Control of Chaos Using Dynamic Sliding Mode Control

A DISSERTATION

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By

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Candidate's Declaration

I hereby declare that this thesis which is being presented as the final evaluation of the dissertation **Control of Chaos Using Dynamic Sliding Mode Control** in partial fulfilment of the requirement of award of Degree of Master of Technology in Electrical Engineering with specialization in System Control, submitted to the Department of Electrical Engineering, Indian Institute of Technology, Roorkee , India is an authentic record of the work carried out during a period from May 2015 to May 2016 under the supervision of Prof. Dr. G. N. Pillai (Department of Electrical Engineering, IIT Roorkee) The matter presented in this report has not been submitted by me for the award of any other degree of this institute or any other institute.

Date:
Place: (Vaibhav Kumar Singh)

Certificate

This is to certify that the above statement made by the candidate is correct to best of my knowledge.

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Vaibhav Kumar Singh

ABSTRACT

This project is focussed on various aspects of sliding mode controller design and suppression of chaos in complex non-linear system using the same. It aims at analysis of methods which can improve the performance of sliding mode controller with respect to various other methods. There are numerous ways which modify sliding mode strategy in different directions to overcome its flaws such as chattering, asymptotic convergence etc. This project keeps its main focus on dynamic sliding mode design which involves addition of an extra dynamics to the system with the primary aim of removing chattering however this addition of dynamics also helps in acquiring desired performance specification from the system such as improvement in transient performance. To eliminate asymptotic convergence of sliding mode , terminal sliding mode has been investigated and modified for better robustness and singularity avoidance. This report presents a comparative analysis of all these upgrades of sliding mode control applied to different chaotic system. Two different control strategies have been developed, firstly for a generalized partially linear system with minimum control input requirement. Secondly, control strategy for any generalized chaotic system using both non-singular terminal as well as dynamic sliding mode concept have been developed and compared with already established results.

Contents

List of Figures

List of Tables

Abbreviations

SMC	**S**liding **M**ode **C**ontrol
DSMC	**D**ynamic **S**liding **M**ode **C**ontrol
TSM	**T**erminal **S**liding **M**ode
FTSM	**F**ast **T**erminal **S**liding **M**ode
NFTSMC	**N**on singular **F**ast **T**erminal **S**liding **M**ode **C**ontrol

Chapter 1

Introduction

1.1 Introduction

Sliding mode control(SMC) strategy is considered as one of the most efficient method for robust controller design specifically for the case of high order non linear systems which are prone to uncertainties in the form of external disturbances as well as parametric uncertainties. The main advantages lies in design simplicity and corresponding good performance produced for a number of different complex systems. It allows low sensitivity to plant parameters variations and disturbances thus removing the need for exact modelling of the plant. Also, the system dynamics is constrained to a lower ordered manifold during sliding hence reducing the complexity level of the problem. This control methodology involves discontinuous control function which are easily implementable using conventional power converters with “on-off” being the only admissible states. Due to above mentioned advantages sliding mode control has been widely researched in past two three decades and is still being applied to a number of problems such as robotics, electric drives, process control, motion control etc.. SMC design can be broken down in three main steps. First is to design a stable sliding manifold, second is to derive a control law which will converge trajectories on to this sliding manifold in a finite time also termed as reaching law and the last step is to design a control action which will keep the system dynamics constrained to this sliding manifold for the time ahead. This phase is termed as sliding phase. The stability of sliding manifold is important as once system enters this manifold its dynamics will be governed by parameters of sliding state which is also the reason of its robustness or invariance toward parametric uncertainties and external disturbances. One can observe that control action so designed is discontinuous as well as it has to involve high frequency switching (theoretically infinite!) which becomes a concern during the practical implementation of controller. The generation of this high frequency control component is also called as chattering effect and can pose serious threat to functioning of actuators. Most of the research in the latter half of past two decades has been on finding out new and efficient ways to reduce or eliminate this chattering effect completely. A numerous methods have been proposed which include boundary layer approach, higher order sliding mode control design, neural network based sliding mode design, dynamic sliding mode control design. As expected every method comes with its own set of pros and cons and choice of any method is completely dependent on what the designer has in his mind.

Dynamic sliding mode control(DSMC) design incorporates selection of a switching surface which is a function of time derivatives of states and controller input signal. It is different than classical SMC as there a conventional switching surface depends on states or error dynamics of the system along with some parameter. It is independent of the input. Further in DSMC, time derivative of actual input signal is used to design the controller dynamics. One can say for purely understanding purposes that actual input signal is passed through an integrator and then applied to the system. The low pass characteristics of the integrator eliminates any kind of high frequency signal and also tends to smoothen out the new input thus preventing a bang- bang type control action. The addition of additional dynamics which can also be termed as a compensator obviously increases the complexity as well as the system no longer slides on manifold with lower order. However rest other advantages of SMC are preserved in this method also in addition with the reduction or removal of chattering effect.

Apart from chattering one other problem faced by both conventional SMC and DSMC is that of asymptotic convergence towards equilibrium point. Both the design strategies tend to have a slower convergence rate when the system states are closer to origin. Thus such type of control is effective only when there are no restriction on the time. In other words, for more strict time or precision limits SMC and DSMC methods are not suited and this particular reason led to the development of specifically designed non-linear switching manifold which guarantees a finite time convergence. Such a surface is known as terminal sliding surface and the control strategy is known as terminal sliding mode control (TSMC). Such control method also yields a better transient performance although there have been many modifications to TSM because of its two main drawbacks. Firstly, TSMC offers improved convergence rate only when the system states are closer to origin. In case when system states are far away from origin the advantage no longer remains. This particular drawback can be removed by using a surface which is combination of both linear and non-linear combination of system states thus providing better convergence irrespective of the position of states. The second main drawback is presence of singularities in the solution. As the non-linear term involves rational power of system states, negative solution creates a problem. Numerous solution have been proposed for this problem including those which focus on avoiding singular points however focus of this project will be on designing such a surface which could eliminate the possibility of existence of singular problems altogether. Such a surface is termed as non-singular terminal sliding mode and has been studied mathematically in chapters ahead.

1.2 Literature Review

To develop understanding of this methodology control of chaos in non-linear system has been taken up as an application. Chaotic system are non-linear system with strong dependence on initial conditions. They are characterized by positive Lyapunov exponent and even a slightest change in initial conditions or parameter values can cause two approximately close trajectories to completely diverge from each other. These complex dynamical systems have been studied a lot with focus on the synchronization of two different chaotic systems[3] [4]. Such synchronization processes find great application in secure communication systems [5]. However in recent times there has been a growing interest and need to control the chaotic phenomenon in such systems. For instance Ott-Gregbogi-Yorke method developed in 1990 developed a closed feedback method to stabilize the system to a periodic orbit or equilibrium point, however the presence of noise might result in divergence from the desired trajectory [6]. In subsequent years many methods have been proposed to control chaos in various systems. Such as self-controlling feedback method were developed in early nineties[7] [8]. However the development of sliding mode control and it's robustness to external disturbance led to wide applicability in a number of control and synchronization problems[9][10][11]. There has been a significant amount of research on sliding mode control and its application to various fields[2][12][13] . The main idea of SMC is to design a discontinuous switching control which brings the trajectory to a sliding surface and keeps it there for time ahead. However, this high frequency switching control may be the cause of many practical problems such as wear and tear of actuators etc. The effect is known as chattering. In the later period many methods have been developed to eliminate this chattering effect for example higher order sliding mode control [14], neural network based sliding mode control [15] adaptive sliding mode control [16] [17] etc. Dynamic sliding mode control was also designed with the motivation of eliminating chattering .Early work can be dated back to 1990 by H. Sira Ramirez[18] which further led to crucial work in further years in this area[19] [20] [21]. DSMC has been also linked to many higher order sliding mode techniques[22][23] Over the years many modification have been reported to SMC theory and have been applied to a lot of fields including chaotic systems[1][24]. For instance, SMC offers asymptotic convergence which has been improved using Terminal Sliding Mode concept where the sliding surface is non-linear combination of system's states[13][25] Consequently, finite time convergence rate is achieved as expected but when compared with its conventional SMC, there is not much gain in convergence performance of TSM if the initial states are far away from the equilibrium point. This problem was subsequently solved using the concept of Fast TSM methods[26][27].However both TSM and FTSM suffered from singularity problem. The singularity problem was overcome by designing a non-singular terminal switching surface[28][29]. In [30], H. Wang has proposed a new TSM surface to avoid the singularity problem and achieve finite time chaos control of multiple input output system. Specifically in [31] study of FTSM methods and their convergence rate has been done and a new non-singular terminal sliding mode(NFTSM) control technique has been proposed. One other less popular modification to SMC was dynamic sliding mode control (DSMC) proposed in 1993 by H. Ramirez [18]. DSMC proposes addition of an extra dynamics to the system, termed as dynamic sliding surface, which not only helps in improving the stability of sliding system by eliminating chattering but also helps in obtaining desired performance from the system[23]. The main characteristic dynamic SMC is that switching surface also depends on the input signal along with the states of the system. The selection of additional dynamics or dynamic sliding surface depends largely on the performance specifications[21][32],.

Chapter 2

Theory And Preliminaries

2.1 Understanding SMC

In this section we will establish the basic notion of sliding mode control technique and analyse various advantages and disadvantages it offers thus leading our discussions towards various ways of improving this method (specifically targeting the problem of chattering reduction). Sliding Mode control theory was motivated from the concept of variable structure system. A dynamical system whose structure varies so as to satisfy or acquire some desired objective. This change in structure is based open an appropriate switching logic. To develop more understanding let us consider a non-linear system in state space form as described in eq. (2.1)

$$\dot{x} = f(x,t) + B(x,t)u(t) + d(x,t) \tag{2.1}$$

Where $x(t) \in \mathbb{R}^{n\times 1}$ represents the state variables of system, $f(x,t) \in \mathbb{R}^{n\times 1}$ is a non-linear vector function, $B \in \mathbb{R}^{n\times m}$ is a full column rank control matrix and $d(x,t) \in \mathbb{R}^{n\times 1}$ represents external disturbances to which the system is exposed.

Assumption 2.1: All parameters and disturbances are constrained with known upper bounds i.e.

$$\|d(x,t)\| \leq D \tag{2.2}$$

Here D represents real positive constant term. In sliding mode control, the main task is to design a suitable control action $u(t)$ such that the system described in equation (2.1) is constrained to slide on a manifold described by

$$s(x,t) = s(\dot{x},t) = 0 \tag{2.3}$$

Where $s^T(x,t) = [s_1, s_2, \ldots, s_m]$ is assumed to be continuous. System described in eq. (2.1) is said to be in sliding mode if manifold (2.2) is reached in a finite time $t = t_0$ and for $t \geq t_0$ system states are constrained to the manifold stated in (2.2).

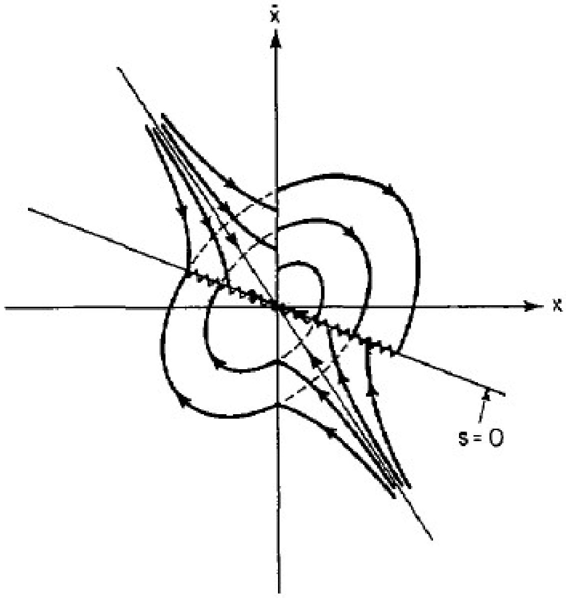

FIGURE 2.1: Phase Trajectories Of Second Order System Using Sliding Mode Control[1]

Definition 2.1: Switching surface $s(x,t) = 0$ is a $(n-m)$ dimensional manifold in R^n determined by $m \times (n-1)$ – dimensional intersection of switching surfaces $s_i(x,t)$. The switching surface is designed such that the system produces desired responses such as stability, tracking etc., when constrained to $s(x,t) = 0$.

Definition 2.2: A sliding mode exists if in the vicinity of switching surface, $s(x,t) = 0$, the tangent or velocity vectors of the state trajectories always point towards the switching surface[2]. An ideal sliding mode exists only when the state trajectory of controlled plant satisfies $s(x,t) = 0$ at every $t \geq t_0$ for some t_0.

The control problem for system in (2.1) is to develop continuous function u_i^+, u_i^- and sliding surface $s(x,t)$ so that the closed loop system exhibits sliding mode on the $(n-m)$ dimensional sliding manifold in presence of the controller

$$u_i(x,t) = \begin{cases} u_i^-(x,t) & s_i(x,t) \leq 0 \\ u_i^+(x,t) & s_i(x,t) \geq 0, i = 1 \dots m \end{cases} \tag{2.4}$$

The design of sliding mode control can be divided in two steps

a. Construction of suitable sliding surface which would yield desired response once the system is constrained to sliding manifold.

b. Design of discontinuous control law which will direct the system trajectories toward sliding surface and eventually will keep it there.

Next step is to guarantee the existence and stability of the sliding surface that is the control must ensure the sliding in finite time and the trajectories must reach the surface at least asymptotically. In general to prove the stability of the method Lyapunov stability criteria is used. A generalized positive definite Lyapunov function is selected whose time derivative must be negative for all instants in the region of attraction. A suitable candidate for Lyapunov function can be

$$V(x,t) = 0.5s^T(t,x)s(t,x) \tag{2.5}$$

Which is clearly a positive definite function. It's time derivative

$$\dot{V}(x,t) = s(t,x)\dot{s}(t,x) < 0 \tag{2.6}$$

In the domain of attraction then all the trajectories will converge to the sliding surface and will stay there for time ahead. The condition described in (2.5) is termed as reachability condition[10]. However a stronger condition for finite time reaching is termed as reachability condition given as equation (2.6) where η is a real positive constant.

$$\dot{V}(x,t) = s(t,x)\dot{s}(t,x) < -\eta \mid s(x,t) \mid \tag{2.7}$$

2.2 Obstacles and Improvements

2.2.1 Order of Sliding Dynamics

Standard sliding mode method can be applied only if the relative degree of the sliding variable is 1 which means that the control action must appear in first total time derivative

$\dot{s}(t,x)$. However many methods such as higher order sliding mode control has been developed and utilized which allow sliding mode even if the relative degree of sliding variable is more than 1.

Definition 2.3: A smooth autonomous SISO system represented by

$$\dot{x}(t) = a(x,t) + b(x,t)u(t) \tag{2.8}$$

With control input as $u(t)$ and output $y(t)$ is said to have relative degree of 1 if the Lie derivative satisfy following condition

$$L_a y = L_a L_b y = \cdots = L_a^{(r-2)} L_b y = 0, L_a^{(r-1)}, L_b y \neq 0, \tag{2.9}$$

2.2.2 Chattering

Theoretically implementation of sliding mode control requires a very high frequency switching control action (almost infinite) to keep the plant dynamics on the sliding surface. Practically implementing such a controller is not feasible due to reasons such as wear and tear of actuators and physical limitation of actuators itself to work on high frequency, excitation of unmodeled dynamics of the system, unavoidable time delays in the system etc. which forces the switching to be finite but of significantly higher frequency value. Due to this finite but high frequency discontinuous control system's trajectories no longer simply slides on the switching surface but instead oscillate in the neighbourhood of the surface. This oscillation is termed as chattering effect. Thus researchers have been continuously working in finding different ways to eliminate or reduce this chattering effect. We will discuss a few in brief

2.2.2.1 Boundary Layer Method

This method utilizes a boundary layer around the switching surface and control action is designed with the aim of smoothing the discontinuity in this layer. One of the way to do so is to select the control action as

$$u = -U \frac{s}{|s| + \varepsilon}, \varepsilon > 0 \tag{2.10}$$

which can also be written as

$$u = -Utanh(s(x,t)) \tag{2.11}$$

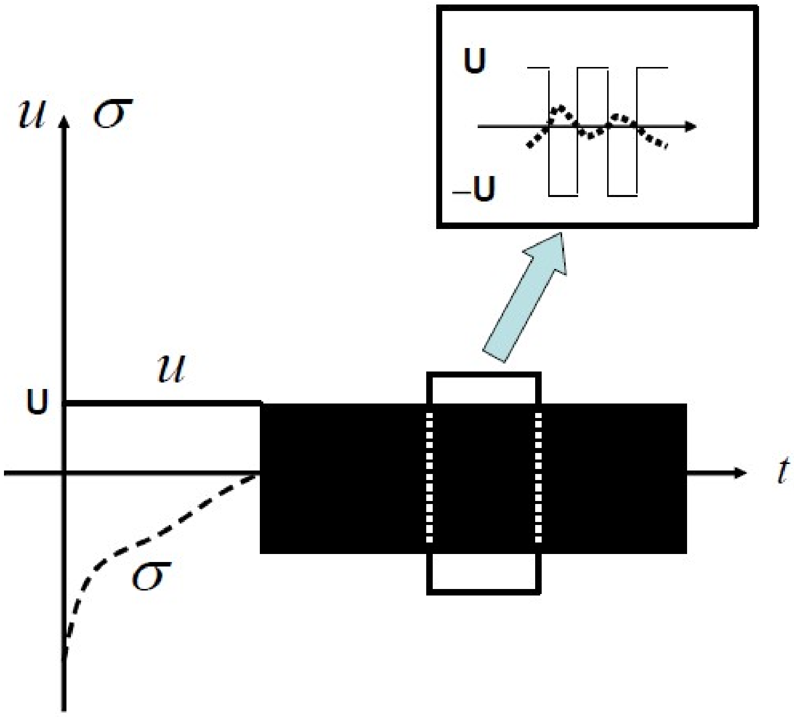

FIGURE 2.2: Chattering in Sliding Dynamics And Controller Plot[2]

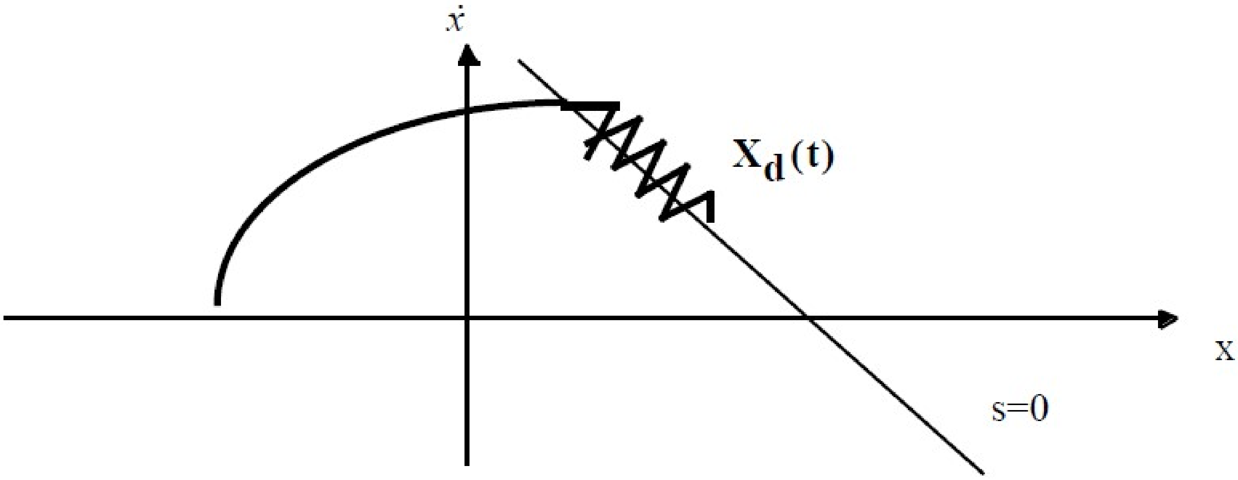

FIGURE 2.3: Sliding Dynamics of A System Showing Chattering

The thickness of boundary layer required to remove or significantly reduce chattering depends on the switching gain which in turn is dependent on the bounds of uncertainties. In simple words, system having large uncertainties will be in need of larger boundary

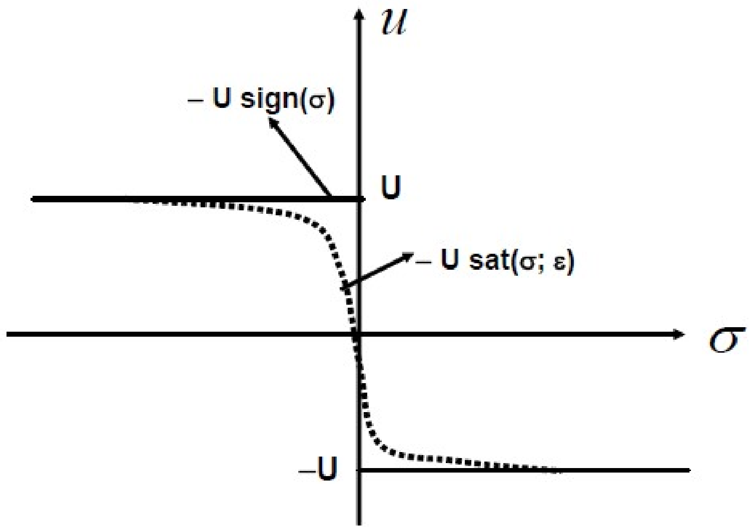

Figure 2.4: Controller Dynamics for Reducing Chattering[2]

layer for removal of chattering. Increasing boundary layer will have two effects. Firstly it will reduce the system accuracy and secondly as we go on increasing width of boundary layer system no longer remains in sliding mode which was the basic intention of control design.

2.2.2.2 Adaptive Sliding Mode Control

The basic idea in adaptive control is to estimate the uncertain plant parameters (or, equivalently, the corresponding controller parameters) on-line based on the measured system signals, and use the estimated parameters in the control input computation. An adaptive controller is a controller that can modify its behaviour in response to changes in the dynamics of the process and the disturbances. Adaptive control can be considered as a special type of nonlinear feedback control in which the stages of the process can be separated into two categories, which can change at different rates. The slowly changing states are viewed as parameters with a fast time scale for the ordinary feedback and a slower one for updating regulator parameters. One of the goals of adaptive control is to compensate for parameter variations, which may occur due to nonlinear actuators, changes in the operating conditions of the process, and non-stationary disturbances acting on the process. As we have discussed in previous section the magnitude of switching gain depends upon bounds of the uncertainty. However this uncertain aspect of the system dynamics will not always remain at its maximum permissible limit. Keeping that fact in mind we can design a controller in two ways. First, we can keep the switching gain at a fixed value such that the sliding surface is stable in terms of Lyapunov stability criteria or secondly we can let the switching gain adapt itself with respect to changing disturbance. In second case the gain doesn't have to remain at large fixed value.

2.2.2.3 Dynamic Sliding Mode Control

A design method proposed earlier to eliminate chattering and to excite minimally the unmodeled dynamics is to introduce compensator dynamics in sliding mode through a new class of switching which has the interpretation of linear operators. In this methodology we consider the switching surface as not merely a hyper surface in the original state space of the plant but a linear operator representable as a linear time invariant dynamic system itself, acting on the states. The sliding system with and added compensator (extra dynamics) is an augmented system which is of higher order than the original system. The designed compensator may not only improve the stability of the sliding system but also yield desired performance and characteristics.

Let us consider the same system as described in [19]

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u \tag{2.12}$$

Where $x_1 \in \mathbb{R}^n$,$x_2 \in \mathbb{R}^m$ and $u \in \mathbb{R}^m$. The matrices are real, compatible and B_2 is a full rank matrix. The switching surface is

$$\zeta = C(x_1) + x_2 \tag{2.13}$$

Here ζ denotes switching surfaces that are linear operators and s denotes the conventional static switching surface and $C(.)$ is a linear operator which has a realization as a dynamic system given by

$$\begin{cases} \dot{z} = Fz + Gx_1 \\ y = Hz + Lx_1 \end{cases} \tag{2.14}$$

Assumption- $C(.)$ has an equal number of poles and zeros (or less zero). Clearly this allows the physical realization of system in (2.13). Thus new switching surface can be written as

$$\zeta = Hz + Lx_1 + x_2 \tag{2.15}$$

And corresponding equivalent control can be calculated using

$$\zeta = Hz + Lx_1 + x_2 = 0 \tag{2.16}$$

$$\dot{\zeta} = HFz + HGx_1 + LA_{11}x_1 + LA_{12}x_2 + A_{21}x_1 + A_{22}x_2 + B_2u_{eq} = 0 \tag{2.17}$$

$$\Rightarrow u_{eq} = -B_2^{-}1[HFz + HGx_1 + LA_{11}x_1 + LA_{12}x_2 + A_{21}x_1 + A_{22}x_2] \tag{2.18}$$

And

$$x_2 = -Hz - Lx_1 \tag{2.19}$$

Thus we can also express eq. (2.18) as

$$u_{eq} = -B_2^{-1}[(HF - LA_{12}H - A_{22}H)z + (HG + LA_{11} - LA_{12}L + A_{21} - A_{22}L)x_1] \tag{2.20}$$

When the system is in sliding mode its dynamics can now be represented by

$$\begin{bmatrix} \dot{z} \\ \dot{x_1} \end{bmatrix} = \begin{bmatrix} (F & G \\ A_{21}H & A_{21} - A_{12}L) \end{bmatrix} \begin{bmatrix} (z \\ x_1) \end{bmatrix} \tag{2.21}$$

And all its pole can be placed by selecting proper value of (F, G, H, L) if the pair (A_{11}, A_{12}) is controllable. In contrast to this if we select a static switching surface as $s = Lx_1 + x_2$, the corresponding dynamics of the same system as given in (2.11) can be represented as

$$\dot{x_1} = (A_{11} - A_{12}L)x_1 \tag{2.22}$$

And all its poles can be placed by selection of proper value of L if the pair (A_{11}, A_{12}) is controllable Another way of utilizing dynamic sliding technique is to use a sliding surface which is not only dependent on the states of the system but also on the input signal to the plant. This new sliding surface can be expressed as

$$\sigma = \dot{s}(x,t) + s(x,t) \tag{2.23}$$

Where $s(t,x)$ represents the static switching surface. A suitable candidate for static switching surface for nth order system represented in controllable canonical form

$$\begin{cases} \dot{x_1} = x_2 \\ \dot{x_2} = x_3 \\ . \\ . \\ . \\ \dot{x_n} = a(x,t) + b(x,t)u \end{cases} \tag{2.24}$$

can be

$$s(x,t) = \sum_{i=1}^{n} C_i x_i \tag{2.25}$$

Where $C_n = 1$ and the coefficient $C_i's$ are selected in such a way that they satisfy a Hurwitz polynomial. For understanding purposes, let's restrict our study to a second order system in which case the static and dynamic sliding surfaces will be given as

$$\begin{cases} s(x,t) = x_2 + Cx_1 \\ \sigma = \dot{s}(t,x) + s(t,x) = \dot{x_2} + (C+\lambda)x_2 + \lambda C x_1 \end{cases} \tag{2.26}$$

It is clear from eq. (2.26) that sliding surface σ will involve control action u and thus in order satisfy the reaching condition with respect to this new switching surface we will encounter the time derivative of control input. One can understand that in dynamic sliding mode control methodology instead of applying a control input directly to the plant we rather use the time derivative of input signal and pass it through an integrator. Since integrators are low pass filters the output so received is free from high frequency components and hence when applied to the plant the overall dynamics in sliding mode is free from chattering.

Chapter 3

Control of Partially Linear Chaotic System

Control of chaos can be broadly classified in two groups

a. Methods in which the trajectories in phase plane are continuously monitored and a feedback process is applied to force them to move in some desired trajectory. Feedback methods do not change the controlled system and stabilize unstable periodic orbits to strange chaotic orbits.

b. Non-feedback methods, in which some other parameters or information about the system is explored and utilized. These methods do change the system to some extent.

The non-feedback methods are less flexible and require more prior knowledge of equation of motion but the need to monitor phase trajectories is eliminated. The control signal can be applied at any time and we can switch from one periodic orbit to another without encountering chaotic behaviour. However transient chaos may be observed while switching. Further we do not need to wait for the system trajectories to close in on a suitable unstable periodic orbit. In this section we will focus on utilizing a single input controller to control a chaotic Lorenz system to one of the unstable equilibrium points

3.1 System Description

Consider a class of partially linear chaotic system as follows

$$\begin{cases} \dot{x_1} = a_1x_1 + b_1x_2 \\ \dot{x_2} = a_2x_1 + b_2x_2 + f(x_1, x_2, x_3) \\ \dot{x_3} = -x_3 + h(x_1, x_2, x_3) \end{cases} \tag{3.1}$$

where x_1, x_2, x_3 represent states of the system while a_1, a_2, b_1, b_2 are real constants and $f(x_1, x_2, x_3), h(x_1, x_2, x_3)$ are nonlinear smooth functions defined on $\mathbb{R}_3 \to \mathbb{R}$. Some of the models which fit in this description are Lorenz system, Lu system, Rucklidge attractor, Lu-Chen system etc. The problem at hand requires a suitable controller design so that the states of the corresponding chaotic system can converge to a desired equilibrium point. Consider the system to be represented by following set of equations

$$\begin{cases} \dot{x_1} = a_1x_1 + b_1x_2 \\ \dot{x_2} = a_2x_1 + b_2x_2 + f(x_1, x_2, x_3) + d(t) + u(t) \\ \dot{x_3} = -x_3 + h(x_1, x_2, x_3) \end{cases} \tag{3.2}$$

$u(t) \in \mathbb{R}$ Represent the controller dynamics and $d(t)$ describes the external disturbances affecting system's behaviour. Let D_0 be a positive constant such that the uncertain aspect of the system is bounded as $\|d(t)\| \leq D_0$ and $\|\dot{d}(t))\| \leq D_1$. For ease of designing and analysis of controller dynamics, let's transform the system into controllable canonical form [15]. Let the control input be composed as follows

$$u(t) = u_1(t) + u_2(t) \tag{3.3}$$

where $u_1(t) = -f(x_1, x_2, x_3)$ which acts as feedback linearisation term

$$\begin{cases} \dot{x_1} = a_1x_1 + b_1x_2 \\ \dot{x_2} = a_2x_1 + b_2x_2 + d(t) + u(t) \\ \dot{x_3} = -x_3 + h(x_1, x_2, x_3) \end{cases} \tag{3.4}$$

Eq. (3.4) can be written as

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \tag{3.5}$$

Using a suitable transformation matrix P we can transform the above set of equation into controllable form

$$\begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} = P^{-1} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{3.6}$$

Then equation (3.6) can be transformed into

$$\begin{bmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \alpha & \beta \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{3.7}$$

Here α, β represent the new real constants after transformation process. The above transformation shifts whole problem to the selection of a proper controller $u_2(t)$ such that the system dynamics described by (3.7) converge to a specified equilibrium point. It will be shown the convergence of the two transformed states in (3.7) will eventually force the convergence of the third state depicted in eq. (3.4).

3.2 Dynamic Sliding Mode Controller Design

Let us define error variables as

$$\begin{cases} e_1 = \bar{x}_1 - \bar{x}_{1d} \\ \dot{e}_1 = \dot{\bar{x}}_1 - \dot{\bar{x}}_{1d} = \bar{x}_2 = e_2 \\ \dot{e_2} = \dot{\bar{x}}_2 = \alpha e_1 + \beta e_2 + d(t) + u_2(t) + \alpha \bar{x}_{1d} \end{cases} \tag{3.8}$$

Here $\bar{x}_{1d}$ denotes the transformed value which can be a static equilibrium point or a periodic cycle or any arbitrary trajectory. The error dynamics for state x_3 can be directly written as

$$\dot{e_3} = h(e_1, e_2, e_3) - \gamma(e_3 + x_{3d}) \tag{3.9}$$

Construct the switching function Such that C must be Hurwitz

$$s(t) = e_2 + Ce_1 \tag{3.10}$$

Then

$$\dot{s} = \alpha e_1 + \beta e_2 + d(t) + u_2(t) + \alpha \bar{x}_{1d} + Ce_2 \tag{3.11}$$

The main aspect of dynamic sliding mode is that the sliding surface is a function of differential of the states. Define a new switching dynamics such that $\sigma = \dot{s} + \lambda s$, where λ must be Hurwitz [1]. Clearly when $\sigma = 0$ then eventually $e_1 \to 0$ asymptotically and hence the system will be stabilized at the desired point in space

$$\sigma = \dot{s} + \lambda s \tag{3.12}$$

$$\dot{\sigma} = (C+\beta+\lambda)(\alpha(e_1+\bar{x}_{1d})+d(t)+u_2(t))+((C+\beta)(\beta+\lambda)+\alpha)e_2+\dot{d}(t)+\dot{u}_2(t) \tag{3.13}$$

By selecting a proper value of dynamic controller can be forced to converge at zero.

$$\dot{u}_2(t) = -(C+\beta+\lambda)(\alpha(e_1+\bar{x}_{1d})+d(t)+u_2(t))-((C+\beta)(\beta+\lambda)+\alpha)e_2-k_1 sgn(\sigma)-k_2\sigma \tag{3.14}$$

And adaptive update law as

$$\dot{k}_1 = -\eta|\sigma| \tag{3.15}$$

Proof: Select the Lyapunov function as

$$V = \frac{1}{2}\sigma^2 + \frac{1}{2\eta}(\grave{k}_1 - k_1)^2 \tag{3.16}$$

Differentiating Lyapunov function with respect to time, we get

$$\dot{V} = \sigma\dot{\sigma} + (\grave{k}_1 - k_1)(-\dot{k}_1) \tag{3.17}$$

Using eq. (3.13)

$$\begin{aligned}\dot{V} = \sigma((C+\beta+\lambda)(\alpha(e_1+\bar{x}_{1d})+d(t)+u_2(t))+((C+\beta)(\beta+\lambda)+\alpha)e_2+\dot{d}(t)+\\ +\dot{u}_2(t))+\frac{1}{\eta}(\grave{k}_1 - k_1)(-\dot{k}_1)\end{aligned} \tag{3.18}$$

From eq. (3.14), (3.15) we can modify above equation as

$$\begin{aligned}\dot{V} &= \sigma((C+\beta+\lambda)d(t)+\dot{d}(t)-k_1 sgn(\sigma)-k_2\sigma)-\frac{1}{\eta}(\grave{k}_1 - k_1)(\dot{k}_1)\\ &\leq (C+\beta+\lambda)D_0|\sigma| + D_1|\sigma| - k_2\sigma^2 + \grave{k}_1|\sigma|\end{aligned} \tag{3.19}$$

Thus if $\grave{k}_1 > (C+\beta+\lambda)D_0 + D_1$ then the reaching condition is satisfied and hence using the controller derived as in eq. (3.14) and adaptive law in eq. (3.15) the trajectories will converge to $\sigma = 0$ and eventually will stay there which will make the error to asymptotically decay to zero.

3.2.1 DSMC of Lorenz System

To prove the feasibility of method described above let us take a chaotic Lorenz system along with the presence of uncertainty in one of the states

$$\begin{cases} \dot{x}_1 = -ax_1 + ax_2 \\ \dot{x}_2 = rx_1 - x_2 - x_1x_3 + d(t) + u(t) \\ \dot{x}_3 = -bx_3 + x_1x_2 \end{cases} \tag{3.20}$$

Where x_1, x_2, x_3 represent the state variables of the system, $d(t)$ denotes the uncertain aspect and u(t) is the dynamic controller which we need to design. a, r and b are real constant parameters. The above system shows chaotic behaviour for the parametric value of $a = 10, r = 28$ and $b = \frac{8}{3}$.As described in section 3.2, eq. (3.3)

$$u_1(t) = -x_1x_3 \tag{3.21}$$

Utilizing (3.21) in (3.20), system dynamics changes into

$$\begin{cases} \dot{x}_1 = -ax_1 + ax_2 \\ \dot{x}_2 = rx_1 - x_2 + d(t) + u(t) \\ \dot{x}_3 = -bx_3 + x_1x_2 \end{cases} \tag{3.22}$$

Selecting the transformation as described in eq. (3.6) where transformation matrix P is

$$P = \begin{bmatrix} a & 0 \\ a & 1 \end{bmatrix} \tag{3.23}$$

We can rewrite eq. (3.22) as

$$\begin{bmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ a(r-1) & a+1 \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{3.24}$$

Describing the error dynamics from equations

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = a(r-1)e_1 + (a+1)e_2 + a(r-1)\dot{x}_{1d} + d(t) + u_2(t) \end{cases} \tag{3.25}$$

Based on the controller derived in section 3.2

$$\begin{aligned} \dot{u}_2(t) = -(C+a+1+\lambda)(a(r-1)(e_1+\dot{x}_{1d}) + u_2(t)) \\ -((C+a+1)(a+1+\lambda) + a(r-1))e_2 - k_1 sgn(\sigma) - k_2\sigma \end{aligned} \tag{3.26}$$

With the selection of this controller and a proper adaptive gain update law it can be show that the error dynamics described in equation (3.25) asymptotically converge to zero. From the results described in next section it can also be verified that the internal error dynamics of the system also converges to zero.

3.2.2 Simulation

For the verification of proposed method, let's take the initial condition as $(x_1(0), x_2(0), x_3(0)) = (3, -4, 2)$ and the desired equilibrium point to be$(x_{1d}, x_{2d}, x_{3d}) = (6\sqrt{2}, 6\sqrt{2}, 27)$ which is unstable .The value of parameters of the switching surface are selected as $C = 15and\lambda = 8$. Assume the disturbance function to be

$$d(t) = 0.5 - sin(\pi x_1)sin(2\pi x_2)sin(3\pi x_3)$$

as in [11].Using the controller given in (3.26) and control of chaotic Lorenz system is achieved. Fig 3.1 shows phase portrait of chaotic Lorenz system given in (3.20). Fig 3.2 shows the errors dynamics after utilization of controller as described in (3.23) .Fig 3.3 and Fig 3.4 shows the states of converging to their corresponding values and the switching surface respectively. Fig. 3.5 depicts the final controller derived.

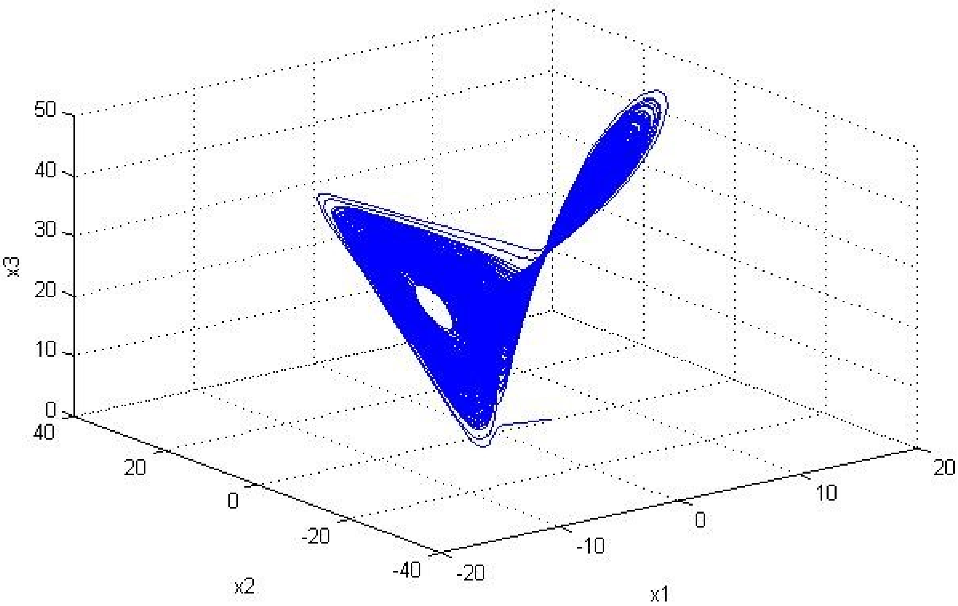

FIGURE 3.1: Phase Trajectory of Chaotic Lorenz System

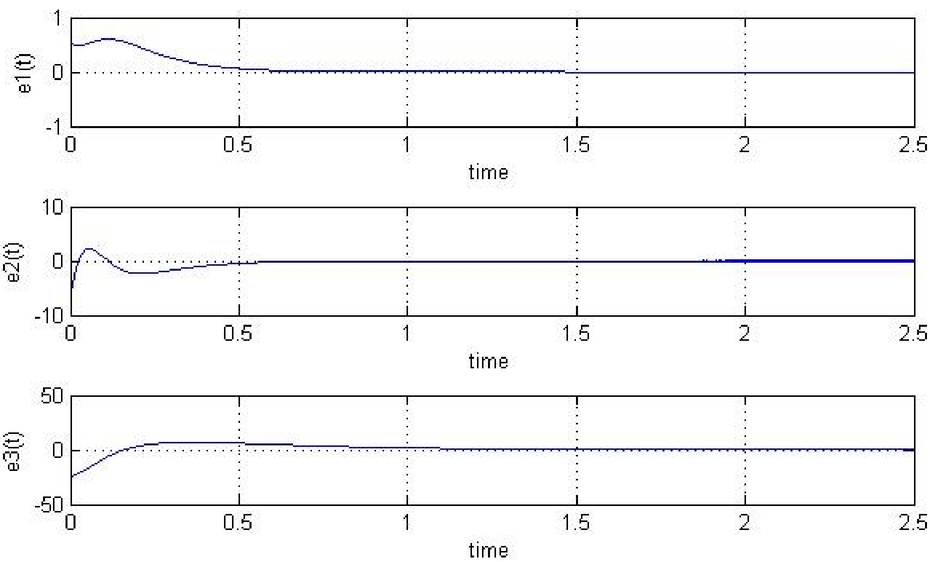

FIGURE 3.2: Error Dynamics For Lorenz System

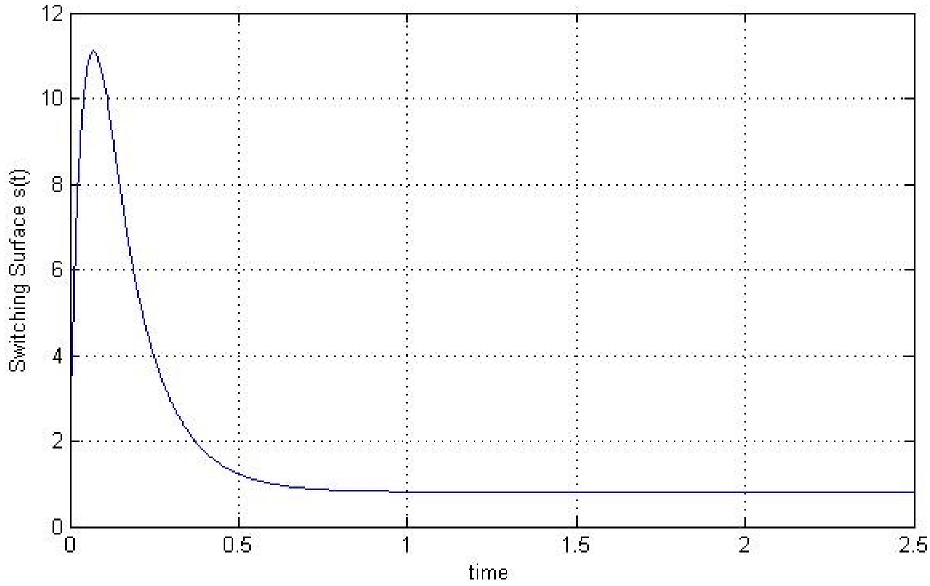

Figure 3.3: Switching Surface $s(t, x)$

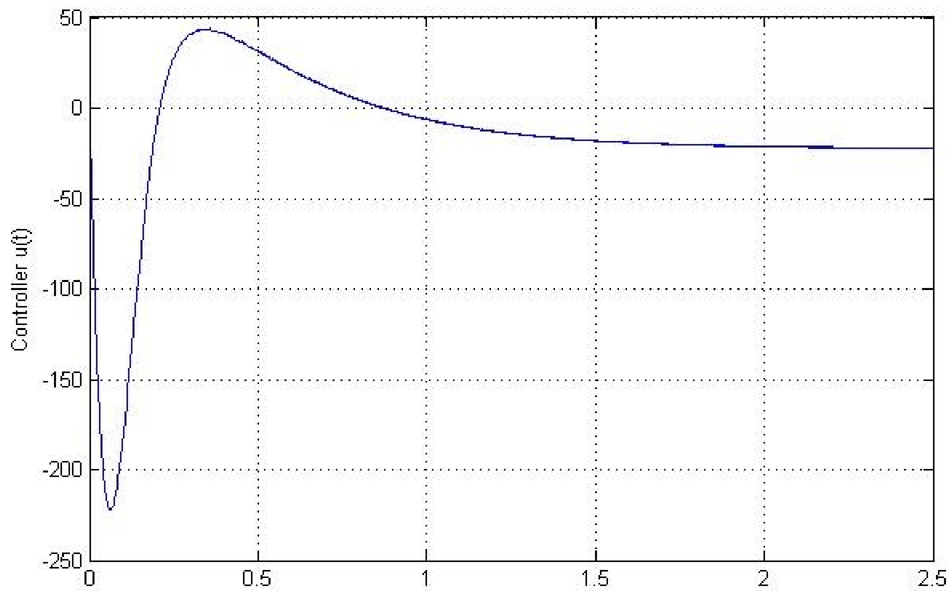

Figure 3.4: Controller Dynamics $u(t)$

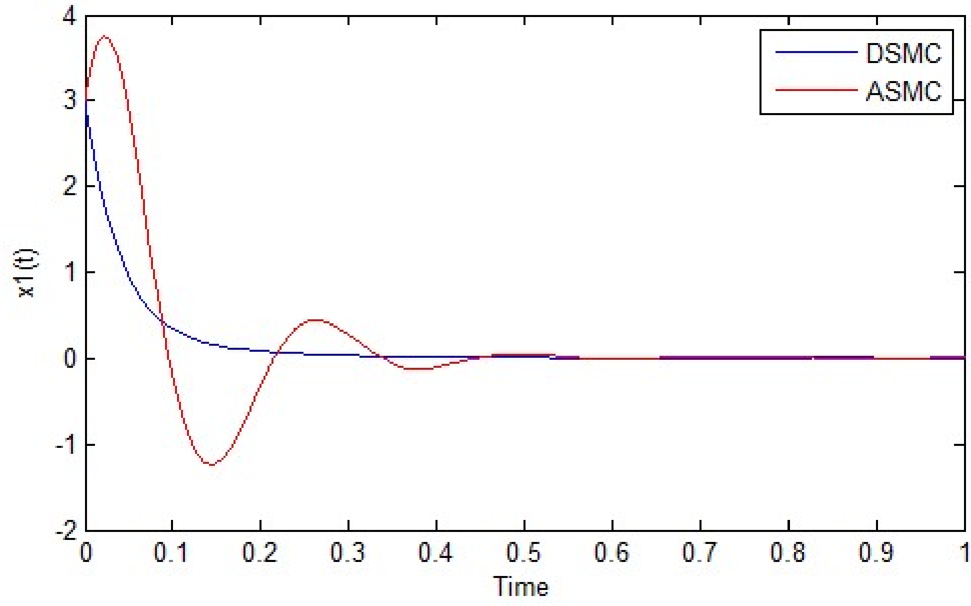

FIGURE 3.5: Comparison between ASMC and DSMC Method- State $x_1(t)$

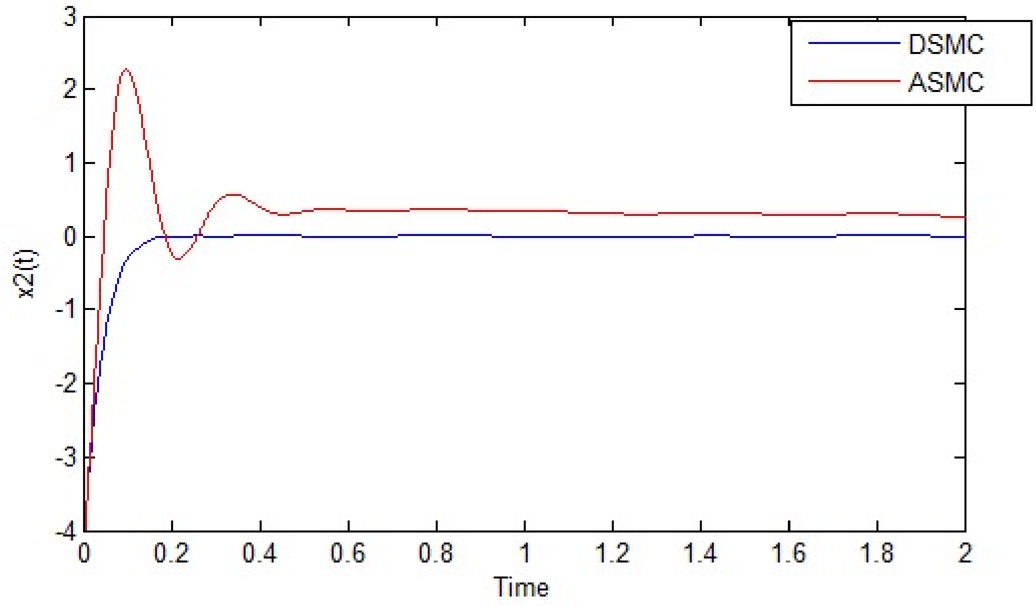

FIGURE 3.6: Comparison between ASMC and DSMC Method- State $x_2(t)$

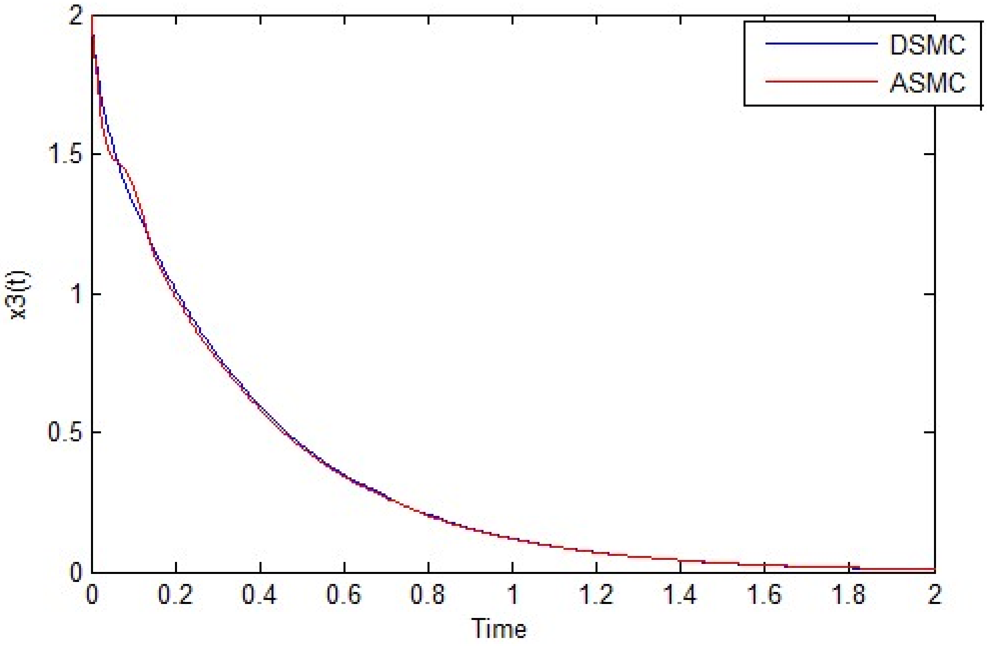

FIGURE 3.7: Comparison between ASMC and DSMC Method- State $x_3(t)$

3.2.3 Analysing Results

Dynamic sliding mode controller design for the control of partially linear chaotic system have been investigated. It can be observed that with the use of a dynamic switching surface instead of conventional static one eliminates chattering completely. A suitable dynamic sliding controller has been designed for chaotic Lorenz system which is stabilizing the system to origin and thus helping in chaos suppression. Further the results have been compared with the adaptive sliding mode control method developed in [17] and significant improvement in transient behaviour and convergence rate of the controlled system can be clearly noticed. However,appropriate transformation of some states of chaotic Lorenz system was needed and hence the controller design process involved two stages. First stage helped in converting the system into partially linear form while the second part of the controller helped in stabilizing the system. Also all the states have been stabilized to an unstable equilibrium point of the chaotic Lorenz system and to the origin.

One can clearly argue about the generality of this algorithm as for transformation process requires feedback linearisation of states and that may be a cause of concerns in some systems but this algorithm is able to produce effective control with minimum input signals and can be applied on higher ordered system as well. It is also to be noted that adaptive gain has been utilized for control signal which not only helps in better performance under noisy environment but also it eliminates the need for any information related to uncertainty bounds and hence improves system's robustness towards unknown noise signals.

Chapter 4

Dynamic Switching Surfaces

One of the crucial aspect in sliding mode control of any system is selection of a proper switching surfaces. Selection of proper surface is important for convergence of the system's states to their respective equilibrium points or trajectories. There are not too many arguments which helps in deciding the parameters of switching surface. However when we go one step ahead and try to use a compensator (or dynamic sliding surface) things change a bit. We can now use this additional dynamics to adjust different performance characteristics to our desires specifications. In this section we will try to analyse and compare the effect of different switching surfaces and corresponding parameters on the control performance of chaotic system.

4.1 System Descriptions and Controller Formulation

4.1.1 Duffing Holmes System

Let us take a system as shown is figure 4.1 which is a simple spring mass system attached to a wall by a spring with stiffness k, frictional resistance r acting between the block and surface and a force F. We have analysed this system many number of times and thus it is well known that depending on the value of friction coefficient r, system will show different behaviour. For example, in absence of friction if a force F is applied and removed immediately the system will keep on oscillating (assuming absence of any other dissipative forces too). However if any kind of dissipative force is present in the system oscillatory behaviour will decay gradually depending upon the overall time constant of the system. However if instead of applying an impulsive force we expose this system to a time varying force and add a non-linearity to system we can generate both periodic and chaotic post transient solutions. One of the ways to add non-linearity is to select a non-linear spring whose restoring force is proportional to the cube of displacement of the block from its equilibrium point i.e. $k = -(ax + cx^3)$ and the time varying forcing function can be chosen as a simple sinusoidal function.

The overall mathematical model of such a system can be derived as equation (4.1).

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = ax_1 + bx_2 + cx_1{}^3 + dcos(\omega t) \end{cases} \tag{4.1}$$

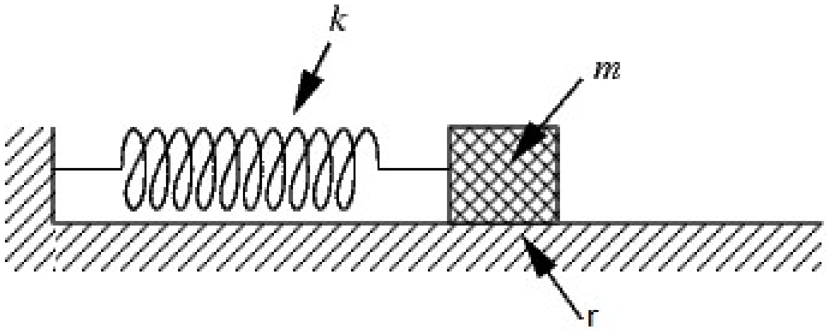

Figure 4.1: Mass Damper System

As stated earlier and further shown in figure 4.2, 4.3 system described above can show both periodic and chaotic behaviour for different values of parametric values.

The two figures show the phase portrait of the system described in equation (4.1) and it can be clearly seen that for a certain set of parameters system shows periodic post transient behaviour as shown in above figures while for slight changes in parameters its behaviour settles to a chaotic attractor also known as strange attractor. The same thing can be verified by the time series graph of the system for the different parametric values. It can be clearly seen that Duffing Holmes system shows one of the most striking characteristic of chaotic system and that is sensitive dependence on initial conditions and parametric values.

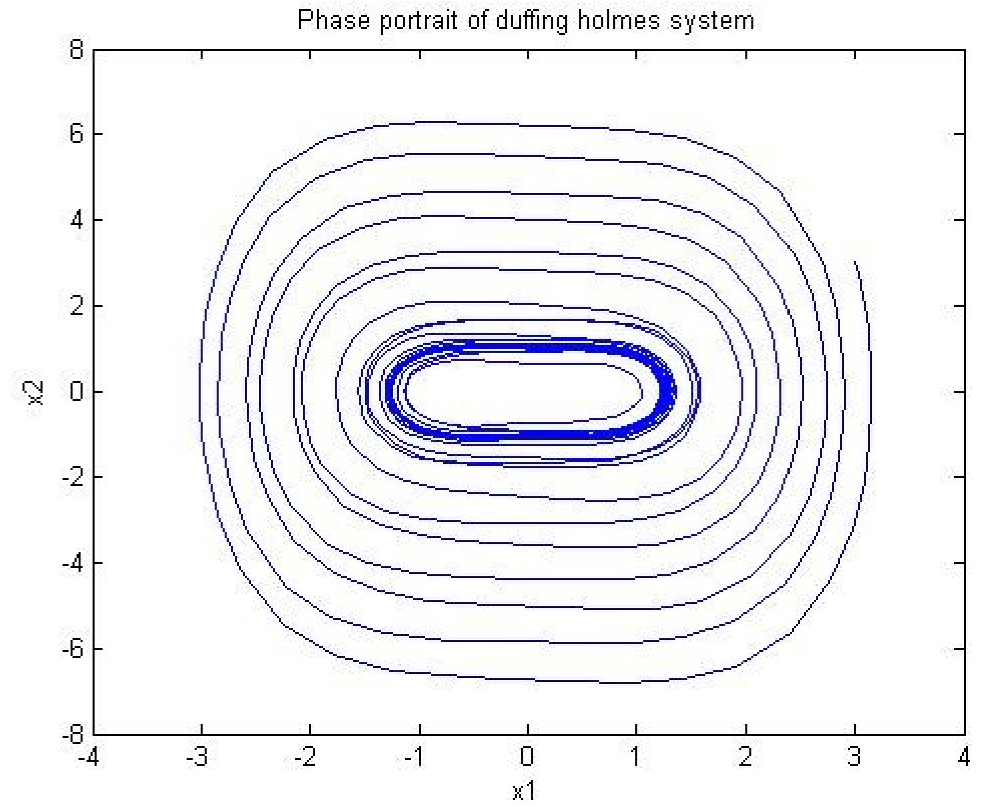

FIGURE 4.2: Periodic Post Transient Behaviour of Holmes System

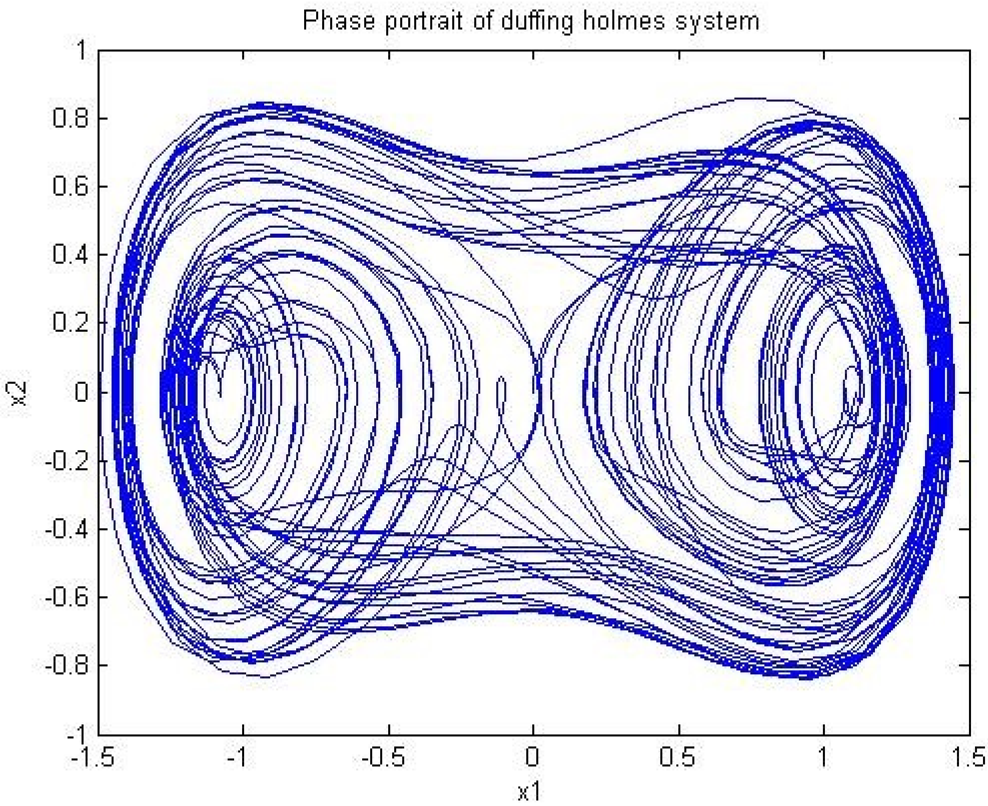

FIGURE 4.3: Chaotic Post Transient Behaviour of Holmes System

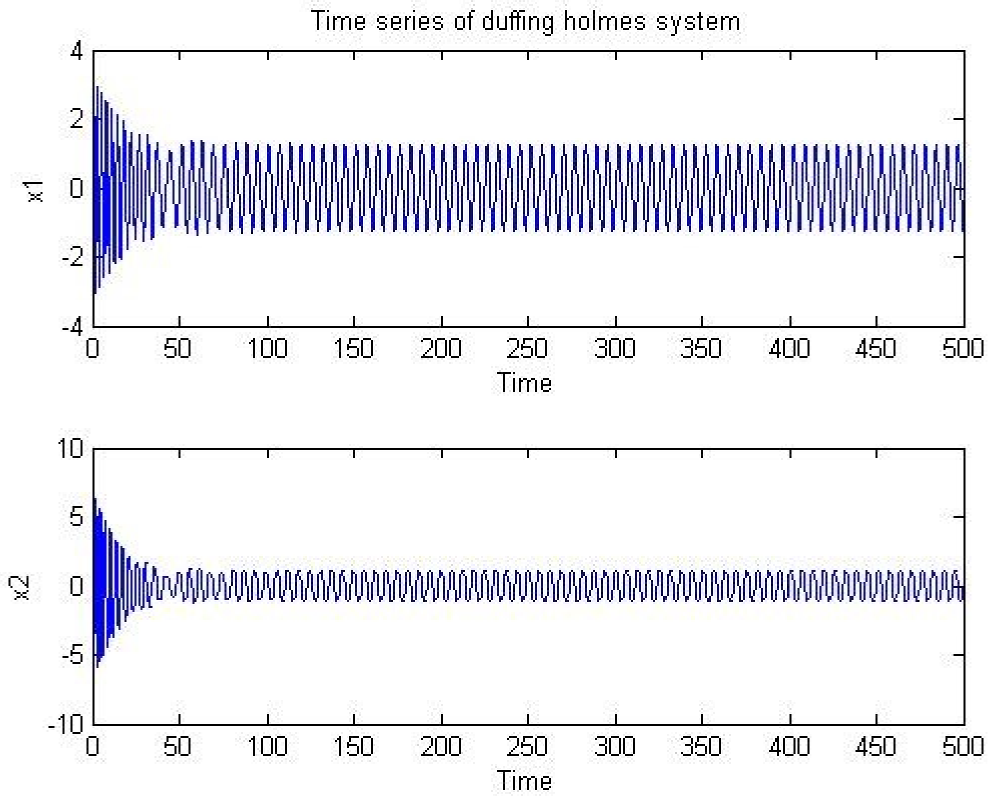

Figure 4.4: Periodic Time Series of Holmes System

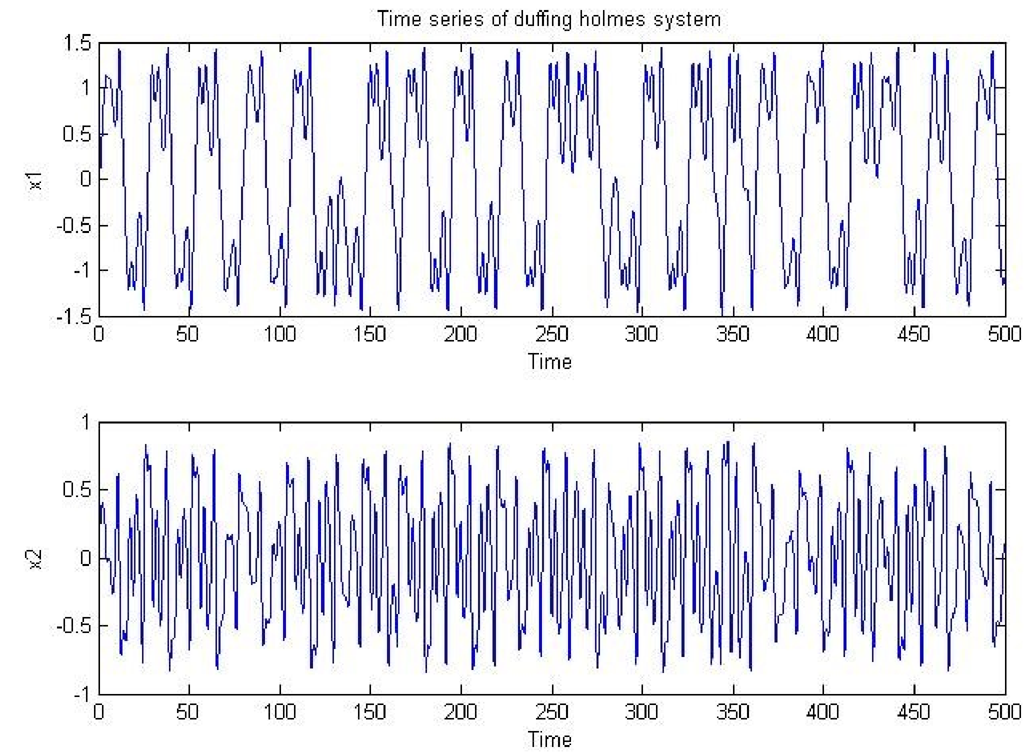

Figure 4.5: Chaotic Time Series of Holmes System

4.1.2 Control Performance For Duffing Holmes System

As we have stated earlier that the concept of dynamic sliding mode utilises the addition of an extra dynamics to the system in such a way that the normal chattering problem of the conventional SMC can be avoided as well as the selection can help in achieving the desired transient and steady state performance, in this section we will try to analyse the effect of different dynamic switching surface on the control of chaotic system. Since our main focus will be on the switching surface we will try to avoid the chattering problem with the help of appropriate modification in the controller dynamics.

The system dynamics can be described by the mathematical model

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = ax_1 + bx_2 + c{x_1}^3 + dcos(\omega t) + u(t) \end{cases} \tag{4.2}$$

As we can see that currently the system is assumed to be free of any kind of external disturbances and parametric uncertainties. The control problem is to design a suitable control signal which will direct the states of the system to origin and keep it there. However the first step in sliding mode control is to select a suitable static switching surface as already described in the previous section. For all the three cases to be discussed ahead the dynamics of static/conventional switching surface will be taken as the linear combination of the states

$$s = c_1x_1 + c_2x_2,\ where\, c_1,\ c_2 > 0 \tag{4.3}$$

Further, the second step is to select a dynamic switching surface which will not only help in the control of given system but as well as will help in obtaining desired steady state and transient performances. Three different types of switching surface has been selected and their names are motivated from the basic controllers present in classical control theory because of the fact that their mathematical structure has a resemblance.

a.) PID Switching – As the name suggest the dynamic switching surface have a proportional, integral and derivative term as shown in (4.4)

$$\sigma = \dot{s} + \lambda_1 s + \lambda_2 + \int sdt \tag{4.4}$$

where s corresponds to conventional surface. The time differentiation of (4.4) will yield

$$\dot{\sigma} = \ddot{s} + \lambda_1 \dot{s} + \lambda_2 s \tag{4.5}$$

Further using the defined static switching surface in terms of system states

$$\dot{\sigma} = c_1 \dot{x}_2 + c_2 \ddot{x}_2 = C_{11} \dot{x}_2 + \lambda 1 c_1 x_1 + C_{12} x_2 + 3c{x_1}^2 x_2 - d\omega sin(\omega t) + \dot{u}(t) \tag{4.6}$$

Constructing the Lyapunov function as

$$V = 0.5 \sigma^T \sigma \tag{4.7}$$

Differentiating (4.7) with respect to time and utilizing the information in (4.5)

$$\dot{V} = \sigma \dot{\sigma} = \sigma (C_{11} \dot{x}_2 + \lambda_1 c_1 x_1 + C_{12} x_2 + 3c{x_1}^2 x_2 - d\omega sin(\omega t) + \dot{u}(t)) \tag{4.8}$$

So as discussed earlier, now the problem shifts to selection of a proper time derivative of control input such that the system states converge to equilibrium points at steady state. One of the valid choice can be

$$\begin{aligned} \dot{u}(t) = -((C_{11} a + \lambda_2 c_1) x_1 + (C_{11} b + C_{12}) x_2 + C_{11} (c x_1^3 + d cos(\omega t) + u(t)) + \\ + 3 c x_1^2 x_2 - d\omega sin(\omega t)) - (\mu + \eta \|\sigma\|^{(\beta - 1)}) \sigma \end{aligned} \tag{4.9}$$

The proposed controller yields a negative definite time derivative of Lyapunov function and hence provides a stable control dynamics.

b.) PD Switching – This type of switching is a subset of that described in PID type with only proportional and derivative terms of conventional sliding surface

$$\sigma = \dot{s} + \lambda_1 s \tag{4.10}$$

The corresponding time derivative control signal has to be selected as

$$\begin{aligned} \dot{u}(t) = -((C_{11} + b) a x_1 + (C_{11} b + \lambda_1 c_1 + a + b^2) x_2 + (C_{11} + 1)(c x_1^3 + d cos(\omega t) + + u(t)) + \\ + 3 c x_1^2 x_2 - d\omega sin(\omega t)) - (\mu + \eta \|\sigma\|^{(\beta - 1)}) \sigma \end{aligned} \tag{4.11}$$

c.) PI Switching – It includes proportional and integral aspects of the conventional switching surface as depicted in the equation

$$\sigma = \lambda_1 s + \lambda_2 \int s dt \tag{4.12}$$

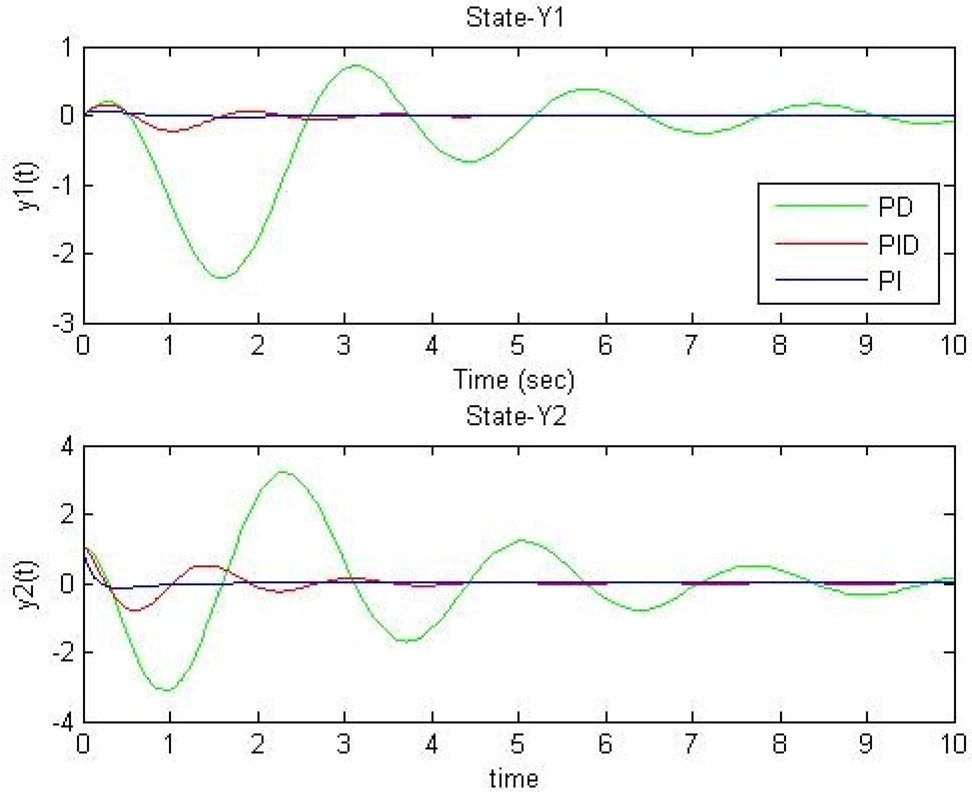

Figure 4.6: Converging System States for Duffing Holmes System

4.1.3 Genesio System and It's Control

On the similar lines as described above a simple third order choatic system known as Genesio system has been controlled using three different switching surfaces.The chaotic Genesio system was first discovered by Genesio and Tesi can be described by following mathematical model[33]. The system shows chaotic behaviour as shown in figures below

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -ax_1 - bx_2 - cx_3 + x_1^2 \end{cases} \tag{4.13}$$

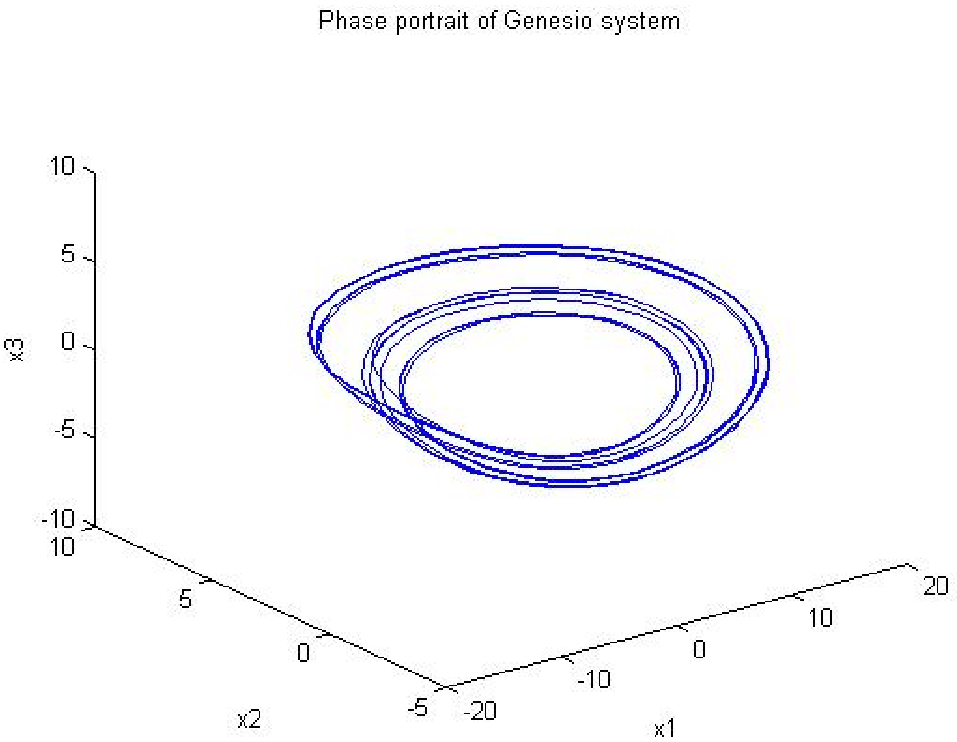

FIGURE 4.7: Phase Portrait for Chaotic Genesio System

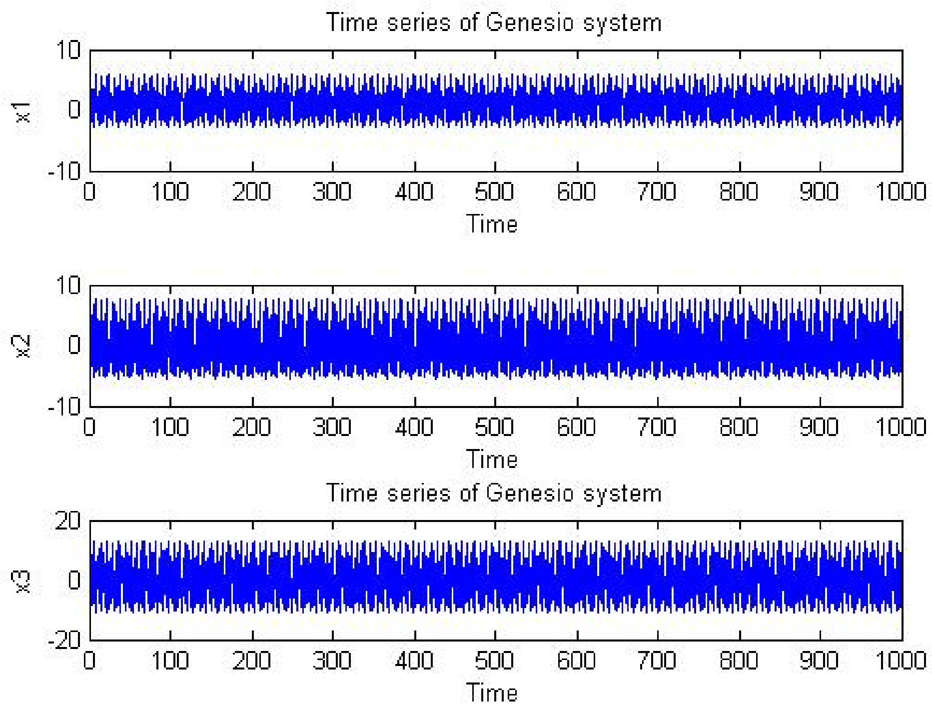

FIGURE 4.8: Chaotic Time Series for Genesio System

A detailed analysis of this system has already been done in [34] and the results are being reflected here to understand the dynamics of this system. It has been observed that for the range $\alpha < 0$ and $\beta < 0$ no bounded solution exists which further implies that for these parametric values there exists no stable attractor. However, for $\alpha > 0$and $\beta > 0$ the system shows a fixed point, limit cycle and chaotic behaviour as we go on varying the values. One of the identified set of parametric value which results into chaotic motion for this system is $a = 6, b = 2.92, c = 1.2$ with the initial condition $x(0) = \begin{bmatrix} 4 & 3 & -4 \end{bmatrix}$ as used in[35] . The same set has been used to derive controllers for three different cases of switching surfaces and since the process is similar the mathematical equation leading to the controller design has been eliminated from this section.

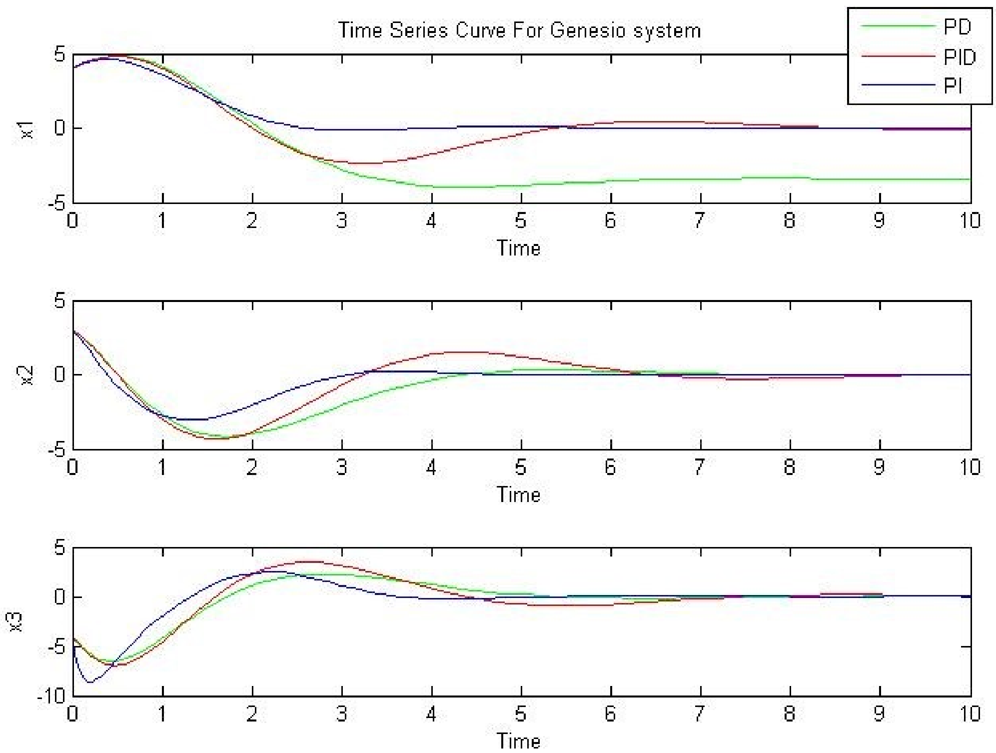

FIGURE 4.9: Converging System States for Genesio System

4.2 Control of General Class of Chaotic System

The performance of sliding mode control design is dependent on the type of switching or sliding surface to which all the states of system are forced to converge using proper controller signal. One of the mostly used sliding surface is linear combination of system states with the parameter of the surface satisfying Hurwitz polynomial. However such a surface can only provide asymptotic convergence to the equilibrium points that means it will take almost infinite time for the states to converge to a specified points. This becomes a drawback when the requirement is restricted highly in terms of reaching time and precision value. In these cases the conventional asymptotic convergence can be a problem when the system states lie close to origin. This issue, however can be solved by designing non linear switching manifold with finite time convergence rate. Terminal sliding mode (TSM) design gives us one such non linear switching surface with fast convergence as well as improved transient behaviour. The TSM was first introduced in [36] and then there have been several application of the same with as many modifications.

4.2.1 Terminal Sliding Mode – A Brief analysis

As Described above TSM utilizes a non-linear switching surface given by following equation [35]

$$s = x_2 + cx_1^{(p/q)} \tag{4.14}$$

Here $p, q > 0$ and both are odd real integers and only real solution is considered so that the term $x_1^{(p/q)}$ remains real under all circumstances. Further, when the states achieve sliding mode i.e. they are on the surface described in (4.14) starting from any given initial condition $x(0)$, the time to reach to origin can be given by yu2002variable

$$t_{final} = \frac{p}{c(p-q)} p/(c(p-q))|x_1(0)|^{((p-q)/q)} \tag{4.15}$$

And the state dynamics during sliding are defined by

$$\dot{x}_1 = -cx_1^{p/q} \tag{4.16}$$

Clearly from eq. (4.15), it can be seen that the total time taken to achieve convergence is finite and depends on the initial condition $x_1(0)$ as well as parameters c, p and q. For a fixed initial condition,t_{final} can be minimized by choosing higher values of c. Further if we select a Lyapunov function as

$$V = 0.5x_1^2 \tag{4.17}$$

Then the corresponding time derivative will yield

$$\dot{V} = x_1\dot{x}_1 = -cx_1^{(p+q)/q} \tag{4.18}$$

Clearly for the dynamics to be stable, time derivative of Lyapunov function must be negative definite and to ensure that the term $(p+q)$ must be even which is assured by the fact that both p and q are odd real numbers. The equation (4.18) also makes it clear that the state $x_1 = 0$ is terminally stable and not necessarily asymptotically stable. Also an interesting fact about the TSM surface given in eq. (4.14) is that although the convergence rate is finite but it does depend on initial condition i.e. convergence rate is faster when system states are closer to the origin and thus TSM might not offer the same advantage over linear switching surface if the system states are far away from the origin. This particular drawback of TSM can be overcome using the concept of Fast Terminal Sliding Mode (FTSM) as used in the paper by Yu and Man. One such surface has been investigated in [35] and can be described by

$$s = x_2 + c_1 x_1 + c_2 x_1^{p/q} \tag{4.19}$$

When the states are far away from origin above surface can be approximated by conventional switching surface and similarly when the states are closer to origin the equation can be approximated by eq.(4.14) thus reducing the convergence time irrespective of the position of system states from the origin. Apart from the drawback discussed above both the methods TSM and FTSM suffer from singularity problem that can be observed when the solution states turn negative and thus tend to shift the problem into complex number domain and thus the switching surface no longer remains in real domain. There have been many methods to overcome this singularity issue, many of which tend to discard singular solution however one particular method completely overcomes the singularity problem by making modifications in switching surface itself [32]. Such strategy has been termed as Non-singular Fast Terminal Sliding Mode (NFTSM) control and it has been proven that such a surface gives fast finite time convergence, strong robustness and singularity avoidance. In the next section a NFTSM switching surface based controller has been designed and the results have been compared with the dynamic PI based sliding surface and non-singular terminal sliding based control structures.

4.2.2 Generalized System Description

Consider a system whose dynamics is defined by following equations

$$\begin{cases} \dot{\mathbf{x}}_1 &= \mathbf{A_{11}x_1} + \mathbf{A_{12}x_2} \\ \dot{\mathbf{x}}_2 &= (\mathbf{A_{21}} + \mathbf{\Delta A_{21}})\mathbf{x_1} + (\mathbf{A_{22}} + \mathbf{\Delta A_{22}})\mathbf{x_2} + \mathbf{f}(\mathbf{x_1}, \mathbf{x_2}) + \mathbf{B_2 u} + \mathbf{d}(\mathbf{x_1}, \mathbf{x_2}) \end{cases} \tag{4.20}$$

where $\mathbf{x_1} \in \mathbb{R}^m, \mathbf{x_2} \in \mathbb{R}^{n-m}$ represents states of the system, $\mathbf{u} \in \mathbb{R}^{n-m}$ denotes the control input to be designed later and $f(x_1, x_2)$ is the nonlinearity present in the system. $\mathbf{A_{ij}}$ represents matrix of suitable order where $i = 1, 2, j = 1, 2$ and $\mathbf{\Delta A_{2j}}$ represents the corresponding uncertainties in the parameters. The external disturbance has been represented by the term $\mathbf{d(x_1, x_2)}$. Most of the chaotic system discovered so far can be described directly or can be transformed to a mathematical model in the form of equation (4.20).

Assumption 4.1: The parametric uncertainties as well as external disturbances in the system are bounded that is

$$\|\mathbf{\Delta A_{2j}}\| \leq q_j \|\mathbf{x_j}\|, \; where \, j = 1, 2 \tag{4.21}$$

$$\|\mathbf{d(x_1, x_2)}\| \leq q_3 \tag{4.22}$$

Where $q_j (j = 1, 2, 3)$ are positive real numbers and $\|.\|$ denotes the norm

Assumption 4.2: Matrix $\mathbf{B_2} \in \mathbb{R}^{(n-m)\times(n-m)}$ is a full rank matrix.

Definition 4.1: Finite Time Stability- Suppose $x(t) \in \mathbb{R}^n$ represents the states of a system. This system has finite time stability if there exists a positive constant T such that $\lim_{(t\to\ T)} \|x(t)\| = 0$ and if for $t \geq T, \|x(t)\| = 0$.

Lemma 4.1: Let $V(t)$ be a continuous positive definite function which satisfies following condition

$$\dot{V}(t) \leq -CV(t) \, \forall \, t \geq t_0, \; V(t_0) \geq 0, \tag{4.23}$$

where $C > 0$ and $0 < \eta < 1$ are two positive constants. Then, for any give $t_0, V(t)$ satisfies following condition

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - C(1-\eta)(t - t_0), \; t_0 \leq t \leq t_1 \tag{4.24}$$

and

$$V(t) \equiv 0 \, \forall t \leq t_1 \tag{4.25}$$

where t_1 can be given by

$$t_1 = t_0 + \frac{V^(1-\eta)(t_0)}{C(1-\eta)} \tag{4.26}$$

4.3 Controller Design

This section deals with the controller design using two different techniques. First we will use Non Singular Fast Terminal Sliding Mode Control (NFTSM) method and later on the concept of dynamics sliding mode (DSM) will be utilized.

4.3.1 Non Singular Fast TSM Controller

Let us select a switching surface as mentioned below

$$\mathbf{s(t)} = \mathbf{C_1 x_1} + \mathbf{C_2 sign(x_1)|x_1|^{\gamma_1}} + \mathbf{C_3 sign(x_2)|x_2|^{\gamma_2}} \tag{4.27}$$

Where $\mathbf{s(t)}$ represents a $(n-m)$ dimensional switching surface with the various matrices described as $C_1 \in \mathbb{R}^{(n-m)\times m}, C_2 \in \mathbb{R}^{(n-m)\times m}, C_3 \in \mathbb{R}^{(n-m)\times(n-m)}$ such that all the elements $C_{ij} > 0.\gamma_1 = diag(\gamma_1)$ and $\gamma_2 = diag(\gamma_2)$ represents two diagonal matrices of order m and $(n-m)$ respectively. Here $\gamma_1 > \gamma_2$ and $0 < \gamma_2 < 1$.

The vectors $\mathbf{sign(x_1)|x_1|^{\gamma_1}}$ and $\mathbf{sign(x_2)|x_2|^{\gamma_2}}$ can be defined as

$$\mathbf{sign(x_1)|x_1|^{\gamma_1}} = \begin{bmatrix} sign(x_{11})|x_{11}|^{\gamma_1} & sign(x_{12})|x_{12}|^{\gamma_1} & \cdots & sign(x_{1m})|x_{1m}|^{\gamma_1} \end{bmatrix}^T \tag{4.28}$$

$$\mathbf{sign(x_2)|x_2|^{\gamma_2}} = \begin{bmatrix} sign(x_{21})|x_{21}|^{\gamma_2} & sign(x_{22})|x_{22}|^{\gamma_2} & \cdots & sign(x_{2(n-m)})|x_{2(n-m)}|^{\gamma_2} \end{bmatrix}^T \tag{4.29}$$

Theorem 4.1: Let all the assumptions to be valid, based on the selection of switching surface in (4.27) the following control input is selected as

$$\begin{aligned}\mathbf{u} = -(\mathbf{B_2})^{-1}\{&\mathbf{A_{21}x_1} + \mathbf{A_{22}x_2} + \mathbf{f(x_1, x_2)} + (\mu + \rho\|\mathbf{s}\|^{\beta-1} + (\mathbf{q_1}\|\mathbf{x_1}\| + \mathbf{q_2}\|\mathbf{x_2}\| + \mathbf{q_3})\|\mathbf{s}\|^{-1})\mathbf{s} \\ &(\mathbf{C_2}\gamma_2|\mathbf{x_2}|^{\gamma_2 - \mathbf{I_{(n-m)}}})^{-1}(\mathbf{C_1}(\mathbf{A_{11}x_1} + \mathbf{A_{12}x_2}) + \mathbf{C_2}(\gamma_1|\mathbf{x_1}|^{\gamma_1 - \mathbf{I_m}})(\mathbf{A_{11}x_1} + \mathbf{A_{12}x_2}))\}\end{aligned} \tag{4.30}$$

where $0 < \beta < 1$ and $\rho, \mu > 0$. Then the system in equation (1) starting from any given initial state will approach the sliding surface $s = 0$ in finite time T and will stay on it for $t > T$.

Proof Consider $V(t)$ to be Lyapunov candidate such that

$$V(t) = 0.5s^T s \tag{4.31}$$

Taking its time derivative and with the help of eq. (4.27) we get,

$$\mathbf{\dot{V}(t) = s^T(C_1\dot{x}_1 + C_2\gamma_1|x_1|^{\gamma_1 - I_m}\dot{x}_1 + C_3\gamma_2|x_2|^{\gamma_2 - I_{n-m}}\dot{x}_2)}$$
$$\mathbf{= s^T(C_1(A_{11}x_1 + A_{12}x_2) + C_2\gamma_1|x_1|^{\gamma_1 - I_m})(A_{11}x_1 + A_{12}x_2) + C_3\gamma_2|x_2|^{\gamma_2 - I_{n-m}}}$$
$$\mathbf{(A_{21} + \Delta A_{21})x_1 + (A_{22} + \Delta A_{22})x_2 + f(x_1, x_2) + B_2u + d(x_1, x_2)))} \tag{4.32}$$

On selecting u as mentioned in eq. (4.30)

$$\mathbf{\dot{V}(t) = s^T(C_3\gamma_2|x_2|^{\gamma_2 - I_{n-m}})\{-(\mu + \rho\|s\|^{\beta-1} + (q_1\|x_1\| + q_2\|x_2\| + q_3)\|s\|^{-1})s}$$
$$\mathbf{+\Delta A_{22}x_2 + A_{21}x_1 + d(x_1, x_2)\} \leq -\alpha(\mu\|s\|^2 + \rho\|s\|^{\beta+1}) \leq -2\alpha\rho V^{(+1)/2)}} \tag{4.33}$$

Thus it is clear that in accordance with lemma 1 and controller in (4.30), the system will reach the sliding surface s=0 in a finite time.

4.3.2 Dynamic Sliding Mode Controller Design

For the same system defined in eq. (4.20) we will design a controller using the concept of dynamic sliding mode. Instead of working on a conventional switching surface, DSMC develops a surface which depends not only the states of the system but also the control dynamics which improves the response of the system as well as helps in removing chattering as will be shown in later sections. Dynamic switching surface is a combination of conventional surface and its integral which resembles to PI (Proportional + Integral) control structure. However a significant point to note is that DSMC also uses asymptotic convergence as conventional SMC. Let the conventional switching surface be defined as

$$\mathbf{s_d = C_1x_1 + C_2x_2} \tag{4.34}$$

where $C_1 \in \mathbb{R}^{(n-m)\times m}, C_2 \in \mathbb{R}^{(n-m)}$.

Then the dynamic sliding surface can be defined as

$$\sigma(\mathbf{t}) = \mathbf{s_d} + \lambda \int \mathbf{s_d} \tag{4.35}$$

Using the time derivative of eq. (4.34)

$$\dot{\mathbf{s}}_{\mathbf{d}} = \mathbf{C_2}\{(\mathbf{A_{21}} + \mathbf{\Delta A_{21}})\mathbf{x_1} + (\mathbf{A_{22}} + \mathbf{\Delta A_{22}})\mathbf{x_2} + \mathbf{f}(\mathbf{x_1}, \mathbf{x_2}) + \mathbf{B_2 u} + \mathbf{d}(\mathbf{x_1}, \mathbf{x_2})\} \\ + \mathbf{C_1}(\mathbf{A_{11}x_1} + \mathbf{A_{12}x_2}) \tag{4.36}$$

$$\dot{\mathbf{s}}_{\mathbf{d}} = \tilde{\mathbf{A}}_{\mathbf{1}}\mathbf{x_1} + \tilde{\mathbf{A}}_{\mathbf{2}}\mathbf{x_2} + \mathbf{C_2}\{\mathbf{\Delta}\tilde{\mathbf{A}}_{\mathbf{21}} + \mathbf{\Delta}\tilde{\mathbf{A}}_{\mathbf{22}} + \mathbf{f}(\mathbf{x_1}, \mathbf{x_2}) + \mathbf{B_2 u} + \mathbf{d}(\mathbf{x_1}, \mathbf{x_2})\} \tag{4.37}$$

where $\tilde{\mathbf{A}}_{\mathbf{1}} = \mathbf{C_1 A_{11}} + \mathbf{C_2 A_{21}}, \tilde{\mathbf{A}}_{\mathbf{2}} = \mathbf{C_1 A_{12}} + \mathbf{C_2 A_{22}}$ and $\mathbf{\Delta}\tilde{\mathbf{A}}_{\mathbf{1}} = \mathbf{A_{21}x_1}, \mathbf{\Delta}\tilde{\mathbf{A}}_{\mathbf{2}} = \mathbf{A_{22}x_2}$

Theorem 4.2: Considering all the assumptions to be valid, based on the selection of static and dynamic switching surface in (4.34) and (4.35) the time derivative of control input is selected as

$$\mathbf{u} = -(\mathbf{B_2 C_2})^{-\mathbf{1}}\{[(\tilde{\mathbf{A}}_{\mathbf{1}} + \lambda\mathbf{C_1})\mathbf{x_1} + (\tilde{\mathbf{A}}_{\mathbf{2}} + \lambda\mathbf{C_2})\mathbf{x_2} + \mathbf{C_2 f}(\mathbf{x_1}, \mathbf{x_2}) + \\ (\mu + \rho\|\sigma\|^{\beta-\mathbf{1}} + \|\mathbf{C_2}\|(\mathbf{q_1}\|\mathbf{x_1}\| + \|\mathbf{x_2}\| + \mathbf{q_3})\|\sigma\|^{-\mathbf{1}})\sigma]\} \tag{4.38}$$

where $0 < \beta < 1$ and $\rho, \mu > 0$. Then the system in equation (4.20) starting from any given initial state will approach the sliding surface $s = 0$ in finite time T and will stay on it for $t > T$.

Proof: Let a new Lyapunov candidate as

$$V(t) = 0.5\sigma^T\sigma, \tag{4.39}$$

where σ has been defined in equation (4.35). Time derivative of (4.39) will provide

$$\dot{\mathbf{V}}(\mathbf{t}) = \sigma(\tilde{\mathbf{A}}_{\mathbf{1}}\mathbf{x_1} + \tilde{\mathbf{A}}_{\mathbf{2}}\mathbf{x_2} + \mathbf{C_2}\{\mathbf{\Delta}\tilde{\mathbf{A}}_{\mathbf{21}} + \mathbf{\Delta}\tilde{\mathbf{A}}_{\mathbf{22}} + \mathbf{f}(\mathbf{x_1}, \mathbf{x_2})\mathbf{u} + \mathbf{d}(\mathbf{x_1}, \mathbf{x_2})] + \\ + \lambda(\mathbf{C_1 x_1} + \mathbf{C_2 x_2})\}) \tag{4.40}$$

On selection of the controller described in theorem 2 and assuming all the assumption to be valid we can reach to the same conclusion as in the proof of NFTSM controller i.e. $\dot{\mathbf{V}}(\mathbf{t}) \leq -\mathbf{2}\alpha\rho\mathbf{V}^{(\beta+\mathbf{1})/\mathbf{2}}$

4.4 Control of Unified Chaotic System

To verify and compare various aspects of the methods discussed in previous section we will take unified chaotic system as an example[36]. The dynamics of the system is defined

Parametric Value	System
$0 \leq \alpha < 0.8$	Generalized Lorenz System
$\alpha = 0.8$	Lü System
$0.8 \leq \alpha \leq 1$	Generalized Chen System

TABLE 4.1: Parameter Value and System Behaviour

by following mathematical model

$$\begin{cases} \dot{x}_1 = (25\alpha + 10)(x_2 - x_1), \\ \dot{x}_2 = (28 - 35\alpha + d_1)x_1 - x_1x_3 + (29\alpha - 1 + d_2)x_2 + d_4 + u_1, \\ \dot{x}_3 = x_1x_2 - ((8+\alpha)/3 + d_3)x_3 + d_5 + u_2 \end{cases} \tag{4.41}$$

The above system got its name from the fact that there are more than one chaotic systems hidden in the dynamics and can be recognized by changing the parameter . The set of equations in (4.41) shows chaotic behaviour for $\alpha \in [0, 1]$. The different chaotic system and the corresponding α has been tabulated below. The system states are represented by vector $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$ and $d_i (i = 1, 2, 3)$ represents the bounded parametric uncertainties and $d_i (i = 4, 5)$ denotes the matched disturbance experienced by the system. As described previously α denotes the system parameters satisfying $0 \leq \alpha \leq 1$ for chaotic behaviour. We will design controller using the concept of NFTSM and DSMC and eventually will compare the results of the two with results published in [36].

NFTSM: Selecting the switching surface as described in eq. (4.27), i.e.

$$s_1(t) = C_1x_1 + C_2 sign(x_1)|x_1|^{\gamma_1} + C_3 sign(x_2)|x_2|^{\gamma_2} \tag{4.42}$$

$$s_2(t) = C_3 sign(x_3)|x_3|^{\gamma_2} \tag{4.43}$$

Here $\gamma_i (i = 1, 2)$ and $C_i (i = 1, 2, 3)$ are positive real numbers with $\gamma_1 > \gamma_2$ and $0 < \gamma_2 < 1$. Using theorem 1 following controller can be derived which will stabilize the system's state in a finite time

$$\begin{aligned} u_1 = &-(C_2\gamma_2)^{-1}\{(C_1 + C_1\gamma_1|x_1|^{\gamma_1 - 1})(25\alpha + 10)(x_2 - x_1)\} - (28 - 35\alpha)x_1 \\ &+ x_1x_3 - (29\alpha - 1)x_2 - (\mu + \rho\|s_1\|^{\beta - 1} + (q_1\|x_1\| + q_2\|x_2\| + q_4)\|s\|^{-1})s \end{aligned} \tag{4.44}$$

$$u_2 = -(x_1x_2 - \frac{(8+\alpha)}{3}x_3 + (\mu + \rho\|s_1\|^{\beta-1} + (q_3\|x_3\| + q_5)\|s_2\|^{-1})s_2) \tag{4.45}$$

Parameter	Value	Parameter	Value
β	0.5	q_4	2
γ_1	7/5	q_5	2
γ_2	3/5	λ	10
$q_i(i=1,2)$	7/5	μ	4
ρ	4	α	0 ,0.8

Table 4.2: Various Parametric Values

DSMC: Using static switching surface as defined in eq. (4.34)

$$s_1 = C_1x_1 + C_2x_2 \tag{4.46}$$

$$s_2 = C_2x_3 \tag{4.47}$$

Corresponding dynamics surface can be developed using eq. (4.35) and with the help of theorem 2 controller dynamics is proposed as

$$\begin{aligned} u_1 = -(C_2)^{-1}\{(\tilde{A}_1{+}_1)x_1 + (\tilde{A}_2{+}_2)x_2 - C_2x_1x_3 \\ +(\mu+\rho\|\|^{\beta-1} + \|C_2\|(q_1\|x_1\| + q_2\|x_2\| + q_4)\|\sigma\|^{-1})\sigma\} \end{aligned} \tag{4.48}$$

$$u_2 = -(x_1x_2 - ((8+\alpha)/3)x_3 + (\mu+\rho\|s_1\|^{\beta-1} + (q_3\|x_3\| + q_5)\|s_2\|^{-1})s_2) \tag{4.49}$$

where $\tilde{A}_1 = -C_1(25\alpha+10)+C_2(28-35\alpha)$, and $\tilde{A}_2 = C_1(25\alpha+10)+C_2(29\alpha-1)$. The different parameter values for both the control methods have been tabulated in table 1

4.5 Simulation

For simulation purposes, unified chaotic system, in previous section, has been taken with two different values of $\alpha, \alpha = 0$ and $\alpha = 0.8$ which represents generalised Lorenz system and Lu system respectively. Initial values are selected as $x_1(0) = 5, x_2(0) = -2, x_3(0) = -5$. External disturbance and parametric uncertainties are

$$d_1 = sin(x_1), d_2 = cos(x_2), d_3 = cos(t) \text{ and } d_4, d_5 = 0.5sin(x_1)sin(x_2)sin(x_3)$$

The switching surface parameters have been chosen as $C_1 = C_2 = C_3 = 1$ for all the cases. Other parametric values used are shown in the table 1. Figure 4.10,4.11,4.12 show the state response of generalized Lorenz system $\alpha = 0$and figure 4.13, 4.14, 4.15 show the state response of Lu system$\alpha = 0.8$ when the controller is designed by using

NFTSM, NTSM and DSMC respectively. It can be seen from the figures that system's states gets converged to their equilibrium values in all case. Both the methods have been compared with NTSM methodology derived in[36] .

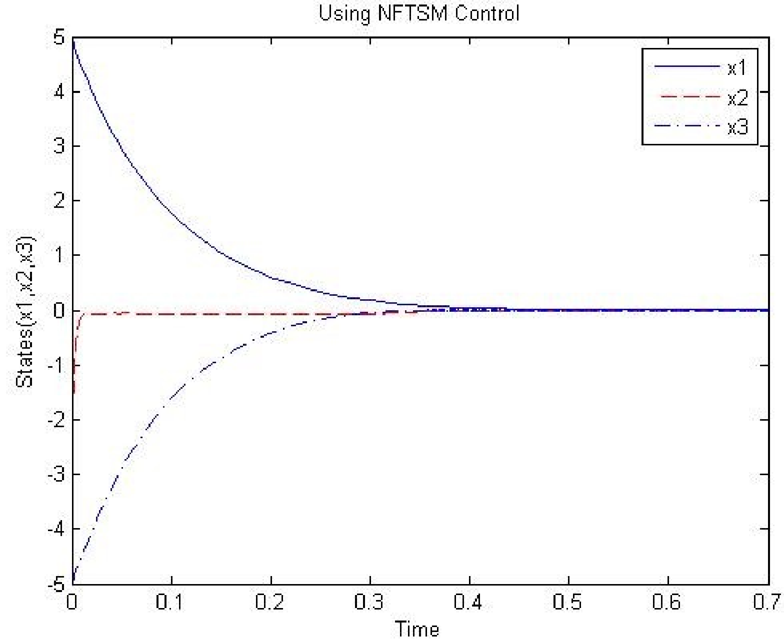

FIGURE 4.10: Converging States of Unified Chaotic System using NFTSM ($\alpha = 0$)

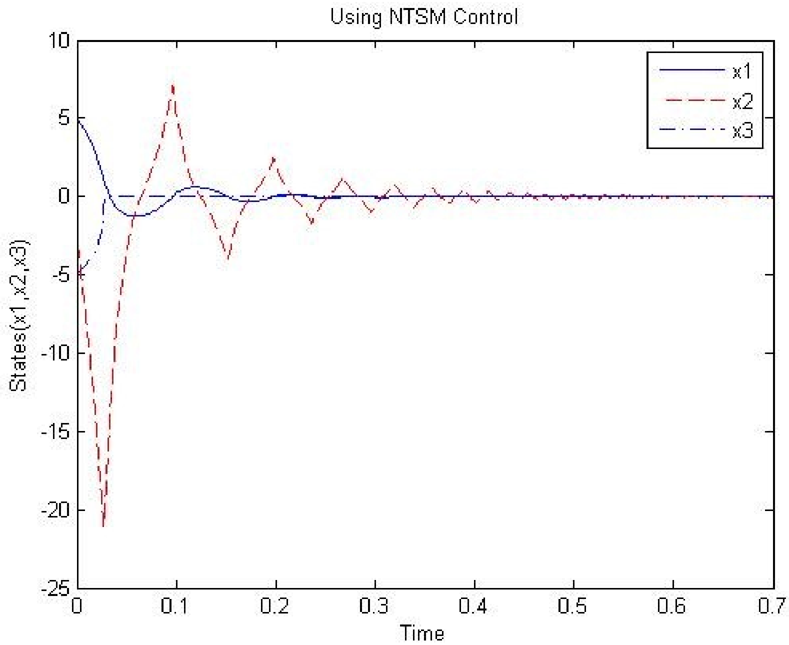

FIGURE 4.11: Converging States of Unified Chaotic System Using NTSM ($\alpha = 0$)

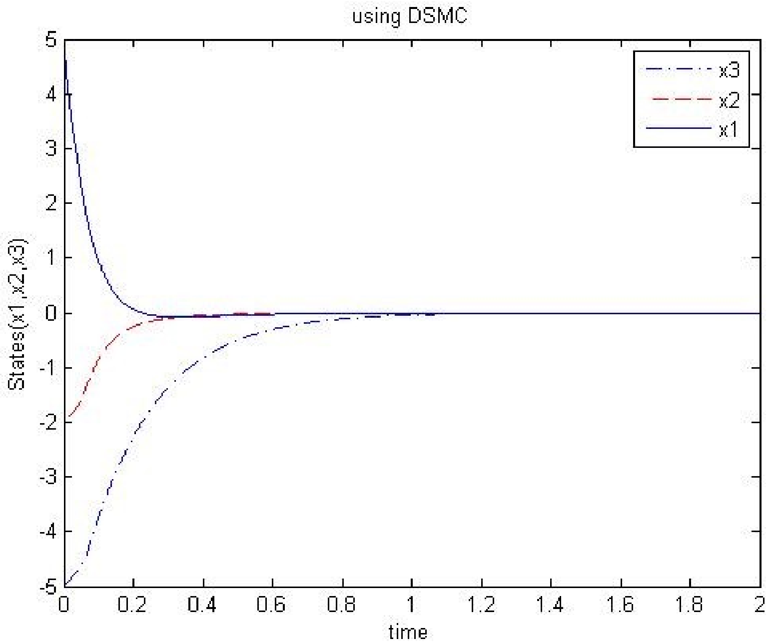

Figure 4.12: Converging States of Unified Chaotic System Using DSMC ($\alpha = 0$)

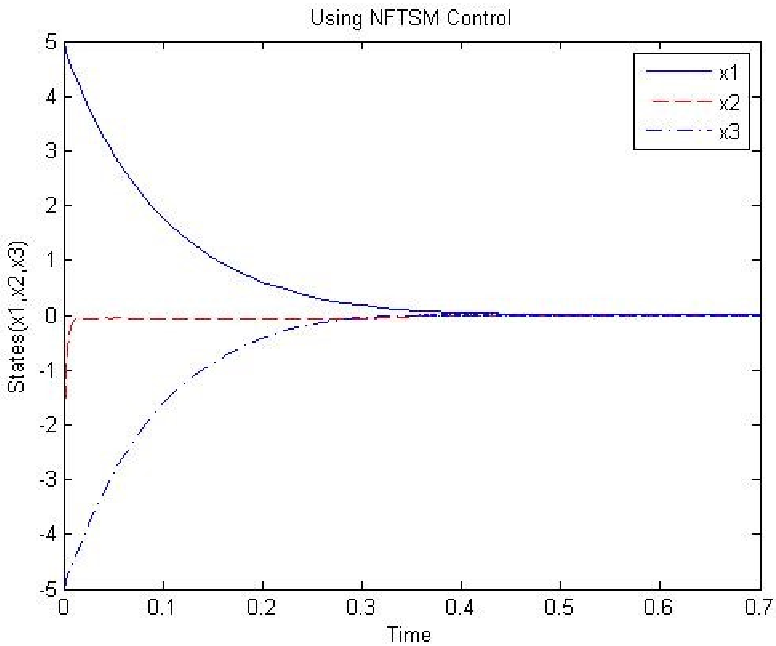

Figure 4.13: Converging States of Unified Chaotic System Using NFTSM ($\alpha = 0.8$)

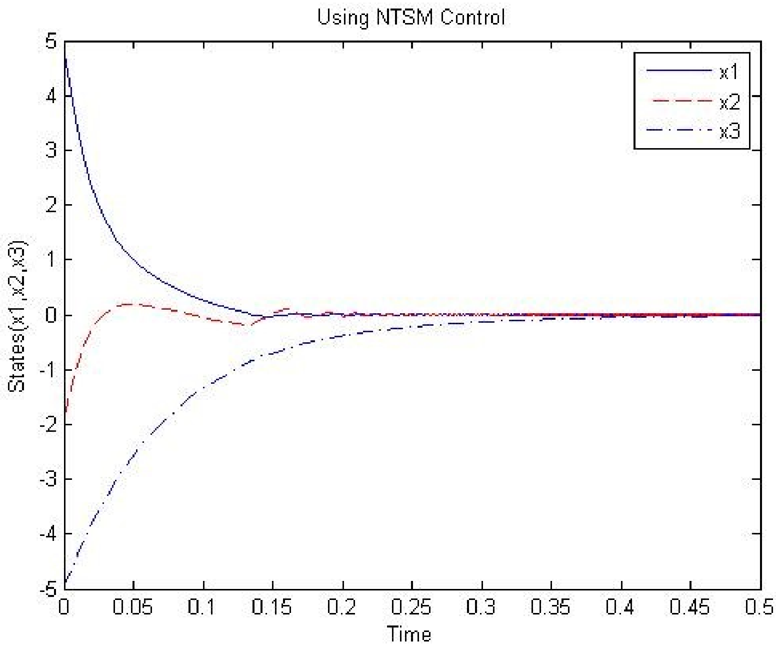

FIGURE 4.14: Converging States of Unified Chaotic System Using NTSM ($\alpha = 0.8$)

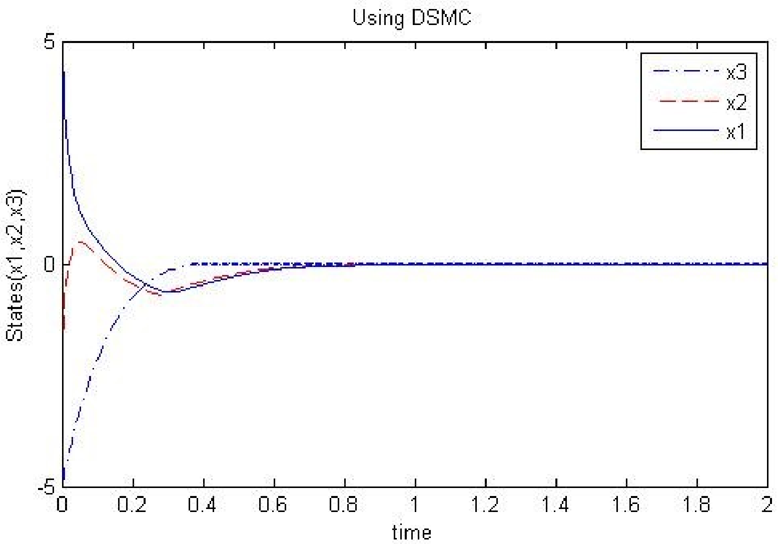

FIGURE 4.15: Converging States of Unified Chaotic System Using DSMC ($\alpha = 0.8$)

4.6 Analysing Result

Firstly, three different switching surfaces and their consequent effects on system output has been observed using chaotic duffing holmes and gensio system. For both convergence rate and better transient performance, PID and PI model sliding surface show better result and also due to integrating term present steady state accuracy is improved while PD based surface shows oscillatory transient performance and poor steady state accuracy if the control input is less than the number of states of the system.To further comment on the speed of response for dynamic switching surface we have utilized non-singular fast terminal sliding mode controller design has been utilized.A DSM as well as NFTSM Control has been developed for a general class of chaotic system and their effectiveness have been compared using already established result of NTSM. The controller demonstrated forces all the state trajectories to converge to their equilibrium state even in the presence of external disturbances and parametric uncertainties for both the system and hence shows strong robustness. Overall the performance of NFTSM is seen much better in terms of convergence time as well as transient performance for both the system. DSMC on the other hand is the slowest among the three in terms of time taken to reach equilibrium point due to its asymptotic convergence property but the choice of its dynamic surface yields a relatively good transient performance in comparison to NTSM method.

Chapter 5

Conclusion

In broader sense this project deals with study of dynamic sliding surface and the advantages obtained by addition of an extra dynamics to the system. Also this project performs the study of terminal sliding mode control and its various modification namely fast terminal sliding and non-singular fast sliding mode control and tries to draw a comparison between controllers obtained from both.

Dynamic sliding mode control offers elimination of chattering from the system due to its ability to work on time derivative of input signal and act as a low pass filter for the same thus removing any high frequency component from the input. In other words this additional dynamics work in similar ways as that of the compensator in classical control theory and hence by changing the structure of dynamic switching surface performance of system state responses can be changed. A smartly selected surface can yield better transient as well as steady state performances than other. However dynamic sliding mode doesn't tackles the issue of asymptotic convergence and hence suffers in convergence rate of response and this fact is evident from the comparative results drawn in chapter 4 where a non-singular fast terminal sliding mode control gives much better convergence rate than dynamic sliding and also in terms of transient behaviour performance of both the system is competitive. Controllers designed using both these concepts show strong robustness towards parametric uncertainty and matched external disturbances and can be utilized for chaos suppression in any complex non-linear system.

Publication

1.Vaibhav Kumar Singh, G.N. Pillai, "Dynamic sliding mode control of partially linear chaotic system", in *2016 39th National Systems Conference (NSC 2015)*, (Dadri,India), December 2015.

2. Vaibhav Kumar Singh, G.N. Pillai, " Nonsingular Fast terminal sliding mode control of general class of chaotic system" submitted in *2016 1st International Conference On Power Electronics, Intelligent Control And Energy Systems(IEEE ICPEICES 2016)*, (to be held in Delhi,India), July 2016.

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