# ASSESSING PROBABILITY OF GREAT EARTHQUAKES IN CENTRAL HIMALAYAN SEISMIC GAP USING SEMI-MARKOV MODEL

**A DISSERTATION**

***Submitted in the partial fulfillment of the***

***requirements for the award of degree***

***of***

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**EARTHQUAKE ENGINEERING**

**(With Specialization in Structural Dynamics)**

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# CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in this dissertation entitled "Assessing *probability of great earthquake in central Himalayan seismic gap using semi-Markov model*" in fulfillment of the requirements for the award of the degree of Master of Technology in Earthquake Engineering with specialization in Structural Dynamics submitted to Department of Earthquake Engineering, Indian Institute of Technology Roorkee is the authentic of my own work carried under the supervision of **Dr.I.D.Gupta**, Honorary Fellow & **Dr.H.R.Wasan**, Emeritus fellow Department of Earthquake Engineering, Indian Institute of Technology Roorkee, Roorkee India. The matter embodied in this dissertation has not been submitted by me for the award of any other degree or diploma.

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# CERTIFICATE

This is to certify that above statement made by the candidate is correct to the best of my knowledge.

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# Abstract

The present work aims at the introduction and application of Markov and semi-Markov models in estimating the waiting times and magnitudes of the great earthquakes in future. These model assumes that the successive earthquakes in same structural discontinuity are not independent events, which means that time and place of future earthquake events are related to previously occurred earthquake in the region considered, as demonstrated by elastic rebound theory. While other most commonly used models such as Poisson model assumes spatial and temporal independence of all earthquakes including great earthquakes i.e., occurrence of one earthquake does not affect the likelihood of a similar earthquake at the same location in the next unit of time. Such models may apply to regions characterized by moderate frequent earthquakes in larger areas. While Markov and semi-Markov models describes the sequences of events more adequately at small regions with great infrequent earthquakes. In this report, these probabilistic models are applied in the central Himalayan region, by considering the sequence of earthquakes to form a stationary Markov chain. The occurrences of earthquakes in these models is described by discrete time and discrete states for the earthquake magnitudes and locations. In Markov and semi-Markov models, the successive states are governed by the transition probabilities depending on the just previous state and not on the history of reaching to that state. The use of probability distributions for earthquake magnitude and inter arrival time as continuous random variables is unable to account for such a dependence. It has been discussed that how this dependence can be utilized to carry out a time dependent seismic hazard analyses. On the basis of seismotectonic characteristics, the study area is divided into four sub regions and the application of these models has been illustrated to predict the probability of the next earthquake in different sub regions as a function of time from the previous earthquake, conditioned on the magnitude and the sub region of the previous earthquake. The results obtained indicate that the next major earthquake can occur in any of the sub regions with almost equally high probability. Thus, the so called central Himalayan seismic gap is expected to be closed in near future.


**Keywords** Markov Process . Semi-Markov Process . Great Earthquakes . Central Himalayan seismic gap . Transition Probability . State Occupancies . Seismic hazard

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**Chapter 1**

# INTRODUCTION

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## 1.1 General Background

In view of the fact that specific prediction of earthquakes is not a reality at present, stochastic modeling of great earthquakes as a function of magnitude, space and time provides a useful tool in seismic hazard analysis and risk studies. A reliable hazard assessment goes a long way in mitigation of seismic risks by earthquake resistant design of structures and keeping necessary preparedness in place. The main motive in seismic hazard analysis includes identifying the earthquake sources, modeling the occurrences of earthquakes on these sources, determining the bedrock motion at the site due to an earthquake's occurrence, evaluating the soil's amplification of the motion at the site and determining the structural response. So, this report mainly concerned with the second step in seismic hazard analysis, modeling of earthquake occurrences. Because of the uncertainties in estimating parameters associated with the underlying physical processes causing occurrence of earthquakes, characterization of process is probabilistic in nature. Recurrence characterization includes estimation of sizes of and holding times between successive great earthquakes at a given location.

From a physical standpoint, occurrences of great earthquakes is governed by a continuous gradual process of accumulation and release of strain energy over a large region. Because of intermittently release of energy, the probability of next great earthquake is expected to be low in the same region. This requires estimating the probability of occurrence of great earthquakes based on magnitude and region of previous earthquake, which clearly shows the dependency of next great earthquake on the previously occurred earthquake in the same region. Also, laboratory representation of elastic rebound theory suggests that the time of occurrence and magnitude of a sequence of earthquakes on given source may not be stochastically independent. Temporal dependencies is demonstration of the latent geophysical mechanism which causes earthquakes. Thus, to describe a unique type of temporal and spatial dependencies in a sequence of earthquake occurrences, Markov and semi-Markov models are used in this report. Earthquake occurrence is examined in three dimensions of time, magnitude and space by determining the joint probability of earthquake occurrence between magnitude and region states. In this report, magnitude and location dependent occurrence probabilities are estimated for the next large earthquake as a function of lapse time since the previous earthquake in central Himalayan region using semi-Markov model.

Various models are available in the literature for characterizing the earthquake recurrence. Conventional extreme value probability distributions, like Poisson's model, normally apply to large geographical areas and are unable to consider the dependency between earthquakes as implied by elastic rebound model. Poisson model is used in the large areas identified with frequent moderate size earthquake occurrence, which consider the earthquake events as independent. Though, probability distributions like Lognormal, Gamma, Weibull and Inverse Gaussian can be used to model the probability of earthquake recurrence times as a function of magnitude, they do not really consider the dependence on the magnitude and location of the previous earthquake. Slip and time-predictable models can predict either the magnitude or the time of the next earthquake for a small source zone from the magnitude of the previous earthquake in the same source. But, it cannot account for the dependence on the magnitude and location of the previous earthquake in a different source zone. The models of earthquake occurrence based on stochastic processes that are characterized by a Markov property can account for temporal and spatial dependences.

A Markov chain is characterized by discrete states of magnitude and location with the time intervals of successive state occupancies governed by the transition probabilities from the previous to the next state. A stochastic process has Markov property such that the conditional property distribution of future states of the earthquake occurrence events for known present state and all past states depend only upon the present state and not any past states, which is called property of memorylessness. The transition to future state is conditional only on the just previous state ( regarded as present state) and not on the history of reaching to that state. Thus, Markov model can be used for the characterization of recurrence of great earthquakes based on uniform and exponential distribution for the recurrence times (holding times) between the earthquakes. Semi-Markov process is generalization of the Markov process which allow sojourn time between transitions to happen randomly based on any kind of distribution functions which rely on present and next visited state. Thus, semi-Markov model provides for distribution of holding time between successive earthquakes by using appropriate probability distributions like Weibull distribution, Lognormal distribution, which in case of Markov model is limited to uniform and exponential distribution. Semi-Markov model defines a stationary discrete-time, discrete-state processes in which occupancy of future successive states are governed by the transition probabilities of a Markov process. The stay in any state(holding time) is described by an integer valued random variable that depends on the present state and on the state to which the process will make next transition.

To develop a Markov model for predicting conditional probabilities of earthquake magnitudes and locations in the study area, a comprehensive earthquake catalog was prepared from different sources, for the period since 1255 to 2015. To quantify the magnitude and location of earthquakes, annual maximum magnitudes of the available data are classified into four magnitude states and the locations are divided into four different sub regions on the basis of seismotectonic characteristics. The transition probabilities among different magnitude and location states are then estimated from the observed data, which are in turn used to estimate the probability of transition from a given initial pair of magnitude and location states to any other target pair as a function of time. For semi-Markov model, earthquake data with moment magnitude $M_w \geq 6$ from year 1803 up to 2015 is considered. The results obtained indicate very high probabilities of occurrence of great earthquakes in all the four sub regions after about 50 years period. Thus, one of the sub regions considered as seismic gap is in fact no different than the other sub regions, as regards the probability of occurrence of large earthquakes in future.

## 1.2 Objective of the Study

Objective of the present study is to apply the Markov and semi-Markov models for assessing the probability of occurrence of great earthquakes ( $M_W \geq 7.5$) in the central Himalayan seismic gap. For this purpose, a study area is taken comprising a west to north-west striking segment of the Himalayan tectonic belt and the southern part of the Tibetan plateau. This area is characterized by very high level of seismicity associated with various tectonic features in the region. Thus, objective of present work is

1. To study the various tectonics features of the Central Himalayan region and its seismicity.

2. To divide the study region into sub-regions which are identified on the basis of distribution of epicenters of earthquakes in the region and thus to identify the seismic gap in the Central Himalayan region.

3. To develop the Markov and Semi-Markov for assessing probability of occurrence of great earthquake in Central Himalayan seismic gap.

4. To conclude from both model about the variation of future seismic activities with time in Central Himalayan seismic gap.

## 1.3 Organization of the Thesis

This dissertation consists of a total of six chapters. Chapter 2 is laying emphasis on literature review, in which works of the other researchers on the same topic is mentioned. This chapter also includes discussion of other models available in the literature for assessing probability of occurrence of earthquakes, explained in brief . In chapter 3, tectonics and seismicity of study region is discussed along with the Zonation of study region into other sub regions by considering the present seismic gap, also information about various sources of earthquake catalog along with its duration is given. In chapter 4, Markov model is explained, which includes about the methodology and results obtained by application of Markov model for assessing probability of occurrence of great earthquakes in central Himalayan seismic gap. In chapter 5, semi-Markov model is explained along with its methodology and results obtained by application of model in the study region. Finally, chapter 6 includes the summary of the Markov and semi-Markov model and conclusions made for the present work, obtained using these two models.

**Chapter 2**

# LITERATURE REVIEW

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In literature, a large number of probabilistic methods for modeling of occurrences of earthquakes are available which are used in probabilistic seismic hazard assessment. Various models are used as per the pertaining physical conditions at the region considered and its seismicity. As the Poisson model ( Cornell 1968; Gardner and Knopoff 1974) is used when the earthquakes occur frequently in a large region, so that the occurrence of next earthquake in the a region can be considered independent of the previously occurred earthquake in the same region. Whereas, Markov and semi-Markov models (Cluff et al. 1980; Herrera et al.2006; Altinok and Kolcak 1999; Nava et al. 2005) are used to characterize the earthquake occurrence processes to include the temporal and spatial dependencies among the successive earthquake events, thus these models are used in the small regions having high seismicity.

Many researchers in the past have used various models present in the literature, to model the earthquake occurrence events according to the underlying physical process and seismicity of region. Some important amongst of them are elastic rebound model (Reid 1910; Richter 1958), Markov models (Vere-Jones and Davies 1966; Knopoff 1971; Vageliente 1973; Veneziano and Cornell 1974; Lomnitz-Adler 1983; Patwardhan et al. 1980), Poisson's models (Brillinger 1982; Lomnitz and Nava 1983) and Slip-predictable models and Time-predictable models ( Shimazaki and Nakata 1980). Recent research as explained by elastic rebound theory suggests that most accurate prediction about future seismic activities requires the prediction of size and location of future earthquake and the time since the previously occurred earthquake in a given region. According to it, the occurrence of a future earthquake of particular size and location depends upon the size, location and time of occurrence of previously occurred earthquake, which shows the spatial and temporal dependencies among the occurrences of seismic activities. Time predictable models correlates the size of an earthquake with the time elapsed since the previously occurred event (Shimazaki and Nakata, 1980), which incorporates the temporal dependencies among the earthquake events (Anagnos and Kiremidjian, 1984). However, it allows for the prediction of time but not the size of the next earthquake. Slip-predictable models correlates the holding time between the earthquakes with the size of an earthquake at the end of time interval. Thus, by knowing the holding time distribution between the earthquakes, amount of strain energy released thus size can be determined.

Vere-Jones (1966) uses continuous time, continuous state Markov process to model aftershocks as sequence of events of decreasing frequency and magnitude. Knopoff (1971) uses stationary Markov process to model stored elastic energy of deformation, main events and aftershocks. Veneziano and Cornell (1974) uses Markov model to show temporal and spatial dependence among earthquake occurrences when shear stress equals static friction stress. Lomnitz-Adler (1983) apply Markov model using simulation to give a simplified representation of the spatial distribution of earthquakes on adjacent faults. Patwardhan et al (1980) describes discrete-state semi-Markov process to show holding time dependence on magnitude of previous and next events. Anagnos and Kiremidjian (1984) uses semi-Markov model for time predictable earthquake sequences applied to Parkfield region and also applied a discrete state time predictable stochastic model with spatial dependency among earthquake events with holding time distribution assuming Weibull probabilistic distribution. Cornell and Winterstein (1986) uses semi-Markov model for combined time and slip predictable model. Cornell and Winterstein (1986) uses semi-Markov model for combined time and slip predictable model.

**Chapter 3**

# TECTONICS AND SEISMICITY OF STUDY REGION

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## 3.1 Study Region

The region chosen for study comprises the central Himalayas covering Nepal and contiguous areas of India and southern Tibet, bound by longitudes 77˚ N and 89˚ N and latitudes 25.5˚E and 32.75˚E. This is among the most active seismic regions of Himalayas, visited regularly (~100 years) by major to great earthquakes. The most recent one after the Bihar-Nepal earthquake of 15.01.1934 with $M_W$ 8.1 is the Gorkha earthquake of 25.04.2015 with $M_W$ 7.8, followed by a very strong aftershock of $M_W$ 7.2 on 12.05.2015. Seismic activity in this area is related to the collision and under thrusting of the Indian plateau beneath the Eurasian plateau.

### 3.1.1 Tectonics of Region

Major tectonic features in the region are the Main Central Thrust(MCT), the Main Boundary Thrust(MBT) and Main Frontal Thrust(MFT). MCT is a major geological fault where the Indian Plate is pushed under The Eurasian Plate along the Himalayas. Indus-Tsangpo suture zone marks the collision between the Indian subcontinent and Eurasia. The fault slopes down to the north and is exposed on the surface in NW-SE direction. The Greater Himalayas is fringed below by MCT and South Tibetan detachment. The lesser Himalayan sequence is fringed by MBT and MCT. The Tethyan Himalayan sequence is fringed below by South Tibetan detachment. The Himalayan Frontal Fault system (MFT) marks the principle present day tectonic displacement zone between the Indian plate and the Himalaya; i.e., at the northern boundary of Indian plate. The Himalaya rides over the Indian plate on a decollement fault that does not cut through the basement. The surface expression of this fault is the discontinuous zone of reverse faulting called MFT between sub-Himalaya and Indian plains and a set of anticlinal ridges and synclinal valleys that accommodates slip on the buried decollement fault by folding. Most structures of Himalaya north of MFT are inactive today, although some part of them particularly MBT have been reactivated by younger deformation accompanying the MFT of India and Nepal and the Salt Range Thrust of Pakistan. The tectonic map of the study area with epicenters of available past earthquakes superimposed is shown in the Fig.1.

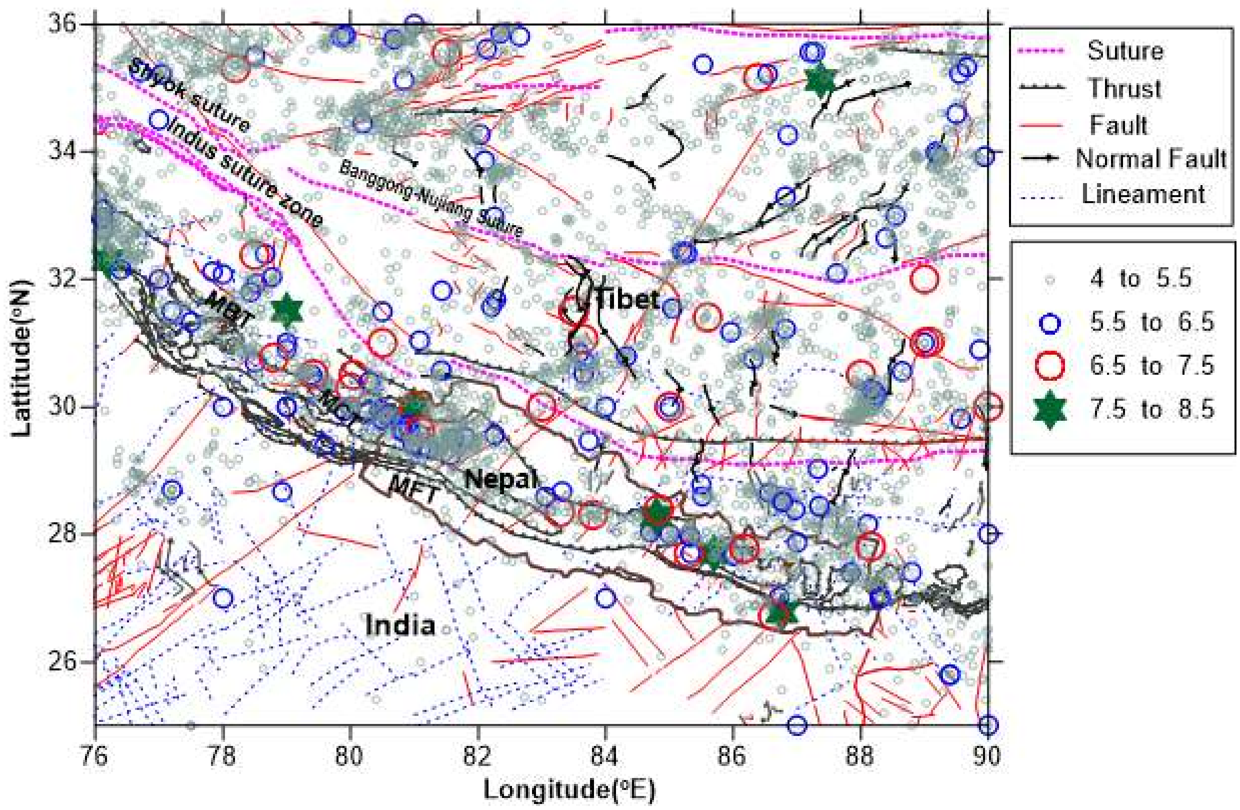



**Fig.1** Major tectonic features in the region of study along with the epicenters of available data on past earthquake *(Sources for preparing this seismotectonic map are taken from GSI: Seismotectonic Atlas of India and its environs)*

### 3.1.2 Seismicity of Central Himalayan Region

Himalayan region is highly seismically active region, whose seismicity is reflected by the frequency of large, medium and small earthquakes. Seismicity of Himalaya is mainly due to the relative high stress buildup because of the continual convergence of the Indian plate beneath the Eurasian plate along a shallow plane, thus causing internal deformation in the earth's crust. Like other parts of Himalaya, central Himalayan region is also concerned with the high seismic activities. Nepal, located in central Himalayan region has experienced great earthquakes in past including Bihar-Nepal earthquake (1934, $M_W$ 8.4) and Gorkha earthquake (2015, $M_W$ 7.8). Central Himalaya lies in the seismic zone IV with PGA value 0.24g. Spatial variation of seismicity in Central Himalayan region is highly non-uniform, with highly concentrated in Western Nepal and fairly scattered in Central Nepal and concentrated in south-east Nepal from where it offsets towards North through major faults. Distribution of epicenters of earthquakes with different magnitude class intervals is shown in Fig 1.

## 3.2 Earthquake Data

In most statistical studies, earthquake are represented by point events in a five dimensional space-time-size continuum. In ordinary earthquake catalogs, the five coordinates are given as longitude and latitude of epicenter, focal depth, origin time and magnitude. There are many other quantities which characterize an earthquake such as fault-plane parameter(or more generally moment tensor components), stress drop, fault rupture length, rupture velocity etc. Statistical studies involving these quantities are few because complete dataset on these is unavailable especially for small or old earthquakes. Usually, earthquake catalog consists of earthquake data classified on the basis of foreshocks, mainshocks, aftershocks and earthquake swarms. For the application of models in this report, earthquake catalog was homogenized to moment magnitude ($M_W$) and declustered to include only mainshocks, obtained by removing the foreshocks and aftershocks. Size of earthquake is taken as moment magnitude of earthquakes ($M_W$).

A catalog of 5719 earthquakes with magnitude $M_W \geq 4$ is compiled ( by Dr.I.D.Gupta) for the study region for the period 1255 to 2015 from various sources, important among which are the Indian Meteorological Department(IMD), International Seismological Center(ISC), United States Geological Survey(USGS), etc. for the instrumental data, and National Oceanic and Atmospheric Administration(NOAA), Disaster Preparedness Network Nepal(DPNET), Oldham(1883), Milne(1911), etc. for the historical earthquakes. For application of Markov model, resulting catalog of 3760 main shocks with magnitude $M_W \geq 4$ is used. For application of semi-Markov model, only main earthquakes with magnitude

$M_W \geq 6$ are chosen from the catalog for the period 1803 to 2015. Available historical records for about past 800 years indicate that large to great earthquake have been occurring regularly in this area. On average, an earthquake of magnitude 7.5 or greater has occurred in the area every 40 years on average. The recent Gorkha earthquake of 25 April 2015 with magnitude 7.8 is the major earthquake after the 1934 Bihar-Nepal earthquake of magnitude 8.1.

To develop the Markov model for occurrence of earthquakes in the region of Fig.1, only the annual maximum magnitudes are considered in the present study. These maximum magnitudes are grouped into four intervals as $4.0 \leq M_W < 5.5$(small), $5.5 \leq M_W < 6.5$(moderate), $6.5 \leq M_W < 7.5$(large) and $7.5 \leq M_W \leq 8.5$(major) to define the magnitude states. For semi-Markov model corresponding magnitude states are defined as $6.0 \leq M_W < 6.5$, $6.5 \leq M_W < 7.0$, $7.0 \leq M_W < 7.5$ and $M_W \geq 7.5$ respectively.

Also, to define the location states for both Markov and semi-Markov model, the study region is divided into four sub-regions designated R1 to R4, as shown in Fig.2. First three sub-regions correspond to the highly seismic Himalayan belt, whereas sub-region 4 to the north is considered to have somewhat lower frequency and maximum magnitude of earthquakes. However, in view of the orogenic history of the region and the ongoing tectonic processes, major earthquakes in the four sub-regions are expected to have strong spatial dependence. The annual maximum magnitudes in the four magnitude states are also plotted in Fig.2. It is seen that sub-region R2 of the Himalayan belt encompassing area of central Nepal has not experienced any major earthquake with magnitude 7.5 or more, and it is thus considered as a seismic gap. On the basis of elastic rebound hypothesis, the gap area is expected to be more prone to a major earthquake in future, if the strain energy has not been released a seismically. Thus, application of the Markov and semi-Markov model is illustrated in the present study to estimate the probabilities of a major earthquake in this gap area as a function of time.

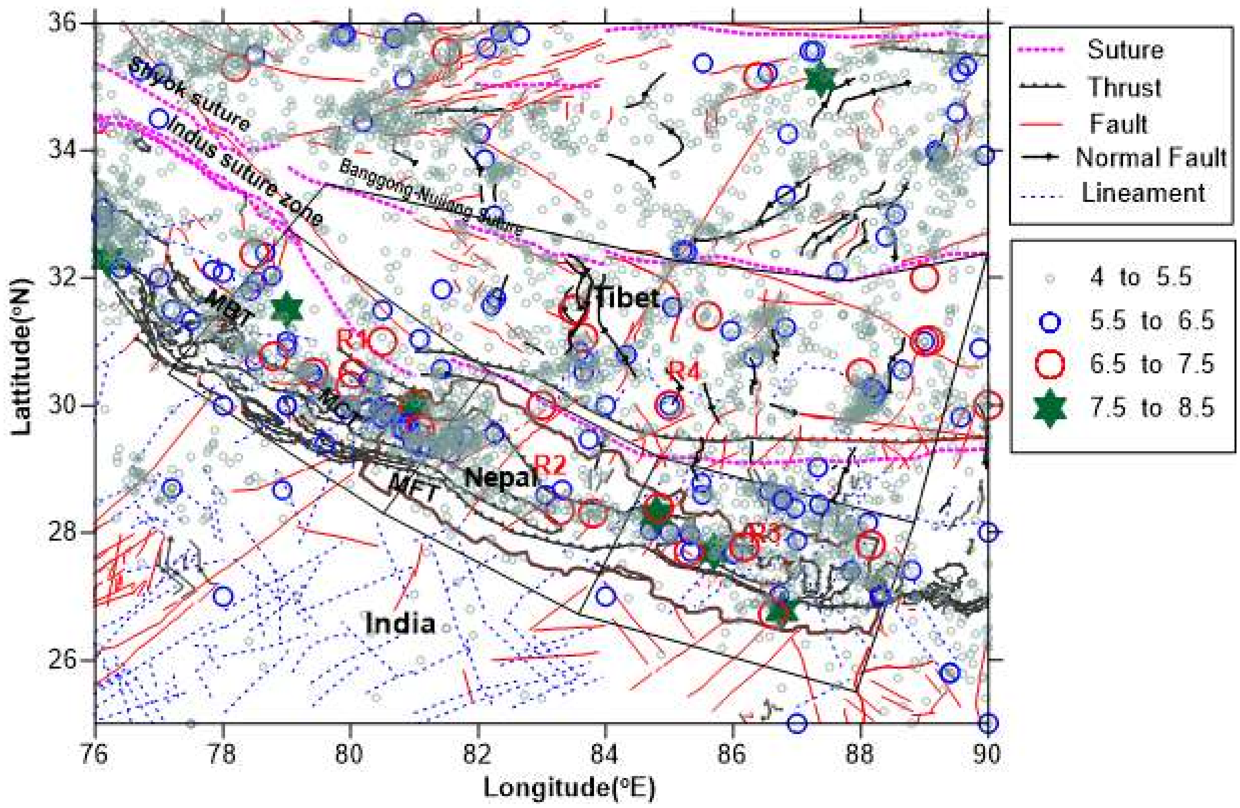



**Fig.2** Four sub-regions to define the location states of the annual maximum magnitudes in the region of study *(Sources for preparing this seismotectonic map are taken from GSI: Seismotectonic Atlas of India and its environs)*

Chapter 4

# MARKOV MODEL

## 4.1 Introduction

The occurrence of earthquakes is represented by a process of strain accumulation interrupted by sudden release intermittently, when strain exceeds a threshold limit specified by shear strength of rocks in Earth's crust. Laboratory representation of elastic rebound theory suggests that the time of occurrence and magnitude of a sequence of earthquakes on given source may not be stochastically independent. Thus to describe a unique type of temporal and spatial dependencies in a sequence of earthquake occurrences, Markov and semi-Markov models are used. Temporal dependency is demonstration of the latent geophysical mechanism which causes earthquake. Markov and semi-Markov models used for earthquake forecasting incorporates two important fundamental postulates:

1. The sequence of occurrences of earthquakes in a particular area is a stochastic process in time.

2. A stochastic process has Markov property such that the conditional probability distribution of future states to which the process will make transition for a given present state and all past states depends only upon the present state and not on any past states. The processes possesses property of memorylessness.

In the application of Markov model two states are considered corresponding to the magnitude of earthquake and region associated with occurrence of earthquake. States corresponding to the earthquake occurrence are considered to be recurrent which means that starting in that state the chain must eventually return to the state at least once. A Markov process is characterized by stationary discrete-state and discrete-time intervals in which the successive state occupancy is governed by a transition probability.

Transition probability completely determines the Markov process. Transition probability represents the probability of going from *i*th state to *j*th state in some steps. For a stochastic process, {X(t),t>0} probability of going from *i*th state to *j*th state in *s* steps is given as

$$p^s(i,j)= P\{X(t+s)=j|\ X(t)=i\} \tag{1}$$

where *i* represents the present state at time *t* and *j* represents the future state after *s* steps from the present state *i*. Moreover, for a process {X(t),t>0} corresponding to a state space E={1,2,3,...,N}

$$P\{X(t+s)=j|\ X(h)=i,\ 0<h\leq t\}=P\{X(t+s)=j|\ X(t)=i\} \text{ for t, s>0 and i, j} \epsilon \text{ E} \tag{2}$$

Mathematically, above equation represents the memoryless property i.e., future states of a stochastic process only depends upon the present state, not on the past states before time *t*.

In Markovian estimation, the transition matrix represents the distribution of a Markov chain at the next step if its current state is given. Matrix element corresponding to *i*th row and *j*th column is denoted by $p_{ij}$, which gives the transition probability from state i to state j

$$p_{ij} = \frac{total\ number\ of\ transitions\ from\ state\ i\ to\ state\ j}{total\ number\ of\ transitions\ from\ state\ i} \tag{3}$$

Transition matrix is used to follow the evolution of system through successive intervals of time. It is non-negative and square matrix and having sum of element within each row equal to one.

$$\sum_{j=1}^{N} \mathrm{p(i,j)} = 1 \tag{4}$$

Also, the transition probability is assumed to be stationary, which allow the transition to future earthquake events in one-step and not changing as the elapsed time increases since the previous state. Thus, the only information needed to describe the process is the initial conditions(a probability mass function for $X_0$) and the transition probabilities, which are used to follow the evolution of system through successive intervals of time. For a stochastic process X={$X_n$, n=0,1,...} with discrete state space E, transition probability is said to be stationary if

$$P\{X_1 = j | X_0 = i\} = P\{X_{n+1} = j | X_n = i\} \tag{5}$$

for any set of states $i_0,...,i_n$ in the state space and $j\epsilon$ E.

## 4.2 Mathematical Formulation

For the application of the Markov model, magnitude states are defined into four classes as given below.

**Table 1** States for magnitude of earthquakes in Markov model

| **Earthquake magnitude ($M_W$) class interval** | **Magnitude States** |
|---|---|
| $4 \leq M_W < 5.5$ | M1 |
| $5.5 \leq M_W < 6.5$ | M2 |
| $6.5 \leq M_W < 7.5$ | M3 |
| $M_W \geq 7.5$ | M4 |

For calculation of the transition probability matrix, the total time length for which the data is available is divided into time intervals of equal size of $\Delta t$, such that every interval has at least one event. If there is more than one earthquake in a particular time intervals, the highest magnitude earthquake in that intervals is used. In each time interval, a magnitude state is then assigned to the earthquake event as per the class intervals defined. In this, $\Delta t$ is chosen as 1 year, so that available earthquake catalog is divided into time intervals of one year, ranging from 1803 to 2015. Now, the maximum magnitude in each class interval is used to assign the magnitude and location states to each earthquake event. However, this results in many empty time intervals of one year. If an empty interval is followed by an interval occupied by M3 or M4 state, an M1 state (lowest) is temporarily assigned to it so that the transition to the following higher state is taken into account. This is done to include the effect of all important earthquakes in the region, even for incomplete part of the catalog. However, for region to region transition, only the complete part of the catalog with continuous data for all the years is used. In this case four region states are considered corresponding to each sub region viz. R1, R2, R3 and R4 as shown in Fig 3. Continuing in this way for all time intervals, a Markov chain is developed for calculation of the transition probability matrix for both the magnitude states and region states. The transition matrix thus obtained for a particular state variable can be used to find the probability of occurrence of a given state during specified future time interval, if the present state is known. If the prediction is made over $n$ time intervals (total duration $n\Delta t$) and if the initial state is $i(0)$, the probability $P$ that the system will evolve through the sequences of $n$ states as $\{i(1), i(2),..., i(n)\}$ can be written as

$$P = p_{i(0)i(1)} \times p_{i(1)i(2)} \times p_{i(2)i(3)} \times \cdots \times p_{i(n-1)i(n)} \tag{6}$$

This can be used to calculate the probability $P_{\mathrm{ij}}$ of occurrence of a particular state $j$ out of $M$ states and given initial state $i$ during time period $n\Delta t$ in future, by first estimating the complementary probability, $P_{ij}^{C}$, of non-occurrence of state $j$, as follows

$$P_{ij}^{C} = \sum_{\substack{i(1)=1 \\ i(1)\neq j}}^{N} \sum_{\substack{i(2)=1 \\ i(2)\neq j}}^{N} \sum_{\substack{i(3)=1 \\ i(3)\neq j}}^{N} \cdots\cdots \sum_{\substack{i(n)=1 \\ i(n)\neq j}}^{N} P \tag{7}$$

The required interval transition probability $P_{\mathrm{ij}}$ can then be obtained as

$$P_{ij} = 1 - P_{ij}^{C} \tag{8}$$

Probability $P_{ij}$ is obtained for both magnitude to magnitude transitions and region to region transitions. Thus overall probability for occurrence of next earthquake of a particular magnitude in a particular region is obtained by calculating the joint probability between magnitude transition probability and region interval transition probability. In doing so, it is assumed that interval transition probability for magnitude is independent from interval transition probability for region.

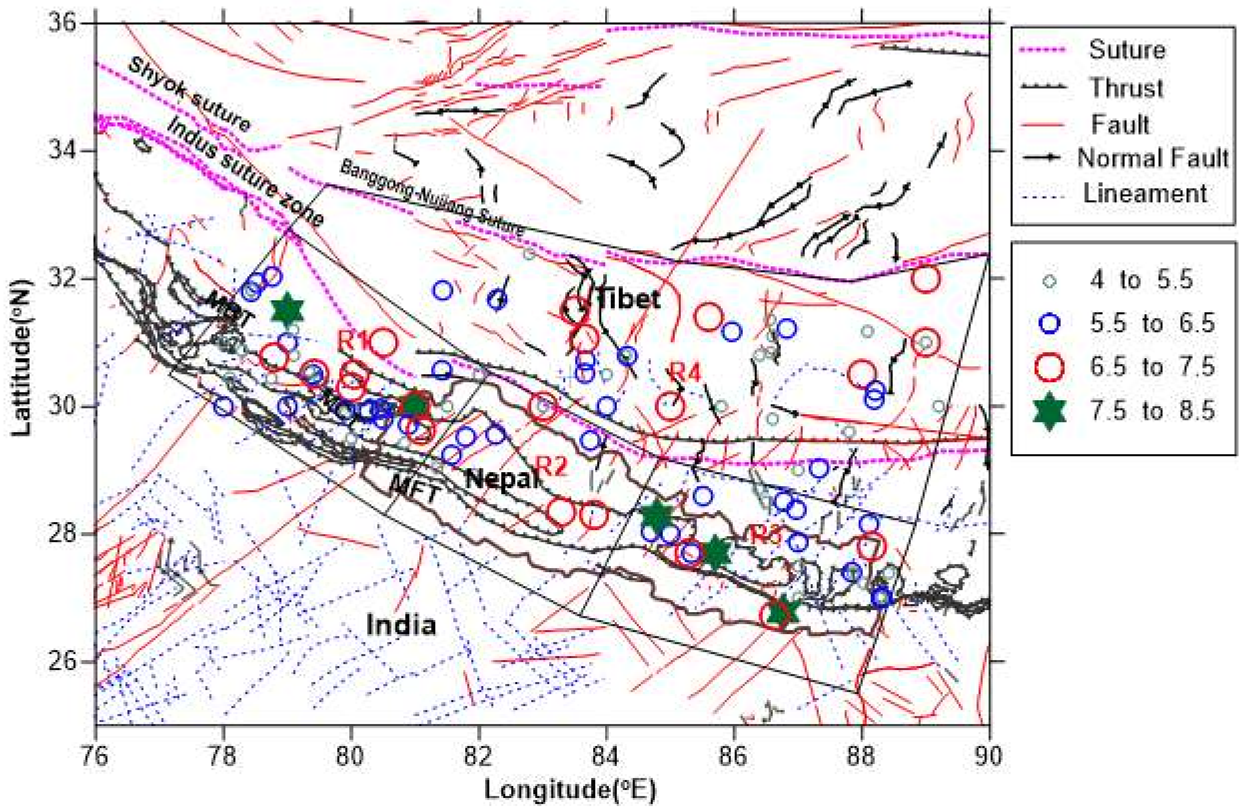



**Fig. 3** Four sub-regions to define the location states of the annual maximum magnitudes in the region of study for the application of Markov model *(Sources for preparing this seismotectonic map are taken from GSI: Seismotectonic Atlas of India and its environs)*

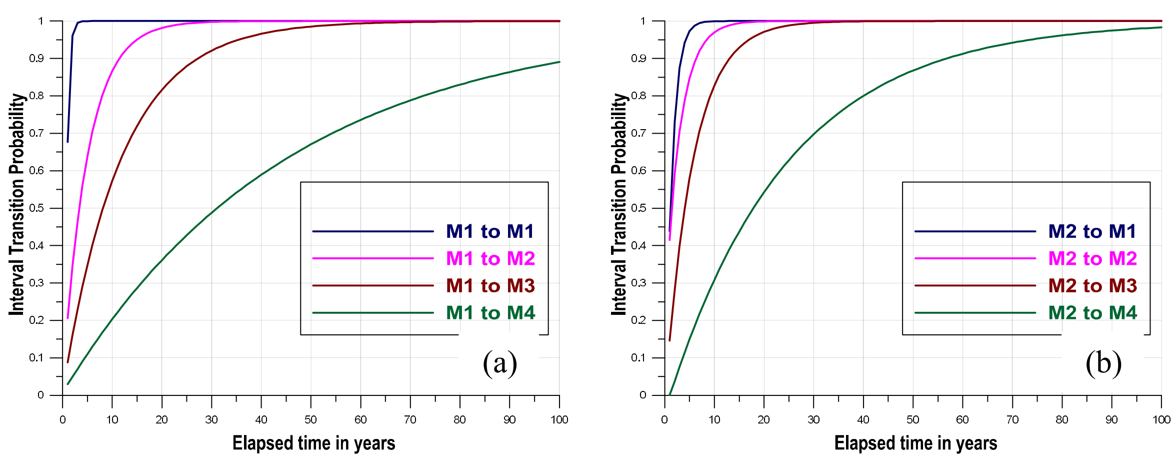
## 4.3 Results and Discussion

To obtain the numerical results for the region of central Himalaya, four magnitude and four location states are considered as mentioned before. Thus a Markov chain is developed for magnitude and location states to calculate the transition probability matrix for both magnitude and region states as explained in section 6. The magnitude and region transition matrices thus obtained are as follows:

$$P_M = \begin{bmatrix} 0.4923 & 0.3077 & 0.1231 & 0.0769 \\ 0.4848 & 0.3636 & 0.1515 & 0 \\ 0.9231 & 0.0769 & 0 & 0 \\ 1.000 & 0 & 0 & 0 \end{bmatrix} \text{ and } P_R = \begin{bmatrix} 0.4583 & 0.2083 & 0.3333 \\ 0.2308 & 0.3846 & 0.3846 \\ 0.5294 & 0.1765 & 0.2941 \end{bmatrix}$$

Interval transition probabilities for occurrence of various magnitude and location states for different initial states are computed from eqns.(4) and (5) using these transitions matrices for future time up to 100 years.

The results for the probability of occurrence of all possible combinations of initial and final magnitude states are given Fig. 4 (a) to (d). As expected from physical considerations, irrespective of the magnitude of the initial state, there is a monotonic increase in the probability with decrease in the magnitude of the final state. Also, the probability of transition from a lower magnitude state to a higher state is in general lower than the corresponding probability from the higher to the lower state (e.g., $P_{13}$ is lower than $P_{31}$). These observations can be explained from the fact that getting a smaller event after a big event is more likely than getting a bigger event after a smaller one. Also, the probability of having the same initial and final states decreases with increase in the magnitude state (e.g., $P_{11}$ is much higher than $P_{44}$). This is because the repetition of smaller magnitudes is more frequent than the larger ones. Thus, the results in Fig.5 can be considered to provide physically realistic probabilities of occurrence of different magnitudes of future earthquakes conditioned on the magnitude of the previous earthquake.



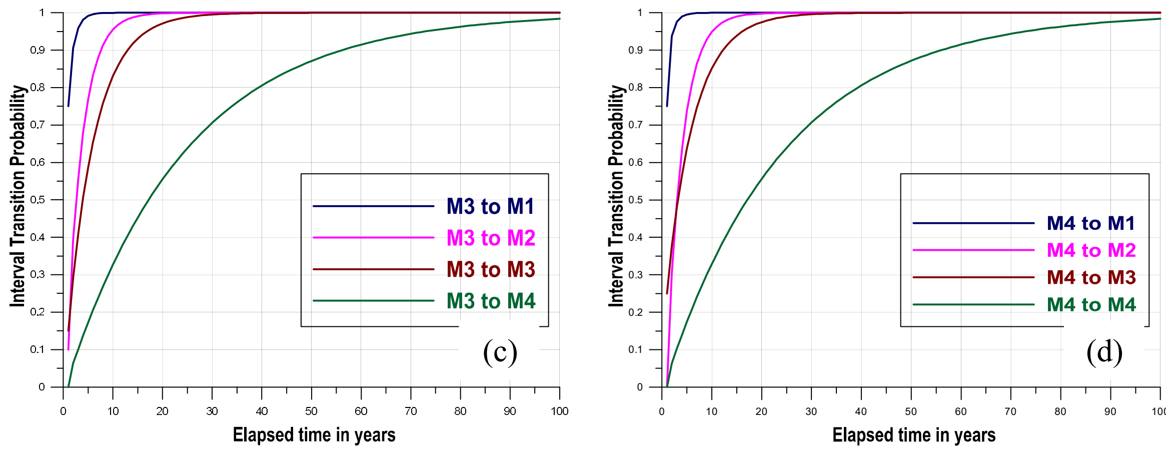



**Fig. 4** Example numerical results on interval transition probabilities from different initial magnitude states to the other states. Panels (a), (b), (c) and (d) in the figure corresponds to initial states as M1, M2, M3 and M4, respectively

Fig. 5 (a) to (d) gives the probabilities of transition for earthquakes from a given region to other regions. The four regions considered are such that, the central region R2 along the Himalaya represents a seismic gap lacking occurrence of major earthquakes (state M4) and characterized by relatively much lower frequency of even smaller earthquakes. The regions R1 and R3 on its left and right sides (Fig.2) are seen to have much higher occurrence rate of smaller earthquakes, with two great earthquakes in region R1 and three great earthquakes in region R3 since 1803. The seismicity in region R4 to the north of main Himalaya also forms a part of the Indian plate boundary, but it accommodates relatively smaller plate motion. This sub-region is also devoid of major earthquakes, but the frequency of smaller magnitude events is comparable to the main Himalayan belt. As the region to region interval transition probabilities are independent of the magnitude states, these are seen to be quite close for transitions to regions R1, R3 and R4, all of which are characterized by almost equally high overall frequency of earthquake occurrences. The transitions to region R2 with much smaller overall frequency are seen to have significantly less probability up to about 50 years in future. However, the interval transition probabilities from region R2 to itself are seen to be slightly higher than those from the other three regions to R2, indicating that consecutive occurrences within R2 are somewhat more likely. Also, from the results in Fig 6 it may be concluded that if the next occurrence has not taken place in any other sub-region for about 50 years, then its occurrence in the gap region R2 is almost equally likely.

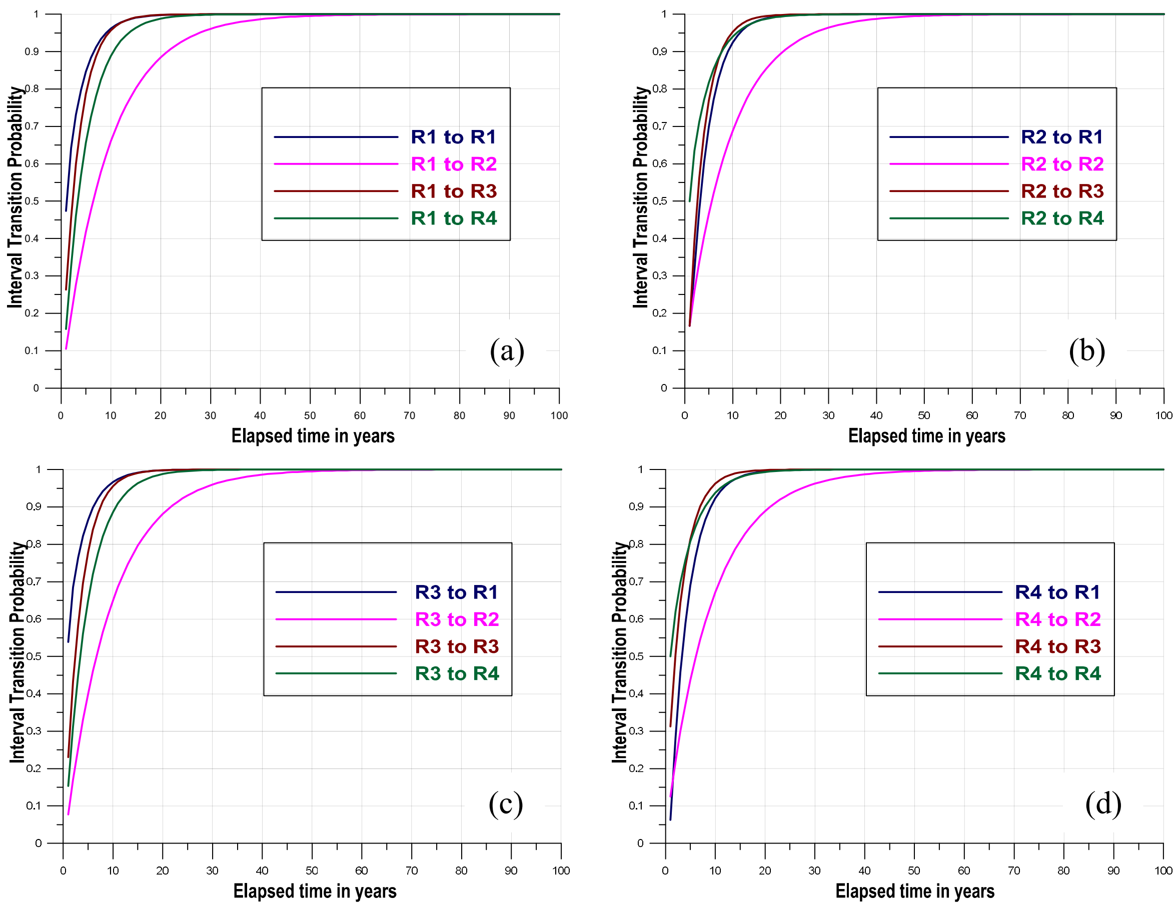



**Fig.5** Example numerical results on interval transition probabilities from different initial regions to all the regions over a time period of 100 years. Panels (a), (b) and (c) in the figure corresponds to initial regions as R1, R2 and M3, respectively

The results in Figs. 3 and 4 can be used for the probabilistic forecasting of the next major earthquake in different sub-region by defining the joint transition probabilities from any specified initial to the final pair of magnitude and region states. For this purpose, magnitude to magnitude and region to region transitions are assumed to be statistically independent. Thus, given an initial region and magnitude combination *($r_0$, $m_0$)*, the transition probability to another magnitude and region combination *($r_1$, $m_1$)* after elapsed time $t$ can be defined using the concept of conditional probability as

$$P(r_1, m_1 \mid r_0, m_0, t) = P_{r_0 r_1}(t) \times P_{m_0 m_1}(t) \tag{9}$$

This has been used to estimate the probabilities of occurrence of a major earthquake (state M4) in various sub-regions from different combinations of initial pair of magnitude and region states. Some typical results are shown in Fig. 7(a) to (d).

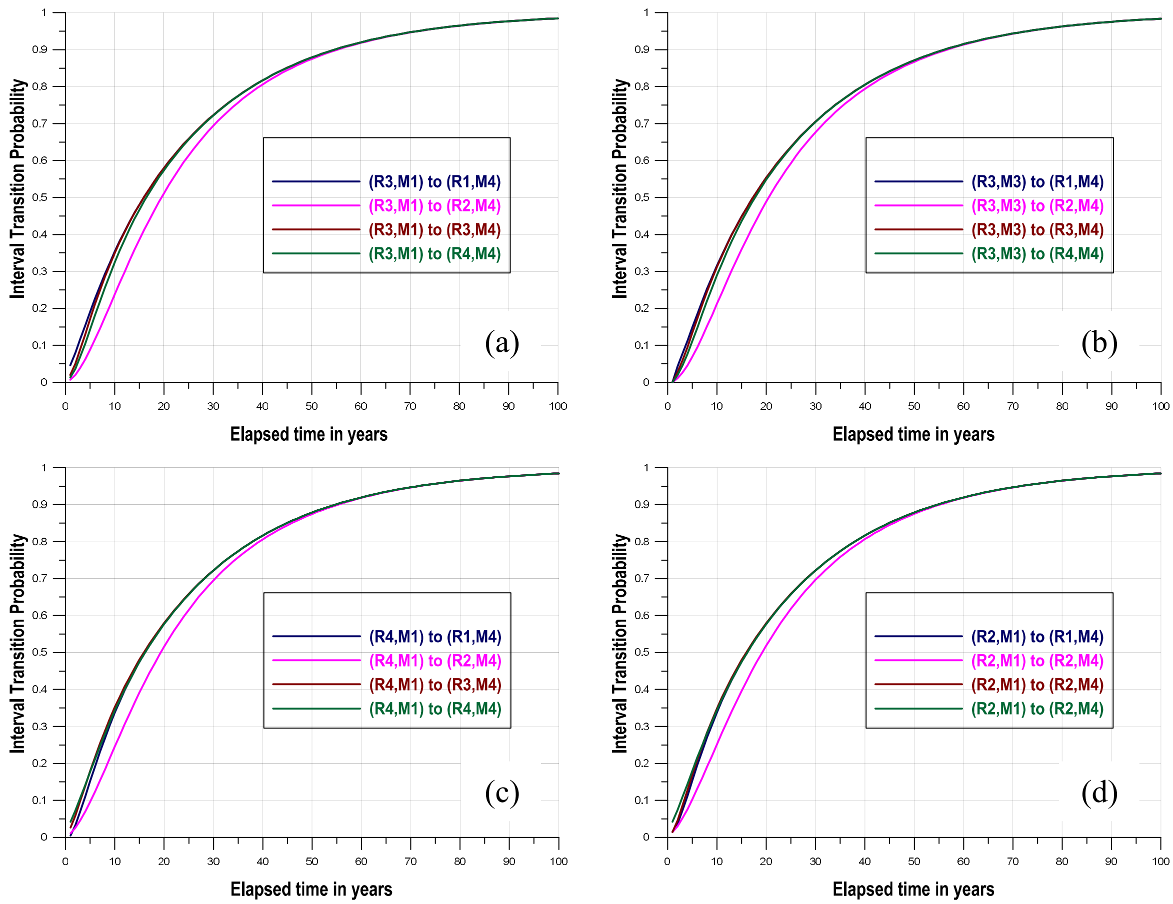



**Fig.6** Probabilities of a major earthquake (state M4) in different sub-regions as a function of time for four typical pairs of initial magnitude and region states

The results in Fig. 5 do not indicate any significant dependence on the choice of the pair of initial magnitude and location states. However, in all the cases, the transition probabilities to the gap region R2 are seen to be significantly smaller than those to other sub-regions for periods up to about 50 years. The transition probabilities to all the sub-regions increase monotonically and become almost the same beyond 50 years of period. Thus as mentioned before, if the next major earthquake has not occurred in any of the sub-regions up to 50 years, the chance of its occurrence in the gap area is also equally high with a probability increasing from around 0.88n at 50 years to around 0.985 at 100 years interval. To get an exact idea about the probabilities of a major earthquake in different sub-regions for smaller time periods, Table-2 gives the average numerical values of all the different combinations of initial states for periods of 10, 20 and 40 years. It is seen that the probability of occurrence of a major earthquake in the gap area is lower by about 8% for smaller period of 10 years, which further decreases to about 5% for 20 years period and to about 2% for the 40 years period.

**Table 2** Typical values of the transition probabilities to combination of states (R2, M4) from various other initial combinations of states for lapse times of 10, 50 and 100 years.

| Target Region | Probability of M4 in future time of | | |
|---|---|---|---|
| | 10 years | 20 years | 40 years |
| R1 | 0.330 | 0.580 | 0.833 |
| R2 | 0.255 | 0.521 | 0.813 |
| R3 | 0.330 | 0.580 | 0.833 |
| R4 | 0.330 | 0.580 | 0.833 |

The results in Figs. 3 ad 4 can also be useful in time-dependent probabilistic seismic hazard analysis(PSHA). The probabilities of transition to smaller magnitude events (e.g., to states M1 & M2 in Fig.3) are seen to reach unity within about 20 years, which is much smaller than the periods of applicability of the obtained PSHA results. Smaller magnitudes can thus be described separately for each source using constant occurrence rates obtained from the recurrence relation based on the available past data for that source only. However, the time dependence of the occurrence rate may be very crucial for the largest magnitudes, because the probability of occurrence may not reach unity for even very large time intervals. Also, the largest earthquakes are expected to have strong spatial dependence, because the next major earthquake is not likely to occur in the same seismic source. Thus, the time-dependent occurrence of such events cannot be described straightaway for each small size seismic source zone used commonly in PSHA estimations. For this purpose, the long term average occurrence rate of great earthquakes for the complete big region is first multiplied by the probabilities of transition to the next great earthquake (e.g., from M4 to M4 in Fig. 3), which gives the occurrence rate for the entire region at different times. These are then distributed among all the sub-regions in proportion to the transition probabilities from the sub-region of the last major earthquake (Fig.4) to get the occurrence rates at different times for each sun-source. Use of these occurrence rates with the Poisson assumption would provide the PSHA estimates as a function of the absolute time since the last occurrence of a large magnitude earthquake. This will be much simpler and more efficient way than using the Monte-Carlo simulations

Chapter 5

# SEMI-MARKOV MODEL

## 5.1 Introduction

Generalization of Markov process is semi-Markov process, which allow sojourn time between transitions to happen randomly based on any kind of holding time distribution functions which rely on present state and next visited state. Semi-Markov models have been used to represent the sequence of large magnitude earthquakes and to characterize spatial and temporal seismic gaps. In Markov model holding time $h_{ij}(t)$, defined as the probability that process stays in state *i* for a time period t before it make transition to state *j*, is exponentially distributed with parameters conditionally on present state *i*. As Markov process is associated with the property of memorylessness, so the exponential is the only memoryless continuous random variable. Every instant is like the beginning of a new random period, which has the same distribution regardless of how much time has already elapsed. The memoryless property makes it easy to reason about the average behavior of exponentially distributed items in queuing systems. However, semi-Markov is not restricted to exponentially distributed holding times. In semi-Markov process, holding time in a given state is identically distributed, conditional on both the current state and the next state, thus providing the greater flexibility in modeling. In semi-Markov model, parameters and distributions have been chosen to assure increasing hazard rates for holding time distributions e.g., Weibull, Lognormal, Gamma distributions, which implies that probability of an earthquake occurrence in near futures increases with time since the last event. The increasing hazard captures some of the characteristics of stress build up and release.

The semi-Markov model defines a stationary discrete-time, discrete-state process in which successive state occupancies are governed by transition probability of the Markov process, which is discussed in previous section 5.1.

## 5.2 Mathematical Formulation

In present model, two states are considered associated with the magnitude of earthquake and the region in which it occurred. The region states viz. R1, R2, R3 and R4 along with the epicenters of the earthquakes with magnitude $M_W \geq 6$ are shown in Fig 7. The set of all values that the state variables can assume forms the state space. Because the state space must have a finite set of states, the state variables cannot take on a range of real numbers, but instead are restricted to discrete values. All the events that can occur and cause a state transition forms the event state. The event set must be finite. The stay in any state known as holding time is described by an positive integer-valued random variable that depends upon on the presently occupied state and the state to which the next transition will be made. Distribution of holding time($t_{ij}$=Tn-$T_{n-1}$) is independent of its behavior before $T_{n-1}$ but may be a function of present state $X(T_{n-1})=i$ and next visiting state $X(T_n)=j$. If all holding times in a semi-Markov chain are equal to a constant value, the chain can be studied as a discrete-time Markov chain. To describe it completely only transition probabilities are needed. If a semi-Markov chain has only one state, all its holding times can only be a function of this one state, hence they are independent and identically distributed. Thus chain is studied as a renewal stream with renewal at each transitions. The basic parameters for the semi-Markov models are defined as below

**1 Transition Matrix**

Transition matrix is obtained by calculating all the transition probabilities $p_{ij}$. Thus, $p_{ij}$ forms the ij$^{th}$ element of the transition matrix, which is discussed in detail in section 5.1. For $N$ number of states, sum of elements of transition matrix along $i$th row must be 1.

$$\sum_{j=1}^{N} \mathrm{p}\,(i,j) = 1 \tag{10}$$

Transition matrix is calculated for both magnitude states and the region states separately.

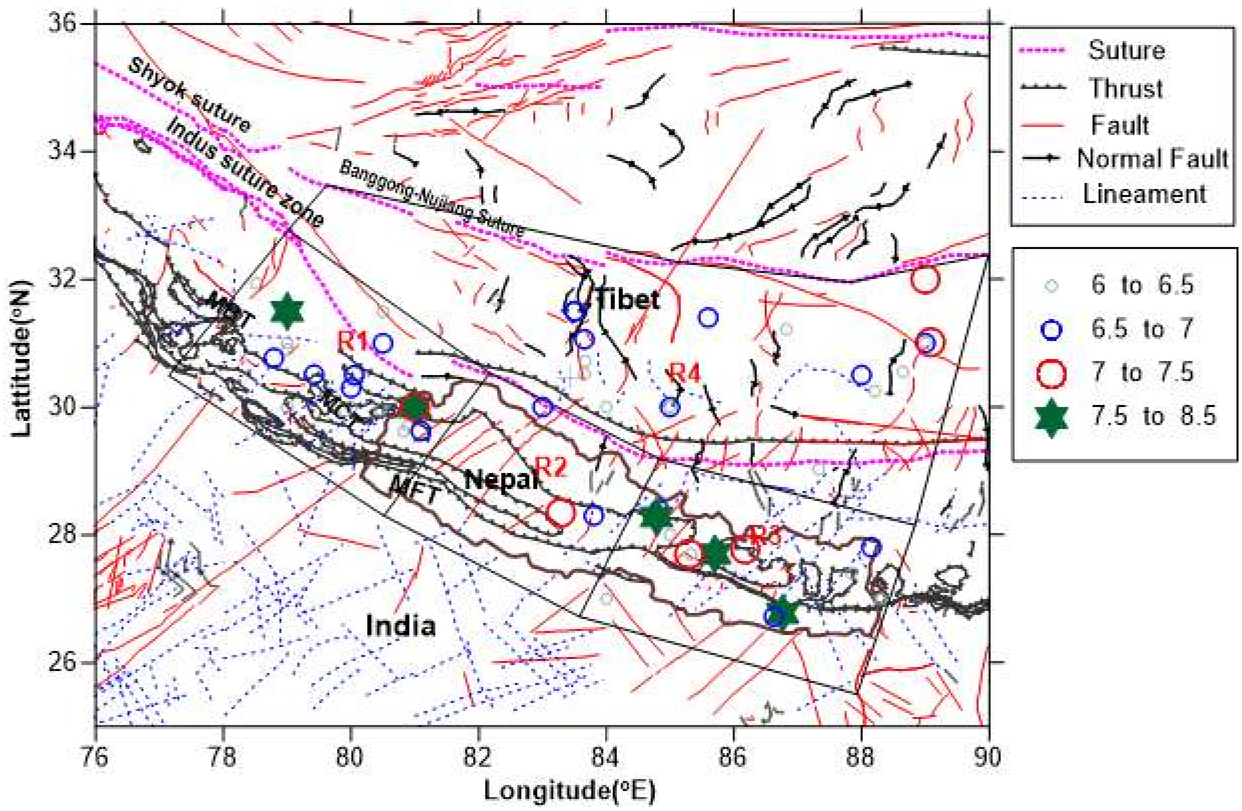



**Fig. 7** Four sub-regions to define the location states of the earthquake magnitudes $M_W \geq 6$ in the region of study for application of semi-Markov model *(Sources for preparing this seismotectonic map are taken from GSI: Seismotectonic Atlas of India and its environs)*

## 2 Holding Time

Before the transition from state *i* to state *j*, the process remains in state *i* which is referred as holding time $t_{ij}$. The holding times are positive, integer-valued random variables. All holding times are finite and each equal to one time unit. The probability mass function $T_{ij}$ in $t_{ij}$ is called the holding time mass function for a transition from state *i* to *j*.

$$P\{t_{ij}=m\}= T_{ij}(m);\ m=1,\ldots,n. \tag{11}$$

Here, n is the number of time intervals. The ij$^{th}$ element of holding time mass function $T_{ij}(m)$ is obtained as

$$P\{t_{ij}=m\} = \frac{\text{number of transitions from state } i \text{ to state } j \text{ during a time interval } m}{\text{total number of transitions from state } i \text{ to state } j} \quad (12)$$

**3 Core Matrix**

The ij$^{th}$ element of the core matrix C(m) is the probability of the joint event in which a system that entered *i* at time t=0 makes its next transition to state *j* after a holding time m.

$$C_{ij}(m) = P((X_n = j|X_{n-1}= i), (t_{ij} = m))$$
$$= P(X_n = j|X_{n-1} = i).P(t_{ij} = m)$$

$$C_{ij}(m) = P_{ij} . T_{ij}(m) \quad (13)$$

where, i,j = {1,2,...,N}, m={1,2,...,n}. N is the total number of states and n represents the total number of time intervals. Number of time intervals are created by equally dividing the range of string which is created by differences between the two successive years, proceeding through whole data. In doing so, a suitable single unit for time interval $\Delta t$ is chosen.

Above equation is represented in form of congruent matrix multiplication form. Thus, core matrix can be obtained by multiplication of corresponding elements of transition matrix P and holding time mass matrix T(m).

$$C(m)= P.*T(m) \quad (14)$$

Sum of the elements of C(m) across the i$^{th}$ row gives the waiting time mass function $w_i(m)$ for the i$^{th}$ state.

$$w_i(m) = \sum_{j=1}^{N} Cij(m) \quad (15)$$

$w_i(m)$ is the probability that waiting time for i$^{th}$ state is equal to m.

The cumulative probability distribution of the waiting time is obtained from

$$\leq w_i(n) = \sum_{m=1}^{n} wi(m) \quad (16)$$

$\leq w_i(n)$ is the probability that the waiting time for the $i^{th}$ state is less than or equal to n. The complementary of $\leq w_i(n)$ is given by

$$Gw_i(n) = \sum_{m=n+1}^{\infty} wi(m) \tag{17}$$

$Gw_i(n)$ is the probability that waiting time for the $i^{th}$ state is greater than n.

## 4 Interval Transition Probability Matrix

The most important statistical parameters of the semi-Markov process are the interval transition probabilities. The probability of a transition from state $i$ to state $j$ in the interval (0,n) requires that the process makes at least one transition during that interval. The process could have made its first transition from state $i$ to some state at time m, where ($0\leq m<n$), and then sum of succession of transitions, it could have made its way to state $j$ at time n. Thus, interval transition probability F(n) is obtained as follows

$$F(n) = Gw(n) + \sum_{m=0}^{n} C(m)F(n-m); \; n=0,1,2,... \tag{18}$$

Since T(0) is equal to 0, so F(n) is obtained for the interval $1\leq m<n$. Thus for any time interval n, F(n) is obtained through a recursive procedure. For n=1, by putting in above formula to obtain F(1), first we have to find F(0). F(0) represents the condition in which the process is in present time t=0, and willing to emerge itself further in future state. Thus, to generalize F(0), if i=j then it is equal to one and zero for other conditions in which process exist. So, F(0) may be represented using Kronecker Delta or Identity Matrix.

$$F_{ij}(0) = \begin{cases} 1, & if\ i = j \\ 0, & if\ i \neq j \end{cases} \tag{19}$$

For the development of semi-Markov model, only earthquake magnitudes $M_W\geq 6$ are considered from the earthquake catalog, ranging from year 1803 to 2015. To develop the transition probability matrix for both magnitude and region states, four magnitude states are defined belonging to the following class intervals for earthquake magnitudes given below

**Table 3** States for earthquake magnitudes in Semi-Markov model

| **Earthquake magnitude ($M_W$) class interval** | **Magnitude states** |
|---|---|
| $6 \leq M_W < 6.5$ | M1 |
| $6.5 \leq M_W < 7$ | M2 |
| $7 \leq M_W < 7.5$ | M3 |
| $M_W \geq 7.5$ | M4 |

Thus, each earthquake event is given a particular magnitude state and region state as per the magnitude class interval defined in above table and the region in which that earthquake event occur. Continuing in above way over whole earthquake data taken for the model, a Markov chain is developed for both magnitude states and region states. Four region states are defined corresponding to each sub-region viz. R1, R2, R3 and R4. Transition matrix so obtained for both states is used further in the process for the calculation of interval transition probability matrix for both states, which will give the probability of occurrence of earthquakes for a particular magnitude state in a particular region.

Next in the process, holding time mass matrix is calculated for both magnitude states and region states. For this first holding time is estimated between two successive earthquake events continuing over whole earthquake data as follows

$$t_{ij} = T_n - T_{n-1} \tag{20}$$

where $T_{n-1}$ is the time of previous earthquake visiting state $i^{th}$ and $T_n$ is the time of next earthquake visiting $j^{\text{th}}$ state

Now, a suitable length of time interval $\Delta t$ is chosen to obtain the $n$ numbers of the equal holding time intervals. Here, in this model $\Delta t$ is taken equal to 5 years and number of holding time intervals are defined on the basis of the range of the holding times $t_{ij}$ of events which is equal to difference between maximum holding time and minimum holding time. In this way, following holding time intervals are developed and an integer is assigned to each interval.

**Table 4** Class intervals for holding times between two successive occurrences of earthquakes

| **Holding time interval (yr.)** | **Integer assigned to each interval** |
|---|---|
| $0 \le t_{ij} \le 5$ | 1 |
| $6 \le t_{ij} \le 10$ | 2 |
| $11 \le t_{ij} \le 15$ | 3 |
| $16 \le t_{ij} \le 20$ | 4 |
| $21 \le t_{ij} \le 25$ | 5 |
| $26 \le t_{ij} \le 30$ | 6 |

Thus, in this case total six intervals are defined ($n$=6) each of which having length equal to 5 years ($\Delta t$=5 years). Continuing over whole earthquake data, each holding time between successive earthquakes is assigned with an integer according to the class intervals for holding time  defined in above table. Thus, ij$^{th}$ element of holding time mass matrix $T_{ij}(m)$ for a particular time interval $m$ is obtained as

$$P\{t_{ij}=m\} = \frac{number\ of\ transitions\ from\ state\ i\ to\ state\ j\ during\ a\ time\ interval\ m}{total\ number\ of\ transitions\ from\ state\ i\ to\ state\ j} \tag{21}$$

Here, m≤n i.e., m=1,2,3...,6. In this way, holding time mass matrix is obtained for both magnitude and region states corresponding to each time interval. Thus a total six mass matrix are obtained for both states.

Now, our next target is reduced to get only interval transition probability matrix. For this, first we have to find the core matrix which is obtained by multiplication of corresponding elements of transition probability matrix and holding time mass matrix for each holding time interval

$$C(m)= P.*T(m)$$

where, m=1,2,3,...,6.

Next, complementary probability distribution $Gw_i(n)$ defined as probability that waiting time for the i$^{th}$ state is less than or equal to n is estimated, which is given as

$$Gw_i(n) = \sum_{m=n+1}^{\infty} wi(m), \text{ for } n=1,2,...,6.$$

where, $wi(m)$ represents the probability that the waiting time for i$^{th}$ state is equal to m for m=1,2,3...,6.

Finally, interval transition probability matrix is obtained using the following recursive procedure

$$F(n) = Gw(n) + \sum_{m=0}^{n} C(m)F(n-m); \; n= 1,2,...$$

Above equation goes for n=1 up to 6. Thus, total six interval transition probability matrices are obtained for each time interval corresponding to both region and magnitude states. For calculate F(1), initially we have to calculate F(0), which is equal to Kronecker Delta or Identity Matrix, as explained in the sec 5.2.

$$F_{ij}(0) = \begin{cases} 1, & if\ i = j \\ 0, & if\ i \neq j \end{cases}$$

Interval transition probabilities so obtained are used to calculate the overall probability of occurrence of earthquake in a particular region for a particular magnitude states. For this we have to determine the joint probability between the region and magnitude interval transition probabilities . For this, it is assumed that region to region transition probabilities are independent from magnitude to magnitude transition probabilities. If the previous earthquake is within region $r_0$ having magnitude $m_0$, then the probability of occurrence of the next earthquake within region $r_1$ having magnitude $m_1$ after $d$ time periods is a conditional probability defined as follows

$$\begin{aligned} P(R=r_1, M=m_1| r_0, m_0, d) &= P(r_1, m_1|r_0, m_0, d) = P(r_1| r_0, d).\ P(m_1|m_0, d) \\ &= F_R\{r_0\ r_1, d\}.F_M\{m_0\ m_1, d\} \end{aligned} \quad (22)$$

Here, it is important to note that the above discussed parameters are calculated separately for both magnitude to magnitude transitions and region to region transitions.

## 5.3 Results and discussion

For semi-Markov model, Markov chain is developed as discussed in the sec 7 to calculate the transition probability matrices for both magnitude and region states. The Transition probability matrices thus obtained are given as below

$$P_M = \begin{bmatrix} .5714 & .25 & .1071 & .0714 \\ .44 & .3889 & .0556 & .1111 \\ .2 & .8 & 0 & 0 \\ .6 & 0 & .4 & 0 \end{bmatrix} \quad P_R = \begin{bmatrix} .45 & 0 & .2 & .35 \\ .33 & 0 & 0 & .6667 \\ .2143 & 0 & .6429 & .1429 \\ .3158 & .1579 & .1053 & .4211 \end{bmatrix}$$

Holding time mass matrix for both region and magnitude states is given below as

**Holding time mass matrices for magnitude states**

$$Tm(1) = \begin{bmatrix} .5625 & 1 & .6667 & 1 \\ 1 & .8571 & 1 & .5 \\ 1 & .5 & 0 & 0 \\ .6667 & 0 & 1 & 0 \end{bmatrix} \qquad Tm(2) = \begin{bmatrix} .3125 & 0 & 0 & 0 \\ 0 & .1429 & 0 & .5 \\ 0 & .5 & 0 & 0 \\ .33 & 0 & 0 & 0 \end{bmatrix}$$

$$Tm(3) = \begin{bmatrix} 0 & 0 & .33 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad Tm(4) = \begin{bmatrix} .0625 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Tm(5) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad Tm(6) = \begin{bmatrix} .0625 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

**Holding time mass matrix for region**

$$\text{Tr}(1)=\begin{bmatrix} 0.4444 & 0.0000 & 0.5000 & 0.8571 \\ 1.0000 & 0.0000 & 0.0000 & 0.5000 \\ 0.6667 & 0.0000 & 0.6667 & 1.0000 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 \end{bmatrix} \quad \text{Tr}(2)=\begin{bmatrix} 0.5556 & 0.0000 & 0.5000 & 0.1429 \\ 0.0000 & 0.0000 & 0.0000 & 0.5000 \\ 0.3333 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}$$

$$\text{Tr}(3)=\begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1111 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix} \quad \text{Tr}(4)=\begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1111 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}.$$

$$\text{Tr}(5)=\begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix} \quad \text{Tr}(6)=\begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1111 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}$$

Interval transition probability matrices for both magnitude and region states as discussed in sec 7, are obtained below as

**Interval transition probability for magnitude state**

$$F_M(1)=\begin{bmatrix} .6071 & .25 & .0714 & .0714 \\ .4444 & .4444 & .0556 & .0556 \\ .2 & .4 & .4 & 0 \\ .4 & 0 & .4 & .2 \end{bmatrix} \quad F_M(2)=\begin{bmatrix} .6348 & .22 & .0490 & .0511 \\ .4513 & .3370 & .0947 & .1169 \\ .2992 & .6278 & .0365 & .0365 \\ .5229 & .26 & .1886 & .0286 \end{bmatrix}$$

$$F_M(3)=\begin{bmatrix} .5555 & .2630 & .1184 & .0631 \\ .5252 & .2842 & .1112 & .0795 \\ .4853 & .3566 & .0789 & .0792 \\ .4950 & .3891 & .0665 & .0493 \end{bmatrix} \quad F_M(4)=\begin{bmatrix} 0.5718 & 0.2624 & 0.1073 & 0.0585 \\ 0.5305 & 0.2862 & 0.1135 & 0.0698 \\ 0.5017 & 0.3011 & 0.1060 & 0.0912 \\ 0.5433 & 0.2919 & 0.0977 & 0.0671 \end{bmatrix}$$

$$F_M(5)=\begin{bmatrix}0.5583 & 0.2766 & 0.1024 & 0.0627\\ 0.5457 & 0.2824 & 0.1067 & 0.0652\\ 0.5366 & 0.2806 & 0.1113 & 0.0714\\ 0.5405 & 0.2780 & 0.1090 & 0.0725\end{bmatrix} \quad F_M(6)=\begin{bmatrix}0.5706 & 0.2669 & 0.1007 & 0.0618\\ 0.5495 & 0.2802 & 0.1051 & 0.0652\\ 0.5421 & 0.2828 & 0.1086 & 0.0665\\ 0.5523 & 0.2754 & 0.1070 & 0.0653\end{bmatrix}$$

**Interval transition probability matrix for region states**

$$F_R(1)=\begin{bmatrix}0.6000 & 0.0000 & 0.1000 & 0.3000\\ 0.3333 & 0.3333 & 0.0000 & 0.3333\\ 0.1429 & 0.0000 & 0.7143 & 0.1429\\ 0.3158 & 0.1579 & 0.1053 & 0.4211\end{bmatrix} \quad F_R(2)=\begin{bmatrix}0.4790 & 0.0474 & 0.2230 & 0.2506\\ 0.3053 & 0.0526 & 0.0684 & 0.5737\\ 0.2635 & 0.0226 & 0.5497 & 0.1642\\ 0.3901 & 0.1191 & 0.1511 & 0.3397\end{bmatrix}$$

$$F_R(3)=\begin{bmatrix}0.4193 & 0.0554 & 0.2466 & 0.2788\\ 0.3950 & 0.1081 & 0.1598 & 0.3371\\ 0.2799 & 0.0335 & 0.5105 & 0.1761\\ 0.3915 & 0.0758 & 0.2027 & 0.3300\end{bmatrix} \quad F_R(4)=\begin{bmatrix}0.3949 & 0.0572 & 0.2795 & 0.2684\\ 0.4003 & 0.0834 & 0.2001 & 0.3162\\ 0.2802 & 0.0365 & 0.4928 & 0.1906\\ 0.3891 & 0.0700 & 0.2422 & 0.2988\end{bmatrix}$$

$$F_R(5)=\begin{bmatrix}0.3761 & 0.0571 & 0.3007 & 0.2662\\ 0.3918 & 0.0677 & 0.2415 & 0.2991\\ 0.2911 & 0.0394 & 0.4650 & 0.2046\\ 0.3812 & 0.0645 & 0.2737 & 0.2806\end{bmatrix} \quad F_R(6)=\begin{bmatrix}0.3649 & 0.0562 & 0.3200 & 0.2590\\ 0.3821 & 0.0639 & 0.2722 & 0.2818\\ 0.2999 & 0.0423 & 0.4485 & 0.2093\\ 0.3718 & 0.0600 & 0.2973 & 0.2709\end{bmatrix}$$

By rearranging above interval transition probability matrix for magnitude states so obtained, magnitude to magnitude interval transition probability by time of occurrence without considering the spatial dimension are shown below in the table. In these tables, the sum of each column is equal to one. The $ij^{th}$ element of this table shows the probability of transition of earthquake of given magnitude state $i$ to a particular magnitude state $j$ during a particular time period.

**Table 5** Magnitude to magnitude interval transition probabilities without considering spatial parameters

| Probability | | | | | | |
|---|---|---|---|---|---|---|
| **Holding Time interval** | 1 | 2 | 3 | 4 | 5 | 6 |
| **Time(Year)** | 1-5 yr | 6-10 yr | 7-15 yr | 16-20 yr | 21-25 yr | 26-30 yr |
| M1 to M1 | .6071 | .6348 | .5555 | .5718 | .5583 | .5706 |
| M1 to M2 | .25 | .22 | .2630 | .2624 | .2766 | .2669 |
| M1 to M3 | .0714 | .0940 | .1184 | .1073 | .1024 | .1007 |
| M1 to M4 | .0714 | .0511 | .0631 | .0585 | .0627 | .0618 |
| M2 to M1 | .44 | .4513 | .5252 | .5305 | .5457 | .5495 |
| M2 to M2 | .44 | .337 | .2842 | .2862 | .2824 | .2802 |
| M2 to M3 | .0556 | .0947 | .1112 | .1135 | .1067 | .1051 |
| M2 to M4 | .0556 | .1169 | .0795 | .0698 | .0652 | .0652 |
| M3 to M1 | .2 | .2992 | .4853 | .5017 | .5366 | .5421 |
| M3 to M2 | .4 | .6278 | .3566 | .3011 | .2806 | .2828 |
| M3 to M3 | .4 | .0365 | .0789 | .106 | .1113 | .1086 |
| M3 to M4 | 0 | .0365 | .0792 | .0912 | .0714 | .0665 |
| M4 to M1 | .4 | .5229 | .4950 | .5433 | .5405 | .5523 |
| M4 to M2 | 0 | .26 | .3891 | .2919 | .2780 | .2754 |
| M4 to M3 | .4 | .1886 | .0655 | .0977 | .1090 | .1070 |
| M4 to M4 | .2 | .0286 | .0493 | .0671 | .0725 | .0653 |

Tables for region to region transition probabilities by time of occurrence without considering dimension of magnitude is given below. This shows the probability of earthquake occurrence in a region during a year if the last earthquake occurrence in a particular region is given. The sum of each column in the table is equal to one.

**Table.6** Region to region interval transition probabilities without considering magnitudes

| Probability | | | | | | |
|---|---|---|---|---|---|---|
| Holding Time interval | 1 | 2 | 3 | 4 | 5 | 6 |
| Time(Year) | 1-5 yr | 6-10 yr | 7-15 yr | 16-20 yr | 21-25 yr | 26-30 yr |
| R1 to R1 | .6 | .4790 | .4193 | .3949 | .3761 | .3649 |
| R1 to R2 | 0 | .0474 | .0554 | .0572 | .0571 | .0562 |
| R1 to R3 | .1 | .2230 | .2466 | .2795 | .3007 | .3200 |
| R1 to R4 | .3 | .2506 | .2788 | .2684 | .2662 | .2590 |
| R2 to R1 | .3333 | .3053 | .3950 | .4003 | .3918 | .3821 |
| R2 to R2 | .3333 | .0526 | .1081 | .0834 | .0677 | .0639 |
| R2 to R3 | 0 | 0.0684 | .2466 | .2795 | .3007 | .3200 |
| R2 to R4 | .3 | .2506 | .2788 | .2684 | .2662 | .2590 |
| R3 to R1 | .1429 | .2635 | .2799 | .2802 | .2911 | .2999 |
| R3 to R2 | 0 | .0226 | .0335 | .0365 | .0394 | .0423 |
| R3 to R3 | .7143 | .5497 | .5105 | .4928 | .4650 | .4485 |
| R3 to R4 | .1429 | .1642 | .1761 | .1906 | .2046 | .2093 |
| R4 to R1 | .3158 | .3901 | .3915 | .3891 | .3812 | .3718 |
| R4 to R2 | .1579 | .1191 | .0758 | .0700 | .0645 | .0600 |
| R4 to R3 | .1053 | .1511 | .2027 | .2422 | .2737 | .2973 |
| R4 to R4 | .4211 | .3397 | .3300 | .2988 | .2806 | .2709 |

Above values shown in the tables, are draw in the form of graphs as given below. The graphs shows the plot of interval transition probabilities for both magnitude and region states with the number of time intervals. Each time interval being equal to 5 years. Thus, prediction is made over a total of 30 years. From the graphs it can be shown that, probabilities are varying randomly with time following a arbitrary trend. But, it is not consistent with the practical understanding of occurrence of earthquakes, as explained by elastic rebound model, which shows that probability should increase with time. It is true, because as the holding time increases for a event to occur, as in this case for occurrence of earthquake, the probability for occurrence of that event also increases. In this report, semi-Markov model is applied only for the past earthquake data, that consists of total of 57 number of earthquakes, which had occurred in the past. So, it is the limitation for the application of model in this report. However, more realistic and practical results may be obtained using simulation, and by choosing parameters and distributions to assure increasing hazard rates for holding time distributions e.g., Weibull, Lognormal, Gamma distributions, which implies that probability of an earthquake occurrence in near futures increases with time since the last event. The increasing hazard captures some of the characteristics of stress build up and release.

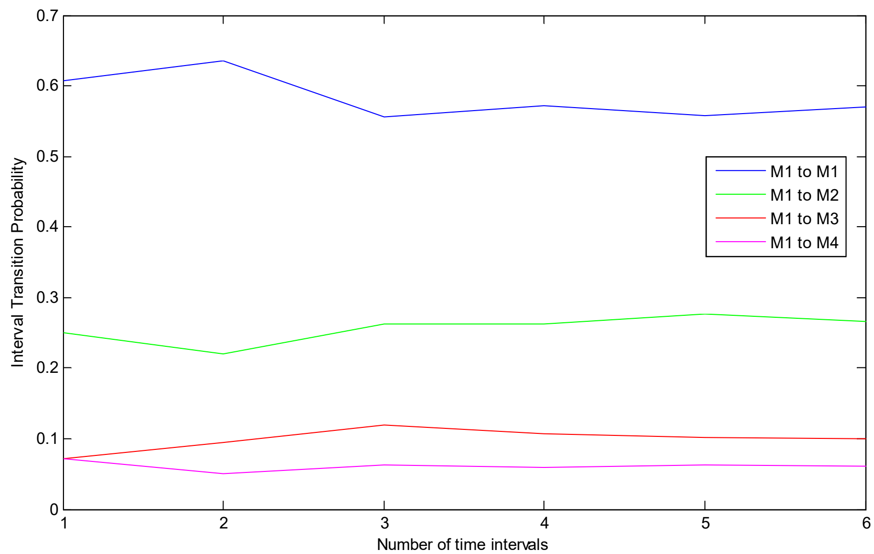



**Fig. 8** Interval transition probabilities for transitions from state M1 to other all magnitude states

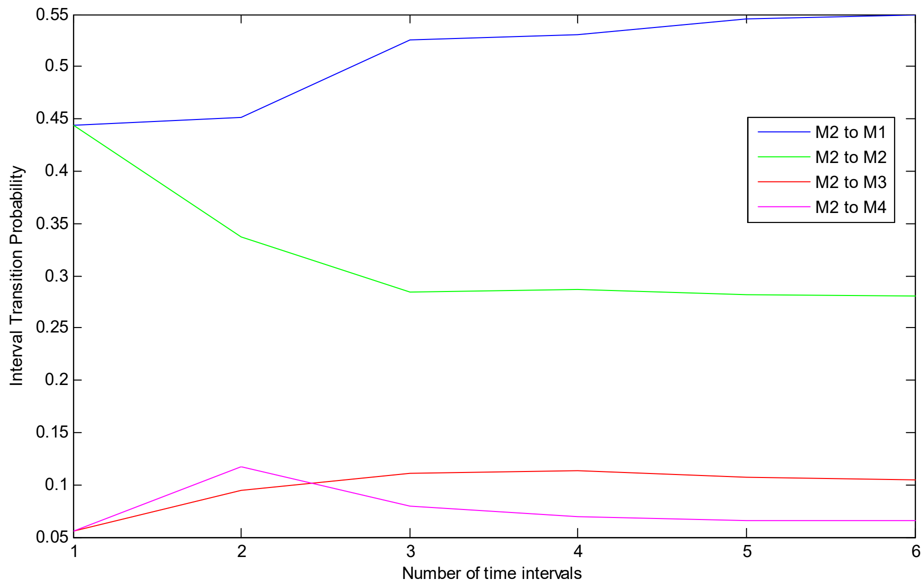



**Fig. 9** Interval transition probabilities for transitions from state 2 to other all magnitude states

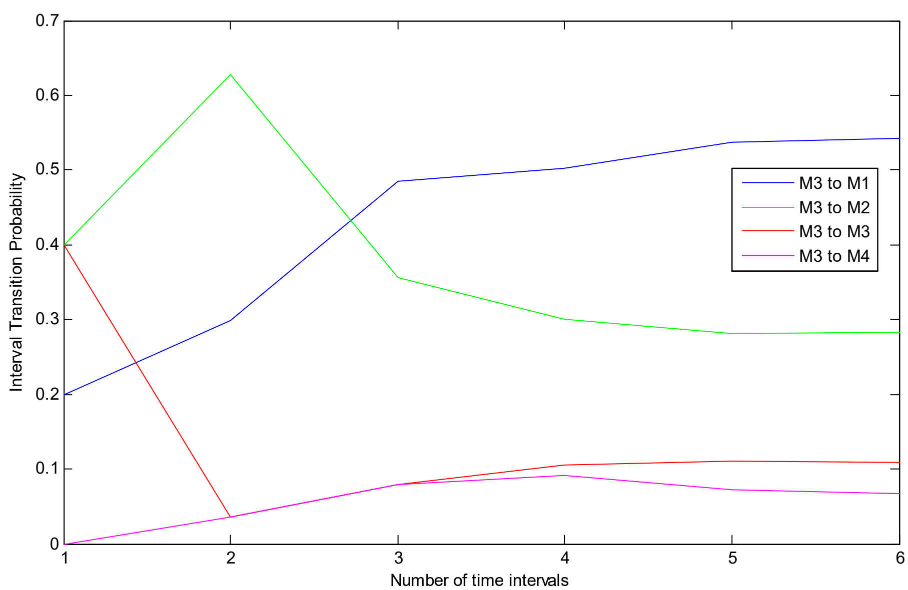



**Fig. 10** Interval transition probabilities for transitions from state M3 to other all magnitude states

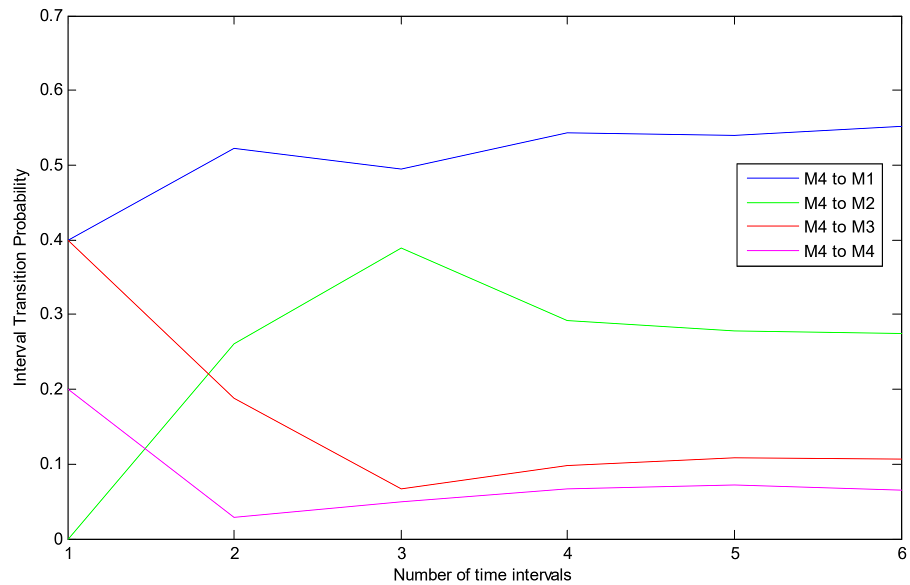



**Fig. 11** Interval transition probabilities for transitions from state M4 to other all magnitude states

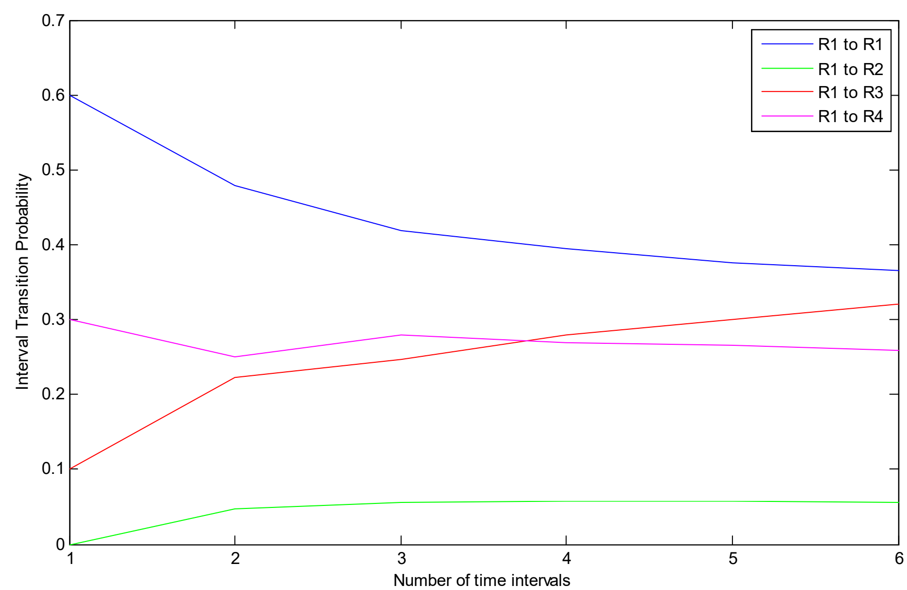



**Fig. 12** Interval transition probabilities for transitions from region state R1 to other all region states

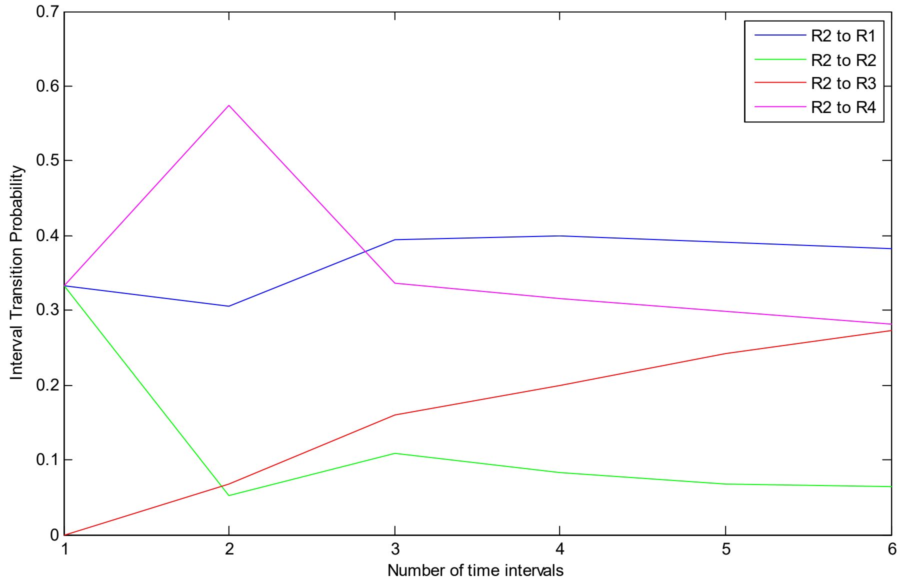



**Fig. 13** Interval transition probabilities for transitions from region state R2 to other all region states

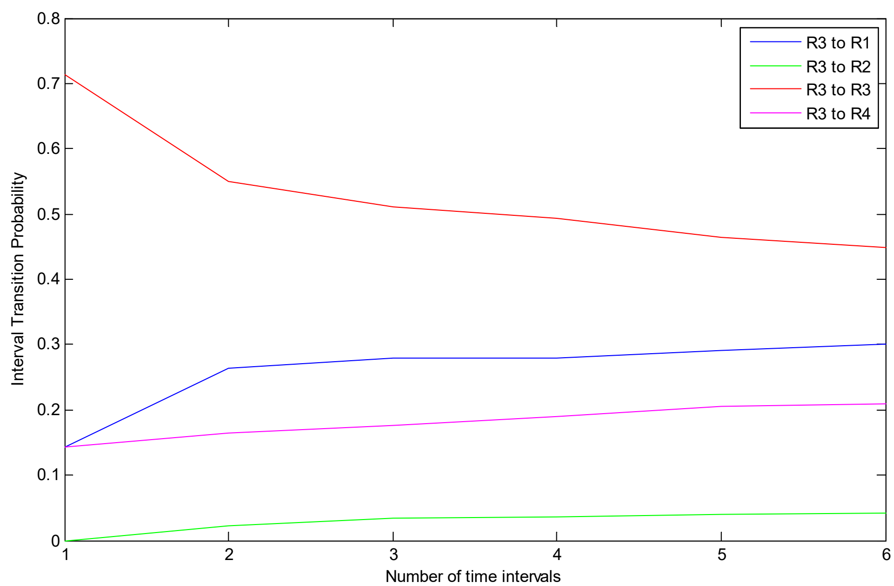



**Fig. 14** Interval transition probabilities for transitions from region state R3 to other all region states

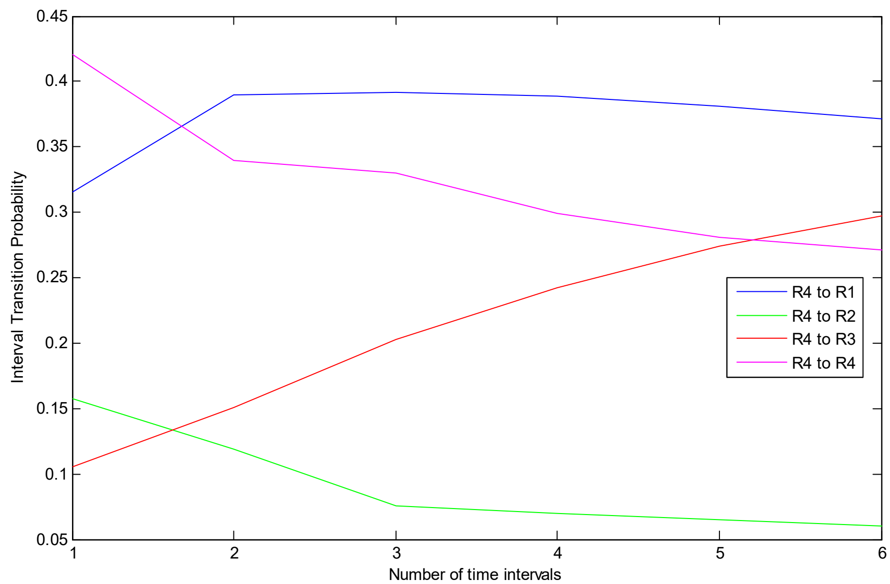



**Fig. 15** Interval transition probabilities for transitions from region state R4 to other all region states

After obtaining the interval transition probabilities for region and magnitude states, time and magnitude of earthquake occurrences can be forecasted for each region in next 30 years, since there are six time intervals, each time interval being equal to five years. Using conditional probability as defined earlier in sec 7. For this, it is assumed that region to region transition probabilities are independent from magnitude to magnitude transition probabilities. If the previous earthquake is within region $r_0$ having magnitude $m_0$, then the probability of occurrence of the next earthquake within region $r_1$ having magnitude $m_1$ after $d$ time periods is a conditional probability defined as follows

$$P(R=r_1, M=m_1| r_0, m_0, d) = P(r_1, m_1|r_0, m_0, d) = P(r_1| r_0, d). P(m_1|m_0,d)$$
$$= F_R\{r_0\ r_1, d\}.F_M\{m_0\ m_1, d\} \quad (23)$$

The last great earthquake occurred in region R3 of magnitude 7.8( state M4) in 2015. Thus, the most probable occurrence of an earthquake within region R2 ( considered as seismic gap, in which no major earthquake had occurred in the past) belonging to a magnitude state of M4, in the next 25 years is .285%.

Some graphs of overall probability for occurrences of major earthquakes (state M4) in seismic gap sub-region R2, for a given initial condition are shown below

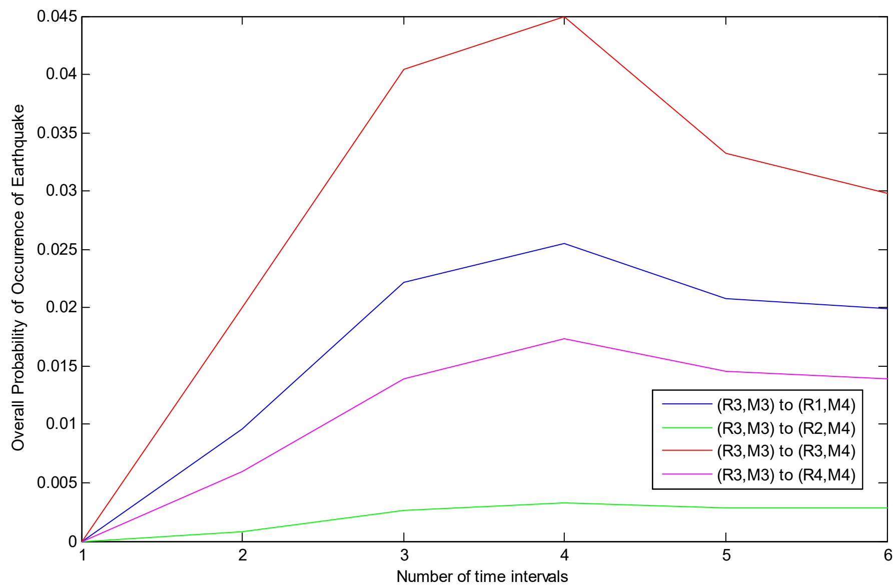



**Fig. 16** Overall interval transition probabilities for transitions from region state( R3,M3) to other all region states and magnitude state M4

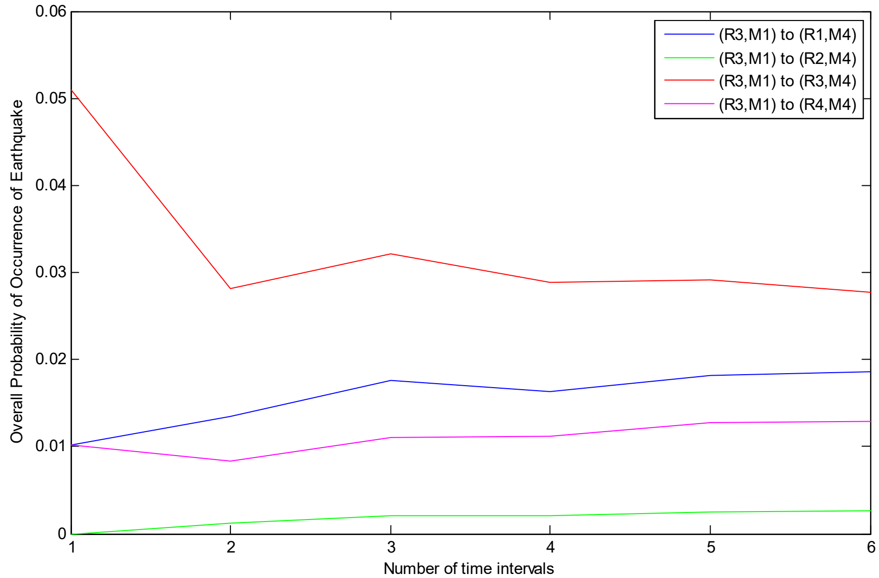



**Fig. 17** Overall interval transition probabilities for transitions from region state( R3,M1) to other all region states and magnitude state M4

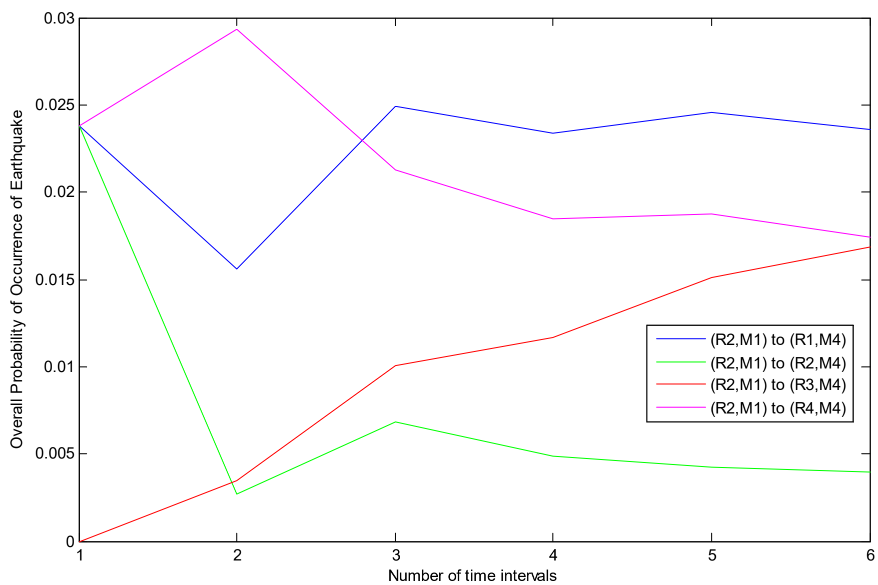



**Fig 18** Overall interval transition probabilities for transitions from region state ( R2,M1) to other all region states and magnitude state M4

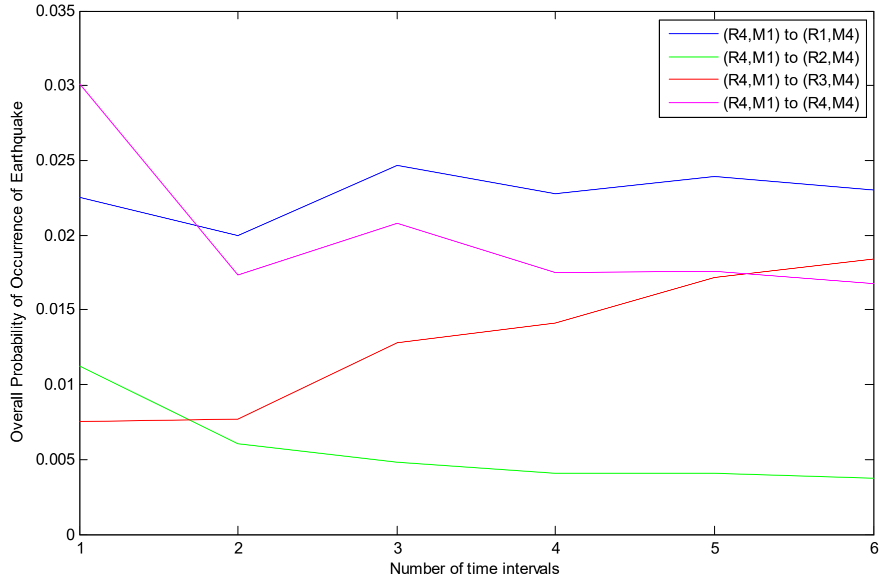



**Fig. 19** Overall interval transition probabilities for transitions from region state( R4,M1) to other all region states and magnitude state M4

Chapter 6

# CONCLUSION

## 9.1 Markov Model

This paper has developed a Markov model for the occurrence of earthquakes in central Himalayan region of Nepal and contiguous areas, the central Nepal of which is represented by a seismic gap. This and the areas to its east, west and north are therefore considered four different sub-regions. Also, by classifying the annual maximum magnitudes in the entire region into four states of a Markov chain, formulation is presented to predict the time-dependent probabilities of occurrence of a major earthquake($M_w \geq 7.5$) in the four sub-regions, conditional on different combinations of the magnitude and sub-region of the just previous earthquake.

Dependence of future earthquakes on both the magnitude and location of the previous earthquake is not possible by other commonly used probability distributions to describe the recurrence intervals of earthquakes. The renewal models used to consider time dependence of occurrence rates of very large magnitude earthquakes since the time of last earthquake are applicable to the same category of earthquakes over a very large region, whereas the proposed Markov model is able to account for the dependence among earthquakes of different magnitudes in adjacent sub-regions.

Illustrative numerical results for periods beyond 50 years indicate the same high probability (which is more than 85%) of occurrence of a major earthquake ($M_W$>7.5) in the gap area as in the other three sub-regions. For smaller periods since the last major earthquake, this probability for the gap area decreases by about 5-8% with decrease in time period.

It has been also explained that how the results on the interval transition probabilities obtained from the Markov model can be useful in carrying out time-dependent probabilistic seismic hazard analysis (PSHA) for the region of study. It has been indicated that the time-dependence need to be considered for the large regional earthquakes only. The transition probabilities for low to moderate magnitude earthquakes reach unity within 10 to 15 years, which is much smaller than the time periods of applicability of the estimated hazard. Thus these earthquakes can be described independently for each seismic source by constant occurrences rates. However, the occurrence rate of major can be defined as a function of time in proportion to the magnitude transition probability since the time of previous major earthquakes. For use in PSHA computations, the occurrence rate at any given time is then divided among various source regions in proportion to the region to region transition probabilities.

## 9.2 Semi-Markov Model

The difference between the Markov model and Semi-Markov model is that of holding time distribution. In Markov model we use exponential distribution of holding times between events, because exponential distribution possesses property of memorylessness. But, semi-Markov is generalized Markovian process. In semi-Markov process, holding time in a given state is identically distributed, conditional on both the current state and the next state, thus providing the greater flexibility in modeling. In semi-Markov model, parameters and distributions have been chosen to assure increasing hazard rates for holding time distributions e.g., Weibull, Lognormal, Gamma distributions, which implies that probability of an earthquake occurrence in near futures increases with time since the last event. The increasing hazard captures some of the characteristics of stress build up and release. Semi-Markov process has the basic Markovian property of one-step memory i.e., the probability that the next earthquake is of a given magnitude depends on the magnitude of previous earthquake. So, the results obtained in this report for semi-Markov model are merely not so accurate because of limitation of past data available on earthquake events. To obtain reliable and practical results, suitable parameters have to be choose ,assuring that probability of occurrences of earthquakes increases with time since last time. Results obtained for overall probability for occurrence of earthquakes, as shown in Fig 15, 16, 17 and 18, shows the probability distribution of events over time domain is varying randomly, not assuring that probability increases with time since last time. To obtain such results, simulation have to be follow with holding time distribution assuming a suitable probability distribution like log-normal etc. The process for obtaining results using simulation is still going on.

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# APPENDIX

List of the available data on early maximum magnitude earthquakes used in the Markov model with the magnitude and location states indicated

| Date | Lon. | Lat | M | Region State | Magnitude State |
|---|---|---|---|---|---|
| 01:09:1803 | 79 | 31.5 | 7.5 | 1 | 4 |
| 1809 | 79 | 30 | 6 | 1 | 2 |
| 28:08:1816 | 81 | 30 | 7.5 | 1 | 4 |
| 29:10:1826 | 85.33 | 27.7 | 6 | 3 | 2 |
| 26:08:1833 | 85.7 | 27.7 | 7.6 | 3 | 4 |
| 05:03:1842 | 78 | 30 | 5.5 | 1 | 2 |
| 11:04:1843 | 80 | 30 | 5.1 | 1 | 1 |
| 27:02:1849 | 88.3 | 27 | 6 | 3 | 2 |
| 05:1852 | 88.3 | 27 | 6.4 | 3 | 2 |
| 18:06:1862 | 88.3 | 27 | 5.1 | 3 | 1 |
| 29:03:1863 | 88.3 | 27 | 5.7 | 3 | 2 |
| 16:12:1865 | 88.3 | 27 | 5.1 | 3 | 1 |
| 23:05:1866 | 85.3 | 27.7 | 7 | 3 | 3 |
| 07:07:1869 | 85 | 28 | 6.4 | 3 | 2 |
| 22:05:1871 | 78.1 | 30.45 | 4.5 | 1 | 1 |

| | | | | | |
|---|---|---|---|---|---|
| 25:09:1899 | 88.3 | 27 | 6 | 3 | 2 |
| 13:12:1902 | 85 | 30 | 6.7 | 4 | 3 |
| 31:03:1904 | 89 | 31 | 6.9 | 4 | 3 |
| 13:06:1906 | 79 | 31 | 6 | 1 | 2 |
| 20:08:1908 | 89 | 32 | 7 | 4 | 3 |
| 17:02:1909 | 87 | 27 | 5.1 | 3 | 1 |
| 14:10:1911 | 80.5 | 31 | 6.5 | 1 | 3 |
| 06:03:1913 | 83 | 30 | 6.5 | 2 | 3 |
| 05:05:1915 | 84 | 30 | 6 | 4 | 2 |
| 28:08:1916 | 81 | 30 | 7.2 | 1 | 3 |
| 28:04:1918 | 82 | 30.5 | 4 | 1 | 1 |
| 24:04:1923 | 87.8 | 29.6 | 4 | 4 | 1 |
| 15:12:1925 | 85.8 | 30 | 4.3 | 4 | 1 |
| 27:07:1926 | 80.05 | 30.5 | 6.5 | 1 | 3 |
| 29:11:1927 | 83 | 30 | 4.3 | 2 | 1 |
| 18:06:1931 | 84 | 30.5 | 4.3 | 4 | 1 |
| 25:03:1932 | 89.2 | 30 | 4.6 | 4 | 1 |
| 18:05:1933 | 80 | 29.5 | 4.6 | 1 | 1 |
| 15:01:1934 | 86.762 | 26.773 | 8.1 | 3 | 4 |
| Date | Lon. | Lat | M | | |

| | | | | | |
|---|---|---|---|---|---|
| 27:05:1936 | 83.283 | 28.345 | 7 | 2 | 3 |
| 20:10:1937 | 78 | 31 | 4.7 | 1 | 1 |
| 29:01:1938 | 87 | 27.5 | 4.8 | 3 | 1 |
| 04:06:1939 | 86.5 | 28.5 | 4.8 | 3 | 1 |
| 10:04:1940 | 81.5 | 30 | 4.8 | 1 | 1 |
| 29:10:1944 | 83.5 | 31.5 | 6.8 | 4 | 3 |
| 04:06:1945 | 80 | 30.3 | 6.5 | 1 | 3 |
| 19:08:1947 | 79.09 | 31.2 | 4.8 | 1 | 1 |
| 05:05:1948 | 78.75 | 30.44 | 4.8 | 1 | 1 |
| 11:08:1949 | 89 | 31 | 4.8 | 4 | 1 |
| 28:05:1951 | 87 | 29 | 4.8 | 4 | 1 |
| 19:11:1952 | 86.6 | 29.8 | 4.8 | 4 | 1 |
| 03:12:1953 | 85.6 | 31.4 | 6.7 | 4 | 3 |
| 04:09:1954 | 83.8 | 28.3 | 6.5 | 2 | 3 |
| 04:08:1955 | 86.4 | 30.8 | 4.8 | 4 | 1 |
| 22:04:1957 | 84.3 | 30.8 | 4.8 | 4 | 1 |
| 31:12:1958 | 79.9 | 29.94 | 6.3 | 1 | 2 |
| 24:12:1961 | 80.83 | 29.43 | 4.8 | 1 | 1 |
| 14:07:1962 | 79.5 | 30.4 | 4.8 | 1 | 1 |
| 27:11:1963 | 79.1 | 30.8 | 4.8 | 1 | 1 |

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|---|---|---|---|---|---|
| 26:09:1964 | 80.46 | 29.96 | 6 | 1 | 2 |
| 12:01:1965 | 87.84 | 27.4 | 6 | 3 | 2 |
| 27:06:1966 | 80.89 | 29.71 | 6.1 | 1 | 2 |
| 02:03:1967 | 86.38 | 28.7 | 5.4 | 3 | 1 |
| 31:05:1968 | 79.92 | 29.91 | 5.3 | 1 | 1 |
| 22:06:1969 | 79.4 | 30.5 | 5.5 | 1 | 2 |
| 12:02:1970 | 81.57 | 29.24 | 5.5 | 2 | 2 |
| 03:05:1971 | 84.328 | 30.789 | 5.5 | 4 | 2 |
| 21:08:1972 | 88.023 | 27.228 | 5.4 | 3 | 1 |
| 02:01:1973 | 88.085 | 31.173 | 5.4 | 4 | 1 |
| 27:09:1974 | 85.512 | 28.594 | 5.7 | 3 | 2 |
| 19:01:1975 | 78.525 | 31.937 | 6 | 1 | 2 |
| 08:09:1976 | 78.764 | 32.033 | 5.5 | 1 | 2 |
| 19:02:1977 | 78.432 | 31.797 | 5.6 | 1 | 2 |
| 10:02:1978 | 84.698 | 28.033 | 5.5 | 3 | 2 |
| 20:05:1979 | 80.27 | 29.932 | 5.8 | 1 | 2 |
| 29:07:1980 | 81.091 | 29.628 | 6.5 | 1 | 3 |
| 28:05:1981 | 78.436 | 31.829 | 5.4 | 1 | 1 |
| 23:01:1982 | 82.284 | 31.675 | 6.3 | 4 | 2 |
| 27:01:1983 | 81.343 | 29.042 | 5.2 | 2 | 1 |

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| 18:05:1984 | 81.793 | 29.52 | 5.8 | 2 | 2 |
| 08:12:1985 | 86.573 | 30.841 | 5.2 | 4 | 1 |
| 20:06:1986 | 86.824 | 31.216 | 6 | 4 | 2 |
| 09:08:1987 | 83.739 | 29.465 | 5.6 | 2 | 2 |
| 20:08:1988 | 86.626 | 26.72 | 6.8 | 3 | 3 |
| 22:05:1989 | 87.858 | 27.381 | 5.3 | 3 | 1 |
| 09:01:1990 | 88.108 | 28.154 | 5.7 | 3 | 2 |
| 19:10:1991 | 78.791 | 30.77 | 6.9 | 1 | 3 |
| 02:06:1992 | 81.904 | 28.938 | 5.4 | 2 | 1 |
| 20:03:1993 | 87.328 | 29.027 | 6.2 | 4 | 2 |
| 23:07:1994 | 86.601 | 31.097 | 5.4 | 4 | 1 |
| 30:07:1995 | 88.21 | 30.246 | 6.4 | 4 | 2 |
| 03:07:1996 | 88.191 | 30.106 | 5.6 | 4 | 2 |
| 05:01:1997 | 80.5 | 29.8 | 5.7 | 1 | 2 |
| 03:09:1998 | 87 | 27.87 | 5.8 | 3 | 2 |
| 28:03:1999 | 79.421 | 30.511 | 6.5 | 1 | 3 |
| 06:09:2000 | 86.97 | 28.38 | 5.5 | 3 | 2 |
| 27:11:2001 | 82.26 | 29.56 | 5.5 | 2 | 2 |
| 04:06:2002 | 81.42 | 30.566 | 5.6 | 1 | 2 |
| 27:05:2003 | 79.337 | 30.556 | 5.3 | 1 | 1 |

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| 11:07:2004 | 83.666 | 30.719 | 6.2 | 4 | 2 |
| 07:04:2005 | 83.655 | 30.517 | 6.3 | 4 | 2 |
| 14:02:2006 | 88.416 | 27.387 | 5.3 | 3 | 1 |
| 22:07:2007 | 78.288 | 30.869 | 5.3 | 1 | 1 |
| 25:08:2008 | 83.652 | 31.061 | 6.7 | 4 | 3 |
| 24:07:2009 | 85.963 | 31.169 | 5.5 | 4 | 2 |
| 26:02:2010 | 86.776 | 28.507 | 5.5 | 3 | 2 |
| 18:09:2011 | 88.154 | 27.804 | 6.9 | 3 | 3 |
| 17:02:2012 | 82.787 | 32.388 | 5.4 | 4 | 1 |
| 30:06:2013 | 81.431 | 31.816 | 5.6 | 4 | 2 |
| 30:03:2014 | 86.558 | 31.357 | 5.4 | 4 | 1 |
| 25:04:2015 | 84.79 | 28.28 | 7.9 | 3 | 4 |