CRITICAL RESPONSE OF MDOF STRUCTURES TO MULTI-COMPONENT SEISMIC EXCITATION

A DISSERTATION

SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF DEGREE

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EARTHQUAKE ENGINEERING

(With Specialization in Structural Dynamics)

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CANDIDATE'S DECLARATION

I hereby declare that the work which is being presented in this dissertation report titled **"Critical response of MDOF structures to multi-component seismic excitation"** presented on behalf of partial fulfillment of the requirement for the award of the degree of **MASTER OF TECHNOLGY** in **EARTHQUAKE ENGINEERING** with specialization in **STRUCTURAL DYNAMICS** submitted in the Department of Earthquake Engineering, Indian Institute of Technology, Roorkee, India**,** under the supervision and guidance of **Dr. I. D. Gupta,** Honorary Fellow, Earthquake Engineering, IIT Roorkee, India.

The matter embodied in this report has not been submitted by me for the award of any other degree or diploma.

Date: 02-05-2016

Place: Roorkee **ABHISHEK KUMAR KANAUJIA**

CERTIFICATION

This is to certify that the above statement made by the candidate is correct to the best of our knowledge and belief.

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ABSTRACT

Critical response is the maximum value of the response which is obtained under certain loading conditions. In present methods of seismic analysis of structures, the earthquake motion is generally considered in horizontal direction acting along the principal axis of the structure. But, in reality, the earthquake ground motion is not so simple, rather, it is much complex having both translational as well as rotational components. The response obtained by the response spectrum analysis of the structures may not give the maximum response. In fact, there are certain assumptions involved in the generation of response spectrum itself, which may not hold good in actual conditions. Moreover, the time histories which are recorded using various instruments are dependent on the orientation of the instrument with respect to the ground motion. So, this orientation dependent time histories when applied to the structures may not give the maximum response

So, the main objective of this project work is to develop such design philosophy by which we will be able to get the maximum response of the structure. For this, a response spectrum shall be constructed which will take into account the orientation dependency and the multi-component nature of the earthquake ground motion.

CHAPTER-1: INTRODUCTION

1.1 Overview

In earthquake resistant design, estimation of peak seismic responses accurately is very important. The distribution of the internal forces and the seismic responses of structures are very complex in nature because the ground motions are multi-directional. The main issue is the uncertainty which is involved in the angle of incidence between the ground motion direction and the axes of reference of the structures. For different angles of incidence, one can get different peak responses using the response spectrum analysis for the given structure and earthquake ground motion record combination. However, the determination of maximum structural response for a given earthquake excitation using time history analysis requires the calculation of critical incident angle which in itself is a very difficult and cumbersome process.

In order to design any structure for earthquake resistance, the most important and necessary step is to determine the peak structural response. Generally, the earthquake ground motions are represented by their projections on a set of reference axes which can be oriented arbitrarily and is usually a set of three perpendicular Cartesian axes. These axes consist of a vertical axis and two other axes lying in the horizontal plane, however, their orientation is selected arbitrarily as per the convenience and simplicity in the analysis. When the axes representing the ground motion records are provided to the designers, there will be a fixed angle, called the critical seismic incident angle, between the reference axes of the structure and the seismic record. The calculated peak response of the structure is dependent on the incident angle and keeps on changing as the incident angle changes.

An earthquake has three components of the ground acceleration due to which the structures are subjected to three dimensional (3D) loading. However, these all three ground acceleration components are not considered in the earthquake resistant design of structures. The reason for this is the sophisticated and rigorous analysis involved and also due to the lack of sufficient knowledge and the absence of any simple method. In present times, most building codes use a design response spectrum which is based on the results computed from a single degree of freedom (SDOF) vibrator subjected to unidirectional ground motion. In order to incorporate the effects of the other components of ground motion, 30% or 40% rule due to “orthogonal effects” is applied.

To include the effect of uncertainty involved in the earthquake ground motion, a design philosophy known as the response spectrum method which gives the deterministic peak values for the purpose of design. Initially, the design response spectrum was constructed by taking the peak responses of a single degree of freedom system subjected to earthquake excitation in one direction only. But, later on, the spectrum was transformed in such a way that it can be applicable to multi degree of freedom systems (SRSS and CQC methods), based on the principle of mode superposition. (Der Kiureghian, 1981). Subsequently, a method was introduced for the calculation of critical incident angle with the use of response spectrum approach (Hernandez and Lopez, 2002; Lopez and Torres, 1997; Wilson et al., 1981; Smeby and Der Kiureghian, 1985). Most of these recent studies are based on the key concept of principal axes of ground motions (Penizen and Watabe, 1975). The principal axes are the directions in which the time histories are uncorrelated, i.e., correlation function being equal to zero. The idea of principal axes of ground motion helps in the approximate determination of the direction of the epicenter. This can be done by the use of “Power Spectral Density Theory”, according to which, the direction having maximum energy will give the rough direction of the epicenter.

Today, if one requires to find out the maximum peak response which is possible for a given structure subjected to a particular ground motion, one has to calculate the response of the structure for every possible value of the input angle using time history analysis. This is a very difficult and time taking process. Hence, in this dissertation, an attempt has been made to suggest a simple method for the calculation of peak structural response using a response spectrum, which we are calling as the critical response spectrum. This response spectrum is believed to behave as an envelope function of the structural response and will provide the ultimate maximum value of the peak response of the given structure. Another important advantage of this method will be that no combination rules will be required to be applied in this case. If one wants a peak value of response of the structure in any particular direction, he must simply apply the critical response spectrum along that particular axis of the structure along which the response is desired.

The main objective of this study is review the existing methods of maximum response calculation and study their advantages and disadvantages. Secondly, to suggest a simple method which is easy to use and is also not based on too many assumptions. A comparative study also has been done between the existing methods and the suggested method to find the merits and demerits. Lastly, some recommendations are also made on the use of the proposed method.

However, this study does not consider the vertical component and the rotational components of the ground motion. Only the translation components of the earthquake ground motion are considered in the analysis. The reason for neglecting the rotational component of ground motion is that it has a low intensity as compared to the translational components of ground motion. Moreover, including it in the analysis will result in a very complex and difficult method. Hence, it is feasible to ignore it in the present context. Whereas, the reasons for neglecting vertical component of earthquake are as given below:

(i) The vertical component is quite weak in comparison to the translational components.

(ii) The structure is subjected to downward loading, so there is already a factor of safety in vertical direction.

(iii) The vertical component is not strong enough to cause the uplift of structure in vertical upward direction as the weight of the structure is acting in the vertical downward direction.

(iv) As we know that the compressive strength of concrete is very high, so the failure in the concrete structures to be crushing failure is very less likely to happen. Most of the failures in the concrete structures during an earthquake are attributed to shear failure.

Considering the above points with respect to the vertical component of the earthquake ground motion, it is neglected in the present analysis.

1.2 Properties of Seismic Excitation

Earthquake ground motion is found to be consisted of three translational components. Out of the three translational components- Two are horizontal and the remaining one is vertical.

One of the two horizontal components is the radial component which is assumed to be acting in the direction of the epicenter from the site whereas the other horizontal component is the tangential component which is perpendicular to the radial component. The radial and tangential components of the ground motion are orthogonal to each other. The radial component is also called the major principal component whereas the tangential component is called as the minor principal component. The vertical component of the ground motion is known as the intermediate component. However, the direction of the horizontal components keeps on varying instantaneously. This is due to the fact that the different soil layers cause reflection and refraction of the earthquake shear waves.

Penizen and Watabe (1995) suggested a method for the identification of the principal directions of the given earthquake excitation. This method involves finding of the covariance of the given time histories which should be equal to zero so the two records become independent or uncorrelated.

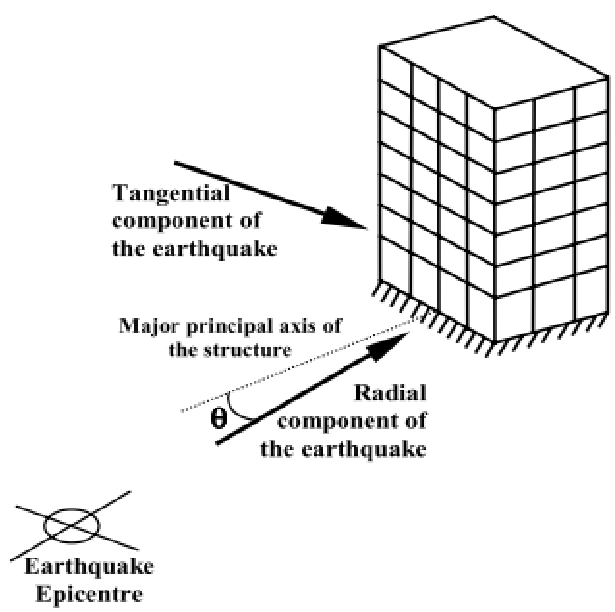

Fig. 1.1: Radial and Tangential Horizontal Components of Earthquakes

(After Zaghlool, Carr and Moss; 2001)

1.3 Independence of Earthquake Components

The basic assumption involved in the study of multi-component seismic response as well as stochastic response of the structures is that the earthquake excitations are independent or uncorrelated. This assumption was also made while deriving the well-known SRSS (Clough and Penizen 1993) and CQC (Wilson et al 1981) combination methods.

The earthquake input in case of the stochastic seismic response is considered as a stationary process and is formulated in the following manner:

$$a_i(t) = \zeta_i(t) \,.\, z_i(t) \quad ; i = x, y, z$$

where, a_i (t) is the ground acceleration at time t,

$z_i(t)$ is a stationary process and

$\zeta_i(t)$ is a deterministic random function.

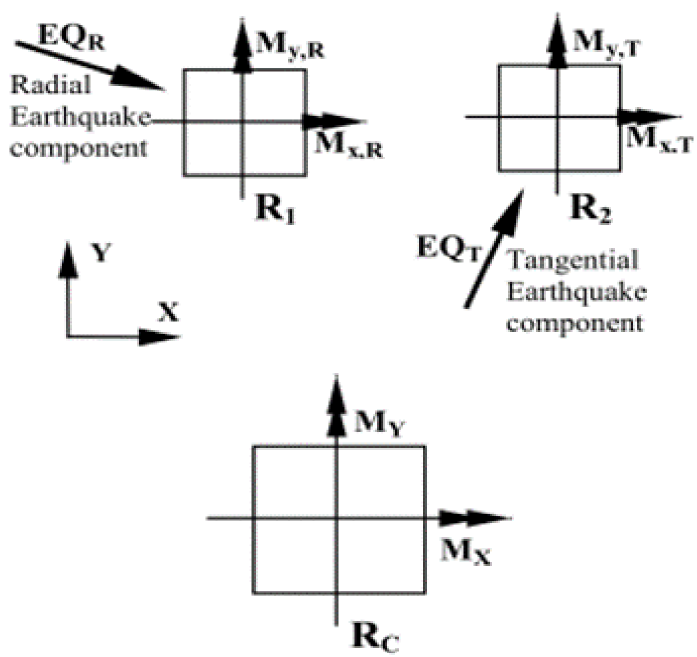

Fig. 1.2: Orthogonally generated Bending Moments in a Column Section

(After Zaghlool, Carr and Moss; 2001)

1.4 Multi-component Seismic Excitations and Combination Rules

Extensive studies have been carried out on the behavior of structures under multi-component earthquake excitations. Most such studies use either elastic modal response analysis (Smeby and Der Kiureghian 1985, Hisada et al 1987, Ger and Cheng 1990, and López and Torres 1997) or are based on probabilistic theories (Rosenblueth and Contreras 1977) due to which they are not suitable for inelastic applications.

Recent codes and soft wares also use the above stated theories in order to evaluate the combined structural response due to orthogonal structural responses (R_C).The structural response is calculated by applying the response spectrum of an earthquake along the major axes of the structure. Then by assuming the two orthogonal responses to be independent of each other, i.e., uncorrelated, the combined structural response can be computed either by using the square root of the sum of squares (SRSS) method,

$$R_C = (R_1^2 + R_2^2)^{1/2}$$

Or by using the λ-percent rule

$$R_C = R_1 + \lambda R_2$$

Where,

R_1 is the peak structural response of the structure subjected to seismic excitation acting along the major structural axis,

R_2 is the response in the direction perpendicular to $R_{1,}$ and

λ is the orthogonal combination factor.

The orthogonal combination factor, λ, generally has a value of 30% as in many codes, such as the AS 1170.4 (1993), ISO 3010:1988 (E), UBC (1997) and Euro code 8 (1994). However, some other codes use percentages for λ as high as 40%.

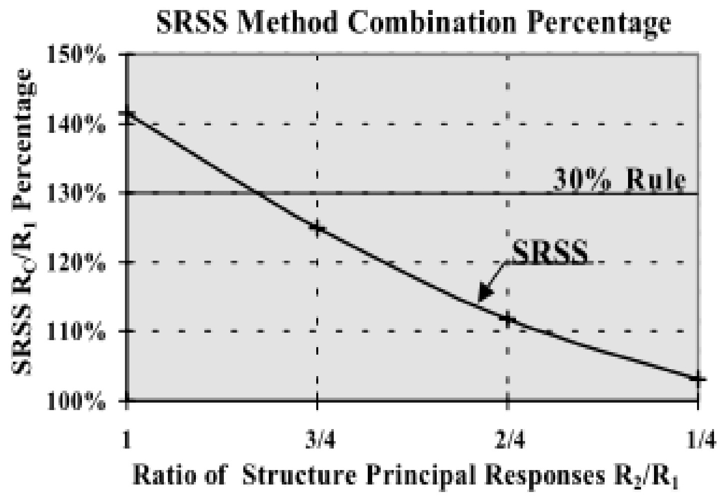

Fig. 1.3: Combination % of SRSS and λ methods

(After Zaghlool, Carr and Moss; 2001)

1.5 Issues Related to Multi-Component Seismic Analysis

Several issues concerned with the multicomponent analysis of the seismic response are listed below:

1) The first issue is the review and evaluation of the existing methods which are used to combine the structural response due to the individual components of earthquake.

2) The second issue deals with analyzing the vertical and other components of earthquake, i.e., estimation of their contribution in the structural response.

Results show that the total base shear calculated by using these combination rules are quite safe. But sometimes, during the inelastic behavior of the structure, these rule may also underestimate the combined response. In case of horizontal components, it has been seen that the SRSS rule is more conservative than the 30 percent rule. So, it is suggested that if the λ-percent rule is to be used then the value of λ to be adopted should be 40 instead of 30 in order to get safer results.

Energy released during an earthquake travels in the form of waves. This energy is recorded in the form of two horizontal and one vertical translation acceleration time histories. However, earthquake can also consist of rotational motion also but these are not recorded and are generally overlooked in the analysis. The effect of the vertical component is quite small as compared with the horizontal components and it is also neglected. Generally, the analysis of a structure is done by applying the two recorded ground motions along the two major structural axes. In this type of analysis the orientation of the structure with respect to angle of incidence of excitation is overlooked.

Article 1620.2.10 of the IBC code (2003) states, "The direction of application of the seismic forces used in the design shall be, that which will produce the most critical load effect in each component. The requirement will be deemed satisfied if the design seismic forces are applied separately and independently in each of the two orthogonal directions." The critical direction requirement will be deemed satisfied by the use of two combination procedures known as the 30 percent and SRSS combination rules.

The above discussion clearly identify several issues that need our attention:

- How to combine the separated response in different directions to obtain overall response?
- What is the critical orientation of the components?
- What is the relative importance of the vertical component?
- What is the relative importance of the weak horizontal component compared to that of the strong horizontal component?
- What is the effect orientation of the sensors recording the strong ground motion which are used in the analysis?

The knowledge of the critical orientation of the various components of earthquake is very important if one wants to find the maximum peak response of the structure. Several studies are being carried on in this regard. According to Penizen and Watabe (1975), the earthquake ground motion are generally uncorrelated along a particular set of directions which are also known as the principal axes. The major and minor principal directions are horizontal and the intermediate principal direction is vertical. The major principal axis is along the direction of the epicenter of earthquake from the structure and the minor principal axis is orthogonal to it.

In this study, a critical response spectrum which contains the resultant of the maximum responses of the structure for each and every value of the time period. These resultants are calculated by taking the vector sum of the maximum responses at a particular value of time period.

CHAPTER-2: REVIEW OF METHODS

2.1 Various Combination Rules for Directional Combination

The commonly used methods for obtaining the peak response quantity of interest for a MDOF system are as follows:

- 30% Rule,
- 40% Rule,
- Square Root of Sum of Squares (SRSS) Method,
- Complete Quadratic Combination (CQC) Method,
- Principal Component Method

2.2 The 30% Rule

This method is given by Rosenblueth and Contreras (1977) and is used by various codes (UBC, 1997; IS: 1893, 2002; IBC, 2009 etc.). The assumption used in this method is that the two horizontal components of the ground motion are uncorrelated. This method gives the critical response by the following equation:-

$$r_c = \sup\left\{0.3r_x + r_y ; r_x + 0.3r_y\right\}$$

Where, r_x is the maximum absolute response in x-direction, and

r_y is the maximum absolute response in y-direction.

2.3 40% Rule

This rule was proposed by Newmark (1975) and is adopted by various design codes. This rule is more conservative than 30% rule and also give slightly higher results than the SRSS method. The equation for the combined response is given as:

$$r_e = \text{larger of } \left\{0.4 r_x + r_y\, ; 0.4 r_y + r_x\right\}$$

The methods discussed above are collectively known as the λ-percent rules as at some places 30% contribution of the orthogonal response is used while at other places 40% contribution can be used. These are highly empirical formulations and do not have strong computational background.

An improvement over the above two methods is the square root of the sum of squares (SRSS) method. It has been observed that the responses calculated by the SRSS method are greater than those calculated from the 30% rule, however, these are quite close enough to the responses calculated from the 40% rule.

2.4 SRSS Method

The maximum structural response is calculated using the SRSS method in the following manner:

$$r_{\max} = \sqrt{r_x^2 + r_y^2}$$

where, r_x is the absolute maximum response in x-direction

r_y is the absolute maximum response in y-direction

The basic assumption in the SRSS method is that there is no correlation between the two horizontal components of ground motion. Another assumption involved in the SRSS method is that the modes (modal frequencies) are considered to be sufficiently well spaced. However, this method will give poor results where the frequencies of major contributing modes are very closely spaced.

The main problem with the SRSS method is that it is based on a number of assumptions and simplifications, as an effect of which the complexity of the problem is overlooked and a very simplified result is obtained which may not be considered as an accurate response.

The methods discussed so far are very simplified methods and cannot be relied upon for calculating the maximum peak structural response. Hence, these methods are not reliable methods for the calculation of the peak structure response subjected to multi-component seismic excitation.

However, many design codes and documents recommend the use of these methods. IS 1893 also recommends the use of these methods for the calculation of response of the structure subjected to multi-components of the earthquake ground motion.

2.5 Combination of Two or Three Components of Motion

As per IS 1893(Part-I):2002

6.3.4.1 When the responses from the three earthquake components are to be considered, the responses due to each component may be combined using the assumption that when the maximum response from one component occurs, the responses from the other two components are 30 percent of their maximum. All possible combinations of the three components (ELx, ELy and ELz) including variations in sign (plus or minus) shall be considered. Thus, the response due to earthquake force (EL) is the maximum of the following three cases:

1) $\pm \mathrm{EL}x \pm 0.3\mathrm{EL}y \pm 0.3\mathrm{EL}z$
2) $\pm \mathrm{EL}y \pm 0.3\mathrm{EL}x \pm 0.3\mathrm{EL}z$
3) $\pm \mathrm{EL}z \pm 0.3\mathrm{EL}x \pm 0.3\mathrm{EL}y$

Where x and y are two orthogonal directions and z is the vertical direction.

6.3.4.2 as an alternative to the procedure in **6.3.4.1** the response (EL) due to the combined effect of the three components can be obtained on the basis of "square root of the sum of squares (SRSS)" that is

$$EL = \sqrt{(EL_x)^2 + (EL_y)^2 + (EL_z)^2}$$

6.3.4.3 When two component motions (say one horizontal and one vertical, or only two horizontal) are combined, the equations in **6.3.4.1** and **6.3.4.2** should be modified by deleting the term representing the response due to the component of motion not being considered.

2.6 Complete Quadratic Combination (CQC) Method

The alternative procedure is the Complete Quadratic Combination (CQC) method.

The maximum response from all the modes is calculated as

$$r_{\max} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} r_x \alpha_{ij} r_y}$$

where r_x and r_y are maximum responses in the x and y-directions, respectively and α_{ij} is correlation coefficient given by

$$\alpha_{ij} = \frac{8(\xi_i \xi_j)^{1/2}(\xi_i + \beta\xi_j)\beta^{3/2}}{(1-\beta^2)^2 + 4\xi_i\xi_j\beta(1+\beta^2) + 4(\xi_i^2 + \xi_j^2)\beta^2}$$

Where, ξ_i and ξ_j are damping ratio in ith and jth modes of vibration, respectively and

$$\beta = \frac{\omega_i}{\omega_j} \qquad \left(\omega_j > \omega_i\right)$$

The range of coefficient, α_{ij} is $0 < \alpha_{ij} < 1$ and $\alpha_{ij} = \alpha_{ij} = 1$.

For the system having the same damping ratio in two modes i.e. $\xi_i = \xi_j = \xi$, then

$$\alpha_{ij} = \frac{8\xi^2(1+\beta)\beta^{3/2}}{(1-\beta^2)^2 + 4\xi^2\beta(1+\beta)^2}$$

2.7 Principal Component Method

To obtain the critical response of the structure directly by the response spectrum superposition analysis for any particular structural axis, one can apply the response spectrum of the major principal component of the ground motion in the desired direction. Penizen and Watabe (1975) showed that the recorded ground acceleration components can be resolved along three principal directions such that these three resolved components of ground acceleration are uncorrelated. The three components of the ground motion consist of two horizontal components which are characterized by the maximum and minimum covariance whereas the third component is the vertical component having intermediate covariance. The component which is having the maximum covariance is called as the 'major principal component' whereas the component having the minimum covariance is called as the 'minor principal component'. The response spectrum of the major principal component is used to get the critical response directly.

It is a general observation that the major principal component is directed towards the epicenter and it does not vary much during the strong motion. Since, there is large uncertainty in the location of epicenter. So, in the present analysis, the principal components are found by diagonalizing the covariance matrix of the two recorded horizontal components of ground acceleration.

$$\begin{matrix} \mu_{xx} & \mu_{xy} \\ \mu_{yx} & \mu_{yy} \end{matrix} \quad \text{with} \quad \mu_{ij} = \frac{1}{T_d}\int_0^{T_d} \left(a_i(t) - \bar{a}_i(t)\right)\left(a_j(t) - \bar{a}_j(t)\right)dt$$

Here $\overline{a_i(t)}$ is the mean value of time-history $a_i(t)$ over duration T_d.

The solution of the above matrix gives two values of the eigen values and corresponding two eigen vectors. The eigen vector corresponding to the higher eigen value will give the direction of the major principal component of ground acceleration whereas the eigen vector corresponding to lower eigen value provides the direction of minor principal component. If the major principal direction makes an angle θ with the x-direction, then the recorded earthquake motion can be resolved along the major principal direction as:

$$a_\theta(t) = a_x(t)\cos\theta + a_y(t)\sin\theta$$

The response spectrum of the above resolved ground motion is termed as the principal component response spectrum. Hence, we have two principal component response spectrums in two mutually perpendicular directions.

But the above direction of the principal component may not coincide with the axes of the structure, due to which it may not yield the maximum value of the response. So, a condition arises in which the spectra for the principal directions are known but the principal directions itself are not known.

Now there are two ways to solve the problem:

(i) One can choose to find the value of critical angle, θ_{cr}, for which the value of response will be maximum.

(ii) θ can be assumed as a random variable and the expected peak response for all possible values of θ can be found out.
For this case, it is assumed that θ is uniformly distributed between 0 and 2π, and the location of epicenter along the existing fault is also uniformly distributed.

In this study, the first approach is adopted to find the maximum value of the structural response.

The expression for the mean peak response is given by the following equation:-

$$\overline{U_r} = \Big\{ \sum_i \sum_j \sum_k \rho_{0.ij}\, \psi_{i.r}{}^{(k)}\, \psi_{j.r}{}^{(k)} \overline{S}_i^{(k)} \overline{S}_j^{(k)} \\ - \rho_{0.ij} [\psi_{i.r}{}^{(1)}\, \psi_{j.r}{}^{(1)} - \psi_{i.r}{}^{(2)}\, \psi_{j.r}{}^{(2)}][\overline{S}_i^{(1)} \overline{S}_j^{(1)} - \overline{S}_i^{(2)} \overline{S}_j^{(2)}] \sin^2\theta \\ - \sum_i \sum_j [\psi_{i.r}{}^{(1)}\, \psi_{j.r}{}^{(2)} + \psi_{i.r}{}^{(2)}\, \psi_{j.r}{}^{(1)}][\overline{S}_i^{(1)} \overline{S}_j^{(1)} - \overline{S}_i^{(2)} \overline{S}_j^{(2)}] \sin\theta \cos\theta \Big\}^{1/2}$$

Where,

$\overline{U}_r$ = maximum structural response in the r^{th} DOF

$\rho_{0.ij}$ = correlation coefficients in x and y-direction

ψ represents the mode shapes of the structure, $\overline{S}$ represents the spectral amplitude corresponding to the various frequencies of the various modes.

The subscript *i* and *j* represents the modes of the structure whereas the subscript *r* represents the DOF of the structure.

The critical angle, θ_{cr}, is obtained by maximizing the above expression for mean peak response with respect to θ;

$$\frac{dU_r}{d\theta} = 0$$

$$\tan 2\theta_{cr} = \frac{\sum_i \sum_j [\psi_{i,r}^{(1)}\,\psi_{j,r}^{(2)} + \psi_{i,r}^{(2)}\,\psi_{j,r}^{(1)}]\overline{S_i^{(1)}}\,\overline{S_j^{(1)}}}{\sum_i \sum_j [\psi_{i,r}^{(1)}\,\psi_{j,r}^{(1)} - \psi_{i,r}^{(2)}\,\psi_{j,r}^{(2)}]\overline{S_i^{(1)}}\,\overline{S_j^{(1)}}} + k\pi \;\; ; \;\; k = 0,1$$

This equation will yield the two values of Θ for which U_r is either maximum or minimum.

Using the values of θ_{cr} obtained from eq. (2) in eq. (1), we will get the maximum value of the response.

CHAPTER-3: CRITICAL RESPONSE SPECTRUM METHOD

In this chapter, the method for the development of the critical response spectrum for a given time history record is discussed. With the use of this critical response spectrum, if we want to calculate the response of the structure in any direction, we can apply this spectrum in that particular direction and get the response. The response thus obtained will be the maximum or the critical structural response for the given earthquake. But before going into the details of the critical response spectrum, some basic concepts of the response spectrum analysis are discussed here.

With a view of seismic analysis and earthquake resistant design of a building, actual time history records are required which may be not available for each and every location. Also, the response of the structure does not only depend on the peak values of ground acceleration but is also dependent on the frequency content of the ground shaking and the dynamic properties of the structure itself. The above stated problem is dealt with the use of response spectrum in the seismic analysis of the structures.

3.1 Response Spectrum

Response Spectrum can be defined as the maximum or peak value of the response (the response quantity may be displacement, velocity or acceleration) of a linear single degree of freedom system having a constant level of damping and excited by a particular ground motion.

The various steps involved in the calculation of a response spectrum from a given time history are discussed below:

(1) Firstly, the acceleration time history for a given earthquake is obtained.
(2) Secondly, the natural time period (T_n) and damping ratio (ξ, usually 5%) is selected.
(3) The maximum response of a SDOF system with the above stated properties is calculated using the following equation:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = m\ddot{x}_g(t)$$

(4) The step 3 is repeated for the different values of fundamental time period of the structure.

Let us consider a SDOF system which is subjected to an earthquake motion, $\ddot{x}_g(t)$

The equation of motion for the given system can be written as:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = -m\ddot{x}_g(t)$$

Substituting the values of;

$$\omega_0 = \sqrt{k/m} \quad \text{and} \quad \xi = \frac{c}{2m\omega_0} \quad \text{and} \quad \omega_d = \omega_0\sqrt{1-\xi^2}$$

The above equation becomes;

$$\ddot{x}(t) + 2\xi\omega_0\dot{x}(t) + \omega_0^2 x(t) = -\ddot{x}_g(\text{t})$$

Using the Duhamel's integral, the maximum value of displacement of the given SDOF system can be expressed as:

$$|x(t)|_{\max} = \left| \int_0^t \ddot{x}_g(\tau)\frac{e^{-\xi\omega_0(t-\tau)}}{\omega_d} ins\omega_d(t-\tau)d\tau \right|_{\max}$$

Hence, the relative displacement spectrum can be obtained as:

$$S_d(\xi, \omega_0) = |x(t)|_{\max}$$

Similarly, the relative velocity spectrum (S_v) and the relative acceleration spectrum (S_a) can be expressed as:

$$S_v(\xi, \omega_0) = |\dot{x}(t)|_{\max}$$

$$S_a(\xi, \omega_0) = |\ddot{x}_a(t)|_{\max} = |\ddot{x}(t) + \ddot{x}_g(t)|_{\max}$$

Now, the pseudo velocity spectrum can be defined as:

$$S_{pv}(\xi, \omega_0) = \omega_0 S_d(\xi, \omega_0)$$

And the pseudo acceleration spectrum is:

$$S_{pa}(\xi, \omega_0) = \omega_0^2 S_d(\xi, \omega_0)$$

The response spectra obtained from the actual time history are shown below:

For example, Northridge Earthquake

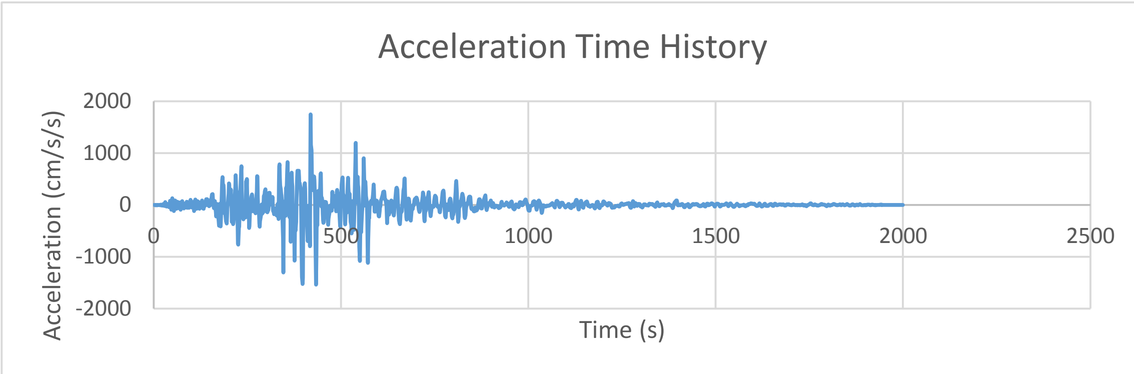

Fig. 3.1: Acceleration Time History for Northridge Earthquake

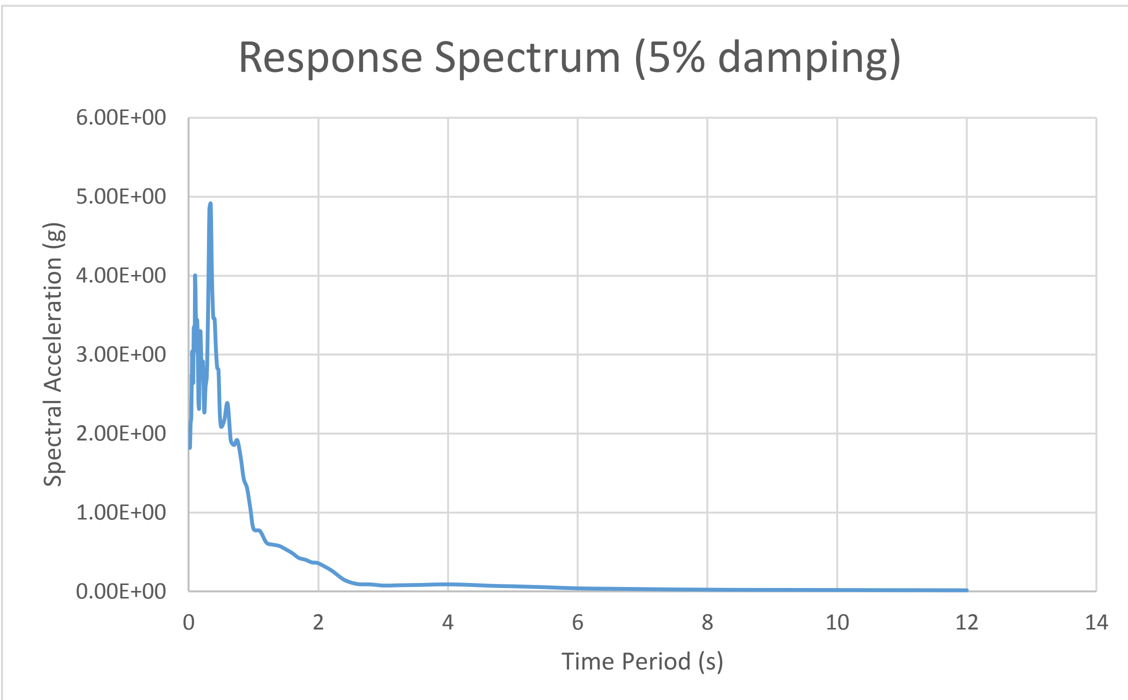

Fig. 3.2: Acceleration Response Spectrum for Northridge Earthquake

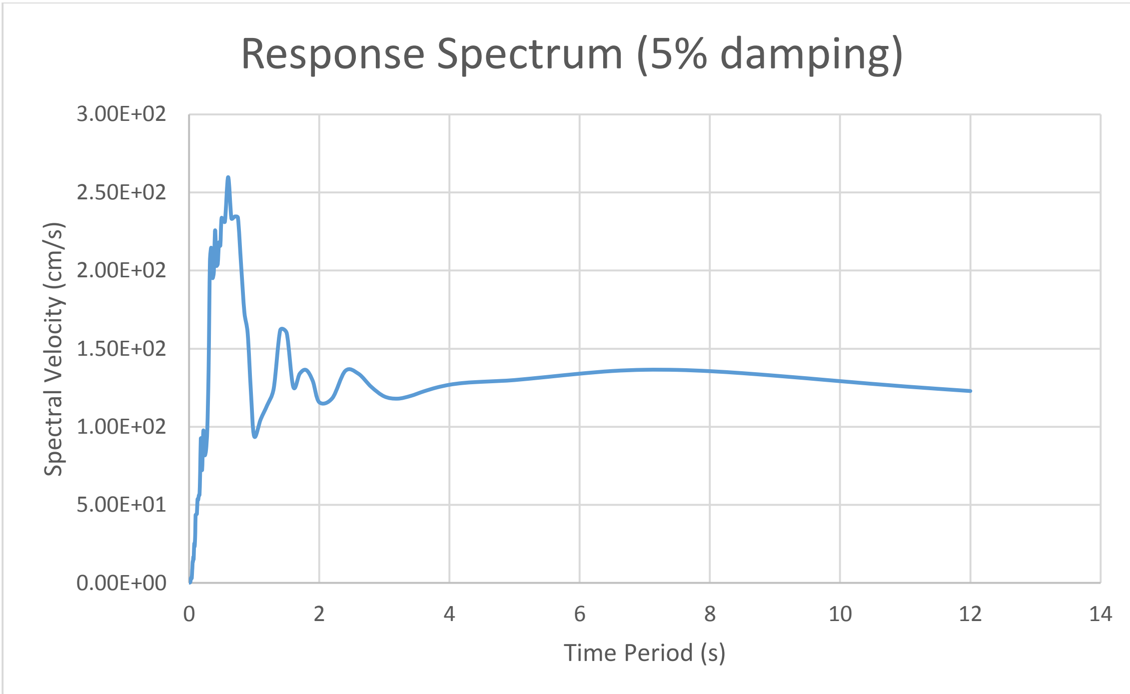

Fig. 3.3: Velocity Response Spectrum for Northridge Earthquake

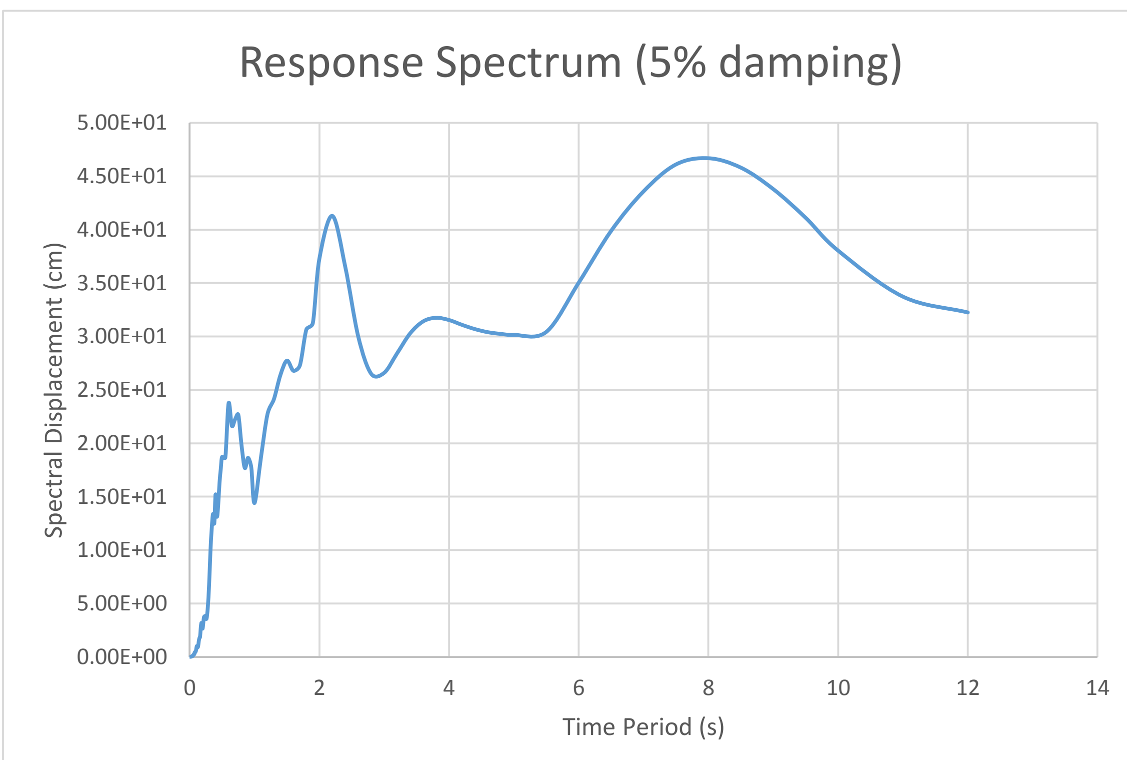

Fig. 3.4: Displacement Response Spectrum for Northridge Earthquake

Response Spectra can also be characterized as a locus of the maximum structural response of a single degree of freedom system for a particular value of damping ratio. It enables us to obtain the peak structural response of a linear system which in turn helps in estimating the lateral forces (base shear) developed in the structure which is a very important parameter for the earthquake resistant design. It is graph with time period on the x-axis and the corresponding response quantity on the y-axis.

But there are certain limitations of the original response spectrum due to which it is not able to predict the peak structural response. In order to overcome these difficulties, some other response spectra have also been defined by various researchers. One of them discussed here is known as the Rotational Independent Spectrum (Rot100).

3.2 Rot100 spectrum

In order to determine ultimate maximum response in any situation i.e. largest response over all possible seismic incident angles, following method is suggested by Boore (2010). It is in tune with NEHRP (2009) recommendations.

1) The two orthogonally recorded time histories are combined to form a single time history having an orientation angle, θ. This is done by using the equation.

$$a_{ORI}(t;\theta) = a_1(t)\cos(\theta) + a_2(t)\sin(\theta)$$

Where, a_1 (t) is the horizontal recorded acceleration time history;

a_2 (t) is the acceleration time history in orthogonal direction; and

θ is the seismic incident angle.

2) By varying the angle θ from 0^0 to 180^0 in the step of 1^0 each time, 181 such different time histories are obtained.

3) The response spectra for the obtained time histories are determined.

4) The spectral values corresponding to each oscillator period and θ are noted down.

5) Now, using the largest spectral values corresponding to each oscillator period, a design response spectrum is developed which is known as the Rot100 spectrum.

3.3 RotI100 spectrum

The Rot100 spectrum does not correspond to single orientation of the time-history. Hence an orientation-independent parameter denoted as RotI100 can be determined as described below:

1) Firstly, the Rot100 spectrum as described above is developed.

2) Secondly, by using the corresponding Rot100 spectrum, the set of SDs for all incident angles for each oscillator period is normalized.

3) The penalty function given by the following equation is computed:

$$penulty(\theta) = \frac{1}{N_{period}} \sum_{i=1}^{h} [SD(\theta, T_i) / Rot100(T_i) - 1]^2$$

4) The angle of rotation corresponding to the minimum value of the penalty function, θ_{pfmin}, is calculated.

5) The response spectra for the time-history corresponding to θ_{pfmin} is RotI100 spectrum.

$$RotI100 = SD(\theta_{pfmin}, T_i)$$

3.4 Critical Response Spectrum

As we have seen till now that all the existing methods for the combination of earthquake response and the calculation of the maximum structural response under multi-component earthquake loading are limited in one manner or the other. Certain assumptions are involved in these methods which may not be true in real sense and are sometimes violated in an actual earthquake event. Hence, a method is required which may be free from the assumptions and may be able to provide an envelope function for the response of the structure which is subjected to multi-component earthquake ground motion. In this study, an attempt has been made to formulate a critical response spectrum for the given earthquake records which is believed to provide the maximum value of the response when it is applied only in that axis of the structure along which the response is required. The results of this method will be compared with the results obtained from the existing methods.

As we known that the direction of the earthquake ground motion keeps on varying instantaneously, so for a particular a time period the orientation of the maximum value of response may not be the same every time. Due to this, the orientation of the peak structural response is not constant and keeps on changing instantaneously. Hence, the response of the structure for a particular time period and orientation angle may not be maximum every time. So, this creates difficulty in calculating the maximum or peak structural response. However, this problem has been solved by the critical response spectrum in which the dependence of response on the orientation angle is eliminated. Hence, the critical response spectrum is able to determine the maximum value of the structural response for each and every value of time period irrespective of the direction in which it occurs.

The steps for obtaining the critical response spectrum for a particular earthquake event are as discussed below:

1) The two horizontal orthogonal time history records for a given earthquake are obtained.
2) The time history records are used to get the response of a SDOF system for a particular value of time period.
3) The response for a particular time period in both the directions are calculated.

4) The maximum value of the response in each direction are selected. The maximum value of the responses is irrespective of the direction in which they are occurring.
5) A resultant time history is obtained by taking the vector sum of the two maximum value of the responses.
6) The response spectrum for the resultant time history is computed. This response spectrum is known as the Critical Response Spectrum.

For example; the graph for the maximum response for the time period 0.78 sec for NORTHRIDGE EATRHQUAKE is shown below:

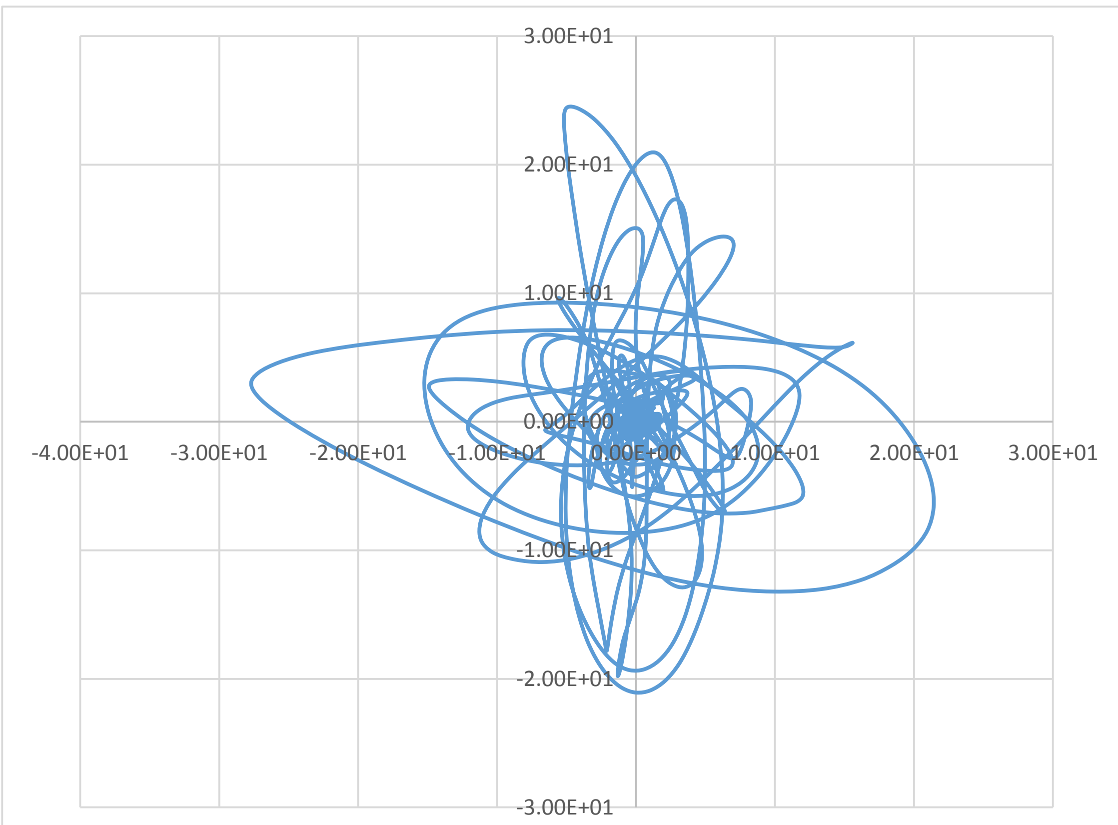

Fig. 3.5: Maximum responses for NORTHRIDGE EARTHQUAKE

The critical response spectrums for the different earthquakes under study are shown below:

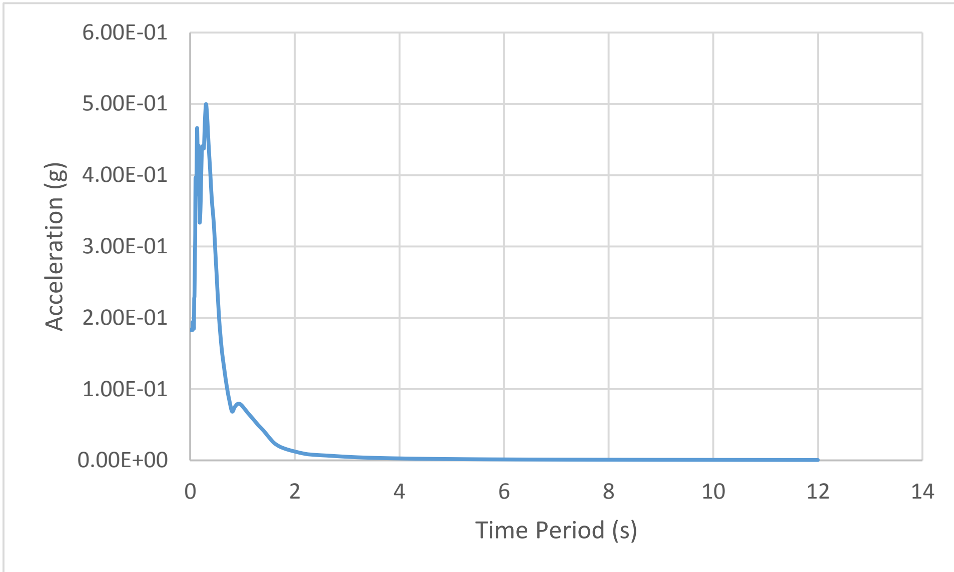

Fig. 3.6: Critical Response Spectrum for CHAMBA EARTHQUAKE

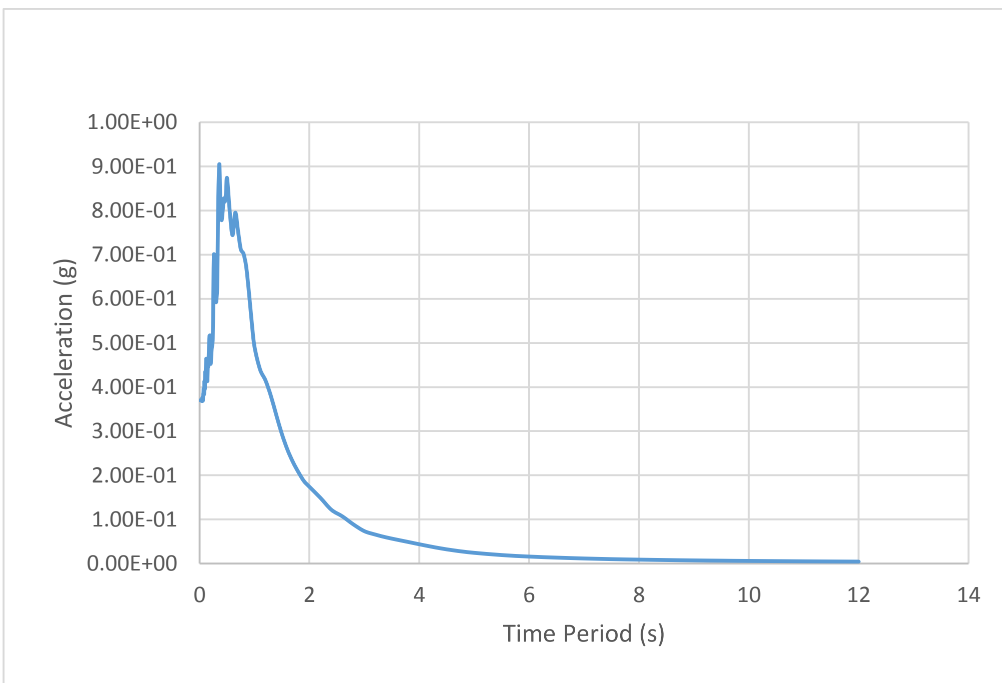

Fig. 3.7: Critical Response Spectrum for CHAMOLI EARTHQUAKE

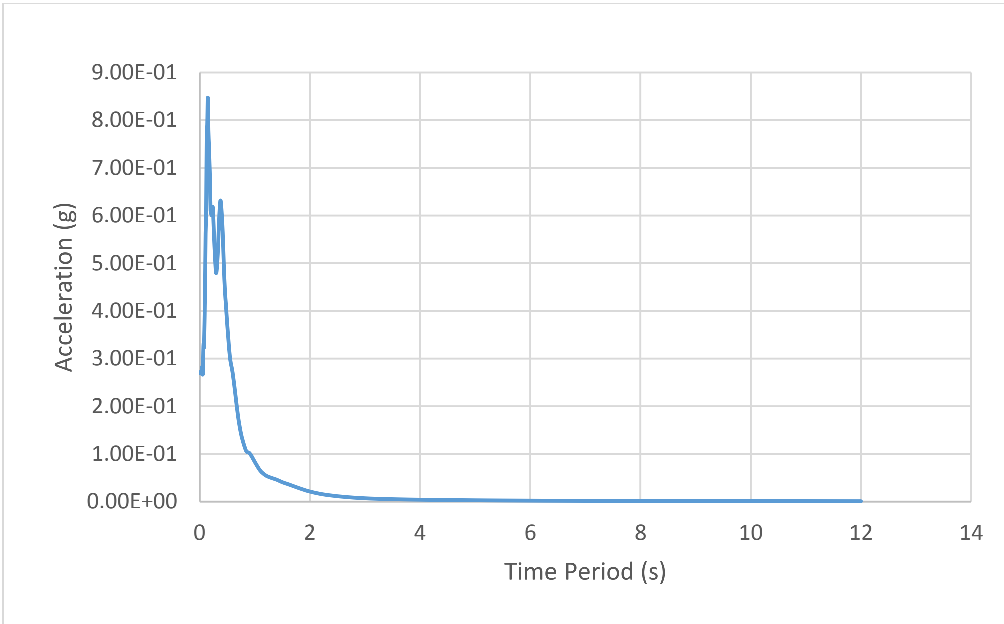

Fig. 3.8: Critical Response Spectrum for KANGRA EARTHQUAKE

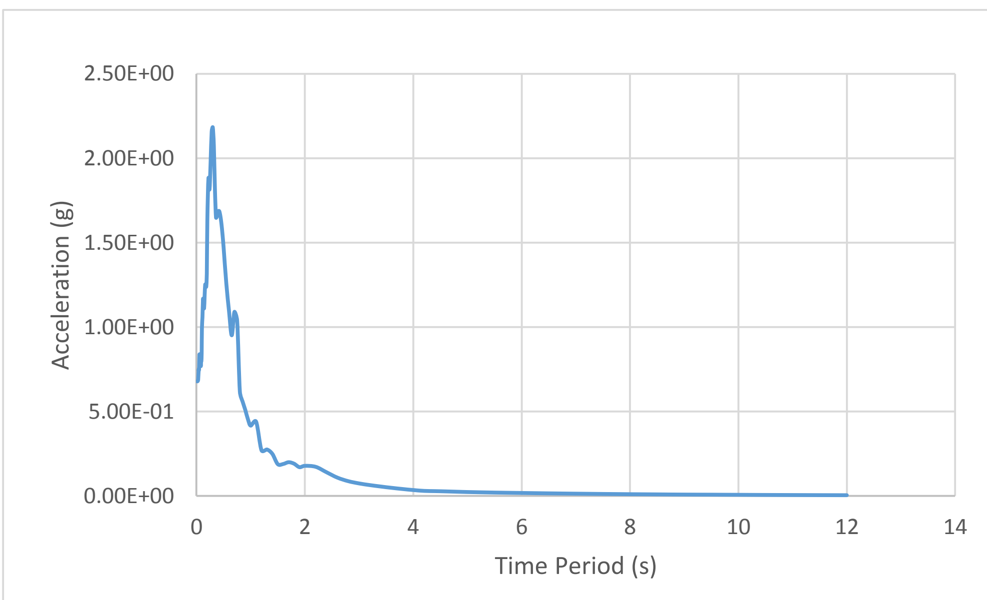

Fig. 3.9: Critical Response Spectrum for LOMA PRIETA EARTHQUAKE

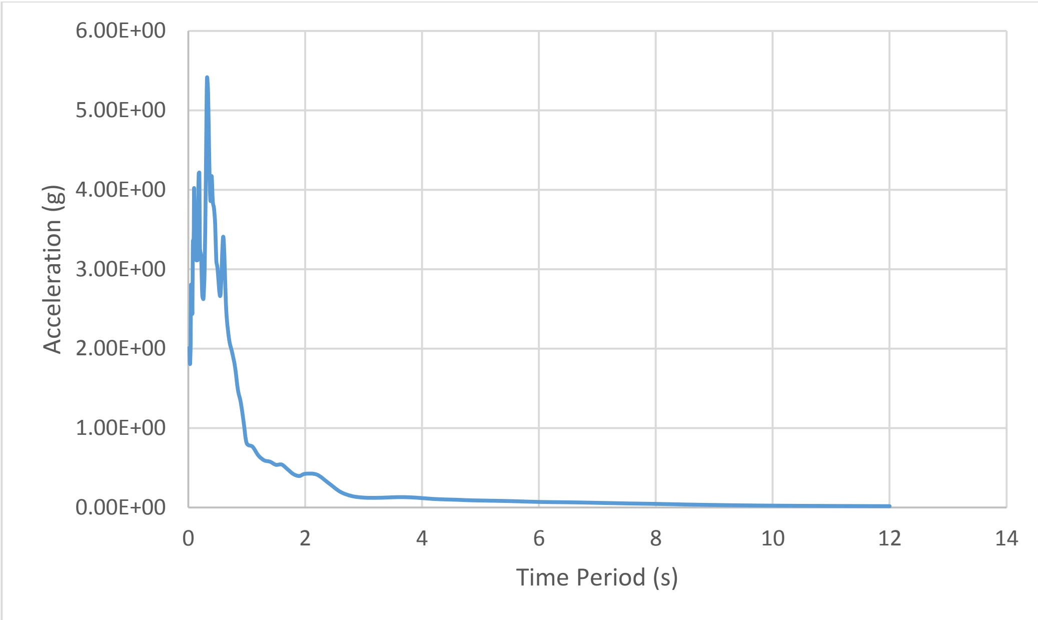

Fig. 3.10: Critical Response Spectrum for NORTHRIDGE EARTHQUAKE

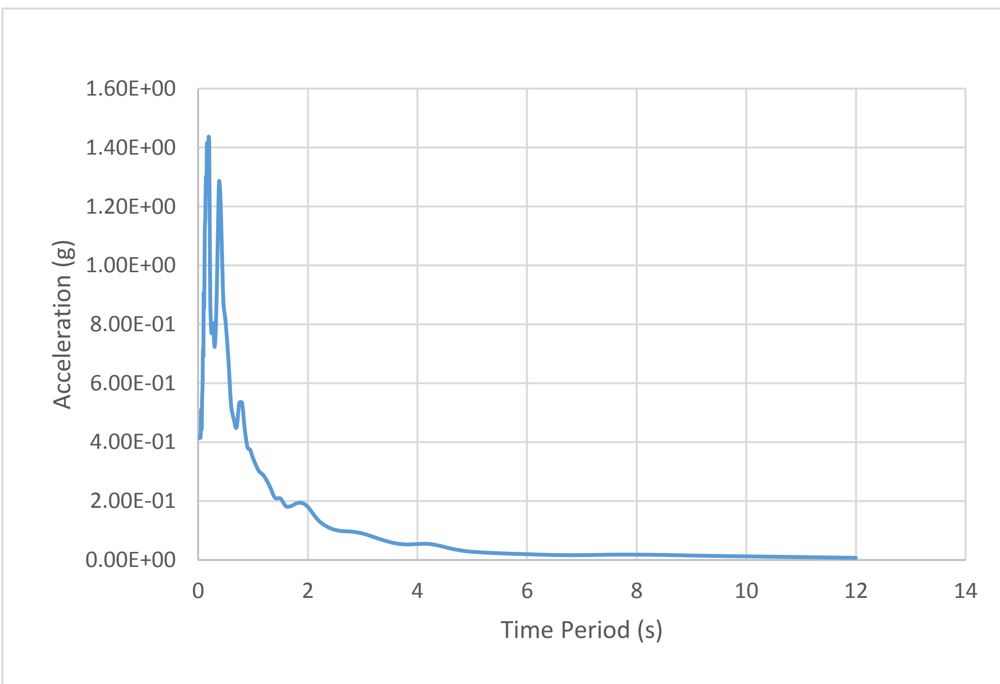

Fig. 3.11: Critical Response Spectrum for TABAS (IRAN) EARTHQUAKE

CHAPTER-4: NUMERICAL ANALYSIS AND RESULTS

4.1 Calculation of Response for MDOF system

A typical MDOF system having 'n' degrees of freedom is shown below:

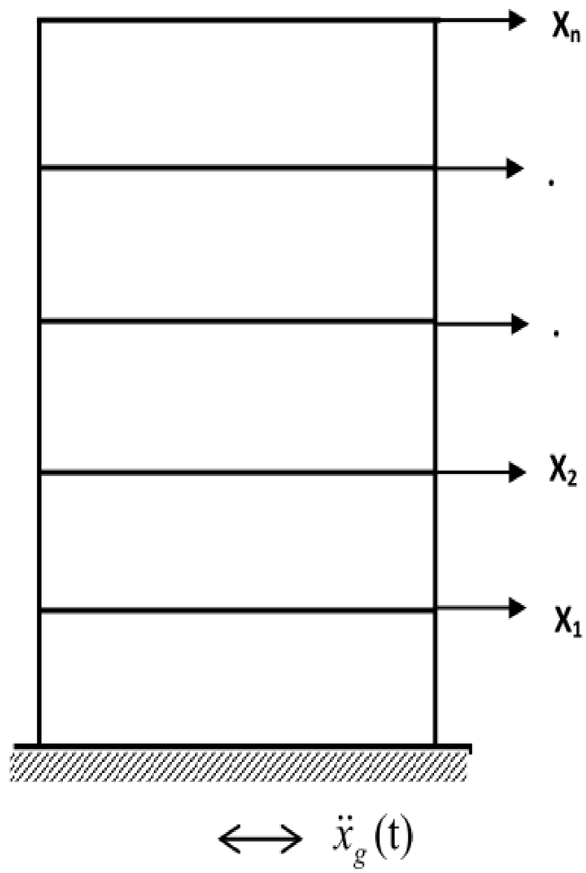

Fig. 4.1 MDOF system with 'n' degrees of freedom

MDOF systems are analyzed by using the modal analysis, i.e., mode superposition methods. The various possible deformation shapes of the given structure subjected to ground shaking are known as the modes of vibration also called as mode shapes. Each mode shape is associated with a particular natural frequency and a time period. The total of different modes for a MDOF system is equal to the number of degree of freedom of the system.

The equation of motion for a MDOF system is given as:

$$[m]\{\ddot{x}(t)\}+[c]\{\dot{x}(t)\}+[k]\{x(t)\}=-[m]\{r\}\ddot{x}_g(t)$$

Where,

$[m]$ = Mass matrix ($n \times n$);

$[k]$ = Stiffness matrix ($n \times n$);

$[c]$ = Damping matrix ($n \times n$);

{r} = Influence coefficient vector ($n \times 1$)

$\{x(t)\}$= relative displacement vector;

$\{\dot{x}(t)\}$= relative velocity vector;

$\{\ddot{x}(t)\}$= relative acceleration vector; and

$\ddot{x}_g(t)$ = earthquake ground acceleration

Hence, the characteristic equation for the undamped system becomes;

$$\{[k]-\omega_i^2[m]\}\phi_i = 0; \qquad i=1, 2, 3, \ldots, n.$$

$$\det\left|\{[k]-\omega_i^2[m]\}\right| = 0$$

Solving the above equation, eigen value and eigen vectors of the system can be known.

ω_i^2 = eigen values of the i^th^ mode

ϕ_i = eigen vector or mode shape of the i^th^ mode

ω_i = natural frequency in the i^th^ mode

So, the displacement response of the structure can be calculated as:

$$\{x(t)\} = [\phi]\{y(t)\}$$

Where,

$\{y\,(t)\}$ represents the modal displacement vector, and

$[\phi]$ is the mode shape matrix;

$[\phi] = [\phi_1, \phi_2, \ldots\ldots\ldots, \phi_n]$

Substituting $\{x\} = [\phi]\{y\}$ and pre-multiply by $[\phi]^T$;

The equation of motion transforms into:

$$[\phi]^T [m][\phi]\{\ddot{y}(t)\} + [\phi]^T [c][\phi]\{\dot{y}(t)\} + [\phi]^T [k][\phi]\{y(t)\} = -[\phi]^T [m]\{r\}\ddot{x}_g(t)$$

On simplifying the above equation, we get

$$[M_m]\{\ddot{y}(t)\} + [C_d]\{\dot{y}(t)\} + [K_d]\{y(t)\} = -[\phi]^T [m]\{r\}\ddot{x}_g(t)$$

Where,

$[\phi]^T[m][\phi] = [M_m]$ = generalized mass matrix

$[\phi]^T[c][\phi] = [C_d]$ = generalized damping matrix

$[\phi]^T[k][\phi] = [K_d]$ = generalized stiffness matrix

We know that $[M_m]$ and $[K_d]$ are diagonal matrices. However, for a classically damped system $[C_d]$ is also a diagonal matrix. Hence, the above equation reduces to:

$$\ddot{y}(t) + 2\xi_i \omega_i \dot{y}(t) + \omega_i^2 y_i(t) = -\Gamma_i \ddot{x}_g(t) \qquad (i = 1, 2, 3, \ldots.., n)$$

$y_i\,(t)$ = modal displacement response in the ith mode,

ξ_i = modal damping ratio in the i^th^ mode, and

Γ_i = modal participation factor for i^th^ mode expressed by

$$\Gamma_i = \frac{\{\phi\}^T [m]\{r\}}{\{\phi_i\}^T [m]\{\phi_i\}}$$

The maximum modal response can be calculated as

$$y_{i,\max} = \left| y_i(t) \right|_{\max} = \Gamma_i S_d(\xi_i, \omega_i)$$

The maximum displacement of the structure in i^{th} mode is given as

$$x_{i,\max} = \phi_i y_{i,\max} \qquad (i = 1, 2, 3, \ldots\ldots, n)$$

The response quantity under interest, (i.e., displacement, shear force, bending moment, etc.) r_i, can be calculated for each mode. However, the final peak response, r_{max} shall be obtained by the combination of the responses for each mode of vibration using the various modal combination rules.

In the next section, a four-storey building is selected and the maximum response of that building for various earthquakes has been calculated using the different methods for the combination of the responses in two different direction and the results are compared with the response obtained from the critical response spectrum method.

4.2 Properties of Building

Building details are as follows:

1. Grade of concrete used is M20 and grade of steel used is Fe415.
2. Floor to floor height is 3.1 m
3. Plinth height above G.L. is 0.55 m
4. Depth of Foundation is 0.65 m below G.L.
5. Slab Thickness is 150 mm
6. External wall thickness is 230 mm and internal wall thickness is 150 mm
7. Size of columns is 300mm X 450mm and size of beams is 300mm X 450mm
8. Live load on floor is 3 kN/m^2 and Live load on roofs is 1.5 kN/m^2.
9. Floor finishes is 1 kN/m^2 and roof treatment is 1.5 kN/m^2
10. Site located in Seismic Zone IV.
11. Building is resting on medium soil.
12. Importance Factor is 1.0
13. Building Frame Type is Special Moment Resisting Frame(SMRF)
14. Density of concrete is 25 kN/m^3 and density of masonry wall is 20 kN/m^3.

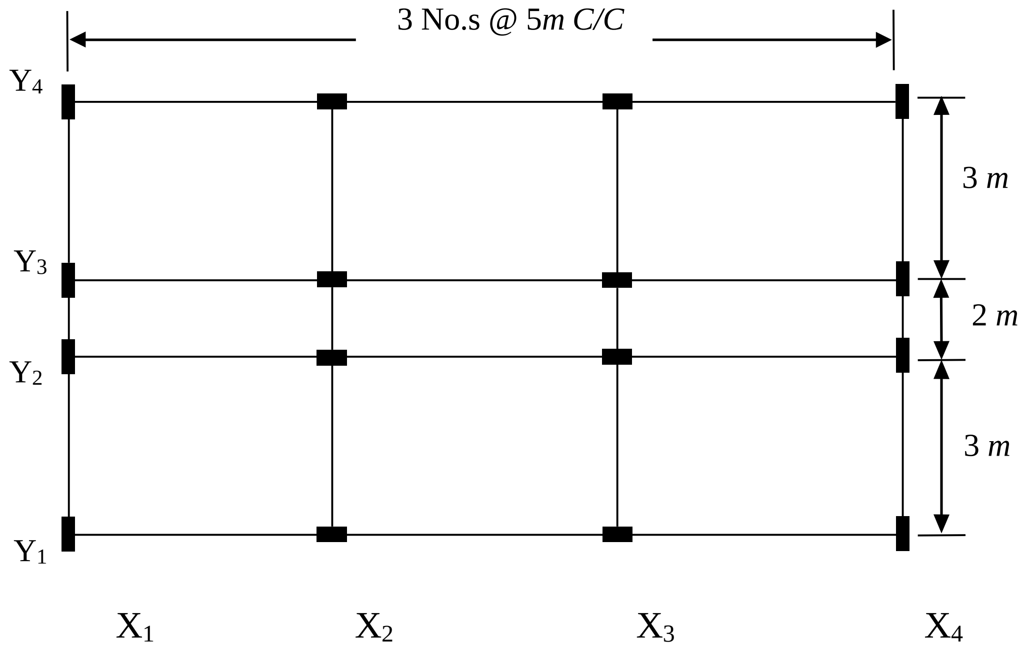

Fig. 4.2: Plan of the building

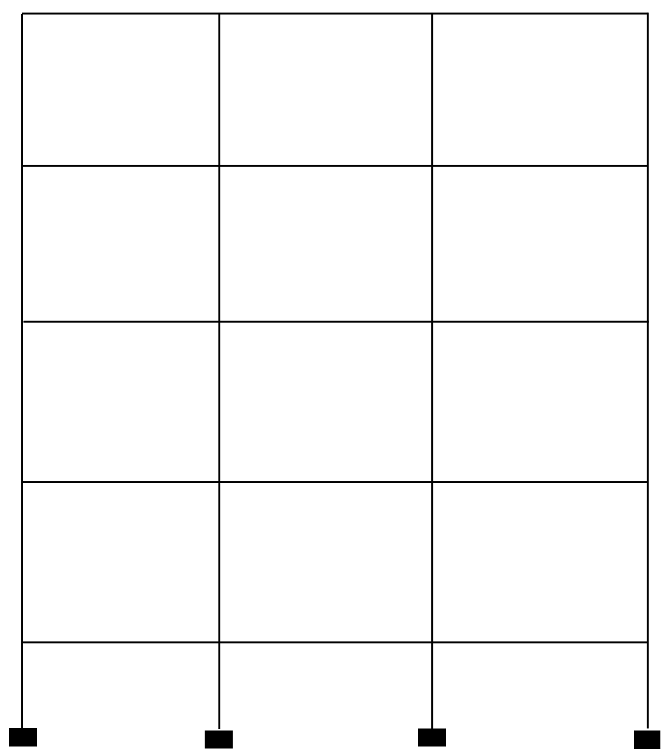

Fig. 4.3: Elevation of building

Table. 4.1: Time Period & Frequencies of the Building

Mode No.	Time Period (T)	Frequency(f)
1	0.78860 s	1.26807
2	0.72562 s	1.37813
3	0.62637 s	1.59649
4	0.25365 s	3.94244
5	0.23742 s	4.21193
6	0.20368 s	4.90963
7	0.14708 s	6.79901
8	0.14100 s	7.09239
9	0.11977 s	8.34940
10	0.10723 s	9.32540
11	0.10590 s	9.44280
12	0.08856 s	11.2922

4.3 Input Data

Table. 4.2: Details of Input Earthquake data

S. No.	Earthquake	Recording Station	Magnitude	Epicentral Distance	Depth
1.	Kangra Earthquake, 1986	Shahpur	5.5	10.46 km	7.00 km
2.	Chamoli earthquake, 1999	Gopeshwar	6.5	8.72 km	21.00 km
3.	Chamba Earthquake, 1995	Chamba	4.7	13.90 km	29.20 km
4.	Loma Prieta Earthquake, 1989	Corralitos	6.9	7.17 km	17.50 km
5.	Northridge Earthquake, 1994	Tarzana	6.7	5.41 km	17.50 km
6.	Tabas (Iran) Earthquake, 1978	Dayhook	7.4	20.63 km	5.80 km

4.4 Results

In the next section, the given four storey building has been subjected to six different earthquakes and the maximum response of the building has been calculated. Further, the response of the structure in two different directions are combined using the various rules for the combination of response of the structure subjected to multi-component earthquake ground motion as discussed before.

The various combination rules used for this purpose are the 30% rule, 40% rule, SRSS method, CQC method and the Principal Component Method. The response calculated by using these methods is then compared with a new method prescribed in this study known as the 'Critical Response Spectrum Method'. For this purpose, a critical response spectrum using the time history data is developed for each earthquake and the peak structural response is calculated by applying this spectrum in that particular direction in which the response is desired. It is seen that the response calculated by the critical response spectrum was the maximum in most of the cases and hence this method also eliminates the requirement to consider the effect of the orthogonal component of the response.

Table 4.3 to Table. 4.14 give the Floor Displacements (in cm) of the structure in *x* and *y*-directions both for various earthquakes used in the analysis calculated by using the described combination rules whereas Table. 4.15 gives the values of the maximum Base Shear (in kN) for different earthquakes.

Fig. 4.4 to Fig. 4.15 shows the Variation of displacement under various combination rules for different earthquakes in *x* and *y*-directions both while Fig. 4.16 shows the variation of the maximum value of the base shear under various combination rules for different earthquakes.

KANGRA EARTHQUAKE

In x-direction:-

Table. 4.3: Floor Displacements (in cm) for Kangra earthquake (in x-direction)

STOREY	X-COMP.	Y-COMP.	30% RULE	40% RULE	SRSS	CQC	PC	CRS
4	0.22	0.44	0.352	0.396	0.491	0.537	1.222	1.855
3	0.19	0.38	0.304	0.342	0.424	0.464	0.607	0.896
2	0.14	0.29	0.227	0.256	0.322	0.351	0.406	0.582
1	0.08	0.16	0.128	0.144	0.178	0.195	0.232	0.311

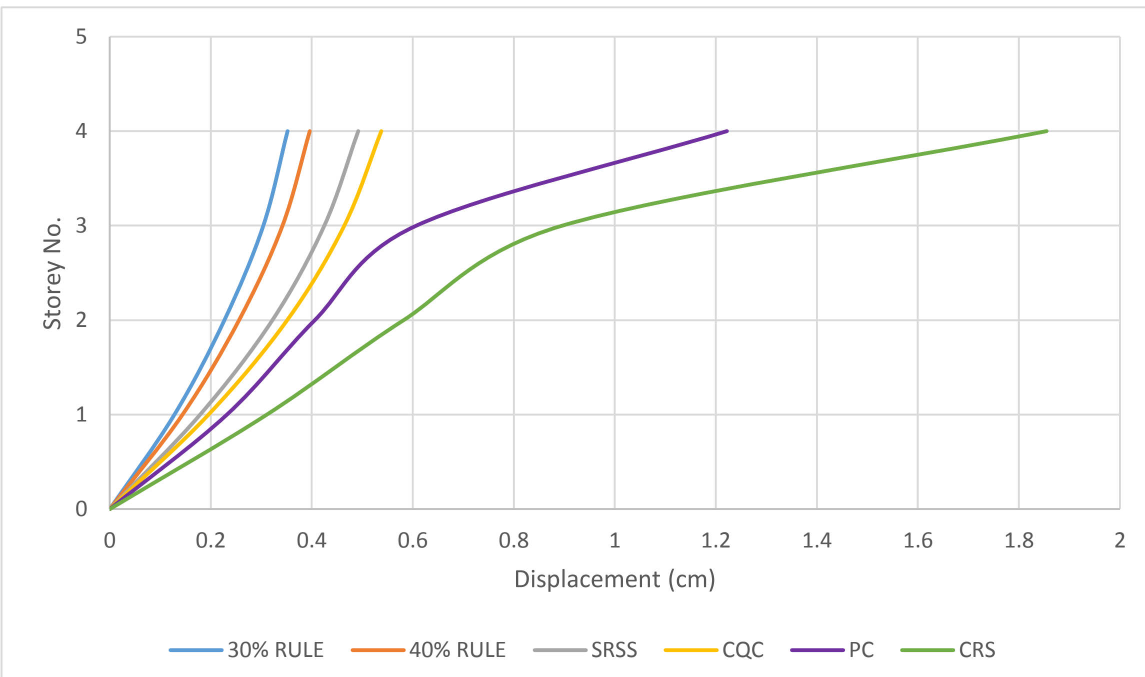

Fig. 4.4: Variation of displacement under various combination rules for Kangra earthquake (in x-direction)

In y-direction:-

Table. 4.4: Floor Displacements (in cm) for Kangra earthquake (in y-direction)

STOREY	X-COMP.	Y-COMP.	30% RULE	40% RULE	SRSS	CQC	PC	CRS
4	0.22	0.4	0.34	0.38	0.456	0.501	1.001	1.52
3	0.19	0.35	0.295	0.33	0.398	0.436	0.498	0.735
2	0.14	0.27	0.221	0.248	0.304	0.332	0.333	0.477
1	0.08	0.15	0.125	0.14	0.17	0.186	0.19	0.255

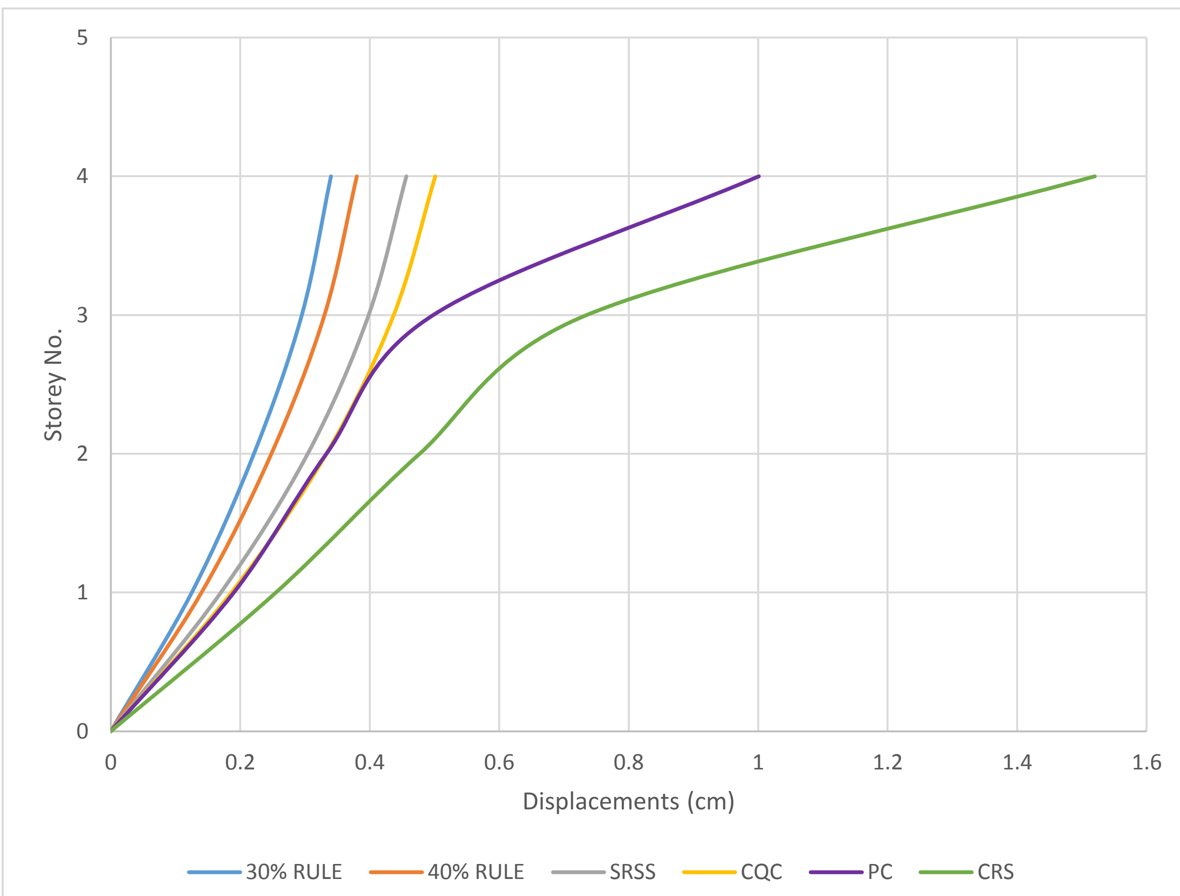

Fig. 4.5: Variation of displacement under various combination rules for Kangra earthquake (in y-direction)

CHAMOLI EARTHQUAKE

In x-direction:-

Table. 4.5: Floor Displacements (in cm) for Chamoli earthquake (in x-direction)

STOREY	X-COMP.	Y-COMP.	30% RULE	40% RULE	SRSS	CQC	PC	CRS
4	1.1	0.93	1.379	1.472	1.440	1.603	6.654	6.9112
3	0.98	0.82	1.226	1.308	1.277	1.422	1.301	1.393
2	0.73	0.61	0.913	0.974	0.951	1.058	0.658	0.682
1	0.39	0.33	0.489	0.522	0.510	0.568	0.413	0.402

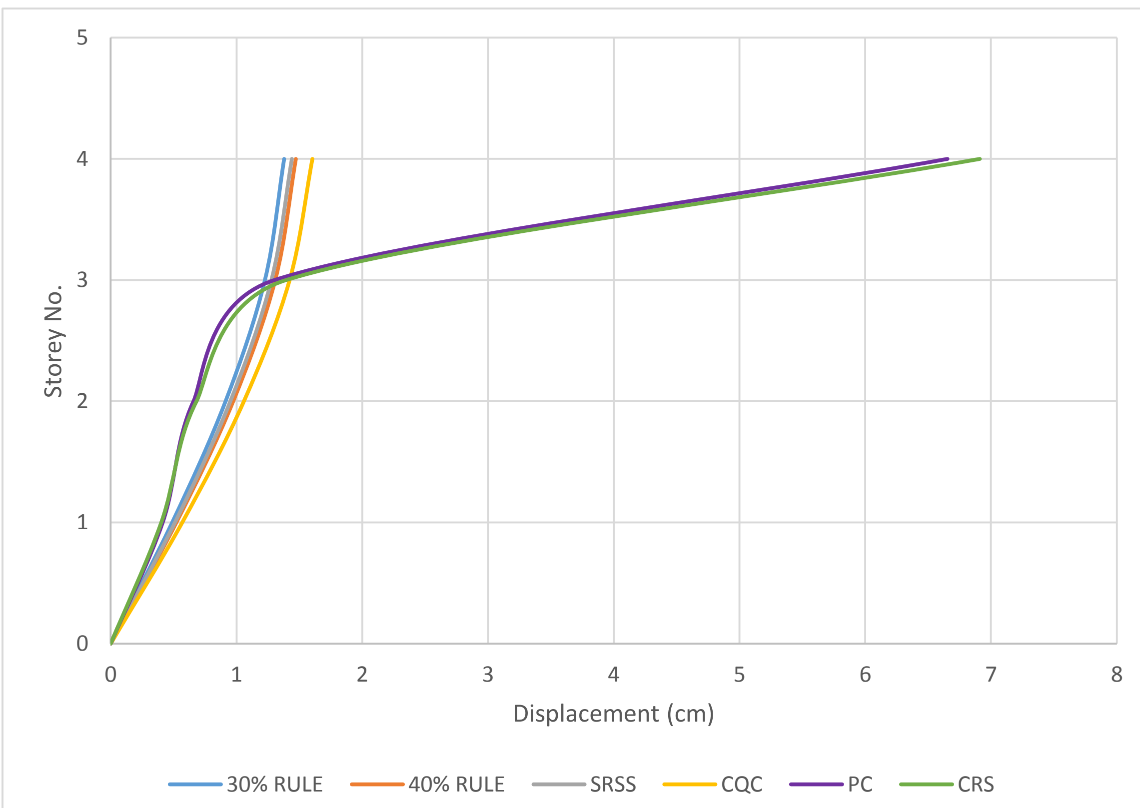

Fig. 4.6: Variation of displacement under various combination rules for Chamoli earthquake (in x-direction)

In y-direction:-

Table. 4.6: Floor Displacements (in cm) for Chamoli earthquake (in y-direction)

STOREY	X-COMP.	Y-COMP.	30% RULE	40% RULE	SRSS	CQC	PC	CRS
4	1.03	0.99	1.327	1.426	1.428	1.592	5.454	5.665
3	0.91	0.88	1.174	1.262	1.265	1.411	1.006	1.142
2	0.68	0.66	0.878	0.944	0.947	1.056	0.54	0.559
1	0.36	0.35	0.465	0.5	0.502	0.559	0.339	0.339

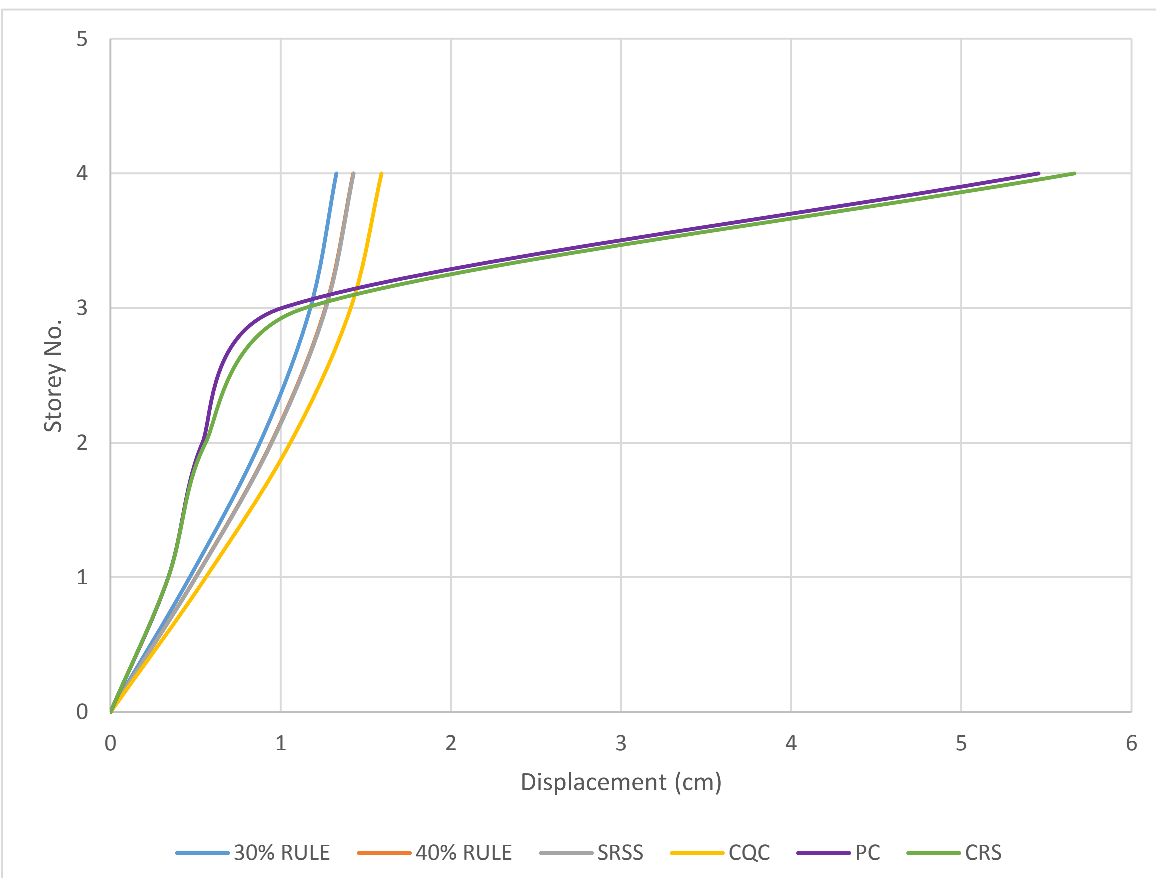

Fig. 4.7: Variation of displacement under various combination rules for Chamoli earthquake (in y-direction)

CHAMBA EARTHQUAKE

In x-direction:-

Table. 4.7: Floor Displacements (in cm) for Chamba earthquake (in x-direction)

STOREY	X-COMP.	Y-COMP.	30% RULE	40% RULE	SRSS	CQC	PC	CRS
4	0.22	0.18	0.274	0.292	0.284	0.316	0.931	1.125
3	0.2	0.16	0.248	0.264	0.256	0.284	0.535	0.589
2	0.15	0.12	0.186	0.198	0.192	0.213	0.291	0.311
1	0.08	0.07	0.101	0.108	0.106	0.118	0.168	0.176

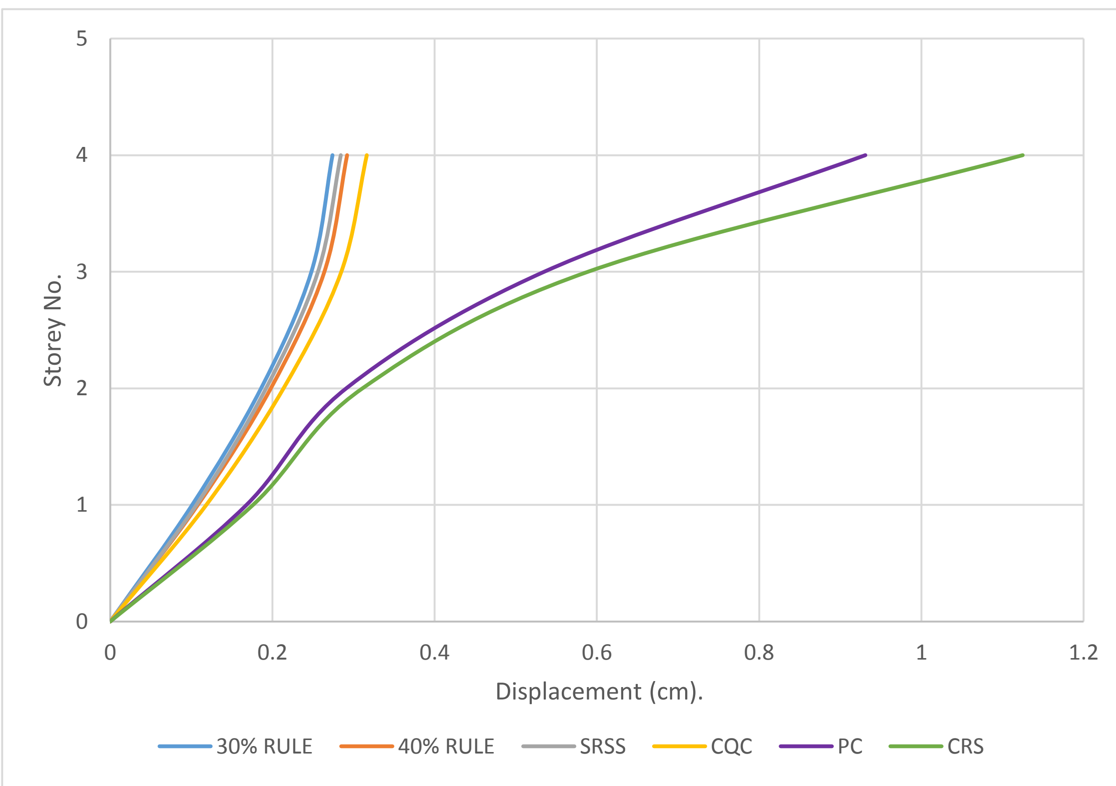

Fig. 4.8: Variation of displacement under various combination rules for Chamba earthquake (in x-direction)

In y-direction:-

Table. 4.8: Floor Displacements (in cm) for Chamba earthquake (in y-direction)

STOREY	X-COMP.	Y-COMP.	30% RULE	40% RULE	SRSS	CQC	PC	CRS
4	0.19	0.18	0.244	0.262	0.261	0.291	0.763	0.922
3	0.16	0.16	0.208	0.224	0.226	0.252	0.439	0.482
2	0.12	0.12	0.156	0.168	0.169	0.189	0.238	0.255
1	0.07	0.07	0.091	0.098	0.098	0.110	0.138	0.144

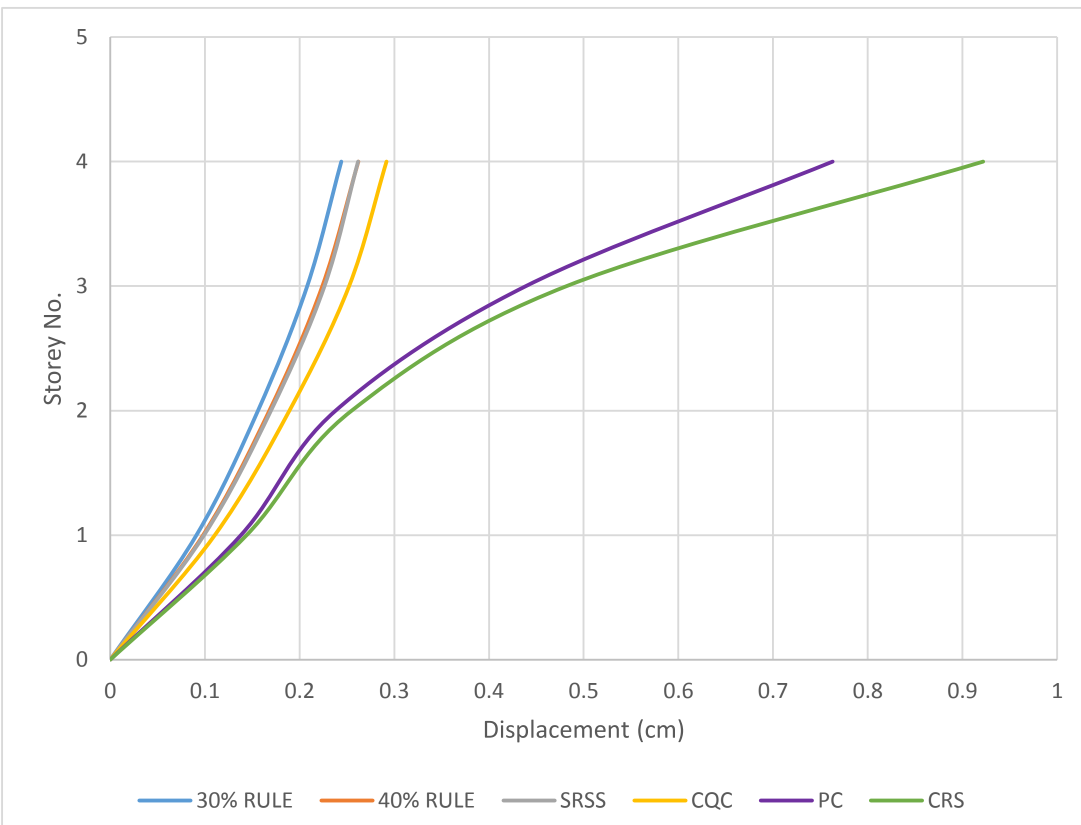

Fig. 4.9: Variation of displacement under various combination rules for Chamba earthquake (in y-direction)

NORTHRIDGE EARTHQUAKE

In x-direction:-

Table. 4.9: Floor Displacements (in cm) for Northridge earthquake

(in x-direction)

STOREY	X-COMP.	Y-COMP.	30% RULE	40% RULE	SRSS	CQC	PC	CRS
4	6.49	5.24	8.062	8.586	8.341	9.279	17.441	21.23
3	5.77	4.64	7.162	7.626	7.404	8.236	5.048	6.04
2	4.32	3.46	5.358	5.704	5.534	6.156	2.891	3.621
1	2.29	1.84	2.842	3.026	2.937	3.267	1.777	2.389

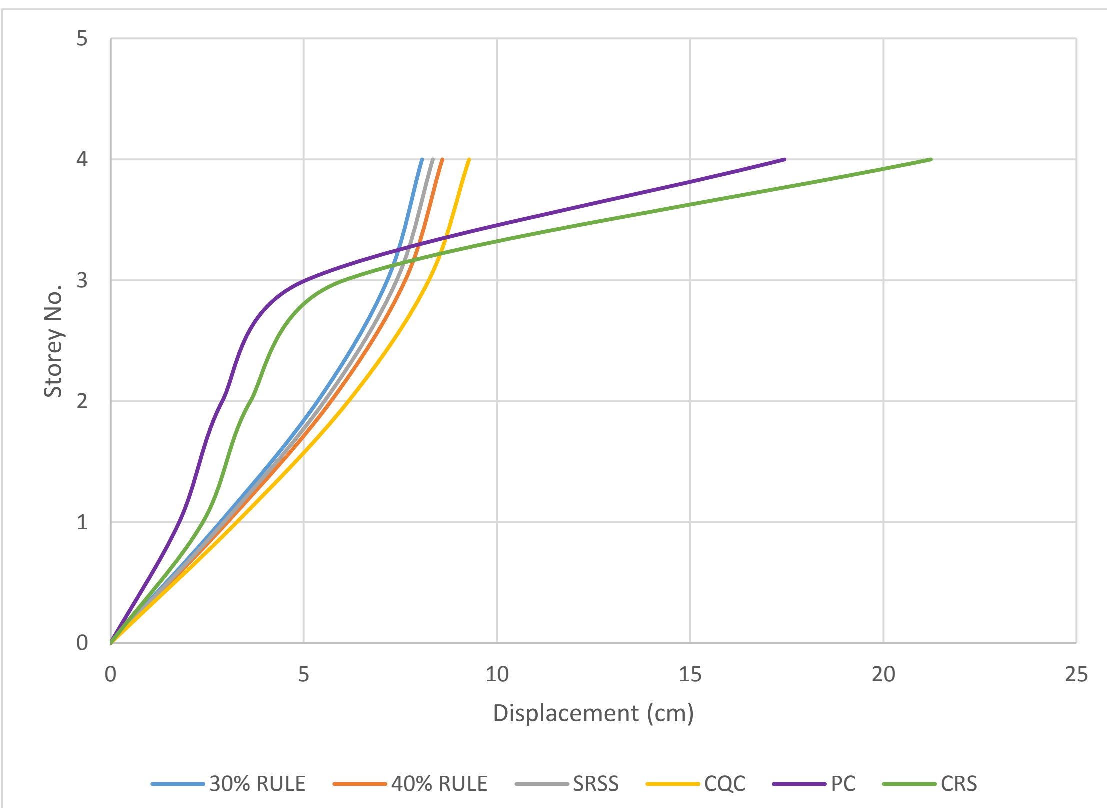

Fig. 4.10: Variation of displacement under various combination rules for Northridge earthquake (in x-direction)

In y-direction:-

Table. 4.10: Floor Displacements (in cm) for Northridge earthquake (in y-direction)

STOREY	X-COMP.	Y-COMP.	30% RULE	40% RULE	SRSS	CQC	PC	CRS
4	5.9	4.58	7.274	7.732	7.469	8.301	14.296	17.406
3	5.22	4.07	6.441	6.848	6.619	7.357	4.138	4.951
2	3.9	3.05	4.815	5.12	4.951	5.503	2.37	2.968
1	2.08	1.62	2.566	2.728	2.636	2.930	1.456	1.958

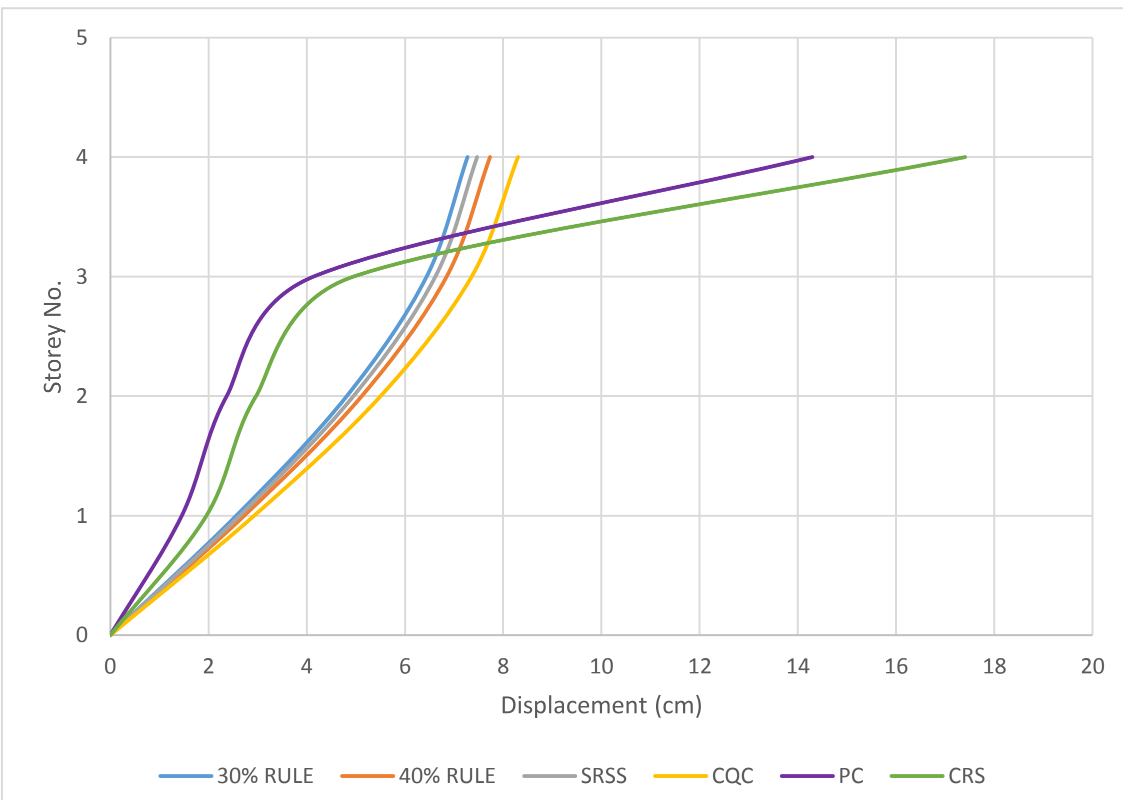

Fig. 4.11: Variation of displacement under various combination rules for Northridge earthquake (in y-direction)

LOMA PRIETA EARTHQUAKE

In x-direction:-

Table. 4.11: Floor Displacements (in cm) for Loma Prieta earthquake (in x-direction)

STOREY	X-COMP.	Y-COMP.	30% RULE	40% RULE	SRSS	CQC	PC	CRS
4	2.55	3.71	3.663	4.034	4.501	4.986	12.439	11.73
3	2.26	3.28	3.244	3.572	3.983	4.412	2.685	3.437
2	1.7	2.45	2.435	2.68	2.982	3.304	1.321	1.445
1	0.91	1.3	1.3	1.43	1.586	1.758	0.736	0.902

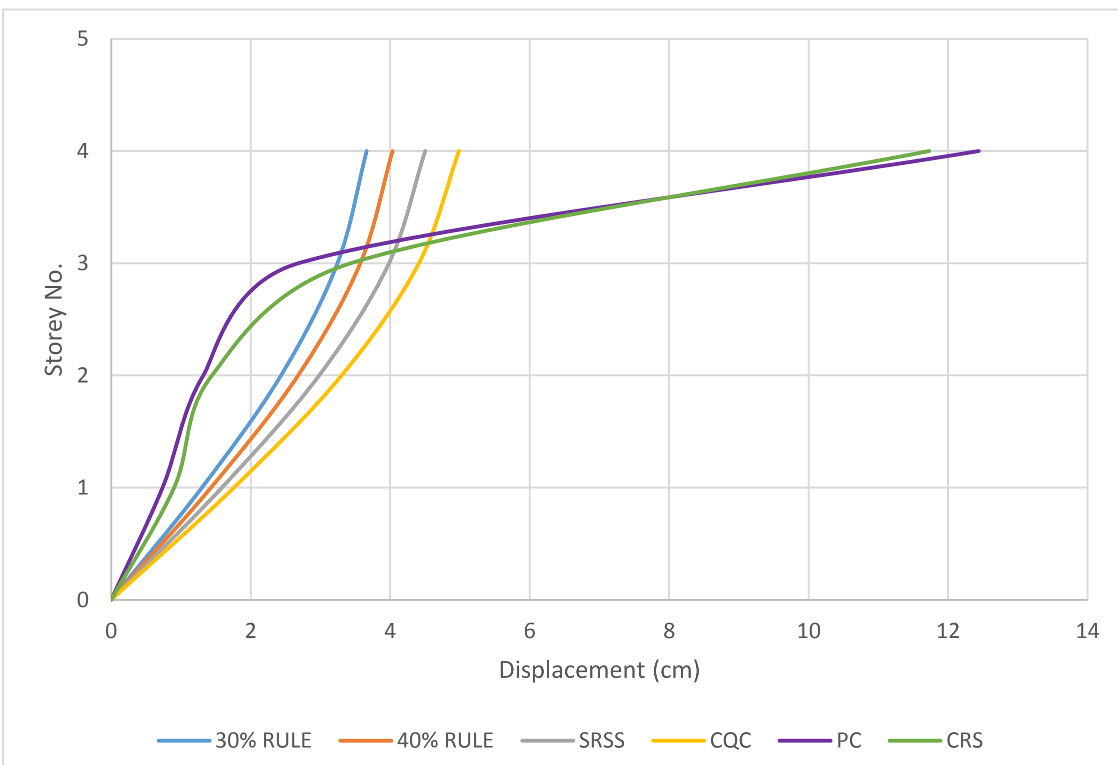

Fig. 4.12: Variation of displacement under various combination rules for Loma Prieta earthquake (in x-direction)

In y-direction:-

Table. 4.12: Floor Displacements (in cm) for Loma Prieta earthquake (in y-direction)

STOREY	X-COMP.	Y-COMP.	30% RULE	40% RULE	SRSS	CQC	PC	CRS
4	3.27	4.4	4.59	5.03	5.482	6.086	10.196	9.022
3	2.9	3.91	4.073	4.464	4.868	5.404	2.198	2.817
2	2.16	2.92	3.036	3.328	3.632	4.032	1.083	1.184
1	1.15	1.54	1.612	1.766	1.922	2.134	0.603	0.739

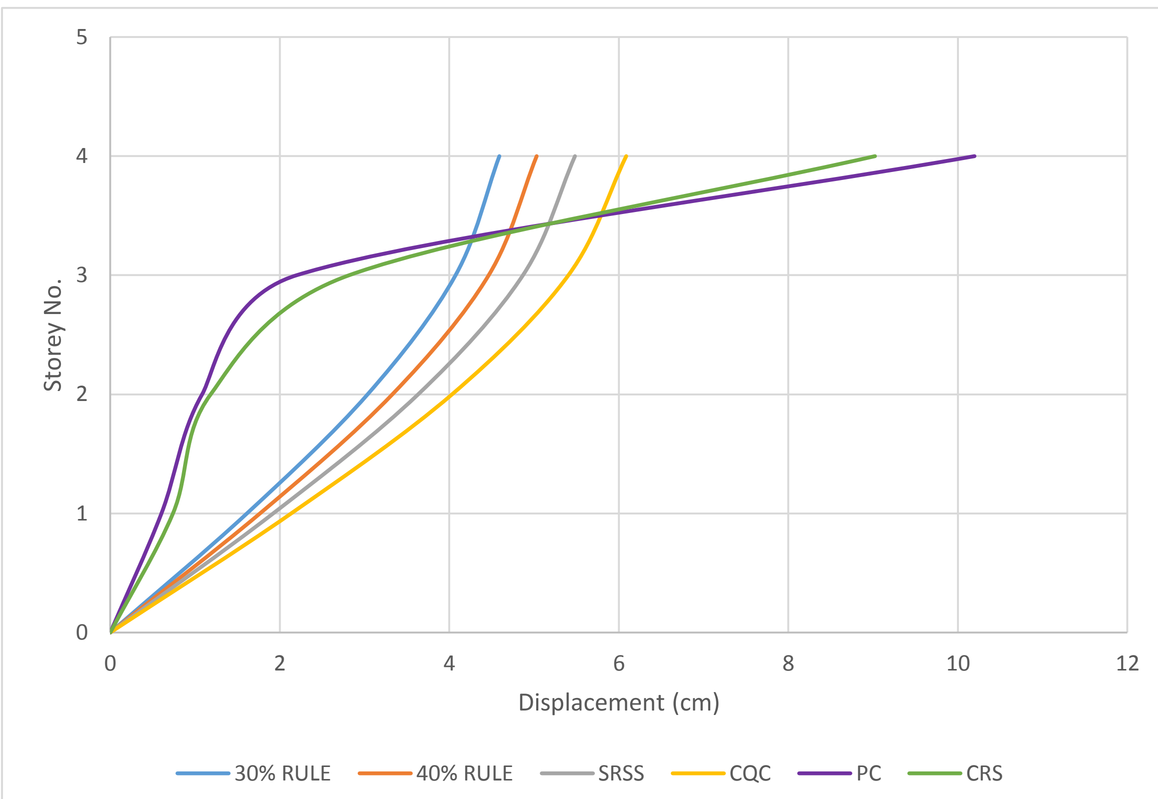

Fig. 4.13: Variation of displacement under various combination rules for Loma Prieta earthquake (in y-direction)

TABAS (IRAN) EARTHQUAKE

In x-direction:-

Table. 4.13: Floor Displacements (in cm) for Tabas (Iran) earthquake (in x-direction)

STOREY	X-COMP.	Y-COMP.	30% RULE	40% RULE	SRSS	CQC	PC	CRS
4	1.16	1.13	1.499	1.612	1.619	1.805	4.477	5.179
3	1.03	1	1.33	1.43	1.435	1.600	1.27	1.549
2	0.77	0.74	0.992	1.066	1.067	1.190	0.766	1.087
1	0.41	0.4	0.53	0.57	0.572	0.638	0.483	0.56

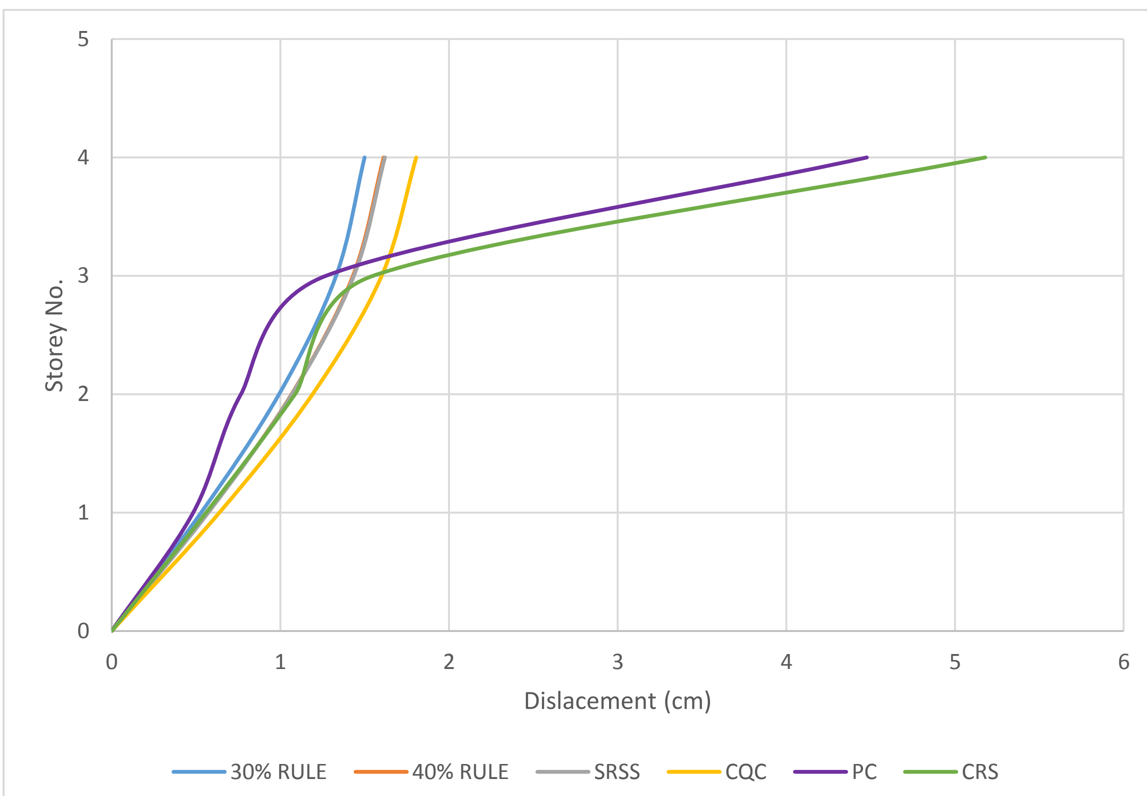

Fig. 4.14: Variation of displacement under various combination rules for Tabas (Iran) earthquake (in x-direction)

In y-direction:-

Table. 4.14: Floor Displacements (in cm) for Tabas (Iran) earthquake (in y-direction)

STOREY	X-COMP.	Y-COMP.	30% RULE	40% RULE	SRSS	CQC	PC	CRS
4	1.04	1.43	1.469	1.612	1.768	1.961	3.672	4.24
3	0.92	1.27	1.301	1.428	1.568	1.739	2.041	2.4
2	0.69	0.95	0.975	1.07	1.174	1.302	1.627	1.891
1	0.37	0.5	0.52	0.57	0.622	0.690	0.696	0.759

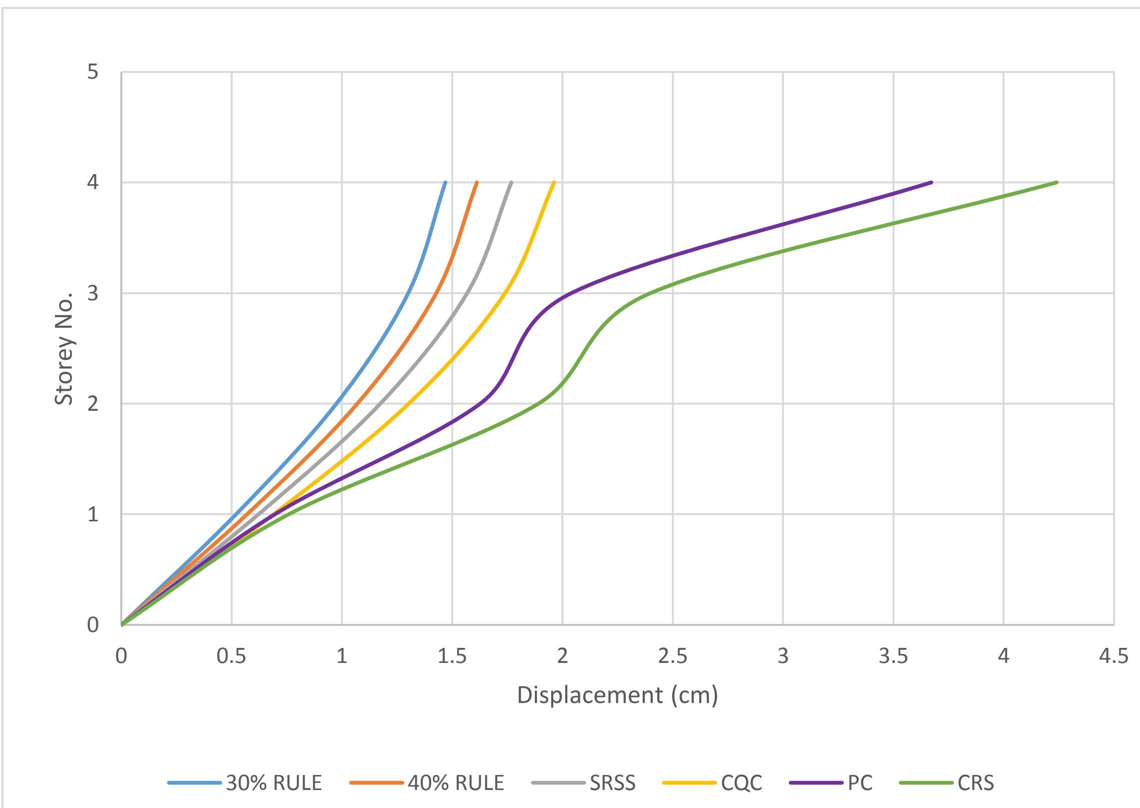

Fig. 4.15: Variation of displacement under various combination rules for Tabas earthquake (in y-direction)

4.5 Results (Base Shear)

Table. 4.15: Base Shear (in kN) for different earthquakes

EQ	X-COMP	Y-COMP	30% RULE	40% RULE	SRSS	CQC	PC	CRC
KANGRA	1818.7	1168.9	2169.37	2286.26	2161.943	2380.947	2588.58	2678.36
CHAMOLI	6044.1	6698.8	8053.74	8723.62	9022.476	10017.66	11524.49	11978.27
CHAMBA	887.1	820.1	1133.13	1215.14	1208.102	1341.635	1534.95	1665.59
LOMA PRIETA	24588.7	24233.9	31858.87	34282.26	34523.7	38350.46	40529.73	41567.62
NORTHRIDGE	32785.4	21769.9	39316.37	41493.36	39354.94	43391.69	45172.78	47943.28
TABAS	8959.5	7761.3	11287.89	12064.02	11853.71	13155	13783.49	15215.39

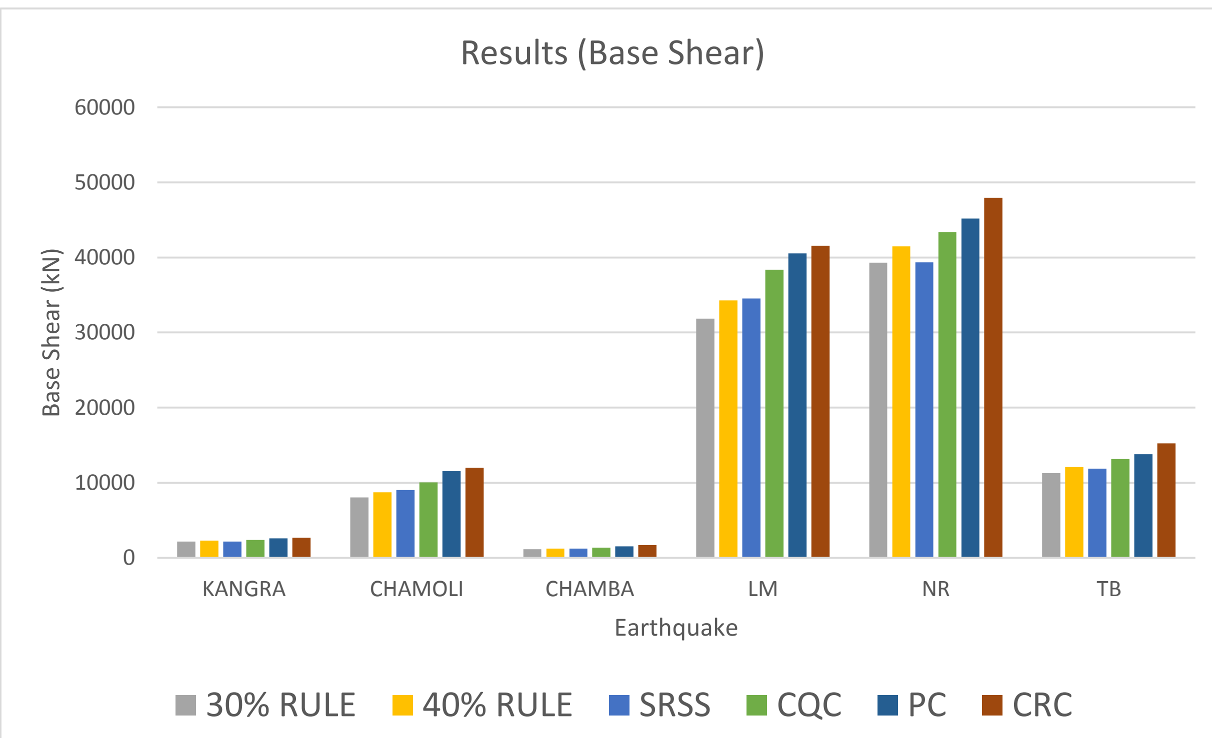

Fig. 4.16: Variation of Base Shear under various combination rules for different earthquakes

4.6 Discussion

As seen from the figures shown above, the maximum structural response is obtained by the critical response spectrum method in most of the cases. It can be concluded that the critical response spectrum method gives safe results and can be used for design purpose. The other methods are giving sufficiently smaller values which means these methods are underestimating the peak structural response as these methods are derived by using quite a large number of assumptions and simplifications. However, the variation of the response of critical response spectrum is similar to the variation of the principal component method. Hence, we can say that the critical response spectrum method is close to reality as principal component method is the most accurate method of response calculation as it involves rigorous calculation and is also free from assumptions and simplifications. However, the critical spectrum method can be considered as a standard method having no assumptions involved.

In most of the cases discussed above, the critical response spectrum method gives the maximum value of the response except for the case of Loma Prieta earthquake in x and y-directions both in which the response of the critical spectrum is slightly less than the response given by the principal component method.

If we consider the results for base shear, the observation concluded is that the base shear obtained by the critical response spectrum is also higher than what is obtained from the other methods. The values of the base shear obtained from the critical response spectrum method are quite close to the values obtained by using the principal component method for all the cases of earthquake. This justifies the authenticity of the Critical Response Spectrum Method for the calculation of the peak structural response.

CHAPTER 5: APPLICATION TO ASYMMETRICAL STRUCTURES

5.1 Introduction

In the previous chapter, the use of the critical response spectrum method has been shown for the symmetrical structures in which the well-defined principal directions exist. However, the question arises that how it can be used to the structures in which no principal directions can be defined as the in these structures the principal directions keep on changing from one point to another. Hence, in this chapter, the application of the critical response spectrum method has been discussed for the structures in which the principal directions are not well-defined. The examples of such structures are the asymmetrical structures, non-rectangular buildings, curved buildings and bridges, arch dams, piping systems etc.

For such complex asymmetrical structures, a direction in which maximum stresses are developed in a particular member at a given time is not constant. So, in order to find the peak response, one has to perform a number of dynamic analyses at different input angles using the time history input. Such an extensive exercise is likely to produce different critical direction for each value of peak response. So, to overcome this problem a solution has been given by this study.

It is apparent that the maximum value of the response cannot be calculated for the structure as a whole as the different members will have their own maximums. So, for the purpose of safety, each member has to be designed for the maximum forces developed in it. In order to get the maximum response, the responses in any two arbitrarily selected orthogonal directions is calculated and the maximum of the above responses is taken as the peak structural response for which the member shall be designed.

Similarly, the critical response methods can be used along the same lines. The critical response spectrum can been applied in any two orthogonal directions and the responses in these directions are noted. The maximum of the above two responses is the maximum value of the critical response.

In this study, a single storey asymmetrical structure has been selected for the analysis. A simple single storey structure was selected and the complete procedure was repeated on that structure and the results were matched with the SRSS and λ-percent rule. Although, for illustration purpose, a simple single storey structure is used but the method can be readily applied in principle for complex asymmetrical also.

5.2 Description of structure:

A very simple structure consisting of the four rectangular columns is taken for study. The structure is asymmetrical in which the center of mass is not lying on the geometric center which means there is torsional irregularity in the structure. The structure is allowed to rotate about its center of mass and also two translational degree of freedom are present.

The columns provided are rectangular in cross-section which are fixed at their base whereas at the top, they are pinned and are connected with a rigid diaphragm.

A cross-sectional view of the structure is shown below:

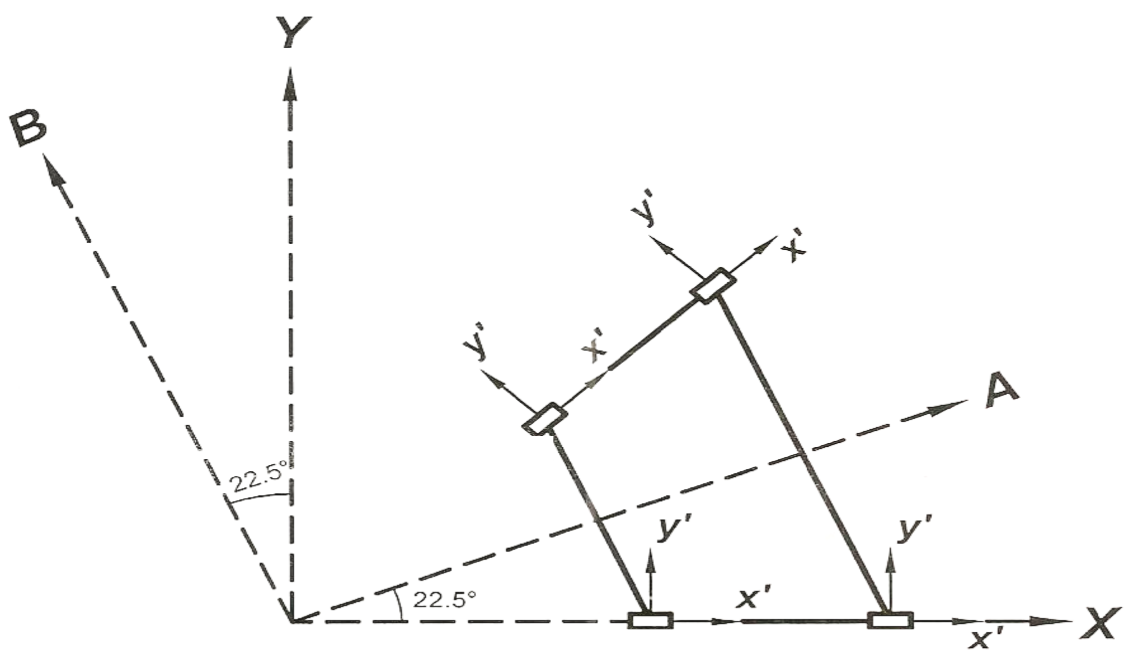

Fig. 5.1: Plan of Asymmetrical Structure

5.3 Properties of the structure

A simple single storeyed building has been modelled using the SAP2000 software.

The building consists of four columns fixed at their base and pinned at their top points in such a way that the rigid diaphragm action has been assigned to the top of the building. The orientation of the columns is such that the two columns are oriented in the x-direction and the other two columns are oriented at 45^0 to the global x- axis. Due to this, the building becomes a asymmetrical building having torsion. The structure is assigned three degree of freedoms, viz. two translations and one rotation about the center of mass. It is evident that the center of mass (C.M) and center of rigidity (C.R.) of the given structure will not be same, with the effect of which there will be two eccentricities in the building 'e_x' and 'e_y'.

Since the building is having torsion, all the degree of freedoms of the building will be coupled. The mass matrix of the structure will be a diagonal matrix having mass corresponding to each DOF at its diagonal elements. The mass matrix of the structure will be of the form:

$$[M] = \begin{bmatrix} m_x & 0 & 0 \\ 0 & m_y & 0 \\ 0 & 0 & J \end{bmatrix}$$

Where,

m_x = mass corresponding to x DOF

m_y = mass corresponding to y DOF

J = mass moment of inertia corresponding to θ DOF

The actual mass matrix of the given structure is given below:

$$[M] = \begin{bmatrix} 39.34 & 0 & 0 \\ 0 & 39.34 & 0 \\ 0 & 0 & 11000.84 \end{bmatrix}$$

The stiffness matrix, as we all know, is a symmetric matrix but in this case the stiffness for different DOF are coupled with each other as the effect of torsion is also included in the analysis.

The stiffness matrix of the given structure will has the terms in which all the degree of freedoms are coupled with each other, i.e., x-DOF is coupled with y-DOF as well as the θ-DOF. So, the stiffness matrix of the given structure will not be a diagonal matrix. It will include the terms depicting the coupling effect of the various DOFs.

The stiffness matrix of the given building will be of the form:

$$[K]=\begin{bmatrix} k_{xx} & k_{xy} & k_{x\theta} \\ k_{yx} & k_{yy} & k_{y\theta} \\ k_{\theta x} & k_{\theta y} & k_{\theta\theta} \end{bmatrix}$$

In the above matrix, the terms k_{xx}, k_{yy} *and* $k_{\theta\theta}$ are uncoupled stiffnesses whereas k_{xy}, k_{yx}, $k_{x\theta}$, $k_{y\theta}$, $k_{\theta x}$ and $k_{\theta y}$ are the terms denoting the stiffness due to coupling effect. The $k_{\theta\theta}$ has been derived by using k_{xx}, k_{yy} and the radial distances of the columns from the center of stiffness whereas the terms kxy and kyx contains the effect of the e_x *and* e_y.

The actual stiffness matrix of the given structure has been shown below:

$$[K]=\begin{bmatrix} 3522.59 & 5762.54 & 655.84 \\ 5762.54 & 13915.50 & 2210.90 \\ 655.84 & 2210.90 & 921410.52 \end{bmatrix}$$

The mode shapes corresponding to the different modes of the building are given below:

<u>Mode-1:</u> T= 1.021 sec; f= 0.979 cyc/s

$$\{\phi_1\} = \begin{Bmatrix} 0.054 \\ 0.1305 \\ 0.00426 \end{Bmatrix}$$

<u>Mode-2:</u> T= 0.644 sec; f= 1.552 cyc/s

$$\{\phi_2\} = \begin{Bmatrix} 0.0287 \\ 0.0683 \\ 0.00853 \end{Bmatrix}$$

<u>Mode-3:</u> T= 1.021 sec; f= 0.979 cyc/s

$$\{\phi_3\} = \begin{Bmatrix} 0.1472 \\ 0.0611 \\ 0.0001 \end{Bmatrix}$$

5.4 Results

Using the above properties of the structure the response of the structure is calculated. The response of the structure includes the displacements of the center of mass of the structure in *x* and *y* directions and the rotation of the building about the center of rigidity. Now, using these responses of the whole structure, the response of the individual column along the local axes of the structure has been calculated.

For the computation of the response, six different earthquakes which are used in the previous chapter are selected here also. The numerical results of the analysis for all the six earthquakes has been shown in the next section of this chapter.

The results depict the maximum displacements of the top of the each column and also the maximum base shear at the base of the column. However, the variation of the different results are on the same lines as the results of the previous chapter. The critical response spectrum gives significantly high value of the response which clearly indicates that the other empirical methods underestimate the maximum response of the structure. Hence, it is evident that the critical response spectrum method gives conservative results and they can be readily used for the design purposes.

In this section, the results of the analysis of the asymmetrical structure is shown. Using, the modal properties of the structure, maximum structural response of the structure has been calculated which is further used to calculate the response of the individual column.

Table. 5.1 to Table. 5.12 gives the values of the displacement of top of the column (in mm) in *x* and y-directions when subjected to different input earthquakes. Fig. 5.2 to Fig. 5.13 shows the variation of displacement under various combination rules for different earthquakes in both *x* and *y*-directions.

Table. 5.13 gives the maximum values of the base shear (in kN) for different earthquakes while Fig. 5.13. shows the variation of base shear under various combination rules for different earthquakes.

KANGRA EARTHQUAKE

In x-direction:-

Table. 5.1: Displacements (in mm) for Kangra earthquake (in x-direction)

COLUMN	X-COMP.	Y-COMP	30% RULE	40% RULE	SRSS	CRS
1	0.7	1.5	1.15	1.15	1.655	2.8
2	0.7	2.2	1.36	1.36	2.308	2.8
3	0.6	1.3	0.99	0.99	1.431	2.4
4	0.6	1.6	1.08	1.08	1.708	2.4

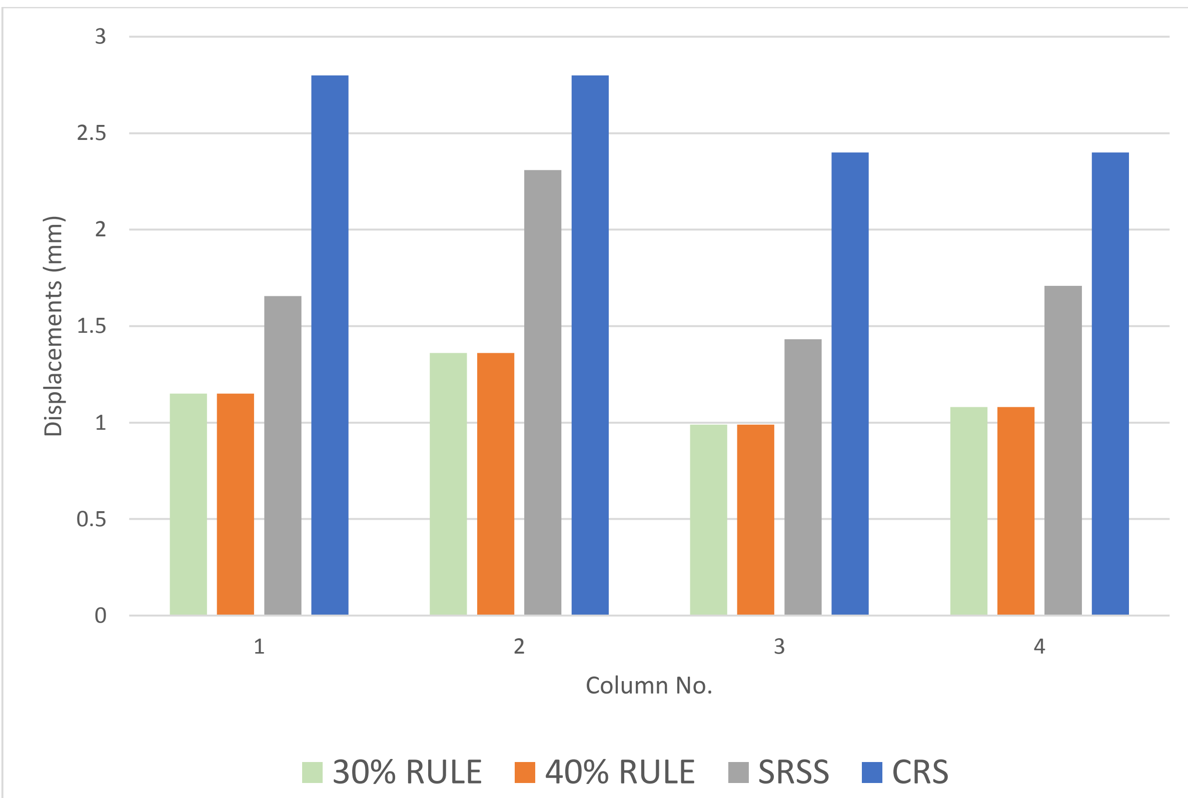

Fig. 5.2: Variation of displacement under various combination rules for Kangra earthquake (in x-direction)

In y-direction:-

Table. 5.2: Displacements (in mm) for Kangra earthquake (in y-direction)

COLUMN	X-COMP.	Y-COMP	30% RULE	40% RULE	SRSS	CRS
1	0.8	1.2	1.16	1.28	1.442	2.7
2	1.1	1.2	1.46	1.58	1.627	3.8
3	0.7	1.1	1.03	1.14	1.303	2.3
4	0.8	1.1	1.13	1.24	1.360	2.8

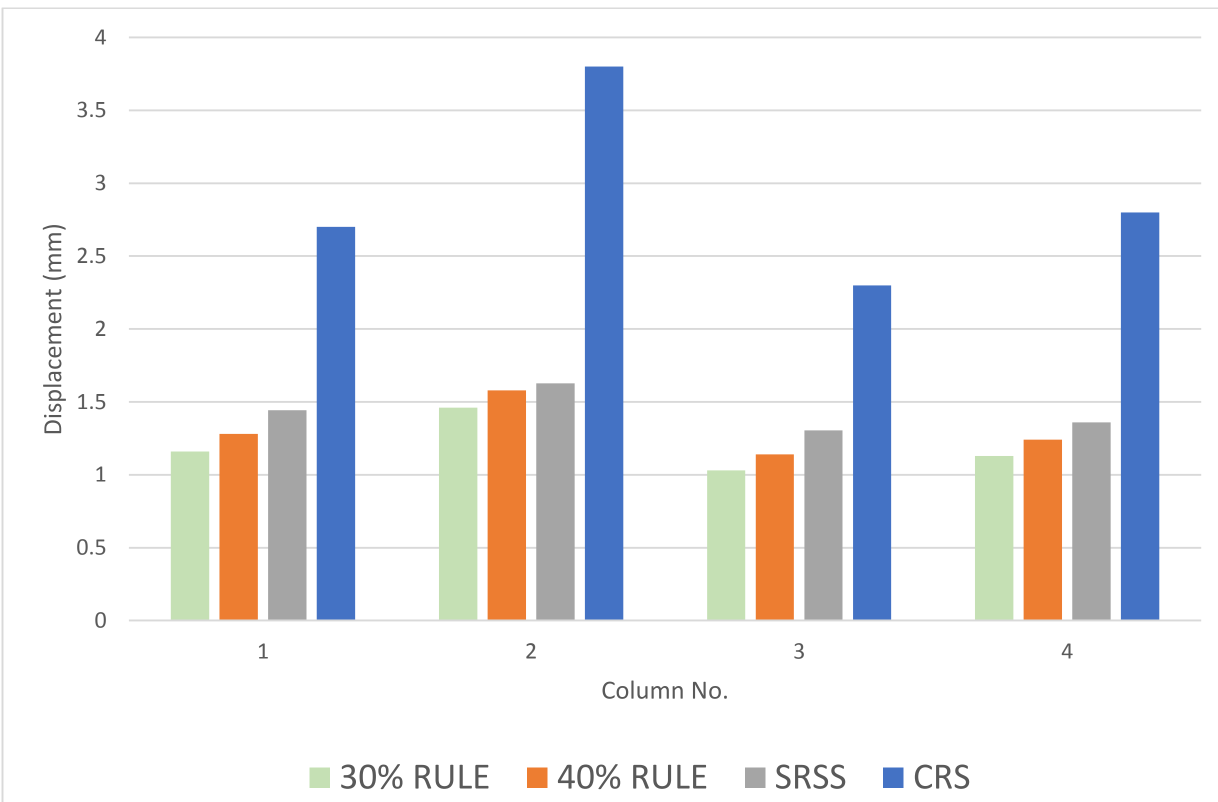

Fig. 5.3: Variation of displacement under various combination rules for Kangra earthquake (in y-direction)

CHAMOLI EARTHQUAKE

In x-direction:-

Table. 5.3: Displacements (in mm) for Chamoli earthquake (in x-direction)

COLUMN	X-COMP.	Y-COMP	30% RULE	40% RULE	SRSS	CRS
1	0.8	1.7	1.31	1.48	1.878	4.1
2	1.1	2.5	1.85	2.1	2.731	4.1
3	0.8	1.3	1.19	1.32	1.526	4
4	0.8	1.8	1.34	1.52	1.969	4.3

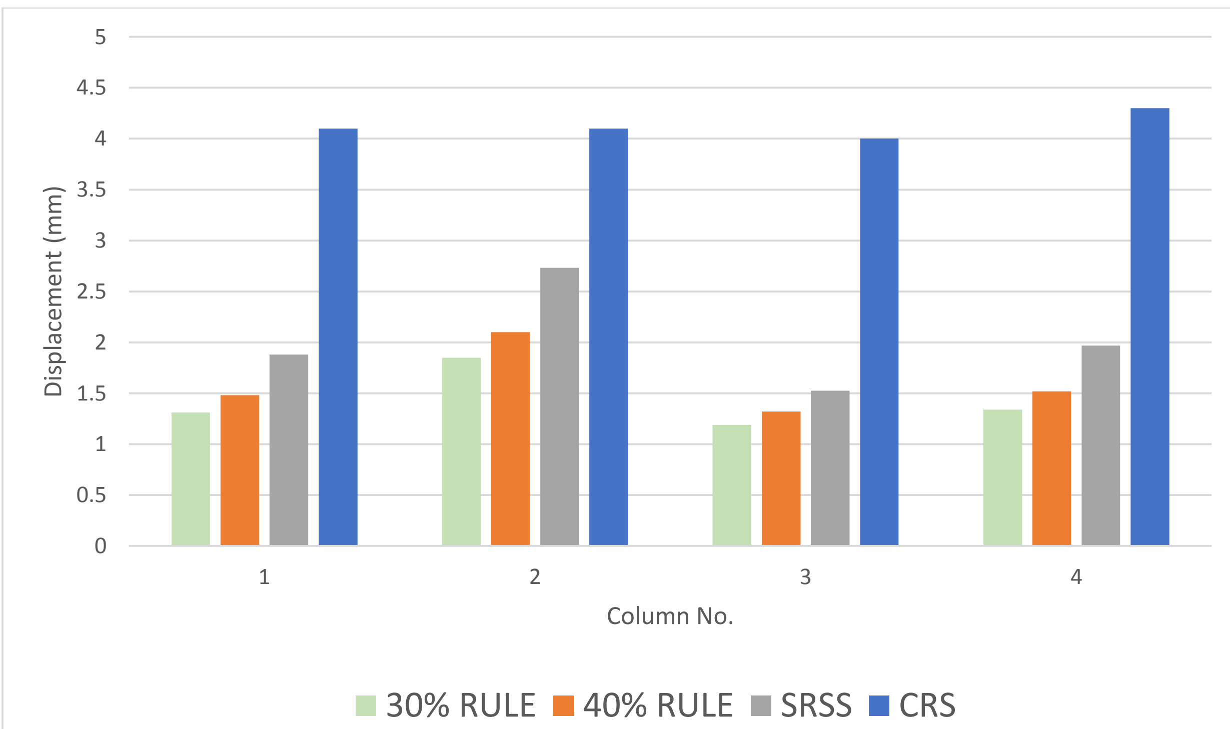

Fig. 5.4: Variation of displacement under various combination rules for Chamoli earthquake (in x-direction)

In y-direction:-

Table. 5.4: Displacements (in mm) for Chamoli earthquake (in y-direction)

COLUMN	X-COMP.	Y-COMP	30% RULE	40% RULE	SRSS	CRS
1	1.7	0.8	1.94	2.02	1.878829	6.4
2	2.5	0.8	2.74	2.82	2.624881	6.83
3	1.3	0.8	1.54	1.62	1.526434	5
4	1.8	1	2.1	2.2	2.059126	6.8

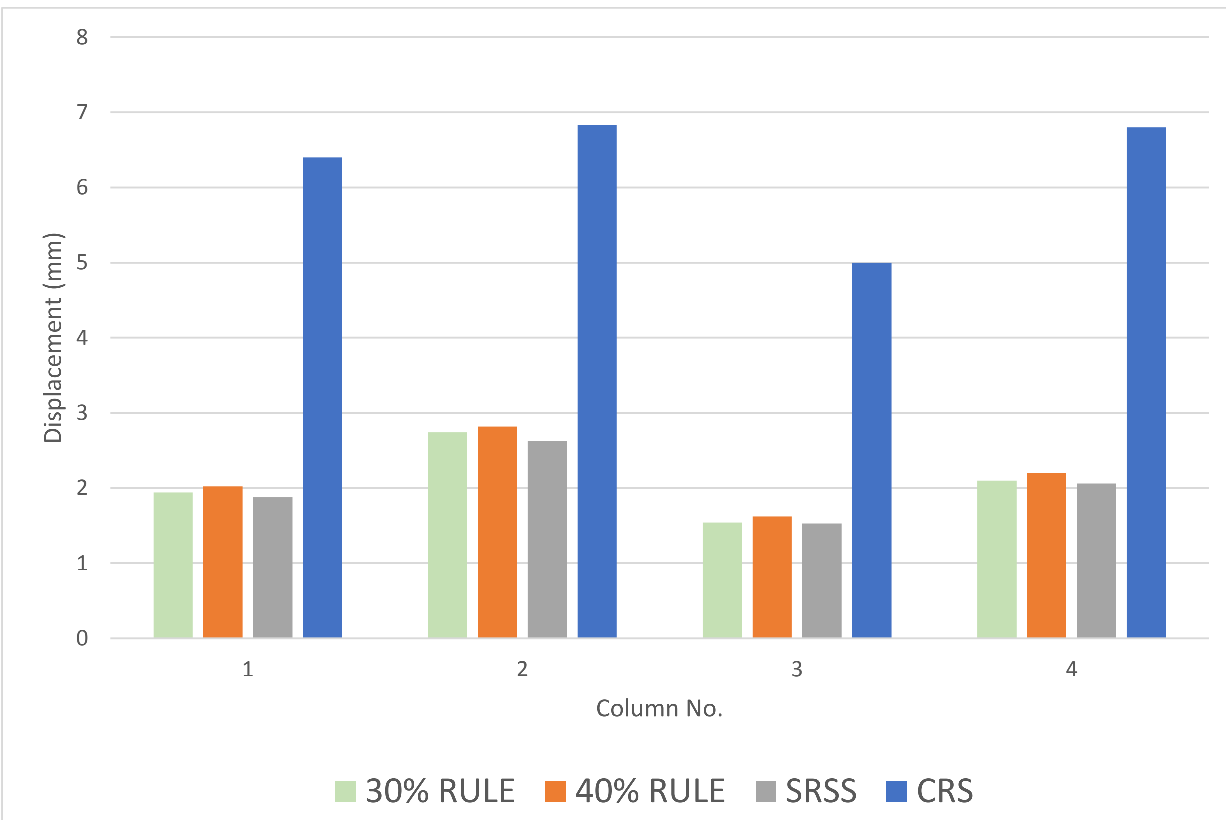

Fig. 5.5: Variation of displacement under various combination rules for Chamoli earthquake (in y-direction)

CHAMBA EARTHQUAKE

In x-direction:-

Table. 5.5: Displacements (in mm) for Chamba earthquake (in x-direction)

COLUMN	X-COMP.	Y-COMP	30% RULE	40% RULE	SRSS	CRS
1	1.1	0.9	1.37	1.46	1.421267	2.4
2	1.1	1.3	1.49	1.62	1.702939	2.4
3	0.9	0.8	1.14	1.22	1.204159	2.1
4	0.8	1.1	1.13	1.24	1.360147	2

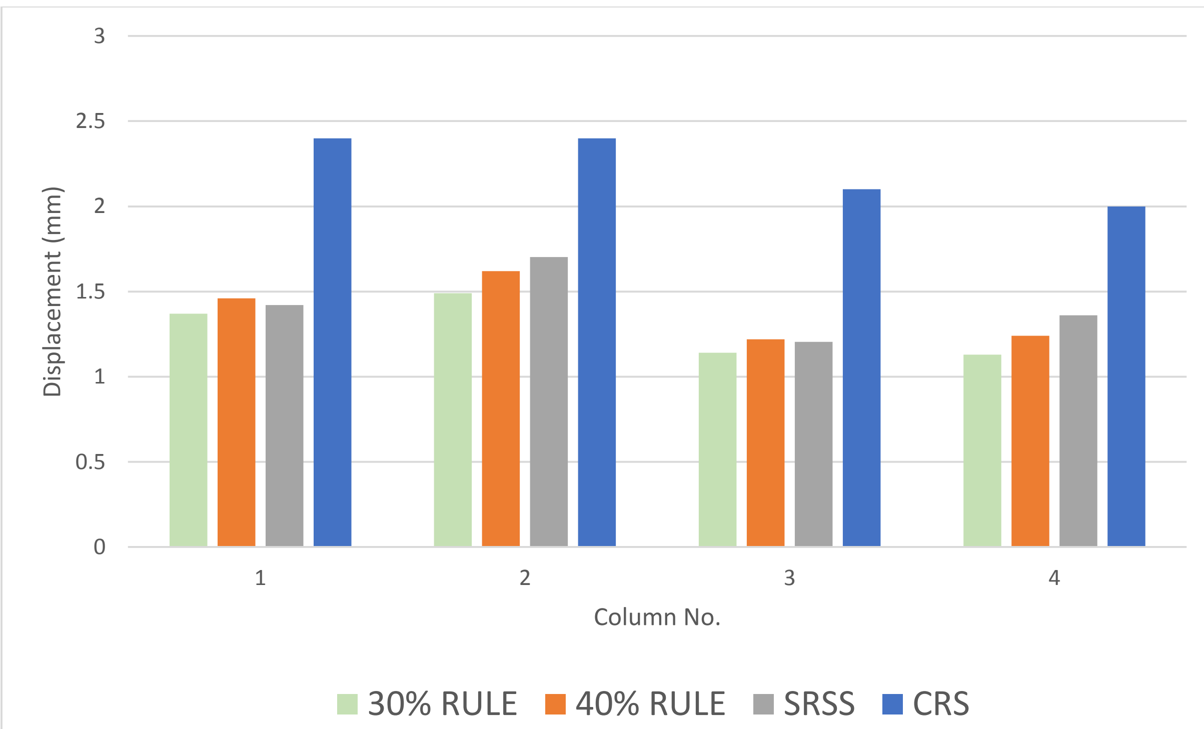

Fig. 5.6: Variation of displacement under various combination rules for Chamba earthquake (in x-direction)

In y-direction:-

Table. 5.6: Displacements (in mm) for Chamba earthquake (in y-direction)

COLUMN	X-COMP.	Y-COMP	30% RULE	40% RULE	SRSS	CRS
1	0.9	0.9	1.17	1.26	1.272	1.9
2	1.2	0.9	1.47	1.56	1.5	2.6
3	0.8	0.7	1.01	1.08	1.063	1.7
4	0.9	0.7	1.11	1.18	1.140	2

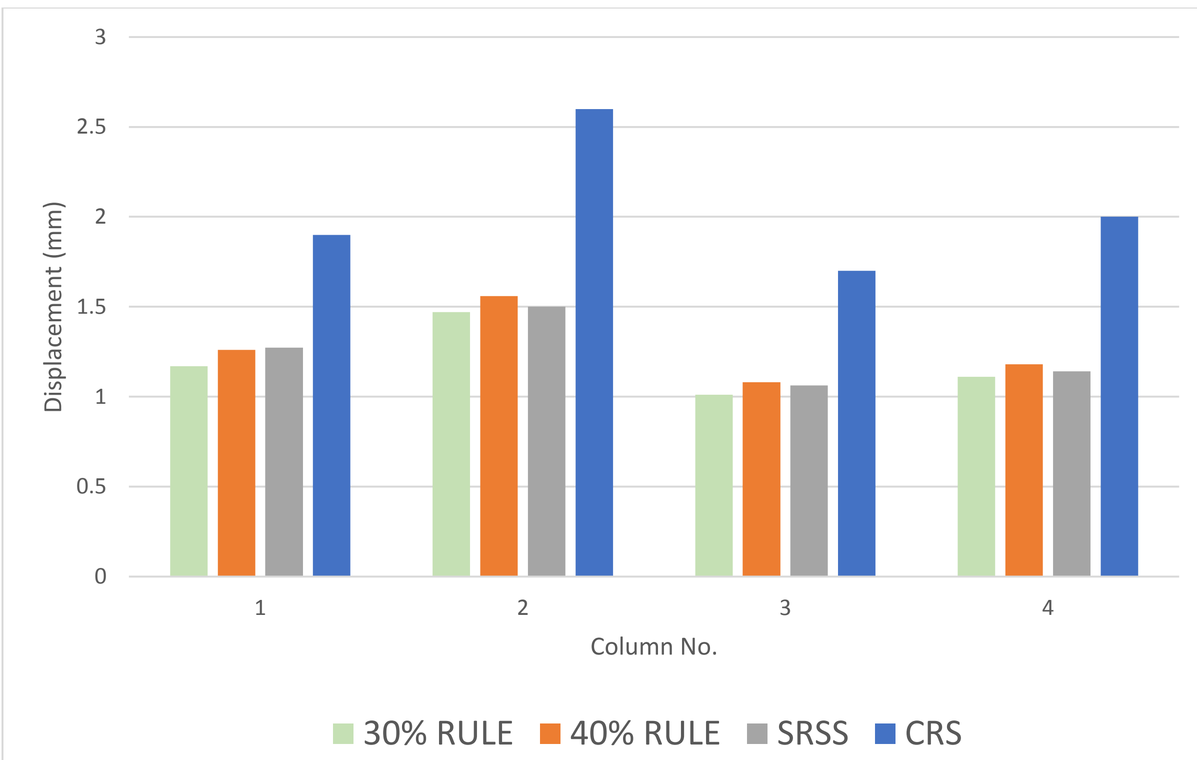

Fig. 5.7: Variation of displacement under various combination rules for Chamba earthquake (in y-direction)

NORTHRIDGE EARTHQUAKE

In x-direction:-

Table. 5.7: Displacements (in mm) for Northridge earthquake (in x-direction)

COLUMN	X-COMP.	Y-COMP	30% RULE	40% RULE	SRSS	CRS
1	13.9	9.1	16.63	17.54	16.613	27.6
2	13.9	13.1	17.83	19.14	19.100	27.6
3	11.7	7.2	13.86	14.58	13.737	23.7
4	11	9.6	13.88	14.84	14.6	22.7

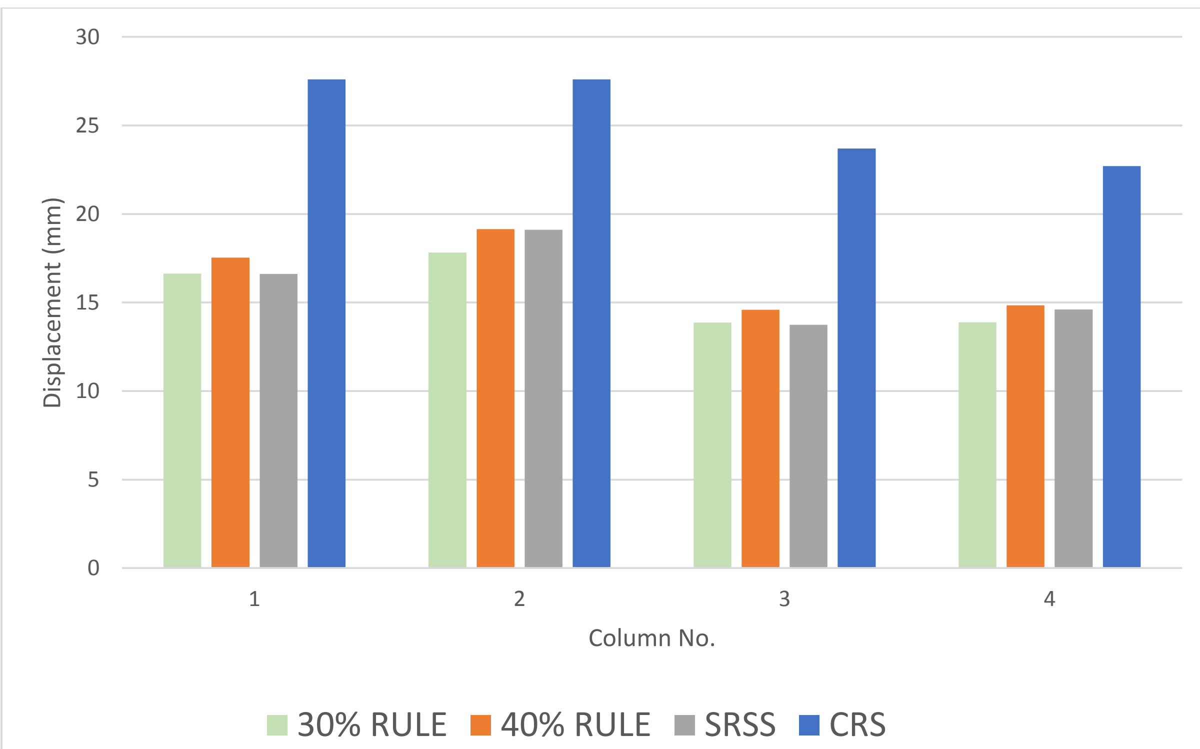

Fig. 5.8: Variation of displacement under various combination rules for Northridge earthquake (in x-direction)

In y-direction:-

Table. 5.8: Displacements (in mm) for Northridge earthquake (in y-direction)

COLUMN	X-COMP.	Y-COMP	30% RULE	40% RULE	SRSS	CRS
1	11.2	6.3	13.09	13.72	12.85029	23.2
2	14.9	6.3	16.79	17.42	16.17714	31.9
3	10.5	5.8	12.24	12.82	11.99542	20.4
4	11.5	6.1	13.33	13.94	13.01768	24.1

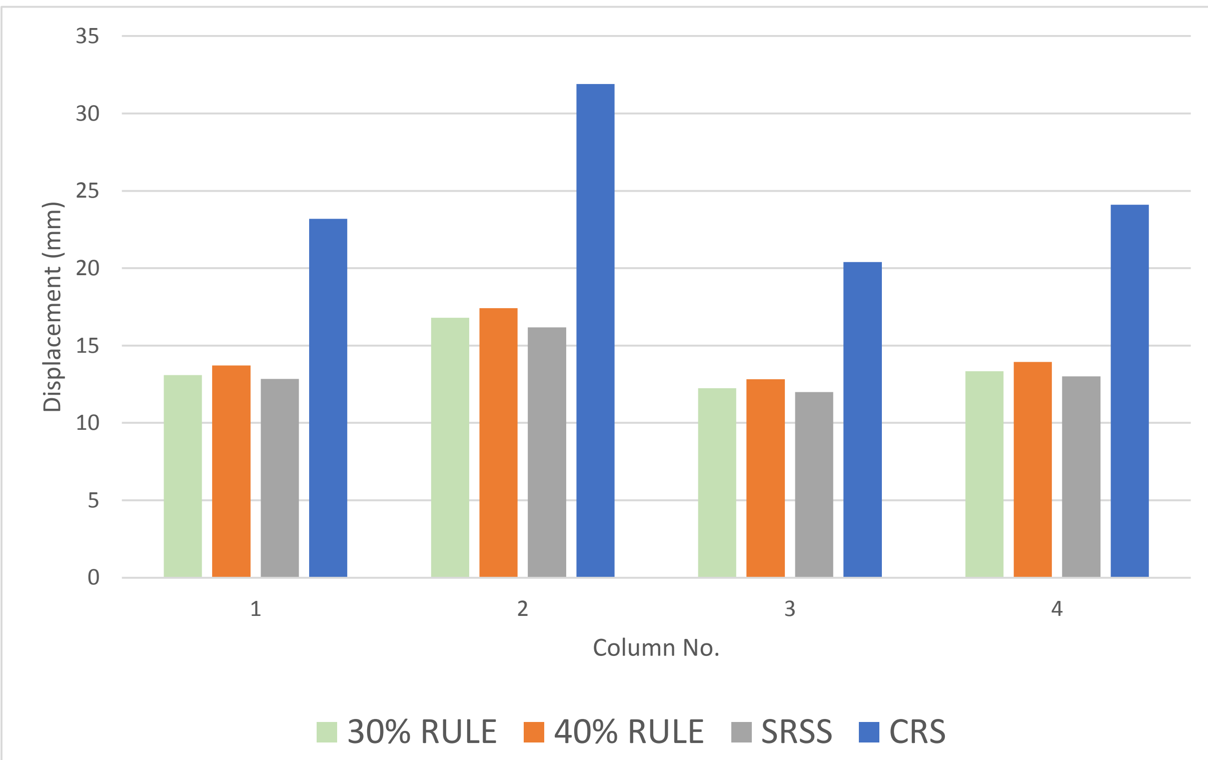

Fig. 5.9: Variation of displacement under various combination rules for Northridge earthquake (in y-direction)

LOMA PRIETA EARTHQUAKE

In x-direction:-

Table. 5.9: Displacements (in mm) for Loma Prieta earthquake (in x-direction)

COLUMN	X-COMP.	Y-COMP	30% RULE	40% RULE	SRSS	CRS
1	5.3	5.6	6.98	7.54	7.710	9.8
2	5.3	8.2	7.76	8.58	9.763	9.8
3	4.8	3.9	5.97	6.36	6.184	8.8
4	4.7	5.6	6.38	6.94	7.310	8.9

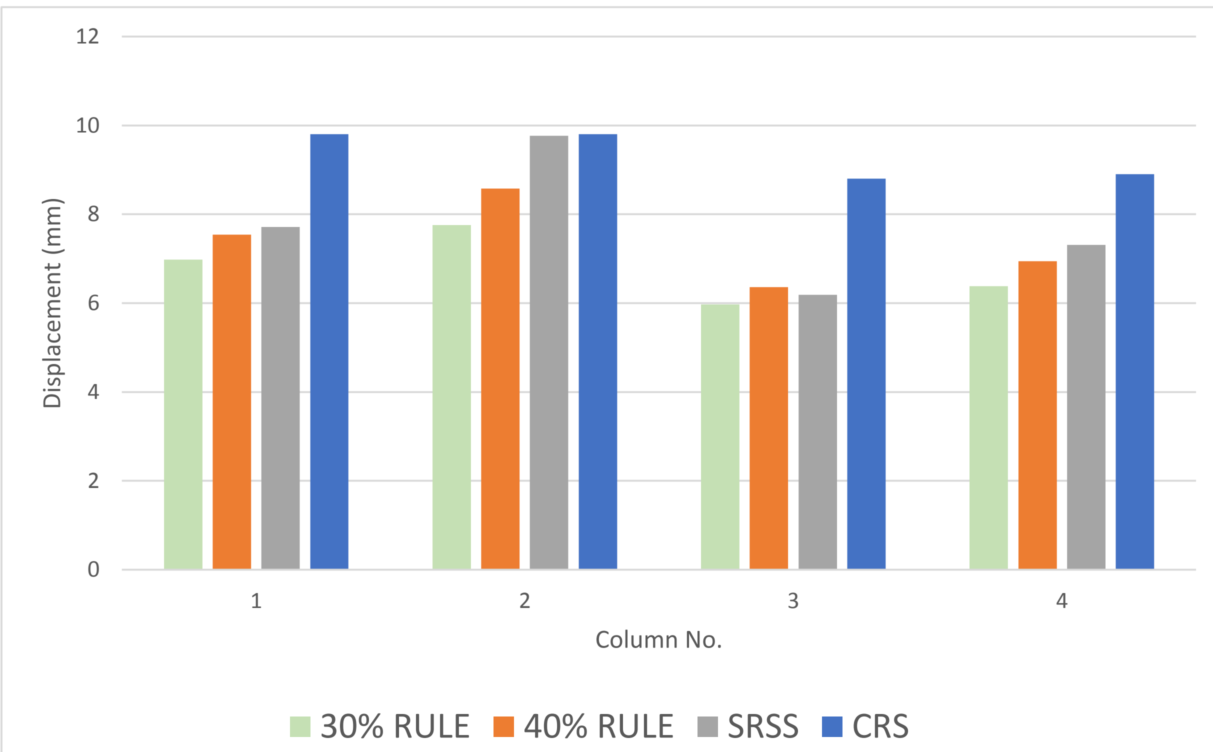

Fig. 5.10: Variation of displacement under various combination rules for Loma Prieta earthquake (in x-direction)

In y-direction:-

Table. 5.10: Displacements (in mm) for Loma Prieta earthquake (in y-direction)

COLUMN	X-COMP.	Y-COMP	30% RULE	40% RULE	SRSS	CRS
1	6.5	2.3	7.19	7.42	6.894	10.7
2	9.3	2.3	9.99	10.22	9.580	15.3
3	5.3	2.5	6.05	6.3	5.860	8.7
4	6.8	2.5	7.55	7.8	7.244	11.2

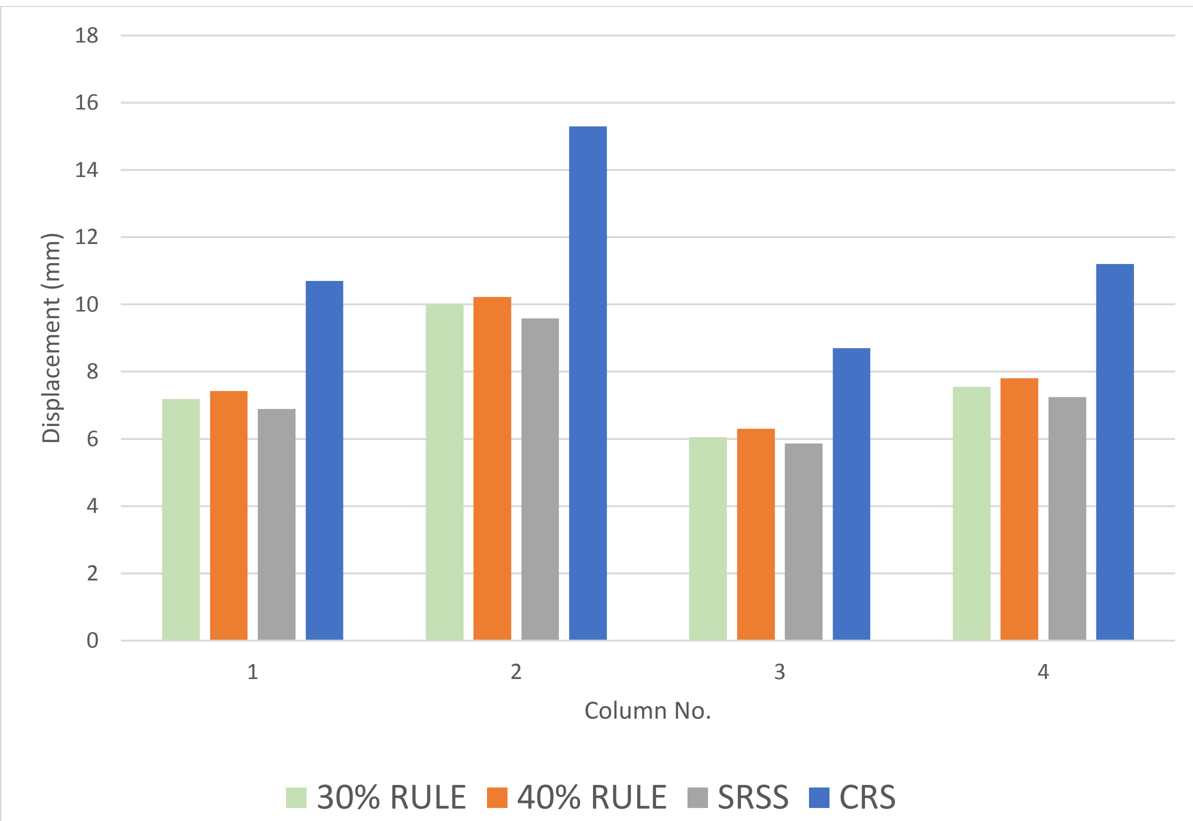

Fig. 5.11: Variation of displacement under various combination rules for Loma Prieta earthquake (in y-direction)

TABAS (IRAN) EARTHQUAKE

In x-direction:-

Table. 5.11: Displacements (in mm) for Tabas (Iran) earthquake (in x-direction)

COLUMN	X-COMP.	Y-COMP	30% RULE	40% RULE	SRSS	CRS
1	1.8	3.4	2.82	3.16	3.847	4.7
2	1.8	4.9	3.27	3.76	5.220	4.7
3	1.6	2.6	2.38	2.64	3.052	4.3
4	1.6	3.5	2.65	3	3.848	4.4

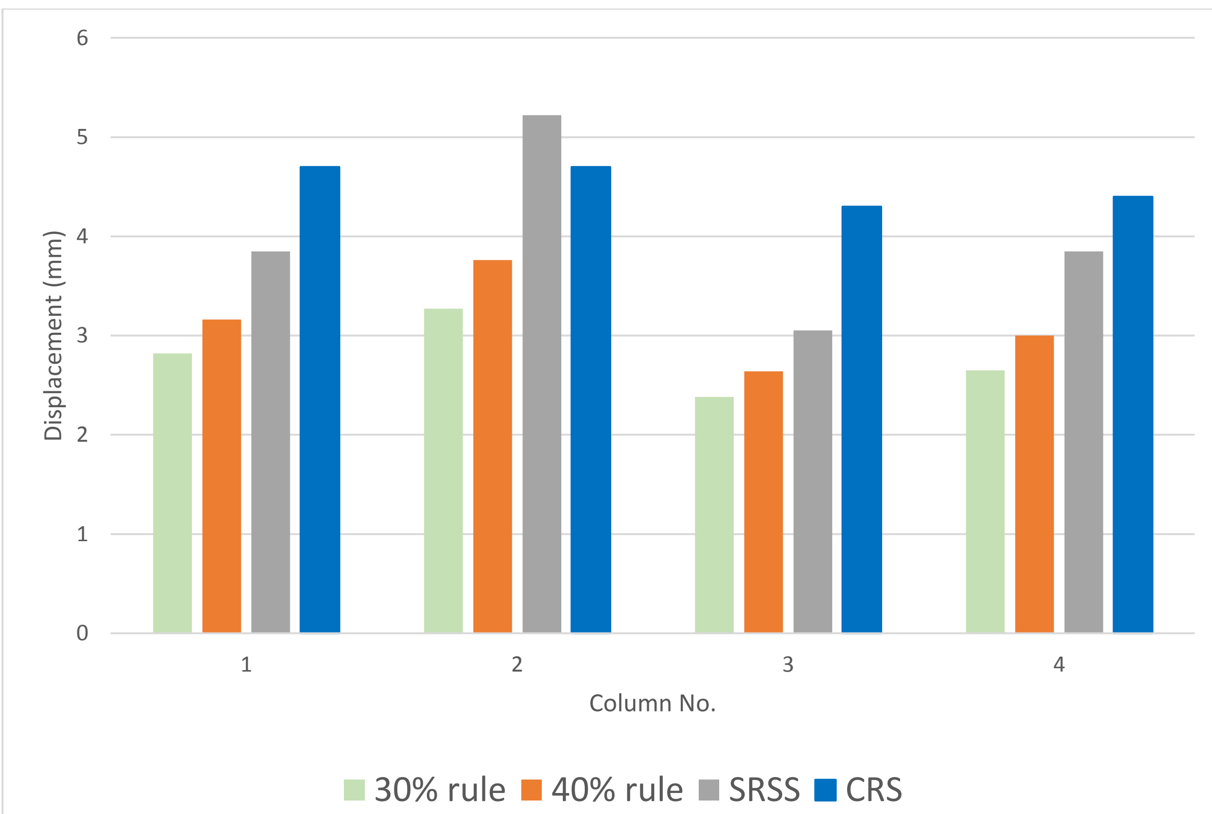

Fig. 5.12: Variation of displacement under various combination rules for Tabas (Iran) earthquake (in x-direction)

In y-direction:-

Table. 5.12: Displacements (in mm) for Tabas (Iran) earthquake (in y-direction)

COLUMN	X-COMP.	Y-COMP	30% RULE	40% RULE	SRSS	CRS
1	2.3	2.2	2.96	3.18	3.182766	5.8
2	3.2	2.2	3.86	4.08	3.883298	8.3
3	1.8	2	2.4	2.6	2.690725	4.6
4	2.4	2.1	3.03	3.24	3.189044	6.1

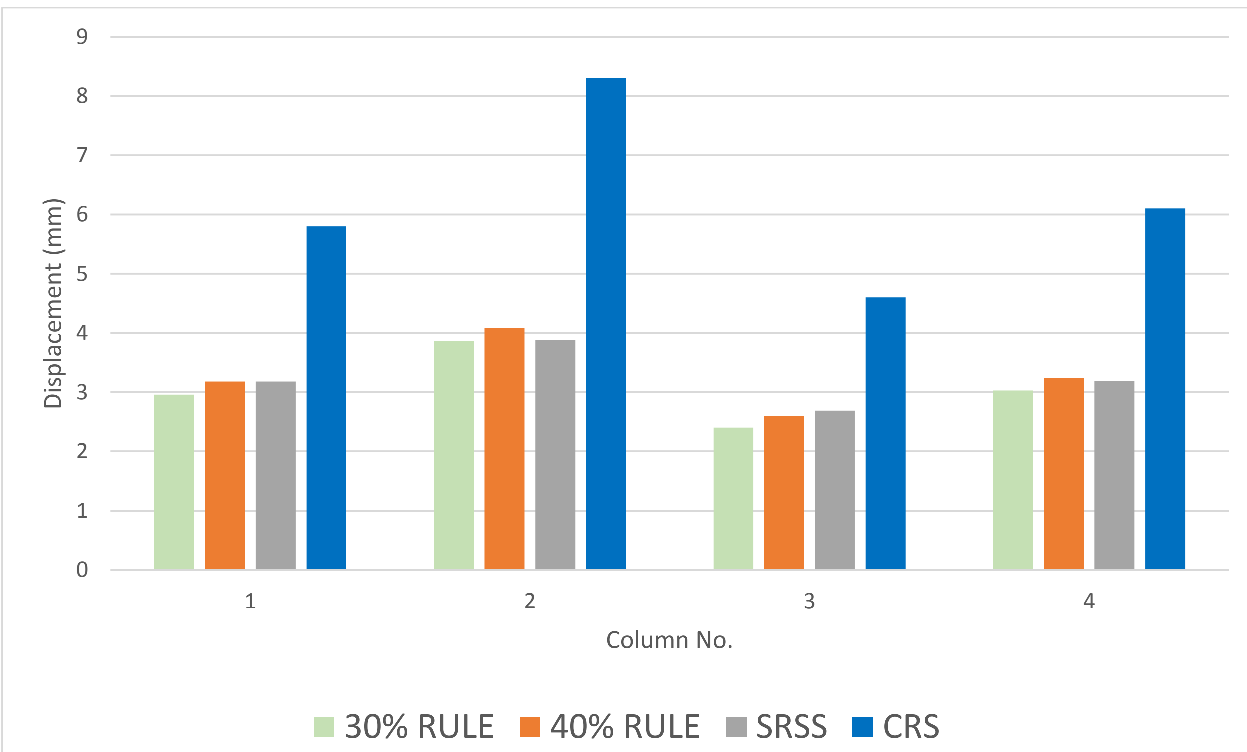

Fig. 5.13: Variation of displacement under various combination rules for Tabas (Iran) earthquake (in y-direction)

5.5 Results (Base Shear)

Table. 5.13: Base Shear (in kN) for different earthquakes

EQ.	X-COMP	Y-COMP	30% RULE	40% RULE	SRSS	CRS
KANGRA	0.943	1.927	1.5211	1.7138	2.145362	3.297
CHAMOLI	4.461	4.461	5.7993	6.2454	6.308807	16.783
CHAMBA	0.849	0.833	1.0989	1.1822	1.189407	1.903
LOMA PRIETA	13.544	19.648	19.4384	21.4032	23.86386	22.613
NORTHRIDGE	27.368	24.346	34.6718	37.1064	36.6297	45.332
TABAS	4.565	6.323	6.4619	7.0942	7.798689	12.586

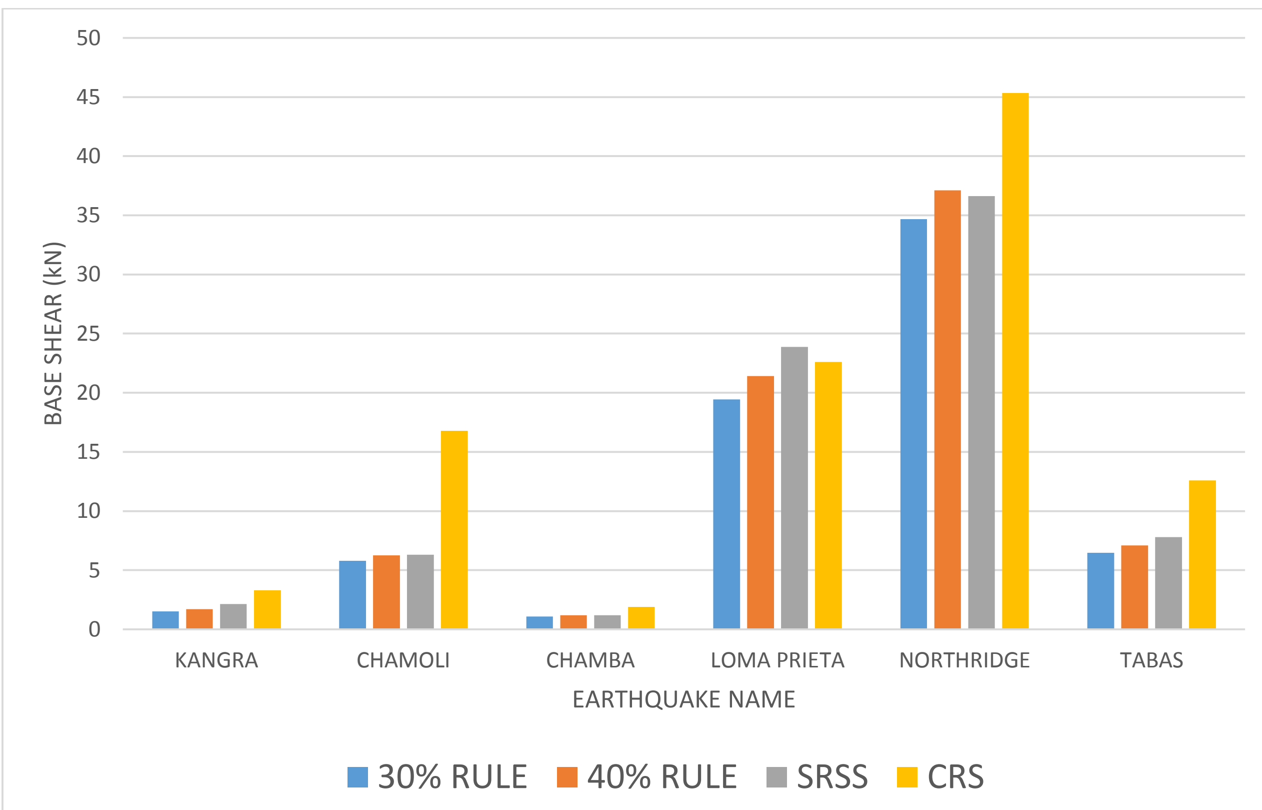

Fig. 5.14: Variation of Base Shear under various combination rules for different earthquakes

5.6 Discussion

As seen from the figures shown above, the maximum structural response is obtained by the critical response spectrum method. It can be concluded that the critical response spectrum method gives safe results and can be used for design purpose. The other methods are giving sufficiently smaller values which means these methods are underestimating the peak structural response as these methods are derived by using quite a large number of assumptions and simplifications. However, the critical spectrum method can be considered as a standard method having no assumptions involved.

In most of the cases discussed above, the critical response spectrum method gives the maximum value of the response except for the case of Column No. 2 in Loma Prieta earthquake (x-direction) in which the response of the critical spectrum is very close to the response given by SRSS method and the other exception is the case of the Column No. 2 in Tabas (Iran) earthquake in x-direction in which the response obtained from the critical spectrum method is lesser than the response of the SRSS methods.

If we consider the results for base shear, the observation concluded is that the base shear obtained by the critical response spectrum is also higher than what is obtained from the other methods. In this case also, the exception is the Loma Prieta earthquake in which the base shear obtained from the critical response spectrum method is lower than the value obtained from the SRSS method.

CHAPTER-6: SUMMARY & CONCLUSION

6.1 Summary

As we know that the distribution of the earthquakes is not uniform as the faults are randomly placed on the surface of the earth due to which the principal direction of the ground shaking is not constant and keeps on changing. Also, the orientation of the structures within a city is also variable, so the chances of the principal axis of the structure being in alignment with the principal direction of ground motion are very rare. However, it is accepted the maximum response will occur when the principal component of the ground motion acts along the principal axis of the structure. Since, this situation does not occur frequently, it is very difficult to find the maximum response of the structure. Moreover, at a particular instant, an earthquake motion consists of both the components but for convenience, we resolve it in two orthogonal directions. So, the exact response is composed of responses due to the effect of both the components of ground motion. In order to incorporate the effects of both component, responses due to individual components are combined in various ways. Most common of them are the percentage rules and the SRSS rules.

Many of the codes use 30-percent combination rules to combine the responses of two orthogonal ground motion components in which the resultant response in calculated by adding 30% of the response in orthogonal direction due to the response in orthogonal direction due to the response of the main direction. Other combination rules that are used generally are SRSS method, CQC method, RMS method, geometrical mean method. However, these methods have not been able to provide satisfactory results.

Hence, a new method for the calculation of maximum response has been proposed in this study which is known as the critical response spectrum method. In this method, a critical response spectrum has been developed by calculating the maximum time history response in x and y directions and then taking vector sum to these two responses to find the resultant response. The resultant response time history is then converted into a critical response spectrum which is believed to behave as an envelope function for the rest of the methods

Most of the earlier methods are based on the concept of combining the response spectra of two components of earthquake in one manner or the other. The most common way of combining the response spectrum is the geometric mean method. But, in this case, direct recorded earthquake data has been used. The critical response has been used to calculate the response of a sample structure which is basically a framed structure of four storeys.

In the context of the present study, it was desired to propose a method which will be able to calculate the critical structural response without being much dependent on the angle of incidence of ground motion, orientation of the structure and certain other assumptions. For this purpose, a study was made on how the critical responses are being calculated using different theories. The various combination rules that are used to combine the orthogonal structural responses of the structure were studied. After evaluating the existing combination rules for their merits and demerits, a new method known as critical response spectrum method was introduced.

Secondly, a four-storeyed concrete structure was selected and its response was calculated using the existing methods as well as the new proposed method. The results of the floor displacements and the base shear for the given structure were compared. On comparing the results, it can be seen that the maximum value of the response has been provided by the critical response spectrum method. The results of all the other methods were lower than the results of the proposed method. Hence, it can be concluded that the critical response spectrum method provides the ultimate maximum response also called as the critical response and its results can be satisfactorily used for the purpose of design.

6.2 Conclusion

In the analysis of the 4-storeyed building under the action of multiple components of earthquake and combination of the response by various combination rules, the following conclusions were drawn:

1. The various existing combination rules are based on certain assumptions and have limitations.
2. The different combination rules do not ensure the maximum response known as critical response for every condition.
3. A combination rule may providing maximum response in some condition need not yield maximum response under different conditions.
4. A seen in the analysis, the principal component does not provide maximum response in all the cases.
5. There is a need of some other method which can provide maximum response for all periods & frequencies of the structure.
6. The response obtained from such a method will ensure maximum response of the structure for every period or frequency of the building.

In order to find the critical response of the structure, a new method is proposed which will ensure the critical response of the structure for any period of the building.

The new proposed method was formulated and was used to find the response of the given structure.

It was seen that the value of the response provided by the critical response spectrum method was maximum for almost all the cases. This means that it is safe to design the structures based on the critical response spectrum as it will ensure a satisfactory design.

6.3 Future Work

- The main objective of the project was to find a method by which a critical response of a MDOF system can be calculated by performing analysis in only one direction.
- The objective has been fulfilled by the Critical Response Spectrum Method.
- However, the critical response spectrum method can be further used for the calculation of critical response for:
    - Asymmetrical Structures
    - Curved Structures
- Since, the earthquake records are not available for each and every site. Thus, a critical design spectrum can be developed by developing the attenuation relationships for a given location.

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