

# **EXPERIMENTAL EVALUATION OF EFFECT OF OVERLAY OF FRP ON THE PERFORMANCE OF RC SLAB**

**A DISSERTATION**

*Submitted in partial fulfillment of the  
requirements for the award of the degree*

*of*

**MASTER OF TECHNOLOGY**

*in*

**EARTHQUAKE ENGINEERING**

(With Specialization in Structural Dynamics)

*By*

**RITESH R. JOSHI**



**DEPARTMENT OF EARTHQUAKE ENGINEERING**

**INDIAN INSTITUTE OF TECHNOLOGY ROORKEE**

**ROORKEE-247667 (INDIA)**

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## CANDIDATE’S DECLARATION

I hereby certify that the work, which is being presented in this Dissertation, entitled “**EXPERIMENTAL EVALUATION OF EFFECT OF OVERLAY OF FRP ON THE PERFORMANCE OF RC SLAB**” submitted in partial fulfillment of the requirements for the award of the degree of Master of Technology in Earthquake Engineering with specialization in Structural Dynamics to the Department of Earthquake Engineering, Indian Institute of Technology Roorkee, India, is an authentic record of my own work carried out under the guidance of **Dr. Pankaj Agarwal**, Professor, Department of Earthquake Engineering, Indian Institute of Technology Roorkee.

The matter embodied in this Dissertation has not been submitted for the award of any other Institute/University.

Date:

Ritesh R. Joshi

Place: Roorkee

(14526019)

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## CERTIFICATE

This is to certify that the above mentioned statement made by the candidate is correct to the best of my knowledge.

**Dr. PANKAJ AGARWAL**

Professor

Department of Earthquake Engineering

Indian Institute of Technology, Roorkee

Roorkee - 247667

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**Ritesh R. Joshi**  
(14526019)

## **ABSTRACT**

Reinforced concrete members become vulnerable to damage due to incorrect design, past earthquake damages, ageing, corrosion etc. Such members need strengthening to enhance their capacity. The current study is about experimental evaluation of effectiveness of Fiber Reinforced Polymers (FRPs) in strengthening of RC slabs. For the study two different types of slabs were fabricated one with full reinforcement in both direction representing conventional slabs and other with no reinforcement in central region in order to simulate removed part of corroded reinforcement except support section. For strengthening purpose two different types of FRPs, Carbon Fiber Reinforced Polymers (CFRP) and Glass Fiber Reinforced Polymers (GFRP) were used, with two different patterns having different widths of FRP strips. FRP strips were bonded on bottom surface of the slab specimens and specimens were tested under central point load with edges kept simply supported load and deflection data was recorded.

In case of unreinforced slab specimens an average strength increase in GFRP is 210% and CFRP strengthened specimens is 210% and 318% respectively. In case of reinforced concrete slab specimens strength increase using GFRP is about 32% and using CFRP is 100%. In case of reinforced concrete slabs strength increase in GFRP strengthened and CFRP strengthened specimens is 32% and 100% respectively.

Further, results obtained from experimental work were used to evaluate effect of strengthening of slabs using FRPs in flat slab system on their performance against lateral loading. For this purpose flat slab building was modeled and analyzed using SAP2000. Results from the analysis shown significant improvement in the performance of flat slab system using both strengthening techniques.

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# INTRODUCTION

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### 1.1 GENERAL

Structures which are exposed to severe environmental conditions, high mechanical loadings, structures which have faulty design and inferior construction quality show poor performance and are most vulnerable to damage during disasters like earthquakes and when these structures subjected to high loadings than that for which they had designed. Thus, it has become present need to modify such structures to enhance their structural performance and durability according to current codal provisions so that in future their vulnerability can be reduced. Also, there are structures, which have undergone damage in past due to earthquakes, heavy loadings etc and need repairing so that these structures can regain their original strength and can be made functional again. Some strengthening techniques which are commonly used for reinforced concrete structures are listed below,

1. Reinforced Concrete Jacketing
2. Steel Jacketing
3. Chemical Grouting
4. Replacement of Reinforcement
5. Use of Fiber Reinforced Polymers etc.

Techniques involving use of composite materials like Fiber Reinforced Polymers (FRP) in retrofitting and repairing works of structures are gaining popularity due to their light weight, high strength, non-corrosiveness, and ease of handling. Ample amount of research has been carried out, particularly on concrete columns and beams reinforced by FRP. There are several studies, simple to use formulae in the literature and codal provisions for strengthening of reinforced concrete columns and beams using FRPs. In case of reinforced concrete slab strengthening most of the work has been carried out on one way slab. In some of those studies, the behavior of one way slab was found to be very similar to that of beams. The usual strengthening method presumes the placing of the FRP sheets bonded on the tensioned side using resins. The sheets are mounted parallel to the long edge of the slabs, the same way as flexural strengthening of beams.

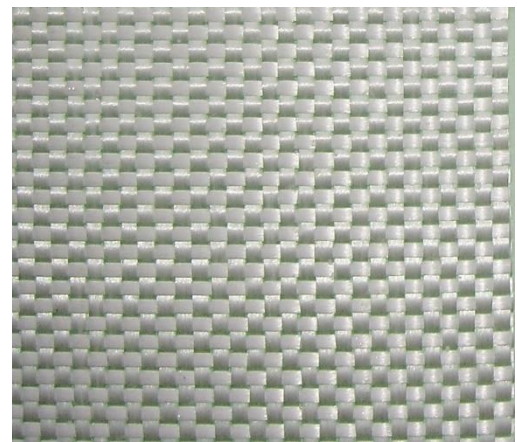
Corrosion of steel reinforcement is one of the biggest problem in construction industry. The problem is common in coastal areas and industries where members are subjected to contact with corrosive environment. Due to corrosion of reinforcement it is no more useful in taking loads and it needs complete removal and replacement. Conventional technique for repair and strengthening involves replacement of corroded reinforcement with new steel reinforcement and then covering it with concrete. But in such case reinforcement again is vulnerable to corrosion in future and will need repair again. In such case strengthening using fiber reinforced polymers proves very beneficial.

### 1.2 FIBER REINFORCED POLYMERS (FRPs):

An FRP is a specific type of two component composite material consisting of high strength fibers embedded in polymer matrix. Some commonly used FRPs for retrofitting are Glass fiber reinforced polymers (GFRP), Carbon fiber reinforced polymer etc.



(a) CFRP



(b) GFRP

Figure 1.1 Different types of FRPs

Strength and stiffness of composite system like FRP comes from strength of fibers in tension and matrix in which they are embedded bind fibers together. FRP composites have high modulus of elasticity, high stiffness, and are resistant to corrosion thus they are commonly used for repair and retrofit. FRPs are most commonly used as tool for external strengthening to upgrade the structural capacity and ductility of reinforced concrete beams and columns in both seismic and corrosive environments. In process of strengthening several layers of FRP are bonded to the finished concrete surfaces in the hoop and longitudinal directions for enhancing

the member ductility, flexural and axial capacity of columns, bridge piers etc. In such case fibers get exposed to tension due to the Poisson's effect which, in turn, results in required hoop stresses. In case of beam FRP strips are attached on bottom surface for flexure strengthening and on side faces to enhance shear capacity. In case of reinforced concrete slabs FRP layers are applied on tension face. One way slabs are strengthened only along longer direction and two way slabs are strengthened in both directions.

**1.2.1 Strength of different FRPs**

FRP materials used in strengthening applications are typically linear elastic up to failure, and do not exhibit the yielding behavior which is displayed by conventional reinforcing steel. This is shown in Figure 1.2 which demonstrates the significant differences in the tensile behavior of FRPs as compared with steel. FRP materials generally have much higher strengths than the yield strength of steel, although they do not yield, and have strains at failure that are often considerably less than steel.

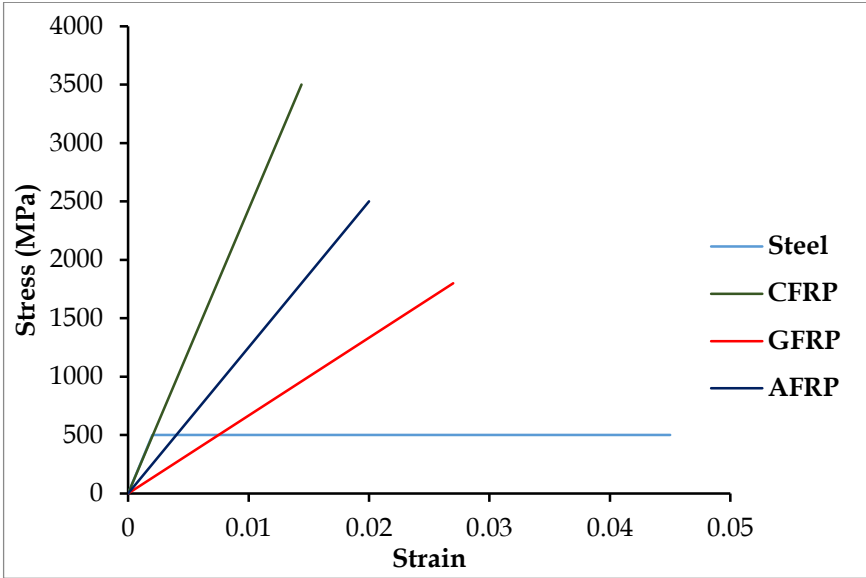


Figure 1.2 Stress strain curve for different FRPs

Table 1.1 Properties of some FRPs

FRP	Youngs' Modulus (MPa)	Ultimate Tensile Strength (MPa)	Poisons' Ratio	Density (g/cc)
Carbon Fibers	242000	1800	0.3	1.6
Glass Fibers	69000	1000	0.25	1.9
Boron	80000	1200	0.23	2.0

### 1.3 OBJECTIVES

Prime objectives of the dissertation are:

1. To study difference between seismic behavior of moment resisting frame building and flat slab building.
2. To study failure of concrete slab specimens under concentrated load with and without reinforcement.
3. To study effectiveness of different FRPs in strengthening of concrete slab specimen with and without reinforcement.
4. To study effect of FRP strengthening of slabs on behavior of flat slab building under lateral loading.

### 1.4 SCOPE OF STUDY

Behavior of moment resisting frame and flat slab building was studied by modeling 8 storied buildings both as frame and flat slab building in SAP 2000. Seismic and non-linear pushover analysis was performed and results for both buildings were compared.

Experimental study was performed on reinforced and unreinforced slab specimens in order to study effectiveness of FRP strengthening. Control specimens of reinforced and unreinforced slab specimens were first tested, and then slab specimens were tested by wrapping two different kinds of FRPs with different patterns on tension face of slab specimens. All specimens were tested under central load applied on area of 100mmx100mm plate and load vs deflection behavior was studied and compared for different specimens. Further, failure patterns of all specimens were studied and compared.

Flat slab building was analyzed by modifying stiffness of the slab to evaluate the effectiveness of FRP strengthening in flat slab building. The modification factor for stiffness was taken from the load-deflection plots of reinforced concrete slab specimens.

## **1.5 ORGANIZATION OF DISSERTATION**

Overview of different chapters of the dissertation is as follows,

**Chapter 1** discusses need of strengthening for the structures and different techniques which can be used for strengthening purposes. It also gives short introduction about FRPs, their use in strengthening purposes, their advantages over conventional techniques.

**Chapter 2** presents various literatures referred for the dissertation. It contains various studies done by researchers and discussion of their results and conclusions.

**Chapter 3** provides an introduction about flat slab systems. It gives comparison of structural behavior of flat slab systems and moment resisting frame systems through the analysis of both models of building using SAP2000 software.

**Chapter 4** is about the yield line analysis of RC slabs. It presents brief elaboration of yield line analysis and its application to RC slabs with different loading and geometric conditions.

**Chapter 5** provides an insight of experimental program. It presents information about different specimens tested, strengthening of those specimens with different FRPs and patterns, test setup etc. It also presents load-deflection curves obtained during tests of specimens and their failure patterns.

**Chapter 6** presents an application of experimental results to the flat slab building model. Flat slab building model with improved stiffness of slab is tested in SAP2000 software and results are compared with the other models presented in chapter 3.

**LITERATURE REVIEW**

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**GENERAL**

Ample amount of research has been done in the field of strengthening of reinforced concrete structural elements like beam, columns using composites like fiber reinforced polymers. Many literatures, codes are available with simplified formulation on the strengthening of beam and columns using FRP composites. In recent years researchers are also trying to use the same approach for reinforced concrete slabs. Many researchers have done experimental and analytical works on strengthening of one way as well as two way slabs using different FRPs in order to strengthen in different failure modes of slabs.

Ebead and Marzouk, 2004 [2] studied experimentally the behavior of two-way reinforced concrete slab with CFRP and GFRP strips. Slab specimen with different percentage of reinforcement for flexural and punching testing were used. The study revealed that an increase of the initial stiffness was achieved for flexural specimens, but decrease in ductility was also observed. Asamoha and Kankam, 2008 [3] performed experimentation on two-way concrete slabs reinforced with steel bars milled from scrap metals. Slabs were tested under a central concentrated load. They observed failure of slab specimens occurred always with combination of flexural and punching shear. Ebead, et al., 2002 [4] performed an experimentation and finite element analysis on strengthened two-way slabs using FRP laminates. At end they develop simple statistical models as replacements for finite element model to explain and predict the ultimate load carrying capacity of the slabs. Teng et. al. 2000 [7] proposed strengthening technique of RC cantilever slabs by bonding GFRP strips on the top surface (the tension side) and extending strips through grooves made in supporting wall. Based on the test results, effectiveness of the method of anchoring FRP strips into the walls through horizontal slots was evaluated. Smith and Kim, 2009 [8] done experiments with one-way reinforced concrete slab with central cutouts strengthened with FRP. FRP strips were applied at different positions on cut outs and observed that different positions give different failure patterns though the final failure in most cases was due to debonding only. Cracks in case of cutouts originate from corner point of cutout. They also correlated the experimental results with analytical model prepared by them.

Michel et al., 2007 [10] done experimental study on punching failure of reinforced concrete slabs. They have given numerical model to evaluate the ultimate punching load capacity of RC slab. Rizk et al., 2011 [11] evaluated the punching shear equation given in Canadian code, by conducting experiments on RC slabs. Study and comparison of different parameters affecting punching shear given in European and Canadian code was also studied by researchers. Statistical regression analysis was then conducted on the experimental data and new equation was proposed which takes into account effect of slab depth and reinforcement ratios. Researchers concluded that reinforcement ratio has significant effect on punching shear strength and it was found proportional to the reinforcement ratio to the power 0.38. Farghaly and Ueda, 2006 [9], done experimental and analytical evaluation of effectiveness of CFRP in increasing the punching shear capacity of slabs and observed significant increase in stiffness and punching shear capacity through strengthening.

Dai and Ueda, 2006 [15] performed experimental study to evaluate bond characteristics of FRP-concrete interface by performing pull out test. The bond characteristics are very important tool in modelling strengthened specimen and computing final capacity and failure analysis of specimen. Researchers also developed non-linear bond stress-slip model in order to compare the test results. Apostolska et al., 2008 [16] done analytical study on seismic performance of flat slab buildings by modelling moment resisting frame building and flat slab building in SAP 2000 and comparing results of modal analysis and non-linear static pushover .

Mosallam and Mosallam, 2003 [3] performed experimental and analytical study on the FRP strengthening on reinforced and unreinforced concrete slabs by testing full scale slab specimens. Study included retrofitting of slabs and repairing of tested slabs with CFRP. Load-deflection curves obtained from tests then studied by researchers to evaluate strength increase in both retrofitted and repaired specimens. The results described the strength increase of 500% in case of unreinforced case and 200% increase in case if reinforced slab specimens. Further, an FEM models of all the specimens were also studied by researchers and compared with experimental results for validation.

Haritos and Hira, 2004 [19] done an experimental investigation on reinforced concrete multi-span RC flat slab bridges with cantilever ends. The study included strengthening of bridge slabs using CFRP system as both laminates and fabrics. Dynamic tests were conducted on



strengthened slab specimens to evaluate performance and effectiveness of strengthening. Researchers quoted the strengthening system as satisfactory in improving performance of slabs. Alkarani and Ravindra, 2013 [18] presented an assessment of punching shear in flat slab buildings. The study elaborated the punching shear mechanism in flat slab systems due to transfer of unbalanced moments through slab-column joint creating unbalanced shear stresses around slab-column joint. Esfahani et al., 2009 [12] performed study on the strengthening of slab-column connection with CFRP sheets. During the study researchers performed tests on RC slabs strengthened with CFRP strips near slab-column connection region under cyclic loading. The experimental results were compared with results obtained from equations given in American and British standards. Researchers found that ACI equation underestimated the load in both unstrengthened and strengthened slabs as the code does not account for effect of flexural reinforcement. Ramanathan, 2008 [20] conducted study on effect of FRP width to spacing ratio on performance of the FRP-concrete bond in one way RC slab by testing RC concrete slab specimens strengthened with CFRP with different width patterns and concluded that more number of narrow closely spaced FRP strips proved more effective in strengthening than wider strips.

## FLAT SLAB SYSTEMS

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### 3.1 INTRODUCTION

Flat slabs are reinforced concrete slabs supported directly by columns without providing beams over columns. Flat-slab building structures have various advantages over conventional beam-column frame buildings as they provide ample amount of the free design of space, shorter construction time, easy to construct, economical, larger clear height, architectural–functional flexibility. Reinforced concrete slabs were developed initially in United States and Europe in beginning of 20<sup>th</sup> century. In order to increase area in shear resistance of flat slabs, slabs can be provided with drop panel or column capital or both. Drop panel is locally thickened section of slab around the column. It also helps in increasing negative moment capacity of slab at support. Column capital is flared portion at top of column, it helps in reducing clear span and total span moments. Sometimes flat slab building are coupled with beams at edges or around the opening, which stiffens discontinuous edges of slab and increases shear capacity of critical exterior slab-column connection. Behavior of flat slab building is different from reinforced concrete framed buildings. Two way slabs in framed buildings span in shorter direction whereas in case of flat slab system slabs span predominantly in longer direction.

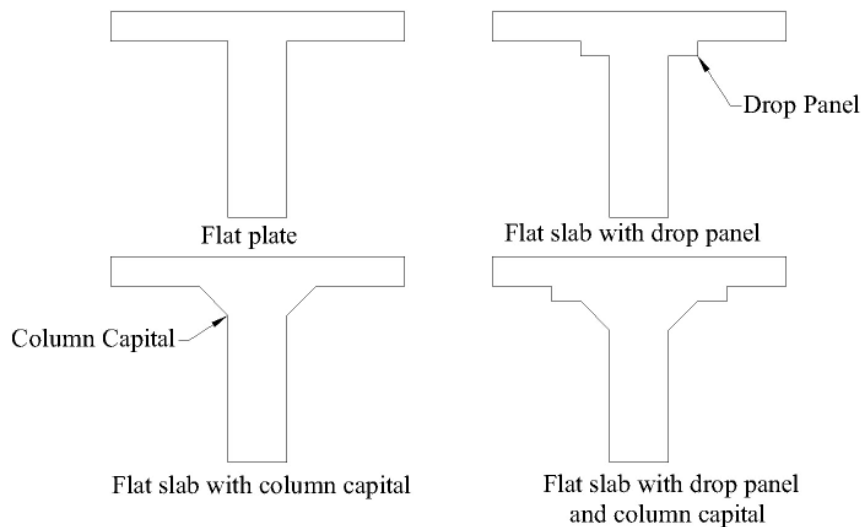


Figure 3.1 Flat slab with different types of supports

Nowadays flat slabs are becoming popular among architects and structural designers. These systems are not only being constructed in low and medium seismicity area but also in high seismicity regions. Since flat slabs have shown very poor performance in past earthquakes, so it has become concern to designers to improve performance of these systems under lateral loading. Fig. shows failure of flat slab buildings during past earthquakes. Flat slab buildings show more flexible behavior as compared to moment resisting frame systems [16], which means they undergo large deformations under seismic loading. During earthquake, slab column connections in flat slab systems experience unbalanced moments. Further these unbalanced moments produce non uniform shear stresses around slab-column connection across critical section. As a result these connections become vulnerable to punching shear failure.

In order to improve performance of flat slab system, conventional way is to combine building with either edge beams, or shear wall or both edge beam and shear wall system. Nowadays use of fiber reinforced polymer (FRP) in reinforced concrete structures for strengthening purposes is gaining popularity. Many researchers are studying effectiveness of fiber reinforced polymers in improving strength of reinforced concrete slabs and slab-column connection. This technique of using FRP is easy to implement in structure, does not add any considerable extra weight to structure and it does not change the aesthetics of the structure.

### **3.1.1 Failures in flat slab buildings**

As in case of flat slab systems there are no beams to support slab and slab is directly resting on columns, all the forces are to be transferred from slab to column through slab-column connection. There are two failure modes which flat slab can undergo

- (i) Flexural Failure
- (ii) Punching shear failure

#### **1. Flexural failure**

If moments at slab-column connection exceeds moment capacity of flat slab then slab fails in flexure. In this case cracks appear on bottom surface of slab before failure as shown in Figure 3.2. This mode of failure comprises of large plastic deformation, thus this is ductile type of failure. Flexural failure is desirable mode of failure in flat slab systems.

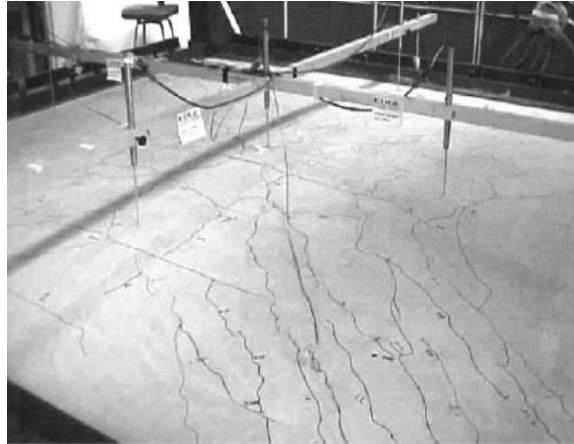


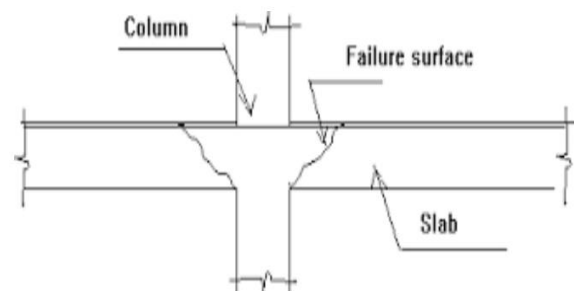
Figure 3.2 Flexural failure in slab

## 2. Punching shear failure

Punching shear failure is caused due to high concentration of shear forces near slab-column connection. In this case of failure cracks propagate diagonally through the slab originating from the column face forming pyramid like structure near connection as shown in Figure 3.3. This type of failure is brittle, as it does not allow large plastic deformations before failure. It is most common failure in flat slab systems.



Figure 3.3 (a) Punching shear failure



(b) Cracks penetrating slab

### 3.1.2 Transfer of unbalanced moment at slab column connection

Shear distribution in flat slab systems due to balanced dead load at slab column connection across the critical section which is at distance  $d/2$  from the column face, where  $d$  is effective depth of slab is uniform. Due to unbalanced gravity forces or lateral loading like wind or

earthquake acting on structure, unbalanced moments generate at slab-column connection [18]. Shear due to this unbalanced moment at critical is non uniform as shown in fig. Combining effect of uniform shear due to balanced gravity loads and non-uniform shear due to unbalanced forces result into non uniform shear distribution at critical section near slab column joint as shown in Figure 2.4.

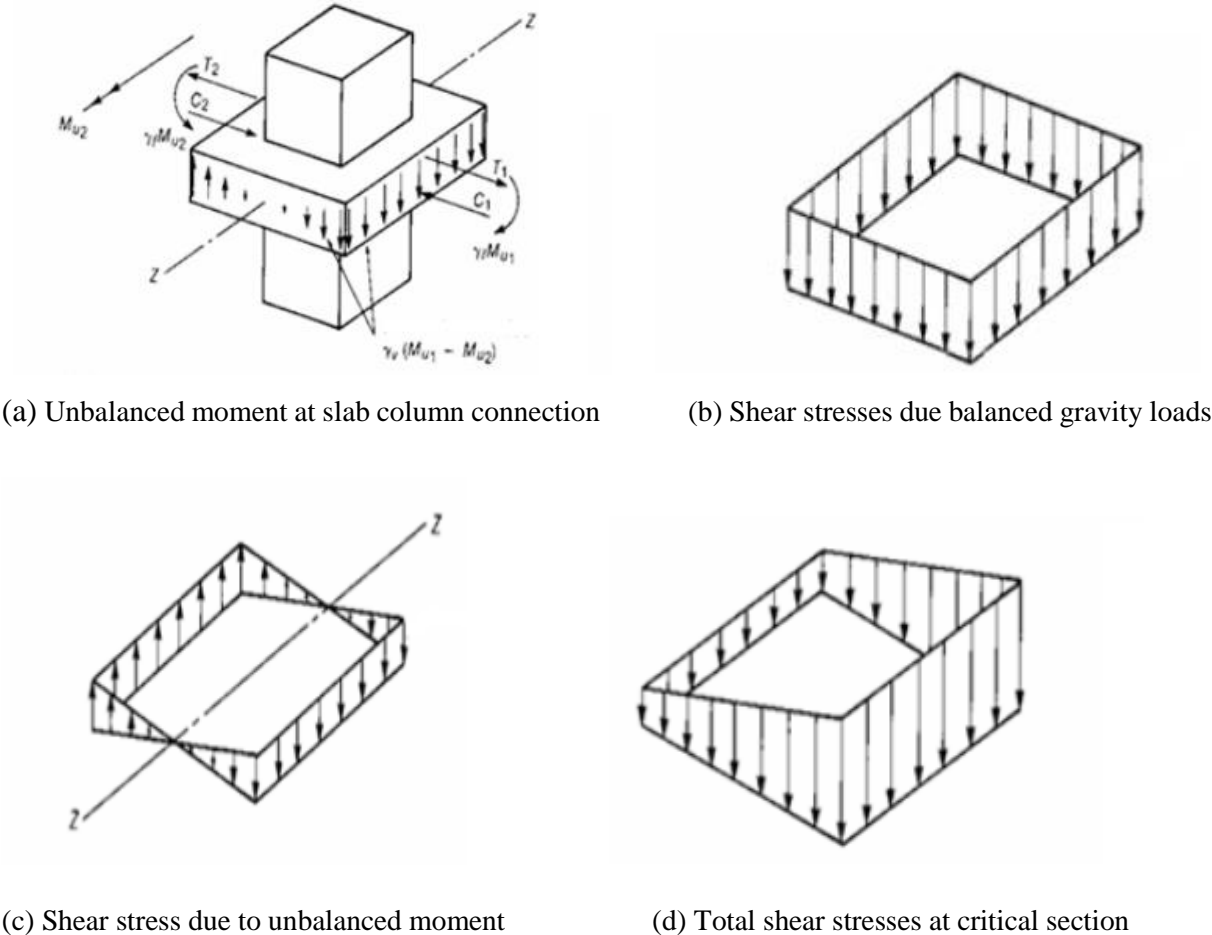


Figure 2.4 Transfer of unbalanced moment at slab-column joint [18]

## 3.2 PERFORMANCE EVALUATION OF FLAT SLAB AND MRF SYSTEMS

### 3.2.1 Seismic analysis

In order to evaluate performance of flat slab system and moment resisting frame system, 8 storey building having plan as shown in Figure 3.6 was analyzed with three different models namely M1, M2 and M3. M1 is reinforced concrete frame system, M2 is purely flat slab system without drops and capitals, and M3 is flat slab system with perimeter beams. Seismic analysis, design and non-linear pushover analysis of all buildings was done using SAP2000. Building was designed according to IS 456-2000.

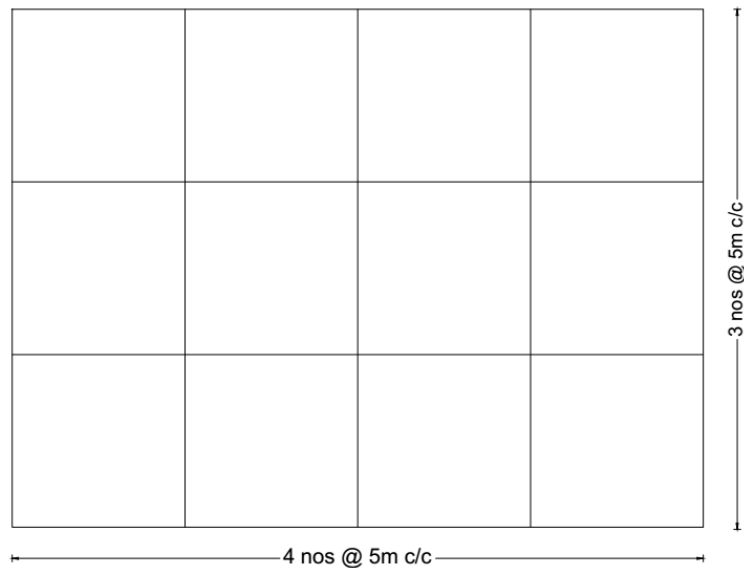


Figure 3.6 Plan of building

#### Model M1

Moment Resisting Frame System

Beam size- 300x300mm

Column size-500x500mm

Slab thickness-150mm

Steel –Fe 415

Concrete-M30

#### Model M2

Purely Flat Slab System

Column size-500x500mm

Slab thickness-150mm

Steel –Fe 415

Concrete- M30

### Model M3

Flat Slab with Perimeter Beams

Column size-500x500mm

Slab thickness-150mm

Perimeter Beams-300x300mm

Steel –Fe 415

Concrete- M30

All models were designed in considering Seismic Zone as IV, and soil type II (Medium soil)

Seismic zone factor for zone IV= 0.24

Results obtained from analysis of these different systems are tabulated below,

Table 3.1 Time periods of model M1

Mode	Direction	Time Period (T)	Frequency (f)
1	Y-axis	1.248 sec	0.802 sec <sup>-1</sup>
2	X-axis	1.227 sec	0.815 sec <sup>-1</sup>

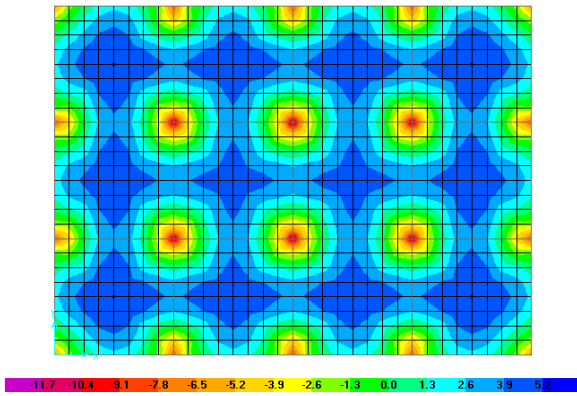
Table 3.2 Time periods of model M2

Mode	Direction	Time Period (T)	Frequency (f)
1	X-axis	2.173 sec	0.460 sec <sup>-1</sup>
2	Y-axis	2.170 sec	0.462 sec <sup>-1</sup>

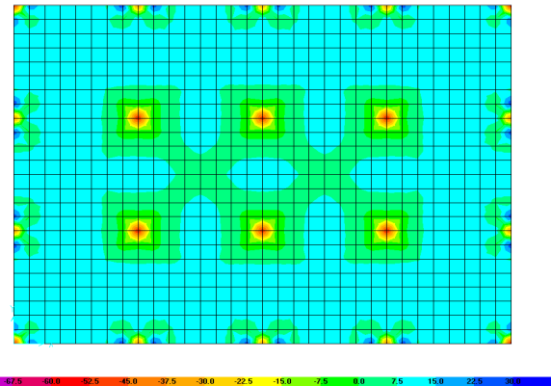
Table 3.3 Time periods of model M3

Mode	Direction	Time Period (T)	Frequency (f)
1	Y-axis	1.803 sec	0.554 sec <sup>-1</sup>
2	X-axis	1.788 sec	0.559 sec <sup>-1</sup>

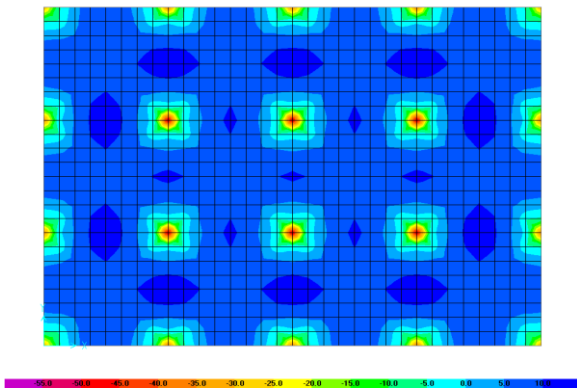
Time period of flat slab system is more as compared to moment resisting frame system and flat slab building with perimeter beams. This is one of the evidence that flat slab systems are more flexible as compared to MRF systems under lateral loading, and thus undergo large deformations.



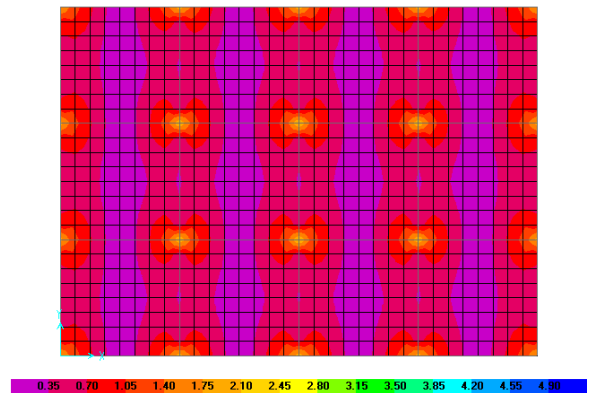
(a)  $M_{\max}$  in M1



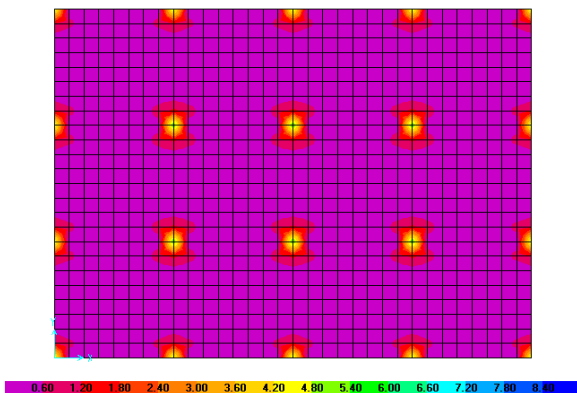
(b)  $M_{\max}$  in M2



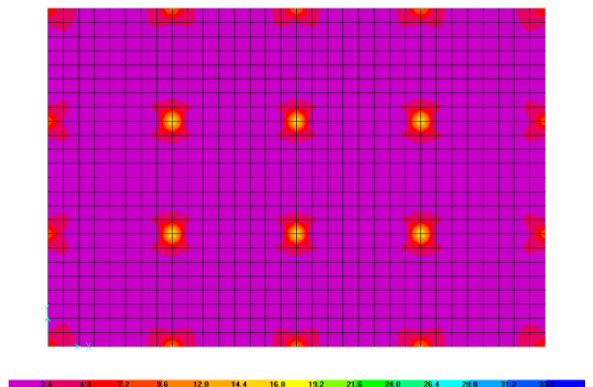
(c)  $M_{\max}$  in M3



(d)  $M_{eq}$  in M1



(e)  $M_{eq}$  in M2



(f)  $M_{eq}$  in M3

Figure 3.7 Moments in slab



Maximum moments at support and mid-section of slab at 7<sup>th</sup> storey of all buildings under load combination 1.5(DL+LL) and EQ-X are listed in following table and are also shown in Figure 3.7 (a), (b) and (c).

Table 3.4. Moments in slab ( $M_{max}$ ) due to gravity loading (DL+LL)

Model	$M_{max}$ at inside support (kNm)	$M_{max}$ at mid section (kNm)	$M_{max}$ at edge support (kNm)
M1	-11.37	3.64	-8.09
M2	-55.98	14.09	-37.83
M3	-52.69	11.87	-36.69

Table 3.5 Moments in slab due to seismic forces (EQ-X)

Model	$M_{max}$ at inside support (kNm)	$M_{max}$ at mid section (kNm)	$M_{max}$ at edge support (kNm)
M1	2.02	0.11	1.87
M2	25.37	0.55	22.76
M3	18.48	0.42	9.23

From above tabulated results it can be observed that moments in flat slab system have very high values as compared to frame system in all cases described above. As compared to flat slab system with edge beams flat slab system has little higher values of moments, which show that providing edge beams also does not prove very effective in reducing moments. It is evident that in flat slab system slab-column connections are subjected to too higher moments, which again contribute to their vulnerability to gravity as well as lateral forces as compared to frame systems.

### 3.2.2 Non-linear static pushover analysis

Non-linear static pushover for all models performed using SAP 2000. Hinges assigned to structural are the default hinges available in SAP 2000 according to the guidelines of FEMA 356. Base shear vs Roof displacement are plotted and compared as given below,

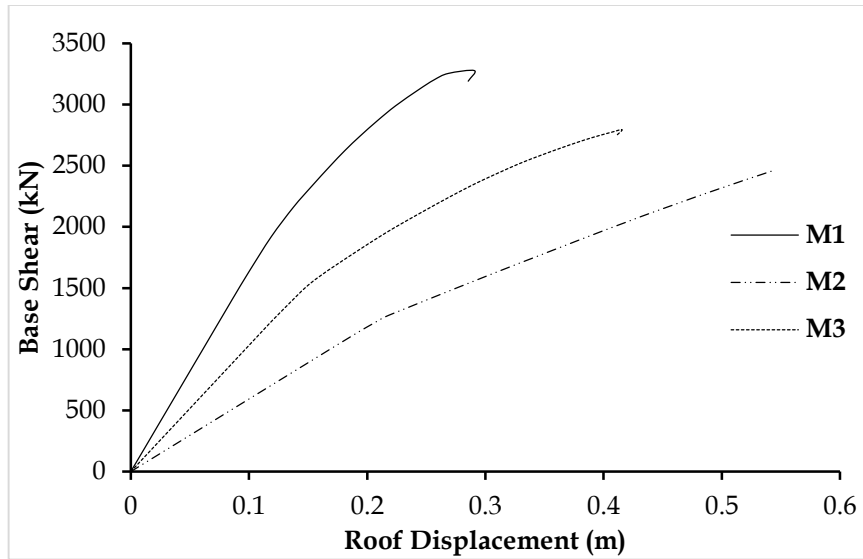


Figure 3.8 Pushover of M1, M2 and M3 in X direction

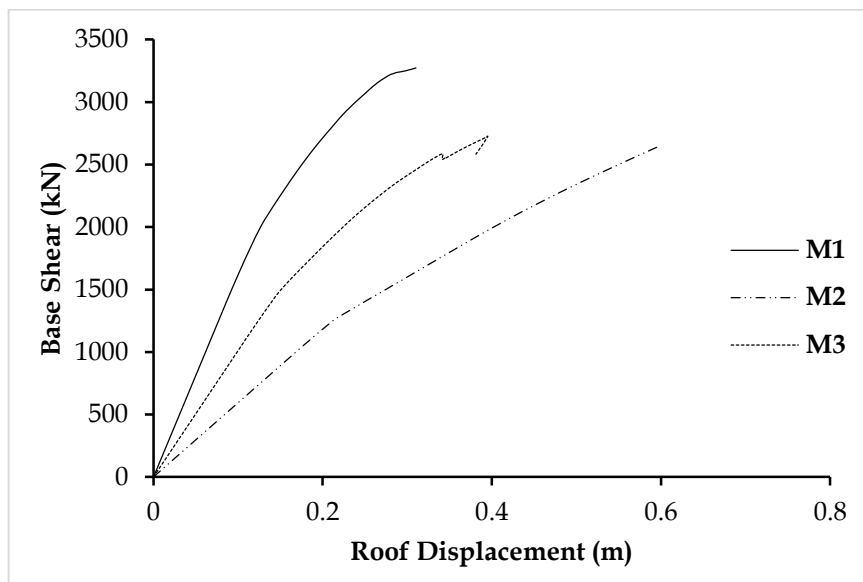


Figure 3.9 Pushover of M1, M2 and M3 in Y direction

It can be observed from above pushover curves that flat slab building is showing more deflection as compared to other models for the same base shear in both X and Y direction that is flat slab building is showing more flexible behavior to lateral loads as compared to RC frame building. This makes flat slab building more vulnerable to damage under lateral loads. As purely flat slab buildings show poor performance as compared to RC frame building as well as flat

slab building with edge beams, performance of such building systems need to be improved. Some conventional measures to improve performance of flat slab buildings include introduction of shear wall provided in specific location to increase stiffness of building. Drops and column capitals are also provided in order to increase punching shear area which leads to increased strength of slab column connection in punching shear.

**YIELD LINE ANALYSIS FOR RC SLABS**

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**4.1 INTRODUCTION**

Most of the methods described in texts, research journals and design codes for calculation of design moments for concrete slabs are based upon elastic theory. While, for calculation of reinforcement for slabs, strength methods are used which in turn take into account inelastic behavior of members at factored load. The same contradiction exists in analysis and design of concrete frame members like beams and columns. Also, elastic analysis gives over-conservative estimate of capacity, which in turn may result into uneconomic design of structural members. In order to eliminate this contradiction plastic analysis was introduced. Plastic analysis helps in eliminating the inconsistency of using elastic analysis and inelastic design procedures. It also permits use of reserved strength of concrete structural members and redistribution of moments which takes place after yielding of slab reinforcement.

Methods for plastic analysis of reinforced concrete frames are tedious and time consuming because of necessity of computing rotation capacities at all each hinge in structure. On the other hand reinforced concrete slabs have large rotation capacities due to much low tensile reinforcement than that of balanced value, thus it becomes easy to perform plastic analysis of reinforced concrete slabs and one of the approach of such analysis is ‘Yield Line Theory’.

Yield line is location along the slab such that, on overloading there would be large inelastic rotation at constant resisting moment. When inelastic rotation occur in slab upon overloading, the resisting moment per unit length measured along yield line is constant, thus it can be said that yield line provides axis for rotation for slab segment. After formation of adequate number of hinges in slab, it undergoes mechanism that is segments of slab between hinges and supports move without increase in load. Number of yield lines required for mechanism depends upon support conditions, loading conditions etc. In figure 4.1 one way slab carrying uniformly distributed load and simply supported at two shorter opposite edges is shown. As it is simply supported on edges, it is free to rotate about those edges. When applied moment reaches value equal to flexural capacity yielding of reinforcement starts and this first starts across the section where bending moment is maximum and results into yield line, as shown in Figure 4.1 (c).

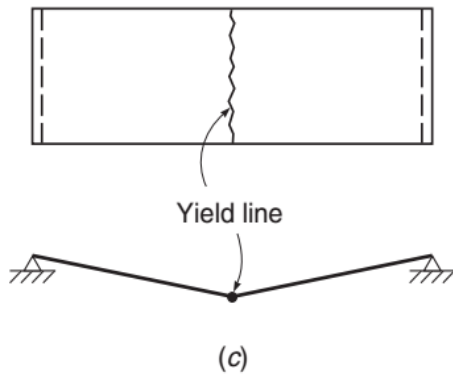
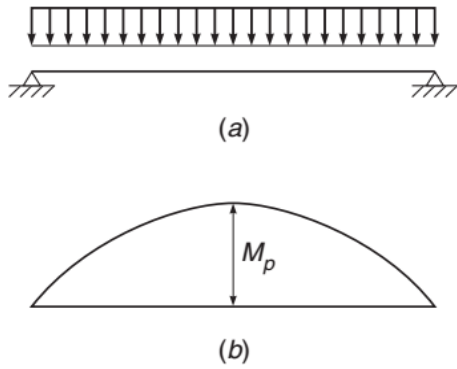


Figure 4.1 Yield lines for SS slab

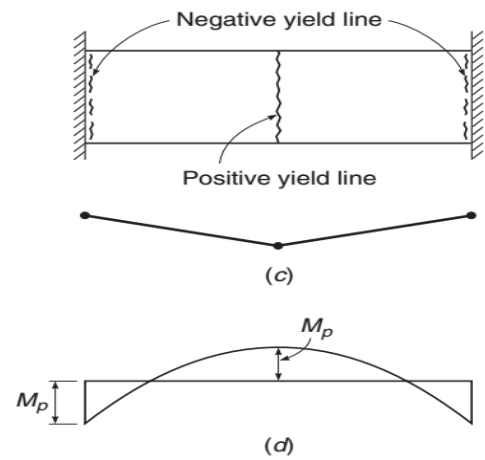
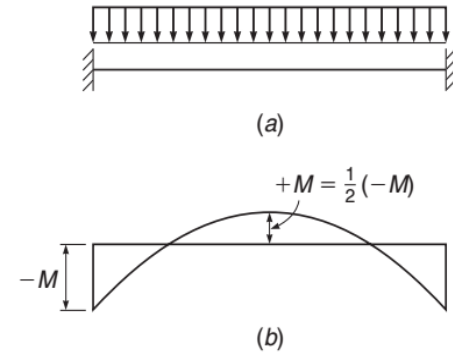


Figure 4.2 Yield lines for slab fixed on two edges.

Second example shown in Figure 4.2 is of one way slab uniformly loaded and fixed on two opposite edges. At initial stage elastic deformations occur in slab, the moment diagram for this is shown in Figure 4.2(b). Further increasing the load supports start yielding as they are highly stressed. On yielding rotations starts occurring at supports at constant plastic moment  $M_p$  but it does not reach mechanism at this point and thus can take loads further. On increasing load further at some point moment at mid span reaches its capacity. At this point yield line forms at this location and slab becomes mechanism as shown in Figure 4.2(c). After formation of mechanism slab undergoes large deflections until collapse takes place. Figure 4.2(d) shows moment diagram just before failure, which comprises of plastic moments about the sections where yield lines form.

## **4.2 UPPER BOUND AND LOWER BOUND THEOREMS OF ANALYSIS**

Yield line theory of plastic analysis is derived from general theory of structural plasticity. Theory of plasticity states that the collapse load of structure lies between two limits, namely upper bound and lower bound of true collapse load. In case of slabs above theorems can be stated as follows:

### **Upper bound theorem**

If for small increment of displacements, the internal work done by slab assuming that the moment along every yield line is equal to the yield moment and that boundary conditions are satisfied, is equal to the external work done by the given load for that small increment of displacement, then that load is the upper bound of the true load carrying capacity.

For given slab and loading, satisfaction of upper bound theorem implies that slab cannot take load higher than given load. Higher load will certainly cause failure, even collapse can occur at lower loads if selected mechanism is incorrect.

### **Lower bound theorem**

If for given external load, it is possible to find a distribution of moments that satisfies equilibrium requirements, with the moment not exceeding the yield moment at any location, and if the boundary conditions are satisfied then the given load is a lower bound of the true load carrying capacity.

If for the given slab and loading, lower bound condition is satisfied then it can surely take the given load whereas it may carry higher loads due to redistribution of moments.

For plastic analysis of structures either lower bound or upper bound theorem is used, and not both.

Yield line method of analysis of slabs is upper bound method and thus failure load calculated with the method can be higher than the actual load causing failure.

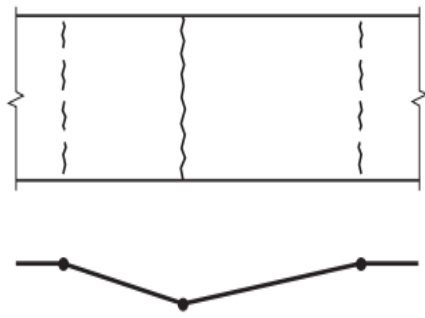
### 4.3 ESTABLISHING YIELD LINES

As discussed in above examples of simply supported slabs, yield lines can be plotted for slabs with different support conditions, loadings, geometry. For simple cases like simply supported, or fixed on opposite edges as discussed above location of yield lines is easy to establish. Yield lines form along maximum bending moment location, and fixed edges where rotation is hindered. When sufficient number of plastic hinges form in slab to result into mechanism, axes of rotation can be located along the support lines and over columns if any. Yield lines divide the slab into segments and these segments rotate about axes of rotation rigidly. Yield lines form the intersection of two segments of slab, thus it is straight line and also passes through the intersection of the axes of rotation of two adjacent slab segment as it should contain point of intersection of axes. Further yield lines need to be distinguished based on whether tension is on top or bottom. For conventions yield lines due to tension on bottom are called *negative yield lines* and yield lines due to tension on top are called *positive yield lines* [17].

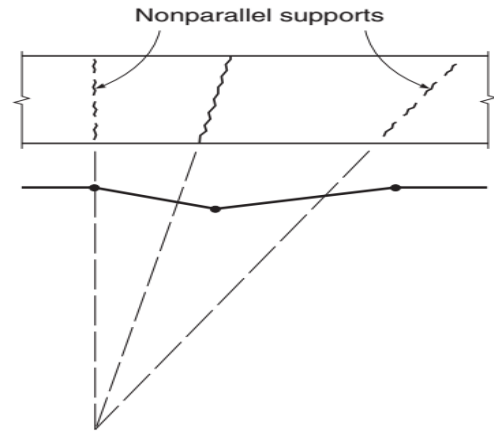
Guidelines for yield lines and axes of rotations:

1. As yield lines represent the intersection of two planes thus these are straight lines.
2. Yield lines represent axes of rotation
3. If edges of slabs are simply supported then they will also establish axes of rotation.
4. If edge is fixed a negative yield line may form which provide constant resistant to rotation.
5. An axis of rotation passes over any column support.
6. If slab is under concentrated load yield lines form under concentrated load radiating outward from the point of application.
7. A yield line between two slab segments must pass through the point of intersection of the axes of rotation of the adjacent slab segments.

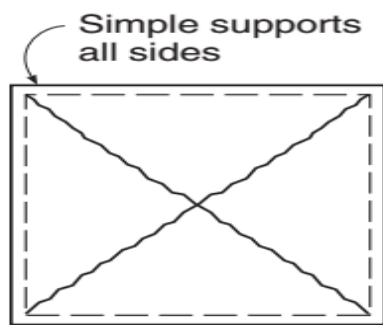
Following figures depict some examples of yield lines for slabs with different geometries, supports and loading conditions:



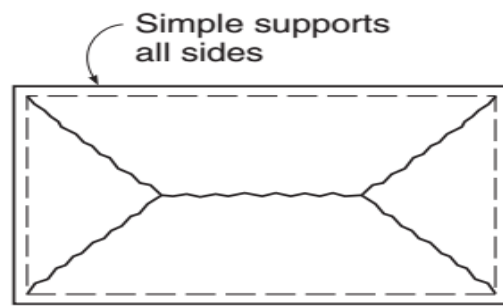
(a)



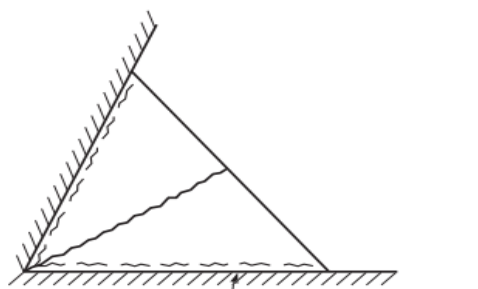
(b)



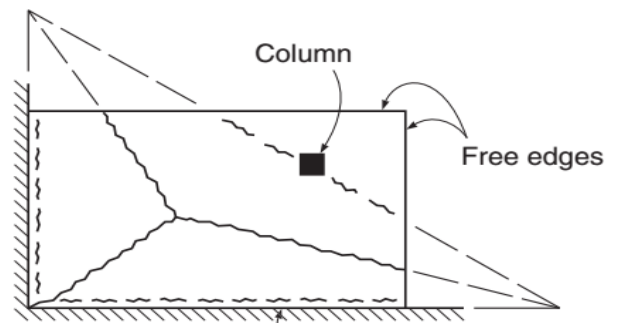
(c)



(d)



(e)



(f)



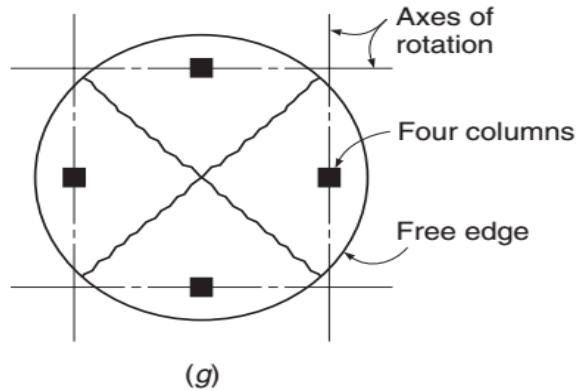


Figure 4.3 Yield lines for different loading and support conditions.

After establishing general pattern of yielding and rotation, next thing is to find out failure load. In order to establish failure load using yield line analysis there are two methods (i) segment equilibrium and other is (ii) virtual work. Both of these methods give an upper bound result, that is true failure load will always be less than that of calculated by these methods. One can use any of the above methods to analyze the slab.

### 1. Segment Equilibrium Method

In this method establishment of failure load is accomplished by equilibrium of different segments of slab. Each segment of the slab divided by yield must be in equilibrium under the action of applied loads, reactions at supports and resisting moments along the yield lines.

Along the yield line twisting moment and shearing forces in most of the cases are negligible as principle moment acts along yield lines thus, during formation of equations only moments are considered along yield lines.

### 2. Virtual Work Method

This method uses principle of virtual work. As the moments and applies loads on slab forms an equilibrium during the formation of yield lines. After formation of yield lines in slab, an increase in load on structure will result into deflect further at constant moment along yield lines. Thus, slab is given virtual deflection and calculation of corresponding rotations is done at various yield lines. Further internal work done by the resisting moments is equated with external work done by applies load for virtual displacement and the relation between resisting moments and applied loads are obtained. Equations are written for plastic

deformations only as elastic deformations are very small compared to plastic deformations. For simplicity of equations unit virtual displacement is applied to slab.

#### 4.4 SLABS UNDER CONCENTRATED LOAD

When concentrated load acts on reinforced concrete slab at interior location, yield lines form nearly in circular pattern. Positive yield lines radiate outward from point of load forming a fan like structure on slab surface as shown in Figure 4.4.

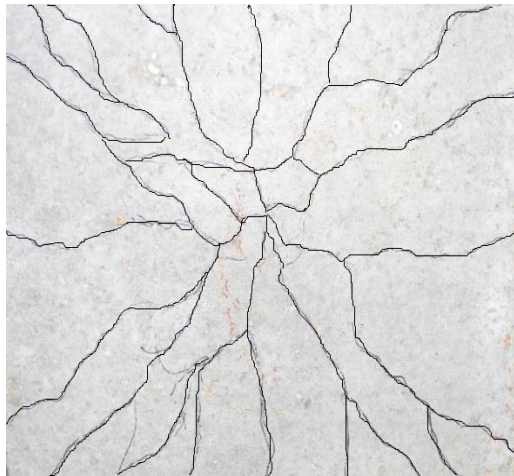


Figure 4.4 Radial yield lines in slabs under concentrated load

Consider radial yield line of radius 'r' radiating from center and forming segment with other radial line through angle  $\beta$  as shown in Figure 4.5. Let 'm' be positive moment of resistance along yield lines and 'm'' be resisting moment at fixed supports if supports are fixed.

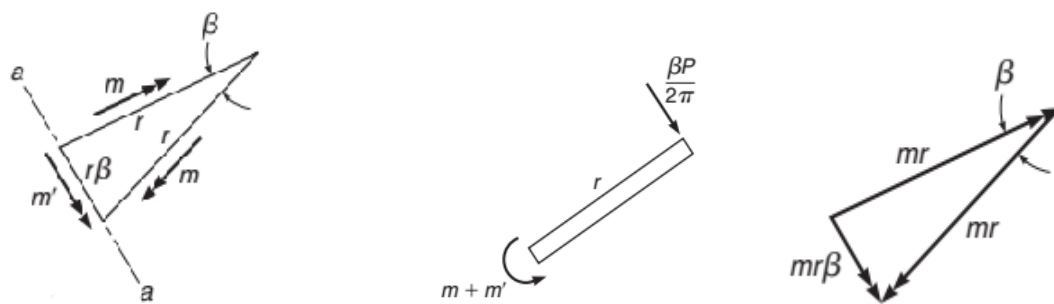


Figure 4.5 Moment vectors acting along radial yield lines

Let P be central applied load,

Taking vector sum of moments  $mr$  acting along radial yield lines we have resultant of  $mr\beta$  acting along  $r\beta$ , thus per unit moment is equal to  $m$  only and this moment acts in same direction as  $m'$ .

Taking moment about a-a axis we have,

$$(m + m')r\beta = \frac{\beta Pr}{2\pi}$$

$$P=2\pi(m+m')$$

Thus, we can estimate collapse load from the above formulae for any shape of slab, knowing resisting moments per unit length of yield lines.

**EXPERIMENTAL PROGRAM**

**5.1 INTRODUCTION**

Two different slab specimens one with reinforcement and other without reinforcement at central portion fabricated for experimental study. In second case reinforcement in middle portion is not provided in order to simulate situation of removed part of corroded reinforcement with side reinforcement unremoved from in support section of slab. For strengthening of these slab specimen in both mode of failures different types of FRPs with different patterns were used. As the slab was two-way FRP layers were applied in both directions with epoxy resin.

**5.2 DETAILS OF TEST SPECIMENS**

Two types of slab specimen dimensions 850mmX800mm one with full steel reinforcement in both direction and other specimen with no reinforcement in central portion. Steel reinforcement of grade Fe 415 was used in both cases with 20mm effective cover.

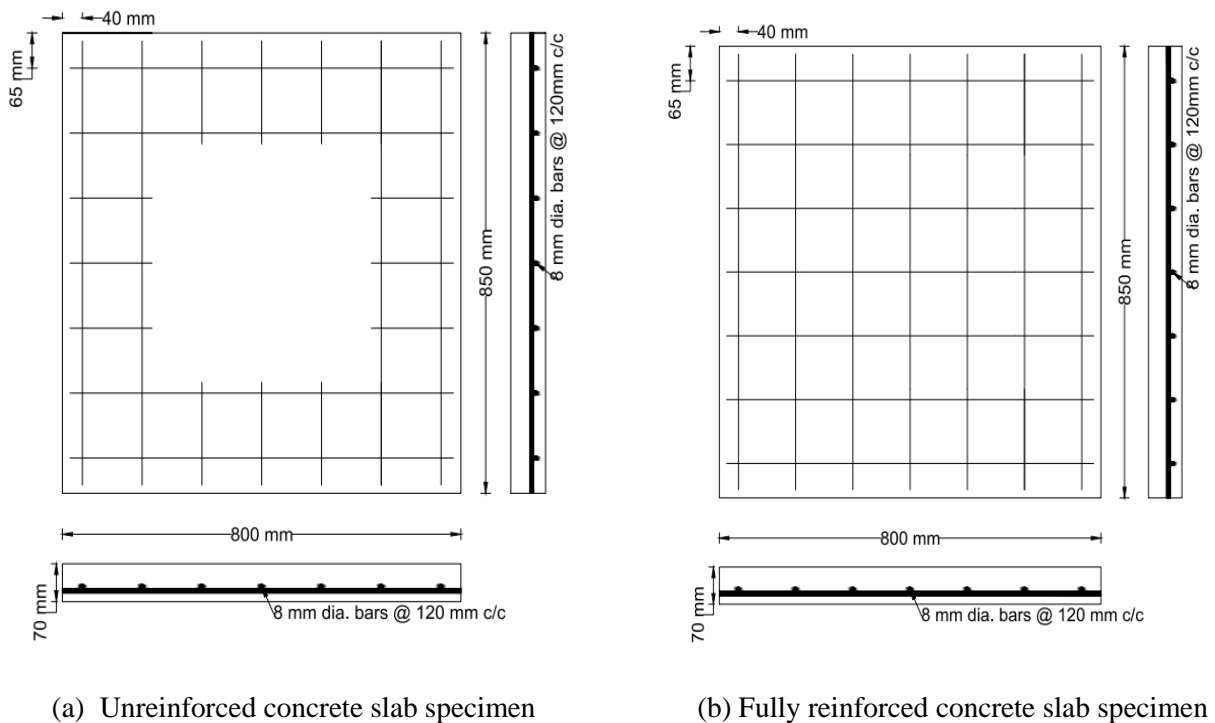
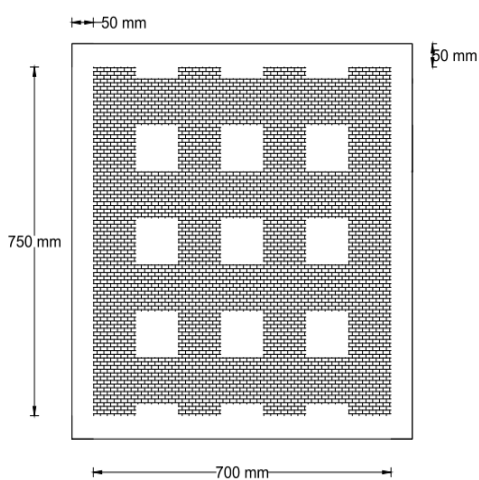


Figure 5.1 Reinforcement details of slab specimens

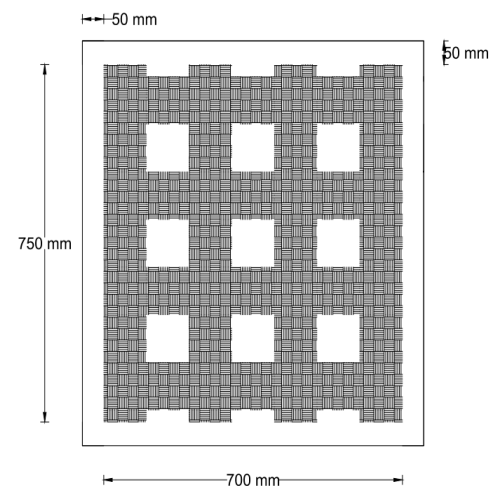
Seven bars were placed at center to center distance of 120mm in both directions. The details of reinforcement in both specimen are shown in Figure 5.1.

All slab specimens were properly fabricated and cured for 28 days before application of FRP. Before application of FRP layers, bottom surface of slab specimens was grinded to make it smooth and remove top cement layer to get hard concrete surface for FRP wrapping. Dust from grinded surface was completely removed before wrapping process to avoid loose contacting due to dust. FRP material was available in form of sheets from which strips of desired shape and size was cut.

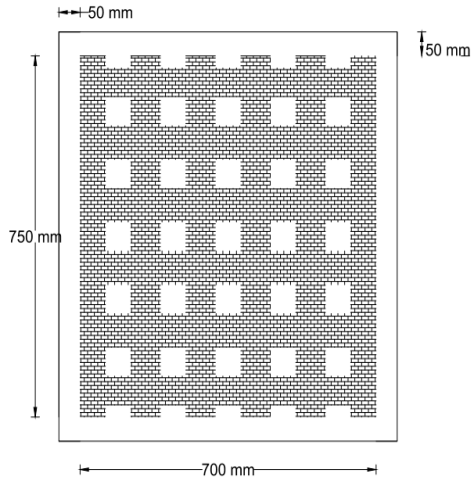
An epoxy used for binding was Dr. Fixit 211 two component adhesive, comprises of base and hardener. Both components has to be mixed in proper quantity and manner. For the given strengthening purpose one is to one ratio of base and hardener was used by volume. Two components were properly mixed to form epoxy resin before application. First markings were made on slab surface to locate FRP strip locations. First layer of epoxy was applied on strip location and allowed it to settle and fill pores on surface. After that second layer of epoxy was applied above layer and then FRP strips were properly placed and adhered with help of roller to ensure no voids between epoxy layer and FRP. As FRP strips were applied in both directions in grid pattern special care was taken at junction of two layers by applying little more epoxy and properly rolling them. After application of FRP specimens were cured for minimum 7 days before performing testing.



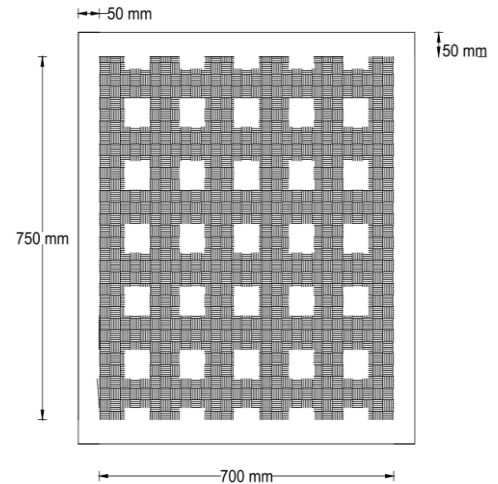
(a) GFRP strengthened specimen with 100mm strips



(b) CFRP strengthened specimen with 100mm strips



(c) GFRP strengthened specimen with 70mm strips.



(d) CFRP strengthened specimen with 70mm strips.

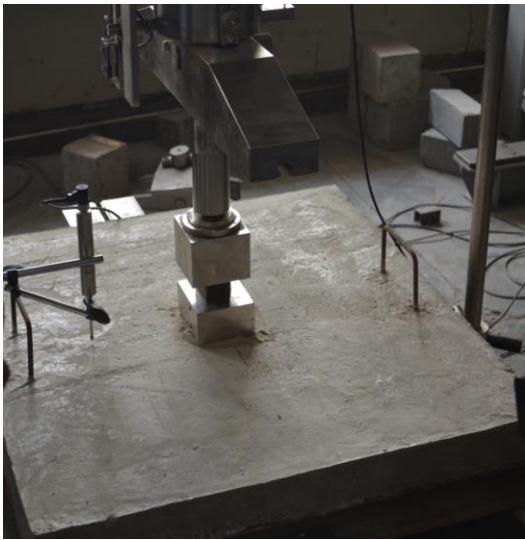
Figure 5.2 Different patterns of CFRP and GFRP wrapping

Table 5.1 Description of different specimens

Specimen ID	Description of Specimen
UC	Unreinforced Control specimen
RC	Reinforced Control specimen
USG-100	Unreinforced specimen strengthened with 100mm GFRP sheets
RSG-100	Reinforced specimen strengthened with 100mm GFRP sheets
USC-100	Unreinforced specimen strengthened with 100mm CFRP sheets
RSC-100	Reinforced specimen strengthened with 100mm CFRP sheets
USG-70	Unreinforced specimen strengthened with 70mm GFRP sheets
RSG-70	Reinforced specimen strengthened with 70mm GFRP sheets
USC-70	Unreinforced specimen strengthened with 70mm CFRP sheets
RSC-70	Reinforced specimen strengthened with 70mm CFRP sheets

### 5.3 TEST SETUP

All the slab specimens including control and strengthened are tested by applying central load on 100mmX100mm area. All specimen were kept simply supported on all four sides. Central deflection corresponding to loading was measured with help of LVDT. The test setup is shown in figure 5.3. The load was gradually increases at the rate of 100 N per second in the equipment. Corresponding load-deflection values were directly recorded in data acquisition system. Load was increased till the failure of the specimens, as load started dropping down.



(a) Loading setup.



(b) Data acquisition system.

Figure 5.3 Test setup

### 5.4 TEST RESULTS AND DISCUSSION

#### 5.4.1 Load-deflection characteristics

Load vs Deflection responses for individual test specimen were obtained from test. An average curve for one type of specimen is then plotted from two sets of data obtained from two testing of two slabs of same type. This is done by taking an average value of two loads corresponding to same deflection. The percentage increase in capacity over control specimen for different FRPs and patterns are also calculated from the load data. Further this load-deflection plot is used to compute energy dissipation for different specimens.

Table 5.2 Peak loads, corresponding deflections and percentage increase in strength of different strengthened specimens

Specimen ID	Peak Load (kN)	Peak Deflection (mm)	% Increase in Load Capacity
RC	32.87	21.25	
RSG-100	43.5	21.83	32.34
RSC-100	70.67	12.37	115
RSG-70	43.5	16.10	32.34
RSC-70	60.38	11.07	83.69
UC	12.82	19.10	
USG-100	40	17.94	212
USC-100	53.46	10.87	317
USG-70	39.60	12.33	209
USC-70	53.93	11.21	320

### 1. Reinforced concrete slab specimens

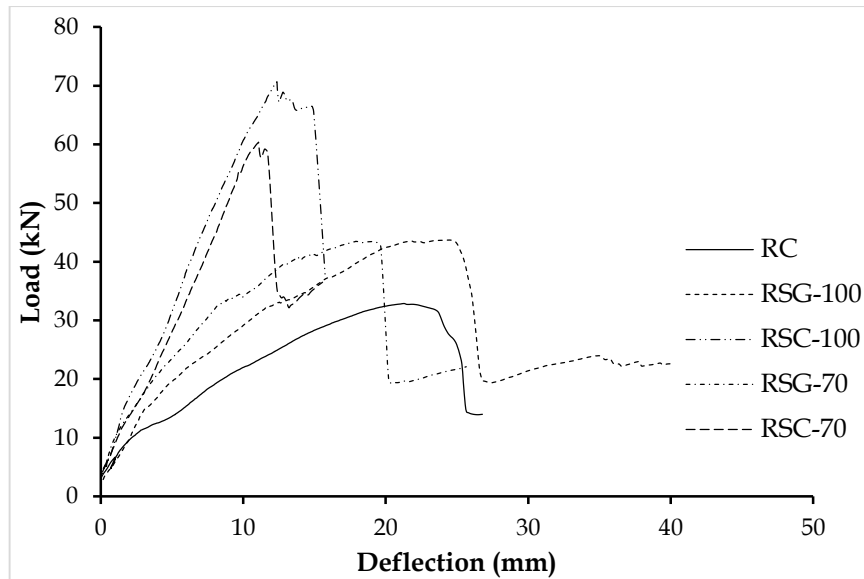


Figure 5.4 Load-deflection curves for reinforced slab specimens

The specimen strengthened with 100mm GFRP strips (RSG-100) showed peak load of 43.5kN which is 32.34% higher than the reinforced concrete control (RC) specimen. RSG-100 specimen shows large post yield deformations. In case of CFRP strengthened specimen (RSC-



100) the peak load is 70.67kN which is 115% increase in load as compared to control specimen. RSC-100 specimen does not show any post yield deformation while it shows sudden drop in load deformation curve. RSG-70 had peak strength of 43.5kN and corresponding deformation of 16.1mm. This is same increase in strength as RSG-100 but it shows less deflection at peak load and ultimate deformation is also less. Thus, stiffness which is the ratio of load and deflection, of the RSG-70 specimen is higher than RSG-100 specimen. Specimen RSC-70 showed peak strength of 60.38kN and corresponding deflection is 11.07mm. The decrease in strength of RSC-70 specimen is due to premature debonding failure of the specimen.

## 2. Unreinforced specimen

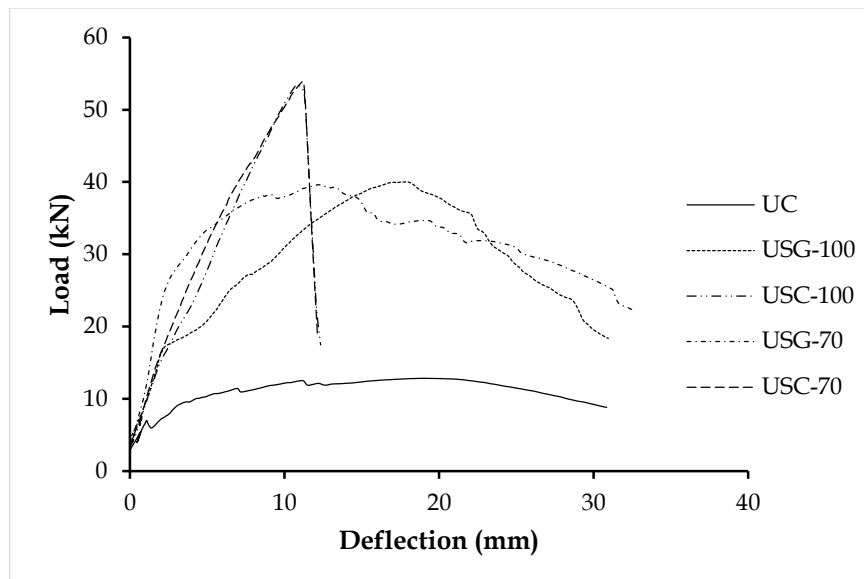


Figure 5.5 Load-deflection curve for unreinforced slab specimen

Unreinforced specimen strengthened with 100mm GFRP strips (USG-100) shows peak strength of 40kN which is 212% more as compared to unreinforced control specimen (UC). USG-100 specimen is showing considerable post yield deformation without sudden drop in load. In case of unreinforced specimen strengthened with 100mm CFRP strips, peak load is 53.46kN and it is showing sudden drop in load before failure. Load increase in case of USC-100 specimen is about 316% more as compared to unreinforced control specimen. Specimen strengthened with GFRP 70 mm strips showed peak load value of 39.6kN. Percentage increase in strength in this case is 208% which is nearly same as USG-100 specimen but peak deflection of USG-70 is less as compared to USG-100 specimen. For USC-70 specimen peak load is 53.93kN which is

percentage increase of 320% over control specimen. It can be also seen that plots for both USC-100 and USC-70 is nearly overlapping. Comparing the stiffness of the specimens, it can be point out that USG-70 specimen has higher stiffness than USG-100 and USC-70 specimen has shown little higher stiffness than USC-100 specimen.

#### 5.4.2 COMPARISON WITH CODAL PROVISIONS

All slab specimens in the experimental study were tested under the central load making slabs more likely to fail in punching shear failure. Also, reinforcement in reinforced slab specimens was adjusted so that punching failure will prevail [9]. There are no direct methods or empirical formulae available in literature or in codes in order to evaluate punching shear strength and behavior of FRP strengthened RC slabs. There are different codal provisions, equations available to calculate the punching strength of the RC slabs. Major factors which affect the punching shear strength are compressive strength of concrete, ratio of reinforcing steel, aspect ratio of concentrated loading area like column, loading plate in our case, effective depth etc. Most of the codes have tried to cover maximum number of these factors to formulate simple equations to use.

ACI 318-02 [21] gives the following formulae which calculate ultimate punching shear strength based on concrete compressive strength and aspect ratio of column-side length but it does not account for reinforcement ratio in slab.

Punching shear strength is considered as smallest of the followings,

$$V_c = 0.083 \left( 2 + \frac{4}{\beta_c} \right) \lambda \sqrt{f'_c} b_o d$$

$$V_c = 0.083 \left( 2 + \frac{\alpha_s d}{b_o} \right) \lambda \sqrt{f'_c} b_o d$$

$$V_c = 0.083 \times 4 \lambda \sqrt{f'_c} b_o d$$

Where,

$b_o$  = rectangular critical perimeter at a distance of  $d/2$  from the face of a column,

$\beta_c$  = ratio of long-to-short sides of the column

$\alpha_s$  = 40 for interior columns, 30 for edge columns, and 20 for corner columns.

$\lambda$ - Modification factor reflecting the reduced mechanical properties of lightweight concrete,  $\lambda=1$  for normal concrete

$d$  is effective depth of slab,  $f'_c$  is concrete compressive strength.

Japan Society of Civil Engineers (JSCE) [22] gives an equation for punching shear calculation which includes effect of concrete compressive strength and aspect ratio of column-side length as well as reinforcement ratio. The equation is as given below,

$$V_c = 0.188\beta_d\beta_p\beta_r\sqrt{f'_c}Ud$$

Where,

$$\beta_d = \left(\frac{1000}{d}\right)^{\frac{1}{4}}, \beta_p = (100\rho)^{\frac{1}{3}}, \beta_r = 1 + \left(\frac{1}{1 + \frac{c}{d}}\right)$$

$U$  = critical perimeter with round corners at a distance of  $d/2$  from the face of a column, which can be given as  $U = 4c + \pi d$

\*Both  $\beta_d$  and  $\beta_p$  should not be assumed to be greater than 1.5.

IS 456-2000 [23] gives an equation for permissible shear stresses in flat slab at critical section near the column. Value of punching load can be calculated by multiplying this shear stress value by area of critical region in punching. The equation is as given below,

$$V_c = k_s\tau_c b_o d$$

Where,

$k_s = (0.5 + \beta_c)$ ,  $\beta_c$  is ratio of shorter side of column to longer side.

$\tau_c = 0.25\sqrt{f_{ck}}$  for limit state method of design and  $\tau_c = 0.16\sqrt{f_{ck}}$  in working stress method of design.

All above codal equations are for steel reinforced concrete slabs and were directly used for calculation of punching shear capacity of reinforced control slab specimen. In order to use these equations to calculate punching shear capacity of FRP strengthened slab specimens, among all

the design variables in equations, effective depth and reinforcement ratio were adjusted to reflect the effect of FRP strengthening on tension face as follows [9],

$$d_{eq} = \left( \frac{A_f E_f h + A_f E_f d}{A_f E_f + A_f E_f} \right)$$

$$\rho_{eq} = \rho_s + \rho_f \left( \frac{E_f}{E_s} \right)$$

The punching shear strength of all FRP strengthened reinforced slab specimens then calculated by substituting above values of  $d_{eq}$ ,  $\rho_{eq}$  for  $d$  and  $\rho$  in different codal equations mentioned above.

Values of different parameters required for punching strength calculation are given as follows,

For slab specimens,

$$E_s = 2 \times 10^5 \text{ MPa}, E_{f(CFRP)} = 2.42 \times 10^5 \text{ MPa}, E_{f(GFRP)} = 6.9 \times 10^4 \text{ MPa}$$

$$h = 70 \text{ mm}, d = 50 \text{ mm}, c = 100 \text{ mm}.$$

$$f_{ck} = 21 \text{ N/mm}^2, f_c' = 16.8 \text{ N/mm}^2$$

$$A_s = 351.68 \text{ mm}^2, A_{f(CFRP)} = 200 \text{ mm}^2, A_{f(GFRP)} = 100 \text{ mm}^2$$

$$\rho_s = 0.0059, \rho_{f(CFRP)} = 0.0033, \rho_{f(GFRP)} = 0.0017$$

Modified  $\rho$  and  $d$  for CFRP strengthened slabs

$$\rho_{eq} = 0.01, d_{eq} = 58.30 \text{ mm}$$

Modified  $\rho$  and  $d$  for GFRP strengthened slabs

$$\rho_{eq} = 0.0071, d_{eq} = 51.80 \text{ mm}$$

Table 5.3 gives comparison of experimental values and values calculated using codal expressions.

Table 5.3 Comparison of experimental and calculated values of punching shear strength

Specimen ID	$V_{u \text{ exp}}$ (kN)	$V_{u \text{ cal}}$ (kN)			$V_{u \text{ exp}}/V_{u \text{ cal}}$		
		ACI	JSCE	IS	ACI	JSCE	IS
RC	32.87	42.21	35.91	32.30	0.78	0.91	1.02
RSG-100	43.50	44.20	40.18	33.89	0.98	1.08	1.28
RSC-100	70.67	51.87	53.43	40.78	1.36	1.32	1.73
RSG-70	43.50	44.20	40.18	33.89	0.98	1.08	1.28
RSC-70	60.38	51.87	53.43	40.78	1.16	1.13	1.48

From above table it can be observed that experimental results are quite comparable with that of results obtained from the codal expression. In can also be seen that results obtained from ACI-318-02 and IS 456-2000 show more deviation from experimental results, it is due to the fact that both expressions do not take in to account effect of tensile reinforcement on punching strength of slabs. While results obtained from expression given by JSCE accounts for effect of flexural reinforcement in slab through factor  $\beta_p$  thus results obtained from this are more comparable with experimental results.

### 5.5 ENERGY DISSIPATION

Energy dissipation is calculated for all specimens in the study to compare overall effectiveness of strengthening technique and values are compared with control specimen and other specimens. Energy dissipation capacity is calculated by computing area under load-deflection plot which gives total energy dissipated during loading till failure. Further cumulative energy dissipation vs deflection is also plotted for different specimens of both reinforced and unreinforced cases in order to show total energy dissipated from starting to each loading step.

Table 5.4 Energy dissipation values for different slab specimens

Specimen ID	Energy Dissipation (kN-mm)
RC	604.54
RSG-100	868.35
RSC-100	694.86
RSG-70	745.57
RSC-70	528.98
UC	340
USG-100	1008.78
USC-100	383.73
USG-70	1031.37
USC-70	396.95

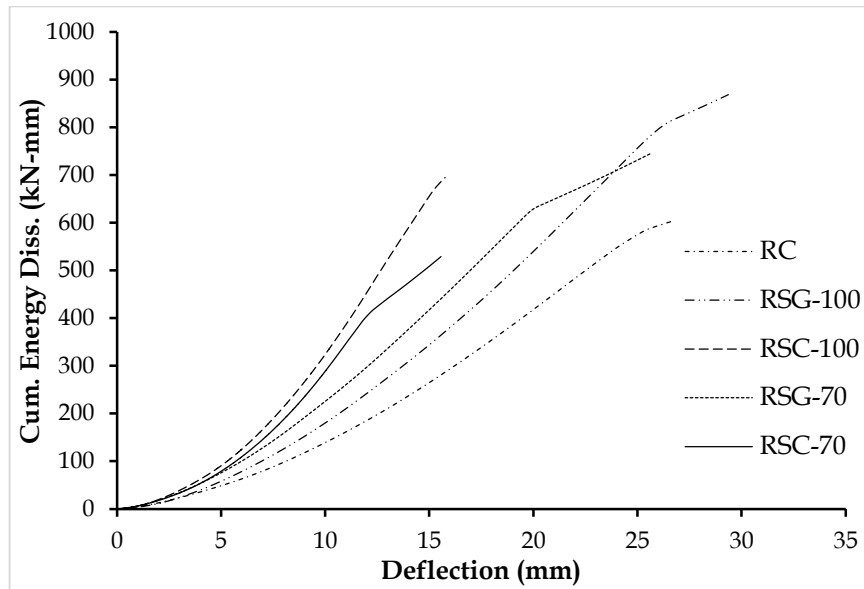


Figure 5.6 Energy Dissipation vs Deflection in reinforced slab specimens

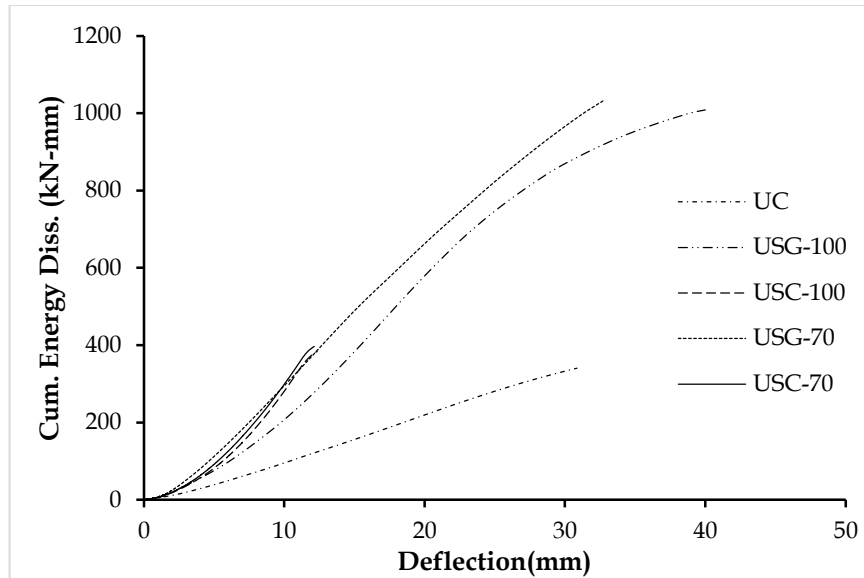


Figure 5.7 Energy Dissipation vs Deflection in unreinforced slab specimens

It can be observed that energy dissipation values of all strengthened specimens are higher than control specimens in both reinforced and unreinforced slab specimens. Also, values are different for different types of FRPs and different patterns used which can be observed from table. For GFRP strengthened specimens energy dissipation values are higher as compared to CFRP strengthened specimens. CFRP has very high modulus of elasticity as compared to GFRP thus failure load in CFRP strengthened slabs are higher as compared to GFRP strengthened specimens. Also CFRP does not allow large deformation of the slab specimens which result in to sudden failure of specimen and less energy dissipation. Thus, GFRP strengthening proves very effective in energy dissipation as compared to CFRP strengthening.

## 5.6 FAILURE AND CRACK PATTERN ANALYSIS

Failure analysis was performed on the tested specimens by observing crack pattern formed after testing as from crack patterns one can assess governing type of failure in the specimen. According to yield line theory of slabs, in case of RC slabs under central point load, radial cracks develop on bottom surface of slab under flexural failure. In case where punching shear dominates, shear failure can be observed in critical region around load along with flexural cracks on bottom (tension face). Some specimens failed in pure punching shear, some specimens failed in flexure-punching mode of failure while some specimens in unreinforced slabs like UC and USG-100 Showed flexure failure without undergoing punching. Thus

specimens strengthened with different FRPs and different patterns showed different modes of failures. Following context describes the failure of different specimens,

### 5.6.1 Reinforced slab specimens

#### 1. RC

In case of RC specimen radial cracks were observed on bottom face along with the shear failure around loaded area as shown in Figure 5.8(b). Flexural cracks on bottom face were radiating from central area and were extended up to sides of the slab as shown in Figure 5.8(c). On top of slab loading plate got punched into concrete and other than this there were no other cracks on top surface of the specimen. Around the central loaded area diagonal shear cracks formed in direction of depth of slab which caused cone failure, which is punching shear failure. In Figure 5.8(d). One can also observe the bending of reinforcement in central portion where load was applied.

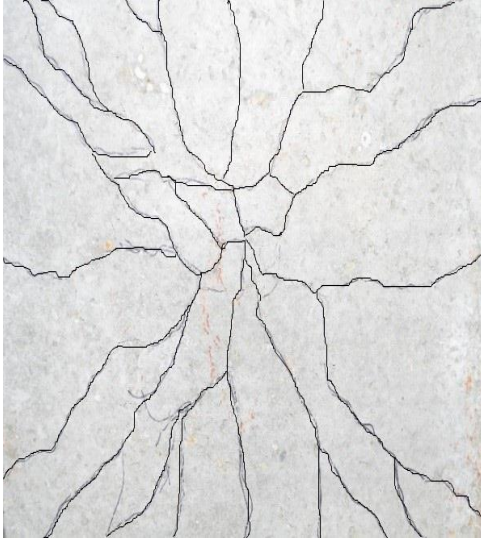


Figure 5.8 (a) Punching on top face



(b) Central punching failure region.





(c) Radial cracks in RC



(d) Bending of reinforcement in central region

## 2. RSG-100

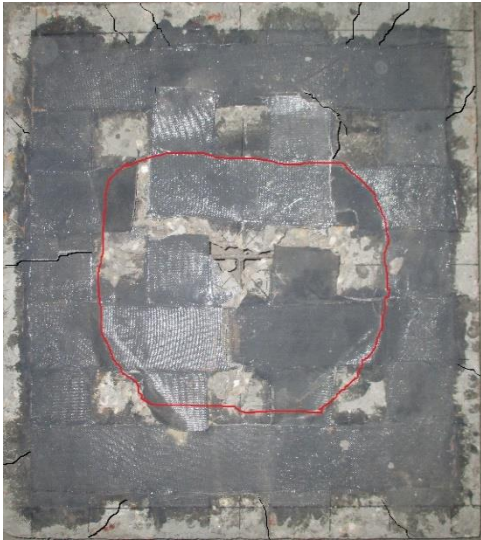


Figure 5.9 (a) Cracks on bottom of RSG-100



(b) Punching on top of RSG-100

In case of RSG- 100 specimen loading plate got punched into the concrete indicating punching shear failure in specimen and there were no other crack on top surface. No delamination of FRP strips observed in this case. On bottom surface central portion concrete came out and fell apart due to punching action and thus failure was limited in central portion only as shown in Figure 5.9(a). Rupture of GFRP strips was observed mainly near central portion along radial flexural cracks which were extending up to sides of slab. Rupture of GFRP indicates that

it completely took part in the resisting load and deflection of slab and full capacity of GFRP got utilized.

### 3. RSC-100



Figure 5.10 (a) Punching on top RSC-100

(b) Cracks on bottom face of RSC-100

In this case also loading plate got punched into concrete causing punching failure and on top surface there were no cracks indicating punching failure. Central portion of concrete came out but it did not fell apart as in cases of RC and RSG-100. Radial hair cracks were also observed on bottom face of specimen but no rupture of CFRP strips took place along these cracks. Delamination of CFRP strips took place in very small area near one corner of specimen and in some strips near central area where concrete came out but FRP strips other than this area remained intact.

### 4. RSG-70

In case of RSG-70 specimen wide cracks were observed on bottom face near central area which were radiating up to sides of slab, which were not observed in case of RSG-100 in which failure was limited in central portion. Along with these wide flexural cracks slab failed in punching also, thus indicating the failure as flexure-punching. Rupture of GFRP strips observed along cracks near central area as well as along some radial cracks near sides of slab as shown in Figure 5.11(b). Concrete at central portion came out but it did not fall apart.

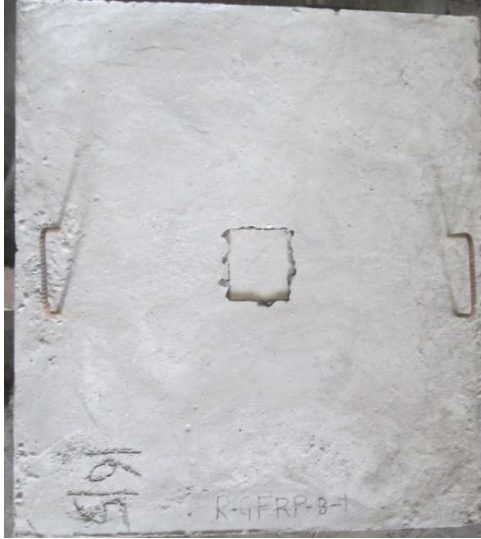


Figure 5.11 (a) Punching on top RSG-70



(b) Cracks on bottom face of RSG-70

## 5. RSC-70



Figure 5.12 (a) Delamination of CFRP strips  
in RSC-70



(b) Cracks on bottom face of RSC-70

This specimen failed completely due to delamination of CFRP strips. Close spacing of strips helped in resisting deflection of slab creating high stresses in FRP system. As CFRP has very high modulus of elasticity so it did not cause rupture of strips but delamination took place at concrete FRP interface. Radial cracks were observed on concrete surface same as in case of RC specimen. There was no punching shear in this case so top face remain intact. Due to

premature debonding failure it showed less increase in strength as compared to RSC-100 specimen.

### 5.6.2 Unreinforced slab specimens

#### 6. UC



Figure 5.13 (a) Cracks on top face of UC

(b) Cracks on bottom face of UC

In this case as middle portion of slab was provided no reinforcement. Under loading cracks started forming on top of slab in central of unreinforced portion. Further diagonal cracks also formed on bottom surface of specimen which can be seen in Figure 5.13(b). Cracks on top surface were wide causing failure of slab along diagonal cracks on top and bottom, which implies flexural failure of slab prior to punching. On bottom side of slab one can see many radial cracks originating from center, but diagonal cracks were wide as compared to other cracks and these cracks divided the slab in triangular portion along them. On bottom face there are cracks along the edges of central unreinforced portion also but concrete did not fall apart from that area along those cracks.

#### 7. USG-100

In this case also failure occurred due to punching shear, load plate got punched into slab. Cracks on top surface also observed during loading as shown in Figure 5.14(a). On bottom face wide diagonal cracks can be observed along which rupture of GFRP strips took place as shown in Figure 5.14(b). Central portion concrete came out and fell apart as GFRP in that area

got ruptured completely. There was also rupture of GFRP strips along the edge line of unreinforced area in central portion which can be seen in Figure 5.14(b).



Figure 5.14 (a) Cracks on top face of USG-100

(b) Cracks on bottom face of USG-100

#### 8. USC-100

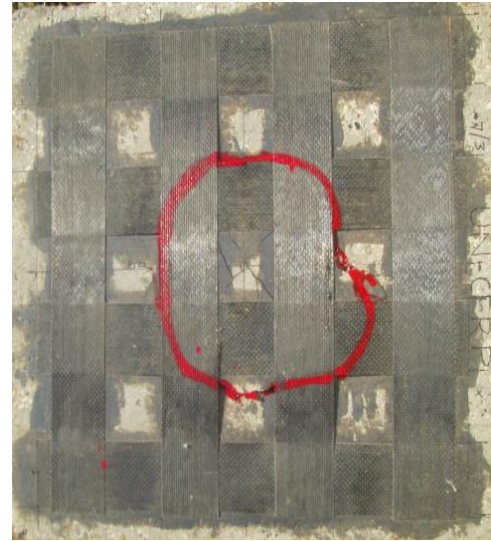


Figure 5.15 (a) Punching on top face of USC-100

(b) Bottom face of USC-100

In this case loading plate got punched into slab causing punching failure but there were no cracks on top face of slab specimen as there were in case of UC and USG-100 specimens. On bottom face concrete came out in central portion as a result of punching failure causing debonding in small portion of some strips around it. There was no sign of radial cracks as there

was in UC and USG-100 specimens on bottom face and failure was limited in central portion only as shown in Figure 5.15(b) indicating pure punching failure.

#### 9. USG-70



Figure 5.16 (a) Cracks on top face of USG-70      (b) Cracks on bottom face of USG-70

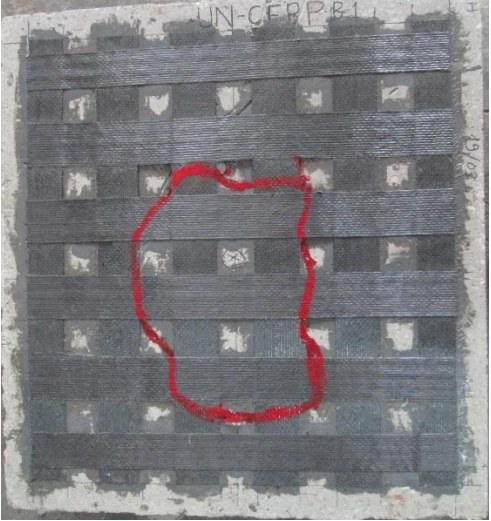
In this case no punching failure observed as there was in above cases. Due to close spacing of GFRP strips, load get transferred effectively to the whole area of slab without getting concentrated in central portion and thus avoiding punching. On top face of slab one can see cracks along the edges of central unreinforced portion along with diagonal cracks. On bottom face radial cracks were observed out of which diagonal cracks are wide and rupture of GFRP strips took place along these lines.

#### 10. USC-70

In this case punching shear failure took place and loading plate got punched into slab. Failure is similar to that was observed in case of USC-100 specimen. There were no cracks on top surface of specimen. On bottom face central portion came out due to punching failure but concrete did not fall apart and there were no flexural cracks suggesting pure punching failure. Delamination in small portion near central failure occurred. There are no radial cracks on bottom face of slab. Only central portion failed and came out as shown in Figure 5.17(b).



Figure 5.17 (a) Punching on top face of USC-70



(b) Bottom face of USC-70

## MODIFIED FLAT SLAB SYSTEM

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### 6.1 INTRODUCTION

As discussed in chapter 3 flat slab systems show poor performance under lateral loads. It was found out that flat slab buildings shows more flexibility as compared to MRF systems and it can be observed from static pushover analysis of flat slab buildings. Flat slab buildings are more vulnerable to damage during earthquake and thus performance of flat slab buildings need to be improved in seismically active areas. In order to improve performance of flat slab buildings, by increasing the strength and stiffness of floor slab by wrapping with FRP composites is proposed. This will be done by modifying the flexural stiffness of floor slab of flat slab building analyzed in chapter 3 with modification factor. Stiffness modification factor will be taken from the load vs deflection curves of tested slab specimens.

### 6.2 STIFFNESS MODIFICATION FACTOR

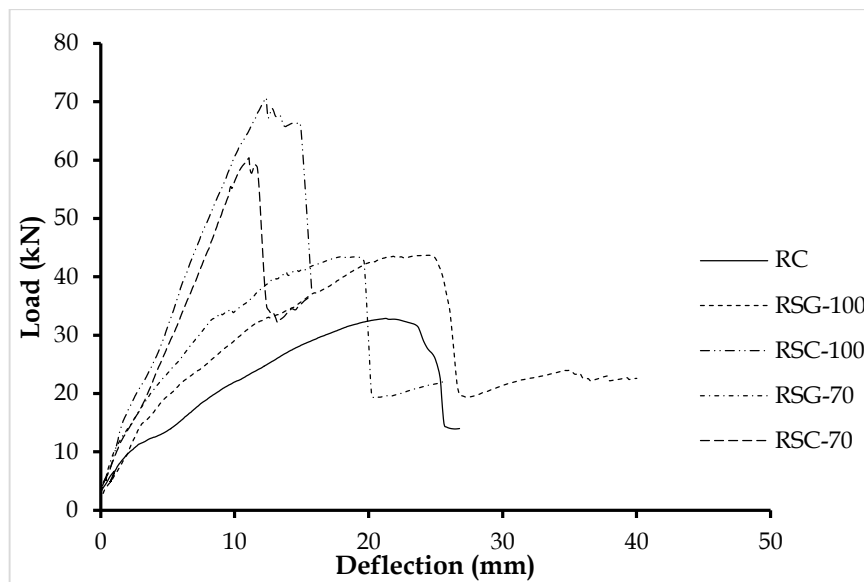


Figure 6.1 Load-Deflection curves for reinforced concrete slabs



Stiffness is slope of load-deflection curve.

$$Stiffness = \frac{Load}{Deflection}$$

For control specimen RC

$$S_{RC} = 1.54 \text{ kN/mm}$$

For GFRP strengthened specimen RSG-100

$$S_{RSG-100} = 2.13 \text{ kN/mm}$$

For CFRP strengthened specimen RSC-100

$$S_{RSC-100} = 5.71 \text{ kN/mm}$$

For GFRP strengthened specimen RSG-70

$$S_{RSG-70} = 2.7 \text{ kN/mm}$$

For CFRP strengthened specimen RSC-70

$$S_{RSC-70} = 5.45 \text{ kN/mm}$$

Taking ratio of  $S_{RSC-70}$  and  $S_{RC}$  to get effective increase in stiffness by CFRP strengthening over control specimen we get

$$S_e = 3.5$$

Taking ratio of  $S_{RSG-100}$  and  $S_{RC}$  to get effective increase in stiffness by GFRP strengthening over control specimen we get

$$S_e = 1.4$$

### **6.3 Modal Analysis Data**

Above factors are used in model of flat slab building (M2) discussed in chapter 3 as flexural stiffness modifiers in slab and new models are analyzed in SAP2000 under seismic loading and for non-linear static pushover.

M4- Model strengthened with CFRP using  $S_e = 3.5$ .

M5- Model strengthened with GFRP using  $S_e = 1.4$

## Time periods of models

Table 6.1 Time periods of model M4

Mode	Direction	Time Period (T)	Frequency (f)
1	X-axis	1.858 sec	0.537 sec <sup>-1</sup>
2	Y-axis	1.856 sec	0.538 sec <sup>-1</sup>

Table 6.2 Time periods of model M5

Mode	Direction	Time Period (T)	Frequency (f)
1	X-axis	1.960 sec	0.510 sec <sup>-1</sup>
2	Y-axis	1.965 sec	0.510 sec <sup>-1</sup>

Table 6.3 Time periods of model M2

Mode	Direction	Time Period (T)	Frequency (f)
1	X-axis	2.173 sec	0.460 sec <sup>-1</sup>
2	Y-axis	2.170 sec	0.462 sec <sup>-1</sup>

Comparing the values of above time with model M2 which is purely flat slab system, it can be observed that, time period of both strengthened buildings M4 and M5 are less than model M2. This means model M4 and M5 buildings are comparatively less flexible under lateral loading as compared to M2, which is one of the indications of improved performance of M4 and M5 over M2 and improved performance of strengthened structures.

### 6.4 NON-LINEAR STATIC PUSHOVER ANALYSIS

Non-linear static pushover for all models performed using SAP 2000. Hinges assigned to structural were the default hinges available in SAP 2000 according to the guidelines of FEMA 356. The results of pushover analysis of model M4 M5 are plotted along with the other models of buildings M1, M2 and M3 in order to compare the performance of all buildings. The pushover curves for all models are shown below

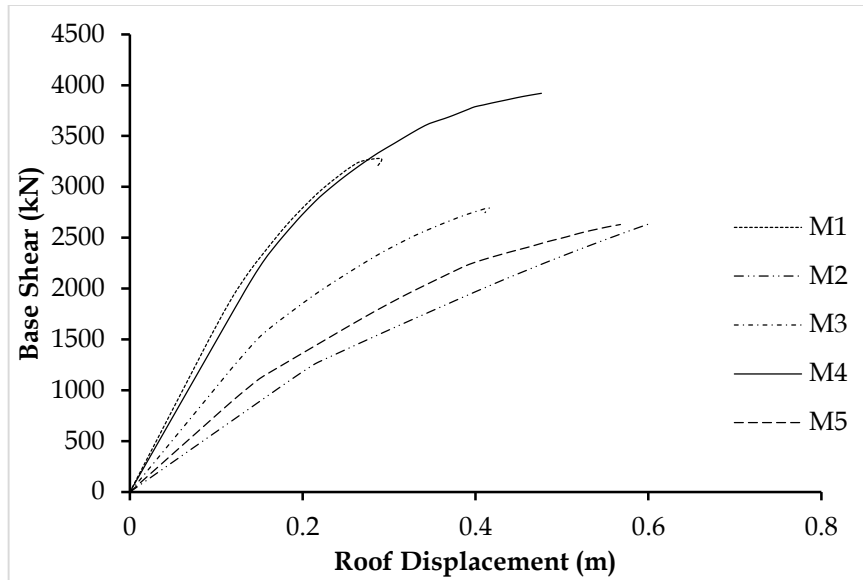


Figure 6.2 Pushover of M1, M2, M3, M4 and M5 in X direction

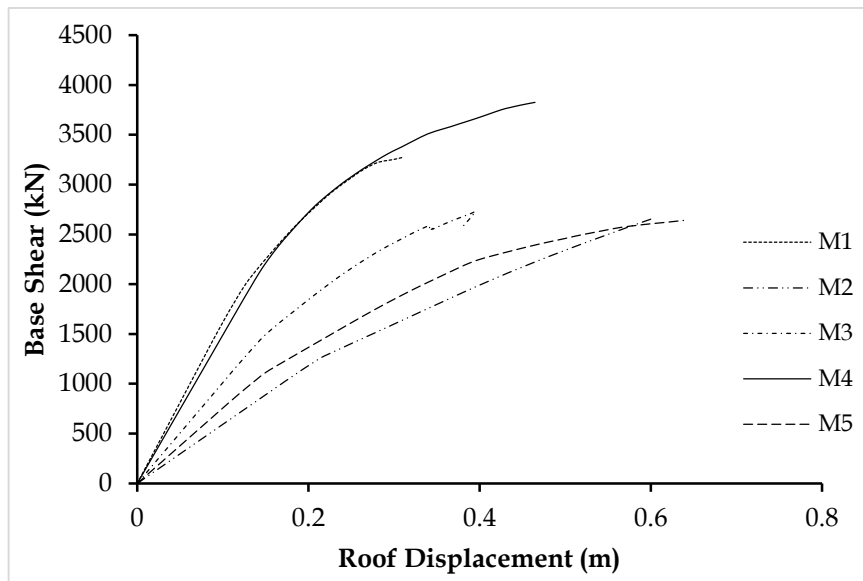


Figure 6.3 Pushover of M1, M2, M3, M4 and M5 in Y direction

It can be observed from above pushover curves plot M5 in both X and Y directions lie above M1 but below M1 and M3 but above M2 which means performance of M5 is poor as compared to M1 and M3 but there is significant improvement in M5 as compared to purely flat slab building M2. In case of plot of M4, it lies above plots of all the models in both X and Y direction which indicates that model M4 has performance better than all other models. Thus, it is evident

from the above results that FRP strengthening of slabs can significantly improve performance of flat slab buildings under lateral loadings.

### 6.5 INTER-STOREY DRIFT COMPARISON

Inter-storey drift is difference of lateral displacement of given storey and the storey below. For all the models variation of inter-storey drift at different stories in X and Y direction under EQ-X (Earthquake force in X direction) and EQ-Y (Earthquake force in Y direction) respectively are plotted and compared as given below,

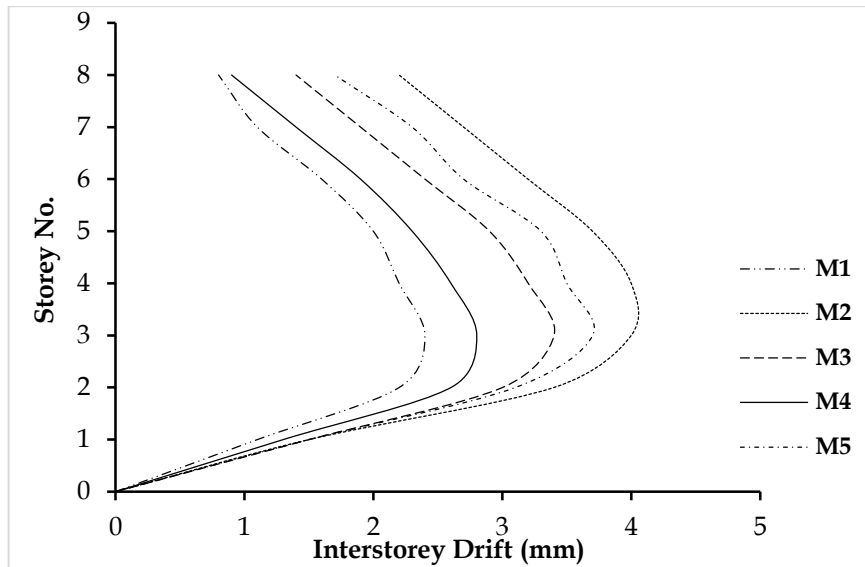


Fig. 6.4 Inter-storey drift of all building models in X direction under EQ-X.

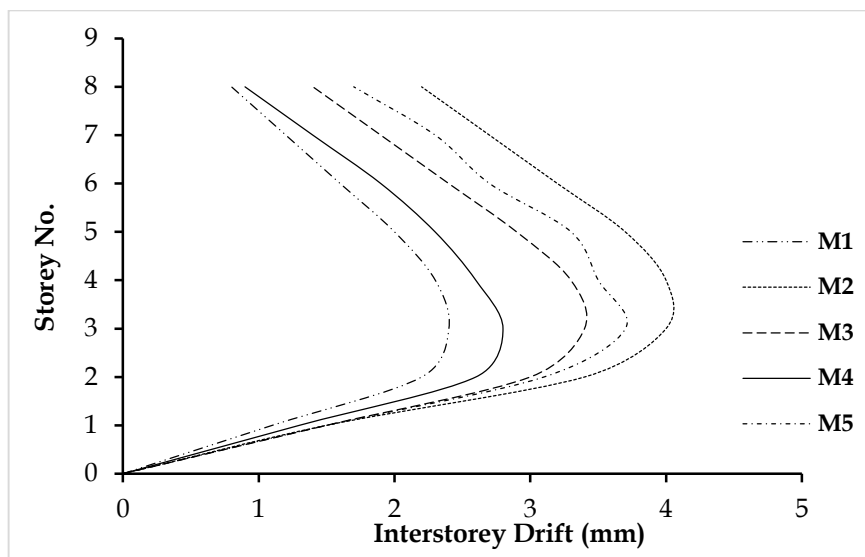


Fig. 6.4 Inter-storey drift of all building models in Y direction under EQ-Y.

Maximum inter-storey drift for all models occur at third storey. It can also be observed that model M2 shows highest inter-storey drift in both directions while model M1 shows lowest inter-storey drift and drift values for model M3 is in between. Plot of model M5 and M4 both fall below plot of M1 which indicates effectiveness of both strengthening techniques in reducing inter-storey drift of flat slab building. Further it can also be observed that model M4 which is strengthened with CFRP has shown very high value of reduction in inter-storey drift as compared to model M5 which is strengthened with GFRP.

## CONCLUSIONS

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1. Difference in behavior of flat slab system and moment resisting frame system was studied by modeling the buildings of both types in SAP2000. After comparing time periods and static pushover curves of different models of the buildings it was concluded that purely flat slab systems show flexible behavior as compared to RC framed buildings under lateral loading.
2. Moments in different building models due to dead load and earthquake load was also compared and it was concluded that purely flat slab building systems are subjected to very high amount of moments at all locations as compared to RC frame building system.
3. Evaluation of effectiveness of different FRPs with different patterns in increasing strength of reinforced and unreinforced concrete slabs was done by conducting experimental program. It was concluded that both GFRP and CFRP proved effective in strengthening slabs. In case of fully reinforced concrete slabs strengthened with GFRP an average increase of 32.34% was observed while in case of CFRP strengthening an average increase of 100% was observed over control specimen. In case of unreinforced specimens strengthened with GFRP increase in strength was by 210% and in case of CFRP strengthened specimens an increase in strength was 320% over the unreinforced control specimens.
4. Comparison of experimental results obtained in RC slab specimens with different codal provisions for punching shear strength was also performed by using modified parameters in order to account effect of FRP strengthening. It was concluded that ACI and IS code show more deviation from experimental results as expressions given in these codes does not account for flexural reinforcements in slab. While expression given by JSCE account for effect of flexural reinforcement and thus results obtained by this expressions were comparable with experimental results.
5. Energy dissipation values for different specimens were also studied which were calculated from load-deflection plots of all specimens. It was concluded that GFRP proved more effective in energy dissipation as compared to CFRP, as CFRP strengthened slab

specimens showed sudden failure due to very high modulus of elasticity of CFRP and chances of debonding failure under some circumstances.

6. In case of GFRP strengthened slabs in both reinforced and unreinforced cases, slab with more number of close strips (RSG-70 and USC-70) showed less deflection as compared to slabs strengthened with wide strips (RSG-100 and USG-100). Thus, it can be concluded that strengthening with more number of narrow closely spaced FRP strips resulted into increased stiffness of slab. While such behavior was not observed in case of CFRP strengthening, as CFRP strips in any case did not undergo rupture but failed in either punching shear or debonding and thus full strength of CFRP could not get utilized prior to failure.
7. In order to study effect of FRP strengthening of slabs in flat slab system, a model of purely flat slab building was modified by increasing flexural stiffness of slabs using results obtained from experimental load-deflection data. After comparing results of seismic analysis and non-linear static pushover analysis of all models it was concluded that strengthened building models show significant improvement in performance over conventional model of flat slab building.
8. Effect of CFRP and GFRP strengthening in RC slab on reduction in inter-storey drift of flat slab building was also studied. It was concluded that both techniques prove effective in reducing inter-storey drift of flat slab building by significant amount. CFRP strengthened flat slab system showed very high reduction in inter-storey drift values as compared to GFRP strengthened system as the stiffness modification factor for CFRP strengthened slab was higher than that of GFRP strengthened slab.

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