ANALYSIS OF CARBON NANOTUBES REINFORCED FUNCTIONALLY GRADED NANO PLATE

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree of

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MECHANICAL ENGINEERING

(With Specialisation in Machine Design Engineering)

By

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CANDIDATE'S DECLARATION

I hereby declare that the work carried out in this dissertation titled "***Analysis of Carbon Nanotubes reinforced Functionally Graded Nano Plate.***" is presented on behalf of partial fulfillment of the requirement for the award of the degree of **Master of Technology** with specialization in **Machine Design** submitted to the department of **Mechanical & Industrial Engineering, Indian Institute of Technology Roorkee, India**, under the supervision and guidance of **Dr. S. P. HARSHA**, Associate Professor, MIED, IIT Roorkee, India.

I have not submitted the matter embodied in this thesis for the award of any other degree or diploma.

Date: May 2016

Place: IIT Roorkee

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CERTIFICATION

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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ABSTRACT

Functionally Graded Material (FGM) comes under category of advanced materials, where properties of material varies in specified direction. Overall properties of FGM is different from their parent materials.

Carbon Nanotubes (CNTs) are strongest material with lesser weight known till now. From this directly structures cannot be developed due to technology limitations. So these properties are utilized by mixing CNTs in different materials.

In this work, CNT reinforced FGM is used. Matrix material is aluminum considered as CNT reinforced Al FGM have lots of engineering applications. CNT reinforced FGM nano plate is considered where CNT volume fraction varies linearly 2% to 0% from top to bottom surface.

Effect of CNTs on FGM have been evaluated using mechanical, thermo mechanical and mode shape analysis. Deflection results for mechanical and thermo mechanical conditions, mode shapes with their natural frequencies has been compared with Al nano plate without CNTs, CNT reinforced Al composite nano plate. Orientation of CNTs are varied from 0 deg to 90 deg in step of 15 deg to find effect of CNTs orientation on FGM plate under mechanical, thermo mechanical conditions and natural frequencies with their mode shapes.

It was observed from the analysis that under mechanical, thermo mechanical and free vibration condition, Al nano plate without CNTs is weakest, while CNT reinforced Al FGM nano plate is slightly weaker then CNT reinforced Al composite nano plate under mechanical loading, while it is reverse in thermo mechanical loading. Similar pattern was observed for natural frequencies. In CNTs orientation study, it was observed that as the orientation varies from 0 deg to 90 deg, strength decrease for mechanical loading, while in thermo mechanical loading theses effects were negligible.

It was observed through this analysis, that CNT reinforced composite is still better option for good strength mechanical application, but FGM is better option where elevated temperature exists. All these observations will help to design FGM plate for specific applications.

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NOMENCLATURE

FGM	Functionally Graded Materials
CNT	Carbon Nanotubes
Al	Aluminum
PM	Powder Metallurgy
Nm	Nano Meter
GPa	Giga Pascal
CAD	Computer Aided Design
CVD	Chemical Vapor Deposition
PVD	Physical Vapor Deposition
STL	Standard Triangulation Language
RPT	Rapid Prototyping
BIAD	Beam Assisted Ion Deposition
SWCNT	Single Wall Carbon Nanotubes
MWCNT	Multi Wall Carbon Nanotubes
FEM	Finite Element Method
MLS	Moving Least Square
EFG	Element Free Galerkin
FSDT	First Order Shear Deformation Theory
TSDT	Third Order Shear Deformation Theory
MLPG	Mess less Local Petrove Galerkin
ECO	Ethylene Carbon Polymer
HOT-SMAC	Higher Order Theory Structural / Micro Analysis Code

σ	Stress
ε	Strain
E	Modulus of Elasticity
v	Poisson’s Ratio
α	Co-efficient of Thermal Expansion
ΔT	Temperature Difference

CHAPTER 1: INTRODUCTION

Materials play an important role for the development of human race. Technologies come to an end, where material requirements are not meet. Therefore a lot of research work is going on for development of new materials to improve human capability. After exhausted study of materials behavior, scientists are able to develop new materials, example: composite materials with well-known mechanical and its physical properties.

In 1980, during the development of space research programme, a Japanese scientist came across a problem when he required a material having sufficient mechanical strength in the thickness of 10 mm and could sustain outside temperature of 2000 K and inside temperature of 1000 K. For resolving this problem, he developed a new material called Functionally Graded Material.

1.1 Type of Materials

Materials are classified as follows

- Pure Metals
- Alloy Metals
- Powdered Metallurgy
- Composite Materials
- Functionally Graded Materials

Pure Metals

Pure metals have very limited application in engineering field due to conflicting property requirements. For an application if we require a material which should be hard as well as ductile, no pure metal available in world for these requirements.

Alloy Metals

To overcome this problem, alloy metals came into existence, where one metal is combined with another metal or nonmetal in molten state. It provides different metal properties to original material. For example, bronze alloy is combination of tin and copper. It is considered as first alloy which was used in Bronze Age, around 4000 BC.

Mixing of two metals which are having different melting temperature cannot be combined to have properties of alloys. Due to this restriction, alloy metals have limited applications to engineering solution.

Powdered Metallurgy (PM)

Powder metallurgy combines two metals which are having different melting temperatures. Here two different metals at powder stage are used to sinter. Materials produced by this process are also called as conventional alloy. But through this process, material properties like alloy, were not able to achieve.

The parts produced through this process were porous and were not having good mechanical strength. In some applications this limitation is advantageous like filter and nonstructural application.

Composite Materials

Different materials are amalgamated in their solid state for combining different material properties. It comes under advanced class of materials. It offers good mechanical properties along with light weight. Wood is an example of natural composites. Nowadays, it has lots of engineering applications.

It has some limitations under extreme conditions like separation of fiber from matrix. For example, under high temperature, separation of two materials which are having different melting temperature.

Functionally Graded Material (FGM)

Functionally graded material comes under advanced materials category. It exhibits changing mechanical property in a defined direction. FGM is available in nature as teeth, bones etc. Here outer surface is hard and keeps on getting softer to inside surface. Idea for development FGM was taken from natures to solve complicated engineering applications.

FGM is developed by mixing of two materials by changing their materials percentage in a particular direction. It is designed and developed based on specific requirements of that application.

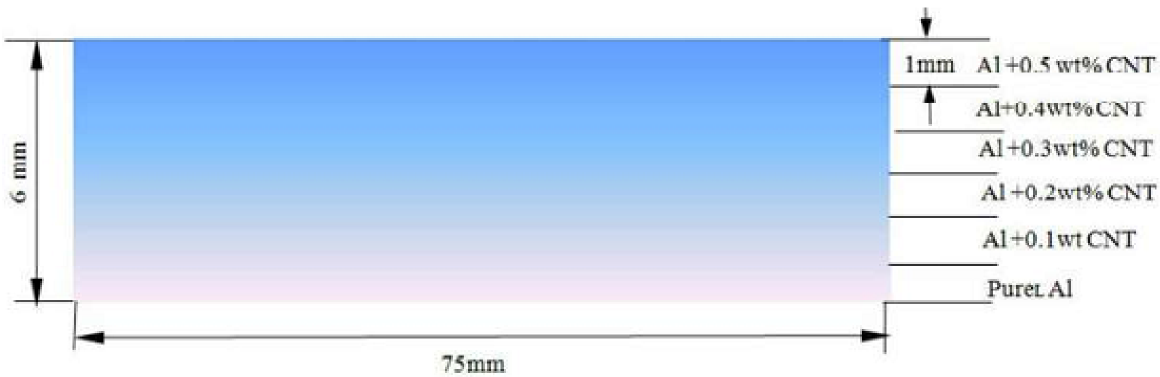


Fig.1.1: Schematic diagram of CNT reinforced FGM [1]

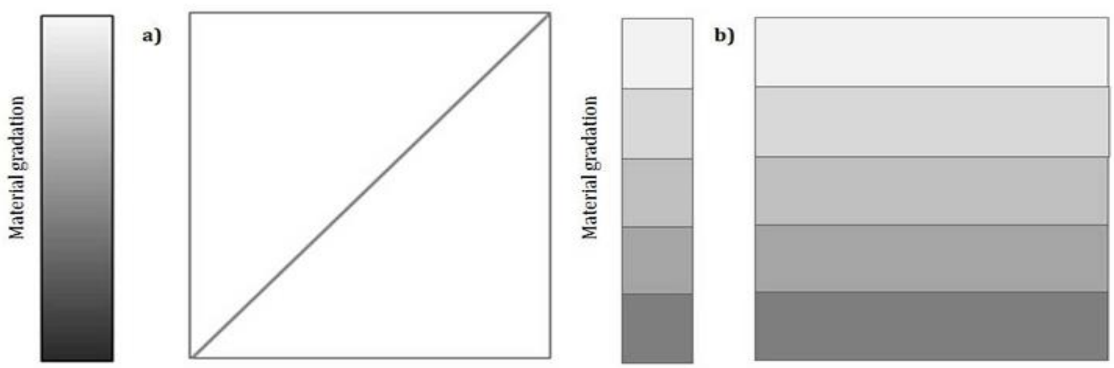


Fig.1.2: Schematic diagram of concept of Gradient [2]

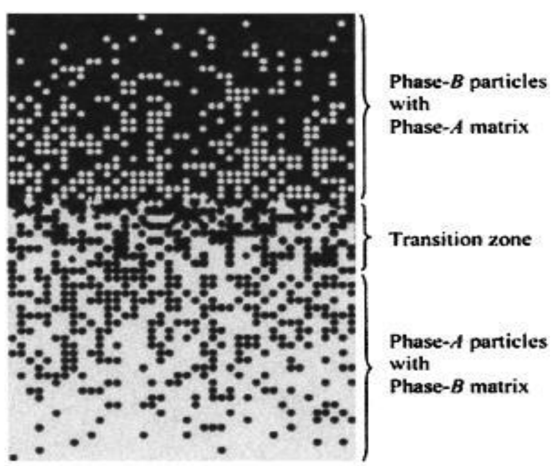


Fig.1.3: Schematic of FGM [2]

1.2 Classification of FGMs

FGM has been classified by many ways, [1]

As per their material combinations, it is classified as:

- Metals & ceramics
- Ceramics & ceramics
- Ceramics & plastics and
- Different materials combinations.

As per their changing compositions, it is classified as:

- Bulk FGMs (varying though out the complete direction)
- Coating FGMs, (vary for particular thickness)
- Connection FGMs (at particular connection only)

As per its varying density, it is classified as:

- Varying property FGMs,
- Composition FGMs,
- Optical FGMs,
- Fine FGMs

And as per application bases, it is classified as:

- Thermal FGMs,
- Medical FGMs,
- Chemical FGMs,
- Electronic FGMs etc.

Table 1.1: FGM types and applications [1]

Sl. No.	FGM Type	Property	Application
1	SiC – Al Alloy	Thermal Resistance and chemical Resistance	Diesel Engine Cylinder, CNG Storage Cylinder
2	Al - SiC	Thermal Resistance & toughness	Combustion Chamber
3	Al Alloy – CNT	Light weight and stiffness	Artificial ligaments, MRI scanner cryogenic Hip Joints,
4	Al - Zirconium	High temperature resistance and high strength	Space Aircraft nose, Aircraft nozzle
5	Ti Alloy- SiC	Chemical resistance, light weight and high strength	Scuba diving cylinders, Oil industries piping

1.3 Advantages and Challenges of FGM

FGM has advantages over other materials, but have some limitation too. That is why it is used for specific applications only.

Advantages

- One of the best advantages of FGM is multi functionality.
- Ability to design for required deflection, corrosion, wears, for different environment.
- Reduced stress concentrations
- Ability to take benefits of different materials.

Challenges

- Mass production of FGMs
- Quality control
- Reduction in cost of FGMs

1.4 Processing of FGMs

There are two types of productions for FGM, one is coating type and other is bulk FGM. Both have different type of processing techniques; few of them are discussed here. [2]

Processing through Deposition

In this technique deposition is performed through vapour. It is mainly for thin FGM. There are physical and chemical depositions through vapor, which are knowns as PVD (physical vapour deposition and CVD (chemical vapour deposition). Sputtering is also a technique used for deposition.

Above techniques for deposition produces dangerous gases as byproduct. It also consumes high energy in these processes.

Other techniques for producing thin FGMs are:

- Plasma spray Techniques
- Electrode Coating,
- BAID (Beam Assisted Ion Deposition),
- High-temperature Self-Propagating Synthesis, etc.

The techniques mentioned above are only for thin FGMs productions, not for bulk FGM. Mass production of FGM requires high energy consumption along with time consumption. So for bulk production of FGMs, these techniques will not be economical.

For bulk production of FGM, following techniques are used.

Powder Metallurgy Technique

Powder metallurgy is used for mixing two different materials. So for FGMs production through powder metallurgy, is done by mixing two different material in their varying weighted ratio in particular direction as per design requirement is done. Then it is sintered to form a part.

This technique is good for bulk production of FGM. Through this technique, FGM is developed in the designed part shape only. It has same limitation as of powder metallurgy products.

Centrifugal Casting Technique

This technique is more like centrifugal casting techniques. Centrifugal force is developed spinning the mold of the part. Varying weighted mixture of two materials is supplied to develop FGMs.

Through this technique, fast and bulk production is easily achieved. Limitation of this technique is that only cylindrical FGMs can be produced.

Rapid Prototype FGM Processing technique

Lot of research work is going on this technique, as it is a fast and an accurate method for the development of FGM. It is almost same as RPT production technique, only two material mixture weighted ratio varies layer to layer. Advantage of this technique is that input is CAD model and two materials. As per design program, it will mix as per design input and will develop product like a 3D printer. Research is going on for development of this kind of machine with reduced cost.

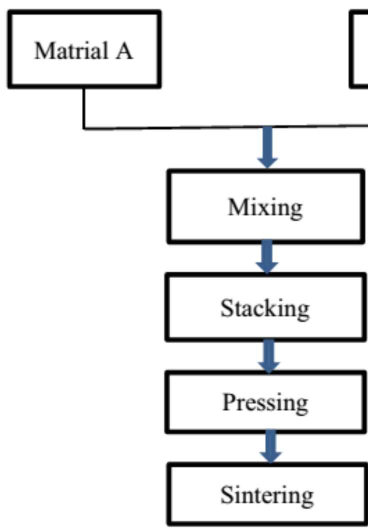


Fig.1.4: RPT processing flow chart [2]

Basic steps involved in this technique:

- Development of CAD Model
- Transferring date from CAD model to Standard Triangulation Language (STL) file,

- Dividing STL in multi layers as 2D.
- Development of part layer wise
- Final finishing and taking out part.

Till now this technique is very costly for FGM production. Therefore this processing technique is used only in costly products and application, like sports car, aerospace etc.

Table 1.2: Overview of processing methods [2]

Sl. No.	Process	Variability of transition function	Versatility in phase content	Type of FGM	Versatility in components geometry
1	Powder stacking	Very Good	Very Good	Bulk	Moderate
2	Sheet lamination	Very Good	Very Good	Bulk	Moderate
3	Wet powder spraying	Very Good	Very Good	Bulk	Moderate
4	Slurry dipping	Very Good	Very Good		Good
5	Jet solidifications	Very Good	Very Good	Bulk	Very Good
6	Sedimentation / centrifuging	Good	Very Good	Bulk	Poor
7	Filtration / slip casting	Very Good	Very Good	Bulk	Good
8	Laser cladding	Very Good	Very Good	Bulk, Coating	Very Good
9	Thermal spraying	Very Good	Very Good	Coating, Bulk	Good
10	Diffusion	Moderate	Very Good	Joint, Coating	Good
11	Directed solidification	Moderate	Moderate	Bulk	Poor
12	Electromechanical gradation	Moderate	Good	Bulk	Good

1.5 FGM Applications

FGMs have lots of application in engineering field. Few of them are discussed here.

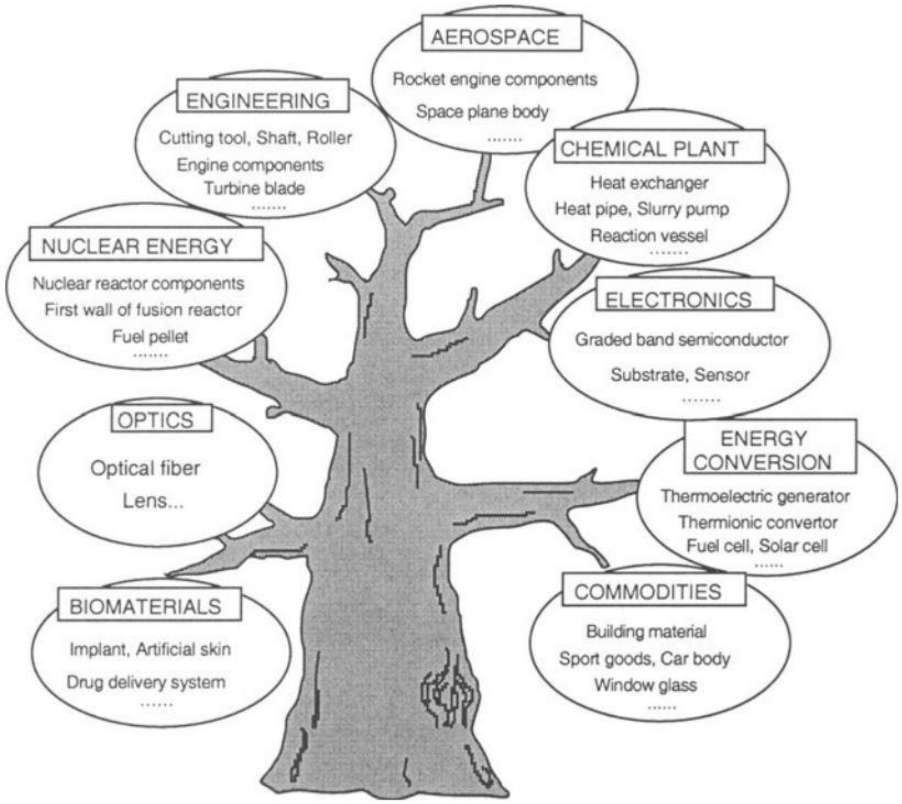


Fig.1.5: FGM application tree [2]

Aero-space

First FGM was developed for this application only. Due to its good mechanical properties under high temperature make it most suitable for aero-space applications. Over the time, development of FGM, it is used in development of space craft body, rocket engine etc. [3]

Fig.1.6: FGM part for aerospace application [3]

Chemical Plant

Due to high temperature resistance and corrosion resistance properties, FGMs have application in chemical industries, as a heat exchanger, heat pipe reaction vessels etc.

Electronics

FGMs are used for development of graded band semi-conductors, sensors etc. [4]

Energy Conservation

It is used in thermo electric generator, fuel cell, solar cell etc. [5]

Commodities

Due to light weight, high stiffness, are used in sports goods, car body, window glasses etc.

Medical

Natural FGMs are like bones, teeth etc. So for replacement, ideal material for solving purpose of these is, FGMs. So in medical field, FGM is used as replacement of bones and teeth. [6]

Fig.1.7: FGM parts for hip replacement [3]

Defence Equipment

In defence application, FGM used to develop armor body of equipment, bullet proof jackets, due to its property that doesn't allow crack to propagate. High strength and light weight properties are very useful for the application of defence equipment.

Energy

Due to its good mechanical property under high temperature, FGM are most suitable in the field of energy. In nuclear power plant, it is used in thermal barrier structures due to its high thermal resistance. [7]

In gas turbine engines, turbine blades are FGM coated that provides thermal barrier.

Optics

Due to its high reflectivity properties along with mechanical strength, it is used in optical fiber, lenses etc.

Engineering

Due to high hardness, good mechanical strength and light weight properties it is used in cutting tools, shaft, rollers, engine components, turbine blade etc.

The application list is increasing day by day due to new improved properties and increase production techniques with reduced cost and high accuracy.

1.6 Carbon Nanotubes (CNT)

Carbon nanotubes are rolled up sheets of graphene. It is a cylindrical nano structure of allotrope of carbon atoms. Carbon nanotubes are the strongest and stiffest materials yet discovered in terms of tensile strength and elastic modulus respectively. [8]

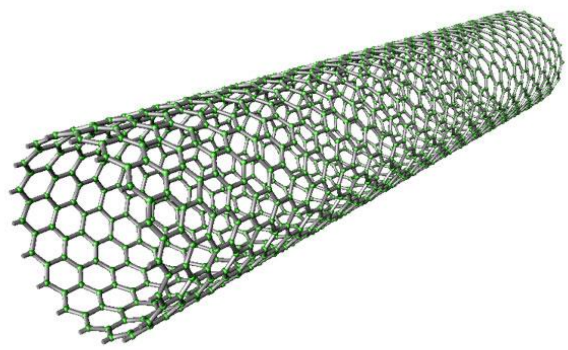


Fig.1.8: Schematic of Nanotube [8]

Carbon naontubes' modulus of elasticity can be upto 1000 GPa, almost 5 times higher to the steel metal. It has approximately 63 GPa tensile strength, 100 times more to steel. All these properties are there along with the density of around 1300 kg/m^3, almost 6 times lighter than steel and half of the Aluminium. [9][10]

Other good mechanical properties it has good thermal conductivity around 3500 $Wm^{-1}K^{-1}$, which is almost 10 times of that copper.

After having these good mechanical properties, still we cannot develop structures, parts etc from this, due to limitation of technology. So these properties are utilized this with mixing in other materials. [11]

CHAPTER 2: LITERATURE REVIEW

Rasheedat M. Mahamood [2] presented over view of Functionally Graded Material (FGM). As per his paper Functionally Graded Material (FGM) comes under advance material class. A wide range of applications and processing techniques are discussed. He discussed about some recent developments in processing techniques of FGMs to reduce production cost and increase quality.

Praven and Reddy [11] studied behavior of metal ceramic FGM plate under non-linear thermo-elastic condition. Using power law function, volume fraction of metal ceramic plate was constituted and dynamic and static response was studied. Temperature response on the plate is discussed. He find out that FGM plate response was not in the range of ceramic and individual metals plate.

Bhavani VS, Jerome TT [5] carried out thermal analysis of FGM beam**.** Stress distribution was obtained by solving close form of thermal equilibrium equations. Exponential law function was used for variation of temperature and elastic constant along the thickness of FGM beam. In this study, two cases are consider, first one where elastic constant variation are same as of temperature variation, and in other one, opposite to each other. He concluded for case one thermal stresses are more compare to case two.

Shyang-Ho C, Yen-Ling C [10] carried out mechanical analysis of FGM plate for transverse loading. A simply supported rectangular FGM plate of moderate thickness, having fix poison ratio and varying elastic constant along the direction of thickness using power law, sigmoid law and exponential law function. Classical plate theory was used for numerical results. Numerical and FEM results are compared for each power law function.

M.K. Singha , T. Prakash , M. Ganapathi [12] also analysed FGM plate for transverse loading using FEM. Simple power law function is used for material to be graded along the thickness. Using the rule of mixture, effective values of properties are find out. He find out that for isotropic material, stresses are linear while for FGM it is highly nonlinear.

Ming Liu, Yuansheng Cheng, Jun Liu [13] considered a different type of FGM sandwich functionally graded less stiff core, and functionally graded top and bottom plate. He carried out high order mode shape analysis. Material is graded using simple power law function. Hamilton principal is used for equations of motions. Results are validated from FEM results. It was observed a good matching of the results.

Qian LF Batra RC, Chen LM [14] used classical plate theory and higher order shear plate theory for deformation of FGM thick plate under thermo mechanical environment. Upper surface is loaded with thermal as well as mechanical load. They concluded that there is 1/3rd maximum deflection of the plate for clamped plate compare to simply supported plate and 40% increase in the stress at the center of the upper surface of the plate**.**

Alshorbagy E, Alieldin SS, Shaat M, Mahmoud FF [15]. carried out deformation of FGM plate under thermo mechanical loading condition using finite element method. FGM plate shows very good mechanical strength under thermal environment, which was not there in isotropic materials. It shows very continuous variation of stress and strain too.

H. Li, J. Lambros, B.A. Cheeseman, M.H. Santare[16] carried out experiments on FGM having a crack to study crack growth in FGM. He developed FGM specimen from ethylene carbon polymer (ECO). ECO have degradation in property under UV light, which has been used for development of FGM. ECO keep on getting stiffer and stronger over exposer time of UV. So speciemen stiffness is varied in one direction during production by varying exposer time of UV to be it as an FGM. Single edge notch test has been carried out on ECO FGM and ECO isotropic material. They observed, fracture toughness increase during crack growth in ECO FGM which was not there in isotropic ECO.

Butcher RJ et. al. [17] measured spatial gradation of young's modulus in functionally grade materials using optical interferometry. Optical measurements were used to extract parameters and results were compared with finite element computations. The advantage of using FGM as opposed to homogeneous joints was also demonstrated.

Cheng and Batra [18] obtained closed form analytical solution of a thermo-elastic elliptic linear plate which is clamped rigidly from the mid-plane. Asymptotic expansion method was used for deformation under thermo mechanical loading. The asymptotic formulations were formulated in the form of the transfer matrix and validated the asymptotic expansion to any desired degrees of numeric accuracy.

Craig et. al. [19] developed Higher Order Theory Structural / Micro Analysis Code (HOT-SMAC) software tool for FGM analysis. Higher order theory was used in this software. It comes with HOTFGM a thermo-mechanical analysis engine. They compared the results with the commercially available FEA codes (e.g., ABAQUS, ANSYS, NASTRAN) and concluded that the accuracy of the codes vary with the approaches.

Livia M and Daniel L [20] investigated the 2-D steady state heat conduction problem for both isotropic, anisotropic, single, composite and non-linear FGMs with the continuity conditions for the thermal environment. They used method of fundamental solutions (MFS) and MFS results found approaching toward and exact solutions.

Fekrar A et. al. [21] used simply supported homogeneous and inhomogeneous FGM plates. The critical buckling temperature was analyzed. For thick plate of high aspect ratio, it was observed that critical buckling temperature is governed by transverse shear deformation as well as material gradient index.

Ferreira et. al. [22] analysed FGM plate using higher order shear deformation theory. He took collocation method to find deformation in the plate. It was concluded that the collocation method took quite less time to solve the problem as compared to MLPG mainly because this scheme did not require any numerical integration. The results were compared with those obtained from the mesh-less local Petrove-Galerkln (MLPG) code and were found to agree.

Kl-Hoon [23] introduced an approach which is based on FEA for FGM objects design. It optimizes the material distribution and geometry both during design. Having modeled and meshed the FEA based software (ANSYS) was used to perform linear elastic analysis to find the stress. Satisfactory results were there after repeating FEA design cycle.

Alieldin et. al. [24] proposed three approaches to find the properties of an FGM plate equivalent to the laminated original composite plate. Computational effort of the elasto-static analysis of that FGM plate is much less than that required for the analysis of the original laminated one.

Mostapha et. al. [25] analysed FGM plate under thermal loading. They derived closed form solution of the FGM plate for critical buckling temperature. For laminated composite plate, stability equations and equilibrium equation were found similar. It was also reported that as aspect ratio increases, critical buckling temperature also got increased, while it is lower for homogeneous plates.

Srinivas G et.al. [26] studied the behavior of FGM plate under thermo-mechanical loading with specific boundary conditions. Convergence studies have also been conducted to check the accuracy of the formulation. Under thermo-mechanical loading, the variation in stress and deflection have been reported. It was also observed that the transverse displacement of the plate under thermal load changes greatly.

Suresh S and Mortensen A [27] investigated the thermo-mechanical deformation of FGM plate. They computed the strains and stresses from experimental measurements of shifts in lattice spacing and also compared with the stress calculated form the known elastic methods. Processing induced stresses have also been measured using the “thermal mismatch stresses" technique.

Vanam et. al. [28] developed FE model considering FSDT plate theory and minimum potential energy principal. They formulated the boundary value problem of the thermo-elastic behavior of FGM plate. Also they investigated the problem considering exact neutral plane position. It was reported that neutral plane of the FGM plate is shifted towards the surface with higher Young’s modulus of the two plate constituents.

Dai et. al. [29] analyzed the FGM plate with the help of mesh-free element free Galerkin (EFG) method for mechanical loading condition. Element connectivity was not required using this method. For constructing the shapes, moving least square (MLS) method was used by them. It was observed that deflection and volume fraction was well differ for two different loading conditions.

Ashraf and Daoud [30] used sinusoidal shear deformation plate theory for deriving the equilibrium and stability equations. For uniform, linear and non-linear thermal loading, thermal deflection analysis was carried out and conclude that critical buckling temperature difference is directly linked to plate aspect ratio. It was observed that effect of temperature for buckling is more for thicker plate compare to thinner plate. Critical buckling temperature was found low in FGM compare to homogeneous material.

Nguyen et. al. [31] coupled FEM and mess free method to develop an efficient element called NS-DSG3. It was observed that it can be used for thick as well as thin plates. They applied this method for various dynamic problem, mechanical problem and thermal bucking problem of FGM plates. Under mechanical and thermal loading, FEM plate behavior studied using few benchmark problems.

Zhu P, Lei ZX, Liew KM [32] used thin to moderate thick CNT reinforced FGM plate. He analyze static and free vibration behavior using first order shear deformation theory. Using FEM, due to CNTs volume fraction, bending, mode shapes with their natural frequency was studies. He concluded that distribution of CNT near to top and bottom faces is more responsible than mid plate for plate stiffness.

Wattanasakulpong N, Ungbhakorn V [33] carried out analysis of CNT reinforced beam for bending, buckling and vibration using analytical techniques. They concluded that for displacement and normal stress results are very much similar for different shear deformation theories. But for shear stress, higher order shear deformation theories fond better.

Alibeigloo A, Liew KM [34] carried out thermo mechanical bending behavior of simply supported rectangular CNT reinforced plate. They used three dimension elasticity theory. They concluded that volume fraction of CNTs plays an important role under thermo-elastic behavior. It is significant for top surface under thermo-elastic environment.

Mehrabadi SJ, Sobhani Aragh B [35] used CNT reinforced FGM cylindrical shell. Under mechanical loading, stresses due to bending behavior were studied. They concluded that at inner surfaces, circumferential and radial stresses decreases due to volume fraction of CNTs. As volume fraction of CNTs keep on decreasing, its effect keep on getting less significant.

Wang ZX, Shen HS [36] used CNT reinforced FGM plate supported on elastic base. They carried out non-linear vibration analysis, due to large amplitude of vibration under thermal conditions. They concluded that due to increasing temperature of elastic foundation base, stiffness decrease, so natural frequencies decreases while increase in non-linear to linear frequencies ratio.

Yas MH, et. al. [37] used single wall carbon nanotubes reinforced cylinder. They carried out static and vibration analysis using three dimension of theory of elasticity. They concluded natural frequencies was high for orientation of CNTs at $\Phi = 2\pi$ & $\pi/6$.

Heshmati M, Yas MH. [38] used multi wall carbon nanotubes reinforced polystyrene FGM beam. They analyses dynamic response of beam, under multi moving load conditions. The authors used Timoshenko beam theory and concluded that due to forced vibration via three dynamic loads, dynamic displacement of beam was smother for increasing velocity.

Rafiee M, Yang J, Kitipornchai S [39] used CNT reinforced FGM beam bonded with piezoelectric layers. They carried out nonlinear thermal bifurcation buckling. They discovered that due to intermediate CNT volume fraction, buckling temperature is not intermediate in some cases and observed increase in buckling load for beam.

Lei ZX, Liew KM, Yu JL [40] used CNT reinforced FGM plate. Under mechanical load, buckling analysis of FGM plate was carried out. The author used the element-free-kp-Ritz theory. They estimated the effective properties of the SWCNTs reinforced FGM plate using micro mechanical model.

Shen HS, Zhang CL [41] used SWCNTs reinforced cylindrical shell. They carried out thermal post buckling analysis under a uniform temperature rise. They concluded CNT plays an important role, as CNT volume fraction increases buckling temperature strength increases along with thermal post buckling strength too.

Ansari R, Faghih Shojaei M [42] used SWCNTs reinforced nano FGM beam. They studied behavior under forced vibration. They used Timoshenko beam theory and von kaman non-linearity in geometry. Hamilton principal was used for driving equations. To reduce non-linear governing equations, they used Galerkin based numerical method.

Shen HS [43] used CNT reinforced cylindrical shells. For analysis of torsional post buckling, under thermal environment, author proposed multiscale model. He found that the structure of shell is weakly stable for torsional post buckling and virtually insensitive.

Most of the research work has been carried out related to analysis of FGM under mechanical loading, thermo mechanical loading and free vibration conditions. People have taken FGM as a plate, cylinder etc for analyzing. Most of the people used various theories for analyzing FGM. People carried out analysis using commercial available software and few of them developed code based on various theories, for FGM analysis.

They considered continuous variation using laminated techniques in which the properties of each layer varies uniformly, and a particular layer have uniform set of properties within it. But in real case, the property distribution is not uniform in a particular layer, but for simplicity average of these properties are taken. This is one of the area which need to be focused.

CNT reinforced FGM is the latest area where a lot of research is going on due to its wide area of applications in aerospace, medical, commodities etc. Effect of CNT on FGM, need to be studied for designing FGM, which needs to be analysed. CNT reinforced FGM to be analyzed and results to be compared with base material plate as well as same materials composite to see effectiveness of CNT reinforced FGM. Effect of CNT's orientation also to be studied in FGM. All above need to be analyzed under mechanical loading, thermo mechanical loading & free vibration conditions.

CHAPTER 3: PLATE THEORY & GOVERNING EQUATIONS

3.1 Plate Theory

There are three major plate theories, which has been used for solving most of the problem. These theories are used based on the accuracy required for the result and computation cost. These theories have been explained below.

Kirchoff's Plate Theory

Kirchhoff's plate theory, is also known as the classical plate theory as it is the simplest and oldest plate theory. This theory considers that straight line, which is normal to the mid plane will be straight and normal to the mid deformed surface and don't go under any stretching in thickness direction, so these assumption neglects transverse shear as well as transverse normal strains. [10]

As per this theory, displacement field equations are:

$$u(x, y, z) = u_0(x, y) - zw_{0,x}$$

$$v(x, y, z) = v_0(x, y) - zw_{0,y}$$

$$w(x, y, z) = w_0(x, y) \quad \text{..............3.1}$$

Where (u_0, v_0, w_0) are displacement at the mid surface point and 'z' is the distance from the mid surface.

The major constrain of using this theory is that it neglects the transverse shear effect, which considerable gives reasonable results for very thin plates. So the classical plate theory will be less reliable for FGM plate analysis.

The First Order Shear Deformation Theory (FSDT)

It is also known as the Reissner Mindlin theory, is considered improved version of classical plate theory. This theory consider the effect of transverse strain which was neglected in earlier theory.

It consider transverse shear strain by allowing the normal to rotate. To overcome, problems related to constant transverse shear stress distribution, a shear correction factor is required. It is find out by comparison of results with the exact results. It involves various parameter like boundary conditions, loading conditions and geometry details etc. [24]

In a functionally graded material (FGM), as material properties continuous variation, due to continuous variation of material properties in a direction, FSDT is efficient for the analysis.

The theory considers transverse shear strain with the following assumptions:

- The transverse normal remain straight after deformation but may not be orthogonal to the mid-surface of the plate.
- The out-of-plane normal stress is equal to zero.
- The layers of the composite plate are perfectly bonded.

As per this theory, displacement field equations are:

$$u(x,y,z) = u_0(x,y) + z\emptyset_x(x,y)$$

$$v(x,y,z) = v_0(x,y) + z\emptyset_y(x,y)$$

$$w(x,y,z) = w_0(x,y) \quad \text{..............3.2}$$

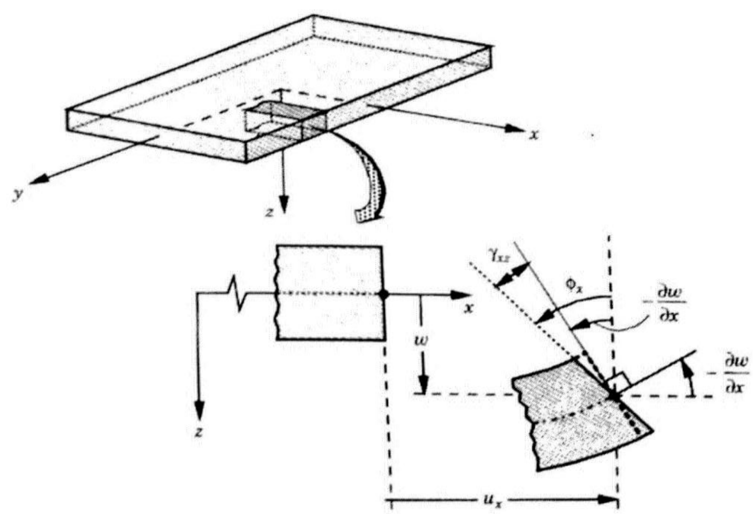


Fig. 3.1: Edge of plate under deformed condition [24]

Where $(u, v, w, \emptyset_x, \emptyset_y)$ are unknown function to find out, (u_0, v_0, w_0) are displacement of a point on the mid surface and 'z' is the distance from the mid surface i.e. of a point on the plane z=0 in the x, y , z direction respectively. $\emptyset_x, \emptyset_y$ is the rotation of transverse normal to the Y axis and $\emptyset_x, \emptyset_y$ is the rotation of transverse normal to the X axis.

The Third Order Shear Deformation Theory (TSDT)

It eliminates the requirement of shear correction factor, which was required in FSDT. As per this theory, mid point plane displacement also considers cubic to the thickness and out of plane displacement. [14]

As per this theory, displacement field equations are:

$$u(x,y,z) = u_0(x,y) + z\emptyset_x(x,y) + z^2\emptyset_x(x,y) + z^3\emptyset_x(x,y)$$

$$v(x,y,z) = v_0(x,y) + z\emptyset_y(x,y) + z^2\emptyset_y(x,y) + z^3\emptyset_y(x,y)$$

$$w(x,y,z) = w_0(x,y) \qquad \text{......3.3}$$

3.2 Governing Equations

Objective of these, for presenting governing equation formulation using first order deformation theory (FSDT) for functionally graded materials plates and to form FEM model for mechanical as well as thermo mechanical analysis. Adding to this, in inherent low consumption cost the FSDT for sufficient accurate response for plate thickness from moderate thick to thin plate.

TSDT is not used for present work as it increase small accuracy in cost of significant increase in computation.

3.2.1 For Mechanical Loading Condition

Stress – Strain Relations

As per generalized Hooks Law, for an isotropic material, the stress strain are given a

$$\varepsilon_{ijM} = \begin{bmatrix} \varepsilon_{xxM} \\ \varepsilon_{yyM} \\ \varepsilon_{zzM} \\ \varepsilon_{yzM} \\ \varepsilon_{xzM} \\ \varepsilon_{xyM} \end{bmatrix} = [C]\sigma = [C] \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{bmatrix} \qquad \text{......3.4}$$

where [C] is a matrix of elastic coefficient given by

$$[C] = \frac{1}{E} \begin{bmatrix} 1 & -v & -v & 0 & 0 & 0 \\ -v & 1 & -v & 0 & 0 & 0 \\ -v & -v & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+v) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+v) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+v) \end{bmatrix}$$

3.2.2 For Thermo Mechanical Loading Condition

Thermal Strain

$$\varepsilon_{ijT} = \begin{bmatrix} \varepsilon_{xxT} \\ \varepsilon_{yyT} \\ \varepsilon_{zzT} \\ \varepsilon_{yzT} \\ \varepsilon_{xzT} \\ \varepsilon_{xyT} \end{bmatrix} = \alpha(z)\Delta T \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Thermo Mechanical Strain

$$\varepsilon_{ij} = \varepsilon_{ijM} + \varepsilon_{ijT}$$

$$\varepsilon_{ij} = \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \\ \varepsilon_{xy} \end{bmatrix} = [C] \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{bmatrix} + \alpha(z)\Delta T \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The expression for stress

$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{bmatrix} = [D] \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \\ \varepsilon_{xy} \end{bmatrix} + \frac{E\alpha(z)\Delta T}{1-2v} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \ldots\ldots 3.5$$

Where

$$[D] = \frac{E}{1-2v} \begin{bmatrix} 1-v & v & v & 0 & 0 & 0 \\ v & 1-v & v & 0 & 0 & 0 \\ v & v & 1-v & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2v}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2v}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2v}{2} \end{bmatrix}$$

Displacement, Strain Field and Temperature Field

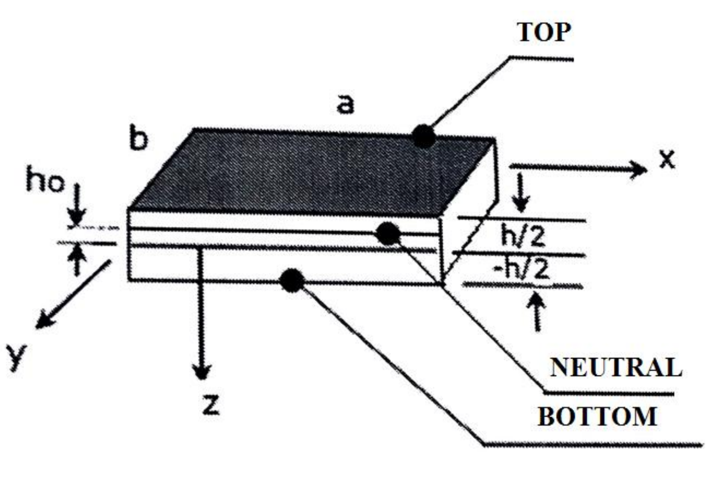


Fig. 3.2: Schematics diagram and dimensions of FGM plate [12]

$$\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{yz}\\ \varepsilon_{xz}\\ \varepsilon_{xy}\end{bmatrix} = \begin{bmatrix}\varepsilon_{xx}^0\\ \varepsilon_{yy}^0\\ \varepsilon_{yz}^0\\ \varepsilon_{xz}^0\\ \varepsilon_{xy}^0\end{bmatrix} + (z+h_0)\begin{bmatrix}\varepsilon_{xx}^1\\ \varepsilon_{yy}^1\\ 0\\ 0\\ \varepsilon_{xy}^1\end{bmatrix} = \begin{bmatrix}\frac{\partial u_0}{\partial x}+\frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\\ \frac{\partial v_0}{\partial y}+\frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2\\ \frac{\partial w_0}{\partial y}+\emptyset_y\\ \frac{\partial w_0}{\partial x}+\emptyset_x\\ \frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}+\frac{\partial w_0}{\partial y}\frac{\partial w_0}{\partial x}\end{bmatrix} + (z+h)\begin{bmatrix}\frac{\partial \emptyset_x}{\partial x}\\ \frac{\partial \emptyset_y}{\partial y}\\ 0\\ 0\\ \frac{\partial \emptyset_x}{\partial y}+\frac{\partial \emptyset_y}{\partial x}\end{bmatrix} \quad \text{......3.6}$$

Temperature distribution

$$T(z) = T_t - \frac{T_t - T_b}{\int_{-h/2}^{h/2}\frac{dz}{k(z)}}\int_z^{h/2}\frac{dz}{k(z)} \quad \text{......3.7}$$

Where T(z) is temperature at particular height of z from neutral axis,

T_t = Top surface temperature

T_b = Bottom surface temperature

3.3 Model Analysis of a Plate

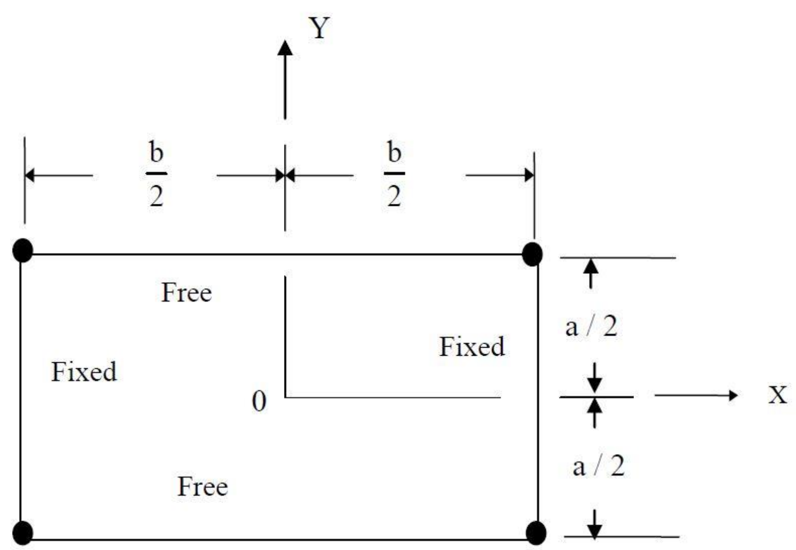


Fig. 3.3: Dimensions of a Plate

$$f_{ij} = \frac{\delta_{ij}^2}{2\pi a^2}\left[\frac{Eh^3}{12\gamma(1-v^2)}\right]^{\frac{1}{2}} \quad ……3.8$$

Where i=1,2,3….

J=1,2,3….

a = Plate length

b = Plate width

h= Plate thickness

δ= small mass

v= Poisson ration

E= modulus of Elasticity

γ= density of plate

CHAPTER 4: CNT REINFORCED FGM MODEL

For analyzing mechanical, thermo mechanical and free vibration conditions, CNT reinforced FGM, base material, Aluminium is considered, because it has lots of engineering application. It will be a replacement of light weight, high strength material.

4.1 FGM Modeling

A CNT reinforced Al FGM nano plate has been used having 600nm width, 600nm depth & 30 nm thickness. This plate is having five layer of 6 nm thickness having different ratio of CNT percentage. Layers having CNT percentage respectively 2%, 1.5%, 1%, 0.5% and 0% CNT from top layer to bottom layer, rest Al alloy. All dimensions are in nano meter.

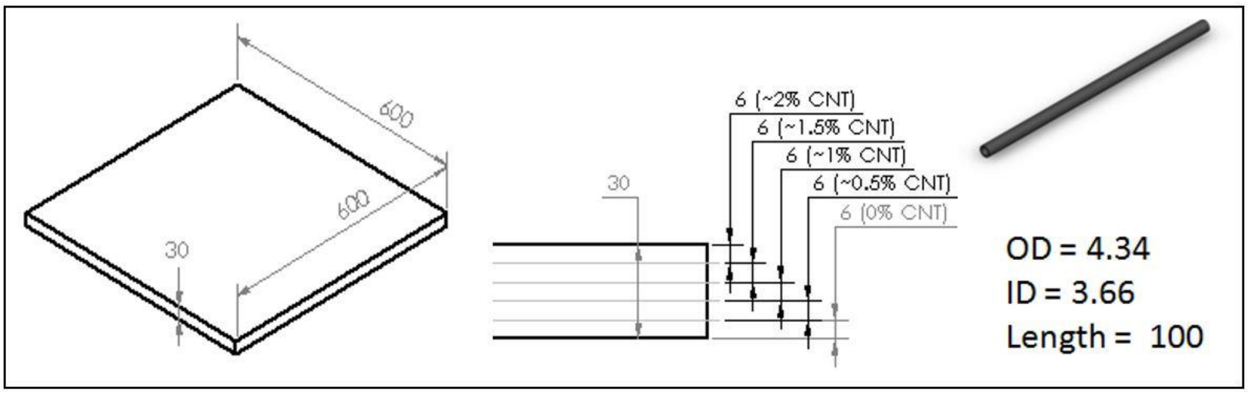


Fig.4.1: Details of Model Dimensions

For this problem long CNT have been used having length of 100nm, OD 4.34nm and ID 3.66nm. So this CNT is of thickness 0.34nm. [44]

Nos. of CNT in each layer has been calculated using following formula:

$$Nos\ of\ CNT = \frac{Volume\ of\ CNT}{(Volume\ of\ Rest\ Material + Volume\ of\ CNT)}\ x\ \%\ CNT\ fraction$$

$$Nos\ of\ CNT = \frac{[\frac{\pi}{4}\{(OD)^2 - (ID)^2\} \times L]\ CNT}{\{(L \times W \times T) Al\ Layer - \frac{\pi}{4}(OD)^2 CNT\} + \left[\frac{\pi}{4}\{(OD)^2 - (ID)^2\} \times L\right] CNT} \times \frac{\%CNT}{100} \qquad \ldots\ldots.4.1$$

Table 4.1: List of CNTs in layers

Layer	% CNT	Nos. of CNT	Arrangement
1	2%	100	5 x 20
2	1.5%	75	5 x 15
3	1%	50	5 x 10
4	0.5%	25	5 x 5
5	0%	0	0
Total	**1%**	**250**	

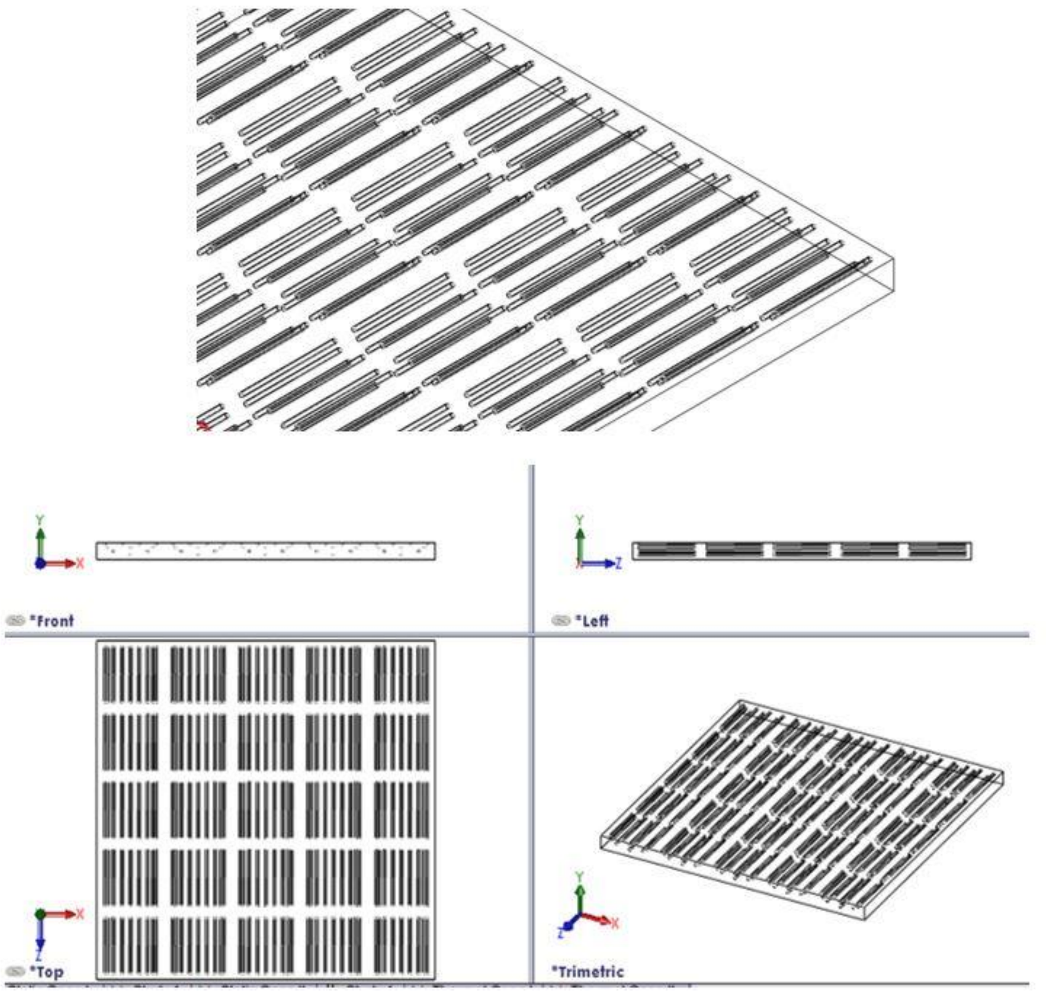


Fig.4.2: Schematic diagram of CNT reinforced FGM

CNT arrangement layer wise is shown as

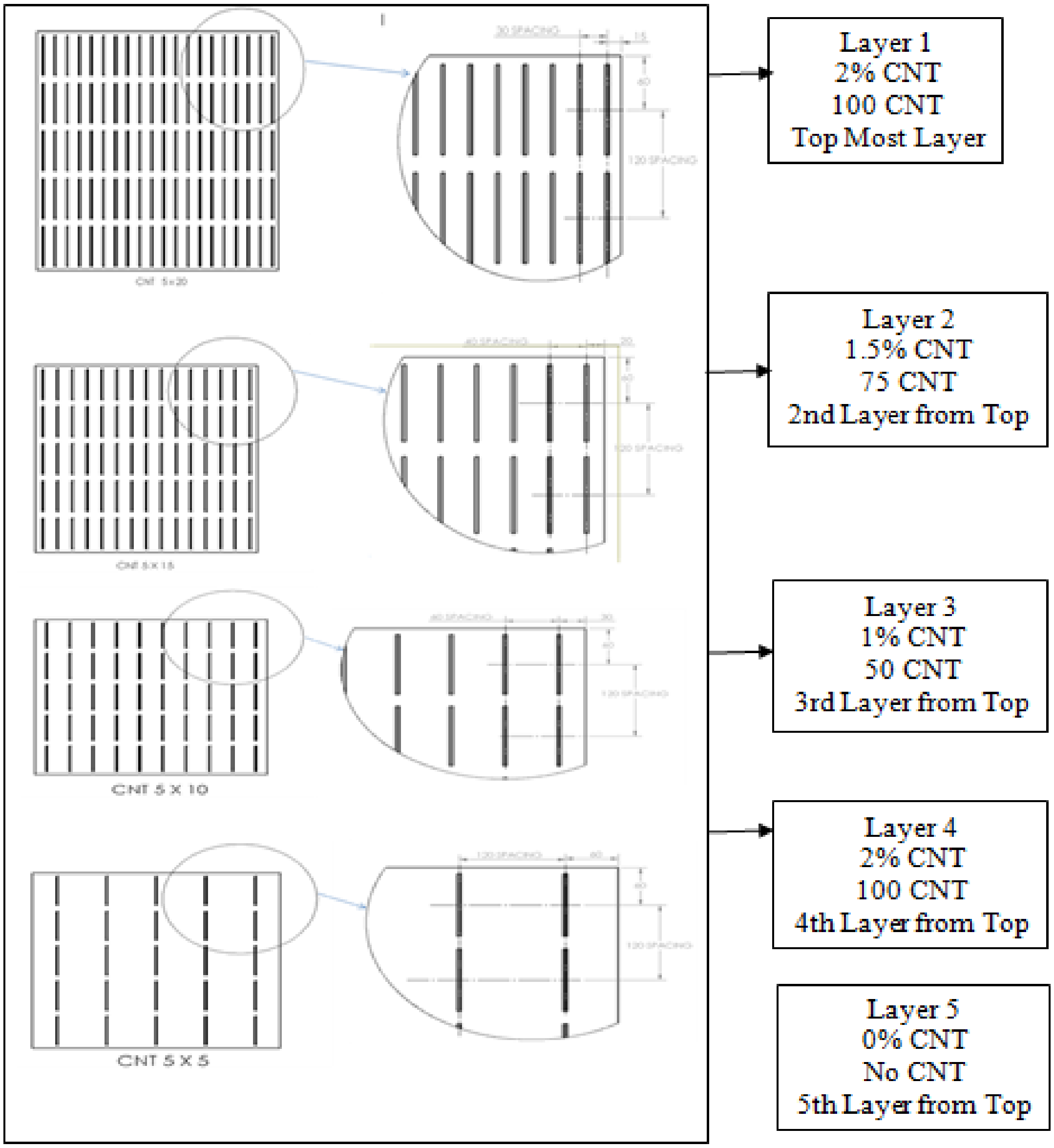


Fig.4.3: Orientation and quantities of CNTs in each layer

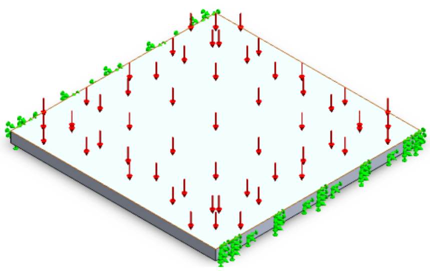
In this, CNTs are assembled in Al plate, having inside cut for CNTs assembly. Bonded contact has been defined between Al plate and CNTs.

Results has been compared within Al plate and 1% CNT Reinforced Al composite [12][13] and CNT reinforced FGM of same dimension with same boundary conditions.

For **composite**, there are five layer having 50 CNTs in each layer. For composite it will be over all 1% CNTs weighted composite. Composite is also having 250 nos. of CNTs as of FGM to compare results, which will give better understand of CNTs role in FGM and composite.

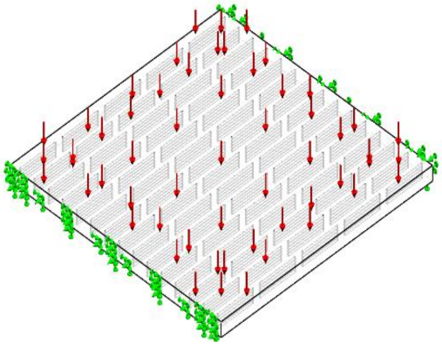
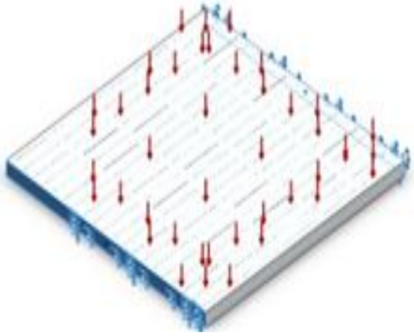
It has been modeled using Solidworks 2015. Analysis have been done using COSMOS Simulation tool from Dassault System.

4.2 Boundary Conditions

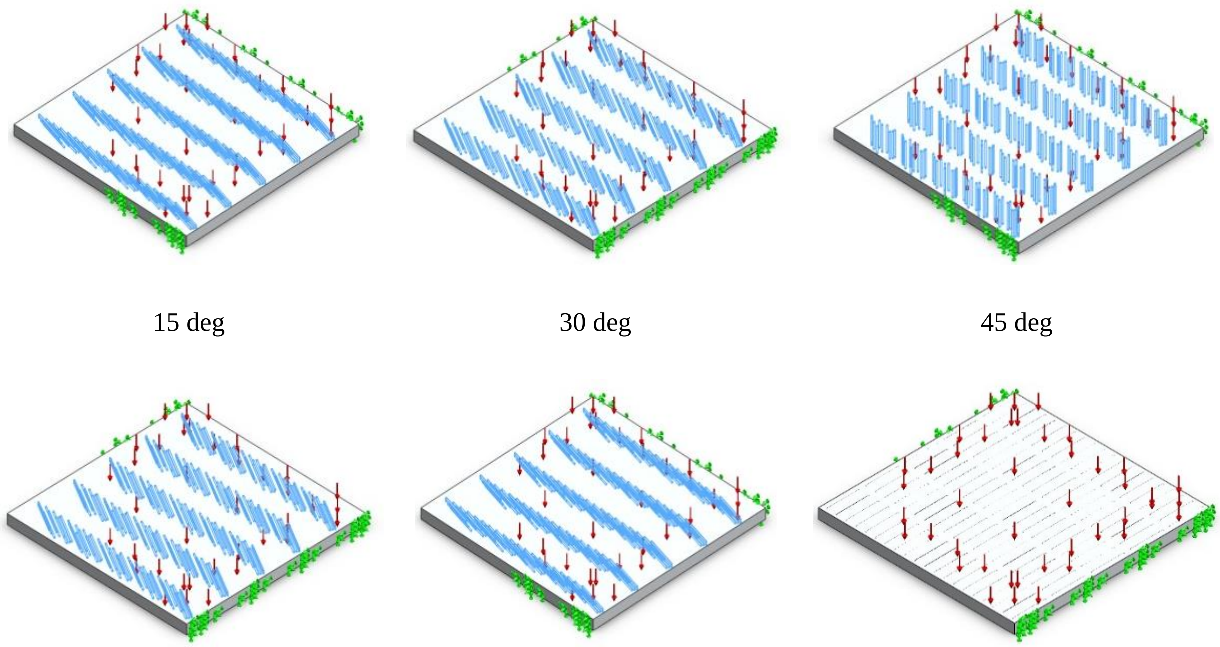
<table>
<tr><td>
Fig. 4.4: Model with Boundary Condition</td><td>Two Side Fixed (As Shown)

Under Mechanical Analysis
Pressure at Top Surface = 100 N/m^2

(Under Thermo Mechanical Analysis)
Pressure at Top Surface = 100 N/m^2
Temp: Upper Surface 380K & Lower surface 300K

Mode Shape Analysis
No Load</td></tr>
<tr><td>
Fig. 4.5: 1% CNT Reinforced Composite</td><td>
Fig. 4.6: CNT Reinforced FGM (0 deg)</td></tr>
</table>

FGM with CNTs various orientation, with boundary condition is shown below



15 deg 30 deg 45 deg

60 deg 75 deg 90 deg

Fig. 4.7: FGM with various CNTs orientation

4.3 Material Properties

In this FGM CNT is reinforced in Al material to form a nano plate FGM. Aluminium and CNT materials properties are tabulated below.

Table 4.2: Al Alloy Properties

Name:	**1060 Al Alloy**
Model type:	Linear Elastic Isotropic
Yield strength:	2.75742 e^7 N/m^2
Tensile strength:	6.89356 e^7 N/m^2
Elastic modulus:	6.9e^{10} N/m^2
Poisson's ratio:	0.33
Mass density:	2700 kg/m^3
Shear modulus:	2.7e^{10} N/m^2
Thermal expansion coefficient:	2.4e^{-5} /Kelvin

Table4.3: CNT Properties

Name:	**Carbon Nano Tube**
Model type:	Linear Elastic Isotropic
Tensile strength:	$2.4e^{10}$ N/m^2
Compressive strength:	$6.8e^{6}$ N/m^2
Elastic modulus:	$1e^{12}$ N/m^2
Poisson's ratio:	0.278
Mass density:	1330 kg/m^3
Shear modulus:	$3.8028e^{11}$ N/m^2
Thermal expansion coefficient:	$2e^{-5}$ /Kelvin

CHAPTER 5: RESULTS & DISCUSSION

Mechanical, thermo mechanical and mode shape analysis has been carried out. Stress, displacement, and their natural frequencies have been captured using finite element analysis. Upper layer and lower layer stress and displacement are also calculated and compared in the results. Behavior of FGM is also studied in this. Percentage variations in stress, strain and displacement have been calculated from the analysis.

Under mechanical, thermo mechanical loading condition and mode shape analysis results are presented here for Al nano plate, CNT reinforced composite, CNT reinforced FGM, CNT reinforced FGN with various angular positions of CNTs.

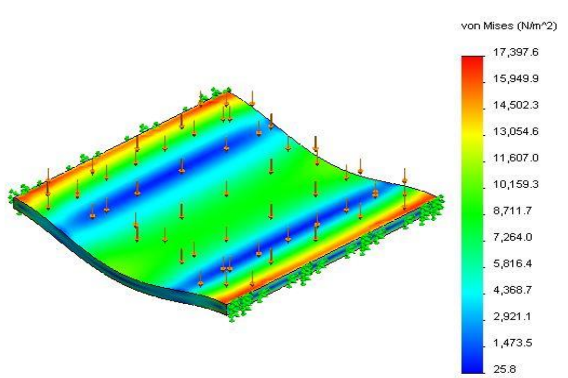
First comparison is there in results for Al nano plate, CNT reinforced composite nano plate & CNT reinforced FGM nano plates in term of displacement, stress and natural frequencies with their mode shapes.

After that comparison of results are there for CNT reinforced FGM nano plate for various angular positions of CNTs in FGM.

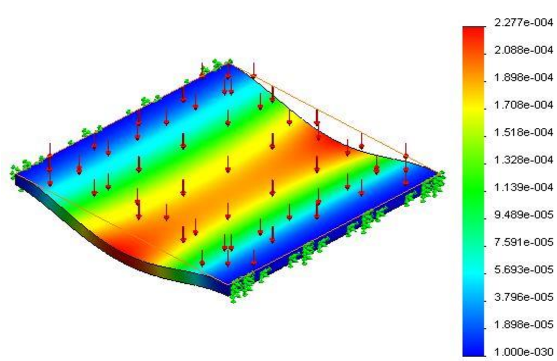
Finally both of the above results are compared to conclude for overall detailing of results.

5.1 Mechanical Loading Conditions

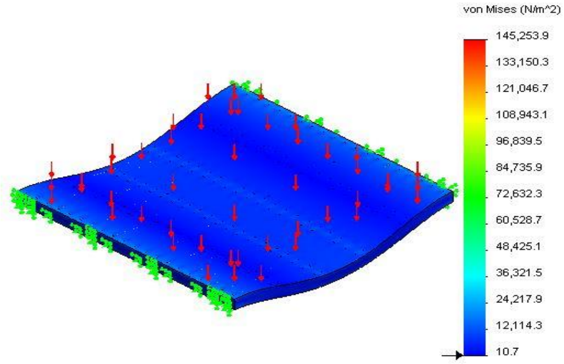
FEA Results for Al, composite & FGM nano plate



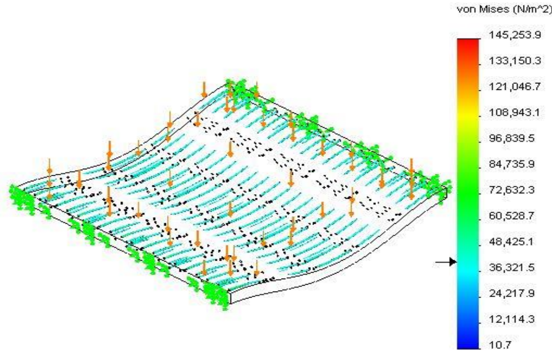
Stress Plot for Al Nano Plate



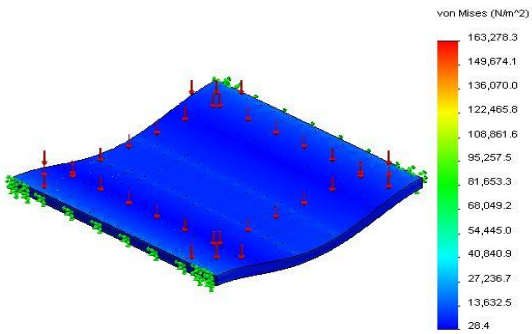
Displacement Plot for Al Nano Plate



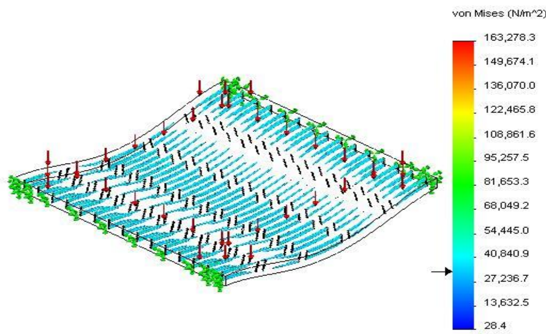
Stress Plot for CNT FGM Nano Plate



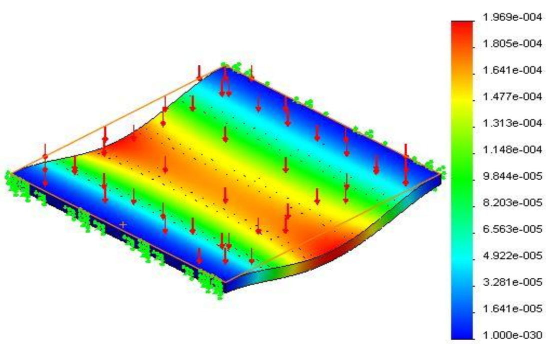
Detailed Stress Plot for CNT FGM Nano Plate



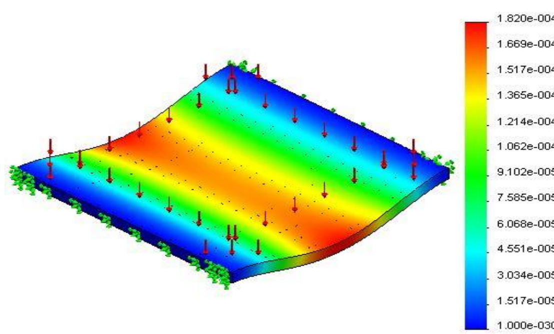
Stress Plot for CNT Composite Nano Plate



Detailed Stress Plot for CNT Composite Nano Plate



Displacement Plot for CNT FGM Nano Plate



Displacement Plot for CNT Composite Nano Plate

Fig. 5.1: Results for Al, composite & FGM plate under mechanical loading

From Al plate FEM results, it is clear that max stress is 17,397 N/m^2 and it is maximum at the support. Displacement results shows maximum displacement is $2.277e^{-4}$ nm at the center portion of the plate at edge. Displacement of the plate varies maximum from center to min at the fix edge of plate as expected.

For FEA stress results of FGM Nano plate, as from figure 5.1 it can be seen that at the surfaces stresses are very less. It varies in the range of 12000 N/m^2, which is lesser than Al plate stresses. But it shows maximum stress 145253.93 N/m^2, which is much higher. To analyse that detailed stress plot was taken and it is shown in detailed stresses plot for CNT FGM nano plate. Stresses above 40,000N/m^2 is shown for FGM plate. And all stresses are at near CNT-Al contact area. It can be understood through basic physics, as when two different strength materials are joined, then maximum resistance is offered by hard materials, which varies from hard material to soft material varies linearly via contact area another reason is because of higher stress concentration at the contact surface due to sudden properties variation, which is shown.

Later stress results of composite Nano plate are shown. As from figure it can be seen that at the surfaces stresses are very less, in the range of 13000 N/m^2, we observed earlier in FGM nano plate. . But it shows maximum stress 163278.3 N/m^2, which higher then FGM nano plate. Same observation was there as FGM nano plate.

In last, displacement plot for FGM nano plate and composite nano plate are shown respectively. FGM nano plate having maximum deflection of 1.969e-4 nm and for Composite nano plate it is 1.82e-4, which is almost 14% lesser to FGM nano plate. Displacement distribution for both the plate is almost similar to each other as well as of Al plate.

Table 5.1: Results for Al, composite & FGM plate under mechanical loading

Description	**Deflection (nm)**	**Max Stress (N/m^2)**
Al Plate	2.277E-04	17397.6
CNT Composite	1.82E-04	163278.3
CNT FGM (Mix)	1.96E-04	145253.9

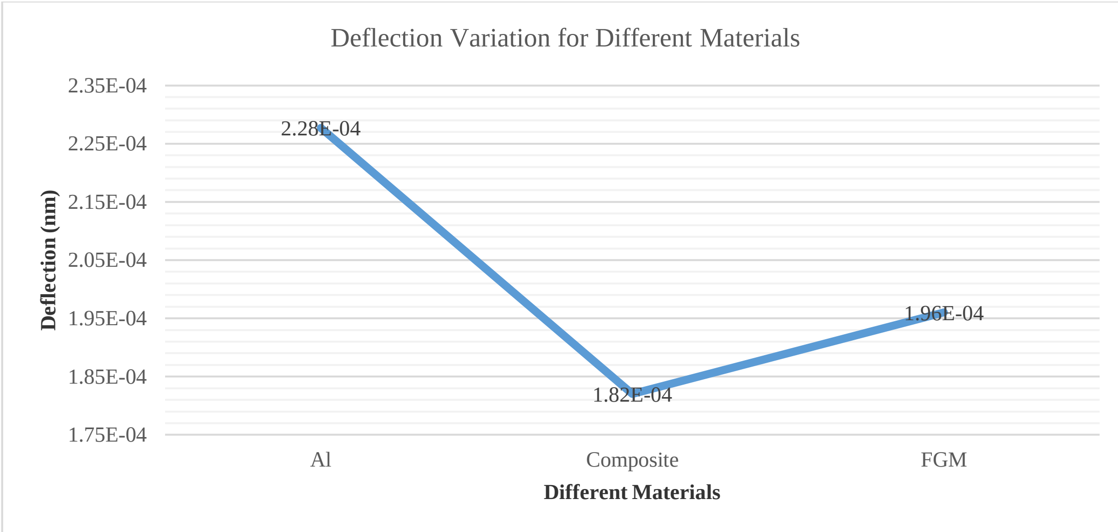


Fig. 5.2: Displacement plot for Al, composite & FGM plate under mechanical loading

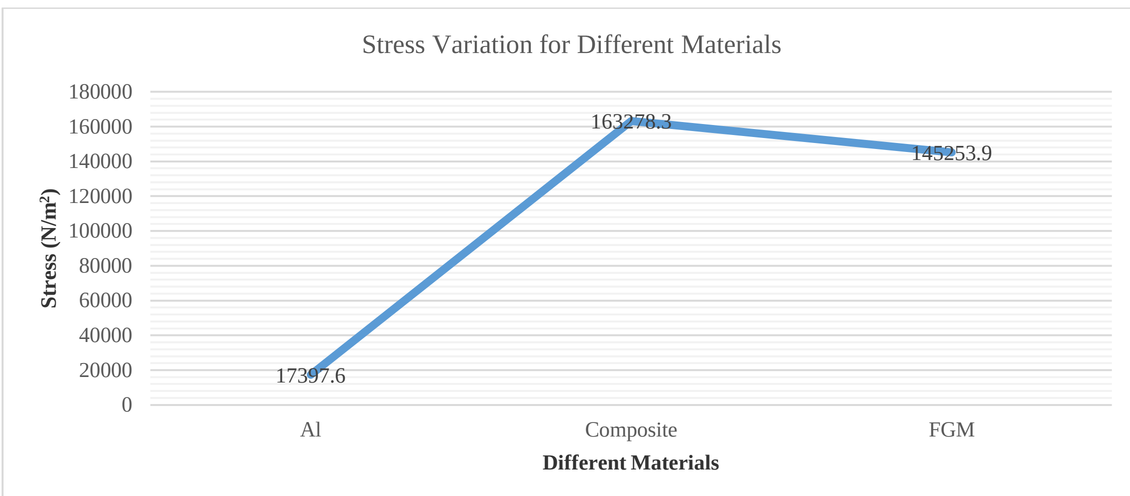


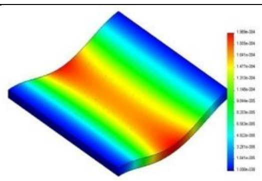
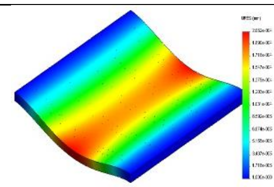
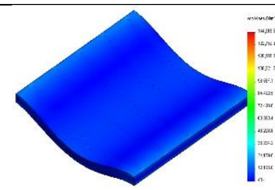
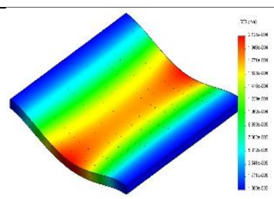
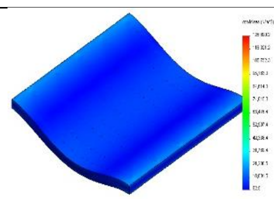
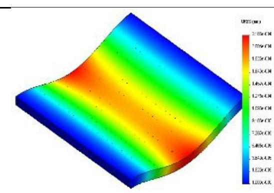
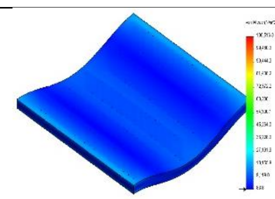
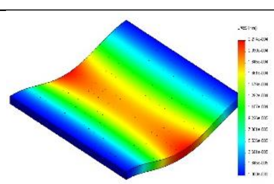
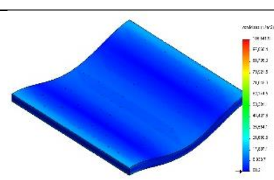
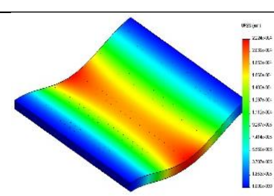
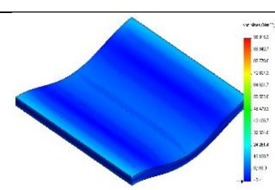
Fig. 5.3: Stress plot for Al, composite & FGM plate under mechanical loading

In fig 5.2 & fig 5.3, max displacement and max stress values are shown respectively for Al, composite and FGM nano plates. In displacement plot, we can see Al is having maximum displacement, after than FGM then lowest displacement is there for composite. While for stress plots its, totally reverse, as Max stress is for composite, after that FGM and then Al nano plate is having minimum stress. It can be explained by basic physics law that maximum resistance offers by material will have minimum deflection, which is shown in above results.

In composite, CNTs are distributed uniformly 1% along the thickness, while in FGM, lower two layer have 0.5% and 0% CNTs, So it may be reason of FGM's weakness over composite.

From the results it can be concluded that in mechanical loading, CNT increases much strength, as can be seen in composite as well as FGM. And for composite and FGM, CNT reinforced FGM plate is slightly weaker compare to CNT reinforced composite plate.

FEA Results for FGM for various orientations of CNTs

Deflection (nm)	Stress (N/m^2)	Deflection (nm)	Stress (N/m^2)
0 deg		15 deg	
 1.9600E-04	 145253.9	 2.1246E-04	 126890
30 deg		45 deg	
 2.1246E-04	 126890	 2.1860E-04	 108516
60 deg		75 deg	
 2.2143E-04	 106542	 2.2241E-04	 96915.3

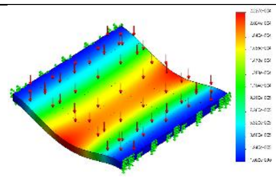
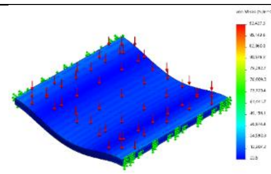
90 deg		
 2.2270E-04	 92427.3	

Fig. 5.4: Results for FGM for various Orientations of CNTs under mechanical loading

Above fig. 5.4 shows the results of deflection and strain for CNT reinforced FGM, where CNTs orientation varying from 0 deg to 90 deg in the step of 15 deg. These figures give the idea of variation in stress and deflection as orientation varies.

From the fig it can be seen that for Orientation of CNTs at 0 deg deflect is 1.96e-4 nm and at 90 deg CNT angular position is 2.227e-4 nm. I can be seen that it is increasing in each cases from 0 deg CNT orientation to 90 deg CNT orientation.

Stress pattern shows reduction in stresses from orientation of CNTs 0 deg to 90 deg. It is expect as earlier explained that as deflection increases, stresses decreases.

Table 5.2: Results for FGM for various Orientations of CNTs under mechanical loading

Orientation	**Deflection (nm)**	**Max Stress (N/m^2)**
0 deg	1.9600E-04	145253.9
15 deg	2.0622E-04	144815
30 deg	2.1246E-04	126890
45 deg	2.1860E-04	108516
60 deg	2.2143E-04	106542
75 deg	2.2241E-04	96915.3
90 deg	2.2270E-04	92427.3

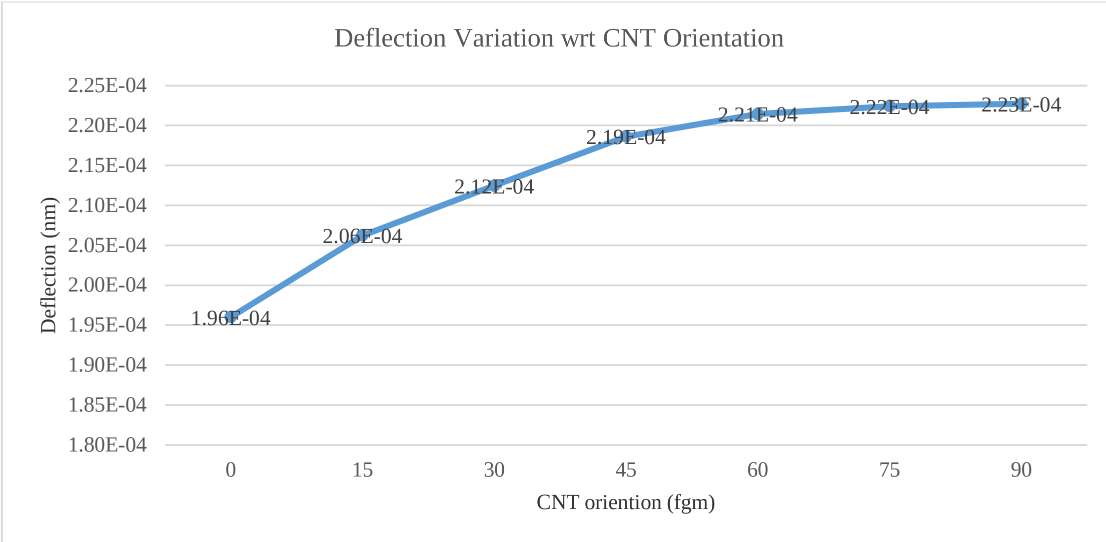


Fig. 5.5: Deflection plot for FGM for various Orientations of CNTs under mechanical loading

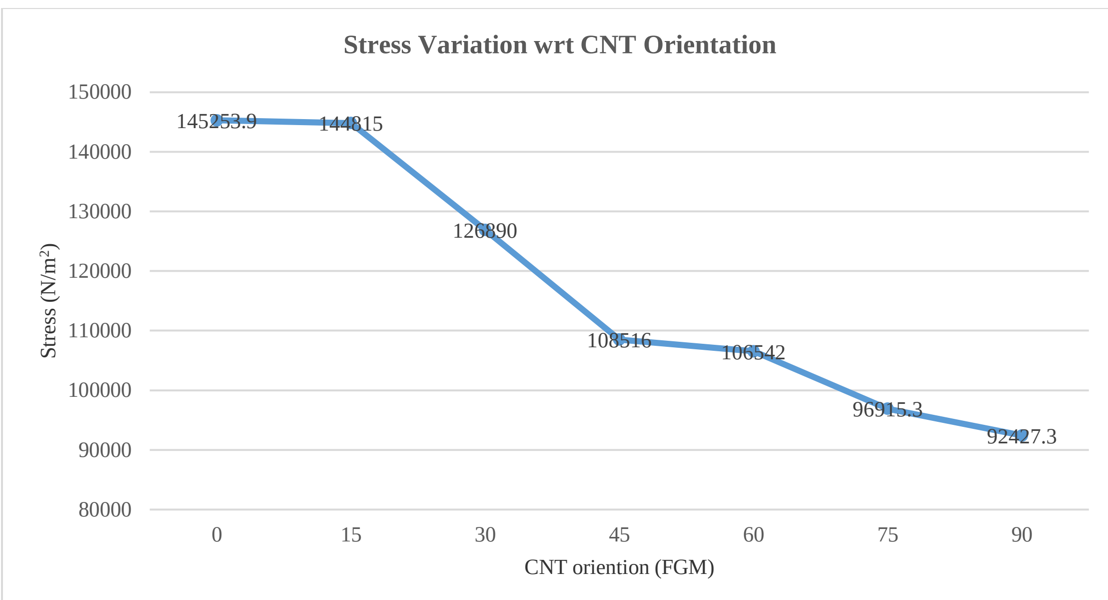


Fig. 5.6: Stress plot for FGM for various Orientations of CNTs under mechanical loading

Above shows table 5.2 and plots of results of stress and deflection, for varying orientations of CNT variation. Fig. 5.5 shows deflection plot, and graph represent smooth increase in deflection form 0 deg to 90 deg orientation of CNTs, But slop of increasing deflection, keep on reducing from 0 deg to 90 deg CNTs orientation in FGM.

Fig. 5.6 shows stress plots and in this graph, stresses are reducing from 0 deg orientation to 90 deg CNTs orientation in FGM. But it is not as smooth as of deflection plot. It may be because of stress concentration as explained earlier.

From the above study, it can be seen for mechanical strength, 0 deg orientation of CNTs in FGM is better, compare to other angle. But others can be utilized where we need moderate strength.

Results for all configurations

Table 5.3: Results for all configurations under mechanical loading

Orientation	Deflection (nm)	Max Stress (N/m^2)
Al Plate	2.277E-04	17397.6
CNT Composite	1.82E-04	163278.3
CNT FGM (Mix)	1.96E-04	145253.9
CNT FGM (0 deg)	1.9600E-04	145253.9
CNT FGM (15 deg)	2.0622E-04	144815
CNT FGM (30 deg)	2.1246E-04	126890
CNT FGM (45 deg)	2.1860E-04	108516
CNT FGM (60 deg)	2.2143E-04	106542
CNT FGM (75 deg)	2.2241E-04	96915.3
CNT FGM (90 deg)	2.2270E-04	92427.3

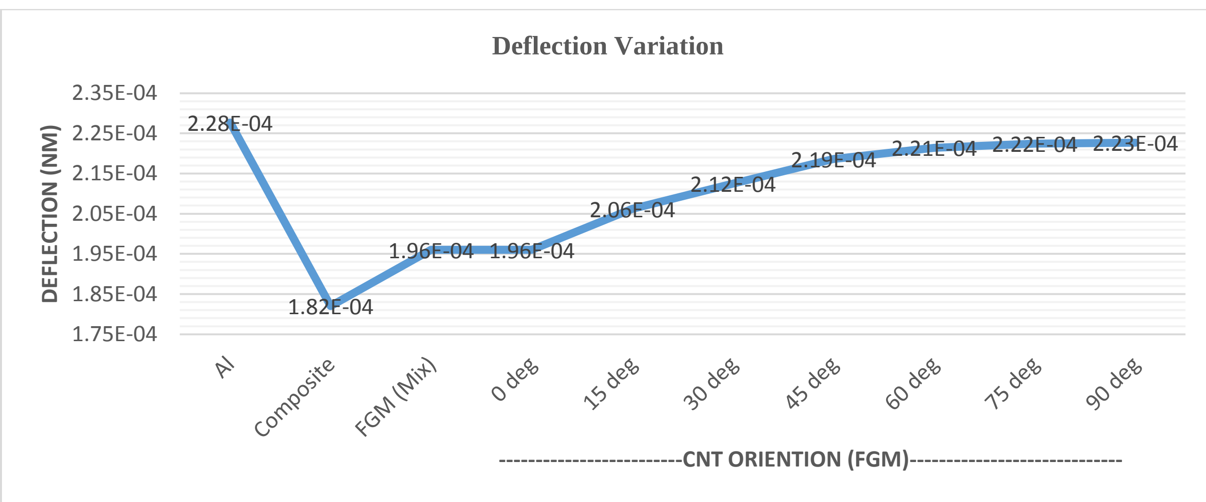


Fig. 5.7: Displacement plot for all configurations under mechanical loading

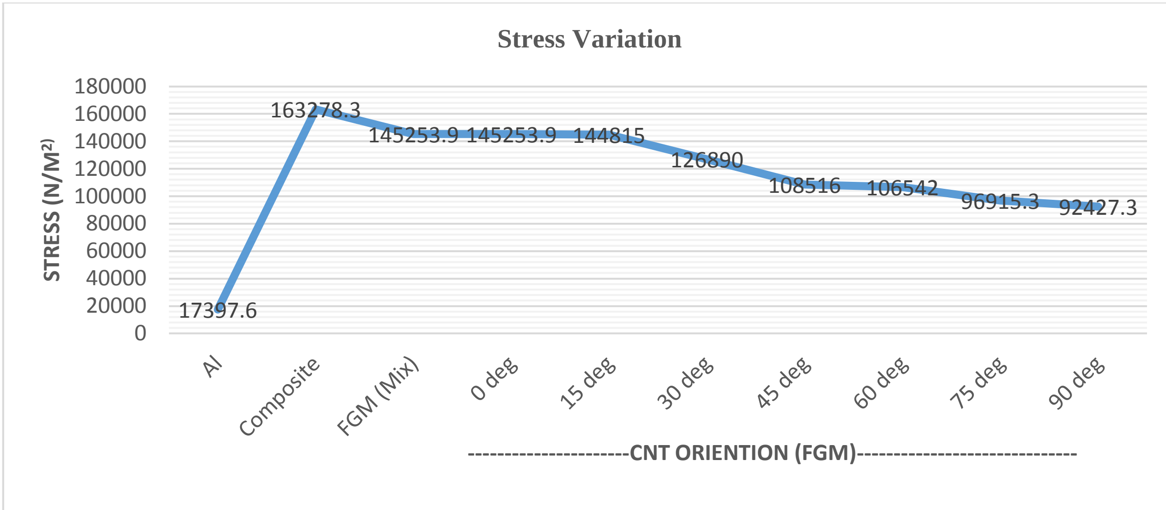


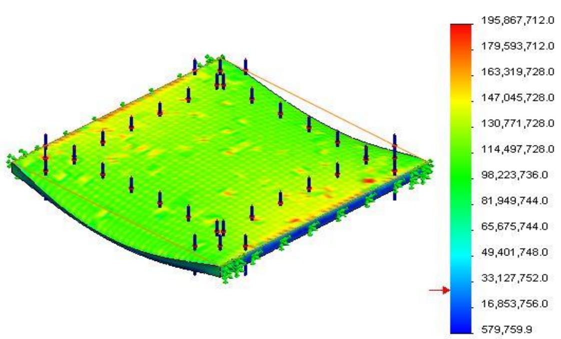
Fig. 5.8: Stress plot for all configurations under mechanical loading

For mechanical loading condition, all the deflection and stress results are tabulated in table 5.3. Deflection and stresses plots are there in fig. 5.7 & fig 5.8 respectively. From the results it can be understood that CNT increases a lot of strength for plate after mixing with Al material.

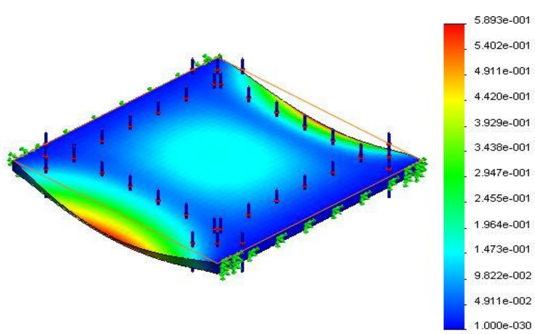
From the results it can be seen that CNT reinforced composite is strongest compare to Al plate and CNT reinforced FGM for all orientation of CNTs, while Al plate is weakest. 0 deg orientation of CNT in FGM is strongest compare to other orientation in FGM.

5.2 Thermo - Mechanical Loading Conditions

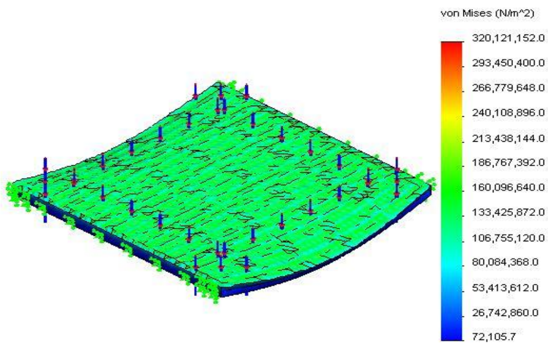
FEA Results for Al, composite & FGM nano plate



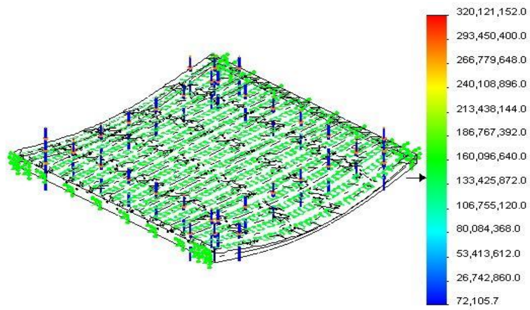
Stress Plot for Al Nano Plate



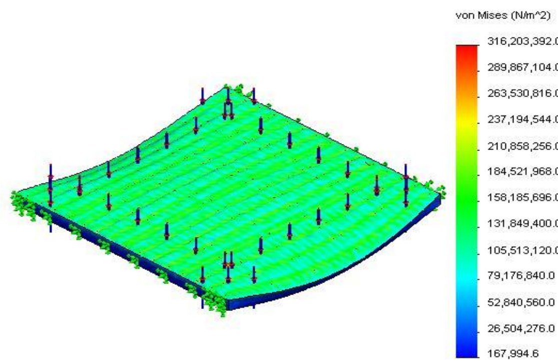
Displacement Plot for Al Nano Plate



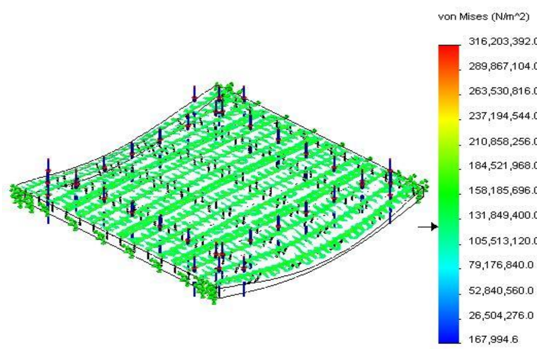
Stress Plot for CNT FGM Nano Plate



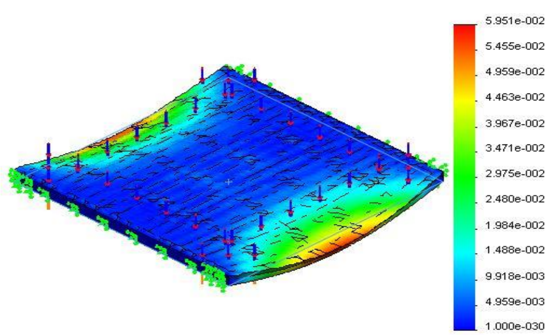
Detailed Stress Plot for CNT FGM Nano Plate



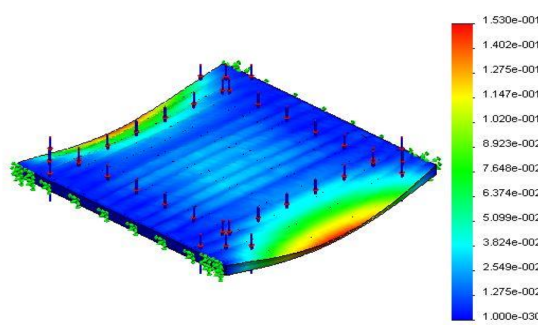
Stress Plot for CNT Composite Nano Plate



Detailed Stress Plot for CNT Composite Nano Plate



Displacement Plot for CNT FGM Nano Plate



Displacement Plot for CNT Composite Nano Plate

Fig. 5.9: Results for Al, composite & FGM plate under thermo mechanical loading

In fig 5.9, in starting, stress and displacement results under thermo mechanical loading for Al nano plate are shown. Under thermo mechanical loading, stresses increases a lot due to thermal stresses. Its shows maximum stresses at the upper surface, near fix edge of the plate which is 195867712.0 N/m^2. Maximum average stresses are there at upper surface due to higher temperature. Max displacement for Al plate is 5.983e-1 nm, which is much higher compare to displacement under mechanical loading, due to thermal loading. These results shows same as of material behavior as under thermo mechanical loading stresses and displacement are more compare to mechanical loading conditions.

For FGM nano plate stress plots are shown later. From this, it can be seen that maximum stresses are there at upper surface where max temperature is applied. Maximum stresses is 320121152.0 N/m^2 at upper surface near the fix edge and max average stress is there at upper surface, as shown.

Later results shows stress plot for composite nano plate. It shows maximum stress is 316203392.0 N/m^2 which is lesser then FGM nano plate. It also shows same pattern as FGM nano plate as average stress is there at upper surface due to higher temperature at that face.

FGM have maximum displacement of 5.951e-2 while composite have maximum displacement of 1.53e-1, much higher to FGM, as shown in last. So under thermo mechanical loading, FGM is much stronger then composite while it was reverse for mechanical loading conditions.

It may be because FGM is having more CNTs on upper layer then composite upper layer, and CNTs show better strength under thermal loading.

Table 5.4: Results for Al, composite & FGM plate under thermo mechanical loading

Description	**Deflection (nm)**	**Max Stress (N/m^2)**
Al Plate	5.893E-01	195867712.0
CNT Composite	1.53E-01	316203392.0
CNT FGM	5.951E-02	320121152.0

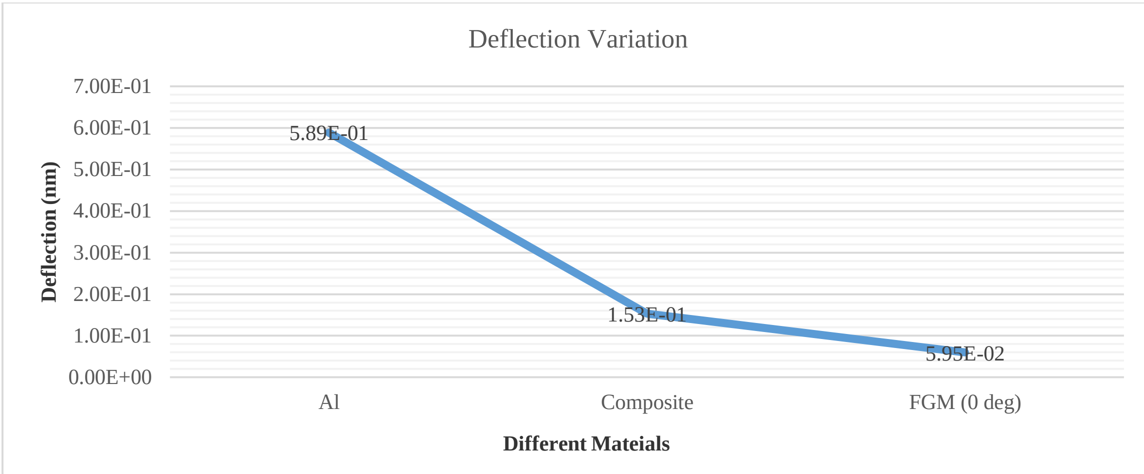


Fig. 5.10: Displacement plot for Al, composite & FGM plate under thermo mechanical loading

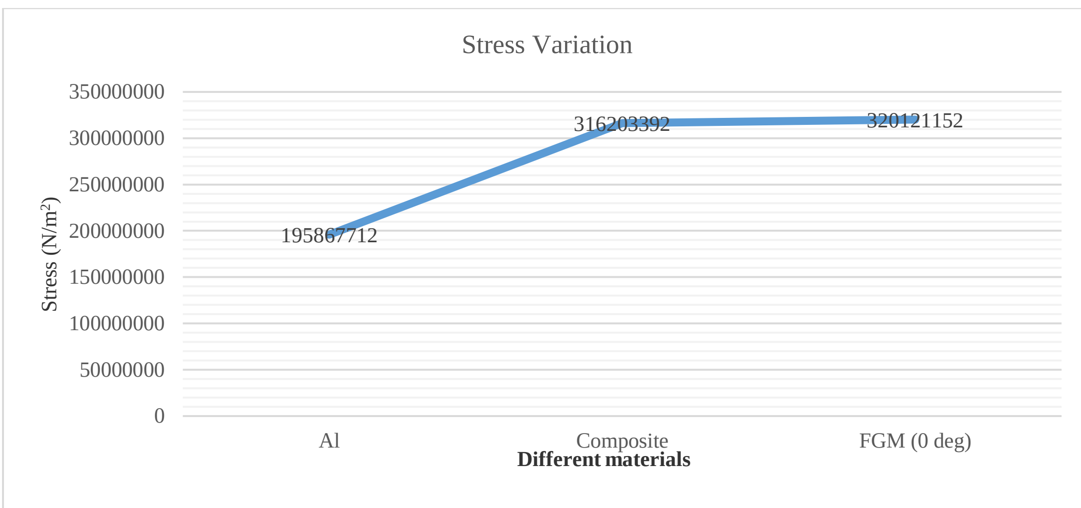


Fig. 5.11: Stress plot for Al, composite & FGM plate under thermo mechanical loading

Stresses and displacement results for Al, composite and FGM nano plates are tabulated in table 5.4. From this table results, maximum displacement and maximum stresses plots for Al, composite and FGM is plotted for comparing in fig 5.10 and fig 5.11

In thermo mechanical loading Al plate, composite and FGM stresses have more stresses than mechanical loading, due to thermal loading is also added in this. Max stresses are higher in FGM and composite compare to Al plate. Max stress in FGM is slightly more compare to composite.

Max stresses are observed at CNT and upper surface of plate, where maximum temperature is applied. It may be because CNT is stronger than Al and max thermal stresses at upper surface due to higher temperature, so in combination it will offer more resistance to applied load. And there is sudden property variation at CNT Al joining area, so it may be because of stress concentration at that area.

Max displacement of FGM plate, max displacement in upper and lower surface in FGM is reduced around 90% compare to Al plate. In case of composite, max displacement of FGM plate, max displacement in upper and lower surface in FGM is reduced around 74% compare to Al plate.

It may be because CNT provides more thermal strength, and FGM has more numbers of CNTs at its upper surface where higher temperature is applied.

From the result it can be concluded that in thermo mechanical loading, CNT reinforced FGM plate is much stronger than Al plate and slightly stronger than CNT reinforced composite plate.

FEA Results for FGM for various orientations of CNTs

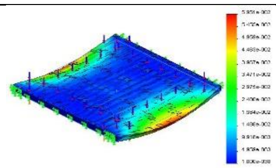
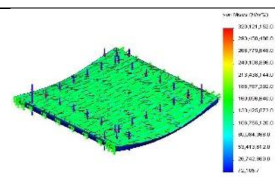
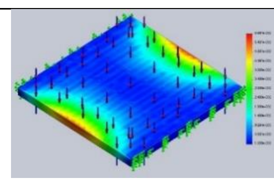
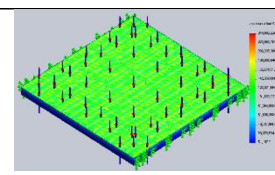
Deflection (nm)	Stress (N/m^2)	Deflection (nm)	Stress (N/m^2)
0 deg		90 deg	
 5.951E-02	 320121152.0	 5.997E-02	 243992224.0

Fig. 5.12: Results for FGM for various Orientations of CNTs under thermo mechanical loading

Under thermo mechanical loading, only two orientation of CNTs in FGM has been analyzed, that is 0 deg and 90 deg only. Deflection and stresses plots are shown in fig 5.12.

From the results, at 0 deg CNT orientation position in FGM, deflection is 5.951e-2 nm and for 90deg orientation, it is 5.997e-2 nm. This deformation change is very less compare to deflection under mechanical loading conditions. Stress changes are also not much effective but shows slight reduction, as expected, as explained earlier. Results also shows not much changes in visual other than their values.

Table 5.5: Results for FGM for various Orientations of CNTs under thermo mechanical loading

Description	**Deflection (nm)**	**Max Stress (N/m^2)**
CNT FGM (0 deg)	5.951E-02	320121152.0
CNT FGM (90 deg)	5.997E-02	243992224.0

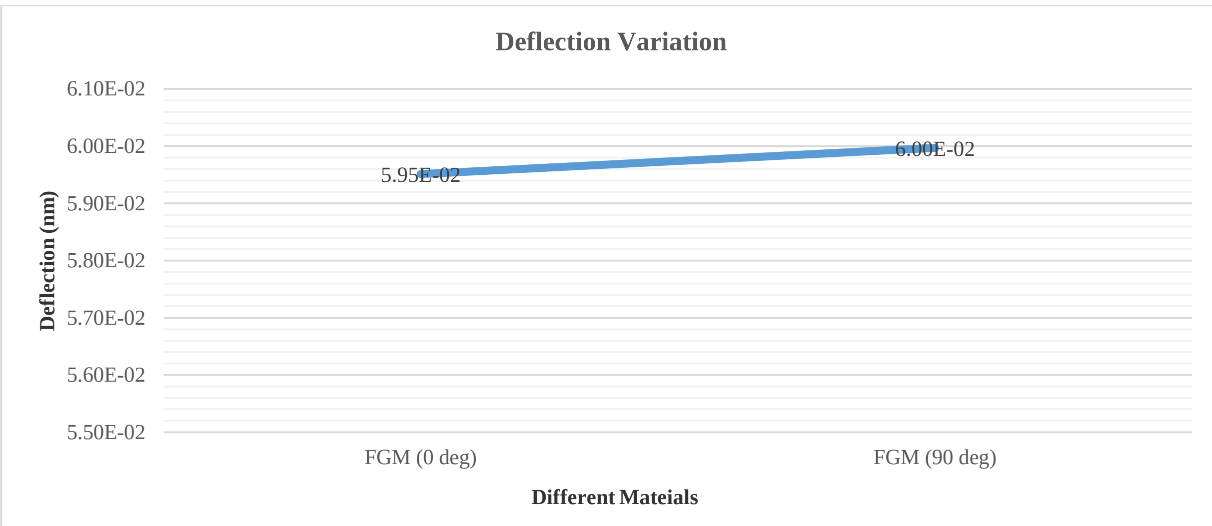


Fig. 5.13: Deflection plot for FGM for various Orientations of CNTs under thermo mechanical loading

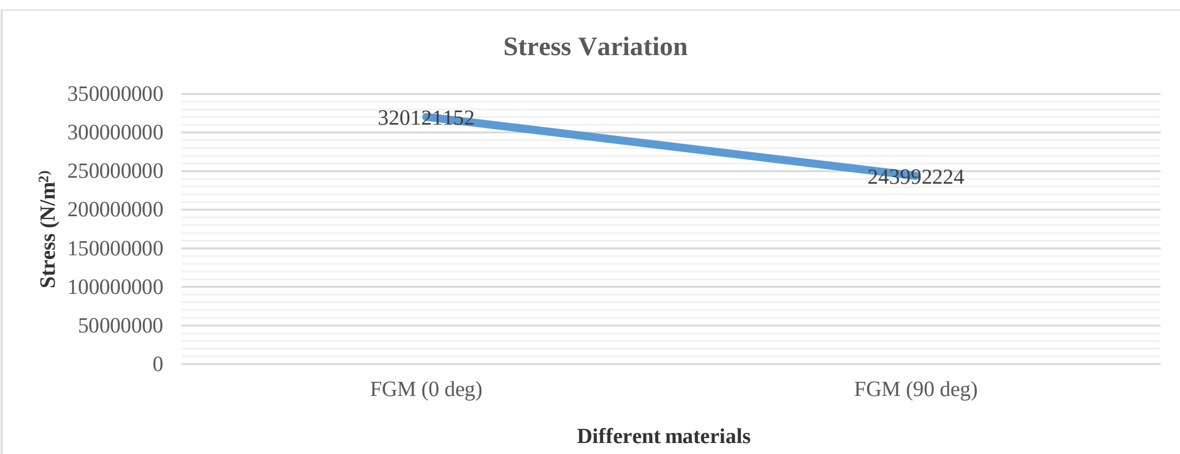


Fig. 5.14: Stress plot for FGM for various Orientations of CNTs under thermo mechanical loading

Under thermo mechanical condition, maximum deflection and maximum stress results are tabulated and graphs are plotted for the same. As explained above strength decreases slight but that is negligible, as it may be because of mechanical load in thermo mechanical loading.

So it can be concluded that effect of orientation of CNTs in FGM, under thermo mechanical loading is negligible or may be because of mechanical load in thermo mechanical loading.

Results for all configurations

Table 5.6: Results for all configurations under thermo mechanical loading

Description	Deflection (nm)	Max Stress (N/m^2)
Al Plate	5.893E-01	195867712.0
CNT Composite	1.53E-01	316203392.0
CNT FGM (0 deg)	5.951E-02	320121152.0
CNT FGM (90 deg)	5.997E-02	243992224.0

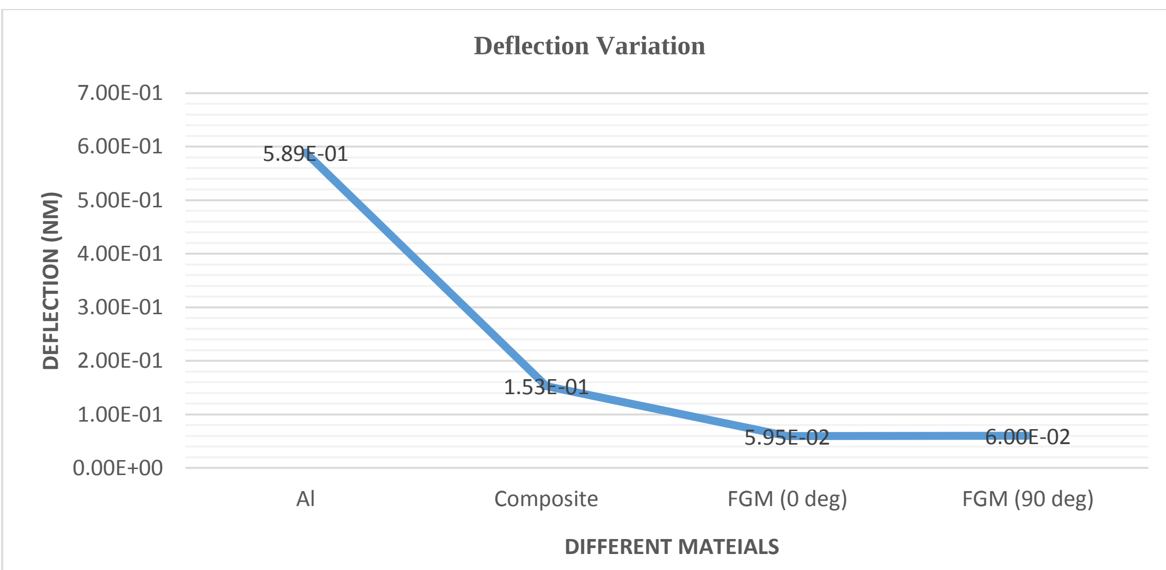


Fig. 5.15: Displacement plot for all configurations under thermo mechanical loading

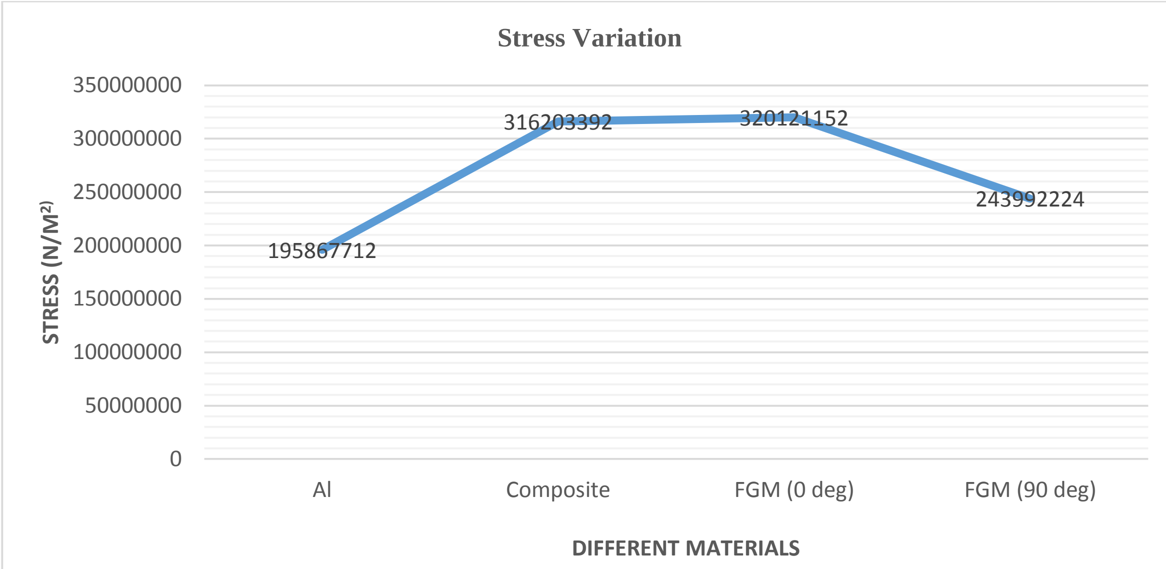


Fig. 5.16: Stress plot for all configurations under thermo mechanical loading

Under thermo mechanical loading, results are tabulate in table 5.6 for all the configurations. Deflection and stress plotted for all the configuration in fig. 5.15 and fig. 5.16 respectively.

From the above results CNT plays in important role for increasing strength of composite and FGM under thermo mechanical loading conditions. But this time, for thermo mechanical loading, FGM shows more strength compare to composite, which was opposite for mechanical loading conditions. Al plate shows poor strength as was expected from this.

Effect of orientation of CNTs in FGM was not effective. So under thermo mechanical loading condition, orientation of CNT are not important during design.

5.3 Free Vibration Conditions

FEA Results for Al, composite & FGM nano plate

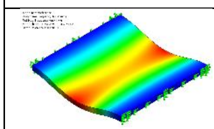
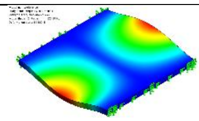
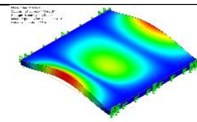
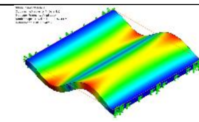
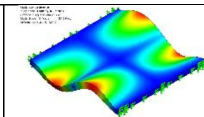
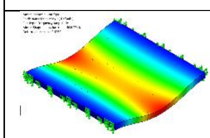
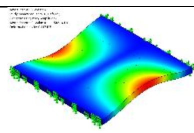
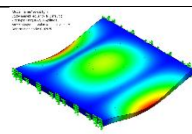
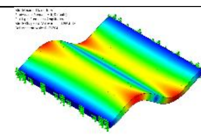
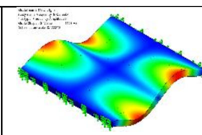
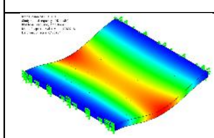
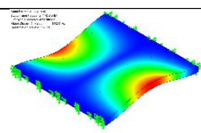
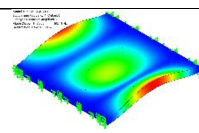
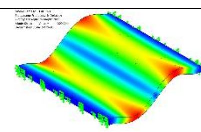
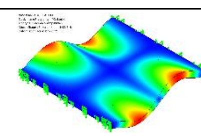
1st Mode (MHz)	2nd Mode (MHz)	3rd Mode (MHz)	4th Mode (MHz)	5th Mode (MHz)
		Al		
 *447.61*	 *525.28*	 *856.54*	 *1207.4*	 *1312.9*
		Composite		
 *506.77*	 *582.14*	 *912.85*	 *1356.4*	 *1458*
		FGM		
 *484.87*	 *560.97*	 *892.13*	 *1300.9*	 *1403.5*

Fig. 5.17: Mode shape results for Al, composite & FGM plate

Fig. 5.17 shows mode shape analysis results for Al, composite and FGM. First five natural frequencies with their mode shapes are presented in above figure. For first mode Al plate have 447 MHz, which is minimum. For composite, it is 506 MHz, which is maximum and 484.87 for FGM, it lies in between Al and composite plate.

These results are as expected as under mechanical loading conditions, composite shows min deflection compare to Al & FGM plate; so it is stiffer, stiffer means more rigid plate, more rigid means maximum natural frequency. While for Al, max deflection- less stiffer – lesser natural frequency. Same explanation for FGM. This pattern is shown for all other natural frequencies too.

Compare mode shapes for all the plates, visually its similar only. It is because that there is not much variation in their natural frequencies.

Table 5.7: Natural Frequency results for Al, composite & FGM plate

	Al	**Composite**	**FGM**
1st (MHz)	447.61	506.77	484.87
2nd (MHz)	525.28	582.14	560.97
3rd (MHz)	856.54	912.85	892.13
4th (MHz)	1207.4	1356.4	1300.9
5th (MHz)	1312.9	1458	1403.5

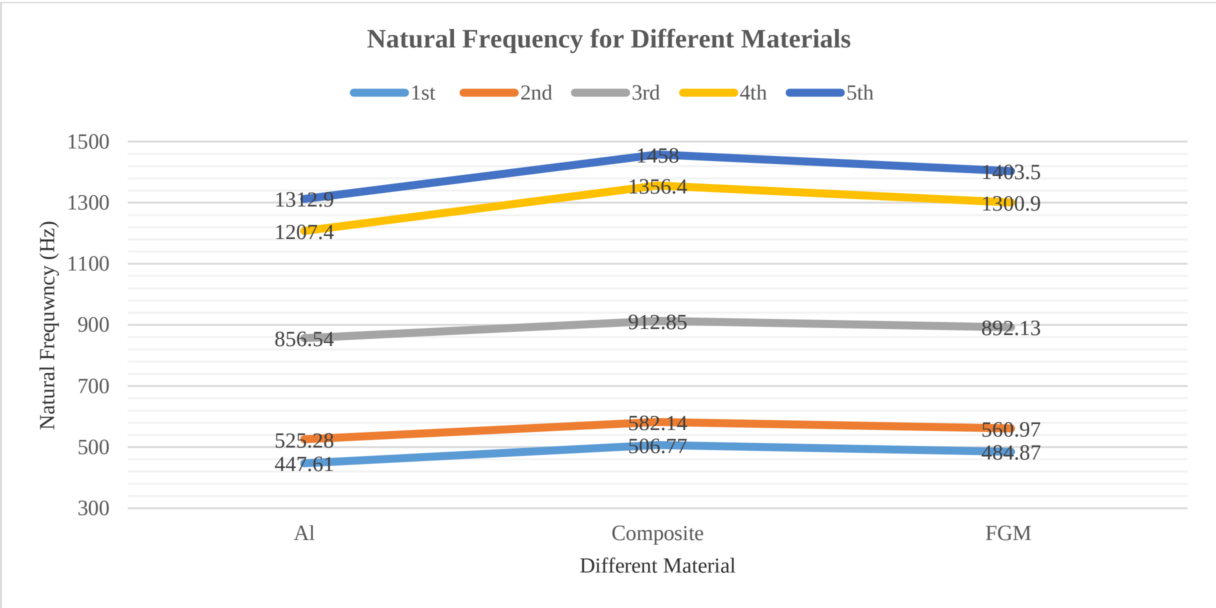
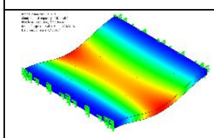
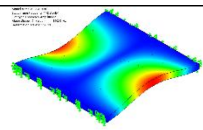
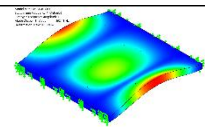
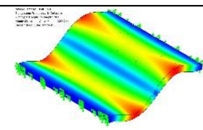
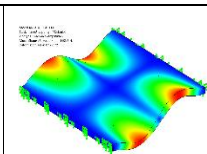
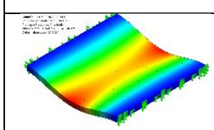
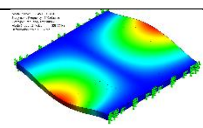
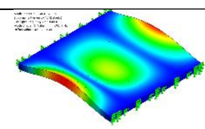
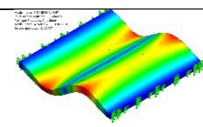
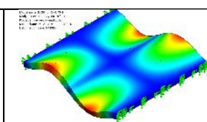
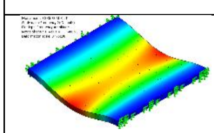
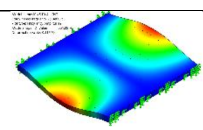
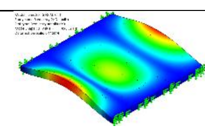
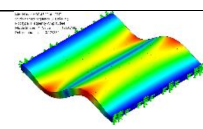
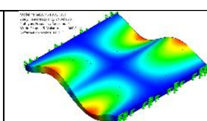


Fig. 5.18: Natural frequency plot for Al, composite & FGM plate

Natural frequency results are tabulated in table. 5.7 and results are plotted in fig 5.18. As expected natural frequency increases from 1st mode to 5th mode. Changes in natural frequencies are as discussed above. But changes variation is decreasing from 1st mode to 5th mode.

So as expected, CNT plays an important role for increasing stiffness of the plate, and for composite is having maximum stiffness.

FEA Results for FGM for various orientations of CNTs

1st Mode (MHz)	2nd Mode (MHz)	3rd Mode (MHz)	4th Mode (MHz)	5th Mode (MHz)
		0 deg Orientation		
 *484.87*	 *560.97*	 *892.13*	 *1300.9*	 *1403.5*
		15 deg Orientation		
 *475.45*	 *553.59*	 *890.47*	 *1288.1*	 *1392*
		30 deg Orientation		
 *468.98*	 *551.33*	 *896.89*	 *1269.7*	 *1380.2*

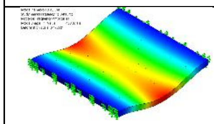
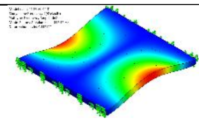
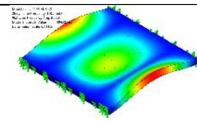
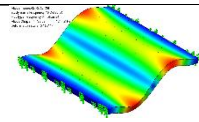
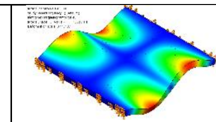
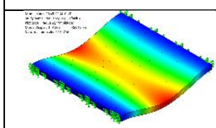
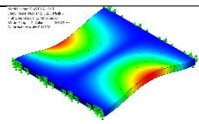
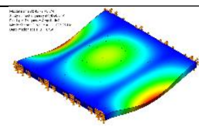
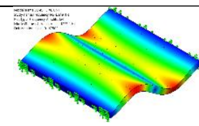
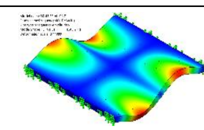
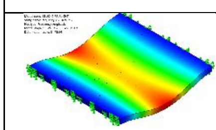
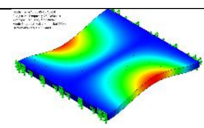
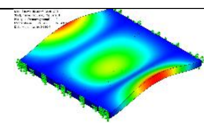
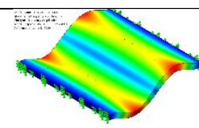
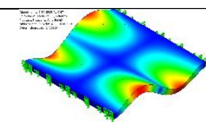
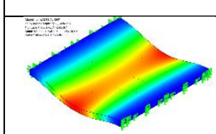
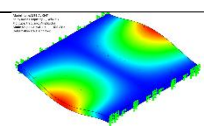
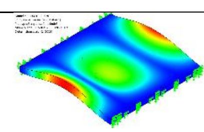
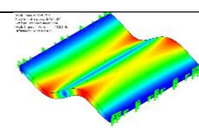
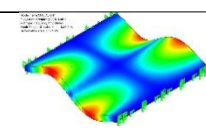
45 deg Orientation				
				
463.07	*543.56*	*894.03*	*1244.8*	*1350.7*
60 deg Orientation				
				
458.99	*540.34*	*896.85*	*1235.1*	*1343.2*
75 deg Orientation				
				
456.75	*536.36*	*895.4*	*1229.4*	*1335.6*
90 deg Orientation				
				
454.36	*532.3*	*895.15*	*1222.9*	*1322.8*

Fig. 5.19: Mode shape results for FGM for various Orientations of CNTs

Above results are there to study the behavior of FGM plate under free vibration conditions, for varying CNTs orientation in FGM from 0 deg to 90 deg in the step of 15 deg as done earlier for mechanical loading.

From the results, it is clear that as orientation varies from 0 deg to 90 deg, Natural frequency decreases. It can be explained as deflection of plate increases, stiffness decreases, so natural frequency decreases, as discussed earlier.

Table 5.8: Natural Frequency results for FGM for various Orientations of CNTs

	0 deg	**15 deg**	**30 deg**	**45 deg**	**60 deg**	**75 deg**	**90 deg**
1^{st} (MHz)	484.87	475.45	468.98	463.07	458.99	456.75	454.36
2^{nd} (MHz)	560.97	553.59	551.33	543.56	540.34	536.36	532.3
3^{rd} (MHz)	892.13	890.47	896.89	894.03	896.85	895.4	895.15
4^{th} (MHz)	1300.9	1288.1	1269.7	1244.8	1235.1	1229.4	1222.9
5^{th} (MHz)	1403.5	1392	1380.2	1350.7	1343.2	1335.6	1322.8

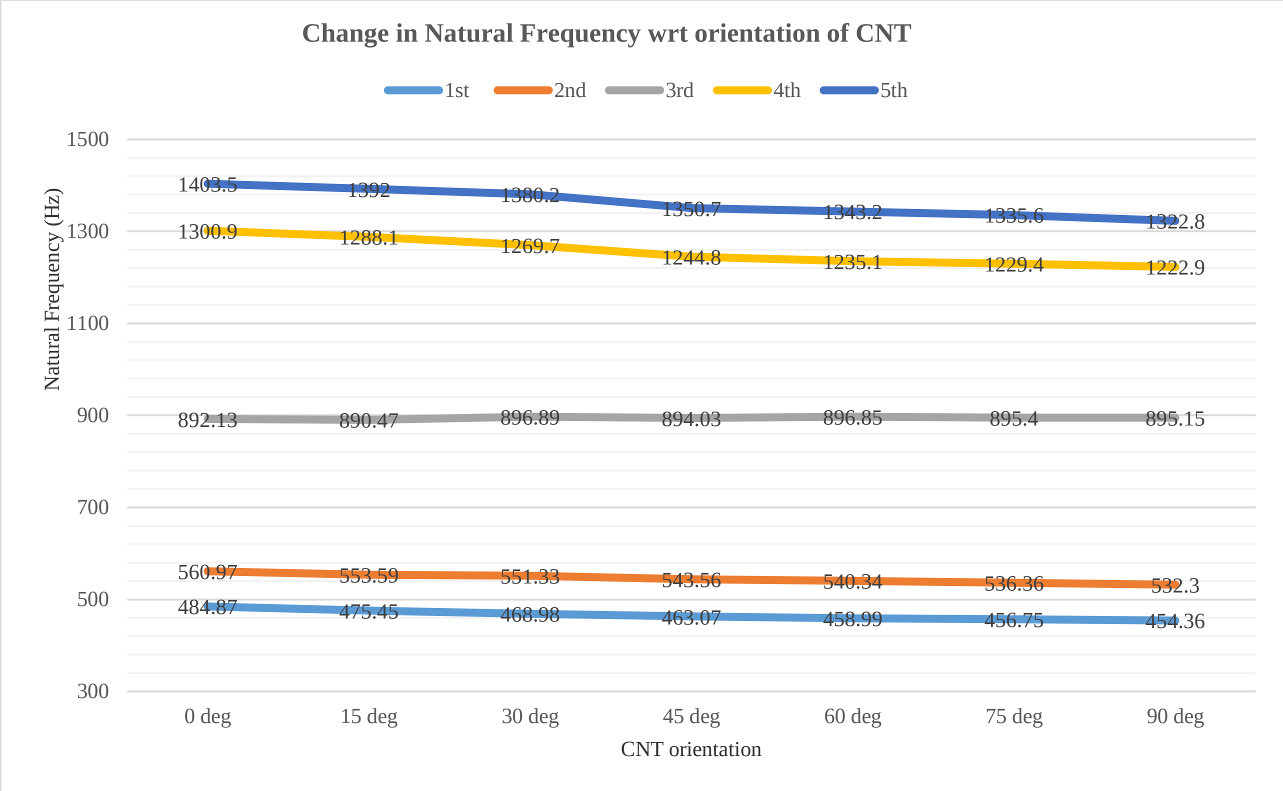


Fig. 5.20: Natural frequency plot for FGM for various Orientations of CNTs

Natural frequency results for various orientation of CNTs in FGM are tabulated in table 5.8, and based on tabled data graphs are plotted in fig. 5.20. As discussed earlier, in the graph it can be seen that natural frequency decrease from 0 deg to 90 deg orientation of CNTs in FGM. There is smooth variation and almost same variation in each step of orientation of CNTs, which keep almost similar for all the frequencies.

Results for all configurations

Table 5.9: Natural frequency stress results for all configurations

	Al	**Composite**	**FGM (deg)**						
			0	**15**	**30**	**45**	**60**	**75**	**90**
1st (MHz)	447.61	506.77	484.87	475.45	468.98	463.07	458.99	456.75	454.36
2nd (MHz)	525.28	582.14	560.97	553.59	551.33	543.56	540.34	536.36	532.3
3rd (MHz)	856.54	912.85	892.13	890.47	896.89	894.03	896.85	895.4	895.15
4th (MHz)	1207.4	1356.4	1300.9	1288.1	1269.7	1244.8	1235.1	1229.4	1222.9
5th (MHz)	1312.9	1458	1403.5	1392	1380.2	1350.7	1343.2	1335.6	1322.8

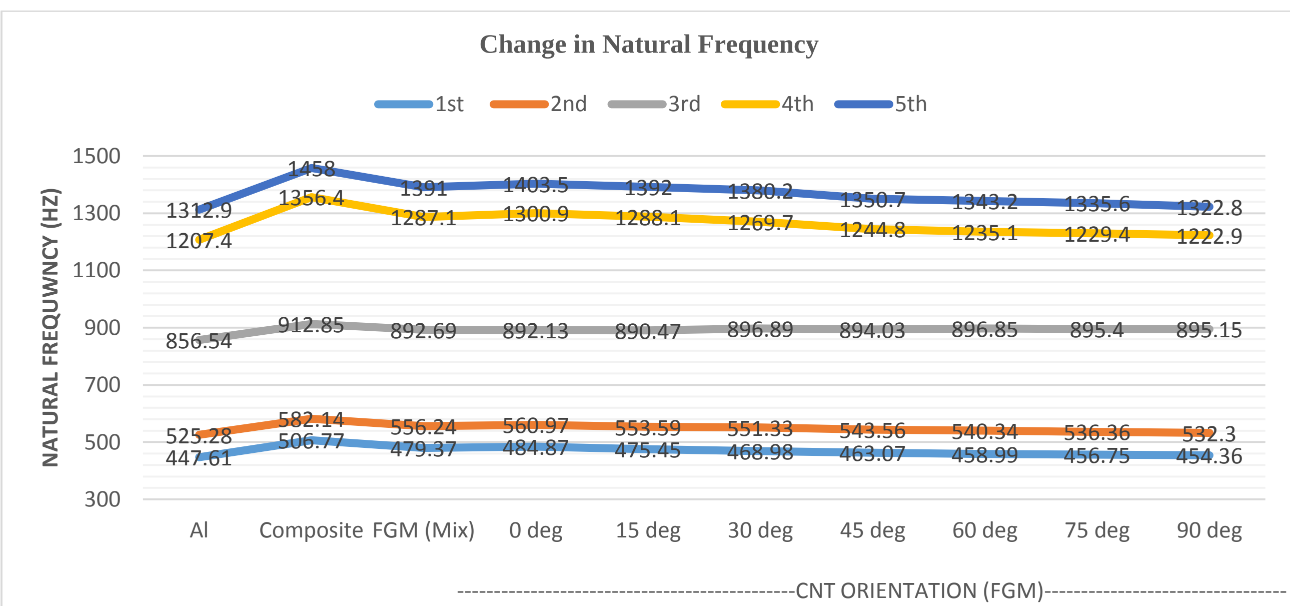


Fig. 5.21: Natural frequency plot for all configurations

For free vibration conditions, all the results are tabulated in table 5.9 and result are plotted in fig 5.21. Results and their pattern is same as discussed earlier. CNT increase stiffness (natural frequency) for composite as well as FGM.

It shows similar results for stiffness as of results under mechanical loading conditions. Composite is having maximum stiffness (natural frequency), while Al plate shows min stiffness (natural frequency) and FGM is having in between composite and Al plate.

CNTs orientation is also important for free vibration condition. At 0 dg orientation of CNT compare to 90 deg orientation of CNT in FGM, provided more stiffness (natural frequency) to the plate.

There is no visual changes in mode shapes for all the configurations.

CHAPTER 6: CONCLUSION & FUTURE SCOPE

CNT reinforced FGM is latest area where lots of research is going on due to its advanced properties in the field of aerospace, defence and bio field etc. Carbon Nanotubes plays an important role in CNT reinforced FGM, due to excellent properties of carbon nanotubes. Effect of CNT has been studied in CNT reinforced FGM for better FGM design.

CNT reinforced FGM plate has been analysed under mechanical loading, thermo mechanical loading and free vibration conditions. Al material is considered as a base material. Results are compare with the base material plate and same materials composite. Orientations of CNTs are varied from 0 deg to 90 deg in the step of 15 deg in FGM, to study the effect of orientation of FGM for all the three loading conditions.

In investigation, following results are concluded:

Under mechanical loading conditions

- CNT plays an important role for increasing strength in composite as well as in FGM.
- CNT reinforced composite is stronger than CNT reinforced FGM. It is because CNTs are uniformly distributed in composite while in FGM it is reducing from top to bottom.
- In CNT reinforced FGM, for CNT orientation varying from 0 deg to 90 deg, in the step of 15 deg, strength decreases. Decreasing strength rate also decreases from 0 deg to 90 deg.

Under thermo mechanical loading conditions

- CNT plays an important role for increasing thermo mechanical strength in composite as well as in FGM.
- CNT reinforced composite is weaker than CNT reinforced FGM. It is because CNTs are more at upper surface than composite. So nos. of CNTs plays important role for high temperature.

- In CNT reinforced FGM, CNT orientation for 0 deg and 90 deg, there is negligible change in strength. So orientation role is negligible under thermo mechanical loading.

Under free vibration conditions

- CNT plays an important role for increase in natural frequency (stiffness) in composite as well as in FGM.
- There is no visual change in mode shapes from changing material.
- CNT reinforced composite is stiffer than CNT reinforced FGM.
- In CNT reinforced FGM, CNT orientation varying from 0 deg to 90 deg, in the step of 15 deg, stiffness decreases.

From the above conclusion it can be said that under mechanical loading conditions and free vibration conditions, CNT reinforced composite is much better than CNT reinforced FGM, where higher mechanical properties are required. For moderate properties, FGM is also can be used. But under thermo mechanical loading condition, FGM is better option than composite.

Future Scope

- Effect of random orientation of CNTs in CNT reinforced FGM need to be studied.
- Multi-wall CNTs, to be considered for analyzing CNT reinforced FGM.
- Effect of various defects like waviness of CNTs in CNT reinforced FGM to be investigated.
- Application based structure to be considered for analyzing CNT reinforced FGM.

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List of Publications

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- *Lalit Kumar, S. P. Harsha* "**Effect of Carbon Nanotubes on CNT Reinforced FGM Nano Plate under Thermo Mechanical Loading**". Procedia Technology-3rd International Conference on Innovations in Automation and Mechatronics Engineering (ICIAME) 2016, GCET, Anand (Gujrat), February 5-6, 2016, volume 23, Pages 130–137. (Elsevier science, Scopus Indexed).

Journal Paper

- *Lalit Kumar, S. P. Harsha* **"Effect of Carbon Nanotubes in Al material as composite and FGM".** Journal of Composites Part B: Engineering (Elsevier) [**Submitted**].