Design of Robust controller for Robotic arm

A DISSERTATION

Submitted in partial fulfilment of the requirements for the award of the degree

Of

MASTER OF TECHNOLOGY

In

ELECTRICAL ENGINEERING

(With specialization in System and Control Engineering)

By

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MAY, 2016

CANDIDATE'S DECLARATION

I hereby certify that this report which is being presented in this dissertation entitled **"Design of Robust controller for Robotic arm"** in partial fulfilment of the requirement of award of Degree of **Master of Technology** in **Electrical Engineering with specialization in *System & Control Engineering***, submitted to the Department of Electrical Engineering, Indian Institute of Technology, Roorkee , India is an authentic record of the work carried out during a period from May 2015 to May 2016 under the supervision of **Dr. Yogesh Vijay Hote**, Department of Electrical Engineering, Indian Institute of Technology, Roorkee. The matter presented in this dissertation has not been submitted by me for the award of any other degree of this institute or any other institute.

Date : 23/05/2016

Place : Roorkee (NIKHIL KP)

CERTIFICATE

This is to certify that the above statement made by the candidate is correct to best of my knowledge.

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ACKNOWLEDGEMENTS

I would like to express my sincere gratitude and indebtedness to my supervisor Dr.Yogesh.Vijay. Hote, Electrical Engineering Department, Indian Institute of Technology, Roorkee for guiding me to undertake this project work on “Design of Robust control for Robotic arm as well as providing me all the necessary guidance and support throughout this work. I would also be thankful to all the staff of Electrical Engineering Department for their constant support and all my friends, Junior and Seniors for their help and encouragement at the hour of need.

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ABSTRACT

The development of automation in the industries involves the use of robots. This increases the productivity in a fast and accurate .which in terms increases the quality of products. There are many researches are going on in the field of robotics. Among that the field involving the robotic arm manipulators are important in the role of growing the industry . Since it reduces the human efforts and increases productivity. As the complexity of the task increases the control of robots increases. It is mainly because, it involves nonlinearities, uncertainties external perturbations etc. This makes the control more complex. More over the dynamic equation of the manipulator is highly nonlinear and input-output coupled. There are many control laws used for the trajectory tracking or tracking desired input. To achieve the required task the controller adjust itself in accordance with the uncertainty or external perturbations. Such kind of controllers are referred as adaptive control. Which adapt to the situation. But these adaptive controllers may not be robust. So there comes the design of robust controller. In this work a robust controller which works based on pole clustering is explained. The boundedness of the system can be set by adjusting the location of poles in the *s* plane. So according to the changes in position of poles the controller performance will change in the specified boundedness.

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CHAPTER - 1

INTRODUCTION

***1.1 Introduction**:* The field of application of robotics has been increasing day by day. And it has already replaced most of the man power in the industries. The main types of robots used in industries are robotic manipulators or simple manipulators. It has wide varieties of applications like material handling, welding, assembly, spray painting, grinding etc.

The use of robots in the industry have emerged in recent year. It have almost replaced most of the human work. As the working areas increases widely the complexity also increases. In order to have the better and accurate performance the controller part should be able to give the desired or required output. The designs of controllers are introduced from the early day for position and velocity tracking. All those designs were done without considering the gravitational effect and without considering the uncertainty in parameter like mass, Coriolis force, friction etc. The consideration of uncertainty in these parameters will make the control of the manipulator more complex since the dynamic equation of the manipulator are highly nonlinear and coupled. One of the mainly used controller technique for this kind of problems was the adaptive control. In adaptive control the system itself adapt to the changes in parameter in the limited bounds. There are many adaptive technique used for this for years. There are many properties for the dynamic equation or torque equation of the manipulator. The exploitation of these properties will leads to different kind of adaptation laws and parameter update law. Adaptive control technique deals with the changes in parameters, but we also have needed the robustness in the control. That means there should be some bounds in which the variation or changes in the system should be handled properly. So there come the needs for robust control. There are many types of robust controller are used in recent years. Here the use or robust control technique using pole clustering have explained.

1.2 Robotic arm*:* A robotic arm is also called as Manipulator. “Degree of freedom of a manipulator” is simply the number of joints. As the degree of freedom increases the complexity in controller design increases. Most of the manipulators used in industries are of multi input/multi output, highly coupled, nonlinear dynamic system. So in order to track the desired trajectory a proper controller should be designed which works in all conditions.

Robotic are basically having higher degree-of-freedom (DOF) positioning devices. They are machines that imitate human movements which give chance the user to locate the position and orientation their tools. A arm having one link and one joint have one degree of freedom. These joints are either translational type or rotational type. Fig.1.1 shown below the robotic manipulator having joints at 1 2, 3 and 4, respectively.

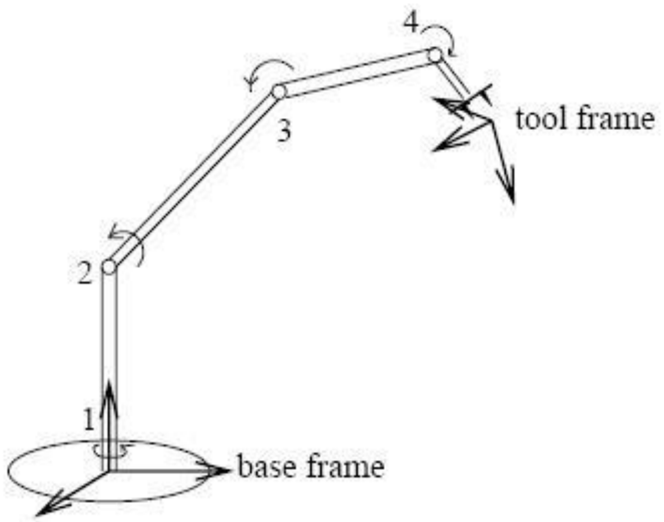

Figure 1.1 Robot Manipulator

Manipulators mobility is lost in the absence of joints. The two adjacent links can be either joined by a revolute or a prismatic joint. Manipulator having either one joint is said to be having one degree of freedom. The movement of one link over other can be in a revelatory or translator if they are having revolute or prismatic joint respectively..

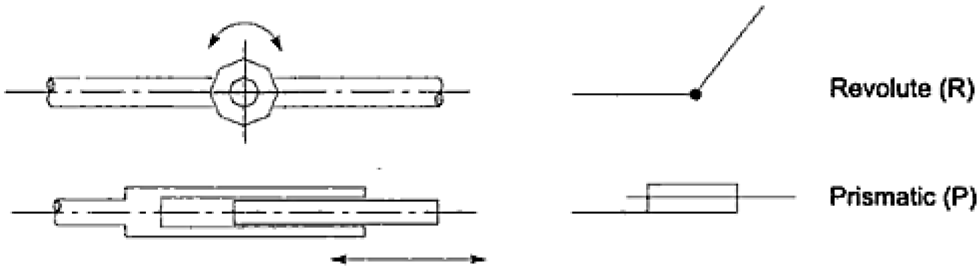

Figure 1.2 Joint types and their symbols

The number of independent movement that an object can perform in three dimensional space is called the number of degree of freedom (DOF). Thus, a rigid body free in space has 6 degree of freedom-3 for position and 3 for orientation. The workspace is that portion of the environment where the manipulators end-effectors can reach. Its size depends on not only on the manipulator structure but on mechanical joint limits also.

Wrist is attached to the endpoint of arm; the sub assembly movements enable the manipulator to orient the end-effectors to perform the task properly. The wrist must possess at least 3-DOF to give three rotations about the three principal axes which is roll (motion in a a direction so that the axis is perpendicular to the end effector), pitch (movement in vertical plane passing through the link), and yaw (movement in a horizontal plane which also passes through the arm) motions.

End-effectors are also called as grippers which is used to grasp the work piece and mainly applicable for holding of materials during the work cycle. The orientation and position of the end effectors position are required to formulate an object in space. The modeling of robot comprises of establishing a special relationship between the manipulator and manipulator object. The motion of each link can be described with respect to a reference coordinate frame; it is suitable way to have a frame attached to the body of each link. The coordinate frame is mapped from one frame to another using rotation and translation of frames.

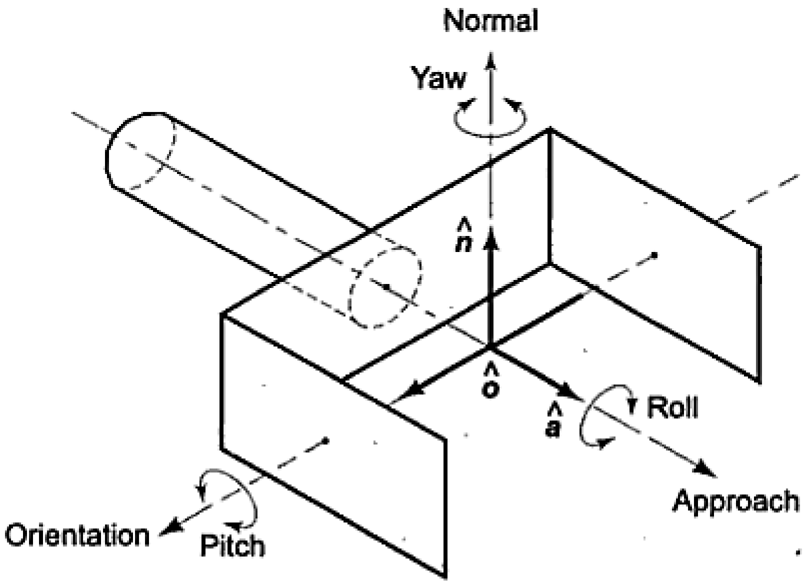

Figure 1.3 Gripper as end effector

1.3 Introduction to Controller Design:

In earlier days PD/PID controllers were used for the control of manipulator. It is capable to track the set point. But in real time situation moreover tracking the set point we are interested in following the trajectory in detail rather than final point tracking. Model based control scheme designed by Hollerbach uses exact model [1], but which was not able to control the manipulator for any changes in uncertainties in the model and environment. Spong, M.W[4] and Slotine, J.J.E proposed adaptive control technique which is able to track the trajectory in the presents of uncertainties. This method uses an adaptive control law. L.L,Slotine Whitcomb and L.Sciavicco proposed an adaptive based controller that converges the tracking error regardless of the persistent existence or non-existence of the trajectory [9].

In order to get the exact tracking of the manipulator in the presence of nonlinearity and uncertainty the use of robust controller is required.

The plant, so called the manipulator is highly nonlinear and along with actuator. The effect of inertia, centrifugal, gravitational, Coriolis forces, hysteresis, backlash etc affects the performance of a manipulator. So in real time application, control of manipulator is a very complex task. In order to achieve high speed and accuracy in operation the control technique need to be improved. The efficiency of performance depend the accuracy in control algorithm as well as dynamic model of robot manipulator. Proportional-Derivative (PD), Proportional-Integral-Derivative (PID), model based control, computed torque control are the conventional control technique. But because of uncertainties in kinematics and dynamic model these control techniques are not able to give perfect trajectory tracking. The other control technique which is commonly is adaptive control. But there is a problem with robustness in this one. So we need to develop an improved control technique which is capable of handling this situation.

In actual practice along with uncertainties, noise and disturbance plays an important role. Disturbance is generally low frequency signal and noise is high frequency signal. Therefore, controller is designed in such a manner that the noise should be eliminated

In this dissertation, first of all modelling of one joint and two joint robots are discussed. Further, effect of integral and noise on pole placement technique is discussed. The analysis has been applied in Robotic arm. Finally, design of robust controller using pole clustering technique has been discussed and it has been applied in Robotic arm.

In the earlier days PD/PID controller are used for the control of manipulators. Even though it is capable of tracking the set point, in real time more over tracking the set point we are interested in tracking the trajectory in which are more used. The model based controls suggested by Hollerbach uses the exact model but it fails to handle the uncertainty in the model or environment. Spong, M.W and Slotine, J.J.E proposed adaptive control technique which can track the trajectory even in the presents of uncertainty. In order to understand the concept of robustness we need to know about the adaptation technique which is a step towards the robust control design.

The study of adaptive control of the manipulator had been introduced by exploiting the properties of the dynamic equation of the manipulator. Which can be mainly classified as inverse dynamic based control and passivity based control method. In the method of know parameter the property of skew symmetry is not exploited here. Instead the cancellation of nonlinearity is done. So that the system behave as linear coupled system.

There are mainly four cases of inverse adaptive control. Spong and Ortega[4], Craig,et,al[3], and Middleton and Goodwin[6], Amestegui et,al[5], So based on the parameter available for the measurement it is further classified into three categories. In the method Craig,et,al the requirement of joint acceleration measurement and updating of adaptation algorithm for ensuring boundedness of the of the invers of the estimated inertia matrix are needed. But Spong and Ortega removed the requirement of the boundedness of the estimated inertia matrix. And the third category Amestegui,et,al follows the same one in second but it uses different parameter law. And finally Middleton and Goodwin removes the requirement of the acceleration measurement but the boundedness of the inverse of the estimated inertia matrix is required.

In the method proposed by Craig,et,al the adaptive implementation is obtained by changing D,C and g with their estimated matrices. Here in this method the author make use of the measurement of acceleration and boundedness of the inverse of the estimated inertia. The main drawback of this is that the requirement of the measurement of acceleration. Which is practically difficult and are very prone to errors. And the estimate of mass inertia matrix should be positive definite. The second drawback mentioned above can be removed by the method explained by Spong and Ortega.

CHAPTER 2:

MODELLING OF ROBOTIC ARM

2.1 Introduction:

Robotic manipulator modelling is important from control point of view. There are mainly 3 types of modelling (i) Kinematic modelling (ii) Inverse kinematics (iii) Dynamic modelling. uThere models are non-linear in nature. Therefore, these models can be converted into transfer function form or state space form. The description of modelling for robotic am as given below.

2.2 Kinematic and Inverse Kinematic modeling:

The link parameter is not variable for given link, where joint parameter is variable according to arm situation**.**

1. Link Parameter:

(a) Link Length- For two axes (i-1) and axis i attached with link (i-1) and link i respectively, there occur a mutual perpendicular, that gives the shortest distance between the two axes. The shortest distance lying on the common normal is defined as the link length and is denoted as a_i.

(b) Link twist - The angle between the projection of axis (i-1) and axis i, on the plane perpendicular to common normal is called as link twist is denoted as α_i.

2. Joint Parameter: A joint has two parameters (θ_i and d_i).and among which one is constant and another is variable. For revolute joint d_i is constant and θ_i is variable, for prismatic joint

θ_i is constant and d_i is variable. Where θ_i is angle the displacement angle of joints and d_i is joint distance.

Frame Assignment to Links:

A manipulator has four joint link parameters and for assigning the orthonormal frame there is a systematic procedure, which was given by Denavit and Hartenberg (1955) and is called as Denavit-Hartenberg (D-H) notation.

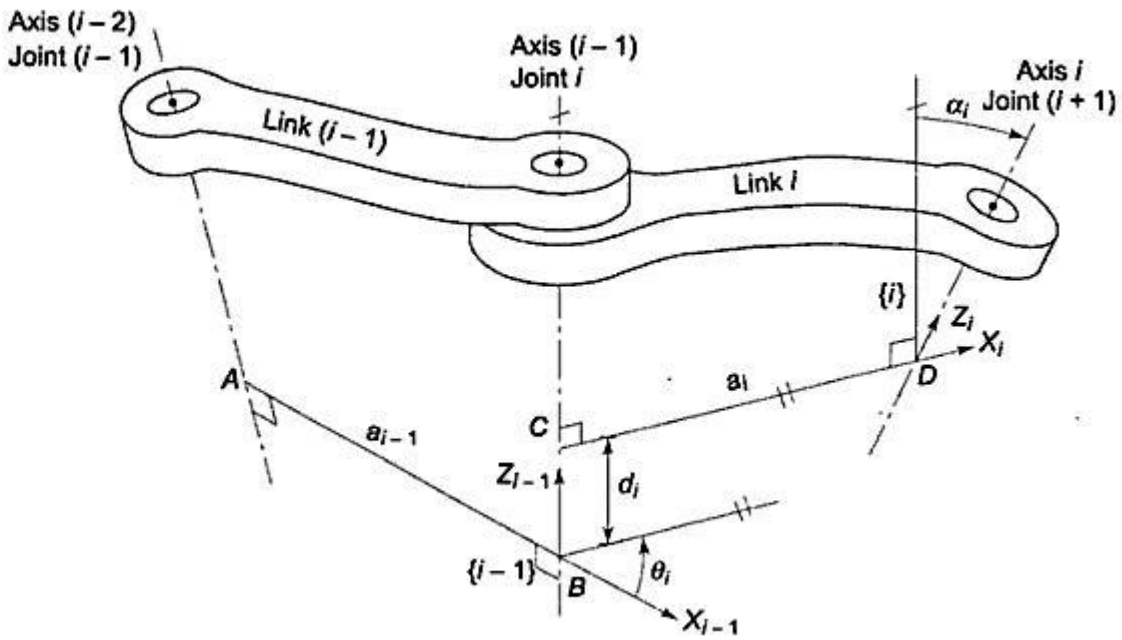

Figure 2.1 D-H convention for assigning the frames

2.3. Dynamic modelling: During the working, a manipulator must accelerate, move at constant speed or decelerate. Time varying torques are applied at the joints (by the joint actuators) to balance out the internal and external forces. The internal forces are caused by the motion (velocity and acceleration) of links. Inertial, Coriolis and frictional forces are some of the internal forces. The external forces are the forces exerted by the environment. It includes the load and gravitational forces. In literature, Newton-Euler and Lagrange-Euler formulation has been used for the dynamic modelling.

A single joint robotic arm is nothing but, a rigid link connected to a actuator. The function of a robotic arm will be different based on the tool attached to its end effector. There should be extreme accuracy in handling the object. For the efficient operation of the system the force and torque applied on the object and the trajectory tracking should be exact. The position or the degree of revolution of the shaft will be based on the input given. The input to the motor is voltage. So the degree of rotation should be first converted into voltage. These voltage will rotate the motor and sensor attached to the end point detect the current position .Based on this an error signal will be produced in order to compensate for the desired trajectory.

2.4 Modelling and simulation:

While doing the modelling part we ignore the effect of gravitation and assume the link is rigid. Both the electrical parameters of motor and mechanical parameter of robot are combined together.

The basic block diagram of a single link robotic arm is shown in fig2.4.

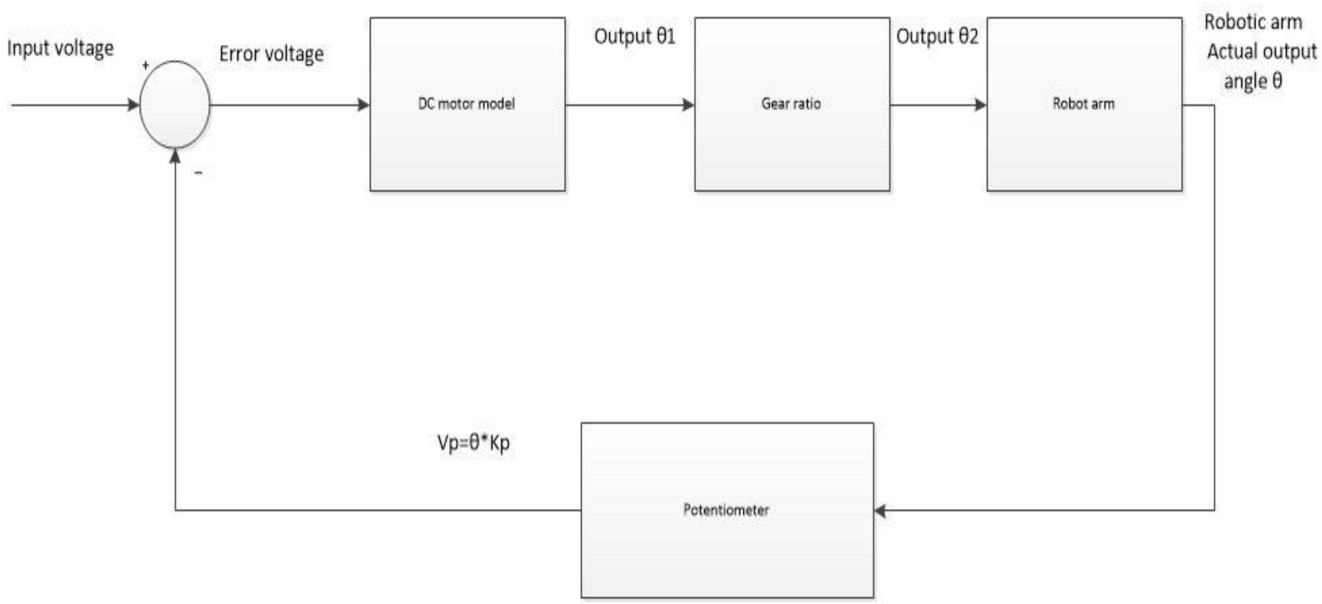

Figure 2.2. Block diagram of one joint robotic arm

And the joint control system diagram is

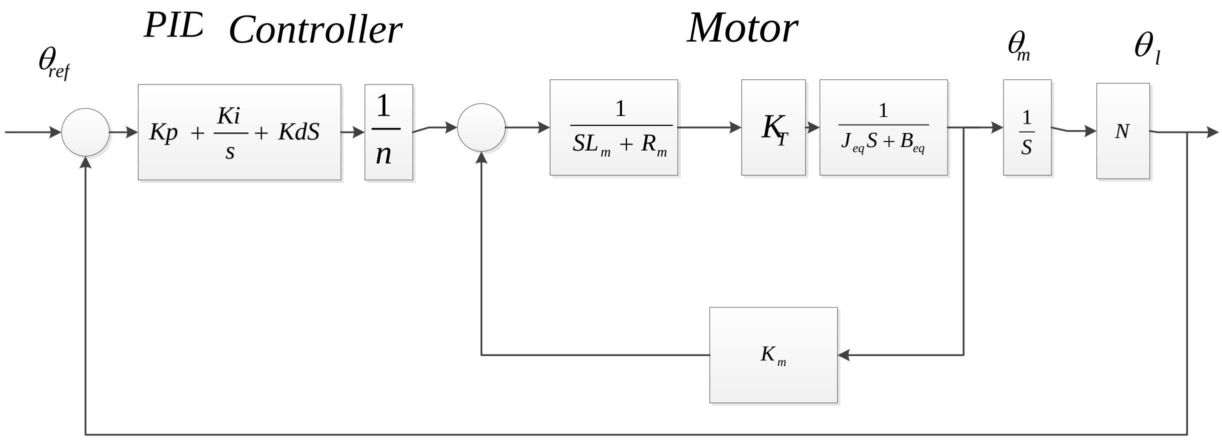

Figure.2.3. Block diagram of robotic arm with PID controller

Derivation of transfer function model:

Let the input to the motor be V_{in} . Then we have,

$$V_{in}(t) = R_m i_a(t) + L_m \frac{di_a(t)}{dt} + e_m(t)$$

$$T_m = K_T i_a(t)$$

$$e_m(t) = K_m \frac{d\theta_m(t)}{dt}$$

$$T_m = J_m \frac{d^2\theta_m(t)}{dt^2} + B \frac{d\theta_m(t)}{dt}$$

$$Jeq = J_m + N^2 J_l$$

$$Beq = B_m + N^2 B_l$$

$$\theta_l = N\theta_m$$

By solving the above equations we find the transfer function of output angle to input voltage as

$$\frac{\theta_l(s)}{V_{in}(s)} = \frac{K_T * N}{[L_m J_{eq} s^3 + (R_m J_{eq} + B_{eq} L_m)s^2 + (R_m B_{eq} + K_T K_m)s]}$$

Where the parameters are ,

V_{in} = armature voltage in volt.

R_m =armature resistance in ohm.

i_a =current through armature in ampere.

L_m = inductance of armature winding in Henry.

e_m =back e.m.f in voltage.

K_m =back e.m.f constant in volt/(rad/sec).

K_T =motor torque constant in N.m/A.

J_{eq} =moment of inertia of motor and robot arm in Kg.m2/rad.

Beq =Viscous friction coefficient of motor and arm in N.m/rad/sec.

θ_l =angular displacement of arm in rad.

θ_m =angular displacement of motor in rad.

N =gear ratio.

An example for this is done using some known values of the parameters. The arm parameters are mass M=4Kg, length L=.2m, and viscous damping constant B=.08 N.sec/rad.

The motor have the parameters as, Vin=12 volt, $J_m = .02 kg.m^2; B_m = .03; K_T = .023 Nm/A;$

$K_m = .023 Vs/rad; R_m = .8 ohm;\ L_m = .23 Henry;$ and gear ratio as 4;

By solving the equations using the parameters given above, we get the transfer function as,

$$G(s) = \frac{\theta_m(s)}{V_{in}(s)} = \frac{.023}{.02913 s^3 + .153 s^2 + .1205 s}$$

The Simulink model of the one link robotic arm is,

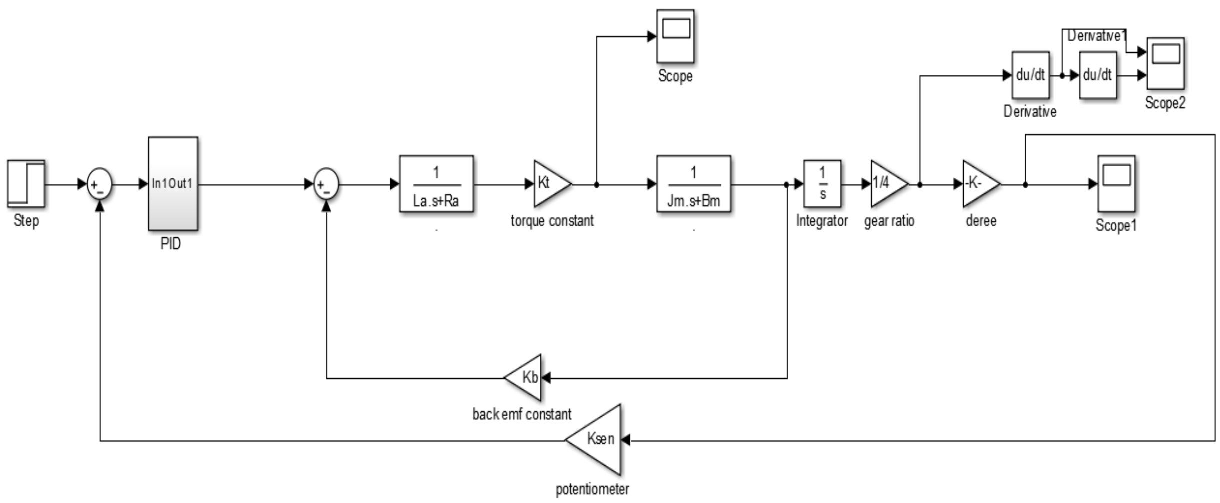

Fig.2.4.: Simulink diagram

The values of PID are determine using Ziegler-Nichols method. And further adjustment in the values of the PID for a input angle of 180 degree.

The results obtain is as shown below. From which we can see that the output follows the desired angle after a particular time.

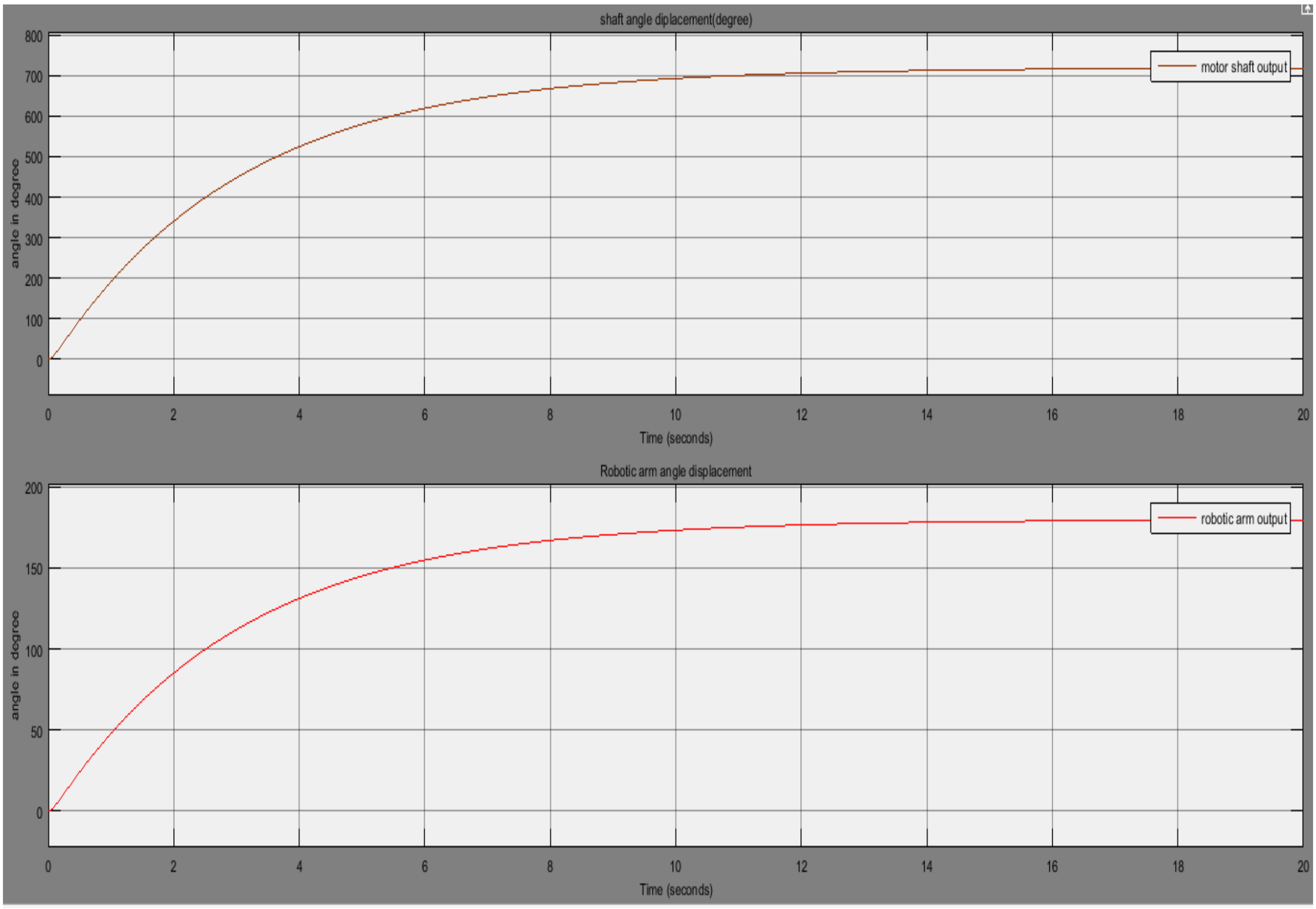

Fig.2.5.: Simulink result of the one link robotic arm.

CHAPTER 3:

STATE FEEDBACK CONTROLLER DESIGN FOR ONE JOINT ROBOTIC ARM WITH NOISE

3.1 Introduction:

Pole placement is widely used technique for stabilization of linear time invariant and time varying system. This pole placement technique is based on state space equation given as

$$\begin{aligned}\dot{X} &= Ax + Bu \\ y &= Cx + Du\end{aligned} \tag{3.1}$$

In above equation, A is system matrix, B is input matrix, C is output matrix and D is coupling matrix. The stability of the system can be determined by calculating eigenvalues of a matrix. If the eigenvalues are lying on the right half of the s-plane, system will be unstable. If the eigenvalues are lying on the left half of the s-plane, then the system is stable. If the eigenvalues are lying far away from the imaginary axis, then the system is more stable, speed of response is more. It is also observed that the conventional pole placement technique is not helpful in reducing steady state error. Therefore, integral based state feedback controller is useful for reducing the steady state error. Further, it is observed that the measurement noise is normally ignored while designing state feedback controller. Designs that ignore noise in a plant are likely to fail when implemented in actual conditions where noise exists. Therefore, one should be careful while dealing with the state feedback controller design. If the noise exists, then moving the closed loop poles far away from the imaginary axis, will also increase the response of the system due to noise which is undesirable. Since the practical system is always affected by noise, one should be careful for placing the poles very far away from the imaginary axis. In this chapter, design of state feedback controller is carried out for model of one joint robotic arm considering state feedback, state feedback with integral controller and state feedback design with noise. This chapter is organised as follows:

3. 2. State feedback controller for one joint robotic arm

3. 3. Integral controlled based State feedback controller for one joint robotic arm

3.4. State feedback controller design for plants with noise

3.5. Application to one joint robotic arm

3.2 State feedback controller design[33]:

Consider the system as

$$\dot{x} = Ax + Bu \tag{3.2}$$

We can convert above system into controllable canonical form via invertible transformation

$$T \in \mathbb{R}^{nxn},\ \text{where}\ z = T^{-1}x;\ \ \mathrm{A}_z = T^{-1}AT, B_z = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} = T^{-1}B \tag{3.3}$$

Where, $\dot{z}$ is in controllable canonical form .

$$A_z = \begin{pmatrix} 0 & 1 & 0 & \ldots \\ 0 & 0 & 1 & \ldots \\ 0 & \ldots & 0 & 1 \\ -a_0 & \ldots & -a_{n-2} & -a_{n-1} \end{pmatrix}, B_z = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \tag{3.4}$$

We can design state feedback as

$$u = -K_z z + v \tag{3.5}$$

To place the poles (for the transformed system).

Since A and $A_z = T^{-1}AT$ have same characteristics polynomials:

$$\det(\lambda I - T^{-1}AT) = \det(\lambda T^{-1}T - T^{-1}AT) \tag{3.6}$$

$$= \det(T)\det(T^{-1})\det(\lambda \mathrm{I} - \mathrm{A}) = \det(\lambda \mathrm{I} - \mathrm{A}) \tag{3.7}$$

The control law:

$$u = -K_z T^{-1}x + v = -Kx + v \tag{3.8}$$

Where, $K = K_z T^{-1}$ places the poles at the desired locations

It is proved in the literature that for a single input LTI system, $\dot{x} = Ax + Bu,$ there is an invertible transformation T that converts the system into controllable canonical form if and only if the system is controllable. The system can be written in block diagram as shown in fig 3.1.

3.3 Design of state feedback with integral control[33,34]:

The designing the state feedback controller by using only the pole placement technique design will give one major disadvantage where large steady state error will be introduced. So, in order to compensate for this problem, an integral control is added where it eliminate the steady state error in the response to the step input. Fig. Shows the block diagram of the system with the integral control added into it. A feedback path from the output has been added via an integrator. The main function of adding an integrator is to increase the system and thus reduces the original finite steady state error to zero. Therefore, a design for zero steady state error for a step input can be obtained.

From the block diagram, let the system matrix with integral control can be written as

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_w(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ \mathrm{C} & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_w(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t)$$

$$y(t) = [C \quad 0] \begin{bmatrix} x \\ x_N \end{bmatrix} \tag{3.9}$$

From above figure, we can write

$$u(t) = -Kx(t) - K_e x_N = - K \quad K_e \begin{bmatrix} x \\ x_N \end{bmatrix} \tag{3.10}$$

The final derivation for the system matrix with the integral control is as follows:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_w(t) \end{bmatrix} = \begin{bmatrix} A - BK & \text{-}BK_e \\ \text{-C} & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_w(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t)$$

(3.11)

$$y(t) = [C \quad 0] \begin{bmatrix} x \\ x_N \end{bmatrix}$$

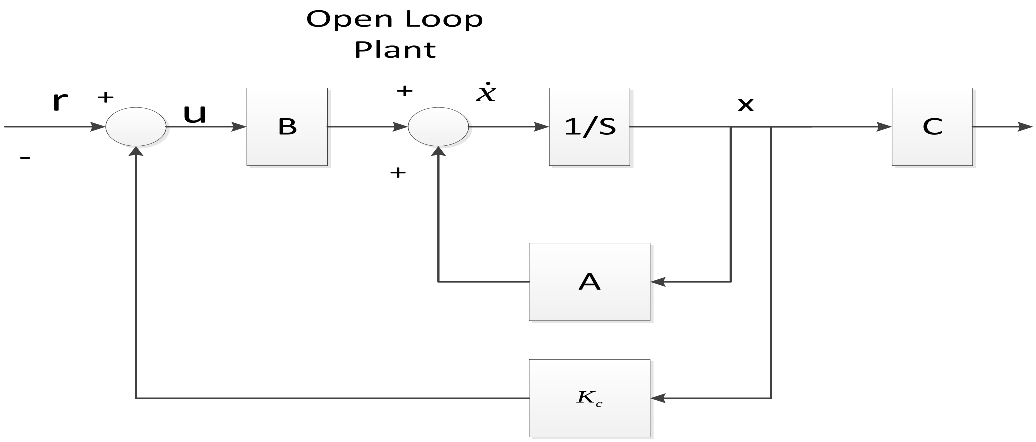

Figure 3.1 The block diagram of full state feedback controller

3.4. Pole placement regulator design for a system with noise [33]:

In previous section, it is found that the design ignore noise while designing full state feedback controller. The design that ignore noise in a plant are likely to fail when implemented an actual conditions when noise exists. Noise can be divided into two categories: measurement noise or the noise caused due to ignored dynamics when modelling a plant. The state equation of a plant with noise vector $x_n(t)$ is the following:

$$\dot{x}(t) = Ax(t) + Bu(t) + Fx_n(t) \tag{3.12}$$

Where F is the noise coefficient matrix. To place the closed-loop poles at desired location while counteracting the effect of noise, a full state feedback regulator is to be designed based on the following control law:

$$u(t) = -Kx(t) - K_n x_n(t) \tag{3.13}$$

Substituting above equation, in ( ), the following state equation of the closed loop system can be obtained:

$$\dot{x}(t) = (A - BK)x(t) + (F - BK_n)x_n(t) \tag{3.14}$$

The above equation implies that the noise vector $x_n(t)$ acts as an input vector for the closed-loop system, whose dynamics matrix as

$$A_{cL} = (A - BK) \tag{3.15}$$

Here, we don't know the exact process by which the noise $x_n(t)$ is generated. We can determine the how the noise affects the plants by deriving the noise coefficient matrix F. Once we know F reasonably, the regulator gain matrix Kn, can be selected such that that the effect of the noise vector xn(t) on the closed loop system is minimized.

3.5. Design of state feedback control law for one joint robotic arm:

Earlier chapter, the state space and transfer function model has been derived for one joint robotic arm.

The state space model of a single joint (One DOF) robot arm and DC motor is given below.

$$\begin{gathered}\frac{d}{dt}\begin{bmatrix}\theta \\ \dot{\theta} \\ i\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 \\ 0 & \text{-b/J} & \text{K}_t\text{/J} \\ 0 & \text{-K}_t\text{/L} & \text{-R}_a\text{/L}\end{bmatrix}\begin{bmatrix}\theta \\ \dot{\theta} \\ i\end{bmatrix} u \\ y = [1 \quad 0 \quad 0]\begin{bmatrix}\theta \\ \dot{\theta} \\ i\end{bmatrix}\end{gathered} \tag{3.16}$$

To simulate and analyze the open loop arm system, considering that the end effector is a part of robot arm. The total equivalent inertia

$$\begin{gathered} b_{equiv} = b + b_{load}\left(\frac{N_1}{N_2}\right)^2, \\ , \\ J_{equiv} = J_m + J_{Load}\left(\frac{N_1}{N_2}\right)^2 \end{gathered} \tag{3.17}$$

The above state space model can be written as

$$\frac{d}{dt}\begin{bmatrix}\theta \\ \dot{\theta} \\ i\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 \\ 0 & -b_{equiv}/J & K/J_{equiv} \\ 0 & -K/L & -R_a/L\end{bmatrix}\begin{bmatrix}\theta \\ \dot{\theta} \\ i\end{bmatrix}u$$
$$y = [1 \quad 0 \quad 0]\begin{bmatrix}\theta \\ \dot{\theta} \\ i\end{bmatrix} \tag{3.18}$$

The various parameters are given in tabular form as

Table 4.1.: Values for various quantity in motor

Symbol	*Quantity*	Values
J_{eqiv}	Equivalent moment of inertia	3.2284×10^{-6}
b_{eqiv}	Equivalent Viscous friction Coefficient	3.5077×10^{-6}
R_a	Armature resistance	4
L_a	Armature inductance	2.75×10^{-6}
K_t	Motor torque constant	0.0274

The transfer function model of the system is given as

$$\dot{x} = \begin{bmatrix}0 & 1 & 0 \\ 0 & -1.087 & 8447 \\ 0 & -9964 & 1.455\times10^{6}\end{bmatrix}x + \begin{bmatrix}0 \\ 0 \\ 3.636\times10^{5}\end{bmatrix}u$$
$$y = [1 \quad 0 \quad 0]x \tag{3.19}$$

The response of the system without closed loop feedback is given as

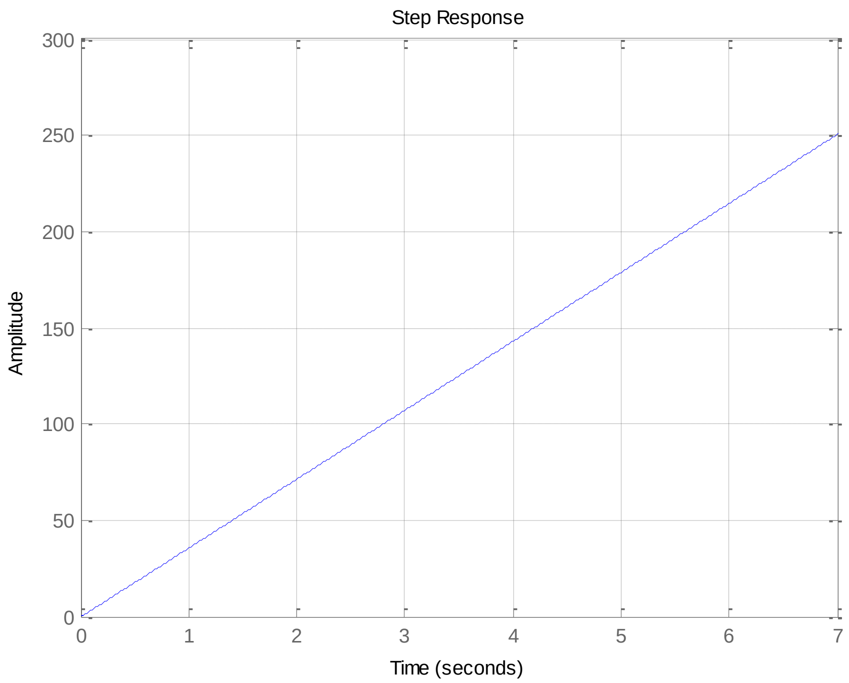

Figure.3.2. Response of the system without feedback

The response of the system with closed-loop feedback when poles are at -10 -10 -10

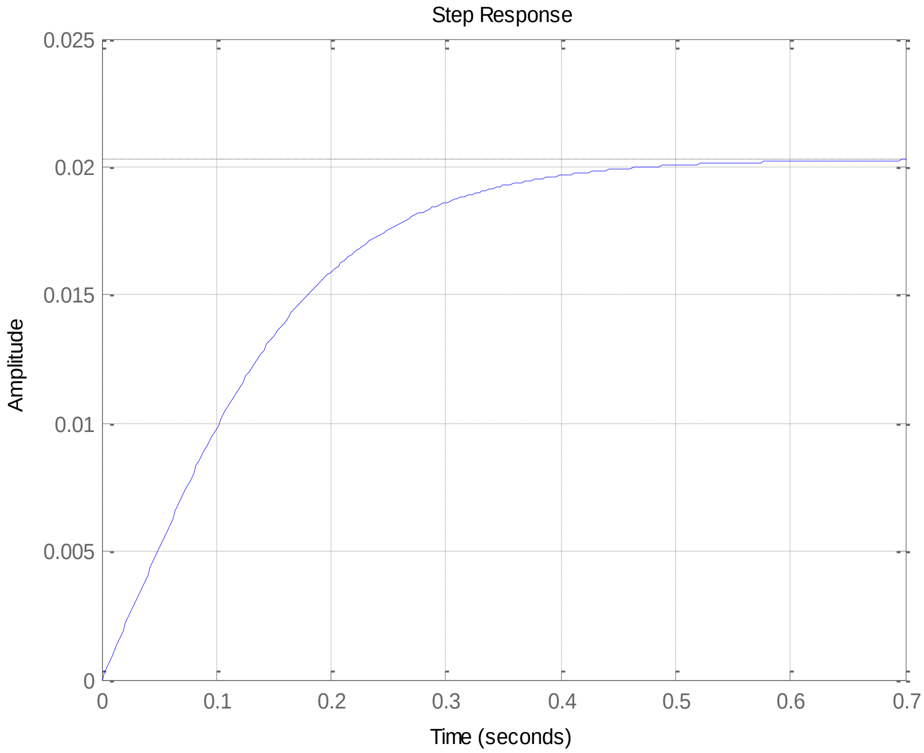

Fig.3.3. Response of the system with feedback

From above figure, it is observed that the system is stabilized but there exist a steady state error. Now in order to reduce the steady state error, we add additional integral controller. Because of integral action, order of integral controller is increased by one. Therefore, system can be represented as

$$\frac{d}{dt}\begin{bmatrix}\theta \\ \dot{\theta} \\ i \\ \omega\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0 \\ 0 & -b_{equiv}/J & K/J_{equiv} & 0 \\ 0 & -K/L & -R_a/L & 0 \\ 1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\theta \\ \dot{\theta} \\ i \\ \omega\end{bmatrix} + \begin{bmatrix}0 \\ 0 \\ 1/L \\ 0\end{bmatrix}u + \begin{bmatrix}0 \\ 0 \\ 0 \\ -1\end{bmatrix}r$$

$$y = [1 \quad 0 \quad 0 \quad 0]\begin{bmatrix}\theta \\ \dot{\theta} \\ i \\ \omega\end{bmatrix} \tag{3.20}$$

The response of the system with integral control can be written as

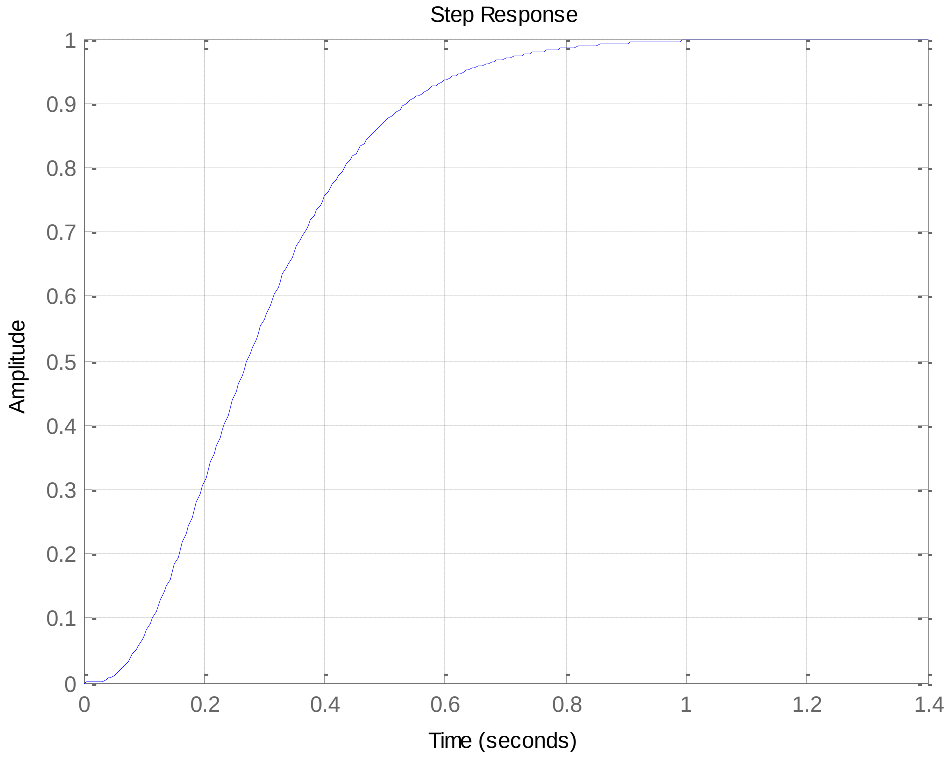

Fig.3.4. Response of the system with integral control.

Now, we consider the effect of noise. Suppose noise coefficient matrix F can be written as

$$[F]=\begin{bmatrix}0.02\\-0.035\\0\end{bmatrix}$$
$$[F]-[B]k_{n1}=\begin{bmatrix}0.02-k_{n1}\\-0.035\\0\end{bmatrix} \tag{3.21}$$

Noise factor will be reduced to zero when $k_{n1}=0.02$. Now we determine effect of noise factor when poles are placed (a) -1 -2 and -3 (b) -10. -20 and -30. And the noise vector is

$$x_n(t)=10^{-5}\sin(100\times t)$$

(a) Poles are at -1 -2 and -3

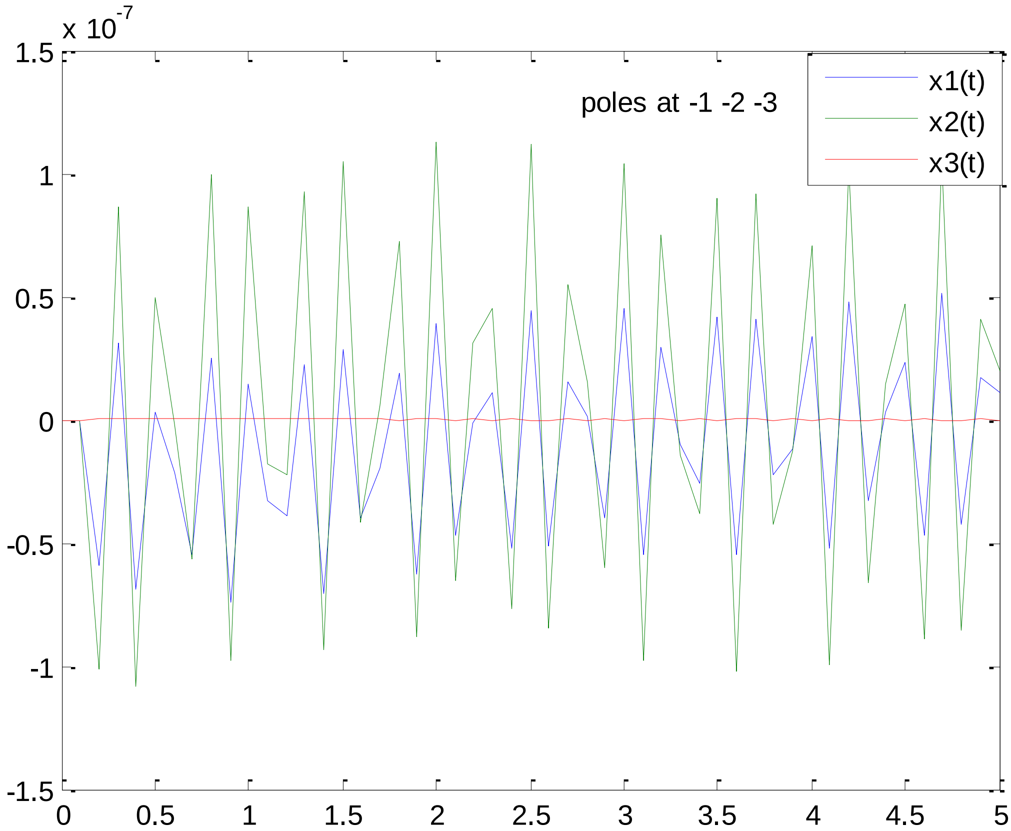

Fig.3.5. Response of the system with poles at -1,-2 and -3

(a) Poles are at -10 -20 -30

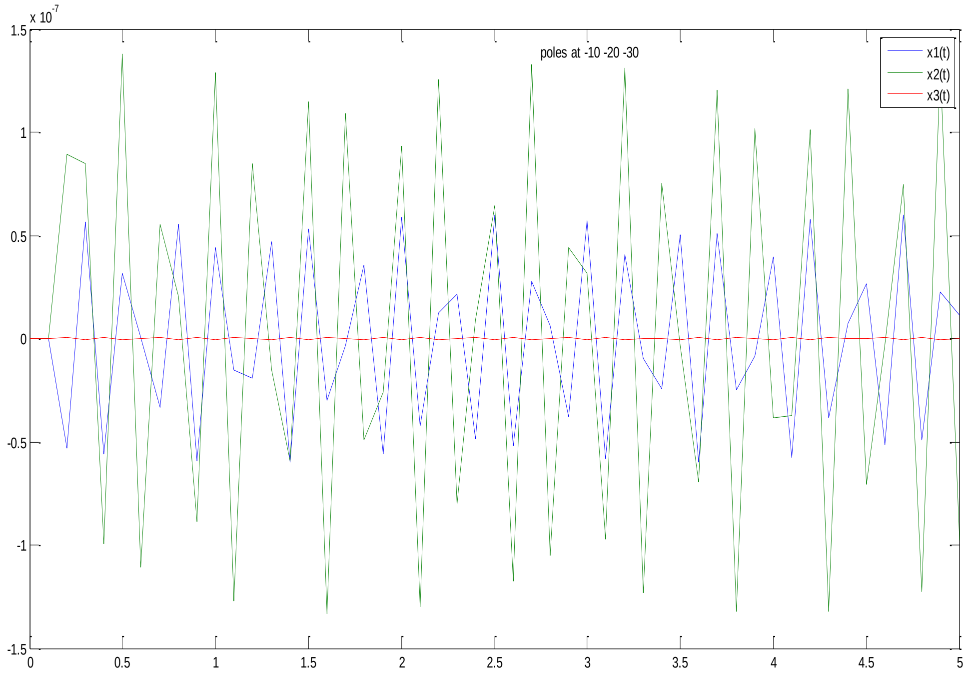

Fig.3.6. Response of the system with poles is at -10,-20 and -30.

CHAPTER 4:

ROBUST CONTROLLER DESIGN FOR ROBOTIC ARM

A robot is a machine designed to execute one or more tasks repeatedly, with speed and precision in any kind of circumstances like parameter changes, uncertainties in the system/surroundings. Basically it has to be versatile for the fulfillment of our desired goals. Hence, there is a need of controller. To meet our desired goals there is a requirement of a robust controller which is defined as “if there is parameter change and any kind of uncertainty in system/surroundings, a controller which can accommodate these kinds of uncertainty and parameter changes is known as a robust controller”.

In designing a control law there is a requirement of good understanding of system model and parameters. Thus, a detailed and correct model of a robot manipulator is needed for this approach as we have seen in the chapter 3. A two-link planar nonlinear robotic system is a well-used robotic system. As mentioned above it is also necessary to consider numerous uncertainties in parameters and modeling.

4.1 Robust control [11]:

The dynamic equation for a manipulator can be written as,

$$\tau_c = M_o(q)\ddot{q}_d + V_o(q,\dot{q})\dot{q} + N_o(q,\dot{q}) \tag{4.1}$$

Generally, in view of possible uncertainties, the terms in the dynamic equation can be decomposed into two parts, one is known parts and another is unknown perturbed part as follows[11]:

$$\begin{aligned} M1 &= M_o + \Delta M1 \\ N1 &= N_o + \Delta N1 \\ V1 &= V_o + \Delta V1 \end{aligned} \tag{4.2}$$

Where M_o, N_o, V_o are the known parts and ΔM, ΔN, ΔV are unknown parts

Torque control law for the system can be shown as:

$$\tau_c = M_o(q)\ddot{q}_d + V_o(q,\dot{q})\dot{q} + N_o(q,\dot{q}) - M_o(q)u \qquad (4..3)$$

Here u has to be designed for the desired disturbance rejection and pole clustering. Since, q_d is the desired trajectory of q so we can define the error as:

$$e = q_d - q \qquad (4.4)$$

Now using the above equations (4.1)-(4.4) we get

$$\begin{aligned}\ddot{e} &= M^{-1}(q)[\Delta M(q)\ddot{q}_d + \Delta V(q,\dot{q})\dot{q} + \Delta N(q,\dot{q}) + M_o(q)u] \\ &= w + E\dot{e} + Fu + u\end{aligned} \qquad (4.5)$$

Where,

$$\begin{aligned}E &= -M^{-1}(q)\Delta V(q,\dot{q}) \\ F &= -M^{-1}(q)\Delta M(q) \\ w &= M^{-1}(q)\Delta N - F\ddot{q}_d - E\dot{q}_d\end{aligned} \qquad (4.6)$$

As we can have the bounded norm,

$$\begin{aligned}\|w\| &< \delta_w \\ \|E\| &< \delta_e \\ \|\tau\| &< \delta_\tau\end{aligned} \qquad (4.7)$$

Then, it leads to the state space equation as:

$$\dot{x} = Ax + Bu + B[0 \quad E]x + BFu + Bw \qquad (4.8)$$

Where,

$$x = \begin{bmatrix} e \\ \dot{e} \end{bmatrix} = \begin{bmatrix} e_1 & e_2 & \dot{e}_1 & \dot{e}_2 \end{bmatrix}'$$
$$A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 \\ I \end{bmatrix} \qquad (4.9)$$

The design objective is to develop a state feedback control law for control 'u'(4.3) as

$$u(t) = -Kx(t) \qquad (4.10)$$

Such that the closed loop system:

$$\dot{x} = (A - BK + B[0 \quad E] - BFK)x + Bw \qquad (4.11)$$

has its poles robustly lie within a vertical strip Ω:

$$\lambda(A_c) \in \Omega = \{s = x + jy | -\alpha_2 < x < -\alpha_1 \le 0\} \qquad (4.12)$$

And degree of disturbance w to the state x as:

$$\|T_{xw}(s)\|_\infty = \|(sI - A_c)^{-1} B\|_\infty \le \delta \qquad (4.13)$$

Where,

$$A_c = (A - BK + B[0 \quad E] - BFK) \qquad (4.14)$$

With the selection of the adjustable scalars ε_1 and ε_2 which are given by

$$\begin{aligned} &(1-\delta_f)/\delta_e > \varepsilon_1 > 0 \\ &(1-\delta_f - \varepsilon_1\delta_e)/\delta > \varepsilon_2 > 0 \end{aligned} \qquad (4.15)$$

There always exists a matrix P>0 satisfying the following Riccati equation:

$$A'_{\alpha 1}P + PA_{\alpha 1} - (1-\delta_f - \varepsilon_1\delta_e - \varepsilon_2/\delta)PBB'P + (\delta_e/\varepsilon_1)I + (1/\varepsilon_2\delta)I + Q = 0 \qquad (4.16)$$

Where,

$$A\alpha_1 = A + \alpha_1 I = \begin{bmatrix} \alpha_1 I_2 & I_2 \\ 0 & \alpha_1 I_2 \end{bmatrix} \tag{4.17}$$

Then, a robust pole-clustering and disturbance rejection control law in(4.3) and (4.10) to (4.13) and (4.14)satisfy the dynamic equation in all admissible perturbations E and F in (4.7) is as[11]:

$$u = -Kx = -rB'Px \tag{4.18}$$

If the gain parameter r satisfies the following two conditions:

$$\begin{aligned} & r \geq 0.5 \\ & 2\alpha_2 P + A'P + PA - (\delta_e / \varepsilon_1)I - [2r(1+\delta_f) + \varepsilon_1\delta_e]PBB'P > 0 \end{aligned} \tag{4.19}$$

Above equations are for α_1 and α_2 are the degree stability and degree decay, respectively.

4.2 Illustration:

Considering only joint link masses for the two-link planar manipulator for simplicity, the system parameters are: link mass m_1 = 2kg, m_2 = 10kg, lengths l_1 = 1m, l_2 = 1m, angular positions q_1, q_2 (rad), applied torques f_1, f_2 (Nm) [11].

Thus, the nominal values of coefficients matrices for the dynamic equation can be found out as[11]:

$$\begin{aligned} M_o(q) &= \begin{bmatrix} 22 + 20\cos q_2 & 10(1+\cos q_2) \\ 10(1+\cos q_2) & 10 \end{bmatrix} \\ V_o(q,\dot{q}) &= -10\sin q_2 \dot{q}_2 \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} \\ N_o(q,\dot{q}) &= g\begin{bmatrix} 12\cos q_1 + 10\cos(q_1+q_2) \\ 10\cos(q_1+q_2) \end{bmatrix} \end{aligned} .$$

The desired value of the trajectory can be given as:

$$q_{d1}(t) = 1.5\cos t + 0.5$$
$$q_{d2}(t) = \cos t + 1$$

Initial states are considered as:

$$q_1(0) = q_2(0) = -2$$
$$\dot{q}_1(0) = \dot{q}_2(0) = 0$$

The state variable is x which is given by:

$$x = [e \quad \dot{e}]'$$

where, $e = q_d - q$

Parametric uncertainties are assumed as:

$$\delta_f = 0.5$$
$$\delta_e = 40$$
$$\delta_N = 10$$

adjustable parameters are calculated and are:

$$\varepsilon_1 = 0.012$$
$$\varepsilon_2 = 0.0015$$

Select Disturbance rejection index δ, the relative stability index α_1, left bound of the vertical strip α_2 as:

$$\delta = 0.1$$
$$\alpha_1 = 0.1$$
$$\alpha_2 = 2000$$

Solving the Riccati equation to get the solution matrix P and the gain matrix with r = 0.2 as:

$$P = \begin{bmatrix} 12693 I_2 & 1584 I_2 \\ 1584 I_2 & 1643 I_2 \end{bmatrix}$$
$$K = rB'P = \begin{bmatrix} 950.1823 I_2 & 985.7863 I_2 \end{bmatrix}$$

Eigenvalues of the closed-loop system matrix A-BK are {-0.9648, -0.9648, -984.8215, -984.8215}

Again using the equation below:

$$\alpha_2 = \alpha_1 + 0.5\bar{\lambda}\{P^{-1/2}[-A'P - PA_{\alpha 1} + (\delta_e / \varepsilon_1)I + (2r(1+\delta_f) + \varepsilon_1\delta_e)PBB'P]P^{-1/2}\}$$

We get $\alpha'_2 = 1873$

The total control law now become[11],

$$\tau_c = M_0\ddot{q}_d + V_0(q,\dot{q})\dot{q} + N_0 - M_0 u$$

$$= \begin{pmatrix} 22+20C_2 & 10(1+C_2) \\ 10(1+C_2) & 10 \end{pmatrix}\begin{pmatrix} -1.5\cos t \\ -\cos t \end{pmatrix} + 10S_2\dot{q}_2\begin{pmatrix} -2 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} \dot{q}_1 \\ \dot{q}_1 \end{pmatrix} + g\begin{pmatrix} 12C_1 + 10C_{12} \\ 10C_{12} \end{pmatrix}$$

$$+ \begin{pmatrix} 22+20C_2 & 10(1+C_2) \\ 10(1+C_2) & 10 \end{pmatrix}(950.18 I_1 \quad 985.7863 I_2)\begin{pmatrix} e \\ \dot{e} \end{pmatrix}$$

For 40% disturbance in mass ie, $\Delta M(q) = .4M_0(q)$,20C% in V, $\Delta V(q,\dot{q}) = .2V_0(q,\dot{q})$ and 20% in N , $\Delta N(q) = .2N_0(q)$ the dynamic equation is simulated.

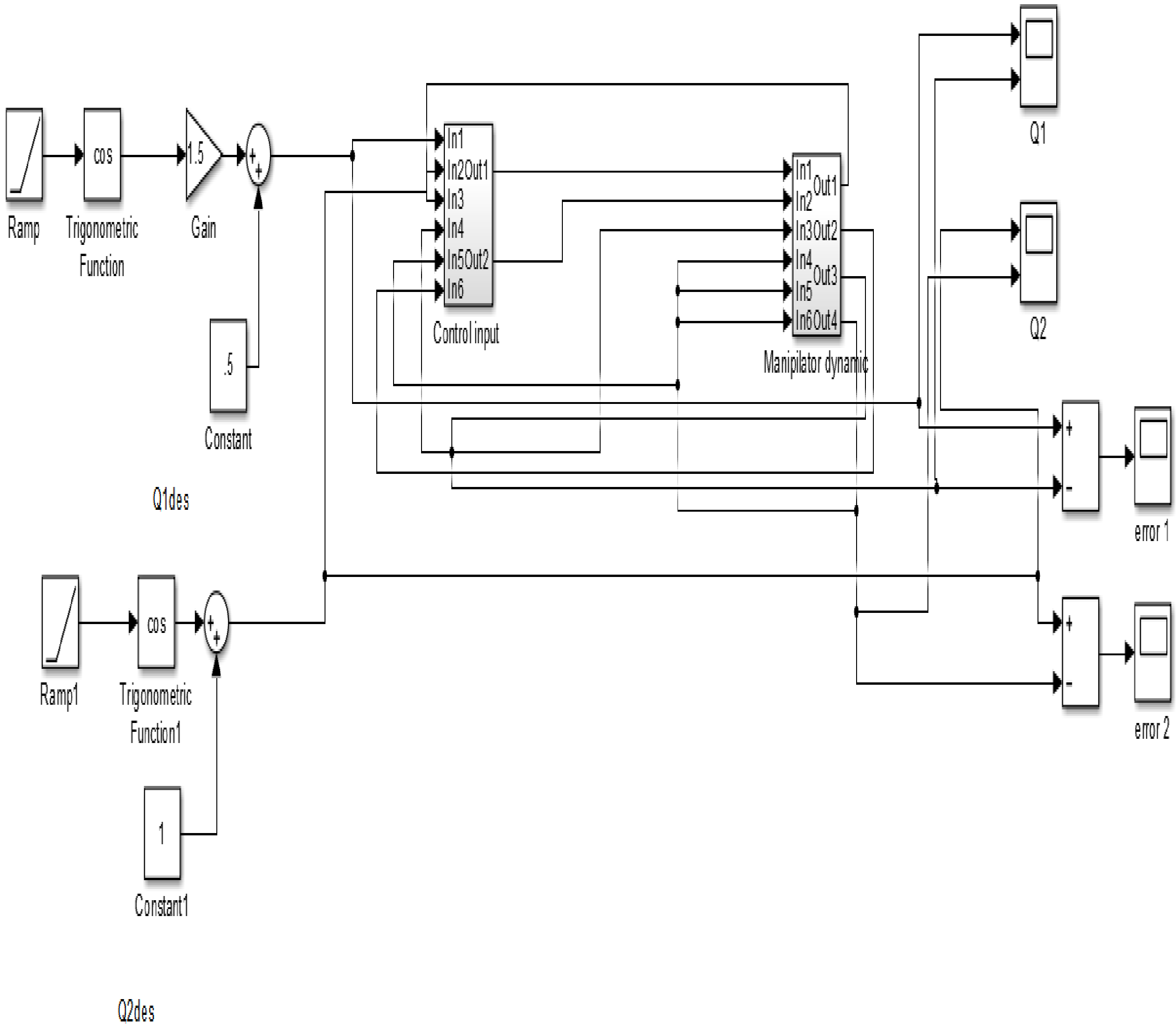

Fig.4.1 Simulink model of the robust controller and manipulator.

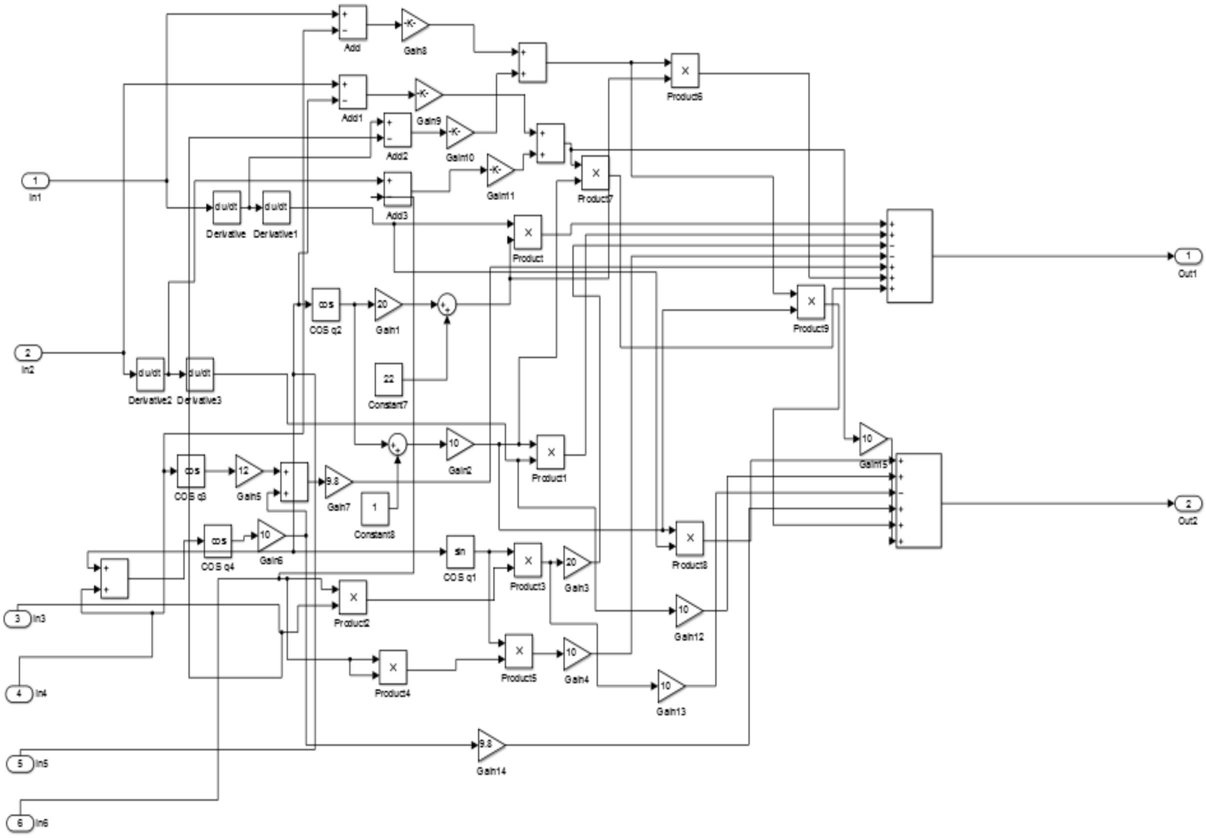

Fig 4.2 Inside block of robust controller

The output of this controller is fed to the dynamic model of the two joint manipulator.

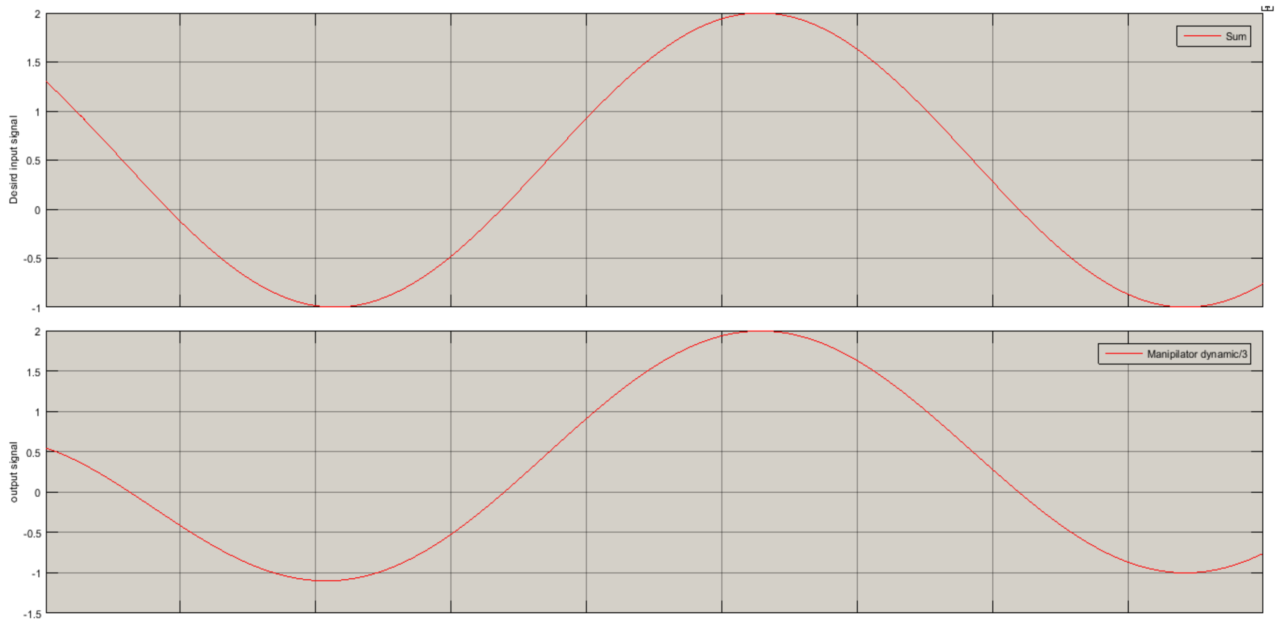

Fig 4.3 q1 and q_desired signal

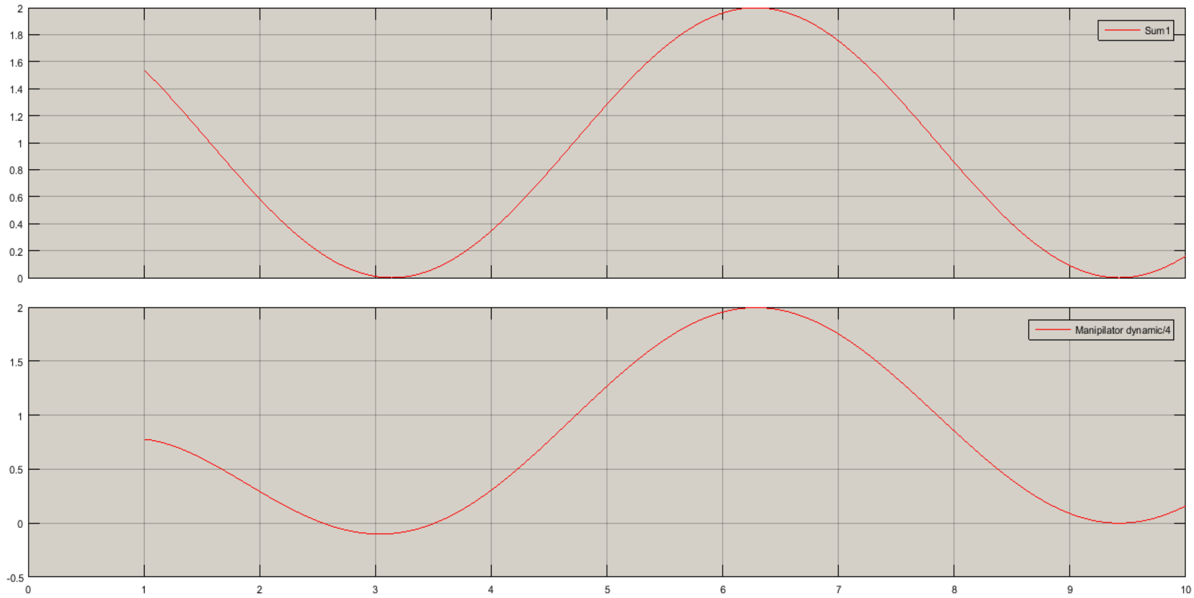

Fig4.4. q2 and q2_desired signal

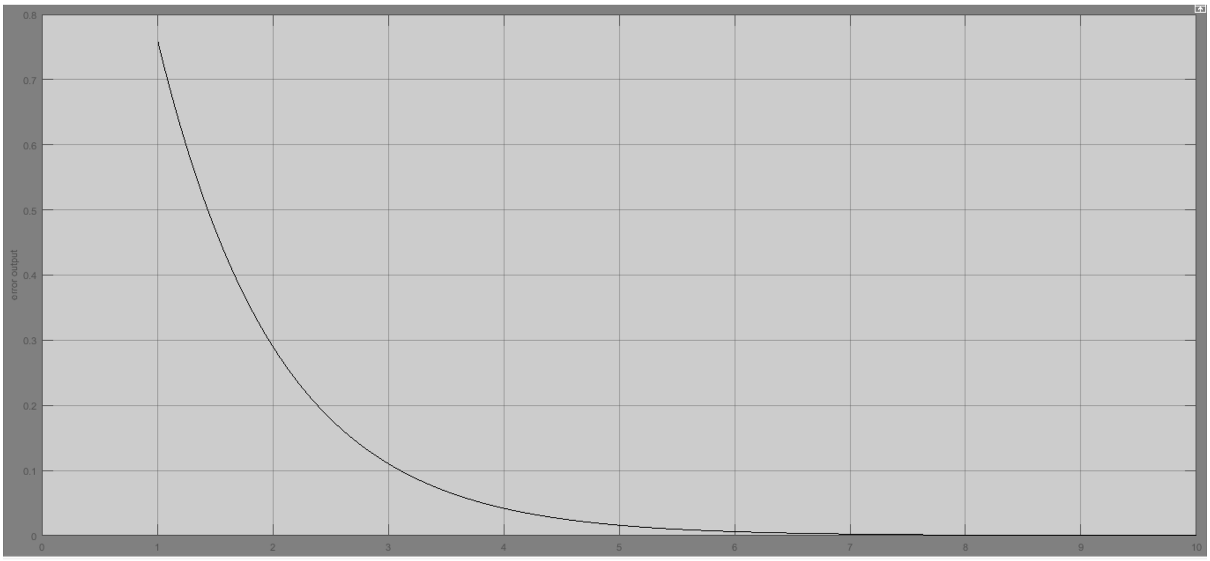

Fig4.5 Error signal of q1.

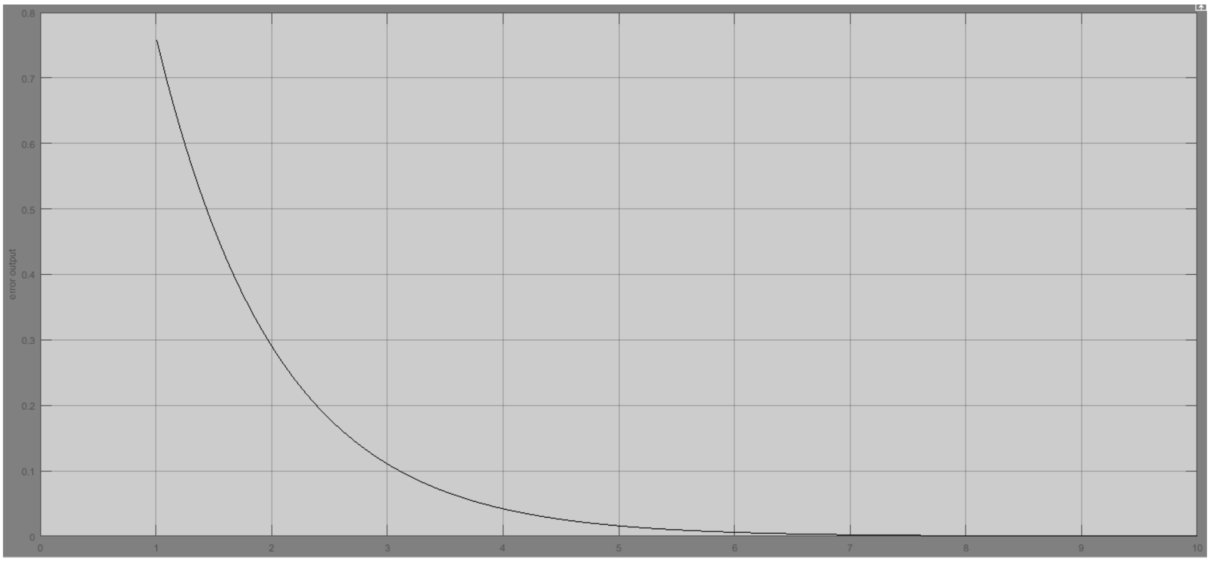

Fig 4.6 Error signal of q2

Chapter 5

CONCLUSION AND FUTURE SCOPE

Through this work the effect of pole placement in the control of a manipulator has been observed. And it has shown that when the distance of poles from the imaginary axis changes that changes will reflect in the performance of the controller. Through robust controller the system is capable of handling the uncertainties or disturbances in the parameters with in suitable bounds. These bounds can be set through the maximum disturbance that can occur in the system. And by suitably selecting the location of poles in the s plane the controller can be designed based on that. Here the width of pole clustering band selected may not be optimal for the robustness. This will further reduces the robustness. So in order to select the optimal strip for the pole clustering some optimisation technique should be developed in the future works.

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