AUTOMATIC TOLERANCING OF MECHANICAL ASSEMBLIES

A DISSERTATION

Submitted in partial fulfillment of the
requirements for the award of the degree
of

MASTER OF TECHNOLOGY

in

MECHANICAL ENGINEERING

(With Specialization in CAD, CAM & Robotics)

By

RAVIKANT MORDIA

DEPARTMENT OF MECHANICAL AND INDUSTRIAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY ROORKEE

ROORKEE-247667 (INDIA)

MAY, 2016

INDIAN INSTITUTE OF TECHNOLOGY ROORKEE

ROORKEE

CANDIDATE'S DECLARATION

I hereby declare that the work carried out in this report titled "**AUTOMATIC TOLERANCING OF MECHANICAL ASSEMBLIES"** is presented on behalf of partial fulfilment of the requirement for the award of the degree of **Master of Technology** with specialization in **CAD, CAM & Robotics** submitted to the department of **Mechanical & Industrial Engineering, Indian Institute of Technology Roorkee, India**, under the supervision and guidance of Dr. Abinash Kumar Swain, Assistant Professor, and Dr. Anil Kumar, Assistant Professor, MIED, IIT Roorkee, India.

I have not submitted the matter embodied in this report for the award of any other degree or diploma.

Date:

Place: Roorkee **(RAVIKANT MORDIA)**

CERTIFICATION

This is to certify that the above statement made by the candidate is correct to the best of our knowledge and belief.

Dr. Abinash Kumar Swain
Assistant Professor
MIED, IIT Roorkee
UK-247667, India

Dr. Anil Kumar
Assistant Professor,
MIED, IIT Roorkee
UK-247667, India

ACKNOWLEDGEMENT

I wish to express my deep sense of gratitude and sincere thanks to my guides **Dr. Abinash Kumar Swain** and **Dr. Anil Kumar** of Mechanical and Industrial Engineering department, IIT Roorkee, for being helpful and a great source of inspiration. I would like to thank them for providing me with an opportunity to work on this excellent and innovative field of research. Their keen interest and constant encouragement gave me the confidence to complete my work. I wish to thank them for their constant guidance and suggestions without which I could not have successfully completed this report.

I would also like to thank **Dr. Dinesh Kumar**, HOD MIED, IIT Roorkee for his constant support during my study. Also I would like to thank all the teaching and non-teaching staff members of the department who have contributed directly or indirectly in successful completion of my dissertation work.

I am very thankful to my parents & all of my friends for their never ending encouragement in bringing out this report.

RAVIKANT MORDIA
CAD, CAM & Robotics
Enrol. No. 14538011

Date:
Place: Roorkee

ABSTRACT

Dimensional and geometric tolerances should be estimated for proper assemblability and design functions as well as manufacturability of mechanical assemblies. Tolerance analysis of mechanical assemblies by manual procedure is easy but it is time consuming and error prone and also more human interaction is required which make the manufacturing cost of the product high. Therefore, automation of tolerance analysis is necessary to reduce the above drawbacks. In this work, dimensional tolerance analysis has been done based on the modified worst case and root sum square methods. The common methods do not consider the sensitivities, so these are applicable only for symmetric tolerances. But the modified models use the sensitivities which make them to handle the asymmetric tolerances. The percentage contributions of the upper and the lower tolerance bounds of the manufactured dimensions or independent variables on the upper and the lower tolerance bounds assembly or dependent variables by using the new relations for both the models are also presented. The procedure is implemented in MATLAB for few examples of linear and non-linear problems.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABBREVIATIONS

X_i	Manufactured or independent variables
X	Assembly or dependent variables
Y	Assembly function
θ	Contact angle
W_i	Signed weighting factor
T_i	Tolerance of i^{th} manufactured/ independent variable or dimension
T_X	Resultant assembly tolerance
$\frac{\partial g}{\partial x_i}$	Sensitivities of assembly function w.r.t. the individual manufactured variables
i	Index for manufactured/ independent variables
j	Index for assembly/ dependent variables
UTL and LTL	Upper and lower tolerance limits of the assembly variable, respectively
UL and LL	Upper and lower tolerance limit of the manufactured variables, respectively
CAD	Computer aided design
CATs	Computer aided tolerance softwares
GDT	Geometric and dimensional tolerances
%WC	Percentage contribution of independent variables using Worst-Case method
%RSS	Percentage contribution independent variables using Root-Sum-Square method

CHAPTER 1: INTRODUCTION

A tolerance is the amount of dimensional variation of parts in a mechanical assembly, which is acceptable/ allowable so that the functional specifications are completely met at the minimum manufacturing cost. By nature, material behavior and manufacturing processes are not exact, so the two manufactured parts cannot be identical in every way. There is some degree of variation will exist in the manufactured parts. The engineers/ designers apply tolerances to the dimensions of the parts at the design stage to prevent undesirable deviations/ variations of assemblies at early stages. Drawings with properly applied tolerances provide the best opportunity for uniform interpretation and cost effectively assembly. Thus tolerance analysis is important not only to verify the ability of parts to assemble but also in the quality of assembled parts.

Tolerance identifications and their values specified manually, will increase the variability in parts or assembly tolerance build-up, which in turn affect the quality of the parts or assembly. Thus automatic tolerancing of the assemblies is necessary to decrease the design complexities and to lower the manufacturing costs of the products/ assembly and also the human interaction will be minimum. Various researchers have given the different methods for tolerance analysis and various CAT (Computer Aided Tolerancing) software have been developed for automatic tolerance specification and accumulation.

There are two types of tolerances on the nominal geometries of the assemblies. These are the dimensional and geometric tolerances. The dimensional tolerances are the size tolerances which can be clearance or interference. The geometric tolerances are the surface tolerances in which control the surface roughness and irregularities. These are the: (1) Form tolerances, (2) Orientation tolerances, (3) Location tolerances, (4) Profile tolerances, and (5) Run-out tolerances. This work presents mathematical model for dimensional tolerance analyses by using the worst-case and the statistical method. Basic purpose of dimensional tolerance analysis is to get a robust design. The process is best when employed at the early design/ development stages, because at earlier stages it is possible to change the nominal geometry of a mechanical assembly. The worst-case (WC) method needs complete interchangeability (100% acceptance rate), i.e. the accuracy of the result obtained from WC method is required to be within the opportunity of the functioning requirement of the assemblies, and the success probability of assembly's is 100 percent. Results of WC method are in more manufacturing cost and it make sure that the assembly specifications are fully met if all of the parts are within the specifications. By contrast, statistical, root sum square (RSS) method require less interchangeability (less than 100% acceptance rate). In RSS method it is assumed that errors in the components being independent and normally distributed. Variance can be used to calculate the assembly preciseness, because the deviations/ variations are identical from the geometry for every statistically independent direction. The calculation process in RSS method is less strict to ensure that most of the products are within the opportunity of the functional requirements of the assembly. However, in RSS method assembly requirements are poorly (less than 100%) satisfied. The results provided by WC method are too pessimistic, while the results provided by RSS method are overly optimistic. These two models are used generally when the specific assembly function (dependent variable or dimension) can be defined as a linear function, in terms of the part dimensions (independent variables) specified on the engineering drawings. As in a journal bearing the clearance of a shaft is an assembly dimension, which is determined by the independent variables which are the bearing and shaft diameters. The spring constant/ stiffness of a helical spring is a dependent variable, while the coil diameter, wire diameters, and the number of turns are independent variables. If we use the methods described above, generally some considerations are taken that limits or bounds of the tolerance associated with the manufactured dimensions are symmetric and bilateral.

These two methods are very popular for tolerance accumulation in the mechanical assemblies because of their simplicity. However the problem with these are: (1) they are useful only for linear assembly functions, (2) symmetric tolerances can only be used, while many of tolerances are asymmetric and unilateral. In this research, first the common practice in linearizing of the assembly function is reviewed. Then modifications of these two methods is presented which can handle asymmetric and unilateral tolerances. This has been done in MATLAB to find upper bound and lower bound by taking three examples. For each of the example bar chart of the percentage contribution of each of the individual part/ component dimension on the assembly variable has been developed using MATLAB. This show the potential influence of each part/ component variations on the assembly variation.

This report is subdivided into following chapters and each chapter is discussed below briefly: Chapter 1 gives an introduction. Chapter 2 describes the literature review of the tolerance analysis of mechanical assemblies with manual and automatic methodologies adopted by researchers in the past. Chapter 3 covers the numerical formulation for the automatic tolerance analysis of the assemblies with symmetric and asymmetric tolerances. Chapter 4 contains the validation of the present model with the available literature results is shown and then results and discussions of the analysis performed in the present work for various other linear and non-linear problems. Chapter 7 concludes the work of this report and gives an outlook for future work followed by references.

CHAPTER 2: LITERATURE REVIEW

In this chapter various literatures related to assembly features, assemblability, tolerance analysis of linear and non-linear problems with unilateral and bilateral, symmetric and asymmetric tolerances of assembly variables, assembly feature recognition, pattern detection, directions of controls, and loop detection are reviewed.

Greenwood and Chase (1988 and 1990) presented the worst case and the root sum square models to solve non-linear problems for dimensional tolerance analysis. They presented these methods for tolerance analysis of impendent variables with respect to the independent variables with symmetric and bilateral limits [5].

Wang and Ozsoy (1990) developed an algorithm for assembly representation and automatic tolerance chain generation for dimensional tolerance analysis at the assembly level. They represented mating relations between the parts and the subassemblies using the properties like: mating paths, mating links, mating characteristics and the mating condition of the each part or subassembly. They represented the mating graphs of parts or subassemblies for tolerance analysis chain generation [23].

Treacy et al. (1990) described a statistical approach for dimensional tolerance buildup of the mechanical assembly. They represented relationships between the assembly-level and the component-level to generate automatic tolerance chain for an assembly or critical dimension. They presented a graphical vision for the distribution of the tolerance ranges of critical dimension with the location and confidence level for that variable by using the beta distribution [22].

Anantha et al. (1996) developed an approach to represent mechanical assemblies which is made of different parts. This approach integrates a philosophy to represent the relationships among the characteristics of the assembly for satisfaction of the geometric constraints. The method expressed in this paper exerts a graphical representation for the geometric configuration of the parts of an assembly for solving the spatial constraints among these parts. In comparison with other approaches to solve a nonlinear equation reducing the geometric relationship [1].

Mullins and Anderson (1998) defined that the recognition of the geometric constraints in a mechanical assembly is required to illustrate the effects of the variations in the dimensions and the tolerances of that assembly. They presented a method for recognition of such assembly constraints in the CAD models of 3D assemblies by using a search algorithm with graphical representation [14].

Whitney (2004) classified mechanical assemblies made of different components into three categories which are the mechanisms, structures, and distributive systems. The mechanism type of assemblies are sub-divided into another two categories which are the parts in a box and the connected parts, while the structures type of assemblies are also sub-divided into another two categories which are the trusses and the shells. He highlighted that the necessary condition for mating two/ more characteristics or features, for the justification of the assemblability of the manufactured parts, is that they should not be over constrained [24].

Shen et al. (2005) described four different type of methods for the purpose of tolerance analyses and compared them. These methods are as: (1) One-dimensional tolerance chart, which deals with the worst-case (WC) tolerance analysis of a mechanical assembly in one direction only at one time but rejects the other possible directions' contribution. (2) Parametric tolerance analysis (Mote Carlo simulation based), which depends on the parametric constraint solving but its implicit disadvantage is that the preciseness of the illustrated results depend on user defined modelling system and it is also unable to corporate all the Y14.5 rules. (3) Kinematic (Vector loop) method, in which for modelling the assembly constraints the kinematic joints are used and it is also unable to corporate all the Y14.5 rules. (4) Tolerance-Map (T-Map) based tolerance analysis, which can model all the geometric tolerances and also their interpretation in exactly 3D circumstances and it is fully consistent with the Y14.5 standard but it may be slightly difficult to use by the designers [20].

Rachuri et al. (2006) developed an Open Assembly Model (OAM), an object oriented interpretation of model of a mechanical assembly. They proposed a unified information model to represent the mechanical assemblies. They represented an object oriented representation model for the electro-mechanical assembly models by using the UML (Unified Modeling Language) [18].

Movahhedy and Khodaygan (2007) presented the worst case and the statistical (root sum square) approaches for asymmetric tolerance analyses of the mechanical assemblies. They developed new models for asymmetric tolerance analysis by considering the signs of the sensitivities because common models do not consider these signs. They also represented the percentage contribution of the individual independent component dimensions on the UTL and the LTL of the critical dimension of the assembly [13].

Shen et al. (2008) addressed minimum or maximum tolerance charts which are popular for tolerance analysis in the components and the assemblies. But this approach is limited to only 1D (One-dimensional) worst-case tolerance accumulation for tolerance analysis in a mechanical assembly. They presented some methodologies to prepare the tolerance charts automatically. For this purpose a CAD (Computer Aided Design) model of the assembly is required as input with the GDT (Geometric and Dimensional Tolerance) specifications. They also presented some approaches for automatic extraction of the dimensions and tolerance loops for the user-defined analysis dimensions, and also the automation of the parts arrangement in the mechanical assemblies consisting with the worst-case analysis [21].

Zhang et al. (2009) described some reasoning rules for solving the component sequence in the mechanical assemblies by techniques based on the product prototypes in a CAD system and they also established hierarchical tree models of the assembly structures. Then on the basis of this, the automatic assembly tolerance type generation and also the assembly tolerance networks construction are presented using the geometric constraint variation theory and the polychromatic set theory [26].

Sambhoos et al. (2009) developed an assembly variant design methodology based on a component relationship model that captures assembly mating relationships at the feature level referred as assembly mating graph. They classified the mating relationships as: the direct relationships, the indirect relationships, and the interference relationships. In this research, the geometric methodologies are used for recognising the direct mating relationships and for identifying the indirect mating relationships and the interference mating relationships, a ray-firing algorithm is used [19].

Murshed et al. (2009) examined the assembly feature attributes and proposed a model for their identification in a consistent manner. For defining the features of an assembly which are implemented independently, this model may be used with a language like- EXPRESS (N-Rep). The model consists slots for the features of the parts of an assembly and their parametric, geometric, structural, kinematic, and mating relations [15].

Murshed et al. (2010) defined an assembly feature tutor in which they developed an interactive system for user defined assembly feature by providing a mechanism. The user provide inputs to the system by example interactivelly and the tutor provides output of assembly definitions written in a language like-EXPRESS. They also implemented a system for identifying the user defined features of the assemblies and developed an algorithm on the basis of the contact pairs for recognition of the part features [16].

Khodagyan and Movahhedy (2010) presented a methodology for automated tolerance design in mechanical assemblies based on fuzzy logic, a mathematical methodology. They developed this approach for asymmetric tolerance analysis for assembly models with variability in the part tolerances by using the WC and the RSS models. They described part tolerances in the form of the fuzzy numbers with their associated function by using the statistical distribution of the independent parts. Thus the assembly tolerances and specifications can be represented as the fuzzy numbers. They also used a fuzzy factor to convert the membership function into the fuzzy intervals which to be used for the model interval analysis. They also presented the upper and the lower bounds of the tolerances for the assembly dimension or the design variable [8].

Li et al. (2011) gave a representation model for tolerance build-up based on polychromatic sets theory (PST) to integrate the GDT design. They addressed some reasoning relations among the integrated and individual colours of PST. They defined two types of tier in their research as: (1) VGC (Variational Geometric Constraints) tier model, in which synthesis matrix of VGC are developed. (2) Tolerance type tier model, in which synthesis matrix of the tolerance type are developed [9].

Cheng and Tsai (2011) developed a new method which makes use of Lagrange multiplier method for optimum statistical tolerances accumulation to minimize the manufacturing costs subjected to the constraint on dimensional chains and the machining efficiency. For employing the proposed method for statistical tolerance analysis and for solving the optimization problems they presented the mutual power and the exponential cost tolerancing models by implementing an algorithmic methodology [3].

Governi et al. (2012) presented an approach for automated tolerance accumulation having the ability of minimizing the manufacturing cost of a component or a whole assembly. This approach is based on the Monte Carlo simulation for computing the statistical distributions of the independent variables to a quality level and makes use of Genetic Algorithm based optimization technique. The procedure described in this research is integrated with a tolerance analysis software eM-TolMate [4].

Zhong et al. (2013) proposed an approach based on ontology for automatic generation the type of assembly tolerances in order to decrease the variability and also to support the semantic conformity in designing the assembly tolerance specifications [27].

Chen et al. (2014) described four methods of 3D tolerance analyses and also presented comparison among them. These methods are as follows: (1) Tolerance-Map: It can model all of the tolerances by using a basis simplex and the areal coordinates. (2) Matrix model: This is traditional and compact using homogeneous matrices integrated with some CATs (Computer Aided Tolerancing Softwares) but the solutions obtained for constraint relationships containing the discriminations are quite problematic. (3) Unified Jacobian Torsor model: It takes the advantage of both the torsor model, appropriate for the tolerance representations and the Jacobian matrix, appropriate for the tolerance accumulation. Its efficiency with regard to computations is good while the constraints relationships among the component of torsor are required to be better for its certainty and effectiveness. (4) Direct linearization method (DLM): This uses the first order Taylor series expansion assembly models which are based on the vector loop of vector by using the vectors to represent either the part dimension or the assembly dimension. In DLM methodology the geometric tolerances are designed as the dimensional tolerance and it is not completely according to the tolerance specifications [2].

Mohan et al. (2014) presented an approach to do some pre-processing steps in the support of the automatic GDT system development by extracting the part features and the assembly specifications for tolerance analyses in mechanical assembly are presented in the form of neutral B-Rep. In that context, first recognising the mating features in the assemblies from the CAD file, next to identify the patterns among that features, then identifying the different direction of controls for each pattern and finally all of the existing loops which are possible for tolerance analysis [11].

Zhang et al. (2014) represented a concise study to the statistical tolerance and clearance analyses of mechanical assemblies and proposed an approach for tolerance analyses by using an effective CAT (Computer Aided Tolerance) analysis software that is VisVSA. The presented approach is based on the Monte Carlo simulation which gives the assurance of the preciseness of the assembly specifications and generating optimal tolerance distribution [25].

Mohan et al. (2014) presented a procedure for generating the direction of controls automatically using the CAD model of the mechanical assemblies or parts. The inputs to the setup is a STEP file of informations about the geometry of components or assemblies. The process of such type of analyses is implemented component by component to a mechanical assembly. The Directions of Controls (DoCs) loops are correlated to each-other by some orientation. Hence, the junction nodes are used to include the orientations with the DoC chains so that they can be joined into a constraint feature graph [12].

Haghighi et al. (2014) investigated a method to develop automatic GDT scheme generation in which identification of critical tolerance loops is required. A tolerance loop is a stack of the dimensions between the faces of the features controlling the assembly conditions. Regarding to this, the first main step is to determine the assembly features which called the mating features. According to their approach the global constraints (relationships of the assembly features), the local constraints (relationships of the component features) and the direction of controls are required to develop the tolerance chains. They presented a methodology for automatic identification of the dimensional loops related to the assembly specifications [6].

Qin et al. (2015) described a based on the description logics to implement the approach for designing the tolerance types automatically. It generates the tolerance type automatically integrating with the CAD system as well as provides an idea about the semantics for exchanging the tolerance informations among the heterogeneous CAD systems. They extended the process with the help of description logic SROIQ (D) for automatic GTZs (Geometric Tolerance Zones) generation in the CAD system [17].

Haghighi at el. (2015) presented a system for automatic GDT (Geometric and Dimensional Tolerance) scheme generation required by machining operation processes. First, they derived the DRFs (Datum Reference Frames) by the fixturing methodology in the every setup, then basic dimensions are determined for the machined features in the setup w.r.t. the derived DRFs. The range of potential geometric uncertainties are illustrated by using the shop datas of machining and operations for each type of tolerances like form tolerances, size tolerances, orientation tolerances, profile tolerances and position tolerances [7].

Litwa et al. (2015) developed a point based method for development of a model to do tolerance analysis automatically by using the existing informations. This approach provides the scope for tolerance accumulation of the assembly models more efficiently and around automatically [10].

CHAPTER 3: METHODOLOGY

In this section mathematical models are presented for automatic tolerance analysis of mechanical assemblies with both symmetric and asymmetric tolerances for linear and non-linear problems.

3.1 LINEARIZED METHOD FOR MEASURING THE ASSEMBLY VARIATION

In this section the worst-case (WC) and the statistical (RSS) models for linear and non-linear problems for tolerance analysis has been reviewed briefly. The basic purpose of reviewing is to provide a background for developing the modified methods. It is assumed that a certain function exists, relating the design variables/ assembly variable to the individual independent dimensions/ variables which are contributing in the assembly. These methods are described below:

3.1.1 WORST-CASE ANALYSIS MODEL

Generally, this method is used for tolerance analysis when several components/ parts are combined together to form an assembly. In WC analysis simply we add or subtract max or min tolerances related to the nominal dimensions of the component/ part to illustrate the possible worst conditions. Thus, such tolerance accumulation makes the assembly at its minimum or maximum acceptable dimension. The laws of probability are taken into account while using this method at least not in the realistic sense. However, in an assembly, it is very rare that all parts/ components will actually be at their minimum or maximum tolerance limits at the same time. Therefore, this is not actually the way of representative the tolerance buildup in a mechanical assembly. WC analysis is used at the only time when really necessary, and the assembly has critical interface with the some other feature of product which cannot be allowed to interfere/ obstruct with or be spaced huge far apart. By critical, it means that either a consumer requirement issue or a safety issue is concerned. The general WC model analysis is simply sum of the all component/ part dimensions at their worst-case minimum or maximum values.

$$\mathrm{X} = \mathrm{X}_1 + \mathrm{X}_2 + \mathrm{X}_3 + \mathrm{X}_4 + \cdots + \mathrm{X}_\mathrm{n}$$

Or, in general form it can be rewritten as:

$$\mathrm{X} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{W}_\mathrm{i}\mathrm{X}_\mathrm{i} \tag{3.1}$$

Where X is the assembly or design variable, X_i is the manufactured variable, and W_i is the signed weighting factor whose value is either -1 or +1 which depends on whether a particular dimension subtracts or adds to get assembly/ critical dimension.

In general, assembly variable or design variable, X, may be defined as a function of individual independent dimensions or the variables X_i, which are contributing in the assembly variation of interest. Usually, this defined function is also called as the assembly/ design function. In reality, tolerances of parts/ components of an assembly are not linearly build up in many cases. Then in that case, the following expression is generally used for defining the design function:

$$\mathrm{X} = \mathrm{g}(\mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3, \ldots \mathrm{X}_\mathrm{n}) \tag{3.2}$$

For non-linear problems, it can be quite challenging to define the design or assembly function. The small changes in the dependent or assembly dimension, in WC tolerance analysis for the non-linear problems, can be defined by using Taylor series (according to Greenwood and Chase):

$$\Delta \mathrm{X} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{\partial \mathrm{g}}{\partial \mathrm{X}_\mathrm{i}} \Delta \mathrm{X}_\mathrm{i} + \frac{1}{2} \sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{\partial \mathrm{g}}{\partial \mathrm{X}_\mathrm{i}} \sum_{\mathrm{j}=1}^{\mathrm{n}} \frac{\partial \mathrm{g}}{\partial \mathrm{X}_\mathrm{j}} \Delta \mathrm{X}_\mathrm{i} \Delta \mathrm{X}_\mathrm{j} + \cdots \tag{3.3}$$

In WC tolerance analysis only first order terms and, absolute values are used. General WC tolerance analysis equation can be expressed as follows:

$$\mathrm{T}_\mathrm{X} = \left|\frac{\partial \mathrm{g}}{\partial \mathrm{X}_1}\right| \mathrm{T}_1 + \left|\frac{\partial \mathrm{g}}{\partial \mathrm{X}_2}\right| \mathrm{T}_2 + \left|\frac{\partial \mathrm{g}}{\partial \mathrm{X}_3}\right| \mathrm{T}_3 + \cdots \left|\frac{\partial \mathrm{g}}{\partial \mathrm{X}_n}\right| \mathrm{T}_\mathrm{n} \tag{3.4}$$

$$\mathrm{X}_{\mathrm{nominal}} \approx \left|\frac{\partial \mathrm{g}}{\partial X_1}\right| \mathrm{X}_1 + \left|\frac{\partial \mathrm{g}}{\partial \mathrm{X}_2}\right| \mathrm{X}_2 + \left|\frac{\partial \mathrm{g}}{\partial \mathrm{X}_3}\right| \mathrm{X}_3 + \cdots \left|\frac{\partial \mathrm{g}}{\partial \mathrm{X}_\mathrm{n}}\right| \mathrm{X}_\mathrm{n} \tag{3.5}$$

Where T_i is the tolerance of i^{th} manufactured/ independent variable or dimension, and $\frac{\partial \mathrm{g}}{\partial \mathrm{X}_\mathrm{i}}$ is the sensitivity. The resultant assembly tolerance T_X will be equals to sum of the part tolerances, by assuming independent dimensions which have symmetric and bilateral tolerances. For a non-linear problem, we must have to assign some certain values of each partial derivative, as these derivatives define some specific sensitivities. Each part dimensions and their tolerance limits will induce these sensitivities on the assembly or critical dimension and their tolerance limits. Sometimes, these sensitivities are necessary for asymmetric tolerances on the assemblies.

It is usually not appropriate doing simple sum of the tolerances for tolerance analysis, because all the tolerances are not bilateral and many geometric and dimensional tolerances require interpretation. If components/ parts are manufactured within specifications, then it is guaranteed that assemblies will be within the specifications by using WC analysis in common practice.

3.1.2 STATISTICAL ANALYSIS MODEL

The statistical tolerance analysis method, which is also known as RSS (Root Sum Square) method, is based on the method approximation of population parameters. In this method probability of all parts/ components, which are being at their extreme range of tolerance is very low, is used. By assuming all the components dimensions to be independent and being normally distributed. Then for approximation of a non-linear assembly function, Taylor series expansion provides following equation for tolerance analysis:

$$\mathrm{T_X} = \sqrt{\left[\left(\frac{\partial \mathrm{g}}{\partial \mathrm{X}_1}\right)^2 \mathrm{T}_1^2 + \left(\frac{\partial \mathrm{g}}{\partial \mathrm{X}_2}\right)^2 \mathrm{T}_2^2 + \left(\frac{\partial \mathrm{g}}{\partial \mathrm{X}_3}\right)^2 \mathrm{T}_3^2 + \cdots + \left(\frac{\partial \mathrm{g}}{\partial \mathrm{X}_\mathrm{n}}\right)^2 \mathrm{T}_\mathrm{n}^2\right]} \quad (3.6)$$

Currently, statistical tolerance analysis model is the most popular method being used now a days. The results, by using this method, are in looser component/ part tolerances, tighter assembly tolerances and also the lower manufacturing cost of the products but computationally it is more complex than the WC method. In this method even if all components/ parts are within the specifications, the defective assembly can also be resulted. That's why it is conceptually unattractive to many designers. However, the probability of occurring this is very less. In fact it is attributed primarily that the manufactured variables which are assumed as independent variables and as normally distributed, are generally not obvious in actuality. The major potentials of RSS method for tolerance analysis are: the ease of automation, and the ability of linearizing very complicated design functions.

3.2 TOLERANCE BUILD-UP AND THE SENSITIVITIES

The sensitivities make known to us that how an assembly function changes according to changes in each of the individual variables. Without considering sensitivity analysis, the non-linear cases like: 1D, 2D and 3D worst-case and statistical cases cannot be expressively understood. A non-linear sensitivity can be calculated form the first term of the Taylor series expansion. In tolerance accumulation analysis the effects of the component tolerances are no similar on the assembly tolerance bounds or limits. The sign of coefficients of the sensitivities do an important role in calculating the tolerance limits of the assembly variable. These sensitivities are the partial derivative of the specified design/ assembly function w.r.t. the independent variables.

$$\mathrm{S_i} = \frac{\partial \mathrm{X}}{\partial \mathrm{X_i}} \tag{3.7}$$

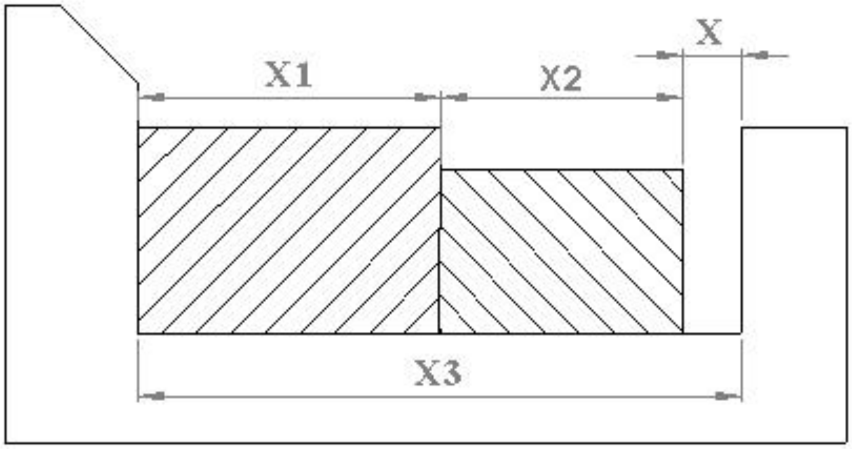

Figure 3.1: A simple assembly having three parts

Let consider a simple assembly, shown in Figure 3.1, to clarify this issue. This assembly consisting of two blocks X_1 and X_2, and a space X_3 in which these blocks have to fit. The gap or clearance dimension, X, is the critical dimension (the assembly variable). The design or assembly function for this assembly is given as:

$$\mathrm{X} = -\mathrm{X_1} - \mathrm{X_2} + \mathrm{X_3} \tag{3.8}$$

The sensitivity is positive for calculating the upper or lower tolerance limits of a mechanical assembly in which effective bounds of the components or parts tolerance are the upper or lower limits, and is negative for lower or upper limits of the parts tolerance. Therefore, the tolerance build-up analysis can be rewritten for the assembly presented in Figure 3.1 as:

$$\mathrm{UB_{\Delta X}} = [-1 \quad -1 \quad +1] \begin{bmatrix} \mathrm{LB_{\Delta X_1}} \\ \mathrm{LB_{\Delta X_2}} \\ \mathrm{UB_{\Delta X_3}} \end{bmatrix} \tag{3.9}$$

$$\mathrm{LB_{\Delta X}} = [-1 \quad -1 \quad +1] \begin{bmatrix} \mathrm{UB_{\Delta X_1}} \\ \mathrm{UB_{\Delta X_2}} \\ \mathrm{LB_{\Delta X_3}} \end{bmatrix} \tag{3.10}$$

3.3 MODIFIED TOLERANCE BUILD-UP MODELS

The tolerance analysis methods which are discussed above are relevant to symmetric tolerances. These methods are not appropriate for asymmetric and unilateral tolerances. The modified methods (WC and statistical method) for tolerance analysis are presented in this section which can handle asymmetric and unilateral tolerances. The above tolerance build-up models are not influenced by the signs of the values of the sensitivities. On the basis of equations (3.4) and equation (3.6) presented in the above models, the relations which consist tolerance limits (upper limit and lower limit) are developed.

The modified relations to estimate the tolerance limits for worst-case method can be expressed on the basis of equation (3.4) as follows:

$$\mathrm{UTL_j} = \frac{1}{2}\sum_{i=1}^{n}\left[\left|\frac{\partial g_j}{\partial X_i}\right|(\mathrm{UL_i} + \mathrm{LL_i}) + \left(\frac{\partial g_j}{\partial X_i}\right)(\mathrm{UL_i} - \mathrm{LL_i})\right] \tag{3.11}$$

$$\mathrm{LTL_j} = \frac{1}{2}\sum_{i=1}^{n}\left[\left|\frac{\partial g_j}{\partial X_i}\right|(\mathrm{UL_i} + \mathrm{LL_i}) - \left(\frac{\partial g_j}{\partial X_i}\right)(\mathrm{UL_i} - \mathrm{LL_i})\right] \tag{3.12}$$

Where the index i is for manufactured/ independent variables, the index j is for assembly/ dependent variable, $\left(\frac{\partial g_j}{\partial X_i}\right)$ is the sensitivity coefficients, UTL and LTL are the upper and the lower tolerance limit of the specified design or assembly variable, respectively.

While using modified equations as expressed above for the analysis of the asymmetric tolerances, the tolerance build-up analysis should be accomplished to calculate the UTL and LTL for the assembly. Equation (3.11) and equation (3.12) can also be rewritten in the general configuration as follows:

$$\mathrm{UTL_j}, \mathrm{LTL_j} = \frac{1}{2}\sum_{i=1}^{n}\left[\left|\frac{\partial g_j}{\partial X_i}\right|(\mathrm{UL_i} + \mathrm{LL_i}) + k\left(\frac{\partial g_j}{\partial X_i}\right)(\mathrm{UL_i} - \mathrm{LL_i})\right]$$

$$k = \begin{cases} +1 \text{ for UL} \\ -1 \text{ for LL} \end{cases} \tag{3.13}$$

The modified relations to estimate the tolerance limits for worst-case method can be expressed on the basis of equation (3.6) as follows:

$$\mathrm{UTL_j} = \sqrt{\frac{1}{2}\sum_{i=1}^{n}\left[\left|\frac{\partial g_j}{\partial X_i}\right|(\mathrm{UL_i} + \mathrm{LL_i}) + \left(\frac{\partial g_j}{\partial X_i}\right)(\mathrm{UL_i} - \mathrm{LL_i})\right]^2} \tag{3.14}$$

$$\mathrm{LTL_j} = \sqrt{\frac{1}{2}\sum_{i=1}^{n}\left[\left|\frac{\partial g_j}{\partial X_i}\right|(\mathrm{UL_i} + \mathrm{LL_i}) - \left(\frac{\partial g_j}{\partial X_i}\right)(\mathrm{UL_i} - \mathrm{LL_i})\right]^2} \tag{3.15}$$

The equations (3.14) and equation (3.15) can be rewritten in general form as follows:

$$\mathrm{UTL_j}, \mathrm{LTL_j} = \sqrt{\frac{1}{2}\sum_{i=1}^{n}\left[\left|\frac{\partial g_j}{\partial X_i}\right|(\mathrm{UL_i} + \mathrm{LL_i}) + k\left(\frac{\partial g_j}{\partial X_i}\right)(\mathrm{UL_i} - \mathrm{LL_i})\right]^2}$$

$$k = \begin{cases} +1 \text{ for UL} \\ -1 \text{ for LL} \end{cases} \tag{3.16}$$

3.4 PERCENTAGE CONTRIBUTIONS OF EACH OF THE INDIVIDUAL COMPONENT TOLERNACE ON A SPECIFIC ASSEMBLY VARIABLE

Tolerance analysis of an assembly presents two graphical representations of the sensitivity and the percentage contribution which show that how specific part tolerances affect the assembly variations. The representation of the sensitivity diagram presents that how much sensitive is a specific assembly variation to the each component dimensional variation. This sensitivity is a measure of potential impact of each of the component dimensional variation on the specific assembly variation. The value of the total contribution of this influence coming from the each of the individual component dimension to the assembly tolerance will be between 1 percent and 100 percent.

$$P_1 + P_2 + P_3 + \ldots + P_n = 100\% \tag{3.17}$$

Percentage contribution (P_i) is the proportion value of each of the individual part dimensions. This percentage contribution is the measure of existing effects of each of the part tolerance on the variations of an assembly variable. Also, these percentage contributions may differ in accordance with the method used for tolerance analysis.

Percentage contribution for worst-case model analysis:

$$\%\mathrm{WC} = \frac{\left|\frac{\partial g_j}{\partial x_i}\right| . T_i}{\sum_{i=1}^{n}\left[\left|\frac{\partial g_j}{\partial x_i}\right| . T_i\right]} \times 100 \tag{3.18}$$

And, percentage contribution for RSS model analysis:

$$\%\mathrm{RSS} = \frac{\left(\frac{\partial g_j}{\partial x_i} . T_i\right)^2}{\sum_{i=1}^{n}\left[\left(\frac{\partial g_j}{\partial x_i} . T_i\right)^2\right]} \times 100 \tag{3.19}$$

Where T_i is the tolerance of i^{th} independent/ manufactured part dimension, and $\left(\frac{\partial g_j}{\partial x_i}\right)$ is the sensitivity.

Modified relationships are used for asymmetric tolerances to define percentage contribution relationships. Hence, the new relations show the percentage contribution of the individual manufactured dimension or the independent variables on the UTL and LTL of the assembly dimension (dependent variable) in the mechanical assemblies. From the equations (3.11), equation (3.12), and equation (3.18) the percentage contribution of the individual independent dimensions on the UTL and LTL of the defined assembly variable can be determined for WC model as follows:

Percentage contribution (%WC) of each of the component tolerances on the upper limit (UTL_j) of assembly variable:

$$\%\mathrm{WC\ on\ UTL_j} = \frac{\left[\left|\frac{\partial g_j}{\partial x_i}\right|(UL_i + LL_i) + \left(\frac{\partial g_j}{\partial x_i}\right)(UL_i - LL_i)\right]}{\sum_{i=1}^{n}\left[\left|\frac{\partial g_j}{\partial x_i}\right|(UL_i + LL_i) + \left(\frac{\partial g_j}{\partial x_i}\right)(UL_i - LL_i)\right]} \times 100 \tag{3.20}$$

Percent contribution (%WC) of each of the component tolerances on the upper limit (LTL_j) of assembly variable:

$$\%\mathrm{WC\ on\ LTL_j} = \frac{\left[\left|\frac{\partial g_j}{\partial x_i}\right|(UL_i + LL_i) - \left(\frac{\partial g_j}{\partial x_i}\right)(UL_i - LL_i)\right]}{\sum_{i=1}^{n}\left[\left|\frac{\partial g_j}{\partial x_i}\right|(UL_i + LL_i) - \left(\frac{\partial g_j}{\partial x_i}\right)(UL_i - LL_i)\right]} \times 100 \tag{3.21}$$

From the equations (3.14), equation (3.15), and equation (3.19) the percentage contribution, on the UTL and LTL of the desired assembly dimension of the manufactured dimensions can be determined for statistical model as follows:

Percent contribution (%RSS) of each of the component tolerances on the upper limit ($\mathrm{UTL_j}$) of assembly variable:

$$\%\mathrm{RSS\ on\ UTL_j} = \frac{\left[\left|\frac{\partial \mathrm{g_j}}{\partial \mathrm{x_i}}\right|(\mathrm{UL_i}+\mathrm{LL_i})+\left(\frac{\partial \mathrm{g_j}}{\partial \mathrm{x_i}}\right)(\mathrm{UL_i}-\mathrm{LL_i})\right]^2}{\sum_{\mathrm{i}=1}^{\mathrm{n}}\left[\left|\frac{\partial \mathrm{g_j}}{\partial \mathrm{x_i}}\right|(\mathrm{UL_i}+\mathrm{LL_i})+\left(\frac{\partial \mathrm{g_j}}{\partial \mathrm{x_i}}\right)(\mathrm{UL_i}-\mathrm{LL_i})\right]^2} \times 100 \tag{3.22}$$

Percent contribution (%RSS) of each of the component tolerances on the upper limit ($\mathrm{LTL_j}$) of assembly variable

$$\%\mathrm{RSS\ on\ LTL_j} = \frac{\left[\left|\frac{\partial \mathrm{g_j}}{\partial \mathrm{x_i}}\right|(\mathrm{UL_i}+\mathrm{LL_i})-\left(\frac{\partial \mathrm{g_j}}{\partial \mathrm{x_i}}\right)(\mathrm{UL_i}-\mathrm{LL_i})\right]^2}{\sum_{\mathrm{i}=1}^{\mathrm{n}}\left[\left|\frac{\partial \mathrm{g_j}}{\partial \mathrm{x_i}}\right|(\mathrm{UL_i}+\mathrm{LL_i})-\left(\frac{\partial \mathrm{g_j}}{\partial \mathrm{x_i}}\right)(\mathrm{UL_i}-\mathrm{LL_i})\right]^2} \times 100 \tag{3.23}$$

The influence of each of the individual component tolerances, using the new relationships, are obtained by the designer on the resultant tolerance allocation will be illustrated.

CHAPTER 4: CASE STUDY

There are different examples are shown in details below to represent the efficiency of modified tolerance analysis models and the new relations for determination of the percentage contributions.

4.1 EXAMPLE 1: COMBUSTION CHAMBER ASSEMBLY

An example of 1-D tolerance analysis is taken from the reference of Movahhedy and Khodaygan is shown in Figure 4.1 which is a combustion chamber assembly. The height of this combustion chamber X is the assembly/ critical dimension.

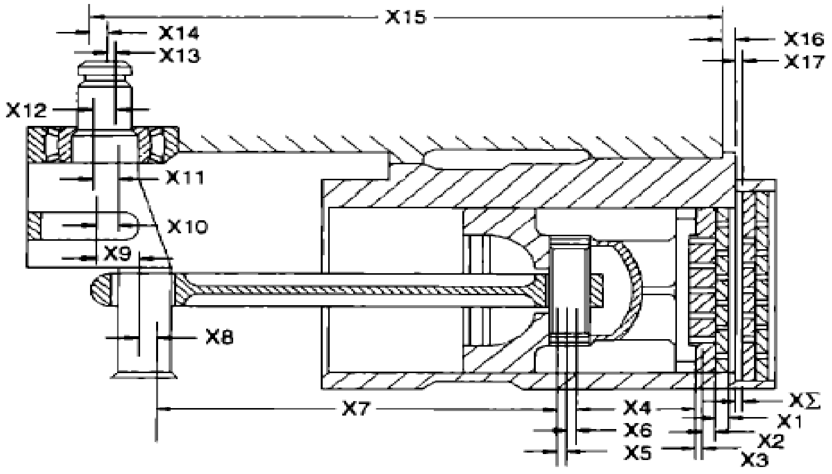

Figure 4.1: Combustion chamber assembly

Assembly function for this assembly can be written as:

$$X\sum = -X_1 - X_2 - X_3 - X_4 + X_5 + X_6 - X_7 + X_8 - X_9 + X_{10} + X_{11} + X_{12} + X_{13} + X_{14} + X_{15} + X_{16} + X_{17} \tag{4.1}$$

Table 4.1 consists of the variables, their tolerance limits and their sensitivities. These sensitivities are the partial derivatives of the assembly function (equation (4.1)) w.r.t. the each of the independent variables. This is the linear equation, therefore sensitivities can be written directly +1 or -1 by seeing the sign of the each variables. There are different types of variables are used in this example such as clearance, interference and eccentricity. The results coming out for tolerance accumulation of a mechanical assembly with asymmetric tolerances of the independent variables can cause the asymmetric tolerance limits for assembly variable or design specification.

Table 4.1: The sensitivities and the tolerances of independent variables for combustion chamber assembly

Variables	Sensitivities	UL (mm)	LL (mm)
X_1	-1	0.04	0.045
X_2	-1	0.045	0.045
X_3	-1	0.03	0.028
X_4	-1	0.041	0.043
X_5	1	0	0.001
X_6	1	0.045	0.047
X_7	-1	0.046	0.045
X_8	1	0.045	0.046
X_9	-1	0.047	0.047
X_{10}	1	0.04	0.04
X_{11}	1	0.15	0.1
X_{12}	1	0.034	0.035
X_{13}	1	0.1	0.1
X_{14}	1	0.039	0.04
X_{15}	1	0.04	0.04
X_{16}	1	0.034	0.036
X_{17}	1	0.045	0.045

The comparison of results is presented in Table 4.2 between reference results and the results obtained by the proposed approach in MATLAB by using modified WC and RSS models for the UTL and LTL design for the assembly variable (X).

Table 4.2: Validation of the UTL and LTL of assembly variable (X∑) by using modified the WC and the RSS models

Tolerance analysis method	**Reference results**		**Presented results**	
	UTL (mm)	**LTL (mm)**	**UTL (mm)**	**LTL (mm)**
Modified WC	0.825	0.779	0.8250	0.7790
Modified RSS	0.336	0.297	0.3362	0.2973

It is clear from the Table 4.2 that the results obtained from the reference and the present approach using MATLAB are similar. This shows the validation of the proposed approach.

The percentage contributions of each of the independent/ the manufactured variables on the UTL and LTL of the design or assembly variable using modified worst-case method are shown in Figure 4.2 and Figure 4.3. Figure 4.4 and Figure 4.5 display the same result using modified RSS method.

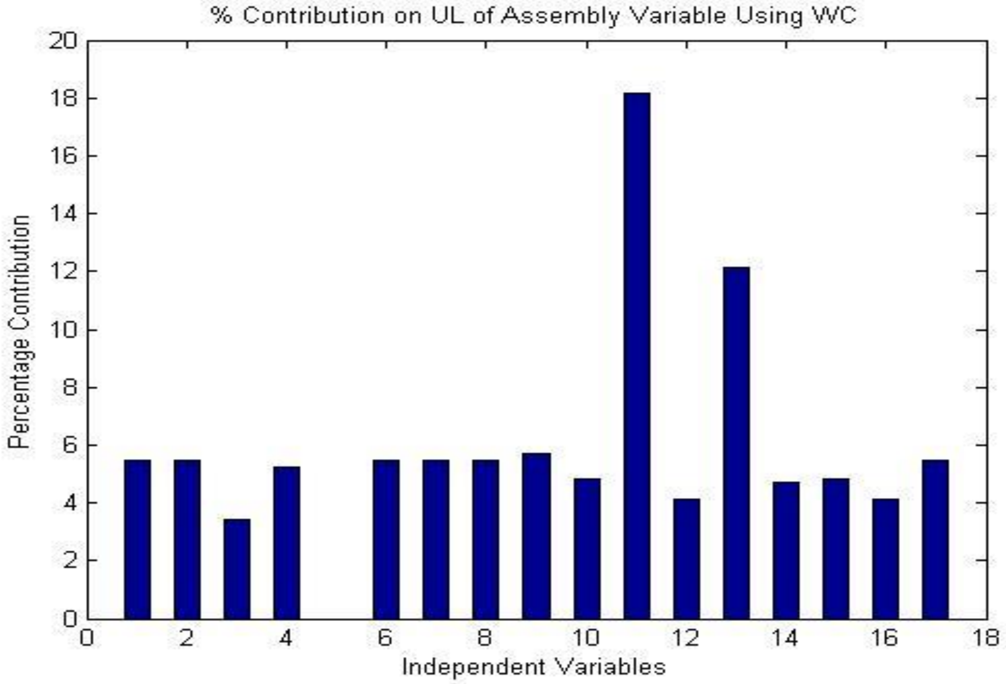

Figure 4.2: Percentage contribution of individual independent variable on the UTL of assembly variable (X∑) using modified WC method

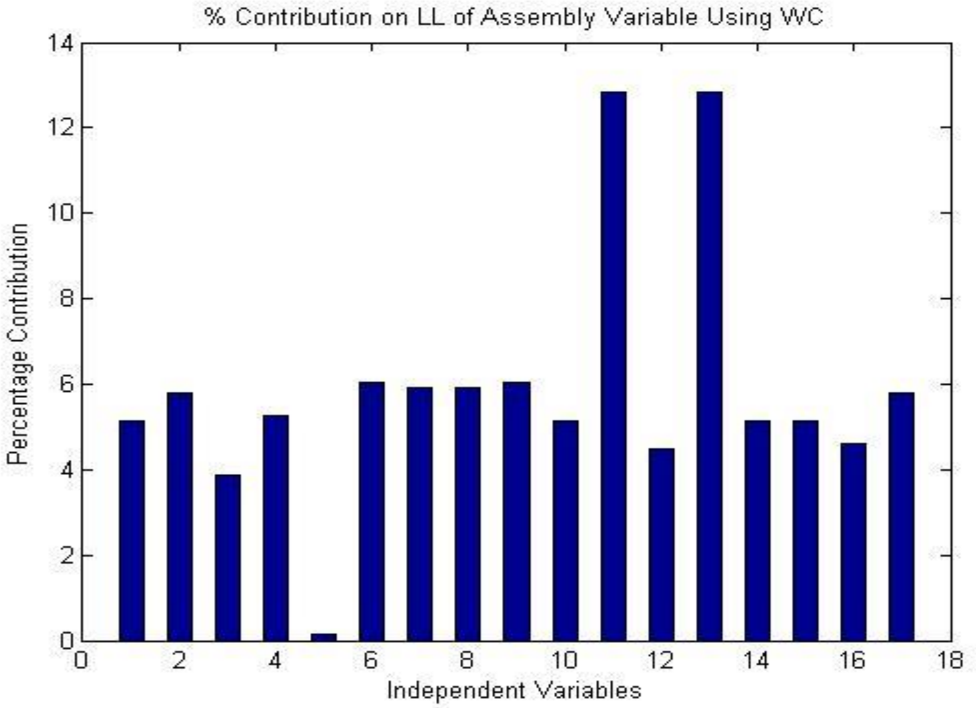

Figure 4.3: Percentage contribution of individual independent variable on the LTL of assembly variable (X∑) using modified WC method

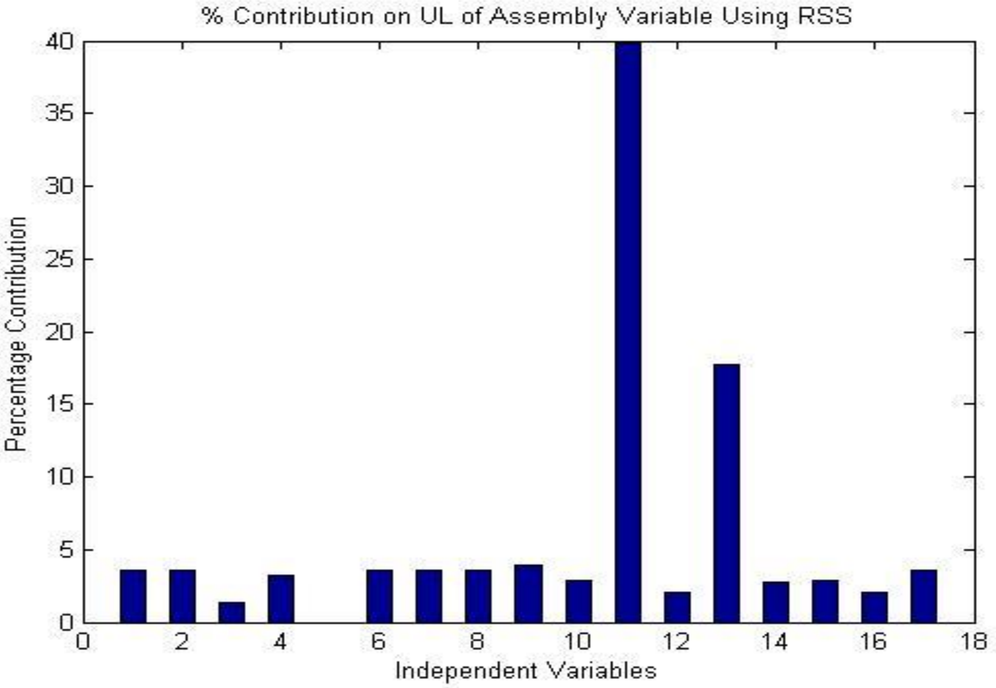

Figure 4.4: Percentage contribution of individual independent variable on the UTL of assembly variable (X∑) using modified RSS method

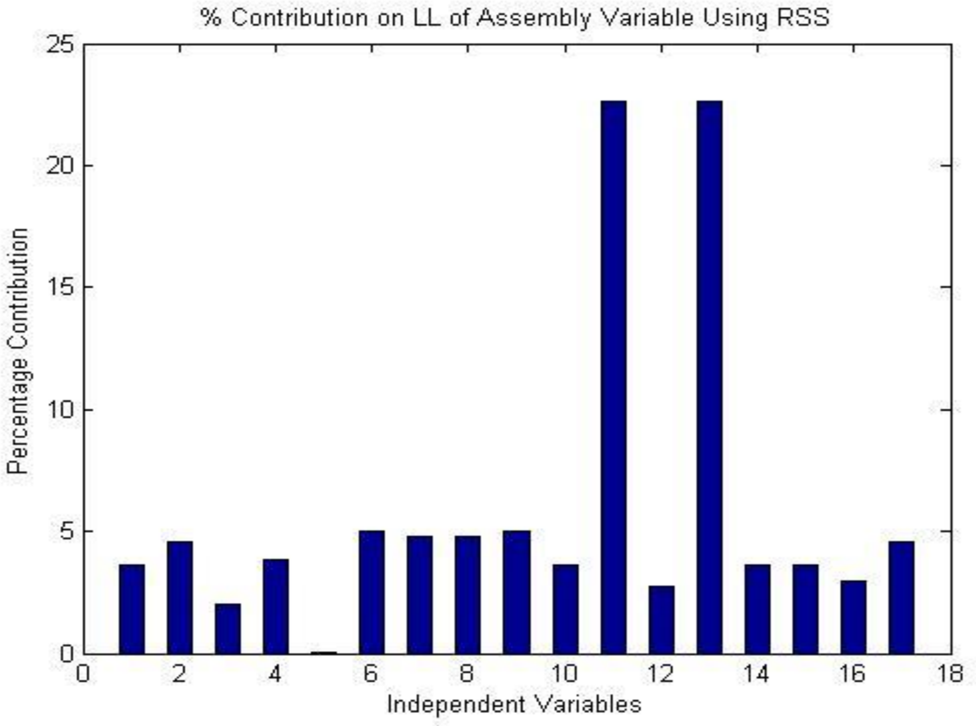

Figure 4.5: Percentage contribution of individual independent variable on the LTL of assembly variable (X∑) using modified RSS method

The comparison results using modified WC and RSS methods are presented in Figure 4.2, Figure 4.3, Figure 4.4, and Figure 4.5 which show the percentage contributions of the individual independent variable on the tolerance limits of design/ dependent variable. It is clear from Figure 4.2 and Figure 4.4 that X_{11} and X_{13} are the critical dimensions to the assembly variable for calculating the upper tolerance limit of that variable while X_5 has less contribution in influencing the assembly variable. Or, we can say that X_{11} and X_{13} are the major contributors while X_1 is lower contributor in influencing the assembly variable. Figure 4.3 and Figure 4.5 are showing the same effects for calculating the lower tolerance limit of the assembly variable. According to Table 4.1 X_{11} and X_{13} have the loosest tolerance limits and X_5 has the tightest tolerance limits. Consequently, X_{11} and X_{13} have highest percentage contribution, while X_5 has smallest percentage contribution. Here, the magnitude of all the sensitivities is equal to 1. Therefore, different percentage contributions are because of the difference in tolerance limits of the variables.

4.2 EXAMPLE 2: THERMOS FLASK

An example of thermos (also called vacuum flask) is shown in Figure 4.6. It is an insulating storage vessel that makes the contents to remain cooler or hotter greatly lengthens over the time than its surroundings. This consists of two flaks which are placed one within another and are joined at the neck. To create partial vacuum the gap between these two flasks is partially evacuated. Because of this vacuum the heat conduction or convection is reduced. Where there is no vacuum are neck and the opening of the flask which cause the most of heat transfer through them. These vacuum flasks (thermos) are used to keep beverages cold or hot domestically for a long periods of time and for many purposes in the industries.

We want to maintain the gap X within some limits so that there can be proper vacuum between two flasks to reduce heat conduction or convection. This X is called the critical dimension or the assembly variable (dependent variable). The other dimensions are X_1 which is the whole thickness of the thermos, X_2 which is the inner flask's thickness, and X_3 which is the outer flask's thickness.

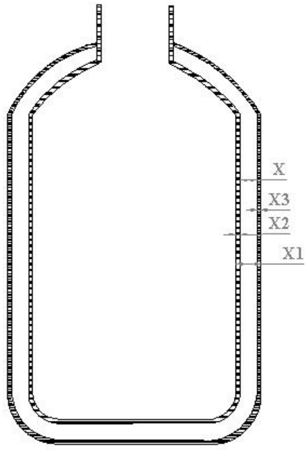

Figure 4.6: Thermos flask

These X_1, X_2, and X_3 are called the manufactured dimensions or independent variables which are given with the tolerances. Therefore, the assembly function can be written as:

$$X = X_1 - X_2 - X_3 \tag{4.2}$$

Table 4.3: Manufactured variables with tolerances and sensitivities for thermos flask model

Variables	**Sensitivities**	**UL (mm)**	**LL (mm)**
X_1	1	0.042	0.039
X_2	-1	0.037	0.036
X_3	-1	0.037	0.036

Table 4.3 consists of the independent variables with their sensitivities and their tolerances. The sensitivities are the partial derivatives of the assembly function (equation (4.2)) w.r.t. each of the independent variables or these can be directly written +1 or -1 from the assembly function for linear cases by seeing the signs of the independent variables. The results which are estimated for upper limit and lower limit for the design or assembly variable (gap between two flasks), X, using modified worst-case and the RSS methods in MATLAB are shown in Table 4.4. It is clear from Table 4.4 that results coming out for tolerance accumulation of an assembly with asymmetric tolerances of independent variables can cause the asymmetric tolerance limits for assembly variable or design specification.

Table 4.4: UTL and LTL of assembly variable (X) by using modified WC and RSS methods

Tolerance analysis method	**UTL (mm)**	**LTL (mm)**
Modified WC	0.114	0.113
Modified RSS	0.0933	0.0922

The percentage contributions of each of the independent/ the manufactured variables on UTL and LTL of the assembly variable (X) using modified worst-case method are shown in Figure 4.7 and Figure 4.8. Figure 4.9 and Figure 4.10 show the same using modified RSS method.

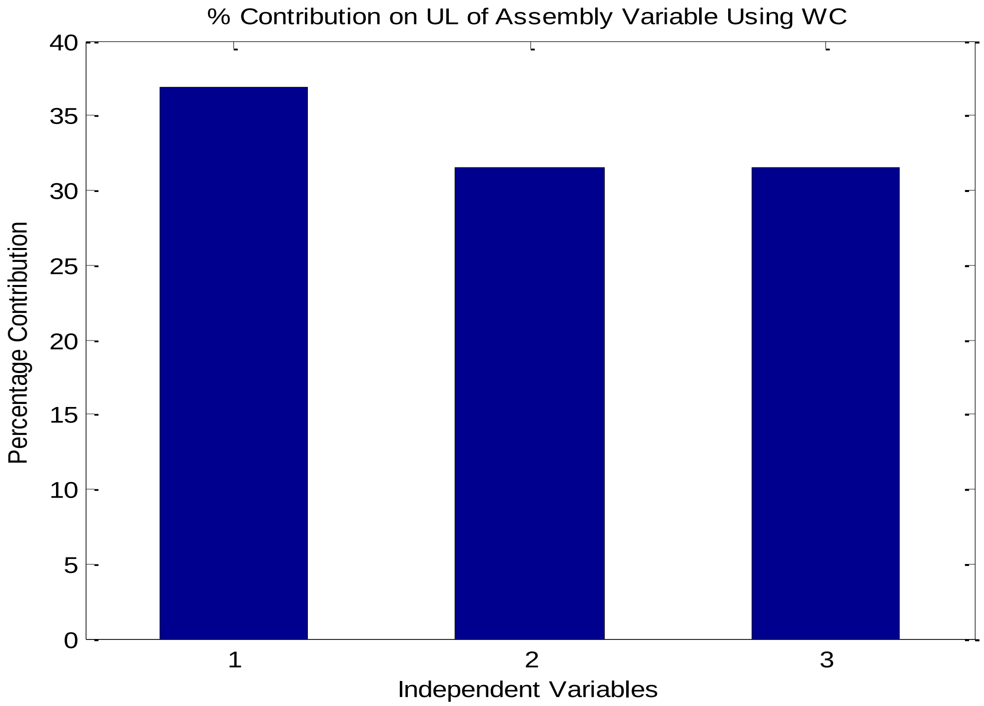

Figure 4.7: Percentage contribution of the individual manufactured variable on the UTL of assembly variable (X) using modified WC method

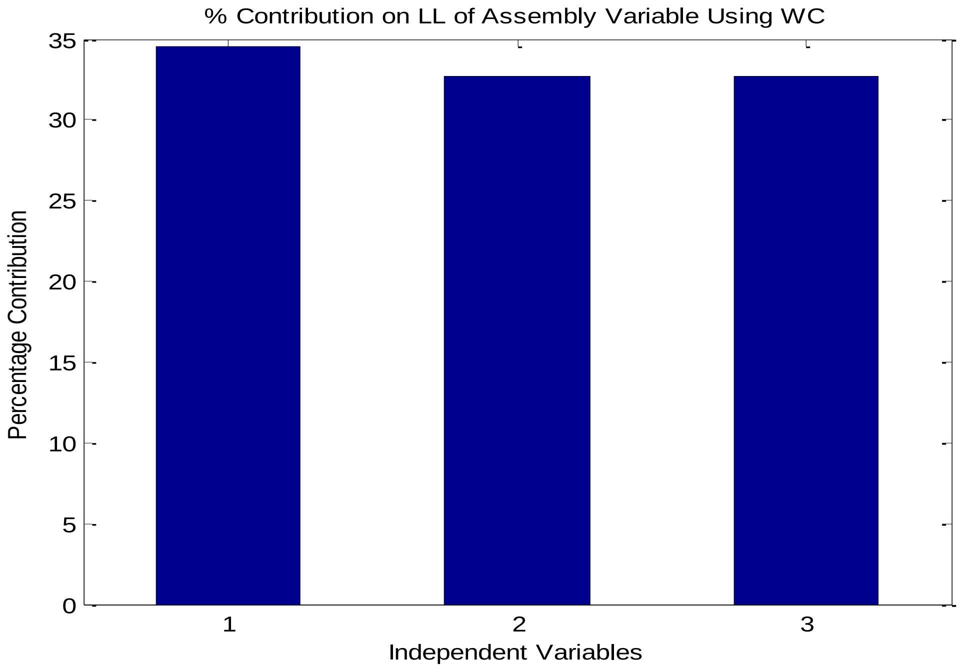

Figure 4.8: Percentage contribution of the individual manufactured variable on the LTL of assembly variable (X) using modified WC method

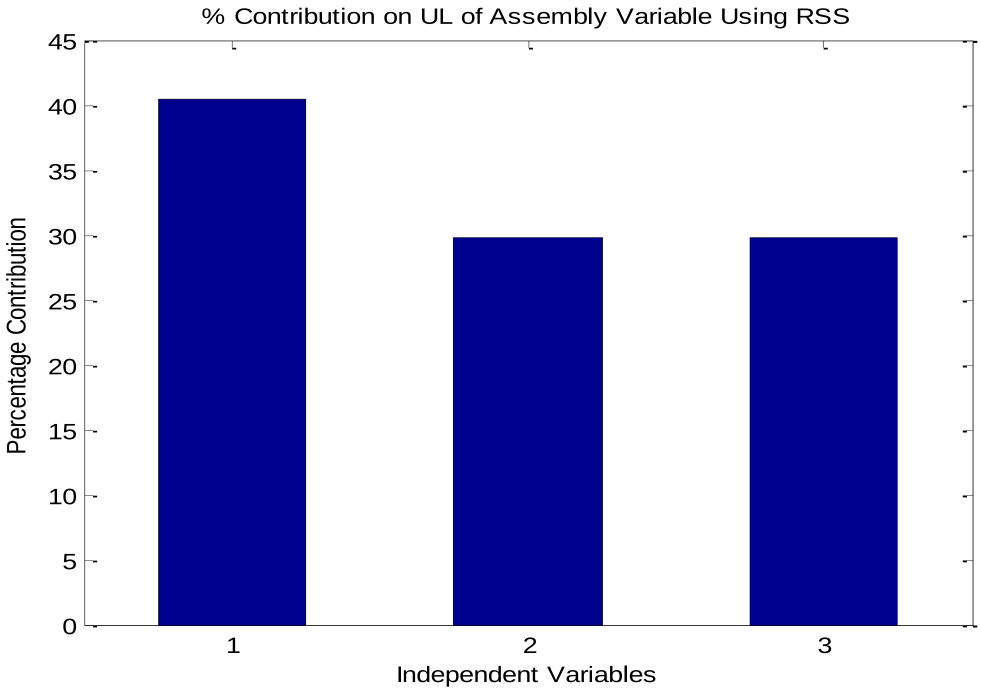

Figure 4.9: Percentage contribution of the individual manufactured variable on the UTL of assembly variable (X) using modified RSS method

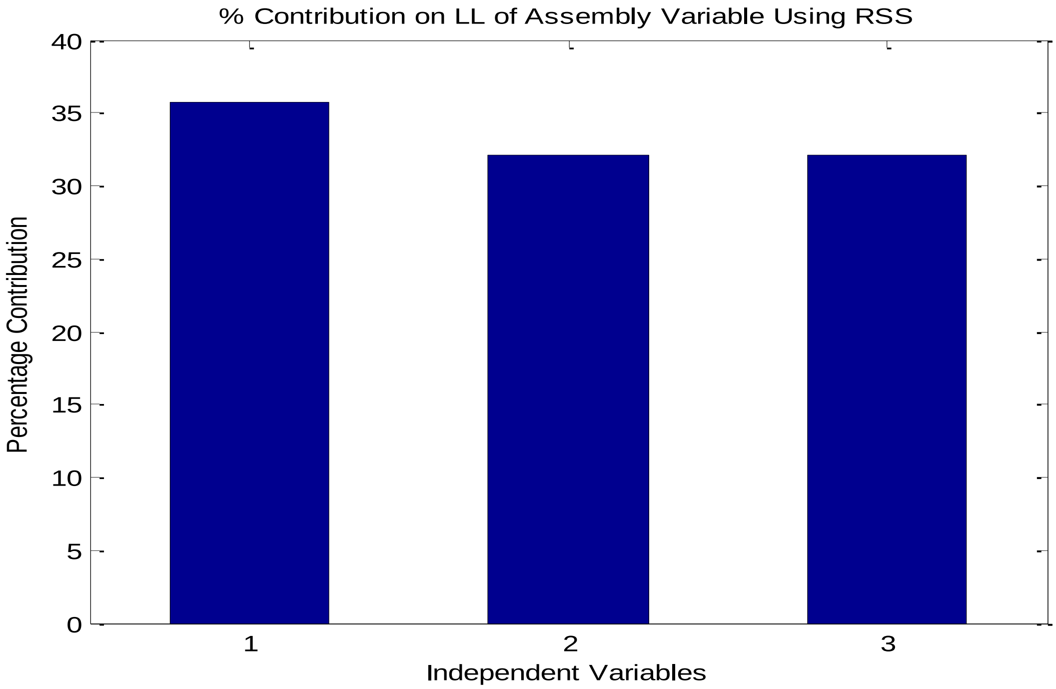

Figure 4.10: Percentage contribution of the individual manufactured variable on the LTL of assembly variable (X) using modified RSS method

The comparison of results coming out by using modified WC and RSS models, the percentage contribution of the independent/ manufactured variables on the tolerance limits design variable which are presented in Figure 4.7, Figure 4.8, Figure 4.9, and Figure 4.10 is similar in both the models. It is clear from Figure 4.7 and Figure 4.9 that X_1 is the judgmental dimension to the assembly variable for calculating the upper tolerance limit of that variable while X_2 and X_3 have less contribution in influencing the assembly variable. Or, we can say that X_1 is the major contributor while X_2 and X_3 are lower contributors in influencing the assembly variable. Figure 4.8 and Figure 4.10 are showing the same results for calculating the lower tolerance limit of that specific assembly variable.

The percentage contribution of the independent variables is determined by the sensitivity of design specification w.r.t. that variable and the tolerance limits of that variable. The magnitude of all the sensitivities in this example is equal to 1. Therefore, different percentage contributions are because of the difference in tolerance limits of the variables. According to Table 4.3, X_1 the loosest tolerance limits and X_1 has the tightest tolerance limits. Consequently, X_1 has highest percentage contribution, while X_2 and X_3 have smallest percentage contribution. Moreover, the asymmetric tolerances of the independent variables is the reason behind the differences in the percentage contribution of that variables on the UTL and LTL of the design/ assembly variable.

4.3 EXAMPLE 3: GEARBOX ASSEMBLY

A gearbox assembly is shown in Figure 4.11. In this example the critical dimension or assembly variable (X) is the gap between the gear-hub and the bushing. For proper working of the gear or to prevent the axial motion of gear, the gap must be less than some value and greater than zero. The other dimensions which are affecting the critical dimension, are X_1, X_2, X_3 X_4, and X_5. The assembly function can be written in the form of independent variables as:

$$X = X_1 + X_2 - X_3 - X_4 - X_5 \tag{4.3}$$

Here X_1 is the breadth of gearbox in the left side, X_2 is the breadth of gearbox in the right side, X_3 is the distance between faces of the gear-hub, X_4 is the thickness of left bushing flange, and X_5 is the thickness of right bushing flange. These independent variables are given with the bilateral symmetrical tolerance limits. The assembly variable (X) can be calculated by using equation (4.3).

The UTL and LTL of assembly variable (X) are presented by using the modified WC and RSS methods in MATLAB.

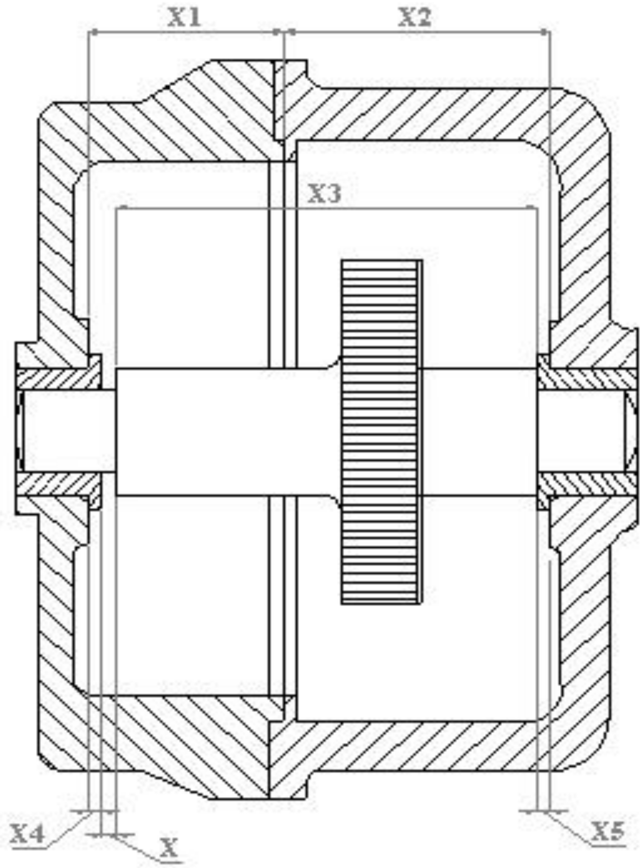

Figure 4.11: Gearbox assembly

Table 4.5: Nominal dimensions of independent variables with their sensitivities, and tolerance limits for the gearbox assembly

Variables	Values (mm)	Sensitivities	UL (mm)	LL (mm)
X_1	40	1	0.086	-0.086
X_2	50	1	0.086	-0.086
X_3	79	-1	0.024	-0.024
X_4	5	-1	0.075	0.075
X_5	5	-1	0.075	0.075

Table 4.5 given above show the independent variables with their nominal values, sensitivities, and tolerance limit which are bilateral. The sensitivities are the partial derivatives of the defined assembly function w.r.t. the each of independent variable respectively. Because this is the example of linear problem, therefore these sensitivities can be directly written as +1 or -1 by seeing the signs of the manufactured variables. The results for the UTL and LTL of assembly variable (the gap between the gear-hub and the bushing), X, by using the modified WC and RSS models in MATLAB are shown in Table 4.6. From Table 4.6, it is clear that the results obtained for UTL and LTL of assembly variable (X) by the modified WC method are the bilateral and symmetrical tolerance limits, and by the modified RSS method these are also symmetrical but unilateral. Hence, the tolerance limits obtained by the both methods are symmetrical because of the symmetrical tolerance of the independent variables.

Table 4.6: UTL and LTL of the gap (X) between the gear-hub and the bushing by using modified WC and RSS methods

Tolerance analysis method	UTL (mm)	LTL (mm)
Modified WC	0.046	-0.046
Modified RSS	0.2307	0.2307

The percentage contributions of each of each of the manufactured/ independent variables on the UTL and LTL of assembly (design) variable by using modified WC method are presented in Figure 4.12, Figure 4.13. The same results are presented by using the modified RSS method in Figure 4.14, and Figure 4.15.

It is clear from Figure 4.12 and Figure 4.14 that X_1 and X_2 are the critical dimensions to the assembly variable for calculating the upper tolerance limit of that variable, while X_4 and X_5 have less contribution and X_3 has the least contribution in influencing the assembly variable (X). Or, we can say that X_1 and X_2 are the major contributors while X_4 and X_5 are the lower contributors, and X_3 is the lowest contributor in influencing the assembly variable. Figure 4.13 and Figure 4.15 are showing the same effects for calculating the lower tolerance limit of the desired assembly variable (X). According to Table 4.5, X_1, X_2, X_4 and X_5 have the loosest tolerance limits and X_3 has the tightest tolerance limits. Consequently, the variables X_1 X_2, X_4 and X_5 have highest percentage contribution, while the variable X_3 has smallest percentage contribution on the assembly variable X.

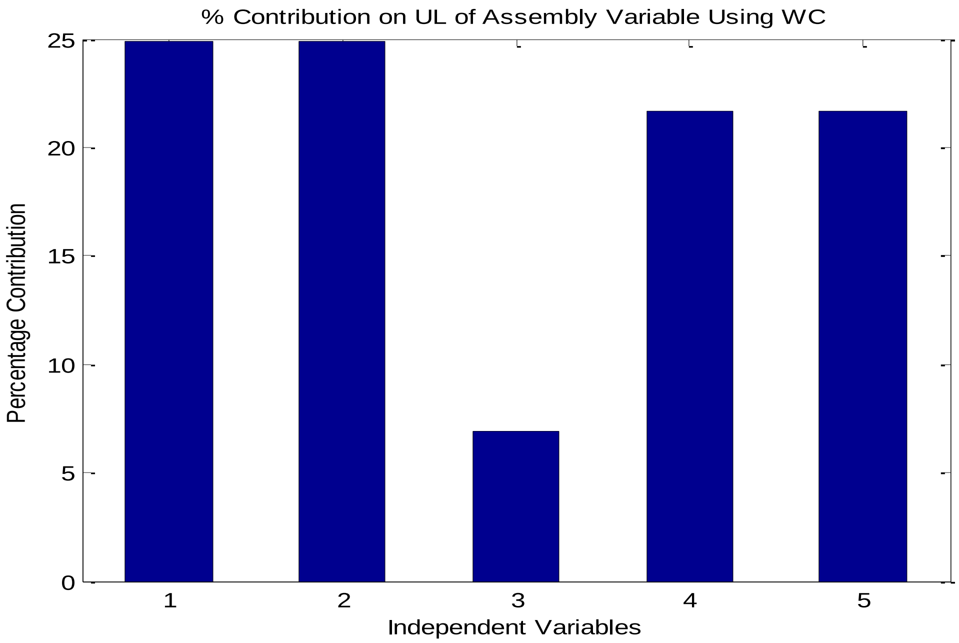

Figure 4.12: Percentage contribution of the individual manufactured variable on the UTL of the gap between the gear-hub and the bushing (X) using modified WC method

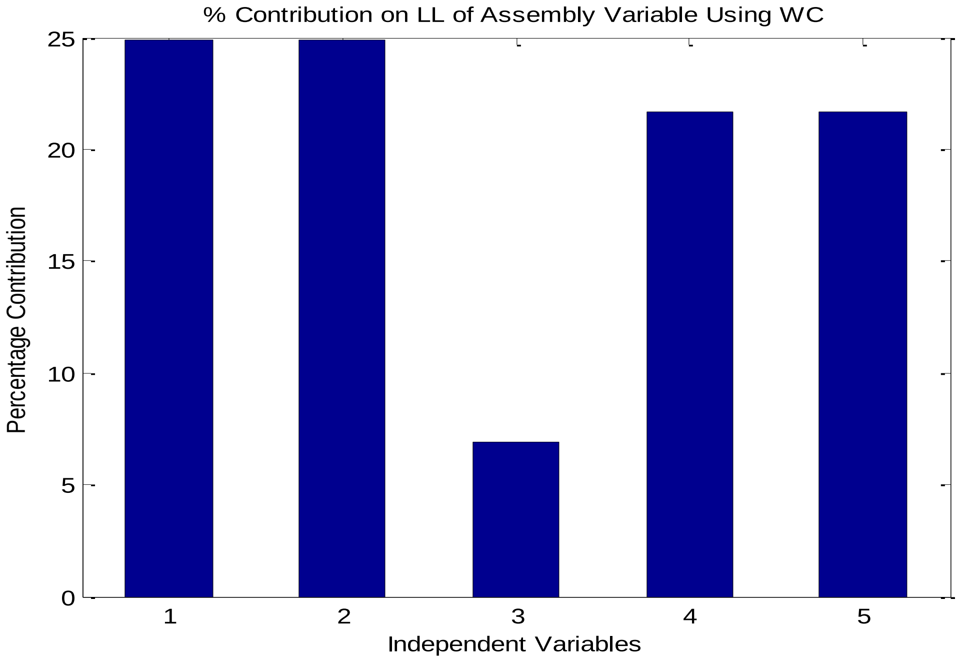

Figure 4.13: Percentage contribution of the individual manufactured variable on the LTL of the gap between the gear-hub and the bushing (X) using modified WC method

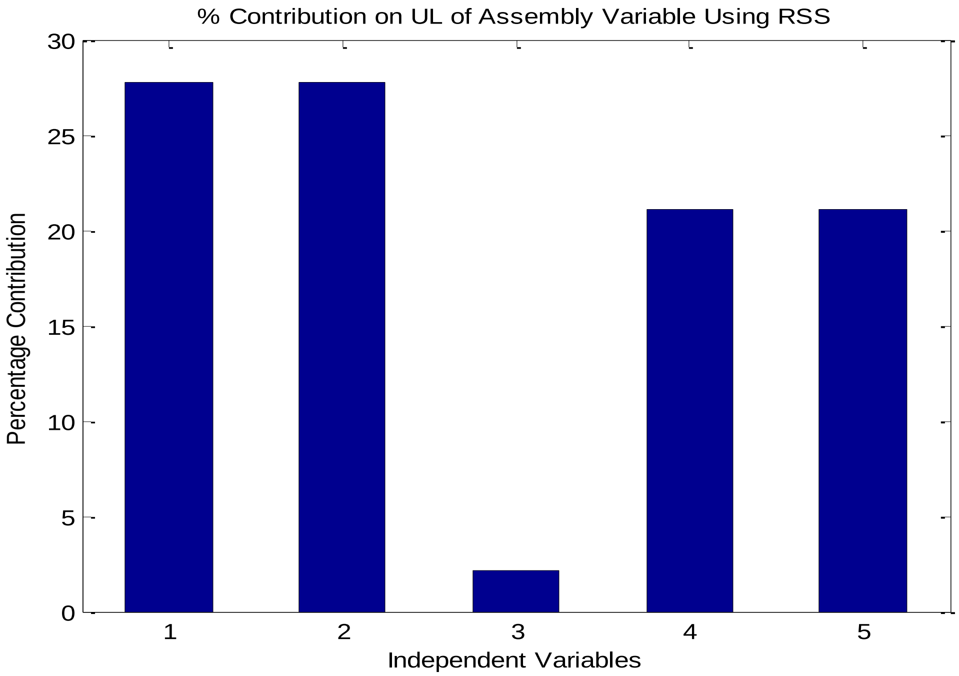

Figure 4.14: Percentage contribution of the individual manufactured variable on the UTL of the gap between the gear-hub and the bushing (X) using modified RSS method

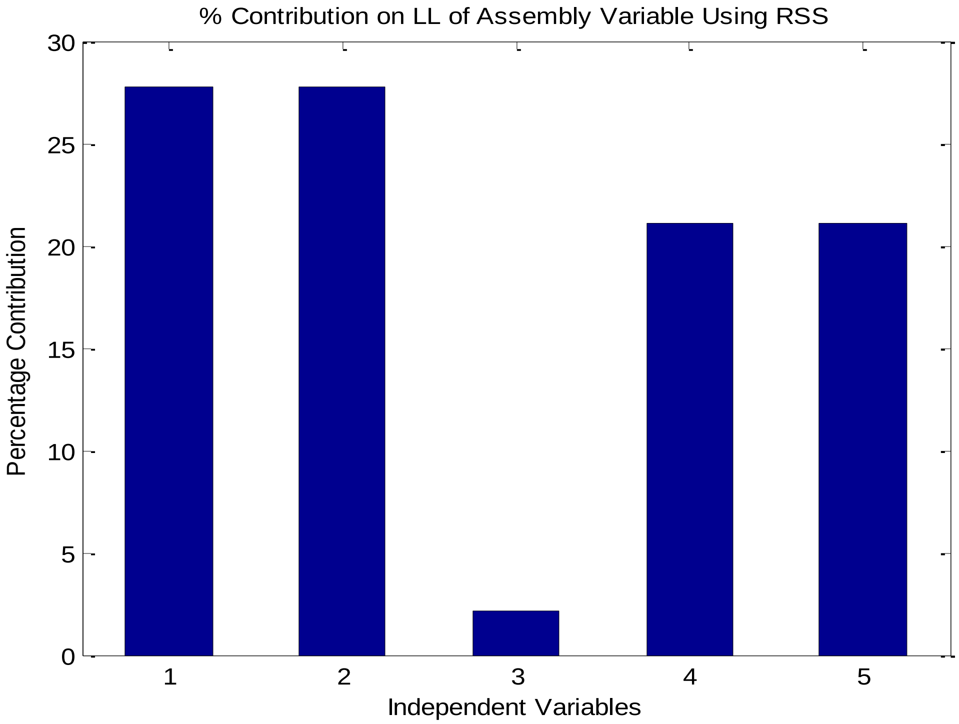

Figure 4.15: Percentage contribution of the individual manufactured variable on the LTL of the gap between the gear-hub and the bushing (X) using modified RSS method

4.4 EXAMPLE 4: ONE-WAY CLUTCH ASSEMBLY

Considering a simple one (single) way clutch assembly model is shown in Figure 4.16 to illustrate the tolerance accumulation for non-linear problems.

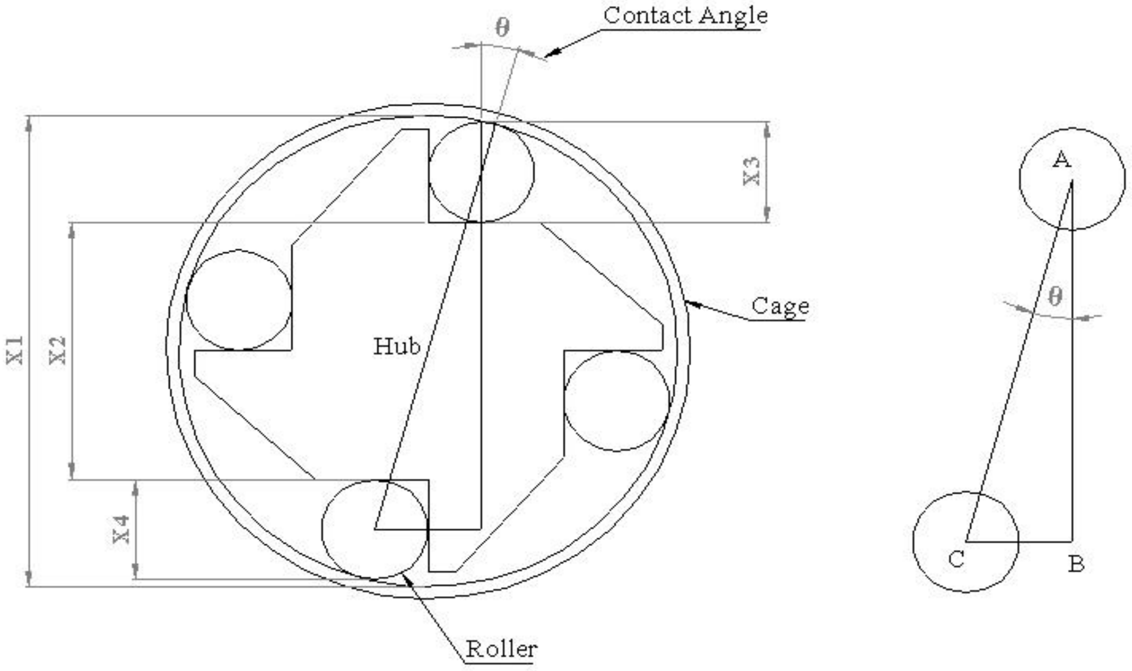

Figure 4.16: One-way clutch assembly

This is basically a 2-D assembly which is made of four parts: rollers, springs, a hub, and a cage. It is called one way assembly because it is designed to rotate only in a single direction. Hub is connected with a shaft which is also connected with some driving system. The rollers slip inner side of the cage when this hub rotates in clockwise direction with respect to the cage. When hub rotates in anticlockwise direction these rollers jam between hub and cage, which causes hub and cage to lock and rotate together. Also, when hub rotates again in clockwise direction, the clutch should be able to free. In this example the design or assembly variable is the contact angle (θ) between the roller and cage which is non-linear. We can define this assembly variable (θ) as non-linear function of the independent variables X_1, X_2, X_3, and X_4 by the trigonometry as follows:

Adjacent AB $= \left(X_2 + \frac{X_3 + X_4}{2}\right)$, and Hypotenuse AC $= \left(X_1 - \frac{X_3 + X_4}{2}\right)$ (4.4)

$$\theta = \mathrm{Cos}^{-1}\left(\frac{X_2+\frac{X_3+X_4}{2}}{X_1-\frac{X_3+X_4}{2}}\right) \tag{4.5}$$

Where X_1 is the diameter of the cage, X_2 is the height of the hub, X_3 is the diameter of the upper roller, and X_4 is the diameter of the lower roller. The nominal values of these variables with their tolerance limits are given in Table 4.7. The dimension of the assembly variable can be estimated from the equation (4.5). The sensitivities for cage, hub, and rollers are calculated automatically by putting the values of the independent variables, by doing partial derivation of the assembly function w.r.t. the each of that variable respectively, as follows:

$$\frac{\partial\theta}{\partial X_1} = 0.1032, \frac{\partial\theta}{\partial X_2} = -0.1039, \frac{\partial\theta}{\partial X_3} = -0.1035, \frac{\partial\theta}{\partial X_4} = -0.1035 \tag{30}$$

Table 4.7: Nominal dimensions, sensitivities, and tolerance limits of independent variables for one-way clutch model

Variables	Values (mm)	Sensitivities	UL (mm)	LL (mm)
X_1	101.60	0.1032	0.155	0.135
X_2	55.29	-0.1039	0.16	0.14
X_3	22.86	-0.1035	0.01	0.015
X_4	22.86	-0.1035	0.01	0.015

Table 4.7 consists of list of independent variables with their nominal dimensions, sensitivities and tolerance limits for one-way clutch assembly. The results obtained for the upper and lower tolerance limits of contact angle (θ), by using modified WC methods in MATLAB are presented in Table 4.8.

Table 4.8: UTL and LTL of the design variable (θ) by using modified WC and RSS methods

Tolerance analysis method	UTL (rad)	LTL (rad)
Modified WC	0.0336	0.0326
Modified RSS	0.0307	0.0307

The percentage contributions of the independent variables on the UTL and the LTL of the tolerance obtained for contact angle by using modified WC method are presented in Figure 4.17 and Figure 4.18. And by using RSS methods the same results are presented in Figure 4.19 and Figure 4.20.

From Figure 4.17 and Figure 4.19 we can say that the variables X_1 and X_2 are the critical dimensions to the contact angle (θ) for calculating the upper tolerance limit of that variable, while X_3 and X_4 have the least contribution in influencing the assembly variable (θ). Or, we can say that X_1 and X_2 are the major contributors while X_3 and X_4 are the lowest contributors in influencing the assembly variable (contact angle). Figure 4.18 and Figure 4.20 are showing the same results for calculating the lower tolerance limit of the contact angle (θ). According to Table 4.7, X_1 andX_2 have the loosest tolerance limits and X_3 and X_4 have the tightest tolerance limits. Consequently, the variables X_1 and X_2 have highest percentage contribution, while the variable X_3 and X_4 have smallest percentage contribution on the contact angle (assembly variable, θ).

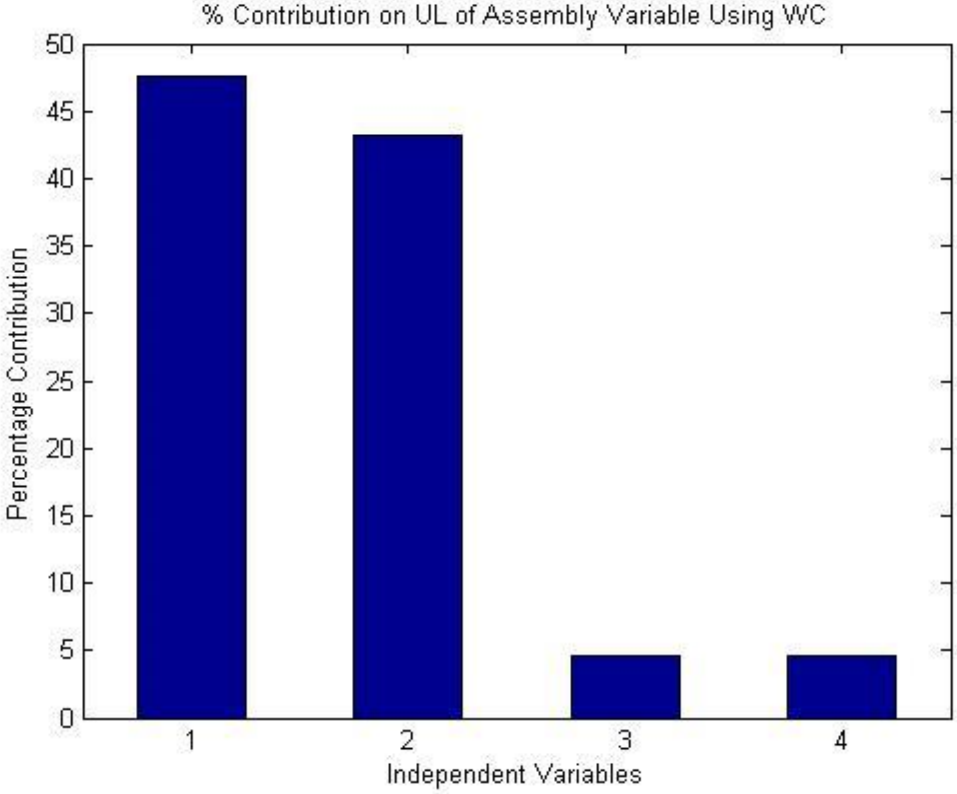

Figure 4.17: Percentage contribution of the individual manufactured variable on the UTL of the contact angle (θ) using modified WC method

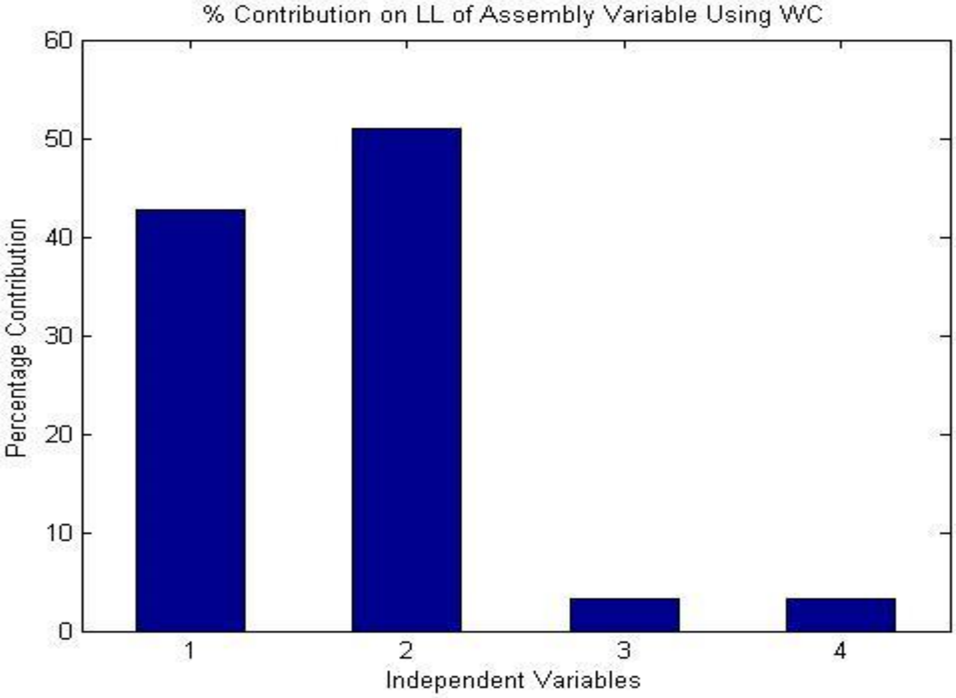

Figure 4.18: Percentage contribution of the individual manufactured variable on the LTL of the contact angle (θ) using modified WC method

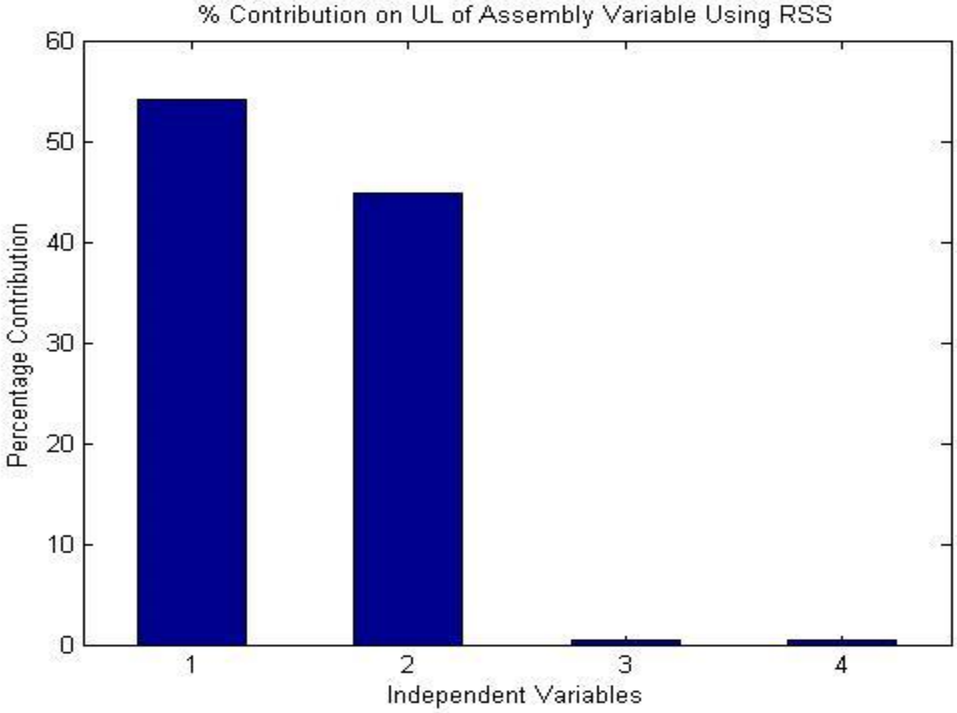

Figure 4.19: Percentage contribution of the individual manufactured variable on the UTL of the contact angle (θ) using modified RSS method

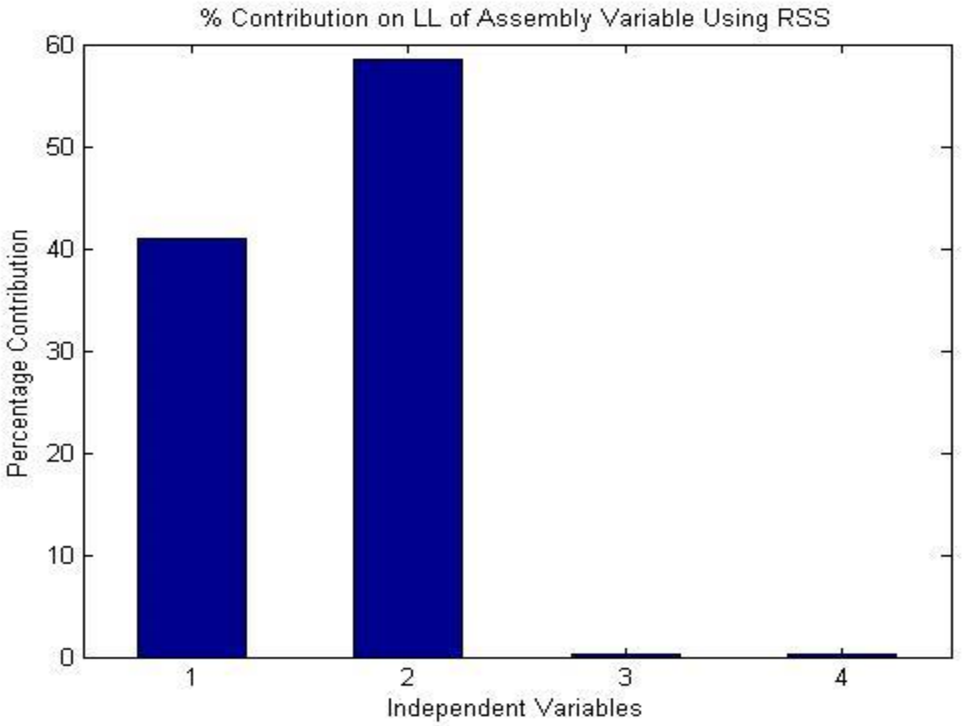

Figure 4.20: Percentage contribution of the individual manufactured variable on the LTL of the contact angle (θ) using modified RSS method

CHAPTER 5: CONCLUSIONS AND FUTURE SCOPE

5.1 CONCLUSIONS

In the presented work, tolerance analysis of the assembly variable has been done in MATLAB by using the worst case and root sum square methods. Firstly, some brief review of the worst case and the statistical analysis has been done on the basis of some literatures. These methods are used when the manufactured variables are given with the bilateral and symmetric tolerances. The sign of sensitivities are not considered in these common methods Therefore, it cannot be determined that how a specific assembly variable changes with changes in the individual dimensions. In this work, modified worst case and root sum square methods are presented for the automatic tolerance analysis for the UTL and the LTL of the assembly variable w.r.t. the manufactured variables with asymmetric tolerances. In these new methods sign of the sensitivities are also considered showing that how sensitive is the critical dimension w.r.t. the variations in the each part dimensions. The percentage contribution of the manufactured dimensions on the critical dimension has been also presented in the form of bar charts for the tolerance limits. These contributions show that how an assembly variable is affected by the each part tolerances.

The proposed approach has been validated with reference Movahhedy and Khodaygan for the combustion chamber assembly which is the case of linear problem. The comparison shows that the results obtained for the upper and lower bounds with asymmetric tolerances of the assembly dimension by the modified relations in MATLAB and the reference results are nearly similar and the percentage of contribution of the individual dimensions on the critical assembly dimension which is the height of the combustion chamber is also presented in the form of bar charts. Another linear problem of thermos flask is also presented in which the critical dimension is the gap between two coaxial flasks. The tolerance limits of the assembly variable and the percentage contribution of each individual dimensions which are given with the linear and asymmetric tolerances, are also presented in this work.

Tolerance analysis for the gap between the gear-hub and the bushing in a gearbox assembly has been done. In this example, tolerance limits of the independent component dimensions are given symmetrical and bilateral. The UTL and LTL obtained by the WC method are symmetrical and bilateral, and by RSS method these are unilateral. This shows that if tolerance limits of independent variables are bilateral may cause of bilateral tolerance limits of the design variable. The percentage contribution of the individual manufactured dimension on the UTL and LTL of assembly variable is shown in the form of bar charts.

Tolerance analysis for a non-linear example of one way clutch model has also been done in this work in which assembly variable is the contact angle between the roller and the cage. Here, the magnitude of the sensitivities of the individual independent variable may differ from 1 because of nonlinearity that affect the assembly variable. It is also shown that how the individual dimension affect the assembly variable by the percentage contribution of those dimensions in the form of bar charts.

Thus, the benefits of the presented approach are that it is easy to automate, and also it can model both the symmetric and the asymmetric tolerance limits.

5.2 FUTURE SCOPE

This work can be further extended in the field of tolerance analysis automatically for the other problems as:

- Automatic tolerance analysis of free-form surfaces.
- Dimensional and Geometric tolerance analysis for the assemblability on the basis of geometric conditions, for design functions and for minimum manufacturing cost using effective tolerance cost model.

REFERENCES

1. Anantha R., Kramer G. and Crawford R. (1996): Assembly Modeling by Geometric Constraint Satisfaction, *Computer Aided Design*, vol. 28, No. 9, pp. 707–722.
2. Chen H., Jin S., Li Z. and Lai X. (2014): A Comprehensive Study of Three Dimensional Tolerance Analysis Methods, *Computer Aided Design*, vol. 53, pp. 1-13.
3. Cheng K.M. and Tsai J.C. (2011): Optimal Statistical Tolerance Allocation of Assemblies for Minimum Manufacturing Cost, *Applied Mechanics and Materials*, vols. 52-54, pp. 1818-1823.
4. Governi L., Furferi R. and Volpe Y. (2012): A Genetic Algorithms Based Procedure for Automatic Tolerance Allocation Integrated in a Commercial Variation Analysis Software, *Journal of Artificial Intelligence*, vol. 5, No. 3, pp. 99-112.
5. Greenwood W.H. and Chase K.W. (1988 and 1990): Worst Case Tolerance Analysis with Nonlinear Problems, *ASME Journal of Engineering for Industry*, vol. 110, pp. 232-235.
6. Haghighi P., Mohan P., Shah J.J. and Davidson J.K. (2014): Automatic Detection and Extraction of Tolerance Stacks in Mechanical Assemblies, *Proceedings of the ASME 2014 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference*, vol. 2, pp. 4143-4152.
7. Haghighi P., Mohan P., Shah J.J. and Davidson J.K. (2015): A Framework for Explicating Formal Geometrical and Dimensional Tolerances Schema from Manufacturing Process Plans for Three Dimensional Conformance Analysis, *Journal of Computing and Information Science in Engineering*, vol. 15, No. 2, pp. 1-13.
8. Khodaygan S. and Movahhedy M.R. (2010): Tolerance Analysis of Assemblies with Asymmetric Tolerances by Unified Uncertainty Accumulation Model Based on Fuzzy Logic, *International Journal of Advanced Manufacturing Technology*, vol. 53, pp. 777–788.
9. Li Z., Zhang Y., Xu L. and Wang J. (2011): A New Method for Automatic Synthesis of Tolerances for Complex Assemblies Based on Polychromatic Sets, *Enterprise Information Systems*, vol. 5, No. 3, pp. 337–358.
10. Litwa F., Gottwald M., Bohn M., Klinger J.F., Walter M., Wartzack S. and Vielhaber M. (2015): Automated Point Based Tolerance Analysis Model Creation for Sheet Metal Parts," *13th CIRP Conference on Computer Aided Tolerancing*, vol. 27, pp. 65-70.

11. Mohan P., Haghighi P., Vemulapalli P., Kalish N., Shah J.J. and Davidson J.K. (2014): Toward Automatic Tolerancing of Mechanical Assemblies: Assembly Analyses, *Journal of Computing and Information Science in Engineering*, vol. 14, No. 4, pp. 1-14.
12. Mohan P., Haghighi P., Shah J.J. and Davidson J.K. (2014): Automatic Detection of Directions of Control in Mechanical Parts," *Proceedings of ASME 2014 International Manufacturing Science and Engineering Conference*, vol. 1, pp. 1-10.
13. Movahhedy M.R. and Khodaygan S. (2007): Tolerance Analysis of Mechanical Assemblies with Asymmetric Tolerances, *SAE Transactions Journal of Materials Manufacturing*, vol. 1, pp. 407-418.
14. Mullins S.H. and Anderson D.C. (1998): Automatic Identification of Geometric Constraints in Mechanical Assemblies, *Computer Aided Design*, vol. 30, No. 9, pp. 715–726.
15. Murshed M., Dixon A. and Shah J. (2009): Neutral Definition and Recognition of Assembly Features for Legacy Systems Reverse Engineering, *ASME, International Design Engineering Technical Conferences & Computers and Information in Engineering Conference*, vol. 5, pp. 615-628.
16. Murshed M., Dixon A. and Shah J. (2010): Assembly Feature Tutor and Recognition Algorithms Based on Mating Face Pairs, *Computer Aided Design and Applications*, vol. 7, No. 3, pp. 319-333.
17. Qin Y., Lu W., Liu X., Huang M., Zhou L. and Jiang X. (2015): Description Logic Based Automatic Generation of Geometric Tolerance Zones, *International Journal of Advanced Manufacturing Technology,* vol. 79, pp. 1221–1237.
18. Rachuri S., Han Y., Foufou S., Feng S.C., Roy U., Wang F., Sriram R.D. and Lyons K. W. (2006): A Model for Capturing Product Assembly Information, *Journal of Computing and Information Science in Engineering*, vol. 6, No. 1, pp. 11–21.
19. Sambhoos K., Koc B. and Nagi R. (2009): Extracting Assembly Mating Graphs for Assembly Variant Design, *ASME Journal of Computing and Information Science in Engineering*, vol. 9, No. 3, pp. 1-9.
20. Shen Z., Ameta G., Shah J.J. and Davidson J.K. (2005): A Comparative Study of Tolerance Analysis Methods, *ASME Journal of Computing and Information Science in Engineering*, vol. 5, No. 3, pp. 247-256.

21. Shen Z., Shah J. and Davidson J. (2008): Automatic Generation of Min/Max Tolerance Charts for Tolerance Analysis from CAD models, *International Journal of Computer Integrated Manufacturing*, vol. 21, No. 8, pp. 869-884.
22. Treacy P., Ochs J.B., Ozsoy T.M. and Wang N. (1991): Automated Tolerance Analysis for Mechanical Assemblies Modeled with Geometric Features and Rational Data Structure," *Computer Aided Design*, vol. 23, No. 6, pp. 444-453.
23. Wang N. and Ozsoy T.M. (1990): Representation of Assemblies for Automatic Tolerance Generation, *Engineering with Computers*, vol. 6, pp. 121-126.
24. Whitney D.E. (2004): Mechanical Assemblies- Their Design, Manufacture, and Role in Product Development, *Oxford Series on Advanced Manufacturing*, vol. 1, pp. 112–140.
25. Zhang H., Zhan G., Li W., Wei Q., Li M. and Tian Y. (2014): The Method of Tolerance Analysis Based on the Monte Carlo, *Advanced Materials Research*, vol. 1039, pp. 140-145.
26. Zhang Y., Li Z. and Zhao L. (2009): Automatic Generation Method of Assembly Tolerance in Large Scale Assembly Design, *Computer Science and Information Engineering*, vol.7, pp. 840-845.
27. Zhong Y., Qin Y., Huang M., Lu W., Gao W. and Du Y. (2013): Automatically Generating Assembly Tolerance Types With an Ontology Based Approach, *Computer Aided Design*, vol. 45, pp. 1253–1275.