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# Agent Base Modeling And Simulation Of Financial Market

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A DISSERTATION  
submitted towards the fulfillment of the  
requirement for the award of the degree of  
MASTER OF TECHNOLOGY  
in  
Computer Science and Engineering

By

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## AUTHOR'S DECLARATION

I declare that the work presented in this dissertation with title "**Agent Base Modeling And Simulation Of Financial Market**" towards the fulfillment of the requirement for the award of the degree of **Master of Technology in Computer Science & Engineering** submitted in the **Department of Computer Science & Engineering, Indian Institute of Technology Roorkee, India** is an authentic record of my own work carried out during the period from **June 2015 to May 2016** under the supervision of **Dr. A.K.Sarje**, Emeritus. Professor, Department of Computer Science and Engineering, Indian Institutes of Technology, Roorkee. The content of this dissertation has not been submitted by me for the award of any other degree of this or any other institute.

DATE: ..... SIGNED: .....

PLACE: ..... (ABHIJEET SHARMA)

This is to certify that the statement made by the candidate is correct to the best of my knowledge and belief.

DATE: ..... SIGNED: .....

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## ABSTRACT

**F**orecasting of economy has been difficult over the last 200 years. Financial market(economy) is a complex system. Which is composed of many small elements which act locally but react globally on whole system. Many economist try to predict the financial market on the basis of different models. Recently a model which is called agent base model has been developed .Agent base model is consisting of the heterogeneous agents which are designed to do specific task locally and they can interact with the other agents of system and the environment of system .Each and every agent shows impact and overall system impact is the conclusive impacts of all agents in system. Agent base modeling simulate the financial market through the interaction with different agents which work on the different behavioral rules assigned to them. In this project we are trying to develop an Agent base model which simulate the financial market in which the agents are represented by investors of market. A program and simulation is developed in C++. In this report we also try to explain the irrationality of different investors which solve to different anomalies of financial market. we will discuss the different dynamics of financial market.

## DEDICATION AND ACKNOWLEDGEMENTS

**F**irstly ,I want to express my thanks to my supervisor DR A.K SARJE for providing me help and useful comments on it.His instinctive thoughts increased the standard and level of the thesis.I am really thankful to him for telling me about the stop point and point where i can get out the important and useful result.His helpful instruction and supportive attitude really helped me to complete the my work.His advice time to time and logical approach helped me with huge extent to complete my work on time. I thank excessively to all staff of the Electronics and computer science department ,IIT ROORKEE for their supportive thoughts and help during my study period.I thank them for the many brainstorming sessions and their guidance in the area of financial markets.

This Dissertation dedicated to DR A.K SARJE .Whose passion of teaching set the new standard of education and training and development or any other endeavor in which one human being seeks to the support the growth and development of another.

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## INTRODUCTION

The dynamics of the financial market are the important subjects because the researchers and experts have been trying to explain and understand the working of financial market and behavior of traders in the market. So agent based modeling is a good approach for the understanding of price formation mechanism and behavior of traders in financial market. There are you kind of contradictory approaches on one hand we have the EMH efficient market hypothesis .Which tells us in market all the available information immediately transmitted in market and reflects in the prices of market and there is rationality in market and rational traders are those traders who perform the market decision On the basis of current available information but on another hand the real approaches which contradict the theoretical approaches in which the assumption are made such as homogeneity, rationality do not correspond to reality .These approaches tell us the traders are not homogeneous but they are heterogeneous and boundedly rational.

### 1.1 Motivation

The financial market has been the very important subject and interesting subject of research among economist and researcher who are interested to understand the financial market. There are the various hardly observable aspects which are very difficult to observe and have a lot impact on the market so it is very important to understand those aspect and research on them. An agent based modeling is the good approach to understand the complex system like financial market[2].

## 1.2 Problem formation and objectives of research

**Objective of research:** Contribution in the study, explanation and understanding in dynamics of market time continuous agent based simulation approach which shows the representation of the financial market which is flexible and behavior of traders which is variable.

**Research question 1:**

What are the relational and variable aspects of stock market which should be taken in consideration while we are studying the stock market? For this we should compare the ASM (artificial stock market) with the theoretical study and we should compare the different ASM to get detail that which aspects should be taken in account while we are studying the financial market.

**Research question 2:**

What is the degree of literature that should reflect from ASM to real market and how we are going to deal with the common and variable aspects? We need an environment in which representation can be done of common and variable aspects which are taken into account.

**Research question 3:**

How we can develop the environment using which one can easily get the information about the common and variable aspects. Which are not easily observable and should be taken in account while you are investing in financial market and not be included in any ASM yet. For doing so we need to create the simulation on model which shows the which common and variable aspects having the real impact in the stock market and that should be taken into account and that are not included yet in any ASM.

**Research question 4:**

What are the additional values which should be added to this research and not present in any existing ASM.

## 1.3 Outline of Thesis

The first section of report includes the introduce about the financial market and introduction of the agent based modeling last part of the first section contains the statement of problem and the challenges of research and the objective of the research The second section of the report includes the details of the dynamics aspect which should be taken into account while we are researching. And what are the different part financial market

and the roles of financial market and their impact on it. The third section includes the literature work which is already implemented Fourth section includes the proposed work. Fifth section include the conclusion and research work which can be done future.

## **1.4 Research Gap**

The first agent model and simulation was done by the LeBron et al. in 1999. And micro simulation was done levy et al. in 2001 .These were the approaches in which the model was like that in which the different agents were competing each other and were interaction each other. The two main feature were present here prior knowledge and ability to handle the large number of user. These model were not for the continues real market means for the continue session and the continues session is the very common session in the stock market so the continuous session was include in model by Harris .In 2009 roll model and call auction models were proposed.After this the continuous model was developed by the Gleston and Milgrom. 0 20.

## BACKGROUND

## 2.1 Market Microstructure

The price of the market is directly determined by the prices formation mechanism .Basically price formed is the direct result of the current execution of orders in the market. And as the result price does not directly depend on the many other factor which are triggered by execution of current orders but somehow influenced by them[4].

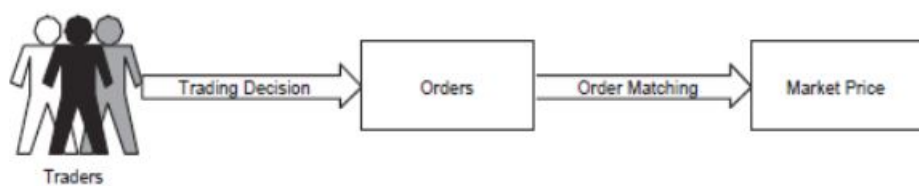


Figure 2.1: Basic Price Formation [3]

The microstructure of the financial market is defined by many factors such as trading rules and trading system which determines who can trade how and when, what can a trader does in the market ,what can the trader knows in the market. Structure in the financial market provides the framework in which the financial market works which include the following related topics:

1. Discovery of price and price formation.

2. Design issues and structure of market.
3. Available information and disclosure of it.

Financial market is also known as black box because of its hardly observable aspects.

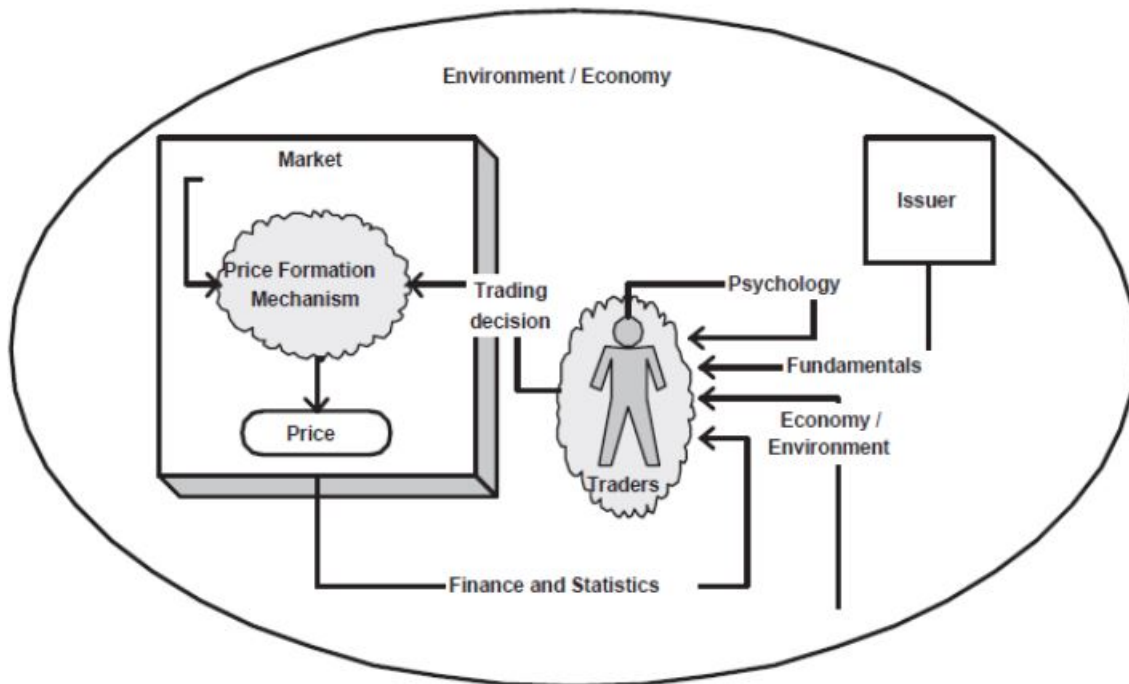


Figure 2.2: High level market dynamics [3]

## 2.2 The Organization of Financial Market

For better understanding the work of financial market we need to focus on the static and invisible side of the stock market. We have to focus on the main factors which describe the market structure and they are following.

### 2.2.1 Traded instruments

The instruments are those objects which are traded in the financial market .They include the many type of contracts and assets. There are two kind of assets real assets which show the physical commodity and financial assets cash flow and ownership on the real assets. Stocks are the financial assets .Every Corporation issued the certain kind of rules and governance criteria which are stated by market rules[5].

### 2.2.2 Orders And Quotes

Willingness to invest in the financial market is known as orders .orders elaborate what to trade how much to trade ,whether to sell or buy .These all information is contained in the most simple form of order which is known as market order. If the orders contain the certain condition which must be satisfied before the order gets executed then these type of orders are known as limit orders. The order will be executed at market price if the price is not specify in the order and that market price will be the valid current market price .Order cannot be executed as at the moment they enter in the market so a book is maintained which known as order book. The order book of the limit order is known as limit order book .If the trader want to buy then he puts the buy order which is known the bid quote. If he wants to sell then the order is known as ask quote. And the difference between the maximum bid and minimum ask quote is known as bid -ask spread. The trader who bid offers the liquidity in market and who ask takes the liquidity from market[5].

### 2.3 Market participants

There is mainly kind of participants in market:

1. Investors
2. Financial traders

The trader who are not the part of market organization they called investors. They may be money manager, corporate fund or individuals. The financial traders are those who endowed the roles in the market and they provide the liquidity in market. They are of two type

1. Brokers
2. Market makers

Brokers primarily handle the order of customers they can put money of investors on behalf of them .The financial traders of are responsible of the orderly market they are known as market maker. Market makers provides the liquidity in market. Each stock can have one market maker or more and liquidity is assigned with each stock. Market maker provides the bid and ask quote of stock if more than one market maker is assigned to stock then they compete with each other to provide the best bid and ask quote.



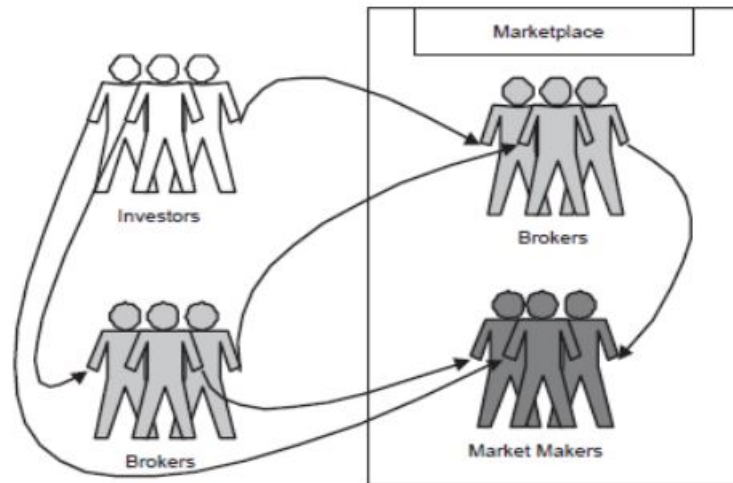


Figure 2.3: Relation of traders [4]

The time of execution of order depends on the strategy of market maker and requirement of market.

## 2.4 Trading Sessions

The trading in stock market can be done only in the trading session. There are two type of trading sessions in the financial market

1. Continuous session
2. Call market

In the call market session all trading requests of stock are taken only on well specified time and then they are aggregated and a single price is set. The main purpose of these session is to Maximize the volume. In the continues session trading can occur at any time when the market is open. The advantage of the call market trading is that the traders in it can easily know about each other and the continuous session is having the advantage the trader are allowed to arrange their orders whenever they want[5].

## 2.5 Execution Systems

There is two kind of execution systems allowed in the financial market

1. Quote driven
2. Order driven

In the quote driven market the market maker is always involved which means that the investors and brokers are not directly allowed the trade with each other. If all the liquidity is supplied from the market maker then market is pure quote driven market. And the order driven market can works without the intermediation of the dealers or the market makers which mean all orders from buyers and sellers can be bought together and can be cleared directly. But most of the markets are not pure order driven or pure quote driven but they hybrid. Market is known as brokered market if the broker actively search for the match of his client[5].

## 2.6 Market Rules

Every financial market has its own set of rules for traders which is known as protocol of market. In the same market rules can be variable for the different stocks so rules in the financial market varies from market to market as well as stock to stock

## 2.7 Price Formation

Price formation mostly depends on the dynamic side of market. So we considered it as the behavioral aspects. The following behavioral aspect affect the process of price formation

## 2.8 Investor's behavior and initiation of order

Order in the financial market can be placed by financial agents or by investor for their own account or to keep liquidity in market. All the instruments which are hold by the traders construct the portfolio. All good traders want to keep the composition of their portfolio good for doing this they do portfolio management .Portfolio management includes the following main tasks

1. Construct policy statement

2. Determine the strategy of investment according the policy statement
3. Portfolio maintenance
4. On time monitoring the needs[5]

### **2.8.1 Policy Statement**

Policy investment is like a road map which tells the investment goal, the constraints and risk involved in the investment. Which depends on whether the investor is long term or short term, experience of investor and the expectation of investor. And policy must be updated on time.

### **2.8.2 Investment strategy**

By keeping the investment objective in the mind the investor can develop the different strategy. For developing the investment strategy investor has to focus on the financial, political economic conditions in order to forecast the future price.

### **2.8.3 Portfolio Maintenance**

Determining the weights of assets, class of assets which is required for each class, finding the difference between in required portfolio and current portfolio, identifying the side and size of order is known as portfolio maintenance.

### **2.8.4 Monitoring**

Monitoring means reconsideration of various aspects on time. Which means investor has to look risk constraint and on the performance of his portfolio with time.

### **2.8.5 Time**

Time factors are very important for the portfolio management and for investors. Which includes the time horizon of market, time to look back on the historical data in order to forecast the price in market, time to update the portfolio, waiting for execution of limit order and time to change the investment objective and reconsideration of strategy.

### 2.8.6 Execution of order

The main task on broker in the market is to receive the order and place in market. But the problems faced by the broker are that which order should be selected for execution and how it will be executed.

Strategy of order selection:

As the order are stored in the order book so the orders are selected one by one and the selection is based on the size of order and the price of order and the order having the high price will also has the high probability of selection.

## 2.9 Mechanism of order execution

Assume the  $B_A$  represents the broker  $A, I_X$  investment from  $X, O_X$  order of Three possibilities which are chosen by the broker to execute he received orders.

1. Cross trade: Execute the order or part of it internally which is represented by

$$O_X^i$$

2. Negotiation: Trying to find the trader who is having the willingness to take other side of selected trader with some improved negotiation.
3. Third one is submitting the order or the part of it for the further execution.

An order can be divided into the many orders according to the decision of the broker which can be done according to improvement in the size or price of order.

## 2.10 Market maker

The following tasks are done by the market maker,

1. For providing the solution over the asynchronous timing with investments the immediacy is supplied to allow the flow in trading.
2. Quoting the bid and ask through which trading can be done
3. Set the bid and ask price and make adjustment in the bid ask spread.
4. Supply the liquidity in market[5].

Market maker try to execute the orders on time if it is possible to execute that order for good functioning of financial market and of that stock .It is market maker who are mainly responsible for providing the liquidity in market or improving the probability of ordering of that stock .Market makers are responsible for motivating the investors for trading of the particular stock by maintaining the new bid and ask quote on time to time. For doing this they first observe the market then they compute the bid and ask quote depends on the type of model they are implementing and then they get the orders from traders and executes them with highest possibility and they calculate the new bid and ask quote which also based on the type of model they are implementing .This is why the market makers are the most important part market.

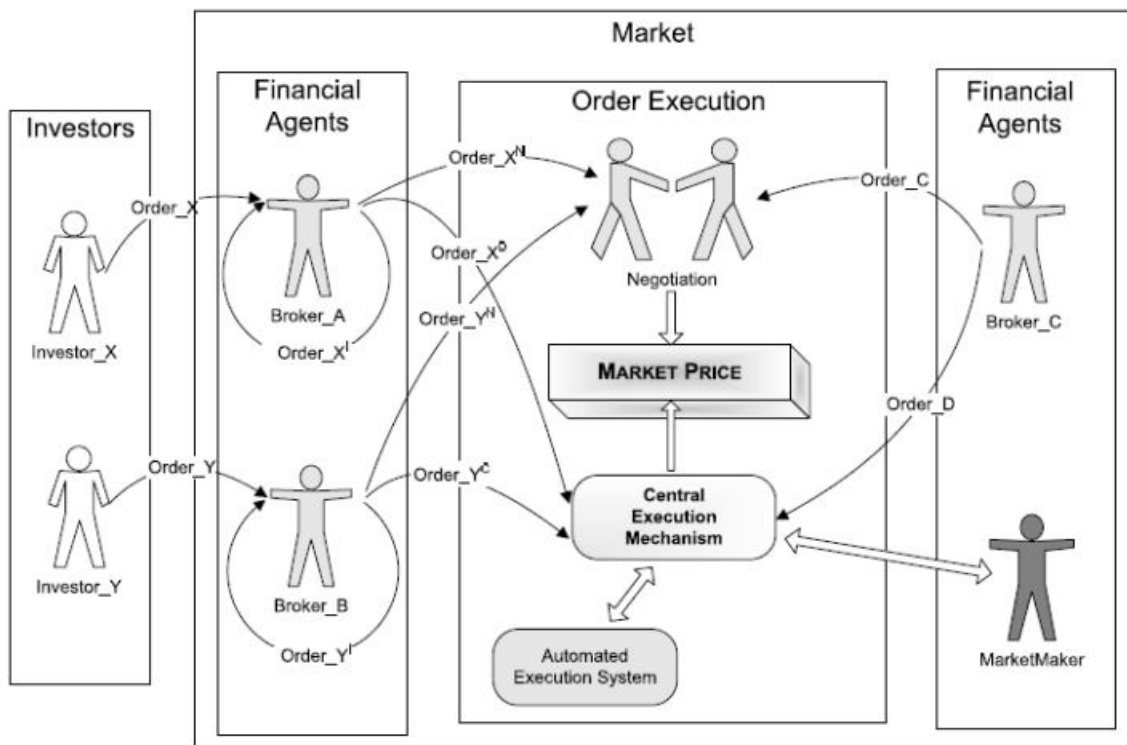


Figure 2.4: Tracking orders [5]

### 2.10.1 Execution of in the call market

In the call market session trading can only be done on the specific time interval so the order in auction are accumulated and executed on a price at some equilibrium and this equilibrium is defined to minimize in excess in demand and maximize the volume.

## 2.10.2 Execution in continuous session

In the continuous session whenever the order is received market maker checks whether the received order matches the current bid or ask quote according to order type .If match occurs then order gets executed otherwise put into the limit order book. There many decision that should be taken by the market maker in the continuous session. *Determining the new quotes:* The New quote must be like that it ensures the liquidity in market ,position of market maker, and reflect the orders of limit order book[8].

*New quote timing:* The decision that here is enough liquidity in market or not so it,Äôs hard to take decision when to change the quote to get new quote[6].

Approach	Formation of price	Behavior of Trader	Efficiency of Market
Theoretical	Easy, Linear	Rational	Average
Empirical	Statistical properties of historical data are analyzed	Does not pay a lot	Anomalies present
Experimental	Assumes real market	Concludes boundedly rational	Anomalies present
Microstructure of market	Analysis of price formation	Learning of market makers, management of inventory	Analysis of efficiency is done with fundamental values
Nonlinear dynamics	Nonlinear	Boundedly rational	Stylized facts are generated
Agent based simulation	Varying	Heterogeneous and boundedly rational	Efficiency depends on the degree of heterogeneity and the setting of ASM

Table 1: Comparative study of different approaches[6]

## 2.11 Agent based approach

### 2.11.1 Origin

The word agent is taken from the Latin word "agree" mean "to do".

### 2.11.2 Economical view

In the financial market the term agent is refers to an individual or to a firm which do the different kind of transaction in market. Agent in the financial market is known as financial agent.

1. In the financial market following facts are accepted and considered true by all physicists and economist.
2. It is not predictable that what is going to happen with price in financial market tomorrow .Means we cannot predicts that whether the price will go up or go down just by looking at past trends or on the basis of current information.
3. Probability of observing the large change in financial market will be higher than on average if market had been so volatile.

At past we had two type of best models both with fatal law. First, is econometric, empirical statistical models they are able to forecast if the things more or less same but if there is great change in this situation they fail. Second one is 'dynamics stochastic general equilibrium this type of models assume the ideal world and they are not useful in the time of crises. So there is one better way of modeling of financial market is the 'agent base modeling'. Agent base modeling is the simulation that is done by computer of different decision making elements which are known as agents and institutions, which are able to interact with each other under the prescribed rules .In financial market agent may a consumer or policy maker or investors and institute can vary from bank to government. These model are not base on the assumption that financial market or economy will move toward the condition of equilibrium .In the agent base modeling of complex system each agent act and react according to its present conditions and situation, according to its environment and rules prescribed for it. Like investor decides to stay in market or sell all stock or buy more stock means the action that will be taken by the investor would depend on his/her current situation and psychology .In the agent base modeling computer keep the track on agents actions and their interaction to see what is happening over the time. Agent based simulation described the nonlinear behavior .So polices maker could simulate the financial market under vary polices and explore their consequences. But it is quite difficult to develop such a model for financial market due to human psychology and adaptability .Agent base modeling is the solution because of the financial market economy as the very complex system. So an artificial financial market is developed in which the different agents work in an cooperative way. .Agent based modeling is useful

when models become so complicated and cannot be solved analytically and they support numerical analysis. Using the different adaptive learning mechanism in agent based modeling agents learn to predict future prices in financial market[6].

## 2.12 Agent based modeling

Agent based modeling and simulation is the new approach to model the complex system which are composed of many elements and those elements are known as agents. These agents are having behavior which is described by rules .Agents can interact with each other and influence the behavior of each other. Agents are having attributes so the behavior of the complete system is the overall composition of the behavior of all elements of system. In the agent based modeling pattern and structure that are derived they are not programmed externally but they come out due to interaction among agents. Heterogeneity and the self-organization of agents are the two important properties of and base modeling and simulation. We can say that the agent based modeling the way to model the complex systems in which each and every agents is organized to work on its own and able to interact with other agents of system with the learning capabilities and adaptively in behavior[6]. The agent based model have three elements:

1. A set of agents and their attributes.
2. Method of interaction.
3. The environment of agents.

## 2.13 Intelligent agent and financial market

The most important part of current financial theory in EMH(Efficient market hypothesis).In which it is assumed that the assets price in the financial market are based in the rational traders who are striving their utility. Which result the past information is not useful in the current price of market. Mathematical models for predicting the market prices are based on homogeneity of traders and equilibrium analysis if something happen with anyone one of this cause of uncertainty in market .In EMH it is assumed that financial market has its own internal dynamics. In the analysis of the market it is found the financial market don't reacted instantaneously but depends on the psychology of traders whether the traders are optimist or pessimist. Agent base approach is the



best suitable way to analysis the internal dynamics of market. In agent based modeling of financial market agents behave in learning and adaptive manners as originally in the financial market traders acts. The behavior of the agents in the financial market is described using the mathematical formula. Traders in the financial market shows the much difference in attitude of risk, time horizon or decision rules. These differences can be represented in form of algorithmic framework. An agent is intelligent agent if it has the rich cognitive structure borrowed from artificial agents. The information diffusion of the market: The prices development in the financial market depends on influences of agents on each other, trading decision of traders and the interaction among the elements of market. The traders take the decision on the basis of available information and the action taken by trader determine the price developments. Which enters again in the decision making system as the newly data. The available information is only available to selected elements which is the filtering step. Different agents receive this information and perceive in it in different manners and react on its according to its own goal which affect the prices development in the financial market. The following factors are the main factor in information diffusion in the market[6].

1. External environment
2. Different data selection cause to the different information transitions.
3. Properties of the agents.
4. Market mechanism.

In agent based system there is many agents which works to meet their design objective. Assume that in the agent based modeling there are many environment states  $S=\{s_1,s_2,\dots\}$  which can influent the agent only partially. Agent can only perform the set action  $A=\{a_1,a_2,\dots\}$  so the agent can be viewed as the function  $S \rightarrow A$ .environment behavior is non deterministic. Which can also be function  $env: S * A \rightarrow p(s)$ .Which is used to map the current environment state to next environment state using the action take from the given set of actions. Agents do not possess the all environment information which it can get after interacting with other agents[6].

## 2.14 Structure of trading environment

There are three main components in the trading environment first one is the instruments which are traded in market, microstructure of the financial market defines price

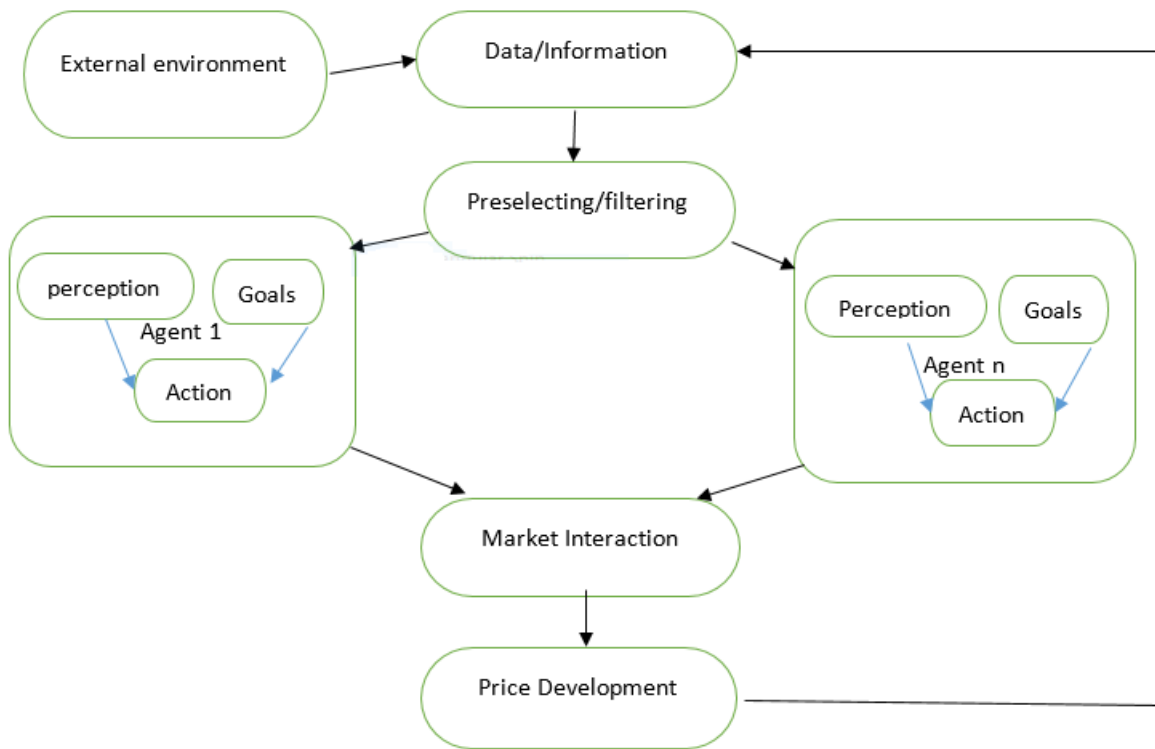


Figure 2.6: Information diffusion in financial market

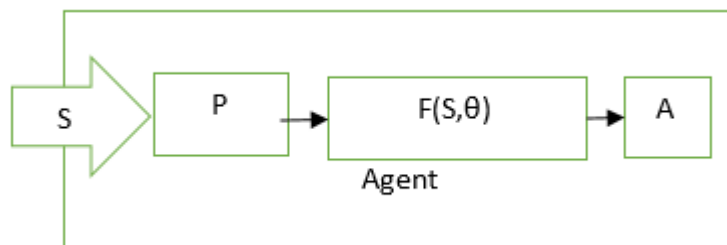


Figure 2.7: Action of agent

formation mechanism and the type of market .The price of instrument depends on many factors Such as type of market, decision of market members and on market maker. The framework of the trading environment consist mainly three components

1. Marketplace
2. Investor's set
3. Source of information

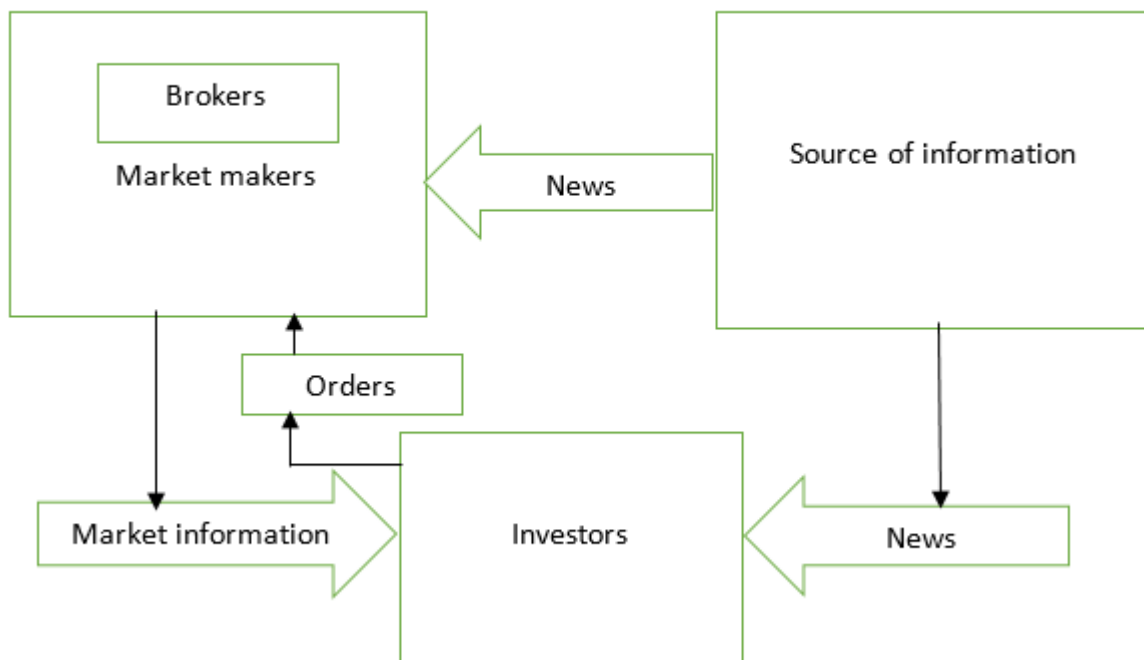


Figure 2.8: The parts of market framework

## 2.15 Behavioral aspect

Its include the following

### 2.15.1 Order placing behavior

For sake of portfolio management process order is put by investor in market. Investors having the below generic behaviors.

1. First they try to understand environment.
2. They interpret the got information and try to put decision of trading as result
3. Then they contact the financial agent for asking him to execute the order

The thing which really matters is the process to get the reason for the trading decision taken is really hardly observable[7]. Two question which are important in the reading behavior

1. How investor can trigger the decision of trading.
2. Which time is the right time to trigger the trading decision?

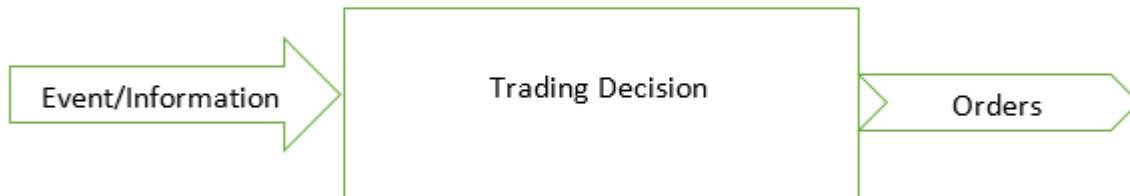


Figure 2.9: Investor's generic behavior

The situations in which the investors probably make the new trading decision.

1. When the new news arrives which is related to fundamental value.
2. When investor reaches at the holding period of his portfolio.
3. The order that he had placed has filled.
4. Market prices changes, change in bid-ask quote[7].

## 2.16 Order execution

Brokers in the financial market work as the intermediates the can execute the order on behalf of investor. In case of auction negotiation is also involved by the brokers. So in the order execution the following tasks are done by the broker

1. They try to understand the environment
2. They get the orders from the investor.
3. Use the particular strategy to decide how and which order to execute.
4. Then they arrange the transactions and notify on them.

Filling the received order is the main problem and the number of possibilities are limited.

1. Use FIFO algorithm to select the order.

2. Select on the base of best probability of execution according to current market condition.
3. Use aggregation and try to execute the orders having similar kind of parameters.

There are the three possibilities which can be used by the brokers

1. Matching the orders internally: Example if broker's limit order book (LOB) contains the orders which are received earlier can be cleared at price which is close enough to the present market price.
2. They can try to negotiate with the other brokers.
3. They can submit to market maker or central execution system[7].

The different negotiation strategies are used by the broker which finally results in a new quote or in acceptance in the quote which is made by other. There are many questions which are need to be considered important while an broker decides to negotiate .They are following

1. What should be the time length of the negotiation?
2. What should be the offered negotiation price?
3. What change should be applied at the negotiation price?

These above condition and questions depends on quote made by the market maker and offered to other to negotiate it on the the initial value of the limit order price. If acceptance happens by the broker on that quote. Then price of transaction is calculated and this will be the new price of market.

## **2.17 Execution of order and role of market maker in continuous market**

These are the following tasks of the market maker

1. Understand the environment.
2. Calculate the bid quote and the ask quote.
3. Take the orders and try to execute them.

- Put the unsatisfied orders in the limit order book.

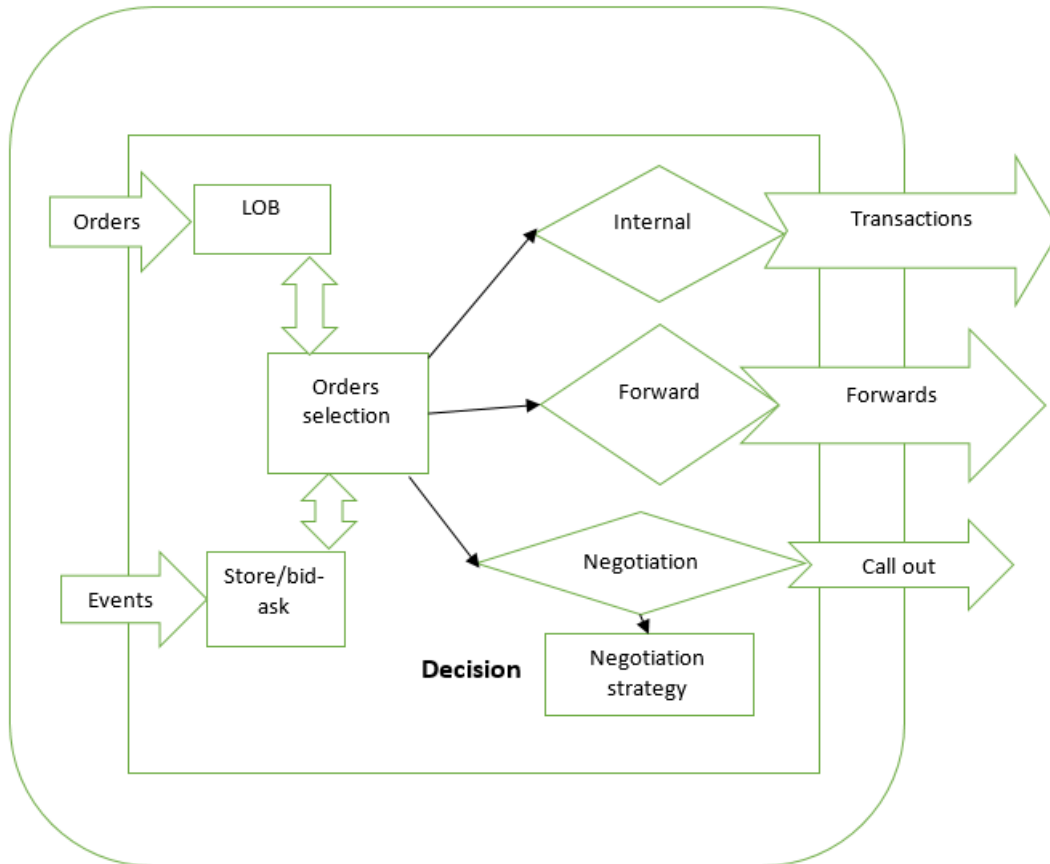


Figure 2.10: Broker's generic behavior[8]

The main question that are the problem and must be answered are following.

- Which time is the proper time to change the bid-ask spread?
- How will we get insurance that orders are providing the liquidity and are able to reflect the condition of market[8].

The market maker reconsider the bid quote and the ask quote at two conditions:

- At the arrival of new order.
- At the time when they observe that the stock which are in their responsibility are not enough liquid. Mean market market get the situation of no orders for that stock for a while.

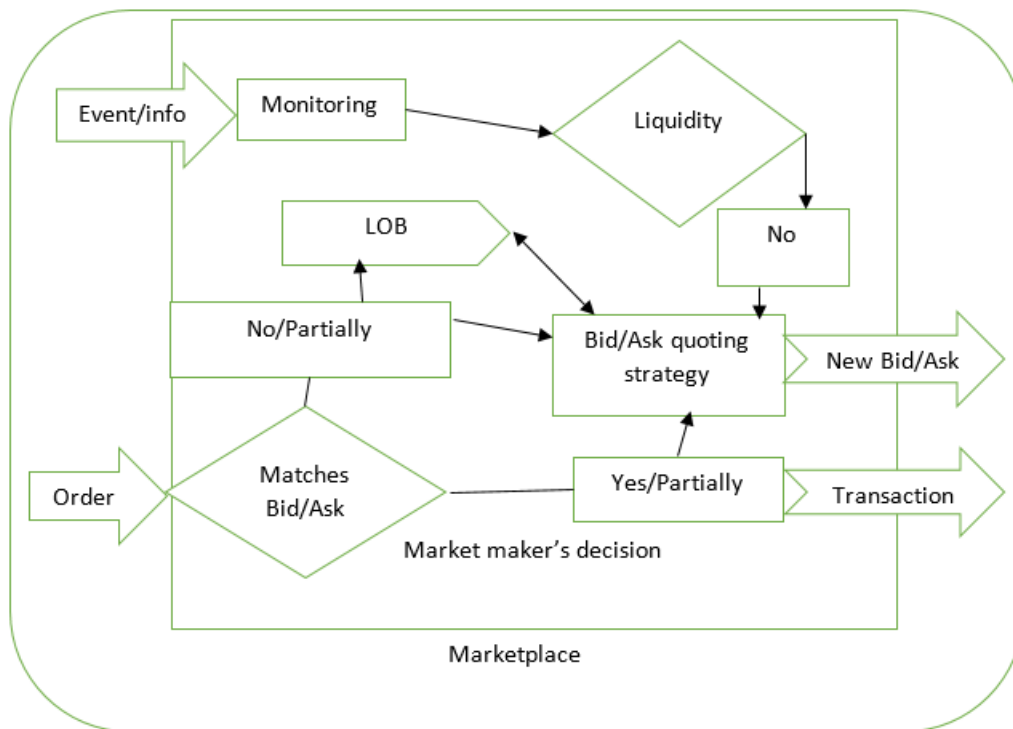


Figure 2.11: Market Maker's behavior[8]

## 2.18 Execution of order based on automation and equilibrium

In the auction market market maker behaves like the auctioneer. And his auctioneer behavior consist of following:

1. Take the trades.
2. On the basis of taken orders calculates the new price of equilibrium.
3. Then orders are executed which match the price.

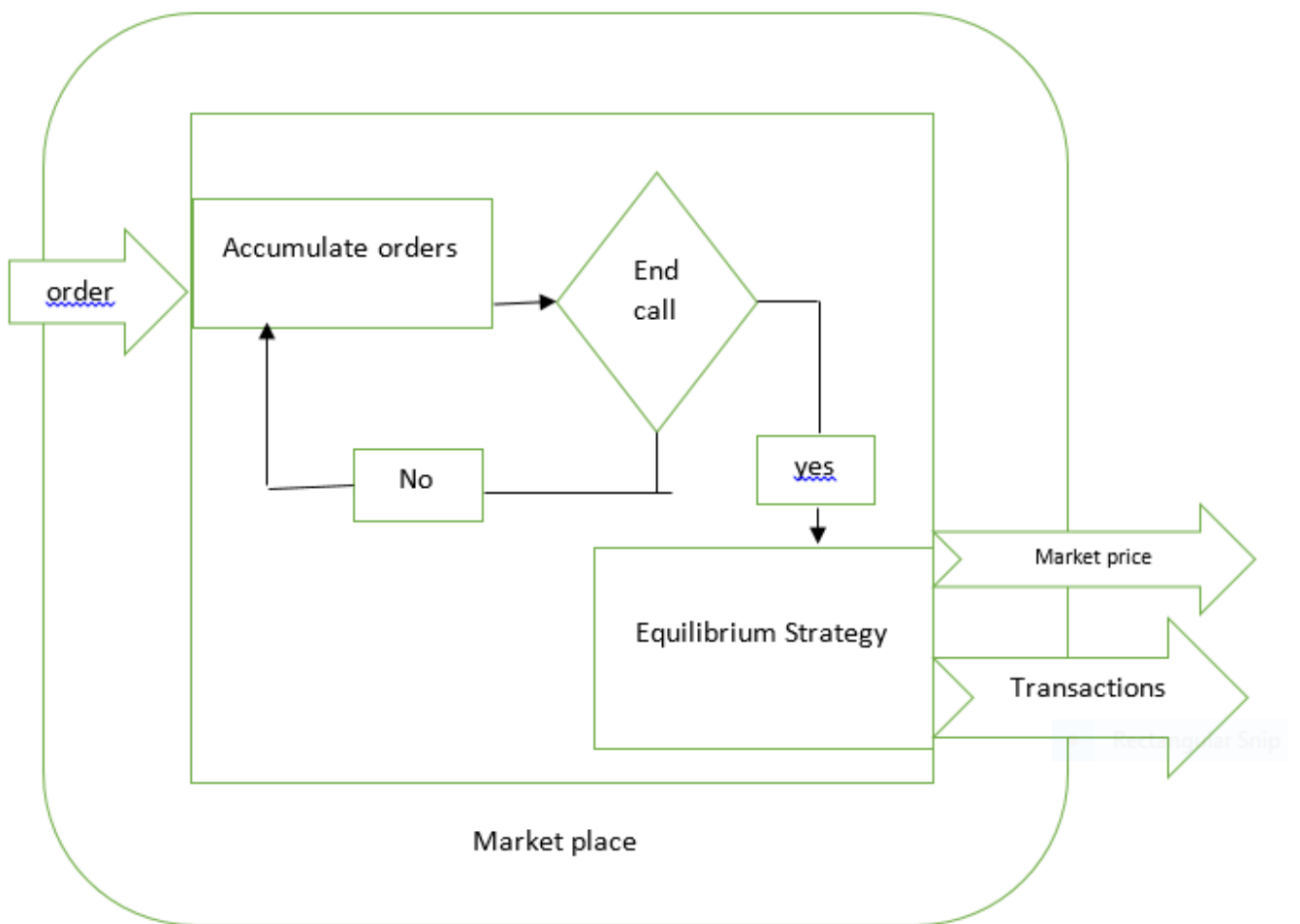


Figure 2.12: Formation of price at equilibrium condition



## LITERATURE REVIEW

### 3.1 Kim Markowitz Model

The market we are having two type of investors "rebalancers" and "portfolio" insurers",and assets are cash and stocks.so if the total wealth is represented by  $W_t$  , $Q_t$  is stocks with number  $P_t$  and  $C_t$  is cash at the time 't' so the total wealth will be represented by following equation.

$$W_t = Q_t P_t + C_t$$

Rebalancers have the aim to have total wealth half in the form of stocks and half in the form of cash. So the target of rebalance is represented by following equation.

$$Q_t P_t = C_t = 0.5 W_t$$

So rebalancers try to stabilize the effect on market:when there is increase in price so this thing induce rebalancers to raise their supply and reduce in the demand and when does the price decreases. This induce the rebalancers increase in demand and decrease in supply.And "portfolio insurers "follow the strategy which guarantee the minimal level of wealth which is called floor 'f' at the insurance expiration date."constant proportion portfolio insurance"(CPPI) is used by them.In this method the value of risky assets is kept constant in proportion which is called "cushion"  $S$ ,so the target of portfolio insurers is following[8].

$$Q_t P_t = k S_t = k(W_t - f)$$

Where  $k$  is chosen greater than 1. And  $f$  is the fraction of initial wealth which remains constant over the duration of insurance plan. Multiple is chosen greater than 1 because it allows the investor to choose his exposure to risky assets. So with increase in prices or decrease in prices set the cushion accordingly and floor remain quite same. In case of falling in the prices fraction of risky assets in the investor's portfolio will also go zero. So only risky less cash remains in the portfolio. Portfolio strategy is contrary to rebalancing strategy suggests the pro cyclical so its destabilize the investor behavior. When does the prices fall then portfolio insurers try to reduce their stock position to protect their floor. And when the prices increase they strive the stock position to realize the additional gain. Each and Every investor reviews his portfolio at each interval of time because the trading is not continuous but it is discrete points in time. Investor rates his current assets position using the prices forecast in the price which is computed accordingly to the current demand and supply [8].

1. If only the buy orders (ask) exist in the market than the investor estimate his prices at 101 percent of the maximum ask price.
2. If only the sell orders (open bids) exist in the market than the investor estimate his prices at 99 percent of the minimum bid price.
3. If both open ask and open bids exists in market then price will be between in open bid and open ask. And the estimation of the new prices will be equal to average of lowest bid price and highest ask of the last period.
4. If neither one of them exists than next period price will be equal previous trading price.

## 3.2 Study of ASM

Study of ASM includes two kind of model [14]:

1. Roll model of financial market
2. Call auction model of financial maker

### 3.2.1 Roll model

Roll model is the model of quote driven financial market. Which is continuous. Market maker first observes the fundamental value then he sets the bid quote and the ask quote at equal distance from that observed fundamental value. According to market maker bid quote is equal to  $FV(\text{fundamental value}) - s/2$  and the ask quote is equal to  $FV + s/2$ . Where  $s$  is known as the spread value. In this model a single trader represents all trader who is responsible for placing the trade of size one continuously with same probability. There are two kind of market role models[14]:

1. Roll which is having the constant FV known as simple roll model.
2. Roll model which is having the variable FV known general roll model.

In the general roll model change in FV occurs by normal distribution having the mean 10000 and standard deviation 1.0. Initial value of FV in both of the model is 100.00. And the value of spread is different for the different experiment which is 0.5, 1.00, 2.00. The motive of the roll model is that .It represents the effects of bid-ask spread on the properties of return with time. There's two kinds of results come out in the experiment[14].

1. Consecutive change in quote
2. Autocorrelation at lag 1 is near to negative 0.5 for simple roll model.

#### 3.2.1.1 Distribution

In the general roll model it is observed that kurtosis of returns found near to 3 .And the skewness is found near to zero[14].

#### 3.2.1.2 Volatile Cluster

In the experiment it is observed that autocorrelation of square of the return is near to 0.2 in case of general roll model. Volatile clustering is done because the fundamental value observed or generated by distribution so in general FV is near to mean and change in the price is very small. And asking the FV far away from the mean cause the large return in measurement[14].

$s$	Length Experiments (minutes)	Nr. Transactions	AC Price	AC Return	AC Squared Return	Skewness Return Series	Kurtosis Return Series	Roll Measure
0.5	5	2165	-0.03	-0.51	0.00	0.00	1.95	0.51
	10	4458	-0.03	-0.53	0.03	0.00	1.94	0.52
	15	6735	-0.01	-0.51	0.01	0.00	1.98	0.51
	30	13887	0.00	-0.50	0.00	0.00	2.01	0.50
	60	26669	-0.01	-0.50	0.00	0.00	1.99	0.50
1.0	5	2386	0.02	-0.50	0.03	0.00	2.04	0.99
	10	4649	0.02	-0.50	0.02	0.00	2.03	1.00
	15	7389	0.01	-0.49	0.00	0.00	2.03	0.99
	30	13836	0.00	-0.49	-0.02	0.00	1.99	1.00
	60	28602	0.00	-0.51	0.01	0.00	2.00	1.01
2.0	5	2179	-0.02	-0.53	0.05	0.00	1.96	2.08
	10	4331	-0.01	-0.51	0.01	0.00	1.98	2.03
	15	6762	0.01	-0.50	0.00	0.00	2.02	1.98
	30	13689	0.01	-0.49	0.00	0.00	2.02	1.98
	60	27373	0.01	-0.50	0.01	0.00	2.01	1.99

Table 2: Result of simple roll model [14]

$s$	Length Experiments (minutes)	Nr. Transactions	AC Price	AC Return	AC Squared Return	Skewness Return Series	Kurtosis Return Series	Roll Measure
1.0	5	2320	0.02	-0.48	0.26	-0.04	2.86	2.22
	10	5136	0.00	-0.49	0.23	0.01	2.90	2.22
	15	6573	-0.01	-0.52	0.28	0.00	3.04	2.28
	30	14193	0.01	-0.50	0.23	0.00	2.95	2.22
	60	26410	0.01	-0.50	0.24	0.01	2.95	2.22

Table 3: Result of general roll model [14]

## PROPOSED APPROACH AND IMPLEMENTATION

### 4.1 Overview

This model determines the value of bid and ask quote which is totally based on the experiments on the real values and the prior knowledge is taking into study with fix uninformed or informed trades. Discrete simulation is used in this model with the probabilistic approach.

#### 4.1.1 Organizational aspects

In this model session are continuous trading session and the execution system is quote driven execution system. It is assumed in model that the value of stock is exogenously generated in market and they are having the fundamental value.  $V_t$  is considered the fundamental value of the stock at time  $t$ . In this model an potential trader is selected from the pool of traders. The market maker considered responsible of the execution of order received at current price which may be the bid price or ask price. Its depends on the order and he is also responsible for providing the liquidity in market. Bid price and the ask price is set as the function which is the function of flow of order and also contains the information of the market. A probability density function is taken by the market maker to get the fundamental value. Which is the probability density function on the range of expected values. There are the prior expectation in the model so PDE and this prior expectation is used to get the initial fundamental value .In the model trading round

having the following steps

1. The probability of the jump process in the fundamental value is calculated.
2. One kind of potential investor is selected from the pool of potential traders.
3. According to selected trader the type of the order is given to the market maker which may be sell, buy or no order.
4. If possible then the order is executed by the market maker and transaction is carried out.
5. The new value of bid price and the ask price is set.

The probability of the jump process in the fundamental value is very low if the fundamental value at time  $t$  is  $V^t$  then the fundamental value at time  $t+1$  will be given by the  $V^{t+1} = V^t + \varpi(0, \sigma)$ . Here  $\varpi(0, \sigma)$  is the normal distribution function having the mean zero and standard deviation  $\sigma$ .

## **4.2 The behavior of the investor in the proposed approach**

There are two kind of investors are taken in this model.

1. Uninformed investors
2. Informed investors

Informed investors are of two kind.

1. Perfectly informed
2. Nosily

Those investors or traders who get the fundamental values correctly them the perfectly informed traders. If the traders get the nosily value then the traders are the nosily traders and that noise value is  $W^t = V^t + \Psi(0, \sigma_w)$ . Here  $\Psi(0, \sigma_w)$  represents the normal distribution with mean zero and variance square of  $\sigma_w$ . The decision whether the informed traders will trade or not it's depends on the observed fundamental value.

### 4.3 Role of the market maker in the proposed approach

1. Get the order and execute them.
2. Update the PDE according to the orders.
3. Set the new value of the bid or ask according to the situation.

### 4.4 Information of the market maker

Initially orders are executed by the market maker on current value of the bid quote or the ask quote it's depends on the types of orders. And the information regarding the fundamental value is diffused by the investors to market maker. Continuous sell orders are the indication that the current bid quote is higher than the fundamental value and continuous buy orders are the indication that current ask price is lower than the fundamental value. Although the nosily traders are also taken into account by the market maker. What will be the range of values, what will be the probabilities and the learning process these all things depend on the prior knowledge of the market maker. Below all information are the set of the prior knowledge of the market maker.

1.  $\alpha$  represents the informed traders in fraction and  $1 - \alpha$  represents the uninformed traders in fraction
2.  $\eta$  is the probability that an uninformed trader will trade.
3.  $V_0$  which represent the initial fundamental value.
4.  $\Psi(0, \sigma_w)$  which is the density function for noise and the  $\varphi(0, \sigma)$  which is the normal density function for the jump.

### 4.5 The probability density estimation

Market maker create a range of the fundamental value which depends on the standard deviation .

$$V_{min} = V_0 + 4\sigma$$
$$V_{max} = V_0 + 4\sigma - 1$$

The interval is of one cent and values are also in cent and probability density function is maintained by the market maker for the whole range. Whenever each time jump process occurred over the fundamental value probabilities are reentered.

### 4.5.1 Initialization

The below given normal distribution is used to initialize the probabilities after jump in fundamental value.

$$Pr(V = V_i) = \int_{V_i}^{V_i+1} N(0, 4\sigma) dx$$

Where  $i \in \{V_0 - 4\sigma, \dots, V_0 + 4\sigma - 1\}$

Density function is n with mean zero and standard deviation .

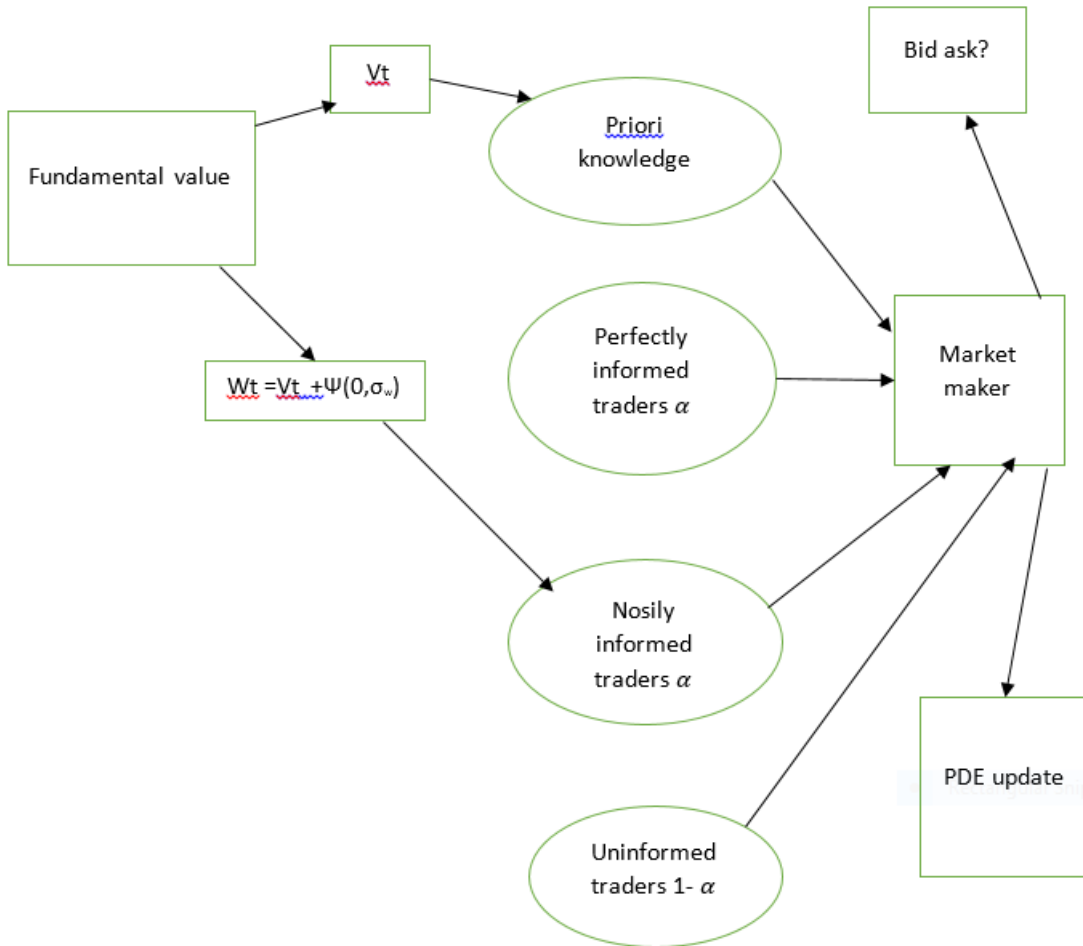


Figure 4.1: Information diffusion



### 4.5.2 Updating of probability density function

At arrival of the new order market maker each time update the probability using the scaling .The new probabilities will be equal to the current probabilities which is the conditional probabilities based on the arrival of that order.

$$P(V = V_i|order) = P(order|V = V_i) * P(V = V_i)/P(order)$$

Type of order

1. Buy
2. Sell
3. No order.

$P(order)$  is the probability of any kind of order.  $P(order|V = V_i)$  is conditional probability of any kind of order at the value  $V_i$ .  $P(order)$  is calculated by the following way

$$P(order) = \sum_{V_i=V_{min}}^{V_{max}} P(order|V_i)P(V = V_i)$$

And the conditional probabilities are calculate by following way

$$P(order | V = V_i) = (1 - \alpha)P(\text{uninformed trader's orders} | V = V_i) + \alpha P(\text{informed trader's orders} | V = V_i)$$

But the probability of the orders from the uninformed trader is constant  $\eta$  .So probability of no order from informed trader will  $= 1 - 2\eta$ .

### 4.5.3 Conditional probabilities in case of perfectly informed investors

The investor always buy if the investor observed that the fundamental value is higher than ask quote. And investors will always sell if they observed the fundamental value lower than bid quote.

$$P(\text{selling} | V = V_i) = (1-\alpha)\eta + \alpha \text{ if } V_i < P_B$$

$$P(\text{selling} | V = V_i) = (1-\alpha)\eta \text{ if } V_i \geq P_B \text{ Where } P_B \text{ is the bid price}$$

$$P(\text{Buying} | V = V_i) = (1-\alpha)\eta \text{ if } V_i \leq P_A$$

$$P(\text{Buying} | V = V_i) = (1-\alpha)\eta + \alpha \text{ if } V_i \geq P_A \text{ Where } P_A \text{ is the ask price}$$

$P(\text{No order}) = 1 - (P_A + P_B)$  which mean there will no informed traders will trade if fundamental value and the bid quote and ask quote propose that the bid quote and the ask quote currently set around the fundamental value. So market maker set the bid ask spread high to give the liquidity to the market.

$$P(\text{No order} | V = V_i) = (1-\alpha)(1 - 2\eta) \text{ if } P_B > V_i > P_A$$

$$P(\text{No order} | V = V_i) = (1-\alpha)(1 - 2\eta) + \alpha \text{ if } P_B \leq V_i \leq P_A$$

#### 4.5.4 Conditional probability in case of nosily informed traders

Nosily informed traders get the incorrect noise fundamental value at place of correct fundamental value so The nosily investor will sell if he observes the noise value lesser

$$P(\text{selling} | V = V_i) = (1-\alpha)\eta + \alpha P(V_i + \Psi(0, \sigma_w) < P_B)$$

$$P(\text{Buying} | V = V_i) = (1-\alpha)\eta + \alpha P(V_i + \Psi(0, \sigma_w) > P_A)$$

$$P(\text{No order} | V = V_i) = (1-\alpha)(1-2\eta) + \alpha P(P_B \leq V_i + \Psi(0, \sigma_w) \leq P_A)$$

than the bid quote, and he will buy if he observes the noise fundamental value higher than ask quote and no order will be placed if he observes the noise value higher than or equal bid quote and lesser than or equal to ask quote.

Bid quote is equal to the expectation of the true value at the condition that the selling order will come.

$$P_B = E[V | \text{Selling}]$$

Ask quote is equal to the expectation of the true value at the condition that the buying order will come.

$$P_A = E[V | \text{buying}]$$

The calculation of expectation is done by computing the all conditional probabilities for positive values.

$$E[V|\text{selling}] = \int_0^{\infty} V P(V = x|\text{selling}) dx$$

$$E[V|\text{buying}] = \int_0^{\infty} V P(V = \text{buying}) dx$$

Which can be computed for the range  $V_{min}$  to  $V_{max}$  like below

$$P_B = \sum_{V_i=V_{min}}^{V_{max}} V_i P(V = V_i|\text{selling})$$

$$P_A = \sum_{V_i=V_{min}}^{V_{max}} V_i P_p(V = V_i|\text{buying})$$

$$E[V|\text{selling}] = \sum_{V_i=V_{min}}^{V_{max}} V_i P(\text{selling}|V = V_i) P(V = V_i) / P(\text{selling})$$

According to bayes's

$P_{\text{selling}}$  is the prior probability of selling

$$P_{\text{selling}} = \sum_{V_i=V_{min}}^{V_{max}} V_i P(\text{selling}|V = V_i) P(V = V_i)$$

$$P_{\text{selling}} = \sum_{V_i=V_{min}}^{V_i=P_b-1} (\alpha + (1 - \alpha)\eta) + P(V = V_i) +$$

$$\sum_{V_i=P_b}^{V_{max}} [(1 - \alpha)\eta P(V = V_i)]$$

$P_B$  or  $P_b$  is bid price is set market maker

$$P_b = 1/P_{\text{selling}} \sum_{V_i=V_{\min}}^{V_i=V_{\max}} V_i P(\text{selling}|V = V_i) P(V = V_i)$$

We know that  $V_{\min} < P_b < V_{\max}$

$$\begin{aligned} P_b &= 1/\text{selling} \sum_{V_i=V_{\min}}^{V_i=P_b-1} V_i P(\text{selling}|V = V_i) P(V = V_i) \\ &+ 1/\text{selling} \sum_{V_i=P_b}^{V_i=V_{\max}} V_i P(\text{selling}|V = V_i) P(V = V_i) \end{aligned}$$

We know that  $P(\text{selling}|V < P_b) = (1 - \alpha)\eta + \alpha$  and  $P(\text{selling}|V \geq P_b) = (1 - \alpha)\eta$

$$\begin{aligned} P_b &= 1/\text{selling} \sum_{V_i=V_{\min}}^{V_i=P_b-1} V_i P(V = V_i) (1 - \alpha)\eta + \alpha \\ &+ 1/\text{selling} \sum_{V_i=P_b}^{V_i=V_{\max}} V_i P(V = V_i) (1 - \alpha)\eta \end{aligned}$$

$$\begin{aligned} \text{Similarly } P_a &= [1/\text{buying} \sum_{V_i=V_{\min}}^{V_i=V_a} V_i P(V = V_i) (1 - \alpha)\eta] \\ &+ 1/\text{buying} \sum_{V_i=pa+1}^{V_i=V_{\max}} V_i P(V = V_i) (1 - \alpha)\eta + \alpha \end{aligned}$$

## 5.1 Results of Proposed Approach

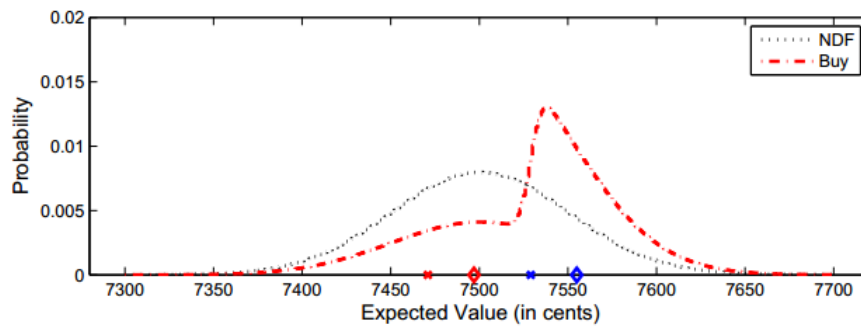


Figure 5.1: Updation in the PDE When  $\alpha = 0.5$  and buy order is received

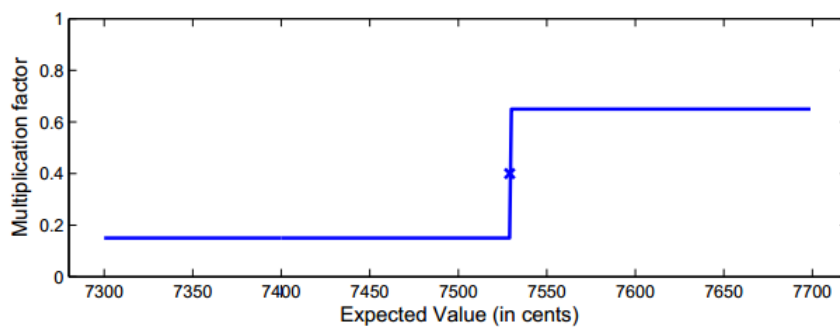


Figure 5.2: Multiplication factor

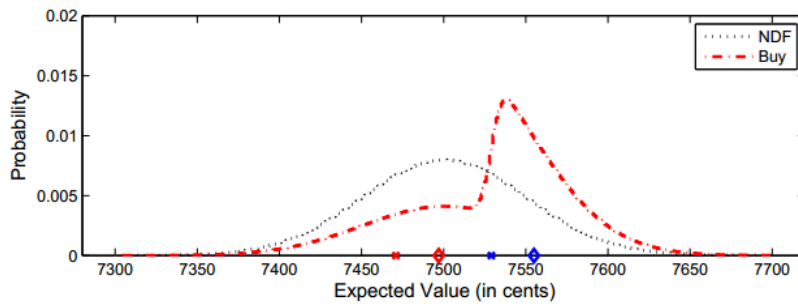


Figure 5.3: Updation in the PDE When  $\alpha == 0.5$  and buy order is received for noisy

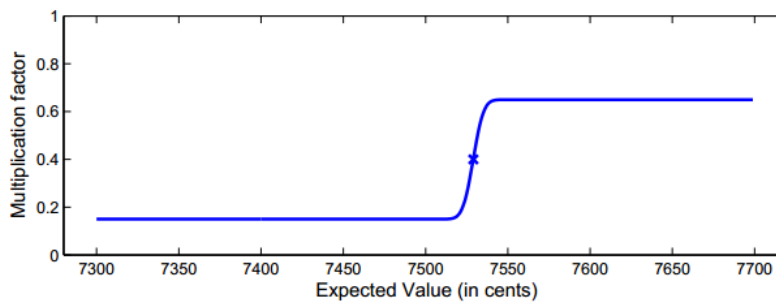


Figure 5.4: Multiplication factor When  $\alpha == 0.5$  and buy order is received for noisy

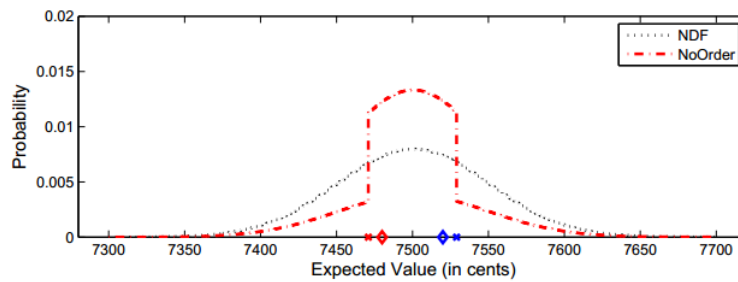


Figure 5.5: Updation in the PDE When  $\alpha == 0.5$  and No order is received with perfect

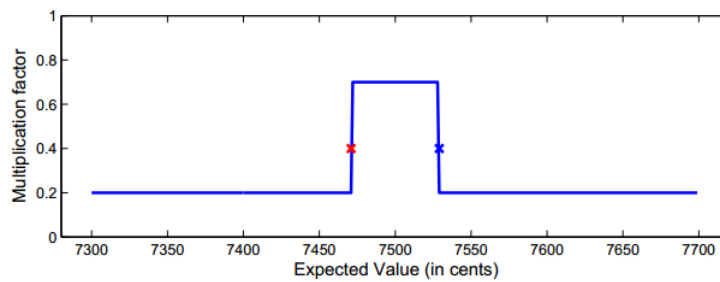


Figure 5.6: Multiplication factor when  $\alpha == 0.5$  and No order is received

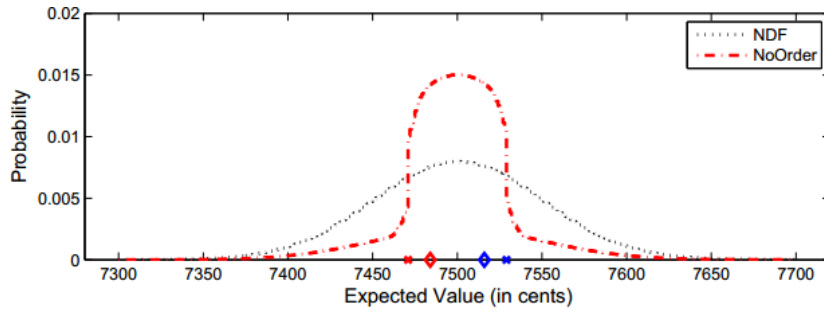


Figure 5.7: Updation in the PDE When  $\alpha == 0.5$  and No order is received for noisy

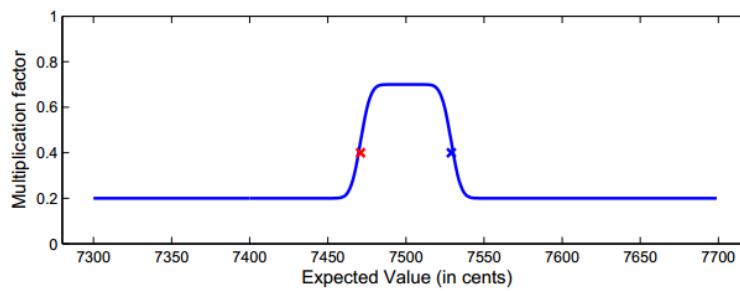


Figure 5.8: Multiplication factor When  $\alpha == 0.5$  and No order is received for noisy

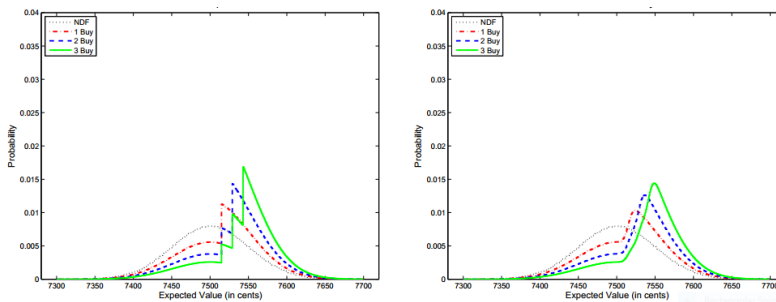


Figure 5.9: Evolutionary path When  $\alpha == 0.25$  and consecutive buy order is received first with perfect and then for noisy

## 5.2 Results in case of central selection of investor

In this type whenever the investor invest each time. He has to decide that whether he will put the order as the informed trader or uninformed trader. The following steps are the description when the EGM model is centralized.

1.  $P_B$  and  $P_A$  are initialized by the maker maker and he made them public.
2. Next order will be shown as the informed investor or uninformed investor it is determined by the representative trader stochastically.

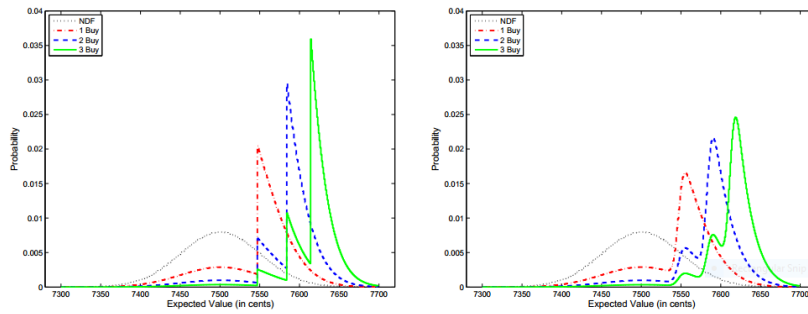


Figure 5.10: Evolutionary path When  $\alpha = 0.25$  and consecutive buy order is received first perfect then for noisy

3. The buy order will be generated or sell, or no order is will be depending on the selected trader.
4. Then the order is executed by the market maker and transaction is carried out if required and updatation in  $P_B, P_A, PDE$  is done by market maker.
5. Then new values of the  $P_B, P_A$  are published.
6. All above steps are repeated.

Jump process is random in time means can happen at any time. In this model the new values are made public so now the traders will trade according to new values. The experiment is done for 10 minutes means for 600 seconds.

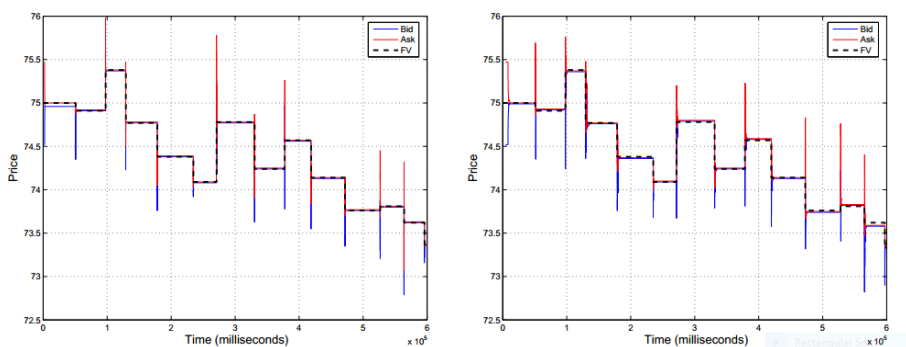


Figure 5.11: Learning behavior of the market maker with 50% perfect in first and 50% noisy in



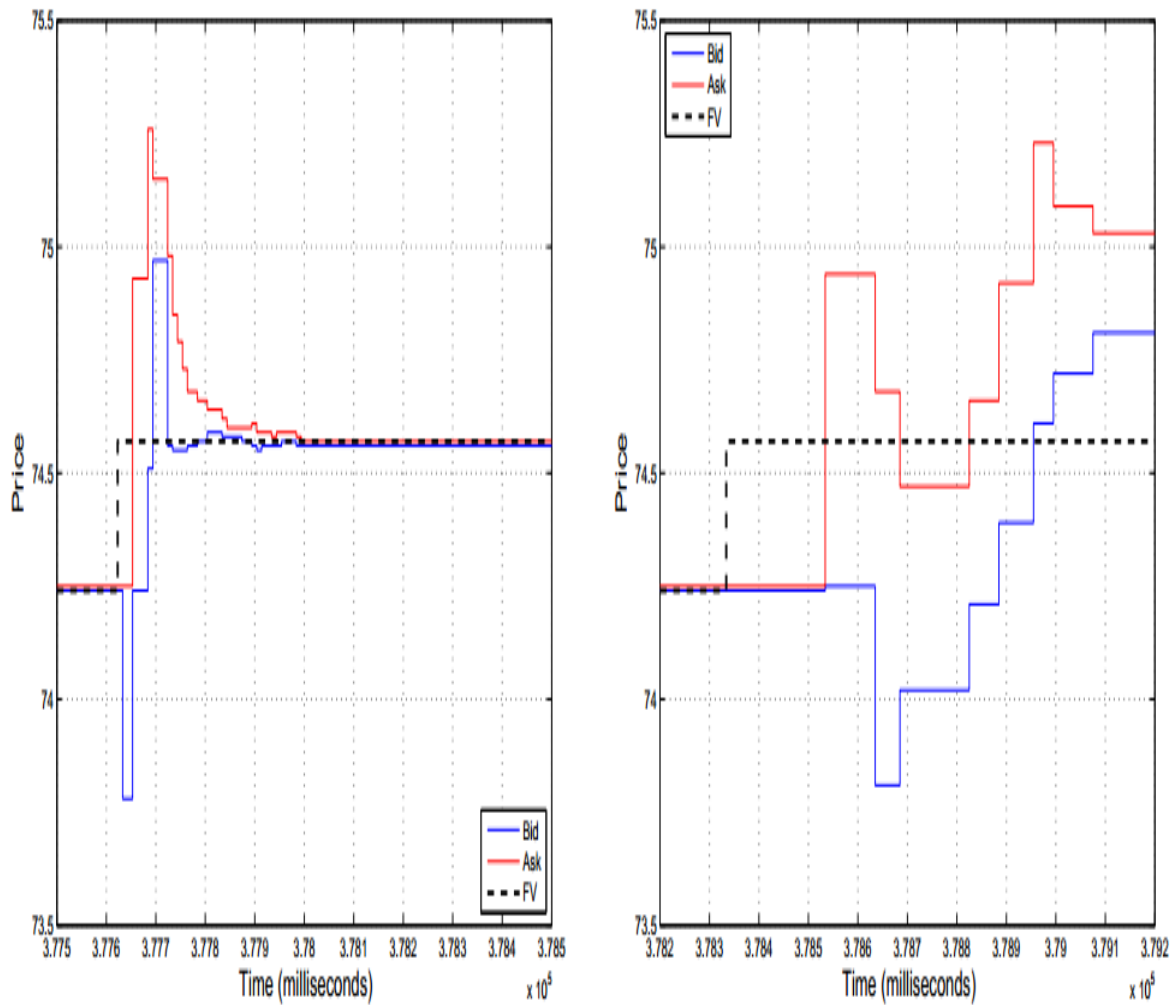


Figure 5.12: Learning behavior of the market maker with 75% perfect in first and 75% noisy in second before and afterwards of jump.

## CONCLUSION

All the research questions which were discussed initially are covered in it. And a better approach is provided to study the financial market's dynamics perfectly. And a time extended agent based approach and simulation is provided with the good representation of financial market's dynamics and the learning behavior of market maker and decision making behavior of the investors.

The mechanism for the order selection and negotiation is provided. Continuous time simulation model is discussed in detail with different conditions Which is very efficiently helps in market prices formation.

## FUTURE WORK

There are many direction in which more research can be done to understand the market dynamics of financial market. There are many hardly observable fields and aspect of the many financial market organization regarding them still there are many possibilities to do research and there is much variety in the thinking or the decision making capabilities of the investor which is still not fully understandable. The framework can be used to design the more number of ASM which would provide the better understanding of the market. There are still many directions in which the more development work can be done to improve the understanding of the dynamics of the financial market .Research can be done in aspect when multiple financial market are working on the single stock. Some assumption can be dropped which are not fully exist in the real market such as the ratio of the informed and uninformed traders to provide the more realistic model for understanding the market and its dynamics closely.

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