CONSOLIDATION INDUCED SOLUTE TRANSPORT THROUGH CLAY DEPOSITS

Ph.D. THESIS

by

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INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
ROORKEE – 247 667 (INDIA)
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A THESIS

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled "**CONSOLIDATION INDUCED SOLUTE TRANSPORT THROUGH CLAY DEPOSITS"** in partial fulfilment of the requirements for the award of the Degree of Doctor of Philosophy and submitted in the Department of Civil Engineering of the Indian Institute of Technology Roorkee is an authentic record of my own work carried out during a period from July, 2010 to June, 2015 under the supervision of Dr. Mahendra Singh and Dr. C. S. P. Ojha, Professors, Department of Civil Engineering.

The matter presented in the thesis has not been submitted by me for the award of any other degree of this or any other Institute.

(RAKESH PRATAP SINGH)

This is to certify that the above statement made by the candidate is correct to the best of our knowledge.

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The Ph.D. Viva – Voce examination of Rakesh Pratap Singh, research scholar has been held on 21-03-2016.

Chairman, SRC **Signature of External Examiner**

This is to certify that the student has made all the corrections in the thesis.

Supervisor **Supervisor** **Head of the Department**

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ABSTRACT

Small strain one dimensional consolidation theory, based on many simplified assumptions is applicable effectively to thin layers only. Theory of large or finite strain one dimensional consolidation takes into account the self weight of soil, variation of void ratio, compressibility and hydraulic conductivity and offers a generalized approach for consolidation of a homogeneous soil type. These attributes make the theory capable of predicting the settlements of soft soils such as the deposits of dredged materials/ mine tailings under self / overburden loads at their disposal sites and also the consolidation settlement of thicker layers of usual soils. This work presents a novel explicit time marching numerical model based on finite volume method with quadratic three point Lagrangian interpolation function. Model takes into account the geometric non linearity of the governing equation and material nonlinearity of the constitutive equations. Unlike the other numerical models, such as finite element method and finite difference method, this model accounts for the continuity of fluid flow (mass conservation) automatically due to conservation specific formulation of the model at discrete control volume level. The conservativeness and boundedness of the numerical scheme makes the model solutions feasible and stable. The accuracy of the model is maintainable to the level of third order. The time step restrictions are not very tight and depend on consolidation induced velocity and the size of the discrete control volume. The boundary conditions of consolidation for drained and undrained boundaries are presented in terms of void ratio. The initial equilibrium distribution of void ratio due to self load and a pre-existing overburden pressure are determined with the help of quadratic interpolation on data of compressibility constitutive relation. Comparison of the model solutions with analytical and other numerical models affirms the accuracy and efficiency of the model. A parametric study on consolidation behaviour of soft soil having initial void ratio ranging from 3.2 to 2.4, shows almost linear relation of settlement and square root of time up to 80% average degree of consolidation. Model has further been tested on experimental results of consolidation of thicker specimens of 40 mm and 70 mm thickness and has been found to work well.

Solute transport through porous media is an important field of research in the context of geoenvironmental issues. The concerned one-dimensional governing equation is also a differential equation of conservation law. An explicit time marching finite volume numerical model for one-dimensional solute transport in rigid porous media is developed on

the pattern of large strain consolidation. The novelty of the model lies in treating the solute concentrations in liquid and solid phases of the media as combined concentration for developing the numerical scheme and segregating it into solid and liquid concentrations during post processing of the solution. The methodology adopted keeps the solute transport equation linear up to solutions and opens the model at the stage of post processing to accommodate variety of sorption isotherms such as linear-equilibrium, nonlinear-equilibrium and nonlinear-nonequilibrium. The model is also set to accommodate the variation of hydrodynamic dispersion with void ratio and decay reaction of first order. The solute concentration boundary conditions taken up are; constant concentration for a boundary with unlimited reservoir, zero concentration gradient for a non-transmitting boundary and constant flux or reservoir boundary condition for a boundary with small well mixed reservoir. The interpolation scheme followed is the quadratic upwinding in general but at critical situations of high gradient or discontinuity the model adopts the exponential upwinding scheme with normalized variables. Model verification and checks through comparative studies with other numerical models show the efficiency of the model and it requires lesser elements to provide an acceptable solution. The model has further been extended to one-dimensional advection with two-dimensional hydrodynamic dispersion. Quadratic interpolation functions for two-dimensional space are derived. The departure from one dimensional interpolation function is found to be only by a small curvature term which can easily be accommodated with exponential upwinding scheme also. Two-dimensional model maintains the accuracy level of third order as well. Comparison of results with exhibits that less number of elements are required in the suggested model as compared to existing linear interpolation models.

Consolidation induced solute transport is important in assessing the spread of contaminants in soft deposits of dredged materials and mine tailings as well as in the compacted clay liners of waste disposal sites. The penultimate chapter of the thesis describes the synthesis of computational modules of large strain consolidation and solute transport through rigid porous media to give a semi-coupled numerical model for consolidation induced solute transport through deforming porous media due to mechanical consolidation. The coupling of two modules requires additional provision of computation of Darcy velocity due to existing hydraulic gradient and the consolidation induced Darcy velocity in consolidation module. Thus computed Darcy velocity is used for computing the solute transport. Consolidation induced velocity is computed on kinematical considerations

on the basis of reduction in void ratio at each time step as calculated during the consolidation. It is obvious that the kinematical provision for consolidation induced advection provides better mass conservation and continuity of fluid flow compared to computations based on dynamic equation of excess pore pressure gradient. The model performance has been tested for four types of problems varying mainly in sorption isotherm. The first one considers the problem of a hypothetical landfill clay liner with linear sorption isotherm, second is about an experimental observation on kaolinite slurry with nonlinear-nonequilibrium sorption isotherm, the third one is regarding organoclay modified bentonite-soil mix liner material and shows the influence of consolidation on design of such a clay liner with nonlinear equilibrium sorption. Fourth problem is related to two-dimensional solute transport in dredged material deposit with linear equilibrium sorption. The comparison of results with other models affirms the efficiency of the present model. It may also be inferred that the consolidation induced solute transport is worth considering while designing a clay barrier systems for waste disposal sites. A limited parametric study on two-dimensional solute transport for only two parameters, the longitudinal/ lateral dispersivities and effective diffusion coefficient, reveals that the dispersivities have almost negligible influence on two-dimensional spread of contaminants, but the influence of effective diffusion is substantial. Finally, it is concluded that the problems of momentum and mass transfer with deterministic approach can be dealt effectively with finite volume formulation with an advantage of automatic mass conservation and complexity level is less than the finite and boundary element based numerical models.

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(RAKESH PRATAP SINGH)

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NOTATION AND ABBRVIATION

ΔA	Differential area element (m^2)
ΔV	Differential volume element (m^3)
a	Lagrangian coordinate in vertical direction (m)
a_v	Coefficient of compressibility (m^2/ N)
c	Courant number
C_c	Coefficient of compression index
C_{cm}	Combined concentration in solid and liquid phase of porous matrix (mg/L)
c_f	Solute concentration in fluid medium (mg/L)
C_k	Hydraulic conductivity index
c_s	Solute concentration in solid medium (mg/ Litre)
c_{v0}	Initial coefficient of consolidation (m^2/ s)
$D*$	Effective diffusion coefficient (m^2/ s)
D_0	Free solution diffusion coefficient (m^2/ s)
D_a	Coefficient of longitudinal hydrodynamic dispersion (m^2/ s)
D_x	Coefficient of transverse hydrodynamic dispersion (m^2/ s)
e	Void ratio of soil at any depth at any time
e_0	Initial void ratio of the soil layer at any depth
e_{H0}	Void ratio at the top of soil
F	A constant for describing Freundlich isotherm
f_{fa}	Longitudinal solute mass flux in fluid (mg/ m^2 s)
f_{fx}	Transverse solute mass flux in fluid (mg/ m^2 s)
f_s	Solute mass flux in solid phase (mg/ m^2 s)
G_s	Specific gravity of soil solids
H	Height of the soil layer (m)
H_w	Height of free surface water above top of the soil (m)
k	Vertical hydraulic conductivity of soil (m/ s)
$k(e)$	Hydraulic conductivity of soil as a function of void ratio (m/ s)
k_0	Initial Vertical hydraulic conductivity of soil (m/ s)
K_d	Partition coefficient of solute for sorption in solids from the liquid phase (ml/g)

k_e	Effective vertical hydraulic conductivity (m/ s)
K_p	A constant for describing Freundlich isotherm (ml/ g)
LL	Liquid limit (%)
m_{v1}	Coefficient of volume compressibility (m^2/ N)
n	Porosity of porous media
N	Frequency of wave to a finite grid on Fourier spectrum (s^{-1})
OMC	Optimum moisture content (%)
P	Peclet number
P_Δ	Local grid Peclet number
PL	Plastic limit (%)
q	Darcy velocity (m/ s)
q_e	Darcy velocity component due to excess pore pressure/ consolidation (m/ s)
q_h	Darcy velocity component due to external hydraulic gradient (m/ s)
q_p	Pre-existing overburden pressure on soil (Pa)
q_u	Pressure increment on soil causing consolidation (Pa)
S	Settlement of soil layer (m)
S_m	Rate of solute mass source/ sink per unit volume (mg/ m^3 s)
s	Sorbed concentration solute in solids (mg/ Kg)
S_∞	Final settlement (m)
S_t	Settlement of top surface at any time t (m)
t	Time (s)
T_v	Time factor
u	Excess pore pressure (N/ m^2)
u_0	Hydrostatic pore pressure (N/ m^2)
U_p	Pore pressure dissipation based degree of consolidation
U_s	Top surface settlement based degree of consolidation
u_w	Total pore pressure in soil matrix fluid (N/ m^2)
v_f	Velocity of fluid in pores in soil matrix (m/ s)
v_s	Velocity of solids in soil matrix (m/ s)
x	Lagrangian coordinate in horizontal direction (m)
z	Material or reduced solid coordinate in vertical direction (m)
α_a	Longitudinal dispersivity (m)
α_x	Transverse dispersivity (m)
γ_s	Unit weight of soil solids (N/ m^3)

γ_w	Unit weight of water (N/ m^3)
λ	Solute decay constant in general (s^{-1})
λ_c	Solute decay constant in porous matrix (s^{-1})
λ_n	Long wave length to a finite grid on Fourier spectrum (m)
λ_s	Sorption rate constant in nonlinear nonequilibrium sorption isotherm
λ_{sc}	Solute decay constant at source (s^{-1})
ξ	Convective coordinate in vertical direction (m)
ρ_d	Dry density of solid phase in the porous matrix (Kg/ m^3)
σ	Total stress in soil (N/ m^2)
σ'	Effective stress in soil (N/ m^2)
$\sigma'(e)$	Effective stress in soil as a function of void ratio (N/ m^2)
τ	Tortuosity factor

Chapter 1

INTRODUCTION

1.1 OVERVIEW

Mounds of dredged materials from waterways, waste of mining industries containing mineral ores, manifest large settlement due to consolidation under self weight. The natural soft clay layers below the foundations of embankments and buildings also pose similar challenge. Proper estimation of primary consolidation settlement, which is based on finite or large strain one dimensional consolidation of soft clays, is a challenging task. The theoretical treatment of this problem involves solution of nonlinear partial differential equation belonging to the category of conservation law. The analytical solution to this equation is available only with certain constraints and the numerical solutions like finite difference, piecewise linear model and finite element also exist in literature. The finite volume method, as a numerical technique of discretization method, works well for solution of all types of conservation laws. It is based on integral formulation and keeps some of the important features similar to the finite element method. The additional feature, i.e., local conservativeness of numerical fluxes at each control volume, makes this method more special and attractive. The present work primarily aims at developing a numerical model of large strain consolidation equation based on the finite volume method with three point Lagrangian interpolation function. The accuracy of the method as well as its versatility in accommodating various initial and boundary conditions has also been assessed. The accuracy of the void ratio based finite strain one dimensional finite volume formulation of consolidation, has been verified with experimental results of consolidation of somewhat thicker specimens of compacted clay. The other part of work describes consolidation induced solute transport. The equations of contaminant/ solute transport with sorption and decay terms also belong to the class of differential equations of conservation laws and can be cast as finite volume formulation. The present work includes the extended application of large strain consolidation to consolidation induced solute transport with one and two dimensional spread in clayey systems followed by validation, advantages and limitations of the proposed model over the existing numerical models.

The issue of the movement of dissolved contaminants into the ground and ultimately to ground water reserves, from waste disposal facilities like engineered landfills, is still significant to researchers. This geo-environmental problem has attracted the interests of many researchers for developing various improvements in waste disposal technologies to control and prevent the ground water contamination. Water in a pool of waste acts not only as carrier of contaminant solutes, but plays also an important role in transformation and degradation of the waste and thus accumulates more of the solutes progressively with time. When the landfills are active and open to precipitation, formation and migration of leachate is enhanced and it becomes important to restrict the egress of leachate below the ground surface to save ground water from contamination. To mitigate the problem of open landfills, seepage barriers are employed to contain the active waste isolated from the ground water system. The engineered landfills are generally made to isolate the waste from the ground system with two barriers, one at the bottom and other at the top. The bottom barrier liner is laid before placing the waste load and the top barrier cap is formed when the waste emplacement fills the landfill fully. Thus the waste is exposed to precipitation till the emplacement is complete. The base seepage barrier has to resist the advection of leachate to ground against the pooled head, subjected to mechanical and chemical stresses as well as the diffusion of contaminants into the ground.

Waste disposal facilities widely use the composite barrier system consisting of an impermeable thin geomembrane (GM) over a compacted clay liner (CCL) with low permeability. Geo-synthetic clay liners (GCL) alone or in addition to CCL are an alternate low cost barrier system. The underlain earthen layer support to impermeable membrane or low permeable GCL improves the integrity of the barrier system that may take care of any defect appearing sooner or later in the GM or GCL. Therefore, in most of the cases composite system of clay liners is the preferred design of seepage barrier at the base of landfills. In many of the areas where soft clay deposits are available locally, the clay would cost low and may be used for CCL. Wastes from mineral processing such as red mud from alumina and crushed mudrocks may be an equivalent substitute where natural soft clay is not available in the nearby region. However, use of such substitutes are still under investigation. Another advantage with use of soft clay as CCL is its adhesiveness which makes a good bonding with the geomembrane that adds to the integrity of the composite barrier.

Fine-grained soils as barrier material are subjected to waste load depending upon the capacity of the landfill. The waste load may vary from heavy to very heavy for ‘simple dump’ to ‘super dump’ landfills. This mechanical load causes the consolidation of the soft clay barriers. It is difficult to observe the consolidation settlement of the CCL as it is concealed well below the mask of waste. However, leachate collection system (LCS) installed below the bottom barriers of landfill can be used for the water balance analysis through the barrier. The consolidation of soft clay liner system is evident from the water balance analysis that identifies the ‘consolidation water’ component.

The consolidation of CCL is not much known particularly the quantitative information is limited, but it has been envisaged that the consolidation certainly influences the contaminant transport through these barriers. It can definitely be inferred that more is the consolidation more will be its contribution to the contaminant transport. The barriers undergo one-way consolidation with the pore fluid movement towards LCS; this consolidation induced advection in some cases may lead to unanticipated transport of contamination. It is difficult to acquire field data on consolidation of clay barriers because of its inaccessible location in the landfill systems. This situation has prompted to mathematical assessment of consolidation and coupled solute transport of clay barriers rather than going for an experimental investigation. The current work is an attempt to investigate the consolidation and coupled contaminant transport through a clay layer by the numerical mathematical model based on finite volume method.

Studies on spreading of contaminants in natural or created open channels employ advection-dispersion theory. The same theory with little extension of sorption and decay is used for analysing contaminant expansion in ground water flow through incompressible granular soils. The theory considers the steady state condition of flow. The engineering practice still employs the above theory for the analysis of contaminant transport through clay barriers. But, the behaviour of clay barriers differ far from granular soils. Clay minerals are more active to sorption due to high ion exchange capacity and have time dependent consolidation characteristics. These properties of clay make the contaminant transport through clay barriers more complicated compared to granular soils. This makes clear distinction between the contaminant transport through rigid porous media (granular soils) and that through deformable porous media (clays).

In the recent past, attempts were made to generalize the theory of contaminant transport in rigid porous media so that it can include the contaminant transport through deformable porous media as well. The governing coupled equations include finite/ large strain consolidation and solute/ contaminant transport. The preliminary analytical solution of these equations have been attempted and are applicable only for limited conditions. In recent past, attempts have been made to solve these equations through finite element method for general conditions. But, the solution is limited to linear sorption model and there is no explicit mention of conservation of mass (pore water and the solute) either at the element level or for the entire system. Alternatively, it has been attempted to tackle the issue by determining consolidation induced velocity at a location and time and use it with the advection-dispersion-Reaction (ADR) equation. The process has been termed as semi-coupled approach. This recent research employs piecewise linear numerical model for large strain consolidation and uses dual Lagrangian framework for solids and fluid elements separately. The model accommodates the solute transport equations with linear or nonlinear sorption isotherm. However solute mass balance has to be observed separately. The solute distribution differs much from assumed linear distribution and requires more number of elements for fluid than that for solid. Furthermore, the assumed linearity of void ratio over the elements also needs relatively more number of elements to get better results regarding consolidation.

The next section describes in brief the work by various researchers concerning the finite/ large strain consolidation of clays and also solute transport in rigid and deformable porous media including the above descriptions. A few relevant works of solute transport in open channel flow with finite volume formulation are also discussed.

1.2 HISTORICAL BACKGROUND

The present section deals with the literature review on finite strain one dimensional consolidation and solute transport through porous media. In particular, it draws attention on mathematical modelling of finite strain consolidation as well as coupled solute transport.

Theoretical investigations on one dimensional consolidation of soils may be grouped into two categories i.e. small strain theories and finite or large strain theories. The pioneer work of small strain theory of one dimensional consolidation due to mechanical loading was introduced by Terzaghi (1923) along with the concept of effective stress. Later, many investigators attempted to generalise this theory by relaxing few of its restrictions such as material linearity, homogeneity, constant compressibility and permeability, incompressibility of solids and fluids, negligible self weight, no creep and small strain. Schiffman and Gibson (1964) developed the governing equation for one dimensional consolidation assuming variable permeability and coefficient of volume compressibility with the depth as per known functions. Davis and Raymond (1965) gave the governing equation for nonlinear consolidation with the assumption of a constant logarithmic relation of void ratio and effective stress. Basak (1979) reported a governing differential equation independent of material linearity which accepts any void ratio-effective stress-permeability relation but assumed the existence of small strain and no creep. Lekha et al. (2002) gave analytical solution of the equations of Basak (1979) for a few particular relations of void ratio-effective stress and void ratio-permeability due to vertical consolidation.

Few analytical solutions to large strain consolidation equation have also been reported, but these solutions assume certain restrictions on material property. Gibson et al. (1967) provide one simplified solution with constant value of a parameter c_F which depends on current value of void ratio during consolidation. Xie and Leo (2004) reported that the one dimensional large strain consolidation equation given by Gibson (1967) in terms of void ratio can be converted into terms of excess pore pressure and gave analytical solution to this equation with assumption of constant coefficient of volume compressibility throughout the consolidation process with reducing void ratio. Xie et al. (2005) presented a semi analytical solution to Schiffman and Gibson (1964)'s differential equation with excess pore pressure as dependent variable. This solution assumes linear variation of permeability and coefficient of volume compressibility with depth. Further, generalised solutions to large strain consolidation have been attempted by various investigators through several numerical techniques as described next.

McNabb (1960) presented a generalised equation for large strain consolidation using void ratio as the governing parameter. The theory uses coordinate system of 'reduced solids'. Restrictions on material linearity is removed in this solution. Mikasa (1965) proposed another theory of large strain consolidation taking the Eulerian strain as governing parameter. Both the theories omitted the movement of solids. Gibson, et al. (1967) combined these two approaches and proposed a governing equation derived in terms of convective (or moving) boundary coordinates, but presented the final equation in terms of material (reduced solids) coordinates. Most of the large strain consolidation studies are in the context of designing the waste ponds for tailings of mining industries. Somogyi (1979) suggested a governing equation of large strain consolidation in terms of excess pore water pressure with respect to reduced material coordinate. Schiffman et al. (1985) proposed an implicit finite difference numerical model to solve the equation of Somogyi (1979). Cargill

(1982, 1983, and 1984) proposed another governing equation for large strain consolidation in terms of void ratio using reduced material coordinates and gave its solution employing explicit finite difference technique. Olson and Ladd (1979) highlighted the limitations of classical one dimensional consolidation and used finite difference solution by taking into account time dependent load, large strains and non linear material properties. The spatial variation of permeability was however neglected. Following this study, Yong et al. (1983, 1984) presented a piecewise linear numerical model. The proposed governing equation is in terms of excess pore pressure and its variation with spatial coordinates and time. The solution procedure is the explicit finite difference technique and spatial variation of permeability is taken into account. This approach requires updating of all static and kinematic variables at each time step. Feldkamp (1989) presented the solution of large strain consolidation equation of Gibson et al. (1967) numerically by finite element method using Galerkin weighted residual approach. Crank-Nicholson time stepping was used with the provision of halving the time step successively for convergence. Townsend and McVay (1990) published a classical research paper that compiles most of the past studies. It concludes that there is good agreement among the results of all the numerical models at quiescent and final consolidation but the differences appear at the stage of filling level. It is further added that the piecewise linear numerical model is most versatile in handling the various boundary conditions. Fox and Berles (1997) presented a numerical model for large strain consolidation and named it as Consolidation Settlement 2 (CS2). The dimensionless piecewise linear finite difference numerical model uses Eulerian (convective) coordinate system and takes into account the self weight of soil, relative velocity of solids and fluids, variation of hydraulic conductivity and compressibility due to consolidation. The constitutive relations of void ratio and effective stress as well as void ratio and hydraulic conductivity may be used in the form of discrete data points. Bartholomeeusen et al. (2002) performed a number of experiments on settling columns. Different heights of columns of river bed sediment were used in this study and settlement was observed due to self weight only. The results were used for comparing the performance of various numerical models. The study concluded that all the predictions overestimate the initial settlements below the level of 1kPa or time up to about 3 days and marked this disagreement to rate or time dependence of void ratio-effective stress correlation. Recently, Ito and Azam (2013) presented solution of large strain consolidation problem in terms of excess pore pressure obtained by the combination of the equation by Koppla (1970) and Somogyi (1980) equations. They used general purpose solver of PDEs named as FlexPDE based on finite element method. The constitutive equations correlating void ratio with compressibility and hydraulic conductivity were chosen from published data on various tailings. The problem geometry was comprised of 10 m high vertical standpipe with impermeable bottom and permeable top for the case of consolidation under self weight. The results show that the predicted settlement of top of tailing with time is in good agreement with the measured data of Jeervipoolvarn et al. (2009a).

Applications of finite volume method are found in abundance in the conservative transport processes through convection and diffusion associated with heat and fluid flow. There are two discretization methods cell-vertex and cell-centred, however the present description focuses on cell-centred finite volume framework. Godunov (1959) suggested a conservative numerical scheme for solving the hyperbolic PDE (Riemann problem) which is the basic scheme that can be taken as first order finite volume method and forms the base for higher order methods. Initial attempts of finite volume method in computational fluid dynamics used the frame of central differencing scheme and linear interpolation for face-values, however the poor performance of the method on transportiveness lead to upwind differencing (Versteeg and Malalasekera, 2007). Later hybrid differencing came into existence due to Spalding (1972). However, Mc Donald (1971) introduced the finite volume method in the field of numerical fluid dynamics. Mac Cormack and Paullay (1972) extended the method to solve the time dependent two dimensional Euler equation of fluid motion and Rizzi and Inouye (1973) presented its further extension for three dimensional flows. The method came more into use for transport processes in heat transfer and fluid dynamics after the Power law differencing scheme by Patankar (1980). Later the higher order interpolation schemes were developed to enhance the accuracy. The convection dominated unsteady transport process during numerical solutions posed the problem of unphysical numerical diffusion and oscillation. For such schemes, various researchers proposed the use of flux limiters and normalised variables to maintain the required boundedness (Waterson and Deconinck, 2007). A few notable such total variation diminishing (TVD) flux limiters are by van Leer (1974, 1977), van Albada et al. (1982), Osherand Chatkravathy (1984), Sweby (1984), Roe (1985), Gaskell and Lau (1988), Koren (1993), Lien and Leschziner (1994), Waterson and Deconinck (1995), Zhou el al. (1995). The various flux limiters are particularly good to a problem but no one is applicable in general to all physical problems.

Normalised variable (NV) schemes use linear, quadratic or cubic interpolations for face values of control volumes, however to ascertain boundedness condition, it switches to linear schemes at the critical location of NV diagram. Leonard (1979) proposed the explicit third order accurate Quadratic Upstream Interpolation for Convective Kinetics (QUICK) scheme for steady flows and QUICKSET (QUICK with Estimated Streaming Terms) for unsteady flows for convective transport. Further Leonard (1987, 1988) improved the QUICK scheme and gave the NV scheme SHARP (Simple High Resolution Program) based on EULER (Exponential Upwinding or Linear Extrapolation Refinement) – QUICK algorithm. Gaskell and Lau (1988) also contributed to further development in this NV method and proposed the alternating direction implicit (ADI) scheme, SMART (sharp and monotonic *algorithm* for realistic *transport*). Zhu (1992) published another quadratic interpolation NV scheme HLPA (Hybrid Linear/ Parabolic Approximation) and Choi et al. (1995) proposed a cubic interpolation scheme, SMARTER (SMART Efficiently Revised); both the schemes and the scheme SHARP give comparable results.

Subsurface and ground water is prone to contamination due to unplanned disposal of municipal wastes, hospital wastes, liquid wastes from industries etc. as well as non-engineered landfills and other repositories. Keshari and Parmar (2006), while discussing on pollution management, mentioned that the water pollution management in developing countries is still quite behind the mark of acceptable level. This has attracted many researchers to work on design of suitable clay liners for waste disposal sites as an effective barrier to keep the supporting ground and the ground water reserves uncontaminated. The two books Scheidegger (1957) and Collins (1961) describe the review of most of the early attempts on the study of flow through porous media and transport including the phenomenon of dispersion. Schlicter (1905) noticed the dispersion of tracer during its transport through ground water flow. After a long gap Wentworth (1948) developed the mathematical theory of dispersion. Gradually, the theory was shaped to *equation of hydrodynamic dispersion* and was finally referred to as *advection-dispersion equation*. The major contributors to this development are Taylor (1953), Scheidegger (1954), Scheidegger (1961), De Josselin de Jong (1958), Ogata(1958), Saffman (1959), Bear (1961), Harlemann and Rumer (1962) and Bachmat and Bear (1964). Seawater intrusion into nearby aquifers remained the prevalent problem which was modelled for solution. These methods used the ground water sand-tank model, plug or piston flow model, and physically analogous models like Hele-Shaw or parallel plate model (viscous fluids) and Electrical Network model (Wang and Anderson, 1995). The analytical solutions to the mathematical models are very restrictive and assume the medium to be homogeneous and isotropic with simplified boundary conditions. The advent of high speed digital computers since 1960 revolutionised the approach of groundwater studies and numerical models became the most favoured choice (Wang and Anderson, 1995).

Buckley and Leverett (1942) presented the equation for water movement in oil reservoirs with the concept of 'immiscible displacement'. The analytical solution to the equation neglects the influence of gravity and capillarity. The numerical methods came into use in 1950s in the field of flow of petroleum in the oil reservoirs overcoming the limitations of analytical solutions of the equation by Buckley and Leverett (1942). West et al. (1954) applied finite difference method to the equations of flow of gas and oil through petroleum reservoirs. A few other contributors to the field of numerical methods are Fayers and Sheldon (1959), Douglas et al. (1958) and Gotterfield et al. (1966). Ground water flow and transport of solutes has been modelled using the numerical techniques finite difference, finite element or methods of characteristics. Redell and Sunada (1970) gave finite difference solution, Pinder and Cooper (1970) used method of characteristics. Few other researchers of relevance are Oster, et al. (1970), Guymon (1970) and Bredehoeft and Pinder (1973). Initial researches assumed the solutes as non-reacting (tracer) and later the reacting solutes showing decay/ sorption/ desorption were also studied. Rubin and James (1973) presented weighted residual finite element method to predict concentration changes of non-conservative solutes during flow through porous media; whereas Lai and Jurinak (1971) as well as van Genuchten et al. (1974) used finite difference method.

Issues of anthropogenic environmental pollution gained importance much earlier in 1960s. In the beginning, the main concern was the industrial discharges of liquid and gaseous pollutants and their adverse effect on surface water and air. Later, ground water contamination also attracted attention of researchers due to pollutants like waste water stabilization plants, sludge lagoons, runoff from barnyard, septic tank leaching fields or soak pits or pit privies, deep well disposal of industrial wastes or effluents of treatment plants, leachates of decomposing solid wastes of open dumps, sanitary landfills, solid waste composting sites, industrial refuge and treatment plant sludge (Zanoni, 1972). Andersen and Dornbush (1967, 1968) performed study on the water quality of test wells constructed in the nearby region of dump area of city Brooking, South Dakota, USA. The study observed adverse influence particularly in the chloride content, which was fifty times more than that of fresh water. Many such other observations, e.g., Cartwright and Sherman (1967), Apgar and Langmuir (1971), Freeze (1972) and Freeze and Cherry (1979) lead the field to further research . Freeze (1972) brought in the numerical analysis of contaminant (non reactive and non dispersive) transport in the ground water flow modelling.

The threat of contamination of ground water reserves was acknowledged extensively by researchers and this prompted them for developing the ways and means to contain the contaminated liquids and gases. The economical and effective earthen/ natural barriers for landfills, waste impoundments etc. have drawn the attention of several researchers. Fuller (1980) suggested the ways to reduce permeability of soil at refuse disposal site to counter the migration of pollutants to ground water. Cartwright et al. (1981) suggested that the size of waste disposal facilities should be guided by the attenuation capacity of the underlying geologic material even though the waste is isolated through a low hydraulic conductivity liner of fine-grained soil. Fine-grained natural cohesive soils exhibit low hydraulic conductivity and therefore have great potential of acting as barriers for containing the migration of contaminants to ground water. But, it is also noteworthy that the geo-materials exhibit large variation in its sorption/ desorption response to various contaminants and wide variation in the representative property, distribution coefficient (K_d). Arnepalli et al. (2010) describes that the usual batch tests may not provide the real field value of distribution coefficient of a contaminant and geo-material-immobilizing agent system. Pathak, et al. (2014) presents the critical review of the issue and correlate K_d with electrical resistivity. Ohrstrom et al. (2002, 2004) studied experimentally the penetration of dye in semiarid plots of different physiographic shapes and inferred that the variable responses can be modelled as a random cascade process. Further, Ohrstrom et al. (2004) experimented on the transport of dye and salt tracers in a plot of sandy loam and observations were recorded at various timings in different space locations. The results provided the preferential flow pattern of the solutes. In unsaturated soils, the transport process is more complicated and the methods based on stochastic probability have been tried. Hamed et al. (2015) proposed diffusion limited aggregation (DLA) model: a random walk model in which model parameters are optimized with genetic algorithm. The uncertainty of diffusive process has also been analysed using semi-infinite probability distribution and Monte Carlo-generated processes (Adrian et al., 2002). Keshari (2014) describes the recent trends in flow through porous media and mentions that the numerical methods in the field are method of characteristics (MOC), finite difference method (FDM), finite element method (FEM), boundary integral equation method (BIEM) and hybrid methods. Eldho and Rao (1997) presented the numerical model for two dimensional contaminant transport through porous media using dual reciprocity boundary element method. Young et al. (2000) proposed Eulerian-Lagrangian boundary element formulation and solution of advection-diffusion equation. The other numerical model by Mategaonkar and Eldho (2011) for two dimensional contaminant transport in an unconfined aquifer is based on mesh free point collocation method. The flow through porous media finds application in modelling the hydrology of a watershed also; tracers are used to capture the information on surface and subsurface flow of water (Singh et al., 2002). Havis et al. (1992) presented the concept of mixing zone depth to analyze the partition of contaminants existing on the ground, infiltrating into the ground and overland flow during precipitation. The analysis of recharging of subsurface ground water systems is one more area where flow through porous media applies. Keshari and Koo (2007) presented the influence of subsurface thermal profile distribution on the ground water flux. The paper describes a finite difference numerical model of convection-diffusion heat transport using Mac Cormack scheme and shows that the temperature profile can affect the ground water flux by ±18%. The studies on flow and contaminant transport through fine-grained soils, however are limited due to complex behaviour of soils attributable to diverse mineralogical character and high specific surface.

The lead role of fine-grained soils in designing the barriers of waste disposal facilities prompted the researchers in the field to look into the issue of bottom liner as well as the top covers. Viswanadham and Rajesh (2009) and Rajesh and Viswanadham (2010) mention the performance of various cover systems subjected to differential settlement artificially in a centrifuge using hydraulic trap-door system. Divya et al.(2012) revealed the influence of geomembrane on deformation behaviour of clay based landfill covers. The landfill design requires insight of settlement behaviour of Municipal Solid Waste (MSW) and the interactive settlement of the supporting liner. Sivakumar Babu et al. (2010, 2013) proposed a constitutive model for prediction of settlement of MSW and also a closure plan of landfills based on reliability analysis of design parameters. Reddy et al. (2013) presented the movement of leachate in the MSW with anisotropic and heterogeneous permeability distributions. Crooks and Quigley (1984) presents the physical model study of a field problem of salt migration from the waste landfill to undisturbed clay situated below. Yeh et al. (1994) analysed the wicking effect in multiple layer clay liners by FEM where the contaminants spread more in lateral directions at interfaces. Srivastva and Brusseau (1996) described the non ideal solute transport/ reactive contaminant using FEM numerical model and explained the spread of solutes through spatial and temporal moments of concentration distribution. Kartha and Srivastava (2006) presented the FEM simulation of solute transport from a landfill in vadose zone and suggested the breakthrough curve and spatial distribution of contaminant. Gillham et al. (1984) described the measured values of diffusion coefficients for two nonreactive and one reactive solutes in various mixes of bentonite and silica sand. Rowe and Booker (1984, 1985 and 1986) developed finite layer theory for solution of single solute transport through non-homogeneous soil as for one, two and three dimensional cases. Acar and Haider (1990) presented an analytical solution for solute transport through clay barriers. All such models are the milestones in the field, but the methods are based on transport of single solute in rigid porous media along with other idealisations and assumptions as required for the mathematical solution such as contaminant concentration to be dilute, sorption characteristic to be linear and reversible.

Physicochemical (osmotic consolidation) and mechanical (effective stress consolidation) behaviour of fine-grained soils influences the flow through it considerably and makes the flow response much different than that through the rigid porous media. The existence of osmotic flow through natural aquitards has been noticed and reported by many researchers including Hanshaw and Zen (1965), Marine and Fritz (1981) and Neuzil (1986). Kemper and van Schaik (1966), Kemper and Rollins (1966), Greenberg et al. (1973), Elrick et al. (1976). Few other references describe the effect of osmotic flow on transport of salt through clay system (aquitard). Detailed study on volume change of clays due to pore fluid concentrations may be attributed to Bolt and Miller (1955), Bolt (1956), Warkentin *et* al.(1957), Aylmore and Quirk (1962), Blackmore and Miller(1962) and Mesri and Olson (1971). Barbour and Fredlund (1989) presented combined influence of osmotic flow and volume change of clays and referred it as *osmotically induced consolidation* or *osmotic consolidation.* Other researches on the chemo-mechanical consolidation are due to Yeung and Mitchell (1993), Kaczmarek and Heuckel (1998), Van Impe et al. (2002), Gens (2010) and Witteveen (2012). The principal focus of these works lies on consolidation of clays due to entry of solutes into the pore fluid causing gradual increase in concentration; however the mechanical consolidation is either ignored or it is limited to small strain consolidation.

Goodall and Quigley (1977) published the observations of pollutants below two landfills founded on silty clays and showed that the migration of the pollutions do not match with the calculated seepage fronts. With these observations, the idea of mechanical consolidation contributing significantly to movement of contamination, got initiated. Few more similar observations affirming the role of mechanical consolidation in solute transport, include Kim, et al. (1997), Bonaparte and Gross (1993), Moo-Young, et al. (2004). However, all these field and laboratory observations provide qualitative information on enhanced leakage flow than the estimated seepage flow through clay barriers, but there is no exclusive field experimental result on the quantity of consolidation induced flow and coupled solute transport.

Schrefler et al. (1994) and Schrefler (2001) have discussed on mechanical consolidation induced contaminant transport and described a sophisticated formulation of heat and multiphase flow through a partially saturated porous media. The proposed theory is based on microscopic thermodynamic balance and macroscopic balance of mass, linear momentum, angular momentum and energy. However, soil deformation has been assumed to be small. Mechanical consolidation induced contaminant transport has been investigated by many researchers in the context of capping of contaminated sediments in waterways to isolate it from the flowing natural water. Loroy, et al. (1996), and Potter, et al. (1994) reported such investigations on consolidation and contaminant transport based on centrifuge and finite element numerical modelling. Gibson, et al. (1995) presented the theoretical development on consolidation coupled solute transport with concept of large strain consolidation. Arega and Hayter (2008) presented the numerical solution to coupled large strain consolidation and contaminant transport for capping of contaminated sediments in water bodies. The solution uses finite difference method for consolidation and finite volume method for solute transport. Alshawabkeh and Rahbar (2006) reported the consolidation induced solute transport by solving Terzaghi's consolidation equation along with solute transport equations using finite difference formulation. The solutions are reported for consolidation and swelling of clays under single and double drainage conditions. Case of a composite barrier overlain by impermeable geomembrane was approximated by single drainage condition which shows faster solute transport and 95% reduction in breakthrough time. All these investigations are remarkable but limited to small strain consolidation.

The governing equations by Gibson, et al. (1995) is the pioneer attempt to describe the finite or large strain consolidation induced solute transport. Smiles (2000) described the water flow in saturated swelling clays and contaminant transport in unsaturated porous medium. The work establishes that the use of material coordinate system makes the analysis and physical interpretation simple. Smith (2000) presented the derivation of large strain consolidation and coupled solute transport equations with linear reversible sorption isotherm. Analytical solutions for a hypothetical landfill case overlying an aquifer was presented. Assuming a quasi-steady-state problem, the solutions were presented for the solute transport through fluid phase only and through fluid and solid phases of the soil matrix. Further, the coupled consolidation and solute transport equations were converted into material and reduced solid coordinate systems for the solution. Peters and Smith (2002) reaffirmed the equations derived by Smith and showed that these equations are equivalent to those of Smiles (2000) for a fully saturated case. Further a solution was presented for a problem of hypothetical landfill liner for the cases of no deformation, small deformation and large deformation. It was concluded that the breakthrough time is lowest with large deformation and highest for no deformation, the small deformation result lies in between. The solutions were obtained by using explicit finite difference numerical method. The analysis was simplified with the assumptions that the self weight of soil is negligible, initial void ratio is constant throughout the space, mechanical dispersion is negligible and diffusion coefficient is constant and independent of void ratio. Fox and co-researchers (2007 a, b) have contributed significantly to the field of consolidation induced solute transport. Fox (2007a, 2007b) presented development of a numerical model (CST1) of solute transport through deforming saturated porous media, the first one describes the numerical model and the companion paper details its verification and checks. Large strain consolidation is dealt with the piecewise-linear model CS2 (author's earlier work, Fox and Berles, 1997), with minor addition of the provisions of time dependent loading, effect of unload/ reload and externally applied hydraulic gradient. Solute transport takes into account the advection, diffusion, longitudinal and transverse dispersion, linear equilibrium sorption and first order decay reactions. The spatial and temporal advective velocity is taken as algebraic sum of seepage velocity contributed by external hydraulic head and the consolidation induced velocity. Motions of fluid and solid elements are considered in two separate Lagrangian fields. The model is very versatile and can accommodate various initial and boundary conditions of consolidation and solute transport. Fox and Lee (2008) extended the model for variable effective diffusion and non-linear, non-equilibrium sorption model and named it as CST2. Lee et al. (2009a, 2009b) described experimental results of consolidation induced solute transport and observed close agreement with simulated results of the numerical model CST2. Lewis et al. (2009) presented an exhaustive parametric analysis of coupled consolidation and solute transport through composite landfill liner system. The analysis uses finite element numerical solution to the coupled equation of large strain consolidation and solute transport. The publications by Fox, Arega and Lewis are the most recent contributions to the development of numerical models for consolidation coupled solute transport through deformable porous media.

1.3 THESIS OUTLINE

The present thesis focuses on the following points.

i. The critical review on numerical and analytical models of one dimensional large strain consolidation and development of finite volume numerical model.
ii. Development of finite volume numerical model of one and two dimensional advection-diffusion equation with variable diffusion, linear, nonlinear equilibrium and non-linear non-equilibrium sorption and first order decay.
iii. Development of finite volume numerical model for consolidation induced solute transport through deformable porous media.
iv. Theoretical parametric analysis of large strain consolidation and consolidation induced two-dimensional solute transport.

All these issues as covered in the chapters are described below.

Chapter 2 contains the critical review of numerical models of Cargill (1982), Arega and Hyter (2008), Fox and Berles (1997) and others for one dimensional large strain consolidation of saturated soil. The chapter further describes;

i. Explicit finite volume formulation of one dimensional large strain consolidation.
ii. Treatment of geometric and material nonlinearity associated with the large strain consolidation equation.
iii. Derivation of drained, undrained and semi-permeable boundary conditions and initial condition of saturated soil in terms of void ratio.
iv. Verification and evaluation of the numerical model by comparing the model solutions with analytical and other numerical methods.
v. Parametric study of consolidation of soft clays.

Chapter 3 is about the experimental study on consolidation of the clay specimens of 20 mm, 40 mm and 70 mm thickness respectively. The 20 mm thick specimen has been tested in usual oedometer and thicker specimens were tested in specially built mould for the purpose. Data of 20 mm specimen is used to characterize the compressibility and hydraulic conductivity of the clay and was utilized for numerical analysis of thicker specimens. The numerical and experimental results are compared to validate the model.

Chapter 4 describes the development of an explicit finite volume model of one dimensional solute transport equations in porous media accommodating the provisions of linear equilibrium, nonlinear equilibrium and nonlinear-nonequilibrium sorption isotherms along with first order decay. Verification of the model has been done by comparing the results with the results of analytical and other numerical models. Problems with the boundary conditions of constant concentration, zero concentration gradient and constant mass flux with known initial condition on solute concentration has been dealt with.

Chapter 5 details with the finite volume model development of one dimensional advection and two-dimensional diffusion and dispersion of solutes in porous media. The considerations of sorption isotherms and first order decay remain same as used in third chapter. The model validation is presented with comparisons of the model results with other numerical models for all the boundary conditions as mentioned earlier.

Chapter 6 deals with the development of fully explicit finite volume model for coupled phenomenon of consolidation and one and two dimensional solute transport, i.e., consolidation induced solute transport in deforming porous media. The numerical model accommodates all the boundary conditions on void ratio for large strain consolidation as stated above and solute transport boundary conditions such as constant solute concentration, zero concentration gradient and constant mass flux (well-mixed reservoir assumption). The validation and advantages of the model are shown by comparing results with other numerical results through various example problems. Further, theoretical parametric study is done on two dimensional solute transport in deforming porous media.

Chapter 7 is the overall summary and conclusion of entire study of this investigation and presents the scope of future studies arising out of this work.

Chapter 2

FINITE VOLUME MODEL OF LARGE STRAIN CONSOLIDATION

2.1 INTRODUCTION

Small strain assumption of Terzaghi's one dimensional consolidation theory restricts its applicability on such soft soils that may likely to undergo substantial vertical deformation due to consolidation. In addition, the other limitations lie with the desertion of the influence of self weight of soil and the change in values of hydraulic conductivity and compressibility with advancement of the consolidation. Basak et al. (1979) derives the governing equation of consolidation for small strain consolidation with any relationship of void ratio-effective stress-permeability. Further, Lekha et al. (1998) presented its general solution for sand drain consolidation and analytical solution for a few particular relationships of void ratio-effective stress-permeability. Zhuang et al. (2005) provided a nonlinear analysis of consolidation with a semi-analytical solution. Nadar et al. (2007) deduced a governing equation of small strain consolidation with two new coefficients (C_n and α) which describe the changes in soil characteristics during the consolidation and gave its numerical solution. However, all these variations to the Terzaghi's theory may give good predictions for small thickness of settled soils but they are not suited for soft soils, new deposits or fills.

Finite or large strain theory of consolidation overcomes few of the limitations of the small strain theory and takes into account the self weight of soil, variable hydraulic conductivity and compressibility and relaxes the small strain assumption. Schiffman and Gibson (1964) derived the governing equation for consolidation assuming the hydraulic conductivity and coefficient of volume change to be the known functions of depth. Davis and Raymond (1965) assuming a constant logarithmic relationship of the void ratio and effective stress, proposed the nonlinear theory of consolidation. Mesri and Rokhsar (1974) presented the governing equation of consolidation and its numerical solution with the assumption that compressibility and hydraulic conductivity vary with void ratio.

The first general theory of finite (or large) strain one-dimensional consolidation was proposed by Gibson et al. (1967). The finite difference numerical model presented by Cargill (1982) uses the one-dimensional non linear theory of finite strain developed in the form of a second order differential equation in terms of material coordinates, to work out the consolidation of soft clays and fills. In the context of consolidation induced solute transport, the nonlinear theory of finite strain has also been developed by Smith (2000), same as the theory of Gibson et al. (1967) and has successively been used in the works of Peters and Smith (2002), Lewis and Smith (2003) and Lewis et al. (2009). These works use the equation in terms of Lagrangian coordinates and provide the analytical and semi analytical solutions with certain limitations and finite element numerical solution with its full generality.

Fox and Berles (1997) presented a piecewise nonlinear numerical model for large strain consolidation settlement (named as CS2). This model, like the Gibson's equation, is based on the mass conservation and instead of deriving a composite differential equation for consolidation it uses the basic principles and ingredients separately to get the settlements due to consolidation and other related derivable physical quantities. Leonard (1988) presents a conservative finite volume formulation with the use of quadratic interpolation for field variables which avoids the stability problems of central differencing schemes and gives quite accurate solutions with much larger grid spacing for the solution of advection-diffusion equation of solute transport in flowing fluids. The above formulation has been followed in this paper to solve the finite strain consolidation equation. It may be noted that the conservation laws are the time dependent system of partial differential equations. One-dimensional finite strain consolidation equation by Gibson et al. (1967) is a nonlinear differential equation representing the conservation law. Such equations are amenable to numerical solution effectively through finite volume formulations. The following sections describe the finite volume formulation of the finite strain one-dimensional consolidation equation and will be referred as FVM here onwards.

2.2 PRELIMINARIES TO GOVERNING EQUATION

2.2.1 Basic Assumptions

The basic assumptions of the theory of one-dimensional finite strain consolidation are:

i. The soil matrix is compressible, but the pore fluid and individual soil particles are incompressible.
ii. The soil is homogeneous as to type and loading is monotonic.
iii. Pore fluid flow velocities are small and governed by Darcy's law.
iv. The soil permeability (k) and vertical effective stress (σ') have the unique relationships with void ratio.

$$k = k(e) \tag{2.1}$$

$$\sigma' = \sigma'(e) \tag{2.2}$$

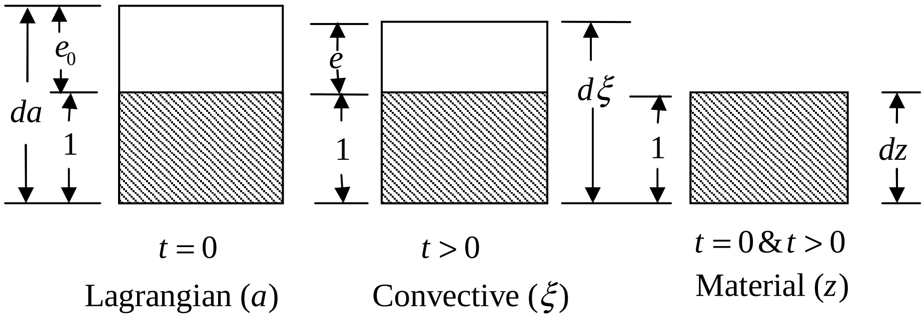


Fig.2.1 Coordinate Systems

2.2.2 Coordinate System

Lagrangian and convective coordinate systems are the measure of soil solids and pore-fluid matrix combine whereas the material coordinates are the measure of only solid particles in the matrix. The Lagrangian coordinates of a consolidating soil matrix starts always at initial boundary, i.e., at time t=0 whereas the convective coordinates starts from current moving boundary at any time after the start of the consolidation for the next time step, i.e., for any time t>0. Thus the values of Lagrangian coordinates and material coordinates are fixed and independent of time while the convective coordinates keep on changing with time.

For the conversion of coordinates from one system to other, the following relationship may easily be deducted (Cargill, 1982). Consider a differential element of soil shown in Fig. 2.1.

$$da = 1 + e_0 \tag{2.3}$$

$$d\xi = 1 + e \tag{2.4}$$

$$dz = 1 \tag{2.5}$$

$$\frac{dz}{da} = \frac{1}{1+e_0} \tag{2.6}$$

$$\frac{d\xi}{da} = \frac{1+e}{1+e_0} \tag{2.7}$$

$$\frac{d\xi}{dz} = 1 + e \tag{2.8}$$

2.3 GOVERNING EQUATION

The governing equation of one-dimensional consolidation, in terms of void ratio (e), hydraulic conductivity *k (e)* and effective stress $\sigma'(e)$ may be given in the material coordinate system as follows (Cargill, 1982). The z-direction has been taken positive against the direction of gravity.

$$\left(\frac{\gamma_s}{\gamma_w}-1\right)\frac{d}{de}\left[\frac{k(e)}{1+e}\right]\frac{\partial e}{\partial z}+\frac{\partial}{\partial z}\left[\frac{k(e)}{\gamma_w(1+e)}\frac{d\sigma'}{de}\frac{\partial e}{\partial z}\right]+\frac{\partial e}{\partial t}=0 \tag{2.9}$$

In the above equation, first term is due to self load of soil and second term is the contribution due to a surcharge load. The terms γ_w, γ_s and t denote unit weight of water, unit weight of soil and time, respectively. Eq. (2.9) may be transformed to the Lagrangian coordinate system (a) using Eq. (2.6) and takes the following form.

$$\frac{1}{1+e_0}\frac{\partial e}{\partial t}=-\frac{\partial}{\partial a}\left[\frac{k}{(1+e)}\left(\frac{\gamma_s}{\gamma_w}-1\right)+\frac{k(1+e_0)}{\gamma_w(1+e)}\left(\frac{\partial\sigma'}{\partial e}\frac{\partial e}{\partial a}\right)\right] \tag{2.10}$$

2.4 FINITE VOLUME FORMULATION

The integration of Eq. (2.10) over the elementary control volume dV and time dt gives,

$$\frac{1}{1+e_0}\int_{CV}\left(\int_t^{t+\Delta t}\frac{\partial e}{\partial t}dt\right)dV=\int_t^{t+\Delta t}\left[\int_{CV}-\frac{\partial}{\partial a}\left[\frac{k}{1+e}\left(\frac{\gamma_s}{\gamma_w}-1\right)+\frac{k(1+e_0)}{\gamma_w(1+e)}\left(\frac{\partial\sigma'}{\partial e}\frac{\partial e}{\partial a}\right)\right]dV\right]dt \tag{2.11}$$

Integrating Eq. (2.11) using Gauss-divergence theorem and one-dimensional consolidation, the following equation may be written for j^{th} control volume element (Fig. 2.3).

$$\frac{1}{1+e_0}[e]_t^{t+\Delta t}\Delta V=-\int_t^{t+\Delta t}\left[\frac{k}{1+e}\left(\frac{\gamma_s}{\gamma_w}-1\right)+\frac{k(1+e_0)}{\gamma_w(1+e)}\left(\frac{\partial\sigma'}{\partial e}\frac{\partial e}{\partial a}\right)\right]_{j-\frac{1}{2}}^{j+\frac{1}{2}}\Delta A dt \tag{2.12}$$

where, volume $\Delta V=\Delta a\Delta A$ and ΔA is the cross-section area of the elementary control volume.

The following generalised scheme on time integral, gives an explicit scheme for θ = 0; a fully implicit scheme for θ = 1 and Crank- Nicolson scheme for θ = 1/2.

$$I_T = \int_t^{t+\Delta t} R_j dt = \left[\theta R_j^{n+1} + (1\text{-}\theta) R_j^n\right]\Delta t \tag{2.13}$$

Now, the explicit scheme for Eq. (2.12) may be written as follows.

$$e_j^{n+1} = e_j^n - \frac{\Delta t}{\Delta a}\left[\begin{array}{l}\left\{\frac{k(1+e_0)}{1+e}\left(\frac{\gamma_s}{\gamma_w}\text{-}1\right) + \frac{k(1+e_0)^2}{\gamma_w(1+e)}\left(\frac{\partial\sigma'}{\partial e}\frac{\partial e}{\partial a}\right)\right\}_{j+\frac{1}{2}}^n \\ -\left\{\frac{k(1+e_0)}{1+e}\left(\frac{\gamma_s}{\gamma_w}\text{-}1\right) + \frac{k(1+e_0)^2}{\gamma_w(1+e)}\left(\frac{\partial\sigma'}{\partial e}\frac{\partial e}{\partial a}\right)\right\}_{j-\frac{1}{2}}^n\end{array}\right] \tag{2.14}$$

Here the superscript ‘n’ denotes time step and subscript ‘j’ denotes the space node as shown in the Fig. 2.3.Finally, Eq. (2.14) may be rearranged as given next.

$$e_j^{n+1} = e_j^n \text{ - } \frac{\Delta t}{\Delta a}\left[\left\{\beta(e^n_{j+\frac{1}{2}}) + \alpha(e^n_{j+\frac{1}{2}})(\frac{\partial e}{\partial a})^n_{j+\frac{1}{2}}\right\} - \left\{\beta(e^n_{j\text{-}\frac{1}{2}}) + \alpha(e^n_{j\text{-}\frac{1}{2}})(\frac{\partial e}{\partial a})^n_{j\text{-}\frac{1}{2}}\right\}\right] \tag{2.15a}$$

where, $\beta(e) = \frac{k(e)}{1+e}\left(\frac{\gamma_s}{\gamma_w}\text{-}1\right)(1+e_0);\ \alpha(e) = \frac{k(e)}{\gamma_w(1+e)}\frac{\partial\sigma'(e)}{\partial e}(1+e_0)^2$ (2.15b)

Eqs. (2.15a) and (2.15b) may calculate values of void ratio at any time with the known prior requisites of initial condition and two suitable boundary conditions on void ratio.

Fig. 2.3 shows the discretization of a compressible layer, the nodes and the control volumes. For calculating the values of void ratio (e) at the control volume faces (*j*+1/2 and *j*-1/2), the following interpolation scheme has been used.

$$e_{j+\frac{1}{2}} = e_{j+1} + \frac{1}{2}\psi(r_{j+\frac{1}{2}})(e_j \text{ - } e_{j+1}) \text{ or } \frac{3}{8}e_j + \frac{6}{8}e_{j+1} - \frac{1}{8}e_{j+2} \tag{2.16}$$

$$e_{j-\frac{1}{2}} = e_j + \frac{1}{2}\psi(r_{j-\frac{1}{2}})(e_{j-1} - e_i) \text{ or } \frac{3}{8}e_{j-1} + \frac{6}{8}e_j - \frac{1}{8}e_{j+1} \tag{2.17}$$

where, $r_{j+\frac{1}{2}} = \frac{e_{j+1} - e_{j+2}}{e_j - e_{j+1}};\ r_{j-\frac{1}{2}} = \frac{e_j - e_{j+1}}{e_{j-1} - e_j}$ (2.18)

and $\psi(r) = (3+r)/4$ (2.19)

This scheme is known as quadratic upstream interpolation of convective kinetics (QUICK) scheme (Leonard, 1988, 1995). The QUICK scheme as above may easily be derived in the above form through the three point Lagrangian interpolation functions for equally spaced nodes. However, in the present case, the flow does not transport a property so, it is not essentially required to restrict to upstream interpolation, the three point Lagrangian interpolation function for the face (*j* +1/2) may be formed with either of the three points (*j, j+1, j+2*) or (*j-1, j, j+1*), covering the face point with negligibly small or no influence on the solutions.

To simulate the nonlinearity of the equation, the nonlinear terms $\alpha(e)$ and $\beta(e)$ are calculated for a sufficient range of void ratio (e) using the available input data correlating void ratio (e), hydraulic conductivity (k) and effective stress (σ'). The terms $\alpha(e)$ and $\beta(e)$ at faces of control volumes (corresponding to $e_{j+\frac{1}{2}}$ and $e_{j-\frac{1}{2}}$) are then interpolated using the quadratic interpolation functions i.e. the three point Lagrangian interpolation function (Burden and Fairs, 2011) given below for the term α at $e_{j+\frac{1}{2}}$ using the values e_j, e_{j+1} and e_{j+2} to maintain consistency.

$$\alpha_{j+\frac{1}{2}} = \alpha_j \prod_{\substack{m=j \\ m\neq k}}^{j+2} \frac{(e_{j+\frac{1}{2}} - e_m)}{(e_k - e_m)} \tag{2.20}$$

In case of given correlations of $e \sim k$ and $e \sim \sigma'$ instead of the discrete point values, the nonlinear terms can be determined using the given correlations directly at control volume faces. Thus, the terms of material non linearity (the constitutive equations to governing equation for material properties k and σ' as nonlinear function of void ratio) and the geometrical non-linearity present in the consolidation equation can be taken care.

The gradients may be approximated as follows.

$$\left(\frac{\partial e}{\partial a}\right)_{j+\frac{1}{2}} = \frac{e_{j+1} - e_j}{\Delta a} \tag{2.21}$$

$$\left(\frac{\partial e}{\partial a}\right)_{j-\frac{1}{2}} = \frac{e_j - e_{j-1}}{\Delta a} \tag{2.22}$$

2.4.1 Initial and boundary conditions

Eqs. (2.15a) and (2.15b) require one initial condition and two boundary conditions for its solution. The initial condition on the void ratio will be the values of void ratios that may be ascertained as consistent with the void ratio and effective stress at the equilibrium pre-existing surcharge. The effective stresses corresponding to pre-existing surcharge at various node points can be calculated and initial void ratios can be interpolated using three point Lagrangian interpolation function on the suitable input data points of compressibility characteristics of the soil. Further, the various possible boundary conditions in terms of void ratio may be deduced as follows (Cargill, 1982).

2.4.1.1 Free draining boundary

For free draining boundary, the excess pore pressure is always zero, thus effective stress equals the total stress and the corresponding void ratio may be interpolated on the input data of void ratio and effective stress.

2.4.1.2 Impermeable boundary

The boundary condition for the impervious strata can be deduced from the fact that there is no flow of pore fluid or soil across such boundary. Let the fluid velocity be v_f and the soil velocity be v_s, then at the boundary;

$$v_f = v_s \tag{2.23}$$

The Darcy's law is usually written in the following form and using the Eq. (2.7) it may be written as;

$$n(v_f - v_s) = -\frac{k}{\gamma_w}\frac{\partial u}{\partial \xi} = -\frac{k}{\gamma_w}\frac{\partial u}{\partial a}\frac{1+e_0}{1+e} \tag{2.24}$$

where, $n = \dfrac{e}{1+e}$, is the porosity.

If u_w, u_0 and u be the total, static and excess fluid pressures, then the following relations may be written;

$$u_w = u_0 + u \tag{2.25}$$

$$\frac{\partial u_0}{\partial \xi} + \gamma_w = 0 \tag{2.26}$$

$$\frac{\partial u_w}{\partial \xi} - \frac{\partial u}{\partial \xi} + \gamma_w = 0 \tag{2.27}$$

Using Eq. (2.7) the above equation may be written as;

$$\frac{\partial u_w}{\partial a} - \frac{\partial u}{\partial a} + \frac{\gamma_w(1+e)}{1+e_0} = 0 \tag{2.28}$$

Fig. 2.2 shows the equilibrium of a soil element. Weight *W* of the pore fluids and the solids in the element will be;

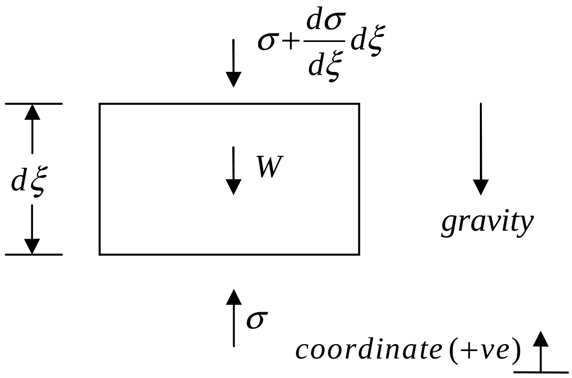


Fig.2.2 Soil element in equilibrium

$$W = e\gamma_w + (1)\gamma_s \tag{2.29}$$

The equilibrium of the soil mixture is given by;

$$\frac{d\sigma}{d\xi} + \frac{e\gamma_w + \gamma_s}{1+e} = 0 \tag{2.30}$$

Using Eq. (2.7) it may also be written as;

$$\frac{d\sigma}{da} + \frac{e\gamma_w + \gamma_s}{1+e_0} = 0 \tag{2.31}$$

The concept of effective stress (σ') relates it with total stress (σ) as follows;

$$\sigma' = \sigma - u_w \tag{2.32}$$

Differentiating Eq. (2.32) with respect to *a* and using Eqs. (2.23), (2.24) and (2.28) the following equation is obtained;

$$(1+e_0)\frac{\partial e}{\partial a} + \frac{\gamma_s - \gamma_w}{\frac{d\sigma'}{de}} = 0 \tag{2.33}$$

Eq. (2.33) gives the value of gradient of the variable void ratio (*e*) at the impermeable boundary and forms one of the Neumann boundary conditions.

2.4.1.3 Semipermeable boundary

This boundary condition is based on the mass conservation that the flow coming out of lower part is equal to the flow into the upper part at the interfacing boundary.

$$[n(v_f - v_s)]_{upper} = [n(v_f - v_s)]_{lower} \tag{2.34}$$

Using Eq. (2.24), the above equation may be written as;

$$\left[k\frac{1+e_0}{1+e}\frac{\partial u}{\partial a}\right]_{upper}=\left[k\frac{1+e_0}{1+e}\frac{\partial u}{\partial a}\right]_{lower} \tag{2.35}$$

The same fluid pressures exist in pore water at the interface boundary and lead to the following equations.

$$[u]_{upper}=[u]_{lower} \tag{2.36}$$

From the effective stress principle (Eq. 2.32);

$$\frac{\partial\sigma}{\partial a}-\frac{\partial u_w}{\partial a}=\frac{\partial\sigma'}{\partial a} \tag{2.37}$$

Eqs. (2.28), (2.31) and (2.37) can give the following equation;

$$\frac{\partial e}{\partial a}=\left[\frac{\gamma_w-\gamma_s}{1+e_0}-\frac{\partial u}{\partial a}\right]\frac{\partial e}{\partial\sigma'} \tag{2.38}$$

Eqs. (2.35), (2.36) and (2.38) form the boundary conditions for a semi-permeable boundary. However, the value $\frac{\partial u}{\partial a}$ for the semi-permeable soil has to be assumed suitably at the interfacing boundary.

2.5 SOLUTION PROCEDURE

Consider a general consolidation problem as shown in Fig. 2.3. A saturated homogeneous type compressible soil layer with a permeable upper boundary and impermeable lower boundary, consolidated under a uniformly distributed pre-existing surcharge pressure (q_p), is subjected to a uniformly distributed pressure $q_u=q_u(t)$. The height of the compressible layer is H and the height of the free water surface above the upper boundary is H_w. Since the time dependent consolidation load in the present work can be monotonic only, it may be a constant load or a step load increasing with time for a practical consolidation situation.

2.5.1 Discretization of compressible layer

For solution over the entire domain, the compressible soil layer of height H may be discretized as shown in the Fig. 2.3. The nodes in addition to boundary points, at which solutions are intended, may be placed at equal distances within the boundaries along with two pseudo nodes (0, m+1) beyond the boundaries as shown in the figure. Here, m is the total number of control volumes or elements. The lower boundary lies midway of the starting nodes (0, 1) and the upper boundary is the middle of the last nodes (m, m+1). Every node represents a control volume of length Δa with limiting faces (j+1/2, j-1/2), where j may vary from 1 to m.

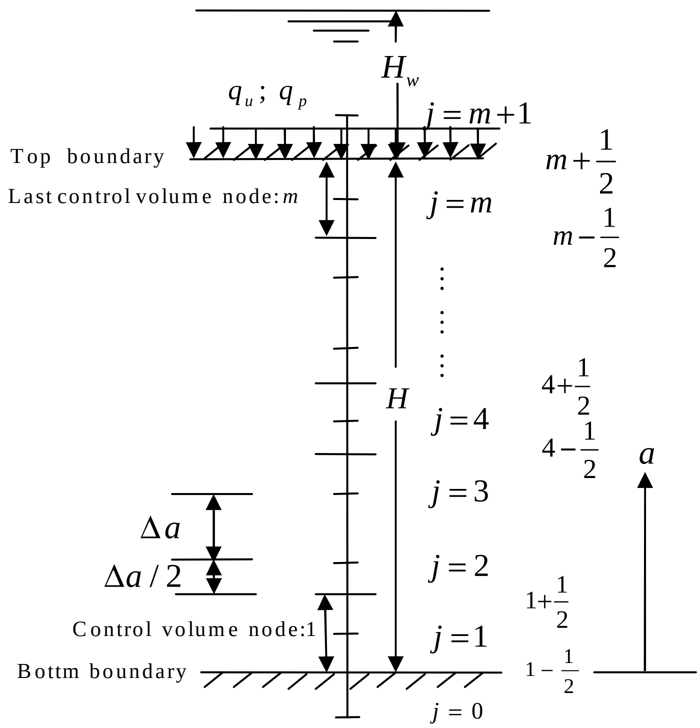


Fig.2.3 Discretization the compressible soil layer

2.5.2 Evaluation of pseudo nodes' values and imposition of the boundary conditions

The local quadratic distribution of void ratio with distance may be expressed as follows (Leonard, 1988).

$$e(\xi) = e_j + \left(\frac{e_{j+1} - e_{j-1}}{2\Delta a}\right)\xi + \left(\frac{e_{j+1} - 2e_j + e_{j-1}}{2(\Delta a)^2}\right)\xi^2 \tag{2.39}$$

Eq. (2.39) at $j=m$ and for $\xi = \frac{\Delta a}{2}$ will give;

$$e_{m+\frac{1}{2}} = e_{UBC} = e\left(\frac{\Delta a}{2}\right) = e_m + \frac{e_{m+1} - e_{m-1}}{4} + \frac{e_{m+1} - 2e_m + e_{m-1}}{8} = -\frac{1}{8}e_{m-1} + \frac{6}{8}e_m + \frac{3}{8}e_{m+1} \tag{2.40}$$

Solving for the value of void ratio (e_{m+1}) at pseudo node;

$$e_{m+1} = \frac{8}{3}e_{UBC} - 2e_m + \frac{1}{3}e_{m-1} \tag{2.41}$$

Where, e_{UBC} is the void ratio at upper boundary.

If the bottom boundary is impermeable, the gradient of void ratio is known through the boundary condition given as Eq. (2.33). Also, the gradient at the lower face of control volume 1, at any time *t*, may be approximated as;

$$\left(\frac{\partial e}{\partial a}\right)_{1-\frac{1}{2}} = \frac{e_1 - e_{0t}}{\Delta a} \tag{2.42}$$

Therefore, the value of void ratio at pseudo node (0) will be;

$$e_{0t} = e_1 - \left(\frac{\partial e}{\partial a}\right)_{1-\frac{1}{2}} \Delta a \tag{2.43}$$

Now, using Eq. (2.40) for i=1, the boundary value of void ratio (e_{LBC}) will be;

$$e_{1-\frac{1}{2}} = e_{LBC} = e\left(-\frac{\Delta a}{2}\right) = e_1 - \frac{e_2 - e_{0t}}{4} + \frac{e_2 - 2e_1 + e_{0t}}{8} = \frac{3}{8}e_{0t} + \frac{6}{8}e_1 - \frac{1}{8}e_2 \tag{2.44}$$

where, e_{LBC} is the void ratio at lower boundary.

Eqs. (2.40, 2.41, 2.43 and 2.44) give the face values of void ratio for the terminal control volumes with inclusion of boundary conditions. The face values of void ratios at all other interior points can be obtained using Eqs. (2.16-2.19). Having known the values of void ratios at all the faces and nodal points as initial condition, the void ratios at next time step is obtainable by Eqs. (2.15a, 2.15b). The above FVM formulation has been implemented through a computer program in FORTRAN-77for the desired solution. Void ratios at various positions are obtained at a required time. Further, from the above solution, the evaluation of the other physical quantities such as excess pore pressure (u), settlement (S), fluid velocity (v_f), velocity of solids (v_s), and degree of consolidation (U_p and U_s) related to the phenomenon of consolidation has been discussed next.

2.5.3 Evaluation of pore pressure

Total vertical stress at a point in the compressible layer (Fig. 2.3) at any time t equals the total pre-existing weight in unit area plus the surcharge load applied and may be calculated by integrating the Eq. (2.31);

$$\sigma(a,t) = \sigma(H,t) + \int_a^H \frac{e\gamma_w + \gamma_s}{1+e_0} da \tag{2.45}$$

where, the first term represents the stress at the upper boundary due to uniformly distributed pre-existing surcharge pressure (q_p)and the applied surcharge pressure (q_u), the second term is the stress due to self load of soil.

The soil property $\sigma' = \sigma'(e)$ can be used to interpolate the values of effective stress corresponding to a solution value of void ratio (e). Eqs. (2.27) and (2.34) of pore water pressures and effective stress principle can be used to get the value of total pore water pressure.

The static pore pressure is determined with the following equation at any nodal point.

$$u_0(a,t) = \gamma_w \{h_1 - \xi(a,t)\} \tag{2.46}$$

where, $h_1=(H-a)+H_w$, is the height of free water surface above the lower boundary (a=0) and ξ is the convective coordinate of that node point. Total pore pressure (u_w) minus the static pressure (u_0) gives the excess pore pressure (u) [Eq. (2.25)]. However, the evaluation of convective coordinates used in Eq. (2.46) is described in the next section.

2.5.4 Evaluation of settlement

The settlement at any point in the compressible soil layer domain can be calculated by subtracting the Lagrangian coordinate and convective coordinate. Thus the following expression gives the settlement.

$$S(a,t) = a(a,0) - \xi(a,t) \tag{2.47}$$

Integrating Eq. (2.7), convective coordinate can be calculated and the above expression can be given as follows.

$$S(a,t) = a(a,0) - \int_0^a \frac{1+e}{1+e_0} da \tag{2.48}$$

The nodal and face values of void ratio obtained through the FVM solution can be used to perform numerical integration of Eq. (2.48) by Simpson’s rule and convective coordinates can be found at all the nodes.

2.5.5 Evaluation of velocity of solid particles and pore fluid velocity

Velocity of solid particles defined by the equation given below can be calculated in terms of Lagrangian coordinates using Eq. (2.7) numerically from data obtained by the solution;

$$v_s = \frac{\partial S(a,t)}{\partial t} \tag{2.49}$$

Eqs. (2.24) and (2.49) gives the relation of pore fluid velocity and excess pore pressure gradient;

$$v_f = -\int_0^h \frac{1}{n(1+e_0)} \frac{\partial e}{\partial t} \tag{2.50}$$

2.5.6 Evaluation of average degree of consolidation

There are two ways to define the average degree of consolidation; one is based on the stress (excess pore pressure) and the other one on the strain (settlement of upper boundary). The stress based average degree of consolidation (U_p) is defined as;

$$U_p = 1 - \frac{1}{qH} \int_0^H u da \tag{2.51}$$

and the strain based average degree of consolidation (U_s) is defined as;

$$U_s = \frac{S(H,t)}{S(H,\infty)} \tag{2.52}$$

The average degree of consolidation U_p can be calculated by numerical integration whereas, U_s can be calculated directly at any node point; however it is evident that the values of U_p and U_s will not be same.

2.6 CONSERVATIVENESS, BOUNDEDNESS AND ACCURACY

The assessment of a finite volume schemes is done on the criteria of the fundamental properties, viz.: conservativeness, boundedness, transportiveness and the accuracy of interpolations (Versteeg et al. 2007). The three point Lagrangian interpolation scheme applied to the consolidation equation has been assessed for these properties.

2.6.1 Conservativeness

Conservativeness is the property of the numerical scheme which ensures conservation of a property (fluid mass in this case). It is established by equating the algebraic sum of fluxes across the domain boundaries with summation of fluxes through all the discrete control volume faces. The linear form of Eq. (2.10) can be written;

$$\frac{1}{1+e_0} \frac{\partial e}{\partial t} = -\frac{\partial q}{\partial a} \tag{2.53}$$

where,

$$q = \left[\frac{k}{(1+e)} \left(\frac{\gamma_s}{\gamma_w} - 1 \right) + \frac{k(1+e_0)}{\gamma_w(1+e)} \left(\frac{\partial \sigma'}{\partial e} \frac{\partial e}{\partial a} \right) \right] \tag{2.54}$$

It may also be noted that q is the Darcy velocity and Eq. (2.53) represents the continuity of pore fluid in a consolidating porous medium (Lewis et al. 2009). The conservativeness of the scheme on finite volume formulation on Eq. (2.53) can be proved easily. Consider the following set of nodal values (q_1, q_2, q_3) control volumes (CV-1, CV-2, CV-3) and pseudo node with variable value $q_4$of a domain shown in Fig.2.4.

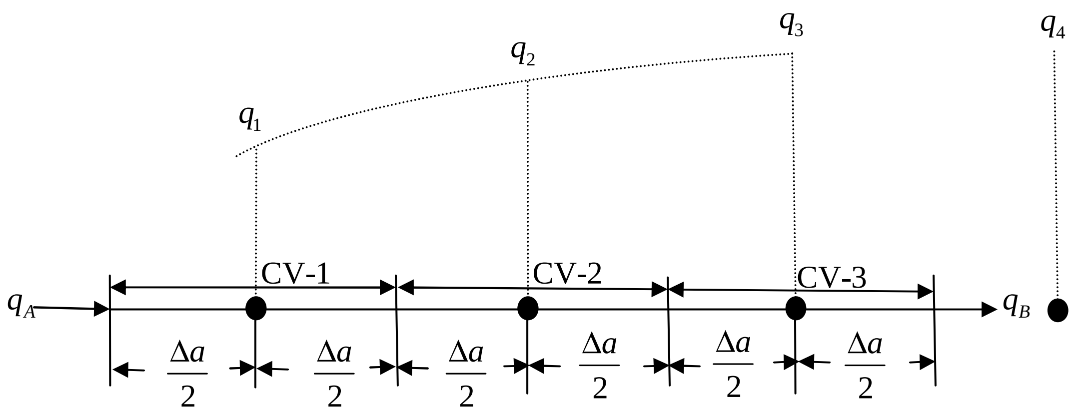


Fig.2.4 A domain of 3 control volumes and nodes with boundaries

Face values of the control volumes fluxes (fluid volume flow per unit area per unit time, i.e., the velocity) can be interpolated using Eqs (2.40) and (2.44) or Eqs (2.16), (2.17), (2.18) and (2.19) in combination. Both approaches will give the same result.

CV-1: left face: q_A; right face: $\frac{3}{8}q_1 + \frac{6}{8}q_2 - \frac{1}{8}q_3$ (2.55)

CV-2: left face: $\frac{3}{8}q_1 + \frac{6}{8}q_2 - \frac{1}{8}q_3$; right face: $\frac{3}{8}q_2 + \frac{6}{8}q_3 - \frac{1}{8}q_4$ (2.56)

CV-3: left face: $\frac{3}{8}q_2 + \frac{6}{8}q_3 - \frac{1}{8}q_4$; right face: q_B (2.57)

It can now be easily established that the algebraic sum of incoming and outgoing fluxes across the boundaries of the domain is equal to the sum of incoming and outgoing fluxes of all control volumes. Thus, the finite volume formulation preserves conservativeness property and gives a consistent algebraic formulation of Eq. (2.12).

2.6.2 Boundedness

If the node values are within the bounds of the face values of control volumes in a finite volume numerical scheme, it is attributed to have boundedness and will give a convergent solution. Consider the control volume CV-2 of Fig. 2.4, it will have boundedness in terms of void ratio (e) for the scheme if following inequalities hold good.

$$e_2 \geq \frac{3}{8}e_1 + \frac{6}{8}e_2 - \frac{1}{8}e_3 \text{ or } 2e_2 \geq 3e_1 - e_3 \text{ or } e_1 \leq e_2 \leq e_3 \quad (2.58)$$

$$e_2 \le \frac{3}{8}e_2 + \frac{6}{8}e_3 - \frac{1}{8}e_4 \text{ or } 5e_2 \le 6e_3 - e_4 \text{ or } e_4 \le 6e_3 - 5e_2 \text{ or } e_2 \le e_3 \le e_4 \qquad (2.59)$$

It may be concluded from Eqs. (2.58) and (2.59) that the finite volume numerical scheme with three point interpolation may lose the boundedness property if e_4 or e_3 is sufficiently high compared to their two predecessors. The right side face values are not guaranteed to be positive with three point interpolations (Leonard, 1979). For such unstable cases, normalised variable approach has been proposed (Leonard, 1988). However, normal problem of consolidation can be solved with numerical convergence and stability using this explicit scheme with a little bit numerical experimentation on number of nodes or the length of control volumes (Δa) and time step (Δt) to maintain the boundedness.

2.6.3 Accuracy

The truncation error in the scheme of finite volume formulation is of third order however, the scheme is effectively third order accurate (even though formally second order accurate as h→0). The enhanced accuracy is attributable to the use of control volume operator average in the formulation (Leonard, 1995).

2.6.4 Time Step Restrictions

The time step restrictions are very tight in quadratic interpolation QUICK schemes. The considerations on time step restrictions for finite grids depend upon local grid Courant number ($c = \frac{q\Delta t}{\Delta a}$).Leonard (1988) opined from the numerical experimentation that the instabilities are avoided if a local Courant number does not exceed 0.2. The maximum Darcy velocity (q) occurs near the draining boundary where the decrease of void ratio is maximum corresponding to a given load increment (q_u) which equals the excess pore pressure developed and dissipates instantly. Thus, for the initial hydraulic conductivity (k_0) the time increment (Δt) may be taken by the following relation.

$$\Delta t \prec \frac{0.2(\Delta a)^2 \gamma_w}{k_0 q_u} \qquad (2.60a)$$

2.7 MODEL VERIFICATION

2.7.1 Problem statement

The performance of the finite volume numerical model has been evaluated through a general one dimensional consolidation problem, as shown in the fig. 2.5. The clay layer is

10 m thick with the overburden pressure q_p. The numerical solution has been obtained for its consolidation due to incremental load q_u. The geotechnical properties of the soft soil are as follows, initial void ratio at the top e_{H0}= 3, the coefficient of volume compressibility m_{v1}= 4 MPa^{-1}, initial hydraulic conductivity k_0= 10^{-9}m/s, G_s= 2.75. The water level above the initial top surface, H_w = 1 m and the pre-consolidation load, q_p = 10 kPa. The load increment, q_u = 100 kPa. The boundary conditions considered are, porous top and impervious bottom (PTIB) and porous top and pervious bottom (PTPB).

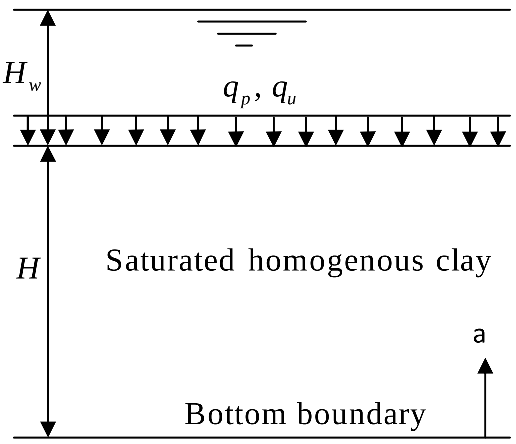


Fig.2.5 Consolidation problem

2.7.2 Analytical solution

Xie and Leo (2004) presented the analytical solution to the above consolidation problem with the following assumptions.

1. The coefficient of volume compressibility (m_{v1}) of the soil remains constant during consolidation.

$$m_{v1} = -\frac{1}{1+e}\frac{de}{d\sigma'} = \text{constant} \tag{2.60b}$$

2. The coefficient of hydraulic conductivity k has the following relationship;

$$\frac{k}{k_0} = \left(\frac{1+e}{1+e_0}\right)^2 \tag{2.60c}$$

where, k_{v0} is the initial coefficient of hydraulic conductivity of the soil at time t = 0.

3. Load increment Q= Q (t) = q_u constant.

The analytical solution for PTIB (porous top and impervious bottom) boundary conditions and the initial condition are given below.

$$u(0,t)=0 \tag{2.61}$$

$$\frac{\partial u}{\partial a}(H,t)=0 \tag{2.62}$$

$$u(a,0)=q_u \tag{2.63}$$

$$u=\frac{1}{m_{v1}}\ln\left\{1+(\exp(m_{v1}q_u)-1\right\}\sum_{m=1}^{\infty}\frac{2}{M}\sin\left(\frac{Ma}{H}\right)\exp(-M^2T_v) \tag{2.64}$$

where, T_v is the time factor given by

$$T_v=\frac{c_{v_0}t}{H^2}\ \text{where,}\ c_{v_0}=\frac{k_0}{m_{v_1}\gamma_w} \tag{2.65a}$$

$$M=\left(m-\frac{1}{2}\right)\pi,\quad m=1,2,3..... \tag{2.65b}$$

The following expression gives the value of settlement at any depth *a* and time *t*.

$$S(a,t)=H\left\{1-\exp(-m_{v1}q_u)\right\}\left\{1-\frac{a}{H}-\sum_{m=1}^{\infty}\frac{2}{M^2}\cos\left(\frac{Ma}{H}\right)\exp(-M^2T_v\right\} \tag{2.66}$$

The top surface settlement S_t and its final settlement S_∞ will be given as,

$$S_t=S(0,t)=H\left\{1-\exp(-m_{v1}q_u\right\}\left\{1-\sum_{m=1}^{\infty}\frac{2}{M^2}\exp(-M^2T_v)\right\} \tag{2.67a}$$

$$S_\infty=S(0,\infty)=H\left\{1-\exp(-m_{v1}q_u)\right\} \tag{2.67b}$$

The average degree of consolidation (*Up*) defined in terms of excess pore water pressure or stress is given by;

$$U_p=1-\frac{1}{q_uH}\int_0^H uda \tag{2.68}$$

The average degree of consolidation (U_s) in terms of settlement of top surface or strain is given by

$$U_s=\frac{S_t}{S_\infty}=1-\sum_{m=1}^{\infty}\frac{2}{M^2}\exp(-M^2T_v) \tag{2.69}$$

The analytical solutions have also been given for the same initial condition but with boundary condition PTPB (porous top and pervious bottom), the excess pore pressure in this case were obtained simply by replacing the total depth of consolidating soil or the drainage path *H* by *H/*2.

2.7.3 Numerical solution

The finite volume formulation described in the paper has the spatial sign convention positive in the direction of gravity and the lowermost boundary of the consolidating layer would be taken as datum. Therefore, the equations used in the numerical solution would have been transformed by using (*H-a*) in place of (*a*) for algebraic equations and the differential terms.

The terms $\beta(\mathrm{e})$ and $\alpha(\mathrm{e})$ of Eq. (2.15) can be written as follows, using Eqs. (2.60b) and (2.60c);

$$\beta(e) = k_0\left(\frac{\gamma_s}{\gamma_w} - 1\right)\left(\frac{1+e}{1+e_0}\right); \qquad \alpha(e) = -\frac{k_0}{\gamma_w m_{v1}} \tag{2.70}$$

Further, the initial condition of the variables void ratio (*e*) and effective stress (σ') are taken as follows, as derived in the analytical solution.

$$e_0 = e(a,0) = e_{H0} - m_{v1}\gamma_w(G_s - 1)(H - a) \tag{2.71}$$

$$\sigma'_0 = \sigma'(a,0) = q_p + \frac{1}{m_{v_1}}\ln\frac{1+e_{H0}}{1+e_{00} - m_{v_1}(G_s - 1)(H - a)} \tag{2.72}$$

The boundary condition for a permeable boundary is given by;

$$e_0(0,t)\, or\, e(H,t) = e_{final} \tag{2.73}$$

The boundary condition for an impervious boundary will be;

$$(1+e_0)\frac{\partial \mathrm{e}}{\partial a} + \frac{\gamma_s\text{-}\gamma_w}{\frac{\mathrm{d}\sigma'}{\mathrm{de}}} = 0 \tag{2.74}$$

The numerical solutions have been obtained for 30 mesh points (m = 30, Δa= 1/3 m) and time increment Δt = 10000 Seconds for both PTIB and PTTB boundary conditions and comparison of numerical and analytical solutions are done.

2.7.4 Comparison of analytical and numerical solutions

The complete solution of a consolidation problem includes distribution of pore pressures along the convective coordinates, variation of average degree of consolidation (strain based U_s and/or stress based U_p) and settlement with time. Fig. 2.6 shows distribution of pore pressures along the convective coordinates calculated by the numerical method and the analytical solution of Xie and Leo (2004) for the boundary conditions PTIB at different values of degree of consolidation (U_s = 0.3, 0.5, 0.7 and 0.9). The corresponding settlement values are shown beside the graph.

Fig. 2.7 shows numerically calculated and analytically given excess pore pressure isochrones for PTTB boundary conditions for different degree of consolidation and settlements.

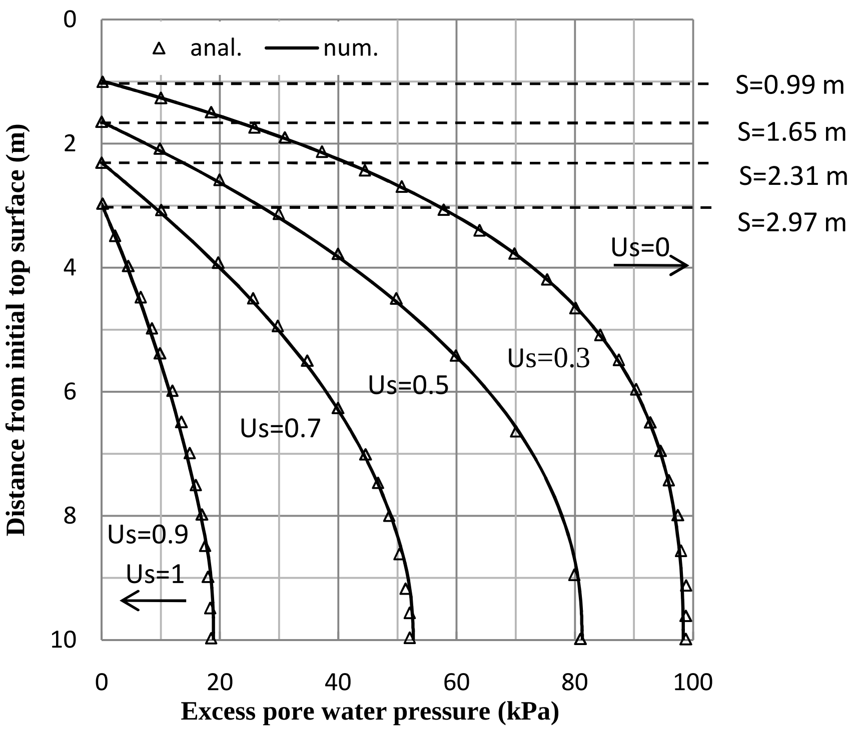


Fig.2.6 Excess pore water pressure isochrones in convective coordinates (PTIB) (Analytical data scaled from Fig.4 of Xie& Leo [2004])

Fig. 2.8 shows the comparison of average degrees of consolidation (U_p and U_s) and settlements with respect to time factor (T_v). It is evident from the Figs. 2.6, 2.7 and 2.8 that the values simulated by the finite volume formulation and the corresponding analytical values show close agreement.

Table 2.1 shows the comparison of FVM and analytical results (Eqs. 2.67a and 2.67b) of settlements of top surface with time in terms of time factor and the errors in percent with respect to analytical results. The maximum error is 1.45% and there is no error in final settlement which infers a good agreement.

Table 2.1 Comparison of top surface settlement (PTIB)

Time factor	0.05	0.102	0.217	0.724	0.853	1.956	2.466	∞
Settlement (m) (Analytical)	0.8279	1.1896	1.7303	2.8488	2.9715	3.2756	3.2909	3.2968
Settlement (m) (FVM)	0.8248	1.1817	1.7101	2.8075	2.9321	3.2534	3.2713	3.2968
Error (%)	0.38	0.66	1.17	1.45	1.33	0.676	0.59	0.0

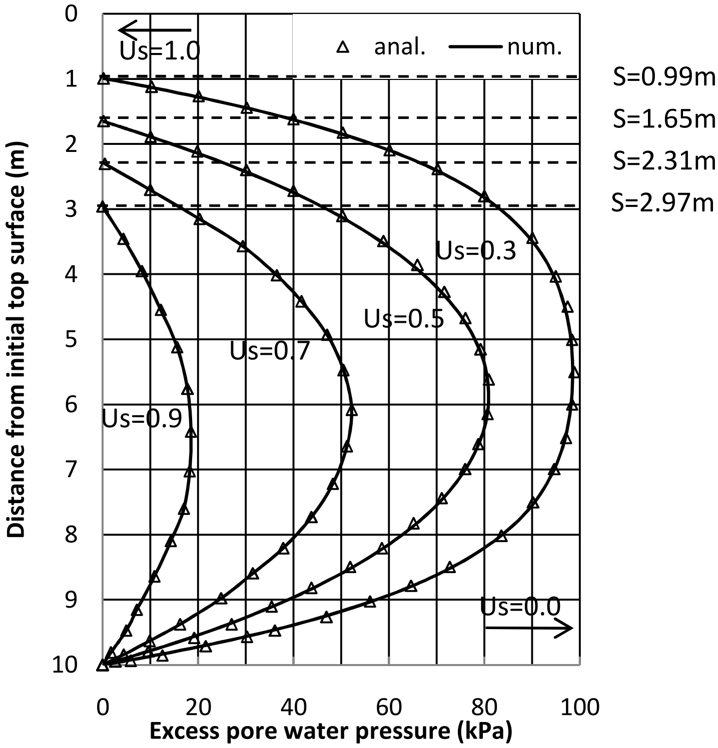


Fig.2.7 Excess pore water pressure isochrones in convective coordinates (PTTB) (Analytical data scaled from Fig.5 of Xie& Leo [2004])

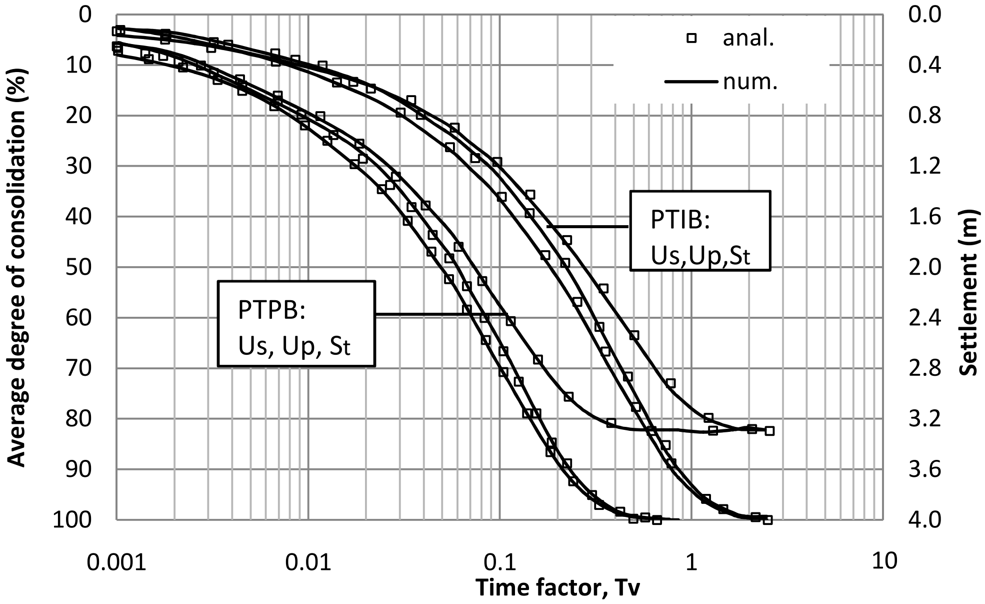


Fig 2.8Average degree of consolidation and surface settlement vs. time (Analytical data scaled from Fig.6 of Xie& Leo [2004])

2.7.5 Comparison with piecewise-linear model CS2

Fox and Berles (1997) describe a piecewise-linear finite difference numerical model for large strain consolidation and named it CS2. The model uses the same constitutive equations like the present FVM model and solves for the consolidation settlement of individual elements due to expulsion of pore water from the element. The pore water flow is calculated with the help of local values of compressibility and hydraulic conductivity characteristics and these are interpolated through linear interpolation using neighbouring coordinates representing void ratios on compressibility and hydraulic conductivity constitutive curves. Whereas, the FVM numerical model uses quadratic interpolation functions and the similar physical condition on void ratio through discretized form of Eq. (2.10). The accuracy is thus enhanced and the solutions of FVM require less number of elements than that for CS2. The FVM solutions of four example problems have been compared with CS2 that affirms the above statement.

Table 2.2 shows all the properties and parameters of these four problems. Problem 1 is a small strain case where strain in soil elements and its self weight are negligible and Terzaghi theory is applicable. Problems 2 and 3 consider large strains in soil elements with negligible self weight. The problem 2 assumes constant coefficient of consolidation (c_v) and maintainable as such with variable hydraulic conductivity following the equation shown in relevant position in the table 2.2. Problem 3 follows the variable coefficient of consolidation (c_v), but another coefficient denoted as c_F as constant which has been achieved through the equation used for hydraulic conductivity as shown in its column in table 2.2. Problem 4 is concerning the large strain and its consolidation due to self weight. The phosphatic clay follows constitutive equations regarding compressibility and hydraulic conductivity as shown in its column. The input data of void ratio, effective stress and hydraulic conductivity, were obtained through respective constitutive equations in sufficient range for all the problems to be used in FVM program for solutions

Table 2.2 Problems' descriptions

Variable	Problem 1 (small strain) c_v=constant	Problem 2 (large strain) c_v=constant	Problem 3 (large strain) c_F = constant	Problem 4 (large strain with self-weight)
No. of elements, m	20,50,100,200	20,50,100	20,50,100	20
e_o	1.0	1.0	1.0	18.8
q_p(kPa)	200	200	200	0.224
q_u(kPa)	0.001	800	200, 800	0.0
G_s	1.0	1.0	1.0	2.71
Initial height of compressible layer, H (m)	20.0	20.0	20.0	6.33
H_w(m)	0.0	0.0	0.0	0.0
Compressibility, a_v (/kPa)	0.001	0.001	0.001	Non-linear $e = 12.19(\sigma'(kPa))^{-0.29}$
Hydraulic conductivity, k(m/s)	Constant 1×10^{-9}	Variable ($k = k_0 \frac{1+e}{1+e_0}$) k_0 =1×10^{-9}	Variable ($k = k_0 \frac{1+e}{1+e_0}$) k_0 =1×10^{-9}	Variable $k(m/s) = 1.41\times10^{-11} e^{4.11}$
c_v (m^2/s)	2.0394×10^{-7}	2.0394×10^{-7}	Variable (equation for c_F)	Variable (equation for c_F)
c_F (m^2/s)	2.0394×10^{-7}	Variable ($c_F = c_v \frac{1+e_0}{1+e}$)	Constant	Variable (equation for c_F)
Boundary conditions	Double drained	Single-drained at either boundary	Single-drained at either boundary	Single drained at top boundary
Final Vertical settlement,S_∞(%)	0.00005	40	10, 40	46

2.7.6 FVM solution and comparisons with CS2

The resulted average degree of consolidation for problem 1 is shown in Table 2.3 for different values of number of elements (20, 50, 100, and 200). The values, at all time factors, are very close to the Terzaghi's solution. However, in case of small strain problem the CS2 values are closer to Terzaghi's solution compared to FVM with more number of elements. It infers that the small strain linear solutions give better results than FVM but with more number of elements, whereas FVM gives sufficiently accurate results with quite lesser elements. This may probably be attributed to linear interpolations which is somewhat advantageous than the quadratic interpolations in case of small strain linear consolidation. Particularly, the pseudo node point needed near the boundary in implementing quadratic interpolation, has been approximated according to quadratic polynomial which may introduce a small error in the solution in case of small scale linear consolidation. This situation can be improved, if the pseudo node point is determined through linear variation of the independent variable. Since the formulation in general deals with large strain non linear consolidation, the linear variation assumption has not been adopted near the boundaries. The better trends in favour of FVM compared to CS2 can be seen in tables 2.4 and 2.5 for large strain consolidation.

Table 2.3 Comparison of solutions of problem 1 (small strain)

Time (days)	Time factor ($c_v\, t/ H^2$)	Average degree of consolidation, U_{avg} (%)								
		Terzaghi Solution	m=20		m=50		m=100		m=200	
			FVM	CS2	FVM	CS2	FVM	CS2	FVM	CS2
28.38	0.005	7.979	7.9174	6.765	7.9369	7.836	7.9499	7.958	7.9677	7.976
56.75	0.01	11.284	11.3902	10.515	11.2539	11.199	11.3190	11.274	11.3113	11.282
283.76	0.05	25.231	25.3411	24.977	25.2520	25.215	25.3005	25.228	25.2290	25.231
567.52	0.1	35.682	35.8135	35.550	35.6438	35.673	35.7058	35.680	35.7321	35.682
851.29	0.15	43.695	43.7508	43.612	43.6699	43.688	43.7239	43.694	43.7765	43.695
1135.05	0.2	50.409	50.3994	50.352	50.4023	50.404	50.4915	50.408	50.4763	50.409
1702.57	0.3	61.324	61.4079	61.304	61.4920	61.324	61.3502	61.324	61.3912	61.324
2838.77	0.5	76.395	76.4466	76.416	76.4838	76.400	76.4712	76.396	76.5132	76.395
4540.19	0.8	88.740	88.9363	88.774	88.9204	88.747	88.7907	88.742	88.9182	88.741
6810.28	1.2	95.803	96.0292	95.828	95.8534	95.808	95.8933	95.805	95.9212	95.804

Table 2.4 Comparison for solutions problem 2 (constant c_v)

Time (days)	Time factor ($c_v t/ H^2$)	Average degree of consolidation, U_{avg} (%)					
		Terzaghi Solution	m=20	m=50		m=100	
			FVM	FVM	CS2	FVM	CS2
113.50	0.005	11.107	10.9968	10.9600	10.829	10.9610	10.926
227.01	0.01	15.619	15.5118	15.5000	15.404	15.5022	15.477
454.02	0.02	22.016	21.9218	21.9219	21.851	21.9244	21.907
908.04	0.04	31.078	30.9989	31.0041	30.953	31.0064	30.996
1589.07	0.07	41.077	41.0094	41.0160	40.979	41.0182	41.013
2270.09	0.1	49.077	49.0156	49.0227	48.993	49.0247	49.022
3405.14	0.15	60.029	59.9727	59.9814	59.960	59.9835	59.984
4540.19	0.2	68.978	68.9249	68.9359	68.921	68.9381	68.942
6810.28	0.3	82.149	82.1061	82.1187	82.113	82.1209	82.127
11350.47	0.5	94.858	94.8391	94.8463	94.848	94.8475	94.852
20430.85	0.9	99.653	99.6505	99.6514	99.652	99.6515	99.652

Table 2.5 Comparison for solutions problem 3 (constant c_F)

Time factor ($c_F t/ H^2$)	Average degree of consolidation, U_{avg} (%)								
	Lee and Sills S_∞=10%	Lee and Sills S_∞=40%	m=20 S_∞= 10%	m=50 S_∞= 10%	m=20 S_∞= 40%	m=50 S_∞= 40%		m=100 S_∞= 40%	
			FVM	CS2	FVM	FVM	CS2	FVM	CS2
0.01	11.315	11.292	11.3322	11.274	11.3322	11.2914	11.274	11.2845	11.282
0.03	19.558	19.488	19.5734	19.540	19.5734	19.5464	19.540	19.5423	19.543
0.06	27.645	27.533	27.6587	27.637	27.6587	27.6389	27.637	27.6361	27.639
0.1	35.683	35.538	35.6951	35.680	35.6951	35.6796	35.680	35.6774	35.682
0.15	43.693	43.522	43.7027	43.694	43.7027	43.6903	43.694	43.6886	43.695
0.25	56.216	56.976	56.2228	56.223	56.2229	56.2159	56.223	56.2149	56.223
0.41	70.333	70.073	70.5156	70.526	70.5156	70.5151	70.526	70.5151	70.525
0.61	81.886	81.682	81.9933	82.008	81.9933	81.9965	82.008	81.9971	82.006
0.81	88.941	88.805	89.0026	89.016	89.0026	89.0067	89.016	89.0074	89.015
1.01	93.248	93.162	93.2835	93.295	93.2835	92.2873	93.295	93.2879	93.294
1.41	97.483	97.451	97.4947	97.501	97.4947	97.4971	97.501	97.4975	97.501
2.01	99.427	99.420	99.4293	99.432	99.4293	99.4302	99.432	99.4303	99.431

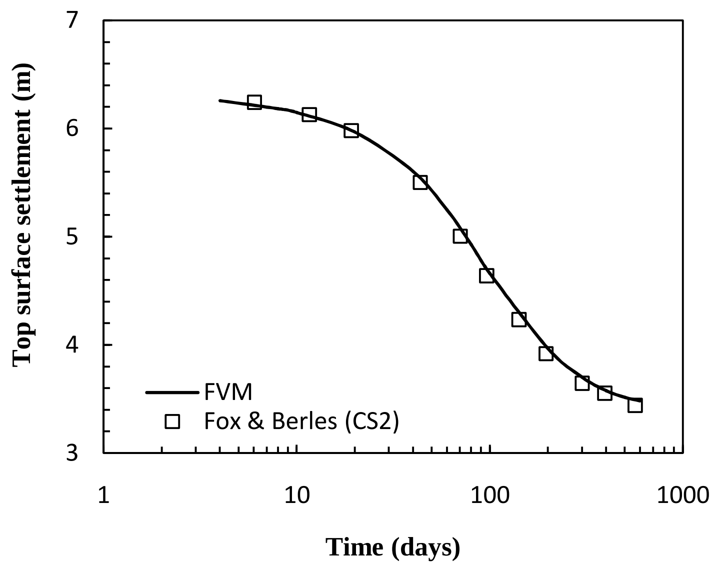


Fig.2.9. Settlement vs time

(CS2 data points scaled from Fig. 5 of Fox and Berles [1997])

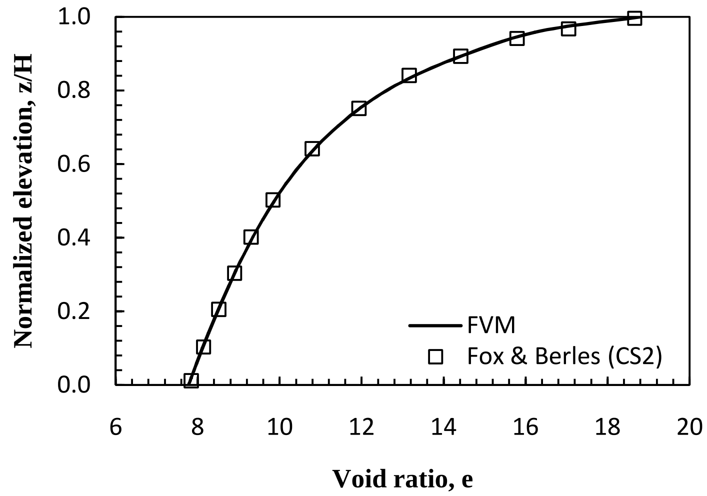


Fig.2.10 Void ratio distribution
[CS2 data points scaled from Fig. 5 of Fox and Berles (1997)]

Problem 4 uses the input data of void ratio, effective stress and hydraulic conductivity as per the relevant nonlinear equations shown in table 2 and the number of elements as 20. The FVM results of settlement with time and distribution of void ratio at 400 days are compared with results of CS2 with 100 elements in Figs. 2.9 and 2.10 and those show close match. All these comparisons depict the advantage of accuracy with quadratic interpolation function.

2.8 PARAMETRIC ANALYSIS

The parametric analysis aims at finding the response of various independent parameters of consolidation of soft soil deposits (soft organic clay), the void ratio of which in natural state ranges from 2.5 to 3.2 (Das, 2010). The variable parameters influencing the consolidation settlement are, void ratio, compressibility, hydraulic conductivity, thickness of compressible soil layer, and applied pressure. The constant parameters are specific gravity of soil solids and unit weight of water. The variation of constant parameters is ignored here and the influences of independent parameters on soil consolation time rate are studied next for the analysis. The constitutive equations used for compressibility and hydraulic conductivity (Taylor, 1948) are given below.

$$\sigma' = \sigma_0'(10)^{\frac{e_0 - e}{C_c}} \tag{2.75}$$

$$k = k_0(10)^{\frac{e - e_0}{C_k}} \tag{2.76}$$

where, C_c is the compression index, C_k is the hydraulic conductivity index, e_0 is the void ratio corresponding to initial effective overburden pressure σ_0' and hydraulic conductivity k_0 and e is the void ratio that corresponds to any effective stress σ and hydraulic conductivity k. The compression index has been approximated as C_c= 1.15 (e_0 – 0.35) by Nishida (1956). Hydraulic conductivity index is taken as $C_k \approx 0.5e_0$ and the initial value of hydraulic conductivity (k_0) is determined approximately from the corresponding curve presented by Travenas et al. (1983). Now, the parameters influencing the consolidation independently are initial void ratio at the top boundary (e_0) at pre-existing load, thickness of soil layer (H) and applied surcharge load. Therefore the effect of variation of these parameters on consolidation has been studied here. The schematic diagram of consolidation problem, as shown in Fig. 2.5, has been followed with boundary conditions are given as permeable top and impermeable bottom. The initial effective overburden pressure (q_p) has been kept constant with a value of 120 kPa. The constant data are the specific gravity of soil (G_s) = 2.3 and unit weight of water = 9.81 kN/ m^3. Equilibrium initial void ratio variations corresponding to overburden pressure 120 kPa at the top of the compressible layer considered for parametric analysis are 3.2, 3.0, 2.8, 2.6, 2.4, 2.2, 2.0, 1.8 and 1.6 and initial hydraulic conductivities corresponding to these are taken as 7.0×10^{-9}, 6.0×10^{-9}, 5.0×10^{-9}, 4.0×10^{-9}, 3.0×10^{-9}, 1.5×10^{-9}, 1.0×10^{-9}, 9.0×10^{-10} and 4.0×10^{-10}. Self load of soil is taken into consideration and the initial void ratio with depth varies in the layer accordingly. The thickness of compressible layer (H) varies from 1 m to 8 m at the rate of 1 m. Incremental applied pressure (q_u) varies as 200, 300, 400, 500, 600, 700 and 800 kPa above the overburden pressure for each thickness of soil layer and initial void ratio. Incremental loads are applied as one time load in the beginning and kept constant throughout the primary consolidation. It is also assumed that the soil is normally consolidated and remains saturated during the consolidation. All combinations of initial void ratio, applied pressure and compressible layer thickness have been worked out and their influence on time rate of consolidation settlement as evaluated by the finite volume formulation and presented here in the next sections.

2.8.1 Effect of initial void ratio

Fig.2.11 shows the influence of initial void ratio for a given thickness of compressible layer (H = 5.0 m) and load increment (q_u=600 kPa). The graph shows the variation of average degree of consolidation (U_s) with square root of time for different void ratios. It is seen that the rate of consolidation decreases with decrease in initial void ratio. While the initial void ratio decreases from 3.2 to 2.4, the rate of the consolidation decreases slowly. However, the successive reduction with each decrease in void ratio is not uniform but it is relatively little more than that for its predecessor. The next reduction of initial void i.e. from 2.4 to 2.2 shows abrupt decrease in the rate of consolidation. The consolidation characteristics of soil with initial void ratio ranging from 3.2 to 2.4 forms one class and moves into other class below the void ratio 1.6 with a transition zone of void ratio from 2.4 to 1.6. It may also be noted that the soft clay shows almost linear relationship between average degree of consolidation and square root of time up to 80% consolidation.

2.8.2 Effect of layer thickness

The influence of compressible layer thickness (H) on consolidation time rate has been found by varying the thickness from 1 m to 8 m and keeping load increment (q_u) and initial void ratio (e_0) constant. Results have shown similar patterns for all the loads (from 200 to 800 kPa), so one of them with q_u = 200 kPa and e_0= 3.2 has been shown in Fig.2.12. This shows that more is the thickness of compressible layer more time is required for consolidation for any additional surcharge load. Also the decreasing gap of curves indicates the successive reduction in the rate of consolidation decreases with decrease in initial void ratio.

2.8.3 Effect of load increment

Fig.2.13 shows the influence of load increment (q_u) and it is evident that this, in general, has relatively less effect on rate of consolidation. Two sets of curves correspond to initial void ratios (e_0) 3.2 and 1.6 for constant thickness of compressible layer (H=5.0 m). However, it may be noticed that the higher initial void ratio has lesser influence of the variation of load increment on consolidation rate compared to that of lower initial void ratio. The consolidation rate decreases with increase in load and the reduction of gap in successive curves at initial void ratio of 1.6 indicates the diminishing influence.

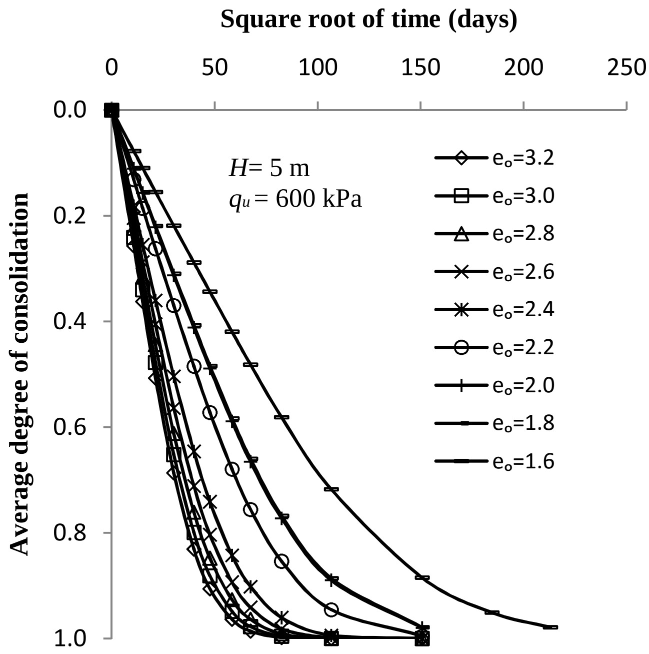


Fig.2.11Effect of initial void ratio on consolidation

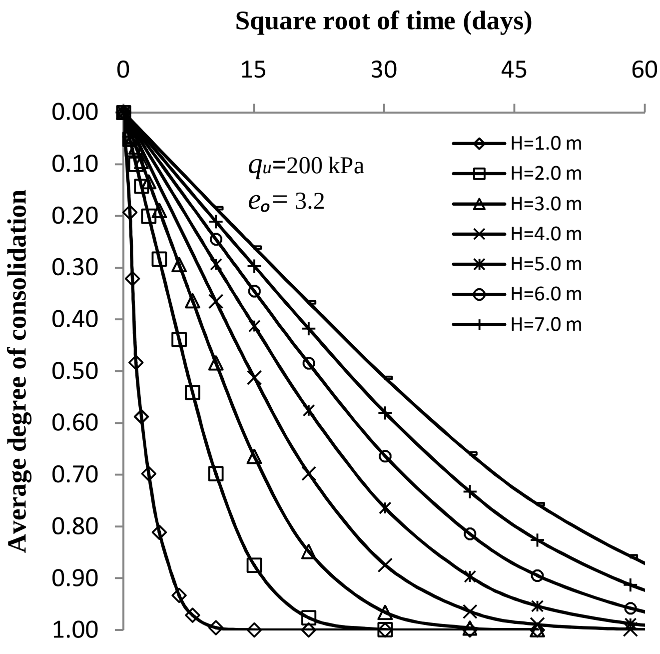


Fig. 2.12 Effect of thickness of compressible layer on consolidation

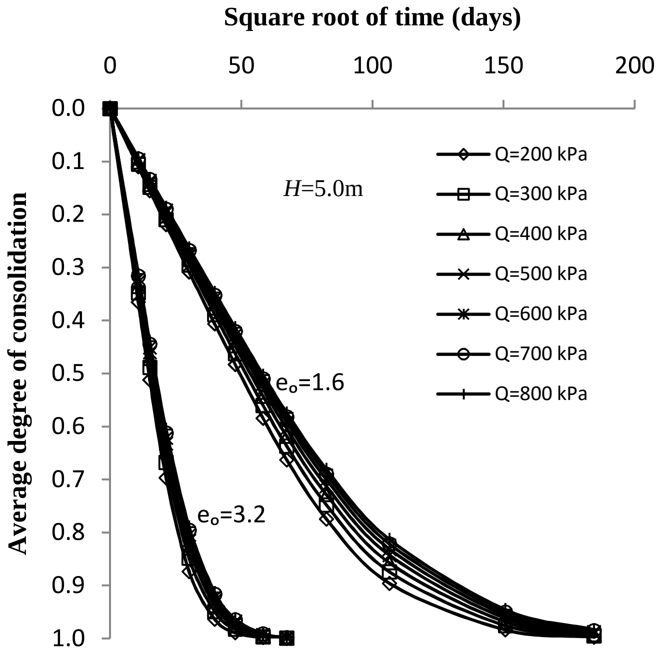


Fig. 2.13Effect of load increment on consolidation

2.9 CONCLUSION

1. The finite volume numerical formulation can be used to solve the large strain one-dimensional consolidation equation, as it falls under the category of conservation law. The spatial distribution of independent variable i.e. void ratio in the practical consolidation problems matches well with the quadratic interpolation functions on void ratio and satisfies boundedness property required in the FVM formulations for convergence.
2. The FVM formulation presented here, maintains third order accuracy and hence it may give sufficiently accurate solutions with relatively lesser number of mesh points.
3. No special treatment is required for the nonlinearity in the explicit form of solution method as the nonlinear terms appearing in the right side of the numerical scheme can be easily evaluated through quadratic interpolation from the known values of independent variable near the face value void ratio at the previous time step.

4. Since the large strain consolidation equation is applicable not only for fresh fills but for the normally or over consolidated soils, the proposed numerical method can handle the consolidation of any clay deposit.
5. The present numerical model is efficient than the linear model and finite difference model whereas the complexity level is much less than FEM based models.
6. The continuity of flow during consolidation is automatic whereas it requires additional care in finite difference or finite element based models.
7. Thus the proposed FVM formulation can be a good alternate to other existing numerical methods used for the solution of consolidation problems.
8. The soft organic clays, with its natural void ratio ranging from 3.2 to 2.5, follow a specific time rate of consolidation characteristic for a given load increment and layer thickness. The proportionality of settlement and square root of time is maintainable almost up to value of 80% degree of consolidation whereas Terzaghi's solution provides that in general only up to 50%.

Chapter 3

EXPERIMENTAL STUDY ON CONSOLIDATION OF THICK SPECIMENS OF CLAY

3.1 PRELIMINARIES

The oedometer tests were conducted on natural black cotton soil sample specimens to assess the compressibility and hydraulic conductivity characteristics of the soil. The same soil sample is used to make thicker specimens and the consolidation tests of these specimens were also performed. For the thick samples, bigger size consolidation cells were used similar to the consolidation test apparatus as described by Lee and Fox (2009) and keeping the loading arrangement as used in oedometer test. Further, the consolidation settlements of thick samples were calculated by the present numerical model using the compressibility and hydraulic conductivity characteristics obtained by simple oedometer test of the soil specimens. The next sections describe the details of the materials, experimental program, methodology and comparison of numerical and experimental results.

3.2 MATERIALS

The soil sample (natural black soil, greyish brown in colour) was taken from Belgaum, Karnataka (India). The index properties of the soil sample were explored in the laboratory and recorded as follows; specific gravity (G_s) of the soil = 2.95; the liquid limit (*LL)* = 79.8 and the plastic limit (*PL*) = 33.7. As per Indian Standard classification system the soil was classified as *CH*. Standard Proctor's compaction test was also done and the optimum moisture content (*OMC*) of soil was found to be 28.4%.

3.3 CONSOLIDATION TEST APPARATUS

Fig. 3.1 shows the schematic arrangement of consolidation apparatus. It uses a rigid wall consolidation cell of 100 mm diameter and 150 mm length. Load is applied through a load plate and piston rod on the specimen. The loading arrangement is as usual that is used in oedometer test i.e. a lever arm, load hanger and different standard weights. Taking into consideration the specimen diameter and load of the plate and piston rod (7.33 N) the influence of oedometer test loads of 25, 50, 100, 200, 400, 800 kN/ m^2 on the present specimen would become 15.2, 29.5, 58.1, 115.3, 229.7, 458.6 kN/ m^2.

3.4 SPECIMEN PREPARATION

The oven dried soil sample was taken and sieved through 4.75 mm aperture sieve. The sample weighing 4.0 Kg was made wet with about 1200 ml water (30% of the weight). Soil and water were mixed in steps so that the mix is thoroughly uniform. Compaction of the wet soil was done in the standard Proctor's mould. Standard method of compaction was followed i.e. three layers compaction and each layer was subjected to 25 blows of the hammer (2.5 Kg) dropping from the height of 0.3 m. Moisture content of the compact was determined and found as 28.34. The prepared compact was sliced to form specimens of thicknesses 20 mm and 30 mm with the diameter of 100 mm. One specimen of 20 mm thickness and 75 mm diameter was also grooved out by the oedometer ring.

3.5 METHODOLOGY

Usual consolidation test on 20 mm thick and 75 mm diameter specimen was performed first. The representative curves between void ratio vs. effective stress and void ratio vs. hydraulic conductivity were drawn. The value of coefficient of consolidation (c_v), the coefficient of compressibility (a_v=$-de/d\sigma'$) and the void ratio (e) at each load were determined. The coefficient of volume compressibility [$m_{v1} = a_v / (1+e)$] was also calculated. The values of c_v, m_{v1} and unit weight of water (γ_w) gave the value of hydraulic conductivity ($k=c_v\, m_{v1}\gamma_w$) corresponding to each void ratio. Compressibility and hydraulic conductivity curves were then plotted. These are the curves that represent the material properties of the subject soil and are used as input data in the present numerical model for predicting the consolidation settlement of any other thickness of soil under any load. Four specimens of diameter 100 mm and thickness 20 mm were first fully saturated that took 10 days time with 1 m head applied across them. The 30 mm thick specimen was kept at 2 m head and that consumed 10 days for full saturation. Five consolidation cells were used to saturate the four specimens of 20 mm thickness and one for the 30 mm specimen. Finally the two specimens of 20 mm were put in one cell to get one 40 mm thick specimen and two 20 mm and one 30 mm specimens were transferred in one cell to form one 70 mm thick specimen. Swelling of the samples on saturation has also been observed, the specimen with 40 mm thickness swells by less than 1 mm and therefore ignored but the height of 70 mm thick specimen raises to 72.3 mm which was taken into consideration while calculating the vertical deformation. The pattern of loading followed was the same as it is done in usual oedometer test, however the stresses developed in the present case is different as mentioned earlier. The saturation status of the specimens was maintained throughout the test by the arrangement shown in figure 3.1.

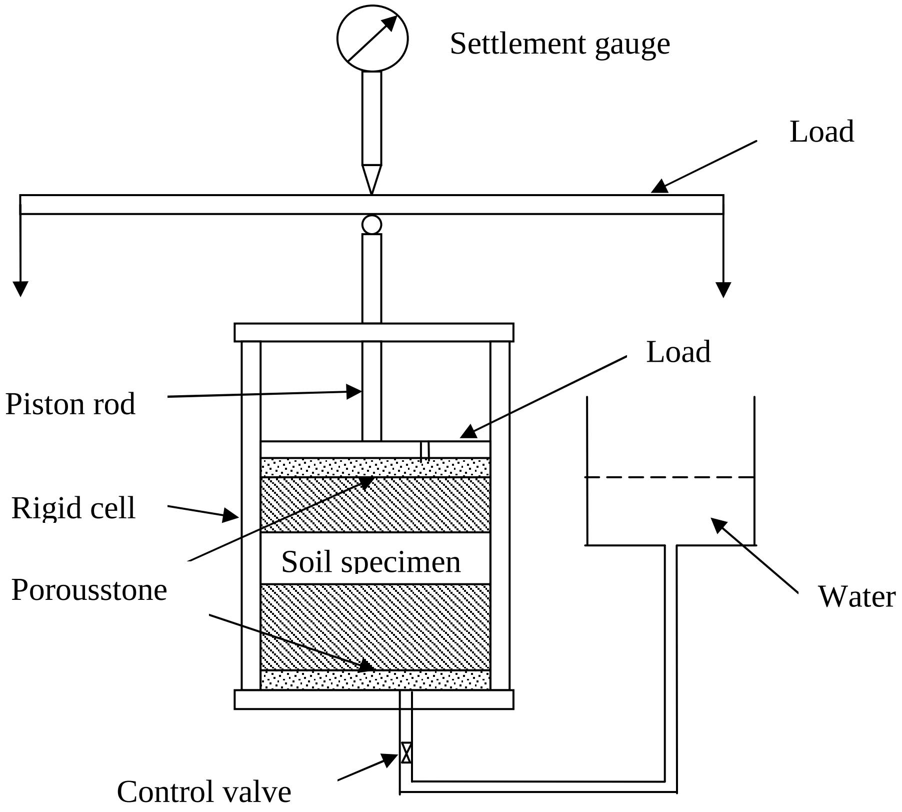


Fig. 3.1 Schematic arrangement of Consolidation Apparatus

Each load was kept for 2880 min. (48 hrs) and the vertical deformations were recorded at the timings 0.25, 1, 4, 9, 16, 36, 60, 120, 240, 1440, 2880 minutes to cover the entire time behaviour of the consolidation settlement during the period.

3.6 COMPARISON OF EXPERIMENTAL AND NUMERICAL RESULTS

3.6.1 Experimental results

The experimental results of settlement of 40 mm and 72.3 mm soil specimens of compacted clay with time and load increments are shown in the Tables 3.1 and 3.2.

3.6.2 Numerical procedure, solution and comparison with experimental results

Table 3.3 shows the data of void ratio, hydraulic conductivity and effective stress. These are derived from the results of oedometer test on 20 mm thick specimen of the black soil and have been used as input values for finite volume analysis. The data are plotted and shown in figs 3.3 and 3.4. These figures provide the characteristic compressibility and hydraulic conductivity curves of the sample soil. The data points are augmented through these curves as shown in the figures and the same are used as input values representing the material properties of the sample soil. The added points, to the laboratory curves of void ratio ~ effective stress and void ratio ~hydraulic conductivity, ranging from void ratio value of 0.97 to 0.73 at an interval of 0.01were marked and the corresponding values of effective stress and hydraulic conductivity were noted.

The initial conditions of the void ratios are obtained considering the self weight of soil consistent with the input data of compressibility curve. The boundary conditions at the top and bottom boundaries are taken as drained as devised in the experiment. Other data as taken for this computation are: specific gravity of soil (2.95), height of specimen (0.04 m and 0.0732 m), head of water on the specimen (0.04 m and 0.099m), unit weight of water (9.81 kN/m^3), number of mesh points (40 and 70), elementary time increments (0.001 min.) and the load increments with time as used during the experiment. The results of the numerical model are shown in table 3.4 and 3.5.

Figures 3.5 and 3.6 present the comparisons of numerical and experimental values of vertical deformations of 40 mm thick and 72.3 mm thick the soil specimens. It is obvious from the figures that the experimental and computed results are very close to each other throughout the time period with changes in the loads. The comparable results of FVM analysis with the laboratory observations on such a small scale, establish the accuracy and efficiency of the method.

Table 3.1 Vertical deformation of 40 mm specimen (experimental results)

Pressure (kN/m^2) ►	15.2	29.5	58.1	115.3	229.7	458.6
Time (min.)▼	Settlement (mm)					
0.00	0.00	0.18	0.40	0.84	1.52	2.41
0.25	0.02	0.20	0.42	0.91	1.58	2.45
1.00	0.03	0.21	0.44	0.92	1.60	2.46
4.00	0.05	0.23	0.46	0.94	1.61	2.47
9.00	0.08	0.25	0.50	0.95	1.65	2.48
16.00	0.11	0.27	0.52	0.96	1.70	2.49
36.00	0.15	0.30	0.58	0.97	1.73	2.54
60.00	0.17	0.33	0.61	0.99	1.80	2.64
120.00	0.18	0.39	0.75	1.17	1.87	2.66
240.00	0.18	0.39	0.80	1.27	2.01	2.80
1440.00	0.18	0.40	0.83	1.52	2.38	3.39
2880.00	0.18	0.40	0.84	1.52	2.41	3.67

Fig3.2Photograph of the experimental set-up (front view)

Table 3.2 Vertical deformation of 72.3 mm specimen (experimental results)

Pressure (kN/m^2) ►	15.2	29.5	58.1	115.3	229.7	458.6
Time (min.)▼			Settlement (mm)			
0.00	0.00	0.34	0.72	1.57	2.84	4.61
0.25	0.01	0.36	0.75	1.59	2.88	4.66
1.00	0.03	0.37	0.76	1.61	2.90	4.68
4.00	0.05	0.39	0.80	1.65	2.94	4.73
9.00	0.07	0.41	0.84	1.69	2.99	4.79
16.00	0.10	0.45	0.88	1.73	3.03	4.84
36.00	0.17	0.49	0.95	1.80	3.14	4.95
60.00	0.21	0.54	1.01	1.86	3.22	5.04
120.00	0.28	0.62	1.13	1.97	3.36	5.21
240.00	0.32	0.68	1.30	2.13	3.58	5.45
1440.00	0.34	0.72	1.56	2.72	4.45	6.49
2880.00	0.34	0.72	1.57	2.84	4.61	6.85

Table 3.3Void ratio (e) ~effective stress (σ′)&hydraulic conductivity (k)

e	σ' (kN/m^2)	k (m/min.)
0.970354	0.00	2.50E-08
0.954433	25.00	1.70E-08
0.933941	50.00	6.36E-09
0.901429	100.00	1.94E-09
0.856453	200.00	1.40E-09
0.794798	400.00	4.96E-10
0.728591	800.00	3.66E-10

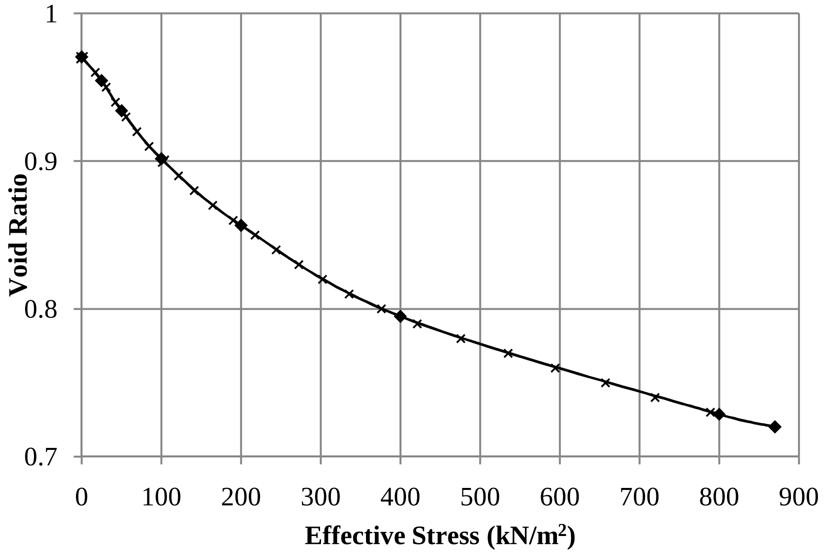


Fig. 3.3 Void ratio vs. Effective stress; ♦ Laboratory points; × Added points

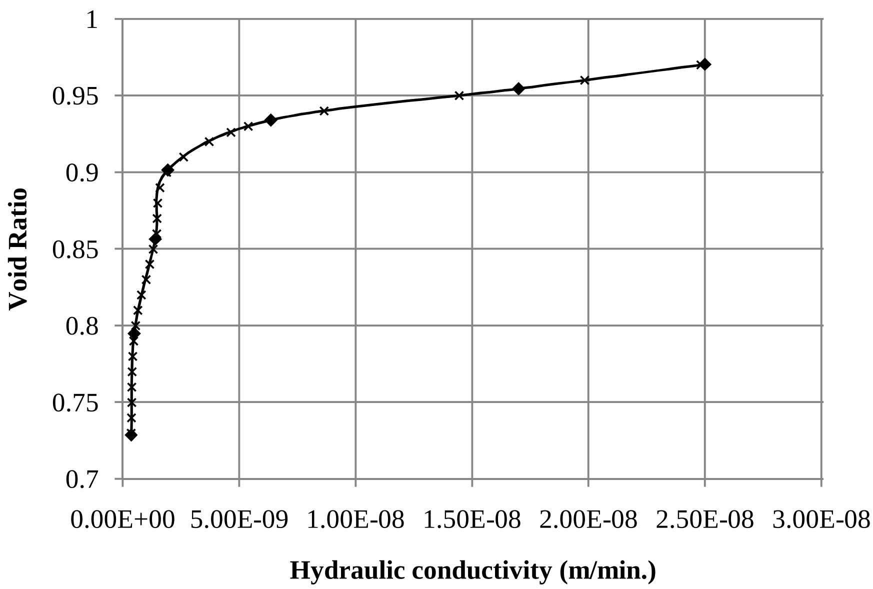


Fig. 3.4 Void ratio vs. Hydraulic conductivity; ♦ Laboratory points; × Added points

Table 3.4 Vertical deformation of 40 mm specimen (numerical results)

Pressure (kN/m^2) ►	15.2	29.5	58.1	115.3	229.7	458.6
Time (min.)▼	Settlement (mm)					
0.00	0.000	0.188	0.398	0.859	1.57	2.52
0.25	0.018	0.207	0.433	0.909	1.63	2.60
1.00	0.030	0.217	0.444	0.917	1.64	2.60
4.00	0.057	0.242	0.474	0.943	1.67	2.61
9.00	0.085	0.267	0.508	0.976	1.71	2.63
16.00	0.112	0.292	0.544	1.01	1.74	2.64
36.00	0.155	0.338	0.614	1.08	1.80	2.69
60.00	0.175	0.366	0.672	1.15	1.84	2.73
120.00	0.187	0.391	0.756	1.26	1.92	2.81
240.00	0.188	0.397	0.825	1.39	2.05	2.94
1440.00	0.188	0.398	0.859	1.57	2.43	3.49
2880.00	0.188	0.398	0.859	1.57	2.52	3.71

Table 3.5 Vertical deformation of 72.3 mm specimen (numerical results)

Pressure (kN/m^2) ►	15.2	29.5	58.1	115.3	229.7	458.6
Time (min.)▼	Settlement (mm)					
0.00	0.000	0.344	0.728	1.57	2.84	4.61
0.25	0.014	0.358	0.750	1.60	2.88	4.66
1.00	0.028	0.371	0.766	1.61	2.90	4.67
4.00	0.055	0.396	0.801	1.65	2.94	4.72
9.00	0.083	0.422	0.837	1.68	2.99	4.77
16.00	0.112	0.448	0.873	1.72	3.04	4.82
36.00	0.168	0.499	0.944	1.79	3.13	4.93
60.00	0.214	0.543	1.010	1.86	3.21	5.02
120.00	0.282	0.616	1.120	1.97	3.37	5.19
240.00	0.329	0.684	1.270	2.14	3.59	5.43
1440.00	0.344	0.728	1.560	2.72	4.45	6.49
2880.00	0.344	0.728	1.570	2.84	4.61	6.84

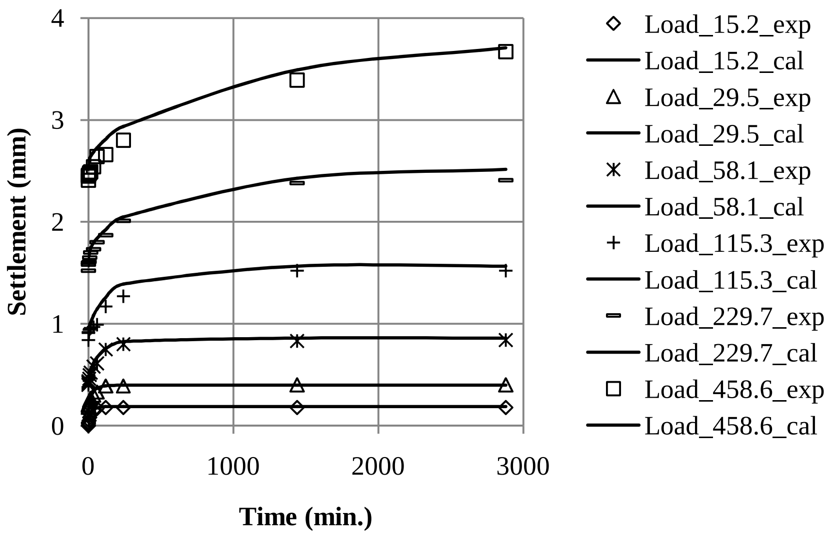


Fig. 3.5 Vertical deformation of 40 mm specimen (Experimental and FVM)

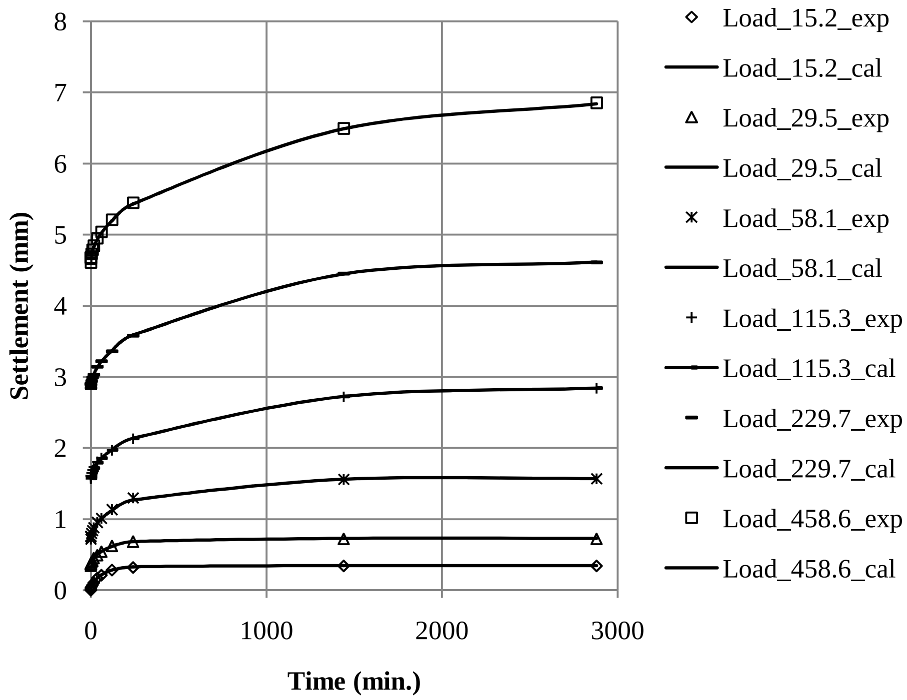


Fig. 3.6 Vertical deformation of 72.3 mm specimen (Experimental and FVM)

3.7 CONCLUSION

This chapter details the experimental set-up and procedure for testing of the relatively thick specimens of soils. This is also shown that usual oedometer test results give the constitutive relations of compressibility and hydraulic conductivity required for the present numerical model of large strain consolidation. The close agreement of numerical and experimental results validates the model.

Chapter 4

FINITE VOLUME MODEL OF ONE DIMENSIONAL SOLUTE TRANSPOERT EQUATIONS

4.1 INTRODUCTION

This chapter contains the description of one dimensional solute transport through rigid porous media, the governing equation and its finite volume numerical model formulation. The governing equation does not have the sorption and decay terms directly but contains the inclusive terms, concentrations in fluid and solid media. The capability of the numerical model to include the linear-equilibrium sorption as well as nonlinear-nonequilibrium sorption is shown. However, the discussions on decay reaction are limited to first order only. Model verification has been done by comparing the results with another numerical model by Fox (2007).

4.2 GOVERNING EQUATIONS &FINITE VOLUME MODEL DEVELOPMENT

The solute transport equation in terms of Lagrangian coordinates may be given as follows (Peters and Smith, 2002, Eq. 43).

$$\frac{\partial}{\partial t}\left\{nc_f J+(1-n)c_s J\right\}=-\frac{\partial}{\partial a}\left\{qc_f-\frac{nD^*}{J}\frac{\partial c_f}{\partial a}\right\} \tag{4.1}$$

where, n=porosity; c_f =solute concentration in fluid medium (mass/ volume); c_s = solute concentration in solid medium (mass/ volume); $J=d\xi/da$ (given by Eq. 2.7); D^*=effective diffusion; $q=n(v_f-v_s)$ =Darcy velocity; v_f = pore fluid velocity in the soil system; v_s = soil velocity. Since the description in this section is limited to rigid porous media, the soil velocity and pore fluid velocity due to consolidation is ignored. Mechanical dispersion of solute transport can be included in the Eq. (4.1) using the coefficient of longitudinal hydrodynamic dispersion (D_a) in place of effective diffusion. Deng et al. (2001) gave analytical method to assess the D_a in open channels based on hydraulic geometry relationship, assuming uniform flow. Later the same work is extended for non-uniform flows also (Deng et al., 2002).The other attempt to handle the diffusion component of solute transport in open channels, Deng et al. (2001) revised the Fick's law component of the theory and proposed the fractional Advection –Dispersion equation along with its finite difference solution. However, for one dimensional solute transport through porous media, coefficient of longitudinal hydrodynamic dispersion (D_a) is modelled by;

$$D_a = D^* + \alpha_a v_f \tag{4.1a}$$

where, α_a is the longitudinal dispersivity and v_f is the pore fluid velocity. Eq. (4.1) is now written as;

$$\frac{\partial}{\partial t}\left\{nc_f J + (1-n)c_s J\right\} = -\frac{\partial}{\partial a}\left\{qc_f - \frac{nD_a}{J}\frac{\partial c_f}{\partial a}\right\} \tag{4.1b}$$

Eq. (4.1b) may also be classified as conservative equation containing a time derivative and a spatial divergent term. The details of finite volume formulation of Eq. (4.1b) are given below.

Let the combined quantity of solute concentration in fluid and solid (C_{cm}) be represented as;

$$C_{cm} = nc_f J + (1-n)c_s J \tag{4.2}$$

Eq. (4.1) is integrated with respect to time within limits as shown over a control volume using Eq. (4.2) and it may be written as;

$$\oint_{CV}\left(\int_t^{t+\Delta t}\frac{\partial}{\partial t}C_{cm}dt\right)dV = \int_t^{t+\Delta t}\left(\oint_{CV}-\frac{\partial}{\partial a}\left(qc_f - \frac{nD_a}{J}\frac{\partial c_f}{\partial a}\right)dV\right)dt \tag{4.3}$$

where, $dV = dA \times da$ = volume of elementary control volume; dA=cross section area of the control volume normal to direction of 'a'; da=length of the control volume. Integrating Eq. (4.3) using Gauss-Divergence theorem for the divergent term, it may be written as;

$$\oint_{CV}\left[C_{cm}\right]_t^{t+\Delta t} dV = \int_t^{t+\Delta t}\left(\oint_{CV}\left(qc_f - \frac{nD_a}{J}\frac{\partial c_f}{\partial a}\right)dA\right)dt \tag{4.4}$$

Now, using definition of time integral Eq. (2.13) for the right hand side, Eq. (4.4)takes the following form of explicit finite volume numerical scheme on a j^{th} control volume (referring to the Fig 2.3);

$$C_{cm_j}^{t+\Delta t} = C_{cm_j}^{t} - \frac{\Delta t}{\Delta a}\left[\left(qc_f - \frac{nD_a}{J}\frac{\partial c_f}{\partial a}\right)_{j+\frac{1}{2}}^{t} - \left(qc_f - \frac{nD_a}{J}\frac{\partial c_f}{\partial a}\right)_{j-\frac{1}{2}}^{t}\right] \tag{4.5}$$

where, superscript ($t+\Delta t$) denotes value of the variable at the time step and similarly the (t); Δt = incremental time step; subscripts ($j+\frac{1}{2}$) and ($j-\frac{1}{2}$) are the designations of upper and lower faces of the j^{th} control volume of discretized field.

4.2.1 Interpolation of face values

Leonard (1988) proposed Quadratic Upstream Interpolation for Convective Kinetics (QUICK) scheme for bulk flow regions along with a modified scheme of same order in terms of normalised variables for critical regions where the QUICK scheme may give undershoots or overshoots. This scheme has been adopted here and its brief description is as follows. The upstream quadratic interpolation function is formed using 3-point Lagrangian interpolation formula;

$$\phi(a)=\frac{(a-a_{j+1})(a-a_{j+2})}{(a_j-a_{j+1})(a_j-a_{j+2})}\phi_j+\frac{(a-a_j)(a-a_{j+2})}{(a_{j+1}-a_{j+2})(a_{j+1}-a_j)}\phi_{j+1}+\frac{(a-a_j)(a-a_{j+1})}{(a_{j+2}-a_j)(a_{j+2}-a_{j+1})}\phi_{j+2} \quad (4.6)$$

For equidistant nodes at an interval (Δa) with the same length of control volume, eq. (4.6) takes the following forms;

$$\text{Upper face, } \phi_{f_u}=\phi_{j+\frac{1}{2}}=\frac{3}{8}\phi_j+\frac{6}{8}\phi_{j+1}-\frac{1}{8}\phi_{j+2} \quad (4.7)$$

$$\text{Lower face, } \phi_{f_l}=\phi_{j-\frac{1}{2}}=\frac{3}{8}\phi_{j-1}+\frac{6}{8}\phi_j-\frac{1}{8}\phi_{j+1} \quad (4.8)$$

where, ϕ is a transported variable; a_{j-1}, a_j, a_{j+1}, a_{j+2} are the locations of the nodes j-1, j, j+1, j+2, j+ 3 (refer Fig 2.3) with the variable values ϕ_{j-1},ϕ_j, ϕ_{j+1}, ϕ_{j+2} respectively; a= the distance of face of control volume j and j-1 above the nodes. Eqs (4.7, 4.8) represent upwinding interpolation for downward flow as they take the influence of two upstream nodes above the faces of control volume and one below it. The above equations cannot be used near the boundaries due to insufficient adjoining nodes, under such situations pseudo nodes may be formed numerically. If quadratic distribution of the variable is assumed, referring the upper boundary in fig.2.3, the variable ϕ may be expressed as follows;

$$\phi(a)=\phi_{m-1}+\frac{\phi_{m+1}-\phi_{m-1}}{2\Delta a}a+\frac{\phi_{m+1}-2\phi_{m-1}+\phi_{m-2}}{2(\Delta a)^2}a^2 \quad (4.9)$$

Thus the boundary node may be expressed as;

$$\phi_m=\phi(\frac{\Delta a}{2})=\phi_{m-1}+\frac{\phi_{m+1}-\phi_{m-1}}{4}+\frac{\phi_{m+1}-2\phi_{m-1}+\phi_{m-2}}{8} \quad (4.10)$$

The pseudo-node ϕ_{m+1} will be;

$$\phi_{m+1}=\frac{8}{3}\phi_m-2\phi_{m-1}+\frac{1}{3}\phi_{m-2} \quad (4.11)$$

Similarly, for the lower boundary one pseudo-node ϕ_0 may be created;

$$\phi_0 = \frac{8}{3}\phi_1 - 2\phi_2 + \frac{1}{3}\phi_3 \qquad (4.12)$$

However, the linear distribution near boundary may also be useful at times and with this the pseudo-nodes will be;

$$\phi_{m+1} = 2\phi_m - \phi_{m-1};\ \phi_0 = 2\phi_1 - \phi_2 \qquad (4.13)$$

The above interpolation functions are applicable in most of the regions of transport process, but near the regions of rapidly changing gradient of the variable, it may be unstable or may return undershoots or overshoots. To overcome this problem, an exponential upwinding, equivalent to the quadratic one, in terms of normalised variables is followed as proposed by Leonard (1988).

$$\overline{\phi}_{f_u} = \frac{\sqrt{\overline{\phi}_{j+1}(1-\overline{\phi}_{j+1})^3} - \overline{\phi}_{j+1}^2}{1-2\overline{\phi}_{j+1}} \qquad (4.14)$$

where, the value of square-root term is taken as positive, the normalised variable is defined as $\overline{\phi} = \dfrac{\phi - \phi_{j+2}}{\phi_j - \phi_{j+2}}$ and thus;

$$\overline{\phi}_{j+1} = \frac{\phi_{j+1} - \phi_{j+2}}{\phi_j - \phi_{j+2}};\ \overline{\phi}_{f_u} = \frac{\phi_{f_u} - \phi_{j+2}}{\phi_j - \phi_{j+2}} \qquad (4.15)$$

Eq. (4.15), in terms of normalized variable, may also be written as;

$$\phi_{f_u} = \phi_{j+\frac{1}{2}} = 0.75 + 0.75(\overline{\phi}_{j+1} - 0.5) \qquad (4.16)$$

Eqs. (4.14) and (4.15) can give the values face values ($\phi_{f_u} = \phi_{j+\frac{1}{2}}$); the lower face values for a node (j) can be calculated by following the above procedure with nodes (j-1, j, j+1) or simply following the fact that upper face of a control volume is the lower face for the next higher control volume.

However, Eq. (4.14) cannot be evaluated for every value of the normalized variable ($\overline{\phi}_{j+1}$). Also, the normalized variable ($\overline{\phi}_{j+1}$) lies in the range (0, 1) in the monotonic region. Inclusion of non-monotonic variable, poses the requirement of otherwise strategies for calculating the face values. The apparent indeterminacy of the Eq. (4.14) near $\overline{\phi}_{j+1}$=0.5, comes out to be 0.75 using L'Hospital's rule as well as gives the same slope value, this in view of Eq. (4.16) becomes the criterion for segregating the smooth and rapidly changing region. The overall scheme is named as (Exponential Upwinding or Linear Extrapolation Refinement) EULER-QUICK algorithm and it is implemented in the following manner (Leonard, 1988).

(i) One system of upstream and downstream is followed throughout in the porous field.

(ii) If $|(\phi_j - \phi_{j+2})| \leq 10^{-5}$ usual QUICK scheme is followed.

(iii) If $|(\phi_j - \phi_{j+2})| > 10^{-5}$ and $|\phi_{j+2} - 2\phi_{j+1} + \phi_j| \leq 0.3|\phi_j - \phi_{j+2}|$; the condition follows the common part of Eq. (3.16) (QUICK) and exponential interpolation thus QUICK is used.

(iv) If $|(\phi_j - \phi_{j+2})| > 10^{-5}$ and $|\phi_{j+2} - 2\phi_{j+1} + \phi_j| > 0.3|\phi_j - \phi_{j+2}|$; the rapid gradient region prevails and depending upon the monotonic or non-monotonic variation of the variable exponential (Eq. 4.14) is followed or otherwise a consistent scheme is used as described next.

(v) If $\bar{\phi}_{j+1} \leq -1$ or $\bar{\phi}_{j+1} \geq 1.5$ (non-monotonic) Eq. (4.16); QUICK is used.

(vi) If $0.35 \leq \bar{\phi}_{j+1} \leq 0.65$ (values near 0.5) Eq. (4.16); QUICK is used.

(vii) If $-1 \leq \bar{\phi}_{j+1} \leq 0$; $\bar{\phi}_{f_u} = 0.375\bar{\phi}_{j+1}$; non-monotonic region (an adhoc arrangement for limited region that avoids unphysical oscillations) is used.

(viii) For the non-monotonic region $1 < \bar{\phi}_{j+1} \leq 1.5$; $\bar{\phi}_{f_u} = \bar{\phi}_{j+1}$ (the equation joins the QUICK at $\bar{\phi}_{j+1} \geq 1.5$) and QUICK is used.

(ix) For the monotonic region ($0 < \bar{\phi}_{j+1} < 0.35$ and $0.65 < \bar{\phi}_{j+1} \leq 1$); away from the common region (item-vi above); exponential upwinding Eq. (4.14) is used.

(x) Finally, the un-normalized face value is found using second part of Eq. (4.15).

The above scheme is applicable to the downward flow and for j^{th} control volume the interpolation of solute concentration at face j+1/2 includes the nodes j+1 and j+2 as the two upstream points and the mesh point j as downstream point. For upward flow, the order is reversed accordingly and the computation requires both types of interpolation modules separately to be used with appropriate case of flow.

4.2.2 Assessment of the Interpolation Scheme

The interpolated face values must satisfy the following three criteria for convergence of the solution (Versteeg and Malalaskera, 2007): (i) Conservativeness and (ii) Boundedness (iii) Transportiveness. First criterion is that the algebraic sum of all the incoming and outgoing flux values through faces of all the control volumes must be equal to the algebraic sum of boundary flux values. The second criterion means that the node values are bounded by their control volume face values. The third one is for a consistent interpolated value that takes care of the direction of the driving potentials of transported variables. The quadratic scheme used here, interpolate the each control volume face values with two adjacent enclosing nodes and one more node above the upper one successively; the conservativeness is therefore maintained. The transportiveness is also maintainable due to use of two upper nodes and one lower node. However, the boundedness is not guaranteed with the QUICK scheme which poses stability problem. Thus, as such the QUICK scheme is conditionally stable (Versteeg and Malalaskera, 2007). The EULER-QUICK algorithm overcomes the stability problem and avoids all critical regions that may cause unphysical oscillations. The entire scheme maintains overall accuracy of third order even though the scheme uses linear interpolations very sporadically in the non-monotonic regions (Leonard, 1988).

4.2.3 Time Step Restrictions

The time step restrictions are very tight in QUICK schemes, particularly in case of unsteady advection and diffusion in an infinite domain. The von Neumann analysis of one-dimensional QUICK scheme proves it easily (Leonard, 1980). However, the considerations on time step restrictions for finite grids results into the following condition (Paollucci and Chenoweth, 1982);

$$c \le \frac{2}{P_\Delta} + \frac{\pi^2}{2N^2} \tag{4.17}$$

where, local grid Courant number $c = \dfrac{q\Delta t}{\Delta a}$; local grid Peclet number $P_\Delta = \dfrac{q\Delta a}{D^*}$; $N\Delta a = \lambda_n$ and λ_n is the long wave length cut-off corresponding to a finite grid on Fourier spectrum. Further, Leonard (1988) opined from the numerical experimentation that the instabilities are avoided if a local Courant number does not exceed 0.2.

4.3 SORPTION, DECAY, BOUNDARY CONDITIONS AND SOLUTION PROCEDURE

The active clayey soils are sensitive to sorption/ desorption for many solutes when comes into contact. Three specific sorption isotherms are followed here that may be described as linear-equilibrium isotherm, nonlinear equilibrium isotherm and nonlinear-nonequilibrium isotherm. The implementation all these isotherms in the present formulation are shown. Initially the composite solute transport (C_{cm}) is calculated with the discrete Eq.

(4.5) and then depending upon the sorption characteristic of the solute and the clay, solute concentration in fluid and solid phase of soil system can be distributed.

The linear and equilibrium sorption isotherm is defined as;

$$c_s = \rho_s K_d c_f \tag{4.18}$$

where, $\rho_s = G_s \gamma_w$= density of solid phase; K_d = partition coefficient and c_f = the equilibrium concentration of solute in the fluid phase. The combined concentration (C_{cm}) as defined in Eq. (4.2) can now be written;

$$C_{cm_i}^{t+\Delta t} = n^{t+\Delta t} c_{f_i}^{t+\Delta t} J + (1 - n^{t+\Delta t}) \rho_s K_d c_{f_i}^{t+\Delta t} J \tag{4.19}$$

Thus, the Eqs (4.18) and (4.19) provide the equilibrium concentrations of solute in the fluid and solid phase of the system at the current time step at a node.

Nonlinear equilibrium Freundlich isotherm ($c_s = \rho_s K_p {c_f}^F$), if used in Eq. (4.1b) directly, introduces geometric nonlinearity in the equation. However the equation is solved in the linear form in the term of combined concentration (C_{cm}) and at the stage of segregation of c_f and c_s the following nonlinear algebraic equation is formed.

$$nc_f \frac{1+e}{1+e_0} + (1-n)\rho_s K_p c_f^F \frac{1+e}{1+e_0} - C_{cm} = 0 \tag{4.19a}$$

Eq. (4.19a) is worked out using Newton-Raphson method (Burden and Fairs, 2011) to get c_f and c_s is calculated using the Freundlich isotherm.

Nonlinear and non-equilibrium sorption isotherm (Travis and Etnier 1981) followed here is expressed as;

$$\frac{\partial s}{\partial t} = \lambda_s (K_p c_f^F - s) \tag{4.20}$$

where, $s = \frac{c_s}{\rho_s}$ =sorbed concentration in the solid phase (mass per unit mass); λ_s=sorption rate constant; K_p and F are the constants describing a Freundlich isotherm. The corresponding equilibrium sorption is defined as $s = K_p c_f^F$ and the Eq. (4.20) represents the imbalance of sorption from this equilibrium. For this typical isotherm, the distribution of solute concentration in fluid and solid phases is obtained by sufficient subdivisions of the main time step and using the Eqs (4.19) and (4.20). The nonlinearity involved here is approximated in piecewise linear manner during the subdivisions of the time step. Let the time step Δt be further subdivided into n_s equal steps. It is assumed that the change in composite concentration during time t to $t+\Delta t$ occurs linearly by equal values ($\frac{C_{cm_j}^{t+\Delta t} - C_{cm_j}^{t}}{n_s}$).

Now the value of $\partial s/\partial t$ at any time t is calculated using Eq. (4.20) and having known $\partial s/\partial t$ the values of $c_{s_j}^{t+\Delta t'}$ and $c_{f_j}^{t+\Delta t'}$ is found as shown in Eq. (4.21). The procedure adopted is presented below.

$$\left.\begin{aligned}
&\Delta t' = \frac{\Delta t}{n_s} \\
&C_{cm_j}^{t+\Delta t'} = C_{cm_j}^{t} + \frac{C_{cm_j}^{t+\Delta t} - C_{cm_j}^{t}}{n_s} \\
&\frac{\partial s}{\partial t} = \lambda_s \left(K_p \left(c_{f_i}^{t} \right)^F - \frac{c_{s_j}^{t}}{\rho_s} \right) \\
&c_{s_j}^{t+\Delta t'} = c_{s_j}^{t} + \frac{\partial s}{\partial t} \Delta t' \rho_s \\
&c_{f_j}^{t+\Delta t'} = \frac{C_{cm_j}^{t+\Delta t'} - (1 - n^{t+\Delta t}) c_{s_j}^{t+\Delta t'} J^{t+\Delta t}}{n^{t+\Delta t} J^{t+\Delta t}}
\end{aligned}\right\} \tag{4.21}$$

where, the known composite concentrations for a node at a time t and $t+\Delta t$ are;

$$C_{cm_j}^{t} = (n^t c_{f_j}^{t} + (1-n^t) c_{s_j}^{t}) J^t;\ C_{cm_j}^{t+\Delta t} = (n^t c_{f_j}^{t+\Delta t} + (1-n^t) c_{s_j}^{t+\Delta t}) J^{t+\Delta t} \tag{4.22}$$

The difference of known new composite concentration at time step ($t+\Delta t$) and the previous time step (t) is divided into number of subdivisions of Δt and further calculations are done through iterations by the set of Eqs (4.21).Where, n_s= the number of subdivisions of the time step (Δt). The current porosity is used in the equations and the variation of porosity during the time step is ignored which is negligibly small.

First order decay is governed by the expression $c_f^t = c_f^0 \exp(-\lambda t)$; where the solute decay constant is represented by λ_c and the source decay constant as λ_{sc}; c_f^t and c_f^0 are the concentration of solute at a time t and at t=0 respectively.

Three types of boundary conditions as envisaged by Danckwerts (1953) are adopted and described as under.

1. Boundary condition on solute concentration: the solute concentration may be assumed to be constant if the inflow boundary has a large solute pool on it or it may be zero if the outflow boundary has a large fluid pool without the solute.

2. Boundary condition on concentration gradient: concentration gradient at the outflow boundary will be zero if solute transfer is restricted through no flow boundary.
3. Boundary condition on mass flux: If a small well mixed reservoir exists over a boundary and its concentration is likely to change with solute transfer, the mass flux at the boundary is constant, i.e., the convective mass flux flowing in at the boundary will be equal to the mass flux flowing out just below the boundary inside the media due to combined effect of advection and diffusion and the representative equation will be;

$$v_f c_0 = v_f c - \frac{D_a}{J}\frac{\partial c}{\partial a} \tag{4.23}$$

where, v_f is the advective velocity, c_0 is the solute concentration in the reservoir, c is the solute concentration at the boundary and D_a is the coefficient of longitudinal hydrodynamic dispersion for one dimensional solute transport.

The entire procedure is coded in the computer language FORTRAN-77 and the problem solved is presented next for the evaluation of the computational model.

4.4 MODEL VERIFICATION

A set of simulations assesses the solute transport in a rigid porous media (no consolidation) with steady flow and constant solute concentrations (boundary condition type I) at the boundaries. Fox (2007) describes the geometry and data of the problem. Fig.4.1 shows the geometry which is initially uncontaminated. The height H=1.0 m, specific gravity of solids G_s= 2.7, porosity n =0.4, dry density ρ_d = 1620 Kg/ m^3, vertical hydraulic conductivity k =2×10^{-8} m/ s and top and bottom boundaries are drained. For this simulation the consolidation part is bypassed, consolidation induced velocity is taken as zero and the seepage velocity due to hydraulic gradient is considered. Table 4.1 shows all the cases, seven in number, with all required data for which the simulations were performed. Linear equilibrium sorption isotherm is followed for the cases with sorption. Longitudinal dispersion has been taken into account by replacing the effective diffusion (D^*) term of Eq. (4.5) by hydrodynamic dispersion coefficient ($D_a = D^* + \alpha_a v_f$) where, α_a is the longitudinal dispersivity and v_f is the pore fluid velocity. Linear-equilibrium sorption followed gives the retardation factor [$R_f = (1 + \rho_d K_d / n) = 1.81$].

Table 4.1Parameters of 1D transport simulations

Simulation	Boundary total head values h_t, h_b (m)	Seepage velocity v_s (m/s)	Effective Diffusion coefficient D^* (m^2/s)	Longitudinal dispersivity α_L (m)	Distribution coefficient K_d (mL/g)	Solute decay constant λ_c (1/s)	Source decay Constant λ_{sc} (1/s)	Peclet Number P
Advection	1.1, 1	5×10^{-9}	0	0	0	0	0	∞
Diffusion	1, 1	0	5×10^{-10}	0	0	0	0	0
Advection+ diffusion	1.1, 1	5×10^{-9}	5×10^{-10}	0	0	0	0	10
Advection+ dispersion	1.1, 1	5×10^{-9}	5×10^{-10}	0.1	0	0	0	5
ADS	1.1, 1	5×10^{-9}	5×10^{-10}	0.1	0.2	0	0	5
ADS+ decay	1.1, 1	5×10^{-9}	5×10^{-10}	0.1	0.2	2×10^{-7}	0	5
ADS+decay+ source decay	1.1, 1	5×10^{-9}	5×10^{-10}	0.1	0.2	2×10^{-7}	1×10^{-8}	5

Solute decay at two levels, i.e., during the flow through and also at source has been considered. First order decay is adopted for the purpose, governed by the expression $c_f^t = c_f^0 \exp(-\lambda t)$; where the solute decay constant is represented by λ_c and the source decay constant as λ_{sc}; c_f^t and c_f^0 are the concentration of solute at a time t and at t=0 respectively.

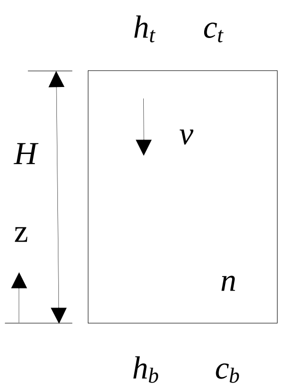


Fig.4.1 Geometry of the rigid porous media

Peclet number P (=$v_f H/D^*$or D_a) varies from zero to infinite as shown in Table 4.1. Solute concentration at top boundary has been given a value of c_o =10 mg/ L, however the solutions are shown in terms of relative concentration (c_f / c_0). Simulations are done with 32 equidistant nodes in the rigid porous field. Fox (2007) gives the solution of this problem with 200 elements with the finite difference program CST1 at the time t=4×10^7s, 4×10^8s and

1.6×10^8s. It has also been shown that the results are in very good agreement with the analytical solutions of Rabideau and Khandelwal (1998).

Figs. 4.2 to 4.4 shows the comparisons of Finite Volume results with the results of Fox (2007) at the instants of time mentioned above for all the cases listed in the Table 4.1. It is evident that the results are quite close except the advective font. Figs.4.2, 4.3 and 4.4 show all the seven cases separately for clarity of presentation of comparisons with the CST1 results at time $t=4\times10^7$ s. The advective font (Fig.4.2) shows the transition of relative solute concentration from 1 to 0 approximately from the elevation 0.9 to 0.7 whereas the CST1 shows this transport instantly at elevation 0.8 which is due to the plug flow assumption for pore fluid velocity based calculation. However the numerical scheme based result presented here are close enough and acceptable and similar trends of advective fonts has also been found as shown in Figs.4.3 and 4.4. The simulations were also performed for higher values of hydraulic gradient (i =1.0 and 10.0) where, the Peclet numbers would be 100 and 1000 for advection + diffusion; for advection + dispersion these would be 9.09 and 9.90. Fig.4.5 ($t=1.0\times10^7$ s) and 4.6 ($t=1.0\times10^6$ s) shows all the curves scaled from CST1 results and finite volume method (fvm) results in one graph. It is evident that the finite volume values are in excellent agreement with Fox (2007). A set of six data sets maintaining a fixed Peclet number P =10 as shown in Table4.2 were also simulated for the same boundary condition of fixed solute concentrations as mentioned above. Retardation factor, for first three cases is 1.0 and for the next three, it is 1.81. The plots are presented between relative concentration and relative elevation (a/H). Fig.4.7 shows the comparison of finite volume results and CST1 results (Fox, 2007) at non-dimensional time factor t^* ($=tv_f/H$) as 0.5; which are in close agreement. The identical results corresponding to a retardation factor with invariant Peclet number shows the uniqueness of solutions of the present computational model. A few more simulations were performed on the data of Fig.4.1 with the constant flux (reservoir/ type III) boundary condition at the top (inflow condition) and zero concentration gradient (type II) at the bottom (outflow condition); defined earlier are

$$v_f c_0 = v_f c - \frac{D_L}{J}\frac{\partial c}{\partial a} \quad at \quad a = (H, t) \tag{4.24}$$

$$\frac{1}{J}\frac{\partial c}{\partial a} = 0 \quad at \ a = (0, t) \tag{4.25}$$

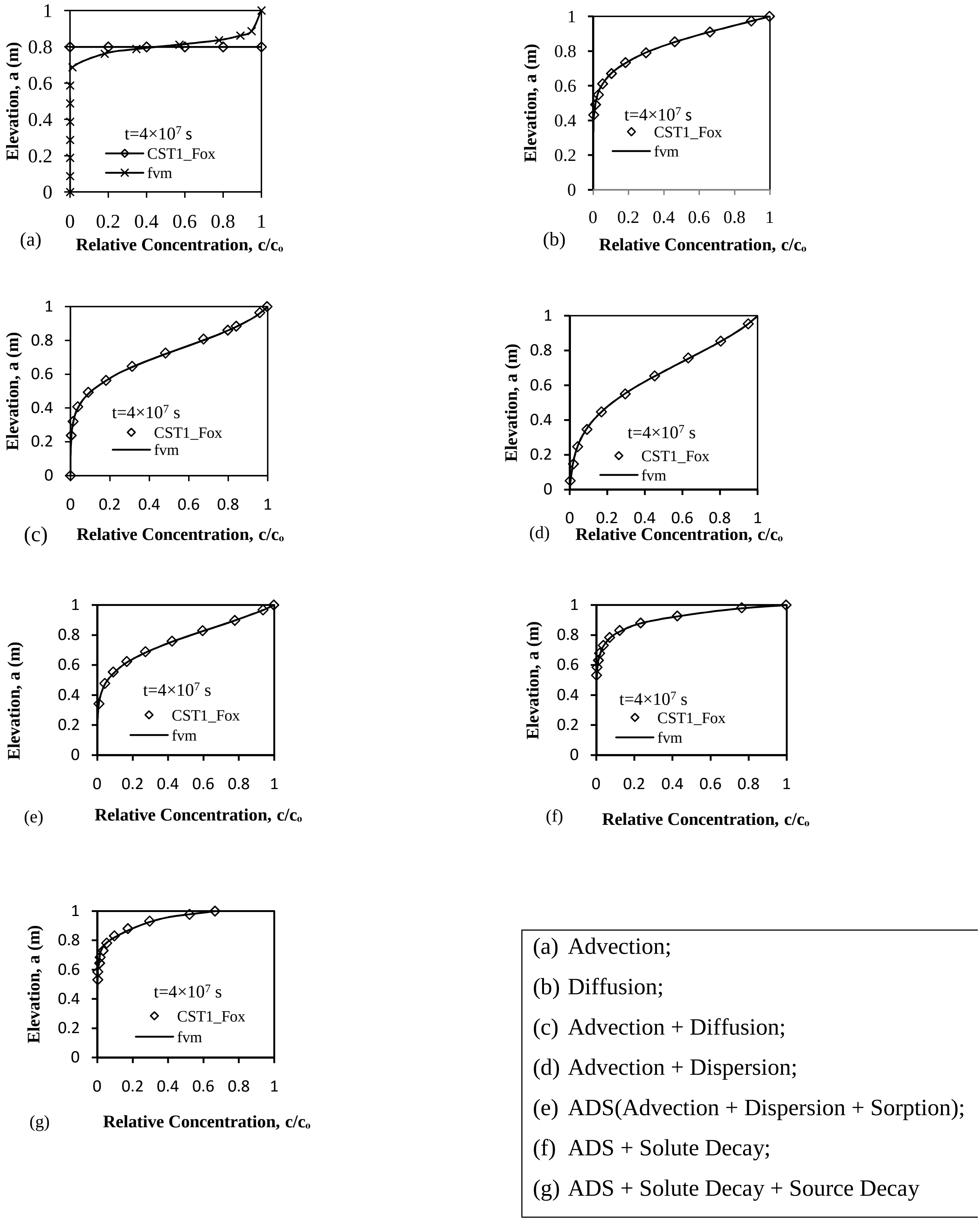


Fig.4.2 Solute concentration profile for 1-d rigid porous media (i=0.1; t=4×10^7 s)

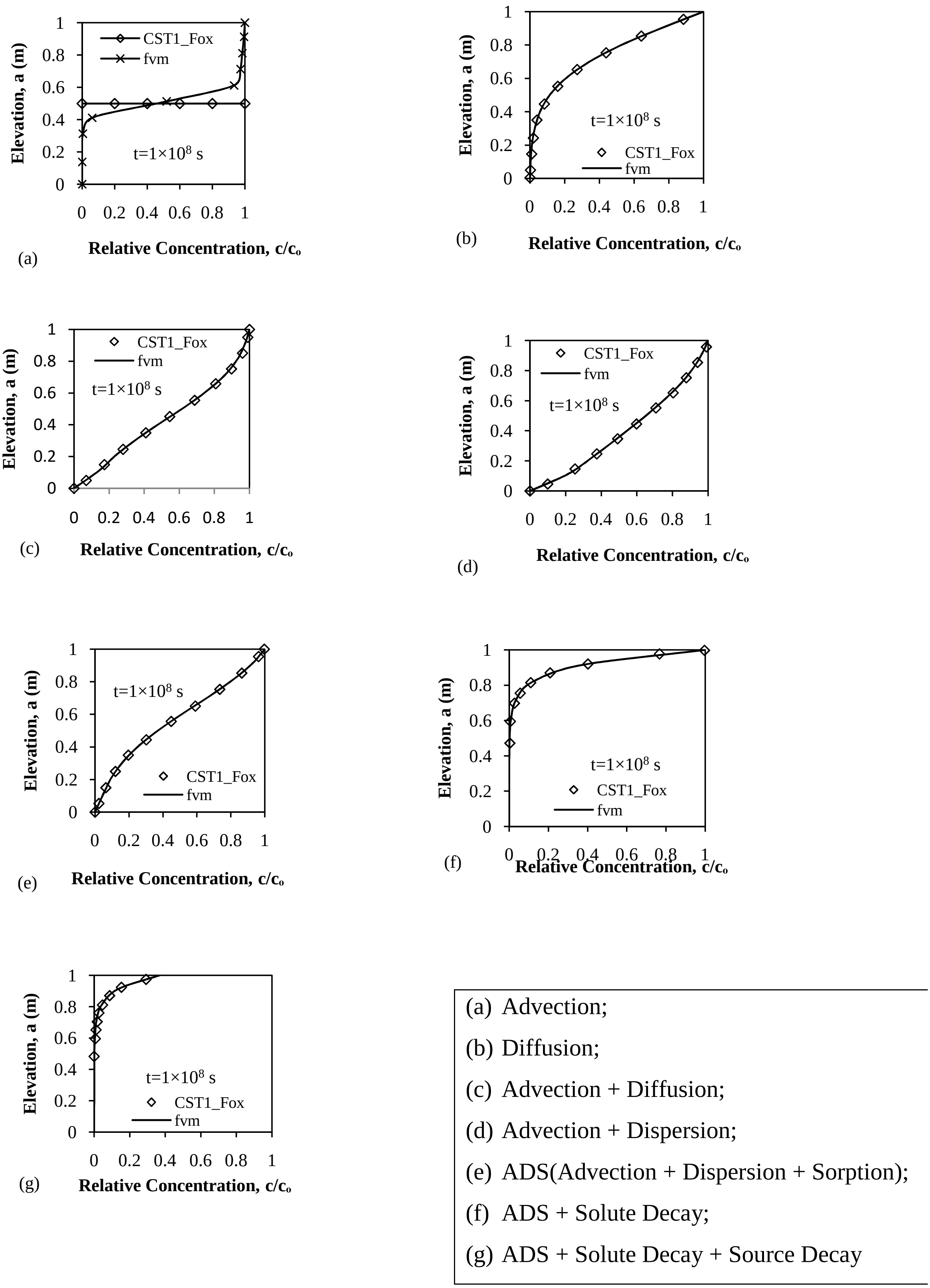
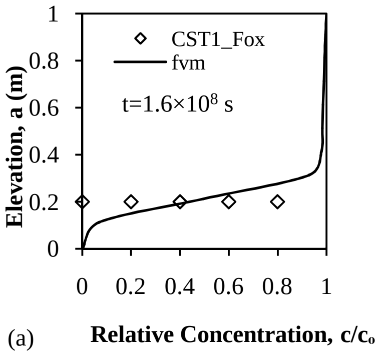
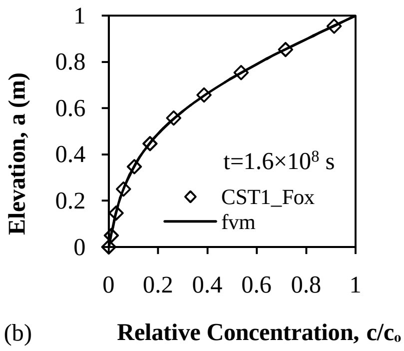
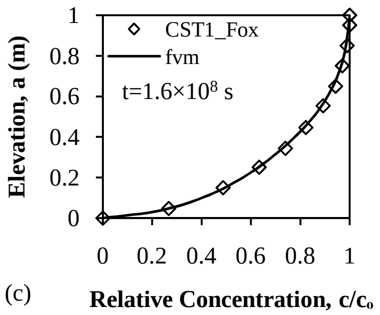
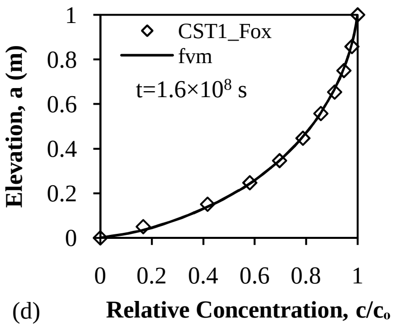


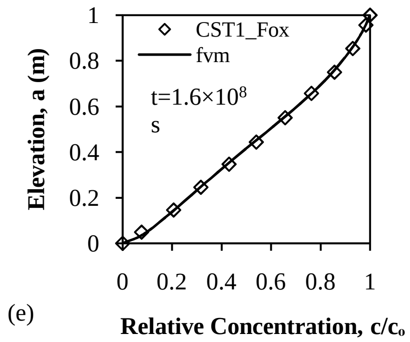
Fig.4.3Solute concentration profile for 1-d rigid porous media (*i*=0.1; *t*=1×10^8 s)

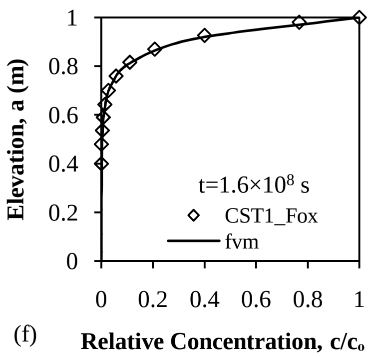


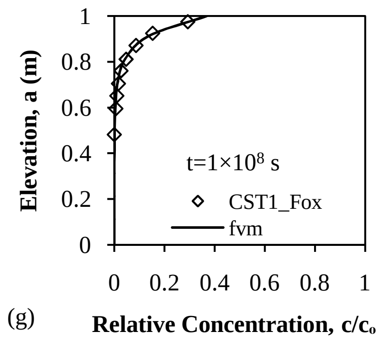












(a) Advection;
(b) Diffusion;
(c) Advection + Diffusion;
(d) Advection + Dispersion;
(e) ADS(Advection + Dispersion + Sorption);
(f) ADS + Solute Decay;
(g) ADS + Solute Decay + Source Decay

Fig.4.4 Solute concentration profile for 1-d rigid porous media (*i*=0.1; *t*=1.6×10^8)

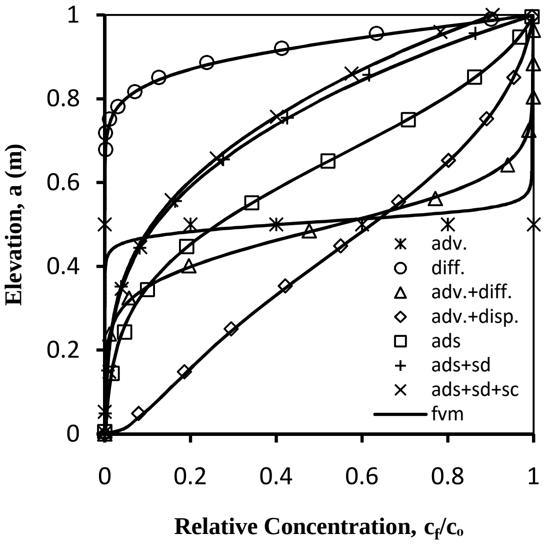


Fig.4.5Solute concentration profile for 1-d rigid porous media (i=1.0; t=1.0×10^7 s)

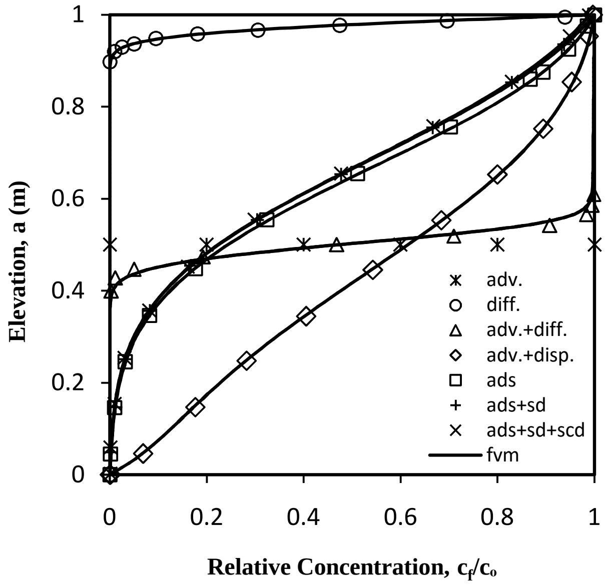


Fig.4.6Solute concentration profile for 1-d rigid porous media (i=10.0; t=1.0×10^6 s)

Table4.2 Parameters for 1-d transport for invariant Peclet number

Sl. No.	Layer height (m)	Total head at top and bottom h_t, h_b (m)	Hydraulic gradient i	Seepage velocity v_f (m/ s)	Effective diffusion coefficient D^* (m^2/ s)	Longitudinal Dispersivity α_L (m)	Longitudinal dispersion coefficient D_a (m^2/ s)	Distribution coefficient K_d (mL/ g)	Retardation factor R_f	Peclet number P
1	1	1.1, 1	0.1	5×10^{-9}	5×10^{-10}	0	5×10^{-10}	0	1	10
2	0.2	0.4, 0.2	1	5×10^{-8}	5×10^{-10}	0.01	1×10^{-9}	0	1	10
3	4	4.8, 4	0.2	1×10^{-8}	8×10^{-10}	0.32	4×10^{-9}	0	1	10
4	1	1.1, 1	0.1	5×10^{-9}	5×10^{-10}	0	5×10^{-10}	0.2	1.81	10
5	0.2	0.4, 0.2	1	5×10^{-8}	5×10^{-10}	0.01	1×10^{-9}	0.2	1.81	10
6	4	4.8, 4	2	1×10^{-8}	8×10^{-10}	0.32	4×10^{-9}	0.2	1.81	10

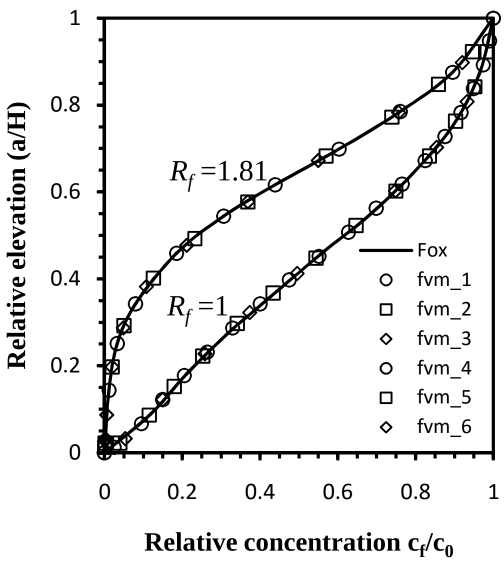


Fig.4.7 Uniqueness of FVM results for 1-d solute transport in rigid porous media

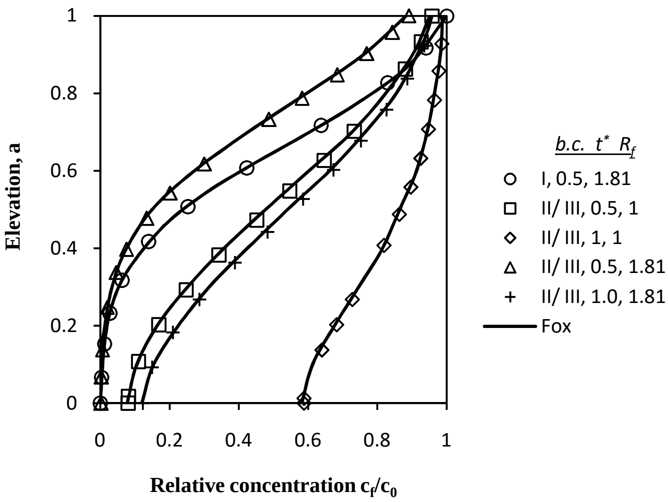


Fig.4.8 Profile of solute concentration for 1-d transport with b.c. type II and type III

Eqs (4.24) and (4.25) are implemented in this computational model with the following approximation of concentration gradient near boundaries (Fig.4.1), where J=1 and e=$e_0$for rigid porous media as given by Eqs (4.26) and (4.27).

$$\frac{\partial c}{\partial a} = \frac{2\left(c_{m-1} - c_{(m-1)+\frac{1}{2}}\right)}{\Delta a}\left(\frac{1+e_0}{1+e}\right)_{i=m-1} \tag{4.26}$$

$$\frac{\partial c}{\partial a} = \frac{c_{1-\frac{1}{2}} - c_1}{\Delta a}\left(\frac{1+e_0}{1+e}\right)_{i=1} = 0 \Rightarrow \left(c_1 = c_{1-\frac{1}{2}}\right)_{i=1} \qquad (4.27)$$

where, c is the solute concentration in fluids at the locations indicated by suffices. Fig.4.8 shows the finite volume results along with the results of Fox (2007) and the close agreement is obvious.

4.5 CONCLUSION

The chapter presents a computational model for one dimensional solute transport in a rigid porous media using the framework of deforming porous media (the consolidating soil) and the descriptions lead to following conclusions.

1. The finite volume computational model is capable to accommodate required boundary conditions, sorption isotherms and decay reactions.
2. The number of node points or elements required for acceptable solutions are relatively quite less, thus resulting in less computational effort.
3. The high Peclet number flows also give quite good results which means the model works well with advection dominated flows also.
4. Highly advection dominated flow with very high Peclet Number ($v_f H/D^*$ or D_a) may follow the plug flow assumption and the proposed finite volume numerical model may not capture it so accurately. However, the solute transport during flow through porous media is seldom advection dominated rather in soils the Reynolds number is less than unity (Kumar and Singh, 1995) and thus flow velocity is always very low. The Peclet Number in such flows is practically supposed to be rather low due to low velocity and significant diffusion during solute transport through clay medium.

Chapter 5

FINITE VOLUME MODEL OF TWO DIMENSIONAL SOLUTE TRANSPORT EQUATIONS

5.1 INTRODUCTION

The hydrodynamic dispersion (mechanical dispersion and molecular diffusion) is the key parameter in solute transport through porous media. When the advective solute transport is not negligibly small consideration of mechanical dispersion becomes inevitable. For a line source of contaminant, the solute transport is modelled with one dimensional advection and two-dimensional hydrodynamic dispersion: longitudinal and transverse. Longitudinal dispersion is the solute movement due to hydrodynamic action in the direction of flow in addition to advection. The mass flux (f_a) in the longitudinal direction due to hydrodynamic dispersion through a porous media and the longitudinal coefficient of hydrodynamic dispersion (D_a) are modelled by the following equations.

$$f_{fa} = nD_a \frac{\partial c_f}{\partial a} \tag{5.1}$$

$$D_a = D^* + \alpha_a v_f \tag{5.2}$$

Where, n = porosity of the porous media; c_f = solute concentration in fluid; a = the vertical direction coordinate as shown in Fig.2.3; D^* = τD_0 effective diffusion coefficient; τ = tortuosity factor (Shackelford and Daniel, 1991); D_0 = free solution diffusion coefficient; α_a = longitudinal dispersivity; v_f = pore fluid velocity or seepage velocity.

Similarly the transverse mass flux (f_x) and transverse hydrodynamic dispersion coefficient (D_x) are modelled by the equations below.

$$f_{fx} = nD_x \frac{\partial c_f}{\partial x} \tag{5.3}$$

$$D_x = D^* + \alpha_x v_f \tag{5.4}$$

Where, α_x = transverse dispersivity; x = the transverse direction coordinate and other terms remains same as described with longitudinal dispersion.

This chapter follows the two dimensional hydrodynamic dispersion solute transport with unidirectional advection as discussed above and includes the details of its finite volume formulation, derivation of two dimensional quadratic interpolation function and verification of the computational model. The basic framework of the model is the deformable porous media, but the discussion here is limited to rigid porous media as a special case of it with no deformation.

5.2 GOVERNING EQUATIONS

Mass conservation in the solid phase of a porous media in Lagrangian coordinates may be given as follows (Peters and Smith, 2002 Eq. 40)

$$\frac{\partial}{\partial t}\{(1-n)c_s J\} - S_m = -\frac{\partial f_s}{\partial a} = 0 \tag{5.5}$$

where, n= porosity; t= time; c_s = solute concentration in solids (mass/ volume); $J=d\xi/da$ (given by Eq. 2.7); S_m=rate of solute mass sink per unit volume in the solid phase; and f_s = solute flux in solid media. Here, the solute flux gradient is zero as in Lagrangian coordinate system the solids are assumed to stay in their control volume and there is no mixing of soils of different locations. Similarly, the mass conservation in fluid phase, due to two-dimensional hydrodynamic dispersion, will be given by the following equation

$$\frac{\partial}{\partial t}\left\{n c_f J\right\} + S_m = -\frac{\partial f_{fa}}{\partial a} - \frac{\partial f_{fx}}{\partial x} \tag{5.6}$$

where, f_{fa}= solute mass flux in fluid phase in the longitudinal a-direction; f_{fx}= solute mass flux in the transverse x-direction; S_m= rate of solute mass source per unit volume in the fluid phase and equal to the rate of solute mass sink in the solid phase. By definition, one dimensional advective mass flux along with the associated hydrodynamic longitudinal and transverse solute mass flux in fluids are expressed as

$$f_{fa} = qc_f - nD_a \frac{\partial c_f}{\partial a} \tag{5.7}$$

$$f_{fx} = -nD_x \frac{\partial c_f}{\partial x} \tag{5.8}$$

where, q= Darcy velocity through porous media. Now combining the Eqs 5.5 to 5.8, two-dimensional governing equation of solute transport in porous media may be written as;

$$\frac{\partial}{\partial t}\left\{nc_f J + (1-n)c_s J\right\} = -\frac{\partial}{\partial a}\left\{qc_f - \frac{nD_a}{J}\frac{\partial c_f}{\partial a}\right\} + \frac{\partial}{\partial x}\left\{nD_x \frac{\partial c_f}{\partial x}\right\} \tag{5.9}$$

The above governing equation can be cast as the finite volume explicit numerical scheme and described next.

5.3 FINITE VOLUME MODEL DEVELOPMENT

Let the combined concentration in a control volume be C_c as defined by the Eq3.2, the above Eq45.9 is written;

$$\frac{\partial C_{cm}}{\partial t} = -\frac{\partial}{\partial a}\left\{qc_f - \frac{nD_a}{J}\frac{\partial c_f}{\partial a}\right\} + \frac{\partial}{\partial x}\left\{nD_x \frac{\partial c_f}{\partial x}\right\} \tag{5.10}$$

Integrating Eq4.10 over a control volume of elementary size Δa×Δx (Fig4.1) and time, it is written as;

$$\oint_{CV}\left(\int_t^{t+\Delta t}\frac{\partial C_{cm}}{\partial t}dt\right)dV = \int_t^{t+\Delta t}\left(\oint_{CV} -\frac{\partial}{\partial a}\left\{qc_f - \frac{nD_a}{J}\frac{\partial c_f}{\partial a}\right\}dV\right)dt + \int_t^{t+\Delta t}\left(\oint_{CV}\frac{\partial}{\partial x}\left\{nD_x\frac{\partial c_f}{\partial x}\right\}dV\right)dt \tag{5.11}$$

Now, using Gauss-divergence theorem for control volume integration and using the definition by Eq2.13 for explicit time integral of RHS, the Eq5.11 may be written as;

$$\begin{aligned}\left(C_{cm}^{t+\Delta t}\right)_{i,j} = \left(C_{cm}^{t}\right)_{i,j} &- \frac{\Delta t}{\Delta a}\left\{\left(qc_f - \frac{nD_a}{J}\frac{\partial c_f}{\partial a}\right)^t_{i,j+\frac{1}{2}} - \left(qc_f - \frac{nD_a}{J}\frac{\partial c_f}{\partial a}\right)^t_{i,j-\frac{1}{2}}\right\} \\ &+ \frac{\Delta t}{\Delta x}\left\{\left(nD_x\frac{\partial c_f}{\partial x}\right)^t_{i+\frac{1}{2},j} - \left(nD_x\frac{\partial c_f}{\partial x}\right)^t_{i-\frac{1}{2},j}\right\}\end{aligned} \tag{5.12}$$

Eq5.12 is the finite volume numerical scheme and can evaluate the composite concentration at next time step with the known values at previous time step. Thus a problem of solute transport can be solved numerically if initial and four boundary conditions of solute concentration are known. The composite concentration can be segregated into solute concentration in fluid and solid phases for a given sorption isotherm as explained earlier in Chapter- 4.

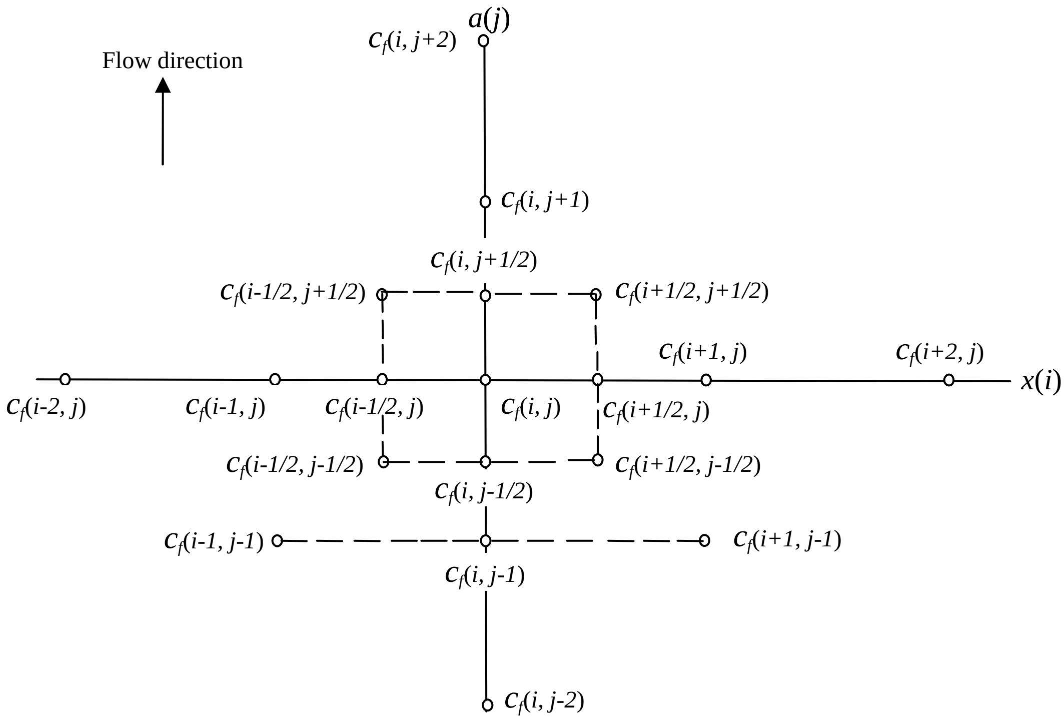


Fig.5.1Two-dimensional control volume of size Δa×Δx

The further requirement of the solution is to assess the face values of the solute concentration on the four faces of the elementary control volume. The face values are approximated by interpolating with upstream bias and the two-dimensional interpolation function is derived next for the purpose.

5.3.1 Two-dimensional interpolation function

The variable solute concentrations on the bottom face [$c_f\,(i$-1/2, j-1/2), $c_f\,(i, j$-1/2), $c_f\,(i$+1/2, j-1/2)] with upstream bias can be approximated with Taylor series about the solute concentration point $c_f\,(i, j$-1), just below it and taking the average of these values. Now, using Taylor's series expansion,

$$\begin{aligned}
& c_f(i-\frac{1}{2}, j-\frac{1}{2}) \\
&= c_f(i-\frac{\Delta x}{2}, j-1+\frac{\Delta a}{2}) \\
&= c_f(i, j-1) - \frac{\partial c_f(i, j-1)}{\partial x}\frac{\Delta x}{2} + \frac{\partial c_f(i, j-1)}{\partial a}\frac{\Delta a}{2} \\
&\quad + \frac{1}{2}\left[\frac{\partial^2 c_f(i, j-1)}{\partial x^2}\frac{(\Delta x)^2}{4} - \frac{\partial^2 c_f(i, j-1)}{\partial a \partial x}\frac{(\Delta a \Delta x)}{4} + \frac{\partial^2 c_f(i, j-1)}{\partial a^2}\frac{(\Delta a)^2}{4}\right] + O(x^3)
\end{aligned} \tag{5.13}$$

Similarly approximating the other points on the bottom face $c_f(i, j\text{-}1/2)$, $c_f(i+1/2, j\text{-}1/2)$ and neglecting the higher order terms $O(x^3)$, the average bottom face value of solute concentration (c_{fb})can be approximated by averaging the values using the following equation by Simpson's rule.

$$c_f(i, j-\frac{1}{2}) = \frac{c_f(i-\frac{1}{2}, j-\frac{1}{2}) + 4c_f(i, j-\frac{1}{2}) + c_f(i+\frac{1}{2}, j-\frac{1}{2})}{6} \tag{5.14}$$

These equations give the average solute concentration at the bottom face as follows.

$$\begin{aligned} c_f(i, j-\frac{1}{2}) = c_f(i, j-1) + \frac{\partial c_f(i, J-1)}{\partial a}\frac{\Delta a}{2} + \frac{1}{8}\frac{\partial^2 c_f(i, j-1)}{\partial a^2}(\Delta a)^2 \\ + \frac{1}{24}\frac{\partial^2 c_f(i, j-1)}{\partial x^2}(\Delta x)^2 \end{aligned} \tag{5.15}$$

Now, using the central difference quotient Eq5.15 is approximated as;

$$\begin{aligned} c_f(i, j-\frac{1}{2}) = \frac{3}{8}c_f(i, j) + \frac{6}{8}c_f(i, j-1) - \frac{1}{8}c_f(i, j-2) \\ + \frac{1}{24}\left\{c_f(i-1, j-1) - 2c_f(i, j-1) + c_f(i+1, j-1)\right\} \end{aligned} \tag{5.16}$$

Eq5.16 is the quadratic interpolation function for evaluating the bottom face value of solute concentration. This interpolation function is same as presented by Leonard (1988). Similarly, the top, left and right face values of solute concentration of the control volume (i, j) can be interpolated with the following expressions.

$$\begin{aligned} c_f(i, j+\frac{1}{2}) = \frac{3}{8}c_f(i, j+1) + \frac{6}{8}c_f(i, j) - \frac{1}{8}c_f(i, j-1) \\ + \frac{1}{24}\left\{c_f(i-1, j) - 2c_f(i, j) + c_f(i+1, j)\right\} \end{aligned} \tag{5.17}$$

$$\begin{aligned} c_f(i-\frac{1}{2}, j) = \frac{3}{8}c_f(i, j) + \frac{6}{8}c_f(i-1, j) - \frac{1}{8}c_f(i-2, j) \\ + \frac{1}{24}\left\{c_f(i-1, j-1) - 2c_f(i-1, j) + c_f(i-1, j+1)\right\} \end{aligned} \tag{5.18}$$

$$\begin{aligned} c_f(i+\frac{1}{2}, j) = \frac{3}{8}c_f(i+1, j) + \frac{6}{8}c_f(i, j) - \frac{1}{8}c_f(i-1, j) \\ + \frac{1}{24}\left\{c_f(i, j-1) - 2c_f(i, j) + c_f(i, j+1)\right\} \end{aligned} \tag{5.19}$$

5.3.2 Solution procedure and other considerations

The solution procedure or the FORTRAN program of one-dimensional solute transport can be extended for two-dimensional case with the only change in interpolation of face value in which the upstream biased transverse curvature term is to be added. The normalised variable EULER-QUICK scheme also admits this small term addition smoothly (Leonard, 1988). Another point of attention is that the node numbering should be advanced in the direction of flow or written in the manner so that two upstream node points are included in the interpolation function similar to the one dimensional description. Sorption and decay reactions are treated in the same way for two dimensional cases as explained in Chapter 4 for one dimensional solute transport.

5.4 MODEL VERIFICATION

To verify the two-dimensional solute transport numerical model, the same problem geometry of rigid porous media as stated in Fig 4.1 is taken up with only alteration in initial distribution of contaminant, which now will be a limited line source and downward flow of fluid spreads the contaminant in two-dimensional space. The initially uncontaminated rigid porous mass has height of 1 m and width as 0.5 m. The flow through porous mass is created under unit hydraulic gradient. Other required properties of the flow through rigid porous system are: Effective diffusion coefficient $D^* = 5\times10^{-10}$ m^2/ s; Longitudinal dispersivity α_a = 0.1m; Transverse dispersivity α_x = 0.01m; Specific gravity of solids G_s = 2.7; Porosity n = 0.4; dry density ρ_d =1620 Kg/ m^3. Drained top and bottom boundaries maintain zero solute concentration throughout the time. Side boundaries follow no flow condition, so the solute mass flux and fluid mass flux both are zero. Initial condition of the contaminant in the rigid porous mass is introduced by injecting the contaminant with a uniform concentration c_0 into the fluid at location x = 0.2475 m; a = 0.5975 m. Bear (1972) mentioned the analytical solution to two-dimensional solute transport in an infinite medium under steady state of flow and also assumed the solute mass as a point source.

$$c(x,a,t) = \frac{M}{4\pi nt\sqrt{D_x D_a}} \exp\left(-\frac{(x-x_0)^2}{4D_x t} - \frac{(a_0 - a - v_f t)^2}{4D_a t} \right) \tag{5.20}$$

Where, M = mass of the injected solute at coordinates (x_0, a_0). Fox (2007) also presented solution to this problem with the numerical model (CST1) using separate Lagrangian coordinate framework for fluid and solid elements but associating the movement of a fluid element relative to a particular solid element. Further, the advective solute transport is evaluated with the plug flow concept and hydrodynamic dispersion of the solute follows finite difference approach at element level. The comparison of the analytical solution and the numerical solution by Fox (2007) with 20000 elements of size 0.005 m × 0.005 m, shows an excellent agreement. For the solution to this problem by the present finite volume numerical model, the rigid porous mass is modelled with 5000 elements of size 0.01 m × 0.01 m.

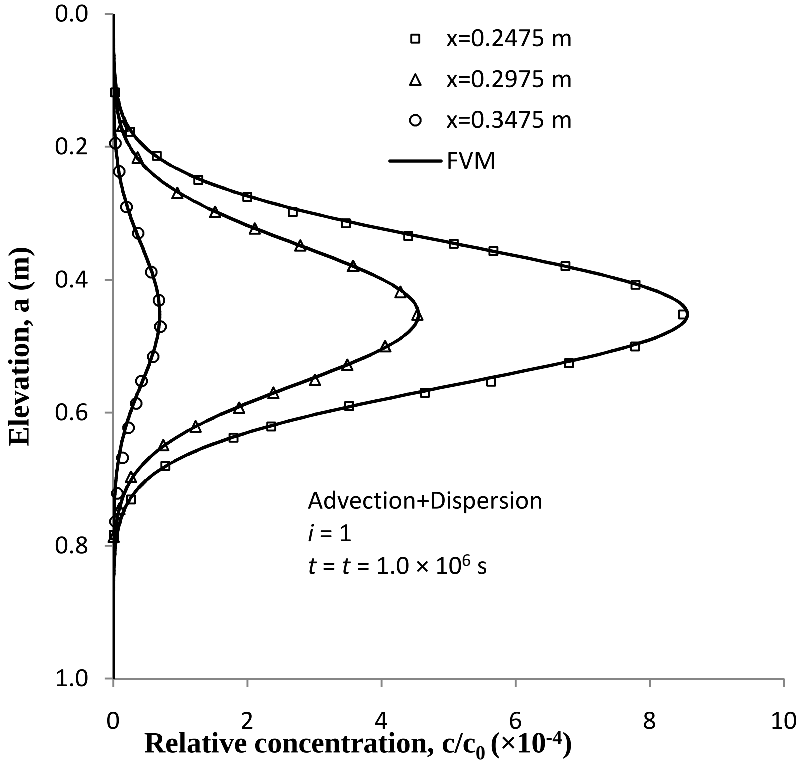


Fig. 5.2 Distribution of relative concentration with elevation

Fig 5.2 and Fig 5.3 shows the comparison FVM solutions and that of Fox (2007). Fig 5.2 shows the horizontal spread of contaminant through distribution of relative concentration of solute (c/c_0) with elevation (a) at horizontal locations x = 0.2475 m, 0.2975 m and 0.3475 m. Fig 5.3 shows the comparison of distribution of relative concentration of solute (c/c_0) with horizontal coordinates at vertical locations a = 0.5475 m, 0.4475 m and 0.3475 m. All these concentration distributions are obtained at time $t = 1.0 \times 10^6$ s.

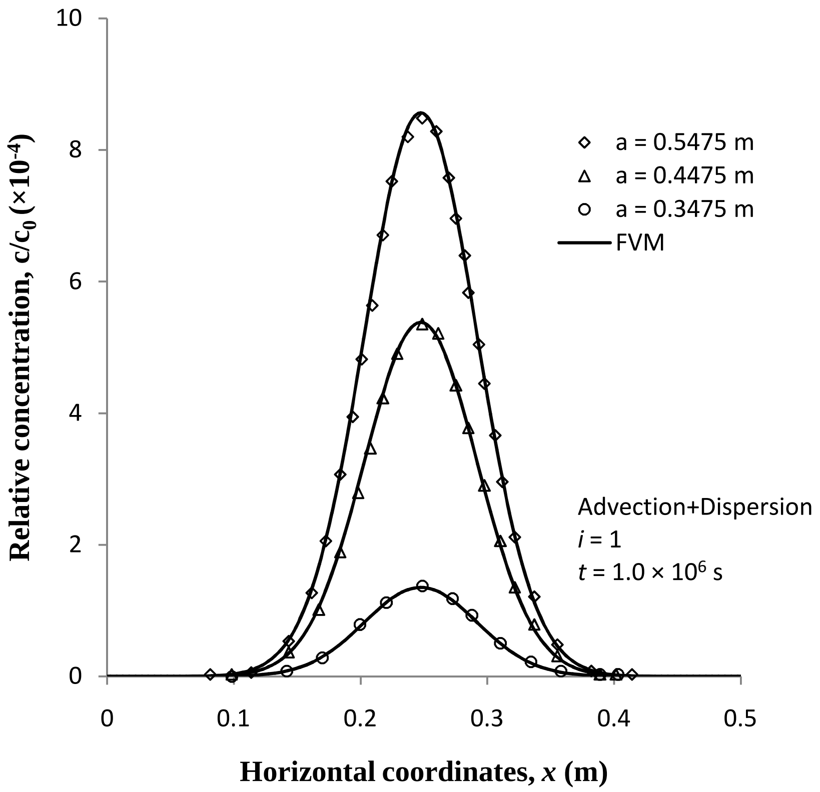


Fig. 5.3 Distribution of relative concentration with horizontal coordinates

The results are in close agreement with the advantage that the FVM numerical model requires relatively less number of elements for the solution for the same level of accuracy.

5.5 CONCLUSION

The focus of this chapter lies primarily on the development of two-dimensional interpolation function for FVM numerical model application. Further testing/ verification are also shown with more number of example problems of flow through deformable (consolidating) porous media in the next chapter. However, with the instant discussions, following conclusions can be drawn.

1. Implementation of two-dimensional interpolation is simpler as it contains only one additional transverse curvature term to the one dimensional interpolation function.
2. The order of accuracy is maintained to third order like the one dimensional interpolation.
3. Number of required elements is relatively less for obtaining an acceptable solution.

Chapter 6

FINITE VOLUME MODEL FOR CONSOLIDATION INDUCED SOLUTE TRANSPORT

6.1 INTRODUCTION

This chapter describes coupling of finite strain consolidation and the resulting advection solute transport along with the flow due to any hydraulic gradient across a saturated soil mass. The finite volume numerical model for finite strain consolidation is used to determine the Darcy fluid velocity due to consolidation and the same is added to flow by hydraulic gradient and the resulting velocity is finally used for one/ two dimensional solute transport in a synchronized manner at each time step. This may be termed as semi coupling as all the involved equations are programmed in separate modules and linked with each other through velocity term of consolidation module. The next few sections describe the entire procedure of the semi coupled numerical model for solute transport through deforming porous media keeping all other attributes such as boundary conditions, sorption, decay etc. to be same as described in earlier chapters about consolidation and solute transport. The comparisons of results of example problems by this model and others, validate this attempt well. Theoretical parametric study on two dimensional solute transports through deforming porous media is also included with sufficient variations in longitudinal and transverse dispersivity and effective diffusion coefficient. At the end, the chapter concludes the performance of the model.

6.2 COUPLING OF FINITE VOLUME MODEL OF CONSOLIDATION WITH ONE AND TWO DIMENSIONAL SOLUTE TRANSPORT

The solute transport computations in deforming (consolidating) soils require inclusion of solute transport module in the program of consolidation module where it computes the consolidation induced Darcy and seepage velocity in addition to the provisions of computation of Darcy/seepage velocity due to a hydraulic gradient. As introduced above, the consolidation induced velocity is added to the flow velocity under any existing hydraulic gradient and once the total advection value is known, the module of solute transport program is called next for computation of solute transport due to advection, dispersion with sorption and decay at that time increment for all elements. The next section details the mathematical procedure adopted for calculation of consolidation and hydraulic gradient induced velocities in soils with varying porosity and hydraulic conductivity along the depth.

6.2.1 Computation of stresses, Darcy velocity and effective hydraulic conductivity

Terzaghi's principle of effective stress defines;

$$\sigma' = \sigma - u_w \tag{6.1}$$

where, σ = total stress and u is the pore pressure which comprised of three components;

$$u_w = u_0 + u_h + u \tag{6.2}$$

where, u_0=hydrostatic pressure; u_h=pressure departure from the hydrostatic due to hydraulic gradient and u=excess pore pressure. The excess pore pressure and the pore pressure due to hydraulic gradient across the soil layer contribute to the Darcy velocity and cause flow of water in the saturated soil field. However, hydrostatic pressure balances the potential head only and does not play any role in the flow of water. Consolidation and hydraulic gradient induced Darcy velocities can be added to get the resultant velocity. Thus;

$$q = q_e + q_h \tag{6.3}$$

$$q_h = -\frac{k}{\gamma_w}\frac{\partial u_h}{\partial a}\frac{\partial a}{\partial \xi} = -\frac{k}{\gamma_w}\frac{\partial u_h}{\partial a}\frac{1+e_0}{1+e} \tag{6.4}$$

where, q=Darcy velocity; q_e=component of Darcy velocity due to excess pore pressure gradient; q_h =component of Darcy velocity due to hydraulic gradient across the soil layer. Hydraulic gradient across the soil layer causes uniform velocity in the soil field through the layers hydraulically connected in series.

The component q_e is determined by the kinematical considerations using Eqs. (2.14 and 2.53). This is advantageous as the pressure based consolidation induced velocity may not be much accurately consistent with the continuity/ conservation of fluid flow and may lead to some discrepancy in conservation of mass of the transporting solute. Integrating the Eq. (2.53) gives the value of Darcy velocity (q_e) due to the consolidation as;

$$q_e = -\int_0^h \frac{1}{1+e_0}\frac{\partial e}{\partial t}\,da \tag{6.5}$$

Where, the discrete point values of the integrand in the right hand side are known through the Eq. (2.14) as given below;

$$\frac{1}{1+e_0}\frac{e_i^{n+1}-e_i^n}{\Delta t}=-\frac{1}{\Delta a}\left[\begin{array}{l}\left\{\frac{k}{1+e}\left(\frac{\gamma_s}{\gamma_w}\text{-}1\right)+\frac{k(1+e_0)}{\gamma_w(1+e)}\left(\frac{\partial\sigma'}{\partial e}\frac{\partial e}{\partial a}\right)\right\}_{i+\frac{1}{2}}^{n}\\-\left\{\frac{k}{1+e}\left(\frac{\gamma_s}{\gamma_w}\text{-}1\right)+\frac{k(1+e_0)}{\gamma_w(1+e)}\left(\frac{\partial\sigma'}{\partial e}\frac{\partial e}{\partial a}\right)\right\}_{i-\frac{1}{2}}^{n}\end{array}\right] \tag{6.6}$$

Thus the Eq. (6.5) is numerically integrated to get the value of Darcy velocity (q_e). It may also be noted that the velocity within a control volume is assumed to be uniform.

The hydrostatic part of the pore pressure is defined as;

$$\frac{\partial u_0}{\partial a}=-\gamma_w\frac{\partial \xi}{\partial a}=-\gamma_w\frac{1+e}{1+e_0} \tag{6.7}$$

$$u_0=-\int_0^a \gamma_w\frac{1+e}{1+e_0}da \tag{6.8}$$

Total stress is the applied pressure plus the pressure exerted by the self load of the soil.

$$\sigma(\xi,t)=\sigma(\xi_T,t)+\int_0^a\frac{\gamma_s+e\gamma_w}{1+e_0}da \tag{6.9}$$

where, σ (ξ, t) = total stress at a location at any time in the consolidating soil; σ (ξ_T, t)=existing load applied at the top of the soil at any time. The solution of Eq. (2.14) yields the spatial distribution of void ratio at a given time; the corresponding effective stresses are interpolated among the input data of soil compressibility and the pore pressures are calculated using Eqs. (6.1 - 6.9). The derivation of Eq. (2.10) considers the hydrostatic pore pressure and excess pore pressure (Cargill, 1982) and the above calculated pore pressure includes these two components only and can be segregated as the hydrostatic pressure is estimated directly through Eq. (6.2). The pressure (u_h) due to hydraulic gradient is taken care separately with the general principles of hydraulic conductivity and the effective hydraulic conductivity for elements of differing values connected in series is estimated using the following equation;

$$k_e=\frac{H}{\sum_1^{m-1}\frac{T_j}{k_j}} \tag{6.10}$$

where, T_j =thickness; k_j= hydraulic conductivity of j^{th} CV; H=total height. The effective conductivity gives the Darcy velocity due to hydraulic gradient across the soil layers by Eq.

(6.4). With the above additions in the large strain consolidation module and then calling the solute transport module, the problems of solute transport in deforming porous media can also be worked out by the present numerical model with different boundary conditions and the next section contains the comparative study and evaluation of the model.

6.3 MODEL VERIFICATION

The computational model has been applied for solutions of solute transport through deformable porous media with decay reactions, linear equilibrium, nonlinear equilibrium and nonlinear nonequilibrium sorption isotherms. Variation of effective diffusion with porosity and cases with one-dimensional longitudinal dispersion and two-dimensional longitudinal and transverse dispersion are also presented.

6.3.1 Solute transport in 1-d compressible porous media

Lewis (2009) and Fox (2007) presented solution of a fictitious problem of a composite liner. The computational model of Lewis (2009) is based the Finite Element Method and that of Fox (2007, CST1) uses the piecewise linear method for the solution of finite strain consolidation equation and solute transport equations in a semi coupled manner. The problem statement is as follows. A single composite liner system composed of a Leachate Collection System (LCS), an impermeable geomembrane, saturated compacted clay liner (CCL) and a drainage layer as shown in Fig 6.1.

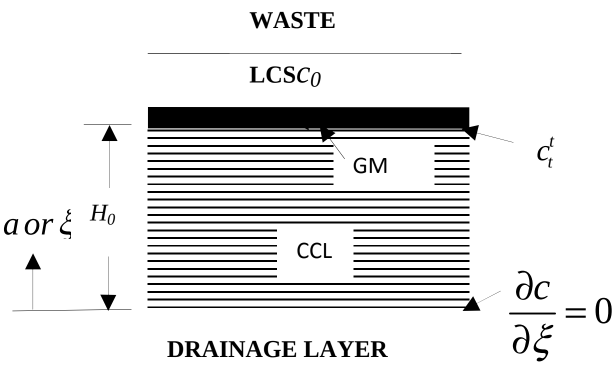


Fig. 6.1 Single composite liner system

The thickness of GM H_{GM}=0.0015 m; a volatile organic compound in the waste with concentration c_0 diffuses through GM and the corresponding diffusion coefficient D^*_{GM}=1×10^{-4} m^2/ y; the CCL is initially uncontaminated; initial thickness of CCL H_0=0.914 m; initial void ratio e_0=0.33; initial vertical hydraulic conductivity k_0=3.74 ×10^{-3} m/ y;

effective diffusion of volatile organic compound in water D^* =0.1 m^2/ y; the compressibility (σ'~e) is linear. Mechanical dispersion, sorption and self weight of the clay is neglected (α_a=0; K_d =0; G=1). The liner system was subjected to a load of 200 kPa/ y for two years and finally the surcharge load reaches to the level of Q=400 kPa.

Lewis (2009) and Fox (2007) presented the solution of the problem for constant values of strain-invariant coefficient of consolidation (c_F^*) defined as

$$c_F^* = \frac{1+e_0}{1+e} c_v = \frac{1+e_0}{1+e} \frac{k}{m_v \gamma_w} \tag{6.11}$$

where, c_v=coefficient of consolidation; m_v = coefficient of volume compressibility.

Hydraulic conductivity is assumed to hold the following relation with void ratio for constant value of c_F^* (Fox, 2007).

$$k_i^t = k_0 \frac{1+e_i^t}{1+e_0} \quad i = 1, 2, 3, ... \tag{6.12}$$

Solute concentration at the top boundary of CCL (c_t^t) is found with the condition of constant diffusion flux but no advection using Eq. (4.24). This gives the relation;

$$\left.\begin{aligned} c_t^t &= \frac{n_{m-1}^t H_{GM} D^* c_{m-1}^t + h D_{GM}^* c_0}{n_{m-1}^t D^* H_{GM} + h D_{GM}^*} \\ h &= \frac{\Delta a}{2} \frac{1+e_{m-1}^t}{1+e_0};\; n_{m-1}^t = \frac{e_{m-1}^t}{1+e_0} \end{aligned}\right\} \tag{6.13}$$

where, Δa = initial distance between the two consecutive nodes or length of a control volume; n_{m-1}^t = porosity of the uppermost control volume. The bottom boundary condition (type-II) is the no flux flow across the boundary as shown in the fig 6.1 and is implemented as $c_1^t = c_{1-\frac{1}{2}}^t$. The linear compressibility relation is expressed as;

$$e_i^t = e_0 - m_v (1+e_0) \sigma_i'^t \tag{6.14}$$

Simulations were performed with 20 elements, for three values of increasing compressibility m_v = (3.82×10^{-5}, 3.82×10^{-4} and 6.37×10^{-4} kPa^{-1}) and corresponding values of decreasing c_F^* = (10, 1 and 0.6 m^2/ y). Fig. 6.2 shows the vertical settlement of CCL with time along with the results Fox (2007) and Fig 6.3 shows the relative concentration of solute at the base (breakthrough curves). The settlements are almost identical whereas the breakthrough curves are in good agreement. It was also observed that the case of maximum compressibility (m_v=6.37×10^{-4}kPa^{-1}; c_F^*=0.6) the void ratio at the bottom becomes almost zero at t = 1.93 y and the program gets unstable and finally terminates.

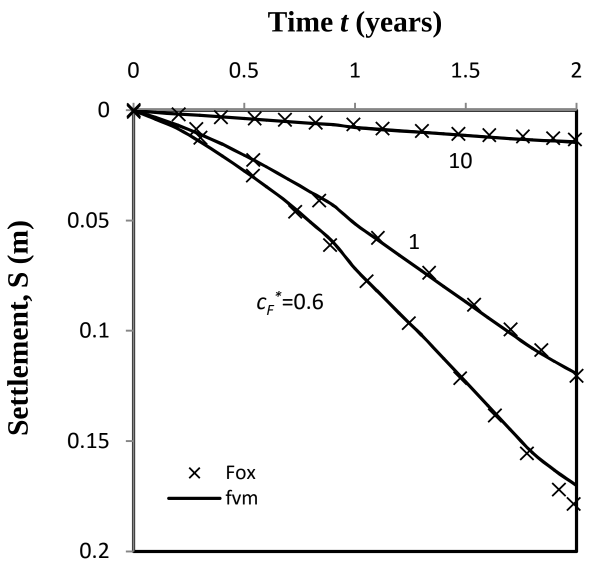


Fig 6.2 Settlement of CCL

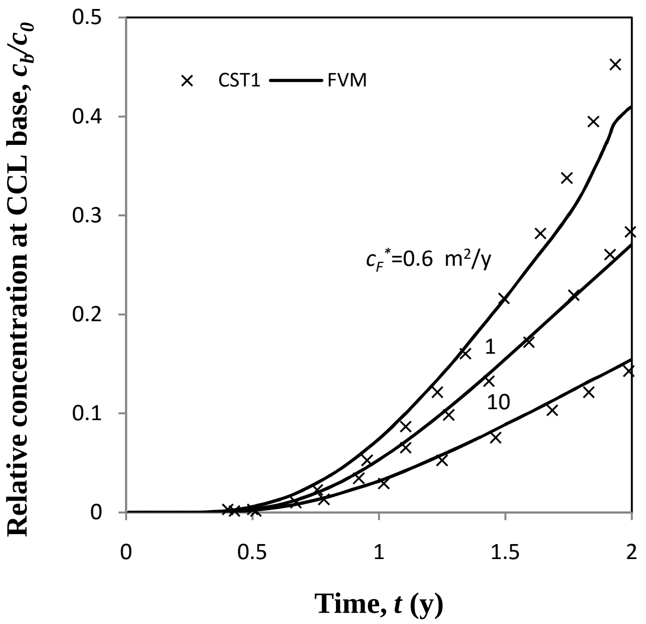


Fig. 6.3 Breakthrough curve for solute transport through CCL

A few more simulations of the same problem stated above are presented to show the influence of longitudinal dispersivity and equilibrium sorption as well as for the comparison of the present model results with that of Fox (2007). Fig 6.4 shows the results with the data c_F^*=1.0 m^2/ y; m_v = 3.82×10^{-4}; no sorption (K_d = 0.0); longitudinal dispersivity α_a= 0.0, 0.1, 0.2 and 0.5 with no consolidation where advective transport is zero and solute mobility is only due to diffusion. It is obvious that the consolidation has considerable influence on solute transport whereas dispersivity has limited impact. The results of present model and that of Fox (2007) show good agreement with each other.

The next comparison Fig 6.5 of the results is regarding the performance of the model with sorption. The simulation shows the effect of equilibrium sorption (K_d = 0.2 mL/ g) on solute transport with and without consolidation. The breakthrough time with sorption is increased as the solute concentration at the base reduces. The results show a close match.

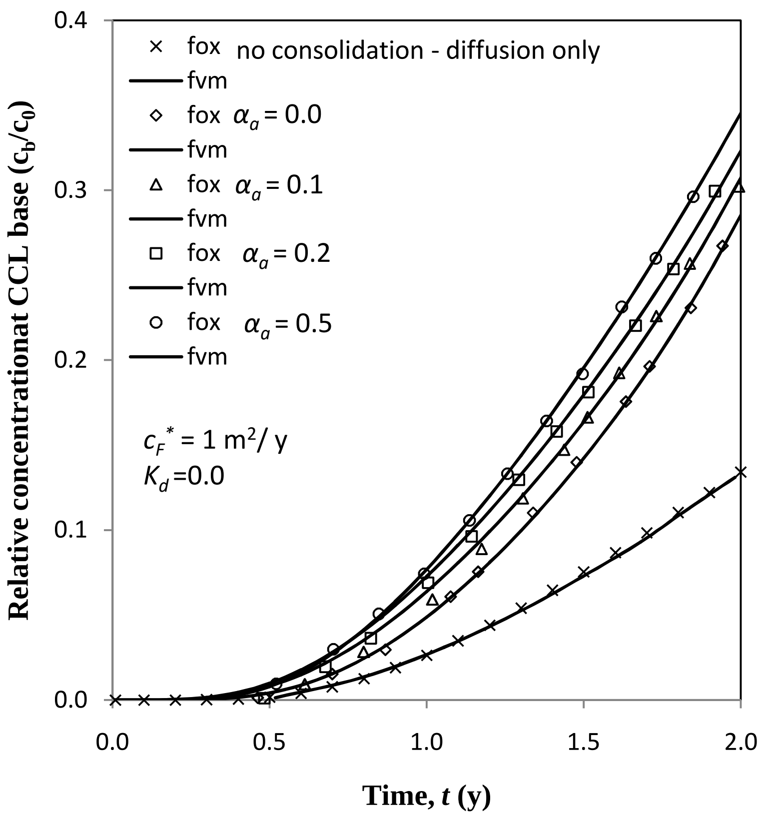


Fig. 6.4 Influence of longitudinal dispersivity and consolidation on solute transport

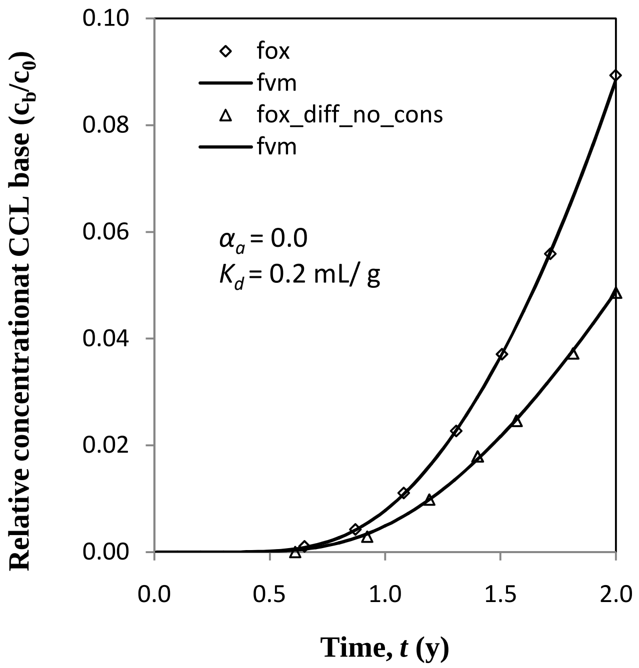


Fig. 6.5 Influence of sorption and consolidation on solute transport

6.3.2 Consolidation induced solute transport through kaolinite slurry

An experimental study for consolidation induced solute transport was conducted on contaminated and uncontaminated specimens of kaolinite slurry (Fox, 2009).

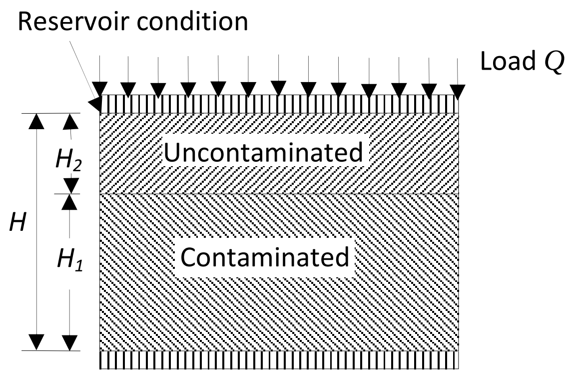


Fig. 6.6 Arrangement of contaminated and uncontaminated soil specimen

Fig 6.6 shows the required descriptions only; however for other experimental details in the cited reference may be seen. The arrangement shown is kept in a special mould with other allied arrangements which provides zero flux condition (for solute mass and pore fluid both) at the bottom and the top boundary condition is the constant flux condition due to formation of a reservoir at the top by the fluid going out of the slurry specimen as the consolidation proceeds. The surcharge loadings are applied in steps as per the schedule

(Q=3.1, 5.6, 10.4, 20.1, 39.5 and 78.4 kPa). Each load is kept for three days. Initially, the height of the contaminated slurry H_1= 50.4 mm; the height of uncontaminated slurry H_2=19.4 and total height H=69.8 mm; void ratio of contaminated specimen=2.48; void ratio of uncontaminated specimen=2.45. Dilute solution of potassium bromide (KBr) was used as contaminant and initial concentration the solute potassium K^+ in the contaminated specimen =234 mg/ L and that of bromide Br^- = 1672 mg/ L uniform throughout.

The compressibility characteristic of the soil follows the relation as given in Eq. (6.15) followed by the relevant details below.

$$e = e_0 - C_c \log \frac{\sigma'}{\sigma'_0} \tag{6.15}$$

where, e_0=weighted average of void ratios uncontaminated and contaminated specimen as per the heights=2.47; C_c = 0.65 (Fox, 2009) and σ'_0=0.92.

The hydraulic conductivity characteristic of the soil used for the simulation follows the relation by Eq. (6.16).

$$e = 8.16 + 0.765 \log k \tag{6.16}$$

The data set based on above constitutive material properties were used as input values. Initial value of void ratio for the entire soil was taken as 2.47, the average void ratio of contaminated and uncontaminated slurry. Boundary condition for void ratio at the top was taken as drained and that at the bottom was taken as undrained. The boundary condition on solute concentration at the top was taken to follow the reservoir condition and it is implemented through the following equation.

$$\left.\begin{aligned} c_r^t &= \frac{v_{f_{m-1}}^t c_{m-1}^t h + D_{m-1}^{*t} c_{m-1}^t}{v_{f_{m-1}}^t h + D_{m-1}^{*t}} \\ h &= 0.5 \Delta a \frac{1 + e_{m-1}^t}{1 + e_0} \\ v_{f_{m-1}}^t &= \frac{q_{m-1}^t}{n_{m-1}^t} \end{aligned}\right\} \tag{6.17}$$

where, c_r^t = concentration of solute in the reservoir.

Effective diffusion (D^*) was assumed to vary with porosity (Fox, 2009) and expressed as;

$$D^* = D_0 (n)^M \tag{6.18}$$

where, D_0 = free diffusion coefficient of solution =20.8×10^{-10} m^2/ s for Br^- and 19.6×10^{-10} m^2/ s for K^+ (Shackelford and Daniel, 1991); M= a constant=1.82 for both solutes (Fox, 2009). Initial conditions of solutes' concentrations in the uncontaminated specimen are uniformly zero in the fluid and solid phases. Sorption isotherm followed was nonlinear and nonequilibrium Eq. (4.20) (Travis and Etnier 1981). The values of constants of Eq. (4.20) were taken as K_p=19.6 mL/gm; F=0.608 and λ=0.005/s. In the contaminated specimen, the reactive solute K^+ has an initial concentration of 234 mg/ L in the fluid medium uniformly. The specific gravity of the soil solids is taken as 2.62. It is assumed that initially the reactive solute was in equilibrium and the concentration of K^+ in the solids $c_s = \rho_s K_p c_f^F$ =407.86 mg/ L. Mechanical dispersion was neglected and dispersivity was taken as zero. The simulations were run with 200 nodes with the time step of 0.25 and for segregation of fluid and sorbed concentration of solutes the time step was further subdivided into 50 divisions.

Figs. 6.7 - 6.9 show the results of consolidation part, the vertical settlement, maximum excess pore pressure with time and the final void ratio along the elevation of the specimen. The comparison of present FVM computation with the CST2 (Fox, 2009) with 200 solid elements and 600 fluid elements and experimental results are also shown. The present simulation gives settlements a little higher and void ratio a little lower compared to CST2 and the measured values. The maximum excess pore pressure shows very close match. The little difference may be attributed to the input values of compressibility and hydraulic conductivity relations that may not be exactly same as used in CST2. However, the results are not out of pattern and are acceptable. Further, the reservoir concentration of K^+ and Br^-, their final distribution in fluids and the final sorbed concentration of K^+ in soil specimen are shown in Figs. 6.10 – 6.12. The comparison of FVM results with CST2 and measured values shows close agreement in Fig 6.8. Initial values of reservoir concentration up to time of six days are little more by the present simulation. Fig. 6.11 shows the final fluid spatial concentrations K^+ and Br^- and the good agreement is evident.

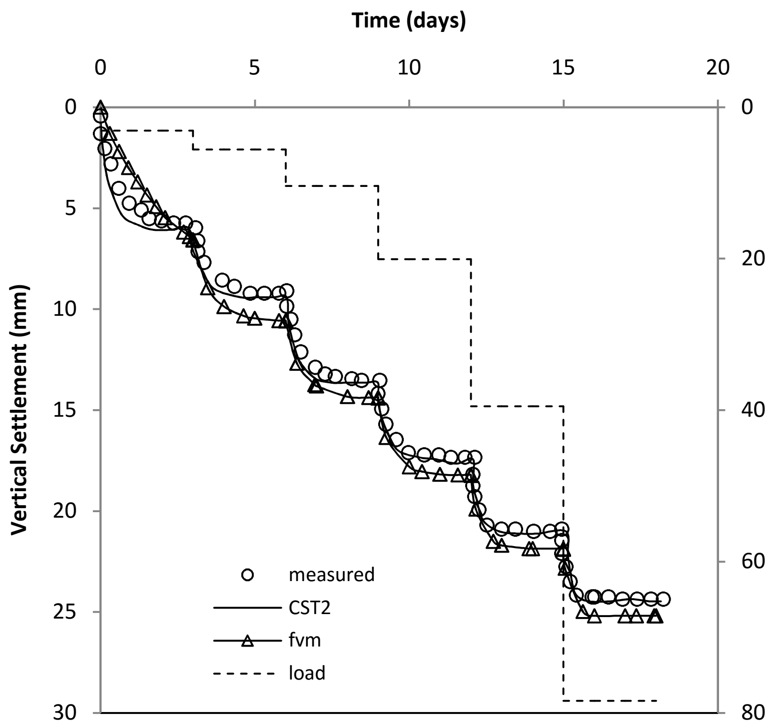


Fig 6.7 Vertical consolidation settlement

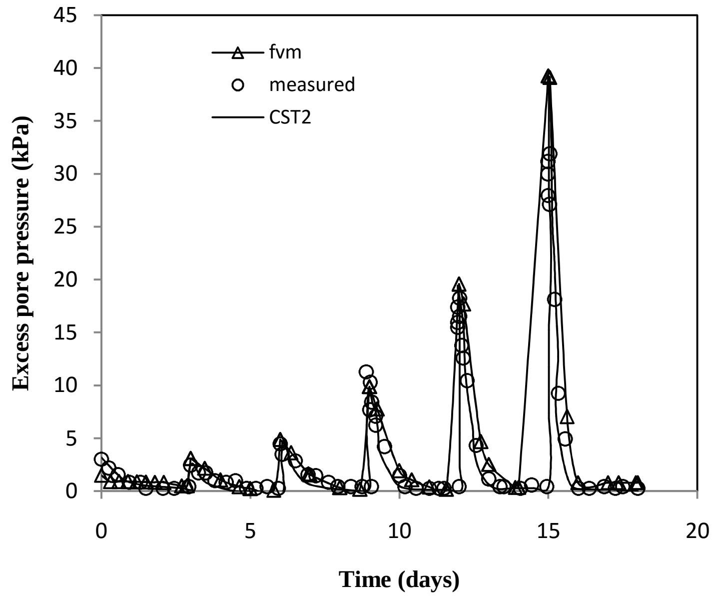


Fig. 6.8 Maximum excess pore pressure

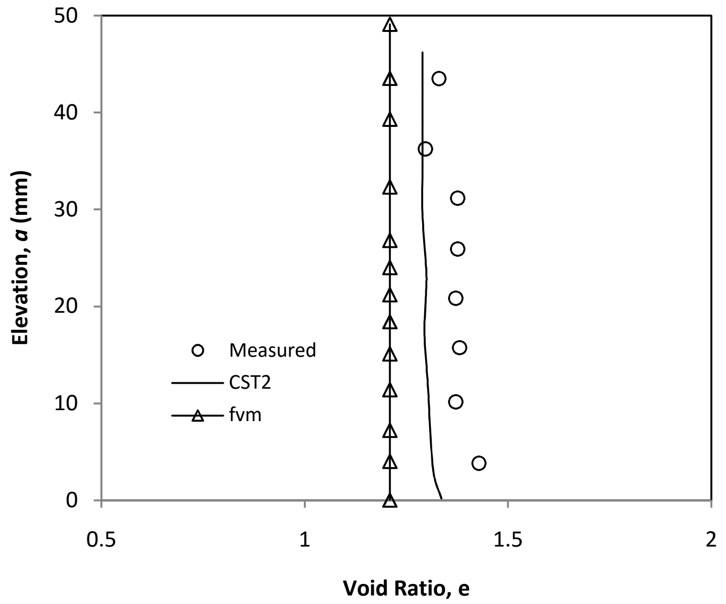


Fig. 6.9 Final void ratio

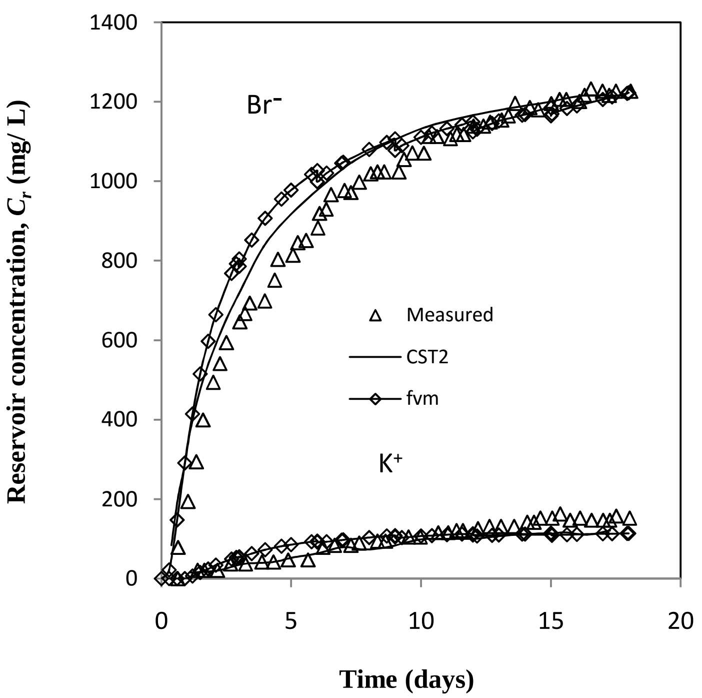


Fig. 6.10 Breakthrough curve of solutes K^+ and Br^-

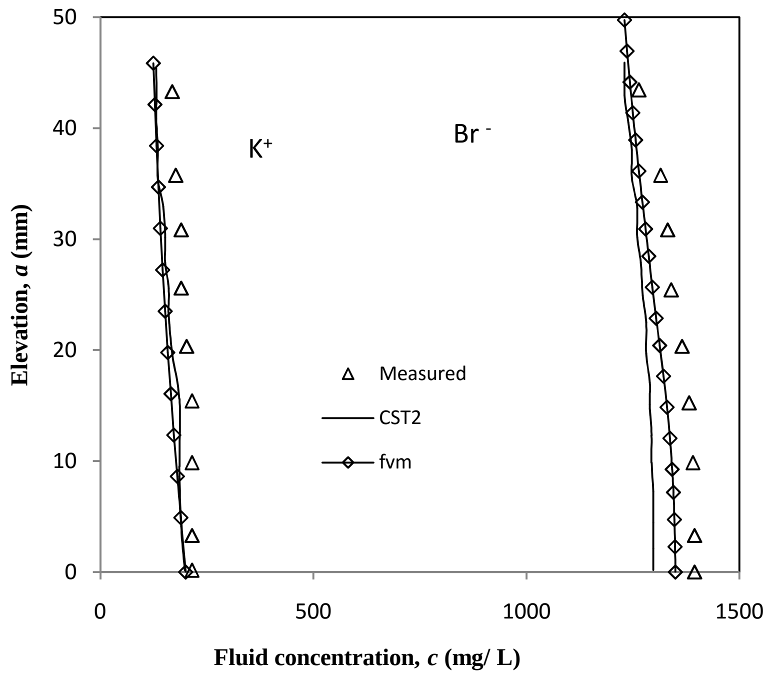


Fig. 6.11Final concentration of solutes in pore fluid

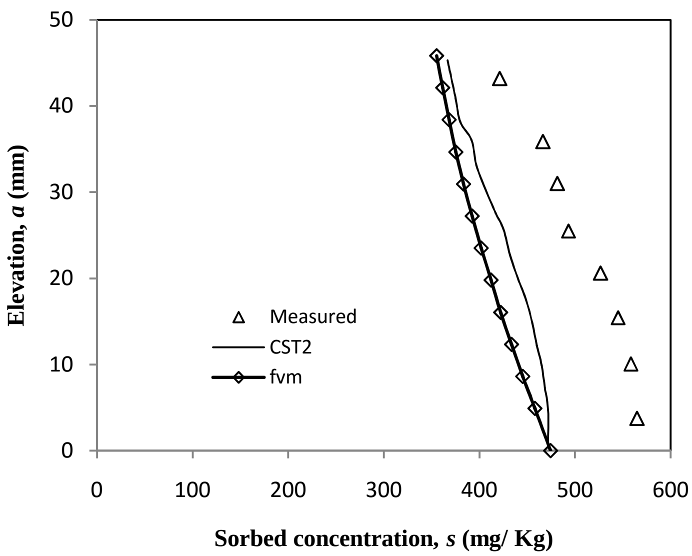


Fig. 6.12Final sorbed concentration in the soil specimen

The experimental results differ somewhat while comparing with the numerical results which may be due to the lower boundary condition. This boundary condition is achieved by saturating the lower layer with the pore fluid of same Br− and K+

concentrations through porous disk and water piping system which is difficult to maintain consistently and vulnerable to temporary lapses. Fig. 6.12 shows the final sorbed concentration of K^+. Experimental results are more but the match of both the numerical results are obvious.

6.3.3 Consolidation induced solute transport through organically modified soil bentonite mix

Earthen barriers of local soils mixed with bentonite and organoclays draws the attention of researchers. Shreedharan and Puvvadi (2013) present the study on various mixes of organoclay and bentonite slurry in water and other organic fluids and this reveals the substantial improvement in compressibility particularly with organic fluids. Further, the study by Shankara et al. (2014) reveals that the retention of Copper and Iron ions increases in sand, fly ash and bentonite mixture. Younus and Sreedeep (2012) mention that fly ash mixed with bentonite up to 70% by weight can give a satisfactory liner material and keep the hydraulic conductivity within the limit of 10^{-7} cm/ s.

Jhamnani and Singh (2009, 2009 and 2010) present the experimental and numerical analysis of potential CCL materials as admixture of organoclay (obtained from synthesis of coco-dimethyl benzyl ammonium chloride and bentonite), natural soil from Delhi, India and bentonite. Authors describe the experimentally obtained properties of five such samples (M1, M2, M3, M4 and M5) and their solute transport performance analysing by finite difference method ignoring the influence of consolidation. This section shows the solute transport analysis of one of the Mix M3 by the present model without consolidation and with consolidation. The properties (Jhamnani and Singh, 2009) of the liner material M3 are; the mix proportion; 15: 10: 75 (by weight of bentonite, organoclay and natural soil); OMC = 35.2; MDD = 1340 Kg/ m^3; Dry density (ρ_d) = 1158 Kg/ m^3; porosity (n) =0.47; hydraulic conductivity (k) = 4.7×10^{-8} cm/ sec. The initial and boundary conditions of concentration of solute in the soil field are given below.

Initial condition: $$c_f(a,0)=0 \tag{6.19}$$

Top boundary condition: $$c_f(0,t)=c_0 \text{ (constant)} \tag{6.20}$$

Bottom boundary condition: $$\frac{\partial c_f}{\partial a}(\infty,t)=0 \tag{6.21}$$

The problem with the above boundary condition is worked by finite difference formulation of the following ADS equation.

$$\frac{\partial c_f}{\partial t} = \frac{D_a}{R}\frac{\partial^2 c_f}{\partial a^2} - \frac{v_f}{R}\frac{\partial c_f}{\partial a} \tag{6.22}$$

where, D_a = 0.02 m^2/ s; mechanical dispersion is neglected; seepage flow (v_f) is very small and R is the retardation factor that has been derived and defined on the basis of Freundlich isotherm ($c_s = \rho_s K_p c_f^{\ F}$) as given below. For the derivation Shackelford (1988) is referred for an average value of K_p by the following expression.

$$K_{pav} = \frac{\int_0^c \frac{\partial s}{\partial c} dc}{\int_0^c dc} = \frac{s}{c} = K_p c^{F-1} \tag{6.23a}$$

$$R = 1 + \frac{\rho_d K_p c_f^{\ F-1}}{n} \tag{6.23b}$$

It may also be noted that $\rho_d = (1-n)\rho_s$, where ρ_s is the density of soil solids and as mentioned earlier ρ_d and n are bulk dry density and porosity of the soil matrix.

Here, the values of constants of Freundlich isotherm for the liner material sample M3 are taken as; K_p= 2.1× 10^{-13} and F = 3.89 with sorbed solute concentration ($s = c_s / \rho_s$) in soils has the unit as mg/ Kg and that in fluid medium is in mg/ L. Fig 6.13 shows the solution by Jhamnani and Singh (2009) that is the distribution of relative concentration of solute in pore fluids with depth up to the value of 1.2 m in the liner after expiry of 50 years.

This problem is solved by the present model without consolidation and with consolidation. The constitutive equations of the soil M3 is assumed to follow the compressibility Eq. (6.15) and hydraulic conductivity Eq. (6.16) with values of associated constants as C_c = 0.65; e_0= 0.92; σ_0' = 100 kPa; k_0 = 4.7×10^{-8} cm/ sec; specific gravity of soil solids G_s=1.158. Top boundary is assumed to be undrained and the bottom boundary as drained. This problem uses nonlinear-equilibrium sorption isotherm i. e. Freundlich isotherm and that introduces a nonlinear equation given below while segregating the known combined solute concentration (C_{cm}) into solute concentration in fluid and solid medium.

$$n c_f \frac{1+e}{1+e_0} + (1-n)\rho_s K_p c_f^F \frac{1+e}{1+e_0} - C_{cm} = 0 \tag{6.24}$$

Eq. (6.24) is solved for c_f using Newton-Raphson method and c_s is calculated to segregate combined concentration C_{cm} into c_f and c_s.

Two simulations were run, one for negligible load increment of 0.1 kPa and other one for an incremental load of 500 kPa at the beginning, next 500 kPa at 5^{th} year and last load increment of 500 kPa at 10th year resulting in cumulative load of 1500 kPa. Results of negligible consolidation and heavy consolidation up to total time of 50 years are shown in Fig 6.13.It is evident from the Fig 6.13 that the present model solution and the solution of Jhamnani and Singh (2009) are in close agreement without considering the consolidation. However, the impact of consolidation is considerable on solute transport that results in much higher concentration throughout the depth and this reduces the breakthrough time considerably.

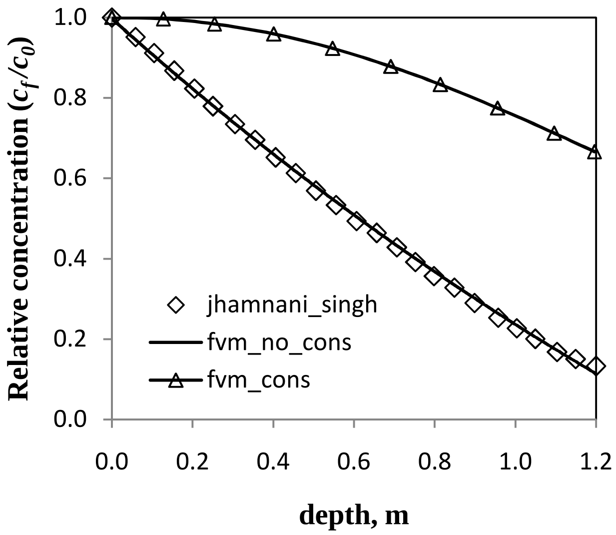


Fig. 6.13 Distribution of solute concentration in pore fluids with depth in liner

6.3.4 Two-dimensional solute transport in compressible porous media

Fig 6.14 shows the hydraulically dredged contaminated sediment impounded in a confined disposal facility (CDF). The bottom of the CDF is lined with an impermeable geomembrane and contains a leachate collection system (LCS) above it. The initial height (H_0) of the dredged slurry is 8.0m. The sediment contains a contaminant tetrachloroethene or perchloroethylene (PEC) within a block of 1m × 1m situated at the location of horizontal coordinates x= 4.5 m to x = 5.5 m and vertical coordinates a = 6.0m to a = 7.0m. The saturated sediment is placed in the CDF suddenly and starts consolidating under self load just after placement. Thus the isolated contaminant starts spreading vertically downwards and horizontally due to consolidation induced advection along with diffusion and makes a case of two-dimensional solute transport in deforming porous media. The material properties and constitutive equations (Fox, 2007) concerning consolidation are given as under.

Specific Gravity of soil solids G_s = 2.78; LL = 112; PL = 56; e_0 =4.34; Compressibility and hydraulic conductivity curves is governed by Eqs. (6.25 and 6.26)

$$\sigma' = \sigma'_0 (10)^{\frac{e_0 - e}{C_c}} \tag{6.25}$$

$$k = k_0 (10)^{\frac{e - e_0}{C_k}} \tag{6.26}$$

where, σ'_0 = 0.946 kPa; C_c = 1.02; k_0 = 2.04 × 10^{-8} m/ s and C_k =1.3.

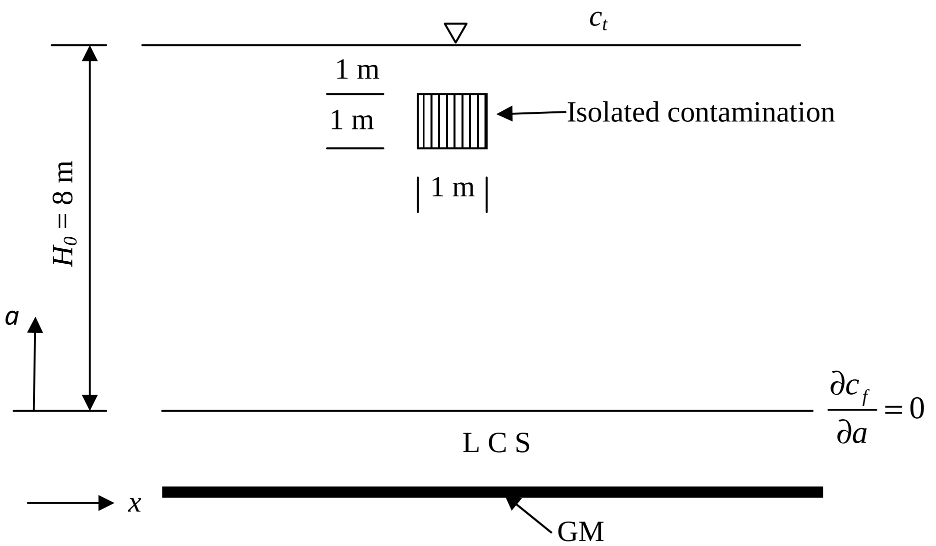


Fig. 6.14 Geometry of impounded slurry in CDF

The transport properties of the solute with respect to porous media are assumed: Diffusion coefficient D^*=6.0×10^{-10} m^2/ s; longitudinal dispersivity α_a = 0.2 m; transverse dispersivity α_x = 0.04 m, first order decay reaction constant λ_c = 0.0 and partition coefficient for linear equilibrium sorption isotherm ($s = K_d c_f$) K_d = 0.8 mL/ g. It is also assumed that height of the water level above ground remains constant (h_t= H_0) and keeps the sediment always saturated. Initial concentration of PCE in the block is c_{f0} = 100 mg/ L and the lateral boundaries are confined to x = 0.0 m to x = 10.0 m. The boundary conditions considered are the following.

(1) The top boundary is always taken as drained and solute concentration is taken as zero.

(2) The bottom boundary condition is taken in two ways i.e. drained in presence of LCS above impermeable geomembrane and undrained if only impermeable geomembrane exists. Both of these consolidation boundary condition lead to zero concentration gradient at the bottom boundary.

(3) One more case that assumes no consolidation (NC) takes the top boundary condition as zero concentration and bottom boundary condition as zero concentration gradient.

(4) The lateral boundaries are assumed as non transmitting boundaries (no mass flux flow across the boundaries) and to have zero concentration gradients.

(5) Initial condition of solute concentration distribution is taken as uniformly 100 mg/ L in the block and the rest of the sediment is contamination free.

Fox (2007), by his computational model CST1, gives the solution of the above problem for no consolidation (NC), singly drained (SD) and doubly drained (DD) conditions. Only the symmetric half portion is modelled with 50 horizontal elements, 120 vertical elements and 360 fluid elements. The present model works out the problem for entire geometry with 80 horizontal elements and 100 vertical elements. Fig 6.15 compares results of present model (FVM) and CST1. The profile of solute concentration in fluids (CPF) with maximum value and that in solids (CPS) on vertical plane at x = 4.95 m and time t = 5.4 years under doubly drained condition.

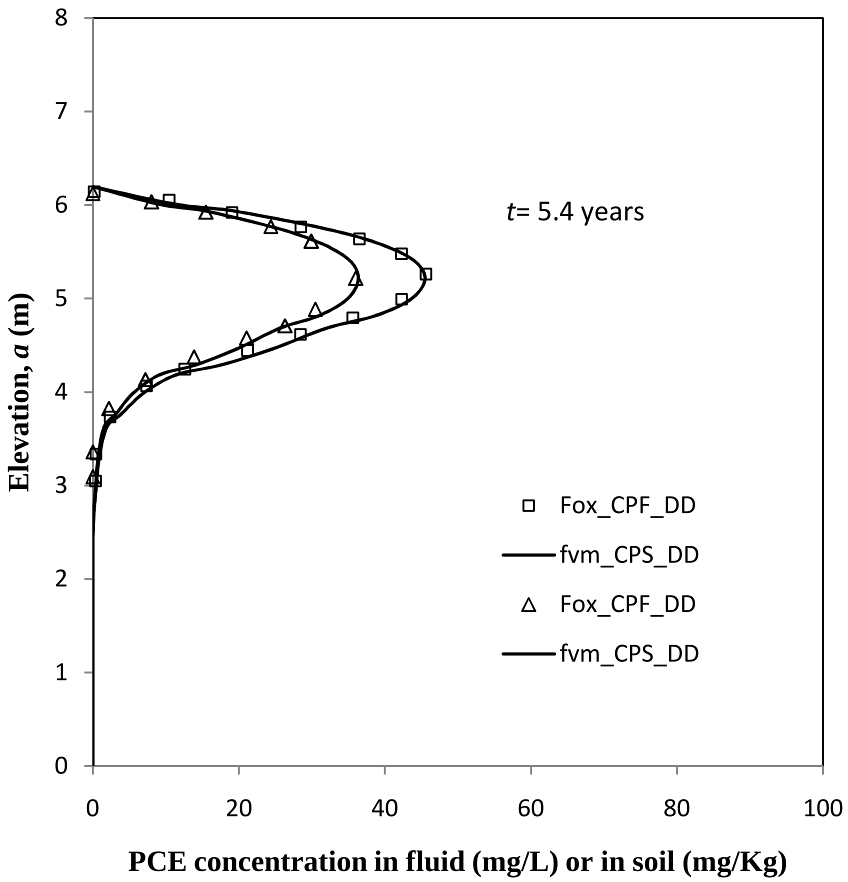


Fig. 6.15 PCE concentration profile in fluids (CPF) and solids (CPS) on a vertical plane (DD)

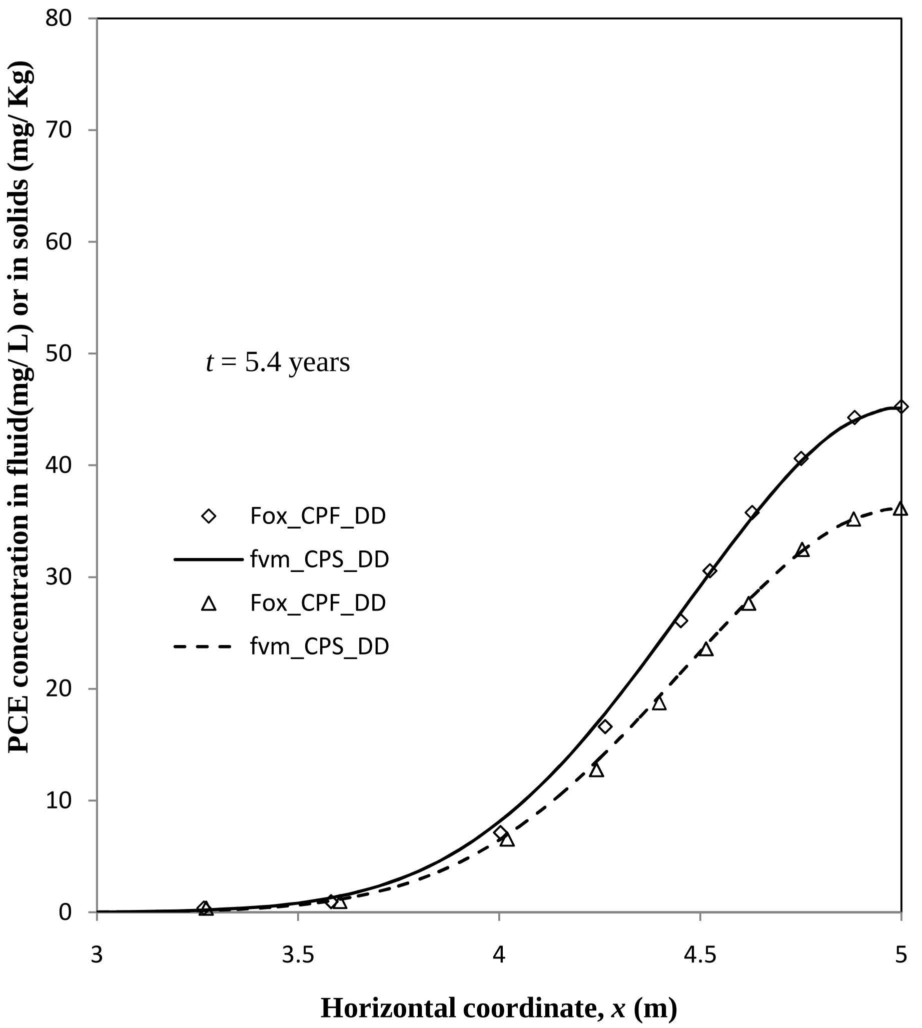


Fig. 6.16 PCE profile in fluids (CPF) and solids (CPS) on a horizontal plane (DD)

Fig 6.16 shows the comparison of PCE concentration profiles in fluid and solid medium on a horizontal plane at a = 5.23 m, time t = 5.4 years.

Fig 6.17 and 6.18 compares concentration profiles of the singly drained (SD) condition on vertical plane (x=4.95m) and horizontal plane (a=6.28m). Fig 6.19 and 6.20 shows the same for the case of no consolidation (NC) at vertical plane (x=4.95m) and horizontal plane (a=6.28m). All the figures show close agreement of the results. The concentration profile in doubly drained case on the vertical plane starts at about the height of 6.0 m and that in case of singly drained case starts at about 7.0. In case of no consolidation (NC) the concentration profile starts at 8.0 m i.e. the height of the sediment deposit at the beginning. This shows the settlement of the top boundary due to singly and doubly drained consolidation and the match of settlements of both the results are obviously evident.

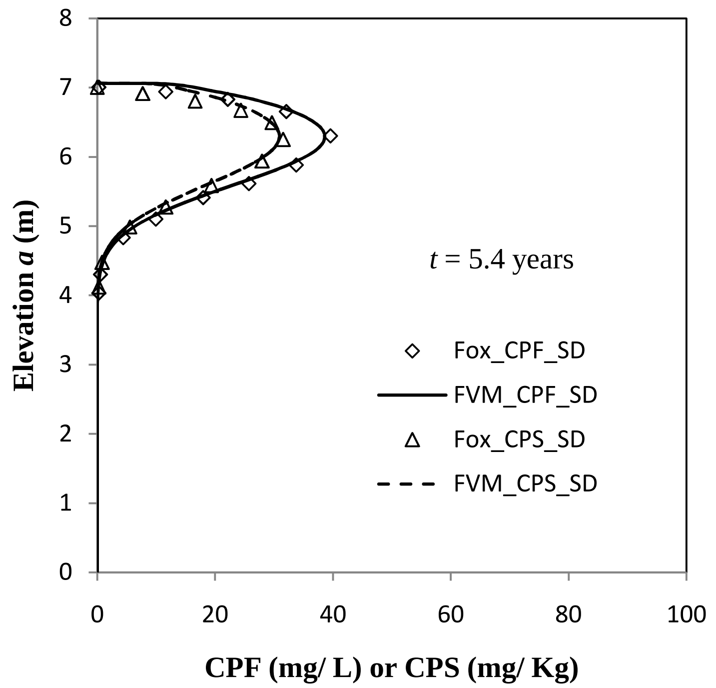


Fig. 6.17 PCE profile in fluids (CPF) and solids (CPS) on a vertical plane (SD)

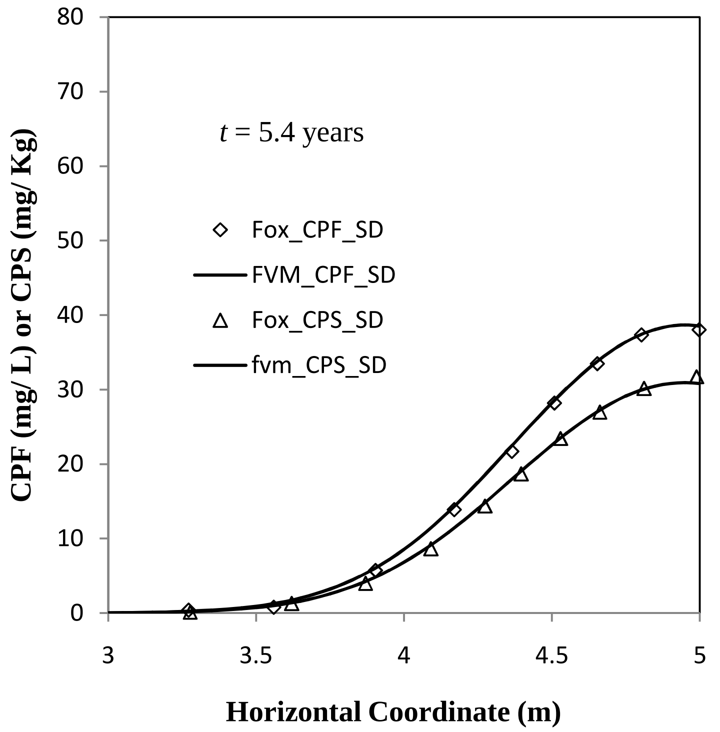


Fig. 6.18 PCE profile in fluids (CPF) and solids (CPS) on a horizontal plane (SD)

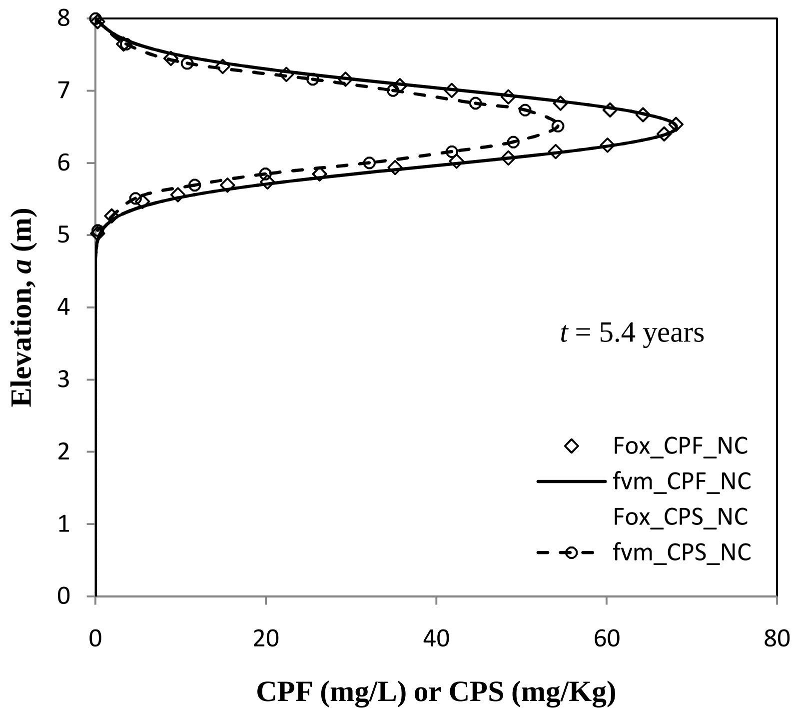


Fig. 6.19 PCE profile in fluids (CPF) and solids (CPS) on a vertical plane (NC)

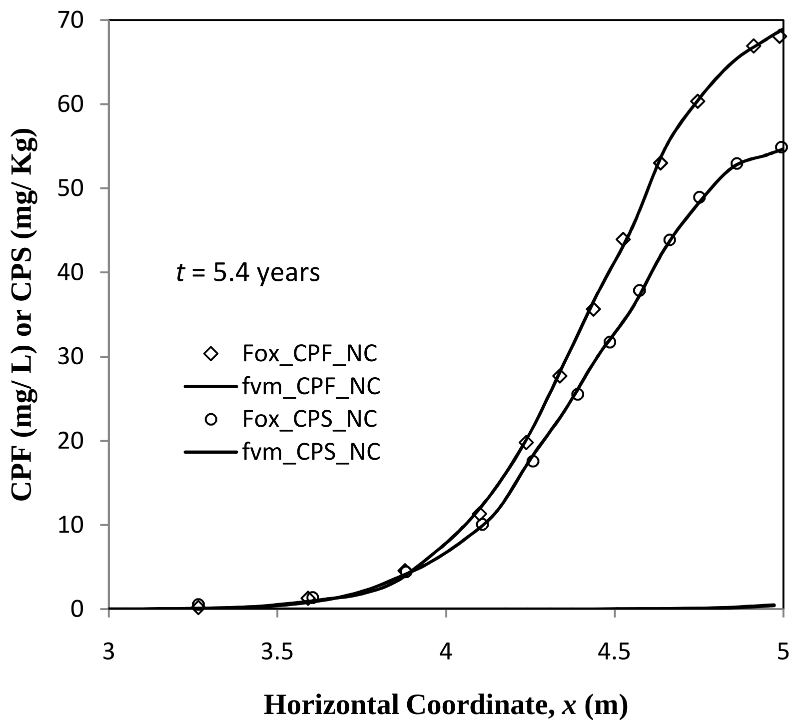


Fig. 6.20 PCE profile in fluids (CPF) and solids (CPS) on a horizontal plane (NC)

6.4 PARAMETRIC STUDY ON TWO DIMENSIONAL SOLUTE TRANSPORT

Two-dimensional solute transport as described in the present work depends on two parameters, the transverse dispersivity and effective diffusion coefficient. The influence of these two parameters, on the spreading of a contaminant in a two dimensional space with time, has been studied. For the purpose, the problem of section 6.3.4 fig. 6.14 is taken up again with singly drained (SD) consolidation under self weight up to time period t = 5.4 years.

6.4.1 Influence of longitudinal dispersivity

Delgado (2007) presents an extensive study on longitudinal and transverse hydrodynamic dispersion coefficients (D_a, D_x). These coefficients depend on molecular diffusion (D_0), tortuosity factor (τ), Peclet number ($P = v_f H / D_a$) and Schmidt number ($S_c = \mu/\rho D_a$). The paper mentions that the value of longitudinal dispersivity (α_a) ranges between 0.1 mm to 10.0 mm referring Freeze and Cherry, 1979 and it further reports that the ratio α_a/ α_x between 5:1 to 100:1 referring Bear and Verruijt, 1987. The present problem deals with water only as the pore fluid and single contaminant, so the change in dispersivity values depends on tortuosity and pore fluid velocity. The pore fluid velocity due to consolidation is always unsteady and non-uniform it is more near drained boundary and decreases with the location towards undrained boundary. Since problem taken is the consolidation of dredged sediment under self load with uniform initial void ratio as 4.34, it is assumed that the longitudinal dispersivity takes uniform values as 10.0 mm, 5.0 mm, 1.0 mm, 0.5 mm, 0.1 mm and corresponding to each the transverse dispersivity (α_x) values are taken as: α_a/5, α_a/25, α_a/50, α_a/75 and α_a/100 and the influence of α_a on two-dimensional solute transport is studied.

Twenty-five simulations were run but the results show that the influence of α_x on solute dispersion in the 2-d field is insignificant it decreases further with decrease in α_x. However, one result is shown here. Fig 6.21 depicts the distribution of solute concentration in pore water on a vertical plane at horizontal coordinate x = 4.975m.This plane have the point of highest concentration. The figure presents the results of α_a = 10.0 mm and α_x = 2.0 mm, 0.4 mm, 0.2 mm, 0.13 mm and 0.1 mm. The influence of this variation is too small to distinguish among the graphs. The numerical values of highest concentrations with decreasing α_T values are 58.9954 mg/L, 59.2997 mg/L, 59.3381 mg/L, 59.3516 mg/L and

59.3574 mg/L. Though this may be inferred that the lower the value of α_x lesser is the spreading of contaminant but it is almost insignificant.

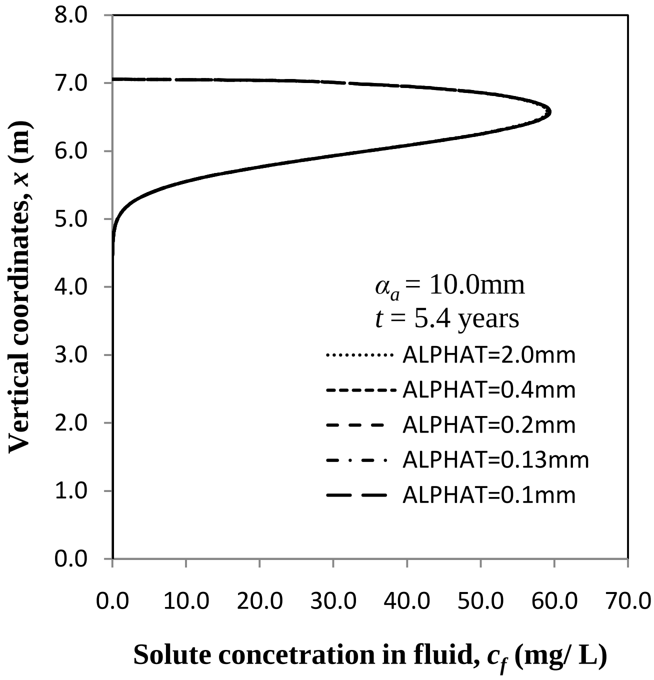


Fig. 6.21Effect of variation of transverse dispersivity on solute transport (vertical plane)

6.4.2 Influence of effective diffusion

Shackelford and Daniel (1991) present a list of molecular diffusion coefficient (D_0) values of many anions and cations that ranges from 5.95 × 10^{-10} to 93.1 × 10^{-10} m^2/ s. It is further expressed there that in soils the effective diffusion coefficient (D^*) becomes less than molecular diffusion coefficients as diffusion is hindered due to presence of soil solids. Effective diffusion can be determined experimentally for a given soil and depends on various parameters such as, surface activity of the soil particles, presence of interfering ions in the fluid, porosity, tortuosity etc. Ramkrishna et al. (2011) present the experimental determination of effective diffusion of Sodium and Sulphate ions in two different soils and correlated the values. Further, Sreedeep and Singh (2008) correlate the effective diffusion with electrical impedance of the soil and shows that the diffusion characteristics of fine grained soils are sensitive to variation of electrical impedance. However, the effective diffusion and molecular diffusion in this study is correlated as $D^*= (1/\tau)\ D_0$ where, τ is the tortuosity factor. Bear (1972) gave a simple empirical correlation of the tortuosity factor with porosity ($\tau = 1/\ n^{0.33}$).

To study of effect of diffusion coefficient on 2d-solute transport, the variations of molecular diffusion coefficient considered here are 6.0×10^{-10}, 1.2×10^{-9}, 2.4×10^{-9}, 6.0×10^{-9} and 9.6×10^{-9} m^2/ s. Other data of the problem of dredged sediment consolidation and initial contamination are kept as it is with dispersivities α_a= 10 mm and α_x= 2.0 mm. Five simulations were run with all these data that resulted into substantial effect of the effective diffusion coefficient on 2d-solute transport. Fig. 6.22 shows the solute concentration distribution on a vertical plane containing highest concentration points of the field. The location of this vertical plane is at horizontal coordinate *x* = 4.975 m. Fig 6.23 depicts the same on the horizontal plane of maximum concentration points. While the location of vertical plane remains same for all diffusion coefficients, the location of horizontal plane varies with increase in molecular diffusion and these locations successively are at vertical coordinates *a* = 6.59 m, 6.54 m, 6.40 m, 5.97 m and 5.69 m.

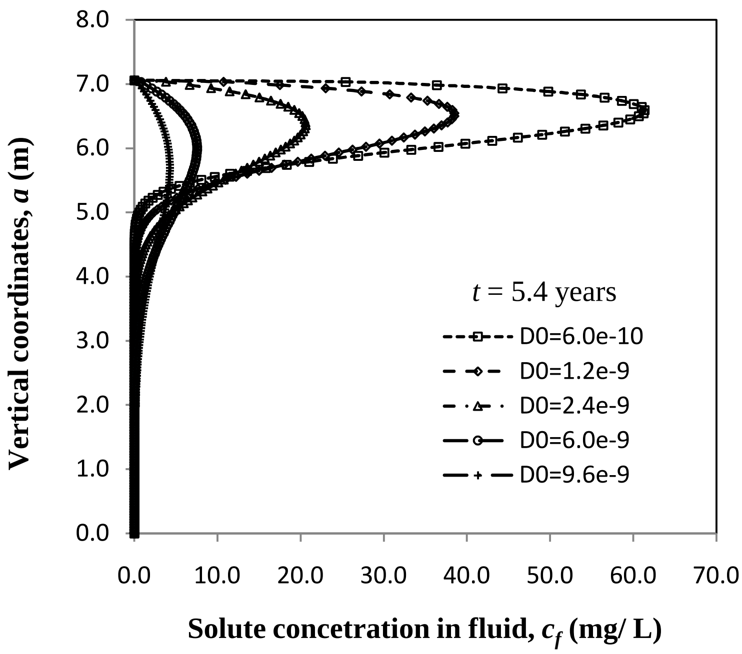


Fig. 6.22 Effect of variation of diffusion coefficient on solute transport (vertical plane)

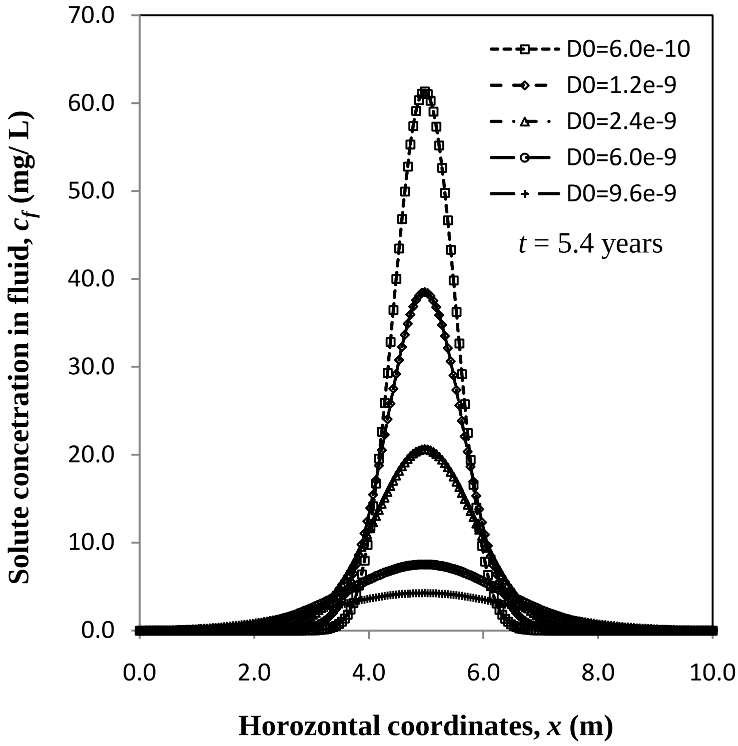


Fig. 6.23Effect of variation of diffusion coefficient on solute transport (horizon. plane)

The extensive effect of effective diffusion coefficient on the spreading of contaminant in a two dimensional field due to diffusion and consolidation is obvious from the figures and the diffusive capacity of a contaminant plays a very vital role in migration of contaminants in the soil field.

6.5 CONCLUSION

The chapter describes the coupling of finite volume numerical models of large strain consolidation and solute transport. The verification of the numerical model results with other recent numerical models' results shows close agreement. The inherent conservativeness of the finite volume method and use of quadratic interpolation function give this model little edge over other methods. Two dimensional extension of the model maintains its accuracy and reduces number of discrete elements required for acceptable solution. The parametric analysis of two dimensional solute transports reveals the importance of effective diffusion that may cause wide spreading of contaminant in two dimensional spaces in a given period of time.

Chapter 7

SUMMARY AND CONCLUSIONS

This thesis is primarily aimed to develop the finite volume numerical models of large strain consolidation, one and two dimensional solute transport through rigid porous media and coupling the models to give a numerical model for solute transport through deforming (consolidating) porous media. The work begins with literature review on finite/ large strain consolidation models, the realm of finite volume method and its applications' areas and lastly reviews the numerical models of solute transport in deforming porous media. The development of numerical models of the one dimensional consolidation, one and two dimensional solute transport and the coupled model with various verification checks successively follows the review work.

The introductory reviews and discussions lead to an inference that the finite volume method suits well for numerical analysis of the conservative equations inheriting the conservativeness property at discrete control volume level and thus in overall solution. It is also obvious that such a numerical model following completely the finite volume method is not available in literature as far as it could be explored here and this has motivated this preset work.

Chapter 2 starts with the introductory review on consolidation followed by preliminaries to the one dimensional consolidation equation, the assumptions, coordinate systems (Lagrangian, convective and material) and their transformations as applicable to the governing equation. The assumptions restrict the application of this model to homogeneous soil type with monotonic loading. The detailed description of numerical model development includes the nonlinear material behaviour of compressibility/ hydraulic conductivity and the geometrical nonlinearity of large/ finite strain consolidation equation. The genesis of the interpolation functions in terms of nodal points from Lagrangian quadratic interpolation functions is explained. The interpolation function in terms of nodal points has been used to approximate the face values of control volumes in terms void ratio and the corresponding approximation of nonlinear terms is done using three point Lagrangian interpolation functions directly to deal with the nonlinearity of the formulation. The boundary conditions encountered in consolidation problems drained/ undrained and semipermeable presented in terms of void ratio which is the independent variable of subject equation. As presented, the solution obtained in terms of void ratio at nodal points at a required point can be used to calculate pore water pressure, settlement, pore fluid velocity, velocity of solid particles and degree of consolidation to evaluate the full picture of the consolidation process. Assessment of the numerical model for stability and convergence is presented with conservativeness and boundedness that shows the acceptability of model with the present scheme and its suitability for all practical distribution of void ratio of a soil field. The accuracy of the model comes out to be of third order with quadratic interpolation and use of control volume face values in the process. Verification of the model is endorsed by close agreement of results with analytical and other numerical models. The proposed model maintains third order accuracy and hence it gives sufficiently accurate solutions with relatively lesser number of mesh points. At the end, the parametric analysis of consolidation of soft clays shows that the soft clays with initial void ratio 3.2 to 2.5 follow proportional relation between average degree of consolidation and square root of time elapsed up to value of 80%. It is further inferred that the lesser is the initial thickness of the soil layer faster is the consolidation.

Chapter 3 is about the experimental study on consolidation of the remoulded specimens of thickness 20 mm, 40 mm and 70 mm of a natural clay sample. The test results of 20 mm thickness provides the compressibility and hydraulic conductivity characteristics of the clay sample that has been used as input data to analyse the consolidation of remoulded soil specimens of other thicknesses by this numerical model. The specimens of 40 mm and 70 mm thickness were consolidated in special moulds and loading arrangement and the experimental results obtained has been used to validate the numerical model. The close agreement of numerical and experimental results endorses the numerical model.

Chapter 4 details the development of finite volume numerical model of solute transport through rigid porous media, it contains the transformation term of Lagrangian to convective coordinate system where its value is unity for a rigid system. The formulation initially developed for the combined concentration of solid and liquid phases, segregates the solution to concentrations of solute in liquid and solid phases through the sorption isotherm. The face values of the control volumes are interpolated with normalised variable scheme with upstream bias as given by Leonard (1988) as the scheme works well at a data set of concentration distribution with discontinuity or sharp gradient maintaining the accuracy of third order even though it uses linear interpolation rarely in non-monotonic regions. The time step restriction is governed by local grid Courant and Peclet Number and avoids the instability if local Courant number is less than 0.2. As mentioned earlier, the model incorporates the sorption isotherm while separating the solute concentration in fluid and sorbed concentration in solids after obtaining the solution in term of combined concentration. Linear equilibrium sorption gives a simple linear equation and directly separates concentrations. Nonlinear-nonequilibrium isotherm is dealt in the segregation with further subdivisions of the time step and during each subdivision of the time step the combined concentration and sorbed concentration is assumed to change linearly. The model follows the first order decay reaction on the solute concentration. The initial condition of solute concentration in rigid porous field is assumed to be known. The chapter explains the three types of boundary conditions of solutes; one is the given solute concentration, second is the zero concentration gradient and third one is reservoir boundary condition. Finally, the model verification check is done it is noted that the finite volume model results, for advection, diffusion, advection + diffusion and with sorption/ decay reactions, are in close agreement with the results of numerical model CST1(Fox,2007) excepting the flows with infinite Peclet Number. This weakness of the model is only hypothetical as the solute transport through porous media is practically never happens without diffusion. In soils, the Reynolds number is less than unity (Kumar and Singh, 1995) and the diffusion contributes significantly to solute transport. However, the accuracy of the model provides the advantage in reduction of number of elements required for the solution.

Chapter 5 presents development of finite volume numerical model of two-dimensional solute transport in rigid porous media due to one dimensional advection and two-dimensional diffusion and dispersion. The two-dimensional control volume of the numerical model contains three discrete points on each face. The quadratic interpolation function for these faces is derived with the help of two-dimensional Taylor's series and averaging the three values on a face. The mathematical procedure is elucidated and its conformity with one dimensional interpolation function is also mentioned. The verification check of the model shows that the requirement of the elements for two-dimensional scheme is also lesser than the piecewise linear numerical model CST1.

Chapter 6 presents the coupling of consolidation and solute transport programming modules coded separately as per the finite volume formulation requirements. The addition to consolidation module for computation of Darcy velocity due to an applied hydraulic gradient and that due to consolidation is also explained. The consolidation induced velocity computation uses the change in void ratio computed at each time step and numerically integrating these to the level of each control volume and it is assumed that the velocity in each control volume is uniform. The resultant Darcy velocity within the porous media is the vector addition of velocities due to hydraulic gradient and consolidation. The verification check part of the model contains four different types of problems, the comparative study and findings are as follows. First problem is regarding the one dimensional solute transport through compressible porous media with a hypothetical compacted clay liner. The reservoir boundary condition at the impermeable top boundary allows only diffusive flux and no advection. The amended top boundary condition is explained. In one case of the problem the void ratio at the bottom reaches near zero at the time nearing 1.9 years and results into instability. This has caused slight difference in results of present model and CST1 as the different softwares may respond differently near instability. Otherwise all other results of the problem are in close agreement. The results also infer that solute transport is least affected by variation in longitudinal dispersivity but significantly influenced by consolidation and breakthrough time in reduced. The second problem shows the performance of the present model on a problem with experimental observations and the results of another numerical model CST2 (Fox, 2009). The comparisons show that the results of the present model sometimes match well with experimental results compared to CST2 and sometimes CST2 results are closer. Problem 3 shows again the influence of consolidation where the present model is applied to CCL made with a mix of organoclay, bentonite and natural soil. The consolidation has substantially reduced the breakthrough time and deserves definite consideration while designing a liner. This problem considers a nonlinear equilibrium sorption isotherm (Freundlich isotherm) and requires the root of a nonlinear algebraic equation that has been worked out in the model using Newton-Raphson Method. Fourth problem shows the performance of the model on two-dimensional solute transport in deforming porous media on a problem of confined disposal facility. This case deals in varied boundary conditions of consolidation i.e. doubly drained, singly drained, and no consolidation for a line source of contaminated dredged slurry. The performance of the model with this problem also establishes that the lesser number of elements give an acceptable results and compares well with the results of CST1. The last section of the chapter shows that the two-dimensional spread of contaminant is mainly governed by the coefficient of effective diffusion of the individual contaminant. The contaminant with lower diffusion coefficient travels down faster whereas the contaminants with higher diffusion coefficient spread laterally.

As the overall conclusion, it is noteworthy that the finite volume method offers a good option as a numerical solution model to all kinds of conservation equations. The method is integration based but simpler than the finite element method/ boundary element method. In particular, the cases of consolidation and solute transport, in finite volume formulation conserve the mass at each discrete control volume level; the requirement of assessment of mass conservation is not required separately. The overall accuracy of the present model is maintained to third order and this makes it better than any linear model. Further, the model is fully capable to handle the sorption isotherms of different types such as linear equilibrium, nonlinear equilibrium, and nonlinear nonequilibrium. The extension of the method to two-dimensional solute transport shows its potential to extend it further to three-dimensional solute transport in deforming or rigid porous media.

The present numerical model of consolidation can deal with only monotonic loading and for non-monotonic loading (swelling problem), it requires the compressibility characteristic Eq. (2.2) to be a functional not a function (Gibson et al., 1967). The explicit finite volume numerical model presented here has not been compared with any such implicit formulation and the performance of such a model has still not appeared in the literature. Further, the potential of the model for extension to three-dimensional case has not been developed here. These are a few issues which form the future scope of study in connection with this work.

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APPENDIX

Flow chart for numerical model

Consolidation

INPUT DATA:
Constants: G_s, γ_w
Geometric & Material Parameters: H, H_w, H_b q_p, LBL
Control Flags: Contaminated soil to uncontaminated (NSOL), No self load (NSL), Uniform initial void ratio (IE0)
Initial Void Ratio: e_{00}
Data Representing Constitutive Eqs: $e \sim \sigma'$; $e \sim k$
Data for contaminated soil to uncontaminated: e_{00} and height of contaminated soil and uncontaminated soil
Consolidation Calculation Data: Number of control volumes (NBDJV), time step, total time, control flags for continuous load (NL), boundary condition (NDRB), number of timings at which output is required with or without load increment.
Required Output Data: Timings at which output is required, load increments at each time
H_b = Water level above bottom of layer
LBL=Number of data points representing constitutive equations

↓

Determine initial void ratio for no load increment and final void ratio for instant initial load.
Calculate nonlinear terms corresponding to each void ratio of input data set.
LOOP1: begins for desired timings of output with or without load increments.
LOOP2: begins on time step.
Calculate void ratio on boundary nodes using boundary condition.
Calculate void ratio for new time increments for all nodes.

↓

Solute Transport

INPUT DATA:
Control flags: One/ two dimensional solute transport, Effective diffusion dependent on void ratio, Sorption isotherm indicator, solute boundary condition, no consolidation, Effective diffusion dependent on tortuosity factor
Solute transport parameters: ρ_d, α_a, α_x, c_t, c_b, λ_c, λ_{sc}, D^*, D_0, M, K_d, K_p, F, λ_s, D_{GM}, Thickness of HDPE liner (TH), initial solute concentration condition

↓

Calculate Darcy velocity due to external hydraulic gradient and consolidation.

↓

Calculate solute concentration at boundary nodes with boundary conditions.
Calculate solute concentration in fluid and solid media at all nodes.

↓

End of LOOP2
Reset final void ratio on desired timings of output with or without load increments.
Calculate excess pore pressure, settlement and degree of consolidation

↓

Output data files
End of LOOP1

PROGRAM-LISTING (F-77)

```
c       **************************************************************
c                           MAIN PROGRAM
c       **************************************************************
c
c       SOLUTE TRANSPORT
c
c       NSTBC=1 boundary conditions on concentration of solute both at top and
c                bottom
c       NSTBC=2 reservoir boundary condition at top and zero gradient at bottom
c       NSTBC=3 flux boundary condition when CCL top is overlain by geomembrane
c                and at bottom gradient is zero
c       NSBCB=1 constant concentration at bottom
c       NSBCB=2 zero concentration gradient at bottom
c       NSORP=1 linear sorption
c       NSORP=2 nonlinear sorption
c       NSL=1 self load of soil is considered otherwise not considered
c       ND1=1 one-dimensional problem
c       ND1=2 two-dimensional problem
c       NTAU=1 tortuosity factor is considered
c       NTAU=2 effective diffusion is taken directly
c       NC=1 no consolidation
c       NSOL=1 case of solute transport from contaminated soil to uncontaminated
c               soil
c       IE0=1 initial void ratio is given and to be used as such
c       IE0=2 for NSOL=1, initial void ratio is given and to be used as such
c       IE0=3 for NSOL=1, initial void ratio to determined by initial load and
c                         self load
c       ISS=1 if initial spread of contaminant is in 1 m breadth
c       ISS=2 if initial spread of contaminant is in at apoint
c       NSTUBC=1 prescribed solute concentration at the top
c       NSTUBC=2 prescribed solute concentration gradient at the top
c       NSTUBC=3 prescribed mass flux or reservoir boundary condition at the top
c       NSTBBC=1 prescribed solute concentration at the bottom
c       NSTBBC=2 prescribed solute concentration gradient at the bottom
c       NSTBBC=3 prescribed mass flux or reservoir boundary condition at the
c                 bottom
c       NSTRBC=1 prescribed solute concentration at the right boundary
c       NSTRBC=2 prescribed solute concentration gradient at the right boundary
c       NSTRBC=3 prescribed mass flux or reservoir boundary condition at the right
c                 boundary
c       NSTLBC=1 prescribed solute concentration at the left boundary
c       NSTLBC=2 prescribed solute concentration gradient at the left boundary
c       NSTLBC=3 prescribed mass flux or reservoir boundary condition at the left
c                 boundary
c       PCT=prescribed concentration at the top (boundary condition)
c       PGT=prescribed concentration gradient at the top (boundary condition)
c       PRT=prescribed reservoir concentration (boundary condition)
c       PCB=prescribed concentration at the top (boundary condition)
c       PGB=prescribed concentration gradient at the top (boundary condition)
c       PRB=prescribed reservoir concentration (boundary condition)
c       PCR=prescribed concentration at the top (boundary condition)
c       PGR=prescribed concentration gradient at the top (boundary condition)
c       PRR=prescribed reservoir concentration (boundary condition)
c       PCL=prescribed concentration at the top (boundary condition)
c       PGL=prescribed concentration gradient at the top (boundary condition)
c       PRL=prescribed reservoir concentration (boundary condition)
c       ND2=1 constant effective diffusion
c       ND2=2 variable effective diffusion with porosity
c       NL=1 continuous load with time stepped into per unit time step
c       NL=2 continuous load with time stepped into per unit time more than the
c            time step
```

```
c      NSL=2 soil is incompressible
c      NDRB=1 top boundary impermeable bottom drained
c      NDRB=2 top boundary drained bottom impermeable
c      NDRB=3 top and bottom both boundaries drained
c      NSOL=1 consolidation of contaminated and uncontaminated samples lain over
c              each other
c      NSOL=2 above condition is not true
c      NNSOL= the number wherefrom the input data for contaminated part starts
c      NNSOL1=the node number where the extent of contaminated slurry ends
c      DGM=diffusion coefficient of geomembrane
c      TH=thickness of geomembrane
c      CFT=initial concentration of solute at the top
c      CFB=initial concentration of solute at the bottom
c      CF0=initial concentration of solute
c      CF1=previous time concentration of solute
c      CF2=current concentration of solute
c      BD=breadth of the soil field (clay liner) or sample
c      HBL=height of the soil field
c      WBL=width of the soil field
c      HCL=height of contaminated layer
c      HUCL=height of uncontaminated layer
c      NBDJV=number of parts of breadth divided for computation
c      NDJV=number of mesh points on the breadth side of the clay liner
c      HT=height of water above top of clay liner
c      HB=height of water above bottom of clay liner
c      CHD0=free solution diffusion
c      CHD1=constant effective diffusion coefficient
c      CHD2=effective diffusion dependent on porosity
c      ALPHAT=coefficient of transverse dispersion
c      ALPHAL=coefficient of longitudinal dispersion
c      CHDA=coefficient of longitudinal hydrodynamic dispersion
c      CHDX=coefficient of transverse hydrodynamic dispersion
c      EG0=initial void ratio
c      EG=current void ratio
c      EN0=initial porosity
c      EN=current porosity
c      CQI=Darcy velocity due to hydraulic gradient
c      CQU=Darcy velocity due to excess pore pressure
c      CQ=total Darcy velocity
c      RK=hydraulic conductivity input values
c      RK1=hydraulic conductivity interpolated values
c      RKEO=overall effective hydraulic conductivity
c      RKEI=interfacial effective hydraulic conductivity
c      DJI=dξ/da
c      CFF1=previous time step value of EN*CF*DJI+(1-EN)*CS*DJI
c      CFF2=current time step value of above quantity
c      CS0=initial solute concentration in soil
c      CS1=previous time step solute concentration in soil
c      CS2=current time step solute concentration in soil
c      AKD=partition coefficient
c      GSBL=specific gravity of soil
c      DS=density of soil
c      DW=density of water
c      GW=unit weight of water
c      GS=unit weight of soil
c      GC=buoyant weight of soil
c      ALAMDAC=solute decay constant
c      ALAMDASC=source decay constant
c      NSORP=1 linear equilibrium sorption
c      NSORP=2 nonlinear nonequilibrium or kinetic sorption
c      NSORP=3 non-linear equilibrium Freundlich isotherm
c      AKP=a constant describing non-linear Freundlich isotherm
c      ANF=a constant describing non-linear Freundlich isotherm
c      ANLAMDA=sorption rate constant
c      INX=horizontal node number at which the contamination is introduced in 2-d
c      JNZ=vertical node number at which the contamination is introduced in 2-d
```

```
c DRD=dry density of soil matrix
c
c SOIL CONSOLIDATION
c
c Q2=current load on the consolidating soil
c NL=1 for continuous loading with certain load
c TTIME=total time of loading
c     E0C=uniform initial void ratio of contaminated layer
c E0UC=uniform initial void ratio of uncontaminated layer
c
c TO AVOID THE MID POINT AS CONTROL VOLUME ALWAYS TAKE NBDIV AS EVEN NUMBER
c
      common DA,DB,DZ,E00,ELL,GC,GS,GSBL,GW,HBL,LBL,NBDIV, NDIV,NBDJV,
     >       NDJV,NFLAG,NNN,NTIME,Q0,Q2,WL,SETT,SFIN,TAU,TIME,TPRINT,
     >       UCON,NNTIME,NST,NL,NBC,NDRB,ND1,ND2,NNL,NSOL,NNSOL,NNSOL1,
     >       ALPHAL,ALPHAT,NSTBC,SL,CFT,CFB,HT,HB,CHD0,CHD1,AKD,ALAMDAC,
     >       ALAMDASC,AM,DW,DS,DGM,TH,RKEO,NSL,HCL,HUCL,E0C,E0UC,NSORP,
     >       NSTUBC,NSTBBC,NSTRBC,NSTLBC,PCT,PGT,PRT,PCB,PGB,PRB,PCR,
     >       PGR,PRR,PCL,PGL,PRL,INX,JNZ,IE0,ISS,NC,TTIME,
     >       A(351),B(351),Z(351),XI(351),ALPHA(351),BETA(351),
     >       DSDE(351),E11(351),EFIN(351),ER(351),ES(351),EFFSTR(351),
     >       F(351),FS(351),FINT(351),PK(351),RK(351),RK1(351),RS(351),
     >       TOTSTR(351),U(351),U0(351),UW(351),VRI(351),DQ(351),
     >       Q1(351),RKEI(351),AKP,ANF,ANLAMDA,IKK,UMAX,NTAU,
     >       CHD2(351),CHDA(351),CHDX(351),CQI(351),CQU(351),CQ(351),
     >       EN0(351),EN(351),EG0(351),EG(351),DVDA(351,351),
     >       CS0(351,351),CS1(351,351),CS2(351,351),CF0(351,351),
     >       CF1(351,351),CF2(351,351),CFF1(351,351),CFF2(351,351),
     >       E11JPLUSHALF(351),E11JMINUSHALF(351),DKQU(351),
     >       FJPLUSHALF(351),FJMINUSHALF(351),AFJPLUSHALF(351),
     >       AFJMINUSHALF(351),BFJPLUSHALF(351),BFJMINUSHALF(351),
     >       EGJPLUSHALF(351),EGJMINUSHALF(351),EG0JPLUSHALF(351),
     >       EG0JMINUSHALF(351),CQJPLUSHALF(351),CQJMINUSHALF(351),
     >       CQUJPLUSHALF(351),CQUJMINUSHALF(351),DRD(351),
     >       CF0IPLUSHALF(351,351),CF0IMINUSHALF(351,351),
     >       CF0JPLUSHALF(351,351),CF0JMINUSHALF(351,351),
     >       CF1IPLUSHALF(351,351),CF1IMINUSHALF(351,351),
     >       CF1JPLUSHALF(351,351),CF1JMINUSHALF(351,351),
     >       PRINT1(351)
      dimension ES2(351),RS2(351),RK2(351)
      dimension ES3(351),RS3(351),RK3(351)
c
      open(unit=17,file='INPUTJ4DD_S',status='OLD')
      open(unit=18,file='OUTJ4DD_19_S',status='OLD')
      open(unit=19,file='OUTJ4DD_20_S',status='OLD')
      open(unit=20,file='INPUTJ4DD_ST',status='OLD')
      open(unit=21,file='OUTJ4DD_19_ST',status='OLD')
      open(unit=22,file='OUTJ4DD_20_ST',status='OLD')
c
c     READ SOIL DATA FOR FOUNDATION LAYER OR SOFT LAYER,
c     IF NSOL IS 1 START CONTMINATED LAYER INPUT DATA AT 301
c
      read(17,*) NST,GSBL,GW,HBL,HT,HB,Q0,LBL,NSOL,NSL,IE0
      if(IE0.eq.1) read(17,*) NST,E00
      if(NSL.ne.1) GSBL=1.0
      if(NSOL.eq.1) then do
      1 J=1,LBL
      read(17,*) NST,ES2(J),RS2(J),RK2(J)
    1 continue
      do 11 J=1,LBL
      ES(J)=ES2(J)
      RS(J)=RS2(J)
      RK(J)=RK2(J)
   11 continue
      if(IE0.eq.2)  read(17,*)  NST,E0C,E0UC,HCL,HUCL
      if(IE0.eq.3) read(17,*) NST,HCL,HUCL
```

```
      endif if(NSOL.ne.1)
      then do 14 J=1,LBL
      read(17,*) NST,ES(J),RS(J),RK(J)
   14 continue
      endif
c
c      CONSOLIDATION CALCULATION DATA
c
      read(17,*) NST,NBDJV,TAU,TTIME,NL,NDRB,NTIME
      do 2 J=1,NTIME
      read(17,*) NST,PRINT1(J),DQ(J)
    2 continue
c
c     SET INITIAL VARIABLES
c
      ELL=0.0; TIME=0.0; DZ=0.0
      UCON=0.0; SETT=0.0; DA=0.0
      SFIN=0.0; VRI=0.0
      NNN=1; NM=1; Q=Q0
      do 3 J=1,NTIME
      Q1(J)=QT+DQ(J)
      QT=Q1(J)
    3 continue
c
c PRINT INPUT DATA OF CONSOLIDATION AND MAKE INITIAL CALCULATIONS
c
      call INTRO_4()
c
c SOLUTE TRANSPORT DATA c
      call INPUT_ST()
c
c PERFORM CALCULATIONS TO EACH PRINT TIME AND OUTPUT RESULTS
c
      if(NL.eq.1) then
      KK=2
      NNTIME=TTIME/TAU
      do 5 J=1,NNTIME+1
      NNN=J
      TIME=TAU* float(NNN)
      Q2=(Q1(NTIME)/float(NNTIME+1))*float(NNN)
      if((TIME-0.001).gt.TTIME) goto 9
      call RESET_4()
      call FDIFEQ_4()
      call STRESS_4()
      TPRINT=PRINT1(KK)
      if(TIME.lt.TPRINT) goto 5
      call DATOUT_4()
      call DATOUT()
      KK=KK+1
    5 continue
      endif
      if(NL.eq.2)  then
      do  7  K=NM,NTIME
      TPRINT=PRINT1(K)
      Q2=Q1(K)
   45  call  FDIFEQ_4()
      call   RESET_4()
      call STRESS_4()
      if(TPRINT.eq.0.0) goto 7
      call DATOUT_4()
      call DATOUT()
    7 continue
      endif
c
    9 stop
      end
```

```
c
c      *******************************************************
       subroutine INTRO_4()
c      *******************************************************
c
c      INTRO PRINTS INPUT DATA AND RESULTS OF INITIAL CALCULATIONS
c      IN TABULAR FORM
c
       common DA,DB,DZ,E00,ELL,GC,GS,GSBL,GW,HBL,LBL,NBDIV, NDIV,NBDJV,
     >        NDJV,NFLAG,NNN,NTIME,Q0,Q2,WL,SETT,SFIN,TAU,TIME,TPRINT,
     >        UCON,NNTIME,NST,NL,NBC,NDRB,ND1,ND2,NNL,NSOL,NNSOL,NNSOL1,
     >        ALPHAL,ALPHAT,NSTBC,SL,CFT,CFB,HT,HB,CHD0,CHD1,AKD,ALAMDAC,
     >        ALAMDASC,AM,DW,DS,DGM,TH,RKEO,NSL,HCL,HUCL,E0C,E0UC,NSORP,
     >        NSTUBC,NSTBBC,NSTRBC,NSTLBC,PCT,PGT,PRT,PCB,PGB,PRB,PCR,
     >        PGR,PRR,PCL,PGL,PRL,INX,JNZ,IE0,ISS,NC,TTIME,
     >        A(351),B(351),Z(351),XI(351),ALPHA(351),BETA(351),
     >        DSDE(351),E11(351),EFIN(351),ER(351),ES(351),EFFSTR(351),
     >        F(351),FS(351),FINT(351),PK(351),RK(351),RK1(351),RS(351),
     >        TOTSTR(351),U(351),U0(351),UW(351),VRI(351),DQ(351),
     >        Q1(351),RKEI(351),AKP,ANF,ANLAMDA,IKK,UMAX,NTAU,
     >        CHD2(351),CHDA(351),CHDX(351),CQI(351),CQU(351),CQ(351),
     >        EN0(351),EN(351),EG0(351),EG(351),DVDA(351,351),
     >        CS0(351,351),CS1(351,351),CS2(351,351),CF0(351,351),
     >        CF1(351,351),CF2(351,351),CFF1(351,351),CFF2(351,351),
     >        E11JPLUSHALF(351),E11JMINUSHALF(351),DKQU(351),
     >        FJPLUSHALF(351),FJMINUSHALF(351),AFJPLUSHALF(351),
     >        AFJMINUSHALF(351),BFJPLUSHALF(351),BFJMINUSHALF(351),
     >        EGJPLUSHALF(351),EGJMINUSHALF(351),EG0JPLUSHALF(351),
     >        EG0JMINUSHALF(351),CQJPLUSHALF(351),CQJMINUSHALF(351),
     >        CQUJPLUSHALF(351),CQUJMINUSHALF(351),DRD(351),
     >        CF0IPLUSHALF(351,351),CF0IMINUSHALF(351,351),
     >        CF0JPLUSHALF(351,351),CF0JMINUSHALF(351,351),
     >        CF1IPLUSHALF(351,351),CF1IMINUSHALF(351,351),
     >        CF1JPLUSHALF(351,351),CF1JMINUSHALF(351,351),
     >        PRINT1(351)
c
c      PRINT PROBLEM  NUMBER AND HEADING
c
       write(18,100)
       write(18,101)
       write(18,102)
       call SETUP_4()
c
c      PRINT SOIL DATA FOR COMPRESSIBLE FOUNDATION
c
       write(18,104)
       write(18,105)
       write(18,106)
       write(18,107)  HBL,GSBL,WL,Q0
       write(18,108)
       write(18,109)
       do 1 J=1,LBL
       write(18,110)  J,ES(J),RS(J),RK(J),PK(J),BETA(J),DSDE(J),
     >                ALPHA(J)
    1 continue
c
c      PRINT CALCULATION DATA
c
   16  write(18,115)
       write(18,116)
       write(18,117)
       write(18,118) TAU
c
c       PRINT TABLES OF INITIAL CONDITIONS
c
       NFLAG=1
```

```
      call DATOUT_4()
      NFLAG=0
c
c    FORMATS
c
  100 format(1H1////9X,60(1H*))
  101 format(9X,48HCONSOLIDATION OF SOFT LAYERS BY FINITE STRAIN --,
     >       6HCF=0.6)
  102 format(9X,60(1H*))
  104 format(/////18(1H*),37HSOIL DATA FOR COMPRESSIBLE FOUNDATION,
     >       17(1H*))
  105 format(//6X,5HLAYER,6X,16HSPECIFIC GRAVITY,4X,11HWATER LEVEL,
     >       9X,7HINITIAL)
  106 format(4X,9HTHICKNESS,8X,9HOF SOLIDS,7X,11HFROM BOTTOM,8X,
     >       9HSURCHARGE)
  107 format(/4X,F8.3,7X,F8.3,2(10X,F8.3))
  108 format(//8X,4HVOID,2X,9HEFFECTIVE,3X,5HPERM-,5X,5HK/1+E)
  109 format(4X,8HI  RATIO,4X,6HSTRESS,3X,8HEABILITY,4X,2HPK,7X,4HBETA,
     >       6X,4HDSDE,5X,5HALPHA)
  110 format(2X,I3,1X,F6.3,6E10.3)
  111 format(/////23(1H*)26HSOIL DATA FOR DREDGED FILL,23(1H*))

  112 format(//5X,5HLAYER,5X,16HSPECIFIC GRAVITY,3X,11HWATER LEVEL,
     >       5X,7HINITIAL,4X,11HUNIT WEIGHT)
  113 format(3X,9HTHICKNESS,7X,9HOF SOLIDS,6X,11HFROM BOTTOM,
     >       3X,10HVOID RATIO,5X,8HOF WATER)
  114 format(/2X,F8.3,8X,F8.3,9X,F8.3,5X,F8.3,7X,F6.2)

  115 format(/////28(1H*),16HCALCULATION DATA,28(1H*))
  116 format(//8X,3HTAU,10X,11HLOWER LAYER,7X,11HLOWER LAYER,7X,
     >       13HDRAINAGE PATH)
  117 format(21X,10HVOID RATIO,8X,12HPERMEABILITY,9X,6HLENGTH)
  118 format(/4X,E11.5,8X,F8.3,9X,E11.5,7X,3HZ =,F8.3)
c
  300 return
      end
c     ************************************************************
      subroutine SETUP_4()
c     ************************************************************
c
c        SETUP MAKES INITIAL CALCULATIONS AND MANIPULATIONS OF INPUT
c        DATA FOR LATER USE
c
      common DA,DB,DZ,E00,ELL,GC,GS,GSBL,GW,HBL,LBL,NBDIV, NDIV,NBDJV,
     >       NDJV,NFLAG,NNN,NTIME,Q0,Q2,WL,SETT,SFIN,TAU,TIME,TPRINT,
     >       UCON,NNTIME,NST,NL,NBC,NDRB,ND1,ND2,NNL,NSOL,NNSOL,NNSOL1,
     >       ALPHAL,ALPHAT,NSTBC,SL,CFT,CFB,HT,HB,CHD0,CHD1,AKD,ALAMDAC,
     >       ALAMDASC,AM,DW,DS,DGM,TH,RKEO,NSL,HCL,HUCL,E0C,E0UC,NSORP,
     >       NSTUBC,NSTBBC,NSTRBC,NSTLBC,PCT,PGT,PRT,PCB,PGB,PRB,PCR,
     >       PGR,PRR,PCL,PGL,PRL,INX,JNZ,IE0,ISS,NC,TTIME,
     >       A(351),B(351),Z(351),XI(351),ALPHA(351),BETA(351),
     >       DSDE(351),E11(351),EFIN(351),ER(351),ES(351),EFFSTR(351),
     >       F(351),FS(351),FINT(351),PK(351),RK(351),RK1(351),RS(351),
     >       TOTSTR(351),U(351),U0(351),UW(351),VRI(351),DQ(351),
     >       Q1(351),RKEI(351),AKP,ANF,ANLAMDA,IKK,UMAX,NTAU,
     >       CHD2(351),CHDA(351),CHDX(351),CQI(351),CQU(351),CQ(351),
     >       EN0(351),EN(351),EG0(351),EG(351),DVDA(351,351),
     >       CS0(351,351),CS1(351,351),CS2(351,351),CF0(351,351),
     >       CF1(351,351),CF2(351,351),CFF1(351,351),CFF2(351,351),
     >       E11JPLUSHALF(351),E11JMINUSHALF(351),DKQU(351),
     >       FJPLUSHALF(351),FJMINUSHALF(351),AFJPLUSHALF(351),
     >       AFJMINUSHALF(351),BFJPLUSHALF(351),BFJMINUSHALF(351),
     >       EGJPLUSHALF(351),EGJMINUSHALF(351),EG0JPLUSHALF(351),
     >       EG0JMINUSHALF(351),CQJPLUSHALF(351),CQJMINUSHALF(351),
     >       CQUJPLUSHALF(351),CQUJMINUSHALF(351),DRD(351),
     >       CF0IPLUSHALF(351,351),CF0IMINUSHALF(351,351),
     >       CF0JPLUSHALF(351,351),CF0JMINUSHALF(351,351),
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     >        CF1IPLUSHALF(351,351),CF1IMINUSHALF(351,351),
     >        CF1JPLUSHALF(351,351),CF1JMINUSHALF(351,351),
     >        PRINT1(351)
c
c     SET CONSTANTS
c
      NDJV=NBDJV+2
      GS=GSBL*GW
      GC=GS-GW
c
c       CALCULATE INITIAL ELL FOR COMPRESSIBLE SOIL LAYER
c
      Z(1)=0.0; A(1)=0.0; XI(1)=0.0
      if(NSOL.ne.1) then
      DZZ=0.0
      NBD=10000*NBDJV
      DABL=HBL/float(NBD)
      EFS=Q0
      do 4 J=1,NBD
      do 1 N=2,LBL
      S1=EFS-RS(N)
      if(S1.le.0.0) goto 2
    1 continue
      V=ES(LBL); goto 3
    2 NN=N-1
      XV=EFS
      if(N.eq.LBL) NN=NN-1
      call   LINTP_4(XV,YV,RS,ES,NN)
      V=YV
    3 TDZ=DABL/(1.0+V)
      EFS=EFS+GC*TDZ
      DZZ=DZZ+TDZ
    4 continue
      ELL=DZZ
      DZ=ELL/float(NBDJV)
c
c         CALCULATE INITIAL COORDINATES AND VOID RATIO FOR COMPRESSIBLE SOIL LAYER
c
      DA=HBL/float(NBDJV)
      Z(2)=Z(1)+(DZ/2.0)
      A(2)=A(1)+(DA/2.0)
      do 5 J=3,NDJV-1
      A(J)=A(J-1)+DA
      Z(J)=Z(J-1)+DZ
    5 continue Z(NDJV)=Z(NDJV-
      1)+(DZ/2.0) A(NDJV)=A(NDJV-
      1)+(DA/2.0) EFS1=GC*ELL+Q0
      do 7 J=1,NDJV EFS=EFS1-
      GC*Z(J) if(EFS.lt.0.0)
      EFS=0.0
      do 8 N=2,LBL S1=EFS-
      RS(N) if(S1.le.0.0)
      goto 9
    8 continue E11(J)=ES(LBL);
      goto 10
    9 NN=N-1
      XV=EFS
      if(N.eq.LBL) NN=NN-1
      call   LINTP_4(XV,YV,RS,ES,NN)
      E11(J)=YV
   10 F(J)=E11(J)
      ER(J)=E11(J)
    7 continue
      if(IE0.eq.1) then
      do 210 J=1,NDJV
      E11(J)=E00
```

```
      F(J)=E11(J)
      ER(J)=E11(J)
  210 continue
      ELL=HBL/(1.0+E00)
      DZ=ELL/float(NBDJV)
      Z(2)=Z(1)+(DZ/2.0)
      do 65 J=3,NDJV-1
      Z(J)=Z(J-1)+DZ
   65 continue
   66 Z(NDJV)=Z(NDJV-1)+(DZ/2.0)
      call INTGRL_4(E11,E11,DA,NDJV,FINT)
      do 211 J=1,NDJV
c     FINT(J)=A(J)*E11(J)/(1.0+E11(J))
      VRI(J)=FINT(J)
      XI(J)=A(J)
      EN0(J)=E11(J)/(1.0+E11(J))
  211 continue
      endif
      call INTGRL_4(E11,E11,DA,NDJV,FINT)
      do 212 J=1,NDJV
c     FINT(J)=A(J)*E11(J)/(1.0+E11(J))
      VRI(J)=FINT(J)
      XI(J)=A(J)
      EN0(J)=E11(J)/(1.0+E11(J))
  212 continue
      endif
      if(NSOL.eq.1)   then
      DA=HBL/float(NBDJV)
      NPART=int(HCL/DA)
      NNSOL1=NPART+1
      A(2)=A(1)+(DA/2.0)
      do 60 J=3,NNSOL1-1
      A(J)=A(J-1)+DA
   60 continue A(NNSOL1)=A(NNSOL1-
      1)+(DA/2.0)
      A(NNSOL1+1)=A(NNSOL1)+(DA/2.0)
      do 161 J=NNSOL1+2,NDJV-1
      A(J)=A(J-1)+DA
  161 continue
  162 A(NDJV)=A(NDJV-1)+(DA/2.0)
      if(IE0.eq.1) then
c
c     CALCULATE AVERAGE DZ FOR TOTAL LAYER
c
      EAA0=(E0C*HCL+E0UC*HUCL)/A(NDJV)
      ELL=A(NDJV)/(1.0+EAA0)
      DZ=ELL/float(NBDJV)
      Z(2)=Z(1)+(DZ/2.0)
      do 61 J=3,NDJV-1
      Z(J)=Z(J-1)+DZ
   61 continue
   62 Z(NDJV)=Z(NDJV-1)+(DZ/2.0)
      do 35 J=1,NNSOL1
      E11(J)=EAA0
   35 continue
      do  38  J=NNSOL1+1,NDJV
      E11(J)=EAA0
   38 continue
      do 111 J=1,NDJV
      F(J)=E11(J)
      ER(J)=E11(J)
  111 continue
      call INTGRL_4(EFIN,E11,DA,NDJV,FINT)
      do 411 J=1,NDJV
c     FINT(J)=A(J)*E11(J)/(1.0+E11(J))
      VRI(J)=FINT(J)
      XI(J)=A(J)
```

```
      EN0(J)=E11(J)/(1.0+E11(J))
  411 continue
      endif
      if(IE0.eq.3) then
      DZZ=0.0
      NBD=10000*NBDJV
      DABL=HBL/float(NBD)
      EFS=Q0
      do  334  J=1,NBD
      do 331 N=2,LBL
      S1=EFS-RS(N)
      if(S1.le.0.0) goto 332
  331 continue V=ES(LBL);
      goto 333
  332 NN=N-1
      XV=EFS
      if(N.eq.LBL) NN=NN-1
      call   LINTP_4(XV,YV,RS,ES,NN)
      V=YV
  333 TDZ=DABL/(1.0+V)
      EFS=EFS+GC*TDZ
      DZZ=DZZ+TDZ
  334 continue
      ELL=DZZ
      DZ=ELL/float(NBDJV)
c
c         CALCULATE INITIAL COORDINATES AND VOID RATIO FOR COMPRESSIBLE SOIL LAYER
c
      DA=HBL/float(NBDJV)
      Z(2)=Z(1)+(DZ/2.0)
      A(2)=A(1)+(DA/2.0)
      do 335 J=3,NNSOL1
      Z(J)=Z(J-1)+DZ
      A(J)=A(J-1)+DA
  335 continue
      do  435  J=NNSOL1+1,NDJV-1
      Z(J)=Z(J-1)+DZ
      A(J)=A(J-1)+DA
  435 continue Z(NDJV)=Z(NDJV-
      1)+(DZ/2.0) A(NDJV)=A(NDJV-
      1)+(DA/2.0) EFS1=GC*ELL+Q0
      do 337 J=1,NDJV
      EFS=EFS1-GC*Z(J)
      if(EFS.lt.0.0) EFS=0.0
      do 338 N=2,LBL
      S1=EFS-RS(N)
      if(S1.le.0.0) goto 339
  338 continue
      E11(J)=ES(LBL); goto 340
  339 NN=N-1
      XV=EFS
      if(N.eq.LBL) NN=NN-1
      call   LINTP_4(XV,YV,RS,ES,NN)
      E11(J)=YV
  340 F(J)=E11(J)
      ER(J)=E11(J)
  337 continue
      call   INTGRL_4(E11,E11,DA,NDJV,FINT)
      do 6 J=1,NDJV
      VRI(J)=FINT(J)
      XI(J)=A(J)
      EN0(J)=E11(J)/(1.0+E11(J))
    6 continue
      endif
      endif
c
```

```
c       CALCULATE FINAL VOID RATIOS FOR SOIL LAYER
c
      if(NSOL.ne.1) then
      C1=GC*ELL; C2=Q0
      S1=C1+C2
      do 18 J=1,NDJV
      S2=S1-Z(J)*GC
      do 16 N=2,LBL
      S3=S2-RS(N)
      if(S3.le.0.0) goto 17
   16 continue EFIN(J)=ES(LBL);
      goto 18
   17 NN=N-1
      XV=S2
      if(N.eq.LBL) NN=NN-1
      call   LINTP_4(XV,YV,RS,ES,NN)
      EFIN(J)=YV
   18 continue
      endif
      if(NSOL.eq.1) then
c
c UNCONTAMINATED LAYER
c
      C1=ELL*GC; C2=Q0
      S1=C1+C2
      do 51 J=NNSOL1+1,NDJV
      S2=S1-Z(J)*GC
      do 52 N=2,LBL S3=S2-
      RS(N) if(S3.le.0.0)
      goto 53
   52 continue EFIN(J)=ES(LBL);
      goto 51
   53 NN=N-1
      XV=S2
      if(N.eq.LBL) NN=NN-1
      call LINTP_4(XV,YV,RS,ES,NN)
      EFIN(J)=YV
   51 continue
c
c     CONTAMINATED LAYER
c
      do 54 J=1,NNSOL1
      S2=S1-Z(J)*GC do
      55 N=2,LBL S3=S2-
      RS(N)
      if(S3.le.0.0) goto 56
   55 continue EFIN(J)=ES(LBL);
      goto 54
   56 NN=N-1
      XV=S2
      if(N.eq.LBL) NN=NN-1
      call LINTP_4(XV,YV,RS,ES,NN)
      EFIN(J)=YV
   54 continue
      endif
c
c CALCULATE INITIAL STRESSES AND PORE PRESSURES FOR SOIL LAYER
c
      do 19 J=1,NDJV
      if(HT.ne.0.0) U0(J)=GW*(HT+XI(NDJV)-XI(J))
      if(HT.eq.0.0) U0(J)=GW*(XI(NDJV)-XI(J))
      U0(J)=GW*(HBL+HT-XI(J))
       EFFSTR(J)=Q0+GC*(ELL-Z(J))
      if(IE0.eq.1) EFFSTR(J)=Q0
       TOTSTR(J)=GC*(ELL-Z(J))+GW*(HBL+HT-XI(J))+Q0
```

```
      UW(J)=TOTSTR(J)-EFFSTR(J)
      U(J)=UW(J)-U0(J)
   19 continue
c
c       ULTIMATE SETTLEMENT FOR COMPRESSIBLE FOUNDATION
c
      call INTGRL_4(EFIN,E11,DA,NDJV,FINT)
      SFIN=VRI(NDJV)-FINT(NDJV)
c
c       CALCULATE FUNCTIONS FOR COMPRESSIBLE SOIL LAYER
c       PERMEABILITY FUNCTIONS
c
      do 28 J=1,LBL
      PK(J)=RK(J)/(1.0+ES(J))
   28 continue
c
c       SLOPE OFPERMEABILITY FUNCTION--BETA
c       SLOPE OF EFF STRESS-VOID RATIO CURVE--DSDE
c
      CD=ES(2)-ES(1)
      BETA(1)=PK(1)*(GSBL-1.0)
      DSDE(1)=(RS(2)-RS(1))/CD
      L=LBL-1
      do 29 J=2,L II=J-
      1; IJ=J+1
      CD=ES(IJ)-ES(II)
      BETA(J)=PK(J)*(GSBL-1.0)
      DSDE(J)=(RS(IJ)-RS(II))/CD
   29 continue
      CD=ES(LBL)-ES(L)
      BETA(LBL)=PK(LBL)*(GSBL-1.0)
      DSDE(LBL)=(RS(LBL)-RS(L))/CD
      do 31 J=2,LBL-1 DE=ES(J)-
      ES(J+1)
      ESJPLUSHALF=ES(J)+(DE/2.0)
      N=J
      XV=ESJPLUSHALF
      call LINTP_4(XV,YV,ES,RS,N-1)
      RSJPLUSHALF=YV
      ESJMINUSHALF=ES(J)-(DE/2.0)
      XV=ESJMINUSHALF if(J.eq.LBL-1)
      N=J-1
      call LINTP_4(XV,YV,ES,RS,N)
      RSJMINUSHALF=YV
c DSDE(J)=(RSJPLUSHALF-RSJMINUSHALF)/(ESJPLUSHALF-ESJMINUSHALF)
    31 continue
c
c       PERMEABILITY FUNCTION TIMES DSDE-- ALPHA
c
      do 33 J=1,LBL
      ALPHA(J)=PK(J)*(DSDE(J))*(1.0/GW)
   33 continue
c
c       COMPUTE VOID RATIO FUNCTION FOR INITIAL VALUES
c
      call VRFUNC_4()
c
      return
      end
c
c
c     ********************************************************
      subroutine INTRO_5()
c     ********************************************************
c
c        INTRO PRINTS INPUT DATA AND RESULTS OF INITIAL CALCULATIONS
c        IN TABULAR FORM
```

```
c
      common DA,DB,DZ,E00,ELL,GC,GS,GSBL,GW,HBL,LBL,NBDIV, NDIV,NBDJV,
     >        NDJV,NFLAG,NNN,NTIME,Q0,Q2,WL,SETT,SFIN,TAU,TIME,TPRINT,
     >        UCON,NNTIME,NST,NL,NBC,NDRB,ND1,ND2,NNL,NSOL,NNSOL,NNSOL1,
     >        ALPHAL,ALPHAT,NSTBC,SL,CFT,CFB,HT,HB,CHD0,CHD1,AKD,ALAMDAC,
     >        ALAMDASC,AM,DW,DS,DGM,TH,RKEO,NSL,HCL,HUCL,E0C,E0UC,NSORP,
     >        NSTUBC,NSTBBC,NSTRBC,NSTLBC,PCT,PGT,PRT,PCB,PGB,PRB,PCR,
     >        PGR,PRR,PCL,PGL,PRL,INX,JNZ,IE0,ISS,NC,TTIME,
     >        A(351),B(351),Z(351),XI(351),ALPHA(351),BETA(351),
     >        DSDE(351),E11(351),EFIN(351),ER(351),ES(351),EFFSTR(351),
     >        F(351),FS(351),FINT(351),PK(351),RK(351),RK1(351),RS(351),
     >        TOTSTR(351),U(351),U0(351),UW(351),VRI(351),DQ(351),
     >        Q1(351),RKEI(351),AKP,ANF,ANLAMDA,IKK,UMAX,NTAU,
     >        CHD2(351),CHDA(351),CHDX(351),CQI(351),CQU(351),CQ(351),
     >        EN0(351),EN(351),EG0(351),EG(351),DVDA(351,351),
     >        CS0(351,351),CS1(351,351),CS2(351,351),CF0(351,351),
     >        CF1(351,351),CF2(351,351),CFF1(351,351),CFF2(351,351),
     >        E11JPLUSHALF(351),E11JMINUSHALF(351),DKQU(351),
     >        FJPLUSHALF(351),FJMINUSHALF(351),AFJPLUSHALF(351),
     >        AFJMINUSHALF(351),BFJPLUSHALF(351),BFJMINUSHALF(351),
     >        EGJPLUSHALF(351),EGJMINUSHALF(351),EG0JPLUSHALF(351),
     >        EG0JMINUSHALF(351),CQJPLUSHALF(351),CQJMINUSHALF(351),
     >        CQUJPLUSHALF(351),CQUJMINUSHALF(351),DRD(351),
     >        CF0IPLUSHALF(351,351),CF0IMINUSHALF(351,351),
     >        CF0JPLUSHALF(351,351),CF0JMINUSHALF(351,351),
     >        CF1IPLUSHALF(351,351),CF1IMINUSHALF(351,351),
     >        CF1JPLUSHALF(351,351),CF1JMINUSHALF(351,351),
     >        PRINT1(351)
c
c      PRINT PROBLEM NUMBER AND HEADING
c
      write(18,100)
      write(18,101)
      write(18,102)
      call SETUP_5()
      do 1 J=1,LBL
      write(18,110) J,ES(J),RS(J),RK(J),PK(J),BETA(J),DSDE(J),
     >              ALPHA(J)
    1 continue

c
c FORMATS c
  100 format(1H1////9X,60(1H*))
  101 format(9X,48HCONSOLIDATION OF SOFT LAYERS BY FINITE STRAIN --,
     >        6HCF=0.6)
  102 format(9X,60(1H*))
  110 format(2X,I3,1X,F6.3,6E10.3)
c
      return
      end
c     ************************************************************
      subroutine SETUP_5()
c     ************************************************************
c
c      SETUP MAKES INITIAL CALCULATIONS AND MANIPULATIONS OF INPUT
c      DATA FOR LATER USE
c
      common DA,DB,DZ,E00,ELL,GC,GS,GSBL,GW,HBL,LBL,NBDIV, NDIV,NBDJV,
     >        NDJV,NFLAG,NNN,NTIME,Q0,Q2,WL,SETT,SFIN,TAU,TIME,TPRINT,
     >        UCON,NNTIME,NST,NL,NBC,NDRB,ND1,ND2,NNL,NSOL,NNSOL,NNSOL1,
     >        ALPHAL,ALPHAT,NSTBC,SL,CFT,CFB,HT,HB,CHD0,CHD1,AKD,ALAMDAC,
     >        ALAMDASC,AM,DW,DS,DGM,TH,RKEO,NSL,HCL,HUCL,E0C,E0UC,NSORP,
     >        NSTUBC,NSTBBC,NSTRBC,NSTLBC,PCT,PGT,PRT,PCB,PGB,PRB,PCR,
     >        PGR,PRR,PCL,PGL,PRL,INX,JNZ,IE0,ISS,NC,TTIME,
     >        A(351),B(351),Z(351),XI(351),ALPHA(351),BETA(351),
     >        DSDE(351),E11(351),EFIN(351),ER(351),ES(351),EFFSTR(351),
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     >        F(351),FS(351),FINT(351),PK(351),RK(351),RK1(351),RS(351),
     >        TOTSTR(351),U(351),U0(351),UW(351),VRI(351),DQ(351),
     >        Q1(351),RKEI(351),AKP,ANF,ANLAMDA,IKK,UMAX,NTAU,
     >        CHD2(351),CHDA(351),CHDX(351),CQI(351),CQU(351),CQ(351),
     >        EN0(351),EN(351),EG0(351),EG(351),DVDA(351,351),
     >        CS0(351,351),CS1(351,351),CS2(351,351),CF0(351,351),
     >        CF1(351,351),CF2(351,351),CFF1(351,351),CFF2(351,351),
     >        E11JPLUSHALF(351),E11JMINUSHALF(351),DKQU(351),
     >        FJPLUSHALF(351),FJMINUSHALF(351),AFJPLUSHALF(351),
     >        AFJMINUSHALF(351),BFJPLUSHALF(351),BFJMINUSHALF(351),
     >        EGJPLUSHALF(351),EGJMINUSHALF(351),EG0JPLUSHALF(351),
     >        EG0JMINUSHALF(351),CQJPLUSHALF(351),CQJMINUSHALF(351),
     >        CQUJPLUSHALF(351),CQUJMINUSHALF(351),DRD(351),
     >        CF0IPLUSHALF(351,351),CF0IMINUSHALF(351,351),
     >        CF0JPLUSHALF(351,351),CF0JMINUSHALF(351,351),
     >        CF1IPLUSHALF(351,351),CF1IMINUSHALF(351,351),
     >        CF1JPLUSHALF(351,351),CF1JMINUSHALF(351,351),
     >        PRINT1(351)
c
c CALCULATE FUNCTIONS FOR COMPRESSIBLE SOIL LAYER
c PERMEABILITY FUNCTIONS
c
      do 28 J=1,LBL
      PK(J)=RK(J)/(1.0+ES(J))
   28 continue
c
c       SLOPE OFPERMEABILITY FUNCTION--BETA
c       SLOPE OF EFF STRESS-VOID RATIO CURVE--DSDE
c
      CD=ES(2)-ES(1)
      BETA(1)=PK(1)*(GSBL-1.0)
      DSDE(1)=(RS(2)-RS(1))/CD
      L=LBL-1
      do 29 J=2,L
      II=J-1; IJ=J+1
      CD=ES(IJ)-ES(II)
      BETA(J)=PK(J)*(GSBL-1.0)
      DSDE(J)=(RS(IJ)-RS(II))/CD
   29 continue
      CD=ES(LBL)-ES(L)
      BETA(LBL)=PK(LBL)*(GSBL-1.0)
      DSDE(LBL)=(RS(LBL)-RS(L))/CD
      do 31 J=2,LBL-1
      DE=ES(J)-ES(J+1)
      ESJPLUSHALF=ES(J)+(DE/2.0)
      N=J
      XV=ESJPLUSHALF
      call LINTP_4(XV,YV,ES,RS,N-1)
      RSJPLUSHALF=YV
      ESJMINUSHALF=ES(J)-(DE/2.0)
      XV=ESJMINUSHALF
      if(J.eq.LBL-1) N=J-1
      call LINTP_4(XV,YV,ES,RS,N)
      RSJMINUSHALF=YV
c DSDE(J)=(RSJPLUSHALF-RSJMINUSHALF)/(ESJPLUSHALF-ESJMINUSHALF)
    31 continue
c
c       PERMEABILITY FUNCTION TIMES DSDE-- ALPHA
c
      do 33 J=1,LBL
      ALPHA(J)=PK(J)*(DSDE(J))*(1.0/GW)
   33 continue
c
      return
      end
c
```

```
c     *******************************************************************
      subroutine    EHALFVALUE_4()
c     *******************************************************************
c
      common DA,DB,DZ,E00,ELL,GC,GS,GSBL,GW,HBL,LBL,NBDIV, NDIV,NBDJV,
     >        NDJV,NFLAG,NNN,NTIME,Q0,Q2,WL,SETT,SFIN,TAU,TIME,TPRINT,
     >        UCON,NNTIME,NST,NL,NBC,NDRB,ND1,ND2,NNL,NSOL,NNSOL,NNSOL1,
     >        ALPHAL,ALPHAT,NSTBC,SL,CFT,CFB,HT,HB,CHD0,CHD1,AKD,ALAMDAC,
     >        ALAMDASC,AM,DW,DS,DGM,TH,RKEO,NSL,HCL,HUCL,E0C,E0UC,NSORP,
     >        NSTUBC,NSTBBC,NSTRBC,NSTLBC,PCT,PGT,PRT,PCB,PGB,PRB,PCR,
     >        PGR,PRR,PCL,PGL,PRL,INX,JNZ,IE0,ISS,NC,TTIME,
     >        A(351),B(351),Z(351),XI(351),ALPHA(351),BETA(351),
     >        DSDE(351),E11(351),EFIN(351),ER(351),ES(351),EFFSTR(351),
     >        F(351),FS(351),FINT(351),PK(351),RK(351),RK1(351),RS(351),
     >        TOTSTR(351),U(351),U0(351),UW(351),VRI(351),DQ(351),
     >        Q1(351),RKEI(351),AKP,ANF,ANLAMDA,IKK,UMAX,NTAU,
     >        CHD2(351),CHDA(351),CHDX(351),CQI(351),CQU(351),CQ(351),
     >        EN0(351),EN(351),EG0(351),EG(351),DVDA(351,351),
     >        CS0(351,351),CS1(351,351),CS2(351,351),CF0(351,351),
     >        CF1(351,351),CF2(351,351),CFF1(351,351),CFF2(351,351),
     >        E11JPLUSHALF(351),E11JMINUSHALF(351),DKQU(351),
     >        FJPLUSHALF(351),FJMINUSHALF(351),AFJPLUSHALF(351),
     >        AFJMINUSHALF(351),BFJPLUSHALF(351),BFJMINUSHALF(351),
     >        EGJPLUSHALF(351),EGJMINUSHALF(351),EG0JPLUSHALF(351),
     >        EG0JMINUSHALF(351),CQJPLUSHALF(351),CQJMINUSHALF(351),
     >        CQUJPLUSHALF(351),CQUJMINUSHALF(351),DRD(351),
     >        CF0IPLUSHALF(351,351),CF0IMINUSHALF(351,351),
     >        CF0JPLUSHALF(351,351),CF0JMINUSHALF(351,351),
     >        CF1IPLUSHALF(351,351),CF1IMINUSHALF(351,351),
     >        CF1JPLUSHALF(351,351),CF1JMINUSHALF(351,351),
     >        PRINT1(351)
c
c       CALCULATE CURRENT FACE VALUES OF FOUNDATION SOIL
c
      if(NDRB.eq.1) then do
      2 J=2,LBL C1=F(NDJV)-
      ES(J) if(C1.ge.0.0)
      goto 3
    2 continue
      DSED=DSDE(LBL); goto 4
    3 II=J-1
      XV=F(NDJV)
      if(J.eq.LBL) II=II-1
      call   LINTP_4(XV,YV,ES,DSDE,II)
      DSED=YV
    4 F(NDJV+1)=F(NDJV-1)-(DA/DSED)*(GC/(1.0+E11(NDJV)))
      endif
      if(NDRB.eq.2.or.NDRB.eq.3)   then
      FJPLUSHALF(NDJV-1)=EFIN(NDJV)
      F(NDJV+1)=(8.0/3.0)*FJPLUSHALF(NDJV-1)-2.0*F(NDJV-1)+
     >          (1.0/3.0)*F(NDJV-2)
      endif
      do 1 J=2,NDJV-3
      if(abs(F(J)-F(J+2)).le.0.00001) then
      FJPLUSHALF(J)=(3.0/8.0)*F(J)+(6.0/8.0)*F(J+1)-(1.0/8.0)*F(J+2)
      endif
      if(abs(F(J)-F(J+2)).le.0.00001) goto 1
      if(abs(F(J+2)-2.0*F(J)+F(J+1)).le.(0.3*abs(F(J)-F(J+2)))) then
      FJPLUSHALF(J)=(3.0/8.0)*F(J)+(6.0/8.0)*F(J+1)-(1.0/8.0)*F(J+2)
      endif
      if((abs(F(J+2)-2.0*F(J)+F(J+1))).gt.(0.3*abs(F(J)-F(J+2)))) then
      FBAR3=(F(J+1)-F(J+2))/(F(J)-F(J+2))
      if(FBAR3.le.(-1.0).or.FBAR3.ge.1.5) then
      FJPLUSHALFBAR2=0.75+0.75*(FBAR3-0.5)
      FJPLUSHALF(J)=F(J+2)+(F(J)-F(J+2))*FJPLUSHALFBAR2
      endif
      if(FBAR3.ge.0.35.and.FBAR3.le.0.65) then
      FJPLUSHALFBAR2=0.75+0.75*(FBAR3-0.5)
```

```
  FJPLUSHALF(J)=F(J+2)+(F(J)-F(J+2))*FJPLUSHALFBAR2
  endif
   if(FBAR3.le.0.0.and.FBAR3.gt.(-1.0)) then
   FJPLUSHALFBAR2=0.375*FBAR3
   FJPLUSHALF(J)=F(J+2)+(F(J)-F(J+2))*FJPLUSHALFBAR2
   endif
   if(FBAR3.gt.0.0.and.FBAR3.lt.0.35) then
  FJPLUSHALFBAR2=(sqrt(FBAR3*((1.0-FBAR3)**3))-FBAR3**2)/
> (1.0-2.0*FBAR3)
FJPLUSHALF(J)=F(J+2)+(F(J)-F(J+2))*FJPLUSHALFBAR2
endif
   if(FBAR3.gt.0.65.and.FBAR3.le.1.0) then
  FJPLUSHALFBAR2=(sqrt(FBAR3*((1.0-FBAR3)**3))-FBAR3**2)/
> (1.0-2.0*FBAR3)
FJPLUSHALF(J)=F(J+2)+(F(J)-F(J+2))*FJPLUSHALFBAR2
endif
   if(FBAR3.le.1.5.and.FBAR3.gt.1.0) then
   FJPLUSHALFBAR2=FBAR3
   FJPLUSHALF(J)=F(J+2)+(F(J)-F(J+2))*FJPLUSHALFBAR2
   endif
   endif
1  continue
2  if(abs(F(NDJV-2)-F(NDJV+1)).le.0.00001) then
   FJPLUSHALF(NDJV-2)=(3.0/8.0)*F(NDJV-2)+(6.0/8.0)*F(NDJV-1)-
  > (1.0/8.0)*F(NDJV+1)
  endif
   if(abs(F(NDJV-2)-F(NDJV+1)).le.0.00001) goto 605
   if(abs(F(NDJV+1)-2.0*F(NDJV-1)+F(NDJV-2)).le.
  >  (0.3*abs(F(NDJV-2)-F(NDJV+1)))) then
   FJPLUSHALF(NDJV-2)=(3.0/8.0)*F(NDJV-2)+(6.0/8.0)*F(NDJV-1)-
  >                  (1.0/8.0)*F(NDJV+1)
   endif
   if(abs(F(NDJV+1)-2.0*F(NDJV-1)+F(NDJV-2)).gt.(0.3*
  >   abs(F(NDJV-2)-F(NDJV+1)))) then
   FBAR3=(F(NDJV-1)-F(NDJV+1))/(F(NDJV-2)-F(NDJV+1))
   if(FBAR3.le.(-1.0).or.FBAR3.ge.1.5) then
   FJPLUSHALFBAR2=0.75+0.75*(FBAR3-0.5)
   FJPLUSHALF(NDJV-2)=F(NDJV+1)+(F(NDJV-2)-
  >                  F(NDJV+1))*FJPLUSHALFBAR2
   endif
   if(FBAR3.ge.0.35.and.FBAR3.le.0.65) then
   FJPLUSHALFBAR2=0.75+0.75*(FBAR3-0.5)
   FJPLUSHALF(NDJV-2)=F(NDJV+1)+(F(NDJV-2)-
  >                  F(NDJV+1))*FJPLUSHALFBAR2
   endif
   if(FBAR3.le.0.0.and.FBAR3.gt.(-1.0)) then
   FJPLUSHALFBAR2=0.375*FBAR3
   FJPLUSHALF(NDJV-2)=F(NDJV+1)+(F(NDJV-2)-
  >                  F(NDJV+1))*FJPLUSHALFBAR2
   endif
   if(FBAR3.gt.0.0.and.FBAR3.lt.0.35) then
   FJPLUSHALFBAR2=(sqrt(FBAR3*((1.0-FBAR3)**3))-FBAR3**2)/
  >                 (1.0-2.0*FBAR3)
   FJPLUSHALF(NDJV-2)=F(NDJV+1)+(F(NDJV-2)-
  >                  F(NDJV+1))*FJPLUSHALFBAR2
   endif
   if(FBAR3.gt.0.65.and.FBAR3.le.1.0) then
   FJPLUSHALFBAR2=(sqrt(FBAR3*((1.0-FBAR3)**3))-FBAR3**2)/
  >                 (1.0-2.0*FBAR3)
   FJPLUSHALF(NDJV-2)=F(NDJV+1)+(F(NDJV-2)-
  >                  F(NDJV+1))*FJPLUSHALFBAR2
   endif
   if(FBAR3.le.1.5.and.FBAR3.gt.1.0) then
   FJPLUSHALFBAR2=FBAR3
   FJPLUSHALF(NDJV-2)=F(NDJV+1)+(F(NDJV-2)-
  >                  F(NDJV+1))*FJPLUSHALFBAR2
   endif
```

```
      endif
  605 continue
      do 5 J=3,NDJV-1
      FJMINUSHALF(J)=FJPLUSHALF(J-1)
    5 continue
      FJMINUSHALF(2)=F(1)
      FJPLUSHALF(NDJV-1)=F(NDJV)
c
c     CALCULATE INITIAL FACE VALUES OF SOIL LAYER
c
      do 6 J=3,NDJV-2
      if(J.eq.NDJV-2) goto 7
      E11JPLUSHALF(J)=(3.0/8.0)*E11(J)+(6.0/8.0)*E11(J+1)
     >                 -(1.0/8.0)*E11(J+2)
    7 E11JMINUSHALF(J)=(3.0/8.0)*E11(J-1)+(6.0/8.0)*E11(J)
     >                 -(1.0/8.0)*E11(J+1)
    6 continue
      E11JPLUSHALF(NDJV-1)=E11(NDJV)
      E11JMINUSHALF(NDJV-1)=(-1.0/8.0)*E11(NDJV-3)+(6.0/8.0)*
     >                        E11(NDJV-2)+(3.0/8.0)*E11(NDJV-1)
      E11JPLUSHALF(NDJV-2)=(-1.0/8.0)*E11(NDJV-3)+(6.0/8.0)*
     >                        E11(NDJV-2)+(3.0/8.0)*E11(NDJV-1)
      E11JPLUSHALF(2)=(3.0/8.0)*E11(2)+(6.0/8.0)*E11(3)-(1.0/8.0)*E11(4)
      E11JMINUSHALF(2)=E11(1)
c
      return
      end
c
c     ***********************************************************************
      subroutine    LINTP_4(X,Y,XVAL,YVAL,IL)
c     ***********************************************************************

      dimension XVAL(100),YVAL(100)
c
c     INTERPOLATE Y FOR GIVEN X USING LAGRANGIAN INTERPOLATION
c
      Y1=(((X-XVAL(IL+1))*(X-XVAL(IL+2)))/
     >((XVAL(IL)-XVAL(IL+1))*(XVAL(IL)-XVAL(IL+2))))*YVAL(IL)
      Y2=(((X-XVAL(IL))*(X-XVAL(IL+2)))/
     >((XVAL(IL+1)-XVAL(IL))*(XVAL(IL+1)-XVAL(IL+2))))*YVAL(IL+1)
      Y3=(((X-XVAL(IL))*(X-XVAL(IL+1)))/
     >((XVAL(IL+2)-XVAL(IL))*(XVAL(IL+2)-XVAL(IL+1))))*YVAL(IL+2)
      Y=Y1+Y2+Y3
c
      return
      end
c
c     *************************************************************
      subroutine FDIFEQ_4()
c     *************************************************************
c
c     FDIFEQ CALCULATES NEW VOID RATIOS AS CONSOLIDATION PROCEEDS BY
c     AN EXPLICIT FINITE DIFFERENCE SCHEME BASED ON PREVIOUS VOID RATIOS.
c     SOIL PARAMETER FUNCTIONS ARE CONSTANTLY UPDATED TO CORRESPOND
c     WITH CURRENT VOID RATIO.
c
      common DA,DB,DZ,E00,ELL,GC,GS,GSBL,GW,HBL,LBL,NBDIV, NDIV,NBDJV,
     >       NDJV,NFLAG,NNN,NTIME,Q0,Q2,WL,SETT,SFIN,TAU,TIME,TPRINT,
     >       UCON,NNTIME,NST,NL,NBC,NDRB,ND1,ND2,NNL,NSOL,NNSOL,NNSOL1,
     >       ALPHAL,ALPHAT,NSTBC,SL,CFT,CFB,HT,HB,CHD0,CHD1,AKD,ALAMDAC,
     >       ALAMDASC,AM,DW,DS,DGM,TH,RKEO,NSL,HCL,HUCL,E0C,E0UC,NSORP,
     >       NSTUBC,NSTBBC,NSTRBC,NSTLBC,PCT,PGT,PRT,PCB,PGB,PRB,PCR,
     >       PGR,PRR,PCL,PGL,PRL,INX,JNZ,IE0,ISS,NC,TTIME,
     >       A(351),B(351),Z(351),XI(351),ALPHA(351),BETA(351),
     >       DSDE(351),E11(351),EFIN(351),ER(351),ES(351),EFFSTR(351),
     >       F(351),FS(351),FINT(351),PK(351),RK(351),RK1(351),RS(351),
     >       TOTSTR(351),U(351),U0(351),UW(351),VRI(351),DQ(351),
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     >        Q1(351),RKEI(351),AKP,ANF,ANLAMDA,IKK,UMAX,NTAU,
     >        CHD2(351),CHDA(351),CHDX(351),CQI(351),CQU(351),CQ(351),
     >        EN0(351),EN(351),EG0(351),EG(351),DVDA(351,351),
     >        CS0(351,351),CS1(351,351),CS2(351,351),CF0(351,351),
     >        CF1(351,351),CF2(351,351),CFF1(351,351),CFF2(351,351),
     >        E11JPLUSHALF(351),E11JMINUSHALF(351),DKQU(351),
     >        FJPLUSHALF(351),FJMINUSHALF(351),AFJPLUSHALF(351),
     >        AFJMINUSHALF(351),BFJPLUSHALF(351),BFJMINUSHALF(351),
     >        EGJPLUSHALF(351),EGJMINUSHALF(351),EG0JPLUSHALF(351),
     >        EG0JMINUSHALF(351),CQJPLUSHALF(351),CQJMINUSHALF(351),
     >        CQUJPLUSHALF(351),CQUJMINUSHALF(351),DRD(351),
     >        CF0IPLUSHALF(351,351),CF0IMINUSHALF(351,351),
     >        CF0JPLUSHALF(351,351),CF0JMINUSHALF(351,351),
     >        CF1IPLUSHALF(351,351),CF1IMINUSHALF(351,351),
     >        CF1JPLUSHALF(351,351),CF1JMINUSHALF(351,351),
     >        PRINT1(351)
c
c        LOOP THROUGH FINITE DIFFERENCE EQUATIONS UNTIL PRINT TIME
c
c        CALCULATE VOID RATIO FOR IMAGE POINT AND FIRST REAL POINT
c
c       LOWER BOUNDARY OF COMPRESSIBLE LAYER
c
      if(NL.eq.1) goto 301
    1 continue
c
c      APPLY BOTTOM BOUNDARY CONDITION
c
  301 if(NDRB.eq.1.or.NDRB.eq.3) then FJMINUSHALF(2)=EFIN(1)
      F0=(8.0/3.0)*FJMINUSHALF(2)-2.0*F(2)+(1.0/3.0)*F(3)
      endif
      if(NDRB.eq.2.and.NSOL.ne.1) then
      do 52 J=2,LBL
      C1=F(1)-ES(J)
      if(C1.ge.0.0) goto 53
   52 continue DSED=DSDE(LBL);
      goto 54
   53 II=J-1
      XV=F(1)
      if(J.eq.LBL) II=II-1
      call   LINTP_4(XV,YV,ES,DSDE,II)
      DSED=YV
   54 F0=F(2)+(DA/DSED)*(GC/(1.0+E11(2)))
      endif
      if(NSOL.eq.1.and.NDRB.eq.2)   then
      do 55 J=2,LBL
      C1=F(1)-ES(J)
      if(C1.ge.0.0) goto 56
   55 continue DSED=DSDE(LBL);
      goto 57
   56 II=J-1
      XV=F(1)
      if(J.eq.LBL) II=II-1
      call   LINTP_4(XV,YV,ES,DSDE,II)
      DSED=YV
   57 F0=F(2)+(DA/DSED)*(GC/(1.0+E11(1)))
      endif
      if(abs(F0-F(3)).le.0.00001) then
      FJMINUSHALF(2)=(3.0/8.0)*F0+(6.0/8.0)*F(2)-(1.0/8.0)*F(3)
      endif
      if(abs(F0-F(3)).le.0.00001) goto 607
      if(abs(F(3)-2.0*F(2)+F0).le.(0.3*abs(F0-F(3)))) then
      FJMINUSHALF(2)=(3.0/8.0)*F0+(6.0/8.0)*F(2)-(1.0/8.0)*F(3)
      endif
      if(abs(F(3)-2.0*F(2)+F0).gt.(0.3*abs(F0-F(3))))then
      FBAR3=(F(2)-F(3))/(F0-F(3))
```

```
     if(FBAR3.le.(-1.0).or.FBAR3.ge.1.5) then
     FJMINUSHALFBAR2=0.75+0.75*(FBAR3-0.5)
     FJMINUSHALF(2)=F(3)+(F0-F(3))*FJMINUSHALFBAR2
     endif
     if(FBAR3.ge.0.35.and.FBAR3.le.0.65) then
     FJMINUSHALFBAR2=0.75+0.75*(FBAR3-0.5)
     FJMINUSHALF(2)=F(3)+(F0-F(3))*FJMINUSHALFBAR2
     endif
     if(FBAR3.le.0.0.and.FBAR3.gt.(-1.0)) then
     FJMINUSHALFBAR2=0.375*FBAR3
     FJMINUSHALF(2)=F(3)+(F0-F(3))*FJMINUSHALFBAR2
     endif
     if(FBAR3.gt.0.0.and.FBAR3.lt.0.35) then
    FJMINUSHALFBAR2=(sqrt(FBAR3*((1.0-FBAR3)**3))-FBAR3**2)/
     > (1.0-2.0*FBAR3)
     FJMINUSHALF(2)=F(3)+(F0-F(3))*FJMINUSHALFBAR2
     endif
     if(FBAR3.gt.0.65.and.FBAR3.le.1.0) then
     FJMINUSHALFBAR2=(sqrt(FBAR3*((1.0-FBAR3)**3))-FBAR3**2)/
     > (1.0-2.0*FBAR3)
     FJMINUSHALF(2)=F(3)+(F0-F(3))*FJMINUSHALFBAR2
     endif if(FBAR3.le.1.5.and.FBAR3.gt.1.0)
     then FJMINUSHALFBAR2=FBAR3
     FJMINUSHALF(2)=F(3)+(F0-F(3))*FJMINUSHALFBAR2
     endif
     endif
607 if(abs(F(2)-F(4)).le.0.00001) then
     FJPLUSHALF(2)=(3.0/8.0)*F(2)+(6.0/8.0)*F(3)-(1.0/8.0)*F(4)
     endif
     if(abs(F(2)-F(4)).le.0.00001) goto 606
     if(abs(F(4)-2.0*F(3)+F(2)).le.(0.3*abs(F(2)-F(4)))) then
     FJPLUSHALF(2)=(3.0/8.0)*F(2)+(6.0/8.0)*F(3)-(1.0/8.0)*F(4)
     endif if(abs(F(4)-2.0*F(3)+F(2)).gt.(0.3*abs(F(2)-F(4))))then
     FBAR3=(F(3)-F(4))/(F(2)-F(4))
     if(FBAR3.le.(-1.0).or.FBAR3.ge.1.5) then
     FJPLUSHALFBAR2=0.75+0.75*(FBAR3-0.5)
     FJPLUSHALF(2)=F(4)+(F(2)-F(4))*FJPLUSHALFBAR2
     endif
     if(FBAR3.ge.0.35.and.FBAR3.le.0.65) then
     FJPLUSHALFBAR2=0.75+0.75*(FBAR3-0.5)
     FJPLUSHALF(2)=F(4)+(F(2)-F(4))*FJPLUSHALFBAR2
     endif
     if(FBAR3.le.0.0.and.FBAR3.gt.(-1.0)) then
     FJPLUSHALFBAR2=0.375*FBAR3
     FJPLUSHALF(2)=F(4)+(F(2)-F(4))*FJPLUSHALFBAR2
     endif
     if(FBAR3.gt.0.0.and.FBAR3.lt.0.35) then
     FJPLUSHALFBAR2=(sqrt(FBAR3*((1.0-FBAR3)**3))-FBAR3**2)/
    > (1.0-2.0*FBAR3)
     FJPLUSHALF(2)=F(4)+(F(2)-F(4))*FJPLUSHALFBAR2
     endif
     if(FBAR3.gt.0.65.and.FBAR3.le.1.0) then
     FJPLUSHALFBAR2=(sqrt(FBAR3*((1.0-FBAR3)**3))-FBAR3**2)/
    > (1.0-2.0*FBAR3)
     FJPLUSHALF(2)=F(4)+(F(2)-F(4))*FJPLUSHALFBAR2
     endif
     if(FBAR3.le.1.5.and.FBAR3.gt.1.0) then
     FJPLUSHALFBAR2=FBAR3
     FJPLUSHALF(2)=F(4)+(F(2)-F(4))*FJPLUSHALFBAR2
     endif
     endif
606 if(NSOL.eq.1) then
     do 75 N=2,LBL
     C1=FJPLUSHALF(2)-ES(N)
     if(C1.ge.0.0) goto 76
```

```
   75 continue
   76 AFJPLUSHALF(2)=ALPHA(LBL)
      BFJPLUSHALF(2)=BETA(LBL); goto 77
   77 NN=N-1
      XV=FJPLUSHALF(2)
      if(N.eq.LBL) NN=NN-1
      call   LINTP_4(XV,YV,ES,ALPHA,NN)
      AFJPLUSHALF(2)=YV
      call   LINTP_4(XV,YV,ES,BETA,NN)
      BFJPLUSHALF(2)=YV
   78 do 78 J=2,LBL
      C1=FJMINUSHALF(2)-ES(J)
      if(C1.ge.0.0) goto 79
   79 continue AFJMINUSHALF(2)=ALPHA(LBL)
      BFJMINUSHALF(2)=BETA(LBL); goto 10
   80 II=J-1
      XV=FJMINUSHALF(2)
      if(J.eq.LBL) II=II-1
      call   LINTP_4(XV,YV,ES,ALPHA,II)
      AFJMINUSHALF(2)=YV
      call   LINTP_4(XV,YV,ES,BETA,II)
      BFJMINUSHALF(2)=YV
      endif if(NSOL.ne.1)
      then do 5 N=2,LBL
      C1=FJPLUSHALF(2)-ES(N)
      if(C1.ge.0.0) goto 6
    5 continue AFJPLUSHALF(2)=ALPHA(LBL)
      BFJPLUSHALF(2)=BETA(LBL); goto 7
    6 NN=N-1
      XV=FJPLUSHALF(2)
      if(N.eq.LBL) NN=NN-1
      call   LINTP_4(XV,YV,ES,ALPHA,NN)
      AFJPLUSHALF(2)=YV
      call   LINTP_4(XV,YV,ES,BETA,NN)
      BFJPLUSHALF(2)=YV
    7 do 8 J=2,LBL
      C1=FJMINUSHALF(2)-ES(J)
      if(C1.ge.0.0) goto 9
    8 continue AFJMINUSHALF(2)=ALPHA(LBL)
      BFJMINUSHALF(2)=BETA(LBL); goto 10
    9 II=J-1
      XV=FJMINUSHALF(2)
      if(J.eq.LBL) II=II-1
      call   LINTP_4(XV,YV,ES,ALPHA,II)
      AFJMINUSHALF(2)=YV
      call   LINTP_4(XV,YV,ES,BETA,II)
      BFJMINUSHALF(2)=YV
      endif
   10 FFPLUS=BFJPLUSHALF(2)*(1.0+E11JPLUSHALF(2))+(AFJPLUSHALF(2)*
     >(1.0+E11JPLUSHALF(2)**2)*((F(3)-F(2))/DA))
      FFMINUS=BFJMINUSHALF(2)*(1.0+E11JMINUSHALF(2))+(AFJMINUSHALF(2)
     >*(1.0+E11JMINUSHALF(2)**2)*((F(2)-F0)/DA))
      if(FFMINUS.gt.FFPLUS) FFMINUS=FFPLUS
      ER(2)=F(2)-((TAU/DA)*(FFPLUS-FFMINUS))
      DKQU(2)=(FFPLUS-FFMINUS)*(1.0/DA)
      if(ER(2).eq.0.0) ER(2)=F(2)
      if(NDRB.eq.3.or.NDRB.eq.1) ER(1)=EFIN(1)
c
c       TOP BOUNDARY OF COMPRESSIBLE SOIL LAYER
c
      if(NDRB.eq.1) then
      do 2 J=2,LBL
      C1=F(NDJV)-ES(J)
```

```
      if(C1.ge.0.0) goto 3
    2 continue
      DSED=DSDE(LBL); goto 4
    3 II=J-1
      XV=F(NDJV)
      if(J.eq.LBL) II=II-1
      call   LINTP_4(XV,YV,ES,DSDE,II)
      DSED=YV
    4 F(NDJV+1)=F(NDJV-1)-(DA/DSED)*(GC/(1.0+E11(NDJV)))
      FJPLUSHALF(NDJV-1)=(-1.0/8.0)*F(NDJV-2)+(6.0/8.0)*F(NDJV-1)
     >                      +(3.0/8.0)*F(NDJV+1)
      endif
      if(NDRB.eq.2.or.NDRB.eq.3) then
      FJPLUSHALF(NDJV-1)=EFIN(NDJV)
      F(NDJV+1)=(8.0/3.0)*FJPLUSHALF(NDJV-1)-2.0*F(NDJV-1)+
     >          (1.0/3.0)*F(NDJV-2)
      endif
      if(abs(F(NDJV-2)-F(NDJV+1)).le.0.00001) then
      FJMINUSHALF(NDJV-1)=(3.0/8.0)*F(NDJV-2)+(6.0/8.0)*
     >                     F(NDJV-1)-(1.0/8.0)*F(NDJV+1)
      endif
      if(abs(F(NDJV-2)-F(NDJV+1)).le.0.00001) goto 608
      if(abs(F(NDJV+1)-2.0*F(NDJV-1)+F(NDJV-2)).le.
     >   (0.3*abs(F(NDJV-2)-F(NDJV+1)))) then
      FJMINUSHALF(NDJV-1)=(3.0/8.0)*F(NDJV-2)+(6.0/8.0)*
     >                     F(NDJV-1)-(1.0/8.0)*F(NDJV+1)
      endif
      if(abs(F(NDJV+1)-2.0*F(NDJV-1)+F(NDJV-2)).gt.
     >  (0.3*abs(F(NDJV-2)-F(NDJV+1))))then
      FBAR2=(F(NDJV-1)-F(NDJV+1))/(F(NDJV-2)-F(NDJV+1))
      if(FBAR2.le.(-1.0).or.FBAR2.ge.1.5) then
      FJMINUSHALFBAR2=0.75+0.75*(FBAR2-0.5)
      FJMINUSHALF(NDJV-1)=F(NDJV+1)+(F(NDJV-2)-
     >                    F(NDJV+1))*FJMINUSHALFBAR2
      endif
      if(FBAR2.ge.0.35.and.FBAR2.le.0.65) then
      FJMINUSHALFBAR2=0.75+0.75*(FBAR2-0.5)
      FJMINUSHALF(NDJV-1)=F(NDJV+1)+(F(NDJV-2)-
     >                    F(NDJV+1))*FJMINUSHALFBAR2
      endif
      if(FBAR2.le.0.0.and.FBAR2.gt.(-1.0)) then
      FJMINUSHALFBAR2=0.375*FBAR2
      FJMINUSHALF(NDJV-1)=F(NDJV+1)+(F(NDJV-2)-
     >                    F(NDJV+1))*FJMINUSHALFBAR2
      endif
      if(FBAR2.gt.0.0.and.FBAR2.lt.0.35) then
      FJMINUSHALFBAR2=(sqrt(FBAR2*((1.0-FBAR2)**3))-FBAR2**2)/
     >                  (1.0-2.0*FBAR2)
      FJMINUSHALF(NDJV-1)=F(NDJV+1)+(F(NDJV-2)-
     >                    F(NDJV+1))*FJMINUSHALFBAR2
      endif
      if(FBAR2.gt.0.65.and.FBAR2.le.1.0) then
      FJMINUSHALFBAR2=(sqrt(FBAR2*((1.0-FBAR2)**3))-FBAR2**2)/
     >                  (1.0-2.0*FBAR2)
      FJMINUSHALF(NDJV-1)=F(NDJV+1)+(F(NDJV-2)-
     >                   F(NDJV+1))*FJMINUSHALFBAR2
      endif
      if(FBAR2.le.1.5.and.FBAR2.gt.1.0) then
      FJMINUSHALFBAR2=FBAR2
      FJMINUSHALF(NDJV-1)=F(NDJV+1)+(F(NDJV-2)-
     >                    F(NDJV+1))*FJMINUSHALFBAR2
      endif
      endif
c     FIMINUSHALF(NDJV-1)=(3.0/8.0)*F(NDJV-2)+(6.0/8.0)*F(NDJV-1)
c    >                      -(1.0/8.0)*F(NDJV+1)
  608 if(NSOL.eq.1) then
      do 17 N=2,LBL
```

```
      C1=FJPLUSHALF(NDJV-1)-ES(N)
      if(C1.ge.0.0) goto 18
   17 continue AFJPLUSHALF(NDJV-1)=ALPHA(LBL)
      BFJPLUSHALF(NDJV-1)=BETA(LBL); goto 19
   18 NN=N-1
      XV=FJPLUSHALF(NDJV-1)
      if(N.eq.LBL) NN=NN-1
      call   LINTP_4(XV,YV,ES,ALPHA,NN)
      AFJPLUSHALF(NDJV-1)=YV
      call   LINTP_4(XV,YV,ES,BETA,NN)
      BFJPLUSHALF(NDJV-1)=YV
   19 do 20 J=2,LBL
      C1=FJMINUSHALF(NDJV-1)-ES(J)
      if(C1.ge.0.0) goto 21
   20 continue AFJMINUSHALF(NDJV-1)=ALPHA(LBL)
      BFJMINUSHALF(NDJV-1)=BETA(LBL); goto 22
   21 II=J-1
      XV=FJMINUSHALF(NDJV-1)
      if(J.eq.LBL) II=II-1
      call   LINTP_4(XV,YV,ES,ALPHA,II)
      AFJMINUSHALF(NDJV-1)=YV
      call   LINTP_4(XV,YV,ES,BETA,II)
      BFJMINUSHALF(NDJV-1)=YV
      endif if(NSOL.ne.1)
      then do 171 N=2,LBL
      C1=FJPLUSHALF(NDJV-1)-ES(N)
      if(C1.ge.0.0) goto 181
  171 continue AFJPLUSHALF(NDJV-1)=ALPHA(LBL)
      BFJPLUSHALF(NDJV-1)=BETA(LBL); goto 191
  181 NN=N-1
      XV=FJPLUSHALF(NDJV-1)
      if(N.eq.LBL) NN=NN-1
      call   LINTP_4(XV,YV,ES,ALPHA,NN)
      AFJPLUSHALF(NDJV-1)=YV
      call LINTP_4(XV,YV,ES,BETA,NN)
      BFJPLUSHALF(NDJV-1)=YV
  191 do 201 J=2,LBL
      C1=FJMINUSHALF(NDJV-1)-ES(J)
      if(C1.ge.0.0) goto 211
  201 continue AFJMINUSHALF(NDJV-1)=ALPHA(LBL)
      BFJMINUSHALF(NDJV-1)=BETA(LBL); goto 22
  211 II=J-1
      XV=FJMINUSHALF(NDJV-1)
      if(J.eq.LBL) II=II-1
      call   LINTP_4(XV,YV,ES,ALPHA,II)
      AFJMINUSHALF(NDJV-1)=YV
      call   LINTP_4(XV,YV,ES,BETA,II)
      BFJMINUSHALF(NDJV-1)=YV
      endif
   22 FFPLUS=BFJPLUSHALF(NDJV-1)*(1.0+E11JPLUSHALF(NDJV-1))+
     >       (AFJPLUSHALF(NDJV-1)*(1.0+E11JPLUSHALF(NDJV-1)**2)
     >*((F(NDJV+1)-F(NDJV-1))/DA))
      FFMINUS=BFJMINUSHALF(NDJV-1)*(1.0+E11JMINUSHALF(NDJV-1))+
     >         (AFJMINUSHALF(NDJV-1)*(1.0+E11JMINUSHALF(NDJV-1)**2)
     >         *((F(NDJV-1)-F(NDJV-2))/DA))
      if(FFMINUS.gt.FFPLUS) FFMINUS=FFPLUS
      ER(NDJV-1)=F(NDJV-1)-((TAU/DA)*(FFPLUS-FFMINUS))
      DKQU(NDJV-1)=(FFPLUS-FFMINUS)*(1.0/DA)
      if(ER(NDJV-1).eq.0.0) ER(NDJV-1)=F(NDJV-1)
      if(NDRB.eq.2.or.NDRB.eq.3) ER(NDJV)=EFIN(NDJV)
c
c     CACULATE ALPHA AND BETA FOR CUURRENT VOID RATIOS
```

```
c
      call VRFUNC_4()
c
c CALCULATE NEW VOID RATIOS FOR REMAINDER NODES OF SOIL LAYER c
      do 47 J=3,NDJV-2
      FFPLUS=BFJPLUSHALF(J)*(1.0+E11JPLUSHALF(J))+(AFJPLUSHALF(J)*
     >(1.0+E11JPLUSHALF(J)**2)*((F(J+1)-F(J))/DA))
      FFMINUS=BFJMINUSHALF(J)*(1.0+E11JMINUSHALF(J))+(AFJMINUSHALF(J)*
     >        (1.0+E11JMINUSHALF(J)**2)*((F(J)-F(J-1))/DA))
      if(FFMINUS.gt.FFPLUS) FFMINUS=FFPLUS
      ER(J)=F(J)-((TAU/DA)*(FFPLUS-FFMINUS))
      DKQU(J)=(FFPLUS-FFMINUS)*(1.0/DA)
      if(ER(J).eq.0.0) ER(J)=F(J)
   47 continue
      if(ER(I).gt.E11(I)) ER(I)=E11(I)
      if(ER(I).lt.EFIN(I)) ER(I)=EFIN(I)
c
c       CALCULATE ER(1) AND ER(NDJV) WITH IMPERMEABLE B. C.
c
      if(NDRB.eq.2) then
      ER(1)=ER(2)
  155 delta1=ER(1)
      do 354 I=1,10
      do 152 J=2,LBL
      C1=ER(1)-ES(J)
      if(C1.ge.0.0) goto 153
  152 continue
      DSED=DSDE(LBL); goto 154
  153 II=J-1
      XV=ER(1)
      if(J.eq.LBL) II=II-1
      call   LINTP_4(XV,YV,ES,DSDE,II)
      DSED=YV
  154 ER0=F(2)+(DA/DSED)*(GC/(1.0+E11(2)))
      ER(1)=(3.0/8.0)*ER0+(6.0/8.0)*ER(2)-(1.0/8.0)*ER(3)
      delta2=ER(1)
      delta=abs(delta1-delta2)
c     if(delta.gt.0.001) goto 155
  354 continue
      endif
      if(NDRB.eq.1) then
      ER(NDJV)=ER(NDJV-1)
  159 delta1=ER(NDJV)
      do 355 I=1,10
      do 156 J=2,LBL
      C1=ER(NDJV)-ES(J)
      if(C1.ge.0.0) goto 157
  156 continue
      DSED=DSDE(LBL); goto 158
  157 II=J-1
      XV=ER(NDJV)
      if(J.eq.LBL) II=II-1
      call   LINTP_4(XV,YV,ES,DSDE,II)
      DSED=YV
  158 F(NDJV+1)=F(NDJV-1)-(DA/DSED)*(GC/(1.0+E11(NDJV-1)))
      ER(NDJV)=(-1.0/8.0)*F(NDJV-2)+(6.0/8.0)*F(NDJV-1)
     > +(3.0/8.0)*F(NDJV+1)
      if(ER(NDJV).gt.E11(NDJV)) ER(NDJV)=E11(NDJV)
      delta2=ER(NDJV)
      delta=abs(delta1-delta2)
c     if(delta.gt.0.001) goto 159
  355 continue
      endif
c
c      RESET NEXT LOOP FOR FOUNDATION SOIL
c
```

```
      do 48 J=1,NDJV
      FS(J)=ER(J)
      F(J)=ER(J)
   48 continue
c
c      CALCULATE SOLUTE TRANSPORT
c
      call FDIFEQ()
      if(NL.eq.1) goto 51
c
c     INCREMENT THE TIME STEP
c
      TIME=TAU*float(NNN)
      NNN=NNN+1
c
c       CALCULATE CURRENT TIME AND CHECK AGAINST PRINT TIME
c
      if(TIME.lt.TPRINT) goto 1
c
   51 return
      end
c     *************************************************************
      subroutine VRFUNC_4()
c     *************************************************************
c
c     VERFUNC CALCULATES FF1 AND FF2 FUNCTIONS FOR CURRENT VOID RATIOS.
c
      common DA,DB,DZ,E00,ELL,GC,GS,GSBL,GW,HBL,LBL,NBDIV, NDIV,NBDJV,
     >       NDJV,NFLAG,NNN,NTIME,Q0,Q2,WL,SETT,SFIN,TAU,TIME,TPRINT,
     >       UCON,NNTIME,NST,NL,NBC,NDRB,ND1,ND2,NNL,NSOL,NNSOL,NNSOL1,
     >       ALPHAL,ALPHAT,NSTBC,SL,CFT,CFB,HT,HB,CHD0,CHD1,AKD,ALAMDAC,
     >       ALAMDASC,AM,DW,DS,DGM,TH,RKEO,NSL,HCL,HUCL,E0C,E0UC,NSORP,
     >       NSTUBC,NSTBBC,NSTRBC,NSTLBC,PCT,PGT,PRT,PCB,PGB,PRB,PCR,
     >       PGR,PRR,PCL,PGL,PRL,INX,JNZ,IE0,ISS,NC,TTIME,
     >       A(351),B(351),Z(351),XI(351),ALPHA(351),BETA(351),
     >       DSDE(351),E11(351),EFIN(351),ER(351),ES(351),EFFSTR(351),
     >       F(351),FS(351),FINT(351),PK(351),RK(351),RK1(351),RS(351),
     >       TOTSTR(351),U(351),U0(351),UW(351),VRI(351),DQ(351),
     >       Q1(351),RKEI(351),AKP,ANF,ANLAMDA,IKK,UMAX,NTAU,
     >       CHD2(351),CHDA(351),CHDX(351),CQI(351),CQU(351),CQ(351),
     >       EN0(351),EN(351),EG0(351),EG(351),DVDA(351,351),
     >       CS0(351,351),CS1(351,351),CS2(351,351),CF0(351,351),
     >       CF1(351,351),CF2(351,351),CFF1(351,351),CFF2(351,351),
     >       E11JPLUSHALF(351),E11JMINUSHALF(351),DKQU(351),
             FJPLUSHALF(351),FJMINUSHALF(351),AFJPLUSHALF(351),
     >       AFJMINUSHALF(351),BFJPLUSHALF(351),BFJMINUSHALF(351),
     >       EGJPLUSHALF(351),EGJMINUSHALF(351),EG0JPLUSHALF(351),
     >       EG0JMINUSHALF(351),CQJPLUSHALF(351),CQJMINUSHALF(351),
     >       CQUJPLUSHALF(351),CQUJMINUSHALF(351),DRD(351),
     >       CF0IPLUSHALF(351,351),CF0IMINUSHALF(351,351),
     >       CF0JPLUSHALF(351,351),CF0JMINUSHALF(351,351),
     >       CF1IPLUSHALF(351,351),CF1IMINUSHALF(351,351),
     >       CF1JPLUSHALF(351,351),CF1JMINUSHALF(351,351),
     >       PRINT1(351)
c
      call EHALFVALUE_4()
c
c COMPRESSIBLE SOIL LAYER c
      if(NSOL.ne.1) then
      do 1 J=3,NDJV-2 do
      2 N=2,LBL
      C1=FJPLUSHALF(J)-ES(N)
      if(C1.ge.0.0) goto 3
    2 continue
    3 AFJPLUSHALF(J)=ALPHA(LBL)
      BFJPLUSHALF(J)=BETA(LBL); goto 1
```

```
    3 NN=N-1
      XV=FJPLUSHALF(J)
      if(N.eq.LBL) NN=NN-1
      call   LINTP_4(XV,YV,ES,ALPHA,NN)
      AFJPLUSHALF(J)=YV
      call LINTP_4(XV,YV,ES,BETA,NN)
      BFJPLUSHALF(J)=YV
    1 continue
      do  4  J=3,NDJV-2
      do 5 N=2,LBL
      C1=FJMINUSHALF(J)-ES(N)
      if(C1.ge.0.0) goto 6
    5 continue
    6 AFJMINUSHALF(J)=ALPHA(LBL)
      BFJMINUSHALF(J)=BETA(LBL); goto 4
    7 NN=N-1
      XV=FJMINUSHALF(J)
      if(N.eq.LBL) NN=NN-1
      call   LINTP_4(XV,YV,ES,ALPHA,NN)
      AFJMINUSHALF(J)=YV
      call LINTP_4(XV,YV,ES,BETA,NN)
      BFJMINUSHALF(J)=YV
    4 continue
      endif
    7 if(NSOL.eq.1) then
c
c     CONTAMINATED LAYER
c
      do 8 J=3,NNSOL1
      do 9 N=2,LBL
      C1=FJPLUSHALF(J)-ES(N)
      if(C1.ge.0.0) goto 10
    9 continue
    10AFJPLUSHALF(J)=ALPHA(LBL)
      BFJPLUSHALF(J)=BETA(LBL); goto 8
   11 NN=N-1
      XV=FJPLUSHALF(J)
      if(N.eq.LBL) NN=NN-1
      call   LINTP_4(XV,YV,ES,ALPHA,NN)
      AFJPLUSHALF(J)=YV
      call LINTP_4(XV,YV,ES,BETA,NN)
      BFJPLUSHALF(J)=YV
    8 continue
      do  11  J=3,NNSOL1
      do 12 N=2,LBL
      C1=FJMINUSHALF(J)-ES(N)
      if(C1.ge.0.0) goto 13
   12 continue
   13 AFJMINUSHALF(J)=ALPHA(LBL)
      BFJMINUSHALF(J)=BETA(LBL); goto 11
   14 NN=N-1
      XV=FJMINUSHALF(J)
      if(N.eq.LBL) NN=NN-1
      call LINTP_4(XV,YV,ES,ALPHA,NN)
      AFJMINUSHALF(J)=YV
      call LINTP_4(XV,YV,ES,BETA,NN)
      BFJMINUSHALF(J)=YV
   11 continue
c
c     UNCONTAMINATED LAYER
c
      do 14 J=NNSOL1+1,NDJV-2
      do 15 N=2,LBL
      C1=FJPLUSHALF(J)-ES(N)
      if(C1.ge.0.0) goto 16
   15 continue
      AFJPLUSHALF(J)=ALPHA(LBL)
```

```
      BFJPLUSHALF(J)=BETA(LBL); goto 14
   16 NN=N-1
      XV=FJPLUSHALF(J)
      if(N.eq.LBL) NN=NN-1
      call   LINTP_4(XV,YV,ES,ALPHA,NN)
      AFJPLUSHALF(J)=YV
      call LINTP_4(XV,YV,ES,BETA,NN)
      BFJPLUSHALF(J)=YV
   14 continue
      do 17 J=NNSOL1+1,NDJV-2
      do 18 N=2,LBL
      C1=FJMINUSHALF(J)-ES(N)
      if(C1.ge.0.0) goto 19
   18 continue AFJMINUSHALF(J)=ALPHA(LBL)
      BFJMINUSHALF(J)=BETA(LBL); goto 17
   19 NN=N-1
      XV=FJMINUSHALF(J)
      if(N.eq.LBL) NN=NN-1
      call   LINTP_4(XV,YV,ES,ALPHA,NN)
      AFJMINUSHALF(J)=YV
      call LINTP_4(XV,YV,ES,BETA,NN)
      BFJMINUSHALF(J)=YV
   17 continue
      endif
c
      return
      end
c
c     ***********************************************************
      subroutine RESET_4()
c     ***********************************************************
c
c     RESET UPDATES PREVIOUS CALCULATIONS TO HANDLE ADDITIONAL
c     DEPOSITIONS OF DREDGED FILLS
c
      common DA,DB,DZ,E00,ELL,GC,GS,GSBL,GW,HBL,LBL,NBDIV, NDIV,NBDJV,
     >       NDJV,NFLAG,NNN,NTIME,Q0,Q2,WL,SETT,SFIN,TAU,TIME,TPRINT,
     >       UCON,NNTIME,NST,NL,NBC,NDRB,ND1,ND2,NNL,NSOL,NNSOL,NNSOL1,
     >       ALPHAL,ALPHAT,NSTBC,SL,CFT,CFB,HT,HB,CHD0,CHD1,AKD,ALAMDAC,
     >       ALAMDASC,AM,DW,DS,DGM,TH,RKEO,NSL,HCL,HUCL,E0C,E0UC,NSORP,
     >       NSTUBC,NSTBBC,NSTRBC,NSTLBC,PCT,PGT,PRT,PCB,PGB,PRB,PCR,
     >       PGR,PRR,PCL,PGL,PRL,INX,JNZ,IE0,ISS,NC,TTIME,
     >       A(351),B(351),Z(351),XI(351),ALPHA(351),BETA(351),
     >       DSDE(351),E11(351),EFIN(351),ER(351),ES(351),EFFSTR(351),
     >       F(351),FS(351),FINT(351),PK(351),RK(351),RK1(351),RS(351),
     >       TOTSTR(351),U(351),U0(351),UW(351),VRI(351),DQ(351),
     >       Q1(351),RKEI(351),AKP,ANF,ANLAMDA,IKK,UMAX,NTAU,
     >       CHD2(351),CHDA(351),CHDX(351),CQI(351),CQU(351),CQ(351),
     >       EN0(351),EN(351),EG0(351),EG(351),DVDA(351,351),
     >       CS0(351,351),CS1(351,351),CS2(351,351),CF0(351,351),
     >       CF1(351,351),CF2(351,351),CFF1(351,351),CFF2(351,351),
     >       E11JPLUSHALF(351),E11JMINUSHALF(351),DKQU(351),
     >       FJPLUSHALF(351),FJMINUSHALF(351),AFJPLUSHALF(351),
     >       AFJMINUSHALF(351),BFJPLUSHALF(351),BFJMINUSHALF(351),
     >       EGJPLUSHALF(351),EGJMINUSHALF(351),EG0JPLUSHALF(351),
     >       EG0JMINUSHALF(351),CQJPLUSHALF(351),CQJMINUSHALF(351),
     >       CQUJPLUSHALF(351),CQUJMINUSHALF(351),DRD(351),
     >       CF0IPLUSHALF(351,351),CF0IMINUSHALF(351,351),
     >       CF0JPLUSHALF(351,351),CF0JMINUSHALF(351,351),
     >       CF1IPLUSHALF(351,351),CF1IMINUSHALF(351,351),
     >       CF1JPLUSHALF(351,351),CF1JMINUSHALF(351,351),
     >       PRINT1(351)
c
c       CALCULATE FINAL VOID RATIOS FOR SOIL LAYER
c
      call INTGRL_4(FS,E11,DA,NDJV,FINT)
```

```
      if(NSOL.ne.1) then
    5 S1=ELL*GC+Q2
      do  8  J=1,NDJV
      S2=S1-Z(J)*GC
      do  6  N=2,LBL
      S3=S2-RS(N)
      if(S3.le.0.0) goto 7
    6 continue
      EFIN(J)=ES(LBL); goto 8
    7 NN=N-1
      XV=S2
      if(N.eq.LBL) NN=NN-1
      call   LINTP_4(XV,YV,RS,ES,NN)
      EFIN(J)=YV
      if(EFIN(J).gt.E11(J)) EFIN(J)=E11(J)
    8 continue
      endif
      if(NSOL.eq.1) then
c
c     UNCONTAMINATED LAYER
c
      C1=ELL*GC; C2=Q2
      S1=C1+C2
      do 51 J=NNSOL1+1,NDJV
      S2=S1-Z(J)*GC
      do 52 N=2,LBL
      S3=S2-RS(N)
      if(S3.le.0.0) goto 53
   52 continue
   53 EFIN(J)=ES(LBL); goto 51
   54 NN=N-1
      XV=S2
      if(N.eq.LBL) NN=NN-1
      call LINTP_4(XV,YV,RS,ES,NN)
      EFIN(J)=YV
   51 continue
c
c     CONTAMINATED LAYER
c
      do 54 J=1,NNSOL1
      S2=S1-Z(J)*GC do
      55 N=2,LBL S3=S2-
      RS(N)
      if(S3.le.0.0) goto 56
   55 continue
   56 EFIN(J)=ES(LBL); goto 54
   57 NN=N-1
      XV=S2
      if(N.eq.LBL) NN=NN-1
      call LINTP_4(XV,YV,RS,ES,NN)
      EFIN(J)=YV
   54 continue
      endif
c
c       ULTIMATE SETTLEMENT FOR COMPRESSIBLE FOUNDATION
c
      call INTGRL_4(EFIN,E11,DA,NDJV,FINT)
      SFIN=VRI(NDJV)-FINT(NDJV)
c
      return
      end
c
c     ************************************************************
      subroutine STRESS_4()
c     ************************************************************
c
c     STRESS CALCULATES EFFECTIVE STRESSES, TOTAL STRESSES AND PORE
```

```
c         WATER PRESSURES BASED ON CURRENT VOID RATIO AND VOID RATIO INTEGRAL.
c
      common DA,DB,DZ,E00,ELL,GC,GS,GSBL,GW,HBL,LBL,NBDIV, NDIV,NBDJV,
     >        NDJV,NFLAG,NNN,NTIME,Q0,Q2,WL,SETT,SFIN,TAU,TIME,TPRINT,
     >        UCON,NNTIME,NST,NL,NBC,NDRB,ND1,ND2,NNL,NSOL,NNSOL,NNSOL1,
     >        ALPHAL,ALPHAT,NSTBC,SL,CFT,CFB,HT,HB,CHD0,CHD1,AKD,ALAMDAC,
     >        ALAMDASC,AM,DW,DS,DGM,TH,RKEO,NSL,HCL,HUCL,E0C,E0UC,NSORP,
     >        NSTUBC,NSTBBC,NSTRBC,NSTLBC,PCT,PGT,PRT,PCB,PGB,PRB,PCR,
     >        PGR,PRR,PCL,PGL,PRL,INX,JNZ,IE0,ISS,NC,TTIME,
     >        A(351),B(351),Z(351),XI(351),ALPHA(351),BETA(351),
     >        DSDE(351),E11(351),EFIN(351),ER(351),ES(351),EFFSTR(351),
     >        F(351),FS(351),FINT(351),PK(351),RK(351),RK1(351),RS(351),
     >        TOTSTR(351),U(351),U0(351),UW(351),VRI(351),DQ(351),
     >        Q1(351),RKEI(351),AKP,ANF,ANLAMDA,IKK,UMAX,NTAU,
     >        CHD2(351),CHDA(351),CHDX(351),CQI(351),CQU(351),CQ(351),
     >        EN0(351),EN(351),EG0(351),EG(351),DVDA(351,351),
     >        CS0(351,351),CS1(351,351),CS2(351,351),CF0(351,351),
     >        CF1(351,351),CF2(351,351),CFF1(351,351),CFF2(351,351),
     >        E11JPLUSHALF(351),E11JMINUSHALF(351),DKQU(351),
     >        FJPLUSHALF(351),FJMINUSHALF(351),AFJPLUSHALF(351),
     >        AFJMINUSHALF(351),BFJPLUSHALF(351),BFJMINUSHALF(351),
     >        EGJPLUSHALF(351),EGJMINUSHALF(351),EG0JPLUSHALF(351),
     >        EG0JMINUSHALF(351),CQJPLUSHALF(351),CQJMINUSHALF(351),
     >        CQUJPLUSHALF(351),CQUJMINUSHALF(351),DRD(351),
     >        CF0IPLUSHALF(351,351),CF0IMINUSHALF(351,351),
     >        CF0JPLUSHALF(351,351),CF0JMINUSHALF(351,351),
     >        CF1IPLUSHALF(351,351),CF1IMINUSHALF(351,351),
     >        CF1JPLUSHALF(351,351),CF1JMINUSHALF(351,351),
     >        PRINT1(351)
c
c      FOR COMPRESSIBLE FOUNDATION
c       CALCULATE XI COORDINATES AND STRESSES
c
      call INTGRL_4(ER,E11,DA,NDJV,FINT)
      do 2 J=1,NDJV
      XI(J)=A(J)-(VRI(J)-FINT(J))
    2 continue W1=HBL+HT
      if(NSOL.ne.1) then
      do 6 J=1,NDJV
      do 3 N=2,LBL
      C1=FS(J)-ES(N)
      if(C1.ge.0.0) goto 4
    3 continue
      EFFSTR(J)=RS(LBL); goto 5
    4 NN=N-1
      XV=FS(J)
      if(N.eq.LBL) NN=NN-1
      call   LINTP_4(XV,YV,ES,RS,NN)
      EFFSTR(J)=YV
    5 if(XI(J).gt.HB) U0(J)=GW*(W1-XI(J))
      if(XI(J).le.HB) U0(J)=GW*(W1-XI(J))
    6 U0(J)=GW*(W1-XI(J))
    7 if(HT.eq.0.0.and.HB.eq.0.0) U0(J)=GW*(W1-XI(J))
c      TOTSTR(J)=GS*(ELL-Z(J))+GW*(FINT(NDJV)-FINT(J)+HT)+Q2
      TOTSTR(J)=GC*(ELL-Z(J))+GW*(W1-XI(J))+Q2
       UW(J)=TOTSTR(J)-EFFSTR(J)
      U(J)=UW(J)-U0(J)
      if(U(J).lt.0.0) U(J)=0.0
    6 continue
    7 if(NDRB.eq.1) U(1)=0.0
      if(NDRB.eq.2) U(NDJV)=0.0
      if(NDRB.eq.3) then
      U(1)=0.0
      U(NDJV)=0.0
      endif
```

```
      endif
      if(NSOL.eq.1) then
c
c      STRESSES IN CONTAMINATED LAYER
c
      do 7 J=1,NNSOL1
      do 8 N=2,LBL
      C1=FS(J)-ES(N)
      if(C1.ge.0.0) goto 9
    8 continue
    9 EFFSTR(J)=RS(LBL); goto 10
   10 NN=N-1
      XV=FS(J)
      if(N.eq.LBL) NN=NN-1
      call   LINTP_4(XV,YV,ES,RS,NN)
      EFFSTR(J)=YV
   11 if(HT.ne.0.0)   U0(J)=GW*(W1-XI(J))
      if(HT.eq.0.0) U0(J)=GW*(W1-XI(J))
      TOTSTR(J)=GW*(W1-FINT(J))+GC*(ELL-Z(J))+Q2
      UW(J)=TOTSTR(J)-EFFSTR(J)
      U(J)=UW(J)-U0(J)
      if(U(J).lt.0.0) U(J)=0.0
    7 continue
c
c      STRESSES IN UNCONTAMINATED LAYER
c
      do 11 J=NNSOL1+1,NDJV
      do 12 N=2,LBL
      C1=FS(J)-ES(N)
      if(C1.ge.0.0) goto 13
   12 continue EFFSTR(J)=RS(LBL);
      goto 14
   13 NN=N-1
      XV=FS(J)
      if(N.eq.LBL) NN=NN-1
      call   LINTP_4(XV,YV,ES,RS,NN)
      EFFSTR(J)=YV
   14 if(HT.ne.0.0) U0(J)=GW*(XI(NDJV)+HT-XI(J))
      if(HT.eq.0.0) U0(J)=GW*(XI(NDJV)+HT-XI(J))
      TOTSTR(J)=GW*(W1-FINT(J))+GC*(ELL-Z(J))+Q2
      UW(J)=TOTSTR(J)-EFFSTR(J)
      U(J)=UW(J)-U0(J)
      if(U(J).lt.0.0) U(J)=0.0
   11 continue
      endif
      if(IKK.eq.2) UMAX=U(1)
      if(NDRB.eq.1) U(1)=0.0
      if(NDRB.eq.2) U(NDJV)=0.0
      if(NDRB.eq.3) then
      U(1)=0.0
      U(NDJV)=0.0
      endif
c
c       CALCULATE SETTLEMENT AND DEGREE OF CONSOLIDATION
c
      SETT=A(NDJV)-XI(NDJV)
      if(SFIN.eq.0.0) UCON=0.0
      if(SFIN.ne.0.0) UCON=SETT/SFIN
c
      return
      end
c
c     ***********************************************************
      subroutine INTGRL_4(EA,EAO,DA,N,F)
c     ***********************************************************
```

```
c
c        INTGRL EVALUATES THE VOID RATIO INTEGRAL TO EACH MESH POINT
c        IN THE MATERIAL
c
      dimension EA(351),EAO(351),F(351)
c
c      BY TRAPEZOIDAL RULE FOR FIRST SEGMENT
c
      F(1)=0.0
      F(2)=F(1)+DA*(EA(2)/(1.0+EAO(2)))
c
c        BY SIMPSONS 1/3 RULE FOR ALL EVEN NUMBERED MESH POINTS
c
    do 3 I=4,N-1,2
     F(I)=F(I-2)+(DA)*(((EA(I-2)/(1.0+EAO(I-2)))+(4.0*EA(I-
  > 1)/(1.0+EAO(I-1)))+(EA(I)/(1.0+EAO(I))))/3.0)
 3  continue

c
c        BY SIMPSONS 3/8 RULE FOR ALL EVEN NUMBERED MESH POINTS
c
     do 4 I=5,N-1,2
     F(I)=F(I-3)+(DA)*((EA(I-3)/(1.0+EAO(I-3)))+(3.0*EA(I-2)/
     >(1.0+EAO(I-2)))+(3.0*EA(I-1)/(1.0+EAO(I-1)))+(EA(I)/
     >(1.0+EAO(I))))*(3.0/8.0)
    4 continue
c
c      BY DIFFERENCES FOR FIRST INTERVAL
c
      F3=(DA)*((EA(3)/(1.0+EAO(3)))+(4.0*EA(4)/(1.0+EAO(4)))
     >    +(EA(5)/(1.0+EAO(5))))/3.0
      F(3)=F(5)-F3
c
c     BY TRAPEZOIDAL RULE FOR LAST SEGMENT

      F(N)=F(N-1)
c
      return
      end
c
c     ***********************************************************
      subroutine INTGRLQ(DCQU,EAO,D1,N,NDR,QR)
c     ***********************************************************
c
c        INTGRL EVALUATES THE VOID RATIO INTEGRAL TO EACH MESH POINT
c        IN THE MATERIAL
c
      dimension DCQU(351),EAO(351),QR(351)
      if(NDR.eq.2) then
c
c      BY TRAPEZOIDAL RULE FOR FIRST SEGMENT
c
      EAO(2)=(EAO(1)+EAO(2))/2.0
      QR(2)=(D1)*(DCQU(2)/(1.0+EAO(2)))
c
c        BY SIMPSONS 1/3 RULE FOR ALL EVEN NUMBERED MESH POINTS
c
      do 3 I=4,N-1,2 QR(I)=QR(I-2)+(D1)*(((DCQU(I-2)/(1.0+EAO(I-2)))+
    >(4.0*DCQU(I-1)/(1.0+EAO(I-1)))+(DCQU(I)/(1.0+EAO(I))))/3.0)
   3  continue

c
c       BY SIMPSONS 3/8 RULE FOR ALL ODD NUMBERED MESH POINTS
c
      do 4 I=5,N-1,2
      QR(I)=QR(I-3)+(D1)*((DCQU(I-3)/(1.0+EAO(I-3)))+(3.0*DCQU(I-2)/
     >(1.0+EAO(I-2)))+(3.0*DCQU(I-1)/(1.0+EAO(I-1)))+(DCQU(I)/
```

```
     >(1.0+EAO(I))))*(3.0/8.0)
    4 continue
c
c      BY DIFFERENCES FOR FIRST INTERVAL
c
      QR3=(D1)*((DCQU(3)/(1.0+EAO(3)))+(4.0*DCQU(4)/(1.0+EAO(4)))
      > +(DCQU(5)/(1.0+EAO(5))))/3.0
      QR(3)=QR(5)-QR3
      QR(N)=QR(N-1)
      QR(1)=0.0
      endif
      if(NDR.eq.1) then
      QR(N-1)=(D1)*(DCQU(N-1)/(1.0+EAO(N-1)))
c
c        BY SIMPSONS 1/3 RULE FOR ALL EVEN NUMBERED MESH POINTS
c
      II1=mod(N,2)
      if(II1.ne.0) then
      do 5 I=N-3,2,-2
      QR(I)=QR(I+2)+(D1)*(((DCQU(I+2)/(1.0+EAO(I+2)))+
     >(4.0*DCQU(I+1)/(1.0+EAO(I+1)))+(DCQU(I)/(1.0+EAO(I))))/3.0)
    5  continue
c
c       BY SIMPSONS 3/8 RULE FOR ALL ODD NUMBERED MESH POINTS
c
      do 6 I=N-4,3,-2
      QR(I)=QR(I+3)+(D1)*((DCQU(I+3)/(1.0+EAO(I+3)))+(3.0*DCQU(I+2)/
     >(1.0+EAO(I+2)))+(3.0*DCQU(I+1)/(1.0+EAO(I+1)))+(DCQU(I)/
     >(1.0+EAO(I))))*(3.0/8.0)
    6 continue
      QR3=(D1)*((DCQU(N-2)/(1.0+EAO(N-2)))+(4.0*DCQU(N-3)/
     >(1.0+EAO(N-3)))+(DCQU(N-4)/(1.0+EAO(N-4))))/3.0
      QR(N-2)=QR(N-4)-QR3
      endif
      do 7 I=N-3,2,-2
      QR(I)=QR(I+2)+(D1)*(((DCQU(I+2)/(1.0+EAO(I+2)))+
     >(4.0*DCQU(I+1)/(1.0+EAO(I+1)))+(DCQU(I)/(1.0+EAO(I))))/3.0)
    7  continue
c
c       BY SIMPSONS 3/8 RULE FOR ALL ODD NUMBERED MESH POINTS
c
      do 8 I=N-4,3,-2
      QR(I)=QR(I+3)+(D1)*((DCQU(I+3)/(1.0+EAO(I+3)))+(3.0*DCQU(I+2)/
     >(1.0+EAO(I+2)))+(3.0*DCQU(I+1)/(1.0+EAO(I+1)))+(DCQU(I)/
     >(1.0+EAO(I))))*(3.0/8.0)
    8  continue
c
c BY DIFFERENCES FOR FIRST INTERVAL c
      QR3=(D1)*((DCQU(N-2)/(1.0+EAO(N-2)))+(4.0*DCQU(N-3)/
     > (1.0+EAO(N-3)))+(DCQU(N-4)/(1.0+EAO(N-4))))/3.0
      QR(N-2)=QR(N-4)-QR3
      QR(N)=0.0
      QR(1)=QR(2)
      endif
      if(NDR.eq.3) then
      II2=mod(N,2)
      if(II2.ne.0) then
      write(*,*) "USE EVEN NUMBER OF ELEMENTS FOR DOUBLY DRAINED CASE"
      stop
      endif
c
c     LOWER HALF
c
      JJ1=N/2
      QR(JJ1)=(D1)*(DCQU(JJ1)/(1.0+EAO(JJ1)))
c
```

```
c         BY SIMPSONS 1/3 RULE FOR ALL EVEN NUMBERED MESH POINTS
c
      JJ2=mod(JJ1,2)
      if(JJ2.ne.0) then
      do 9 I=JJ1-2,3,-2
      QR(I)=QR(I+2)+(D1)*(((DCQU(I+2)/(1.0+EAO(I+2)))+
     >(4.0*DCQU(I+1)/(1.0+EAO(I+1)))+(DCQU(I)/(1.0+EAO(I))))/3.0)
    9  continue
c
c        BY SIMPSONS 3/8 RULE FOR ALL ODD NUMBERED MESH POINTS
c
      do 10 I=JJ1-3,2,-2
      QR(I)=QR(I+3)+(D1)*((DCQU(I+3)/(1.0+EAO(I+3)))+(3.0*DCQU(I+2)/
     >(1.0+EAO(I+2)))+(3.0*DCQU(I+1)/(1.0+EAO(I+1)))+(DCQU(I)/
     >(1.0+EAO(I))))*(3.0/8.0)
   10 continue
      endif
      do 11 I=JJ1-2,2,-2
      QR(I)=QR(I+2)+(D1)*(((DCQU(I+2)/(1.0+EAO(I+2)))+
     >(4.0*DCQU(I+1)/(1.0+EAO(I+1)))+(DCQU(I)/(1.0+EAO(I))))/3.0)
   11  continue
c
c        BY SIMPSONS 3/8 RULE FOR ALL ODD NUMBERED MESH POINTS
c
      do 12 I=JJ1-3,3,-2
      QR(I)=QR(I+3)+(D1)*((DCQU(I+3)/(1.0+EAO(I+3)))+(3.0*DCQU(I+2)/
     >(1.0+EAO(I+2)))+(3.0*DCQU(I+1)/(1.0+EAO(I+1)))+(DCQU(I)/
     >(1.0+EAO(I))))*(3.0/8.0)
   12  continue
c
c       BY DIFFERENCES FOR FIRST INTERVAL
c
      QR3=(D1)*((DCQU(JJ1-1)/(1.0+EAO(JJ1-1)))+(4.0*DCQU(JJ1-2)/
     > (1.0+EAO(JJ1-2)))+(DCQU(JJ1-3)/(1.0+EAO(JJ1-3))))/3.0
      QR(JJ1-1)=QR(JJ1-3)-QR3
c
c      UPPER HALF
c
      QR(JJ1+1)=(D1)*(DCQU(JJ1+1)/(1.0+EAO(JJ1+1)))
c
c         BY SIMPSONS 1/3 RULE FOR ALL EVEN NUMBERED MESH POINTS
c
      do 13 I=JJ1+3,N-1,2
      QR(I)=QR(I-2)+(D1)*(((DCQU(I-2)/(1.0+EAO(I-2)))+
     >(4.0*DCQU(I-1)/(1.0+EAO(I-1)))+(DCQU(I)/(1.0+EAO(I))))/3.0)
   13  continue
c
c        BY SIMPSONS 3/8 RULE FOR ALL ODD NUMBERED MESH POINTS
c
      do 14 I=JJ1+4,N-1,2
      QR(I)=QR(I-3)+(D1)*((DCQU(I-3)/(1.0+EAO(I-3)))+(3.0*DCQU(I-2)/
     >(1.0+EAO(I-2)))+(3.0*DCQU(I-1)/(1.0+EAO(I-1)))+(DCQU(I)/
     >(1.0+EAO(I))))*(3.0/8.0)
   14  continue
c
c       BY DIFFERENCES FOR FIRST INTERVAL
c
      QR3=(D1)*((DCQU(JJ1+2)/(1.0+EAO(JJ1+2)))+(4.0*DCQU(JJ1+3)/
     > (1.0+EAO(JJ1+3)))+(DCQU(JJ1+4)/(1.0+EAO(JJ1+4))))/3.0
      QR(JJ1+2)=QR(JJ1+4)-QR3
      QR(N)=QR(N-1)
      QR(1)=QR(2)
      endif
c
      return
      end
```

```
c      ***********************************************************
       subroutine DATOUT_4()
c      ***********************************************************
c
c       DATOUT PRINTS RESULTS OF CONSOLIDATIO CALCULATIONS AND BASE
c       DATA IN TABULAR FORM
c
       common DA,DB,DZ,E00,ELL,GC,GS,GSBL,GW,HBL,LBL,NBDIV, NDIV,NBDJV,
     >        NDJV,NFLAG,NNN,NTIME,Q0,Q2,WL,SETT,SFIN,TAU,TIME,TPRINT,
     >        UCON,NNTIME,NST,NL,NBC,NDRB,ND1,ND2,NNL,NSOL,NNSOL,NNSOL1,
     >        ALPHAL,ALPHAT,NSTBC,SL,CFT,CFB,HT,HB,CHD0,CHD1,AKD,ALAMDAC,
     >        ALAMDASC,AM,DW,DS,DGM,TH,RKEO,NSL,HCL,HUCL,E0C,E0UC,NSORP,
     >        NSTUBC,NSTBBC,NSTRBC,NSTLBC,PCT,PGT,PRT,PCB,PGB,PRB,PCR,
     >        PGR,PRR,PCL,PGL,PRL,INX,JNZ,IE0,ISS,NC,TTIME,
     >        A(351),B(351),Z(351),XI(351),ALPHA(351),BETA(351),
     >        DSDE(351),E11(351),EFIN(351),ER(351),ES(351),EFFSTR(351),
     >        F(351),FS(351),FINT(351),PK(351),RK(351),RK1(351),RS(351),
     >        TOTSTR(351),U(351),U0(351),UW(351),VRI(351),DQ(351),
     >        Q1(351),RKEI(351),AKP,ANF,ANLAMDA,IKK,UMAX,NTAU,
     >        CHD2(351),CHDA(351),CHDX(351),CQI(351),CQU(351),CQ(351),
     >        EN0(351),EN(351),EG0(351),EG(351),DVDA(351,351),
     >        CS0(351,351),CS1(351,351),CS2(351,351),CF0(351,351),
     >        CF1(351,351),CF2(351,351),CFF1(351,351),CFF2(351,351),
     >        E11JPLUSHALF(351),E11JMINUSHALF(351),DKQU(351),
     >        FJPLUSHALF(351),FJMINUSHALF(351),AFJPLUSHALF(351),
     >        AFJMINUSHALF(351),BFJPLUSHALF(351),BFJMINUSHALF(351),
     >        EGJPLUSHALF(351),EGJMINUSHALF(351),EG0JPLUSHALF(351),
     >        EG0JMINUSHALF(351),CQJPLUSHALF(351),CQJMINUSHALF(351),
     >        CQUJPLUSHALF(351),CQUJMINUSHALF(351),DRD(351),
     >        CF0IPLUSHALF(351,351),CF0IMINUSHALF(351,351),
     >        CF0JPLUSHALF(351,351),CF0JMINUSHALF(351,351),
     >        CF1IPLUSHALF(351,351),CF1IMINUSHALF(351,351),
     >        CF1JPLUSHALF(351,351),CF1JMINUSHALF(351,351),
     >        PRINT1(351)
c
c PRINT CONDITIONS IN COMPRESSIBLE FOUNDATION
c
       if(NFLAG.eq.1)   write(18,100)
       if(NFLAG.eq.0)   write(18,108)
       write(18,101)
       write(18,102)
       do 16 J=1,NDJV
       K=NDJV+1-J
       write(18,103) A(K),XI(K),Z(K),E11(K),ER(K),EFIN(K)
   16  continue
       write(18,104)
       write(18,105)
       do 17 J=1,NDJV
       K=NDJV+1-J
       write(18,103) XI(K),TOTSTR(K),EFFSTR(K),UW(K),U0(K),U(K)
   17  continue
       write(18,107) TIME,UCON
       write(18,110) SETT,SFIN
       write(18,112) WL
       write(19,*) TIME,SETT,U(1)
c
c FORMATS
c
  100 format(/////14(1H*),34HINITIAL CONDITIONS IN COMPRESSIBLE,
     >        11H FOUNDATION,13(1H*))
  101 format(//9X,5(1H*),13H COORDINATES ,5(1H*),13X,5(1H*), >
     13H VOID RATIOS ,5(1H*))
  102  format(/9X,1HA,11X,2HXI,10X,1HZ,7X,8HEINITIAL,8X,1HE,8X,
     >        6HEFINAL)
  103  format(5(F11.4,1X),F11.4)
  104  format(//15X,5(1H*),10H STRESSES,5(1H*),7X,5(1H*),
     >        16H PORE PRESSURES,5(1H*))
```

```
  105 format(/7X,2HXI,8X,5HTOTAL,5X,9HEFFECTIVE,6X,5HTOTAL,6X,
     >        6HSTATIC,6X,6HEXCESS)
  106 format(/////19(1H*),34HINITIAL CONDITIONS IN DREDGED FILL,
     >        19(1H*))
  107 format(//10X,7HTIME = ,E10.4,5X,26HDEGREE OF CONSOLIDATION = ,
     >        F10.6)
  108 format(/////14(1H*),34HCURRENT CONDITIONS IN COMPRESSIBLE,
     >        11H FOUNDATION,13(1H*))
  109 format(/////19(1H*),34HCURRENT CONDITIONS IN DREDGED FILL,
     >        19(1H*))
  110 format(/10X,13HSETTLEMENT = ,F10.4,5X,19HFINAL SETTLEMENT = ,
     >        F10.4)
  111 format(/10X,27HBOTTOM BOUNDARY GRADIENT = ,F10.4)
  112 format(/10X,27HWATER LEVEL ABOVE BOTTOM = ,F10.4)
c
  300 return
      end
c
c     ********************************************************
      subroutine INPUT_ST()
c     ********************************************************
c
      common DA,DB,DZ,E00,ELL,GC,GS,GSBL,GW,HBL,LBL,NBDIV, NDIV,NBDJV,
     >        NDJV,NFLAG,NNN,NTIME,Q0,Q2,WL,SETT,SFIN,TAU,TIME,TPRINT,
     >        UCON,NNTIME,NST,NL,NBC,NDRB,ND1,ND2,NNL,NSOL,NNSOL,NNSOL1,
     >        ALPHAL,ALPHAT,NSTBC,SL,CFT,CFB,HT,HB,CHD0,CHD1,AKD,ALAMDAC,
     >        ALAMDASC,AM,DW,DS,DGM,TH,RKEO,NSL,HCL,HUCL,E0C,E0UC,NSORP,
     >        NSTUBC,NSTBBC,NSTRBC,NSTLBC,PCT,PGT,PRT,PCB,PGB,PRB,PCR,
     >        PGR,PRR,PCL,PGL,PRL,INX,JNZ,IE0,ISS,NC,TTIME,
     >        A(351),B(351),Z(351),XI(351),ALPHA(351),BETA(351),
     >        DSDE(351),E11(351),EFIN(351),ER(351),ES(351),EFFSTR(351),
     >        F(351),FS(351),FINT(351),PK(351),RK(351),RK1(351),RS(351),
     >        TOTSTR(351),U(351),U0(351),UW(351),VRI(351),DQ(351),
     >        Q1(351),RKEI(351),AKP,ANF,ANLAMDA,IKK,UMAX,NTAU,
     >        CHD2(351),CHDA(351),CHDX(351),CQI(351),CQU(351),CQ(351),
     >        EN0(351),EN(351),EG0(351),EG(351),DVDA(351,351),
     >        CS0(351,351),CS1(351,351),CS2(351,351),CF0(351,351),
     >        CF1(351,351),CF2(351,351),CFF1(351,351),CFF2(351,351),
     >        E11JPLUSHALF(351),E11JMINUSHALF(351),DKQU(351),
     >        FJPLUSHALF(351),FJMINUSHALF(351),AFJPLUSHALF(351),
     >        AFJMINUSHALF(351),BFJPLUSHALF(351),BFJMINUSHALF(351),
     >        EGJPLUSHALF(351),EGJMINUSHALF(351),EG0JPLUSHALF(351),
     >        EG0JMINUSHALF(351),CQJPLUSHALF(351),CQJMINUSHALF(351),
     >        CQUJPLUSHALF(351),CQUJMINUSHALF(351),DRD(351),
     >        CF0IPLUSHALF(351,351),CF0IMINUSHALF(351,351),
     >        CF0JPLUSHALF(351,351),CF0JMINUSHALF(351,351),
     >        CF1IPLUSHALF(351,351),CF1IMINUSHALF(351,351),
     >        CF1JPLUSHALF(351,351),CF1JMINUSHALF(351,351),
     >        PRINT1(351)
c
      read(20,*) NST,ND1,ND2,NSORP,NSTBC,NC,NTAU
c
c        READ SOLUTE TRANSPORT DATA FOR CLAY LINER OR DREDGED SEDIMENT
c
      read(20,*) NST,DS,ALPHAL,ALPHAT,CFT,CFB,ALAMDAC,ALAMDASC
      if(ND2.eq.1) read(20,*) NST,CHD1
      if(ND2.eq.2) read(20,*) NST,CHD0,AM
      if(NSORP.eq.1) read(20,*) NST,AKD
      if(NSORP.eq.2) read(20,*) NST,AKP,ANF,ANLAMDA
      if(NSORP.eq.3) read(20,*) NST,AKP,ANF
c
c     CALCULATE CONSTANTS
c
      if(ND1.eq.1) then
      if(NSTBC.eq.3) then
      read(20,*) NST,DGM,TH
      endif
```

```
      if(NSTBC.eq.1) then
      CF0(1,1)=CFB
      read(20,*) NST,CL,CSS
      do 1 J=2,NDJV-1
      CF0(1,J)=CL
    1 continue
      CF0(1,NDJV)=CFT
c     CF0(1,25)=1.0
      if(NSTBC.eq.3)   CF0(1,NDJV)=0.0
      do 2 J=1,NDJV
      CF1(1,J)=CF0(1,J)
    2 continue
      do  3  J=1,NDJV
      CS0(1,J)=CSS
    3 continue
      do 4 J=1,NDJV
      CS1(1,J)=CS0(1,J)
    4 continue
      do 6 J=2,NDJV-1
c
c     EQUILIBRIUM SORPTION WITH LINEAR ISOTHERM
c
      DJI=(1.0+F(J))/(1.0+E11(J))
      DRD(J)=(GSBL*(GW/9.81))/(1.0+ER(J))
      if(NSORP.eq.1.and.AKD.eq.0.0) then
      CFF1(1,J)=EN0(J)*CF1(1,J)*DJI+((1.0-EN0(J))*DJI*CS1(1,J))
      endif
      if(NSORP.eq.1.and.AKD.ne.0.0) then
      CFF1(1,J)=EN0(J)*CF1(1,J)*DJI+((1.0-EN0(J))*DRD(J)*
     >          AKD*DJI*CF1(1,J))
      endif
c
c     EQUILIBRIUM SORPTION WITH NONLINEAR ISOTHERM
c

      if(NSORP.eq.2.or.NSORP.eq.3) then
      endif if(NSTBC.eq.2.and.NSOL.eq.1)
      then DJI=(1.0+ER(J))/(1.0+E11(J))
      S=AKP*(CF1(1,J)**ANF)
      CS1(1,J)=DRD(J)*S CS0(1,J)=DRD(J)*S
      CFF1(1,J)=EN0(J)*CF1(1,J)*DJI+((1.0-EN0(J))*CS1(1,J)*DJI)
      endif
    6 continue
c
c     CONTAMINATED LAYER
c
      do 20 J=1,NNSOL1
      CF1(1,J)=CFB
      CF0(1,J)=CFB
c
c       EQUILIBRIUM SORPTION WITH NONLINEAR ISOTHERM
c
      if(NSORP.eq.2.or.NSORP.eq.3) then
      DJI=(1.0+ER(J))/(1.0+E11(J))
      S=AKP*(CF1(1,J)**ANF)
      CS1(1,J)=DRD(J)*S CS0(1,J)=DRD(J)*S
      CFF1(1,J)=EN0(J)*CF1(1,J)*DJI+((1.0-EN0(J))*CS1(1,J)*DJI)
      endif
c
c       EQUILIBRIUM SORPTION WITH LINEAR ISOTHERM
c
      if(NSORP.eq.1) then
      DJI=(1.0+ER(J))/(1.0+E11(J))
      S=AKD*CF1(1,J)
```

```
      CS1(1,J)=DRD(J)*S
      CS0(1,J)=DRD(J)*S
      CFF1(1,J)=EN0(J)*CF1(1,J)*DJI+((1.0-EN0(J))*CS1(1,J)*DJI)
      endif
   20 continue
c
c     UNCONTAMINATED LAYER
c
      do 21 J=NNSOL1+1,NDJV
      DJI=(1.0+ER(J))/(1.0+E11(J))
      CF1(1,J)=CFT
      CF0(1,J)=CFT
      CS1(1,J)=0.0
      CS0(1,J)=0.0
c     CFF1(1,J)=EN0(J)*CF1(1,J)*DJI
   21 continue
      endif
c
c     EQUILIBRIUM SORPTION WITH LINEAR ISOTHERM
c
      if(NSORP.eq.1) then
      do 22 J=NNSOL1+1,NDJV
      DJI=(1.0+ER(J))/(1.0+E11(J))
      CFF1(1,J)=EN0(J)*CF1(1,J)*DJI+((1.0-EN0(J))*DRD(J)*
     >          AKD*DJI*CF1(1,J))
   22 continue
      endif
c
c       EQUILIBRIUM SORPTION WITH NONLINEAR ISOTHERM
c
      if(NSORP.eq.2.or.NSORP.eq.3) then
      DJI=(1.0+ER(J))/(1.0+E11(J))
      S=AKP*(CF1(1,J)**ANF)
      CS1(1,J)=DRD(J)*S
      CS0(1,J)=DRD(J)*S
      CFF1(1,J)=EN0(J)*CF1(1,J)*DJI+((1.0-EN0(J))*CS1(1,J)*DJI)
      endif
      endif
      if(ND1.eq.2) then
      read(20,*) NST,NSTUBC,NSTBBC,NSTRBC,NSTLBC,ISS
      read(20,*) NST,NBDIV,WBL
      NDIV=NBDIV+2
      DB=WBL/float(NBDIV)
      B(1)=0.0
      B(NDIV)=WBL
      B(2)=B(1)+DB/2.0
      do  30  I=3,NDIV-1
      B(I)=B(I-1)+DB
   30 continue
c
c       READ THE PRESCRIBED INITIAL BOUNDARY VALUES
c
      if(NSTUBC.eq.1)  read(20,*)  NST,PCT
      if(NSTUBC.eq.2)  read(20,*)  NST,PGT
      if(NSTUBC.eq.3)  read(20,*)  NST,PRT
      if(NSTBBC.eq.1)  read(20,*)  NST,PCB
      if(NSTBBC.eq.2)  read(20,*)  NST,PGB
      if(NSTBBC.eq.3)  read(20,*)  NST,PRB
      if(NSTRBC.eq.1)  read(20,*)  NST,PCR
      if(NSTRBC.eq.2)  read(20,*)  NST,PGR
      if(NSTRBC.eq.3)  read(20,*)  NST,PRR
      if(NSTLBC.eq.1)  read(20,*)  NST,PCL
      if(NSTLBC.eq.2)  read(20,*)  NST,PGL
      if(NSTLBC.eq.3)  read(20,*)  NST,PRL
      read(20,*) NST,C0,S0,XCO,ZCO
      do 875 J=1,NDJV
      DRD(J)=(GSBL*(GW/9.81))/(1.0+ER(J))
```

```
  875 continue
      do 11 J=1,NDJV
      do 12 I=1,NDIV
      CF0(I,J)=0.0
   12 continue
   11 continue INX1=(XCO-
      (DB/2.0))/DB INX=INX1+2
      JNZ1=(ZCO-(DA/2.0))/DA
      JNZ=JNZ1+2 INX2=((XCO+1.0)-
      (DB/2.0))/DB INX3=INX2+2
      JNZ2=((ZCO+1.0)-(DA/2.0))/DA
      JNZ3= JNZ2+2
      if(ISS.eq.1) then
      do 31 I=INX,INX3
      do 32 J=JNZ,JNZ3
      CF0(I,J)=C0
   32 continue
   31 continue
      endif
      if(ISS.ne.1) then
      CF0(INX,JNZ)=C0
      endif
      do 13 I=1,NDIV do
      14 J=1,NDJV
      CF1(I,J)=CF0(I,J)
   14 continue
   13 continue
      do  15  J=1,NDJV
      do 16 I=1,NDIV
      CS0(I,J)=0.0
   16 continue
   15 continue
      if(ISS.eq.1) then
      do 131 I=INX,INX3
      do 132 J=JNZ,JNZ3
      CS0(I,J)=DRD(J)*S0
  132 continue
  131 continue
      endif
      if(ISS.ne.1) then
      CS0(INX,JNZ)=DRD(J)*S0
      endif
      do  17  I=1,NDIV
      do 18 J=1,NDJV
      CS1(I,J)=CS0(I,J)
   18 continue
   17 continue
c
c      LINEAR EQUILIBRIUM SORPTION
c
      if(NSORP.eq.1) then
      do 27 I=2,NDIV-1 do
      28 J=2,NDJV-1
      DJI=(1.0+ER(J))/(1.0+E11(J))
      CFF1(I,J)=EN0(J)*CF1(I,J)*DJI+(1.0-EN0(J))*CS1(I,J)*DJI
   28 continue
   27 continue
      endif
c
c       INITIAL EQUILIBRIUM SORPTION WITH NONLINEAR ISOTHERM
c
      if(NSORP.eq.2) then
      do 25 I=2,NDIV-1 do
      26 J=2,NDJV-1
      DJI=(1.0+ER(J))/(1.0+E11(J))
```

```
      DRD(J)=(GSBL*(GW/9.81))/(1.0+ER(J))
      S=AKP*(CF1(I,J)**ANF)
      CS1(I,J)=DRD(J)*S
      CS0(I,J)=DRD(J)*S
      CFF1(I,J)=EN0(J)*CF1(I,J)*DJI+((1.0-EN0(J))*CS1(I,J)*DJI)
   26 continue
   25 continue
      endif
      endif
      call INTRO()
c
      return
      end
c
c     ********************************************************
      subroutine INTRO()
c     ********************************************************
c
c     INTRO PRINTS INPUT DATA AND RESULTS OF INITIAL CALCULATIONS
c     IN TABULAR FORM
c
      common DA,DB,DZ,E00,ELL,GC,GS,GSBL,GW,HBL,LBL,NBDIV, NDIV,NBDJV,
     >       NDJV,NFLAG,NNN,NTIME,Q0,Q2,WL,SETT,SFIN,TAU,TIME,TPRINT,
     >       UCON,NNTIME,NST,NL,NBC,NDRB,ND1,ND2,NNL,NSOL,NNSOL,NNSOL1,
     >       ALPHAL,ALPHAT,NSTBC,SL,CFT,CFB,HT,HB,CHD0,CHD1,AKD,ALAMDAC,
     >       ALAMDASC,AM,DW,DS,DGM,TH,RKEO,NSL,HCL,HUCL,E0C,E0UC,NSORP,
     >       NSTUBC,NSTBBC,NSTRBC,NSTLBC,PCT,PGT,PRT,PCB,PGB,PRB,PCR,
     >       PGR,PRR,PCL,PGL,PRL,INX,JNZ,IE0,ISS,NC,TTIME,
     >       A(351),B(351),Z(351),XI(351),ALPHA(351),BETA(351),
     >       DSDE(351),E11(351),EFIN(351),ER(351),ES(351),EFFSTR(351),
     >       F(351),FS(351),FINT(351),PK(351),RK(351),RK1(351),RS(351),
     >       TOTSTR(351),U(351),U0(351),UW(351),VRI(351),DQ(351),
     >       Q1(351),RKEI(351),AKP,ANF,ANLAMDA,IKK,UMAX,NTAU,
     >       CHD2(351),CHDA(351),CHDX(351),CQI(351),CQU(351),CQ(351),
     >       EN0(351),EN(351),EG0(351),EG(351),DVDA(351,351),
     >       CS0(351,351),CS1(351,351),CS2(351,351),CF0(351,351),
     >       CF1(351,351),CF2(351,351),CFF1(351,351),CFF2(351,351),
     >       E11JPLUSHALF(351),E11JMINUSHALF(351),DKQU(351),
     >       FJPLUSHALF(351),FJMINUSHALF(351),AFJPLUSHALF(351),
     >       AFJMINUSHALF(351),BFJPLUSHALF(351),BFJMINUSHALF(351),
     >       EGJPLUSHALF(351),EGJMINUSHALF(351),EG0JPLUSHALF(351),
     >       EG0JMINUSHALF(351),CQJPLUSHALF(351),CQJMINUSHALF(351),
     >       CQUJPLUSHALF(351),CQUJMINUSHALF(351),DRD(351),
     >       CF0IPLUSHALF(351,351),CF0IMINUSHALF(351,351),
     >       CF0JPLUSHALF(351,351),CF0JMINUSHALF(351,351),
     >       CF1IPLUSHALF(351,351),CF1IMINUSHALF(351,351),
     >       CF1JPLUSHALF(351,351),CF1JMINUSHALF(351,351),
     >       PRINT1(351)
c
c      PRINT PROBLEM NUMBER AND HEADING
c
      if(ND1.eq.1) then
      write(21,100)
      write(21,101)
      write(21,100)
c
c       WRITE SOLUTE TRANSPORT DATA FOR CLAY LINER
c
      write(21,100)
      write(21,102)
      write(21,100)
      write(21,103)
      if(ND2.eq.1) write(21,104) CHD1,ALPHAL
      if(ND2.eq.2) write(21,104) CHD0,ALPHAL
c     write(21,*)  "**************DIFF.  H.G  i=0.2  Rf=1.81**************"
      write(21,*)  "INITIAL  CONC.  OF  SOLUTE  AT  THE  UPPER  BOUNDARY=",CFT
      write(21,*) "INITIAL CONC. OF SOLUTE AT THE LOWER BOUNDARY=",CFB
```

```
      write(21,*) "TIME STEP=",TAU
      write(21,*) "NUMBER OF NODES=",NDJV
      write(21,100)
      write(22,105)
      endif
      if(ND1.eq.2) then
      write(21,100)
      write(21,106)
      write(21,100)
      write(21,100)
      write(21,102)
      write(21,100)
      write(21,107)
      write(21,108) CHD1,ALPHAL,ALPHAT
      write(21,100)
c
c       WRITE SOLUTE TRANSPORT DATA FOR CLAY LINER
c
      write(21,*)  "INITIAL  CONC.  OF  SOLUTE  AT  THE  UPPER  BOUNDARY=",CFT
      write(21,*)  "INITIAL  CONC.  OF  SOLUTE  AT  THE  LOWER  BOUNDARY=",CFB
      write(21,*) "TIME STEP=",TAU
      write(21,*)  "NUMBER  OF  NODES  X-DIRECTION=",NDIV
      write(21,*)  "NUMBER  OF  NODES  Y-DIRECTION=",NDJV
      endif
c
c    FORMATS
c
  100 format(////60(1H*))
  101 format(9X,35HSOLUTE TRANSPORT THROUGH CLAY LAYER)
  102 format(//37HCALCULATION DATA FOR SOLUTE TRANSPORT)
  103 format(/2X,19HEFFECTIVE DIFFUSION,25HLONGITUDINAL DISPERSIVITY)
  104 format(/10X,F1.15,15X,F5.2)
  105 format(/8X,4HTIME,12X,5HCb/C0/)
  106 format(9X,51HTWO DIMENSIONAL SOLUTE TRANSPORT THROUGH CLAY LAYER)
  107 format(/2X,19HEFFECTIVE DIFFUSION,25HLONGITUDINAL DISPERSIVITY,
     >            23HTRANSVERSE DISPERSIVITY)
  108 format(/10X,E1.15,15X,F5.2,15X,F5.2)
c
  300 return
      end
c
c     **********************************************************************
      subroutine    FACEVALUE()
c     **********************************************************************
c
      common DA,DB,DZ,E00,ELL,GC,GS,GSBL,GW,HBL,LBL,NBDIV, NDIV,NBDJV,
     >        NDJV,NFLAG,NNN,NTIME,Q0,Q2,WL,SETT,SFIN,TAU,TIME,TPRINT,
     >        UCON,NNTIME,NST,NL,NBC,NDRB,ND1,ND2,NNL,NSOL,NNSOL,NNSOL1,
     >        ALPHAL,ALPHAT,NSTBC,SL,CFT,CFB,HT,HB,CHD0,CHD1,AKD,ALAMDAC,
     >        ALAMDASC,AM,DW,DS,DGM,TH,RKEO,NSL,HCL,HUCL,E0C,E0UC,NSORP,
     >        NSTUBC,NSTBBC,NSTRBC,NSTLBC,PCT,PGT,PRT,PCB,PGB,PRB,PCR,
     >        PGR,PRR,PCL,PGL,PRL,INX,JNZ,IE0,ISS,NC,TTIME,
     >        A(351),B(351),Z(351),XI(351),ALPHA(351),BETA(351),
     >        DSDE(351),E11(351),EFIN(351),ER(351),ES(351),EFFSTR(351),
     >        F(351),FS(351),FINT(351),PK(351),RK(351),RK1(351),RS(351),
     >        TOTSTR(351),U(351),U0(351),UW(351),VRI(351),DQ(351),
     >        Q1(351),RKEI(351),AKP,ANF,ANLAMDA,IKK,UMAX,NTAU,
     >        CHD2(351),CHDA(351),CHDX(351),CQI(351),CQU(351),CQ(351),
     >        EN0(351),EN(351),EG0(351),EG(351),DVDA(351,351),
     >        CS0(351,351),CS1(351,351),CS2(351,351),CF0(351,351),
     >        CF1(351,351),CF2(351,351),CFF1(351,351),CFF2(351,351),
     >        E11JPLUSHALF(351),E11JMINUSHALF(351),DKQU(351),
     >        FJPLUSHALF(351),FJMINUSHALF(351),AFJPLUSHALF(351),
     >        AFJMINUSHALF(351),BFJPLUSHALF(351),BFJMINUSHALF(351),
     >        EGJPLUSHALF(351),EGJMINUSHALF(351),EG0JPLUSHALF(351),
     >        EG0JMINUSHALF(351),CQJPLUSHALF(351),CQJMINUSHALF(351),
     >        CQUJPLUSHALF(351),CQUJMINUSHALF(351),DRD(351),
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     >        CF0IPLUSHALF(351,351),CF0IMINUSHALF(351,351),
     >        CF0JPLUSHALF(351,351),CF0JMINUSHALF(351,351),
     >        CF1IPLUSHALF(351,351),CF1IMINUSHALF(351,351),
     >        CF1JPLUSHALF(351,351),CF1JMINUSHALF(351,351),
     >        PRINT1(351)
c
c      CALCULATE THE PREVIOUS TIME FACE VALUES OF SOLUTE CONCENTRATION
c      FOR THE ELEMENTARY VOLUME AROUND MESH POINTS
      if(ND1.eq.1) then
      do 1 J=2,NDJV-3
      if(abs(CF1(1,J)-CF1(1,J+2)).le.0.00001) then
      CF1JPLUSHALF(1,J)=(3.0/8.0)*CF1(1,J)+(6.0/8.0)*CF1(1,J+1)
     >                  -(1.0/8.0)*CF1(1,J+2)
      endif
      if(abs(CF1(1,J)-CF1(1,J+2)).le.0.00001) goto 1
      if(abs(CF1(1,J+1)-2.0*CF1(1,J)+CF1(1,J-1)).le.
     >  (0.3*abs(CF1(1,J-1)-CF1(1,J+1)))) then
      CF1JPLUSHALF(1,J)=(3.0/8.0)*CF1(1,J)+(6.0/8.0)*CF1(1,J+1)
     >                  -(1.0/8.0)*CF1(1,J+2)
      endif
      if((abs(CF1(1,J+1)-2.0*CF1(1,J)+CF1(1,J-1))).gt.
     >(0.3*abs(CF1(1,J-1)-CF1(1,J+1)))) then
      CF1BAR3=(CF1(1,J+1)-CF1(1,J+2))/(CF1(1,J)-CF1(1,J+2))
      if(CF1BAR3.le.(-1.0).or.CF1BAR3.ge.1.5) then
      CF1JPLUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
      CF1JPLUSHALF(1,J)=CF1(1,J+2)+(CF1(1,J)-
     >                  CF1(1,J+2))*CF1JPLUSHALFBAR2
      endif
      if(CF1BAR3.ge.0.35.and.CF1BAR3.le.0.65) then
      CF1JPLUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
      CF1JPLUSHALF(1,J)=CF1(1,J+2)+(CF1(1,J)-
     >                  CF1(1,J+2))*CF1JPLUSHALFBAR2
      endif
      if(CF1BAR3.le.0.0.and.CF1BAR3.gt.(-1.0)) then
      CF1JPLUSHALFBAR2=0.375*CF1BAR3
      CF1JPLUSHALF(1,J)=CF1(1,J+2)+(CF1(1,J)-
     >                  CF1(1,J+2))*CF1JPLUSHALFBAR2
      endif
      if(CF1BAR3.gt.0.0.and.CF1BAR3.lt.0.35) then
      CF1JPLUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
     >                  (1.0-2.0*CF1BAR3)
      CF1JPLUSHALF(1,J)=CF1(1,J+2)+(CF1(1,J)-
     >                  CF1(1,J+2))*CF1JPLUSHALFBAR2
      endif
      if(CF1BAR3.gt.0.65.and.CF1BAR3.le.1.0) then
      CF1JPLUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
     >                  (1.0-2.0*CF1BAR3)
      CF1JPLUSHALF(1,J)=CF1(1,J+2)+(CF1(1,J)-
     >                  CF1(1,J+2))*CF1JPLUSHALFBAR2
      endif
      if(CF1BAR3.le.1.5.and.CF1BAR3.gt.1.0) then
      CF1JPLUSHALFBAR2=CF1BAR3
      CF1JPLUSHALF(1,J)=CF1(1,J+2)+(CF1(1,J)-
     >                  CF1(1,J+2))*CF1JPLUSHALFBAR2
      endif
      endif
    1 continue
    2 CF1(1,NDJV+1)=(8.0/3.0)*CF1(1,NDJV)-2.0*CF1(1,NDJV-1)+
     >              (1.0/3.0)*CF1(1,NDJV-2)
      if(abs(CF1(1,NDJV-2)-CF1(1,NDJV+1)).le.0.00001) then
      CF1JPLUSHALF(1,NDJV-2)=(3.0/8.0)*CF1(1,NDJV-2)+(6.0/8.0)*
     >                        CF1(1,NDJV-1)-(1.0/8.0)*CF1(1,NDJV+1)
      endif
      if(abs(CF1(1,NDJV-2)-CF1(1,NDJV+1)).le.0.00001) goto 605
      if(abs(CF1(1,NDJV+1)-2.0*CF1(1,NDJV-1)+CF1(1,NDJV-2)).le.
     >  (0.3*abs(CF1(1,NDJV-2)-CF1(1,NDJV+1)))) then
      CF1JPLUSHALF(1,NDJV-2)=(3.0/8.0)*CF1(1,NDJV-2)+(6.0/8.0)*
```

```
     >CF1(1,NDJV-1)-(1.0/8.0)*CF1(1,NDJV+1)
     endif
     if(abs(CF1(1,NDJV+1)-2.0*CF1(1,NDJV-1)+CF1(1,NDJV-2)).gt.(0.3*
     >abs(CF1(1,NDJV-2)-CF1(1,NDJV+1)))) then
      CF1BAR3=(CF1(1,NDJV-1)-CF1(1,NDJV+1))/(CF1(1,NDJV-2)-
     >         CF1(1,NDJV+1))
      if(CF1BAR3.le.(-1.0).or.CF1BAR3.ge.1.5) then
      CF1JPLUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
      CF1JPLUSHALF(1,NDJV-2)=CF1(1,NDJV+1)+(CF1(1,NDJV-2)-
     >                        CF1(1,NDJV+1))*CF1JPLUSHALFBAR2
      endif
      if(CF1BAR3.ge.0.35.and.CF1BAR3.le.0.65) then
      CF1JPLUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
      CF1JPLUSHALF(1,NDJV-2)=CF1(1,NDJV+1)+(CF1(1,NDJV-2)-
     >                        CF1(1,NDJV+1))*CF1JPLUSHALFBAR2
      endif
      if(CF1BAR3.le.0.0.and.CF1BAR3.gt.(-1.0)) then
      CF1JPLUSHALFBAR2=0.375*CF1BAR3
      CF1JPLUSHALF(1,NDJV-2)=CF1(1,NDJV+1)+(CF1(1,NDJV-2)-
     >                        CF1(1,NDJV+1))*CF1JPLUSHALFBAR2
      endif
      if(CF1BAR3.gt.0.0.and.CF1BAR3.lt.0.35) then
      CF1JPLUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
     >                  (1.0-2.0*CF1BAR3)
      CF1JPLUSHALF(1,NDJV-2)=CF1(1,NDJV+1)+(CF1(1,NDJV-2)-
     >                        CF1(1,NDJV+1))*CF1JPLUSHALFBAR2
      endif
      if(CF1BAR3.gt.0.65.and.CF1BAR3.le.1.0) then
      CF1JPLUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
     >                  (1.0-2.0*CF1BAR3)
      CF1JPLUSHALF(1,NDJV-2)=CF1(1,NDJV+1)+(CF1(1,NDJV-2)-
     >                        CF1(1,NDJV+1))*CF1JPLUSHALFBAR2
      endif
      if(CF1BAR3.le.1.5.and.CF1BAR3.gt.1.0) then
      CF1JPLUSHALFBAR2=CF1BAR3
      CF1JPLUSHALF(1,NDJV-2)=CF1(1,NDJV+1)+(CF1(1,NDJV-2)-
     >                        CF1(1,NDJV+1))*CF1JPLUSHALFBAR2
      endif
      endif
  605 continue
      do 2 J=2,NDJV-2
      CF1JMINUSHALF(1,J+1)=CF1JPLUSHALF(1,J)
    2 continue
      endif
c
      if(ND1.eq.2) then
c
c      INTERPOLATIOS IN I-DIRECTION
c
      do 4 I=2,NDIV-3
      do 5 J=2,NDJV-3
      CURVETI=(1.0/24.0)*(CF1(I+1,J+1)-2.0*CF1(I+1,J)+CF1(I+1,J-1))
      if(abs(CF1(I,J)-CF1(I+2,J)).le.0.00001) then
      CF1IPLUSHALF(I,J)=(3.0/8.0)*CF1(I,J)+(6.0/8.0)*CF1(I+1,J)
     >             -(1.0/8.0)*CF1(I+2,J)+CURVETI
      endif
      if(abs(CF1(I,J)-CF1(I+2,J)).le.0.00001)   goto   5
      if(abs(CF1(I+1,J)-2.0*CF1(I,J)+CF1(I-1,J)).le.
     >(0.3*abs(CF1(I-1,J)-CF1(I+1,J)))) then
      CF1IPLUSHALF(I,J)=(3.0/8.0)*CF1(I,J)+(6.0/8.0)*CF1(I+1,J)
     >             -(1.0/8.0)*CF1(I+2,J)+CURVETI
      endif
      if((abs(CF1(I+1,J)-2.0*CF1(I,J)+CF1(I-1,J))).gt.
     >(0.3*abs(CF1(I-1,J)-CF1(I+1,J)))) then
      CF1BAR3=(CF1(I+1,J)-CF1(I+2,J))/(CF1(I,J)-CF1(I+2,J))
      if(CF1BAR3.le.(-1.0).or.CF1BAR3.ge.1.5) then
      CF1IPLUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
```

```
      CF1IPLUSHALF(I,J)=CF1(I+2,J)+((CF1(I,J)-
     >                   CF1(I+2,J))*CF1IPLUSHALFBAR2)+CURVETI
      endif
      if(CF1BAR3.ge.0.35.and.CF1BAR3.le.0.65) then
      CF1IPLUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
      CF1IPLUSHALF(I,J)=CF1(I+2,J)+((CF1(I,J)-
     >                   CF1(I+2,J))*CF1IPLUSHALFBAR2)+CURVETI
      endif
      if(CF1BAR3.le.0.0.and.CF1BAR3.gt.(-1.0)) then
      CF1IPLUSHALFBAR2=0.375*CF1BAR3
      CF1IPLUSHALF(I,J)=CF1(I+2,J)+((CF1(I,J)-
     >                   CF1(I+2,J))*CF1IPLUSHALFBAR2)+CURVETI
      endif
      if(CF1BAR3.gt.0.0.and.CF1BAR3.lt.0.35) then
      CF1IPLUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
     >                   (1.0-2.0*CF1BAR3)
      CF1IPLUSHALF(I,J)=CF1(I+2,J)+((CF1(I,J)-
     >                   CF1(I+2,J))*CF1IPLUSHALFBAR2)+CURVETI
      endif
      if(CF1BAR3.gt.0.65.and.CF1BAR3.le.1.0) then
      CF1IPLUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
     >                   (1.0-2.0*CF1BAR3)
      CF1IPLUSHALF(I,J)=CF1(I+2,J)+((CF1(I,J)-
     >                   CF1(I+2,J))*CF1IPLUSHALFBAR2)+CURVETI
      endif
      if(CF1BAR3.le.1.5.and.CF1BAR3.gt.1.0) then
      CF1IPLUSHALFBAR2=CF1BAR3
      CF1IPLUSHALF(I,J)=CF1(I+2,J)+((CF1(I,J)-
     >                   CF1(I+2,J))*CF1IPLUSHALFBAR2)+CURVETI
      endif
      endif
    5 continue
    4 continue
      do 6 I=2,NDIV-3
      do 7 J=2,NDJV-3
      CURVETJ=(1.0/24.0)*(CF1(I+1,J+1)-2.0*CF1(I,J+1)+CF1(I-1,J+1))
      if(abs(CF1(I,J)-CF1(I,J+2)).le.0.00001) then
      CF1JPLUSHALF(I,J)=(3.0/8.0)*CF1(I,J)+(6.0/8.0)*CF1(I,J+1)
     >             -(1.0/8.0)*CF1(I,J+2)+CURVETJ
      endif
      if(abs(CF1(I,J)-CF1(I,J+2)).le.0.00001)   goto   7
      if(abs(CF1(I,J+1)-2.0*CF1(I,J)+CF1(I,J-1)).le.
     >(0.3*abs(CF1(I,J-1)-CF1(I,J+1)))) then
      CF1JPLUSHALF(I,J)=(3.0/8.0)*CF1(I,J)+(6.0/8.0)*CF1(I,J+1)
     >             -(1.0/8.0)*CF1(I,J+2)+CURVETJ
      endif
      if((abs(CF1(I,J+1)-2.0*CF1(I,J)+CF1(I,J-1))).gt.
     >(0.3*abs(CF1(I,J-1)-CF1(I,J+1)))) then
      CF1BAR3=(CF1(I,J+1)-CF1(I,J+2))/(CF1(I,J)-CF1(I,J+2))
      if(CF1BAR3.le.(-1.0).or.CF1BAR3.ge.1.5) then
      CF1JPLUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
      CF1JPLUSHALF(I,J)=CF1(I,J+2)+((CF1(I,J)-
     >                   CF1(I,J+2))*CF1JPLUSHALFBAR2)+CURVETJ
      endif
      if(CF1BAR3.ge.0.35.and.CF1BAR3.le.0.65) then
      CF1JPLUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
      CF1JPLUSHALF(I,J)=CF1(I,J+2)+((CF1(I,J)-
     >                   CF1(I,J+2))*CF1JPLUSHALFBAR2)+CURVETJ
      endif
      if(CF1BAR3.le.0.0.and.CF1BAR3.gt.(-1.0)) then
      CF1JPLUSHALFBAR2=0.375*CF1BAR3
      CF1JPLUSHALF(I,J)=CF1(I,J+2)+((CF1(I,J)-
     >                   CF1(I,J+2))*CF1JPLUSHALFBAR2)+CURVETJ
      endif
      if(CF1BAR3.gt.0.0.and.CF1BAR3.lt.0.35) then
      CF1JPLUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
     >                   (1.0-2.0*CF1BAR3)
```

```
      CF1JPLUSHALF(I,J)=CF1(I,J+2)+((CF1(I,J)-
     >                   CF1(I,J+2))*CF1IPLUSHALFBAR2)+CURVETJ
      endif
      if(CF1BAR3.gt.0.65.and.CF1BAR3.le.1.0) then
      CF1JPLUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
     >                   (1.0-2.0*CF1BAR3)
      CF1JPLUSHALF(I,J)=CF1(I,J+2)+((CF1(I,J)-
     >                   CF1(I,J+2))*CF1JPLUSHALFBAR2)+CURVETJ
      endif
      if(CF1BAR3.le.1.5.and.CF1BAR3.gt.1.0) then
      CF1JPLUSHALFBAR2=CF1BAR3
      CF1JPLUSHALF(I,J)=CF1(I,J+2)+((CF1(I,J)-
     >                   CF1(I,J+2))*CF1JPLUSHALFBAR2)+CURVETJ
      endif
      endif
     7continue
     6continue
c
c      BOUNDARY CONDITIONS AT THE TOP
c
c      PRESCRIBED CONCENTRATION AT THE TOP
c
      if(NSTUBC.eq.1) then
      do 17 I=1,NDIV
      CF1JPLUSHALF(I,NDJV-1)=PCT
      CF1(I,NDJV+1)=CF1JPLUSHALF(I,NDJV-1)
   17continue
c
c      LATERAL BOUNDARIES' CONDITIONS (ZERO FLUX)
c

      do 18 J=1,NDJV
      CF1(NDIV+1,J)=CF1(NDIV-1,J)
      CF1IPLUSHALF(NDIV-1,J)=CF1(NDIV,J)
c      CF1IPLUSHALF(NDIV-1,J)=(3.0/8.0)*CF1(NDIV+1,J)+
c     >                  (6.0/8.0)*CF1(NDIV-1,J)-(1.0/8.0)*CF1(NDIV-2,J)
      CF1(NDIV,J)=CF1IPLUSHALF(NDIV-1,J)
   18 continue
      endif
      if(NSTUBC.eq.2) then
c
c      PRESCRIBED CONCENTRATION GRADIENT (GENERALLY ZERO) AT THE TOP
c
      do 19 I=1,NDIV
      CF1JPLUSHALF(I,NDJV-1)=CF1(I,NDJV-1)
      CF1(I,NDJV+1)=CF1JPLUSHALF(I,NDJV-1)
c       CF1(I,NDJV+1)=CF1(I,NDJV-1)
c      CF1IPLUSHALF(I,NDJV-1)=(3.0/8.0)*CF1(I,NDJV+1)+
c     >                  (6.0/8.0)*CF1(I,NDJV-1)-(1.0/8.0)*CF1(I,NDJV-2)
      CF1(I,NDJV)=CF1IPLUSHALF(I,NDJV-1)
   19 continue
c
c      LATERAL BOUNDARIES' CONDITIONS (ZERO FLUX)
c
      do 20 J=1,NDJV
      CF1IPLUSHALF(NDIV-1,J)=CF1(NDIV-1,J)
      CF1(NDIV+1,J)=CF1IPLUSHALF(NDIV-1,J)
c       CF1(NDIV+1,J)=CF1(NDIV-1,J)
c      CF1IPLUSHALF(NDIV-1,J)=(3.0/8.0)*CF1(NDIV+1,J)+
c     >                  (6.0/8.0)*CF1(NDIV-1,J)-(1.0/8.0)*CF1(NDIV-2,J)
      CF1(NDIV,J)=CF1IPLUSHALF(NDIV-1,J)
   20 continue
      endif
      if(NSTUBC.eq.3) then
c
c      RESERVOIR BOUNDARY CONDITION AT THE TOP
c
```

```
      VS1=abs(CQ(NDJV))/EN(NDJV)
      HH=0.5*DA*(1.0+EG(NDJV-1))/(1.0+EG0(NDJV-1))
      do 21 I=1,NDIV
      CF1(I,NDJV)=(VS1*PRT*HH+CHDA(NDJV-1)*CF1(I,NDJV-1))/
     >             (VS1*HH+CHDA(NDJV-1))
      CF1JPLUSHALF(I,NDJV-1)=CF1(I,NDJV)
      CF1(I,NDJV+1)=PRT
   21 continue
c
c     LATERAL BOUNDARIES' CONDITIONS (ZERO FLUX)
c
      do 22 J=1,NDJV
      CF1IPLUSHALF(NDIV-1,J)=CF1(NDIV-1,J)
      CF1(NDIV+1,J)=CF1IPLUSHALF(NDIV-1,J)
c      CF1(NDIV+1,J)=CF1(NDIV-1,J)
c     CF1IPLUSHALF(NDIV-1,J)=(3.0/8.0)*CF1(NDIV+1,J)+
c    >                  (6.0/8.0)*CF1(NDIV-1,J)-(1.0/8.0)*CF1(NDIV-2,J)
      CF1(NDIV,J)=CF1IPLUSHALF(NDIV-1,J)
   22 continue
      endif
      do 10 J=2,NDJV-1 CURVETI=(1.0/24.0)*(CF1(NDIV-1,J+1)-
      2.0*CF1(NDIV-1,J)+
     >        CF1(NDIV-1,J-1))
      if(abs(CF1(NDIV-2,J)-CF1(NDIV+1,J)).le.0.00001) then
      CF1IPLUSHALF(NDIV-2,J)=(3.0/8.0)*CF1(NDIV-2,J)+(6.0/8.0)*
     >                     CF1(NDIV-1,J)-(1.0/8.0)*CF1(NDIV+1,J)+CURVETI
      endif
      if(abs(CF1(NDIV-2,J)-CF1(NDIV+1,J)).le.0.00001) goto 10
      if(abs(CF1(NDIV+1,J)-2.0*CF1(NDIV-1,J)+CF1(NDIV-2,J)).le.
     >  (0.3*abs(CF1(NDIV-2,J)-CF1(NDIV+1,J)))) then
      CF1IPLUSHALF(NDIV-2,J)=(3.0/8.0)*CF1(NDIV-2,J)+(6.0/8.0)*
     >                    CF1(NDIV-1,J)-(1.0/8.0)*CF1(NDIV+1,J)+CURVETI
      endif
      if(abs(CF1(NDIV+1,J)-2.0*CF1(NDIV-1,J)+CF1(NDIV-2,J)).gt.(0.3*
     >abs(CF1(NDIV-2,J)-CF1(NDIV+1,J)))) then
      CF1BAR3=(CF1(NDIV-1,J)-CF1(NDIV+1,J))/(CF1(NDIV-2,J)-
     >         CF1(NDIV+1,J))
      if(CF1BAR3.le.(-1.0).or.CF1BAR3.ge.1.5) then
      CF1IPLUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
      CF1IPLUSHALF(NDIV-2,J)=CF1(NDIV+1,J)+(CF1(NDIV-2,J)-
     >                        CF1(NDIV+1,J))*CF1IPLUSHALFBAR2+CURVETI
      endif
      if(CF1BAR3.ge.0.35.and.CF1BAR3.le.0.65) then
      CF1IPLUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
      CF1IPLUSHALF(NDIV-2,J)=CF1(NDIV+1,J)+(CF1(NDIV-2,J)-
     >                        CF1(NDIV+1,J))*CF1IPLUSHALFBAR2+CURVETI
      endif
      if(CF1BAR3.le.0.0.and.CF1BAR3.gt.(-1.0)) then
      CF1IPLUSHALFBAR2=0.375*CF1BAR3
      CF1IPLUSHALF(NDIV-2,J)=CF1(NDIV+1,J)+(CF1(NDIV-2,J)-
     >                        CF1(NDIV+1,J))*CF1IPLUSHALFBAR2+CURVETI
      endif
      if(CF1BAR3.gt.0.0.and.CF1BAR3.lt.0.35) then
      CF1IPLUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
     >                  (1.0-2.0*CF1BAR3)
      CF1IPLUSHALF(NDIV-2,J)=CF1(NDIV+1,J)+(CF1(NDIV-2,J)-
     >                        CF1(NDIV+1,J))*CF1IPLUSHALFBAR2+CURVETI
      endif
      if(CF1BAR3.gt.0.65.and.CF1BAR3.le.1.0) then
      CF1IPLUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
     >                  (1.0-2.0*CF1BAR3)
      CF1IPLUSHALF(NDIV-2,J)=CF1(NDIV+1,J)+(CF1(NDIV-2,J)-
     >                        CF1(NDIV+1,J))*CF1IPLUSHALFBAR2+CURVETI
      endif
      if(CF1BAR3.le.1.5.and.CF1BAR3.gt.1.0) then
      CF1IPLUSHALFBAR2=CF1BAR3
      CF1IPLUSHALF(NDIV-2,J)=CF1(NDIV+1,J)+(CF1(NDIV-2,J)
```

```
  > CF1(NDIV+1,J))*CF1IPLUSHALFBAR2+CURVETI CF1IMINUSHALF(NDIV-
   1,J)=CF1IPLUSHALF(NDIV-2,J)
   endif
   endif
10 continue
   do 11 I=2,NDIV-1
   CURVETJ=(1.0/24.0)*(CF1(I+1,NDJV-1)-2.0*CF1(I,NDJV-1)+
  >        CF1(I-1,NDJV-1))
   if(abs(CF1(I,NDJV-2)-CF1(I,NDJV+1)).le.0.00001) then
   CF1JPLUSHALF(I,NDJV-2)=(3.0/8.0)*CF1(I,NDJV-2)+(6.0/8.0)*
  >                   CF1(I,NDJV-1)-(1.0/8.0)*CF1(I,NDJV+1)+CURVETJ
   endif
   if(abs(CF1(I,NDJV-2)-CF1(I,NDJV+1)).le.0.00001) goto 11
   if(abs(CF1(I,NDJV+1)-2.0*CF1(I,NDJV-1)+CF1(I,NDJV-2)).le.
  >  (0.3*abs(CF1(I,NDJV-2)-CF1(I,NDJV+1)))) then
   CF1JPLUSHALF(I,NDJV-2)=(3.0/8.0)*CF1(I,NDJV-2)+(6.0/8.0)*
  >                   CF1(I,NDJV-1)-(1.0/8.0)*CF1(I,NDJV+1)+CURVETJ
   endif
   if(abs(CF1(I,NDJV+1)-2.0*CF1(I,NDJV-1)+CF1(I,NDJV-2)).gt.
  >(0.3*abs(CF1(I,NDJV-2)-CF1(I,NDJV+1)))) then
   CF1BAR3=(CF1(I,NDJV-1)-CF1(I,NDJV+1))/(CF1(I,NDJV-2)-
  >         CF1(I,NDJV+1))
   if(CF1BAR3.le.(-1.0).or.CF1BAR3.ge.1.5) then
   CF1JPLUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
   CF1JPLUSHALF(NDIV-2,J)=CF1(I,NDJV+1)+(CF1(I,NDJV-2)-
  >                       CF1(I,NDJV+1))*CF1JPLUSHALFBAR2+CURVETJ
   endif
   if(CF1BAR3.ge.0.35.and.CF1BAR3.le.0.65) then
   CF1JPLUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
   CF1JPLUSHALF(I,NDJV-2)=CF1(I,NDJV+1)+(CF1(I,NDJV-2)-
  >                       CF1(I,NDJV+1))*CF1JPLUSHALFBAR2+CURVETJ
   endif
   if(CF1BAR3.le.0.0.and.CF1BAR3.gt.(-1.0)) then
   CF1JPLUSHALFBAR2=0.375*CF1BAR3
   CF1JPLUSHALF(I,NDJV-2)=CF1(I,NDJV+1)+(CF1(I,NDJV-2)-
  >                       CF1(I,NDJV+1))*CF1JPLUSHALFBAR2+CURVETJ
   endif
   if(CF1BAR3.gt.0.0.and.CF1BAR3.lt.0.35) then
   CF1JPLUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
  >                  (1.0-2.0*CF1BAR3)
   CF1JPLUSHALF(I,NDJV-2)=CF1(I,NDJV+1)+(CF1(I,NDJV-2)-
  >                       CF1(I,NDJV+1))*CF1JPLUSHALFBAR2+CURVETJ
   endif
   if(CF1BAR3.gt.0.65.and.CF1BAR3.le.1.0) then
   CF1JPLUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
  >                  (1.0-2.0*CF1BAR3)
   CF1JPLUSHALF(I,NDJV-2)=CF1(I,NDJV+1)+(CF1(I,NDJV-2)-
  >                       CF1(I,NDJV+1))*CF1JPLUSHALFBAR2+CURVETJ
   endif
   if(CF1BAR3.le.1.5.and.CF1BAR3.gt.1.0) then
   CF1JPLUSHALFBAR2=CF1BAR3
   CF1JPLUSHALF(I,NDJV-2)=CF1(J,NDJV+1)+(CF1(I,NDJV-2)-
  >                       CF1(I,NDJV+1))*CF1JPLUSHALFBAR2+CURVETJ
   CF1JMINUSHALF(I,NDJV-1)=CF1JPLUSHALF(I,NDJV-2)
   endif
   endif
11 continue
   do  8  I=2,NDIV-2
   do 9 J=2,NDJV-2
   CF1IMINUSHALF(I+1,J)=CF1IPLUSHALF(I,J)
   CF1JMINUSHALF(I,J+1)=CF1JPLUSHALF(I,J)
 9 continue
 8 continue
   endif
   EGJPLUSHALF(NDJV-1)=EG(NDJV)
   EG(NDJV+1)=(8.0/3.0)*EGJPLUSHALF(NDJV-1)-2.0*EG(NDJV-1)+
  >           (1.0/3.0)*EG(NDJV-2)
```

```
      EG0JPLUSHALF(NDJV-1)=EG0(NDJV)
      EG0(NDJV+1)=(8.0/3.0)*EGJPLUSHALF(NDJV-1)-2.0*EG(NDJV-1)
     >                       +(1.0/8.0)*EG0(NDJV-2)
      do 3 J=3,NDJV-3
      EGJMINUSHALF(J)=(3.0/8.0)*EG(J-1)+(6.0/8.0)*EG(J)-(1.0/8.0)*
     >                  EG(J+1)
      EG0JMINUSHALF(J)=(3.0/8.0)*EG0(J-1)+(6.0/8.0)*EG0(J)-(1.0/8.0)*
     >                   EG0(J+1)
      EGJPLUSHALF(J)=(3.0/8.0)*EG(J)+(6.0/8.0)*EG(J+1)-(1.0/8.0)*
     >                  EG(J+2)
      EG0JPLUSHALF(J)=(3.0/8.0)*EG0(J)+(6.0/8.0)*EG0(J+1)-(1.0/8.0)*
     >                  EG0(J+2)
    3 continue
    4 EGJMINUSHALF(NDJV-2)=(3.0/8.0)*EG(NDJV-3)+(6.0/8.0)*EG(NDJV-2)-
     >                      (1.0/8.0)*EG(NDJV-1)
      EG0JMINUSHALF(NDJV-2)=(3.0/8.0)*EG0(NDJV-3)+(6.0/8.0)*EG0(NDJV-2)-
     >                       (1.0/8.0)*EG0(NDJV-1)
      EGJPLUSHALF(NDJV-2)=(3.0/8.0)*EG(NDJV-2)+(6.0/8.0)*EG(NDJV-1)-
     >                     (1.0/8.0)*EG(NDJV+1)
      EG0JPLUSHALF(NDJV-2)=(3.0/8.0)*EG0(NDJV-2)+(6.0/8.0)*EG0(NDJV-1)-
     >                      (1.0/8.0)*EG0(NDJV+1)
c
      return
      end
c
c     ***********************************************************************
      subroutine    FACEVALUE_1()
c     ***********************************************************************
c
      common DA,DB,DZ,E00,ELL,GC,GS,GSBL,GW,HBL,LBL,NBDIV, NDIV,NBDJV,
     >       NDJV,NFLAG,NNN,NTIME,Q0,Q2,WL,SETT,SFIN,TAU,TIME,TPRINT,
     >       UCON,NNTIME,NST,NL,NBC,NDRB,ND1,ND2,NNL,NSOL,NNSOL,NNSOL1,
     >       ALPHAL,ALPHAT,NSTBC,SL,CFT,CFB,HT,HB,CHD0,CHD1,AKD,ALAMDAC,
     >       ALAMDASC,AM,DW,DS,DGM,TH,RKEO,NSL,HCL,HUCL,E0C,E0UC,NSORP,
     >       NSTUBC,NSTBBC,NSTRBC,NSTLBC,PCT,PGT,PRT,PCB,PGB,PRB,PCR,
     >       PGR,PRR,PCL,PGL,PRL,INX,JNZ,IE0,ISS,NC,TTIME,
     >       A(351),B(351),Z(351),XI(351),ALPHA(351),BETA(351),
     >       DSDE(351),E11(351),EFIN(351),ER(351),ES(351),EFFSTR(351),
     >       F(351),FS(351),FINT(351),PK(351),RK(351),RK1(351),RS(351),
     >       TOTSTR(351),U(351),U0(351),UW(351),VRI(351),DQ(351),
     >       Q1(351),RKEI(351),AKP,ANF,ANLAMDA,IKK,UMAX,NTAU,
     >       CHD2(351),CHDA(351),CHDX(351),CQI(351),CQU(351),CQ(351),
     >       EN0(351),EN(351),EG0(351),EG(351),DVDA(351,351),
     >       CS0(351,351),CS1(351,351),CS2(351,351),CF0(351,351),
     >       CF1(351,351),CF2(351,351),CFF1(351,351),CFF2(351,351),
     >       E11JPLUSHALF(351),E11JMINUSHALF(351),DKQU(351),
     >       FJPLUSHALF(351),FJMINUSHALF(351),AFJPLUSHALF(351),
     >       AFJMINUSHALF(351),BFJPLUSHALF(351),BFJMINUSHALF(351),
     >       EGJPLUSHALF(351),EGJMINUSHALF(351),EG0JPLUSHALF(351),
     >       EG0JMINUSHALF(351),CQJPLUSHALF(351),CQJMINUSHALF(351),
     >       CQUJPLUSHALF(351),CQUJMINUSHALF(351),DRD(351),
     >       CF0IPLUSHALF(351,351),CF0IMINUSHALF(351,351),
     >       CF0JPLUSHALF(351,351),CF0JMINUSHALF(351,351),
     >       CF1IPLUSHALF(351,351),CF1IMINUSHALF(351,351),
     >       CF1JPLUSHALF(351,351),CF1JMINUSHALF(351,351),
     >       PRINT1(351)
c
c     CALCULATE THE PREVIOUS TIME FACE VALUES OF SOLUTE CONCENTRATION
c     FOR THE ELEMENTARY VOLUME AROUND MESH POINTS
      if(ND1.eq.1) then
      do 1 J=3,NDJV-2 if(abs(CF1(1,J+1)-CF1(1,J-
      1)).le.0.00001) then
      CF1JMINUSHALF(1,J+1)=(3.0/8.0)*CF1(1,J+1)+(6.0/8.0)*CF1(1,J)
     >                     -(1.0/8.0)*CF1(1,J-1)
      endif
      if(abs(CF1(1,J+1)-CF1(1,J-1)).le.0.00001) goto 1
      if(abs(CF1(1,J-1)-2.0*CF1(1,J)+CF1(1,J+1)).le.
```

```
     >  (0.3*abs(CF1(1,J+1)-CF1(1,J-1)))) then
      CF1JMINUSHALF(1,J+1)=(3.0/8.0)*CF1(1,J+1)+(6.0/8.0)*CF1(1,J)
     >                      -(1.0/8.0)*CF1(1,J-1)
      endif
      if((abs(CF1(1,J-1)-2.0*CF1(1,J)+CF1(1,J+1))).gt.
     >(0.3*abs(CF1(1,J+1)-CF1(1,J-1)))) then
      CF1BAR3=(CF1(1,J)-CF1(1,J-1))/(CF1(1,J+1)-CF1(1,J-1))
      if(CF1BAR3.le.(-1.0).or.CF1BAR3.ge.1.5) then
      CF1JMINUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
      CF1JMINUSHALF(1,J+1)=CF1(1,J-1)+(CF1(1,J+1)-
     >                      CF1(1,J-1))*CF1JMINUSHALFBAR2
      endif
      if(CF1BAR3.ge.0.35.and.CF1BAR3.le.0.65) then
      CF1JMINUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
      CF1JMINUSHALF(1,J+1)=CF1(1,J-1)+(CF1(1,J+1)-
     >                      CF1(1,J-1))*CF1JMINUSHALFBAR2
      endif
      if(CF1BAR3.le.0.0.and.CF1BAR3.gt.(-1.0)) then
      CF1JMINUSHALFBAR2=0.375*CF1BAR3
      CF1JMINUSHALF(1,J+1)=CF1(1,J-1)+(CF1(1,J+1)-
     >                      CF1(1,J-1))*CF1JMINUSHALFBAR2
      endif
      if(CF1BAR3.gt.0.0.and.CF1BAR3.lt.0.35) then
      CF1JMINUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
     >                   (1.0-2.0*CF1BAR3)
      CF1JMINUSHALF(1,J+1)=CF1(1,J-1)+(CF1(1,J+1)-
     >                      CF1(1,J-1))*CF1JMINUSHALFBAR2
      endif
      if(CF1BAR3.gt.0.65.and.CF1BAR3.le.1.0) then
      CF1JMINUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
     >                   (1.0-2.0*CF1BAR3)
      CF1JMINUSHALF(1,J+1)=CF1(1,J-1)+(CF1(1,J+1)-
     >                      CF1(1,J-1))*CF1JMINUSHALFBAR2
      endif
      if(CF1BAR3.le.1.5.and.CF1BAR3.gt.1.0) then
      CF1JMINUSHALFBAR2=CF1BAR3
      CF1JMINUSHALF(1,J+1)=CF1(1,J-1)+(CF1(1,J+1)-
     >                      CF1(1,J-1))*CF1JMINUSHALFBAR2
      endif
      endif
    1 continue
c
      CF10=(8.0/3.0)*CF1(1,1)-2.0*CF1(1,2)+(1.0/3.0)*CF1(1,3)
      if(abs(CF1(1,3)-CF10).le.0.00001) then
      CF1JMINUSHALF(1,3)=(3.0/8.0)*CF1(1,3)+(6.0/8.0)*
     >                     CF1(1,2)-(1.0/8.0)*CF10
      endif
      if(abs(CF1(1,3)-CF10).le.0.00001) goto 705
      if(abs(CF10-2.0*CF1(1,2)+CF1(1,3)).le.
     >  (0.3*abs(CF1(1,3)-CF10))) then
      CF1JMINUSHALF(1,3)=(3.0/8.0)*CF1(1,3)+(6.0/8.0)*
     >                     CF1(1,2)-(1.0/8.0)*CF10
      endif
      if(abs(CF1(1,3)-2.0*CF1(1,2)+CF10).gt.(0.3*
     >   abs(CF1(1,3)-CF10))) then
      CF1BAR3=(CF1(1,2)-CF10)/(CF1(1,3)-CF10)
      if(CF1BAR3.le.(-1.0).or.CF1BAR3.ge.1.5) then
      CF1JMINUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
      CF1JMINUSHALF(1,3)=CF10+(CF1(1,3)-CF10)*CF1JMINUSHALFBAR2
      endif
      if(CF1BAR3.ge.0.35.and.CF1BAR3.le.0.65) then
      CF1JMINUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
      CF1JMINUSHALF(1,3)=CF10+(CF1(1,3)-CF10)*CF1JMINUSHALFBAR2
      endif
      if(CF1BAR3.le.0.0.and.CF1BAR3.gt.(-1.0)) then
      CF1JMINUSHALFBAR2=0.375*CF1BAR3
      CF1JMINUSHALF(1,3)=CF10+(CF1(1,3)-CF10)*CF1JMINUSHALFBAR2
```

```
      endif
      if(CF1BAR3.gt.0.0.and.CF1BAR3.lt.0.35) then
      CF1JMINUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
      > (1.0-2.0*CF1BAR3)
      CF1JMINUSHALF(1,3)=CF10+(CF1(1,3)-CF10)*CF1JMINUSHALFBAR2
      endif
      if(CF1BAR3.gt.0.65.and.CF1BAR3.le.1.0) then
      CF1JMINUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
      > (1.0-2.0*CF1BAR3)
      CF1JMINUSHALF(1,3)=CF10+(CF1(1,3)-CF10)*CF1JMINUSHALFBAR2
      endif
      if(CF1BAR3.le.1.5.and.CF1BAR3.gt.1.0) then
      CF1JMINUSHALFBAR2=CF1BAR3
      CF1JMINUSHALF(1,3)=CF10+(CF1(1,3)-CF10)*CF1JMINUSHALFBAR2
      endif
      endif
  705 continue
      do 2 J=2,NDJV-2
      CF1JPLUSHALF(1,J)=CF1JMINUSHALF(1,J+1)
    2 continue
    3 CF1JPLUSHALF(1,NDJV-1)=CF1(1,NDJV)
      CF1JMINUSHALF(1,2)=CF1(1,1)
      endif
c
      if(ND1.eq.2) then
c
c      INTERPOLATIOS IN I-DIRECTION
c
      do 4 I=2,NDIV-3
      do 5 J=2,NDJV-3
      CURVETI=(1.0/24.0)*(CF1(I+1,J+1)-2.0*CF1(I+1,J)+CF1(I+1,J-1))
      if(abs(CF1(I,J)-CF1(I+2,J)).le.0.00001) then
      CF1IPLUSHALF(I,J)=(3.0/8.0)*CF1(I,J)+(6.0/8.0)*CF1(I+1,J)
     >             -(1.0/8.0)*CF1(I+2,J)+CURVETI
      endif
      if(abs(CF1(I,J)-CF1(I+2,J)).le.0.00001)   goto   5
      if(abs(CF1(I+1,J)-2.0*CF1(I,J)+CF1(I-1,J)).le.
     >(0.3*abs(CF1(I-1,J)-CF1(I+1,J)))) then
      CF1IPLUSHALF(I,J)=(3.0/8.0)*CF1(I,J)+(6.0/8.0)*CF1(I+1,J)
     >             -(1.0/8.0)*CF1(I+2,J)+CURVETI
      endif
      if((abs(CF1(I+1,J)-2.0*CF1(I,J)+CF1(I-
     1,J))).gt.>(0.3*abs(CF1(I-1,J)-CF1(I+1,J)))) then
      CF1BAR3=(CF1(I+1,J)-CF1(I+2,J))/(CF1(I,J)-CF1(I+2,J))
      if(CF1BAR3.le.(-1.0).or.CF1BAR3.ge.1.5) then
      CF1IPLUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
      CF1IPLUSHALF(I,J)=CF1(I+2,J)+((CF1(I,J)-
     >                   CF1(I+2,J))*CF1IPLUSHALFBAR2)+CURVETI
      endif
      if(CF1BAR3.ge.0.35.and.CF1BAR3.le.0.65) then
      CF1IPLUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
      CF1IPLUSHALF(I,J)=CF1(I+2,J)+((CF1(I,J)-
     >                   CF1(I+2,J))*CF1IPLUSHALFBAR2)+CURVETI
      endif
      if(CF1BAR3.le.0.0.and.CF1BAR3.gt.(-1.0)) then
      CF1IPLUSHALFBAR2=0.375*CF1BAR3
      CF1IPLUSHALF(I,J)=CF1(I+2,J)+((CF1(I,J)-
     >                   CF1(I+2,J))*CF1IPLUSHALFBAR2)+CURVETI
      endif
      if(CF1BAR3.gt.0.0.and.CF1BAR3.lt.0.35) then
      CF1IPLUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
     >                   (1.0-2.0*CF1BAR3)
      CF1IPLUSHALF(I,J)=CF1(I+2,J)+((CF1(I,J)-
     >                   CF1(I+2,J))*CF1IPLUSHALFBAR2)+CURVETI
      endif
      if(CF1BAR3.gt.0.65.and.CF1BAR3.le.1.0) then
      CF1IPLUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
```

```
     >                    (1.0-2.0*CF1BAR3)
      CF1IPLUSHALF(I,J)=CF1(I+2,J)+((CF1(I,J)-
     >                   CF1(I+2,J))*CF1IPLUSHALFBAR2)+CURVETI
      endif
      if(CF1BAR3.le.1.5.and.CF1BAR3.gt.1.0) then
      CF1IPLUSHALFBAR2=CF1BAR3
      CF1IPLUSHALF(I,J)=CF1(I+2,J)+((CF1(I,J)-
     >                   CF1(I+2,J))*CF1IPLUSHALFBAR2)+CURVETI
      endif
      endif
    5 continue
    4 continue
c
c       INTERPOLATIOS IN J-DIRECTION
c
      do 6 I=2,NDIV-3
      do 7 J=2,NDJV-3
      CURVETJ=(1.0/24.0)*(CF1(I+1,J)-2.0*CF1(I,J)+CF1(I-1,J))
      if(abs(CF1(I,J+1)-CF1(I,J-1)).le.0.00001) then
      CF1JPLUSHALF(I,J)=(3.0/8.0)*CF1(I,J+1)+(6.0/8.0)*CF1(I,J)
     >             -(1.0/8.0)*CF1(I,J-1)+CURVETJ
      endif
      if(abs(CF1(I,J+1)-CF1(I,J-1)).le.0.00001)   goto   7
      if(abs(CF1(I,J-1)-2.0*CF1(I,J)+CF1(I,J+1)).le.
     >(0.3*abs(CF1(I,J+1)-CF1(I,J-1)))) then
      CF1JPLUSHALF(I,J)=(3.0/8.0)*CF1(I,J+1)+(6.0/8.0)*CF1(I,J)
     >             -(1.0/8.0)*CF1(I,J-1)+CURVETJ
      endif
      if((abs(CF1(I,J-1)-2.0*CF1(I,J)+CF1(I,J+1))).gt.
     > (0.3*abs(CF1(I,J-1)-CF1(I,J+1)))) then
      CF1BAR3=(CF1(I,J)-CF1(I,J-1))/(CF1(I,J+1)-CF1(I,J-1))
      if(CF1BAR3.le.(-1.0).or.CF1BAR3.ge.1.5) then
      CF1JPLUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
      CF1JPLUSHALF(I,J)=CF1(I,J-1)+((CF1(I,J+1)-
     >                   CF1(I,J-1))*CF1JPLUSHALFBAR2)+CURVETJ
      endif
      if(CF1BAR3.ge.0.35.and.CF1BAR3.le.0.65) then
      CF1JPLUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
      CF1JPLUSHALF(I,J)=CF1(I,J-1)+((CF1(I,J+1)-
     >                   CF1(I,J-1))*CF1JPLUSHALFBAR2)+CURVETJ
      endif
      if(CF1BAR3.le.0.0.and.CF1BAR3.gt.(-1.0)) then
      CF1JPLUSHALFBAR2=0.375*CF1BAR3
      CF1JPLUSHALF(I,J)=CF1(I,J-1)+((CF1(I,J+1)-
     >                   CF1(I,J-1))*CF1JPLUSHALFBAR2)+CURVETJ
      endif
      if(CF1BAR3.gt.0.0.and.CF1BAR3.lt.0.35) then
      CF1JPLUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
     >                   (1.0-2.0*CF1BAR3)
      CF1JPLUSHALF(I,J)=CF1(I,J-1)+((CF1(I,J+1)-
     >                   CF1(I,J-1))*CF1IPLUSHALFBAR2)+CURVETJ
      endif
      if(CF1BAR3.gt.0.65.and.CF1BAR3.le.1.0) then
      CF1JPLUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
     >                   (1.0-2.0*CF1BAR3)
      CF1JPLUSHALF(I,J)=CF1(I,J-1)+((CF1(I,J+1)-
     >                   CF1(I,J-1))*CF1JPLUSHALFBAR2)+CURVETJ
      endif
      if(CF1BAR3.le.1.5.and.CF1BAR3.gt.1.0) then
      CF1JPLUSHALFBAR2=CF1BAR3
      CF1JPLUSHALF(I,J)=CF1(I,J-1)+((CF1(I,J+1)-
     >                   CF1(I,J-1))*CF1JPLUSHALFBAR2)+CURVETJ
      endif
      endif
    7 continue
    6 continue
```

```
c
c      BOUNDARY CONDITIONS AT THE TOP
c
c      PRESCRIBED CONCENTRATION AT THE TOP
c
       if(NSTUBC.eq.1) then
       do 17 I=1,NDIV
       CF1JPLUSHALF(I,NDJV-1)=PCT
       CF1(I,NDJV+1)=CF1JPLUSHALF(I,NDJV-1)
   17 continue
c
c      LATERAL BOUNDARIES' CONDITIONS (ZERO FLUX)
c

       do 18 J=1,NDJV
       CF1(NDIV+1,J)=CF1(NDIV-1,J)
       CF1IPLUSHALF(NDIV-1,J)=CF1(NDIV,J)
c      CF1IPLUSHALF(NDIV-1,J)=(3.0/8.0)*CF1(NDIV+1,J)+
c     >                  (6.0/8.0)*CF1(NDIV-1,J)-(1.0/8.0)*CF1(NDIV-2,J)
       CF1(NDIV,J)=CF1IPLUSHALF(NDIV-1,J)
   18 continue
       endif
       if(NSTUBC.eq.2) then
c
c      PRESCRIBED CONCENTRATION GRADIENT (GENERALLY ZERO) AT THE TOP
c
       do 19 I=1,NDIV
       CF1JPLUSHALF(I,NDJV-1)=CF1(I,NDJV-1)
       CF1(I,NDJV+1)=CF1JPLUSHALF(I,NDJV-1)
c       CF1(I,NDJV+1)=CF1(I,NDJV-1)
c      CF1IPLUSHALF(I,NDJV-1)=(3.0/8.0)*CF1(I,NDJV+1)+
c     >                  (6.0/8.0)*CF1(I,NDJV-1)-(1.0/8.0)*CF1(I,NDJV-2)
       CF1(I,NDJV)=CF1IPLUSHALF(I,NDJV-1)
   19 continue
c
c      LATERAL BOUNDARIES' CONDITIONS (ZERO FLUX)
c
       do 20 J=1,NDJV
       CF1IPLUSHALF(NDIV-1,J)=CF1(NDIV-1,J)
       CF1(NDIV+1,J)=CF1IPLUSHALF(NDIV-1,J)
c       CF1(NDIV+1,J)=CF1(NDIV-1,J)
c      CF1IPLUSHALF(NDIV-1,J)=(3.0/8.0)*CF1(NDIV+1,J)+
c     >                  (6.0/8.0)*CF1(NDIV-1,J)-(1.0/8.0)*CF1(NDIV-2,J)
       CF1(NDIV,J)=CF1IPLUSHALF(NDIV-1,J)
   20 continue
       endif
       if(NSTUBC.eq.3) then
c
c      RESERVOIR BOUNDARY CONDITION AT THE TOP
c
       VS1=abs(CQ(NDJV))/EN(NDJV)
       HH=0.5*DA*(1.0+EG(NDJV-1))/(1.0+EG0(NDJV-1))
       do 21 I=1,NDIV
       CF1(I,NDJV)=(VS1*PRT*HH+CHDA(NDJV-1)*CF1(I,NDJV-1))/
     >             (VS1*HH+CHDA(NDJV-1))
       CF1JPLUSHALF(I,NDJV-1)=CF1(I,NDJV)
       CF1(I,NDJV+1)=PRT
   21 continue
c
c      LATERAL BOUNDARIES' CONDITIONS (ZERO FLUX)
c
       do 22 J=1,NDJV
       CF1IPLUSHALF(NDIV-1,J)=CF1(NDIV-1,J)
       CF1(NDIV+1,J)=CF1IPLUSHALF(NDIV-1,J)
       CF1(NDIV,J)=CF1IPLUSHALF(NDIV-1,J)
   22 continue
       endif
```

```
      do 10 J=2,NDJV-1
      CURVETI=(1.0/24.0)*(CF1(NDIV-1,J+1)-2.0*CF1(NDIV-1,J)+
     >        CF1(NDIV-1,J-1))
      if(abs(CF1(NDIV-2,J)-CF1(NDIV+1,J)).le.0.00001) then
      CF1IPLUSHALF(NDIV-2,J)=(3.0/8.0)*CF1(NDIV-2,J)+(6.0/8.0)*
     >                    CF1(NDIV-1,J)-(1.0/8.0)*CF1(NDIV+1,J)+CURVETI
      endif
      if(abs(CF1(NDIV-2,J)-CF1(NDIV+1,J)).le.0.00001) goto 10
      if(abs(CF1(NDIV+1,J)-2.0*CF1(NDIV-1,J)+CF1(NDIV-2,J)).le.
     >  (0.3*abs(CF1(NDIV-2,J)-CF1(NDIV+1,J)))) then
      CF1IPLUSHALF(NDIV-2,J)=(3.0/8.0)*CF1(NDIV-2,J)+(6.0/8.0)*
     >                   CF1(NDIV-1,J)-(1.0/8.0)*CF1(NDIV+1,J)+CURVETI
      endif
      if(abs(CF1(NDIV+1,J)-2.0*CF1(NDIV-1,J)+CF1(NDIV-2,J)).gt.(0.3*
     >abs(CF1(NDIV-2,J)-CF1(NDIV+1,J)))) then
      CF1BAR3=(CF1(NDIV-1,J)-CF1(NDIV+1,J))/(CF1(NDIV-2,J)-
     >         CF1(NDIV+1,J))
      if(CF1BAR3.le.(-1.0).or.CF1BAR3.ge.1.5) then
      CF1IPLUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
      CF1IPLUSHALF(NDIV-2,J)=CF1(NDIV+1,J)+(CF1(NDIV-2,J)-
     >                        CF1(NDIV+1,J))*CF1IPLUSHALFBAR2+CURVETI
      endif
      if(CF1BAR3.ge.0.35.and.CF1BAR3.le.0.65) then
      CF1IPLUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
      CF1IPLUSHALF(NDIV-2,J)=CF1(NDIV+1,J)+(CF1(NDIV-2,J)-
     >                        CF1(NDIV+1,J))*CF1IPLUSHALFBAR2+CURVETI
      endif
      if(CF1BAR3.le.0.0.and.CF1BAR3.gt.(-1.0)) then
      CF1IPLUSHALFBAR2=0.375*CF1BAR3
      CF1IPLUSHALF(NDIV-2,J)=CF1(NDIV+1,J)+(CF1(NDIV-2,J)-
     >                        CF1(NDIV+1,J))*CF1IPLUSHALFBAR2+CURVETI
      endif
      if(CF1BAR3.gt.0.0.and.CF1BAR3.lt.0.35) then
      CF1IPLUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
     >                   (1.0-2.0*CF1BAR3)
      CF1IPLUSHALF(NDIV-2,J)=CF1(NDIV+1,J)+(CF1(NDIV-2,J)-
     >                        CF1(NDIV+1,J))*CF1IPLUSHALFBAR2+CURVETI
      endif
      if(CF1BAR3.gt.0.65.and.CF1BAR3.le.1.0) then
      CF1IPLUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
     >                   (1.0-2.0*CF1BAR3)
      CF1IPLUSHALF(NDIV-2,J)=CF1(NDIV+1,J)+(CF1(NDIV-2,J)-
     >                        CF1(NDIV+1,J))*CF1IPLUSHALFBAR2+CURVETI
      endif
      if(CF1BAR3.le.1.5.and.CF1BAR3.gt.1.0) then
      CF1IPLUSHALFBAR2=CF1BAR3
      CF1IPLUSHALF(NDIV-2,J)=CF1(NDIV+1,J)+(CF1(NDIV-2,J)-
     >                        CF1(NDIV+1,J))*CF1IPLUSHALFBAR2+CURVETI
      CF1IMINUSHALF(NDIV-1,J)=CF1IPLUSHALF(NDIV-2,J)
      endif
      endif
   10 continue
      do 11 I=2,NDIV-1 CURVETJ=(1.0/24.0)*(CF1(I+1,NDJV-1)-
      2.0*CF1(I,NDJV-1)+
     >        CF1(I-1,NDJV-1))
      if(abs(CF1(I,NDJV-2)-CF1(I,NDJV+1)).le.0.00001) then
      CF1JPLUSHALF(I,NDJV-2)=(3.0/8.0)*CF1(I,NDJV-2)+(6.0/8.0)*
     >                   CF1(I,NDJV-1)-(1.0/8.0)*CF1(I,NDJV+1)+CURVETJ
      endif
      if(abs(CF1(I,NDJV-2)-CF1(I,NDJV+1)).le.0.00001) goto 11
      if(abs(CF1(I,NDJV+1)-2.0*CF1(I,NDJV-1)+CF1(I,NDJV-2)).le.
     >  (0.3*abs(CF1(I,NDJV-2)-CF1(I,NDJV+1)))) then
      CF1JPLUSHALF(I,NDJV-2)=(3.0/8.0)*CF1(I,NDJV-2)+(6.0/8.0)*
     >                   CF1(I,NDJV-1)-(1.0/8.0)*CF1(I,NDJV+1)+CURVETJ
      endif
      if(abs(CF1(I,NDJV+1)-2.0*CF1(I,NDJV-1)+CF1(I,NDJV-2)).gt.
     >  (0.3*abs(CF1(I,NDJV-2)-CF1(I,NDJV+1)))) then
```

```
      CF1BAR3=(CF1(I,NDJV-1)-CF1(I,NDJV+1))/(CF1(I,NDJV-2)-
     >         CF1(I,NDJV+1))
      if(CF1BAR3.le.(-1.0).or.CF1BAR3.ge.1.5) then
      CF1JPLUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
      CF1JPLUSHALF(NDIV-2,J)=CF1(I,NDJV+1)+(CF1(I,NDJV-2)-
     >                       CF1(I,NDJV+1))*CF1JPLUSHALFBAR2+CURVETJ
      endif
      if(CF1BAR3.ge.0.35.and.CF1BAR3.le.0.65) then
      CF1JPLUSHALFBAR2=0.75+0.75*(CF1BAR3-0.5)
      CF1JPLUSHALF(I,NDJV-2)=CF1(I,NDJV+1)+(CF1(I,NDJV-2)-
     >                       CF1(I,NDJV+1))*CF1JPLUSHALFBAR2+CURVETJ
      endif
      if(CF1BAR3.le.0.0.and.CF1BAR3.gt.(-1.0)) then
      CF1JPLUSHALFBAR2=0.375*CF1BAR3
      CF1JPLUSHALF(I,NDJV-2)=CF1(I,NDJV+1)+(CF1(I,NDJV-2)-
     >                       CF1(I,NDJV+1))*CF1JPLUSHALFBAR2+CURVETJ
      endif
      if(CF1BAR3.gt.0.0.and.CF1BAR3.lt.0.35) then
      CF1JPLUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
     >                  (1.0-2.0*CF1BAR3)
      CF1JPLUSHALF(I,NDJV-2)=CF1(I,NDJV+1)+(CF1(I,NDJV-2)-
     >                       CF1(I,NDJV+1))*CF1JPLUSHALFBAR2+CURVETJ
      endif
      if(CF1BAR3.gt.0.65.and.CF1BAR3.le.1.0) then
      CF1JPLUSHALFBAR2=(sqrt(CF1BAR3*((1.0-CF1BAR3)**3))-CF1BAR3**2)/
     >                  (1.0-2.0*CF1BAR3)
      CF1JPLUSHALF(I,NDJV-2)=CF1(I,NDJV+1)+(CF1(I,NDJV-2)-
     >                       CF1(I,NDJV+1))*CF1JPLUSHALFBAR2+CURVETJ
      endif
      if(CF1BAR3.le.1.5.and.CF1BAR3.gt.1.0) then
      CF1JPLUSHALFBAR2=CF1BAR3
      CF1JPLUSHALF(I,NDJV-2)=CF1(J,NDJV+1)+(CF1(I,NDJV-2)-
     >                       CF1(I,NDJV+1))*CF1JPLUSHALFBAR2+CURVETJ
      CF1JMINUSHALF(I,NDJV-1)=CF1JPLUSHALF(I,NDJV-2)
      endif
      endif
   11 continue
      do  8  I=2,NDIV-2
      do 9 J=2,NDJV-2
      CF1IMINUSHALF(I+1,J)=CF1IPLUSHALF(I,J)
      CF1JMINUSHALF(I,J+1)=CF1JPLUSHALF(I,J)
    9 continue
    8 continue
      endif
  605 EGJPLUSHALF(NDJV-1)=EG(NDJV)
      EG(NDJV+1)=(8.0/3.0)*EGJPLUSHALF(NDJV-1)-2.0*EG(NDJV-1)+
     >           (1.0/3.0)*EG(NDJV-2)
      EG0JPLUSHALF(NDJV-1)=EG0(NDJV)
      EG0(NDJV+1)=(8.0/3.0)*EGJPLUSHALF(NDJV-1)-2.0*EG(NDJV-1)
     >                     +(1.0/8.0)*EG0(NDJV-2)
      do 3 J=3,NDJV-3
      EGJMINUSHALF(J)=(3.0/8.0)*EG(J-1)+(6.0/8.0)*EG(J)-(1.0/8.0)*
     >                EG(J+1)
      EG0JMINUSHALF(J)=(3.0/8.0)*EG0(J-1)+(6.0/8.0)*EG0(J)-(1.0/8.0)*
     >                 EG0(J+1)
      EGJPLUSHALF(J)=(3.0/8.0)*EG(J)+(6.0/8.0)*EG(J+1)-(1.0/8.0)*
     >                EG(J+2)
      EG0JPLUSHALF(J)=(3.0/8.0)*EG0(J)+(6.0/8.0)*EG0(J+1)-(1.0/8.0)*
     >                EG0(J+2)
    3 continue
    4 EGJMINUSHALF(NDJV-2)=(3.0/8.0)*EG(NDJV-3)+(6.0/8.0)*EG(NDJV-2)-
     > (1.0/8.0)*EG(NDJV-1)
     EG0JMINUSHALF(NDJV-2)=(3.0/8.0)*EG0(NDJV-3)+(6.0/8.0)*EG0(NDJV-2)-
     > (1.0/8.0)*EG0(NDJV-1)
     EGJPLUSHALF(NDJV-2)=(3.0/8.0)*EG(NDJV-2)+(6.0/8.0)*EG(NDJV-1)-
     > (1.0/8.0)*EG(NDJV+1)
     EG0JPLUSHALF(NDJV-2)=(3.0/8.0)*EG0(NDJV-2)+(6.0/8.0)*EG0(NDJV-1)-
```

```
     >                    (1.0/8.0)*EG0(NDJV+1)
      EGJMINUSHALF(NDJV-1)=(3.0/8.0)*EG(NDJV-1)+(6.0/8.0)*EG(NDJV-2)-
     >                    (1.0/8.0)*EG(NDJV-3)
      EG0JMINUSHALF(NDJV-1)=(3.0/8.0)*EG0(NDJV-1)+(6.0/8.0)*EG0(NDJV-2)-
     >                     (1.0/8.0)*EG0(NDJV-3)
      EGJPLUSHALF(2)=(3.0/8.0)*EG(2)+(6.0/8.0)*EG(3)-(1.0/8.0)*EG(4)
      EG0JPLUSHALF(2)=(3.0/8.0)*EG0(2)+(6.0/8.0)*EG0(3)-(1.0/8.0)*EG0(4)
      EGJMINUSHALF(2)=EG(1)
      EG0JMINUSHALF(2)=EG0(1)
c
      return
      end
c
c     ***********************************************************************
      subroutine   LINTP(X,Y,XVAL,YVAL,IL)
c     ***********************************************************************

      dimension XVAL(351),YVAL(351)
c
c     INTERPOLATE Y FOR GIVEN X USING LAGRANGIAN INTERPOLATION
c
      Y1=(((X-XVAL(IL+1))*(X-XVAL(IL+2)))/
     >((XVAL(IL)-XVAL(IL+1))*(XVAL(IL)-XVAL(IL+2))))*YVAL(IL)
      Y2=(((X-XVAL(IL))*(X-XVAL(IL+2)))/
     >((XVAL(IL+1)-XVAL(IL))*(XVAL(IL+1)-XVAL(IL+2))))*YVAL(IL+1)
      Y3=(((X-XVAL(IL))*(X-XVAL(IL+1)))/
     >((XVAL(IL+2)-XVAL(IL))*(XVAL(IL+2)-XVAL(IL+1))))*YVAL(IL+2)
      Y=Y1+Y2+Y3
c
      return
      end
c
c     *************************************************************
      subroutine FDIFEQ()
c     *************************************************************
c
c     FDIFEQ CALCULATES NEW VOID RATIOS AS CONSOLIDATION PROCEEDS BY
c     AN EXPLICIT FINITE DIFFERENCE SCHEME BASED ON PREVIOUS VOID RATIOS.
c     SOIL PARAMETER FUNCTIONS ARE CONSTANTLY UPDATED TO CORRESPOND
c     WITH CURRENT VOID RATIO.
c
      common DA,DB,DZ,E00,ELL,GC,GS,GSBL,GW,HBL,LBL,NBDIV, NDIV,NBDJV,
     >       NDJV,NFLAG,NNN,NTIME,Q0,Q2,WL,SETT,SFIN,TAU,TIME,TPRINT,
     >       UCON,NNTIME,NST,NL,NBC,NDRB,ND1,ND2,NNL,NSOL,NNSOL,NNSOL1,
     >       ALPHAL,ALPHAT,NSTBC,SL,CFT,CFB,HT,HB,CHD0,CHD1,AKD,ALAMDAC,
     >       ALAMDASC,AM,DW,DS,DGM,TH,RKEO,NSL,HCL,HUCL,E0C,E0UC,NSORP,
     >       NSTUBC,NSTBBC,NSTRBC,NSTLBC,PCT,PGT,PRT,PCB,PGB,PRB,PCR,
     >       PGR,PRR,PCL,PGL,PRL,INX,JNZ,IE0,ISS,NC,TTIME,
     >       A(351),B(351),Z(351),XI(351),ALPHA(351),BETA(351),
     >       DSDE(351),E11(351),EFIN(351),ER(351),ES(351),EFFSTR(351),
     >       F(351),FS(351),FINT(351),PK(351),RK(351),RK1(351),RS(351),
     >       TOTSTR(351),U(351),U0(351),UW(351),VRI(351),DQ(351),
     >       Q1(351),RKEI(351),AKP,ANF,ANLAMDA,IKK,UMAX,NTAU,
     >       CHD2(351),CHDA(351),CHDX(351),CQI(351),CQU(351),CQ(351),
     >       EN0(351),EN(351),EG0(351),EG(351),DVDA(351,351),
     >       CS0(351,351),CS1(351,351),CS2(351,351),CF0(351,351),
     >       CF1(351,351),CF2(351,351),CFF1(351,351),CFF2(351,351),
     >       E11JPLUSHALF(351),E11JMINUSHALF(351),DKQU(351),
     >       FJPLUSHALF(351),FJMINUSHALF(351),AFJPLUSHALF(351),
     >       AFJMINUSHALF(351),BFJPLUSHALF(351),BFJMINUSHALF(351),
     >       EGJPLUSHALF(351),EGJMINUSHALF(351),EG0JPLUSHALF(351),
     >       EG0JMINUSHALF(351),CQJPLUSHALF(351),CQJMINUSHALF(351),
     >       CQUJPLUSHALF(351),CQUJMINUSHALF(351),DRD(351),
     >       CF0IPLUSHALF(351,351),CF0IMINUSHALF(351,351),
     >       CF0JPLUSHALF(351,351),CF0JMINUSHALF(351,351),
     >       CF1IPLUSHALF(351,351),CF1IMINUSHALF(351,351),
     >       CF1JPLUSHALF(351,351),CF1JMINUSHALF(351,351),
```

```
     > PRINT1(351)
      dimension HC(351)
c
c SET BOUNDARY CONDITIONS ON SOLUTE TRANSPORT AND SOLVE THE BOUNDARY NODES c
      if(ND1.eq.1.or.ND1.eq.2) then
      do 875 J=1,NDJV
      DRD(J)=(GSBL*(GW/9.81))/(1.0+ER(J))
  875 continue
c
c CALCULATE CURRENT HEIGHT OF SOIL LAYER AND USE EG c
      call INTGRL_4(FS,E11,DA,NDJV,FINT)
      do 2 J=3,NDJV-1 XI(J)=A(J)-(VRI(J)-
      FINT(J))
    2 continue
      do 6 J=1,NDJV
      EG0(J)=E11(J)
      EG(J)=FS(J)
      EN(J)=EG(J)/(1.0+EG(J))
    6 continue
c
c        CALCULATE THE HYDRAULIC CONDUCTIVITY AT EACH NODE
c
      if(NSOL.ne.1) then
      do 107 J=1,NDJV do
      108 N=2,LBL
      C1=EG(J)-ES(N)
      if(C1.ge.0.0) goto 109
  108 continue
      RK1(J)=RK(LBL); goto 107
  109 NN=N-1
      XV=FS(J)
      if(N.eq.LBL) NN=NN-1
      call LINTP_4(XV,YV,ES,RK,NN)
      RK1(J)=YV
  107 continue
      endif
      if(NSOL.eq.1) then
c
c       HYDRAULIC CONDUCTIVITY AT NODES OF CONTAMINATED LAYER
c
      do 110 J=1,NNSOL1
      do 111 N=2,LBL
      C1=EG(J)-ES(N)
      if(C1.ge.0.0) goto 112
  111 continue
      RK1(J)=RK(LBL); goto 110
  112 NN=N-1
      XV=F(J)
      if(N.eq.LBL) NN=NN-1
      call LINTP_4(XV,YV,ES,RK,NN)
      RK1(J)=YV
  110 continue
c
c       HYDRAULIC CONDUCTIVITY AT NODES OF UNCONTAMINATED LAYER
c
      do 113 J=NNSOL1+1,NDJV
      do 114 N=2,LBL
      C1=EG(J)-ES(N)
      if(C1.ge.0.0) goto 115
  114 continue
      RK1(J)=RK(LBL); goto 113
  115 NN=N-1
      XV=F(J)
      if(N.eq.LBL) NN=NN-1
      call LINTP_4(XV,YV,ES,RK,NN)
```

```
      RK1(J)=YV
  113 continue
      endif
      HTA=HBL+HT
      HTB=HB
c       if(NSOL.eq.1.and.NSTBC.eq.2) HT=HBL-XI(NDJV)
c
c       CALCULATE DARCY VELOCITY DUE TO HYDRAULIC GRADIENT
c
      THS=0.0
      do 800 J=2,NDJV-1
      HC(J)=DA*((1.0+EG(J))/(1.0+EG0(J)))
      THS=THS+HC(J)
  800 continue
      RCF=0.0
      do 801 J=2,NDJV-1
      RCF=RCF+(HC(J)/RK1(J))
  801 continue
      RK1EI=THS/RCF
      if(NDRB.eq.3.or.NDRB.eq.1)     CQUC=RK1EI*((HTB-HTA)/THS)
      if(NDRB.eq.2.and.HTA.gt.HTB) CQUC=0.0
      if(NDRB.eq.3.and.HTA.lt.HTB) CQUC=RK1EI*((HTB-HTA)/THS)
c     if(CQUC.lt.0.0)   CQUC=0.0
      do 206 J=1,NDJV
      CQI(J)=CQUC
  206 continue
      if(NDRB.ne.3) then
      do 4 J=1,NDJV
      CQI(J)=0.0
    4 continue
      endif
c*******************************************************
c     do 904 J=1,NDJV
c     CQI(J)=-0.007*EN(J)
c  904 continue
c*******************************************************
      call STRESS_4()
      call RESET_4()
c
c DARCY VELOCITY AT NODAL POINTS DUE TO EXCESS PORE PRESSURE HEAD DUE TO
c    CONSOLIDATION
c
  415 call INTGRLQ(DKQU,E11,DA,NDJV,NDRB,CQU)
      if(NDRB.eq.2) then CQUJPLUSHALF(NDJV-
      1)=CQU(NDJV-1) CQUJMINUSHALF(NDJV-
      1)=CQU(NDJV-2) CQUJPLUSHALF(2)=CQU(2)
      CQUJMINUSHALF(2)=CQU(1)
      do 13 J=3,NDJV-2
      CQUJPLUSHALF(J)=CQU(J)
      CQUJMINUSHALF(J)=CQU(J-1)
   13 continue
      endif
      if(NDRB.eq.3) then
      J2=((NBDJV+2)/2) do
      205 J=2,NDJV-1
      if(J.le.J2) then
      CQU(J)=-CQU(J)
      endif
      if(J.gt.J2) then
      CQU(J)=CQU(J)
      endif
  205 continue
      do 306 J=2,NDJV-1
      if(J.eq.J2) then
      CQUJPLUSHALF(J)=0.0
      CQUJMINUSHALF(J)=CQU(J)
```

```
       CQUJPLUSHALF(J+1)=CQU(J+1)
       CQUJMINUSHALF(J+1)=0.0
       endif
       if(J.gt.J2+1) then
       CQUJPLUSHALF(J)=CQU(J)
       CQUJMINUSHALF(J)=CQU(J-1)
       endif
       if(J.lt.J2) then
       CQUJPLUSHALF(J)=CQU(J+1)
       CQUJMINUSHALF(J)=CQU(J)
       endif
  306  continue CQU(1)=CQUJMINUSHALF(2)
       CQUJPLUSHALF(NDJV-1)=CQU(NDJV-1)

       CQUJMINUSHALF(NDJV-1)=CQU(NDJV-2)
       CQU(NDJV)=CQUJPLUSHALF(NDJV-1)
       endif
       if(NDRB.eq.1) then
       CQUJPLUSHALF(NDJV-1)=0.0
       CQUJMINUSHALF(NDJV-1)=-CQU(NDJV-1)
       do 312 J=2,NDJV-2
       CQU(J)=-CQU(J)
  312  continue
       do 313 J=2,NDJV-2
       CQUJPLUSHALF(J)=CQU(J+1)
       CQUJMINUSHALF(J)=CQU(J)
  313  continue
       CQU(1)=CQUJMINUSHALF(2)
c      CQUJPLUSHALF(NDJV-1)=CQU(NDJV-1)
       CQU(NDJV)=CQUJPLUSHALF(NDJV-1)
       endif
c
c     TOTAL DARCY VELOCITY
c
       do 5 J=2,NDJV-1
       CQJPLUSHALF(J)=CQUJPLUSHALF(J)+CQI(J)
       CQJMINUSHALF(J)=CQUJMINUSHALF(J)+CQI(J)
    5  continue
       do 14 J=1,NDJV
       CQ(J)=CQU(J)+CQI(J)
   14 continue
c
       if(ND2.eq.1) then
       do 15 J=1,NDJV
       CHDA(J)=CHD1+(ALPHAL*abs(CQ(J))/EN(J))
       if(ND1.eq.2) CHDX(J)=CHD1+(ALPHAT*abs(CQ(J))/EN(J))
       if(NTAU.eq.1) CHDA(J)=CHD1/(1.0/(EN(J)**0.33))+
      >                  (ALPHAL*abs(CQ(J))/EN(J))
       if(NTAU.eq.1) CHDX(J)=CHD1/(1.0/(EN(J)**0.33))+
      >                  (ALPHAT*abs(CQ(J))/EN(J))
   15  continue
       endif
       if(ND2.eq.2)  then
       do 16 J=1,NDJV
       CHD2(J)=CHD0*(EN(J)**AM)
       CHDA(J)=CHD2(J)+(ALPHAL*abs(CQ(J))/EN(J)) if(ND1.eq.2)
       CHDX(J)=CHD2(J)+(ALPHAT*abs(CQ(J))/EN(J))
   16  continue
       endif
       endif
c
c      ONE DIMENSIONAL SOLUTE TRANSPORT
c
       if(ND1.eq.1) then
c
c     EULER-QUICK ALGORTHIM
c
```

```
      if(NDRB.eq.2) call FACEVALUE_1()
      if(NDRB.eq.1.or.NDRB.eq.3) call FACEVALUE()
c
c      SOLUTE TRANSPORT FROM TOP BOUNDARY
c
      EGJPLUSHALF(NDJV-1)=EG(NDJV)
      EG(NDJV+1)=(8.0/3.0)*EGJPLUSHALF(NDJV-1)-2.0*EG(NDJV-1)+
     >            (1.0/3.0)*EG(NDJV-2)
      EGJMINUSHALF(NDJV-1)=(3.0/8.0)*EG(NDJV-2)+(6.0/8.0)*EG(NDJV-1)-
     > (1.0/8.0)*EG(NDJV+1)
     EG0JPLUSHALF(NDJV-1)=EG0(NDJV)
     EG0(NDJV+1)=(8.0/3.0)*EG0JPLUSHALF(NDJV-1)-2.0*EG0(NDJV-1)+
     >            (1.0/3.0)*EG0(NDJV-2)
      EG0JMINUSHALF(NDJV-1)=(3.0/8.0)*EG0(NDJV-1)+(6.0/8.0)*EG0(NDJV-2)-
     >                       (1.0/8.0)*EG0(NDJV+1)
      ENJPLUSHALF=EGJPLUSHALF(NDJV-1)/(1.0+EGJPLUSHALF(NDJV-1))
      ENJMINUSHALF=EGJMINUSHALF(NDJV-1)/(1.0+EGJMINUSHALF(NDJV-1))
      DJI=(1.0+EG(NDJV-1))/(1.0+EG0(NDJV-1))
      DJJPLUSHALF=(1.0+EGJPLUSHALF(NDJV-1))/(1.0+EG0JPLUSHALF(NDJV-1))
      DJJMINUSHALF=(1.0+EGJMINUSHALF(NDJV-1))/
     >              (1.0+EG0JMINUSHALF(NDJV-1))
      if(NSTBC.eq.1) then
      CF1(1,NDJV)=CFT
      CF1JPLUSHALF(1,NDJV-1)=CF1(1,NDJV)
      CF1(1,NDJV+1)=CFT
      endif
      if(NSTBC.eq.2) then
      CF1(1,NDJV+1)=2.0*CF1(1,NDJV)-CF1(1,NDJV-1)
      if(CF1(1,NDJV+1).lt.0.0) CF1(1,NDJV+1)=0.0
      CF1JPLUSHALF(1,NDJV-1)=CF1(1,NDJV)
      endif
      if(NSTBC.eq.3) then
      DJJ=(1.0+EG(NDJV))/(1.0+EG0(NDJV))
      DAA=DA
      RCF5=(DGM*CFT/TH)+((2.0*EN(NDJV)*CHDA(NDJV)/(DJJ*DAA))*
     >                   CF1(1,NDJV-1))
      RCF6=(DGM/TH)+(2.0*EN(NDJV)*CHDA(NDJV)/(DJJ*DAA))
      CF1(1,NDJV)=RCF5/RCF6
      CF1JPLUSHALF(1,NDJV-1)=CF1(1,NDJV)
      CF1(1,NDJV+1)=(8.0/3.0)*CF1JPLUSHALF(1,NDJV-1)-2.0*CF1(1,NDJV-1)+
     >              (1.0/3.0)*CF1(1,NDJV-2)
      endif
  604 FQCFJPLUSHALF=CQJPLUSHALF(NDJV-1)*CF1JPLUSHALF(1,NDJV-1)
      FDJPLUSHALF=(ENJPLUSHALF*CHDA(NDJV-1)/DJJPLUSHALF)*
     >((CF1(1,NDJV+1)-CF1(1,NDJV-1))/(DA))
      FQCFJMINUSHALF=CQJMINUSHALF(NDJV-1)*CF1JMINUSHALF(1,NDJV-1)
      FDJMINUSHALF=(ENJMINUSHALF*CHDA(NDJV-1)/DJJMINUSHALF)*
     >              ((CF1(1,NDJV-1)-CF1(1,NDJV-2))/(DA))
      CFF2(1,NDJV-1)=CFF1(1,NDJV-1)-(TAU/DA)*((FQCFJPLUSHALF-
     >                FDJPLUSHALF)-(FQCFJMINUSHALF-FDJMINUSHALF))
      if(CFF2(1,NDJV-1).lt.0.0) CFF2(1,NDJV-1)=0.0
      if(NSORP.eq.1) then
      if(AKD.eq.0.0) then
      CF2(1,NDJV-1)=(CFF2(1,NDJV-1)-((1.0-EN(NDJV-1))*
     >                CS1(1,NDJV-1)*DJI))/(EN(NDJV-1)*DJI)
      endif
c
c     DEDUCT LOSS OF SOLUTE DUE TO SOLUTE DECAY AND SOURCE DECAY
c
      if(AKD.ne.0.0) then
      DL1=EN(NDJV-1)*CF1(1,NDJV-1)*DJI*(1.0-exp(-ALAMDAC*TAU))
      DL2=EN(NDJV-1)*CF1(1,NDJV-1)*DJI*(1.0-exp(-ALAMDASC*TAU))
      CFF2(1,NDJV-1)=CFF2(1,NDJV-1)-DL1-DL2
      CF2(1,NDJV-1)=CFF2(1,NDJV-1)/((EN(NDJV-1)*DJI)+((1.0-EN(NDJV-1))*
     >              DRD(NDJV-1)*AKD*DJI))
      CS2(1,NDJV-1)=DRD(NDJV-1)*AKD*CF2(1,NDJV-1)
```

```
      endif
      endif
      if(NSORP.eq.2) then
      CFN2=CF1(1,NDJV-1)
      CSN2=CS1(1,NDJV-1)
      NA=50
      TAUN=TAU/float(NA)
      CFFN=(CFF2(1,NDJV-1)-CFF1(1,NDJV-1))/float(NA) do
      602 J=1,NA
      if(CFN2.le.0.0) DSDT=0.0
      if(CFN2.le.0.0) goto 608
      DSDT=ANLAMDA*(AKP*(CFN2**ANF)-(CSN2/DRD(J)))
  608 CSN2=CSN2+DSDT*TAUN*DRD(J)
      CFFN1=CFF1(1,NDJV-1)+CFFN
c if(((1.0-EN(I))*DJI*CSN2).gt.CFFN1) CSN2=0.0
c if(CSN2.lt.0.001) CSN2=0.0
      CFN2=(CFFN1-((1.0-EN(NDJV-1))*DJI*CSN2))/EN(NDJV-1)*DJI
  602 continue
      CF2(1,NDJV-1)=CFN2
      CS2(1,NDJV-1)=CSN2
      if(CF2(1,NDJV-1).lt.0.0)    CF2(1,NDJV-1)=0.0
      if(CS2(1,NDJV-1).lt.0.0)    CS2(1,NDJV-1)=0.0
      endif
c
c        FOR EQUILIBRIUM FREUNDILICH ISOTHERM BISECTION METHOD FOR CF
c
      if(NSORP.eq.3) then
      DVALUE=0.2
      if(CFF2(1,NDJV-1).lt.0.000001) then
      CF2(1,NDJV-1)=0.0
      CS2(1,NDJV-1)=0.0
      DVALUE=0.000001
      endif
      CR0=CF1(1,NDJV-1)
      do 701 while(ABS(DVALUE).gt.0.00001)
      call BISFVALUE(EN(NDJV-1),ER(NDJV-1),E11(NDJV-1),CR0,
     > CFF2(1,NDJV-1),DRD(NDJV-1),AKP,ANF,FVALUE,GVALUE)
     CRN=CR0-(FVALUE/GVALUE)
      DVALUE=(CR0-CRN)
      CR0=CRN
      CF2(1,NDJV-1)=CRN
      CS2(1,NDJV-1)=DRD(NDJV-1)*AKP*CF2(1,NDJV-1)**ANF
  701 continue
      endif
      if(NSTBC.eq.1) CF2(1,NDJV)=CFT
c
c       SOLUTE TRANSPORT FROM BOTTOM BOUNDARY
c
      EGJMINUSHALF(2)=EG(1)
      EGJPLUSHALF(2)=(3.0/8.0)*EG(2)+(6.0/8.0)*EG(3)-(1.0/8.0)*EG(4)
      EG0JMINUSHALF(2)=EG0(1)
      EG0JPLUSHALF(2)=(3.0/8.0)*EG0(2)+(6.0/8.0)*EG0(3)-(1.0/8.0)*EG0(4)
      ENJPLUSHALF=EGJPLUSHALF(2)/(1.0+EGJPLUSHALF(2))
      ENJMINUSHALF=EGJMINUSHALF(2)/(1.0+EGJMINUSHALF(2))
      DJI=(1.0+EG(2))/(1.0+EG0(2))
      DJJPLUSHALF=(1.0+EGJPLUSHALF(2))/(1.0+EG0JPLUSHALF(2))
      DJJMINUSHALF=(1.0+EGJMINUSHALF(2))/(1.0+EG0JMINUSHALF(2))
      if(NSTBC.eq.1) then
      CF1(1,1)=CFB
      CF1JMINUSHALF(1,2)=CF1(1,1)
      CF10=(8.0/3.0)*CF1JMINUSHALF(1,2)-2.0*CF1(1,2)+(1.0/3.0)*CF1(1,3)
      endif
      if(NSTBC.eq.2.or.NSTBC.eq.3) then
      H=DA*((1.0+EG(2))/(1.0+EG0(2)))
      VS=abs(CQ(1))/EN(1)
      CF1(1,1)=CF1(1,2)
      CF10=CF1(1,2)
```

```
      CF1JMINUSHALF(1,2)=CF1(1,2)
      endif
  606 FQCFJPLUSHALF=CQJPLUSHALF(2)*CF1JPLUSHALF(1,2)
      FDJPLUSHALF=(ENJPLUSHALF*CHDA(2)/DJJPLUSHALF)*
     >((CF1(1,3)-CF1(1,2))/(DA))
      FQCFJMINUSHALF=CQJMINUSHALF(2)*CF1JMINUSHALF(1,2)
      FDJMINUSHALF=(ENJMINUSHALF*CHDA(2)/DJJMINUSHALF)*
     >              ((CF1(1,2)-CF10)/(DA))
       CFF2(1,2)=CFF1(1,2)-(TAU/DA)*((FQCFJPLUSHALF-FDJPLUSHALF)-
     > (FQCFJMINUSHALF-FDJMINUSHALF))
      if(CFF2(1,2).lt.0.0) CFF2(1,2)=0.0
      if(NSORP.eq.1) then
      if(AKD.eq.0.0) then
      CF2(1,2)=(CFF2(1,2)-((1.0-EN(2))*CS1(1,2)*DJI))/(EN(2)*DJI)
      endif
c
c        DEDUCT LOSS OF SOLUTE DUE TO SOLUTE DECAY AND SOURCE DECAY
c
      if(AKD.ne.0.0) then
      DL1=EN(2)*CF1(1,2)*DJI*(1.0-exp(-ALAMDAC*TAU))
      DL2=EN(2)*CF1(1,2)*DJI*(1.0-exp(-ALAMDASC*TAU))
      CFF2(1,2)=CFF2(1,2)-DL1-DL2
      CF2(1,2)=CFF2(1,2)/((EN(2)*DJI)+((1.0-EN(2))*DRD(2)*AKD*DJI))
      CS2(1,2)=DRD(2)*AKD*CF2(1,2)
      endif
      endif
      if(NSORP.eq.2) then
      CFN2=CF1(1,2)
      CSN2=CS1(1,2)
      NA=100
      TAUN=TAU/float(NA)
      CFFN=(CFF2(1,2)-CFF1(1,2))/float(NA)
      do 603 J=1,NA
      if(CFN2.le.0.0) DSDT=0.0
      if(CFN2.le.0.0) goto 607
      DSDT=ANLAMDA*(AKP*(CFN2**ANF)-(CSN2/DRD(2)))
  607 CSN2=CSN2+DSDT*TAUN*DRD(2)
      CFFN1=CFF1(1,2)+CFFN
      CFN2=(CFFN1-((1.0-EN(2))*DJI*CSN2))/EN(2)*DJI
  603 continue
      CF2(1,2)=CFN2
      CS2(1,2)=CSN2
  604 endif
  605 if(NSORP.eq.3) then
      DVALUE=0.2
      CR0=CF1(1,2)
      if(CFF2(1,2).lt.0.000001)   then
      CF2(1,2)=0.0
      CS2(1,2)=0.0
      DVALUE=0.000001
      endif
      do 702 while(ABS(DVALUE).gt.0.00001)
      call BISFVALUE(EN(2),ER(2),E11(2),CR0,CFF2(1,2),
     >DRD(2),AKP,ANF,FVALUE,GVALUE)
      CRN=CR0-(FVALUE/GVALUE)
      DVALUE=(CR0-CRN)
      CR0=CRN
      CF2(1,2)=CRN
      CS2(1,2)=DRD(2)*AKP*CF2(1,2)**ANF
  702 continue
      endif
c
c        SOLUTE TRANSPORT VALUES AT NEXT TIME STEP FOR INTERIOR NODES
c
      do 7 J=3,NDJV-2
      DJI=(1.0+EG(J))/(1.0+EG0(J))
      ENJPLUSHALF=EGJPLUSHALF(J)/(1.0+EGJPLUSHALF(J))
```

```
      ENJMINUSHALF=EGJMINUSHALF(J)/(1.0+EGJMINUSHALF(J))
      DJJPLUSHALF=(1.0+EGJPLUSHALF(J))/(1.0+EG0JPLUSHALF(J))
      DJJMINUSHALF=(1.0+EGJMINUSHALF(J))/(1.0+EG0JMINUSHALF(J))
      FQCFJPLUSHALF=CQJPLUSHALF(J)*CF1JPLUSHALF(1,J)
      FDJPLUSHALF=(ENJPLUSHALF*CHDA(J)/DJJPLUSHALF)*
     >((CF1(1,J+1)-CF1(1,J))/(DA))
      FQCFJMINUSHALF=CQJMINUSHALF(J)*CF1JMINUSHALF(1,J)
      FDJMINUSHALF=(ENJMINUSHALF*CHDA(J)/DJJMINUSHALF)*
     >((CF1(1,J)-CF1(1,J-1))/(DA))
      CFF2(1,J)=CFF1(1,J)-(TAU/DA)*((FQCFJPLUSHALF-FDJPLUSHALF)-
     > (FQCFJMINUSHALF-FDJMINUSHALF))
      if(CFF2(1,J).lt.0.0) CFF2(1,J)=0.0
      if(NSORP.eq.1) then
      if(AKD.eq.0.0) then
      CF2(1,J)=(CFF2(1,J)-((1.0-EN(J))*CS1(1,J)*DJI))/(EN(J)*DJI)
      endif
      if(AKD.ne.0.0) then
c
c        DEDUCT LOSS OF SOLUTE DUE TO SOLUTE DECAY AND SOURCE DECAY
c
      DL1=EN(J)*DJI*CF1(1,J)*(1.0-exp(-ALAMDAC*TAU))
      DL2=EN(J)*DJI*CF1(1,J)*(1.0-exp(-ALAMDASC*TAU))
      CFF2(1,J)=CFF2(1,J)-DL1-DL2
      CF2(1,J)=CFF2(1,J)/((EN(J)*DJI)+((1.0-EN(J))*DRD(J)*AKD*DJI))
      CS2(1,J)=DRD(J)*AKD*CF2(1,J)
      endif
      endif
      if(NSORP.eq.2) then
      CFN2=CF1(1,J)
      CSN2=CS1(1,J)
      NA=50
      TAUN=TAU/float(NA)
      CFFN=(CFF2(1,J)-CFF1(1,J))/float(NA)
      do 601 IR=1,NA
      if(CFN2.le.0.0) DSDT=0.0
      if(CFN2.le.0.0) goto 609
      DSDT=ANLAMDA*(AKP*(CFN2**ANF)-(CSN2/DRD(J)))
  609 CSN2=CSN2+DSDT*TAUN*DRD(J)
      CFFN1=CFF1(1,J)+CFFN
      CFN2=(CFFN1-((1.0-EN(J))*DJI*CSN2))/EN(J)*DJI
  601 continue
      CF2(1,J)=CFN2
      CS2(1,J)=CSN2
      if(CF2(1,J).lt.0.0)   CF2(1,J)=0.0
      if(CS2(1,J).lt.0.0)   CS2(1,J)=0.0
      endif
      if(NSORP.eq.3) then
      DVALUE=0.2
      CR0=CF1(1,J)
      if(CFF2(1,J).lt.0.000001)   then
      CF2(1,J)=0.0
      CS2(1,J)=0.0
      DVALUE=0.000001
      endif
      do 703 while(ABS(DVALUE).gt.0.00001)
      call BISFVALUE(EN(J),ER(J),E11(J),CR0,CFF2(1,J),
     > DRD(J),AKP,ANF,FVALUE,GVALUE)
      CRN=CR0-(FVALUE/GVALUE)
      DVALUE=(CR0-CRN)
      CR0=CRN
      CF2(1,J)=CRN
      CS2(1,J)=DRD(J)*AKP*CF2(1,J)**ANF
  703 continue
      endif
    7 continue
c
      if(NSTBC.eq.1) then
```

```
      CF2(1,NDJV)=CFT
      CF2(1,1)=CFB
      CS2(1,1)=DRD(2)*AKD*CF2(1,1)
      endif
      if(NSTBC.eq.2) then VS1=abs(CQ(NDJV))/EN(NDJV)
      VS2=abs(CQ(NDJV-1))/EN(NDJV-1)
      VS3=abs(CQ(NDJV-2))/EN(NDJV-2)
      HH=0.5*DA*(1.0+EG(NDJV-1))/(1.0+EG0(NDJV-1))
      if(TIME.eq.0.0) then
      CRVR=0.0
      VR=0.0
      endif
      CRVR=CRVR+(VS1*CF1(1,NDJV)+
     > CHDA(NDJV)*((CF1(1,NDJV-1)-CF1(1,NDJV))/HH))*EN(NDJV)*TAU
      VR=VR+VS1*EN(NDJV)*TAU
      CR=CRVR/VR
      CF2(1,NDJV)=CR
      CF2(1,1)=CF2(1,2)
      endif if(NSTBC.eq.3)
      then
      DJJ=(1.0+EG0(NDJV))/(1.0+EG0(NDJV))
      DAA=DA
      RCF5=(DGM*CFT/TH)+((2.0*EN(NDJV)*CHDA(NDJV)/(DJJ*DAA))*
     > CF2(1,NDJV-1))
       RCF6=(DGM/TH)+(2.0*EN(NDJV)*CHDA(NDJV)/(DJJ*DAA))
       CF2(1,NDJV)=RCF5/RCF6
      CF2(1,1)=CF2(1,2)
      endif
c
c      LOSS OF SOLUTE IN THE TOP POOL AND BOTTOM POOL (VOID RATIO FACTOR IS NOT
c      APPLICABLE)
c
      CF2(1,1)=CF2(1,1)*exp(-ALAMDASC*TAU)
      CF2(1,NDJV)=CF2(1,NDJV)*exp(-ALAMDASC*TAU)
      if(NSORP.eq.1) CS2(1,1)=DRD(2)*AKD*CF2(1,1)
c
c        RESET NEXT LOOP FOR SOLUTE TRANSPORT IN CLAY LINER
c
      do 10 J=2,NDJV-1
      CFF1(1,J)=CFF2(1,J)
      CF1(1,J)=CF2(1,J)
      CS1(1,J)=CS2(1,J)
   10 continue
      CF1(1,1)=CF2(1,1)
      CF1(1,NDJV)=CF2(1,NDJV)
      CS1(1,1)=CS2(1,1)
      CS1(1,NDJV)=CS2(1,NDJV)
      endif
c
c      TWO DIMENSIONAL SOLUTE TRANSPORT
c
      if(ND1.eq.2) then
c
c      SOLUTE TREANSPORT FROM TOP BOUNDARY
c
      if(NSTUBC.eq.1) then
c
c      PRESCRIBED CONCENTRATION AT THE TOP
c
      do 17 I=1,NDIV
      CF1(I,NDJV)=PCT
      CF1JPLUSHALF(I,NDJV-1)=CF1(I,NDJV)
      CF1(I,NDJV+1)=PCT
   17 continue
c
c     LATERAL BOUNDARIES' CONDITIONS (ZERO FLUX)
```

```
c
      do 18 J=2,NDJV-1
      CF1(NDIV+1,J)=CF1(NDIV-1,J)
      CF1(NDIV,J)=CF1(NDIV-1,J)
      CF1IPLUSHALF(NDIV-1,J)=CF1(NDIV,J)
      CF1I0=CF1(2,J)
      CF1(1,J)=CF1(2,J)
      CF1IMINUSHALF(2,J)=CF1(1,J)
   18 continue
      endif
      if(NSTUBC.eq.2) then
c
c        PRESCRIBED CONCENTRATION GRADIENT (GENERALLY ZERO) AT THE TOP
c
      do 19 I=1,NDIV
      CF1(I,NDJV+1)=CF1(I,NDJV-1)
      CF1(I,NDJV)=CF1(I,NDJV-1)
      CF1IPLUSHALF(I,NDJV-1)=CF1(I,NDJV)
   19 continue
c
c       LATERAL BOUNDARIES' CONDITIONS (ZERO FLUX)
c
      do 20 J=2,NDJV-1
      CF1(NDIV+1,J)=CF1(NDIV-1,J)
      CF1(NDIV,J)=CF1(NDIV-1,J)
      CF1IPLUSHALF(NDIV-1,J)=CF1(NDIV,J)
      CF1I0=CF1(2,J)
      CF1(1,J)=CF1(2,J)
      CF1IMINUSHALF(2,J)=CF1(1,J)
   20 continue
      endif
      if(NSTUBC.eq.3) then
c
c      RESERVOIR BOUNDARY CONDITION AT THE TOP
c
      VS1=abs(CQ(NDJV))/EN(NDJV)
      HH=0.5*DA*(1.0+EG(NDJV-1))/(1.0+EG0(NDJV-1))
      do 21 I=1,NDIV
      CF1(I,NDJV)=(VS1*PRT*HH+CHDA(NDJV-1)*CF1(I,NDJV-1))/
     >(VS1*HH+CHDA(NDJV-1))
      CF1JPLUSHALF(I,NDJV-1)=CF1(I,NDJV)
      CF1(I,NDJV+1)=PRT
   21 continue
c
c       LATERAL BOUNDARIES' CONDITIONS (ZERO FLUX)
c
      do 22 J=2,NDJV-1
      CF1(NDIV+1,J)=CF1(NDIV-1,J)
      CF1(NDIV,J)=CF1(NDIV-1,J)
      CF1IPLUSHALF(NDIV-1,J)=CF1(NDIV,J)
      CF1I0=CF1(2,J)
      CF1(1,J)=CF1(2,J)
      CF1IMINUSHALF(2,J)=CF1(1,J)
   22 continue
      endif
      if(NDRB.eq.2) call FACEVALUE_1()
      if(NDRB.eq.1.or.NDRB.eq.3) call FACEVALUE()
c
c        CALCULATE NEXT TIME STEP CONCETRATION NEAR TOP BOUNDARY LOCATION
c
      EGJPLUSHALF(NDJV-1)=EG(NDJV)
      EG(NDJV+1)=(8.0/3.0)*EGJPLUSHALF(NDJV-1)-2.0*EG(NDJV-1)+
     >            (1.0/3.0)*EG(NDJV-2)
      EGJMINUSHALF(NDJV-1)=(3.0/8.0)*EG(NDJV-2)+(6.0/8.0)*EG(NDJV-1)-
     > (1.0/8.0)*EG(NDJV+1)
     EG0JPLUSHALF(NDJV-1)=EG0(NDJV)
     EG0(NDJV+1)=(8.0/3.0)*EG0JPLUSHALF(NDJV-1)-2.0*EG0(NDJV-1)+
```

```
     >            (1.0/3.0)*EG0(NDJV-2)
      EG0JMINUSHALF(NDJV-1)=(3.0/8.0)*EG0(NDJV-1)+(6.0/8.0)*EG0(NDJV-2)-
     > (1.0/8.0)*EG0(NDJV+1)
     ENJPLUSHALF=EGJPLUSHALF(NDJV-1)/(1.0+EGJPLUSHALF(NDJV-1))
     ENJMINUSHALF=EGJMINUSHALF(NDJV-1)/(1.0+EGJMINUSHALF(NDJV-1))
     DJI=(1.0+EG(NDJV-1))/(1.0+EG0(NDJV-1))
     DJJPLUSHALF=(1.0+EGJPLUSHALF(NDJV-1))/(1.0+EG0JPLUSHALF(NDJV-1))
     DJJMINUSHALF=(1.0+EGJMINUSHALF(NDJV-1))/
     >(1.0+EG0JMINUSHALF(NDJV-1))
      do 23 I=2,NDIV-1
      FQCFJPLUSHALF=CQJPLUSHALF(NDJV-1)*CF1JPLUSHALF(I,NDJV-1)
      FDJPLUSHALF=(ENJPLUSHALF*CHDA(NDJV-1)/DJJPLUSHALF)*
     >((CF1(I,NDJV+1)-CF1(I,NDJV-1))/(DA))
      FQCFJMINUSHALF=CQJMINUSHALF(NDJV-1)*CF1JMINUSHALF(I,NDJV-1)
      FDJMINUSHALF=(ENJMINUSHALF*CHDA(NDJV-1)/DJJMINUSHALF)*
     >((CF1(I,NDJV-1)-CF1(I,NDJV-2))/(DA))
      if(I.eq.(NDIV-1)) then
      FDIPLUSHALF=(ENJPLUSHALF*CHDX(NDJV-1))*
     >             ((CF1(NDIV+1,NDJV-1)-CF1(NDIV-1,NDJV-1))/(DB))
      endif
      if(I.lt.(NDIV-1)) then
      FDIPLUSHALF=(ENJPLUSHALF*CHDX(NDJV-1))*
     >             ((CF1(I+1,NDJV-1)-CF1(I,NDJV-1))/(DB))
      endif
      if(I.eq.2) then
      FDIMINUSHALF=(ENJMINUSHALF*CHDX(NDJV-1))*
     >             ((CF1(I,NDJV-1)-CF1I0)/(DB))
      endif
      if(I.gt.2) then
      FDIMINUSHALF=(ENJMINUSHALF*CHDX(NDJV-1))*
     >             ((CF1(I,NDJV-1)-CF1(I-1,NDJV-1))/(DB))
      endif
      CFF2(I,NDJV-1)=CFF1(I,NDJV-1)-(TAU/DA)*((FQCFJPLUSHALF-
     > FDJPLUSHALF)-(FQCFJMINUSHALF-FDJMINUSHALF))+
     > (TAU/DB)*(FDIPLUSHALF-FDIMINUSHALF)
      if(NSORP.eq.1) then
      if(AKD.eq.0.0) then
      CF2(I,NDJV-1)=(CFF2(I,NDJV-1)-((1.0-EN(NDJV-1))*
     >CS1(I,NDJV-1)*DJI))/(EN(NDJV-1)*DJI)
      CS2(I,NDJV-1)=DRD(NDJV-1)*AKD*CF2(I,NDIV-1)
      endif
      if(AKD.ne.0.0) then
c
c        DEDUCT LOSS OF SOLUTE DUE TO SOLUTE DECAY AND SOURCE DECAY
c
      DL1=EN(NDJV-1)*DJI*CF1(I,NDJV-1)*(1.0-exp(-ALAMDAC*TAU))
      DL2=EN(NDJV-1)*DJI*CF1(I,NDJV-1)*(1.0-exp(-ALAMDASC*TAU))
      CFF2(I,NDJV-1)=CFF2(I,NDJV-1)-DL1-DL2 CF2(I,NDJV-
      1)=CFF2(I,NDJV-1)/((EN(NDJV-1)*DJI)+
     >((1.0-EN(NDJV-1))*DRD(NDJV-1)*AKD*DJI))
      CS2(I,NDJV-1)=DRD(NDJV-1)*AKD*CF2(I,NDJV-1)
      endif
      endif
      if(NSORP.eq.2) then
      CFN2=CF1(I,NDJV-1)
      CSN2=CS1(I,NDJV-1)
      NA=50
      TAUN=TAU/float(NA)
      CFFN=(CFF2(I,NDJV-1)-CFF1(I,NDJV-1))/float(NA)
      do 611 IR=1,NA
      if(CFN2.le.0.0) DSDT=0.0
      if(CFN2.le.0.0) goto 619
      DSDT=ANLAMDA*(AKP*(CFN2**ANF)-(CSN2/DRD(NDJV-1)))
  619 CSN2=CSN2+DSDT*TAUN*DRD(NDJV-1)
      CFFN1=CFF1(I,NDJV-1)+CFFN
      CFN2=(CFFN1-((1.0-EN(NDJV-1))*DJI*CSN2))/EN(NDJV-1)*DJI
  611 continue
```

```
        CF2(I,NDJV-1)=CFN2
        CS2(I,NDJV-1)=CSN2 if(CF2(I,NDJV-1).lt.0.0)
        CF2(I,NDJV-1)=0.0 if(CS2(I,NDJV-1).lt.0.0)
        CS2(I,NDJV-1)=0.0 endif
   23 continue
c
c       SOLUTE TREANSPORT FROM BOTTOM BOUNDARY
c
      if(NSTBBC.eq.1) then
c
c       PRESCRIBED CONCENTRATION AT THE BOTTOM
c
      do 24 I=1,NDIV CF1(I,1)=PCB
      CF1JMINUSHALF(I,2)=CF1(I,1)
      CF1J0=CF1(I,1)
   24 continue
c
c       LATERAL BOUNDARIES' CONDITIONS (ZERO FLUX)
c
      do 25 J=2,NDJV-1
      CF1(NDIV+1,J)=CF1(NDIV-1,J)
      CF1(NDIV,J)= CF1(NDIV-1,J)
      CF1IPLUSHALF(NDIV-1,J)=CF1(NDIV,J)
      CF1I0=CF1(2,J)
      CF1(1,J)=CF1(2,J)
      CF1IMINUSHALF(2,J)=CF1(1,J)
   25 continue
      endif
      if(NSTBBC.eq.2) then
c
c        PRESCRIBED CONCENTRATION GRADIENT (GENERALLY ZERO) AT THE BOTTOM
c
      do 26 I=1,NDIV
      CF1(I,1)=CF1(I,2)
      CF1J0=CF1(I,1)
      CF1IMINUSHALF(I,2)=CF1(I,1)
    26 continue
c
c       LATERAL BOUNDARIES' CONDITIONS (ZERO FLUX)
c
      do 27 J=2,NDJV-1
      CF1(NDIV+1,J)=CF1(NDIV-1,J)
      CF1(NDIV,J)= CF1(NDIV-1,J)
      CF1IPLUSHALF(NDIV-1,J)=CF1(NDIV,J)
      CF1I0=CF1(2,J)
      CF1(1,J)=CF1(2,J)
      CF1IMINUSHALF(2,J)=CF1(1,J)
   27 continue
      endif
      if(NSTBBC.eq.3) then
c
c       RESERVOIR BOUNDARY CONDITION AT THE BOTTOM
c
      VS1=abs(CQ(1))/EN(1)
      HH=0.5*DA*(1.0+EG(2))/(1.0+EG0(2))
      do 28 I=1,NDIV
      CF1(I,1)=(VS1*PRT*HH+CHDA(2)*CF1(I,2))/
     >(VS1*HH+CHDA(2))
      CF1JPLUSHALF(I,2)=CF1(I,1)
      CF1I0=PRT
    28 continue
c
c       LATERAL BOUNDARIES' CONDITIONS (ZERO FLUX)
c
```

```
      do 29 J=2,NDJV-1
      CF1(NDIV+1,J)=CF1(NDIV-1,J)
      CF1(NDIV,J)= CF1(NDIV-1,J)
      CF1IPLUSHALF(NDIV-1,J)=CF1(NDIV,J)
      CF1I0=CF1(2,J)
      CF1(1,J)=CF1(2,J)
      CF1IMINUSHALF(2,J)=CF1(1,J)
   29 continue
      endif
c
c        CALCULATE NEXT TIME STEP CONCETRATION NEAR BOTTOM BOUNDARY LOCATION
c
      EGJMINUSHALF(2)=EG(1)
      EGJPLUSHALF(2)=(3.0/8.0)*EG(2)+(6.0/8.0)*EG(3)-(1.0/8.0)*EG(4)
      EG0JMINUSHALF(2)=EG0(1)
      EG0JPLUSHALF(2)=(3.0/8.0)*EG0(2)+(6.0/8.0)*EG0(3)-(1.0/8.0)*EG0(4)
      ENJPLUSHALF=EGJPLUSHALF(2)/(1.0+EGJPLUSHALF(2))
      ENJMINUSHALF=EGJMINUSHALF(2)/(1.0+EGJMINUSHALF(2))
      DJI=(1.0+EG(2))/(1.0+EG0(2))
      DJJPLUSHALF=(1.0+EGJPLUSHALF(2))/(1.0+EG0JPLUSHALF(2))
      DJJMINUSHALF=(1.0+EGJMINUSHALF(2))/(1.0+EG0JMINUSHALF(2))
      do 30 I=2,NDIV-1
      FQCFJPLUSHALF=CQJPLUSHALF(2)*CF1JPLUSHALF(I,2)
      FDJPLUSHALF=(ENJPLUSHALF*CHDA(2)/DJJPLUSHALF)*
     >((CF1(I,3)-CF1(I,2))/(DA))
      FQCFJMINUSHALF=CQJMINUSHALF(2)*CF1JMINUSHALF(I,2)
      FDJMINUSHALF=(ENJMINUSHALF*CHDA(2)/DJJMINUSHALF)*
     >((CF1(I,2)-CF1J0)/(DA))
       if(I.eq.(NDIV-1)) then
      FDIPLUSHALF=(ENJPLUSHALF*CHDX(2))*
     >             ((CF1(NDIV+1,2)-CF1(NDIV-1,2))/(DB))
      endif
      if(I.lt.(NDIV-1)) then
      FDIPLUSHALF=(ENJPLUSHALF*CHDX(2))*
     >             ((CF1(I+1,2)-CF1(I,2))/(DB))
      endif
      if(I.eq.2) then
      FDIMINUSHALF=(ENJMINUSHALF*CHDX(2))*
     >             ((CF1(2,2)-CF1I0)/(DB))
      endif
      if(I.gt.2) then
      FDIMINUSHALF=(ENJMINUSHALF*CHDX(2))*
     >             ((CF1(I,2)-CF1(I-1,2))/(DB))
      endif
      CFF2(I,2)=CFF1(I,2)-(TAU/DA)*((FQCFJPLUSHALF-
     >                FDJPLUSHALF)-(FQCFJMINUSHALF-FDJMINUSHALF))+
     >                (TAU/DB)*(FDIPLUSHALF-FDIMINUSHALF)
   30 continue
      if(NSORP.eq.1) then
      if(AKD.eq.0.0) then
      CF2(I,2)=(CFF2(I,2)-((1.0-EN(2))*
     > CS1(I,2)*DJI))/(EN(2)*DJI)
      CS2(I,2)=DRD(2)*AKD*CF2(I,2)
      endif
      if(AKD.ne.0.0) then
c
c        DEDUCT LOSS OF SOLUTE DUE TO SOLUTE DECAY AND SOURCE DECAY
c
      DL1=EN(2)*DJI*CF1(I,2)*(1.0-exp(-ALAMDAC*TAU))
      DL2=EN(2)*DJI*CF1(I,2)*(1.0-exp(-ALAMDASC*TAU))
      CFF2(I,2)=CFF2(I,2)-DL1-DL2
      CF2(I,2)=CFF2(I,2)/((EN(2)*DJI)+
     >((1.0-EN(2))*DRD(2)*AKD*DJI))
      CS2(I,2)=DRD(2)*AKD*CF2(I,2)
      endif
      endif
      if(NSORP.eq.2) then
```

```
      CFN2=CF1(I,2)
      CSN2=CS1(I,2)
      NA=50
      TAUN=TAU/float(NA)
      CFFN=(CFF2(I,2)-CFF1(I,2))/float(NA)
      do 621 IR=1,NA
      if(CFN2.le.0.0) DSDT=0.0
      if(CFN2.le.0.0) goto 629
      DSDT=ANLAMDA*(AKP*(CFN2**ANF)-(CSN2/DRD(2)))
  629 CSN2=CSN2+DSDT*TAUN*DRD(2)
      CFFN1=CFF1(I,2)+CFFN
      CFN2=(CFFN1-((1.0-EN(2))*DJI*CSN2))/EN(2)*DJI
  621 continue
      CF2(I,2)=CFN2
      CS2(I,2)=CSN2
      if(CF2(I,2).lt.0.0)   CF2(I,2)=0.0
      if(CS2(I,2).lt.0.0)   CS2(I,2)=0.0
      endif
c
c        CALCULATE NEXT TIME STEP CONCETRATION AT INTERNAL LOCATIONS
c
c
c        LATERAL BOUNDARIES' CONDITIONS (ZERO FLUX)
c
      do 322 J=3,NDJV-2
      CF1(NDIV+1,J)=CF1(NDIV-1,J)
      CF1(NDIV,J)=(3.0/8.0)*CF1(NDIV+1,J)+
     >(6.0/8.0)*CF1(NDIV-1,J)-(1.0/8.0)*CF1(NDIV-2,J)
      CS1(NDIV,J)=DRD(J)*AKD*CF1(NDIV,J)
      CF1I0=CF1(2,J)
      CF1(1,J)=(3.0/8.0)*CF1I0+
     >          (6.0/8.0)*CF1(2,J)-(1.0/8.0)*CF1(3,J)
  322 continue
      do  31  J=3,NDJV-2
      do 32 I=2,NDIV-1
      DJI=(1.0+EG(J))/(1.0+EG0(J))
      DJJPLUSHALF=(1.0+EGJPLUSHALF(J))/(1.0+EG0JPLUSHALF(J))
      DJJMINUSHALF=(1.0+EGJMINUSHALF(J))/(1.0+EG0JMINUSHALF(J))
      FQCFJPLUSHALF=CQJPLUSHALF(J)*CF1JPLUSHALF(I,J)
      FDJPLUSHALF=(ENJPLUSHALF*CHDA(J)/DJJPLUSHALF)*
     >((CF1(I,J+1)-CF1(I,J))/(DA))
      FQCFJMINUSHALF=CQJMINUSHALF(J)*CF1JMINUSHALF(I,J)
      FDJMINUSHALF=(ENJMINUSHALF*CHDA(J)/DJJMINUSHALF)*
     >             ((CF1(I,J)-CF1(I,J-1))/(DA))
      if(I.eq.(NDIV-1)) then
      FDIPLUSHALF=(ENJPLUSHALF*CHDX(J))*
     >             ((CF1(NDIV+1,J)-CF1(NDIV-1,J))/(DB))
      endif
      if(I.lt.(NDIV-1)) then
      FDIPLUSHALF=(ENJPLUSHALF*CHDX(J))*
     >             ((CF1(I+1,J)-CF1(I,J))/(DB))
      endif
      if(I.eq.2) then
      FDIMINUSHALF=(ENJMINUSHALF*CHDX(J))*
     >             ((CF1(2,J)-CF1I0)/(DB))
      endif
      if(I.gt.2) then
      FDIMINUSHALF=(ENJMINUSHALF*CHDX(J))*
     >             ((CF1(I,J)-CF1(I-1,J))/(DB))
      endif CFF2(I,J)=CFF1(I,J)-
      (TAU/DA)*((FQCFJPLUSHALF-
     >                FDJPLUSHALF)-(FQCFJMINUSHALF-FDJMINUSHALF))+
     >                (TAU/DB)*(FDIPLUSHALF-FDIMINUSHALF)
      if(CFF2(I,J).lt.0.0) CFF2(I,J)=0.0
      if(NSORP.eq.1) then
      if(AKD.eq.0.0) then
      CF2(I,J)=(CFF2(I,J)-((1.0-EN(J))*
```

```
     > CS1(I,J)*DJI))/(EN(J)*DJI)
       CS2(I,J)=0.0
       endif
       if(AKD.ne.0.0) then
c
c        DEDUCT LOSS OF SOLUTE DUE TO SOLUTE DECAY AND SOURCE DECAY
c
      DL1=EN(J)*DJI*CF1(I,J)*(1.0-exp(-ALAMDAC*TAU))
      DL2=EN(J)*DJI*CF1(I,J)*(1.0-exp(-ALAMDASC*TAU))
      CFF2(I,J)=CFF2(I,J)-DL1-DL2
      CF2(I,J)=CFF2(I,J)/((EN(J)*DJI)+
     >((1.0-EN(J))*DRD(J)*AKD*DJI))
      CS2(I,J)=DRD(J)*AKD*CF2(I,J)
      endif
      endif
      if(NSORP.eq.2) then
      CFN2=CF1(I,J)
      CSN2=CS1(I,J)
      NA=50
      TAUN=TAU/float(NA)
      CFFN=(CFF2(I,J)-CFF1(I,J))/float(NA)
      do 631 IR=1,NA
      if(CFN2.le.0.0) DSDT=0.0
      if(CFN2.le.0.0) goto 639
      DSDT=ANLAMDA*(AKP*(CFN2**ANF)-(CSN2/DRD(J)))
  639 CSN2=CSN2+DSDT*TAUN*DRD(J)
      CFFN1=CFF1(I,J)+CFFN
      CFN2=(CFFN1-((1.0-EN(J))*DJI*CSN2))/EN(J)*DJI
  631 continue
      CF2(I,J)=CFN2
      CS2(I,J)=CSN2
      if(CF2(I,J).lt.0.0)   CF2(I,J)=0.0
      if(CS2(I,J).lt.0.0) CS2(I,J)=0.0
      endif
   32 continue
   31 continue
c
c       APPLY BOUNDARY CONDITIONS TO GET NEW BOUNDARY VALUES
c
      if(NSTUBC.eq.1) then
c
c      PRESCRIBED CONCENTRATION AT THE TOP
c
      do 117 I=1,NDIV
      CF2(I,NDJV)=PCB
  117 continue
c
c LATERAL BOUNDARIES' CONDITIONS (ZERO FLUX)
c
      do 118 J=NDJV,NDJV
      CF2(NDIV,J)=CF2(NDIV-1,J)
      CF2(1,J)=CF2(2,J)
  118 continue
      endif
      if(NSTUBC.eq.2) then
c
c        PRESCRIBED CONCENTRATION GRADIENT (GENERALLY ZERO) AT THE TOP
c

      do 119 I=1,NDIV
      CF2(I,NDJV+1)=CF2(I,NDJV-1)
      CF2(I,NDJV)=CF2(I,NDJV-1)
  119 continue
c
c       LATERAL BOUNDARIES' CONDITIONS (ZERO FLUX)
c
      do 120 J=NDJV,NDJV
      CF2(NDIV,J)=CF2(NDIV-1,J)
```

```
        CF2(1,J)=CF2(2,J)
  120  continue
       endif
       if(NSTBBC.eq.3) then
c
c         RESERVOIR BOUNDARY CONDITION AT THE BOTTOM
c
       VS1=abs(CQ(NDJV))/EN(NDJV)
       HH=0.5*DA*(1.0+EG(NDJV-1))/(1.0+EG0(NDJV-1))
        do 121 I=1,NDIV
        CF2(I,NDJV)=(VS1*PRT*HH+CHDA(NDJV1)*
      >CF2(I,NDJ-1))/(VS1*HH+CHDA(NDJV-1))
   121 continue

c
c         LATERAL BOUNDARIES' CONDITIONS (ZERO FLUX)
c
       do 122 J=NDJV,NDJV
       CF2(NDIV,J)=CF2(NDIV-1,J)
       CF2(1,J)=CF2(2,J)
  122  continue
       endif
       if(NSTBBC.eq.1) then
c
c         PRESCRIBED CONCENTRATION AT THE BOTTOM
c
       do 124 I=1,NDIV
       CF2(I,1)=PCB
  124 continue
c
c         LATERAL BOUNDARIES' CONDITIONS (ZERO FLUX)
c
       do 125 J=1,1
       CF2(NDIV,J)=CF2(NDIV-1,J)
       CF2(1,J)=CF2(2,J)
  125  continue
       endif
       if(NSTBBC.eq.2) then
c
c          PRESCRIBED CONCENTRATION GRADIENT (GENERALLY ZERO) AT THE BOTTOM
c
       do 126 I=1,NDIV
       CF2(I,1)=CF2(I,2)
  126 continue
c
c         LATERAL BOUNDARIES' CONDITIONS (ZERO FLUX)
c
       do 127 J=1,1
       CF2(NDIV,J)=CF2(NDIV-1,J)
       CF2(1,J)=CF2(2,J)
  127  continue
       endif
       if(NSTBBC.eq.3) then
c
c         RESERVOIR BOUNDARY CONDITION AT THE BOTTOM
c
       VS1=abs(CQ(1))/EN(1)
       HH=0.5*DA*(1.0+EG(2))/(1.0+EG0(2))
       do 128 I=1,NDIV
       CF2(I,1)=(VS1*PRT*HH+CHDA(2)*CF2(I,2))/
       > (VS1*HH+CHDA(2))
  128 continue
c
c         LATERAL BOUNDARIES' CONDITIONS (ZERO FLUX)
c
       do 129 J=1,1
       CF2(NDIV,J)=CF2(NDIV-1,J)
       CF2(1,J)=CF2(2,J)
```

```
  129 continue
      endif
      do 422 J=2,NDJV-1
      CF2(NDIV,J)=CF2(NDIV-1,J)
      CF2(1,J)=CF2(2,J)
  422 continue
      do  230  I=1,NDIV
      do 231 J=1,NDJV
      if(CF2(I,J).lt.0.0)CF2(I,J)=0.0
      if(CS2(I,J).lt.0.0)CS2(I,J)=0.0
      CF1(I,J)=CF2(I,J)
  231 continue
  230 continue
      do  232  I=2,NDIV-1
      do 233 J=2,NDJV-1
      if(CFF2(I,J).lt.0.0) CFF2(I,J)=0.0
      CFF1(I,J)=CFF2(I,J)
      CS1(I,J)=CS2(I,J)
  233 continue
  232 continue
      endif
c
      return
      end
c
c     ***********************************************************
      subroutine DATOUT()
c     ***********************************************************
c
c         DATOUT PRINTS RESULTS OF CONSOLIDATIO CALCULATIONS AND BASE
c         DATA IN TABULAR FORM
c
      common DA,DB,DZ,E00,ELL,GC,GS,GSBL,GW,HBL,LBL,NBDIV, NDIV,NBDJV,
     >        NDJV,NFLAG,NNN,NTIME,Q0,Q2,WL,SETT,SFIN,TAU,TIME,TPRINT,
     >        UCON,NNTIME,NST,NL,NBC,NDRB,ND1,ND2,NNL,NSOL,NNSOL,NNSOL1,
     >        ALPHAL,ALPHAT,NSTBC,SL,CFT,CFB,HT,HB,CHD0,CHD1,AKD,ALAMDAC,
     >        ALAMDASC,AM,DW,DS,DGM,TH,RKEO,NSL,HCL,HUCL,E0C,E0UC,NSORP,
     >        NSTUBC,NSTBBC,NSTRBC,NSTLBC,PCT,PGT,PRT,PCB,PGB,PRB,PCR,
     >        PGR,PRR,PCL,PGL,PRL,INX,JNZ,IE0,ISS,NC,TTIME,
     >        A(351),B(351),Z(351),XI(351),ALPHA(351),BETA(351),
     >        DSDE(351),E11(351),EFIN(351),ER(351),ES(351),EFFSTR(351),
     >        F(351),FS(351),FINT(351),PK(351),RK(351),RK1(351),RS(351),
     >        TOTSTR(351),U(351),U0(351),UW(351),VRI(351),DQ(351),
     >        Q1(351),RKEI(351),AKP,ANF,ANLAMDA,IKK,UMAX,NTAU,
     >        CHD2(351),CHDA(351),CHDX(351),CQI(351),CQU(351),CQ(351),
     >        EN0(351),EN(351),EG0(351),EG(351),DVDA(351,351),
     >        CS0(351,351),CS1(351,351),CS2(351,351),CF0(351,351),
     >        CF1(351,351),CF2(351,351),CFF1(351,351),CFF2(351,351),
     >        E11JPLUSHALF(351),E11JMINUSHALF(351),DKQU(351),
              FJPLUSHALF(351),FJMINUSHALF(351),AFJPLUSHALF(351),
     >        AFJMINUSHALF(351),BFJPLUSHALF(351),BFJMINUSHALF(351),
     >        EGJPLUSHALF(351),EGJMINUSHALF(351),EG0JPLUSHALF(351),
     >        EG0JMINUSHALF(351),CQJPLUSHALF(351),CQJMINUSHALF(351),
     >        CQUJPLUSHALF(351),CQUJMINUSHALF(351),DRD(351),
     >        CF0IPLUSHALF(351,351),CF0IMINUSHALF(351,351),
     >        CF0JPLUSHALF(351,351),CF0JMINUSHALF(351,351),
     >        CF1IPLUSHALF(351,351),CF1IMINUSHALF(351,351),
     >        CF1JPLUSHALF(351,351),CF1JMINUSHALF(351,351),
     >        PRINT1(351)
c
c       PRINT CONDITIONS IN COMPRESSIBLE FOUNDATION
c
      write(21,100)
      write(21,*) TIME
      write(21,101)
c
      if(ND1.eq.1) then
```

```fortran
      do 1 J=1,NDJV
      L=NDJV+1
      LL=L-J
      write(21,*) XI(LL),CF1(1,LL),CS1(1,LL)/DRD(J)
    1 continue
      write(22,*)   TIME,CF1(1,1)/CFT
      endif
      if(ND1.eq.2) then
      write(21,*) '***************VERTICAL***************************'
      do 3 I=1,NDIV
      do 2 J=1,NDJV
      L=NDJV+1
      LL=L-J
      write(21,*) XI(LL),I,LL,CF1(I,LL),CS1(I,LL)/DRD(J)
    2 continue
    3 continue
      write(21,*)'******************HORIZONTAL*********************'
      do 5 J=1,NDJV
      do 4 I=1,NDIV
      write(21,*) B(I),J,I,CF1(I,J),CS1(I,J)/DRD(J)
    4 continue
    5 continue
      endif
c
c    FORMATS
c
  100 format(//5HTIME=)
  101 format(// 12X,2HA1,12X,3HCF2)
c
  300 return
      end
c     *********************************************************************
      subroutine BISFVALUE(ENT1,ENEW,ENI,CFT1,CFFT1,DST1,AKPT1,FT1,
     >                      FVALUE,GVALUE)
c     *********************************************************************
      DJIDA=(1.0+ENEW)/(1.0+ENI)
      FVALUE=ENT1*CFT1*DJIDA+(1.0-ENT1)*DJIDA*DST1*AKPT1*(CFT1**FT1)-
     >       CFFT1
      GVALUE=ENT1*DJIDA+(1.0-ENT1)*DJIDA*DST1*AKPT1*FT1*
     >       (CFT1**(FT1-1.0))
      return
      end
```

List of Publications

The following papers have been published on the basis of the work presented in this thesis.

Journal:

Singh, R. P., Singh, M. and Ojha, C. S. P., “An Experimental Study on Consolidation of Compacted Clays”, International Journal of Geotechnical Engineering, Vol. 8 (1), pp. 112-117, 2014.

Singh, R. P., Singh, M. and Ojha, “Finite Volume Approach for Finite Strain Consolidation”, International Journal for Numerical and Analytical Methods in Geomechanics, Accepted.

Conference:

Singh, R. P., Singh, M. and Ojha, “Finite Strain Theory of Consolidation of Clays: Finite Volume Approach”, Proceedings of IGC – 2012, Paper F-630.

Singh, R. P., Singh, M. and Ojha, “Explicit Finite Volume Approach to Solute Transport through Porous Media”, Proc. IGC-2013, Paper 4-TH-15.