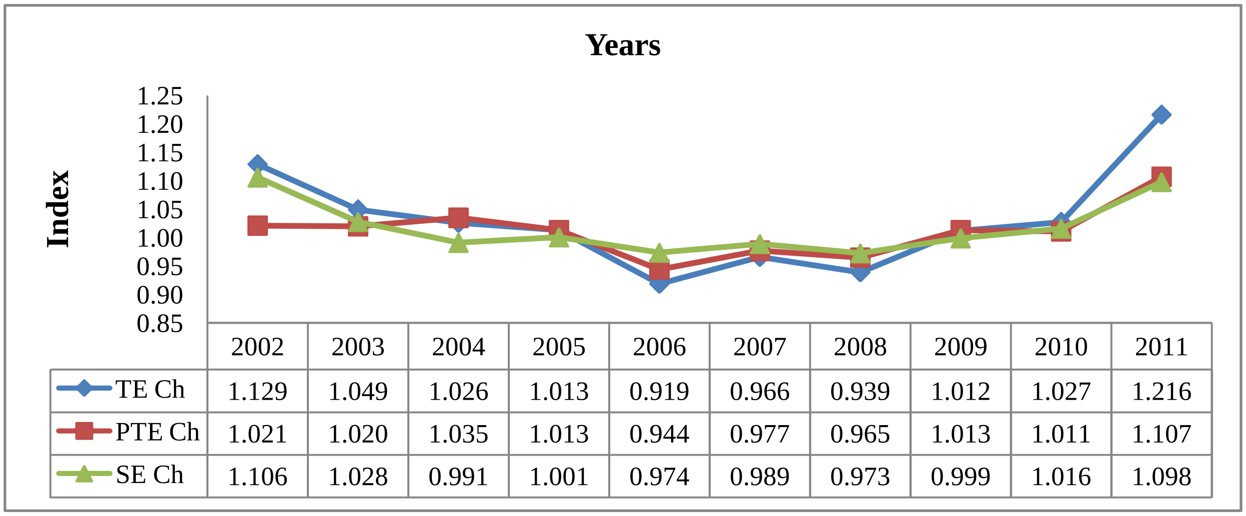
	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
◆ TE Ch	1.129	1.049	1.026	1.013	0.919	0.966	0.939	1.012	1.027	1.216
■ Tech. Ch	0.896	0.981	1.061	1.022	1.156	1.128	1.140	1.047	0.996	0.845
▲ TFP Ch	1.012	1.029	1.089	1.036	1.062	1.090	1.070	1.059	1.023	1.027

Figure 7.3 demonstrates that TFP change indices observed relatively less fluctuations across years when compared to technical efficiency and technological change indices. It seems to be due to the fact that in some years, technical efficiency change and technological change indices moved in the opposite direction and cancelled the effect of each other, to a greater extent, thus making the TPF change curve smoother. Figure 7.3 also shows that there was technical regress in

years 2002, 2003, 2010 and 2011. The technical change was depicting negative growth due to less investment in innovation and technological up-gradation by the government in public hospitals.

In order to know the contribution of PTE change and scale efficiency change to the overall technical efficiency change, TE change index is decomposed into PTE change index and SE change index. The estimated results indicate that on average, PTE increased by 1% per annum; while SE grew by 1.6% annually. This implies that SE is the main driving factor in the growth of TE. Figure 7.4 shows that SE change has the main contributing factor in TE change. The PTE shows a decline in nine hospitals over the years, while only four hospitals have experienced decline in SE. Average PTE shows progress in seven years; while it evinces regress only in three years.

Figure 7.4: Change in TE and its Components for the Entire Period



	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
◆ TE Ch	1.129	1.049	1.026	1.013	0.919	0.966	0.939	1.012	1.027	1.216
■ PTE Ch	1.021	1.020	1.035	1.013	0.944	0.977	0.965	1.013	1.011	1.107
▲ SE Ch	1.106	1.028	0.991	1.001	0.974	0.989	0.973	0.999	1.016	1.098

Table 7.4 shows the hospital-wise average TFP change, technical efficiency change and technical change indices. It is obvious from the table that hospital H15 achieved the highest TFP growth of 16.70% per annum, followed by hospital H3 (13.20%). Nine hospitals achieved TFP growth ranging from 11.20% for H21 to 6.80% for H5 and H27 both. Ten hospitals achieved TFP growth ranging from 5% for H24 to 1.10% for H19. Three hospitals achieved TFP growth ranging from 0.8% for H14 to 0.1% for H1. The remaining three hospitals H10, H13 and H20 experienced negative TFP growth, which is attributed mainly to the negative growth in the technical change index. Hospital H13 achieved the highest TFP regress of 3.5%, followed by 0.4% for H10 and H20 both. Those hospitals, which experienced high TFP growth, are also the ones that have

experienced significant technological progress. Maximum technological progress (6.80%) was estimated for H5, followed by H22 (6.20%) and H7 (5.70%). The maximum technological regress was observed in H20 (2.20%), followed by H16 (1.10%).

Table 7.4: Hospital-wise TECh, TECHCh and TFPCh Indices for the Entire Period

Code	TECh	TECHCh	TFPCh
H1	0.992	1.009	1.001
H2	1.023	1.016	1.039
H3	1.106	1.023	1.132
H4	1.049	1	1.049
H5	1	1.068	1.068
H6	1.048	1.047	1.097
H7	1.02	1.057	1.078
H8	1.029	1.015	1.045
H9	0.988	1.019	1.006
H10	1	0.996	0.996
H11	1.06	1.035	1.097
H12	1.094	0.989	1.082
H13	0.968	0.997	0.965
H14	1	1.008	1.008
H15	1.116	1.045	1.167
H16	1.104	0.989	1.092
H17	1.008	1.011	1.019
H18	0.983	1.034	1.016
H19	0.977	1.035	1.011
H20	1.019	0.978	0.996
H21	1.082	1.028	1.112
H22	1.03	1.062	1.094
H23	0.995	1.038	1.033
H24	1.01	1.04	1.05
H25	1.008	1.014	1.021
H26	0.977	1.039	1.015
H27	1.046	1.021	1.068
Mean	1.026	1.022	1.049

Sources: *author's calculation*

Since productivity growth represents the net effect of technical efficiency change (TE change) and technical change, it is perhaps the best way to examine the productivity growth by considering profiles under which most hospitals fall. Hospitals fit into the seven categories as given below:

Frontier Hospitals: One of the case of this category is that when hospitals are relatively technical efficient and the technology frontier shift outward (TECHCh >1). These hospitals can be termed as technological innovators. They determine the efficiency levels of other hospitals, since they have shifted the frontier in all periods. The hospitals, namely, H5, H10 and H14 are technically efficient in every year and experienced 2.20% average technological progress.

Inefficient, but Improving in Efficiency: Among the sample hospitals that fit in this category are H2, H3, H4, H6, H7, H8, H11, H12, H15, H16, H17, H20, H21, H22, H24, H25 and H27. Among these 17 hospitals, 5 hospitals have significant progress in technical efficiency ranging from 8.20% to 11.60%. Among the remaining 12 hospitals, 8 have progress, ranging from 2% to 6% and the remaining 4 have progress ranging from 0.80% to 1.90%.

Inefficient and becoming more Inefficient: The hospitals that fit in this profile are H1, H9, H13, H18, H19, H23 and H26. Hospital H13 depicts a typical situation of dominant average efficiency losses (-3.20%), followed by H19 and H26 (-2.30% each) in the entire period.

Increasing Productivity due to Dominant Technical Change: Hospitals H1, H2, H3, H5, H6, H7, H8, H9, H11, H14, H15, H17, H18, H19, H21, H22, H23, H24, H26 and H27 fit in this profile, because all of these hospitals experienced TFP growth mainly due to positive technical change.

Increasing Productivity due to Technical Efficiency Change: Two hospitals H12 and H16 show a slight regress in technical change but improvement in technical efficiency. It indicates that the productivity improvements in these hospitals are mainly due to improvement in technical efficiency.

Productivity Regress due to decline in Technical Change: Three hospitals H10, H13 and H20 show regress in their annual average TFP. These hospitals show regress in technical change, but improvement in technical efficiency. This clearly implies that fall in productivity in these hospitals is mainly due to technical regress.

Hospitals having Productivity Regress: Hospitals H10 and H20 have productivity regress due to technical regress, while the hospital H13 has negative TFP growth due to both adverse technical change and decline in technical efficiency.

Analysis of the results shows that there has been a consistently positive growth in the TFP, which is, by and large, equally contributed by the technological change and the technical efficiency change, though the magnitude of change varies significantly across hospitals and years. The technical progress showed by certain hospitals suggests that they have made some innovations with regards to both physical and human capital that may improve output by way of improving the quality of healthcare services. The production frontier shifted due to improvement in their healthcare technologies embodied in equipment and machines or due to skill formation and motivation among the healthcare workforce, including doctors. We have taken number of beds as a proxy variable for capital input and number of doctors and paramedical staff as labour input. Number of beds was used as an input variable by most of the earlier studies (see Table 7.1), however, it is a poor proxy for capital and cannot fully capture the effect of overall capital investment in the hospitals. Technological progress in a hospital, among others, depends on the availability of modern healthcare technology, complementarity of inputs, institutional changes and government transfer policy. Changes in all these factors affect the level of productivity and efficiency in the hospitals. Our findings indicate that technical progress and efficiency change indices observe high magnitude of variations across years, probably due to the fact that technology improvement in the hospitals requires huge investment which may not occur on yearly basis. Therefore, these two components of the TFP growth (catch up and frontier shift) moved in the opposite direction in some hospitals and reduced the TFP growth rate by mutually cancelling the effect of each other. Several hospitals experience technical regress (for example H10), which may imply that these hospitals are operating on the production possibility frontier closer to the origin than further away from it. These hospitals may improve their TFP by investing in modern medical technologies.

Most of the hospitals have observed positive growth in technical efficiency change indices which imply that they are catching up over the period. However, there are a few hospitals, which observe regress in the technical efficiency (for example, H13). These hospitals may improve their TFP by following the best managerial practices of their peer hospitals. As discussed above, progress/regress in technical efficiency in the hospitals is mostly driven by the progress/regress in PTE change. Since, average PTE change is observed much lower than the average SE change, it suggests that overall technical efficiency may be increased by improving the managerial efficiency (better conversion of inputs into outputs) of the hospitals. Under-utilized inputs may be efficiently used in the healthcare services production process by healthcare promotion strategies, such as, social mobilization and awareness building among the local community.

7.6.3 Region-wise TFP and its Components

Region-wise comparison of TFP and its components are given in Table 7.5. The results show that average annual TFP growth in Garhwal region was observed slightly higher (5.2%) than that in Kumaon region (4.9%). Both technical efficiency change (2.8%) and technical change (2.7%) contributed to the productivity growth in Garhwal Region; while in the Kumaon region, the contribution of technical efficiency change was much higher (3.1%) than that of technical change (1.8%). Relatively higher TFP growth in Garhwal region may be due to variation in the number of people served per hospital. For instance, in Garhwal region each hospital served for a population of 4.20 lakhs with 109 beds and 21 doctors per hospital, while in Kumaon region, each hospital served for a population of 3.25 lakhs with a bed strength of 94 and 21 doctors per hospital [150].

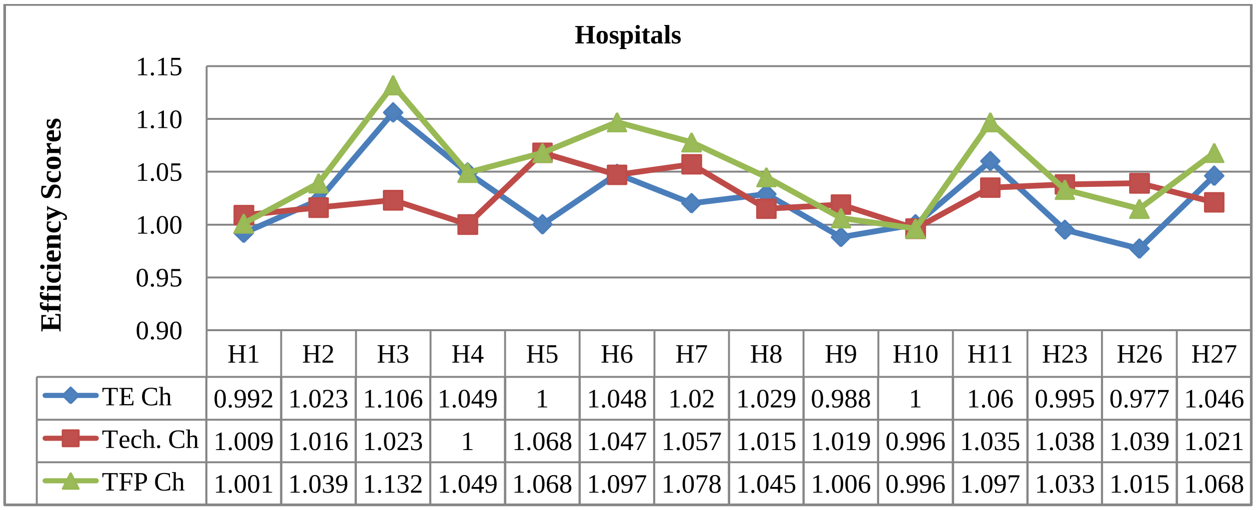
Table 7.5: Region-wise TE, Technological and TFP Changes for the Entire Period

Garhwal Region				Kumaon Region			
Code	TE Ch	Tech. Ch	TFP Ch	Code	TE Ch	Tech. Ch	TFP Ch
H1	0.992	1.009	1.001	H12	1.094	0.989	1.082
H2	1.023	1.016	1.039	H13	0.968	0.997	0.965
H3	1.106	1.023	1.132	H14	1	1.008	1.008
H4	1.049	1	1.049	H15	1.116	1.045	1.167
H5	1	1.068	1.068	H16	1.104	0.989	1.092
H6	1.048	1.047	1.097	H17	1.008	1.011	1.019
H7	1.020	1.057	1.078	H18	0.983	1.034	1.016
H8	1.029	1.015	1.045	H19	0.977	1.035	1.011
H9	0.988	1.019	1.006	H20	1.019	0.978	0.996
H10	1	0.996	0.996	H21	1.082	1.028	1.112
H11	1.060	1.035	1.097	H22	1.030	1.062	1.094
H23	0.995	1.038	1.033	H24	1.010	1.040	1.050
H26	0.977	1.039	1.015	H25	1.008	1.014	1.021
H27	1.046	1.021	1.068	Mean	1.031	1.018	1.049
Mean	1.028	1.027	1.052				

Sources: *Author’s calculation*

In Garhwal region, out of 14 hospitals, three hospitals experienced regress in the technical efficiency (H1, H23 and H26); two hospitals recorded no change; and rest of the hospitals achieved progress in the technical efficiency. As far as technological change is concerned, one hospital (H10) observed regress and another hospital (H4) achieved no change, while remaining hospitals experienced progress in the technical change. In Kumaon region, out of 13 hospitals, three hospitals (H13, H18, and H19) observed regress, one hospital (H14) witnessed no change, and remaining hospitals achieved progress in the technical efficiency. In case of technical change index, four hospitals observed negative growth, while all other hospitals achieved positive growth. A perusal of Table 7.5 reveals that on an average, hospitals located in the Garhwal region achieved TFP growth higher than that of their counterparts in Kumaon region. Relatively lower TFP growth in the hospitals of the Kumaon region was due to technical change rather than technical efficiency change. In order to improve their TFP, these hospitals have to invest in modern medical technology embodied in equipment and machines along with investment in skill formation of medical and paramedical staff.

Figure 7.5: Trends between TECh, TECHCh and TFPCh in Garhwal Region.

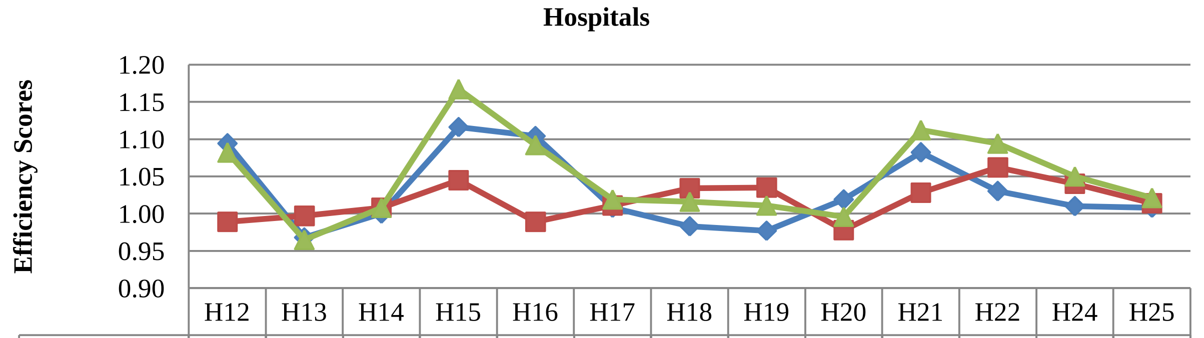


	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	H11	H23	H26	H27
TE Ch	0.992	1.023	1.106	1.049	1	1.048	1.02	1.029	0.988	1	1.06	0.995	0.977	1.046
Tech. Ch	1.009	1.016	1.023	1	1.068	1.047	1.057	1.015	1.019	0.996	1.035	1.038	1.039	1.021
TFP Ch	1.001	1.039	1.132	1.049	1.068	1.097	1.078	1.045	1.006	0.996	1.097	1.033	1.015	1.068

Figure 7.5 demonstrates that in all the hospitals TFP growth is observed except H10. The TFP progress is mainly due to technological progress. It seems to be due to the fact that in most of the hospitals, technical efficiency change and technological change indices moved in same positive directions and due to this the TFP growth is observed. Figure 7.5 also shows that there was technical regress for the hospital H10, and thus TFP regress is observed. In H1, H9, H23 and H26,

technical efficiency regresses are observed, while due to technological progress, TFP progress is observed.

Figure 7.6: Trends between TECh, TECHCh and TFPCh in Kumaon Region



	H12	H13	H14	H15	H16	H17	H18	H19	H20	H21	H22	H24	H25
◆ TE Ch	1.094	0.968	1	1.116	1.104	1.008	0.983	0.977	1.019	1.082	1.030	1.010	1.008
■ Tech. Ch	0.989	0.997	1.008	1.045	0.989	1.011	1.034	1.035	0.978	1.028	1.062	1.040	1.014
▲ TFP Ch	1.082	0.965	1.008	1.167	1.092	1.019	1.016	1.011	0.996	1.112	1.094	1.050	1.021

Figure 7.6 demonstrates that TFP change indices observed positive growth across hospitals when compared to technical efficiency and technological change indices. It seems to be due to the fact that in some hospitals, technical efficiency change and technological change indices moved in the opposite direction and cancelled the effect of each other. It seems to be due to the fact that in five hospitals, namely, H12, H16, H18, H19 and H20 technical efficiency change and technological change indices moved in opposite directions and due to this the observed TFP growth TFP growth is lower than the hospitals in Garhwal region. Figure 7.6 also shows that TFP regress for hospital H13. It is due to the regress in both technical efficiency and technology. Technical efficiency regress is observed in H13, H18, and H19 during the study period.

7.6.4 Area-wise TFP and its Components

In order to know whether TFP in the public hospitals varies across plain and hill areas of the state, we classify hospitals into two categories, namely, plain/semi-plain and hilly areas. Out of 13 districts in Uttarakhand, Haridwar and Udham Singh Nagar are in plain areas, Dehradun and Nainital are in semi-plain areas and rest 9 districts are in hilly areas. Table 7.6 shows that as against 6.5% average annual TFP growth in hospitals of plain/semi-plain areas, average TFP growth in hospitals of hilly areas is estimated to be only 3.7% per annum. The contributing factor in TFP progress in plain/semi-plain areas is technical efficiency change (4.1%) rather than technology change (2.2%), while in hilly areas, it is just reverse. In hilly areas, it is technical change that has higher contribution (2.3%) to the TFP growth than technical efficiency change (1.4%). This implies that the hospitals located in the hill region have to improve their technical efficiency which can be done through better conversion of inputs into outputs and by adjusting the size of the hospitals to the optimum level.

Table 7.6: Area-wise TE, Technological and TFP Changes for the Entire Period

Plain/Semi-Plain				Hilly			
Code	TE Ch	Tech. Ch	TFP Ch	Code	TE Ch	Tech. Ch	TFP Ch
H6	1.048	1.047	1.097	H1	0.992	1.009	1.001
H7	1.020	1.057	1.078	H2	1.023	1.016	1.039
H8	1.029	1.015	1.045	H3	1.106	1.023	1.132
H9	0.988	1.019	1.006	H4	1.049	1	1.049
H10	1	0.996	0.996	H5	1	1.068	1.068
H11	1.060	1.035	1.097	H17	1.008	1.011	1.019
H12	1.094	0.989	1.082	H18	0.983	1.034	1.016
H13	0.968	0.997	0.965	H19	0.977	1.035	1.011
H14	1	1.008	1.008	H20	1.019	0.978	0.996
H15	1.116	1.045	1.167	H23	0.995	1.038	1.033
H16	1.104	0.989	1.092	H24	1.010	1.040	1.050
H21	1.082	1.028	1.112	H25	1.008	1.014	1.021
H22	1.030	1.062	1.094	H26	0.977	1.039	1.015
Mean	1.041	1.022	1.065	H27	1.046	1.021	1.068
				Mean	1.014	1.023	1.037

Sources: *Author's calculation*

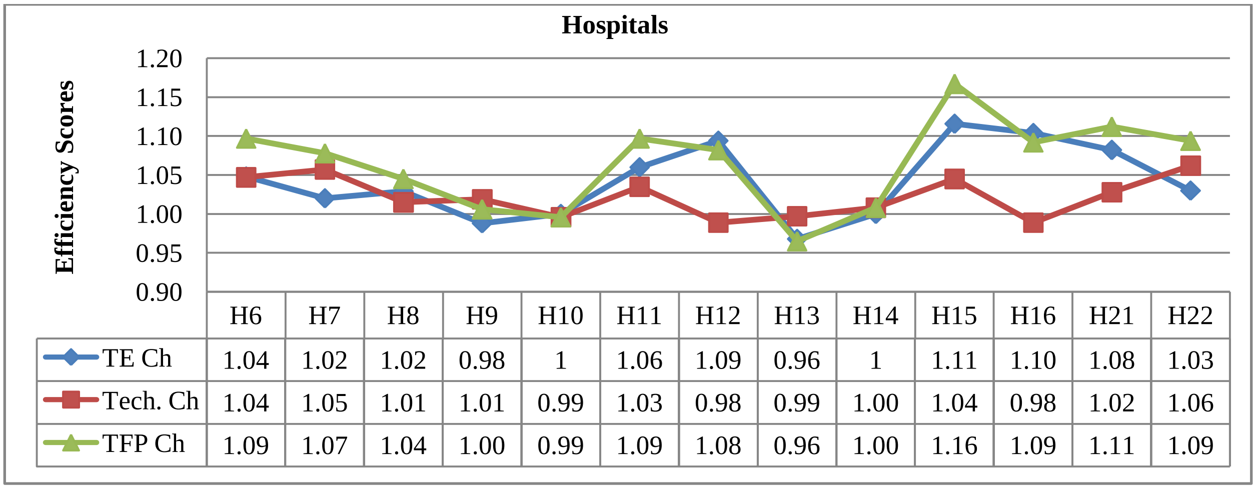
Table 7.6 also shows that 11 out of total 13 hospitals located in the plain/semi-plain areas achieved positive change in the TFP indices. It is also found that 9 hospitals obtained positive growth in both technical efficiency and technical change indices; 4 hospitals observed technical regress and 2 hospitals efficiency regress, while 2 hospitals were on the efficiency frontier. In hilly areas, out of 14 hospitals, 13 achieved TFP progress. Further, 8 hospitals experienced progress and 5 hospitals regress in the technical efficiency. In case of technical change, it is observed that out of 14 hospitals, 12 achieved progress, one hospital regress and another one no change in technology during the entire period.

Table 7.6 also demonstrates that on average, performance of hospitals located in the plain/semi-plain areas of the state has been better than that of the hospitals located in the hilly areas. High density of population and better transport facility in plain areas could be one of the reasons for higher TFP growth of hospitals in these areas; whereas hospitals in hilly areas are having locational disadvantages due to thin population and inadequate transport facilities. For instance, in plain/semi-plain area each hospital served for a population of 4.79 lakhs, with 114 beds and 22 doctors per hospital; while in a hilly area, each hospital served for a population of only 2.78 lakhs, with 90 beds and 19 doctors per hospital [150].

A perusal of Table 7.6 reveals that although technical efficiency is the driving factor in the TFP growth of the plain/semi plain areas, there are a few hospitals, which have either experienced negative or no growth in the technical efficiency (for example H9). These hospitals have to follow the managerial practices of their peer hospitals to improve technical efficiency. Four hospitals of the plain/semi plain areas, which have observed negative growth in the technical change index, have to make technological advancement to improve their TFP. In hilly areas, five hospitals, which are found to have negative growth in technical efficiency, may come closer to the production possibility frontier by following the best practices of their peer hospitals. In case of technical change, we observe that except for one hospital (H20) which achieved negative growth and another one (H4) which experienced no change, all other hospitals in the hill areas attained the positive growth in the technical change index. These findings suggest that hospitals located in the plain/semi plain areas should focus more on improving the technology to increase TFP, whereas hospitals in the hill areas should emphasize more on improving the technical efficiency to increase TFP.

Figure 7.7 demonstrates that in plain/partially plain areas TFP growth is observed in all the hospitals except H10 and H13. Except these two hospitals, all other hospitals experienced TFP growth during the entire study period. The regress in technology and no change in technical efficiency is the main cause of the TFP regress of H10 and the regress in both technical efficiency and technology is the cause of TFP regress for H13. For the hospitals H12 and H16, the TFP progress is observed mainly due to higher positive growth in technical efficiency. The TFP growth in all other hospitals, except H10, H12, H13 and H16 are due to the progress in technical efficiency as well as technology.

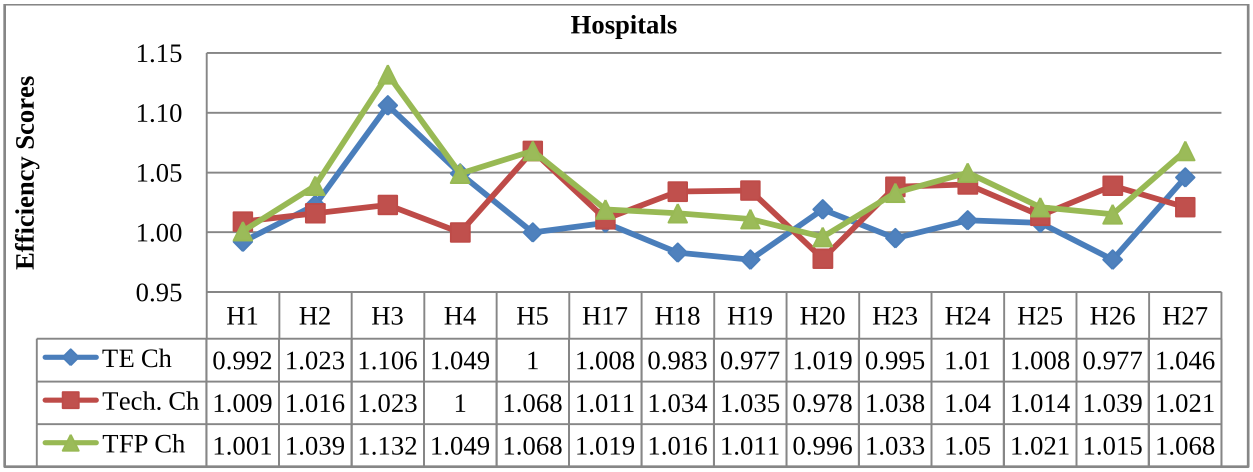
Figure 7.7: Trends in TECh, TECHCh and TFPCh in Plain/Partially Plain Areas



	H6	H7	H8	H9	H10	H11	H12	H13	H14	H15	H16	H21	H22
TE Ch	1.04	1.02	1.02	0.98	1	1.06	1.09	0.96	1	1.11	1.10	1.08	1.03
Tech. Ch	1.04	1.05	1.01	1.01	0.99	1.03	0.98	0.99	1.00	1.04	0.98	1.02	1.06
TFP Ch	1.09	1.07	1.04	1.00	0.99	1.09	1.08	0.96	1.00	1.16	1.09	1.11	1.09

Figure 7.8 demonstrates that TFP growth is observed across hospitals except H20. The TFP growth in hilly areas is observed mainly due to the progress in technology. The technical efficiency regress is observed in many hospitals during the study period. The TFP growth in the hilly areas is mainly due to the lower technical efficiency regress and relatively higher technological progress indices. Figure 7.8 also shows that the progress in technology is observed in all the hospitals of hilly region in the entire period. Technical efficiency regress is observed in H1, H18, H19, H23 and H26 during the study period.

Figure 7.8: Trends in TECh, TECHCh and TFPCh in Hilly Areas



	H1	H2	H3	H4	H5	H17	H18	H19	H20	H23	H24	H25	H26	H27
TE Ch	0.992	1.023	1.106	1.049	1	1.008	0.983	0.977	1.019	0.995	1.01	1.008	0.977	1.046
Tech. Ch	1.009	1.016	1.023	1	1.068	1.011	1.034	1.035	0.978	1.038	1.04	1.014	1.039	1.021
TFP Ch	1.001	1.039	1.132	1.049	1.068	1.019	1.016	1.011	0.996	1.033	1.05	1.021	1.015	1.068

7.7 Summing Up

In this chapter, we apply DEA based MPI approach to measure the efficiency and TFP growth and its sources in 27 public sector hospitals of Uttarakhand for the period from 2001 to 2011. The main findings of the efficiency and TFP analysis are summarized as below:

1. On an average, technical efficiency in the hospitals for the entire study period was estimated to be 71.40%, indicating that an average hospital could increase 28.60% of its outputs with the existing level of inputs.
2. The decomposition of technical efficiency into pure technical efficiency and scale efficiency shows that technical inefficiency in the hospitals was mainly due to inefficient conversion of inputs into output (average PTE=77.50%) rather than scale inefficiency (average SE = 91.30%).
3. Result of productivity analysis reveals that on average, TFP in the hospitals has grown at a rate of 4.9% per annum during the period under study.
4. The productivity growth was driven by both the catch up and innovation (frontier shift). However, contribution of technical efficiency change (2.6%) to the TFP growth was observed slightly higher than that of technical change growth (2.2%).
5. Region-wise analysis of TFP growth demonstrates that on average, TFP growth in Garhwal region was observed slightly higher (5.2%) than that in Kumaon region (4.9%).
6. Both technical efficiency change (2.8%) and technical change (2.7%) contributed to the productivity growth in Garhwal Region; while in the Kumaon region, the contribution of technical efficiency change was much higher (3.1%) than that of technical change (1.8%).
7. Further, the results show that the TFP growth was relatively higher in the hospitals located in the plain/semi-plain areas than that located in the hill areas of the State.
8. The results also conclude that in most of the years, technical efficiency change and technical change indices did not move in the same direction in some hospitals and therefore positive impact of one component on the TFP growth was largely cancelled by the negative impact of the other, probably due to the fact that technology improvement in the hospitals requires huge investment which may not occur on yearly basis.

7.8 APPENDIX 7A: Distance Functions used to Calculate MPIs.

$$\left.\begin{aligned}
&[D^{t}\left(X^{t},Y^{t}\right)]^{-1} = \underset{\phi,\lambda}{Max}\,\phi \\
&s.t.,\ \phi\, y_{r}^{t} - \sum_{j=1}^{n} \lambda_{j}^{t} y_{rj}^{t} \le 0, \forall r = 1,\dots,s \\
&\qquad \sum_{j=1}^{n} \lambda_{j}^{t} x_{ij}^{t} \ \le\ x_{i}^{t} \qquad \forall i = 1,\dots,m \\
&\qquad \lambda_{j}^{t} \ge 0 \qquad\qquad\qquad \forall j = 1,\dots,n
\end{aligned}\right\} \tag{7.27}$$

$$\phi \text{ is unrestricted in sign}$$

$$\left.\begin{aligned}
&[D^{t+1}\left(X^{t+1},Y^{t+1}\right)]^{-1} = \underset{\phi,\lambda}{Max}\,\phi \\
&s.t.,\ \phi\, y_{r}^{t+1} - \sum_{j=1}^{n} \lambda_{j}^{t+1} y_{rj}^{t+1} \le 0,\ \forall r = 1,\dots,s \\
&\qquad \sum_{j=1}^{n} \lambda_{j}^{t+1}\, x_{ij}^{t+1} \qquad \le\ x_{i}^{t+1} \quad \forall i = 1,\dots,m \\
&\qquad \lambda_{j}^{t+1} \ge 0 \qquad\qquad\qquad\qquad \forall j = 1,\dots,n
\end{aligned}\right\} \tag{7.28}$$

$$\phi \text{ is unrestricted in sign}$$

$$\left.\begin{aligned}
&[D^{t}\left(X^{t+1},Y^{t+1}\right)]^{-1} = \underset{\phi,\lambda}{Max}\,\phi \\
&s.t.,\ \phi\, y_{r}^{t+1} - \sum_{j=1}^{n} \lambda_{j}^{t} y_{rj}^{t} \le 0,\ \forall r = 1,\dots,s \\
&\qquad\quad \sum_{j=1}^{n} \lambda_{j}^{t}\ x_{ij}^{t} \ \le\ x_{i}^{t+1} \qquad \forall i = 1,\dots,m \\
&\qquad\quad \lambda_{j}^{t} \ge 0 \qquad\qquad\qquad \forall j = 1,\dots,n
\end{aligned}\right\} \tag{7.29}$$

$$\phi \text{ is unrestricted in sign}$$

$$\left.\begin{aligned}
&[D^{t+1}\left(X^{t},Y^{t}\right)]^{-1} = \underset{\phi,\lambda}{Max}\,\phi \\
&s.t.,\ \phi\, y_{r}^{t} - \sum_{j=1}^{n} \lambda_{j}^{t+1} y_{rj}^{t+1} \le 0,\ \forall r = 1,\dots,s \\
&\qquad\qquad \sum_{j=1}^{n} \lambda_{j}^{t+1} x_{ij}^{t+1} \ \le\ x_{i}^{t},\ \forall i = 1,\dots,m \\
&\qquad\qquad \lambda_{j}^{t+1} \ge 0 \qquad\qquad\quad \forall j = 1,\dots,n
\end{aligned}\right\} \tag{7.30}$$

$$\phi \text{ is unrestricted in sign}$$

Appendix 7B: Full Name of Selected Hospitals.

Code	District Name	Hospital Name
H1	Pauri	District Male Hospital
H2	Pauri	District Female Hospital
H3	Pauri	Base Hospital Srinagar
H4	Pauri	Combines Hospital Srinagar
H5	Pauri	Combines Hospital Kotdwar
H6	Dehradun	Doon Hospital Dehradun
H7	Dehradun	Female Hospital Dehradun
H8	Dehradun	SPS Hospital Rishikesh
H9	Haridwar	HMG Hospital Haridwar
H10	Haridwar	CR Female Hospital
H11	Haridwar	Combined Hospital Roorkee
H12	Nainital	B.D Pandey Male Hospital
H13	Nainital	B.D Pandey Female Hospital
H14	Nainital	Base Hospitla Haldwani
H15	Nainital	Female Hospital Haldwani
H16	Nainital	G.B Pant Hosp. Nainital
H17	Almora	District Hospital Male
H18	Almora	District Hospital Female
H19	Almora	Combined Hospital Ranikhet
H20	Almora	Base Hospital
H21	US Nagar	L.D Bhatt hospital Kashipur
H22	US Nagar	District Hospital Rudrapur
H23	Tehri	Combined Hospital Narendnagar
H24	Pithoragarh	DH male Pithoragarh
H25	Pithoragarh	DH Female Pithoragarh
H26	Chamoli	District Hospital Gopeshwar
H27	Uttarkashi	District Hospital Uttarkashi

Table 7C.1: Descriptive Statistics of Input and Output Variables for the Period 2001 to 2011

		Inputs			**Outputs**			
Years	**Values**	**Input 1**	**Input 2**	**Input 3**	**Output 1**	**Output 2**	**Output 3**	**Output 4**
2001	Min	30	3	9	5044	520	59	40
	Max	212	40	111	390762	9413	3969	2180
	Mean	82.19	14.85	40.93	64874.44	3970.19	649.52	418.89
	SD	43.37	8.63	25.05	75551.13	2407.20	960.49	454.95
	Sum	2219	401	1105	1751610	107195	17537	11310
2002	Min	30	4	10	5500	545	35	27
	Max	212	41	114	352749	22898	3269	2095
	Mean	82.19	15.85	41.85	66174.74	4650.96	595.63	412
	SD	43.37	8.63	25.31	67200.04	4371.38	763.18	502.80
	Sum	2219	428	1130	1786718	125576	16082	11124
2003	Min	30	5	8	5198	515	48	46
	Max	212	42	108	265140	12614	3776	2503
	Mean	82.19	18.44	40.78	68465.07	4602.74	563.85	425.41
	SD	43.37	9.13	24.78	56258.39	3103.25	776.93	515.85
	Sum	2219	498	1101	1848557	124274	15224	11486
2004	Min	30	5	10	10866	659	49	60
	Max	212	42	112	265864	12660	4304	2390
	Mean	82.19	18.44	41.93	75530.89	4918.48	661.63	443.67
	SD	43.37	9.13	25.05	61606.22	3270.34	858.79	559.72
	Sum	2219	498	1132	2039334	132799	17864	11979
2005	Min	30	6	11	14925	1018	45	56
	Max	252	42	112	283762	12937	4405	2284
	Mean	91.93	19.44	42.85	81308.19	5292.67	671.93	470.89
	SD	56.16	8.76	24.42	59702.57	3450.21	869.12	515.87
	Sum	2482	525	1157	2195321	142902	18142	12714
2006	Min	30	6	11	17458	1390	42	64
	Max	252	42	108	320951	15154	4352	2304
	Mean	91.93	19.78	42.81	89562.15	6009.96	620	512.22
	SD	56.16	8.87	23.76	66215.27	4025.67	868.47	490.18
	Sum	2482	534	1156	2418178	162269	16740	13830
2007	Min	30	6	11	20586	1689	52	73
	Max	252	41	108	355425	18168	4262	2414
	Mean	91.93	19.70	42.67	95664.15	7084	640.85	533.15
	SD	56.16	8.78	23.53	73224.07	4513.85	859.32	496.84
	Sum	2482	532	1152	2582932	191268	17303	14395
2008	Min	30	6	11	17119	1138	73	137
	Max	252	41	108	488195	20560	4042	2534

	Mean	92.52	19.70	42.67	106315.8	7816	647.11	610.70
	SD	55.71	8.78	23.53	94239.06	5159.04	834.68	510.95
	Sum	2498	532	1152	2870527	211032	17472	16489
2009	Min	30	6	11	16565	734	84	189
	Max	290	45	108	587519	21599	3982	2644
	Mean	93.93	19.93	42.85	114200.70	8411.78	642.07	654
	SD	60.19	9.17	23.91	110449.60	5524.92	829.82	522.53
	Sum	2536	538	1157	3083418	227118	17336	17658
2010	Min	30	6	11	13499	601	123	249
	Max	402	55	105	644696	21119	3941	2789
	Mean	99.26	20.41	42.81	129910.60	8267.41	659.44	721.44
	SD	75.49	10.28	23.63	129560.40	5562.97	827.04	559.79
	Sum	2680	551	1156	3507587	223220	17805	19479
2011	Min	30	7	11	16982	485	142	324
	Max	402	55	105	715221	22111	4128	2834
	Mean	102.04	20.56	42.85	139930.30	8426.18	668.07	764.37
	SD	74.93	10.24	23.58	139715.90	6015.55	839.12	592.59
	Sum	2755	555	1157	3778117	227507	18038	20638

Source: Directorate of Medical Health and Family Welfare, Government of Uttarakhand, Dehradun, India

Table 7C.2: Observed Input-Output Data for the Years 2001 to 2011

	Inputs			Outputs			
Code	Number of Beds	Number of Doctors	Number of PMS	Outdoor Patients	Indoor Patients	Major Surgeries	Minor Surgeries
2001							
H1	132	16	58	76582	3412	1119	1180
H2	30	4	10	10419	2806	343	195
H3	100	13	44	42911	3087	111	122
H4	45	8	36	44616	1760	68	123
H5	104	12	13	62692	3668	356	218
H6	212	40	97	189858	8599	714	747
H7	111	16	51	41886	8816	1119	1181
H8	100	18	54	111015	6832	592	319
H9	70	18	51	68968	3040	3969	387
H10	38	3	9	12178	3582	911	508
H11	100	12	37	63983	3221	209	441
H12	63	20	45	62526	1765	295	167
H13	56	6	21	24219	8430	468	319
H14	150	34	111	390762	9413	3650	2180
H15	45	15	30	17507	1807	174	40
H16	38	3	18	5044	520	115	74
H17	59	21	40	86976	2356	390	401
H18	69	8	32	16265	2862	519	532
H19	45	12	33	53123	3805	185	110
H20	144	20	73	36867	2723	452	362
H21	66	10	37	39700	3180	154	238
H22	44	18	19	46560	2213	59	273
H23	40	11	12	21075	1467	98	139
H24	120	23	53	90821	3751	209	135
H25	62	5	14	17921	4665	234	426
H26	68	18	49	54616	5250	770	303
H27	108	17	58	62520	4165	254	190
2002							
H1	132	17	60	84555	3530	698	200
H2	30	5	10	15829	2289	280	116
H3	100	14	45	54572	3737	118	164
H4	45	9	37	48207	1672	35	68
H5	104	13	14	75502	4067	320	116
H6	212	41	98	352749	9914	809	744

H7	111	17	52	43044	9591	1100	1475
H8	100	19	55	112874	8513	828	352
H9	70	19	52	74366	3843	2560	1494
H10	38	4	10	12308	3312	1633	574
H11	100	13	38	75845	3518	224	398
H12	63	21	45	61125	1622	188	91
H13	56	7	22	22677	8432	408	226
H14	150	35	114	163294	22898	3269	2095
H15	45	16	32	14610	1457	165	27
H16	38	4	18	5500	545	150	56
H17	59	22	40	90456	3196	188	273
H18	69	9	32	15158	3103	523	678
H19	45	13	34	60130	3698	136	169
H20	144	21	72	43535	3214	536	294
H21	66	11	38	39700	3180	205	181
H22	44	19	20	75110	2957	120	405
H23	40	12	13	21413	1699	97	207
H24	120	24	54	84924	4208	292	107
H25	62	6	15	31995	4927	450	232
H26	68	19	50	49078	2951	500	243
H27	108	18	60	58162	3503	250	139
2003							
H1	132	18	58	91794	3645	616	225
H2	30	6	10	16172	2337	461	177
H3	100	17	44	59578	3768	120	215
H4	45	14	36	50288	1786	48	6
H5	104	11	13	89272	8530	395	91
H6	212	42	97	265140	12614	647	838
H7	111	18	51	49274	10279	925	1770
H8	100	17	54	119818	9361	593	368
H9	70	20	51	79324	3882	1480	892
H10	38	5	8	15852	3952	1968	708
H11	100	18	37	66291	3017	196	461
H12	63	26	45	114980	2349	199	168
H13	56	8	21	21394	8278	397	315
H14	150	36	108	181608	9952	3776	2503
H15	45	17	30	26806	3011	275	136
H16	38	6	18	5198	515	85	140
H17	59	23	40	91014	2533	263	201
H18	69	10	32	23868	2838	433	572

H19	45	16	33	61477	3225	200	108
H20	144	28	73	47298	3953	685	296
H21	66	14	37	37421	2334	50	61
H22	44	22	19	72717	2502	109	316
H23	40	24	12	20090	1486	71	215
H24	120	25	53	108343	4496	160	298
H25	62	7	14	18534	4928	438	177
H26	68	20	49	53434	3327	169	111
H27	108	30	58	61572	5376	465	78
2004							
H1	132	18	59	164995	4239	852	437
H2	30	6	11	16667	2457	849	183
H3	100	17	45	60614	5012	90	182
H4	45	14	37	62013	2694	315	261
H5	104	11	14	90986	5614	347	143
H6	212	42	98	265864	12522	689	904
H7	111	18	52	53400	11075	1719	2090
H8	100	17	55	128350	9877	656	351
H9	70	20	52	69934	3479	1154	149
H10	38	5	10	18813	4622	1766	840
H11	100	18	38	93614	3735	242	594
H12	63	26	46	124292	2522	207	134
H13	56	8	22	23260	8381	429	490
H14	150	36	112	175625	12660	4304	2390
H15	45	17	31	27421	4144	503	193
H16	38	6	19	10866	659	119	160
H17	59	23	41	68547	1175	451	86
H18	69	10	33	13326	2559	398	372
H19	45	16	34	55059	3283	117	204
H20	144	28	74	54026	4980	718	407
H21	66	14	38	39360	1960	102	60
H22	44	22	20	80732	3104	66	231
H23	40	24	13	23889	2348	49	134
H24	120	25	54	173056	4665	267	332
H25	62	7	15	19821	4140	443	287
H26	68	20	50	55414	4086	156	135
H27	108	30	59	69390	6807	856	230
2005							
H1	132	18	59	106907	4590	792	537
H2	30	7	11	24790	2288	952	233

H3	100	17	45	64421	6510	99	172
H4	45	14	37	69451	3241	385	284
H5	104	13	19	98663	5649	370	235
H6	252	42	98	283762	12355	589	984
H7	111	16	52	64472	12349	1699	1890
H8	150	17	55	148952	10574	676	411
H9	70	20	52	76188	3715	1094	249
H10	30	6	12	18330	4142	1816	920
H11	100	18	40	100592	3575	252	624
H12	63	26	46	138910	2755	257	214
H13	65	22	32	25163	10376	489	360
H14	206	38	112	183798	12937	4405	2284
H15	45	17	31	33649	4896	524	183
H16	38	7	19	14925	1018	219	210
H17	74	23	42	88151	1456	521	102
H18	69	13	34	15903	2015	408	392
H19	90	18	36	69071	4089	187	194
H20	200	30	75	58543	3045	828	387
H21	66	14	38	66403	4301	98	56
H22	44	22	22	121135	3434	82	305
H23	40	24	13	30732	1933	45	184
H24	120	25	54	119693	4764	257	412
H25	62	8	15	26376	5660	383	307
H26	68	20	50	61949	4527	163	245
H27	108	30	58	84392	6708	552	340
2006							
H1	132	20	60	105086	5397	762	607
H2	30	7	11	24379	2633	892	325
H3	100	18	45	83054	8093	179	212
H4	45	14	37	75715	3815	355	324
H5	104	15	22	111720	6486	410	357
H6	252	42	98	320951	15154	619	894
H7	111	18	52	66928	15011	1829	1790
H8	150	20	55	175312	12189	586	441
H9	70	18	52	88657	3894	134	305
H10	30	6	12	25231	5759	1796	1020
H11	100	18	40	93301	3963	241	584
H12	63	26	46	106628	2180	187	321
H13	65	22	32	36996	11352	421	430
H14	206	38	108	210154	12573	4352	2304

H15	45	17	31	43951	3761	534	203
H16	38	7	19	17458	1390	203	231
H17	74	24	42	107880	1686	431	342
H18	69	11	33	23165	2213	368	432
H19	90	20	37	81798	4857	207	264
H20	200	34	74	65180	4159	788	407
H21	66	14	38	78385	3516	106	64
H22	44	20	22	130635	4528	88	355
H23	40	22	15	24197	1904	42	234
H24	120	25	54	132005	6363	243	380
H25	62	9	15	27492	6143	342	352
H26	68	19	48	57939	4183	143	249
H27	108	30	58	103981	9067	482	403
2007							
H1	132	20	60	93837	5320	796	582
H2	30	7	11	25030	3125	632	415
H3	100	18	45	98251	9773	209	262
H4	45	14	37	81591	4106	332	404
H5	104	15	22	127901	9374	398	407
H6	252	41	96	355425	16549	579	824
H7	111	18	52	73770	18168	1932	1810
H8	150	20	55	188944	13700	563	419
H9	70	18	52	89040	3664	1124	425
H10	30	6	12	32189	6907	1816	983
H11	100	18	40	92022	4825	307	631
H12	63	26	46	105670	2879	181	343
H13	65	22	32	39552	12619	371	428
H14	206	38	108	236803	14259	4262	2414
H15	45	17	31	50737	7123	514	253
H16	38	7	19	20586	1689	213	241
H17	74	24	42	105363	2215	381	363
H18	69	11	33	25642	2825	372	382
H19	90	20	37	82193	5162	237	304
H20	200	34	74	62498	5602	724	392
H21	66	14	38	88436	5345	96	73
H22	44	20	22	145606	6509	74	285
H23	40	22	15	26512	1863	52	251
H24	120	25	54	141960	7064	231	392
H25	62	9	15	28949	7197	351	382
H26	68	18	48	59412	4580	128	268

H27	108	30	56	105013	8826	428	462
2008							
H1	132	20	60	104810	5830	712	542
H2	30	7	11	25937	2900	592	495
H3	100	18	45	132746	16633	248	281
H4	45	14	37	63754	2287	296	434
H5	104	15	22	136794	10634	369	457
H6	252	41	96	488195	17626	609	864
H7	111	18	52	81678	20560	2014	1941
H8	150	20	55	201830	14889	583	629
H9	70	18	52	94632	3255	1214	575
H10	30	6	12	41912	7791	1964	1023
H11	100	18	40	97731	5748	427	761
H12	63	26	46	110893	3432	231	403
H13	65	22	32	46360	11127	341	518
H14	206	38	108	240693	13972	4042	2534
H15	45	17	31	55890	8197	484	343
H16	38	7	19	17119	1138	213	381
H17	74	24	42	116948	3402	372	431
H18	69	11	33	21612	2944	346	432
H19	90	20	37	91547	5842	307	386
H20	200	34	74	64073	6876	682	643
H21	66	14	38	93139	5744	126	137
H22	60	20	22	166950	7289	103	318
H23	40	22	15	26508	1686	73	271
H24	120	25	54	150742	8044	264	451
H25	62	9	15	33433	7447	268	409
H26	68	18	48	60349	5553	147	318
H27	108	30	56	104252	10186	445	512
2009							
H1	132	20	60	100547	5635	741	553
H2	30	7	11	22905	2749	538	545
H3	100	18	45	138845	19070	328	431
H4	45	14	37	75415	2022	146	473
H5	104	15	22	149328	12306	342	482
H6	290	45	100	587519	17542	682	984
H7	111	20	53	83719	21599	2244	2024
H8	150	20	55	187916	15334	541	664
H9	70	18	52	83860	2721	1156	625
H10	30	6	12	42718	7942	1768	983

H11	100	18	40	108980	7584	461	648
H12	63	26	46	122597	4298	262	429
H13	65	22	32	45865	11098	318	496
H14	206	38	108	251094	15761	3982	2644
H15	45	17	31	74389	9047	354	381
H16	38	7	19	16565	734	179	403
H17	74	24	42	137567	3249	386	509
H18	69	11	33	29317	3158	375	622
H19	90	20	37	87644	7349	329	486
H20	200	34	74	62799	6226	712	682
H21	66	14	38	118853	8170	206	189
H22	60	20	22	184762	9296	121	342
H23	40	22	15	29218	2131	84	286
H24	120	25	54	146413	8427	291	472
H25	62	9	15	29503	7613	188	413
H26	68	18	48	63511	6169	181	349
H27	108	30	56	101569	9888	421	543
2010							
H1	132	20	60	125653	5476	762	632
H2	30	7	11	20490	2375	428	562
H3	100	18	45	135384	19633	341	472
H4	54	15	37	79503	2113	168	482
H5	104	16	22	222001	12080	389	538
H6	402	55	100	644696	18029	982	1283
H7	111	20	55	102046	21119	2395	2218
H8	150	20	55	217159	14627	628	761
H9	70	18	52	115126	3880	1108	725
H10	30	6	12	41755	6681	1618	1023
H11	106	18	40	288308	5624	482	768
H12	63	26	46	122574	4836	276	549
H13	65	22	32	46211	5305	327	536
H14	206	38	105	322798	15177	3941	2789
H15	52	17	31	79154	10156	298	438
H16	38	7	19	13499	601	168	439
H17	74	24	42	125492	3484	349	528
H18	69	12	33	26019	3211	346	641
H19	90	20	37	84540	7692	429	528
H20	200	34	74	62475	5548	783	646
H21	66	14	38	115816	9969	187	249
H22	60	20	22	158376	10884	142	386

H23	40	22	15	39786	1577	123	366
H24	120	25	54	149952	9066	352	496
H25	62	9	15	33855	6902	192	458
H26	78	18	48	31776	6738	203	362
H27	108	30	56	103143	10437	388	604
2011							
H1	132	20	60	130310	6092	793	657
H2	30	7	11	18134	2112	447	583
H3	100	18	45	169823	20348	422	542
H4	54	15	37	105207	3078	182	473
H5	104	16	22	187659	12607	401	638
H6	402	55	100	715221	20006	1202	1423
H7	111	23	55	106535	22111	2413	2541
H8	150	20	55	206844	15583	662	772
H9	70	18	52	108594	2987	1149	865
H10	30	7	13	30426	2065	952	763
H11	106	18	40	271514	5103	471	741
H12	63	26	46	148385	5035	283	584
H13	65	22	32	44652	9483	314	616
H14	206	38	105	330964	14714	4128	2834
H15	52	17	31	70406	10576	318	513
H16	38	7	19	16982	485	187	464
H17	74	24	42	111013	3092	358	549
H18	69	12	33	26469	3552	326	628
H19	90	20	37	103482	7775	457	586
H20	200	34	74	61659	4847	862	711
H21	66	14	38	158385	9885	208	324
H22	125	20	22	233711	10711	238	426
H23	50	22	15	35292	1301	142	389
H24	120	25	54	135234	9444	351	528
H25	62	9	15	34394	7003	183	442
H26	78	18	48	81946	6889	218	428
H27	108	30	56	134876	10623	371	618

Table 7C.3: Technical Efficiency (2001 to 2011)

Code	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Mean
H1	0.652	0.632	0.697	1	0.783	0.671	0.671	0.487	0.443	0.465	0.601	0.646
H2	0.749	0.756	0.729	0.739	0.816	0.720	0.720	0.589	0.605	0.599	0.940	0.724
H3	0.366	0.541	0.521	0.530	0.674	0.687	0.687	0.879	0.930	1	1	0.710
H4	0.528	0.663	0.715	0.876	0.864	0.809	0.809	0.619	0.676	0.622	0.856	0.731
H5	1	1	1	1	1	1	1	1	1	1	1	1
H6	0.592	1	1	0.942	0.930	0.944	0.944	1	1	0.782	0.949	0.917
H7	0.824	0.861	0.901	0.872	1	0.873	0.873	0.880	0.816	0.948	1	0.895
H8	0.713	0.895	1	1	1	1	1	0.998	0.919	0.877	0.953	0.941
H9	1	1	0.901	0.732	0.714	0.692	0.692	0.639	0.545	0.728	0.882	0.775
H10	1	1	1	1	1	1	1	1	1	1	1	1
H11	0.560	0.768	0.637	0.805	0.849	0.705	0.705	0.550	0.576	1	1	0.741
H12	0.405	0.587	1	1	0.946	0.626	0.626	0.681	0.689	0.777	1	0.758
H13	1	1	1	1	1	0.910	0.910	0.659	0.645	0.391	0.723	0.839
H14	1	1	1	1	0.864	0.855	0.855	0.686	0.694	0.732	1	0.881
H15	0.333	0.255	0.626	0.753	0.788	0.548	0.548	0.767	0.865	0.914	1	0.672
H16	0.223	0.232	0.213	0.308	0.390	0.390	0.390	0.336	0.351	0.368	0.600	0.346
H17	0.623	0.909	0.908	0.679	0.665	0.673	0.673	0.613	0.665	0.666	0.675	0.704
H18	0.564	0.612	0.549	0.308	0.274	0.351	0.351	0.259	0.360	0.313	0.474	0.401
H19	0.787	0.905	0.959	0.823	0.582	0.564	0.564	0.495	0.474	0.479	0.622	0.659
H20	0.211	0.307	0.302	0.335	0.288	0.266	0.266	0.211	0.194	0.178	0.253	0.256
H21	0.456	0.537	0.435	0.420	0.773	0.738	0.738	0.698	0.856	0.887	1	0.685
H22	0.744	1	1	1	1	1	1	1	1	1	1	0.977
H23	0.557	0.563	0.484	0.594	0.579	0.382	0.382	0.334	0.350	0.478	0.532	0.476
H24	0.501	0.502	0.717	1	0.715	0.696	0.696	0.611	0.563	0.551	0.553	0.656
H25	0.849	1	0.769	0.662	1	0.857	0.857	0.765	0.767	0.826	0.917	0.843
H26	0.675	0.514	0.590	0.614	0.631	0.486	0.486	0.428	0.425	0.388	0.535	0.525
H27	0.398	0.434	0.512	0.590	0.58	0.588	0.588	0.478	0.429	0.477	0.623	0.518
Mean	0.641	0.721	0.747	0.762	0.767	0.705	0.705	0.654	0.661	0.683	0.803	0.714

Source: Author’s Calculations

Table 7C.4: Pure Technical Efficiency (2001 to 2011)

Code	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Mean
H1	0.976	0.640	0.745	1	0.803	0.704	0.704	0.506	0.475	0.536	0.606	0.699
H2	1	1	1	1	1	1	1	1	1	1	1	1
H3	0.366	0.542	0.529	0.549	0.688	0.746	0.746	1	1	1	1	0.742
H4	0.770	1	0.876	1	0.935	0.856	0.856	0.624	0.684	0.631	0.908	0.831
H5	1	1	1	1	1	1	1	1	1	1	1	1
H6	0.927	1	1	1	1	1	1	1	1	1	1	0.933
H7	1	1	1	1	1	1	1	1	1	1	1	1
H8	0.776	0.926	1	1	1	1	1	1	0.956	0.967	0.963	0.963
H9	1	1	0.933	0.740	0.759	0.696	0.696	0.750	0.687	0.778	0.891	0.912
H10	1	1	1	1	1	1	1	1	1	1	1	1
H11	0.560	0.794	0.641	0.855	0.875	0.726	0.726	0.640	0.612	1	1	0.766
H12	0.544	0.59	1	1	1	0.731	0.731	0.735	0.745	0.784	1	0.805
H13	1	1	1	1	1	1	1	0.836	0.802	0.451	0.784	0.898
H14	1	1	1	1	1	1	1	1	1	1	1	1
H15	0.401	0.327	0.663	0.769	0.797	0.587	0.587	0.862	0.927	0.962	1	0.717
H16	0.414	0.447	0.253	0.458	0.519	0.489	0.489	0.355	0.381	0.396	0.608	0.437
H17	0.850	0.910	0.915	0.681	0.689	0.718	0.718	0.703	0.767	0.669	0.678	0.754
H18	0.684	0.737	0.570	0.346	0.304	0.385	0.385	0.324	0.470	0.418	0.479	0.464
H19	0.983	1	1	0.870	0.586	0.580	0.580	0.524	0.529	0.528	0.622	0.709
H20	0.304	0.320	0.382	0.425	0.352	0.354	0.354	0.357	0.329	0.290	0.288	0.341
H21	0.465	0.584	0.484	0.474	0.777	0.806	0.806	0.714	0.858	0.897	1	0.715
H22	1	1	1	1	1	1	1	1	1	1	1	1
H23	0.932	0.623	0.553	0.636	0.724	0.410	0.410	0.334	0.353	0.509	0.532	0.547
H24	0.513	0.504	0.726	1	0.739	0.714	0.714	0.641	0.598	0.659	0.599	0.673
H25	0.903	1	0.780	0.673	1	0.939	0.939	0.851	0.823	0.831	1	0.885
H26	0.684	0.531	0.592	0.616	0.652	0.523	0.523	0.502	0.489	0.435	0.540	0.553
H27	0.472	0.435	0.555	0.647	0.727	0.808	0.808	0.651	0.585	0.616	0.672	0.634
Mean	0.760	0.774	0.785	0.805	0.812	0.769	0.769	0.737	0.743	0.754	0.821	0.775

Source: Author’s Calculations

Table 7C.5: Scale Efficiency (2001 to 2011)

Code	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Mean
H1	0.669	0.988	0.936	1	0.975	0.954	0.954	0.963	0.933	0.868	0.992	0.930
H2	0.749	0.756	0.729	0.739	0.816	0.720	0.720	0.589	0.605	0.599	0.940	0.724
H3	0.999	0.999	0.986	0.965	0.979	0.921	0.921	0.879	0.930	1	1	0.962
H4	0.686	0.663	0.816	0.876	0.924	0.945	0.945	0.992	0.990	0.986	0.943	0.888
H5	1	1	1	1	1	1	1	1	1	1	1	1
H6	0.639	1	1	0.942	0.930	0.944	0.944	1	1	0.782	0.949	0.921
H7	0.824	0.861	0.901	0.872	1	0.873	0.873	0.880	0.816	0.948	1	0.895
H8	0.918	0.967	1	1	1	1	1	0.998	0.961	0.907	0.990	0.976
H9	1	1	0.965	0.988	0.942	0.994	0.994	0.852	0.793	0.936	0.990	0.950
H10	1	1	1	1	1	1	1	1	1	1	1	1
H11	0.999	0.967	0.995	0.942	0.971	0.971	0.971	0.859	0.940	1	1	0.965
H12	0.746	0.995	1	1	0.946	0.857	0.857	0.927	0.925	0.991	1	0.931
H13	1	1	1	1	1	0.910	0.910	0.788	0.804	0.868	0.923	0.928
H14	1	1	1	1	0.864	0.855	0.855	0.686	0.694	0.732	1	0.881
H15	0.829	0.779	0.945	0.979	0.989	0.934	0.934	0.890	0.933	0.950	1	0.924
H16	0.537	0.519	0.845	0.671	0.751	0.798	0.798	0.948	0.922	0.929	0.987	0.791
H17	0.734	0.999	0.993	0.996	0.966	0.938	0.938	0.871	0.868	0.995	0.996	0.934
H18	0.825	0.831	0.963	0.889	0.902	0.912	0.912	0.799	0.766	0.750	0.990	0.867
H19	0.801	0.905	0.959	0.946	0.993	0.972	0.972	0.944	0.896	0.907	0.999	0.934
H20	0.692	0.961	0.791	0.789	0.817	0.753	0.753	0.591	0.589	0.613	0.880	0.748
H21	0.981	0.920	0.898	0.886	0.995	0.915	0.915	0.979	0.997	0.989	1	0.952
H22	0.744	1	1	1	1	1	1	1	1	1	1	0.977
H23	0.598	0.903	0.876	0.933	0.800	0.931	0.931	1	0.993	0.940	0.999	0.900
H24	0.976	0.996	0.987	1	0.967	0.975	0.975	0.953	0.942	0.835	0.923	0.957
H25	0.940	1	0.985	0.984	1	0.913	0.913	0.898	0.932	0.994	0.917	0.952
H26	0.986	0.967	0.998	0.998	0.967	0.929	0.929	0.852	0.869	0.893	0.991	0.944
H27	0.843	0.998	0.923	0.912	0.798	0.728	0.728	0.734	0.733	0.775	0.927	0.827
Mean	0.841	0.925	0.944	0.937	0.937	0.913	0.913	0.884	0.883	0.896	0.975	0.913

Source: Author’s Calculations

Table 7C.6: Returns to Scale (2001 to 2011)

Code	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
H1	DRS	IRS	DRS	CRS	DRS	DRS	DRS	DRS	DRS	DRS	IRS
H2	IRS	IRS	IRS	IRS	IRS	IRS	IRS	IRS	IRS	IRS	IRS
H3	CRS	IRS	IRS	DRS	DRS	DRS	DRS	DRS	DRS	CRS	CRS
H4	IRS	IRS	IRS	IRS	IRS	IRS	IRS	DRS	DRS	IRS	IRS
H5	CRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS
H6	DRS	CRS	CRS	DRS	DRS	DRS	DRS	CRS	CRS	DRS	DRS
H7	DRS	DRS	DRS	DRS	CRS	DRS	DRS	DRS	DRS	DRS	CRS
H8	DRS	IRS	CRS	CRS	CRS	CRS	CRS	DRS	DRS	DRS	DRS
H9	CRS	CRS	IRS	IRS	DRS	IRS	IRS	DRS	DRS	DRS	DRS
H10	CRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS
H11	CRS	IRS	IRS	DRS	DRS	DRS	DRS	DRS	DRS	CRS	CRS
H12	IRS	IRS	CRS	CRS	DRS	DRS	DRS	DRS	DRS	IRS	CRS
H13	CRS	CRS	CRS	CRS	CRS	DRS	DRS	DRS	DRS	DRS	IRS
H14	CRS	CRS	CRS	CRS	DRS	DRS	DRS	DRS	DRS	DRS	CRS
H15	IRS	IRS	IRS	IRS	IRS	DRS	DRS	DRS	DRS	DRS	CRS
H16	IRS	IRS	IRS	IRS	IRS	IRS	IRS	DRS	DRS	DRS	IRS
H17	IRS	DRS	IRS	IRS	DRS	DRS	DRS	DRS	DRS	IRS	DRS
H18	DRS	DRS	DRS	DRS	DRS	DRS	DRS	DRS	DRS	DRS	IRS
H19	IRS	IRS	IRS	IRS	DRS	DRS	DRS	DRS	DRS	DRS	IRS
H20	DRS	DRS	DRS	DRS	DRS	DRS	DRS	DRS	DRS	DRS	DRS
H21	IRS	IRS	IRS	IRS	IRS	IRS	IRS	IRS	DRS	DRS	CRS
H22	IRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS	CRS
H23	IRS	IRS	IRS	IRS	IRS	IRS	IRS	CRS	DRS	IRS	DRS
H24	DRS	DRS	IRS	CRS	DRS	DRS	DRS	DRS	DRS	DRS	DRS
H25	DRS	CRS	IRS	DRS	CRS	DRS	DRS	DRS	DRS	DRS	IRS
H26	IRS	IRS	IRS	DRS	DRS	DRS	DRS	DRS	DRS	DRS	DRS
H27	DRS	IRS	DRS	DRS	DRS	DRS	DRS	DRS	DRS	DRS	DRS

Source: Author’s Calculations

Table 7C.7: Total Factor Productivity Change (2001 to 2011)

Code	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Mean
H1	0.735	1.016	1.736	0.695	0.934	0.900	1.091	0.969	1.200	1.050	1.001
H2	0.975	1.042	1.093	1.213	0.969	0.950	1.113	1.010	1.027	1.024	1.039
H3	1.179	0.929	1.060	1.173	1.234	1.192	1.471	1.089	1.007	1.066	1.132
H4	0.951	0.927	1.314	1.134	1.100	1.089	0.823	1.158	0.879	1.228	1.049
H5	1.057	1.542	0.823	0.871	1.016	1.209	1.087	1.108	1.174	0.95	1.068
H6	1.617	0.806	1.004	0.994	1.113	1.118	1.339	1.087	0.964	1.115	1.097
H7	1.18	1.134	1.128	1.123	1.081	1.208	1.132	0.945	0.978	0.914	1.078
H8	1.072	1.103	1.065	0.980	1.046	1.084	1.084	0.961	1.060	1.006	1.045
H9	1.019	0.808	0.840	1.059	1.105	1.105	1.065	0.896	1.281	0.969	1.006
H10	1.114	1.220	0.923	1.020	1.173	1.089	1.126	0.958	0.936	0.584	0.996
H11	1.073	0.754	1.380	1.060	0.929	0.998	1.081	1.091	2.075	0.942	1.097
H12	0.957	1.743	1.071	1.132	0.768	0.994	1.062	1.099	1.044	1.172	1.082
H13	0.905	0.943	1.029	0.872	1.111	1.112	0.882	0.997	0.533	1.581	0.965
H14	0.878	0.864	1.020	0.870	1.106	1.098	1.021	1.043	1.207	1.028	1.008
H15	0.816	1.964	1.274	1.186	0.882	1.661	1.135	1.185	0.957	1.021	1.167
H16	0.791	0.961	1.617	1.254	1.151	1.148	1.006	1.023	1.088	1.057	1.092
H17	1.051	0.985	0.798	1.153	1.158	0.973	1.118	1.177	0.933	0.918	1.019
H18	1.203	0.973	0.644	0.883	1.483	1.028	0.933	1.392	0.900	0.986	1.016
H19	1.053	0.961	0.946	0.789	1.123	1.018	1.121	1.024	0.995	1.131	1.011
H20	1.093	0.939	1.194	0.812	1.016	0.968	1.088	0.951	0.956	0.998	0.996
H21	0.923	0.779	1.041	1.794	1.093	1.148	1.058	1.307	1.054	1.182	1.112
H22	1.454	0.950	1.079	1.343	1.142	1.154	0.980	1.134	0.935	0.897	1.094
H23	1.045	0.967	1.257	0.997	0.790	1.064	1.028	1.084	1.329	0.877	1.033
H24	0.974	1.233	1.548	0.720	1.088	1.073	1.071	0.986	1.038	0.953	1.050
H25	1.085	0.874	0.866	1.228	1.081	1.169	1.035	1.022	0.907	1.012	1.021
H26	0.716	1.041	1.128	1.113	0.959	1.085	1.066	1.078	0.807	1.282	1.015
H27	0.855	1.090	1.231	1.073	1.282	0.998	1.053	0.972	1.042	1.149	1.068
Mean	1.012	1.029	1.089	1.036	1.062	1.090	1.070	1.059	1.023	1.027	1.049

Source: Author’s Calculations

Table 7C.8: Technical Efficiency Change (2001 to 2011)

Code	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Mean
H1	0.969	1.103	1.434	0.783	0.857	0.832	0.872	0.909	1.052	1.292	0.992
H2	1.009	0.965	1.013	1.104	0.882	0.836	0.978	1.027	0.991	1.569	1.023
H3	1.480	0.963	1.017	1.271	1.019	1.039	1.232	1.058	1.075	1	1.106
H4	1.256	1.079	1.226	0.985	0.937	0.991	0.772	1.092	0.919	1.376	1.049
H5	1	1	1	1	1	1	1	1	1	1	1
H6	1.688	1	0.942	0.988	1.015	1.036	1.023	1	0.782	1.214	1.048
H7	1.045	1.046	0.968	1.147	0.873	1.005	1.003	0.928	1.162	1.055	1.020
H8	1.255	1.117	1	1	1	1	0.998	0.921	0.955	1.087	1.029
H9	1	0.901	0.812	0.976	0.968	1.003	0.921	0.853	1.337	1.211	0.988
H10	1	1	1	1	1	1	1	1	1	1	1
H11	1.371	0.830	1.263	1.055	0.831	0.917	0.850	1.047	1.737	1	1.060
H12	1.447	1.704	1	0.946	0.662	0.903	1.204	1.012	1.127	1.287	1.094
H13	1	1	1	1	0.910	0.927	0.782	0.978	0.607	1.848	0.968
H14	1	1	1	0.864	0.990	0.988	0.812	1.012	1.055	1.365	1
H15	0.766	2.458	1.202	1.047	0.695	1.352	1.035	1.128	1.057	1.094	1.116
H16	1.042	0.920	1.441	1.267	1.002	1.045	0.825	1.045	1.047	1.631	1.104
H17	1.458	0.999	0.748	0.980	1.011	0.878	1.036	1.086	1.001	1.015	1.008
H18	1.085	0.897	0.561	0.891	1.281	0.942	0.782	1.392	0.870	1.514	0.983
H19	1.149	1.060	0.858	0.707	0.969	0.915	0.959	0.958	1.011	1.299	0.977
H20	1.459	0.984	1.109	0.858	0.926	0.866	0.916	0.917	0.917	1.426	1.019
H21	1.178	0.809	0.966	1.841	0.955	1.034	0.916	1.226	1.036	1.127	1.082
H22	1.344	1	1	1	1	1	1	1	1	1	1.030
H23	1.009	0.861	1.226	0.976	0.659	0.970	0.902	1.049	1.366	1.112	0.995
H24	1.002	1.428	1.395	0.715	0.975	0.990	0.885	0.922	0.979	1.004	1.010
H25	1.177	0.769	0.861	1.511	0.857	0.972	0.917	1.003	1.078	1.109	1.008
H26	0.761	1.149	1.040	1.027	0.771	0.931	0.945	0.994	0.913	1.379	0.977
H27	1.092	1.180	1.152	0.982	1.015	0.846	0.959	0.897	1.113	1.305	1.046
Mean	1.129	1.049	1.026	1.013	0.919	0.966	0.939	1.012	1.027	1.216	1.026

Source: Author’s Calculations

Table 7C.9: Technological Change (2001 to 2011)

Code	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Mean
H1	0.758	0.921	1.210	0.887	1.089	1.082	1.251	1.065	1.141	0.813	1.009
H2	0.966	1.080	1.079	1.098	1.098	1.137	1.137	0.983	1.037	0.653	1.016
H3	0.796	0.965	1.043	0.923	1.211	1.147	1.194	1.030	0.937	1.066	1.023
H4	0.757	0.859	1.072	1.151	1.174	1.099	1.066	1.060	0.957	0.893	1
H5	1.057	1.542	0.823	0.871	1.016	1.209	1.087	1.108	1.174	0.950	1.068
H6	0.958	0.806	1.066	1.006	1.096	1.080	1.309	1.087	1.233	0.918	1.047
H7	1.130	1.085	1.165	0.980	1.238	1.202	1.128	1.019	0.841	0.867	1.057
H8	0.854	0.987	1.065	0.980	1.046	1.084	1.086	1.044	1.110	0.926	1.015
H9	1.019	0.898	1.034	1.085	1.141	1.102	1.157	1.050	0.958	0.800	1.019
H10	1.114	1.220	0.923	1.020	1.173	1.089	1.126	0.958	0.936	0.584	0.996
H11	0.782	0.908	1.092	1.004	1.118	1.088	1.272	1.042	1.194	0.942	1.035
H12	0.661	1.023	1.071	1.196	1.160	1.100	0.882	1.085	0.926	0.910	0.989
H13	0.905	0.943	1.029	0.872	1.221	1.199	1.128	1.019	0.878	0.855	0.997
H14	0.878	0.864	1.020	1.007	1.118	1.112	1.257	1.031	1.144	0.753	1.008
H15	1.064	0.799	1.060	1.133	1.269	1.229	1.097	1.051	0.906	0.934	1.045
H16	0.759	1.045	1.122	0.990	1.149	1.099	1.219	0.979	1.039	0.648	0.989
H17	0.721	0.986	1.068	1.176	1.145	1.108	1.079	1.084	0.933	0.905	1.011
H18	1.109	1.085	1.149	0.991	1.158	1.091	1.194	1	1.034	0.651	1.034
H19	0.917	0.907	1.102	1.116	1.159	1.112	1.170	1.069	0.984	0.871	1.035
H20	0.749	0.954	1.076	0.947	1.097	1.118	1.187	1.036	1.042	0.700	0.978
H21	0.783	0.963	1.077	0.975	1.145	1.111	1.155	1.067	1.017	1.049	1.028
H22	1.082	0.950	1.079	1.343	1.142	1.154	0.980	1.134	0.935	0.897	1.062
H23	1.036	1.123	1.025	1.022	1.199	1.097	1.139	1.033	0.974	0.789	1.038
H24	0.972	0.863	1.110	1.008	1.116	1.083	1.209	1.069	1.061	0.950	1.040
H25	0.922	1.137	1.005	0.813	1.261	1.202	1.128	1.019	0.841	0.913	1.014
H26	0.940	0.906	1.084	1.084	1.245	1.166	1.127	1.084	0.884	0.930	1.039
H27	0.783	0.924	1.069	1.092	1.263	1.179	1.097	1.084	0.936	0.88	1.021
Mean	0.896	0.981	1.061	1.022	1.156	1.128	1.140	1.047	0.996	0.845	1.022

Source: Author’s Calculations

Table 7C.10: Pure Technical Efficiency Change (2001 to 2011)

Code	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Mean
H1	0.655	1.165	1.342	0.803	0.876	0.844	0.851	0.939	1.130	1.131	0.954
H2	1	1	1	1	1	1	1	1	1	1	1
H3	1.480	0.976	1.039	1.253	1.084	1.037	1.294	1	1	1	1.106
H4	1.299	0.876	1.142	0.935	0.916	0.954	0.764	1.095	0.923	1.439	1.017
H5	1	1	1	1	1	1	1	1	1	1	1
H6	1.078	1	1	1	1	1	1	1	1	1	1.008
H7	1	1	1	1	1	1	1	1	1	1	1
H8	1.192	1.080	1	1	1	1	1	0.956	1.012	0.996	1.022
H9	1	0.933	0.793	1.025	0.917	1.066	1.011	0.916	1.133	1.145	0.989
H10	1	1	1	1	1	1	1	1	1	1	1
H11	1.417	0.807	1.334	1.023	0.830	0.916	0.962	0.956	1.634	1	1.060
H12	1.085	1.695	1	1	0.731	0.919	1.094	1.014	1.052	1.276	1.063
H13	1	1	1	1	1	1	0.836	0.959	0.562	1.738	0.976
H14	1	1	1	1	1	1	1	1	1	1	1
H15	0.816	2.027	1.159	1.037	0.736	1.395	1.053	1.076	1.037	1.039	1.096
H16	1.079	0.565	1.815	1.132	0.942	0.976	0.744	1.074	1.039	1.535	1.039
H17	1.070	1.006	0.745	1.011	1.041	0.880	1.114	1.090	0.873	1.013	0.978
H18	1.077	0.774	0.607	0.878	1.267	0.925	0.908	1.452	0.888	1.147	0.965
H19	1.017	1	0.870	0.674	0.989	0.911	0.992	1.009	0.998	1.179	0.955
H20	1.052	1.194	1.112	0.828	1.004	0.973	1.039	0.920	0.881	0.993	0.994
H21	1.256	0.829	0.98	1.639	1.038	1.012	0.875	1.203	1.045	1.115	1.080
H22	1	1	1	1	1	1	1	1	1	1	1
H23	0.668	0.888	1.151	1.138	0.566	0.964	0.845	1.057	1.442	1.046	0.945
H24	0.982	1.441	1.377	0.739	0.967	0.978	0.918	0.933	1.103	0.909	1.016
H25	1.107	0.780	0.862	1.487	0.939	0.987	0.918	0.967	1.010	1.203	1.010
H26	0.776	1.114	1.041	1.06	0.802	0.911	1.052	0.976	0.888	1.242	0.977
H27	0.922	1.276	1.165	1.123	1.112	0.842	0.957	0.898	1.052	1.092	1.036
Mean	1.021	1.020	1.035	1.013	0.944	0.977	0.965	1.013	1.011	1.107	1.010

Source: Author's Calculations

Table 7C.11: Scale Efficiency Change (2001 to 2011)

Code	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Mean
H1	1.478	0.947	1.069	0.975	0.978	0.986	1.024	0.969	0.931	1.142	1.040
H2	1.009	0.965	1.013	1.104	0.882	0.836	0.978	1.027	0.991	1.569	1.023
H3	1	0.987	0.979	1.015	0.940	1.002	0.952	1.058	1.075	1	1
H4	0.967	1.231	1.074	1.054	1.022	1.039	1.010	0.998	0.996	0.956	1.032
H5	1	1	1	1	1	1	1	1	1	1	1
H6	1.565	1	0.942	0.988	1.015	1.036	1.023	1	0.782	1.214	1.040
H7	1.045	1.046	0.968	1.147	0.873	1.005	1.003	0.928	1.162	1.055	1.020
H8	1.053	1.034	1	1	1	1	0.998	0.963	0.944	1.091	1.008
H9	1	0.965	1.024	0.953	1.056	0.940	0.911	0.931	1.180	1.058	0.999
H10	1	1	1	1	1	1	1	1	1	1	1
H11	0.967	1.029	0.947	1.031	1.001	1.001	0.883	1.095	1.064	1	1
H12	1.334	1.005	1	0.946	0.905	0.983	1.101	0.999	1.071	1.009	1.030
H13	1	1	1	1	0.910	0.927	0.935	1.020	1.079	1.063	0.992
H14	1	1	1	0.864	0.990	0.988	0.812	1.012	1.055	1.365	1
H15	0.939	1.213	1.037	1.010	0.945	0.969	0.983	1.048	1.019	1.052	1.019
H16	0.966	1.627	0.794	1.119	1.063	1.071	1.109	0.973	1.007	1.062	1.063
H17	1.362	0.994	1.004	0.969	0.971	0.998	0.930	0.996	1.147	1.001	1.031
H18	1.007	1.159	0.923	1.014	1.011	1.018	0.861	0.959	0.980	1.32	1.018
H19	1.130	1.060	0.986	1.050	0.979	1.004	0.967	0.949	1.012	1.102	1.022
H20	1.387	0.824	0.997	1.035	0.922	0.89	0.882	0.997	1.041	1.435	1.024
H21	0.938	0.976	0.986	1.123	0.920	1.022	1.047	1.019	0.991	1.011	1.002
H22	1.344	1	1	1	1	1	1	1	1	1	1.030
H23	1.510	0.970	1.065	0.857	1.165	1.006	1.067	0.993	0.947	1.063	1.053
H24	1.021	0.992	1.013	0.967	1.008	1.013	0.965	0.988	0.887	1.104	0.994
H25	1.063	0.985	0.999	1.016	0.913	0.985	0.999	1.037	1.067	0.922	0.997
H26	0.981	1.032	1	0.969	0.960	1.021	0.899	1.019	1.028	1.110	1.001
H27	1.185	0.925	0.988	0.874	0.912	1.005	1.003	0.999	1.058	1.195	1.010
Mean	1.106	1.028	0.991	1.001	0.974	0.989	0.973	0.999	1.016	1.098	1.016

Source: Author’s Calculations

Chapter 8

Findings, Conclusions and Recommendations

8.1 Introduction

The performance is implied in every economic activity and primarily stands for producing more and more outputs from less and less inputs. It can be defined by the two terms, namely, efficiency and productivity. Productivity is evaluated by the ratio of output(s) to the inputs(s) and efficiency by the ratio of observed output/input to optimal output/input. Assessment and monitoring of performance of any organization is critical to check the degree to which inputs are utilized in the process of obtaining desired outputs. It is in this context that this study has been carried out to examine the efficiency and productivity of public sector hospitals of Uttarakhand.

This study is based on both cross-sectional and time series data collected from the Directorate of Medical Health and Family Welfare, Government of Uttarakhand, Dehradun, India. The cross-sectional data for the year 2011 are used for the efficiency evaluation of the hospitals while for the productivity measurement of the hospitals, panel data for the years 2001 to 2011 are used.

8.2 Major Findings and Conclusions

The major findings and conclusions of the study are given in two sections as cross sectional analysis and time series analysis. The cross sectional analysis is carried out by using three different DEA models. The time series analysis is carried out using DEA based MPI approach.

8.2.1 Cross Sectional Analysis

The cross sectional analysis using basic DEA models, non-oriented model and NSM model are discussed in this section.

8.2.1.1 Analysis by Using Basic CCR and BCC Models

1. The results of basic CCR models reveal that only 10 hospitals attain the maximum degree of overall technical efficiency. All the other remaining 26 hospitals are inefficient.

2. The average OTE score works out to be 0.772, which implies that an average 22.80% of the technical potential of hospitals is not in use, implying that these hospitals have the scope of producing 22.80% more outputs with the same level of inputs.
3. The results of basic BCC model reveal that only 16 hospitals are found efficient with average PTE score 0.810. The average SE score of hospitals is 0.955. This shows that the hospitals are 19 % pure technical inefficient and 4.5% scale inefficient
4. It is noted that the six hospitals, namely, H2, H6, H21, H28, H31 and H36 are overall technically inefficient but pure technically efficient. This shows that these hospitals are able to convert their inputs into outputs with 100% efficiency, but their OTE is not 100% due to their scale-size disadvantages.
5. Two hospitals, namely, Female Hospital Dehradun (H7) and Combined Hospital Roorkee (H14) with highest peer count (17) are likely to be the hospitals that are efficient with respect to a large number of factors and are probably a good example of "global leader". Another two efficient hospital, namely, Combines Hospital Kotdwar (H5, number of peer counts =12) and CR Female Hospital Haridwar (H12, number of peer counts =11) also appear in the reference sets of the inefficient hospitals and they may be the role model for the inefficient hospitals to improve their performance.
6. The RTS analysis shows that 38.88% of the hospitals operate at IRS, and 30.56% at DRS. Most of the hospitals found operating at DRS are larger in size, while those operating at IRS are relatively smaller in size. To attain the efficiency frontier, the hospitals on DRS have to reduce their scale-size and those on IRS have to increase their scale-size.
7. The input-output slack analysis shows that on average, number of beds, number of doctors and number of paramedical staff show slacks 8.33, 2.36 and 4.69 respectively.
8. The target setting results show that number of major surgeries has significant scope to expand. This can be expanded by using the proper referral system for secondary and tertiary hospitals so that these hospitals may not have to spend time and resources on providing primary healthcare or treating minors ailments.
9. The region-wise comparison shows that the hospitals of Garhwal region perform better than the hospitals in Kumaon region. Further, on an average, hospitals in Garhwal and Kumaon

regions have to increase their outputs by 20.10% and 25.90% respectively with the given level of inputs.

10. Category-wise comparison of hospitals shows that District male/female hospitals are performing better (80.00% OTE) than combined and base hospitals (73.60% OTE).
11. Area-wise comparison of efficiencies of the hospitals show that the hospitals of plain/ partially plain area are found to perform better than the hospitals located in hilly areas.
12. The post DEA analysis is done to know the robustness of efficiency scores and to assess the effect of various background variables on the efficiency scores. Results of sensitivity analysis show that the efficiency scores are robust and no outlier is found on the frontier even after the exclusion of the most efficient hospitals.
13. Tobit regression analysis shows that hospitals located in the plain/semi-plain region of the state performed better than their counterparts located in the hilly region.

8.2.1.2 Analysis by Using Slack Based Model

The results are also obtained by slack based model (a non-oriented model) using the same cross-sectional data of 36 government hospitals of Uttarakhand for the year 2011. CCR and BCC model give the results either towards input reduction or towards output augmentation. On contrast SBM model give the results in both input reduction and output augmentation. Thus, these results suggest the required improvements in inputs and output both.

1. The results of SBM model suggest that 10 (27.78%) out of 36 hospitals have the maximum degree of OTE.
2. The average value of OTE indicates that about 46% of the technical potential of hospitals is not in use. It implies that public hospitals of the state have the scope of producing 46% more outputs with their existing level of inputs.
3. The results of SBM-VRS model show that out of 36 hospitals, 18 (50%) are pure technical efficient as they efficiently convert their inputs into outputs. However, out of 18 pure technical efficient hospitals, 8 hospitals are technically inefficient due to scale-size effect.

4. The hospital H19 has the lowest SE score (0.173), implying that it can improve its OTE by enhancing its scale size.
5. The target setting results show that all the inputs have the significant scope of reduction and outputs have significant scope of augmentation. On average, inefficient hospitals have to reduce 12.57% of beds, 13.16% of doctors, 14.04% of paramedical staff, and to expand 17.53% of out-door patients, 66.55% of in-door patients, 208.23% of major surgeries and 110.73% of minor surgeries if they want to operate at the level of efficient hospitals.
6. Results of Jackknifing analysis suggest that the efficiency scores are stable even after the exclusion of the most efficient hospitals. In addition, Spearman's rank correlation of coefficients shows that the rankings of hospitals are stable.

8.2.1.3 Analysis by Using New Slack Model

Since the CCR model does not account for slacks and SBM model does not have the radial properties of DEA model, we use NSM DEA model to evaluate the OTE of hospitals. This model deals directly with input-output slacks. The model satisfies the monotone decreasing property with respect to slacks. It also satisfies the other properties of radial DEA model, such as, unit invariance and translation invariance in inputs for the output-oriented model. The dual of the model reveals that all multipliers become positive, i.e., all input and output variables are fully utilized in the performance assessment of the hospitals.

The key findings by using NSM Model are summarized as follows:

1. The results obtained by NSM Model reveals that out of 36 hospitals only 2 (5.56%) are efficient.
2. The average OTE (53.20%) indicates that 46.80% of the technical potential of hospitals is not in use, implying that these hospitals have the scope of producing 46.80% more outputs with the same level of inputs.
3. The hospitals H7 and H12 form efficiency frontier as they scored OTE equal to one.

4. H12 is found to be the most efficient as it is the reference set for the largest number of hospitals.
5. Combined Hospital Padampuri (H21) is found to be the most inefficient hospital as it has the least OTE score 24.30%. This hospital has to increase its number of out-door patients, number of in-door patients, number of major surgery and number of minor surgeries by 311.21%, 311.31%, 311.65% and 311.03% respectively with the same level of inputs.
6. The hospitals of Garhwal region perform better than the hospitals in Kumaon region. On average, hospitals in Garhwal and Kumaon regions have to increase their output by 42.20% and 52% respectively with the given level of inputs.
7. The target setting results show that number of indoor patients has significant scope to expand. This can be expanded by providing better facilities in the hospital, including medical and non-medical staff.
8. Sensitivity analysis results show that the NSM efficiency scores are robust. Removal of efficient hospitals H7 and H12 gives no high influence on the mean OTE, whereas removal of H12 has somewhat high influence on its own efficiency. So, it may be treated as an outlier.
9. Hospital H5 becomes efficient after removal of H12. It means that H5 has the structure similar to H12; it becomes inefficient due to the existence of H12.
10. Super Efficiency scores of the efficient hospitals H7 and H12 are 1.217 and 1.009 respectively.

8.2.2 Panel Data Analysis

The results of efficiency, TFP growth and its components measured by DEA based MPI approach, in 27 public sector hospitals of Uttarakhand for the period from 2001 to 2011 are summarized. The main findings of the efficiency and TFP analysis are as below:

1. On an average, technical efficiency in the hospitals for the entire study period was estimated to be 71.40%, indicating that an average hospital could increase 28.60% of its outputs with the existing level of inputs.
2. The decomposition of technical efficiency into pure technical efficiency and scale efficiency shows that technical inefficiency in the hospitals was mainly due to inefficient conversion of inputs into output (average PTE=77.50%) rather than scale inefficiency (average SE = 91.30%).
3. On average, TFP in the hospitals has grown at a rate of 4.9% per annum during the period under study.
4. The TFP growth was driven by both the catch up and innovation (frontier shift). However, contribution of technical efficiency change (2.6%) was observed slightly higher than that of technical change growth (2.2%).
5. Region-wise analysis of TFP growth demonstrates that on average, TFP growth in Garhwal region was observed slightly higher (5.2%) than that in Kumaon region (4.9%).
6. Both technical efficiency change (2.8%) and technical change (2.7%) contributed to the productivity growth in Garhwal Region; while in the Kumaon region, the contribution of technical efficiency change was much higher (3.1%) than that of technical change (1.8%).
7. Further, the results show that the TFP growth was relatively higher in the hospitals located in the plain/semi-plain areas than that located in the hill areas of the State.
8. The results also conclude that in most of the years, technical efficiency change and technical change indices did not move in the same direction in some hospitals and therefore positive impact of one component on the TFP growth was largely cancelled by the negative impact of the other, probably due to the fact that technology improvement in the hospitals requires huge investment which may not occur on yearly basis.

8.3 Recommendations

The following recommendations are made on the basis of the findings of the study.

1. The hospitals for which the value of efficiency score has been less than one, improved performance could result from diffusion of new technical knowledge, improved managerial practices, and better use of inputs.
2. In order to increase output slacks in an inefficient hospital, augmentation of output parameters is recommended. There is great potential for inefficient hospitals to augment their practice patterns, making it similar to the best practice hospital in order to significantly improve their efficiency and productivity.
3. Scale-wise analysis indicates that the small and medium sized hospitals are more efficient than the larger sized hospitals. As the scale size of the hospitals increases, they move towards the DRS. Most of the small and medium sized hospitals are operating on IRS. In addition, the low PTECh is a major concern for larger-sized hospitals. This indicates that the small and medium sized hospitals are more efficient in resource utilization as compared to larger-sized hospitals.
4. Capacity building of doctors and supporting staff through regular training and skill formation programs should be made. Incentives should also be offered to consistently better performing doctors and staffs. They may be rewarded in public to motivate other staff members. Systematic assessment of their periodical work must be made and considered at the time of promotion.
5. Most of the hospitals are found to be operated either at CRS or IRS, their efficiency and productivity can be increased by expanding their scale-size.
6. The performance of hospitals located in plain/partially plain areas is better than the hospitals situated in hilly areas. As private hospitals/nursing homes in the hilly areas are quite rare, most of the patients depend on the government hospital services. Therefore, for improving the performance of these hospitals, these hospitals must be located at appropriate places where most of the people of that region be connected. Competent doctors and all the facilities must be available in the hospital so that people may have no need to go far from these hospitals, and ambulance facility should also be available in the hospital.

7. Performance can be improved by recruiting motivated and trained health workers and capacity building of existing staff through training programs. Some motivational policies such as promotions, and performance based reward system should be introduced.
8. Efforts are required to be made in the direction of transfer policy to the competent doctors in the hilly and remote areas.
9. The questionable performance hospitals have to be evaluated on a case-by-case basis taking into account the management structure, prevailing technology, staff relations, location and environment in order to identify those with the potential for improvement.
10. The positive effect of background variables on efficiency reveals that the hospitals could improve their performance by enhancing the input beds, doctors and paramedical staff, which requires the scrapping the damage beds and inducting the new ones. The development of the public health system is not only inadequate, but also the available facilities are not of quality, particularly in hilly and remote areas.
11. Use of more advanced technologies, better maintenance of available infrastructure and recruitments of new competent doctors may improve the performance of the hospitals.
12. It is found that the TFP regress in some hospitals is mainly due to negative technological change. In order to achieve the technological progress, the hospitals should increase the use of latest technological resources and create awareness among the people regarding the available facilities in the hospitals.
13. Productivity of public healthcare system of the state could be improved by reallocated the doctors and paramedical staff from inefficient hospitals to efficient ones; improving the human capital base of inefficient hospitals through training, social mobilization and motivation programmes; and investing in new medical technology embodied in medical machines and equipment.
14. Since, average PTE change is observed much lower than the average SE change in the hospitals, it is suggested that overall technical efficiency may be increased by improving the managerial efficiency of the hospitals.

15. Hospitals located in the plain/semi plain areas should focus more on improving the technology to increase TFP, whereas hospitals in the hill areas should emphasize more on improving the technical efficiency to increase TFP.
16. In order to improve the productivity growth, superior quality of services, and economics of scale, better technology and integrated practices should be explored.
17. Keeping in view the global competition, it is recommended that the government of Uttarakhand should develop a mechanism to evaluate its hospitals performance on yearly basis.
18. Among all three models (CCR, SBM and NSM), NSM computes the actual impact of slacks on efficiencies instead of minimum impact. Thus NSM model is recommended for measuring the efficiency of the organizations such as hospitals.

8.4 Directions for Future Research

DEA yields relative performance assessment among the public sector hospitals of Uttarakhand. Further interstate comparison of healthcare and productivity analysis is possible by using different combinations of hospitals of the states. DEA can be used as a selection tool for further detailed hospital-wise study, such as private and public, small and big hospitals. Interstate comparisons can be made to know the performance level of Indian public sector hospitals. The new slack model can be modified to deal with non-discretionary and ordinal factors to measure the efficiency of the DMUs under environmental factor constraints.

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List of Publications

Paper in Journals

1. **Sandeep Kumar Mogha**, Shiv Prasad Yadav and S.P. Singh, (2012), "Performance Evaluation of Indian Private Hospitals Using DEA Approach with Sensitivity Analysis", *International Journal of Advance in Management and Economics*, Vol. 1, No. 2, pp. 1-12.
2. **Sandeep Kumar Mogha**, Shiv Prasad Yadav and S.P. Singh, (2014), "New Slack Model Based Efficiency Assessment of Public Sector Hospitals of Uttarakhand: State of India", *International Journal of Systems Assurance Engineering and Management*, (Springer), Vol. 5, No. 1, pp. 32–42.
3. **Sandeep Kumar Mogha**, Shiv Prasad Yadav and S.P. Singh, (2014), "Estimating Technical and Scale Efficiencies of Private Hospitals Using a Non-Parametric Approach", *International Journal of Operational Research*, Vol. 20, No. 1, pp. 21.40.
4. **Sandeep Kumar Mogha**, Shiv Prasad Yadav and S.P. Singh, "Total Factor Productivity Growth in Private Sector Hospitals of India: An Assessment through MPI Approach", *Global Business and Economic Review*, (Accepted).
5. **Sandeep Kumar Mogha**, Shiv Prasad Yadav and S.P. Singh, "Estimating Technical Efficiency of Public Sector Hospitals of Uttarakhand (India)", *International Journal of Operational Research*, (Accepted).
6. **Sandeep Kumar Mogha**, Shiv Prasad Yadav and S.P. Singh, "Technical Efficiency and Productivity Growth in Public Sector Hospitals of Uttarakhand (India)", *International Journal of Systems Assurance Engineering and Management*, (Accepted).
7. **Sandeep Kumar Mogha**, Shiv Prasad Yadav and S.P. Singh, "Slack Based Measure of Efficiencies of Public Sector Hospitals in Uttarakhand (India)", *Benchmarking: An International Journal*, (Accepted).
8. **Sandeep Kumar Mogha** and Shiv Prasad Yadav, "Application of Differential Evaluation for Data Envelopment Analysis", *International Journal of Data Science*, (Accepted).

Papers in Conference Proceedings

1. **Sandeep Kumar Mogha**, Shiv Prasad Yadav and S.P. Singh, "Technical and Relative Efficiency Assessment of Some Private Sector Hospitals in India*", Int. Conf. on Soft Computing for Problem Solving* (Soc.Pros.) held in Dec. 20-22, 2011 at IIT Roorkee, AISC 130, pp 657-666.
2. Pravesh Kumar, **Sandeep Kumar Mogha** and Milli Pant, "Differential Evaluation of Data Envelopment Analysis", *Int. Conf. on Soft Computing for Problem Solving* (Soc.Pros.) held in Dec. 20-22, 2011 at IIT Roorkee, AISC 130, pp. 311-319.
3. **Sandeep Kumar Mogha**, Shiv Prasad Yadav and S.P. Singh, "SBM-DEA Model Based Efficiency Assessment of Public Sector Hospitals in Uttarakhand, India", *Int. Conf. on Soft Computing for Problem Solving* (Soc.Pros.-2013) held in Dec. 26-28, 2013 at Noida Extension center IIT Roorkee, Vol. 258, pp 467-480.
4. **Sandeep Kumar Mogha**, Shiv Prasad Yadav and S.P. Singh, (2014), "Efficiency Assessment of Secondary Level Public Hospitals of Uttarakhand (India)", *12th International Conference on Data Envelopment Analysis*", April, 14-17, 2014, Malaya University, Kula Lumpur, Malaysia.

Papers Presented in Conference

1. **Sandeep Kumar Mogha** and Shiv Prasad Yadav, "Relative Efficiency of Private Hospitals in India: An Application of DEA", *National Conference on Modeling, Computational Fluid Dynamics and Operations Research*, Feb. 4-5, 2012, Department of Mathematics BITS Pilani, Rajasthan.
2. **Sandeep Kumar Mogha** and Shiv Prasad Yadav, DEA Based Performance Assessment of Government Hospitals in Uttarakhand (India)", *International Conference on Optimization Modelling and Applications*, Nov. 29, 2012 to Dec. 1, 2012, Department of Mathematics Delhi University.