CONTROL OF MAGNETIC LEVITATION SYSTEM

A DISSERTATION

Submitted in partial fulfillment of the
requirements for the award of the degree

Of

MASTER OF TECHNOLOGY

In

ELECTRICAL ENGINEERING

(With specialization in System and Control Engineering)

By

ARINDAM GHOSH

(14530004)

DEPARTMENT OF ELECTRICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY

ROORKEE-247667, (INDIA)

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CANDIDATE'S DECLARATION

I hereby certify that this report which is being presented in the seminar entitled **"CONTROL OF MAGNETIC LEVITATION SYSTEM"** in partial fulfilment of the requirement of award of Degree of **Master of Technology** in **Electrical Engineering with specialization in *System & Control Engineering***, submitted to the Department of Electrical Engineering, Indian Institute of Technology, Roorkee , India is an authentic record of the work carried out during a period from May 2015 to May 2016 under the supervision of **Dr. Yogesh Vijay Hote**, Department of Electrical Engineering, Indian Institute of Technology, Roorkee. The matter presented in this dissertation has not been submitted by me for the award of any other degree of this institute or any other institute.

Date : 23/05/2016

Place : Roorkee **(ARINDAM GHOSH)**

CERTIFICATE

This is to certify that the above statement made by the candidate is correct to best of my knowledge.

Dr. Yogesh Vijay HOTE
Associate Professor
Department of Electrical Engineering,
Indian Institute of Technology,
Roorkee-247667, India

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I would also be thankful to all the staff of Electrical Engineering Department for their constant support and all my friends, for their help and encouragement at the hour of need.

Date: 23/05/2016
Place: Roorkee

ARINDAM GHOSH

ABSTRACT

In this thesis, a non- linear mathematical model of the Magnetic Levitation System has been presented. This modelling consists of the electromagnetic and the mechanical subsystems. The complete system consists of a ferromagnetic ball suspended in a voltage controlled magnetic field. First of all in this thesis, the state space model has been derived from the system equations. It is found that this non-linear model is inherently an unstable one. Therefore, the linear and non-linear state space controllers have been proposed in order to stabilise the system. In this, linear controller has been designed by linearizing the model about an equilibrium point while the non–linear controller has been designed using the feedback linearization technique where the system is linearized using a non-linear state space transformation and a different choice of co-ordinate systems. The linear and non-linear controllers used have been simulated using MATLAB environment.

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CHAPTER - 1

MAGNETIC LEVITATION SYSTEM

1.1) Introduction:

This is a method in which an object can be suspended in mid air only with the help of magnetic forces. The magnetic force produced counter balances the weight of the levitating object.

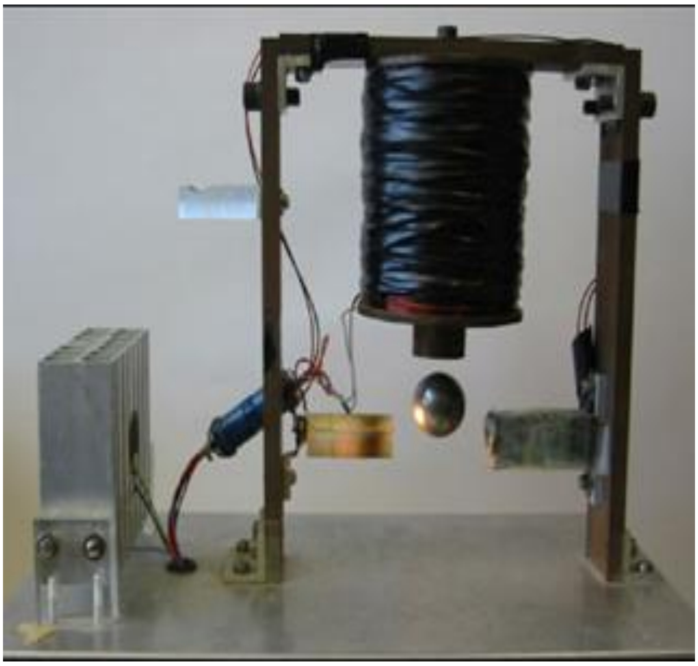

Figure 1.1: Magnetic Levitation System

This strange phenomena has got a wide variety of applications like maglev transportation, magnetic globes, magnetic guns, magnetic floaters, contactless melting etc.

The principle of magnetic levitation is based on the two simple fundamental laws of electromagnetic induction. They are :-

• FARADAY'S LAW
• LENZ'S LAW

In this thesis various types of control schemes have been implemented. The magnetic force produced by the current carrying coil is used to counterbalance the weight of the levitating object. However practically the magnetic force produced by the electromagnetic coil is very much sensitive to system disturbances thus causing it to be unstable. The various types of designed controllers have been analysed for system robustness, stability and output response. The principle function of the controller is to maintain static as well as dynamic equilibrium between the levitating objects weight and the electromagnetic force developed even in the presence of other disturbances. The various advantages of magnetic levitation are as follows:-

Less wear and tear involved since there is no contact between the various parts.

Losses in the system are minimum since there is no friction involved hence producing much higher system efficiency.

Irrespective of the fact that magnetic levitation systems are inherently non linear and highly unstable most of the designs are carried out taking into consideration the linearized models about a particular equilibrium point. In this case with more and more deviation from the equilibrium point the system performance starts to deteriorate. Thus in order to obtain good performance within a large operating range the non- linear model needs to be considered.

The method of feedback linearization is a modern approach to obtain good responses over a large operating range. This method helps to design the control law in such a manner so as to cancel out the non-linear complex parts and then pole placement method is carried out to attain the desired level of stability according to the user needs.

1.2) Simple demonstration of magnetic levitation.

The figure shown below consists of a coil carrying current and a metal ring. The current flowing through the coil is responsible for producing magnetic field . Whenever there is a change in the current, there is a corresponding change in the magnetic field. Thus eddy currents are induced in the metal ring and the corresponding magnetic field is produced in such a manner so as to oppose the cause producing it. In this case like poles are developed in the near ends of the coil and metal ring and hence there is a repulsive force acting between them which is responsible for lifting the ring. Thus magnetic levitation can be achieved.

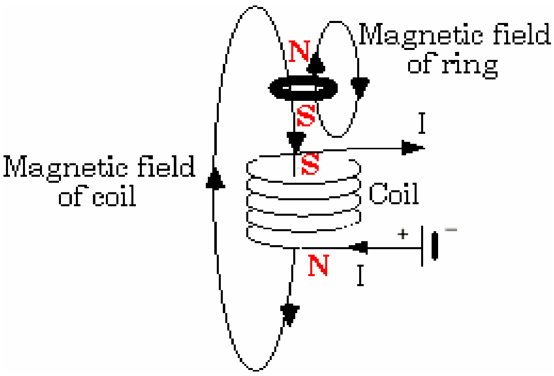

Figure.1.2 (a): Effect of magnetic field- (Ref no-3)

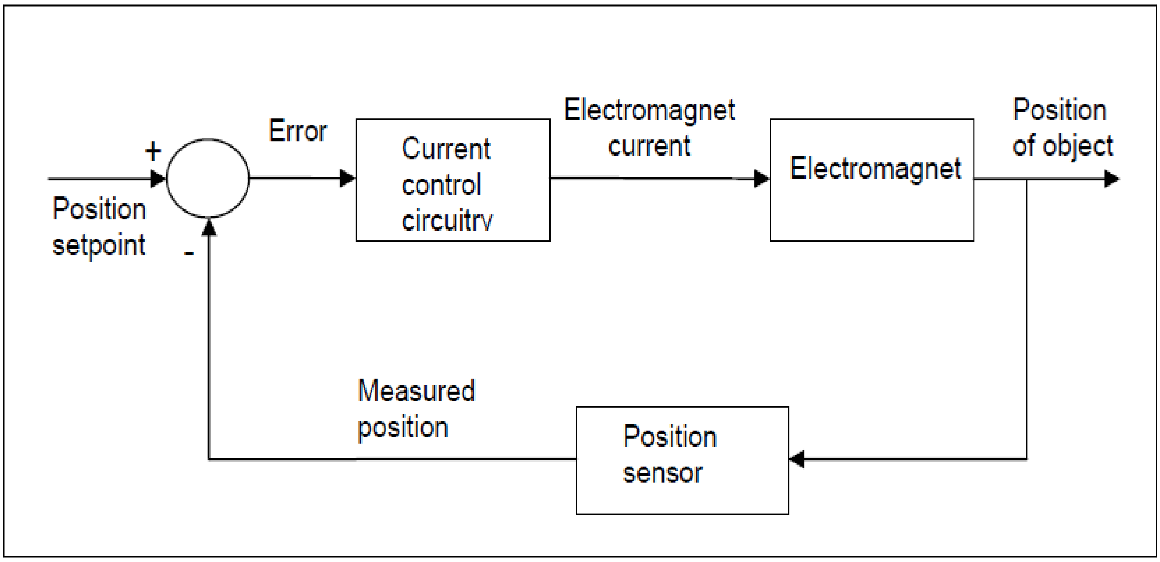

Figure:1.2 (b)- Diagram showing the basic control circuitry of magnetic levitation system - (Ref no – 7)

1.3) LITERATURE REVIEW

A good amount of research has been focussed on the Control of Magnetic Levitation System (MLS). They are being used in a wide variety of applications such as very high speed Maglev trains , magnetic gun, magnetic floaters, frictionless bearings etc. Magnetic Levitation Systems (MLS) are in general highly non-linear and open loop unstable systems. This feature of MLS with its inherent non-linearities make the modelling and controlling part very challenging indeed. Various dynamic models of MLS have been proposed over the the past years and various control strategies have also been used comparing their performances. Both linear and non-linear techniques have been used. The linear system model has a limitation; it only works well over a small region around the operating point.

[1] Wong T. Obtained an approximate linear model of MLS containing an open loop pole in the R.H.S of S-plane. He suggested the use of a linear phase lead compensator in order to stabilize the system about the equilibrium point.

[2] W.Barie and J.Chiasoson in 1996, proposed the transformation of non-linear MLS into a linear one via state and feedback transformations using explicit algorithm. This algorithm allowed to compute explicitly the linearizing state co-ordinates and feedback for any non-linear system.

[3] Ying Shing Shao in 2001, suggested system linearization using virtual pole cancellation and phase lead compensator to design the controller of unstable non-linear MLS.

[4] Bonivento, Gentili and Marconi proposed that a saturated feedback can be successfully used in oreder to solve the problem of positioning a ball in a vertical magnetic field while rejecting some external disturbances. Thus designing an internal model based regulator in presence of some input constraints providing a robust solution with respect to the uncertain parameters.

[5] Valer Dolga and Lia Dolga in 2007, obtained a non-linear model of MLS and proposed the system linearization principle (Fourier series expansion and then preserving only the first order terms while discarding the remaining higher order terms) in order to linearize the original system.

[6] Here the author obtained a non-linear model of MLS and a Lyapunov based stability analysis was used for position control of a magnetically levitated permanent magnet.

[7] Naumivic M.B. Veselic B.R. obtained an approximate linear model of MLS and suggested the use of two linear controllers (proportional derivative and phase lead) and shared that the MLS can be stabilized by an appropriate selection of parameters using classical approach.

CHAPTER-2

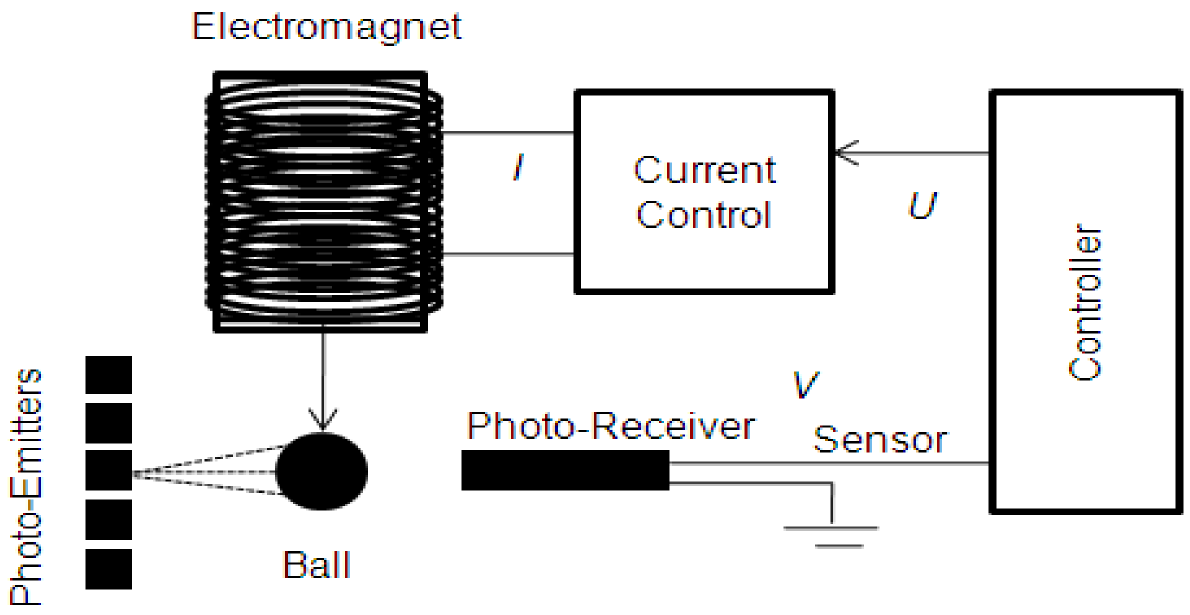

Figure:2 - Schematic figure of a Magnetic Levitation system (Ref no – 11)

2.1) Model Description

In this set up an optical sensor is used to determine the balls position and the coil is acting as electromagnetic actuator. Here we use a controller to vary the amount of current in the circuit and thus the electromagnetic force is adjusted to balance the weight of the ball. Thus the ball will be levitating in an equilibrium state. But the above system is non-linear unstable system that requires a good stabilized control.

Modelling:

This can be subdivided into broadly two categories:-

i. Electromagnetic Modelling
ii. Mechanical Modelling

Electromagnetic Modelling:-

The electromagnetic force produced by the current can be described by KVL

$e(t) = V_R + V_L$

$$e = iR + \frac{dL(x)}{dt} i \ldots \ldots \ldots \ldots \ldots (1.1)$$

Where e = applied voltage to the coil

I = current in the coil

R= coil resistance

L = inductance of the coil.

Mechanical Modelling:-

The net force acting on the ball can be described by Newton's 3rd law of motion.

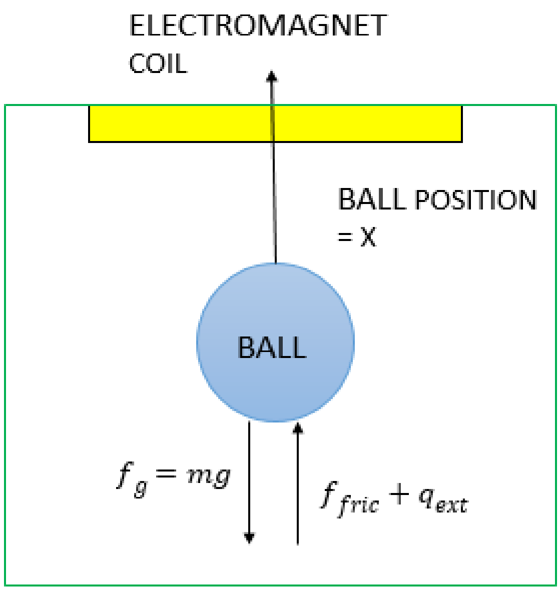

Figure:2.1(a) - Free Body diagram of Magnetic Levitation system (Ref no – 11)

$$f_{net} = f_g - f_{mag} - f_{fric} -.q_{ext} \text{..................................(1.2)}$$

$$m\frac{d^2x}{dt^2} = mg - K\left(\frac{i}{x}\right)^2 - f\frac{dx}{dt} - q_{ext}$$

$$m\frac{d^2x}{dt^2} = mg - K\left(\frac{i}{x}\right)^2 - f\frac{dx}{dt} - q_{ext} \text{ ....(1.3)}$$

Where : m = mass of the levitating ball

f = Coefficient of friction

f_{net} = net force acting on the levitating ball

f_g = force due to gravity on the ball

f_{fric} = frictional force due to air or medium .

K = magnetic force coefficient

q_{ext} = External disturbance force acting (assumed direction is upwards)

$f_1 = \frac{f}{m}$ *and* $q = \frac{q_{ext}}{m}$

Non-Linear Model:-

On the basis of the basis of the equations described above the non-linear model of the magnetic levitation system can be described by the following differential eqns.

$$v = \frac{dx}{dt} \text{ .... (1.4)}$$

$$e = iR + \frac{dL(x)}{dt}i$$

$$m\frac{d^2x}{dt^2} = mg - K\left(\frac{i}{x}\right)^2 - f\frac{dx}{dt} - q_{ext}$$

From eqn(1.5) we can conclude that $L(x)$ is a non-linear function which depends on the position of the ball ‘x’.

Here we can consider the approximation that the inductance varies in an inverse manner with respect to the position of the ball.

$$L(x) = L + \frac{L_e\, x_e}{x} \text{ ....(1.5)}$$

Where L = constant inductance of coil when the ball is absent.

L_e = additional inductance due to the presence of the ball.

x_e = equilibrium position of the ball

$$e(t) = Ri + \frac{d}{dt}\left(L + \frac{L_e x_e}{x}\right)i$$

$$e(t) = Ri + L\frac{di}{dt} - \left(\frac{L_e x_e}{x^2} i\right)\frac{dx}{dt}$$

Now on putting $K = \frac{L_e x_e}{2}$ we get finally,

$$\boldsymbol{e(t) = Ri + L\frac{di}{dt} - 2K\left(\frac{i}{x^2}\right)\frac{dx}{dt}} \quad \text{...(1.6)}$$

Now the system is represented in state space form :

Consider $x = X_1$, $v = X_2$ *and* $i = X_3$

Therefore the final non-linear system in state space model is represented as follows:-

$$\dot{X}_1 = \boldsymbol{x_2} \quad \text{...(1.7)}$$

$$\dot{X}_2 = \boldsymbol{\frac{d^2x}{dt^2} = g - \frac{k}{m}\left(\frac{x_3}{x_1}\right)^2 - f_1 x_2 - q} \quad \text{...(1.8)}$$

$$\dot{X}_3 = \boldsymbol{-\frac{R}{L}x_3 + \frac{2K}{L}\left(\frac{x_3\, x_2}{x_1^2}\right) + \frac{e}{L}} \quad \text{...(1.9)}$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} \boldsymbol{x_2} \\ \boldsymbol{g - \frac{k}{m}\left(\frac{x_3}{x_1}\right)^2 - f_1 x_2 - q} \\ \boldsymbol{-\frac{R}{L}x_3 + \frac{2K}{L}\left(\frac{x_3\, x_2}{x_1^2}\right)} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{\frac{1}{L}} \end{bmatrix}\boldsymbol{e}$$

$$\dot{X} = a(x) + b(x)u$$

$$y = [1 \quad 0 \quad 0] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \quad \text{...(1.10)}$$

2.2) Sensors used in magnetic levitation

Various methods can be used to sense the position of the levitating object.

Optical method :-

A beam of light is placed at the bottom end of the electromagnet and the light is received or detected in the other side. When the distance between the levitating object and the electromagnet decreases more amount of light is obscured and thus the controller limits the the magnitude of current flowing through the coil. When the distance between the levitating object and the electromagnet increases the sensor receives more amount of light and the controller correspondingly increases the magnitude of current flowing through the coil.

For good sensing the light source and the light sensor should be properly aligned. The rate at which the amount of light received by the sensor increases or decreases should follow a linear trend as the levitating object comes closer or moves further away from the electromagnet. Thus the shape of the floating object is also an important factor of concern in this method.

Capacitive method : -

In this method a metal plate is introduced in the space between the electromagnet and the suspended object. The value of the capacitance between the suspended object and the metal plate is measured. This measured value of capacitance is used to find out the distance between the two. Here the capacitance always varies in a linear fashion and it is independent on the object shape.

Hall Effect sensor : -

In this method two Hall sensors are placed on the two ends of the electromagnet, one at the top and the other at the bottom end. This kind of sensor produces an output voltage whenever there is magnetic flux linking it. The corresponding output ports of the two sensors are connected to the two ends of a differential amplifier.

For instance when levitating object is not present both the sensors produce equal voltage as output and the corresponding output of the differential amplifier is zero.When the levitating object approaches closer or moves farther away from the electromagnet there is a change in the amount of magnetic flux received by the sensor placed at the bottom end of the electromagnet while the flux received by the Hall sensor placed at the other end of the electromagnet remains constant. Thus the differential amplifier will produce a corresponding output voltage which is used to measure the distance between the object and the electromagnet. The signal is then sent to the controller to control or limit the magnitude of current flowing through the coil.

Thus the basic system consists of a current control circuit which is used to vary the magnitude of current within a certain maximum limiting value. There must be a sensor circuit which is used to determine the position of the suspended object along with an active source of supply. However it has been seen that oscillations occur in the suspended ball which is caused by the lag of phase between the electromagnet and the current controller circuit due to the large value of inductance present in the magnetic coil. Physically it is seen that the current control circuit responds very slowly with respect to the change in distance of the levitating ball from the electromagnet. Thus in order to obtain good and stable responses the phase lag between the two systems must be neutralised which is possible by adding a phase lead part to it. Thus in respect of control systems it is not sufficient only to measure the position of the levitating ball or object the velocity of the ball i.e. the rate of position change value is to be taken into consideration as well.

CHAPTER -3

Simulink model of Magnetic Levitation System

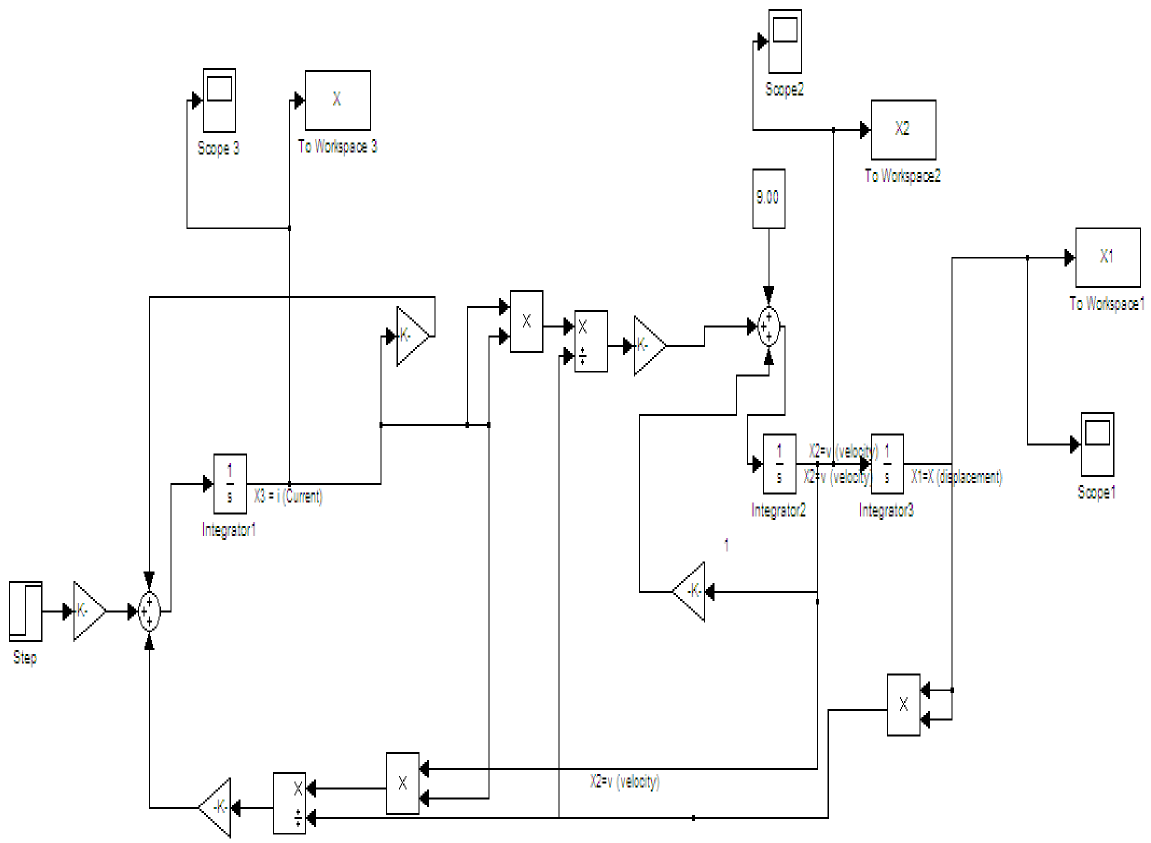

Figure: (3.1) - Non Linear open loop simulink model of the magnetic levitation system

The above figure shows the Simulink block diagram of the non-linear system:-

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\\ \dot{X}_3\end{bmatrix} = \begin{bmatrix} x_2 \\ g - \frac{k}{m}\left(\frac{x_3}{x_1}\right)^2 - f_1 x_2 - q \\ -\frac{R}{L}x_3 + \frac{2K}{L}\left(\frac{x_3\, x_2}{x_1^2}\right)\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \frac{1}{L}\end{bmatrix} e$$

$$y = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}X_1\\ X_2\\ X_3\end{bmatrix}$$

Responses of the non-linear unstable magnetic levitation model

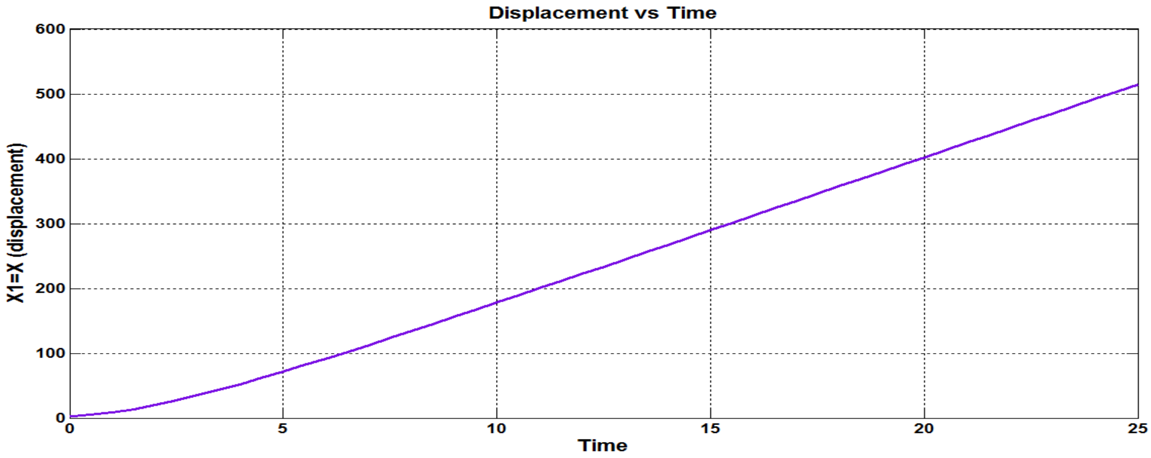

Figure: 3.1 (a)-Displacement vs Time

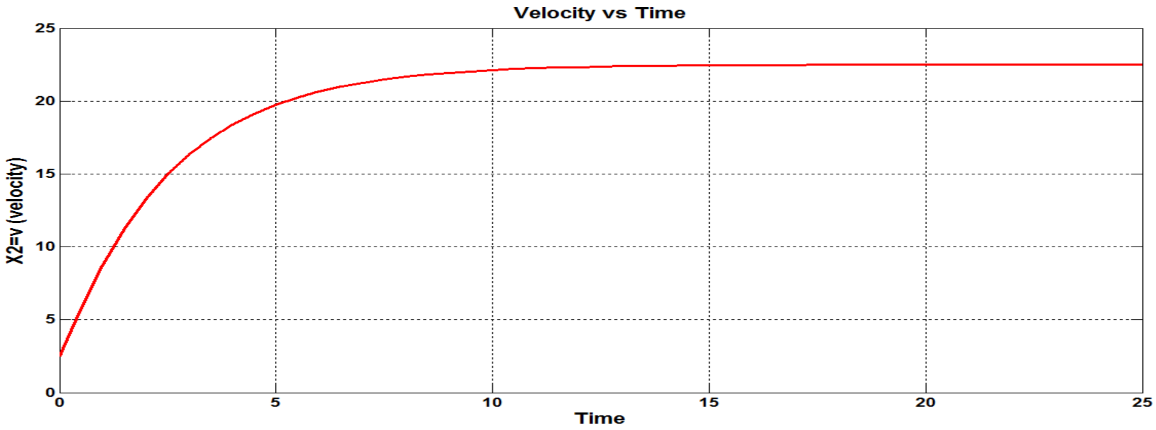

Figure: 3.1 (b)-Velocity vs Time

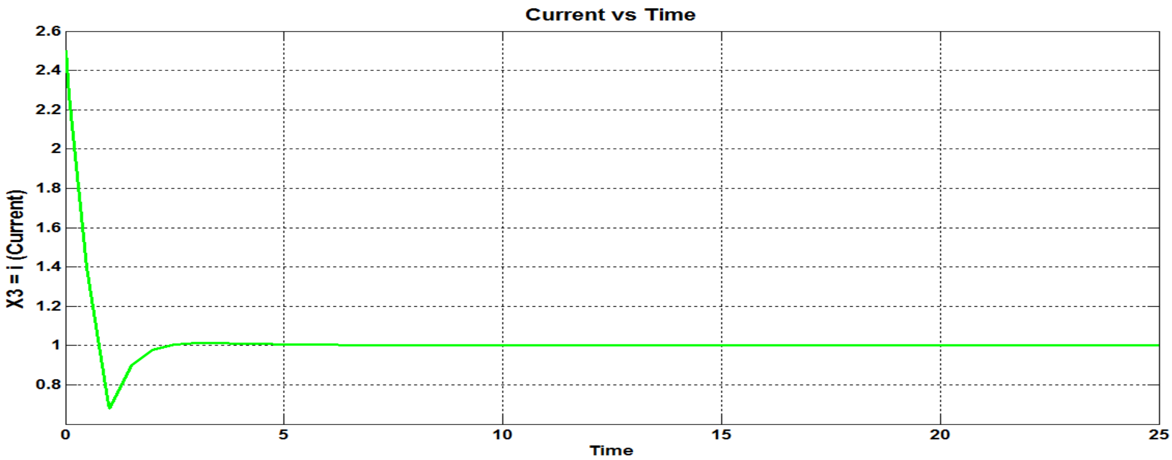

Figure: 3.1 (c)- Current vs Time

3.2) Derivation of a linear model using Jacobian matrix method

Here, linear model of the above non-linear system using the Jacobian matrix method is derived which is given below.

At the equilibrium point $\dot{X}_1 = 0,\ \dot{X}_2 = 0,\ \dot{X}_3 = 0$,.

$\Rightarrow\ x_{2e} = 0$

$$A_{sys} = \begin{bmatrix} \frac{\partial w_1}{\partial x_1} & \frac{\partial w_1}{\partial x_2} & \frac{\partial w_1}{\partial x_3} \\ \frac{\partial w_2}{\partial x_1} & \frac{\partial w_2}{\partial x_2} & \frac{\partial w_2}{\partial x_3} \\ \frac{\partial w_3}{\partial x_1} & \frac{\partial w_3}{\partial x_2} & \frac{\partial w_3}{\partial x_3} \end{bmatrix} \quad (3.1)$$

Where $w_1 = x_2$ ……………………………… (3.2)

$$w_2 = g - \frac{k}{m}\left(\frac{x_3}{x_1}\right)^2 - f_1 x_2 - q \quad (3.3)$$

$$w_3 = -\frac{R}{L} x_3 + \frac{2K}{L}\left(\frac{x_3\, x_2}{x_1^2}\right) \quad (3.4)$$

$$A_{sys} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2Kx_{3e}}{m{x_{1e}}^3} & -f_1 & -\frac{2K\, x_{3e}}{m\, {x_{1e}}^2} \\ 0 & \frac{2Kx_{3e}}{L\, {x_{1e}}^2} & -\frac{R}{L} \end{bmatrix} \qquad B_{sys} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}$$

$$C_{sys} = [1 \quad 0 \quad 0]$$

3.3) Assumption of Parameter values:-

Let: R = 1 Ω

m = 0.2 Kg

x_e = 0.3 metres

K = 0.5 L = 0.6 H q = 0.8 f_1 = 0.8

On substituting the above assumed parameter values we get,

$$A_{sys} = \begin{bmatrix} 0 & 1 & 0 \\ 65.34 & -0.4 & -33 \\ 0 & 11 & -1.667 \end{bmatrix} \qquad B_{sys} = \begin{bmatrix} 0 \\ 0 \\ 1.667 \end{bmatrix}$$

The eigenvalues of A_{sys} are as follows:

$\boldsymbol{\lambda}$ = 0.3640;

$\boldsymbol{\lambda}$= -1.2155 +17.255j

$\boldsymbol{\lambda}$= -1.2155-17.255j

Since we can see that one pole is lying to the left half of s-plane so the above system is an unstable one.

Now converting the above state space model into the transfer function form. Finally we get the transfer function of the approximate linearized magnetic levitation system as shown below.

$$\mathbf{G_m(S)} = \frac{\mathbf{55.01}}{\boldsymbol{S^3 + 2.067S^2 + 298.3S - 108.9}} \qquad (3.5)$$

The Simulink block diagram of the linearized model is as shown below:-

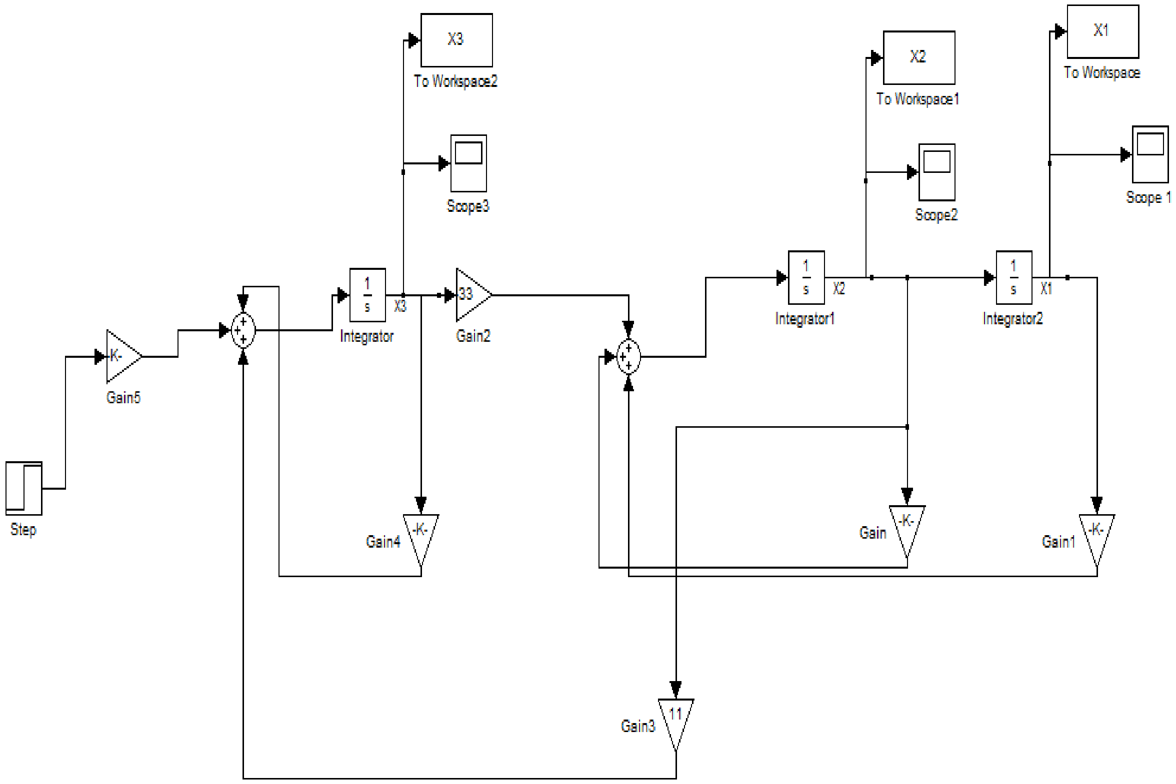

Figure: (3.4) - Linearised unstable open loop simulink model of magnetic levitation system

The responses of the linearised unstable magnetic levitation model is given below:

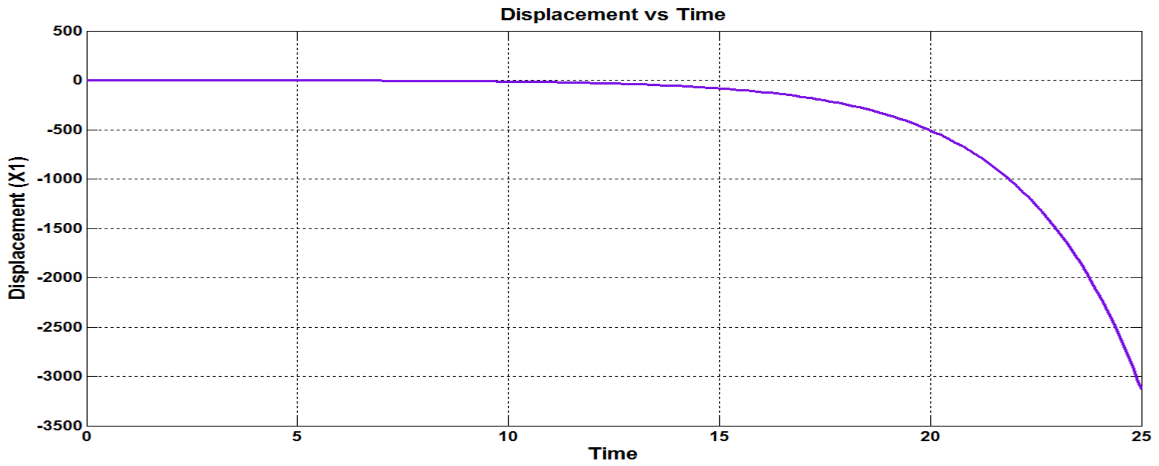

Figure: 3.4 (a)- Displacement vs Time

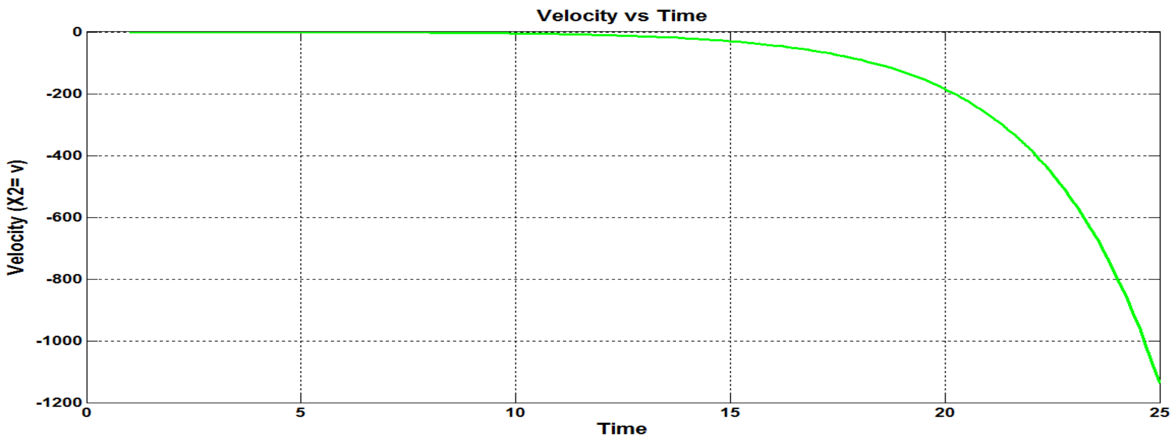

Figure: 3.4 (b)- Velocity vs Time

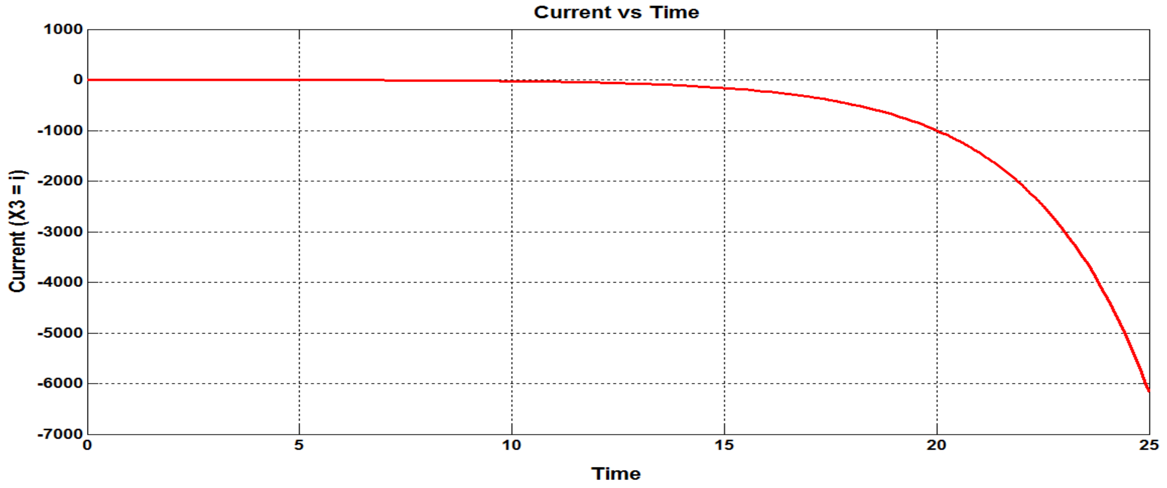

Figure: 3.4 (c)- Current vs Time

Pole placement technique:-

Using this method the poles can be placed at the desired locations according to the user needs say at (-1,-2,-3).

$$P = \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix} \qquad \text{..........(3.6)}$$

Here the MATLAB command used as ,

k = acker (A, B, P)

$\Rightarrow \quad k = [-6.7605 \quad 5.2517 \quad 2.3593]$..........(3.7)

So control law used is :

$\boldsymbol{e = 6.7605x_1 - 5.2517x_2 - 2.3593x_3}$..........(3.8)

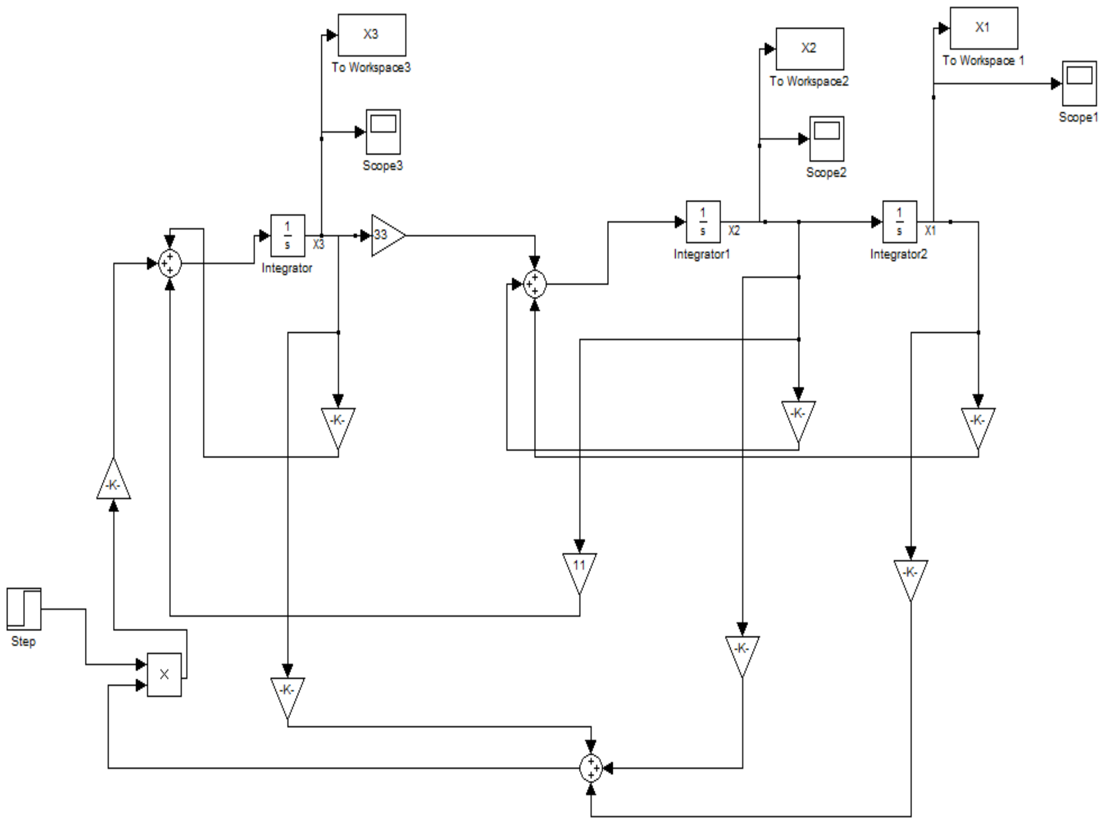

Figure:(3.5) - Simulink block diagram of the linear closed loop system (using linear feedback law)

Responses for poles at (-1,-2,-3)

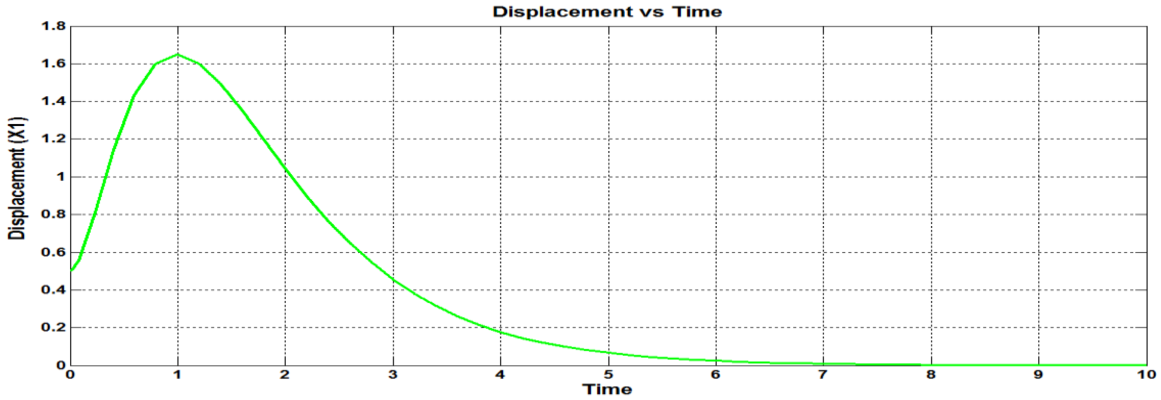

Figure: 3.5 (a) – i: Displacement vs Time

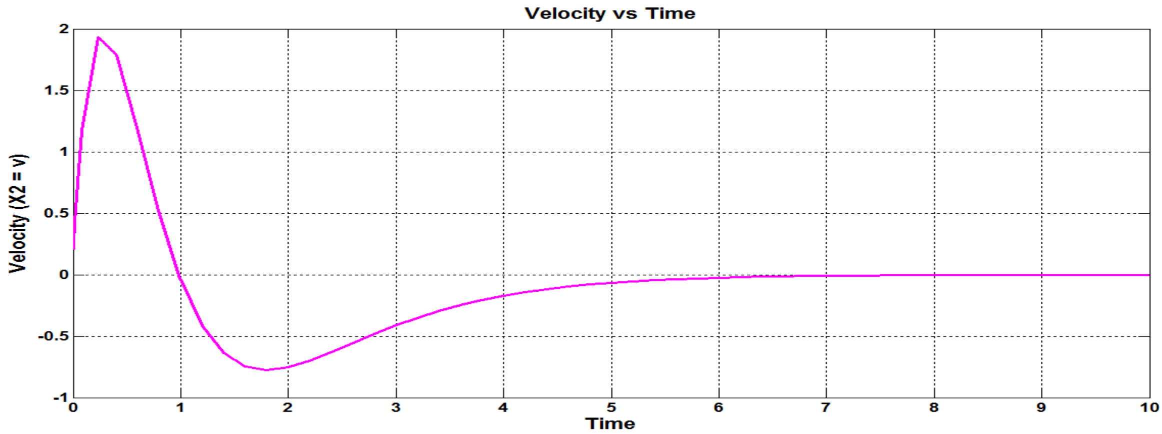

Figure: 3.5 (a) – ii: Velocity vs Time

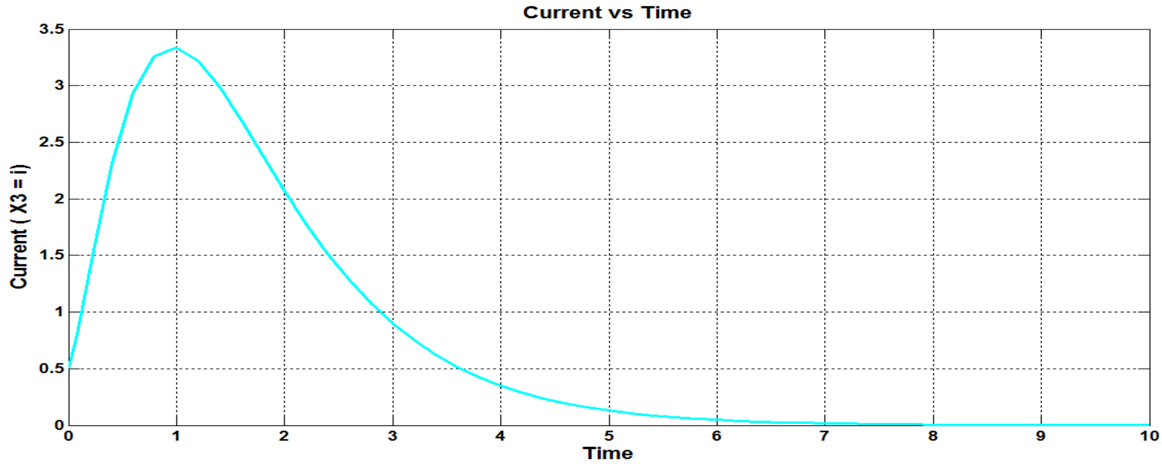

Figure: 3.5 (a) – iii: Current vs Time

Responses for poles at (-3.5,-5,-6)

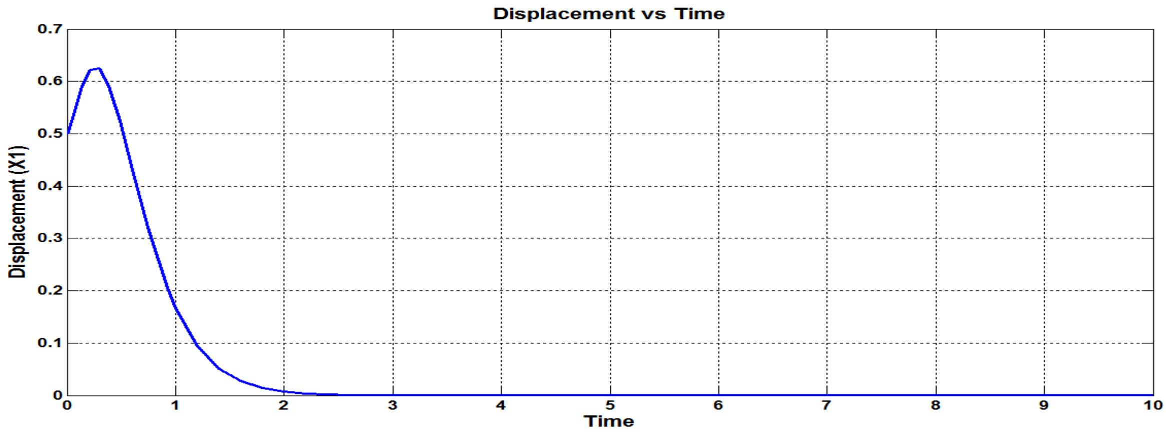

Figure: 3.5 (b) – i: Displacement vs Time

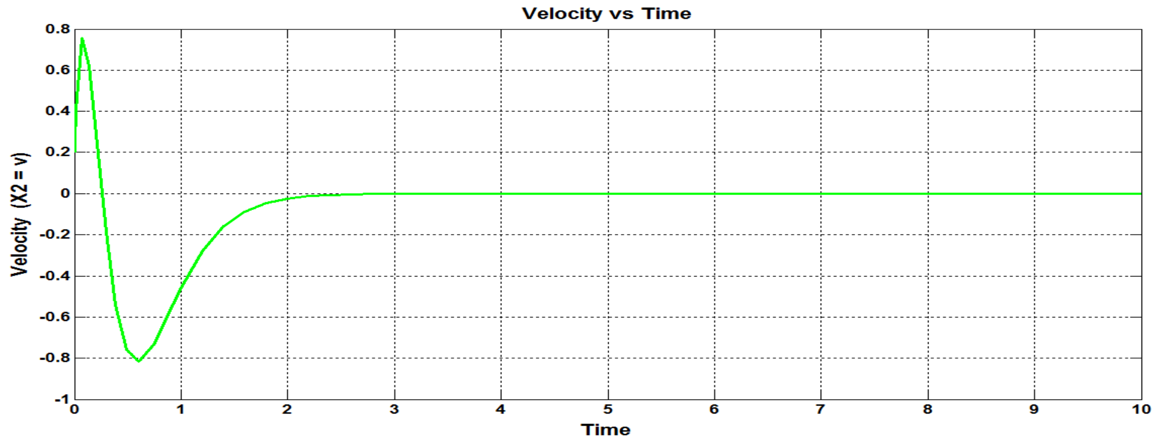

Figure: 3.5 (b) – ii: Velocity vs Time

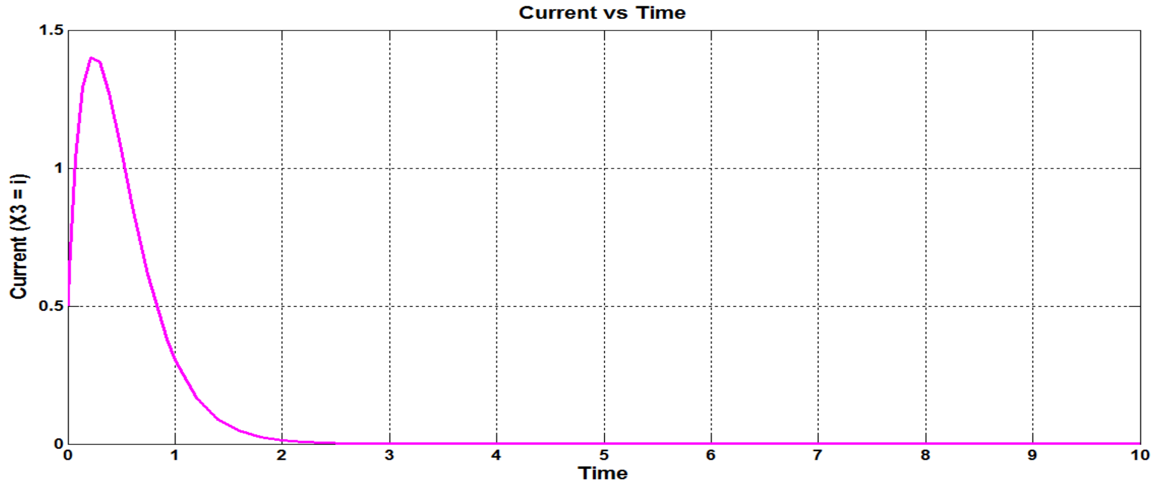

Figure: 3.5 (b) – iii: Current vs Time

Responses for poles at (-6,-9,-11)

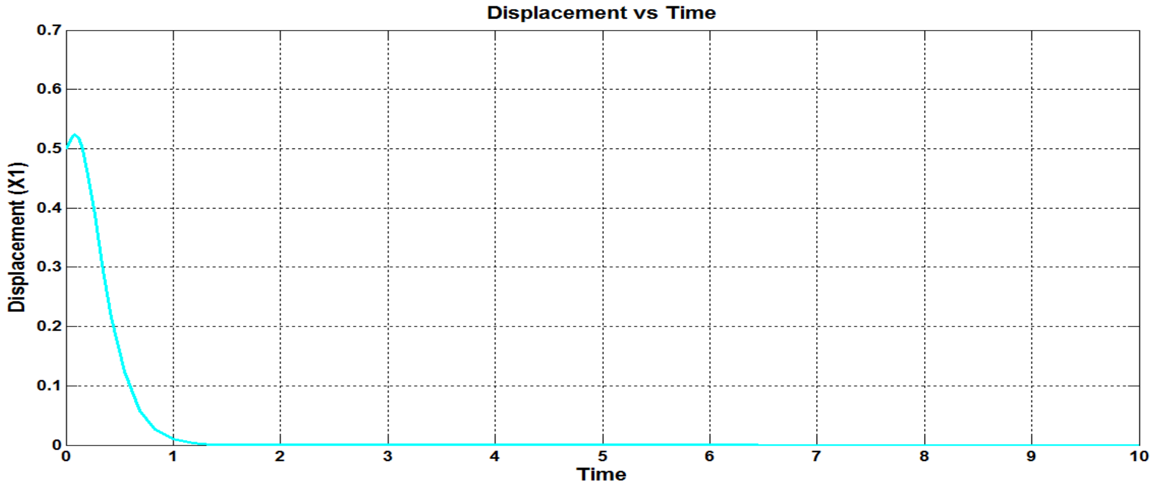

Figure: 3.5 (c) – i: Displacement vs Time

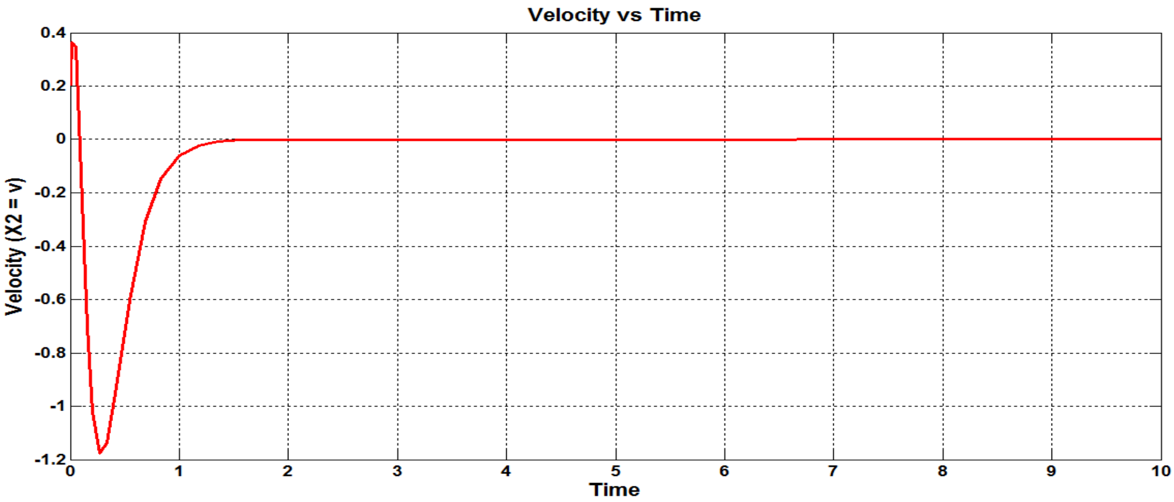

Figure: 3.5 (c) – ii: Velocity vs Time

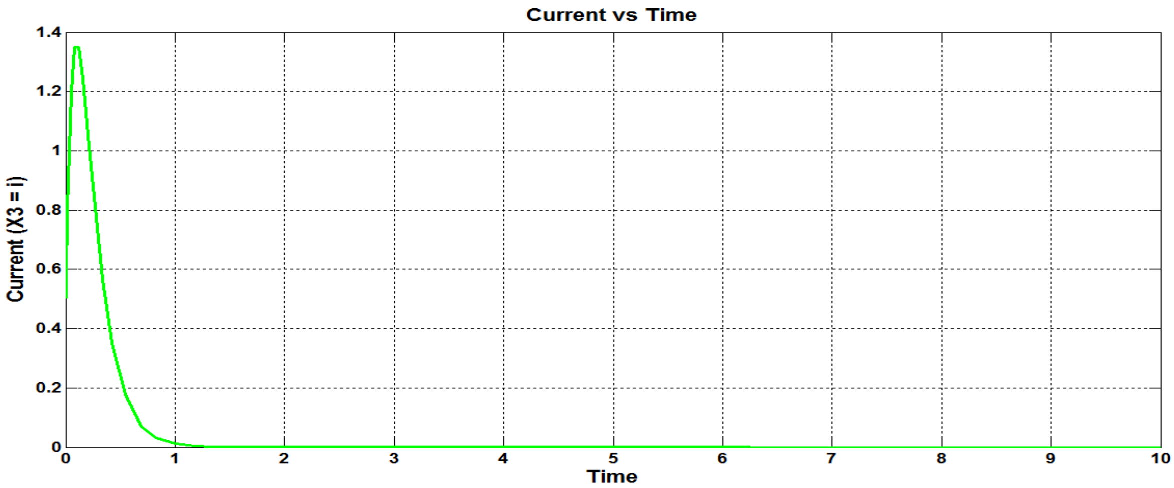

Figure: 3.5 (c) – iii: Current vs Time

3.6) Phase Lead Compensated Controller

In this method the magnetic levitation system is stabilized about an working or equilibrium point. It can easily be seen that the open loop system is inherently unstable and stabilization cannot be achieved by changing the system gain. The open loop uncompensated system contains an unstable pole in the right hand side of s-plane. A phase lead compensator can be used in order to cancel out the unstable pole and stabilize the system. The pole of the lead compensator is placed far away from the origin in the left hand side of s-plane so that this pole has the minimum effect on the system root locus. The transfer function of the phase lead compensator is of the form as shown below:-

$$R(s) = K\frac{(S+Z)}{(S+P)}$$

Transfer function of phase lead compensator :

$$R(s) = 10\frac{(S-0.3640)}{(S+100)} \qquad \text{where K=10 (assumed)}$$

$$= \frac{10S-3.640}{S+100} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.9)$$

$$G_m(S) = \frac{55.01}{S^3 + 2.067S^2 + 298.3S - 108.9} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.10)$$

So the overall transfer function after using the phase lead compensator is:

$$T(s) = \frac{550.1S+200.20}{S^4 + 102.1S^3 + 505S^2 - 162.265S - 10890} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.11)$$

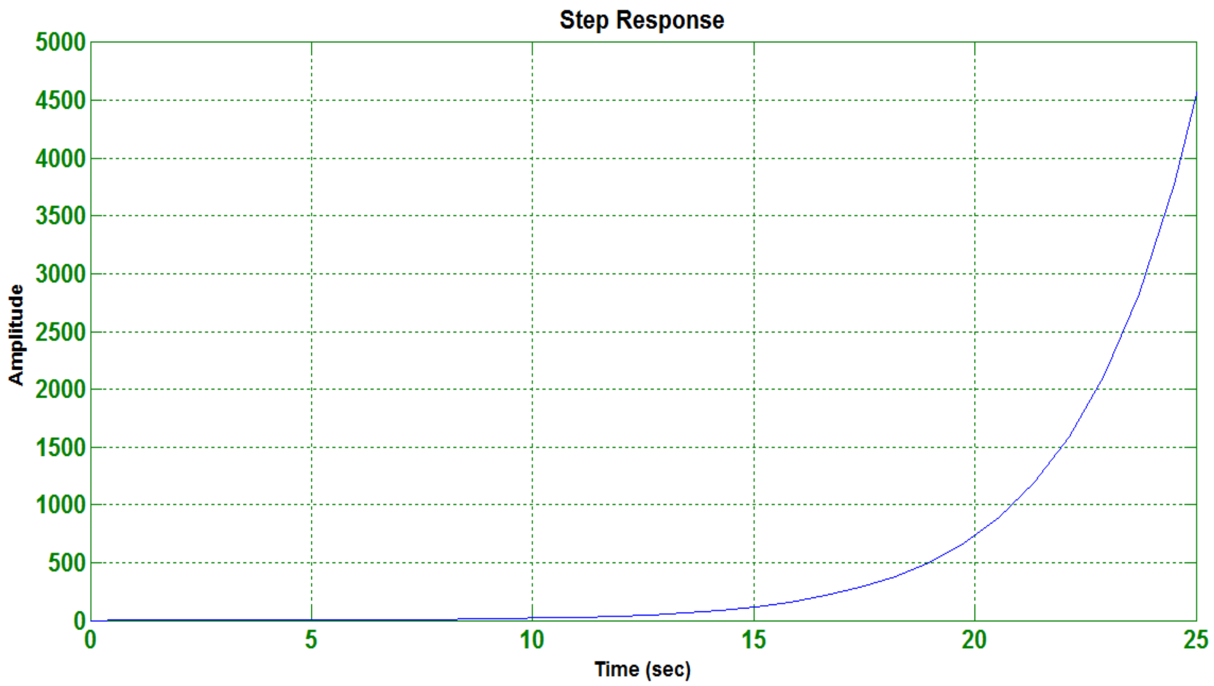

Figure :3.6 (a) - Step Response of uncompensated system

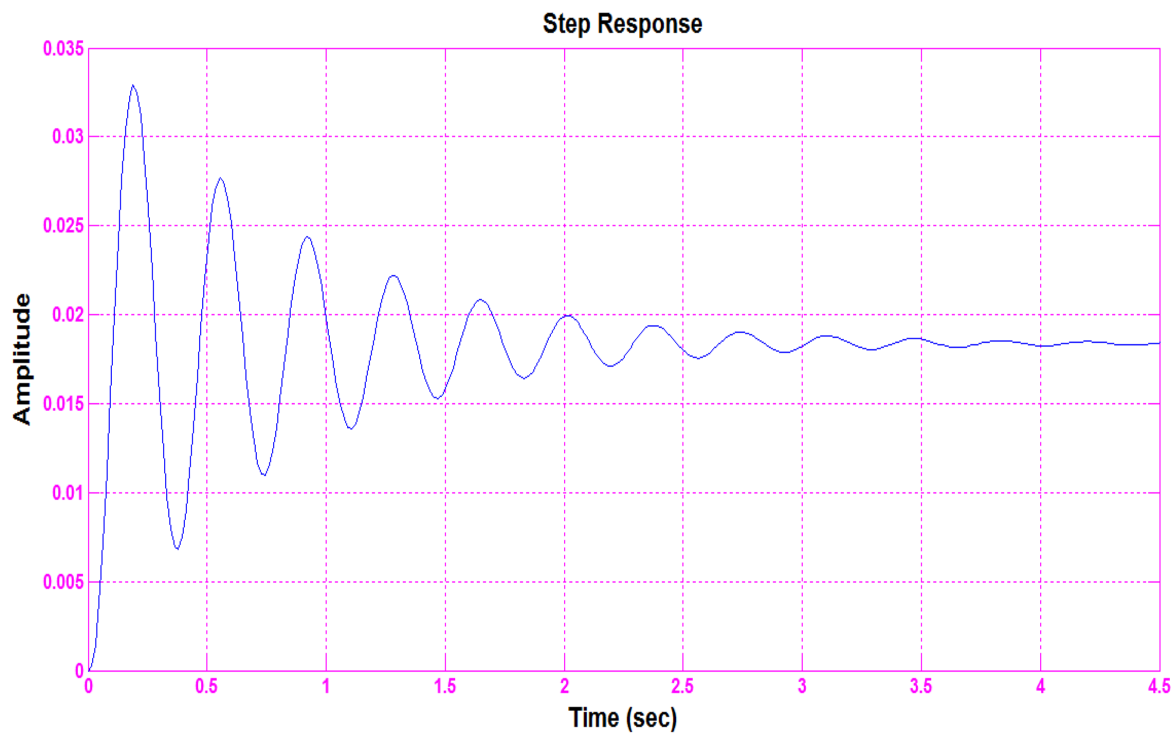

Figure :3.6 (b) - Step Response after using phase lead compensator

3.7) Using PD Compensator

$$G_m(S)H_m(S) = \frac{55.01\,DS + 55.01P}{S^3 + 2.067S^2 + 298.3S - 108.9} \quad \ldots\ldots(3.12)$$

Where P = Proportional gain

D = Derivative gain

$1 + G_m(s)H_m(s) = 0$

$S^3 + 2.067S^2 + 298.3S - 108.9 + 55.01P + 55.01DS = 0$

$S^3 + 2.067S^2 + (298.3+55.01D)S + (55.01P - 108.9) = 0 \quad \ldots\ldots(3.13)$

Routh Table

S^3	1	298.3 + 55.01D	0
S^2	2.067	55.01P – 108.9	0
S^1	350.985 – 26.61P + 55D	0	0
S^0	55.01P – 108.9	0	0

For the above system to be a stable one all the elements of the first row of the Routh table should be positive and of the same sign.

⇒ **P > 1.979**……………………………………………………………………...(3.14)

Also 350.985 – 26.61P + 55D > 0

⇒ 350.985> 26.61P – 55D

P < 13.189 + 2.066D…………………………………………………………......(3.15)

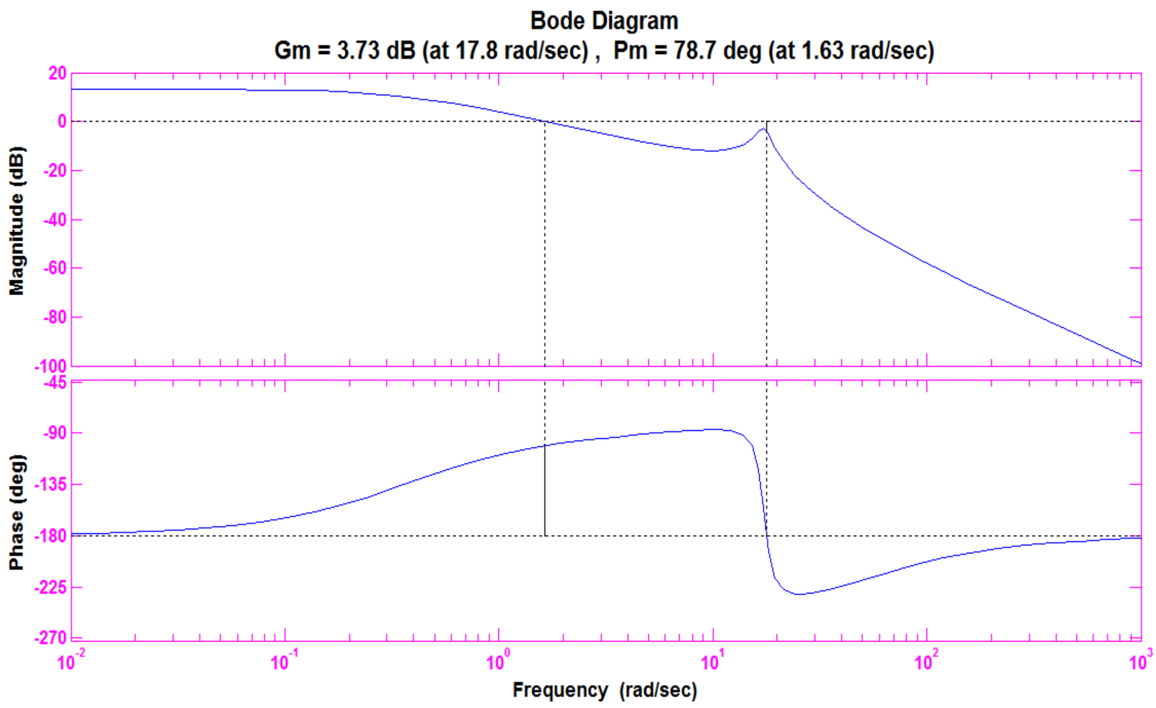

Figure :3.7 (a) - Gain and phase plots for D = 0.2 and P = 9.0 (Stable system)

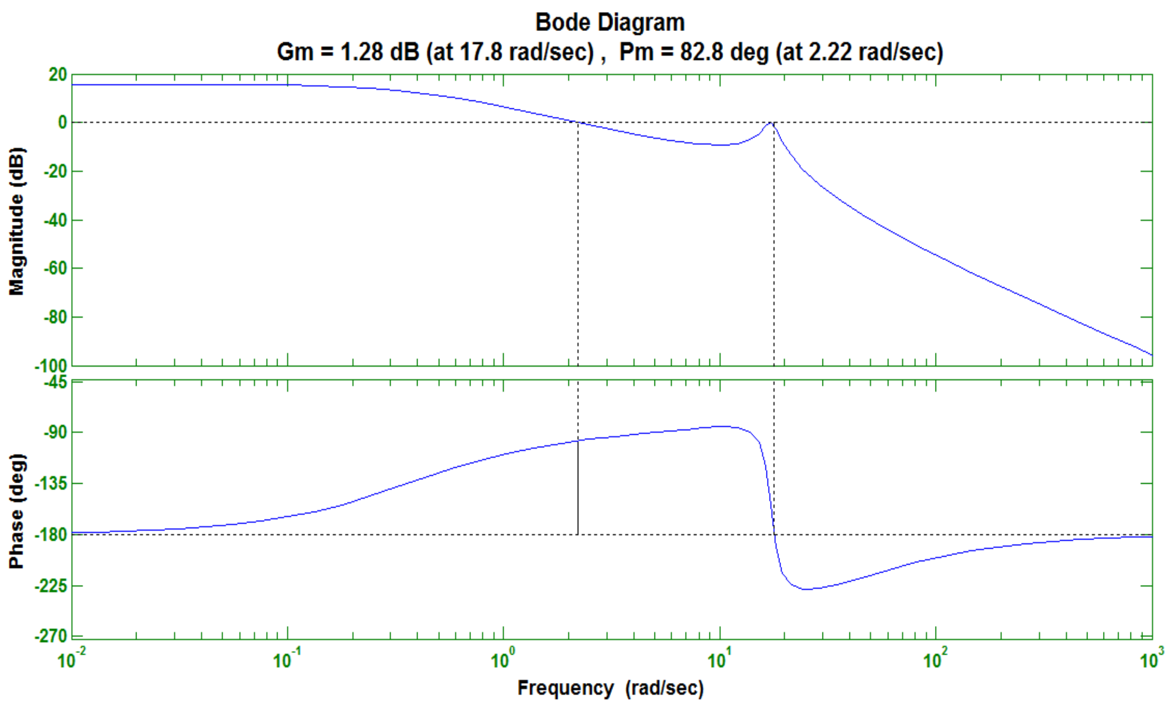

Figure :3.7 (b) - Gain and phase plots for D = 0.3 and P = 12.0 (Stable system)

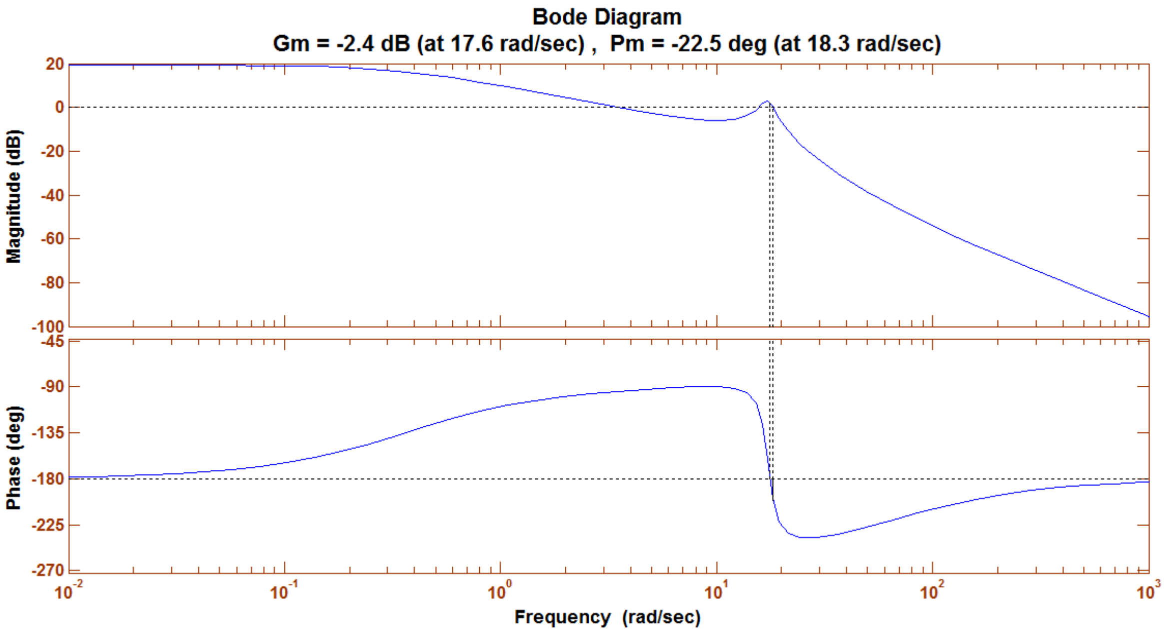

Figure :3.7 (c) - Gain and phase plots for D = 0.3 and P = 18.0 (Unstable system)

3.8) PID CONTROL OF MAGNETIC LEVITATION SYSTEM

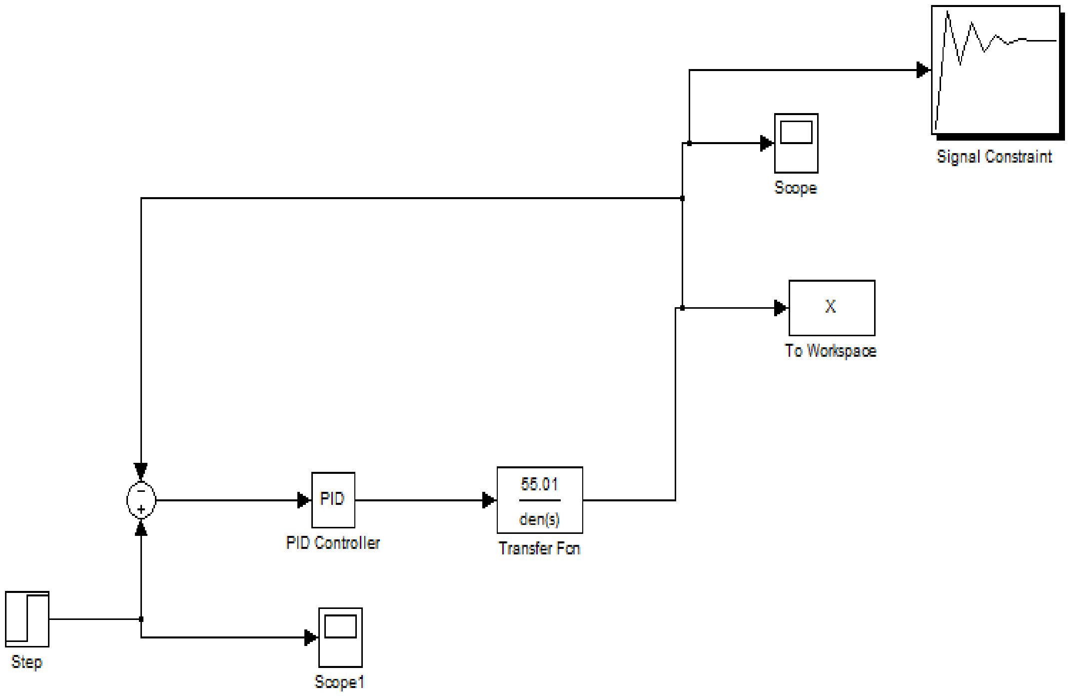

Figure :3.8 (a)-Closed loop linear model using PID controller

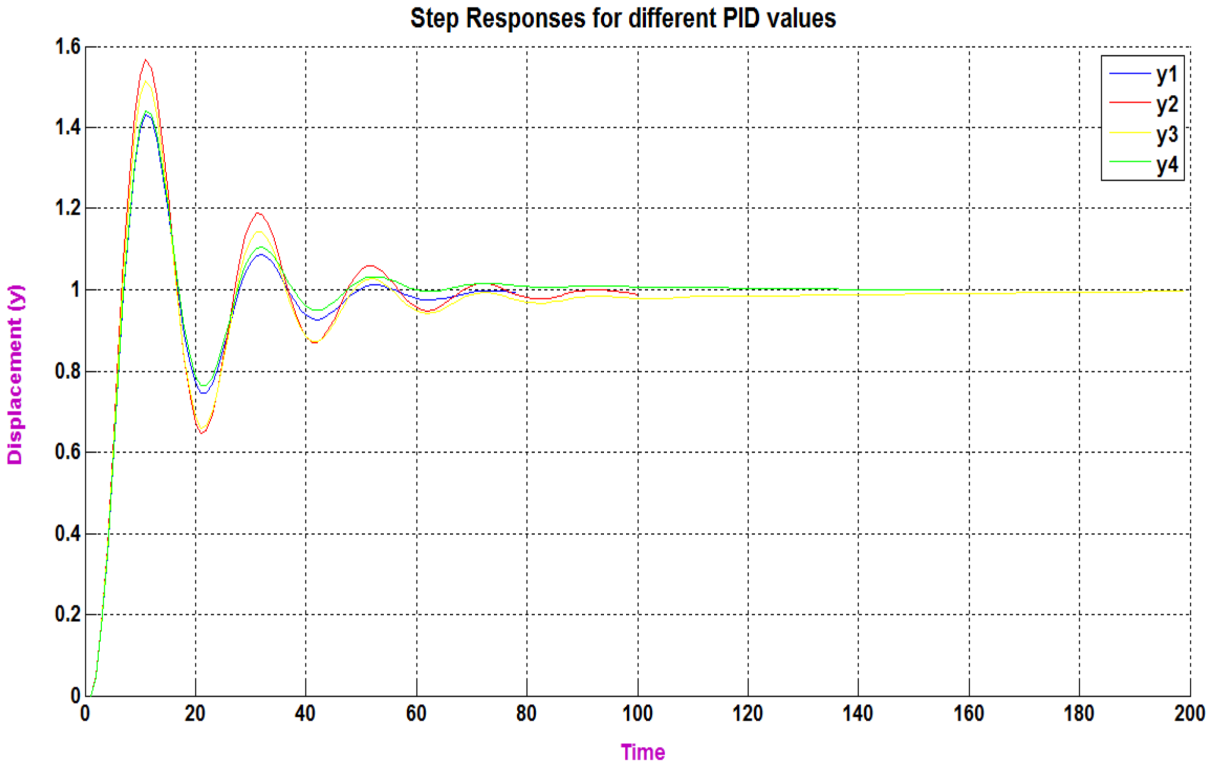

Figure :3.8 (b)- Step responses for different PID values

For the above figure, the table below describes the different PID parameter values along with their step response names.

PID VALUES	***RESPONSE CURVE***
➢ **Kp = 2.5 Ki = 1.2 Kd = 2.0**	**y1**
➢ **Kp = 5.0 Ki = 1.2 Kd = 2.0**	**y2**
➢ **Kp = 2.5 Ki = 1.2 Kd = 4.0**	**y3**
➢ **Kp = 2.5 Ki = 2.6 Kd = 2.0**	**y4**

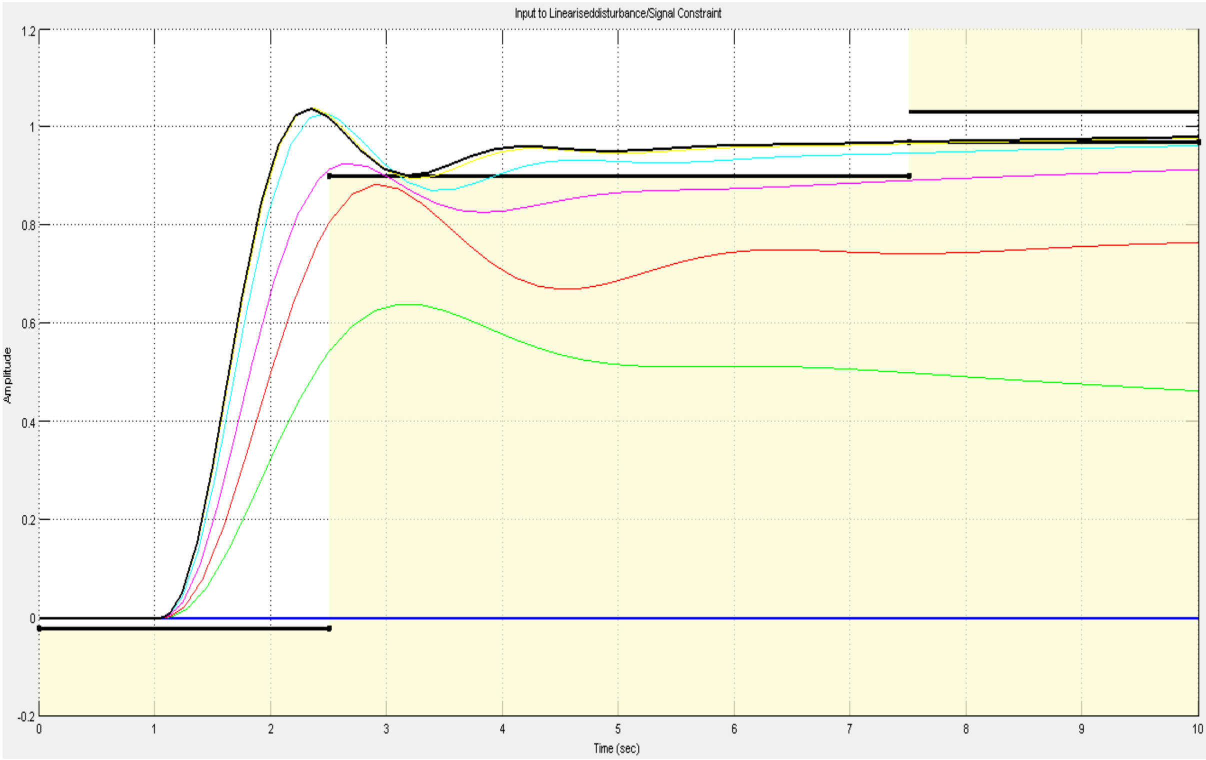

Figure :3.8 (c)- Output Responses obtained on performing optimization of PID values

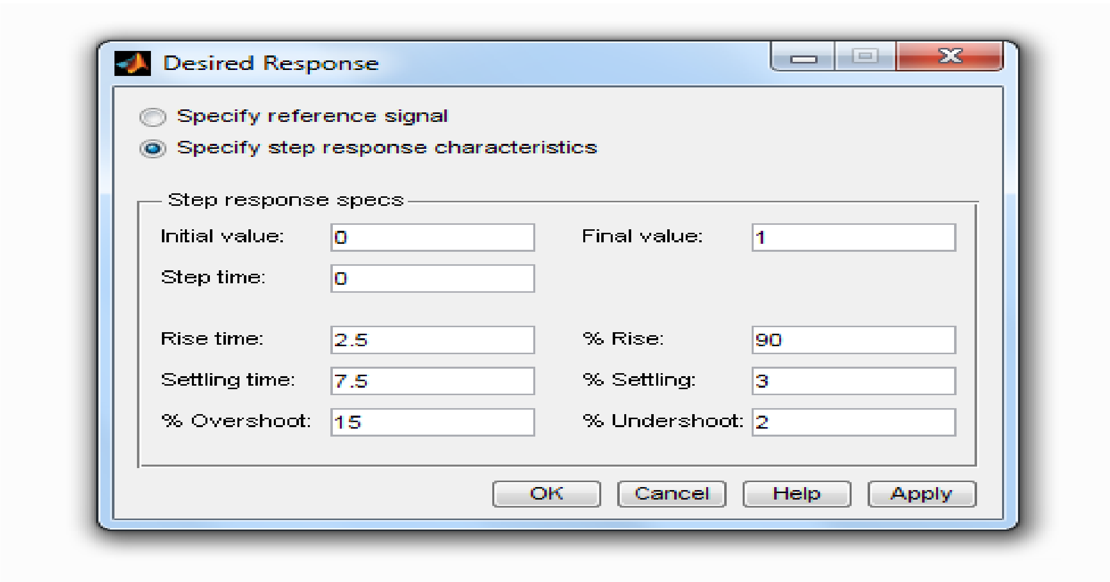

Figure :3.8 (d)- Block showing parameter values of desired response

```
                                   max                     Directional  First-order
 Iter  S-count          f(x)    constraint    Step-size    derivative   optimality  Procedure
    0        1             0         143.3
    1       14             0         62.12        0.152             0        0.374
    2       21             0         17.61       0.0834             0      0.00398
    3       28 -1.48992e-017         2.775        0.159   -1.49e-017        0.0681   Hessian modified
    4       35             0        0.4327        0.153    1.49e-017       0.0147
    5       42             0       0.02995       0.0949             0       0.0111   Hessian modified
    6       49             0     0.0001147       0.0225             0      0.00152   Hessian modified
Successful termination.
Found a feasible or optimal solution within the specified tolerances.

kd =

    0.1837


ki =

    0.1043


kp =

    0.6033
```

Thus the optimum values to get the desired response are as follows:-

Kp = 0.6033 **Ki = 0.1043** **Kd = 0.1837**

3.9) Responses to various types of input waveforms

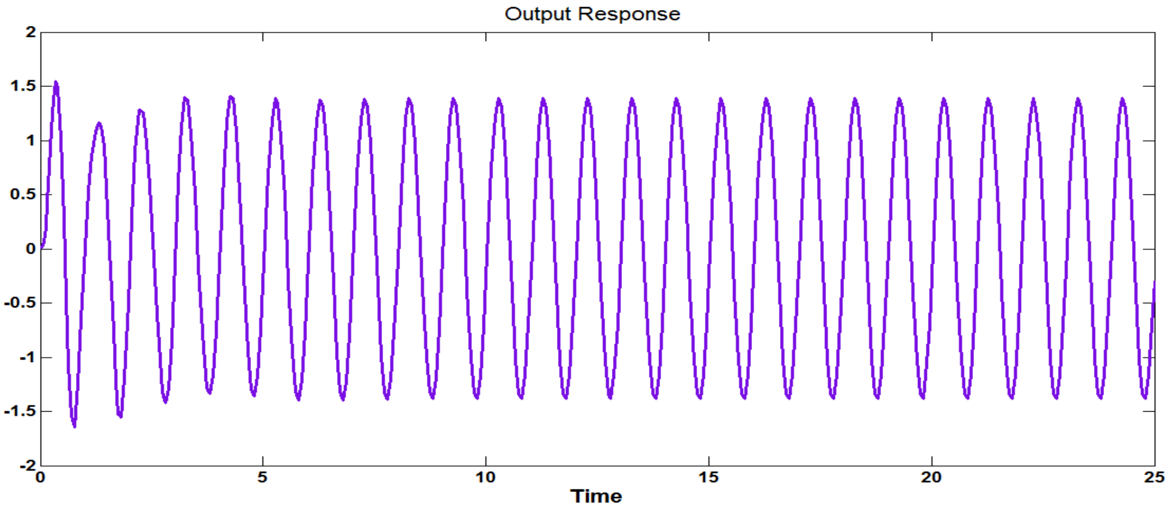

Fig:3.9(a)- Output Response to Sinusoidal input waveform

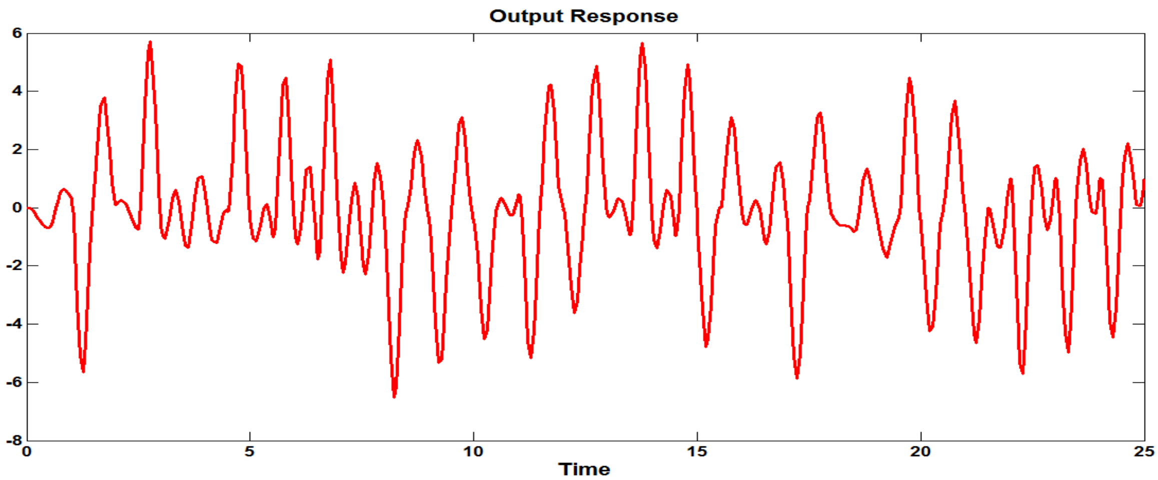

Fig:3.9(b) - Output Response to Square type input waveform

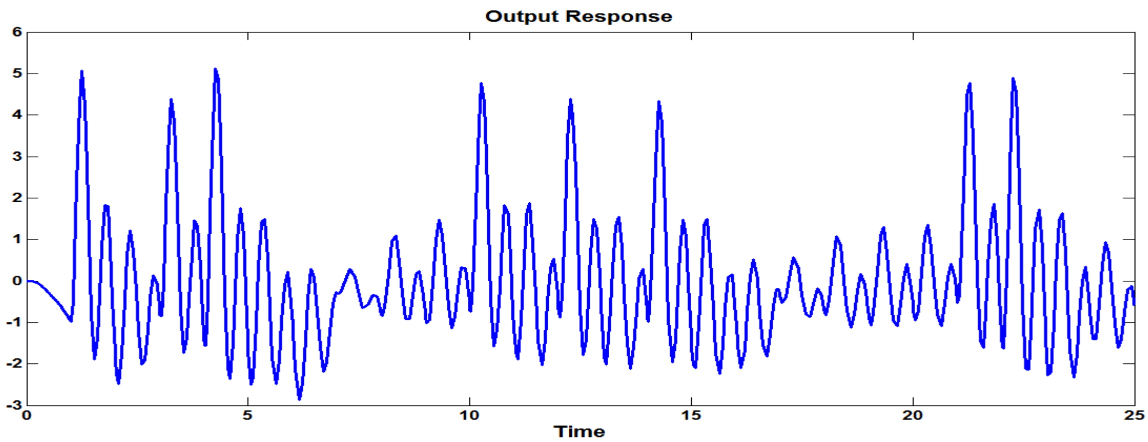

Fig:3.9(c)- Output Response to Sawtooth type input waveform

CHAPTER - 4

4.1) Using Feedback Linearization technique:

Feedback linearization is an approach which involves change of variables and designing a proper control signal so that the complex non-linear parts are cancelled out resulting in a much simpler and equivalent linear system. Thus enabling linear control methods to be applied to the resulting new system.

- This approach is different from Jacobian linearization method because feedback linearization is achieved by the exact transformation of states, their feedback and linear approximations of dynamics.

- The non-linear system is simplified through a different choice of co-ordinate systems and other type of state representation.

Applications

- High speed aircrafts, robotic control in industries, helicopters, various biomedical devices , control of vehicles etc.

Advantages:

- The non-linear control signal helps to reduce the complex system to a much simpler and linear one thus making it easier for analysis purpose.

- The non-linear control input leads to the global asymptotical stability of the resulting linear behaviour.

Disadvantages:

- The real system might be different from the nominal system. Thus the behaviour might be non-linear and hence different kinds of results can be obtained on analysis of the derived linear system as compared to the original one.

- This technique is not applicable for all systems.

4.2) Relative degree of a system:

The relative degree of a system can be determined from the following definition:-

$$y^m = L_a^m c(x) + L_b L_a^{m-1} c(x) u \qquad \text{……(4.1)}$$

where'm' is the relative degree of system when,

$L_b L_a^d c(x) = 0$ where 0 < d < m-1

And $L_b L_a^d c(x) \neq 0$ where d=m-1

The given general form of a single input single output system (SISO) system is:-

$$\dot{X} = a(x) + b(x)u$$

$$y = c(x)$$

$$\dot{y} = \frac{\partial c}{\partial x}\dot{X}$$

$$= \frac{\partial c}{\partial x}[a(x) + b(x)u]$$

$$= \frac{\partial c}{\partial x}a(x) + \frac{\partial c}{\partial x}b(x)u$$

$$= L_a c + (L_b c)u \qquad \text{……(4.2)}$$

Where : $L_a c = \frac{\partial c}{\partial x}a(x)$ and $L_b c = \frac{\partial c}{\partial x}b(x)$

Now let us assume that the input signal 'u' does not affect or influence the 1st derivative of O/P. i.e. $L_b c = 0$ and $\dot{y} = L_a c$. Then the next derivative of O/P is,

$$\ddot{y} = \frac{d}{dt}\dot{y} = \frac{d}{dt}(L_a c) = \frac{\partial (L_a c)}{\partial x}\dot{X}$$

$$= \frac{\partial (L_a c)}{\partial x}[a(x) + bu]$$

$$= L_a(L_a c) + L_b(L_a c)u$$

$$= L_a^2 c + L_b(L_a c)u$$

$$||$$

$$0$$

From the principle of mathematical induction it can be said that,

$\dot{y} = (L_a c)$

$\ddot{y} = L_a^2 c$

.

.

.

.

.

$y^m = L_a^m c(x) + L_b L_a^{m-1} c(x) u$

It is assumed that the 'm[th]' derivative of the output is affected by the input signal 'u'.

Then m = Relative degree of the system.

In other words ,the output should be differentiated a number of times until the input appears in the equation. The value of the derivative upto which we have arrived is the degree of the system.

4.3) Input/Output Linearization:

It has already been shown that the given non-linear system can be transferred to the form,

$\dot{y} = (L_a c)$

$\ddot{y} = L_a^2 c$

.

.

.

.

$y^m = L_a^m c(x) + L_b L_a^{m-1} c(x) u$

Now if introduce $P(x) = L_a^m c$ and $Q(x) = L_b (L_a^{m-1} c)$ then ,

$y^m = P(x) + Q(x) u$

The input signal is chosen in such a way that the nonlinear parts of the system are compensated or cancelled out i.e.

$$y^m = P(x) + Q(x)u = G \quad \text{...(4.3)}$$

$$\Rightarrow \; \boldsymbol{u} = \frac{\mathbf{1}}{\boldsymbol{Q(x)}}[\boldsymbol{G} - \boldsymbol{P(x)}] \quad \text{...(4.4)}$$

4.4) Representation in controllable canonical form:

Thinking in a different way it can be said that a new state vector has been defined which comprises of the derivatives of the output.

$$S_1 = y = L_a^0 c$$

$$S_2 = \dot{y} = L_a^1 c$$

.

.

$$S_m = y^{m-1} = L_a^{m-1} c$$

$$S_{m+1} = y^m = P(x) + Q(x)u = G$$

So the state space representation of the system is,

$$\begin{bmatrix} \dot{S}_1 \\ \dot{S}_2 \\ \dot{S}_3 \\ \cdots . \\ \dot{S}_m \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & \cdots & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & 0 & \cdots & \cdots & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ \cdots \\ S_{m-1} \\ S_m \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \cdots \\ 1 \end{bmatrix} G \quad \text{...(4.5)}$$

$$y = [1 \quad 0 \ldots \ldots \ldots \ldots \ldots \ldots \quad 0] \begin{bmatrix} S_1 \\ S_2 \\ \cdots . \\ S_{m-1} \\ S_m \end{bmatrix} \quad \text{...(4.6)}$$

4.5) Algorithm for feedback linearization

Step -1:

Find the relative degree 'm' of the nonlinear system

$L_b[L_c^d c(x)] = 0$ for d < m-1

$L_b[L_a^{m-1} c(x)] \neq 0$ for d=m-1

Step -2:

Make the 'm' state transformations .

$$S = \gamma(x) = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_{m-1} \\ \gamma_m \end{bmatrix} = \begin{bmatrix} L_a^0 c \\ L_a^1 c \\ \cdots \\ \cdots \\ L_a^{m-1} c \end{bmatrix}$$

Step -3:

Define the new input so that the states are linearized.

$P(x) = L_a^m c$ ……………………………………………………………………………………………..(4.7)

$Q(x) = L_b(L_a^{m-1} c)$ … (4.8)

$$u = \frac{1}{Q(x)}[G - P(x)]$$

4.6) Implementation of feedback linearization to the non-linear model of magnetic levitation system :

Step -1

To determine the relative degree of the system.

(For d=0) $L_b L_a^0 c = L_b\ c$

$$= \begin{bmatrix} \frac{\partial x_1}{\partial x_1} & \frac{\partial x_1}{\partial x_2} & \frac{\partial x_1}{\partial x_3} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}$$

= 0

(For d=1)

$$L_b L_a c = L_b \begin{bmatrix} \frac{\partial x_1}{\partial x_1} & \frac{\partial x_1}{\partial x_2} & \frac{\partial x_1}{\partial x_3} \end{bmatrix} \begin{bmatrix} x_2 \\ g - \frac{K}{m}\left(\frac{x_3}{x_1}\right)^2 - f_1 x_2 - q \\ -\frac{R}{L}x_3 + \frac{2K}{L}\left(\frac{x_3\ x_2}{x_1^2}\right) \end{bmatrix}$$

$= L_b(x_2)$

$$= \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}$$

= 0

(For d=2)

$L_b L_a^2 c = L_b L_a L_a c$

$= L_b L_a (x_2)$

$$= L_b \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ g - \frac{K}{m}\left(\frac{x_3}{x_1}\right)^2 - f_1 x_2 - q \\ -\frac{R}{L}x_3 + \frac{2K}{L}\left(\frac{x_3\ x_2}{x_1^2}\right) \end{bmatrix}$$

$$= L_g \left[g - \frac{k}{m}\left(\frac{x_3}{x_1}\right)^2 - f_1 x_2 - \mathrm{q} \right]$$

$$= \begin{bmatrix} \frac{2Kx_3^2}{mx_1^3} & -f_1 & -\frac{2Kx_3}{mx_1^2} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}$$

$$= -\frac{2K\ x_3}{mL\ x_1^2} \neq 0$$

⇛ Degree of the given system = 3

4.7) *Feedback State Transformation*

$\dot{S}_1 = S_2$……………………………………………………………………………(4.9)

$\dot{S}_2 = S_3$…………………………………………………………………………..........(4.10)

$\dot{S}_3 = P(x) + \mathrm{Q}(x)u$ = G……………………………………………………...........(4.11)

Where $P(x) = L_a^3 c$ and $\mathrm{Q}(x) = L_b L_a^2 c$ ……………………………………………(4.12)

⇛ $\mathbf{Q}(\boldsymbol{x}) = -\frac{\mathbf{2Kx_3}}{\mathbf{mLx_1^2}}$ (already derived before) ………………………………………(4.13)

$$P(x) = L_a^2 L_a c$$

$$= L_a^2 \,[1 \quad 0 \quad 0] \begin{bmatrix} x_2 \\ g - \frac{K}{m}\left(\frac{x_3}{x_1}\right)^2 - f_1 x_2 - q \\ -\frac{R}{L} x_3 + \frac{2K}{L}\left(\frac{x_3\, x_2}{x_1^2}\right) \end{bmatrix}$$

$$= L_a^2 (x_2)$$

$$= L_a \,[0 \quad 1 \quad 0] \begin{bmatrix} x_2 \\ g - \frac{k}{m}\left(\frac{x_3}{x_1}\right)^2 - f_1 x_2 - q \\ -\frac{R}{L} x_3 + \frac{2K}{L}\left(\frac{x_3\, x_2}{x_1^2}\right) \end{bmatrix}$$

$$= L_a \left[g - \frac{k}{m}\left(\frac{x_3}{x_1}\right)^2 - f_1 x_2 - q \right]$$

$$= \begin{bmatrix} \frac{2Kx_3^2}{mx_1^3} & -f_1 & -\frac{2Kx_3}{mx_1^2} \end{bmatrix} \begin{bmatrix} x_2 \\ g - \frac{K}{m}\left(\frac{x_3}{x_1}\right)^2 - f_1 x_2 - q \\ -\frac{R}{L} x_3 + \frac{2K}{L}\left(\frac{x_3\, x_2}{x_1^2}\right) \end{bmatrix}$$

$$= \frac{2K\, x_2 x_3^2}{m\, x_1^3} + \frac{2KR\, x_3^2}{mL\;\, x_1^2} - \frac{4K^2\, x_2 x_3^2}{mL\quad x_1^4} + \frac{f_1 K}{m}\left(\frac{x_3}{x_1}\right)^2 + f_1^2 x_2 \; + f_1(q-g)$$

$$\Rightarrow \boldsymbol{P(x)} = \frac{2K\, x_2 x_3^2}{m\, x_1^3} + [\frac{f_1 K}{m} + \frac{2KR}{mL}]\left(\frac{x_3}{x_1}\right)^2 - \frac{4K^2\, x_2 x_3^2}{mL\quad x_1^4} + f_1^2 x_2 + f_1(q-g) \qquad .(4.14)$$

Now using state feedback,

$$u = \frac{1}{Q(x)}\,[G - P(x)]$$

Where $\quad G = \dot{S}_3$

$$=[\frac{f_1 qmL}{2K} - \frac{GmL}{2K} - \frac{gmLf_1}{2K}]\frac{{x_1}^2}{x_3} + \frac{L\, x_2 x_3}{x_1} - \frac{2K\, x_2 x_3}{x_1^2} + (R + \frac{f_1 L}{2})x_3 + \frac{mL{f_1}^2}{2K}\,\frac{x_2 x_1^2}{x_3} \ldots..(4.15)$$

Here the control input 'u' is the input voltage to the coil i.e. 'e' $\quad$ ($u \equiv e$)

$$\Rightarrow e = \left[\frac{f_1 qmL}{2K} - \frac{GmL}{2K} - \frac{gmLf_1}{2K}\right]\frac{x_1^2}{x_3} + \frac{Lx_2x_3}{x_1} - \frac{2Kx_2x_3}{x_1^2} + \left(R + \frac{f_1 L}{2}\right)x_3 + \frac{mLf_1^2}{2K} - \frac{x_2 x_1^2}{x_3} \;..(4.16)$$

The system obtained after state transformation is as shown below.

$$\begin{bmatrix}\dot{S}_1\\ \dot{S}_2\\ \dot{S}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}S_1\\ S_2\\ S_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ G\end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.17)$$

This is the control canonical form. Thus it becomes very easy to design the feedback gains according to the desired closed loop pole locations.

On putting the values of the various parameters we get,

$$\boldsymbol{e = -\frac{0.12 G x_1^2}{x_3} + \frac{0.6\, x_2 x_3}{x_1} - \frac{x_2 x_3}{x_1^2} + \frac{0.019\, x_2\, x_1^2}{x_3} - \frac{0.432\, x_1^2}{x_3} + 1.12\, x_3} \ldots\ldots\ldots\ldots\ldots(4.18)$$

$$\boldsymbol{e = -\frac{0.12\, Gx^2}{i} + \frac{0.6\, Vi}{x} - \frac{Vi}{x^2} - \frac{0.432\, x^2}{i} + \frac{0.019\, vx^2}{i} + 1.12\, i} \ldots\ldots\ldots\ldots. (4.19)$$

The above equation describes the required control law.

After the state transformation ,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Now if we want to place the closed loop poles at our desired locations say (-M_1, - M_2, - M_3)

$$\Rightarrow \qquad P = \begin{bmatrix} -M_1 \\ -M_2 \\ -M_3 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.20)$$

$$k = [M_1 M_2 M_3 \quad (M_1M_2+M_2M_3+M_3M_1) \quad (M_1+M_2+M_3)\ ] \ldots\ldots\ldots\ldots(4.21)$$

$$\Rightarrow G = -\,M_1M_2M_3\,S_1 \ -(M_1M_2+M_2M_3+M_3M_1)\,S_2 - (M_1+M_2+M_3)S_3 \ldots\ldots\ldots\ldots\ldots\ldots(4.22)$$

$$S_1 = \gamma_1(x) = L_a^0 c = x_1 = y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.23)$$

$$S_2 = \gamma_2(x) = L_a^1 c$$

$$= [1 \quad 0 \quad 0] \begin{bmatrix} x_2 \\ g - \frac{K}{m}\left(\frac{x_3}{x_1}\right)^2 - f_1x_2 \ - q \\ -\ \frac{R}{L}x_3 + \frac{2K}{L}\left(\frac{x_3\,x_2}{x_1^2}\right) \end{bmatrix}$$

$$= x_2$$

$$\Rightarrow\ S_2 = x_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (4.24)$$

$$S_3 = \gamma_3(x) = \ L_a^2 c$$

$$= L_a(x_2)$$

$$= [0 \quad 1 \quad 0] \begin{bmatrix} x_2 \\ g - \frac{K}{m}\left(\frac{x_3}{x_1}\right)^2 - f_1x_2 \ - q \\ -\ \frac{R}{L}x_3 + \frac{2K}{L}\left(\frac{x_3\,x_2}{x_1^2}\right) \end{bmatrix}$$

$$\Rightarrow \quad S_3 = g - \frac{K}{m}\left(\frac{x_3}{x_1}\right)^2 - f_1x_2 - q \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.25)$$

Finally on calculating,

$$G = -M_1M_2M_3\,x_1 - (M_1M_2 + M_2M_3 + M_3M_1)\,x_2 - 9(M_1 + M_2 + M_3) + 2.5(M_1 + M_2 + M_3)\left(\frac{x_3}{x_1}\right)^2 + 0.4(M_1 + M_2 + M_3)\,x_2 \quad \ldots\ldots\ldots(4.26)$$

$$V = -M_1M_2M_3\,x - (M_1M_2 + M_2M_3 + M_3M_1)\,v - 9(M_1 + M_2 + M_3) + 2.5(M_1 + M_2 + M_3)\left(\frac{i}{x}\right)^2 + 0.4(M_1 + M_2 + M_3)\,x_2 \quad \ldots\ldots\ldots(4.27)$$

So finally if the closed loop poles are placed at the desired locations of ($-M_1$ $-M_2$, $-M_3$) the value of 'G' is substituted to get the final control law as :

$$e = \frac{0.12\,(M_1M_2M_3)\,x_1^3}{x_3} + \frac{[0.12\,(M_1M_2+M_2M_3+M_1M_3) - 0.048\,(M_1+M_2+M_3) + 0.0192]\,x_1^2x_2}{x_3} + \frac{[1.08\,(M_1+M_2+M_3)-0.432]\,x_1^2}{x_3} + \frac{0.6\,x_2x_3}{x_1} - \frac{x_2x_3}{x_1^2} + [1.12-0.3(M_1+M_2+M_3)]\,x_3 \quad \ldots\ldots(4.28)$$

$$e = \frac{0.12(M_1M_2M_3)x^3}{i} + \frac{[0.12\,(M_1M_2+M_2M_3+M_1M_3) - 0.048\,(M_1+M_2+M_3) + 0.0192]\,x^2v}{i} + \frac{[1.08\,(M_1+M_2+M_3)-0.432]\,x^2}{i} + \frac{0.6\,vi}{x} - \frac{vi}{x^2} + [1.12-0.3(M_1 + M_2 + M_3)]\,i \quad \ldots\ldots(4.29)$$

- So for desired poles at (-1, -2, -3) the derived final control law is :

❖ $$e = \frac{0.72x^3}{i} + \frac{1.0512\,x^2v}{i} + \frac{6.048\,x^2}{i} + \frac{0.6\,vi}{x} - \frac{vi}{x^2} - 0.68i \quad \ldots\ldots(4.30)$$

- For desired poles at (-3, -5, -9) the derived final control law is :

❖ $$e = \frac{16.2x^3}{i} + \frac{9.6432\,x^2v}{i} + \frac{17.928x^2}{i} + \frac{0.6\,vi}{x} - \frac{vi}{x^2} - 3.98i \quad \ldots\ldots(4.31)$$

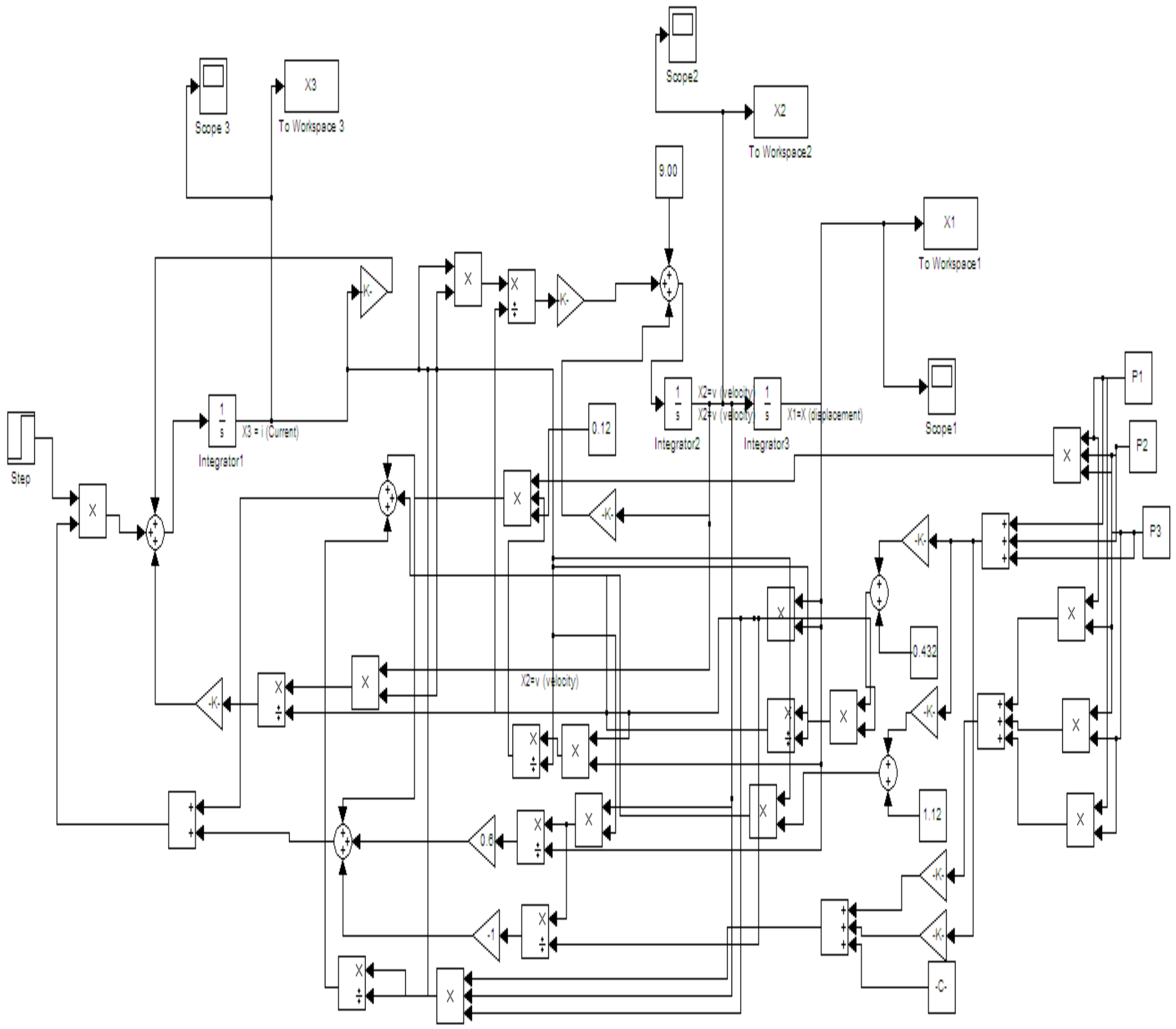

Figure:(4.8)- Stablised Complete Simulink model of Magnetic Levitation system (considering friction and external disturbances)

4.9) Responses for poles at (-1,-2,-3)

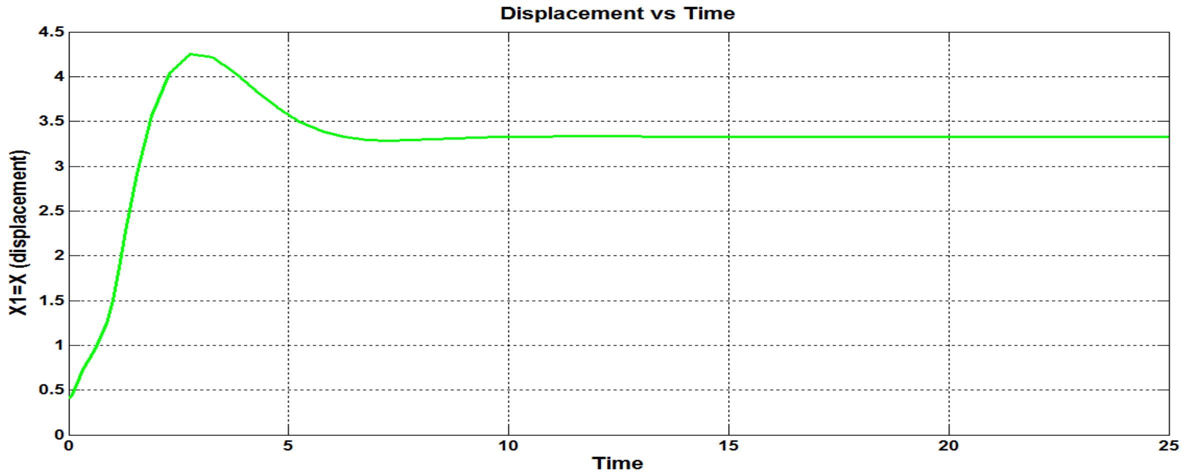

Figure: 4.9 (a) – i: Displacement vs Time

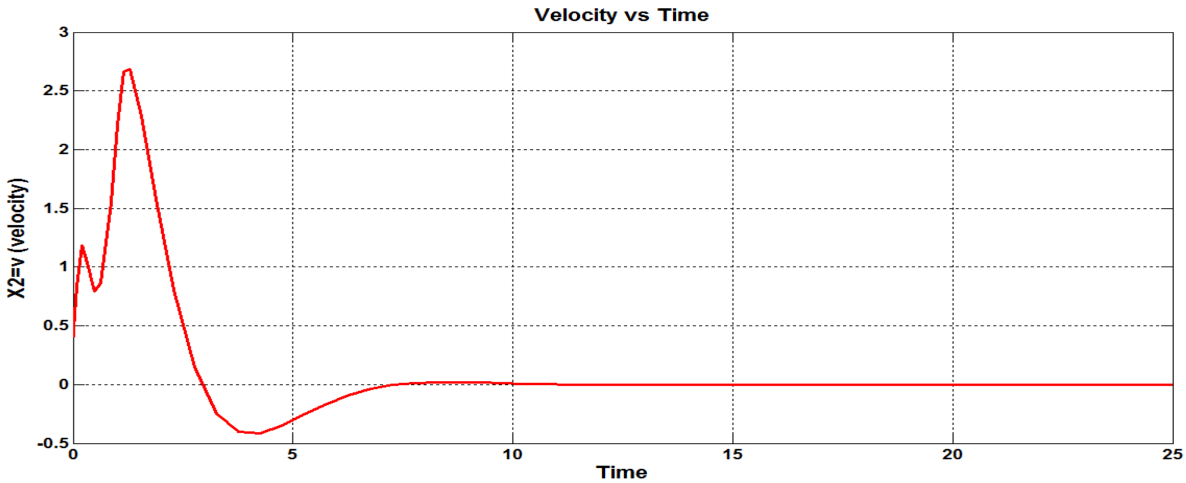

Figure: 4.9 (a) – ii: Velocity vs Time

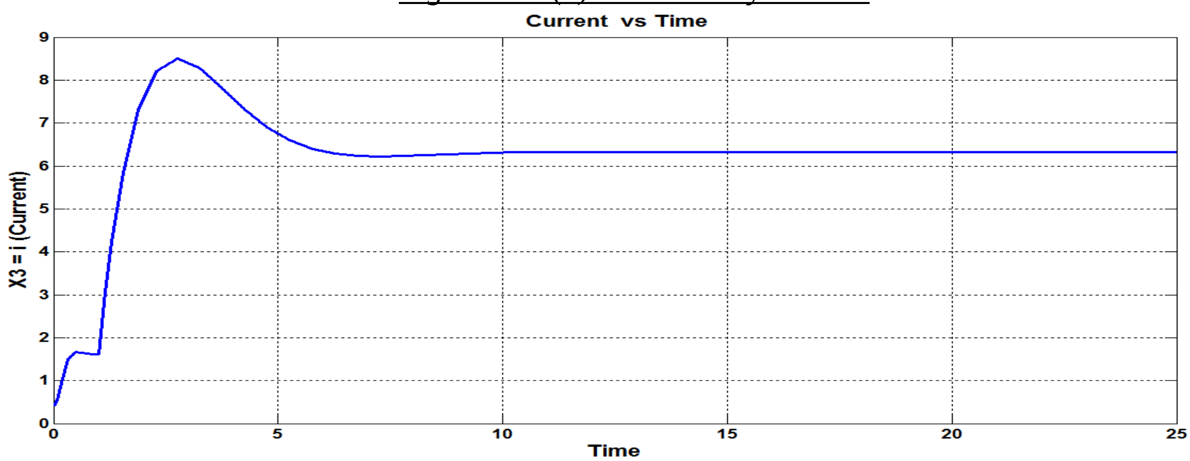

Figure: 4.9 (a) – iii: Current vs Time

Responses for poles at (-3.5,-5,-6)

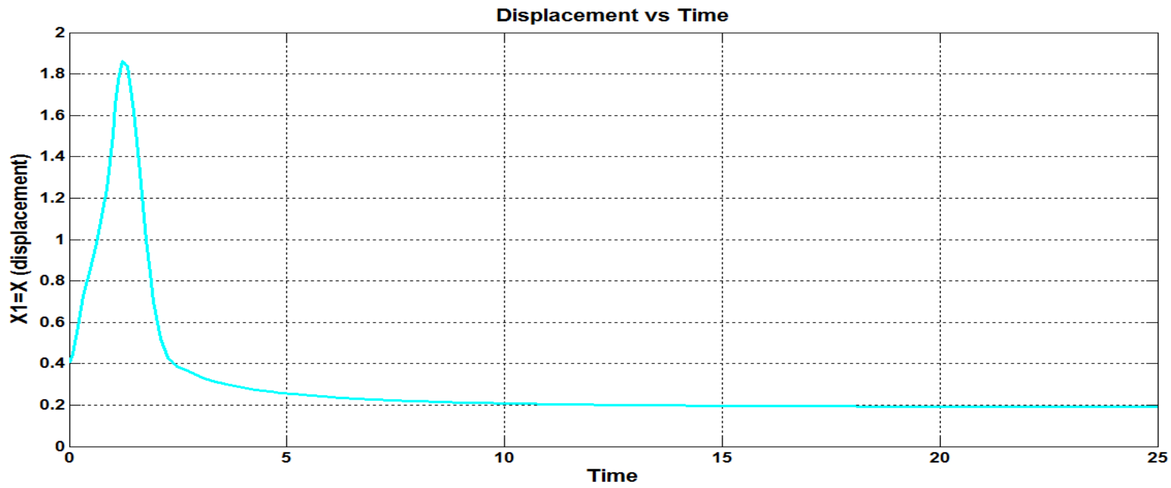

Figure: 4.9 (b) – i: Displacement vs Time

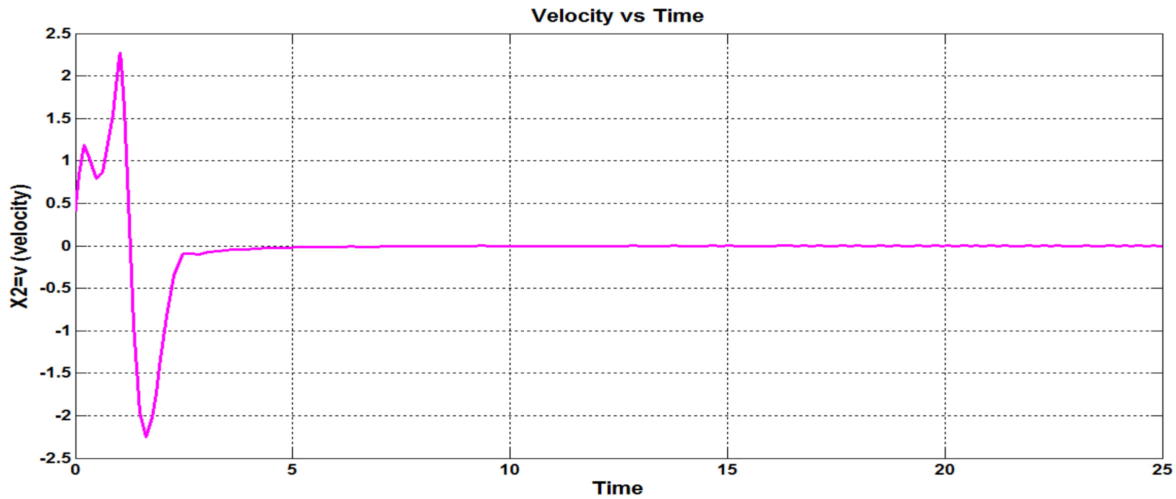

Figure: 4.9 (b) – ii: Velocity vs Time

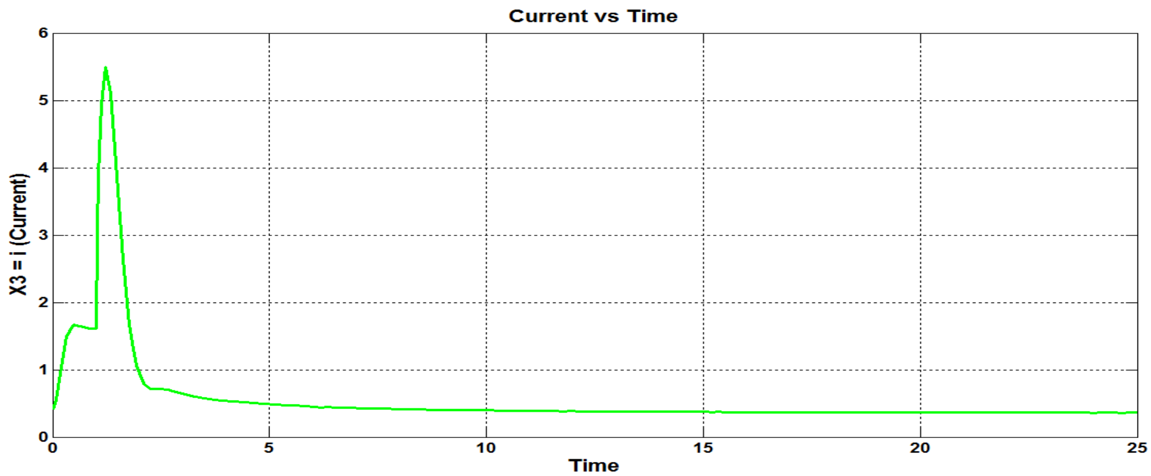

Figure: 4.9 (b) – iii: Current vs Time

Responses for poles at (-8,-9,-11)

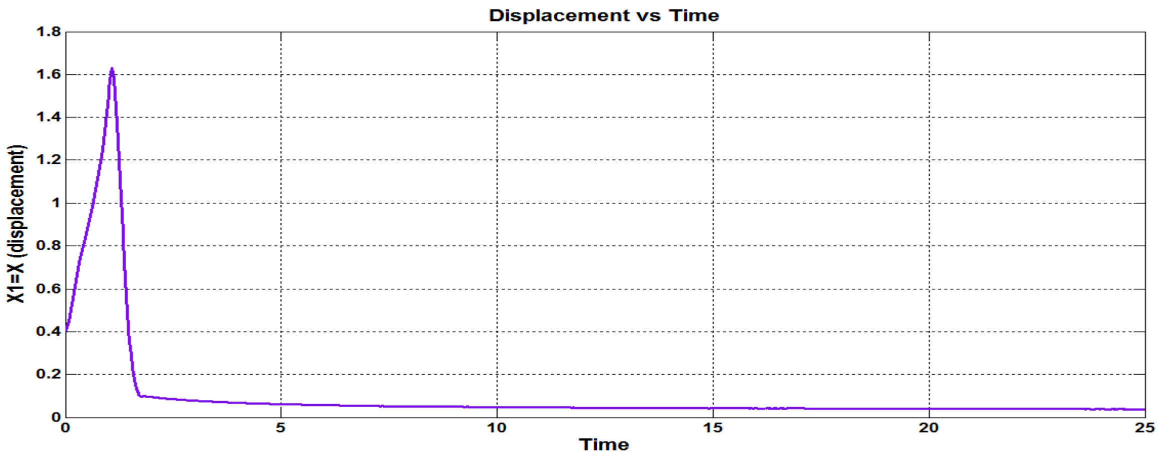

Figure: 4.9 (c) – i: Displacement vs Time

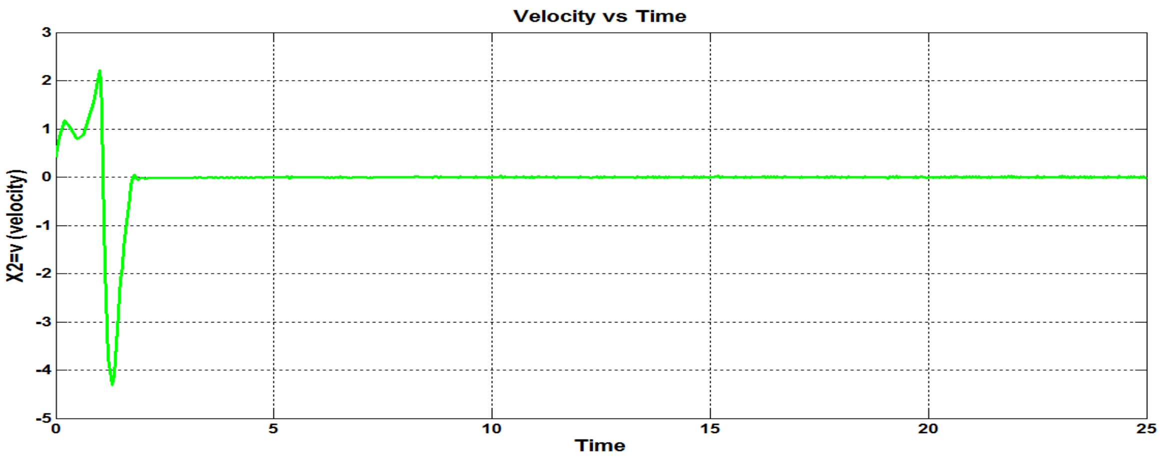

Figure: 4.9 (c) – ii: Velocity vs Time

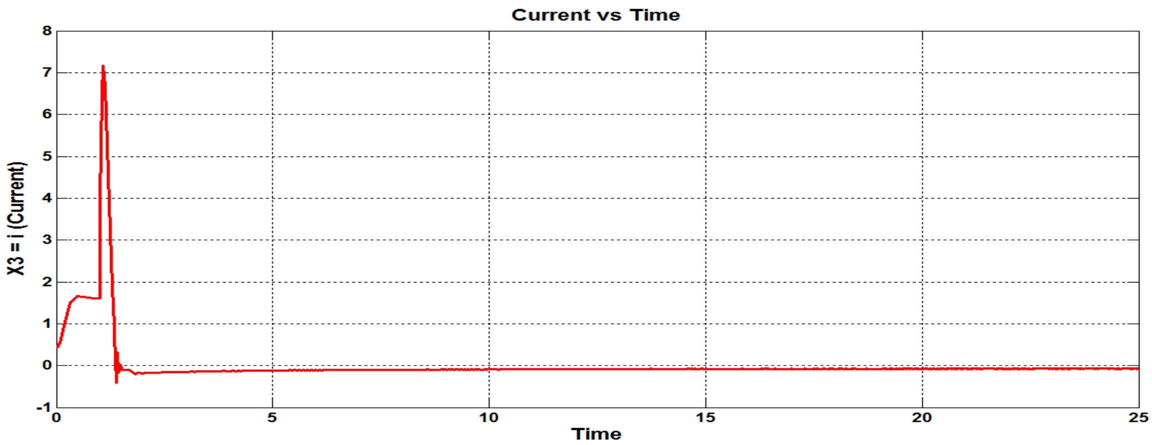

Figure: 4.9 (c) – iii: Current vs Time

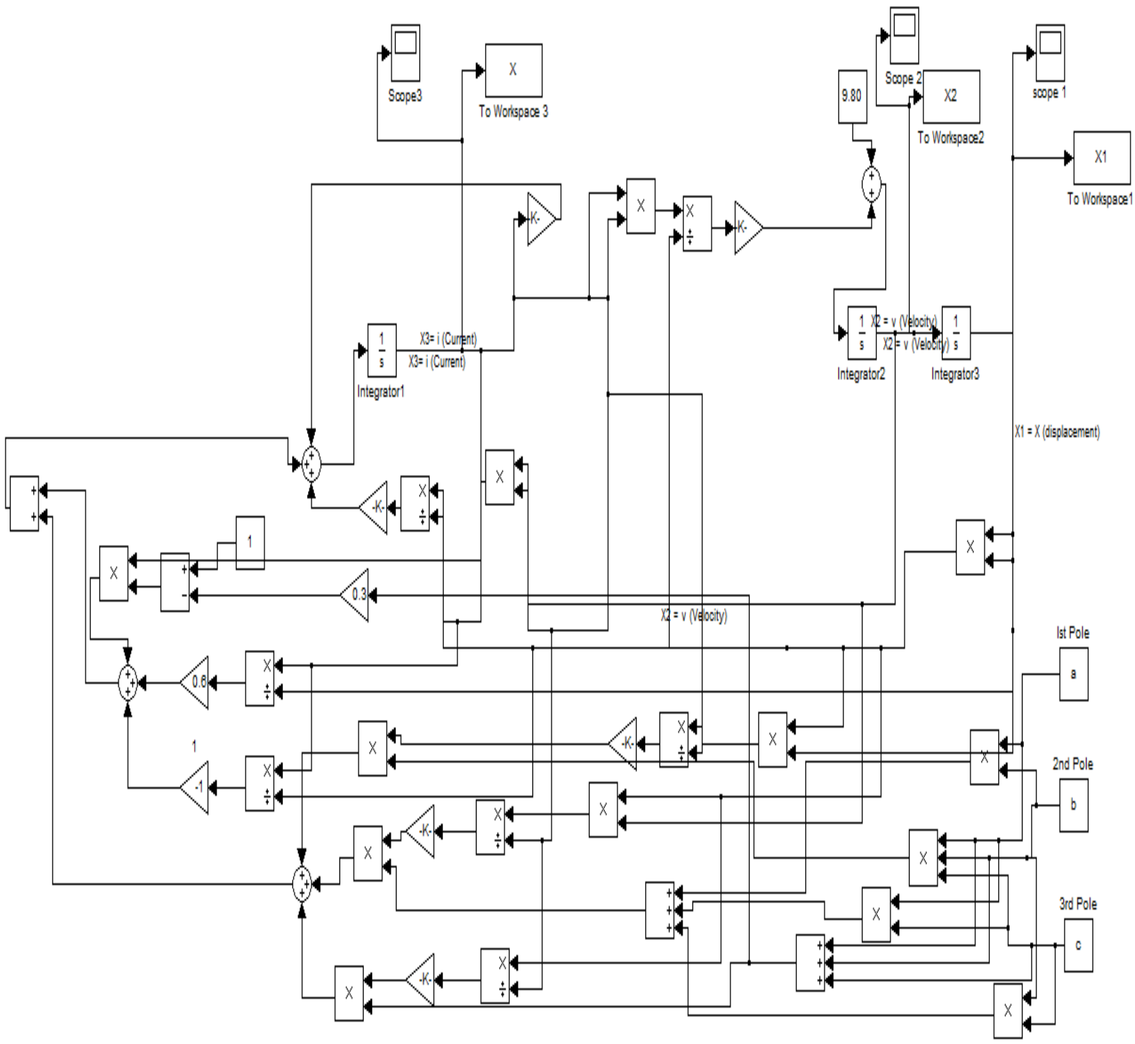

Figure : (4.10) - Stablised Simulink model of complete system (without air friction and external disturbance)

4.11) Control law of simplified system (without friction and external disturbance)

The final control law for the simplified system without considering air friction and external disturbance force is shown as follows:-

$$e = \frac{0.12\,(M_1 M_2 M_3)\,x_1^3}{x_3} + \frac{0.12\,(M_1M_2+M_2M_3+M_1M_3)\,x_1^2 x_2}{x_3} + \frac{1.176\,(M_1+M_2+M_3)\,x_1^2}{x_3} + \frac{0.6\,x_2 x_3}{x_1} - \frac{x_2 x_3}{x_1^2} + (1\text{-}0.3(M_1+M_2+M_3)\,x_3 \quad \ldots\ldots\ldots\ldots(4.32)$$

$$\boldsymbol{e} = \frac{0.12(M_1M_2M_3)x^3}{i} + \frac{0.12\ (M_1M_2+M_2M_3+M_1M_3)\ x^2v}{i} + \frac{1.176\ (M_1+M_2+M_3)\ x^2}{i} + \frac{0.6\ vi}{x} - \frac{vi}{x^2} + (1\text{-}0.3(a+b+c))\ i \ldots\ldots\ldots\ldots (4.33)$$

- So for desired poles at (-1, -2, -3) the final derived control law is :

❖ $$\boldsymbol{e} = \frac{0\ .72x^3}{i} + \frac{1.32\ x^2v}{i} + \frac{7.056\ x^2}{i} + \frac{0.6\ vi}{x} - \frac{vi}{x^2} - 0.8i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.34)$$

- For desired poles at (-3, -5, -9) the final derived control law is :

❖ $$\boldsymbol{e} = \frac{16.2x^3}{i} + \frac{10.44\ x^2v}{i} + \frac{20x^2}{i} + \frac{0.6\ vi}{x} - \frac{vi}{x^2} - 4.1i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.35)$$

4.12) Responses for poles at (-1,-2,-3)

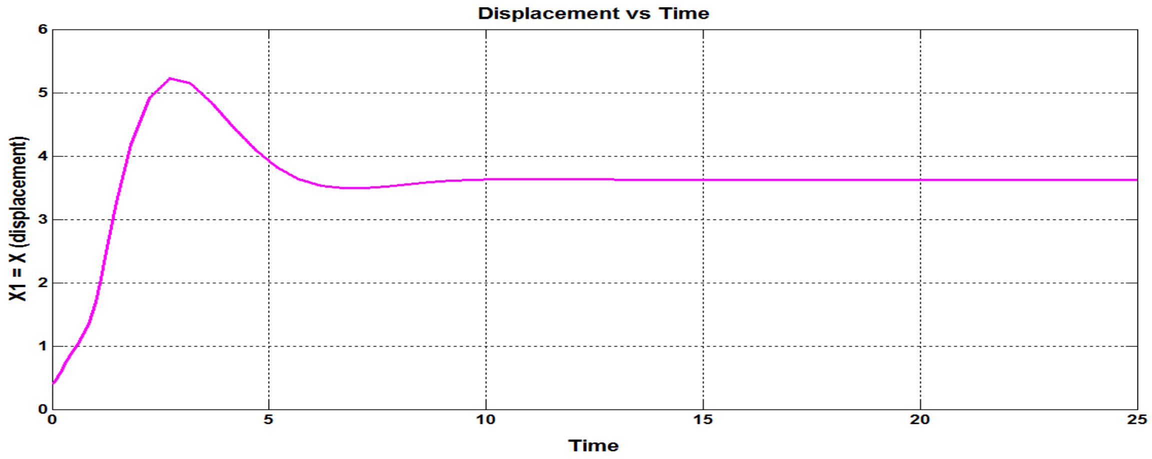

Figure: 4.12 (a)- Displacement vs Time

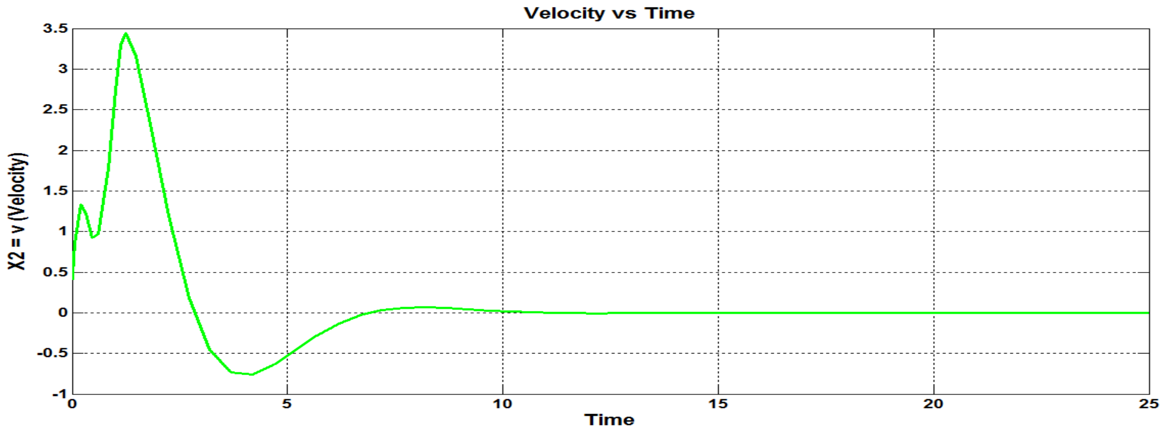

Figure: 4.12 (b)- Velocity vs Time

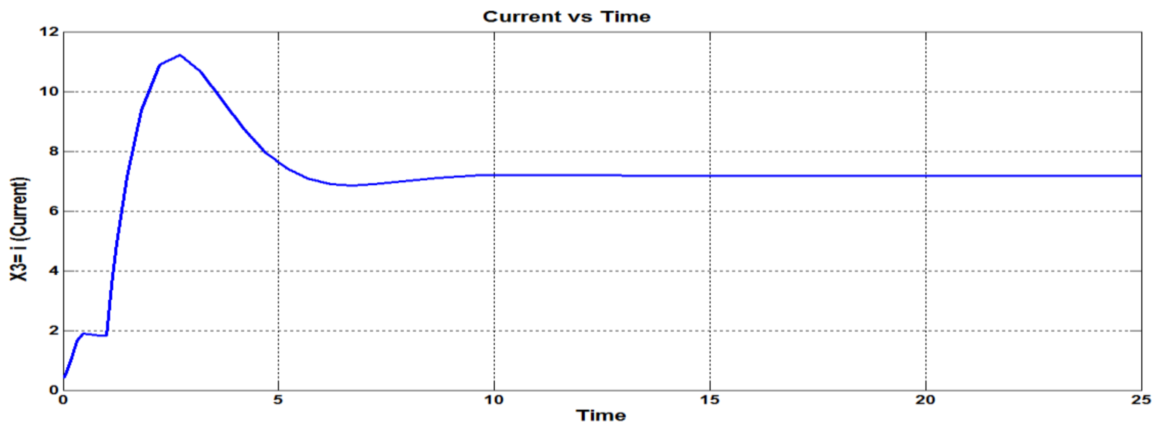

Figure: 4.12 (c)- Current vs Time

CHAPTER - 5

CONCLUSION

To conclude we can say that the Jacobian method of linearization is an exact representation of the given non-linear magnetic levitation system only at the equilibrium points given by(x_{1e}, x_{2e} x_{3e}). As a result the control strategy based on a linearized model may yield unsatisfactory performance and robustness at other operating points. Some other types of linear controllers like PD compensator helps to improve the speed of response. PID control helps to improve the system steady state performance. Thus the use of these linear controllers helps to improve the system response upto a certain level but their range of operation is quite limited. Their performance deteriorates with distance from the equilibrium point. On the other hand feedback linearization is a non-linear control technique that produces a linear model which is an exact representation of the original non-linear model over a large set of operating conditions.

Disturbance rejection using feedback linearization is much better as compared to the other linear methods of control. Thus the technique of feedback linearization provides a much better and robust control over a large range for the magnetic levitation system taken into consideration in this thesis.

SCOPE OF FUTURE WORK

The non-linear model of the magnetic levitation system can be developed by taking various approximations for the inductance of the coil. The model shown in this report is based on the assumption that the coil inductance varies in an inverse manner with respect to the ball position. Thus there can be a different dynamic model of the magnetic levitation system and different method for calculating the inductance. The results obtained after the use of the feedback linearization controller are valid for a large region of operation but it is not global. The control law is not well defined if the initial states are at singularity points. This cannot bring the system to the stable equilibrium point. Thus some other better non-linear control methods such as Sliding mode control , LQR based control, or H_∞ method be applied which may provide good system response in a global manner.

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