GENERALIZED THREE PHASE UNBALANCED LOAD FLOW WITH RENEAWALE GENERATIONS

A DISSERTSATION REPORT

Submitted in partial fulfillment of the requirements for the award of the degree

Of

MASTER OF TECHNOLOGY

In

ELECTRICAL ENGINEERING

With Specialization In

POWER SYSTEMS

Submitted by

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MAY, 2016

CANDIDATE'S DECLARATION

I hereby declare that this report which is being presented in the seminar entitled **GENERALIZED THREE PHASE UNBALANCED LOAD FLOW WITH RENEWABLE GENERATIONS** in partial fulfillment of the requirement of award of Degree of Masters of Technology in Electrical Engineering with specialization in **Power System Engineering** submitted to the Department of Electrical Engineering, Indian Institute of Technology, Roorkee, India is an authentic record of this report carried out during a period from June 2015 to May 2016 under the supervision of **Dr. N. P. Padhy,** Professor, Department of Electrical Engineering, Indian Institute of Technology, Roorkee.

The matter presented in this seminar has not been submitted by me for the award of any other degree of this institute or any other institute.

Date:

Place: Roorkee **(Udit Kumar Singh)**

CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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Acknowledgement

This work was carried out at the Department of Electrical Engineering, Indian Institute of Technology, Roorkee, under the supervision of Dr. N. P. Padhy.

I would like to express our sincerest gratitude to Prof. N.P. Padhy for his guidance and support throughout my work. Without him I will never be able to complete my work with this ease. He was very patient to hear my problems that I was facing during the project work and finding the solutions. I am very much thankful to him for giving his valuable time for me.

My special thanks to all my friends who did give their time for MATLAB Coding and providing me good company. And I would like to thank my friends who helped me in completing my report in time.

I thank God for giving me the chance to pursue my academic goals and a wonderful, complete life full of amazing people.

Finally, hearty thanks to my family member for their sacrifice that encouraged me to proceed with the work.

Udit Kumar Singh

Abstract

The technique for the power flow solution of the distribution system with renewable generation is presented in this paper work. The load flow problem is solved by the methods one is direct approach for distribution system and the second is the fortescue equivalent admittance matrix approach for distribution load flow. The first load flow approach is based on building two matrices in taking into account the different characteristics of the distribution system. The two developed matrices are BIBC and BCBV matrix. The matrix giving the relation between bus current injections to the branch currents is known as BIBC matrix and the matrix giving the relation between branch current to the bus voltage is known as BCBV matrix. The above two matrices provides the direct approach solution for load flow when combined together.

In the second method the phase admittance matrix is converted into fortescue coordinate. Then injected current is calculated from the given power. From this current the voltages are calculated. The power is calculated from this voltage and current. Error in the power is checked if it is in within the limit then the load flow is completed. The two different renewable generation models discussed briefly. The renewable generation is accommodated in the load flow with simply by considering PQ model.

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Chapter 1

Introduction

As the techniques of the electricity generation are becoming more and more advance the renewable power sources becoming the major shareholder in the power generation. The renewable energy sources have all most negligible effect on the environment. Many more advantages can be achieved by this renewable energy sources such as reliability of the system will improve, voltage of the system will improve etc. The electrical power system has two major parts one is transmission and the other is distribution. The first one is a balanced system where as the second one is unbalanced.

The steady state behavior of any system is determined by the fundamental calculation of the system. This calculation is known as “steady state power flow” also “load flow”. This gives the voltage and angle at each node with the information of generation and loading at some nodes. The load flow techniques are widely used in industry. The ‘Newton Raphson’

method is widely used in the transmission system. The transmission system has the mesh type structure. It has parallel paths and many paths from the generating nodes to the loading nodes. From the computation point the Newton Raphson method is very complex in case of large networks due to the size of the Jacobian matrix. In case of transmission system the approximation can be made to decouple the real and reactive power the magnitude of the voltage and the angle. By the approximations the Jacobian can be made to a constant matrix which results in the method named fast decoupled Newton Raphson method. It is considered as a great improvement in comparison to Newton Raphson method in lot of the cases. This report focuses on distribution power flow solution. The origin of a distribution system is as substation at which the conversion of electric power takes place from transmission voltage level to the costumer usable voltage level. The structure of the distribution system is typically radial one. The fast decoupled Newton Raphson method is not suitable for most of the distribution systems because of its radial nature and higher (R/X) ratios. As the power flow solution is a very fundamental calculation it finds application in many areas like power system planning and operation, power system optimization, distribution automation. The distribution automation needs hundreds of power flow solution to meet the requirement of capacitor placement at desired nodes, service restoration and the reconfiguration of network. Modeling of the system should be done in such a manner such that it reflects the actual behavior of the component and the algorithm should be efficient and robust.

We further can say that the power flow solution of the power system is the best and elementary tool to analyse the power system. All the important actions of the power system such as switching, state estimation, Var planning and Network optimization depends upon the load flow analysis to meet the future demand. Due to rising new technologies and advantages of digital devices great revolution in this field is going on. Nowadays lot of techniques are there to study the power flow solution such as Gauss Siedel method, Fast decoupled method and Newton Raphson method but these methods are suitable for transmission system. These methods are not suitable for distribution system because the distribution system has some characteristics different from the transmission system. Some of them are given as:-

- The structure is weakly meshed and radial;
- R/X ratio is high ;
- The operation is unbalanced with multiphase;
- Distributed and unbalanced load;
- The nodes and branches are very large;
- The values of reactance and resistance are high.

To take care of the above problem we need a robust and efficient load flow technique. The power flow solution methods which are used in the transmission system are not able to meet the criteria of robustness and performance in case of distribution system because of the previously described characteristics of the distribution system. The considered assumptions to simplify in fast decoupled load flow [1] are not useful in the distribution system. So the unique power flow solution is needed. Any good distribution load flow must take care of above described characteristics. The literature [2] suggests number of load flow algorithms designed especially for distribution system. These derived methods from transmission systems are based on mesh type structure. When we talk about the distribution system even Gauss implicit Z matrix method is not able to fully utilize the weakly meshed and radial nature of distribution system. Hence a solution is required whose size is proportional to the number of buses. Other load flow algorithms are also anticipated like Newton Raphson, fast decoupled load flow and phase decoupled method [3].

The work presented in this paper uses the previously mentioned characteristics of the distribution system and forms the two matrices and the approach is known as 'Direct approach'.

Some new ideas are presented in the recent research. Some data manipulation or special data format are needed to calculate the special characteristics of the distribution system. A compensation based technique is mentioned in [6] is used to solve distribution power flow problem. In the advanced version the use of branch power flow rather than branch currents presented

The two methods are discussed in this report one is "Direct Approach for the Distribution Load Flow" and the other is "Fortescue Pi Equivalent Admittance Matrix Approach to Power Flow solution." The first one is based on two matrices BIBC and BCBV. These two matrices provide the direct approach solution for load flow when combined together. In the presented method the time taking procedures as Jacobian matrix, admittance matrix, forward/ backward substitution and LU factorization are avoided. In the second method is based on the fortescue equivalent matrix development from the phase admittance matrix.

Chapter 2

Unbalanced Three-Phase Line

The power flow solution/Load flow study of any radial unbalanced distribution power system is very important because of unbalanced 3phase, 2 phase and 1 phase distributed lines. The effect of mutual impedances also considered in the calculation of the voltage magnitude and angle of the voltage at each node. These systems are also affected by the nearby objects such as telephonic line which creates interference with these lines and result in unwanted noise problem in the both of the system which needs extra care to deal with it. To analyze mathematically a three phase unbalanced distribution system model is considered. In the below shown fig 2.1 a section of three phase line is displayed and the parameters can be derived by the Carson and Lewis [2] technique. The self and mutual impedances of a four wire line section having three phase and one neutral can be given by a 4x4 matrix.

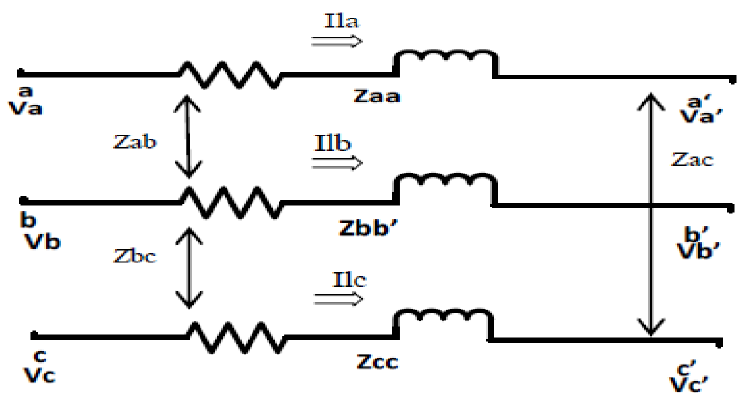


Fig.2.1. Typical three phase feeder

Above descried matrix of order 4, 4 is given below. It considers the self-impedance term and mutual impedance term and the effect of ground wire of the distribution system.

$$[Z_{abcn}] = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{an} \\ Z_{ba} & Z_{bb} & Z_{bc} & Z_{bn} \\ Z_{ca} & Z_{cb} & Z_{cc} & Z_{cn} \\ Z_{na} & Z_{nb} & Z_{nc} & Z_{nn} \end{bmatrix} \tag{2.1}$$

The above equation can be made of order 3, 3 matrix on application of Kron's reduction technique by considering the effect of ground wire. It is expressed as:

$$[Z_{abcn}] = \begin{bmatrix} Z_{aa-n} & Z_{ab-n} & Z_{ac-n} \\ Z_{ba-n} & Z_{bb-n} & Z_{bc-n} \\ Z_{ca-n} & Z_{cb-n} & Z_{cc-n} \end{bmatrix} \tag{2.2}$$

From the above figure the voltages at sending end of the feeder can be related to the receiving end voltages of the feeder by the given below relationship. The phases which are absent will be treated by placing zero for their respective entries in the matrix equation.

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} - \begin{bmatrix} Z_{aa-n} & Z_{ab-n} & Z_{ac-n} \\ Z_{ba-n} & Z_{bb-n} & Z_{bc-n} \\ Z_{ca-n} & Z_{cb-n} & Z_{cc-n} \end{bmatrix} \begin{bmatrix} I_{Aa} \\ I_{Bb} \\ I_{Cc} \end{bmatrix} \tag{2.3}$$

For example let say there are two phases only. These may ab,bc and ca. In this situation the raw and column for that phase will carry zero entries at their respective places. The relationship equation for two phases is represented as given below.

$$\begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} V_A \\ V_B \end{bmatrix} - \begin{bmatrix} Z_{aa-n} & Z_{ab-n} & Z_{ac-n} \\ Z_{ba-n} & Z_{bb-n} & Z_{bc-n} \end{bmatrix} \begin{bmatrix} I_{Aa} \\ I_{Bb} \end{bmatrix} \tag{2.4a}$$

$$\begin{bmatrix} V_b \\ V_c \end{bmatrix} = \begin{bmatrix} V_B \\ V_C \end{bmatrix} - \begin{bmatrix} Z_{ba-n} & Z_{bb-n} & Z_{bc-n} \\ Z_{ca-n} & Z_{cb-n} & Z_{cc-n} \end{bmatrix} \begin{bmatrix} I_{Bb} \\ I_{Cc} \end{bmatrix} \tag{2.4b}$$

$$\begin{bmatrix} V_a \\ V_c \end{bmatrix} = \begin{bmatrix} V_A \\ V_C \end{bmatrix} - \begin{bmatrix} Z_{aa-n} & Z_{ab-n} & Z_{ac-n} \\ Z_{ca-n} & Z_{cb-n} & Z_{cc-n} \end{bmatrix} \begin{bmatrix} I_{Aa} \\ I_{Cc} \end{bmatrix} \tag{2.4c}$$

For consideration let say there is single phases only. These may a,b and c. In this situation the raw and column for that phase which are absent will carry zero entries at their respective places. The relationship equation for single phase is represented as given below.

$$V_{ia} = V_{ja} + Z_{aa-n} \times I_{ij-a} \tag{2.5a}$$

$$V_{ib} = V_{jb} + Z_{bb-n} \times I_{ij-b} \tag{2.5b}$$

$$V_{ic} = V_{jc} + Z_{cc-n} \times I_{ij-c} \tag{2.5c}$$

Chapter 3

Direct Approach for Distribution Load Flow

The two matrices BIBC and BCBV are the backbone of this method. BIBC is the abbreviation of bus injection to branch current and the BCBV is the abbreviation of branch current to bus voltage. This method operates on these two matrices and current injection. In this section, the development procedure will be described in detail. When we deal with distribution network the current injection methods are widely used and these are more practical. The power taken by any load at bus i in the complex form can be expressed as.

$$S_i = (P_i + jQi) \quad \text{i} = 1......N \tag{3.1}$$

The solution for the injected current at bus i for k-th iteration is expressed as follow:

$$I_i^k = I_i^r(V_i^k) + jI_i^i(V_i^k) = \left(\frac{P_i + jQ_i}{V_i^k}\right)^* \tag{3.2}$$

In the above equation V_i^k represents the voltage at bus i for k-th itteration and I_i^k stands for the injected current at bus i at k-th itteration. The real part of the injected current at bus i at k-th iteration is given by $I_i^r$and the imaginary part of the injected current at bus i at k-th iteration is given by I_i^i.

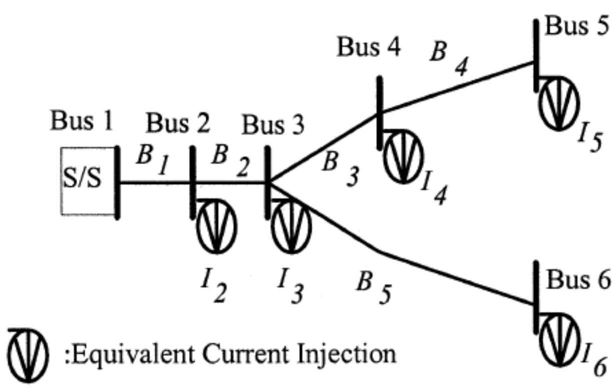


Fig.3.1. Simple distribution system

3.1 Relationship Matrix Development

The figure 3.1 is taken into consideration for the purpose of the developments of the various relationship matrices. By using the equation (3.2) the conversion from complex power injected at a particular bus to the current injected at particular bus can be completed. The relationship matrix can be developed from the above procedure. The matrices relating the bus current to the branch current is derived on application of Kirchhoff's Current Law (KCL) in the distribution system.

The relation between current in the branch and currents injected into the bus can be expressed as given below. Let's consider the currents in the branches B_1,B_3 and B_5. These are expressed in terms of currents in the branches as follows.

$$\begin{aligned} B_1 &= I_2 + I_3 + I_4 + I_5 + I_6 \\ B_3 &= I_4 + I_5 \\ B_5 &= I_6 \end{aligned} \tag{3.1.1}$$

The above equations can be represented in the matrix form which is the purpose. The calculations will take place in the matrix form.

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} \tag{3.1.2}$$

The generalization of the above equation (3.1.2) is as follows

$$[B] = [BIBC][I] \tag{3.1.3}$$

'BIBC' is the current injected into bus to current in the branch matrix. This BIBC matrix is a upper triangular matrix with all the rows and column 0 and +1 entries. In the similar manner the branch current and bus voltage can be developed. From figure 3.1 the equations for node 2, 3 and 4 can be written as

$$\begin{aligned} V_2 &= V_1 - B_1 Z_{12} \\ V_3 &= V_2 - B_2 Z_{23} \\ V_4 &= V_3 - B_3 Z_{34} \end{aligned} \tag{3.1.4}$$

Where

V_i - voltage of bus i

Z_{ij} -impedance of the branch connecting node i and j

From the equation (3.1.4) all the voltages can be expressed in terms of V_1

$$V_4 = V_1 - B_1 Z_{12} - B_2 Z_{23} - B_3 Z_{34} \tag{3.1.5}$$

From the equation (3.1.5) we can observe that voltage of each node can be represented in terms of the branch impedance and voltage of the node 1. The other node voltage can be obtained by above discussed equation. The above equation can be represented in the matrix form by considering all the nodes.

$$\begin{bmatrix} V_1 \\ V_1 \\ V_1 \\ V_1 \\ V_1 \end{bmatrix} - \begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} Z_{12} & 0 & 0 & 0 & 0 \\ Z_{12} & Z_{23} & 0 & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & Z_{45} & 0 \\ Z_{12} & Z_{23} & 0 & 0 & Z_{36} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} \tag{3.1.6a}$$

The generalisation of the equation (3.1.6) is as follows:

$$[\Delta V] = [BCBV][B] \tag{3.1.6b}$$

The ‘BCBV’ represents its usual meaning.

3.2 Development of Algorithm

From equation (3.1.6) the algorithm for BIBC matrix is formed as given below:

a) The distribution system having m branches and n nodes the BIBC matrix has mx(n-1) dimensions

b) If a line section B_k is located between bus i and bus j, copy the column of the i -th bus of the BIBC matrix to the column of the j -th bus and fill a +1to the position of the k-th row and the j -th bus column.

c) Repeat procedure (b) until all line sections are included in the BIBC matrix.

From (3.1.5) a building algorithm for BCBV matrix can be developed as follows.

d) For a distribution system with m -branch section and n-bus, the dimension of the BCBV matrix is (n-1) x m.

e) If a line section B_k is located between bus I and bus j copy the row of the i-th bus of the BCBV matrix to the row of the j-th bus and fill the line impedance Z_{ij} to the position of the j-th bus row and the k-th column.

f) Repeat procedure (e) until all line sections are included in the BCBV matrix.

The above presented algorithm can be easily to the line section or nodes having more than single phases. Let's consider an example of line section between bus 'i' and bus 'j' having three phases. The 3x1 vector will be formed corresponding the B_i branch current and +1 in the BIBC matrix. A 3x3 matrix (identity) will be formed. In the same manner the three phase line between bus i and bus j of the distribution feeder, the BCBV will have an impedance matrix of order 3x3 of Z_{ij} matrix.

We can observe that algorithm for 'BIBC' and 'BCBV' matrices are same. The above two matrices can be in accommodated in the same program because these two matrices are very similar. The space occupied for computation purpose is reduced. The time taken to the data preparation is saved because the algorithm is developed on the basis of classical 'bus branch oriented' data base. The above presented method can easily be incorporated in the previously existent distribution automation system.

3.3 Solution Technique Development

During the development of the 'BIBC' and 'BCBV' matrices the different characteristics of distribution system structure is taken care. The load flow problem is solved by building two matrices in taking into account the different characteristics of the distribution system. The two developed matrices are BIBC and BCBV matrix. The matrix giving the relation between bus current injections to the branch currents is known as BIBC matrix and the matrix giving the relation between branch current to the bus voltage is known as BCBV matrix. The above two matrices provides the direct approach solution for load flow when combined together. Combining (3.1.5) and (3.1.6), the relationship between bus current injections and bus voltages can be expressed as

$$\begin{aligned} [\Delta V] &= [BCBV][BIBC][I] \\ [\Delta V] &= [DLF][I] \end{aligned} \tag{3.3.1}$$

And the solution for distribution load flow can be obtained by solving (3.3.2) iteratively

$$I_i^k = I_i^r(V_i^k) + jI_i^i(V_i^k) = \left(\frac{P_i + jQ_i}{V_i^k} \right)^* \tag{3.3.2a}$$

$$\left[\Delta V^{k+1}\right]=\left[DLF\right]\left[I^{k}\right] \quad (3.3.2b)$$

$$\left[V^{k+1}\right]=\left[V^{0}\right]+\left[\Delta V^{k+1}\right] \quad (3.3.2c)$$

Number of arithmetic operation is almost proportional to N^3 in the factorization of LU. It will take major portion of computational time for factorization of LU for large value of N number of nodes. If we can avoid the LU factorization the major portion of computational time can be saved. The above two matrices provides the direct approach solution for load flow when combined together. In the presented method the time taking procedures as Jacobian matrix, admittance matrix, forward/ backward substitution and LU factorization are avoided. From the solution techniques described before, the LU decomposition and forward/backward substitution of the Jacobian matrix or the Y admittance matrix are no longer necessary for the proposed method.

3.4. Considering Loops in the Network

This method can deal with weakly messed network means little number of loops in the system. The distribution network with one loop is shown in the given below figure. These loops may be because of tie switches or some other means. The modification in the various matrices is discussed in the preceding sub sections.

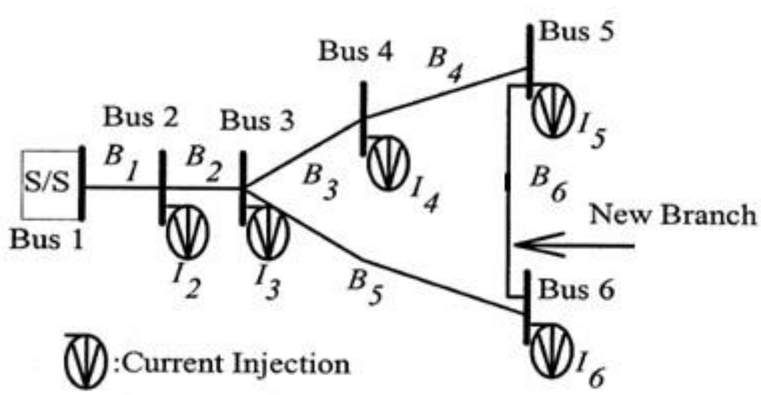


Fig 3.2 Simple distribution system with one loop.

3.4.1 Modification for BIBC Matrix

The branch forming loop is connected between the nodes 5 and 6. The current injected into the bus will not be influenced by the presence of the loop. So for the BIBC matrix we have to take care of the new branch. On the consideration of the new line the equation for the modified current will be given by:

$$\begin{aligned} I_5' &= I_5 + B_6 \\ I_6' &= I_6 - B_6 \end{aligned} \tag{3.4.1.1}$$

The BIBC matrix will be given as:

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 + B_6 \\ I_6 - B_6 \end{bmatrix} \tag{3.4.1.2}$$

Equation (3.4.1.2) can be rearranged as

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} B_6 \\ -B_6 \end{bmatrix} \tag{3.4.1.3}$$

And the modified BIBC matrix can be obtained as

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ B_6 \end{bmatrix} \tag{3.4.1.4}$$

Generalization of the above matrix results in the given below matrix

$$\begin{bmatrix} B \\ B_{new} \end{bmatrix} = \left[BIBC \right] \begin{bmatrix} I \\ B_{new} \end{bmatrix} \tag{3.4.1.5}$$

The modification in the steps to build up the algorithm is given below:

The step b is modified to B as follows

B)—Let the connection between bode i and node j is provided by the line B_k then the modification is to copy the bits of the i th node column to the k-th node column and multiply with minus one to the bits of j-th column and fill the +1in place of j-th row and k-th column.

3.4.2 Modification for BCBV Matrix

Apply KVL in the newly formed loop shown in the figure 3.4. The KVL equation can be written in terms of branch current and branch impedance.

$$Z_{34}B_3 + Z_{45}B_4 + Z_{56}B_6 - Z_{36}B_5 = 0 \tag{4.2.1}$$

From the equation (4.2.1) and (3.1.6a), the new BCBV matrix is

$$\begin{bmatrix} V_1 \\ V_1 \\ V_1 \\ V_1 \\ V_1 \\ 0 \end{bmatrix} - \begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{12} & 0 & 0 & 0 & 0 & 0 \\ Z_{12} & Z_{23} & 0 & 0 & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & 0 & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & Z_{45} & 0 & 0 \\ Z_{12} & Z_{23} & 0 & 0 & Z_{36} & 0 \\ 0 & 0 & Z_{34} & Z_{45} & -Z_{36} & Z_{56} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \end{bmatrix} \tag{4.2.2}$$

Generalized modified equation in the matrix form can be written as

$$\begin{bmatrix} \Delta V \\ 0 \end{bmatrix} = \left[BIBC \right] \begin{bmatrix} B \\ B_{new} \end{bmatrix} \tag{4.2.3}$$

The modification in the development of the algorithm step e for the buildup of BCBV matrix is given as follows:

E1. Let B_k is the line connecting the two buses results in the formation of the messed network. The BCBV matrix can be modified to the desired matrix by adding a new row in the BCBV matrix without loop connection. The generalized equation by applying KVL can be written as:

$$\sum_{t=1}^{m} Z_t B_t \tag{4.2.4}$$

In the above equation n_l represents the lines in the loop, Z_l is the impedance of the line and B_l is the current flowing through that line.

3.4.3 Modification for Solution Techniques

From the equation (3.4.1.4), (4.2.2) and (3.3.1), equation (3.3.1) can be rearranged as given below as

$$\begin{bmatrix} \Delta V \\ 0 \end{bmatrix} = \left[BCBV\right]\left[BIBC\right]\begin{bmatrix} I \\ B_{new} \end{bmatrix}$$
$$\begin{bmatrix} \Delta V \\ 0 \end{bmatrix} = \begin{bmatrix} A & M^T \\ M & N \end{bmatrix}\begin{bmatrix} I \\ B_{new} \end{bmatrix} \tag{4.2.5}$$

The final developed algorithm for the distribution network system having little number of loops by applying Kron's reduction to equation (4.2.5) can be written as:

$$\Delta V = [A - M^T N^{-1} M][I]$$
$$\Delta V = [DLF][I] \tag{4.2.6}$$

From the above discussion it is clear that the this method can be used for the distribution network systems which are weakly messed structure because the changes in the different matrices such as BIBC,BCBV and DLF are required , other procedure is the same.

Chapter 4

Fortescue Equivalent Admittance Matrix Approach

4.1 Theory of Fortescue Transformation

The equation presented below is proved by C.L. Fortescue in1918 in a seminal paper. According to this equation any number of unbalanced phases can be represented or transformed into n-1 balanced phase of different sequences and one zero sequence system by the given equation. This is called Fortescue transformation.

$$\begin{bmatrix} V_a^0 \\ V_a^1 \\ V_a^2 \\ V_a^n \end{bmatrix} = \frac{1}{n} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & a & a^2 & a^{n-1} \\ 1 & a^2 & a^4 & a^{2(n-1)} \\ 1 & a^{n-1} & a^{2(n-1)} & a^{(n-1)(n-1)} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_n \end{bmatrix} \qquad (4.1)$$

$V_a^0 ....V_a^{n-1}$ Voltages in fortescue domain;

$V_a ,.......V_n$ Voltages in phase domain;

Because the transformation is non-singular it is proved that by using inverse n x n transformation the phase domain quantities can be recovered from the Fortescue domain quantities. We deal with up to three phases so the transformation for n=3,n=2,n=1 will be discussed briefly.

Three phase system to Fortescue coordinate transformation is done by $T_{ph_3}^{F_3}$ using $a_3 = e^{j\frac{2\Pi}{3}}$ The quantities voltage and current can be recovered from fortescue domain to phase domain by using inverse transformation $T_{F_3}^{ph_3}$

$T_{F_3}^{ph_3}$ and $T_{ph_3}^{F_3}$ are given as follows

$$T_{ph_3}^{F_3} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a_3 & a_3^2 \\ 1 & a_3^2 & a_3 \end{bmatrix}, T_{F_3}^{ph_3} = \left(T_{ph_3}^{F_3}\right)^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a_3^2 & a_3 \\ 1 & a_3 & a_3^2 \end{bmatrix} \qquad (4.2)$$

Three phase electric current through an element is expressed as

$$I_{ph_3} = Y_{ph_3} V_{ph_3} \tag{4.3}$$

Transformation of voltage and current into fortescue coordinate system can be written as

$$\begin{aligned} T_{F_3}^{ph_3} I_{F_3} &= Y_{ph_3} T_{F_3}^{ph_3} V_{F_3} \\ I_{F_3} &= (T_{F_3}^{ph_3})^{-1} Y_{ph_3} T_{F_3}^{ph_3} V_{F_3} \end{aligned} \tag{4.4}$$

From the above equation the three phase admittance matrix in the fortescue domain is given by

$$Y_{F_3} = T_{ph_3}^{F_3} Y_{ph_3} T_{F_3}^{ph_3},\ ph_3 \in \{abc\} \tag{4.5}$$

The above when applied to the three phase symmetrical element results in 3x3diagonal matrix The fortescue coordinate will have only 0and 1 sequences in case of two phase system because of n=2

Hence the fortescue transformation matrices are given by

$$T_{ph_2}^{F_2} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & a_2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, T_{F_2}^{ph_2} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{4.6}$$

The admittance matrix for two phases is transformed into fortescue coordinates as follows.

$$Y_{F_2} = T_{ph_2}^{F_2} Y_{ph_2} T_{F_2}^{ph_2},\ ph_2 \in \{abc\} \tag{4.7}$$

In case of two phases the admittance matrix is given by.

$$Y_{ph_2} = \begin{bmatrix} Y_{ii}^{F_3} & Y_{ij}^{F_3 \leftarrow F_2} \\ Y_{ji}^{F_3 \leftarrow F_2} & Y_{jj}^{F_2} \end{bmatrix} \tag{4.8}$$

Applying fortescue transformation, the admittance matrix in fortescue coordinate for two phases is given by

$$Y_{F_2} = \begin{bmatrix} y_s + y_m & 0 \\ 0 & y_s - y_m \end{bmatrix} \quad (4.9)$$

On application of fortescue to the one phase nothing will change in comparison to the phase coordinate system because for the single phase system the fortescue transformation matrix is a 1x1 matrix.

4.2 Application of Fortescue Coordinate System to an Unbalanced and Unsymmetrical Network

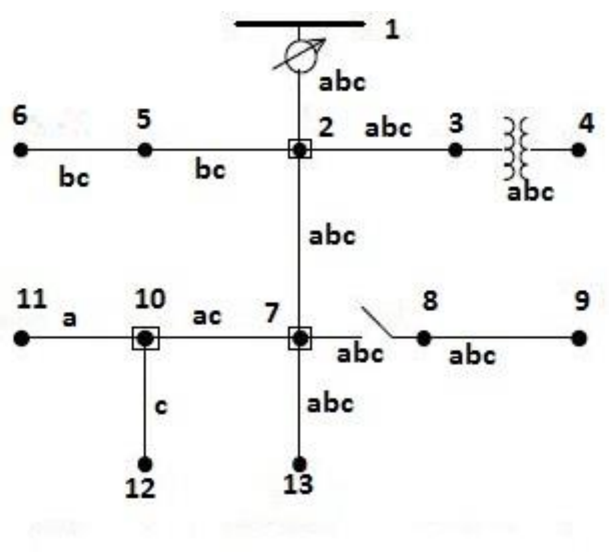


Fig 4.1 IEEE 13 bus distribution system

The unbalanced and unsymmetrical nature of distribution network is typically because of the asymmetrical equipment such as single phase and two phase laterals from the three phase bus and the unbalanced load connected to the bus. The asymmetry also arises because of the nontransposed lines.

In the above figure the IEEE 13 bus distribution system is shown. It is a good example of unbalanced distribution system. The information regarding the phase type of the lines is provided for each line. The nodes which are connected to the different number of branches are called the phase transition node (PTN). All phase transition nodes are identified and marked by a square surrounding it. There are three PTNs in the figure. These are bus 2,7and 10. The bus number 2 of phase type abc is connected to the bus 5 by a line of phase type bc. The bus number 7 of phase type abc is connected to the bus number 10by line phase type ac. The bus number 10 is connected to the line of phase type a and c.

The branches which are connected to the similar type of phases at both the ends the phase to fortescue coordinate transformation is done by simply using standard fortescue transformation matrices. The selection of proper transformation matrix depends on the branch phase type. The matrices of equation number (4.1) are used for the three phase branches. The branches which are connected to at least one phase transition nodes must be treated carefully for the formation of error free bus admittance matrix so that the power flow is good.

4.3 Fortescue Pi Equivalents

4.3.1 Three Phase to Two Phase Branch

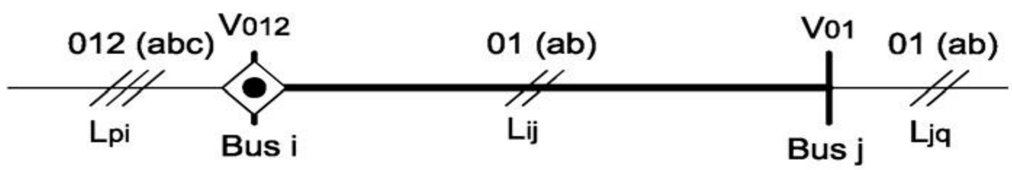


Fig 4.2 Three phase to two phase line

Table 4.1 $T_{F_3}^{ph_2}$ and $T_{ph_2}^{F_3}$ sub-matrices for various branches

ph_2	$T_{F_3}^{ph_2}$	$T_{ph_2}^{F_3}$
ab	$\begin{bmatrix} 1 & 1 & 1 \\ 1 & a_3^2 & a_3 \end{bmatrix}$	$\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a_3 & a_3^2 \end{bmatrix}^T$
bc	$\begin{bmatrix} 1 & a_3^2 & a_3 \\ 1 & a_3 & a_3^2 \end{bmatrix}$	$\frac{1}{3}\begin{bmatrix} 1 & a_3 & a_3^2 \\ 1 & a_3^2 & a_3 \end{bmatrix}^T$
ca	$\begin{bmatrix} 1 & a_3 & a_3^2 \\ 1 & 1 & 1 \end{bmatrix}$	$\frac{1}{3}\begin{bmatrix} 1 & a_3^2 & a_3 \\ 1 & 1 & 1 \end{bmatrix}^T$

Three phases to two phase lateral is shown in the above figure. Bus i is connected to three phase and two phase type branches. Bus j is connected to two phase type branch. The voltage at bue i and j is expressed in fortescue coordinate. The lines are expressed in phase coordinates as well as in fortescue coordinates. The line L_{ij} is connected at bus i where the line phase type and bus phase type is not same. Remaining the entire element in the above figure bus and branch type are same. At bus i the third order fortescue domain is required because it is a three phase bus. At bus j the second order fortescue domain is required because it is a two phase bus. For the application of Kirchhoff's current law, same order of fotescue formulation is must for all the current at bus i. the current contribution of line L_{ij} at bus i must be calculated in F_3 domain although the line is two phase line. The derivation for the fortescue equivalent is given below.

$$\begin{bmatrix} I_i^{F_3} \\ I_j^{F_2} \end{bmatrix} = \begin{bmatrix} \underset{3\times3}{Y_{ii}^{F_3}} & \underbrace{Y_{ij}^{F_3 \leftarrow F_2}}_{3\times2} \\ \underbrace{Y_{ji}^{F_2 \leftarrow F_3}}_{2\times3} & \underset{2\times2}{Y_{jj}^{F_2}} \end{bmatrix} \times \begin{bmatrix} V_i^{F_3} \\ V_j^{F_2} \end{bmatrix} \quad (4.3.1.1)$$

a) Submatrix $Y_{ii}^{F_3}$ **:** The self-admittance sub-matrix is derived in the fortescue domain by transforming phase current into fortescue domain current and the phase voltage into fortescue domain voltage by considering the current contribution.

$$\begin{aligned} \underset{2\times1}{I_{i,i}^{ab}} &= \underset{2\times2}{Y_{ii}^{ab}} \underset{2\times1}{V_i^{ab}} = \underset{2\times2}{Y_{ii}^{ab}} \underset{2\times3}{T_{F_3}^{ab}} \underset{3\times1}{V_i^{F_3}} \\ \underset{3\times2}{T_{ab}^{F_3}} \underset{2\times1}{I_{i,i}^{ab}} &= \underset{3\times2}{T_{ab}^{F_3}} \underset{2\times2}{Y_{ii}^{ab}} \underset{2\times3}{T_{F_3}^{ab}} \underset{3\times1}{V_i^{F_3}} \\ \underset{3\times1}{I_{i,i}^{F_3}} &= \underset{3\times2}{T_{ab}^{F_3}} \underset{2\times2}{Y_{ii}^{ab}} \underset{2\times3}{T_{F_3}^{ab}} \underset{3\times1}{V_i^{F_3}} \end{aligned} \quad (4.3.1.2)$$

Hence the equivalent sub matrix in fotescue coordinate is given by

$$\underset{3\times3}{Y_{ii}^{F_3}} = \underset{3\times2}{T_{ab}^{F_3}} \underset{2\times2}{Y_{ii}^{ab}} \underset{2\times3}{T_{F_3}^{ab}} \quad (4.3.1.4)$$

The general form of the given equation can be expressed as

$$\underset{3\times3}{Y_{ii}^{F_3}} = \underset{3\times2}{T_{ph_2}^{F_3}} \underset{2\times2}{Y_{ii}^{ph_2}} \underset{2\times3}{T_{F_3}^{ph_2}}, \; ph_2 \in \{ab, bc, ca\} \quad (4.3.1.5)$$

For the line connected between three node phase bus to two phase bus the all possible phase combinations are given in the table 4.1.

b) Submatrix $Y_{ij}^{F_3 \leftarrow F_2}$ **:** The mutual admittance submatrix is derived in the fortescue domain by transforming phase current into fortescue domain current and the phase voltage into fortescue domain voltage by considering the current contribution.

$$\begin{aligned} \underset{2\times1}{I_{i,j}^{ab}} &= \underset{2\times2}{Y_{ij}^{ab}} \underset{2\times1}{V_{j}^{ab}} = \underset{2\times2}{Y_{ij}^{ab}} \underset{2\times2}{T_{F_2}^{ab}} \underset{2\times1}{V_{j}^{F_2}} \\ \underset{3\times2}{T_{ab}^{F_3}} \underset{2\times1}{I_{i,j}^{ab}} &= \underset{3\times2}{T_{ab}^{F_3}} \underset{2\times2}{Y_{ij}^{ab}} \underset{2\times2}{T_{F_2}^{ab}} \underset{2\times1}{V_{j}^{F_2}} \\ \underset{3\times1}{I_{i,j}^{F_3}} &= \underset{3\times2}{T_{ab}^{F_3}} \underset{2\times2}{Y_{ij}^{ab}} \underset{2\times2}{T_{F_2}^{ab}} \underset{2\times1}{V_{j}^{F_2}} \end{aligned} \tag{4.3.1.6}$$

Hence the equivalent sub matrix in fotescue coordinate is given by

$$\underbrace{Y_{ij}^{F_3 \leftarrow F_2}}_{3\times2} = \underset{3\times2}{T_{ab}^{F_3}} \underset{2\times2}{Y_{ij}^{ab}} \underset{2\times2}{T_{F_2}^{ab}} \tag{4.3.1.7}$$

The general form of the given equation can be expressed as

$$\underbrace{Y_{ij}^{F_3 \leftarrow F_2}}_{3\times2} = \underset{3\times2}{T_{ph_2}^{F_3}} \underset{2\times2}{Y_{ij}^{ph_2}} \underset{2\times2}{T_{F_2}^{ph_2}},\ ph_2 \in \{ab, bc, ca\} \tag{4.3.1.8}$$

c) Submatrix $Y_{ji}^{F_2 \leftarrow F_3}$ **:** The mutual admittance submatrix is derived in the fortescue domain by transforming phase current into fortescue domain current and the phase voltage into fortescue domain voltage by considering the current contribution.

$$\begin{aligned} \underset{2\times1}{I_{j,i}^{ab}} &= \underset{2\times2}{Y_{ji}^{ab}} \underset{2\times1}{V_{i}^{ab}} = \underset{2\times2}{Y_{ji}^{ab}} \underset{2\times3}{T_{F_3}^{ab}} \underset{3\times1}{V_{i}^{F_3}} \\ \underset{2\times2}{T_{ab}^{F_2}} \underset{2\times1}{I_{j,i}^{ab}} &= \underset{2\times2}{T_{ab}^{F_2}} \underset{2\times2}{Y_{ji}^{ab}} \underset{2\times3}{T_{F_3}^{ab}} \underset{3\times1}{V_{i}^{F_3}} \\ \underset{2\times1}{I_{j,i}^{F_2}} &= \underset{2\times2}{T_{ab}^{F_2}} \underset{2\times2}{Y_{ji}^{ab}} \underset{2\times3}{T_{F_3}^{ab}} \underset{3\times1}{V_{i}^{F_3}} \end{aligned} \tag{4.3.1.9}$$

Hence the equivalent sub matrix in fotescue coordinate is given by

$$\underbrace{Y_{ji}^{F_2 \leftarrow F_3}}_{2\times3} = \underset{2\times2}{T_{ab}^{F_2}} \underset{2\times2}{Y_{ji}^{ab}} \underset{2\times3}{T_{F_3}^{ab}} \tag{4.3.1.10}$$

The general form of the given equation can be expressed as

$$\underbrace{Y_{ij}^{F_2 \leftarrow F_3}}_{2\times3} = \underset{2\times2}{T_{ph_2}^{F_2}} \, \underset{2\times2}{Y_{ji}^{ph_2}} \, \underset{2\times3}{T_{F_3}^{ph_2}}, \; ph_2 \in \{ab, bc, ca\} \tag{4.3.1.11}$$

d) Submatrix $Y_{jj}^{F_2}$: This self sub-matrix will not be influenced by the phase mapping at bus i so it can be obtained from the equation number (6) because this node has the same number of branches coming to it and going out of it.

4.3.2 Three Phase to Single Phase Branch

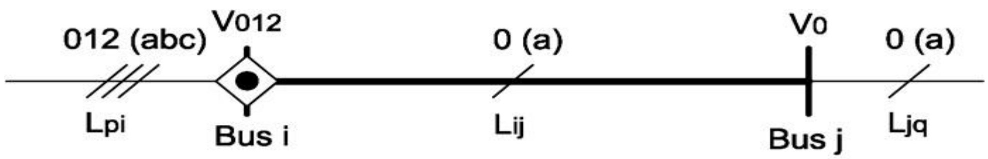


Fig 4.3Three phase to single phase line

In the figure 6 the bus i is a three phase bus and bus j is a single phase bus so we need to deal it carefully. At bus i the third order fortescue domain is required because it is a three phase bus. At bus j the first order fortescue domain is required because it is a single phase bus. For the application of Kirchhoff's current law, same order of fotescue formulation is must for all the current at bus i. the current contribution of line L_{ij} at bus i must be calculated in F_3 domain although the line is single phase line. The derivation for the fortescue equivalent is given below.

$$\begin{bmatrix} I_i^{F_3} \\ I_j^{F_1} \end{bmatrix} = \begin{bmatrix} \underset{3\times3}{Y_{ii}^{F_3}} & \underset{3\times1}{Y_{ij}^{F_3 \leftarrow F_1}} \\ \underset{1\times3}{Y_{ji}^{F_1 \leftarrow F_3}} & \underset{1\times1}{Y_{jj}^{F_1}} \end{bmatrix} \begin{bmatrix} V_i^{F_3} \\ V_j^{F_1} \end{bmatrix} \tag{4.3.2.1}$$

The derivation of these submatrices is given in the following sections.

Table 4.2 $T_{F_3}^{ph_1}$ and $T_{ph_1}^{F_3}$ sub-matrices for various branches

ph_1	$T_{F_3}^{ph_1}$	$T_{ph_1}^{F_3}$
a	$\begin{bmatrix}1 & 1 & 1\end{bmatrix}$	$\frac{1}{3}\left(T_{F_3}^{a}\right)^T$
b	$\begin{bmatrix}1 & a_3^2 & a_3\end{bmatrix}$	$\frac{1}{3}\left(T_{F_3}^{c}\right)^T$
c	$\begin{bmatrix}1 & a_3 & a_3^2\end{bmatrix}$	$\frac{1}{3}\left(T_{F_3}^{b}\right)^T$

a) Submatrix $Y_{ii}^{F_3}$ **:** The self-admittance sub-matrix is derived in the fortescue domain by transforming phase current into fortescue domain current and the phase voltage into fortescue domain voltage by considering the current contribution.

$$\begin{aligned}
&\underset{1\times1}{\boldsymbol{I}_{i,i}^{a}} = \underset{1\times1}{\boldsymbol{Y}_{ii}^{a}}\underset{1\times1}{\boldsymbol{V}_{i}^{a}} = \underset{1\times1}{\boldsymbol{Y}_{ii}^{a}}\underset{1\times3}{\boldsymbol{T}_{F_3}^{ab}}\underset{3\times1}{\boldsymbol{V}_{i}^{F_3}} \\
&\underset{3\times1}{\boldsymbol{T}_{a}^{F_3}}\underset{1\times1}{\boldsymbol{I}_{i,i}^{a}} = \underset{3\times1}{\boldsymbol{T}_{a}^{F_3}}\underset{1\times1}{\boldsymbol{Y}_{ii}^{a}}\underset{1\times3}{\boldsymbol{T}_{F_3}^{a}}\underset{3\times1}{\boldsymbol{V}_{i}^{F_3}} \\
&\underset{3\times1}{\boldsymbol{I}_{i,i}^{F_3}} = \underset{3\times1}{\boldsymbol{T}_{a}^{F_3}}\underset{1\times1}{\boldsymbol{Y}_{ii}^{a}}\underset{1\times3}{\boldsymbol{T}_{F_3}^{a}}\underset{3\times1}{\boldsymbol{V}_{i}^{F_3}}
\end{aligned} \tag{4.3.2.2}$$

Hence the equivalent sub matrix in fotescue coordinate is given by

$$\underset{3\times3}{Y_{ii}^{F_3}} = \underset{3\times1}{T_{a}^{F_3}}\underset{1\times1}{Y_{ii}^{a}}\underset{1\times3}{T_{F_3}^{a}} \tag{4.3.2.3}$$

The general form of the given equation can be expressed as

$$\underset{3\times3}{Y_{ii}^{F_3}} = \underset{3\times1}{T_{ph_1}^{F_3}}\underset{1\times1}{Y_{ii}^{ph_1}}\underset{1\times3}{T_{F_3}^{ph_1}},\ ph_1 \in \{a,b,c\} \tag{4.3.2.4}$$

For the line connected between three node phase bus to two phase bus the all possible phase combinations are given in the table 4.2.

b) Submatrix $Y_{ij}^{F_3 \leftarrow F_1}$ **:** The mutual admittance submatrix is derived in the fortescue domain by transforming phase current into fortescue domain current and the phase voltage into fortescue domain voltage by considering the current contribution.

$$\begin{aligned} \underset{1\times1}{\boldsymbol{I}_{i,j}^{a}} &= \underset{1\times1}{\boldsymbol{Y}_{ij}^{a}} \underset{1\times1}{\boldsymbol{V}_{j}^{a}} = \underset{1\times1}{\boldsymbol{Y}_{ij}^{a}} \underset{1\times1}{\boldsymbol{T}_{F_1}^{a}} \underset{1\times1}{\boldsymbol{V}_{j}^{F_1}} \\ \underset{3\times1}{\boldsymbol{T}_{a}^{F_3}} \underset{1\times1}{\boldsymbol{I}_{i,j}^{a}} &= \underset{3\times1}{\boldsymbol{T}_{a}^{F_3}} \underset{1\times1}{\boldsymbol{Y}_{ij}^{a}} \underset{1\times1}{\boldsymbol{T}_{F_1}^{a}} \underset{1\times1}{\boldsymbol{V}_{j}^{F_1}} \\ \underset{3\times1}{\boldsymbol{I}_{i,j}^{F_3}} &= \underset{3\times1}{\boldsymbol{T}_{a}^{F_3}} \underset{1\times1}{\boldsymbol{Y}_{ij}^{a}} \underset{1\times1}{\boldsymbol{T}_{F_1}^{a}} \underset{1\times1}{\boldsymbol{V}_{j}^{F_1}} \end{aligned} \tag{4.3.2.5}$$

Hence the equivalent sub matrix in fotescue coordinate is given by

$$\underset{3\times1}{Y_{ij}^{F_3 \leftarrow F_1}} = \underset{3\times1}{T_{a}^{F_3}} \underset{1\times1}{Y_{ij}^{a}} \underset{1\times1}{T_{F_1}^{a}} \tag{4.3.2.6}$$

The general form of the given equation can be expressed as

$$\underset{3\times1}{Y_{ij}^{F_3 \leftarrow F_1}} = \underset{3\times1}{T_{ph_1}^{F_3}} \underset{1\times1}{Y_{ij}^{ph_1}} \underset{1\times1}{T_{F_1}^{ph_1}}, \, ph_1 \in \{a,b,c\} \tag{4.3.2.7}$$

c) Submatrix $Y_{ij}^{F_3 \leftarrow F_1}$: The mutual admittance submatrix is derived in the fortescue domain by transforming phase current into fortescue domain current and the phase voltage into fortescue domain voltage by considering the current contribution.

$$\begin{aligned} \underset{1\times1}{\boldsymbol{I}_{j,i}^{a}} &= \underset{1\times1}{\boldsymbol{Y}_{ji}^{a}} \underset{1\times1}{\boldsymbol{V}_{i}^{a}} = \underset{1\times1}{\boldsymbol{Y}_{ji}^{a}} \underset{1\times3}{\boldsymbol{T}_{F_3}^{a}} \underset{3\times1}{\boldsymbol{V}_{i}^{F_3}} \\ \underset{1\times1}{\boldsymbol{T}_{a}^{F_1}} \underset{1\times1}{\boldsymbol{I}_{j,i}^{a}} &= \underset{1\times1}{\boldsymbol{T}_{a}^{F_1}} \underset{1\times1}{\boldsymbol{Y}_{ji}^{a}} \underset{1\times3}{\boldsymbol{T}_{F_3}^{a}} \underset{3\times1}{\boldsymbol{V}_{i}^{F_3}} \\ \underset{1\times1}{\boldsymbol{I}_{i,j}^{F_1}} &= \underset{1\times1}{\boldsymbol{T}_{a}^{F_1}} \underset{1\times1}{\boldsymbol{Y}_{ij}^{a}} \underset{1\times3}{\boldsymbol{T}_{F_3}^{a}} \underset{3\times1}{\boldsymbol{V}_{i}^{F_3}} \end{aligned} \tag{4.3.2.8}$$

Hence the equivalent sub matrix in fotescue coordinate is given by

$$\underset{1\times3}{Y_{ij}^{F_1 \leftarrow F_3}} = \underset{1\times1}{T_{a}^{F_1}} \underset{1\times1}{Y_{ji}^{a}} \underset{1\times3}{T_{F_3}^{a}} \tag{4.3.2.9}$$

The general form of the given equation can be expressed as

$$\underset{3\times1}{Y_{ji}^{F_1 \leftarrow F_3}} = \underset{1\times1}{T_{ph_1}^{F_1}} \underset{1\times1}{Y_{ji}^{ph_1}} \underset{1\times3}{T_{F_3}^{ph_1}}, \, ph_1 \in \{a,b,c\} \tag{4.3.2.10}$$

d) Submatrix $Y_{jj}^{F_1}$ **:** This self sub-matrix will not be influenced by the phase mapping at bus i so it can be obtained from the equation number (4.2) because this node has the same number of branches coming to it and going out of it.

4.3.3 Two Phase to Single Phase Branch

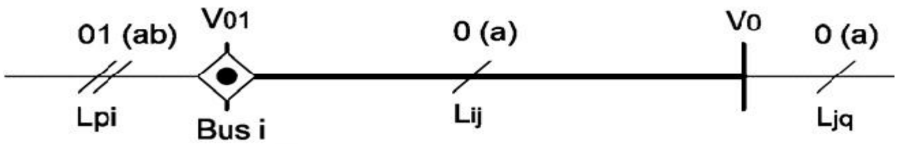


Fig.4.4 line model for two phase to single phase

The above fig (4.4) represents the line section for i as two phase and bus j as single phase the line in between these nodes is a single phase line. For the transformation of the phase domain admittance matrix to the fotescue domain admittance matrix the fortescue transformation matrices is given in the table.

$$\begin{bmatrix} I_i^{F_2} \\ I_j^{F_1} \end{bmatrix} = \begin{bmatrix} \underset{2\times2}{Y_{ii}^{F_2}} & \underset{2\times1}{Y_{ij}^{F_2\leftarrow F_1}} \\ \underset{1\times2}{Y_{ji}^{F_1\leftarrow F_2}} & \underset{1\times1}{Y_{jj}^{F_1}} \end{bmatrix} \begin{bmatrix} V_i^{F_2} \\ V_j^{F_1} \end{bmatrix} \tag{4.3.3.1}$$

a) Submatrix $Y_{ii}^{F_2}$ **:** The self admittance submatrix is derived in the fortescue domain by transforming phase current into fortescue domain current and the phase voltage into fortescue domain voltage by considering the current contribution.

$$\underset{2\times2}{Y_{ii}^{F_2}} = \underset{2\times1}{T_a^{F_2}}\,\underset{1\times1}{Y_{ii}^{a}}\,\underset{1\times2}{T_{F_2}^{a}} \tag{4.3.3.2}$$

The general form of the given equation can be expressed as

$$\underset{2\times2}{Y_{ii}^{F_2}} = \underset{2\times1}{T_{ph_1}^{F_2}}\,\underset{1\times1}{Y_{ii}^{ph_1}}\,\underset{1\times2}{T_{F_2}^{ph_1}},\ ph_1 \in \{a,b,c\} \tag{4.3.3.3}$$

Table 4.3 shows all possible combinations for a two-phase branch connected between two and single phase nodes.

Table 4.3 $T_{F_2}^{ph_1}$ and $T_{ph_1}^{F_2}$ sub-matrices for various branches

F_2	ph_1	$T_{F_2}^{ph_1}$	$T_{ph_1}^{F_2}$
01(ab)	a	$[1 \quad 1]$	$\frac{1}{2}\left(T_{F_2}^{a}\right)^T$
01(ab)	b	$[1 \quad -1]$	$\frac{1}{2}\left(T_{F_2}^{b}\right)^T$
01(bc)	b	$[1 \quad 1]$	$\frac{1}{2}\left(T_{F_2}^{b}\right)^T$
01(bc)	c	$[1 \quad -1]$	$\frac{1}{2}\left(T_{F_2}^{c}\right)^T$
01(ca)	a	$[1 \quad 1]$	$\frac{1}{2}\left(T_{F_2}^{a}\right)^T$
01(ca)	c	$[1 \quad -1]$	$\frac{1}{2}\left(T_{F_2}^{c}\right)^T$

b) Submatrix $Y_{ij}^{F_2 \leftarrow F_1}$ **:** The mutual admittance submatrix is derived in the fortescue domain by transforming phase current into fortescue domain current and the phase voltage into fortescue domain voltage by considering the current contribution.

$$\underset{2\times1}{Y_{ij}^{F_2 \leftarrow F_1}} = \underset{2\times1}{T_{a}^{F_2}} \underset{1\times1}{Y_{ij}^{a}} \underset{1\times1}{T_{F_1}^{a}} \tag{4.3.3.4}$$

The general form of the given equation can be expressed as

$$\underset{2\times1}{Y_{ij}^{F_2 \leftarrow F_1}} = \underset{2\times1}{T_{ph_1}^{F_2}} \underset{1\times1}{Y_{ij}^{ph_1}} \underset{1\times1}{T_{F_1}^{ph_1}}, \; ph_1 \in \{a,b,c\} \tag{4.3.3.5}$$

c) Submatrix $Y_{ji}^{F_1 \leftarrow F_2}$: The mutual admittance submatrix is derived in the fortescue domain by transforming phase current into fortescue domain current and the phase voltage into fortescue domain voltage by considering the current contribution.

$$\underset{1\times2}{Y_{ji}^{F_1\leftarrow F_2}} = \underset{1\times1}{T_a^{F_1}}\,\underset{1\times1}{Y_{ji}^{a}}\,\underset{1\times2}{T_{F_2}^{a}} \tag{4.3.3.6}$$

Generalized expression for the above equation is given by

$$\underset{1\times2}{Y_{ji}^{F_1\leftarrow F_2}} = \underset{1\times1}{T_{ph_1}^{F_1}}\,\underset{1\times1}{Y_{ji}^{ph_1}}\,\underset{1\times2}{T_{F_2}^{ph_1}},\ ph_1 \in \{a,b,c\} \tag{4.3.3.7}$$

d) Submatrix $Y_{jj}^{F_1}$ **:** This self sub-matrix will not be influenced by the phase mapping at bus i so it can be obtained from the equation number (4.2) because this node has the same number of branches coming to it and going out of it.

4.3.4 Treatment for Special Type of Transformer

The simple type of transformer connection will not create problem. The can be dealt in the same manner as the other nodes or buses. The problem will occur when we will come across the special type of transformer connection such as open delta and open wye. This type of transformer connection present in the distribution system network will require a careful treatment. In the distribution network system this special type of transformer connection is used to supply the load of three phase connected to the branch of two phase. The relation in current and voltage in phase domain is given by the below equation with admittance matrix.

$$\begin{bmatrix} I_i^{ph_2} \\ I_j^{ph_3} \end{bmatrix} = \begin{bmatrix} \underset{2\times2}{Y_{ii}^{ph}} & \underset{2\times3}{Y_{ij}^{ph}} \\ \underset{3\times2}{Y_{ji}^{ph}} & \underset{3\times3}{Y_{jj}^{ph}} \end{bmatrix} \begin{bmatrix} V_i^{ph_2} \\ V_j^{ph_3} \end{bmatrix} \tag{4.3.4.1}$$

The self admittance submatrces Y_{ii}^{F} and Y_{jj}^{F} can be obtained from the section first equations directly. The mutual admittance matrices Y_{ij}^{F} and Y_{ji}^{F} will be obtained from the three phase to two phase fortescue transformation explained in the previous sections.

4.4 Steps for the Power Flow Calculation

The generalized fotescue equivalent admittance method can be used in any matrix based power flow solution. When we go for the phase mapping, the self admittance sub matrices will not be affected. The steps for the power flow solution are discussed as follows:

a) From the given data of the distribution network system find out the phase transition nodes for the careful treatment of these.

b) Build up fortescue equivalent admittance Y^F

c) Factories this fortescue equivalent admittance Y^F into lower and upper matrices Y_l^F and Y_u^F respectively.

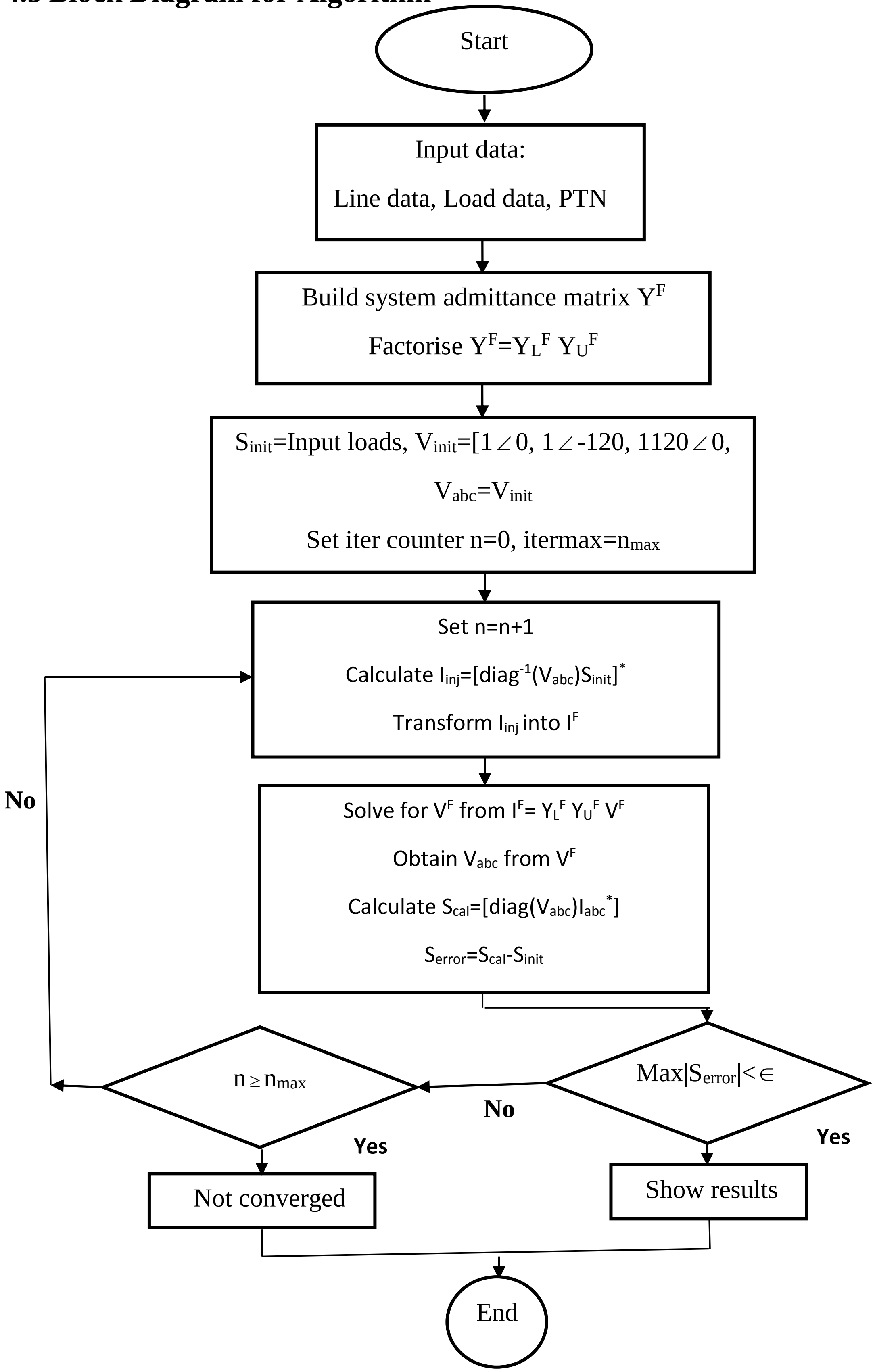
d) For the first iteration take the values of all the voltages at 1.0pu and angle 0 for phase a,-120 for phase b and 120 for phase c.

e) Calculate the injected current at the buses where the power injected is given by the relation $I_{inj}^{abc} = \left(\frac{S_{inj}^{abc}}{V_{abc}} \right)^*$. By using fotescue transformation transform this phase domain current into fortescue domain current.

f) Calculate the voltage in the fotescue domain by using relation $I_{inj}^F = Y_L^F Y_U^F V^F$ convert this fotescue domain voltage V^F into phase domain and calculate power $S_{cal}^{abc} = V^{abc}(I^{abc})^*$ and calculate power error by $S_{err} = S_{cal} - S_{init}$.

g) If the error is below a specified value the stop the solution otherwise go to the step e and repeat the above procedure.

4.5 Block Diagram for Algorithm



Chapter 5

Load Flow with Renewable Generation

There are many types of non-conventional or renewable energy sources. There are wind turbine generation system, fuel cell, photovoltaic generating system etc. The brief description of the renewable generation sources is given in the preceding section.

5.1 Wind Turbine Generating System

In the below given figure (5.1a) and (5.1b) the connection of the wind generation system to the distribution system is shown. Figure (5.1a) shows the connection of the wind turbine generation system connected to the distribution system network with the interface transformer included in the distribution network system. Figure (5.1b) shows the connection of the wind turbine generation system connected to the distribution system network with the interface transformer included in the wind turbine generation system.

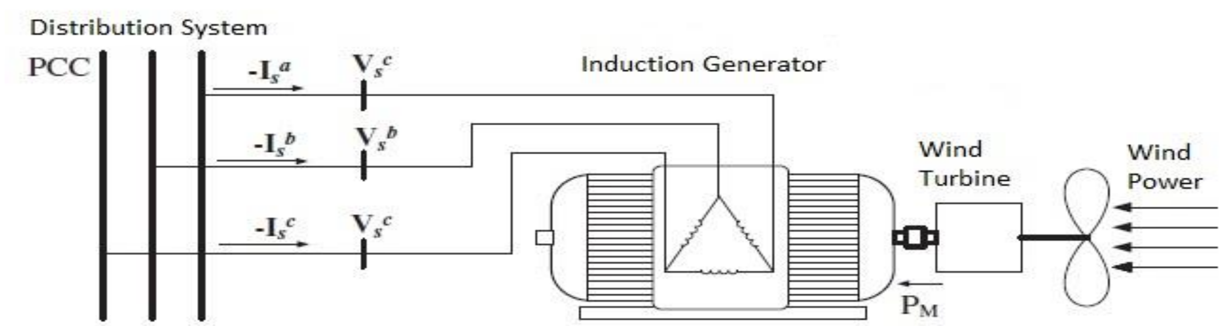


Fig 5.1a the interfacing transformer is considered in distribution system

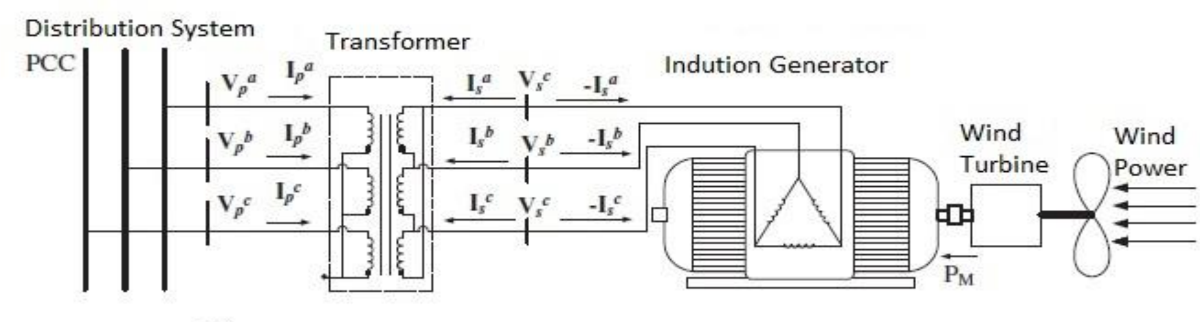


Fig 5.1bThe interfacing transformer is considered in WTG system

The modelling of the wind turbine system is of two types one is ‘PV’ model and the second one is ‘PQ’ model. The voltage controlled is less preferred as compared PQ model. The power developed by the wind turbine is given by the equation given below. The equation represents the relation of speed of the blade tip and the speed of the wind.

$$P_T = \frac{1}{2}\rho A v^3 C_p(\lambda) \tag{5.1.1}$$

Where P_T - Power developed by turbine

ρ - Density of the air in kg/m^3

A - Swept area in m^2

v - Wind speed in m/s

C_p - Power coefficient

The manufacturer provides the power speed characteristic graph from where the power output for a particular wind speed can be obtained. The reactive power (Q) whether specified or can be calculated from the given power factor by the equation given below:

$$Q = -P\tan(\cos^{-1}\phi) \tag{5.1.2}$$

5.2 Photovoltaic model

The function of the photovoltaic system is to convert sun light energy into electrical energy. The photovoltaic system is made by connecting the PV cells in the form of groups. The regulation of voltage and tracking of MPP (maximum power point) is done by the electronic converters.

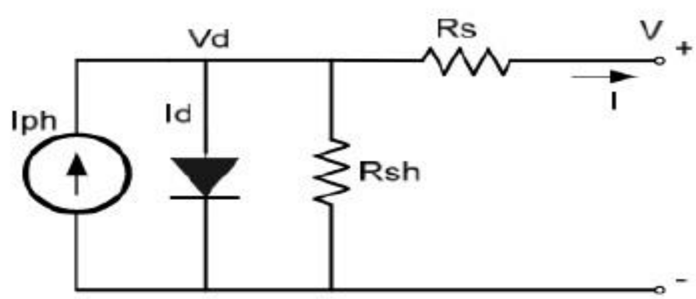


Fig 5.2 Photovoltaic model

Figure (5.2) is the representation of a practical equivalent photovoltaic panel. A current source is connected in parallel with the diode with series connected resistor. The magnitude of the current depends on the intensity of the radiation and the temperature.

The relation in voltage and current is given by below equation:

$$I = I_{ph} - I_0\left(e^{\left(\frac{q(V+IR_s)}{aN_skT}\right)} - 1\right) - \frac{(V + I.R_s)}{R_p} \tag{5.2.1}$$

Where

I_0- Reverse saturation current of the diode

q- Charge of the electron (1.60217646 $\times$ 10-19)

T- Temperature of the diode pn junction

k- Boltzmann constant (1.3806503 $\times$ 10-23 J/K)

N_s-Number of cells connected in series in the panel

a- Ideality constant for the diode

I_{ph}-Current produced by the light

R_s- series resistance

R_p- parallel resistance

The equation for the power of the photovoltaic cell is given by:

$$P = V\left[I_{ph} - I_0\left(e^{\left(\frac{q(V+IR_s)}{aN_skT}\right)} - 1\right) - \frac{(V + I.R_s)}{R_p}\right] \tag{5.2.2}$$

The power obtained from the above equation can be used in the load flow of the distribution system network. The bus will be treated as PQ bus with Q set to zero.

In the presented work the PQ model of wind turbine is used directly and the positive P with appropriate Q is included in the distribution load flow.

Chapter 6

Results and Discussion

The load flow for unbalanced distribution system is presented with two different approaches with renewable generations. The effect of mutual impedances is considered in the unbalanced distribution system and the voltage drop due to mutual impedances. Here the solution for the different number of bus system is presented in unbalanced phase basis with and without renewable generation. The result for each bus voltage magnitude and angle obtained from MATLAB is presented in the tabular form.

6.1Result with direct approach

Table 6.1 Result of 8 bus radial distribution system

Bus No.	Phase	Voltage(pu)	Angle(rad)
1.	A	1.0	0.000
1.	B	1.0	-2.0944
1.	C	1.0	2.0944
2.	A	0.9825	0.0033
2.	B	0.9703	-2.0901
2.	C	0.9667	2.0931
3.	B	0.9646	-2.0896
3.	C	0.9638	2.0934
4.	C	0.9616	2.0895
5.	C	0.9685	2.0931
6.	C	0.9642	2.0932
7.	A	0.9687	.0030
8.	B	0.9609	2.0930

Table 6.2 Result of 13 bus radial distribution system

BUS NO.	PHASE A		PHASE B		PHASE C	
	VOLTAGE (pu)	ANGLE (Rad)	VOLTAGE (pu)	ANGLE (rad)	VOLTAGE (pu)	ANGLE (rad)
1.	1.0	0.000	1.0	-2.0944	1.0	2.0944
2.	.9886	-0.016	.9864	-2.0934	.9875	2.0931
3.	.9861	-0.020	.9859	-2.0965	.9845	2.0839
4.	.9812	-0.033	.9812	-2.1020	.9821	2.0801
5.	-	-	.9791	-2.1501	.9802	2.0711
6.	-	-	.9758	-	.9786	2.0687
7.	.9782	-0.049	.9723	-2.1701	.9764	2.0515
8.	.9710	-0.063	.9681	-2.1832	.9699	2.0394
9.	.9654	-0.091	-	-	.9645	2.0321
10.	-	-	-	-	.9627	2.0271
11.	.9691	-0.121	.9568	-2.1902	.9582	2.0125
12.	.9584	-0.162	-	-	-	-

6.2 Results with fortescue equivalent admittance approach

Table 6.3 Result of 8 bus radial distribution system

Bus No.	Phase	Voltage(pu)	Angle(rad)
1.	A	1.0000	0.0000
1.	B	1.0000	-2.0944
1.	C	1.0000	2.0944
2.	A	0.9852	0.0243
2.	B	0.9733	-2.0911
2.	C	0.9677	2.0941
3.	B	0.9666	-2.0856
3.	C	0.9658	2.0944
4.	C	0.9644	2.0845
5.	C	0.9625	2.0931
6.	C	0.9602	2.0932
7.	A	0.9787	0.0550
8.	B	0.9579	-2.0934

Table 6.4 Result of 13 bus radial distribution system

BUS NO.	PHASE A		PHASE B		PHASE C	
	VOLTAGE (pu)	ANGLE (Rad)	VOLTAGE (pu)	ANGLE (rad)	VOLTAGE (pu)	ANGLE (rad)
1.	1.000	0.000	1.000	-2.0944	1.000	2.0944
2.	.9887	-0.017	.9854	-2.0944	.9852	2.0941
3.	.9865	-0.022	.9847	-2.0955	.9845	2.0839
4.	.9823	-0.035	.9824	-2.1131	.9832	2.0810
5.	-	-	.9774	-2.1521	.9785	2.0732
6.	-	-	.9746	-	.9743	2.0657
7.	.9774	-0.048	.9731	-2.1621	.9747	2.0605
8.	.9745	-0.064	.9662	-2.1802	.9678	2.0354
9.	.9664	-0.094	-	-	.9645	2.0312
10.	-	-	-	-	.9581	2.0284
11.	.9592	-0.134	.9564	-2.1912	.9557	2.0113
12.	.9484	-0.151	-	-	-	-

6.3 Results with fortescue equivalent admittance approach with renewable generation

Table 6.5 Result of 8 bus radial distribution system

Bus No.	Phase	Voltage(pu)	Angle(rad)
1.	A	1.0000	0.000
1.	B	1.0000	-2.0944
1.	C	1.0000	2.0944
2.	A	0.9825	0.0033
2.	B	0.9703	-2.0901
2.	C	0.9657	2.0920
3.	B	0.9646	-2.0878
3.	C	0.9645	2.0944
4.	C	0.9626	2.0885
5.	C	0.9674	2.0931
6.	C	0.9652	2.0932
7.	A	0.9897	0.0023
8.	B	0.9511	2.0930

Chapter 7

Conclusion and Future Work

The technique for the power flow solution of the distribution system with renewable generation is presented in this paper work. The load flow problem is solved by the methods one is direct approach for distribution system and the second is the fortescue equivalent admittance matrix approach for distribution load flow. The first load flow approach is based on building two matrices in taking into account the different characteristics of the distribution system. The two developed matrices are BIBC and BCBV matrix. The matrix giving the relation between bus current injections to the branch currents is known as BIBC matrix and the matrix giving the relation between branch current to the bus voltage is known as BCBV matrix. The above two matrices provides the direct approach solution for load flow when combined together. In the second method the phase admittance matrix is converted into fortescue coordinate. Then injected current is calculated from the given power. From this current the voltages are calculated. The power is calculated from this voltage and current. Error in the power is checked if it is in within the limit then the load flow is completed. The renewable generation is accommodated in the load flow with simply by considering PQ model.

Hence we can say that this is an efficient and robust method presented in this dissertation report. It can be seen from the results that large distribution systems can be handled suitably.

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Appendix

(A)Data sheet of 8 bus unbalanced radial distribution system

Table1. Load data:

S.NO.	BUS NO.	PHASE(A)	PHASE(B)	PHASE(C)
1	2	0.519+0.250i	0.259+0.126i	0.515+0.250i
2	3	0	0.259+0.126i	0.486+0.235i
3	4	0	0	0.324+0.157i
4	5	0	0	0.226+0.109i
5	6	0	0	0.145+0.070i
6	7	0.486+0.235i	0	0
7	8	0	0.267+0.129i	0

Table2. Line data (self-impedance):

S.NO.	FROM	TO	PHASE(A)($x10^{-4}$)	PHASE(B) ($x10^{-4}$)	PHASE(C)($x10^{-4}$)
1	1	2	7.74+3.33i	7.74+3.33i	7.74+3.33i
2	2	3	0	12.9+5.55i	12.9+5.55i
3	2	5	0	0	3.87+1.665i
4	2	7	3.87+1.665i	0	0
5	3	4	0	0	2.58+1.11i
6	3	8	0	5.16+2.22i	0
7	5	6	0	0	6.45+2.775i

Table3. Line data (mutual impedance):

NBR	AB ($x10^{-4}$)	BC ($x10^{-4}$)	CA ($x10^{-4}$)
1	2.58+1.11i	2.58+1.11i	2.58+1.11i
2	0	4.3+1.85i	0
3	0	0	0
4	0	0	0
5	0	0	0
6	0	0	0
7	0	0	0

(B)Data sheet of 13 bus unbalanced radial distribution system

Table4. Line data for 13 bus distribution systems:

S.No.	From Bus	To Bus	R	X
1.	1	2	0.00148	0.00287
2.	2	3	0.00044	0.00124
3	3	4	0.00028	0.00078
4	4	8	0.00160	0.00310
5	8	9	0.00029	0.00083
6	9	10	0.00053	0.00151
7	10	11	0.00059	0.00166
8	9	12	0.00038	0.00107
9	12	13	0.00037	0.00104
10	4	5	0.00060	0.00167
11	5	6	0.00034	0.00097
12	6	7	0.00032	0.00092

Table5. Bus data for 13 bus distribution system:

Bus No.	P	Q	V
1	0.0000	0.0000	1.053
2	4.730	1.550	1.00
3	1.270	0.410	1.00
4	0.350	0.110	1.00
5	4.380	1.440	1.00
6	2.110	0.690	1.00
7	0.420	0.130	1.00
8	4.730	1.550	1.00
9	1.270	0.410	1.00
10	0.350	0.110	1.00
11	4.380	1.440	1.00
12	2.110	0.690	1.00
13	0.420	0.130	1.00