LOCATION OF FAULTED LINE SECTION IN DISTRIBUTION SYSTEM

A DISSERTATION
Submitted in partial fulfillment of the
requirements for the award of the degree
of
Master of Technology
in
ELECTRICAL ENGINEERING
(With specialization in Power System)

By

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May 2016

Candidate's Declaration

I hereby declare that the work presented in this Master's thesis entitled **LOCATION OF FAULTED LINE SECTION IN DISTRIBUTION SYSTEM** submitted towards completion of the requirements for the award of the degree of M.Tech. (Electrical Engineering with specialization in Power System) in the Department of Electrical Engineering of the Indian Institute of Technology Roorkee, India is an authentic record of my own work carried out during the period from May 2015 to May 2016 under the esteemed guidance of Dr. Biswarup Das Professor and Dr. Vinay Pant Associate Professor Department of Electrical Engineering, Indian Institute of Technology, Roorkee, India. The matter embodied in this dissertation has not been submitted by me for the award of any other degree of this or any other Institute/University.

Date:
Place: (Deepika Chhetija)

Certificate

This is to certify that the above statement made by the candidate is correct to best of my knowledge.

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Acknowledgements

A dissertation or any publication can reach to the acme only by the proper guidance, cooperation and kind words of the individuals involved.

I would like to express my sincere gratitude to my supervisor, **Dr. Biswarup Das**, Professor and **Dr. Vinay Pant**, Associate Professor Department of Electrical Engineering, Indian Institute of Technology Roorkee, Roorkee, for providing me an opportunity to work under their guidance. Their continuous guidance with enriched knowledge, useful criticism, regular encouragement and invaluable suggestions at every stage of the present work has proved to be extremely beneficial to me.

I wish to express my profound gratitude to **Dr. S. P. Srivastava**, Professor, Head, Department of Electrical Engineering, Indian Institute of Technology Roorkee, Roorkee, for providing all the facilities which have made it possible for me to complete this work. I am also indebted to all the faculty members of Department of Electrical Engineering, who have taught me during last two years. I thank the entire teaching, non-teaching faculty of Department of Electrical Engineering and all the well-wishers who in any manner directly or indirectly have put hands in any part of this work.

I extend my sincere gratitude to **Mr. Akhilesh Mathur** for their invaluable support, help and encouragement. I am thankful to IIT Roorkee for providing me all sorts of facilities for study and peaceful life inside campus.

Last but not the least, it is owed to the blessings of my family and God that I have come up with this work in due time.

DEEPIKA CHHETIJA

ABSTRACT

To locate fault in power distribution system various methods are proposed which are iterative and time consuming in nature. Therefore, to speed up the restoration process and to improve reliability at consumer end a non-iterative and fast approach is presented to locate the fault in power distribution system (PDS). This method is applicable to all types of faults. It requires voltage and current measurement at substation node after occurrence of fault and bus impedance matrix of healthy system. Current and Voltage at local substation can be expressed as resistance of fault and location of fault by bus impedance matrix, solution of which yields to fault location. This approach of fault location is applicable to unbalanced system as it is developed in phase domain. It is assumed that distribution system network topology and parameters are known through which bus impedance matrix is developed. To calculate the pre fault bus voltages and currents of each bus unbalanced three phase load flow is utilized which is different from the conventional methods used for three phase balanced transmission system. It requires BIBC (bus injection to branch current) and BCBV (branch current to bus voltage) matrix and a simple matrix multiplication to acquire the appropriate results. Therefore, it eliminates the need of forward/backward substitution of the Jacobian matrix or admittance matrix used in the transmission system load flow methods. Utilizing load flow results bus impedance matrix is developed and then algorithm for fault location is applied to calculate the actual fault location. The method presented is fast, accurate and robust for load variations.

Contents

List of Figures

List of Tables

1 Introduction

Due to large growth of electrical power systems number of transmission and distribution lines and their length is increased. These lines are prone to various types of faults due to storms, lightning, short circuits, insulation breakdown, freezing rain etc. System stability, security and reliability is affected by the occurrence of fault. The large amount of current flows through the network which can damage the equipments if not interrupted. Thus, it is important to provide protection schemes to maintain power quality and minimize the damages.In this regard it is important to find the exact location of fault for the reliable restoration process. The term fault is defined as:

Fault – It is basically a physical condition due to which a device, an element or a component fail to perform in a desired manner, as short circuit or a broken wire. It involves short circuit between phases and ground or between energized phases. A fault can be bolted connection or may have some impedance in the fault connection. The term “fault” is often used synonymously with the term “short-circuit” defined as:

Short circuit – A contact or an abnormal connection between the two points of various potential whether made deliberately or accidently termed as short-circuit.

To provide the uninterrupted power supply to consumers it is necessary to isolate the faulty section and restore the system by the protective schemes. After occurrence of fault, restoration can be accomplished more quickly if fault location is estimated accurately.In transmission system digital distance relays performs the function of detecting the fault, tripping and determining the fault location. To improve accuracy and speed of distance relays in transmission system various algorithms are applied. These algorithms cannot be used directly in distribution system due to unbalance nature of PDS. Distribution system fault section detection techniques are taking heed as utilities are competing with each other and working in deregulated scenario to readily provide uninterrupted power supply to consumers.

The faults occurring in a power system can be classified as balanced faults and unbalanced faults. In balanced or symmetrical faults, all the phases are equally affected by the fault and in unbalanced or unsymmetrical faults, the balanced state of the network is disturbed. The power system faults can be categorized as:

1. **Symmetrical faults**

Three phase fault – These type of fault causes due to tower falling, equipment failure or line breaking and touching the remaining phases. In reality, this type of fault is not so common and only 5% of faults in power system are three phase faults [2].

2. **Unsymmetrical faults**

(i) Shunt type faults:

a) L-G fault (Single line to ground fault),

b) L-L fault (Line to Line fault),

c) L-L-G fault (Double-line-to-ground).

Table 1.1: Probability of Occurrence of Fault

S.No.	**Type of fault**	**Probability**
1	L-G	70%
2	L-L	15%
3	L-L-G	10%
4	Three phase fault	5%

(ii) Series type faults:

a) Open conductor fault.

Open conductor fault – When one or two phases of a balanced system opens it creates unbalance in system. It occurs due to storm or if fuses or isolator operates only on single phase leaving remaining phases connected. Faulted phase current falls and voltage increases due to occurrence of open conductor fault.

Distribution feeder has load taps, many laterals, balanced and unbalanced load and different type of conductors.Due to the unbalanced nature of distribution systems, methods used for transmission system fault location cannot be applied directly. Accurate fault location methods in power distribution system are required to identify faulted section, decrease system restoration and outage time, maintain the system stability and therefore to improve reliability.

1.1 Organization Of Thesis

Chapter 1: This chapter comprises the introduction of the work done, including definition and types of fault. It explains the need of fault location methods for power distribution system.

Chapter 2: In this chapter literature survey is done. It includes the various methods proposed and their drawbacks which shows the requirement of presented fault location method.

Chapter 3: A direct approach to solve the unbalanced system load flow is studied in this chapter. It provides the system bus voltages and currents of system having two phase,three phase as well single phase line and laterals.

Chapter 4: A technique for locating the faulty section, fault distance and fault resistance is described. It is a non-iterative, easier to implement and fast method which overcomes the difficulties mentioned in literature review.

Chapter 5: Case studies to validate the methods studied in chapter 3 and chapter 4 are carried out and obtained results are presented in this chapter. Different types of faults having different values of fault resistances are taken to validate the method.

Chapter 6: This chapter includes the conclusion and future scope of the work done.

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2 Literature Review

To provide continuous and reliable power supply to consumers, it is necessary to repair the faulted lines in shortest possible time. Various methods have been proposed to locate fault in distribution system. The ancient method of fault location was visually investigating the line[1]. The method involved patrolling and inspecting the line with or without binoculars. To reduce the line length that must be inspected sectionalizing is done. Surge-operated targets are placed on line towers, and tracer currents have been used to further assist in locating faults. These approaches are expensive, slow, inaccurate and are not safe during nasty weather conditions.

Next proposed approaches for locating fault in power distribution system uses line terminal voltage and current. This approach is divided into two categories. High frequency components of currents and voltage caused by fault are used in first category due to which the travelling waves of current and voltage start between the terminal bus and fault point[2]. Accuracy is high of these methods but implementation is very difficult and complex.

The methods in second category use the fundamental frequency components of current & voltage at the line terminals & line parameters & loads as well[3]. In this approach line impedances are calculated as seen from the line terminals. Fault distance from line terminals are evaluated using the calculated impedances. Drawback of this method is that multiphase taps & the dynamic nature of loads are not described in it.

Another technique of fault distance measurement also uses the fundamental frequency components of voltage and current at line terminals, but applied to radial distribution system having only shunt faults [4][5]. This approach is generally applicable for the systems having non homogeneous lines which can either have capacitor banks or dynamic loads. The main disadvantage is that it does not include the laterals present in distribution system.

In [6], to find the fault location collection of voltage sag data is done at various points of feeder, whereas in [7] voltage sag data which is simulated are compared with the sag data which is recorded. Recorded sag data is obtained at another location for obtaining the most probable fault location. This method is convient only for the feeder node faults.

An another method utilizes genetic algorithm & heuristic to match voltages & simulated current at some fault conditions with that of recorded one to locate the faulty section[8]. Actual location of fault by this method requires an outage call from customer and customer location. Therefore, it is a slow and inaccurate approach.

Another technique used in [9] employs fault-path-current concept which is an iterative short circuit analysis. In this method to acquire the fault path currents short circuit simulation is done iteratively based on presumed location of fault and then are compared with the actual measured values.

The authors of [10] proposes an algorithm of fault location for radial distribution system having single-line-ground fault which is again an iterative approach, updates fault current and location in every iteration. In [11] method used in [10] is extended for all types of fault with same iterative approach.

Another method utilizes direct circuit analysis for fault location of radial distribution system prone to LG and LL faults[12][13]. These methods still require updated voltage and current for assumed faulty section in each iteration

It is evident from literature survey that fault location in power distribution system is time consuming as most of the approaches require iterative short circuit analysis and others are complex to implement.

A non-iterative approach presented in [14] is implemented in this thesis to calculate the distance of fault location in the faulty section which is fast, less complex and accurate. It is applicable to all types of faults occur in distribution system & for different values of fault resistances.

To implement the method proposed in [14] of determining the actual fault location Z bus of system is required. Prefault voltage and current needs to be calculated to form the system Z bus to change the constant power load into constant impedance load.

Due to unbalanced nature of power distribution system traditional load flow methods such as Newton-Raphson & Gauss-Seidel technique used in transmission system cannot be appllied. Therefore, an algorithm proposed in [15] is applicable for calculating the healthy system bus voltage and current.

This algorithm requires only the conventional bus-branch oriented data which is used by most utilities. It is applicable to three phase unbalanced distribution system which contains single as well as multiple phase lines and laterals.

3 Distribution System Load flow

Electrical distribution systems contain large number of nodes and branches, unbalanced and distributed loads, wide ranged resistance and reactance values are either radial or weakly meshed by structure. Due to above mentioned characteristic of power distribution system, conventional load flow approaches used for transmission system, as Gauss-Seidel and Newton-Raphson methods does not give the appropriate result in the distribution system applications. A 'novel but classic' method proposed in [15] for distribution system load flow requires the conventional bus branch data as input which is used by most utilities. Load flow is solved directly by taking advantage of topological characteristics of system. The bus injection to branch current matrix (BIBC) matrix & the branch current to bus voltage matrix (BCBV) matrix are developed to solve the load flow problem of unbalanced distribution system.

3.1 Unbalanced Three-Phase Line Model

Fig. 3.1 shows a three-phase line section between bus i and j.

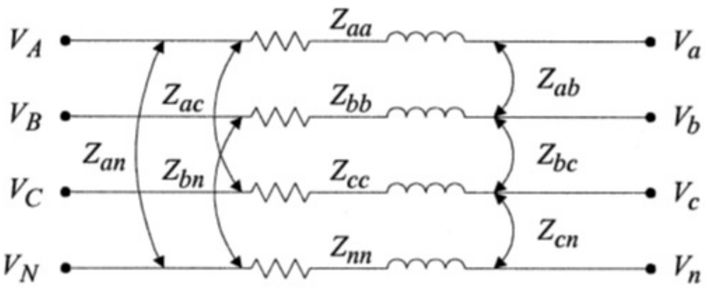

FIGURE 3.1: Three Phase Line Section Model

Z matrix of unbalance three phase line section takes account of self and mutual coupling effects can be written as

$$\begin{bmatrix} Z_{abc} \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \tag{3.1}$$

After Kron's reduction (4x4) matrix is reduced to (3x3) as shown in (3.2). Effect of neutral and ground wire is still included

$$\begin{bmatrix} Z_{abc} \end{bmatrix} = \begin{bmatrix} Z_{aa-n} & Z_{ab-n} & Z_{ac-n} \\ Z_{ba-n} & Z_{bb-n} & Z_{bc-n} \\ Z_{ca-n} & Z_{cb-n} & Z_{cc-n} \end{bmatrix} \tag{3.2}$$

The relation of bus voltages and branch currents in Fig. 3.1 can be defined by,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} - \begin{bmatrix} Z_{aa-n} & Z_{ab-n} & Z_{ac-n} \\ Z_{ba-n} & Z_{bb-n} & Z_{bc-n} \\ Z_{ca-n} & Z_{cb-n} & Z_{cc-n} \end{bmatrix} \begin{bmatrix} I_{Aa} \\ I_{Bb} \\ I_{Cc} \end{bmatrix} \tag{3.3}$$

If any of the three phases failed to present, then null entries are inserted at corresponding rows and column.

3.2 Algorithm Development

The current injections equivalent to the two matrices named Bus-Injection to Branch-Current (BIBC) and Branch-Current to Bus-Voltage (BCBV) are determined to develop the described method.

The complex load S_i for the bus i can be defined as,

$$S_i = (P_i + jQ_i) \tag{3.4}$$

The solution of the k^{th} iteration of the corresponding equivalent current injection is

$$I_i^k = I_i^r + jI_i^i = (P_i + jQ_i)/V_i^k \tag{3.5}$$

Where, I_i^k is the equivalent current injection and V_i^k is bus voltage of bus i at the k^{th} iteration. I_i^r is a real part and I_i^i is an imaginary part of the equivalent current injection of bus at the k^{th} iteration.

3.2.1 Relationship Matrix Developments

A distribution system shown in Fig. 3.2 contains 6 buses and 5 branches. At each bus current injections are calculated using power injections by equation (3.5). Kirchhoff's Current Law (KCL) can be used for finding the relationship between the branch currents and calculated bus injection current in the given network

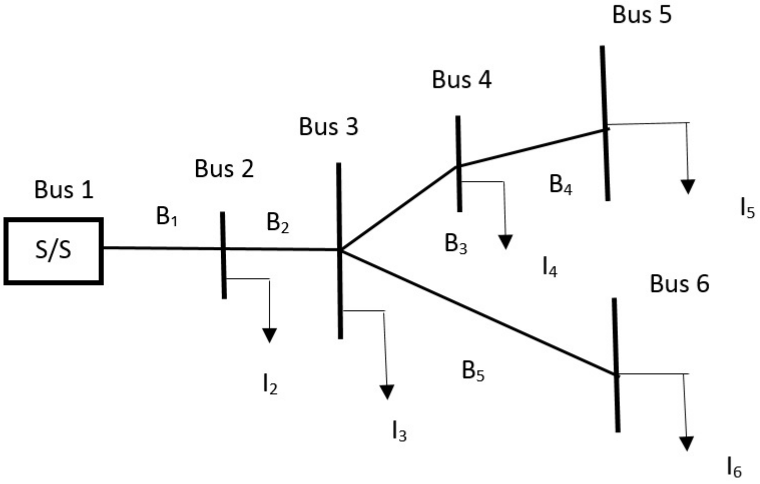

Figure 3.2: Simple Distribution System

The branch currents can be written in terms of current injections at each bus. Applying KCL to above system branch currents B_1, B_4, B_5 can be expressed as

$$\begin{aligned} B_1 &= I_2 + I_3 + I_4 + I_5 + I_6 \\ B_4 &= I_5 \\ B_5 &= I_6 \end{aligned} \tag{3.6}$$

Therefore, the relation between the branch currents and bus current injections can be defined as

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} \tag{3.7}$$

So, in general, the above Equation (3.7) can be expressed as

$$[B] = [BIBC][I] \tag{3.8}$$

Where, BIBC is a constant matrix and also an upper triangular matrix which can hold the values of +1 and 0 only. To develop the BCBV (branch-current to bus-voltage) matrix of system shown in fig (3.2) relationship between branch current matrix and bus voltage is obtained by equation (3.3). E.g., the voltages at the buses no. 2, 3, and 4 are given as,

$$V_2 = V_1 - B_1 Z_{12} \tag{3.9}$$

$$V_3 = V_2 - B_2 Z_{23} \tag{3.10}$$

$$V_4 = V_3 - B_3 Z_{34} \tag{3.11}$$

where, V_i = voltage at i^{th} bus

Z_{ij} = Impedance of the line between bus i & j . equation (3.11) can be rewritten after Substituting (3.9) and (3.10) into (3.11)

$$V_4 = V_1 - B_1 Z_{12} - B_2 Z_{23} - B_3 Z_{34} \tag{3.12}$$

From (3.12), it is evident that bus voltage can be written in terms of substation voltage, line parameters and branch currents. Similarly, second bus voltage can be expressed in same manner.

Therefore, the relationship between the branch currents and bus voltages can be expressed as

$$\begin{bmatrix} V_1 \\ V_1 \\ V_1 \\ V_1 \\ V_1 \end{bmatrix} - \begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} Z_{12} & 0 & 0 & 0 & 0 \\ Z_{12} & Z_{23} & 0 & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & Z_{45} & 0 \\ Z_{12} & Z_{23} & 0 & 0 & Z_{36} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} \tag{3.13}$$

In general, Equation (3.13) can also be written as

$$[\Delta V] = [BCBV][B] \tag{3.14}$$

where, BCBV matrix is the branch-current to bus-voltage (BCBV) matrix.

3.2.2 Building Formulation

Observing (3.7), BIBC matrix can be developed by the following building algorithm:

1. Size of BIBC matrix of m- branch & n- bus section distribution system is m x (n-1). An array of size of BIBC matrix is initialized with null values.

2. Between the bus i and j for a line section B_k, i^{th} bus column of BIBC matrix is copied to j^{th} bus column of matrix and a +1 is filled to the k^{th} row and j^{th} bus column position.

3. Method (2) is repeated till all the line sections present in the system are included.

From (3.13), BCBV matrix building algorithm can be developed as follows:

1. Size of BCBV matrix of n- bus and m- branch section distribution system is (n-1) x m. An array of size of BCBV matrix is initialized with null values.

2. For a line section B_k located between bus i and j, i^{th} bus row of BCBV matrix is copied to j^{th} bus row and line impedance Z_{ij} is filled to the j^{th} bus row and k^{th} bus column position.

3. Procedure (2) is repeated till all the line sections present in the system are included.

For three phase line section

The methodology described above for BIBC and BCBV matrix building, can be extended for system having multiphase line section. If line section located between bus i and j is three phase then +1 in BIBC matrix will be a 3x3 null matrix and likewise in BCBV matrix line impedance Z_{ij} will be 3x3 impedance matrix. Branch current B_i is a 3x1 vector

3.2.3 Solution Technique Developments

The two matrices BCBV and BIBC are developed depends upon the physical structure of the distribution system. The BIBC matrix develops relation between branch currents and bus injections. Any variation in bus injections will cause change in branch currents and that can be obtained directly by BIBC matrix. Similarly, as BCBV represents the relation between branch current and bus voltage, bus voltage variations corresponding to branch currents can be evaluated directly using BCBV matrix.

The relationship between bus voltages and bus current injections is depicted in equation (3.15) by combining (3.8) and (3.14),

$$[\Delta V] = [BCBV][BIBC][I] = [DLF][I] \tag{3.15}$$

And the distribution system load flow solution can be obtained by solving (3.16) iteratively,

$$\begin{aligned} I_i^k = I_i^r + jI_i^i = (P_i + jQ_i)/V_i^k \\ [\Delta V^{k+1}] = [DLF][I^k] \\ [V^{K+1}] = [V^0] + [\Delta V^{k+1}] \end{aligned} \tag{3.16}$$

Hence, for solving load flow only the DLF matrix is required. Thus, this technique reduces considerable amount of time and computational resources, due to which it is suitable for online operation as well.

4 Method Implemented For Fault Location

It is revealed from the literature review that available methods of fault location for distribution system requires time consuming iterative approach or iterative short circuit analysis. Implemented method does not require iterative voltage and current updates and reduces the need of iterative short circuit analysis. Unbalances in PDS is also taken into consideration. Different types of fault with various fault resistance values are considered. Presented method also accommodate the load variations. Therefore, this approach of fault location for unbalanced power distribution system is easy to implement, less time consuming and accurate.

After application of fault location algorithm multiple fault points are achieved which have same fundamental values of current and voltage at receiving end substation. Among all obtained possible fault points, only one is the actual fault location. Therefore, to determine the actual fault location an index is calculated defined in [16], lowest value of which gives the desired location of fault.

Thus it can be stated that objective of the applied algorithm of fault location is

- To eliminate the necessity of iterative short-circuit analysis and current & voltage updates.
- To include unbalances in the system, consider all types of fault and also account for variations of load.
- To determine the real location of fault among all possible fault location.
- Easier implementation, more accuracy and better speed of fault location calculation for radial distribution system.

Assumption

- Network configuration and parameters are known.
- Method is based on fundamental frequency.
- Fault type, including the faulted phase is already known.
- The shunt capacitances of feeder are ignored in the model simulated in PSCAD.

4.1 Fault Location Method For Radial Distribution System

A radial distribution system consists of 3-phase, 2-phase and 1-phase loads and laterals. In this section, a method will be presented which is applicable to radial distribution system. For determining the fault locations, a direct short circuit analysis in which current and voltage measurements are utilized at substation site. A simple radial system is shown in the Fig. 4.1. Depending on the number of phases a *Bus* can have one or more than one nodes but the *Node* can have a single phase connection only.

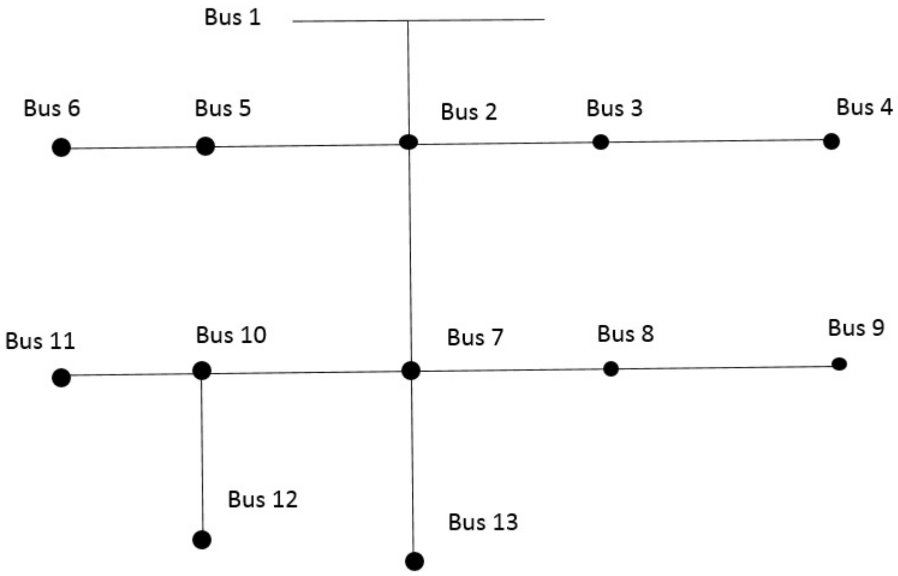

FIGURE 4.1: Radial Distribution System

4.1.1 Steps Followed To Implement Fault Location Method

- Bus impedance matrix is formulated for unbalanced distribution system. To include the load of each bus in Z Bus, prefault voltage and currents have to be calculated using load flow analysis of an unbalanced 3-phase power distribution system.

- At the assumed location of fault fictitious nodes are added depending upon fault type. If single phase then a single fictitious node, for L-L fault two nodes and for L-L-L fault three fictitious nodes are included.

- Then the driving point impedance at fictitious node and transfer impedance between fictitious node and existing node is acquired which are function of fault location.

- Substation currents measured are treated as current injections, then equations linking the current injections and substation voltage through bus impedance matrix are obtained, solution of which leads to the location of fault distance.

Fig 4.2 represents the different types of faults contains single-line-to-ground faults (LG), line-to-line faults (LL) and 3-phase faults (LLL), Inter-phase fault resistance and ground resistance are marked in figure.

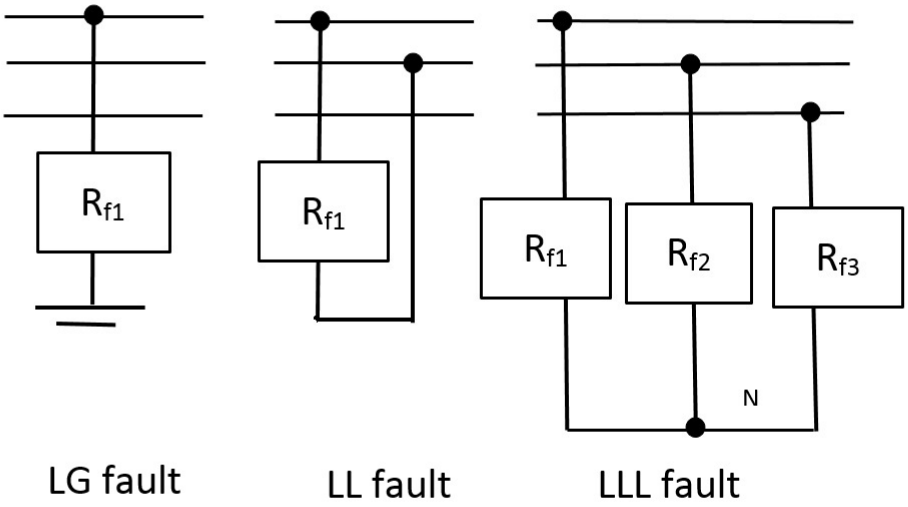

Figure 4.2: Illustration Of Fault Type

Fictitious nodes added are depicted as r_1, r_2, r_3. Three phase feeder is taken in fig 4.2 can have all types of fault as shown in fig 4.2. Whereas only LG fault can occur on single phase feeder. Different fault location algorithms are presented for different types of fault.

4.1.2 Derivation of Driving & Transfer Point Impedance

A three phase feeder section of a power distribution system is shown in fig 4.3 existing between node p and q. Remaining part of system is omitted. The following notations are utilized:

n: Total number of nodes available in the system excluding fault nodes r_1, r_2, r_3.

$p_1, p_2, p_3, q_1, q_2, q_3$: nodes of the three phase sample feeder;

z_1, z_2, z_3: Total self-impedance of the feeder between nodes $p_1 - q_1, p_2 - q_2$ and $p_3 - q_3$ respectively

z_{12}, z_{23}, z_{31}: total mutual impedance between different phases of the feeder

m: per unit fault distance from node p_1, p_2 *or* p_3 to the fault point

$[Z_0]$: bus impedance matrix of the entire prefault distribution system in phase domain, excluding fictitious node r_1, r_2, r_3. $[Z_0]$ will be of size n by n

$[Z]$: A phase domain bus impedance matrix of the entire system, including the fictitious fault nodes. [Z] will be of size (n+3) by (n+3)

Z_{kl}: element in the k^{th} row and l^{th} column of [Z]

$E_{p_1}, E_{p_2}, E_{p_3}, E_{q_1}, E_{q_2}, E_{q_3}$: Voltage at node p_1, p_2, p_3 *and* q_1, q_2, q_3 respectively

I_1, I_2, I_3: Current of branch $p_1 - r_1, p_2 - r_2$ and $p_3 - r_3$

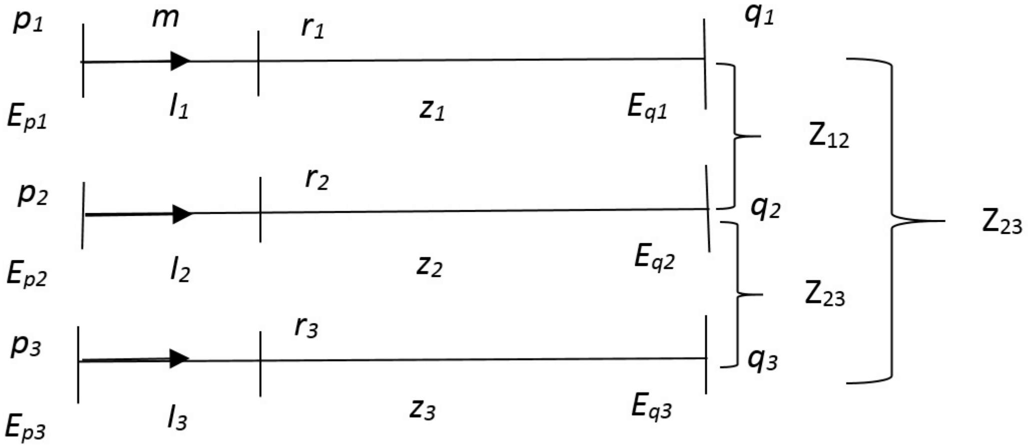

FIGURE 4.3: Section Of A Power Distribution System

The fault nodes are numbered as $r_1 = n+1, r_2 = n+2, r_3 = n+3$. Matrix $[Z_0]$ can be determined readily. It is also shown that the [Z] bus matrix is equivalent with the first n rows and n columns of $[Z_0]$ matrix and remaining other rows and columns of [Z] can be derived from driving and transfer point impedances of fault nodes being added. As explained in Appendices A.1, the driving and transfer point impedance of [Z] related to the fault nodes can be obtained as

$$Z_{kri} = B_{ki} + C_{ki}m \quad i = 1, 2, 3 \tag{4.1}$$

$$Z_{rirs} = A_{is}\_0 + A_{is}\_1m + A_{is}\_2m^2 \quad i = 1, 2, 3 \ and \ i \neq s \tag{4.2}$$

$$Z_{riri} = A_{ii}\_0 + A_{ii}\_1m + A_{ii}\_2m^2 \quad i = 1, 2, 3 \tag{4.3}$$

where:

Z_{kri}: transfer impedance between node k and fault node r_i
Z_{rirs}: transfer impedance between fault node r_i and r_s
Z_{riri}: driving point impedance at fault node r_i

$B_{ki}, C_{ki}, A_{is}\_0, A_{is}\_1, A_{is}\_2, A_{ii}\_0, A_{ii}\_1, A_{ii}\_2$ are the constants that are determined by the network parameters.

Equations derived for transfer and driving point impedance are valid for either single, two or three phase feeder as shown in the appendices. The nitty-gritty is that transfer and driving point impedances are represented as function of fault distance.

Node voltage vector comprising of all node voltages and current injection vector consists of all node injected currents. Node voltage vector is equal to the current injection vector multiplied by bus impedance matrix.

All node injected currents are assumed to zero except fault node r_i. Therefore, k^{th} node voltage is equal to the product of injected current at node r_i and transfer impedance Z_{kri} This relation is utilized for development of fault location algorithm.

4.2 Derivation Of Fault Location For L-G Fault

An L-G fault is considered as shown in Fig. 4.2. Let us define the nodes at the substation as k_1, k_2 and k_3 and suppose k_1 is the node corresponding to the faulty phase. Let [Z] define for the bus impedance matrix of the distribution network with added fault nodes but excluding the resistances of the fault, and also excluding the source impedance. [Z] can be determined as illustrated in above section. (4.1)–(4.3) will be true even now, except that the constants which are defined in the equations, will differ in case of the source impedance is included. Let us develop [W] to define the bus impedance matrix with added fault resistance. We will determine [W], with expressing the element W_{kl} in the k^{th} row and l^{th} column of [W]. After addition of R_{f1} at node r_1, [W] can be achieved by updating [Z] as

$$[W] = [Z] - \frac{Z(:, r_1)[Z(:, r_1)]^T}{Z_{r1r1} + R_{f1}} \tag{4.4}$$

where, Z(: r_1) is r_1^{th} column of [Z] and with T symbolizes vector/matrix transpose.

In a radial distribution system the source is located only at substation. So, during the fault the voltage at node k1 at the substation is obtained as

$$E_{k1} = W_{k1k1}I_{k1} + W_{k1k2}I_{k2} + W_{k1k3}I_{k3} \tag{4.5}$$

Where, I_{k1}, I_{k2}, I_{k3} are measured currents during the fault at the substation. It follows from (4.4) and (4.5) that

$$E_{k1} = Z_{k1k1}I_{k1} + Z_{k1k2}I_{k2} + Z_{k1k3}I_{k3} - \frac{1}{Z_{r1r1} + R_{f1}} \times (Z_{k1r1}^2 + Z_{k1r1}Z_{k2r1} + Z_{k1r1}Z_{k3r1}) \tag{4.6}$$

By separating real and imaginary parts in the equation above leads two real equations. The location of the fault can now be obtained by the solution of a quadratic equation. So, estimation of fault resistance is done thereafter.

4.3 Derivation Of Fault Location For L-L Fault

An L-L fault, which could be between phase A-B or B-C or C-A,is considered as shown in Fig. 4.2. Assuming the substation nodes corresponding to faulted phase are k_1 and k_2 and bus impedance matrix is denoted as [M] including fault resistances. [M] can be represented as

$$[M] = [Z] - \frac{[Z(:,r_1) - Z(:,r_2)][Z(:,r_1) - Z(:,r_2)]^T}{Z_{r1r1} + Z_{r1r2} - 2Z_{r1r2} + R_{f1}} \tag{4.7}$$

The voltage at substation node K_1 during the fault can be obtained as

$$E_{k1} = M_{k1k1}I_{k1} + M_{k1k2}I_{k2} + M_{k1k3}I_{k3} \tag{4.8}$$

Equation (4.8) can be rewritten as equation (4.9) based on (4.7)

$$\begin{aligned} E_{k1} &= Z_{k1k1}I_{k1} + Z_{k1k2}I_{k2} + Z_{k1k3}I_{k3} \\ &- (Z_{k1r1} - Z_{k1r2})\frac{(Z_{k1r1} - Z_{k1r2})I_{k1} + (Z_{k2r1} - Z_{k2r2})I_{k1} + (Z_{k3r1} - Z_{k3r2})I_{k1}}{Z_{r1r1} + Z_{r1r2} - 2Z_{r1r2} + R_{f1}} \end{aligned} \tag{4.9}$$

Z_{rr} and Z_{kr} are function of fault location, thus by substituting the values and separating it into real and imaginary parts a quadratic equation in m is achieved. Solution of which yields to multiple fault section with fault location having same electrical distances. Alternatively, at faulted node K_2 another equation similar to (4.9) can be written. Adding both results a complex equation which is solved for the fault location (m) and fault resistance (R_f).

4.4 Fault Section Estimation

To find out the real location of fault among the obtained fault points, same type of fault is simulated at different possible fault locations and then for each faulty section voltage samples at recieving end substation are stored. This data is termed as saved data.To find out actual faulty section an index is defined explained below.

4.4.1 Actual Fault Location using the recorded voltage samples at the beginning of feeder

After fault occurence voltage samples at the beginning of feeder are recorded from installed data loggeres available at substation. This recoreded data is then compared with the saved data which is the obtained from simulation of fault at different fault points. The one with the highest matching is real location of fault. To determine the matching between saved and recorded data one index is defined as follows

$$index = J = \frac{1}{N}\sum_{i=1}^{N} Er_i^2 \tag{4.10}$$

$$Er_i = |V_r(t_i) - V_{sk}(t_i)|$$

$$Minimise\ J \tag{4.11}$$

where,

$V_r(t_i)$: ith sample of post fault voltage for actual fault (recorded data)
$V_{sk}(t_i)$: ith sample of voltage for simulation of fault at kth possible fault point (stored data)
N: number of samples

5 Results And Discussion

5.1 Flow Chart Of Steps Followed To Calculate Fault Location

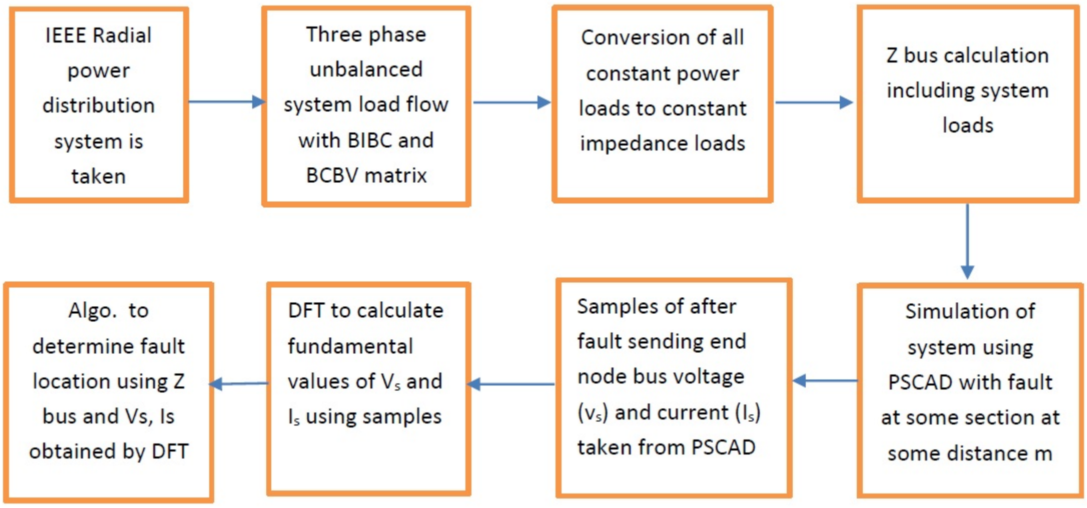

Figure 5.1: Flow Chart

The work done has been carried out in the following manner:

- Three phase unbalanced distribution system load flow using proposed technique in [15] on 16 bus system data. Results of voltage of each bus are compared by simulating the same system in PSCAD.

- Fault section estimation by described algorithm on 5 bus system for both LG and LL fault for different values of fault resistance and different locations.

- Fault section estimation on IEEE 36 bus system data for various types of fault and fault resistance.

5.2 Case Studies

5.2.1 Results of Load Flow of An Unbalanced 3- Phase 16 Bus Distribution System

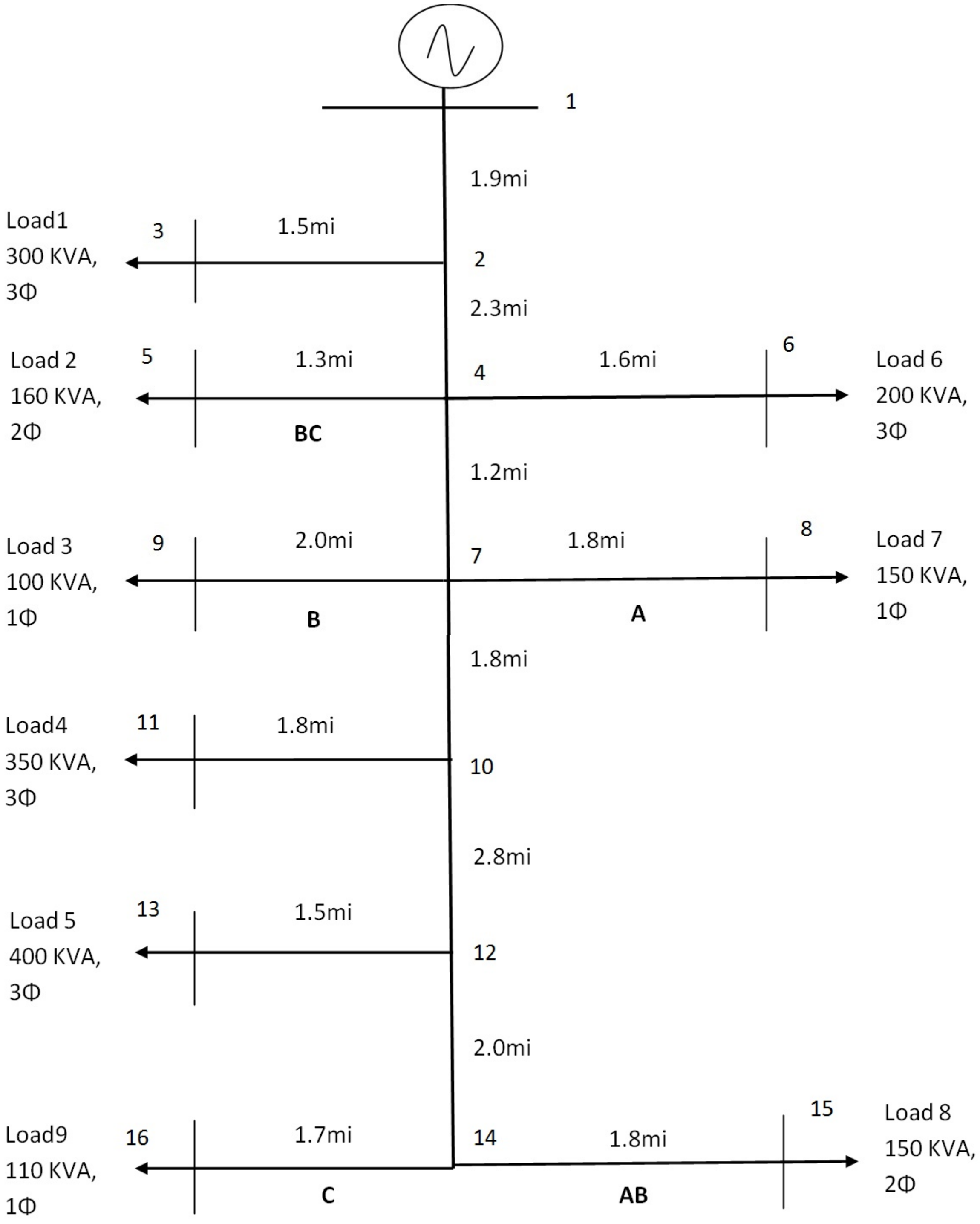

FIGURE 5.2: 16 Bus Unbalanced Radial Distribution System

The presented three phase load flow algorithm is implemented using MATLAB for an unbalanced 16 bus distribution system, the same system is simulated in PSCAD and results obtained are compared. The study will use a sample 12.47 kV overhead power distribution system consists of 3-phase, 2-phase & 1-phase loads and laterals as shown in the Fig. 5.2. E.g. the main feeder supplies load 7 (which is a phase-A load) through a single phase lateral tap.

The labeling of ratings of the load, phases and feeder length in miles is done. A constant power factor of value 0.9 is assumed for all loads. The literals which have single or double phase are indicated and remaining are three phases. The data of feeder impedance are specified in Appendix A.2.

Fig. 5.3 shows the result of load flow of given system. All the three phase voltages obtained at each bus are depicted. Bus 5 contains only B and C phase voltage and bus 8 contains A-phase voltage.

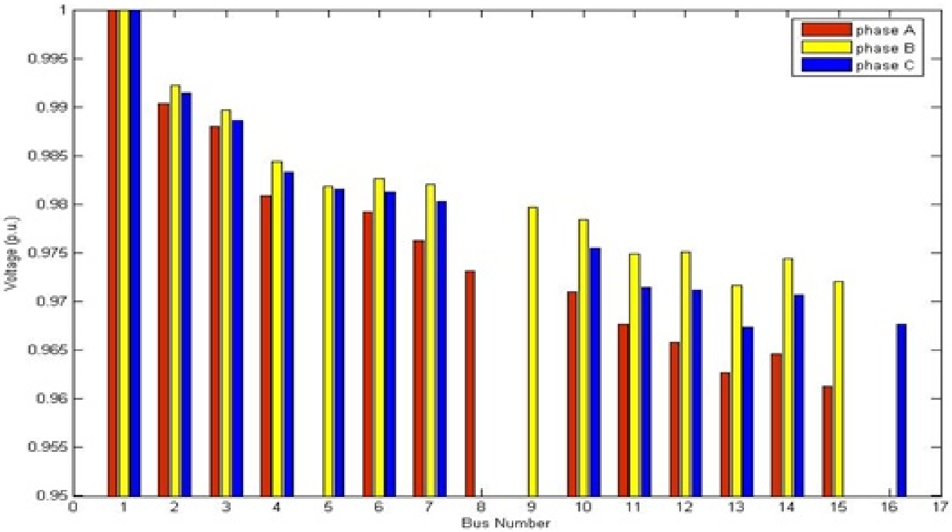

Figure 5.3: Load Flow Results

Table 5.1 depicts the comparison of results obtained from MATLAB and PSCAD of phase A bus voltage of 16 bus system. As bus 5 (two phase BC), bus 9 (single phase B) and bus 16 (single phase C) does not contain A phase, thus not included in Table 5.1.

Table 5.1: Comparison of Phase A Voltage of 16 Bus Distribution System

Bus Number	MATLAB Voltage(KV)	PSCAD Voltage(KV)
1	7.1996	7.1994
2	7.1301	7.1380
3	7.1135	7.1214
4	7.0616	7.0794
6	7.0497	7.0674
7	7.0288	7.0519
8	7.0059	7.0340
10	6.9906	7.0129
11	6.9669	6.9889
12	6.9534	6.9750
13	6.9308	6.9520
14	6.9450	6.9664
15	6.9208	6.9430

Table data shows that the error between voltages obtained from MATLAB and PSCAD is negligible.

Plot is drawn between the voltage obtained from MATLAB and PSCAD, to the bus number, as shown in table.

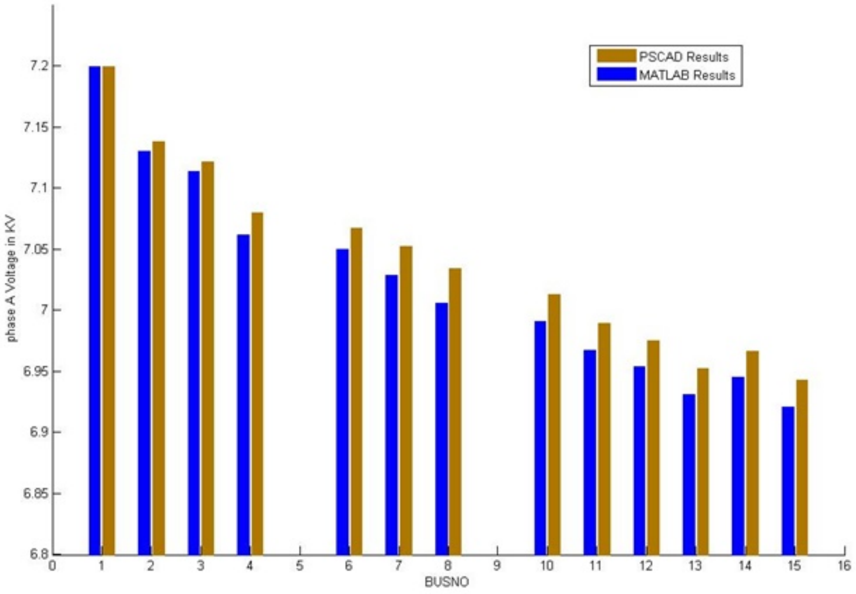

Figure 5.4: Comparison of MATLAB and PSCAD Results

It can be seen in Fig 5.4 that voltage obtained are almost same in magnitude. Blue bar represents the MATLAB results and the brown one is for PSCAD results.

5.2.2 Results for Fault Location in 5 Bus Test System

A three phase 5 bus test system is studied shown in Fig. 5.5.

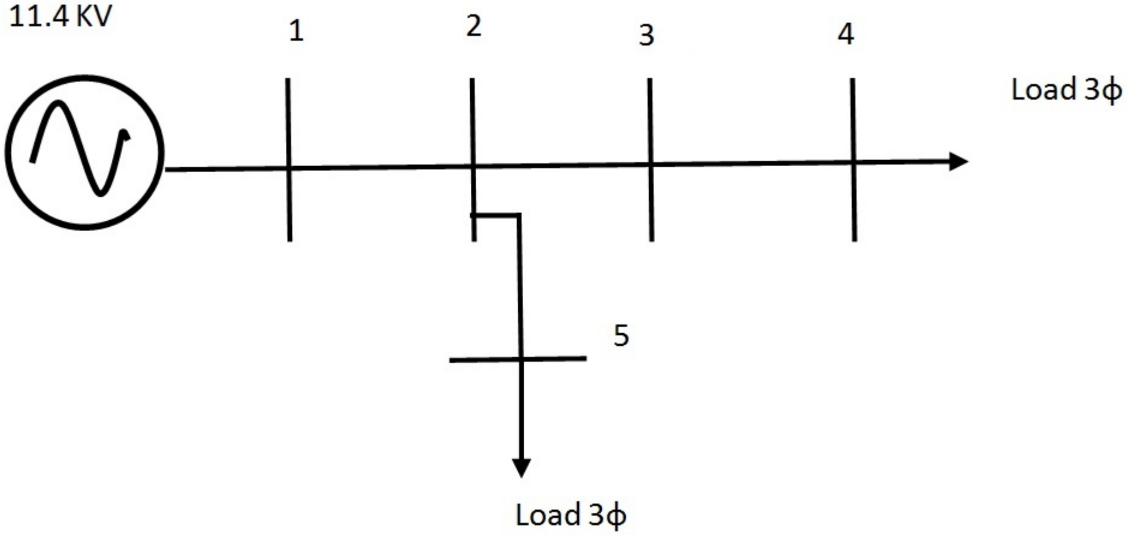

FIGURE 5.5: 5-bus test system

All the healthy system bus voltage and current are calculated by applying load flow algorithm described in Chapter 3. Constant impedance load at bus 4 and bus 5 is calculated by using load flow results. A 5 bus test system has Base KV as 11.4 kV and Base MVA as 1.0 MVA. Three phase line impedances, bus data and line data are given in Appendix A.3

Case I: LL and LG fault at section 3-4

An LL and LG fault is simulated at section 3-4 for different values of fault location and fault resistance. Results obtained by the algorithm discussed in Chapter 4 for different values of m and R_f are shown in Table 5.2.

Table 5.2: Results of fault location for fault at section 3-4

Fault Type	Actual m (p.u.)	Actual R_f (Ω)	Esti. Error in m (%)	Esti. Fault Resi. (Ω)
LG	0.5	5	0.08	5.05
	0.4	10	0.06	10.0
LL	0.5	5	0.04	5.00
	0.4	10	0.06	10.07

Case II: LL and LG fault at section 2-3

An LL and LG fault are simulated at section 2-3 for different values of fault location and fault resistance. In this case two possible fault points are obtained in two sections 2-3 and 2-5 respectively due to same electrical distance. Table 5.3 depicts the results for this case.

Table 5.3: Results of fault location for fault at section 2-3

Fault Type	Actual m(p.u.)	Actual R_f (Ω)	Esti. Error in m (%)	Esti. Fault Resi. (Ω)	Esti. Fault Section
LG	0.5	5	0.02	5.05	2-3
			0.02	5.05	2-5
LL	0.5	5	0.04	5.00	2-3
			0.05	5.05	2-5

Further to investigate the actual fault section same type of fault is simulated at different obtained possible locations and samples of voltage at beginning of feeder are collected for each section. The index defined in Chapter 4 is then calculated for both the possible sections of fault. for section 2-3 index has value of 0.1318 and for section 2-5 index is 27.5417. Therefore, the actual fault location is in section 2-3 at 0.5 p.u. distance from node 2.

5.2.3 Results For Fault Location of IEEE 36 Bus Test System

IEEE 36 bus radial distribution system [17] as shown in Fig. 5.6 is simulated in PSCAD to simulate various types of fault at different fault location. All the lines are three phase whereas loads connected are single as well as three phase. Base MVA as 2.5 MVA and Base KV as 4.8 KV are taken. Bus data and line data are given in Appendix A.4.

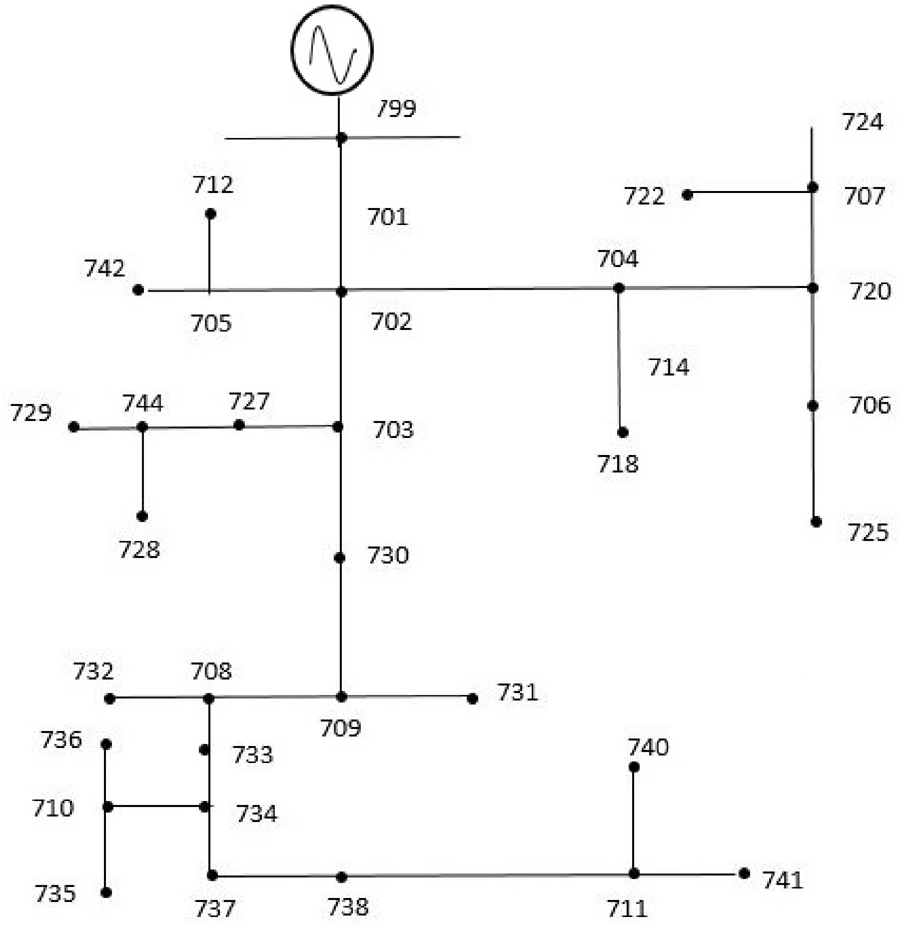

FIGURE 5.6: 36 Bus Test System

Three phase unbalanced load flow algorithm gives result of prefault system voltage and current by which the values of load impedances are calculated. Then Z bus of the prefault system are calculated.

Using the Z bus of system, fault voltage and fault current at substation node, method discussed in chapter 4 for fault location is applied. Table 5.4 shows the result for fault location for different types of fault at different locations having different values of fault resistance.

Table 5.4: Results Of Fault Location for Different Types of Fault with Different Values of R_f

Fault Section	Fault Type	Actual m (p.u)	Actual Rf (Ω))	FL Esti. Error (%)	Esti. Fault Resi. (Ω)
6-31	A-G	0.5	5	0.01	5.05
	A-G	0.8	10	0.05	10.08
	A-B	0.5	5	0.03	5.07
	A-B	0.7	10	0.03	10.06
28-29	A-G	0.5	5	0.08	5.06
	A-G	0.8	10	0.04	10.02
	A-B	0.5	5	0.05	5.09
	A-B	0.7	10	0.03	10.08

Multiple possible fault points are obtained which have the equal values of fundamental voltage and current at the beginnening of feeder. Table 5.5 shows that after applying the fault location algorithm different possible fault points exists. At each possible fault point the same type of fault is simulated at obtained value of m and then voltage samples at begining of feeder are stored. Using recorded and stored values of voltage at beginning of feeder defined index is calculated. The section with minimum value of index is the actual location of fault.

Table 5.5: Estimation Results By Fault Location Algorithm On 36 Bus Test System

Fault Type	Actual Fault Section	Actual m (p.u)	Possible Fault Points	Calc. Fault Location	Index
Ag	704-720	0.5	704-720,744-729,744-728	0.50,0.254, 0.181	0.564,9.075,27.053
Bg	707-722	0.4	707-722,708-709,709-731	0.405,0.418, 0.381	1.245,27.075,123.05
AB	744-727	0.5	704-727,704-718,705-742	0.503,0.535, 0.234	0.789,58.075,223.05
AB	730-709	0.6	730-709,714-718,704-720	0.60,0.615, 0.289	0.356,7.352,56.05

Therefore, it can be revealed from the obtained results that the presented method is applicable to different types of fault having different values of fault resistance and is accurate to determine faulty section as well as fault location.

6 Conclusion & Future Scope

To locate fault in overhead power distribution network a non-iterative technique is studied. To implement the same technique load flow of distribution system is required to calculate the prefault system bus voltages and currents. Due to unbalanced laterals and loads present in distribution system conventional methods used for transmission system load flow cannot be applied. Therefore, to solve the distribution system load flow a direct method is implemented which uses two matrices developed by the topological characteristics of PDS. The BIBC matrix is bus injection to branch current whereas BCBV matrix is branch current to bus voltage matrix. Combining these two matrices load flow problem is solved directly.

After calculation of prefault voltages and currents Z bus is formed including loads present in system. Algorithm discussed of fault distance calculation is applied for radial system using Z bus and the recorded sending end post fault bus voltage and current fundamental phasors. Fault location algorithm gives the multiple fault location in various sections which has the same fundamental voltage and current at receiving end substation.

Among various fault points to find the actual location same type of fault is simulated at every fault point and by comparing voltage samples at substation with the recorded voltage samples, actual fault location is determined.An index J is defined which gives the matching value. Real location of fault is which have minimum index value. Fault resistance is also calculated. The method implemented is fast, accurate and robust for load variations.

Further the work done can be extended for LLG and LLLG fault as well as the unbalance distribution system which have remote in-feed system. Fault type is assumed to be known in this thesis work, it can also be determined applying different algorithms.

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Appendix A

A.1 Transfer & Driving Point Impedance [14]

A.1.1 Derivation of the Transfer Impedance between Node k and Fault Node r_i : Z_{kri}

Referring to Fig. 4.3 & assuming that all the sources in the network are removed and then we inject 1-A current at node k. We obtain:

$$E_{p1} - E_{r1} = m(z_1 l_1 + z_{12} I_2 + z_{13} I_3) \tag{A.1}$$

$$E_{p1} - E_{q1} = z_1 l_1 + z_{12} I_2 + z_{13} I_3 \tag{A.2}$$

The voltage at fault node r is therefore obtained as:

$$E_{r1} = E_{p1} - m(E_{p1} - E_{q1}) \tag{A.3}$$

which does not explicitly contain any mutual impedance between phases. Hence the transfer impedance between node k and fault node r_1 is

$$Z_{kr1} = Z_{kp1} - m(Z_{kp1} - Z_{kq1}) \tag{A.4}$$

By defining

$$B_{k1} = Z_{kp1} \tag{A.5}$$

$$C_{k1} = -(Z_{kp1} - Z_{kq1}) \tag{A.6}$$

Equation A.4 can be written as:

$$Z_{kr1} = B_{k1} + C_{k1} \tag{A.7}$$

which is a function of the fault location. The transfer impedance between node k and other fault nodes r_2 and r_3 can be derived similarly. As a result the general form for the transfer impedance between node k and the fault nodes can be acquired as shown in (4.1)

A.1.2 Derivation of the Driving Point Impedance at Fault Node r_i : Z_{riri}

Let us assume that all of the sources in the network are removed and now we inject 1-A current at node r_1. From Fig. 4.3 it can be attained that

$$Z_{r1r1} = Z_{p1p1} + m(z_1 - 2Z_{p1p1} + 2Z_{p1q1})c + m^2(Z_{p1p1} + Z_{q1q1} - 2Z_{p1q1} - z_1) \quad (A.8)$$

$$A_{11_0} = Z_{p1p1} \quad (A.9)$$

$$A_{11_1} = (z_1 - 2Z_{p1p1} + 2Z_{p1q1}) \quad (A.10)$$

$$A_{11_2} = (Z_{p1p1} + Z_{q1q1} - 2Z_{p1q1} - z_1) \quad (A.11)$$

by proper substitution & (A8) can be shown to be

$$Z_{r1r1} = A_{11_0} + A_{11_1}m + A_{11_2}m^2 \quad (A.12)$$

which is a function of the fault location .

The driving point impedance at other fault nodes can be derived similarly. The general form thus takes the form of (4.3).

A.1.3 Derivation of the Transfer Impedance between Fault Node r_i and Fault Node r_s : Z_{rirs}

Assuming all the sources are removed present in the network and injecting 1-A current at node r_1 following equations are obtained

$$E_{r2} = E_{p2} - m(z_2I_2 + z_{12}I_1 + z_{23}I_3)$$

$$E_{p2} - E_{q2} = z_2I_2 + z_{12}I_1 + z_{23}I_3 + (1-m)z_12 \quad (A.13)$$

From (A.12) and (A.13)

$$E_{r2} = E_{p2} - m(E_{p2} - E_{q2}) + m(1-m)z_{12} \quad (A.14)$$

By utilizing (A.4) the node voltages are expressed in terms of bus impedance matrix elements as:

$$E_{p2} = Z_{p2p1} - m(Z_{p2p1} - Z_{p2q1} \tag{A.15}$$

$$E_{q2} = Z_{q2p1} - m(Z_{q2p1} - Z_{q2q1)} \tag{A.16}$$

Substituting (A.15) and (A.16) in (A.14) leads to the voltage at node r2 & that is the transfer impedance Z_{r1r2} as:

$$Z_{r1r2} = Z_{p1p2} + m(z_{12} - 2Z_{p1p2} + Z_{p1q2} + Z_{q1p}2) \ + m^2(Z_{p1p2} + Z_{q1q2} - Z_{p1q2} - Z_{q1p2} - z_{12} \tag{A.17}$$

By defining

$A_0 = Z_{p1p2}$

$A_1 = z_{12} - 2Z_{p1p2} + Z_{p1q2} + Z_{q1p2}$

$A_2 = Z_{p1p2} + Z_{q1q2} - Z_{p1q2} - Z_{q1p2} - z_{12}$

Equation (A.17) becomes

$$Z_{r1r2} = A_0 + A_1 m + A_2 m^2 \tag{A.18}$$

which is a function of the fault location m.The transfer impedance between other fault nodes can be deduced similarly.

A.2 parameters of the 16 bus system used for load flow

The feeder impedance matrices in ohms/mile are given as follows. For the main feeder the impedance matrix is:

$$Z = \begin{bmatrix} 0.3465 + 1.0179i & 0.1560 + .5017i & 0.1580 + .4236i \\ 0.1560 + 0.5017i & 0.3375 + 1.0478i & 0.1535 + 0.3849i \\ 0.1580 + 0.4236i & 0.1535 + 0.3849i & 0.3414 + 1.0348i \end{bmatrix}$$

For the three-phase lateral the impedance matrix is:

$$Z = \begin{bmatrix} 0.7526 + 1.1814i & 0.1580 + 0.4236\mathrm{i} & 0.1560 + 0.5017i \\ 0.1580 + 0.4236i & 0.7475 + 1.1983i & 0.1535 + 0.3849i \\ 0.1560 + 0.5017i & 0.1535 + 0.3849i & 0.7436 + 1.2112i \end{bmatrix}$$

For the two-phase lateral the impedance matrix is:

$$Z = \begin{bmatrix} 1.3294 + 1.3471i & 0.2066 + 0.4591i \\ 0.2066 + 0.4591i & 1.3238 + 1.3569i \end{bmatrix}$$

For the single-phase lateral the impedance is:

$$Z = \begin{bmatrix} 1.3292 + 1.3475i \end{bmatrix}$$

A.3 Parameters of 5-bus test system used for fault location

A.3.1 Impedance of line section in ohm/mile

$$Z = \begin{bmatrix} 0.7526 + 0.1814i & 0.1580 + 0.4236i & 0.1560 + 0.5017i \\ 0.1580 + 0.4236i & 0.7475 + 1.1983i & 0.1535 + 0.3849i \\ 0.1560 + 0.5017i & 0.1535 + 0.3849i & 0.7436 + 1.2112i \end{bmatrix}$$

A.3.2 Bus Data

Bus Number	P_A (KW)	Q_A (KVAR)	P_B (KW)	Q_B (KVAR)	P_C (KW)	Q_C (KVAR)
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	145	28	643	100	643	100
5	145	28	643	100	643	100

A.3.3 Line Data

Line Number	From Bus	To Bus	Length of Line
1	1	2	500
2	2	3	500
3	3	4	600
4	2	5	600

A.4 Parameters of the IEEE 36 bus system used for fault location

A.4.1 Phase Impedance Matrices in ohm/mile

- Configuration 721

$$Z = \begin{bmatrix} 0.2926 + 0.1973i & 0.0673 - 0.0368i & 0.0337 - 0.0417i \\ 0.0673 - 0.0368i & 0.2646 + 0.1900i & 0.0673 - 0.0368i \\ 0.0337 - 0.0417i & 0.0673 - 0.0368i & 0.2926 + 0.1973i \end{bmatrix}$$

- Configuration 722

$$Z = \begin{bmatrix} 0.4751 + 0.2973i & 0.1629 - 0.0326i & 0.1234 - 0.0607i \\ 0.1629 - 0.0326i & 0.4488 + 0.2678i & 0.1629 + 0.0326i \\ 0.1234 - 0.0607i & 0.1629 - 0.0326i & 0.4751 + 0.2973i \end{bmatrix}$$

- Configuration 723

$$Z = \begin{bmatrix} 1.2936 + 0.6713i & 0.4871 + 0.2111i & 0.4585 + 0.1521i \\ 0.4871 + 0.2111i & 1.3022 + 0.6326i & 0.4871 + 0.2111i \\ 0.4585 + 0.1521i & 0.4871 + 0.2111i & 1.2936 + 0.6713i \end{bmatrix}$$

- Configuration 724

$$Z = \begin{bmatrix} 2.0952 + 0.7758i & 0.5204 + 0.2738i & 0.4926 + 0.2123i \\ 0.5204 + 0.2738i & 2.1068 + 0.7398i & 0.5204 + 0.2738i \\ 0.4926 + 0.2123i & 0.5204 + 0.2738i & 2.0952 + 0.7758i \end{bmatrix}$$

A.4.2 Bus Data

Node	Ph-A (KW)	Ph-A (KVAR)	Ph-B (KW)	Ph-B (KVAR)	Ph-C (KW)	Ph-C (KVAR)
701	140	70	140	70	350	175
712	0	0	0	0	85	40
713	0	0	0	0	85	40
714	17	8	21	10	0	0
718	85	40	0	0	0	0
720	0	0	0	0	85	40
722	0	0	140	70	21	10
724	0	0	42	21	0	0
725	0	0	42	21	0	0
727	0	0	0	0	42	21
728	42	21	42	21	42	21
729	42	21	0	0	0	0
730	0	0	0	0	85	40
731	0	0	85	40	0	0
732	0	0	0	0	42	21
733	85	40	0	0	0	0
734	0	0	0	0	42	21
735	0	0	0	0	85	40
736	0	0	42	21	0	0
737	140	70	0	0	0	0
738	126	62	0	0	0	0
740	0	0	0	0	85	40
740	0	0	0	0	42	21
742	8	4	85	40	0	0
744	42	21	0	0	0	0

A.4.3 Line Data

Line Number	Node A	Node B	Length (ft.)	Config.
1	701	702	960	722
2	702	705	400	724
3	702	713	360	723
4	702	703	1320	722
5	703	727	240	724
6	703	730	600	723
7	704	714	80	724
8	704	720	800	723
9	705	742	320	724
10	705	712	240	724
11	706	725	280	724
12	707	724	760	724
13	707	722	120	724
14	708	733	320	723
15	708	732	320	724
16	709	731	600	723
17	709	708	320	723
18	710	735	200	724
19	710	736	1280	724
20	711	741	400	724
21	711	740	200	723
22	713	704	520	723
23	714	718	520	724
24	720	707	920	724
25	720	706	600	723
26	727	744	280	723
27	730	709	200	723
28	733	734	560	723
29	734	737	640	723
30	734	710	520	724

31	737	738	400	723
32	738	711	400	723
33	744	728	200	724
34	744	729	280	724
35	799	701	1850	721